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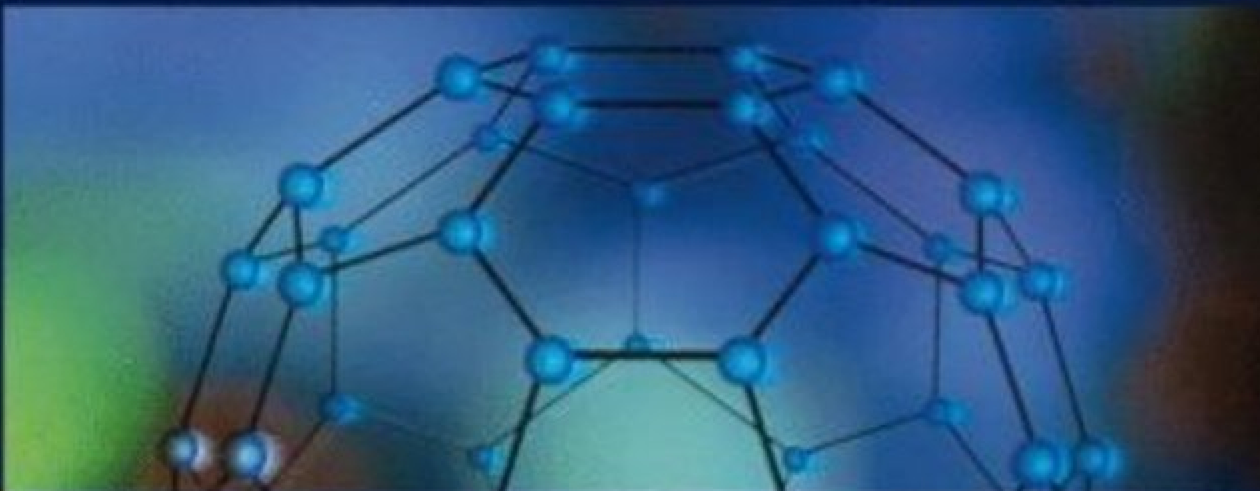

Ari Ben-Menahem

VOLUME 1
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Historical Encyclopedia of Natural and Mathematical Sciences

Springer

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Volume 1

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Professor Ari Ben-Menahem
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Volume 4

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To make this general synthesis possible, we found it expedient to write a large number of monographs on various subjects which emphasize the interrelations between environmental, economic, social, cultural, political and scientific 'events' (e.g. the history of epidemics is needed to correctly estimate the evolution of medical ideas).

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We have interspersed our history with a narration of general intellectual climate and of major social, cultural, political, economic and environmental events: science does not develop in a social vacuum and every man of science needs a modicum of food and other amenities in order to do his work; if called to arms and killed in battle his activities come to an end; if he is an empirical scientist, his opportunities will depend upon the laboratory or observatory to which he has been admitted or which he was able to fashion, and his freedom to pursue his work will be limited by the good or bad will of administrators or fellow workers. Yet nobody can completely control his spirit; he may be helped or hindered, but his scientific ideas are not determined by social factors. Honest men of science and mathematics have often continued activities detrimental to their material interests.

In this treatise, I have tried to draw a map of science, technology and great ideas that would be as accurate and complete as possible, yet sufficiently free from unnecessary details and sufficiently condensed so as not to obstruct the general view.

The book was composed through intensive work during 1991–2008. I had planned to present it to my readers just at the turn of the millennium, but unexpected difficulties prevented me from this symbolic gesture.

The diverse sources used in my work are listed in the bibliography and sometimes in the text itself. Whenever possible, the data was cross-checked between different sources.

January 2009 Ari Ben-Menahem

About the Author

Ari Ben-Menahem (Schlanger) was born in Berlin, Germany and came to Israel in 1934.

He earned his M.Sc. degree in physics from the Hebrew University of Jerusalem (1954) and his Ph.D. degree from the California Institute of Technology (1961). He has been a Professor of Mathematics and Geophysics at the Weizmann Institute of Science in Israel since 1964 and a visiting Professor at MIT, Stanford University and the University of Paris. During 1970–1995 he was the incumbent of the Ayala and Sam Zacks Chair of Geophysics at the Weizmann Institute.

Prof. Ben-Menahem also served as Chief Seismologist of the Government of Israel, member of the European Seismological Commission, US National Research Council Fellow and Director of Adolpho Bloch Geophysical Observatory.

Between 1958 and 2008 he published over 150 papers including graduate textbooks (Springer 1981, Dover 2000). He pioneered the birth of modern seismic source elastodynamics based on his observation of wave radiation from

finite rupturing faults with subshear velocity (1960). He introduced the fundamental concepts of Directivity (1959), and Potency (1965) from which the Moment Tensor was derived. He unraveled (1975) the mystery of the Tunguska bolide explosion of 1908.

In this comprehensive tour de force, Prof. Ben-Menahem – a polymath equally versed in pure and applied science, and the humanities – rises to the challenge of tracing the tapestry of human thought and action throughout history and up to our own age, placing it in the context of our niche in the universe.

Alternating between fine details and bird's-eye vistas, this Encyclopedia encompasses man's science and artifice, illuminating his connections with his environmental, economic, sociopolitical and cultural development.

Acknowledgments

I would like to express heartfelt thanks to our family members: Dr. Shahar Ben-Menahem, assisted me in the formulation of essays dealing with modern theoretical physics, contemporary pure mathematics and the nature and evolution of scientific discovery and invention. He is also the text editor of chapters 4 and 5 of this encyclopedia. I am deeply indebted to him for dedicating long time and great efforts to the completeness, accuracy and streamlining of the book.

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My secretary, Sarah Fliegelman, word-processed the manuscript, until her untimely death in 2002. The camera-ready text was then prepared by Denis Simakov, Ph.D. and Olga Shomron, M.Sc.

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My teachers throughout my student years deserve my gratitude for opening to me the doors of the world of science; At the Hebrew University I owe tribute to Profs. G. Racah, A.H. Fraenkel, Y. Levitski, M. Schiffer and C.L. Pekeris; At the California Institute of Technology I had the privilege of being taugth by Profs. F. Press, H. Benioff, O. Taussky, A. Erdelyi and R.P. Feynman.

At the Weizmann Institute I benefited from the friendship of Profs. D. Vofsy, J. Gillis and S. Freier.

But, first and foremost, I am honored to pay my homage to my father Moshe Ben-Menahem (Schlanger): poet, writer and educator, who set me on the course of my scientific career through his imperative maxim: "Always think like a philosopher, see like an artist, and feel like a poet."

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Why Study the History of Science?

There are a number of compelling reasons for studying the history of science:

1. The story of the development of science can be used as a vehicle for helping *teaching science* to non-scientists. In students, it awakens their critical sense; in working scientists and other thinkers, it tempers the tendency to assume that their original contributions are antecedent-free.

2. From a purely historical perspective, the study of the history of science helps us analyze the development of civilization, to understand man and understand the deeper significance of science. The history of ancient and medieval science is at least as useful as that of modern science. He who knows only one of these histories does not really know the history of science, nor does he know the history of civilization.

3. To project into the future, the present is obviously not enough, and we must depend heavily on historical perspective. This is because the recapitulation of the work of previous generations puts in context the present state of science. Moreover, our civilization in particular is essentially different from earlier ones, because our knowledge of the world and of ourselves is deeper, more precise, and more certain; because we have gradually learned to disentangle the patterns, structures and forces of nature, and because we have contrived, by strict obedience to their governing laws, to capture them and to divert them to the gratification of our own needs. Thus our civilization, more so than previous ones, cannot be understood without proper comprehension of the history of science.

4. Geography and history are two necessary bases of man's education; just as some knowledge of geography removes his provincialism with regard to space – that is, teaches him that things are not necessarily better in his own village, in his own metropolis or in his own country than elsewhere – even so, a knowledge of history is the only way of removing this provincialism with regard to time as well – that is, of making him realize that things are not necessarily better in his day than in earlier or, maybe, in later ones. One has to look at things as they really were perceived in their own time: scientists of the past did *not* think about their own work the way we do; e.g. Kepler did *not* deduce his famous three laws by merely studying the data of Tycho Brahe; much of the spirit of the Middle Ages and the Greek World suffused Kepler's thinking – things that we now no longer associate with planetary motions, were on his mind.

5. As emphasized by physicist Steven Weinberg, the history of science shows us that in their final form the laws of nature are *culture-free*

and permanent, in spite of the fact that many cultural and psychological influences enter into scientific work. In the limit, cultural influences are refined away. Aside from inessentials like the mathematical notation we use, the laws of physics as we understand them now – and thus those of all other sciences, which can be reduced to them in the final analysis – are nothing but a description of reality. When a "revolution" in science replaces one well-verified theory with a better one – e.g. classical mechanics with *relativistic* and/or *quantum* mechanics – the old theory continues to be both (approximately) valid and useful, within its own, more restricted domain.

On this view, physical theories are like fixed points, toward which we are attracted. Starting points may be culturally determined, paths may be affected by personal philosophies, but the fixed point is there nonetheless. It is something toward which any physical theory moves; when we get there we know it, and then we stop. The *final theory* toward which we are moving will be (if reached) a theory of unrestricted validity, a theory applicable to all phenomena throughout the universe - a theory that, when finally attained, will be a permanent part of our knowledge of the world, encompassing all previous successful theories as special limiting cases.

6. The history of science teaches us that it is those ideas that were most successful, of which we should be especially wary, i.e. great heroic ideas of the past can weigh upon us, preventing us from seeing things in a new light and from a fresh perspective. Pioneers are beginners: they cannot be expected to complete their task; it is not their business to complete it. Consider the following example (again due to Weinberg):

   In 1915, Einstein, in the formulation of his theory of gravitation, assumed (in addition to the principle of equivalence) that the equations of the General Theory of Relativity are 2^{nd} order partial differential equations (PDEs); i.e. involve only rates of change in time and space and the rates at which these rates change – but no higher order of change [like the Maxwell equations, the diffusion equation, etc.].

   He could have made them 4^{th} order PDEs – but he did not. Today, this theory of his is regarded a field theory that provides an *approximation valid in the limit of large distances* (much larger than the *Planck length* of 10^{-33} cm). If one supposes that there are terms of higher order derivatives, such terms play no significant role at these large distances, and are thus fairly irrelevant to astronomical (or even most cosmological) observations. We now realize that there must – for many reasons which Einstein could not have foreseen – be such corrections, and that they will be necessary in order to study matter, fields and spacetime at the *Planck scale*.

Guide to the Reader

The main body of this multilateral Encyclopedic History consists of biographies interspersed with essays, articles and events. The biographies are arranged chronologically, within the domains of six consecutive epochal chapters. There are about 2070 detailed biographies of scientists, thinkers, engineers, explorers, inventors and associated creative minds who, in one way or another, left their mark on the history of science and technology in the fields of: *mathematics*, *philosophy*, *logic*, *physical and environmental sciences* (physics, chemistry, astronomy, earth and space sciences, cosmology), *life sciences* (biology, medicine, physiology, botany, zoology, biochemistry), associated engineering disciplines, and *social sciences* (economics, psychology, sociology, anthropology, linguistics etc.).

The total number of scientists, thinkers and other creative individuals featured in the treatise is about 3000.

The book includes the names of some 1700 *inventors* over a period of 2500 years. Of these, 300 belong to the period 350 BCE – 1900 and the rest to the 20^{th} century alone.

The articles (380 in number) summarize the time-evolution of ideas in the above leading fields of science, technology, mathematics and philosophy. In addition, I have included historical environmental events that impacted civilization and also important politico-historical events that affected science, technology and world-order. The reader will also find many useful tables and some 20 'Science Progress Reports' (SPR) dealing with scientific setbacks.

The biographies and articles of this encyclopedia are interspersed with many *quotations*, gathered from the wit and wisdom of sages, savants and scholars throughout the ages from antiquity to modern times.

These quotations are to be found within the text in chapter 6 and under the heading "worldviews" where they are attached to some fifty selected individuals from Socrates to Feynman.

In these quotations, man reflects on himself, his condition, his times and on science proper through the thoughts of scientists, philosophers, humanists, poets, theologists, statesmen, and other miscellaneous mortals.

Our list of biographies is arranged in strict chronology. We must however remember that history is full of uncertitudes. Even when we have managed to arrange our facts in chronological order, we are not sure that the antecedents have influenced the consequents. But at any rate, we are sure that the consequents have not influenced antecedents, and historical certainty is so rare that, when we find it, we must stick to it as closely as possible.

Chapter 6 is dedicated to the second half of the 20^{th} century. Had I included here biographies of contemporary scientists together with proper essays on their respective research achievements (and considering the exponential growth of science during 1950–2005), this chapter alone would have grown into a gigantic size of some 5000 pages. The time needed for the completion of this task would require me to delay the presentation of the encyclopedia by at least five years. To circumvent this difficulty, I have decided to present a general layout of the major research topics and current front-lines, supplemented with tables, timelines and a few key essays.

Credits for Originality of Ideas and Inventions

Observations and measurements provide data that accumulate from epoch to epoch. These form the stuff of hypotheses and theories that in turn suggest new observations. Ultimately we arrive at a major synthesis. These also accumulate, building toward the goal of an ever more inclusive and simple conceptual structure. Connectivity involves the influences of one epoch, one school, one scholar on others, leading to a significant advance in the science. It is a sad but true fact that historical relevance accrues not to the originator of the idea or a fact but to the person providing the connections. e.g. credit for relevance goes not to Aristarchos of Samos, who first proposed, eighteen centuries before Copernicus, that the earth moves around the sun, since that insight was lost. Rather, the connectivity of Copernicus to Galileo, Kepler and Newton was seminal to eventual progress.

It has been said that: "credit in science goes to the man who convinced the world, not to the man to whom the idea first occurs".

My policy in this treatise has been to give credit wherever credit in due, i.e. also to the originator of an idea or a fact, irrespective of whether he made the connection or not. In fact, I made it my business, as much as I was able, to search for the *true originators.*

Credits for Sources of Information

I have borrowed factual information from a vast number of books, articles and encyclopedias, but the creative synthesis of all these sources into a final mold is mine alone, and the responsibility for accuracy and relevance rests on my shoulders. I believe that in presenting the finished material to my readers, I have saved them the trouble and effort of reading libraries of books.

The cited bibliography is sufficient, although no effort was made to make it complete: first, there is a limit to what a single author can reach and read. Then, even from his chosen sources, not everything is suitable nor available to the common reader. However, in this day and age, when the Internet is within reach of many people, all those who wish to, can update their knowledge on a particular subject, person or event. The length of my reviews of a person's life-work was not intended to be a measure of his or her greatness and importance.

Selection of Persons

In a work of this kind, it is not easy to know where to draw the line between inclusions and omissions. While it is relatively easy to determine the most important scientists and thinkers, it is more difficult to agree upon personalities of the second and lower orders. Very often, the inclusion of one scientist entails the admittance of a number of others whose merits were of the same stature. Thus, while it is impossible that I have overlooked any really important personality, it is probable that I have included a few whom it would be better to omit. Clearly, no selection would please everybody. As a rule, however, I have adhered to the following selection criteria:

- Not to mention a person unless there is something special to say of his or her activity (discovery, book, etc.).

- Name people who took the first step in the right direction, however simple it may seem in retrospect.

- Take into consideration the opinion of both contemporaries and later scholars about the contributor.

- Consider the impact and influence of a person's actions, writings and ideas upon the history of science, in both short and long range perspectives.

It was my original intent to deal only with the history of pure science, *but it is often difficult, if not impossible, to draw the line between pure science and the applications.* Sometimes the applications were discovered first and the principles deduced from them; sometimes the converse; but in any case pure and applied science grow together. Yet a line must be drawn somehow, for while the number of pure scientists is relevantly small, that of physicians, teachers, engineers, and other practitioners have always been considerable. My rule is to *speak of a physician, an engineer, or a teacher only if he added something definite to our knowledge*, or if he wrote treatises which were sufficiently original and valuable, or if he did his task in such a masterly way as to introduce new professional standards.

Selection of Data and Evaluation of Scientists

Objective material and scientific facts (equations, laws, rules, discoveries, inventions etc.) are relatively easy to choose, formulate and explain. Human facts are not as clear-cut and are often highly capricious and evanescent. It is thus a difficult mission to choose a few of the achievements amongst a great many.

My account of each personality has been as brief is possible but it is sometimes much easier to indicate a great achievement than a much smaller one, and thus some of the notes devoted to second-rate personalities are much longer than one would expect. This does not matter in itself, *but the reader is warned not to try to measure the importance of a person by the length of the note devoted to him or her; there is no relation between the two.*

I may have made accidental errors in my choice, by omission or commission, but I do not believe that I have made systematic errors. Nevertheless, a certain bias may have been introduced by my *linguistic preferences* (English, French, German, Hebrew), my *scientific specialization* (physics, mathematics, chemistry, geophysics, history, philosophy), *education* (Israel, Sweden, USA) and *origins* (Eastern Europe, Germany and Israel).

I have tried to be as concrete as possible, that is to say, to indicate the specific achievements or contribution in the clearest and briefest manner. That was never easy, often difficult, sometimes impossible. Even as in our own day, there were a number of people in the Middle Ages who attained considerable prestige and rendered undeniable services, yet of whom it can not be said positively who did this or that. In such cases, where the influence was of an indefinite nature, I have been obliged to be vague. In a few other cases I have been reduced to a similar vagueness by my ignorance.

Names of Persons

In the transliteration of names not originally written in the Latin alphabet (e.g. Japanese, Persian, Greek, Chinese) I have followed Sarton whose motto was: 'consistency and simplicity', and who wrote foreign names in such a form that the original written form might be easily reconstructed and found in the dictionary.

Permanent Surnames are of relatively modern origin; they did not exist at all in the Middle Ages. There are many ways of naming a person, and much ambiguity is thus caused. I quoted all the names that each person was known under and selected one of these as the best. When mentioning that person I have always used that name.

The Muslim names are especially difficult. I have given, whenever I could, a large part of their names, not necessarily the whole of them, because this involve a genealogy of indefinite length. In general I have tried to select a name which would be convenient and as characteristic as possible. The case of Chinese and Japanese names poses another difficulty, because, according to their customs, names are not only changed during life, but even after death. and many men are best known under a posthumous name. Hebrew names and words were used in accordance with the 'Jewish Encyclopedia'.

Following George Sarton, I have translated the Greek termination *os* and $o\nu$ by os and on. This has the distinct advantage of distinguishing Greek from Latin writers: Epicuros, Epictetos – Lucretius. Likewise, we write: Miletos, Herodotos, Nicomachos, Eudoxos, Menaichmos, Appolonios, Heironymos, Pappos, Herophilos, Aristarchos, Euhemeros, Aratos, Hipparchos, Ctesibios, Erasistratos, Zenodoros, Diodoros, etc.

Activity Intervals of Scientists

The period of activity is centered in the age of peak creative activity, usually between 30–50 years of age. There are however many exceptions to this rule, especially when the age of greatest prestige is different from that of greatest activity, or because the person's life was cut short. Indeed, it takes a considerable time before the activity of a truly original mind is properly appreciated.

Apart from the class of outstanding persons, whose biographies are known in sufficient detail, the exact period of 'scientific fertility' of most scientists is not accurately known prior to the 17^{th} century. Thus, in the absence of contradictory information, I have considered as a man's prime the year when he became 40 years of age, the nearest to the intellectual climax of most men. It is probable that the greatest scientific discoveries were made, and the most pregnant resolutions taken by men younger than forty, but the accomplishment of their work took considerable time and the maturing of their thought extended over many years.

Classification of Chronology

Every classification has the disadvantage of introducing artificial discontinuities in the flow of life. It must necessarily happen that contemporaries are dealt with in two successive chapters, because one was active at the end of one century and the other at the beginning of the following century; they were flourishing almost at the same time, but at different sides of the cut. This drawback is unavoidable, but is not really objectionable, unless the reader is unaware of it.

The efforts of classification are most disadvantageous in the *Middle Ages*. Not everyone seems to realize that these ages lasted about a 1000 years, and that their development, far from being monotonous, was exceedingly varied. Moreover, brutal political vicissitudes introduced discontinuities in many countries. Under these conditions, any classification of Middle Ages is quiet artificial and no natural classification would be applicable to all sciences and all nations. For that reason the Middle Ages were left undivided and unclassified between 529 CE–1583 CE.

It should be kept in mind that no epoch in history begins or ends sharply at a given year and chronological dividing lines are made mostly for the sake of convenience.

Background Events
(Wars, Revolutions, Social Upheaval and Natural Disasters)

Although our center of interest is the evolution of scientific facts and ideas, general history is always in the background. Not only do the different sciences interact among themselves, but there is also a constant interaction between science and all other intellectual developments, as well as social, natural and economic phenomena. These events often interfere with the accomplishment of science but sometimes stimulate or are even stimulated by it.

Wars and revolutions are not essentially different from natural catastrophes such as earthquakes, volcanic eruptions, floods or epidemics; they are almost as impersonal and uncontrollable. For most men these catastrophes are by far the most important events, and this is natural enough, since their welfare is often radically affected by them. Galileo's or Newton's discoveries did not lower the price of food or shelter, at least not with sufficient suddenness to be perceptible. For us, on the contrary, these discoveries which must sooner or later transform man's outlook and, so to say, magnify both the universe and himself, are the cardinal events of the world's history. All the catastrophes and upheavals, caused either by the untamed forces of nature or by the irrepressible folly of men, are nothing but accidents. They interrupt, upset and sometimes enhance man's essential activity but, however formidable, they do not and cannot dominate it.

* * *

To give our history its full heuristic value, it was not sufficient to retrace the progress of the human mind. It was also necessary to record the regressions, the sudden halts, the mishaps of all kinds that have interrupted its course. The history of errors is extremely useful: for one thing, because it helps us to better appreciate the evolution of truth; also because it enables us to avoid similar mistakes in the future; and lastly, because the errors of science are, to some extent, of a relative nature. Some of the accepted and well-established truths of today will perhaps be considered tomorrow as very incomplete truths; and there are even precedents for the perceived errors of yesterday eventually becoming approximate or partial truths of today. Similar rehabilitations frequently occur en route to be the "fixed points" mentioned above, and the results of historical research often oblige us to admire and honor people who have been misunderstood and despised in their own time.

Thus, although we undertook to explain the progress of scientific thought, it is clear that we cannot properly explain that progress without giving at least a brief account of the intellectual delusions which often delayed our advance or threatened to sidetrack it.

Moreover, to correctly appreciate the scientific ideas of any people, we must consider them not only from our point of view, but also from their own, however wrong the latter may seem. Thus it is necessary to outline the development of some pseudo-sciences, such as astrology, alchemy, and physiognomy.

It should be noted that it is not always easy to distinguish a pseudo-science from one which is sound but imperfect; in some cases it is almost impossible; we can do it now with reference to the past, but it is not certain that we can always do it with regard to the present.

For these reasons I have included in my book a condensed history of astrology, alchemy and other delusions. In addition I have included some 20 articles called '*Science Progress Reports*' in which follies of scientific *regression* are expounded.

I have made no attempt to tell that history with any completeness, for the history of error is, of its very nature, infinite. Besides, as I am bent upon explaining the progressive – not the regressive – tendencies of human civilization, I have kept these fallacies in the background where they belong. Indeed, since they never represented the main current of human endeavor, but were rather like undertows, it would be equally wrong to ignore them altogether or to attach too much importance to them.

As the scope of my field of study is immense, both in time and size, errors are unavoidable; in spite of severe precautions taken by myself and the editors, some errors will regretfully still be present. I hope that in time, we shall be able to weed them out.

Ari Ben-Menahem — Rehovot, Nov 04, 2008

Pre-Science

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In the beginning God created the heaven and the earth.
And the earth was without form, and void;
and the darkness was upon the face of the deep.
And the spirit of God moved upon the face of the waters.
*And God said, "**Let there be light!**" and there was light.*

Genesis 1, *1-3*

* * *

בראשית ברא אלהים את השמים ואת הארץ
והארץ היתה תהו ובהו וחשך על פני תהום ורוח
אלהים מרחפת על פני המים ויאמר אלהים יהי
אור ויהי אור

* * *

In principio creavit Deus caelum et terram.
Terra autem erat inanis et vacua;
et tenebrae super faciem abyssi;
Et spiritus Dei ferebatur super aquas.
Dixitque Deus, "fiat lux!", e facta est lux.

(Vulgata)

ca 14,000 Mya

"Give me matter and motion and I will make the world"

(René Descartes, 1637)

Big Bang A model for the history of the universe stating that it began in an infinitely dense and hot state and has been expanding ever since a creation "event", which took place sometime between 13 and 14 billion years ago. The theory is now widely accepted since it explains three of the most significant observations in cosmology: the expanding universe, the cosmic microwave background radiation and the origin and abundancies of the light elements.

A class of models of the early universe that include a thermodynamic phase transition (and subsequent re-heating and a brief exponential expansion) at fraction of second after the Big bang, are known as *inflationary universe* models.

This event, if it happened, released enormous energy, stored until then in the vacuum of space-time. The causal horizon of the universe expanded, temporarily, much faster than the speed of light. This scenario may be able to account satisfactorily for the present spatial flatness of the universe and its uniformity, and some other features.

ca 12,800 Mya Formation of the first stars in the universe.

ca 4700 Mya Our sun is thought to have formed from a small cloud of gas and dust (produced by earlier stars) which, after lingering for billions of years in one of the Milky Way's spiral arms, succumbed to its own gravitational pull and collapsed. Most of the material in this cloud was drawn into a central body. Thus, our sun has a composition nearly identical to that of the original cloud. It contains 99 percent hydrogen and helium, with about 1 percent comprising the remaining elements.

A small amount of the matter in the cloud ended up in a nebular disk around the newly formed sun. This material gravitationally aggregated into planets, moons, asteroids, and comets. The compositions of these objects are very different from that of the sun. This difference is the result of extensive chemical and physical differentiation: those elements contained in dust grains were largely retained through incorporation into the objects making up the planetary system; those elements in gaseous form were largely driven off by the particles and radiation streaming forth from the sun thus, most of the lighter elements (especially hydrogen and helium) not incorporated into the sun, became part of the larger outer planets.

In those parts of the disk close to the hot sun (which formed earth and the terrestrial planets) only the least volatile of the chemical compounds were in solid form. What little primordial hydrogen survived was locked inside chemical compounds (most notably water and organics). In the more distant reaches of the disk, the abundance of elements heavier than helium was low.

Because of this, the comets and planets of the outer solar system have a chemical composition quite different from that of the asteroids and planets of the inner solar system.

ca 4600 Mya Earth[1] was formed by the accretion of rocky masses. As this mass increased, the temperature rose dramatically and the rocky solid material melted and separated. Eventually the dense material, mostly iron and nickel, sank to form the central core while the lighter, mostly silicate material rose to the surface to form the planet's crust.

As earth cooled, it was bombarded by objects from space.[2] Perhaps the

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[1] The third planet from the sun is one of the most geologically active planets in the solar system, with large volcanoes and great mountain chains. Water is abundant, as vapor in the atmosphere and as liquid and ice on the surface (and subterranean liquid as well). Earth is the only planet in the solar system with an average temperature between the freezing and boiling points of water. Its most unique characteristic is that it supports an amazing diversity of life.

Oceans cover 70 percent of earth's surface. It they were spread out evenly, earth would be under 4 km of water. More than 80 percent of all *photosynthesis* (the process by which plants convert sunlight into chemical energy) takes place in the oceans, making them the principal habitat of life on earth. Ocean water contains many minerals leached or dissolved out from rocks and soil and carried to the oceans by rivers. Sodium chloride, or common table salt, composes 3.5 percent of ocean water.

Fresh water, containing almost no salt, is essential for most living things not found in the oceans. The sources of fresh water is rain, which comes from pure water vapor evaporated from the oceans. The proportion of fresh water to ocean water is small. Only a little over 2 percent of the total amount of water on or near earth's surface is fresh water. Of this fresh water, about 80 percent is frozen in glaciers at the poles.

[2] On July 4, 2005, NASA's spacecraft *Deep Impact* (launched Jan 12, 2005) plunged a self-guided copper slug (having a mass of 370 kilograms) into Comet Temple 1, thus delivering to its nucleus' surface the equivalent of 4.5 tons of TNT. Just minutes after the impact the flyby probe passed the nucleus at a close distance of 500 km, taking pictures of the crater position, the ejecta plume, and the entire cometary nucleus. The entire event was photographed by earth-based telescopes and orbital observatories, such as the *Hubble*, *Chandra*, *Spitzer* and

largest of these objects was the size of Mars, blasting a cloud of debris into orbit that would eventually become the moon. Before the bombardment ceased, about 3.8 billion years ago, 35 impact basins larger than 300 km across were formed on the moon. The larger and more massive earth probably experienced hundreds of comparable impacts. Some would have formed gigantic impact basins with diameters equal to half the width of the continental United States.

Earth is the only inner planet that has liquid water on its surface, although Mars and Venus may have surface ice particles now. As earth cooled, a primitive atmosphere was formed through a volcanic process known as outgassing. This is the release of gases from the interior, including water vapor, hydrogen, nitrogen, and carbon dioxide. As the water vapor in the atmosphere increased, it condensed and fell as rain. Eventually, enough water collected to form our oceans.

ca 3960 Mya Oldest rocks (discovered 1989 CE).

ca 3900 Mya Advent of life; oldest fossils of primitive cells (simple algae and other single-celled organisms) have been found in rocks in Greenland, South Africa and Australia.

ca 3500 Mya Organic carbon compounds; primitive virusoid cells.

ca 3300 Mya Sedimentary rocks already teaming with life.

ca 3000 Mya Organisms capable of *photosynthesis*.

ca 2600 Mya Evidence for the existence of the geomagnetic field, at that epoch.

ca 2500 Mya Tectonic plates clash in India (studied by 1989 CE).

XMM-Newton. In addition, the impact was observed by *cameras* and *spectroscopes* on board Europe's *Rosetta* spacecraft, which was about 80 million km from the comet at the time of impact. Rosetta could then determine the composition of the gas and dust cloud kicked up by the impact.

Analysis of data from this mission disclosed that comets (formed about 4.5 billion years ago) probably carried to earth *water* and *organic* compounds (the two necessary ingredients of life) early in the planet's history, when strikes by asteroids, meteors and comets were common.

Table 0.1: Geological Time

	Millions of years ago
Precambrian Time	
Archean Era	4600–2500
Proterosoic Era	2500–570
Phanerozoic Time	
Paleozoic Era	
Cambrian Period	570–505
Ordovician Period	505–438
Silurian Period	438–408
Devonian Period	408–360
Carboniferous Period	360–286
Permian Period	286–245
Mesozoic Era	
Triassic Period	245–208
Jurassic Period	208–144
Cretaceous Period	144–66.4
Cenozoic Era	
Tertiary Period	
Paleocene Epoch	66.4–57.8
Eocene Epoch	57.8–38.6
Oligocene Epoch	38.6–23.7
Miocene Epoch	23.7–5.3
Pliocene Epoch	5.3–1.6
Quarternary Period	
Pleistocene Epoch	1.6–0.01
Holocene Epoch	0.01–0

ca 2000 Mya *Free oxygen*[3], a by-product of photosynthesis by algae[4], began

[3] Much of earth's initial atmosphere, and its water were probably outgassed early in the planet's history. Carbon dioxide, nitrogen, hydrogen, water vapor, and other *volatile* gases (i.e. that evaporate at relatively low temperatures) comprised earth's early atmosphere. These elements may have come both from the rocky material on earth and from the volatiles-rich material from the outer solar system.

For the first billion years, oxygen was only a small component of earth's atmosphere. Oxygen accumulated in the atmosphere only after photosynthesis began producing it faster than it was lost by chemical combination with other gases and metals. Most of the carbon dioxide in earth's early atmosphere was removed not by plant life but by chemical combinations with calcium, hydrogen, and oxygen, forming limestone (calcium carbonate) in the oceans and other compounds.

to accumulate in the atmosphere.

ca 1600 Mya Primitive plant life.

ca 1300 Mya *Eukaryotes* cells with nuclei, e.g. amoeba; tremendous proliferation and diversification of life.

ca 1000 Mya First *multicellular* animal phylum: e.g sponges (no nervous system nor muscle fiber).

ca 750 Mya First '*Snow-ball earth*' episode. One of four climatic reversals that ended in ca 580 Mya. (The theory was first suggested in 1964 and accepted in 1997).

ca 700 Mya Advent of the cycle of disruption and unification of the earth's crust known as *continental drift*; Remains of the earliest known large-bodied animals in rocks, most belonging to corals and jelly-fish phylum.

The interaction among the plates of the lithosphere is called *plate tectonics*. Much of earth's landscape has been shaped by plate tectonics. As oceanic plates have been created and subducted and the continents have collided and broken apart, mountains have been built, rift valleys formed, ocean ridges and trenches created, and volcanoes constructed.

Most activity occurred along the edges of the plates as they move in relation to one another. Plate tectonics began with plate recycling and was driven by convective cooling of earth's interior. The continental lithosphere was made largely of rocks such as granite that are less dense than the mantle. So, while oceanic plates were forced back into the mantle, the continents stayed afloat and remained at the surface, although they have drifted together and broken apart many times. By 80 million years ago, most of the continents we know today were isolated and had begun moving toward their current positions.

In recent times humans have been generating carbon dioxide faster than plants or the oceans can take it up. At the same time, we are cutting down forests that use carbon dioxide and burning the wood, releasing more carbon dioxide. Earth's atmosphere is now about (by volume) 78 percent nitrogen, 20 percent oxygen, 1 percent argon, and 1 percent water vapor, with CO_2 making up only 385 ppm (seasonally averaged). The average atmospheric surface pressure is about 15 pounds per square inch (1 kg/sq cm).

[4] *The web of life*: In the single-celled algae *Emaliana huxlexi*, the shield-like structures on its surface are composed of calcium carbonate, which goes to form chalk when the cell dies. Blooms of *Emaliana* cover large areas of the ocean. They perform a vital role by removing carbon dioxide from the air and producing dimethyl sulphide, which acts to nucleate clouds.

The term "continental collision" conjures up visions of high-speed crashes, but the continents move only up to 10 cm per year, and it takes millions of years to build a mountain range. Around 250 million years ago, when North America collided with Africa, the ensuing large-scale crustal shortening generated the Appalachian Mountains. Erosion has worn them down, but they were once comparable to the present-day Himalayas. The Himalayas, however, were formed only about 35 million years ago when India plowed into Southern Asia. The large-scale horizontal shortening that resulted built up the highest mountains on today's earth.

ca 660 Mya Hard-shelled animals in the sea: organisms large enough to leave clear *fossil* evidence started to evolve at this time. Oldest known multicellular animal fossils.

Collision Events and Geological History

After the earth formed, its bombardment by 'cosmic bullets' such as comets and asteroids (protoplanets) did not cease. Accretion, the process of planetary building, continued, and to a tiny extent it continues to this day.

The accretion by the early earth of comets *through collisions played a key role in the history of life on earth. The comets brought the water of the oceans and the carbon-enriched materials that would someday form the biomass of the planet. Indeed, trillions of comets in the* Oort cloud*, billions of objects in the* Kuiper belt*, and millions in the asteroid belt are reservoirs for potential earth-crossing objects.*

Of all the terrain on earth that underwent the period of heavy bombardment, none has survived the effects of erosion, plate recycling, and plate tectonics. Yet, evidence remained that the earth has been hit by asteroids and comets: In the last 100 million years, earth has been hit by well near a 1000 objects capable of producing craters with diameters in the range 10–150 km. About a 100 known impact structures *on earth range in diameter from 2 to over 150 km. Only a few of the largest impact structures are older than 500 million years.*

The earth's history is divided into periods of time based on the types of fossils present in sedimentary rocks formed during those times. The time before 570 million years ago, the Pre-Cambrian era, *lacked life-forms with hard parts that formed fossils easily. All large organisms were soft-bodied and lived in the ocean. Fossils of primitive cells have been found inside rocks in South Africa and Australia that are 3500 million years old.*

However, organisms large enough to leave clear fossil evidence did not evolve until about 600 million years ago. During the Paleozoic era, *from 570 to 245 million years ago, hard-bodied plants and animals exploded in abundance, and colonized the land. The first amphibians arose, and ferns and conifers dominated the forests. The* Mesozoic era, *from 245 to 65 million years ago, was the time when dinosaurs were abundant land animals. Mammals also existed, but were of lesser importance. Later in this era, the first flowering plants occurred.*

The Mesozoic era of the dinosaurs is in turn divided into three periods based on the dominance of different species: the Triassic *(245–180 million years ago), the* Jurassic *(180–144 million years ago), and the* Cretaceous *(144–66 million years ago). The earth's continents began to assume a recognizable configuration only during the Cretaceous period. The continuing processes of erosion, plate-tectonics, volcanism and cratering created changes in the surface of the earth.*

The fossil record testifies to global mass extinction *of species in the oceans and on land. Such events occurred about 11 million, 35 million, 66 million, 91 million and 250 million years ago, correlating with sedimentary layers that contain the element iridium, which is 10,000 more abundant in most meteorites than in the earth's crust. The iridium could have come either from volcanic eruption from deep in the mantle or from an impact of an asteroid.*

Other evidence suggests that large impacts threw enormous amounts of dust into the atmosphere, thus blocking out much of the sunlight for a few months or even years, and possibly igniting great forest fires.

In the mass extinction about 65 million years ago, 75 percent of the existing plant and animal species disappeared in less than a few million years, ending the age of the dinosaurs and ushering in the age of the mammals. The Cenozoic era, *since the extinction of the dinosaurs 65 million years ago, is the age dominated by mammals. The first grasses developed, and the landscape took on its present appearance. In comparison, human civilizations have developed only during the last six thousand years.*

Some believe that this extinction was triggered by the impact that formed the Chicxulub Crater[5]*, releasing energy of the order of* 10^8 *megatons of TNT [1 MT* $= 4.2 \times 10^{22} erg$*]. Others point to the outgassing that accompanied the formation of the Deccan Plateau about 66 million years ago as the cause of that mass extinction.*

ca 543 Mya The *Cambrian evolutionary explosion*:

Sudden appearance of *complex life*[6]; most of the animal groups now on earth appeared in the fossil record (and can also be genomically traced today) a relatively short time; possible causes:

- Level of atmospheric oxygen rose to a critical point so that oxygen dissolved in sea-water, finally achieved sufficient levels to support large arrays of active animal life.

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[5] It is hidden under the sediments on the coast of the Yucatan Peninsula. The crater was detected when instruments measuring variations in the earth's gravitational and magnetic fields showed a circular structure about 180 km wide and possibly a larger concentric structure 300 km wide. The asteroid that made the crater is estimated to have been about 10 km across, having an estimated mass of 10^{18} g and hitting the earth with geocentric velocity of 40 $\frac{\text{km}}{\text{sec}}$. Its radius of total destruction was about 5000 km.

[6] Until recently, scientists had thought that the so-called "*Cambrian explosion*" lasted for 20–30 million years. A new study (1994), however, revealed that the period probably lasted only about 5 million years, telling us that evolution can operate extraordinarily rapidly when conditions are right.
The researchers determined the new time scale for the Cambrian explosion by analyzing volcanic rock from the early Cambrian period; by dating *zircon crystals* within these rocks, they were able to determine that the *Cambrian period began* about 543 million years ago and the *Cambrian explosion began* about 530 million years ago, and terminated about 525 million years ago. The body plans that evolved in the Cambrian served as blueprints for those seen today. Few new major body plans have appeared since that time. All the evolutionary changes since the Cambrian period have been mere variations on those basic themes. Are these blueprints truly the optimal solutions to the problems of survival and reproduction (reached through an early fast bout of natural selection before development congealed)? Or are they just random combinations of characters assembled by accidents of history? And just how random is biological evolution? – Science, at this point, has no answer to these questions.

- Sunlight radiation on earth reached a critical level, increasing the transparency in oceans and atmosphere.
- It is during this period that the *eye* evolved: the primitive Cambrian life-forms were able to process visual imagery for the first time.

ca 420 Mya Earliest life on the land. Life probably originated in the seas, to which the majority of invertebrate groups are still restricted. Life on land involved major adaptations for these creatures that originated and lived in the oceans. The modifications included changes necessary for protection against dessication, new methods of support in air as opposed to the more buoyant water, breathing oxygen as opposed to extracting it from the water, new sources of food and water, and new reproductive mechanisms to ensure fertilization in the absence of water. Colonization of rivers and lakes was only slightly less formidable, for it involved development of mechanisms to prevent dilution of body fluids that, in all animals, contain dissolved salts precisely adjusted to the osmotic balance of sea water.

Land dwelling, in spite of its problems, offered all the advantages of an empty environment. Because of the delicate interdependence of all living things, it is not surprising that both plants and animals seem to have colonized the land at about the same time during the Silurian and Devonian. The invasion of the land almost certainly involved the earlier invasion of fresh waters. Many living groups, which are essentially marine, contain a few freshwater colonists (clams and crustaceans, for example), but only the plants and three major groups of animals (snails, arthropods, and vertebrates) have become fully established on the land.

ca 250 Mya A volcanic[7] eruption in Hawaii that lasted for a million years. Some evidence for encounter with a large bolide, which caused mass extinction of life on earth.

ca 230 Mya Mammal fossils.

ca 200 Mya Breakup of Pangea; opening of the Atlantic.

ca 180 Mya Separation of African and South American continents.

[7] Volcanism occurs when magma, or melted rock, beneath the surface of a planet or satellite breaks through the surface. On earth, an example of a volcanic land-form is a stratovolcano, a cone-shaped mountain built up by alternating layers of lava and volcanic ash.

ca 65 Mya A large object from outer space rammed into the earth. Extinction of dinosaurs may alternatively be linked to megavolcanic eruptions.

ca 40 Mya Primitive apes.

ca 22 Mya *Hominoid primates* split from Old-World monkeys to a line eventually leading to humans.

ca 6–4.5 Mya Our ancestral ape diverged from chimps and gorillas (we share 98.4 % of our DNA with chimps and 97.7 % with gorillas). Thus, a full 6 million years of evolution separate the minds of modern humans and chimpanzees. We know nothing about the environment in which the ancestral ape lived, as it appears to have left no stone tools.

4.8–4.5 Mya Appearance of the *upright bi-pedal* hominids, known as *Australopithecus (A.) ramidus* (4.5 Mya), *A. anamensis* (4.2 Mya), *A. afarensis* (3.5 Mya), *A. africanus* (2.5 Mya), *A. robustus* (2.2 Mya) and *Homo habilis*[8] (2 Mya).

ca 2 Mya Volcanic eruption (VEI = 8, Energy $\simeq 2 \times 10^6$ MT) shaped the Yellowstone landscape. The lava covered more than 2600 km^2.

ca 1.8–0.4 Mya *Homo erectus* – The first large-brained proto-humans (900–1200 ml); spread out of Africa to explore the Near East and Asia ('*Peking man*', '*Java man*'); begins to develop *local* characteristics. By 500,000 ya the descendants of homo erectus reach Europe and *fire* is tamed. *Ice age* (Pleistocene) began around 1.75 Mya with formation of ice sheets in high latitudes. Glaciation began about 1.2 Mya.

ca 700,000 ya Latest Geomagnetic field reversal.

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8 Modern anthropology has confirmed their existence in Africa through the discoveries of the Leaky family. In 1959 **Louis Leaky** (1903–1972, USA) discovered the skull of *Australopithecus Robustus* in Tanzania and the remains of *Homo Habilis* (1964). His wife **Mary Leaky** (1913–1996, USA) found (1976) fossil footprints of one of our upright ancestors. Homo habilis was confined to East Africa. His larger brain (700 ml), fueled by eating more meat, may have made room for the feature known as "Broca's area" – a region connected with *speech perception*; speech enables complex information, history and legends, to be passed and lies to be told. It encouraged invention of words - an important social tool; enabled group *decisions* to be made, intentions to be communicated, memories shared and passed on.

For further reading, see: Mithen, S.: *The Prehistory of the Mind*. Phoenix, Thames and Hudson, 1996. 357 pp.

ca 400,000–40,000 ya *Homo Sapiens*: Wooden tools, artifacts and weapons give evidence of a hunting-gathering life style. This is the earliest form of our own species.

By 150,000 years ago a new actor had appeared in Europe and the Near East, *Homo neanderthalensis*, popularly known as Neanderthal man. He had the propensity to use tools, and could hunt large game. Like the other characters of this act, the Neanderthals had to cope with frequent and dramatic changes of scenery: their's was a period of the ice ages witnessing ice sheets repeatedly advanced and then retreated across Europe, and with them vegetation changed from tundra to forest and back. Yet even with such changes, the action seems rather monotonous. His tools seem to be very finely crafted, they are all made of either stone or wood. Although unmodified pieces of bone and antler were used, no carving of these materials took place.

At first glance, Neanderthal remains appear primitive and crude, rather like *Homo erectus* and quite different from modem humans. Their arm and leg bones were, in fact, approximately twice as thick as ours, suggesting their immense strength and the rugged conditions of their existence. Otherwise, their bodies were strikingly modern. They had prominent noses, long faces with sloping foreheads and big skulls. Their average brain capacity (1400–1500 cc) actually exceeded that of modem humans – although the configuration of parts of the brain was different. The speech areas of the Neanderthal brain were not as developed as ours and the forebrain was smaller.

The Neanderthals were the first humans to live in Ice Age conditions, surviving by hunting the largest and most formidable Pleistocene mammals – the mammoth, woolly rhinoceros, and wild cattle. They competed with large wolves and lions in an extremely harsh Ice Age environment.

Neanderthals lived in caves, forming large family groups, and used stone axes, bone tools, bows and arrows; pregnancy lasted one year; they took care of their sick and aged.

Climatic Changes[9] *in the Earth's History*

The earth has gone through alternate periods of cold and warm climates. Some changes extended over millions of years, others through a couple of centuries. The causes of these changes were diverse, the leading phenomena being:

- *Changes in volcanic activity (with diminution of activity leading to rise of global temperature) modulating the amounts of infrared-trapping of CO_2 and albedo-decreasing volcanic dust in the atmosphere.*
- *Variation of solar output and activity*

  Contrast between the zones of low and high pressure is apparently controlled to some extent by variations of solar activity. When sunspots become more numerous, pressure increases in the areas where it is already high, and decreases in those where it is already low. In the temperate storm belts, a high sunspot number tends to be associated with low pressure, great storminess, and heavy rainfall.

  Since in the interior of the continents and in tropical regions a good deal of rain is associated with thunderstorms, this suggests that rainfall maxima should coincide with maxima of sunspots, and there is other evidence that on the whole the total rainfall over the land area is greatest when sunspots are most numerous[10]*. Additionally, the sun overall light output has varied over its history. Thus it is thought that during the Cambrian bio-evolutionary explosion, the sun was significantly dimmer than it is today, and 'greenhouse gases' such as CO_2, played an important role in maitaining life-friendly temperatures on our planet.*

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[9] Further readings:

- Brooks, C.E.P., *Climate Through the Ages*, Dover Publications: New York, 1970, 395 pp.
- Graedel, T.E. and P.J. Crutzen *Atmosphere, Climate and Change*, Scientific American Library: New York, 1995, 196 pp.
- Lamb, H.H., *Climate: Present, Past and Future*, Methuen: London, 1977.

[10] The short record of the low-level stage of the Nile can only be relied upon between 640 and 1400 CE, but during this period it presents considerable similarity to the sunspot curve (which can be read off ancient tree-rings). Thus we have

Sunspot maxima	620	840	1077	1200	1370	CE
Nile low-level stage	645	880	1100	1225	1375	CE

The maxima of water levels in the low-water stage of the Nile apparently follow sunspot maxima by intervals of from five to forty years.

- *Bolide impacts (collisions with objects on earth-crossing orbits).*
- *Astronomical* causes (changes in *eccentricity* of the earth's orbit, *obliquity* of the ecliptic, *precession* of the equinoxes etc.). These may have caused the succession of glacial and interglacial periods.

  Seasonal weather changes on earth are largely a matter of *geometry.* Earth follows a nearly circular orbit. The tiny difference of three percent between its greatest and smallest distance from the sun does not account for the range of temperatures between the seasons. The explanation lies in earth's axis, which is tipped 23.5 degrees off the perpendicular to the plane of its orbit. When the northern hemisphere tips toward the sun, the north pole is continuously lighted and days are longer everywhere north of the equator. There is also more radiative heat generated by insolation in the hemisphere. The sun's rays reach the northern hemisphere at less of an angle, with less filtering by the atmosphere, so on average the surface absorbs more heat each hour. For part of the year more heat is gained each day than is lost at night through re-radiation.

  Seasons change each time earth moves a quarter of the way around its orbit. The longest day, on about June 21, is called the *summer solstice* in the northern hemisphere. Three months later, on about September 23, day and night are of the same length. This is the *autumnal equinox.*

  The obliquity of the ecliptic appears to have reached a maximum et about 8150 BCE and to have decreased steadily since that date. Also, about 8500 BCE the earth was farthest from the sun (aphelion) in the northern winter, whereas it is now farthest from the sun in the northern summer. Both these factors would cause an appreciably greater seasonal range of insolation in the ninth millennium BCE than at present.

  Variations in earth's orbital *eccentricity* (the degree to which an orbit deviates from a circle) and in the *obliquity* of its axis (tilt of the axis with respect to the plane of orbit) vary on time scales of 10,000 and 40,000 years, respectively. Changes in the orientation of earth's axis, called precession, occur over a 25,800-year period.

  One of the most dramatic effects of these changes is ice ages. Quasi-periodic ice ages, during which the polar ice caps advanced halfway to the equator, occurred over the past million years and dramatically reshaped landscapes.

  Climate changes caused by earth's orbital and axial characteristics have also influenced the formation of deserts. Climate models that adjust for the precession and nutation of the axis suggest that about 10,000 years ago the Sahara Desert of Africa had ample rainfall, rich vegetation, and monsoons. But there may be other influences. While earth's climate *has been stable since the last ice age 10,000 years ago, analysis of ice core samples from Greenland shows that some earlier abrupt changes in climate lasted only decades or centuries: At one time, the average temperature plunged 14° C in a decade, and the cold snap lasted for 70 years.*

- *Short-range variations [especially variations in rainfall] are due to changes in the general circulation in the atmosphere which may have no external cause. They result from the interaction of winds, ocean currents, and floating ice fields.*

  The atmosphere, oceans, continents, and glaciers are the principal motors of the climatic system. They have a stabilizing effect on the temperatures at the surface of the earth: a slightly different distribution of heat can modify these effects.

  Because of the nonlinear nature of these interactions, under favorable conditions comparably small causes may have disproportionately large effects.

  Climatic changes tend to be synchronized globally, but not generally in the same sense or magnitude in all places. So there can be damp weather in Western Europe, but drought in North America.

  When warm interglacial climate changes into cold glacial climate, the precipitation of large areas change from rain to snow. The latter does not return back to the water of the oceans but stays on the land locked up in ice sheets. This results in lowered sea levels. In general, any change in climate that alters the amount of ice on the land will effect the level of the oceans correspondingly (so called eustatic changes in sea-level).

  During the peak of the last glaciation, much water was stored in the form of ice on Greenland, northern Canada, Antarctica, and high mountain ranges. As a result, sea levels were far below what they were today.

  A rise of sea level may be due to one and more of three causes:

  - *A decrease in depth of part of the sea floor, compensated by the decrease in the elevation of part of the land area.*
  - *An increase in the volume of sea water without change of mass, owning to a decrease in density.*
  - *The actual addition of water to the oceans.*

The mean depth of the oceans is approximately 4000 meters. Taking the coefficient of thermal expansion of water as 0.00015 for one degree centigrade, we find that an increase of temperature by 1° would raise the mean level of the ocean's surface by 0.55 meters. Thus, a rise of the *mean temperature of the whole mass of the oceans by 3° would raise the general level about 1.7 meters.*

The main way in which water can be added to the oceans is through the melting of the ice sheets in Greenland and the Antarctic. Their area is about (1.55×10^7) *km*^{2} *and the average thickness of the ice is nearly 1500 meters. If all this ice were melted it would raise the general level of the ocean by between 43 and 58 meters. The area occupied by the ocean is about* $(3.62 \times 10^8$ *km*$^2)$*, or 23 times the area occupied by ice, so that in order to raise the sea level by 3 meters, it would be necessary to melt off some 70 meters of ice. Because of the difference of density between glacier-ice and water, we may put the actual figure at 76 meters.*

Now, we know that even in the much less intense warm period of the high Middle Ages, the boundaries of the Greenland ice sheet retreated appreciably, which implies a corresponding diminution of thickness, so that in the prolonged warm period of the Climatic Optimum a lowering of the average level of the ice sheets by 76 m is quite possible. These two factors, increase of ocean temperature *and* increase in the mass of water*, appear to be quite competent between them to raise the general level of the oceans by 3 m, the greater part of this being due to the melting of ice.*

We have evidence that sea levels rose at a rate of 1 meter per century and reached its maximum during the Climatic Optimum *between 5500 and 3000 BCE, after which the rate gradually diminished and by 500 BCE it had reached nearly its present level.*

At present, the level of the world oceans is rising by 1 millimeter/year (= one meter per millennium). Scientists have calculated that a complete melting of ice caps would cause sea level to rise 60 meters.

Even if only the ice in the Antarctic will melt $(20 \times 10^6$ *km*$^3)$ *it will cause a sea-level rise of 6 meters. Under these conditions, all large coastal cities will disappear into the ocean, one third of France would be covered by water, and so would much of England. All these catastrophes could become a reality if the content of* CO_2 *in the air were to double*[11]*.*

[11] Indeed, a UN panel of climatologists has recently (2006) warned the world scientific community and world governments that a global warming by more than 3 deg is expected within the next century, and they adduced evidence that greenhouse gas emission — including human-caused CO_2 — were strongly implicated. The consequent melting of Antarctic ice sheets may then raise the ocean level by 5–10 m, causing a global flooding catastrophe of unpredictable proportions. However, these conclusions remain quite controvertial and politicized.

The Gaia[12] Hypothesis[13]

It often seems obvious that life on earth is at the mercy of powerful non-biological forces like volcanic eruptions, storms, climatic changes and earthquakes. On the other hand, much of the earth's surface is covered by a layer of life, and everywhere on earth the influence of living organisms had an effect.

Recently there has emerged a theory, known as the Gaia Hypothesis. *It is based on the idea that, over the long run of geological time, life may control the powerful physical forces on which it depends for its own good.*

The essential idea of the Gaia Hypothesis is analogous to the concept governing the thermostat in one's home, or the thermostat in one's brain. One's thermostat at home may be set to $18.5°C$ in order to keep a comfortable living environment. When the temperature falls below this, the furnace is switched on. When the temperature in the house reaches the target, the furnace is switched off. Something more complicated, but with similar effect, goes on in our bodies. If our body temperature deviates very far from a narrow range, we die. The human body has a number of self-regulatory, or homeostatic, mechanisms, which monitor and stabilize such quantities as: temperature, acidity, and concentrations of hormones.

The conditions for life as we know it to exist also require a relatively narrow range of circumstances in our terrestrial environment. How does life modify the physical and chemical conditions of the earth? Some examples of regulations of the environment, according to Gaia are:

- *Where does the oxygen come from? Small amounts emanate from volcanic activity, but usually it is combined with other elements, e.g., the compounds as CO_2 and H_2O. The earth's original atmosphere contained almost no oxygen, it was the advent of photosynthesis some 2.5 billion years ago that was responsible for the presence of abundant oxygen in the atmosphere (presently 20%, by volume). Initially it was the liberated oxygen combined with oxidizable minerals such as iron, leaving a sedimentary record of red bands that tells us that a new atmospheric chemistry was being brought about by life. Other geological evidence suggests that oxygen levels on earth have been, within a factor of roughly*

[12] Gaia = Greek Goddess of the earth.

[13] Originated in the 1970's by James Lovelock and Lynn Margulis. For further reading, see: Lovelock, J.E., *Gaia: A New Look at Life on Earth*, Oxford University Press: Oxford, 1987, 157 pp.

two, at near-present values for the past billion years, during which complex multi-cellular life arose.

If oxygen were to reach a level of 35% of atmospheric gas composition, fires would occur whenever a lightening bolt hit humid vegetation. The planet would be in serious danger of burning up. What has kept oxygen from building up to dangerous levels? Why has it gone from nearly zero to 20%, and then stopped? One possible answer is the biological production of methane by bacteria. A short-lived molecule, methane can combine with oxygen to produce CO_2 *and water, thus stabilizing oxygen concentrations.*

- *We know that climate has changed a great deal in the past, producing episodes of glaciation as well as short-term warming and cooling episodes. However, climate change might have been much more extreme. At least for the past billion years it is unlikely that the earth's mean temperature was more than 15 degrees warmer or 5 degrees colder than it is today. Earlier temperatures are very uncertain.*

  Astrophysicists calculations suggest that the sun emitted perhaps 25% less radiant energy some 4 billion years ago, than it does today. Calculations also suggest that under this faint early sun, the earth should have been a frozen ball. However, life arose under these conditions, and there is geological evidence of flowing water from this time. It has been suggested that a kind of greenhouse warming was in effect at that time, involving such gases as methane, ammonia, and carbon dioxide, and that this is evidence of a kind of Gaian planetary temperature control mechanism.

  And why hasn't the planet overheated, since the sun has increased in luminosity over the past 4 billion years? ***Lovelock*** *and* ***Margulis*** *argue that life solved this problem too. A warming earth stimulated greater plankton production, removing* CO_2 *from the atmosphere. When the plankton died they sank to the ocean floor, forming sediments, and thus removed* CO_2 *from the system.*

  Moreover, a warmer planet has more rain, which means more erosion and more nutrient runoff to the oceans. This also stimulates phytoplankton growth, again removing CO_2 *from the atmosphere. Thus, Gaia maintains a fairly constant climate as the sun heats up.*

  Life exerts other influences over the chemistry of the planet: methane and ammonia exist in their present abundances because bacteria continually regenerate them by decomposing organic matter. Perhaps life regulates the physical and chemical environment of the planet so as to maintain suitable planetary conditions for the good of life itself. If so, then the planet can be thought of as a single, integrated, living entity with self-regulating abilities.

The adherents of Gaia have offered the following alternative formulations to their hypothesis:

"The Gaia hypothesis says that the temperature, oxidation state, acidity, and certain aspects of the rocks and waters are kept constant, and that this homeostasis is maintained by active feedback processes operated automatically and unconsciously by the biota."

(James Lovelock 1979)

"The Gaia hypothesis states that the lower atmosphere of the earth is an integral, regulated and necessary part of life itself. For hundreds of millions of years, life has controlled the temperature, the chemical composition, the oxidizing ability, and the acidity of the earth's atmosphere."

(L. Margulis and J. Lovelock 1976)

"The biota have effected profound changes on the environment of the surface of the earth. At the same time, that environment has imposed constraints on the biota, so that life and the environment may be considered as two parts of a coupled system."

(Watson and Lovelock 1983)

"The Gaia hypothesis states that the temperature and composition of the earth's atmosphere are actively regulated by the sum of life on the planet."

(Sagan and Margulis 1983)

But perhaps the most succinct observation was made by James Lovelock:

"People have the attitude that 'Gaia will look after us'. But that is wrong. Gaia will look after herself. And the best way for her to do so might be to get rid of us."

ca 300,000 ya First appearance of the *Neanderthal man*.

ca 130,000–11,000 ya *Homo Sapiens Sapiens*; First seen in South Africa and the Near East[14] and joined a cast that continued to include the Neanderthals and archaic Homo Sapiens. He grinded pieces of bones to make harpoons and made tools from materials other than wood or stone. Around 60,000 ya, Homo Sapiens Sapiens built *boats* and then made the very first crossing to Australia. He entered Europe at about 40,000 ya, bringing with him a host of new tools made of new materials, including bone end ivory. During the interval 40,000–30,000 ya the Neanderthals of Europe were trying to mimic the new type of blade tools that Homo Sapiens Sapiens was making, but the Neanderthals soon faded away, leaving Homo Sapiens Sapiens alone on the world stage. Modern man (known as *Cro-Magnon*) immigrated to Siberia and North America (ca 11,000 BCE) and domesticated the *dog* (descendant of the Asian Wolf) at about 9000 BCE.

ca 100,000 ya Homo Sapiens Sapiens developed symbolic thinking manifested in linguistic speech capacity.

ca 75,000 ya *Tuba volcano eruption* (Indonesia); VEI = 8 (Energy $\simeq 6 \times 10^5 MT$).

ca 50,000 ya An iron-nickel *asteroid* about 50 m in diameter hit flat-lying sedimentary rocks and blasted out a crater about 1200 m wide and about 450 m deep in Arizona USA; known as the *Barringer Crater*. (Energy ca 20 MT.)

- Oil lamps, made from carved stone and using animal fat as fuel, found in Mesopotamia and Europe.

- Cave painting made with manganese oxide paints found in Europe, Middle East and Africa.

[14] His traces were discovered in Israel (1988).

ca 40,000–10,000 BCE Preagricultural Cro-Magnon man lived by hunting and gathering. He was an excellent artist and craftsman in stone, bone, wood, and antler. His application of new and innovative technology allowed him to spread throughout the world.

His achievements were: First blade tools (34,000 BCE); early engravings (32,000 BCE); Asian hunters cross the Bering Strait land bridge to populate the Americas (30,000 BCE); first tent-like structure constructed with mammoth bones and tusks; first man-made sculpture (26,000 BCE); first known spear thrown (20,000 BCE); first sewing needle (20,000 BCE); cave paintings in France and Spain (18,000 BCE); Dogs (descendants of the Asian Wolf) domesticated in the Middle East. People inhabiting caves in what are now Israel and Jordan used notches in bones to record *sequences of numbers.*

ca 35,000–20,000 ya *Emergence of Proto-Mathematics in Africa.* Mathematics initially arose from a need to count and record numbers. Proto-Mathematics existed when no written record was available. It is believed that there has never been a society without some form of *counting* or *tallying* (i.e. matching a collection of objects with some easily-handled set of marking like stones, knots or notches on wood or bone).

The earliest evidence of a numerical recording device was found in a cave in the Lebembo Mountains, on the border of Swaziland in South Africa (ca 35,000 ya). It was a baboon bone with 29 clearly visible tally-mark notches, used to record the passage of time. Then, archaeological excavations (1962) on the Shores of Lake Edward (one of the farthest sources of the Nile, on the borders of Uganda and Zaire) unearthed a bone, ca 20,000 years old known as the *Ishango Bone*. The markings on it consist of a series of notches arranged in three distinct columns. Certain underlying numerical patterns may be observed in each column. It seems to have been devised by people who had a number system based on 10, as well as a knowledge of multiplication and of *prime numbers.*

It is conjectured that the bone markings represent of a system of sequential notation – a record of different *phases of the moon.* This they apparently needed to maintain their regular lake-shore lifestyle as a hunter-gatherer neolithic society: migration between dry and rainy seasons, festivities, religious rituals and other activities dictated by their special economic and religious needs.

It is now believed that the creation of a complicated system of sequential notation was well within the inherent capacity of early man to reason and conceptualize. Moreover, the close link between mathematics and astronomy was enhanced by the need felt by early man to record the passage of time, out of curiosity as well as for practical needs. This, in turn, was translated by him into *observations* of the changing aspects of the moon.

From the existing evidence of the transmission of Ishango tools (notably harpoon heads) northward up to the frontiers of *Egypt*, the Ishango numeration system may have traveled as far as Egypt and influenced the development of its own number system – the earliest decimal-based system in the world.

Table 0.2: Classification and nomenclature of periods in the evolutionary track of earth and man; Categories: *Geological* (G); *Climatical* (C); *Historical* (H); *Archaeological-Anthropological* (A)

Pleistocene (G):	the last 2 million years (up to 10,000 ya)
Holocene (G):	the past 10,000 years
Paleolithic (A):	1,000,000 ya → 8000 BCE
Mesolithic and Neolithic (A):	8000 → 4500 BCE
Chalcholithic (A):	4500 → 3200 BCE
Bronze Age (A):	ca 3100 → 1200 BCE (in Middle East)
Iron Age (A):	ca 1200 → 500 BCE (in Middle East)
Israelite period (H):	ca 1200 → 587 BCE (in Middle East)
Climatic Optimum (C):	ca 5500 → 3000 BCE (in Europe)
Ice Ages (C):	1,750,000 ya → 10,000 ya

Post-glacial world – the dawn of civilization

ca 10,000 BCE End of most recent *Ice Age* (*Pleistocene*) and beginning of the *Holocene* age: glaciers melted and retreated and climate was stabilizing. Vegetation began to dry up and water became scarce in the high grasslands, taking with it the herds on whose survival the hunting nomads depended. People were moving to the river valleys, planting crops and domesticating animals. Consequently they created towns and cities. In North Africa, for example, people were moving to the Nile Valley where African peoples mixed with people from the East. Thus, these post-glacial climatic changes impacted the decline and rise of civilizations throughout the Holocene[15].

With the rapid development of the postglacial warming, rivers were swollen enormously, particularly in spring and summer, by the melting ice; gravel and sand were deposited in great quantities along the river courses, lakes formed and sometimes quickly burst or rapidly silted up. The landscape was changing rapidly. But the greatest change for the human population and the animals they hunted was the disappearance of the open plains, as the forest advanced north in Europe. Man seems to have adapted himself more successfully than the animals. The ranges of both moved northward in Europe, Asia and North America, but some species among the animals were lost, probably due to their reduction by Man.

Other significant changes in the landscape were brought about by *the rise of sea levels, proceeding over some thousands of years at the rate of about 1 m a century.*

ca 9000 BCE Earliest beginnings of *agriculture*; domestication of the *dog.* Beginning of rapid rise in sea levels. Floods.

ca 9000–8000 BCE Urban culture in the Anatolian highlands founded by refugees from the flooded lowlands.

ca 8000–7700 BCE Final break up of the Scandinavian Ice Sheet. Highly organized Neolithic civilizations gradually begin to develop in a number of river valleys and alluvial plains in latitudes 20° to 40° N, where irrigation could be most easily arranged and provided seemingly reliable intensive crops.

[15] **F. Hoyle** and **N.C. Wickramasinge** speculated that sometime between 9000 and 8000 BCE, a cometary mass of ca 10^{10} tons impacted the Tasman and China Seas, causing a sudden global warming of 10 °C. This, they said, abruptly ended the Ice Age and created the necessary conditions for an advanced human culture.

Thus Neolithic man changed from food-hunter and food-gatherer into food-producer, domesticating animals and plants. This agrarian revolution centered mainly in the Middle East and the lower hills and adjacent plains of the Zagros Mountains.

- Agriculture in Mesopotamia (Northern Iraq) with farming of *wheat* and *barley*
- *Potatoes* and *beans* cultivated in Peru; *rice* in Indochina.
- Polynesians in the East Indies and Australia begin to spread out over islands of the South Pacific.
- First city-states appear in Mesopotamia and the Near East, including *Jericho* in Israel (population ca 2000, area $\simeq$ 40,000 m^2).
- *Clay tokens* in Mesopotamia used to tally shipments of grain and animals – the basis of a forthcoming first system of numerals and writing.
- The *Maya* make *astronomical* inscriptions and constructions in Central America.
- Sheep domesticated in Persia (Iran).
- Domestication of the bee.

ca 7000–6000 BCE Rise of temperatures continued and moisture increased. By 6000 BCE all of Western Europe was occupied by a rich forest of oak, cedar and elm. With the stabilization of the seas and coastlines, mankind dared to venture forth and seek the fertile valleys. First settlers in *Greece* (ca 7000 BCE).

The *pig* and *water buffalo* domesticated in China and East Asia; the *chicken* domesticated in South Asia; *Sugarcane* cultivated in New Guinea; *Flax* was grown in Southwest Asia; *Maize*, *squash* and *beans* were grown in Mexico; *Mortar* was used with *sun-dried bricks* in Jericho; invention of the *potter's wheel* in Asia Minor (ca 6500 BCE) heralded the production of clay pottery; use of *Woven cloth* was invented in Anatolia; *Cattle* was domesticated in Asia Minor (from a wild ox called auroch, now extinct). This ended the age of the *hoe* (began 8000–7000 BCE) in agriculture [*Genesis*, **3**, 19]. The earliest direct evidence for water craft (ca 6500 BCE) is a wooden paddle found at Star Carr, England.

ca 6300 BCE The llama and alpaca domesticated in Peru. Advent of picture-writing; Copper knives used in the Middle East. An *alcoholic drink* was made from grain.

ca 6000–3000 BCE *Age of revolutionary inventions*: With the availability of cheap tools the way was clear for a rapid expansion of agriculture and technology. The plow was devised; the wheeled cart was in use on land and the sailboat on water; the inclined plane and lever were common implements.

Copper ores were smelted; bronze was alloyed; bricks and pottery were fired. Canals and ditches were dug; grains were fermented; orchards were planted.

At the same time a systematic urban life became clearly outlined. There was an accurate solar calendar; eclipses were predicted. There were adequate methods of accounting and measuring. There was a definite recording of information and transmission of knowledge from one individual to another and from one generation to the next. Discoveries of that period are unrivaled in their impact on human progress. Even today's science cannot match them in fundamental importance to the basic well-being of man.

ca 5600 BCE The great *Mesopotamia flood* (Deluge); an uncertain date for the earliest documented natural catastrophe in the annals of mankind. It first echoed in the Sumerian story about **Ziusudra**, who by building an ark saved himself from the great flood sent by the gods. The narrative (which derived from folk memories of a giant cataclysm, actually experienced in recent geological times) evolved down the millennia and passed from nation to nation. Ziusudra became **Ut-Napishtim** of the Assyrian Gilgamesh Epic[16], **Noah** of the Hebrew Bible, and eventually the **Deukalion** of the Greeks. The most remarkable parallels between the Bible and the entire corpus of cuneiform inscriptions from Mesopotamia are found in the deluge accounts of the Babylonians and Assyrians on one hand, and the Hebrews on the other. It seems that these two versions are independent and refer to an actual event of some kind. According to the *Gilgamesh Epic* and the Sumerian source, the flood was accompanied by a storm of extraordinary magnitude probably issuing from the Persian Gulf, that ravaged for a week or so. The *Biblical* version[17], on the other hand, tells about simultaneous torrential downpour

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[16] Fragments from the Gilgamesh Epic were discovered in the Nineveh library of King Ashurbanipal and date from ca 700 BCE. However, other examples of tablets of this epic date from about 1700 BCE. The story was probably composed at about 2000 BCE. Yet many of the episodes included in the epic have prototypes in the *Sumerian* language which are much older than 2000 BCE.

[17] The total number of flood legends has been estimated at ca 100. They are found in India, southern Asia, the East Indies, Mesopotamia, Polynesia, New Guinea, Australia, China, Europe and North and South America. These widespread legends of a great flood reflect dim folk-memory of the inundation of former coastal plains with the rise of world sea-level in the post-glacial period as the ice age glaciers melted.

from heaven *and* a flow of subterranean waters, the whole occurrence lasting for about a year.

The specific cause of the great flood is uncertain; two competing scenarios were recently[18] suggested:

- Large fragments of a *comet* crashed into seas around the world. The resulting impact triggered violent earthquakes and severe flooding of surrounding coastlines.
- The melting of the Eurasian Ice Sheet (began at ca. 12,500 BCE) raised the level of the ocean some 115 m above the shoreline of the Black Sea, eventually causing the sea water to burst through the narrow Bosporus valley into the lake, racing over beaches and up rivers, destroying or chasing all life before it[19].

  The flood rapidly created a human diaspora[20] that spread as far as Western Europe, Central Asia, China, Egypt and the Persian Gulf. These Black-Sea people could well have been the *proto Sumerians*, who developed civilization in Mesopotamia. These people could be responsible for the spread of farming into Europe, and the advances in agriculture and irrigation in Anatolia and Mesopotamia.

  Associated with the Mesopotamian flood is the biblical account of an Ark, constructed of gofer wood, or cypress, smeared without and within with pitch, or bitumen, to render it water-tight. Today's biblical scholars and shipbuilders agree that the Ark was a very suitable device for shipping heavy cargoes and floating upon the waves without rolling or pitching.[21] It was thus admirably suited for riding out the tremendous storms in the year of the flood.

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[18] The astronomer **Edmond Halley** suggested (1688) that the biblical flood might have been due to a shock of a *comet*. Newton's successor, **William Whiston** (1696) supposed that flood was due to the close earth approach of a comet that broke the earth's crust via tidal forces and released subterranean waters. With the help of water released from the comet's tail and the atmosphere, an enormous tide was created.

[19] **W. Ryan** and **W. Pitman**: "*Noah's Flood*", Simon and Schuster N.Y., 1998, 319 pp.

[20] *Genesis* **11**, 8: "So the Lord scattered them abroad from thence upon the face of all the earth: and they left off to built the *city*" – the advent of *civilization*!

[21] Assuming the *cubit* to be 45.7 cm long, the Ark would have been 137.2 m long, 22.8 m wide and 13.7 m high. Noah's Ark was said to have been the largest seagoing vessel ever built until the late 19^{th} century, when giant metal ships were first constructed. Its length to width ratio of 6 : 1 provided *excellent stability* on the high seas.

ca 5000 BCE Animal domestication and land cultivation in South-East China. It spread from there to all of South-East Asia, but stopped short of Australia where the aborigines continued to be gatherers and hunters. Rice had a major impact on the mode of living of the peoples of Asia and was a direct cause of their fast rate of reproduction.

ca 5000 BCE *Irrigation* invented in Mesopotamia and China; nuggets of metals (gold, silver, copper) were used as ornaments or for trade; first alphabet began to develop; *Sumerians* enter Mesopotamia after the flood.

ca 4895 BCE Crater Lake formed in North America by a volcanic eruption ($VEI = 7, \quad E \simeq 5 \times 10^4$ MT).

ca 4800 BCE Sailing boats in Eridu, Sumer.

ca 4400–4200 BCE Neoglaciations and severe climatic depressions on *a global scale*. Several episodes of climatic deterioration were manifested through glacial advance, increased rainfall, decline of average temperatures, rise of sea level and major flooding.

1. Origins – Splendor of the Simple

4200 BCE–529 CE

Calendars, or the Conquest of Time

Wheels and Numerals

Papyri and Clay

Birth of Science in Ionia

Blossom in Alexandria

Personae

Gilgamesh 82 · Ahmes 104 · Hammurabi 105 · Moshe 107 · Bezalel ben Uri 108 · Chou Kung 117 · Amos of Tekoa 120 · Archilochos of Pharos 143 · Zoroaster 143 · Jeremiah 143 · Thales of Miletos 145 · Lao Tsu 150 · Buddha (Siddhartha Gautama) 151 · Anaximander of Miletos 151 · Confucius (Kung Chiu) 152 · Pythagoras of Samos 159 · Theodoros of Samos 176 · Xenophanes of Colophon 176 · Eupalinos of Megara 177 · Mandrocles of Samos 177 · Hecataeos of Miletos 178 · Nabu-rimannu 178 · Heraclitos of Ephesos 178 · Alcmaeon of Crotona 179 · Panini 181 · Parmenides of Elea 181 · Leucippos of Miletos 182 · Anaxagoras of Clazomenae 182 · Zeno of Elea 183 · Ezra 185 · Empedocles of Acragas 186 · Herodotos 187 · Bryson of Heraclea 188 · Philolaos of Tarentum 188 · Hanno the Carthaginian 189 · Hippocrates of Chios 190 · Melissos of Samos 190 · Protagoras of Abdera 193 · Antiphon the Sophist 194 · Socrates 195 · Meton of Athens 200 · Thucydides of Athens 201 · Hippias of Elis 201 · Hippocrates of Cos 202 · Theodoros of Cyrene 204 · Democritos of Abdera 204 · Theaitetos of Athens 206 · Archytas of Tarentum 207 · Plato 207 · Eudoxos of Cnidos 213 · Thymaridas 221 · Kiddinu 221 · Aristotle 222 · Menaichmos 230 · Heracleides of Pontos 232 · Xenocrates of Chalcedon 232 · Alexander the Great 234 · Praxagoras of Cos 267 · Callippos 268 · Deinocrates of Rhodes 268 · Antimenes 268 · Eudemos of Rhodes 268 · Aristaios the Elder 268 · Pytheas of Massilia 268 · Zeno of Citium 270 · Epicuros of Samos 272 · Hieronymos of Cardia 272 · Autolycos of Pitane 273 · Ptolemy I

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Science 'Progress Reports'

Worldviews

Environmental Events that Impacted Civilization

ca 5600 BCE	The Great Mesopotamian Flood
2500–2000 BCE	Major climatic changes in Afro-Eurasia linked to encounter of Taurid meteor stream with earth; Megalithic constructions around the world
ca 2180–2130 BCE	Prolonged drought over the Eastern Mediterranean and adjacent lands
ca 1800 BCE	Ecological collapse of the irrigation system in Mesopotamia
ca 1628 BCE	Paroxysmal eruption of Tera Volcano near Crete
ca 1200–850 BCE	Major climatic changes in Eurasia; Drought disrupted agriculture in lands of the Eastern Mediterranean. The resulting Indo-European migration caused the collapse of the Hittite kingdom
ca 400 BCE	Malaria epidemic in Greece
79 CE	Eruption of Mount Vesuvius
ca 250–550 CE	Prolonged drought and epidemics (malaria) hastened the decline and fall of the Roman Empire. Simultaneous drought on the steppes of Central Asia drove barbarians to seek new lands

Political and Religious Events that Impacted Science, Philosophy and World Order

ca 1800 BCE	Hebrews conceive the idea of *Monoteism*
1286 BCE	Hittite-Egyptian Battle of *Kadesh* on the Orontes River
1230 BCE	Emergence of the *Hebrew Mosaic laws* – mankind first 'bill of rights'
1200–200 BCE	Creation of the *Hebrew Bible*
853 BCE	Battle of *Karkar* on the Orontes River
583–331 BCE	Rise and fall of the Persian Empire
508 BCE	*Democracy* born in Athens
490–479 BCE	*The Persian Wars* – a decisive and permanent influence upon development of Western culture and civilization
431–404 BCE	*Peloponnesian Wars*
390–220 BCE	*Celtic-Roman Wars*
339 BCE	*Battle* of *Amphissa*: Greek becomes a Macedonian protectorate
331 BCE	The Greco-Persian battle of *Arbela*
213 BCE	*'Burning of the books'* in China; construction of the Great *Wall of China* (214 BCE)
202 BCE	The Scipio-Hannibal *Battle of Zama*
146 BCE	Greece made a Roman province
52 BCE	*Battle of Alesia*, birth of Gallicio-Roman civilization
48 BCE	The Roman burn the *Alexandrian Library*

63 CE	Israel comes under Roman rule
391 CE	Christian Emperor Theodosas I ordered the burning of the remnants of the Alexandrian Library
325 CE	*Council of Nicaea* marks the rise to power of the *Christian Church* in the Mediterranean World
ca 400–500 CE	End of compilation and canonization of the *Talmud*
451 CE	The *Hun Invasion*
476 CE	'Official' end of the Roman Empire
529 CE	Emperor Justinian closed the *Athenian Academy*

* * *

"*Chymystry, or Chemistry is the art of separating metals, see Alchemy*".

(From a glossary, 1670)

* * *

"*Alchemy was never at any time anything different from chemistry*".

(Justus von Liebig, 1865)

* * *

"*Do you believe that the sciences would ever have arisen and become great if there had not beforehand been magicians, alchemists, astrologers and wizards, who thirsted and hungered after recondite and forbidden powers?*"

(Friedrich Nietzsche, 1886)

* * *

"*Chemistry emerged from alchemy as astronomy from astrology and physics from philosophy*".

(George Sarton, 1950)

The Beginning of Science[1]

"Our forefathers had no other books but the score and the tally".

William Shakespeare, 'Henry 6', pt. 2, **iv***, 7 (1591)*

We shall perhaps never have any adequate information on the most critical period of man's history, when he was slowly emerging out of the darkness.

Science began thousands of years before man learned to write. There certainly was science before there were scientists but no one knows who first kindled a fire, who invented the earliest stone implements, who invented the wheel, developed the bow and the arrow, or tried to explain the rising and setting of the sun. Without articulate language man would have remained an animal. Without writing, the transmission and preservation of knowledge were impossible. It is probable that the above-mentioned breakthroughs involved collaboration; thousand of men, each big step forward being finally secured by exceptional genius. The evolutions leading to each of these fundamental discoveries were exceedingly slow – so slow that the people who took part in them were utterly unaware of them. Genius was then required only from time to time to clinch the results obtained by the unconscious accumulation of infinitesimal efforts, to secure what was gained and prepare another slow movement in the same general direction.

These major advances were among man's first attempts to understand and control the things he saw around him.

Through careful observations of recurring relationships in their environment they began to know nature, and because of nature's dependability, they found they could make predictions that would give them some control over their surroundings.

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[1] Derived from the Latin verb *scire* = to know, to discern, to distinguish. The word *scientist* was coined by **William Whewell** of Cambridge in 1840. For further reading, see:

- Sarton, G., *Ancient Science and Modern Civilization*, University of Nebraska Press, 1954, 111 pp.
- Clark, G., *Space, Time and Man*, A Prehistoric Review, Cambridge University Press, Canto edition, 1994, 165 pp.

When our own particular subspecies, Homo sapiens sapiens, emerged some 40,000 year ago he was already a rather advanced user of tools: spears for hunting, axes for killing animals and for fashioning wood, knives and scrapers for working with materials — these were some of man's earliest tools. He soon devised tools to help him make tools, something no animal before him had done.

These first tools, man's earlies technology, had a great impact on his way of life; weapons enabled him to change his diet from a mostly vegetable one to a reasonably steady diet of animal flesh supplemented with fruits and vegetables. His tools enabled him to fashion cloths and dwellings, so that he was able to live in cold climates. Dwelling gave him another alternative to living in caves or in the open.

Man's bodily evolution reached its present form about twenty-five thousand years ago. Since then there has been a continuous unfolding of his mental capacities. Even at that time the Cro-Magnons in France were already paying attention to life far beyond the bare necessities of food. Spirited inscriptions of women and animals were being made on the walls of their caves. Expressive creativity was an early human trait.

For all but the most recent 10,000 years of his existence, man was a hunter-gatherer; he did not raise plants or animals for his own use, but went out and got them from the wild, when he needed them. Then — about 10,000 years ago — came man's most significant technological discovery: the cultivation of plants[2]*. In terms of its impact on society, agriculture is undoubtedly the most far-reaching of all man's technological innovations. A number of other technological innovations accompanied agriculture, the domestication and raising of animals, the making of pottery, and the weaving of cloth.*

The elaborate civilizations of Mesopotamia and Egypt depended for their birth and development on a settled agriculture practiced by Neolithic people in these regions since 8000 BCE. Without it, they could not have sustained either the increased population or the specialization of urban life. Once secured, these societies spawned a remarkable series of technological advances. The 4th *millennium BCE saw the invention of the plow, the wheel, the sailing boat, and methods of writing. Stone tools gave way to those of copper and bronze, that came into use ca 2500 BCE.*

Although yet devoid of the scientific method as we know it, the technical knowledge of these early societies was rather sophisticated; the pyramids remain one of the finest engineering feats of all time and the mathematics used

[2] The biblical narrative about man's *expulsion from Paradise* (*Gen* **3**, 23) may hint to his transition from food-gathering to food-producing at the beginning of the Holocene.

by priests provided for the development of other sciences. Trade and warfare were the main stimuli of technological advance in the second millennium BCE. The greatest technological event was the mastery of iron by Hittite smiths.

After centuries of experience, the annual nature of the river flood created the need to match human agricultural activity to the natural cycle – irrigation and ploughing came to pass. This most fundamental innovation in the history of man brought civilization into being because if was the instrument of surplus*. Communities were capable of supporting those who are not food producers.*

In order for early man to change from the primitive hunting and gathering way of life to a settled agricultural existence he needed to learn the concept of time*: settled farmers needed to understand the movement of the moon and the sun in order to regulate their agricultural production. Settled farming means trade, trade means communities (villages or towns) which in turn mean the specialization of trades, craftsmanship and art, language, laws and a primitive writing are not far away.*

Science, therefore, began with the conquest of time and distance. That is to say, with the quest of the knowledge needed to keep track of the seasons and to find man's whereabouts in the world he inhabited.

We now know that the social achievements of mankind before the beginning of the written record include far more important things than the perfection of axes and arrowheads. Several discoveries into which he blundered many millennia before the dawn of civilization in Egypt and Sumer, are especially significant: he learned to herd instead of to hunt, he learned to store grain to consume when there were no fruits to gather and he collected bits of meteoric iron. The sheep is an animal with seasonal fertility and cereal crops are largely annual. The recognition of the passage of time now became a primary necessity of social life.

In learning to record the passage of time, man learned to measure things. He learned to keep account of past events. He made structures on a much vaster scale than any of which he employed for purely domestic use.

The arts of writing[3]*, architecture, numbering, and in particular geometry, which was the offspring of star lore and shadow reckoning, were all by-products of man's first organized achievement – the construction of the* calendar*.*

Science began when man started to plan ahead for the seasons, because this demanded an organized body of continuous observations and a permanent record of their recurrence.

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[3] In fact, a symbolic system of notation (primitive writing) seems to have been in use by prehistoric cave-painters! Ergo, necessary elements of civilized cultures were in place by 35,000 BCE.

In the age of satellite communication, atomic clocks, and cheap almanacs, we take time for granted. Before there were any clocks or even simpler devices like the hour-glass or the clepsydra for recording the passage of time, mankind had to depend on the direction of the heavenly bodies (sun by day and stars by night). He learned to associate changes in vegetation, the mating habits of animals, and the recurrence of drought or floods, with the rising and setting of bright stars and star clusters immediately before sunrise or in evening twilight.

When the agrarian revolution reached its climax in the dawn of city-life (ca 8000 BCE), a technique of timekeeping emerged as its pivotal achievement. What chiefly remain to record the beginnings of an orderly routine of settled life in cities are the vast structures which bear eloquent witness to the primary social function of the priesthood as custodians of the calendar: The temple *with its corridor and portal placed to greet the transit of its guardian star or to trap thin shaft of light from the rising of setting sun; the* obelisk *or shadow clock; the* Pyramids *facing equinoctial sunrise or sunset, the pole and the southings of the bright stars in the zodiac; the great stone circle of* Stonehenge *with its sight-line pointing to the rising sun of the summer soltice – all these are first and foremost almanacs in architecture.*

Nascent science and ceremonial religion had a common focus of social necessity in the observatory-temple of the astronomer – priest. That we still divide the circle into 360 *degrees, that we reckon fractions of a degree in minutes and seconds, remind us that men learned to measure angles before they had settled standards of length or area. Angular measurement was the necessary foundation of time keeping. The social necessity of recording the passage of time forced mankind to map out the heavens. How to map the earth came later as unforeseen result.*

Science rests on the painstaking recognition of uniformities in nature. In no branch of science is this more evidence than in astronomy, the oldest of the sciences, and the parent of the mathematical arts. Between the beginning of city life and the time when human beings first began to sow corn or to herd sheep, ten of twenty thousand years – perhaps more – may have been occupied in scanning the night skies and watching the sun's shadow throughout the seasons. Mankind was learning the uniformities which signalize the passage of the seasons, becoming aware of an external order, grasping slowly that it could only be commanded by being obeyed, and not as yet realizing that it could not be bribed. There is no hard and sharp line between the beginnings of science and what we now call magic. Magic is the discarded science of yesterday. The first priests were also the first scientists and the first civil servants.

The Sumerian Heritage (ca 4000–1800 BCE)

The first civilizations[4] *grew up when nomadic hunters and seed-gatherers began to farm the land, and so were able to form settled communities. Where the soil was rich, they could grow enough food to support non-producers like craftsmen and administrators. Probably the earliest civilization grew up in the fertile region between the rivers Tigris and Euphrates — in the lower part of an area known as Mesopotamia*[5].

The first settlers came here about 4000 *BCE, soon after the soil deposited by the rivers had dried enough for farming. The* Sumerians *were among the earliest invaders who probably came from the highlands at present day Turkey and Iran (scholars, however, do not know the exact origin of their racial or language group). They learned to drain the swamps, and to make bricks from mud. They farmed, dug canals, and raised livestock. On an agriculture economy they developed a true city life, so they must have understood the technique of specialization or division of labor.*

Their small settlements grew into cities and city-states. The more powerful city-states conquered their neighbors and created small kingdoms, including Kish*,* Lagash*,* Nippur*,* Umma*,* Uruk *and* Ur*. Each of these had fine buildings, streets, temples, palaces, fortifications, public water supplies and drainage over an average area of one square kilometer. They built a great network of irrigation canals, domesticated animals and their society included priests, soldiers, tradesmen and engineers. To this end they developed a government that handled both secular and religious matters, a trained army with armor, weapons of war, and carts for transportation.*

The Sumerian cities were united in a loose league. They shared a common culture, they spoke similar languages, and they worshiped the same gods. A typical Sumerian city-state consisted of the city proper and as much of the surrounding population as the city walls could accommodate in times of crisis. The focal point of each city was a great platform raised above the surrounding residential areas. On it were erected the many-storied temples (ziggurates) which became a hallmark of Babylonian monumental architecture[6].

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[4] From the Latin: *civitas* for "city-state".

[5] From the Greek: "between the rivers".

[6] Many Sumerian traditions lie behind Bible legends: the *Tower of Babel* was almost certainly one of the ziggurates (its height could reach 100 m). The *Flood* is described in the Sumerian *Epic* of *Gilgamesh*. Archaeologists have shown that a great flood overwhelmed the area thousands of years ago. For further reading,

Writing, in the strict sense of the term, was first invented and employed in Mesopotamia about 3100 BCE. If it was not actually originated by the Sumerians, it was at any rate very soon taken over by them to represent their language, and it became one of the most important ancient writing systems.

The Sumerian wrote by pressing a split bamboo reed (stylus) on wet clay which was then baked until it was hard. The end used was triangular in shape. The writer did not scratch the lines of his picture, but in making a single line he impressed one corner of the tip of the stylus into the soft clay, and then raised it again to impress another line in the same way. Owing to the oblique tilt of the stylus, as well as its shape, each line thus made was wider at one end than at the other, and hence appeared triangular or wedge-shaped. Finally, every picture or sign written with such a stylus came to be made up of a group of wedge-shaped lines. We therefore call the system cuneiform[7] *(from the Latin* cuneus *= wedge) writing. Pictures made up of these wedge lines became more and more difficult to recognize, especially as speed of writing increased. All resemblance to earlier pictures finally disappeared.*

Most of the earliest examples of Sumerian writing that we have found are business records. But school texts appeared almost at once as means of educating specialists in the new technique. Literary texts and historical records followed in due course.

The transition from the picture stage to the phonetic *stage was early made. Sumerian writing finally possessed over 560 signs, but each of these signs represented a syllable or a word, that is a* group *of sounds; the Sumerian system never developed an* alphabet of letters *which made up the syllables. That is, there were signs for syllables, but no signs for the letters which made up such syllables.*

The Sumerian clay tablets reveal the sophistication of the Sumerian civilization as well as their degree of knowledge of mathematics, astronomy and medicine. The clay records show us that in measuring time the Sumerian scribe began a new month with every new moon, and he made his year of 12 of these moon-months. Since 12 such months fall short of making up a year, the scribe slipped an extra month whenever he found that he had reached the end of his calendar-year a month or so ahead of the seasons. As in Egypt,

see: Kramer, S.N., *History Begins at Summer*, Doubleday Anchor Books: New York, 1959, 247 pp.

[7] The decipherment of the cuneiform was pioneered in 1802 by **Georg Friedrich Grotefend** (1775–1853, Germany). His work was continued by **Henry Creswicke Rawlinson** (1810–1895, England), and by the mid-1800's the cuneiform code was broken. Since 1802, several hundred thousands cuneiform tablets have been recovered and placed in museums.

the years themselves were not numbered, but each year was named after some important event occurring in the course of the year.

The Sumerian system of numerals was not based on tens, but had the unit 60 *as a basis. The leading unit of weight which they used was the mina, divided into 60 shekels (the* mina *had the weight of a pound).*

The Sumerian writing spread to Elam (Iran) and Egypt by 3000 *BCE, to Crete (2200 BCE), India (2000 BCE) and to the Hittites (1500 BCE). [China invented writing independently by* 1300 *BCE and the Mayans by* 700 *CE].*

The Sumerian age of small, independent city-states which began at about 3000 *BCE, lasted until* 2500 *BCE. Such cities as Ur, Uruk, and Lagash waged local wars. Each city ruled its neighboring areas at various times. But the advantages of civilization could not be contained within the confines of the Sumerian city-states. By about* 2500 *BCE, cities on the Sumerian model had appeared throughout the area from the Tigris-Euphrates Valley to the eastern coast of the Mediterranean. The people of these newer cities were not Sumerian but Semitic. The Semites, a number of different people speaking similar languages, seem to have originated from the Arabian peninsula where they had been nomads who lived by breeding animals. As Arabia turned desert, Semites moved in several waves into the eastern Mediterranean and the parts of Mesopotamia north of the Sumerians. By* 2300 *BCE, the old Sumerian cities were merely the core of a much larger civilization that embraced the entire Fertile Crescent and whose population was primarily Semitic. The region first occupied by the Semites north of Sumer was finally called* Akkad *and the leading Semitic settlers there bore the name of* Akkadians*. Akkad occupied a very strong commercial portion on the main road from the Two Rivers to the eastern mountains and its trade brought it prosperity.*

The Semitic King Sargon I (ca 2300 BCE) was first to unite the city-states of Mesopotamia into a single empire, and the Sumerian civilization was gradually absorbed by the Semitic people. By 1800 BCE the Sumerians lost all political power, but the union of Sumerians and Semites furnished the base on which the Babylonian civilization developed[8].

[8] Recent archaeological evidence disclosed that by 1800 BCE the accumulated *salinization* of the soil due to irrigation reached a level where food production was insufficient for the growing population. This man-made ecological disaster was probably one of the major factors in the demise of the Sumerian civilization.

Climate and Civilization

Climate has been subject to change since the earliest times known to geologists. Because geology is a relatively young science, however, this fact has been recognized for only about a century; and at first only the larger fluctuations, the extremes of Ice Ages and Interglacial, were recognized. But soon geologists found evidence that ice sheets, both in their expansion and recession, were subject to interruption — that is, neither advances nor retreats proceeded smoothly and linearly, but each of them from time to time interrupted by a reversal of the primary trend. Thus, paleo-ecological, geological and archaeological evidence has been accumulated in favor of climatic fluctuations in Europe, in the form of damped oscillations, of gradually diminishing amplitude and duration, over the past 10 *to* 15 *thousand years.*

In the Mediterranean Basin, the Near East and northern and central Africa the evidence is more subtle and difficult to detect. However, the relation in time between the subtropical and European climate changes were clarified due to intensive research in the past four decades. A synchronism was established between the period of advance and growth of glaciers in Europe to glacier advance in central-east African mountains and the pluvial period in the Mediterranean Basin. It was also found that fluctuations that were too small to leave clear geological evidence, could still be large enough to produce highly significant ecological effects, which may be reflected in archaeological evidence.

The 8000 *years between the end of the last ice-age (breakdown of the Scandinavian ice-sheet) and the time of Christ saw a sequence of substantial changes, including some thousands of years that were warmer, and in low latitudes, moister than now. Such shifts in climate may have been manifested through long summer drought, repeated rains and river floods, damaging gales and long severe frost. Glaciers may have melted or expanded, blocking mountain passages previously used for traffic. World sea-level rose and fell with the gain or loss of water from the world's glaciers. In primitive economies such effects could be disastrous and may have affected the course of history.*

A particularly sharp change seems to have occurred in the interval 1200–500 BCE. This history overlaps the rise and fall of various civilizations and migrations of the so-called 'barbarian' cultures of Northern and Central Europe and Asia. The driving forces behind many historical upheavals could well have been famine, epidemics and wars caused by droughts and floods and the arrest of naval commerce lines caused by stormy seas.

At the present time we have more or less a complete record of human history in South-Western Asia since about 5200 BCE for which we have the following data sources:

- *Instrumental records and old weather journals going back at most for 300 years.*
- *Literary records: accounts of floods, droughts, severe winters, and great storms.*
- *Traditions (such as that of the Deluge which can sometimes be correlated with other data.*
- *Fluctuations of lakes and rivers, glaciers and other natural indices of climate, which can often be connected with historical events or dated by laminated clays.*
- *Arguments from migration of people, for which climatic reasons may be assigned with some determined probability. To this we may add the waxing and waning of civilizations.*
- *The rate of growth of trees, as shown by annual rings of tree-growth, which can be correlated with the annual rainfall.*
- *Geological evidence — great advances or retreats of glaciers, growth of peat-bogs, succession of floras, etc., which can sometimes be approximately dated.*

The evidence afforded by racial migrations depends on the principle that during a period of increased rainfall there is a movement of peoples from regions which are naturally moist to regions which are naturally dry, while during the drier periods the direction of movement is reversed.

It has been shown, for example, that emigration from Europe to the United States depended on the rainfall. In order that this principle may be used to determine the course of climatic variations, certain conditions are necessary. First, there must be large areas which are on the borderline between aridity and complete desert; these areas must be mostly too dry for extensive agriculture, but with sufficient resources to support under average conditions a large nomadic population, while a succession of dry years renders them almost uninhabitable.

In close proximity to this arid region there must be a fertile well-watered plain, with a long and accurately dated history. During dry periods the nomads are driven from their homes by lack of water, but they find little difficulty in moving from point to point, and the sedentary agriculturists of the neighboring plain generally find them irresistible. It is only in Asia that these conditions are fulfilled in perfection, the rich plains of the Tigris and Euphrates, the site of a long succession of civilized states, having on the one *side the semi-deserts of Arabia and Syria, on the other side a great dry region extending eastwards and north-eastwards as far as China. We should expect a period of decreased rainfall to initiate a series of great migrations spreading out from the dry regions and recorded in the history of the Mesopotamian states as the invasions of barbarians.*

The history of Egypt does not provide such a complete a record, because the desert on either side of the Nile valley is too dry, even under favorable conditions, to support a nomadic population sufficiently large to have made any impression on the might of ancient Egypt. The Hyksos conquest of about 1800 *BCE is the main exception, but the Hyksos themselves probably came out of Asia. The invasions of China from the west provide some evidence of climatic fluctuations in the east of Asia.*

Man's awareness of the fickleness of climate can be traced back to the dawn of history and may have existed long before that in tales carefully transmitted from generation to generation, which even now survive enshrined in myth and legend. In contrast, the view, regarded as scientific, which was widely taught in the earlier part of this century, that climate was essentially constant apart from random fluctuations from year to year, was at variance with the attitudes and experience of most earlier generations. It has also had to be abandoned in face of the significant changes that occurred in many parts of the world.

Among the earliest written reports of climatic fluctuations are the inscriptions recording the yearly levels of the Nile flood, some of them from around 3000 BCE. Other inscribed tablets, or steles, show a reasoned awareness of the liability of the Nile to fluctuations lasting some years.

In numerous ancient writings and legends we may distinguish knowledge of certain climatic catastrophes, which changed the face of the world and from which recovery was either long delayed or never fully established. The commonest of these accounts, usually of the distant past, are those that relate to a great flood, of which NOAH's flood is perhaps the best known example.

Climatic oscillations lasting up to 100 years or so, may be due in part to changes in the general circulation of the atmosphere and in part due to interaction of ocean currents and floating ice-fields.

Because of the nonlinear nature of these interactions, under favorable conditions, comparatively small causes may have disproportionally large effects.

One must also remember that while major climatic changes tend to be nearly synchronous globally, they are not generally in the same sense or magnitude in all places: so there can be damp weather in Western Europe, and simultaneous drought in North America.

One of the consequences of a short term climatic change which affected and afflicted mankind during its entire history is the drought condition resulting from a significant shortage in water storage due to lack of precipitation.

4226 BCE Time-reckoning may have began in Egypt with the institutioning of the first calendar with a 365-day year, broken into 12 thirty-day months plus 5 days for festivals.

4000–3200 BCE *Chalcolithic Age*: The copper-stone age, just before the dawn of history, during which man first learned to create metal tools and ceremonial objects. Emergence of houses, towns and geometric designs on pottery. This transition from the Prehistoric to the Historic Age was everywhere a slow and gradual one. *Civilization* slowly appeared in the *Near East* while the men of the Late Stone Age in *Europe* continued to live without metal, government, writing, large ships and many other creations of civilization. Beginning at about 2000 BCE, civilization gradually and slowly diffused toward Europe via the Aegean world as it received all the above benefits from the nations of the Near East.

ca 4000 BCE Advent of *horseback riding* in the uplands between the Dnieper River and the Carpathian Mountains, first *sail-propelled boats*; bricks are fired in kilns in Mesopotamia; first recording of *star-constellations*. Invention of the plough. Sea-level rise began to slow down, but the slow rise will continue to ca 2000 BCE.

4000–3000 BCE First urban civilization in the world appeared in the region between the Mediterranean and India during the so-called post-glacial optimum[9].

ca 4000 BCE Early metallurgists in the foothills and semiarid plains of Anatolia[10] and Iran discovered sources of copper other than the native metal. None of these ores superficially resembles copper itself and cover a

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[9] Caused by cessation of volcanic activity and slight increase in solar radiation in the middle and high latitudes.

[10] Neolithic man was acquainted with various *minerals*: In the cave painting in France and Spain, he employed *ocher* (hydrated Fe_2O_3) and *pyrolusite* (MnO_2) as pigments. Women used *rouge* (Fe_2O_3) and *eye shadow* (PbS) before the 6^{th} millennium BCE. Beads of malachite [$Cu_2(OH)_2CO_3$] and other minerals and

wide range of colors: green malachite, blue azurite [$Cu_3(OH)_2(CO_3)_2$], red cuprite (Cu_2O), gray chalcocite (Cu_2S) yellow chalcopyrite ($CuFeS_2$), and purple bornite (Cu_2FeS_2). The first reduction of malachite to metal must have been an accidental discovery, probably in a pottery oven. The propagation of copper from sulfide ores was a still more extraordinary discovery, because a much more sophisticated technique is required.

3784, Oct 20 BCE First record of a solar eclipse was made in India (according to the **Rig Veda**). The next four records of solar eclipses in human history were: 2136, Oct 10 BCE (China); 1375, May 13 BCE (Ugarit); 1178, April 16, BCE (Homer's *Odyssey*, Greece).

ca 3580 BCE Great eruption of mount *Vesuvius* (VEI=6).

ca 3500 BCE The introduction of the *wheel*[11] in Mesopotamia during the period of establishment of the city-states. It took two forms: a stone potter's wheel and a cartwheel made from a solid piece of wood[12]. A simple picture of a solid-wheeled vehicle was found at *Uruk*. The royal standard of Ur, dating from 2750 BCE, shows carts with solid wooden wheels.

ca 3500 BCE *Indo-European proto language*, spread by nomadic *horseback* herders from the Eurasian steppes, reached the Near-East. Their language will give rise to the Indo-European languages, including the branches called *Germanic* (English, German), *Italic* (Latin, French), *Slavic* (Russian), *Indo-Iranian* (Sanskrit), *Baltic* (Lithuanian), *Celtic* (Gaelic), and Greek, Albanian, Armenian, and Anatolian (Hittite). Linguists and anthropologists tend to

pieces of copper and gold were among their items of personal adornment. In the region from the Anatolian plain to the edge of the Iranian desert, neolithic man could find nuggets of gold and copper in stream beds, where the native metal was concentrated by the flowing water. In northern Iraq, copper beads have been found that date from 8500 BCE. By 6000 BCE, neolithic people in the region of the Fertile Crescent had abandoned their nomadic life as hunters to become grain growers and stock raisers. They had also mastered some of the secrets of fire and clay, for pottery makes its first appearance at this time and the skills of potter were indispensable to the first metallurgists.

[11] Was the invention of the wheel a result of man's keen observation of the ease with which the dung-beetle (scarab) was maneuvering the dung ball? We know that this beetle was once held sacred by the ancient Egyptians.

[12] First literary mention in the biblical story of Joseph (*Genesis* **45**, 21, 27), relating to ca 1650 BCE.

believe that this *mother tongue*[13] spread into Europe by farmers. Even if a farmer's offsprings had moved only 15 km from the family farm to set up farms of their own, the resulting move of agriculture could have diffused throughout Europe from Anatolia in about 1500 years, carrying the Indo-European language with it.

Moreover, cultural anthropologists[14] are now able to trace *the migration of people* throughout the world in the last 10,000 years as having started with the growth of population in a location between the Black Sea and the Caspian Sea.

ca 3500 BCE Earliest known use of *Bronze*[15] in Sumer. The period in history between the *Stone Age* and the *Iron Age* (ca 1100 BCE) became known as the *Bronze Age*[16]. The origin of bronze had been different in different places, but it is reasonable to suppose that it was first discovered by the accidental smelting of mixed ores of copper and tin, such as known to occur.

Bronze Age flourished in the *Aegean world* during 2900–2000 and continued during 2000–1100 in the *Cretan*, *Minoan* and *Mycenaean* civilizations.

ca 3500–3000 BCE Mesopotamia springs into *history* with the *city-states* of Sumer and Akkad; advent of pictographic *writing*; emergence of societies in the Great Valleys with social and material benefits (Egypt, China, India,

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[13] The Bible (*Genesis* **11**) associates the story of the *Tower of Babel* with the confusion of languages and the *dispersion of the races throughout the world*. Clay tablets written in Sumerian indicate that Sumerians believed that there was a time when all mankind spoke one and the same language, and that it was *Enki*, the Sumerian god of wisdom, who confounded their speech.

[14] **Luigi Luca Cavalli-Sforza** and **Francesco Cavalli-Sforza**, *The Great Human Diasporas*, Addison-Wesley, 1995.

[15] *Bronze*: an alloy of *copper and tin*. The oldest alloy known to man. It replaced stone and soft copper and was used as the primary material for weapons and cutting tools. It was commonly used to cast containers such as cups, urns, and vases. Men also shaped bronze into battle-axes, helmets, knives, shields, swords, and ornaments. In contradistinction, *Brass* is an alloy of *copper and zinc* and was probably discovered later by melting copper ore that also contained a small amount of zinc.

[16] The bronze age is not a particular period of time; some areas had their bronze age early, others had it late, and some skipped it altogether. The Bronze Age in any region usually overlapped on earlier Stone Age and a later Iron Age, because people did not stop using one material all at once. The Biblical reference to *Tuval-cain* as a master forger of bronze tools (*Genesis* **4**, 22) is probably associated with the early Bronze Age III in the Near East (ca 2400 BCE).

Mesopotamia); introduction of the *wheel* in Mesopotamia and its use in hauling carts and making pottery (ca 3300 BCE).

Glacier advances around 3000–2800 BCE severely disturbed Neolithic agricultural economy in central Europe and Asia, causing the spread of people to Western Europe. Climate deteriorated on a global scale and glacier's movement caused major flooding in Mesopotamia and Egypt.

ca 3400 BCE Development of *hieroglyphic writing*. Egyptians developed their *number system*[17] to the point where they could record numbers as large as 100,000,000. Thus, written numbers preceded any known form of written

[17] The invention of hieroglyphics took many years, maybe even centuries. Hieroglyphs were carved or "printed." The *hieratic* was the first cursive form of hieroglyphics, developed much later. It was a quicker and more convenient way of recording an agreement, conveying a message, or making a calculation with numbers than by drawing hieroglyphs.
As the need for keeping track of large numbers arose, the Egyptians developed a grouping system in order to perform more complex operations. In this system some number n is selected for the base and symbols are adopted for $1, n, n^2, n^3$, and so on. Then any number was expressed by adding these numbers.
Their grouping system had a base of ten. Their symbols were powers of ten:

- 1: The symbol for a staff or rod.
- 10: A heel bone or arc.
- 100: A rope coil or scroll symbol.
- 1,000: The lotus flower.
- 10,000: A pointing finger.
- 100,000: A burbot (fish).
- 1,000,000: An astonished man.
- 10,000,000: A sun on the horizon.

They had no mathematical notation, except the symbol for total. There were no plus, minus, multiplication, or division signs. No square root signs, zeros, or decimal points were used. If they wanted to indicate a sign, they had to write it out in hieroglyphics. Although they didn't have mathematical notation, theretwice-times table and two-thirds-table enabled them to do a significant amount of mathematical problems for their time.

words. Numerals probably preceded words in the Orient and in the Americas as well.

ca 3200 BCE Metal-molding were practiced at Sumer to make copper and bronze axes. Egyptians used papyrus to write on. Sailing ships were used in Egypt. A *ziggurat* in Ur (Mesopotamia), 12 m high, shows that the Sumerians were familiar with columns, domes, arches and vaults.

3200 BCE Earliest evidence of *political structuring* in Egypt: administrators are appointed to ensure a regular water supply to the fields. Egypt is developing into a sophisticated centralized civilization. Astronomy, picture-writing and mathematics evolve and develop as necessities from the demands of canal-building, regular harvesting, granary building, taxation system and security (metal weapons).

3300 BCE Evidence for *copper industry* in the Alps found in 1991 with the discovery of the Austrian Alps 'Iceman' buried in snow. This pushes back the copper age by some 1000 years.

ca 3200 BCE Egyptians made the earliest recorded sea voyage during the reign of Pharaoh **Sneferu**. It was commemorated in a hieroglyphic inscription which recorded the "bringing of 40 ships of one hundred cubits with cedar wood from Byblos". It was the need for timber, for their temples, palaces and ships, that occasioned the voyage to Byblos, the port for timber hewn in the great cedar groves of the Lebanon mountains.

ca 3000 BCE The first glass was made by man in the form of a glaze on ceramic vessels.

ca 3000 BCE Breakup of a large comet produced a *zodiacal light*[18] so prominent that it was confused with the *Milky Way*. It is mentioned repeatedly in ancient literature.

ca 3000 BCE *First large-scale irrigation system* in Egypt and Mesopotamia. It began with the discovery of farmers that they could exploit the natural irrigation afforded by rains and floods in order to raise crops during most of

[18] *Zodiacal light*: sunlight reflected off dust that lies in the ecliptic plane (zodiac). This dust is believed to be fine debris from long-lost comets and is studied by astronomers using space-borne telescopes sensitive to infrared (heat) radiation. Sometimes, when an active comet sheds a great deal of material, the zodiacal light becomes very bright. Such was the case e.g. on the night of 07 June 1843 in South Africa, when it resulted from debris of comet *Encke*. It is currently so faint that it barely even merits mention in textbooks.

the year. By this means the same patches of land were cultivated repeatedly. Farmers were then led to invent the *plow* that oxen could pull. This in turn helped farmers produce much more food than they needed for their families. The food surpluses enabled more and more people to give up farming and more to the cities. Classes of builders, craftsman, merchants priests, clerks, miners, smelters, transporters and officials began to appear – and systems of writing were invented.

ca 3000 BCE Earliest written reports of *climatic fluctuations* in Egyptian inscriptions, on tablets or steles, recording the yearly levels of the Nile flood.

ca 3000 BCE The Chinese used the binary system in their arithmetic calculations. Hieroglyphic numerals in Egypt.

ca 2900 BCE Earliest archaeological evidence for the existence of the city of *Yafo*[19], on the Mediterranean coast of Israel. It was the scene of the ancient legend of Andromache and Perseus. The part of Yafo is mentioned in Old Testament (*Josh* **19**, 46; *Jonah* **1**, 3; *Ezra* **3**, 7; *Chron.* II. **16**). The name is also mentioned in the tribute lists of the Egyptian king Tethmosis III in the 15^{th} century BCE.

Yafo was probably under the control of the Phoenicians until the Persian period. It was brought under Israeli control by the Maccabees (164 BCE). Pompey made it a free city (63 BCE), but Caesar restored it to the Israelis (47 BCE).

As the only harbor in the Israeli coast between Egypt and Carmel, Yafo was of great commercial importance throughout history. The modern *Tel Aviv-Yafo* with a population of ca 360,000 is the largest city in Israel.

ca 2800 BCE The earliest known book of medicine, the *Great Herbal* of Emperor **Shen Lung**. There is a Babylonian physicians seal of about the same date.

[19] Other names are *Joppa* (Greek) and *Jaffa* (Arabic). The Hebrew name could have originated from the Hebrew word *Yafé* (= beautiful) or *Japeth* (= son of Noah).

Origins of the Egyptian Civilization

Egyptian civilization did not emerge spontaneously as a full-grown discipline. It had its note in former African civilizations: Carbon dating of the remains of barley and einkorn wheat found at Kubbaniya, near Aswan in Upper Egypt, shows the beginning of agriculture that existed around 16,000 BCE, and this evidence is supported by the large concentrations of agricultural implements from around 13,000 BCE.

Moreover, the discovery of recent archaeological artifacts of neolithic communities in Egypt, indicate that they may have belonged to groups from the once fertile Sahara region who were forced to migrate eastward as the desert spread. So, the culture and people of Egypt initially originated in the heartlands of Africa. This is borne out by the historian **Diodorus**, *who wrote (ca 50 BCE) that the Egyptians "are colonists sent out by the Ethiopians... And the large part of the customs of the Egyptians are Ethiopian".*

Egyptian mathematics[20] (3100–1250 BCE)

Due to the more advanced economic development of Babylonia, the mathematics of ancient Egypt never reached the level attained by Babylonian mathematics: Babylonia was located on a number of great caravan routes, while Egypt stood in semi-isolation. Nor did the relatively peaceful Nile demand such extensive engineering and administrational efforts as did the more erratic Tigris and Euphrates. But the veneration that the Egyptians had for their dead and the unusually dry climate of the region, led to the preservation of many papyri and objects that would otherwise have perished.

The origins of the urban revolution that transformed Egypt into one of the great ancient civilizations are the gradual development of effective methods of flood control, irrigation and marsh drainage which contributed to a significant increase in agricultural yield. Clearly, a prerequisite for such innovations required organization that sprang out of cooperation among preexisting scattered settlements. Thus, prior to the emergence of the highly centralized government of Pharaonic Egypt, a form of communal village nucleations may

[20] For further reading, see:

- Van der Waerden, B.L., *Science Awakening*, P. Noordhoff: Groningen, Holland, 1974, 306 pp.
- De Camp, L.S., *The Ancient Engineers*, Ballantine Books: New York, 1963, 450 pp.
- Gillispie, C.C. and M. Dewachter (Editors), *Monuments of Egypt*, The Complete Archaeological Plates from the Description De L'Egypt. Princeton Architectural Press, 1994.
- David, R.A., *The Egyptian Kingdoms*, Elsevier: Phaidon, 1975, 152 pp.
- Neugebauer, Otto, *The Exact Sciences in Antiquity*, Dover, New-York, 1969, 240 pp.
- Joseph, G.G., *The Crest of the Peacock*, Princeton University Press, 2000, 455 pp.
- Wilson, A.M., *The Infinite in the Finite*, Oxford University Press, 1995, 524 pp.
- Hogben, L., *Mathematics for the Million*, W.W. Norton and Company, London, 1993, 649 pp.

have come into existence as an institutional back-up for these agricultural innovations.

Between 3500 and 3000 BCE, the separate agricultural communities along the banks of the Nile were gradually united, first to form two kingdoms – Upper and Lower Egypt – which were brought together (ca 3100 BCE), as a single unit by ***Menes****, who came from Nubia (part of present-day Sudan). Menes forced a long line of Pharaohs, 32 dynasties in all, who ruled over a stable society for the next 3000 years.*

Up to 1350 BCE, the territory of Egypt covered not only the Nile valley but also parts of Israel and Syria. Control over such a wide expanse of land required an efficient and extensive administrative system. Censuses had to be taken, taxes collected, and large armies maintained. Agricultural requirements included not only drainage, irrigation and flood control, but also the parceling out of scarce arable land among the peasantry and the construction of silos for storing grain and other produce.

As Egyptian civilization matured, there evolved other pursuits requiring practical arithmetics and mensuration: financial and commercial practices demanded numerical facility. This evolving numerate culture, serviced by a growing class of scribes and clerks led, in turn, to the construction of calendars and the creation of a standard system of weights and measures. Finally, this practical mathematical culture culminated in the construction of ancient Egypt's longest lasting legacy – the Pyramids.

It is possible to distinguish three different notational schemes of numeration used in ancient Egypt: hieroglyphic *(pictorial),* hieratic *(symbolic) and* demotic *(popular). The hieratic notation was employed in both the Moscow and Rhind papyri . The demotic variant was a popular adaptation of the hieratic notation and became important during the Greek and Roman periods of Egyptian history.*

The hieroglyphic *system of writing was a pictorial script where each character represented an object. Special symbols were used to represent each power of 10 from 1 to 10^7. With these symbols, numbers were expressed in the decimal system e.g.* $17509 = 1(10^4) + 7(10^3) + 5(10^2) + 0(10^1) + 9(10^0)$. *No difficulties arose from not having a symbol for zero or place-holder in this number system. Addition and subtraction posed few problems: In adding two numbers, one made a collection of each set of symbols that appeared in both numbers, replacing them with the next higher symbol as necessary. Subtraction is merely the reversal of the process of addition, with decomposition achieved by replacing a larger hieroglyph with ten of the next lower symbol.*

The hieratic *representation was similar in that it was additive and based on powers of ten, but it was far more economical (fewer symbols). While this* *notation was more taxing on memory, its economy, speed and greater suitability for writing with pen and ink (for writing on paper in contradistinction to hieroglyphs carved on stone or metal) caused the gradual replacement of hieroglyphs.*

These papyri reveal the state of mathematics in Egypt from about 3100 BCE to about 1100 BCE.

One consequence of their numerical system is the additive character of the dependent arithmetics. Multiplication and division were usually performed by a succession of doubling operations depending on the fact that any number can be represented as a sum of powers of 2.

This Egyptian process of multiplication and division eliminated the necessity of learning a multiplication table. It is also so convenient on the abacus that it persisted as long as that instrument was in use.

Thus, this method required prior knowledge of only addition and "two-time" table.

In a modern variation of this method, still popular among rural community in Russia, Ethiopia and the Near East, there are no multiplication tables and the ability to double and halve numbers (and to distinguish odd from even) is all that is required.

Whether the Egyptians knew it or not, their method is based on the unique binary representation of every number.

In modern notation, if N *is a positive integer, it can be written as*

$$N = a_r 2^r + a_{r-1} 2^{r-1} + \cdots + a_1 2^1 + a_0,$$

where $a_k = 0$ *or* 1. *Suppose that* N *and* M, *whole numbers, are to be multiplied; then* $NM = \sum_i 2^i M$, *where the sum includes only these terms for which* $a_i = 1$. *Then to calculate* $2^i M$, *just double* M, *then double the result and continue to double until* $2^i M$ *is reached. Therefore, if we can add and double numbers, we can also multiply them. The Egyptians (lacking computers) still faced one little problem: how to find, quickly, the non-zero coefficients in the binary representation of* N. *Their ingenious trick was to half the multiplicand* N *(to the nearest integer* $[\frac{N}{2}]$*) and keep halving the result until unity is obtained. Then, consider only the odd number in this sequence!*

This ancient method of multiplication provides the foundation of Egyptian arithmetic. It was widely used, with some modifications, by the Greeks and continued well into the Middle Ages in Europe.

In the inverse process of division, fractions had to be introduced whenever the quotient was not an integer. But since their method of writing numerals did not allow any unambiguous way of expressing fractions, they tackled the problem in a quite ingenious way.

Modern historians and mathematicians have tried to discover the rules that the Egyptians actually followed in performing calculations with fractions. From all cases available in the table of the Papyrus Rhind (ca 2000–1800 BCE), ***Neugebauer*** *recorded that the Egyptians actually used simple algebraic rules such as (in modern notation)*

$$\frac{2}{n} = \frac{1}{2}\frac{1}{n} + \frac{3}{2}\frac{1}{n} \qquad \textit{or} \qquad \frac{2}{n} = \frac{1}{3}\frac{1}{n} + \frac{5}{3}\frac{1}{n}$$

to obtain results such as

$$\frac{2}{3} = \frac{1}{6} + \frac{1}{2}, \qquad \frac{2}{5} = \frac{1}{15} + \frac{1}{3}.$$

In general, they were concerned with the representation of a rational numbers as the sum of unit fractions, i.e.

$$\frac{m}{n} = \frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_k}.$$

This has suggested numerous problems, many of which are still unsolved, and continue to suggest new problems, so that interest in Egyptian fractions is as great as it has ever been. One such problem suggested by ***Paul Erdös*** *concerns the Diophantine-like equation*

$$\frac{4}{n} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}, \qquad n > 1,$$

where $x,\ y,\ z$ and n are positive integers [e.g. $n = 4,\ x = 2,\ y = 3,\ z = 6$]. Examples of other fractions are

$$\frac{2}{7} = \frac{1}{5} + \frac{1}{13} + \frac{1}{115} + \frac{1}{10465},$$

$$1 = \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \frac{1}{15} + \frac{1}{21} + \frac{1}{27} + \frac{1}{35} + \frac{1}{63} + \frac{1}{105} + \frac{1}{135}.$$

A well-known riddle associated with Egyptian fractions is the following: A dying rich man, who owned 11 cars, willed 1/2 of them to his oldest daughter, 1/4 to his middle daughter, and 1/6 to his youngest daughter. But the *problem arose how to divide his 11 cars in strict accordance with the will. A car-dealer offered his help by* lending *the heirs a brand-new identical vehicle so that each daughter could now receive a whole car: the oldest 6, the middle 3 and the youngest 2. After the heirs had driven off, one car remained for the dealer to reclaim!*

The problem really solved here was to express $\frac{n}{n+1}$ *as a sum of three Egyptian fractions:*

$$\frac{n}{n+1} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}.$$

In the above story $n = 11,\ a = 2,\ b = 4,\ c = 6$. *Interestingly, for* $n = 11$, *there is another solution (and story) with* $a = 2,\ b = 3,\ c = 12$ *because* two *subsets of the divisors of 12 (1, 2, 3, 4, 6, 12) add to 11 (2+3+6=1+4+6=11). The inheritance problem is related to* pseudo perfect *numbers, defined as numbers equal to a sum of a subset of their divisors.*

Thus, the main Egyptian mathematical accomplishment was the invention of a system of notation that made it possible to express the result of an arbitrary division of integers. However, this process was cumbersome and did not result in a unique expression for a fraction, which in turn made it difficult to compare the size of numbers. They used $17/12$ *for* $\sqrt{2}$ *and symbols for plus, minus, equality and unknowns appeared in their arithmetic.*

With this limitation of their calculational capabilities, the Egyptians could not have solved quadratic equations. Clearly, advances in algebra are dependent on an efficient notation for numbers and on systematic methods for computing with them.

As opposed to continued fractions, *unit fractions are of relatively little use. In fact, they probably set back the development of Egyptian mathematics. However, they do provide fertile ground for numerous unsolved problem in* Diophantine analysis.

Operating with unit fractions is a singular feature of Egyptian mathematics, and is absent from almost every other mathematical tradition.

*Egyptian algebra is referred to as '*rhetorical algebra*': the rules devised by mathematicians for solving problem about numbers were expressed* verbally *and consisted of detailed instructions about what was to be done to obtain the solution to a problem.*

In time, the prose form of rhetorical algebra gave way to the use of syncopated algebra *where abbreviations for recurring quantities and operations* *were introduced. Traces of such algebra are to be found in the works of* **Diophantos***(ca 250 CE), but it achieved its fullest development in the work of Hindu and Arab mathematicians during the first millennium CE.*

During the past five centuries there has developed 'symbolic algebra' where, with the aid of letters and signs of operation and relation ($+, -, \times, \div, =$)*, problems are stated in such a form that the rules of solution may be applied consistently. This evolution had to await the development of a positional number system which allowed numbers to be expressed concisely and with which operations could be carried out efficiently.*

An ancient mathematical document dated ca 1850 BCE and known as the "Moscow Papyrus" reveals the state of knowledge of ancient Egyptian mathematicians.

The papyrus was discovered in 1890 in the Necropolis of Dra Abul Negga in Egypt and it was acquired in 1912 by the Moscow Museum of Fine Arts. It consists of 25 mathematical problems and their solutions. The solutions of the geometrical problems are basically arithmetical, offering no trace of a proof in the Euclidean sense.

A second document from the same time is the "Rhind Papyrus", now in the British Museum. It reveals a highly original procedure for operating with fractions. The Egyptians used a decimal system of notation, but had no procedure which was in the nature of a general proof. The geometrical problems were mensurational. The document was found at Thebes and purchased in 1858 by the English archaeologist Alexander Henry Rhind.

The above sources reveal that Egyptian algebra did not go far beyond linear equations in one unknown and pure quadratic equations in one unknown. Their algebra had some symbolism such a symbols for plus, minus and equals.

In geometry they advanced only as far as was required for computing simple land areas and granary volumes. Thus, the area of a circle was taken equal to that of a square of side 8/9 of the diameter (leading to the value $\pi \sim 3.16$*). The volume of a right cylinder was obtained as a product of the base area by the length of the altitude. They also knew that the area of any triangle is given by half the product of the base and altitude.*

An army of some 100,000 laborers working for a period of 30 years built for them the great pyramid of Gizeh at about 2700 BCE. It undoubtedly involved some mathematical and engineering problems. Indeed, problem 14 of the Moscow Papyrus reveals that they knew the general formula

$$V = \frac{1}{3}h(a^2 + ab + b^2),$$

giving the volume of a frustum of a square pyramid of the height h and the sides a and b of the bases.

The geometry of the Egyptians was merely an applied arithmetic in the sense that areas and volumes were determined according to certain approximate calculational rules[21]*. However, nowhere does a systematic derivation of these rules occur. Looking at Egyptian mathematics as a whole, one sees only rules for calculation without any motivation, but calculation is not the same as mathematics.*

Astrology was, in Egypt, the prelude to astronomy*. The stars were observed that they might be duly worshiped. The importance of their first visible appearances at dawn (for the purposes of both practical life and ritual observance) caused them to be systematically noted. The length of the year was accurately fixed in connection with the annually recurring Nile flood, while the curiously precise orientation of the Pyramids afforded a lasting demonstration of the high degree of technical skill in watching the heavens attained in the 3^{rd} millennium BCE. The constellational system, in vogue among the Egyptians, appeared to have been of native origin, but they contributed little or nothing to the genuine progress of astronomy.*

The ancient Egyptians bequeathed to us the idea which is at the heart of our calendar*. Unlike the Babylonians, Greeks and early Romans, they based their calendar upon the* sun *alone. As the earliest great farming civilization, Egypt was dependent upon the annual flood of the Nile which brought water and rich silt to the river's flood plain. Life in Egypt was controlled by the seasons, and hence by the sun. The moon played no part in the calendar.*

The Egyptian year had 12 months, each of 30 days, plus an extra 5 days at the end of the year. These 5 days were associated with the birthday of the greatest gods of the Egyptian Pantheon and were given over to celebrations. Thus the year was 365 days long. The Egyptians made no attempt to force their calendar to keep step with the actual seasons (as we do by adding leap-days, or a leap-month). Instead, they accepted that the seasons would gradually become later and later w.r.t. the calendar, in a cycle that would take 1460 years to complete. The Egyptian checked the relation of their calendar to the natural year not by observing the equinoxes and solstices but

[21] The two major achievements of Egyptian geometry were the approximation to the area of the circle, and the derivation of the rule for calculating the volume of a truncated pyramid. Some historian claimed that they also found the correct formula for the surface area of a hemisphere.

by the heliacal rising of Sirius, the Dog-star. This was the first sighting each year of Sirius in the morning sky just before sunrise.

ca 2770 BCE Egyptian introduced the *calendar* based on a year of 365 days. The Egyptians recognized that the point on their horizon where the sun rises or sets moves from day to day; in the spring the sunrise and sunset points move north along the horizon until they reach a northern limit at the time when daylight last longest. Then they move south until they arrive at a southern limit when the period of daylight is shortest. By counting the number of sunrises or sunsets from either one of these points until that point is next reached they found the number to be about 365. They noted independently that the Dog star, Sirius, reappeared in the eastern sky just before sunrise after several months of invisibility.

They also discovered that the annual flood of the Nile river came soon after Sirius reappeared; again they counted 365 days between two such consecutive events. So at some point[22] they began using this event to fix their calendar and came to recognize a year of 365 days made up of 12 months, each 30 days long and an extra 5 days added at the end for festivals. But the year length is actually 365 days, 5 hours, 48 minutes and 46 seconds (about a quarter of a day) and so, their calendar slowly drifted into error.

Important observations of the sky were also made by people in other parts of the ancient world, especially in India, China and Mesopotamia. These early observers realized that it was easier to describe the location of a particular object in the night sky if the stars were divided into recognizable groups, called *constellations*.

2750 BCE **Gilgamesh**, legendary king of Uruk, Sumer.

2700–2500 BCE Building of the early big Pyramids[23] during the 3^{rd} and 4^{th} Old Kingdom Dynasties in Egypt.

[22] The Egyptian calendar is known to have accurately matched the seasons with dates in 139 CE. Their calendar gradually went into and out of alignment with the seasons with a period of about 1455 years. Knowing this, astronomers have speculated that the year of 365 days was institutioned at ca 4226 BCE.

[23] The word *Pyramid* appears for the first time in the *Ahmes Papyrus* (ca 1630 BCE) and is believed to have originated either from the Egyptian *piromi* or *pyros* = grain, as in "granary". The Greeks obtained it from the Egyptians.

At the beginning of the 3^{rd} Dynasty, **Imhotep**[24] (ca 2700 BCE) designed for King Djoser the first great stone structure built by man – the Step-Pyramid at Saqqarah [h. 60 m; base: 121×109 m^2]. This was an experiment, using a new architectural form and new building materials and construction technique with dressed stone.

The first smooth-sided Pyramid was built at Meidum at about 2600 BCE [h. 92 m; base: 144×144 m^2; inclination: $51°51'$].

The *Pyramid of Cheops*, the largest and most massive of the Pyramids, stands at Gizah (west bank of the Nile river, outside Cairo). Built by Prince **Hemon**, a son of King Sneferu and cousin to Cheops (Khufu), sometimes during 2600–2500 BCE [originally 146.5 m tall and base of 230.4 m with inclination $51°51'$. Some 2.5 million stones were used for its construction.][25]. According to Herodotos, some 100,000 workers[26] were engaged in the construction at any given time over a period of 30 years.

The great Pyramids were constructed on a common geometrical plan: the perimeter of the four sides, which face exactly to the north, south, east and

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Because of the pyramidal form of a flame, the word was thought by medieval and Renaissance writers to come from the Greek word pyr for fire (as in "pyrotechnic"). However, the word *prism* is from the Greek *prizein* = to saw, hence something sawed off.

[24] According to **Manetho** (ca 280 BCE). **Imhotep**, architect, engineer, writer, statesman and physician, is honored in medicine as the first physician known by name. He served as *vizier* (prime minister), and after his death was elevated to the status of a god. He was worshiped for his healing powers, the only scientist ever to have become a god. The Greeks identified him with their own god of healing, Asclepius: temples were built to Imhotep, and bronze statuettes of him have been preserved. A statue of him stands today in the Hall of Immortals in the International College of Surgeons in Chicago.

Imhotep is the first figure of a *universal man* to stand out clearly from the mists of antiquity; probably of the caliber of **Archimedes** and **Leonardo da Vinci**, to be produced by nature only once in some 2000 years.

Apart from Manetho's book there is no real history of Imhotep and his royal master Djoser. However, a papyrus of Ptolemaic times relate how the kingdom was afflicted by famine for several years because the Nile failed to rise. Djoser accordingly took counsel with Imehotep, who explained that Khnum, the god of the Cataracts, was wrath. So the king deeded lands for temples to the god, and all was well. This story provides the kernel of the biblical story of Joseph and the seven lean years.

[25] Relative error in the right angle at corners not exceeding 1/27,000.

[26] Replaced every 3 months.

west, has the same ratio to the height as the ratio of the circumference to the radius of a circle, i.e. $2 \times 3\frac{1}{7}$ or 2π. This was *no* coincidence: The symbol of the sun-god Ra was a circle. So, when Ra rose every morning over the land of Egypt, he was greeted by a shining golden image of himself. The squareness and level of the base have an average error that is less than 10^{-4} of a side.

The rays of Sirius (whose rising announced the Egyptian New Year and the flooding of the Nile river), were perpendicular to the south face at transit, and shone down the ventilating shaft into the royal chamber while building was in progress. The Pyramids, built when α-Draconis was the pole-star[27], had interior passages aligned in the direction of that star as it passed the meridian of the Pyramid. Thus, the ancients, believing that the motion of the heavenly bodies held the secret of man's fate, spared no effort to unlock this secret by a systematic study of the heavens.

ca 2679 BCE Chinese astronomers record an observation of a Nova.

ca 2600 BCE Egyptians build the first *stone-paved* highway – a 60 km long road which carried the materials for the pyramids. At about the same time they invented the *wooden saw* to cut granite.

First recorded seagoing voyage. Egyptian sailors travel to Byblos in Phoenicia (today's *Lebanon*) in search of cedarwood.

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[27] During 4480–2330 BCE the sun rose on the spring equinox in the constellation *Taurus* and during 10,970–8810 BCE in the constellation *Leo*. This led some scholars to speculate that the *Sphinx* (Lion-faced!) was built during the age of Leo. Geological studies of the *Sphinx* (1991) by a team of American geologists from Boston University, headed by **Robert Schoch**, concluded that the Sphinx would seem to date from around 7000 to 5000 BCE, during the Neolithic rainy period. This is a direct challenge to Egyptologists who maintain that the Sphinx was excavated out of solid rock around 2500 BCE at the time of Pharaoh Khafre. It means that social organization and technology needed to create this Sphinx had to exist prior to the 1^{st} Dynasty kingdoms – a revolutionary idea unaccepted by most Egyptologists today.
It is believed that the Gizah-complex pyramids (Khufu, Khafre, Menkaura) were oriented according to the stars, i.e. the three pyramids were apparently deliberately related to each other in a unified geometric design, forming an integral part of the ritual basis behind the Egyptian understanding of death and the afterlife. For example, if a line is drawn linking the center points of the two larger pyramids then the third and smaller pyramid is off set. The three then seem to create a pattern on the ground corresponding to the three stars of *Orion's Belt* in the sky.

ca 2500 BCE Early Minoan civilization in Crete. Indus Valley civilization founded in India.

Egyptian priests began to develop and codify medical practice, including primitive surgical procedures. They gain their understanding of the human body by preparing *mummies*. Papyri from that time tell how to set bones, the pumping function of the heart, the pulse and the prescription of medications and diets. Mummies were in part produced by *saponifying* the flesh of the corpse with *natron* (a mixture of Na_2CO_3, $NaHCO_3$, NaCl and Na_2SO_4). *Chinese* probably developed practice of acupuncture by this time.

2500 BCE Egyptians and Mesopotamians had developed a sophisticated society operating with essential tools: civil engineering, astronomical measurement, water lifting machineries, writing and mathematics, primitive metallurgy, and the *wheel*.

ca 2500 BCE The knowledge of agriculture, of the potter's art, and of the use of copper reached northern China, by way of Central Asia, either from Russian Turkestan or from Persia. Thus the initial (Neolithic) civilization of China came indirectly from the ancient centers in Western Asia – from either the lower Indus lands or from lower Mesopotamia.

From these areas cultural currents equally passed into Europe. Subsequently to this remote phase, however, Chinese civilization seems to have developed in almost complete independence of influence from the West. Europe and China developed along independent lines because of geographical conditions (distance, mountain and desert barriers). Both Europe and China stood at the terminals of a steep-desert belt where rainfall was sufficient for cultivation; hence they were geographically equipped to support relatively dense populations.

ca 2500–2000 BCE Hugh climatic changes in the Aegean, Anatolia, Near and Middle-East, Egypt, North Africa and large parts of Asia. May be linked to the encounter of the *Taurid meteor stream* (including asteroids and active comets) with earth. This is the age of *megalithic constructions* around the world such as the Pyramids of Egypt and the Stonehenge[28] in England.

[28] For further reading, see:

- Hoyle, Fred, *On Stonehenge.* W.H. Freeman, 1977, 157 pp.
- Niel, F., *The Mysteries of Stonehenge.* Avon, 1975, 208 pp.

ca 2500 BCE *The Egyptian sea-expedition to Punt*[29] (in the reign of the *Pharaoh Sahure*) in quest of incenses and cosmetics ingredients.

As the power and luxury of Egypt increased, so did its need for rare, costly and exotic materials. From the mines on the Sinai peninsula Egyptian ships brought turquoise, malachite and copper. From equatorial Africa they brought the incenses frankincense and myrrh, and antimony – an ingredient of rouge, one of the most highly prized cosmetics of the day.

These products became so prodigiously expensive as they traveled slowly up the continent, passing through the hands of one entrepreneur after another, that the Egyptians were eventually impelled to send their own fleets through the Red Sea to the sources of these goods, in order to trade directly with the inhabitants.

ca 2500 BCE The final disappearance of the elephant, giraffe and rhinoceros from Egyptian territory. Elephants continued to exist in Syria and Mesopotamia up to ca 1000 BCE.

ca 2500 BCE Coastal Indians at Chicama (Peru) engaged in agriculture.

ca 2400 BCE Sumerians develop positional notation for numbers with base 60.

ca 2400 BCE *Bitumen*, a form of oil that seeps from the ground, was used in Mesopotamia to make boats watertight. This is reflected in the Biblical story of Noah and the Deluge [*Gen* **6**, 14].

ca 2316 BCE Chinese record an observation of a comet.

ca 2300 BCE The earliest surviving *map* is one inscribed on a baked clay tablet from *Mesopotamia*. It is a map of the city-state *Lagash*.

ca 2300 BCE The observational work of the Chinese resulted in an accurate calendar in which the year was $365\frac{1}{4}$ days long.

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[29] The Land of Punt, "The Sacred". Various locations have been suggested: Perhaps on the borders of Somalia and Ethiopia; could be identical with the Biblical *Ophir* (*I Kings* **9**, 28; **10**, 11; **22**, 49; *II Chron* **8**, 18; **9**, 10). Equatorial Africa was the source of ebony and other rare woods, ivory, gold and silver. Prized above all were the incenses, *frankincense* and *myrrh*. The latter, a gum resin burned in vast quantities in the temples, and also widely used as an unguent, perfume and embalming agent. The tree which exudes the myrrh resin, *Commiphora myrrha*, grows extensively inland from the port of Zeila.

ca 2200 BCE The custom of *horseback riding* finally diffused to the Middle-East from its origin in the forested uplands between the Dnieper River and the Carpathian Mountains[30].

2180–2130 BCE A Dark Age in the history of ancient Egypt. Earliest known great famine in Upper Egypt, due to drought caused by a severe failure of the annual floods of the Nile. A second famine, less severe, occurred between 2002 and 1991 BCE.

This prolonged drought occurred more or less simultaneously over the entire Eastern Mediterranean and adjacent lands. The dire famine was caused by failure of the rains over the central and eastern African sources of the Nile[31]. The crisis shattered a weakened central government utterly unable to

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30 *Horseback riding* began at ca 4000 BCE in the Cucuteni-Tripolye culture, which flourished from about 4500 to 3500 BCE in the forested uplands between the Dnieper River and the Carpathian Mountains.

Horseback riding, by bringing distant cultures into contact, seems to have stimulated both trade and war, and provided a possible mechanism for the dispersal of early Indo-European dialects.

An eastward dispersion by the first riders would have encountered only small and scattered human resistance. Dispersal to the west would have been much more complex because it would have encountered the well-established agricultural societies of Copper Age Europe.

Archaeological data and theoretical models of migration tend to support the theory that such movements took place, first to the east, and then to the west, between 3500 and 3000 BCE. In all these developments the horse played a critical role.

It took a very long time for the custom of riding to diffuse southward into the Middle East. When horses finally did appear there around 2200 to 2000 BCE, they were used first as draft animals attached to battle carts, and eventually, to drive the war chariots. It was as a chariot animal that the horse trotted onto the pages of history, two millennia after it had first been broken to the bridle.

31 The volume of water in the Nile depend on the rainfall over Central Africa (*White Nile*) and the summer monsoon rains over East-African Highlands (*Blue Nile*). Thus the most important source of information as to the variations of rainfall in Africa is provided by the levels of the River Nile.

As is well known, the Nile commences in *Lake Victoria*, in Central Africa, and flows to *Lake Albert* as the Victoria Nile. From there it continues as the Bahr-el-Jebel, becoming known as the White Nile after the junction of the Sobat River. At Khartoum it receives the Blue Nile, and near Berber the Atbara River, both of which originate in the mountains of Abyssinia. From the junction of the Blue Nile to the Mediterranean, a distance of 2900 km, it receives no appreciable accession of water. The level of the Nile passes through an extremely regular cope with the problem, decimated the Egyptian people and brought about a general decline in material culture[32].

Outside Egypt, the drought affected civilization throughout the Eastern Mediterranean Basin: it contributed to the destruction of the Akkadian Empire. The second drought (2002–1991 BCE) contributed to the downfall of the Third Dynasty of Ur.

2159 BCE According to accounts of the time, the court astronomers in China, Hi and Ho were beheaded when they failed to predict on eclipse.

2100 BCE Egyptians record star configurations and base upon it a 24-hour day.

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annual variation; the water is at its lowest in April or May, it rises slowly and irregularly in June and the first half of July, but rapidly and steadily in the latter half of July and the first half of August, remaining high during September and commencing to fall rapidly in October.

[Herodotos tells us: "*When the Nile overflows, the country is converted into a sea, and nothing appears but the cities, which look like islands in the Aegean*".]

The regular annual flood is the source of fertility of Egypt, without it the whole land would be a barren desert, and hence the levels of the flood have been recorded annually, probably for some thousands of years.

The significance of both high and low levels of the Nile is as follows: The *White Nile* drains a large area of equatorial Africa which has a considerable annual rainfall distributed fairly evenly throughout the year; moreover, it passes through two large lakes, Victoria and Albert, which further regulate the flow. Hence the *White Nile*, above its junction with the Sobat River, *discharges an almost constant volume of water throughout the year*.

The *Blue Nile*, the *Atbara*, and the *Sobat River*, on the other hand, originate in Abyssinia, which receives the greater part of its monsoonal rainfall in the summer months. It is these rivers which *supply most of the waters of the annual flood*. For this reason the level of the Nile during the stage of low water reflects the general rainfall of equatorial Africa, while the flood levels represent the monsoon rainfall of the eastern highlands. Since the rainfall in the equatorial belt is associated with *low pressure* (which is very closely connected with the intensity of the *general circulation of the atmosphere*) it shows a closer relation to the rainfall of Europe than the monsoon rainfall in the eastern highland (governing the floods).

32 When a similar situation arose in the middle of the 2^{nd} millennium BCE, it was met by a stronger and more experienced government which succeeded to manage the economy through a sequence of 7 years of abundance and 7 lean years that followed (*Gen* **41**, 29–57).

2000 BCE Babylonian arithmetic evolved into *rhetorical algebra*: quadratic equations are solved and cubic and biquadratic equations were discussed.

ca 2000 BCE *Ziggurates* in Mesopotamia serve as platforms for astronomical observations. The *shaduf*, a device for astronomical observations raising water from one level to another with a bucket, appears in Mesopotamia.

The palace of *Minos in Crete* has light and air shafts and interior bathrooms with their own water supply.

Mesopotamian traders journey as far east as India. Egyptians trade with Nubia, Ethiopia, and Crete.

Mathematics and Astronomy in Mesopotamia[33]

(ca 3200–300 BCE)

"In Babylon, in Babylon
They baked their tablets of the clay;
And year by year, inscribed thereon
The dark eclipses of their day;

They saw the moving finger write
Its 'Mene, Mene', on their sun
A mightier shadow cloaks their light,
And clay is clay in Babylon".

Alfred Noyes (1922)

The Babylonians[34] *played a large part in laying the foundations of our science. From the beginning of the* 2^{nd} *millennium BCE they catalogued and classified with meticulous care everything that came under their observation, and this body of information was passed on to be recognized or revised by the generations following.*

Archaeological excavations of the ruins of the ancient cities of Babylonia during 1840–1940, *revealed a complex civilization which flourished more than* 4000 *years ago in the fertile land watered by the Tigris and Euphrates rivers. It was a civilization characterized by prospering agricultural settlements, crisscrossed by a network of canals, whose purpose was to reclaim swamps and feed parched areas. Numerous individuals were occupied with law, religion, science, art, architecture, trading, teaching and engineering. Large palaces, sculptures, metal bass-reliefs, copper and bronze figures, painted pottery and other artifacts of great antiquity were recovered.*

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33 The data used in this article was assembled form the groundbreaking works of the Assyriologist **Otto Neugebauer** (1899–1990) [*The Exact Sciences in Antiquity*, Dover 1969] and **B.L. Van der Waerden** (1903–1996) [*Science Awakening*, Kluwer 1988]. We have also consulted the works of the mathematicians **Morris Kline** (1908–1992) [*Mathematical Thought from Ancient to Modern Times*, Oxford Univ. Press 1990] and H.L. Resnikof and R.O. Wells [*Mathematics in Civilization*, Dover 1984].

34 "Babylonian" refers to all cultures of the cuneiform users in Mesopotamia, after the city that was the center of many of the empires that occupied the region between the Tigris and the Euphrates rivers.

Most significant of all the finds, however, were the thousands upon thousands of clay tablets unearthed at Nineveh, Assur, Nippur and other cities, bearing the written records of the economic, juridical, educational and scientific phases of daily life, previously known to us only through Biblical allusions.

Lacking papyrus and having little access to suitable stone, they resorted to clay as a writing medium. The inscription was pressed into a wet clay tablet by stylus. The finished tablet was then baked in an oven or sun-dried to a time-resisting hardness that resulted in a permanent record. The cuneiform writings incised on the tablets include among a whole gamut of subjects, astronomical observations that were used principally for the timing of religious festivals.

Underlying the commercial, monetary and astronomical systems employed by the Babylonians, were their achievements in mathematics, made known to us through the continuing study and decipherment of the cuneiform inscriptions on the ancient tablets.

Their numerical system was a mixed one: numbers below 60 were written in the decimal system, but number above 60 were written according to the sexagesimal system. Thus

$$1\cdot 60^2 + 0\cdot 60 + 1\cdot 60^0 = 101 \quad (sexagesimal),$$

$$= 3\cdot 10^3 + 6\cdot 10^2 + 0\cdot 10^1 + 1\cdot 10^0 = 3601 \quad (decimal).$$

Likewise

$$524,549(decimal) = 2\cdot 60^3 + 25\cdot 60^2 + 42\cdot 60 + 29 = (2\cdot 25\cdot 42\cdot 29).$$

Fractions could also be represented in this system, e.g.

$$\frac{1}{2} = (30)60^{-1} = 0;\ 30$$

$$\frac{1}{8} = (7)60^{-1} + (30)60^{-2} = 0;\ 7,30$$

$$532\frac{3}{4} = (8)60 + (52)60^0 + (45)60^{-1} = 8,52;\ 45$$

$$\frac{1}{64} = (56)60^{-2} + (15)60^{-3} = 0;\ 0,56,15$$

$$1029 = (17)60^1 + (9)60^0 = 17,9.$$

To divide, say, 1029 by 64 the Babylonians evaluated the product $1029\times\frac{1}{64}$. *The answer* $16;4,41,15$ *can be converted to the decimal base*

$$16 + 4(60)^{-1} + (41)60^{-2} + (15)60^{-3} = 16.078125.$$

As used by the early Babylonians, the sexagesimal number system lacked two basic features of our modern decimal system: there was no zero and there was no "sexagesimal point". The resulting ambiguity (on existing clay tablets) due to the lack of the zero can often be resolved only by a careful study of the context. After 300 BCE a special symbol was introduced to denote the unfilled position. However, the lack of the sexagesimal point did not impede their computational ability, and this is why their astronomy and algebra were far superior to that of their Egyptian contemporaries.

Most of the mathematical tablets have been classified either as 'table texts' or as 'problem texts'. The latter appear in many cases to have been school texts illustrating rules for the solution of problems. The table texts were ever-present aids both for instructional problems and for practical use. They include tables of reciprocals, squares, square roots, cubes, cube roots *and multiplication tables. With these tables at their disposal for the numerical calculations involved, the Babylonians developed many ingenious rules and methods for the solution of a wide variety of mathematical problems. These tablets are cogent evidence of a high degree of skill and originality on the part of the Babylonian mathematicians.*

Their tablets reveal that they knew the Pythagorean theorem *more than a thousand years before the Greeks. Indeed, the Pythagorean triplet*

$$(3456)^2 + (3367)^2 = (4825)^2$$

was incised on Plimpton 322 (1500–1600 BCE). This example leads us to believe that they were familiar with the general solution *of $a^2 + b^2 = c^2$ in integers:*

$$a = 2pq, \quad b = p^2 - q^2, \quad c = p^2 + q^2$$

for which the above triplet is obtained with $p = 64, \quad q = 27$. They also prepared numerical tables for c^n with

$$n = 1, 2 \;...\; 10, \qquad c = 9, 16, 10, 225.$$

The Babylonians could solve linear and nonlinear equations with one unknown [e.g. $ax = b, \;\; x^2 = a, \;\; x^2 \pm ax = b, \;\; x^3 = a, \;\; x^2(x+1) = a$], and systems of equations with 2 unknowns such as

$$x \pm y = a, \quad xy = b;$$

$$x \pm y = a, \quad x^2 + y^2 = b.$$

Furthermore, the following formulae were known to them by 300 BCE:

$$(a+b)^2 = a^2 + 2ab + b^2,$$
$$(a+b)(a-b) = a^2 - b^2,$$
$$1 + 2 + 4 + \cdots + 2^{n-1} = 2^n - 1,$$
$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{1}{3}(1+2n)(1+2+3+\cdots+n).$$

The Babylonians were able to solve different types of quadratic equations, e.g. (in today's notation)

$$x^2 + bx = c, \quad b > 0, \quad c > 0 \quad \therefore \quad x = \sqrt{\left(\frac{b}{2}\right)^2 + c} - \frac{b}{2},$$

$$x^2 - bx = c, \ b > 0, \ c > 0 \quad \therefore \quad x = \sqrt{\left(\frac{b}{2}\right)^2 + c} + \frac{b}{2}.$$

To handle the more general case $ax^2 - bx = c$, *they multiplied throughout by* a *to get* $(ax)^2 - b(ax) = ac$ *and then substituted* $y = ax,\ e = ac$ *to obtain their standard form* $y^2 - by = e, \quad e > 0$. *They also solved higher order equations such as* $ax^4 - bx^2 = c$ *and* $ax^8 - bx^4 = c$ *by treating them as if they were 'hidden' quadratics in* x^2 *and* x^4, *respectively.*

Babylonian handled cubic equation of the form $x^3 = c$ *with the help of cube root tables, and equations of the form* $x^2(x+1) = c$ *with help of* $(n^3 + n^2)$ *tables. There are even a few examples in Babylonian algebra of the solution of a set of equations in three unknowns*

$$x^2 + y^2 + z^2 = 1400, \quad x - y = 10, \quad y - z = 10 \qquad (x, y, z) = (30, 20, 10).$$

Babylonian geometry is intimately related to practical mensuration, though their geometry was chiefly of algebraic character.

They were familiar with proportionalities arising from parallel lines, the Theorem of Pythagoras, the area of a triangle and of a trapezoid, volumes of a prism and of a cylinder. They used the poor approximations $3r^2$, $6r$ *for the respective area and perimeter of a circle of radius* r, *and wrong formulae for volumes of cone and pyramid frustums.*

Of special interest is the value that the Babylonians have assigned to $\sqrt{2}$ *in their sexagesimal system, that is equivalent to* 1.414 213 562, *to 10 decimal* *figures. This again, could not have been achieved without the availability of some algorithm, probably the same one known to the Greeks*[35].

The Babylonians could also calculate areas of triangles and quadrilaterals and volumes of prismatically-shaped canals. Finally, the Babylonians knew the laws of similarity of triangles and the ensuing rules of proportion, ergo – they knew the fundamentals of trigonometry.

In ancient times, astronomy was the sole means of regulating the calendar and thereby determining the proper time for annual agricultural activities such as crop planting and land irrigation. In addition, stars and other celestial bodies were worshiped as gods, and the study of their motion formed

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[35] Babylonians gave approximations to square roots of nonsquare numbers such as $\frac{17}{12}$ for $\sqrt{2}$ and $\frac{17}{24}$ for $\frac{1}{\sqrt{2}}$. Apparently they were aware of the approximation formula

$$a + \frac{h}{2a+1} < \sqrt{a^2+h} < a + \frac{h}{2a}, \qquad 0 < h < a$$

(used later by Archimedes for his calculation of π). Historians of mathematics now believe that the Babylonians had an *algorithm* through which they could extract square roots accurate to 10 decimal digits. Take 27, for instance. Since $5^2 = 25 < 27$ and $6^2 = 36 > 27$, it is expected that $\sqrt{27}$ will lie somewhere between 5 and 6. They then noticed that since $5 < \sqrt{27}$, $\frac{27}{5}$ must be *greater* then $\sqrt{27}$. They then guessed that a better approximation to $\sqrt{27}$ than either 5 or $\frac{27}{5}$ would be the average

$$\sqrt{27} \approx \frac{1}{2}\left(5 + \frac{27}{5}\right) = 5.2,$$

which yields $5.2 \times 5.2 = 27.04$. This process can now be repeated, with the next step giving

$$\sqrt{27} = \frac{1}{2}\left[5.2 + \frac{27}{5.2}\right] = 5.19615,$$

where now $(5.19615)^2 = 26.99997$. Thus, two application of their method yields a result of better than six-figure accuracy.

With a method as powerful as this, the Babylonians had no fear of square roots. Nowadays, digital computers use this iterative process (Hero's algorithm) $y_{n+1} = \frac{1}{2}\left(y_n + \frac{x}{y_n}\right)$ for a successive approximation of the solution $y = \sqrt{x}$ of $y^2 - x = 0$. The Babylonians could have arrived at the same result through the following reasoning: Let the positive number a_1 be a guess that is too small, $a_1 < \sqrt{x}$. It then follows that $a_1\sqrt{x} < x$ and consequently $a_1 < \sqrt{x} < \frac{x}{a_1}$. The *mean* of these estimates, namely $a_2 = \frac{1}{2}\left(a_1 + \frac{x}{a_1}\right)$ can be shown to be closer to $\sqrt{x}$ than either a_1 or $\frac{x}{a_1}$. Therefore, a_2 can be taken as a *new* estimate of $\sqrt{x}$. With $a_1 = 1$, the process has to be repeated just 4 times in the sexagesimal system to obtain the value for $\sqrt{2}$ found in the Babylonian texts!

part of the religious duties owed them. In this sense astronomy was initially a technical branch of religion that served as one of several channels of communication between the priestly hierarchy and the god they served. Therefore, the astronomical study of these deified objects was virtually identical with the study and prediction of their paths of motion through the heavens. Astronomy was therefore exclusively mathematical, and mathematics thereby became the servant of religion which in return fostered its development and ensured its prestige for thousands of years.

The practical component of religion which applied the achievements of mathematical astronomy to the welfare of mankind was astrology, the ancient art of divining the fate of human beings from the configurations and motions of the planets and stars.

Already in 3200 BCE Sumerian priests made astronomical observations on watch towers and prepared maps and time tables of star-motions.

In 3000 BCE, astrology was already well developed in Babylonia. It was founded on the Babylonian's identification of personal deities with the planets[36] *Mercury (Nebo), Venus (Ishtar), Mars (Nergal), Jupiter (Marduk), Saturn (Ninib), as well as moon (Sin) and the sun (Shamash). The movements of these heavenly bodies, were regarded as representing the activity of the corresponding gods. If one could correctly 'read' the heavenly motion of these divinities, one would know what they were aiming to bring about on earth.*

Their astrology was based on the fundamental assumption that all events on earth are influenced by the stars. In particular we can trace back to them, by way of the Hebrews, the origin of our present seven-day week associated with the Sun, Moon and the five planets[37] *Mars, Mercury, Jupiter, Venus,*

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[36] Before the regular movement of *planets* about the sun was known, they seemed to the ancients as stars wandering the heavens. Hence the name (Greek) *planan*, to wander. Latin used *planetae* to mean wandering stars.

We know today that to be called a planet, an object must have a mass less than about $\frac{1}{10}$ of its sun.

[37] If at that time there had been any knowledge of the planets *Uranus*, *Neptune* and *Pluto*, discovered in 1781, 1846 and 1930 respectively, our week might today consist of 10 days instead of seven!

The planetary week presents a strange combination of ideas from different cultures. From Babylon came the doctrine of the influence of the stars on man's destiny, from the Alexandrian Greeks came the mathematical astronomy that placed the planets in a certain order of distance from the earth, and then on these foundations the late Hellenistic astrologers, who were familiar with the ancient cult of the magical number seven, constructed a purely Pagan week. By the *and Saturn that they discovered. Indeed, the separation of these planets from the so-called 'fixed stars', was one of their greatest achievements. Many believe that the week had its origin in the above seven "stars" visible to the naked eye which traverse the celestial zodiac. (Of these seven, only the sun, of course, is actually a* star *in the modern scientific meaning of the word.)*

The motion of the bodies in the solar system is viewed against the background of the fixed stars. From the earliest times the various star formations were identified with familiar creatures and objects whose forms appeared to be similar to the patterns traced by the stars. Of principal importance were those constellations that lie, when viewed from earth, behind the path of motion of the planets, moon and sun. Since all these bodies move in nearly the same plane – the plane of the ecliptic[38] *– their motion against the background of the stars appears to take place in a relatively narrow band. This imaginary zone of the heavens, bounded by two circles equidistant from the ecliptic plane and separated by about 18°, is the* zodiac[39]*. The zodiac is partitioned into 12* equal signs[40] *each comprising 30° in the ecliptic plane. Each sign*

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end of the third century CE, the Christians, who had previously adhered to the Jewish seven-day week in which the days simply had numbers and not names, began to be influenced by the astrological beliefs of converts from Paganism and changed over to the planetary week. The stars were no longer regarded as deities but as demons capable of affecting the fate of man. At the same time, the oriental worship of the Sun-god Mithra was extremely influential in the Roman world. This led to the substitution by pagans of the *dies Solis* (the Sun-day) for the *dies Saturnis* (the Saturn-day) as the first day of the week. This change appealed to the Christians, who had long observed Sunday as the first day of the week. All the names of the days can be traced back to the Roman (or equivalent Norse) planetary gods: Monday (Moon = Luna); Tuesday (Tiw = Mars, god of war); Wednesday (Odin = Mercury); Thursday (Thor = Jupiter, god of thunder); Friday (Frigga = Venus); Saturday (Saturn); Sunday (Sun).

In 1901, the following inscription was found scratched on the wall of a dining-room in Pompei (79 CE): SATVRNI, SOLIS, LVNAE, MARTIS, IOVIS, VENERIS. This gives the days of the week in the order still adopted at the present time, with the omission of Wednesday, which is no doubt an accidental error.

[38] Ecliptic – the sun's apparent path in the heavens, so-called because eclipses can occur only when the moon crosses it.

[39] From the Greek *zoon* = a living thing.

[40] The *duodecimal* system, based on 12, allows thirds, quarters and sixths to be expressed very simply, in contradistinction to the decimal system where a third cannot be represented exactly, but only as a repeating decimal fraction. It is no coincidence that we have 12 hours of day, 12 hours of night, 12 months in the

is associated with a constellation that lies in the zodiacal band from which the sign draws its name. These signs are: Capricorn (Goat), Saggitarius (Archer), Scorpio (Scorpion), Libra (Balance), Virgo (Virgin), Leo (Lion), Cancer (Crab), Gemini (Twins), Taurus (Bull), Aries (Ram), Pisces (Fish), Aquarius (Water bearer).

No one could deny that the Babylonians were the fathers of astronomy. The little rainfall in Mesopotamia and its clear unpolluted sky enabled almost continual watch of the motion of the planets by the naked eye over many centuries on end[41]*. Their astronomers noted every single phenomenon with such great care that they were able to notice even the changes caused by the precession of the equinoxes*[42]*. Eclipses of the sun, moon, and stars were so carefully described that part of the ancient chronology has now been unambiguously determined by just such occurrences.*
In fact, the basis for predicting eclipses, which rests on a period of 6585 *days (known as the Saros*[43]*), was discovered by the Babylonians.*

Through their continuing astronomical observations, the Babylonians knew that the apparent slow rotation of the heavens (due to the precession of the earth's axis), gave rise to an equally slow change in the position at which the sun appears to rise each year on the vernal equinox relative to the

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year and 12 tribes of Israel. Also, the 12 signs of the zodiac divide into 4 groups of signs associated with fire, air, earth and water respectively. It is as easy to test a number for divisibility by 2, 3, 4, 6, 8, 12, 16, 24 in base 12 as it is to test for divisibility in base 10 by 2, 5, 10, 20 etc. These were important advantages when calculation itself was a subtle art and difficult to learn.

The sexagesimal system, based on 60, has been used for scientific calculations. Because $60 = 5 \times 12 = 6 \times 10$ it has the advantages of bases 10 and 12 combined. In 1944, *the Duodecimal Society* was established in New York State as a voluntary non-profit organization. Its aims were "to conduct research and education of the public in mathematical science, with particular relation to the use of *base twelve* in numeration, mathematics, weights and measures, and other branches of pure and applied science". This society proposed to add the letter X to represent 10 and E to represent 11.

41 It is an extraordinary fact that modern astronomers have not yet been able to accumulate a series of astronomical observations as long as the Babylonians'. The longest known series of modern observations – those at Greenwich – was begun in 1750.

42 Although it has not been established that the Babylonian astronomers were aware of precession as a regular phenomenon.

43 After the lapse of this period, eclipses of the sun and the moon recur under almost identical circumstances except that they are displaced about 120° westward on the earth.

zodiacal constellations. This phenomenon, the precession of the equinoxes[44], *was viewed as responsible for the catastrophic fall and subsequent rise of successive "ages" of the world. When the vernal equinoctial sun passed from one zodiacal constellation to the next, an Age ended and a new one began. At this time, (so they believed) violent and cataclysmic events are to be expected as the transcendental power guiding our world passes from one constellation to the next.*

In the period before 4000 *BCE the sun rose in Gemini, during* 4000–1800 *BCE it remained in Taurus, then in Aries (*1800 *BCE*–400 *CE). It will remain in Pisces until* 2740 *and then move into Aquarius*[45].

[44] The proximate reason for it and the estimate of nearly 26,000 years for this great rotation of the heavens was first given by the Greek astronomer **Hipparchos of Nicaea** (180–135 BCE). The *physical mechanism* that actually causes this precession is due to *tidal torques* applied upon the earth's equatorial bulge by the solar and lunar gravitational fields. This insight, along with a theoretical explanation of the 26,000 year period, had to await modern science and the age of Newton.

[45] The image of the Bull (Taurus) is repeated in ancient myths and religions, from the Bull worship of Apis-Osiris in Egypt and Zeus the Bull carrying off Europe, to Hercules' defeat of the Cretan bull and Jason's triumphal capture of the Golden (ram's) Fleece. The Hebrew calendar starts at 3760 BCE, the year of the world's creation according to the Hebrew tradition. This date was moved by James Ussher (1650) to the transition zone between the ages of Taurus and Gemini.

The Scribes

The difficulty of writing on clay necessitated a long period of schooling. The regular schools were attached to the temples and were therefore at some distance from one another. However, scribes who could be depended upon to teach were scattered everywhere, even in the small towns. Just as in the Middle Ages an expert craftsman would take under his protection some young boy as an apprentice to whom he taught his trade, so most of the scribes had some youths who were ambitious to enter the profession. The scribe "adopted" his apprentice as his own son, and the relationship lasted until the young man was able to enter the profession as a regular member. Such private tutoring would be quite sufficient to prepare scribes for a business life. But only schools that were located in the vicinity of great temples had facilities for the study of the sciences and literature, and prepare one to become a priest or a 'scientist' (kings usually kept scribes at court to copy manuscripts and write official letters).

Besides taking dictation, all students in the temple schools had a certain amount of arithmetic (the four operations). It was follows by instruction in the higher branches qualifying for the different professions.

In addition to temple libraries, used mainly for instruction, kings kept their own royal libraries. We see them sending their own scribes throughout the land for the purpose of collecting all the important works gathered in the temples. Thus, Assyrian kings collected clay tablets in huge libraries.

The library of Ashurbanipal (reigned 668–627 BCE), discovered at Nineveh *in* 1864*, contained tablets dealing with religion, literature, medicine, history and other subjects. Indeed, these kings did more than merely collect, index and recopy the material found in ancient libraries. After their scribes had copied texts written in the Sumerian language, they* retranslated *this whole mass of material into the vernacular and adopted it to the needs of the time. It must have required an immense amount of time and a great number of learned scholars to bring this work to completion.*

The royal courts at this time must have been centers of culture as notable as those of the patrons of science during the Renaissance. Notwithstanding their great array of learning, the translators *must have had trouble of interpreting the old texts, for the language had been dead more than a thousand years. It is an achievement of modern science that we can now correct some of the translations made in those days. Moreover, scholars put in possession of such a large number of bilingual texts could immediately tackle the decipherment of Sumerian.*

2nd Millennium BCE Renewed predominance of warm summers in temperature latitudes. Drier climate; fair sailing weather around the coasts of Europe even in latitudes $50^\circ - 65^\circ N$, making possible the trading exploits of the Bronze Age people and later the Phoenicians.

Ships underwent improvement and *sea battles* took place toward the end of the millennium.

Glass bottles appeared in Egypt. Chinese bronze urns and vases appeared under the Shang dynasty. The earliest form of *steel* appeared in Asia Minor under the rule of the Hittites.

Water-clocks were invented in Egypt in about 1400 BCE. Around 1200 BCE, the know-how of Hittite iron-smiths (scattered with the destruction of the Hittite empire, and kept secret by them for hundreds of years) began to diffuse into Eastern Europe. By 1100 BCE this lore was absorbed by the Assyrian iron-smiths who developed a technology for mass production of iron tools, especially *iron blades*. The subsequent production of effective swords, axes and *iron plowshares* led to greater crop yields and boosted the Assyrian military potential.

Early *food technology*, including the preservation of fish by drying was developed by the Phoenicians and the Greeks around 1100 BCE.

2000–1650 BCE *The Cretan Age*. The peak of the Aegean civilization centered on the island of Crete.

The Hittites[46] (ca 1900–700 BCE)

There have been many vast movements of populations across and within Asia Minor. Numerous invasions from East, West and North, made its territory the scene of incessant conflicts, and the blending place of diverse cultures, races, and religions. Arian, Mongolian and Semitic masses were either attracted to Asia Minor by its wealth or driven there by the pressure of stronger hordes behind.

During 12 centuries the history of Asia Minor was practically the history of the rise and decline of the powerful Hittites. They left their impressive monuments from Smyrna to the Euphrates and from Boghaz-Keui to Aleppo. For centuries they contended on equality with the powers of the Nile and the Euphrates, and for a thousand years Asia Minor under their leadership held the balance of power in antiquity. They saved Asia Minor from being completely Asiaticized so that as a result its history has throughout been bound up with that of Europe as much as with Asia. They carried oriental (especially Mesopotamian) culture, technology and art Westward. They overthrew the Amorite empire of Babylon; they annihilated the Egyptian power in Asia; they held the dreaded Assyrians in check far centuries; they exercised an important economic influence by their control of the rich mineral resources of Asia Minor.

In the 15th century BCE they engaged the attention of the Pharaohs in several military expeditions. In the 14th century BCE they attained the zenith of their power in an empire of federated states under their leadership. For two centuries they were the dominant power in West Asia.

However, these exhaustive wars led to the eclipse of the Hittite Empire: by the 9th century, a renewed pressure of the Phrygians and the renewal of the struggle with Assyria rendered the decline of the Hittites irrevocable: they vanished, scattered by wakes of migrations from Europe.

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[46] The Hittites mentioned in the Old Testament were probably an offshoot or remnant of the Anatolian Hittites who became separated from the main body and had remained in ancient Israel. **Abimelech** (*Sam I*, **26**, 6) and **Uriah** (*Sam II*, **11**) belonged to these Israeli Hittites, and **Ezekiel 16**; 3, 45 is to be explained in the same way. The origin of the Hittites is obscure, but their kingdom was mentioned already in 1900 BCE by a colony of merchants in Capadocia (on the Anatolian plateau) who meticulously recorded their business transactions in cuneiform writings on clay tablets. The Hittites language, found on clay tablets, was deciphered only in the 1960's.

The Hittites were possibly the first people to make iron *of superior quality. Other kingdoms came to the Hittites as supplicants of their proficiency in iron making (ca 1300 BCE). Nevertheless, the use of iron in the old world was on a very small and restricted scale until after the downfall of the Hittite Empire, as their metal craftsmen dispersed throughout the Middle East. Consequently, metal-smiths began to make technical discoveries that led to the transformation of iron, from a metal inferior to bronze, to one which was destined to become a universal replacement for it.*

ca 1800 BCE Ecological collapse of the *irrigation system* in southern Mesopotamia due to salinization of cultivated soils: food supply to the growing population of the city-states was hampered gradually, leading eventually to malnutrition, epidemics and mass-migration. This man-made ecological disaster was probably one of the major factors in the demise of the Sumerian civilization.

ca 1800 BCE The *Hebrews* conceive the idea of *monotheism*: evolutionary universe of *one* origin, *one* and only *one* supreme cause.

ca 1765 BCE Severe prolonged drought in China.

ca 1750 BCE Invention the *war-chariot* in China; Due to its *shaft* and *balance-point*, the new chariot gained speed and mobility, thus increasing its military power. This new weapon revolutionized the art of war with an effect similar to the appearance of the tank and war-plane in Europe in the first half of the 20^{th} century. From China, the war-chariot arrived in the 'Fertile Crescent' to become a decisive weapon in the armies of Egypt and Mesopotamia.

ca 1750–1650 BCE Age of the Biblical Patriarchs and Matriarchs: Abraham and Sarah, Isaac and Rebecca, Jacob, Leah and Rachel.

ca 1700 BCE A form of printing with movable type was invented by an unnamed printer of ancient Crete in the Minoan age: a single baked-clay disk 15 cm in diameter was found buried deep in the ruins of a palace at Phaistos on Crete; the disk is covered on both sides with spiraling arrays of 241 symbols constituting 45 different syllabic signs. A decipherment identified the signs' language as an ancient form of Greek that predates even Homer. The symbols were printed by a set of punches, one for each of the 45 signs (and *not* scratched into the clay by hand, as was true of most ancient writing on clay). The Minoan printing, however, died out because it was syllabary rather than alphabetic and therefore clumsy and ambiguous; it could be read by few people and used for only very particular kind of texts, perhaps only tax lists and royal propaganda. To make if efficient would have required technological advances that did not occur until later, like the creation of paper, an alphabet, improved ink, metals and presses.

ca 1700 BCE The neolithic astronomical analog computer that is *Stonehenge*, was completed[47]. It was built from stones weighing as much as 35 tons each, to keep track of the progress of lunar eclipses. The position of a stone marked the day of the summer solstice when the sun rises farthest north along the eastern boundary of the horizon. In contradistinction to the sidereal or Pyramid year, the Stonehenge year was solar, and corresponds to what is now called the *tropical year*. Because of the *Precession of the Equinoxes*, the average solar year is not exactly the same as the sidereal year.

By the time Stonehenge was erected, stoneage people had observed the sky for more than 10,000 years. No doubt they recognized the constellations and the paths of the sun and the moon. If the religion of these people was concerned with the worship of the sun and the moon as divinities, eclipses would be events of great importance. Successful predictions ahead of time would confer power and prestige to those who understood how the predictions might be made. Perhaps this is why paleolithic people dragged 35 ton stones over 300 km to erect this mysterious monument.

The people who built Stonehenge are known today as the *Beaker People*; their remains have been found all over Europe and the British Isles. The earliest remains from north-west France date from 5500 BCE. They left thousands of stones in shapes of lines, circles, spirals and ovals covering the Atlantic coast of Brittany and most of Britain and Ireland. Most of the stones are large, some gigantic ranging up to 300 tons or more (*megaliths*). Stonehenge differs from the stone alignments and circles because it has stones placed on top of two supports to form 3-stone arches called *trilithons*. I was discovered in 1974 that the axis of the trilithons points to the first rays of the midsummer sun in 2045 BCE!

ca 1630 BCE The first phase of the decline and eventual demise of the Minoan civilization due to the paroxysmal eruption of the Thera volcano

[47] Construction began at ca 2500 BCE.

(Santorini)[48], which lies some 100 km north of Crete. It may have been the site of Plato's Atlantis as told in the 'Critias'. This, in turn, is based on documents passed on to Solon by priests in Egypt on his visit there ca 500 BCE.

This natural disaster changed the whole course of civilization in the Eastern Mediterranean. The Minoan language and culture and the power of Crete, which was dominant until then, thereafter gave way to Mycanean civilization of the Greek mainland.

ca 1630 BCE **Ahmes** (ca 1680–1620 BCE, Egypt). A scribe who wrote the *Rhind*[49] *Papyrus*. Ahmes claims not to be the author of the work, but only a scribe. He says that the material comes from an earlier work of about

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[48] The Greek archaeologist **Spyridon Nikolaou Marinatos** (1900–1974) excavated (1967) an ancient port city on the Island of Thera. Under the pumice he brought to light a settlement which had close links the Minoan culture. His findings place Thera alongside with Mycenae and Knossos for our understanding of the prehistoric people of the Aegean.

The severe drought in Egypt, managed by Joseph (*Gen* **41**, 29-57) may be linked to the aftereffects of the Thera event. Some historians and Biblical scholars link the Thera event to the '*plagues of Egypt*' and the *Exodus* of the Hebrews (led by Moses), which they place at ca 1500 BCE during the reign of Ahmose I (18^{th} dynasty). Biblical allusions can be found in *Exodus* **10**, 21–22; **13**, 21; *Psalm* **46**, 1–8; *Jeremiah* **47**, 2–4.

According to archaeological dating, the Thera explosion ($36.25°N$ $25.25°E$) occurred sometime between 1450–1500 BCE, whereas dendrochronology and radiocarbon dating support the earlier time window 1600–1650 BCE for the eruption. On account of its great intensity (VEI=6), the effects of Thera could have affected the Nile delta at a distance of almost 1000 km. Plume height could have reached 36 km and the displacement volume is estimated at 30 km^3, forming a caldera of 480 m deep and with an overall area of 83 km^2.

After the Thera explosion came a great displacement of the surviving population in search of arable land. Their exodus took them to Greece, Italy, Sicily, North Africa, Egypt and the Levant. A remnant of this 'diaspora' settled on the southern coastal strip of the Mediterranean and were known as the Philistines. The Hebrew prophet **Amos** (fl. 765–750 BCE) refers to this remote migration (*Amos* **9**, 5–7) and links the event with a description of volcanism and inundations (tsunami).

[49] **Alexander Henry Rhind** (1833–1863, Scotland). Egyptologist, went to Thebes for health reasons. He then became interested in excavating and purchased the papyrus in Egypt (1858), donating it later to the British Museum (1863). He died of tuberculosis in the same year.

The papyrus was published in 1927. It is about 6 m long and about 30 cm wide.

2000 BCE. The papyrus is one of our chief sources of information on Egyptian mathematics; it contains 87 problems of the four operations, solution of equations, progressions, volume of granaries and more. Nothing is known of Ahmes except for his own comments in the papyrus.

Some of the problems that appear in the Rhind did not apply to the real life in Egypt. This shows a genuine interest in mathematic for mathematics sake, characteristic of a society made of mathematics oriented minds.

ca 1600 BCE **Hammurabi** promulgated his famous code in Babylonia. It includes regulations on medical fees and penalties for malpractice.

ca 1600 BCE A primitive form of the *Greek language* was inscribed on fire-baked clay tablets found at *Pylos*, on mainland Greece, and at *Knossos*, on Crete. No other European language comes close to claiming such longevity.

1600–1550 BCE Unknown Egyptian physicians completed the oldest known medical document: *Smith Papyrus* contains 48 clinical descriptions of surgical cases, including injuries to head, spine, and chest.

Egyptian physicians wrote *Ebers Papyrus*: encyclopedic work that lists remedies for many diseases, including deformative arthritis, and conjunctivitis.

1600–1100 BCE *The Mycenaean Age*: As the fleets of Egypt and Crete pushed their commerce with the mainland of Greece, they naturally entered the southern bays, and especially the Gulf of Argos. Here, in the plain of Argos, behind the sheltered harbor, the Cretan nobles, migrating to the mainland, established their settlements.

1500 BCE The Egyptians invented the well-sweep with counterpoise (shadoof)[50] for irrigating the fields. Introduced later into Assyria.

ca 1500 BCE The first vessels entirely of *glass* were made in Egypt and Mesopotamia. Oldest *sundial*.

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[50] Usually made by erecting two pillars, some 2 m high, joined near the top by a short beam. Over this, a long pole is balanced, which has at one end a vessel to hold water and at the other end a counterpoise. A man standing at the water edge fills the receptable by dipping, raises it, and empties it into an irrigation channel. With this device, a man can raise about 2400 liter to a height of 2 m in a day.

ca 1500 BCE Appearance of *gold ingots* in Egypt. Their *weight* was certified by the government of the kingdom and represented the maximum exchange values[51].

1500–1100 BCE Origins of the oldest *phonetic alphabet* in the Sinai peninsula[52]. The Semites who lived in the Land of Israel developed from it an alphabet writing. They used signs to show the consonants of syllables, just as the Egyptian did, and invented their own set of characters to stand for consonants in their language. No direct links with the Egyptian writing were found.

Egyptian voyagers, at behest of Queen Hatshepsut (d. 1481 BCE), journeyed down Red Sea to Punt (probably present-day Somaliland) in search of myrrh-trees.

1479 BCE *Battle of Megiddo*, the first major battle in history. Thutmosis III conquers the Land of Israel, Phoenicia and Syria.

1375 BCE, May 03 Literary evidence from Ras Shamra Ugaritic Tablets of a total eclipse of the sun observed in Ugarit city-state. This may be the earliest record of a solar eclipse that we posses.

ca 1350 BCE Emergence of a system of mathematical notation in China (Shang dynasty) which used nine numerals and the place-value principle. This was about a 1000 years before the Hindu number system, and was the earliest instance of the use of the place-value principle, after Babylon.

1350 BCE Multiplication tables appeared in Mesopotamia. (Its development lasted some 400 years.) Decimal numerals were used in China.

ca 1350 BCE Great emigration from Arabia due to extended drought.

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[51] Material *symbols* of goods exchanged in transactions between individuals or from an individual to a group, have existed since very early in prehistory. They are attested by the bronze and iron *arrowheads* which the inhabitants of Gaul used for this purpose 5000 years BCE. It was called *obeliskos*. Other forms of non-metallic money were found in Africa, Oceania and South America (e.g. feathers and beaks of birds, shells, etc.).

[52] According to *Hebrew* tradition [as revealed in the book of **Ex.**: *16*, 14; *24*, 4; *32*, 15–16; *34*, 28, and the book of **Deut.**: *10*, 4; *27*, 3; *31*, 9, 22, 34], the protosinaic version of the Hebrew alphabet was invented by **Moses** in Sinai. The phonetic alphabet was based on symbols for sounds, not things or syllables and is the ancestor of all modern Western alphabets.

ca 1300 BCE Great eruption of Bronze age people from the Hungarian plain due to floods.

1286 BCE The Hittite under **Muwatallish** defeated Egypt under Rameses II in the decisive battle of *Kadesh* on the Orontes. The following peace treaty (1271 BCE) determined the final borders of the Land of Israel (*Num.* **34**, 1–12).

ca 1240 BCE Egyptians under Rameses II (1301–1235) dug a proto-Suez canal from Lake Timsah (the Nile) to the Red Sea.

The first canal at Suez to link the Mediterranean and the Red Sea may be a case in which climate played a part; it was done approximately when the world sea-level[53] reached its highest post-glacial stand, and the project may therefore have suggested itself just because it then, for the first time, looked feasible. It is believed to be during the reign of **Sestoris I** (ca 1980 BCE) that a fresh water canal was dug from the Nile delta to the Red Sea near where Suez now stands. The fact that **Rameses II** was able to build his canal (or perhaps put the earlier canal in order again), fits the concept of sea-level being specially high in the latter end of the very long warm epoch.

The canal fell into decay and was restored for the first time by Pharaoh **Necho II** (ca 600 BCE), and completed a century later by the Persian conqueror **Darius**. It was reopened around 100 BCE by the Roman emperor Trajan, and again for the last time in 7^{th} century CE. A century later it was finally abandoned after being blocked for military reasons.

ca 1230 BCE *Exodus* of the Hebrews from Egypt under the leadership of Moses (Hebrew: **Moshe**). According to biblical tradition Moses gave the Israeli nation a written code of law[54] that spelled out the new relationship of man to man, man to state, and man to God.

The laws of the Torah (the five books of Moses) regarding man's relation to man constitute mankind's first "*bill of rights*". These laws boldly assert that man's freedom is his supreme right. He has the right to personal liberty,

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[53] The rise of the sea-level, proceeded over some thousands of years at a rate of 1 *meter per century*. It continued until about 2000 BCE.
The account of the exodus of the Israelites from ancient Egypt (ca 1230 BCE; *Exodus* **13**, 17–**14**, 31) indicates that the isthmus of Suez was narrowed at that time by an area penetrated by a tongue of the Red Sea, in which the sand of the desert was awash, sometimes as tidal shallows but sometimes rather more deeply.

[54] The Greeks had no written laws until the time of **Lycurgos** (ca 700 BCE). A written judicial code was totally unknown to the Egyptians until 300 BCE.

free speech, and private property. Charges against him must be made in open court where he has a right to defend himself. The Torah recognizes no class distinction before the law. Slaves in ancient Israel were treated more humanely than slaves in the United States in 1800 CE. Slave-trading, as practiced by Christians until the 19^{th} century CE was unthinkable to the Jews a millennium before Jesus.

Although the Mosaic laws pertaining to man's relations to man and man's relations to the state provided the Hebrews with a workable framework to govern themselves, it was the ideas in the Torah pertaining to man's relations to God which assisted the Hebrews most in carrying out their mission; by making God spiritual instead of material, they were free to speculate of the nature of God himself. This permitted them to attain a higher concept of deity than was possible for the Pagan Greeks.

In the history of all other people, first comes the state, and then comes the law; this held true for the Babylonians, the Greeks and the Romans. With the Jews – first came the Torah, the law to shape the future state, and 200 years later came the state. This Mosaic Magna Carta saved the Jews from straying into detours and oblivion, and prepared them for their special destiny in the history of mankind.

ca 1220 BCE **Bezalel ben Uri**. Master craftsman and chief artificer of the Sanctuary tent and its furnishings. Constructed and built the *tabernacle*, *ark*, *altars*, and the *ephod* [*Ex* **31–39**, I *Chron* **2**, 20, II *Chron* **1**, 5]. His *brasen altar* survived onto the days of King Solomon[55] (ca 950 BCE).

1201–1198 BCE **Joshua** in Jericho and Gibeon; singular astronomical and geophysical events over the ancient Near-East[56] during the Israeli conquest of Canaan:

[55] There is little in the biblical data itself to suggest that ancient Israel has a class of professional architects. The construction of buildings and monuments appears to have been the responsibility of craftsmen or master masons. Even these seem to be of non-Israelian origin, such as *Hiram*, the Phoenician master craftsman from Tyre who was in charge of Solomon's Temple in Jerusalem (*I Kings* **7**, 13; *2 Chron* **2**, 13-14).

[56] Hasegawa, I. (1980) Catalogue of ancient and naked-eye comets, *Vistas in Astronomy* **24**, 59–102.
Sekanina, Z. and D.K. Yeomans (1984), Close encounters and collisions of comets with earth, *Astr. J.* **89**, 154–161.
Ben-Menahem, A. (1992), Cross-dating of Biblical history via singular astronomical and geophysical events over the ancient Near-East, *Q. J. Roy. Astr. Soc.* **33**, 175–190.

(1) Comet Halley apparition 1198 BCE, May 11, or the apparition of another comet brighter than the 6^{th} magnitude in 1201 BCE (**Josh** *5*, 13). It may have been seen by **Joshua bin-Nun** prior to the assault of the Israelites on Jericho (**Josh** *5*, 13). Described as bearing the shape of a flaming sword.

(2) Long day with extra light hours due to "bright nights" phenomenon following a close encounter of Apollo-type asteroid with the earth. This could explain: "*And the sun stood still, and the moon stayed...*" (**Josh** *10*, 13).

ca 1200 BCE The Hittite kingdom (began ca 1750 BCE) came to an end as a result of the great *Aegean migration*, of which the *Homeric War* against Troy was an incident.

ca 1200 BCE *Indo-European Invasion* was disturbing the entire Eastern Mediterranean world[57]. The whole of Asia Minor had been overrun by another wave of new Indo-European, who came behind the Greeks and crossed the Hellespont from Europe. The most important of these were the *Phrygiens* and the *Armenians*.

The *Hittite Empire*, lying directly in their path, was completely crushed and disappeared. Many of its communities peregrinated beyond the Mediterranean.

The Egyptian monuments of this time reveal these sea-wanderers very vividly. Besides the *Philistines*, who were fleeing from Crete, the monuments show us the sea-roving *Achaeans* who combined with the other displaced people to invade Egypt in the last declining days of the Egyptian Empire. This was apparently a second group of Achaeans, who had remained in Asia Minor after the invasion of Greece by their earlier kindred. Forced out by the Indo-European invasion, this second group of Achaeans joined with other fleeing

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[57] Modern scholars link this mass-migration to sharp *climatic changes* during 1200–850 BCE. A northward displacement of the arid zone in Europe and Asia caused the rise and fall of civilizations and southward migration of people in Northern and Central Europe and Asia, even as late as 500 BCE. The ensuing extended drought was sufficiently severe to disrupt agriculture in Crete, Greece and the whole Eastern Mediterranean. The Greek countryside was apparently depopulated for a time before the Dorians moved in from the north. Refugees wandered from the drought-stricken land to Egypt and southwest coasts of Asia Minor.

The *atmospheric circulation patterns* which caused this drought may have continued rather prominent until about 850 BCE, e.g., a drought lasting $3\frac{1}{2}$ years occurred in the days of Ahab (King) (reigned 874–853 BCE; *I Kings*, 17–18).

Asia Minor peoples to seek a new home in Egypt. Among these were the *Sardinians* and the *Etruscans*. The Cretan Philistines and the Sardinians had in their possession, for the first time in the ancient world, a long two-edged bronze-made dagger, to be known henceforth as the *sword*[58]. The main body of the "*sea-people*", as the Egyptian monuments call them, kept together for the purpose of invading and conquering Egypt.

All the great powers of the Ancient Near East were threatened by this vast Indo-European movement[59], which stretched from the Balkan Peninsula eastward to the upper Euphrates. Its front had set in motion before it a wave of fleeing Aegeans and Asia Minor people, who were mostly pre-Indo-Europeans. It was this wave of refugees that crossed the sea and began to break upon the shores of the eastern and southeastern Mediterranean, from the Nile Delta to the harbors of Phoenicia. The onset of these sword bearing northerners shook the Egyptian Empire to its foundations. Eventually, however, this wave was driven back by Rameses III.

Consequently, the *Cretan Philistines* settled on the coast of Israel, some of the *Achaenas* migrated to Greece, while the *Etruscans* sailed far westward around the heel of Italy. The *Sardinians* settled in Sicily. Thus the Indo-European invasion of the Eastern Mediterranean ushered a new age in the *Western Mediterranean*.

The Iron Age

Metal forms a large part of the earth on which we live. The earth's crust is made up of about 8 per cent aluminum, 5 per cent iron, and 4 per cent calcium. Potassium sodium and magnesium also occur in large amounts.

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[58] This elongation of the Egyptian bronze dagger into a heavier weapon took place in the north after the discovery of tin in *Bohemia*.

[59] *The Table of Nations* (*Genesis* **10**) provides a schematic arrangement of the major cultural divisions of mankind, specifying ancient nations and their great cities. It derives from Mesopotamian sources reflecting the Near-East population toward ca 1100 BCE. The figure behind the hero **Nimrod** is the Assyrian Emperor **Tukulti-Ninurta I** (reigned 1234–1200 BCE), known for his conquest of Babylon and his building projects. The genealogical lists actually reflect political and economical filiation, as well as cultural affinities.

Ancient man knew and used many native metals. Some metals, such as gold, silver, copper and tin[60]*, occur naturally and are easy to work. They have been used to make object for ornaments, plates, and utensils as early as 3500 BCE. Thus, gold objects showing a high degree of culture have been excavated at the ruins of the ancient city of Ur in Mesopotamia. The attached evolutional timetable reflects the order by which metals were discovered since the end of the stone age.*

As early as the beginning of the Bronze Age (ca 3000 BCE), some people in the Middle East began to make tools by beating and hammering iron from meteors. They decorated many of these skillfully made implements. The oldest pieces of such iron in existence are Egyptian sickle blades and crosscut saw, thousands of years old. During the Bronze Age, however, most craftsmen continued to use primitive tools of the late stone age, because metal was expensive. Only kings and warriors could afford if. Indeed, a dagger, resurrected (1926) from the tomb of Tutankhamun (ca 2000 BCE) has an iron blade almost untarnished.

How could Egyptian craftsman, 3000 ears ago, achieve such a startling feat? Iron was a metal which at that time was hardly used. It was considered inferior to other metals: it was soft and would not harden. It was also easily oxidable by the atmosphere. There were few iron objects that survived these 3000 years and those left show severe deformation.

Around 1400 BCE wrought iron was first produced by the Hittites in Northern Anatolia: they burned iron ore with wood and removed most impurities by repeated hammering. Hittite dominated Asia Minor from ca 1900–1200 BCE, establishing a great empire.

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[60] Human history reflects the activity of metals; the fact that metals were discovered in the order: gold, silver, copper, tin, iron is strongly tied to the chemical properties of metals. One such property is their *tendency to loose electrons*; This tendency is greatest for the metals on the lower left side of the periodic table. Very active metals, such as sodium, lose their electrons easily; they are unlikely to be found free in nature. In contrast, noble metals, such as gold, are often found free in nature. The larger pieces of free gold near the surface of the earth may have been found during "gold rushes". When heated, compounds of inactive metals yield the free metal. We may imagine, therefore, prehistoric people discovering free metals when building fires in areas rich in metal ores. Since it is easier to extract less active elements from ores (oxides of metals) than it is to extract more active elements, we can see why less active metals were discovered first. Iron is more active than copper, lead and tin. We have been able to make *stainless steel* (80% iron, 12% chromium, 8% nickel) since 1790 CE, when the even more active metal *chromium* was discovered. In the early 1800's very active metals such as *potassium* and *lithium*, were discovered.

Events were however to confer about iron a new and almost everlasting life: At the eastern end of the Mediterranean, in the 2^{nd} *millennium BCE, trade in metals thrived between ports like Tyre and Sidon and the ancient Aradus (Arwad) port on the coast of Syria. Copper and other metals were being mined in quantities in Cyprus and Anatolia and supplied to metal workers in the cities of Egypt, Crete and Mesopotamia. But this established order was suddenly disrupted.*

Historically, the Eastern Mediterranean has always been a linking passage from Europe to India, Central Asia and Afghanistan, through which the circulation of metals had come to have increasing importance. In ca 1200 BCE, it was invaded by the Sea People-waves of migrating masses from the north – who disrupted the established trade routes which followed the supplies of tin from their distant sources.

And without tin there is no bronze! An age which lasted for 2000 years was strangled and it never revived.

The whole material balance tipped: bronze became scarce and iron, which had been used until then mainly for ornaments, begun to appear with increasing regularity in the archaeological record.

Out of the ashes of the bronze age cultures there slowly emerged the cultures of the iron age.

Nevertheless, the use of iron in the old world was on a very small and restricted scale until after the downfall of the Hittite Empire under the heavy blows inflicted upon it by the Sea-People. Its iron craftsmen dispersed through the entire Middle-East and interacted with metal-smiths of other nations. New technical discoveries were made which led to the transformation of iron, from a metal inferior to bronze, to one which was destined to become a universal replacement for it.

The basic technology, successful for a thousand of years in smelting copper, failed when applied to iron; the melting point of iron is much higher, more than $1500\,^{\circ}C$*. Such temperatures were unobtainable. At the temperatures which were possible, iron was reduced from its ore without ever becoming liquid, or separated from the slag. The result was a spongy mess of slag, embedded with grains of iron called the bloom. By hammering the bloom and driving out the slag, a blacksmith could actually obtain a bar of almost pure wrought iron. To keep the iron meltable enough to work, the smith kept reheating it in his forge. This produced a subtle but important change – the repeated contact with white hot charcoal caused small amounts of carbon to combine with a surface layer of iron. This blend of carbon iron was much harder then pure wrought iron – it is steel.*

Having discovered the method of steeling (carburizing) iron, the early smiths made a further advance – they found that if steeled iron was cooled suddenly like quenching, it became even harder, or quite brittle. Finally they found that if this hardened steel was reheated, it lost its brittleness but retained its hardness – this process came to be called tampering.

These three vital discoveries transformed the properties and potential of iron. About 1100 BCE true iron working began in the Middle East and from there spread over much of Asia, Africa and Europe. Iron ores were much more plentiful than copper and with the diffusion of the techniques of steeling iron in the first millennium BCE the Iron Age gained momentum. Craftsmen abandoned the crude tools of the Bronze Age and made wider use of iron, including plows and weapons. This age continues to the present day.

In Europe the Iron Age began at about 600 BCE at Noricum in the East Alps: In the village of Halstad in the Austrian Alps, excavations (1864) unveiled a cemetery. Buried with the dead was a whole culture of metal, including weapons and ornaments made of iron. This iron metallurgy had spread to Noricum from the Hittite empire. After about 400 years the center moved to the Celtic lands, and especially to Spain, where the Celtic smiths developed the Catalan iron furnace[61]*. Soon all of Western Europe was put under the anvil. In the energetic hands of these creative people, the iron technology laid the political and economical foundation of European civilization.*

Iron is a symbol of the confident positive commitment by man everywhere to a new metal and to a new age. It is a milepost of that time by which the making of iron has become a central preoccupation, a determinant of success or failure of national survival[62]*, and became something not just to*

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[61] The new metal was first used for weapons; then for the *hoe* and the axe and pick of the farm and mine; lastly, for improved tools; the iron of early classical Greece was not suitable for ploughshares, and was evidently a very inferior metal compared with the properly hardened and tempered material that served the *Roman legions* about the beginning of the Christian era.

The best steel known to the Romans was the so-called '*Seric iron*'. It was a high-carbon crucible steel, made in round cakes, about 10 cm in diameter, which reached Rome via Abyssinia from Southern India.

The defensive armor and shield of the Greek hoplite were made of bronze, but his main weapons were a 3 m spear tipped with iron and a short, straight iron sword.

[62] E.g. the word iron is mentioned in the Old Testament 76 times, and duly represents the impact of this metal in the world of the Hebrews during 1200–500 BCE. Israel became acquainted with it through the *Philistines*, who had long known to work the metal. The raw material was brought in by Tyrians, mainly from Spain, though it was found also in the Lebanon range (*Jer* **11**, 4; *Deut* **4**,

make but to emulate. It entered all languages as a synonym for strength and resolution: before the iron age, man was as strong as an oak, now they seek to become iron-willed! That change in metaphor signalized a further step away from agriculture and towards industry, and iron changed the quality if not the form of a vast number of things in both.

It means better tools and better weapons, sharper axes, cut down more trees to make more fuel. It cleared more forest for more cultivation to support larger populations and when it came to blood and iron – superior sword won decisive wars.

Without it there would be no bridges, no engines, no skyscrapers: iron is the top root of our material civilization.

20; **8**, 9; *I Kings* **8**, 51). Out of iron, the blacksmiths made axes, hatches, sickles knives, swords, spears, bars, chairs, fetters, nails, hoes, pens, plows and sledges.

Iron metallurgy – signposts of Progress
(4000 BCE–1600 CE)

ca 4000 BCE First reduction of *copper* in Anatolia and Iran, Egyptians mine copper ores and smelt them. End of Stone Age and begin of the *Copper Age*.

ca 4000–3500 BCE Egyptian and Sumerians smelt *silver and gold*; Egyptians mine and process *iron*, used mainly for utensils. Metal mirrors in Egypt. Egyptians and Babylonians make extensive use of *bronze*.

ca 3500 BCE Discovery of *lead*.

ca 3000 BCE End of the Copper Age. Begin of Bronze Age.

ca 2500 BCE Chaldeans in Ur (Mesopotamia) join sheets of *gold* by soldering.

ca 2000 BCE *Copper* bar from Nippur (weight 41.5 kg, length 110.35 cm) is the earliest standard measure.

ca 1500 BCE *Gold* nuggets in Egypt serve as weight standards.

ca 1400 BCE Hittites of Anatolia first produced *wrought iron*: they burned iron ore with wood and removed most impurities by repeated hammering.

ca 1100–800 BCE Begin of Iron Age.

ca 1000 BCE *Steel* was being made in the Middle-East and India: bars of iron were steeled by hand labor with hammer and anvil or by roasting with charcoal. In this way the iron was mixed with small amounts of carbon that made it harder and stronger.

ca 600 BCE Iron-Age began in Europe at *Noricum*.

ca 315 BCE The earliest existing treatise on *minerals* written by the Greek philosopher **Theophrastos**.

ca 300 BCE Chinese first produced *cast-iron*. Better furnaces produced higher temperatures which could melt the iron completely. The product was stronger than wrought iron. Their technique was, however, unknown in Europe until 1380 CE.

ca 200 BCE Chinese develop form of cast iron.

ca 77 CE **Pliny the Elder** (Rome) writes about ores in his *Historia Naturalis*.

ca 450 CE Chinese learn to make *steel* by forging together cast and *wrought iron*.

ca 800 CE Blast furnaces for making *cast iron* are built in *Scandinavia*.

ca 1000 CE *Celtic* iron technology in Spain; the Catalan iron furnaces.

ca 1262 CE **Albertus Magnus** (Germany) wrote about minerals in his treatise *De Mineralibus*.

1380–1389 CE *Cast iron* becomes generally available in Europe: Tall furnaces were built and water-power was harnessed to produce stronger blast of air than that achieved by hand-operated bellows. Furnaces could now be operated at temperatures high enough to produce molten iron.

ca 1540 CE **Vannocio Biringuccio** of Siena (Italy) issued his treatise *Pirotechnica*, describing production techniques of brass and bronze in Europe.

1556 CE **Georgius Agricola** (Germany) published *De re metalica*, a systematic treatise on mines and metallurgy.

ca 1200 BCE The quest of *Jason and the Argonauts* for for the *Golden fleece* describes an expedition out beyond the Golden Horn over the Euxine Sea to present-day Armenia to seize the source of gold. It was extracted by the laborious process of washing the river sands through the fleece of sheep, with gold nuggets left clinging to the oily fibers.

ca 1200 BCE *The Phoenicians*[63], who lived along the coast of the Mediterranean Sea, developed a system of 22 signs to form an alphabet that was structurally related to Semitic and Egyptian. It has signs for consonant sounds, but not vowel sounds. Early Phoenician writing consists partly of pictographic forms and partly of geometric or diagrammatic signs. Possibly, the Phoenicians based their alphabet on the earlier Semitic alphabet. The Phoenician alphabet spread throughout Western Asia along the caravan routes. It passed down the Euphrates to Persia, and, penetrating to the frontiers of India, even furnished the East Indian people with their Sanskrit alphabet.

ca 1200–400 BCE *Chavin de Huantar*. Pre-Inca culture in the central Andes. Pottery, weavings and impressive stone buildings.

1122 BCE Severe drought in China.

ca 1100 BCE **Chou Kung**. Chinese statesman and mathematician. He is accepted as the author of the first dialogue contained in the Chou-Pi, one of the oldest of the ancient Chinese scientific treatises. It deals principally with calendrical problems and thus with astronomy and mathematics. We have evidence here of knowledge of mensurational geometry, the *Pythagorean theorem*, *elements of trigonometry* and some instruments for astronomical measurements.

Chinese astronomers determined the obliquity of the ecliptic (the angle between the earth's orbit and its equatorial plane) to a few minutes of arc.

ca 1000 BCE The Chinese developed the *counting board*, the forerunner of the *abacus*.

ca 1000 BCE Legendary *Thule* civilization in the Gobi region destroyed by a *natural catastrophe* of unknown origin. Survivors migrate to Agarthi and Shamballah.

ca 1000–700 BCE Beginning of the *Iron Age*[64]. Iron was already known to man in prehistoric days, but it remained a rarity until the *Hittites* discovered

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63 The origin of this alphabet is attributed to the Phoenician **Cadmus** (son of Agenor, king of Tyre), who is said to have brought 18 letters to Boetia (ca 1313 BCE).

64 The three ages of metals:
The *Copper Age*: from the 4^{th} millennium to about 2000 BCE;
The *Bronze Age*: from about 2000 to 1000 BCE;
The *Iron Age*: from about 1000 BCE to the modern *Age of Steel*.

it in northeastern Asia Minor. From the 13^{th} century onward, the Hittite kings distributed iron throughout the Near East. It was therefore in the first centuries of the Age of Iron that the *Assyrians* were preparing for Western conquests, and their success was largely due to the use of this metal in warfare. Thus, the Assyrian forces were the first large armies completely equipped with weapons of iron. The bulk of the Assyrian army was composed of archers, supported by heavy-armed spearmen and shield bearers.

Assyria had without doubt learned much from the skillful horsemen of *Mitani*. The famous horsemen and chariotry of Nineveh became the scourge of the East. For the first time, too, the Assyrians employed the battering-ram and formidable siege machinery. The sun-dried brick walls of the Asiatic cities could thus be battered down or pierced, and no fortified place could long repulse the assaults of the fierce Assyrian infantry.

Under the influence from the Hittite art, the sculptors of Assyria learned to tell the story of the king's valiant exploits in elaborate *stone pictures* cut in flat relief on great slabs of *alabaster*.

The Assyrian armies had marched westward and had crossed the Euphrates by 1300 BCE. They had looked upon the Mediterranean by 1100 BCE, but for more than 350 years after this the kings of Assyria were unable to conquer and hold this western region against the strong alliance of Arameans, Hebrew and Phoenician kingdoms.

986 BCE, Dec. Perihelion passage apparition of comet Halley. May have been witnessed by King David in Jerusalem. [*Chron I* **21**, 16; *Psa* **18**, 13–15].

950 BCE The valued $\pi = 3$ was used by the Israelites in constructions associated with King's Solomon Temple in Jerusalem. [*I Kings* **7**, 23; *Chronicles II* **4**, 2].

Chinese chariots had wheels with spokes. Iron mines in Italy.

ca 950 BCE The Queen of Sheba arrives in Jerusalem to visit King Solomon (ca 984–928 BCE) and establish commercial relations with the Hebrew Kingdom.

ca 900 BCE *The Etruscans* settled on the western coast of Italy, north of the Tiber. The earliest of them had arrived in consequence of the breakdown of the Hittite Empire[65] and brought with them an *oriental* civilization. They introduced the chariot, the arch in building and an alphabet, and were

[65] The eastern origin of the *Etruscans* has been proved (1926) by the discovery of an Etruscan cemetery on the Greek island of Lemnos.

therefore not illiterate like their predecessors in Italy. The Etruscans brought also from the East much skill as craftsmen. In Italy they found copper, and in the course of time they developed the finest bronze industry in the ancient world of that period. Their goldsmiths too were unrivaled by any in the older countries.

The Etruscans invented gladiatorial games, drained the marshes, plied the seas with commerce, traversed the heartland of Europe with goods, and founded a religion built on fornication, death and hellfire. The senior trinity of their gods consisted of a holy father, a virgin mother, and an immaculately begotten daughter. In Etruscan theology, the dead went first to purgatory, for judgment, where, if found guilty, their souls were damned to various degrees of torment, the ultimate punishment being eternal hellfire. [In the 13^{th} century CE, these concepts seeped into Christianity via the *Divina Commedia* of Dante, who was steeped in Etruscan mythology.]

When the Greeks arrived in Italy to plunder, trade, and colonize, those Etruscans who survived the encounter acquired Greek culture. The Roman Kingdom founded by Romulus in the 8^{th} century BCE, was conquered by the Etruscans in the 6^{th}. Though their rule was brief, the Etruscans did nevertheless influence Roman culture more profoundly than the Sumerians influenced the Babylonian; two and a half centuries of Etruscan rule (ca 750–500 BCE) left their mark on Rome, always afterwards discernible in architecture, religion, organization, city planning and roads. Many Etruscans continued to live in Rome and Latium, and in the days of Roman splendor, some of the greatest families of Rome were of Etruscan descent and were proud of it.

After their expulsion from Rome, the Etruscan continued as a powerful and highly civilized federation, although surrounded by dangerous enemies. They finally lost their territories to Gauls, Samnites and the Romans.

876 BCE A symbol for zero was used in India. The first known reference to this symbol.

853 BCE The *Battle of Karkar*; one of the biggest iron-battles of the ancient world through which a grand alliance of Aramean, Hebrew and Phoenician kingdoms stemmed the advance of the Assyrians along the Eastern Mediterranean coast to Egypt. The historic military encounter at Karkar (ca 100 km NNW of Hamah, on the Orontes River; today's Qarqur) involved some 200,000 men and 6000 chariots on either side.

After the fall of *Mitani*, *Egypt* and the *Hittite* empires, there still remained the powerful mercantile civilizations of the Western Semites – the line of harbor towns on the Phoenician coast, the Aramean city of Damascus under

Ben Hadad II, and the Israeli kingdom of Ahab[66], which blocked their way to Egypt. When Assyria, under **Shalmaneser III** were ready to strike, Ahab and Ben Hadad II were ready and the might of Assyria clashed with the massed strength of twelve buffer states. When all was over, the Assyrians were dealt a stunning defeat that set their timetable for conquest back a hundred years. This battle "bought" Israel an extra century until the kingdom was finally dissolved and the *Ten Tribes* dispersed and lost by the Assyrian deportations of 740–700 BCE.

842–771 BCE Great drought in China.

831 BCE, Aug. 15 A total eclipse of the sun in Southern Judea at *midday*. The Hebrew prophet **Amos of Tekoa** (fl. 765–750 BCE) rendered a vivid account of this event[67] (*Amos* **8**, 9; **5**, 8).

ca 800 BCE Egyptians used techniques of *tanning* hides to make leather and *hardening* of leather with alum. They were using *sundials* with six time divisions to tell time. The sundial was introduced in Greece by the 6^{th} century BCE.

ca 800 BCE Phoenicians established trade routes to *Gadir*, on the Atlantic coast of Spain. By this time, Phoenician refused to allow ships other than their own to sail through the strait of Gibraltar to Atlantic coasts of Europe or Africa.

ca 800 BCE The Greeks borrowed Phoenician symbols and modified them to form the *Greek alphabet*. They came into contact with Phoenician traders and learned from them the idea of writing individual sounds of the language. Since the Phoenician alphabet included more consonants than the Greek needed for their language, they used the extra consonants for vowel sounds: thus the Phoenician *aleph* (meaning *Ox*) became *alpha*, the Phoenician *beth* (meaning *house*) became *beta*, etc. The Greeks later modified the

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66 During the reign of King Ahab (874–853), there occurred a drought in Israel which lasted for $3\frac{1}{2}$ years (*I Kings* **17–18**).

67 The zone of totality fell within the southern boundaries of Judea. The moment of greatest darkness was almost exactly at midday. Such phenomena are very rare at any given locality. The partial eclipse of June 15, 763 BCE was not as impressive in Judea, since it was not total within the borders of Israel and greatest darkness occurred in the early morning hours.
Amos may have witnessed the 831 BCE event in his early youth or could have been told of it. [*Solar and Lunar Eclipses of Ancient Near East From* 3000 BCE *to* 0 *with Maps* Kudlek, M. and E.H. Mickler (eds.), Verlag, Butzon and Bercker Kevelaer, 1971].

shapes of these letters, adding and dropping some letters to form the 24-letter Greek alphabet of today.

8^{th} century BCE Chinese astronomy reaches its peak with outstanding quality and quantity of observations[68]. The earliest reliable observation report a total solar eclipse on July 8, 709 BCE.

776 BCE The first *Olympiad* celebrated in Olympia, Greece. The Olympic games took place 292 times, the last occasion being in 393 CE. Their importance was such that Olympiads were the basis of *Greek chronology* and their organization and development were subject to an established and unchanging ritual and continued for almost 1200 years.

767 BCE First recorded worldwide plague (*Amos* **4**, 10).

ca 720 BCE The *Latin alphabet* was borrowed by the Romans from that of the Greeks, probably through the Etruscans (who moved to central Italy from somewhere in the Eastern Mediterranean sometimes after 1000 BCE, carrying the Greek alphabet with them). The Romans learned the alphabet from the Etruscans, and gave it much the same form we use today. The early Roman alphabet had about 20 letters, and gradually gained 3 more. The letters J, U and W of the English alphabet were not added until the Middle Ages.

Capital letters were the only forms used for hundred of years, and were finally perfected by 114 CE when *sculptors* carved the inscriptions on a memorial column built in honor of the emperor Trajan. Lower-case letters gradually developed from capitals by *scribes* who copied books, using rounded letters (*uncials*) that were easier to form than some capitals.

711 BCE, Mar. 14 A partial eclipse of the sun, visible in Jerusalem. The prophet **Isaiah I** (fl. 740–685 BCE) and the Judean king **Hezekiah** (reigned 715–687 BCE) are linked to this event in the unique biblical story that is threaded in three books [*Kings II* **20**, 8–11; *II Chron* **32**, 31; *Isa* **13**, 10; **38**].

Hezekiah, king of Judea, was a patron of learning and caused great public works to be undertaken; he fortified Jerusalem and improved its water-supply.

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[68] About 800 BCE (Homer's time), the Greeks believed that the mass of the earth, surrounded by the river *Okeanos*, filled up the lower half of the sphere of the universe, while the upper half was out above it, and that *Helios* (the sun) extinguished its flames each evening by bathing in the deep waters of the ocean and lit them again in the morning.

To this effect, the Shiloa tunnel[69] was chiseled in ca 700 BCE [*Chron II* **32**, 3–4, 30; *Kings II* **20**, 20; *Isa* **22**, 9–11].

700 BCE The Assyrians introduce the *aqueduct*. Carved bone or ivory dentures with gold braces worn by Etruscans of Northern Italy.

668 BCE **Ashurbanipal**, king of Assyria, established a *library* at his capital *Nineveh* (destroyed ca 612 BCE).

ca 650 BCE The Lydians of Asia Minor introduced the *first standard coinage* of the Western world.

It has been the custom for centuries in the Egyptian and Babylonian empires to *stamp* ingots with some mark giving authority of the value of the metal. This however did not ensure a standard quality to the precious metal, and therefore did not make the ingots more freely exchangeable. However, some time in the 8^{th} century BCE someone discovered the *touchstone*, a flinty river stone (schist), used to assay the quality of the gold ingot.

Herodotos tells us that the Lydians cut the top surface of the stone flat, leading it matt. If gold were rubbed on this matt surface it would make scratch marks. Pure gold would leave yellow marks, gold mixed with silver, white ones, and gold mixed with copper, red marks. It gave the rulers of Lydia, starting with **King Gyges** (685 BCE), the ability to ensure a standard quality to their money by easily detecting forgery. The mark of the King's mint was now evidence of purity, weight and acceptability.

In the 6^{th} century BCE a smaller unit of exchange, the Lydian *stater* was produced. It was made of a gold-silver alloy called *electrum* and punch-marked by the issuing mint. Within a century a set of coins, each one a

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[69] An underground *S*-shaped aqueduct, 513 meters long. The rock was pierced simultaneously at both ends and the workmen met in the middle – a triumph of precision engineering.
Herodotos tells us of a similar water project undertaken in Samos by the Greek engineer **Eupalinos of Megara** (fl. 520 BCE). He built water conduits during the rule of Polycrates (ca 530–522 BCE). The remains of the tunnel were found in 1882; it is about 1000 m long and 1.75 m high and wide, at the bottom of the tunnel there is a trench, about 60 cm wide and reaching at the south end a depth of 8.3 m, wherein the clay pipes were embedded.
The Hezekiah and the Eupalinos tunnels were *started at both ends*. How did the engineers solve the mathematical problems involved? We may guess that they had instruments to measure azimuths and difference of levels. The problem involved was solved *theoretically* for the first time by **Hero** in his treatise on *dioptra* (ca 50 BCE).

fraction of the stater, had been issued. When *Croesus of Lydia* introduced the first standard imperial coinage (550 BCE), Lydian money was already known for its high and unchanging standard. When **Cyrus** conquered Lydia (546 BCE) he issued the first Persian coins.

The economic revolution following the development of coinage was enormous. Before this, the only money was the bullion weighted out at the time of a transaction. Small farmers and artisans were forced to barter. The introduction of coins that petty producers could use brought the mass of the population the benefits of money, since the craftsman was no longer condemned to "eat his wages". He could buy products with his earnings, opening up new markets for fellow artisans. The small farmer was free to follow the aristocrats into specialized farming for an export market.

Money is perhaps man's most useful discovery, next to fire and the wheel. As the use of coinage spread, it had two fundamental effects. The first was *political*: money issued by a central mint had a unifying effect on the users. The mark of the government on the coin was present in every transaction. Its presence defined the boundaries of governmental authority, and its value mirrored the health of the economy and political stability of the country. The second was *cultural*: with the growth of international trade grew the exchange of technological innovations and scientific ideas.

But money was a mixed blessing, and in its wake followed usury, mortgages, and *debt slavery*. Money was power. Anything could be reduced to *abstract numbers*: the value of a pot, a jar of oil, a plot of land, a slave, could all be expressed by exact number of coins, as could the wealth and worth of any citizen. Numbers seemed to have magical powers. Money invested at interest could even multiply itself without any effort on the part of the lender.

As the money economy developed, so did chattel *slavery*. Such an institution, based on the sale of slaves, is impossible without the free exchange of money. Slavery threatened either to enchain the small producers themselves or to undercut their livelihood, and so devalued productive activity. To Greek slaveholders, work was something done by slaves, thus in itself degrading. Only detached thought is worthy. As slavery separated thought from action, so did a new trend in Greek philosophy glorify abstract reason while denigrating physical observation. Slavery also undercut the development of technology that required and fed observation: slave-owners did not need labor-saving devices.

The Greek philosophers in the 6^{th} century witnessed the effects of money on the Greek states, and some of them extrapolated from the power of numbers in society to the idea that numbers rule the universe as well.

650 BCE Assyrians compile Mesopotamian medical knowledge in collection of *clay tablets* at the Royal Library in Nineveh. Over 300 medications and such diseases as paralytic stroke and rheumatism are described. The diseases of tuberculosis, gonorrhea and leprosy first described with accuracy.

The Greeks[70]

"The isles of Greece, the isles of Greece!
Where burning Sappho loved and sung,
Where grew the arts of war and peace,
Where Delos rose, and Phoebus sprung!

Eternal summer gilds them yet,
But all, except their sun, is set".

George Gordon Byron (1788–1824)

Originally a shepherd people, the Greeks had migrated to the Greek peninsula in about 1900 *BCE out of central Europe, possibly from the Danube. They were one of several peoples speaking Indo-European languages who migrated southward at this time. These include, among others, the Germanic, Celtic, Latin and Iranian peoples, and the Aryans who in the same millennium invaded and conquered Northern India. The Greeks found their new home too arid to satisfy their needs and soon turned to the sea for their livelihood, learning the secrets of navigation, and their metal working skills, from the Minoans.*

By about 1600 *BCE, the Greeks were established throughout the Aegean area in independent city-states and the* Mycenaean civilization *had begun to take shape.*

70 For further reading, see:

- Bowra, C.M., *Classical Greece*, Time-Life International: The Netherlands, 1970, 192 pp.
- Wittle, T., *The World of Classical Greece*, William Heinemann Medical Books: London, 1971.
- Heath, T., *A History of Greek Mathematics*, Oxford at the Clarendon Press, vols I-II, 1921, 446+586 pp.
- Cohen, M.R. and I.E. Drabkin, *A Source Book in Greek Science*, McGraw-Hill Book Company: New York, 1948.
- Schrödinger, Erwin, *Nature and the Greeks*, Cambridge University Press, 1996, 172 pp.

Imaginative, curious, with an innate appreciation of the beautiful and a genuine love of adventure they became fishermen, pirates and merchants, trading with the nearby island of Crete, where the Minoan civilization was just reaching its peak.

With the decline of the Minoans, following the Thera natural catastrophe, the westward migration of the Phoenicians and the comeback of the New Kingdom in Egypt, Mycenaean traders sought new markets in the Black Sea. During 1150–900 *BCE, the Dorians moved into the Greek Peninsula in a series of destructive waves and wiped out the Mycenaean ruling classes together with their entire civilization. The centuries preceding and following* 1000 *BCE witnessed in that part of the world a tremendous upheaval caused by the introduction of iron, complicated migrations, and widespread turbulence. This was the Dark Age that preceded the dawn of Greek culture. (In this respect that Dark Age resembles the Christian Middle Ages; both were periods of unconscious assimilation and preparation.)*

By 800 *BCE a new urban society was formed, and during* 800–600 *BCE, more than* 100 *colonies were founded, from the Black Sea, virtually a Greek lake, to Marseille (Massillia). They controlled the coast of Thrace and the entire littoral of Asia Minor, where they took over the trade previously enjoyed by the Phoenicians.*

They made Miletos, on the Aegean shore, the richest of their cities, and a port of departure for expeditions to the Black Sea. They colonized the eastern sea-board of the Adriatic, southern Italy, Sicily and Cyrene on the coast of Libya[71].

By the middle of the 7^{th} *century, they have established themselves in Egypt, founding the port of Naucratis at one of the now extinct mouths of the Nile. To this port, which later became the city of Alexandria, they brought merchandise which was paid for by Egyptians with gold from Sudan.*

During this period of trade and colonization, the small Greek villages of the Aegean evolved into strong self-sufficient city-states which transformed from monarchies to aristocracies. It is in these Greek cities of Asia Minor, particularly Miletos, that western philosophy was born.

Indeed, between 600–400 *BCE, there occurred a unique phenomenon in the annals of mankind: People with Greek names, living in the Greek colonies on islands and inlets of the Eastern Aegean Sea (Ionia) suddenly decoupled themselves from the bondage of myths and became aware that the universe*

71 In ca 650 BCE, **Colaeos**, Greek merchant, discovered the Straits of Gibraltar for Greece. *Massillia* was founded by Greeks from the Ionian city of *Phocaea*.

around them is knowable and exhibits an internal order ("cosmos"). They began to look for regularities, laws, rules, predictions – science was born.

And they found that everything is made of 'atoms', that the earth is only a 'planet' going around the sun, that stars are far away, that light travels with finite speed, that the earth was a sphere, and more.

Why did this happen in Samos, Miletos, Ephesos, Elea, Abdera and Cnidos and not in India, Egypt, or in the Maya and Aztec societies?

They lived in an open, free, decentralized, mercantile, pluralistic island society, situated on the cross roads of Africa, Asia and Europe, under the influence of the great cultures of Egypt and Mesopotamia. Not far to the east, roamed the voices of the Hebrew prophets, poets and sages in praise of unity, law and order in the universe.

Moreover, widespread literacy and writing and the free communication of thoughts through debates made inquiry and accumulation of knowledge feasible.

And finally, the umbilical cord to mainland Greece, where art, architecture and literature have been flourishing since 800 *BCE. It is this Greek connection that finally brought the decline of the Ionian awakening. The complex amalgam of city-states was inherently unstable: rivalry between individual states and alliances made each of them vulnerable. With the emergence of the powerful Macedonian kingdom, the majority of independent city-states disappeared. Greek culture was carried away to Italy (Tarentum, Syracuse), Africa (Cyrene) and as far east as the Oxus and Indus rivers, and a new imperial age, the Hellenistic period, begun.*

The Romans (300 BCE–476 CE) did not develop a civilization of seaboard city-states like the Greek. Rome was a warrior-agricultural community, like Sparta (the least intellectual of the Greek states). Commerce was forbidden to the senators of Rome, whilst the merchants submitted to the values of their society, aspiring to become the owners of farming land.

The Romans therefore lacked the quantitative and spatial thinking of the merchant-traveler, rendering them weakest in the mathematical sciences. The spirit of pragmatism and utilitarianism which pervaded the Roman world was alien to the air of disinterested and free creativity so essential to all developments of science and art. Thus they did not add a great deal to science. Their contribution lay in the fields of organization: public medical service, buildings, roads and aqueducts, Julian calendar, Roman Law, etc. They had no mathematicians and astronomers of note, and consequently science declined during their sojourn in history.

Notwithstanding the surpassing brilliance of the Greeks, there occurred earlier (ca 1800 BCE) a historical event which had a direct bearing on the *development of science, albeit* 3000 *years later. It was the* intellectual revolution *born with the introduction of the concept of monotheism by the ancient Hebrews.*

It freed men from a fearful subordination to the forces of nature by positing a supernatural cosmic power; the idea of one God emancipated men from their terror of many evil demons, apparitions and fiendish hobgoblins.

Apart from fusing religion and morality, monotheism ultimately became a stimulus to science*, because it suggested a unitary, consistent pattern within which everything in nature functioned. It promoted the idea of order, consistency, and meaning in the universe – all waiting for man to explore and understand.*

The idea of One God contained within itself the concept of a central cause, a prime reason for things, and the search for that reason in the analyses of sacred writings, or experimental ventures, or the detached observation of physical phenomena. For once cosmic unity is accepted, universal consistencies, regularities, and interrelations follow.

Another major contribution of the ancient Hebrews concerns their apprehension of time as a linear cosmic trend; history is presented as a unified process confirming a master plan of divine significance, which began with the creation. Prior to the rise of Christianity, with the exception of Philo and Seneca, only the Hebrews and the Zoroastrian Iranians have thought of history, man's fate included, as progressive rather than cyclic. This is made clear in every Book of the Old Testament (with the exception of Ecclesiastes) and is rather strongly emphasized in the Book of Daniel.

In contradistinction, all *other ancient civilizations, and especially the Greek scholars Plato, Aristotle, Pythagoras, the Stoics and certain Neoplatonic philosophers, believed in the doctrine of eternal recurrence and cyclic time, in which the world was destined to be destroyed and created anew in a cyclic pattern. The linear concept of time had a profound effects on western thought. Without it, it would be difficult to conceive of the ideas of* progress *or* evolution.

Curiously enough, both Judaism and Christianity, each in its own way, and for different reasons, did not encourage their believers to develop science for its own sake, and the world had to wait for the late Renaissance *and Spinoza to continue the intellectual revolution of the Hebrews.*

The same source that kindled the spirit of the great natural philosophers during 600–400 *BCE, has earlier endowed humanity with equally great poetry. As happened thousands of years later, during the European Renaissance, art preceded science by some* 200 *years: In Ionia, perhaps on the island of Khios,* **Homer** *sang his great epics, the* Iliad *and the* Odyssey*, at ca 750 BCE. [Not* *far away, in Judea, the contemporary Hebrew prophet,* **Isaiah**, *was reciting his Biblical visions*[72]*.]*

These two great epics bring European literature into existence with a bang; its echoes are still reverberating. However, there must have been a long history behind the Homeric poems, since works of such massive scale and great sophistication do not come out of nothing. The common origins of Greek science, philosophy and literature are to be sought in the heritage of the Indo-European family of peoples which later created the corresponding separate cultures.

Moreover, in the Homeric epics themselves, we can already discern the buds of later philosophical apprehension of nature and of man: the Greek people stand before us alive with all their skills, cunning and above all, their inquisitiveness and ingenuity which later developed into a keen sense for detailed physical observation. The Greek culture, through all of its transformations, is the manifestation of one and the same spirit.

In Greece, humanity approached for the first time the riddles of nature and man as problems that can be rationally solved.

We can feel the harmonious equilibrium of these two elements in the following lines from the conclusion of book VIII of the Illiad [in the translation of **Alexander Pope** *(1688–1741, England), during 1715–1720].*

[72] Greek literature reflects the vital impact of the stars on the life of an agricultural and seafaring people. **Homer** noted the Bears, Bootes, the Pleiads and Hyades, and the star Sirius, all by the names we use today. [These are also mentioned in the Bible: Sirius = *Hadre Theman*; Orion = *Kesil*; Venus = *Mazzaroth*; Hyades = *Ayish*; Pleiads = *Kimah*.] The Greeks in Homer's time used Ursa Major to navigate by.

"The troops exulting sat in order round,
And beaming fires illumined all the ground.

As when the moon, refulgent lamp of night,
O'er heaven's pure azure spreads her sacred light,
When not a breath disturbs the deep serene,
And not a cloud o'ercasts the solemn scene,
Around her throne the vivid planets roll,
And stars unnumber'd gild the glowing pole,
O'er the dark trees a yellower verdure shed,
And tip with silver every mountain's head:
Then shine the vales, the rocks in prospect rise,
A flood of glory bursts from all the skies:
The conscious swains, rejoicing in the sight,
Eye the blue vault, and bless the useful light.
So many flames before proud Ilion blaze,
And lighten glimmering Xanthus with their rays.
The long reflections of the distant fires
Gleam on the walls, and tremble on the spires.
A thousand piles the dusky horrors gild,
And shoot a shady lustre o'er the field.
Full fifty guards each flaming pile attend,
Whose umber'd arms, by fits, thick flashes send,
Loud neigh the coursers o'er their heaps of corn,
And ardent warriors wait the rising morn".

The Greeks themselves declared that they found in Egypt and in Babylonia the material for their geometry and astronomy[73]*. Thales and Pythagoras, Democritos and Eudoxos: all of them are reported to have traveled to Egypt and to Babylonia, in search of elements of value in alien cultures.*

From the Egyptians, the Greeks learned their multiplication and their computations with fractions, which they then developed further. The Greeks may also have taken from the Egyptians the rules for the determination of areas and volumes [for the Greeks such rules did not constitute mathematics, it merely led them to ask: how does one prove this?]

It is, however, on Babylonian science and mathematics that the Greeks based their own. The two cultures met around 604 BCE, at the beginning

[73] In his posthumous dialogue *Epinomis*, **Plato** says of the relation of the Greeks to the old cultures of the Orient: "*Whatever Greeks acquired from foreigners is finally turned by them into something nobler*".

of the reign of the Chaldean Nebuchadnezzar, when cultural and commercial communications were opened in both directions. In addition to the mathematical heritage, the Greeks received the sun-dial and the 12 *hours of the day.*

The political equilibrium was disturbed in 540 *BCE, when Cyrus II, the Great, subjected the entire Orient to Persian domination. The Ionian cities, which had come to the aid of his opponent Croesus, had to pay heavy tributes. Many Ionians left the country: the Phocaens, for example, established the town of Elea, in Italy, which was destined to play an important part in the history of philosophy. It was also at this time that Pythagoras migrated from Samos to Croton. The center of gravity of the world of mathematics and philosophy moved from Ionia to Italy.*

But it was not long before the Persian empire reestablished economic and cultural links with the Greeks. Ionian artisans and artists took part in the construction of the palace of Darius. The sculptor ***Telephanes of Phocia*** *worked for Darius and for Xerxes. The Greek physician* ***Democedes of Croton*** *lived at the court of Darius.*

The great Persian kings Cyrus and Darius were very tolerant. They did not interfere with the cultures and the religions of subject peoples (a reference to the Bible is in order here). Babylonian stellar rituals continued to exist. The observation of the moon and the planets by the Babylonian priest-astronomers, were continued systematically during the Persian regime. Without these carefully dated observations, the later flowering of Babylonian theoretical astronomy during the era of the Seleucids, the successors of Alexander the Great, would have been impossible. The Greeks also showed interest in these observations.

Callisthenes*, who accompanied Alexander the Great to Babylon, sent his uncle Aristotle, upon his request, Babylonian observations.* ***Hypsicles****, a Greek astronomer of the* 3^{th} *century, calculated the times of rising and setting of the Zodiacs in the Babylonian manner, because Greek geometry of the sphere was not yet able to solve this problem.* ***Hipparchos*** *(150 BCE) made use of Babylonian observations and periods of the moon, which Ptolemy could use* 300 *years later – practically without corrections.*

From all this it is apparent that even during the period of flowering of their own astronomy, the Greek were glad to learn from the Babylonians in any respect in which the latter had advanced beyond them. This certainly applied as well to the initial period of Greek mathematics, when the Babylonians were already in possession of a highly developed algebra and geometry, which the Greek had not yet acquired.

The natural point at which fruitful contacts between East and West could take place at the beginning of the 6^{th} *century BCE, was the flourishing commercial town of Miletos on the coast of Asia Minor, the most important center of Ionian culture.*

The Greeks invented democracy *more than* 2000 *years before any modern Western nation took the first steps toward it; they invented* philosophy *and the* theater *and organized competitive athletics; they invented* political theory, biology, zoology, *and* atomic theory[74].

The Greeks created philosophy; *without philosophy we would have no science, and the attempt to arrive at truth of any sort would remain largely a matter of fantasy and whim.*

Chinese *civilization invented the movable type, gunpowder, the rocket, the magnetic compass, the seismoscope, and made systematic observations and chronicles of the heavens.* Hindu *mathematicians invented the zero – the key to comfortable arithmetic and therefore to quantitative science.* Aztec *civilization developed a far better calendar than that of the European civilization; they were better able, and for longer periods into the future, to predict where the planets would be.*

But non of these civilizations had developed the skeptical and inquiring method of science. All that came from ancient Greek. The development of objective thinking *by the Greek appears to have required a number of specific cultural factors. First was the assembly, where men first learned to persuade one another by means of rational debate. Second was a maritime economy that prevented isolations and parochialism. Third was the existence of a wide spread Greek-speaking world around which travelers and scholars could wander. Fourth was the existence of an independent merchant class that could hire its own teachers. Fifth was a literary religion not dominated by priests, and sixth was the persistence of these factors for a thousand years.*

Greek mathematics was a brilliant step forward. Greek science, on the other hand was riddled with error:

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[74] Yet, in spite of their great achievements in science, art and literature, their *technology* stood still through the entire millennium of their creative existence (the inventions of Archimedes were never applied to the needs of daily life)! Why?

The answer is quite simple: almost all their manual work was done by *slaves*. The Greeks despised work of any kind and therefore lacked the motivation for the invention and applications of labor-saving tools.

- *Believed that* vision *depends on a kind of radar that emanates from the eye, bounces off what we are seeing, and returns to the eye (Ignoring the fact that we cannot see in pitch darkness!).*
- *Believed that* heredity *was carried by semen alone, the woman being a more passive receptacle (Ignoring the fact children resemble their mothers!).*
- *Believed that the* horizontal motion *of a thrown rock somehow lifts it up, so that it takes longer to reach the ground than a rock dropped from the same height at the same moment.*
- *Believed that* planetary orbits *are exactly circular.*
- *Did not believe in interrogating nature by doing* experiments*, (Exceptions:* ***Eratosthene****'s measurement of the earth's diameter or* ***Empedocles****' experiment demonstrating the material nature of air). In a society is which manual labor was thought fit only for slaves, the experimental method did not thrive.*

What Ionia and ancient Greece provided is not so much inventions or technology or engineering, but the idea of systematic inquiry, the notion that nature, rather than capricious gods, govern the world. Water, air, earth, and fire – all had their turn as candidate "explanations" of the nature and origin of the world. Each such explanation was deeply flawed in its details. But the mode of explanation, an alternative to divine intervention, was productive and new[75].

Indeed, in contrast to the static civilizations of the great Eastern river valleys – Tigris, Euphrates and Nile – the Greeks created in the restless turbulence of their tiny city-states that impatient rhythm of competition and innovation that has been the distinguishing characteristic of Western civilization ever since.

[75] This approach of the *pre-Socratics* was (beginning in about 4^{th} century BCE) quenched by Plato, Aristotle, and then Christian theologians. If the brilliant guesses of the atomists about the nature of matter, the plurality of worlds, the vastness of space and time – had been treasured and built upon, if the innovation technology of Archimedes had been taught and emulated, if the notion of invariable laws of nature that humans must seek out and understand had been widely propagated – we would perhaps be living by now in a different world.

The Hebrews and their Bible[76]

Whereas the Greeks were interested chiefly in positive knowledge, the Hebrew's main concern were morality and apocalypses, though they were also keen observers of nature. The Hebrews had freed man's mind from magic by tying him with ethics to a moral God; the Greeks had freed man's mind from magic by tethering him with reason to a relative truth[77].

The Hebrews introduced the notion of monotheism, which may be considered as a scientific hypothesis. The earliest known Hebrew prophet after Moses, to expound monotheism to the people of Israel in their own land, was ***Amos*** *(fl. 765 BCE). The ethical theory of the world and the conception of a God, unique, conscientious, and just are published under his name in the Book of Amos. The ethical side of Hebrew monotheism was reinforced by the prophets* ***Hoshea*** *(fl. 740 BCE),* ***Isaiah*** *(fl. 720 BCE),* ***Micha*** *(fl. 720 BCE),* ***Zephaniah*** *(fl. ca 630 BCE),* ***Jeremiah*** *(fl. 626–586 BCE),* ***Nahum*** *(fl. 620 BCE),* ***Habakkuk*** *(fl. 615 BCE),* ***Ezekiel*** *(fl. 610–580 BCE),* ***Haggai*** *(fl. 520 BCE),* ***Zachariah*** *(fl. 520 BCE)* and ***Joel*** *(fl. 370–340 BCE).*

The Bible is also the main source-book on Hebrew law and history. The only part of the Pentateuch which can be dated with any precision is the 5^{th} *and last book called Deuteronomy. The kernel of this book was discovered in Solomon's Temple in Jerusalem in* 621 *BCE. It inspired King Joshia's revolutionary decision to centralize all formal worship in one place, Jerusalem. The book of* ***Samuel*** *was probably composed around* 600 *BCE. It deals with*

[76] For further reading, see:

- Jacobus, M.W. Ed, *A New Standard Bible Dictionary*, Funk and Wagnalls Company: New York and London, 1926, 965 pp.
- Thompson, J.A., *Handbook of Life in Bible Times*, Inter-Varsity Press, Leicester, England, 1996, 384 pp.
- *Harper's Encyclopedia of Bible Life*, Castle Books, 1996, 423 pp.
- Bowker, John, *The Complete Bible Handbook*, Barnes and Noble, New York, 2005, 544 pp.
- Rogerson, John, *Atlas of the Bible*, Phaidon, Oxford, 1985, 237 pp.

[77] Jewish law became tied to religious thought, and Greek law became tied to philosophical precepts. The Romans went a step beyond the Jews and the Greeks by totally separating their civil law from both religion and philosophy.

the history of the early kings Saul and David (fl. 980 BCE). The Books of ***Judges*** *and* ***Kings*** *are the product of the* 6^{th} *century, but were based upon early manuscripts.*

The Jews had two unique characteristics as ancient writers; They were the first to create consequential, substantial and interpretive history: They were fascinated by their past from very early times. They believed that they were a special people who had not simply evolved from an unrecorded past but had been brought into existence, for certain definite purposes, by a specific series of divine acts. They saw it as their collective business to determine, record, comment and reflect upon these acts.

No other people has ever shown so strong a compulsion to explore their origins. This passion for aetiology, the quest for explanations, broadened into a more general habit of seeing the present and future in terms of the past. The Jews wanted to know about themselves and their destiny. They wanted to know about God and his intention and wishes. Since God, in their theology, was the sole cause of all events, and thus the author of history, and since they were the chosen actors in his vest dramas – the record and study of historical events was the key to the understanding of both God and man.

The Jews developed the power to write terse and dramatic historical narrative 500 years before the Greeks, and because they constantly added to their historical records they developed a deep sense of historical perspective which the Greeks never attained. In the portrayal of character, too, the Biblical historians achieved a degree of perception and portraiture which even the best Greek and Roman historians could never reach. The Jews were not interested and did not believe in impersonal forces. They were less curious about the physics of creation *than any other literate race of antiquity. They turned their back on nature and discounted its manifestations except in so far as they reflected the divine-human drama. The notion of vast geographic or economic forces determining history was quite alien to them. There is much natural description in the Bible, some of astonishing beauty, but it is stage-scenery for the historical play.*

The second unique characteristic of ancient Jewish literature is the verbal presentation of human personality and its full range of complexity. The Jews were the first race to find words to express the deepest human emotions, especially the feeling produced by bodily or mental suffering, anxiety, spiritual despair and desolation, and the remedies for these evils produced by human ingenuity – hope, resolution, confidence in divine assistance the consciousness of innocence or righteousness, pertinence sorrow and humility [e.g. Psalms: *22, 23, 39, 51, 90, 91, 103, 104, 130, 137, 139]. Wisdom texts produced by the Jews are more observant of human nature and more ethically consistent than their precursors and models in the entire ancient Near East. Thus* Ecclesiastes, *is quite without equal in the ancient world, with which the Greeks alone could complete.*

This book is counted among the four great skeptical dramas of history along with Prometheus vinctus (Aeschylos); Hamlet (Shakespeare), and Faust (Goethe). Yet not even the Greeks produced a document so mysterious and harrowing as the book of Job, written in Israel in 5^{th} *century BCE. This great essay in theodicy and the problem of evil has fascinated and baffled both scholars and ordinary people for more than two millennia*[78]*, and of all the books of the Bible it has most influenced other writers.*

Job is a formidable work of Hebrew literature, written in a sustained level of powerful eloquence. Its burden is to show that suffering is not necessarily retributive and that the justice of God is impenetrable. It is crammed with natural history in poetic form, presenting a fascinating catalogue of organic, cosmic and meteorological phenomena. In chapter 28, for instance there is an extraordinary description of mining in the ancient world. Through this image, a view is presented of the almost unlimited scientific and technological potentials of the human race, and this is then contrasted with man's incorrigibly weak moral capacities[79]*.*

The Bible opens to us a window in time that extends from ca 2000 *BCE to ca* 300 *BCE*[80]*. It abounds with detailed descriptions as well as terse allusions to eclipses, comet apparitions, bolide impacts, paroxysmal volcano eruptions, major earthquakes, tsunamis, floods, and a host of meteorological*[81]*, botanical,*

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78 **Thomas Carlyle** called it "one of the grandest things ever written with pen".

79 The author of Job delivers the following message: There are two orders in creation – the *physical* and the *moral* order. To understand and master the physical order of the world is not enough: man must come to accept and abide by the moral order, and to do so must acquire the secret of Wisdom, and this knowledge is something of an altogether different kind from, say, mining technology. Wisdom came to man, as Job dimly perceived, not by trying to penetrate God's reasoning and motives in inflicting pain, but only through obedience, the true foundation of moral order: "And he said to man: "Behold, the fear of the Lord, that is wisdom: and to depart from evil is understanding" [*Job* **28**, 28].

80 The Old Testament was canonized *gradually*, book by book, over a period of ca 1000 years. However, the final stages of canonization of the *Pentateuch* and most of the books of the *Prophets* began at the time of the scribe **Ezra** (fl. 458 BCE) and ended ca 200 BCE. The time of canonization of the *Writings* is uncertain, but believed to end by the first century BCE. A standard text of the Hebrew Bible appeared by about 150 CE.

81 Earth's *weather patterns* and the *hydrological cycle* were known to the Hebrews [*Job* **26**, 8; **28**, 23–26; *Eccl* **1**, 6; *Psalms* **146**, 16–17]. *Job* [**38**, 19–20] wonders

Table 1.1: Prophets, philosophers, poets and scientists in the Golden Age of Eurasian cultures (800–200 BCE)

BCE, ca	Hebrews	Greeks	Eastern
800–750	Isaiah; Amos	Homer	—
620	Jeremiah	—	Zoroaster
597–570	Ezekiel	Thales	Lao Tsu
535–520	Zachariah	Pythagoras	Buddha
500	Malachi	Heraclitos	Confucius
440–400	Ezra; Nehemiah	Socrates; Democritos	—
390	—	Plato	—
340	Jonah; Joel	Aristotle	—
300	Ecclesiastes	Euclid	—
230	—	Archimedes	—

zoological, medical[82]*, geological, geographical, astronomical and other data*

on the nature of light and states that it is an element in a state of *motion*(!). The *earth* is floating in space, surrounded by a thin form-fitted layer – our atmosphere [*Job*, **26**, 7].

82 Ancient peoples regarded *disease* as something aside from the regular working of nature. To them disease was the expression of disfavor or hostility on the part of the god, and in the nature of *punishment*. As a consequence, the treatment of disease in the early days lay in the hand of the *priests* (*Lev* **13**), whose concern was to appease the angry Jehovah; diagnosis could be disregarded and medication naturally played a minor part. Diseases might be averted by repentance, e.g. Jeroboam's paralysis cured [*I Kings* **13**, 4–6], Hezekiah's illness cured [*II Kings* **20**, 5], Miriam's leprosy healed [*Nu* **12**, 14], plague checked by Aaron's use of incense [*Nu* **16**, 47].

Anatomy known to Biblical writers was a thing of shreds and patches – only very few parts of the body are mentioned in the Bible, and these references are vague and general. Something, however, was learned of comparative anatomy by the examination of animals slaughtered in sacrifice [*Deut* **18**, 3; *Lev* **3**, 10; **8**, 17; *Ezek* **21**, 21]. More accurate knowledge of the anatomy of the human body to be obtained by dissection was totally forbidden [*Nu* **18** 16].

Physicians were recognized as a distinct class [*II Chron* **16**, 12; *Job* **13**, 4], but their intervention met with the disapproval of the priests. There are several allusions to medical matters in *Proverbs* [**17**, 22; **20**, 30].

Preventive medicine: The Mosaic laws enforcing public and personal cleanliness were of great hygienic value [e.g. circumcision, ban upon sexual perversions (*Lev* **15**), sexual inversion (*Lev*, **18**), sanitation of camp life (*Deut* **23**, 13) etc.] These rules invested the figure of a good and virtuous woman with that peculiar halo of respect which has been preserved by all highly civilized nations down to the present time. The institution of the *Sabbath day of rest* was perhaps the most distinctive and beneficial of all the Mosaic provisions for the physical and moral well-being of the Hebrew people.

Medicines were largely used [*Prov* **17**, 22; *Jer* **30**, 13; **46**, 11; *Ezek* **47**, 12]. Moses was learned in all the wisdom of the Egyptians, and among the other branches of this lore, medicine was largely cultivated. Assyria and Babylon were learned in the medical science of that day. Thus, King Asa had medical treatment for his feet [*II Chron* **16**, 12], King Joram went back to be healed of his wounds [*II Kings* **8**, 29]. Of details of treatment and *materia medica* we have:

- *Balm* used for the treatment of wounds [*Jer* **8**, 22].
- *Caperberry* and *Mandrake* [*Ec* **12**, 5; *Gen* **30**, 14] used as aphrodisiac.
- *Ointments*: *Oil* was used in dressing wounds, and in anointing the sick [*Isaiah* **1**, 6]. *Wine* was used as a stimulant for gastric disturbances [*Pro* **31**, 6]. A *poultice of figs* [*II Kings* **20**, 7].
- *Antidotes*: An indication to the knowledge of the use of antidotes for poison

concerning the natural history of the ancient Near East.

No archaeological discovery has ever been made that contradicts or controverts historical statements in the scriptures, and most of the narratives relating to geophysical and astronomical events, are found today to bear a

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is given in *II Kings* **4**, 38–41. Here the bitter fruit of the poisonous *colocynth* has been mixed in the pot with the stew. **Elisha** neutralized its effects by *meal* thrown into the pot. In modern times chemists have been able to show the neutralizing effect of the protein of the meal, when cooked, upon the poisonous alkaloid of colocynth.

There is no mention in the Bible of any anesthetics or analgesic drug treatment.

Diseases and *epidemics* are repeatedly mentioned. The great epidemic of the East are cholera, bubonic plague, small pox, typhoid, typhus, measles, and venereal diseases [*Ex* **9**, 9; *Deut* **28**, 27; *Nu* **11**, 33; **14**, 37; **16**, 47; **21**, 27; **25**, 1–9]. The plague that followed the capture of the Ark by the Philistines [*I Sam* **5**, 6–**6**, 21] killed 50,070 of the Hebrews; it would seem that contagion from the Ark was still possible and it was not without reason that the Ark was quarantined for 20 years.

Of the Assyrian army against Ethiopia 185,000 died in one night [*II Kings* **19**, 35]. Such sudden mortality suggests either cholera or plague. A pestilence of 70,000 mortality is recorded in *I Chron* **21**, 14 and *Ps* **91**, 5–10. *Leprosy* is abundantly mentioned [*Lev* **13**, 1–17; *Ex* **4**, 7; *Num* **12**, 10; *II Kings* **5**, 27; **7**, 3; **15**, 5].

Individual cases of disease cited: *Infantile paralysis* [*II Sam* **4**, 4]; *Dysentery* [*II Chron* **21**, 18]; *Homicidal melancholia* [*I Sam* **16**, 14; **18**, 10–11; **19**, 9–24; **20**, 33; **22**, 17]; *Inscaity* [*Dan* **4**, 33]; *Dwarfism* [*Lev* **21**, 29]; *Gigantism* [*Deut* **4**, 11; *I Sam* **17**, 1; *II Sam* **21**, 16–20]; *Alcoholism* [*Gen* **9**, 20; **19**, 33; *II Sam* **11**, 13; *I Kings* **20**, 16 etc.]. *Heart disease* [*Deut* **28**, 65]; *Sunstroke* or *meningitis* or *pernicious malaria* [*II Kings* **4**, 18; *Ps* **121**, 6]; *Pneumonia* [*II Sam* **12**, 15–18].

Circumcision is the only surgical procedure mentioned in the Bible. There is a reference to the *roller bandage* in the treatment of fractures [*Ezek* **30**, 22].

Diseases of the eye were common and numerous in the East. This is largely due to the presence of *trachoma* and of gonorrheal ophthalmia [*Gen* **27**, 1; *I Sam* **4**; *I Kings* **14**, 4]. Thus, Leah was tender-eyed, probably the result of *trachoma* [*Gen* **29**, 17].

Woman's diseases are mentioned [*Lev* **15**, 19–24; **15**, 25–30]. Sterility was regarded as a great calamity [*Gen* **18**, 11; **20**, 17; **25**, 21; *I Sam* **1**, 5; *II Sam* **6**, 23; *II Kings* **4**, 4].

Obstetrics: The suffering of child-birth is the penalty for the sin of **Eve** [*Gen* **3**, 16]. Midwives in the Near East from the earliest times have conducted deliveries upon the obstetric chair [*Ex* **1**, 16]. The obstetric chair is mentioned by ancient Greek authors. Rachel offers the use of her knees in lieu of an obstetric chair, as a symbol that the child borne by her maid is her own [*Gen* **30**, 3].

rather exact account of real natural phenomena. Several examples are:

- *Volcanism [Ex.* **19**, *16;* **20**, *18; Deut* **4**, *11; Jud* **5**, *5; Micha* **1**, *3–4; Psa* **97**, *3–5]. It is the oldest known description of a volcanic eruption based on a very careful observation. In addition to the pillars of cloud and fire, the account contains no fewer than seven features, each characteristic of a volcanic explosive eruption: noise, flames, smoke, quaking, summit cloud, electrical discharges and darkness.*
- *Eclipses. The events:* **Sept. 30, 1131 BCE** *[Jud* **5**, *20],* **Aug. 15, 831 BCE** *[Amos* **8**, *9],* **June 15, 763 BCE** *[Amos* **5**, *8; Isa* **24**, *19–23],* **May 09, 594 BCE** *[Ezek* **8**, *14–16],* **Sept. 21, 582 BCE** *[Ezek* **30**, *18;* **32**, *7–8],* **Mar. 28, 517 BCE***[Zach* **14**, *6],* **Mar. 01, 357 BCE** *[Joel* **3**, *3–4],* **July 04, 336 BCE** *[Joel* **3**, *3–4], are among he earliest documented descriptions.*
- *Comet apparitions.* **1230 BCE** *[Exod* **13**, *21],* **May 11, 1198 BCE** *[Josh* **5**, *13],* **Dec. 02, 986 BCE** *[I Chron* **21**, *16],* **July 28, 616 BCE** *[Ezek* **1**, *4; Hab* **3**, *4–12].*
- *Earthquakes.* **2100 BCE**, *the destruction of Sodom and Gomorrah [Gen* **19**], **ca 1210 BCE**, *the fall of the walls of Jericho [Psa* **114**, *3–8],* **759 BCE** *[Amos* **1**, *1; Zach* **14**, *4–5; II Chron* **26**, *16–23].*
- *Severe climatic changes* causing *floods, drought* (famine), *epidemics* and *mass-migration [Gen* **6–9**; *I Kings* **17**–**18**; *Gen* **41**, *29–57; Jer*[83] **18**, *14; Proverbs* **25**, *13].*

The creation of the universe ex nihilo *(from nothing) is described in the first chapter of Genesis. Genesis tells how God created the universe by organizing a preexisting chaos, "the waters": Genesis* is *silent about where the initial chaos came from since the priests who wrote the account had no interest in the question. Their concern was how God created* order *in the universe.*

Next to cosmogony, treated in the first chapter of Genesis, (Table 1.2) *the following two chapters relate to anthropology – the origin of human civiliza-*

[83] A careful reading of the Bible reveal around 600 BCE more frequent snow than now and that it lay longer into the summer on the heights of the Lebanon, possibly throughout the year.

tion. The story of man's expulsion from paradise alludes to the transition from food-gathering to food-producing society[84] *(ca 10,000–8000 BCE).*

The outburst of intellectual energy that characterizes the extraordinary phenomenon of Hebrew prophecy during 750–350 *BCE, had its counterpart in an akin occurrence in the East: the foundation of new philosophies of religions by* ***Confucius***, ***Buddha***, ***Lao Tsu*** *and* ***Zoroaster***, *all of whom flourished during the* 6^{th} *century BCE. These spiritual leaders had the capacity to persuade their fellow-men to adopt new conception of the universe and of themselves; they thus radically transformed the intellectual atmosphere of their time.*

[84] When the glaciers still covered parts of Northern Europe, the whole area extending from the Atlantic Ocean to Iran was virtually a gigantic garden, providing ample food for animals and man ('Paradise'). As climate became drier, abundance of edible plants diminished and man was forced to produce a great part of his food: domesticate animals and use the plough to cultivate the land in the valleys of the great rivers.

Table 1.2: Dating of key events in the Book of *Genesis*, consistent with discoveries of modern science up to 2000 CE

Event	Thousands of years ago	Reference
Origin of the universe	ca 14,000,000	"In the beginning..." (**1**, 1)
Undecoupled matter and radiation; light was continually emitted and screened		"...darkness was on the face of the deep" (**1**, 2)
Light – transparent universe; Decoupling of matter from radiation; recombination of plasma to neutral hydrogen atoms	ca 13,999,700	"Let there be light."(**1**, 3) "...divided light from darkness."(**1**, 4)
Formation of primitive stars	ca 13,100,000	"Let there be light in the firmament of the heaven" (**1**, 14)
Primitive plant-life on earth	ca 1,600,000	"Let the earth bring fourth grass." (**1**, 11)
Primitive life at sea	ca 1,000,000	"Let the waters bring forth abundantly the moving creature that hath life." (**1**, 20)
Evolutionary explosion of life an earth	ca 540,000	"Let the earth bring forth the living creature..." (**1**, 24)
Appearance of hominides	ca 4,500	"Let us make man..." (**1**, 26)
Branching off of Homo Sapiens from the Neanderthals; The biblical *Eve*	ca 200	"And Adam knew Eve his wife; and she conceived..." (**4**, 1)
Man's expulsion from "Paradise" – past glacial warming; advent of agriculture	ca 11	"...sent him forth from the garden of Eden, to till the ground..." (**3**, 23)
The Mesopotamia Flood	ca 7.6	"...the water of the flood were upon the earth" (**7**, 10)
Advent of civilization	ca 6.2	"...scattered them abroad upon the face of all the earth" (**11**, 8)

648 BCE, Apr. 06 **Archilochos of Pharos**. A Greek poet. Described a solar eclipse on that date, which he may have watched in Pharos on Thasos (Cycledes Islands).

644 BCE Windmills in Neh, Persia.

630 BCE **Zoroaster** (Zarathustra, ca 660–583 BCE, Persia). Philosopher and prophet. Preached ethical monotheism, insisting on moral values, especially truth, justice and agricultural labor. Founded the Zoroastrian religion. Cyrus the Great and Darius the Great, spread his religion throughout their empire. After Alexander the Great conquered Persia, Zoroastrianism began to die out.

Zoroaster was born in the vicinity of Lake Urmiyah in Azerbaijan, in northern Persia. At the age of 30 he came out of the wilderness to preach his new religion.

626–574 BCE **Jeremiah** (Yirmiahu, 644–574 BCE). The last pre-exilic Hebrew prophet. Historian and biblical author and editor. Lived at the time of the decline and fall of the Hebrew monarchy[85]. He came of a priestly family descendant from Moses (the *Shiloh* line).

After the discovery of the Deutronomic Code in the days of King Josiah (622 BCE), Jeremiah (perhaps with the assistance of his scribe Baruch ben Neriyah) set forth to compile the final version of Deutronomy on the basis of ancient texts. It is believed[86] that he compiled the books of Joshua, Judges, Samuel and Kings, and the Book of Lamentations. Jeremiah died in Egypt.

ca 600 BCE *Phoenicians* sent by the Egyptian King Necho II (d. 593 BCE) sailed around Africa. These voyages took about two years each, going

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[85] He witnessed six major political events in the Near East; which deeply impacted his ideology:

- 625 BCE; Scythians overrun Western Asia; Judah escaped serious damage;
- 612 BCE; *Nineveh* fell to the Babylonians; end of Assyrian Empire;
- 609 BCE; King Josiah killed at Megiddo while attempting to oppose the northward march of Egypt;
- 605 BCE; Egypt defeated by Babylon in the battle of Carcemish; Judah comes under Babylonian rule;
- 597 BCE; Babylonian captured Jerusalem and deported King Yehoiachin to Babylon;
- 587 BCE; Babylonian destroyed Solomon's Temple, sacked Jerusalem and deported many Judeans to Babylon; end of Hebrew monarchy in Israel.

[86] Freedman, R.E., *Who Wrote the Bible*, 1987.

through the Red Sea, around Africa, and back to Mediterranean by way of Strait of Gibraltar.

600–500 BCE Cooler weather, increase of rainfall and flooding in Europe and Middle-East. Confirmed by Herodotos' description of the climate of the Northern Black Sea. Evidence in the Old Testament.

ca 600 BCE Completion of the *Etemenanki Ziggurate Tower*[87] in the ancient city of Babylon[88] in honor of Marduk, the Babylonian Jupiter. After several destructions and rebuilding, it reached its final form under Nebuchadnezzar II. Then it towered skyward for nearly 100 meters with seven stages, pinnacling with a brilliant blue-glazed temple for Marduk. It was covered with Enameled bricks in colorful patterns, as if it was clothed in the scaly skin of some monstrous reptile. Some anthropologists believe that the tower was meant as a landing platform to facilitate the gods' descent to earth from the heaven to which they have vanished. Thus the Tower of Babel was built for communication with the gods.

ca 600 BCE According to Herodotos, Phoenician seamen circumnavigated Africa from east to west. Their enterprise, however, bore no results; the opening-up of the oceanic route from Europe to India, the East Indies and China, awaited the discoveries of Portuguese navigators about 2100 years later.

600–300 BCE (**Thales to Euclid**) Development of postulational thinking and the foundation of systematic logical structures in mathematics.

Mathematics, as we understand it today encompasses any activity that arises out of, or directly generates, concepts relating to numbers or spatial configurations together with some logic.

In trying to understand things of the physical universe, the ancients repeatedly encountered certain basic *patterns*; these were patterns of *form* (such as the shapes and paths of the astronomical bodies), patterns of *arrangements* (such as the symmetrical arrangements of the limbs of living creatures), or patterns of *relation*. To this last group belong the orderings in man's minds of sound, which we call *music*, the ordering in man's minds of forms and color,

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[87] Some Biblical scholars connect it to the *Tower of Babel* (*Genesis* **11**) and the *Confusion of Tongues* which may have echoed the labor troubles during the building of Etemenanki. We know that when Alexander the Great set out to rebuild the tower, he put 20,000 men to work. It stands to reason that the original tower must have required many more laborers than that.

[88] 80 km south of today's Baghdad.

which we call *art*, the ordering in man's minds of words, which we call *poetry*, or the ordering in man's minds of thought, which we call *philosophy*. To the study of these and all other such pattern the Greek called *Ta mathémata*, which means: '*what is learnable*' (*manthanein means to learn*) *and also from the word mathésis, which means* '*the teaching*'. *Thus* '*mathematics*' *essentially stands for those things that can be learned and at the same time also taught. The idea that the Greeks wished to convey by the choice of this word is that only he who can truly learn can truly teach, i.e. the genuine teacher differs from the pupil only in that he can learn better and that he more genuinely wants to learn. In the last analysis, in all teaching, the teacher learns the most.*

6^{th} century BCE an important epoch in the history of humanity. It was an age when **Buddha** was searching for a path to enlightenment in *India*, **Confucius** was teaching new rules for society in *China*, *Ionian* philosophers were initiating a tradition of scientific thinking in *Greece*, and when the exiled *Hebrews* in Babylon were collecting the messages of their prophets in a holy scripture. On the other flank of Mesopotamia another religious movement was initiated by an *Iranian* prophet, **Zoroaster**, who preached his message of cosmic strife between the God of Light and the principle of evil.

ca 585 BCE **Thales of Miletos** (624–548 BCE). The first known Greek philosopher, scientist and mathematician. Was the first to attempt an explanation of the world in terms of its observable nature rather than by mythology. This meant that his conclusions could be subjected to rational arguments about whether they were right or wrong.

None of his writing survived, so it is difficult to determine his views and to be certain about his mathematical discoveries. With him began the study of geometry and scientific geography and astronomy in Greece. The invention of geometry as an abstract mathematical theory supported by rigorous deductive proofs was one of the turning points of scientific thinking. It led to the creation of mathematical models for physical phenomena.

Thales taught the *sphericity of the earth*, the *obliquity of the ecliptic* and the *causes of eclipses*. His visit to Egypt and his exposure there to the heritage of Babylonian astronomy may have aroused his interest in these subjects.

Thales was an Ionian, but possibly had some Phoenician blood in his veins. During middle life Thales engaged in commercial pursuits, which took him to Egypt. He resided there and studied the physical sciences with the Egyptian priests, and upon his return founded the Ionian School of Astronomy and Philosophy. Thales is credited with the prediction of the *year* of the solar eclipse of May 28, 585 BCE. Estimated the height of an Egyptian pyramid by measuring its shadow when his own shadow was equal to his height.

Thales noticed that matter comes in three forms – liquid, solid and gas. He investigated the phenomenon that amber, when rubbed, gained the property of attracting light objects. He also knew about the power of *lodestone* to attract iron[89].

His great contribution to mathematics is in fitting the Babylonian and Egyptian empirical geometrical rules in a logically connected abstract system through which one advances by means of demonstration from theorem to theorem. Specifically, he is credited with five theorems of elementary geometry:

(1) A circle is divided into two equal parts by its diameter;

(2) The base angles of an isosceles triangle are equal.

(3) The angles between two intersecting straight lines are equal.

(4) Two triangles are congruent if they have two angles and one side equal[90].

(5) An inscribed angle in a semicircle is a right angle.

Some idea of the advancement in astronomy made by the Greeks can be gleaned from the fact that Thales taught the *sphericity of the earth*, the obliquity of the ecliptic, and the causes of eclipses. According to **Diogenes Laertius**[91] (fl. 222–235 CE) he was first to determine the length of the year. It is fair to assume that he borrowed much of his information from Egypt, though the basis for predicting eclipses rests on a period of 6585 days, known as the *saros*, discovered by the Chaldeans. Thales thought that all things are derived from the single element *water* and that the solid earth was afloat on the world-encircling flat ocean.

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89 This appears to have been familiar to the Greeks as early as 800 BCE, and is mentioned by Homer. However, the property of orientation, in virtue of which a freely suspended magnet points approximately to the geographical north and south, is not referred to by any. European writers before the 12^{th} century, thought it is said to have been known to the Chinese at a much earlier period. The oxide Fe_3O_4 occurred plentifully in the district of *Magnesia* near the Aegean Coast; hence the name *magnes* or the *Magnesian stone* given to if by Greeks.

90 He applied this theorem to determine the distance between two ships at sea through measurements made on board one of them: In general, to find the distance from A to the inaccessible point B, one erects in the plane a normal AC to AB, of arbitrary length, and determines the midpoint D. In C one constructs a line CE perpendicular to CA, in a direction *opposite* to that of AB, and one extends it to a point E, collinear with D and B. Then CE has the same length as AB.

91 Biographer of the Greek philosophers, from the town of Laerte in Cilicia (or the Roman family of Laertii). Flourished during the reign of Alexander Severus and his successors. He was an Epicurean.

His conclusion that water is the original substance may seem fantastic on the surface, but it becomes far more plausible if one examines it more closely. Water is the only substance that is known to man without difficulty in the three states, solid, liquid and gaseous; it is not difficult to connect clouds, fogs, dew, rain, hail with the ice and snow found in the mountains and the water of the sea and rivers. Water seems to occur everywhere in one state or another; would it be overbold to imagine that it may occur also in hidden forms? Moreover, without water no life is possible, but as soon as water appears, there may be life.

While the Jews were postulating the moral unity of the cosmos, Ionian philosophers, of whom Thales was the first, were postulating its material unity.

While giving the honor of being effectively the first scientist to Thales, one recognizes that Thales was heir to an intellectual tradition whose origins are obscure. Strangely enough, his fame rests mainly on an achievement that we are now obliged to discredit, though its genuineness was accepted as a cast-iron belief until our own day.

Nevertheless, Thales is the father of a new breed of thinkers who have taken upon themselves the task of trying to understand the nature of the physical universe. This endeavor and the various activities arising from it are collectively labeled as *science*.

After Thales, philosophy rapidly began to flourish. More philosophers appeared, with a succession of different explanations of the world. The philosophers who belong to this period (ca 550–450 BCE) are generally known as *pre-socratists*.

ca 580–320 BCE At Laureion (modern Lávrion) in Southern Attica, galena rich in *silver* was mined by the Athenian. The lumps of rock from the mine were picked over by hand and worthless pieces discarded. The ore was then crushed in mortars to facilitate further parting of galena from gangue, and it was washed on sloping trays to effect some separation of placer action. Only after these preliminary concentration steps was the galena reduced to metal by heating it with *charcoal*.

The lead so produced contained 850 g silver per ton and was parted by *cupellation* – air blown over the molten metal converted the lead to litharge (PbO), which was absorbed by a bone ash hearth, leaving a residue of pure silver.

In the 5^{th} century BCE, the annual yield of silver from Laureion was about 2.5 tons. Following the battle of Marathon, Themistocles persuaded the Athenians to use this revenue to build ships for defense against Xerxes. The mines later provided the silver for the Golden Age of Pericles. Against these splendid achievements the balance sheet of history must set the abominable mistreatment of slaves who worked the mine and smelter.

Origins of Philosophy and Metaphysics

The word 'philosophy'[92] *comes from the Greek, philos = love, sophia = science or wisdom. One may loosely define philosophy as a rational method of examining a body of knowledge that provides us with a reasoned framework within which to think. "An unusually stubborn attempt to think clearly" (William James). Plato defined philosophers as those who "are able to grasp the ethernal and immutable, setting their affections on that which in each case really exist."*

The fundamental problem of philosophy is that of the relation between existence and thought. All philosophical tendencies divide up into materialists and idealists.

Philosophy started as a criticism of religious beliefs, by seeking reasons for natural phenomena. Two opposed camps sprang up which still persist to his day: religion on one hand and science on the other. Eventually religion had to invent some kind of 'science' for itself (theology) to justify its existence. Philosophical ideas are of two kinds: Materialism considers that there is noting beyond natural things and seeks to explain them on the basis of science (including even religion). Early form of materialism sought to explain all natural phenomena by mechanical laws.

IDEALISM *starts by assuming the existence of supernatural and divine forces. Idealists imagine things and presuppose the existence of 'spirits'.*

RELIGION *is a combination of beliefs and cult-practices which subordinates human life to a divine super-order. Alternatively it can be defined as a body of knowledge (dogma or revelation) imposed from without (e.g. Bible,*

[92] The sages of Greece used to be called *sophoi* ("wise men"), but Pythagoras thought the word too arrogant and adopted the compound *philosophoi* ("lovers of wisdom"), whence "philosopher", one who courts or loves wisdom.

Koran, Gita etc.). The believer is expected to accept this revelation without question.[93]

The pseudo-science which seeks to give foundation to religion by borrowing from philosophical argumentation is known as Theology.

Syncretism – *The interworking of two or more cultural perspectives into one system.*

Eclectism – *Choosing according to taste, without internal framework of a genuine understanding of function.*

Rationalism *is a philosophical theory which tends to recognize reason as the* unique *source of true knowledge; contrary to* empiricism *which makes perception this source of knowledge.*

Other related concepts are:

Metaphysics *A branch of philosophy that seeks to understand reality beyond what we know from our sense perception, i.e. to seek answers to the question of* why *there are patterns in nature any* why *they are describable in mathematical schemes. It lies outside the scope of physics.*

Fundamental questions of metaphysics:

- *What is real?*
- *How is knowledge possible?*
- *How are the mind and body related?*
- *How can we find truth?*
- *What is the nature of the universe?*
- *How can we reconcile personal freedom and scientific determinism?*

93 There are *exoteric* religions (Judaism, Christianity, Islam, Baha'ism, etc.), *esoteric* religions (Sufism, Lurianic Kabbalah, Gnosticism, etc.) and even *secular* religions (Marxism). *Western* religions are more concerned with the "Manifest" (creation, nature, ultimate purpose of the cosmos and of men). *Eastern* religions are more concerned with the "Unmanifest" (transcendental) and how to attain it.

Teleology*: The study of the evidence of design or* purpose *in nature based on the* belief *that such characteristics exist and are indeed apparent.*

Epistemology*: The theory of knowledge. A philosophical discipline that investigates origin, nature, methods, and limits of human knowledge.*

The two great founders of modern physics, Albert Einstein and Max Planck produced by their contributions to epistemology a revolution in the theories of knowledge that is almost as profound in its impact on philosophy.

System*: a set of interrelated structures that may consist of an hierarchic arrangement of subsets or subsystems.*

Essentialism*: a "realistic" philosophical position that dates back to the idealism of Plato and Aristotle, was then placed into religious scholastic argument by Thomas Aquinas, and finally put into modern methodological form by Karl Popper (1957). Essentialism is concerned with understanding the potential, or universal, meaning of the descriptors of a class or set; it takes an opposite view of that of* nominalism*, which believes that such descriptors are only labels of characteristics. The concept of essentialism, as first proposed, argued that those properties which are common to the many individuals of a set, a class, a group, have a reality exceeding that of their mere occurrence as parts of the individual members of such sets, classes, and so on.*

This position, then, allows one to search within natural groupings for common plans, designs, and patterns, with the expectancy that such patterns will be understandable by analogy. In recent years, essentialism has been considered somewhat sterile, at least in the natural sciences, because it focused on universals of meaningless generality. Plato, on the other hand, used this position to search within nature for order, regularity, and perfection.

Historicism: the approach to science – particularly to science that is historical, as is geology in large part – which assumes that historical prediction (or retrodiction) is a principal aim, attainable by discovering the patterns, laws, and trends that underlie the evolution of history.

570 BCE **Lao Tsu** (604–510 BCE, China). Philosopher. The reputed founder of the religion called *Taoism*. In his *Book of Tao* he preaches non-assertion, the being without desire, and submission to the universal Tao (the way). The Tao represents the characteristic or behavior that makes each thing in the universe what it is. The word is also used to mean reality as a whole, which consists of all the individual "ways". The ideas of Tao were partly a reaction against *Confucianism*. According to Confucianism, people can live a good life only in a well-disciplined society that stresses attention to ceremony, duty, and public service. The Tao ideal, on the other hand, is a person who avoids conventional social obligations and leads a simple, spontaneous, and meditative life close to nature. The Taoist search for knowledge of nature has led many believers to pursue various sciences, such as astronomy and medicine.

Lao Tsu was born in Honan and flourished at Lo-Yang, capital of the Chou dynasty. He died at an unknown place.

ca 563–483 BCE **Buddha (Siddhartha Gautama)**. Founder of Buddhism. Born in southern Nepal to a rich and powerful royal family. At age 29 he became overwhelmed with the conviction that life was filled with suffering and unhappiness and he abandoned his wife and infant son to seek religious enlightenment as a wandering monk.

After traveling throughout northeastern India for about six years, Gautama experienced enlightenment. He believed he had discovered a way for man to escape his unhappy existence. His disciples called him Buddha, which means *Enlightened One*.

To break the eternal cycle of death and rebirth, Buddha preached to follow a code of ethics composed of the Middle Way and the '*Eightfold Way*', the latter consisting of 8 'commandments'.

ca 560 BCE **Anaximander of Miletos** (611–547 BCE). Ionian astronomer[94], geographer and philosopher. One of the first to give a naturalistic rather than a mythological explanation of natural phenomena. He anticipated the theory of evolution by stating that animals came originally from a moist environment, and man evolved from aquatic animals. Anaximander taught (and may have discovered) *the obliquity of the ecliptic*, and is believed to have produced the first map of the known world (although he claimed the earth to be *cylindrical in shape*).

Anaximander introduced into Greece the *gnomon* (for determining the soltices) and the *sundial*.

He was a citizen of Miletos and a companion or pupil of Thales, both of them being pioneers of the exact sciences among the Greeks. His pupil, **Anaximenes of Miletos** (585–525 BCE) was first to suggest that the moon

[94] From the Greek $\alpha\sigma\tau\rho o\nu$ = star; $\nu\epsilon\mu\epsilon\iota\nu$ = to classify or arrange.

receives its light from the sun, and correctly understood (ca 550 BCE) *eclipses* of the moon as being caused by the shadow of the earth, cast by the sun. From this he inferred that the earth is roughly *circular* in section (a disc). He went further than his teacher in completely ignoring the mythical elements in the natural laws. He taught that, with age, the earth broke down by its own weight, thus causing the motion of earthquakes.

ca 551–479 BCE **Confucius (Kung Chiu)**. The most influential philosopher in Chinese history. The name Confucius is a Latin form of his title Kung-fu-tzu, which means Great Master Kung. At his death he was largely unknown throughout China. His disciples spread his teachings. In a work referred to as the *Annals of Lu*, his native state, Confucius has sketched a history of 242 years from 722 to 481 BCE. Therein he recorded 36 eclipses of the sun. His description of the 720 BCE eclipse is one of the earliest astronomical publications on record.

Ancient Eastern Philosophy (ca 1500 BCE–500 CE)

Philosophy is the search for knowledge and for the means to express it, and Eastern philosophy has many forms of both.

Eastern philosophy is a vast collection of philosophical and religious ideas that derive from the ancient cultures of India, China, Persia, Japan, Korea, Tibet, etc, and from many different traditions and forms of thought that shaped the development of the East from earliest recorded times; it is a multifaceted set of ideas that reflect the complex societies they grew out of.

Unlike Western philosophy, the Eastern tradition does not attempt to distinguish clearly between philosophy and religion. While Western philosophy has always been more concerned with truth, logic, reason and independence, the search for knowledge in Eastern philosophy has always been more holistic, and less scientific; where Western science has sought absolute truth in rationality, Eastern philosophy has sought-complete enlightenment via reflection.

Western philosophy has always tended to over-emphasize the individual and individual things. In contradistinction, Eastern thought resolutely believes in the interconnectedness of all things and of the need to escape from the limits of individualism, which is connected to materialism and bodily pleasures. In contrast to the Western idealization of power, money and science, Eastern traditions have honored the thinker, the sage, the poet and the mystic. They declare the reality of the unseen world and venerate the call of spiritual life.

In *India*, history, myth, religion and philosophy interacted within a very long tradition, dating back to ca 3000 BCE. There are two main schools, *Hinduism* and *Buddhism*. Hinduism derived from ancient Vedic religions brought to India by people who called themselves *Aryans*. They arrived in India in waves between 2000 and 1000 BCE.

Since there was no written language in India before the 11^{th} century BCE, the Vedas were transmitted by word of mouth in the form of cryptic poetry. The *Rig-Veda*, the oldest of them all, was created during 1500–900 BCE. The invading Aryans created the cast system.

The main thing about Hinduism and Hindu philosophy is that they do not claim one master, one truth and one revealed wisdom, but many. This diversity is due to the very mixed ethnic groups and invasions of India throughout its history, and to its varied climate, geography and customs.

India has been at the crossroads of trade, culture and religion for thousands of years, continually absorbing, adapting and redefining its own beliefs. As in other regions of the world, the great spiritual upheaval in India during the Golden Age 800–200 BCE (see Table 1.1), derived from the fusion of national cultures with foreign influences.

Developing over many centuries through hymns and texts, the Vedas (meaning "knowledge") became a living, unified and complex religion and philosophy. The fullest philosophical expression of the most dominant strand within Hinduism are the *Upanishads* (800–300 BCE). This philosophy was not a philosophy of the mind but a philosophy of life in which the individual had to live the problem of trying to find deliverance. It is concerned with seeking self-enlightenment through the search for identity and salvation through a higher ultimate reality which lies behind wordly appearances.

Buddhism is opposed to the philosophy of the Vedas. It is less strictly philosophical, being more concerned with practical liberation from suffering. Buddhism avoided the ritual and elitism of the Vedic approaches, teaching in the common languages of the day and supporting an egalitarian approach to Enlightenment. It is not interested in questions about the ultimate nature of reality, but simply in the down-to-earth business of achieving *nirvana*, or the escape from suffering.

Unlike Hinduism, Buddhism asserts that there is no unique individual self[95]*: the 'self' is an illusion created by the combination of mental and physical activities:* meditation *is a way of approaching the mind as pure mind to get past the illusion of self.* Consciousness *is the combination of the many forms and states of being which are like a stream of impressions, ideas and sensations.* Mind *links these states that are thought of as the* 'person'*, but only mind exist through* rebirth*.*

Buddhism did not fare well in India itself, but became a very important force in many parts of Asia and the East, particularly China[96] *and Japan. However, Chinese culture, although Eastern, is markedly different from Hindu. What fundamentally distinguishes Chinese culture is its sense of harmony, interconnectedness, language and continuity. China has a cult of the Old in which* tradition *is everything. Also, the Chinese* language *is much more allusive and dependent on context than Romance languages and is not easily amenable to logical philosophy.*

Chinese culture is one of the most ancient and self-contained we know of, and its longevity is one of the key features that mark it out as unique. Its philosophy, rather like Hindu philosophies, shows a complexity and unity which incorporates many diverse veins and philosophical strategies that stretch over thousands of years. In philosophical terms, there is a period known as the time of the hundred Schools (400–200 BCE) *in which many different philosophical positions were advanced, but they almost all relied on tradition and past ideas for authority.*

Chinese culture does not depend on the idea of God to whom everyone is answerable, nor do its original myths talk about a supernatural creation. Chinese culture is secular and philosophical in the broadest sense.

From the earliest days of Chinese civilization, traditional ways of thinking were passed from generation to generation. Traditional ideas were treated with the kind of respect that is often accorded to religion. Also, the language of ancient China remained unchanged throughout the centuries, so that Confucius can be understood today in much the same way as 2500 years ago.

To sum up, the characteristics of Chinese civilization are these:

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95 Western *postmodern* philosophy often seems to claim that the self is fiction [e.g. **Jacques Lacan**, 1901–1981].

96 Buddhism was brought to China by various Hindu missionaries traveling along the silk route and the channels of trade. The influence of Buddhism began during the reign of *Emperor Ming* (58–75 CE).

- *The oldest continuous culture in the world (ca 5000 years old), and fundamentally a product of an agricultural society.*
- *Secular and multi-religious civilization.*
- *Its language is an important unifying factor, and expresses a particular way of thinking.*
- *Confucian philosophy, or ethics, underpins most of later Chinese civilization and culture.*
- *Religion and philosophy are closely interwoven and therefore difficult to separate out.*
- *Much Chinese philosophy extolled the virtues of wise rulers and a well-governed people.*
- *The family, and familial relations, are the basis of Chinese society, with women at the bottom of the pile.*

The three over-arching philosophies are: Confucianism, Taoism *and* Buddhism, *but they all interact with one other. Chinese philosophy is not about accumulating facts, but about elevating human nature. It was basically humanistic – it thought of man as being the center of the universe. It is not divorced from daily ordinary activities, yet it is very concerned with leaving this world behind in achieving nirvana (Buddhism).*

Confucians seem to believe in a predetermined fate. Next to the Confucian Analects, *the Tao Te Ching (the 'way of the Power', supposedly by* ***Lao Tsu****), is the most famous work of Chinese philosophy.*

Tao (the 'way') means the Universal Path. It is the force that governs the universe, the ultimate reality which cannot be described, but which is the origin of all things. The Taoists did not think much of Confucius trying to make the world a better place. Because of their position on the forces of nature and the ultimately unchanging reality *that lay behind all change, the Taoists generally did not believe in progress and science. Holding that the conquest of the world, comes invariably from doing nothing, they wanted to escape from the world, rather than challenging it. Thus Taoists aimed at a life of simplicity and harmony with Tao, not being ruled by intellect, but by a natural power, the Te. However, the Confucians held that the paradoxical sayings of the Taoists did not add up to anything other than the avoidance of complex realities.*

Although Buddhism ideas of no-self, withdrawal from the world and self-liberation seem to be the very opposite of Confucian social thinking, it managed to implant itself in China, in a culture that was practical, secular and not very fond of foreign things.

Perhaps the condition of the Chinese peasantry was seriously grim and something that promised definite release would seem attractive. Whatever the reasons, the introduction of Buddhism to China was a cultural revolution with a long-term impact.

***Bodhidharma** (ca 460–534 CE) brought Chán (Zen)*[97] *which later became important in Japan. Chán emphasized meditation and study. The introduction of Buddhism to Japan in the* 6^{th} *century CE, was the crucial step in the development of Japanese philosophy. This led eventually to the formation of the Japanese form of* Zen Buddhism*, which epitomizes the special character of the Japanese Way.*

Japanese Ways of thinking about the world are concerned with the clarity and precision of images, rather than with formal logic. Japanese philosophy can be seen in its art, its calligraphy, its ritual, in the order and interconnectedness of its culture. The key ideas of Zen are:

- *Genuine Enlightenment is* instantaneous*. Preparation may be necessary, but true realization is a total experience.*
- *Zen involves action through non-action, working towards a result in which no-self in exercised.*
- *Enlightenment and ordinary experience are related, but scriptures, texts and theory do not provide the path to nirvana. One does not have to retreat to the mountain to find Enlightenment.*
- *Zen enters into everything, calling for a* mastery between mind and body*, a sense of being through doing which transcends the act.*
- Meditation *leads to the intuitive experience which transcends ordinary reality. It can be described as finding one's true nature. One aims for the losing of 'body-mind' to achieve a non-conceptual awareness.*
- *The enlightened nature of the whole world is that '*nothingness*' that we must recognize as the essential emptiness of all things. Things do not have a meaning in themselves, but only in relation to other things.*

[97] *Zen*, the Sanskrit for *meditation*.

ca 550 BCE *The Temple of Artemis at Ephesos* was one of the largest and most complicated temples built in ancient times. Designed by the architect **Cherisphron** and his son, **Metagenes** in the Greek city of Ephesos (on the Aegean coast of Asia Minor), and dedicated to the Greek goddess Artemis. It was entirely marble, except for its tile-covered wooden roof, and its foundation measured 115×55 m^2. It had 106 columns, about 12 meters high, in a double row around an inner space. Cherisphron invented a new method to transport the huge columns. His son improved the method of transportation of columns and of putting them into place.

The temple was burned down in 356 BCE, and another one like it was built on the same foundation by **Deinocrates of Rhodes**, the Greek architect of the age of Alexander the Great. Goths burned down the second temple in 262 CE. Only the foundation and parts of the second temple remain.

The earliest clues, indicating to the existence of *Ephesos* are found in the 14^{th} century BCE Hittite documents. There, they were mentioning the city of *Apasas* around the Miletos region, where some artifacts belonging to the *Mycenaean* era were found. The city was colonized by the Ionians in the 11^{th} century BCE. Its ruins lay today about 80 km south of Izmir. During the 6^{th} centuries BCE, the city came under control of **Croisos**, king of Lydia and later under the rule of the Persian king **Cyros**.

Artemis, the most sacred goddess in Anatolia, had a long history of evolution going back thousands of years. The earliest forms of Artemis statue were found in Catalhoyuk and Hacilar, Central Anatolia, where its earlier name was Kybele. Then, the same goddess earned a wide respect and acceptance from Rome to Mesopotamia and even Arabia. Arabs named her *Lat*, Egyptians – *Iris*, Romans – *Diana* and Ionians – *Artemis*.

Artemis descended from the primitive Sumerian goddess of nature. She always remained the *virgin-mother of all life* and an embodiment of fertility and production power of the earth. The cult of Artemis survived till the edict of **Theodosius I** (394 CE) closed the pagan temples. Consequently, the material of the Temple of Artemis was quarried extensively for the construction of the nearby Cathedral of St. John Theologos. The Ecumenical Council of Ephesus (431 CE), headed by Emperor **Theodosius II**, officially replaced the cult of Artemis by the cult of the Virgin Mary. It was thus elevated from the rank of mother of the Mesiah to that of god's mother; thus closing the cycle of the ancient Sumerian virgin cult one full circle.

During 1866–1874, Ephesos was first excavated for the British Museum by the British Architect **John Turtle Wood** in an effort to locate the Temple of Artemis. After many explorations, Wood struck the actual pavement of the temple on Dec 31, 1869.

In 1904–1906 another British mission, directed by **David George Hogarth** (1862–1927) renewed the excavation. He established the chronology of its famous shrine from remnants, and on reaching deep levels, found some significant treasure dating from 650–550 BCE. The ivory and gold objects showed that Ionian Helenism, from its beginning, had borrowed freely from oriental traditions.

Of the Temple of Artemis even ruins were not found: there is absolutely nothing remaining above the ground. When the Byzantic emperors prohibited Pagan cults, the goddess was dethroned and the temple itself was taken to pieces. Vestiges from it can be found in the city buildings erected during the Christian epoch or in St. Sophia in Constantinople, to where many columns were taken.

The apostle Paul stayed in Ephesos but was driven away by the supporters of the cult of Arthemis. The apostle John stayed and died there. Alexander the Great was received by the Ephesonians in 334 BCE and established a democratic government in the city. During the summer of 33 BCE, Mark Antony was at Ephesos with Cleopatra, assembling the forces used against Octavians.

538–331 BCE *Rise and fall of the Persian Empire*. **Cyrus the Great**[98] (ca 580–530 BCE), a Zoroastrian, conquered Babylonia (539) and founded the ancient Persian Empire, which extended from the Indus to the Mediterranean, and from the Caucasus to the Indian Ocean. Good roads, with stations for royal messengers, made possible regular communications within the empire. A canal was dug from the Nile to the Red Sea by **Darius I, the Great** (reigned 521–486 BCE). A revolt of the Ionion Greeks in Asia ended in the fall of Miletos (494), but the war against the European Greeks was unsuccessful (battle of *Marathon*, 490 BCE). The son of Darius I, **Xerxes I** (ca 519–465 BCE) ascended to the throne in 485 BCE. In 483 BCE he collected the largest army that was even known before him, over 180,000 men drawn from all parts of the empire, with an immense fleet that the Phoenicians had assembled for him. He used a double line of ships to form two bridges across the Hellenspont, and cut a canal through the isthmus of Mount Athos Peninsula.

In 480 BCE Xerxes sent his warriors across the Hellenspont and invaded Greece. He won a victory at *Thermopylae*, entered Athens and burned all houses and temples. But his fleet was crushed at the sea battle of *Salamis*

[98] He permitted the Jews to return to Jerusalem (538 BCE) from their captivity in Babylonia (586 BCE), and rebuild the Solomon Temple. The first wave of ca 50,000 people returned in 537 BCE and the next in 458 BCE (Ezra), and 445 BCE (Nehemiah). The Second Temple in Jerusalem was built during 519–516 BCE.

(480). A Greek offensive began in 465 BCE, and after the peace treaty of 446 BCE, Persia was on the decline.

In 401 BCE, **Cyrus the Younger** employed 10,000 Greek mercenaries in a civil war against his brother. But Cyrus and the Greek commanders were killed, and the 'ten thousand' were stranded in a strange country without a leader. They chose **Xenophon** (430–355 BCE) who lead them during a five month's retreat (401–400 BCE) from Cunaxa on the Euphrates through Kurdistan and the highlands of Armenia and Georgia to Trapezus (Trebizond) on the Black Sea, a trek of 2410 kilometers. Xenophon described this march in his *Anabasis*. Although a soldier and a man of action, Xenophon is best known as an historian and a disciple of Socrates.

ca 535 BCE **Pythagoras of Samos** (ca 580–500 BCE). A Greek philosopher and mathematician who clothed his wisdom in a mysterious oracular form, and made momentous and lasting contributions to mathematical knowledge. In particular, he introduced the notions of *axiom* and *proof* into geometry: the very terms *mathematics*, *theory*[99] and *philosophy*, as they are known today, were originated by the Pythagoreans.

Pythagoras, the second person (after Thales) to be mentioned by name in the history of mathematics, was the founder of the semimystical Pythagorean brotherhood. First to furnish a logical demonstration to the theorem that was named after him, although he did not in fact discover it himself. Pythagoras discerned the role played by numbers in music i.e. that harmony depends upon numerical ratios.

To Pythagoras the pure relations of *numbers* in arithmetic and geometry were the changeless reality behind the shifting appearances of the sensible world. In contrast to the Ionians, he taught that reality can be known not

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[99] From the Greek *theoria* = a viewing, from *theoros*, from *thearos* (spectator), from *theasthai* (to behold) – whence also *thea-tron* = place for beholding things (the *theater*). It then diffused into Latin as *theoria*, French: *théorie* (1496), Spanish: *teoria* (1580), English: *theory* (1597), German: *theorie* (1700), and Russian: *teoriya* (1720).

Pythagoras proclaimed that the pursuit of disinterested knowledge is the greatest purification, and that the highest kind of life is the *theoretical* or contemplative. It is noted that in Greek, *theorein* is used for the contemplation of a spectacle such as the olympic games or the contemplation of truth; *theorema* may mean a spectacle but also speculations; *theoria* is a viewing or a theory. However, our words: theorem, theory, theoretical, have lost the early concrete senses and preserved only the abstract ones.

through sensory observation, but only through *pure reason*, which can investigate the abstract mathematical forms that rule the world.

Pythagoras was born on the Aegean Island of Samos, not far from Miletos. According to an old tradition recorded by Aristotle and his pupil Aristoxenes, Pythagoras had made extensive journeys to practically all Oriental countries, probably going as far as India. He remained in Babylon for a few years, during which time he learned from the Magi the theory of numbers, the theory of music and astronomy, and was initiated by the priests into oriental mysticism. This lore and wisdom served later as the basis for the teaching and learning of the Pythagorean school.

Returning home after years of wandering, he found Samos under the tyranny of Polycrates and much of Ionia under Persian dominion, and accordingly migrated to the Greek port of Crotona, located in the boot of southern Italy. There he founded a brotherhood of believers among the aristocrats of that city. In addition to being an academy for the study of philosophy, mathematics and natural science, it developed into a closely knit brotherhood with secret rites and observances[100]. This order later spread from Croton to a number of Greek cities in Italy and seemed to have played an important role in the political life of these cities.

In time, the political power and aristocratic tendencies of the brotherhood became so great that the democratic forces of southern Italy destroyed the buildings of the school and caused the society to disperse. Pythagoras fled to Metapontum, Lucania, where he died, maybe through murder by his pursuers. The brotherhood, although scattered, continued to exist for at least two more centuries.

What distinguished the Pythagoreans from other mystery rites is that mathematics formed a part of their religion. Their doctrine proclaimed that number was the essence of all things and that God has ordered the universe by means of numbers. God is unity, the world is plurality and it consists of contrasting elements. It is harmony which restores unity to the contrasting parts and which moulds them into a cosmos. Harmony is divine, it consists of numerical ratios. Whosoever acquires full understanding of this number-harmony, becomes himself divine and immortal.

[100] He preached the immortality and the transmigration of the soul (it is still the belief of many sects in India). After a testing period and after rigorous selection, the initiates of this order were allowed to hear the voice of the Master behind a curtain, but only after some years, when their souls had been further purified by music and by living in purity in accordance with their regulations, were they allowed to see him.

Music, harmony and numbers – these three are indissolubly united, according to the doctrine of the Pythagoreans. All three are among the essential elements of the Pythagorean system of education and of its path for the elevation of the soul. It was from this mystical doctrine that the exact science of the later Pythagoreans developed.

Not much is known about the true achievements of Pythagoras in theory of numbers, geometry and astronomy. Our sources are mainly secondary: the Neo-Pythagoreans **Nicomachos of Gerasa** (ca 100 CE), **Iambilichus** (300 CE) and the 3 arithmetical books of **Euclid**. It is also impossible to separate the contributions of Pythagoras himself from those of his followers.

The Pythagoreans introduced the *perfect numbers* [equal to the sum of their true divisors, e.g. 6, 28, 496, 8128][101], *amicable numbers* [each of which equals the sum of the proper divisors of the other, e.g. 284 and 220], *figurative numbers* [sequences of numbers, each of which is a partial sum of a certain arithmetical progression[102]]. Pythagoreans have been given credit for a rule for determining numerical solutions of the indeterminate equation $x^2 + y^2 = z^2$. Apparently Pythagoras adapted from the Babylonians various remarkable ideas about numbers and about their mystical significance. His disciples continued the investigations in a more systematic manner and built them into a logically consistent form[103].

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101 The general rule of their formation was proved by **Euclid**: when the sum $1 + 2^1 + 2^2 + \cdots + 2^{n-1} = p$ is a prime number, then $2^{n-1}p$ is a perfect number. Thus, $2^{n-1}(2^n - 1)$ is an *even* perfect number if 2^{n-1} is a prime (*Mersenne Prime*). **Euler** proved that this formula gives *all* even perfects.

Euclid's formula leads to all kind of weired and beautiful properties of perfect numbers [e.g. the sum of the reciprocal of all divisors of any perfect number always equals 2]. It is not known if there is an odd perfect number nor if their number is infinite. The 30^{th} perfect number was discovered in 1985, with $n = 216,091$, having 130,099 digits.

As for *amicable numbers*, the Pythagorean brotherhood regarded 220 and 284 as symbols of friendship. Indeed, we find in *Gen* (**32**, 14) that Jacob gave Esau a gift of 220 goats as a token of friendship! More than 1000 amicable pairs are now known, the largest of which has 152 digits in each pair.

102 Triangular numbers $1, 3, 6, 10, 15, \ldots$, $a_n = 1 + 2 + 3 + \cdots + n = \frac{1}{2}n(n + 1)$;
Square numbers $1, 4, 9, 16, 25, \ldots$, $a_n = 1 + 3 + 5 + \cdots + (2n - 1) = n^2$;
Rectangular numbers $2, 6, 12, 20, \ldots$, $a_n = 2 + 4 + 6 + \cdots + 2n = n(n + 1)$;
Pentagonal numbers $1, 5, 12, 22, \ldots$, $a_n = 1+4+7+\cdots+(3n-2) = \frac{1}{2}n(3n-1)$.
The names of these sequences stem from the fact that they can be arranged in arrays with the corresponding geometrical shapes.

103 It is believed that Pythagoras knew the identity $n^2 + \left(\frac{n^2-1}{2}\right)^2 = \left(\frac{n^2+1}{2}\right)^2$. From the writings of later Greek mathematicians we gather that Plato (ca 380

In geometry, the Pythagoreans were acquainted with the cube, the tetrahedron and the dodecahedron. The faces of a dodecahedron are regular pentagons. The diagonals of such a pentagon form a star-pentagon, the pentagramma, which served as a distinctive mark among the Pythagoreans. The construction of the star-pentagon led to the quadratic equation $x^2 = a(a-x)$, associated with the '*golden section*', which the Pythagoreans knew how to solve (as did the Babylonians).

However, the greatest contribution of the Pythagoreans to mathematics was the '*Pythagoras theorem*'[104], the first truly great theorem in mathematics. It occupies a central position in Euclidean geometry, since it governs the metric of the Euclidean space. This theorem led Pythagoras to the establishment of the concept of the *irrational numbers* $\sqrt{2}, \sqrt{3}, \ldots$ etc. and *their incommensurability* with the integers[105].

BCE) was familiar with $(2n)^2 + (n^2-1)^2 = (n^2+1)^2$ and Proclos (ca 460 CE) with

$$(2n+1)^2 + \left[\frac{(2n+1)^2-1}{2}\right]^2 = \left[\frac{(2n+1)^2-1}{2}+1\right]^2.$$

104 The origins of this theorem are obscure. It is, however, established that the Egyptians and the Babylonians were familiar with the Pythagorean theorem as early as 2000 BCE, and were also acquainted with the general parametric representation for primitive Pythagorean triplets (**Plimpton** *322*). They used $3\frac{1}{8}$ for π [Egyptians used $\pi = 4\left(\frac{8}{9}\right)^2$]. The Pythagorean triplet $(3,4,5)$ was known to the Egyptians and the Chinese. Moreover, it has recently been claimed that the Chinese had a *geometrical proof* of the theorem as far back as 1100 BCE! Did *Pythagoras prove the Theorem*? (For further reading, see: Swetz, F.J. and T.I. Kao, *Was Pythagoras Chinese*? The Pennsylvania State University Press: Pennsylvania, 1977, 75 pp.) Although it was attributed to him by various writers [**Proclos** (ca 460 CE), **Plutarch** (1^{th} century CE), **Cicero** (ca 50 BCE), **Diogenes Laertius** (2^{nd} century CE), and **Athenaeus** (ca 300 CE), no one of them lived within, say, 5 centuries of Pythagoras. Not only are we not positive that the proof is due to Pythagoras at all, but we are still more in doubt as to the line of demonstration that he may have followed.

105 The Pythagoreans had an analytic proof by contradiction that $\sqrt{2}$ is irrational. This proof appears in Aristotle's *Prior Analytics*. It goes as follows: Suppose $\sqrt{2}$ is rational; then it can be expressed as a fraction in lowest terms, say $\frac{n}{m} = \sqrt{2}$, where n and m are integers which have no common factor except 1. Then $n = m\sqrt{2}$, or if we square both sides of this equation $n^2 = 2m^2$. Thus n is an integer whose square is even and by the identity $(2k+1)^2 = 4k(k+1)+1$, n is also even. Setting $n = 2r$ and substituting in the above equation we find $(2r)^2 = 2m^2$, or $2r^2 = m^2$. But this shows that m is an integer whose square

The appearance of the *Pythagorean Theorem* marks the first known intellectual leap from the confines of empirical speculation to the limitless bounds of deductive reasoning. This is a milestone of mathematical accomplishment in the early history of the human race. In fact, it is actually the most profound such accomplishment, judged by its level of intellectual achievement and its eventual consequences for mankind.

From the earliest times, man has perceived his environment in terms of vertical and horizontal – a tree grows vertically to the horizontal plane of the earth, a person stands vertical to the surface that supports him. These vertical-horizontal relations were manifestations of the action of gravity on terrestrial objects. No doubt, man rapidly learned to use this relationship to his advantage; thus for maximum efficiency and security, the supporting pole for a shelter should be placed vertically to the ground. While nature supplied the prime example of perpendicularity, man must have soon realized that elements of the vertical realm always met with elements of the horizontal realm in the same visual pattern, forming what we know as a *right angle*. Once this concept was grasped, the potential of this union could be utilized in a wider variety of human endeavors, such as a stringing of an arrow to a bow or erecting permanent structures of wood and stone.
Perpendicularity could now be translated from the fixed vertical-horizontal constraints of nature and used to human benefit. Some of the first '*scientific instruments*' incorporated in their functioning the use of right angles. With an understanding of perpendicularity, a pole fixed in the ground could become a gnomon. The Pyramids of Egypt stand in mute testimony to the surveying applications of the right angle. Both the construction of the Egyptian plumb bed level and the rope loop square were based on an understanding of the right angle and the right triangle.
Eventually, an empirically based formulation of the *relationship of perpendicularity* emerged in the form of a rule that related the three sides of a right triangle – the Pythagorean theorem.

Numbers were thought of as made up of units, which was a suitable measure as long as they were integers. But numbers like $\sqrt{2}$ could not be accounted for in this way and were called irrational, which in Greek meant measureless rather than bereft of reason. In order to overcome this difficulty, the Pythagoreans invented a method of finding these elusive numbers through a sequence of approximations where the irrational number aimed at

is even; hence m is even, which contradicts the statement that n and m have no common factor except one.

is the limit of the process. This is the construction of continued fractions[106] through which we can reach rational approximation as close as we like to the *limit*. This feature is indeed the same as that involved in the modern conception of limit. Pythagoras was probably the first to represent number by length and produce geometric processes to prove identities and solve quadratic equations. Despite their theoretical disdain for the senses, the Pythagoreans did in practice make accurate observations of nature. Thus, in astronomy, Pythagoras himself taught that the earth is spherical, rotates on its axis and revolves around a central fire. He also believed that the motion of celestial bodies obey certain quantitative laws. His fantastic theories of the heavens were based on his very real discoveries of the laws of musical harmonies and of the regular polyhedra.

Myth and Number – Our Pythagorean Heritage

Greek philosophers, and in particular Pythagoras, endowed natural numbers with an almost magical character. They actually believed that the formal statement of facts expressed in terms of natural numbers are natural laws. Pythagoras singled out the triangular array of 10 points which he called tetraktys. This pattern is encoded in the 4^{th} *in a series of triangular numbers (Fig. 1.1)* $a_{n+1} = a_n + n + 1 \quad (a_0 = 0)$

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[106] $1+\frac{1}{1}=2; \qquad 1+\cfrac{1}{1+\frac{1}{1}}=1\tfrac{1}{2}; \qquad 1+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{1}}}=1\tfrac{2}{3};$

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{1}}}}=1\tfrac{3}{5};$$

$$1+\cfrac{1}{1+\cfrac{1}{+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{1+\cdots}}}}}=\tfrac{1+\sqrt{5}}{2}=\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\cdots}}}}.$$

(a)

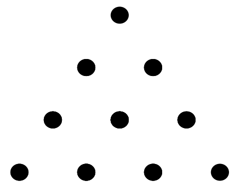
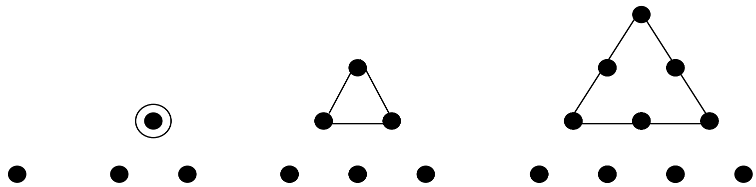


Fig. 1.1: Triangular numbers

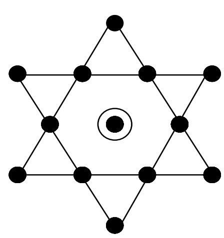
Thus, $a_n = 1, 3, 6, 10, ..., \frac{1}{2}n(n + 1)$ *are pictured as*

(b)



In mythical lore, the natural numbers 1 was called monad, the origin of all numbers. The dyad 2 was the first feminine number and represented the first stage of creation. The number 3, the first masculine number, represented the second stage of creation. The sum of the first feminine and the first masculine number, 5, represented man, microcosm, harmony, love, and health. Inanimate life was represented by the number 6. The tetraktys 10, represented the cosmos and macrocosmos. Two interlocking tetraktys created the "Star of David" in which 12 evenly spaced dots, representing the signs of the zodiac, surround a 13th *representing the "source of all being",*

(c)



In retrospect, we can deduce that the prescientific mind found, in the mystical mode of expression, a concise way to convey the kernel of meaning in *a mass of observations about the natural world. For example, the number 6 does seem to arise most frequently in inanimate forms such as snowflakes and other crystals. One the other hand, the number 5 characterizes living forms such as the starfish and certain forms of radiolaria.*

Number and geometry also lies at the basis of many sacred structures such as Stonehenge *and the* Pyramids. *In the latter case the "Egyptian triangle" 3, 4, 5 had sacred significance used in some of the key proportions of the Pyramid of Cheops. The significance of this triangle arose from the fact that the celestial sphere can be represented as a circle divided into 12 equal segments. The line can then be folded up to a 3, 4, 5 right triangle with a perimeter of 12 units.*

In ancient tradition, the square, *by its axial geometry symbolizing the directions of the compass, represented the earth and the dimensions of space while the circle, symbolizing the celestial sphere, represented the realm of the heavens and the dimension of time. Thus, ancient* mathematics, architecture, astronomy *and* music *may have been entwined to form a holistic view of the cosmos.*

It can be said that an attempt was made to bring heaven down to earth and replicate it at all scales and to synchronize space and time. The extant mathematics of the ancient world and the surviving artifacts and structures that comprise the archaeological record, serve as evidence for the number myth of those ancient cultures.

The Pythagorean philosophy, however, went beyond the link of number and geometry. They studied four subjects which they believed to be different aspects of one unifying entity: arithmetica *(the theory of numbers),* geometry, music, *and* spherics *(mathematical astronomy). The unifying theme was Number, namely* positive integers. *The relationship between number and musical intervals was one of their first discoveries.*

If a stretched string of length, say, 12, sounds a certain note, the tonic, *then it sounds the* octave *if the length is reduced to 6. If sounds the fifth (do to sol) if the length is reduced to 8, and the fourth (do to fa) if reduced to 9. So Harmony is Number.*

There follows a study of means. *The fourth is the* arithmetic *mean of the tonic and octave,* $9 = \frac{1}{2}(12 + 6)$, *while the fifth is their* harmonic *mean,* $\frac{1}{8} = \frac{1}{2}(\frac{1}{12} + \frac{1}{6})$, *since its pitch is half-way between theirs.*

There also follows a study of proportion. *The fifth is to the tonic as the octave is to the fourth, and the criterion of such proportionality is found in* $8 \cdot 9 = 12 \cdot 6$. *Since we may write this as* $9 \cdot 8 = 12 \cdot 6$, *we also have that the fourth is to the tonic as the octave is to the fifth, etc. The study of means and proportion was an important ingredient of Pythagoreanism.*

The Pythagorean relationship between music and spherics is less convincing. The intervals between the seven "planets" – the Moon, the Sun, Venus, Mercury, Mars, Jupiter and Saturn – correspond to the seven intervals in the musical scale. This explains the Celestial Harmony, and shows that the Heavens too are essentially Number. However, this mystic nonsense played a most important role in the history of science, *as we shall soon see.*

The direct relation between number and spherics, without music as a middleman, was also known to Pythagoras from his travels in Egypt. The simple gnomon *instrument, which he saw there, exemplifies the Pythagorean synthesis of spherics, geometry and arithmetica.*

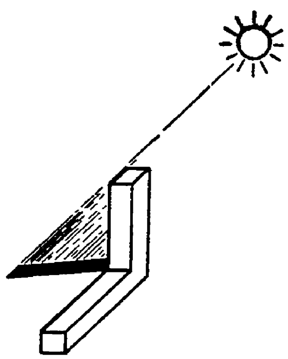


Fig. 1.2: The gnomon sundial

The gnomon (Fig. 1.2) is an L-shaped movable sundial used for scientific studies. It rests on one leg; the other is vertical. The length and direction of the shadow is measured at different times of the day and year. If the shadow falls directly on the horizontal leg at noon (when the shadow is shortest), that leg points north. The noon shadow changes length with the seasons – minimum at summer solstice and maximum at winter solstice. The sunrise shadow is perpendicular to the horizontal leg during the vernal or autumnal equinox. Thus the gnomon is a calendar, a compass and a clock.

Pythagoras knew the world was a sphere – the gnomon measures latitude, it measures the obliquity of the ecliptic, etc. Here we have Solar Astronomy with Number (measurements) as the basis.

In all such shadow measurements the geometry of similar triangles *and of right triangles is essential. A generation before Pythagoras, Thales of Miletos (a commercial center near Samos) also went to Egypt, studied mathematics* *and started a school of philosophy. It is sometimes said that Pythagoras was one of his students. Plutarch tells the story that Thales determined the height of the Great Pyramid by comparing the length of the shadows cast by the Pyramid and by a vertical stick of known length.*

The Pythagoreans used the word gnomon *also in another connection: while the triangular numbers were important as the sums of consecutive numbers, the Pythagoreans also loved the* squares *1, 4, 9, 16,... being the sum of successive* odd *numbers:*

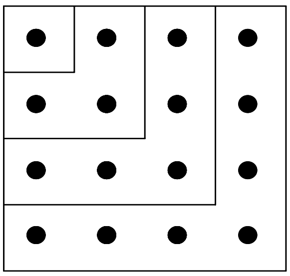


Fig. 1.3: Gnomons

16=1+3+5+7 etc. The odd numbers the Pythagoreans called gnomons. *It follows at once that if m is odd, and if m^2 is thought of as a gnomon of side $\frac{1}{2}(m^2+1)$, then*

$$m^2 + \left[\frac{1}{2}(m^2-1)\right]^2 = \left[\frac{1}{2}(m^2+1)\right]^2$$

which links number and the gnomon geometry *through the Pythagorean Theorem.*

It is irrelevant whether or not Pythagoras "discovered" the Pythagorean Theorem. He may have actually learnt it from Egyptian "rope stretchers" who used it in their construction of the Great Pyramid (2700 BCE). It is believed, however, that he was first to prove *it, using the fact that the altitude from the right-angle vertex divides the hypotenuse into two triangles that are similar to the original triangle.*

Thus, to the Pythagoreans, the right-handed triangle, the squares built upon its sides, the square numbers and the 'astronomical shadow' were all aspects of the same thing.

But not all was wine and roses in the kingdom of the Pythagoreans: when they discovered that $2a^2 = c^2$ has no solution in positive integers[107]*, or equivalently that the hypotenuse and side of a 45° right-triangle are incommensurable, it undermined their entire philosophy. For if number (i.e. positive integers) cannot even explain a 45° triangle, what becomes of much more far reaching claims.*

At a late date a new embarrassment arose: the Pythagoreans knew of 4 regular polyhedra, and they associated these with 4 "elements". The tetrahedron was fire, the cube was earth, the octahedron was air, and the icosahedron, was water. But ***Hippasos***[108] *(ca 500 BCE), a member of the society, discovered the* 5^{th} *regular polyhedron, the dodecahedron.*

The "$\sqrt{2}$*-crisis" of Greek mathematics lead to paradoxes and contradictions, but it also served as a strong motivation for the transition from naive mathematics to rigorous mathematics*[109]*. The task was left to* ***Eudoxos*** *(ca 370 BCE) and* ***Euclid*** *(ca 300 BCE), who expelled Number from geometry*[110] *placing integers were they belong – in the theory of numbers.*

The Pythagorean said that number is everything, but, aside from the analysis of music, they did not make a good case for this assertion.

If we ask whether modern physical scientists believe that the world can be best understood numerically, the answer is in the affirmative. But here "numbers" are no longer confined to integers; they also include real numbers, vectors, complex numbers, and other generalizations.

The founders of modern physical science at the dawn of the 17^{th} *century,* ***Galileo*** *and* ***Kepler*** *did not have a rigorous theory of real numbers, but they had the practical equivalent, namely, decimal fractions which the Greek did not have. The formulation of the laws of nature in terms of ordinary differential equations [***Newton***], and in terms of partial differential equations [***Euler***;* ***D'Alambert***;* ***Fourier***;* **Cauchy**; **Poisson**; **Navier**; ***Maxwell***], appeared to further weaken the role of integers in nature and to strengthen that of real numbers. But even here we may note that while the variables in an equation are continuous, the order of the equation, and the number of variables in it are integers! Furthermore, the case for Old Pythagoreanism,*

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107 From a modern point of view, $\sqrt{2}$ is a *Dedekind Cut* – a class of ordered pairs of rational numbers. It is totally "man-made" according to **L.Kronecker** (1861).

108 Supposed to have been drowned for divulging this "secret" of the Pythagorean brotherhood.

109 A similar situation arose in the 19^{th} century, where the paradoxes of the Fourier Series motivated a rigorous functional analysis.

110 Restored back to geometry by **Fermat** (1629) and **Descartes** (1637).

namely – the fundamentality of integers (not only mathematically) in nature is even stronger as the following examples will show:

- ***Galileo*** *(1590) found that during successive seconds from the time of which it starts falling, a body falls through distances proportional to the odd integers and that the total distance fallen is proportional to the square of time. Here we have square numbers arising as sums of the odd Pythagorean gnomons.*

- ***Kepler*** *(an avid Pythagorean who really believed in the Harmony of the Spheres) sought for years to find accurate numerical laws for astronomy. He finally discovered (1618 CE) his important* Third Law*, stating that the squares of the periods of the planets are proportional to the cubes of their mean distances from the sum.*

- *Even before* Newton's Principia *(1687) it was known to* ***Robert Hooke****,* ***Christopher Wren****, and others that the kinematic laws of Galileo and Kepler imply that each planet has an acceleration toward the sun which is inversely proportional to the square of distance from the sun. Hence the power of (-2) in the universal Law of Gravitation.*

- *Inspired by Newton's Law of Gravitation,* ***Coulomb***[111] *(1785) determined, with a torsion balance, that electrostatic forces were also inverse square. Many years later it was shown by* ***Maxwell*** *that mathematically, the only law of force which would behave in this way is one whose divergence in zero – that is, one that falls radially in such a way as to just compensate for the increase in the area of a spherical shell with its radius; this area increases with the square of the radius because we live in a space of three dimensions.*

- *From a similar inverse-square law due to* ***Ampere*** *(1822), and from other experimental results,* ***Maxwell*** *(1865) was led to the electromagnetic wave equations. While the dependent and independent variables here are both continuous, the number of independent variables (4), and the number of dependent variables (6) is fundamental, as became clear from the Einstein-Minkowski space-time structure (1905-1908).*

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[111] **Henry Cavendish** (1773) had already observed the same law by another method. The experiment was repeated by **Maxwell** a hundred years later. They showed that the field inside a charged hollow conductor is zero. The Cavendish-Maxwell experiment not only suggested that the exponent(-2) is *exact*, but that the reason for this is that the dimensionality of space is an integer.

- **Proust**'s *Law of Definite Proportions (1799)* and **Dalton**'s *Law of Multiple Proportions (1808) in chemistry directly imply an Atomic Theory of matter. The integral rations in the second law exclude any other interpretation. Further, it appears that chemical affinity involves integers directly.*

- *In exact analogy, the* **Romé de l'Isle** *(1772) Law of Constant Angles* and **Haüy**'s *Law of Rational Indices (1784) for crystals, directly imply that a crystal consists of an integral number of layers of atoms. Again, the integral ratios in the second law exclude any other interpretation. Further, there is a direct relationship between number and form, e.g., the six-sided symmetry of frozen H_2O.*

- *The ratio of the two specific heats of air is 7/5 and of Helium is 5/3. While phenomenological theory (thermodynamics) cannot explain these integral ratios at all, the atomic theory explains them easily (***Boltzmann***). By a similar argument Boltzmann explained the Dulong-Petit Law for the specific heat of solids.*

- ***Faraday's*** *Law of Electrolysis (1834) states that the weight of the chemical deposited during electrolysis is proportional to the current and time. If chemical weight is atomic, then this law implies that electricity is also atomic. Such electric particles were called electrons by Stoney (1891).*

- *In 1814* ***Joseph von Fraunhofer*** *invented the diffraction grating. A glass plate is scratched with a large number of parallel, uniformly spaced fine lines. This produces an optical spectrum: since parallel beams of a given wavelength, shining through the successive apertures on the glass, will be diffracted only into those directions where the successive beams have path lengths that differ by an integral number of wavelengths.*

- *The simplest spectrum is that of hydrogen. The wavelengths of its lines have been accurately determined. In 1885* **Balmer** *found that these wavelengths are expressible by a simple formula involving integers.*

- ***Pieter Zeeman*** *(1896) discovered that the lines of a spectrum are altered by a magnetic field, and* **H. A.** ***Lorentz*** *at once devised an appropriate theory. The radiating atoms contain electrons whose oscillations produce the spectrum by electromagnetic radiation. The frequency of the oscillations (and therefore also their wavelength) is changed by the action of the magnetic field upon the electrons.*

- *From Maxwell's Equations and thermodynamics,* ***Ludwig Boltzmann*** *(1884) derived Stefan's Law of Radiation (1879). This states that a blackbody radiates energy at a rate proportional to the fourth power of its absolute temperature. Although electromagnetism and thermodynamics are both theories of continua, the real point of the law is the* exponent. *Here again the exponent 4 is said to be exact and, in fact, even a casual examination of Boltzmann's derivation shows that this exponent equals the number of independent variables in the wave equation – the three of space and one of time. Just as* $2 = 3 - 1$ *so does* $4 = 3 + 1$ *here.*

- ***Max Planck*** *(1900) found it necessary to assume that energy is radiated discretely in* quanta and ***Albert Einstein*** *(1905) used them to explain photoelectricity. Thus* discrete matter (atom) implies both *discrete electricity* (electron) and *discrete energy* (quantum).

- *Emergence of the 4-dimensional space-time continuum of* ***Albert Einstein*** and ***Minkowski*** *(1905-1908). In this theory, particular importance is attached to vectors with four components. One such vector is a space-time displacement. Another is the momentum-energy vector, three components of momentum and one of energy. A skew-symmetric tensor in this four-dimensional world has six components – four things taken two at a time. The most important example is the electromagnetic field – three components of electric field, and three of the magnetic field.*

Note that the Pythagoreans also considered 4 to be especially important because it was related both to properties of the tetrahedron and fire. Tetrahedron has two special properties: it is the smallest polyhedron, and it has the same number of vertices and faces (i.e. it is self-dual). Both of the properties follow from the fact than its number of vertices is one more than the dimensionality of space. If Pythagoras could be alive today and deeply versed in modern physics, he could argue "I told you so" on two counts:

(1) *The number 4 is as important to me for the same simple reason that it is important to Albert Einstein, Minkowski, Stefan and Boltzmann:* $4 = 3 + 1$.

(2) *Fire is radiation heat and light, and that is electromagnetic: your 6 components of this fields are obtained by taking the 4 dimensions of space-time, 2 at a time. So likewise, my 6 edges of the tetrahedron join the 4 vertices 2 at a time, and also are the intersections of the 4 faces, 2 at a time.*

- ***Mendeleev's*** *Periodic Table of the chemical elements (1869). If the elements are listed in order of their atomic weights, then chemical, spectroscopic, and some other physical properties recur periodically. But there were many imperfections and many questions arose. Tellurium weighs more than iodine. But if placed in the table in that order these elements clearly fall into the wrong groups. Again, the position of the rare earths and the numerous radioactive decay products were not clear. The rare gases were entirely unanticipated.*

  Further, the table is not strictly periodic but has periods of length 2, 8, 18, and 32. Why these periods should all be of the form $2n^2$ *was not clear. Indeed, how could it be – for what can mere* weight *have to do with these other properties?*

  But the experiments of **C.G. Barkla** *(1877-1944, England) by X-ray scattering (1911) and the experiments of* ***E. Rutherford****, in the same year, by alpha particles, lead to an atomic model – a miniature "solar" system with the light, negatively charged electrons bound to a heavy, positively charged nucleus by inverse-square Coulomb forces.*

  In 1913 **Niels Bohr** *assumed that the hydrogen atom had this (simplest) Rutherford structure – one proton as a nucleus and one electron as a satellite. With the use of Planck's* $E = h\nu$*, he deduced the Balmer formula with great precision. However, he had to assume that the electron could have a stable orbit only if its angular momentum were an integral multiple of* $h/2\pi$*. That is,*

  $$mvr = nh/2\pi$$

  with m *the electron's mass,* r *the orbit's radius,* v *the electron's velocity, and* h *Planck's constant. The integer* n*, the* principal quantum number*, made no sense in the theories then in vogue, but its acceptance was forced by the remarkable accuracy of the theory's predictions.*

  Thus, 1913 was a good year for Old Pythagoreanism. ***Soddy*** *and* ***Fajans*** *found that after radioactive emission of an alpha particle (charge +2) the resulting element is two places to the left in the periodic table, whereas emission of a beta particle (charge -1) results in a daughter element one place to the right. Together with the earlier results this Displacement Law makes it clear that* atomic number*, not atomic weight, is the important factor. This integer is the positive charge on a nucleus, the equal number of electrons in that atom, and the true place in the table of elements.*

  In 1912 **von Laue** *made the suggestion that a crystal would act like a diffraction grating for radiation of a very short wavelength.*

Henry Moseley *(1913) used von Laue's suggestion to measure the (very short) wavelengths of X-rays. Optical spectra, like chemical behavior, are due to the outer electrons in an atom, and thus have a periodic character. But X-ray spectra are due to the inner electrons, and these electrons are influenced almost solely by the charge on the nucleus. Moseley's photographs show a most striking monotonic variation of the X-ray wavelengths with atomic number.*

Atomic number at once cleared up most of the difficulties. But what about $2n^2$*?*

In 1923, **L. *de Broglie*** *combined relativistic invariance of four-vectors with Planck's* $E = h\nu$*. The energy* E *and the time associated with the frequency* ν *are merely single components of two four-vectors. The remaining three components of momentum and of space, respectively, must be similarly related. Thus a particle of momentum* mv *should have a wavelength* λ *given by*

$$\lambda = \frac{h}{mv}.$$

When this is applied to Bohr's

$$mvr = nh/2\pi$$

one obtains

$$n\lambda = 2\pi r.$$

Thus the matter wave has exactly n *periods around the circumference of the orbit and the interpretation of the stability of the electron's discrete orbitals (quantum orbits) is that it constitutes an azimuthal standing wave.*

This conception was refined in the ***Schroedinger****'s Wave Equation (1926). Here there are three quantum numbers* n*,* l*, and* m *corresponding to the dimensionality of space. In polar coordinates the wave functions corresponding to given* l *and* m *are spherical harmonics (not quite "Harmony of the Spheres" – but very close to it). It further develops that the integer* l *can equal* $0, 1, 2, ..., n-1$ *while* m *can equal* $-l, -l+1, ..., l-1, l$.

For n = 4, for example, we have 16 possible 'Gnomon-like' states (Fig. 1.4):

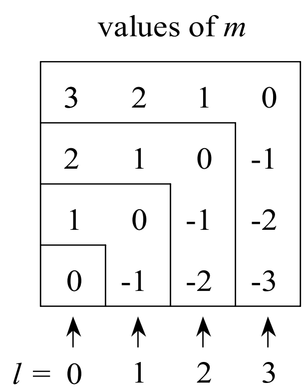

Fig. 1.4: Quantum ‘Gnomon-like’ states

But a fourth quantum number was already waiting in the wings. In 1925 ***Uhlenbeck*** *and* ***Goudsmit*** *discovered the spin of the electron. This gives rise to a fourth quantum number m_s which can take on two possible values. When this fourth "integer coordinate" is added, we obtain the $2n^2$ states which correlate with the periods in the periodic table of the elements. But we must distinguish two different types of "harmonies" here. In one atom an electron can transition from state to state; thus giving rise to the spectrum. This is the first "harmony." On the other hand, as we go through the periodic table, adding one new electron each time, the new electrons will also fill successively available distinct quantum states according to the* Pauli Exclusion Principle *(1925) combined with Coulomb energetics criteria resulting from inter-electron repulsion. This gives rise to the periodic table – the higher-level "harmony."*

*Pythagoras, if living today, would be delighted with nuclear "magic" numbers (**Mayer**), "strangeness" (**Gell-Mann**) of quarks, charmed quarks, the "eightfold way," all the numerology associated with the elementary particle Zoo and the strong links of string theory with number theory.*

The Earth as a Sphere (550–530 BCE)

One may wonder: how did Thales and Pythagoras reach the bold conclusion that the earth is a sphere? They may have observed that the surface of the sea is not flat but curved, for as a distant ship approaches one first sees the top of its mast and sail and the rest appears gradually. The circular edge of the shadow cast in an eclipse of the moon would also suggest a spherical (or disk) shape of the earth, yet this observation implies an understanding of eclipses that had not yet been attained in the 6^{th} century BCE.

It is more probable that as soon as the Babylonian hypothesis of a flat earth (common in the Old Testament) had been rejected, the sphericity of the earth was postulated*, on insufficient experimental grounds. This fundamental Pythagorean idea was an act of faith rather than a scientific calculation, but it made the theory of eclipses possible. In turn, the development of that theory and the observations that suggested it, repeatedly confirmed the initial assumption. Does not every scientific hypothesis start that way?*

The dogma of spherical perfection and its cosmological consequences may be considered the kernel of early Pythagorean science. It was postulated that the celestial bodies are of spherical shape and that they move along circular path in uniform motion.

532 BCE **Theodoros of Samos**. Greek architect and engineer, to whom many inventions are ascribed: level, square rule, key. He is said to have introduced bronze casting from Egypt into Greece. When the foundations of the Temple of Ephesos were laid down, he used various means to solidify the marshy ground.

ca 530 BCE **Xenophanes of Colophon** (ca 580–485 BCE). Greek poet and philosopher. The first thinker of Greek culture to advance the idea of one, true, eternal, supreme God (in opposition to the ideas of the gods of the poets and the popular cults), to whom he attributed the shape of a *sphere*. A contemporary of Pythagoras, he relied principally on the Miletian school and his main concern was in studying the phenomena of nature.

Xenophanes was the first of the early philosophers to focus upon the geological time scale, recognizing the significance of fossils as remnants of former life and correctly inferring that sedimentary rocks originated as sediments deposited on the sea bottom. Moreover, he concluded that such rocks and fossils must be of great age, considering them as witnesses of periodical submergences of the dry land. Xenophanes was not particularly loved by some of the erudite establishment of Greek philosophers, most of whom had deduced that there was no beginning and no end to the earth.

Born in Ionia (Asia Minor), his religious rigor caused him to leave his native country and lead a migratory life: he resided for a time in Sicily, at Zancle and Catana and afterwards established himself in southern Italy, at Elea, where he founded the Elean school of philosophy.

The idea of God's sphericity was later echoed in the words of **Aristotle** (354 BCE): "*...there can only be God, the same from all sides... Otherwise the various parts would be superior and inferior to each other, and this is impossible. Hence such a universal homogeneity of God implies that he has the shape of a sphere...*".

530–520 BCE **Eupalinos of Megara** (c. 570–510, Greece). Architect and engineer. Built an aqueduct and a water-supply system for Megara. Constructed a tunnel of 1100 m under 300 m high Mount Castro on the island of Samos to supply water. The tunneling was started from the opposite sides and driven into the center[112].

512 BCE **Mandrocles of Samos**. Greek civil engineer. The first known bridgemaker.

When Darius I (king of Persia, 550–486 BCE) made his expedition against the Scythians (ca 514 BCE), he ordered Mandrocles to build a bridge across the Bosporos to enable his immense army to pass into Europe. Mandrocles was able to satisfy him by building a *floating bridge of boats*. The Greek borrowed ideas from the Egyptians, the Babylonians, and the Phoenicians, much as these people in their time had borrowed ideas from each other.

510–507 BCE **Darius I** conquered northwestern India. He then sent the Greek **Scylax of Caryanda** on exploratory voyage down the Indus. Scylax sailed to its mount and then followed the coast to the Red Sea.

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[112] As in the case of the *Hezekiah Siloam Tunnel* in Jerusalem, dug ca 710 BCE by Hebrew engineers. The tunnel was about 533 m long. An inscription commemorating its completion was discovered in 1880 CE, and provides the information that it was dug from both ends simultaneously [see *II Kings* **20**, 20; *2 Chron* **32**, 3-4; *Neh* **3**, 16].

ca 510 BCE **Hecataeos of Miletos** (ca 550–475 BCE). Greek traveler, map-maker and historian. Drew the first recognizable map of the Mediterranean world. Wrote *Periegesis* (tour round the world), widely used by Herodotos and other writers.

Opposed Ionian revolt (500 BCE) against Persia and, after the Ionians were defeated (494), was appointed Ionean ambassador to negotiate terms of peace with Artaphenes.

508 BCE *Democracy in the form of isonomia (equality before the law) was born in Athens.* To break the power of the noble clans, which were connected with the old hereditary tribes, **Cleisthenes** reformed the constitution of Athens; a new system of government was created, called *demokratia* (the people in power). With the economic boom that followed the *Persian wars* (490–479 BCE), the nature of the Athenian system of government changed from conservative democracy based on agriculture into full popular sovereignty based on commerce.

ca 500 BCE A toy regular dodecahedron of Etruscan origin found in 1885 near Padua.

ca 500 BCE **Nabu-rimannu** (Naburimanus). Leading Babylonian astronomer. Employed a special sign for a true *zero* in his astronomical tables for the calculation of a new moon and eclipses. Known to the Greeks and mentioned by Pliny and Strabo.

ca 500 BCE Quill pens were introduced in Europe and the Middle East.

ca 500 BCE **Heraclitos of Ephesos** (ca 540–475 BCE). Ionian philosopher, first to discover the concept of *Natural law*. To sum his philosophical system in one sentence is to say that universal law and order, manifests itself through a state of perpetual change. His philosophy has three main themes:

(1) Nothing is permanent except change. All phenomena are in a state of continuous transition from non-existence to existence and vice versa. Where there is no strife, there is decay. Things lack identity and possess only the attributes of *becoming* but not *being*. In his own words and letters: "πάντα ρεῖ καῖ ούδὲν μὲνεῖ" (all things flow, nothing abides). One can discern here a crude form of the principle of Relativity.

(2) The process of change takes place in accordance with a deep Universal Reason (λογ́ος = Logos), i.e. a rational principle that dominates nature. This law is the only constant thing amidst the cosmic motion. This law was and will be forever: cosmic motion is the eternal reality.

(3) Cosmic history runs in repeating cycles, each beginning and ending in fire: from chaos to cosmos and vice versa. Contemporary Buddhist philosophy taught that the universe is periodically created and destroyed.

ca 500 BCE **Alcmaeon of Crotona**. Greek Pythagorean philosopher and writer on medical subjects. A pupil of Pythagoras. He was concerned with the internal causes of diseases. He divided these causes into disorders of environment, of nutrition and of lifestyle. His book is lost and only a few fragments of his writings have remained. He is the first person known to have dissected human cadavers for scientific purposes. Discovered the *optical nerve*, the *Eustachian tube*[113] [two millennia before **Bartolomeo Eustachi** (1564)], the origin of *sperm*, and gave explanations of *sleep*. He performed physiological experiments and knew that the *brain* is the central controlling organ of the body and the seat of the intellect.

Alcmaeon held that health and disease are respectively an equilibrium and a rupture of equilibrium of the organism, and that everything in nature is a conflict between opposites.

490–479 BCE *The Persian Wars*: A series of military conflicts which had a decisive and permanent influence upon the development of Western culture and civilization.

- 490 BCE, Sept. 12. *Battle of Marathon*: Army of Militiades (11,000 Athenians and Palataeans) defeated much larger Persian force under Darius I (549–486 BCE), turning back Persia's second invasion of Greece.
- 480 BCE. *Battle of Thermopylae*: Small force of 300 Spartans and 700 Thespians led by King Leonides bravely held strategic pass against invading Persian army, 180,000 men strong, under King Xerxes I (519–465 BCE). The Persians burned Athens to the ground, but its citizens fled to Salamis and the Peloponnese.

– *Battle of Salamis*: The greatest naval encounter of the ancient world. Celebrated Greek naval victory in which Themistocle's (525–460 BCE) fleet sinked about 200–300 Persian ships in the narrow straits of the Island of Salamis.

[113] Auditory passage (3.8 cm long) made of bone and cartilage, and lined with mucous membrane. The tube connects the middle-ear to the throat: it allows air to pass through it. Swallowing helps open the tube and thus equalize the air pressure on the inner side of the eardrum to the air pressure on the outside. The tube also allows mucous formed in the middle ear cavity to escape into the throat.

- 479 BCE. *Battle of Plataea*: The Spartan general **Pausanios** won a decisive victory over Persian forces led by **Mardonias**.
- *Battle of Cape Mycale*: Greeks won a naval victory against Persian fleet. This ended any further threat of invasion by Persia.

These events, some of the most momentous in the history of the world, marked the victory of Europe over Asia and enabled Greece to become that which it had to be.

As soon as the Persian retreated, the Athenian rebuilt their city on an even grander scale. They used their large navy to bring former Milesian trading positions (on the Black Sea, in Syria and in Egypt) under their own control and gradually established a prosperous economic empire. Athenians of the 5^{th} century, not only dominated the Aegean but produced one of the richest eras in the history of Western civilization. This era is known as the *Golden Age of Greece*.

In addition to its most sublime resources of the human intellect, the Greek civilization also drew its strength from more mundane endorsements. *Silver* financed the city-state of Athens, its trade and its commerce. From the mines at *Laureion*, at the southern tip of Attica, a stream of silver flew through the Athenian treasury. The raising up of a commercial power in Greece, able to throw back the advancing tide of barbarism that threatened to extinguish its arts, literature and science, was due in some indispensable measure to the silver mines at *Laureion*.

ca 480 BCE "Optical telegraph"[114] operating in Greece; Using two torches (one in each hand), letters of the alphabet were simulated by various positions to code plain-language messages transmitted from one hilltop to another in succession.

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[114] During the French Revolution (1794 CE), **Claude Chappe**(1763–1805 CE) converted this old idea into a reality by establishing a nation-wide *semaphore* (a word devised by Chappe from the Greek for "bearing a sign") visual telegraph. With the National Assembly's backing he built a series of 22 towers over the distance of 240 km between Lille and Paris. Each tower was equipped with a pair of telescopes, one pointing in each direction, and with a two-arm semaphore. Each arm could assume 7 clearly visible angular positions, making 49 combinations that were assigned to the alphabet, numerals and other symbols. It only took 2 to 6 minutes to transfer a message over 240 km whereas riding couriers would have needed 6 hours. Depressed by illness and by mounting claims of Plagiarism, Chappe committed suicide by throwing himself down the well in his hotel.

Greeks also used sound signals (trumpets, drums, shouting), smoke signals, mirror reflections and beacon fires. At about the same time Persia had a form of Pony express.

ca 480 BCE **Panini** (ca 520–460 BCE, India). A Sanskrit grammarian who gave a comprehensive and scientific theory of phonetics, phonology, and morphology. In his treatise called *Astadhyayi*, Panini distinguished between the language of sacred texts and the usual language of communication. Panini gave formal production rules and definitions to describe Sanskrit grammar. The construction of sentences, compound nouns etc. is explained as ordered rules operating on underlying structures in a manner similar to modern theory.

Panini should be thought as the forerunner of modern formal language theory used to specify *computer languages*.

ca 470 BCE **Parmenides of Elea** (ca 504–456 BCE, southern Italy). Greek philosopher. Founder of the Elean school and one of the great pre-Socratic thinkers. Down to recent times, philosophy has accepted fundamental concepts from Parmenides, notwithstanding considerable modifications and combinations with other ideas.

He was the originator of the doctrine of *being*, which he developed in opposition to the doctrine of *becoming* of **Heraclitos**. He also initiated the distinction between the sensible world (the world known by the senses) and the intelligible world (the world known by the mind). It was he who first assumed an indestructible *substance* and used it as a basis for his speculations (although he did not formulate its concept). He was among the first to distinguish between scientific truth and popular opinion. In this way Parmenides influenced **Empedocles**, **Leucippos** and **Democritos**, the Sophists and **Plato**. **Hegel** was not the last philosopher who followed Parmenides by founding metaphysics upon logic.

Parmenides shaped a principal characteristic of the Greek mind, which is significant in Greek philosophy, science and art – his preference for *unity, composure, and the comprehension of limits and contours*. His longing for unity made him suspicious of the senses; his emphasis on composure made him deny change, and the need of limits made him conceive of the unchanging world as a *spherical* form and repudiate the idea of infinite, or empty space. Specifically he asserted that *void* was unnecessary for the description of the world.

Parmenides was born in *Elea*, a Greek colony in southern Italy; probably a disciple of **Xenophanes** and **Ameinias**, a Pythagorean. He resided for some years in Athens (ca 450 BCE) where **Socrates** met him and learned much from the aged philosopher. He was one of the first Greek philosophers to express his thoughts in poetry.

ca 460 BCE **Leucippos of Miletos** (ca 490–430 BCE). Greek philosopher and founder of the Abderan school. One of the early mathematicians to investigate the squaring of the circle. He was founder of the doctrine of *atomism*, and all modern physicists may be regarded as the followers of his way of thinking that led to immense results in science and practical life. His theory that the Universe is composed of an infinite number of small indivisible particles (which he called *atoms*) remained a philosophical idea until 1808 CE. It has undergone many and important modifications, but has maintained its validity even after the "indivisible" atoms could be split.

Leucippos maintained that atoms are separated and distinguished from one another by *Non-beings* (empty space). He stated that atoms are *imperceptible*, *individual particles that differ only in shape and position*. Things come into existence by virtue of motion of these atoms in space and their accidental coming together.

All Leucippos' works, among which the books *Megas Diacosmos* (the Great Order of the Universe) and *Peri Nou* (On Mind), were most famous, are lost. He was a contemporary of Zeno, Empedocles and Anaxagoras. His fame was so completely overshadowed by that of Democritos (who subsequently developed the theory into a system) that his very existence was denied by Epicuros, but Aristotle expressly credits him with the invention of atomism.

Nothing is known of his life, and even his birthplace is uncertain.

ca 460 BCE **Anaxagoras of Clazomenae** (500–428 BCE). The last philosopher of the Ionian school. Maintained that nature is a work of design and order, driven by reason. He assumed that things are made up of an immense number of tiny "*seeds*" of different kind of matter. These seeds (today's atoms) never change, but they exist mixed together in different combinations; apparent changes in matter are simply recombinations of the changeless seeds. For these recombinations to occur, *motion* is needed. He thus paved the way for atomic theory. In astronomical history, Anaxagoras is remembered for correctly explaining eclipses and for his cosmology.

Anaxagoras was first in attempting to give scientific account of celestial objects and events such as eclipses, meteors, rainbows and the sun which he described as a mass of blazing metal. The heavenly bodies, according to him, were masses of stone, formed from the earth and ignited by rapid rotation. He is known to have made the first attempts to square the circle[115].

[115] It is thought that Anaxagoras worked on solving it while he was in prison for having claimed that the sun is a giant red-hot metal rather than a deity, and that the moon shines by its reflected light.

Anaxagoras was born at Clazomenae (30 km west of today's Izmir). He came to Athens from Ionia in 480 BCE and brought with him the spirit of scientific inquiry. But the ignorant polytheism of the time could not tolerate such explanations, and he was imprisoned (450 BCE) on the charge of contravening the established dogmas of religion. It required all the eloquence of **Pericles** to secure his acquittal. Even so he was forced to leave Athens in 434 BCE. He died in Lampsacos.

Anaxagoras was a contemporary of **Euripides** and **Socrates**. The concept of reason in nature was taken up again by **Aristotle**, on whose scientific work he exerted much influence.

ca 460 BCE **Zeno of Elea** (ca 490–430, southern Italy). Philosopher. Pupil of Parmenides. Expressed eight paradoxes contrasting continuity in space and time with discreteness. These paradoxes had a profound influence on the later development of the notion of infinitesimals, and assert that motion is impossible. The Greeks could not break away from their intuitive notion that the sum of an infinite number of positive quantities is infinitely large, even if the quantities become infinitely small, and that the sum of an infinite numbers of quantities of zero measure is zero. Thus infinitesimals were excluded from Greek mathematics, and science had to wait another 2000 years to resolve these paradoxes.

The *first paradox* is that of the runner of the race track. He can never reach the end of his course, Zeno tells us, since he must first cover half the remaining distance, and then after he has done so, half of the remaining distance, and then half of that, and so on. Each new distance to be covered is half of the distance just covered, and is therefore a finite distance, and yet there are an infinite number of these (finite) half-distances. Hence, Zeno concludes, an infinite time would be required to reach the goal.

The *second paradox* is the one about Achilles and the tortoise. Achilles is trying to catch up with the slowly moving tortoise, which he trails by a short distance. But he can never reach the tortoise, because whenever he comes to the point where the tortoise was, he will find that the tortoise has moved ahead some small distance. It will always take Achilles some time to cover the distance between himself and the tortoise, and the tortoise will always move some distance ahead during that time, Zeno argued[116].

[116] Essential fallacies in Zeno's arguments were exposed by Aristotle in his *Physics*. In modern times, an acceptable treatment of the paradoxes has been given along lines similar to Aristotle's, but with the benefits of the precision of the mathematical theory of continuity and infinite sums.

It is clear to us today, thanks to well-established mathematical principles, that the first two paradoxes are based on the misconception by Zeno regarding

In the *third paradox*, Zeno claims to prove the impossibility of any motion whatsoever. He states that if what is moving is always in the *now* – is always in the instantaneous present – then motion is impossible. As an example, Zeno uses an arrow in flight. At a given instant the arrow is in a fixed position and occupies just the space which corresponds to its physical volume. It occupied a different region of space in its past, and will occupy a different space in the future. Hence, at any given instant the arrow is motionless and therefore indistinguishable from a motionless arrow in the same position. How then do we know that it is moving? Since any period of time is a sum (albeit infinite) of its component instants, the arrow is motionless throughout the period. Zeno concludes from the paradox that all motion is impossible[117].

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the value of an infinite sum of terms. It can be readily shown that the sum $\sum_{n=0}^{n=\infty} q^n$, where q is a positive number less than one and n takes the integral values from zero to infinity, is equal to $\frac{1}{1-q}$. In the first paradox, Zeno asserted that the runner must first go $\frac{1}{2}$ the remaining distance, then $\frac{1}{2}$ of that distance, which is $\frac{1}{4}$, then $\frac{1}{2}$ of that, which is $\frac{1}{8}$, and so on, giving a total distance of $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots=\sum_{n=1}^{n=\infty}\left(\frac{1}{2}\right)^n$. This sum is equal to 1 by the above formula, since q is $\frac{1}{2}$ in this case, and the first term, $\frac{1}{2}$ to the zeroth power, which is equal to one, is not included in the series. Hence, in this case the sum of an infinite number of terms is finite, and not infinite as Zeno supposed.

The solution of the problem of Achilles and the tortoise is similar to that of the runner. Suppose that Achilles' and the tortoise's running speeds are such that while Achilles runs a given distance the tortoise runs only q times that distance, and that Achilles is X meters behind the tortoise when we first consider the two. While Achilles advances X meters, the tortoise advances qX meters. Then while Achilles covers this distance between them of qX meters, the tortoise moves $q(qX)$, or Xq^2 meters, and so on. In order to catch up with the tortoise, Achilles will have to run through an infinite number of intervals; i.e., he will have to run $X+Xq+Xq^2+Xq^3+Xq^4+\cdots$ meters, which is equal to $X\cdot\sum_{n=0}^{n=\infty} q^n=\frac{X}{1-q}$ meters. Thus, if X is 25, and q is $\frac{1}{10}$, Achilles will have to run $25\left(\frac{1}{1-\frac{1}{10}}\right)=25\left(\frac{10}{9}\right)$ meters in catching up with the tortoise. Of course, if q is 1, Achilles is running only as fast as the tortoise, and will never catch up with him. This is indicated by the fact that $\frac{1}{1-q}\to\infty$ as $q\to 1$. Incidentally, when $q=\frac{1}{10}$ it is easy to see why the sum of an infinite number of terms, $\sum_{n=0}^{n=\infty}\left(\frac{1}{10}\right)^n$, may be finite in value. Each new term that we add gives a digit 1, one more decimal place further to the right than its predecessor, giving us a sum of the form 1.11111. . . . No matter how many terms are added, we never increase the digit from 1 to 2 in any decimal position.

117 The concept of the continuity of a line, which was introduced into mathematics by Dedekind in 1872, gives us a solution of the third paradox. According to this concept a straight line, or generally any continuous region, contains as many

The argument of the arrow paradox can be further amplified in the following manner. The moving arrow occupies different sets of spatial points at different instants. Yet, since it must be stationary at any instant, how can it ever move to the set of points which it occupies at some other instant?

458–428 BCE **Ezra ben Seraya ha-Kohen** the scribe[118]. Founder of the postexilic viable Judaism. Edited and canonized the Five Books of Moses

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points in any finite interval as we wish to denumerate. By giving a line this property of consisting of as many points as desired, it is found that the line will consistently have the properties of being continuous. In contrast, a discrete line is one which has a finite number of points between any two given points. Thus, we ask, if the arrow is at rest *now* at one point (instant) on the time-axis line, how does it ever get to the next instant?. The answer is that such a question is ill-defined, since there is no "next" point. For between the point in question, and any point that is designated as the "next point", there can be any desired number of intervening points. To quote Bertrand Russell's discussion of the problem (1929): "*The solution lies in the theory of continuous series: we find it hard to avoid supposing that, when the arrow is in flight, there is a next position occupied at the next moment; but in fact there is no next position and no next moment, and when once this is imaginatively realized, the difficulty is seen to disappear*".

118 At the beginning of the Persian period (538 BCE) the Jewish exiles in Babylon were free to return to their own land and there was the first exodus of Babylonian exiles in the days of Cyrus (538–529 BCE). These people built the second Temple in Jerusalem (ca 515 BCE). The Persian rule lasted until 332 BCE, when it was overthrown by Alexander the Great. During the two centuries of Persian domination, the Jewish community in Israel, with its center at Jerusalem, in spite of varied hindrances, gained new life; the Law was codified, the Temple-worship fully organized, and the work of collecting and arranging the sacred books of the Old Testament well begun.

Under the long reign of Artaxerxes I (465–424 BCE), the 'second return' of the Jews (458 BCE) under the aegis of Ezra and **Nehemiah** during their two visits to Jerusalem (445 and 432 BCE) took place.

If not for the actions of Ezra and Nehemiah, Judah would have disintegrated, Jerusalem faded out of Jewish consciousness, and there would be no motivation for exilic Jews to exist as Jews. Dying memories and tempting assimilation would hurl them out of Jewish history.

Ezra and Nehemiah created a new age for the Jews; now that the era of prophecy was over, the time has come to transubstantiate prophetic ideology into practical policies. Where the function of the prophets has been to universalize the Hebrew concept of God and give mankind a universal ethic, the function of Ezra and Nehamiah was to formulate ideas that would preserve the Jews as Jews.

(Pentateuch), thus fusing the most important of the then-extant Mosaic documents into what is now known as the Torah. This act of canonization cleared the way for the compilation of the Old Testament. His exegesis on the Torah and the major reforms he instituted in Judaic practice may be seen as the beginning of classical Judaism.

Ezra was a scribe in the court of the Persian King Artaxerxes in his summer capital city of Susa. It is there that he heard of the plight of the Jews and the sad state of Judaism in Jerusalem. He successfully petitioned the king to let him organize a second Zionade, which proved successful beyond all expectations.

Ezra and Nehemiah introduced three innovations to strengthen the Jewish identity of the returnees from Babylon: a ban on intermarriage with any Gentile, a stress on nationalism, and a canonization of scripture. These were destined to shape the character of the Jews and chart their course through the next two millennia.

450 BCE **Empedocles of Acragas** (ca 490–435 BCE). Greek philosopher, statesman, and physician. A scientific thinker, precursor of the physical scientists. First to try to identify principles of motion. He believed that light moves in space with a finite velocity.

Empedocles was born in Acragas (Agrigentum) on the south coast of Sicily of a distinguished family, then at the height of its glory. His grandfather was victorious in the Olympian chariot race in 496 BCE. Like his teacher Parmenides, he was steeped in Pythagorean tradition. He tried to combine this with the more naturalistic philosophy and science of the Milesians. Fragments of two treatises, one entitled *On Nature* and the other *Purification*, are extant.

The doctrine of the four elements: water, fire, air and earth, which dominated the popular thinking about nature for more than 2000 years, was probably originated by Empedocles. He is credited with founding the first great medical school. His legendary death is supposed to have taken place by falling into the crater of Mount Etna (this has been a source of inspiration for many poets, among them: Mattew Arnold and Friedrich Hölderlin). He has been celebrated by followers of Mazzini as the democrat of antiquity par excellence.

Empedocles tried to reconcile the doctrine of the permanence of being (Eleatics) with the doctrine of change and motion (Heraclitos). His four elements are eternally brought into union and eternally part from each other. The different proportions in which these four indestructible and unchangeable matters are combined with each other determine the different organic structures produced. It is in the aggregation and segregation of elements thus arising that Empedocles, like the atomists, finds the real process which corresponds to what is popularly termed growth or decay. Nothing new comes or can come into being; the only change that can occur is a change in juxtaposition of element with element.

Empedocles believed that light travels very fast, but not infinitely fast. Demonstrated that nature does not allow the creation of a macroscopic vacuum ("horror vacui").

450 BCE **Herodotos** (ca 484–425 BCE). Greek historian[119], called the *Father of History*. He undertook to write the history of the world up to his own time, yet he limited his 9 books to the rise of the Persian Empire, the Persian invasions of Greece in 490 and 480 BCE, the heroic fight of the Greeks against the invaders, and the final Greek victory. He wholly omitted the histories of Phoenicia, Carthage and Etruria, three of the most important states existing in his day. Even the Trojan war is not mentioned in his writings.

Herodotos included many stories which he did not believe[120], because they made his account more interesting. Historians even now cannot disentangle fact from legend in his work. Yet some of his keen observations are remarkable: while traveling through the lower Nile River valley, his observations led him to reason that the Nile delta must have been made from a series of floods. It then quickly followed, by his reasoning, that if a single flood were to lay down only a thin layer of sediment, it must have taken many thousands of years to build up the Nile delta.

He was born at *Halicarnassos*, today *Bodrum*, in Asia Minor. During his youth he traveled widely in Greece, the Middle East and North Africa. The things he learned in his travels formed the materials of his histories. In about 447 BCE he visited Athens, and three years later settled in the colony of Thurii which Pericles was then founding in southern Italy. Nothing is known of the rest of Herodotos' life.

His visits to the cultural centers of Persia, Babylon, Egypt and Greece, and his attempts to describe the whole evolution of the non-Greek peoples of the Persian Empire (including the Egyptian and the Babylonians) make his books a valuable source of information on the state of science and technology in the ancient world.

In Herodotos' writings, the existence of *China* is recorded for the first time in the European literature: According to him, a Greek named **Aristeas** (ca

[119] His history was written about 450 BCE; first printed in Latin translation in 1474; the Greek text first printed in 1502; first translated into English in 1709.

[120] In Egypt, for example, he was shown a temple in which the priests put out food for the god every night. The food was always gone in the morning, a fact which they presented to Herodotos as proof of the god's existence. "*I saw no god*", he commented, "*but I saw many rats around the base of the statute*".

6^{th} or 7^{th} century BCE) claimed to have journeyed across Central Asia as far as the Djungarian Basin and the Altai mountains. There he heard of the Chinese as a settled and prosperous people who dwelt by a never-frozen sea[121].

ca 450 BCE **Bryson of Heraclea**[122] Mathematician. Discovered that the area of a circle is a limit to the increasing areas of inscribed polygons and to the decreasing areas of the circumscribed polygons, and as the number of sides of those two series of polygons is increased, their areas approach closer to the area of the circle on both sides of it. The method was actually applied by Archimedes, who measured the areas of two inscribed and circumscribed polygons of 96 sides each and reached the conclusion that

$$3.141 \approx 3\frac{10}{71} < \pi < 3\frac{1}{7} \approx 3.142.$$

ca 450 BCE **Philolaos of Tarentum** (*or Croton*). Greek mathematician and philosopher of the Pythagorean school. The first to propound the doctrine of the *motion of the earth*. He arrived at this conclusion on the basis of the

121 When Alexander the Great overthrew the Persian Empire (329 BCE) he did not venture beyond Bukhara and Southern Turkestan and did not attempt further conquests in Central Asia. The first efforts of China itself to establish relations with the West were made by the Emperior Wu Ti (of the Han dynasty) in 128 BCE. He sent an embassy to the Yuer-Chi (nomadic people) whose court lay near Bukhara; it provided China with the *geographic knowledge* on which it based the imperialistic policy in Central Asia. It let also to the introduction into China of the *vine* which the Greek brought into Bukhara and Samarkand. The Chinese exchanged this for gold and *silk*, then unknown in the Greco-Roman world during the first century BCE. In the second century CE, some Roman sailors actually reached China directly by sea, by rounding Cape Comorin in Southern India and passing through the Strait of Malacca.

122 In his book *"Ancient Science Through the Golden Age of Greece"* (1952), **George Sarton** made the observation that the mathematical genius of Greece manifested itself through the ideas of men who were not mathematicians in the restricted sense of today; they were philosophers and sophists who realized the fundamental importance of mathematics and tried to understand it as well as possible. They came from many parts of the Greek world, widely distributed across Hellas as was the artistic or literary genius: **Zenon** hailed from Magna Grecia, **Hippocrates** from Ionia, **Democritos** from Thrace, **Hippias** from Peloponnesos, **Theodoros** from Cyrenaica, **Bryson** from the Black Sea, **Antiphon** from Athens (the only one from that city) and **Archytas** from Sicily. This burst of mathematical creativity was not restricted to any locality – it was the genius of Greece.

following observations: the Sun, Moon, Venus, Mercury, Mars, Jupiter, Saturn – all travel slowly *across the stars* from West to East. On the other hand, the star pattern carries the whole lot daily from East to West.

This reversal, which spoils simplicity, could be removed if the view of the earth as the static center of the universe is abandoned. The new scheme of Philolaos was based on two elements:

(1) The earth is revolving around a central fire, making a small circle once every 24 hours. This accounted for the daily motion of the stars, sun, moon and planets without the need to assume that the earth was spinning about its axis (to explain why the fire was not seen from earth, it was necessary to assume further that its inhabited part was always facing outward, away from the fire).

(2) Seven spheres, carrying the sun, moon and 5 planets respectively, rotate *slowly* in the same direction as the earth around the central fire; the outermost crystal sphere of the stars is fixed.

This fantastic scheme was revolutionary in the sense that it treated the earth as a planet.

Philolaos was perhaps the first to suggest the earth's spherical shape. He inferred this from the circular shape of the earth's shadow cast on the moon during lunar eclipse. Philolaos and the Pythagoreans described the motion of the heavenly bodies by a rough but simple scheme that could be called a *theory*, in contrast with the more accurate working *rules* that were developed in Babylon. As a machine for making predictions, this first Greek system of uniform revolutions was hopelessly inaccurate; but as a frame of knowledge it was indeed superior: it gave a feeling that the heavenly scheme of things makes sense.

ca 450 BCE Development of the 12 constellations of the zodiac in Mesopotamia, recognizing the importance of the *plane of the ecliptic*.

ca 450 BCE **Hanno the Carthaginian**. Phoenician navigator and explorer. Made exploring and colonizing voyage down African west coast, reaching Gambia, Sierra Leone and perhaps Cameroon, or even Gabon. Hanno had no real successors until the Portuguese in the Middle Ages. They were to take over 40 years to accomplish what Hanno had achieved in a single voyage of only a few months.

Hanno described, among other things, the gorilla. Phoenician navigators are believed to have reached the Atlantic Ocean, sailing as far as Cornwall, England to the north where they established tin mines. They also circumnavigated Africa, to the south.

About the same time, another Carthagenian, **Himilco**, sailed out out Massalia (Marseille). In his voyage, which lasted four months, Himilco sailed northward round Spain. He reached the coasts of Brittany and Cornwall and thus firmly established a Phoenician monopoly of the flourishing trade in Cornish tin between Brittany and the Mediterranean[123].

450 BCE **Hippocrates of Chios** (ca 470–410 BCE). Greek mathematician and excellent geometer. Among the inventors of the method of 'reductio ad absurdum'. Was first to use letters in geometrical figures. Devised a method for the quadrature of certain lunes. He was the author of the first systematic treatise in geometry, and proved numerous geometrical theorems[124]. Though his work is now lost, it must have contained much of what **Euclid** later included in Books 1 and 2 of the *Elements*. Hippocrates' book also included geometrical solutions to quadratic equations and early methods of integration.

ca 445 BCE **Melissos of Samos** (ca 480–415 BCE). Philosopher and statesman. Disciple of *Parmenides* and a contemporary of Zeno. Commanded the fleet of Samos in victory over Athens, but after defeating Pericles was himself defeated (442–440 BCE).

The last member of the Eleatic school of philosophy, differing from Parmenides in maintaining the spatial infinity of the universe.

The extant fragments of his work defend Eleaticism against **Empedocles**' doctrine of the four elements, against the Atomists' belief in a void, against **Anaximenes**' derivation of the world from its original matter by rarefaction

[123] The *Phoenicians* emerged as a significant maritime power after the collapse of the Mycenaens in the 12^{th} century BCE, although by this date they had long been involved in trade between Mesopotamia and Egypt. In the civilized East Mediterranean they established extensive commercial contacts, but no colonies, with the possible exception of Kition (Larnaka) in Cyprus.
Indeed, it was only in the central and West Mediterranean outside the areas directly controlled by the Greeks, that the Phoenicians founded a network of colonies. The most powerful western colony was Carthage, founded in 814 BCE. The Phoenicians were the most adventurous merchants and explorers of the ancient world (*Ezechiel* **27**); in the early 6^{th} century BCE a Phoenician fleet reputedly circumnavigated Africa.

[124] Contributed to the old problem of the *duplication of the cube*, by reducing it to the finding of two mean proportionals between one straight line and another twice as long. In modern language we would say that Hippocrates has reduced the solution of a cubic equation to that of two quadratic equations.

and condensation, and against **Anaxagoras**' assumption of the reality of heat and cold.

The Sophists

* * *

"Man is the measure of all things, of those which are – that they are, of those which are not – that they are not".

* * *

"As to the gods, I cannot say whether they exist or not. Many things prevent us from knowing, in the first place the obscurity of matter, then the brevity of human life".

(Protagoras of Abdera)

The political and economical unrest in the interim period between the Persian wars (490–478 BCE) and the Peloponnesian wars (431–404 BCE) created a social climate of skepticism, disillusionment, uncertainty and despair. Intellectuals were perplexed by the conflicting doctrines of **Heraclitos**, **Parmenides**, **Anaxagoras** *and* ***Empedocles***. *Where could the truth be found? Is there any truth? And if there is any, can mortal man attain it?*

The most perplexing question of all was this one – to whom should one entrust the education of ones growing sons? The need for teachers was felt acutely, and it was now satisfied by a new class of them who were called

Sophists[125]*: professional teachers of grammar, rhetoric, dialectics, and eloquence who taught young men to behave themselves, to be wise and happy; the first to require students to pay for instructions.*

For nearly a century the sophists held almost a monopoly of liberal education and contributed much to advancement of learning and popularization of science. They practiced their mission in introducing the common man to a higher standard of culture – imparting to him the values and skills of sophisticated philosophical inquiry and communication and tasks enabling him to live more useful and better life.

The sophists build their philosophy on the doctrine of the relativity of truth. At first they where influenced mainly by ***Heraclitos*** *and his followers who rejected the idea of absolute truth and interpreted reality as a process of constant change. The leading sophist philosopher,* **Protagoras** *applied the idea of change to epistemology (the study of what knowledge is and how it is obtained). He concluded that knowledge and truth are both dependent on judgment by the individual.*

The Pythagorean concept of unchanging, mathematical relationships as the essence of reality and the Eleatic principle that nature obeys laws of permanent Being, that the laws of physics are fixed and permanent[126]*, were propositions analyzed critically by the Sophists. They inquired whether such*

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[125] From Greek $\sigma o \phi \iota \sigma \tau \eta' \varsigma$ = man of wisdom; the name given by the Greeks about the middle of the 5^{th} century BCE to certain teachers of a superior grade who, distinguishing themselves from philosophers on one hand and from artists and craftsmen on the other, claimed to prepare their pupils, not for any particular study or profession, but for civil life. Most of the Sophists were good men, yet other more conspicuous were moneymakers and hypocrites. As time went by, the number of bad teachers increased in numbers and the name Sophist acquired gradually a negative connotation.

For about 2300 years the Sophists have been despised and unjustly discredited as unscrupulous distorters of facts. It was **Friedrich Nietzsche** (1883–1887 CE) who rehabilitated them, and since then their contribution to philosophy can no longer be disregarded. Their foremost critic and adversary, **Socrates**, condemned their practice of accepting financial remuneration for teaching philosophy, stating that the search for scientific knowledge should never be debased in this way. **Plato** and **Aristotle** followed suite and induced posterity to condemn them as dishonest thinkers whose practices degenerated into hair-splitting of words, frivolous argumentativeness, and heuristic discussions designed to confuse the issue. Consequently the name *Sophist* which had earlier been applied to all philosophers (including even Socrates and Plato) became an opprobrious term.

[126] The Greek word for *physics* means unchanging nature.

unchanging laws of nature could be accepted as the basis for laws governing man. In other words: are there genuine moral laws, norms for evaluating human behavior, comparable to the laws which govern physical nature?

On the basis of their premise that knowledge was at best relative, they concluded that truth was unattainable. Because of his belief in the unavailability of absolute truth, the sophists turned his concern to the art of debate, to technique of convincing or converting ones opponent. Instead of trying to make truth prevail, the Sophist was interested only in winning an argument. Thus the Sophist's deep interest in grammar was motivated by the desire to manipulate it to serve his personal ends.

ca 450–410 BCE **Protagoras of Abdera**[127] (ca 485–410 BCE) Greek philosopher. Known as the first of the *Sophists*. One of the creators of Greek *rhetoric* and the *science* of *language*. He is credited with being first grammarian, distinguishing parts of speech, tenses and modes. He wrote numerous books, of which only four small fragments survived. Protagoras based his entire philosophy on the concept of *relativity of truth*. His doctrine can be summarized as follows: "Man is measure of all things..."[128]

- Sensation is the only source of knowledge. Man is capable of knowing what his sense tell him about what he perceives, not the thing itself perceived. Thus, sense knowledge is incomplete and not to be trusted, and man is wiser to be skeptic about everything.

- The subjectivity of sensation implies not only the *relativity of truth* (what is true for you is true only for you and what is true for me is true only for me) but also the *relativity of morals* (what is right for me may be wrong for you). From this follows, a most remarkable corollary, the

[127] *Abdera*, at the northern end of the Aegean Sea, was an ancient flourishing city. It gave birth to Democritos, Anaxarchos and Protagoras; it was the cradle of *atomic theory*.

[128] Some philosophers have interpreted this dictum as referring to man as an *individual*, while others have interpreted it as referring to a man in the *generic sense*, hence that ethical conduct and justice depend upon the moral codes of groups instead of the moral practices of individuals. In any case, either point of view had an enduring impact on the history of philosophy down to contemporary times.

doctrine of *equal rights* for all mankind, including women and slaves: the doctrine that each individual must decide for himself as to the validity of any proposition, leads to a principle of *social reform*, for it places all persons on an equal footing as judges of the truth. Protagoras therefore demanded that sweeping social and political reforms be instituted by democratic means.

- Human character can be improved by *education*, especially during childhood years. This capability is implied by the doctrine of the relativity of truth.

- God existed for those who wanted to believe in God.

Protagoras was born in Abdera, and when he reached the age of thirty he set out on his travels all over Greece as well as Sicily, lecturing and teaching. He lived and taught in Athens and became a friend of **Pericles** (495–429 BCE). In 411 BCE Protagoras was forced to flee Athens under sentence of death after having been accused of impiety. His books were publicly burned[129] (411 BCE). The ship that was carrying him to freedom was wrecked and he perished.

Another leader of the sophist school was **Gorgias** (ca 483–375). Born in Sicily, he settled in Athens (427 BCE) and supported himself by the practice of oratory and teaching rhetoric. He died at Larissa in Thessaly. He was the author of a lost work *On Nature or the Non-Existent*. Gorgias is the central figure in the platonic dialogue *Gorgias*.

440–410 BCE **Antiphon the Sophist** (480–411 BCE). An Athenian orator, who flourished at the same time as Socrates. He was a sophist, interested in many sciences. He made an early attempt to square the circle and invented a new method for the solution of that old problem – the *method of exhaustion*: Antiphon suggested that a simple regular polygon, say a square, be inscribed in a given circle: Then an isosceles triangle could be built on each side, its vertex being on the circumference. A regular octagon would thus be constructed, and continuing in the same manner one would easily construct regular polygon of $16, 32, 64, \ldots$ sides. Now it is obvious that the areas of each of those regular polygons approaches in the limit the area of the circle,

[129] This is the first recorded example of book burning. It suggests that there was already an established book trade in Athens at that time. The last event of this kind is the one ordered by Hitler on May 10, 1933 in the Third German Reich.

thus *exhausting* the area of the circle[130]. Since the area of each polygon can be exactly expressed in terms of repeated square roots of integers (i.e., the polygon can be "squared"), and since these areas increase in such a way that they are bounded by the said limit, the Sophists claimed that the limit itself is also squareable.

Antiphon was involved in an anti-democratic revolution which failed and, despite his profession as a writer of defense speeches, his brilliant speech failed to save his life when he was tried for treason, and he was executed.

440–399 BCE **Socrates** (469–399 BCE, Greece). Philosopher; One of the great Greeks who fashioned the traditional Western thinking system. Teacher of Plato. He embodied the revolt of Greek common sense against the intellectual extravagances of the early philosophers. Although he distrusted science, few men have contributed more to its development. His method of investigation prepared the elaboration of the method of inductive science. It was characterized by: insistence upon *clear definition*, use of *induction*, incessant war against vagueness of thought, deep sense of duty, *reasoned skepticism* (the very skepticism of the scientist who refuses to believe a thing until it has been proved to him).

Socrates held that scientific knowledge about the external world is not enough since it provides us only with universals, principles which hold true for us all in common. What is needed is self-knowledge, gained through self-examination; the principal value of scientific knowledge is simply to gain better understanding of oneself to enable one to live a better life. His question "who am I?" led him to the secret of self-control "Know thyself" (*gnothi-seauton*).

He was concerned with putting ethics on a firm basis so that the persuasive skills of the *Sophists* could no longer sway society. **Plato**, with his strong totalitarian tendencies, developed the notion of 'ideal forms' which was imposed on the world of his thinking. Later, **Aristotle** tightened up the system and showed its application to science. Plato took over Socrates' ideas and methods, reshaped them, added some of his own, and then gave them back to Socrates in the dialogues: what we call the 'Socratic method' is as expressed in Plato's writing, when the method is put into the mouth of Socrates by Plato.

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[130] In modern notation we would write for a unit circle: $S_{2^n} = 2^n \sin\left(\frac{\pi}{2^n}\right)$, $n = 2, 3, 4, \ldots$, with $\lim_{n\to\infty} S_{2^n} = \pi$. This method was criticized by **Aristotle** on the grounds that no matter how many times the number of sides of each polygon is doubled, the area of the circle can never be completely used up. The method of exhaustion was later perfected by Archimedes and led to an early determination of an approximation for π.

It consists of the '*endless search for truth through asking questions*'. To this we attribute the immense progress of *science*. This obsession with 'search' means that the quest may be more important than the destination itself, which is why Socrates was not at all bothered when his discourses reached no conclusion. The '*endless search*' has a great merit because it means the sort of divine dissatisfaction which is the *essence of progress*. The apparent advance of Western Civilization compared to some others have been due to this 'search' component of intellectual effort.

Socrates wrote nothing of his own. Most of our information about his life and teachings comes from the writings of **Xenophon**, **Plato** and **Aristotle**.

Socrates was born in a village on the slopes of Mount Lycabettus (20 minutes walk from Athens) and lived in Athens. Upon the outbreak of the Peloponnesian War (431), Socrates, then 38 years of age, was called to service as a haplite (private, with shield and sword), and distinguished himself in battle. He had an enthusiastic following among the young men of Athens, but the general public mistrusted him because of his unorthodox views on religion and his disregard of public opinion. Inevitably, Socrates made enemies among influential Athenians. He was brought to trial, charged with corrupting the young and showing disrespect for religious traditions.[131] Sentenced to death, he refused several opportunities to escape from prison, and carried out the sentence by drinking a cup of hemlock poison.

During his youth Socrates said to have met the aging **Parmenides** and learned much from him. According to Parmenides, the world as we know it is merely an illusion; the ever changing multiplicity we observe is merely the *appearance* of a static, all-embracing Being. Why them bother with the working of the world (i.e. science) when they are nothing but an illusion?

Socrates accepted this antiscientific attitude: Reality was an illusion. This had negative effects on him and his successor Plato. During their lifetime a few significant advances were made in mathematics, but, only because this was considered timeless and abstract, and thus thought to be in some way connected with the ultimate reality of Being. Fortunately their successor Aristotle drew philosophy back toward reality.

[131] The modern political philosopher **Leo Strauss** (1899–1973) in his book *Socrates and Aristophanes* (1966) shockingly admits (contrary to generations of liberal professors who have taught him as a martyr to the First Amendment) that the prosecution of Socrates was not entirely without point. Strauss argues that philosophy cannot really construct a rational basis for ethics and therefore has a tendency to promote *nihilism* in mediocre minds and they must be prevented from being exposed to it. Thus, philosophy (contrary to mythology) — matters.

It is most ironic that, although Socrates and his followers disdained science as pointless and considered it a waste of time, the most beneficial effects of his legacy have in fact been in the field of science. But Socrates, himself, sought to make ethics, politics and other social matters the subject of his scientific inquiry. He hoped to discover universal laws and truths (as in mathematics) which would put these subjects on an absolute basis and so rescue them from the manipulations of people like the Sophists.

The antiscientific attitude that developed with Socrates was to cast a blight on philosophy for centuries to come. Largely as a result of Socrates' antiscientific attitude, the few great scientific mind of the ancient Greek world worked *outside* philosophy. **Archimedes** (in physics), **Hippocrates** (in medicine), and to a certain extent **Euclid** (in geometry) were isolated from philosophy and thus from any developing tradition of knowledge and argument.

Ancient Greek scientists knew the earth went round the sun, knew it was round, and even calculated its circumference. They observed electricity and were aware that the earth had a magnetic field. Outside the "universal wisdom" of philosophy, such factual bits of knowledge were isolated oddities. The fact that philosophy came of age under the aegis of an antiscientist must count as one of the great misfortunes of human learning. The mental energy expended in the Middle Ages calculating the number of angels who could stand an the head of a pin might instead have been directed toward the solution of real-world problems.

Worldview I: Socrates

* * *

"The unexamined life is not worth living".

* * *

"Only those who have lived an evil life hope that death is the end of everything for them".

* * *

"Knowing nothing, what could I write down?"

* * *

"Do as you please, since whatever you do you will regret it".

(When someone asked him whether he should be married or not)

* * *

"The body is forever wasting our time with its demands".

* * *

"I know that I don't know".

* * *

"We go our separate ways I to die and you to live. Which one is better God alone knows".

* * *

"This discovery of yours will create forgetfulness in the learner's souls, because they will not use their memories; they will trust to the external written characters and not remember of themselves. The specific which you have discovered is an aid not to memory, but to reminiscence, and you give disciples not truth, but only the resemblance of truth; they will be hearers of many things and will have learned nothing...; they will have the show of wisdom without the reality".

(Lamenting the effects of writing
on the memory and the soul of the learner)

* * *

437 BCE Hospital, possibly earliest, was build in Sri Lanka (Ceylon).

ca 435 BCE The sculpture **Pheidias**[132] (ca 500-430 BCE, Greece) completed the *statue of Zeus* at *Olympia*, Greece, the most famous statue in the ancient world and one of the *seven wonders* of the world. The statue, 12 meters high, showed Zeus on his throne. Zeus' robe and ornaments were made of *gold*, and his flesh of *ivory*; he had a wreath around his head and held a figure of victory in his right and a scepter with an eagle in his left. Thousands who came to Olympia for the *Olympian Games* admired this gold and ivory figure.

The statue was toppled during the great earthquake of July 21, 365 CE, some 800 years after its erection. It was later removed to Constantinople (ca 429 CE) where it still could be seen for another century until its final demolition by the Christian authorities.

432 BCE **Meton of Athens**. Greek astronomer. Discovered 19-year *Metonic cycle* of solar years and synodic months.

In 432 BCE, Meton and **Euctemon** in Athens, made the first accurate solstitial observations. These observations enabled them to determine the length of the seasons with greater precision. They introduced in that same year a new cycle, called the *Metonic cycle*[133], a period of 19 solar years, equivalent to 235 lunar months; this implied a year of ca $365\frac{5}{19}$ days, that is $365^d 6^h 18^m 56^s$. (It is just $30^m 10^s$ longer than the Mean tropical year of $365^d 5^h 48^m 46^s$.) In the next 300 years the margin of error was steadily reduced, until Hipparchos (130 BCE) arrived at the figure $365^d 5^h 55^m 12^s$. Thus the accuracy of Meton was of the order of approximately 1:17,000, and that of Hipparchos 1:100,000.

431–404 BCE *Peloponnesian War*. A series of three wars fought between the city-states of Athens and Sparta.

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[132] **Pheidias** son of Charmides, universally regarded as the greatest of Greek sculptures. Born in Athens and studied under Agelades of Argos. According to Plutarch he was made an object of attack by the political enemies of Pericles, and died in prison at Athens. Pheidias was commissioned by Pericles to execute the greatest of the city's monuments. Among his notable works were sculptures on the *Parthenon* and the *Propylaea*, statue of *Athene Parthenos*, and statue of an *Amazon at Ephesos*.

[133] Our knowledge of this observation is obtained from a papyrus (now in the Louvre) called the *papyrus of Eudoxos*. It is probably a notebook of a student who flourished in Alexandria in ca 193–190 BCE.

The Peloponnesian league, consisting of Sparta and its allies, attacked the Athenian empire because it feared the growing power of Athens. The war ended in victory for Sparta which gained the support of Persia, helped subjects of Athens to revolt and forced the city to surrender.

Sparta was a military nation with good army but no fleet; Athens was a maritime power with a strong navy and a week army. Since her land defenses were almost invulnerable and ample supplies could be imported by sea, Athens could neither be brought to battle by land nor starved into submission. Fighting a defensive battle on land and offensive war at sea, she should have been able to defeat Sparta without great difficulty. During the first year of war Athens was successful both on land and sea, but her defensive policy necessarily led to the Athenians being crowded and besieged within their city walls.

Disaster struck in 430 BCE: A plague epidemic, starting in Ethiopia and traveling to Egypt, was carried across Mediterranean by ships to the Piraeus and Athens. It ravaged the city, killing as much as $\frac{2}{3}$ of the population and breaking the morale. The plague then broke out in the ships, decimating the naval expedition force and killing **Pericles** (429 BCE).

The plague of Athens undoubtedly contributed to the downfall of the Athenian empire. By killing so large a number, by demoralizing the capital and, above all, by destroying the fighting power of the navy, the plague prevented Athens from striking a decisive blow at Sparta. It thus provides a striking example of the effect of disease upon history.

431–400 BCE **Thucydides of Athens** (460–395 BCE, Greece). One of the greatest historians of all times. The father of scientific historiology. His history of the Peloponnesian War (of which he had been a witness, 431–404 BCE), is a literary and scientific classic. His description of the annular solar eclipse of Athens on August 3, 431 BCE, is the first *detailed* description of an eclipse (solar crescent, visibility of certain stars). In his history he described also the plague which lay waste to Athens in 430–425 BCE.

Thucydides was born in Athens. During the Peloponnesian War he commanded part of the Athenian fleet. He failed to relieve the siege of Amphipolis, and was exiled for 20 years. During his exile, he visited all parts of the Greek world.

430–420 BCE **Hippias of Elis** (460–ca 400 BCE). Greek sophist. Younger contemporary of Socrates. Introduced the first curve beyond the circle and the straight line later termed *quadratrix of Deinostratos*. It can be used to trisect an angle[134] and to square the circle (although Hippias probably did not know it). The quadratrix is the first curve known that cannot be constructed with straightedge and compass.

Hippias lectured in Athens on mathematics, music, astronomy, history, poetry and politics. He traveled all over Greece, giving public lectures and teaching, being a kind of a wandering sophist whose activities were dominated by love of fame and money. He was ready to discuss any subject but was especially interested in mathematics and science. His aim was not to give knowledge, but to provide his pupils with the weapons of argument in discussions on all subjects alike.

430–380 BCE **Hippocrates of Cos** (460–377 BCE, Greece). The father of medicine; one of the greatest clinical physicians of all times. His writings later provided scientific and ethical basis for modern Western medicine. He emancipated medicine from superstition, systematized the empirical knowledge which had accumulated in Egypt and in the schools of Cnidos and Cos, and founded inductive and positive medicine.

Hippocrates introduced the elements of the *scientific method*: he argued careful and meticulous observation: "Leave nothing to chance. Overlook nothing. Combine contradictory observations. Allow yourself enough time".

His principles of medical science formed the basis for the medical theory developed in the 19^{th} century. The *Hippocratic oath*, named for him, gave the medical profession a sense of duty to mankind which it never lost. Hipparchos maintained that the laws of nature could be discovered by studying facts and applying reason to them. He showed that disease had only natural causes,

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[134] The function is $r = \frac{2a}{\pi}\frac{\phi}{\sin\phi}$, where $r(\phi)$ is the polar equation for a curve with polar coordinates $\{r, \phi\}$. Consider a Cartesian x–y system, with a circle of radius a about its origin, and let ϕ be measured from the positive x-axis counterclockwise. From a general point $\{r, \phi\}$ in the first quadrant draw a line normal to the x-axis; divide the line in the ratio 2:1, with the larger section toward the curve. Draw a parallel to the x-axis through the point of division of the line. Connect the origin with the point of intersection of that parallel with the curve. The angle between this new radius-vector and the x-axis is $\frac{\phi}{3}$. **Deinostratos** was the brother of **Menaichmos**.

The impossibility of trisecting a general angle with straightedge and compass tended to direct attention away from problems involving the trisectors of angles. This helps account for the late appearance of the following delightful theorem: "*The adjacent pairs of the trisectors of the angles of a triangle meet at the vertices of an equilateral triangle*".

This was discovered only in 1904 by the Anglo-American geometer **Frank Morley** (1860–1937) and the theorem now bears his name.

and took treatment of disease out of the hands of religion. He treated his patients with proper diet, fresh air, change in climate, and attention to habits and living conditions.

Before the invention of the thermometer, he charted the temperature courses of many diseases. He recommended that physicians be able to tell, from present symptoms alone, the probable past and future course of each illness. He stressed honestly the limitations of the physician's knowledge, (confiding to posterity that more than half of his partners were killed by the diseases he was treating). His options of course were limited; the drugs available to him were chiefly laxatives, emetics, and narcotics. He objected to the use of strong drugs without careful tests of their curative values.

Hippocrates also used surgery, but only as a last resort. His period was one of great intellectual development, and he certainly brought to bear upon medicine the same influences which were at work in other sciences by such contemporaries as **Socrates**, **Herodotos**, **Democritos**, and **Plato**[135].

He was born on the island of Cos of a family of priest-physicians. Democritos of Abdera was one of his masters. Hippocrates began his medical studies at Cos and Cnidos[136]. He traveled extensively, and taught and prac-

[135] Considerable further advances were made in classical times through the fall of Rome. While the medicine in the Islamic world flourished, what followed in Europe was truly a dark age. Much knowledge of anatomy and surgery was lost. Reliance on prayer and miraculous healing abounded. Secular physicians became extinct. Chants, potions, horoscopes, and amulets were widely used. Dissection of cadavers was restricted or outlawed, so those who practiced medicine were prevented from acquiring firsthand knowledge of the human body. Medical research came to a standstill.

[136] The great doctors recognized the spiritual side of healing; when all else failed, they were quite happy for their patients to attend one of the many *asclepieia* – temples to the patron god of physicians, *Asclepios* (*Aesculapis* to the Romans and realy the Egyptian **Imhotep** in a Greek dress), and his daughters **Hygeia** (health) and **Panacea** (healing). [Asclepios would often be accompanied by a snake, the *dracon* – hence the medical symbol of a snake rounding a staff.]

Pilgrims to the temples relaxed among beautiful surroundings and read inscriptions on marble pillars that told of the miraculous cures performed by god. Then they would bed down for the night in the sacred hall, where Asclepios would supposedly appear as they slept, to give them a 'dream drug' or even perform 'dream surgery'. These rites were derived from Egyptian models. Thanks to them, a large number of clinical observations were concentrated in the temples, especially in Cos and Cnidos.

Without those abundant clinical cases such as were afforded by the Asclepieia, ticed his profession at Athens, Thrace, Thessaly, Delos and his native island. He died at Larissa in Thessaly.

ca 425 BCE **Theodoros of Cyrene**. (ca 465–399 BCE) Mathematician and philosopher. Showed that the noninteger roots $\sqrt{3}$ up to $\sqrt{17}$ were irrational. The proof of the irrationality of $\sqrt{2}$ is quoted by **Aristotle** (384–322 BCE), but its originator is unknown. It is possible that $\frac{1}{2}(\sqrt{5}-1)$, which is the ratio of a side to a diagonal of a regular *pentagon*, was the first known irrational.

Theodoros was born in Cyrene, which was then a flourishing Greek colony just south of Greece on the North African coast. He was the teacher of both Plato and **Theaitetos**, starting his life as a philosopher and then switching to mathematics.

Plato stated in *Theaitetos* that Theodoros discussed the irrationality of $\sqrt{2}$, $\sqrt{3}$ ... and stopped at $\sqrt{17}$.

ca 420 BCE **Democritos of Abdera** (ca 460–370 BCE). A Greek physical philosopher and mathematician. The father of *Materialism* . Synthesized the ideas of Parmenides and Heraclitos and is the intellectual forerunner of the modern philosophers Locke and Descartes. Archimedes claimed that Democritos (ca 410 BCE) stated that the volume of pyramid on any polygonal base is 1/3 that of a prism with the same base and altitude[137].

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the progress of medicine would have been considerably slower. Thus, the Asclepieia were the cradles of Greek medicine, and they help to account for the extraordinary richness of the Hippocratic collection, which themselves inherited and continued Egyptian tradition.

The Greek believed in the existence of 4 fluids, or humors, within the body, the *balance* of which was vital for health. The humors correspond to the 4 elements, and had the same qualities. They were also associated with particular parts of the body: Air $\leftrightarrow$ blood; Water $\leftrightarrow$ brain; Fire $\leftrightarrow$ liver; Earth $\leftrightarrow$ spleen; the corresponding humors were: blood, phlegm, yellow bile; black bile.

[137] Although Democritos could have hardly render a rigorous demonstration of this theorem, he still have *guessed* the result following two logical steps:

1. A prism can be dissected into a sum of prisms all having triangular bases, and, in turn, a prism of this latter sort can be dissected into 3 triangular pyramids having, in pairs, equivalent bases and equal altitudes. It follows that the crux of Democritos' problem is to show that two pyramids of the *same* height and *equivalent bases* have equal volumes (this was later demonstrated by Eudoxos of Cnidos, using the method of exhaustion).

2. If two pyramids with equal heights and equivalent bases (same area but not necessarily same form) are cut by planes parallel to the bases and dividing the

A pupil of Anaxagoras and Leucippos, a friend of Philolaos, and an admirer of the Pythagoreans. Studied in Egypt the mathematical and physical systems of the ancient schools. Diodoros Siculus tells us that he lived to be ninety years old.

Taught that mechanical relationships (arrangements of atoms) account for various characteristics of nature i.e. mechanistic causes account for all phenomena. Even morality, the soul, and all mental life are reducible to mechanistic terms with physical imperceptible atoms as their basic structure.

Spiritual reality does not exist: mechanistic Materialism is complete, self-sufficient and self-contained.

Atoms, owing to their non-sensory nature and small size, can only be thought, not directly observed. Nevertheless, these imperceptible atoms account for all observable phenomena of nature which are manifest to the senses. But sense experiment produces multiplicity of options because people receive and interpret sensory phenomena from different perspectives. Knowledge derived from the senses is therefore *relative to the person from whose experience it originates*. Therefore one can never use the senses to attain *truth* (knowledge of metaphysical reality[138]).

In his own words: "*Opinions say hot and cold, but the reality is atoms and empty space*". He believed that at every moment stars are colliding and new worlds are rising out of 'Chaos' by the selective aggregation of atoms, that there is no design and that the universe is a machine. He vaguely anticipated the notions of conservation of matter and energy and was first to discover the correct formulae for the volumes of a pyramid and a cone.

Democritos thus stated that all material is built out of indivisible atoms moving about in the microvacuum. He also believed that space is infinite, having always existed, and that the number of atoms is infinite. Records state that Democritos ranked intellectually equal if not superior to Plato and that his works, now vanished, were as comprehensive as those of Aristotle.

After Democritos, his school rapidly passed into near oblivion and its followers diverted in into *Sophism*.

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height in the same ratio, then the corresponding sections formed are equivalent. Therefore the pyramids contain the same infinite number of equivalent plane sections, and hence must be equal in volume.

[138] Thus, 2300 years before Planck, Albert Einstein, Bohr, Born and Feynman, Democritos already foreshadowed the ontological difficulties that *quantum physics* posed to 20^{th} century physics.

ca 400 BCE Magnetic needles were used on Chinese ships. The mariner's compass is said to have been known to the Chinese as early as 1100 BCE, though it was not introduced into Europe until ca 1000 CE. Prior to 1600 CE nobody knew the true reason for the orientation of the needle.

ca 400 BCE The diseases of malaria and gout first identified or described with accuracy.

400 BCE Diminished rainfall caused endemic malaria in ancient Greece, undermining people's vitality.

400 BCE Oil was drilled on one of the Greek islands.

390 BCE **Theaitetos of Athens** (ca 415–369 BCE). A mathematician of great stature of the time of Plato. Studied with Theodoros of Cyrene and at the Academy. First to study the *octahedron* and the *icosahedron*. He fell on the battlefield (369 BCE) in a war between the Athens-Sparta alliance against the Theban army. One of Plato's dialogues is dedicated to his memory.

Theaitetos introduced the exact concept of *commensurability in length* and proposed that line segments, which produce a square whose area is an integer, but not a square number, are incommensurable with the unit of length. He was the first to write on the 5 *regular polyhedra* and the first to construct them, and finally, he formulated the theory of proportions. Since Euclid's Book X contains a detailed mathematical development of matters briefly indicated in Plato's dialogue, it is believed that a number of Euclid's propositions on the subjects of commensurability and proportion were originated by Theaitetos.

390–220 BCE *The Celtic-Roman Wars*. By about 400 BCE, Etruscan power had already started to wane, largely as a result of the occupation of most of Northern Europe by the *Celts* (ancestors of the Bretons, Welsh, Scots, Irish and the French). One group of Celts, the *Gauls*[139], crossed the Alps into Italy and pushed the Etruscans out of the Po Valley. From here they attacked Etruscan cities further south. The presence of the Celts in Northern Italy blocked the north-south trade routes and slowly choked the Etruscan economy. In 391 BCE, the Etruscan asked Rome for help and a military encounter between the two archenemies of the Etruscans became inevitable.

On July 18, 390 BCE, 70,000 Celts under Brennus defeated the Roman army at Allia and captured Rome. They sacked and burned the city and held all of it except the capitol. The Celts could have, there and then, destroyed the city-state, changing at one fell swoop the course of worlds history, but

[139] The origin of the district names of *Galicia* in Spain, Turkey and Poland derived from *Gaul*.

they sold their victory for ransom gold and withdrew after seven months. This they would dearly regret 65 years later. Indeed, during that time-interval, the Romans improved their weapons, military skill, discipline and administration.

In 225 BCE, a grand army of 70,000 Celts was crushed by an equal-sized Roman army under Marcellus at Telamon. *The Battle of Clastidium* (220 BCE) finally removed the Celtic treat on Rome and eventually caused the Celts to leave Italy.

387–360 BCE **Archytas of Tarentum** (ca 428–347 BCE). Greek philosopher, scientist, mathematician and distinguished in the administration of civic affairs. Occupies a high place among the versatile savants of the ancient Greek world. An intimate friend of Plato who quoted him as an example of the perfect ruler, the philosopher-king who combines practical sagacity with high character and philosophic insight. It was through him that Plato received his initiation in the exact sciences and Pythagorean philosophy. Aristotle wrote a special treatise '*On the Philosophy of Archytas*'.

He is described as the 8^{th} leader of the Pythagorean school, and was a pupil of Philolaos.

He was elected seven times to command the army. Under his leadership, Tarentum fought with unvarying success against the Messapii, Lucania, and even Syracuse. After a life of high intellectual achievement and uninterrupted public service, he drowned while on a voyage across the Adriatic.

In mathematics, he was the first to draw up a methodical treatment of *mechanics* with the aid of geometry and for that reason he is sometimes called the *founder of mechanics*. He first distinguished *harmonic* progression from *arithmetical* and *geometrical* progressions. He contributed many original theorems to geometry and new ideas to the study of music and acoustics. For, besides computing the numerical ratios for the new musical scales by means of systematic applications of the arithmetic and the harmonic means, he also laid the number-theoretical foundations for the theory of music which is found in Euclid's "*Sectio Canonics*".

Archytas is said to have been the inventor of a kind of *flying machine*, a wooden pigeon, balanced by a weight suspended from a pulley, and set in motion by compressed air escaping from a valve.

387–347 BCE **Plato** (427–347 BCE). One of the most influential thinkers in Western culture. Regarded as a father of traditional Western philosophy, who gave civilization a powerful thinking method. Philosopher and mathematician. Rejected the experimental method with ardor and contempt. In his view, no precise study of the ever changing phenomena in the natural universe was possible, and it was only in the philosophic theory of forms and in the science of pure mathematics that absolute knowledge could be attained. These contemplative disciplines of the intellect dealt with objects *timeless* and *invariant*, known independently of experience and existing logically prior to the material world, which could at most be merely an approximation to eternal forms or ideas.

Plato posed the three most basic questions of philosophy:

- How can man discover the truth?
- What is the origin of the Universe?
- What is the purpose of human life?

The answers Plato gave to these questions laid the foundation to a system of philosophy called 'Objective Idealism', according to which all things are the mere shadows of *ideas*[140]. Ideas are eternal, while things are transitory: true knowledge comes neither through perception nor reason but only via 'inspiration' arriving from beyond. We must look on this world as only the image, the shadow of an invisible system; the universe we see is based on ideal forms, which are imperfectly embodied in various objects. Since these ideal forms are ideas, they cannot be perceived by the senses, but only uncovered by the use of reason, guided by critical use of logic.

In his dialogue *Timaeus* Plato gave an account of his ideas about the creation of the universe; he maintained that every theory was of necessity based on ideas originated by the scorching intellect, *Science* is never able to reach any conclusion with absolute certainty. All the phenomena that we can perceive with our senses, the objects of physical science, provide only a picture of transcendental world of ideas , which represented for Plato the *real* world. Whereas the physical world is subject to continuous changes and consists of passing phenomena, the actual reality of the world of ideas is permanent. *Plato thus refused to accept the absolute character of science.*

Plato admitted in *Timaeus* that science may have a certain amount of precision. But in that case it must use *mathematics*, which was for him the natural language of science. Deeply influenced by the Pythagorean school, Plato realized the powerful creative force of mathematical formalism. Mathematics is somehow in the middle between the world of ideas and the observable

[140] One of Plato's strong opponents was **Antisthenes of Athens** (c. 444-365 BCE), founder of the *Cynic* school of Greek philosophy. He was a faithful student of Socrates and was present at the latter's death. Ridiculing Plato's doctrine of Ideas he said: "I am able to see the horse, but I cannot see his horse-ness".

world. His emphasis on the paramount importance of mathematics for science is widely considered one of the most important and far-reaching contributions to the development of human thought.[141]

In the *Timaeus* Plato formulates a cosmology and creation story consistent with this concept of human knowledge. At the beginning of time, a beneficent creator used the eternal ideas or forms to mold *preexisting, chaotic matter*[142] (like Pythagoras, Plato believed that the ultimate basis of these forms was mathematical[143] and geometrical). The creator molded matter into approximations of these ideal shapes, creating a universe ruled by eternal mathematical laws, laws which humans can deduce through reason. *These eternal mathematical laws are the true reality*, while the changeable universe we see is mere appearance – the observation of nature is thus unreliable. Ergo men cannot know truth by means of science.

Plato emphasizes the *ethical* implication of this distinction: the ideal forms are the source of all good, while base, earthy matter is the source of the world's evil. The mundane, changeable world of everyday life cannot be used to understand the eternal, perfect, and unchangeable heavens. The most perfect motion, circular motion, occurs only in heaven, not on earth. Plato thus developed a new mode of thinking about the universe and creation. Against the traditional appeal to authority, Plato counterposes the power of human reason. But he *attacks observation as a route to knowledge* and strictly separates the worlds of thinking and doing, the spirit and the flesh, the heavens and earth. He thereby created a mathematical myth, a formidable barrier to the development of science.

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141 The views of Democritos on the atom were generally accepted as a dogma by 19^{th} century scientists. But Planck's observations on thermal radiation (1900 CE) where difficult to reconcile with the prevailing notion of the atomic structure of matter, thereby reviving Plato's notion in science, with strong emphasis on the belief that *mathematical laws* underline the structure of matter.

142 In this sense, Plato's creation story holds important similarities with the story of *Genesis* **1**.

143 In Book 5 of his *Laws*, Plato gives $7! = 5040$ as the population of an ideal city: it has 58 proper divisors (without 1 and 5040), which makes for efficient division of the population for purposes of taxes, land distribution, war and so on. (Plato was not aware that 7560 and 9240 have 62 proper divisors each – the maximum possible number of 4 and fewer digits). In Book 12, Plato cites 3 and 18 as the most difficult sums to roll with three dice: they are the only sums that can be made in only one way, 1–1–1 and 6–6–6. Since there are $6^3 = 216$ equally probable ways of rolling three dice, the probability of making 3 or 18 is $\frac{1}{216}$.

Plato conceived his view of the universe as consistent with, and reinforcement for, his concept of the ideal society. In that society, outlined in the dialogue of *The Republic*[144], all thought is to be done by philosopher-kings, aided by a small elite of guardians. No one else has political or social rights. As ideas and matter, heaven and earth, are separated at creation, so guardians and philosopher-kings must be separated from those who work: slaves are to work without thinking, and philosopher-kings are to think without working. As the creator gave eternal mathematical laws to the universe, so the philosopher-king give laws to society[145].

His real name was **Aristocles** and Plato was a nickname (broad-shouldered). After the death of **Socrates** (399 BCE) he left Athens and

[144] From the vantage-point of the turn of the 21^{th} century Plato can be regarded as *centralist*, *totalitarian* and *authoritarian*. His rigid rules, harsh judgments, high degree of righteousness and category boxes remind us of the symptoms of certain totalitarian regimes in our own times.

In his *Republic*, Plato suggested that society is to be ruled by a special class called *Guardians*. These are originally to be soldiers who take over government. They, in turn, are divided into *Rulers*, who make policy decisions and the *Auxiliaries* (police etc.) who carry out the policies. Ordinary people are to have no say in government whatsoever. The Guardians are a sort of hereditary caste who are to be bred on strictly scientific lines. Families and private wealth are distractions and are to be abolished. The *State* is to come first. There is to be *censorship* in the arts and in the materials allowed into education. Nothing that might threaten the State is to be permitted. The whole purpose of education is to produce a small 'elite of Guardians'. *Breeding* is to be arranged through special marriage festivals. It is thus not surprising that there are strong echoes of Plato in the Marxists approach to state and government and that the Nazi party in Germany had as one of its official aims the production of 'Guardians of the highest Platonic ideals'. But this is where the similarity ends. Plato was a good philosopher who meant well and did not seek power for himself. Moreover, he was *not* advocating bully-boy fascism and was against the rule of wealth. What he wanted was competence.

[145] Plato's *Republic*, a rejection of Athenian democracy, was modeled on Sparta, where a small body of landlords ruled over a mass of rightless serfs, or helots. Sparta had defeated Athens in the 30-year long Peloponnesian War, begun in the year of Plato's birth, 428 BCE. Deprived of its colonies in the wake of defeat, Athens erupted in social conflict as rich landholders battled freeholders and artisans. To protect themselves from the growing demands for abolition of debts and land distribution, the landholders sought to combat political democracy and to erect a hierarchical society. Plato became the theoretician of this new society, rationalized in *The Republic* and justified by the cosmology expounded in *Timaeus*.

traveled widely for several years through the ancient world. In 387 BCE he returned to Athens and founded there a school of philosophy and science known as the *Academy* (after the Greek hero Academus) which lasted under various forms until 529 CE. It can be considered as the first university. Subjects such as astronomy, biological sciences, mathematics and political science were investigated at the Academy. While the influence of **Socrates**[146] (ca 469–399) in the development of mathematics was negligible (if not actually negative) that of Plato was substantial, since the Academy became the mathematical center of the world. His '*Theory of Forms*' [147] is a remarkable precursor of modern *physical* theories on two counts: (1) the notion that everything can be reduced to geometry, a view held by Descartes and in a different way by Einstein; (2) the notion that the elements (atoms of Democritos) are made of still smaller basic entities ('triangles' in his language, which are evidently what in modern physics are called nuclear or elementary particles or even quarks). Plato believed that the planets move in circular orbits around a stationary earth.

In addition, Plato accepted the Pythagorean doctrine that the world is ultimately intelligible in terms of numbers. Altogether therefore, his method is that of mathematical modeling of the physical world, which is the aim of mathematical physics today. In mathematics proper, Plato introduced rigorous definitions (straight line; plane surface) and began the study of the *golden section*. He gave a new rule to find square numbers which are the sum of two squares $\left[(2n)^2 + (n^2-1)^2 = (n^2+1)^2\right]$ and the five regular solids are named after him.

It must be remembered, however, that the Pythagorean school and Plato fostered the idea that rational order and harmony lie at the base of all things – quite independently of the human mind. This and similar thoughts were organized in a systematic philosophical edifice.

Plato's concept of eternal mathematical laws is two-sided. The belief in such laws and the search for them has been immensely important to science.

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[146] This great philosopher championed clear thinking with careful definitions, and condemned the astronomers for their wild conjectures. Thus he probably helped astronomy towards becoming an inductive science that extracts its data from observations.

[147] Before Plato, Philosophers like **Parmenides** and **Heraclitos** had been wrestling with the problem of *change*. Heraclitos believed that all was change: you can never stem into the same river twice. Parmenides believed that there was an unchanging inner core. Plato put both of these views together with his theory of 'inner forms' or 'essence'. Those were absolute and fixed while the surficial matter could change.

But scientists have had two contrasting views about what these laws are: One view follows Plato and believes that the laws truly rule the universe, that the universe is the embodiment of abstract mathematics. The other, quite different view, is that mathematical laws are *descriptions* of physical processes and patterns of nature – the reality is a process *described* by mathematics, the language of exact science.

It took mankind another 1600 years to liberate itself from Plato's total reliance on the regularity and perfection of the material universe and advocate systematic observations, experimentation and the reliance on mathematical rigor.

Plato Versus Democritos

The concept of the atom was first proposed by Democritos and Leucippos (ca 460–420) BCE. They considered their hypothetical atom the smallest indivisible unit of matter, eternal and indestructible. The atoms of Democritos were all of the same substance, but had different sizes and different shapes.

Plato finally rejected the idea of the atom proposed by Democritos and Leucippos. For him the smallest parts were geometrical forms: *Those of the earth he compared with cubes, of fire with tetrahedra, of air with octahedra, and of water with icosahedra. Moreover, he did* not *consider these smallest part indivisible. The elements can be transformed*[148] *into each other, they*

[148] Anaximander (ca 560 BCE) was the first to envisage the possibility of the *transformation* of one primary substance into another. **Heraclitos** (ca 500 BCE) assumed that *fire* is the basic element, which is both matter and moving force. **Werner Heisenberg** (1954) pointed out that if the word *fire* is replaced by *energy*, modern atomic physics is in some way extremely close to the doctrine of Heraclitos: Energy – or more precisely, quantum fields – is in fact the substance from which all things, and even the vacuum itself, are considered to be made in modern physics; energy is what moves (the famous $\pi'\alpha\nu\tau\alpha \quad \chi\omega\rho\epsilon\iota$), and even in classical (pre-quantum) physics energy is transmutable into motion, heat, light and tension. Einstein's Special Relativity, of course, taught us that *energy* and *mass* (the latter quantifying *inertia* – the tendency *opposing* motion) are, in a deep sense, inter-convertible.

can be taken apart and new regular solids can be formed by them; for Plato, the form *was more fundamental than the* substance *of which it was the form.*

The ideas of Democritos prevailed in physics and chemistry until the turn of the 20th century, soon to be modified through the advent of quantum physics: **Planck**, ***Heisenberg***, **Pauli** *and other leading atomic physicists were greatly influenced and inspired in their philosophy of science by Plato and Neoplatonism. They considered themselves to be much nearer to Plato and the Pythagoreans then to the materialistic view of Democritos. The elementary particles (such as* protons, neutrons, electrons, mesons, quarks *and the like) are not eternal and indestructible units of matter, but rather, interconvertible (in certain allowed combinations) via absorption and release of motion-energy. Furthermore, one of these particles is the photon – a particle of light ("pure energy").*

As modern physics has indeed shown, these particles can be transformed into each other (in the laboratory or in natural radioactive processes) through collisions or instability. Such events support the idea that all particles are made from the same substance, namely dynamical quantum fields, of which a particle is but one manifestation. These modern views resemble in a remarkable way those of Plato as expressed in Timaeus. *In the last analysis, the basic building blocks of matter a-la-Plato are not substance but* mathematical forms.

In modern atomic physics elementary particles are also considered as forms, albeit of a much more complicated nature. In Greek philosophy these forms were considered as static; *modern physics stresses their* dynamic *nature. There are in few fields so many strikingly similar ideas, despite basic differences, as are encountered when comparing Greek philosophy with modern theories of the atom.*

384–100 BCE The concept of 'parallelogram of velocities' was known to **Aristotle** (384–322 BCE), **Archimedes** (287–212 BCE) and **Hero of Alexandria** (ca 100 CE, author of '*Automata*', first book on robots).

373 BCE A major earthquake followed by a devastating sea wave emanated from the Corinthian Gulf (38.3 °N, 22.1 °E).

ca 370–355 BCE **Eudoxos of Cnidos** (ca 408–355 BCE, Asia Minor). A distinguished Greek mathematician, astronomer, orator, legislator, philosopher, geographer, and medical man. The method of Exhaustion, credited to him, is a mathematical scheme similar to the integral calculus, although formally quite different. It is the Platonic school's answer to the paradoxes of Zeno. The method assumes the infinite divisibility of magnitudes and has, as a basis, the proposition: "*If from any magnitude there be subtracted a part not less than its half, from the remainder another part not less than its half, and so on, there will at length remain a magnitude less than any preassigned magnitude of the same kind*".

The method is rigorous but sterile, because it does not enable one to calculate the result. Archimedes however, discovered a procedure which he established by the method of exhaustion. His idea was to cut up his area, or volume, into a very large number of parallel plane strips, or thin parallel layers and (mentally) hang these pieces at one end of a given lever in such a way as to be in *equilibrium* with a figure whose content and centroid are known. With our modern method of limits this method can be made perfectly rigorous, and is identical with present day integration.

Eudoxos was the founder of scientific astronomy. He constructed a mathematical model of motions of heavenly bodies, with 33 concentric spheres rotating around a stationary earth. The combination of these motions succeeded in imitating the actual motions of the sun, moon and even planets across the fixed stars. His work was the first astronomy which broke away from philosophical speculations and sought to build a mathematical model to fit observations.

Eudoxos studied mathematics with Archytas in Tarentum, and medicine with Philistrium on the island of Sicily. When he was 23 years old, he went to Athens to learn philosophy and rhetoric. He was so poor that he had to live in the harbor-town Piraeus, a walk of two hours each way from Plato's Academy. Some years later his friends enabled him to undertake a journey to Egypt. From Agesilaos, King of Sparta, he received a letter of recommendation to the Pharaoh Nectanebus. In Egypt he learned astronomy from the priests of Heliopolis and he made observations himself in an observatory, situated between Heliopolis and Cyzicus on the sea of Marmara, which attracted a large number of pupils.

Around 365 BCE Eudoxos came once more to Athens with his pupils. He held discussions on philosophical questions with Plato who did not agree with some of his views and ideas. He died in his native town of Cnidos on the Black Sea, highly renowned and honored.

The story of Greek Infinitasphobia

The Pythagoreans could not accept $\sqrt{2}$ *as a number, but no one could deny that it was the diagonal of the unit square. Consequently, they believed that geometrical quantities must be treated separately from numbers or, rather, without mentioning any numbers except rationals. Greek geometers thus developed ingenious techniques for precise handling of arbitrary lengths in terms of rationals, known as the* theory of proportions *and the* method of exhaustion.[149] *The rejection of irrational number by the Greeks was just part of a general rejection of infinite processes.*

Aristotle *said that infinity exists only potentially, not in actuality. In fact, until the late 19*th *century*[150] *most mathematicians were reluctant to accept infinity as more than 'potential'. The infinitude of a process was understood as the possibility of its indefinite continuation without its eventual completion. (e.g. the paradoxes of Zeno; ca 450 BCE)*

Thus, the Greek accepted that the sequence of natural numbers 1,2,3,... is a potential infinity. They also understood, that one can generate a sequence of rational numbers according to a definite rule that approaches $\sqrt{2}$ *in the limit*[151]*, yet they were afraid to draw the final logical conclusion even in the face of the* geometrical *evidence that such limits do indeed exist.*

Nevertheless, the fear of infinity forced the Greek mathematicians to seek the infinite in the finite*, that is, devise a calculus devoid of infinitesimals, through which they could successfully compute areas, volumes and lengths of arcs. This was perhaps the Platonic school's answer to the paradoxes of Zeno. The names of the persons who achieved this feat were* **Antiphon** *(480–411*

149 When theses techniques where reconsidered in the 19^{th} century by **Dedekind**, he realized that they provided an *arithmetical* interpretation of irrational quantities after all. It was then possible to reconcile the apparent conflict between arithmetic and geometry.

150 With the exception of the 18^{th} century, the great age of the infinitesimals; then, no barrier between mathematics and physics was recognized, since the leading physicists and the leading mathematicians were the same people.

151 The first mathematical process we would recognize as infinite was devised by the Pythagoreans via the recurrence relations $x_{n+1} = x_n + 2y_n$; $y_{n+1} = x_n + y_n$, for generating integer solutions of the equations $x^2 - 2y^2 = \pm 1$. It is likely that those relations arose from an attempt to understand $\sqrt{2}$ and it is easy to see that $\lim_{n\to\infty}(\frac{x_n}{y_n}) \to \sqrt{2}$.

BCE), **Eudoxos** *(ca 408–355 BCE) and* **Archimedes** *(287–212 BCE) The method was advanced in two stages:*

- *Theory of proportions: enables length to be treated as precisely as numbers while only admitting rational numbers. The objectives of* ***Eudoxos*** *was to be able to compare two ratios to determine whether they are equal, and if they are not, which is larger in a way that is equally valid for ratios of commensurable and incommensurable magnitudes. This resulted in the statement: "The ratio $\frac{a}{b}$ exists when whole numbers m and n can be found such that $ma > b$ and $nb > a$". [i.e. $\frac{1}{m} < \frac{a}{b} < n$].* ***Euclid*** *included this statement among his 'definitions' in book V.* **Archimedes***, who used it extensively, regarded it as an axiom. He attributed it to* ***Eudoxos****, and it is now known as the Eudoxos-Archimedes axiom. Its virtue is that it excludes not only zero but any idea of infinity – either the infinitely large or the infinitely small.*

  Eudoxos went on to state a criterion for two ratios to be equal (in modern terminology): "If a, b, c, d" are four given magnitudes, then $\frac{a}{b} = \frac{c}{d}$ iff, given any two positive integer, then:

  $$\begin{array}{l}(1)\ ma > nb \text{ implies } mc > nd\\ (2)\ ma = nb \text{ implies } mc = nd\\ (3)\ ma < nb \text{ implies } mc < nd\end{array}$$

  If the magnitudes are commensurate, (2) is sufficient, both ratios being equal to the rational number $\frac{n}{m}$. The subtlety of the theory lies in (1) and (3), because (2) never holds for incommensurable magnitudes. Out of two inequalities the condition for equality somehow emerges. Eudoxos goes on to deal with unequal ratios by asserting that if two numbers m and n exist such that $ma > nb$ and $mc < nd$ then $\frac{a}{b} > \frac{c}{d}$.

  The above three Eudoxian assertions (or axioms) provided a firm foundation from which to extend the Pythagorean treatment of the ratios of whole numbers to deal with incommensurable magnitudes, while avoiding the notion of irrational numbers on the one hand and providing the basis for a generalization into the method of exhaustion, on the other.

  The theory of proportions was so successful that it delayed the development of the theory of real numbers for 2000 years! This was ironic, because the theory of proportions can be used to define irrational numbers just as well as can lengths[152].

[152] Any *arithmetic* approach to $\sqrt{2}$, whether by sequences, decimals, or continued fractions, is infinite and therefore less intuitive. Until the 19^{th} century this seemed a good reason for considering geometry to be a better foundation

For the measurement of the circle, Archimedes needed to calculate the value of $\sqrt{3}$*. He did it by a clever uses of the method of proportions in the following way (modern notation):*

$$2^2 = 3 \times 1^2 + 1 \qquad \therefore \quad \left(\frac{2}{1}\right)^2 = 3 + \frac{1}{1^2}$$

$$7^2 = 3 \times 4^2 + 1 \qquad \therefore \quad \left(\frac{7}{4}\right)^2 = 3 + \left(\frac{1}{4}\right)^2$$

$$26^2 = 3 \times 15^2 + 1 \qquad \therefore \quad \left(\frac{26}{15}\right)^2 = 3 + \left(\frac{1}{15}\right)^2$$

$$97^2 = 3 \times 56^2 + 1 \qquad \therefore \quad \left(\frac{97}{56}\right)^2 = 3 + \left(\frac{1}{56}\right)^2$$

$$362^2 = 3 \times 209^2 + 1 \qquad \therefore \quad \left(\frac{362}{209}\right)^2 = 3 + \left(\frac{1}{209}\right)^2$$

$$1351^2 = 3 \times 780^2 + 1 \qquad \therefore \quad \left(\frac{1351}{780}\right)^2 = 3 + \left(\frac{1}{780}\right)^2$$

Therefore

$$\frac{2}{1} > \frac{7}{4} > \frac{26}{15} > \frac{97}{56} > \frac{362}{209} > \frac{1351}{780} = 1.7320510... > \sqrt{3}$$

Likewise, the numerical identities

$$5^2 = 3 \times 3^2 - 2 \qquad \therefore \quad \left(\frac{5}{3}\right)^2 = 3 - \frac{2}{3^2}$$

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for mathematics than arithmetic. Then the problems of geometry came to a head, and mathematicians began to fear geometric intuition as much as they previously feared infinity. There was a purge of geometric reasoning from the textbooks and an industrious reconstruction of mathematics on the basis of numbers and sets of numbers.

Indeed, **Dedekind**(1872) introduced a partition of the positive ratios into sets L, U such that a member of L is less than any member of U. This definition of a positive real number, now known as *Dedekind cut* [e.g. $L(\sqrt{2}) = \{r|r^2 \leq 2\}$; $U(\sqrt{2}) = \{r|r^2 > 2\}$ with r rational], gives a complete and uniform construction of all real numbers as points on a line, using just the rationals. This is an explanation of the *continuous* in terms of the *discrete*, finally resolving the fundamental conflict in Greek mathematics.

$$19^2 = 3 \times 11^2 - 2 \qquad \therefore \quad \left(\frac{19}{11}\right)^2 = 3 - \frac{2}{11^2}$$

$$71^2 = 3 \times 41^2 - 2 \qquad \therefore \quad \left(\frac{71}{41}\right)^2 = 3 - \frac{2}{41^2}$$

$$265^2 = 3 \times 153^2 - 2 \qquad \therefore \quad \left(\frac{265}{153}\right)^2 = 3 - \frac{2}{153^2}$$

$$989^2 = 3 \times 571^2 - 2 \qquad \therefore \quad \left(\frac{989}{571}\right)^2 = 3 - \frac{2}{571^2}$$

$$3691^2 = 3 \times 2131^2 - 2 \qquad \therefore \quad \left(\frac{3691}{2131}\right)^2 = 3 - \frac{2}{2131^2}$$

lead to the inequalities

$$\frac{5}{3} < \frac{19}{11} < \frac{71}{41} < \frac{265}{153} < \frac{989}{571} < \frac{3691}{2131} = 1.7320506... < \sqrt{3}$$

To 6 significant figures, we therefore find by Archimedes' method

$$\sqrt{3} = 1.732050...$$

To calculate the ratio of the circumference of a circle to its diameter, Archimedes began by trapping the circle between an exterior hexagon and an interior hexagon. His result, in terms of $\sqrt{3}$, is

$$3 < \frac{\text{circumference}}{\text{diameter}} < 2\sqrt{3} = 3.461...$$

Successively halving the angle subtended by the polygon side at its center, there appear regular figures having 12, 24, 48, 96 ... sides[153]*. Using no more than proportions and the Pythagorean theorem, Archimedes obtained with a 96-gon his best value*[154] *for π*

$$3.1408 = 3\frac{10}{71} < \frac{\text{circumference}}{\text{diameter}} < 3\frac{1}{7} = 3.1428.$$

[153] Today we would have written:

$$2^k n \sin\left(\frac{\theta}{2^k}\right) < \pi < 2^k n \tan\left(\frac{\theta}{2^k}\right), \qquad \theta = \frac{\pi}{n}$$

when n is the original number of sides, doubled k times. (Archimedes took $n = 6$, $k = 4$).

[154] This *may* have been known to the ancient Egyptians who built the Great Pyramid of Gizah: the ratio of the perimeter of this pyramid to its height was 2×3.142 which falls right in between Archimedes' two limits.

It correctly compares to $\pi \approx 3.14159$ *to 5 figures. The value of* $3\frac{1}{7}$ *was used by most European mathematicians until 1573, when Valentinus Otho used* $\pi = \frac{355}{113} = 3.141592...$ *That Archimedes arrived at his result without trigonometry, and without decimal (or other positional) notation is an illustration of his tenacity.*

- *The* method of exhaustion *is a generalization of the theory of proportions. Just as an irrational* length *is determined by the rational lengths on either side of it, more general unknown quantities (such as arcs, for example) become determined by arbitrary close approximation using known elements. Examples (as expounded in book XII of Euclid's elements) are an approximation of the* circle *by inner and outer polygons and an approximation of the* pyramid *by stacks of prisms.*

  In both cases the approximating figures are known quantities, on the basis of the theory of proportions and the theorem that the area of a triangle $= \frac{1}{2}$ *base* $\times$ *height.*

  "Exhaustion" does not mean using an infinite sequence of steps to show (in the case of the circle) that area is proportional to the square of the radius. Rather, one shows that any disproportionality can be refuted in a finite *number of steps. This is typical of the way in which exhaustion arguments avoid mention of limits and infinity.*[155] *In the case of the pyramid, one uses elementary geometry to show that stacks of prisms approximate the pyramid arbitrarily closely. Then exhaustion shows that the volume of a pyramid, like that of a prism, is proportional to base* $\times$ *height. Finally there is an argument to show that the constant of proportionality is* $\frac{1}{3}$.

  Archimedes used the method of exhaustion to calculate the area of a circle. He was able to show that $S = \lambda r^2$ *(*S *= area;* r *= radius), with*

$$\lambda = 32\sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2}}}}} = 3.140...$$

In Euclid, both the infinite and the infinitesimal are deliberately excluded[156].

155 **Euclid** (330–260 BCE) avoided even the method of exhaustion itself in his derivation of the area of a polygon, using clever *dissection* arguments.

156 Had the Greeks accepted the concept of infinity, they would have had to accept the notion that the whole is no greater than some of it's parts, since infinity, plus whatever number you please, or plus or times or minus infinity, can still be infinity. This Aristotle was *not* ready to accept.

Archimedes, *working in the tradition of Aristotle and Euclid , asserted that an infinitesimal, if it existed, would be a number greater than zero, which nevertheless remained less then 1, say, no matter how (finitely) many times it was added to itself. But such a number* could not exist *since even a very small non-zero number becomes arbitrary large if it is added to itself enough times. So, on the one hand Archimedes rejected infinitesimals, but on the other hand, being also an engineer and physicist, he* used infinitesimals *to solve problems in the geometry of parabolas. Then, since infinitesimals "do not exist", he gave "rigorous" proofs of his results using the method of exhaustion, relying on indirect arguments and purely finite constructions.*

The method of exhaustion, is a rigorous but sterile method: once a formula is known, the method may furnish an elegant tool for establishing it. How then, did Archimedes discoverer the formulas which he so neatly established by the method of exhaustion?

*The above question was finally answered in 1906, with the discovery by Heiberg, in Constantinople, of a copy of Archimedes' long lost treatise '*Method, addressed to Eratosthenes*'. In it, Archimedes reveals that his deepest results were found using dubious infinitary arguments, and only later proved rigorously. Because, as he says, "It is of course easier to supply the proof when we have previously acquired some knowledge of the questions by the method, than it is to find it without any previous knowledge".*

Archimedes, nevertheless, made the most elegant application of the method of exhaustion, coming nearest to actual integration, in his quadrature of a parabola*. In a letter to his fried* ***Dositheos****, he mentioned that he had shown that 'any segment bounded by a straight line and a section of a right-handed cone is $\frac{3}{4}$ of a triangle, which has the same base with the section and equal height.' We now call the section of right-angled cone a* parabola[157]*.*

The concepts that Archimedes needed for proofs in geometry – the theory of proportions and the method of exhaustion – had already been supplied by Eudoxos. It was, however, Archimedes' phenomenal insight and technique that lifted him head and shoulders above his contemporaries. Indeed, even

157 A canonical parabola $y = \lambda x^2$ is cut by the line $y = b$; the area of the triangle is $b\sqrt{\frac{b}{\lambda}}$; the area of the parabolic sector is

$$2\int_0^{\sqrt{\frac{b}{\lambda}}} (b - \lambda x^2)dx = \frac{4}{3}b\sqrt{\frac{b}{\lambda}} = \frac{4}{3} \times (\text{area of triangle}),$$

as calculated by Archimedes.

with the limitations imposed on him through Aristotle's fear of the infinitesimals, he was not to be surpassed for the next 1900 years.

370–350 BCE **Thymaridas** (ca 400–350 BCE). Phytagorean mathematician known for his solution of the special system of linear equations

$$\begin{aligned} x + x_1 + x_2 + \ldots + x_{n-1} &= S \\ x + x_1 \qquad\qquad\qquad &= a_1 \\ x + \quad x_2 \qquad\qquad &= a_2 \\ x + \qquad x_3 \qquad &= a_3 \\ \vdots \qquad\qquad\qquad\qquad &\vdots \\ x + \qquad\qquad x_{n-1} &= a_{n-1} \end{aligned}$$

The solution $x = \frac{(a_1+a_2+\ldots+a_{n-1})-S}{n-2}$, is known by the name "*flower of Thymaridas*". In this line of development, the Phytagoreans carried forward the development of Babylonian algebra and transformed it into geometrical algebra. Thymaridas also wrote on prime numbers. Little is known about his life except for the fact that he was apparently a rich man, who for some unknown reason, fell into poverty.

367 BCE **Kiddinu** (Cidenas). A Babylonian astronomer from the city of Sippar, known for its school of astronomy. His name is mentioned by Greek and Roman historians and appeared also on clay tablets. He may have been the originator of lunar theory, some 200 years before Hipparchos[158].

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[158] Cidenas is mentioned by **Strabo** and **Pliny**. He was rescued from a long obscurity, to take his place as one of the greatest ancient astronomers: The Jesuits **Epping** and **Kugler** deciphered in 1881 a Babylonian tablet of about 100 BCE, entitled '*The Lunar Computation-Table of Kiddinu*'. They found in it just the same length of the lunar (synodic) month from New Moon to New Moon, that Ptolemy afterwards attributed to **Hipparchos**.
In 1920 it was discovered that Kiddinu had recognized the *tropical and the sidereal years*. Thus he was acquainted with, and was probably the discoverer of, the *precession of the equinoxes*, on which Hipparchos' fame had mainly

357–336 BCE Astronomical evidence for the date of the prophecy of **Joel**.

Joel is one of the few biblical writers who clearly alludes to *total lunar and solar eclipses*. His statement in **2**, 31, "*The sun shall be turned into darkness and the moon into blood*" – is unique in the Old Testament.

Total eclipses of the sun, because of their extreme rarity in any one region, are very useful for chronological dating. The sudden and intense *darkness* which occurs when the whole sun is obscured by the moon is an awe-inspiring sight, not easily forgotten. Joel's reference to the moon is more direct; the association of a total eclipse of the moon with *blood* is often mentioned in early literature; the moon often glows a deep red color during totality of a lunar eclipse on account of sunlight scattered in the earth's atmosphere.

The originality of Joel's prophecy suggests that Joel himself witnessed total eclipses of moon and sun.

As lunar obscurations of this type are frequent (on the average about one every three years at any given place), it is not improbable that Joel saw several of these in his lifetime. Consequently, he would not need to draw on the experience of others.

Astronomical calculations show that from 1130 BCE until almost 310 BCE only two obscurations of the sun could have been complete anywhere in Israel.

The dates of these events in terms of the Julian Calendar are 357 BCE, March 01 and 336 BCE, July 04. Both eclipses would be very striking in Judah; the 357 eclipse would be total at about 1:15 p.m. with the sun at altitude of about 45 degrees. The later eclipse (336) would be even more impressive: it would reach totality within a few minutes of noon and the sun would be at an altitude of 80 degrees (almost at zenith), duration of totality of each event being about three minutes. Total lunar eclipses visible through Israel occurred within two years of each event. It would therefore appear that the astronomical evidence favors a time window 357–356 BCE for the prophecy of Joel.

354–322 BCE **Aristotle** (384–322 BCE). A Greek philosopher, educator, encyclopedic scientist and a mastermind in all fields of human inquiry: he wrote on physics, metaphysics, ethics, logic, politics, art, biology, zoology

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rested!

The astronomical data which Kiddinu determined (or at least employed) are incredibly accurate: for the sun's motion relative to the moon he erred by a single second; for a certain other motion of the sun from the node, his value is 0.5 second too great, and is actually better than that on which our principal modern tables of eclipses are based. Kiddinu had at his disposal the records of every eclipse of the moon visible in Babylon for at least 300 years.

and astronomy. He formalized the gathering of scientific knowledge. While it is difficult to point to one particular theory, the *total* result of his compilation of knowledge was to provide the fundamental basis of science for a thousand years. Tightened up the *Socratic method* and showed its application to science. Made important contributions by systematizing deductive logic.

His conception of nature stood in marked contrast to those of Pythagoras and Plato in that it grossly underestimated the importance of mathematics. Logic, on the other hand, was overrated.

Aristotle found Plato 'ideas' ridiculous and considered the *senses* as the only sources of truth. His teachings about ethics was that the goal of life was happiness. He was a student and disciple of Plato in the Academy (and later an educator and protegé of Alexander the Great).

Established in 334 BCE the Lyceum of Athens[159]. He was perhaps the world's first great biologist, but his views on physics and astronomy were rather confused[160]. In his book *Physics* (stemming from the Greek word 'nature') he defined philosophy of nature as a study of things that change. But although Aristotelian physics was wrong, it was an essential precursor of modern physics.

Aristotle was primarily a philosopher and biologist, but he was thoroughly au courant with activities of the mathematicians. He made no lasting impact on the field, although like his teacher, he contributed indirectly to the development of mathematics.

Aristotle was among the first to rightly describe the heart as the center of a system of blood vessels. However, he thought that the heart was both the seat of intellect (the brain was not identified as a seat of intellect until over a century later) and a furnace that heated the blood. He considered this warmth to be the vital force of life because the body cools quickly at death. Aristotle also erroneously theorized that the "furnace" was ventilated

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159 A member of Aristotle's school or a follower of Aristotle (Aristotelian) is known as *peripatetic*; it is an adjective pertaining to the philosophy or method of teaching of Aristotle, who conducted discussions while walking about in the Lyceum (from the Greek *peri* = around and *patein* = walk). The Lyceum was a school of philosophy and rhetoric near the temple dedicated to the god of shepherds, **Apollo Lyceus**.

160 In a letter to his friend Solovine, Albert Einstein said (1948): "*Certain things from the philosophical writings of* **Aristotle** *were actually deceptive. If they had not been so obscure and so confusing, this kind of philosophy would not have held its own very long. But most men revere words that they cannot understand and consider a writer whom they can understand to be superficial*".

by breathing. Moreover, he did not think that there was a direct connections between arteries and veins.

Aristotle supported the view that vacuum could not exist ("*horror Vacui*" – nature abhors vacuum). He believed that object would travel with infinite speed in vacuum, but did not believe that actual infinity could exist. He did believe, however, that all of space is filled with the four elements (fire, earth, water, air) and with "ether". The terms 'ether' and 'vacuum' would be largely viewed as synonymous for the next 2200 years.

Aristotle set out to construct an actual physical model of the universe. All motion, according to Aristotle, is either rectilinear, circular, or a combination of the two, because these are the only "simple movements". Upward motion is motion toward the center, downward motion leads away from it, while circular motion is movement around the center. Celestial bodies revolve in circles. He devised a scheme that allowed the concentric spheres of Eudoxos to rotate in practice as well as in theory. His cosmology required a large number of spheres for its elaborate machinery to operate.

Aristotle had a strong sense of religion and placed much of his belief in the existence of God on the glorious sight of the starry heavens. He delighted in astronomy and gave much thought to it. In supporting the scheme of concentric spinning spheres, he gave a dogmatic reason: *the sphere is the perfect solid shape*; and this prejudiced astronomical thinking about orbits for centuries. The heavens then, are the region of perfection, of unchangeable order and circular motions.

For ages Aristotle's writings were the only attempt to systematize the whole of nature[161]. They were translated from language to language, carried from Greece to Rome to Arabia, and back to Europe centuries later, to be copied and printed and studied and quoted as the authority. Long after the crystal spheres were discredited and replaced by eccentrics, those circles were spoken of as spheres; and the medieval schoolmen returned to crystal spheres in their short-sighted arguments, and believed them real.

The distinction between the perfect heavens and the corruptible earth remained so strong that Galileo, 2000 years later, caused great annoyance by showing mountains on the moon and claiming the moon was earthly; and even he, with his understanding of motion, found it hard to extend the mechanics of downward fall to the circular motion of the heavenly bodies.

Aristotle did much to set science on its feet. His whole teaching was a remarkable life's work of vast scope and enormous influence. At one extreme

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[161] As **Renan** observed, "*It is largely by the titles of his books that Aristotle has dominated the human mind; the labels of his books remained for two thousand years the divisions of knowledge itself*".

he catalogued scientific information and listed stimulating questions; at the other extreme he emphasized the basic problems of scientific philosophy, distinguishing between the *true physical causes* and *imaginary schemes to save the phenomena*.

He was first to realize that the economic set-up gives rise to social inequalities and hence to social conflicts. *Slavery*, however, seemed justified to him because it was 'necessary' to society.

Aristotle was born at Stagira in Chalcidice and grew up at the Macedonian court, where his father was the king's physician. In 367 BCE, at the age of 18, he went to the academy at Athens to study with Plato and remained there until the latter's death 20 years later. In 343 BCE Aristotle returned to Macedonia as tutor of Alexander the Great, then a lad of 13. When Alexander died at the age of 33, his conquests had spread Greek culture and learning throughout most of the known world. Thus Aristotle's association with the young prince, which lasted four years, was to have far-reaching effects on history in general and his own life in particular.

Aristotle was identified with the Macedonian rulers of Athens and their supporters, and upon Alexander's death in 323 BCE, he fled the city to Chalcis, on the Aegean Island of Euboea, to escape the outbreak of anti-Macedonian sentiment. He died a few months later, leaving a body of writings whose importance in the history of Western thought cannot be overestimated[162].

The works of Aristotle fall under three headings:

(1) dialogues and other works of popular character;

(2) Collection of facts and material from scientific research;

(3) systematic works.

The works on the second group include 200 titles, most in fragments collected by Aristotle's school and used as research. Some may have been done at the time of Aristotle's successor **Theophrastos**. The systematic treatises

[162] It is reported that Aristotle writings were held by his student **Theophrastos**, who succeeded Aristotle in leadership of the Peripatetic School. Theophrastos' library passed to his pupil **Neleos**. To protect the books from theft, Neleos' heirs concealed them in a vault, where they were damaged somewhat by dampness, moths and worms. In this hiding place they were discovered about 100 BCE by **Apellicon**, a rich book lover, and brought to Athens. They were later taken to Rome after the capture of Athens by Sulla in 86 BCE. In Rome they soon attracted the attention of scholars, and the new edition of them gave fresh impetus to the study of Aristotle and of philosophy in general. This collection is the basis of the works of Aristotle that we have today.

of the third part were not, in most cases, edited and published after his death from unfinished manuscript. Aristotle's systematic treatises may be grouped as follows:

- *Logic*

  The heart of his logic is the *syllogism* (e.g.: All men are mortal; Socrates is a man; therefore Socrates is mortal). The syllogistic form of logical argumentation dominated logic for 2000 years.

- *Physical works*

  1. Physics (explains change, motion, void, time)
  2. On the Heavens (structure of heaven, earth, elements)
  3. On Generation (through combining material constituents)
  4. Meteorology (origin of comets, weather, disasters)

  Empty space is an impossibility. *Elements* are not composed of geometrical figures (as taught by Plato and Pythagoras). *Time* depends for its existence upon motion and on man's counting mind if there were no mind to count, there could be no time.

- Psychological works

  1. On the Soul (explains faculties, sciences, mind, imagination)
  2. On Memory, Reminiscence, Dreams, and Prophesying.

- *Works on natural history* (mostly on animals)

- *Philosophical works*

  1. Metaphysics (substance, cause, form, potentiality)
  2. Nicomachean Ethics (soul, happiness, virtue, friendship)
  3. Eudemain Ethics
  4. Magna Moralia
  5. Politics (best states, utopias, constitutions, revolutions)
  6. Rhetoric (elements of forensic and political debate)
  7. Poetics (tragedy, epic poetry)

The prodigious activity of Aristotle marks the climax of the golden age of Greece. The very existence of his works not only reflects his encyclopedic genius, but is also testimony that a large amount of scientific research had already been accomplished by his time. He had a deep mathematical knowledge, but that knowledge was happily balanced by a very extensive acquaintance with every branch of natural history. Thus, his philosophy was naturally more experimental, more inductive, than that of Plato[163].

Aristotle is one of the founders of the inductive method. He was first to conceive the idea of *organized research*, and himself contributed considerably to the organization of science by his systematic survey and classification of knowledge of his time[164]. He classified the sciences into two broad categories: *theoretical* (knowledge and speculation) and *practical* (action) sciences. The first group was in turn divided into three subgroups: philosophy, physics and mathematics. He took pains to elucidate the fundamental principles of each science in particular and of science in general.

Aristotle's theories were *physical*, not metaphysical, in nature. Superstition and astrological predictions were noticeably absent. Although his cosmology of concentric spheres and circular, geocentric, heavenly motions was quickly superseded by Ptolemy's epicyclic motions, Aristotle's views on natural phenomena were mostly unchallenged for the next 2000 years and his influence lasted until the beginning of the 18^{th} century.

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163 In a detail of Raphael's fresco *The School of Athens*, **Plato** and **Aristotle** are depicted in a way which symbolizes their approach to knowledge. Aristotle gestures toward the earth; Plato points his finger to the heavens. Aristotle looked to nature for answers; Plato searched for the ideal.

164 The Aristotelian method of problem-solving hinges on the Socrates doctrine of 'endless search for the truth'. The notion that there *is* an ultimate truth and that we can seek to get nearer and nearer to that truth, is what has driven science along. We look at a complex phenomenon in the belief that at the end there will be the simplest of 'ultimate truths'. In practice, the problem is first subjected to *analysis*; the *cause* is identified. When the cause is removed, the problem is solved. The method presupposes that the cause (or causes) can be identified, and then, when eventually found – can be removed.

Worldview II: Aristotle

* * *

"To live alone one must be either a beast or a god".

* * *

"Wisdom is not a wisdom at all, if it comes too late."

* * *

"Plato and truth are both clear to us; but it is a sacred duty to prefer truth."

* * *

"The mathematical sciences particularly exhibit order, symmetry, and limitation; and these are the greatest forms of the beautiful."

* * *

"Inferiors revolt in order that they may be equal, and equals that they may be superior."

* * *

"Injustice arises when equals are treated unequally and when unequals are treated equally."

* * *

"Wicked men obey from fear; good men from love."

* * *

"Philosophy is the science which considers truth."

* * *

"The whole is more than the sum of its parts."

* * *

"It was through the feeling of wonder that men now and at first began to philosophize."

* * *

"Art not only imitates nature, but also completes its deficiencies."

* * *

"Dignity does not consists in possessing honors, but in deserving them."

* * *

353 BCE *The Mausoleum at Halicarnassos*: A great marble tomb erected by Queen Artemisia in memory of her husband King Mausolos. The Greek architects **Satyros** and **Pythios** designed the tomb and the sculptors **Bryaxis**, **Leochares**, **Scopas**, and **Timotheos** carved the decorated band on the building. The tomb was about 41 meters high. It has a rectangular basement beneath a colonnade formed by 36 columns. A stepped pyramid rested on the colonnade, and a statue of Mausolos in a chariot stood on top of the pyramid. Only pieces of the building and its decorations remain.

350 BCE **Menaichmos** (ca 380–320 BCE). Mathematician. Pupil of Eudoxos. Geometry teacher of Alexander the Great. Is reputed to have discovered the conic sections, later to be known as the *ellipse*, *parabola* and *hyperbola*. He derived some of their properties. The names were supplied by **Apollonios of Perga** (262–200 BCE). Menaichmos lived mostly in Alexandria, and made a fundamental study of the conic sections.

Menaichmos was especially interested in the old problem of the duplication of the cube[165]. He found two ways of solving the reduced quadratic equations

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[165] Three classic problems emerged in the 5^{th} century BCE:

- the squaring of the circle;
- the trisection of an angle;
- the duplication of a cube (the *Delian* problem).

Hippocrates of Chios and Menaichmos were especially interested in the third of these problems; Hippias of Elis found an ingenious solution of the second by means of the curve invented by him, the *quadratrix*. Deinostratos applied it to the solution of the first problem.

The origin of the third problem is associated with a story: In 430 BCE a great plague struck Athens, killing **Pericles** (429 BCE). The Athenians appealed to the oracle at Delos to provide a remedy. The oracle said that Apollo was angry because his cubical altar was too small. If it were doubled, the plague would end. The Athenians had a new altar built that was twice the original in length, breadth, and height. The plague became worse because the god wanted the *volume* of the cube doubled, and the Athenians has octupled it. The plague went on until 423 BCE. The Delian problem went on until the 19^{th} century.

Around 320 CE, **Pappos** declared that it was impossible to solve any of the classic problems under the Platonic restrictions (straightedge and compass only), although he did not offer a proof of this assertion. In the 19^{th} century, each problem was shown incapable of any solution that met the Platonic requirement:

- **Pierre Wentzel** (1837) supplied a rigorous proof that an angle cannot be trisected with an unmarked straightedge and collapsing compass.
- **Ferdinand Lindemann** (1882) showed that π is a transcendental number, implying that the circle cannot be squared.

of Hippocrates of Chios by determining the intersection of two conics – two parabolas in the first case, a parabola and a rectangular hyperbola in the second.

ca 350 BCE *Mayan arithmetic and numerical notation.* In order to escape rapidly mounting calendric chaos, the Mayan priests devised a simple numerical system, including the concept of *zero*, which even today stands as one of the brilliant achievements of the human mind. Numbers were denoted by bar-and-dot glyphs; the dot · has a numerical value of 1 and the bar – a numerical value of 5, and by varying combinations of these two symbols, the numbers from 1 to 19 were written. The glyph for *zero* was a symbol that looks roughly like a half-closed eye,

$$\begin{array}{ccccc}
 & \dot{1} & \ddot{2} & \dddot{3} & \ddddot{4} \\
\overline{5} & \dot{\overline{6}} & \ddot{\overline{7}} & \dddot{\overline{8}} & \ddddot{\overline{9}} \\
\overline{\overline{10}} & \dot{\overline{\overline{11}}} & \ddot{\overline{\overline{12}}} & \dddot{\overline{\overline{13}}} & \ddddot{\overline{\overline{14}}} \\
\overline{\overline{\overline{15}}} & \dot{\overline{\overline{\overline{16}}}} & \ddot{\overline{\overline{\overline{17}}}} & \dddot{\overline{\overline{\overline{18}}}} & \ddddot{\overline{\overline{\overline{19}}}}
\end{array}$$

Mayan bar-and-dot notation was simpler than Roman notation: to write the numbers from 1 to 19 in Roman notation, it is necessary to employ the symbols I, V and X, and the processes of addition and subtraction (VI is V plus I, but IV is V minus I). In the Mayan system there is only one arithmetic process, that of addition.

The Maya employed a second notation comparable to our Arabic notation in which different types of *human heads* represented the numbers from 1 to 13, and zero.

In writing bar-and-dot numbers above 19 the Maya used a positional system of numeration. In our decimal system, the positions to the left of the decimal point increase by tens. In the Mayan vigesimal system the values of the positions increase by *twenties* from bottom up. (An exception is made in counting *time*, when, the third position is 18 instead of 20 times the second.)

- It was shown that the Delian problem required the construction of a line whose length is the *cube* root of 2. (With a straightedge and a compass, it is only possible to construct square roots.).

Thus, for example, the numbers $10,951 = 11 \cdot 20^0 + 7 \cdot 20^1 + 7 \cdot 20^2 + 1 \cdot 20^3$ (right) and $806 = 6 \cdot 20^0 + 0 \cdot 20^1 + 2 \cdot 20^2$ (left), where written as:

		$\cdot$	1×20^3
$\cdot\cdot$	2×20^2	$\overset{\cdot\cdot}{-}$	7×20^2
(figure of an 'eye')	0×20^1	$\overset{\cdot\cdot}{-}$	7×20^1
$\overset{\cdot\cdot}{-}$	6×20^0	$\overset{\cdot}{=}$	11×20^0

The largest number found in the codices is 12,489,781.

ca 350 BCE **Heracleides of Pontos** (ca 388–315 BCE). Greek philosopher and versatile and prolific writer on mathematics, music and physics. Made a step toward the heliocentric idea: "*The stars of Mercury and Venus, make their retrograde motions and retardations about the rays of the sun, forming by their courses a wreath or crown about the sun itself as center*". He clearly recognized that the earth rotates on its axis.

Heracleides was born at Heraclea in Pontos. Studied philosophy at Athens under Speusippos, Plato and Aristotle (Plato, on his departure for Sicily, left his pupils in charge of Heracleides). The latter part of his life was spent at Heraclea.

339–314 BCE **Xenocrates of Chalcedon** (ca 385–314 BCE). Mathematician and philosopher. Disciple of Plato. Head of the Athenian Academy for 25 years. Continued Plato's policy of excluding from the Academy the applicants who lacked geometric knowledge. Saying to one of them: "*Go thy way for thou haste not the means of getting a grip of philosophy*".

Xenocrates wrote a great many treatises, all of which are lost. It is known that he tried to solve the earliest problem of combinatorial analysis: calculating the number of syllables that could be formed with the letters of the alphabet (According to **Plutarch** that number was 1,002,000,000,000).

At the time of Plato's death, his sister's son **Speusippos**, succeeded him as the head of the school (347–338). **Aristotle** and his friend Xenocrates decided to leave, accepting the invitation of a fellow student, Hermeias, ruler of Atarneus (opposite Lesbos). Aristotle then married Pythias, who was Hermeias niece and adopted daughter. Hermeias also recommended Aristotle to Philip as a tutor to his son, Alexander (the Great).

Xenocrates advanced the idea of *indivisible atoms of time*. The idea of temporal atomicity does not necessarily imply that there must be gaps between successive instants. The essential criterion for atomicity is that there is a limit to the division of any duration into constituent parts. In other words, time would be like a line which can be divided into a finite number of adjacent segments with no intervals between them. It would mean that, from the temporal aspect, there are minimal-time processes in nature, no process occurring in less than some shortest unit of time, or *chronon*[166].

339 BCE *The Fourth Sacred War*: Philip II of Macedonia defeated an alliance of Greece with Thebes near Amphissa. Philip conquered Greece and turned the Greek peninsula into a Macedonian protectorate.

The Macedonian army used the *phalanx* offensive battle formation. It was made up of heavily armed infantry troops formed in tight ranks for the attack. The troops carried long spears and protected themselves with overlapping shields. The depth of each formation ranked from 8 to 12 ranks. The phalanx had great striking power, but no flexibility. It needed support from lighter troops and cavalry.

338 BCE, August The decisive *battle of Chaeronea* where Philip of Macedonia crushed the allied Greek citizen armies. Athens finally lost its independence, as foreseen by **Demosthenes** (384–322 BCE). End of the classical period of Greece[167].

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[166] *Modern* speculations concerning the chronon have often been related to the idea of a smallest natural length. One suggestion was that this is given by the effective diameter of the proton and electron, that is to say about 10^{-15} meters. If this were a shortest natural length and we divided it by the fastest possible speed, that of light in *vacuo* (3×10^8 meters per second), the resulting interval of time would be about 10^{-23} – 10^{-24} seconds. A time of this order characterizes the normal decay processes of nucleons, i.e. processes involving the so-called 'strong interactions' between protons and neutrons, and also the lifetime of the most transient elementary particles. It is therefore possible that if the chronon exists it is of this order of magnitude. A purely theoretical unit of length much shorter than 10^{-15} meters can, however, be constructed from the three fundamental constants, G, h and c (constant of gravitation, Planck's constant, and velocity of light). It is $\sqrt{(Gh/c^3)}$, and is of the order of 10^{-34} meters. If this is divided by the velocity of light, it gives a time of the order of 10^{-43} seconds. It is possible that this time, sometimes called the 'Planck time', might be the chronon.

[167] Athens made peace, but Demosthenes kept up his opposition. Later he defended his policy in his speech *On the Crown*, considered as the most nearly perfect speech in history. He poisoned himself (322 BCE), when the last Greek effort to win freedom was a failure.

334 BCE **Alexander the Great** (356–323 BCE). One of the greatest conquerors in the annals of history. Hellenized Western Asia and orientalized Eastern Europe. Pupil of Aristotle. Attacked the Persian empire with an army of Macedonians and Greeks. Within 12 years he conquered an area almost the size of the United States, founding Greek cities and establishing Greek garrisons everywhere he went.

By spreading Greek civilization, ideas and language into the very heart of Asia, Alexander profoundly affected the history of the world, and Greek culture rapidly became world culture. Moreover, the cultural intercourse among ancient peoples was so stimulated by his conquests, that the various cultures of the ancient world began to congeal into a universal culture to which all contributed but in which the Greek element predominated. This was the first large scale contact between East and West.

Alexander's scientific curiosity was apparently great, and his and Aristotle's initiative are probably responsible for the first *scientific expeditions* and the first attempts to organize science on a large scale. In 332 BCE Alexander founded in Egypt the city of Alexandria, which was to become one of the greatest centers of learning and of cultural diffusion between the East and the West.

334–ca 146 BCE *The Hellenistic Era*. Marks the culmination of Greek science and its influence on the civilized ancient world, mainly from its center in Alexandria.

The word 'Hellenism' signifies that culture which drew its vitality from the classical Hellenic culture. It spread over the entire empire of Alexander the Great and coated with a thin layer the original ancient cultures of the conquered nations. The Hellenic influence penetrated by means of the Greek *language*, *religion*, *art*, *literature*, *philosophy*, *science*, *architecture*, and *political* ideas. Hellenism spread to Rome in the beginning of the 2^{nd} century BCE and challenged Judaism as of the 3^{rd} century BCE.

But Jewish culture in Israel was firmer and more consolidated than other cultures and was thus capable of arresting the advance of Hellenism. In fact, the rigorous encounter of the two rivals at the time of the Maccabees (164 BCE) marks the first sign of the Hellenistic retreat in the east.

Hellenism, though, was more successful in the Jewish diaspora in the east and in the Aegean world, where the Bible was translated into Greek (*septuaginta*) and where Greek became the spoken language even in the synagogue. Eventually, the Hellenization of diaspora Jewry expedited the spread of Christianity among the Jewish masses in Asia Minor and elsewhere. The Sassanic Kingdom that rose in Persia in the 3^{rd} century CE, finally put an end to Hellenism in the entire Orient.

The roots and fruits of Hellenistic philosophy

1. Historical background

Throughout history, philosophy and religion both influenced and were influenced by economical, political and social conditions. This is true in particular for the transition era between the downfall of the Hellenic city-state regime and the rise of Christianity. We list below the main political events in this time-window:

339 BCE *Greece becomes a Macedonian protectorate.*

323 BCE *Death of* **Alexander the Great***; Beginning of Hellenistic age.*

275 BCE *Alexander's Empire finally divided under the rule of three dynasties: Antigonides (Macedonia), Ptolemies (Egypt) and the Seleucides (East).*

188 BCE *Peace of Apamea: Greek cities lose their independence.*

148 BCE *Macedonia becomes a Roman province.*

133–27 BCE *Decline of the Roman Republic. A century of incessant violence and confusion. Fall of the Hellenistic states.*

27 BCE *Beginning of the Roman Empire under* **Augustus***. End of the Hellenistic age; order and stability are restored.*

180 CE *Death of* **Marcus Aurelius***; Beginning of the decline of the Roman Empire.*

325 CE *The Council of Nicaea: rigidification of the Christian Church.*

415 CE *End of the Alexandrian School.*

451 CE *The Hun Invasion of Europe.*

476 CE *End of the Western Roman Empire.*

529 CE **Justinian** *closes the Athenian schools of Philosophy.*

During 322–148 BCE, Greece was subject, not only to political decline and the confusion of the wars, but also to very serious social stress. Consequently, the condition of the majority grew steadily worse. In all parts of Greece, and in the Aegean islands, many people were living at the very edge of a subsistence level, and the impression of personal insecurity was very general. Sparta, in particular, underwent three successive violent social revolution during 250–200 BCE.

Thus, in spite of their wealth and almost unlimited opportunities, the Hellenistic states eventually wasted their resources in a series of inconclusive wars among themselves. As a result, between 200 BCE and 31 BCE all the Hellenistic world except Macedonia and Persia was annexed piecemeal by Rome, a new militaristic power that had risen, at first almost unnoticed, in the Western Mediterranean.

But the Greek culture that had been forged in the classical era and that had spread in Hellenistic times to most of the rest of the civilized world was not suppressed by the Romans. It survived and spread to the West. Although Latin was spoken in Rome itself and in the western provinces, Greek remained the language of the educated classes of the East. Throughout the life of the Roman Empire, Rome itself was never more than the political capital. Alexandria remained the most active commercial city of the empire and Athens the center for scholars, yet most of the scientific work was done in Alexandria. After the western half of the Roman Empire, including Rome itself, fell to barbarian invaders in the fifth century CE, the Hellenistic world survived for more than 1,000 years as the Byzantine Empire, with its new capital in the Greek city of Constantinople.

How did philosophy fare in this era of great political changes?

The earlier Milesian Greek philosophers, had called attention to cosmic order and the beauty of nature. Later, the monist ***Parmenides of Elea*** *stressed the power of reason and thought, whereas* ***Heraclitos of Ephesos****, precursor of the philosophy of becoming, had alluded to the constancy of change and the omnipresence of divine fire, which illuminates all things.*

A deeper understanding of man *himself came with* **Socrates** *(469–399 BCE), symbol of the philosophical man, who personified* Sophia *and* Sapientia *(Greek and Latin: "wisdom"). From Socrates emanated the philosophical system of the* Cynics *and the* Skeptics[168]*. Thus, although Socrates did not found any philosophical school, and in fact had no intention of doing so, his*

[168] **Socrates** said that the only thing he knew was that he did not know anything. However, he did at least believe that knowledge was possible, and, he was bent on acquiring some. He took a positive attitude towards inquiry and the possibility of learning.

teaching were indirectly responsible for the systems of thought sponsored by groups of his disciples. His heritage lasted for many centuries and left its mark upon the medieval and modern philosophies of the Western World.

In spite of its political downfall and its poverty, Athens was still the focus of philosophical teaching. The four main schools were: the Academy *(*Plato*), the* Lyceum *(*Aristotle*), the* Garden *(*Epicuros*), and the* Porch *(*Stoa*). To these must be added the unorganized efforts of the* Cynics *and the* Skeptics*.*

The successors of ***Plato*** *(427–347 BCE) as heads of the Academy were:* ***Speusippos*** *(347–339 BCE),* ***Xenocrates*** *(339–315 BCE),* ***Polemon*** *(315–270 BCE),* ***Crates of Athens*** *(270–264 BCE),* ***Arcesilaos*** *(264–240 BCE),* ***Lacydes of Cyrene*** *(241–224 BCE),* ***Teleclos*** *(224–216 BCE),* ***Erandros the Phocian*** *and* ***Hegesinos of Pergamum****.*

The love of philosophy was so diffuse in the Greek population that the Athenian schools were not enough to satisfy it. Provincial schools were needed in Megara*,* Eretria*,* Cyrene*, and probably other places.*

Aristotle's leadership of the Lyceum *lasted only thirteen years (335–323 BCE). It was then headed in succession by* ***Theophrastos of Eresos*** *(328–286 BCE),* ***Straton of Lampsacos*** *(286–268 BCE),* ***Lycon of Troas*** *(268–225 BCE),* ***Ariston of Iulis****. The golden age of the Lyceum lasted less that 70 years (335–268 BCE).*

The death of ***Alexander the Great*** *(323 BCE) and* ***Aristotle*** *(322 BCE) marked the beginning of a period of extreme social and political disturbances, the character of which Aristotle failed to foresee, and which radically affected the course of philosophical thought. From this time onwards, the political importance of the Greek city-states rapidly declined.*

With Athens no longer the center of worldly attraction, its claim to cultural prominence passed on to other cities – to Rome, to Alexandria, and to Pergamum. The Greek polis gave way to larger political units; local rule was replaced by that of distant governors. The earlier distinction between Greek and Barbarian was destroyed; provincial and tribal loyalties were broken apart, first by Alexander and then by Roman legions.

The loss of freedom by subject peoples further encouraged a deterioration of the concept of the freeman and resulted in the rendering of obligation and service to a ruler whose moral force held little meaning. The early intimacy of order, cosmic and civic, was now replaced by social and political disorder and traditional mores gave way to transient values. Amid so much uncertainly and confusion, many felt the need to preserve some firm foundation. At that time no religion *had power or prestige enough to satisfy such a demand, and the result was an enormous popular interest in* philosophy*.*

Hellenistic philosophy and religion reflected the great problem of the age: What part could an individual play in a society in which individual effort seemed useless? In a small city-state during the Golden Age the individual played an important role in his community; he could find meaning and purpose in his life and his work. But in the busy and cosmopolitan Hellenistic world a man was lost in a community as large as civilization itself. Often he lived in a massive city with a population of hundreds of thousands where the highest endeavors of an individual might pass unnoticed, or still worse, serve no purpose. In such a society the values of an earlier time – such as duty to the state, honesty, justice, piety, and personal honor – often seemed to have little meaning.

It was inevitable that, under these conditions, there should occur an almost immediate movement away from the recent tradition of Plato and Aristotle. Those philosophers were certainly too difficult, and in a sense too conventional, to satisfy the new popular demands. Their work was too full of intricate logical argument, it demanded too high a devotion to abstract thought, to provide a possible basis for any popular creed.

***Aristotle** in particular, in his writing on ethics and politics, had tended to take for granted as the background of his argument just those political conditions and conventions of conduct which, so soon after his death, had ceased to exist.*

He addressed himself to the ordinary well-educated citizen of the (now) old-fashioned Greek city-state; he had not, and probably would not have wished to claim that he had, any moral message for mankind at large in a period of chaos. He and Plato were philosophers for intellectuals; and for this reason, though their prestige and their fame remained unassailable, the philosophy of the new period was provided by others, and was quite different from theirs.

Perhaps the clearest indication of the changed atmosphere of philosophy can be found in the changing conception of "happiness," the goal of life, and consequently of the means proposed for achieving it. For one thing, this question came to be the dominant concern of philosophy, at the expense of those epistemological and metaphysical inquiries which, for Plato and Aristotle at least, had been no less important and absorbing. But also the topic itself was very differently treated.

Aristotle, in his Nicomachean Ethics, had taken the conventional – one would almost like to say, the sensible – view that the well-being of the individual was determined in large part by the circumstances in which he lived, by the activities in which he engaged, by the achievements which could be counted to his credit. He would certainly have recommended an active, indeed a masterful, part in the public affairs of one's community. But it is clear that, if so, the well-being of the individual is dependent in part upon external affairs; and in the precarious post-Aristotelian world, it seems to have been felt that individual happiness, thus conceived, was itself intolerably precarious. Accordingly, almost every later school agreed in the attempt to maintain that happiness, rightly conceived, must lie in the sole power of the individual himself. The attempt to maintain this, however understandable, led at times to an extremity of paradox.

Even so, it seems to have been tacitly agreed that some conception of happiness must be worked out which would ensure that, at least in theory, it could be attained quite independently of shifting, perilous and uncontrollable circumstances. The resulting tendency was, strongly and persistently, towards some sort of philosophy of "non-attachment" – towards a kind of strategic withdrawal, as it were, from a world which no one now could believe, as Aristotle did, to be manageable by careful and enlightened individual effort.

Out of this new reality there arose four systems of philosophy: Epicureanism, Stoicism, Cynism and Skepticism. The first three were primarily moral doctrines, philosophies of life, far more earnestly occupied with the actual predicament of man than with any merely theoretical questions and they were concerned in particular to teach, in a world that was too often dangerous and deceptive, the secret of individual well-being. And even the Skeptics were apt to recommend their skepticism as offering a relief from anxiety – as a kind of restful acquiescence in the single conviction that no exclusive faith or doctrine whatever would ever be proved, so that all intellectual struggles must be ultimately vain.

The pursuit of knowledge passed from Aristotle to the distinguished scientists of the Hellenistic age. The philosophers took up instead the pursuit of virtue, of happiness, or – in this at least they were all agreed – of security.

Table 1.3 lists most of the important persons that were active in the various philosophical schools in the Greco-Roman world in the millennium prior to the close of the Athenian schools by Justinian.

Table 1.3: Greek Schools of philosophy and their post - Aristotelian cults and creeds *(470 BCE–530 CE)*

PHILOSOPHER	*LIFE-SPAN BCE*	*FLOURISHED BCE*	*SCHOOL*
Xenocrates of Chalcedon	—	*339–314*	*Platonic*
Theophrastos of Ephesos	*373–286*		*Peripatetic*
Eudemos of Rhodes		*325*	"
Antisthenes of Athens	*444–365*		*Cynic*
Diogenes of Sinope	*412–323*		"
Crates of Thebes	*365–285*		"
Demetrius		*40–75 CE*	"
Theodoros of Cyrene	*465–399*		*Hedonist*
Aristippos of Cyrene	*435–355*		"
Bion of Borysthenes	*325–255*		"
Euhemeros of Messene		*311–298*	"
Hegesias of Cyrene		*283*	"
Anniceris of Cyrene	*350–283*		"
Menipass of Gadera		*250*	"

Table 1.3: (Cont.)

PHILOSOPHER	*LIFE-SPAN BCE*	*FLOURISHED BCE*	*SCHOOL*
Epicuros of Samos	*341–270*		*Epicurean*
Metrodoros of Lampsacos	*330–278*		"
Hermarchos of Mytilene (Lesbos)		*270*	"
Zeno of Sidon		*150*	"
Phaedrus of Rome	*140–70*		"
Titus Lucretius Carus	*98–55*		"
Polystratos		*250*	"
Diogenes Laertius		*222–235 CE*	"
Pyrrhon of Elis	*365–275*		*Skeptic*
Anaxarchos of Abdera	*217*	*ca 320*	"
Timon of Phlias	*320–230*		"
Arcesilaos of Pitane	*315–240*		"
Carneades of Cyrene	*214–129*		"
Lucian of Samosata	*120–185 CE*		"
Clitomachos	*187–109*		"
Aenesidemos of Cnossos (Alexandria)		1^{st} *BCE*	"
Sextus Empiricus		*200 CE*	"

Table 1.3: (Cont.)

PHILOSOPHER	*LIFE-SPAN BCE*	*FLOURISHED BCE*	*SCHOOL*
Zeno of Citium	*336–265*		*Stoic*
Cleanthes of Assos	*331–233*		"
Chrysippos of Soli	*280–207*		"
Zeno of Tarsos		*204*	"
Diogenes of Babylon	*240–152*		"
Antipater of Tarsos	*185–119*	*150*	"
Panaetios of Rhodes	*185–110*		"
Posidonios of Apamea	*135–51*		"
Boethos of Sidon	*119–55*		"
Arios Didymos of Alexandria	*63 BCE–10 CE*		"
Gaius Musonius Rufus of Volsinii	*25–101 CE*		"
Epictetos of Hierapolis	*55–135 CE*		"
Marcus Aurelius Antoninus	*121–180 CE*		"
Ouintus Tertullian	*155–225 CE*		"
Antiochus of Ascalon		*130–120*	*Eclectic*
Marcus Tullius Cicero	*106–43*		"
Quintus Sextius	*70 BCE–10 CE*		"

Table 1.3: (Cont.)

PHILOSOPHER	*LIFE-SPAN*	*FLOURISHED*	*SCHOOL*
Sotion of Alexandria	*ca 40 BCE–30 CE*		*Electic*
Lucius Annaeus Seneca	*4 BCE–65 CE*		"
Plutarch	*46–127 CE*		"
Favorinus	*80–150 CE*		"
Numenius of Apamea	*150–200 CE*		*Neo-Pythagorean*
Atticus	*150–200*		*Neo-Platonist*
Plotonius	*204–270*		*Neo-Platonist*
Ammonius Saccas	*206–268*		*Neo-Pythagorean*
Porphyrius	*234–305*		*Neo-Platonist*
Iambilichus	*260–330*		"
Aurelius Augustinus	*354–430*		*Platonist*
Proclos of Byzantium	*410–485*		*Neo-Platonist*
Boethius	*480–524*		*Stoic*

2. The Cynics (ca 400 BCE–200 CE)

* * *

"I am a citizen of the world".

"There is no need to rebel because everyone is already free".

(Diogenes, ca 350 BCE)

* * *

"Truly, if I were not Alexander I would wish to be Diogenes".

*(**Alexander the Great**, ca 330 BCE)*

The Cynics rejected all social conventions and believed in individual self-sufficiency, independence and self-control, virtue as the only good and withdrawal from the corrupt world.

It is especially desirable to understand the essentials of cynicism because that school of thought developed into Stoicism, the major philosophy of the Stoics which lasted for centuries and left its mark upon the medieval and modern philosophies of the Western world.

The Cynic school of Greek Philosophy can be traced back to **Socrates** *and* **Antisthenes***(c. 444–365 BCE), who was one of Socrates' immediate pupils and is generally considered the founder of the sect. The word cynic means 'doglike' in Greek, because Antisthenes accentuated Socrates' tendency to live in the simplest fashion and to disregard many of the social conventions and amenities.*

Antisthenes' most famous disciple was ***Diogenes of Sinope*** *(c. 412–323 BCE), on the Black Sea, whose excesses of austerity have become proverbial.*

Diogenes, an anarchist who lived in great poverty, proclaimed the necessity of self-sufficiency, austerity, and shamelessness and made an aggressive display of his contempt of conventions; But he did not add anything new to Antisthenes' teachings, just dramatized and advertised them. From a popular conception of the intellectual characteristics of the school comes the modern *sense of "cynic", implying a sneering disposition to disbelieve in the goodness of human motives and a contemptuous feeling of superiority.*

What are then the general principles of this school in its internal and external relations as forming a definite philosophical unit? The importance of these principles lies not only in their intrinsic value as an ethical system, but also in the fact that they form the link between Socrates and the Stoics, between the essentially Greek philosophy of the 4^{th} century BCE, and a system of thought which has exercised a profound and a far-reaching influence an medieval and modern ethics.

Antisthenes imbibed from Socrates the fundamental ethical precept that virtue, not pleasure, is the end of existence. He was, therefore, in the forefront of that intellectual revolution *in the course of which speculation ceased to move in the realm of the physical and focused itself upon human reason in its application to the practical conduct of life.*

"Virtue," says Socrates, "is knowledge"; in the ultimate harmony of morality with reason is to be found the only true existence of man. Antisthenes adopted this principle in its most literal sense, and proceeded to explain "knowledge" in the narrowest terms of practical action and decision, excluding from the conception everything except the problem of individual will realizing itself in the sphere of ordinary existence.

Just as in logic the inevitable result was the purest nominalism, so in ethics he was driven to individualism, to the denial of social and national relations, to the exclusion of scientific study and of almost all that the Greeks understood by education; This individualism he and his followers carried to its logical conclusion. The ordinary pleasures of life were for them not merely negligible but positively harmful inasmuch as they interrupted the operation of the will. Wealth, popularity and power tend to dethrone the authority of reason and to pervert the soul from the natural to the artificial. Man exists for and in himself alone; *his highest end is self-knowledge and self-realization in conformity with the dictates of his reason, apart altogether from the state and society. For this end, disrepute and poverty are advantageous, in so far as they drive the man back upon himself, increasing his self-control and purifying his intellect from the dross of the external. The good man (i.e. the wise man) wants nothing: like the gods, he is self-sufficing; "let men gain wisdom – or buy a rope"; he is a citizen of the world, not of a particular country.*

With all its defective psychology, its barren logic, its immature technique, cynicism emphasized two great and necessary truths: firstly, the absolute responsibility of the individual as the moral unit, and, secondly, the autocracy of the will. These two principles are sufficient ground for our gratitude to these "athletes of righteousness" (as Epictetos calls them). Furthermore they are profoundly important as the precursors of Stoicism.

Finally it is necessary to point out two flaws in the Cynic philosophy. In the first place,the content of the word "knowledge" is never properly developed. "Virtue is knowledge"; knowledge of what? and how is that knowledge related to the will? These questions were never properly answered by them.

Secondly they fell into the natural error of emphasizing the purely animal side of the "nature," which was their ethical criterion. Avoiding the artificial restraints of civilization, they were prone to fall back into animalism pure and simple. Many of them upheld the principle of community of wives; some of them are said to have outraged the dictates of public decency.

It was left to the Stoics to separate the wheat from the chaff, and to assign to the words "knowledge" and "nature" a saner and more comprehensive meaning.

3. The Skeptics (330 BCE–200 CE)

* * *

"Every man of science is somewhat of a Cynic*, because he does not accept words and conventions at their fare value, and of a* skeptic*, because he refuses to believe anything without adequate proof".*

(George Sarton, 1955)

* * *

*"*Skepticism *produces happiness, because by having no dogmatic beliefs you become free from worry".*

"By Skepticism *we arrive first at suspension of judgment, and second at freedom from disturbance".*

(Sextus Empiricus, c. 200 CE)

Skepticism was a philosophical movement in ancient Greece that elevated doubt into an overall thought-principle claiming that it is impossible ever to arrive at the knowledge of truth, either through the senses or by the mind.

The skeptics tried to weaken man's confidence in observation and reason as trustworthy guides to understanding the world. They believed that man can be certain of the nature of his observations, but he cannot be sure that these observations reflect the real world. Therefore, man must withhold judgment if he wants happiness and mental peace.

According to them, the best thing to do was to believe in nothing, to give nothing away, and to feel as little emotion as possible; the only way to live a virtuous life was to dispense with philosophy altogether and avoid asking such questions, since there were no any answers to them. All knowledge is untrustworthy and nothing can ultimately ever be proven. No two views are alike because no two men are alike. Hence – there can be no single truth understood by man.

The philosophical school of Skepticism was a direct result of the competition between the conflicting views of the diverse schools of thought. The Skeptics believed that by suspending judgment, they could avoid a feeling of insecurity arising from possible error. Their most potent rivals were the Stoics who claimed that Skepticism was self-contradictory, since, if nothing can be known, how could this alleged fact itself be known?

The origins of Skepticism can be traced to the earliest days of Greek philosophy when the inclination towards bold assertion was accompanied by a contrary tendency towards questioning, doubt, and sometimes despair. Among the pre-Socratics ***Xenophanes of Colophon*** *(560–478 BCE) and* ***Empedocles*** *(490–430 BCE) expressed occasionally the gloomy feeling that, in the welter of conflicting doctrines, it was really impossible to find any assertion deserving of full belief.*

A basis for skepticism may be found in Socratic irony, a technique devised by Socrates (he started from the assumption that he knew nothing about the truth); a similar approach is evident during the Socratic period in ***Plato****'s dialogues, which arrive at no definite conclusions, in accordance with the Socratic method.*

Eventually however, skepticism itself became a doctrine in the hands of ***Pyrrhon of Elis****, who died at nearly ninety years of age in about 275 BCE. Pyrrhon had served as a soldier with Alexander the Great, and had campaigned with him as far as India.*

Another philosopher present on that occasion was ***Anaxarchos of Abdera*** *(fl ca 320 BCE), a follower of* **Democritos** *(460–370 BCE). Democritos' view of the insufficiency of mere sense perception had been given a definitely* *skeptical emphasis by his followers, and it may thus be that Pyrrhon derived from Anaxarchos the first impulse towards general questioning and doubt.*

Distinguished among later leaders of the Academy, and also a skeptic, was **Carneades** *(214–128 BCE) who made a great stir on a visit to Rome (156 BCE) by giving a series of public lectures, in the first of which he forcefully expounded the views of Plato and Aristotle on Justice, while in the second lecture refuted everything he had said in the first.*

The skeptics apparently differed over the proper guide to personal conduct. Some believed that a man can best decide how to act by calculating the most practical course of action. Others believed he should follow local laws and customs. Under ***Arcesilaos*** *and* **Carneades** *the skeptics philosophy spread through the Athenian Academy. No writings of Skeptics survived. Summaries of Skeptic doctrines were presented in the writings of* **Cicero** *and* ***Sextus Empiricus****, a Roman physician of the 200's CE.*

4. The Epicureans (300 BCE–500 CE)

* * *

"Live unknown".

"Death is nothing to us: for that which is dissolved is without sensation, and that which lacks sensation is nothing to us".

"It is not possible to live pleasantly without living prudently and honorably and justly, nor again to live a life of prudence, honor, and justice without living pleasantly".

"Self-sufficiency is the greatest of all riches".

"The ultimate purpose of philosophy is to lead man to the good life of true happiness".

"If God listened to the prayers of men, all men would quickly have perished".

(Epicuros, ca 306 BCE)

* * *

"Epicuros questions are yet unanswered: Is God willing to prevent evil, but not able? Then he is impotent. Is he able but not willing? Then he is malevolent. Is he both able and willing? Whence then is evil?"

(David Hume, 1765)

Epicureans believed that man could attain the 'good life' by seeking moderate pleasures and avoiding pain. Pleasure can best be gained by living in accordance with prudence, moderation, courage, and justice, and by cultivating friendship. Death should not be feared because good and evil lie in sensation, and death deprives man of sensation.

Epicureanism can be traced back to discussions of the Epicurean Society at Athens in 306 BCE. There had been a few temporary meetings of these philosophers in Mitylene and Lampsacos. Thereafter the society met in the gardens of Epicuros, and were therefore known as "Philosophers of the Garden."[169]

[169] The *Garden* was like the Porch in many respects; their resemblance was perhaps due to common Oriental origins and even more so to the similarity of their functions. As far as can be judged from the fragments relative to it and to its founder, the *Garden was more informal and simpler than the other schools*; life was generally frugal but animated and quickened with regular feast days which brought the fellows more closely together; women were admitted to the fellowship.

Epicuros, who regarded philosophy as medicine for the soul, wrote three hundred books to set forth his views. The basic principles of his philosophy were derived chiefly from two sources: Cyrenaic Hedonism, and the physics, metaphysics, and psychology of Democritos. From the Cyrenaics he accepted the theory that pleasure is the sole good. He agreed with Aristippos that pleasure consists of a gentle motion (the modern equivalent of pleasant emotion), and with Theodoros that the goal of life is an optimistic disposition. From Anniceris came the concept of the great value of friendship, and from Hegesias the conclusion that life's principal objective is to avoid or escape from suffering.

Among post-medieval thinkers, the following were influenced, in one way or another, by the ethics, physics or the metaphysical philosophy of Epicuros' during the past four centuries: **Michel de Montaigne** *(1533–1592);* ***Pierre Gassendi*** *(1592–1655);* ***Isaac Newton*** *(1642–1727);* ***La Mettrie*** *(1709–1751);* ***Helvetius*** *(1715–1771);* ***Immanuel Kant*** *(1724–1804);* ***Jeremy Bentham*** *(1748–1832);* ***James Mill*** *(1773–1836);* ***John Stuart Mill*** *(1806–1873);* ***Herbert Spencer*** *(1820–1903);* **Ernest Renan** *(1823–1892);* ***Karl Marx*** *(1818–1883);* ***Niels Bohr*** *(1885–1963);* ***Werner Heisenberg*** *(1901–1976).*

5. Cyrenaic Hedonism

Hedonism is the belief that pleasure is the highest good in life. The name hedonism comes from a Greek word meaning pleasure. In contradistinction, the Epicureans believed that men should seek pleasures of the mind rather than pleasures of the body, in direct antithesis to everything that the Cynic stood for.

Both philosophical systems developed out of the teachings of Socrates; while the Cynics emphasized the Socratic concept of virtue, the Cyrenaics stressed the Socratic principle that happiness results from the practice of virtue. Whereas the former argued that virtue itself is happiness, the latter taught that the virtuous man is he who knows how to achieve happiness or has the talent necessary for achieving it. Although both philosophies agreed that happiness is a state of mind, for the Cynic the term meant the serenity and mental security which a self-sufficient man experiences, whereas for the Cyrenaic it referred to that satisfaction which follows upon the fulfillment of physical appetite.

Both schools derived their basic concepts from Socratic philosophy and their leaders have been included among the Lesser Socratics. Moreover, both schools developed into more sophisticated thought – the Cynics into Stoicism, and Cyrenaic Hedonism into Epicurean Hedonism.

Just as basic concepts of Stoicism have been perpetuated in Christian doctrine, and in the philosophies of Kant, Spinoza, and other leading philosophers, so the fundamental views of Hedonism have been incorporated in the philosophies of the Utilitarians, Freudians, and some Darwinians.

The founder of Cyrenaic Hedonism, **Aristippos of Cyrene** (435–355 BCE) was influenced by the teaching the Sophist **Protagoras** and subsequently became a disciple of Socrates. His philosophy reflects both sources, particularly the Protagorean doctrine of relativism and the Socratic belief that virtue is a sine qua non of happiness. Since the Cyrenaics equated pleasure with happiness, they concluded that pleasure is man's highest attainable good.

Moreover, according to the Cyrenaic Hedonists, since pleasures are of a single kind, namely, physical satisfactions, there should be no attempt to designate any of them as inferior or superior to the others, the only discernible difference among them being their intensity or duration. The Cyrenaics' theory that pleasures lack qualitative distinction, that they differ only quantitatively from one another, is known as 'quantitative Hedonism'. A corollary of this doctrine is the conclusion that pleasure itself is never evil, that only the laws and customs of the community designate some as good, others as morally bad – the central thesis of ethical relativism.

How did the Cyrenaic Hedonists define this experience of pleasure, which they accepted as man's highest good? Aristippos described it as a sensation of gentle motion in contrast to the violent motion of painful experience – but it must be understood that the Cyrenaics did not use the term motion in the sense of merely physical movement. Their interpretation came much closer to the modern conception of emotion.

They held that there are two basic emotions, the emotions of pleasure and of pain. The emotion of pleasure, or the sensation of gentle motion, resembles the pleasant feeling of the hungry person immediately after he has satisfied his appetite. The Cyrenaics ascribed no significance to the state of apathy, the absence of emotion, which the Cynics regarded as a worthy goal.

Since the Cyrenaic Hedonists believed pleasure to be the only good, they ignored all scientific, mathematical, and cultural pursuits except those with useful applications. In this sense they could be classified as Utilitarians, for they devoted themselves to the study of logic because of its practical utility.

The Utilitarian view of epistemology (the study of how man obtains knowledge) colored their entire philosophy. Accepting the doctrine of Protagoras, they agreed that human knowledge is limited to sensations and does not extend to the real objects to which the sensations correspond – a basic thesis of Protagorean relativism.

Consequently, man is aware of nothing but his own subjective states of feeling; for Aristippos, therefore, feeling is the only valid criterion of truth. The feelings which we experience are the essence of our existence. During our span of life, we should experience as much pleasure as possible; therefore, Aristippos insisted that we must pursue the pleasures of the moment, for tomorrow we shall die. We must enjoy immediate experience inasmuch as the future does not lie in our hand.

Virtue, then, is the means whereby we can achieve pleasure and increase our capacity for enjoyment. But the indiscriminate gratification of pleasure is to be eschewed; at this point in the philosophy of Aristippos, the influence of Socrates is discernible. The sage, or wise man, while enjoying pleasure, remains in control of it. Aristippos claimed that the Cynics erred in seeking independence by abstaining altogether from pleasure, for "not he who abstains, but he who enjoys without being carried away, is master of his pleasures."

***Diogenes Laertius**, discussing the Cyrenaic philosophers, emphasized their view that man's proper goal is to control pleasure and never to be controlled by pleasure. In order to achieve the highest ends of life, man must retain mastery over his experience by means of adaptation to circumstances, self-control, wisdom, the curbing of momentary desires, and an optimistic outlook and temperament.*

*Other leading Cyrenaic Hedonists included Aristippos' son, **Aristippos the Younger**, and daughter, **Arete**; **Theodoros**, the atheist; **Anniceris**; **Euhemeros**; and **Hegesias**.*

Theodoros contended that the specific momentary gratification of pleasure is an inconsequential matter and that, consequently, the primary objective should be to develop an optimistic cheerful attitude toward life. In other words, the wise man knows that true happiness can be found only within the mind of the individual – as a result of an appropriate inner mental disposition.

Anniceris raised Cyrenaic Hedonism to a higher level by designating friendship, gratitude, piety, and aid to others as the true sources of pleasure. Euhemeros contributed the theory that men who had achieved distinction in their lifetime became divine beings in the hereafter. Finally, Hegesias, "the death-counselor," introduced the doctrine of eudaemonistic pessimism, the theory justifying suicide as a way out of the pain and suffering which dominate the lives of the vast majority of human beings.

According to Hegesias, since the frustration of human desires is a universal experience, man should prefer death as a happy, pain-free alternative. Thus, ironically, the philosophy which set out to promote the pursuit of pleasure became a self-defeating philosophy as a consequence of critical deficiencies in its rationale.

6. The Stoics (ca 300 BCE–200 CE)

* * *

"Of all existing things – some are in our power, and other are not. In our power are: thought, desire, will to chose and will to avoid, and, in a word, everything which is our own doing. Things not in our power include the body, property, reputation, office, and, in a word, everything which is not our own doing. Things in our power are by nature free, unhindered, untrammeled; things not in our power are weak, servile, subject to hindrance, dependent on others".

"It is not death or hardship that is fearful, but the fear of hardship and death".

"Death, the most dreaded of evils, is therefore of no concern to us; for while we exist, death is not present, and when death is present, we no longer exist".

(Epictetos, ca 100 CE)

* * *

"Death, as birth, *is a mystery of nature: a composition out of the same elements, and a decomposition into the same. You shall disappear in that which*

produces you, be received back into its seminal principle by transmutation".

"Think of the universal substance, *of which you have a very small portion; and of the* universal time, *of which a short interval has been assigned to you".*

"Constantly regard the universe *as one living being, having one substance and one soul; and observe how all things have reference to one perception, the perception of this one living being, and how all things are the cooperating causes of all things which exist".*

"What then is that which is able to conduct a man? One thing and only one, philosophy*".*

"My city and my country, so far as I am Antoninus, is Rome, but so far as I am a man, it is the world".

"It is one of the acts of life, this act by which we die".

(Marcus Aurelius Antoninus, ca 170 CE)

Stoicism is composed of three parts: Ethical, social and metaphysical, with ethics being by far the most important part of the doctrine. The core of Stoic Philosophy lies in the view that there can be no authority higher than reason. *Its overriding objective was the* attainment of peace, *through an attitude of self-control of one's passions, self-sufficiency and utter indifference both to pleasure and pain.* Virtue *(which the stoics equated with* wisdom *and obedience to* reason*) is the only necessary prerequisite to* happiness.

The stoic view was that all that occurs should be accepted *without any stirring whatever of emotion or appetite. The only good is virtue, the only evil is* vice *(which the stoics equated with failure to control one's passions). Virtue and vice were held to consist, respectively, primarily in right and wrong* *disposition of the will. But the will (it was assumed) was wholly and unalterable under the control of the individual. Everything that did not fall within the sphere of his absolute control, was to be regarded with indifference – pain and pleasure along with the rest.*

According to Stoic philosophy, since passions are a disease of the soul, the individual must expel them completely from his personality; he must never allow his feelings to deteriorate into passions, which are both unnatural and irrational. Man conquers the world only by overcoming his own impulses. External things cannot control him unless he, by an act of his own will, permits them to do so. A person must maintain an unconquerable will, which surrenders to no man. Man should act in accordance with nature because nature is reason – the same reason which every man recognized as the highest part of himself. Everything that occurs has its place in nature's grand design. Their essential worldview can succinctly be described as follows[170]*:*

"The world as our reason presents it to us, that is to say the world of nature, is all the reality there is. There is nothing "higher." And nature itself is governed by rationally intelligible principles. We ourselves are part of nature. The spirit of rationality that imbues us and it (and that is to say, everything) is what is meant by God. As thus conceived, God is not outside the world and separate from it, he is all-pervadingly in the world – he is, as it were, the mind of the world, the self-awareness of the world.

Because we are at one with nature, and because there is no higher realm, there can be no question of our going anywhere "else" when we die – there is nowhere else to go. We dissolve back into nature. It is through the ethics evolved from this belief that Stoicism achieved its greatest fame and influence.

Because nature is governed by rational principles there are reasons why everything is as it is. We cannot change it, nor should we desire to. Therefore our attitude in the face of our own mortality, or what may seem to us personal tragedy, should be one of unruffled acceptance. In so far as our emotions rebel against this, our emotions are in the wrong. The Stoics believed that emotions are judgments, and therefore cognitive: they are forms of "knowledge", whether true or false. Greed, for instance, is the judgment that money is a pre-eminent good and to be acquired by every available means – a false judgment. If all our emotions are made subject to our reason they will embody none but true judgments, and we shall then be at one with things as they actually are."

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[170] Brayan Magee, "The Story of Philosophy", DK Publishing Book, 1998, New York.

The social *component of Stoicism is based on the recognition that* friendship *is natural, hence good. Although Stoics were indifferent to nationalistic loyalties, they had faith in the social value of friendship between individuals. Common brotherhood and common legal prescriptions were prized as natural laws, two of which – justice and brotherly love – are innate in human nature. Marriage is an accepted institution insofar as it is infused with moral spirit. The Stoics viewed man as a citizen of the world. Their cosmopolitan spirit was reflected also in their defense of the natural rights of slaves. The Stoic social philosophy harmonized with Christian ideals and its point of view was buttressed by the rise of Christianity.*

Stoic metaphysics and philosophy of religion was closely allied with ethical naturalism; piety was identified with knowledge, and religious obedience with universal laws of nature. Stoic religious naturalism acquired pantheistic characteristics, emphasizing the belief that God and nature are one, although it differentiated the essential nature of God from the world of nature. Their metaphysics, which was decidedly Heraclitean, posited fire as the fundamental principle in things. It was their view that man's fate is determined by the mechanistic laws governing all natural phenomena. They linked religion closely with philosophy. Although they permitted recourse to polytheistic spirits as media for the worship of God, they held steadfast to a monotheistic doctrine and made their belief in one supreme, universal Deity (as creator and sustainer of the universe) the foundation of their moral philosophy.

Three types of Stoics were common, each emanating from its respective source: (1) philosophers, (2) statesmen, and (3) poets. Consequently, the Stoics remained sympathetic to the central themes of their religion. Their point of view was optimistic about nature, but pessimistic about man's moral insight. Stoic pessimism was vividly reflected in their acceptance of suicide (taedium vitae) *as a permissible solution to extreme exigencies of life. This practice seemed to them a symbol of moral strength, evidence that they could choose to be indifferent to life itself.*

Accordingly, they felt justified in ending life whenever the natural course of events made such a course appear to be an appropriate way of dealing with insufferable problems. Zeno committed suicide, as did Cleanthes, who deliberately starved himself to death. It may be noted that the Stoics were not always consistent in their ideology. Thus they adhered to the doctrine of self-preservation as the most fundamental law of nature, yet this doctrine was precisely contrary to their belief in suicide as a rational solution.

The Greek name for Stoicism, Stoa Poikile *(Painted Porch), referred to the portico in Athens where the early adherents held their meetings. The movement was founded by* ***Zeno of Citium*** *(c. 340–265 BCE), who was a disciple of the Cynic* **Crates***, and taught the first Stoic groups in that center.*

Among his eminent contemporary Stoics were two heads of the Stoic school: ***Cleanthes of Assos*** *in Troas (c. 303–232 BCE), who had also been a student of Crates and was the purported author of a monotheistic hymn (Hymn to the Most High) to Zeus; and* ***Chrysippos of Soli*** *(280–206 BCE), who integrated many of the Stoic doctrines and classified their terminology. Two students of Chrysippos, namely,* ***Zeno of Tarsos*** *and* ***Diogenes of Babylon****, succeeded him as leaders of Stoic philosophy in the third century BCE.*

The Stoics of the middle period (second and first centuries BCE) modified the doctrines of early Stoa to take into account the doctrines of the Platonic philosophers of the Academy and the Aristotelian Peripatetics. The founder of this form of Stoicism was ***Panaetios of Rhodes*** *(c. 180–110 BCE) who was greatly influenced by some of the views of Plato even though he disagreed with the fundamental Platonic principles expounded by Carneades, head of the Academy. Subsequent leaders of the Stoics during this middle period were* ***Antipater of Tarsos*** *and* ***Boethos of Sidon*** *(d. 119 BCE).*

Later Stoa, which prevailed during the first two centuries of the Christian era, was dominated by two groups of Roman philosophers, one concerned chiefly with Stoic interpretations of reality, the other with applied morality, that is, a religious belief in God's relationship to and interest in mankind and the universe. To this period belonged ***Arios Didymos of Alexandria*** *(63 BCE–10 CE), author of commentaries on Greek philosophical works;* ***Lucius Annaeus Seneca****, tutor and for some years an advisor to Nero, who condemned him to death in 65 CE;* **C. Musonius** ***Rufus of Volsinii****;* ***Epictetos of Hierapolis*** *(c. 50–138 CE), the most articulate Stoic exponent of moral philosophy; and the Roman emperor* **Marcus Aurelius Antoninus** *(121–180 CE).*

During the period when Christian institutions were developing (230–1450 CE), elements of Stoic moral theory were known and used in the formulation of Christian and Muslim philosophical doctrines of man and nature, of the state and society, and of law and sanctions – e.g. in the works of **Boethius** *(524–525 CE),* ***Isidore of Seville*** *(ca 560–636 CE), and* ***John of Salisbury*** *(1120–1180 CE).*

If the influence of Stoic doctrines during the Middle Ages was largely restricted to the resolution of problems of social and political significance, it remained for the Renaissance*, in its passion for the rediscovery of Greek and Roman antiquity, to provide a basis for the rebirth of Stoic views in logic, epistemology, and metaphysics, as well as the documentation of the more familiar Stoic doctrines in ethics and politics. The thinkers associated with this revival are:* ***Pietro Pompanazzi*** *(1462–1525 CE);* ***Desiderius Erasmus*** *(1466–1536);* ***Thomas More*** *(1478–1535);* ***Huldrych Zwingli***

(1484–1531); ***Philipp Melanchton*** *(1497–1560);* **Michel de Montaigne** *(1533–1592);* ***Pierre Charron*** *(1541–1603);* ***Justus Lipius*** *(1547–1606).*

The leading thinkers of the seventeenth century were predominantly rationalistic. Reason was the faculty that distinguished man from the beast, and the triumphs of seventeenth century science proved that reason could be trusted. And so the conclusion was drawn that the man of reason could know and understand the world into which he was born if he made the right use of his mind.

This optimistic attitude was reflected in the growing belief in "natural law." The idea of a law of nature that served as a standard of moral behavior for all men at all times in all places originated with the Stoics and was developed by the medieval Schoolmen. During the Renaissance and the Reformation this idea went into eclipse, but the discovery of "laws of nature" like Kepler's laws of planetary motion helped to revive it in the seventeenth century.

Cicero had given the idea classic formulation: "There is in fact a true law – namely, right reason – which is in accordance with nature, applies to all men, and is unchangeable and eternal. By its commands this law summons men to the performance of their duties; by its prohibitions it restrains them from doing wrong."

This law was implanted in the minds of men by God himself. Its content was hazy, but it was understood to include respect for life and property, good faith and fair dealing, giving each man his due. These principles could always be discovered by reason, just as reason could discover the proof of a geometrical proposition.

The influence of Stoic philosophy on 17^{th} and 18^{th} centuries thinkers is reflected through the works of: ***Francis Bacon*** *(1561–1626);* ***Hugo Grotius*** *(1583–1645);* **Rene Descartes** *(1596–1650);* ***Blaise Pascal*** *(1623–1662);* ***Baruch Spinoza*** *(1632–1677) and* ***Montesqieu*** *(1689–1755).*

Along with its rivals, Stoicism enabled the individual to better order his own life and to avoid the excesses of human nature that promote disquietude and anxiety. It was easily the most influential of the schools from the time of its founding through the first two centuries CE, and it continued to have a marked effect on later thought.

Unfortunately, the Stoics paid little, if any, attention to science and favored divination (manteia) and astrology; on the ethical plane their doctrines were too abstract, cold, impersonal; this explains the ultimate victory of Christianity over Stoicism, for the Christians put a new emphasis on love, charity, and mercy.

Christianity in general, in spite of striking contrasts with Stoicism, has found elements within it that parallel its own position. As the Stoic, for example, feels safe and protected in the rational care of some immanent Providence, so the Christian senses that a transcendent though incarnate and loving God is looking after him. And in general, Stoicism has played a great part throughout the ages in the theological formulation of Christian thought as well as in the actual realization of the Christian ideals.

Contemporary philosophy has borrowed from Stoicism, at least in part, its conviction that man must be conceived as being closely and essentially connected with the whole universe. And contemporary Humanism still contains some obviously Stoic elements, such as its belief in the solidarity of all peoples based upon their common nature, and in the primacy of reason. It is perhaps just because Stoicism has never become a full-fledged philosophic system that, many centuries after the dissolution of the Stoic school, fundamental themes of its philosophy have emerged again and again, and many have become incorporated into modern thinking.

Stoic ethics have always been widely found to be impressive and admirable, even by people who do not wholly go along with them. They are not easy to practice – but perhaps it is bound to be a characteristic of any ethics worthy of the name that they are difficult to put into practice. They had an unmistakable influence on Christian ethics, which were beginning to spread at the time when Seneca, Epictetos, and Marcus Aurelius were writing. And, of course, to this very day the words "stoic" and "stoicism" are in familiar use in our language, with perhaps grudgingly admiring overtones, to mean "withstanding adversity without complaint".

There must be many people now living who – even if they have never consciously formulated this fact to themselves – subscribe to an ideal in ethics which is essentially the same as that of the Stoics.

The fact that in recent centuries the best available school education in many European countries was based on the study of Latin literature had, as one of its side-effects, that many generations of well educated European males absorbed some of the values of Stoicism.

7. The Eclectics (100 BCE–200 CE)

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“Truth itself is hidden in obscurity; I only wish I could discover the truth as easily as I can expose falsehood”.

“Frivolity is inborn, conceit acquired by education”.

“Any man is liable to err, only a fool persists in error”.

“To be ignorant of what occurred before you were born is to remain always a child. For what is the worth of human life, unless it is woven into the life of our ancestors by records of history?”

“Law is the highest reason implanted in Nature, which commands what ought to be done and forbids the opposite”.

“We must conceive of this whole universe as one commonwealth of which both gods and men are members”.

“Mankind must pray to God for fortune but obtain wisdom for themselves”.

“Men are sprung from the earth, not as its inhabitants and denizens, but to be as it were the spectators of things supernal and heavenly, in the contemplation whereof no other animal participates”.

"The study and knowledge of the universe *would somehow be lame and defective, were no practical results to follow".*

"That long time to come when I shall not exist has more effect on me than this short present time".

(Marcus Tullius Cicero, ca 50 BCE)

The Eclectics advocated a philosophical system which would bring together and integrate the best ideas of all the philosophers. Like their predecessors, both the Skeptics and the Eclectics regarded philosophy as a practical inquiry, for they were concerned primarily with ethics, the philosophy of life, *a philosophy which can be implemented. This system which amalgamated the best doctrines available, was established mostly by Roman philosophers, including particularly* **Cicero** *(106–43 BCE) and* **Seneca** *(4 BCE–65 CE).*

The Eclectics did not take refuge in Skeptical defenses to maintain a formidable philosophical position. Yet, like the Skeptics, they too developed a reactionary view seeking to establish acceptable doctrines to cope with the conflicting philosophical situation which prevailed at that time.

The following four leading schools of philosophy in Athens competed for adherents: (1) the Stoa *group of the Stoics; (2) the* Epicurean *"Philosophers of the Garden"; (3) the* Academy *(Platonists); and (4) the* Lyceum, *consisting of the Peripatetic philosophers accepting the views of Aristotle. The Skeptics supported none of these schools; in contrast the Eclectics sought truths in all of them and attempted to unify them into a single integrated philosophical system.*

It will be recalled that the Skeptics of the Middle Academy regarded probability *as the best criterion of truth. The Stoics preferred* consensus gentium, *that is, the universal opinion of mankind derived from the laws of nature, such as the law of self-preservation. Cicero and other Roman philosophers joined these two criteria with the concept of innate ideas (a concept which the modern philosopher Descartes later used as a foundation stone of his philosophy). In ethics,* **Cicero** *vacillated between the* Stoic *virtue of self-control and the* Peripatetic *view of happiness as the product of satisfying purposive activity. In the field of physical science, he showed no interest. In political philosophy, he blended the ideas of Platonism, Aristotelianism, and Stoicism. (The Eclectics had revived and emphasized ancient ideas which had been neglected and* *underemphasized, but they did not actually contribute any important original ideas to philosophy.)*

8. Gnostic and Gnosticism (100 CE–500 CE)

Alexander the Great's military campaigns were instrumental in expanding Greek cultural influence into what is now Egypt, the Middle East, and out into India. Almost inevitably this brought Greek culture into contact with many other cultures that adhered to very different religious and philosophical traditions.

Following Alexander's death in 323 BCE, the Greek empire collapsed and political power in the Mediterranean shifted from the east to the west, from Greece to Rome. In 146 BCE, Greece became a Roman colony and Rome was about to begin the cultural dominion over Europe and beyond that was to last until the barbarian invasions some 500 years later.

The status of the Land of Israel as a Roman colony was a source of resentment among certain members of the many radical Jewish sects that existed at this time. It was also the place where Greek, Judaic, and Persian philosophers met each other. From this cultural melting pot there arose a number of spiritual movements.

Gnosticism *is a blanket term used to embrace various dualistic esoteric cosmologies and spiritual teachings which developed in the syncretic Hellenistic civilization in the centuries immediately after Christ. The name designates a wide assortment of sects that promised salvation through an occult knowledge that they claimed was revealed to them alone. Scholars trace these salvation religions back to such diverse sources as* Jewish *mysticism,* Hellenistic *mystery cults,* Iranian *religious dualism (*Zoroastrianism[171]*),* Babylonian *and* Egyptian *mythologies, astrology and pagan religions. The definition of* Gnosis[172] *was already present in earlier Greek philosophy.*

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171 Zoroastrianism — ancient Persian religion that viewed the universe as an eternal struggle between forces of light and darkness. (Still has many adherers today. The German philosopher Nietzsche have been sympathetic to some of its doctrines.)

172 *Gnosis*, from the Greek $\gamma\nu\omega\sigma\iota\zeta$ = knowledge, insight, enlightenment (it is pronounced with a silent "G", No-sis). The essential knowledge that comes from *within.* A gnostic may be part of a tradition (Sufi, Kabbalistic, Christian, etc.), but each person always interprets that tradition in an individual way;
Complete comprehension which comes from both rational and intuitive means.

Gnosticism, Religion and Philosophy

Gnosticism is the teaching based on Gnosis, the knowledge of transcendence arrived at by way of interior, intuitive means, as opposed to episteme, which is knowledge in the more mundane sense. The Gnostics believed that knowledge, not blind faith, held the key to the mysteries of life, but they thought that knowledge came from spiritual insight, rather than from scientific study.

This knowledge is hidden (esoteric) and only a few may possess it. In other words, Gnostics claimed to have secret knowledge about God, humanity and the rest of the universe of which the general population was unaware.

Gnosticism thus rests upon specific personal religious experience, an experience that does not lend itself to the language of theology or philosophy, but which is instead closely affinitized to, and expresses itself through, the medium of myth. The truths embodied in these myths are of a different order from the dogmas of theology or the statements of philosophy.

The Gnostics taught that:

(1) *The physical world is ruled by evil Archons, led by a lesser "god" (known as Demiurge, after Plato), the deity of the Old Testament, who hold captive the spirit of humanity. But human bodies, although their matter is evil, contain within them a divine spark (pneuma) that fell from the Source, or Nothingness from which all things came. Knowledge (gnosis) enables the divine spark to return to the Source from whence it came. Thus, salvation is achieved through knowledge.*

    Gnosticism held that human beings consist of flesh, soul, and spirit (the divine spark), and that humanity is divided into classes representing each of these elements. The purely corporeal (hylic) lacked spirit and could never be salved; the Gnostics proper (pneumatic) bore knowingly the divine spark and their salvation was certain; and those, like the Christians, who stood in between (psychic), might attain a lesser salvation through faith. Such a doctrine may have inspired extreme asceticism (as in the Valentinian school) or extreme licentiousness (as in the sect of Caprocrates and the Ophites). The influence of Gnosticism on the later development of the Jewish Kabbalah and heterodox Islamic sects such as the Ismailis is much debated.

(2) *The Good God is a transcendental spiritual being, who is utterly alien to this world, and had nothing to do with its creation.*

Experience based not in concepts and percepts, but in the sensibility of the heart; The idea that there is a special hidden knowledge that only a few may possess.

(3) *The Savior – whether Christ, Seth, the Thought (Ennoia) of God, or some other figure – is an emissary of the transcendental God who has descended into this lower world to confer gnosis on those able to receive it (the gnostic race).*

In stark difference to many orthodox religions or secular worldviews, Gnosis holds that human's "natural" state in this world is that of contingency, dissatisfaction, bondage and exile. At best, human beings, confined and enclosed within gnostic "box" of the empirical natural world, can live lives of temporary relief and fleeting pleasures. True peace and fulfillment evade them. Salvation can be achieved only by contemplation (theoria), not by faith (orthodox Christianity).

Christian ideas were quickly incorporated into these syncretic systems by ***Simon Magus*** *(fl. 30 CE),* ***Marcion of Sinope*** *(85–160 CE),* ***Capocrates*** *(fl. 140 CE),* ***Ptolemaeus*** *(fl. 140 CE),* ***Valentinus*** *(ca 100–175 CE) and* ***Basiliades of Alexandria*** *(fl. 135 CE). The largest of them, organized by Valentinus and Basiliades, were a significant rival to Christianity. Other known early Gnostics were* ***Numenius of Apamea*** *(fl. 150 CE), a Platonic philosopher, and* ***Mani*** *(216–276 CE) who founded the syncretic religion of Manichaeism with roots in Zoroastrianism, Jewish Christianity and Buddhism. Much of early Christian doctrine was formulated in reaction to this movement by* ***Irenaeus*** *(ca 130–200 CE),* ***Clement of Alexandria*** *(ca 145–213 CE),* ***Tertullian*** *(160–225 CE),* ***Hippolytus*** *(170–230 CE) and* ***Plotinos*** *(fl. 255 CE).*

By the second century CE, many very different Christian-Gnostic sects had formed within the Roman Empire at the eastern end of the Mediterranean. Some Gnostics worked within Jewish Christian and mainline Christian groups, and greatly influenced their beliefs from within. Others formed separate communities. Still others were solitary practitioners.

There does not seem to have been much formal organization among the Gnostics during the early centuries of the Christian movement. As mainline Christianity grew in strength and organization, Gnostic sects came under increasing pressure, oppression and persecution. They almost disappeared by the 6^{th} *century. The only group to have survived continuously from the* 1^{st} *century CE into modern times is the Mandaean sect of Iraq and Iran who can trace their history continuously back to the original Gnostic movement.*

Many new emerging religions in the West have adopted some ancient Gnostic beliefs and practices. By far, the most successful of these is the Church of Jesus Christ of Latter-day Saints– the LDS or Mormon church, centered in Salt Lake City, UT.

Gnosticism began with the same basic, pre-philosophical intuition that guided the development of Greek philosophy — that there is a dichotomy between the realm of true, unchanging being, and ever-changing becoming. However, unlike the Greeks, who strived to find the connection between the overall unity of these two 'realms', the Gnostics amplified the difference, and developed a mytho-logical doctrine of humankind's origin in the realm of Being, and eventual fall into the realm of darkness or matter, i.e. Becoming.

This general Gnostic myth came to exercise an influence on emerging Christianity, as well as upon Platonic philosophy[173]*, and even, in the East, developed into a world religion (Manichaeism) that spread across the known world, surviving until the Middle Ages.*

Gnosticism, in its turn, was influenced by:

1. Hellenistic-Jewish *speculation regarding the figure of Sophia (in Hebrew,* Hokmah*) or divine Wisdom, a personified female creator deity — midrashic (rabbinic) interpretation of the first few chapters of Genesis (specifically, Gen* ***2***–*6)*

2. *A particular form of baptismal ritual. Baptism was a common practice at this time (the biblical John the Baptist being by no means unique), but the Sethians believed their baptismal water to be of a celestial nature, a Living Water identical with the spiritual Light, and enabling the ascent of the soul; and so were critical of the ordinary water baptism*

3. *The developing* Christ-doctrine *(Christology) of the early church, especially the identification of Christ with the pre-existent Logos (the creative power or emanation of God)*

4. Neopythagorean *and* Middle Platonic metaphysics*, such as the emanation of divine beings from a single transcendent Absolute, and the understanding of the cosmos as the reflection of the spiritual world.*

[173] Long before the advent of Gnosticism, **Plato** had posited two contrary World Souls: one "which does good" and one which has the opposite capacity. It meant to him that this cosmos, like the human soul, possesses a rational and an irrational part, and that it is the task of the rational part to govern the irrational. Clearly, this conjecture flew in the face of everything that the ancient thinkers believed about the cosmos — i.e., that it was divine, orderly, and perfect. A common solution, among both Platonists and Pythagoreans, was to interpret the "evil" soul as Matter, which is the opposite of the truly divine and unchanging Forms. Then, the purpose of the "god" soul is to bring this "disorder" under the control of reason, and thereby maintain an everlasting (but not *eternal*) cosmos.

5. *Babylonian and Chaldean Astrology; the identification of the planets with deities, and the postulation of a number of a celestial heaven pertaining to each; all of which determines human destiny and induces a sort of fatalism.*

6. *The Pythagorean tradition with its pairs of opposites (light and dark, good and evil, spirit and matter, etc.)*

7. *Zoroastrian dualism*

8. *Ancient Egyptian thought, specifically Memphite and Heliopolitan theologies*

In the 20th century, there began a renewed interest in Gnostic ideas; Gnosticism has been treated by several modern authors, philosophers and psychologists:

- ***Carl Jung*** *(1875–1961, Switzerland), drew upon Gnostic motifs in his theoretical work and explained the Gnostic faith from psychological viewpoint.*

- ***Hans Jonas*** *(1903–1993, Germany and USA) interpreted it from an existential viewpoint. Indeed Gnosticism, as an intellectual product, is grounded firmly in the general human act of reflecting upon existence. The Gnostics were concerned with the basic questions of existence or "being-in-the-world" (Dasein) – that is: who we are (as human beings), where we have come from, and where we are heading, historically and spiritually. These questions lie at the very root of philosophical thinking; but the answers provided by the Gnostics go beyond philosophical speculations toward the realm of religious doctrine and mysticism.*

- ***Eric Voegelin*** *(1901–1985, Germany and USA) held the view that totalitarian ideologies were caused by Gnostic impulses, including Communism and Nazism.*

331 BCE, Sept. 20 Eleven days before the great *battle of Arbela*[174] (Oct. 01, 331 BCE, the last of the Greco-Persian wars and the end of Darius' Persian

[174] **Pliny** implies that at Arbela the moon was eclipsed two hours after sunset, while at Sicily the full moon was at sunset. **Ptolemy** also cites this eclipse and uses Pliny's data to calculate the distance between Carthage and Arabela.

Empire), there occurred a lunar eclipse[175] just as **Alexander**'s army crossed the Tigris. Greek astronomers used it to determine the difference in *longitude*[176] of two cities from the different *times* read in clocks when the eclipse began. One of the cities was *Carthage* (36°54′N, 10°16′E; now Tunisia) and the other *Arbela* (36°12′N, 44°01′E; now Arbil, Iraq). Sadly, the clocks were wrong, the difference in longitude was overestimated, and maps of the region were severely distorted.

ca 330 BCE **Praxagoras of Cos** (ca 360–290 BCE). Greek physician; the first to study the *pulse* and the first to point out the distinction between *arteries* and *veins*, although he thought that arteries are hollow tubes that carry *air* throughout the body. Indeed, the word artery is derived from the Greek *arteria* = windpipe [since it was noticed that no blood is in them after death, it was assumed the arteries carried air].

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[175] In modern times, astronomical determination of longitude at an observing station is done by fixing the exact instant of a star's meridian passage. To determine the longitude of the station, one needs to determine only the *Greenwich-time* of the transit of the star over the local meridian. The difference in time between the star's meridian passage at the said station and its meridian passage at Greenwich is converted into the difference in longitude of the two stations by using the knowledge that the earth rotates at a rate of 15° in each hour of sidereal time. In practice, radio signals are used, where any reference longitude (not necessarily Greenwich) can serve.

[176] The plane of the earth's orbit and the plane of the moon orbit intersect along a line called the *line of nodes*. The line of nodes passes through the earth and is pointed in a particular direction in space. Lunar eclipses can occur only when both the sun and the moon are in or very near the line of nodes, because only then do the sun, earth, and moon lie along a straight line at a full moon.

When the moon is full and near one of its *nodes*, the *umbra* of the earth will cover its face completely, producing a *total lunar eclipse*. Somewhat further from the node, only part of the umbra will fall on the moon – a *partial lunar eclipse*. Still further from the node, only the *penumbra* may fall on the moon. If the moon is more than about 10° from the node, there is no eclipse. As the moon and sun will be near the nodes only twice a year, usually not more than two lunar eclipses occur in one year.

The moon's hourly motion in the sky is a little greater than its own diameter. As the width of the earth's shadow at the moon's distance is more than two and a half times the moon's diameter, the whole moon may be in shadow (totality) for over an hour; shadow may cover part of the moon for about two hours. Lunar eclipses are visible at the same moment from every part of the earth's surface, wherever the moon is above the horizon at the time.

330–310 BCE **Callippos** (ca 370–310 BCE). Astronomer. Made accurate determinations of the length of the seasons and constructed a 76 year cycle to harmonize the solar and lunar years, which was used by all later astronomers.

330–290 BCE **Deinocrates of Rhodes**. Greek architect. Designed for Alexander the Great the new city of Alexandria and for Ptolemy I Soter the library and museum.

Deinocrates designed the *new* temple of Artemis at Ephesos and constructed the vast funeral pyre at Hephaestion.

325 BCE **Antimenes** (Greece) devised the first system of insurance mentioned in history. With a premium of 8 percent per annum he guaranteed owners against the loss of their slaves.

ca 325 BCE **Eudemos of Rhodes**. The first historian of mathematics on record. A pupil of Aristotle, a friend of Theophrastos and a member of the Lyceum. Among the writing ascribed to him (but lost), were histories of arithmetic, geometry, and astronomy. Only fragments have come to us, yet his work was the main source out of which whatever knowledge we possess of pre-Euclidean mathematics has trickled down. The appearance at this time of a historian of mathematics and astronomy proves that so much work had already been accomplished in these fields that a historical survey had become necessary.

ca 320 BCE **Aristaios the Elder**. Mathematician. Marks the transition between the age of Aristotle and the Age of Euclid . Wrote a treatise on Conics regarded as *loci*, anticipating Euclid's book on the same subject. He defined the different kind of conics as sections of cones with acute, right, and obtuse angles. In a second book entitled *Comparison of the five figures* (regular solids) he proved the remarkable proposition that "*the same circle circumscribes both the pentagon of the dodecahedron and the triangle of the icosahedron when both solids are inscribed in the same sphere*"[177].

ca 320 BCE **Pytheas of Massilia**. A Greek explorer-mariner, geographer, and astronomer. The first Southern European to visit the northern seas. He was also the first person to work out the position of the true North (realizing that the North Star is not directly above the North Pole), to *link the alternation of tides with the phases of the moon* and to invent an accurate

[177] This is truly on unexpected result! For who could have foreseen that the faces of two different regular solids are equally distant from the center of the sphere enveloping them? Thus the icosahedron and dodecahedron have a special relation not shared by the other three solids.

method of determining latitude with a calibrated sundial. At the time when Alexander the Great had carved himself an empire as big as the United States – territory stretching from the eastern Mediterranean to the banks of the Indus – the Greeks, astonishingly, had given no more than a passing glance to the North and West. It was not until some years after Alexander's death in 323 BCE that a Greek first ventured into the chill waters of the Arctic Circle.

Sometime near 320 BCE, Pytheas set out from his native city, Marseilles, slipped by a blockade set up by the Carthaginian navy at Gibraltar and headed northward to Britain and beyond on an extensive voyage of the order of 11,000 km. He visited the Cornish tin mines and circumnavigated Britain, which he described as triangular in shape with three unequal sides (its perimeter he reckoned at about 4000 km). Thus he had occasion to observe the considerable tides on its coasts, and the daily regression of the times of high water, parallel to that of the time of the moon's transit. He then went to make a 6-day crossing of the North Sea to 'Thule' (Norway). Turning back to a further commercial objective, Pytheas sailed along the coast of Europe to the estuary of the Vistula on the Baltic coast.

His writings have been lost, and are known only through quotations and allusions by later authors. Also, since Pytheas was acting as a commercial spy, his mission was cloaked in a certain amount of secrecy. Greek and Roman geographers, among them **Eratosthenes**, **Polybius** and **Strabo**, were incredulous about his geographical discoveries, and scoffed at them [they found it hard to believe that lands in the latitude of Britain could be habitable – knowing nothing of the Gulf Stream and its effect on the climate of western Britain].

However, geographers would depend for centuries on Pytheas' data about northern countries. Later discoveries showed that he was telling the truth about what he had seen. The countries visited by Pytheas were not visited by any subsequent authority during more than 200 years. Pytheas' accurate determinations of latitude were adopted by Ptolemy, and became the basis of the Ptolemaic map of the Western Mediterranean.

312 BCE *Via Appia* (Appian way). The most famous of Roman roads. A high-road leading from Rome to Campania and lower Italy, whose construction began in 312 BCE by the censor **Appius Claudius Caecus**. It originally ran only as far as Capua (200 km), but was successively prolonged to Beneventum (268 BCE), Venusia (190 BCE), Tarentum, and Brundisium. Initially covered by gravel and later by stone; altogether, the road was made of multiple layers of durable materials, the top layer composed of a mixture of concrete, rubble and stones set in mortar.

Not only was the road usable by troops in all weather, it was crucial to building commercial interests and sustaining cultured links and political control over the provinces. The road was administered under the empire by a curator of praetorian rank, as were the other important roads in Italy.

Over time, several roads were built to link Rome with other cities and colonies, including the *Via Flaminia*, which headed north to link Italy with the Latin colony of Ariminum. In all, the Roman road system covered 80,000 km and crossed through 30 countries.

The Via Appia was the world's first all-weather road system built to facilitate warfare. Following their defeat in the *Samnite Wars*, particularly their humiliation at the Battle of the *Caudine Forks* along the rocky Apennines (321 BCE), the Roman military began to develop more effective attack formations and better transportation routes through uneven terrain.

A large number of milestones and other inscriptions relating to its repair at various times are known.

The first aqueduct brought pure water into Rome.

ca 310 BCE **Zeno of Citium** (336–264 BCE). Philosopher of Phoenician descent. Founded the *Stoic*[178] school of philosophy in Athens. Most of its early representatives were not Greeks but Asians. Stoicism began in Greece and then spread to Rome and flourished for more than 500 years until it was finally harmonized with the spirit of Christianity by some Fathers of the Church.

The early Stoics were interested in logic and natural philosophy as well as ethics. The later Stoics, especially **Seneca** (4 BCE–65 CE), **Marcus Aurelius** (121–180) and **Epictetos** (ca 60–135) emphasized ethics.

Zeno was born in Citium on the Island of Cyprus. The commercial activities of his family first took the young man to Athens (314 BCE) and there he developed an interest in philosophy. He abandoned trade and eventually set up a school of his own.

Fragments of 26 books written by him are extant, but most of his works have been lost.

According to Zeno, nature is strictly ruled by law; the laws in conformity with which the world runs its course emanate from some supreme authority that governs history in all its details. Everything happens for some reason in a pre-ordained manner. The supreme or divine agency is thought of not as something outside the world, but running through it, like moisture seeping

[178] Derived from *Stoa*, a painted roofed colonnade (portico) on the north side of the market place of Athens, which the celebrated painter Polygnotos had adorned with frescoes representing scenes from the Trojan war. There Zeno taught his disciples.

through sand. God is thus immanent power, part of which lives within each human being. (This kind of view has become famous in modern times through the philosophical writings of **Spinoza**, who was influenced by the Stoic tradition.) Men's lives are guided by Provident Reason, against which it is futile to resist and to which the wise man willingly submits with indifference to life's vicissitudes.

It was Stoicism, not Platonism, that filled men's imaginations and exerted the wider and more active influence upon the ancient world at some of the busiest and most important times in all history. And this was chiefly because it was above all a practical philosophy, a rallying point for strong and noble spirits contending against odds. Its concepts, values, and codes of honor have infiltrated Western Culture more deeply than we commonly recognize.

Although Stoicism was by no means dormant through the Middle Ages, the great period of its revival began with the Renaissance and lasted until the beginning of the 19^{th} century. Stoic morality inspired **Shakespeare**, **Schiller**, **Spinoza**, **Kant** and many leaders of the French Revolution.

Stoicism is symptomatic of the subtle infusion of the Asian soul into the wearied civilization of the Greeks overlords during the decay of Greece after the death of **Alexander the Great** (323 BCE). While this boy-emperor dreamed of spreading the Greek culture through the Orient, he ended up importing into Europe oriental cults and faiths along the new lines of communication opened up in the wake of his victorious armies. He had underrated the inertia and resistance of the Oriental mind, and depth of the Oriental culture.

The Oriental spirit of apathy and resignation found a ready soil in decadent and despondent Greece. The introduction of the Stoic philosophy into Athens by the Phoenician merchant Zeno was but one of a multitude of Oriental influences.

Both Stoicism and Epicureanism – the apathetic acceptance of defeat, and the effort to forget defeat in the arms of pleasure – were theories as to how one might yet be happy though subjugated or enslaved. When the glory had departed from Athens, she was ripe for Zeno and Epicuros.

The Romans, coming to despoil Hellas in 146 BCE, found the rival schools of Stoicism and Epicureanism dividing the philosophical field. Having neither leisure nor subtlety for speculation themselves, they brought back these philosophies with their other spoils to Rome.

Stoics agreed that space has no edge and rejected the Aristotelean outer boundary of the universe. They proposed instead a system consisting of a star-filled cosmos surrounded by a starless extracosmic void extending to infinity. The Stoic astronomical scheme endured in various guises for more than 2000 years until the first quarter of the 20^{th} century, when the existence of the galaxies beyond the Milky Way was established beyond dispute.

Cleanthes (310–232 BCE), a devoted disciple of Zeno, studied under his master for 19 years. Upon Zeno's death, he assumed the directorship of the school for 31 years. **Chrysippos** (ca 280–207 BCE) succeeded Cleanthes as director of the Stoic school. He was born in Soli, Cilicia, Asia Minor, and went to Athens in 260 BCE.

310–301 BCE **Epicuros of Samos** (341–270 BCE). Greek philosopher. Promoted the concept of Democritos that matter is made of atoms and adopted the principle of *conservation* of *matter*. Established in Athens a freethinking school that embraced much of the atomistic philosophy. He rejected the gods as the controlling forces of the natural world, invoked physical causes whenever possible, and taught that sense perceptions form the basis of all knowledge. Epicuros also outfitted the atomistic world with a comprehensive theory of ethics, claiming the highest good to be pleasure taken wisely and in moderation, leading to life of maximum freedom from physical and mental pain. The atomistic system, taught in this practical form – unfortunately easily misunderstood as a hedonistic doctrine – gained the acceptance of multitudes throughout the known world.

Educated Romans, skeptical of the mythological religion, turned to the teachings of Epicuros. For more than seven centuries, three before and four after Christ, Epicureanism flourished. Others – first the Platonists, then the Stoics, and finally the Christians – reviled Epicuros as an atheist and attacked his doctrine with bitter hostility.

Epicuros was born on the Island of Samos, seven years after the death of Plato. At the age of 18 he went to Athens, where the Platonic school was flourishing under the lead of Xenocrates, and stayed there for a year. Stimulated by the perusal of some writings of Democritos, he began to formulate a doctrine. In 307 he returned to Athens and established there a philosophy school. He lived there for the rest of his life.

The arena for his teaching was a garden which he purchased. There he passed his days as the loved and venerated head of a remarkable and unique society of men.

310–272 BCE **Hieronymos of Cardia** (ca 350–246 BCE). Greek historian and general. Took active parts in the wars that followed the death of Alexander the Great. He served successively Eumenes of Cardia, Antigonos and Demetrios in various administrative and diplomatic posts, including that of governor of Boeotia and superintendent of the asphalt beds of the Dead Sea.

Hieronymos wrote a history of the *Wars of the Successors* from the death of Alexander (323 BCE) to the death of Pyrrhos (272 BCE). His work was used by **Arrian**, **Diodoros** and **Plutarch**. In the last part of his work he made an attempt to acquaint the Greeks with the character and early history of the Romans.

ca 310 BCE **Autolycos of Pitane**[179] (ca 340–280 BCE). Astronomer and mathematician. An older contemporary of Euclid, representing the transition period between the Hellenic school of mathematics and the Alexandrian Age. Author of the earliest Greek mathematical text-book that has come to us entire. His first treatise, entitled *On the moving sphere*, deals with the geometry of the sphere (poles, great circles, etc) and many of its prepositions are used by Euclid in his *Phaenomena*.

Another work of Autolycos, in two books, *on the rising and settings* (of stars) is more astronomic and implies observations. In these books, propositions follow one another in logical order; each proposition is clearly enumerated with reference to lettered figures, then proved[180] – a Euclidean form, before Euclid!

In a third book (now lost) Autoclycos criticized Eudoxos' hypothesis of concentric spheres on the grounds that it did not account for the planets being at different distances from the earth at different times. He wondered how the theory could be reconciled with the changes of the relative size of the sun and the moon and with the variations of brightness of the planets, especially Mars and Venus. Autolycos, however, could not resolve the difficulty[181].

The practical value of his books was immediately realized by mathematical astronomers, who transmitted them from generation to generation with special care. Their preservation was facilitated and insured by the fact that they were eventually included in a collection called "Little Astronomy" (in

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179 A harbor facing Lesbos, near Assos (where Aristotle taught).

180 Some propositions, however, are not proved; that is, they are taken for granted. This suggests that Autolycos' books had been preceded by at least one other book, now lost. Indeed, in his *Sphaerics*, **Theodosios of Bithynia** (ca 100 BCE) gives the proof of theorem unproved by Autolycos.

181 He assumed that all the stars were supposed to be on a single sphere. Any three stars are vertices of a spherical triangle, the sides of which are great circles. Measuring the distance between two stars on that sphere is equivalent to measuring the angle which that side of the triangle subtends at the center of the earth. All such problems are solved now by means of spherical trigonometry, but trigonometry had not yet been invented in Autolycos' time and he tried to obtain geometric solutions.

contradistinction to Ptolemy's *Almagest*, the "Great collection"). "Little Astronomy" was transmitted in its integrity to the Arabic astronomers, and became in Arabic translation, parts of homogeneous collections, each helping the other to survive.

300–291 BCE **Ptolemy I (Soter)** (called **Soter**, i.e., preserver) built the *library and Museum of Alexandria* and made it a haven for scholars. The library may have contained 700,000 'volumes' (rolls) and its Museum ('home of the muses') was a publicly funded research institute.

Soter (366–282 BCE, king 323–285 BCE) was a general in the army of Alexander the Great and one of his successors. He made Alexandria his capital and the foremost city of the world.

His son, **Ptolemy II** (called **Philadelphus**, 308–246 BCE, king 285–246 BCE) built a canal from the Red Sea to the Nile and made Alexandria the center of Hellenistic culture. Built a lighthouse on Pharos.

His son, **Ptolemy III** (called **Euergetes**, i.e., benefactor, king 246–221 BCE) replaced the Macedonian calendar with Egyptian solar year. He was a liberal patron of the arts and added many books to the library of Alexandria.

The Alexandrian School[182] (300 BCE–415 CE)

Alexandria was founded by Alexander the Great about the time (332 BCE) when Greece, in losing her national independence, lost also her intellectual supremacy. The city was in every way admirably adapted for becoming the new center of the world's activity and thought. Its situation brought it into commercial relations with all the nations lying around the Mediterranean, and at the same time rendered it as the main communication link with the wealth and civilization of the East. These natural advantages were augmented to a great extent by the care of the sovereigns of Egypt.

In 304 *BCE* ***Ptolemy I Soter*** *began to draw around him from various parts of Greece a circle of men eminent in literature, art and science. To these he gave every facility for the prosecution of their learned researches. Under the inspiration of his friend* **Demetrios of Phaleron** *(350–283 BCE), the Athenian orator, statesman and philosopher, this Ptolemy laid the foundations of the great Alexandrian library and originated the keen search for all written works, which resulted in the formation of a collection such as the world has seldom seen.*

He also built, for the convenience of his men of letters, the Museum, in which they resided, studied and taught. This museum, or academy of science, was in many respects not unlike a modern university. It probably included dormitories for the men of science, their assistants and disciples, assembly rooms, roofed colonnades and open-air study for discussion, laboratories, an observatory, botanical and zoological gardens. Its scientific development owed much to its royal patrons and even more so to ***Straton***[183] ***of Lampascos*** *(Latin: Strato Physicus, ca 340–268 BCE) who was called to Alexandria by*

[182] For further readings, see:

- Sarton, G., *Hellenistic Science and Culture*, Dover: New York, 1993, 554 p.
- Sarton, G., *Ancient Science Through the Golden Age of Greece*, Dover: New York, 1993, 646 pp.

[183] Greek Peripatetic philosopher; known for his doctrine that all substances contain void; tempered Aristotle's interpretation of nature by insisting on *causality* and materialism and denial of supernatural forces at work in nature. Pupil and successor of the botanist **Theophrastos** (372–288) as head of the Peripatos (ca 288 BCE). He also expanded on Aristotle's *Physics*, noticing that falling objects (e.g. rainwater off a roof) *accelerate* as they reach the ground rather than falling at a steady rate as Aristotle foretold.

Ptolemy. Straton was indeed the real founder of the Museum for he brought to it the intellectual atmosphere of the Lyceum, and it was thanks to him that it became not a school of poetry and eloquence, but an institute of scientific research. He was deeply interested in the study of nature and thus was nicknamed hoi physicos (the physicist). Under the distant influence of Aristotle and his own master, Theophrastos, he realized that no progress is possible except on a scientific basis and stressed the physical tendencies of the Lyceum. He remained in Egypt about 15 years, being finally recalled to Athens in 288.

The patronage of the Museum was passed on to the son and grandson of Ptolemy I Soter.

***Ptolemy II Philadelphus** [whose librarian was the celebrated **Callimachos** (305–240 BCE)] bought up all Aristotle's collection of books, and also introduced a number of Jewish and Egyptian works. Among these appears to have been a portion of the Septuagint.*

***Ptolemy III Euergetos** largely increased the library by seizing on the original editions of the dramatics laid up in the Athenian archives, and by compelling all travelers who arrived in Alexandria to leave behind a copy of any work they possessed.*

The Alexandrian Renaissance was mainly accomplished by these three kings within the first half of the third century. It was a magnificent attempt to continue and develop, under new conditions, the old Hellenic culture.

*Much was done at the Museum during the first century of its existence. Mathematical investigations were led by **Euclid**, **Eratosthenes** (who was first to measure the size of the earth with remarkable precision), and **Apollonios** (who composed the first book on conics). **Archimedes** flourished in Syracuse, but may have visited Alexandria and was certainly influenced by its mathematical school.*

*Alexandria was an ideal place for astronomical syncretism; Greek, Egyptian and Babylonian ideas could mix freely – there were no established traditions and representatives of various races and creeds could and did actually meet. Astronomical observations were made by **Aristarchos** and **Timocharis**, and a little later by **Conon of Samos** (ca 245 BCE).*

*The anatomical investigations carried out in the Museum by **Herophilos** resulted in an elaborate survey of the human body on the basis of dissections.*

*During 250–150 BCE, the scholars and critics of the Alexandrian library set to work to establish the texts of the classical Greek authors, equip them with commentaries and select the list of the principal figures in each literary genre. **Homer** and **Hesiod** were singled out from the immense body of early epic poetry as the great masters.*

Among the lyric poets nine were nominated for immortality – ***Pindar***, ***Bacchylides***, ***Sappho***, ***Anacreon***, ***Stesichoros***, ***Ibycos***, ***Alcman***, ***Alaeos***, *and* ***Simonides***. *The crowned heads of the tragic stage were* ***Aeschylos***, ***Sophocles***, *and* ***Euripides***. *Of the comic stage they chose* ***Aristophanes***, ***Eupolis***, *and* ***Caritanos***. *There were lists of best historians and philosophers, and also a list of ten greatest Attic orators,* ***Demosthenes*** *prominent among them. The Alexandrian term for the canonized authors was hoi enkrithentes (the admitted). The Roman later expressed the idea with the word classicus, meaning: "belonging to the highest class of citizens".*

The period of creative activity of the library lasted only about 150 *years; this period was also that of greatest commercial prosperity. After the* 2^{nd} *century BCE, the library declined and fell into somnolence. At the time of its climax it may have contained* 700,000 *"rolls". It was by far the largest one of antiquity and found no equal perhaps until the* 10^{th} *century, when very large collection of books became available in the Muslim world, both East in Baghdad and West in Cordova. Decadence was rapid during the* 2^{nd} *century CE and there is good reason for believing that many books were taken to Rome. Under Aurelian (Emperor 270–275 CE) the Museum and the mother library ceased to exist.*

Yet, the scientific tradition was carried on by ***Hero*** *(ca 100 CE),* ***Menelaos*** *(ca 100 CE),* ***Ptolemy*** *(ca 150 CE),* ***Diophantos*** *(ca 250 CE),* ***Pappos*** *(ca 300 CE), and* ***Hypatia*** *(ca 415 CE).*

Alexandrian science, however, did not have the same significance to the Hellenistic world as it does to ours; true, the above-mentioned scholars developed theories and performed some remarkable experiments that foreshadowed the scientific achievements of the 17^{th} *and* 18^{th} *centuries. Yet, experimentation with physical phenomena was to them only an aspect of philosophy, a speculative pastime not intended to impact day to day life.*

The principle of the steam engine, for instance, was understood but used only for tricks of magic. The astronomer ***Aristarchos*** *demonstrated that the earth was round and that the planets revolved around the sun.* ***Eratosthenes*** *measured the circumference of the earth and came within* 300 *km of modern measurements. But neither was interested in publicizing the discoveries. Most of the Hellenistic world, including scholars, continued to believe that the world was flat and that it was the center of the universe.*

The Hebrew Torah (5 books of Moses) was translated[184] *into Greek during the* 3^{rd} *century BCE, in the reign of Ptolemy II Philadelphus. Other books*

184 The ancient Greeks hardly paid any attention to the nearby Hebrews and vice versa. In Hellenistic times, this situation was reversed, because Jews and Greeks were sharing the same environment in Egypt. However, the large and influential Jewish colony of Alexandria was losing its command of the Jewish language. Ac-

of the Old Testament were translated later, many of them in the 2^{nd} *century BCE — the last one, Koheleth (Ecclesiastes), not until about* 100 *CE (the original was produced in the period 250–168 BCE).*

While important contributions to optics, mechanics, and medicine are to be credited to the Alexandrian culture, its supreme achievements are all related directly or indirectly to the discovery of a scientific basis for earth survey. Alexandria was a center of maritime trade. It was also a cosmopolitan product of Greek imperialism, and thereafter a cultural center of the Roman Empire. The art of navigation grew out of the mariner's practice of steering his course by the heavenly bodies. Thus, for example, knowledge of latitude arose from noticing the changing elevation of the Pole star in coastal sailing northward and southward across the Mediterranean or beyond the Pillars of Hercules.

Knowledge of longitude came from the arts of war, where estimates of long east-west distances depended on information from campaigns. Scientific geography was in part a by-product of imperial expansion. Brilliant innovation in mathematics arose in close relation to the same group of problems: the trigonometry of Archimedes and Hipparchos, the algebra of Theon and Diophantos can be traced to the inadequacies of Platonic geometry and Greek arithmetic for handling the large-scale measurements which Alexandrian geodesy and astronomy entailed.

The principal discoveries which form the basis of the Ptolemaic system may be arranged under 4 *headings:*

- *Measurements of the size of the earth.*
- *Construction of universal star maps with latitude and longitude as 'coordinates'.*
- *Introduction of latitude and longitude for terrestrial cartography.*
- *First estimates of the distances of the moon and sun from the earth.*

300–250 BCE Irrigation by means of the *sakya* was introduced by Greek engineers into Egypt. It is a rude water-wheel, with earthen pots on an endless chain running round it, worked by one or two bullocks. [In Northern India

cording to the story told in Greek by the Jew **Aristeas**, the librarian Demetrios of Phaleron explained to King Ptolemy II the need for translating the Torah into Greek. This allegedly resulted in the *septuagint*.

it is termed the *harat*, or *Persian wheel*]. With one such water-wheel a pair of oxen can raise water up to 6 m, and keep from 5 to 12 acres irrigated throughout an Egyptian summer. This primitive contrivance is still in use today in Egypt and India.

Archimedes is credited with the invention of the *Archimedes screw* (ca 250 BCE) which raises water as the handle is turned: the water flows from one spiral of the inclined screw to the next and finally come out of the upper end of the screw. The devise is still used today; except that a Diesel engine turns the screw.

300 BCE–100 CE The former cities of *Palmyra* and *Petra* (now in the deserts of Syria and Jordan) flourished, cultivating the vine and olive without much recourse to artificial irrigation. This seems to imply not only a higher water table than now but a climate that supplied more dependable rain. The cities were abandoned at ca 100 CE.

304–291 BCE **Herophilos of Chalcedon** (ca 335–280 BCE, Alexandria). Greek physician and surgeon. First scientific anatomist. Founder of the Alexandrian school of anatomy. One of the first to conduct post-mortem examinations and public dissections[185].

Discovered: prostate gland, duodenum, ovaries, fallopian tubes. Made the first methodical study of the brain (describing its ventricles), the liver, the spleen, the retina. First to time the pulse, to distinguish between arteries and veins and between sensory and motor nerves. Determined that the seat of *reasoning* is in the brain, not the heart.

The Alexandrian doctors also discovered *Ligature*, the tying off of blood vessels. This enabled them to perform operations that had before been impossible. They removed bladder stones, repaired hernia and performed amputations.

Herophilos was born in Chalcedon (Asia Minor) and flourished under Ptolemy Soter (366–282 BCE). His bold anatomical investigations were carried through in the *Museum*. Through his ambitious program of anatomical research, he conducted an elaborate survey of the human body on the basis of dissections. As this was done *systematically* for the first time, the men in charge were bound to make many discoveries.

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185 The Ptolemaic rulers allowed the dissection of human corpses, a *taboo* among the Greeks.

It was claimed by the Roman encyclopedist Celsus (1^{st} century CE) and by Church fathers (who were eager to discredit Pagan science) that the Alexandrian anatomists obtained permission to dissect the bodies of living men, in order to have a better understanding of the functions of the organs.

His pupil **Philinos of Cos** (fl. 250 BCE) founded the so-called *Empirical School of Medicine* that rejected dogma and based its practice on experience, clinical cases and analogy.

ca 300 BCE **Euclid**[186] **of Alexandria** (ca 330–260 BCE). Greek mathematician who founded the mathematical school of Alexandria. The university opened ca 300 BCE and Euclid may have been invited from Athens, to head its school of mathematics.

Euclid is the most prominent mathematician of antiquity best known for his treatise on geometry *The Elements* (13 books). The long lasting nature of *The Elements* makes Euclid the leading mathematics teacher of all times. Indeed, the treatise became the center of mathematical teaching for 2000 years.

We are ignorant of the dates of his birth and death, of his parentage, teachers and the residence of his early years. **Proclos** (412–485 CE), the authority for most of our information regarding Euclid, states in his commentary on the first book of the 'Elements' that Euclid lived at the time of Ptolemy I (**Soter**), King of Egypt, who reigned from 323 to 285 BCE, that he was younger than the associates of Plato, but older than **Eratosthenes** (276–196 BCE) and **Archimedes** (287–212 BCE).

The question has often been raised as to the extent to which Euclid, in *The Elements*, is a compiler rather than a discoverer. To this question no satisfactory answer can be given. The general agreement is that Euclid must have made great advances beyond his predecessors both in geometry and number theory.

Five propositions in the Elements are of special interest:

(1) Propositions 12 and 13 of Book II are recognized as the *law of cosines* for plane triangles. They adumbrate the concern with trigonometry that was shortly to blossom in Greece.

(2) *Euclid's algorithm*, which gives a very simple and efficient method for the determination of the greatest common divisor (g.c.d.) of two numbers – a basic method of elementary number theory. This is found in the seventh book of 'Elements'. This method is essentially that of converting a fraction into a *continued fraction*. It therefore constitutes the earliest important step in the development of the concept of a continued fraction.

(3) The idea of the *prime-factorization theorem* as well as the lemma used in proving it – are found in 'Elements' Books VII and IX.

[186] His name reads *Eucleides*.

(4) The proof of the *infinity of primes* is found in Proposition 20, Book IX.

(5) $2^{n-1}\,(2^n - 1)$ is perfect if $(2^n - 1)$ is prime. [Final proposition, Book IX].

If we take Egyptian and Babylonian efforts into account, Euclid's *Elements* is the climax of more than a thousand years of research. Granted that many discoveries were made before him, he was the first to build a synthesis of all the knowledge obtained by others and himself and to put all the known propositions in strict logical order. Thus, although he was seldom a complete innovator, he did much better and on a larger scale what other geometers had done before him. Those propositions which cannot be ascribed to others were probably discovered by Euclid himself and their number is considerable. The arrangement is to a large extent Euclid's own; he created a monument which is marvelous in its inner beauty, clearness and durability.

Consider Book I, explaining first principles, definitions, postulates, axioms, theorems and problems. It is possible to do better at present, but it is almost unbelievable that anybody could have done as well 2300 years ago. The most amazing part of Book I is Euclid's choice of *postulates*. In particular, the choice of postulates 5 is, perhaps, his greatest achievement, the one which has done more than any other to immortalize the word "*Euclidean*": "*... if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which the angles are less than two right angles*".

The first consequence of Euclid's decision to accept this as a postulate[187], was the admirable concatenation of his *Elements*. The second was the endless attempts which mathematicians through the ages made to correct him.

The third consequence is illustrated by the list of alternatives to the 5^{th} postulate. Some bright men thought that they could rid themselves of the postulate and succeeded in doing so, but at the cost of introducing another one (explicit or implicit) equivalent to it[188]. Thanks to his genius, Euclid

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187 **George Sarton**'s remark concerning the greatness of this postulate is adequate here: "*A person of average intelligence would say that the proposition is evident and needs no proof; a better mathematician would realize the need of a proof and attempt to give it. It requires extraordinary genius to realize that a proof was needed yet impossible. There was no way out, then, from Euclid's point of view, but to accept it as a postulate and go ahead*".

188 "*If a straight line intersects one of two parallels, it will intersect the other also*". (Proclos)

"*Given any figure there exists a figure similar to it of any size*". (John Wallis)

"*Through a given point only one parallel can be drawn to a given straight line*" (John Playfair) saw the necessity of this postulate and selected intuitively the simplest form of it.

The 4^{th} consequence, and the most remarkable, was the eventual creation of non-Euclidean geometries.

Euclid flourished in Alexandria under the first Ptolemy and possibly under the second. Two anecdotes help to reveal his personality. It is said that the King (Ptolemy I) asked him "*if there was in geometry any shorter way*

"*There exists a triangle in which the sum of the three angles is equal to two right angles*". (Legendre)

"*Given any three points not in a straight line there exists a circle passing through them*". (Legendre)

All these men proved that the 5^{th} postulate is not necessary if one accepts another postulate rendering the same service. The acceptance of any of those alternatives would, however, increase the difficulty of geometrical teaching. It is clear that a simple exposition is preferable to one which is more difficult.

On the other hand, mathematicians like **Saccheri**, **Lambert** and **Gauss** argued that inasmuch as the 5^{th} postulate cannot be proved, there is no obligation to accept it, and if so, it can deliberately be rejected.

The first to build a new geometry on an opposite postulate was **Nicolai Ivanovitch Lobachevsky** (1793–1850), who assumed that through a given point more than one parallel can be drawn to a given straight line or that the sum of the angles of a triangle is less than two right angles. The discovery of a non-Euclidean geometry was made at about the same time by **Janos Bolyai** (1802–1860). Sometime later, another geometry was outlined by **Bernhard Riemann** (1826–1866), who was not acquainted with the writings of Lobachevsky and Bolyai and made radically new assumptions. In Riemann's geometry, there are no parallel lines and the sum of the angles of a triangle is greater than two right angles. The mathematical teacher **Felix Klein** (1847–1925) showed the relationship of those geometries: Euclid's geometry refers to a surface of zero curvature, in between Riemann's geometry on a surface of positive curvature (like the sphere) and Lobachevsky's applying to a surface of negative curvature. To put it more briefly, he called the Euclidean geometry parabolic, because it is the limit of elliptic (Riemann's) geometry on one side and of the hyperbolic (Lobachevsky's) geometry on the other.

It would be foolish to give credit to Euclid for pan-geometrical conceptions; the idea of a geometry different from the common-sense one never occurred to his mind. Yet, when he stated the fifth postulate, he stood at the parting of the ways. His subconscious prescience is astounding; there is nothing comparable to it in the whole history of science.

than that of the Elements, and he answered that there was no royal road to geometry". The other anecdote is equally good. "*Someone who had began to read geometry with Euclid when he had learned the first theorem asked him*, '*But what shall I get by learning these things*?' *Euclid called his slave and said*: '*Give him an obol* (*silver coin*), *since he must gain out of what he learns*' ".

A Brief History of Geometry[189] (3000 BCE–1900 CE)

Geometry Before the Greeks

The beginnings of geometry are shrouded in the mists of pre-history. This stage is sometimes called "subconscious geometry." Later, humans came to recognize certain principles, such as the fact that the circumference and diameter of circles are always in the same ratio. This stage is the "scientific geometry."

Geometry as a science may have begun in Egypt, where the rulers had need to measure the areas of fields in order to assess taxes on them. (The word "geometry" means "earth measurement.") The Moscow papyrus *(19th cen. BCE) and the* Rhind papyrus *(17th cen. BCE) make it clear that the Egyptians had a significant amount of geometrical knowledge at least 4000 years ago, and perhaps much earlier than that. (The great Pyramid of Gizah*

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[189] To dig deeper, consult:

- Resnikoff, H.L. and R.O. Wells, Jr., *Mathematics in Civilization*, Dover: New York, 1984, 408 pp.
- Ghyka, M., *The Geometry of Art and Life*, Dover: New York, 1977, 174 pp.
- Huntley, H.E., *The Divine Proportion*, Dover: New York, 1970, 186 pp.
- Dodge, C.W., *Euclidean Geometry and Transformations*, Dover: New York, 1972, 295 pp.
- Durell, G.V., *Modern Geometry*, MacMillan, 1947, 145 pp.
- Brannan, D.A., M.F. Esplen and J.J. Gray, *Geometry*, Cambridge University Press, 2002, 497 pp.
- Coxeter, H.S.M., *The Beauty of Geometry*, Dover, New York, 1999, 274 pp.
- Hilbert, D. and S. Cohn-Vossen, *Geometry and Imagination*, Chelsea Publishing Co.: New York, 1952, 357 pp.
- Zwikker, C., *Advanced Geometry of Plane Curves*, Dover, 1963, 299 pp.

was built nearly 5000 years ago.) Many of the methods used to calculate areas and volumes were only approximately correct. One of the most remarkable results in the Moscow papyrus is a correct procedure for calculating the volume of a truncated pyramid.

Wherever it may have appeared first, it is clear that some significant geometry was developed – probably independently – in Egypt, Mesopotamia (Babylonia), China, India, and perhaps in other places and cultures as well. Wherever it developed it seems likely that this development came about to meet the practical needs of surveying, engineering, and agriculture. Geometry remained empirical and utilitarian until the Greeks made it into a science which could be, and was, studied independently of its practical applications.

Geometry in Greece

The Greeks of the Classical Period (600–300 BCE) not only increased the quantity of geometric knowledge, they changed the very nature of the subject, and of mathematics in general. Some Greek mathematicians traveled to Egypt and Babylonia and learned what was known of geometry in those places. They "transformed the subject into something vastly different from the set of empirical conclusions worked out by their predecessors. The Greeks insisted that geometric fact must be established, not by empirical procedures, but by deductive reasoning; geometrical truth was to be obtained in the study room rather than in the laboratory." They created what we may call "demonstrative geometry," whose truths are supported by (deductive) proofs rather than only by inductive evidence.

This work began with **Thales** *in the first part of the 6th century BCE. Thales, who had traveled in Egypt, is the first known individual with whom the use of deductive methods in geometry is associated. This axiomatic-deductive method is the cornerstone of modern mathematics, which thus may be truly said to have begun with the classical Greeks.*

The next important name is that of **Pythagoras**, *who may have studied under Thales. He founded the celebrated Pythagorean school, a brotherhood knit together with secret and kabbalistic rites and observances and committed to the study of philosophy, mathematics, and natural science. The Pythagorean belief that everything is explainable by numbers may be regarded as one of the origins of the quantitative emphasis in modern science. Pythagoras may have been the first to provide a proof of the theorem named* *for him, but the result had been understood by many peoples for many centuries. Demonstrative geometry was considerably advanced by Pythagoras and his followers.*

Several attempts were made to unite all of the truths of mathematics into a single chain which could be deduced from explicitly-stated "self-evident" assumptions (or axioms). And then, about 300 BCE, **Euclid** *produced his epoch-making effort, the Elements, a single deductive chain of 465 propositions neatly and beautifully comprising plane and solid geometry, number theory, and Greek geometrical algebra. From its very first appearance this work was accorded the highest respect, and it so quickly and so completely superseded all previous efforts of the same nature that now no trace remains of the earlier efforts. The impact of this single work on the future development of geometry has been enormous and is difficult to overstate. Euclid's Elements remained the standard textbook in geometry until the early years of the 20th century.*

After Euclid the most exceptional of the Greek mathematicians were **Archimedes** *and* **Apollonios**. *Archimedes (287–212 BCE), universally acclaimed as the greatest mathematician of antiquity and among the greatest of all time, wrote extensively on geometry. He proved that the value of π must lie between $3\frac{10}{71}$ and $3\frac{1}{7}$. He discovered and established the relationship between π and the area of a circle. He found and proved formulas for the volume and surface area of a sphere (using methods which anticipated the development of integral calculus).*

Apollonios (262–190 BCE) wrote a number of works, most of which have been lost. We do have his monumental work on parabolas, hyperbolas, and ellipses, The Conic Sections, which has been called one of the greatest scientific works of antiquity.

Arabic and Hindu Contributions

The five or six centuries following the fall of Rome (in the 5th century CE) is often referred to as the Dark Ages of Europe. Leadership in the world of mathematics during this period passed to the Arabs and the Hindus. It has been said that the greatest mathematical contribution of the Arabs was the preservation and transmission of Greek achievements to the modern period. But it is important to note that the Arabs and Hindus made many important contributions of their own. The most important of these are in the areas of numeration, computation, algebra, and trigonometry.

Noteworthy geometrical achievements include the work of ***Brahmagupta*** *(fl. 630 CE) on cyclic quadrilaterals,* ***Abu al-Wafa*** *on constructions,* ***Omar Khayyam*** *on geometric solutions of cubic equations, and the work of* **Nasir al-Din al Tusi** *on Euclid's parallel postulate.*

The Modern Period

The modem period has seen much activity in geometry, including the creation of a range of new kinds of geometry, namely: Projective Geometry, Coordinate Geometry, Differential Geometry, Non-Euclidean Geometry, and Topology. The first three of these arose in the 17th century. Projective geometry results from the application of the concept of a "projection" to geometry. Coordinate geometry results from the application of algebra to geometry. Differential geometry results from the application of calculus to geometry. The previously existing geometry is sometimes called "synthetic geometry" to distinguish it from these newer branches of the subject.

Projective geometry grew out of the efforts of Italian artists in the 15th century who created a geometrical theory called "mathematical perspective" to aid them in the production of realistic paintings. Later this theory was taken into mathematics and considerably expanded by several, led by ***Girard Desargues*** *and* **Pascal**. *Descriptive geometry, which is related to projective geometry, was created and extended by several mathematicians:* ***Gaspard Monge***, ***Jean Victor Poncelet***, *and others in the 18th and 19th centuries.*

Analytic geometry, also called coordinate geometry or Cartesian geometry, was one of the great mathematical achievements of the 17th century – a century which also saw the creation of calculus and foundations laid in the theory of probability. Created independently by **Rene Descartes** *and* ***Pierre de Fermat***, *coordinate geometry is the fruitful marriage of algebra and geometry. It made possible the use of algebra as a tool for solving geometric problems (and vice versa).*

Differential geometry is that geometry which uses calculus as a tool to investigate the properties of curves and surfaces. The most prominent names in the history of differential geometry are ***Gaspard Monge*** *(1746–1818),* ***Gauss*** *(1777–1855), and* **Bernhard Riemann** *(1826–1866).*

Non-Euclidean Geometry was created by ***Gauss***, ***Nicolai Ivanovitch Lobachevsky*** *(1793–1856), and* ***Janos Bolyai*** *(1802–1860).* **Bernhard Riemann** *extended their researches. Non-Euclidean geometry is obtained* *by replacing Euclid's parallels postulate by one of its contradictory forms, and deducing theorems from this new set of axioms.*

It took unusual imagination to entertain the possibility of a geometry different from Euclid's, for the human mind had for two millennia been bound by the prejudice of tradition to the firm belief that Euclid's system was most certainly the only way to geometrically describe physical space, and that any contrary geometric system simply could not be consistent. One result beyond the world of mathematics was the doubt which the discovery of non-Euclidean geometries cast upon human ability to know the truth about anything.

Topology, sometimes described as "rubber-sheet geometry," is the study of those properties of objects which are not altered by stretching or bending (or more generally, by "continuous deformation"). It began in the 19th century as a branch of geometry. It has been a major area of mathematical research in the 20th century and has come to be regarded as a fourth division of mathematics, along with algebra, geometry, and analysis. Among the many important contributors to topology are **Riemann** *and* ***Henri Poincare*** *(1854–1912).*

Other recent developments in geometry include "The Erlangen Programm" of ***Felix Christian Klein****, the abstract spaces of* ***Maurice Frechet****, and the "Grundlagen" of* **David Hilbert**.

The 19th century also saw the serious investigation of spaces of dimension greater than three. **Cayley**, ***Grassmann***, *and* ***Riemann*** *were important contributors.*

Perfect and Amicable Numbers

The ancients considered numbers with the property $\sigma(n) - n = n$, *or* $\sigma(n) = 2n$, *where* σ *is the sum of all divisors (including* 1 *and* n*). The first two examples* $6 = 1+2+3$ *and* $28 = 1+2+4+7+14$ *were noticed already by the Pythagoreans and had a strong appeal for mystics, since the creation took*

6 days, and there are about 28 days in a lunar month. For this reason they were named perfect numbers by the Pythagoreans. Two other perfect numbers known to the Greeks were 496 and 8128. ***Euclid*** *proved that* $2^{p-1}(2^p-1)$ *is perfect whenever* 2^p-1 *is prime.*

If we denote $M = 2^p - 1$, $n = \frac{1}{2}M(M+1)$, *then* $\sigma(n) = 2n$. *To see this, list all the possible factors of* n. *Certainly each of* $1, 2, 2^2, ...2^{p-1}$ *is a factor. It is a hypothesis of the theorem that the factor* $2^p - 1$ *is prime, but as many more factors again can be obtained by multiplying it by each factor in the above list of powers of 2. This exhausts the possibilities:* n *has no further factors. We have then two sets to add:*

$$S_1 = 1 + 2 + 2^2 + \cdots + 2^{p-1} \equiv 2^p - 1$$

$$S_2 = (1 + 2 + 2^2 + \cdots + 2^{p-1})(2^p - 1) = S_1(2^p - 1)$$

$$\therefore \quad S_1 + S_2 = (2^p - 1) + (2^p - 1)(2^p - 1) = 2^p(2^p - 1) = 2n$$

This is the sum of the factors. Why did it come out $2n$ *instead of* n*? Because we neglected to delete* n *itself as one of the factors when we formed* S_2. *Thus the sum of the proper factors, including* 1 *but not including* n, *is* n, *and* n *is therefore a perfect number.*

The fact that $2^p - 1$ *was prime had to be used in the proof, which brings us back to the problem of the primality of the Mersenne numbers. We know already that for* $2^p - 1$ *to be prime it is necessary that* p *be prime, but not sufficient;* $2^{11} - 1$ *is composite* $(= 23 \cdot 89)$ *and therefore* $2^{10}(2^{11} - 1)$ *is not perfect.*

Euler *proved*[190] *the inverse: all even perfect numbers have the Euclid form. That is, an even number is perfect if and only if it is of the form*

$$N = 2^{p-1}(2^p - 1),$$

where $2^p - 1$ *is prime. Thus, the knowledge of even perfect numbers is equivalent to the knowledge of Mersenne primes.*

As for odd perfect numbers, not even one has ever been found. If one exists (as it well might) it is fairly large, at any rate greater than 10^{300}.

The only perfect numbers known to the Greeks were

$$N_2 = 2(2^2 - 1) = 6$$

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190 Let $2^n q$ be perfect, where q is odd and $n > 0$. Then $2^{n+1}q = (2^{n+1} - 1)s$, where s is the sum of all the divisors of q. Thus $s = q + d$, where $d = q/(2^{n+1} - 1)$. Hence d is a divisor of q, so that q and d are the only divisors of q. Hence $d = 1$ and $q = 2^{n+1} - 1$ is a prime.

$$N_3 = 2^2(2^3 - 1) = 28$$

$$N_5 = 2^4(2^5 - 1) = 496$$

$$N_7 = 2^6(2^7 - 1) = 8128 = 1^3 + 3^3 + 5^3 + 7^3 + 9^3 + 11^3 + 13^3 + 15^3$$

Four additional perfect numbers were discovered by the end of the 18^{th} century:

$$N_{13} = 33,550,336$$

$$N_{17} = 8,589,869,056$$

$$N_{19} = 137,438,691,328$$

$$N_{31} = 2,305,843,008,139,952,128 \quad (Euler,\ 1772)$$

Perfect numbers have the following remarkable properties

- *The last digit of every even perfect number*[191] *is always 6 or 8.*
- *Except 6, the sum of digits* $\equiv 1(mod \quad 9)$.
- *The sum of the reciprocal of all divisors is 2.*
- *Every even perfect number* $2^{p-1}(2^p - 1)$ *is a sum of cubes of* 2^k *odd numbers, when* $k = \frac{p-1}{2}$, *except for* $n = 2$. *Indeed, let*

$$S_1 = 1 + 2 + 3 + \cdots + m = \frac{m(m+1)}{2},$$

$$S_2 = 1^2 + 2^2 + 3^2 + \cdots + m^2 = \frac{m(m+1)(2m+1)}{6},$$

$$S_3 = 1^3 + 2^3 + 3^3 + \cdots + m^3 = \frac{m^2(m+1)^2}{4} = S_1^2.$$

[191] For $p = 2$ we have the number 6. If $p > 2$, then p is a prime of the form $4k+1$ or $4k+3$. If $p = 4k+1$, then $2^{p-1} = 2^{4k} = 16^k$, and the last digit of 2^{p-1} is obviously 6, while $2^p - 1 = 2^{4k+1} - 1 = 2 \cdot 16^k - 1$ and the last digit is obviously 1. Thus, the last digit of the product $2^{p-1}(2^p - 1)$ is 6. If $p = 4k+3$, then the number $2^{p-1} = 2^{4k+2} = 4 \cdot 16^k$ has the last digit 4, while the last digit of 2^p is 8, hence the last digit of the number $2^p - 1$ is 7, and, consequently, the number $2^{p-1}(2^p - 1)$ (as the product of two numbers, one with the last digit 4, and the other with the last digit 7) has the last digit 8.

Then

$$\begin{aligned} S &= 1^3 + 3^3 + 5^3 + \cdots + (2m-1)^3 = \sum_{i=1}^{m} (2m-1)^3 \\ &= \sum_{i=1}^{m} (8m^3 - 12m^2 + 6m - 1) \\ &= 8 \cdot \frac{m^2(m+1)^2}{4} - 12 \cdot \frac{m(m+1)(2m+1)}{6} + 6 \cdot \frac{m(m+1)}{2} - m \\ &= m^2(2m^2 - 1). \end{aligned}$$

If $m = 2^k$, $S = 2^{2k}(2^{2k+1} - 1)$. *The even perfect numbers are of this form for* $n = 2k + 1$.

Amicable Numbers

The ancients, with their quest for the mystic interpretation of numbers, came across number pairs *related by the property that the sum of the aliquot divisors of either number equals the other number,*

$$\sigma(m) - m = n, \quad \sigma(n) - n = m$$

or

$$\sigma(m) = \sigma(n) = n + m.$$

Pythagoras, for example, knew of the smallest pair[192] *(220; 284), since*

$$\sigma(220) - 220 = 1 + 2 + 4 + 5 + 11 + 20 + 22 + 44 + 55 + 110 = 284$$

$$\sigma(284) - 284 = 1 + 2 + 4 + 71 + 142 = 220.$$

[192] So may have **Jacob** (*Genesis* **32**, 14) when he presented Esau with 200 she-goats plus 20 he-goats, to secure his friendship.

The Pythagoreans named such numbers amicable. Other pairs are (1184; 1210), (17,296; 18,416).

Amicable numbers intrigued the Greeks, Arabs and many other since antiquity[193]*. The mathematician* ***Thabit Ibn Qurra al-Harrani*** *(ca 855 CE) found that prime numbers of the form*

$$p = 3 \cdot 2^m - 1; \qquad q = 3 \cdot 2^{m-1} - 1; \qquad r = 9 \cdot 2^{2m-1} - 1$$

generate the amicable pairs $2^m pq$, $2^m r$. *Clearly, the permissible values of* m *(such that* p, q, r *are prime) follow from the relations*

$$\sigma(2^m pq) = (2^{m+1} - 1)(p+1)(q+1) = 2^m(pq + r)$$

$$\sigma(2^m r) = (2^{m+1} - 1)(r+1) = 2^m(pq + r).$$

It is thus required that for any prime of the form $q = 3 \cdot 2^{m-1} - 1$, *also* $p = 2q+1$ *and* $r = 2(q+1)^2 - 1$ *be primes. Consequently, beyond the small values of* m

$m = 2$	$p = 11$	$q = 5$	$r = 71$
$m = 4$	$p = 47$	$q = 23$	$r = 1,151$
$m = 7$	$p = 383$	$q = 191$	$r = 73,727,$

these amicable numbers become vary large, following the exponential growth of permissible values of m. *Many other amicable numbers, generated by different algorithms, are also known.*

Whether or not there are infinitely many amicable pairs is unknown, although it was proved (1977) that $\sum(\frac{1}{m})$ *is convergent if* m *runs over all members of amicable pairs.*

[193] One can ask for numbers such that $\sigma(n) = kn$. Although k seems to be as large as we wish, it was conjectured that $k = O(\log \log n)$.

The Ancient Maya[194] (1200–300 BCE)

About 1000 *BCE a strange civilization emerged in the lowland jungles of the Yucatan area in Central America. The people who created it, the Maya, were not city dwellers like most early civilized people; they lived in small farming villages. But they built great ceremonial centers – complexes of pyramid-like temples and palaces which were the focus of their religion and political life.*

Some time during the fourth or third centuries BCE, their priests devised a vigesimal (base 20) positional numerical system, which embodied the concept of zero – a notable abstract intellectual achievement. Elsewhere, this mathematical concept is known to have been developed only in the late Babylonian civilization. The Mayan calendar was more sophisticated and complicated than either the Gregorian or the Julian calendar. Apparently, the Maya did not attempt to correlate their calendar accurately with the length of the solar year or lunar months. Rather, their calendar was a system for keeping track of the passage of days and for counting time into the past or future. Among other purposes, their calendar was useful for predicting astronomical events, for example, the position of Venus in the sky[195].

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[194] For further reading, see:

- Von Hagen, V.W., *The World of the Maya*, Mentor Books: New York, 1962, 224 pp.
- Morley, S.G., *The Ancient Maya*, Stanford University Press: Stanford, CA, 1956, 507 pp.

[195] Using the simplest equipment, the Mayas calculated the length of the solar year with an accuracy equal to that of modern astronomy, and devised correction formulae to adjust the discrepancy between the true year and the calendar year which is handled by our leap-year correction. They worked out an accurate lunar calendar and calculated the synodical revolution of Venus, in each case devising means for correcting the accumulated error.

The ancient civilizations of Mesopotamia employed a positional system of arithmetic, but it seems to have been in existence for centuries before the concept of zero diffused to them from the Hindu world. The ancient civilizations of the Mediterranean arena derived many of their cultural features from the civilizations of the Middle East, but they did not take over the well-developed system of computation. This sort of mathematical complex did not penetrate Western Europe until the time of the Arab invasion in the Early Middle Ages, several centuries after the Maya had developed their own accurate and flexible system.

To specify completely a particular date, the Maya made use of what is called the 'long count', a perpetual tally of the days that had elapsed since a particular date about 3000 *years ago in the past. This system is analogous to that of the 'Julian day', but the starting date was not meant to be that of "the beginning" – it was merely a fiducial, from which days could be counted. The significant point of the 'long count' is that it employed a* vigesimal *number system (i.e.* based on 20*) which included a zero and consequently simplified their arithmetic (at that time the zero was unknown in Europe).*

Mayan astronomers could predict eclipses of the sun*. This was achieved without the instruments upon which modern astronomers depend (Mayan temples were sufficiently high to obtain clear lines of sight from their summits to distant points on the horizon: a pair of crossed sticks was set up on top of a pyramid. From this fixed observation point, the places where the sun, moon or planets rise or set was noted with reference to some natural fiducial feature on the horizon. When the heavenly body under observation rose or set behind this same point for the second time, it had made one complete synodical revolution).*

It must be noticed that the Mayan precise predictions of eclipses, the position of the moon in the sky and the position of Venus, were all done by arithmetic*: they counted a certain number and subtracted some numbers, and so on. There was no discussion of what the moon was, or even of the idea that it revolved. They just calculated the time when there would be an eclipse, or when the moon would rise at the full, and nothing more.*

In about 1200 *CE, the Maya deserted their centers and the old stable way collapsed*[196]*; the Spaniards who came in the* 1500*'s met little resistance from*

The basis for the Maya calendric system was the 260-day *tzolkin*, with its named and numbered days, and the 365-day *haab*, with its named months and numbered positions within each month. These two meshed calendars repeated cyclically, and could return to a given starting point only after a lapse of 52 years. The inscriptions for longer time periods were based on the vigesimal (base 20) system. Using these longer periods, the Maya counted the elapsed time since the hypothetical starting date of their chronology. They also reordered in the inscription the fact that the stela was erected on a certain *numbered and named* day of the tzolkin which occupied a certain position in a particular month of the haab; the accompanying supplementary series recorded lunar information for the date.

196 It is now believed that over-exploitation of farming land ruined the agrarian resources of the Maya. The ravishes of famine finally put an end to the Maya civilization. A steadily growing population and limited fertile corn-growing land around the royal centers, forced the Maya peasants to intensify food production by disregarding *fallow* (resting periods which the land needs to regain its fertil-

the decadent remnant, weakened by internecine fighting.

ca 300 BCE **Euhemeros of Messina**. Greek philosopher and mythographer from Sicily. The author of *Hiera anagraphe* (Sacred History). First to try to link mythical beings and events with historical fact, explaining the gods as distorted representation of ancient warriors and heroes. His name lives on in the term *Euhemerism*.

Flourished at the court of Cassandros (358–297 BCE; Regent of Macedonia 316–306 BCE, King 306–297). He was said to have sailed down the Red Sea and across the Arabian sea and to have reached an Indian Island called Panchaia, where he found sacred inscriptions which he described in his book. Therein he emphasized the historical origins of myths – an attempt to rationalize mythology (i.e. Greek religion).

283 BCE The sculptor **Chares of Lindos** completed the *Colossos of Rhodes* – a huge bronze statue built near the harbor of Rhodes on the Aegean Sea, in honor of the sun god Helias to commemorate the survival of the people of Rhodes after a year-long siege by the Macedonians (303 BCE). The statue stood 37 meters tall (about as high as the Statue of Liberty) and Chares worked on it 12 years, using stone blocks and 6.8 ton of iron to support the hollow statue. Its cost was defrayed by selling the siege machines left behind by Demetrius I Poliorcetes.

The Colossos was thrown down by the great earthquake of 227 BCE, but its remains lay on the spot for centuries, until they were sold for scrap by the Arabs in 653 CE.

ca 280–240 BCE **Aristarchos of Samos** (310–230 BCE). Alexandrian Greek mathematician and astronomer. The exponent of a sun-centered universe who made pioneering efforts to determine the sizes and distances of the sun and the moon. Appeared to have been the first to combine *trigonometrical theory* and a mathematical model of the heavens, with some simple

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ity) and also causing massive *soil erosion* by farming the hillsides around the royal centers. Thus, the Maya could not grow enough food to meet the needs of the population, which dwindled from 3 million (ca 800 CE) to about 50,000 (ca 950 CE). Malnutrition finally caused iron-deficiency anemia (discovered in skulls exhumed from graves). This is an example of human-induced ecological disaster – a punishment for living in disharmony with nature.

physical measurements, to investigate the metrical relationships of the earth, moon and sun. In these calculations, trigonometry is used for the first time as a *basic tool*. In his study he assumes certain properties of the *trigonometric functions* ($\sin\alpha/\sin\beta < \alpha/\beta < \tan\alpha/\tan\beta$ for $0 < \beta < \alpha < \frac{\pi}{2}$), which must have been known in his time. This indicates a rather advanced state of knowledge in this branch of mathematics.

Aristarchos proposed an improved scheme of motion for the celestial bodies by making two simplifying suggestions:

(1) The earth spins – and that accounts for the apparent daily motion of the stars (others had made this suggestion before).

(2) The earth moves round the sun[197] in a yearly circular orbit, and the other planets do likewise – that accounts for the apparent motions of the sun and the planets across the fixed stars.

His book on the subject has not come down to us, but his idea was quoted by Archimedes and referred to by Plutarch.

This simple scheme failed to catch on[198]. Tradition was against it and the Alexandrian school was reluctant to accept such a radical alternative: earth

[197] Although it was merely an idea, not backed up by observational evidence, Aristarchos anticipated the great discovery of Copernicus in 1514. Copernicus could not have known of Aristarchos' doctrine, since Aristarchos' work was not published till after Copernicus' death.

On this account, Aristarchos was accused of impiety by the Stoics, just as Galileo, in later years, was accused by the theologians. His heliocentric scheme was abandoned and was not revived until 1800 years later.

[198] Instead of the bold suggestion of making the earth spin and move round a central sun, the school of Alexandria devised the new theories of *Eccentrics* and *Epicycles*:

A stationary central earth remained the popular basis, but spinning concentric spheres made the model too difficult. Instead, the slightly uneven motion of the sun around its "orbit" could be accounted for by a single eccentric circle. According to this model, the sun is carried around a circular path by a radius that rotates at constant speed. *The observer on earth is off-center*, so that he sees the sun moves unevenly – as it does – faster in December, slower in June. The eccentric scheme for a planet was somewhat more elaborate: each planet is carried at the end of a radius that rotates at constant speed, but this whole circle – center, radius and planet – revolves once a year round the eccentric earth. This added a small circular motion to the large main one, producing the planet's epicycloidal track. The daily motion with the whole star pattern was superimposed on this.

An alternative scheme to produce the same effect assumed that the earth remained fixed at the center of the main circle. The planet, while going around moving around an orbit raised *mechanical* objections that seemed even more serious in later ages, and it immediately raised a great astronomical difficulty. If the earth moves in a vast orbit, the pattern of fixed stars should show parallax changes during the year. None was observed, and Aristarchos could only reply that the stars must be almost infinitely far off compared with the diameter of the earth's orbit. Thus he pushed the stars away to far greater distances and released them from being all on one great sphere!

Aristarchos also made the hypothesis that the moon receives its light from the sun.

In his only surviving short treatise, *On the Magnitudes and Distances of the Sun and the Moon* (with a commentary by Pappos), he devised most ingenious schemes for determining distances to the sun and the moon and the relative sizes of the earth, moon and sun. His method of estimating the relative lunar and solar distances is geometrically correct, though the instrumental means at his command rendered his data erroneous.

These methods were refined in ca 140 BCE by **Hipparchos of Nicaea** (180–110 BCE) who obtained an *earth-moon* distance of 385,000 km and a *sun-earth* distance of about 8 million km.

276 BCE **Aratos of Soli** (315–245 BCE). Greek poet of the Alexandrian school, who in his poem *Phenomena* (= $\phi\alpha\iota\nu o\mu\epsilon\nu\alpha$) gives a systematic account of the stars, including the early fruits of Greek observation since Thales.

Aratos described 43 constellations: 19 in the north, the 12 of the zodiac, and 12 in the south. He also named 5 individual stars: Arcturus, Stachys (Spica), Protrugater (Vendemiatrix in Virgo), Sirius and Procyon. He described the Pleiads and his account of the polar star sheds an interesting light on the state of astronomical understanding in the 3^{rd} century BCE.

Aratos recognized the earth as the center of the heavens, which turn about two poles – one visible, the other hidden. His mention of a slight shifting of

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this circle, was simultaneously rotating with a small radius-arm around an instantaneous center on the circumference of the main circle. The motion around the small sub-circle (epicycle) was at a steady rate, once in 365 days. To an observer in space, the two circular motions combine to produce an epicycloidal pattern.

Though these schemes use *circles*, they were described more grandly in terms of spinning spheres and sub-spheres. For many centuries, astronomers thought in terms of such *motions of the heavenly spheres* – the spheres growing more and more real as Greek delight in pure theory gave place to naive insistence on authoritative truth.

the axis suggests a suspicion of precession, which was not actually discovered (except perhaps by Kiddinu) until a century later.

Aratos was born in Cilicia. He was invited (ca 276 BCE) to the court of Antigonos Gonatas of Macedonia, where he wrote the *Phenomena*. He then spent some time with Antiochos I of Syria, but subsequently returned to Macedonia, where he died. Although Aratos was ignorant of astronomy, his poem attracted the attention of **Hipparchos**, who wrote commentaries upon it. Amongst the Romans it enjoyed a high reputation; **Cicero** translated it, **Ovid** mentioned him, **Virgil** imitated it, and **Paul** quoted from it (**Acts** *17*, 28).

270 BCE *The Pharos lighthouse of Alexandria.* A landmark in the development of both navigation and architecture. One of the seven wonders of the ancient world. Designed by the Greek architect **Sostratos**; it rose to the height of 134 meters and stood on the island of Pharos off the coast of Alexandria, guiding ships into the city's harbor. The structure rose from a stone platform in 3 sections: the bottom section was square, the middle eight-sided and the top circular. A fire on top provided light.

It stood for over 1000 years before being toppled by the great earthquakes of April 07, 796 and Aug. 08, 1303.

250–550 BCE Decline and fall of the Roman Empire. Decreased rainfall in Rome. Condition of warmth and drought undermined the agriculture and favored epidemic diseases (such as malaria) that gradually weakened the population of the Roman Empire. Simultaneously, increasing drought on the steppes of Central Asia drove the barbarians to seek new lands, tending to cut the trade caravans route between Rome and China.

ca 250 BCE **Erasistratos of Ceos** (Iulis) (ca 304–2 BCE). Greek physician and anatomist of the Alexandrian school. Founder of physiology and comparative anatomical pathology. His main anatomical discoveries concern the brain, the heart, and the nervous and vascular systems. Credited with being the first to distinguish between motor and sensory nerves. Distinguished between cerebrum and cerebellum, traced cranial nerves in the brain itself and investigated the relation of muscles to motion. Traced veins and arteries to the heart and named the trachea and tricuspid valve of the heart. Was close to the discovery of the circulation of the blood[199]. Conducted post-mortem dissections.

[199] Erasistratos proposed that the liver used food to make blood, which was delivered to the other organs by the veins. He believed that the arteries contained air, not blood. This air (*pneuma*), a living force, was taken in by the lungs, which transferred it to the heart. The heart transformed the air into a "vital

Erasistratos was born at Iulis, the main city of Ceos, one of the Cyclades, close to the mainland of Attica (todays Zia). Was educated at Athens and began his career as assistant of Herophilos. He continued the latter's investigations, but was more interested in physiology and the applications of physical ideas to the understanding of life.

ca 250 BCE **Ctesibios**. Greek inventor. The 'Edison' of Ptolemaic Alexandria. He built the first *pump*; it consisted basically of a cylinder with a plunger (valve) inside. As the plunger was moved up and down, it created a pressure that could be used to pump water. Invented the *water clock* or *clepsydra*; it has water dropping like tears into a funnel from the eyes of a statue. A float mechanism raised another human figure with a pointer which indicated the hours on a vertical cylinder. Once in 24 hours the figure descended to the bottom of the column by a siphon mechanism. The siphon outflow worked a water wheel which very slowly rotated the cylinder dial, making a complete rotation in a year. The graduation of the cylinder was adapted to the varying lengths of the hours throughout the seasons.

Ctesibios also built musical instruments worked by pneumatic machinery, a hydraulic pipe organ, the musical keyboard and the *metal spring*, made from bronze plates and used in a catapult.

ca 250 BCE – The earliest preserved examples of our present number symbols 1, 2, 3, 4, 5, 6, 7, 8, 9, are found in some stone columns erected in India by King Aśoka. These early specimens do not employ positional notation.

ca 250 BCE **Archimedes of Syracuse**[200] (ca 287–212 BCE). Mathe-

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spirit" that was carried by the arteries to the other organs. George Sarton, in his "*Hellenistic Science and Culture*" (1959) warns against modern interpretation of *physiological* facts: "*... one may easily misinterpret Erasistratos' ideas, which we know to be his only through Galen, and Galen's phrasing may suggest to us some ideas that did not exist in his mind, let alone in the mind of Erasistratos. It is almost impossible for us to put ourselves back in their situation, and relatively easy to interpret their ideas in terms of our knowledge*".

[200] For further reading, see:

- Hollingdale, S., *Makers of Mathematics*, Penguin Books: London, 1989, 437 pp.
- Stillwell, J., *Mathematics and its History*, Springer Verlag: New York, 1991, 371 pp.
- Dijksterhuis, E.J., *Archimedes*, Princeton University Press, 1987.
- Weil, A., *Number Theory, an Approach Through History*, Birkhäuser, 1983.

matician, physicist, astronomer, engineer and inventor, the greatest universal intellect of ancient times. The creator of statics, hydrodynamics and mathematical physics. Impressed his contemporaries and the bulk of posterity by his practical inventions. Considered for almost two thousand years to be the archetype of the inventor and of the mechanical wizard.

His methods anticipated the *integral calculus* 2000 years before Newton and Leibniz. Discovered the mathematical law of the lever, screw[201] and pulley and the basic laws of hydrostatics. Invented ballistic machines, a hydraulic organ, cranes, burning mirrors, compound pulleys, fulcrums[202], and the like.

He showed[203] that the value of π is between $3\frac{1}{7}$ and $3\frac{10}{71}$. This discovery made it possible for him to solve many problems involving the area of circles

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- Dörrie, H., *100 Great Problems of Elementary Mathematics, Their History and Solutions*, Dover Publications: New York, 1965, 393 pp.

- Beiler, A.H., *Recreations in the Theory of Numbers*, Dover, 1966, 394 pp.

[201] Although Archimedes discovered the *principle of the screw*, ordinary wood screws, as a small fixing device, originated only in the 16^{th} century!
Saws, hammers, nails, chisels and drills all date from the Bronze and early Iron Ages. Many types of modern tools originated even earlier, in the Neolithic period, about 8000 years ago. In fact, there is only one tool that would puzzle a Roman and a medieval carpenter – a *screwdriver*! Ancient screws were large wood contraptions, used for raising water. One of the earliest devices that used a screw to apply pressure was a Roman clothes press; presses were also used to make olive oil and wine. The Middle Ages applied the same principle to the printing press and to that fiendish torturing devices, the *thumbscrew*. The inventor of the hand-held screwdriver remains unknown, but the familiar tool does not appear in carpenters' toolboxes until after 1800. There was not a great call for screwdrivers, because screws were expensive. They had to be painstakingly made by hand and were used in luxury articles like clocks. It was only after 1850 that wood screws were available in large quantities.

[202] He allegedly said: "Give me a fulcrum on which to rest and I will move the earth; give me a place to stand on, and I will move the world".

[203] His method for calculating π is based on the observation that the circumference of a circle is greater than the perimeter of the inner regular n-polygon and less than the perimeter of the outer regular polygon. In modern notation: $n \sin \frac{180}{n} < \pi < n \tan \frac{180}{n}$. For $n = 6$ (hexagon), $3 < \pi < 2\sqrt{3} = 3.46\ldots$. For $n = 96$, calculations yield values which are close to the Archimedean bounds.

and the volume of cylinders, cones and spheres[204]. He also proved that the area of the *spherical cap* is equal to the area of a circle, the radius of which is equal to the cord connecting the 'pole' of the cap to its rim.

Archimedes invented an enumeration system that was more workable with large numbers than were the Greek and the Roman systems. Using this, he calculated the number of grains of sand he thought it would take to fill the universe[205]. He also used methods which, in essence, are used now in modern calculus ("Method of Equilibrium"). He was probably the greatest mechanical genius until Leonardo da Vinci (1452–1519, Italy). Since he made some of man's basic scientific discoveries, some historians call him "the father of experimental science". In his own time he was best known for his many inventions, but he looked upon his inventions as play, and considered mathematics his real work.

The work of Archimedes is characterized by extreme originality and directness as compared with the work of Euclid and Apollonios. The geometry of Archimedes is chiefly a geometry of measurements, while that of Apollonios is rather a study of forms and situations. He realized quadratures of curvilinear plane figures and quadratures and cubatures of curved surfaces by a method more general than the method of exhaustion, and which can be regarded as an anticipation of the integral calculus. He studied paraboloids, hyperboloids, ellipsoids and polyhedra[206]. He summed $\sum n^2 = \frac{1}{6}n(n+1)(2n+1)$, and

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204 Archimedes stated that the volume of any sphere is 4 times that of a cone with base equal to a great circle of the sphere and with height equal to the radius of the sphere – a statement that amounts to saying that $V = \frac{4}{3}\pi r^3$. He also stated that a cylinder with base equal to a great circle of the sphere and with height equal to the diameter of the sphere is equal to $1\frac{1}{2}$ times the sphere – a statement that amounts to the same thing. These facts are stated in yet another form, known as *Archimedes' theorem*: the ratios of the volume of a cone, half-sphere and a cylinder, all of the same height and radius are 1:2:3 (i.e. $\frac{1}{3}\pi r^2 h$, $\frac{2}{3}\pi r^2 h$ and $\pi r^2 h$ respectively).

205 The largest number given a name by the Greeks was a *myriad*, which we call 'ten thousand' (10^4). Archimedes began by thinking of *myriad of myriads*, and he referred to numbers from 1 to 10^8 as *numbers of the first order*, numbers from 10^8 to 10^{16} as *numbers of the second order*, etc. He continued in this way until he got to the *numbers of order* 10^8. Archimedes then estimated that 10^{63} grains of sand were needed to fill the entire universe. Current estimates of the size of the observable universe in terms of Archimedes' grain-size units, require no more than 10^{93}.

206 *Archimedean polyhedra*: According to Pappos, Archimedes invented a class of *semi-regular* polyhedra, known also as 'facially' regular. This means that every face is a regular polygon, though the faces are not all of the same kind. How-

solved cubic equations by means of the intersection of two conics. [His problem, to cut a sphere by a plane so that the two segments shall be in a given ratio, leads to a cubic equation $x^3 + c^2 b = cx^2$.]

To Archimedes is credited the famous 'Cattle Problem'[207] (*Problema Bowinum*). He must have known about the Pell Equation $u^2 - Nv^2 = 1$ since

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ever, the faces must be arranged in the same order around each vertex, and all the solid angles of the polyhedron are equal. Thirteen such solids exist. One of them, the *truncated icosahedron*, is formed by truncating the vertices of an icosahedron such as to leave the original faces hexagons. It is therefore enclosed by 20 *hexagonal* faces belonging to the icosahedron, and 12 *pentagonal* faces belonging to the coaxial dodecahedron. It has altogether 32 faces, 60 vertices and 90 edges. The dihedral angles are $138°11'(6–6)$, $142°37'(6–5)$. All Archimedean solids are inscribable in a sphere.

Chemists assumed they knew everything about pure carbon and its manifestations as graphite and diamond. But in 1985 they discovered that 60 carbon atoms can arrange themselves at the vertices of a truncated icosahedron to form the most symmetric molecule possible in 3-dimensional space. This molecule was given the name *buckminsterfullerine*, in honor of Richard Buckminster Fuller's geodesic dome.

207 *The sun god had a herd of cattle consisting of bulls and cows, one part of which was white, a second black, a third spotted, and a fourth brown.*

Among the bulls, the number of white ones was one half plus one third the number of the black greater than the brown; the number of the black, one quarter plus one fifth the number of the spotted greater than the brown; the number of the spotted, one sixth and one seventh the number of the white greater than the brown.

Among the cows, the number of white ones was one third plus one quarter of the total black cattle; the number of the black, one quarter plus one fifth the total of the spotted cattle; the number of spotted, one fifth plus one sixth the total of the brown cattle; the number of the brown, one sixth plus one seventh the total of the white cattle.

What was the composition of the herd?

If we use the letters X, Y, Z, T to designate the respective number of the white, black, spotted, and brown bulls and x, y, z, t to designate the white, black, spotted, and brown cows, we obtain the following *seven* equations for these *eight* unknowns:

$$\begin{aligned} X - T &= \tfrac{5}{6}Y, & x &= \tfrac{7}{12}(Y + y), \\ Y - T &= \tfrac{9}{20}Z, & y &= \tfrac{9}{20}(Z + z), \\ Z - T &= \tfrac{13}{42}X, & z &= \tfrac{11}{30}(T + t), \\ & & t &= \tfrac{13}{42}(X + x). \end{aligned}$$

After some algebraic manipulations, one is led to so-called *Pell equation* $u^2 - 4729494\,v^2 = 1$, where v must be divisible by 9314. Stupendous feats he used it to obtain a very good approximation to $\sqrt{3}$. Indeed,

$$1351^2 - 3 \cdot 780^2 = 1, \qquad \text{so } \left(\frac{1351}{780}\right)^2 - 3 = \frac{1}{780^2}.$$

Therefore

$$\sqrt{3} \approx \frac{1351}{780} = 1.732,051,282\ldots,$$

whereas

$$\sqrt{3} = 1.732,050,805\ldots.$$

Archimedes was born in Syracuse, then a Greek colony. He went to school in Alexandria, Egypt, then the center of Greek learning. He spent the rest of his life back in Syracuse, Sicily.

When Syracuse was attacked by the Roman forces under Marcellus, The Romans met an enemy in the form of a 75-years-old mathematician, supplied with unexpected and powerful weapons. Archimedes actively participated in the defense of the city. He directed the use of systems of pulleys, levers, cranes and other devices which he had invented long before, as practical demonstrations of his theories. The approaching Roman were knocked down by a very effective artillery, which discharged long-range projectiles. The ships of the Roman fleet were sunk by huge cranes. Finally the ships were incinerated by "burning mirrors"[208]. Thus Archimedes forced the withdrawal of the Romans and was the principal factor in the 3-year delay of the fall of Syracuse.

Details of his death are variously told, but all accounts agree that he was killed at the age of 75 by a blundering soldier of the invading Roman army. Marcellus, overwhelmed with grief at the news of his death, attempted to make amends by protecting and even honoring any who could claim relationship to Archimedes. **Cicero** informs us that when he was quaestor in Sicily (75 BCE), he saw Archimedes' tombstone which, though neglected, still showed the incised figure of a sphere inscribed in a cylinder. The theory epitomized by this figure was considered by Archimedes to be his greatest achievement. His greatness can be measured by the fact that the scholars of the 17^{th} century,

of calculation have been performed throughout the ages to obtain integer solution to this equation. Finally, H. Williams, R. German and C. Zarnke obtained the first exact solution in 1965, using an IBM 7040 electronic computer. For a good discussion of the problem, see Vardi (1998): *Archimedes' cattle problem*, Amer. Math. Monthly **105**, no. 4, pp. 305-319; or http://www.jstor.org/.

208 The feasibility of this last contrivance has been demonstrated experimentally (1747) by the naturalist **George Buffon** and again (1973) by the Greek engineer **Ioannis Sakkas**.

living almost 2000 years after him were the first to perceive the subtleties of his computational methods. In their hands, Archimedes' writings were of prime influence in the evolution of the calculus.

On Archimedes

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"The works of Archimedes are without exception, monuments of mathematical exposition; the gradual revelation of the plan of attack, the masterly ordering of the propositions, the stern elimination of everything not immediately relevant to the purpose, the finish of the whole, are so impressive in their perfection as to create a feeling akin to awe in the mind of the reader."

(Thomas Heath, 1921)

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"One cannot read Archimedes' complicated accounts of his quadratures and cubatures without saying to oneself, "How on earth did he imagine those expedients and reach those conclusions?"

(George Sarton, 1952)

'Do Not Touch My Circles'

Rome completed the domination of Italy in 338 BCE (battle of Trifanum). During the next two centuries it vanquished the Carthaginian Empire (3 Punic wars, 264–146 BCE) and reduced Macedonia to a Roman province (4 Macedonian wars, 215–148 BCE). In 286 CE, Diocletian divided the Roman Empire, which marked the start of its long decline.

The rise and fall of Alexandria (as a center of learning) runs almost concurrently to that of Rome; founded by Alexander the Great in 332 BCE, it soon became the world capital of the sciences, edified by the great minds of ***Menaechmos***, ***Euclid***, ***Apollonios***, ***Eratosthenes***, ***Hero***, ***Ptolemy***, ***Diophantos*** *and* ***Pappos***.

It was just a matter of time before the two cultures ran into a collision course; around 180 BCE, Alexandria entered the Roman sphere of influence and in 80 BCE the city passed formally into Roman jurisdiction. There, Julius Caesar dallied with Cleopatra in 47 BCE while the great Alexandrian Library lay burned in ashes; there his example was followed by Antony, for whose favor the city paid dearly to Octavian, who placed over it a prefect from the imperial household. Alexandria seemed from this time to have regained its old prosperity, commanding, as it did, an important granary of Rome. This latter fact, doubtless, was one of the chief reasons which induced Augustus to place it directly under imperial power. In 215 CE the emperor Caracalla visited the city; and, in order to repay some insulting satires that the inhabitants had made upon him, he commanded his troops to put to death all youth capable of bearing arms. This brutal order seems to have been carried out even beyond the letter, for a general massacre was the result.

Notwithstanding this terrible disaster, Alexandria soon recovered its former splendor, and for some time longer was esteemed the first city of the world after Rome. But its importance no longer sprang from Pagan learning, but as a center of Christian theology and Church government. Its great library was repeatedly sacked and burned by the Roman vandals in 273, 295 and 391 CE.

It is a great fallacy to believe that the Romans had attained an advanced level in the sciences, the arts, law[209]*, architecture and engineering. Like later*

[209] It was not inherent superiority that facilitated the spread of Roman law throughout Europe, for it had borrowed heavily from both Semitic and Greek concepts. What made Roman law so widely acceptable was that people could borrow its legal concepts without finding a Roman god or a doctrinaire trap tied to the end of the paragraph. Even the Jews could borrow from Roman law without

empires in history, the Romans enslaved peoples whose cultural level was far above their own. They not only ruthlessly vandalized their countries, but also looted them, stealing their art treasures, abducting their scientists and copying their technical know-how, which the Romans' barren society was rarely able to improve on; the light of their culture was borrowed from the Greek and their other enslaved colonies.

Roman engineering was void of all subtlety; long after Heron of Alexandria (ca 150 BCE) had designed water clocks, water turbines and two-cylinder water pumps, and had written works on these subjects, the Romans were still describing the performance of their aqueducts (and charging the users) in terms of the cross-section of the flow (!), as if the volume of the flow did not also depend on its velocity.

Nevertheless, the Romans did develop the use of concrete and the arch in the building of bridges, roads, and aqueducts, creating a series of civil and military engineering projects that surpassed in scale any since those of the Assyrians.

Roman engineering, like that of the earlier watershed empires, was based upon intensive applications of rather simple principles, with plenty of raw materials, cheep labor, and time. Rome possessed abundant supplies of brick, stone, and timber. Labor was especially cheap because the swift extension of the empire had crowded Rome with thousands of slaves.

The Romans devoted more of their energy and capital to useful public works than did their predecessors. They built roads, harbor works, aqueducts, temples, forums, town halls, arenas, baths and sewers. A magnate, governor, or emperor might be a scoundrel in other respects, but he believed that he gained honor by presenting the people with some useful or entertaining public work. Under the republic this was a method of bidding for votes; and, even after elections had ceased in the early Empire, the tradition lived on.

The Roman contribution to science was mostly limited to butchering antiquity's greatest mathematicians, burning the library of Alexandria, and slowly

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fear of becoming beholden to its Pagan deities.

Another reason for the acceptance of Romanism throughout Europe was the nature of the Roman people. Though they were cruel, they were free from prejudice, with the noted exception of their antipathy to Christians. They massacred people and gave them citizenship with equal impartiality, disregarding race, creed, color, or previous conditions of servitude. The Roman formula for tying a conquered nation to its victorious chariot was based on four basic principles – annex the land by the sword, connect it to Rome roads, bind its people with citizenship, and govern them with secular laws.

stifling the sciences that flourished in the colonies of their empire. The Naturalis Historia by ***Pliny the Elder*** *(23–73 CE) is an encyclopaedic compilation which is generally regarded as the most significant scientific work to have come out of Rome; and it demonstrates the Roman's abysmal ignorance of science when compared to the scientific achievements of their contemporaries at Alexandria, even a century after the Romans had sacked it.*

The Roman contribution to mathematics was almost nihil. Their greatest mathematician ***Poseidonius*** *(135–51 BCE), friend and teacher of Cicero and Pompey who calculated the circumference of the earth with high accuracy, was a Syrian who studied in Athens. But whatever the value of* π *used by Poseidonios, it was high above Roman brows. The Roman architect and military engineer* ***Marcus Vitruvius Pollio*** *(flourished ca 25 BCE), in De Architectura, used the value* $\pi = 3\frac{1}{8}$, *the same value the Babylonians had used at least* 2000 *years earlier.*

The alleged Roman achievements are largely a myth. What they really excelled in was warefare and blood baths. Thus, during the second Punic war, the Romans sent an expeditionary force under Marcus Claudios Marcellus[210] *(268–208 BCE) to Sicily in* 214 *BCE to "convince" the king of Syracuse to sever his alliance with Carthago. However, Roman brute force, assaulting the city of Syracuse by land and sea, ran into some smart scientific engineering. Using machines that applied the secrets of the lever, the pulley and the principle of the mechanical advantage, Archimedes was master-minding the defense of city walls for three years before the city finally fell to the Roman cut-throats (212 BCE). Inside the city was the 75-year old Archimedes, engaged in the solution of some geometrical problem.* ***Plutarch*** *tells us that a soldier came up to him and bade him follow to Marcellus, but he would not go until he had finished his problem and worked it out to the proof. "Do not touch my circles!" said the savant. Thereupon the soldier became enraged, drew his sword and slew the savant.*

The Greeks were interested in man as a creature who by nature desires to know; the Romans were interested in man for what they could get from him by force or persuasion – through the Roman legions or Latin oratory. As ***Alfred North Whitehead*** *put it "The death of Archimedes at the hands of a Roman soldier is symbolical of a world change of the first magnitude... No Roman lost his life because he was absorbed in the contemplation of a mathematical diagram".*

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[210] Killed near Venusia, Italy, by Hannibal's forces.

247–168 BCE *Collision of the Greco-Roman and the Phoenician and Judean cultures*: Hebrew scriptures were translated into Greek during 285–247 BCE by a group of 72 Alexandrian Jews (*Septuagint*). In 216 BCE, Hannibal (247–183 BCE) stood with his army in front of the gates of Rome. **Ecclesiastes** was composed sometime between 245 and 168 BCE. The Book of **Daniel** was written during 175–164 BCE, probably close to the revolt of the Maccabees[211] in 168 BCE.

ca 245 BCE **Conon of Samos**. Greek astronomer and mathematician. The friend of Archimedes, who survived him. Wrote a work on astronomy, which contained a collection of the observations of solar eclipses made by the Babylonians. He also investigated the question of the number of points of intersection of two conics, and his researches probably formed the basis of the 4^{th} book of the *Conics* by Apollonios of Perga. Considered the inventor of the curve known as the *Spiral of Archimedes*.

Conon became court astronomer to Ptolemy III Euergetes at Alexandria; he is known in connection with the *Coma Berenices*[212].

240 BCE, Aug. 02 Historical record of a Chinese observation of a Perihelion passage of *Comet Halley*. No observations of this comet prior to this date has yet been identified in the ancient Chinese records.

This event has been recently (1981) ascertained by actually integrating the comet's equations of motion backward in time, including planetary and non-gravitational perturbations.

235 BCE **Eratosthenes of Cyrene** (276–197 BCE). A Greek mathematician and astronomer. Librarian of the Museum of Alexandria (235 BCE). Devised an ingenious way to measure the circumference of the Earth: he noticed

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211 Antiochus IV Epiphanes began the persecution of the Jews by declaring Judaism illegal (168 BCE). Under **Jehudah Ha'macabee** (Judas Maccabaeus), the Jewish army, outnumbered 5:1, defeated the Seleucian forces in four decisive battles and gained religious independence (164 BCE). After a series of wars, the Jews gained political independence (142 BCE) and established the *Hasmonean Kingdom*.

212 Berenice II, wife of Ptolemy III Euergetes had dedicated her hair in a temple as an offering to secure the safe return of her husband from his Syrian war. It disappeared from the temple. To flatter the queen, Conon declared that it has been placed among the stars in the constellation *Coma Berenices* (Hair of Berenice).

that when the sun was overhead at Aswan (Syene), it was about 7.2 degrees from vertical in Alexandria (sun rays arrive almost parallel to both places)[213]. This simple observation sufficed to yield the desired result in terms of the distance Aswan–Alexandria (ca 800 km). His deduced length for the earth's circumference is only 160 km in excess of the present accepted value. Invented the so-called *sieve of Eratosthenes* to obtain the prime numbers, and an instrument to solve the duplication of the cube. He was the founder of scientific chronology of ancient Greece and was also a distinguished philologist.

Eratosthenes also measured the tilt of the earth's axis (relative to the ecliptic) with great accuracy and compiled a star catalogue. He became blind at old age and is said to have committed suicide by starvation.

ca 230 BCE **Apollonios of Perga** (262–200 BCE). Greek geometer and astronomer of the Alexandrian school. Founder of Greek mathematical astronomy, which used geometrical models to explain planetary theory. Flourished in the reigns of Ptolemy Euergetes and Ptolemy Philopater (247–205 BCE). His treatise *Conic Sections*[214] gained him the title of *The Great Geometer*. All his other treatises have perished, and we have only their titles handed down, with general indications of their contents, by later writers, especially **Pappos** (ca 300 CE).

Apollonios was born in southern Asia Minor. As a young man he went to Alexandria, studied under the successors of Euclid and remained there for a long time. Later he visited Pergamum in western Asia Minor, where there was a recently founded university and library patterned after that of Alexandria. He returned to Alexandria and died there.

His book on conic sections[215], comprising 8 volumes and some 400 propositions, completely superseded the earlier works on the subject by Menaechmos

[213] Syene – a place on the upper Nile River near the Tropic of Cancer (lat. $23\frac{1}{2}\,^\circ$N). The observation was made at the time of summer soltice (June 21), when the sun's noon rays were nearly perpendicular to the earth's surface, so as to shine upon the floor of a deep, vertical well. On the *same* soltice day, the sun's *noon rays* were observed at Alexandria, located approximately on the *same* meridian.

[214] For further reading, see:

- Somerville, D.M.Y., *Analytical Conics*, Bell and Sons: London, 1949, 310 pp.
- Heath, T.L. (Editor), *Apollonius of Perga: Treatise on Conic Sections*, W. Heffer and Sons: Cambridge, 1961, 254 pp.

[215] In fact, Book 8 of *Conics* is lost while 5 to 7 only exist in Arabic translation. However, we know something of his other work from the writings of others. Thus we know that he obtained an approximation for π better than $\frac{22}{7} > \pi > \frac{223}{71}$ known to Archimedes. He showed that parallel rays of light are *not* brought to

and Euclid. The names *ellipse*, *parabola* and *hyperbola* were coined by Apollonios. In Book II one finds the harmonic properties of poles and polars and theorems concerning the products of segments of intersecting chords which constitute the basic elements of today's projective geometry. Book V is the most remarkable and original of the extant books. It treats *normals* considered to be maximum and minimum line segments drawn from a point to the curve. The construction and enumeration of normals from a given point are dealt with. The subject is pushed to the point where one can write down the Cartesian equations of the *evolutes* of the three conics!

Appolonios stated the problem of constructing a circle tangent to three given circles, where the given circles are permitted to degenerate independently into straight lines or points. This is now known as the *problem of Appolonios*[216]. It had attracted many mathematicians, among them **Viète**, **Euler**, and **Newton**.

Many attempts to restore the lost works of Apollonios were made by leading mathematicians, among them **Vièta**, **Snellius**, **Fermat** and others. Indeed, the prodigious activity surrounding the study and reconstitution of his works effectively placed Apollonios in the direct line of the invention of analytic geometry and made him a powerful force in the modern development of mathematics.

220–200 BCE **Dionysodorus** (ca 250–190 BCE). Mathematician. Solved the problem of the cubic equation using the intersection of a parabola and a hyperbola. He also is believed to have invented a conical sundial.

218 BCE **Archagathos**. The first Greek physician whose name is preserved as having migrated to Rome from the Peloponnesos; but there were probably others before him.

When Greece was made a Roman province (146 BCE), the number of such physicians who sought their fortunes in Rome must have been very large. The bitter words of Marcus Porcius Cato (234–149 BCE), who disliked them as he did other representatives of Greek culture, are evidence of this.

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a focus by a spherical mirror (as had been previously thought) and discussed the focusing properties of a *parabolic mirror* (proved by Diocles).

216 One of the first solutions, applying the new Cartesian geometry was given by Descartes' pupil **Elizabeth of Bohemia** (1618–1680), daughter of Elector Palatine Frederick V of Bohemia.

Probably the most elegant solution is that furnished by the French artillery officer and professor of mathematics, **Joseph Diaz Gergonne** (1771–1859).

214 BCE The construction of *the Great Wall of China*[217] began by emperor Shih Huang Ti of the *Ch'in* dynasty to safeguard the country against invasion from the north. He built the wall by linking together shorter walls erected by earlier times.

This defensive structure extends over 2400 km in northern China from Kansu in the west to the Yellow Sea in the east, over 22 degrees of longitude (98° to 120°E). It defines the historical boundary between China and Mongolia. Legend has it that over million workers died during the construction and their bodies were thrown into the wall. It is the longest fortified line ever built.

The wall is carried over valleys and mountains, and in places over 1300 meters above sea-level. It stand about 8 meters high. Towers from 11 to 12 meters high were build into the wall every 180–270 meters. It tapers from a width of 8 meters at the base to about 5 meters at the top. Its sides are made of earth, brick and stone. The top is paved with bricks that are set in lime, forming a roadway for horsemen. The Great Wall was built by hand and took hundreds of years to complete. Under *Sui* dynasty (589–610 CE), it was expanded, and during the Ming dynasty (1368–1644) it was repaired. Later rulers neglected it.

213 BCE '*Burning of the books*'. In China, Emperor Shin Huang-Ti proclaimed that everything was to begin with his reign. He then issued an order to burn all books. 400 scholars were burned alive for having disobeyed the imperial command. Some books were bricked up in walls and upon the collapse of the Chin dynasty in 206 BCE, were recovered from their hiding places, but many others were lost.

Chang Tsang (ca 210–152 BCE), soldier, statesman, and scholar, reconstructed the earlier *K'iu-ch'ang* (whose fragments escaped the book-burning) into the greatest mathematical work of antiquity, the *K'iu-ch'ang suan-shu* (Arithmetical Rules in Nine Sections). Tradition places the origin of the early work in the third millennium BCE. The latter book contains 246 problems classified into 9 sections. Among the topics involved are: 'Euclid's algorithm', proportion, extraction of square and cube roots, plane and solid mensuration and the right triangle.

210–190 BCE **Diocles of Carystos** (ca 240–180 BCE). Mathematician. Wrote *On burning mirrors* which *proves* the focusing property of a parabolic mirror for the first time. Largely ignored by later Greeks, it had a large

[217] The name by which that country is known to other nations, comes from a dynasty which ruled the land for only 14 years (221–206 BCE).

influence on the Arab mathematicians, in particular on Alhazen. Latin translations of Alhazen, from about 1200 CE, brought the properties of parabolic mirrors to the West. Diocles also studied the Cissoid (named after him) as part of an attempt to duplicate the cube. He also studied the problem of Archimedes to cut a sphere in such a way that the volumes of the segments shall have a given ratio.

ca 200 BCE Indirect evidence for the use of the *zero* in India.

ca 200 BCE Romans adopted and improved on an earlier Etrurian notation of numbers.

Roman numerals were written in terms of certain capital letters of the Latin alphabet (but not *successive* letters, in the manner practiced by the Syrians, Hebrew, and Greeks).

All Roman numerals are written with seven basic symbols. These are $I(1)$, $V(5)$, $X(10)$, $L(50)$, $C(100)$, $D(500)$, and $M(1000)$. There is no zero. All other numbers are written by combining these seven symbols. Roman numerals are written from left to right, using the principle of addition: e.g., $2713 = MMDCCXIII$; $MMM = 3000$. A smaller numeral (or composite number) appearing *before* a larger numeral indicates that the smaller number is *subtracted* from the larger one, e.g., $84 = XXCIV$.

It is simple to add and subtract with Roman numerals, but the system is inconvenient and clumsy for other types of calculations.

The Roman-numerals system was the most popular form of writing numbers until the widespread use of Arabic numerals began in the late 1500's. Today, the former is used to number the faces of clocks, to list important topics in outlines, to number prefatory book pages, and to record dates on monuments and public buildings.

187 BCE Plague occurred throughout Egypt, Syria and Greece. Reported by **Pliny**.

180 BCE **Zenodoros** (ca 200–140 BCE). Greek mathematician. Wrote a treatise on *isometric figures*, some fragments of which were preserved in the writings of **Theon of Smyrna** (ca 125 CE) and **Pappos of Alexandria** (ca 300 CE). Zenodoros proved that the circle has a greater area than any regular polygon of equal periphery[218]. He showed that among polygons with equal

[218] The dual statement, that of all curves enclosing a given area the circle possesses the shortest perimeter, was already known to **Aristotle** (384–322 BCE); in his book *De Caelo* he writes: "*Of lines which return upon themselves the line which bounds the circle is the shortest*".

perimeter and an equal number of sides, the *regular polygon* has the greatest area, and also studied the volume of a solid figure with a fixed surface area.

ca 180 BCE **Shimon Ben-Sirah** (ca 240–175 BCE, Jerusalem). Savant, scribe and author of one of the books of the Apocrypha entitled: *The Wisdom of ben-Sirah*, or *The Book of ben-Sirah*; a proverbial wisdom in praise of both *secular knowledge*[219] and traditional religious wisdom (Torah).

Ben-Sirah lived in Jerusalem at the time of **Simon II the Just** (d. 190 BCE). The Hebrew original of this book was considered lost until 1896, when a great part of it was found in a cellar of the Ezra Synagogue in Cairo. The book was translated in Egypt into Greek in 132 BCE by the author's grandson and the Christian Church preserved the book in Greek.

164 BCE Babylonian clay tablets (in the British Museum) documented observation of *comet Halley* apparition in the region of the Pleiades in the constellation Taurus. Recent calculation confirm its appearance in the month of November of that year.

161–127 BCE **Hipparchos of Nicaea** (**Rhodes**) (180–110 BCE). A Greek astronomer and mathematician and one of the greatest astronomers of antiquity. He fixed the chief data of astronomy – the lengths of the tropical *and sidereal years*, of the various months, and of the *synodic periods of the five planets*; determined the obliquity of the ecliptic and the moon's path; the position of the sun's apogee; the eccentricity of earth's orbit, and the moon's horizontal parallax. His borrowings from the Babylonian experts appear to have been numerous, but were doubtless independently verified. His supreme merit, however, consisted in the establishment of astronomy on a sound geometrical basis. His acquaintance with trigonometry, a branch of science initiated by him, together with his invention of the *planisphere*, enabled him to solve a number of elementary problems.

Hipparchos recognized the existence of the *precession of the equinoxes*, which for him was just an empirical fact, and so it remained until 1543. He became aware of it when he compared his own observations with those of earlier astronomers. Thus, he may have known that Alpha Draconis was the pole star for the Egyptian civilization about 3000 BCE.

To an earthbound observer, the sun's annual trace across the celestial sphere (of "fixed" stars) intersects the celestial equator at two points. At

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[219] e.g., he advices: "*Honor thy physician. His knowledge allows him to walk with raised head, and gains for him the admiration of princes. If you fall ill, cry to the Lord, but also call for the physician, for a sensible man does not neglect the remedies which the earth offers*".

each of those points the sun is directly over the equator at noon and the length of the day and night are nearly the same everywhere on earth. This happens twice a year: once at the vernal (spring) equinox on March 20 or 21, as the sun moves *north*; the other at the autumnal equinox on Sept. 22 or 23, when the sun moves *south*. Also on those days, the sun rises precisely in the east and sets in the west, at all latitudes. The beginning of the year was reckoned in days from the start of spring, – the day when the sun crosses the celestial equator from south to north in its path around the ecliptic.

The ancient astronomers, through their amazingly careful observations, had discovered that the celestial sphere of "fixed stars", seems to turn gradually from west to east with respect to the nodal line connecting the two equinoctial points. (An observer outside the solar system would instead see a *westward* drift of the equinoctial points.) **Hipparchos** discovered this phenomenon and reported its magnitude to be about $36''$ of arc per year (the true value is $50''$).

He also recognized that the existence of this precession of the equinoxes allowed two different definitions of the year – either the time between spring equinoxes, bringing the sun back into the same positional relationship to the earth's spatial orientation, or the slightly longer time (by about 20 min) for the sun to return to exactly the same observed position with respect to the fixed stars. These times are known as the *tropical year* and the *sidereal year*, respectively.

Since 20 minutes is about $\frac{1}{26{,}000}$ year, it follows that the vernal equinox revolves once every 26,000 years. In the days of Hipparchos, the equinox had moved to constellation Aries, and now it is in constellation Pisces. The equinox will continue to move westward, reaching the constellation Taurus – the location it held in the earliest days of the Zodiac – around the year 23,000 CE. For the same reason, the earth's axis will not always point to Polaris. In the days of the Egyptians the polar star was near Thuban (Alpha Draconis), a bright star in the constellation Draconis. About 6000 years from now the axis will point to Alpha Cephei. In 14,000 CE Vega will be the polar star.

Around 160 BCE, Hipparchos built an observatory on the island of Rhodes. In 130 BCE, he made an observation at Rhodes, from which he obtained a remarkably accurate estimate of the *earth-moon distance*. His method had been suggested by **Aristarchos**, about 150 years earlier. The method involves a clear understanding of the positional relationship of sun, earth, and moon. First, he knew that sun and moon subtended almost exactly the same angle α at the earth[220]. Hipparchos measured this angle to be 0.553° ($\approx \frac{1}{103.5}$ radian);

[220] Which is why total solar eclipses are both *possible* and *very brief*.

he also knew what Aristarchos before him had found — that the sun is far more distant than the moon.

Hipparchos used this knowledge in an analysis of an *eclipse of the moon by the earth*: Assume that centers of the sun, earth and moon (in this order) are collinear, and that the rays coming from the extreme edges of the sun and tangent to the earth, cut the moon's circular orbit at two points A and B. Let the angle subtended between these two boundary rays be α radians. The moon passes through the shadow from A to B, and from the measured time that passage took, Hipparchos deduced that the angle subtended at the earth's center by the arc BA was $\measuredangle AOB = 2.5\alpha$. The rest is simple geometry: if the distance from the earth's center to the moon is D, the length of the arc AB is about $\overline{AB} = 2R_{_E} - \alpha D$ ($R_{_E}$ = earth's radius). Also $\overline{AB}/D = 2.5\alpha$. With $\alpha = \frac{1}{103.5}$, Hipparchos found $D/R_{_E} \approx 59$.

Hipparchos also discovered an ingenious method of calculating the mean duration of a synodic month; by measuring the exact interval between a lunar eclipse recorded by the Babylonians and one observed by himself, he found that 4267 full moons (lunations) occurred in the interval of 126,007 days + one hour = 3,024,169 hours. Therefore $\frac{3{,}024{,}169}{4267}$ hrs = $29^d : 12^h : 44^m : 3.3^s = 29.53^d$ is the synodic period[221] !

By ca 120 BCE, Hipparchos compiled the first comprehensive star catalogue, which listed the coordinates and brightnesses of 850 stars. During the course of this pioneering work, he compared his star positions with earlier records dating back to Aristarchos' time. Hipparchos soon noted systematic differences that led him to conclude that the north celestial pole had shifted slightly over the preceding century. This led him to the discovery of the precession of the equinoxes.

While compiling his star catalogue, Hipparchos established a system to denote the brightest of stars. His system is the basis of the magnitude scale used by astronomers today: the brightest star he saw in the sky he called *1st magnitude star*. The dimmest visible star he called a *6th magnitude star*. To stars of intermediate brightness he assigned intermediate numbers on this logarithmic scale of 1 to 6. With only a few refinements, the same system is used today.

The erratic configuration of the naked-eye stars has remained essentially unchanged since the times of the first records; all the stars that Hipparchos described can be found, with the same brightnesses and practically at the same places, in the contemporary sky. In fact, there has been no gross alteration in the constellations for millennia before his time.

221 According to Pliny, Hipparchos also observed a new star (probably a Nova) in the year 134 BCE.

The whole sky has been arbitrarily divided into 12 areas which differ greatly in angular size and shape. Each area embraces a "*constellation*", or group of stars, and is known by a mythical or a semi-mythical name. More than half the constellations were recognized and mentioned by Hipparchos. Some data of Hipparchos' star catalogue may be preserved in the star catalogue contained in Ptolemy's *Almagest*. The remaining constellations, which Hipparchos did not observe, lie in the Southern Hemisphere and were not named until the Age of Exploration in the 16^{th} and 17^{th} centuries. The constellations listed by Hipparchos were known by the Greek equivalents of their present Latin names. They were not invented by Greeks but came to Greece from the earlier civilization of Mesopotamia, and their names are found in Euphratean tables of about 600 BCE, which embody ideas of an even remoter age.

Hipparchos made great advances in both mathematics and astronomy. He was a careful observer, made new instruments and used them to measure star positions. He practically invented *spherical trigonometry* for use in his studies of the sun and moon. He constructed the first celestial globe on record. He used and probably invented the *stereographic projection*. In Rhodes (but also in Alexandria), he made an immense number of astronomical observations with amazing accuracy. He noted the existence of misty patches in the heavens which he called 'nebulae'. Yet he *did not reject the geocentric system*, and is therefore responsible for its long predominance.

Hipparchos' works are lost, and it is possible that their loss was partly the result of the fact the Ptolemy's great book, the *Almagest* (ca 150 CE) superseded them and made them superfluous. What we know of Hipparchos we know almost exclusively from Ptolemy, who quoted him often, sometimes verbatim. It is strange to think of two men separated by a barrier of 300 years – yet working as if the second was the immediate disciple of the first.

Hipparchos and Ptolemy rejected the ideas of Aristarchos of Samos (280 BCE) who had anticipated the Copernican system (1514), because they did not tally sufficiently with the observations. Their objections were of the same nature as Tycho Brahe's at the end of the 16^{th} century; a sufficient agreement between observations and the heliocentrical system became possible only when Kepler replaced circular trajectories by elliptic ones (1609).

Lunar Theory[222]*, Part I*

The celestial objects that have received the greatest attention throughout the long history of astronomy are the moon and the sun. In fact, the first astronomical phenomenon to be understood was the cycle of the moon. On a monthly basis, we observe the phases of the moon *as a systematic change in the amount of the moon that appears illuminated; the lunar phases are a consequence of the motion of the moon and the sunlight that is scattered from its surface. Half of the moon is illuminated at all times, but to an earthbound observer, the percentage of the bright side facing him depends on the location of the moon w.r.t. the sun and the earth. When the moon lies between the sun and the earth, none of its bright side faces the earth, thus producing the new-moon phase. Conversely, when the moon lies on the side of the earth opposite the sun, all of its lit side faces the earth, producing the full moon. At a position between these extremes, an intermediate amount of the moon's illumination is visible.*

The cycle of the moon through its phases requires about $29\frac{1}{2}$ *days, a period called the* synodic month, *a cycle that was the basis for the first Roman calendar. However, this is not the true period of the moon's revolution, which takes only about* $27\frac{1}{3}$ *days and is known as the* sidereal month. *The reason for the difference of* 2 *days each cycle is this: as the moon orbits the earth, the earth-moon system also moves in orbit around the sun. Consequently, even after the moon has made a complete revolution around the earth with respect to the background stars, it has not yet reached its new-moon starting phase, which is directly between the sun and earth. This additional motion takes another* 2 *days*[223].

The lunar year, consisting of 12 *synodic months, contains only* 354 *days; its conclusion consequently anticipates that of the solar year by* 11 *days, and passes an integer number of times through the whole cycle of the seasons in about* 34 *lunar years. It is therefore obviously ill-adapted to the computation of time, and all nations who have regulated their months by the moon have*

[222] To dig deeper, see: Moulton, F.R., *An Introduction to Celestial Mechanics*, Dover: New York, 1970, 436 pp.

[223] The synodic period of the moon is given by the kinematic equation: $\tau_s = \frac{2\pi}{\omega_M - \omega_E}$, where $\omega_E = \frac{2\pi}{365.256}$ is the earth's orbital mean angular velocity relative to the stars (sidereal year) in radians/day, and $\omega_M = \frac{2\pi}{27.32166}$ is the mean angular velocity of the moon in its orbit, relative to the background stars (sidereal month).

employed some method of intercalation by means of which the beginning of the year is retained at nearly the same fixed place in the seasons.

In the early ages of Greece the year was regulated entirely by the moon. ***Solon*** *(ca 639–559 BCE) divided the year into* 12 *months, consisting alternately of* 29 *and* 30 *days, the former of which were called* deficient *months, and the latter* full *months. The first expedient adopted to reconcile the lunar and solar years was the addition of a month of thirty days to every* 2^d *year. Since the difference of* $7\frac{1}{2}$ *days was still too great to escape observation, occasional corrections were made as they became necessary; but since these corrections were left to the care of incompetent persons, the calendar soon fell into great disorder, until a new division of the year was proposed by* ***Meton***[224] *(ca 432 BCE), which was immediately adopted in greater Greece.*

The ancient astronomers recognized the much greater complexity of the moon's motion as compared with that of the sun. Thus they found from observations of eclipses that the length of the synodic month is not constant, but varies within a few hours on either side of the mean $29^d : 12^h : 44^m : 3\frac{1}{3}^s$. *In addition, there were many irregularities in the moon's motion in various parts of its course. We know today that these irregularities are satisfactorily explained in the framework of the Newton-Kepler theory. But the ancient astronomers tried and failed to explain these phenomena by means of an eccentric circle alone (as they did in the case of the sun). They were obsessed with the idea that heavenly bodies whose courses were "ordained by God" could have no other than* perfect *movements, which to their minds was synonymous with* circular *movements. They thus represented the moon's motion by an ingenious device – a combination of circular motions.*

While the day, the month and the year have clear astronomical origins, the week *is a period of* 7 *days, having no reference whatsoever to celestial*

[224] The Athenian astronomer **Meton** (fl. 432 BCE), who took the length of the year as $365\frac{1}{4}$ days, discovered that 19 solar years expressed in days, hours, etc., contain *almost* exactly 235 synodic months (lunations). This is known as Meton's *19 solar year cycle*.

After the period of 19 years the full moons again occur on the same days of the solar year. Since $235 = 19 \times 12 + 7$, the period of Meton consists of 12 years containing 12 months each, and 7 years containing 13 months each, and these last are chosen to form the 3^{rd}, 5^{th}, 8^{th}, 11^{th}, 13^{th}, 16^{th}, and 19^{th} years of the cycles.

Calculations from modern data show that 235 lunations are 6939 days + 16.5 hours, while 19 solar years are 6939 days + 14.5 hours. The relation between integral numbers of months and years expressed by *Meton's rule* therefore deviates only 2 hours from the truth. Since 19 *Julian years* make 6939 days + 18 hours, the relation errs only 1.5 hours when the Julian year is used.

motions. It might have been suggested by the phases of the moon, or by the number of planets known in ancient times, an origin which is rendered more probable from the names universally given to the different days of which it is composed. (The idea of the Sabbath, however, with its social, religious and moral connotations, is exclusively Hebrew in origin.)

The plane of the moon's motion is inclined to the ecliptic, on the average by $5°8'$. *These two planes intersect along a* line of nodes: *The moon's* nodes *are the points where its orbit intersects the plane of the ecliptic. The node where the moon crosses the ecliptic from south to north is called the* ascending node. *The other, where the moon crosses from north to south is the* descending node *[eclipses can occur only when the moon is very near one of the nodes].*

The moon's orbit is an ellipse of average eccentricity $e = 0.055$. *The position along the orbit where the moon is nearest to us is called* perigee. *In the farthest position the moon is at its* apogee. *The line connecting the apogee and the perigee is the line of* aspides.

The complexity of the moon's motion is caused by gravitational action of the sun on the earth-moon system. Some of the principal perturbations are:

(1) *Evection: periodic change in the orbital eccentricity of the moon with a period of* 31.8 *days.*

(2) *Variation: an effect that makes new and full moons occur too early and half-moons too late.*

(3) *Annual equation: a perturbation in the moon's annual equation of motion due to changes in the sun's attraction, caused in turn by variations in the earth-sun distance throughout the year.*

(4) *Retrogression of the moon's nodes: a precessional motion of the moon's orbital plane due to solar attraction. It results in the backward motion of the nodes along the ecliptic [similar in effect to the precession of the equinoxes]. A complete sidereal revolution of the ascending node takes place in about* 18.61 *years.*

(5) *The inclination of the moon's orbit* varies periodically between $4°59'$ and $5°18'$.

(6) *The progression of the line of aspides* causes the whole orbit of the moon to rotate and turn once upon itself every 8.85 *years.*

The nodical *month*[225] *(27.21222 days) is somewhat variable due to the attraction of the sun, earth's equatorial bulge and the planets. It is defined as the time interval between successive passages of the moon through a given*

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225 Also called '*draconitic*' because of the ancient belief that a dragon was supposed to swallow the sun at a total solar eclipse.

node and is a very important period in eclipse theory. There are also: sidereal *month (relative to the stars; 27.32166 days on the average),* tropical *month (relative to vernal equinox: 27.32156 days),* anomalistic *month (relative to perigee; 27.55460 days) and* synodic *month (relative to sun; 29.53059 days).*

Along with understanding the moon's phases, the early Greeks also realized that eclipses *are simply shadow effects; the moon is eclipsed when it moves within the shadow of the earth, a situation that is possible only during the full moon phase. A lunar eclipse takes place only if a full moon phase occurs when the moon lies in the plane of the ecliptic. During the total lunar eclipse, the circular shadow of the earth can be seen moving across the disk of the full moon. When totally eclipsed, the moon will still be visible as a coppery disk, because the earth's atmosphere bends and transmits some long-wavelength light (red) into its shadow*[226]*. A total eclipse of the moon can last up to 4 hours and is visible to* anyone *on the side of the earth facing the moon.*

Lunar eclipses are one of the oldest known celestial phenomena, reported in historical sources as early as 2283 BCE (the eclipse associated with the Mesopotamian town of Ur). A Chinese eclipse is again mentioned in 1136 BCE. From the beginning of the 8^{th} *century BCE, the number of eclipses observed in Mesopotamia and in the Mediterranean region has been continually growing, with later additions from the rest of Europe.*

Chaldeans found empirically that one lunar eclipse was generally followed by another at an interval of 223 synodic months, or 18 years + 11 days + 8 hours (the saros*). The explanation of this fact is as follows: The moon, which was in opposition at the time of any one eclipse, is necessarily in opposition again after a whole number of lunations. This however is not sufficient for an eclipse to take place; it is also necessary for the moon to be back in the ecliptic, that is to say at one of the nodes of its orbit. The returns of the moon to its nodes are governed by the draconic revolution of 27.2122 days, whereas the return of a state of opposition is regulated by the synodic revolution (or*

[226] The apparent paradox of the simultaneous visibility above the horizon of the setting sun *and* the eclipsed rising moon (or vice versa) was already noticed in ancient times. Since the sun, the earth, and the moon are in a straight line during the eclipse, at least approximately, and since furthermore the effect of parallax is to *lower* the moon by nearly one degree for an observer who sees it near the horizon, this effect appears inexplicable at first. But on the one hand atmospheric *refraction* lifts up the apparent sun and moon by over half a degree each, which compensates the effect of parallax, and on the other hand the center of the moon may be situated above the center of the earth's shadow at the time of observation. The simultaneous visibility is a fleeting effect, however, and never lasts more than a few minutes.

lunar month) of 29.5306 *days. Since the saros is a period after which eclipses recur, it must represent the least whole number of both synodic and draconic revolutions. It does indeed turn out that* 223 *lunations have the same total length as* 242 *draconic revolutions within an accuracy of* 51 *minutes.*

But this is not yet the entire story! Owing to its high orbital eccentricity, the moon is capable of being over 6 *degrees fast or slow when it is half-way between its perigee and apogee. This would be more than enough to make the eclipse impossible. But through a very remarkable accident it happens that* 223 *synodic months make up nearly* 239 *anomalistic revolutions (to within* 5 *hours), so that the moon returns essentially to the same point of its orbit relative to perigee*[227]*. Since the saros is* 8 *hours longer than an integral number of days (6585), the eclipsed moon will not be seen at zenith* 18 *years* 11 *days later from the same region of the earth but from a region about* 120° *further west. But after* 3 *saroses, an eclipse is visible in approximately the same longitude as the first one (There is also a slight shift in latitude).*

The Hebrew prophets were also apparently able to foretell the occurrences of eclipses, and in view of the ignorance and credulity of the masses, threatened them with coming disaster – such as darkening of the sun, moon, or stars – as a punishment for their sins[228] *[e.g., Isaiah (***13***, 10–11); Amos (***8***, 9)].*

Hipparchos found that the moon moved most rapidly near a certain point in its orbit and most slowly near the opposite point. He could reconcile this motion with a virtual moon moving uniformly in a circle in which the earth was displaced from the center by $\frac{1}{20}^{th}$ *of the radius of the orbit. (Virtual eccentricity* $e = 0.05$*). He also discovered the* 9*-year period of the motion of the line of aspides (ca 130 BCE).*

Hipparchos then set forth to determine the other elements of the moon's orbit in order to be able to predict its motion at all times. To this end he used

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227 $223 \times 29.53059 = 6585.32157^d = 29^d 12^h 44^m 2.8^s$ (synodic) $\times 223$
$239 \times 27.55460 = 6585.55494^d = 27^d 13^h 18^m 35^s$ (anomalistic) $\times 239$
$242 \times 27.21222 = 6585.35724^d = 27^d 5^h 5^m 40^s$ (draconic, nodical) $\times 242$
$19 \times 346.620 = 6585.4^d$ (*eclipse years*).
Note that because of the *regression of the nodes*, the time between successive passages is less than one year. It is known as an *eclipse year*, equal to 346.6 mean solar days; 19 eclipse years contain 18 years and $11\frac{1}{3}$ days. During about 1200 years, there are 29 lunar eclipses and 41 solar eclipses: 10 of the latter are total.

228 The saros method does not, however, enable one to calculate the character and details of an eclipse, or even the place on the earth's surface where such an eclipse will be visible. For that one needs strictly accurate mathematical methods, using spherical trigonometry, and the laws of planetary motions.

lunar eclipses. Each eclipse gave a moment at which the longitude of the moon was different by $180°$ *from that of the sun, which he could readily calculate. Assuming the* mean *motion of the moon to be known and the perigee to be fixed, 3 eclipses observed at different points of the orbit would render 3 longitudes, that could be used to determine 3 other unknowns, namely: the mean longitude at a given epoch, the eccentricity and the position of the perigee.*

Thus, by taking three eclipses separated at short intervals, all observations could be reduced to the same epoch[229] *and knowing the mean motion and perigee, the three unknown elements could be eliminated.*

A second triplet of eclipses at as remote an epoch as possible, was then used to redetermine the three elements in order to calculate the annual variation.

*Besides the contribution of lunar eclipses to astronomy, they also played an important part in chronology. Eclipses were often reported in early historical sources as "*epitheton ornans et constans*" of many great events. According to the superstitions of the ancients, lunar eclipses were considered as evil omens, due no doubt to the bloody color of the totally eclipsed moon. In this way some historical events, especially battles, were directly influenced by the appearance of a lunar eclipse.*

Some historical lunar eclipses are:

Oct. 02, 731 BCE *A partial eclipse visible in Babylon (Ptolemy). Time intervals were measured with the aid of* clepsydra *(water clocks); the only useful* timed *observations are of eclipses occurring close to sunrise or sunset, where the effect of drift of clepsydra is minimal.*

July 16, 523 BCE *In the* 7^{th} *year of the reign of Cambyses, one hour before midnight in Babylon, the moon was eclipsed from the north over half of its diameter (***Ptolemy***).*

Aug. 27, 413 BCE *This eclipse retarded the retreat of the Athenian army under Nicias from Sicily and caused its defeat by the Syracusians (****Plutarch****).*

Aug. 19, 366 BCE *This eclipse, explained by the prophet Miltas of Dion's army, decided its departure for Sicily in order to overthrow the local tyrant Dionisios (****Plutarch****).*

Sept. 20, 331 BCE[230] *This eclipse happened eleven days before the battle of Arbela in which Alexander triumphed over Darius (****Plutarch****).*

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229 Same vernal equinox position, such that no correction for precession and nutation are necessary.

230 This may well allude to the lunar eclipse referred to by the prophet **Joel** (*3*, 3–4): "*The sun shall be turned into darkness and the moon into blood*". Indeed, the moon often glows a deep red color during totality on account of sunlight

Sept. 01, 218 BCE *Gaulish mercenary troops were greatly alarmed by this eclipse so that Attalus, king of Pergamos, had to dismiss them (**Polybios**).*

Apr. 30, 174 BCE *A partial eclipse visible in Alexandria (**Ptolemy**). At this period, times were measured to no better than the nearest third of an hour.*

Sept. 02, 172 BCE *On the eve of the battle of Pydna the eclipse predicted by the Roman tribune, C. Sulphicius Gallus, took place (**Livy**).*

Feb. 22, 72 CE *An example of a 'horizontal' eclipse: the rising sun and setting eclipsed moon were simultaneously visible (**Pliny**).*

Sept. 14, 927 CE *Eclipse timed with 'clock stars' at Baghdad.*

Mar. 01, 1504 CE ***Columbus**, knowing in advance that this eclipse should happen, gained the reputation of a prophet among the Indians who, in consequence, supplied provisions to the Spanish expedition.*

159–137 BCE **Carneades of Cyrene** (214–129 BCE). Greek *Skeptic* philosopher of the Middle Academy. One of the earliest to develop the doctrine of *logical probabilism* , which held that certainty is unattainable and that *probability* is the only guide to belief and action. He claimed that the truth of an idea can only be probable, not certain; one discovers probable truth by means of critical analysis, synthesis and comparison.

Carneades was the founder and director of the Third or New Platonic Academy in Athens. Little is known of his life. In 155 BCE, together with **Diogenes the Stoic** and **Critolaos the Peripatetic**, he was sent on an embassy to Rome. On this occasion he delivered two speeches on successive days, one in favor of justice, the other against it. His powerful reasoning excited speculations, and the elder Cato insisted that Carneades and his companions be sent back to Athens. Thus was Greek philosophy introduced in Rome.

ca 150 BCE **Seleucus of Babylon**. Astronomer. Born in Mesopotamia and lived in Seleuceia on the Tigris. There he obtained some knowledge of Greek astronomy. His enduring achievements are:

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being scattered in the earth's atmosphere. [The same description was given to a total lunar eclipse visible in Germany on Nov. 9, 1128 CE.] The closest total *solar* eclipse answering the above biblical narrative may have occurred in Judea on *July 04, 336* BCE at midday with a duration of totality of ca 3 minutes.

- The only supporter in antiquity of Aristarchos' *heliocentric hypothesis*. Went beyond Aristarchos in declaring the heliocentric hypothesis to be true while Aristarchos merely treated it as an hypothesis.

- Tried to account for the *oceanic tide* by the resistance opposed to the moon by the diurnal rotation of the earth's atmosphere. His conclusions are erroneous, but they showed the independence and originality of his mind.

150–120 BCE **Hypsicles of Alexandria** (ca 180–120 BCE). Mathematician. Wrote a treatise on regular polyhedra. He is essentially the author of Book 14 of Euclid's *The Elements* which deals with inscribing regular solids in a sphere. In this work Hypsicles proves some results due to Apollonios.

Hypsicles also wrote *On the Ascension of Stars* in which he was first to divide the Zodiac into 360 degrees.

146 BCE *Greece made a Roman province.* The battle of *Zama* (202 BCE) established Rome as the strongest military power in the Mediterranean[231], and almost immediately the eastern states began to court Roman assistance in their numerous wars. Within two years the Roman became embroiled in the complex affairs of the Hellenistic East, and during the course of the 2^{nd} and 1^{st} centuries they gradually gained control over the entire Mediterranean world.

As the power of Egypt declined (ca 200 BCE), the Greek states around the Aegean asked Roman protection against Macedon and the Seleucid Empire. The Roman took firm action and freed Greece from foreign domination. Rome expected Greece to accept its dictation in foreign affairs. The Greeks, however, wanted little to do with their barbarian "liberators" once the threat to their independence had been removed. When Rome continued to intervene in the affairs of the *Achaean League*, the Greek revolted.

Goaded into fury by what they considered Greek ingratitude, the Romans forgot their love for Greek culture long enough to conduct a savage retaliatory campaign in the Aegean area – the first of many – in 146 BCE. By the time the Roman armies had slashed their way up the Greek peninsula, Macedon had been recognized as a Roman province (148 BCE); the ancient city of Corinth had been leveled; other Greek cities, including Athens, had been plundered; and thousands of young Greeks had been transported to Italy as slaves.

140 BCE Alchemy said to have begun in China.

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231 Before the Punic Wars the rulers of the Hellenistic states regarded Rome as a remote western city, hardly worthy of notice.

136 BCE, Apr. 15 Total eclipse observed in Babylonia after sunrise; Venus, Mercury, Jupiter, Mars and stars above the horizon were seen. A remarkable accurate description is contained in an astronomical diary (there is no comparable account until the 18^{th} century) – testimony to the observational skill which the Babylonian astronomers had achieved.

100 BCE Chinese began to use negative numbers. They also discovered that a magnet orients itself toward the North Pole, but did not use magnets for navigation at sea until the 10^{th} century. Their physicians formed accurate theory of the circulation of blood.

100 BCE **Andronicos of Cyrrhestes (Cyrohus)**. Greek astronomer. Architect of *Tower of Winds*, known in the Middle Ages as *Lantern of Demosthenes*; a tower in Athens bearing a weather vane, eight sundials and a water clock.

ca 100 BCE **Philo of Byzantium**. Mechanical inventor. Author of a sort of encyclopedia of applied mechanics. Invented many war engines, pneumatic machines and the so-called *Cardan's suspension* of gimbals[232].

100–70 BCE **Zenon of Sidon** (ca 150–70 BCE, Greece). Epicurean mathematician and mathematical philosopher. First considered the possibility of non-Euclidean geometry. Also discussed the principle of induction. Made deep criticism of Euclid. For example he claimed that Euclid's first proposition assumes that two straight lines *can* intersect in at most one point but Euclid does not have this as an axiom. He attacked Euclid's proof of the equality of right angles on the ground that it presupposes the existence of a right angle.

Zenon was born in Sidon, now Saida in Lebanon. In 79–78 BCE, **Cicero** (then 27 years old) was obliged by bad health to travel and he attended in Athens the lectures of Zenon.

100–70 BCE **Poseidonios of Apamea** (135–51 BCE). Greek Stoic philosopher, astronomer, geographer and encyclopedist. Teacher of Cicero. Born in Apamea (Syria), studied in Athens, settled in Rhodes and died in Rome. Estimated the circumference of the earth and its distance to the sun to a much better accuracy than Hipparchos and Ptolemy but was still far from

232 A little treatise *On the Seven Wonders of the World* is attributed to Philo, although some scholars believe that it belongs to the 6^{th} century CE.

the true values[233]. During his extensive travels he collected a large amount of geographical information, observed earthquakes and volcanoes, and recorded the elevation of a new volcanic islet among the Lipari Islands. He was *first to explain the tides by the joint action of the sun and the moon*, and to call attention to spring and neap tides.

90–50 BCE **Asclepiades[234] of Bithynia** (124–ca 35 BCE, Rome). Greek Physician. The most eminent of the earlier Greek physicians at Rome, a

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[233] The Stoic astronomer **Cleomedes** (ca 150–200 CE) in his extant book *Theory of Revolutions of the Heavenly Bodies* gives a striking description of the method used by Poseidonios to calculate the earth's circumference. The method consists of three steps:

• Choosing two stations that lie on the same *meridian* with known distance between them: Rhodes and Alexandria are separated by 5000 stadia (1 stadion = 160 meters). Peseidonios relied on mariners' estimates of a straight course across the Mediterranean.

• On a spherical earth, the angle subtended at the center by the meridian between the stations is equal to the angle between the respective horizons (tangents) at the stations, or alternatively, by the difference between the *zenith-distances* (normals) at the stations. This angle is also equal to the difference of *latitude* between the two stations.

• A certain distant star sends parallel rays of light toward earth: the difference between the zenith-distances of the star when measured at its *meridian transit* from two stations is the number of degrees in the arc of the earth's circumference between the two stations. Since the star *Canopus* (the brightest star after Sirius in the sky) grazed the horizon (zenith distance = 90°) at Rhodes, the difference in latitude between Alexandria and Rhodes was {90° minus zenith distance at Alexandria}. The figure that Poseidonios obtained for the circumference of the earth was 250,000 stadia, within a few percent of the actual figure of 40,000 km. **Strabo** (63 BCE–24 CE) quoted a figure of 180,000 stadia. This last figure was adopted by Ptolemy, and this and other errors of Ptolemy were the basis of **Columbus**' belief that India was near. Had he known the true distance, possibly he never would have sailed. . . .

[234] Named after **Asclepios**, a Greek mortal physician, later to be elevated to the rank of god of medicine according to Homer's Illiad. He was the son of Apollo, educated by the centaur Chiron, and killed by lightening sent by Zeus, for bringing the dead back to life. A cult of Asclepios as a hero originated in the Thessalian town of Tricca. When it extended to Epidauros, Asclepios was honored as a god in the classical period. From there the cult spread to many other places (to Athens in 420 BCE); the sanctuary on Cos became very famous, and was the first of many temples of Asclepios. About 280 BCE, the cult was introduced in Rome, as a result of an epidemic.

friend of Cicero. He was born at Prusa in Bithynia (Asia Minor), traveled much when young, and seems at first to have settled at Rome as a rhetorician. In that profession he did not succeed, but he acquired great reputation as a physician. Opposed the Hippocrates (fl. 430–380 BCE) theory of disease and taught that disease is caused by a disturbance in the particles that make up the body. His remedies were baths, diet and exercise. Credited with being the first to distinguish between acute and chronic diseases. Pioneered human treatment of mental disturbances. He recommended the use of wine, and in every way strove to render himself as agreeable as possible to his patients. His pupils were very numerous, and the school formed by them was called the *Methodical*.

His system was his own, though founded upon the *Epicurean* philosophical creed; on the practical side it conformed closely to the *Stoic* rule of life, thus adapting itself to the leanings of Roman in the later times of the republic.

The *Romans* cannot be said to have at any time originated or possessed an independent school of medicine. They had from early times a very complicated system of superstitious medicine, or religion, related to disease and the cure of disease, borrowed from the Etruscans. Though the saying of Pliny that the Roman people got on for 600 years without doctors was doubtless an exaggeration, it must be accepted for the broad truth which it contains. When the medical profession appears, it is as an importation from Greece.

87 BCE Babylonian clay tablets (in the British Museum) documented observation of *comet Halley* apparition. The observation agrees with calculated perihelion passage-time in the month of August of that year. The subsequent apparition of Oct. 12 BCE was also reported. Later observations reported the apparitions of: 451, 684, 760, 837, 912, 989, 1066, 1145, 1222 CE.

81–46 BCE **Marcus Tullius Cicero** (106–43 BCE, Rome). Roman political eclectic-stoic philosopher, historian, orator and statesman whose writings had considerable influence on the formation of subsequent societies. He is most noted for his eclectic exposition of general scientific knowledge and philosophy, by which he aimed to arouse an appreciation of Greek culture in the minds of Romans. Key works[235]: *On the Republic* (54–51 BCE); *Stoic Paradoxes* (46 BCE). One of the great Latin stylists. Through his orations, letters and books influenced medieval and post-medieval cultural heritage and affected the thinkings of **Petrarch**, **Erasmus**, **Copernicus**, **Voltaire** and **John Adams**.

Cicero was born in Arpinum of a well-to-do-family. He studied law, oratory, Greek literature and philosophy in Rome and in Greece. He won his

[235] Cicero, *Selected Papers*, Penguin Books, 1976, 272 pp.

first fame and riches as a defense lawyer. His chief role as prosecutor was in representing the people of Sicily against Gaius Verres, robber-governor of the island.

Gained the office of praetor (66 BCE) and consul (63 BCE). As consul, he crushed the conspiracy of **Cataline**[236] against the Republic; his great speech at this time was his "First Oration Against Cataline" ["*Quo usque tandem abutere, Cataline, patientia nostra?*"]. He was banished from Rome during 58–57 BCE by Caesar, which caused him to write essays on philosophy and political theory[237]. As leader of the Senate in 44 BCE, he launched a great attack on Mark Antony for which he payed with his life a year later.

Cicero ranks among the greatest of ancient writers. He was responsible for developing a style in Latin prose that has become the basis of literary expression in the languages of Europe.

[236] **Lucius Sergius Catalina** (108 – 62 BCE)

[237] **Thadaeus Zielinsky**, Russian philologist, asserted that of all Julius Caesar's achievements none was as important as the fact that Caesar, by compelling Cicero to retire to the country, forced the latter to state his philosophy in writing.

Worldview III: Cicero

* * *

"I only wish I could discover the truth as easily as I can expose falsehood".

* * *

"We must conceive of this whole universe as one commonwealth of which both gods and men are members".

* * *

"Philosophy is the art of life".

* * *

"That long time to come when I shall not exist has more effect on me than this short present time, which nevertheless seems endless".

* * *

"Frivolity is inborn, conceit acquired by education".

* * *

"Fame lives in great things, but dignity lives in humility".

* * *

"While I breath I hope (Dum spiro spero)".

* * *

"By teaching we learn (Docendo discimus)".

* * *

"We are not born just for our own sake".

* * *

"Any man is liable to err, only a fool persists in error".

* * *

"To be ignorant of what occurred before you were born is to remain always a child. For what is the worth of human life, unless it is woven into the life of our ancestors by the records of history?"

* * *

"The whole passion ordinarily termed love *is of such exceeding triviality that I see nothing that I think comparable with it".*

* * *

"No one is so old as to think he cannot live one more year".

* * *

"Peace is achieved by victory, not by compromise".

* * *

"Nature abhors vacua".

* * *

"Rightly defined philosophy is simply the love of wisdom".

* * *

"It is not by muscle, speed, or physical dexterity that great things are achieved, but by reflection, force of character and judgment".

* * *

"If you pursue good with labor, the labor passes away but the good remains; if you pursue evil with pleasure, the pleasure passes away and the evil remains".

* * *

"The greater the difficulty, the greater the glory".

* * *

"If you have a garden and a library, you have everything you need".

* * *

"The more laws, the less justice".

* * *

"A home without books is a body without soul".

* * *

"We should not be so taken up in the search for truth, as to neglect the needful duties of active life; for it is only action that gives a true value and commendation to virtue".

* * *

"Justice consists in doing no injury to men; decency – in giving them no offense".

* * *

"Knowledge which is divorced from justice, may be called cunning rather than wisdom".

* * *

"Laws are silent in time of war".

* * *

"No one can give you better advice than yourself".

* * *

"No one was ever great without some portion of divine inspiration".

* * *

"The wise are instructed by reason*; ordinary minds by* experience*; the stupid by* necessity *and the brutes by* instinct*.".*

* * *

"The enemy is within the gates: it is with our own luxury, our own folly, our own criminality that we have to contend".

* * *

"To each his own".

* * *

"Never go to excess, but let moderation be your guide".

* * *

"Nothing is too hard for him who loves".

* * *

"Ask not what your country can do for you, but rather what you can do for your country".

* * *

"Cannot people realize how large an income is thrift?"

* * *

"What is more delightful than leisure devoted to literature?
That literature I mean which gives us the knowledge of the infinite greatness of nature, and, in this actual world of ours, of the sky, the land, the sea".

* * *

"It might be pardonable to refuse to defend some men, but to defend them negligently is nothing but crime".

* * *

70–55 BCE **Titus Lucretius Carus** (Lucretius, 98–55 BCE, Rome). Roman Epicurean philosopher, poet, and scientist. Author of the philosophical poem *De rerum Natura*[238] (On the Nature of Things), in which he gave an account of the total Greek knowledge of his time, together with a few prophetic views (such as a vague anticipation of the theory of natural selection). Lucretius' chief purpose was to vindicate the rights of reason against superstition.

Nothing certain is known about the life of this natural philosopher. His sole poem was published posthumously by Cicero. Virgil studied it, Ovid admired it, and many authors of didactic poems since have tried to imitate it. No one, however has equaled his passionate sincerity, his fearless logic or his breadth of vision. He was not a popular poet and was not read during the Middle Ages; his present-day fame dates from the Renaissance. The astonishing fact is that, without a true scientific method, Lucretius at least hinted at many important physical discoveries of modern times – his physical theory was superior to any which existed up to the 17^{th} century.

For example, he discusses the spontaneous unpredictable instability through which laminar flow turns into turbulent flow due to a slight perturbation.

Lucretius was an ardent exponent of the concept of plurality of worlds. In his poem he wrote: "*Nature is not unique to the visible world; we must have faith that in other regions of space there exist other earths, inhabited by other people*".

60–21 BCE **Siculus Diodoros**. Greek historian. Born at Agyrium in Sicily; lived in the times of Julius Caesar and Augustus. He traveled in Egypt between 60–57 BCE and spent several years in Rome. His writing consisted of forty books of which only fifteen are still extant. His books contain important information on various astronomical phenomena, such as comets and eclipses.

The Egyptian Calendar merited his comment (ca 50 BCE): "*They have made special arrangement concerning the months and the years. For they do not reckon days by the moon, but by the sun, putting 30 days in the months, and adding 5 days and a quarter to those of the twelve months, and thus they fill out the yearly circle*".

52 BCE *The Battle of Alesia.* The decisive battle of Caesar against a confederacy of Gallic tribes led by **Vercingetorix**, which changed the course of Roman history and Western civilization. While besieging Vercingetorix

[238] Lucretius (Titus Lucretius Carus), *On the Nature of the Universe*, Translated by R.E. Latham, Penguin Books: New York, 1951, 275 pp.

in Alesia (Alise St. Reine, near Dijon), a Roman army, 60,000 strong, was itself surrounded. Outnumbered 5:1, Caesar managed to turn a desperate struggle into a complete victory, chiefly due to his own courage, cunning and leadership, and the endurance of his men. Caesar's victory and the subsequent colonialization of Gaul as a Roman province, carried Roman influence beyond the circum-Mediterranean sphere, into the heart of Europe. A new Gallico-Roman civilization evolved, carrying the heritage of Rome long after the empire fell to the Germanic tribes.

ca 40 BCE *Wooden ball-bearings* for 4-wheeled wagon, found at Dejbjerg, Jutland.

The Julian Calendar (46 BCE)

The year is either astronomical or civil. The solar astronomical year is the period in time in which the earth performs a revolution in its orbit about the sun, or passes from any point of the ecliptic to the same point again; it consists of $365^d : 5^h : 48^m : 46^s$ *in terms of mean solar time*[239].

The civil year is that which is employed in chronology, and varied throughout history among different nations, both in respect to the season at which it commences and in its subdivisions. As far as the sun's apparent motion alone is concerned, the regulation of the year and its subdivision into days and months may be effected without much trouble. The difficulty, however, shows up when one seeks to reconcile solar and lunar periods, or to make the subdivisions of the year depend on the moon, and at the same time to preserve the correspondence between the whole year and the seasons. In the arrangement of the civil year, two objects are therefore sought: first, the equable distribution of the days among 12 *months; and secondly, the preservation of the beginning of the year at the same distance from the solstices or equinoxes.*

[239] What is generally meant by the word 'year', particularly so far as our calendar is concerned, is correctly termed *tropical year*. It is defined as the time elapsed between two successive crossings by the sun of the celestial equator at the *vernal equinox*. The tropical year has a length of $365^d 5^h 48^m 46^s$, or 365.242 mean solar days, where the *mean solar day* is the time of a complete turn of the earth w.r.t. *sun* (corrected for all known irregularities). Because one calendar year is exactly 365 days there is an excess of almost 6^h, or one quarter of a day, per tropical year. By adding a day (Feb. 29) each leap year, this excess is largely corrected. The exact duration of *the apparent solar day* (i.e. time elapsing between two consecutive meridional transits of the sun) is not constant and changes from day to day. This irregularity of solar day relative to sidereal day is a result of several circumstances, chief among which are: (1) the *obliquity of the ecliptic* (the sun's apparent annual motion is *not* in the same plane as its apparent diurnal rotation relative to the fixed stars), due to the earth's tilt relative to the (ecliptic) plane of its circumsolar orbit; (2) *eccentricity of the earth's orbit* (varying translational velocity of the earth in its orbit in conformity with Kepler's second law). An average value of the apparent solar day for the entire year is known as the *mean solar day*. *The Equation of Time* is the misnomer for the number of minutes which, on any particular day of the year, must be *added* to the apparent (true) solar time to give the mean time. Clearly, the *Equation of Time* will apply to the analog time-difference as monitored by *sundial* (solar time) versus *mechanical clocks* (mean solar time).

Now, as the year consists of 365 days and a fraction, and 365 is a number not divisible by 12, it is impossible that the months can all be of the same length and at the same time include all the days of the year. Also, because of the fractional excess of the length of the year above 365 days (explicitly: 0.242 days), the years cannot all contain the same number of days, if the epoch of their commencement is to remain fixed. But as the day and the civil year must begin at the same instant, the extra hours cannot be included in the year till they have accumulated to a whole day. As soon as this has taken place, an additional day must be given to the year.

At the time of ***Julius Caesar*** *(100–44 BCE), the civil equinox differed from the astronomical by 3 months, so that the winter months were carried back into autumn and the autumnal into summer. Caesar ordered a major reform of the calendar: With the advice and assistance of his Greek astronomer* ***Sosigenes*** *he abolished the lunar year and the intercalary month and regulated the civil year entirely by the sun. He fixed the mean length of the year at* $365\frac{1}{4}$ *days and decreed that every fourth year should have 366 days, the other years each having 365.*

In order to restore the vernal equinox to the 25^{th} *of March (the place it occupied in the time of Numa) he ordered two extraordinary months to be inserted between November and December in the current year, the first to consist of 33 days and the second of 34 days. The intercalary month of 23 days fell into that year so that the ancient year of 355 days received an augmentation of 90 days, having altogether 445 days. By means of this device, the calendar was realigned with the seasons. The Romans called it: "the last year of the confusion". The first Julian year commenced with the* 1^{st} *of January of the* 46^{th} *year before the birth of Christ, and the* 708^{th} *from the foundation of the city.*

Insofar as the distribution of the days through the months Caesar ordered that January, March, May, July (which he named after himself), September and November be given 31 days each and the other months thirty each, excepting February which in common years should have only 29, but every fourth year 30 days. His successor, Augustus Caesar, to gratify his vanity, removed a day from February and added it to his month, August, which now had also 31 days. Then, in order that three months of 31 days might not come together, September and November were reduced to thirty days and one day each given to October and December. Curiously enough, the intercalary day, added to February every fourth year, was inserted in the calendar between the 24^{th} *and the* 25^{th} *day, such that nominally, February had 28 days even on that year. In spite of the reform, the year was still too long by* $11^m 14^s$*, which amounted to a whole day in 128 years. Thus, in the course of a few centuries, the equinox retrograded appreciably towards the beginning of the year: when* *the Julian calendar was introduced, the equinox fell on March* 25^{th}*, but by* 1582*, it had retrograded to the* 11^{th}*.*

A lunar year is defined as 12 *new moons, and is about* 11 *days shorter than a tropical year. Because the earth's orbit is not quite circular, the apparent angular velocity of the sun is not strictly uniform. Also because the sun's apparent motion in the sky is not along the celestial equator, the component of its velocity parallel to the equator is variable. Therefore, for the purpose of time-reckoning, a 'mean sun' is defined which moves at the rate equal to the average of that of the true sun. The difference between mean solar time and the apparent solar time (as given by any form of sundial) is called 'the equation' of time. It may be either positive or negative.*

The interval between successive transits of the same star across the meridian is the sidereal day, which is about 4 *minutes shorter than a mean solar day. A sidereal year is the time elapsed between successive occupations of exactly the same orbital point with reference to the fixed stars. It is longer by* 20^m23^s *(mean solar) than the tropical year because of the precession of the equinoxes.*

37–15 BCE **Marcus Vipsanius Agrippa** (63–12 BCE, Rome). Soldier, statesman, engineer and builder. Accomplished great public works (aqueducts, sewers). One of the most eminent builders of the Roman world. As a builder he ranks with Rameses II and Nebuchadnezzar II.

Born into an obscure Roman family, he studied at Apollonia, (a Greek city on the Adriatic coast, opposite the heel of Italy). With him studied Gaius Octavius, the future Augustus. They became lifelong friends.

Agrippa later became praetor and won victories in Gaul and Germany. As consul in 37 BCE, he commanded Octavianus' fleet against Pompeius in Sicily waters. Later he commanded the whole Octavianist fleet at the decisive battle of Actium. In these naval campaigns he used his own inventions of two devices which gave him military advantage: One was a collapsible tower, which could be quickly raised from the deck when a ship neared the enemy. The other was a grapnel that could be shot from a catapult to catch another ship and pull it close for boarding.

Between these wars, he build a new water system for Rome, including two new aqueducts, 130 water-distributing stations, 300 large cisterns, and 500 fountains. He even took a boat ride through the Cloaca Maxima – the great sewer – to direct its renovation. He built the first public bath in Rome, another bridge over the Tiber, a series of temples and porticoes, and the Pantheon. The rest of Agrippa's life was spent on military and diplomatic missions. Amidst these activities he built roads in Gaul, Spain and Syria.

The dependence of Augustus on Agrippa was so great that in order to bind him closer, he persuaded Agrippa to divorce his first wife and marry Augustus' niece Marcella, then to divorce Marcella and wed his daughter, the promiscuous Julia. In addition, Augustus made his stepson Tiberius marry Vipsania, Agrippa's oldest daughter[240].

32 BCE–8 CE **Hillel the Elder** (80 BCE–8 CE, Israel). Tannaic scholar, moralist and logician. Founder of the scientific *Mishna*. President of the Sanhedrin. First to formulate clearly the *Seven Middot* (rules) of inductive reasoning for properly deriving new concepts from old through the use of logic (*Tosefta*, Sanhedrin **7**, 11; *Avot d'Rabbi Nathan* **37**, 10). He argued that if a deduction could be shown to stem logically from a divine proposition, then the deduction had to be as divine as the source. He thus devised the intellectual apparatus for an orderly evolution of divine principles.

Modern scholars have shown that Hillel's syllogisms went beyond those of his Greek masters, approximating the methods used in modern logic today[241].

Hillel came from Babylonia in search of a higher education (40 BCE) and lived at Jerusalem in the time of King Herod. Though hard pressed by poverty, he applied himself to study in the academies of Shemaaiah and Avtalion[242].

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[240] After Agrippa's death, Tiberius' mother Livia prevailed upon him to divorce Vipsania and marry the amorous Julia. As Tiberius loved Vipsania, the experience soured him for life, and he became a morose and miserly emperor. (To most upper-class Romans, marriage was more a matter of business than it is with us. They traded wives back and forth as liberally as movie stars.)
Agrippa left several other children, most of whom came to violent ends. A son of one of these children became the emperor Caligula, while one of Caligula's sisters was the dreadful Agrippina, mother of Nero.

[241] For instance, one of his rules, known as *Binyan Av*, is almost identical to that of **John Stuart Mill**'s (1843) "*Method of Agreement*". Hillel's rules were used by rabbis to discover new laws of Scripture in much the same way that Mill's *inductive reasoning* was used by scientists 18 centuries later to obtain new corollaries from a given natural law.
The seven rules seem to have been first laid down as abstract rules by the teachers of Hillel though they were not immediately recognized by all as valid and binding. Hillel collected them as current in his day and amplified them.

[242] Having no financial means, he is said to have climbed the roof of an academy to eavesdrop on a class. One day the roof caved in, and Hillel fell into the

Thoroughly familiar with Greek literature, thought, and science, he taught his rabbinic pupils to be the keepers of a viable Judaism.

Hillel filled his leading position as head of the Sanhedrin for forty years. His descendants, with few exceptions, remained at the head of Judaism in Israel until the beginning of the 5^{th} century. Hillel's ancestry was tracked back to King David. He is especially noted for the fact that he gave a definite form to the Jewish traditional learning, as it has been developed and made into the ruling and conserving factor of Judaism in the latter days of the Second Temple, and particularly in the centuries following the destruction of the Temple.

Hillel lived in the memory of posterity chiefly as the great teacher who enjoined and practiced the virtues of charity, humility, and true piety. His proverbial sayings strongly affected the spirit both of his contemporaries and of his succeeding generations. In his *Maxims* (*Avot* **1**, 12) he recommends the love of peace[243] and the love of mankind beyond all else.

His charity towards men is given its finest expression in the answer which he made to a proselyte who asked to be taught the commandments of the Torah in the shortest possible form: "*What is unpleasant to thyself that do not do to thy neighbor; – this is the whole Law, all else is but its exposition*".

This allusion to the scriptural injunction to love one's neighbor (*Lev* **19**, 18) as the fundamental law of religious morals, became in a certain sense a commonplace of Pharisaic scholasticism. For the Pharisee who accepts the answer of Jesus regarding that fundamental doctrine which ranks the love of one's neighbor as the highest duty after the love of God (*Mark* **12**, 33), does so because as a disciple of Hillel, the idea is familiar to him. Paul also (*Gal* **5**, 14) learned this in the school of Rabban Gamliel.

9 CE *The Varian Disaster*: Roman army under Varus defeated in Germany by Arminius. Three Roman Legions (out of 28 overall) were annihilated in the Teutoburg Forest. It led to the abandonment of Germany east of the Rhine and thus saved Teutonic Civilization from absorption by Rome. It is considered as Rome's greatest defeat.

This defeat put an end to Augustus' plans for the conquest of Germany to the Elbe and established the Rhine as the future border between Latin and German territory. Augustus discontinued his conquest because of financial

classroom, thus becoming, presumably, history's first drop-in. Impressed with such a thirst for knowledge, the academy granted him a scholarship.

243 Hillel, *unlike* the great Rabbi Akiva, was a totally apolitical figure, which explained how he could survive under the rule of the terrible Herod.

difficulties involved in replacing the lost legions and levying enough additional forces to subdue Germany permanently.

31–65 CE **Lucius Annaeus Seneca** (4 BCE–65 CE). A Roman statesman, author and an *eclectic-stoic philosopher*.

Seneca was born at Cordova, Spain, into a distinguished Roman family. He became prominent in political and literary life in Rome. Later he became tutor and advisor to emperor Nero. Nero accused him of plotting his death and forced him to commit suicide.

The Stoic philosophy, which started with *Zeno of Citium* (ca 335–265 BCE), spanned a period of some 500 years. One of the principal issues which remained a central interest in this philosophy is the problem of determinism and free will: nature is strictly ruled by law which emanates from a supreme authority that impregnates it. The universe ends in a pristine fire and evolves all over again in an eternal repetitious cycle.

In modern times, Stoa was revived through the philosophical writings of **Spinoza**, who was strongly influenced by this tradition. Among the surviving works of Seneca there is an essay on *Naturalium Quaestionum*. We read there a statement that could have been written today:

"*The day will come when diligent research over long periods will bring to light the mysteries of nature which now lie hidden. A single lifetime, even though entirely devoted to the sky, would not be enough for the investigation of so vast a subject... And so this knowledge will be unfolded only through long successive ages. The day will yet come when our descendants will be amazed that we did not know things that are so plain to them... Many discoveries are reserved for ages still to come, when memory of us will have been perished. Our universe is a sorry little affair unless it has in it something for every age to investigate... Nature does not reveal her mysteries once and for all*". [**Book** *7*].

The above essay is a collection of physical, astronomical, geographical, geological, and meteorological questions explained from the atomistic point of view. His account of the earthquake of Feb. 5, 63 CE is the earliest detailed report of its kind. Seneca was one of the first hydrologists: he noted that because of what is now called the hydrologic cycle, the constant flow of rivers into the sea does not cause the ocean to overflow, nor does the addition of fresh water dilute the saltiness of the sea. Water is evaporated from the ocean, falls as rain, and collects in rivers to return to the sea[244].

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[244] This was already known to the author of the book of *Ecclesiastes* (ca 330 BCE), for it is written: "*All the rivers run into the sea; yet the sea is not full*".

Seneca noted the magnification of objects seen through water-filed transparent vessels (and his friend, the Emperor Nero may have been the first to use a monocle, employing an *emerald lens* to view events in the coliseum.

Seneca was the first to express a belief in the progress of knowledge (not the progress of humanity!); this idea of progress is unique in ancient literature.

Worldview IV: Seneca

* * *

"Philosophy is the love of wisdom and the endeavor to attain it".

* * *

"What a despicable thing is man, unless he rises above the human condition!"

* * *

"Any deviation by nature from the existing state of the universe is enough for the destruction of mankind".

* * *

"Success comes to the common man, and even to commonplace ability; but to triumph over the calamities and terrors of mortal life is the part of a great man only... Toil summons the best men".

* * *

"Nothing is so deceptive as human life, nothing is so treacherous. Heaven knows! not one of us would have accept it as a gift, were it not given to us without our knowledge".

* * *

"There is nothing after death, and death itself is nothing... greedy time and chaos engulf us altogether".

* * *

"Go on through the lofty spaces of high heaven and bear witness, where thou ridest, that there are no gods*".*

* * *

"No great genius *has ever existed without a touch of madness".*

* * *

"A great fortune is a great slavery".

* * *

"It takes the whole life *to learn how to live, and what will perhaps make you wonder more – it takes the whole of life to learn how to die".*

* * *

"It is not the man who has little, but the man who craves more, that is poor".

* * *

"Philosophy... molds and constructs the soul, guides our conduct, shows us what we should do and what we should leave undone; it sits at the helm and direct our course as we waver amid uncertainties. Without it, no one can live fearlessly or in peace of mind".

* * *

*"*Liberty *cannot be gained for nothing. If you set a high value on liberty, you must set a low value on everything else".*

* * *

"Men do not care how nobly they live, but only how long, although it is within the reach of every man to live nobly, but within no man's power to live long".

* * *

"You cannot escape necessities, but you can overcome them".

* * *

"Leisure without study is death".

* * *

"I respect no study, and deem no study good, which results in money-making".

* * *

"When savants have appeared, sages have become rare".

* * *

31 BCE, Sep 2[245] *Battle of Actium*: a decisive great naval battle off the coast of Greece between Rome and an Eastern-Hellenistic coalition for the dominance of the Mediterranean world and culture.

The fleet of **Octavian** (under command of Agrippa) defeated the combined fleets of **Marcus Antonius** and **Cleopatra**; The fleets met outside the Gulf of Arta, each over 200 ships strong. Antony's heavy battleships endeavored to close and crush the enemy with their artillery; Octavian's light and mobile craft made skillful use of skirmishing tactics. During the engagement, Cleopatra suddenly withdrew her squadron and Antony slipped away behind her, but his fleet was set on fire and thus annihilated. Anthony committed suicide and Cleopatra followed suite (Aug 29, 30 BCE) and Egypt thereafter became a Roman Province. Octavian was granted supreme power by the Senate and the title of *Augustus* ('Exalted').

Cleopatra (69–30 BCE) was the last member of the Ptolemaic Dynasty to rule Egypt. She was one of the most talented and ambitious woman rulers in history. Set herself to restore the power of the Ptolemies and become an Hellenistic monarch with Roman aid. To this end she seduced the leading Roman lords Julius Caesar and Mark Anthony and when they failed to realize her dream, planned to use Octavian, who had enough sense to refuse to yield to her charms. Imperious will, masculine solidness, relentless ambition and luxurious profligacy made this tragic beautiful Macedonian woman a unique historical figure.

28 BCE–1638 CE A continuous record of sunspot activity is kept by Chinese astronomers. First recorded observation of this data made in 165 BCE.

ca 25 BCE **Marcus Vitruvius Pollio** (Vitruvius). Roman architect and engineer. In his writings sound is described as a vibratory motion of the air[246]

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[245] On the very same day there occurred a major earthquake in Israel (ca 32 °N, 35.5 °E) which caused great destruction and numerous casualties in Judea, Qumran, Massada and Herod's winter palace in Jericho. It is mentioned in *Josephus* (AJ **15**, 5), *Mark* (**13**, 8) and *Mathew* (**24**, 2–7; **27**, 51). Among the building destroyed by the tremor was the most ancient *synagogue* in the world, built during 75–50 BCE by the Hasmonean queen Shlomzion or one of her sons, near Herod's winter palace.

[246] The possibility that sound exhibits analogous behavior to water waves was emphasized earlier by the Greek philosopher **Chrysippos of Soli** (280–207 BCE), with antecedents dating back to **Pythagoras** (ca 550 BCE), who speculated that air motion is generated by a vibrating body. The wave interpretation is also consistent with **Aristotle**'s (384–322 BCE) statement to the effect that air

and some study is made of *architectural acoustics*. Very little is known about him, save that he had worked for the state as an artillery engineer.

25 BCE–23 CE **Strabo** (ca 63 BCE–24 CE). Greek geographer and historian. Became famous for his 17 volume *Geography*, which described all parts of the known world of his time. These volumes are the best known source of geographical information about the Mediterranean countries at the beginning of the Christian Era. It summarized the works of the Greek mathematical geographers to obtain the total size of the earth, estimated the inhabited fraction, and gave a description of the climatic zones of the globe. He also wrote a lengthy history that is now lost.

Strabo was born in Amasia in Pontos (a Hellenized city, and the royal residence of the kings of Pontos). Some of his ancestors were Hellenic, others of Asiatic origin, but Strabo himself was by language and education thoroughly Greek. He studied in Rome and Alexandria and traveled in Arabia, southern Europe and northern Africa. During 25–24 BCE he accompanied the prefect of Egypt, Aelius Gallus, on his expedition to Upper Egypt. In Alexandria he remained a long time, amassing material, and studying astronomy and mathematics in its famous library.

Because he lived in an active part of the globe, Strabo believed that the crust of the earth was in a constant state of flux. He pointed out that earthquakes and volcanic eruptions make the land move vertically and cause the oceans to invade the land. The rivers, by flowing over the land, erode it and transport soil to the sea. Because of the eroded material is almost always deposited close to the shore, the oceans do not fill up so rapidly as one might expect. Strabo taught that the landscape is sculpted by the wind as well as by running water. Applying his geographic knowledge to problems in the social sciences, he pointed out that the ocean plays an important role in the development of civilization.

Strabo appears to be the first who conceived a complete geographical treatise as comprising of four divisions: mathematical, physical, political and historical geography. He endeavored, however imperfectly, to keep all these objects in view.

ca 20 BCE Chinese documents serve as the earliest known *sunspot records* made by the naked eye[247].

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motion is generated by a source "*thrusting forward in like manner the adjoining air, so that the sound travels unaltered in quality as far as the disturbance of the air manages to reach*".

247 From 1605 to 1611, **Johannes Fabricius**, **Thomas Harriot**, **Christoph Scheiner** and **Galileo Galilei**, among others, began *telescopic* studies of

Comets[248] in the Greco-Roman World – The Advent of Astrophysics

The shape of comets and the manner of their appearance has in all generations been a source of baffled wonderment: the sun *and* moon *are far brighter,* aurorae *more impressive, and* eclipses *more startling. Yet it is* comets*, with their modest radiance and infrequent visits, that have commanded more concern.*

In the year 467 BCE, a meteorite fell at Aegospotami, in Thrace (on the European side of the Dardanelles), and a comet was seen in the same year. The meteorite fell during day time: it was brown in color and the size of a wagon load. This event made a great impression on Greek thought, and probably influenced ***Anaxagoras*** *(ca 500–428 BCE). His attempts to give a scientific account of eclipses, meteors, comets, rainbows and the sun resulted in a theory that the heavenly bodies were masses of stone torn from the earth and ignited by rapid rotation.*

Aristotle *(384–322 BCE), in his Meteorologia, believed comets would form when the sun, or planets, warmed the earth, causing the evaporation of dry, warm exhalations from the earth itself.*

In his Naturalium Quaestionum (63 CE), a work devoted primarily to meteorology and astronomy, ***Seneca*** *(4 BCE–65 CE) included newer cometary ideas of the Greek from the 4^{th} to the 1^{st} century BCE. He presented the opinion of* ***Epigenes*** *(a follower of Aristotle, fl. 4^{th} century BCE) and* ***Apollonios of Myndos*** *(a contemporary of Seneca), as well as his own ideas.*

Epigenes' ideas were but those of Aristotle slightly modified. The theory of Apollonios of Myndos is, however, close to the modern view:

"... A comet is a separate star like the sun and the moon. In shape it is not compressed into a ball, but loose and elongated; it has no definite course, but

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sunspots. These records, as the German astronomer **Samuel Heinrich Schwabe** announced in 1843, displayed a prominent periodicity of roughly 10 years in the number of observed sunspot groups. By the 20-th century **George Ellery Hale** of the Mount Wilson Observatory in California found those dark surface irregularities to be the seat of intense magnetic fields, with the strength of several thousand gauss.

[248] For further reading, see: Yeomans, D.K., *Comets*, Wiley: New York, 1991, 485 pp.

it passes across the upper regions of the cosmos and becomes visible only on reaching the lower part of its course... The comets are many and various, and different in size and color... Some of them are blood-soaked and terrifying, ominous of future bloodshed...".

Seneca himself concluded that comets were not sudden fires but were among nature's permanent creations, and while their orbits generally differed, the two comets seen during his age had circular orbits much like the planets. Seneca suggested that comets moved in closed orbits, traveling in a uniform manner and disappearing only when they passed beyond the planets. While his views did little, in a quantitative way, to advance understanding of the phenomenon, his rejection of the prevailing Aristotelian theory inspired eventual rethinking of the nature of comets which, unfortunately, had to wait some 1500 *years. However, Seneca's belief is astrology and divination planted, with Aristotle, the seeds of superstition. This seed was to flourish under the guidance of Pliny and Ptolemy.*

***Pliny the Elder** (23–79 CE) did not acknowledge the contemporary views of Seneca, and gave more credence to comets as portents , viewing comets as terrifying apparitions by noting the disasters that followed a few cometary returns.*

***Joshua ben Hannania** (35–117 CE)*, a Hebrew astronomer and *Tanna* was probably the first man in recorded history to establish the *periodicity* of a comet. *[Babylonian Talmud,Horayoth 10^a.]*

***Ptolemy of Alexandria** (ca 94–172 CE), last of the great astronomers of antiquity, in his tetrabiblos regarded comets as mysterious signs that provoke discord among men and give rise to wars and other evils. Moreover, he expanded somewhat on the astrological implications that Pliny had outlined. Because of his great reputation, the more rational views of Seneca were quickly submerged, and in the* 15 *centuries that followed, Ptolemy's guidelines were used repeatedly to correlate cometary apparitions and terrestrial disasters. Even during the Renaissance, the bulk of cometary literature was superstitious in nature.*

The final break with Aristotelian tradition came on Mar. 19, 1681, *with the telescopic observation of **Isaac Newton** of the comet of* 1680*. Although the physical nature of comets was still largely unknown, Newton had essentially solved the problem of their dynamical behavior.*

0–50 CE The first treatises on the "*Divine Art*" appeared in Alexandria, Egypt, containing the earliest chemistry.

20–50 CE **Philo of Alexandria** (Judaeus, 20 BCE–50 CE). Jewish philosopher. Spent his whole life in Alexandria, where he was born. A contemporary of Jesus and Paul, and deeply versed in both Hellenistic and Judaic cultures.

Though we know little of Philo's own life, his numerous extant writings give the fullest information as to his views of the universe and his scientific aims, and so enable us to estimate his position and importance in the history of thought.

Although Philo borrowed much from Greek philosophy (Plato, the Stoics), he was highly modern and by far ahead of his time. For in his commentary on Genesis he refers to the history of creation in these words: "*It would be a great naivety to believe that the universe was created in time. The right thing to say is that the existence of time is conditioned by the existence of the universe since the movement of stars determines the nature of time*". Thus, 1900 years before Einstein, Philo claimed that time is not absolute, but is inherent property of the material universe and had no meaning prior to the creation of matter.

26–29 CE **Jesus (Joshua) of Nazareth** (7 BCE–30 CE, Israel). A central figure in the Christian religion. Jewish wandering preacher and social reformer. His *image* had a decisive influence on the development and history of Western culture, and through it upon the entire of humanity.

He taught that man must relinquish his earthly riches and treat his fellow men with love and mercy, even passively accepting the pains and insults inflicted on him. He proclaimed the imminence of the *Kingdom of God* as a relief to the poor in particular. His followers called themselves *Nazarenes*[249], regarded him as the anointed descendant of King David (or *Messiah*), foreseen as a redeemer in biblical prophecy.

It is clear from Jesus' own assertions that he did not aim to found a new religion (*Matt* **5**, 17). He observed scrupulously all Jewish feasts, taught Jewish doctrines and ethics. In fact, his whole system of ethics, sometimes even down to the very expressions he used, were derived from current Jewish Pharisee teachings[250].

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[249] Originally a Jewish sect, living near the Jordan river, whose members objected to the rituals in the Temple.

[250] The widely held belief that Christianity introduced a new conception of morality into the world is, on the basis of historic record, an overstated one; the notion

Jesus (a diminutive of **Joshua**) was born[251] in Nazareth, Israel. He met no strong opposition from the authorities until he and his twelve closest associates traveled to Jerusalem, probably in 30 CE, to preach during the Passover. Like many rural prophets, Jesus was enraged by the worldliness of the capital and denounced the priests of the Temple as hypocrites. Jerusalem was crowded with pious Jews who had come from all parts of Judea for the Passover. Both Jewish and Roman authorities were prepared to take action against any rural preacher who might ignite a popular demonstration and necessitate another Roman purge.

Upon Jesus' visit to the Temple, a riot broke out which caused his arrest by Roman soldiers, his consequent trial before the Roman procurator, **Pontius Pilate**, and his crucifixion[252].

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that the Jews believed in a God of Vengence and that Christianity first projected a God of Love is from an historical point of view quite untenable. Jewish writings *before* and *during* the time of Jesus prove this conclusively. There are innumerable references in Jewish writings, centuries before Jesus, which condemn hatred, cruelty, envy, and, conversely, which glorify truth, love of man, gentleness, generosity of spirit and forgiveness. The ethics of Jesus were *totally* Jewish, and they were derived from the Mosaic commandment: "*Love the neighbor as thyself*" (*Leviticus* **19**, 18). To cite only a few examples: The Book of Proverbs, written many centuries before the Christian movement arose, wished to establish the moral truth that: "*The reasonable man is noble, he glories in pardoning injury*" (*Prov*). **Philo of Alexandria**, a contemporary of Jesus, exhorted: "*If you ask pardon for your sins do you also forgive those who have trespassed against you. For remission is granted for remission*".

[251] The birth of Jesus is linked to *star of Bethlehem*. This event is believed to be a conjunction of Saturn and Jupiter *in Pisces*, occurring once in 900 years. Thus, Jesus was probably born on 07 September, 7 BCE. The Last Supper happened on April 06, 30 CE and the crucifixion on Friday, April 07, 30 CE.
The conjunction hypotheses goes back to **Johannes Kepler**, who observed another such conjunction from his observatory in Hradcyn, near Prague, on 17 December 1603. Kepler, in turn, drew his theory from commentary on the book of **Daniel** by **Don Isaac Abrabanel** (1497).

[252] It was tragically inevitable that Jesus should have suffered the same fate as several other "messiah" before him who were called by their followers "*King of the Jews*" [e.g., Judah of Galilee, 6 BCE; Theudas in 44 BCE and Benjamin, "the Egyptian", in 60 BCE]; the preaching of Jesus and the devoted following that clustered around him, although pacifist in nature, were also considered inimical to the security of the Roman state in Judea. To Roman ears, not used to this mystic conception of a savior, it sounded very much like high treason against the Emperor in Rome. Whatever the title "King of the Jews", by which

The teaching of Jesus might have been forgotten with the crucifixion had not a group of his followers soon appeared in Jerusalem proclaiming that Jesus had been the Messiah, that he had come back from the dead, that he had appeared to many of his disciples and commissioned them to carry on his teachings, and after 40 days he had returned to God[253]. Although these first *Christians* considered themselves practicing Jews, they were generally regarded as heretics by the other Jews of that city.

The community of *Jewish Christians* virtually disappeared after the Roman suppression of the Bar-Kochba revolt (135 CE). But by this time Christianity had already been established among the Gentiles, primarily by **Paul**.

40–60 CE **Saul of Tarsos** (Paul, Paulus, ca 4 BCE–64 CE). Transubstantiated Jesus from a Jewish preacher into a Christian redeemer and succeeded in marketing this image to the Pagans[254]. Paul stood for the direct admission of Pagans into Christianity without a prior introduction into Judaism. He taught that Jesus' death atoned for human sin, that faith in Jesus' divinity redeems without observance of the Torah (5 books of Moses), and the abrogation of the laws and rituals of the Torah[255]. It was hatred of the law (Torah) and his inability to respect it[256] that led Saul to become Paul and preach divine redemption from original sin by exploiting the death of Jesus. Paul was a Jew by birth, a Roman by citizenship, and a product of Greek

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his followers called him, may have signified to Jesus, to the Romans it probably appeared as a challenge to imperial authority.

253 Messianic movements usually emerged throughout Jewish history after major national disasters or during long periods of oppression and persecution. It is thus very likely that after the destruction of the second Temple by the Romans (70 CE), the disciples of Jesus, in their great despair of the redemption of Israel, modified the character of his teachings, altered the story of his life and death and his ideas in order to facilitate the marketing of his ideology in the Greco-Roman Paganic world.

254 see:

- Maccoby, H. *The Mythmaker — Paul and the Invention of Christianity*, Barnes and Noble, 1986, 237 pp.

255 After the death of Jesus, the leadership of the Christian sect gravitated toward two men. One was **James**, the brother of Jesus, who tried to keep Christianity within the fold of Judaism but failed. The other was Paul. James admitted Pagans to the new Christian sect only after their conversion to Judaism.

256 **Nietzsche** (1888) denoted Christianity as being *immoral* from the outset because its object is to free the individual from the burden of Jewish tradition.

culture. Whereas Jesus was a messiah-intoxicated Jew who died a Jew, Paul was a Christ-intoxicated Jew who died a Christian. Whereas Jesus, like the Jews, had taught that man could earn God's grace through repentance and righteousness, Paul the Christian taught that salvation could only be obtained through the dead Christ.

Paul rejected the Jewish establishment and law, basing his objection upon the premise that man is *devoid of free will* and is too weak to follow percepts and commandments; man can only be saved by identifying himself with the sacrifice of the crucified Jesus. Thus man is redeemed from the original sin and becomes immortal too. By rejecting free will Paul liberated his followers from the yoke of observance of the Biblical commandments[257].

For two decades (50–70 CE), the teachings of James and Paul competed as the true creed of Jesus. The destruction of Jerusalem (70 CE) selected the victor. Both Sadducee Judaism and Jamesian Christianity perished in that holocaust. And just as Pharisee Judaism rose out of the rubble, so did Pauline Christianity. Both were universalist religions in outlook, tailored for the Jewish exile, the latter for Pagans at large.

During the Hellenistic era, the Jewish way of life made a great impression on many Greeks and Romans. They liked the non-sexualized symbols of Judaism and respected the dignity of the Jewish God. They admired the Jews for not indulging in the Bacchanalian revelry so common in those days among the Pagans, and they envied the devotion of the Jewish people to spiritual, family, and scholastic ideas rather than materialistic goals. In the two-century span 100 BCE–100 CE, thousands of Sabbath candles flickered in Grecian and Roman homes – so many in fact that **Seneca** (4 BCE–65 CE) noted this phenomena by remarking that Jewish customs were everywhere so prevalent that the Romans were in danger of being swallowed up by them[258].

[257] The Catholic Church could not accept this idea since free will is at the base of any establishment based on law and order. Thus the Church was forced to return to the premise that man has free will and therefore must accept the laws of the Church. Free will is one of the fundamental principles of the Bible; a man devoid of free will cannot be moral since he cannot differentiate between good and evil. In fact, any human culture must, by its own nature, assume that man has a limited free will or else any interaction between people is impossible.

[258] This and the heavy causalities of the Roman Legions in Israel during the War of Independence (66–70 CE), raised a wave of hatred toward Jews and their culture in the intellectual circles of Rome. It is indeed regrettable that the writings of a great moralist like **Seneca** and a great historian like **Tacitus** are fouled up by an irrational venomous hatred directed toward Jewish values, customs and religion.

Jewish virtue and ideology did indeed threaten to undermine the Pagan nations and might have done so if it had not been for the Christian sect, which began to proselytize more actively than did the Jews themselves.

In the first century CE, over 10 percent of the population of the Roman Empire was Jewish – 7 million out of 70 million. Of these 7 million professing the Jewish faith, only an estimated 4 million were Jewish by virtue of centuries of descent; the rest were converted Pagans or of converted Pagan descent. The rate of conversion would have been even greater but for two factors: the rigorous dietary laws, and the necessity for circumcision. In Paul's time, the early Christians sect dropped these two requirements, and the Pagans flocked to the Christian religion, whose entrance specifications were less demanding than the Jewish.

Paul was born in the Greek-speaking city of *Tarsos*, in Asia Minor (Turkey), to a Jewish family, wealthy enough to purchase Roman citizenship. He received traditional Hellenistic education, for he later wrote in polished Greek and was well acquainted with Greek philosophy.

In the city of Tarsos he led the life of an ordinary artisan, being a tent-maker by trade. Three things characterized him and affected his life: a body subject to some serious ailment, a gift for oratory and public disputation, and a devotion to Judaism so great as to make him impatient with its slow progress in *converting the Pagans*. He must have spent years in wondering why the ideals of Judaism, obviously so noble, failed to be accepted by the Pagans and sometimes were not lived up to by the Jews. He finally made up his mind to go to Jerusalem and study Judaism at its source (ca 35 CE) under Rabban Gamliel the Elder. Soon after his arrival, he was exposed to ideas of the followers of Jesus. He saw in Jesus a great attraction to the Pagan world. He decided to speak of him, not only as a messiah[259] who had come to redeem

[259] The period in which Jesus and Paul lived was unique in all Jewish religious experience; it marked the peak development of *Messianism* among Jews. The idea of *saviors* was already well established in the 5^{th} century BCE (*Nehemiah* **8**, 27). Because it was a time of acute suffering, much of the Jewish apocalyptic literature of that period was permeated with the expectation of the imminent coming of the Messiah. The prophets **Ezekiel** and **Enoch** had spoken awesomely of the *Last Judgment* and of the *Resurrection*, both of which had now become associated in the popular mind with the era of the Messiah. It was the tradition among Jews that the Messiah would come when the affliction of the Jewish people became unendurable.

Suffering, insecurity, and the helplessness of the Jews under the iron heel of Rome made them abandon faith in their own strength and efforts. They yearned increasingly for a supernatural redeemer who would have the invincible power to bring the enemies of Israel to justice and who would usher in the era of the

the world (as the original followers of Jesus thought), but also as an ideal, a divine personality whose example could influence each human being, whether Jew or Gentile, and lead him to perfection.

Paul then traveled through many parts of the Diaspora, especially Asia Minor and Greece, offering Judaism devoid of Jewish ceremonial and ritual together with a new non-historical image of Jesus. Being rejected by the Jews, he turned his attention almost exclusively to the Pagans. Yet to the very end he never denied his own Jewishness. On his last appearance in Jerusalem (ca 60 CE) he visited the Temple and observed all the other Jewish customs. When, however, it became known that he had violated Jewish traditions while on his travels, he was arrested and, later condemned by the Romans and crucified at Rome during Nero's persecution of the Christians in 64 CE.

What happened to Paul's teachings? – Most of the people who listened to him remembered only the connection between believing in Jesus and meriting God's mercy and ultimate resurrection. They skipped the middle part – the need to live godly lives. Thus, faith in Jesus came to be the only and entire basis of the religion adopted by the Pagan Christians, while the Judeo-Christians, who lived in Israel, continued to observe Jewish law.

But it was the *Pagan Christian* attitude which won in the end. Many Pagans who had admired Judaism either openly or secretly now had a chance to adopt a form of Judaism which appealed to them. It is not strange that Christianity spread most easily and rapidly in those cities in which Jews had lived for a long time, so that people were already acquainted with their life and religious ideas. Nevertheless, for some centuries still, it was not at all certain whether Judaism or Christianity would make the greater number of converts.

Kingdom of God on earth. The prophets, beginning with **Isaiah**, had articulated this desire and dream, which they expected would bring everlasting peace and happiness not only on the Jewish people but for all mankind. They had a vision of an ideal ruler who once more would raise up the fallen kingdom of David whose departed glories so haunted and tormented the Jews of later days.

The Silent Century (0–100 CE)

It has always been an unfailing source of astonishment to the historical investigator of Christian beginnings that there is not one single word from the pen of any Pagan writer of the first century of our era which can in any fashion substantiate the story recounted by the Gospel writers of the New Testament. The very existence of Jesus seems unknown! In other words – historical research has made it obvious that there is no way to get at the *historical events which have produced the Biblical picture of Jesus with more than a degree of probability.*

For lack of historians? There were many historians just then and some of them the most illustrious of all time – ***Tacitus****,* ***Plutarch****,* ***Livy****, the two* **Plinys**, **Philo** *and* ***Josephus****, among others; and besides these many man of literary note such as* ***Seneca****,* ***Martial****,* ***Juvenal****,* ***Epictetos*** *,* ***Plotinos*** *and* **Porphyrios**, ***Virgil****, Horace and* **Ovid**. *These were all men of great intellect, and deeply interested in the doctrines and morals of their day. Some of them held high office and therefore knew their world (Pliny the Elder was procurator of Spain, Pliny the Younger was governor of Bithynia, Josephus was governor of Galilee, Seneca was the brother of Gellio, Proconsul of Achaia at precisely the time Paul is said to have preached there). In fact, we find nothing like divinity ascribed to Christ before 141 CE.*

There is thus only one way to explain away this 'silent century': Christianity (religion and Church) are creations of the third and fourth centuries. By the third century all the science, philosophy and mythology of Greece had disappeared, mostly in flames. Rome was now the dominant power. The Romans, however, lacked the Hellenic love for learning; they had no use for philosophy. In fact they drove out philosophers. Power was their god and conquest their vocation.

And so, when the Empire declined, they finally fell, having no inner light to guide them nor inner strength to sustain them. While a few intellectuals remained, the masses sunk in abysmal ignorance, poverty and want. They were thus ripe for the mythology of Christ.

In this lies another contributing factor – the economic one. All mass movement are security inspired, being more interested in bread then philosophy, especially masses that have never known philosophy. So, as Roman prosperity vanished, the masses found themselves in desperate straits, ripe for a 'New Deal'. Christianity offered it. It was, then, the coincidence of these factors that furnished the mental soil for Christianity.

ca 43 CE **Pomponius Mela**. Earliest Roman geographer. His *De Situ Orbis* is the earliest description of the ancient world written in Latin. He divides the earth into the climatic zones: North Frigid, North Temperate, Torrid (equatorial), South Temparate, South Frigid.

Pomponius was born in southern Spain in the days of Emperor Claudius. The first edition of Mela was published at Milan in 1471.

ca 50 CE The Greek seaman **Hippalos** used the south-west monsoon to steer a direct course from the Persian Gulf to Western India. He showed that sea routes might be followed, shorter and quicker than the old coastwise sailing. Since the monsoons blow from the south-west in summer and from north-east in winter, they could be utilized both for outward and homeward voyages.

ca 50 CE *Romans* used *coal* as a fuel in Northern Europe

ca 50 CE **Aulus Cornelius Celsus**. Roman writer. Compiler of an encyclopedia on agriculture, medicine, military science, law, and philosophy of which only the portion on medicine is extant. It was one of the first medical works to be printed (1478).

Celsus, a Roman patrician, appears to have studied medicine as a branch of general knowledge, which he practiced on his friends and dependents, but not as a remunerative profession. His work *De Medicina* was one of a series of treatises intended to embrace all knowledge proper for a man of the world. It was not meant for physicians, but the whole body of medical literature belonging to Hippocratic and Alexandrian times is ably summarized, and the knowledge of the state of medical science up to and during the times of the author is thus conveyed in a way which cannot be obtained from no other source [e.g., it contains first description of sewing an artery and of heart disease and also describes hernia operation].

The influence of Celsus commenced in the 15^{th} century, when his works were first discovered and printed.

60–78 CE **Pedanios Dioscorides of Anazarba** (20–90 CE). A Greek physician and alchemist[260] who wrote the first catalogue of drugs and their

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[260] *Alchemy* – a pseudoscience of obscure origin. Sought a *philosopher's stone*, thought capable of changing base metals into gold, and the *elixir of life* that would preserve youth indefinitely. Alchemy was academically accepted by **Robert Boyle**, **Isaac Newton** and **G.W. Leibniz**.

recipes. Described the processes of *crystallization*, *sublimation* and *distillation* of substances. Known also as Dioscorides of Cilkia (Asia Minor).

Dioscorides was the author of the herbal *De Materia Medica* which remained for 1500 years the authority in botany and pharmacopeia. It describes more than 500 plants and 35 animal products. Ninety of the plants he mentions will still be in use in the 20^{th} century. The Italian physician and botanist **Pietro Andrea Mattioli** published (1544) an Italian version of De Materia Medica.

63 CE *Israel comes under Roman rule* which ends only with the disintegration of the Roman Empire (395 CE). In a series of four revolts [66–70; 114–117; 132-135; 351 CE], the majority of the Jews in Israel perished[261] and the rest were dispersed all over the Empire. With Judea largely depopulated, the centers of Jewish culture and learning were in the Galilee and in Babylonia and later shifted to Jewish communities around the Mediterranean.

66 CE **Josephus Flavius** (Yoseph ben Matitiahu; 37–100 CE, Israel and Rome) described the apparition of comet Halley as a star resembling a sword. This he considered as an omen for the fall of Jerusalem (70 CE).

70–110 CE **Rabban Gamliel II** (ca 30–117 CE, Yavne). Mathematical astronomer and Tanna. A great-great grandson of Hillel.

Gamliel worked out the problem of the *moon's visibility*[262] and is said to have possessed an instrument resembling an astrolabe or a telescope by means

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[261] **Josephus** in *The Jewish Wars* estimated the Roman force besieging Jerusalem in 70 CE at 80,000 soldiers [Jerusalem was defended by no more than 23,000 Jewish soldiers]. In comparison, **Alexander the Great** used 35,000 men to carve out his vast Empire. **Caesar** had fewer than 60,000 legionaries with which to conquer Gaul and to invade Britain. **Hannibal** had no more than 50,000 soldiers when he crossed the Alps to defeat the Romans.

Tacitus estimates 600,000 defenseless Jewish civilians were slain in the aftermath of the siege.

[262] The following episode, related in the Mishna [*Rosh Hashana* **2**, 8; $\mathbf{24}^b$ and $\mathbf{25}^a$], illustrates the great confidence that Gamliel had in his astronomical calculations of the moon's visibility: Once two witnesses reported that they saw the moon in the East *before* sunrise and in the West *after* sunset. When they came to Yavne, (some 30 km south of today's Tel-Aviv) Gamliel accepted their evidence (needed to establish the definite *consecration* (Kidush ha-Hodesh) of the new moon), while the other members of the Calendar Council rejected it. Those who considered the evidence false, claimed that the moon cannot be seen within an interval of 24 hours both *before* sunrise and *after* sunset. They argued that since the moon is *East* of the sun throughout the first half of the month, and West of

of which he could estimate a distance of 2000 cubits (about 1120 meters), as well as measure depths and height.

70–110 CE **Plutarch**[263] (ca 46–127 CE). Greek historian, biographer, philosopher and essayist. Wrote a large number of essays and dialogues on philosophical, scientific and literary subjects. His philosophical standpoint was eclectic and he frequently attacked both Stoics and Epicureans. He is best known for his *Parallel Lives*, biographies of eminent Greeks and Romans, generally composed in pairs, one Greek and one Roman, followed by a comparison between the two.

Plutarch was born at Chaeronea in Boeotia, Greece. He studied philosophy in Athens. In his travels through Greece, Italy, and Egypt he spent much time studying and collecting facts on men he wrote. He combined academic studies with multifarious civic activities. Through friendships formed on an official visit to Rome, he was made procurator of Achaea by Hadrian, and he also held a priesthood at Delphi. Yet much of his time he devoted to his school at Chaeronea and to his literary works.

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it during the second half, the *New Moon* cannot be seen in the morning at the eastern horizon until *after sunrise*, and in the evening at the western horizon until *after sunset*.

However, the *Waning Moon* can only be seen in the morning *before sunrise* and in the evening *before sunset*. If, therefore, witnesses testify to having seen the moon *before* sunrise and *after* sunset, their evidence cannot be true – unless it refers to the waning moon in the morning and the new moon in the evening. If one accepts this argument, one is in for yet a new obstacle; there must be a certain *minimum angular distance* between the sun and the moon for the latter to be visible. This leads to a situation where between morning [when the moon must have been in that minimum angular distance *behind* (or to the west of) the sun] and in the evening [when the moon must have got that minimum angular distance *ahead of* (or to the east of) the sun], the moon must have covered double that angular distance. But at the rates at which the sun and the moon are traveling, **this would be impossible**!

Hence the objection of the majority of the Calendar Council members to the evidence of the witnesses.

Rabban Gamliel, however, who by his *calculations* concluded that the new moon was indeed visible on the evening in question, nevertheless accepted the witnesses' evidence, on the assumption that what they believed to have seen in the morning was a piece of cloud that looked like a very thin crescent of the waning moon.

263 For further reading, see: Grant, M., *The Ancient Historians*, Barnes and Noble Books: New York, 1970, 486 pp.

The ancient solar eclipse of March 20, 71 CE was documented in his writings and therefore bears his name. In his essay on life of the Spartan general **Lysander**, Plutarch mentioned that in a year corresponding to 467 BCE, a meteorite fell at Aegospotami, in Thrace, on the European side of the Dardanelles. Before the stone fell, a vast fiery body was seen in the heavens for 75 days continually (a comet) "*but when it afterwards came down to the ground there was no fire to be seen, only a big stone*".

The sun's corona, observed during total eclipses, was first reported by Plutarch.

ca 75–130 CE The composition of the *Evangelions*[264] and the advent of the *Christian*[265] *Church*.

Within a century of the death of Jesus, Christianity became a force in history through the rise of a new institution. In the words of **Alfred Loisy** (1857–1940): "*What Jesus proclaimed was the Kingdom of God, and what arrived was the Church*". In the name of Jesus, this new Church reversed many of his policies to gain larger membership. To achieve this, it ingested many of the ideas of *Essenism*, which has also perished with the fall of Jerusalem.

Jesus, like the Pharisee rabbis, was a layman who rejected priesthood, founded no Church and sought no institutionalized hierarchy. The Church, however, realizing it needed a devoted hierarchy for future growth, dragged in through the back door the priesthood Jesus has thrown out of the front door. The Church institutionalized its creed and established an elaborate organization of judges and tribunals. In effect, the historical Jesus stands closer to **ben Zakkai**'s Judaism, which also rejected Essenism, than to *Pauline Christianity*, which absorbed it.

76–116 CE **Cornelius Tacitus** (55–120 CE, Rome). Historian. Lived through the reigns of 9 emperors (Nero, Galba, Otho, Vitellius, Vespasian, Titus, Domitian, Nerva and Trajan). The son-in-law of Julius Agricola and a friend of Pliny the younger. His most important books, the *Histories* and the *Annals*, cover periods for which our other sources are scanty.

79 CE, Aug. 24 The eruption of Mount Vesuvius buried the towns of Pompeii, Herculaneum and Stabiae. **Pliny the Elder** (b. 23 CE), admiral of the Roman fleet that anchored offshore during the event, died of gas poisoning from the volcano, and his young nephew, **Pliny the Younger** (61–113 CE) lived to tell us about it.

264 From the Greek *evangelion* = *good news*; the "official" biographies of Jesus; any of the four gospels of the *New Testament*.

265 From the Greek *christos* = "*the anointed*", *messiah*.

79 CE **Pliny the Elder** (Gaius Plinius Secundus) (23–79 CE). Writer, civil servant, soldier and encyclopedist, perhaps the only Roman on record, to die of scientific curiosity. Holds a place of exceptional importance in the history of Western culture and its diffusion.

Born in Novum Comum in Northern Italy, he begun his career in the army, in Germany, and later held a number of procuratorships in Gaul, Africa and Spain (70–75). On his return to Rome he devoted his talents to writing. Seven works are known but only one, the *Natural History*[266], survived. It is an encyclopedic account of the state of science, superstition, art and technology in the first century CE, and contains material from works no longer extant. Hence its unique value for our assessment of early imperial science and technology. Its derivation from over 2000 earlier texts, makes it *the* major source for ancient beliefs about every form of useful knowledge[267] – from agriculture, architecture and astronomy to geography, metallurgy and zoology. Pliny's work was an extraordinary catalog of truths, half-truths, myths, and outright nonsense. He was more encyclopedist than a scientist; his knowledge was almost exclusively derived from the writing of others rather than from personal observations.

With the decline of the ancient world and the loss of the Greek texts on which Pliny had so heavily depended, the *Natural History* became a substitute for a general education. In the European Middle Ages many of the larger monastic libraries possessed copies of the work; these and many abridged versions ensured Pliny's place in European literature. His authority was unchallenged, partly because of a lack of more reliable information and partly because his assertions were not and, in many cases, could not be tested.

The first attack on Pliny's work – **Niccolò Leoniceno**'s tract on the errors of Pliny – was published in Ferrara in 1492. Thereafter, Pliny's influence diminished, as more writers questioned his statements. By the end of the 17th century, the *Natural History* had been rejected by the leading scientists.

Up to that time, however, Pliny's influence, especially on nonscientific writers, was undiminished; he was, for example, almost certainly known to William Shakespeare and John Milton. Although Pliny's work was never again accepted as an authority in science, 19th-century Latin scholars conclusively demonstrated the historical importance of the *Natural History* as one of the greatest literary monuments of classical antiquity. It is still of value to those who wish an honest resumé of 1^{st}-century Rome.

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266 Pliny the Elder (Gaius Plinius Secundus), *Natural History* (A selection), Penguin Books: England, 1991, 400 pp.

267 Pliny coined the word '*albumen*' for egg white.

80–110 CE **Joshua ben Hannania ha-Levi** (35–117 CE, Israel). Astronomer, mathematician and *Tanna*. One of the leading scholars of the 2^{nd} generation of the *Tannaim*. First man in recorded history to establish the *periodicity* of a comet [*Horayoth* $\mathbf{10}^a$].

Joshua was born in Jerusalem to a Levite family and probably served in the second Temple for some years before its destruction (70 CE).

He was a disciple of **Yohanan ben Zakkai** and assisted in the removal of the latter from Jerusalem to Yavne during the Roman siege. He traveled extensively by sea and by land to Rome, Alexandria, Asia Minor and Babylonia for either political, academic and business reasons. He founded his own private academy at Pekiin (upper Galilee) and was also a member of the Sanhedrin at Yavne [*Shabbat* $\mathbf{75}^a$].

85–117 CE **Epictetos of Hierapolis** (ca 55–135 CE). Greek *Stoic* philosopher. Left no writings, his philosophy is known through his *Discourses* and the *Encheiridion* (handbook) of his pupil **Flavius Arrianus** (the historian of Alexander the Great).

Epictetos taught that nothing is ours besides our will. We should not demand that events happen as we want but instead want them to happen as they do. A wise, divine Providence governs all things, so that what seem to be calamities are really parts of a divine scheme that orders everything for the best. Only foolish men are upset by events they cannot control. The body which accompanies us is not, strictly speaking, ours but belongs to things outside us.

Epictetos was a native of Hierapolis (southwest of Pergia, Asia Minor). His name is merely the Greek word for "acquired", his real name being unknown. He was lame and of weakly health. Originally a slave in Rome at the time of Nero, he was freed by his master and taught philosophy in Rome until expelled (89 CE) with other philosophers by Emperor Domitian. He then settled for the rest of his life in Nicopolis (Greece). He never married.

89 CE Emperor Domitian[268] and the Roman Senate expelled from Rome all philosophers, mathematicians and astrologers.

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268 Titus Flavius Domitianus (51–96 CE), Vespasian's second son, inherited the mantle of Empire at the age of 29. He was a man beset by suspicions and fear, and terrorized Rome for 15 years. Domitian ferreted out opponents in the Army and Senate and had them executed. He was killed at last by a member of his own household. After his death, the Senate ordered Domitian's name to be removed from all public places and refused to give him a state burial.

ca 90–120 CE **Hero** (**Heron**) **of Alexandria**. Greek mathematician, physicist and inventor. An important geometer and worker in mechanics.

His geometrical treatises which have survived in Greek are entitled respectively: *Definitiones*, *Geometria*, *Geodesia*, *Stereometrica*, *Mensurae*, *Liber Geoponicus* and *Metrica*. *Metrica* deals with areas of triangles, quadrilaterals, regular polygons of between 3 and 12 sides, surfaces of cones, cylinders, prism, pyramid, sphere etc. Methods for approximating square and cube roots are shown. For the area of a circle of diameter d, he gave the formula $\frac{11}{14}d^2$ $\left(\text{based on } \pi = 3\frac{1}{7}\right)$, in the name of **Archimedes** and **Euclid**[269]. Hero also proved the expression for the area of a triangle in terms of its sides, and determined the distance between Rome and Alexandria by observation of the same lunar eclipse in both places and drawing the analemma for Rome.

Akin to the geometrical works is *On the Dioptra*, a remarkable book on land-surveying. The *Pneumatica* describes siphons, "Hero's fountain", "penny-in-the-slot" machines, a fire-engine, a water-organ and arrangements employing the force of steam. He also wrote 3 volumes on *Mechanics*[270] and a book on *Water clocks.* In his *Catoptris* (on reflecting surfaces) we encounter the first minimum principle in physics. He reasons that when a ray of light is reflected in a mirror, the *path* (not time!) actually taken from the object to the observer's eye is shorter than any other possible path. This proposition was obtained from a generalization of the observed fact that when light travels from one point to another, its path is a straight line, that is, it travels the shortest possible *distance* between these points.

Hero's *Siegecraft* (*Belopoiika*) tells about several designs for catapults. One is the crossbow; the others are conventional darts and stone throwers. In *Dioptra* he describes a surveying system which did not seem to have come into use, being probably too far ahead of its time.

Some of Hero's inventions seem to have been for the benefit of the priesthood of Alexandria, to enable them to awe their worshipers. Of all his inventions, the one most pregnant with future possibilities was his steam engine

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[269] **Euclid** in his *Elements*, or other writings known to us, never gives the computation of the area of the circle, nor of other areas or volumes.

[270] In this book he was mainly concerned with *mechanical advantage* which he achieved via the lever, the compound pulley, the wedge and the gear train. He showed how, with such devices, the force applied and the distance moved vary inversely according to the mechanical advantage. In fact, Hero came close to discovering the modern technical concept of "*work*" (= force × displacement). He also described cranes, methods of raising large building stones by means of tongs and keys and screw presses.

or *aeolipile*. It worked on the *reaction principle* of the rotary lawn sprinkler[271]. He had in it nearly all the elements needed to make a working steam engine of either the reciprocating or the turbine type, yet none of the devices he described under this category was such a steam engine. He simply could not attack a problem that he did not know to exist. But even so, Hero's technological accomplishments are remarkable.

Although Hero's books were not widely read in later centuries, they were never altogether forgotten. The engineers of the Renaissance would later study Hero with lively interest. In the 17^{th} century, when Europeans began to harness the power of steam, they remembered Hero's achievements.

95–136 CE **Akiva ben Yosef** (ca 50–136 CE, Israel). Scholar, philosopher and jurist; perhaps the greatest Jewish savant in the past two millennia. Through his admirable systematization and scientification of the accumulated lore of oral law (which until his time was only a subject of knowledge), he brought it into methodic arrangement, basing it on sound logical foundations and turning it into a "survival-kit" which marked the path of Judaism for next 1800 years. He is the man to whom Judaism owes prominently its activity and its capacity for development.

If the older Halakah (oral ruling) is to be considered as the product of the internal struggle between Phariseeism and Zaducceeism, the Halakah of Akiva must be conceived as the result of the external contest between Judaism on one hand and the Hellenism and Hellenistic Christianity on the other.

Akiva no doubt perceived that the intellectual bond uniting the Jews – far from being allowed to disappear with the destruction of the Jewish state – must be made to draw them closer together than before. He pondered also on the nature of that bond. The Bible could not fill the place alone; for the Christians also regarded it as a divine revelation. Still less could dogma serve the purpose, for dogmas were always repellent to Judaism, whose very essence is development and the susceptibility to development.

His first act in this direction was the final canonization of the Bible (inclusion of the books of *Song of Songs*, *Ester* and *Kohelet* and the rejection of the Apocrypha (the 14 books of the *Septuagint* included in the *Vulgate*). His underlying motive was to disarm Christians, especially Jewish Christians, who drew their "proofs" from the Apocrypha, and at the same time emancipate the Jews of the Diaspora from the domination of the Septuagint, the errors and inaccuracies in which frequently distorted the true meaning of Scripture, and were even used as arguments against the Jews by the Christians. Thus,

[271] Newton's third law was applied 18 centuries before it was formulated!

under Akiva's guidance, his pupil **Aquila** gave the Greek-speaking Jews a faithful translation of the Bible which they could use in their synagogues.

But this was not sufficient to obviate all threatening danger. It was feared that the Jews might still become entangled in the net of Grecian philosophy, and even in that of Gnosticism and must therefore be provided with some counterpoise to the intellectual influence of the non-Jewish world.

To this end, Akiva developed a remarkable method of reconciling the unchangeable legal and ethical code of the Holy Scriptures with the dynamical needs of the ever-developing Judaism.

As a speculative philosopher, Akiva brought new insight into man-God relationship. Next to the transcendental nature of God, he insists on the freedom of will, to which he allows no limitations ["*everything is foreseen; but freedom of will is given to every man... the divine decision is made by the preponderance of the good and bad in man's actions*"][272]. In opposition of the Christian insistence on God's love, Akiva upholds God's retributive justice elevated above all chance or arbitrariness.

Akiva came of humble parentage in the town of Lod, Israel. He began as an illiterate shepherd and learned to read and write together with his son

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[272] Akiva here laid bare the solid rock on which the whole structure of Rabbinic Judaism was founded: Man *apparently* acts according to his free will, but in fact realizes, unknowingly, the divine plan. Nowhere is this idea expounded with greater lucidity then in the words of Joseph to his brothers [*Gen*, **45**, 5–8]:

"*Now therefore be not grieved, not angry with yourselves, that ye sold me hither: for God did send me before you to preserve life. For these two years hath the famine been in the land; and yet there are five years, in which there shall neither be earing nor harvest. And God sent me before you to preserve you a posterity in the earth, and to save your lives by a great deliverance. So now it was not you that sent me hither, but God: and he hath made me a father to Pharaoh, and lord of all his house, and a ruler throughout all the land of Egypt.*"

Akiva's succinct and ambiguous statement attempts *no solution* of the contradiction between divine foreknowledge and human free will, but affirms both and rests in the assurance that He who judges is good. Judgment takes account of what man *has done*. Into these few lines Akiva compressed well nigh the whole Judaic philosophy of religion expressing the fundamental conviction that divine foreknowledge and human freedom are equally well real and true, though human wisdom could not intellectually reconcile them. Akiva spoke for the *practical* needs of the religious and ethical consciousness of man, not for theoretical satisfaction of the inquiring mind.

at an age of 25 or so. He then studied in the local academy for 13 years before becoming a teacher himself. It is known that in 95–96 CE Akiva had already attained great prominence. He had made numerous journeys within and outside the Roman Empire (Spain, Babylonia, Asia Minor, Sicily, Phartia, North Africa, Athens).

In 132, the prohibition of Jewish observances by Hadrian were accompanied by a decree forbidding the study of the Torah. This he saw as a major threat to the very survival of the Jewish nation for which a Jew must risk dying for. Risking capture by the Roman soldiers he continued to study and to teach. Consequently he was arrested and executed (136 CE).

97–103 CE **Sextus Julius Frontinus** (ca 35–103 CE, Rome). Soldier and writer on subjects of military science, land surveying and water supply systems. City praetor (70 CE), Governor of Britain (75–78 CE), Trajan's water commissioner (97 CE). His chief work *De Aquis Urbis Romae* contains a history and description of the water-supply of Rome. He also wrote a theoretical treatise on military science (*De re Militari*), and a treatise on land surveying.

The Roman were, of course, not the first folk to build aqueducts. However, the Roman aqueducts were distinguished from the earlier ones mainly by their size and number. Sections of their arcades are still to be seen around within the former limits of their empire. Most Roman aqueducts took the form of gravity-powered open channels. They sometimes used the *inverted siphon* (i.e., U-shaped pipe higher at the intake than at the outlet) instead of aqueduct bridges to cross deep valleys. Frontinus tells us that water was conveyed to Rome by nine aqueducts from sources in a spur of the Apennines, some 25 km to the east[273].

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[273] The first aqueduct brought pure water into Rome as early as 312 BCE. At the beginning of the Christian era there were six.
Modern estimates of the total volume delivered to Rome runs from 0.3 to 1.2 million cubic meters/day. About half of this enormous supply was required for the *public baths*. The baths of Caracalla, dating from about 200 CE were capable of accepting 1600 bathers at a time. Those of Diocletian, built about 80 years later, had no less than 3000 rooms. The bath, accompanied Roman civilization wherever it penetrated, and certain places became famed for the curative power of their mineral-impregnated waters. The population of Imperial Rome is estimated at one million. As nearly as we can estimate, Babylon, Nineveh, Athens, Syracuse, Carthage, Alexandria, Antioch, Capua and the Republican Rome had all at their height, harbored somewhere from 250,000 to 500,000 people. Probably larger cities were impractical because of the difficulty of bringing food from a distance to feed their populations. Roman roads

The water system consisted of arcades (aqueduct bridges), conduits and tunnels, altogether totaling about 450 km, of which only one-ninth were on arches. They were built mainly of concrete. The pipes that branched off from the main aqueducts were made of lead (poisonous!). Most of what we know of these matters comes from **Frontinus**' book. He bossed a small staff of engineers, surveyors, clerks, and a crew of 700 government slaves, including inspectors, foremen, masons, plumbers and plasterers. Frontinus bitterly complained of the frauds that had taken place under his predecessors; he was shocked to find secret, illegal pipes running to irrigated fields, shops, and even whorehouses.

Aside from the abuse of the water system by grafters and water steelers, the system suffered from natural causes, which made its upkeep a heavy responsibility to a conscientious bureaucrat like Frontinus. The water channels were always cracking and leaking, caused by the settling of the piers of the arcades. Moreover, the Romans did not understand *thermal expansion*. Hence the expansion and contraction of a straight concrete channel several km long, between a hot summer day and a cold winter night, was enough to crack the cement. As the water was heavily charged with mineral salts, the leakage built up thick limestone concretions around the piers.

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made it possible to concentrate more people in one metropolis. Hence, Imperial Rome, and – later, for similar reasons – medieval Constantinople, Baghdad, Anuradhapura (Ceylon), and Hangchow (China) all approached or exceeded the million mark. There remained about 200 liters of water per head for the million inhabitants, the same amount that is used today by a citizen of New York or London. Thus, in 1954, four aqueducts sufficed for the needs of modern Rome. In its cleanliness, sanitation and water supply, Rome was much more akin to 20^{th} century London and New York than to medieval Paris or 18^{th} century Vienna. A 17^{th} century Londoner existed in conditions which would hardly have been tolerated by first century Roman. Public health and sanitation were more advanced in the year 300 CE than they were to be again until the middle of the 19^{th} century. The great drainage system, the *cloaca Maxima*, was built in the 6^{th} century BCE, to drain an area of marsh which later became the site of the Forum Romanum. The Cloaca gradually assumed the function of a modern sewer and its plan was copied elsewhere in Italy and the empire. The modern water closet was not devised until well over a 1000 years after the fall of Rome, but the ruins of Pompeii and Harculaneum (destroyed 79 CE) have revealed an elaborate system of waterworks connected with *flushing closets*. Public lavatories, uncomfortably hard to find in the present-day city, were common place in Rome during the 1^{th} century CE. The best known of these, a palatial building fitted with marble urinals, was erected by Vespasian in about 70 CE.

Nevertheless, Frontinus was very proud of his aqueducts as is evident from his own words: "*With such an array of indispensable structures carrying so many waters, compare, if you will, the idle Pyramids or the useless though famous, works of the Greeks*!"

ca 98 CE **Menelaos of Alexandria**. Greek mathematician. Author of *Sphaerica*, a work extant in Hebrew and Arabic, but not in Greek. In it he proves the theorems on the congruence of spherical triangles, and describes their properties in much the same way as Euclid treats plane triangles. In it are also found the theorem that the sum of the three angles in a spherical triangle exceeds two right angles. Two other celebrated theorems of his deal with plane and spherical triangles[274].

ca 100 CE **Nicomachos of Gerasa** (ca 60–120 CE). Neo-Pythagorean mathematician, astronomer, physicist and philosopher. Wrote *Arithmetike eisagoge* (Introduction to Arithmetic), the first work on the theory of numbers, in which arithmetic was treated quite independently of geometry. This work exerted a powerful influence over the study of arithmetic for more than a millennium.

The principal scene of his activity was the city of Gerasa (today in Jordan). Nicomachos is our main source of information on Pythagorean mathematics. He told in a manner intelligible to every one, and in an entertaining way, about triangular, square, rectangular and polygonal numbers, about prime numbers and geometric progressions, all illustrated by numerous examples, but never accompanied by proofs. He quoted the first four perfect numbers: 6, 28, 496, 8128 and noted that the last figure was either 6 or 8.

ca 100 CE The diseases of pneumonia, diabetes, tetanus and diphtheria first identified or described with accuracy.

ca 100 CE **Mary** (**Maria**) **the Jewess**. Alchemist. Invented chemical apparatus for *distillation* (the *ambix* or *alembic*) and for *sublimation* (the *kerotakis*; a hot-ash bath). A water-bath was later named *bain-marie* in her honor. Her writings combine practical techniques, mystical imagery, and theoretical ideas.

105 CE The Chinese discovered paper.

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274 *Lemma of Menelaos*: "*If the three lines constituting a plane triangle be cut by a straight line, the product of the three segments which have no common extremity is equal to the product of the other three*". There exists a corresponding theorem for spherical triangles.

106 CE Chinese and Roman traders meet in central Asia. Chinese silk and spices are exchanged for gems, precious metals, glassware, pottery and wine.

ca 110 CE **Marinos of Tyre**. Greek geographer and mathematician. Regarded as the founder of *mathematical geography*. Predecessor of Ptolemy, who acknowledged his great obligations to him. His chief merits were that he assigned to each place its proper *latitude* and *longitude*[275], and introduced improvements in the construction of his maps. He also carefully studied the works of his predecessors and the diaries of travelers. His geographical treatise is lost.

117–138 CE **Soranos of Ephesos** (Alexandria and Rome). Greek physician. Practiced medicine in Alexandria and later in Rome during the reigns of Trajan and Hadrian. Founder of obstetrics and gynecology. Influenced medical practice in these fields until the 15^{th} century. His works included *On Midwifery and the Diseases of Women* and *On Acute and Chronic Diseases*. He was the chief representative of the school of physicians known as "*methodists*". It is notable that the *speculum*, an instrument invented in modern times, was used by Soranos[276].

120–150 CE **Claudios Ptolemy** (**Ptolemaeos**) **of Alexandria** (ca 85–165 CE). Astronomer, mathematician, geographer, physicist and chronologist. One of the greatest astronomers and geographers of ancient times. Introduced geocentric cosmology that was not seriously challenged for 1400 years. His observations and theories are preserved in a 13-volume work which he entitled '*Mathematike Syntaxis*' (Mathematical composition). Because of the admiration this work won, it became known as the *Almagest*, a combination Greek-Arabic term meaning *the greatest*.

Ptolemy developed his astronomical system largely from the ideas of **Hipparchos** (150 BCE) and made a critical reappraisal of the planetary records. He collected the works of Hipparchos and his predecessors, added his own observations, evolved a first-class theory and left a masterly exposition that dominated astronomy throughout the Dark Ages. He rejected the idea that the earth moves and placed a motionless earth at the center of the universe. Around it went the moon, sun and planets at various speeds. The stars were brilliant spots of light in a concave dome that arched over everything. Against this stellar background, Ptolemy traced the motions of the planets and worked out the theory of each of them.

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[275] Marinos was, in effect, the first to invent the concept of a *coordinate system* and *coordinates of a point* with respect to it.

[276] In fact, specimens of still earlier date, showing great mechanical perfection, have been found among the ruins of *Pompeii*.

The positions of the sun, moon, and planets, relative to the fixed stars, had been mapped with angles measured to a fraction of a degree. He could therefore elaborate the system of eccentric crystal spheres and epicycles and refine its machinery, so that it carried out past motions accurately and could grind out future predictions with success. He devised a brilliant mathematical machine, with simple rules but complex details, that could 'save the phenomena' with centuries-long accuracy. In this he neglected the crystal spheres as moving agents; he concentrated on the rotating spoke (or radius) that carried the planet around, and he provided sub-spokes and arranged eccentric distances.

The general picture was this: the heavens of the stars is a sphere turning steadily round a fixed axis in 24 hours; the earth must remain at the center of the heavens – otherwise the star pattern would show parallax changes; the earth is a sphere, and it must be at rest for various reasons – e.g., objects thrown into the air would be left behind a moving earth. The sun moves round the earth with the simple epicyclic arrangement of Hipparchos, and the moon obeys a more complicated scheme.

His system was a gorgeously complicated system of *main circles* and *subcircles*, with different *radii*, *speeds*, *tilts*, and different amounts and directions of *eccentricity*[277]. Like a set of mechanical gears the system grounded out accurate predictions of planetary positions for year after year into the future, or back into the past. And like a good set of gears, it was based on simple principles: *circles with constant radii*, rotations with *constant speeds*, *symmetry*, *constant tilts* of circles, with the earth fixed in a constant position[278].

In the *Almagest*, Ptolemy described a detailed scheme for each planet and gave tables from which the motion of each heavenly body could be read off.

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277 For the motion of the planets, Ptolemy found that he could not save the sacred principle of uniform motion even with a simple epicyclic motion; there were residual discrepancies between theory and observation, so he not only moved the earth off-center but also moved the center of uniform rotation to a new point, called the *punctum equans*. This point enabled him to say that there exists, after all, a point in space where an observer could enjoy the illusion that the planet's motion is of uniform speed.

278 Though this now seems as an unreal model, it is still fashionable as a method of analysis. Adding circle on circle in Greek astronomy corresponds to our use of a series of sines (projected circular motions) to analyze complex motions. Physicists today use such *Fourier Analysis* in analyzing time-series such as musical sounds, tides, etc. In principle, planetary motion can be represented to any desired accuracy, provided one is allowed to compare enough circular motions (Fourier components).

The book was copied (by hand), translated from Greek[279] to Latin, to Arabic and back to Latin as high culture moved eastward and then back to Europe. It served for centuries as a guide to astronomers and a handbook for navigators. It also provided basic information for astrologers.

The methodic excellence of the *Almagest* ensured the supremacy of the Ptolemaic system until the 16^{th} century, in spite of abundant criticism which became more acute as observations increased in number and precision.

Clearly, Hipparchos and Ptolemy were backward in two respects: they rejected the heliocentric ideas of **Aristarchos** and the ellipses of **Apollonios**. In retrospect, this can be understood as resulting from a combination of two factors: since heliocentricity did not lead to greater simplicity or precision, they lacked the conviction to shake off the prejudices of their own environment.

Ptolemy (Claudios Ptolemaeos) was born in Ptolemais Hermeiu, a Greek city of the Thebais, upper Egypt, and flourished in Alexandria. His influence upon later times (until the middle of the 16^{th} century) is second only to that of Aristotle. Assuming that the bulk of his facts, methods, and principles was derived from Hipparchos (in most cases Ptolemy admits it), the credit of his masterly exposition of his subjects and his mathematical treatment of them, belongs to Ptolemy himself. It seems difficult to conceive of two men separated by almost full three centuries as close collaborators, yet their fame is inseparable.

Ptolemy pointed out that the earth is round and that gravity is directed toward its center. In his *Syntax* there is a catalog of 1022 stars found in 48 constellations, each with its own celestial latitude and longitude. He discovered the irregularity of the moon's motion (evection).

In a book entitled '*Optics*' he discussed the *refraction of light*: He realized that it provided an explanation of why bodies immersed in water seem to be nearer to the surface than we find them to be when we try to grasp them.

The attempt to find a correction for "atmospheric refraction", led Ptolemy to make the first extensive physical experiments in which numerical measurements were recorded to discover the amount of bending. In his experiments

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[279] The world in which Ptolemy lived was a Roman world, whose intellectual ideas was still predominantly Greek.

By this time, the top culture of the West was Greek, not Latin. Greek was the language of science and philosophy while Latin was the language of law, administration and business. Marcus Aurelius wrote his *Meditations* in Greek. Ptolemy and Galen would not have been able to write in Latin, even if they wished to do so. The remarkable thing about the Roman empire, from an intellectual point of view, was its *bilingualism*; every educated man in the West was supposed to know two languages, Greek as well as Latin.

he studied the passage of light from air to water, from air to glass, glass to water and vice versa. His results were not accurate enough to display the simple geometrical rules of refraction.

Ptolemy's geography includes a theory of *map projection*; places are listed with their *longitudes* and *latitudes*. However, in his map of the world he exaggerated the land mass from Spain to China, and underestimated the size of the ocean. This mistake later encouraged **Columbus** to undertake his famous voyage of discovery in 1492.

As a mathematician, Ptolemy devised new geometrical proofs and theorems. He obtained, using chords of a circle and an inscribed 360-gon, the approximations: $\pi = 3\frac{17}{120} = 3.14166$, $\sqrt{3} = 1.73205$.

His serious treatment of *astrology* helped to spread that superstition[280].

132 CE **Zhang Heng** (78–139, China). Mathematician, astronomer and geographer. Invented the first *seismoscope* (132) to record earthquakes. Heng's device was in the shape of a cylinder with 8 dragon heads around the top, each with a ball in his mouth. Around the bottom were 8 frogs, each directly under a dragon head. When an earthquake occurred, a ball fell out of the dragon's mouth into a frog's mouth, making a noise and giving a rough indication of the direction of the initial impulse.

ca 150 CE **Rabbi Nehemiah** (2^{nd} century, Land of Israel). Hebrew scholar and mathematician. Author of the *Mishnat ha-Middot*, the earliest

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280 Along with the *Almagest*, Ptolemy wrote an astrological treatise by the name of *Tetrabiblos* which is a compilation of Chaldean, Egyptian and Greek folklore and earlier writings. The book remained a standard work until our own day. In that it was even more successful than the *Almagest*, for the simple reason that astronomy being a science was bound to develop and change, while modern astrology being a superstition is essentially the same as the ancient one. The *Almagest* is published anew from time to time for students of the history of science, but has no practical value; on the other hand, new editions of the Tetrabiblos are still issued for the guidance of practicing astrologers.

Many scholars have claimed that the same man could not possibly be the author of the rational *Almagest* and of the *Tetrabiblos*. They forget that astrology was the *scientific religion* of Ptolemy's day. At a time when the old mythology had become untenable, the sidereal religion had gradually taken its place in the minds of men who were loyal to Pagan tradition as well as scientifically minded. Stemming from Greek astronomy and Chaldean astrology, it was a compromise between the popular religion and monotheism; it was a kind of scientific pantheism endorsed by men of science as well as by philosophers, especially the neo-Platonists and Stoics.

Hebrew treatise on mathematics known to us. This book includes his own contributions, as well as those of Greek mathematics. It exerted influence on Persian-Arabic mathematics, especially Al-Khowarizmi. Nehemiah was concerned with the determination of the calendar.

Mishnat ha-Middot became known in modern times through the translation and publication of two manuscripts. The first manuscript, *Cod. Hebr. 36*, in the Münich Library, containing 5 chapters dealing with plane and solid geometry, appeared in a German translation in 1862. The other, *Ms. Hebr. c. 18* of the Oxford Bodleian Library, found more recently, is a fragment consisting of two leaves which contain parts of the 1^{st}, 2^{nd} and 5^{th} chapters already known and, in addition, the beginning of a 6^{th} chapter, hitherto unknown. The fragment, on vellum, contains explanations of the size and the construction of the Tabernacle.

Although Nehemiah had the courage to give the value $3\frac{1}{7}$ for π and offer a different explanation of the Biblical text (**Kings I** *7*, 23), the Talmud restored the canonical value of 3.

He was one of the late pupils of **Rabbi Akiva** (ca 50–134).

The Astrologers

Astrology is the ancient art of divining the fate of men from indications given by the position of the stars (sun, moon and planets). The belief in a connection between the heavenly bodies and the life of man has played an important part in human history. For long ages astronomy and astrology were identified. This combination can be traced back to the earlier phases of Babylonian history (3000 BCE) and directly or indirectly spread through them to other nations. It came to Greece in about 350 BCE and reached Rome before the opening of the Christian era. With the introduction of Greek culture into Egypt, both astronomy and astrology were actively cultivated in the region of the Nile during the Hellenistic and Roman periods. Astrology was further developed by the Arabs from the 7^{th} *to the* 13^{th} *century, and in Europe of the* 14^{th} *and* 15^{th} *century, astrologers exerted dominating influences at court.*

Astrology is based on a theory of divine government of the world. Starting with the indisputable fact that the fertility of the soil is dependent upon the sun shining in the heavens as well as upon the rain that comes from heaven, and taking into consideration the damage done by storms and inundations (to both of which the Euphratean Valley was almost regularly subject), the conclusion was drawn that all the great gods had their seats in the heavens.

In the early age of culture, known as the Nomadic stage (which predated the agricultural stage) the popular moon and sun cults produced a sect of priests (corresponding to the "scientists" of a latter day) who perfected a theory of a complete accord between phenomena observed in the heavens and occurrences on earth. The movements of the sun, moon and five planets were regarded as representing the activity of 7 gods. The system was then extended to include prominent and recognizable fixed stars and constellations. All heavenly phenomena were meticulously remote and correlated with climatic, royal and public events. In this way a mass of traditional interpretations of all kinds of observed phenomena was gathered, and once gathered became a guide to the priests for all times.

Astrology at this stage was centered almost exclusively in the public welfare and the person of the king, i.e., the movements and position of the heavenly bodies point to such occurrences as due of public import and affect the general welfare. The individual interests are not in any way involved, and we must descend many centuries and pass beyond the confines of Babylonia and Assyria before we reach the phase which in medieval and modern astrology is almost exclusively dwelt upon individual horoscope.

In the hands of the Greeks and the Hellenists both astronomy and astrology were carried far beyond the limits attained by the Babylonians, and it is indeed a matter of surprise to observe the harmonious combination of the two fields – a harmony that seems to grow more complete with each age, and that is not broken until we reach the threshold of modern science in the 16^{th} *century. The endeavor to trace the horoscope of the individual from the position of the planets and stars* at the time of birth *represents the contribution of the Greeks to astrology. The system was carried to such a degree of perfection that later ages made but few additions of an essential character.*

The system was taken up almost bodily by the Arab astronomers. It was embodied in the Kabbalistic lore of Jews and Christians, and through these and other channels came to be the substance of the astrology of the Middle Ages. It thus formed a pseudo-science which was placed on equal footing *with astronomy. Moreover, under Greek influences, the scope of astrology was enlarged until it was brought into connection with practically all of the known sciences: botany, chemistry, zoology, mineralogy, anatomy and medicine; colors, metals, stones, plants, drugs and animals were associated with the planets* *and placed under the guidance and protection of one or other of the heavenly bodies. Indeed, the entire realm of the natural sciences was translated into the language of astrology with the single avowed purpose of seeing in all phenomena signs indicative of what the future has in store.*

The fate of the individual, as that feature of the future which had a supreme interest, led to the association of the planets with parts of the body, e.g.: the right ear is associated with Saturn*, the left ear with* Mars*, the right eye (male) with the* sun *and the left eye with the* moon*, the pubis to* Scorpion*, the breasts to* Cancer*, the thighs to* Sagittarius*, etc. Not only was the fate of the individual made to depend upon the planet which happened to be rising at the time of birth or of conception, but also upon its local relationship to a special sign or to certain signs of the Zodiac. With human anatomy thus connected with the planets, with constellations, and with single stars –* medicine *became an integral part of astrology.*

A favorite topic of astrologers of all countries has been the immediate end of the world. As early as 1186 *the earth had escaped one threatened cataclysm of the astrologers. This did not prevent others astrologists from predicting universal deluge in the year* 1524 *– a year as it turned out, distinguished for drought.*

Astrology did not stop after the acceptance of the Copernican system; the postulates of astrology are independent of whether the sun or the earth is the center of our planetary system. ***Kepler*** *himself drew horoscopes.* ***Tycho Brahe****,* ***Gassendi*** *and* ***Huygens*** *subscribed to that superstition.*

George Sarton*, one of the leading historians of science in the* 20^{th} *century and an erudite deeply versed in the European cultural heritage had this to say on astrology and astrologers (1959):*

"America is leading the world in astronomy, and we have every right to be proud of that, but if we be honest, we cannot accept praise for our astronomers without accepting full blame for our astrologers. There are more astrologers than astronomers in America and some of them, at least, earn considerably more than the latter; the astrological publications are far more popular than the astronomical; almost every newspaper has an astrological column which has to be paid for and would not be published at all if a large number of people did not want it.

Astrology was perhaps excusable in the social and spiritual disarray of Hellenistic and Roman days; it is unforgivable today... We would be indulgent to Ptolemy, who had innocently accepted the prejudices endemic in his age and could not foresee their evil consequences, but the modern diffusion of astrological superstitions deserves no mercy".

The Heritage of Ancient Astronomy[281] (3000 BCE–150 CE)

A practical acquaintance with the elements of astronomy is indispensable to the conduct of human life. Hence it was most widely diffused among uncivilized peoples, whose existence depended upon immediate and unvarying submission to the dictates of external nature.

Having no clocks, they regarded instead the face of the sky; the stars served them for almanacs. They hunted and fished, sowed and reaped in correspondence with the recurrent order of celestial appearances. But these, to the untutored imagination, presented a mystical as well as a mechanical aspect. Thus, familiarity with the heavens developed at an early age, through the promptings of superstition, into a fixed system of observation. In China, Egypt, and Babylonia, strength and continuity were lent to this native tendency by the influence of a centralized authority; considerable proficiency was attained in the art of observation, and from millennial stores of accumulated data empirical rules were deduced by which the scope of prediction was widened and its accuracy enhanced. But no genuine science *of astronomy was founded until the Greek sublimed experience into theory.*

Already in the 3^{rd} *millennium BCE,* equinoxes *and* solstices *were determined in China by means of culminating stars. This is known from the orders promulgated by the emperor Yao about* 2300 *BCE, as recorded in the* Shu Chung *[a collection of documents already ancient at the time of Confucius (551–478 BCE)], and Yao was merely the renovator of a system already long established. There is no certainty that the Chinese were then capable of predicting eclipses, but they were probably acquainted with the* 19*-year cycle by which solar and lunar years were harmonized. They made observations in the meridian, regulated time by water-clocks and used measuring instruments of the nature of quadrants.*

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[281] For further reading, see: **Agnes Mary Clerke** (1842–1907), *History of astronomy*; Pannekoek, A., *A History of Astronomy*, Dover: New York, 1989, 521 pp.

In 1100 BCE, ***Chou Kung****, as able mathematician, determined with surprising accuracy the obliquity of the ecliptic, but his attempts to estimate the sun's distance failed hopelessly as being grounded on belief in the flatness of the earth.*

Circles were divided into $365\frac{1}{4}$ *parts, so the sun described daily one Chinese degree, and the equator began to be employed as a line of reference, concurrently with the ecliptic, probably in the* 2^{nd} *century BCE. Both circles, too, were marked by star-groups more or less clearly designated and defined.*

Cometary records go back in China to 2296 BCE; they are intelligible and trustworthy from 611 BCE onward.

In Egypt the stars were observed that they might be duly worshiped. The importance of their first visible appearances at dawn (for the purposes of both practical life and ritual observance) caused them to be systematically noted. The length of the year was accurately fixed in connection with the annually recurring Nile flood, while the curiously precise orientation of the Pyramids afforded a lasting demonstration of the high degree of technical skill in watching the heavens attained in the 3^{rd} *millennium BCE. The constellational system, in vogue among the Egyptians, appeared to have been of native origin, but they contributed little or nothing to the genuine progress of astronomy.*

Babylonian science lacked the vital principle of growth imparted to it by their successors. From them the Greeks derived their first notions of astronomy: They copied the Babylonian asterisms, appropriated the Babylonian knowledge of the planets and their courses, and learned to predict eclipses by means of the saros.

Records dating from the reign of Sargon of Akkad (3800 BCE) imply that even then the varying aspects of the sky had long been under expert observation. It may be taken as certain that the heavens described by ***Aratos of Soli*** *(315–245 BCE, Greek poet) represented approximately observations made some 2500 years earlier, by the Babylonians, in or near north latitude 40°. In the course of ages, Babylonian astronomy, purified from the astrological taint, adapted itself to meet the most refined needs of civil life. The decipherment of Babylonian clay tablets supplied detailed knowledge of the methods practiced in Mesopotamia in the* 2^{nd} *century BCE. They show no trace of Greek influence, and were the improved outcome of an unbroken tradition. The Babylonian astronomers were not only aware that Venus returns in almost exactly 8 years to a given starting-point in the sky, but they have established similar periodic relations (with periods 46, 59, 79 and 83 years respectively) for Mercury, Saturn, Mars and Jupiter. They were accordingly able to fix in advance the appropriate positions of these objects with reference to ecliptical stars, which served as fiducial points for their determinations. In* *the Ephemerides published year by year, the times of new moon were given, together with the calculated intervals to the first visibility of the crescent, from which the beginning of each month was reckoned. The dates and circumstances of solar and lunar eclipses were predicted, and due information was supplied as to the forthcoming risings and settings of the sun, conjunctions and oppositions of the planets. The Babylonians knew the inequality in the daily motion of the sun, but misplaced by* $10°$ *the perigee of its orbit. Their sidereal year was* $4\frac{1}{2}$m *too long*[282]*, and they kept the ecliptic stationary among the stars, making no allowance for the shifting of the equinoxes. On the other hand, it has been recognized (1900) that* ***Hipparchos*** *had borrowed from Chaldea the lengths of the synodic, sidereal, anomalistic and draconitic months.*

A steady flow of knowledge from East to West began in the 7^{th} *century BCE. The Babylonian* ***Berossos*** *founded a school about* 640 *BCE in the Island of Cos, and perhaps counted* ***Thales of Miletos*** *(624–546 BCE) among his pupils.* ***Pythagoras of Samos*** *(fl. 540–510 BCE) learned on his travels in Egypt and the East to identify the morning and evening stars, to recognize the obliquity of the ecliptic, and to regard the earth as a sphere freely poised in space. The tenet of its axial movement was held by many of his followers; in an obscure form by* ***Philolaos of Croton*** *(ca 450 BCE), and more explicitly by* ***Ecphantos*** *and* ***Hicetas of Syracuse*** *(*4^{th} *century BCE), and by* ***Heracleides of Pontos*** *(a disciple of Plato in 360 BCE).*

A genuine heliocentric system, developed by ***Aristarchos of Samos*** *(fl. 280–264 BCE), was described by* ***Archimedes*** *in his Arenarius, only to be set aside with disapproval. The long-lived conception of a series of crystal spheres, acting as the vehicles of the heavenly bodies, and attuned to divine harmonies, seems to have originated with Pythagoras himself.*

The first mathematical theory of celestial observations was devised by ***Eudoxos of Cnidos*** *(408–355 BCE). The problem he attempted to solve was to combine uniform circular movements so as to produce the resultant effects actually observed. With this end in view, the sun and moon and the* 5 *planets were accommodated each with a set of variously revolving spheres, with their numbers totaling* 27*. The Eudoxian system, after it had been further elaborated and modified by* ***Aristotle*** *and* ***Apollonios of Perga*** *(fl. 250–220*

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[282] Yet, their observations were amazingly precise; *they computed the length of the year with a deviation less than 0.001 percent from the current value* and their figures relating to the motions of the sun and the moon have only 3 times the margin of error of the 19^{th} century astronomers, armed with mammoth telescopes. Their observations were verifiable, and enabled them to make precise predictions of astronomical events.

BCE), held sway for 1800 *as the characteristic embodiment of Greek ideas in astronomy.*

Greek astronomy culminated in the school of Alexandria. Soon after its foundation, ***Aristyllos*** *and* ***Timocharis*** *(ca 320–260 BCE) constructed the first catalogue giving star-positions as measured from a reference-point in the sky. This fundamental advance rendered inevitable the detection of precessional effects.* ***Aristarchos of Samos*** *made his observations at Alexandria (280–264 BCE). His general conception of the universe was comprehensive beyond that of any of his predecessors.* ***Eratosthenes*** *(276–196 BCE), a native of Cyrene, was summoned from Athenes to Alexandria by Ptolemy Euergetes to take charge of the royal library. He determined the obliquity of the ecliptic*[283] *at* $23°51'$ *(a value too large by* $5'$*), and introduced an effective method of arc-measurement.*

Among the astronomers of antiquity, two great names stand out with unchallenged pre-eminence; ***Hipparchos*** *and* ***Ptolemy*** *entertained the same large designs; they worked on similar methods, and, as the outcome, their performances fitted so accurately together that between them they re-made celestial science. Hipparchos fixed the chief data of astronomy – the lengths of the tropical and sidereal years, of the various months, and of the synodic periods of the five planets, the obliquity of the ecliptic and of the moon's path, the place of the sun's apogee, the eccentricity of its orbit, and the moon's horizontal parallax, all with appropriate accuracy. His loans from Chaldean expertise appear to have been numerous indeed, but were doubtless independently verified. His supreme merit, however, consisted in the establishment*

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283 The Greek astronomers must have noticed the correlation between the march of the seasons, the yearly variations in the sun's altitude angle above the horizon, the length of day and the changes in distribution of sunlight. Knowing the yearly relative motion of the sun and the earth, and the inclination of the earth's equatorial plane to the sun rays (ecliptic), they must have understood that when the North pole points away from the sun, the northern hemisphere experiences *winter*, and when the North pole points toward the sun, that hemisphere experiences *summer*.

Furthermore, they must have associated the *summer soltice* with the sun's vertical rays striking $23\frac{1}{2}°$ north latitude, and the *winter soltice* with the sun's vertical rays striking $23\frac{1}{2}°$ south latitude. Likewise they knew that at both *equinoxes*, the sun's vertical rays strike the equator. It is questionable however, whether they understood the physical aspect of the situation, namely that rays striking at low angle must traverse more of the atmosphere than rays striking at a higher angle and thus are subjected to greater depletion by scattering and absorption.

of astronomy on a sound geometrical basis. His acquaintance with trigonometry, a branch of science initiated by him, enabled him to solve a number of elementary problems, and he was thus led to focus special attention upon the position of the equinox, as being the common point of origin for measures both in right ascension and longitude. Its steady regression among the stars became manifest to him in 130 *BCE, on comparing his own observations with those made by* ***Timocharis*** *a century and a half earlier, and he estimated it to be not less than* $36''$ *per year. An interval of* 250 *years elapsed before the constructive labors of Hipparchos obtained completion in Alexandria. The Ptolemaic system was, in a geometrical sense, defensible; it harmonized fairly well with observations, although physical reasoning had not been extended to the heavens. To the ignorant it was recommended by its conformity to crude common sense, while to the learned it appealed by the wealth of ingenuity expended in bringing it to perfection.*

Nevertheless, the Ptolemaic model of the universe was wrong. It survived for so long in part because its ad hoc machinery was marvelously adaptable; it survived also because it took for granted elements of physics which were not *calculable at that time and so had to be accepted, willy nilly. But it failed in the end for the same reasons: its calculable results were found wanting, its incalculable bases were seen to be unfounded, and its increasingly baroque architecture was no longer to the mathematical taste of a new generation of thinkers.*

122 CE Romans build *Hadrian's Wall* in Britain to defend against northern tribesmen. The wall, built mainly of stone, runs 115 km from Tyne to Solway.

138–165 CE **Marcion of Sinope** (ca 100 CE – ca 165 CE, Asia Minor and Rome). Created *Gnostic*[284] *Christianity* by a syncretism of Gnosticism

[284] Judaism had never sought to convert outsiders and attract recruits so long as circumcision and ritual food restrictions were enforced. Christianity might have remained a sect of unorthodox Jews, had not one of its adherents set himself to broaden the basis for membership. Paul of Tarsus, a hellenized Jew and Christian, removing these external obstacles, made Christianity universally acceptable.

Still, to the hellenized citizens of the Empire it would not do that Christ should be the son of the God of the Jews. This blemish was avoided by Gnosticism, a syncretic movement that arose at the same time as Christianity. According and Paulian Christianity (Marcion's Church). Although later condemned as a heretic over his unorthodox views, Christianity kept parts of Gnosticism at Nicaea (325 CE) and his replacement theology has infiltrated into the Church of today.

At the beginning, after all the apostles died, the leaders who replaced them were mostly Gentile pagans. These Gentiles had comparatively little understanding of the Old Testament Scriptures unlike the Jewish apostles who had been exposed to the teaching of the Law and the Prophets since birth. This caused a shift in focus to the New Testament (written by fellow converts) and the elimination of anything Jewish. It was very true of the moral codes which Jesus exalted his followers to obey.

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to Gnosticism the sensible, material world was created by Yahweh, who was really a minor deity, having fallen out with the supreme godhead and thereafter practiced evil. At last the son of the supreme god came to live among men in the guise of a mortal, in order to upturn the false teaching of the Old Testament. These, along with a dose of Plato, were the ingredients of Gnosticism. It combines elements of Greek legend and Orphic mysticism with Christian teaching and other eastern influences, rounding it off with an eclectic admixture of philosophy, usually Plato and Stoicism. The Manichaean variety of later Gnosticism went so far as to equate the distinction between spirit and matter with the antithesis of good and evil. In their contempt for things material they went further than the stoics had ever ventured. They forbade the eating of meat and declared sex in any shape or form to be altogether a sinful business. From their survival for some centuries it seems proper to infer that these austere doctrines were not practiced with complete success.

While orthodox Christians believe that Creation (physical world and men) became corrupt with Adam and 'the Fall', Gnostics believe that Creation was corrupt to begin with, and that the God of Israel (who created the material universe) was a totally different God from the Father spoken in the gospel of Christ.

All this ran contrary to the Hebrew Scriptures. In Hebrew thought, the chief virtue is in oneness, wholeness, "shalom".There is *one* God, Yahweh. There is *one* world consisting of the heavens, the earth, and sheol under the earth, and all creation is good. Each human being is wonderfully made as one cohesive unit. Salvation is found in living life in covenant relationship with Yahweh, and salvation is experienced in immediate time, as well as in the future.

In Hebrew thought, the Messiah is a human being, raised up from the people, chosen by God and anointed to serve and cause redemption to come to the people of Israel. In Gnostic thought, the savior is god, sent down to earth by the main good god to teach all.

It was Marcion who then caused the Church to virtually ignore the Law, the Hebrew Prophets, all the Apostles (except the Gnostic John), and even Jesus himself and focus full attention to Paul and his writing on *grace*.

Marcion was born at the Black Sea port of Sinope to a pagan family. Around 139 CE, he traveled to Rome and converted to Christianity. As a wealthy ship-owner, he made large contributions to the Church and became a respected member in the Christian community.

Marcion's reference was always to the teaching of Paul (the only apostle whom he trusted). He held the Gnostic idea that the whole creation is faulty, being the creation of a lesser god, thus containing no element of the divine. In this he was influenced by *Persian dualism* (*Zoroastrianism*).

Marcion believed that the lawgiver of the Old Testament was the bad God and that no good could be found in the Old Testament and that after Jesus Christ, the Law was obsolete. Jesus has come to free man from the Law. Marcion, therefore, rejected the entire Old Testament, and viewed the Law as opposed to grace.

All this put Christians in a quandary; they wanted the moral authority of the Old Testament, yet they just did not want to follow it.

For several generations, Marcion's Church survived. His anti-Jewish, pro-Paul churches spread throughout the Roman Empire and became a threat to the Messianic faith.

As Christianity became more firmly established, its hostility to the religion of the Old Testament grew fiercer. The Jews, it held, had failed to recognize the Messiah announced by the prophets of old, and therefore must be evil. From Constantine onwards anti-semitism became a respectable form of Christian fervor, though in fact the religious motive was not the only one. It is odd that Christianity, which had itself been suffering appalling persecution, should, once in power, turn with equal ferocity on a minority that was just as steadfast in its beliefs.

156 CE A census estimated the Chinese population of over 50 millions (compared with 70 millions estimated by scholars for the population of the Roman Empire).

160–190 CE **Claudius Galen (Galenus)** (129–200, Greece and Rome). Anatomist, physician, philosopher. The greatest physician of antiquity after Hippocrates, and the father of experimental physiology. Discovered a large number of new facts in the fields of anatomy, physiology, embryology, pathology, therapeutics and pharmacology. Galen made various physiological experiments, e.g., to determine the mechanism of respiration and pulsation, the function of the kidneys, of the cerebrum, and of the spinal cord. He proved experimentally that arteries contain and carry blood (and not air as had been taught up to his time). He also knew that the heart set the blood in motion, but he failed to discover how the blood circulates through the human body (Harvey, 1628), although he came very close to it. He was first to use the pulse as a diagnostic aid.

In his writings (over 500 works, of which only 83 are extant), Galen systematized and unified Greek anatomical and medical knowledge and practice. It was used until the end of the Middle Ages. He dissected numerous animals, but very few human bodies. His writings contain many errors which were accepted by physicians and scholars throughout the Middle Ages because his authority was unquestioned.

Galen was born in Pergamum, Asia Minor. His father, Nicon, gave him his early education. He began his study of medicine at the age of 16 in his native city, and continued his medical studies in Smyrna and Alexandria. In 158 CE he returned to Pergamum and was appointed surgeon of gladiators. From there he went to Rome (161 CE), where he enjoyed great success as a physician. In Rome he gave lectures in the public theater and performed experiments with animals before large audiences. He later became court physician under Marcus Aurelius (169 CE).

161–180 CE **Marcus Aurelius Antoninus** (121–180, Rome). Emperor and *Stoic Philosopher*[285]. Author of *Meditations* (written in *Greek*), a collection of percepts of practical morality. The bulk of his reign was spent in efforts to ward off the attacks of the Germanic tribes across the upper Danube (166–180). He visited Egypt and Athens (176). Worn out at the age of 59, Aurelius died near Vienna, in the midst of a campaign.

He was master of the Empire during one of the most troubled period of its history[286]. His imperial administration, lasting 19 years, was marked by

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[285] Founded by **Zeno** (336–264 BCE) in Athens and spread to Rome. Stoicism remained dominant through the early Roman empire (31 BCE–192 CE) and claimed among its chief exponents the statesman **Seneca** (4 BCE–65 CE), and the slave **Epictetos** (60–135). But it had to compete with mystical tendencies which found expressions in astrology, the oriental religions (Egyptian *Isis* and Persian *Mithras*), and *Christianity* by which it was finally assimilated.

[286] In the years from the accession of **Marcus Aurelius** (160) to the accession of **Diocletian** (284), basic weaknesses became apparent in the government, in the army, in the economy, and in the society. It was an age of moral corruption – the last effulgence of a dying culture. The military victories of Aurelius gave the Roman civilization two hundred more years of life, in which Christianity might rise to strength so that the collapse of the political order would not mean the destruction of Western civilization.

prudent and generous reforms at home, conceived in a human spirit[287], and by decisive victories on the Parthian and German frontiers; and in all these the Emperor himself was the moving force. The comprehensiveness of his legal and judicial reforms is very striking: slaves, heirs, women and children were benefited.

During his reign, the atmosphere of Roman society was heavily charged with the popular Greek philosophy to which Christianity was diametrically opposed. His upbringing was such that he felt that the policy of the Flavian emperors was the only logical solution to the problems of Roman society. Since the Christians taught a unity which transcended that of the Roman empire, they were regarded as antagonistic to the existing political and social organism. In this age of decadence, the Stoic philosophy held together the civil social order of imperial Rome, and taught *thinking men* the nature of true freedom, which is not dependent upon swords and laws. It was a philosophy imported from Greece blended with the high old Roman virtue, the sense of piety and honesty and office.

Aurelius' *Meditations*, not generally known until late in the 16^{th} century, were written in the midst of public business, and on the eve of battles (166–168) on which the fate of the empire depended – hence their fragmentary appearance, but hence also much of their practical value and even their charm. His thoughts represent a transitional movement, and it is difficult to discover in them anything like a systematic philosophy.

He held the view of **Anaxagoras** – that God and matter exist independently, but that God governs matter. The soul of man is most intimately united to his body, and together they make one animal which we call man;

His Stoic philosophy, on the other hand, prepared the way for the acceptance of Christianity in the dying classical world. The era of competent and conscientious emperors ended with his death in 180 CE. During the century that followed, the army placed a succession of ruthless, uncouth, and politically inept provincial generals on the imperial throne, known as barrack-emperors. Under them the civil service came to be stuffed with semi-literate peasants with little understanding of Greco-Roman political and cultural traditions.

[287] Compelled by the force of depraved public opinion, to be present at the gladiatorial shows and receive the salutes of the poor wretches below in the arena, he refused to look at the slaughter. Detesting these inhuman displays, he read books, or gave audience during the course of the spectacle, and for that he was jeered by the crowd for his aversion. When, in an hour of great public peril, he recruited gladiators in the city to fill the ranks of the decimated legions, the mob threatened to rise against their savior, crying that he designed to turn them all into philosophers by depriving them of their sport.

and so deity is most intimately united to the world or the *material universe*, and together they form one whole.

The goal of life is not happiness, but tranquility or equanimity. This condition of mind can be obtained only by "living conformably to nature", i.e., one's *whole* nature. Consequently, man must cultivate the four chief virtues, each of which has its own sphere: *wisdom* (knowledge of good and evil), *justice* (giving to every man his due), *fortitude* (the enduring of labor and pain), *temperance* (moderation in all things). Man should not yield to the persuasion of the body, when they are not conformable to the rational principle which must govern. This legislative faculty within a man which can be looked from one point of view as conscience and from another as reason, must be implicitly obeyed. He who obeys it will attain a tranquility of mind; nothing can irritate him, for everything is according to nature. As much as life is a composition of elements, death is a decomposition into the same, and altogether not contrary to the reason of our constitution.

The morality of Marcus Aurelius cannot be said to be new when it was given to the world. What gives his sentences their enduring value and fascination is that they are simply the records of his *practice*, not a saintliness of the cloister but the wisdom of the man of the world;

As a Roman Stoic, Aurelius had little understanding of all Eastern cultures, which he held in contempt.

ca 180–211 CE **Clement of Alexandria** (**Clemens Alexandrinus**; ca 150–215 CE; Athens and Alexandria). Greek Father of the Church. With his disciple **Origen of Alexandria** (ca 185–254 CE) laid the groundwork for welding of Christian and Hellenistic thought. Argued that Christianity could profitably utilize pagan Greek philosophy and learning since the latter is based on natural reason and therefore pointed toward truth. Philosophy and secular learning in general could be used to interpret Christian wisdom and therefore philosophy *and* science could be used to understand the Holy Scriptures. This attitude was a compromise between the rejection of pagan learning and its full acceptance[288].

Clement was born of heathen parents in Athens, but lived in Alexandria, where he converted to Christianity. Succeeded his teacher **Pantaenus** as head

[288] This had already been advocated by **Philo of Alexandria** a century earlier, who sought to reconcile the revealed religion of the Pentateuch with philosophical reason as influenced by Plato, Aristotle, Neo-Pythagoreans, Cynics and Stoics. On the other hand, the Latin ecclesiastic writer **Tertullian** (ca 155–225 CE) spoke for the rejectionists when he asked: "What indeed has Athens to do with Jerusalem? What concord is there between the Academy and the Church?"

of Catechetical school in Alexandria. He left Alexandria during persecutions of Emperor Severus (ca 201 CE) and visited Cappadocia, Jerusalem and Antioch.

The recognition that Christianity could not turn away from Greek learning was an essential precondition for the scientific revolution during the late Renaissance.

186 CE Major volcanic eruption at Taupo, New-Zealand.

190 CE Chinese mathematicians use powers of ten.

ca 200 CE Compilation and codification of the *Mishna*, the early oral interpretation of the *Torah* (law of Moses); a collective endeavor of about 350 savants (known as *Tannaaim*). Its formulation extended over a period from 300 BCE to 200 CE when it was committed to writing. The *Tannaaim* labored to formulate a new set of laws which would reinterpret the ancient Mosaic concepts to the sons of Israel living in a Pagan world. As long as the second Temple existed, the rituals, ceremonies and observances, sacrifices, commands and prohibitions made the *Torah* a living spirit of Israel. It was both a state law and religious fountainhead, the guide to daily conduct and the basis for family and social structure of all the adherents to the Covenant[289].

But with the sudden advent of the overbearing and hostile Caesarian Empire, which culminated with the destruction of Jerusalem, the binding ties of Jewish life changed from national institutions, like a land and a government, to religious institutions, like the synagogue and the regulations of everyday life. A thousand practical problems arose that called for immediate solutions: problems concerning marriage and divorce and other aspects of family life; concerning personal hygiene and ritual parity; concerning civil and ceremonial law; dietary obligations and sacrificial cults; concerning the observation of holidays and festivals; the keeping of the Sabbath; the treatment of illness; the care of the poor; and so on.

It was then that the achievements of the spiritual leaders over more than 500 years of the Second Commonwealth were crystallized into a definite rules of conduct under the editorship of **Rabbi Yehuda ha-Nasi**. He summed it up in logical order into a volume of six books known as the *Mishna*, which became a companion to the Bible. Moreover, together with the later *Talmud*,

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[289] Hebrew: *repetition*, *oral learning*. Outdoor teaching was practiced in Israel and Babylonia ancient academies. Neither teacher nor pupils carried notebooks, as all lessons were committed to memory. The lesson was a discourse, after which the pupils asked questions or engaged in discussions.

it proved to be a time-capsule which metamorphosed the lost land and independence into spiritual values sufficient to maintain and nourish the Jews in exile for almost 18 centuries – a unique phenomenon in the annals of mankind.

According to the traditional view, the canon of the Old Testament closed with the work of **Ezra** (458 BCE). He was followed by the *sofrim* (scribes), to the *Maccabean age* (167 BCE), and these again by the *zugoth* (pairs; the reputed heads of the Sanhedrin), down to the *Herodian age* (150–30 BCE). The codification process culminated with in **Hillel** (80 BCE–8 CE) and **Shamai**, the founders of the two great rival schools, and to this famous pair the work of collecting the *Halakah* ("legal decisions") has been ascribed. It was Hillel who gave *Mishna* its scientific foundation.

The ensuing period of the *Tannaaim* ("teachers"; about 10–220 CE) is that of growth of the *Mishna*. The best known representatives of the said generations are listed in Table 1.4.

The *Mishna* was created amidst one of the most tragic and bloody periods in the history of the Jewish nation, through which it lost major parts of its homeland. The *Tannaaim* witnessed three great national disasters [the war for Jewish independence, 66–70 CE; the Diaspora rebellion, 114–117 CE; Bar-Kochba revolt, 132–135 CE] in which a total of $2\frac{1}{4}$ millions perished (out of 3 millions) over a period of 100 years.

With the codification of the *Mishna*, Jewish destiny shifted from Rome to Parthia.

Table 1.4: Leaders of the five generations of Tannaaim

I. (40–90 CE)	**Rabban Gamliel the Elder** (ca 30 BCE–52 CE) (teacher of the apostle Paul; astronomer) **Rabban Yohanan ben Zakkai** (7 BCE–77 CE) (founder of the Academy of learning at Yavne[290])
II. (90–115 CE)	**Rabban Gamliel II** (ca 30–117 CE) **Rabbi Eliezer ben Horkanos** **Rabbi Joshua ben Hannania** (35–117 CE) (astronomer) **Rabbi Eleazar Ben Azariah**
III. (115–135 CE)	**Rabbi Akiva ben Yosef** (55–136 CE) (fixed the official text of the canonical books) **Rabbi Ishmael** **Rabbi Tarfon** **Rabbi Yossi ha-Glili**
IV. (135–170 CE)	**Rabbi Shimeon Bar-Yohai** (80–160 CE) **Rabbi Meir** **Rabbi Nehemiah** **Rabbi Yohanan ha-Sandlar** **Rabbi Nathan of Babylon** **Rabbi Yossi ben-Halafta**
V. (170–210 CE)	**Rabbi Yehuda ha-Nasi** (137–210 CE) (known as the *Patriarch*; brought the *Mishna* into essentially its present shape; a close friend of Emperor **Caracallah** (Marcus Aurelius Severus Antoninus, 211–217 CE).

[290] *Massada* and *Yavne* have come to symbolize two antithetical aspects of Jewish history, the former that of a resistance, the latter that of a surrender. The spirit of *Massada* permeated the first act through which the Hebrews took on Canaanites, Philistines, Assyrians, Egyptians, Babylonians, Greek and Romans, scrapping their way through defeats and victories in a struggle for national survival.

The spirit of *Yavne* permeates the second act, representing the Jewish response called for by new world order. The secret of Jewish survival is summed up in Jewish ability to select the right weapon at the right time.

During the next 300 years the *Mishna* was supplemented by many recorded discussions or commentaries, contributed by Babylonian as well as Israeli scholars. Some of these were legalistic, some philosophic, some folklorist, some allegorical. These later writings known as the *Gemara* (Aramaic: *learning*), were intended to expound the Mishna and to facilitate the understanding of its difficult passages.

200–250 CE **Shmuel the Astronomer** (Yarchinai, 165–254 CE, Babylonia). Jewish jurist, educator, astronomer and physician. He became the director of the Mesopotamian Academy of Nehardea. Opposed the medical superstitions of the day and introduced more rational methods. His astronomical interests were centered upon the *Hebrew Calendar*, which he improved.

ca 250 CE **Diophantos of Alexandria** (206–290 CE). Greek mathematician. Author of *Arithmetica*[291], one of the greatest mathematical treatises of ancient times. It is a masterly exposition of algebraic analysis, so thorough and complete in its time, that most previous works in its field ceased to be of interest and passed into oblivion. In the *Arithmetica*, algebraic methods advanced to a peak of achievement which was not to be surpassed before the 16^{th} century. Following the rediscovery of a manuscript of Diophantos in the late Renaissance (1570 CE), **R. Bombelli** (1572 CE) included many problems of the *Arithmetica* in his *Algebra*.

In 1621, **Bachet de Meziriac** (1581–1638 CE, France) published an edition of the *Arithmetica* which contained the Greek text as well as his Latin translation. Bachet's edition was made famous by the notes written in the margin of a copy of his book by **Fermat**. Since that day, Fermat's notes have stimulated a prodigious output in the theory of numbers.

Diophantos solved indeterminate equations of second and higher degrees[292] in integers by ingenious devices (but no general method). Used *abbreviations*

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[291] The *Arithmetica* is a collection of 130 problems giving numerical solutions of determinate equations (those with a unique solution), and indeterminate equations. The method for solving the latter is now known as Diophntine analysis. Only 6 of the original 13 books survived.

Diophantos was always satisfied with a rational solution and did not require a whole number. He did not deal in negative solutions and one solution was all he required to a quadratic equation. In fact, most of the *Arithmetica* problems lead to quadratic equations. Although he did not use sophisticated algebraic notation, he did introduce an algebraic symbolism that used an abbreviation for the unknown.

[292] Diophantos never dealt with the simpler *linear* equations. This was left to **Aryabhata** (476–550 CE) and **Brahmagupta** (598–678 CE) in India.

for powers of numbers and for relationships and operations throughout the six surviving books of the "Arithmetica". He was familiar with the rules of powers combination, equivalent to our laws of exponents, and he had special names for the reciprocals of the first six powers of the unknowns, quantities equivalent to our negative powers.

Thus the identity

$$(a^2 + b^2)(c^2 + d^2) = (ac + bd)^2 + (ad - bc)^2 = (ac - bd)^2 + (ad + bc)^2,$$

which played important roles in Medieval Algebra and modern trigonometry, appears in the work of Diophantos. Various new properties of numbers were discovered by him: e.g., no number of the form $8n + 7$ can be the sum of three squares. As far as notation is concerned, Diophantos has a good claim to be known as the 'Father of Algebra'.

The few details of the personal life of Diophantos available to us, are all contained in his epitaph. It is said to have been composed shortly after his death by a close friend. It states that the $\frac{1}{6}^{th}$ of his life was spent in childhood (14 years), after a $\frac{1}{12}^{th}$ more had elapsed he grew a beard (at 21), that when a $\frac{1}{7}^{th}$ more had passed he married (at 33), and that five years later his son was born (at 38). The son lived to half his father's age and 4 years later the father died (84).

A commentary on his work (first six books out of the original 13) written by **Hypatia** (d. 415), daughter of **Theon of Alexandria**, is the ultimate source of all extant manuscripts and translation of the *Arithmetica*. The remainder of the work was probably lost before the 10^{th} century.

Book I of the *Arithmetica* opens with the following dedication: "*Knowing, my most esteemed friend Dionysios, that you are anxious to learn how to investigate problems in numbers, I have tried, beginning from the foundations on which the science is built up, to set forth to you the nature and power subsisting in numbers. Perhaps the subject will appear rather difficult, inasmuch as it is not familiar* (*beginners are, as a rule, too ready to despair of success*)*. But you, with the impulse of your enthusiasm and the benefit of my teaching, will find it easy to master. For eagerness to learn, when seconded by instruction, ensures rapid progress*".

253 CE **Plotinos** (204–270 CE, Rome). Philosopher. The most important representative of *Neoplatonism*. Under Ammonius Saccas he became imbued with the eclectic spirit of the Alexandrian school. Having accepted the Platonic metaphysical doctrine, he applied to it the Neo-Pythagorean principles and the Oriental doctrine of Emanation. The results of this introspective mysticism were collected by him in a series of 54 treatises arranged in 6 *Enneads* which constitute the most authoritative exposition of Neoplatonism.

The principal doctrine of Plotinos states that God is a *transcendental principle* who creates the universe out of himself in a process which is timeless and eternal. Although present in all objects of creation, he is distinct from them, supreme, above and before all things. God cannot be categorized or classified as spiritual, material, soul substance, or in any other category, but must be regarded as *One* who, without possessing specific attributes, yet creates all things.

God is the absolute permanent One (as **Parmenides** taught), but the universe as his creation comprises a changeable plurally (as **Heraclitos** taught). The physical and spiritual universe (living and nonliving matter) is the by-product of the One, embracing **Anaxagoras**' *Nous*, as well as **Plato**'s concepts of ideas, substance, and matter as emanations from God. The ideas, inasmuch as they are immanent in the *Nous*, are not subject to error. But the soul, being a product of the *nous*, is only its image, and hence is fallible.

Plotinos was born of Roman parents at *Lycopolis*, Egypt. At Alexandria he attended the lectures of Ammonius Saccas. He joined the Persian expedition of Gordian III (242 CE), with the object of studying Persian and Hindu philosophy on the spot. After the assassination of Gordian (244 CE), he was obliged to take refuge in Antioch, whence he made his way to Rome and set up as a teacher there. He soon attracted a large number of pupils. The Emperor Gallienus and his wife Salonina were also among his enthusiastic admirers, and favored his idea of founding a Platonic Commonwealth (Platonopolis) in Campania, but the opposition of Gallienus counselors and the death of Plotinos prevented the plan from being carried out.

268–301 CE **Porphyrios (Malchos)** (ca 234–305 CE, Tyre and Rome). Greek scholar, historian, logician and *Neoplatonic*[293] *philosopher.* Exerted great influence on Neoplatonic trends in philosophy and theology in the Middle Ages. Contributed indirectly to mathematics (in his commentary on Aristotle) through the *Tree of Porphgry*[294] (now called "*binary tree*"): categories are split into two mutually exclusive and exhaustive parts on the basis of a property

[293] *Neoplatonists* resolutely reaffirmed the *Pythagorean* conviction that mathematics had a decisive heuristic value as regards man's search for the patterns of the physical world.

[294] A *connected graph* is a set of points (vertices) joined by line segments in such a way that a path can be found from any point to any other point. If there are no circuits, a connected graph is called a *tree*. In 3D, real trees, crystals, river tributaries, brittle solid cracks and electric discharges are few examples of natural *trees*. Sets of 2–12 points have respectively 1, 1, 2, 3, 6, 11, 23, 47, 106, 235, 551 topologically distinct *trees*.

possessed by one part but not the other. This prepared the use of genealogical trees and the division of a subject matter into hierarchic categories.

Porphyrios was born at Tyre. After studying grammar and rhetoric under Longinus he went to Rome (262 CE) and studied under Plotinos (262–268 CE). He then lived a few years in Sicily and returned to Rome (273 CE). His most distinguished pupil was **Iambilichus**. When advanced in years, he married Marcella, a widow with seven children and an enthusiastic student of philosophy.

Porphyrios was a violent opponent of Christianity and defender of Paganism. He wrote numerous books on a great variety of subjects, among them a biography of Pythagoras, comments on Aristotle, history of philosophy and history proper. He dated the Book of Daniel to the time of Antiochus Epiphanes.

269 CE The great library at the Alexandria Academy was partially burned. The Queen of Palmyra, Septimia Zenobia, captured Egypt.

ca 300 CE **Iambilichus** (ca 260–330 CE, Syria). Neoplatonic Greek philosopher. Follower of Plotinos and Porphyrios. Born in Chalcis, Syria to a rich illustrious family.

Although lacking in originality, he gave a more systematic application of Pythagorean number-symbolism. He went beyond the Pythagoreans in making mathematics a principle for *all* that can be observed in the cosmos. (*"I believe we can attach mathematically everything in nature and in the world of change"*).

Iambilichus defined mathematics as the "prognostic science of nature". He stated that his search for causes, or the causal approach to nature, consisted "in positing mathematical things as causes" from which the objects in the perceptible world arise. His was the Pythagorean belief that only what was possible in mathematics was possible in the structure of nature, and nothing could exist that implied a mathematical impossibility. He formulated a program that had a ring strongly reminiscent of some aspirations of twentieth-century physics.

The Origins of Chemistry[295] (0–400 CE)

Chemistry, or the study of the composition of substances, had its origin about the beginning of the Christian era in the Hellenistic-Egyptian city of Alexandria, and was probably the result of blending together of material from two sources: (1) the speculative philosophy of the Greeks, and (2) the Egyptian practical arts of working in metals and glass, the dyeing of tissues and the falsification of precious metals and gems.

Alexandria has a mixed population of native Egyptians, Greeks, Syrians, and Jews, but was essentially Greek in culture. It contained a temple of the god Serapis, two libraries and the Museum (or University) and, in later times, the Christian Church of St. Mark and the famous Pharos or lighthouse, 152 *m high. One library, said to contain* 700,000 *books, was destroyed by fire in* 47 *BCE. The Museum was mainly interested in classical literature, philosophy, mathematics, and medicine.*

Under the merger of the above two streams of knowledge, the technical arts gradually assumed a new form, and the result was the "divine" or "sacred" art of making gold or silver. This contained the germ of chemistry, and during the first four centuries a considerable body of positive, practical chemical knowledge came into existence – the early chemistry.

The Egyptian technique, handed down in the workshops, is described in the papyri of Leyden and Stockholm, discovered at Thebes in 1828. *It is written in Greek and dates probably from about* 300 *CE, although much of the material is probably derived from older Egyptian sources. It summarizes technical information on metallurgy, dyeing, and imitation of precious stones.*

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[295] For further reading, consult:

- Partington, J.R., *A History of Chemistry*, 4 Volumes, Macmillan and Company: New York, 1961.
- Partington, J.R., *A Short History of Chemistry*, Dover Publications: New York, 1989, 415 pp.
- Moore, F.J., *A History of Chemistry*, McGraw-Hill Book Company: New York, 1939, 447 pp.
- Farber, E., *The Evolution of Chemistry, A History of its Ideas, Methods, and Materials*, Ronald Press Company: New York, 1952, 349 pp.
- Leicester, H.M., *The Historical Background of Chemistry*, Dover Publications: New York, 1971, 260 pp.

The earliest of the true chemical treatises, written in Greek at Alexandria during the first 4 centuries CE, speak clearly of the artificial production of gold and silver and the imitation of valuable dyes. The earliest name for chemistry is the "divine art"; the name chemeia *appears about 250 CE and seems to derive from the Egyptian word* chemi*, meaning "black or burnt", or "Egyptian", or both. There were Greek treatises on the divine art in existence as early as the first century CE. Practical operations and apparatus were invented about the same time by* ***Maria the Jewess****, who described apparatus for distillation.*

The most copious author who wrote a kind of chemical encyclopedia was ***Zosimos of Panapolis*** *in Egypt, who lived about 250–300 CE, and first used the name* chemieia*. His books contain interesting descriptions and illustrations of chemical apparatus and experiments (solution, filtration, fusion, sublimation, distillation, etc.) and several chemical substances and reactions*[296]*. During his time there arose belief in transmutation of metals.*

300–1000 CE **Tiahuanaco**. Andean Empire (Peru and Bolivia). A dominant pre-Inca civilization which built large stone buildings decorated with carvings of animals and geometric figures. It had a strong understanding of science, especially of astronomy and medicine. The remains of a great ceremonial center are still to be seen on the Altiplano in Bolivia, near Lake Titicaca (Puma Punku Temple) at an altitude of 4200 m. Their architectural megalithic stone work presupposes a social organization, a strong central government which could direct the use of manpower into non-food-producing channels on such a large scale. All this must have been done by a large supply of workers with a long technical tradition.

The empire declined rapidly after 1000 CE as a result of a major climatic change, causing a drought that lasted for 80–100 years.

The livelihood of this one-million people empire depended on a sophisticated agricultural system of raised fields interlaced by water channels. The water served as solar collectors, which kept away the killing frost during night by radiating infra-red heat over the ground crops. Due to the prolonged drought, water could not be replenished and the frost controlled the environment of the valley.

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[296] E.g., he explains the burning of limestone to form quicklime ($CaCO_3 \Leftrightarrow CaO + CO_2$). The Alexandrian chemists were very near to recognition of gases.

ca 320–350 CE **Pappos of Alexandria**. Greek geometer and scientist. The greatest mathematician of the final period of ancient science and the last mathematical giant of antiquity. One of his theorems is cited as the basis of modern *projective geometry*. A prolific writer on mathematical and scientific subjects. His main work (340 CE) is "*Synagoge*" (*Collection*), in 8 books. It is an account of previous research, equally valuable for the historical information it contains and for the additional explanations. It includes the definition of conic sections by means of the directrix, involution of points, and the theory of center of gravity.

His surviving books form one of the richest sources of information about ancient mathematics and exerted a stimulating force upon the greatest scholars of the 17^{th} century[297]. Building upon his teachings, **Descartes**, **Fermat**, **Pascal** and **Desargues** opened the way for the development of modern mathematics.

Ptolemy's gigantic efforts were followed by lull of more than a century. So much so that when Pappos, the next great mathematician, appeared he felt obliged to prepare a summary of earlier books. Pappos was not a teacher like Euclid or Ptolemy, but a learned man who was familiar with the whole Greek mathematics and tried to summarize it in his own peculiar way. He was a good commentator because he was on the level with his greatest predecessors and was able to add ingenious theorems and problems of his own, but he was not very methodical. He had taken notes on the mathematical classics, invented and solved problems, and then classified them in eight books. Each book is preceded by general reflections which give to that group of problems its philosophical, mathematical and historical setting.

Book I covered arithmetic (and is lost) while Book II is mostly lost but the remaining part deals with large numbers. In Book III he gives a construction of the arithmetic, geometric and harmonic means. Book IV contains properties of curves including the *spiral of Archimedes* and the *quadratrix of Hippias*.

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297 Pappos was among the first keen observers of mathematical patterns in nature. Thus he noticed that the bees use the regular hexagon exclusively for the shape of the cells in the honeycomb:

"*Though God has given to men the best and most perfect understanding of wisdom and mathematics, he has allotted a partial share to some of the unreasoning creatures as well... This instinct is specially marked among the bees. They prepare for the reception of the honey the vessels called honeycombs, with cells all equal, similar and adjacent, and hexagonal in form*".

It is of interest to note that **Pliny the Elder** reported inaccurately in his *Natural History* (ca 75 CE) on the rectangular and circular shape of honeycombs.

Book VII includes the famous *Pappos' problem*: "*given several straight lines in a plane, to find the locus of points, such that when straight lines drawn from it to the given lines at a given angle, the products of certain segments shall be in a given ratio to the product of the remaining ones*". This problem exercised Descarte's mind and caused him to invent the method of coordinates explained in his *Géométrie* (1637). It acted like a seed lying dormant for more than 1300 years and then producing the flowering of analytic geometry.

The final Book VIII is mechanical and largely derived from Heron of Alexandria. This book may be considered as the climax of Greek mechanics and helps us to realize the great variety of problems to which Hellenistic mechanicians addressed themselves.

The whole *Collection* is the culmination of Greek mathematics. Little was added to it in the Byzantine age and the Western world, having lost its knowledge of Greek together with its interest in higher mathematics, was not able to avail itself of all the riches which Pappos has put together.

The ideas collected or invented by Pappos did not stimulate Western mathematicians until very late, but when they finally did, they caused the birth of modern mathematics – analytical geometry and projective geometry. That rebirth, from Pappos' ashes, occurred within four years (1637–1640). Thus was modern geometry connected immediately with the ancient one as if nothing had happened between.

Table 1.5: The Greatest Greek Mathematicians

Name	Life-span	Major contribution
Thales of Miletos	624–548 BCE	The first geometer
Pythagoras	580–500 BCE	Notions of 'Axiom' and 'proof' in geometry
Hippasos of Metapontum	490–430 BCE	Irrationality of $\sqrt{2}$; Dodecahedron
Zeno of Elea	490–430 BCE	Infinity and Infinitesimals
Antiphon the Sophist	480–411 BCE	Method of 'Exhaustion'
Hippocrates of Chios	470–410 BCE	Use of letters in figures; Reductio ad absurdum
Theodoros of Cyrene	470–410 BCE	Irrationality of $\sqrt{3}$; $\sqrt{17}$; $\frac{1}{2}(\sqrt{5}-1)$
Archytas of Tarentum	428–347 BCE	Arithmetic; Harmonic and geometric series; Theory of music
Theaetetos of Athens	415–369 BCE	Incommensurability; Regular polyhedra
Eudoxos of Cnidos	408–355 BCE	Approximate 'integration' procedure ('Exhaustion')
Menaichmos	380–320 BCE	Discovery of conic sections
Euclid	330–260 BCE	Plane geometry; Platonic solids; 'E. algorithm'
Aristarchos of Samos	310–230 BCE	Father of Astronomy; Trigonometry
Archimedes	287–212 BCE	Father of Mathematical Physics

Table 1.5: (Cont.)

Name	Life-span	Major contribution
Eratosthenes	276–197 BCE	Applied geometry; Number Theory
Apollonios	262–200 BCE	Properties of conic sections (circle, ellipse, parabola, hyperbola)
Zenodoros	200–140 BCE	Isometric figures
Hipparchos of Nicaea	180–110 BCE	Spherical trigonometry; Father of trigonometry
Heron of Alexandria	60–120 CE	Areas and volumes; Engineering
Menelaos	65–130 CE	Spherical trigonometry; 'M. Theorem'
Ptolemy of Alexandria	85–165 CE	Mathematical astronomy; Law of sines
Diophantos of Alexandria	206–290 CE	Father of algebra
Pappos	350–410 CE	Advent of projective geometry

Avantgarde Chinese Mathematics (300 BCE–1303 CE)

Unlike the early Greeks, who were interested in formal logic, the practical minded Chinese were at heart applied mathematicians and numerical analysts. To them, the primacy of arithmetical operations was their main concern. The notation of a number, in the Pythagorean sense, as an atomic *and invisible entity did not trouble them. Neither did Chinese mathematicians have to face the dilemma of the irrational numbers, which perplexed the Greeks. Their 'dialectical logic' is best exemplified in their method of solving equations. A solution is assumed beforehand, and the answer is obtained to the desired accuracy by performing as many iterations as are necessary. Thus, they invented the 'rule of double false position' (ca 300 BCE), one of the oldest methods of approximating a real root of an equation*[298].

For over a thousand years, the Chinese used this method, refining and extending it, passing it to the Arabs, who in turn passed it on to the Europeans. The method still continues to have basic applications in modern numerical analysis under the name of the chord or chain rule.

*Other notable contributions of Chinese mathematics which, at the time of their conception were yet unknown (and remained so until the 14th century) were associated with the solution of a system of simultaneous linear equations. Although composed sometime between 300 BCE to 200 CE it nevertheless failed to lead to the concept of determinants before the 17th century (**Seki Kowa**, Japan, 1683).*

As an example consider a single linear equation in one unknown $ax+b=0$. *Let* g_1 *and* g_2 *be two preliminary guesses for the value of* x *and let* f_1 *and* f_2 *be the errors arising from these guesses; Then* $ag_1+b=f_1$; $ag_2+b=f_2$. *From these we obtain by simple manipulation*

$$a(g_1-g_2)=f_1-f_2; \qquad b(g_2-g_1)=f_1g_2-f_2g_1$$

and hence

$$x=-\frac{b}{a}=\frac{f_1g_2-f_2g_1}{f_1-f_2}=\frac{1}{f_1-f_2}\begin{vmatrix} f_1 & f_2 \\ g_1 & g_2 \end{vmatrix}$$

The method can be easily extended to system of simultaneous equations. It is known as the 'rule of double false position'. The later transmitted it, through India, to the Arabs, who in turn passed it to the West. Other important contributions are:

- *The concept of negative numbers appears for the first time in China near the beginning the Christian era.*

[298] Variants of this method are to be found in *Babylonian* mathematics, in *Alexandrian* mathematics (Heron's method), and in *Hindu* mathematics.

- *The development of an algorithm for extracting square and cube roots was first explained in the Chiu Chang, and elaborated and refined by* ***Sun Tsu*** *(ca 300 CE). The 13^{th} century mathematicians extended the algorithm to the extraction of roots of any order. The Arab mathematician* **Al-Kashi** *(ca 1400 CE) and later European adopted the method.*

- *The development of numerical methods of solving high-order equations in the 13^{th} century – methods similar to those associated with* ***Horner*** *and* ***Ruffini****, at the beginning of the 18^{th} century (****Ch'in Chiu-Shao****, 1247).*

  Thus, to solve the equation $x^2 + 252x - 5292 = 0$, *Ch'in first established that there is a root between 19 and 20. He then used the transformation* $y = x - 19$ *to obtain the equation* $y^2 + 290y - 143 = 0$, *with a root between 0 and 1. His final approximate solution is* $x = 19 + \frac{143}{1+290}$.

  Similarly, to solve the equation $x^3 - 574 = 0$ *he set* $y = x - 8$ *to obtain* $y^3 + 24y^2 + 192y - 62 = 0$, *yielding as an approximate root* $x = 8 + \frac{62}{1+24+192} = 8 + \frac{2}{7}$.

  Indeed, 'Horner's method' must have been well known in medieval China; it was used by several Sung mathematicians for the numerical solution of cubic and even quartic equations. The unknown quantity in these equations was represented by a monad and the zero by a little circle, (the Chinese zero may have come directly from India with Buddhism or it may have been imported later by Muslims). Red and black ink were used respectively to represent positive and negative numbers.

  A mathematician, ***Li Yeh*** *(1178–1265), wrote treatises in 1248 and 1259 involving problems on quadrilaterals and circles, with their solutions. Instead of using red and black colors to designate positive and negative numbers, Li differentiated the latter by drawing diagonal strokes across them.*

- *Pascal's triangle of binomial coefficients was known in China as early as 100 CE. Chinese mathematicians used it to solve numerical equations of higher degree.*

- *The value of π estimated by* ***Liu Hui*** *(ca 200 CE) and* ***Tsu Chung Chin*** *(ca 400 CE), namely*

  $$3.141\,592\,6 < \pi < 3.141\,592\,7,$$

  (by successive application of the Pythagorean theorem to polygons with up to 24576 sides!), remained the must accurate values for a thousand years.

- *A method of solving system of simultaneous linear equations (up to 5 unknowns), which is basically a variant of a method developed by **Gauss** 1500 years later. It appeared in a text 'Nine Chapters on the mathematical Arts' by an unknown author.*

- ***Sun Tsu** (c 300 CE) and **Chin Chiu Shao** (ca 1250) devised a novel approach to the solution of* indeterminate equations *of the first degree. It culminated with the* Chinese Remainder Theorem *of elementary number theory. Their general solution predated the work of **Euler** and **Gauss** by the five hundred years.*

  Indeterminate analysis arose from problem in Calendar making and astronomical calculations. All Calendars need a beginning. A Calendar constructed during the Wei dynasty (220–65 BCE) took as its starting-point the last time that winter solstice coincided with the beginning of a lunar month and was also the first day of an artificial 60-day cycle, known as chia tsu. *The objective was to locate exactly the number of years (measured in days) since the beginning of the calendar.*

  Other practical problems of indeterminate analysis arose from engineering and military applications, and architecture.

  To restate the problem of the calendar in modern symbolic notation, let y be the number of days in a tropical year, N the number of years since the beginning of the calendar, d the number of days in a synodic month, r_1 the number of days in the 60 day cycle between winter solstice and the last day of the preceding chia tsu, *and r_2 the number of days since the beginning of the lunar month. The number of years since the beginning of the calendar can then be calculated from*

  $$yN \equiv r_1 (mod\ 60) \equiv r_2 (mod\ d).$$

  More complex alignments, including planetary conjunctions, were built into models for estimating the beginning of Calendars, and as early as the 5^{th} century CE Chinese astronomers solved a set of 10 linear congruences.

Through history China has been relatively isolated from other cultural centers due to the natural barriers such as the Himalayas and the Central Asian plains. Yet, these geographical barriers were not sufficient to exclude all contacts:

By 2^{nd} century CE trade over the Silk Routes from China to the West was at its height, and along with the goods went ideas and techniques. In the centuries to come, the Classical civilization of both East and West would suffer invasions small and large, culminating in Mongol hegemony over vast *stretches of the Eurasian plains, which both served as instruments for diffusion and led to the convergence of ideas and technological practices. In examining the dissemination of Chinese mathematics, one needs to look at the Hindu and Arab connections.*

While there is little evidence of Chinese science in any of the extant Hindu texts, we have evidence of from 7th century on that translations were made of Hindu astronomical and mathematical texts which were mentioned in the records of the Sui and Tang dynasties. The texts contain sections on Hindu numerals and operations, and sine tables. A surviving Chinese block-print text contains Hindu numerals, including the use of a dot to indicate zero.

There is evidence of Chinese diplomats posted at the Court of the Guptas in India in the middle of the 1st millennium CE. A number of Chinese Buddhist scholars made their pilgrimage to holy places in India, bringing back many texts for translation.

The Arab connection is better documented: there are a number of reports of political and diplomatic links between the Arab world and China to supplement trade relations. Arab travelers (including ***Ibn Battuta****, ca 1350 CE), gave detailed accounts of Chinese society and science.*

Chinese mathematics may have made specific borrowings from Arab sources: it is possible that trigonometric methods used in astronomy may have been transmitted through Arab and Hindu contacts. In constructing a calendar in the 14th century, ***Kou Shou Ching*** *used spherical trigonometric methods which seem to have Arab origins.*

Avenues of direct transmission of mathematical lore from China to the West did probably exist: as early as the 3rd century BCE, Chinese silk and fine ironware were to be found in the markets of Imperial Rome. And a few centuries later a whole gamut of technological innovations found their way slowly to Europe. Thus, it is reasonable to expect that mathematical knowledge from China diffused westwards to Europe, there perhaps to remain dormant during Europe's Dark Ages, but coming to life once more with the cultural awakening of the Renaissance.

Finally, during the late 17th and 18th centuries, Europe became aware of the Chinese intellectual heritage and there began an 'East-West Passage' of scientific ideas through the Jesuit connection.

The question is often asked as to why did modern science evolved in the West and not in China? The answer is quite simple. In the absence of the concept of a divine being who acted to legislate what went on in the natural world and whose decrees formed inviolate 'laws' of nature, Chinese and Japanese science was condemned to a curious stillbirth. It is interesting *to ponder whether science would have flourished in medieval and Renaissance Europe were it not for the Judeu-Christian theology.*

Although differences in scientific progress between East and West can indeed be traced to theological differences, other factors are also responsible: Eastern philosophy lacked the ingredient of reductionism*, whereby the properties of a complicated system are understood by studying the behavior of its component parts.*

Eastern philosophy emphasized holistic *interconnectedness of physical things. The ability to dissect natural systems has been crucial to the progress of science in the West. On the other hand, the notion that the whole is more than the sum of its parts arrested the motivation of Eastern thinkers to know something without knowing everything!*

Finally, in the East (especially in China) stability was much more prized than freedom and the rulers had a powerful vested interest in not being challenged. Thus, the continuous interest in science in the West may be sought in the Western tendency to be dissatisfied with the status quo*. In China, a philosopher like Thomas Aquinas would have become an unchallenged authority; in Europe, his system was questioned within a generation after his death.*

Mathematics in Ancient India[299] (320–500 CE)

During the Gupta dynasty, Hindu mathematics and astronomy reached its zenith. The Hindus had long been interested in these subjects and surpassed even the Greeks of the Hellenistic period in some branches of Mathematics. Using abstract principles of algebra, the Hindus could cope with much more difficult concepts than found in the visible *Greek demonstrations of geometry.*

[299] For further reading, see: Joseph, G.G., *The Crest of the Peacock*, Princeton University Press, 2000, 455 pp.

Greek algebra was rudimentary, but the Hindus invented the concept of negative quantities, solved quadratic equations, and calculated the square root of 2.

In 1881, *in a sequestered village called Bakhshali (on the north-west border of India), a farmer came upon an old manuscript, known today as The Bakhshali manuscript. The content of this document proved to be mathematical, ranging over topics in arithmetic, algebra and mensurational geometry. It was written in an old form of Sanskrit and its content were composed not later than the* 4^{th} *century CE.*

Most of the illustrative problems are of the type requiring the solution of linear equations or of an indeterminate equation of the second degree. The solutions display the writer's knowledge of average value, and series, and a considerable skill in operation with fractions. Consistent use is made of the positional decimal system, where a heavy dot serves the purpose of a zero. Calculations involve positive and negative numbers, the sign (+) *is used to denote a negative number or subtraction, and multiplication is often indicated by juxtaposition.*

The Hindus summed (algebraically!) arithmetical and geometrical progressions, and solved commercial problems in simple and compound interest, discount and partnership, mixture and cistern problems, similar to those found in modern texts. They admitted negative and irrational numbers and recognized that a quadratic has two formal roots. They also showed remarkable ability in indeterminate equations, and were perhaps the first to devise general methods in this branch of mathematics. Unlike ***Diophantos****, who sought only one rational solution to an indeterminate equation, the Hindus endeavored to find all possible solutions especially for linear indeterminate equations, which Diophantos did not treat at all. The Hindu work on indeterminate equations reached Western Europe too late to exert any beneficial influence. The Hindus were not proficient in Geometry. Postulational developments were nonexistent. Their geometry was largely empirical and generally connected with mensuration.*

In trigonometry, the Hindus introduced the notion of the sine and calculated[300] *tables of 24 sines, progressing by intervals of* $3^\circ 45'$.

300 The sine of $3^\circ 45' = 225'$ was considered equal to the arc of $225'$. The other sines up to sin $86^\circ 15'$ were calculated by the identity

$$\sin(n+1)\alpha = 2\lambda \sin(n\alpha) - \sin(n-1)\alpha,$$

where $\alpha = 225'$, $\lambda = \cos\alpha$, and λ was approximated by the number $\left(1 - \frac{1}{450}\right)$.

The Hindu astronomers knew that the earth was round and that it rotated and they had some understanding of gravitation. Their most famous invention, however, was the system of the so-called 'Arabic numerals', which first appeared in India in the 3^{rd} *century BCE.*

By the time of the Guptas, Hindus were using a sort of decimal system. The zero first appeared in the works of scholars who were familiar with the Hindu numerals. It is significant that Christian Europe did not know of Hindu-Arabic numerals until the 12^{th} *century and did not use them extensively until the* 16^{th}*. The two civilizations were too far apart, and contacts between them were too infrequent, for even such obviously useful ideas to travel quickly from one region to the other.*

At about 412 CE the Gupta state began to decline, chiefly because of internal disorders. By the 6^{th} *century the Gupta kingdom had collapsed. During the next six centuries India suffered a succession of internal wars and foreign invasions, and the overwhelming influence of Brahman priests made it difficult for secular rulers to exercise effective political control. When a new Hindu Empire finally emerged, it was ruled by the Muslims, not the Hindus.*

Nevertheless, Hindu mathematics continued to flourish up to 1400 CE with considerable achievements in the field of algebra, trigonometry and infinite series, in contradistinction to the Greeks who excelled in geometry.

321 CE The edict of the Emperor **Flavius Valerius Aurelius Constantinus**[301] (274–337 CE), known as **Constantine**[302], enacted that magistrates, citizens and artisans were to rest from their labors 'on the venerable day of the Sun, namely *Sunday*, the first day of the week. In the same cen-

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301 Proclaimed emperor in 306 CE; won his first campaign against his rival Maxentius (312 CE) by virtue of the fact that Christians sided with him, and that Christian centurions led his legions. Constantine set aside all persecution of Christians and admitted them to his court at the very beginning of his reign. When he defeated the Eastern Roman Emperor Licinius, he established full equality for Christians, and shortly thereafter he began a cautious repression of the Pagans. In 329 CE, he began a series of legal actions which expressed his hatred for the Jews. In the year 330 CE, all people of the Roman Empire were forbidden (under death penalty) to convert to Judaism. The death penalty was also specified for all Jews who taught the *Torah* to gentiles or encouraged gentiles to embrace Judaism. The death penalty was prescribed for any Jew who married a gentile. All intermarriage was forbidden, unless a Jew converted to Christianity. Judaism was referred to in imperial pronouncements as the *secta nefaria* ("abominable religion"), or the *secta feralis* ("mournful religion") or the "bestial religion". As a final touch, Constantine forbade any Jew to set foot in Jerusalem.

So long as the early Christians were persecuted by Imperial Rome, the Christians did not turn upon the Jews. They needed them too desperately: Christians were saved from death by Jewish bribes; and thousands of Christians were brought out of slavery to Jews. During these early years, Christians were regarded as Jews by Jews – a condition that lasted into to 4^{th} century CE in some cities of Asia Minor. It was only when Christianity became the official state religion of the Roman Empire that the separation of Jew and Christian was complete, and *anti-Semitism* became one of the foundation stones of the new religion. By the time of Emperor Constantine, Christianity had won the tacit approval if not the total commitment of the most viable and thoughtful section of the Roman Empire's population. The need for Jewish help that had been lessening over the 3^{rd} century CE, now ceased.

Thus, Christianity was established in the Western world in the holy hatred of Judaism – a Hatred that would exact from the Jew suffering beyond description, untold millions of lives, and a river of blood.

It is of no credit to the Christian Church that it erased the years of Jewish-Christian brotherhood, so that the murderous attitude of so many Christians toward Jews, in the centuries that followed Rome's acceptance of Christianity, might not be softened by any memory of Jewish mercy and loving kindness toward Christians.

302 Hadas, M., *Imperial Rome*, Time-Life International: The Netherlands, 1966, 190 pp.

tury, Christmas was assigned to 25 December, because on that date each year was celebrated the birth of the sun to a new life after the winter soltice. Easter, being a lunar feast in origin, retained a variable date.

325 CE *The Council of Nicaea* (Asia Minor) marks the rigidification of the Christian Church. Although the Julian calendar removed much of the confusion of civil reckoning of time, and was adopted throughout the Roman Empire, the *moon* continued to cause complications. The feasts of the Jewish and Christian years were fixed by the moon; the Passover was set by the date of the full moon in the month of which the 14^{th} day (from new moon) fell on or after the *vernal equinox*. The date of Easter, in turn, depended on the date of Passover: most Christians wanted Easter to be the Sunday following the 14^{th} day of the moon (those who placed it *on* the 14^{th} day were regarded as heretics, and called *Quartodecimans*).

The problem of the date of Easter was officially settled at the Council of Nicaea: Easter Day was to be (and still is) the first Sunday after the 14^{th} day of the moon that occurs on, or directly after, the vernal equinox. At that time, March 21 was the date of the vernal equinox. (It was on March 25 in 46 BCE, the date of the Julian reform, but slipped back to the 21^{st} on account of the accumulated discrepancy between the Julian year and the tropical year.) Thus the date of Easter is set by luni-solar reckoning, and its fluctuations by more than a month from year to year are a good illustration of the complication involved. At any rate, the possibility of an occasional coincidence with the Jewish Passover was avoided.

At the bottom of these 'calendar-exercises' was the desire of the Christian Church to separate the Jews from the Christians, segregate them, make Judaism illegal and finally make Judaism disappear altogether.

The fact was that in 325 CE Judaism still appealed to a great many people among the Pagans as well as among Christians. There were many Christians who respected the Synagogue and its traditions as institutions of Mother Religion. Since Christianity was a movement among the lower classes of the empire's society, the more cultured Pagans had more respect for Judaism which had for centuries been a part of the Roman world.

Thus, the bishops prevailed upon the emperor to prevent Pagans from becoming converts to Judaism, and also take away from the Jews one of the political privileges which they had long enjoyed. Many of the Christian clergy, who now stood close to the Roman government, urged the emperors to deal harshly with Judaism. But the Roman government had a great deal of respect for old Roman law, and the law had always permitted Judaism to be practiced. As the Church acquired power, it began to use unfair methods which have flagrantly contradicted the great ideas for which Christianity stands.

The council of Nicea marks an important change in the attitude of the Church towards the Jews. Until now the rivalry for converts had been based upon an idealogical basis. Now, however, the Church took up the fight and carried it on with the assistance of the government. The immediate results were the litigations of empire-wide anti-Jewish laws, religions and economical harassments and violent persecutions, especially in Israel (351 BCE). Many of the Talmudic scholar there had to flee the country and escape to Babylonia.

355–390 CE **Oribasius of Pergamum** (ca 325–400 CE). Greek physician. Personal physician of Emperor Julian. Compiled a medical encyclopedia. Reputed discoverer of the *salivary glands*.

358 CE **Hillel III** (286–365, Israel). Reformer of the Hebrew Calendar. Jewish patriarch in Israel from 330 to 365 BCE. Political persecution made it impossible for the dispersed Jewish communities to communicate each year with the Judean Sanhedrin for the determination of the fasts and feasts. Hillel III fixed the calendar for the whole Diaspora and for all time to come in 358 CE.

Hebrew Mathematics (200 BCE–500 CE)

The Talmud, which literary means "the study", is the name given to a work of many volumes, written partly in Hebrew and partly in Aramaic, which embodies the teaching and opinions, on religious and sociological matters, of the ancient Jewish sages during a period of some 700 years (200 BCE *to 500* CE*). In addition it contains the theological, theosophical and philosophical views, as well as the moral and ethical maxims, of those sages. It is further interwoven with many anatomical, physiological, medical and anthropological observations of scientific character. The Talmud consists of distinct portions:*

(1) The Mishna contains the teachings transmitted from generation to generation by word of mouth. It represents the legal traditions (based on the written law of the Pentateuch) of some 310 eminent scholars over 14 generations (zugoth and Tanaaim) dating from the time of **Simon the Just** *(fl. ca 200* BCE*) to that of* **Yehuda ha-Nasi** *(137–210* CE*),*

who compiled and edited it. It was canonized sometimes during 210–220 CE.

(2) *The Gemara consists of commentaries on the Mishna. It represents the discussions and disputations of the Amoraaim, who were the directors and members of the Babylonian or Jerusalem academies (220–470* CE*). They numbered about 2000 savants, over a span of 7 generations.*

Each school of Amoraaim, that of Babylonia and that of Israel, expounded the Mishna in somewhat different manner from the other: The Talmud Bavli ("Babylonian Talmud") was compiled in the Mesopotamian academies of Nehardea, Sura, and Pumbedita. Its compilation began by **Rav Ashi** *(375–427* CE*) and it was canonized in 525* CE*. The Talmud Jerushalmi ("Jerusalem Talmud") was compiled by the Israeli academies. It is much smaller in size and scope and its discussions do not exhibit the same dialectical acumen as is shown in the Babylonian edition.*

While the primary substance of the Talmud *lay in expounding and developing civil and religious law, a considerable acquaintance with the various branches of* mathematics *was mandatory in connection with such legislation.*

Thus, the determination of the calendar, legislation regarding the sowing of fields, as well as certain other religious observances, demanded a knowledge of branches of mathematics such as algebra, geometry and trigonometry. None of these sciences are dealt with in any systematic manner in the Talmud, but scientific problems are alluded to casually in relation to the legal questions under consideration. Hence we find that statements of mathematical and astronomical interest are scattered about, in a haphazard way, throughout the many thousands of pages of the numerous treatises comprising the Babylonian and Jerusalem Talmuds and their various addenda.

The astronomical facts directly or indirectly mentioned in the Talmud show that the participants of the discussions must have possessed a considerable ingenuity and skill in mathematical computations and geometrical constructions. In fact, as far as astronomical lore is concerned, the Talmud reflects the best definitive knowledge available in the 5^{th} *century* CE*. The following example will substantiate this claim:*

In the field of pure mathematics, the knowledge of the Hebrews equaled that of the Babylonians, Egyptians and Romans, but certainly did not equal that of the Greeks in their geometrical speculations. Not only did they not produce mathematicians on the level of Thales, Pythagoras, Euclid and Archimedes, but there is no evidence whatever that they discovered anything original in mathematical theory.

While the Greeks studied mathematics for its own sake and felt a strong craving to speculate and discover new mathematical facts, the Talmudic sages were satisfied with applying (with varying levels of skill) what simple mathematical tools they possessed to the various practical problems with which they had to deal in their exposition of civil and religious laws. For this reason and others, the foundations *of the classical physical sciences were laid during* 1550–1750 *without direct Jewish participation.*

Thus, for example, the Tannaaim and the Amoraaim used the Babylonian value of $\pi = 3$ *as late as the* 5^{th} *century CE while Archimedes in the* 3^{rd} *century BCE (as well as the Egyptians before him!), had given more exact values. Likewise, they used the Pythagoreans' approximation*[303] $\sqrt{2} \approx 1.4$, *which they needed in their calculations of areas.*

In the field of applied mathematics, *however, the Hebrew scholars exhibited great ingenuity and originality, which led them, serendipitously, to the brink of great discoveries in 5 distinct areas:*

I. Problems of the Hebrew Calendar[304]

Before the departure of the Israelites from Egypt (ca 1250 BCE) their year commenced at the autumnal equinox; *but in order to solemnize the memory of their deliverance, the month of Nisan (or Aviv) in which that event took place, and which falls about the time of the* vernal equinox, *was afterwards regarded as the beginning of the ecclesiastical or legal year. In civil affairs, and in regulation of the Jubilees and sabbatical years, the Jews still adhere to the ancient calendrical year, which begins with the month of Tishri, about*

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303 $\sqrt{2} > \sqrt{\frac{49}{25}} = \frac{7}{5}$ [an *upper bound* is obtained by *Heron's approximation* $\sqrt{a^2+b} \leq a + \frac{b}{2a}$ for $a = 1$, $b = 1$, leading to $\sqrt{2} < \frac{3}{2}$]. They knew that 1.4 is not the exact value for $\sqrt{2}$ but did not bother to estimate the difference [Talmud Yerushalmi, *Erubin* **2, 1**] because further accuracy was not necessary for their applications.

304 For further reading, see:

- Feldman, W.M., *Rabbinical Mathematics and Astronomy*, Hermon Press, 1978, 239 pp.
- Reingold E.M. and H. Dershowitz, *Calendrical Calculations*, Cambridge University Press, 2001, 422 p.

the time of the autumnal equinox. After their dispersion, the Jews found it necessary to utilize the astronomical rules and cycles of the more enlightened heathen, in order that their religious festivals might be observed on the same day in all the countries through which they were scattered.

The canon of the Old Testament was closed with the work of ***Ezra*** *(fl. 458 BCE). He was followed by the Sopherim (scribes), and the sages of the Talmud (200 BCE–500 CE). While the primary work of the Talmudic savants lay in expounding and developing the civil and religious law of the Bible, a considerable acquaintance with the various branches of science was necessary in connection with such legislation.*

The determination of the calendar required sound knowledge of astronomy, since not only were the Jewish festivals fixed on given days of the lunar month, but they also depended on the position of the sun. Passover, for instance, which begins on the 15^{th} *day of the month of Nisan, must also occur in the month of the wheat harvest or Aviv (Deut* **16**, *1). The month of Tishri must take place in the autumn. Further, the position of the moon in relation to the sun, as well as its height above the horizon at any moment, had to be computed mathematically, for the purpose of ascertaining whether the new moon could be visible at that given moment. This procedure was necessary for the purpose of fixing the beginning of every new month.*

In about 200 CE, ***Mar Shmuel*** *proposed to adopt the Metonic cycle of* 19 *solar years, by which the Jewish lunisolar year is regulated to the present day. The determination of the beginning of a month by the Phase Method prevailed until* 358 CE, *when it was replaced by the Fixed Calendar Method which makes use of the Mean Conjunction, or Molad (literally, "birth"), to determine the beginning of the month.*

The Phase Method operated as follows: to determine whether the moon is visible at any moment, and to ascertain the exact moment of true conjunction, it is necessary to know the moon's true position (celestial latitude and longitude) at that moment. First, the moon's mean longitude was determined. Then, the Talmudic rabbis were confronted with the ascertainment of the moon's mean longitude at sunset, or at any given interval after sunset on a given day. (The time of sunset is calculated by first finding the sun's longitude on the day in question, and using the longitude to compute the sun's declination on the same day.)

Prior to Kepler's time, the above calculations were not based on the elliptic motion of the planets. Instead, the moon's path was constructed by means of eccentric circles with epicycles, a device which however ingenious from a mathematical standpoint, is a purely fantastic one, devoid of any physical basis. In such a theory, the moon's apparent diameters when nearest to and furthest from the earth, should be in a proportion of about $2:1$ *instead* *of about 1.14: 1, which is the actual value. Moreover, complicated as the epicyclic theory was, it became still more involved if one tried to explain a few of the numerous irregularities of the lunar orbit.*

The theory of Keplerian motion is not only simple, but has the merit of being true. Indeed, the calculation of solar and lunar longitudes on the basis of an elliptical orbit is extremely simple.

Having found the moon's longitude, the Talmudic rabbis next had to ascertain whether in that position in the heavens the new moon would be visible *in the neighborhood of Jerusalem. For as the beginning of the month was fixed on the accredited evidence of witnesses, who reporting having seen the new moon soon after sunset on a certain day, it was the duty of the Calendar Council not only to test their evidence by stringent cross-examination, but also to ascertain by* mathematical calculations*, whether the moon could in fact be seen at that particular moment, at the particular place from which the witnesses came. (This involves laborious and detailed calculations which were ingeniously perfected in the Middle Ages by* ***Maimonides****, although they were somewhat inaccurate.) The council sat in Jerusalem on the* 30^{th} *of each Hebrew month to receive witnesses. If after cross-examination and further confirmation of their evidence by mathematical calculations, the Council concluded that the new moon was indeed seen by the witnesses at the time they mentioned, that day was proclaimed to be the* 1^{st} *of the month without any further delays.*

A month of 29 days was called a defective month *and one with 30 days was called a* full month*. It was possible for 3 months to be consecutively full if (owing to unfavorable atmospheric conditions) the new moon was not visible. If however, no witness appeared for 3 months, then the beginning of the next month was determined by calculations alone. No year was allowed to have more than 8 or less than 4 full months so that no lunar year lasted more than 356 or less than 352 days.*

When the Court was satisfied that a new moon had actually been seen, the President declared the new moon to be consecrated and news spread to people outside Jerusalem by means of bonfires on the top of the mountains. When the awaited signal was observed at neighboring mountains, similar fire signals were lit, and thus information was rapidly transmitted to distant places.

This method was continued until deliberate attempts to confuse the Jews were made by the Samaritans, who maliciously lit signals at improper times. Therefore, these fire signals were altogether abolished toward the end of the 2^{th} *century* CE *by* ***Yehuda ha-Nasi*** *(137–210* CE*), and couriers were sent instead (seven times a year) to convey the tidings. As these could not reach the more distant places before the* 31^{st} *day of the old month, the residents of those places used to celebrate two days for their holidays because they were* *not sure whether the first of the month was the* 30^{th} *or the* 31^{st} *day. In the case of New Year (Rosh Hashana), two days were (and still are) kept even in Jerusalem, in case witnesses arrived late in the day.*

Note that the beginning of the month was not fixed by the moment of the true conjunction but by the moment the crescent of the new moon was seen — which was at least 18 *hours after the conjunction.*

In the construction of the Fixed Calendar, numerous details require attention. The calendar is dated from the creation, which is considered to have taken place in the year 3761 BCE[305]*. The year is luni-solar with the following structure: as the average length of the synodic month is about* $29\frac{1}{2}$ *days, and as a calendar month must have an integral number of days and must start at the same hour of the day — viz.,* 6 *p.m. — it has been agreed to make a civil month consist alternatively of* 30 *days (full month) and* 29 *days (defective month). Thus, Nisan (1), Sivan (3), Av (5), Tishri (7), Kislev (9) and Shvat (11) are full, while Iyar (2), Tamuz (4), Elul (6), Marheshvan (8), Teveth (10) and Adar (12) are defective in an ordinary regular year consisting of* 354 *days =* 50 *weeks +* 4 *days. This arrangement must, however, be modified in a number of ways due to the following disparities:*

- *The astronomical lunar year exceeds the civil lunar year by*

$$12 \times [29^d : 12^h : 44^m : 3\frac{1}{3}^s - 29^d : 12^h] = 8^h : 48^m : 40^s.$$

  Hence it is necessary, at fixed intervals, to add a day to the ordinary year by making one of the defective months full, giving rise to an ordinary full year of 355 *days. The defective month chosen is Marheshvan. Such a procedure, if adopted, say, every* 3^{rd} *year, necessarily overbalances the disparity, and hence at other (longer) fixed intervals, the excess so introduced is adjusted by subtracting a day from one of the full months, making that particular year only* 353 *days. Such a year is called an ordinary defective year, and the full month chosen for this purpose is Kislev.*

- *The Hebrew calendar, being based on the Metonic cycle of* 19 *years, adopts a fictive mean solar year of length*

$$\frac{235}{19} \times [29^d : 12^h : 44^m : 3\frac{1}{3}^s] = 365^d : 5^h : 55^m : 25\frac{25}{57}^s,$$

305 From Biblical, Talmudic and other traditional lore the period between Adam and the Exodus amounted to 2448 years, and between the latter and the destruction of the 2^{nd} Temple there elapsed another 1380 years, making a total of 3828 years; and as the destruction of the Temple is considered to have taken place in the year 68 BCE, the above fiducial follows.

which exceeds the tropical year by 6.6 minutes. With respect to this year, the lunar year of 12 lunations falls short by $10^d : 21^h : 6^m : 45\frac{25}{57}^s$*. This amounts to* $\frac{10^d : 21^h : 6^m : 45\frac{25}{57}^s}{29^d : 12^h : 44^m : 3\frac{1}{3}^s} = \frac{7}{19}$ *of a synodic month.*

Hence, if in the course of every cycle of 19 years 7 extra months are intercalated, making 12 ordinary years of 12 months each (= 144 lunations), and 7 leap years of 13 months each (= 91 lunations), the sun and the moon will have (almost) exactly the same relative mean positions in the heavens at the end of the cycle as at its beginning (it would be exactly right if one could neglect the 6.6^m *discrepancy between the Hebrew solar year and a tropical year*[306]*). The order of the leap years, in each cycle of 19 years, is*

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306 The 'solar year' of $365^d : 5^h : 55^m : 25.44^s$ is an *arbitrary* figure designed to make the solar year *exactly* $\frac{1}{19}$ of 235 lunations. Since it is about 6.6 minutes *longer* than the present value of the tropical year, the accumulated difference during the 1500 years that the Fixed Calendar has been in use, is about 7 days. Hence, the beginning of Passover (which according to Biblical injunction should fall in the *first spring month*, viz., during the 30 days between the 21^{st} of March and 19^{th} of April) has been shifted forward by a complete week. Indeed, already in 1929 the Passover began on April 25, i.e., 6 days later than the Biblical limit. In order to remedy this it is necessary to *reform the Hebrew Calendar by introducing a different cycle with a different sequence of leap years*. This is done in the following way:

$$\frac{\text{present length of tropical year} - \text{present length of astronomical lunar year}}{\text{present length of synodic month}} \approx$$

$$\frac{10^d : 21^h : 10^s}{29^d : 12^h : 44^m : 3^s} = \frac{939610}{2551443}.$$

Converting this ratio into a continued fraction, we get

$$\cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{17 + \ldots}}}}}}.$$

If we stop at the dotted line, the value of the fraction becomes $\frac{123}{334}$, which means that very approximately the excess of a tropical year over a lunar year is $\frac{123}{334}$ of a synodic month. Hence, if in the course of every cycle of 334 years we introduce 123 leap years, the excess would be practically wiped out. Indeed, such an arrangement would make the 334 modified civil years only about 39 minutes less than 334 tropical years, and it would take about 12,500 years (instead of about 200 years as it does now) for the difference to accumulate to one day.

As $334 = 19 \times 17 + 11$, the order of the leap years could continue as at present

$\{3, 6, 8, 11, 14, 17, 19\}$. *A Hebrew year,* $Y = x + 3761$, *is leap if the residue of* $\frac{Y}{19}$ *is a member of the above series, where* x *is the Gregorian year*[307]. *The intercalary year is inserted as Adar bet ("*2^{nd} *Adar"), just after Adar aleph ("*1^{st} *Adar").*

The Hebrew month begins at 6 p.m. of the day on which the moon is in conjunction (Molad). Hence, if the Molad of any given month is known, that of the next month is ascertained by adding an average lunation $L = 29^d : 12^h : 44^m : 3\frac{1}{3}^s$. *Thus, if the Molad of any given month is known, (say, day number =* M_1*) then that of any other month (*M_2*),* n *months after or before the given month, is found from the relation:* $M_2 = M_1 \pm nL$.

The most important Molad of each year is that of the month of Tishri which determines the New Year Day of that year. Having found the Molad of Tishri of the first year of any given cycle, the Molad of Tishri of any year in that cycle easily follows [e.g., the year 5606 was the first of the 296^{th} *cycle, and the mean new moon appertaining to the* 1^{st} *of Tishri for that year was Oct. 01, 1845,* $15^h : 42^m : 43\frac{1}{3}^s$*].*

There are five separate occasions which necessitate the postponement of the New Year Day by one or even two days. These postponements depend on the exact moment at which a Molad occurs, and necessarily entail a lengthening of the year by one or two days (with the consequent lengthening of one or two months in that year). A full embolismic (leap) year may have up to 385 days, in order that certain festivals may fall on proper days of the week for their due observance.

Note that whereas the accuracy of the phase method hinges on the determination of the appearance of the crescent of the new moon, the accuracy of the Fixed Calendar Method depends on the interval between true and mean conjunction, which is at most 15 hours (positive or negative). In one respect, however, the Phase Method was more accurate: the lunar year was made to keep pace with the solar year by intercalating an extra month whenever there was an actual need for it (as found by the size of the discrepancy between the date of Passover and that of the calculated, or observed, vernal equinox) instead of intercalating at regular intervals – as is done under the Fixed Calendar Method. But on the whole, the advantages of the Fixed Calendar Method considerably outweigh its disadvantages.

The Jewish Calendar was progressively improved during the ages by three distinguished astronomers:

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for 17 complete Meton cycles of 19 years, as well as the first 11 years of the 18^{th} cycle. After that, a new 343 year's cycle would have to begin.

307 We ignore here the Gregorian calendar's own slight fluctuations about the earth's (current) astronomical tropical year.

- ***Rabban Gamliel II*** *(30–117 CE)*
- ***Shmuel*** *(165–254 CE)*
- ***Hillel III***, *(286–365 CE)*

In the year 357 CE, political persecutions made it impossible for the dispersed Jewish communities to communicate each year with the Judean Sanhedrin for the determination of the fasts and feasts. Hillel III thus fixed the calendar for the whole Diaspora and for all time to come in 358.

II. General Astronomy and Cosmology

Although there was no Hipparchos or Ptolemy amongst them, the Rabbis certainly produced several astronomers of considerable merit: ***Yohanan ben Zakkai*** *(7 BCE–77 CE);* ***Gamliel II*** *(ca 30–117, Yavne);* ***Joshua ben Hannania*** *(35–117);* ***Shmuel*** *(165–254);* ***Eliezer Hisma***[308] *(ca 60–120);* ***Yohanan ben Gudgada*** *(ca 60–120);* ***Jossi ben Halafta***[309] *(ca 95–150);* ***Nathan of Babylon*** *(ca 90–160);* ***Abayei***[310] *(278–338);* ***Hillel III*** *(286–365).*

The physical nature of comets and their orbits was unknown until Newton showed them to move in ellipses or in parabolas. It is therefore not surprising that the Talmudic astronomer ***Shmuel*** *(ca 200 CE) acknowledged his ignorance of the nature of these celestial bodies (Berachot* **58**^{b}*). It is, however, remarkable that although, until the middle of the* 17^{th} *century, even the most civilized nations did not know the period of any comet, the period of at least*

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308 Rabban Gamliel II was rebuked by Rabbi Joshua ben Hannania for having refused to promote his pupils Eliezer Hisma and Yohanan ben Gudgada in spite of their great excellence. Consequently, Gamliel sent for them offering them promotion, but in their great humility they refused to accept it. He then sent for them a second time and said: "*Do you suppose I am offering you greatness? It is slavery that I am offering you since it is said* (*I Kings* **12**, 7) *that a king is a servant of his people*".

309 **Jossi ben Halafta** defined the differential positions of the sun in the heavens at different times of the year as well as the relation between the ecliptic and the equator [*Erubin* **56**^{a}].

310 **Abayei** understood the motion of the planets very well and knew the 28 year Solar Cycle [*Berachot* **59**^{b}].

one comet was known to **Joshua ben Hannania** *(ca 85* CE*), for he said [Horayoth* $\mathbf{10}^a$*]:*

"A star appears every 70 *years and leads ships astray" (by causing the captains to steer by it erroneously). Apparently Joshua possessed some data on the apparitions of comet Halley during his lifetime and before his time [87 BCE, 12 BCE, 66* CE*].*

The biblical account of the creation of the universe *is presented in* Genesis **1***, 1–4; it is very incomplete but highly attractive, because it is pervaded by a breath from primitive times.*

The Talmudic scholars noticed immediately that in the biblical cosmology, the existence of light apart from the sun is presupposed. Their way of interpreting the biblical revelation is not inconsistent with the current Big Bang theory:

- *"Rabbi Abahu said: God used to create universes and destroy them, recreate them and destroy them again..." (Genesis Rabba* **9***).*
- *"Rabbi Judah said in the name of Rav: When God created the universe, it kept expanding until God ordered it to halt, and it stopped" (Hagiga* **12***).*
- *"The universe is made in the form of a sphere" (Yerushalmi, Avoda Zara* **3***).*
- *"Rabbi Eleazar said: The light that God created on the first day, is used by man to observe the Universe from one end to the other" (Hagiga* **12***).*

Another important astronomical problem was the determination of sidereal periods *on the basis of* synodic periods*.*

Synodic period *is defined as the time of orbital revolution of a planet (or asteroid, or earth's moon) w.r.t. the sun-earth line. In practice, it is measured as the time interval between two successive conjunctions or between two successive oppositions. At their conjunctions, Mercury and Venus are not visible, but by interpolations carried out over many years (or even centuries), these times have seen computed, and the synodic periods are known with great accuracy. Because of the* elliptical *orbits of earth and other planets (and asteroids) about the sun – and of the moon about the earth – the times of conjunction or opposition do not occur with exact regularity.*

The synodic periods result from the difference between sidereal periods of the planets and that of the earth. The more distant the planet is from the sun, the slower it travels and the longer is its sidereal period (the time required for a planet to complete one orbit around the sun). The inner planets gain a lap on the earth between similar conjunctions, whereas the outer planets lose a lap between oppositions.

The relation between sidereal and synodic period can be developed by determining the fraction of a lap gained or lost each day. Thus (ignoring non-uniform angular speeds attributed to orbital ellipticities), we denote by $\frac{1}{S_i}$ *the fraction of its orbit that a planet travels in one day, by* $\frac{1}{E}$ *the fraction of its orbit the earth travels in one day, and by* $\frac{1}{S_y}$ *the fraction of a lap that the planet gains or loses on the earth each day. Here* S_i *and* E *denote the mean sidereal years of the given planet and earth, respectively, while* S_y *denotes the planet's mean synodic period.*

We find in the Talmud (Shabat **75**^{a}*; Genesis Raba* **10***, 4) that the calculated sidereal periods of the planets Venus, Mars, Jupiter and Saturn are respectively* 10 *months,* $1\frac{1}{2}$ *years,* 12 *years and* 30 *years. Clearly these periods could not have been directly observed, nor precisely calculated before the times of Kepler, Galileo and Newton. The only data that the Talmudic sages could have used were the synodic periods of the five planets, known to* ***Hipparchos*** *(ca 150 BCE). With the aid of these, the sidereal periods can be evaluated arithmetically, using the simple approximate relations based on additivity of angular speeds:*

$$\frac{1}{S_i} = \frac{1}{E} + \frac{1}{S_y} \qquad \text{(Mercury, Venus)}$$

$$\frac{1}{S_i} = \frac{1}{E} - \frac{1}{S_y} \qquad \text{(Mars, Jupiter, Saturn)}$$

where

S_i = *sidereal period (m days, say)*
S_y = *synodic period*
E = *365.2422 mean solar days (tropical year).*

The following table compares modern values with the results quoted by the Talmudic savants:

Planet	*Synodic period (mean solar days)*	*Sidereal period (tropical years)*	
		Talmud (ca 450 CE)	*Modern*
Venus	*584*	*0.8*	*0.615*
Mars	*780*	*1.5*	*1.881*
Jupiter	*399*	*12*	*11.865*
Saturn	*378*	*30*	*29.650*

III. The Eulerian limit $\frac{1}{e} \approx \left(1 - \frac{1}{n}\right)^n$, n large [Euler 1748].

The Talmudic scholars needed to know how to divide an inheritance among ten daughters and one son in a rational way such that the son is not deprived [Jerusalem Talmud, Ketuboth **6***, 6; Babylonian Talmud, Nedarim* **39***].*

If each daughter took an equal share of ten percent, the poor son would be left empty handed. So, ***Rabbi Yehuda ha-Nasi*** *(fl 180 CE) devised the following scheme: the first daughter takes $\frac{1}{10}$ of the property value, the second daughter receives $\frac{1}{10}$ of the remaining property, etc. Finally, the son gets what is left after all ten daughters collected their shares. Clearly, the share of the k^{th} daughter is $\frac{1}{10}(1 - \frac{1}{10})^{k-1}$, $k = 1, 2, ..., 10$, while the son gets*

$$\left(1 - \frac{1}{10}\right)^{10} = 0.348678440 > \frac{1}{3}.$$

Now, the Talmud says that: "...the son receives a little bit more than one third and the daughters receive together a little bit less than two-thirds...".

There is no way that the Rabbis in the 2^{nd} century CE could have calculated the value of $(1 - \frac{1}{10})^{10}$ with the mathematical tools available in the Hellenistic period. Although they were acquainted with the contemporary Alexandrian mathematics of **Heron** *(90–120 CE),* **Nicomachos***(100 CE), and the earlier results of* **Euclid** *(300 BCE),* **Archimedes** *(250 BCE) and* **Apollonios** *(230 BCE), this feat was beyond the capability of even Greek mathematics.*

My guess is, therefore, that the result $(1 - \frac{1}{10})^{10} > \frac{1}{3}$ was derived either empirically or geometrically (e.g. using Euclid's method of dividing a line segment into n equal parts).

Only in the 17th century, with Napier's logarithms (1614) and Newton's Binomial Theorem (1676), could one see that

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = \lim_{n\to\infty}\left[1+1+\frac{n(n-1)}{2!n^2}+\frac{n(n-1)(n-2)}{3!n^3}+\cdots\right]$$
$$= 2+\frac{1}{2!}+\frac{1}{3!}+\cdots = 2.718281828459045...$$

and

$$\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n = \frac{1}{e} = 0.36787944...$$

IV. Probability – Prior and Posterior Probabilities – Bayes' Theorem (1736)

The factor that ensured the primacy of Jewish thinkers in pioneering ways of thinking about chance in practical situations was that Orthodox Jews were forbidden to gamble. The Greeks might have discovered probability, for they were addicted to dice-throwing. But the dice they used were made from the astragalus bones of a sheep (what would be the knuckle-bones if sheep had knuckles). These were very irregular in shape, so that dice were biased and there was not an equal likelihood of each side falling uppermost. Because of the astragalus' imbalance, it was not a random device, so it was impossible for the statistical law of large numbers ('If the number of tests of an event is large, then the proportion of successes in the tests is close to the likelihood of the event') to manifest itself. By contrast, the elaborate system of casting lots used by Jewish priests did exactly what it was devised to do; that is, gave everyone an equal chance of being chosen.

The historical origin of the lottery system is the annual choice of one of two goats to be the 'scapegoat', carrying the heavy load of sin into the wilderness as the Goat of Azazel. (The other goat was sacrificed). Since the choice here was between only two participants, it would be demonstrable that the chances of being chosen were equal for both. This idea could be easily extended to include the whole group of temple priests who were daily caught up in casting lots for all sorts of purposes. The concept of an equal chance of being chosen in a 'fair' lottery would be the prime motivation of the whole system. In turn, the notion of equal probabilities, and the fact that such chances would be shown by the equal numbers of votes for each participant over a period, would naturally emerge. [And indeed, the statistical law of large numbers, the *formulation of which was credited to* ***Jacob Bernoulli*** *(1709), actually was first declared by Rabbi* ***Isaac Aramah*** *(1420–1484, Spain)].*

Consider the following example (Babylonian Talmud: Yevamoth **4***): A widow married her brother-in-law three months after her husband's death and there were no signs that the widow was pregnant prior to her second marriage. Six months after the marriage she gave birth to a child. Was the child full-term baby (the first husband being the father) or was it premature (the second husband being the father)? The question was relevant to the child's inheritance rights. If the mother was already pregnant by the first husband when she married his brother (no visible sign of pregnancy) it would affect the child's right to inherit from the brother. If would also invalidate the second marriage and complicate the widow's right to support. Thus a good deal hinged on the court's decision. Two cases must therefore be weighted against each other:*

a. *The deceased husband is the father (full-term baby) while the woman showed no signs of pregnancy. In ancient Israel the probability of the first premise was* $\frac{9}{10}$ *while the probability of the second premise was* $\frac{2}{10}$. *The product of these probabilities is* $\frac{9}{10} \times \frac{2}{10} = \frac{18}{100}$.

b. *The second husband is taken to be the father with probability of* $\frac{1}{10}$. *Since in this case the widow would show no sign of pregnancy with certainty, the overall probability here is* $\frac{1}{10} \times 1 = \frac{1}{10}$.

The judges rightly rejected a third possibility that the two partners breached the law by having intercourse within the 3-month waiting period. This would invalidate a levirate marriage and work against the interests of the widow herself and her offspring. The probability that the widow would intentionally harm herself is very low.

The comparison is therefore between a probability of 18 chances in 100 that the deceased husband is the father, as against only 10 in 100 that his brother fathered the child. The first or deceased husband is nearly twice as likely as his brother to be the father.

In Europe, it was only in 1736 that this kind of double-barreled question could be asked, and answered precisely. A scientific formula was worked out by ***Thomas Bayes****. (His theorem states that if there is no ground to believe one of a set of alternative hypotheses rather than another, their prior probabilities are equal. When in addition, posterior evidence is available, then in retrospect the most probable hypothesis is the one that would have been most likely to lead to that evidence).*

The rabbis solved the problem much earlier, but expressed the argument in words, not numbers. They also thought of the analysis as a way to solve moral and legal problems, not as an end in itself. The degree of precision that they aimed at was quite adequate for this.

Working with whole numbers, we can verify the rabbinical analysis by means of Bayes' theorem. His method was as follows: work out the relative probabilities (in this case 18 and 10), add these to give the total probability, 28; then divide each relative probability by this total. Using his theorem, we can confirm that the court decision was correct. The chances that the deceased husband was the father are 9 in 14; those that his brother was the father are 5 in 14. The relative probabilities remain the same, *but we now know the* absolute probabilities *as well. The chances are similar in both cases (but not quite 50:50). They still favor the deceased husband as father by about* $6\frac{1}{2}$ *in 10, against about* $3\frac{1}{2}$ *in 10 in favor of his brother as the father.*

There is no suggestion here that the Talmud *discussion anticipated Bayes' theorem. It lacks the clarity of Bayes' analysis. We must remain content with relative, not absolute probabilities. None the less, the rabbis understood the logic underlying this analysis. They recognized the need for an estimate of prior and posterior probabilities, of evaluating (if only in words) the different levels of credibility of different hypotheses, and the need for some method of establishing these results.*

V. Game Theory

Although modern game theory proper began in the 1920's, its roots are 2000 years old. Indeed, in the Mishna *(*Seder *'Nashim',* Masehet *Kethubot* **10**, *4) we find a* marriage contract problem: *a man has three wives whose marriage contracts specify that in the case of his death they receive* $d_i = 100$, *200 and 300 Dinars respectively, with* i *ranging over* $1, 2, 3$. *How should the value of the property be divided among the three women?*

The Mishna stipulates the divisions as follows:

Value of estate, E	*Contract specification*		
	wife I	*wife II*	*wife III*
	$d_1 = 100$	$d_2 = 200$	$d_3 = 300$
100	$33\frac{1}{3}$	$33\frac{1}{3}$	$33\frac{1}{3}$
200	*50*	*75*	*75*
300	*50*	*100*	*150*

When $E = 100$, *equal division makes a good sense, because when the estate does not exceed the smallest contract debt* $\min(d_i)$, *any amount of debt to one person that goes beyond the entire estate might well be considered irrelevant – you cannot get more than there is.*

The case $E = 300$ *is based, apparently, on a different principle of proportional division. The figures for* $E = 200$ *look mysterious: but whatever they mean, they do not fit any simple extension of either equal or proportional division.*

The principle behind the Talmud solution for $E = 200$ *is based upon a famous Mishna ruling (Baba Metzia 2a) which states: "Two hold a garment; one claims it all, the other claims half. Then the one is awarded* $\frac{3}{4}$, *the other* $\frac{1}{4}$*." The principle is clear: the lesser claimant concedes half the garment to the greater one. It is only the remaining half that is at issue; this remaining half is therefore divided equally (this is quite different from the proportional division). The relevancy to the marriage contract problem is immediate: When the estate value* $E \leq min(d_i) = 100$, *all claimants divide it equally. As E grows, this continues until all have received* $\frac{1}{2}d_1 = 50$. *At this point the lesser claimant stops receiving payments and each additional payment is divided equally between the remaining claimants until* $E = d_2 = 200$, *for which case the algorithm leads to the division 50, 50+25, 50+25.*

A common rational for all three cases is not apparent. Consequently, this particular Mishna has baffled Talmudic scholars for two millennia. In 1985, it was recognized [Aumann[311] *R.J., and M. Maschler, J. Economic Theory* **36**, *195–213] that the Talmud had anticipated the modern theory of Cooperative games.*

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311 Won the 2005 Nobel Prize for Economy for his contribution to the theory of games.

Of course, it is unlikely that the sages of the Mishna were familiar with the general notion of a coalitional game. But it can be shown that the ruling fits well with other Talmudic principles *which are independent of game theory. However, without modern Game Theory, it is unlikely that one could have hit on this retroanalysis.*

VI. Combinatorics – The Josephus Problem

We know from the Mishna (Yoma **2***, 2), Jerusalem Talmud (Yoma* **22***, 2) and the Babylonian Talmud (Yoma* **2***, 1), that the priests of the second Temple [515 BCE–70 CE] used to win their various daily service-jobs by the following* 'lottery' *arrangement: they stood in a circle, each pointing one or two fingers toward a man in charge at the center. This man would then announce a number (usually 100 or 60), which was larger than the total number of participating priests, and then* count fingers *in a specified direction from a certain fiducial person, ending the count of the preassigned number at the winner.*

The Latin writer ***Hegesippus*** *(340–397 CE) tells us that the Jewish historian* ***Josephus*** *(37–100 CE) saved his life by knowing the solution to a simpler* deterministic *problem. According to his account, after the Roman captured Yodfat (67 CE), Josephus (the Northern commander of the Jewish revolt) and 40 other warriors took refuge in a cave. His companions were resolved to die rather than fall into the hands of the Romans. Josephus and one friend, not wishing to die, yet not daring to dissent openly, feigned to agree. Josephus even proposed an arrangement by which the deaths might take place in an orderly manner: the men were to arrange themselves in a circle; then every third man was to be killed until but one was left, and he must commit suicide. Josephus and his friend placed themselves in places 16 and 31, thus guaranteeing their survival.*

There is no closed-form solution to the Josephus problem for the general case, not even a recurrence relation.

369–375 CE *Hospital* of St. Basil is built in Caesarea, Israel (369). It heralded start of hospital-building movement by Christians. Charity hospital with 300 beds for plague victims was set at Edessa in the Eastern Roman Empire (375 CE).

378, Aug 09 CE *Battle of Adrianople* (480 km west of Constantinople) between the Romans and the Visigoths. The Visigoths defeated and killed the Emperor Valens. It signifies the beginning of the collapse of the Roman Empire under the pressure of the Barbarian's demographic invasion. Two years earlier, Valens allowed them to cross the Danube and settle in lower Moesia. Faced with the unprecedented problem of these refugees, the Roman government bungled the administration, failed really to disarm the Goths and ultimately had to fight a two-year war with them.

This defeat of the Roman infantry by mounted warriors forecast the revolution in the art of war which determined the military, social, and political development of Europe throughout the Middle Ages.

390–405 CE **Eusebius** (**Sophronius**) **Hieronymus** (Jerome, ca 345–420 CE). Christian Latin scholar and writer. Translated the Old Testament directly from the Hebrew, with the aid of Jewish scholars. The translation is known as the *Vulgate*.

Hieronymus was born to wealthy Christian parents at Strido (Dalmatia) and received a grammatical rhetorical and philosophical education at Rome. In 373 he went on a pilgrimage to the Holy Land. In 385 CE he settled at Bethlehem and engaged in literary activity for the remaining years of his life.

His knowledge of Greek, Latin and Hebrew and his excellence as a scholar made him the ideal erudite for translation of the bible.

393 CE Chinese observed a supernova in the constellation Scorpio. It remained visible for eight months.

397–426 CE **Aurelius Augustinus** (Augustine, 354–430 CE, Numidia). Platonist philosopher and metaphysician who had a marked influence on **Pascal**, **Descartes**, **Leibniz** and **Kant**. Although concerned principally with matters of religious doctrine, and although his conclusions might be classified as highly motivated, his philosophy of history enunciated a doctrine that was impressed upon the consciousness of Western civilization until the time that **Hegel**'s dialectic and **Comte**'s positivistic approaches shed new lights on the possibilities of history. Thus Augustine's influence on subsequent thought is not confined to theology. As a link between the Pagan and the Christian world, he reinterpreted the ideals of antiquity, and in a sense created the modern soul with its conflicts and its unfathomable depths.

To science, Augustine contributed the first serious elaboration on the concept of *time* since Aristotle. He emphasized the *subjective character of time* as being part of the mental experience of man, and placed *historical time* as intermediary between *subjective time* and *physical time* (the same subjective interpretation of time is found in Kant, who makes it a form of understanding). This approach led Augustine to foreshadow the Cartesian doctrine that the only thing that one cannot doubt is what one thinks. One of the features of time that puzzled Augustine was the difficulty of defining it.

Augustine was born at Tagaste in the African province of Numidia (now Constantine, Algeria), a son of a Pagan father and a pious Christian mother. He received a thoroughly Roman education and at twenty went to Rome together with his mistress and their young son. A little later we find him in Milan where he made his living as a teacher of rhetoric. On the religious side he was a Menichaean during this period. But in the end, the continued pressure of remorse and a scheming mother brought him within the orthodox fold. In 387 he was baptized, and ordained a priest in 391. He returned to Africa, became Bishop of Hippo in 396 and so remained until his death.

Augustine lived in the period of the disintegration of the Roman world, and his time was marked by the fall of the Western Empire[312]. (In fact, Rome was sacked by Alaric's Goths (410) during his own lifetime.) It was the end of the twilight era between the decline of Hellenistic Paganism and the advent of the Judeo-Christian one-way view of time in the philosophy of history. Thoroughly impregnated with the Judaistic notion of creation out of nothing, Augustine set himself to counter the Greek pantheistic view, for which God is the world, and adopt the Creator of the Old Testament, a God outside the world, a timeless spirit, not subject to causality or historical development; when he created the world, he created time along with it[313].

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312 Large-scale trade had collapsed, and slavery had disintegrated too, as supplies dried up and the population fell. The empire was a society based on impoverished serfs, bound to the land and raising subsistence foodstuffs for powerful landlords: virtually the entire population was reduced to a level of not much above slaves. Diocletian took the road of repression with savage and widespread persecutions. When this failed, Constantine took the other road of merger, converting to Christianity and almost immediately subordinating the Church to imperial rule. Many Christians revolted against the idea of alliance with the empire they had fought so long. But many more saw the advantages to the Church of a new powerful friend. Augustine was the most prominent of the latter. He formulated the ideology of the new alliance of Church and state, an alliance that would shape the next 1000 years of Western history.

313 To the Greeks, it would have seemed quite absurd that the world could be conjured up out of nothing. If God created the world, he is to be thought of as the master builder who constructs from raw materials that are already there. That something could come from nothing was alien to the scientific temper of the Greek mind.

ca 400 CE Invention of the Candle[314] (Lat, *Candela*, form *candera*, to glow). Relatively few candles were used in the home until about the 14^{th} century, however they were an important symbol of the *Christian* religion. The best candles were made of beeswax and were used chiefly in church rituals because the bee was regarded as a symbol of purity. But because beeswax was expensive, crude tallow candles had to be used by the common people. Tallow was smelly and smoky. The candles dripped badly and generally gave a feeble light.

The energy effective in the form of light emitted by a source of light through a given surface per unit time per unit solid angle is known as the *luminous intensity* of one *candle-power*. In 1948 a modern unit was adopted and named the *candela* (cd). One candela is equal to $\frac{1}{60}$ of the luminous intensity per square centimeter of a blackbody radiation at a temperature at which platinum solidifies (2042 degrees Kelvin).

ca 400 CE Chinese mathematician **Sun Tsu** in his *Suan Ching* (Mathematical Manual) began the study of indeterminate equations of the first degree.

400–500 CE End of compilation and codification of the *Jerusalem Talmud*[315] (ca 400 CE) and the *Babylonian Talmud* (ca 500 CE). It is an encyclopedia of knowledge which incorporates the collective endeavor of about 2000 scholars (known as *Amoraaim*[316]) over a period of 300 years (7 generations) both in Israel and Babylonia (219–500 CE).

The *leading* scholars in the *Babylonian academies* are listed in Table 1.6.

Table 1.6: Leading scholars of the Hebrew Babylonian academies (Amoraaim)

1st generation	**Rav** (Rabbi Abba bar Ayvu)	(175–247 CE)
	Shmuel the Astronomer	(165–254 CE)
2nd generation	**Rabbi Huna**	(218–298 CE)
	Judah bar Ezechiel	(ca 210–300 CE)
	Rabbi Hisda	(217–310 CE)

[314] The biblical 'candlestick' (*ner* in Hebrew) had a wick of flax which was saturated with oil in the container. Although there is no direct description of a lamp in the Bible, it was certainly not made of solid fatty or waxy matter.

[315] Hebrew: *teaching*, *learning*; the name *Gemara* is used for the *Talmud* without the *Mishna*.

[316] Aramaic: *speakers*, *interpreters*.

Table 1.6: (Cont.)

3rd generation	**Raba bar Nahmani**	(261–321 CE)
	Rav Joseph bar Hamma	(ca 260–323 CE)
	Rabbi Zeira	(ca 250–325 CE)
4th generation	**Abayei**	(278–338 CE)
	Raba	(278–352 CE)
5th generation	**Rabbi Pappa**	(ca 300–372 CE)
6th generation	**Rav Ashi**	(352–427 CE)
	Rabbina I	(334–422 CE)
7th generation	**Rabbi Tosfaah**	(ca 410–474CE)
	Rabbina II	(ca 415–475 CE)

The final editing of the Babylonian Talmud began with Rav Ashi and Rabbina I and ended with Rabbi Tosfaah and Rabbina II. By the year 500 CE, no new material was added to the Talmud. The last of the Babylonian *Amoraaim* was Rabbina II. The Parthian *Arsacid* Empire existed during 129–226 CE. The new *Sassanian* Persian Empire followed through 226–542 CE.

The *Jerusalem Talmud* was created by 5 generations of scholars. Their leading members flourished in the academies of Tiberias, Zippori (Sepphoris), Caesarea and Lydda (Table 1.7).

Table 1.7: Leading scholars of the Jerusalem Talmud

1st generation	**Hiyya the Great**	(Tiberias; 155–240)
	Yannai	(Zippori; 165–250)
	Hoshaya Rabba	(Caesarea; ca 180–240)
	Hanina bar Hama	(Zippori; 155–240)
2nd generation	**Yohanan bar Naphha**	(Zippori and Tiberias; 190–279)
	Shimon ben Lakish	(Zippori and Tiberias; ca 195–280)
	Eleazar ben Pedat	(Tiberias; ca 210–289)
3rd generation	**Hiyya bar Abba**	
	Abahu	(Caesrea; fl. 300)
	Ammi	(Tiberias; ca 220–305)

Table 1.7: (Cont.)

	Assi	(245–315)
4th generation	**Yossi**	(Tiberias; d. 351) [*Moed Katan* **25**]
	Jeremiah	(Tiberias; d. 350)
	Hillel III	(Zippori; 286–365)
5th generation	**Yona**	(Zippori and Tiberias; fl. 375)
	Yossi bar Avin	(Tiberias; fl. 375)
	Tanhuma bar Abba	(Tiberias; fl. 390)

By the time Rome succeeded in driving the Jewish community of Israel into poverty, ineffectiveness and obscurity, the Jewish community in Babylon has awakened from a slumber of centuries and was ready to carry on. In their relative safety, under the Parthian rule, they established schools and led an active intellectual life, continuing the traditions of Judaism that begun and developed in Israel.

The downfall of the *Hasmoneans* (37 BCE), the defeat of the independence war against the Romans (70 CE) and the failure of the Bar-Kochba insurrection (135 CE), were three occasions where the Babylonian Jewish population received a large influx of Israeli refugees. But during the intervening periods, too, a steady exchange of scholars went between the two centers. Before 135 CE, Babylonian scholars went to the Israel academies (e.g., Hillel, 40 CE) to complete their education and staying there. The opposite example is that of **Rav** (219 CE) who had been a pupil of **Yehuda ha-Nasi** and returned to Babylonia to establish the Academy of Sura.

Despite war and other misfortunes, the Israel academies kept up their studies. The completion of the *Mishna* (ca 200 CE) served a further incentive for discussions and applying the laws and customs of Jewish life. More teachers arose, and more students came to their academies; every town had its school for children, and it was the Mishna which now became a sort of textbook, every sentence of which gave rise to discussions. In this very fashion, the *Jerusalem Talmud* was created during 200–400 CE. However, the persecutions of Constantine brought about the decay of the Israeli schools and there was an imminent danger that the oral tradition, achieved by generations of great scholars and sages was likely to be forgotten. Thus, following the riots of 351 CE it was decided to *collect and arrange* the existing material, which was never quite completed. This task fell to the 5th generation of Israeli scholars in Tiberias and the process finally ended in 429 CE.

At the beginning of the 5^{th} century **Rabbi Ashi** took in hand the *arranging* of the framework of the Babylonian Talmud. Under **Rabbi Tosfaah** and **Rabbina II**, heads of the Academy of Sura[317], the recension of the Babylonian Talmud became practically complete (499 CE). As the Babylonian schools decayed, Talmudic learning was assiduously pursued outside its oriental home and some Babylonian Talmudists apparently reached the West. After the compilation of the Talmud, the commentaries and addenda have never ceased up to the present days. In the Middle Ages, the philosopher **Maimonides** (fl. 1180), the commentator **Rashi** (fl. 1080), and the codifier **Joseph Karo** (1488–1575) were among those who brought about a renaissance of Talmudic study in Western Europe.

Although the Talmud is an academic product and may be characterized in the main as a report of the discussions of the schools, it also sheds a flood of light on the culture of the people outside the academies. The Talmud thus became an important source for the history of civilization, discussing the most varied branches of human knowledge – astronomy and medicine, mathematics and law, anatomy and botany, and *furnishing valuable data for the history of science too.*

The peculiar form of the Talmud is due to the fact that it is composed almost entirely of individual sayings and discussions on them, this circumstance being a result of its origin: the fact that it sought especially to preserve the *oral tradition* and the *transaction of the academies*, allowed the introduction only of the single sentences which represented the contributions of teachers and scholars to the discussions.

The Talmud preserved and fostered in the Diaspora, for many centuries and under most adverse external conditions, the spirit of deep religion and strict morality. Moreover, it had an exceedingly stimulating influence upon the intellectual powers of the Jewish people, which were then directed toward other department of knowledge. The Talmud gradually became an intellectual activity having no ulterior object in view – a model of study for the sake of study.

The excessive legalism which pervades the Talmud was the scholarship of the age. Talmudic discussions are sometimes more exhibition of dialectic skill, but it is this predilection for casuistry that impelled Jewish scholars of the Middle Ages to study or translate the learning of the Greeks. *This trend played a necessary part in the development of European science and philosophy* and benefited common humanity.

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[317] The other two great academies in Babylon were *Nehardea* and *Pumbedita*. In Israel, learning flourished in Caesarea, Tiberias, Zippori, Usha, Shefaraam and Lydda.

Table 1.8: HEBREW ASTRONOMERS $^{(A)}$ AND MATHEMATICIANS $^{(M)}$ (70–365 CE)

fl.			
50 CE	Yohanan ben Zakkai (A)	07 BCE–77 CE	Israel
70 CE	Gamliel II (A)	30–117 CE	Israel
85 CE	Joshua ben Hannania (A)	35–117 CE	Israel
150 CE	Nehemiah (M)	120–180 CE	Israel
180 CE	Yehuda ha-Nasi (M)	137–210 CE	Israel
200 CE	Shmuel (A)	165–254 CE	Babylon
315 CE	Abayei (A)	278–338 CE	Babylon
340 CE	Hillel III (A)	286–365 CE	Babylon

409–418 CE **Pelagius** (354–420 CE, Rome and Jerusalem). Early British theologian. Revived the Ionian's idea of a nature distinct from the human will, a nature whose working and processes can be learned by *observation*. He taught that nature is ruled by a process that all can see, not by punishment devised by a capricious God.

Born in Ireland (his name graecized from the Cymric 'Morgan'), he came to Rome in ca 405 and by 409 CE refuted Augustinian doctrines of predestination and total depravity, asserting freedom of will. After the sack of Rome by the Goths (410), he crossed to Africa and met Augustine. He proceeded to the Land of Israel (410), where he was accused of heresy, but acquitted by the Synod of Jerusalem (415). Innocent I called upon him to abjure his teachings and later excommunicated him (417), banishing him from Rome in 418. Pelagius expounded his worldviews at a point on the time scale midway between Thales and Copernicus. His disciple, Julian, exclaimed: "*The merit of one single person is not such that it could change the structure of the universe itself*". Words like these were spoken for the first time in a millennium, and will not be spoken for yet another millennium when the Ionian methods again became accepted wisdom.

Pelagius thus planted a seed that would blossom only at the time of Copernicus. But before the rise of the scientific worldview would occur, the two central concepts of medieval cosmology had to be overthrown – the idea of a decaying universe, finite in time and space, and the belief that the world could be known merely through reason and authority.

5th century Chinese solved problems with arithmetical series.

415 CE Since 300 BCE the city of *Alexandria* reigned as the world's foremost center of science, engineering and philosophy. There lived and created **Euclid** (mathematics), **Eratosthenes** and **Aristarchos** (astronomy), **Dionysios of Thrace** (ca 100 BCE) (language), **Herophilos** (physiology), **Hero** (engineering), **Apollonios** (mathematics), **Diophantos** (mathematics), **Archimedes** (mechanics), **Ptolemy** (astronomy and geography) and **Hypatia**[318] (mathematics and astronomy). Famous for its research facilities, observatory, Museum and above all its legendary library in which the accumulated lore of the ancient world lay encapsulated in the form of some 700,000 papyrus scrolls.

In 389 an edict of Emperor Theodosius I ordered the destruction of the Serapeum, and its books were pillaged by the Christians. But as often happens in history, the destruction of books was followed by the destruction of people. In March 415, **Hypatia** (b. 370 CE), the first woman mathematician to be mentioned in the annals of mathematics and one of the first martyrs of science, was barbarously killed by Christian fundamentalists in Alexandria.

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[318] Her father, **Theon of Alexandria** (ca 335–395 CE) was a professor of mathematics and astronomy. Further details about her are found in Osen, L.M., *Women in Mathematics*, Massachusetts Institute of Technology Press: Cambridge, 1988, 185 pp.

History of Biology and Medicine[319] *I – Ancient Time*

From early times, perhaps predating the appearance of modern humans, people must have had and passed on knowledge about plants and animals to increase their chances for survival. For example, they had to know how to avoid (or sometimes use) poisonous plants and animals and how to track, capture and butcher different species of animals. They had to know which plants could be prepared to make good nets or baskets. In this sense, biology predates the written history of humans.

Agriculture requires specialized knowledge on plants and animals. Ancient Oriental people knew about the pollination of date palm from a very early point of time. In Mesopotamia they knew that pollen could be used in fertilizing plants. A business contract of the Hammurabi period (c. 1800 BCE) mentions flowers of the date palm as an article of commerce.

In India texts described some aspects of bird life. In Egypt the metamorphosis of insects and frogs was described. Egyptians and Babylonians also knew of anatomy and physiology in various forms. In Mesopotamia, animals were sometimes kept in what can be described as the first zoological gardens.

However, superstition often blended with facts. In Babylon and Assyria organs of animals were used in prediction, and in Egypt medicine included a large amount of mysticism.

The biological sciences emerged from traditions of medicine and natural history reaching back to the ancient Greeks. For example, the idea of biological evolution was supported in ancient times, notably among Hellenists such as ***Anaximander****,* ***Empedocles****,* ***Democritos*** *and his student* ***Epicuros****. As early as 400 BCE the Greek atomists taught that the sun, earth, life, humans, civilization, and society emerged over aeons without divine intervention.*

Later on, in the Greek and the Hellenistic worlds scholars became more interested in empiricism. ***Aristotle*** *is one of the most prolific natural philosophers of Antiquity. He made countless observations of nature, especially*

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[319] For further reading, see:

- *Timetables of Medicine*, Black Dog & Leventhal Publishers, 2000, 72 pp.
- Sutcliffe, J. and N. Duin, *A History of Medicine*, Barnes and Noble: New York, 1992, 255 pp.
- Mayr, Ernst, *The Growth of Biological Thought: Diversity, Evolution and Inheritance*, Harvard University Press, 1982.

the habits and attributes of plants and animals in the world around him, which he devoted considerable attention to categorizing. Aristotle's successor, ***Theophrastos*** *(c. 300 BCE), wrote a series of books on botany,* History of Plants*, which survived as the most important contribution of antiquity to botany, even into the Middle Ages.*

In ancient Rome, ***Pliny the Elder*** *was known for his knowledge of plants and nature. The Roman medical writer* ***Dioscorides*** *provided important evidence on Greek and Roman knowledge of medicinal plants. Around 60 BCE the Roman atomist* ***Lucretius*** *wrote the poem* On the Nature of Things *describing the development of the living earth in stages from atoms colliding in the void as swirls of dust. Later,* ***Claudius Galen*** *became a pioneer in medicine and anatomy.*

Table 1.9 lists 14 leading thinkers in ancient Greek and Rome whose influence extended over 2200 years into the Middle Ages and up to the 19th *century.*

Table 1.9: LEADING THINKERS IN THE LIFE-SCIENCES (560 BCE–450 CE)

Name	*fl.*	*Specialization*
Anaximander of Miletos	*ca 560 BCE*	*(ET)*
Alcmaeon of Crotona	*ca 500 BCE*	*(M)*
Empedocles of Acragas	*ca 450 BCE*	*(ET)*
Democritos of Abdera	*ca 420 BCE*	*(ET)*
Hippocrates of Cos	*ca 430–390 BCE*	*(M)*
Aristotle	*ca 354–322 BCE*	*(B), (A)*
Theophrastos	*ca 328–286 BCE*	*(BO)*
Epicuros of Samos	*ca 310–301 BCE*	*(ET)*
Herophilos of Chalcedon	*ca 304–291 BCE*	*(M), (A)*
Asclepiades of Bithnia	*ca 90-50 BCE*	*(M)*
Lucretius	*ca 70-55 BCE*	*(ET)*
Pliny, the Elder	*ca 50-70 CE*	*(B)*
Pedanios Dioscorides	*ca 60–78 CE*	*(M), (BO)*
Claudius Galen	*160–190 CE*	*(M), (A)*

Key:

B = *Biology*

A = *Anatomy*

M = *Medicine*

ET = *Evolutionary Thinking*

BO = *Botany*

Herbalism and Medicine

All human societies have medical beliefs that provide explanations for, and responses to, birth, death, and disease. Throughout the world, illness has often been attributed to witchcraft, demons, averse astral influence, or the will of the gods, although the rise of scientific medicine in the past two centuries has altered or replaced many historic health practices.

There is no actual record of when the use of plants for medicinal purposes first started, although the first generally accepted use of plants as healing agents were depicted in the paintings discovered in the Lascaux *caves in France, which have been radiocarbon dated to between 13,000–25,000 BCE.*

Over time and with trial and error, a small base of knowledge was acquired within early tribal communities. As this knowledge base expanded over the generations, tribal culture developed into specialized areas. These 'specialized jobs' became what are now known as healers or shamans.

Medical information contained in the Edwin Smith Papyrus *date as early as 3000 BCE. The earliest known surgery was performed in Egypt around 2750 BCE.* ***Imhotep*** *in the 3rd dynasty is credited as the founder of ancient Egyptian medicine and as the original author of the Edwin Smith papyrus, detailing cures, ailments and anatomical observations. The Edwin Smith papyrus is regarded as a copy of several earlier works and was written circa 1600 BCE. It is an ancient textbook on surgery and describes in exquisite detail the examination, diagnosis, treatment, and prognosis of numerous ailments.*

Additionally, the Ebers papyrus *(c. 1550 BCE) is full of incantations and foul applications meant to turn away disease-causing demons and other superstition. In it there is evidence of a long tradition of empirical practice and observation. The Ebers papyrus also provides our earliest documentation of a prehistoric awareness of tumors.*

Medical institutions are known to have been established in ancient Egypt since as early as the 1st Dynasty. By the time of the 19^{th} Dynasty their employees enjoyed such benefits as medical insurance, pensions and sick leave. Employees worked 8 hours per day.

The earliest known physician is also credited to ancient Egypt: ***Hesyre****, "Chief of Dentists and Physicians" for King Djoser in the 27^{th} century BCE. Also, the earliest known woman physician,* ***Peseshet****, practiced in Ancient Egypt at the time of the 4^{th} dynasty. Her title was "Lady Overseer of the Lady Physicians." In addition to her supervisory role, Peseshet graduated midwives at an ancient Egyptian medical school in Sais.*

In 2001 archaeologists studying the remains of two men from Mehrgarh, Pakistan, made the discovery that the people of Indus Valley Civilization, even from the early Harappan periods (c. 3300 BCE), had knowledge of medicine and dentistry. Later research in the same area found evidence of teeth having been drilled, dating back 9,000 years.

Ayurveda (the science of living), the Vedic system of medicine originating over 3000 years ago, views health as harmony between body, mind and spirit. Its two most famous texts belong to the schools of Charaka and Sushruta. According to Charaka, health and disease are not predetermined and life may be prolonged by human effort. Sushruta defines the purpose of medicine to cure the diseases of the sick, protect the healthy, and to prolong life.

The student of Ayurveda was expected to know ten arts that were indispensable in the preparation and application of his medicines: distillation, operative skills, cooking, horticulture, metallurgy, sugar manufacture, pharmacy, analysis and separation of minerals, compounding of metals, and preparation of alkalis. The teaching of various subjects was done during the instruction of relevant clinical subjects. For example, teaching of anatomy was a part of the teaching of surgery, embryology was a part of training in pediatrics and obstetrics, and the knowledge of physiology and pathology was interwoven in the teaching of all clinical disciplines.

China also developed a large body of traditional medicine. Much of the philosophy of traditional Chinese medicine derived from empirical observations of disease and illness by Taoist physicians and reflects the classical Chinese belief that individual human experiences express causative principles effective in the environment at all scales. These causative principles, whether material, essential, or mystical, correlate as the expression of the natural order of the universe.

Thus, already during the reign of the ***Yellow Emperor*** *(2696–2598 BCE), the influential* ***Neijing Suwen*** *(Basic Questions of Internal Medicine) was composed. This was expanded and edited by* ***Wang Ping*** *during the Tang dynasty, and again revisited by an imperial commission during the 11^{th} century CE. The result is the extant representation of the foundational roots of traditional Chinese medicine. Acupuncture was advocated during the Chin dynasty by* ***Huang-fu Mi*** *(215-282 CE).*

Most of our knowledge of ancient Hebrew medicine during the 1^{st} millennium BCE comes from the Old Testament of the Bible which contain various health related laws and rituals, such as isolating infected people (Leviticus ***13****:45-46), washing after handling a dead body (Numbers* ***19****:11-19) and burying excrement away from camp (Deuteronomy* ***23****:12-13).* ***Max Neuberger****, writing in his "History of Medicine" says:*

> *"The commands concern prophylaxis and suppression of epidemics, suppression of venereal disease and prostitution, care of the skin, baths, food, housing and clothing, regulation of labor, sexual life, discipline of the people, etc. Many of these commands, such as Sabbath rest, circumcision, laws concerning food (interdiction of blood and pork), measures concerning menstruating and lying-in women and those suffering from gonorrhea, isolation of lepers, and hygiene of the camp, are, in view of the conditions of the climate, surprisingly rational."*

As societies developed in Europe and Asia, belief systems were replaced with a different natural system. The Greeks, from ***Hippocrates****, developed a humoral medicine system where treatment was to restore the balance of humors within the body.* Ancient Medicine *is a treatise on medicine, written roughly 400 BCE by Hippocrates. Similar views were espoused in China and in India. In Greece, through* ***Galen****, until the Renaissance, the main thrust of medicine was the maintenance of health by control of diet and hygiene.*

Anatomical knowledge was limited and there were few surgical or other cures, doctors relied on a good relation with patients and dealt with minor ailments and soothing chronic conditions. But they could do little when epidemic diseases, growing out of urbanization and the domestication of animals, then raged across the world.

Metallurgy in China – The Missed Opportunity

During 300 BCE–450 CE China grew into a powerful empire. Its culture flourished and great technological advancement were made. The invention of cast-iron (ca 300 BCE) was a feat in which the Chinese had pioneered, some 1700 years ahead of the Europeans!

The military technology which built the Great Wall (ca 214 BCE) went far beyond the achievement of the Wall itself. Recently (1975) Chinese archaeologists discovered that weapons (swords, spears and arrowheads) dating from the second century BCE were made of sophisticated alloys of 15 different metals. The major ingredients are copper, tin and lead, but also other which have come to use in the West in our own time (Aluminum, Titanium, Vanadium, Cobalt etc).

A different kind of advanced technology is represented by this set of intricate casting: They are the trigger-release mechanism of the weapon that armed the emperor Chin troops in holding the barbarians at bay on the Great Wall – the crossbow*; this is a weapon which will not appear in the West for another 1400 years! Clearly, the exploitation of cast iron by the Chinese in ca 300 BCE marks one of the most significant advances in all history of metals.*

We know that the main barrier to the wider use of iron, after the collapse of the Bronze age, was that it could not be melted. Iron workers could not therefore cast objects in iron as their predecessors had cast them in Bronze. This inability meant drudgery at the anvil and frustrated the further use of iron in Europe for more than a millennium. However, unknown to Europe, that particular threshold had already been crossed in the East.

An iron blast furnace of remarkable proportions, weighting ca 25 tons, dating from before the dawn of the Christian era, was discovered in China (1975). No one outside China handled iron in this quantity until the Industrial Revolution in Britain. The furnace operated with temperatures in excess of 1400° C. The Chinese then liquefied iron, treating it as they treated bronze by pouring it into moulds. With this they made ploughshares and haws.

But casting was only part of the Chinese achievement. Cast iron was high in carbon, which made it too brittle for useful tools.

By 450 CE, Chinese metallurgists knew how to remove the carbon on the surface of cast iron and create a steel jacket *around the cast iron case. It was this discovery which finally made it possible for the Chinese to use iron so comprehensively so early in history.*

How did they attain and maintain the required high temperature in their furnaces? The answer is in more efficient bellows; the more air one blows into the furnace in a unit of time – the hotter it gets.

Whereas in the West blacksmith's bellows operated on a vertical principle, the Chinese system was horizontal. Very large bellows could be suspended from an overhead beam and driven by waterwheels or by animal power. The Chinese control of air supply to the furnace was superior to that in the West. In addition, Chinese used double acting box bellows: *the handle operates two pistons, in upper and lower chamber of the box; simple flat valves control the* *air-flow such that air is pumped both on the pull and the push action. It supplies twice as much air into the furnace by continuous flow. This was the key to the Chinese success in high-temperature furnace.*

But the more evidence that emerges for the technological lead that China established over the West in the early Iron Age, the more intriguing becomes her failure to sustain it. All the conditions seem to exist for China to lead the world into an industrial and scientific revolution 1300 years before it occurred in the West.

Historians think that the reasons were political and social. China was an extremely conservative society in which stability was much more prized than freedom and where rules had a powerful vested interest in not being challenged. China's belief that it was a universe in itself led to dangerous contempt to development elsewhere. The rise of a stifling bureaucracy anesthetized innovation. The iron masters of the Han period who were well placed to launch a power-driven industrialized revolution went on to making plows and haws. China lost an advantage to the West that was not yet regained.

From the first through the thirteenth centuries, as Europe passed from late antiquity through the Dark Ages, science in China flourished. It kept pace with Arab science, even though geographic isolation deprived Chinese scholars of the ready-made base that Greek culture provided their Western counterparts.

The Chinese made brilliant advances in subjects such as descriptive astronomy, mathematics, and chemistry. But they never acquired the habit of reductive analysis in search of general laws that served Western science so well from the seventeenth century on. They consequently failed to expand their conception of space and time beyond what was attainable by direct observation with the unaided senses. The reason was their emphasis on the holistic properties and harmonious relationships of observable entities, from stars to trees to grains of sand.

Unlike Western scientists, they had no inclination to search for abstract codified law in nature. Their reluctance was stimulated to some degree by the historic rejection of the Legalists, who attempted to impose rigid, quantified law during the transition from feudalism to bureaucracy in the fourth century BCE.

But of probably greater importance was the fact that the Chinese steered away from the idea of a supreme being who created and supervises a rational, law-governed universe. If there is such a ruler in charge, it makes sense – Western sense at least – to read a divine plan and code of laws into physical existence. If, on the other hand, no such ruler exists, it seems more appropriate to search for separate rules and harmonious relations among the diverse entities composing the material universe.

In summary, it can be said that Western scholars but not their Chinese counterparts hit upon the more fortunate metaphysics among the two most available to address the physical universe.

ca 435 CE **Martianus Minneus Felix Capella** (North Africa). Latin encyclopedic compiler. His comprehensive treatise *Satyricon* was a complete encyclopedia of the liberal culture of the time, and was in high repute during the Middle Ages. A passage in book 8 contains a very clear statement of the *heliocentric system of astronomy.* It has been supposed that **Copernicus**, who quotes[320] Capella, may have received from this work some hints towards his own new system.

The author's chief sources were Varro, Pliny, Solinus, Aquila Romanus and Aristides Quintillanus.

Capella was a native of Madaura in Africa. He appears to have practiced as a lawyer at Carthago.

ca 450 CE **Proclos of Byzantion** (410–485, Alexandria and Athens). Philosopher, mathematician and historian of Greek science. Chief representative of the later Neoplatoists. Head of Plato's Academy in Athens. The most illustrious philosopher and teacher of his time. Discovered an equivalent postulate to Euclid's 5^{th}; it states that if a straight line intersects one of two parallels, it will intersect the other also.

He seems to have been the first to state explicitly the fact that the three altitudes of any triangles are concurrent (it is not in Euclid, although Archimedes implies it).

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320 In the words of Copernicus: "*Therefore it seems to me that it would be wrong to ignore certain facts well-known to Martianus Capella, who wrote an encyclopedia, and some other Latins. He believed that Venus and Mercury do not go around the earth like other planets, but turn around the sun as their center and therefore cannot go further away from the sun than the sizes of their orbits permit. What else does this mean but that the sun is the center of their orbits and that they turn around him?... Therefore we do not hesitate to state that the moon and the earth describe annually a circular orbit placed between the outer and inner planets round the sun, which rests immobile in the center of the world*".

Proclos was born in Constantinople, but brought up at Xanthus in Lycia. After studying grammar and philosophy at Alexandria, he proceeded to Athens. There he attended the lectures of the Neoplatonists Plutarch and Syrianus, and about 450 CE succeeded the latter in the chair of philosophy. As an ardent upholder of the old Pagan religion, Proclos incurred the hatred of the Christians, and was obliged to take refuge in Asia Minor. After a year's absence he returned to Athens, where he remained until his death.

Although possessed of ample means, Proclos led a most temperate, even ascetic life, and employed his wealth in generous relief of the poor. His great literary activity was chiefly devoted to the elucidation of the writings of Plato, but he also wrote treatises in the fields of mathematics and astronomy.

451 CE *The Hun Invasion.* The Huns, a nomadic Mongol tribe headed by Attila, established an empire in Eastern Europe in a region extending from the Danube River to the Baltic Sea, and from the Rhine River to the Caspian Sea. During 441–447 they looted the provinces of southeastern Europe and forced the East Roman Empire to pay a yearly tax[321]. In 450, Attila demanded Honoria, sister of Emperor Valentinian III, as his bride, and half the West Roman Empire as her inheritance. Valentinian refused. Attila stormed into Gaul (now France), but a combined Roman and Visigoth army repulsed him near Troyes. He retreated east to the Rhine River and invaded Italy in 452, capturing and destroying many cities north of the Po River. But famine and plague forced his troops to withdraw.

Although he seriously threatened the East and West Roman Empire, he was unable to win decisive victories over them. His kingdom collapsed soon after his death in 453.

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[321] One theory holds that the westward movements of the Huns started because of virulent *smallpox* in Mongolia; the disease traveled with them, was communicated to the Germanic tribes upon whom the Huns were pressing and, in turn, infected the Romans who were in contact with the Germans.

Downfall of the Roman Empire – plague-ridden decadence

The causes of this fall have been argued by historians for many years past; some of these causes are connected with disease.

Although in its cleanliness, sanitation and water supply, Rome was much more akin to 20th century London and New York, Romans, like 17th century Londoners, did not know the cause of disease. This lack of essential knowledge rendered the magnificent health measures of Imperial Rome entirely useless during the long years of her decline.

Imagine Rome as a bloated spider sitting in the center of its web. This web, in the height of Roman expansion stretched from the Sahara in the south to the borders of Scotland in the north, from the Caspian Sea and the Persian Gulf in the east to the western shores of Spain and Portugal. To the north and west lay the oceans; to the south and east, vast unknown continents in which dwelt less civilized people, Africans, Arabs, the savage tribes of Asia. Beyond, in the dim shadows, lay the older civilizations of India and China.

The long land frontiers were manned by garrisons scattered at strategic points; from these frontier garrisons stretched back the filaments of the spider's web, the sea routes from Africa and Egypt, the straight, legionary-made roads, all of which led to Rome. Along these very roads crept the micro-organisms of foreign disease, reaching the center of a highly civilized organization lacking the most rudimentary means of combating infection.

Given a conjunction of circumstances such as this, it is little wonder that the story of the last centuries of Roman power is a long tale of plague:

It started in the 1st century BCE; a very severe type of malaria *appeared in the agricultural district around Rome and remained a problem for the next 500 Years. All fertile land of market gardens which supplied the city with fresh vegetables, went out of cultivation; the small farmers who tilled it added to the overcrowding of Rome, bringing infection with them. It caused the live-birth rate of the Italo-Roman to fall steeply at the time when the birth-rate throughout the empire was rising. Moreover, chronic illness and ill-health caused by untreated malaria decreased life-expectancy and enervated the nation. Possibly malaria rather than decadent luxury imported from the East, accounted for the slackness of spirit which characterized the later years of Rome.*

The first of the great epidemics occurred about the year 79 CE, shortly after the eruption of Vesuvius. This may have been fulminating malaria *compounded by an epidemic of* anthrax*, resulting in the large-scale destruction of livestock. The infection was raging destructively in the cities of Italy. For about a century there was much sickness (mostly malaria); then came the* plague of Orosius *(125 CE), a famine-plague sequence, for the sickness was preceded by an invasion of* locust *which destroyed large areas of crops. The pestilence that followed killed hundred of thousands in Numidia and on the north coast of Africa. The plague then passed to Italy.*

Forty years later there followed the plague of Antoninus *(164–189 CE) through which the Roman Empire was swept by three separate waves of plague. It began by causing great mortality among the legions under the command of Avidius Claudius, who had been sent to repress a revolt in Syria (164–166 CE). The plague accompanied this army homewards, spreading throughout the countryside and reaching Rome in 166 CE. It rapidly extended into all parts of the known world, causing many thousands of deaths.*

The plague of Antoninus caused the first crack in the Roman defense lines. Until 161 CE the empire continually expanded and maintained its frontiers. In that year a Germanic barbaric horde (the Marcomanni *from Bohemia and the* Quadi *from Moravia) forced the north-eastern barrier of Italy. Owing to the disorganization produced by the plague, it was not until 169 CE that the whole weight of the Roman army was thrown against the invaders. Possibly the failure of this invasion was as much due to the legions carrying plague with them, thus infecting the Germans. One of the victims of the plague was the Roman emperor,* **Marcus Aurelius** *(d. 180 CE).*

A new wave of epidemic originated in Ethiopia (ca 250 CE) and spread, via Egypt, to the Roman colonies in North Africa. It engulfed all countries from Egypt in the south to Scotland in the north. It advanced with appalling speed, decimating the entire Roman empire.

It lasted no less than 16 years and indisputably changed the course of history; wide areas of farmland throughout Italy reverted to waste; some thought that the human race could not possibly survive. But despite warfare in Mesopotamia, on the eastern frontiers and even in Gaul, the empire managed to survive this catastrophe; but by 275 CE the legions had fallen back from Transylvania and the Black Forest to the Danube and the Rhine, and so dangerous did the position seem to have become that Aurelian decided to fortify the city of Rome itself.

Throughout the next 3 centuries, while Rome slowly collapsed under the pressure of the Goths and Vandals, there were recurrent outbreaks of a similar plague. Gradually the evidence became less exact, degenerating into a *generalized story of war, pestilence and famine, as the darkness descended over Rome and her mighty empire collapsed.*

476 CE *Goths* under Odovacar deposed Romulus Augustus; the "official" end of the Western Empire.

Since Attila's death in 453 CE, Germanic army commanders governed Italy behind the facade of puppet emperors. Odovacar, the leader of a band of German mercenaries, tired of the pretense and became king of Italy. All Roman territories west of the Adriatic were now under barbarian rulers.

Although this was just a symbolic act, the passing of the Latin-speaking part of the Empire under Germanic rule was a watershed in history; the Latins became barbarized and the Greek became orientalized. Thus, the two halves of the Roman world drifted apart, preparing the way for the later split between Eastern and Western Europe.

For a while the East, on account of its superior stability saved itself from the Germans, but could not repel the next great attack, that of the Arabs. The "Fall of the Empire" was not a political episode of the year 476; it was a deadly crisis in the history of civilization.

Origins of the European Civilization

On Assyrian monuments the contrast between Asu = (the land of the rising sun) and ereb = (the land of the setting sun), is frequent [erev = evening in Hebrew; Genesis **1***, 5]. These names were probably passed on by the Phoenicians to the Greeks, and gave rise to the names of Asia and Europe. The earliest mention of Europe is in the Homeric Hymn to Apollo. The distinction between Europe and Asia is found, however, in Aeschylos in the* 5^{th} *century BCE.*

In places where the names originated, the intervention of the sea clearly marked the distinction between Europe and Africa. As the knowledge of the world extended, the difficulty of fixing the land boundary between Europe and Asia caused uncertainty in the application of the two names, but never obscured the necessity for recognizing the distinction. Even in the 3^{rd} *century BCE, Europe was regarded by Eratosthenes as including all that was then known of Northern Asia. But the character of the physical features and climate finally determined the fact that what we know as Europe came to be occupied by more or less populous countries in intimate relation with one another, but separated on the east by unpeopled (or very sparsely peopled) areas from the countries of Asia.*

Within the limits thus marked out on the east and on other sides by the sea, the climatic conditions are such that inhabitants are capable of and require a civilization of essentially the same type, based upon the cultivation of European grains. Those inhabitants have led a common history in a greater measure than those of any other continent, and hence are more thoroughly conscious of their dissimilarities from, than of their consanguinity with, the peoples of the east and the south. Within these geographical limits, the tradition of the Roman Empire, and above all the organization of the Catholic Church gave to the European nations, and the states based upon them, a homogeneity which without them could not have survived. The history of Europe is the history of this civilization and of the forces by which it was produced, preserved and developed.

Broadly speaking, European civilization may be traced to four principal origins: (1) The Aegean civilization (Hellenic and pre-Hellenic); (2) the Roman Empire; (3) Christianity; (4) the break-up of the Roman Empire by the Teutonic invasions. All these forces helped in the development of Europe as we now know it.

To the Aegean civilization [whether transformed by contact with Rome, and again transformed by the influence of Christianity, or rediscovered during the classical Renaissance] Europe owes the characteristic qualities of its thought and of its expression in literature, art and the sciences. From republican Rome it largely drew its conceptions of law and of administrative order. From the Roman Empire it inherited a tradition of political unity which survived (in visible form, though but as a shadowy symbol) until the last Holy Roman emperor abdicated in 1806.

Yet more does Europe owe to Christianity, basically a Hebrew religion, modified by contact with Greek thought and powerfully organized on the lines of the Roman administrative system. The Roman Church remained a reality when the Roman Empire had become little more than a name. Indeed, throughout the period of chaos and transformation that followed the collapse of the Roman Empire, the Church remained the most powerful instrument for giving to the heterogeneous races of Europe a common culture and a certain sense of common interests.

The history of Europe, then, might well begin with the origins of Greece and Rome, and trace the rise of the Roman Empire and the successive influence upon it of Hellenism and Christianity.

ca 490 CE **Tsu Ch'ung-Chi** (430–501) and his son, whose joint book is now lost, found the remarkable rational approximation 355/113 to π, correct to 6 decimal places. They further discovered that $3.1415926 < \pi < 3.1415927$. This rational approximation was not rediscovered in Europe until 1573. The precision of π achieved by the Tsu's seems not to have been surpassed until about 1420 when the astronomer **Al-Kashi** of Samarkand found π correctly to 16 decimal places. Western mathematicians did not surpass the Tsu approximation until around 1600.

ca 499–530 CE **Aryabhata the Elder** (476–550, India). The first great Hindu mathematician and astronomer. The author of '*Aryabhatiya*' (written in verse, 499 CE), the first Hindu astronomical text to contain a section devoted to mathematics. It is a summary of Hindu mathematics up to that time and includes rules for computational procedures, the extraction of square and cube roots, the solution of quadratic equations, the sum of powers of the first n natural numbers [e.g., he gave the formula $\sum n^2 = \frac{1}{6}n(n+1)(2n+1)$], arithmetic progressions[322] and the trigonometric sine function. It gives formulas for the areas of a triangle and a circle which are correct, but the formulas for the volumes of a sphere and a pyramid are wrong.

Aryabhata gave $62,832/20,000 = 3\frac{177}{1250} = 3.1416$ as an approximate value of π [it may have come from some earlier Greek source or from calculating the perimeter of a regular inscribed polygon of 384 sides][323]. His work contained one of the earliest attempts at the general solution of a linear indeterminate equation by the use of continued fractions. Thus he found the integral solutions of the linear indeterminate equation $ax + by = c$ (a, b, c integers). He also solved the indeterminate quadratic equation $xy = ax + by + c$ by a method later reinvented by **Euler**.

As an astronomer he taught that the apparent rotation of the heavens was due to the axial rotation of the earth and explained the cause of eclipses of sun and moon.

Aryabhata was born near the present day city Patna on the Ganges.

500 CE *Polynesians* began to inhabit the islands of Hawaii.

ca 500 CE **Joannes Philoponus** (John the Grammarian). Philosopher. Flourished in Alexandria. Contested the Aristotelian ideas on motion and vaguely anticipated the concept of *inertia*. He denied that bodies of greater weight fall more quickly, referring to *experiment*. Nor did he accept the Aristotelian doctrine about the impossibility of vacuum.

Philoponus applied Severus' unity of soul and matter to cosmology. Reviving the Ionian ideas, he argued that these same ideas apply to the heavens and the earth; stars are neither divine nor perfect, beings but material bodies on fire. The heavens are not unchanging but governed by the same changes as are earthly objects. In supporting his assertion that stars are lighted by fire, he pointed to their obvious differences in color: this shows, he said, they cannot be simple bodies, made of pure ether (the rarefied medium filling the heavens), as dualism claimed, since we know that on earth different material produce different colorations in fire, implying that thus the stars must be composed of different materials, like these on earth. Thus, Philoponus hit on the basis of *spectrography*, which centuries later allowed scientists to figure out what the stars are made of.

His work, however, could not have led to an early revival of science, for he lived during the collapse of the Mediterranean civilization. The trade

[322] It contains the rule for summing an arithmetic series from the p-th term on.

[323] At about the same time, the scholars of the *Talmud* still used the Biblical approximation $\pi = 3$ for all practical purposes of mensuration (*Eruvin*, 14).

that demanded and supported scientific research was disrupted by Byzantine attempts at reconquest and its futile battles with Persia.

Joannes Philoponus represents the last flicker of ancient science.

ca 500 CE Chinese began the use of *gunpowder*.

ca 500 CE The *Zero* symbol is introduced in India.

Zero – The Mathematics of Nothing[324]

* * *

"Thirty spokes are made one by holes in the hull,
By vacancies joining them for a wheels' use.
The use of clay in moulding pitchers
Comes from the hollow of its absence;
Doors, windows, in a house,
Are used for their emptiness:
Thus we are helped by what is not
To use what it".

Lao Tsu, 570 BCE

* * *

"Each act of creation could be symbolized as a particular product of infinity and zero. From each such product could emerge a particular entity of which the appropriate symbol was a particular number".

S. Ramanujan (1887–1920)

[324] Includes quotations from: "zero" by Charles Seife. Penguin Books, 200, New York.

* * *

"The zero is the most important digit. It is a stroke of genius, to make something out of noting by giving it a name and inventing a symbol for it".

Bartel Leendret van der Waerden (1903–2000)

Riddle: "The Babylonians invented it, the Greeks banned it, the Hindus worshiped it, the Church used it to fend off heretics; once harnessed, it became the most important tool in mathematics; starting as an Eastern philosophical concept, its struggle for acceptance in Europe ended with its apotheosis as the mystery of the black hole. Today it lies at the heart of one of the biggest scientific controversies of all time, the quest for the theory of everything. Its companion concepts are infinity and void, and it has been an ever-present threat to modern physics".

What is it? It is a number known as Zero.

The first symbol for zero was introduced by the Babylonians in about 300 BCE out of the practical need to give a sequence of digits a unique meaning[325] *Yet, their symbol for zero (two slanted wedges) was merely a symbol for a blank space on the cuneiform, a placeholder; it did not really have any numerical value of its own. It was a digit, not a number.*

Like the Babylonians, the Mayans (ca 300 BCE) had a place-value system of digits using the vigesimal (base 20) system, and needed a zero to keep

325 To see how this need arose, consider the representation of different numbers in their sexagesimal (base 60) system; Today, we would write (*with* zero)

$$\begin{aligned} 61 &= 1\cdot 60^1 + 1\cdot 60^0 &&\equiv 11\\ 3601 &= 1\cdot 60^2 + 0\cdot 60^1 + 1\cdot 60^0 &&\equiv 101\\ 3660 &= 1\cdot 60^2 + 1\cdot 60^1 + 0\cdot 60^0 &&\equiv 110 \end{aligned}$$

But *without* the placeholder 0, all three numbers would be written as 11. (The Babylonian actually used the wedge symbol

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instead of the Arabic 1).

track of what each digit meant. But the Mayans went one step further than the Babylonians (and the Egyptians) in their solar calendar – they started numbering days with the number zero; the first day of the month was day zero, the next day was 1, and so forth. This is conceptually equivalent to placing zero at the origin of the number-line, giving it both a cardinal and an ordinal value.

Most ancient people believed that only emptiness and void were present before the universe came to be. There was always a fear that at the end of time, disorder and void would reign once more. Thus, to the ancients, zero represented that void and its odd mathematical properties[326] *were inexplicable, as shrouded in mystery as the birth of the universe.*

In the Greek universe, created by Pythagoras, Aristotle and Ptolemy, there was no zero. To begin with, the framework of the Pythagorean universe was controlled by ratios and the zero, with it odd properties "would punch a hole in the neat Pythagorean world order". For that reason zero (void, vacuum) was totally excluded from Greek science and mathematics. Because of this, the West could not accept zero for nearly 2000 years! Pythagorean doctrine became the centerpiece of Western philosophy: all the universe was governed by ratios and shapes; the planets moved in heavenly spheres that made music as they turned. Since infinity (the inverse of zero) was also rejected, the number of those spheres, according to Aristotle and Ptolemy, must be finite – the cosmos was finite in extent and entirely filled with matter: there was no infinite and there was no void.

Moreover, since the Greek rejected the number zero, *they could not refute Zeno's paradox ('Achilles and the tortoise')*[327]*. This rejection is the biggest failure in Greek mathematics, and it is the only thing that kept them from discovering the calculus. This state of affairs continued into the Dark Ages; As the medieval thinkers imported the philosophy and science of the ancient, they inherited the ancient prejudices: a fear of the infinite and a horror of the void. "The fear of the void was so great that Christian scholars tried to fix the Bible*[328] *to match Aristotle rather than vice versa".*

In the 4^{th} century BCE, Alexander the Great marched with his troops from Babylon to India, carrying with him the gospel of the Babylonian system

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[326] $0+a=0$; $0\times a=0$; $a/0=\infty$ $(a\neq 0)$; $\infty\cdot 0=$ *indeterminate*; $0/0=$ *indeterminate*.

[327] They did not believe that adding infinite terms could lead to a finite result $[\lim\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^n}\right)=2,\ n\to\infty]$. To them, terms in an infinite series did not have a *limit* or a destination.

[328] The Judeo-Christian story of creation was Semitic, and Semites did *not* have a fear of the void.

of numbers and with it the Babylonian 'place-holder zero'. "Unlike Greece, India never had a fear of the infinity and the void". Consequently, India, as a society that actively explored the void and the infinite – embraced the mathematical zero. Moreover, India transformed it, changing its role from a mere placeholder to a number.

So, sometimes around 500 CE the Hindus moved to a base-10 system representing all numbers, negative *as well as* positive *in the number line. No longer did zero sit to the right of nine – it had a position in the number line that was all its own, just in the middle between +1 and -1. "Numbers were stripped of their geometrical significance; mathematician no longer had to worry about mathematical operations making geometric sense*[329]*. This was the birth of what we now know as* algebra*".*

When the Arabs conquered India (ca 710 CE), they learned quickly all about Hindu numerals and their zero. They accepted it and soon rejected Aristotle.

Christianity initially rejected zero, but trade would be soon demanding it. The man who introduced zero to the West was **Leonardo of Pisa** *(Fibonacci) in his book Liber Abaci (1202 CE). After many vicissitudes, and thanks to the persistence of both the Muslims and the Hindus, the zero and its companion, infinity, have by 1430 arrived throughout Western Europe.*

In the century that followed, **Nicholas of Cusa** *["Terra no est centra mundi", 1440 CE] and* **Nicolaus Copernicus** *(1543 CE) cracked open the nutshell universe of Aristotle and Ptolemy. The Church was now under attack both from the inside (***Luther***, 1517) and the outside. For if the universe were infinite, then there could be no center. How could earth, then be the center of the universe?*

"Zero became a heretic!"

The Catholic Church stroke back by re-rejecting the void and the infinite. It did so with the establishment of the Jesuit order (1530 CE), the Inquisition (1543 CE), and the Index of forbidden books. "This counter-Reformation *was the Church's attempt to rebuild the old order by crushing the new ideas".*

In the meantime the scientific revolution was on its way, but its first harbingers were somewhat ambiguous: For example, **Descartes** *(being himself a Jesuit) "would bring zero to the center of the number line, yet, being*

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[329] Negative number did not have necessarily to mean a negative area. **Brahmagupta** (ca 630 CE) gave rules for dividing numbers of the same or mixed polarity ($\frac{\pm}{+} = \frac{-}{-} = +, \qquad \frac{\pm}{-} = \frac{-}{+} = -$), but he wrongly asserted that $\frac{0}{0} = 0$ and failed to attach any sensible meaning to $\frac{1}{0}$.

indoctrinated with the Aristotelian philosophy, denied the existence of the vacuum (1637)".

Not for long. It was Galileo's secretary, ***Evangelista Torricelli****, who proved this was not true by creating the first vacuum (1643). Aristotle was defeated, and scientists stopped fearing the void and began to study it. Then an unexpected help for the Church came from within the scientific camp:* **Pascal***, a devout Jansenist, combined mathematical theory (probability theory), with zero and infinity to* prove *(?!) the existence of God.*

Then, from deep within the scientific world emerged a new language – the Calculus of Newton and Leibniz: the most powerful mathematical method ever by dividing by zero and adding an infinite number of zeros together – the language of nature!

The Church, however, refused to admit defeat. Its last battle was fought via an Irish bishop, ***George Berkeley*** *(1734). He ridiculed infinitesimals, arguing that Newton himself could not explain how his infinitesimals disappeared when squared. He charged the mathematicians of his day that they just accepted the fact because making them vanish at the right time gave the correct answer. Berkeley's attack forced mathematicians to re-examine the foundation of analysis. There followed 200 years of intense efforts by the best minds in Europe. The result was a rigorous calculus as we know it today.*

This process was started by ***D'Alambert*** *(the idea of the* limit*; 1754) and* ***Lagrange*** *(1797), in what we call today 'the Arithmetization of the Calculus'. It was then followed by the French geometer* **Poncelet** *(point at infinity in projective geometry; 1822), and by* ***Gauss*** *(1827) and* ***Riemann*** *(1854) in their new differential geometry. The concept of infinity was finally harnessed by the studies of* **Cantor** *(1872).*

While infinity and zero have been inseparable essential mathematical entities for over 2500 years, these concepts became truly relevant to physicists only in the 20^{th} century: In thermodynamics *zero became an uncrossable barrier; in Einstein's GTR clockrates vanish and densities diverge in black holes and cosmological solutions; in quantum mechanics zero-point motion is an infinite source of energy present even in the deepest vacuum.*

Toward the end of the 20^{th} century, physicists began to realize that the zero of the vacuum might explain the lumpiness of the universe. Since the vacuum everywhere in the universe is seething with a quantum foam of virtual particles, the fabric of the universe is filled with infinite zero-point energy. Under the right conditions this energy is able to push objects around.

Moreover, "zero might also hold the secret of what created the cosmos. Just as the nothingness of the vacuum and the zero-point energy spawn particles, they might spawn universes. Perhaps the universe is just a quantum *fluctuation on a grand scale. Zero might hold the secret to our existence – and the existence of an infinite number of other universes".*

"Zero is so powerful because it unhinges the laws of physics. It is the zero hour of the Big Bang and the ground zero of the black hole. It not only holds the secret to our existence, it will also be responsible for the end of the universe."

510–530 CE **Anthemios of Tralles** (ca 474–534 CE). Mathematician and architect. Best known for the *Hagia Sophia* at Constantinople. Described the construction of an ellipse with a string fixed at two foci. His famous book *On Burning Mirrors* describes the focusing properties of a parabolic mirror.

ca 510–524 CE **Anicus Manilus Severinus Boethius** (480–524 CE, Italy). Roman philosopher, statesman and writer on mathematics. Played a role in the history of mathematics because his writings on geometry and arithmetic remained standard texts in the monastic schools for centuries.

Boethius was born in Rome into an aristocratic Christian family, during the reign of Odoacar. After the death of his father, a consul, (487 BC) he was taken under the patronage of men of the high nobility. He befriended Theodoric, Ostrogoth ruler of Rome, who made him consul (510), and later head of government and court services (520). But his good fortune did not last: accused of conspiring against Theodoric[330], he was arrested, imprisoned at Pavia, and finally executed without trial. His work *De consolatione philosophiae* (the Consolation of Philosophy) was written in prison. In the year 996, Otto III, ordered the bones of Boethius to be placed in the church of S. Pietro in Ciel d'Oro within a splendid tomb, for which Gerbert, afterwards Pope Silvester II, wrote an inscription.

The very meager works of Boethius came to be considered as the height of mathematical achievement, and thus well illustrate the poverty of the subject in Christian Europe during the Middle Ages: For the *Geometry* consists of nothing but the statements of the propositions of Book I, and a few selected propositions of Books III and IV of Euclid's *Elements*, and the *Arithmetic* is founded on the tiresome and half mystical work of Nicomachos of four

330 When Boethius published his works on philosophy, contemporaries assumed they were about the occult sciences, and he was accused of being an astrologer and magician.

centuries earlier. With these works, and his writings on philosophy, Boethius became the founder of Medieval scholasticism.

510–538 CE **Severus of Antioch** (465–538 CE). Greek monk – theologian and prelate. Undermined the base of the dominant ideology of the Church by attacking the notion that soul and body are separate. Revived *causality*[331] – the idea that one event leads to other events, that man and nature can be understood as historical phenomena, autonomous of divine intervention. Severus sees human beings as *processes* whose individuality is based on their *history* – their parentage, their education, their actions and moral decisions – which shape them to what they are. Evil arises historically, from people's relations with one another in society – rather than from the inherent sinfulness of man or matter. Therefore – to combat evil, society must change. This justified revolution. But if the affairs of humanity can be understood in terms of historical causes and effects, then the *world as a whole* can be understood in the same way.

Severus thus revived the battle between an *evolving world* and the one created once and for all.

Severus lived as a monk in the land of Israel; became a leading exponent for Monophysitism. He became a confidant of Emperor Anastasius who made him patriarch of Antioch (512 CE); fled to Egypt on accession of Justin I (518). He then became formulator and head of the Monophysite movement in Egypt and Syria.

ca 510–540 CE **Eutocius of Ascalon** (ca 480–540 CE). Lived as a monk in the city of Ascalon in Israel. Wrote commentaries on works of Archimedes and Apollonius. Does not appear to have done any original work.

520 CE **Bodhidharma** (d. 528, India and China). Founded in China the *Zen* (Ch'an) school of Buddhist meditation. It is not a philosophy or religion in the proper sense. It has nothing to teach and no rituals. Zen is a method of self-restraining that leads to understanding of reality. Its basic idea is that a person can discipline his mind so that he comes into touch with the inner workings of his being. He aims to grasp *intuitively* what he cannot grasp *rationally*. This larger *awareness* cannot be taught and each person must find it for himself.

Bodhidharma was born in Southern India. In 520 CE he came by sea to Canton, China, amongst Hindu refugees. He settled in the monastery of Shaolin near Lo-Yang, where he founded his school. He realized that scholastic and

[331] The Platonic-Augustinian worldview was *anti-historical*; God created the universe once and for all. Cause and effect was excluded – things happen because God willed it, and thus science could not begin to take root.

philosophical Buddhism had become dominant in China and that the schools of Buddhism had failed to grasp the reality of its basic ideas. This led him to promulgate Zen.

Against rational knowledge, on which Western Science is based, Zen has placed intuitive knowledge that is not enkindled by mathematical formulae and scientific treatises. Zen preaches that segmented knowledge, as science offers through its emphasis on rationality, is hardly wisdom. Zen believes that in order to penetrate the nature of man, a language beyond the confines of man must be employed. Man cannot *know himself* through his own language. He must resort to the language of universal scope, that of *no-knowledge*. Perhaps wisdom is the artful way in which rational knowledge, intuitive knowledge and no-knowledge can be integrated and applied.

529 CE Byzantine emperor Justinian closed the *Academy* (founded by Plato in 387 BCE) and the *Lyceum* (founded by Aristotle in 335 BCE) in Athens. This move was made to defend the state religion, Christianity, from what was then perceived as Pagan influence. Under Rome the status of science sank to a point where philosophers, astronomers, and charlatan magicians often were grouped together and stigmatized, even outlawed as *mathematicians*.

The end of the Roman Empire[332] is traditionally placed in the year 476 CE in which Odovacar set aside the titular ruler, Romulus Augustus; but

[332] Nobody knows precisely what caused the fall of Rome. There is, however, a consensus among contemporary historians that it was a result of a series of contingent events. Major among them were:

- *Social inequality* - the impoverishment of the masses by an economic system which enriched a small propertied minority.
  Whereas the Western Empire went under, the Eastern sector endured as Byzantine civilization for a very long time. In the East, much more of the land was owned by peasant proprietors than in the West, and therefore a correspondingly larger proportion of the total agricultural yield went to them. In the West, however, a landed aristocracy had a stranglehold on government administration and used its power and connections to funnel cash into its own coffers, creating a class of idle rich.
  The empire was already structurally flawed in the extreme by the 3^{rd} century. The taxes levied to maintain the army were massive, and they fell largely on the poor. The Roman rulers also managed to ruin the middle class. It was this class that had held the culture of the ancient world together, and by the 4^{th} century, it was going under. By the 5^{th} century it was gone, and it did not reappear in Italy until the rise of the mercantile families of the High Middle Ages.
- *Bankruptcy of the State*: By the 3^{rd} century nearly every denarius col-

from an intellectual point of view other dates have greater significance. The Greek schools of philosophy at Athens had lost much of the vitality they enjoyed during the days of Aristotle, but they continued to serve as a focus of scholarly activity until their dissolution in 529 CE. This year marked also the founding of the monastery at *Monte Casino*, a coincidence which may be taken as symbolic of the shift in interest from secular learning to religious activity.

The attitude of the early Church fathers to natural science had been characteristically expressed by **Lucius Caecilius Firmianus Lactantius** (ca 240–320 CE) [the "Christian Cicero" and tutor (at Trier) to Crispos, the son of Constantine], when he wrote:

"*To search for the causes of things; to inquire whether the moon is convex or concave; whether the stars are fixed in the sky, or float freely in the air; of what size and what material are the heavens; whether they be at rest or in motion; what is the magnitude of the earth; on what foundations is it suspended or balanced; to dispute and conjecture upon such matters is just*

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lected in taxes was going into military and administrative maintenance, to the point that the state was drifting toward bankruptcy. The standing army rose from 300,000 troops in 235 CE to about 600,000 in 300 CE. This caused further debasement of the coinage and enormous inflation. Rome's policy of geographic and military expansion, became nonviable. By the time of the 5^{th} century, Rome was an empire in name only.

• *Spiritual and intellectual collapse.* For centuries, the aim had been to hellenize or romanize the rest of the population - to pass on the learning and ideals of Greco-Roman civilization. But as the economic crisis deepened, a new mentality arose among the masses, and based on *religion*, which was hostile to the achievements of higher culture. The new "intellectual" effort were designed to cater to the masses, until intellectual life was brought down to the lowest common denominator, primitive forms of life finally drowning out the higher ones. i.e. the gradual absorption of the educated classes by the masses and the consequent simplification of all functions of political, social, economical and intellectual life – the barbarization of the ancient world.

Religion played critical role in these developments: by the 3^{rd} century there was an attitude among many Christian that education was not relevant to salvation, and that ignorance had a positive spiritual value. Consequently, there was a sharp increase in mysticism and a belief in knowledge by revelation. The cognitive ability of comparing different viewpoints or perspectives had disappeared by the 6^{th} century. Even by the 4^{th} century, what little that had survived from Greek and Roman philosophy was confused with magic and superstition. In fact, the study of Greek (and therefore of *science* and philosophy) was completely abandoned.

as if we chose to discuss what we think of a city in a remote country, of which we never heard but the name".

Eusebius of Caesarea (ca 260–339 CE), adviser to the Emperor Constantine, one of the most learned men of the age, explained that it was not through ignorance of natural philosophy that he had turned from science, but through contempt for the uselessness of its activity, devoting himself to the direction of souls to better things.

Science Progress Report No. 1

The Earth Becomes Flat Again

"For a man, in those days, to have had an idea that his ancestors hadn't had, would have brought him under suspicion of being illegitimate".

Mark Twain, 1889 (1835–1910)

In the first centuries of the Christian era, mathematics was not in a flourishing state. It was suspect because of its close connection with heathen philosophy. Many even considered it the work of the devil, since the soothsayers and the astrologers often called themselves mathematicians.

The year 529 *marks the end of ancient mathematics and the beginning of the 'Dark Ages' in the history of science. When in* 527 *Justinian became emperor in the East, he evidently felt that the Pagan learnings of the Platonic Academy in Athens and other philosophical schools at Athens were a threat to orthodox Christianity. Hence in* 529*, the philosophical schools were closed and the scholars dispersed.*

As a result, there began a migration of scholars into Persia, where they established, under King Chosroes, the 'Athenian Academy in Exile'. Henceforth, the seeds of Greek science were to develop in Near and Far Eastern countries, until, some 600 *years later, the Christian world was in a more receptive mood.*

Mathematics did not disappear entirely from Europe in 529*. The spirit of mathematics languished, however, and was replaced by barren theological scholasticism.*

As far as the earth and the heavens were concerned, the Ptolemean geocentric cosmology reigned supreme. The actual architecture of the world was thought to approximate Aristotle's scheme of crystal spheres. But this purely descriptive scheme left an obvious epistemological void, which the theologians moved in to fill. The early leaders of the Christian Church insisted on a literal interpretation of the relevant Biblical passages, and the earth became flat again[333]*.*

[333] In 1993, the supreme religious authority of Saudi Arabia, issued an edict (fatwa), declaring that the world is flat. The edict also stated that anyone of the round persuasion does not believe in God and should be punished.

Among many ironies, the lucid evidence that the earth is a sphere, accumulated by Ptolemy, was transmitted to the West by astronomers who were Muslim and Arab; It the 9^{th} century, they named Ptolemy's book (in which the sphericity of the earth is demonstrated), the *Almagest*, "The Greatest".

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2. *Slumber and Awakening*

529 CE–1583 CE

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1337–1453 CE	The *Hundred Year War*; Battle of *Agincourt* (1415)
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1492–1498 CE	Expulsion of the Jews from Spain and Portugal: Iberian culture sank back into the Middle Ages and never recovered
1507–1595 CE	Ca 50 million natives of South and Central America Perished from smallpox and typhus pandemic in the wake of the Spanish conquest

* * *

“*O sweet spontaneous* earth,
How often have the doting fingers
    of prurient philosophers *pinched and poked thee,*
    has the naughty thumb of science *prodded thy beauty;*
How often have religions *taken thee*
    upon their scraggy knees, squeezing and buffeting thee
    that thou mightest conceive gods.
But true to the incomparable couch of death–
    thy rhythmic lover,
Thou answerest them only with spring”.

E.E. Cummings (1923)

Monasteries and Monks or,– the case of the ignorant copyists (529–1100)

The period of history which lies between ancient times and modern times, from the fall of the Western Roman Empire to the fall of Constantinople and the end of the Eastern Roman Empire, is generally known as the Middle Ages.

It was a period when the removal of the strong, central government of Rome left Europe in chaos. The mighty empire was fragmented into small kingdoms, and in many places rule was by local lords, each of whom exercised power only in the immediate vicinity of his own castle.

In Europe, this was a time of hardship and poverty. With the lack of wealth and consequently of people able to act as patrons, there was a decline in learning. Gradually however, nationalistic feelings grew and strong kings began to shape countries out of their lands. The rise of the feudal system created a structure on which secular governments could be based.

The drop in literacy in the Roman Empire was particularly sharp after the 3^{rd} *century. There was a decline in the availability of texts, and the period saw a basic cultural shift, an expansive loss of awareness of past achievements in the writing of history, as well as in philosophy and literature. Even by 400 CE, works by Cicero were difficult to find, and by the end of the* 6^{th} *century, the very few leading intellectuals of the Latin West who did exist, such a* **Gregory of Tours**[1] *(538-594), could barely write coherent sentences. From 600 to 1000, most people forgot to read and think, and, in fact, forgot that they had forgotten. There was an inability to approach texts critically, even among the leaders of European culture. Scholarship consisted of collecting quotes and facts, and the reasoning used by these scholars in their own works bore little resemblance to the classical texts they admired. Real scholarly debates and understanding, genuine logical interaction, did not appear until the* 11^{th} *century. Thus the proverbial lights went out in Western Europe.*

One may therefore ask: how did the Phoenix rise from its ashes 600 years later? To answer this question one must consider what the sources of cultural preservation were between roughly 500 and 1100 BCE, *and what difference these made for the cultural reawakening of Western Europe that began toward the end of the* 11^{th} *century and then lurched unevenly toward the Italian Renaissance and the Scientific Revolution.*

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[1] He was ignorant of the rules of grammar, his spelling was faulty, his syntax shaky, and his arguments elementary. He wrote in the vernacular language of his time.

During the 6^{th} and 7^{th} centuries, monasteries, especially Irish ones, began to stow away the nuggets of intellectual achievement from Roman civilization, and, to a lesser extent, that of Greece. By 700 BCE, *while Europe was sacked by Goths, Arabs, and Vikings, a few scholars as the* ***Venerable Bede*** *(672–735), preserved the knowledge of the classics, carrying the seeds of Western life through the grim winter of the Dark Ages. In the 7^{th} century alone, 200 new monasteries were founded in Gaul. For 300 years, Irish monasteries produced a series of remarkable men who exerted a profound influence on thought and letters in Western Europe. Their missionary work extended to Scotland and the Continent, and disciples flocked to these regions. Monasteries such as those founded by the Irish monks* ***Columba*** *(521–597) and* ***Aidan*** *(590–651) became important study centers.*

In the monasteries, monks were busy in the scriptorium transcribing old manuscripts[2]. Most of them did not understand the implications of the texts they were copying. Passages got transcribed without any inquiry as to whether they made sense, or contradicted other authorities. In fact, many Benedictine monks had entered the monasteries as children, handed over by their parents. The scriptoria thus became the loci of cultural preservation, but the copying of manuscripts was more a manual training than an intellectual one: calligraphy rather then philosophy.

Moreover, monasteries, from the 4^{th} century on, were not schools for sacred study, but, rather, of ascetic practice. Monks and clerks regarded philosophy as a source of heresy. By 500, the monastic ideal included the notion that learning was incompatible with Christian culture. The idea of the monastic school was to break with classical learning, and to teach the "science" of ascetic contemplation instead. Thus, it appears that the intellectual disciplines of distinction, definition, and dialectic were lost to the readers of the Dark Ages. Their apprehension of the world rested on myth and magic, and the prevalent mind-set was one of symbols, analogies, and images. Both Columba and Aidan renounced classical learning, implanting instead the desire to meditate on Scripture.

By 800, then, written civilization has disappeared, and only a tiny elite had access to intellectual culture. And yet, this warped form of classical preservation did serve an important purpose. Manuscripts were copied, libraries were accumulated, and the later dispersion of this material made the

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[2] Supposedly, the monastic built-in schools taught the old curriculum of the *trivium* (grammar, rhetoric, and dialectic) and the *quadrivium* (arithmetic, geometry, music, and astronomy). However, the anglo-saxon clergy, knew little or no Latin and they were in fact, as ignorant as the majority of laymen. By 600, even leading intellectuals were not able to think in the sense that the people of antiquity or of the 12^{th} century were able to do.

subsequent revival of learning possible. When the cognitive revolution *(that began in the late* 11^{th} *century) finally took hold, the ancient material was at least available, now to be looked at through new eyes.*

So the texts had to be there for a 12^{th} *century renaissance to occur, and the monks of the Dark Ages managed to preserve them, even if they did not understand them very well and despite the fact that their own purposes were ascetic rather then intellectual.*

Finally, the steady growth of power and wealth within the church provided another unifying force, and gave some men leisure to pursue lives of scholarship and study.

The great monasteries founded throughout continental Europe during the Middle Ages preserved ancient learning. These were inhabited by communes of monks and nuns who lived by the so-called 'rule' of the monastery. The greatest 'rule' was that of ***St. Benedict of Nursia*** *(480–542) who founded the monastery of Monte Cassino in Italy at about 529. Under the guidance of Pope Gregory VII, some monks became scholars and teachers. At a time when few laymen could read or write, they preserved much classical learning which would otherwise have been lost*[3]*. Monks prayed for the souls of the dead, and had practical duties such as caring for the sick and feeding the poor. The religious houses – both male and female – provided almost all the medical skill available then. The monastic libraries preserved much classical learning, but books disapproved of by the church, were kept hidden away or destroyed.*

In the Germanic kingdom, the Benedictine monasteries exerted powerful influence. Their emphasis on obedience to higher authority helped hold the church together at a time when any sort of centralization was hard to achieve. In many parts of Europe the Benedictines introduced valuable new techniques, such as stone masonry and organizing agriculture around the large estate. Thus they served as nuclei for the growth of towns, for they were usually more prosperous than the neighboring countryside.

Monasteries spread rapidly throughout most of the Germanic kingdoms, often more rapidly than organized dioceses and parishes.

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[3] It was the monks who *preserved* the cultural heritage of the ancient Greeks by copying and recopying the old parchments to save them from decaying. It was the Christian Church, however, who decided what should be copied and what should not. These decisions affected mainly works of literature and philosophy. Thus, the Church held the power to censor and exterminate any piece of cultural item which in its opinion threatened the Christian dogma.

ca 530 CE **Simplicius**. Greek philosopher. Disciple of Ammonius and Damascius. Author of commentaries on some of Aristotle works.

A native of Cilicia and one of the last of the Neoplatonists. When in 529, the school of philosophy at Athens was disendowed and the teaching of philosophy forbidden, the scholars **Damascius**, Simplicius, **Priscianus** and four others resolved in 531 or 532 to seek the protection of Chosroes, king of Persia. They returned to Greece when Chosroes, in his peace treaty with Justinian (533 BCE), expressly stipulated that the seven philosophers should be allowed "*to return to their own homes, and to live henceforward in the enjoyment of liberty of conscience*".

Simplicius was not an original thinker, but his remarks are thoughtful and intelligent and his learning is prodigious.

530–540 CE **Dionysius Exiguus** (497–540, Rome). Chronologist. A Christian monk, introduced in ca 525 the method of dating years of the Christian Era. He chose its first year to be the year of Christ's birth (the *actual* time of that event was a few years earlier). Since he started from year 1 and not year 0, the previous year is called 1 BCE.

Prior to this date, Near-Eastern, Greek and Roman Calendars were all related to local fiducials, centered on current monarchs or specific events such as the *fall of Troy* (1183 BCE, recommended by Eratosthenes), the great *earthquake* at the time of the prophet Amos (Amos I, 759 BCE), the *olympic games* (Greece, 776 BCE, 772 BCE etc.), the *founding of Rome* (A.U.C.=ab urba condita, ca 754 BCE), the *battle of Salamis* (480 BCE) and other examples.

The era most commonly used in Dionysius' day was the *Diocletian era*, which the Christians called the *era of martyrs*. It began on August 29, 284. Dates were often established with reference to the Roman consuls.

The Christian era was not adopted at once; in the Byzantine Empire, Dionysius' reform was not accepted at all. Byzantines numbered their years with reference to the 15 years indiction-cycles[4] and to the creation of the world. The Christian era was introduced in Russia only in the time of Peter the Great. The habit of counting pre-Christian years with reference to the Christian era is a recent innovation.

530–548 CE **Cosmas of Alexandria** (**Indicopleustes**). Greek navigator and explorer of the Indian Ocean. (The surname is inaccurate since he never

[4] **Diocletian** fixed a 15-year assessment of property tax. It was used as a chronological unit in ancient Rome and incorporated in some medieval systems.

reached India proper; further, it is doubtful whether Cosmas is a family name, or merely refers to his reputation as a cosmographer.)

In his early days he had sailed the Red Sea and the Indian Ocean, visiting Abyssinia and Socotra and apparently also the Persian Gulf, coast of western India and Ceylon. He subsequently became a monk, and at about 548 CE, in the retirement of a Sinai cloister, wrote a work called *Topographia Christiana*.

According to Cosmas' map the earth is a rectangular plane, covered by the vaulted roof of the firmament, above which lies the heaven. In the center of the plane is the inhabited earth, surrounded by an ocean, beyond which lies the paradise of Adam. His was probably the oldest Christian map. In it the sun revolves round a conical mountain to the north – round the summit in summer, round the base in winter, which accounts for the difference in the length of the day. Cosmas is believed to have been a Nestorian.

537 CE Dedication of the church *Hagia Sophia*[5] in Constantinople (now Istanbul), the finest and most famous example of Byzantine architecture (East Roman) in the world. It was built as a Christian cathedral by **Justinian I, the Great**, (482–565 CE) between 532 and 537 CE. The architects employed were **Anthemias of Tralles** and **Isidoros of Miletos**.

The main problem they had to solve was that of carrying the dome (which measures 56 m high and 33 m across) on four arches. The building itself is 76 m from east to west and 72 m from north to south. The four arches formed a square on a plane, and between them were built spherical pendentives, which, overhanging the angles, reduced the center to a circle on which the dome was built. During the earthquake of 555 CE, this dome fell down, and when rebuilt was raised higher and pierced round its lower part with 40 circular-headed windows, which give an extraordinary lightness to the structure. This was done between 558 and 563 CE. After 1453, when the Turks conquered the city, the building was used as a mosque. Since 1953, Hagia Sophia has served as a museum.

The inside appearance is one of great space, height and richness. The rare and costly building materials were brought from many parts of the Roman Empire. The marble-lined walls have many colors and designs. Mosaic decorate the vaults, and beautiful pictures decorate the walls.

542–594 CE Waves of *Bubonic Plague* over Europe, Asia and Africa. Known as the *Plague of Justinian*, it began in lower Egypt and from there spread to Israel, Asia Minor, Italy, the North coast of Africa and Gaul. Many millions died over the whole Roman world.

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[5] A Greek phrase meaning 'Holy Wisdom'.

ca 550 CE **Asaf ha-Rofeh**; **Asaf Yudaeus** (Israel and Babylonia). Physician and medical scholar from the pre-Arabian period. Known for his extant "*Book of Medicine*". Although the book is, in the main, based on the teachings of the Greek Hippocratic school, it was modified to suit Near-Eastern climate, national tastes and habits, with important Talmudic and ancient Israeli sources. The book was originally written in Hebrew and later translated into Greek, Latin and Arabic. It serves today as a source of ancient medicine (terminology, prognostics, diagnostics, hygiene, diet, pathology, anatomy, pharmacology etc.).

ca 550 CE **Olympiodoros (the Younger) of Alexandria** (ca 510–570 CE). A Greek Neoplatonic philosopher. Maintained Platonic tradition in Alexandria after suppression of Athenian school (529 CE) by Justinian. Wrote commentaries on Plato and Aristo. Euclid's and Heron's discovery that light (reflected from a plane surface) takes the path of least time prompted him to say in his *Catoptrica*:

"Nature does nothing superfluous or any unnecessary work."

In medieval times it was commonly accepted that nature behaved in this manner. Indeed, **Leonardo da Vinci** said: "Nature is economical and her economy is quantitative". Later on, **Robert Grosseteste** said: "Nature always acts in the mathematically shortest and best possible way."

ca 550 CE *The map of Madaba.* The oldest true geographical map extant. It is a mosaic map of Israel, discovered in Moab in 1896.

550–580 CE **Varahamihira** (505–587, India). Astronomer. Produced a revised version of the Indian Calendar. His works on mathematical astronomy include a number of trigonometrical identities, apparently derived by him for the first time:

$$\sin x = \cos(\frac{\pi}{2} - x); \qquad \sin^2 x + \cos^2 x = 1; \qquad 1 - \cos 2x = 2\sin^2 x.$$

Also, extending the work of Aryabhata the Elder, he gave more accurate sine tables. He also produced a Pascal's triangle for the binomial coefficients C_r^n.

550–580 CE **Alexander of Tralles** (Trallianus, ca 525–605 CE). Byzantine physician. Practiced in Rome; author of major work on pathology and therapy circulated in Greek, Arabic and Latin (*Libri duodecim de re medica*) into the 16^{th} century. Born at Tralles in Lydia.

560–760 CE Decline and eradication of the *Moche civilization* (since 100 CE) in Northern Peru due to destruction of their water irrigation canals by repeated El-Ninio weather systems.

ca 600 CE The first documentary evidence of *Chess*. There is supportive evidence that places it in North India in 455 CE. Some historians place the origin of Chess even prior to the invasion of India by Alexander the Great in 326 BCE. From India, Chess spread to Persia and China.

600–650 CE A dry period preceded the great wave of Arab outburst out of Arabia.

600–800 CE Arabs carved themselves an empire from the Indian Ocean to the Atlantic within one century.

622–633 CE **Isidore of Seville or Isidorus Hisplensis** (ca 560–636, Spain). Encyclopedist and historian whose books served to keep alive, throughout the Dark Ages, some little knowledge of the antique culture and learning. One of the most important links between the learning of antiquity and the Middle Ages. His chief works were:

- *Chronica* – a history extending to his own time.
- *Originum sive etymologiarum* – an encyclopedia in 20 volumes which deals with geography, law, medicine, natural history, architecture, etc. It was based on available knowledge of Greek learning, borrowed from **Pliny the Elder**, **Suetonius** and a wide range of other authorities, to whom credit is not given. It is an unsystematic, uncritical treatise, merely reproducing at second hand the substance of antique sources. Yet even this inadequate encyclopedia was very influential in early Medieval times. It contributed to the acceptance of astrology in Medieval Europe.

Isidore was educated in a monastery and became archbishop of Seville (602–636). He was of Jewish birth[6] and was made a Saint of the Catholic Church.

[6] It is a fact that the founders of Christianity, many of the Church Fathers (including the first three Popes), and a considerable number of the Catholic Saints, were of Jewish origin. So was Saint Isidore of Seville, one of the most commanding and devout figures in Spanish Church History of the Visigothic period. Yet, it is these apostates out of spite, that generated the greatest hate against the Jews. In his polemic book *De fide Catholica... contra Judaeos* (630) he endeavored to 'prove', using biblical quotations, the 'wrong' religious perception of Jews.

622 CE, July 16 (**Friday**) *Commencement of the Muslim Calendar*, on the day of the flight of **Muhammad**[7] (570–632 CE) from Mecca to Medina

[7] Founder of *Islam*. In 595 CE, he married a wealthy widow, who had employed him as a camel-driver for her caravans. It is at this point, while traveling throughout Arabia, that Muhammad came into contact with Jews and Judaism and began to reflect on the differences between the naive pagan pantheism of his people and the lofty religious concept of the Jews.

The Islamic religion is embodied in the *Koran* (from the Arabic work "reading"). Muhammad rejected the concepts of Virgin Birth and Trinity, and insisted with the Jews that God was one, needing neither a family nor companions. He was also repelled by the Christian worship of saints, which he viewed as idolatry, and like the Jews banned all statue worship. The predominant non-Arabic figure in the Koran is **Moshe**, not Jesus.

Of the six basic tenets of Islam, four are derived from Judaism (belief in the immortality of the soul; belief in one invisible God; the belief in a God-sent prophet; the belief in the Book as the revealed text) and two from Christianity (Judgment day; total surrender of the human will to God). The correspondence is Jehovah-Allah; Moses-Muhammad; Torah-Koran; "God's grace"-Islam.

The mosque, like the church, was modeled on the synagogue. In the matter of prayers, forbidden foods, circumcision, hygiene, marriage and divorce, and the study of sacred Scripture, Muhammad and his successors also followed Jewish models faithfully. This may explain why Arabic and Jewish culture during the first 5 centuries of Islam showed such a remarkable affinity, and why in Spain, North Africa and Babylonia they went through an almost parallel development.

Whether from conviction or expediency, Muhammad, at the outset of his prophetic career, addressed his preachings almost exclusively to the Jews of Arabia. He borrowed much of the narrative material as well as the doctrine for the *Koran* from the *Talmud* and *Midrash*. In composing his poetic *Suras* he used the well-known Jewish Bible stories of Adam, Abraham, Lot, Joseph, Moshe, Saul, David, Solomon, Elijah, Job and Jonah.

When Muhammad first raised the standard of militant Islam in Mecca, he was persecuted and had to flee (622 CE). He went to Medina, where he expected that a large part of the Jewish population would accept him as the Prophet of Allah since, as he saw it, their religion was the same as his. He posed as a prophet of Israel and preached to the Jews of Medina in a style he thought rabbinical. Furthermore, he made Jerusalem his *Kibla* (the direction in which he turned his face while at prayer).

But when the Jews of Medina, who were well acquainted with their sacred writings, proceeded to expose his brand of *Torah* as fraud, Muhammad turned furiously away from them after many acrimonious disputes with their rabbis. He then changed his *Kibla* from Jerusalem to Mecca.

(*Hegira*). The years of the Hegira are purely lunar, and always consist of 12 lunar months, commencing with the approximate new moon, without any intercalation to keep them to the same season w.r.t. the sun, so that they retrograde through all seasons in about $32\frac{1}{2}$ years. They are also partitioned into cycles of 30 years, 19 of which are common years of 354 days each, and the other 11 are intercalary years having an additional day appended to the last month. The mean length of the year is therefore $354\frac{11}{30}$ days, or $354^d : 8^h : 48^m$, which upon division by 12 gives $29\frac{191}{360}$ days or $29^d : 12^h : 44^m$, as the time of a mean lunation. This differs from the astronomical mean lunation by only 2.8 seconds, and leads to an error that amounts to a day in about 2400 years.

625 CE In *Chiku Suan Ching* (Continuation of Ancient Mathematics), Chinese mathematician **Wang Hsiao Tung** presented approximate solutions of algebraic equations of the third degree with practical applications to problems encountered by engineers, architects and surveyors.

628 CE **Brahmagupta of India** (598–670 CE). Astronomer and mathematician. Head of an astronomical observatory at Ujein. Wrote a book on

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Jews arrived in Arabia as early as the days of King Solomon (900 BCE), when his ships sailed the seas in search of new markets. During the period of the *Hasmonean Kingdom* (142–64 BCE), Israel became too small to hold the growing Jewish population, and numerous Jews went forth to seek homes in Syria, Egypt and the western coast of Arabia. It is also possible that when the Nabatean Arabs invaded Israel (as they did on several occasions) they took away some captives who eventually made their way further south. The defeat by Rome (70 CE), and the ill effects of the subsequent rebellions, increased the Jewish population in this part of the Eastern world.

By the middle of the 5^{th} century Jews were to be found as far as the Kingdom of Yemen. The Jews helped found the city of Yathrib (Medina), which, by that time, had become the largest, most important city in Arabia, its 10,000 Jews constituting the majority of the population. Far from the centers of Jewish culture in Israel and Babylonia, the Arabian Jews possessed little learning. They did know and revere the Bible and therefore were known to the pagans around them as "the People of the Book". Their belief in the unity of God, their higher personal morality, their dignified observance of Jewish feast and fast days, their rest from work on the Sabbath and their refusal to permit a fellow Jew, even of a different tribe, to remain in slavery, left a deep impression upon their neighbors. In this way, their presence and example prepared the mind of the heathen Arabs for the acceptance of a higher form of religion.

Thus Judaism in Arabia served as a foundation for Islam in the same way that Diaspora Judaism in the Roman Empire had served as a foundation for early Christianity.

algebra and trigonometry. Brahmagupta also wrote an important work on astronomy in 21 chapters *Brahma-sphuta-siddhanta* (628) (The Opening of the Universe). His understanding of the number systems was far beyond others of his period. He developed some algebraic notation. In his trigonometry he gave not only Heron's formula for the area A of a triangle in terms of the three sides but also the remarkable extension for the area of a cyclic quadrilateral having sides a, b, c, d and a semiperimeter $s = \frac{1}{2}(a+b+c+d)$, namely[8]

$$A = \sqrt{(s-a)(s-b)(s-c)(s-d)}.$$

For the sides of right-angled triangle, Brahmagupta gave the two sets of values $\{2mn, m^2-n^2, m^2+n^2\}$ and $\{\sqrt{m}, \frac{1}{2}(\frac{m}{n}-n), \frac{1}{2}(\frac{m}{n}+n)\}$, values that he probably obtained from Greek sources.

Among other mathematical topics taught by Brahmagupta were rules for elementary operations with positive and negative numbers, progressions and Diophantine equations of the first and second degree.

He perhaps used the method of continued fractions to find integral solution to indeterminate equations of the type $ax + c = by$.

Brahmagupta also solved quadratic indeterminate equations of the type $ax^2 + c = y^2$ and $ax^2 - c = y^2$. For example he solved $8x^2 + 1 = y^2$, obtaining the solutions:

$$(x, y) = (1, 3), (6, 17), (35, 99), (204, 577), (1189, 3363), \ldots$$

For the equation $11x^2 + 1 = y^2$ Brahmagupta obtained the solution

$$(x, y) = (3, 10), (161/5, 534/5), \ldots$$

He also solved $61x^2 + 1 = y^2$, which has

$$(x, y) = (226\ 153\ 980, \quad 1\ 766\ 319\ 049)$$

as its smallest solution.

[8] Brahmagupta wrongly believed that it held good for any quadrilateral. The Greek mathematician **Hero** had, however, pointed out that the area of the general quadrilateral is not determined by the four sides alone. Indeed, medieval mathematicians discovered the general formula

$$A^2 = (s-a)(s-b)(s-c)(s-d) - abcd\cos^2\left(\frac{A+C}{2}\right),$$

where A,C is a pair of opposite vertex angles of the quadrilateral.

He gave explicit expressions for the sum of squares of the first n natural numbers as $n(n+1)(2n+1)/6$ and the sum of cubes of the first n natural numbers as $(n(n+1)/2)^2$. No proofs are given so we do not know how Brahmagupta discovered these formulas.

Brahmagupta derived an interpolation formula which he used to compute values of sines. The rule is equivalent to the *Newton-Stirling* interpolation formula to second order differences.

The great Hindu mathematicians (500 BCE–1400 CE)

There are many differences between Greek and Hindu mathematics. In the first place, the Hindus who worked in mathematics regarded themselves primarily as astronomers, and thus Hindu mathematics remained largely a handmaiden to astronomy; with the Greeks, mathematics attained an independent existence and was studied for its own sake. Also, due to the caste system, mathematics in India was cultivated almost entirely by the priests; In Greece, mathematics was open to anyone who cared to study the subject. Again, the Hindus were accomplished computers but mediocre geometers; the Greeks excelled in geometry but cared little for computational work. Even Hindu trigonometry, which was meritorious, was arithmetical in nature; Greek trigonometry was geometrical in character.

The Hindus wrote in verse and often clothed their works in obscure and mystic language; the Greeks strove for clarity and logic in presentation. Hindu mathematics is largely empirical, with proofs or derivations seldom offered; an outstanding characteristics of Greek mathematics is its insistence on rigorous demonstration. Hindu mathematics is of very uneven quality, good and poor mathematics often appearing side by side; the Greek seemed to have an instinct which led them to distinguish good from poor quality and to preserve the former while abandoning the latter.

In the period between the demise of the Greek world and the rise of Islam, it was India that occupied the center of the mathematical stage. We know that there was some mathematical activity during the first millennium BCE, *but we have no texts earlier than the* 5^{th} *century CE. The degree of influence of Greek, Babylonian, and Chinese mathematics on Hindu mathematics, and vice versa, is still an unsettled matter, but there is ample evidence that influence in* *both directions was appreciable. One of the pronounced benefits of the Pax Romana was the diffusion of knowledge between East and West, and from a very early date India exchanged diplomats with both the West and the Far East. From about 450 CE until ca 1500 CE, India was subjected to numerous foreign invasions: Huns, Arabs (8^{th} century)and Persians (11^{th} century). During this period there were several Hindu mathematicians of prominence.*

Hindu mathematics and astronomy reached its zenith during the period of the Gupta dynasty (320–550 CE). The Hindus had long been interested in these subjects and surpassed even the Greeks of the Hellenistic period in some branches of Mathematics. Using abstract principles of algebra, the Hindus could cope with much more difficult concepts than found in the visible Greek demonstrations of geometry. Greek algebra was rudimentary, but the Hindus invented the concept of negative quantities, solved quadratic equations, and calculated the square root of 2.

Hindu astronomers during the Gupta period were interested in predicting the positions of the planets for astrological purposes, having a deep belief in the cyclic nature of the world. Thus, for example they believed that it takes the planets 4,320,000 years to return to their identical positions. This time span they called Maharyuga (great yuga). According to the Hindus, once in about 4 million years, the planets come into grand conjunction (roughly in line as seen from Earth). According to two pieces of Sanskrit from the Bhagavad-Gita this occurred on Friday, 18 February 3102 BCE.

Furthermore, the Hindus believed that the entire universe went through alternating periods of 'awake' and 'sleep' every 1000 Maharyuga (= 4.32 billion years) just like the oscillating universe in modern cosmology. Aryabhata's task was to fit the Greek description of planetary motions developed by Apollonios and Claudius Ptolemy at Alexandria, into a Hindu religious settings.

The degree of influence of Babylonian algebra on the pre-Diophantine geometric algebra of the Greeks, on Diophantos himself, and on algebra in India, has been considerable. This may have been brought about through trade relations and by frequent foreign invasions of India throughout the first millennium CE, when Hindu algebra and trigonometry were being developed mostly by **Aryabhata the Elder** *(476–550 CE),* **Brahmagupta** *(598–670 CE),* **Mahavira** *(ca 850 CE),* **Bháskara** *(1114–1185 CE) and* **Madhava** *(ca 1400 CE).*

Nevertheless, there is sufficient evidence today, from a number of independent sources, to support the claim that original ideas, methods and computational systems in the fields numeration, algebra, trigonometry and even mathematical analysis (!), originated for the first time in the minds of Hindu *mathematicians throughout 19 centuries before the High Renaissance in Europe. Let us summarize their achievements, one by one:*

- `Algebra:` *Permutations and combinations, notations for indices and laws of their operations, including fractional powers. Binomial theorem. Extraction of roots. Solution of indeterminate equations of the first and second degree. Solutions of Pell's equation.*

- `Trigonometry:` *First use of the sine function as we know it today. Basic trigonometric identities and construction of sine tables. Second-order interpolation to compute intermediate functional values. Infinite power-series expansions of trigonometric functions (without the use of infinitesimal calculus). Power series for* π.

- `Geometry:` *Discovery of the formula for the circumradius of a cyclic quadrilateral.*

- `Advent of mathematical calculus:` *by the* Kerala *school of astronomy (**Narayana**, **Madhava** and **Nilakantha**). This incorporates the concept of* instantaneous motion *of the moon at a given point in time; they calculated this quantity from the formula (in modern notations)*

$$u' - u = v' - v \pm e(\sin w' - \sin w) \tag{1}$$

where

$$\begin{aligned} u &= \text{moon's true longitude,} \\ v &= \text{moon's mean longitude,} \\ w &= \text{moon's mean anomaly} \end{aligned}$$

all at a particular time; (u', v', w') *are the same entities after a specific time interval;* e *is the orbital eccentricity at this stage. The Hindu mathematicians recognized the important conceptual step that for short time intervals*

$$(\sin w' - \sin w) \approx (w' - w)\cos w \tag{2}$$

In modern notations (1) and (2) are written as

$$\delta u = \delta v \pm e \cos w(\delta w) \tag{3}$$

***Bháskara** then made the decisive step of isolating the fundamental mathematical idea of the* differential *from its astronomical manifestation. Clearly, equation (2) implies in modern notations that*

$$d(\sin w) = \cos w dw.$$

He noted that when a variable attains the maximum value, its differential vanishes.

He also gave us the trigonometric identities

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta.$$

- `Present-day numerals, decimal place-notation and first use of the zero. Representation of unknown quantities and negative signs.`

  The Hindu-Arabic decimal numeral system $(0, 1, 2, 3, 4, 5, 6, 7, 8, 9)$ *is named after Hindus, who invented it, and after the Arabs, who transmitted it to Western Europe. This was the most significant achievement of Hindu mathematics during the European Dark Ages. Final maturation of this notation into a place-value system probably took place around 500 CE.*

  The last and most difficult step was the promotion to full membership of a tenth numeral: a round symbol for zero – or sunya (which means 'empty'), as the Hindus called it. Confusion about the status of this mysterious numeral persisted for centuries as 'a symbol that merely causes trouble and lack of clarity'. How, it was asked, could a symbol which means 'nothing', when placed after another numeral, enhance its value tenfold?

  The Hindus were not proficient in geometry. Rigid demonstration were unusual in their works, and postulational development were nonexistent. Their geometry was largely empirical and generally connected with mensuration. Yet some of their algebraic results in geometry are remarkable. Among these are the area A *and diagonals* $\{D_1, D_2\}$ *of a cyclic quadrilateral having sides* (a, b, c, d) *and semiparameter* s*, given by* ***Brahmagupta****:*

$$A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

$$D_1^2 = \frac{(ab+cd)(ac+bd)}{ad+bc}; \qquad D_2^2 = \frac{(ac+bd)(ad+bc)}{ab+cd}$$

$$D_1 D_2 = ac + bd$$

  In algebra, Brahmagupta solved quadratic equations and allowed for the possibility of negative solutions (665 CE).

  In his astronomical treatise Khanda Khadyaka (665 CE) he derived an interpolation formula for the sine function which in modern notation is *equivalent to the Newton-Stirling interpolation formula*[9] *up to second order differences,*

$$f(a + xh) = f(a) + x\frac{\triangle f(a) + \triangle f(a - h)}{2} + x^2\frac{1}{2!}\triangle^2 f(a - h)$$

where $\triangle$ *is the first-order forward-difference operator,* $\triangle^2$ *is the second-order difference operator.*

- *`First use of continued fractions`.* ***Aryabhata****, in particular, used them to solve linear indeterminate equations.*

Narayana Pandit *(1340–1400) obtained a rule to calculate approximate values of a square root of a non-square number. He did this by using an indeterminate equation of the second order,* $Nx^2 + 1 = y^2$*, where* N *is the number whose square root is to be calculated. If* x *and* y *are a pair of roots of this equation with* $x < y$ *then* $\sqrt{N}$ *is approximately equal to* y/x*. To illustrate this method Narayana took* $N = 10$*. He then found the solutions* $x = 6$*,* $y = 19$ *giving for* $\sqrt{10}$ *the approximation* $19/6 = 3.16666666666666666667$*, which is correct to two decimal places. Narayana then gave the solutions* $x = 228$*,* $y = 721$ *which render the approximation* $721/228 = 3.16228070175438596949$*, correct to four places. Finally Narayana gave the pair* $x = 8658$*,* $y = 227379$ *which yield the approximation* $227379/8658 = 3.16227766227766227777$*, correct to eight decimal places. Note for comparison that* $\sqrt{10}$ *is, correct to 20 places, 3.16227766016837933320.*

Madhava of Sangamaramma *(1350–1420), astronomer and mathematician, clinched the mathematical analysis trend began by his predecessors Bháskara and Brahmagupta. By giving a* geometric *derivation to infinite power series expansion of circular and trigonometric functions, as well as a finite-series approximation to them, he discovered the sine and cosine power series, about 300 years before* **Isaac Newton***. These series made their first appearance in Europe (1676) in a letter written by Newton to the secretary of the Royal Society, Henry Oldenburg.*

The series are

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9 With the aid of this formula one interpolates the sines of intermediate angles from a sine table. The interval is h and $\triangle\theta$ is the residual angle.

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \tag{4}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \tag{5}$$

Newton was not aware of their earlier discovery. Using his calculus (which obviously was unknown to Madhava) he started from the definite integral representation

$$\sin^{-1} y = \int_0^y \frac{dt}{\sqrt{1-t^2}}$$

and used the binomial series

$$(1+a)^p = 1 + pa + \frac{p(p-1)}{2!}a^2 + \frac{p(p-1)(p-2)}{3!}a^3 + \dots$$

with $a = -x^2$, $p = -1/2$ *to obtain (after term by term integration)*

$$\sin^{-1} y = x = y + \frac{1}{2}\frac{y^3}{3} + \frac{1}{2}\frac{3}{4}\frac{y^5}{5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}\frac{y^7}{7} + \dots$$

Assuming $y = a_0 + a_1 x + a_2 x^2 + \dots$ *and substituting this into the r.h.s. of the above equation, the coefficients* $\{a_0, a_1, a_2, \dots\}$ *can successively be obtained by comparing with the coefficients on the l.h.s. This yields*

$$y = \sin x = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \dots$$

Madhava also gave a series for the inverse tangent function, $\tan^{-1} x$*, three centuries before* ***James Gregory*** *(1667):*

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots \qquad x \le 1 \tag{6}$$

He went on to anticipate ***Brook Taylor*** *(1712) with his expansions for* $h \ll 1$

$$\sin(x+h) \simeq \sin x + h\cos x - \frac{1}{2}h^2 \sin x \tag{7}$$

$$\cos(x+h) \simeq \cos x - h\sin x - \frac{1}{2}h^2 \cos x \tag{8}$$

Now Madhava put $x = 1$ into (6) to obtain

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \dots \tag{9}$$

and he also put $x = \frac{1}{\sqrt{3}}$ into (6) to obtain

$$\pi = \sqrt{12}\left[1 - \frac{1}{3 \cdot 3} + \frac{1}{5 \cdot 3^2} - \frac{1}{7 \cdot 3^3} + \dots\right]. \tag{10}$$

From these results he calculated π correct to 11 decimal places

$$\pi = 3.141\ 592\ 653\ 59$$

which can be obtained from (10) by taking only 22 terms.

Perhaps even more impressive is the fact that Madhava gave a remainder term for his series. He thus improved the approximation of the series for $\pi/4$ by adding a correction term R_n to obtain

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \dots \frac{1}{2n-1} \pm R_n$$

Madhava gave three forms of R_n which improved the approximation, namely

$$R_n = \frac{1}{4n} \qquad \text{or}$$

$$R_n = \frac{n}{4n^2 + 1} \qquad \text{or}$$

$$R_n = \frac{n^2 + 1}{4n^3 + 5n}.$$

There has been a lot of work done in trying to reconstruct how Madhava might have found his correction terms. The most convincing argument is that they come as the first three convergents of a continued fraction which can itself be derived from the standard Hindu approximation to π, namely 62832/20000.

Madhava's outstanding contribution in the field of infinite series expansion of circular and trigonometric functions with the finite series approximations to them, predates European work on the subject by two to three hundred years. Historians claim that the method used by Madhava to derive the above results amounts to term by term integration. Thus, this was a decisive step toward modern classical analysis.

635 CE The value of $\pi = 3.1415927$ was given in decimal notation in the official history of the Sui dynasty.

638 CE *The Muslims conquered Jerusalem* and allowed Jews to resettle there. Upon their conquest of Babylonia (642), Jewish life entered a new cultural phase: an inevitable fusion between Greco-Arab and Jewish-Babylonian civilizations. Aramaic, which had been the Jewish vernacular since the first return of the Exiles (538 BCE), gave way to Arabic, although Hebrew continued to be the language of prayer and religious study.

640–646 CE The remains of the great Alexandria library were burned to ashes by the Saracens and the Arabs (previously pillaged in 48 BCE, 269, 273, 295, 389 and 415). The story is told that the precious manuscripts in the great library were used by the Arabs as fuel to heat the baths of the city.

ca 650 CE The first definite trace of the Hindu numerals outside of India. **Severus Sebokht** praises the Hindus for "… *their subtle discoveries in this science of astronomy, discoveries that are more ingenious than those of the Greeks and the Babylonians; their valuable methods of calculation; and their computing that surpasses description. I wish only to say that this computation is done by means of nine signs*".

From this it is clear that the Hindu numerals had somehow reached the monastic schools of Mesopotamia.

650–690 CE **Paul of Aegina** (Paulus Aegineta, ca 625–690 CE). Greek surgeon. His *Epitomae medicae libri septem* contained nearly everything known of medicine in his time and greatly influenced Arab physicians.

725 CE The Chinese invented the escapement of the *mechanical clock*.

732 CE **Charles Martel**, ruler of the Franks, defeated the Moslems at *Tours*, thus halting their northward advance into Europe. They finally retreated over the Pyrenees in 759. It was the first unsuccessful effort of Islam to take over Western civilization. At this very time, civilization was passing from the *thalassic* to the *oceanic* stage. The Mediterranean was losing its hegemony as a cultural center of the world for cities were widely established throughout Europe to the Atlantic coastline.

732–735 CE **Bede** (672–735, England). Anglo-Saxon scholar, historian, naturalist and theologian. Father of English history. Writer about the calendar, the shape of the earth, and the tides, giving a correct account in each case.

Bede was born at Jarrow, England. Ordained (703); associated with the monastery of St. Paul at Jarrow throughout his life. Taught Greek, Latin, Hebrew and theology. Concluded (732) his ecclesiastical history of England. It may be said that his works, scientific, historical and theological, practically sum up all the learning of Western Europe in his time. Though Bede makes no pretensions to originality, freely taking what he needed, and (what is very rare in medieval writers) acknowledging what he took, still everything he wrote is informed and impressed with his own special character and temper.

746–749 CE Plague devastated Constantinople and spread into Greece and Italy. Ca 200,000 die.

ca 760 CE **Yehudai ben Nachman Gaon** (ca 700–770, Babylonia). Scholar, legalist and perhaps the *Talmud redactor*. Head of the Sura Academy (760–764), when the *Abbasides* were in power. Composed a digest of the Talmudic law, the *Halakhot Psukot* (Law as Decided) to answer fundamental questions at a time when few copies of the Talmud were available.

Yehudai was already old and blind when he was appointed Gaon of Sura, but he radically changed the Jewish world. As Jews spread through the Christian and Muslim Empires, there were more and more legal questions. The Geonim of Babylonia, enjoying the respect of the Abbasid leadership, were recognized as the world's Jewish legal authorities, and communities sent them their questions. Yehudai was the first Gaon to answer such questions systematically. These questions and answers were called *Responsa*. Yehudai was pushing for the authority of the Talmud throughout the Jewish world.

773 CE Hindu works translated into Arabic: A set of astronomical tables was taken to Baghdad and translated from the Sanskrit into Arabic (on the caliph's command) by **Al-Fazari**. It is probable that the *Hindu numerals*, including the zero, were made known in Baghdad at this time.

781–804 CE **Alcuin** (732–804, Babylonia). Monk and scholar. Organized a reform of the Latin language, Roman Alphabet and education under Emperor **Charlemagne**[10] (742–814). A prominent figure of the *Carolingian*

[10] **Charlemagne** inherited his throne as king of Franks (768). He overrode the claims of rivals and relatives, subdued the Saxons, conquered Lombardy, and finally organized an empire that included northern Italy, France and most of modern German and Eastern Europe. As an ally of the Pope and a passionate Christian, Charlemagne was shocked by the decay of Christian learning. He was dismayed by the crude Latin in the letters he received even from bishops. The Carolingian Renaissance that he sparked was a Latin Renaissance. In his famous edict (789), written by Alcuin, Charlemagne ordered: "*In each bishopric*

Renaissance. He transmitted to the ignorant Franks the knowledge of Latin culture which had existed in England since the time of Bede.

Alcuin's main achievement was the *reformation of the Latin script*: Before the age of Charlemagne, letters were strung together without space between words, without periods, commas, or paragraphing. With the Carolingian reform in script, came now the practice of separating words by empty space between them. This helped prevent ambiguities in meaning and so preserved the pure text.

Alcuin was born at Eboracum (York). He was educated at the cathedral school of York. Upon his visit to Rome (781) he met Charlemagne, who persuade him to come to Aachen to organize a reform of language and education. Charlemagne's rich library in his palace at Aachen became a cultural center drawing scholarly Christian refugees from the Moors in Spain, and even from the distant islands of Ireland. He ordered every school to have a *scriptorium*. From monasteries in Germany, Italy and Bulgaria came manuscripts to be copied in Reformed Latin in the scriptorium. Alcuin had the knowledge and the taste to devise new standards. Charlemagne had the administrative power and the will to enforce them. At his school of Calligraphy in the monastery of St. Martin's in Tours, Alcuin taught his reformed script.

700 years later, when movable type came to Europe, letters were fashioned on the model of Alcuin *Carolingian Minuscule*. Long after other monuments of Charlemagne's empire have crumbled, the pages of books published today remain a vivid reminder of the power of well-designed written word. What we call the Roman alphabet is really Alcuin's alphabet.

789 CE **Charlemagne** (Charles the Great, ca 742–814). Roman emperor and king of the Franks; ordered that *schools*[11] should be established in every diocese. The main effect of these schools was to elevate the education of the rude and ignorant priesthood of the age, restore Latin to its position as a literary language, and introduce a correct system of spelling and an improved

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and each monastery let the psalms, the notes, the chant, calculation and grammar be taught and carefully corrected books be available."

[11] From ancient times until the first half of the 19^{th} century, most people – even in advanced societies – never attended school. In ancient Greece, for example, only the sons of citizens could attend school, and most Greek residents were not citizens. During the Middle Ages (400–1500's), the Roman Catholic Church ran cathedral and monastery schools in Europe, chiefly to train young men for the priesthood. Widespread development of *public schools* began in the early 1800's, when government leaders in many countries acted on the belief that a nation's progress depends on educated citizens. By the mid-1800's, the United States and many European countries had established public school systems.

handwriting. The manuscripts of the time are accurate and artistic, copies of valuable books were made, and the texts were purified by careful collation. At that time, no further result ensued from this measure, since the available knowledge was to remain stagnant for a long time to come.

Charlemagne introduced a new system of *weights* and *measures*, introduced a new *calendar*, reformed the coinage, and condemned medical superstitions.

790–880 CE The Chinese introduced the ancestor of *paper money*. Appeared first as bank drafts (790) on money deposited; it could be exchanged for hard cash at a later date. Real paper money i.e. printed paper money used as a medium of exchange, was first used (880) in Szechuan Province in China.

800–1000 CE Vikings assaulted France and England and penetrated deep into Russia. Feudalism settled over Europe as the continent sank into its cultural nadir. Islam rose to its intellectual zenith. Capetian dynasty founded in France, and Saxon in Germany.

ca 800 CE **Mashallah** (ca 750–817, Egypt). Astronomer and astrologer. One of the earliest medieval astronomers. Egyptian Jew. Wrote: *De Scientia Motus Orbis* (translated from Arabic by Gerhardo of Cremona) which was very popular in the Middle Ages.

830–860 CE **Yaqub al-Kindi**, **Alkindus** (ca 800–873, Baghdad). Mathematician, philosopher and encyclopedic scholar. The first Arab philosopher. Made a deep study of Aristotle from the Neo-platonic point of view. Relatively few of his numerous works are extant. They deal with mathematics, physics, music, medicine, pharmacology and geography. He wrote four books on the use of the Hindu numerals. Many translations from the Greek into Arabic were made or revised by him or under his direction. He considered alchemy as an imposture.

Al-Kindi made significant contributions to the theory of knowledge and being: His metaphysics recognized five primary substances – matter, form, motion, place and time – from whose interactions the universe is formed. He was first to apply mathematics not only to the physical world but also to Materia Medica, where he calculated the effect of medicines from the proportions and qualities represented in the various mixtures.

In his manuscript '*on Deciphering Cryptographic Messages*', al-Kindi was perhaps the first author to discuss how contents of a scrambled message can be revealed simply by analyzing the *frequency* of the characters in the ciphertext. The technique is known today as *frequency analysis*[12]

Al-Kindi was born in Basra and flourished in Baghdad, under al-Mamun. A son of a South Arabian governor, he was given the best possible education at Basra and Baghdad. His life was spent in the service of the court as tutor, astrologer, translator and editor of many Greek philosophical works. He was persecuted during the orthodox reaction led by al-Mutawakkil (847–861) and died in Baghdad.

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[12] The method is not valid for short texts (less than 100 letters, say). Longer texts are more likely to follow standard frequencies, although this is not always the case. The following table of relative frequencies in English, is based on passages taken from newspapers and novels with total sample length of ca 100,000 alphabetic characters:

Letter	Percent	Letter	Percent	Letter	Percent	Letter	Percent
a	**8.2**	**h**	**6.1**	o	**7.5**	v	1.0
b	1.5	**i**	**7.0**	p	1.9	w	2.4
c	2.8	j	0.2	q	0.1	x	0.2
d	**4.3**	k	0.8	**r**	**6.0**	y	2.0
e	**12.7**	**l**	**4.0**	**s**	**6.3**	z	0.1
f	2.2	m	2.4	**t**	**9.1**		
g	2.0	**n**	**6.7**	u	2.8		

Thus, the arrangement of the alphabet in order of decreasing frequency is {e,t,a,o,i,n,s,h,r,d,l,u,c,m,w,f,y,g,p,b,v,k,j,x,z,q}.
Clearly, the frequency test fails for the short sentence:
'From Zanzibar to Zambia and Zair, ozone zones make zebras run zany zigzags.'
Since the distribution of frequencies depends on the particular language used in ciphertext, it can be used to identify the language. For example, in German, the letter 'e' has the extra ordinary high frequency of 19 percent. In Italian, there are 3 letters with a frequency greater than 10 percent, and nine letters with frequency less than 1 percent.
If the correlation is sympathetic with English, but the plain text does not reveal itself immediately, the code breaker will focus on *pairs of repeated* letters. In English, the most common repeated letters are 'ss', 'ee', 'tt', 'ff', 'll', 'mm' and 'oo'. Other clues may come from the identification of words containing just one, two or three letters. The only one-letter words are: *a* and *i*. The commonest two-letter words are: *of, to, in, it, is, be, as, at, so, we, he, by, or, on, do, if, me, my, up, an, go, no, us, am.* The most common three-letter words are *the* and *and*.

The Translators

Except for two of his logical treatises, Aristotle's work had been unknown before 1100 CE – buried and forgotten, together with the works of Archimedes, Euclid, the atomists, and the rest of Greek science. What little knowledge survived had been handed down in sketchy, distorted versions by the Latin compilers and the Neoplatonians. Insofar as science is concerned, the first 600 years of established Christendom (500–1100 CE) was a glacial period.

During 600–800, the Muslims on their way from the Arab peninsula through Mesopotamia, Egypt, and Spain have picked up the wreckage of Greek science and philosophy in Asia Minor and in Alexandria, and carried it in a circumambient and haphazard fashion into Europe. From 1100 onwards the fragments of works of Archimedes and Hero of Alexandria, of Euclid, Aristotle and Ptolemy found their way into Christendom.

How devious this process of Europe's recovery of its own past heritage was, may be gathered from the fact that some of Aristotle's scientific treatises, including his Physics*, had been translated from the original Greek into Syriac, from Syriac into Arabic, from Arabic into Hebrew and, finally, from Hebrew into medieval Latin. With Euclid, Aristotle, Archimedes and Ptolemy recovered, science could start again where it had left off a millennium earlier.*

From about 750–1100 CE, Arabic *was the scientific and progressive language of mankind.*

During the period 950–1580 *a 'life line' of science was constantly in operation: Ancient Greek, Hindu, Persian and Arabic lore were steadily flowing to Western Europe via Constantinople, Baghdad, North Africa, Sicily and Spain.* Transmission *is as essential as discovery. For if the results of Ptolemy's investigations had been lost in transit, it would almost be as if they had never existed!*

At the time when Western Europe was weak, its cities almost non-existent and its scholars limited to the study of theology, both Byzantium and Islam had well-organized bureaucratic states, large commercial cities and eminent scholars. About the year 1000*, the Byzantine empire gained control over Asia Minor and the entire eastern Mediterranean, and was at the apex of its power. This political ascendancy was accompanied by intellectual revival. Although not universities of the Western type, schools of philosophy and law were established in Constantinople, where professors were paid regular salaries and held high positions at the imperial court. These scholars however, spent most of their energy copying and commenting on ancient texts, which served* *to preserve many books that otherwise would have been lost. These works include the translation of Plato into Latin. No original work was done.*

In spite of the felt influence of Byzantium in Western Europe (especially in Italy), Western scholars preferred to acquire the Greek-Arabic-Latin translations done in Europe than avail themselves of the more direct Greek-Latin translation that was available in Byzantium. Because Western translators were primarily interested in Aristotle, they overlooked the opportunity to increase the stock of Platonic works available in Latin.

Nevertheless, some important texts, such as the advanced works of Euclid, would have been unknown to Western medieval scholars, had it not been for the efforts of the translators at Constantinople.

During the 11^{th} *century, the Latin and the Greek churches drifted apart. Finally, in* 1054*, pope and patriarch excommunicated each other and the two churches broke off relations in a split that never healed.*

Outside the Byzantine Empire, intellectual activity was going on in the Abbasid Caliphate.

The lands of the Caliphate, which reached its peak of power and wealth under ***Harun ar-Rashid*** *during* 706–809*, stretched from Morocco to the Indus River, from the steppes of central Asia to the Sudan. In this vast territory, which was traversed by all the important east-west trade routes, there were dozens of populous and prosperous cities, of which Baghdad was the largest and the richest.*

This city attracted books and scholars just as it did merchandise and traders. The Abbasids did even more than the Ommiads to transform their empire into a center of scholarship. Hundreds of Greek works, especially on philosophy, science and mathematics, were translated into Arabic and much was learned from Persian and Jewish sources.

Chinese scholarship had little influence, but the Abbasids borrowed many ideas from the Hindus, notably the system of arithmetic notation *and what we call the* Arabic numerals*.*

By the 9^{th} *century, Muslim scholars had assimilated the work of their predecessors and were beginning to make original contributions of their own. From* 900 *to* 1200*, the most important work done anywhere in the world in mathematics, astronomy, physics, medicine and geography, was done in Muslim countries.*

Much Arabic scholarship merely added details to support established scientific theories [accurate observations of star positions; many stars still bear

Arabic names today[13]*]. However, their most remarkable contribution was in physics and algebra.*

Chemistry among the Arabs was cultivated principally by ***Jabir ibn Hayyan Geber*** *(ca* 721*–ca 803), court physician to Caliph Harun ar-Rashid and author of large number of works on alchemy. He enjoyed high reputation with later alchemists and his theory of mercury and sulfur as fundamental substances contributed to later theory of the phlogiston. Jabir was aware that addition of sal* ammoniac *(ammonium chloride, said to have been first derived from camel stables near the Egyptian temple of Jupiter Ammon) to nitric acid enables it to dissolve gold, a fact of considerable metallurgical importance.*

In physics they performed interesting experiments in reflection and refraction of light. In mathematics, besides greatly simplifying arithmetical operations through the use of the new Arabic figures, they carried trigonometry far beyond the Greek accomplishments. And their work in algebra was even more impressive, for they fashioned a whole mathematical discipline out of a few hints provided by the Greeks. Most notable was the illustrious figure of ***al-Khowarizmi*** *(780–850).*

Muslim interest in mathematics and the natural sciences had a decisive influence on the course of Western science. The Byzantines tended to neglect these subjects, and little was known about them in Western Europe.

The Chinese had great technical skill, but they developed no general theories. Hindus, after a promising start, lost their interest in mathematical and scientific problems.

Thus, the Muslim world was the only region that was both actively interested in science and close enough to Western Europe to touch off a revival of scientific interest there.

Western European scholars made their first attempts to recover ancient scientific texts by going to the Muslims of Spain and Sicily. Only after the revival was well underway, did they begin to seek manuscripts in Constantinople.

The Moore invasion of Spain began in 711 *and they conquered almost all of the Visigoth Kingdom by* 718*. The Moores (mostly Berbers from North Africa, converted to Islam), had a more advanced culture than did most of*

[13] E.g.: *Algol*, *Altair*, *Antares*, *Aldebaran*, *Betelgeuse*, *Deneb*, *Sabik*, *Rigel*. The Arabs also generated the terms: *Zenith*, *Nadir*, *Alchemy*, *Algebra*, *Algorithm*, *Alcohol*, *Alkali*.

The word *Zero* probably came from *ziphirum*, a Latinized form of the Arabic word *sifr*. *Sifr* is the translation of the Hindu word sunya (void or empty).

medieval Europe. The Muslims carried with them the Greco-Indian heritage of mathematics, medicine and other fields of study. They also preserved many of the writings of the ancient Greek, Roman and Middle Eastern civilizations.

About the time of ***Gerbert*** *(980), the Greek classics in science and mathematics began to filter into Western Europe. A period of transmission followed during which the ancient learning preserved by the Muslim culture was passed on to the Western Europeans. This took place partly through Latin translations made by Christian scholars traveling to Muslim centers of learning.*

The conquest of Toledo by the Christians in 1085 *was followed by an influx of Christian scholars to that city to acquire Muslim learning. Other Moorish centers in Spain were infiltrated and the* 12^{th} *century became, in the history of mathematics, a century of translators. One of the earliest Christian savants to engage in this pursuit was the English monk* ***Adelard of Bath*** *(1075–1160).*

By the middle of the 12^{th} *century, the scholars of the West had absorbed the Latin classics and the Roman law. Searching about for new materials, they seized upon the rich stone of learning from the East (acquired partly through the Crusades, which brought Christianity into close contact with the East) which had never been translated into Latin. Though the scholars were deeply interested in astronomy and astrology, they had only a few brief texts of those subjects, while the Greeks and Arabs had scores of volumes. The same applied to mathematics and physics – all the really advanced works were in the Eastern languages; in Latin there were only elementary textbooks.*

To correct this deficiency, the Western scholars began the task of translating into Latin the works of Aristotle and the scientific writings of the Greeks and Arabs. The difficulties were formidable – there were no grammars, no lexicons, none of the scholarly apparatus we take for granted in learning foreign languages. The Romans had never been particularly interested in science and had not developed a scientific vocabulary. Consequently, even when a translator knew the meaning of a Greek or Arabic word, he had trouble finding a Latin equivalent. Moreover, many of the important texts had been corrupted by repeated translation.

However, by ca 1250, *they had translated into Latin almost all the works of Aristotle and a great mass of other material.*

The West had acquired the philosophy and science of the East – just in the nick of time, for during the 13^{th} *century, both Byzantium and the Arab world were shattered by civil wars and foreign invasions. Though they made a partial recovery later on, they were never again the intellectual centers they had been in the early middle ages. The scientific tradition which the Greeks had originated and the Arabs had preserved, might have been lost had it not been for the efforts of the translators of the* 12^{th} *and* 13^{th} *centuries.*

The diffusion of Arabic learning into northern Europe was largely due to two factors:

(1) the influence of Jewish physicians (themselves educated in the Moorish schools of Spain and imbued with intellectual independence) who founded the medieval schools of medicine;

(2) the development of scientific navigation before the tradition of the Moorish schools had been finally extinguished. Two features of Catholic tradition and organization forced the universities of Western Christendom to open their doors to the Moorish learning. One was the humanitarian ideology which prompted the monastic orders to found hospitals and seek the assistance of experienced physicians.

The other, which encouraged monks like ***Gerhardo of Cremona*** *or* ***Adelard of Bath*** *to visit the schools of Spain during the Moorish occupation, was the social function of the priesthood as custodians of the calendar.*

In conformity with their role as timekeepers, the Augustinian teaching had endorsed astronomy as a proper discipline of Christian education. So, although the patristic influence was mainly hostile to pagan science, clerical education was not completely immune to influence from non-Christian world.

ca 830 CE **Muhammad Ibn Musa al-Khowarizmi** (780–850). Mathematician, astronomer and geographer. The first to use *zero* as a place holder in positional base notation. Blended the accumulated Greco-Hindu mathematical achievements into the systematic treatise *Hisab al-j'abr w'al-muqabala*[14] ("science of transpositions and cancellations"). The book was introduced to Spain by the Arabs and became known in Western Europe only after the conquest of Spain (1085–1118). The name of the branch of mathematics to which the treatise is devoted, *algebra*, is derived from the title of this work. Al-Khowarizmi's influence upon writers in the field of algebra can be traced directly or indirectly for more than 700 years after his death.

Al-Khowarizmi recognized the value of the *Hindu numerals* (known in Baghdad by that time) and wrote a small book explaining their use[15]. [This

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[14] The Arabic word '*al-j'abr*' means "the bonesetting". Only later it came to acquire the connotation of the setting of the parts of an equation.

[15] Some historians believe that Al-Khowarizmi visited India at about 825 CE and was influenced there by the works of **Brahmagupta**. On his return (ca 830

book was translated into Latin by Adelard of Bath (ca 1120) under the title *Liber Algorismi de numero Indorum*].

The Hindu numerals, adopted by the Arabs were brought to Spain at about 900 CE. From there they diffused to the rest of Europe by traders in the Mediterranean area and by scholars who attended the universities in Spain. It finally came into general use in Europe by the invention of the printing press in the mid 1400's.

In the complete absence of axiomatic foundations and of proofs dependent upon them, al-Khowarizmi's algebra evidenced the author's marked preference for Hindu over Greek methods.

Al-Khowarizmi was born in Persia in the city of Khowarezm (Khiva). He may have been of Zoroastrian descent and had acquired his early knowledge of Hindu mathematics and astronomy from Zoroastrian clergy. He was encouraged by the Caliph al-Mamun (who succeeded Harun ar-Rashid) to join his court at Baghdad in the early 9^{th} century to participate in the work of his newly organized Bayt-al hikma (House of Wisdom). Al-Mamun constructed two astronomical observatories for his new academy, and he made strenuous efforts to obtain extant scientific works for it. In his eagerness to collect scientific manuscripts, he went so far as to send a mission to the Byzantine Emperor Leon for the purpose of securing Greek works. The writings collected by al-Mamun were immediately translated into Arabic by the scholars of his academy. It was in this scholarly atmosphere that al-Khowarizmi worked and reached the peak of his activities.

Al-Khowarizmi also constructed a set of astronomical tables that was to remain important in astronomy for the next five centuries. His *geographical* work is not less important: he helped measure the length of one degree of latitude at the latitude of Baghdad (91 km), and used astronomical observations to determine the latitude and longitude of 1200 important places on the earth's surface, including cities, lakes and rivers. He incorporated these and additional findings in his book *"The Image of the Earth"*.

CE) he brought with him the Hindu numeral system, which he later expounded in his books. However, the Hindu forms described by al-Khowarizmi were not used by the Arabs. The Baghdad scholars evidently derived their forms from some other source, possibly from Kabul in Afghanistan, where they may have been modified in transit from India. **David Eugene Smith** (1860–1944), a historian of mathematics, raised the possibility that the numerals were taken from Egypt to Afghanistan by the Lost Tribes of Israel after being exiled there by the Assyrians (721 BCE). [*Cabul* is a Biblical name: *I Kings* **9**, 13; *Joshua* **19**, 27.] This assumption is strengthened by the Hebrew origin of some of the numeral names in the 10^{th} century: *arbas* (4), *quimas* (5), *temenias* (8).

840–880 CE **Abu-Máshar** (**Albumazar**; 805–885, Baghdad). Muslim astrologer and astronomer. Was born at Balkh, flourished at Baghdad, and died at Wasid in Central Asia. His principal works are: *De Magnis Conjunctionibus* (Latin translation); *Introductorium in Astronomiam*, and *Flores Astrologici*.

Second[16] to offer a rational explanation (although incorrect) of the *causes of ocean tides*, not by invoking supernatural agencies but in terms of the perceived physical nature of the universe: His works, being translated into Latin already in the 12^{th} century, circulated widely in Europe. **Robert Grosseteste** (ca 1200) adopted Albumazar's theory of the tides. The correct gravitational theory was first given by **Newton** (1687).

ca 848 CE First important medieval *medical school* at Salerno, Italy, gained notice. It helped stimulate medical advances between 9^{th} and 11^{th} centuries.

ca 850 CE **Mahavira** (fl. 850 CE). Indian mathematician from Mysore, Southern India, where he lived at the court of one of the Rashtrakuta monarchs. His book *Ganita Sara Samgraha* contributed to the history of *indeterminate equations* and general polynomial equations of the first and second degree. Yet his claim to fame is in giving our rule for dividing one fraction by another in the same words which a school-teacher might use today:

> *"Make the denominator the numerator and then multiply"*

namely

$$\frac{a}{b} : \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}.$$

855 CE **Johannes Scotus Erigena** (ca 810 – ca 877, Ireland and France). Philosopher and Theologian. Born in Ireland and educated in Irish schools. Was called by Charles II the Bald (843) to head the Schola Palatina (court school) at Paris. One of his more important works is *De divisione naturae* (on the division of nature), which was condemned by the Church because of pantheistic learning. It was an attempt to reconcile Neoplatonistic and Church philosophy of creation[17], i.e. to demonstrate rationally all the truths of the

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[16] He was preceded in this respect by **Seleucus the Babylonian** (fl. 150 BCE).

[17] From Plato to Erigena philosophers had been explaining the universe as a union of *ideas* (or forms) and *matter*. **Plato** thought of ideas as existing before things. **Aristotle** taught that forms existed in things but were distinct from matter. The Christian taught that ideas or forms existed in the mind of God and moulded matter into things of the universe. All these philosophers have been called *realists* since they taught that ideas or forms are real things existing independently of whether or not they ever come into contact with matter.

Christian faith. He was a more rigorous thinker than Augustine – the technical quality of the argumentation is higher, and his intellectual points drive deeper. One of his arguments was to the effect that since God is unknowable and undefinable, man cannot expect, with his little and limited brain, to understand God or to comprehend his ways. Although, not an original idea (*Job* **25**; *38–39*), this insight was generalized by **Kant** into the point that it is impossible for a human being to understand its own nature.

Erigena taught that God created the world out of nothing or 'out of himself', the causeless first cause. Before God created the world, he had a complete pattern of the world in his mind. Then, as a light radiated from its source, so the world was radiated from God. Consequently, the universe and God are one, but God is more than the universe: God is in his creation and his creation is in him. The universe is a unity. *The universe is "an expression of the thought of God"*. And therefore cannot exist apart from him.

Erigena was the only large-scale systematic philosopher in the West during a period of 600 years (5^{th} to 11^{th} century).

855 CE **Thabit Ibn Qurra al-Harrani** (836–901, Mesopotamia). Physician, mathematician and astronomer. One of the greatest translators from Greek into Arabic, and a founder of a school of translators. Improved the theory of *amicable numbers* and generalized Pythagoras' theorem to an arbitrary triangle (as did Pappos).

Thabit was born in Harran and lived in Baghdad where he obtained his mathematical training. He returned to Harran but his liberal philosophies led to a religious court appearances when he had to recant his 'heresies'. To escape further persecution he left Harran and was appointed court astronomer in Baghdad. A hitherto unknown Arabic translation of Thabit of a work by **Archimedes** was discovered (1919) which contains a construction of a regular *heptagon* (7 sides).

His grandson **Ibrahim Ibn Sinan Ibn Qurra** (908–946 CE, Baghdad) was also a known mathematician. He worked on the quadrature of the parabola, where he introduced a method of integration more general than that of Archimedes. He also studied the geometry of shadows.

858 CE **Amram ben Sheshna Gaon** (ca 810–875, Babylonia). Talmudic scholar. Composed the oldest extant Jewish book of prayers. Head of the Sura Academy. In his book he introduced for the first time a systematic and logical arrangement of liturgy for the whole annual cycle as well as the pertinent laws. It is known as the *Seder* or *Siddur*. Amram's sources, in addition to the Talmud, were the works of the Geonim and the rites of the Babylonian Academies. The *Seder* had a wide influence in Europe and the Middle East. It enjoyed a very wide circulation and was extensively quoted by leading scholars of Spain, France and Germany. It served as a basis for later orders of prayers.

865–950 CE Decline of the *Khazar state* influenced by a climatic change.

868 CE The first printing press (block printing) appeared in China.

877–910 CE **Muhammad al-Battani**; **Albategnius** (ca 858–929, Mesopotamia). The greatest Muslim astronomer. Rather than using geometrical methods as **Ptolemy** had done, al-Battani used trigonometrical methods. His main work is an astronomical treatise with tables (“*De scientia stellarum*”, “*De numeris stellarum et motibus*”), which was extremely influential until the Renaissance. Discovered the motion of the sun's apogee.

He made astronomical observations of remarkable range and accuracy from 877 on. His tables contain a catalogue of fixed stars for the years 880–881. From his observations at *al-Raqqa* he was able to correct some of Ptolemy's results, previously taken on trust. He compiled two tables of the sun and the moon, subsequently accepted as authoritative. He found that the longitude of the sun's apogee had increased by $16°47'$ since Ptolemy; that implied the discovery of the motion of the solar apsides and of a slow variation in the equation of time. He determined many astronomical coefficients with great accuracy: *Precession*, $54.5''$ a year; *inclination of the ecliptic*, $23°35'$. He proved the possibility of annular eclipses of the sun.

The third chapter of his astronomy book is devoted to trigonometry, where (perhaps independently of **Aryabhata**, ca 500), he introduced the regular use of *sines* with a clear awareness of their superiority over the *Greek chords*. He completed the introduction of our *cotangents* and *tangents* and gave a table of cotangents by degrees. He knew the relation between the sides and angles of a spherical triangle which we express by the formula

$$\cos a = \cos b \cos c + \sin b \sin c \cos .A$$

Al-Battani was born near Harran, flourished at al-Raqqa (on the Euphrates) and died near Samarra.

880–890 CE **Eldad ha-Dani** (ca 830–890). Jewish traveler and philologist. Native of South Arabia who journeyed in Egypt, Mesopotamia, North Africa and Spain. Spent several years at Kairawan in Tunisia and died on a visit to Cordova, Spain. He is the supposed author of a travel-narrative (in Hebrew) on the question of the Lost Ten Tribes. His story is supported by the known Hebrew physician of his own time, **Zemah Gaon**, the rector of the Academy of Sura, Mesopotamia (889–898 CE). Eldad is quoted as an authority on linguistic difficulties by leading medieval grammarians and lexicographers. The *Book of Eldad ha-Dani* was first published in Mantua (1480) and had since been translated into several languages.

880–920 CE **Abu Kamil Shuja** (al-Hasib) (850–930 CE, Egypt). Mathematician. Worked on integer solutions of equations. He also gave the solution of a 4^{th} degree equation and of a quadratic equation with irrational coefficients. Abu Kamil's work was one of the sources of **Fibonacci's** books.

880–930 CE **Itzhak ben Shlomo Israeli the elder**[18] (832–932 CE, North Africa). Jewish physician, medical scholar of consequence and philosopher. Regarded as father of medieval Jewish Neoplatonism. Author of scientific and philosophical treatises renowned among Latin scholastics. His works, written in Arabic and translated into Latin, were studied and admired by **Albertus Magnus**, **Vincent of Beauvais** and **Thomas Aquinas**. His medical treatises "On Fever" and "On Diet" remained authoritative for some 500 years.

Israeli was born in Egypt and served as court physician to two Fatimide caliphs in al-Qayrawan (today in Tunisia). He died in Kairawan. His medical ideas, enunciated one thousand years ago, sound astonishingly modern:

- "*The most important problem of the doctor is how to prevent illness...*".
- "*The majority of diseases are cured by the help of Nature without the aid of a doctor...*".
- "*If you can cure a patient by means of a diet, forbear to prescribe drugs...*".
- "*Hold forth the prospects of recovery of patients, even if you are not sure of them yourself, so that at least you shall second the efforts of Nature to care them*".

ca 900 CE The diseases of smallpox and measles first identified or described with accuracy.

ca 900–925 CE **Abu Bakr Muhammad ibn Zakariya al-Razi**, known in the West as **Rhazes** (ca 865–925, Persia). Physician, medical scholar and philosopher. Considered the greatest physician of the Islamic world[19]. Chief physician of hospitals in Rayy and Baghdad; believed in atomist theory

[18] His Arab name was **Abu Yaqub ibn Suleiyman al-Israeli**.

[19] As in the parallel case of the Roman conquest of Greece, the superior culture of the conquered race asserted its supremacy over their Arab conquerors. After the Muslim conquests became consolidated, and learning began to flourish, schools of medicine, often connected with hospitals and schools of pharmacy, arose in all

of nature. Author of numerous treatises in medicine, especially a survey of Greek, Syrian and early Arabic medicine and a treatise on smallpox and measles. Some of his works were translated into Latin and had great influence on medical science in the Middle Ages.

Rhazes realized that fever could be a defense against disease. *Continens Liber*, one of his 200 or so treatises, gained him fame.

Al-Razi was also a skilled practical alchemist. He talked about *soda* (natron), common salt, kali (potash), salpeter, vitriol, arsenic, magnesia and mercury.

900–930 CE **Muhammad al-Farabi**; **Alpharabius** (ca 870–950, Aleppo). Philosopher, musicologist and encyclopedist. Continued the harmonization of Greek philosophy with Islam (begun by al-Kindi), preparing the way for Ibn Sina. He wrote a number of commentaries on Aristotle (physics, meteorology, logic) and his own work on the classification and fundamental principles of science. Al-Farabi was conversant with the whole range of scientific thought of his day.

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the chief seats of Muslim power. At Damascus, Greek medicine was zealously cultivated with the aid of *Jewish* and *Christian* teachers. In Baghdad, under the rule of Harun al-Rashid and his successors, a still more flourishing school arose, where numerous translations of Greek medical works were made. At the same time Arabs became acquainted with Indian medicine, and Indian physicians lived at the court of Baghdad. The Islamic rulers in Spain were not long behind those of the East in encouraging learning and medical science. In that country much was due to the *Jews*, who had already established schools in places which were afterwards the seat of Muslim dominion.

Thus, Arabian medicine was in the main Greek medicine, modified to suit other climates, habits and national tastes, and with some important additions from Oriental sources. The greater part is taken from Hippocrates, Galen, Dioscorides and later Greek writers. The *Latin* medical writers were necessarily unknown to the Arabs. In anatomy and physiology the Arabians distinctly went back; in surgery they showed no advance upon the Greeks; in practical medicine nothing new can be traced, except the description of certain diseases (e.g., smallpox and measles) unknown or imperfectly known to the Greek). The only real advance was in pharmacy and the therapeutical use of drugs. By their relation with the farther East, the Arabs became acquainted with valuable new remedies which have held their ground till modern times. Also, their skill in *chemistry* enabled them to prepare new chemical remedies, and form many combinations of those already in use.

They produced the first pharmacopeia, and established first apothecaries' shops.

He was born near Farab, Turkestan, of a Turkish family. He studied in Baghdad and flourished chiefly in Aleppo. Died in Damascus.

913–942 CE **Saadia (ben Yosef) Gaon** (882–942, Babylonia). Philosopher, philologist and trained mathematician. Creator of the Jewish philosophy of religion.

Equally versed in Arabian culture, Biblical and Talmudic scholarship, Christian dogma, Hindu and Greek philosophy, and the doctrines of Zoroaster and the Manicheans, he attempted to reconcile rationalism with the Jewish faith and render a rationalistic interpretation of Talmudic law. Saadia criticized Platonic cosmology, refuted gnostic doctrines and tried to reconcile the freedom of man with the all-embracing knowledge of God.

His major philosophical work, *Kitab al-Amanat wa-al Itiqadat* (The Book of beliefs and opinions), makes ample references to Biblical and Talmudic authority, but in addition draws on medicine, anatomy, mathematics, astronomy and music. His work reflects the mathematics of his day (which he had thoroughly mastered), and in his systematic theology there were already present some of the methods and the processes of thought which characterized 19^{th} and 20^{th}-century mathematics.

In Saadia's writings, one finds the process of *abstraction*, the use of the syllogism including some interesting logical devices as "proof by contradiction". There are also certain modern logical concepts such as the formation of the unit class consisting of a sole element[20]. Furthermore, there is the realization of the central role that *existence* and *uniqueness* theorems must play within a theory.

He composed the first Hebrew dictionary, prayer book and grammar, and was the founder of scientific Hebrew philology. His Arabic translation of the old Testament, with commentary, remained the standard translation to this day.

Saadia Gaon *influenced the future course of Judaism*; he removed the threat of *Karaism* and thus re-established the supremacy of Jewish tradition, saved Jewish unity and fought successful battles against the alluring influence of Muhammadian philosophy. All this he achieved through the richness of his knowledge, the depth of his learning, the keenness of his intellect and the ruggedness of his personal courage.

Saadia was born in Fayyum, Egypt and educated there. He went to Israel and then to Babylonia, where he was appointed head of the academy in Sura

[20] Such logical concepts have become standard since **Russel** and **Whitehead** (1910).

in 928. But he soon came into conflict with the exilarch and was banished. He was reinstated a Gaon of Sura close to the end of his life, and died there.

ca 925–930 CE **Aharon ben Moshe ben Asher** (ca 890–950, Tiberias[21], Israel). Hebrew grammarian and scholar. The last and foremost of a Karaite[22] family of *Masoretes* that was active in Tiberias for five generations [Asher I (fl. 785); Nehemiah (fl. 820); Asher II (fl. 855); Moshe (fl. 890); Aharon (fl. 925)], standardizing the Biblical text and its chanting.

The *Masoretes* were Jewish scholars who annotated and attempted to remove errors from the Old Testament. They began their work in the 2^{nd} century CE, and their principal effects took place in the 5^{th} to the 12^{th} centuries CE. Their final text (the *Masorah*) included the rows markings that had been omitted from previous texts, as well as explanatory annotations. Prior to the *Masoretes*, there was no rigorous system for ensuring textual accuracy, nor is it certain that absolute literal accuracy was always deemed necessary.

Aharon was the first systematic Hebrew grammarian. He laid the foundations of the Hebrew Grammar in his works *Mahberet ben Asher* (ca 925) and *Dikdukei ha-Te'amim* (Grammar of the Vocalization). These were original collection of grammatical rules and masoretic information. In 930 CE he produced corrected and annotated edition (so-called *Aleppo Codex*) of the Old Testament including diacritical marks and marginal notations that serve as precise directions of vocalization, accentuation and chanting the Hebrew text. His standard will be compared and followed for generations prior to invention of printing (1450 CE) and will bear tremendous influence on the world of Biblical grammar and scholarship.

930 CE The *Althing*, the world' first parliament, was set up by the settlers of *Iceland*. A civil war in the 13^{th} century put an end to democracy in the Icelandic commonwealth, and the country came under Norwegian rule.

940–980 CE **Shabbethai ben Avraham Donnolo** (913–983, Southern Italy). Jewish physician and writer on medicine. Wrote the first book on

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[21] Between the two great destructive earthquakes of 18 Jan 749 and 05 Dec 1033, the city of Tiberias served as a center of Hebrew language and the Biblical Masorah. We know that already by 895 CE, Tiberians had a complex system of punctuation and vocalization, which began to form there in ca 650 CE.

[22] **Saadia Gaon** attacked the Karaites in general and **Ben Asher** in particular, considering them to be outsiders. It was unthinkable to him that traditional "normative" Jews world accept the work of a Karaite. Nevertheless, being a Karaite did not disqualify Aharon ben Asher in the eyes of **Maimonides** and all subsequent generations of leading Jewish scholars.

pharmacology in Europe after the fall of the Western Empire and before the influence of Arabian medicine began to be felt.

Donnolo was born in Oria, Italy. When 12 years of age, he was made prisoner by Arab pirates, but was ransomed by his relatives at Otranto, Southern Italy. He turned to medicine for livelihood, studying the sciences of the Greeks, Arabs, Babylonians and Hindus.

His book *Sefer ha-Yakar* (Precious Book) contains practical directions for preparing medicinal prescriptions. His medical science is based on Greco-Latin sources.

940–980 CE **Hasdai Ibn Shaprut** (ca 915–990, Spain). Physician, medical researcher and statesman. Physician to the caliphs Abdurrahman III and IV. Experimented with drugs and invented cares for a number of diseases. Gained reputation of being an excellent diplomat, and on several occasions helped maintain peace between Muslims and Christians. Established communication with the Jewish Kingdom of Khazar.

Hasdai was born in Jaen, Spain and died in Cordova.

ca 960–990 CE **Albucasis** (936–1013, Spain). Physician. One of the greatest surgeons of the Middle Ages. Born in Cordova. Wrote *Al-Tasrif*, the first illustrated book of surgery and manipulations of spinal deformities. It also contains the earliest description of haemophilia.

ca 960 CE **Abu Hassan al-Uqlidisi** (ca 920– ca 980). Mathematician. Wrote the earliest known text offering direct treatment of *decimal fractions*.

968, Dec. 22 CE The earliest description of the solar corona during a total eclipse visible in Europe and the Near-East, and observed at Constantinople. It was only in the 18^{th} century that the phenomenon began to be studied in detail.

The Hebrew Golden Age of Reason[23] (900–1600)

With the conquest of southern Spain by the African Moors (711 CE), a new era of Arab-Jewish *culture began. The Jews had enjoyed equal treatment in Arab lands under the enlightened rule of the Mohammadian Caliphs. Accordingly, as soon as the Moors had established their first foothold in Spain, the Jews began to arrive in great numbers from all parts of the Islamic world. Every encouragement was given them to develop their own religious communal life.*

This held especially true during the 10^{th} *century in the reign of the enlightened Umayyad Caliphs* ***Abd ar-Rahman*** *(891–961) and his son* ***Al-Hakim II*** *(929–996) who made Cordova the most important center of learning in Europe. It was a time marked by liberality of mind and the advancement of the sciences and the arts.*

Despite Muhammad's attempts to suppress Greek cultural influences among the Arabs, they nonetheless persisted; Greek-Arab *civilization reached its most brilliant development during this period. The Jewish intellectuals became enthusiastic co-workers of the learned Arabs in every branch of knowledge and cultural creativity.*

When the Jews began to emigrate from Spain into Southern France, Italy and other Mediterranean countries, they brought with them into those still backward Christian lands elements of the superior Greek-Arab-Hebrew *culture.*

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[23] For further reading, consult:

- Barnavi, Eli (ed), *A Historical Atlas of the Jewish People*, Kuperard: London, 1998, 299 pp.
- Ausubel, Nathan, *Pictorial History of the Jewish People*, Crown Publ.: New York, 1968, 346 pp.
- Feuerstein, Emil, *The Genius of the Jew*, Udim Publishers: Tel-Aviv, Israel, 1975, 164 pp.
- Johnson, Paul, *A History of the Jews*, Harper Perenial: New York, 1988, 644 pp.
- Roth, C., *The Jewish Contribution to Civilization*, The Phaidon Press: Oxford, 1943, 369 pp.
- Gribett, J., *The Timetables of Jewish History*, Simon and Schuster: New York, 1993, 808 pp.

The historical fact seems somewhat paradoxical: the Jews and the Arabs who were nurtured by the Orient were, in a cultural sense, the first Europeans. They planted the intellectual seed of Western civilization on the continent.

In Arab lands, where there had existed a free intermingling of many cultures, there had blossomed a rich and unique Jewish culture. It has spread to Babylonia after its conquest in 642 by Muhammad in the course of his holy war. In the centuries of Babylonian Jewry's declining preeminence as the world center of Jewish learning, there emerged a galaxy of highly gifted philosophers, scholars, poets and scientists whose writings showed the impact of Greek thinking. In a way, this Jewish Renaissance was a resurgence of Hellenism. The Jews, who during their Greco-Roman period had inveighed against the Epicureans, Aristotelians and Platonists now opened their minds to the ideas of these philosophers and tried to reconciliate their teachings with Jewish theology.

Moreover, unheard-of occupations became respectable Jewish professions; they became astronomers, mathematicians, alchemists, architects, translators, finance ministers and international businessmen. And yet, though the door to assimilation was wide open, they stayed in the house of Judaism.

How had Hellenism found its way back into Jewish life in an Arabic world? The simple fact is that in rescuing Greek works for the Arabs, the Jews became imbued for the first time with the true essence of Greek philosophy and science. As the early Christians had no use for the writings of the heathen Greeks, and the invading barbarians had no use for the Greek language, most of the former were lost and the latter forgotten.

Greek literary and scientific works, however, survived in Syriac translations and in the libraries of wealthy and cultured Jews and unconverted Roman pagans. When the Arabs heard of this wealth of knowledge, they encouraged its translation into Arabic, and the task fell mainly to the Jews, the cosmopolitans of that age, who spoke Hebrew and Arabic, Greek and Latin, Syriac and Persian with equal facility.

These 'Channels to Europe', the transmission of Greek Science and humanism to Europe, were reopened by the Jews in the 8^{th} *century, and the work continued through 1400. Their first translations were from the Greek and the Syriac into Arabic, but soon they began to translate Greek and Arabic works into Hebrew, and finally Hebrew literature and philosophy into Arabic. A two-way cultural communication had been established. It soon included a third partner.*

The enlightened crowned heads of Europe heard of these Jewish achievements and invited Jewish scholars, linguists, and translators to come to their capitals to translate the works of the Greeks and the Arabs, as well as their own Hebrew literature, into Latin, at the time the international language of

European scholarship. So, for instance, Frederick II [King of the Romans (1212), King of the Germans (1215), King of Jerusalem (1229)] appointed Jewish scholars to teach Hebrew at the University of Naples.

Now that there was no danger of being absorbed into Hellenism (the Greeks vanished, but the Jews survived), they began to examine more closely the wisdom of the Greeks. They had opened a Pandora box of reason and looked at everything with the new glasses of rational scrutiny. The result was inevitable – a split between faith and reason developed. Into the breach rushed the conservatives to explain that reason and faith were but opposite sides of the same coin, and the liberals to prove that they were incompatible. A new tension developed out of which grew science and philosophy.

Numerous were the thinkers, philosophers, astronomers, mathematicians, jurists, physicians, nautical scientists, linguists, grammarians and biblical exegetes that the fecund Jewish culture milieu produced in Spain, France, Holland, Italy, North Africa and Babylonia during the millennium 700–1700 *CE. Of these, the following names (Table 2.1) can be directly associated with the accumulated wisdom and knowledge that opened the floodgates of modern science; unfortunately, some of these names did not receive the recognition they deserve. Many of their fate sakes which are not of this list, or any other list for that matter, will always remain the 'unknown soldiers' in the service of science.*

The Jewish Age of Reason took the same course that, centuries later, was taken by its Christian counterpart. The Age of Reason in Europe, born by the 18^{th} *century with the French encyclopedists, collapsed in the* 20^{th} *century revolutionary age of totalitarianism. The Jewish Age of Reason, born in the* 8^{th} *century with the great Talmudists, collapsed in the* 16^{th} *century revolutionary age of Reformation. Slowly the pendulum swung back to faith as the people rejected the mechanistic Jehovah of the rationalist philosophers and responded to the humanistic Jehovah of the Romantics. By the time the Muhammadian Empire collapsed, the Jews had made the transition back to faith, which was to sustain them in Europe's gethos. By shear irony of fate, the Jews stepped into their 'private' Middle Ages just when the rest of Europe emerged from it. The two cultures would meet again in the* 19^{th} *century.*

The tradition of preserving the text of scripture and its vocalization is known as Masorah and the tradition of text is called Masoretic. The people who were active in this work are known as Masoretes. This work of handling the past over to the future was neither simple nor quickly accomplished; it took a number of generations of scholars to do it. Moreover, these scholars thereby laid the foundation for the study of Hebrew grammar. For a discussion of words and how they are to be read in various connections was bound to lead to the fixing of grammatical rules. The Israeli Masoretes, down to the

10^{th} *century, did not go that far; the real beginning of Hebrew grammar was made by their followers in Babylonia and Spain.*

The 10^{th} *century marked a turning point in Hebrew and Biblical studies. Original masoreti researches reached their high water-mark in Israel in the works of* ***Ben-Asher****, a famed contemporary of* ***Saadia Gaon*** *(882–942). The vowel-system became definitely established, with the Tiberian system predominating. Studies of Hebrew then began to direct their attention to purely grammatical problems, without regard to their implication for biblical exegesis.*

Occasional grammatical observations are to be found already in the Talmud and he Midrashim. The masoreti notes and comments, likewise, contain a number of significant grammatical remarks. Considerable grammatical material is found in the Sefer Yetzirah, an anonymous ancient Kabbalistic work. But it was not until ***Saadia*** *that Hebrew grammar was treated as an independent science, and not merely as an aid to the clarification of biblical texts. This versatile scholar laid the foundation for the scientific movement in Hebrew philology. Saadia was prompted to undertake the task of writing Hebrew grammar because he was irked by the ignorance of the language and by the disregard for grammatical accuracy among the Hebrew writers and poets of his day.*

As long as the writers confined themselves to conventional themes, such as liturgical compositions and legal discourses, the available vocabularies, idioms and word-forms in the Bible and the Talmud were adequate and could be readily employed as vehicles for self-expression. When, however, the writers began to deviate from conventional themes, whether because of the influence of the Arabic culture of medieval Spain or as a result of independent creative urges, a departure from the stereotyped linguistic pattern became essential. New words had to be coined and new word-forms had to be constructed. Since Hebrew was not the vernacular of the people, ignorance of grammar was proving most serious handicap and threatened to corrupt the language. Even some of the grammarians of that period fell into glaring etymological errors.

As time went on, the philological movement initiated by Saadia gathered momentum, especially under spur of Arabic philological pursuits, and the urge for literary and religious expression in Hebrew. The rise of the Karaite sect (toward ca 790), which rejected rabbinic tradition as expressed in the Talmud and emphasized diligent scrutiny of the Bible as a basis for its tradition, was also a significant factor in focusing the attention of Hebrew scholars on a more searching study of the language of the Bible. The knowledge of Hebrew grammar, consequently, became a vital need.

Grammatical accuracy served as a criterion for the recognition of the mer-

Table 2.1: Prominent Medieval Hebrew Thinkers, Savants and Scientists

Name	Life-Span	Vocation	Country
Itzhak Israeli	850–950	philosopher and physician	North Africa
Saadia Gaon	882–942	philosopher and grammarian	Babylonia
Aharon ben Asher	890–950	grammarian	Israel
Shabbethai Donnolo	913–982	physician and pharmacist	Italy
Yehudah Hayyuj	940–1005	philosopher, grammarian, lexicographer	Spain
Jonah ibn Janah	990–1050	grammarian, philosopher, physician	—
Shlomo Ibn-Gabirol (**Avicebron**)	1021–1058	philosopher, poet	Spain
Rashi	1040–1105	Biblical and Talmudic exegete	France
Avraham bar Hiyya (**Savasorda**)	1065–1136	mathematician, astronomer	Barcelona and Provence
Yehudah Halevi	1071–1141	philosopher-poet, physician	Spain, Israel
Avraham Ibn-Ezra	1089–1167	scientist, physician, philosopher	Spain
Avraham Halevi	1110–1180	philosopher, translator, physician	Spain
Moshe ben Maimon (**Maimonides**)	1135–1204	philosopher, physician	Spain and Egypt
David Kimhi	1160–1235	linguist, grammarian, lexicographer	—
Yaacov Anatoly	1194–1256	translator, philosopher, physician	France, Italy
Moshe ben Nachman	1194–1270	philosopher and physician	Spain, Israel
Meir of Rothenburg	1215–1293	Talmudist and jurist	Germany
Yaacov Ibn Tibbon (**Prophatius**)	1236–1307	astronomer and physician	France
Levi ben Avraham	1240–1315	astronomer, mathematician	France
Moshe de Leon	1245–1305	philosopher	Spain
Levi ben Gershon (**Gersonides**)	1288–1344	mathematician, astronomer, physician	Perpignan and Avignon

Table 2.1: (Cont.)

NAME	LIFE-SPAN	VOCATION	COUNTRY
Yosef Caspi	1297–1340	philosopher and grammarian	France
Immanuel Bonfils	1310–1385	mathematician, astronomer, physician	Tarascon
Hisdai Crescas	1340–1411	philosopher and logician	Spain
Yosef Albo	1380–1444	philosopher and Talmudist	Spain
Itzhak Aramah	1420–1494	mathematician, logician and Talmudist	Spain
Yehudah Ibn Verga	1430–1499	astronomer, mathematician	Spain and Portugal
Itzhak Abravanel	1437–1508	philosopher, statesman and Talmudist	Spain, Italy
Shmuel Zarfati	1450–1519	physician	Italy
Avraham Zacuto	1450–1515	astronomer, mathematician	Spain
Yehudah Abravanel (**Leone Ebero**)	1460–1530	philosopher, physician, astronomer	Italy
Pedro Nuñes	1492–1577	mathematician and cartographer	Portugal
Garcia da Orta	1500-1560	botanist and physician	Goa
Amatus Lusitanus	1511–1568	physician, medical scientist	Portugal, Italy
Azariah dei Rossi	1511-1578	savant and physician	Italy
Yehudah Liwa of Prague	1512–1609	philosopher and Talmudist	Bohemia
Moshe Isserles	1525–1572	philosopher, codifier, Talmudist	Poland
Avraham Ortelius	1527–1598	cartographer and geographer	Antwerp
Itzhak Luria Ashkenazi	1534–1572	philosopher and mystic	Israel
Avraham Zacuto II	1576–1642	medical researcher and physician	Amsterdam
Yosef Delmedigo	1591–1655	astronomer, mathematician, philosopher, linguist and physician	Italy

its of literacy and religious composition, and grammatical knowledge constituted a badge of honor and the measure of Jewish learning and scholarship. Interest in Hebrew grammar was, therefore, not confined to professional grammarians, but gained vogue among poets, philosophers and even statesmen. The celebrated poet, statesmen and Talmudist ***Shmuel ha-Nagid*** *(993–1056), the eminent physician and poet* ***Yehudah Halevi*** *(1071–1141), the brilliant philosopher and poet* ***Shlomo Ibn-Gabirol*** *(1021–1058), the poet and scientist* ***Avraham Ibn Ezra*** *(1089–1167), the grammarian* ***Jonah Ibn Janah*** *(995–1050) and others, all concerned themselves with Hebrew grammatical problems to a greater or lesser degree and wrote about them.*

The study of Hebrew grammar as an independent science was pursued with zeal and profundity by Saadia's immediate successors. An important lexical work of ***Menahem ben Saruk*** *(910–970), entitled Mahberet, inaugurated Hebrew grammatical research on Spanish soil and provoked a vehement attack by a pupil of Saadia,* ***Dunash ben Labrat*** *(920–990). Dunash advanced views which already forecast the triliteral theory*[24]*, later scientifically and systematically expounded by* ***Yehudah Hayyuj*** *(ca 975 CE). Although* ***Dunash*** *had a glimpse into the operation of these phonetic principles, Hayyuj is undoubtedly entitled to the credit of being the first grammarian to observe*

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24 The stem (root) of a word is determined by removing the prefixes, infixes and suffixes. In the majority of cases the residual number of radicals clearly amount to three. In good many instances, however, the number of these radicals seems to be reduced to two and even to one [e.g. in *wa-tofehu* (1 *Samuel* **28**, 24), "and she baked it", the f is the only stem letter that remains after the removal of prefixes an suffixes]. Hence, all the predecessors of **Hayyuj**, with the exception of **Dunash**, who on occasion had an insight into triliteral basis of the Hebrew stems, operated with the idea of the biliteral and even uniliteral stems. These grammarians failed to recognize the special character of *assimilated stems*. In Hebrew, as in other Semitic languages, stems from which nouns and verbs are derived consist of consonants and meaning depends on these consonants. The vowel sounds, which in Indo-European languages are generally on a par with the consonants as regard their essential role in the stem or root, play a minor part in the Hebrew word; they merely serve to indicate different shading of the inherent meaning. A change in vowels will transform an active into a passive verb, or a verbal into a nominal form.

Thus, the consonantal stem *ktb* will yield such forms as *katab* (wrote), *koteb* (writes), *katub* (written), *ketab* (script) and a lots of other forms, in all of which the concept of "writing" is inherent. These basic consonants, which are never dropped in all the modifications of the word, were designated by the Medieval Hebrew grammarians the *shoresh* (radix, root). It therefore became customary among Christians grammarians to refer to these letters as "radicals" or "root letters".

the overall pattern of triliterality in Hebrew language as a design in which the structure of nouns and verbs fitted in a perfect mosaic fashion.

Another significant contribution by the medieval grammarians is the arrangement of the paradigms, that is, model of verbs and their various inflections in the different conjugations*. Once a nature of a given consonantal stem is understood, it is possible to construct a long series of derivative forms, all having the same basic meaning. The various semantic shadings and modifications, as well as the differences in person, number and gender, are indicated by mere changes in vowels or by additions of prefixes and suffixes as the case may be. There are seven most common conjugations in biblical Hebrew and are known as* binyanim *(literally in Hebrew, building forms). In all formations of the various* binyanim *the basic meaning is more or less in evidence. Consequently, an understanding of the nature of* binyanim *opens up the wide vistas of the language and provides an insight into its linguistic pattern. Equipped with this understanding it is possible to get along on a comparatively small basic vocabulary and to manipulate the language with relative ease and facility.*

Saadia was the first to attempt the arrangements of paradigms. He, however, confined himself only to two conjugations. The standard arrangement of the seven conjugations is to be credited to ***Moshe Kimhi*** *(ca 1130–1190). His brother* ***David Kimhi*** *(ca 1160–1235) assumed the role of a "gleaner after the reaper", whose task was to compile and present succinctly and simply the voluminous scientific findings of their predecessors. Kimhi's works mark the closing of the "Golden Era" of Hebrew medieval philology.*

The reaction which set in against the works of Maimonides toward the end of David Kimhi's days, marked the waning of all rationalistic pursuits, including grammatical research, and of any studies outside of the Talmud. Furthermore, the deterioration of the Jewish economic and political position in South-Western Europe, resulting largely from persecution and from the internal rivalries and quarrels within the Jewish communities, led to a serious decline among the Jews in general cultural interests and activities.

The Jewish people, faced by an accelerated rise in jealousies and prejudices among the Christian populace and in persecutions and repressions by the Catholic Church, sought shelter in their own shell, as it were. Their interest shifted accordingly from the luxury of pursuing poetry, philosophy and grammar to the necessity of fortifying themselves by means of a better understanding of Judaism and a stronger faith in their religious destiny. This interest found its outlet in the study of the Talmud and of the mystic philosophy of the Kabbalah, which began to flourish around that period.

Most of the Jewish scholars of the subsequent generations regarded the study of grammar as a waste of time, and some even saw in it lack of piety.

Even the study of the Bible began to be regarded as of secondary importance and was dwindling to such an extent that a German rabbi of the 17^{th} *century complained that there were rabbis in his generation "who had never in their lifetime seen a text of the Bible".*

As interest in Hebrew grammar waned among Jewish scholars, it began to flourish among Christians. The Renaissance, which initiated in Italy, grew apace and crossed the Alps into the Netherlands, France and Germany, reached its peak during 1450–1550. This intellectual movement brought in its wake the Reformation which split the Christian world into two warring camps.

One of the main motives of the Reformation was to break the shackles of the Church's authority and to reassert the right of individuals to search on their own for the word of God, without the Church as intermediary. They therefore turned to the study of Hebrew as the master key to the Judaic background of Christianity, in order to discover for themselves the pristine meaning of the Bible and the original principles of the Christian faith. Many of the early leaders of the Reformation achieved considerable proficiency in Hebrew and familiarity with its literature.

The center of interest in Hebrew grammar and lexicography accordingly shifted from Jewish to Christian scholars. The father of Hebrew grammar among the Christians was the humanist ***Johannes Reuchlin*** *(1455–1522). The Christian grammarian, whose work enjoyed widest currency and influence, was* ***Wilhelm Gesenius*** *(1786–1842). The contribution of the Christian scholars to Hebrew grammar was considerable. They resumed the comparative study of Hebrew and Arabic and extended these studies to include the other Semitic languages. However, being unconversant with the source materials of the Golden Age Hebrew grammar, and being handicapped by the tendency to restrict themselves to the language of the Bible, their work was robbed of progressive and dynamic value.*

There was a renewed spurt of zeal for the study of the Hebrew language and Hebrew grammar among the Jews during the Jewish enlightenment movement in the first half of the 19^{th} *century. Only two grammarians of that period made any notable and original contribution to the science of Hebrew grammar, namely* ***Shmuel David Luzzato*** *(1800–1865) and* ***Simhah Pinsker*** *(1801–1864).*

Transmission of the Bible

Once canonized, the Five Books of Moshe became divine. Thereafter, no changes, additions, or deletions were permitted, and the job of maintaining the *text was entrusted to a class of scribes known as Masoretes. The* 20^{th} *century discovery of the Dead-Sea Scrolls, which yielded Old Testament manuscripts dating 200* BCE, *show what an excellent job these scribes did in preserving the original text.*

The accuracy of the present-day Hebrew version of the Old Testament is a result of the fastidious care with which the Sopherim and the Masoretes transmitted it. The Sopherim copied manuscripts of the Hebrew Scriptures from about 300 BCE *until 500 CE. According to the Talmud, they came to be called "Sopherim" because, in their endeavor to preserve the text from alteration or addition, they counted the number of words in each section of Scripture, as well as the number of verses and paragraphs.*

During this time, there were two general classes of manuscript copied, the synagogue rolls and private copies. Even the private copies, or "common copies" of the Old Testament text, which were not used in public meetings, were preserved with great care. For the synagogue rolls, however, there was a very elaborate set of rules for the copyist. The manuscript had to be prepared by a Jew, written on the skins of clean animals and fastened together with strings taken from clean animals. Every skin was to contain a certain number of columns, equal throughout the codex. The length of each column was to be no less than 48 and no more than 60 lines. The breadth was to be 30 letters. The ink was to be prepared according definite special recipe. An authentic copy was to be used from which to copy, and the transcriber was not to deviate from it in the least.

No word or letter, nor even an iota (yod), was to be written from memory. The scribe was to examine carefully the codex to be copied. Between all of the consonants of the new copy, a space of at least the thickness of a hair or thread had to intervene. Between every parashah, or section, there was to be a breadth of nine consonants. Between every book, there was to be three lines.

During the period 500–900 CE, the text of Hebrew Bible was standardized by the Masoretes, who were also very careful in the transmission of the text. They counted every letter and marked the middle letter and middle word of each book, of the Pentateuch and of the whole Hebrew Bible, and counted all parashas (sections), verses, and words for every book. These procedures were a manifestation of the great respect they had for the sacred Scriptures, and secured their minute attention to the precise transmission of the text.

The Torah scroll, used for reading in the synagogues, contains only consonants of the Hebrew words; there are no vowel indications such as are to be found in printed books. There are hardly even paragraph divisions to be seen, and no sentence markings.

How then did people know how to read and where to stop? They were *in the same position as one would be if English had no vowels and one came across the letters "ct". Only the sense of the sentence would tell one whether to read it "cut", "cot" or "cat". A word of three Hebrew letters like* שכל *might be read* sachal *(he acted cleverly), or* sechel *(good sense), or* shikel *(lost his children), or* shekol *(that all). The meaning depends upon the vowels, according to which consonants are read.*

Until the 6^{th} *century, people learned this from teachers and the division into phrases, sentences, paragraphs and chapters was handed down by tradition. But that certainly was not a satisfactory situation when troubles prevented people from studying, when the Jews were scattered all over the world and teachers were few, and when Hebrew had ceased to be the spoken language by any part of the people.*

The Jews, both in Babylonia and Israel, learned a lesson from the Muhammedians and from certain Christian groups who lived near them. These people, confronted with the same problem in their Arabic and Aramic languages, had invented little signs to indicate the vowels. Jewish scholars in Babylonia began to use a variety of lines and dots written above *the letters of the Hebrew alphabet to show how they were to be read.*

Certain scholars in Israel, at the same time, suggested another set of signs, written mostly below *the line of letters. The Israeli system turned out to be the easier and more efficient, and was soon adopted universally by all Jews. These scholars were very careful to follow the* traditional pronunciation *of the words, and then wrote under them the sign appropriate to indicate that tradition. They also used another set of signs, below or above the words, to indicate the phrases and stops within the sentence. These are signs now used as musical notes for chanting. For the chanting, too, was traditional, and the new signs now fixed the chant for the future.*

Their most important use, however, was as commas, colons, and periods rather than musical notes.

Finally, the same Israelis wrote footnotes *to the words of the Bible into which mistakes had crept,* they counted every word and letter, *all with the object of handling the traditional readings of the sacred books down to the future exactly as they had been handed down to them from the past.*

To sum up, the Masoretes introduced a complete system of vowel pointings and punctuation for the text. Because of their high regard for faithfulness to the text in transmission, wherever they felt that corrections or improvements should be made, they placed them in the margin. They retained certain marks of the earlier scribes relating to doubtful words and offered various possibilities as to what they were. Among the many lists they drew up was one containing *all the words that occur only twice in the Old Testament.*

A number of texts of the Masoretes are still extent:

- *Cairo Geniza fragments* (6^{th} *to* 9^{th} *century CE).*
- *The Cairo Codex (895 CE), copied by* ***Moshe ben Asher****. Includes the Former and Latter Prophets. Found in the Karaite synagogue in Cairo.*
- *The St. Petersburg Codex (916 CE), containing only the Latter Prophets.*
- *Aleppo Codex (930 CE), copied by* ***Aharon ben Moshe ben Asher****; It used to be a complete copy, but was partially destroyed in a synagogue fire in 1948. Used by* ***Maimonides****.*
- *The British Museum Codex (950 CE). It is a complete copy of the Pentateuch.*
- *The St. Petersburg Codex (1008 CE). It is the largest complete manuscript of the entire Old Testament, on which the ben Asher family worked for five generations. Copied by the masorete* ***Shmuel ben Yaacov****.*
- *The Reuchlin Codex (1105 CE). Copied by* ***Moshe ben Naphtali*** *(890–940, Tiberias), a rival of ben Asher. Now in Karlsruhe, Germany.*
- *The Dead Sea Scrolls (200* BCE*– 70 CE). The earliest copies of Old Testament books, discovered (1947) in a cave at Qumran, near the Dead Sea. Yet when the scroll of Isaiah was examined and compared with the codex from 1000 years later it was found that the two texts are almost identical.*

  This shows the accuracy with which the Bible was copied by the scribes through the ages. Indeed, the attention which the Bible has received in the search for the true text, in exegesis, hermeneutics and commentary, exceeds by far that devoted to any other work of literature. Nor is this interest disproportionate, because it has been the most influential of all books.

969–985 CE **al-Quhi** (940–1000 CE, Persia). Astronomer and mathematician. Worked in Baghdad. Built an observatory in Shiraz to study the planets. Worked chiefly in geometry and considered problems leading to quadratic and cubic equations.

968 CE Drought in Africa that caused low Nile in Egypt, resulted in ca 600,000 deaths.

970 CE Al-Azhar University founded in Cairo, Egypt.

970 CE **Abu al-Wafa al-Buzjani** (940–998, Baghdad). Persian astronomer and mathematician. Contributed considerably to the development of trigonometry. He was probably the first to show the generality of the sine law relative to spherical triangles. He gave a new method of constructing sine tables, his value of sin $(30')$ being correct to 8 decimal places. He knew relations equivalent to ours for $\sin(\alpha \pm \beta)$ (though in an awkward form) and to $2\sin^2\frac{\alpha}{2} = 1 - \cos\alpha$, $\sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$. He was first to use the tangent function, calculated a table of tangents and introduced the secant and cosecant. His trigonometric tables are accurate to 8 decimal places (converted to decimal notation) while Ptolemy's were only accurate to 3 places. Worked at the observatory in Baghdad (from 959).

ca 975 CE **Yehudah ben David Hayyuj** (ca 940–1005, Spain). Perhaps the most important grammarian of the Hebrew language. Solved the mystery of the Biblical Hebrew verb. He demonstrated that, despite the way they looked, all Biblical verbs consisted of three letters (root), Hence he derived the rules surrounding the Biblical verbs and explained the difficult Hebrew nouns. His work ended and decided century-old debates on Hebrew grammar and paved the way for modern analysis of Hebrew. He also wrote a book, cited by **Ibn Ezra**, explaining for the first time linguistically difficult verses in the Old Testament. Hayyuj's work spread rapidly throughout the Jewish world.

Hayyuj was born in Fez and arrived in Cordova, Spain in 960 CE at the time of the famous dispute between Menahem ben Saruk and Dunash ben Labrat and sided with ben Saruk. Little is known about his life.

980 CE **Gerbert of Aurillac** (ca 945–1003, France). Mathematician and natural scientist. A remarkable Medieval Figure. One of the first Christians to study in the Muslim schools in Spain. May have brought with him to Christian Europe the Hindu-Arabic numerals (without the zero)[25]. He is said to have

[25] He went to Spain in 967 and obtained his knowledge in the convent of Santa Maria de Ripoll, a well-known center of learning near Barcelona. There is con-

constructed, terrestrial and celestial globes, a clock, and perhaps an organ. He was considered a profound scholar and wrote on astrology, arithmetic and geometry[26].

Gerbert was born in Auvergne, France. He became teacher at Reims; archbishop of Reims (991); befriended by emperors Otto II and III; archbishop of Ravenna (998). Elected to the papacy (999) as Sylvester II.

ca 985 CE **Muhammad al-Karkhi** (ca 950–1029, Baghdad). Mathematician. Among the last of the real contributors to mathematics in the city of the caliphs. His first work of note was on arithmetic, the *Kafi fil Hisab* (ca 1010), and drawn largely from Hindu sources. His second book was on algebra[27], called the *Fakhri* (ca 1020), named after his patron Fakhr al-Mulk, the grand vizier of Baghdad at the time.

His algebra is largely based on Diophantos. Gave complete solutions of quadratic equations with proofs, and the reduction of equations of the type $ax^{2p} + bx^p = c$ to quadratic equations. Treated addition and subtraction of radicals (e.g.: $\sqrt{8} + \sqrt{18} = \sqrt{50}$), and summation of series such as $\sum_1^n n^2$, $\sum_1^n n^3$.

"Of course, America had often been discovered before, but it had always been hushed up."

Oscar Wilde

986–1006 CE Norse discovery, exploration and settlement in North America, as attested in old Scandinavian sagas from the 12^{th} and 13^{th} centuries,

siderable evidence to support the belief that the monks in this cloister obtained their knowledge of these numerals through mercantile sources which were in communication with the East, rather than through any Moorish channels in Muhammadean Spain.

[26] Gerbert improved the abacus by labeling separate beads.

[27] Al-Karkhi gave an approximation of radicals: $a + \frac{h}{2a+1} < \sqrt{a^2+h} < a + \frac{h}{2a}$; $0 < h \leq a$. He also gave a solution in rationals to $x^3 + y^3 = z^2$, namely

$$x = \frac{n^2}{1+m^3}, \qquad y = mx, \qquad z = nx$$

where (m, n) are arbitrary rational numbers.

the account of the German priest **Adam von Bremen** (ca 1075), and excavations in Newfoundland (1961). According to these sources the following story can be reconstructed:

Bjarni Herjolfsson sighted (986 CE) the North American continent, probably in the region of Frobisher Bay [63.30°W–66°W], at the southern end of Baffin Island.

Leif Eriksson (ca 980–1025), a Norse explorer, led the first European expedition to the mainland of North America in ca 1000 CE. (He was the son of **Eric the Red**, who established the first settlement in Greenland.) Returning from Norway to Greenland, he was driven onto the American coast, which he called *Vinland* (Wineland), probably in Newfoundland. Some scholars believe Eriksson sailed further south, and that Vinland was near Cape Cod.

Thorfinn Karlsefni set out from Greenland in 1003 with three ships to settle in Vinland. He and his party spent three winters on the American continent. How long the Norsemen continued to visit America is an open question. The last definite mention is for 1189 CE, but there is some reason to believe that they came at least as far as southern Labrador for ship's timber as late as 1347. After this date the Vinland colonies declined[28].

The Viking[29] Invasions in the Medieval Warm Period (MWP) (787–1066)

Scandinavia developed in isolation until ca 200 CE. But the combined effect of population pressure (over population), tribal warfare and a global

[28] It is believed, that the Greenland colonies were so weakened by the *Black Death* (1346–1361) and by failure of supplies from enfeebled Norway that they could not withstand Eskimo attacks. The last Viking settlers disappeared in the 15^{th} century and Greenland became unknown country until rediscovered by **John Davis** (1550–1605; England) in 1585. It is thought that the Viking settlements maintained sporadic contact with "*Vinland*" (part of the coast of Canada or Newfoundland), and so the Black Death may have entirely altered the history of North America.

[29] From the Icelandic (Old Norse): vik = bay, inlet; vikingr = sea-rover, pirate; viking = predatory voyage.

climatic warming period (ca 600–1150 CE) disturbed the balance of marginal Scandinavian farming, causing the Norsemen to set off on 'land taking' and voyages of plunder in Western Christendom. Their ships spread out across the North Atlantic to Iceland, Greenland and North America; they emerged into the Mediterranean; they sailed down the great rivers of Central Asia, setting up the Russian state, and reaching Constantinople; they won control of Northern Britain and Normandy. Finally, in 1066, a family of Norse descent won the crown of England.

The viking aggressive expansion was facilitated by a number of factors:

- *The development of the Viking ship: they were expert seamen and they had perfected a type of long, shallow-draft ship which was very effective for raiding. With its flexible hull and its keel and sail, this boat was far superior to vessels used by other people at the time. They used a sail when the wind was astern, but they were usually propelled by oars. Equipped with such craft, the vikings could swoop down from the sea and loot a whole river valley before the local troops, moving slowly over bad roads, could be mobilized to repel them. (Note that the men who sailed these ships had neither compass nor loadstone and had no way of reckoning longitude!)*

- *Improved metallurgical techniques enabled them to construct better weapons, such as their war axes.*

- *The political weakness of Europe at that time: the Frankish Empire was disintegrating; the British Isles were split into small, warring kingdoms and the Slavs in Eastern Europe were politically disorganized. Thus, the continent lay open to any group of determined men bent on marauding and looting.*

- *Recent studies of the oxygen isotopic record, tree-ring data (dendrochronology), history of glacier waxing and waning and historical data of North Atlantic drift-ice led to the conclusion that during the MWP, temperatures were typically above normal. Data from North America and Peru support a global Warm Period consistent with the MWP. [There are several competing theories to explain the climatic changes experienced during the MWP. These include: sunspot variation, volcanic eruptions, changes in the large-scale ocean current conveyor belt, and to a lesser extent – changes in the earth's albedo. It is likely that each of these mechanisms played a role].*

  The warm climate during the MWP allowed the viking migration to flourish. Decreased drift-ice posed less hazards to sailors, and warmer climate would also result in a greater harvest in Iceland. Thus, the *warmer climate brought the vikings in increasing numbers to Greenland and Iceland.*

The participation of Scandinavia in the viking expansion through Europe was as follows: (there was never a mass-migration!)

(1) Norwegians raided Scotland, Ireland, France;

(2) Danes raided the British Isles, France, the Low Counties. Though the Scandinavian raiders called themselves vikings, the rest of Europe usually referred to them as Northmen. In their first serious raids against Ireland early in the ninth century they rapidly occupied the east coast. They met somewhat stiffer resistance from the Anglo-Saxons in England, but by 870 they had subdued all the Anglo-Saxon kingdoms except the southern state of Wessex. They had already begun to attack the Frankish lands. Year after year the Northmen pushed their long ships up the Rhine, the Seine, and the Loire to collect tribute and loot from towns and monasteries. Finally, an especially strong band of raiders forced the West Frankish king to cede them the land at the mouth of the Seine. This outpost, founded about 911, became the nucleus of Normandy, the most famous of the viking states.
In a show of bravado, the Northmen even sailed their ships into the Mediterranean and plundered a few coastal cities in Spain, southern France, and Italy.

(3) Swedes moved across Slavdom to Byzantium: They began to push down the Russian river valleys toward Constantinople. They had known this route for a long time, and eastern goods and eastern coins had been common in Scandinavia long before the great raids began. In the late ninth and tenth centuries, however, the Swedes began to settle in Russia and to bring the scattered Slavic population under their control. The fortified trading posts where the vikings settled soon burgeoned into towns; the most famous of them was Kiev, which became the capital of a large principality. Once they had gained a footing in Russia, the vikings, with typical boldness, turned their eyes south to Constantinople. Their attacks on the imperial city were unsuccessful, but they did manage to wrest a favorable commercial treaty from the emperor. These early viking princes and warriors gave the eastern Slavs their first effective political organization; in fact, the word Russia itself probably comes from Rus, the name of a Swedish tribe.

Although the vikings shot around the Northern Hemisphere, terrorizing and plundering vast swaths of territories with the rapacity of a Ghengis Khan, they were also traders, explorers and settlers: their sites stretched from Russia to Newfoundland, and they became the first Europeans to set foot in the Americas.

The pagan vikings learned rapidly from the more civilized people they attacked; they quickly adopted the Christian religion. As they became more civilized, they gave up their cruelty and their savage love of destruction and shifted from piracy to peaceful and productive commerce. They disturbed the old way of life in medieval Europe and their invasion finally helped create a new Europe.

Table 2.2: The Viking Timeline

787	*First recorded appearance of vikings in England (Dorset)*
795	*First viking raid on Ireland*
798	*Vikings raid the Isle of Man*
834	*Vikings sack Utreaht*
836	*Vikings sack Antwerp*
840	*Viking settlers founded the city of Dublin in Ireland*
844	*Vikings raid Seville, Spain*
845	*Vikings sailed up the river Seine and destroy Paris (again in 856)*
848	*Vikings took Bordeaux*
851	*Vikings sailed up the Elbe and burned Hamburg (again 880)*
859	*Vikings sailed through the strait of Gibraltar and attacked coastal cities in Spain, Southern France and Italy; defeated the Moors in Morocco (again in 900)*
860	*Norwegian vikings discovered Iceland (colonized 874)*
862	*Swedish Russ vikings founded Novgorod in Russia*
867	*Danish vikings established a kingdom in York, England*
by 870	*Viking subdued all Anglo-Saxon kingdoms except the southern state of Wessex*
871	*Vikings took London*
879	*Rurik established Kiev*
881	*Vikings sacked Aachen*
911	*Viking chief Rollo was granted Normandy by the Franks*
941	*The vikings attacked Constantinople*
981	*Viking leader Eric the Red discovered Greenland*
986	*Viking ships sailed in Newfoundland waters*
1000	*Leif Eriksson explored the coast of North America*
1003	*Vikings explorer Thorfinn Karlsefni attempted to establish settlements in North America*
1013	*The Danes conquered England*
1015	*The vikings abandoned the Vinland settlement on the coast of North America*
1016	*The Danes under Knut rule England*
1066	*William duke of Normandy defeated the Saxon king Harold at the Battle of Hastings*

990–1007 CE **Ibn Yunus** (ca 940–1009, Cairo). Astronomer and mathematician. Prepared improved astronomical tables based on his observations at the Cairo observatory. Improved the values of astronomical constants (inclination of the ecliptic, $23°35'$; longitude of the sun's apogee, $86°10'$; solar parallax, $2'$; precession, $51.2''$ a year). Introduced the trigonometric formula $\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)]$. Ibn Yunus described 40 planetary conjunctions accurately and 30 lunar eclipses used by Simon Newcomb (1876) in his lunar theory.

ca 1000 CE Collapse of the Andean *Tiahuanaco Empire* due to a prolonged drought lasting some 80 years.

1000–1030 CE **Alhazen (Ibn al-Haitham)** (965–1039, Cairo). Optician, physician, mathematician and astronomer. Gave the first correct explanation of vision, showing that light is reflected from an object into the eye. One of the greatest students of optics of all times. He went beyond Philoponus and any reliance on the speculative method of ancient natural philosophy. He started from systematic, repeated experiments, which were arranged to yield quantitative measurements, and from these he developed hypotheses expressed in mathematical form. These were inspired guesses as to the physical relationships underlying various sets of measurements. If an hypothesis was seen to fit the measurements, further experiments were devised to see if the proposed relationship could accurately predict new measurements. He was thus a pioneer of the *scientific method*, which he used to demolish the old optics of Ptolemy and establish the framework for a science of light.

The Latin translation of Alhazen's main work, the *Optics* (Kitab al-manazir) exerted a great influence upon Western science (**Roger Bacon**, **Peckham**, **Vitelo**, **Alberti**, **Kepler**, **della Porta**) and showed a progress in experimental methods. Alhazen conducted research with spherical and parabolic mirrors, studying magnifying power of concave and convex lenses, spherical aberration, catoptrics[30], dioptrics and reflection[31]. He also studied

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[30] The following problem in catoptrics is known as *Alhazen's problem*, states: Given a light source and a spherical mirror, find the point on the mirror where the light will be reflected to the eye of an observer. This can be reduced to the planar geometrical construction: from two points in the plane of a circle, draw lines meeting at a point of the circumference making equal angles with the normal at that point. It leads to an equation of the 4^{th} degree. Alhazen solved it by the aid of an hyperbola intersecting a circle. In a similar way he also solved the cubic equation $x^3 + c^2b = cx^2$, which results from the *Archimedean problem* of dividing a sphere by means of a plane into two segments at a given voluminal ratio to one another.

[31] He investigated reflection from non-planar surfaces (concave, convex-spherical,

the phenomena of atmospheric refraction (attempted to measure the thickness of the atmosphere on the basis of his observation that the twilight only ceases or begins when the sun is 19° below or above the horizon).

Alhazen gave a correct account of the structure of the eye (although he considered the lens as the sensitive part), contested Plato's idealism which made the eye a *source* of illumination, and appeared to have recognized the eye as what we now call *camera obscura*. Alhazen seems to have been well acquainted with the projection of images of objects through small apertures, and to have been the first to show that the arrival of the image of an object at the retina, corresponds with the passage of light from an object through an aperture in a darkened box.

In mathematics, Alhazen solved problems involving congruences, using what is now called *Wilson's Theorem* [if p is prime, then $(p-1)!+1$ is divisible by p].

He was born in Basra, and there he acquired such a scientific reputation that he was called to Cairo by the Caliph to apply his knowledge to the use of the waters of the Nile for the irrigation of lower Egypt. After examining the situation (as well as the failures of his predecessors), he realized that the feat was impossible. Fearing the anger of the Caliph, Alhazen feigned madness, for which he was confined and his property confiscated. Upon the death of the Caliph, he regained his liberty and property – as well as the opportunity to continue his study of science.

1006 CE The eruption of the volcano *Merapi* believed to have destroyed the Hindu-Javanese state of *Mataram*.

1010–1040 CE **Abu ar-Rayhan al-Biruni** (973–1048). Arabian scholar, historian and writer on mathematics, astronomy and medicine. His *History of India* (ca 1030 CE) became the principal introduction of Hindu numeration for the Arabs. Biruni transmitted Hindu knowledge to the Muslims and vice versa. Gave a clear account of Hindu numerals (positional notation) and the sum of a geometrical progression. Simplified the stereographic projection. Made accurate determinations of latitudes, geodetic measurements, and specific gravities of precious stones and metals. Remarked that the speed of light is immense relative to that of sound. Explained the works of natural springs and artesian wells in terms of the hydrostatic principle of communicating vessels.

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cylindrical, etc.) and finally formulated *Alhazen's law of reflection*, namely: that the incident ray, the normal of the point of reflection, and the reflected ray, all lie in the same plane.

Biruni was probably born to *Persian* parentage in Khwarizm (Khiva)[32], and was a Shi'ite in religion. He devoted his youth to the study of history, chronology, mathematics, astronomy, philosophy and medicine. For some years he lived in India, where he remained for a few years, teaching Greek philosophy and learning the Hindu language. In 1017 he was taken by Mahmud of Ghazni to Afghanistan, where he remained for the rest of his life.

1010 CE **Ibn Sina (Avicenna)** (980–1037, Persia). Physician, astronomer, philosopher and encyclopedist. Wrote a 5-volume treatise (the *Qanun* = Canon) on Greek and Arabic medicine that dominated the teaching of medicine in Europe until the 17^{th} century. Wrote many philosophical works, among them a philosophical encyclopedia (*Kitab al-Shifa*), and made a profound study of various physical questions. He observed that if the perception of light is due to the emission of some sort of particles by the luminous source, the speed of light must be finite. Many of his writings are devoted to mathematical and astronomical subjects.

He abandoned the various creation myths and studied geological formations in order to understand the origin of the present day earth; he correctly concluded that nearly all land today was once under water, that sedimentary rocks formed under water, and that the land was subsequently lifted by earthquakes.

Ibn Sina was born at Afshana, near Bukhara[33], and died in Hamadhan.

1020–1050 CE **Jonah Ibn Janah** (ca 990–1050, Spain). Leading Hebrew grammarian, philologist, logician, poet and physician. Wrote books on grammar and medicine. In such works as *al-Mustalhak* and *Kitab at-Tenkiyeh* established Hebrew syntax, grammar and lexicon. His work is of permanent value. A huge number of scholars from 11^{th} through 13^{th} centuries quoted his work, and it became the basis for understanding Hebrew grammar and philology. Ironically, Rashi and his students apparently did not know his work.

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[32] He may have been born at *Byrun* in the valley of the Indus.

[33] He is said to be of Jewish origin [see A. Soub: Avicenna, *Prince des Medicins*, Paris 1935]. There is, indeed, nothing improbable in this; for near Bukhara, the Jews had been settled from time immemorial. But, whether it is so or not, it is a fact that a large proportion of Avicenna's writings reached Europe through the medium of Jewish scholars and translators who worked in Spain, Italy, and Provence, and whose activity was important and influential in the earlier stages of the Renaissance, The material on which they worked was partly Avicenna's own, partly that of the series of scholars who carried on and developed his tradition.

Ibn Janah was born in Cordova, studied in Lucena under Itzhak ibn Gikatilla. In Cordova he studied medicine, which provided him with a livelihood throughout his life.

ca 1025 CE **Guido of Arezzo** (ca 995–1050 CE, Italy). The father of modern music. Music teacher and a Benedictine monk in the monastery of Pomposa. Invented the *musical alphabet*: the four-line staff, the F cleft and the first 6 notes of the scale, *ut* (later renamed *do*), *re*, *mi*, *fa*, *sol*, and *la*. These reforms made the teaching of music much easier. In his theoretical writings he assumes a thoroughly practical tone, which differs greatly from the clumsy scholasticism of his contemporaries and predecessors.

Of his life little is known. He taught singing at Pomposa and invented there his educational method, by means of which, according to his own statement, a pupil might learn within 5 months what formerly it would have taken him 10 years to acquire. Envy and jealousy, however, were his only reward, and he was compelled to leave his monastery. He moved to Arezzo, and there, about 1030, received an invitation to Rome from Pope John XIV. He obeyed the summons, and the pope himself became one of his most proficient pupils. He died as Prior of Avellana.

1040–1058 CE **Shlomo Ibn-Gabirol**, **Avicebron** (1021–1058, Spain). Poet[34]-philosopher and thinker of striking originality. His philosophical system had powerful impact on medieval Christian thinkers, as well as on Spinoza and Schopenhauer. A man of broad vision and keen penetration who saw further than the ordinary poet and felt deeper than the ordinary philosopher, and even cultivated science in his effort to grapple with riddle of the universe and of man's existence in it. Influenced by the Neoplatonism of Plotinus (253 CE) and the pseudo-Empedoclean writings, Ibn-Gabirol's doctrine contains not only certain teachings not to be found in these sources, but others irreconcilable with Neoplatonism.

Ibn-Gabirol, unlike other medieval philosophers, pursued his philosophical studies regardless of the claims of religion, keeping his speculations free from every theological admixture. In fact, his work shows a total and absolute independence of Jewish religious dogma; not a verse of the Bible nor a line from the Rabbis is cited. For this reason Ibn-Gabirol exercised little influence upon his Jewish successors (though this may be accounted for on the ground of the predominance of *Aristotelianism* from the 12^{th} century) and was accepted by the scholastics as a non-Jew. The odor of heresy which clung

[34] In the days of Ibn-Gabirol, the art of versification was not limited to emotional subjects. Law, medicine, and even mathematics were considered legitimate themes for the poet.

to him prevented Ibn-Gabirol from exercising a great influence upon Jewish thought; his theory of *emanation* was irreconcilable with the Jewish doctrine of creation, and the tide of Aristotelianism turned back the slight current of Ibn-Gabirol's Neoplatonism.

The "*Source of Life*", his chief work in philosophy was translated from the Arabic into the Latin in the middle of the 12^{th} century by **Dominicus Gundissalvus**, archdeacon of Segovia, with the assistance of a converted Jewish physician, Ibn Daud (afterwards called Johannes Hispalensis). Henceforth, the *Fons Vitae* as it is called in Latin, became a work to be reckoned with in the world of scholasticism. And just as Ibn Sina was corrupted into **Avicenna** and Ibn Rushd into **Averroes**, so Ibn-Gabirol traveled down the ages under the disguise of **Avicebron**[35].

The main themes of the *Fons Vitae* are:

(1) All that exists is constituted of matter and form (i.e., physics and geometry). This includes all created things, spiritual or corporeal. The various species of matter being but varieties of the universal matter, and similarly all forms being contained in one universal form. Ibn-Gabirol's many arguments in proof of the universality of matter is among his most original contributions to philosophy.

(2) Everything that exists may be reduced to three categories: the first substance, God; matter and form (the world); the divine will as intermediary and cause of the union of matter and form. This will is above the distinction of form and matter; it is neither attribute nor substance.

Ibn-Gabirol was born in Malaga and lived in Saragossa. Little is known of his life there except that his residence was embittered by strife. Envy and ill-will pursued him, which accounts for the pessimistic strain underlying his work. Life finally became unbearable in Saragossa, and he fled. He thought of leaving Spain, but remained and wandered about. After years of wandering and ill-health, he died in Valencia.

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[35] In the course of time and after much *recopying*. It was not until 1846 that the author's true identity was discovered. The Dominican and Franciscan scholars who fought about his philosophy had no idea he was a Jew and celebrated as a writer of religious hymns used in the Synagogue. Ibn-Gabirol nowhere betrays his Judaism in *Fons Vitae*, and so for centuries, he marched through the philosophical schools of medieval Europe, some taking him as a Christian and some for a Muhammadian. The most zealous of the champions of Ibn-Gabirol's theory of the universality of matter is **Duns Scotus** (1265–1308), through whose influence the basal thought of the *Fons Vitae*, the materiality of spiritual substances, was perpetuated in Christian philosophy, influencing later philosophers even down to **Giordano Bruno**, who refers to "*The Moor Avicebron*".

He had written twenty books, most of which were lost. In his great metaphysical poem, *The Royal Crown*, he renders the following poetic description of the *big-bang*:

> "*Calling unto the void and it was cleft,*
> *And unto existence and it was urged,*
> *And to the universe and its was spread out*".

1041–1049 CE **Pi Sheng** (fl. 1022–1063, China). Alchemist. Invented *typography* – the printing with movable type. His type was made of a baked-clay-and glue amalgam. Its usefulness was limited by China's own non-alphabetic writing system.

1045–1087 CE **Constantine the African** (ca 1020–1087, Italy). Also known as **Constantinus Africanus**. Translator. Latin scholar. Born at Carthage. Spent most of his life in the Benedictine monastery of *Monte Cassino*, translating into Latin Arab works on Greek medicine, philosophy and Aristotelian physics.

Constantine was aptly called "*magister orientis at occidentis*" and was indeed one of the great intermediaries between East and West. First to transport the medical literature of the Arabs into the Western world. The results of his activity considerably stimulated the hunger of European scholars for Greco-Arabic knowledge.

1054 CE, July 04 Chinese astronomers observed a supernova explosion (the creation of a neutron star that occurred some 5000 years ago) in the Crab nebula, in our galaxy. It was visible for 22 months[36].

1055 CE The Arabs introduced the Hindu decimal system into Spain.

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[36] Other reported observation of *supernova* are:

- 1006 CE: reports from Switzerland and Arab sources; the only known observation outside the Far-East before the Renaissance.
- 1572 CE: European reports on supernova of varying brightness in the constellation of Cassiopeia. First detected by **Maurolycus** and **Schüler** on Nov. 06, 1572. Seen until Feb. 1574. The amount of energy released in a supernova explosion is awe-inspiring. The star will shine as brightly as ten thousand million suns, and the total energy given out during the outburst is greater that released by the Sun over its entire lifetime.
- 1604 CE, Oct. 8–9: Reported from Europe, Japan and Korea. Seen by **Tycho Brahe**, **Kepler** and **David Fabricius**.

ca 1060 CE **al-Zarqali** (**Arzachel**, 1028–1087). Arab astronomer; flourished in Toledo, Spain. Compiled the *Toledan Astronomical Tables*. Suggested that planetary orbits are *elliptical*.

1064–1072 CE Drought in Africa caused a 7-year failure of Nile flooding, resulting in a wide-spread famine.

1066 CE *Battle of Hastings*; William of Normandy (the 'Conquerer') with a motley of invasion forces defeated Harold II; Western civilization was thus introduced to England.

1070–1105 CE **Rabbi Shlomo Itzhaki** (1040–1105, Troyes, Northern France), known by the acronym RASHI. Commentator on the Old Testament and the Talmud. The greatest teacher and educator that the Jewish nation had for the past millennium. His commentary on the Talmud influenced Jewish destiny; his subsequent commentary on the Bible influenced Christian destiny[37].

Unlike Maimonides (1135–1204) and Gersonides (1288–1344), both ardent followers of Aristotle and both rationalists (who believed that the Bible was not to be taken in its strict literal meanings but should be interpreted freely and allegorically), Rashi took a diametrically opposite point of view[38]. His commentary was written with such warmth and humanity, in such clear He-

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[37] The most profound Jewish influence on European civilization has been exercised through the Bible. The clergy studied it, and, through them, its ideas penetrated into the minds of the people. European law and morals, beliefs and hopes for the future derived largely from what the Christian called the Old Testament. Many Christian reformers of the Middle Ages based their doctrines on interpretations of the Bible text. During the first thousand years of the Christian era, when relationships between Jews and Christians were friendly and direct, such influence came through personal contact and, therefore, have been left unrecorded. **Nicolas de Lyra**, of the early 14^{th} century, wrote a Bible commentary that was admittedly influenced by that of Rashi. He quoted Rashi constantly, either in approval or refutation.

The German humanist **Johannes Reuchlin** (1455–1522), after studying the work of de Lyra, took up the cause of Jewish humanism in his bitter conflict with the Dominicans. One of Reuchlin's ardent supporters was **Martin Luther** (1483–1546); in his monastic cell Luther read both de Lyra and Reuchlin and the fury of the coming *Reformation* took shape in his mind. When Luther translated the Bible, as part of his religious reform movement, he leaned heavily upon Lyre's commentary and thus, indirectly, upon Rashi.

[38] Except for his exegesis of the *Song of Songs*.

brew, so artfully interspersed with French vernacular[39] (where Hebrew lacked the precise words), that it became loved as literature as much as it was revered as Scripture. He laid down an undeviating rule: "*Scripture must be interpreted according to its plain natural sense, each word according to the context*".

Rashi was born in Troyes. He worked his way through the Yeshivas (Talmudic academies) in Germany as a wandering student. After graduation he settled in his home town where he founded an academy of his own which attracted scholars from all over the world.

Rashi was the right man for the times. Life in the 11^{th} century Europe no longer related to many percepts of the Talmud. The people did not understand Aramaic, did not understand the phraseology, and did not understand its application to modern life. There was a need for a *universal Talmud* which could be understood without interpreters. It was this need that Rashi served.

In the 15^{th} century, classic Talmudic learning split off in two directions. The Italian and German schools, continuing the former Babylonian traditions, led to an affirmation of the *past*. The Spanish schools, resurrecting the Greek tradition, led to inquity into the *future*. The former produced a few more brilliant scholars whose influence died with them; the latter produced *philosophers* like Maimonides and Spinoza, whose influence lived after them.

ca 1074 CE **Omar Khayyam**[40] (ca 1044–ca 1123, Persia). Mathematician, astronomer, freethinker and poet. One of the greatest mathematician of medieval times. His reform of the Persian calendar achieved extraordinary accuracy. The first mathematician to study and classify cubic equations and to employ conic sections in their solutions[41]. His standard work on algebra, written in Arabic, and other treatises of a similar character, raised him at once to the foremost rank among the mathematicians of that age and induced Sultan Malik-Sāh to summon him in 1074 to institute astronomical observations on a large scale, and to aid him in his enterprise of reforming the calendar and calculating the length of the solar year with great accuracy.

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[39] As over 3000 of the French words he used have disappeared from the language, Rashi's writings have become important source book on medieval French.

[40] His full name is: **Ghiyath ud Din Abu'l Fatah Omar bin Ibrahim al-Khayyami**. 'Al-Khayy'am' means 'the tent-maker', but this occupation was most likely far back in his ancestry, his immediate forebears having been a literary family.

[41] The algebra of Khayyam is *geometrical* solving linear and quadratic equations by methods appearing in Euclid's *Elements*. He devised a geometrical method to solve cubic equations by intersecting a parabola with a circle.

Far ahead of his time in mathematical methods, Omar supported his algebraic solutions by geometrical constructions and *proofs*. He is also known for his critical treatment of Euclid's parallel postulate, showing him to be a forerunner of **Saccheri**'s ideas (1733) that finally led to the creation of non-Euclidean geometry[42].

Omar's scientific fame is nearly eclipsed by his still greater poetical renown, which he derives from his 'rubais' (quatrains). In these he appears as a radical free thinker forcibly protesting against the narrowness, bigotry and austerity of the orthodoxy. He has often been called the Voltaire of the East. His articulation of an inexorable fate, which dooms to eternal oblivion, still rings true today as the sentence of a universe that has been set in motion by the initial conditions of the 'Big Bang' cosmology:

> "*With Earth's first clay They did the Last Man knead,*
> *And there of the Last Harvest sow'd the Seed*:
> *And the first Morning of Creation wrote*
> *What the Last Dawn of Reckoning shall read*".

His phenomenal rise to fame in all parts of the civilized world began in 1859 with the anonymous publication of a hundred of his four-lined verses in a book entitled the *Rubaiyatt of Omar Khayy'am*. It was translated rather freely into English by Edward FitzGerald (1809–1883), an Irish writer and student of Iranian philology. Within 50 years after the appearance of that edition, more than 300 English editions were published, and within 70 years after 1859, more than 1300 works connected with the *Rubaiyatt* have appeared.

Omar Khayyam died in Naishapur. He once said that his tomb would be located in a spot where the north wind would scatter rose petals over it. In 1884, William Simpson, a traveling artist of the *Illustrated London News*, visited Naishapur and found the tomb of Omar. It was just outside a rose garden. Boughs hanging over the garden had dropped many blossoms on the grave. He plucked from these a few of the hips still hanging on the bushes. These seeds were planted successfully in the Kew Botanical Gardens, and on Oct. 7, 1893, one of these rose trees was transplanted to FitzGerald's graveside in a little English churchyard at Boulge, Suffolk.

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[42] **Alhazen** (ca 1000 CE) presented a "proof" of Euclid 5^{th} (parallel) postulate, but like the Greeks before him, he also ran into problems with circular reasoning. **Khayyam** was not satisfied with this proof and presented one of his own. **Saccheri** (1697) assumed the postulate false and tried to reach a contradiction. In doing so, he unknowingly derived many of the theorems of non-Euclidean geometry. Others, such as **Legendre** and **Lambert**, improved upon Saccheri's work, but fell into the trap of circular reasoning themselves, while striking perilously close to discovering non-Euclidean geometry.

1090 CE **Shen Kua** (1030–1093, China). Government official, engineer and astronomer. Wrote *Meng chi pi t'an* (Dream Pool Essays) which contain first reference to the *magnetic needle*, first account of relief maps, a rather accurate explanation of *fossils*, and other valuable scientific contributions in medicine, optics, astronomy, cartography and mathematics.

He entered imperial government service (1063); appointed commissioner for prefectural civil and military affairs in Yen-chou province (1077); banished from office after defeat of his troops by the Khitan tribes (1081).

1092 CE **Su Sung**, built an *astronomical clock* at the imperial observatory at K'ai Feng: a cross between the water-clock and the spring driven clock. It was powered by a water-wheel which advanced in step by step motion and its time-keeping ability could be adjusted by weights. It struck a gong to indicate the passing hours by a system of bamboo revolving and snapping springs. An astronomical check on timekeeping was made by a sighting tube pointed to a selected star.

1096–1291 CE *The Crusades*[43]. Christian military expeditions to free the Holy Land from the 'infidel' Muslims (Saracens). The men who fought in them came from Western Europe. There were 8 expeditions: 1096–1099; 1147–1149; 1189–1192; 1201–1204; 1217–1221; 1228–1229; 1248–1254; 1270[44]. Jerusalem changed hands several times (1099, 1187, 1243, 1244).

The first Crusade (proclaimed by the Council of Clermont on Nov 26, 1085) captured much of the Holy Land from the Saracens. Later Crusades were less successful, and in the 4^{th} the Crusaders, unable to pay the Venetians for shipping, were persuaded to sack the Christian city of Constantinople instead. This event was disastrous to Greek culture: quantities of works of art and manuscripts were lost forever. Moreover, the ensuing Latin domination

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[43] The word *Crusade* comes from the Latin *crux*, meaning cross. Members of the expedition sewed the symbol of the cross on their tunics. (For further reading, see: Benvenisti, M., *The crusaders in the Holly Land*, Israel University Press, Jerusalem, 1970, 408 pp. and Prawer, J., *A History of the Latin Kingdom of Jerusalem*, Mosad Bialik, 1971, 561+654 pp.)

[44] An additional one in 1212, known as *The Children's Crusade*, ended tragically: some 50,000 children from France and Germany went on a long march south to the Mediterranean sea. Many perished of hunger and cold. Other were sold into slavery. None reached the Holy Land.
During the first Crusade (Sept – Dec 1097), a severe outbreak of plague afflicted Egypt and Israel. About 100,000 died. A second wave broke out in 1218.

(1204–1261) was a terrible blow from which the Byzantine culture was never able to fully recover.

Another disaster which the returning crusaders inflicted on Europe was an epidemic of *leprosy* which reached its peak towards the middle of the 13^{th} century. It was finally controlled by isolating lepers in special houses.

Christians were aroused to organize the Crusades primarily by religious faith. But the expeditions were also part of a larger effort by the Europeans to increase their power, territory and riches (by the time of the first Crusade, the Christians had already retaken southern Italy and Sicily and had put the Muslims on the defensive in Spain).

In addition, the Crusades were in the common interests of kings *and* popes in Europe: Pope Urban II, for example, viewed the first Crusade as an opportunity to win glory for the Church[45], while at the same time help to reduce

[45] The Crusades were not simply a war against Islam; they were also, though less openly, a war against the Jews. As it turned out, the Crusades were not a profitable business – neither morally nor financially (except for Venice!). It is chiefly in that age that the seeds of the most virulent form of anti-Semitism were sown. In 1096, the Crusaders massacred and tortured many thousands of Jews in the Rhine Valley. Later (1099) they burned Jews alive in Jerusalem and killed 70,000 Muslims and Jews in one week. These outrages were repeated in Europe during the 2^{nd} and 3^{rd} Crusades, when they occurred in England as well (1189–1190). These anti-Jewish outbreaks of 1096 and 1146 are so important that they may be considered turning points in the history of the people of Israel. Before that time Jewish persecutions had been exceptional in Western Europe. They now became more frequent. Moreover, they were the cause of anti-Jewish legislation by Innocent III and other rulers. The enormous amount of money needed for the Crusades were partly obtained by special taxation of the Jews and confiscation of their goods.

During the Crusades, the persecution of the Jews tended to become more vicious and more petty; in 1215 a *yellow badge* was enforced upon the Jews by the Fourth Lateran Council, under Innocent III. The segregation of the Jews into ghettos, voluntary at first (for mutual protection), obligatory later, was almost a natural consequence of that branding. But one cannot persecute people without reason, and new reasons were needed all the time. It was hoped that a critical examination of the Jewish writings, and especially the *Talmud*, would provide the justification for their treatment as outcasts. Gregory IX ordered the confiscation of all the Talmuds (1240) on the allegation that it consists of the main source of Jewish antagonism to Christendom. A great number of Talmuds and other Hebrew books were burned in Paris (1242). The tide of persecution once started could not be stemmed. Evil begets evil:

In Spain, a vigorous anti-Jewish propaganda was carried on by the Dominicans.

warfare among European kings and nobles.

On the other hand, some crusaders hoped to win glory, wealth, and new lands, whereas merchants, such as those in the Italian seaports of Genoa and Venice, joined in search of new markets.

In 1291, after the last Crusade, the Muslims seized Acre (now Akko, Israel), the last Christian foothold.

By this time Europeans were losing interest in the Holy Land. Europe was turning its attention westward, to the Atlantic Ocean and beyond. [In 1492, the Spaniards drove the Muslim Moors out of Europe, and in the same year Columbus sailed to the New World.]

The expeditions to the Holy Land prepared Europe for expansion into America. Europeans acquired new tastes in food and clothing. Their desire to travel increased. They learned how to make better ships and better maps, and they learned new ways to wage war. The Crusades quickened the pace of progress of Western Europe by bringing profit and prosperity to Italian trading cities.

1100–1123 CE **Avraham bar Hiyya ha-Nasi**; **Savasorda**[46] (ca 1065–1136, Barcelona and Provence). Mathematician, astronomer and philosopher. The author of the first major book of mathematics written in Hebrew. Introduced Arab trigonometry to the West. He was one of the leaders of the movement which caused the Jews of Provence, Spain and Italy to become transmitters of Muslim science to the Christian West. He helped Christian scholars to translate scientific works from Arabic into Latin. His book *Hibbur*

In 1263 they obliged the great Jewish physician, Moshe ben Nahman, to defend his faith in a public disputation at Barcelona, against the Jewish renegade Pablo Christiani. Pablo seems to have had the worst of it, but this only added fuel to the fire, and the persecution was renewed with increased vigor. Once more the Talmud was scrutinized and attacked.

In England things were hardly better. The financial success of some Jews excited the hatred and covetousness of their Christian neighbors. More and more vexations were piled upon them, and the hostile feelings which they inspired accumulated throughout the 13^{th} century, gathering more and more intensity. The Spanish method of persuasion was tried in 1280 when they were obliged to attend Dominican sermons, but this could only make matters worse. The almost inevitable climax occurred in 1290: Edward I ordered all the Jews to leave England before All Saints Day, their immovable property being confiscated to the crown's profit. Sixteen years later the Jews were expelled from France.

[46] He held the honorary title *captain of the guard*, the Arabic name of which was condensed into **Savasorda**.

ha-meshiha ve-ha-tishboret (1116) was translated by Plato of Tivoli in 1145 under the title of *Liber embadorum*. It contains a complete solution of the quadratic equation $x^2 + b = ax$, showing that it has two roots. The book exerted a deep influence upon the development of Western mathematics and was used by Leonardo of Pisa as the foundation for his text-book on Hindu arithmetic, geometry and trigonometry.

Avraham also composed an encyclopedia, treating mathematics, astronomy and optics, and Sefer-ha-ibbur (1122) – the oldest Hebrew treatise specifically devoted to calculation of intercalation. Avraham was one of the creators of Hebrew scientific language.

While Christianity and Islam met each other on the battlefield, Avraham bar Hiyya, called by his fellow Jews "the prince", took a leading part in promoting spiritual interchange between the representatives of the Christian and Arabic civilizations.

Through his translations, Muslim trigonometry, and more specifically the sine and tangent functions, were introduced into the Latin world. Although starting as an abacist (he wrote a treatise on the abacus before his Arabic contacts), he later became one of the earliest algorists.

1100–1500 CE *Christian pilgrimage*[47] *in medieval times*. Journeys of devotion to the Holy Land, Rome and other sanctified places.

The medieval Church adopted the custom of pilgrimage from the ancient Church. The young Germanic and Romance nations did precisely as the Greek and Romans had done before them, and the motives of these journeys (now much more difficult of execution in the general decay of the great world-system of commerce) remained much the same. They were undertaken to the honor of God, for purposes of prayer or in quest of assistance, especially health. But the old causes were reinforced by others of equal potency.

The medieval Church was even more profoundly convinced than its predecessor that the miraculous power of Deity is attached to the body of saints and their relics. But the younger nations – French, English and German – were scantly endowed with saints; while, on the other hand, the belief obtained that the home-countries of Christianity, especially Rome and Jerusalem, possessed an inexhaustible supply of these sanctified bodies. Far more important consequences, however, resulted from the fact that the medieval mind associated the pilgrimage with the forgiveness of sins, an idea foreign to the ancient Church. The pilgrimage became an act of obedience. The place to be visited

[47] From the French *pelegrin*, Latin *peregrin*, meaning: to travel. (Similarly *saunter* is derived from *saintre terre*, meaning: stroll.)

was not specified, but the pilgrim lay under the obligation, wherever he went, to offer his prayers to the tombs of the saints.

As the system of indulgences developed, a new motive came to the fore which rapidly overshadowed all others: to obtain the indulgence which was vested in the respective church or chapel. By the close of the Middle Ages there were thousands of churches in every Western country, by visiting of which it was possible to buy an almost indefinite number of indulgences. A system was thus formed through which a repentant sinner could buy himself out of guilt and obtain total absolution. Simultaneously, the opportunity was offered of acquiring an indulgence for the souls of those already in purgatory! Consequently, during the whole period of medievalism, the number of pilgrims was perpetually on the increase, and so were the accumulated riches of the Church.

The pilgrimage to Rome received their greatest impetus through the inauguration of the so-called *Year of the Jubilee* (1300 CE), on which the bull of Boniface 8^{th} promised Plenary indulgence to every Christian who should visit the churches of Peter and Paul on 15 days during the year. This placed the pilgrimage to Rome on a level with the crusades – the only mode of obtaining a plenary indulgence. The success of the papal bull was indescribable: in the year of the Jubilee, on the average, 200,000 strangers were present in Rome during the day, the greatest number of pilgrims coming from Southern France.

In the years 1350, 1390, 1423, 1450, 1475, 1500, the troops of pilgrims again came steaming into Rome to obtain the cherished dispensation.

Next to Rome, the shrine of St. James of Compostella (Santiago, Spain) became the most favored devotional resort, and a pilgrimage road from France to that resort was known as the *Way of St. James*.

Science benefited from pilgrimage in a number of ways: it enabled exchange of ideas and information between various cultural centers. It also accelerated the fusion and diffusion of knowledge from the East into Europe.

1109–1140 CE **Yehudah ben Shmuel Halevi** (1071–1141, Spain and Jerusalem). Philosopher-poet and physician. Presented his *philosophy of history* in general and of the history of the people of Israel, in particular, in his famous book *Kitab Al Khazari* (1139), written in Arabic and translated into Hebrew under the title *Sefer Ha'Kuzari* (Book of the Khazar). It is a profound meditation on the interrelations between the histories of Hebrews and non-Hebrews and an acute demarcation between philosophy and religion.

According to Halevi, the knowledge of God is attained not through rationalistic discussions or theoretical philosophical scholasticism (which only lead to controversies and disagreements), but via personal inner *experience*[48] such as experienced by the Hebrew prophets.

He maintains that the universe was created by intentional God's will out of nothing, a belief which is at the common root of the three great religions. The laws of nature were established *arbitrarily* by God and are manifested both in the physical world *and in history*. He rejected Epicurean metaphysics which maintained that the universe was created by chance.

Halevi seeked to free Judaism from a dependence on the intellectualization of creed and Aristotelianism which he reproached for subjecting the Deity to necessity and for becoming incompatible with the idea of a personal God. Platonic tradition seemed more fitting to him, for he was inclined to regard God as a principle of form that moulds the eternal material principle. He maintained that Judaism does not center in the person of its founder as the other religions do but in the people to whom the Torah has been given. Jewish history is the work of Divine Providence which he regarded as the continuation of the Divine creative activity. Halevi became the first philosopher to speculate on the meaning of the Diaspora in Jewish destiny; in his view, the successful conclusion of the Diaspora and the restitution of the Land of Israel would herald the redemption of not only the Jews but of all mankind.

Halevi was born in the city of *Tudela* (*not* Toledo), Northern Spain. His well-to-do parents sent him to best schools where he studied algebra, grammar, Arabic, astronomy and poetry. He left his home town at an early age (ca 1086) and wandered to Southern Spain, visiting Cordova and Granada. He studied the Talmud at the academy of *Lucena*, and medicine at the medical school of *Cordova*. Finally, at the age of 24 he established a successful medical practice in *Toledo* and married into a prominent family in that city. As the persecution of the Jews reached an intolerable level, Halevi left Christian Spain (1109), gave up his career as physician and abandoned his family to take up the life of a wandering poet.

With his friend, the poet-scientist-physician, Avraham Ibn Ezra, he roamed the cities of Muslim Spain for many years and at least once, visited North Africa. (The two poets became related by the marriage of Halevi's daughter with Ibn Ezra's son.) In 1140 Halevi decided to make a pilgrimage to Jerusalem, despite the dangers to travelers in those days and the risk of going to Israel while the crusading movement was in progress. He arrived in Alexandria on Sept. 8, 1140, and apparently died in Egypt some six months later. Legend has it that at the very moment when he knelt to kiss the ground at the gate of Jerusalem, an Arab horseman hurled his spear at the prostrate figure and cut short the poet's life.

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[48] In *The Varieties of Religious Experience* (1902), **William James** expresses these very ideas.

1116–1142 CE **Adelard of Bath** (1075–1160, England). Philosopher, translator and mathematician. English monk, who in 1120 studied in Spain and traveled extensively through Greece, Syria and Egypt. He is credited with Latin translations of Euclid's '*Elements*' and of al-Khowarizmi's astronomical tables. Adelard had to run physical risks in his acquisition of Arabic learning: to obtain the jealously guarded knowledge, he disguised himself as a Muhammadan student.

1120–1160 CE **Ibn Zuhr (Avenzoar)** (ca 1090–1162, Spain). Physician. A Muslim of a Jewish descent. Greatest clinician of the Western Caliphate. Described pericarditis, mediastinal abscesses, surgery for cataracts, kidney stones. His most important work, *Taysir* (Aid to Health), was influential throughout Europe in Latin and Hebrew translations[49]. Avenzoar was born in Seville.

1134 CE A Sicilian coin bearing this date serves as the earliest known example of the official use of the Arabic numerals in the West.

ca 1140–1164 CE **Avraham** (**ben Meir**) **Ibn Ezra** (1089–1167, Spain and Western Europe). One the greatest Jewish savants of the Middle Ages. A roving scholar who developed unusually rich literary and scientific activity in his roaming existence under the stress of circumstances, and who wrote works of lasting importance: poet, philosopher, physician, mathematician, astronomer, chronologist, Hebrew philologist, grammarian and biblical commentator. He wrote on the history of numbers, the calendar, astronomy and the astrolabe.

Ibn Ezra translated astronomical works from the Arabic (al-Biruni and al-Khowarizmi) and wrote at least five books on mathematics and astronomy (some of which were translated from the Hebrew into Latin!) [*Sefer-ha-Ehad* – peculiarities of numbers; *Sefer-ha-Mispar* on arithmetic and combinatorics; *Luhot* – astronomical tables; *Sefer-ha-Ibbur* – on the Calendar; *Klei-ha-Nehoshet* – on the astrolabe]. He adopted the positional decimal system for integers (with place values from left to right) of the Hindus, using the first nine Hebrew letters for the numerals 1–9 (with the tens to the left of the units etc.), and denoting the zero by the special sign $\circ$ or $\bar{\circ}$ (*galgal*=wheel in Hebrew). This lore he spread all over Western Europe, sixty years before Fibonacci. He was thus instrumental in propagating Eastern mathematics and astronomy into Italy, France and England. He was fitted for this mission through the versatility of his learning.

Ibn Ezra was born in Tudela, northern Spain, and afterwards settled in Cordova, where he practiced medicine and raised a big family, winning for

49 Translated from Hebrew into Latin (1280) by a Paduan physician.

himself a name of a poet and a thinker. At about 1137 he emigrated from Spain and led a wandering life of a lonely exile for nearly three decades; he visited Northern Africa, Egypt, the land of Israel, Mesopotamia and India, and then returned to Europe and lived in Rome (1140), Salerno (1141), Mantua (1145), Verona (1146), Lucca (1148), Beziers (1156), London (1158), Narbone (1160). Longing to see his old Spanish home, he left Rome, but died on the way at Calahorra.

Ibn Ezra was one of the greatest Biblical commentators of the Middle Ages. He was also one of the forerunners of modern Biblical criticism, and much admired by **Spinoza** on that account. (His conclusion that the Book of Isaiah contains the sayings of two prophets, has been confirmed by modern criticism.)

1144 CE **The Norwich blood libel**. Concoction of the first ritual murder fantasy - a pernicious medieval superstition which would encourage the most virulent Jew-hatred in subsequent centuries[50]. Invented in Norwich, England, following the murder of a Christian boy just before Easter, who would later be venerated as medieval's Europe first child martyr. The crime was then attributed without any evidence to local Jews. They were accused of crucifying him in mockery of the passion of Jesus, and the fantasy gained acceptance because people wanted a local saint to work miracle cures and the Norwich clergy realized that his shrine would enhance the city's standing on the pilgrim route.

1145–1149 CE **Robert of Chester** (England). Mathematician, astronomer and alchemist. Translated al-Khowarizmi's book into Latin using '*Algebra*' for 'al-jabr' and 'Algorism' for the author's name. From this is

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[50] Similar accusations spread across England and to the Continent. A famous blood libel in Italy is the *Trent episode* (1473) which resulted in the burning of all the Jews of the city, with the approval of the Pope. The Church revoked its accusations in the Trent libel only as late as 1965!

Throughout the Middle Ages and into the modern era, blood libels have abounded in the Christian world and frequently provoked persecutions and massacres of the Jews. Even in the late 19^{th} century there were notorious blood libel cases: *Tisza-Eszlar*, Hungary (1882); *Corfu*, Greece (1891); *Xanten*, Germany (1891); *Blondes*, Russia (1900); *Konitz*, Germany (1900).

As late as 1911 the Russian government sought to exploit popular antisemitism by putting on trial a poor Jewish artisan in Kiev, Mendel Beilis, who was eventually acquitted (the *Beilis trial*, 1913). The Czarist minister of justice stopped at nothing to prove the charge, but he failed. Even his handpicked judge and jury being unable to find Beilis guilty. The affair brought ridicule upon Russia. German Nazi publications also used the blood libel to whip up antisemitism.

derived today's word *algorithm*, meaning: a method of calculating in any particular way. Introduced the Latin word *sinus* (1149) into trigonometry.

The Hindus have given the name 'jiva' to the 'half-cord' in trigonometry, which the Arabs took over as 'jiba'. In the translation he confused it with the Arabic word 'jaib' meaning 'bay' or 'inlet' – which he translated into the Latin 'sinus'.

He lived in Spain during 1141–1147 and in London about 1147–1150.

ca 1145 CE **Gerhardo of Cremona** (1114–1187, Italy). A most industrious translator who translated into Latin over 90 Arabian works, among which were Ptolemy's *Almagest*, Euclid's *Elements*, al-Khowarizmi's algebra and the works of Aristotle.

Gerhardo was perhaps the greatest of all translators. He was born in Cremona, Lombardy and died in Toledo. Being anxious to read the Almagest, which was not yet available in Latin, he went to Toledo, where he studied Arabic and carried on a prodigious activity as translator until the time of his death.

ca 1150 CE **Bháskara** (1114 – ca 1185, India). Also known as *Bhaskaracharya* (the Teacher). Hindu astronomer and mathematician. In his book *Sidd'hanta-siromani* he presented algorithms for solving Diophantine problems (including the Pell equation[51] $y^2 = px^2 + 1$), problems in permutations and combinations, operations with zero (excluding division by zero) and rules for employing positive and negative numbers.

Gave the remarkable identities

$$\sqrt{a \pm \sqrt{b}} = \sqrt{\frac{1}{2}\left(a + \sqrt{a^2 - b}\right)} \pm \sqrt{\frac{1}{2}\left(a - \sqrt{a^2 - b}\right)}.$$

These identities are also found in Book X of **Euclid**'s 'Elements', but are given there in an involved language which is difficult to comprehend. He also gave the trigonometric identities

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

Bháskara represents the peak of mathematical knowledge in the 12^{th} century. It shows an understanding of the number system and solving equations which was not to be reached in Europe for several centuries.

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[51] A complete theory for this equation was worked out by Lagrange in 1766–1769. Bháskara studied the Pell equation for $p = 8, 11, 32, 61, 67$. For $p = 61$ he found the solutions $x = 1776, 319, 049$; $y = 22, 615, 390$.

1154 CE **Muhammad al-Idrisi** (1100–1166, Spain and Sicily). An Arab cartographer, geographer and traveler who lived in Sicily, as a court geographer of King Roger II. He was born in nowadays Ceuta, Spain and died in Sicily.

Al-Idrisi was educated in Cordova. Later he traveled far and wide in connection with his studies and then flourished at the Norman court in Palermo.

In 1154 al-Idrisi issued a large map of the "inhabited earth" (as far as it was known to the author) and an accompanying book, named *Geography.* Taken together, they were named *Kitab Rudjar* ("Roger's Book"). This book was based on reports of emissaries which Idrisi sent to observe and describe various countries and regions including Scandinavia, Germany, France, Italy, Syria, Asia Minor and Egypt.

In his book he divided the "inhabited earth" into seven "climates" beginning at the equinoctial line, and extending northwards to the limit at which the earth was supposed to be rendered uninhabitable by cold. Each climate is then divided by perpendicular lines into eleven equal parts, beginning with the western coast of Africa and ending with the eastern coast of Asia. The whole world is thus formed into seventy-seven equal square compartments. The geographer begins with the first part of the first climate, including the westernmost part of the Sahara and a small (north-westerly) section of the Sudan (of which a vague knowledge had now been acquired by the Moslems of Barbary), and thence proceeds eastward through the different divisions of this climate till he finds its termination in the Sea of China. He then returns to the first part of the second climate, and so proceeds till he reaches the eleventh part of the seventh climate, which terminates in north-east Asia, as he conceives that continent. The inconveniences of the arrangement (ignoring all divisions, physical, political, linguistic or religious, which did not coincide with those of his "climates") are obvious.

His major contribution lies in medicinal plants as presented in his other books, specially *Kitab al-Jami-li-Sifat Ashtat al-Nabatat.* He studied and reviewed all the literature on the subject of medicinal plants and formed the opinion that very little original material had been added to this branch of knowledge since the early Greek work. He, therefore, collected plants and data not reported earlier and added this to the subject of botany, with special reference to medicinal plants. Thus, a large number of new drugs plants together with their evaluation became available to the medical practitioners. He has given the names of the drugs in six languages: Syriac, Greek, Persian, Hindi, Latin and Berber.

Al-Idrisi, later on, also compiled another geographical encyclopedia, larger than the former entitled *Rawd-Unnas wa-Nuzhat al-Nafs* ("Pleasure of men and delight of souls") also known as *Kitab al-Mamalik wa al-Masalik.*

Apart from botany and geography, Idrisi also wrote on fauna, zoology and therapeutical aspects. His work was soon translated into Latin and, especially, his books on geography remained popular both in the East and the West for several centuries.

Though Idrisi was in such close relations with one of the most civilized of Christian courts and states, we find few traces of his influence on European thought and knowledge.

1155 CE A map of western China *printed* in China. It is the oldest known printed map.

12–13th centuries Chinese mathematicians knew how to solve problems involving equations of the second and higher degrees.

1157 CE **Frederick Barbarossa** founded the holy Roman Empire, abolished by **Napoleon Bonoparte** (1806)

1160–1195 CE **Muhammad Ibn Rushd**; **Averroës** (1128–1198, Spain). Philosopher, physician and jurist. An outstanding Muslim philosopher who deeply influenced Christian philosophy in various ways. His main contribution was to free Aristotelian studies from the distorting influence of Neo-Platonism. He believed, as did Aquinas later, that God's existence can be proved on rational grounds alone. As for the soul, he held with Aristotle that it is not immortal.

He wrote three commentaries[52] on the works of Aristotle (as he knew them through Arabic translations. Most of these are preserved in Hebrew translation, or in Latin translation from the Hebrew) in which he tried to reconcile philosophy (reason) with religion (faith) by the doctrine of *twofold truth*: the one (more esoteric) for the philosophers, the other more concrete and literal (theological) for the masses ("teach the people what they can understand"). The success of the commentaries had caused orthodox revulsion everywhere – first among the Muslims in Spain, and finally in Christendom. Consequently, he was violently attacked by the Muslim clergy, his doctrine condemned and his books burned. In the eyes of the Church, Ibn Rushd came to be regarded as the arch-infidel, and the greatest enemy of the faith.

He wrote a medical encyclopedia (1160). His theories of the evolution of pre-existent forms, and of the intellect, anticipated modern concepts.

[52] A "commentary" was a medieval form of publishing one's views on a definite subject. To write a commentary on Aristotle meant to compose a philosophic and scientific encyclopedia, using Aristotle's writings as a framework and guide. Such commentary might be original or not, but its being labeled a "commentary" does not in itself justify any presumption one way or the other.

Ibn Rushd was born in Cordova of a line of qadis. He studied there law and medicine, amongst other subjects, and was a qadi in Seville and later in Cordova. In 1182 he became physician to the caliph in Marrakesh, but in 1194 he was banished to Cordova for holding philosophical views instead of contending himself with the faith. In 1198 Ibn Rushd was forgiven and recalled to Morocco, where he died soon thereafter.

Ibn Rushd was the greatest and the last of the Muslim philosophers. Although his influence on the members of his own faith was insignificant, he made a tremendous stir in the minds of men for centuries. His originality must be understood in a relative sense: it appeared chiefly in his way of interpreting anew the teaching of the wise men who had come before him. His philosophy was essentially a return to the purer Aristotelianism, a return to positivism or *scientific philosophy* which was largely stimulated (as most philosophical systems are) by opposite tendencies – mysticism and pragmatism.

1160–1200 CE **Moshe ben Maimon**; **Maimonides**; "RAMBAM" (1135–1204). Philosopher, physician, astronomer and mathematician. The leading Jewish savant and philosopher of the Middle Ages.

For Muslim scholars, as for medieval Christians, science was merely one aspect of philosophy. Aristotle, the great authority on science for both religions, was thought of primarily as a philosopher, and one of the chief intellectual problems of the Middle Ages was to reconcile his philosophy with the truths of religion. Moshe ben Maimon was the most prominent figure of this category, who exerted great influence on the Catholic scholastic philosophers [in particular upon Thomas Aquinas (1225–1274)].

Born in Cordova, Spain, his family was forced by persecution to leave their home in 1148 when he was thirteen. For the next ten years they drifted from place to place; at some time they crossed to Morocco and by 1158 were settled in Fez. It is not known exactly what Maimonides was doing during that time, but we may be sure that he seized every opportunity of increasing his knowledge; he studied theology, philosophy, and medicine. In 1165 the family sailed to Acre, where they traveled to Jerusalem and to Cairo. They established themselves at the end of 1165 in Fustat, where he made his living as a physician. He later became physician to Salah-al-din, and to the latter's son. He fell ill in 1200 and probably ended medical practice or reduced it considerably, being then able to devote the remnant of his life to writing. He died in Cairo at the age of 69; his remains were carried to Tiberias, Israel, where his tomb may still be seen.

Maimonides was at one and the same time an Arabic physician and a Jewish theologian. This implied no contradiction, yet his life was a double life. His activity was prodigious, for he was a physician and astronomer, a

Talmudist, a rabbi, and a philosopher. He was at the same time an ardent Aristotelian, a lover of reason and science, and an orthodox and pious Jew. He wrote many works on astronomy, medicine, logic and law, and devoted his philosophy to the reconciliation of Aristotelian philosophy with the Jewish theology – of *reason* with *faith*.

To this end, he gave an allegorical interpretation of Biblical anthropomorphism and concreteness, steering clear of both mysticism and religious skepticism; for example, he explained prophetic visions as psychical experiences, and Jewish laws and customs from the point of view of comparative ethnology. He insisted that human perfection is inseparable from knowledge, and that the *acquisition of knowledge is one of the highest forms of religion*. To that extent Maimonides was the champion of *science against Biblical* "*fundamentalism*". He thus gave a tremendous stimulus to philosophical studies, and helped check Kabbalistic extravagances as well as theological obscurantism[53].

Maimonides' medical writings have deeply influenced Muslim, Jewish and Christian physicians. In his astronomical works he rejected eccentric move-

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[53] One of the distinguished anti-Maimonidean was **Moshe ben Nahman**.

Another famous anti-Maimonidean was **Moshe de Leon** (1245–1305, Spain), a Jewish occultist, and probable author or editor of the Kabbalistic compilation, the so-called *Sefer ha-Zohar* (Book of Splendor: title derived from **Daniel** *12*, 3).

The Zohar is a mystical commentary on the Torah. It deals with such subjects as astronomy, cosmogony, physiognomy, psychology, demonology, etc. It is derived from many sources: Neo-Platonism, Neo-Pythagoreanism, gnosticism, Hinduism, and the mysticism of many nations.

The recrudescence of Kabbalism at the end of the 13^{th} century, and the appearance of the Zohar, are partly explained as reactions against the excessive intellectualism fostered by Maimonides and his followers. The triumph of Jewish philosophy has repressed other fundamental tendencies beyond endurance – these emotional, sexual, and mystical needs which are normal constituents of human nature. The Zohar completed the establishment of the Kabbalah as a definite body of doctrine, and put it on a level of equality with the two other great streams of Jewish thought, the *rabbinical* and the *philosophical* (Maimonidean).

The influence of the Zohar was immense: while containing admirable thoughts and some sound knowledge, it was adulterated by many superstitions and extravagances. It stimulated magical tendencies and as such has strong attraction for unbalanced minds, whose destruction it helped to complete. But its influence did not stop there: every Jewish thinker was more or less affected by it.

One of the central theories of the Kabbalah is the *theory of emanation*, which is the result of reconciling the Aristotelian notion of eternity of the world with the Biblical dogma of creation.

ments as contrary to Ptolemaic views. In fact, Maimonides carried these views with him to Egypt in 1165. Their transmission to Eastern Islam was largely due to him and to his disciple **Joseph ben Yehudah Ibn Aqnin**. Thus, the most advanced astronomical lore was diffused in the East by Jews.

The subtitle 'astronomer', may seem a little ambitious and to claim more for Maimonides than he would have claimed for himself. Yet, in his philosophical synthesis and even in his rabbinical commentaries, he had been obliged to investigate the scientific knowledge available in his time in order to understand the universe. This meant primarily cosmogony, and, as far as science was concerned, astronomy. Although he was not a practical or creative astronomer, he was deeply versed in both the Eastern and Western astronomical literature of his time.

He did however made a significant contribution to science proper, namely to the concept of *time*; in his work *The Guide to the Perplexed* (1190) he wrote (in Arabic): "*Time is composed of time-atoms, i.e., of many parts, which on account of their short duration cannot be divided... An hour is, e.g., divided into* 60 *minutes, the minute into* 60 *seconds, the second into* 60 *parts and so on; at last after ten or more successive divisions by sixty, time-elements are obtained which are not subjected to division, and in fact are indivisible...*". He thus concluded that these were 60^{10} or more such time-atoms in one hour![54]. The notion of Maimonides that time is composed of time-atoms and that the universe would exist only for one of them, were not for the continual intervention of God, was also held by Descartes.

In the field of mathematics proper he will remembered among the first who claimed clearly and unequivocally that π *is not rational* and that it can only be *approximated* by a ratio of integers. He further stated, although without proof, that squaring the circle by rules of Greek geometry is impossible[55].

Maimonides condemned astrology in the most uncompromising manner, describing it as a system of superstitions. In this he was too far ahead of his

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[54] In todays jargon such unit is known as *chronon*. If indeed space-time is discrete (quantized), then the scale must be very small to agree with experimental observations. Indeed, the present smallest directly observable division of a second, which is better than 1×10^{-13} sec, is coming close to Maimonides' division of $60^{-10} \sim 5 \times 10^{-15}$ sec.

[55] In his exegesis to the MISHNA (*Eruvin* **1**,5). **Archimedes** found the upper bound $\frac{22}{7}$ for π in the 3^{rd} century BCE. **Adrian Anthoniszoon** (1583) found the lower bound $\frac{333}{106}$. The irrationality of π was established by **Lambert** (1767). **Legendre**, in his *Elements of Geometry* (1794) proved the irrationality of π (and π^2) more rigorously. Finally, **Lindemann** (1882) proved that π is not algebraic, thus closing the lid on the 2400 year long quest for the circled square.

time to be understood. The world according to Maimonides is one indivisible whole, rational through and through; for without rationality, there could not be any knowledge, or morality, or religion. But he rejected the Aristotelian view of the eternity of matter and accepted the account of Genesis. In this he reconciled reason and faith, for he believed that both the world *and* faith derive from God[56]. This gives us a measure of his rationalism – he was ready to carry it to any extent compatible with his creed – but no further.

The influence of Maimonides was tremendous and far reaching in space and time. It affected not only the Jews, but also the Christians, chiefly the Dominicans (while **Ibn Gabirol** inspired the Franciscans). It can be strongly detected in **Spinoza**, and even in **Kant**.

In spite of the violent antagonism raised by Maimonides' work[57], he succeeded in his main object, the Aristotelization of Jewish philosophy. To be sure, he did not convince those who rejected every kind of philosophy. But aside from these, the ulterior development of Jewish philosophy was mainly a contest between Aristotelians and Neo-Platonians (most of these Kabbalists), a contest strikingly similar to the one which was waged at the same time in the Latin world.

The history of Maimonidism, like that of Averroism, is essentially the history of a continuous battle against fundamentalism on the one hand and mysticism (Kabbalism, or its Muslim and Christian equivalents) on the other. And is not this the very battle fought by men of science? For this reason alone, Maimonides would deserve a place of honor in the history of science.

1160–1173 CE **Benjamin ben Jonah of Tudela**, in Navarra (ca 1120–1175, Spain). Jewish traveler who visited, more than a century before Polo, all the known countries of the 12^{th} century. Slowly and carefully he went through Southern Europe, North Africa, the Near East, Arabia, Persia, India, Ceylon and penetrated up the frontiers of China. The record of his journeys is one of the most valuable travel accounts of Medieval times. He left Saragossa in 1160 and came back by way of Egypt and Sicily to Castile in 1173. His book was first published in Constantinople in 1543, translated into Latin in 1575, and into English in 1625.

1161 CE **Avraham ben David Halevi**, **Ibn Daud**, "RABED"[58] (1110–

[56] If one identifies the Biblical story of creation with the contemporary Big Bang theory, then the synthesis of Maimonides does not contradict the ideas of modern cosmology!

[57] In 1234, copies of the works of Maimonides had been destroyed by Christian authorities at the instigation of the conservative rabbis of southern France.

[58] Acronym: *Rabbi Avraham ben David*. Also known as **Johannes Avendahut Hispanus**.

1180, Spain). Philosopher, historian, physician, translator, astronomer and Talmudist. Author of books on astronomical measuring instruments. First Jewish philosopher to draw systematically on Aristotelian thought. Translated Hebrew, Greek, and Arabic literature into Latin. Introduced Arabic numerals and the concept of the *zero* into European mathematics. The RABED was born in Cordova and died in Toledo, a martyr of the Inquisition.

1170–1185 CE **Petahyah of Regensburg** (ca 1130–1195, Germany). Traveler. Visited Eastern Europe and Western Asia. His travelogue (1595), *Sibuv Ha'olam* (Journey of the World) was apparently written by a pupil, based on his notes or on dictation.

Starting from his home-town Regensburg (Ratisbon), Petahyah traveled through Bohemia, Poland, Southern Russia, Armenia, Kurdistan, Khazaria, Persia, Babylonia, Syria and Israel (1180).

Rise of the European Universities (1050–1582)

The economic, political and religious revival in Western Europe after 1000 *stimulated the growth of towns, especially in Italy and Flanders. During the* 11^{th} *century, Italian shipping in the Mediterranean increased steadily. With the Byzantine Empire weakening and the Muslim Caliphate breaking up, the Italian merchants had little competition. In addition, the Italian monopolized the trade in oriental goods for Western markets, and these markets were becoming steadily more profitable, thanks to the general increase in prosperity and security throughout the West. The great seaports of Venice, Pisa and Genoa flourished. As the Italian merchants carried their wares north through France and Germany, they stimulated the growth of other trade centers along the routes they traveled. Simultaneously, the towns of Flanders found their nourishment in their textile industry. By the* 12^{th} *century the Flemish textile towns of Ghent, Brugs and Ypres rivaled the flourishing seaports of Italy in wealth and population.*

Outside Italy and Flanders old towns were expanding and new towns were springing up, and rational division of labor between town and country was set up. Eventually towns gained freedom for their people and a separate system of municipal government.

The economical revival was accompanied by political and religious revivals. Slowly Christianity became less and less a matter of external observance, more and more a matter of strong internal conviction. A great wave of popular piety swept through Europe in the 11^{th} *and* 12^{th} *centuries, changing the whole character of European society. This revival continued unabated into the* 12^{th} *century. The men of this era, with much energy and originality, laid the foundations for a new architecture, a new literature, and a new system of education. All these activities drew on the Church for intellectual inspiration and material support, and in many of them Church men played a leading role.*

From 1000 CE on, there had been a great wave of church-building in Western Europe. These cathedrals were larger and more beautiful than any that had been built before in Europe. The cathedrals and monasteries soon became centers for theological studies. But 12^{th} *century scholars had a wider range of interests such as Latin classics, Roman law and especially Canon law, which opened the road to high office in the church.*

The cathedral and monastery schools developed slowly into universities. The oldest was ***Salerno*** *(ca 1050)*[59]*. The next was the University of* ***Bologna****, founded in 1088. It has existed as a law school since 890. The* ***University of Paris***[60]*, founded in 1167, also developed from a pre-existing theological school. Many other universities evolved from church schools, such as* ***Oxford*** *(1167),* ***Padua*** *(1222),* ***Cambridge*** *(1231),* ***Montpellier***[61] *(1220),*

59 The University of Salerno developed gradually from a medical school. One of the founders of this school was **Shabbethai ben Avraham ben Joel Donnolo** (913–982, Italy), a physician and pharmacist, and one of the earliest Jewish writers on medicine (in Hebrew). Donnolo wrote at the crossroads of the Greco-Latin and Arab cultures. The Salerno school is said to have taught in Hebrew, Latin and Arabic, and the Jewish element appears to have been important among the students and the professors. The reputation of the school was great till the 13^{th} century.

60 During its period of glory – the 13^{th} century – the leading teachers in the University of Paris were almost all foreigners: **Roger Bacon** (England), **Albertus Magnus** (Germany), **Thomas Aquinas** (Italy) and others. This was less surprising than it seems at first view, for the University of Paris was not a Parisian or a French institution; it was neither regulated nor subsidized by town or country; its ideals were Christian ideals, and it was controlled by an international organization, the Catholic Church. It is true the situation was far from being the same in the following century, especially when the popes were "captive" in Avignon under the thumb of the French Kings; Indeed, in the 14^{th} century the main teachers of Paris are no longer foreigners, but Frenchmen.

61 The *medical* school of Montpellier existed since early in the 12^{th} century. It owed its foundation largely to Jewish teachers, themselves educated at the Moorish

Salamanca *(1243) and the* ***Sorbonne***[62] *(1257).*

These universities were founded largely to serve the professions. They provided the first unified teaching of law, medicine and theology. The medieval Latin term 'Universitas' (from which the English word "university" is derived) was originally employed to denote any community or corporation. In the course of time, probably toward the latter part of the 14^{th} *century, the term began to be used exclusively for a community of teachers* and *scholars recognized and sanctioned by civil or ecclesiastical authority or by both. Other customary names were '*studium generale*' and '*universitas studii*'.*

The university at its earliest stage of development, appears to have been simply a scholastic gild – a spontaneous combination of teachers, of scholars (both called 'students'!), or of both combined, and formed probably on the analogy of the trades gilds, and the gild of aliens in foreign cities.

At Bologna, for example, the university was a corporation of the students. They hired the teachers and controlled the school's politics: the many foreign students there, felt that they were being cheated by the Italian boardinghouse keepers, and sometimes by their professors as well. Their union kept down the price of food and lodging and made sure that teachers covered the adequate amount of material in their lecture courses.

At Paris, the chief problem was to determine at what point the student was entitled to set himself up as a teacher. So the teachers themselves formed a union to which they admitted only students who had passed a rigorous examination. They also collected fees from the students and directed the policies of the university.

Medieval science, as taught at the universities, was based on authority and developed its ideas through formal logic rather than through observation and experimentation, and it was contaminated by the wishful thinking of philosophers and magicians. Nevertheless, it was an attempt to explain the physical

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schools of Spain, and imbued with the intellectual independence of the Averroists. Its rising prosperity coincided with the decline of the school of Salerno. Montpellier became distinguished for the practical and empirical spirit of its medicine, as contrasted with the dogmatic and scholastic teaching of Paris and other universities.

62 Founded by **Robert de Sorbon** (1201–1274) in Paris. It started as a modest establishment which accommodated seven priests charged with the duty of teaching theology gratuitously. To this was added a college of preparatory studies. In 1257 a site was given by King Louis IX in the heart of the Latin Quarter. Destined originally for poor students, the Sorbonne soon became a meeting-place for all the students of the University of Paris.

world and find ways of summing up seemingly unrelated phenomena by general laws. Sometimes, meaningful questions were asked about the nature of the universe, and from that questioning our modern science has evolved.

The attitude of the Church toward the universities was ambiguous. Although it favored these organizations, it was suspicious of the new learning, which carried philosophical doctrines that diffused from the Pagan Greeks through the Arab commentators on Aristotle. Since any corporate group had to be recognized by some higher authority, the Church saw obvious advantage in patronizing associations of scholars: it could more easily control the new learning and use the professors as experts to examine the suspect doctrines.

The famous scholars of the 13^{th} *century were university professors, and university graduates filled high offices in the Church and important positions in secular governments. Thus, during the* 13^{th} *century, the universities took over the intellectual leadership of Western Europe.*

Control of the scholars gradually passed to permanent bodies of administrators. During the Renaissance, in the 15^{th} *century, the universities helped direct the revival of interest in Greek and Roman learning. Discussion of a new method of inquiry (the so-called 'scientific method') began in the universities in the late* 13^{th} *and* 14^{th} *centuries, and came to fruition in Western Europe after 1600. The new method was essentially a combination of two elements: careful observation and controlled experimentation on the one hand, and rational interpretation of results by the use of mathematics on the other.*

In the 13^{th} *and* 14^{th} *centuries, a small but increasing number of scholars began to question existing explanations of astronomical problems. Many of these were Franciscans, inspired by their founder's sensitive feeling for nature. Stimulated by the current study of Greco-Arabic science, a group of teachers at Oxford and Paris began to apply mathematical reasoning to problems of physics and astronomy, such as accelerated motion. Their speculations were continued by professors at the University of Padua in the* 15^{th} *and* 16^{th} *centuries. At Padua, a center of medical training for 3 centuries, the proper method of studying nature was vigorously debated in the course of arguments about Aristotle. Medieval universities had kept interest in science alive, and the first faint beginning of the scientific revolution may be seen in Oxford, Paris and Padua.*

Other known universities were established in the following chronological order: ***Prague*** *(1348),* ***Cracow*** *(1364),* ***Vienna*** *(1364),* ***Heidelberg*** *(1385),* ***Leipzig*** *(1409),* ***Basel*** *(1460),* ***Uppsala*** *(1477),* ***Copenhagen*** *(1479),* ***Königsberg*** *(1544),* ***Jena*** *(1558),* ***Leyden*** *(1575),* ***Edinburgh*** *(1582),* ***Utrecht*** *(1634),* ***Harvard*** *(1636),* ***Yale*** *(1701),* ***Göttingen*** *(1737),* ***Princeton*** *(1746),* ***Columbia*** *(1754),* ***Moscow*** *(1755),* ***Berlin*** *(1809).*

The idea of the university was one of the most important medieval contributions to modern civilization. Prior to their establishment, schooling was mainly the privilege of the wealthy (private tutoring by individual teachers) or through instruction of novices by priests.

It must be remembered, however, that institutions of higher learning existed already 1500 *years prior to the first European university, in the form of the Athenian or Alexandrian Academies, and later as the religious schools (Yeshivot) of the Jews in Babylon, North Africa, Spain and France and also at the Muslim Al-Azhar University in* ***Cairo*** *(established 970).*

It is remarkable that the Chinese did not develop similar institutions: they did not have the conviction that people can dominate nature and they were not interested in the scientific method. Consequently, their theories remained divorced from observation and experimentation.

The Friars (ca 1200)

The growing dissatisfaction of town people with the clergy in Southern Europe, the influx of new ideas along the Eastern trade routes, and the nucleation of free thinkers in circles of the new universities, increased the heresy in the Church. By 1207, *the church saw no other way than to suppress this movement by sheer force. The heretic movement then went underground or dispersed to England and Scandinavia, to escape the claws of the Inquisition.*

*In another vein, the church responded in an internal reform movement through the establishment of the Franciscan and Dominican Orders [**Francis of Assisi** (1182–1226,* Italy*) and **Dominic of Caleruega** (1170–1221, France)]. These orders were founded in the first quarter of the* 13^{th} *century, and they played a prominent part in the scientific revolution. So many of their members contributed to the development of science and philosophy that it is impossible to understand the thought of their times and of subsequent centuries without paying considerable attention to them.*

The Franciscan Order was founded in 1210, *and the Dominical Order in* 1215. *While the majority of Christian monks shunned the people and spent their lives in secluded monasteries, the Friars went to the people. They established themselves in growing towns where they could reach the multitudes. They were not attached to monasteries, but simply to their Order, and wandered about as missionaries. They soon realized that it would be impossible to influence the masses without controlling the education of the leaders, and tried to obtain chairs in the universities, notably in Paris. Thus the Franciscans established themselves in Paris in* 1219, *and obtained a chair of theology in the University about* 1232. *The Dominicans played a very important role in the development of medieval universities. (However, the organization of the Inquisition was also essentially the work of Dominicans.) They established themselves in Paris in* 1217 *and obtained the first chairs of theology in* 1229 *and* 1231.

The spiritual attitude of these two orders were almost antagonistic. The Franciscans spoke to the heart, continuing the Platonic *tendencies of early Christianity. They had less intellectual curiosity but more intellectual freedom.*

The Dominicans, on the other hand, spoke to the head, and made it their special task to develop Aristotelian *philosophy. It is not a mere matter of chance that Roger Bacon (1214–1292) was a Grey Friar (Franciscan), and Thomas Aquinas (1225–1274) a Black Friar (Dominican).*

The Dominicans established schools, and the graduates of these schools soon dominated the faculties of theology of European universities. Both Orders met with immediate success. Thousands of men joined their ranks during the 13^{th} *century.*

The creation of these two orders had a decisive influence on the intellectual development of Christendom. It is impossible to explain the vicissitude of medieval philosophy from this time on without paying attention to the complications and the new incentives and rivalries caused by the existence of these two militant organizations.

They took part in the immense educational expansion which was then taking place in Western Europe. To understand properly the educational revolution *of the* 13^{th} *century one must try to imagine the creation of some* 15 *universities in four countries at a time when such opportunities were unknown except in a very few places.*

There is perhaps a touch of irony to the fact that the movements that sprang out of a need to suppress 'heresy', carried with it the seed of the forthcoming scientific revolution through men like ***Roger Bacon*** *(Franciscan friar) and* ***Giordano Bruno*** *(Dominical friar).*

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The Inquisition (1231–1834)

Up to a point, Church influence had been for the public good. She preserved a limited peace in times of strike, tried to impose a code of human behavior, and acted as school-mistress. The Church harnessed and nourished intellect, taught and provided administrators, lawyers, physicians, encouraged and preserved architecture. literature, and art. But, although creative work might be encouraged, creative thought, so essential to the development of science, was sternly repressed, suppressed and oppressed. The doctrine of persecution formed an integral part of medieval Christianity. Those whose written and spoken thoughts did not follow the rigid line permitted by the Church stood in danger of persecution as heretics.

Pope Gregory IX created in 1231 *a special court to investigate, punish and eliminate so-called 'heretics'. It was known as the Inquisition*[63]*. Dominican and Franciscan friars served as Inquisitors. The Inquisition operated chiefly in France, Germany, Italy and Spain. It later followed the Spaniards and the Portuguese in their overseas conquests to Central and South America.*

The censure of books was established in 1502 *by Ferdinand and Isabella as a state institution. All books had to pass through the hands of the bishops; in* 1521 *the Inquisition took upon itself the examination of books suspected of Lutheran heresy. In* 1547 *the Suprema produced an Index of prohibited books, drawn up in* 1546 *by the University of Louvain. In* 1558 *the penalty*

[63] The first 'Inquisition' was established by **Augustine** (414 CE) when he promulgated a doctrine to justify the persecution of heretics by church *and* state, working together. He mobilized the church's own vast resources to hound the leaders of the *Donatist* heresy. [A powerful sect which arose in the Christian church of Northern Africa at the beginning of the 4^{th} century, named after **Donatus of Bagoi**, executed in 350 CE.] Donatists led an open revolt against Rome which the imperial legions could not defeat. Donatist peasants and agricultural workers terrorized landlords, tax collectors, and creditors, liberated slaves, destroyed rent rolls and land titles, and unraveled the fabric of Roman rule. The Donatists were crushed by the combination of *Catholic inquisition* and imperial force. This civil war and the ensuing schism in the church, helped the invading Vandals, after their sack of Rome to conquer North Africa (430 CE), and enabled them win over the enserfed population of the empire. The Donatist finally disappeared prior to the Saracen onslaught two centuries later.

of death and confiscation of property was decreed against any bookseller or individual who should keep in his possession condemned books.

After the expulsion of the Jews under Isabella the Catholic (1492) and the Moriscoes (Spanish Moors) under Philip III (1609), the Inquisition attacked especially Catholics descended from infidels, the Marranos and Conversos. As late as 1715, *a secret association was discovered in Madrid, consisting of twenty families, having a rabbi and a synagogue, and was prosecuted with the utmost rigor. A great number of people were sent to the galleys, or burnt, for having returned to their ancestral religion, on the flimsiest evidence.*

It is estimated that 30,000 'heretics', mostly Jews, were burned at the stakes in 2000 auto da fé public executions or tortured to death in the Inquisition chambers during its overlong 600 *year reign, most of them in Portugal and Spain. About* 100,000 *were punished after being imprisoned. Among the burned were* ***Jan Hus*** *(1415) and* ***Giordano Bruno*** *(1600). One of the most brutal and bloodthirsty Inquisitors was the Dominican friar Tomas de Torquemada*[64] *(1420–1498).*

In 1632 ***Galileo Galilei*** *was summoned to appear before the Inquisition, following the publication of his book: "A Dialogue on the Two Principal Systems of the World". After a long trial, which included threats of physical torture, Church officials forced him to publicly renounce his belief in the Copernican theory, and sentenced him to an indefinite prison term. Before he published his Dialogues, he became blind. He lived only five more years, but the Inquisition constantly watched him in his home during that time. The prosecution of free thinkers did not end with Galilei and continued with full vigor throughout the* 18^{th} *century: The naturalist* ***José Calvigo y Faxarcho*** *(1730–1806), the mathematician* ***Benito Bails*** *(1730–1797) and the poet* ***Tomas de Iriarte*** *were prosecuted as "philosophers", under a special papal bull (1738).*

The Inquisition died late and hard: Napoleon, on his entry into Madrid (December 1808), at once suppressed the Inquisition, and on the 12^{th} *of February,* 1813 *it was declared to be incompatible with the constitution, in spite of the protests of Rome. The census of books was abolished earlier in* 1812. *Ferdinand VII restored the Inquisition (July 21, 1814) on his return from exile,*

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64 He was himself a descendant of Marranos(!) Universally hated, he was in constant dread of being poisoned or assassinated, but always traveled with a bodyguard of 250 men. He viewed with alarm the heresies sweeping Europe and felt that to save the purity of Spanish Catholicism, the threat of incipient heresy posed by worldly Marranos, relapsed Muslims, and cynical Christians would have to be stamped out before it was too late. Torquemada elevated the *auto-da-fé* into a masterpiece of showmanship, deliberately planned to resemble the popular concept of the Last Judgment.

but it was impoverished and almost powerless. In 1816 *the pope abolished torture in all tribunals of the Inquisition. The inquisition was again abolished as a result of the Liberal resolution of* 1820, *but was restored in* 1823 *after the French military intervention under duke d'Angoulême.*

The last auto-da-fé of the Spanish Inquisition took place in Valencia, Spain in 1826. It finally disappeared on the 15^{th} *of July* 1834, *when Queen Christina allied herself with the Liberals. It was not however till the* 8^{th} *of May* 1869 *that the principle of religious liberty was proclaimed in the Iberian peninsula; and even since then it has been limited by the constitution of* 1876, *which forbids the public celebration of dissident religions.*

The Abacus

Early man used the fingers of his two hands to represent numbers. In this way it is comparatively easy to count up to ten. The next major step, taken by the early river-valley civilizations, was to represent numbers by means of pebbles arranged in heaps of ten. This, in turn, led to the development of the abacus[65] *, or counting frame through which arithmetic problems could be solved.*

This device, in its simplest form, consists of a tray covered with dust or sand in which a number of grooves are made, or of a wooden board with grooves cut in it. A number is represented by pebbles (or beads) put in the grooves; as many pebbles are put in the first groove as there are units in the number to be represented, as many in the second groove as there are ten, and so on. Objects are counted by placing, for each object, a pebble in the first groove. As soon as there are ten pebbles in that groove, they are removed, and a single pebble is placed in the second groove, and so on.

[65] Greek: $\alpha\beta\alpha\xi$, a slab.

The abacus is one of the hallmarks of early civilization. Since it was in use in so many widely disparate cultures, it is believed to have been invented independently in several centers. It was to be found throughout the Mediterranean world in the first millennium BCE. ***Herodotos*** *(ca 450 BCE) remarks that the Egyptians reckoned with pebbles, bringing the hand from right to left, while the Greeks proceeded in the opposite direction!*

On the other side of the world, the Spaniards would later find in South America abaci which were in common use in the pre-Columbian civilizations of Mexico and Peru. The Aztec form of abacus consisted of a set of parallel rods stuck into a piece of wood, on to which beads could be threaded. Several different types of abacus were in use in Rome: the instrument is mentioned, for instance, by ***Pliny***, ***Juvenal*** *and* ***Cicero***. *Some Roman abaci were quite elaborate, being provided with a number of additional grooves to facilitate the addition of fractions.*

An early form of abacus using bamboo rods instead of pebbles or beads was used in China at the time of Confucius (it survived in Korea until quite recently). The modern form of Chinese abacus, the su pan, came into general use about the 12^{th} *century. There is some evidence that it was introduced into China from Rome (!). The Chinese abacus was in turn introduced into Japan in the* 15^{th} *or* 16^{th} *century. Today the Japanese abacus, in the hands of a skilled operator, is certainly a most impressive instrument*[66]. *It can be used to add, subtract, multiply, divide, and to calculate square roots and cube roots.*

There can be little doubt that, viewed in the perspective of human history, the humble abacus is the second most significant aid to calculation that has ever been invented. For many centuries it was in sole possession of the field; even today, more people probably compute with the aid of abacus than in any other way – to say nothing of the still larger number who rely on their fingers.

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[66] After World War II there has been a marked revival of interest in the abacus in Japan, and operational techniques have been greatly simplified and improved. The Japanese Chamber of Commerce and Industry sponsored an *Abacus Research Institute* and a *Central Committee of Abacus Operators*. This latter body lays down methods of training, formulates standards of performance and awards certificates of proficiency at various levels.

On November 12, 1946, a contest was staged between a Japanese abacus operator and an American operator of an electric desk calculating machine. The contest covered five types of calculation involving the four basic arithmetic operations, each being judged on speed and accuracy. The abacus' victory was complete, its operator winning by 4:1.

1180 CE **Alexander Neckam** (1157–1217, England). Man of science and encyclopedist. In his books *De Naturis rerum* and *De utensilibus* Neckam has preserved to us the earliest *European* notices of the nautical use of the *magnetic needle*. It was probably in Paris, the chief intellectual center of his time, that Neckam heard how a ship can be guided in murky weather or on starless nights by a needle (previously placed upon a magnet) that would revolve on a pivot until it settled into a northern orientation.

Neckam, the foster-brother of King Richard I, was born in London. He was educated at St. Albans Abbey school, and began to teach as schoolmaster of Dunstable. Later he resided several years in Paris, where by 1180 he had become a distinguished lecturer of the university. By 1186 he was back in England. Having become an Augustinian canon, he was appointed abbot of Cirencester in 1213.

The Compass (ca 1080–1269)

The early history of the compass is very obscure. It would seem that Chinese have known the fundamental property of a magnetic needle for a considerable time, but have applied it chiefly for occult and pseudo-scientific purposes. The first clear mention of the magnetic needle in any literature occurred in ca 1070 *by* ***Shen Kua*** *(1030–1093), a Chinese author, mathematician, astronomer, and instrument maker*[67]*. The earliest Chinese mention of the use of a magnetic needle for navigation refers to the period* 1086–1099*, where it was used by Muslim sailors between Canton and Sumatra. Thus although the Chinese were first to perceive the directional property of the magnetic needle, they failed to apply it to any rational purpose.*

It is not surprising that the origin of the discovery cannot be dated, if one considers that the first pilots who had the wit to make use of the needle

[67] His books also contain the earliest Chinese example of summation of a progression, and the earliest description of printing with movable type. We know, however, that as early as 1037 a 'south-pointing needle' was submitted to the Sung emperor Jen Tsung.

to direct their course, had no reason to publish their discovery, and on the contrary had every inducement to keep and transmit it as a trade secret. Since maritime trade between the Far East on the one hand, and India, Persia, Arabia, and Africa on the other was a Muslim monopoly, we may assume that this great discovery was made probably around 1080 *CE. Considering its origin, it is curious that the earliest reference to it outside of China, are found not in Arabic or Persian writings, but in French and Latin ones.* **Neckam** *(1180) does not speak of the compass as of a novelty, and* **James of Vitry** *(1219) describes it as having come from India. The earliest Muslim references are in* 1228*.*

It would seem that the Muslims attached more importance to the southern *end of the needle, owing to the fact for Muslims in Syria and Asia Minor, the southern end pointed roughly toward Mecca.*

Italian sailors were among the first to use the compass, and such use necessarily led to gradual improvements. The first technical description of a compass was given by **Peter the Stranger** *in 1269.*

The Triple Point

The web of Renaissance was extremely complex, because there were incessant conflicts in many directions.Three great waves of creative wisdom came from the Orient: The first, and the most fundamental of all, came from Egypt and Mesopotamia *(3100–1650* BCE*); the second, of incalculable pregnancy, came from* Israel *(800–300* BCE*), though it influenced science only in an indirect way; the third, came from* Arabia *and from* Persia *(750–1250 CE).*

It took Islam only 90 *years (622–712 CE) to bring under its rule a large belt of the world, all the way from Central Asia to the Far West [Damascus (635); Jerusalem (637); Egypt (641); Persia (642); Spain (712)]. A new Muslim civilization was formed from the fusion of Arabic and Persian cultures. Under* *the guidance of a series of Abbasid caliphs who had a passion for knowledge – the new civilization developed with incredible speed and efficacy: From the Hebrews they took, with very few modifications, Semitic monotheism and morality; from the Hindus and Persians they learned arithmetic, algebra, trigonometry, iatrochemistry; from the Greeks logic, geometry, astronomy and medicine. With the aid of the Christian Syrians they translated the whole Greek scientific heritage into Arabic.*

The cultural importance of Islam lies in the fact that it finally brought together the two great intellectual streams which had flowed independently in ancient times. Previous attempts had failed; Jews *and* Greeks *had mixed in Alexandria but, in spite that Jews learned the language of the Greeks and that Philo (fl. 40 CE) had made a deep study of both traditions, there has been no real fusion. The* Christians *had not succeeded any better, because of their excessive devotion to the new Gospel, which reduced everything else to futility in their eyes. Now, for the first time, Semitic religion and Greek science actually combined in the minds of many people all over the world.*

The contacts between Muslims and non-Muslims was generally friendly for the Muslims who treated their subjects with tolerant condescension. Under their patronage, many important works were published in Arabic by non-Muslims: Sabians, Christians and Jews. Down to the 12^{th} *century, Arabic was the philosophical and scientific language of the Jews [e.g., the earliest Hebrew grammars were composed in Arabic;* **Maimonides** *wrote his* Guide to the Parplexed *in Arabic].*

However, the major part of the activity of Arabic-writing scholars consisted of the translation of Greek works and their assimilation; the Arabic-writing scientists elaborated algebra *and* trigonometry *on Greco-Hindu foundations; they collected abundant* astronomical observations *and their criticism of the Ptolemaic system helped to prepare the astronomical reformation of the* 16^{th} *century; they enriched enormously our* medical experience*; they were the distant originators of modern* chemistry*; they improved the knowledge of* optics*, and* meteorology*; their geographical investigations extended from one end of the world to the other.*

During 1000–1050 *there was a splendid mathematical school in Cairo, made famous by the astronomer,* **Ibn Yunus** *and the physicist* **Ibn al-Haitham***;* **al-Karkhi** *was working in Baghdad,* **Ibn Sina** *in Persia,* **al-Biruni** *in Afghanistan. These mathematicians and others, were not afraid to tackle the most difficult problems of Greek geometry; they solved cubic equations by the intersection of conics, they investigated the regular heptagon* ($n = 7$) *and enneagon* ($n = 9$)*, developed spherical trigonometry, Diophantine analysis.*

Nothing of this sort existed in the West at that time: some little treatises *on the calendar, on the cite of the abacus, on Roman duodecimal fractions and geometry on the pre-Pythagorean level – a truly pitiful state.*

We may say then, that from ca 750 *CE to ca* 1100 *CE, the Arabic-speaking people (including Jews and some Christians) were marching at the head of mankind with Arabic being the international language of science, and almost the only key to the new expanding culture. However, Oriental supremacy ended about* 1100 *CE; the power and knowledge of the Latin world was growing faster and faster; whereas the main task of the Arabic scientists was already completed, and after that time the relative importance of Muslim culture declined steadily.*

By the middle of the 13^{th} *century Islam was already on the downward path, while Latin Christendom had finally realized the richness of the Greco-Arabic knowledge and made gigantic efforts to be allowed to share it: Consequently, Christians and Jews were feverishly pouring out the Greco-Arabic learning from the Arabic vessels into the Latin and Hebrew ones. But the Christians were far ahead of the Jews in this new stage of transmission. By the end of the* 12^{th} *century, the main body of Greco-Arabic knowledge was already available to Latin readers. By the end of the* 13^{th} *century, there was little of real importance in the Arab scientific literature which they were not aware of.*

At first the Eastern Jews and those of Spain were much better off than the Christians, for the whole of Arabic literature was open to them without effort. But in the 12^{th} *century the scientific life of Judaism began to move from Spain across the Pyrenees, and in the following century it began to decline in its former haunts. By the middle of the* 13^{th} *century a great many Jews had already been established so long in France, Germany, and England, that Arabic had become a foreign language to them. Up to this period the Jews had been generally ahead of Christians, and far ahead; now for the first time the situation was reversed. Indeed, the Christians had already transferred most of the Arabic knowledge into Latin; the translations from Arabic into Hebrew were naturally far less abundant, and hence the non-Arabic-speaking Jews of Western Europe were in a position of political inferiority (the crusades had caused many anti-Semitic persecutions and the Jews of Christendom were everywhere on the defensive).*

The gravity of the change is well illustrated by the appearance in the 14^{th} *and following centuries of an increasing number of translations (e.g., of medical works) from Latin into Hebrew. Thus the stream of translations which had been running from East to West was again reversed in the opposite direction. Note that a curious cycle had been completed, for the source of these writings was Greek; their Arabic elaborations had been translated into Latin and had inspired new Latin treatises; these treatises were now translated into Hebrew. From East to East via the West!*

But other cycles were even more curious. In the 14^{th} century and later, Arabic, Persian, and Latin writings which were ultimately of Greek origin were re-translated into Greek. For example, the most popular logical textbook of the Middle Ages, the Summulae logicales *of Peter of Spain (Pope John XXI), was not only translated into Hebrew, but also into the very language from which its main sustenance had been indirectly derived. From Greek to Greek via Arabic and Latin!*

During the 12^{th} century the three civilizations which exerted the deepest influence upon human thought and which had the largest share in the molding of the future, the Jewish, the Christian, and the Muslim, were remarkably well balanced; but that state of equilibrium could not last very long, because it was due to the fact that the Muslims were going down while the two others were going up. By the end of the 12^{th} century it was already clear (that is, it would have been clear to any outside observer, as it is to ourselves) that the Muslims would soon be out of the race, and that the competition would be restricted to the Christians and the Jews.

Now the latter were hopelessly jeopardized by their political servitude and by the jealous intolerance (to put it mildly!) of their rivals. Moreover, for the reason explained above, the main sources of knowledge were now less available to them than to their persecutors. This went much deeper than it seems, for whenever an abundant treasure of knowledge becomes suddenly available to a group of people, it is not only the knowledge itself that matters, but the stimulation following in its wake. The Jews were steadily driven into the background, and in proportion as they were more isolated, they tended to increase their isolation by devoting their attention more exclusively to their own Talmudic studies.

Towards the end of the 13^{th} century, the Christians enjoyed the political and intellectual hegemony. The center of gravity of the learned world was in the West and it has remained there until our own days (West = Europe + the United States).

Of course Muslim and Jewish efforts went on and both faiths produced many great men in the following centuries, yet the Western supremacy continued to wax until a time was reached, in the 16^{th} century, when the expanding civilization was so deeply Westernized that the people – even those of the Orient – began to forget its oriental origins, and when the very conception of Muslim and Jewish science almost disappeared.

The final result of science are, of course, independent of the people who discovered them. After the 16^{th} century, when science was finally disentangled from theology, the distinction between Jewish, Christian, and Muslim science ceased to be justified, but it keeps its historical value. In spite of his deep Jewishness and of his abundant use of Jewish sources, we do not count Spinoza *any more as a Jewish philosopher in the same sense that we count Maimonides or Levi ben Gershon; he is one of the founders of modern philosophy, one of the noblest representatives of the human mind, not Eastern not Western, but the two unified.*

1178, June 25 Apollo asteroid (2 km in diameter) of the *Taurid stream* impacted the moon, creating the *Giordano Bruno Crater*.

1195–1215 CE **Yehudah ben Shmuel Ben Kalonymus 'He-Hasid'** ('The Pious'; ca 1150–1217, Germany). Philosopher and moralist. A central figure of the school of Jewish *Hassidism* in Western Europe. From his center at Regensburg he led the Hassidic movement, which heralded the Hassidic branch of Judaism in the 18^{th} century in Central and Eastern Europe.

Teaching the religious value of self-restraint, Yehudah is the author (ca 1200 CE) of *Sefer Hassidim* (Book of the Devout) and *Sefer ha-Kavod* (Book of Glory). These books achieved enormous popularity among plain folk and its vogue persisted for 800 years, down to modern times! Throughout the ages, his numerous disciples strove diligently and ardently to hasten the redemption of Israel, first through Hassidism and eventually via Zionism.

Yehudah denied all possibility of human understanding of God; Man must fulfill his religious duties, as they are prescribed in the Bible, without reasonable knowledge of the Almighty, but, by purification, obedience to ceremonial life and ascetics, he may obtain union with God that is beyond reasoning.

He was born in Spier, Germany and lived most of his life in Regensburg. It is believed that Jewish mysticism diffused from Babylonia to Italy and from there by the Kalonymus family to Germany.

The Kalonymus family began in Italy, where in the 8^{th} and 9^{th} centuries, they were a primary force (along with the **Anav** family) in establishing an academy in Rome. Most of the family was induced by Emperor Charles the Bald to move to Mainz, where they established a school of rabbinic learning.

Many of the family were wiped out in the Rhineland Massacres during the First Crusade. Other family members became major figures in the creation of the Hassidic Ashkenaz movement in the 13^{th} century, among them **Shmuel he-Hassid**, his son **Yehudah he-Hassid** and his son **Eleazar of Worms** (1160–1238).

ca 1199–1202 CE Severe drought in Africa caused several year of low Nile. More than 100,000 perished.

ca 1200 CE Widespread fires on the South Island of New Zealand, created by a comet impact.

1200–1235 CE **David Kimhi**[68] (1160–1235, France). Linguist, grammarian, lexicographer, philosopher and biblical exegist. The most accomplished of a family of Hebrew linguists and exegets in Narbonne, Provence, who were among the founders of Hebrew scientific philology and grammar.

David Kimhi summarized the conclusions arrived by the earlier Spanish grammarians and presented a scientific analysis of the Hebrew language which did much for its study later both by Jews and Christians. He also wrote a commentary on the greater part of the Bible which is considered next in importance to those of Rashi and Ibn Ezra.

His father, **Joseph Kimhi** (1105–1170), a native of southern Spain, emigrated to Narbonne, Provence. [The Provence was the bridge physically and intellectually, between Spain and Central Europe. Many of the important literary products of the Jews of Spain were worked over by those of the Provence and they found their way into the thoughts of Jews of Northern and Central Europe.] Introduced a new grammatical classification of the *stems of verbs*, and translated Arabic treatises into Hebrew.

Moshe Kimhi (ca 1130–1190), was first to introduce the now usual sequence in the enumeration of stem-forms of verb conjugation.

David was the pupil and brother of Moshe, and eclipsed the fame of both his brother and father.

1200–1250 CE *Mongol* outbreak into China and Europe correlated to drought or moist conditions in Mongolia.

1202–1247 CE *Mathematical revival*. This period witnessed the activities of five outstanding mathematicians: **Fibonacci**, **Nemorarius**, **al-Hasan al-Marrakushi**, **Ch'in Chiu-Shao**, and **Li Yeh**. These men represent four different countries: Italy, Germany, al-Magrib, and China. The main accomplishments of this period were:

- The publication of *Liber abaci* (1202) marked the beginning of European mathematics.
- The diffusion of Hindu-Arabic methods into Europe, by Fibonacci and by many minor mathematicians.

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[68] known as the 'RADAK', acronym of *RA*bbi *DA*vid *K*imhi.

- **Fibonacci**'s interpretation of a negative solution as a debt (1225).
- His problems of Diophantine analysis (1225).
- General proof of the fundamental theorem of stereographic projection, by Nemorarius.
- Contributions to the theory of numbers by **Fibonacci** and **Nemorarius**.
- Publication of astronomical tables in Marseilles and London (ca 1231–1232). The needs, which brought these tables into being were astrological rather than astronomical.
- The 'Jami' of **al-Hasan al-Marrakushi** (1229), which was the most elaborate trigonometrical treatise of the Western caliphate. It was also the best medieval treatise on practical astronomy, on gnomonics and the best explanation of graphical methods.
- Introduction of astronomy into the Latin world by **Michael Scot** (1217).
- Translation of the Almagest into Hebrew by **Yaacov Anatoli** (ca 1231).
- Development of the *t'ien yuan shu* by **Ch'in Chiu-Shao** and **Li Yeh** (1247–1248).
- Numerical solution of equations of any degree by **Ch'in Chiu-Shao** (1247).

1204–1248 CE *Political events*:

- 1204. Constantinople taken and sacked by the Crusaders. It was almost a death blow to Greek culture.
- 1214. Battle of Bouvines (near Lille), won by Philip Augustus over the emperor Otto IV, supported by English and Flemish contingents.
- 1238–1241. Tatar invasion of Russia, Hungary, Bohemia, and Poland. Kiev and Cracow destroyed, Pesth besieged. The Mongol tide was stopped by the battle of Wahlstatt (near Legnitz) in 1241.
- 1236–1248. The Moors were slowly driven out of Spain. Cordova reconquered by the Christians in 1236 and Seville in 1248. By the middle of the century the Moors were restricted to the little Kingdom of Granada.

1202–1220 CE **Leonardo of Pisa (Pisano) or Fibonacci** (1170–1250, Italy). The most skilled mathematician of the Middle Ages who played an important role in reviving ancient mathematics and made significant contributions of his own. His book, *Liber abaci* (1202), played an influential role in spreading knowledge and use of the Hindu-Arabic numerical system into Europe.

Liber abaci was the first complete and systematic explanation of the Hindu numerals by a Christian writer; and also, the first complete exposition of Hindu and Muslim arithmetic. Leonardo, however, gave more rigorous demonstrations than the Muslims. (It is apparent that he had a good knowledge not only of Muslim, but also of Greek mathematics, largely derived from Latin translations of Euclid, Archimedes, Heron and Diophantos.)

Liber abaci contains sections on Roman and Indian numerals and on finger counting. Later chapters are devoted first to commercial calculations and then to puzzles and recreational mathematics – including the famous 'rabbit problem' which lead to the Fibonacci sequence. He deals with approximating square roots, cube roots and problems on volumes, in which he takes π to be $3\frac{1}{7}$.

In his later book *De practica geometriae* (1220) he obtained the value 3.141818 using Archimedes' 96-sided polygonal representation of the circle. This book also contains various geometrical and arithmetical problems, notably an extension of the Pythagorean proposition to solid geometry and the rule to calculate the volume of a pyramid frustum.

Fibonacci used algebra to solve geometrical problems, which was a novelty in Christendom. In his book *Flos*, Fibonacci considered, among other things, the equation $x^3 + 2x^2 + 10x = 20$. Showing that there is no rational solution, he gave an approximate solution, using the sexagesimal number system. In *Liber Quadratorum* (1225), a work on indeterminate analysis, he emerged as the outstanding number theorist between Diophantos and Fermat. Thus, this book contain a solution in integers of $x^2+y^2 = z^2$, and a proof of the theorem that the difference of two biquadrates is not a square. His works were quite beyond the abilities of contemporary scholars, and he certainly appears as the lone mathematical beacon of his time in Europe.

The transition from the Roman number scheme was surprisingly slow, because computation with the abacus was quite common and in this case the advantages of the new scheme are not nearly so apparent as in calculations with pen and paper today. For three centuries there was a keen competition between '*abacists*' and '*algorists*' and the latter won only close to 1500.

Thus came to an end the saga of the round zero, which started its slow wheeling from India (876 BCE) through Persia and Arabia (825).

It is remarkable that the extremely tedious arithmetic of Roman numerals[69], lacking as it does the zero and positional notation, did not induce the Europeans to invent the zero earlier.

Fibonacci[70] was born in Pisa. His father was called 'Bonaccio' (a nickname with the ironical meaning of "a good stupid fellow"). To Leonardo himself, another nickname, Bigollone (dunce, blockhead), seems to have been given. The father was a secretary in one of the numerous factories erected on the southern and eastern coasts of the Mediterranean by the warlike and enterprizing merchants of Pisa. Leonardo was educated at Bugia on the Barbary coast and taught by a Muslim master; later he traveled in Sicily, Syria and Egypt, where he made the acquaintance of the Hindu-Arabic methods. The influence of Greek, Arabian and Indian mathematics may be clearly discerned in his methods. He returned to Italy in 1202 and published *Liber abaci*, which procured him access to the learned and refined court of the emperor Frederick II.

The Fibonacci Sequence

One of the problems appearing in Liber abaci *is as follows: Each month, the female of any pair of mature rabbits gives birth to a pair of rabbits (of different sexes). Two months later, any female of a new pair gives birth to a pair of rabbits, and continues to do so each successive month. Find the number of rabbits at the end of the year if there was one pair of rabbits in the beginning of the year. This problem leads to a sequence, wherein the terms are the numbers of pairs of rabbits present in successive months* $1, 1, 2, 3, 5, \ldots, x, y, x + y, \ldots$. *This sequence has become known as the Fibonacci sequence and it occurs in an astonishing number*[71] *of unexpected situations. It has applications to art,*

69 The reader who questions this characterization should try multiplying 57,498 by 837 in Roman numerals!

70 The nickname **Fibonacci** was given by the 18th century French mathematician **Guillaume Libri**.

71 One of the distinguishing characteristics of ordinary honeybees is a system of controlled reproduction: It seems that early in her career a queen bee goes on a spree, collecting sperm from eager males. The queen produces many eggs, and

to the propagation of bees, to phyllotaxis (arrangement of leaves), florets of composite flowers, population genetics, ray optics and various other topics in mathematics and physics[72].

Biologists have tried to explain the peculiar prevalence of Fibonacci num-

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it is the general rule that unfertilized eggs hatch into males and fertilized eggs into females. Thus male honeybees do not have fathers. The queen bee is able to store the collected sperm for months and even years and, upon information supplied by her attendants, she can regulate the gender of the offsprings to meet the needs of the hive. Female bees are undoubtedly the superior sex; they do everything; the male's only function is in his role in the production of the prized female.

Let us trace the ancestry honeybees of either gender, on the assumption that each male contributes a single sperm during his lifetime, and each female produces one unfertilized egg per generation (we ignore the rate of female mortality). Denote by a_n the total number of males *and* females in the n^{th} generation. Since any female has both a father and a mother, the number of *females* in a generation is simply the number of bees in the previous generation. On the other hand, since only females have fathers, the number of *males* in a generation is just the number of females present in the previous generations. Hence $a_n = a_{n-1} + a_{n-2}$ for $n > 2$.

While on the subject of bees, there are $a_{n+2} = a_{n+1} + a_n$ paths by which the bee can crawl over hexagonal cells in the hive [one path to cell 0, two paths to cell 1, three to cell 2, five to cell 3, and so on], yielding again the Fibonacci sequence.

72 One encounters the Fibonacci sequence, for example, in ray-optics and information theory:

- A light-ray impinges upon a stack of m planar glass plates welded together, such that at each interface (except the upper boundary) the ray may either be reflected or transmitted. It is shown that for $m = 2$, the number F_n of paths by which a ray can be reflected n times in the system, is governed by the difference equation $F_{n+1} = F_n + F_{n-1}$, yielding the sequence of reflections 1, 2, 3, 5, 8, ... for $n = 0, 1, 2, \dots$.
- Imagine a signaling system that has only two signals s_1 and s_2 (e.g., the dots and dashes in telegraphy or "low" and "high" voltage [light-intensity] levels in a binary digital electronic [optical] telecommunication system). Messages are transmitted over some channel by first coding them into sequences of these two signals. Suppose s_1 requires exactly t_1 units of time and s_2 exactly t_2 units of time to be transmitted. Let N_t denote the *number of possible message sequences* of duration t. A message of duration t must end in either s_1 or s_2, but there are N_{t-t_1} possible messages to which a last s_1 may be appended and N_{t-t_2} possible messages to which a last s_2 may be appended. Therefore $N_t = N_{t-t_1} + N_{t-t_2}$. We consider the special case $t_1 = 1$, $t_2 = 2$ (one signal takes twice as long to be transmitted over the channel as the other). Then $N_t = N_{t-1} + N_{t-2}$.

bers in phyllotaxis. Symmetry may play a major role, because symmetry maintains a mechanical equilibrium of a stem, gives the leaves the best exposure to light, and supports the regular flow of nutrients. However, science is still far from a satisfactory explanation.

The Fibonacci numbers appear as the coefficients of the power series expansion of

$$(1-x-x^2)^{-1} = a_0 + a_1 x + a_2 x^2 + \cdots,$$

$$a_0 = 1, \quad a_1 = 1, \quad a_2 = 2, \quad a_3 = 3, \ldots.$$

The n^{th} term of the sequence a_n, obeys the difference equation $a_n = a_{n-1} + a_{n-2}$ with $a_0 = 1, a_1 = 1$; or $a_{n+1}a_{n-1} = a_n^2 + (-1)^{n+1}$, $n \geq 2$. Its solution is

$$a_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

*[**De Moivre**, 1730; **Binet**[73], 1843.] Both numerators and denominators of the convergents of the continued fraction representation of the Golden[74] Ratio $\frac{1+\sqrt{5}}{2}$, are formed from the sequence of Fibonacci numbers: $\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \ldots$. It would seem that nature strives to approximate the famous golden ratio.*

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73 **Euler** and **Daniel Bernoulli** were also in possession of this result a century earlier. However, Binet did make the discovery independently, and he has so little claim to mathematical fame that it won't hurt these giants to let Binet take the credit for this crumb.

74 Was the Fibonacci sequence known to the Greeks? The Greek theater at Epidaurus (ca 300 BCE) was designed by an architect called **Polyelitos** (not the sculptor). The main objective there was to ensure perfect acoustics; and although most of the stage buildings have long since disappeared, the auditorium is so well preserved that the acoustics remain extremely good. From the point of view of numerology is it only a coincidence that the numbers of rows and of seats below and above the *diazoma* (azimuthal passage across the auditorium) are respectively 34 and 21? These are two successive numbers in the Fibonacci sequence. Their ratio $\frac{34}{21} = 1.6190\ldots$ approximates the true value of the Golden Ratio: $\frac{1+\sqrt{5}}{2} = 1.6180\ldots$.

*The Fibonacci sequence also appears in different parts of mathematics. For instance, in the Euclidean algorithm for finding the greatest common divisor (g.c.d.) of two given positive integers, a certain number of successive divisions is required. In 1844, **Gabriel Lamé** proved, by using some properties of the Fibonacci sequence, that the number of divisions required to find the g.c.d. of two positive integers is never greater than 5 times the number of (decimal) digits in the smaller number.*

*The Fibonacci sequence has also played a prominent role in determining the primeness or compositeness of some "astronomically" large numbers. Indeed, **Edouard Lucas** (1842–1894, France) employed certain divisibility properties of the Fibonacci numbers to establish a test for the primality of any given Mersenne number. The literature on the ubiquitous Fibonacci sequence and its many properties is incredibly large and continues to grow. The interesting relations seem, like the geometry of the triangle, to be inexhaustible. In 1963, a group of Fibonacci-sequence enthusiasts founded the Fibonacci Association and began publication of a journal, The Fibonacci Quarterly.*

Man's yearning for a unifying principle aside, 'universal truths' are often so extraordinary that they entice the faithful to push into realms where the law does not belong: Fibonacci sequences are not omnipresent; for all the fruit blossoms that have pentagonal symmetry, there are tulips, lilies and hyacinths that do not. Nature does work in many *strange ways, not just one.*

1206–1260 CE *The Mongols*. Under the leadership of **Jenghis Khan** (1167–1227), and his successor **Kublai Khan** (1216–1294), the united Mongol and Kirghiz tribes carved out one of the biggest empires in history, stretching east-west from Japan to the Caspian Sea, and north-south from Siberia to Tibet.

In the Middle Ages the Mongols were a group of nomadic tribes living in Central Asia. They were herdsmen and fierce warriors. Temujin, later called Jenghis Khan, organized the wandering inhabitants of the steppe along rigid military lines.

Throughout the years 1206–1218, the Mongols swept over China, Azerbaijan, Georgia, and Northern Persia[75]. This conquest brought the Mongolian power to the frontiers of the Muslim state of Khorezum, which Jenghis Khan

[75] It was suggested that the outbreak of the Mongols was stimulated by drought or moist conditions in Mongolia at that time.

attacked and destroyed. In 1220 and 1221 he took and totally destroyed Bukhara, Samarkand, and Gurgan, then the centers of Muslim-Persian culture. In 1223 the Mongols defeated a strong force of Russians and Cumans.

After Jenghis' death, Mongol armies overrun and conquered southern and central Russia and then invaded Poland and Hungary (1237–1240).

In 1241 the Mongols defeated the Poles and Germans in the battle of Liegnitz in Silesia, while another army defeated the Hungarians. But because of political complications arising from the death of the Great Khan, the Mongols withdrew from Western Europe. Subsequently, they settled on the lower Volga, where a Tatar state was organized under the name Golden Horde.

Kublai Khan completed the conquest of China and founded the Yuan dynasty there. During 1245–1253, the Mongols continued to ravage Mesopotamia and Armenia. In 1258, Hulagu captured and sacked Baghdad and executed the caliph. He then invaded Syria and took Aleppo. Finally, in 1260, the Mamluks of Egypt secured a victory over the Mongols at Ain Jalut. This checked the Mongol advance and saved Egypt, the last refuge of Muslim culture.

Kublei Kahn was too great an administrator not to realize the need for mathematicians and astronomers, and it is typical of the cultural mixture organized by him that his scientific assistants were drawn from many nations. In 1263 he appointed **Isa the Mongol** (in Chinese, Ai-hsieh) the head of the astronomical board. In 1267 a new calendar was devised for Kublai Kahn by the Persian **Kamal al-Din al-Farisi**, who also introduced new astronomical instruments.

1209–1229 CE The *Albigensian Crusade* mounted by Pope Innocent III[76] against the entire population of the Albigens[77]. The Albigense, in the south of France, was then the most populous, the most technically, socially, and economically advanced part of Europe. Its population was largely *Gnostics*[78] and *Arian Christians*, and a sanctuary for Jews who were persecuted almost

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76 He also reinstated a prohibition against the owning or reading of Bibles by anyone other than the clergy, under penalty of death.

77 They took their name from the town of *Albi* in Languedoc in Southern France. There and in Northern Italy the sect acquired immense popularity. The movement was condemned at the Council of Toulouse (1119) and by the Third and Fourth *Lateran Councils* (1179, 1215).

78 *Gnosticism* usually refers to an esoteric cult of divine knowledge (a synthesis of Christianity, Judaism, Greek philosophy, Hinduism, Buddhism, and the mystery cults of the Mediterranean), which flourished during the 2^{nd} and 3^{rd} centuries CE, and had influence on early Christianity.

everywhere else in Europe. All of these groups had a high percentage of literacy and read the Bible, which was prohibited by the Vatican.

The slaughter of the Albigensians sealed the fate of the brilliant Provencal culture. In the reign of Louis IX, the County of Toulouse passed under the Capetian administration and the royal domain was extended to the Mediterranean.

Hunted down by the Inquisition and quickly abandoned by the nobles of the district, the Albigensians became more and more scattered, hiding in forests and mountains, and only meeting surreptitiously. The sect was finally exhausted and could find no more adepts in a district which, by fair means or foul, had arrived at a state of peace and political and religious unity.

1215 CE *Magna Carta*. English barons force King John to agree to a statement of their rights.

1218–1288 CE Period of great storminess in the North Sea, causing catastrophic floods on the coasts of Holland: 1228 (100,000 victims); 1282 (200,000 people drowned under 2 meters of water); Dec. 14, 1287 (50,000 victims).

The coasts of the Netherlands are exposed to the strong tides of the North Sea and have been *slowly sinking*: Since Roman times they have sunk 2 meters! At high tide, the sea is several meters above the land. This sinking, in turn, is caused by subsidence of the land under the weight of huge quantities of *sediments* transported by the *rivers* to the North Sea, and is accentuated by the building of gigantic *dikes*.

ca 1220–1240 CE **Ibn al-Baitar** (ca 1197–1248, Spain and Syria). Botanist, pharmacist, herbalist and physician. Left Spain to travel in North Africa (1219), in Bulgaria (1220), Tunis, Tripoli (1221–1223), Egypt (1224), Damascus (1237), Cairo (1238). Considered greatest botanist and pharmacologist in Islam. Traveled extensively in Greece, Egypt and Asia Minor in search of medical herbs, which he later described in his comprehensive treatise. Contemporary of **Grosseteste** and **Anatoli**, which together represent the three main streams of culture in the first half of the 13^{th} century.

Al-Baitar was born in Malaga, Spain and died in Damascus.

ca 1220–1230 CE **Jordanus Nemorarius** (ca 1185–1237, Germany). Mathematician. The founder of the medieval Christian school of mechanics and first to correctly state the law of the inclined plane. Second only to Fibonacci as a medieval mathematician.

The first one widely to use letters to represent general numbers, although his practice had little influence on subsequent writers. He wrote several books on arithmetic, algebra, geometry, and astronomy. His book *De triangulis* is a geometric work in four books containing topics such as the centroid of a triangle, curved surfaces and trisection of an angle. In his *Tractatus de numeris datis* he solved problems in which a given number is to be divided in some stated fashion.

Nemorarius was a contemporary of Fibonacci. The Arabic *sifr* was introduced by him into Germany as the word *cifra*, from which the word *cipher* was later derived[79].

In his arithmetical treatises he showed that $n(n+1)$ is neither a square nor a cube $(n \neq 0, -1)$. His book on algebra contains problems leading to quadratic equations, using letters to replace numbers. Finally, his geometrical treatises deal with the determination of the center of gravity, and contain the first general demonstration of the fundamental procedure of *stereographic projection*. Here, he projected a globe on a plane tangent to the North Pole, and showed how circles are projected into circles (going beyond Ptolemy who had proven it only in special cases).

Nemorarius influenced the rebirth of *mechanics* and his writings include the germs of concepts like *impetus*, *statical moment*, *principle of virtual displacements*, *angular velocity*, *mechanical advantage of a lever*, concepts which culminated hundreds of years later in the discoveries of Galileo, Newton, and J. Bernoulli (1717).

Little is known for certain about his life, and even that is controversial. Most historians believe, however, that Nemorarius was born in Westphalia, joined the Dominican order in Paris (1220) and died at sea on the homeward journey from the Holy Land.

1220–1235 CE **Robert Grosseteste** (1168–1253, England). Ecclesiastical statesman, bishop and one of the foremost mathematician and physicist of his age. In these fields of thought he anticipated some of the most striking ideas to which Roger Bacon subsequently gave wider currency. Thus, he understood, far ahead of his time the principles and methods of empirical science. In 1220, influenced by the writings of Augustine and the scientific method of Islam, he presented a concept of an infinite isotropic Euclidean universe. In *De luce seu de inchoatione* and *De motu corporali et luce* he suggested that light was the first form of matter and propagated isotropically in all directions according to mathematical laws.

In *De generatione sonorum* he asserted that sound was a vibratory motion, propagating through the air from the source to the receiving ear. In his

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79 The word *zero* probably comes from the Latinized form of *zephirum* of the Arabic *sifr*, which in turn is a translation of the Hindu *sunya*, meaning “void” or “empty”. On the other hand, the Arabic *sifr* could have originated from the Hebrew verb *safor* meaning “to count”.

commentaries on Aristotle's *Physics* he presented views on proper scientific methodology, encouraging the search for general principles, the testing of hypotheses against observations and the use of mathematics.

Grosseteste was greatly influenced by the Latin translation of Alhazen's writings on optics, and in his own work on optics he suggested that *rainbows* are caused by the refraction of sunlight inside clouds. He made Latin translations of many Greek and Arabic scientific writings.

He was born at Stradbrook in Suffolk to humble parents and received his education at Oxford (1199–1209) and Paris (ca 1210), where he became proficient in law, medicine, theology and the natural sciences. He settled in Oxford as a teacher and was pre-eminent among his contemporaries for his knowledge in the natural sciences. He was chancellor of Oxford University from 1215 to 1221. In 1235, after a severe illness, he relinquished his teaching and research career and became the bishop of Lincoln (for this he is also known as **Robert of Lincoln**). In this capacity he was deeply involved in the struggle to maintain the independence of the English clergy against the alliance of King Henry III and pope Innocent IV.

Robert Grosseteste was born at the decisive moment when Greek and Arabic science became accessible in Latin versions. Due to his unusual interest in the study of languages and in the sources of knowledge, he became interested in science and the scientific method; and he paid particular attention to optics, which he regarded as the key to an understanding of the physical world. He was conscious of the dual approach to knowledge by means of induction and deduction. This approach to science was not far removed from that of Aristotle, but Robert gave it a clarity and sharpness that allowed him and his 14^{th} century successors to make the first moves towards the creation of modern experimental science.

It is certain that the Bishop of Lincoln placed an emphasis upon experimentation which has been regarded as characteristic only of later periods. Indeed, **Roger Bacon**, his most famous pupil, said of him that he neglected the books of Aristotle for his own experiments, and with the aid of other authors and scientists, treated *independently* scientific questions (e.g., the study of the *rainbow*) which had occupied Aristotle.

1220–1250 CE **Vincent of Beauvais** or VINCENTIUS BELLOVACENSIS (ca 1190–1264, France). Dominican friar and encyclopedist. His *Speculum Majus* is the greatest encyclopedia written before the 18^{th} century.

Vincent entered the Dominican order at Beauvais (ca 1217) and held the post of "reader" at the monastery of Royaumont near Paris (ca from 1250). He was also chaplain to the court of Louis IX of France.

He labored on his encyclopedia during 1220–1244. It contains three parts: *Speculum Naturale* – a vast summary of all the natural history known to Western Europe towards the middle of the 13th century; *Speculum Doctrinare* – a summary of all scholastic knowledge of the age; *Speculum Historiale* – the history of Christianity and the evolution of Christian theology.

Vincent undertook a systematic and comprehensive treatment of all branches of human knowledge of his time. In the preparation of this colossal work he was helped in the purchase of books by his royal patron Louis IX.

1225–1235 CE **Michael Scot** (ca 1175–1235, Scotland, Spain and Italy). Translator, mathematician and magician. Studied at Oxford and Paris. Devoted himself to mathematics, philosophy and theology. Ordained as a priest. Went from Paris to Bologna, Palermo and Toledo (1217). There he acquired a knowledge of Arabic. This opened to him the Arabic version of Aristotle. He was one of the savants in the Napolitan court of Frederick II and at the instigation of the latter translated Aristotle into Latin.

His own books on astrology, alchemy and the occult are mainly responsible for his popular reputation. [He appears in the *Inferno* of Dante, *Canto* **20**, 115–117, among the magicians and soothsayers.]

1229–1262 CE **Al-Hasan al-Marrakushi** (Morocco). Mathematician and astronomer. His book, the *Jami* (1229), was the most elaborate trigonometrical treatise of his age, the best medieval treatise on practical astronomy (instruments and measuring methods), on gnomonics, and the best explanation of geographical methods.

1231–1256 CE **Yaacov (ben Abba Mari) Anatoli** (1194–1256, France and Italy). Translator, philosopher and physician. His translations from Arabic into Hebrew of the works of Aristotle, Averroes and Ptolemy contributed to the evolution of the ideas of the Renaissance.

A pupil and son in law of Shmuel Ibn Tibbon. He was invited to Naples (1231) by the enlightened ruler Frederick II (Emperor, 1212–1250) to serve as a physician, and under this royal patronage and in association with **Michael Scot**, made Arabic learning accessible to Western readers.

ca 1240–1250 CE **Dogen** (1200–1253, Japan). Philosopher. The most famous exponent of *Zen* Buddhism in Japan. His work *Shobogenzo* is the first Buddhist text written in Japanese. It is said to be the first truly philosophical work in Japan.

1240–1242 CE *The Talmud on trial* – the Paris disputation[80] (June 12, 1240) and the first public burning of the Talmud (June 6, 1242) under the Bull of Pope Gregory IX.

On a certain summer Saturday, while the Jews were in the synagogue, police surrounded their homes and carried off what books they could find. Then a trial was arranged in Paris: the prisoner was a book, the *Talmud*. The prosecution was led by a renegade Jew, Nicholas Donin; the defense by a group of rabbis led by **Yehiel of Paris** (ca 1192–1268). The judges were a number of bishops; and the queen-mother (of Louis IX) presided. Donin was given every opportunity to poke fun at the Talmud by pointing out the naive tales and legends which it contained, and to twist a number of its statements into attacks upon Christianity. Rabbi Yehiel did not have the right to point out equally naive ideas to be found in Christian literature. The judges being what they were – the cause of the defense was lost from the start.

Consequently, the Talmud was condemned to be an evil, a dangerous book to Christianity, and ordered to be burnt in a great public ceremony arranged by the Dominicans. In June 6, 1242, twenty-four cartloads of copies were burnt in the public square in Paris. To leave no loophole for the Jews, Donin persuaded pope Gregory IX to issue a Bull, a first of its kind, for the burning of the Talmud *everywhere*, and to establish inquisitions and censors over other Jewish writings, a practice which tormented the Jews for centuries.

Yehiel and his pupils came to Israel (1259) and established an academy at Akko.

1241 CE The Mongols withdrew from central Europe following the death of Ogadai Khan.

1245 CE **Yehudah ben Shlomo ha-Kohen**, **Ibn Matka**. Translator and encyclopedist. Wrote an encyclopedic treatise in Arabic including such subjects as astronomy, logic, mathematics, metaphysics, philosophy and natural history. His sources were Aristotle, Ptolemy and Euclid. It was later translated into Latin under the title *Inquisito sapientiae*. He was born in Toledo, Spain. Corresponded with Duke Frederick II of Babenberg, who invited him to Toscana, Italy, to participate in his translation project.

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[80] Characteristic of the age, the disputations were called by the medieval Church "*Tournaments for God and Faith*". But there was little of the chivalrous element in these so-called "tournaments", the position of the combatants being so flagrantly unequal. Almost always the verdict was against the Jewish debaters, with the direct consequences to the practice of their religion, to themselves and to their fellow-Jews.

Table 2.3: TRANS-ASIATIC JOURNEYS (1245–1307)

Within half a century the innumerable risks of such journeys were faced and successfully overcome by the following 14 persons (not counting the many less known or anonymous ones who traveled in their wake independently):

- Giovanni Pian del Carpine (1245–1247)
- Hayton the Elder and his brother Sempad (1251–1254)
- William of Rubruquis (1253–1255)
- Niccolo, Maffeo and Marco Polo (1272–1295)
- Giovanni da Montecorvino (1272–1307)
- Bar Sauma (1280–1287)
- Ricoldo di Monte Croce (1288–1301)
- Buscarello de Ghizolfi
- Al-Juwaini
- Chang Te
- Yeh-lu Hsi-liang

1245–1247 CE **Giovanni Pian del Carpine** (1182–1252, Italy). The first of the great travelers from Europe to Asia. A Franciscan Friar who crossed Asia to contact the Mongols on behalf of Pope Innocent IV (in line with a policy of opening diplomatic intercourse with a power whose alliance might be invaluable against Islam).

In his forward journey, the Friar crossed a distance of some 5000 km (Lyons–Karakorum) in 463 days [April 16, 1245–July 22, 1246]. He left Lyons, traveling by a northern route through Bohemia, Poland and the Ukraine. In November 1247, Carpine delivered the Great Khan's reply to the Pope. It was, to say the least, discouraging:

... "*You must come yourself at the head of all your kings and prove to Us your fealty and allegiance, And if you disregard the command of God and disobey Our instructions, We shall look up on you as Our enemy. Whoever recognizes and submits to the Son of God and the Lord of the World, the Great Khan, will be saved. Whoever refuses submission will be wiped out*".

Though not yielding an immediate panacea for the medieval world's ills, Carpine's trail-blazing journey – undertaken when the Friar had already turned 60 years of age – had opened up the first real dialogue with the East. Moreover, Carpine was an astute observer, and his account of his travels, *Historia Mongolorum*, furnished Europe with the first insightful glimpse into Eastern culture.

1245–1260 CE **Albertus Magnus** (1206–1280, Germany). Naturalist and philosopher, known as the 'Doctor Universalis'. One of the most learned men in the Middle Ages, and the first serious naturalist since the Plinys who taught men to think again. He knew neither Greek nor Arabic, and his only sources were Latin translations. He wrote a series of encyclopedic treatises, presenting the entire body of knowledge: natural science, logic, rhetoric, mathematics, ethics, economics, politics and metaphysics. His influence was great, but did not last long because his works were too eclectic and too superficial. Nevertheless, his writings remained for centuries a treasury of information. He had the courage to reject *some* superstitious ideas (but he continued to believe in many others). He advocated that a visible object alters the medium between the object and the eye, and this alteration is propagated to the eye. He studied insects, whales, and polar bears, and gave a fairly complete description of German mammals and birds. He also made original contributions in biology, zoology, chemistry and geology.

Albertus excelled all his contemporaries in the width of his learning. In his scientific works he was a faithful follower of **Aristotle** as presented by Jewish, Arabian and Western commentators; in voluminous commentaries on the writings of Aristotle he comprehensively documented 13^{th}- century European knowledge of the natural sciences, mathematics and philosophy. He also engaged in alchemy, although his works express doubts about possibility of transmutation of the elements, and gave a detailed description of the element arsenic.

He was born Albrecht von Bollstädt in Lauingen, Swabia, Germany; studied in Padua and joined the Dominican order in 1223. He taught theology and philosophy in various schools, especially is Paris, until 1254, and was bishop of Ratisbon during 1260–1262. He died at Cologne.

1246–1286 CE **Meir of Rothenburg** ("MAHARAM", 1215–1293, Germany). Talmudist, jurist and poet. Foremost Rabbi of Western Germany and greatest German legalist in the 13^{th} century. A religious leader who exercised much influence on subsequent development of Judaism. He wrote glosses to the Talmud (*tosaphot*) and many *Responses* of the utmost value for historical research.

Maharam was born in Worms. He studied in Wuerzburg, Mainz, and France. He witnessed the famous *Talmud Disputations* of 1240 and saw the burning of the 24 cartload of parchments in 1242. He wrote an elegy about their destruction which is still used in Jewish Ashkenazic communities.

After the Talmud-burnings, he returned to Germany and settled in Rothenburg. Although only in his early thirty's, he was already recognized as a leading Talmudic authority. Communities began to send him legal questions. He wrote almost a thousand responsa, thus having a tremendous influence on shaping custom and lifestyle. His modification of synagogue and home rituals became the accepted forms for the Ashkenazic world.

The Maharam lived in Rothenburg for forty years, thus gaining the title "Meir of Rothenburg". He wrote commentaries and discussions on 18 tractates of the Talmud.

In 1286 he left Germany on account of severe persecution by Rudolf I of Habsburg (1218–1291), King of Germany and emperor of the Holy Roman Empire. He was arrested[81] in Italy, returned to Germany and then thrown into prison in Alsace, where he eventually died.

[81] The reason given was that, as a Jew, the Rabbi was a serf of the emperor's treasury; by leaving, he was depriving the treasury of a source of income: Rudolf demanded a tremendous sum as ransom, which the Jews were ready to pay. But the Rabbi declined the deal, fearing that the precedent would lead to extortion in other cases. After his death, the emperor refused to release the body for Jewish burial unless the ransom is paid. Finally, after 14 years a rich man named **Alexander Suesskind Wimpfen** paid the ransom in full on the condition that, upon his death, he would be buried by the Rabbi's side. Maharam was laid to rest on Feb. 7, 1307 and Wimpfen is buried by his side.

1247–1259 CE **Ch'in Chiu-Shao** (ca 1202–1261). Chinese mathematician. He wrote *Mathematical treatise in Nine Sections* (1247). It contains simultaneous integer congruences, the Chinese Remainder Theorem, algebraic equations, areas of geometrical figures and linear simultaneous equations. Developed methods of successive approximations to solve numerical polynomial equations of any degree, a method not discovered in Europe until the 19^{th} century. Ch'in invented a method of solving these numerical equations which is substantially identical with the Ruffini-Horner procedure[82]. The unknown quantity in these equations was represented by a monad and the zero by a little circle, like ours (the Chinese zero may have come directly from India with Buddhism or it may have been imported later by Muslims). Red and black ink were used respectively to represent positive and negative numbers. A contemporary mathematician, **Li Yeh** (1178–1265), wrote treatises in 1248 and 1259 involving problems on quadrilaterals and circles, with their solutions. Instead of using red and black colors to designate positive and negative numbers, Li differentiated the latter by drawing diagonal strokes across them.

1250 CE **Johannes Campanus of Novara** (d. 1296, Italy). Mathematician and astronomer. A chaplain of Pope Urban IV. Translated Euclid's *Elements* from Arabic into Latin.

ca 1250–1272 CE **Nasir al-Din al-Tusi** (1201–1274, Persia). Mathematician, astronomer, physicist, physician and philosopher. One of the greatest mathematicians of medieval times.

In geometry, he made first attempts to determine whether Euclid's parallels postulate can be derived from the other Euclidean postulates. In trigonometry he presented the law of sines and began to separate trigonometry from astronomy. His work in these two areas may have influenced the further advances of Regiomontanus (1464) and Saccheri (1733). In 1259 al-Tusi began the construction of a major astronomical observatory at Marãgha[83], where

[82] Thus, to solve the equation $x^2 + 252x - 5292 = 0$, Ch'in first established that there is a root between 19 and 20. He then used the transformation $y = x - 19$ to obtain the equation $y^2 + 290y - 143 = 0$, with a root between 0 and 1. His final approximate solution is $x = 19 + \frac{143}{1+290}$. Similarly, to solve the equation $x^3 - 574 = 0$ he set $y = x - 8$ to obtain $y^3 + 24y^2 + 192y - 62 = 0$, yielding as an approximate root $x = 8 + \frac{62}{1+24+192} = 8 + \frac{2}{7}$. Indeed, '*Horner's method*' must have been well known in medieval China; it was used by several Sung mathematicians for the numerical solution of cubic and even quartic equations.

[83] The means to construct the observatory were given to him by Hulagu. It was an institute for scientific research, with a rich library. Under al-Tusi's direction, Marãgha became the outstanding astronomical center of the time and one of the leading scientific centers of the world. Mathematicians, astronomers and

he used his self-made quadrants to observe star positions. During 1256–1265 he conducted observation which served as a basis for his astronomical tables (1272). His criticism of Ptolemaic astronomy was an additional step toward the Copernican reform.

Al-Tusi's discussion of the 5^{th} postulate of Euclid was taken up later by **Girolimo Saccheri** (1733). In other words, the history of non-Euclidean geometry can be traced back through Saccheri to one of Nasir al-Din's writings. His independent textbook of trigonometry, plane and spherical, can be considered the climax of a long Greek-Hindu-Arabic tradition. Some spherical problems were solved by implicit considerations of polar triangles. This treatise was almost equivalent to the Latin treatise composed by Regiomontanus two centuries later.

Al-Tusi was born in Khurasan. Under Mongol occupation (1256) he became astrologer to the Mongol chief, Hulagu Khan, and went with the Khan when the latter sacked Baghdad in 1258. He resided in Marãgha from 1259 until almost the end of his life. In 1274 he went to Baghdad and died there.

The work of al-Tusi was continued by his pupil, the astronomer and physician **Qutb al-Din al-Shirazi** (1236–1311, Persia). In 1281 the latter suggested an alternative planetary model to that of Ptolemy, making more use of uniform circular motions. He was the first to give a satisfactory qualitative account of the *rainbow* (excepting, of course, the colors). He explained it by the study of passage of a ray of light through a transparent sphere (drop of water): the ray is reflected once and refracted twice. This explanation is essentially similar to that of Descartes. Further details on al-Shirazi's correct theory are found in a comment by his student **Kamal al-Din al-Farisi** (ca 1260–ca 1320; Tabriz), on the *Optics* of Alhazen. Both, however, lacked the quintessence of physical theory – a *quantitative explanation*.

Al-Shirazi was born in Shiraz. He belonged to a family of learned men, and received part of his medical training from his father and uncles. He traveled extensively in the Middle East, partly in the service of the Khans of Persia. He finally settled down in Tabriz and died there.

1251–1254 CE **Hayton the Elder** (d. 1271, Armenia). Trans-Asian traveler, King of Little Armenia. With his brother Sempad, made a journey to Mongol capital Karakorum, the record of which by one of his courtiers was one of the earliest written accounts of Mongolian geography and ethnology.

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instrument makers gathered there from both near and distant countries (Caucasus, Morocco, China). The main task of the Marãgha astronomers was the compilation of new tables. Unfortunately this center did not long survive its founder.

1252–1273 CE **Thomas Aquinas** (1225–1274, Italy). Theologist. One of the first Christian scholars to recognize the '*light of reason*' as an independent source of knowledge. Aquinas was one of the leading scholars of Aristotelian physics of his day and was responsible for the general acceptance of Aristotelian physics[84] throughout Europe. In his attempt to reconcile religion and science, Aquinas set himself the task of harmonizing 631 questions between Christian and classical science. He made some efforts to appeal to common sense and the natural word but in most of his cases the desired answer was simply assumed: Faith always got priority over Reason.

By using Aristotle as a mental catalyst, he and **Albertus Magnus** contended that there is no conflict between faith and reason. In this sense he followed Maimonides and Ibn-Rushd, and did for Christianity what the latter had done for Judaism and Islam respectively, a century before him. He wrote 21 volumes of mental gymnastics to defend the doctrines of the Catholic Church. These are still studied today in Catholic seminaries.

Aquinas was born near Aquino, Italy, in the province of Naples. He attended the University of Naples in 1239 and then joined the Dominican Order. His family opposed the idea so strongly that his brothers seized him as he was traveling to France, and imprisoned him for a year. After he was released, he left for Paris in 1245 to study under Albertus Magnus, and then followed the latter to Cologne. He ended his studies in 1259 and taught at the papal court from 1259 to 1268. He went to Italy in 1272 to organize the Dominican school of theology in Naples, and died on his way to Rome.

1253–1255 CE **Rubruquis** (Willem van Ruysbroeck, Flanders). Flemish Franciscan Friar and traveler to the East before Marco Polo. He was sent by Louis IX of France to Mongolia as an unofficial representative in 1253. He left Constantinople, sailed to the Crimea, crossed South Russia and a large part of Asia, and finally reached the Mongolian capital Karakorum in April 1254. He left the capital in August, and reached Tripoli, Syria, a year later. He was the first to describe the Caspian as an inland sea. His travel account was praised and used by **Roger Bacon**.

1256–1312 CE **Yaacov ben Machir Ibn Tibbon** (1236–1312, Provence). Also known as **Prophatius**. Astronomer, physician and translator. A distinguish medical man who worked in the medical faculty of the University of Montpellier. One of the most famous astronomers of medieval times. His works, translated into Latin, were quoted by Copernicus and Kepler. He compiled new astronomical tables for the longitude of Montpellier and the year beginning on March 1, 1300.

[84] Although Aristotelian physics was wrong, it was an essential precursor of modern physics.

Invented a new instrument, the *quadrans novus*, meant to be of the same service as the astrolabe. Yaacov was one of the greatest translators of scientific works from Arabic into Hebrew (and Latin). Among his translations: Euclid's *Elements* and Ptolemy's *Almagest*.

He was born in Marseilles, studied at Lunel, and flourished at Montpellier, where he died.

The Jewish family of **Ibn Tibbon** comprised at least four generations of authors and translators, instrumental in the infusion of Arabic learning into Europe. The head of this clan was **Yehudah Ibn Tibbon** (1120–1190); born in Granada, Spain and immigrated to Lunel, in Southern France, where he practiced medicine and became a famous physician in the King's court. Through his translations from Arabic into Hebrew, he saved the books of the great Jewish thinkers from oblivion, and through this also enlarged the vocabulary of the Hebrew language to include scientific concepts and terms.

His son **Shmuel Ibn Tibbon** (1150–1232), physician and philosophic writer, was born in Lunel and died in Marseilles. He received a thorough education in Arabic and Jewish literature, and in all the secular knowledge of his age. Later he lived in several cities of Southern France and traveled to Spain and Alexandria (1210–1213). He translated Aristotle, Ibn Rushd and Maimonides into Hebrew. His son **Moshe Ibn Tibbon** (ca 1200–1283), born in Marseilles, was a physician and a prolific translator of Arabic works on philosophy, mathematics, astronomy, and medicine. His brother was the father of **Yaacov Ibn Tibbon**. Finally, Moshe's two sons, **Yehudah** and **Shmuel Ibn Tibbon**, were also famous translators.

ca 1260 CE *The invention of gunpowder*[85]. Probably made by alchemists in Western Europe, but it is impossible to say where or by whom.

[85] Ordinary gunpowder is an explosive mixture of finely powdered *salpeter* (potassium-nitrate, KNO_3, also known as nitre or neter), *sulphur* and wooden *charcoal*. The proportions of the constituents (by weight) and the main products of the combustion correspond roughly with the equation $2KNO_3+S+3C=K_2S+N_2+3CO_2$, which by atomic weights yields the ratio $202:32:36 \approx 6:1:1$. Other gaseous products include CO_2, H_2S, CH_4, H_2, and the residue contains potassium carbonate (K_2CO_3) and sulphate (K_2SO_4). The reaction is ignited at about 300°C and creates a heat of explosion of ca 750 cal/g, together with ca 250 cc/g permanent gases, at 0° and 760 mm. The temperature of the explosion reaches 2200°C, or even higher. Under these conditions, a *strongly confined gunpowder* produces a shattering effect.

Of course, sulphur and charcoal have been known from time immemorial. The 'invention' of gunpowder thus essentially implies a *knowledge of pure salpeter and of the proper mixture*. The word salpeter = sal petrae, or its equivalent, niter, has been taken to mean many different things. By niter (nitrum, natron), Fuming, fiery and flaming composition of many kinds ("wild fire") were known and used before the introduction of gunpowder. These consisted of mixtures of such ingredients as charcoal, sulphur, resins, fats, and natural petroleum, pitches and bitumens. The celebrated *Greek fire*, used in the Byzantine period, was probably a mixture of this kind. It is easy to imagine that enterprizing alchemists would not fail to try the effect of adding familiar mineral materials to these mixtures, and so eventually the particular virtue of *salpeter* would be recognized.

Recipes for pyrotechnics and explosives are found in the works of several men: (1) *Liber ignium* (ca 1300) by **Marc the Greek** (Marcus Graecius, unknown author of a collection of recipes who flourished during 1260–1300); (2) *Kitab al-furusiya* by **al-Rammah** (ca 1260–1295); (3) *Opus tertium* by **Roger Bacon**[86], composed 1266–1267. Therein he describes inflammable mixtures, such as were used for fireworks, and an explosive one which was probably gunpowder. It is clear from all these sources that the said authors were not the inventors, although it is evident that gunpowder was discovered within Bacon's lifetime.

On the other hand, there is no evidence at all that the Chinese discovered gunpowder (let alone firearms). [They did however use pyrotechnic devices comparable to the Greek fire in the battles of 1161 and 1162, and again at the battle of 1232 against the Mongols.] The discovery of gunpowder depends on the isolation and purification of potassium nitrate. It was necessary to distinguish that kind of saltpeter from others which were of no use for this special purpose, and to sufficiently purify it from the impurities which compromised its usefulness.

In the light of this situation it is very probable that gunpowder was actually made, in diverse places, before 1300 CE, at the end of a *process of slow and gradual evolution*.

It should be remembered that the great invention which revolutionized the world was not, after all, that of saltpeter, nor even gunpowder, but the

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the ancients meant any alkaline salt, such as potash (KNO_3), soda ($NaNO_3$) or sodium carbonate ($NaCO_3$) [used by the early Egyptians for embalming, cleansing, curing meat, smelting ores, and as medicine]. The Hebrew word for neter occurs in **Proverbes** *25*, 30 and **Jeremiah** *2*, 22.

86 There is no reason for ascribing the invention of gunpowder to him. His epistola is apocryphal, and the cipher supposed to contain the recipe of gunpowder, has no reference whatsoever to a known manuscript. He may have known something of gunpowder, and he was acquainted with various inflammable and pyrotechnic materials; but there is no reason for believing that he knew the explosive properties of gunpowder, nor that he purified and isolated saltpeter.

application of gunpowder to the propulsion of missiles. When gunpowder was invented in ca 1280, nobody understood as yet the implication of that invention.

1261–1275 CE **Yang Hui** (ca 1238–1296, China). Mathematician. Wrote books which use *decimal fractions* (in the modern form) and gave the first account on *Pascal's triangle*.

1263–1270 CE **Moshe ben Nahman**; **Nahmanides**; RAMBAN (1194–1270, Spain and Israel). Talmudist, philosopher, physician, Kabbalist, and one of the few biblical commentators that have withstood time's test.

In his *Commentary on Genesis* (1267), Nahmanides renders an amazingly prescient description of the early moments of the universe, which almost reads like a quotation from Steven Weinberg's *The First Three Minutes* (1977). Translated from the Hebrew text, it reads:

"*At the briefest instant following creation all the matter of the universe was concentrated in a very small place, no larger than a grain of mustard. The matter at this time was so thin, so intangible, that it did not have real substance. It did have, however, a potential to gain substance and form and to become tangible matter.*

From the initial concentration of this intangible substance in its minute location, the substance expanded, expanding the universe as it did. As the expansion progressed, a change in the substance occurred. This initial thin incorporeal substance took on the tangible aspects of matter as we know it. From this initial act of creation, from this ethereally thin pseudosubstance, everything that has existed, or will ever exist, was, is, and will be formed".

Commenting further on the first chapter of *Genesis*, Nahmanides reached the conclusion that prior to the creation of the universe, neither time nor space existed; the creation of the universe brought with it not only the time in which it flows, but also the space into which it expands. Thus, Nahmanides and cosmologists discuss the events of the Big Bang in uncannily similar terms.

Like other biblical scholars before him, Nahmanides had a well-developed understanding of the phenomena that produced the day-night cycle on earth. He summarized his knowledge: "*On the earth both evening and morning are always present. There are on earth at every moment ever changing places where it is morning and in the places opposite them it is evening*".

This reveals quite an exact comprehension of the illumination of the sun's light on a spherical earth, at time in the history of mankind when most of Western humanity dreaded falling off the edge of a flat earth.

He was born in Genova, Aragon, and became chief rabbi of Catalonia. In 1263, he was obliged by James of Aragon (king from 1213 to 1276) to discuss publicly at Barcelona the respective merits of Christianity and Judaism with the apostate and renegade Pablo Christiani, a Dominican. The discussion lasted four days and turned in Moshe's favor, but soon afterwards the Dominicians caused him to be exiled from Aragon, and the Talmud to be censored[87].

After spending a few years in Castile and Southern France, he moved in 1266 to the Holy Land; he lived in Jerusalem and Acre (Akko) and is buried at Haifa.

1266 CE **Theodoric Borogoni of Lucca** advocated the use of narcotic-soaked sponges to put surgical subjects to sleep.

1266–1278 CE **Roger Bacon** (1214–1292, England). Philosopher and scientist. One of the founders of the experimental method and a leading figure in the development of science during the Middle Ages. His writings and experiments helped lay the foundations for the scientific revolution that occurred in Europe during the 1500's and 1600's.

Advocated direct observations, some experimentation and the application of mathematics to the various sciences ("*Mathematics is the door and key to the sciences*"). In that he paved the way to the emancipation of men's intellect from total reliance on the search for regularity and perfection in the material universe as advocated by the Pythagorean school and by Plato. He asserted that the temptation to search for patterns of regularity in nature must be balanced by the equally powerful inclination of the human mind to seek simplicity and mathematical definiteness.

Bacon attached to *logic* only a subordinate importance. For him the main avenue to knowledge was not logic, but linguistic and mathematical ability. Logical ability he thought to be almost instinctive, but languages and mathematics had to be learned. He was not an original mathematician, and his knowledge of mathematics was very limited. But he was imbued with the Platonic idea of the transcendental importance of mathematics, and he helped to spread that idea. He was equally convinced of the practical utility of mathematics in almost every study, and explained it at great length.

His most important mathematical contribution is the application of geometry to *optics*. He performed experiments with mirrors and lenses, chiefly burning lenses, and foresaw vaguely both the compound microscope and the

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- Maccobi, H. *Judaism on Trial: Jewish – Christian Disputations in the Middle Ages*, London, 1993.

telescope. He had some understanding of spherical aberration and of its correction by the use of paraboloidal and hyperboloidal surfaces. In opposition to **Grosseteste** he claimed that the passage of light through a medium cannot be instantaneous. *Gunpowder* was discovered probably within Bacon's lifetime, but there is nothing to indicate that he was the inventor. He had great interest in *alchemy*, although he made a distinction between practical and speculative alchemy.

Bacon explained to the Pope the importance of undertaking a complete and accurate survey of the world. He indicated the possibility of reaching the Indies by sailing westward from Spain, and assumed that these countries were considerably nearer than they in fact are, as did Columbus later. Bacon made few experiments himself, and in a sense he never understood the experimental method as we now understand it. Like his countryman **Francis Bacon**, more than three centuries later, he laid down many percepts which he never followed himself. To be sure, some of the experiments proposed by him were fantastic, and in spite of his own efforts to separate experimentation from magic, he did not always succeed. However, in his time, even to speak of the superior value and indispensability of experimentation was an immense achievement; the essential soundness of his experimental point of view is proved by his readiness to sacrifice theories to facts and to confess his own ignorance.

Bacon realized the supreme importance of the texts written in Greek, Hebrew and Arabic, and insisted upon the necessity of studying these languages. He attached great importance to the study of the Bible in its original form, and to this end he advocated the study of Hebrew and Greek. He believed that all knowledge is explicitly or implicitly set forth in the Bible.

Bacon was born at Ilchester, Somerset. He studied in Oxford under Grosseteste. Before 1236 he went to Paris, and later seemed to have traveled to Italy.

Bacon returned to Paris, lectured there as a regent master of the university, and returned to Oxford in 1251. Sometime between 1251 and 1257 he joined the Franciscan Order, but soon got into difficulties with his superiors in matters of censorship of his writings. Happily for him, he was ordered (1266) by Pope Clement IV to send him copies of his works, as secretly as possible. This papal request, made over the head of Bacon's superiors, made him anathema to them. After the pope's death (1278) Bacon was condemned for teaching "suspected novelties" and imprisoned from 1278 to 1292. He died soon afterwards.

1268–1286 CE **William of Moerbeke** (ca 1215–1286, Greece and Italy). Scholar, Orientalist, Philosopher and translator of scientific and philosophical texts from the Greek into Latin. One of the most distinguished men of letters of the 13^{th} century. Held intellectual intercourse with **Aquinas**, **Witelo** and

Campanus. His translations were influential in his day, when few competing translations were available, and are still respected by modern scholars.

He translated the works of **Aristotle**, **Hero of Alexandria**, **Archimedes** and **Proclos**.

ca 1269 CE **Peter Peregrinus** (the Stranger) **of Maricourt**. French physicist, perhaps one of the greatest of medieval times. Discovered the fundamental properties of magnets: the two kind of poles, their attractions and repulsions, magnetization by contact, breaking of a magnetic needle into smaller ones, and the exertion of magnetic force through water and glass. He even traced the lines of force on a loadstone from one pole to another. One can find in his writings a vague suggestion of terrestrial magnetism.

He described floating and pivot compasses, provided with a fiducial line and a circle divided into 360 degrees.

Peregrinus was in the army of Crusaders which besieged Lucera in 1269. He was a teacher of Roger Bacon, who payed him the highest tributes in the *Opus tertium* and in the *Opus maius*. According to Bacon, Peregrinus was a recluse who devoted himself to the study of nature, and was able to work metals.

1269, June 19 Louis IX of France decreed that all Jews must wear a *yellow badge*. The Jews of Europe continued to wear the badge for the next 529 years; on Feb 15, 1798, the Jews of Rome were allowed to remove the badge. However, in 1935, the Germans renewed the practice of the badge, branding their victims prior to their final extermination.

1270–1290 CE **Zerahia (ben Itzhak ben Shaaltiel) Chen of Barcelona** (ca 1230–1292, Spain and Italy). Physician, philosopher and translator. Translated Greek medical and philosophical writings from Arabic into Hebrew, especially the writings of Aristotle. Lived in Barcelona during 1277–1290, where most of his translations work was done.

1271–1295 CE **Marco Polo** (1254–1324, Italy). The greatest medieval traveler, who traced a route across the whole longitude of Asia, naming and describing kingdom after kingdom which he had seen with his own eyes. The first traveler to reveal China in all its wealth and vastness, and tell the West about Tibet, Burma, Laos, Siam, Japan, Sumatra, Nicobar and Andaman, India, Ceylon, Abyssinia, Zanzibar, Madagascar, Siberia and the Arctic Ocean.

In 1271, Marco Polo (then 17) and his father and uncle[88] sailed from Venice to Acre (now Akko, Israel). From there they rode camels to the Persian port

[88] Nicolo (Marco's father) and his brother Maffeo Polo, Venetian Jewel traders in the Black Sea, embarked in 1254 on a travel to Central Asia. After spending

of Hormuz, and from there on camel back across the deserts and mountains of Asia. More than three years after leaving Venice, they reached Kublai Khan's summer palace in Shang-tu. During 1274–1292 the Polos stayed in the Chinese empire. Young Marco served the Kahn on many official tours of the kingdom, taking detailed notes during his travels. On their way back, the Polos sailed from Southern China (Zaitun) in the South China Sea to what is now Singapore then around the southern tip of India. They crossed the Arabian Sea and the Gulf of Oman to Hormuz. From there they traveled overland to the Turkish port of Trebizond and over the Black Sea to Constantinople.

The Polos returned to Venice in 1295 after covering a total distance of 24,100 km. In 1296, the Genoese captured and jailed Marco Polo for a year. In prison he met a popular writer, Rustichello of Pisa and dictated to him his story. Polo told about Kublai Khan's prosperous, advanced empire. He described the Khan's *communication system*, which consisted of a network of some 10,000 courier stations throughout the Kingdom; horseback riders relayed messages from one station to another, covering some 700 km per day.

Polo commented on many Chinese customs, such as mining and the use of coal as fuel (coal had not yet been used in Europe). He also marveled at the Chinese use of paper money, which bore the seal of emperor. Europeans still traded with heavy coins made of copper, gold or lead.

Marco Polo's book *Description of the World* was completed in 1298. The book may have helped bring to Europe such Chinese inventions as the *compass*, *papermaking*, *paper money, printing* and *explosives* (which according to Polo were in use in China already in 1237).

Printing had not yet been invented in Europe, and so scholars copied Polo's book by hand. It soon became the most widely read book in Europe. Historians believe it may have influenced many explorers including Christopher Columbus. Yet, in Polo's time and through the century that followed, people did believe most of his stories.

It is told that when asked, on his deathbed, whether his reports were true or not, Marko Polo replied: "I have not told you even half of what I saw".

ca 1272 CE **Alfonso X, El Sabio**; '**the learned**' (reigned 1252–1284, Spain), ordered the compilation of new astronomical tables to replace the

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three years in Bokhara they proceeded to China, meeting Kublai Khan in 1266 in Peking. They reached Acre in 1269, bearing letters to the Pope from the Mongol ruler, only to find out that Clement IV had died the previous year and that the Cardinals had not yet elected a successor. And so they returned to Venice, where Nicolo's wife had died in his absence, leaving a son – Marco, who was then 15 years old.

Toledan tables which had been edited by al-Zarquli two centuries before in the same city. The so-called *Alphonsine Tables* were prepared in 1272 under Alfonso's direction, in Toledo, by **Yehudah ben Moshe ha-Kohen** (Jewish physician and astronomer), and **Itzhak ha-Hazzan**[89] (Jewish astronomer, constructor of instruments, and translator from Arabic into Spanish). The original tables are lost, but the introduction is extant, and thus we have a definite idea of the nature of these tables.

The scientific fame of Alfonso is based mainly on his encouragement of astronomy. He was brought up in an intellectual atmosphere which was impregnated with Muslim and Jewish influences. He completed the incorporation of the University of Salamanca in 1254, and gathered around him a number of Jewish and Christian scholars to continue the transmission of Muslim knowledge to Christendom and the translation of Arabic writings into Spanish. Within this framework, he ordered the translation into Castilian of the Quran, of Talmudic and Kabbalistic writings, and of a number of astronomical treatises. Thus Alfonso was one of the greatest intermediaries between Arabic and European knowledge (on the other hand it must be confessed that he showed but little ability as a king and statesman!).

The Alfonsine Tables became known in Paris only in 1292. In their Spanish form they could hardly exert any influence outside of the Iberian peninsula. They owed their immense popularity to the Latin versions, which were printed for the first time in 1483. They were superseded by the *Rudolphine Tables* computed by Kepler in 1627.

1275–1289 CE Severe drought depleted the agricultural resources of the corn-growing Anasazi Indians in Mesa Verde, Colorado (USA today), causing then to migrate far from their ancient cultural centers.

1272–1307 CE **Giovanni da Montecorvino** (1247–1328, Italy). Trans-Asian traveler and diplomat. Franciscan prelate. Missionary in Armenia and Persia (ca 1280). Papal emissary to Il-Khan of Persia (1289). Founded first Catholic mission in India and China; reached Peking (1294). First archbishop of Peking and patriarch of the Orient (1307).

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[89] He is also called in the Spanish documents **Itzhak Ibn Sid**, and was the main collaborator of King Alfonso. He flourished in Toledo ca 1263–1277, made observation of eclipses in 1263–1266, and invented or improved various astronomical instruments. He is said to have introduced two periods of 49,000 and 7,000 years respectively for the precession and the "*trepidation*" (a fictitious periodic variation in the precession of the equinox based on an error in Ptolemy's determination of the precession. It figured in astronomical tables until the time of Copernicus). One recognized here a Kabbalistic influence, in the form of sabbath and jubilee extended to astronomy.

1275–1305 CE **Raymond Lully** (Ramon Llull, 1235–1316 CE). Catalan scholar, linguist, poet, philosopher, alchemist and mysticist. Originated symbolic logic. His principle philosophical concern was with attempting to reconcile the seeming contraries of life, such as art and science, rationalism and mysticism, abstract philosophy and practical life.

His *Ars Generalis* (1275) intended to serve as a basis for all sciences and as a key to invention and discovery. This work was much admired, even several hundred years later by Giordano Bruno and Leibniz.

In his main work *Ars Magna*[90] (1305–1308) he set out his theosophical attempt to encompass all knowledge in a Neoplatonic schema; he broke away from the scholastic system, criticized *fraudulent* alchemy and made himself some contributions to chemistry[91] and mathematics. Because of his great learning he has often been called *Doctor Illuminatus*.

Lully was born and reared at Palma on the Spanish island of Majorca, where Christian civilization was in close contact with both Jewish and Arabic lore. He was the first Christian scholar to study the Kabbalah, which he regarded as a divine science and a true revelation of the rational soul. He was a great linguist and obtained (1311) the consent of the Council of Vienna for teachers of Hebrew and Arabic to be admitted to the papal schools and the great universities.

Lully traveled throughout Asia Minor and North America, attempting to convert Muslims. According to legend he was stoned to death at Bugia (Tunisia).

1275–1310 CE **Levi ben Avraham ben Hayim** (ca 1240–1315, France). Natural philosopher, astronomer and mathematician. Recognized heat as a form of motion four centuries before **Robert Boyle** (1675).

Levi was born in Villefranche-de-Confluent to an illustrious rabbinic family. He studied at Perpignan and Montpellier and was instructed in astronomy and mathematics by **Yaacov ibn Tibbon**. He was the grandfather of **Levi**

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[90] Lully, searching for the way to break through the stranglehold which Scholasticism had upon science, used the Kabbalah and the works of the Spanish Jew *Avraham ibn Latif* (1220-1290) as a basis for his book *Ars Magna*. Avraham merged Kabbalah, Aristotelianism, mathematics, and natural science into a unified system. His works were translated into Latin and caught the attention of Lully.

[91] He is credited with the discovery of *ammonia* and supported the importance of *sulfur*. His work makes him also as one of the precursors of *symbolic notation* and *combinatorics*.

ben Gershon. His book *Loyat Chen* (adornment of grace) is a comprehensive encyclopedia of the sciences known in his day.

Levi lived in poverty, seeking a living through the teaching of science and foreign languages. Persecuted by the rabbinic establishment because of his rationalistic interpretation of the Bible[92], he was forced to wander in Provene. He died in Arles.

Medieval Cosmology

In 1227 CE the Bishop of Paris, Etienne Tempier, issued 219 condemnations of all professors who dared to place limits on the power and scope of the supreme being.

Pierre Duhem (1906), the French historian of science showed that the condemnation stands as a landmark in the history of cosmology, and that the year 1277 could serve as a fiducial for the birth of modern science for the following reason:

In the Middle Ages, Arabs, Jews and then Christians, in their philosophical and theological studies, adopted the Aristotelian system of concentric celestial spheres. The Arabs created the primum mobile*, an outer sphere that transmitted motion to all other spheres and was itself driven by divine will. The high Middle Ages became an age of scribes translating Arab and Greek manuscripts. The new knowledge exceeded the scope of the monastery and cathedral schools, and communities of learned scholars founded the universities. Students flocked to these centers of learning, and the translated works of Aristotle, Euclid, Ptolemy, Galen, and other sages of the ancient world revealed vistas of knowledge that exalted the power of the human mind. The earth-centered universe of Ptolemy was especially congenial to the way medieval scholars thought. Once it was introduced to Europe in the 12th century*

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[92] Levi endeavored to render rational explanations to biblical miracles. e.g. he interpreted *Joshua* **10**, 12–14 as Joshue's *prayer* onto God to grant him enough strength to reach Gibeon before sunset and then arrive at the valley of Ayalon before moonset. The Biblical narrator then confirms that his prayer was indeed accepted.

(by way of translation from the Arabic texts of the ancient Greeks), it swept through the universities like wildfire.

But ecclesiastical authorities grew alarmed. The wholesale adoption of ancient beliefs about the nature of the universe threatened to transform Christian doctrine beyond recognition. Granted that the earth rests at the center of the universe, and granted that the heavens consist of rotating spheres; but to go further and assert that even God, if he so willed, could not move the earth, or could not create other worlds than the earth, as taught by professors in the school of art, controverted the tenets of sacred doctrine. Tempier's condemnations were aimed at the new Greek learning that was being taught in the academies. His main objection seemed to be that by talking about laws of nature, the science faculties were somehow limiting the power of God.

By proclaiming Aristotle's fallibility and by undermining his already besieged finite anthropocentric system*, the bishop's condemnation alerted scholars and divines to the need for a system more compatible with the idea of* omnipotent and omniscient supreme being*. Thus, scholars of the high and late Middle Ages were inspired to seek a more ample system to accommodate the works of a supreme being of greater power and extent.*

In any case, the Ptolemaic universe was quickly amalgamated with Christian thought. The Ptolemaic universe fitted the preexisting view of a moral universe in which man occupied a middle place, with hell beneath his feet and heaven above. The spheres of the stars and planets were thus between man and heaven. Volcanoes provided glimpses into the underworld, and the blue of the daytime sky was a reflection of the glory of heaven. Demons walked at night, when the heavenly glow was blocked by shadows – further proof of the validity of this cosmology. Thomas Aquinas' doctrine of the right of scientific reason to operate according to its own rules within the larger framework of the Christian faith only strengthened the idea of the earth-centered cosmology; it went hand in hand with the Christian faith. The medieval universe, then, combined the best of both worlds: the rational, phenomenon-oriented astronomy of the Greeks and the secure and emotionally satisfying spiritual interpretation of life offered from the earliest times by mythology. No wonder church and secular authorities of the late Renaissance were so reluctant to abandon this synthesis.

ca 1280 CE **Nathan ben Eliezer ha-Meaati** (of Cento, Italy). Translated medical Arabic work into Hebrew. Lived in Rome 1279–1283.

1280–1288 CE **Rabban bar Sauma** (ca 1220–1294, China). Trans-Asiatic traveler and diplomat. Born in Peking. Started on a pilgrimage to Jerusalem but settled in Armenia, becoming a Nestorian Christian monk (1243). He gained fame as ascetic and became a Nestorian prelate and later traveled to Baghdad. Became a European ambassador to Arghun Khan, the Mongol ruler of Persia. The purpose of this was to form an alliance with the chief states of Christendom against the Mameluke power. In 1287, Arghun Khan sent him back with letters to the Byzantine Emperor, the pope and the Kings of France and England, to gain support for a crusade against Muslims in the Holy Land. His Asian travels were then narrated in the book, giving a picture of medieval Europe at the close of the crusading period, painted by a keenly intelligent, broad-minded and statesmanlike observer.

ca 1280 CE **Peter Olivi** (Pierre fils de Jean Olivi, 1248–1298, France). Physicist and theologian. Anticipated the concept of *inertia*. Further developed the '*impetus theory*' of motion (outlined in ca 500 CE by **Joannes Philoponus**), and asserted that after an object is placed into motion by an initial *impetus*, it can continue to move even though the propellant force is no longer applied (this counters Aristotelian tradition, in which motion required the *continued* action of an external motive force).

Olivi was born in Sérignan, Languedoc, and joined the Franciscan Order in Beźiers (1260). He studied in Paris and died in Narbonne.

1281 CE A typhoon sank the entire fleet of *Kubla Khan* just as it landed in Kyushu, Japan, aborting the possible occupation of Japan by the Mongols. This event is the source of the Legend of the Divine Wind – the *Kamikaze*.

1285–1289 CE *Spectacles*[93] (lens for myopia) were invented in Venice [called 'Occhiali']. Popularized by **Alessandro della Spina** (d. 1313), a

[93] It is well known that **Goethe** was no friend to spectacles, of which he said (1830): "*Whenever a stranger steps up to me with spectacles on his nose, a discordant feeling comes over me, which I cannot master. It annoys me so much, that on the very threshold it takes away a great part of my benevolence, and so spoils my thoughts, that unconstrained natural play of my own nature is impossible... It always seems to me as if I am to serve strangers as an object for strict examination, and as if with their armed glances they would penetrate my most secret thoughts and spy out every wrinkle of my old face. But while they thus endeavor to make my acquaintance, they destroy all fair equality between us, as they prevent me from compensating myself by making theirs. For what do I gain from a man into whose eyes I cannot look when he is speaking, and the mirror of whose soul is veiled by glasses that dazzle me?*".

Dominican at Santa Caterina monastery of Pisa, and **Slavino degl' Armati** (d. 1317). Their production spread to the south Netherlands, where they were called 'brillen'. The earliest mention of spectacles was in 1316.

1286 CE Appearance of *Sefer ha-Zohar* (The Book of Splendor): an allegorical, symbolic and mystic commentary on the Torah. Building on earlier Kabbalistic works, the *Zohar* deals with the nature of God, the nature and design of human soul; the mysteries of Creation, the Messiah and redemption.

There are two way of dealing with the problems of *religion*. One is the way of the *philosophers*, like Saadia and Maimonides, who use reason to arrive at whatever religious truth is humanly comprehensible. This method results in a system of religious thought which is called *theology*.

Obviously, theology cannot appeal to everybody, for it is the product of fine-spun reasoning and systematic logic, requiring acute *intellectual powers* and large resources of *knowledge*. The masses of the people could not be interested in this method of describing their religion. Moreover, when difficult times come upon a religious group, it wants a warm faith, some assurance that God is near and friendly, not a theology which depends upon cold logic.

The method of *mysticism* was, therefore, more suitable for the Jews of the 13^{th} and 14^{th} centuries. For to the mystics God was a matter of daily experience in direct and intimate contact. They had no objection to speaking of him in human terms. No wonder that mysticism grew among the Jews in Spain in direct proportion to the somber difficulties of their life as Jews. It is not surprising, therefore, that the *Zohar* became the chief expression of Jewish mysticism. Apart from the tremendous impact it had on Jewish life, it has also influenced Christian theology. It carried the Jewish Messianic fervor over more than 700 years under the guise of Hasidism and eventually Zionism.

Nobody knows for sure who wrote *Sefer ha-Zohar*. Some believe that parts of it were composed by **Avraham ben Shmuel Abulafia** (1240-1291, Spain). It is however certain that its final editor was **Moshe ben Shem-Tov de Leon** (1245–1305, Spain).

1288–1301 CE **Ricoldo di Monte Croce** (1242–1320, Italy). Near-Eastern traveler and Dominican missionary. Traveled extensively in Israel, Mesopotamia, Asia Minor, Armenia and Persia, and described his journeys in a detailed *Itinerarius*. As a traveler and observer his merits are conspicuous.

1290 CE *Expulsion of the Jews from England*[94]. They were readmitted on Feb 4, 1657, under **Cromwell**.

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[94] *A mass-expulsion of* 16,000 *people by Edward I*.
Jews first arrived in England in 1066 at the invitation of William the Conqueror, who depended on their capital to forge a strong English state. King William

For the next 484 years, the mass-expulsion of Jews by European monarchies was often exercised; e.g.: **France** [1394, 1615]; **Italy** and **Sicily** [1492-3, 1569]; **Germany** [1450]; **Russia** [1727]; **Vienna** [1670] **Prague** [1774].

1291–1455 CE **Ugolino** *and his brother* **Vadino Vivaldo**. Genoese explorers. Headed the first expedition in search of an ocean way from Europe to India. They left Genoa in May 1291 in two galleys. The galleys were well armed, and sailed down the Moroccan coast to a place called Gozora (Cape Nun), in 28°47′N, after which nothing more was heard of them. In 1315, **Sorleone de Vivaldo**, son of Ugolino, undertook a series of distant wanderings in search of his father, and even penetrated to the Somali coast. In 1455 another Genoese seaman, **Antoniotto Uso di Mare**, sailing in service of Prince **Henry the Navigator** of Portugal, claimed to have met, near the mouth of the Gambia River, with the last descendant of the survivors of the Vivaldo expedition. He was told that the two galleys had sailed to the sea of Guinea; in that sea one was stranded, but the other passed on to a place on the coast of Senegal, where the Genoese were seized and held in captivity.

Rufus, successor to William, even forbade Jews to convert to Christianity because that would "rid him of valuable property and give him only a subject". At one time the Jews sent a deputation begging permission to leave the country. It was refused, because they were still too useful to the treasury. Slowly their wealth was drained out of them, until by the end of the 13^{th} century their financial value was nil. Besides, the Lombards, Italian bankers, had come to England and had taken the place of the Jews in England's economic life. Under the vigorous urging by the Church, the kingdom felt it could get along without them and expelled them. As Jews became essential again to English economy, they began to resettle there under Cromwell in 1657.

A Jewish community lived in *Oxford* since 1090 CE. On Sept 3, 1189, many Jews were killed in a riot that broke during the coronation of Richard the Lion-Hearted. On Jan 31, 1253, Henry II forbade Jews in England to build new synagogues. After the expulsion, their synagogue became a tavern (1309) and is now a part of Christ-Church. Their cemetery (in use 1231–1290) is now part of the Botanical Gardens and of Magdalen College.

Time Reckoning in Antiquity

In the ancient Near East and the world of Greece and Rome, all systems of time reckoning used cues provided by the periodic motions of the earth, sun and moon, which functioned as time-standards. Two parallel systems of time-reckoning were recognized. The first, known already to dynastic Egypt, consisted, like our modern one, of 24 hours of equal length and used by astronomers ('equinoctial hours' or 'equal-day hours').

In the second, known as 'seasonal hours' or 'temporal hours' [which Herodotos attributed to the Babylonians] the period of daylight is divided into 12 hours and the night into 12 hours or 'watches'. Consequently, the length of the hour depended on latitude and season.

The interval between two successive returns of a fixed point on the sphere to the meridian is called the sidereal day, *and* sidereal time *is reckoned from the moment when the vernal equinox passes the meridian, the hours being counted from* 0 *to* 24*. Clocks and chronometers regulated to sidereal time are only used by astronomers, to whom they are indispensable, as the sidereal time at any moment is equal to the* right ascension *of any star when just passing the meridian. For ordinary purposes solar time is used.*

The solar day, as defined by the successive returns of the sun to the meridian, does not furnish a uniform *measure of time, owing to the slightly variable velocity of the sun's motion and the inclination of the orbit to the equator, so that it becomes necessary to introduce an imaginary* mean sun *moving in the equator with uniform velocity. The* equation of time *is the difference between apparent (or true) solar time and the mean solar time. The latter is that shown by clocks and watches used for ordinary purposes. Mean time is converted into apparent time by applying the equation of time with its proper sign. (As the equation of time varies from day to day, it is necessary to take this into account, if apparent time is required for any moment different from noon.)*

While it has, for obvious reasons, become customary in all civilized countries to commence the civil day at midnight, astronomers count the day from noon, being the transit of the mean sun across the meridian. The ancient astronomers, although they left us ample information about their dials, water or sand clocks, and similar timekeepers, are very reticent as to how these were controlled. ***Ptolemy****, in his* Almagest*, states nothing whatever to show how the time was found when the numerous astronomical phenomena which he recorded took place.*

However, ***Hipparchos*** *gives a list of 44 stars scattered over the sky at intervals of right ascension equal to exactly one hour, so that one or more of them could be on the meridian at the commencement of every sidereal hour. (It has been shown that the right ascensions assumed by Hipparchos agree within about* $15'$*, or one minute of time, with those calculated back to the year 140 BCE from modern star-places and proper motions.) The accuracy which, it thus appears, could be attained by the ancients in their determinations of time was far beyond what they seem to have considered necessary, as they only recorded the hours of astronomical phenomena (e.g. eclipses, occultations) without ever giving minutes.*

The Arabs had a clearer perception of the importance of knowing the accurate time of phenomena, and on observing lunar eclipses, never failed to measure the altitude of some bright star at the beginning and end of the eclipse. A sketch of the principal method of determining time is as follows:

In the spherical triangle ZPS *between the zenith* (Z)*, the pole* (P) *and the star* (S)*, the side* $ZP = 90^\circ - \phi$ *(*ϕ *being the latitude),* $PS = 90^\circ - \delta$ *(*δ *being the declination) and* $ZS = \zeta = 90^\circ -$ *observed altitude. The angle* $ZPS = t$ *is the star's hour angle or, in time, the interval between the moment of observation and the meridian passage of the star. We have then*

$$\cos t = (\cos\zeta - \sin\phi\sin\delta)/\cos\phi\cos\delta.$$

This formula can be made more convenient for the use of logarithms by putting $\zeta + \phi + \delta = 2S$*, which gives*

$$\tan^2\left(\frac{1}{2}t\right) = \sin(S-\phi)\sin(S-\delta)/\cos S\cos(S-\zeta).$$

According as the star was observed west or east of the meridian, t *will be positive or negative.*

The sidereal time $= t + \alpha$*, (where* α *is the right ascension) and* δ *are taken from an ephemeris. If the sun had been observed, the hour-angle* t *would be the apparent solar time. The latitude observed must be corrected for refraction, and in the case of the sun also for parallax.)*

Time in antiquity was kept by sundials, sand-clocks or water-clocks which were the only mechanical time-recorders. Of these, the sand clock is the earliest timekeeper independent of celestial bodies, but is inefficient for measuring time for more than a limited duration.

A combination of water-clock and mechanical clock appeared for the first time in China (1092), but totally mechanical clocks (that is, clocks equipped with a weight drive setting a train of wheels in motion with a simple oscillatory escapement) appeared for the first time in Western Europe in the late 13^{th} *century.*

The English word 'clock' is related to the French word 'cloche', meaning a bell. Bells played an important part in medieval life, and it is probable that mechanisms for ringing them, made of toothed wheels and oscillating levers, paved the way for the invention of mechanical clocks. The earliest public clock known on the European continent outside Italy was constructed at the Strasbourg cathedral in 1352. *The builder (unknown) worked at it for two years. Nothing remained of that old clock except the rooster, preserved in the Frauenhaus, Strasbourg.*

Chaos and Misery[95] (1300–1450)

The 13^{th} *century represented the acme of scholastic thought and medieval culture. Contributions of the time to literature, philosophy, science, and the pure and applied arts rose to the highest levels since antiquity. But what advance could be anticipated during the* 14^{th} *century, during which the decline in learning was accelerated by war and pestilence?*

Yet, strangely enough, the century seems to have been responsible for more striking additions to science and technology than was its predecessor. Gunpowder, the compass, spectacles and mechanical clocks may have been adumbrated before 1300, but it was the 14^{th} *century which saw their effective introduction into European civilization.*

In mathematics *the graphical representation of functions, and in* physics *the revival of the concept of impetus or inertia and the suggestion of new laws for the motion of a freely falling body, are among the outstanding examples of the development of scientific thought of the time.*

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[95] For further study, see:

- Tuchman, Barbara W., *A Distant Mirror*, Ballantine Books: New York, 1978, 677 pp.

In the 14^{th} *century there was hardly a place, certainly not a country in Europe which was ever free for a whole year from war, plague, or starvation. No place was ever delivered from fear for a single day, for to the natural fears caused by physical calamities, wars, revolts, and general lawlessness were added the artificial fears bred by superstitions and unreason. One could possibly escape natural miseries, but no one could escape the nightmares of his own mind: devils, witches and evil eye.*

The Church dominated everything and every person. Every action or idea had its theological aspect; every form of nonconformism, or protest was construed as heresy. Thus, anything concerning the church would affect everything else.

From 1305 *to* 1377 *the papal court was removed from Rome to Avignon, in Southern France, to escape the turmoil which was raging in Italy (the so-called 'Babylonian Captivity'). This was followed by the Great Schism (1378–1417), dividing Western Christendom into two hostile parties which excommunicated each other (*popes *in Rome and* anti-popes *in Avignon). All this shattered the spiritual basis of Europe; people began to wonder and doubt, and the Church was so weakened that it was unable to resist the greater and lasting Schism which was to occur a century later. The Great Schism was, indirectly, one of the main causes of the Reformation.*

The most obvious cause of the troubles of the last medieval centuries was economic depression*; by* 1300*, Western Europe had reached the limit of its capacity to produce food and manufacture goods. There were no more reserves of fertile land to bring into cultivation. For many years there was no significant increase in industrial output; most towns barely held their own (the Italian towns fared better due to increased Mediterranean trade on account of the Crusades). Until Europe found new markets and new forms of production, economic stagnation created a climate hostile to innovation and efforts to cooperate for the common welfare. Each individual, community or class was eager to preserve the monopolies and privileges that guaranteed it some share of the limited wealth available.*

Economic weakness led to weakness of most governments. Rulers were always short on money, for the old taxes brought in less and less and it was very difficult to levy new ones. Salaries of government officials were low and most officials supported themselves by taking fees, gifts, and bribes from private citizens. Secular rulers had neither the ability to make realistic plans for the welfare of their people, nor the authority to impose such remedies as they did devise. Most secular rulers could think only of increasing their revenues by conquering new lands or robbing their own wealthy classes[96]*. Such a policy solved no problems; it merely postponed them for the victor*

[96] In France, for example, Philip the Fair (Philip IV, reigned 1285–1314) expelled

and aggravated them for the vanquished. With governments discredited by futile and costly wars, many men lost faith in their political leaders and turned to rebellion and civil war.

The leaders of revolt were members of the landed nobility, who still had wealth and influence though they had lost their old rights of feudal government. Their main purpose was to preserve their privileges or to direct government revenues to their own pockets. The other classes fared no better than the nobles. The bourgeoisie were inept at running their own municipal government.

The townsmen split into factions – old families against new families, rich against poor – and the faction in power tried to ruin its opponents by discriminatory taxation. As a result, local self-government collapsed in town after town. As for the peasants, they were far more restive and unhappy than they had been in the 13^{th} *century. With no new lands to clear and no new jobs to be had in the towns, they had little hope of improving their lot. Some of them managed to ease the burden of payments and services to landlords by renegotiating their leases or by moving from one estate to another. The peasants rebelled in country after country, killing landlords, burning records, and demanding that payments for their lands be lowered or abolished altogether.*

Feudal institutions were thus breaking down everywhere with lords ground between the royal yoke on the one hand, and the growing independence and insubordination of the townspeople on the other. The feudal world was on the decline. Its two pillars, the papacy and the kingship, were crumbling.

The main series of wars was the Hundred Years' War (1337–1453) between England and France. Those wars were fought on French territory, aggravated by civil wars among the French people, and caused much misery. It has been estimated that 1/3 of the French population was destroyed; the destruction of wealth was at least as great.

One of the main causes of chaos and misery was the existence of mercenary troops, recruited by captains from among the most lawless and restless elements of many nations. These companies were hired by princes or towns when the latter needed military assistance, and discharged when the need was over. In times of peace between two campaigns, however, these companies, drawing no pay, robbed and tormented the peasants, and sacked or blackmailed the cities.

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the *Jews* (1306) and confiscated their money. He then cooperated with Pope Clement V (French too) and the Inquisition to abolish the wealthy *Order of the Knights Templar* and made their treasury a section of the royal finance administration (1307–1312).

The effects of economic depression, political confusion and religious uncertainty were intensified by terrible outbursts of the bubonic plague, known as the Black Death (1347–1400). The panic caused by the plague drove the sorely tried people of Western Europe into emotional instability. It is no accident that the bloodiest peasant rebellions *and the most senseless* civil wars *took place after the plague, and that the* witchcraft delusion, *unknown in the early Middle Ages, then reached its height. Innocent men and women were falsely accused of practicing black magic. The rationalism and confidence in the future that had been so apparent at the height of medieval civilization were gone.*

Concurrent with the economical regression of Western Europe, there appeared a new military danger in the East – the Turks. The Turks arose in Central Asia in the 6^{th} *century CE. They converted to Islam in the* 9^{th} *and* 10^{th} *centuries, and in the* 11^{th} *century began to attack the Byzantine Empire. Under Osman I, the Turks advanced towards the heart of the Empire through gradual infiltration. Later, in a series of military campaigns (1317–1340), they completed the conquest of Anatolia and the Black-Sea ports with the exception of Constantinople. In* 1345, *these* Ottoman Turks *first crossed into Europe, and by* 1400, *under Bayazid I, they were in partial control of the Balkans, challenging the Mediterranean commercial lines of Venice and Genoa, and had Constantinople under siege.*

Another Turkish Muslim tribe, under Timur (Tamerlane, ca 1336–1405), began (1370) a long series of raids and wars, conducted with incredible energy and ferocity. Timur conquered, destroyed, and decimated many of the leading cities of his time: Isfahan (1387), Edessa, Moscow (1395), Delhi (1398), Aleppo, Damascus, Baghdad (1401), Bursa, Smyrna (1402). In many places he built pyramids of human skulls to serve as warnings. His expeditions and conquests influenced the whole of Asia and Eastern Europe, either directly or indirectly through the nomadic populations which were hustled out of their usual territories, or through the sedentary populations which were uprooted and violently displaced.

The Timurian invasions caused a fantastic reshuffling of Asiatic cultures, because whatever blows Timur delivered were transmitted from one end of Asia to another with incredible speed due to the extreme mobility of nomadic tribes. Thus, the Arabic cultures of the Middle East [already weakened by the destruction of Baghdad by Hulagu (1258), the Black Death (1348, 1381), and the Turkish Hegemony] suffered upheavals and decline in 1393 *when Timur conquered and sacked Mesopotamia and Baghdad (its population nearly exterminated!).*

Consequently, the destruction of the Eastern Islamic world begun by

Jenghis Khan in the first half of the 13^{th} *century, was completed by Timur*[97] *a century and a half later. It never recovered from this calamity.*

In 1400 *Timur invaded Anatolia and challenged Bayazid for the leadership of the Turkish peoples. A military confrontation was inevitable: on July* 20, 1402, *the two armies met at Angora (Ankara), with nearly one million combatants on either side. The Ottomans were completely defeated, and Bayazid was captured. Fortunately for Europe, Timur turned east, to conquer China, and died in* 1405 *before embarking on this mission.*

The Ottoman Turks, however, made a rapid recovery: by 1453, *Constantinople was taken and soon became the Turkish capital.*

The misery that befell the population of Europe was amplified in the case of the Jews. In 1306 *they were expelled from France, in* 1348 *from Germany, in* 1349 *from Hungary, in* 1394 *from Provence, in* 1421 *from Austria, in* 1492 *from Spain, in* 1495 *from Lithuania, and in* 1497 *from Portugal.*

1295–1318 CE **Rashid al-Din Fadlullah** (1247–1318, Persia). Physician, theologian and historian. Of Jewish descent. Physician to the Mongol sovereigns of Persia in Tabriz. Vizier of the empire (from 1298). Composed encyclopedic universal history *Jami al-tawarikh* ('Collection of chronicles').

Held office under the Mongol sultan Abaqa Khan and his successors Ghazan, Uljaytu and Abu Said. The envy aroused by his great wealth and grandiose benefactions enabled his enemies to procure first his deposition from office and then his execution on the charge of having poisoned Uljaytu.

His great history was begun as a history of the Mongols at the invitation of Ghazan Khan, who put the state archives at his disposal, and continued as a universal history, for Uljaytu. The work is notable for impartiality, clarity of style, and the wide range and authority of its sources.

[97] Timur's raids also finished the destruction of the irrigation system which had made the Mesopotamia of earlier days a paradise. It is said that in Sassanian times (before the Muslim conquest) a squirrel could travel from Seleucia to the Persian Gulf without ever having to come to ground. The canals and ditches without which the earth lost its fertility were neglected or destroyed, and gradually disappeared. Timur completed the job.

1297–1307 CE **John Duns**[98] **Scotus** (1265–1308, Scotland). Philosopher. One of the earlier forerunners of the scientific method. A Franciscan monk. Probably born in the Scottich village of Duns. He was educated at Oxford and the University of Paris. He taught theology at Oxford (1301) and at Paris (1303), and died at Cologne. He is most noteworthy for his criticism of the views of Augustine and Aquinas, for which he suffered great personal ridicule; his name was disparaged by his opponents who, after his premature death, publicly burned his books and distorted the meaning of his doctrine.

Duns Scotus emphasized separation of philosophy from theology, of reason from faith, of independent thought from dogma. Contrary to his own expectations, the movement generated by his thought (*Scotism*) led away from the Church rather than toward it. His insistence on demonstrative proof (perhaps because of a mathematical background in his Oxford education) led him to a demarcation between rationalism and empiricism that has followers among modern philosophers. He taught that philosophy must be regarded as a science, within which logic is given a scientific realm of its own. He anticipated aspects of *Gestalt psychology*, *Gegenstands theory* and *Existentialism*. In several of his views, Duns Scotus was inspired by **Shlomo Ibn-Gabirol**'s *Fons Vitae* (Source of Life, ca 1050) which influenced many Franciscans.

In spite of his great success as a teacher, both in London and Paris, he was removed from Paris to Cologne (1308) due to jealousy and in the same year died of apoplexy (according to some tradition he was buried alive).

Since the middle of the 19^{th} century, Duns Scotus has been ranked among the important thinkers of the Middle Ages. His influence has continued into modern philosophy down to the present.

William of Ockham was one of his disciples.

1298–1338 CE Ritual-murder accusations (*blood libels*) and *Host-desecration* in libels lead to massacres of Jews in Southern and Central Germany in the name of God and for the sake of their pockets. In 1298, more than 15,000 Jews were killed and 146 communities destroyed in six months. Among the communities affected were Röttingen, Würzburg, Nüremberg and Bavaria. Again, during 1336–1338, 10,000 more were killed and 120 communities were destroyed.

The pattern had been set, and in the ensuing two centuries *ritual-murder accusations* against the Jews reached epidemic proportions throughout the

[98] The stubborn opposition of Scotists to classical studies of the Renaissance gave rise to the use of the word *dunce* for pedant or blockhead. It signifies the age-old contempt in which his posterity has held the man who dared criticize Aquinas.

continent. By the 15^{th} century, ritual-murder accusations had died out, although they were briefly revived in the 17^{th} century Poland and late czarist Russia. *The stealing-of-the-Host* hysteria reached its height in 14^{th}-century Germany. By the end of the century, Host-stealing accusations also died out. The Jews had began to flee Germany, and the rulers, seeing the economies of their duchies stagnate, quickly stopped the canard by hanging those who spread such false accusations. The Jews were invited to return, with assurances that such charges would never again be brought against them again.

The Germans, perhaps because they were still closest to the barbaric strain, which had nursed them, were the most barbaric in their persecutions. Most of the anti-Jewish measures are popularly attributes to the entire Middle Ages were of German-Austrian origin, and grew only on German soil. Here the ritual-murder charges, the Host-desecration libels, and Black Death accusations were used to whip the population into a frenzy by sadists and fetishists.

It was here in Germany that the cheating of the Jews reached its noblest and purest forms. Local German princes enticed Jews to their realms under sacred promises to protect them and solemnly gave them liberal charters, swearing on the cross they meant it all, only to rob them later of their wealth, confiscate their land, and then sell them protection, gangster style. One can marvel that in spite of it all, the Jewish spirit survived, and Jewish culture life continued. Talmudic learning still exerted its power, something realized by Jean-Jacques Rousseau, who wrote in his *Social Contract* (1762):

"*Through it alone* [*the Talmud and its ritualistic legislation*] *that extraordinary nation so often subjugated, so often dispersed and outwardly destroyed, but always idolatrous of its Law, has preserved itself unto our days... Its moves and rituals persist and will persist to the end of the world...*".

1299 CE The city of Florence passed an ordinance prohibiting the use of the new Hindu numerals (i.e. 1,2,3,4,5,6,7,8,9,0) since they were more easily altered (e.g. by changing 0 to 6 or 9) than Roman numerals or numbers written out in words[99].

ca 1300 CE **Dante Alighieri** (1265–1321, Italy). Poet. Urged a reform in the Julian Calendar for its being out of step with the tropical year by more than a week.

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[99] As late as the end of the 15^{th} century, the Mayor of Frankfurt ordered his officials to refrain from calculating in Hindu numerals. Even after the decimal numeral system was well established, Charles XII of Sweden (1682–1718) tried in vain to ban the decimal system and replaced it with a base 64 system, for which he devised 64 symbols!

1303 CE **Chu Shih-Chieh** (ca 1270–1330, China). Mathematician. In a text marking the peak of Chinese mathematics, Chu presented an iteration scheme for solving equations (used up to degree 14) rediscovered by Horner and Ruffini. It also contains *Pascal's triangle* and gives explicit formulas for summation of series of polygonal numbers.

1304–1310 CE **Dietrich (Theodoric) of Freiberg** (ca 1250–1311, Germany). Dominican scholar, optician, meteorologist and philosopher. In his book *De iride* (On the rainbow), he reports on his experiments with globes of water and correctly explains many aspects of rainbow formation. According to Dietrich, the rainbow arises due to two refractions and one reflection of solar rays. The first refraction occurs when the light ray first enters a raindrop, the reflection occurs within the raindrop, while the second refraction occurs as the ray exits the raindrop. He explains the less-frequent secondary arcs as resulting from *double* reflection of light rays entering near the bottom of the raindrop[100].

Dietrich had also made simple experiments on the dispersion of light by crystals, some 362 years before Newton.

Dietrich was a Teutonic member of the Order of Preachers. He had been a professor of theology in Germany, and the author of at least 30 works on metaphysics and optics, of which about a dozen are now lost. From 1285 to 1296 he was the Dominican master of the large province including Germany, Austria and the Low Countries. He then earned the degree of Master of Theology at Paris (1297). In 1304 he was sent as German elector to the General Chapter of the Order; and in this year at Toulouse he undertook, at the instigation of the Master-General, Aymeric de Plaisance, his work on the rainbow. In 1310 he went to Paris as Master of Theology. After 1311 there is no further information about him, from which one may conclude that this is approximately the year of his death.

1305 CE Spain banned the study of all science. Among the first victims of the *Inquisition* were Christian scientists. Since this was done *before* the Renaissance could gain a foothold in that country, no major scientific discovery had been made in the Iberian Peninsula.

[100] It is remarkable that Dietrich's theory of the rainbow was proposed at about the same time by the Persian **Qutb al-Din al-Shirazi** (1281). A complete explanation of the main rainbow could only be given after the genesis of colors had been correctly explained by Newton in 1672, and this was done by Newton himself in 1704. So Dietrich and al-Shirazi went just as far as it was possible to go without knowledge of the exact law of refraction and the *dispersion* of light.

1306–1322 CE **Estori ben Moshe ha-Parhi** (1280–1355, France and Israel). Physician, translator, geographer and natural historian. First scientific topographer of Israel. His family originated in Florenzia, Andalusia, hence the name Parhi [= flower in Hebrew]. He studied in Montpellier under his relative **Yaacov ben Machir Ibn Tibbon**. When the Jews were expelled from Montpellier (1306) he moved to Barcelona, Toledo and finally to Egypt. He then proceeded to Israel (1313) and established himself in Beth-Shan in the Jordan Valley, near the Sea of Galilee. He spent some seven years (1313–1320) in studying Israelian topography. This work he completed in 1322 and described in his book *Kaftor u-Perah*. In it, he was the first to try to identify systematically Biblical, Talmudic and Arabic place names. His work is valuable also from the point of view of archeology and natural history (botany, zoology, history).

While in Barcelona, ha-Parhi translated into Hebrew medical and astronomical treatises. His astronomical knowledge was derived from Avraham bar Hiyya and Yaacov ben Machir. He was acquainted with the writings of Hippocrates, Galen, Aristotle and Ptolemy.

1316–1343 CE **Levi ben Gershon**, or **Leo de Banelis**, or **Gersonides**; known also as RALBAG (1288–1344, Perpignan and Avignon, France). Mathematician, astronomer, physicist, philosopher and physician. One of the leading mathematicians and astronomers of the European Middle Ages. Was first to apply the algebraic algorithm of mathematical induction, if not actually inventing it. Foreshadowed Copernicus in his firm objection to the Ptolemaic system. Stressed the importance of the sine function; emphasized the importance of plane trigonometry and developed it, including the *law of sines*. In his book 'Sefer ha-mispar' ('Book of Numbers') he calculated, for the first time, the number of permutations of n objects, taken r at a time, and the number of combinations of n objects taken r at a time[101]. He proved his results by the *method of induction*[102] (1321), a hitherto unknown principle, first used by him.

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[101] Levi's results, in modern notation, are: $P_{n,n} = n!$, $P_{n,r} = n(n-1)\cdots(n-r+1)$, $C_{n,r} = \frac{P_{n,r}}{P_{r,r}}$. He also found that $\sum_n n^3 = (\sum_n n)^2$, and proved that the numbers 2^n, 3^m differ by more than a unit, except for $(n,m) = (1,0),(1,1),(2,1),(3,2)$. This latter theorem has connections with Fermat's Last Theorem through the so called "ABC conjecture" – a topic of great interest among number theorists.

[102] The first to *formulate* the principle of mathematical induction was **Francesco Maurolico** (1575).

The *Principle of Mathematical Induction*: If for a given assertion $P(n)$ we can prove that:

1°. The assertion is true for $n = 1$;

2°. If it is true for index $n = k$, then it is also true for index $n = k + 1$;

Levi invented a new astronomical instrument bearing the name *cross-staff*. [It is a long graduated staff with a short perpendicular crosspiece, later called *transversary*. To measure an angle, e.g., the altitude of the sun, the observer holds the staff in his hand and, sighting from one end of it, moves the crosspiece until one end touches the horizon and the other the center of the sun.] It remained for centuries one of the main tools of navigation.

In physics he completed the invention of the *camera obscura* (Alhazen, 1026) in the sense that he rationalized it, popularized it, and tried to account for the dimensions of the images. He then used the camera and his cross-staff to determine more exactly the variation in the apparent diameters of sun and moon. These determinations were important elements in the comparison of conflicting planetary theories.

Levi realized the prodigious distances of the stars from the earth: the stars may be considered as points, their rays as parallel, and the size of their images in the camera depends only on the size of the aperture. He explained how to observe the progress of lunar and solar eclipses. He compiled astronomical tables (1320) and wrote a treatise explaining in detail the motion of stars and planets, criticizing the Almaghest and the leading medieval Arab astronomers, whose observations and calculations he corrected. As his astronomical writings were soon available in Latin and were very well received, he influenced Western astronomy strongly and rapidly.

Then the assertion is true for every positive integer n.

Consider for example the inequality $2^n < n^{10} + 2$. It is found to be true for all $n < 59$, but $2^{59} \approx 5764 \times 10^{14} > 59^{10} + 2 \approx 5111 \times 10^{14}$. It is not hard to prove that the inequality is *false* for all $n > 59$.

Another example, this time from geometry, also serves to show how intuitional induction may lead us astray: n points are marked arbitrarily on the circumference of a circle, each joined to all others by straight lines. What is the maximum number of regions formed altogether? Simple counting yields for the first five cases the sequence 1, 2, 4, 8, 16 and one would have liked to stop explicit construction there, hoping that $a_n = 2^n$. But alas! The next term for $n = 6$ is 31, breaking the convenient pattern. The general formula, obtained via induction, is

$$\frac{1}{24}(n^4 - 6n^3 + 23n^2 - 18n + 24) \equiv n + \binom{n}{4} + \binom{n-1}{2}.$$

It yields the following sequence for $n = 1, 2, \ldots, 12$:

$$1 \quad 2 \quad 4 \quad 8 \quad 16 \quad 31 \quad 57 \quad 99 \quad 163 \quad 256 \quad 386 \quad 562 \quad \cdots$$

Pascal borrowed the idea from Maurolico and clarified it in his *Traité du triangle arithmétique* (1655) [*Bull. Amer. Math. Soc.* **16**, 70–73, 1910; *Amer. Math. Mont.* **24**, 199–207, 1917].

Yet, he shared with his Arabic, Latin, and Greek contemporaries and predecessors the inability to use the zero and the positional decimal notation, and used no numerals except Hebrew letters.

He was born at Bagnols in Languedoc, the maternal great grandson of **Nahmanides**. Little is known of his life. His family had been distinguished for piety and exegetical skill, but though he was known in the Jewish community for his commentaries on certain books of the Bible, he never seemed to have accepted any rabbinical post; the freedom of his opinions may have put obstacles in the way of his advancement. He is known to have been at Avignon and Orange during his life. Part of his writings consists of commentaries on the portions of Aristotle then known. Some of these were printed in the early Latin editions of Aristotle's works.

His most important treatise, which secured him a place in the history of science, is entitled *Milhamot Adonai* (The Wars of God), and was thirteen years in the making (1316–1329). A portion of it, containing an elaborate survey of astronomy as known to the Arabs, was translated into Latin in 1342 at the request of Pope Clement VI. Medieval astronomers have used the astronomical instruments invented by him. On account of his boldness and the suspicion of heresy that clung to him, *Milhamot Adonai* was a forbidden book[103]. There is evidence that Spinoza was influenced by the writing of Gersonides.

1320–1330 CE **Maximus (Manuel) Planudes** (1260–ca 1330). Prominent Byzantine scholar and mathematical writer. Author of numerous works, among them two books on the *Arithmetic* of Diophantos and arithmetic based upon Hindu-Arabic numerals. Planudes possessed a knowledge of Latin at a time when Rome and Italy were regarded with hatred and contempt by the Byzantines. By his translation, he paved the way for the introduction of the Greek language and literature into the West.

He seems to have been among the first of the mathematicians to use the word *million*[104], which is not found anywhere before the 13^{th} century.

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103 Levi ben Gerson was the intellectual product of Greek-Arabic thought that streamed from Babylonia, Alexandria and Spain. Both the content and method of his religious writings were influenced by his training and outlook as mathematician, astronomer and philosopher. The orthodox followers of **Rashi** rejected his daring discussions of religious matters.

104 One of the most striking features of ancient arithmetic is the rarity of large numbers. There are exceptions, as in some of the Hindu traditions, in the records of some of the Babylonian tablets, and in the *Sand Reckoner* of Archimedes, with its number system extending to 10^{63}, but these are all cases in which the élite of the mathematical world were concerned; the people, and indeed the substantial

Planudes was born at Nicomedia in Bitynia (Asia Minor), but the greater part of his life was spent in Constantinople, where as a Greek Orthodox monk he devoted himself to study and reading. At one time he was appointed (1327) ambassador to Venice.

His arithmetic is of value chiefly as showing the influence of Baghdad upon the mathematical thought of Constantinople.

1320–1340 CE **Yosef Caspi ben Aba Mari** (Don Bonafous de Largentra, 1297–1340, France). Philosopher, grammarian and exegete. Maintained that natural phenomena can sometimes violate the laws of nature over small intervals of time[105]; he used this argument to render a *logical explanation* to biblical miracles.

Caspi was born in L'Argentiere[106], Southern France and died in Tarascon. He traveled much, visiting Spain, Majorca and Egypt. He was one of the most prolific writers of his age.

In his *Will and Testament*, which he wrote in 1332, he described his travels and his studies, shedding much light on the literary and religious conditions of the period as well as expressing his personal views on philosophy and theology. He wrote no less that twenty-nine works (most of which are still extant in manuscripts), mainly on philosophy and logic.

ca 1325 CE *Invention of firearms*. The use of cannons and guns[107] implied not only the availability of quality gunpowder, a task for the alchemist, but

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mathematicians in most cases, had little need for or interest in numbers of any considerable size. By the 15^{th} century the 'million' was known to the Italian arithmeticians, and first appeared in print in 1478. Until the World-War of 1914–1918 taught the world to think in *billions* (10^9) there was not much need for number names beyond millions (except in physics and astronomy).

105 *Example*: In hindsight we can say today that classical physics fails to explain the *shining of the sun* (!) and that the sun shines by a quantum 'miracle' namely, by *proton tunneling*: there is an electrostatic repulsion barrier preventing hydrogen nuclei from fusing to form Helium. Indeed, the probability of fusion per encounter between two hydrogen nuclei in the sun is shown to be 10^{-434}. However, with tunneling, probability rises to 10^{-20}, but since there are ca 10^{57} hydrogen nuclei in the sun's core, fusion happens to each hydrogen nuclei some of the time. i.e. the 'miracle' occurs to *some* nuclei at any given time.

106 Hence the Hebrew equivalent: *Caspi* = made of silver.

107 The list of munitions at Windsor Castle in 1330 mentions a "large ballista" called *Lady Gunhilda*, which gives us our *gun*. *Mortar* was an alchemist tool. The word *cannon* is derived from the Hebrew-Greek *qaneh*, *kanna* = reed, since

also the making of guns, a task for the armorer, blacksmith and founder. It hinged on the progress made in the 13^{th} and 14^{th} centuries by the workers in brass and bronze. The art of the founder, however, remained a very subtle one, full of pitfalls and of secrets. As with gunpowder, it is unknown where or by whom the discovery was made that exploding gunpowder could exert a propelling force and drive a body through the air. We do, however find an illustration of a primitive gun in a manuscript of 1327; moreover, there was a powder-works at Augsburg in 1340, and both gunpowder and cannons were being manufactured in England in 1344, and probably earlier.

The use of explosive power for the propulsion of projectiles, and the inventions of guns and mortars, probably originated in Germany[108] during the third decade of the 14^{th} century – at about the same time in various places. By the end of the 14^{th} century, firearms were being manufactured all over Europe. By the middle of the 15^{th} century, artillery would become a decisive weapon against feudal stone castles and the traditional curtain walls of towns.

The abundance of references to the cannon in accounts of various sorts is very remarkable. They are mentioned in the archives and chronicles of England, France, Spain, Italy, Germany, Flanders, etc. Those early firearms were very inefficient and did not make much impression, either military or psychological. The revolution brought about by their use was very gradual; it was not fully realized until the 16^{th} century. Indirect evidence of that slowness is provided by the continued popularity of armor. The day would come when armor would no longer protect the bodies of men and horses, but that day was still very distant, and in fact the period 1400–1550 was the golden age of armor.

1325–1354 CE **Muhammad Ibn Battuta** (1304–1369, Morocco). One of the greatest travelers in medieval times. During his 29 years of travel he covered, at the very least, some 120,000 thousand kilometers by land and sea

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cannons had the form of a long hollow tube.

The ballista was not necessarily a gun, and could be used for throwing stones and inflammable substances. We know for sure that cannons were used by the English at the siege and capture of Calais in 1347, but they could have been used as early as 1327 by Edward III in his war against the Scots.

108 The discovery of gunpowder and firearms is sometimes attributed to a legendary monk known as '**Berthold der Schwarze**' who flourished in Freiburg im Breisgau, and made his invention in 1380. Since firearms, not to speak of gunpowder, had already been invented by that time, he may at most have introduced some improvements. Since there is no evidence of this either, we are led to assume that he was the symbolic incarnation of the popular conception of a satanic environment characterized by flashes of fire and the smell of brimstone.

(not counting detours), and visited all the Islamic lands, India, China, Africa, Siberia, Russia, the Balkan and Spain. On his return the Sultan of Morocco provided him with a secretary, who transcribed and corrected his manuscripts.

He had seen more of the known world than any man before him, and much more than the most indefatigable travelers since his time; even the list of the regions and cities he visited resembles a gazetteer. His point of view was never that of a geographer or a historian. He was not interested in nature, but very much so in people, yet his account has considerable geographical and historical value, natural history included.

Ibn Battuta was born at Tangier (Tunisia) to the Berber tribe of the Lawata (Tunisia) into a respected family of scholars and Islamic judges (qais). At the age of 21, after finishing his education, he set out to make the pilgrimage to Mecca. Along the way, the young man studied under well-known scholars of Islam. These studies qualified him to become a judge.

In 1326, Ibn Battuta completed his first pilgrimage to Mecca. But instead of returning home, he decided to see as many parts of *Dar al-Islam* as possible, vowing never to travel the same road twice. In 1333, Ibn Battuta arrived in India after traveling through much of west Asia. Here too, he was well-received by the sultan of India. The sultan honored him with feasts and gifts and gave him an important position as grand judge of the capital. After seven years in India, the sultan appointed the traveler as ambassador to China.

Even this famed traveler was greatly impressed by China: "*China is the safest, best regulated of countries for a traveler. A man may go by himself on a nine-month journey, carrying with him a large sum of money, without any fear. Silk is used for clothing even by poor monks and beggars. Its porcelains are the finest of all makes of pottery and its hens are bigger than geese in our country.*"

He was surprised by the well-established Muslim community he found in China's ports. China's first mosque was built 350 years before his arrival. Muslim merchants had come to live permanently in China to manage the far end of their businesses. They had grown wealthy, built mosques and developed into a thriving community.

After his return in 1354 he was appointed a qadi in Fez and died there.

1327–1335 CE **Richard of Wallingford** (1292–1336, England). Mathematician contributed to astronomy, horology[109] and trigonometry, but is best known for the *astronomical clock* he designed (1327).

[109] To dig deeper, see:
- North J., *God's Clockmaster: Richard of Wallingford and the Invention of Time*, Oxford Books, 2004.

Richard was born, the son of a blacksmith, at Wallingford in Berkshire (now Oxfordshire) in England. He studied 15 years at Oxford University before becoming abbot of St Albans. The clock was completed about 20 years after his death (1356) by William of Walsham, but was destroyed during Henry VIII reformation and the dissolution of St Albans abbey (1539).

Richard also designed and constructed a calculation device, known as *equitorium* (which he called *Albion*). This could be used for astronomical calculations such as lunar, solar, and planetary longitudes and could predict eclipses.

He died from what was then thought to be leprosy (although it might have been syphilis, scrofula or tuberculosis).

1327–1367 CE **Francesco Petrarca, Petrarch** (1304–1374, Italy). The most remarkable man of his time: Poet, and first true reviver of learning in medieval Europe. Exerted great influence upon his contemporaries.

Although not a man of science himself, he was instrumental in bringing about the downfall of scholastic philosophy, a prerequisite for the development of experimental science. He sharply criticized astrology, alchemy, Aristotelianism and Averroism.

Petrarca is considered to be the '*first modern man*'; he was intensely anti-medieval and the first to consider the Middle Ages as *dark ages*. With him began the return to secular ideas – a rebirth of interest in the secular culture of the ancients, in both the arts and sciences, and a break with the clerical traditions of the Middle Ages. Petrarca is rightly called the father of a new *Humanism*[110].

He opened for Europe a new sphere of mental activity. By bringing the men of his own generation into sympathetic contact with antiquity, he gave a decisive impulse to that European movement which restored to freedom, self-consciousness, and the faculty of progress to the *human intellect*. He was the first man to collect libraries, to accumulate coins, to advocate the collection of classical manuscripts. For him the authors of the Greek and Latin world were living men, and the rhetorical epistles he addressed to Cicero, Seneca, and Varro prove that he dwelt with them on terms of sympathetic intimacy.

• Watson, E., *The St Albans Clock of Richard of Wallingford*, Antiquarian Horology, 1979, 372–384 pp.

[110] The medieval science was dominated by preoccupations with *God*. The Renaissance thinkers were more interested in *man*. From this circumstance the new cultural movement derives the name *Humanism*.

Petrarca regarded the orator and the poet as teachers, bound to complete themselves by education, and to exhibit to the world an image of perfected personality in prose and verse of studied beauty.

The life of Petrarca was that of a spectator rather than a participant. He did not actually take part in the administration of government, and did not ever practice a profession, marry or fight a battle.

Petrarca was born in Arezzo to a notary that was banished from Florence. The impoverished family wandered to France and settled in Carpentras. During 1320–1326 Petrarca studied law at Bologna. In 1326 he settled in Avignon and began an ecclesiastical career (he never went further than the minor orders). There he came to the attention of Cardinal Giovanni Colonnas and his brother, the Bishop Giacomo, who where Petrarca's reliable patrons for many years. Through their financial support he improved his social standing, and lead the good life of a carefree traveler and bibliophile (he was a keen collector, hunter and discoverer of manuscripts, including Greek texts he could not read himself).

During 1330–1336 he traveled extensively in France, Germany, Italy, Spain and England, and in 1341 was crowned Poet Laureate by Robert of Naples on Capitoline in Rome. During 1343–1361 he served on papal political missions as ambassador to Naples, Milan, Prague and Paris. He had two children by an unknown woman (son 1336; daughter 1343) which he eventually legitimatized. After 1361 he wandered restlessly about Italy, invited by churchmen, princes and friends. Wherever he found himself, he wrote incessantly, composing new works and polishing old ones, releasing them when he thought them sufficiently elegant to circulate. In 1368 he settled in Arquà[111] with his daughter and son-in-law and died there four years later.

1328–1349 CE *The Oxford Calculators*: a group of four Merton Collage scholars consisting of **Thomas Bradwardine** (1290–1349), **William Heytesbury**, **Richard Swineshead** (also known as Richard Suiseth) and **John Dumbleton**.

These skillful mathematicians and logicians were first to differentiate *kinematics* from *dynamics*. Their studies emphasized *kinematics* and included the concept of *instantaneous velocity*. They were the first to enunciate the *mean speed theorem* which states that a body traveling at constant velocity will cover the same distance in the same time as an accelerated body if its velocity is half the final speed v_f of the accelerated body with constant acceleration a. They were able to demonstrate this theorem even without Galileo's

[111] A village in the Euganean hills overlooking the Adriatic. On the 18^{th} of July 1374, his people found the old poet and scholar dead among his books in the library of his little house.

formulation,

$$\bar{v} = \frac{d}{t} = \frac{\frac{1}{2}at^2}{t} = \frac{1}{2}v_f.$$

Thomas Bradwardine (1328) discussed the issue of the hypothetical free fall of bodies in void and concluded that two bodies of the same material but different size, will fall with the same terminal velocity, contradicting the Aristotelian view that the heavier body falls faster. He was afterwards raised to the high offices of chancellor of Oxford University and professor of divinity. From being chancellor of the diocese of London, he became chaplain and confessor to Edward III, whom he attended during his wars in France.

In 1349 he was appointed archbishop of Canterbary, but died of the plague at Lambeth forty days after his consecration. At about 1350, Richard Swineshead (Suiseth) became the first person to show that a sum of an infinite series may converge to a finite number; specifically, he showed that

$$\sum_{n=1}^{\infty} \frac{n}{2^n} = 2,$$

thus braking the Zeno curse which hang over mathematics for two millennia. (At about the same time the Frenchman **Nicole Oresme** demonstrated the *divergence* of the *harmonic series*.)

1330 CE **William of Ockham** (1285–1349, England). Natural philosopher and logician. One of the most profound speculative minds of the scholastic period. His logico-metaphysical system foreshadowed the trend of science and natural philosophy prevailing today. Followed to its conclusion, his secular attitude would adumbrate our present predisposition towards observation, experiment and theory. Encouraged stronger emphasis on observations, and the separation of science and theology. Advocated the building of a theory on the least number of simple premises necessary for explanation of the facts – "Pluritas non est pondera sine necessitate", which is known as "Ockham's Razor" (principle of economy).

This principle, traceable back to Aristotle and the Jewish *Talmud*, has become one of the foundations of modern science. Thus, when two, otherwise equally successful, theories, compete for explaining a set of observations, the advantage goes to the one theory which rests on the least number of premises.

In accordance with the scholastic legacy of Augustine (354–430) and Thomas Aquinas (1225–1274), the theory of knowledge in all its diverse aspects was completely dependent on theology and ontology. Science in the modern sense of the word was practically unknown, and what did exist was totally subservient to the Church. It is without doubt the inalienable merit of this Franciscan scholar that he sundered anew the calamitous union of faith and reason. The 'razor' clearly entails the rejection of the Platonic conception that universals exist apart from and prior to so-called real things.

Ockham emphasized the distinction between statements referring to language and statements referring to things. Denying independent reality to mere abstractions, he anticipated the phenomenalist and positivist schools and **Mach**'s economy of thought.

We may say that Ockhamism was a powerful factor both in the disintegration of medieval thought and in the slow elaboration of modern science. It became one of the main guides used by scientists to choose between alternative views. It asserts that there is no special virtue in elaborate and complex explanations, as opposed to a few, simple hypotheses that imply the observed facts.

Ockham was born in Southern England. He joined the Franciscans and eventually became prominent in that religious order. Ockham studied at Oxford University and then taught theology. In 1324, Pope John XXII summoned him to Avignon, France, to answer charges of heresy. Ockham remained in Avignon for four years. In 1328 he fled from Avignon to the protection of Louis of Bavaria, who was the Holy Roman emperor and an enemy of the Pope. Ockham lived in Münich, Germany from 1330 until his death.

1332 CE Millions perished in India and China by an outbreak of Bubonic plague.

1337–1345 CE Parching drought with consequential famine in Central Asia and China may have caused and accelerated the migration of rodents carrying the bubonic plague westward into Europe.

Diseases and History[112]

"He that is in the field shall die with the sword; and he that is in the city, famine and pestilence shall devour him."

Ezekiel **7**, 15 (ca 590 BCE)

[112] For further reading, see:

- Karlen, Arno, *Men and Microbes*, Touchstone Book: New York, 1995, 266 pp.
- Cartwright, F.F., *Disease and History*, Dorset Press: New York, 1972, 248 pp.

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"Typhus, plague, cholera, typhoid, dysentery had decided more campaigns than Alexander, Caesar, Hanibal, Napoleon, and the inspector general of history. The epidemics get the blame for defeat, the generals for victory. It ought to be the other way around."

Hans Zinsser

For 10,000 years, since the first hunter-gatherers settled in villages, infections had killed more people than war, natural disasters and famine.

Disease is as old as life. Infection was already ubiquitous when higher organisms left their first fossil traces, some 500 million years ago. Dinosaur bones 250 million years old have marks of bacterial infection, as do the remains of mastodons and saber-toothed tigers. Evidence has accumulated to support the idea that all *complex cells evolved through the merger of simpler ones, in what began as parasitism and ended in symbiosis.*

Indeed, cellular invasions can be witnessed today that resemble those theorized for the past. The adaptation of parasite and host goes through stages called epidemic, endemic, and symbiotic. A germ entering a virgin population – one that is unfamiliar and has few defenses against it – often causes acute diseases in people of all ages. This is the classical picture of an epidemic: *if it involves much of the world, it is called* pandemic. *The survivors are usually left with improved defenses against reinfection; over generations, additional defenses may develop. The disease eventually becomes* endemic, *a widespread, lower-grade infection or routine childhood disease.*

For several millions years, the main causes of human deaths were accidents and wounds. With the coming of sedentary life, nutrition and longevity declined, famine and infection became the leading causes of death. For the next 10,000 years, it was common for microbes to strike people down at every stage from infancy to late life.

Farming, timbering, animal domestication, irrigation, pollutions and other traumas to natural ecosystems have created new breeding grounds for malaria, and such intensional diseases as dysentery and cholera. Plowing, irrigation and fertilizer invited other diseases into Neolithic settlements. Domesticated animals brought new diseases to Neolithic humans. Virtually, every step our ancestors took to increase and vary food supply invited more infections. Furthermore, when they had turned to farming 5000 years ago, a diet heavy in carbohydrates caused their size and health to decline.

Malaria *was not very important until humans created villages; than it became one of the most influential diseases in human history. (It still kills million children each year in Africa and another million in the rest of the world. Together, malaria, schistosomiasis and tuberculosis cause more sickness and death world-wide than any other three infectious diseases). For thousands of years, malaria, sleeping sickness, helminth infections and diarrheal diseases limited the size of human populations.*

The population explosion of the Bronze age, 6000 years ago, took city dwellers beyond a critical threshold. By the late Neolithic, irrigated fertile plains around the world supported cities as large as 100,000 people. These first appeared in the valleys of Tigris and Euphrates in Mesopotamia, then along the Nile in Egypt, the Indus in India, and the Yellow River in China.

Urban masses became sufficiently large and dense to support 'crowd diseases' (zymotic), what in other species are called 'herd diseases'. For the first time, infection became humanity's chief cause of death. (despite a few respites, this would remain true in the West until 20^{th} *century.) The reason epidemics did not take hold until urban times is simply the conditions imposed by numbers. While nomads were not free of infection, their most common diseases were chronic, not acute. People did not live densely packed together, aiding transmission of germs from one person to another. Their settlements were sufficiently far apart, and travel was sufficiently limited, to keep outbreak of diseases localized.*

Furthermore, most bacterial and viral infections with epidemic potential leave survivors temporarily or permanently immune. When such disease did jump from an animal to nomads or villagers, they flashed through the population; soon most of people in a community were either dead or immune. The microbes, having run out of susceptible hosts died off. Only years or generations later could the germ attack successfully again depending on new crop of susceptibles and another accident of reintroduction.

When farmers and villagers began crowding into cities, this immunologically virgin mass offered a feast to germs lurking in domesticated animals, wastes, filth, and scavengers. Countless people were sickened and killed by previously unknown epidemics – smallpox, measles, mumps, influenza, scarlet fever, typhus, bubonic plague, syphilis, gonorrhea, and common cold.

Plague became epidemic only 3000 years ago, when people began to live in large settlements. In the years 1490–1920 *typhus* killed more people than armies had. Thus, from their beginnings until the 20^{th} century, cities have been pestholes. Only when towns became big cities did massive die-off become regular part of human life.

Diseases associated with civilization is older than written history, for civilization of a kind existed before the earliest records were kept. The earliest known textbook of medicine, the Great Herbal *of the Emperor* ***Shen Lung****, dates from about 3000* BCE*, and there is a Babylonian physician's seal of approximately the same date in the Welcome Historical Medical Museum, London. Epidemic fevers are mentioned in the* Ebers Papyrus*, found in a tomb at Thebes (1862) and dated about 1500* BCE.

In the Old Testament (book of Exodus*), there is an account of a plague which smote Egypt about 1500* BCE*. The war-pestilence sequence is well described in* ***Samuel I****. The disease spread throughout Israel, bringing death (ca 1141* BCE*) to about 50,000 people. The plague of Athens (430* BCE*) provides a striking example of the effect of disease upon the course of history. The pestilence is supposed to have started in Ethiopia; from there it traveled to Egypt and was carried across the Mediterranean by ship to Piraeus and Athens. This plague undoubtedly contributed to the downfall of the Athenian empire.*

In general, foci of infections have developed in Mediterranean, Europe, Egypt, Mesopotamia, India and China. Each area, with its own climate, ecosystem, and germs, had a distinctive set of infections, to which people adapted with a distinctive complex of immune defenses. But commerce*,* war *and* travel *caused epidemics to spread across the world and merge all local foci into one. There were more people, bigger armies, better transport.*

Thus, for example, the conquest of South and Central America by the Spaniards in the 16^{th} *century ended with disastrous consequences for the Amerindians: In 1568, less than 50 years after Cortés arrived in Mexico, its total population was decimated from 30 million to mere 3 million. According to one estimate, smallpox alone killed 18 million people in Mexico in the* 16^{th} *century. In Peru, the Inca population sank from 8 million to 1 million. In North America and Australia, the native suffered similar slaughter by the European microbes.*

In the centuries after 1500, exploration, technology, and diseases gave Europe control over much of the world. By about 1700, most of its own diseases had been domesticated to endemics; the traffic of unfamiliar microbes was unidirectional: from Old World to the New. Pandemics, which began with *the Black Death, were now a regular feature of human life and the movement of hosts and pathogens all over the world would keep increasing.*

By the late 18^{th} *century, the Industrial Revolution was under way, and urban population soared. Social changes had never run at such a pace. The shift from nomadism to farming had taken millennia; the rise of industry and the megalopoli spanned only a couple of centuries. From the Neolithic to 1820, world population rose from 5 million to about 1000 million; much of the increase came at the end of the span. After population surges such as those of the Roman era and the Late Middle Ages, famine and plague had erased much of the gains. The growth that began in the* 17^{th} *and* 18^{th} *centuries was unique; it continued at an ever faster pace, and it ran out of control to this day.*

During 1951–1993 new diseases had their first appearance or recognition. Among them Ebola fever, Legionnaires' disease, AIDS. On the other hand, limited or controlled revival became widespread (e.g. Cholera, Diphtheria, Malaria, Syphilis, Tuberculosis). Even in places where they are controlled, they break out lethally when natural disaster, social chaos, or war disrupts modern defense against them.

Table 2.4 lists some historically significant epidemics since the first recorded worldwide plague of 767 BCE.

1332–1364 CE **Ibn al-Shatir** (1306–1375, Damascus). Astronomer. Made valuable astronomical observations with instruments of his own design, and criticized the accepted astronomical theories. He fully realized the need for continued and precise observations if one would discover the true motions of heavenly bodies. He determined the obliquity of the ecliptic[113] at Damascus in 1363/4 to be $23°31'$. (The correct value extrapolated from the present one is $23°31'19.8''$.)

1335 CE **Yaácov ben Asher** (1270–1343, Germany and Spain). One of the great codifiers of Jewish law. Fused three generations of European Talmudic thought into one great code. Through subsequent centuries this

[113] The value obtained by **Ibn Yunus** (990) was $23°35'$. That obtained by **William of Saint-Cloud** (fl. in Paris 1292–1296) for 1290 was $23°34'$ (the value for that year computed by means of LeVerrier's formula is ca $23°32'30''$).

Table 2.4: Historically significant epidemics

Year	Location	Disease	Comments
ca 1350 BCE	Asia Minor, Hittite Empire	unknown	
ca 1235 BCE	Egypt	Bubonic plague	*Exodus* **12**, 29
ca 1050 BCE	Israel	Bubonic plague	*Samuel* **5**, 9-12
767 BCE	Europe and Mediterranean world	Bubonic plague	First recorded world-wide epidemic
480 BCE	Persian army of Xerxes	Dysentery	Herodotos
430 BCE	Ethiopia, Egypt, Athens	Scarlet fever	Thucydides
187 BCE	Egypt, Syria, Greece	unknown	Pliny
79–88	Egypt, Syria, Italy, Rome	Bubonic plague	
165–189	Roman Empire	Smallpox	Galen
251–270	Roman Empire, Rome	Smallpox, measles and malaria	caused mass conversion to Christianity
542–594	Europe, Asia, Africa	Bubonic plague	Old world pandemic; began in Ethiopia and Egypt; millions perish
735–736	Japan	Smallpox	Facilitated spread of Buddism
746–749	Constantinople, Greece, Italy	Bubonic plague	200,000 perish
1097	Near East (1^{st} crusade)	Typhoid fever	100,000 die
1218	Egypt		67,000 die
1235	England		40,000 die
1332–1370	Europe and Asia	Bubonic plague	ca 50 million die; 'Black Death' pandemic;
1408–1551	England	Bubonic plague	("Sweating Disease")
1494–1495	Europe	Syphilis	

Table 2.4: (Cont.)

YEAR	LOCATION	DISEASE	COMMENTS
1507–1595	South and Central America	Smallpox, typhus	ca 80 million natives die
1528–1530	Italy	Typhus	More than 100,000 die
1591	Philippines	Smallpox	
1600	Russia	Bubonic plague	500,000 perish
1590–1711	European cities (Italy, England, France)	Bubonic plague	ca 2.5 million perish
1760–1799	North Africa, Egypt, Syria	Bubonic plague	ca 1.2 million perish
1812–1813	Napoleon's army	Dysentery	ca 500,000 die
1817–1893	Worldwide (5 waves)	Cholera	ca 10 million perish in the pandemic; spread by contaminated water
1851–1855	England	Tuberculosis	250,000 die
1889–1890	Worldwide	Influenza	ca 3 million die; moves with a speed of trains and steamships
1898–1923	China, India, North Africa and South America	Bubonic plague	ca 20 million die
1917–1920	Europe and Asia	Influenza, typhus	ca 25 million die
1921–1923	India	Cholera	ca 1 million die
1926–1930	India	Smallpox	ca 500,000 die
1981–	Worldwide	AIDS	

code formed the cornerstone of Jewish legislation. Moreover, it played a direct and vital part in the creation of the legal system of Western civilization[114]

Ben Asher was born at Cologne, Germany. Due to local persecution of Jews he was forced to flee, with his father (1303), through Savoy and Provence to Barcelona. About 1314 he was living in poverty and sickness in Toledo, and in fact he endured privation throughout his life, but declined a position as a rabbi. Through his father, a great Talmudist in his own right, he became acquainted with the works of the Franco-German scholars; in Spain he became acquainted with the Spanish Talmudists, and his subsequent wandering through Western Europe familiarized him with the customs of its varied communities.

His life work, which revealed a profound knowledge of Jewish literature, was his *Arba Turim* (The Four Columns), after the four columns of Jewels on the breast place of the high priest. It was intended to supply coordinated information for the average Jew, and therefore concerned itself only with those laws which were still in force after the destruction of Temple. Ben Asher started this monumental undertaking when he was a young man. The laws are grouped in the form of connected arguments, recording all the decisions of earlier rabbis down to his own. They terminate in a decision, and the author usually makes his father the decisive authority. Yaacov writes very objectively and never tries to force an opinion upon the reader. The work became so popular that it was regarded as "the people's possession and people's lawbook of the entire Jewish world". Many commentaries on it were written, and glosses were frequently added to it; it eventually became the basis of the *Shulhan Aruch*.

Ben Asher's code answered the need of the times because it combined the rich strands of French, German, and Spanish learning into one tapestry of

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114 When the Jews arrived in England, the English method of settling legal disputes was through trial by combat. The Jews, on the other hand were accustomed to judicial procedure base on evidence, examination of witnesses and impartial judges. Thus, they demanded and were granted the right to use Talmudic guidelines in dispute with Christians. As early as the 2^{nd} century CE Talmudic law had specified that in property disputes the verdict of 12 men, agreed upon the litigants would be legally binding on both parties. After a century, even the Anglo-Saxons found the Jewish method of settling disputes better then trial by combat. By the 13^{th} century this "Jury" method found its way into British common law.

The famed due process of law concept, so firmly embedded in the Fifth and Fourteenth Amendments of the American Constitution, and derived from the Magna Carta, stems from the 10^{th} century interpretation of the Talmud. This Talmudic concept of due process of law was stated most succinctly by Maimonides several decades before the signing of the Magna Carta.

European Talmudism. His lucid writing, his logical arrangement of subject matter, his encyclopedic knowledge of the entire gamut of Talmudic development over 300 years, and his clever way of presenting dissenting opinion while pointing a way out of the jungle of dissent, made his *Four Columns* the most popular and definitive code.

1337–1453 CE The *Hundred Year War*. A struggle between England and France for control of France that consisted of a successions of wars broken by truces and treaties. It extended over the reign of five English and five French kings and culminated in the *battle of Agincourt* (1415), where Henry V won one of the most famous victories in English history with an army that was the best-trained and best equipped fighting force since the Roman legions (this battle was immortalized by **Shakespeare** in his historical play *Henry V*).

The English won most of the battles, but the French won the War. By the time the war ended in 1453, England had lost all its territory on the continent of Europe, except Calais, which was regained by France in 1558.

Firearms were first used in the Hundred Year's War. *Artillery* came into use during 1335–1345 and the major battle involving firearms was on Aug 26, 1346: Edward II of England invaded France with 10,000 man and defeated some 20,000 Frenchmen at Crécy.

1338–1353 CE **Giovanni dei Marignolli** (ca 1285–1357, Italy). Traveler Franciscan Friar. Left a journal of his travels in China, India, Ceylon, Persia, Mesopotamia, Syria and Jerusalem (1352). He was sent on a mission to the court of Emperor Togon Temür of China by Pope Benedict XII (1338), reaching Peking (1342), where he remained for four years. He returned to Arignon (1353), delivering a letter from the Mongol-Chinese Emperor to Pope Innocent VI. Later (1354), Marignolli became chaplain to Emperor Charles IV.

ca 1340–1377 CE **Immanuel ben Yaacov Bonfils** (14^{th} century, France). Physician, mathematician and astronomer. Taught and wrote in Orange and Tarascon. Published the highly regarded astronomical tables, *Kanfe Nesharim* (Wings of Eagles; *Exodus* **19**, 4; *Isaiah* **6**, 2) [which were translated into Latin in 1406 and commented upon in Greek], primarily as an aid in the determination of the Hebrew calendar.

Manuscripts of 15 of his works have survived to our time [nine of them are in the National Library of Paris]. One of them, *Derech Hilluq*, contains an exposition of an arithmetic which consistently employs the *decimal system* of notation for integers and fractions, and positive and negative exponents. It also includes an algorithm for root extraction.

Immanuel of Tarascon thus took his place in the line of the contributors to the development of the decimal system.

Mathematics in the Medieval world[115] (800–1500 CE)

A. Jewish contribution (1100–1500 CE)

Talmudic scholars in South-Western Europe were among the leading mathematicians in the Middle Ages (see Table 2.1).

Bar Hiyya *was a key figure in the transmission of mathematics to the Christian West, translating texts from Arabic and Hebrew into Latin. In his book Liber embadorum he presented a complete solution of the quadratic equation*

$$x^2 + b = ax,$$

showing that it has two roots. The book exerted deep influence upon the development of Western mathematics and was used by Leonardo of Pisa as the foundation for his text books on arithmetic, geometry and trigonometry.

Levi ben Gershon *was one of the leading mathematicians and astronomers of the European Middle Ages. In his book 'Sefer ha-Mispar' ('Book*

[115] For further reading, see:

- Lindberg, D.C. (Editor), *Science in the Middle Ages*, University of Chicago Press: Chicago, 1978, 549 pp.
- Grant, E. (Editor), *A Source Book in Medieval Science*, Harvard University Press: Cambridge, 1974, 864 pp.
- Jones, Terry and Alan Ereira, *Medieval Lives*, BBC Books, 2006, 224 pp.
- Rundle, D. Ed., *The Hutchinson Encyclopedia of the Renaissance*, Helicon Publishing, 1999, 434 pp.
- Sarton, G., *Six Wings*, (Men of Science in the Renaissance), Indiana University Press: Bloomington, 1957, 318 pp.
- Dales, R.C., *The Scientific Achievement of the Middle Ages*, University of Pennsylvania Press: Philadelphia, 1973, 182 pp.
- Johnson, Paul, *The Renaissance*, The Modern Library, 2000, 196 pp.

of Numbers'), he calculated, for the first time, the number of permutations, $P_{n,r}$, of n objects, taken r at a time.

$$P_{n,r} = n(n-1)\dots(n-r+1), \qquad P_{n,n} = n!.$$

He then calculated the number of combinations, $C_{n,r}$, of n objects taken r at a time.

$$C_{n,r} = C_{n,n-r}, \qquad C_{n,r} = \frac{P_{n,r}}{P_{r,r}}.$$

He also found that

$$\sum_{n=1}^{N} n^3 = \Big(\sum_{n=1}^{N} n\Big)^2,$$

and proved that the numbers 2^n, 3^m differ by more than a unit, except for $(n,m) = (1,0), (1,1), (2,1), (3,2)$.

He proved his results by the Principle of Mathematical Induction, hitherto an unknown principle, first used by him in 1321. As we now formulate this method of the proof, a property $S(n)$ of natural numbers n is proved to hold for all n if we can prove $S(1)$ (the base step) and, for arbitrary n, $S(n) \Longrightarrow S(n+1)$ (the induction step).

***Francesco Maurolico** (1575) was the first to formulate the principle of induction. **Pascal** (1655) borrowed the idea from Maurolico and clarified it in his treatise Tracté du triangle arithmetique.*

Levi foreshadowed Copernicus in his firm objection to the Ptolemian system. Stressed the importance of the sine function, emphasized the importance of plane trigonometry and developed it, including the law of sines.

***Aramah** was active during 1470–1494 as a philosopher, mathematician and Talmudic scholar. He was first to formulate the statistical law of large numbers (1470) in his book 'Akedat Itzhak' (published in Saloniki: 1522). There he stated, over two centuries ahead of **Jacob Bernoulli** (1770):*

> *"Ordinary lots due to chance are without any tendency to one side or the other... They are not a 'sign', for matters of this kind are not established unless they are found many times... The casting of a lot indicates primarily a reference to chance."*

Aramah was born in Zamorah, Spain and served as a head of rabbinical academies in various Jewish learning centers. He was expelled from Spain (1492) and died in Naples, Italy. His philosophical system was influenced by Aristotle and Maimonides, and deals specifically with such major questions as faith and reason. Aramah became popular and influential and his thinking represents the mainstream of Jewish medieval philosophy.

Immanuel Bonfils *calculated highly regarded astronomical tables, employed the decimal system of notation for integers, fractions and positive and negative exponents (1340 CE). He also developed an algorithm for root extraction.*

Avraham Ibn Ezra *wrote five books on mathematics and astronomy, including research on arithmetic, number theory, combinatorics and astronomical tables. He adopted the positional decimal system (1140 CE) for integers with place-values from left to right, and denoting the zero by a special sign of the wheel ('galgal'* in Hebrew*). In his book 'Ta'hbulah' he discussed the Josephus problem. In his Sefer-ha-Mispar* (book of numbers) *we find the rule of summation of an arithmetical progression: "Whoever would know how great the sum of the numbers is which follow one another in a series to a certain number, multiply this by its half increased by $\frac{1}{2}$." That is*

$$S_n = n\left(\frac{n}{2} + \frac{1}{2}\right) = \frac{n(n+1)}{2}.$$

Ibn-Gabirol *(fl 1040–1058) was a philosopher of striking originality, whose philosophical system had powerful impact upon medieval Christian thinkers as well as on* **Spinoza** *and* **Schopenhauer**. *His chief work, the 'Source of Life' was translated from the Arabic into the Latin in the middle of the* 12^{th} *century by* **Dominicus Gundissalvus**, *archdeacon of Segovia, under the name 'Fons Vitae'. And just as Ibn Sina was corrupted into* ***Avicenna*** *and Ibn Rushd into* ***Averroes***, *so Ibn-Gabirol traveled down the ages under the disguise of* ***Avicebron***.

Since his work shows a total and absolute independence of Jewish religious dogma, he exercised little influence upon Jewish thought. His doctrine of creation is at the base of the Lurianic cosmology which itself was later found to be coherent with modern cosmology.

Maimonides *was first to claim clearly that π is not rational (1160 CE) and can only be approximated by ratio of integers. He further stated (without proof) that squaring the circle by rules of Greek geometry is impossible.*

In his book Mishneh Torah ("Repetition of Teaching" the most distinguished code of Jewish law), Maimonides discusses the mathematical theory of visibility of the moon, needed for ascertaining the beginning of the month ('consecration of the moon'). Therein he developed a simple approximation method of ascertaining whether the moon was visible from the West point. In this connection he writes (Hilchoth Kiddush Hachodesh, Chapter ***11***, *1-17):*

> *"Lest any mathematician think that the approximate methods sometimes described are due to my ignorance of the more accurate*

> *mathematical ones, let him dismiss such an idea from his mind. I have only employed such methods when I was convinced by comparison with the results obtained by the strictly mathematical - but more laborious - methods that the calculation of the moon's visibility is not appreciably affected by such approximations. He will further find that any appreciable positive or negative differences incidental to such approximate methods cancel themselves out, with the consequence that sufficiently accurate results are finally obtained without the labor of a long calculation which might frighten away the non-mathematical reader".*

No modern applied mathematician could have issued a better apologia for approximate methods them did Maimonides 800 years ago!

Maimonides made a significant contribution to the physical concept of time. In his work The Guide to the Perplexed (1190) he wrote (in Arabic):

"Time is composed of time-atoms, i.e., of many parts, which on account of their short duration cannot be divided... An hour is, e.g., divided into 60 *minutes, the minute into* 60 *seconds, the second into* 60 *parts and so on; at last after ten or more successive divisions by sixty, time-elements are obtained which are not subjected to division, and in fact are indivisible...".*

He thus concluded that there were 60^{10} *or more such time-atoms in one hour!*

The notion of Maimonides that time is composed of 'time-atoms' is known today in the parlance of physicists as chronon. *If indeed space-time is discrete (quantized), then the scale must be very small to agree with experimental observations. Indeed, the present smallest directly observable division of a second, which is better then* 1×10^{-13} *sec, is coming close to Maimonides' division of* $60^{-10} \sim 5 \times 10^{-15}$ *sec.*

Levi ben Avraham *was a natural philosopher, astronomer and mathematician. Recognized heat as a form of motion, 400 years before* ***Robert Boyle*** *(1675 CE).*

Levi was the grandfather of Levi ben Gershon. He lived in poverty, making a living through the teaching of sciences and foreign languages. Persecuted by the rabbinic establishment because of his rationalistic interpretation of the Bible, he was forced to wander in Provance (Narbonne, Beziers). He died in Arles.

Yosef Caspi*, in his book Will and Testament (1332), maintained that natural phenomena can sometimes violate the laws of nature over a small interval of time. (He used this argument to render the logical explanation to biblical miracles). This idea was ahead of its times by more than 600 years.*

Yehudah ibn Verga *invented (Lisbon, 1457) a new instrument to determine the sun's meridian and wrote a number of books on mathematics and astronomy. He died in the dungeons of the Portuguese Inquisition (1499).*

Zacuto*'s astronomical tables, maritime charts and new astrolabe played an important role in the Spanish and Portuguese discoveries, especially in the voyages of Columbus and Vasco da Gama.*

Zacuto's achievements in astronomy were many: his astrolabe of copper, the first of its kind (previously they had been made of wood), enabled sailors to determine the position of the sun with greater precision; his astronomical tables, based on the Alphonsine tables, were an improvement on the latter. They permitted sailors to ascertain latitudes without recourse to the meridian of the sun, and to calculate solar and lunar eclipses with greater accuracy.

Columbus used Zacuto's tables on his voyages, and on one occasion they were instrumental in saving him and his crew from certain death. Knowing from the Zacuto tables that a lunar eclipse was imminent, Columbus threatened the natives that he would deprive them of the light of the moon as well as of the sun. (A copy of the tables, with Columbus' notes, is preserved in Seville). Zacuto's astronomical work Ha-Hibbur ha-Gadol (the Hebrew original is extant in several manuscripts) enjoyed a wide reputation during his lifetime.

Yehudah Abravanel *(fl. 1490–1520 CE) was a mathematician, astronomer, physician and one of the great philosophers of the Renaissance. In 1497 he drew attention to the periodic conjunction of Jupiter and Saturn, occurring about every 20 years. The sign of the Zodiac in which they occur changes from one conjunction to the other. This study was consulted by* ***Johannes Kepler*** *(1603) and prompted him to advance the hypothesis that the 'star of Bethlehem' was indeed a conjunction of Jupiter and Saturn in Pisces in 7* BCE.

B. Muslim contribution (800–1400 CE)

Hindu work on mathematics and astronomy during 200–1400 CE spread westward, reaching the Arabs who, in turn, absorbed, refined and augmented what they received, before transmitting the results to Europe. By far, the greatest contribution of the Arabs was to pursue a process of creative synthesis in which they blended a variety of earlier mathematical traditions including the Babylonian, Greek, Indian, Persian and Chinese.

This they did with an openness of mind and clear understanding of need in mathematics (and other sciences) to balance empiricism and theory. It is most apparent in their astronomical tables, their algebraic approach to applied mathematical problems, the popularization of our present-day numerals, the first systematic treatment of trigonometry and the bringing together of the geometric and algebraic approaches to the solution of equations.

The astronomers and mathematicians who contributed mostly to this program were: ***al-Kharki*** *(950–1029);* ***al-Khowarizmi*** *(780–850);* ***abu-Mahsar*** *(805–885);* ***al-Kindi*** *(815–873);* ***ibn-Qurra*** *(826–901);* ***al-Harrani*** *(836–901);* ***al-Battani*** *(850–929);* ***al-Wafa*** *(940–998);* ***Ibn Yunus*** *(940–1009);* ***al-Haitam*** *(965–1039);* ***al-Biruni*** *(973–1048);* ***Omar Khayyam*** *(1048–1126);* ***al-Marakushi*** *(1190–1265);* ***al-Tusi*** *(1201–1274);* ***al-Shirazi*** *(1236–1311);* ***al-Shatir*** *(1306–1375)* and ***al-Kashi*** *(1360–1436).*

The Arabs played a seminal role in transmitting mathematics to Western Europe, setting the stage for the development of modern mathematics. But they were not just custodians of Greek learning and transmitters of knowledge. They brought together two different mathematical strands - the algebraic *and* arithmetic *traditions so evident in the mathematical cultures of Babylonia, India and China, and the* geometric *traditions of Greece and the Hellenistic world. The intertwining of these strands had already begun with the latter Alexandrian mathematicians* ***Heron****,* ***Diophantos*** *and* ***Pappos****, who had absorbed much of their mathematics from Babylon and Egypt. They could not, however, break loose from the constrains imposed by the straight jacket of Greek mathematical tradition.*

It was left to the Arabs to bring together the best of both traditions. In doing so, they provided us with an efficient system of numeration*, in which calculations, were no longer tied to mechanical devices, an* algebra *which was both practical and rigorous, a* geometry *which was no longer an intellectual pastime, and a* trigonometry *freed from its ties to astronomy to become indispensable tool in fields as diverse as optics and surveying.*

Even before the beginning of Arab rule, knowledge of Hindu numerals had spread westwards. Christian sects, particularly the Nestorians and Syrian Orthodox denominations, needed to calculate an accurate date for Easter, and various astronomical texts were examined with this in mind, In fact, it was a problem that continued to occupy mathematicians, including ***Gauss****, down to the* 19^{th} *century. There is also the possibility, given the thriving commercial relations between Alexandria and India, that the Hindu numeral system reached the shores of Egypt as early as the* 5^{th} *century CE. It would have been regarded as a useful* commercial device *rather then a tool for scientific and astronomical calculations since Alexandrian scientists continued to use the die-hard Babylonian sexagesimal system.*

***Fibonacci** (1170–1250), who was first introduced to Hindu numerals by his Arab teachers, quickly recognized the enormous advantage of the Hindu system. The change was, however, a slow process, primarily because the abacus remained popular for carrying out calculations, and traders and others engaged in commercial activities were reluctant to adopt the new system which was difficult to comprehend.*

Before Muhammed, the Arabians wrote out all numbers in words. Less then a century after the Hegira (622 CE) their empire extended from India to Spain, including North Africa, Southern Italy and large parts of Western Asia. In 775 the Islamic Empire split into a Western Kingdom with its capital at Cordova and an Eastern Kingdom centered on Baghdad. Both kingdoms rapidly developed a rich culture, absorbing intellectual nourishments from the Greek, Jewish, Persian and Hindu worlds. The 9^{th} *century was the Golden Age of Arab mathematics. Baghdad, with its fine library and lavishly equipped astronomical observatory, became the new Alexandria.*

*The first Arabic arithmetic known to us is that of **al-Khowarizmi**, who is believed to have visited India. On his return, about 830* CE, *he wrote his algebra treatise, which was founded on the work of **Brahmagupta**. The Hindu numerals, adopted by the Arabs were brought to Spain at about 900 CE. From there they diffused to the rest of Europe by traders in the Mediterranean area and by scholars who attended the universities in Spain. It finally came into general use in Europe by the invention of the printing press in the mid 1400's. His second book on algebra also shows little originality.*

*Although **al-Khowarizmi** is often hailed as the 'Father of Algebra', in two respects he looked backwards rather than forward: he rejected both algebraic symbolism (his treatment is entirely rhetorical) and the Hindu acceptance of negative roots and negative coefficients of equations.*

***Omar Khayyam** went beyond al-Khowarizmi to investigate cubic and even some quadratic equations. However, he mistakenly believed that the general cubic equation could not be solved algebraically, but that geometrical methods, involving the use of conic sections, were necessary. He also did not recognize negative numbers.*

Arab mathematicians addressed problems of inheritance posed by Islamic law; geometric solutions of cubic equations, introduction of 6 basic trigonometric functions and the construction of highly detailed trigonometric tables with the aid of various interpolation procedures.

*One of their greatest scholars was **Nasir al-Din al-Tusi**, mathematician, astronomer, physician and philosopher. In geometry, he made first attempts to determine whether Euclid's parallels postulate can be derived from the other Euclidean postulates. In trigonometry he presented the law of sines and began to separate trigonometry from astronomy. His work in these two* *areas may have influenced the further advances of* ***Regiomontanus*** *(1464) and* ***Saccheri*** *(1733).*

In 1259 al-Tusi began the construction of a major astronomical observatory at Marãgha, where he used his self-made quadrants to observe star positions. During 1256–1265 he conducted observation which served as a basis for his astronomical tables (1272). His criticism of Ptolemaic astronomy was an additional step toward the Copernican reform.

Some highlights from the works of other Arab mathematicians of the 9^{th} *and* 10^{th} *century are:*

- ***Al-Harrani*** *(855 CE) proved that if*

$$p = 3 \cdot 2^n - 1, \quad q = 3 \cdot 2^{n-1} - 1, \quad r = 9 \cdot 2^{2n-1} - 1,$$

  and if p, q *and* r *are primes, then* $2^n pq$ *and* $2^n r$ *are amicable numbers.*

- ***Abu al-Wafa*** *(970 CE) Contributed considerably to the development of trigonometry. He was probably the first to show the generality of the sine law relative to spherical triangles*[116]*. He gave a new method of constructing sine tables, his value of sin* $(30')$ *being correct to 8 decimal places. He knew relations equivalent to ours for* $\sin(\alpha \pm \beta)$ *(though in an awkward form) and to*

$$2\sin^2\frac{\alpha}{2} = 1 - \cos\alpha, \qquad \sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}.$$

  He calculated a table of tangents and introduced the secant and cosecant.

- ***Al-Kharki*** *(985 CE) gave a solution in rationals to* $x^3 + y^3 = z^2$ *namely*

$$x = \frac{n^2}{1+m^3}, \quad y = mx, \quad z = nx,$$

  where (m, n) *are arbitrary rational numbers.*

- ***Ibn Yunus*** *(990 CE) Prepared improved astronomical tables based on his observations at the Cairo observatory. Improved the values of*

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[116] It is hard to determine who discovered the planar *sine law*

$$a/\sin A = b/\sin B = c/\sin C.$$

It is quite certain that **Ptolemy** knew it. It may however been rediscovered by Abu al-Wafa or his disciple **Abu Nasr Mansur** (970–1036 CE).

astronomical constants (inclination of the ecliptic, $23°35'$; longitude of the sun's apogee, $86°10'$; solar parallax, $2'$; precession, $51.2''$ a year). Introduced the trigonometric formula

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)].$$

Ibn Yunus described 40 planetary conjunctions accurately and 30 lunar eclipses used by Simon Newcomb (1876) in his lunar theory.

- ***Alhazen** (1000 CE) solved the following problem: given a light source and a spherical mirror, find the point on the mirror where the light will be reflected to the eye of an observer. This can be reduced to the planar geometrical construction: from two points in the plane of a circle to draw lines meeting at a point of the circumference and making equal angles with the normal at that point. It leads to an equation of the 4^{th} degree. Alhazen solved it by the aid of an hyperbola intersecting a circle. In a similar way he also solved the cubic equation* $x^3 + c^2b = cx^2$, *which results from the* Archimedean problem *of dividing a sphere by means of a plane into two segments at a given ratio to one another.*

The Muslim contribution to the development of mathematics, viewed as a whole, exhibits a nice blending of Greek, Babylonian and Hindu influences. While making small advances, some of their achievements, when viewed against the scientifically sterile backdrop of the rest of the world of the time, seem greater then they really were. Yet, there remains the outstanding fact that they served admirably as custodians of much of the world intellectual possessions, which were transmitted to the later Europeans after the Dark Ages had passed.

The debt of the West is twofold. First, the Islamic scholars collected, preserved and translated the Classical Greek mathematical texts. Secondly, they adopted the fully developed Hindu system of numeration, which was in due time transmitted to the West and eventually to the whole world. Although the new system had become known in the West by the year 1000, it took several centuries to displace the Roman number-language in Western Europe and the Ionic Greek number-language in the Byzantine Empire. We must remember that most arithmetical calculations were performed on an abacus or a counting-frame; only the results needed to be recorded on paper, and for this the Roman or Greek numerals were quite adequate. Indeed, both the medieval abacus and the counting-frame were direct physical analogues of the Hindu decimal system of numeration, with the symbol for zero corresponding *to the empty column. The battle between the 'abacists' (the supporters of the old Roman numbers) and the 'algorists' (who advocated the new system of al-Khowarizmi) continued for centuries. As late as 1299 the city of Florence issued an edict prohibiting the commercial use of the Hindu-Arabic numerals; they were thought to be too easy to falsify on accounts.*

The loss of Toledo by the Moores to the Christians (1085 CE) was followed by an influx of Christian scholars to that city to acquire Muslim learning. Other Moorish centers in Spain were infiltrated and the 12^{th} *century became, in the history of mathematics, a century of translators. Another center of interaction and transmission was the island of Sicily.*

The location and political history of Sicily made that island a natural meeting ground of East and West. Sicily started as a Greek colony, became part of the Roman Empire, linked itself with Constantinople after the fall of Rome, was held by the Arabs for about 50 years in the ninth century, was recaptured by the Greeks, and then taken over by Normans. During the Norman regime the Greek, Arabian, and Latin tongues were used side by side, and diplomats frequently traveled to Constantinople and Baghdad. Many Greek and Arabian manuscripts in science and mathematics were obtained and translated into Latin. This work was greatly encouraged by the two rulers and patrons of science, Frederick II (1194–1250) and his son Manfred (ca. 1231–1266).

Among the first cities to establish mercantile relations with the Arabic world were the Italian commercial centers at Genoa, Pisa, Venice, Milan, and Florence. Italian merchants came in contact with much of Eastern civilization, picking up useful arithmetical and algebraical information. These merchants played an important part in the dissemination of the Hindu-Arabic numeral system.

C. Christian contribution - Arithmetic comes of age (1200–1600 CE)

After the absorption of the Hindu-Arabic number system, and the stabilization of the numeral forms, the time was ripe for the standardization of the four fundamental operations – addition, subtraction, multiplication and division. All these evolved slowly from the times of ancient Egypt and Babylon, and their final form became common in Europe not before the dawn of the 17^{th} *century.*

Addition: The operation has not changed much since Hindu-Arabic numerals began to be used, as is evident from ***Bháskara*** *(ca 1150) in his Lilavati.*

The "carrying" process dates from the time when a counter was actually carried on the line abacus to the space or line above.

The evolution of the arithmetical procedure can be followed in the writings of ***Fibonacci*** *(1202),* ***Pacioli*** *(1494),* ***Gemma Frisius*** *(1540),* ***Recorde*** *(1542) and* ***Digges*** *(1572). The name of the operation has had its vicissitudes: writers used 'aggregation', 'composition', 'collection', 'assembling', 'joining' and 'summation'.*

Subtraction: The name of the process since Fibonacci (1202) went through the phases: extraction, detraction, subduction, deduction, rebating and diminishing. Only in the 19^{th} *century (!) did subtraction become common in England and America. There have been, throughout the past nine centuries about five different processes of subtraction, some of which are practical today. Among them, the plan of simple borrowing (e.g. in the operation*

$$\begin{array}{r} 42 \\ -\underline{27} \\ 15 \end{array}$$

the computer says: "7 from 12 is 5, 2 from 3 is 1"). This plan is very old and goes back to ***Avraham ibn Ezra*** *(ca 1140). The computer always begins at the right and looks ahead to take care of the borrowing. This feature is Oriental and used in the work of* ***al-Khowarizmi*** *(ca 825).*

Multiplication: We know little about the methods of multiplication used by the ancients. The Egyptians probably made some use of the duplation plan, which accounts for the presence of the chapter on duplation in so many books on the Renaissance period. Indeed, even a good a mathematician as ***Stifel*** *(ca 1525) still multiplied by successive duplation according to the scheme*

$$\begin{array}{rl} 1 \cdot 42 = & 42 \\ 2 \cdot 42 = & 84 \\ 4 \cdot 42 = & 168 \\ 8 \cdot 42 = & 336 \\ 16 \cdot 42 = & 672 \\ \hline 31 \cdot 42 = & 1302 \end{array}$$

The basic idea here is the expansion of one of the multipliers in the binary system $(31 = 1.2^0 + 1.2^1 + 1.2^2 + 1.2^3 + 1.2^4)$. *Our common form appears in Pacioli's Suma (1484) together with seven other plans of multiplication.*

The oldest multiplication-tables are found already in Babylonian tablets, where they appear in columns. The square form is found in the arithmetic[117] *of* ***Boethius*** *(ca 510) and* ***Bede*** *(ca 710), and became common in the* 13^{th} *century, as is evident from the writings of* ***Jordanus Nemorarius*** *(1225).*

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117 *Arithmetic*: from the Greek *arithmeein* - to number, to count.

Many of the 16^{th} *century writers found it necessary to urge their pupils very strongly to learn the tables, showing that the custom was relatively recent in countries where the abacus had only just been abandoned.*

Division: Defined as an unknown number R*, satisfying the equation* $a = bR$ *when* a *and* b *are given. The oldest form of division (known also as 'partition' in the* 13^{th} *century) is the one used by the Egyptians. The process was based upon duplation and mediation: e.g. to divide* 19 *by* 8*, they prepared a list of multipliers of* 8 *by* 2, 1, $\frac{1}{4}$, $\frac{1}{8}$*, namely* 16, 8, 4, 2, 1 *and then selected from these numbers these which have* 19 *for their sum. Since* $19 = \left[2 + \frac{1}{4} + \frac{1}{8}\right] \times 8$, *the result of the division is* $\frac{19}{8} = 2\frac{3}{8}$ *(in modern notation).*

It is impossible to fix an exact date for the origin of our present algorithm of long division, partly because it developed gradually over some 600 *years from the time of* ***al-Khowarizmi*** *(ca 825).* ***Gerbert*** *(ca 980) introduced the operation of 'long division'. It may be, illustrated by the simple case of* $\frac{900}{8}$*. The process consists of dividing* 900 *by* $10 - 2$ *(2 being the complement of the divisor) and runs essentially as follows:*

$$
\begin{array}{l|l}
900 & 10 - 2 \\
\underline{900 - 180} & 90 + 18 + 3 + 1 + \frac{1}{2} = 112\frac{1}{2} \\
\quad 180 & \\
\quad \underline{180 - 36} & \\
\qquad 36 & \\
\qquad \underline{30 - 6} & \\
\qquad\ 6 + 6 = 12 & \\
\qquad\qquad \underline{10 - 2} & \\
\qquad\qquad\ 2 + 2 = 4, \qquad \frac{4}{8} = \frac{1}{2} &
\end{array}
$$

The method was refined by ***Calandri*** *(ca 1491) who was first to exhibit it in a printed book. With the opening of the* 17^{th} *century it began to replace other methods.*

Roots: ***Theon of Alexandria*** *(ca 390) used sexagesimals and Euclid's identity* $(a+b)^2 = a^2+2ab+b^2$ *to extract the square root of an integer. From Greece the method passed over to the Arabs and Hindus, with no particular improvement.*

Robert of Chester *(England) introduced (1145–9) the current mathematical terms: 'Algebra' (for 'al-jabr' of* ***al-Khowarizmi****), 'algorism' (algorithm), 'sine' (trigonometry). The Hindus have given the name 'jiva' to the* *'half-cord' in trigonometry, which the Arabs took over as 'jiba'. In the translation he confused it with the Arabic word 'jaib' meaning 'bay' or 'inlet' – which he translated into the Latin 'sinus'.*

One of the first, and certainly the most influential, of the medieval textbooks on the new arithmetic was the Liber abaci*, which was completed in 1202 and revised in 1228. Its author,* ***Leonardo of Pisa*** *(ca 1175–1250), is better known as* ***Fibonacci*** *('son of good nature'). His father was a Pisan merchant who also served as a customs officer in North Africa. The young Leonardo traveled widely and learned the Arabic methods from a Muslim teacher. His extended trips to Egypt, Sicily, Greece, and Syria brought him in contact with Eastern and Arabic mathematical practices. Thoroughly convinced of the practical superiority of the Hindu-Arabic system, Fibonacci published his* Liber abaci *soon after his return home. This work contains a large collection of problems which served later authors as a storehouse for centuries.*

Liber abaci *was the first complete and systematic explanation of the Hindu numerals by a Christian writer; and also, naturally the first complete exposition of Hindu and Muslim arithmetic. Leonardo, however, gave more rigorous demonstrations than the Muslims. (It is apparent that he had a good knowledge not only of Muslim, but also of Greek mathematics, largely derived from Latin translations of Euclid, Archimedes, Heron and Diophantos.)*

Liber abaci *contains sections on Roman and Indian numerals and on finger counting. Later chapters are devoted first to commercial calculations and then to puzzles and recreational mathematics – including the famous 'rabbit problem' which lead to the Fibonacci sequence. He deals with approximating square roots, cube roots and problems on volumes, in which he takes* π *to be* $3\frac{1}{7}$.

The 14^{th} *century was a relatively mathematically barren one. It was a century of* Black Death*, which swept away more than a third of the population of Europe, and in this century the* Hundred Years' War*, with its political and economic upheavals in Northern Europe, got well under way.*

Four significant events influenced the evolution of mathematics and science in Western Europe from the 15^{th} *century onward:*

- *The collapse of the Byzantine Empire, culminating in the fall of Constantinople to the Turks (1453). Refugees flowed into Italy bringing with them treasures of Greek civilization. Many Greek classics, hitherto known only through the often inadequate Arabic translations, could now be studied from original sources.*

- *The invention of printing (1440). It enabled the dissemination of knowledge at an unprecedented rate.*

- *The discovery of America (1492) and the consequent circumnavigation of the Earth.*
- *The ruin of the Arab centers of learning and the virtual imprisonment of the Jews in their ghettos left Christians alone at the center stage of science for the next 300 years. The Jews will return with great vigor, following their emancipation in the Napoleonic era.*

The mathematical activity in the 15^{th} *century was largely centered in the Italian cities and in the central European cities Nüremberg, Vienna and Prague. It concentrated on arithmetic, algebra and trigonometry and flourished principally in the growing mercantile cities under the influence of trade, navigation, astronomy and surveying. With the interest in education and tremendous increase in commercial activity, hosts of popular textbooks in* arithmetic *began to appear.*

New ideas in the 14^{th} *and the* 15^{th} *century were advanced by 'three Nicolas':*

1370	*Nicole Oresme*	*1323–1382*	*France*
1440	*Nicolas of Cusa*	*1401–1464*	*Germany*
1485	*Nicolas Chuquet*	*1445–1488*	*France*

Another important mathematician, belonging to the same period is ***Johannes Müller (Regiomontanus)****. Being both astronomer and mathematician he advanced* spherical trigonometry *(1470).*

1344 CE **Gregorio da Rimini** (d. 1358, Italy). Augustinian Hermit, theologian and Ockhamist. Studied in Rimini and Paris (1323–1329). In 1351 he left Paris and returned to the Augustinian house in Rimini. Died in Vienna. Gregorio addressed the issues of continuity and infinity, discussing such problems as whether an infinite spiral can exist on a finite body.

1347–1351 CE The *Black Death* epidemic of bubonic plague wreaked havoc throughout Europe and Asia killing about 50 million people. One of History's greatest natural disasters. It started in 1338 near Lake Issyk-Kul (ca 78 °N, 43 °E, south of Alma-Ata) in one of the zones in which it lied endemic, and thence it spread out, eastward into China, south to India and west along the trade routes to reach the Crimea some eight years later. In 1347 it appeared in Cyprus and by the following year had spread to France, Italy, Germany and England. In 1349 it was in Poland, Scandinavia and Scotland and in 1351 it ravaged Russia[118].

About 25 million Europeans died – a depletion of about one quarter of the continent's population. The Black Death caused a great shortage in labor and wages rose enormously. The result was to strengthen the position of the workers and to hasten the end of the feudal system on which medieval society was based.

In spite of the Calamity, the second half of the 14^{th} century was not essentially different from the first, as far as the history of science and learning is concerned. It is certain that the plague caused the untimely death of many scholars or potential scholars, but apparently other scholars took their place and continued their work.

The medieval pandemic did not strike out of the void; earlier outbreaks were recorded. One of these began in Arabia, reached Egypt (542 CE), fatally weakened the Roman Empire of Justinian and moved on across Europe to England and Ireland, which it laid waste in 664 CE. One of its parting flourishes was the Great Plague of London (1665), which seemed to have died out in the 17^{th} century. Finally came the pandemic which started in 1892 in Yunnan and reached Bombay in 1896, killing some six million people.

Bubonic plague is endemic to certain areas in the world: Uganda, Western Arabia, Kurdistan, Northern India and the Gobi Desert. From time to time it erupts there from a minor, localized epidemic. Far more rarely it breaks its bounds and surges forth as one of the great pandemics.

In the endemic state, the bacillus *Pasteurella Pestis* exists in the bloodstream of an animal or the stomach of a flea such as *Xenopsylla Cheopsis*, which in turn resides in the hair of some rodents. In 1338 it was the tarbagan, or *Manchurian marmot*, a beguiling squirrel-like creature much hunted for its skin.

To disturb the harmless steady-state existence of the bacillus, something had to happen to make the rodents leave their habitat. It is believed that extreme *ecological environments and climatic changes* such as locust, earthquakes, floods and prolonged droughts (known to occur during 1333–1337 in the plains watered by the rivers Kiang and Hoai, in Honan and the mountains of Ki-Ming-Chan) can affect rodent migration. Even without this incentive, an increase of rodent population can put too great a strain on the available supplies of food.

At all events a massive exodus took place and it was above all *rattus rattus*, the tough, nimble, by nature vagabond, black rat which made the move. By

118 It could have been carried earlier into Europe by the armies of Jenghis Kahn.

the middle of the 14^{th} century rats abounded in Europe, probably having been imported originally in the boats of the returning Crusaders.

The Black Death must have seemed to be of supernatural origin, a punishment inflicted by higher power upon unknown sinners for unknown crimes. Culprits were sought: nobles, cripples and Jews in turn came under suspicion. The Jews, in particular, were suspected of purposely spreading plague by contaminating wells. Their persecution started at Chillon on Lake Geneva (1348) and rapidly spread to Basel, Bern, Freiburg and Strasbourg. Jews were herded into large wooden buildings and burned to death. At Strasbourg over 2000 are said to have been hanged on a scaffold set up in the Jewish cemetery. So bitter did the persecution become that Pope Clement VI issued two Bulls declaring Jews to be innocent. Many fled from Western Europe into east Germany and Poland. The Black Death intensified the medieval Christian tradition of the scapegoat-Jew.

The Paris Quartet (1350–1370)

In the middle of the 14^{th} century there arose at the University of Paris a quartet of brilliant scholastic philosophers, two of whom are sometimes referred to as initiators of the modern age of science.

*One of them was **Jean Buridan**, whose opposition to Aristotelian doctrines made him one of the founders of modern dynamics. One historian of science, **Pierre Duhem**, places the precise line separating ancient from modern science at the time (ca 1350) when Buridan applied the theory of impetus to the heavens, thus breaking the ancient distinction between terrestrial and celestial motions. It has been claimed that from Buridan, **Galileo** borrowed the idea of momentum, **Descartes** borrowed the principle of the quantity of motion, and **Leibniz** took the doctrine of vis viva (kinetic energy). While such claims must be properly qualified, and although the notion of inertia has roots in antiquity, nevertheless Buridan was by any reckoning an outstanding figure in the history of science.*

*A second member of the quartet, a close associate of Buridan, was **Nicole Oresme**. He was hailed as a man whose contributions to physics, astronomy, and mathematics marked the beginning of the modern period in experimental science. The holder of more precursorship claims, advanced by modern* *admirers, than any other medieval scientist, he has variously been regarded as a forerunner of Copernicus, Galileo, and Descartes. His appreciation of* the relativity of motion *led him to advance arguments in favor of the diurnal rotation of the earth; but more important, probably, were his contributions to mathematics. His introduction of* generalized powers *in algebra, his anticipation of the* graphical representation of functions *in analytic geometry, and his study of* instantaneous rates of change*, all show the originality and power of his "hunches". In particular, his observation that the rate of change of a variable quantity is least at its maximum value (ca 1360), played a key role in the* calculus.

Albert of Saxony *(1316–1390), a third member of the scientific quartet at Paris and transmitter of this science to Vienna, wrote on logic, physics, mathematics, and geology. His work on the void, on centers of gravity, and on terrestrial erosion may well have been used later by* ***Leonardo da Vinci***.

Albert studied at Prague and then at Paris. He was the rector of the University of Paris (1353–1362), the founder and first rector of the University of Vienna (1365–1366), and bishop of Helmstädt (1366). Albert helped spread the nominalist logic of ***William of Ockham*** *and the ideas of* ***Bradwardine***, *Oresme and others. He was mainly a transmitter of good mathematical ideas but did not contribute his own work to those.*

The 4^{th} *member of the closely-knit Parisian group was Albert's associate* ***Themo Judaei*** *(Themon son of the Jew; fl. 1349–1361). Author of commentaries on physics, astronomy, and meteorology which were also probably used by Leonardo.*

Themo was born in Westphalia, and spent part of his youth in Münster. After converting to Christianity he proceeded to Paris and studied at the Sorbonne, passing his final examination in 1349. He became a prominent teacher at the University of Paris and trained many students. His discussion of the rainbow *is very elaborate, comparable to that given by* ***Dietrich of Freiberg***, *and superior to that given by* **de Dominis** *(1611).*

So intimately was the work of the four men interwoven that it is virtually impossible to disentangle the literary threads to determine the original authorship in cases in which all of them wrote similar works on the same topic. All four of them were, of course, strongly influenced by Aristotle; but during the 13^{th} *and* 14^{th} *centuries a reaction had set in against servile acceptance of Peripatetic teachings. There arose an increasing awareness of the need for a mathematical treatment of physics and a definite inclination to regard quantitative formulations as adequate explanations of natural phenomena. Emphasis on the metaphysical "why" was being shifted, in scientific discussions, to the physical "how". This attitude foretold for advances in understanding of physical phenomena.*

The Buridan school was primarily responsible for the triumph of the mathematical over the philosophical approach to physical problems.

Later, the teachings of Buridan, Oresme, Albert of Saxony, and Themo found their way to other centers of learning. Paris and Oxford in particular were at the time closely related by scholarly ties, and Buridanian doctrines flourished in England as well as France.

ca 1350 CE **Jean Buridan**; **Joannes Buridanus** (1295–1358, France). Post-Aristotelian philosopher of the Ockhamist school of Paris and a methodological contributor to science. In a commentary on the *Physics* of Aristotle he showed that Aristotelian physics with its *unmoved movers*, its *natural* and *violent* motion etc., was empty verbiage. He made considerable advances in the study of motions, momentum, acceleration, and the theory of falling bodies, and came very close to formulating Newton's Law of Inertia.

These ideas, clearly expressed by Buridan, had been slowly taking shape at Merton College and at the Sorbonne during the 14^{th} century, and contributed to the eventual downfall of Aristotelian physics and cosmogony.

Buridan insisted that scientific truth is not absolute, like mathematical truth, but has degrees of certitude. The kind of certainty Buridan had in mind consisted of indemonstrable principles that formed the basis of natural science but are derivable from inductive generalization and accepted because they have been observed to be true in many instances, and to be false in none. Moreover, Buridan regarded these inductively generalized principles as conditional because their truth is predicted on the assumption of "common course of nature". This was a profound assumption that effectively eliminated the effect on science of unpredictable, divine interventions and with it the need to worry about miracles in the pursuit of natural philosophy.

Miracles could no longer affect the validity of natural science. Nor could chance occurrences that might occasionally impede or prevent the natural effects of natural causes. On this basis Buridan proclaimed that "*for us the comprehension of truth with certitude is possible*". Using reason, experience, and inductive generalizations, he sought to "save the phenomena" in accordance with the principle of *Ockham's Razor* – that is, by the simplest explanation that fits the evidence[119]

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[119] The widespread use of the principle of simplicity was a feature typical of me-

Buridan studied in Paris under William of Ockham. He was a professor of philosophy in the University of Paris and its rector in 1327. An ordinance of Louis XI, in 1473, prohibited the reading of his works.

1350–1370 CE **Nicole Oresme** (1323–1382, France). One of the greatest men of science of the 14^{th} century who meditated and theorized on motion, infinity and the continuum – all concepts of which are fundamental to modern mathematics. A bishop who wrote numerous works in both French and Latin on scholastic, political, scientific and mathematical problems. Introduced notions that correspond to the idea of rational (fractional) exponents as well as a concept similar to that of a *function*. Invented the basic idea of coordinate geometry before Descartes (1350). Foreshadowed Galileo in his *mathematical* formulation of uniformly accelerated motion. His translations of Aristotle served to popularize science.

While accepting the Ptolemaic system of the Universe, Oresme, in a commentary (1370) on Aristotle's *De Coelo et Mundo*, attributed the apparent daily motion of the stars to the rotation of the earth about its axis. He argued that the earth's rotation is compatible with other observed astronomical phenomena, such as eclipses of the sun and conjunctions and oppositions of the planets.

In his unpublished *Algorismus proportionum* he was first to introduce (1360) *fractional exponents*, which were not widely used until the 17^{th} century. In his tract *De uniformitae at difformitate intensionum* (1350) he introduced the idea of graphical representation of functions, as the independent variable was permitted to take on small increments, thus foreshadowing Cartesian coordinate geometry through the logical equivalence between tabulated values and their graphical display. A century after Oresme's tract was written, it was extant in several printings, and in this way came to the attention of **Descartes**.

Among other contributions of Oresme was his proof, apparently the first in the history of mathematics, that the *harmonic series* is divergent[120].

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dieval natural philosophy. It was also characteristic of science in the 17^{th} century, as when **Johannes Kepler** declared that "*it is the most widely accepted axiom in the natural science that Nature makes use of the fewest possible means*".

120 His proof was:

$$\frac{1}{2} + (\frac{1}{3} + \frac{1}{4}) + (\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}) + \cdots + \frac{1}{n} + \cdots > \frac{1}{2} + \frac{2}{4} + \frac{4}{8} + \frac{8}{16} + \cdots \to \infty.$$

This was shown again by **Pietro Mengoli** (1626–1686, Italy), who also demonstrated that the harmonic series with *alternating signs* converges to $\{\log_e 2\}$.

Oresme was born in Normandy. In 1348 he was a student in the College of Navarre at Paris, of which he became head in 1356. In 1361 he was named dean of the cathedral of Rouen. He was advisor to Charles V of France, who appointed him bishop of Lisieux (1377).

1360 CE **Henry de Vick of Württemburg** built the first mechanical clock (with iron movement) for King Charles V of France.

1362 CE One of the North-Sea greatest stormtides. Destroyed the Island of *Strand* and the city of *Ronghold*.

1368 CE In China, the **Ming** dynasty (militantly Chinese and exclusive) replaced the Mongol Yuan dynasty (cosmopolitan and tolerant). Thus ended for centuries the European prospects of Christianizing China, and also of obtaining allies in Eastern and Central Asia with whom to crush the Muslim hosts of the Near East. After a century of intercourse with the West, China became again a remote, inaccessible country.

ca 1375 CE **Narayana Pandit** (1340–1400, India). Mathematician. Gave a rule to calculate the approximate value of a non-square root number N, using the equation $Nx^2 + 1 = y^2$. He thus calculated $\sqrt{10}$, correct to 20 decimal places.

1375–1440 CE **Avraham** and **Yehudah Crescas**[121] (or **Cresques**) (ca 1330–1387; 1360–1440, Majorca, Barcelona and Portugal). Father and son. The greatest cartographers of their age, nautical instrument makers and founders of the Majorcan school of cartography. **Avraham** issued the famous *Catalan map of the world* (1375). Discarding Ptolemy, it shows India (for the first time!) as a peninsula.

In the second half of the 14^{th} century, Jewish cartographers and instrument makers (astrolabes, compasses, etc.) were active in Majorca under the patronage of the kings Peter III and IV of Aragon. The maps were originally intended for the use of seamen navigating in the Mediterranean and along the coasts of the Atlantic, but in the course of time they were extended to the mainland and ultimately developed into maps of the whole world, as then known.

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[121] Personal or family name common among the Jews of Southern France and Catalonia. It comes from the Latin verb *crescere* (= to grow, increase) and is a French form of *Joseph* (= may God increase, *Gen* **30**, 24). Bearers of the name include **Hisdai Crescas**, the above Majorean map-makers, **Crescas Elijah** (physician to Pedro IV of Aragon), and the astrologer **Cresques de Vivers**.

Avraham, who held the title *magister mappamundorum et buxolarum* was in the service of the infante John of Aragon, who in 1381 presented the world map to king Charles VI of France. He was originally from Barcelona.

His son, Jehuda Crescas, collaborated for a while with his father in Majorca and after his father's death (1387) continued as cartographer and compass maker for the kings John I and Martin I of Aragon. During the riots in Spain (1391), about 50,000 Jews in Majorca were massacred and Yehudah was forcibly converted to Christianity to save his life. His name was changed to Jayme Ribes. After staying in Majorca for a while, where he was known as *Io Jucu buscoler* ("Jew of the map"), he moved to Barcelona (1420), migrated (1427) to Portugal, where he instructed Prince Henry's pilots in their art.

1391 CE **Geoffrey Chaucer** (ca 1340–1400, England). The greatest English poet of the Middle Ages. Wrote *A Treatise on the Astrolabe*, describing the construction of the astrolabe and its use in calculating the position of a star.

The astrolabe is an instrument that ancient astronomers used to measure the angles of celestial bodies above the horizon. It consists of a metal disc suspended from a frame so that the disk remains vertical. The disk has sights for observing a star, and graduations for measuring its elevation. The astrolabe became obsolete with the invention of more modern instruments, such as the *sextant*.

A reduced version of the astrolabe was the *quadrant*, which is shaped like a quarter of a pie with scales marked on its curved edge.

ca 1400 CE **Madhava of Sangamaramma** (1350–1425, India). Astronomer and mathematician. Discovered the series expansion equivalent to the Maclaurin expansion of $\sin x$, $\cos x$ and $\tan^{-1} x$, about 300 years before they were discovered in Europe (Newton 1676, Gregory 1677, Taylor 1715). This outstanding contribution was a decisive step toward modern classical analysis.

1405–1433 CE **Zheng He (Cheng Ho)** (1371–1435, China). Chinese mariner and navigator, '*Admiral of the Western Seas.*'

China's most famous navigator and explorer. For 28 years he traveled to the West, totaling more than 50,000 km (in seven expeditions, and visiting some 30 countries and territories including: Java, Sumatra, Vietnam, Siam, Singapore, Cambodia, Philippines, Ceylon, Yemen, Arabia, Bangladesh, and Somalia).

China has a very old seafaring tradition. Chinese ships had sailed to India as early as the Han Dynasty. Chinese sailors had an important invention to help them – the compass. The compass, or “south pointing spoon,” started out as a fortune-telling instrument.

Along the Silk Road, warfare and chaos were commonplace. The dangers of travel were increased by roving armies, bandits and unpredictable governments. Each small kingdom along the route taxed merchants, making their goods more expensive to the buyer. None of this decreased demand for the goods of the Silk Road trade. But in did propel merchants to search for other ways to transport their wares.

Merchant ships had always traveled the seaways between China and Egypt. But the disturbances in the interior of the continent, combined with improvements in shipbuilding and navigation, gave merchants all the reasons they needed to sell their camels and invest in ships. The seas were safe by comparison. Ships were faster than camels and could carry more cargo. In addition, the countries on the coasts of the Indian Ocean were generally far removed from the warring Mongols and the rebellions against them.

After the Mongols were overthrown in 1368, the emperor of the new Ming Dynasty[122] wanted to assert Chinese power. Because China was no longer part of a land empire that stretched from Asia to Europe, the emperor turned to the sea. He decided to build a navy. The Chinese made elaborate plans that would not be fulfilled for many years. A shipyard was built at the new capital of Najing (Nanking). Thousands of varnish and tung trees were planted on nearby Purple Mountain to provide wood for shipbuilding. The emperor established a school of foreign languages to train interpreters. While all this was going on, the man who would lead the navy was still an infant.

Chinese shipbuilders also developed fore-and-aft sails, the stern post rudder, and boats with paddlewheels. Watertight compartments below decks kept the ship from sinking. Some boats were armor plated for protection. All these developments made long distance navigation possible.

At the orders of the then emperor, Cheng Zu, a vast fleet set sail in July, 1405 from Liujia Harbor near Suzhou on a distant voyage. The purpose was to establish relations with foreign countries, to expand trade contacts and to look for treasures to satisfy the desire of the sovereign for luxuries. The man who was given charge of the fleet was Zheng He, a eunuch. Under his command was a vast fleet of 62 ships manned by more than 27,800 men, including sailors, clerks, interpreters, officers and soldiers, artisans, medical men and meteorologists.

On board the ships were large quantities of cargo including silk goods, porcelain, gold and silver ware, copper utensils, iron implements, cotton

[122] The Ming dynasty lasted until 1644. Mongol incursions and Japanese sea pirate attacks occurred during 1522–1566.

goods, mercury, umbrellas and straw mats. The fleet sailed along the coast of Fujian, down south to Zhancheng and, after crossing the South China Sea, reached such places as Java and Sri Lanka. On the way back, it sailed along the west coast of India and returned to the home port in 1407.

Between 1405 and 1433, Zheng He had, over a period of 28 years, been ordered eight times to act as envoy to countries lying to the west of China. Each time he had under his command a big fleet and a staff of more than 20,000 men. His fleets had sailed in the South China Sea and the Indian Ocean. They had gone further south to Java in today's Indonesia. Sailing then in a northwest direction, they had visited Yemen, Iran and Mecca and further west to today's Somalia in East Africa. All this had taken place about half a century before Columbus' voyage to America.

On each voyage Zheng He was acting as the envoy and commercial representative of the Ming court. No matter what country he visited, he called on the ruler of the land, presenting to him valuable gifts in token of China's sincere desire to develop friendly relations and inviting the host sovereign to send emissaries to China. Wherever he was, he made a careful study of the customs and habits of local residents. Showing them due respect, he bartered or dealt with them through consultation and negotiation on the basis of equality and mutual benefit. In this way, he obtained large quantities of pearls and precious stones, corals, ivory and dyestuffs for the Chinese emperor. He also brought back several kinds of rare and precious animals such as giraffe, lion, ostrich and leopard.

In ancient India, Chinese sailors made a good impression on the local people by observing local trading customs and practices such as clapping hands to clinch a deal in full view of others and never going back on it.

When he visited Sri Lanka on his third voyage, Zheng He offered a quantity of gold and silver, Buddhist ceremonial vessels and silk-knit religious pennants to local temples on whose ground steles were set up to mark the occasion of his visit.

Wherever he went, he was warmly received. At Zhancheng, the king of the land, in full royal regalia, came in person on elephant back with 500 cavalrymen to meet him at the wharf and then take him back to the palace. On the way they were greeted by local inhabitants who blew trumpets made of coconut shells and performed national dances at a solemn and joyous ceremony. Even today, people in Somalia and Tanzania look upon Ming China as a symbol of the traditional friendship between their own country and China. In Thailand today, there are places named after Zheng He's childhood name Sanbao (tree treasures) such as Sanbao Harbor and Sanbao Pagoda. Malacca of Malaysia is known also as the City of Sanbao. At Java in Indonesia, there is the Sanbao Temple. In Calicut (Kozhikode today) of India, there is an inscribed tablet set up in Zheng He's memory.

On his first voyage overseas, the largest ship in the fleet had a length of 400 feet and a width of 180. Manned by more than 200 sailors and able to accommodate 1,000 passengers, it was equipped with nine masts which flew 12 big sails. This was probably the largest sea-going vessel of the day[123]. Other vessels might not be of the same size but on an average each one was able to carry aboard four to five hundred passengers.

Many of the navigational problems encountered were solved in a rational, scientific way. For instance, the way fresh water was collected and stored, the stability of the hull and its buoyancy, the making of sea charts and the use of navigational apparatuses like the compass. This accounted for the fact that in spite of terrible storms, this fleet had ploughed the waves day and night in full sail.

It is generally believed that Zheng He had the largest, most advanced fleet in the world in the 15^{th} century. On each of his 8 voyages, Zheng He kept a detailed logbook and made many charts which were later collected in what was called Zheng He's Nautical Charts, which was the first of its kind in the world. From this we can say that China in those days probably led the world in the technology of ship-building and the science of navigation.

Zheng He was born in 1371 in Kunyang, a town in southwest Yunnan Province. His family, named Ma, were part of a minority group known as the Semur. They originally came from Central Asia and followed the religion of Islam. Both his grandfather and father had made the Muslim pilgrimage to Mecca. Zheng He grew up hearing their accounts of travel through foreign lands.

Yunnan was one of the last strongholds of Mongol support, holding out long after the Ming Dynasty began. After Ming armies conquered Yunnan in 1382, Zheng He was taken captive and brought to Nanjing. The eleven year old boy was made a servant of the prince who would become the Emperor.

When his prince seized the Chinese throne from a nephew, Zheng He fought well on his behalf. As a result, Zheng He became a close confidant of the new emperor and was given an important position at court.

When Zheng He came back from his seventh voyage in 1433, he was sixty-two years old. He had accomplished much for China, spreading the glory of the Middle Kingdom to many countries that now sent tribute and ambassadors to the court. Though he died soon afterward, his exploits had won him fame.

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[123] In comparison, the flagship of Columbus, the *Santa Maria* was only 75 ft $\times$ 25 ft in size.

Plays and novels were written about his voyages. In such places as Malacca and Java, towns, caves, and temples were named after him.

However, a new Ming emperor had come to the throne. His scholar-officials criticized Zheng's achievements, complaining about their great expense. China was now fighting another barbarian enemy on its western borders and needed to devote its resources to that struggle. When a court favorite wanted to continue Zheng He's voyages, he was turned down. To make sure, the court officials destroyed the logs that Zheng He had kept. We know about his voyages only from the pillar and some accounts that his crew members wrote.

Thus, China abandoned its overseas voyages. It was a fateful decision, for just at that time, Portugal was beginning to send its ships down the west coast of Africa. In the centuries that followed, European explorers would sail to all parts of the world. They would establish colonies in Africa, America, and finally in the nations of East Asia. China would suffer because it had turned its back on exploration. Zheng He had started the process that might have led the Middle Kingdom to greater glory. Unfortunately the rulers of the Ming Dynasty refused to follow his lead.

The British submarine engineer and historian Gavin Menzies gave a seminar on March 15, 2002 to the Royal Geographical Society in London to support his theory that Zheng He beat Columbus by more than 70 years in discovering America. Using evidence from maps dated before Columbus' trip, and astronomical maps traced back to Zheng He's time, Menzies claimed that the Zheng he should be honored as the first discoverer of America.

The Voyagers (1405–1550)

"There will come a time in future ages when the ocean will loosen the chains of the universe and a vast land will appear, new worlds will be seen and Iceland will not be the end of the earth."

Lucius Annaeus Seneca, '*Medea*' (4 BCE–65)

Almost concurrently with the collapse of the Byzantine Empire and the 'printing revolution', the energy accumulated during the long intellectual hibernation of the middle ages suddenly burst forth in yet another endeavor – the geographical revolution.

The European 'discovery' of the Indian Ocean in the 15^{th} *and* 16^{th} *centuries was only new to them as Western Europeans. The Ancient Egyptians had traded on the African shores of the Indian Ocean. The Arabs began to settle it in the* 8^{th} *century CE: Mogadishu (ca 720), Sofala (ca 780), Madagascar (*9^{th} *century). These were the bases from which their subsequent explorations began.*

Chinese junks were in the harbor of Malacca when Albuquerque arrived there. Chinese texts as early as 860 *CE describe the south coast of the Gulf of Aden and the Somali coast. Malindi (Mo-Lin) was known at about* 1060 *and the Zanzibar coast (Tsheng-Pa) and Madagascar at about* 1178. *Chinese coins and porcelain dating from the* 7^{th} *century onwards (the majority from the Sung dynasty 960–1279) are so abundant on the east coast of Africa that they must have been used in payment for goods.*

In the early 15^{th} *century the great Chinese voyages to the Indian Ocean began and they are as well documented as any in the West. There were seven expeditions in all. In the first (1405–1407), the Emperor Cheng Zu commissioned* 62 *junks carrying* $37,000$ *men under the command of the Imperial Palace Eunuch Cheng Ho. They set sail from Liu-Chia Kang in the province of Suchow, reached Indo-China, Java, Sumatra, Ceylon and Calicut.*

In the second expedition (1407–1409) they reached Siam, Cochin and other parts on the west coast of India. The third, based on Malacca, voyaged in the East Indies and the southwest coast of India and Ceylon.

On the fourth voyage, based on Ceylon, they explored the Bengal coast, the Maldive Islands, and reached Hormuz.

On the fifth voyage the fleet divided into squadrons: A Pacific squadron reached the Ryukyu Islands and Brunei in northwest Borneo, while squadrons in the Indian Ocean explored the coast between Hormuz and Aden, Mogadishu in Somalia, Malindi and the coast further south.

The sixth expedition (1421–1422) was an amplification of the previous one, visiting 36 *states between Brunei and Zanzibar.*

The last expedition (1431–1433) covered the areas north and west of Java as far north as Jiddah in the Red Sea. It was the last great venture of the Ming Navy. From now on the inward-looking Confucians faction was in the ascendant. It regarded things brought from beyond the seas as superfluous. Among them were the tusks of elephants, rhinoceros horns, pearls and aromatics from Africa and, above all else, spices.

It is more than possible that they got as far as the southern Atlantic and some of their captains may have doubled the Cape and seen the West African shore. Thus, upon the death of the last Ming Emperor, ended the age of Chinese maritime glory. Could they discover America ahead of Columbus?

In 1400, *Europeans knew scarcely more about the earth than the Romans had. The oceans around the continent were still an impenetrable barrier. In fact, many people in those days believed that further south the sea grew boiling hot, or that ships could fall off the edge of the earth.*

The man who helped overcome such fears, at least among his own men, was ***Prince Henry the Navigator*** *(1394–1460), the third son of King John of Portugal from an English mother. Henry was a soldier, not a sailor, and did not himself take part in any journeys of discovery. As a driving force he had no peer in the history of navigation, blending his vision with the skill of his sailors and the wisdom of medieval scholars.*

In 1415 *Henry took part in the campaign in which Spain captured the North African port of Ceuta from the Moors. A substantial link between Moorish science and the medieval universities of Christendom already existed when the growth of mercantile navigation renewed the impetus to astronomical discovery. From* 1419 *onwards, Henry dispatched a series of expeditions down the western coast of Africa and spent most of his time organizing and financing the exploration fleets. About* 1439 *he retired to Sagre, in southern Portugal, where he erected an observatory, and founded a college of navigation. He assembled a team of geographers and navigators, and developed ships suitable for voyages of exploration.*

For 40 *years he devoted himself to cosmographical studies, while equipping and organizing expeditions which won for him the title of* ***Henry the Navigator****. For the preparation of maps, nautical tables and instruments, he enlisted Jewish cartographers and astronomers of the* **Zacuto** *and* **Crescas** *families, employing them to instruct his captains and assist in piloting his vessels. The development of astronomy once more became part of the everyday life of mankind.*

The motives that prompted the Europeans, rather than the Chinese or Muslims, to 'discover' the rest of the world were both religious (convert the heathen and weaken Islam by placing Christian allies in the Muslim rear) and economic (need to find precious metals to pay for Eastern spices).

A third cause, not less important, was the challenge of fresh ideas that took hold in Europe during the Renaissance. The learning that was brought to Europe by the fleeing savants of Byzantium spread rapidly, with the aid of printing. It boosted the questioning of established dogmas in religion, art and science. Scholarship began to develop independently of the church, and the human rather than divine in life and art was underlined; the well-rounded *individual* became an ideal. This gave impetus to explorers and reformers alike.*

By the late 15^{th} century a restless, energetic, and bold seafaring population was scattered along Europe's Atlantic Coastline. Resourceful sailors, fishermen, and merchants had developed techniques and the ships for making long voyages. They had religious and economic motives strong enough to overcome their superstitious fears of what lay beyond unknown waters[124]*, and their governments were often ready to back them. Europe needed Asia more than Asia needed Europe, and Europeans believed that it would not be too difficult to reach Asia by sailing West.*

Ptolemy in the 2^{d} century had underestimated the size of the globe and had overestimated the span of Asia, and the geographers of the 15^{th} century, accepting his miscalculations, were convinced that Japan and China lay only a few thousand miles West of Europe. The journey seemed possible, and the psychological environment was favorable for an age of discovery.

After Henry's death, his grandnephew, King John II, speeded up the effort to find an all-water route to India that would short-circuit the Venetian-Arab monopoly. Within ten years (1487–1497), two voyages took place which raised Portugal from a nation of peasants, fishermen and seafaring adventurers to a great maritime power. By 1488 **Bartholomeu Dias** *(1450–1500) rounded the Cape of Good Hope and in 1497,* **Vasco da Gama** *(1469–1524) followed his route and with four ships reached Calicut, India in* 1498. *In* 1500, *a larger fleet under* **Pedro Álvars Cabral** *(1467–1520) touched the coast of Brazil, and headed for India in da Gama's wake.*

Once the Portuguese had overcome the natural hazards of the sea journey from Europe to India, they moved purposefully to establish a commercial dominance in south-east Asia. King Manoel sent a permanent force to India, led by **Francesco de Almeida** *(1450–1510) in* 1505. *On the way, Almeida took Kilwa and razed Mombasa, before setting himself up as Governor-General of India, based at Cochin. His son Laurenco reached Ceylon, an important source of spices, and the Maldives, an island chain in the Indian Ocean.*

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[124] When seamen left Portugal for the unknown, their wives cried at their departure, dressed in black as if they were becoming widows. They spoke of *saudade* or "regret of absence". This word which best explains the soul of the *Fado* (fate), the traditional song of Portugal, has no exact translation in any other language (the English *longing*, the French *nostalgia*, and the German *sehnsucht* are part of it). One may call it "sweet sadness" – a longing for something very dear but impossible to achieve. *Fado* is the expression for this feeling. Today, the lady singers of the *Fado* are always dressed in black, lights are muted and the public listens in silence.

In 1509, off Diu Island, a small force of 19 ships under the command of Almeida totally vanquished the combined Muslim fleet of 100 vessels. It was the end of Arab power east of Aden.

Almeida was succeeded as Governor-General of India by one of the great empire-builders of Portuguese history, ***Alfonso d'Albuquerque*** *(1453–1515), in 1509. He seized Goa, on the Indian mainland, captured Malaca to control the trade between the Spice Islands and the Indian Ocean and secured Hormuz to dominate the entrance of the Persian Gulf. The capture of these strategic points opened before the Portuguese the seaway to China and Japan. Indeed, the first Portuguese mission reached China in 1514 and then Japan in 1542.*

In 1557, in recognition of the assistance that they rendered to the Chinese in exterminating Chinese pirates, the Portuguese were granted a lease on the peninsula of Macau.

The voyagers had made large profits, but the cost to the Portuguese government of equipping fleets soon ate up those profits. Italian, German and Flemish bankers soon dominated the Portuguese trade, and the spices that arrived at Lisbon were sent on directly to Antwerp. The burden of the empire was already proving heavy when Portugal fell into the grip of Spain in 1580.

Before da Gama's successful voyage, a Genoese navigator, ***Christopher Columbus*** *(1446–1506) had persuaded Queen Isabella of Spain to finance an expedition to find a westward route to India. Columbus was using the ancient maps of Ptolemy, according to which the size of the Atlantic was underestimated. Finally, in 1492 he sailed across the Atlantic Ocean and discovered the Indies, believing them to be his goal. Columbus never did realize he had found a whole new continent.*

However, a later adventurer, ***Amerigo Vespucci*** *(1451–1512) [director of the Medici branch bank in Seville], claimed that he had sailed on both Spanish and Portuguese voyages and described what he saw in letters that were widely read throughout Europe. In one, he referred to the great southern continent in the west as 'Mundus Novus' (New World). A German mapmaker,* ***Martin Waldseemüller****, labeled the two new continents in 1507, by the name 'America' in honor of Vespucci.*

In 1513, the Spanish explorer ***Vasco Núñez de Balboa*** *(1475–1519) crossed the Isthmus of Panama and became the first European to see the Eastern Pacific. Balboa is known as the European discoverer of the Pacific because he realized that it was a great unknown sea.*

The first round-the-world voyage was the inspiration of a Portuguese navigator, ***Ferdinand Magellan***[125] *(Femão de Magalhaes; Fernando de Mag-*

[125] It took **Magellan** to prove that the world was round, but as early as 350 BCE

allanes, 1480–1521). Out of favor in his own country, Magellan offered his services to Spain, whose ruler, Emperor Charles V, provided a fleet for the voyage.

Magellan set sail in 1519 *with five ships, aiming for the southern tip of South America, where he was sure there was a route to the Western Ocean. Despite tremendous storms, Magellan found his route – the strait that now bears his name – and sailed into a calm sea which he named 'Pacific'; he had lost one ship, and one had turned for home. Magellan sailed across the Pacific to the Philippines, where he was killed in a skirmish with the natives. The epic voyage was completed by* ***Juan Sebastian del Cano*** *with one ship, returning by way of the Cape of Good Hope in* 1522*, the first ship to sail around the world. In that year Europe has begun to cast a web of communication and influence around the earth.*

During the next four centuries that web was to draw all of the world under the influence of European civilization.

The Leading Edge of Western Civilization (1300–1600)

In spite of all the chaos and misery that afflicted Europeans during 1300-1450 CE, science and technology prevailed. Why? Although the scholarly work of the period was mostly unoriginal it was important in the sense that it was useful to men with ideas of their own. Columbus drew most of his notions about geography from books written in the fourteenth and early fifteenth centuries. And not all the work was unoriginal. In philosophy there was a sharp attack on the system of Thomas Aquinas which freed scholars, to some extent, from their adherence to the Aristotelian ideas that had been incorporated into Thomas' theology. Once Aristotle's ideas had been challenged, there could be wider speculation on scientific questions, especially on explanation of motion

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the idea was generally accepted by the *Greeks*.

and acceleration. The studies through which Galileo revolutionized the science of physics were based on problems raised by fourteenth-century scholars.

More important than any specific achievement was the very fact that interest in scientific problems persisted. Up to the end of the Middle Ages, Western scholars, relying largely on the work of the Greeks and Muslims, had made no outstanding contribution to scientific knowledge. But they were remarkably persistent and kept working on scientific problems after other peoples had given up. The Greeks and Muslims eventually lost interest in science, as did the Chinese, who had their own independent scientific tradition. But from the twelfth century on, there were always some scholars in the West who were interested in science, and this long-term devotion led, in the end, to the great discoveries of the early modern period. Men like Copernicus and Galileo were trained in universities that used the methods and the books of the later Middle Ages.

No one has ever given a completely satisfactory explanation of this continuing interest in science. Certainly Westerners were paying more attention to the things of this world during the later Middle Ages and less attention to the aims of the Church. But Chinese society was far more secular, yet the Chinese, in the long run, fell behind the Europeans. Perhaps more important was the Western tendency to be dissatisfied with the status quo*, a tendency that was especially evident in the crucial years between 1300 and 1600. In China, a philosopher like Thomas Aquinas would have become an unchallenged authority; in Europe his system was questioned within a generation after his death. Europeans respected authority, but they always felt that authoritative treatises needed to be reinterpreted. Finally, there was a curious patience with details, a willingness and an enthusiasm to work very hard for very small gains.*

These qualities also explained some of the advances in technology *that were made in the last medieval centuries. These are:*

- *The development of firearms.*
- *The invention of printing (ca 1446).*
- *Ocean shipping.*
- *The invention of the mechanical clock (ca 1352).*

In the field of firearms the Europeans capitalized on a technique known to other peoples. The Chinese, for example, were probably the first to discover gunpowder, and they had cannons about as early as the Europeans. But Chinese guns were never very efficient, and the Chinese never developed an army that was primarily dependent on firearms. The Europeans carried *their experiments with cannons much further than the Chinese. Although the European guns were not very good - they were as apt to kill the men who fired them as those against whom they were aimed - they had become fairly reliable by the end of the fifteenth century. The military significance of this development is obvious. It reduced the power of local feudal lords by making their castles untenable; conversely, it increased the power of kings and great princes like the Duke of Burgundy, for they were the only ones who could afford the expensive new weapons.*

The development of firearms caused a rapid growth in other branches of technology. In order to make gun barrels that would not burst under the shock of an explosion, much had to be learned about metallurgy. And in order to make gun barrels that were truly round and hence could deliver the full effect of the charge, better metalworking tools and more precise instruments had to be developed. Better techniques in using metals led to greater demands for metals, and this in turn stimulated the mining industry. The miners of Germany (including Bohemia and Austria), supplying the chief source of metals for Europe, learned to push their shafts deeper and devise ways of draining off underground water. Increased use of metals and greater skill in mining in the long run transformed European industry. To take the most famous example, pumps operated by a piston traveling in a cylinder were developed in order to remove water from mines; it was this kind of pump that eventually furnished the model for the first steam engine.

The invention of printing *in the fifteenth century also owed much to developments in metallurgy. The essential element in printing was the use of movable type, and good type in turn depended on the availability of a metal that would take the exact shape of the mold into which it was poured. Thanks to their knowledge of metallurgy, the Germans succeeded in developing an alloy that expanded as it cooled, so that it fitted the mold exactly and gave sharp, clear impressions.*

Another technical advance of Western Europe in the latter Middle Ages was in ocean shipping. *Here there was at first more patient experimentation than striking discoveries. By the end of the thirteenth century the sailors of Western countries had ships that could tack against the wind and were seaworthy enough to survive the storms of the Atlantic. The navigators of the period could find their latitude, though not their longitude, by star and sun sights; they knew that the earth was round and that the distance to the rich countries of the East was not impossibly great. Very little more was needed for the great voyages of discovery except practice, and during the fourteenth and fifteenth centuries daring men were mastering the art of oceanic navigation. French and Spanish seamen had discovered the Canary Islands at least by the fourteenth century, and by 1400 the Portuguese had pushed down to the bend in the African coast, claiming Madeira and the Cape*

Verde Islands along the way.

These voyages illustrate the point that was made earlier: Europeans were no more skillful or intelligent than other peoples, they were simply more persistent or more aggressive. During the same years in which the Europeans were making their first sorties into the Atlantic, the Chinese were sending expeditions into the Indian Ocean. There they found rich kingdoms, ancient civilizations, and profitable sources of trade. In contrast, the Europeans discovered only barren islands and the fever-stricken coast of Africa. Yet the Chinese abandoned their explorations because they, or at least their rulers, were satisfied with what they had at home. The Europeans persisted, though it was almost two centuries before they reached the thriving trading centers of the East or the treasures of Mexico and Peru.

Not as striking as the early voyages, but almost as significant, was the invention of the mechanical clock. The first clocks, which appeared in the fourteenth century, were not very accurate, but they were soon improved by the discovery of the principle of the escapement - that is, the system by which the train of gears moves only a precise distance before it is checked and then released to move the same distance again – thus ensuring uniformity of the time-keeping motions. Crude as the first clocks were, they modified, in the long run, the mental outlook of the Western peoples.

For several centuries one of the sharpest differences between the West and the rest of the world lay in attitude toward precise measurement, especially the precise measurement of time. Western civilization has come to be dominated by the clock and the timetable, and Westerners have had little sympathy with people who have escaped this domination.

1410 CE **Hisdai ben Avraham Crescas** (1340–1411, Spain). Philosopher. The first European thinker to establish the feasibility of *infinite magnitude* and *infinite space*, thus paving the way for the modern conception of the Universe. A pioneer in his criticism of Aristotle and in his revival of the views of pre-Aristotelian Greek philosophers.

Crescas rejected Neo-Platonism and Aristotelian physics and metaphysics. He stated that "*there are no other worlds*" than the one system in which the earth is situated. This inspired such Christian thinkers as **Nicolas of Cusa** (1401–1464), **Giordano Bruno** (1548–1600), **Marsilio Ficino**[126]

[126] Translated (1484) the complete works of **Plato** from Greek into Latin. His

(1433–1499) and **Pico della Mirandola** (1470–1533). He deeply influenced **Spinoza**, who was indebted to Crescas for his concept of the Universe.

Crescas launched a sharp criticism of Maimonides and Gersonides because of their efforts to reconcile Judaism with Greek philosophy. He refuted the Aristotelian conception of one finite Universe and believed in the *unification of forces of nature* (including magnetic attraction!). These ideas he expounded in his principal work on *Or Adonai* (Light of God) to the completion of which he devoted many years of his life.

Like almost all Jewish philosophers in the Middle Ages, Crescas developed his philosophical system in the face of persecution and imminent personal danger. He endured much personal persecution during the first Spanish Inquisition, but still defended his views with a spiritual originality and courage uncommon in the history of the Middle Ages.

He was born in Barcelona and was denounced and victimized there, imprisoned and fined (1367), despite the recognition of his innocence. He moved and settled in Saragossa (1390). Crescas became an authority on Jewish law and ritual tradition, and often intervened diplomatically on behalf of his co-religionists in Aragon and the neighboring kingdoms. In 1401–2, at the request of Charles III (the Noble), he spent some time in Pamplona. A son of Crescas suffered a martyr's death in Barcelona in the anti-Jewish riots of 1391. Afterwards he received permission (1393) from the King of Aragon to marry a second wife, since his first wife was unable to have any more children. He died at Saragossa.

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teaching, translations, and Platonic interpretations formed the basis of a thriving literary culture which formed the pillar of European thought over subsequent ages.

1413–1414 CE *The Tortosa Disputation*, the most remarkable of its kind ever held[127]. It took place in the presence of the anti-pope[128] Benedict XIII, many cardinals, bishops and a vast audience. It began Feb. 7, 1413 and lasted 21 months in 69 sessions. The principal matter debated was whether the Messiah had already arrived or not. The renegade Jew Joshua Lorki opened for the Christians and the learned Vidal Benveniste for the Jews. The disputations degenerated into the usual interpretations and misinterpretations of Biblical and Talmudic passages. Naturally, the Jews were declared ignominious losers by the anti-pope, the Talmud was condemned and a ban was placed on the study of the Talmud by Jews.

The Jewish defenders counted among them the philosopher **Joseph Albo** (1380–1444), who rigorously presented the Jewish viewpoint of the Talmud. He attained popularity among medieval scholars (both Christians and Jews alike) for his book *Sefer-He-Ikkarim* (Book of root principles, 1425; published, 1485). It is important to the general philosophy of religion because it establishes criteria whereby the primary fundamental doctrines may be distinguished from those of secondary importance.

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127 Other well-known disputation book place in Paris (*Yehiel of Paris*, 1240) and Barcelona (*Nahmanides*, 1263). A modern chapter of medieval disputation occurred at the McGill University, Montreal on 31 January 1961 between the Ambassador of Israel to Canada. **Yaacov Herzog** (1921–1972) and the British historian **Arnold Toynbee** (1889–1975). Toynbee declared publicly (and in his books) that the entire life of the Jewish Diaspora from 132 years after Jesus of Nazareth until our own day was a fossilized relic of an obsolete culture that no longer had the right to exist. This indictment was lodged against the Jewish people from an academic rostrum of world repute, with an air of quiet certainty, without any anti-Semitic wrapping, relying solely on the authority of pure philosophical objectivity. The gauntlet had been thrown down before the entire Jewish people and the truth of history, and the young ambassador picked it up on the public platform, confronting the professor as equal in McGill University. At the end of a conclusive and convincing demonstration, Toynbee was compelled to admit defeat and apologized for his analogy.

128 Most Christians did not consider him the legitimate pope; only Spain recognized his claim to the papacy. Consequently, he thought he could add to his reputation by focussing the eyes of all Christians on an elaborate attempt which he would conduct to refute Judaism. Nowadays we would call it a propaganda trick to the anti-Jewishness for personal advancement. The Jews, of course, did not want to take part in anything of the kind. They knew that, far from being a free and open debate, in which they would have an equal right with their Christian opponents to speak their mind, this was but another attempt to humiliate Judaism. But there was no refusing the invitation; threatened with fines, imprisonment and expulsion, they were forced to appear.

Albo stated that three principles are basis to every revelational religion: a belief in God, the concept of divine revelation and divine retributive justice. His object was to show that all religions grow out of the same ideas; only after they grow out of these roots do various religions begin to differ from one another. The branch principles which make up Judaism are purer and more in harmony with philosophy than the branch principles which compose Christianity.

Maimonides in the 12^{th} century had formulated the principles of Judaism in thirteen articles; Albo reduced them to three.

Albo set the example of minimizing Messianism in the formulation of beliefs. Though he fully maintained the Mosaic authorship of the Law and the binding force of tradition, he discriminated between essential and the non-essential in the practices and beliefs of Judaism.

1413–1436 CE **Filippo Brunelleschi** (1377–1446, Italy). The greatest architect of the Early Italian Renaissance. His interest in mathematics led to his invention of *linear perspective* as plane representation of objects in three-dimensional space. The first formal account of the laws of perspective was given by **Leon Battista Alberti**[129] (1404–1472, Italy), the *Universal man* of the Early Renaissance. Alberti, mathematician, architect, painter and writer, is also credited as the originator of scientific cryptography.

The art of the *Renaissance* derived its influence from two main sources. The first was classical: the use once again, after an interval of almost a millennium, of the forms generally applied in Greek or Roman art. The second was the application of the newly discovered technique of perspective. This device gave the artist access to graphic and mathematical rules that enabled him to reproduce on paper or any other flat surface, with scientific accuracy, the appearance of 3-dimensional reality. The examples provided by classical art were of interest above all to architects, who could draw inspiration from monuments of classical antiquity still in existence.

When, relatively early on, the new movement came to be called *Renaissance*, the influence on it of Greek and Roman antiquity was also the most

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[129] **Alberti** studied the representation of 3-dimensional objects and wrote the first general treatise *Della Pictura* on the laws of perspective (1435). He also worked on maps and collaborated with **Toscannelli** who supplied **Columbus** with the maps for his first voyage.
For further reading, see:

- Gadol, J., *Leon Battista Alberti, Universal Man of the Renaissance*, University of Chicago Press: Chicago, 1973, 266 pp.

obvious. Its proponents considered themselves heirs to the traditions of classical art, and consciously sought the 'rebirth' of its forms, or at least of its spirit. They rejected completely the artistic achievements which had succeeded those of Greece and Rome. They were led to this attitude partly by admiration for classical art but mainly by a deep-rooted conviction that art, like science, had its own laws and that these had been discovered and applied by the artists and craftsmen of ancient Greece and Rome.

Perspective, the second source of influence on the art of the Renaissance, was in fact the most striking of a whole series of revolutionary discoveries. It was a decisive element in the development of the arts because it enabled the artist to show, by means of a realistic sketch – a form which everyone could understand – how a proposed work might look like when finished.

Brunellechi's knowledge of Roman construction principles combined with an analytical and inventive mind permitted him to solve an engineering problem that no other man of the 15^{th} century could have solved – the design and construction of a *dome* (cupola) for the huge crossing of the unfinished *Cathedral of Florence*. The problem was staggering, since the space to be spanned was 44.5 meters in diameter, much too large to permit construction with the aid of traditional wooden centering. Nor was it possible, because of the plan of crossing, to support the dome with buttressed walls

Brunellechi seems to have begun work on the problem about 1417 and erected this revolutionary structure between 1420 and 1436. With exceptional ingenuity he not only discarded traditional building methods and devised new ones[130] but also invented much of the machinery that was necessary for the job (e.g. lifting construction materials into position).

Although he might have preferred the hemispherical shape of Roman domes, Brunellechi raised the center of his dome and designed it around an *ogival section* (ovoid shape) which is inherently more stable, as it reduces the outward *thrust* around the dome's base. To reduce the *weight* of the structure to a minimum, he designed a thin *double shell* (the first in history) around a skeleton of 24 ribs, of which the 8 major ones are visible on the exterior.

Finally, in almost paradoxical fashion, he anchored the structure at the top with a *heavy lantern*. This lantern, although adding to the weight of the dome, has the curious effect of *stabilizing* the entire structure, since without the pressure of the weight the ribs have a tendency to tilt outward from the center, spreading at the top (the cupola is about 113 meter high).

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[130] To avoid the need for reinforcements or scaffolding, Brunellechi used techniques of brickwork taken from antiquity, but the *form* of the dome is pointed and therefore *Gothic* in style.

Alberti's Cipher (1535)

Codes and cyphers are methods of writing a message so that only persons with a key can read it. The science of making and breaking of codes is known as cryptography.[131]

*An early substitution cipher is found in the Old Testament (**Jeremiah 25**:26 and **51**:41). There, the prophet wrote Sheshach for Babel* (Babylon). *The second letter of the Hebrew alphabet (b) was replaced by the second-to-last letter (sh), and the twelfth letter (l) was replaced by the twelfth-to-last letter (ch). (This cipher is known as Athbash, meaning that the first letter a of the Hebrew alphabet corresponds to the last letter th, the second from the beginning .b, corresponds to the second from the end, etc.) Another Biblical example is found in **Isaiah 7**:6 where Tabeal is written for Remaliah. In this cipher, known as Albam, the first letter a is replaced by the* 12^{th}, *the second, b, by the* 13^{th}, *etc.*

*The Spartans wound a belt in a spiral around a stick, wrote a message along the length of the stick, and unwounded the belt. In this transposition cipher, no one could read the message unless he had a stick exactly the right size. **Caesar** (50* BCE*) used a simple substitution cipher; each letter of the plain text was replaced by the letter three positions to the right in the normal alphabet.*

***Gabriel de Lavinde** wrote the first manual on cryptography (1379). **Sico Simonetta** wrote the first treatise on cryptoanalysis (1474). **John Trithemius** (d. 1516) was an important writer on cryptography.*

In every language, the letters of a lengthy plain text exhibits a predictable frequency; In English the letter that occurs most often is e, and the next in order of frequency is t. The rest, grouped in order of decreasing recurrence are: a, o, i, n, s, h, r, d, l, u, c, m, w, f, y, g, p, b, v, k, s, j, x, z, q.

The drawback of an elementary substitution cipher is that it can be cracked simply by analyzing the frequency with which each symbol occurs. If, on the basis of frequency count, the cryptoanalyst can decipher the nine most common letters e, t, a, o, i, n, s, h, r - he has generally broken 70 percent of

[131] For further reading, see:

- Singh, Simon, *The Code Book*, Anchor Books: New York, 1999, 411 pp.

the cipher. The most modern of code-breaking techniques are based on the age-old method of frequency analysis[132].

*All the single letters must be a or i. Doubling letters are ee, oo, ff, ss, etc. The common words of two letters are (roughly arranged in the order of their frequency) of, to, in, it, is, be, by, or, as, at, an, so etc. The common words of three letters are: the, and (in great excess), for, are, but, all, not, etc.; and of four letters - that, with, from, have, this, they, etc. [**Edgar Allan Poe** (1843) made use of these statistics in his tale The Gold Bug].*

Alberti came up with an ingenious scheme to sabotage a frequency count with his Polyalphabetic substitution cipher in which more than one alphabet is used during encryption. His idea, the basis of modern cryptography, makes use of the so called vigenere table and a keyword agreed upon by the communicating parties.

Say the keyword is LOVE and the plaintext message is SEND MORE MONEY. The sender would then write (Table 2.5)

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keyword:    L O V E   L O V E   L O V E L    x-axis
plaintext:  S E N D   M O R E   M O N E Y    y-axis
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The sender, starting from the left, finds in the table the letter corresponding to the coordinates X=L, Y=S, which is the letter D. He then moves to the next pair (O, E) which gives him the letter S, etc. The ciphertext then reads DSIHXCMIXCIIJ. Decryption is the inverse process: (L, D) yields S, by moving on the row L to the letter D and reading the letter S on the x-axis.

The polyalphabetic cipher belongs to the family of homophonic ciphers which equalize the frequencies of the ciphertext letters since a given letter of the alphabet will not always be encrypted by the same ciphertext letter. Consequently, it cannot be described by a single set of ciphertext alphabet corresponding to a single set of plaintext alphabet. The equalization is effected in such a way that the number of ciphertext symbols assigned to a plaintext letter is determined by the frequency of that letter: a(8), b(1), c(3), d(4), e(12), f(2), g(2), h(6), i(7), j(1), k(1), l(4), m(2), n(6), o(7), p(2), q(1), r(5), s(6), t(10), u(3), v(1), w(2), x(1), y(1), z(1).

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[132] If it had not been for frequency analysis, **Mary Queen of the Scots** might have kept her head (1587). She used a simple substitution cipher to write her perfidious correspondence, interspersing the letters with meaningless symbols (nulls). Nevertheless, **Francis Walsingham**, the founder of the British Secret Service, managed to weed out the nulls and do a frequency count of the remaining symbols, thus breaking her code.
Had she known the work of **Alberti** (1535), she might have avoided the chopping block.

Table 2.5: Polyalphabetic Substitution Cipher

Key Word Letters

		A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	→ x-axis
	A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	
	B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	
	C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	
	D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	
	E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	
	F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	
	G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	
	H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	
M	I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	
e	J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	
s	K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	
s	L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	
a	M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	
g	N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	
e	O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	
	P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	
L	Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	
e	R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	
t	S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	
t	T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	
e	U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	
r	V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	
s	W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	
	X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	
	Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	
	Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	
	↓																											
	y-axis																											

The beauty of the Alberti's cipher is that the table of cipher alphabets - can be made public[133]*, so long as the keyword is kept secret, without jeopardizing the security of the cipher. After Alberti, cryptography continued to develop through the works of* ***Giovanni Battista della Porta*** *(1563),* ***Blaise de Vigenère***[134] *(1586) and* ***Francis Bacon*** *(1613).*

Cardinal Richelieu *(Armand-Jean du Plessis, 1585–1642) invented a grille; he would place a card with holes in it over a sheet of paper, write his secret message in plain language in the holes, then fill in the rest of the paper to look like an innocent letter. Only a man with an identical grille could read the secret message.*

For 300 years Alberti's polyalphabetic cipher was thought to be invulnerable, but then, ***Friedrich Kasiska*** *(1865, Prussia), discovered a few intrinsic weaknesses, overlooked by the sender but exploited by the wily eavesdropping cryptoanalyst. He found, for example, that if a sequence of plaintext letters, that come up more than once, happens to be enciphered each time by the same keyword letters, identical ciphertext results.*

For example, in the message SEND MORE MONEY, the key-letter sequence LO, twice enciphers the plaintext sequence MO into XC. If bits of the ciphertext repeat often enough, the cryptoanalyst can figure out the length of the keyword and, hence, the number of cipher alphabets employed. Then it is just a matter of cataloging which ciphertext letters came from which cipher alphabet. For each cipher alphabet, a frequency count will reveal the plaintext letters.

Thus, careless use of the best cipher can compromise its security, making codebreaking much easier in practice than in theory. Diplomatic and military communications often begin and end with characteristic pleasantries ("Greetings!", "Respectfully yours", etc.) which are footholds for the analyst. Certain proper names (especially ones that are overly long) can also give the show away[135]*. Information can often be coaxed out of the enemy*[136]*. In this*

[133] Modern cryptographers, drawing on innovative mathematical methods, have been able to take this trend to an amazing extreme: *both* the encryption method and the key itself can be made public without compromising the cipher. In other words, the power to encipher a message is not the same as the power to decipher it.

[134] A French diplomat (1523–1596). His book *Traicte des Chiffres ou Secretes d'Escrire* (1586), earned him the title: 'father of modern cryptography'.

[135] In WWII, German communications spoke in cipher of the *Wehrmachtnachrichtenverbindungen* (a 32-letter word).

[136] In May 1942, the American high command knew that a vast Japanese naval force was going to strike soon, but did not know where. Japanese radio dis-

day and age, when cryptography is increasingly computerized, breakdowns in technology can have severe consequences[137].

Mathematics and Art – the story of perspective: Brunellechi (1413) to Chasles (1852)

Although Hellenistic painters could create an illusion of depth in their works, there is no evidence that they understood the precise mathematical laws which govern perspective.

***Alhazen** was first to give correct explanation of vision, showing that light scattered from an object into the eye (ca 1000 CE). Although he did not apply*

patchers referred again and again to AF. To find out, American intelligence agents instructed the U.S garrison at Midway to radio Pearl Harbor that it was running out of water. Shortly afterwards, the Americans intercepted a Japanese dispatch that reported a water shortage at AF. This helped to win the great naval battle of Midway.

137 In October 1985, the Reagan administration learned from intelligence sources that Egypt was lying about the whereabouts of the four terrorists who had hijacked the Italian cruise ship *Achille Lauro* and murdered an American citizen aboard. Contrary to what Egypt had publicly stated, the hijackers were still on Egyptian soil, preparing to leave the country quietly by air via an EgyptAir Boeing 737 jetliner. The Pentagon counterterrorist experts came up with a plan for intercepting the civilian getaway plane with F-14 Tomcats. Reagan, then aboard Air Force One, ordered the reluctant Defense Secretary, Weinberger (on flight in a different military plane), via an open *uncoded* shortwave radio channel, to proceed with the daring mission. An amateur radio operator overhead every word and immediately tried to sell the story to CBS News, who chose not to report the president's order. A few hours later the EgyptAir jet was forced down.

his ideas to paintings, the Renaissance artists later made important use of his optics.

By the end of the 13^{th} *century* ***Giotto*** *was painting scenes in which he was able to create the impression of depth by using certain rules which he followed: he inclined lines above eye-level downwards as they moved away from the observer, lines below eye-level were inclined upwards as they moved away from the observer, and similarly, lines to the left or right would be inclined toward the center. Some of his last works suggested that he may have come close to the correct understanding of linear perspective. A timeline history of linear perspective is given bellow:*

1413 ***Brunellechi*** *gave the first correct formulation of linear perspective. He understood that there should be a single vanishing point to which all parallel lines in a plane (other than the plane of the canvas) converge. He correctly calculated the relation between the actual length of an object and its length in the picture, depending on its distance behind the plane of the canvas.*

1435-6 *Using both the principles of geometry and the science of optics,* ***Alberti*** *was the first to give the mathematical description of perspective needed for a proper understanding of painting: setting up a system of triangles between the eye and the object, he gave a precise concept of proportionality which determines the apparent size of an object in the picture relative to its actual size and distance from the observer.*

1447 ***Lorenzo Ghiberti*** *applied Alberti's principles to the design of his bronze doors.*

ca 1470 *In his treatise 'On perspective for painting',* ***Piero della Francesca*** *gave a complete account of the geometry of vision, proving theorems which relate to the perspective of plane figures, prisms and more complicated objects such as a human head, the decoration on the top of columns and other more difficult shapes. To this end he measured both width and height along different axes. In fact, he was using a coordinate system (!), computing the correct perspec-*

tive position of many points of the 'difficult shape' from which the correct perspective of the whole object could be filled in.

1490 ***Leonardo da Vinci*** *was first to study the converse problem of perspective: given a picture drawn in correct linear perspective – compute where the eye must be placed to see this correct perspective*[138].

1525 *In his book 'Unterweisung der Messung mit dem Zirkel und Richtscheit',* **Albrecht Dürer** *made an important addition to the theory of the Italian School by stressing the importance of light and shade in depicting correct perspective. He also invented a variety of mechanical tools which could be used to draw images in correct perspective.*

1639 ***Desargues*** *laid the foundations to projective geometry in his treatise 'Brouillon project d'une atteinte aux evenemens de recontres du cone avec un plan' in which he was motivated by the problem of finding the perspective image of a conic section. He dealt with the properties of pencils of straight lines and ranges of points lying on a straight line, and used them to investigate the properties of conics. The modern term "point at infinity" appears for the first time in this treatise.*

1673 ***Philippe de la Hire*** *introduced the cross-ratio of 4 points. In his book 'Conic sections' (1685) he expounded a projective approach to conics.*

1719 ***Brook Taylor*** *published 'New principles of linear perspective'. The work gave the first general treatment of vanishing points. The*

[138] He realized that a picture painted in correct linear perspective only looked right if viewed from one exact location. Thus, for a painting on a wall, many people would *not* view it from the correct position. Indeed, for many paintings it would be impossible for someone viewing them to have their eye in this correct point, as it may have been well above their heads.

phrase 'linear perspective' was coined by Taylor in this work and he defined the vanishing point of a line, not parallel to the plane of the picture, as the point where a line through the eye parallel to a given line intersects the plane of the picture. He also defined the vanishing line of a given plane, not parallel to the plane of the picture, as the intersection of the plane through the eye parallel to the given plane, with the plane of the picture. The main theorem in Taylor's theory of linear perspective is that the projection of a straight line not parallel to the plane of the picture, passes through its intersection with the picture plane as well as through its vanishing point.

Taylor readdressed mathematically the inverse problem, which is to find the position of the eye necessary in order to see the picture from the viewpoint that the artist intended.

Taylor's work was an important step towards the theory of descriptive and projective geometry as developed by ***Gaspard Monge*** *(1799),* ***Jean-Victor Poncelet*** *(1822) and* ***Chasles*** *(1852).*

ca 1420 CE **Jamshid al-Kashi of Samarkand** (d. 1436). Arab mathematician and astronomer. Noteworthy for the accuracy of his computations, especially in connection with the solution of equations by Horner's method and his practice of using decimal fractions, both of which he derived from the Chinese. Calculated π correctly to 16 decimal places. No mathematician approached this accuracy until the late 16^{th} century.

Al-Kashi found a patron in the prince **Ulugh Beg**[139] (1393–1449), grandson of the Mongol conqueror Tamerlane. At Samarkand, where he held his court, Ulugh Beg had built an observatory (1428), and al-Kashi joined the group of scientists gathered there. With his death, the account of Arab mathematics was closed, since the cultural collapse of the Muslim world was more complete than the mere political disintegration of the Empire.

[139] **Muhammed Targai Ulugh Beg** and his teacher **Quadi Zada** (1364–1436) compiled trigonometric tables to a high degree of accuracy (8 to 12 decimal places). Ulugh was eventually put to death at the instigation of his own son. After the death of Al-Kashi, Quadi Zada became the director of the Samarkand Observatory. He computed the sine of one degree to an accuracy of 10^{-12} in decimals.

However, Arab mathematics had a crucial impact on modern science: the mathematical tradition handed over to the Latin world in the 12^{th} and the 13^{th} centuries, was richer than that with which the unlettered Arabic conquerors had come into contact in the 7^{th} century.

1427–1444 CE **Leonardo Bruni** (Aratino, ca 1370–1444, Italy). Humanist, historian and scholar. Promoter of Greek learning. Born in Arezzo. Secretary to the papal Chancery under Innocent VII and John XXII. Chancellor of Florentine Republic. A key figure in ushering the Renaissance of ancient Greek lore in art, literature and science.

1430–1466 CE **Piero della Francesca** (1406–1492, Italy). Painter and mathematician. Recognized as one of the most important painters of the Renaissance. Three of his books survived[140]: Abacus treatise (*Trattato d'abaco*), short book of the five regular solids (*Libellus quinque corporibus regularibus*) and on perspective for painting (*De perspectiva pingendi*). In the first he came up with some entirely original 3-dimensional problems involving two of the *Archimedean polyhedra* (truncated tetrahedron and cuboctahedron). Four more Archimedean appear in the second book: the truncated cube, the truncated octahedron, the truncated icosahedron and the truncated dodecahedron (all these modern names are due to **Kepler**, 1619). Piero appears to have been the independent rediscoverer of these six solids. Moreover, the way he describes their properties makes it clear that he has in fact invented the notion of truncation in its modern mathematical sense.

On perspective for painting is the first treatise to deal with the mathematics of *perspective*[141]. Piero was determined to show how this technique is firmly based on the science of vision (as it was understood in his time). He accordingly started with a series of mathematical theorems, some taken from the optical work of Euclid (possibly through medieval sources) but some original to Piero himself.

Though non of Piero's mathematical work was published under his own name in the Renaissance, it seems to have circulated quite widely in manu-

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[140] His pupil, the Franciscan **Eva Luca dal Borgo**, usurped his master's books and after Piero's death published them as his own works! This we know from "*Lives of the Artists*" of Giorgio Vasari (1511–1572). Vasari says that Piero was regarded as "*a great master of the problems of regular polygons, both arithmetical and geometrical... Made an intense study of painting and perspective... acquired an intimate knowledge of Euclid...*". Vasari added that he was prevented by his blindness (which overtook him through an attack of catarrh at the age of 60) from publishing his written mathematical manuscripts.

[141] A technique for giving an appearance of the 3^{rd} dimension in 2-dimensional works such as painting or sculptured reliefs.

script form and became influential through its incorporation into the works of others. Thus, much of Piero's algebra polyhedra appears in Pacioli's *Suma* (1494), much of his work on the Archimedean polyhedra appears in Pacioli's *De divina proportione* (1509), and the simpler parts of Piero's perspective treatise were incorporated into almost all subsequent treatises on perspective addressed to painters.

1440–1446 CE **Lorens Janszoon Coster of Haarlem** invented printing with movable cast-metal types (typography).

1440–1450 CE **Nicholas of Cusa** (Nicolaus Cusanus, Nicholas Krebs; 1401–1464, Germany). Theologian and scholar in the 'twilight zone' between medieval and modern times. Broke with scholasticism while it was still the orthodox system, and proposed scientific ideas ahead of his time. Although his knowledge of mathematics and natural philosophy did not go beyond the Greeks, he responded to their ideas, and reached conclusions not found among the savants of antiquity:

- Challenged, the Augustinian finite universe. Revived the Ionian cosmological model of an *infinite* universe in which all heavenly bodies are essentially alike. Seriously considered that the earth might be rotating daily on its axis due to an initial impetus imparted to it at the beginning of time. Maintained that it might be equally possible to build an astronomical theory on the basis of the earth moving around the sun (1440). But like **Oresme** before him (1370), he did not work out a new theory.
- Proposed to reform the calendar following a method later adopted (1582) by Pope Gregory XIII.
- Recommended timing the pulse rate as an aid in medical diagnosis (1450).
- Gave the first known description in the West of a hygrometer (1450).
- Suggested that plants grew by assimilation of water.
- Believed in the 'impetus theory' of motion of a spherical ball: the motive force is due to an initial impulse and lessens gradually.
- Vaguely anticipated the concept of *infinitesimals*, and had some intuitive feel for the procedure of the *limit*[142]. However, measured by modern standards his accomplishments are not exciting. Nevertheless, he contributed

[142] Through his futile efforts to square the circle and trisect an angle, he constructed the following scheme: a unit circle O is intersected by a diameter AB. At A a tangent is drawn normal to AB, and a point T is marked on it. At B, the diameter is extended by one radius to a point S. The points S and T are joined and the line TS intersects the circle at D, closest to T. Draw from D a perpendicular to AB at F. Denoting $\theta = AOD$, $\varphi = AST$ we find, from the

to the study of *infinity*, studying the infinitely large and the infinitely small. (He considered the circle as the limit of regular polygons.)

- Asserted that the truth can never be known in total, although it can be approached ever closer through scientific reasoning. God can be apprehended by intuition, i.e. an exalted state of the intellect in which all limitations disappear.

Nicholas became a great transitional figure between the worldview of the Middle Ages and that of the Renaissance. Like everyone in his time, he cast his thought as a continuation of tradition, yet his ideas initiated the fall of the entire cosmology and social outlook that held sway since Augustine. Nicholas' ideas were in truth a rebirth of ancient Greek learning, but not that of Plato and Aristotle, which had indeed never been rejected by the medieval thinkers. It was instead a revival of the Ionian methods of exactly 2000 years earlier.

In his work Nicholas returned to the central idea of **Anaxagoras** – an infinite, unlimited universe. In contrast to Ptolemy's finite cosmos circumscribed by concentric spheres with earth as their center, he argued that the universe has no limits in space, no beginning or ending in time, no center, and no single immobile place of rest, including the earth; God is not located outside the finite universe, he is everywhere and nowhere, transcending space and time.

Furthermore, since reality is infinite in its complexity, knowledge can only be a series of improved approximations, unifying ever expanding realms of experiences. In this sense learning will never lead to the *final* truth. There can be no "theory of everything". This open-ended theory of knowledge led Nicholas to the unavoidable conclusion that there is no final authority. While conservative in form, his ideas undercut the basic notions of hierarchy – social and cosmic – entrenched since the days of Plato.

He was born in the city of Cues (Latin: Cusa) on the Moselle, in the archbishopric of Trier (Treves), the son of a poor fisherman. He took his doctor's degree in law at the University of Padua in 1424. Failing his first case, he abandoned the legal profession and took the holy orders. He rose rapidly in the Church, finally becoming a Cardinal in 1448. In 1450 he was appointed bishop of Brixen in the Tyrol. In 1459 he acted as governor of Rome during the absence of his friend, Pope Pius II.

Although one of the great leaders in the reform movement of the 15^{th} century, Nicholas of Cusa's interest for later times lies in his philosophical

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similarity of the triangles TAS and DFS, $\overline{AT} = \frac{3\sin\theta}{2+\cos\theta} = 3\tan\varphi$. For small θ, $\overline{AT} \approx \theta \approx 3\varphi$; the length $\overline{AT}$ of the tangent segment AT is approximately equal to the arclength $\widehat{AD}$, whose angle θ is thus approximately trisected by constructing the angle φ.

much more than in his political or ecclesiastical activity. The novel ideas mentioned above are propounded in his principal work *De docta ignorantia* (On learned ignorance, 1440). His chief philosophical doctrine was taken up and developed more than a century later by **Giordano Bruno** (1584).

1443–1482 CE **Paolo (dal Pozzo) Toscanelli** (1397–1482, Italy). Physician, mathematician, geographer, astronomer and philosopher. A Florentine savant who collected information and recorded data in the natural sciences and channeled it further to whoever sought his advice or seemed to him worthy of it. He sought out travelers and adventurers from distant part of the earth and ferreted out from their rodomontades some nuggets of truth.

As the leading geographer of Italy, he prepared new charts and maps for the Medici's sea captains. He recalculated the earth's diameter, as had the ancients, with the same goal of finding the length of a degree of latitude. Based on his calculations (which were wrong) he encouraged **Columbus** in the idea that China and India could be reached by sailing across the Atlantic.

Toscanelli recorded observations of comets; the one of 1456 was later named comet Halley. Proclaimed (1450) the feasibility of sailing from Europe to Asia across the Atlantic he estimated the distance between the two continents at roughly 4900 km. In 1468 he traced in the cathedral of Florence the famous meridian line, which was to serve for determining the dates of the movable feasts of the church.

He never left his native city of Florence. But in front of him in his study there stood a globe showing countries still unexplored, and oceans not yet traversed, and strange races of mankind. From the confusing mass of fantastic reports that reached him, and from his measurements, made with inadequate resources, he drew with increasing certainty the conclusion that it must be possible to reach India by a westward route. He could no longer search for that passage himself, but he wanted to see it found by bolder and younger men, and gave them maps and advice. When **Columbus** began to dream of the untraveled sea route to India, he sought the advice of Toscanelli who sent him (1482) a chart of the ocean with an accompanying letter including all the fragments of his knowledge. This information increased the confidence of Columbus in his ability to cover the distance that an experienced sea captain could easily traverse.

He collaborated with **Nicholas of Cusa** and disseminated the latter's new cosmology, linking it to the emerging observational science.

Toscanelli exercised decisive influence over the young **Leonardo da Vinci**. He turned Leonardo's gaze to the skies, explaining to him the nature of the heavenly bodies, according to the new worldview of Nicholas of Cusa, thus propagating through him a conception of science that was wholly secular.

1446–1454 CE **Johannes Gensfleish zur Laden zum Gutenberg** (1397–1468, Germany). Inventor. Perfected in Mainz a printing press from movable metal type and conducted the first large-scale printing-office in the modern sense. By 1448, at the latest, he was using accurate metal type through which he could produce in sufficient quantity the print of the Latin *Vulgate Bible*. By 1454, Gutenberg printed 300 copies of the so-called *Gutenberg Bible*. The names of **Johann Fust** (1400–1465) and his son in law **Peter Schöffer** (1425–1502), both from Mainz, recur in some sources as possible collaborators.

Gutenberg said: "With my 26 soldiers of lead I will conquer the world". And he did. He died blind and penniless.

Gutenberg did much more than invent the printing press, and much less than invent printing. He played a major practical and symbolic role in independently reinventing (in a greatly improved form and within more receptive society) a printing technique previously developed in Minoan Crete (ca 1700 BCE) and by 1041 CE in China[143] by **Pi Sheng**.

1450–1475 CE **Ali al-Qalasadi** (1412–1486, Spain). The last of the Moorish mathematicians of Spain. [The name al-Qalasadi means the Upright, or Versed in the Law.] Made original contributions to the theory of numbers. Introduced a new radical sign and a sign of equality, and proposed a system of ascending continued fractions. Computed $\sum n^2$, $\sum n^3$ and used the method of successive approximation to determine square roots. Although these were all known to be discovered earlier, some of these may have been discovered independently by al-Qalasadi. He wrote several books on arithmetic and one on algebra.

Al-Qalasadi was born in Basta, a Moorish city in Spain, and remained there until the Christians captured it. He then traveled through the Islam world and died in Tunisia.

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143 By the year 858 CE, China was printing books. But most Chinese printers carved or otherwise wrote out a text on a wooden block instead of assembling it – letter by letter as Gutenberg did. Why did Gutenbergian printing take off while Minoan and Chinese did not? Coming up with an invention itself may be the easy part; the real, obstacle to progress may instead be a particular society's capability to utilize the invention. Other premature inventions include *wheels* in pre-Columbian Mexico (relegated to play toys because Mexican Indians had no draft animals!) and Cro-Magnon *pottery* from 25,000 BCE (what nomadic hunter-gatherer really wanted to carry pots?). The first made automobile driven with an internal-combustion engine was built in 1863, but no motor vehicle came along for decades, because the public was content with horses and railroads.

1450–1461 CE **Georg von Peurbach** (1423–1461, Austria). Mathematician and astronomer. Due to him (and Regiomontanus) European scholars became well acquainted with Arab trigonometry. Peurbach wrote on astronomy and gave tables of ecliptic calculations. He observed Halley's comet (June 1456) and wrote a report on his observation. He made further observations of comets and recorded the lunar eclipse of 1457. He published further tables checked by his own eclipse observations, and devised astronomical instruments.

Peurbach studied under Nicholas of Cusa. He became a professor of mathematics of Vienna, making this university the mathematical center of his generation.

1450–1600 CE The *Renaissance period* in music; The leading composers are:

- Josquin Despres 1440–1521
- Giovanni De Palestrina 1525–1594
- Orlando di Lassus 1532–1594
- Giovanni Gabrieli 1554–1612
- Claudo Monteverdi 1567–1643

1450–1850 CE 'Little Ice-Age' in Europe; advance and recession of glaciers.

Books and Guns

In the beginning there was only the spoken word. Then, to entrust his thoughts to a more lasting medium than mere memory, man took to drawing pictures representing things. Perhaps the oldest picture-script originated at 4000 BCE *in Mesopotamia. Its images – bird, ox, ear of barley – were scratched into soft clay tablets, then baked hard for preservation.*

But such writing was a cumbersome affair, mainly used for priestly documents and public records and literature depended almost totally on word-of-mouth transmission. The quick Mediterranean mind, awakening to a new culture, demanded a better way of harnessing the spoken language; shortly before 1000 BCE*, the* Phoenicians *– swift seafarers, sharp traders and good record keepers – began breaking spoken sound into their basic elements, and shuffling the resulting "letters" to form words – thus creating the world's oldest alphabetic writing. Soon the alphabet was seized upon by the* Greeks *who gave letters more convenient shapes and added the still-missing vowels.*

No sooner had man taught himself to spell then a new problem arose: what to write on? Leather, tree bark, leaves and wax tablets had all proved unsatisfactory.

In Egypt, for some 2500 BCE*, texts had been inscribed on brittle sheets made from the pith of a Nile Delta water plant,* papyrus*. The use of this material gradually spread through the Mediterranean world. Usually, several papyrus sheets were glued together to form a* scroll *that could accommodate a lengthy text (one* 40*-meter scroll containing the picture-script of Pharaoh Rameses II in still extant). But what a clumsy thing to read! The scroll, wrapped around a wooden stick, had to be held in the right hand, while the left slowly unwound it to reveal the next column of writing. Nevertheless, the royal library at* Alexandria *is believed to have had no fewer than* 700,000 *scrolls. Relatively fragile, papyrus invited rivalry.*

In wealthy Pergamum*, on the coast of Asia Minor, scribes wrote on specially prepared sheep, goat or calf skins. This fine pellucid stationary, tougher than papyrus and foldable, came to be known as* parchment*. Shortly after* 1 *CE, an unknown* Roman *scribe with a sense of compactness took a stack of thin parchment sheets, folded them, and fastened them together at the spine. Thus, the* book *was born. The earliest promoters of books were Roman Christians; to them it was essential to preserve the Scripture in the most lasting medium – and parchment did not wilt when handled. Moreover, when one wanted to hunt up a reference, chapter and verse, a book was a lot handier than a scroll.*

So it came about that, all through Europe's Dark Ages, an army of monks, ensconced behind monastery walls, hand-copied the torn and shredded writings of the past on sturdy parchment sheets. Without their toil, the literary and scientific glories of ancient Greek might have been lost forever; it frequently took years to finish copying a thick tome.

Meanwhile, in distant China, in the year 105 *CE an inventor by the name* **Tsai Lun** *devised paper, a writing surface that could be produced cheaply from wood, rags, or other substances containing cellulose. For* 600 *years this invention remained a closely guarded secret of the East. It was not until some Chinese papermakers were captured by the Arabs in the battle of Samarkand (7 CE) that the secret was divulged. Arab manuscripts written on paper survive from the* 9^{th} *century, and in the* 12^{th} *century the industry was established among the Moors in Spain, and also among the Moorish subjects of the Norman Kingdom of Sicily. From there it spread to the Christians in Spain and Italy, and in the* 14^{th} *century to Germany and elsewhere, though down to the close of the Middle Ages the most important paper-making centers were in north Italy.*

In 1439*, a stubbornly determined German craftsman,* **Johannes Gutenberg***, began experimenting with a substitute for handwriting; if he could cast the letters of the alphabet in reusable metal type, then arrange them, in a mirror pattern, into words, lines and columns on an even-surfaced plate, an imprint taken from this plate would make one page. In place of one painstakingly handwritten book, he would be able to run off on his "press" as many imprinted books – exact copies of each other – as he wished.*

Laboriously, Gutenberg put together his first page plates, each one composed of more than 3700 *signs and letters. With the help of a hand-worked wooden press that he had adapted from the wine press of his native Rhineland (and which remained unchanged for the next* 350 *years), he started printing in a rented workshop in Mainz. It took three years to turn out some* 190 *copies of the Gutenberg Bible (1454).*

With this remarkable invention, book prices dropped 80 *percent overnight, and learning to read became worthwhile among the Christians. A mere* 50 *years after Gutenberg's exploit, every major European country except Russia was printing its own books. It was as if floodgates had been opened. Some* $520,000$ *new titles were published in the* 16^{th} *century,* 1.25 *million in the* 17^{th}, 2 *million in the* 18^{th}*, and* 8 *million in the* 19^{th}*. Today, more than three billion titles come off the presses in a single year, adding up to an estimated* 5 *billion individual books.*

With the invention of printing came the invention of engraving: Woodcuts and copperplates did for the graphic arts exactly what printing did for letters. Works of art could be diffused and standardized. The two inventions, *printing and engraving, were of immense importance for the development of knowledge. Printing made possible the publication of mathematical and astronomical tables which could be depended upon; engraving, the publication of books with illustrations representing plants, animals, anatomical or surgical details, chemical apparatus, etc. One good figure is more revealing than many pages of text; the use of illustrations obliged the author to be more precise than he could have been, or wished to be, without them.*

The new technology was symbolized by the publication of illustrated treatise by ***Vannoccio Biringuccio of Siena*** *(1540),* ***Georgius Agricola*** *(1556),* ***Lazarus Ercker*** *(1574), all of which included a wealth of information on mining, metallurgy, chemistry, the founding of guns and bells, the making of weapons and gunpowder, the casting of alloys such as type metal, the coining of money, and many other arts and crafts. This suggested that, thanks to printing and engraving, the Renaissance was a vigorous age of stocktaking and encyclopedism as well as of invention. Every bit of knowledge could now be garnered and preserved forever. Words and images were immortalized.*

The Chinese gave us paper. Phoenicians brought forth our alphabet. To Rome we owe the format of the book; to Germany, the art of printing from movable type. Britain and the United States perfected book production. Today, 30,000 *finished books roll of high-speed presses in the U.S. alone in just one hour, and we find it hard to visualize the bookless world of our forebears.*

As is the case with many other inventions, it is impossible to determine the contribution of a particular individual to the development of typography and its commercial exploitation. This illustrates the fact that an invention is sometimes not the creation of an individual but a social product, multiple and cumulative.

The previous Chinese inventions of block printing and paper, are linked with the beginning of typographical printing in Western Europe. Xylography, or wooden block printing, originated in China in the early 8^{th} *century. Its transmission to the west [probably during the century from* 1250 *to 1350] suggested the next crucial step of cutting up an old block into constituent letters.*

When the art of writing, xylography, and all the subsidiary arts of illuminating, decorating and binding manuscripts, books, pictures etc., were at their apex (and had long passed out of the exclusive hands of the monasteries into the hands of students and artisans) – the art of typography was invented.

The invention of the movable type alone could not have brought about the 'printing revolution' in the absence of paper, ink and type. Printing on *parchments (sheep skin) or on vellum (calf skin) would require* 170 *calf skins or* 300 *sheep skins for a single copy of the Bible!*

Paper manufacture was introduced, as mentioned above, in Spain during the 12^{th} *century by the Arabs, who had themselves received the technique from China. It spread slowly during the next two centuries to much of Europe [to Germany by 1390]. In Europe, the chief raw material was old rags. By the time of Gutenberg's youth, paper was plentiful and sold for approximately one sixth of the price of parchment. The oil base for an* ink *suitable for metal type was used by Flemish artists in the early* 15^{th} *century.*

Finally, the type *[the mirror image of each of the letters of the alphabet] required skills connected with delicate forms of metallurgy, namely: those of the metal engraver and the designer of coins and medals, of the goldsmith adept at casting small objects, of craftsmen who made punches for stamping letters on bells, and of manufacturers of pewter vessels.*

We must imagine that in many places in Europe, ingenious artisans experimented with type, inks, papers and presses; that many parallel efforts were made to replace the scribe by a mechanical device. The actual invention of printing, namely, the dramatic fusion of familiar techniques into a workable process, probably occurred independently in several places, but first organized as an industry in Mainz by ***Gutenberg****,* ***Fust*** *and* ***Schöffer****. By* 1500 *the presses all over Europe had issued more than* 9 *million books representing over* 30,000 *titles.*

The immediate contribution of printing to learning was that it halted the propagation of error incurred by successive generations of scribes, and restored the great texts of the past to their original integrity. Printing gave scholars all over Europe identical texts to work with*, referring precisely to a particular word in a particular line on a particular page. Printing turned intellectual work as a whole into a cooperative instead of a solitary human activity; printing thus enlarged the amount of intellectual effort applied to individual problems.*

The effects of this novel concentration of brain power were noticeable, especially in the exact sciences. Thus, following Copernicus' publication of his book in 1543*, it gradually drew the best minds in Europe into a cooperative study of the controversial problem, and a solution was found* sooner *than it would have otherwise. Scientific research*[144] *acquired through this tool* a public forum*, a published exchange of novel results and ideas controlled by*

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144 The *immediate* effect of printing on science was not especially favorable, for those works printed most often were old-fashioned. A publisher is more keenly aware of what men wish to read than of the obligation to supply them with what is best for them to read.

cooperative critical examination and the repetition of experiments. Printing made books easier to acquire, freed students from compiling their own dictionaries and reference books, and freed their minds from barren memorization. It accelerated the diffusion of ideas. The reformation spread with the same astonishing rapidity as printing itself; it could not have done so without it.

The printing press spread the news at a hitherto unthinkable speed, increased the range of human communication, broke down isolation. The broadsheets and brochures were not necessarily read by all the people on whom they exercised their influence; rather, each printed word of information acted like a pebble dropped into a pond, spreading its ripples of rumor and hearsay. The printing press was only the ultimate source of the dissemination of knowledge and culture; the process itself was complex and indirect, a process of dilution and diffusion and distortion, which affected ever increasing numbers of people, including the backward and illiterate.

Even three or four centuries later, the teaching of Marx and Darwin and the discoveries of Einstein and Freud, did not reach the vast majority of people in their original, printed text, but through second and third hand sources, through hearsay and echo.

Printing assumed a double role through its promise of enlightenment and popular education on the one hand, and being a weapon of revolution, hostile to the status quo, on the other. This is why the censorship *of books appeared very soon after the invention of printing, and was practiced by both secular and ecclesiastical authorities. Banning and* burning *of books were designed to maintain political as well as religious orthodoxy. By* 1560*, censorship of books of all forms was prevalent in Western Europe.*

No less important was the invention of the gun*. After the invention of the rocket in* 1232*, the Chinese performed some further experiments in the military use of explosive power. They put it into tubes of bamboo, making the first 'Roman candles'.*

Early Roman candles had alternate packing of loose and compressed powder, so that as the powder burned down from the muzzle, the solid lumps were thrown out and burned as they flew. This was as close as the Chinese came to the invention of the primitive gun, which probably took place in Germany not later than 1326*. Some of the earliest guns were made of wood strengthened by iron hoops, or of copper and leather.*

Guns soon evolved into cannon and hand guns. The latter were at first small cannons latched to poles, which the gunners held under their arms like lances at rest. Cannons evolved into long guns for direct fire and very short guns, called mortars, for high-angle fire. For a time, balls of iron or lead were used in hand guns and balls of stone in cannon. Iron cannon balls soon *replaced those of stone, carrying as they did more kinetic energy for a given bore. Now cannons had to be made stronger and of smaller bore.*

The gun soon brought the feudal system tumbling down. This was not accomplished by shooting holes in the armor of knights; the early hand-guns were not as effective as all that. They did not completely displace the crossbow until the 16^{th} *century, and they did not put the armor makers out of business until the* 17^{th} *century,* 300 *years after guns came into use. What the early gun did was to knock down the walls of the castles where the local lordlings had dominated the countryside and defied their king. By shattering the feudal castle, just as it had the walls of Constantinople, the cannon prepared the way for the era of sun-kings ruling by divine rights. The towering medieval fortresses with their draw-bridges and moats became mere relics.*

Meanwhile, the hand-gun in its turn improved until it outshone the cannon. As the flintlock musket, it became cheap enough for any citizen to own, simple enough for him to use, and deadly enough to enable him to face regulars. Then the stage was set for fall of kings and the setting up of republics.

1452 CE The University of Paris required that students must read the first 6 books of Euclid to get a master's degree.

1453 CE, May 29 *The Turks captured Constantinople*[145] to the sound of heavy gun salvo, used for the first time in history in a major war. The Sultan Mahomet II was able to deploy against the astonished Byzantines 14 batteries, each of several great bombards, plus 56 smaller cannons of various types. Most spectacular of all were two enormous guns which fired stone balls nearly 1 meter in diameter and weighing over 400 kg. The guns required two oxen each and more than 1000 men to drive them from Adrianople, where they were cast, to the Bosphorus. The great guns took 2 hours to load and could fire only few times a day. Many Greek-speaking scholars escaped to the West. They brought with them classical manuscripts in Greek along with the ability to translate it into Latin. This may be considered as the beginning

[145] Founded by Constantine the Great, through the enlargement of the old town of Byzantium in 328 CE. Famous in history as the capital of the Roman empire in the East for more than eleven centuries (330–1453) and as the capital of the Ottoman empire since 1453. Rivals only Athens, Rome and Jerusalem in respect of influence over the course of human affairs. The University of Constantinople was founded by Theodosius II (401–450) in 425.

of the *Renaissance in science*[146], which lagged behind the Italian Renaissance by some 150 years.

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[146] The *Renaissance* may be divided into the *Humanistic period* (1453–1600, death of Giordano Bruno), and the *Natural Science period* (1600–1690, publication of Locke's *Essay on the Human Understanding*, which marked the beginning of the Enlightenment). Philosophy, during the Humanistic period was *man-centered*, emphasizing the place of man in the universe, while that during the Natural Science period was *cosmos-centered*. In both periods, philosophers turned their attention from theological studies of heaven, the life to come, God and Church, and supernatural things – to the study of man and nature, the earthly needs of man, nature's relationship to man and scientific methodology. Even the language of the Church, Latin, was discarded by the academic world for the various national languages, as the power of the Church declined. Rome was loosing mastery over the Church as new centers of religion appeared in Wittenberg (Martin Luther), Geneva (John Calvin), and London.

Scientific inquiry developed in all directions. While the universities of Paris and Oxford remained as great centers of learning, new institutions were established in Vienna, Heidelberg, Prague, and throughout Italy as well as Protestant Germany. During the 17^{th} century the scientific point of view became dominant in England, France, and the Netherlands. Both groups, the Humanists and the scientists, accepted new points of view – new methods of inquiry, new knowledge, new standards for the new man in his new Universe. There was a difference in their approach and emphasis, for the Humanists sought to revive Hellenic culture, while the scientists relied mainly upon empirical observation and rational methods of discovery.

Among the most influential philosophers adhering to the strictly scientific school were: **Galileo Galilei** (1564–1642), **Francis Bacon** (1561–1626), **Hugo Grotius** (1583–1620), **Thomas Hobbes** (1588–1679), and **Isaac Newton** (1642–1727). The approach of **René Descartes** (1596–1650) differed markedly from both Humanist and scientific philosophers.

The period of Renaissance philosophy (1453–1690) saw the rebirth of Greco-Roman culture, the revival of an independent spirit of learning, the renewal of interest in the humanities, and (with the downfall of Scholasticism) the termination of subserviency of philosophy to theology, and the authority of the Church. Philosophy developed in a natural progression free from the yoke of ecclesiastical dogmatism.

Men in the 15^{th} century developed an interest in nature, but most turned to the Greek writings rather than add new knowledge. It is true that **Leonardo da Vinci** (1452–1519) found that sound and light follow certain rules, but he did not publish his results and had little influence on his contemporaries.

1457–1503 CE **Itzhak ben Yehudah Abravanel (Abrabanel)** (1437–1508, Portugal, Spain, Italy). Jewish statesman, philosopher and biblical exegete. The last scholar-statesman who occupied an important position in the financial administration of both Spain and Portugal. Assisted in financing the Columbus first expedition and led Jewish exodus out of Spain (1492).

He was born in Lisbon of an ancient family which claimed descent form the royal house of David. Like many of the Spanish Jews he united scholarly tastes with political ability. He held a high place in the favor of King Alphonso V, who entrusted him with the management of important state affairs. On the death of Alphonso (1481), his counselor and favorites were harshly treated by his successor John II, and Abravanel was compelled to flee to Spain (with his son Yehudah), where he held for eight years (1484–1492) the post of a minister of state under Ferdinand and Isabella. When the Jews were banished from Spain in 1492, no exception was made in Abravanel's favor. He afterward resided in Naples, Corfu and Monopoli and in 1503 removed to Venice, where he held office as a minister of state till his death in 1508.

His repute as a commentator on the Scriptures is still high. He was one of the first to see that for Biblical exegesis it was necessary to reconstruct the social environment of olden times, and he skillfully applied his practical knowledge of statecraft to the elucidation of the books of *Samuel*, *Kings* and *Daniel*[147].

Abravanel rejected philosophical rationalism, drawing on contemporary social and political realities to elucidate Messianic Biblical passages. His political theory is expounded in his commentary on *Deuteronomy* and *Judges*: the monarchy, according to him is a human (not a divine) institution. He advocated Constitutional form of government, yet he denied right of subjects to rebel.

1457–1480 CE **Yehudah Ibn Verga** (ca 1430–1499, Spain and Portugal). Astronomer, mathematician, chronicler and Kabbalist. Wrote a number of books on astronomy and mathematics. Invented a new instrument to determine the Sun's meridian, described in his book *Kli-ha-Ofek*, (Lisbon, 1457).

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147 In his commentary on the book of Daniel (1497) entitled "The Wells of Salvation" (Maa'yanot ha'Yeshuah), the author indulges in an *astrological* study, discussing in detail the significance attributed by the Jews to the periodic conjunctions of *Jupiter and Saturn* which occur about every 20 years. The sign of the zodiac in which they occur changes from one conjunction to another. This study was consulted by Juhannes Kepler (1603) and prompted him to advance the hypothesis that the 'Star of Bethlehem' was indeed a conjunction of Jupiter and Saturn in Pisces in 7 BCE.

Ibn Verga came from Seville and was a relative of Itzhak Abravanel. There he was active in maintaining an understanding between the Marranos and the Jews. When the Inquisition was introduced into Spain it desired him to betray the former. When Jews were expelled from Andalusia (1483) he moved to Northern Spain, but had to flee again to Lisbon (1492). He lived there for several years until he was imprisoned, perishing as a martyr in the Inquisition dungeons (1499).

Ibn Verga wrote a history of the persecution of the Jews; his work, in turn, was the basis of the historical work *Shevet Yehudah* (The Rod of Yehudah) compiled and published (1550) by his relatives **Shlomo Ibn Verga** (ca 1460–1524) and his son **Yosef Ibn Verga** (ca 1495–1559). This work has special importance in the annals of Jewish historical thought: it is one of the outstanding achievement of the Hebrew literature of the Renaissance.

1463–1476 CE **Johannes Müller**, or **Regiomontanus** (1436–1476, Germany). Astronomer and mathematician. Together with **Georg von Peurbach** (**Purbach**) (1423–1461) revived observational astronomy in the 15^{th} century[148]. His observations of the great comet of January 1472 and his major work on comets (published by Johannes Schöner in 1531) provided several techniques for determining a comet's parallax, as well as determining a comet's position and size[149].

Müller published *De Triangulis*, the first systematic European treatise on spherical trigonometry, from which Copernicus borrowed heavily (without acknowledgement!) in his own chapter on trigonometry. It was the earliest work treating trigonometry as a substantive science[150]. Müller made his own astro-

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[148] They jointly undertook a reform of astronomical observations rendered necessary by the errors they detected in translations of Ptolemy's tables. This need arose in connection with navigation and the reform of the old Julian Calendar. From **Roger Bacon** in the 13^{th} century to **Petrus Ramus** in the 16^{th}, there have been outstanding individuals and schools who realized that Aristotelian physics and Ptolemaic astronomy had to be put out of the way before a new departure could be made. When Müller completed the commentaries on Ptolemy which Purbach had begun, he realized the need to put astronomy on a new basis by 'ridding posterity of ancient tradition'.

[149] Already **Levi ben Gershon** suggested in 1328 that Ptolemy's parallax method for the moon be applied to determine the distance to a comet.

[150] *Regiomontanus' problem* (1471): "*At what point on the earth's surface does a perpendicularly suspended rod appear largest*?" The point of this problem becomes obvious when phrased in the more vivid form: "*From what distance will a statue on a pedestal appear largest to the eye*?". Clearly this occurs when the angle α subtended at the eye is widest.

nomical observations and prepared a nautical almanac for the aid of Spanish and Portuguese navigation, including tables of trigonometric functions.

Müller was born at Königsberg, the son of a miller[151]. He was a child prodigy, who at the age of 12 published the best astronomical yearbook for 1448, and at 15 received a bachelors degree from the University of Vienna (and was asked by the Emperor Frederick III to cast a horoscope for the imperial bride). At 16 he became the pupil and associate of Peurbach in Vienna and was appointed to the faculty at 21 (his association with Peurbach lasted through 1452–1461). Upon Peurbach's death in 1461, he took over his teacher's condensation and explication of **Gerard of Cremona's** 12^{th} century translation of Ptolemy's *Almagest*. In 1462 he traveled to Italy with Cardinal Bessarion. At Rome he learned Greek and studied Ptolemy in the original. Although this work was completed sometime before 1463, it was not published until 1496.

He left Rome in 1468 to return to Vienna, and there was summoned to Buda by Matthias Corvinus, King of Hungary, for the purpose of collating Greek manuscripts at a handsome salary. But he convinced his royal patron that Ptolemy could no longer be relied on, and that it was necessary to put astronomy on new foundations by patient observations, making use of such recent inventions as the *corrected sundial* and the *mechanical clock*. Matthias agreed and in 1471, Regiomontanus left for Nuremberg where, under the patronage of Johann Bernard Walther (1430–1504) he installed the *first European observatory*, for which he partly invented the requisite instruments. In 1475 he was summoned to Rome by Pope Sixtus IV to aid the reform of the calendar (which was getting out of step with the solar year), and there he died within the year, most likely of the plague (some contemporary accounts attribute his death to poisoning).

If the events surrounding the death of Regiomontanus seem strange, the events surrounding the fate of his effects were even more peculiar. After his

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Assume for simplicity that the x-axis goes through the foot of the pedestal $(0, 0)$ and the eye $(\xi, 0)$, while the ends of the statue $(0, h)$, $(0, h+L)$ lie on the y-axis. Simple geometrical considerations show that

$$f(\xi) = \sin\alpha = \xi L[\xi^2 + h^2]^{-1/2}\left[\xi^2 + (h+L)^2\right]^{-1/2}.$$

The extremum condition $f'(\xi) = 0$ then yields $\xi = \sqrt{h(h+L)}$, corresponding to $\sin\alpha = \frac{L}{L+2h}$. At this distance, the best view is guaranteed. It can be shown that the points $(0, h)$, $(0, h+L)$, $(\xi, 0)$ lie on a *circle* that is tangent to the x-axis at ξ.

151 Königsberg means 'King's Mountain' in German. In Latin, Regiomonte translates as 'from the royal mountain'.

death, his books, papers, and instruments were acquired by Walther. Walther used the instruments to continue the observations that he and Regiomontanus had started, but the books and papers were locked up and access to them denied to one and all. After Walther's death in 1504, the executors of Regiomontanus' estate apparently started selling his books, all the while denying that they were doing so. Several law suits were initiated over the remains of the estate. Finally, most of the remaining books and papers were acquired by Johannes Schöner, who published a few of them in the 1530's and the 1540's.

The manuscripts and notes of Regiomontanus' last years are lost, and there remain only scant indications of the reform of astronomy that he planned. But we know that he had paid special attention to **Aristarchos**' heliocentric system, as a note on one of his manuscripts shows. And much earlier he, too, had noted that the sun ruled the motion of the planets.

Towards the end of his life, he wrote on a piece of paper enclosed in a letter the words: "*It is necessary to alter the motion of the stars a little because of the motion of the earth*". The wording, seems to indicate that the '*motion of the earth*' here refers to its annual revolution round the sun. In other words, Regiomontanus has arrived at the same conclusion as Aristarchos and Copernicus, but was prevented from finalizing his work by his untimely death. He died at 40, when Copernicus was three years old.

1470–1494 CE **Itzhak ben Moshe Aramah** (1420–1494, Spain). Philosopher, mathematician and Talmudic scholar. First formulated the *statistical law of large numbers* (1470) in his book *Akedat Yitzhak* (Binding of Isaac); It was published in Salonici (1522), over two centuries ahead of **Jakob Bernoulli** (1770). Therein, Aramah stated:

> "Ordinary lots due to chance are without any tendency to one side or the other... They are not a 'sign', for matters of this kind are not established unless they are found many times... The casting of a lot indicates primarily a reference to chance."

Aramah was born in Zamorah, Spain and served as head of rabbinical academies in various Jewish learning centers. He was expelled from Spain (1492) and died in Naples, Italy. His philosophical system was influenced by Aristotle and Maimonides, and deals specifically with such major questions as faith and reason. Aramah became popular and influential, and his thinking represents the mainstream of Jewish medieval philosophy.

1473–1497 CE **Avraham ben Shmuel Zacuto** (1450–1515, Spain and Portugal). Astronomer, mathematician and historian. His astronomical tables, maritime charts and new astrolabe played an important role in the Spanish and Portuguese discoveries, especially in the voyages of Columbus and Vasco da Gama.

Zacuto's achievements in astronomy were many: his astrolabe of copper, the first of its kind (previously they had been made of wood), enabled sailors to determine the position of the sun with greater precision; his astronomical tables, based on the Alphonsine tables, were an improvement on the latter. They permitted sailors to ascertain latitudes without recourse to the meridian of the sun, and to calculate solar and lunar eclipses with greater accuracy.

Zacuto was born in Salamanca, Spain. His ancestors were Jewish exiles who had come to Castile from France in 1306. He attended the university of his native city, where he specialized in astronomy. Subsequently he became a professor at the universities of Salamanca and Saragossa. There he wrote his major astronomical work (1473–1478), and engaged in research and writing until 1480. He later removed to Gata, where he wrote a treatise on solar and lunar eclipses.

In 1492, when the Jews were expelled from Spain, Zacuto emigrated to Portugal, where he was appointed court astronomer to King John II and later to his successor Manuel I. Here he was engaged in fitting and instructing the expedition of Vasco da Gama. Gama himself consulted Zacuto in Lisbon before he set sail in 1497.

The great services rendered by Zacuto did not protect him, however, from the persecution inaugurated by Manuel at the instigation of the infamous royal couple Ferdinand and Isabella of Spain; he and his son were forced to seek safety in flight. He reached Tunis (1498), where he lived until the Spanish invasion in 1509. He then fled to Turkey, residing there for the remainder of his life.

The work of Zacuto was continued by his pupil and associate **Joseph Vecinho** (Vizino), who supplied Columbus with a translation of Zacuto's astronomical tables. Vecinho was later sent by king John II of Portugal to the coast of Guinea, there to measure the altitude of the sun by means of the astrolabe as improved by **Yaacov ben Machir Ibn Tibbon**.

1477–1514 CE **Donato Bramante** (1444–1514, Italy). Architect and painter. Evolved a style known as the *High Renaissance*. The greatest architect of his generation.

Bramante was born in Urbino and trained as a painter by **Francesca, Piero della**. Arrived in Rome (1499) to build the *Tempietto*[152] of St. Pietro in Montorio (1502). Worked in Milan (1477–1499) and Rome (1499–1514).

[152] So called because it has the aspect of a small pagan temple from antiquity.

Patronized by the Sforza family. Worked with **Leonardo da Vinci** on problems of the Milan Cathedral. Bramante designed the Basilica of St. Peter (begun 1506). He strongly influenced Pope Julius II city-plan for Rome (from ca 1508).

Even though Bramante was called unlettered (as were Leonardo), because he was ignorant of Latin and Greek, he must have acquired considerable learning, however fragmentary. His theoretical writings, have all been lost. His insatiable thirst for experiment and for new knowledge forced him away from convention in his works to a multiplicity of attitudes and expressions. Perhaps these characteristics indicate a certain dissatisfaction, an inner melancholy, or a deep sense of solitude. He apparently never married or had children. In unceasing experiments in his work, he may have been seeking a remedy for his incurable restlessness.

1481 CE **Meshullam ben Menahem of Volterra** (ca 1441–1500, Italy). Jewish goldsmith and traveler. Sailed from Naples, Italy (May 1481) via Rhodes[153] to Alexandria. Visited Cairo and went through the Sinai Desert to Gaza, Hebron, Jerusalem, Jaffa, Damascus and Beirut. From there he sailed back via Rhodes, Crete and Korfu to Venice (October 1481). His travelogue was first published in Vienna in 1882. The manuscript is an important historical document concerning the cultural and socio-political background of life in the Eastern Mediterranean and the Holy Land at the end of the 15^{th} century.

1480–1484 CE **Felix Fabri** (1441–1502, Germany). Dominican friar traveler and writer; one of the most distinguished learned writers of the 15^{th} century.

Fabri (Faber, Schmid) was born in Zürich and died in Ulm, Germany, where he spent most of his life. He made his early studies under the Dominicans at Basle and Ulm and graduated as master of sacred theology. He became head preacher of the Preaching Order at Ulm (1477–1478). Made two pilgrimages to the Holy Land (1480, 1483–1484) taking the route Ulm - Memmingen - Innsbruck - over the Alps into Italy - Venice - by ship to Corfu - Crete - Cyprus - Syria - Holy Land - Sinai - Egypt. Wrote two accounts of his travels , one in German and second in Latin. The Latin version is very complete and accurate in its description of places visited. The journal of his second pilgrimage made Fabri one of the most distinguished writers of his time. This work was reproduced by the Stuttgart Literary Society in three

[153] During 1428–1485, the Popes forbade all Italian ship owners to transport Jews to the Holy Land on board their vessels. This explains why Jewish travelers had to zigzag their way by hopping from port to port in different ships or go overland through Turkey.

octavo volumes (1843–1849) under the title: "*Fr. Felicis Fabri Evagatorium in Terrae Sanctae, Arabiae at Aegyptis peregrinationen*".

1482–1519 CE **Leonardo da Vinci**[154] (1452–1519, Italy). Painter, sculptor, architect, musician, civil and military engineer, inventor and philosopher. The father of modern aviation and anatomy. Student of the natural sciences (anatomy, botany, astronomy, geology, and physics), who sought to discover the mathematical laws governing all observable phenomena.

Through his teacher **Paolo Toscanelli**, his thinking developed towards the modern scientific method: a conception of science that was wholly secular and in no way based on religious doctrines or philosophy. In his thinking as in his actions, the gap between spirit and matter, theory and practice, was finally bridged in reality. While philosophers from Philoponus to **Nicholas of Cusa** had recognized the unity of the world, they had remained abstract thinkers. In Leonardo, the craftsman, scientist, and inventor are merged into one. Liberated philosophically by the new infinite cosmology, and liberated economically by widespread social change that had weakened the authoritarian hierarchy, he went far beyond his predecessors – he observed the *whole world*.

Leonardo put into practice Nicholas' idea that knowledge must derive from observations, and linked it with the necessity of mathematical description. He emphasized that "*there is no certainty in science where one of the mathematical sciences cannot be applied*". But he emphatically rejected the Platonic idea of mathematics as the master of science. In Leonardo's method, experiments lead to the hypothesis of 'rules of nature', mathematical rules whose utility is as an aid to human beings in their lives. He applied this method on a scale not equaled before him.

Leonardo recorded his ideas about art, engineering and science in his *notebooks*, which include about 4200 pages. He wrote his notes backwards, such that they can be read only with a mirror. Many of his ideas and designs were far ahead of their time, but because his writings remained largely uncirculated, and were extremely difficult to decipher[155], he exerted little or no

[154] For further reading, see:

- Vallentin, A., *Leonardo da Vinci, The Tragic Pursuit of Perfection*, Viking Press: New York, 1938, 561 pp.
- Hart, I.B., *The World of Leonardo da Vinci*: *Man of Science, Engineer and Dreamer of Flight*, Viking Press: New York, 1961, 374 pp.

[155] His method of mirror writing on little notes which he often inserted in any blank space that was available, had to do with a number of independent factors: First,

influence on subsequent generations of scientists. By the time Leonardo's scientific and technical investigations became widely known, other people had come up with many of the same ideas. Uneducated[156], incapable of adequate expression in words of the ideas flashing through his extraordinary mind, his notes are often incoherent and without conclusions. Thus he never developed his ideas systematically, seldom formulated scientific laws or principles, and did not make any notable scientific discoveries. He *did* however develop the empirical side of the scientific method[157], and must be credited with an impressive number of inventions, such as: *parachute*, *helicopter*, *construction crane*, the *wheelbarrow*, a *hodometer*, a *lens grinder and polisher*, a *flying machine*, a *compass*, *flexible chain or sprocket drive*, and *ball bearings*.

His wide variety of suggestions for the fields of *military engineering and hydraulics* is amazing: he could produce engines for offense and defense, and equipment for military excavations, mining, supply, and ordnance. *Projectile throwers*, *aerial bombs*, *catapults*, *mine detectors*, *military bridges*, *firearms* capable of imparting spin to bullets, are among his inventions or improvements on existing devices.

In hydraulics (a subject of endless fascination to him throughout his life) he was generations ahead of his time, especially in the field of *canal engineering*. He also devised innumerable ways of employing hydraulic power, such as *water lifts*, *ventilators*, *screws*, *pumps* and *water shells*. (Yet, there is no proof that all the machines he sketched were his own invention.) He stated that the sun does not move (some fifty years ahead of Copernicus), though savants of his day believed that the sun revolves around the earth.

Together with **Andreas Vesalius of Brussels** (1514–1564) he is the

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paper was not as cheap then as it is now and people did not like to waste it as we do. Second, there was no patent office and secrecy was the only method of protection.

[156] The distinguished historian of science and longtime Leonardo scholar, **George Sarton**, remarks soberly that in comparison to his contemporaries, he quoted very little from the great minds of his day, and indeed seems, for all his natural brilliance, to have been comparatively unlettered.

[157] Leonardo studied various building problems experimentally. Using small-scale models, he investigated how the weight that vertical pillars and horizontal beams could support, varied with their thickness and their height or length. His experiments led him to the results that the carrying power of a pillar, of given material and height, varied as the cube of its diameter, and that the carrying power of a beam was directly proportional to its thickness and inversely proportional to its length.

The bones and joints of animals he considered to be lever systems operated by the forces of their muscles.

founder of modern anatomy. Leonardo was fearless in his research, at a time when ecclesiastical and magical taboos still intimidated his contemporaries. In his anatomical dissections his often restated belief in the primary value of observation, and his willingness to repeat his dissections rather than content himself with one single example, establish his drawings as important scientific documents. But again, his anatomical drawing and researches remained hidden for centuries.

Leonardo was born near the village of Vinci, near Florence, as the illegitimate son of Piero da Vinci (a legal specialist) and a peasant girl. During the late 1460's Leonardo became an apprentice to the painter and sculptor **Andrea del Verrocchio**. During 1478–1482, Leonardo had his own studio in Florence, and he left Florence in about 1482 to become court artist for Lodovico Sforza, the Duke of Milan. There Leonardo stayed for 17 years, being employed as a military and civil engineer and a sculptor. He painted *The Last Supper* in 1495. During his Milan years he began to produce scientific drawings.

In 1499, the French overthrew Sforza and forced him to flee Milan. Leonardo too left the city, and visited Mantua and Venice before returning to Florence. The Florentine government hired him and Michelangelo to decorate the new hall for the city council. While working on this project, Leonardo painted the *Mona Lisa*, a portrait of Lisa del Giocondo, the young wife of a Florentine merchant. The *Mona Lisa* became famous because of the mysterious smile of the subject[158]. In 1517 Leonardo settled in France at the invitation of King Francis I, and spent his final two years near Tours.

Leonardo was one of the greatest heroes of the Renaissance, whose activities marked a climax of the Italian Renaissance as well as the beginning of the French one. Leonardo moves us deeply, first because of the Oriental proclivities that are one of the aspects of his mysterious genius, and second because of his scientific tendencies. He was a man without academic learning, whose attention, therefore, was not focused on books but rather on nature. He was concerned with new discoveries, not with rediscoveries.

At the end of the 15^{th} century, a new way of thinking opened up great vistas of learning and beauty. The wonderful phenomenon of the Renaissance was not merely a revival of classical culture; it was a change in the whole outlook of thinking men, who demanded escape from the tyranny of dogmatism,

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[158] "*Women did not arouse in him any feelings of desire. He was therefore all the more curious about the mysterious character of reproduction and generation. He could consider the sexual act as part of the endless flux of growth, decay and rebirth which formed for him the most fascinating and fundamental of all intellectual problems*". (Kenneth Clark).

from limitations of thought imposed by the Church. It was the Renaissance that finally broke the Church's stranglehold upon science and medicine.

Worldview V: Leonardo da Vinci

* * *

“He never turns back who has found his star.”

* * *

“Our life is made by the death of others.”

* * *

“Necessity is the mistress and guide of nature. Necessity is the theme and the inventress, the eternal curb and law of nature.”

* * *

“Science is the observation of things possible.”

* * *

“The common sense is that which judges the things given to it by other senses.”

* * *

"The Sun does not move."

* * *

"A bird is an instrument working according to mathematical law, which is within the capacity of man to reproduce." (1505)

* * *

"There is no higher or lower knowledge, but one only, flowing out of experimentation."

* * *

"Among all the studies of natural causes and reasoning, Light chiefly delights the beholder; and among the great features of mathematics the certainty of its demonstrations is what preeminently tends to elevate the mind of the investigator: Perspective, therefore, might be preferred to all the discourses and systems of the human learning." (1497)

* * *

"He who loves practice without theory is like the sailor who boards ship without a rudder and compass and never knows where he may cast."

* * *

"No human investigation can be called real science if it cannot be demonstrated mathematically."

* * *

1484 CE **Nicolas Chuquet** (ca 1445–1500, France). Mathematician. Published (1484) a manuscript treatise entitled *Triparty en la science des nombres*. Although the work was not printed until 1880, it had considerable contemporary influence. In the third part of his book Chuquet devised an exponential notation in which the power of the unknown is indicated by an exponent [e.g., $6x+12x^3+4+7x^{-2}$ were written in the form $.6.^1+.12.^3+.4.^0+.7.^{2.m}$]. Chuquet also produced what is, in effect, a small table of *logarithms* to base 2. In his work, *negative numbers* appear for the first time and *zero* is used.

Chuquet was born in Paris, qualified in medicine and practiced in Lyon.

Table 2.6 lists important Chinese, Hindu and Medieval mathematicians.

Eclipses, Occultations and Maps

If noon *(when the sun lies directly over the meridian on which one is located) at a place B on earth occurs y hours* after *noon at A, B is $15y$ degrees west of A. If noon occurs at B y hours* before *A, B is $15y$ degrees east of A. A difference in longitude is the* hour angle *difference between the local transits of the sun across the two* meridians *(the lines joining the north and south points of the horizon at the two locales).*

Simple as this may appear, it was a very difficult mental leap to make in ancient times! Until there were wheel-driven clocks, there were no portable devices with which to compare solar time at two different places. Although the later Alexandrians had very elaborate water-clocks *of much greater delicacy than the crude hour-glasses of earlier periods (and certainly no less accurate than the first medieval clocks which were driven by weights) they had no means of a maintaining a continuous record of time over a long journey.*

Yet, ***Eudoxos*** *(fl. 370 BCE),* ***Kiddinu*** *(fl. 367 BCE),* ***Aristarchos*** *(fl. 280 BCE),* ***Eratosthenes*** *(fl. 235 BCE),* ***Hipparchos*** *(fl. 150 BCE), and* ***Ptolemy*** *(fl. 150 CE) – all understood that certain astronomical events can be used in lieu of accurate clocks to determine longitude. Equipped with neither chronometers, nor radio signals, they offered a way of using natural*

Table 2.6: Important Chinese, Hindu and Medieval Mathematicians

Name	Life-Span	Major Contribution
	China	
Liu Hui	fl. 263 CE	Negative numbers; Solution of linear indeterminate equations. Suggested Cavalieri's principle to find accurate volumes .
Tsu–Chung–Chi	430–500	$\pi = 3.1415926\ldots$
Chin–Chiu–Shao	1202–1261	Chinese Remainder Theorem
Chu–Shih–Chieh	1270–1330	'Horner's Method'; Pascal's Triangle
	India	
Unknown	ca 500	Decimal position number-system; *Zero* symbol
Aryabata the Elder	476–550	Indeterminate equations and continued fractions
Varahamihira	505–587	Hindu Calendar; Trigonometric identities
Brahmagupta	598–670	Area of cyclic quadrilateral; Quadratic equations; Negative solution of equations
Khadyaka	610–680	Interpolation formula for sine function
Mahavira	fl. 850	Linear and nonlinear indeterminate equations
Bháskara	1114–1185	'Pell Equation'; Operations with *zero*; Early combinatorics; Idea of the *differential*
Narayana Pandit	1340–1400	Accurate calculation of square roots
Madhava	1350–1425	First *power series* expansion of a function

Table 2.6: (Cont.)

Name	Life-Span	Major Contribution
	Middle East	
Al-Khowarizmi	780–850	Algebraic manipulations
Omar Khayyam	1044–1123	Geometrical solutions of cubic equations; Forerunner of non-Euclidean geometry
	Western Europe	
Avraham bar Hiyya ha-Nasi	1065–1136	First complete solution to the quadratic equation
Leonardo of Pisa (Fibonacci) (I)	1170–1250	Indeterminate equations;; Fibonacci sequence; Approximations to cubic equations
Levi ben Gersion (Gersonides) (J)	1288–1344	*Mathematical Induction* (1321); Permutations; $\sum n^3 = (\sum n^2)^2$
Nicole Oresme (F)	1323–1382	Forerunner of *Coordinate Geometry*; Fractional exponents; Harmonic series
Nicolas of Cusa (G)	1401–1464	Forerunner of the concepts: *limit*, *infinity*, *infinitesimals*; Calendar Reform
Pietro Della Francesca (I)	1406–1492	Three-dimensional *perspective*
George von Puerbach (G)	1423–1461	Trigonometric tables
Johannes Müller (Regiomontanus) (G)	1436–1476	Modern Trigonometry
Johann Widman (G)	1462–1498	Algebraic symbolism
Nicolas Chuquet (F)	1445–1488	First modern algebraist: negative numbers, fractions, exponents; The idea of logarithms (1484)

I = Italian; J = Jewish; F = French; G = German *phenomena visible from many parts of the earth such as eclipses of the moon or the occultation of a planet or star by the moon's disk.*

The method, based on the lunar eclipse, works as follows. Let the moon enter the earth's shadow cone at local time t_1 *as reckoned at location* A_1 *(relative to the local noon as measured on a sun-dial, say), and let it likewise enter the same cone at local time* t_2 *as reckoned by the local timekeeping at location* A_2*. The longitude difference between the two locations is then* $15°(t_2 - t_1)$*. The mariner who possesses an almanac giving the local time at which an eclipse (or other astronomical event) will occur at one place, can therefore obtain his longitude by recording the local time of its occurrence where he is*[159]*.*

Aside from lunar eclipses, the moon's course displays other circumstances which were highly portentous; as it moves along its orbit, it may block the visibility of other heavenly bodies, in particular planets which move near the ecliptic and thus also near the moon's orbit. Thus, during an occultation of Mars, the planet will disappear behind the moon's disk.

People observed these occultations and were able to know when they would occur at a particular place. Anyone who possessed an hour-glass at another place could watch for them and record the interval which elapsed between an occultation (or an eclipse) and that day's local noon.

Latitude could be determined by either sighting the elevation of a star at night (with a sextant) or the sun at daytime. Thus true maps could in principle be constructed from the knowledge of both latitude and longitude of places.

Occultations and conjunctions of planets were reported by Greek, Chinese, Arab and European astronomers throughout history; some examples are:

- ***May 04, 357*** *BCE: Occultation of Mars visible in Greece during the lifetime of Aristotle, and one of the very few astronomical observations which Aristotle himself is known to have made. He renders a very careful description in De Caelo* ***2****, 12.*
- ***Sept. 04, 241*** *BCE: Close conjunction of Jupiter and the star* δ *Cnc (Ptolemy, Almagest* ***11****, 3).*

[159] **Columbus** used this technique to fix the longitude of *Jamaica* (1504), and was disappointed to find that it lay not as far west as he expected. Earlier, he looked out for a port of anchor at *Haiti* for observing the lunar eclipse on January 13, 1493. Allegedly, **Vespucci** found the difference of longitudes between Venezuela and Cadiz in 1499 by observation of a lunar eclipse.

- ***Feb. 12, 73*** *CE: Chinese report on occultation of β Sco by Jupiter (disappeared Feb. 12; reappeared Feb. 16).*
- ***Sept. 13, 1170*** *CE: Conjunction of Mars and Jupiter. During the night of* 12–13 *Sept. the planets passed within a minute of arc of one another, so that the unaided eye would not be able to resolve them.*
- ***July 18, 1273*** *CE: Mercury occulted by the moon. Observed in Egypt for half an hour.*
- ***Jan. 09, 1591*** *CE: Jupiter totally occulted by Mars. Observed by Kepler at Tübingen, Germany.*

Unfortunately, eclipses or occultations occur so rarely as to be almost useless for ships on the move. Nevertheless, ***Hipparchos*** *believed that an extensive series of observations should be carried out in order to ascertain, by mathematical and astronomical means alone, latitudes and longitudes of a large number of places. To facilitate such a survey, he prepared tables of lunar eclipses, and tables to aid in the determination of latitudes, but the practical difficulties of the undertaking were too great and the work was never completed. In fact, throughout antiquity the total number of places whose position had thus been accurately determined probably does not exceed half a dozen, if that*[160].

The navigators of ***Columbus*** *and* ***Magellan****, versed in Muslim astronomy, were able to define the position of America on the world's map. So knowing exactly where the planets are located was a matter of some practical importance in the period of the Great Navigations, when* ***Copernicus*** *and* ***Kepler*** *showed that their positions can be calculated more accurately and far more simply if we reject the commonsense-view of the priestly astronomers.*

1485–1488 CE **Ovadiah Yareh of Bertinoro** (ca 1435–1515, Italy). Traveler and Mishna commentator. Sailed (1485) from Naples to Sicily and

[160] **Pliny** speaks of an eclipse of the sun that was seen in *Campagnia* between the 7^{th} and 8^{th} hours and in *Armenia* between the 11^{th} and 12^{th}, indicating a difference in longitude of 4 hours, or 60°. The actual distance is no more than half of this. Much greater accuracy was attained by the Arabs in their calculations of longitude, and some of their figures were passed on to the Western world in astronomical tables during the 12^{th} and 13^{th} centuries.

from there to Egypt[161] (1487). Then went through the Sinai Desert to Jerusalem (1488). Described his perilous journey in a series of letters (1488–1490) to his family in Northern Italy. Composed his famous exegesis to the Mishna (1490–1505). Died in Jerusalem and was buried in a cave in the Mount of Olives.

1486–1489 CE **Johannes Widman** (1462–1498, Germany). Mathematician. Introduced the symbols (+) and (−) to denote addition and subtraction, respectively. Author of *Behennde und hüpsche Rechnung auf allen Kauffmanschafften* (1489). This marks the beginning of *algebraic symbolism*. He considered computation with irrational numbers and polynomials to be part of algebra.

Widman attended the University of Leipzig, graduating in 1482. His Masters Degree was awarded in 1485 and he then taught at the University of Leipzig on arithmetic and algebra.

Although the Germans did not enrich algebra during the Renaissance with great inventions, as did the Italians, they still cultivated it with great zeal.

1492 CE, Jan. 23 First printing of the Pentateuch.

1490–1520 CE **Yehudah Abravanel** (LEONE EBREO) (1460–1530, Portugal, Spain, Italy). One of the great philosophers of the Renaissance, physician, mathematics and astronomer. An outstanding figure of the period of transition between the Middle Ages and the Renaissance. He lived not only at the conjunction of two eras but was also in contact with three cultures – Jewish, Spanish and Italian.

Yehudah was born in Lisbon. He fled (with his father Itzhak) to Spain (1483) and practiced medicine there. After the expulsion (1492) he left with his father for Naples and worked there as a physician. Upon the conquest of Naples by the French (1496) he moved to Geneva, but returned to Naples and lived there (1503–1521) as court physician to the Spanish viceroy. He lectured on mathematics and astronomy at the Universities of Naples and Rome.

His most famous work *Dialoghi di Amore* (1535) is a landmark in the history of aesthetics, metaphysics and ethics. He maintained that true happiness is the union of human intellect with the Divine intelligence, and that it is directly connected to aesthetic enjoyment. There is a pantheistic strain in Abravanel's philosophy, but he always emphasized his orthodox Judaism, and tries to reconcile his pantheistic feelings with the biblical concept of God. The book was translated to most European languages and exerted great influence.

[161] When the ship reached Palermo, Ovadiah met (1487) with **Meshullam of Volterra**, who was on his second voyage to Egypt.

It was one of the most widely read books in Shakespeare's England. One of its heritages is the concept of *Platonic love*.

1492 CE, Aug. 3 **Christopher Columbus**[162] (Cristoforo Colombo, 1451–1506, Italy), *Admiral of the Ocean Sea*, sailed from the Spanish port of Palos with a crew of 89 men on board three small and fragile ships – the *Niña*, *Pinta* and the *Santa Maria* (a 24 meter long carrack of about 280 tons). On Oct. 12, he completed his first Atlantic crossing and landed on a Caribbean island that he named San Salvador, which he believed to be an island of the Indies, near Japan or China. He discovered Cuba (Oct. 27) and Hispaniola (Dec. 6). On March 4, 1493 he reached Lisbon after a tempestuous recrossing of the Atlantic.

Columbus' ambition was to find a short sea route to the Indies. He used a map given to him by the Florentine geographer **Paolo Toscanelli** (1397–1442, Italy), in which Japan lay only 4400 kilometers west of Lisbon.

Columbus navigated by dead reckoning: his navigators knew just enough celestial navigation to measure latitude from the North star. The navigation tools at his disposal were: compass needle, sandglass, quadrant and ephemerides (tables listing positions of stars and planets at given times). It was virtually impossible for him to use celestial navigation for the measurement of longitude (i.e. east-to-west distance) because there were no chronometers. To find latitude (i.e. north-to-south location) the height of the sun at its meridian (noon) was measured, or in the Northern hemisphere – the height of the North star. Sailors used an instrument called a *quadrant* for making these measurements; but the quadrant was so difficult to use on a rolling ship that reliable measurements could be made only through repeated land-based observations.

Given the primitive state of navigational instruments, the *ship's fix* in Columbus' day was usually only a rough approximation. It was necessary to know three elements: time, speed, direction. Based on these, the route

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[162] Some historians believe that Columbus was of Jewish origin, possibly a marrano or secret Jew. Five of his crew, as well as his interpreter **Luis de Torres** (who knew Hebrew, Chaldaic and a little Arabic) were known to be *Marrano* Jews (among them **Maistre Bernal**, the official doctor of the expedition, and **Marco**, the surgeon), and he used the Alfonsine Tables compiled by Jews.
There is bitter irony to the fact that the wealth that Ferdinand and Isabella confiscated (1492–1497) from the expelled Jewish population (ca 200,000), was used to finance the journeys of discovery of Columbus, Magellan, Cabral, da Gama and others. Thus, both Jewish material and intellectual resources were instrumental in the discovery of the New World. (History repeated here what had previously occurred during the Crusades!).

taken from the point of departure could be marked on the chart. *Direction* was given by the compass: the horizon was divided into 32 points instead of 360°, which means that directions were measured roughly. Accuracy was further reduced by the fact that no account was taken either of declination (difference between geographic north and magnetic north which varied from place to place) or of the shifting of the needle caused by nearby metal objects. It was relatively easy to record successive changes in course, but much more difficult to calculate the effects of drifts or the movement of currents.

Speed was a matter of guesswork, estimating by looking at the flow of water along the hull.

Time was measured with a *sandglass* which was turned every half hour (every 8 turns of the sandglass – that is, every 4 hours – the old watch was replaced and fresh crewmen took their place. Each watch passed on the compass course to the next watch and a new helmsman began his work). Since the time of noon was essential, and since the hour of midday became progressively later as the ships sailed westward, it was necessary periodically to synchronize the sandglass with the sun. Columbus did this on a weekly basis, using a method that probably was accurate within a quarter of an hour.

It is estimated that Columbus' first voyage cost $14,000 in today's currency. (A few years after the voyage, the 'poorest' of Spain's 13 dukes had an annual income that was about 5 times greater than all the funds raised for Columbus' voyage.) This suggest that his expedition was, in modern terms, a relatively low-budget operation.

Columbus was born in Genoa. The family name was Colombo. He called himself Cristóbal Colón after he settled in Spain. His father, Domenico, was a wool weaver. Columbus was sent to the University of Pavia, where he devoted himself to mathematics, astronomy, geometry, and cosmography. He had settled in Lisbon about 1479, marrying the daughter of the governor of Porto Santo in Madeira Island. There he spent some time studying his late father-in-law's collection of navigational works, talking to sailors about their voyages and making charts.

Like Aristotle and Eratosthenes, who had successfully calculated the circumference of the earth more than 1600 years earlier, Columbus knew that the earth was a sphere. Most of his reading seems to have been medieval. It was Marco Polo's overestimation of the east-west extent of Asia, and of the distance of Japan from the Asian mainland, that led him to believe that the voyage westward from Europe to Japan would be less than 3000 nautical miles. Indeed, he called the natives of the Bahamas "Indians", certain that he had arrived in Asia, in the realm of the Great Khan.

On Sept. 25, 1493 he went on his *second voyage* with the objective of the colonization of Hispaniola (today's Haiti and the Dominican Republic). He set out with 3 great carracks, 14 caravels and 1200 men – soldiers, farmers, missionaries, civil servants, and assorted fortune hunters. During his voyage he discovered the Lesser Antilles – from Dominicana to the Virgin Islands, including Puerto Rico and Jamaica. He returned with his fleet to Cadiz on June 11, 1496.

He embarked again, on his *third voyage*, on May 30, 1498, with 6 ships, crewed by pressed men and released criminals, the only men who could be found. The new lands discovered during this expedition was Trinidad and what is now Venezuela (= little Venice). From this voyage he was sent home in chains, but later reinstated in the Queen's favor.

After this Columbus made his *last voyage*, sailing on March 9, 1502, once more presumably in search of a way to Asia, and en route discovered Martinique, the mainland of Honduras and Costa Rica.

He had no means of knowing that the Pacific was not more than 500 kilometers to the west![163] He reached Seville on September 7, 1504, only to die two years later, a disillusioned and broken man.

1492–1498 CE *The New Exodus*[164]: expulsion of the Jews from Spain un-

[163] Columbus drew most of his ideas about geography from books written in the 14^{th} and early 15^{th} centuries. The cosmography in these books was based on erroneous maps and imaginary concepts that propagated from Greek and Roman writings

[164] In the course of the years 1355–1482, about 60,000 Jews were killed in massacres and riots and 20,000 forcibly baptized into Christianity under Spanish rule. On the eve of the expulsion there still remained about 200,000 Jews in the Iberian Peninsula. Of these ca 25,000 perished while seeking a new home; ca 50,000 remained in Spain as Marranos and Conversos; about 55,000 eventually reached Turkey; some 35,000 finally settled in France and Holland; 10,000 went to Italy; 5,000 to South America, and 20,000 to North Africa and Egypt.
Jews played a leading part in the cultural and economic life of Spain during the Roman and Islamic periods. When the Christians wrestled Spain from the Moors (1212), the Jews brought the splendor of the Islamic civilization to Christian Spain. By virtue of their learning and sophistication they left an imprint of humanism on that country. They rose to great positions of power, many attaining high ranks of nobility. Resentment against "outsiders" as "insiders" smoldered for a century, then erupted into an anti-Jewish movement popularly known as the "*Second Reconquest*", a movement to force the Jews to "give" Spain back to the Christians. It climaxed in the great conversion drives (1391) when many thousands were forcibly baptized. Thus, these "*New Christians*", the flower of Jewish aristocracy and intelligentsia, entered the service of the Church (becoming bishops, archbishops, cardinals) and began to dominate

der complex and chilling circumstances which laid the foundation for modern racist holocausts in Europe.

1494–1509 CE **Luca Pacioli** (ca 1455–1517, Italy). Mathematician and Franciscan friar. Compiled an overview of most mathematical methods handed over from the Middle Ages in his book *Summa de arithmetica, geometria, proportioni et proportionalita* (1494). This work, freely compiled from many sources, purported to be a summary of the arithmetic, algebra and geometry of the time. It contains little of importance not found in Fibonacci's *Liber abaci* (1202), but does employ superior notation. Methods are presented for solving equations of the first and second degree, accompanied

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Spanish intellectual life. It was this "Jew in Christian cloths" who became the villain, with disastrous results for the Jews and Spain. The cry was raised by the "Old Christians" that the "New Christians" were not loyal to the Church. They held that *limpieza de sangre* (purity of blood), not mere ability, should determine one's fitness for Church office. So persistent was that feeling that the Inquisition was introduced (1480) to determine blood purity; the stage was set for the Grand Inquisitor Tomas de Torquemada and the *Auto-da-fé* ("act of faith") – the rite of purification by being burned alive. [His grandfather, Alvar Fernandez de Torquemada married a recently baptized Jewess; this genealogical fact throws psychopathological light on the grandson's ruthlessness toward Jews.]

Torquemada asked for the expulsion of the all Jews. No charge was brought against them other than that they were not Catholic. The 50,000 Jews who chose to stay in Spain by conversion to Christianity became the new Marranos. But because the all-prevailing effects of *limpieza* they did not soar to intellectual eminence as the Jews and the Marranos of the two preceding centuries – the intellectual lights in Spain went out.

In the 2005 Internet text of the *Catholic Encyclopedia* we read the following lines under the item 'Tomas de Torquemada':

"During Torquemada's office (1493–1498) 8,800 suffered death by fire and 9,654 were punished by other ways. Whether Torquemada's ways of ferreting out and punishing heretics were justifiable is a matter that has to be decided by an inquiry into their necessity for preservation of *Christian* Spain." (sic!)

The contemporary Spanish chronicler, **Sebastian de Olmedo** calls Torquemada "the hammer of heretics, the light of Spain, the savior of his country, the honor of his order."

History tells us that after 1498, Spain sank back into the Middle Ages [see Science Progress Report No. 5]. The persecution by the Inquisition and the following expulsion of the Jews, stripped Spain of most of its intellectual, scientific and economic powers. It never recovered, and after Torquemada was not worth saving.

by many problems leading to such equations. It includes algorithms for multiplication of integral numbers written in Hindu-Arabic numerals, extraction of square roots, and the first printed description of *double entry bookkeeping*.

Pacioli's second book, *De Divina Proportione* (1509), included figures of regular solids on plates engraved by his friend **Leonardo da Vinci**.

Pacioli traveled extensively and taught at various places.

1495–1512 CE **Alesandro Achillini** (1463–1512, Italy). Philosopher, anatomist and surgeon. One of the first to dissect a human body. In his anatomical writings he described the veins of the arm, the seven bones of the tarsus, the fornix, ventricles, and infundibulum of the brain, and the trochlear nerve. He also described the ducts of the submaxillary gland before **Thomas Wharton** (1656), and two of the three ossicles of the ear (the malleus and incus).

Achillini was a lecturer both in medicine and in philosophy at Bologna and Padua.

1495 CE Epidemic of *syphilis* sweeps Europe.

1496–1500 CE The diseases of syphilis and cholera first identified or described with accuracy.

1497 CE **Amerigo Vespucci** (1451–1512, Italy). Merchant, pilot and adventurer, who gave his name of **Amerigo** to the new world as *America* although he had no share in the first discovery of the American continent. Yet, due to some odd combination of circumstances, America was named after him.

He was born at Florence. His father was notary, and his uncle, to whom he owed his education, was a scholarly Dominican and a friend of Savonarola. He studied natural philosophy, astronomy and geography and later was placed as a clerk in the great commercial house of the Medici, then the ruling family in Florence. He resided in Seville, Spain throughout the period 1492–1496 as an agent of the Medici, and certainly did not make the 1497 voyage, which he claimed he had made. Whether or not he took part in the expeditions of 1499 (for Spain), 1501 and 1503 (for Portugal) is yet an open question.

The connection of the new world with Vespucci, is derived from an alleged letter written by him from Lisbon (march of April 1503) to Lorenzo Piero Francesco di Medici, the head of the firm under which his business career had been mostly spent, describing his alleged Portuguese voyage of March 1501–September 1502. The original Italian text is lost, but a Latin translation is extant. According to this letter, Vespucci reached the mainland of America on a Spanish expedition 8 days before **Giovanni Gaboto** (John Gabot, June 16^{th} against June 24^{th}, 1497).

A second letter, written from Portugal (September 1504) to his old schoolmate Piero Sodorini, was made available in Latin translation to Martin Waldseemüller, professor of cosmography in St. Dié University. This man wrote a book *Cosmographiae Introductio* (St. Dié, 1507) in which he suggested that the newly discovered 4^{th} part of the world should be called "*America, because Americus discovered it*"[165]. His name was first accepted for South America, and gradually came into use for North America as well.

1498–1519 CE **Shmuel Zarfati**[166] (called **Gallo**, ca 1450–1519, Italy). One of the greatest physicians of his age; Pioneer of blood-transfusion in modern medicine. Originated from France. When the Jews were expelled from Provence (1493–1500), Shmuel was assured by King Louis XII that he personally would not be molested. He nevertheless preferred to emigrate (1498) and settled in Rome, where he became famous for his cures. There he was granted special privileges by Pope Alexander VI. He represented the Jewish community at the coronation of Pope Julius II (1503), and became the latter's personal physician (1504). His reputation was greatly enhanced (1511), when he successfully predicted the recovery of the Pope from a serious illness at a time when all the Pope's other physician had given up hope. He was permitted to treat Christian patients and was granted full rights of residence; He and his family were exempted from wearing the Jew badge. In 1515 he became physician of Giuliano de Medici.

Shmuel's son, **Joseph Zarfati** (called **Josiphon**, **Giosifante**, or **Giuseppe Gallo** by Christian writers, ca 1470–1527) was a physician, philosopher, poet, mathematician and an accomplished linguist (Latin, Greek,

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[165] In 1492, **Christopher Columbus** had no idea that he reached the Western Hemisphere. He thought that the islands he explored were part of the Indies. He first set foot on the mainland of America on his third voyage in 1498. When Vespucci claimed that he had discovered the new continent, or the New World, as it was called in 1497, Columbus did not dispute his claim.

[166] **Zarfati, Zarefati, Sarfaty**: An illustrious group of families of rabbinic Talmudic scholars, originating from France, some of which descended from **RASHI** (1040–1105). For the next 700 years these families issued Rabbinic scholars, Talmudists, physicians, poets, mathematicians and linguists which spread from France to Spain, Netherlands, Italy, Turkey, Israel and North Africa.
Other famous physicians from this family were:

- **Yaacov ben Shlomo Zarfati** (ca 1365–1425, Avignon, France).
- **Itzhak Zarfati** (ca 1485–1550, Italy). Physician to Pope Clement VII.
- **David Zarfati de Pina** (ca 1630–1700, Amsterdam). Noted physician, poet and preacher.

Hebrew, Aramaic, Arabic). The Pope extended to him the privileges that had been accorded to his father; these were confirmed by Pope Leo X and Pope Clement VII (1524).

The latter part of his life was dogged with misfortune. An unfaithful servant ran off with his fortune, he was falsely accused of being a spy, he was beset by robbers, and he escaped the siege of Rome in 1527 only to fall victim of the plague and die outside of the city of Vicovaro.

1498–1510 CE **Gaspar da Gama** (1444–1510, India and Portugal). Jewish navigator and explorer. Participated in the Portuguese naval expedition of **Vasco da Gama**, **Cabral**, **Nicolau Coelho** and **Francisco d'Almeida**.

He was born in Posen, Poland and became a traveler. He made his way to Jerusalem and then Alexandria, was taken prisoner and sold as a slave in India, where he obtained his freedom and entered the service of the ruler of Goa. There, he took on the name Yusuf' Adil. We do not know what his original name was.

When the Portuguese explorer Vasco da Gama arrived off Angediva in 1498, he was greeted in a friendly fashion by this long-bearded European on behalf of his master. Vasco da Gama self-righteously seized the Jew and compelled him to embrace Christianity under the baptismal name of Gaspar da Gama. He was also known as Gaspar d'Almeida and Gaspar de las Indias.

As a Catholic, Gaspar da Gama became the pilot of Vasco da Gama's fleet. He successfully guided the ships through treacherous Indian waters and was brought back to Portugal.

In Lisbon, Gaspar was granted a pension by the King, who employed his linguistic ability in subsequent Portuguese naval expeditions. In 1500 he accompanied Cabral on his voyage in Western waters and was with Nicolau Coelho when he first stepped ashore in Brazil.

On the return voyage he met Amerigo Vespucci (the Tuscan explorer after whom America is named) at Cabo Verde and was consulted by him.

In 1502, he went to India once more with Vasco da Gama and again in 1505 with Francisco d'Almeida. He took part in the latter's expedition against Calicut in 1510, when he may have died.

1500 CE *Hindu-Arabic* numerals finally superseded Roman numerals for most computational purposes in Europe.

1500 CE The Portuguese navigator and explorer **Pedro Álvars Cabral** (1467–1520) discovered *Brazil*. After da Gama's return, King Manuel I of Portugal sent Cabral in command of a fleet of 13 vessels to establish trade with India. Taking a westward course, he was carried by wind and current to the coast of Brazil and took possession of it in name of Portugal. They named the country for the red dyewood, called *Brazilwood*, that they found there.

The 16th ***century:*** final collapse of the *cultural balance* of the Old World when Europe opened the Americas, and then explored the rest of the world's habitable coastlines, using the oceans as highways for their commerce and conquests. For the first time, one civilization gained such superiority as to upset the fourfold balance that existed since 1000 BCE between the four distinctive civilized traditions of Greece, the Middle East, India and China.

Those four major civilizations were set apart by different cultural traditions, and by distinctive religious and philosophical world views, all of which found their initial expressions before the end of the sixth century BCE.

The relationship between the four major civilizations may be thought of as an equilibrium. Any serious disorders might influence other parts of the system, but not until 1500 CE did any one civilization gain such superiority as to upset the fourfold balance of the whole. This balance did, however, encounter a number of jolts:

- First, **Alexander the Great** pushed Greek civilization far beyond its original borders.
- Then, *Hindu Buddhism* advanced along the silk road into the heart of China.
- But the *Hellenization of the Middle East*, like the *Indianization of China* did not last and was soon either repudiated or absorbed into native concepts.
- Next came the explosive conquest of *Islam*, first across the Middle East, North Africa and Spain, and then into India, Eastern Europe and Central Asia.

The Amateur Mathematicians of the Renaissance[167]

(1500–1600)

The Renaissance was a golden age of part-time naive mathematicians.

Before 1500*, the three components of modern scientific method – the logical, the experimental and the mathematical – developed in isolation. The underlying reason was that the aims, intellectual interests and social positions of the professional groups with which they were connected, were too diverse to permit fruitful communication among them.*

By 1500*, the gaps between these groups had narrowed rapidly due to printing: the cultural ideas of humanism influenced all educated men. Fashion forcefully imposed newly translated texts on their attention, while an increasing number of scholars learned Greek and began to study Greek science in its original source. An important phase of this reappropriation of Greek antiquity was the recovery of understanding of the works of* ***Pappos****,* ***Apollonios****,* ***Diophantos****,* ***Hero*** *and above all those of* ***Archimedes****, which appeared in Latin translation in* 1543*. With these came the growing emphasis on the cultural and scientific importance of mathematics. Humanist educational practice stressed mathematics at the expense of logic. The professional teaching of mathematics spread to the universities and there was hardly a Renaissance writer who did not remind his readers of the sentence inscribed over the door of the Platonic Academy: "Let no one unskilled in geometry enter here".*

In this atmosphere emerged persons who had skills in many fields of knowledge.

The artist-engineer ***Leonardo da Vinci*** *(1452–1519, Italy) was such a 'universal man', and being interested in everything, he was naturally interested in mathematics. Although he did not have much of a mathematical education, he tried to square the circle.*

Albrecht Dürer *(1471–1528, Germany), originated what we call today: 'descriptive geometry', because as an artist he was interested in perspective and the proportions of the human form.* ***Francois Vieta*** *(1540–1603 ,*

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[167] To dig deeper, read:

- Coolidge, J.L., *The Mathematics of Great Amateurs*, Oxford University Press: London, 1949, 211 pp.
- Ore, O., *Cardano the Gambling Scholar*, Dover Publications: New York, 1965, 249 pp.

France) was a Royal Privy Counselor and mathematician. ***Joseph Scaliger*** *(1540–1609, Netherlands)was a philologist at the University of Leyden. He was an amateur mathematician whose name is associated with the Gregorian Calendar.*

The monk and Lutheran preacher ***Michael Stifel*** *(1487–1567), was for a time professor of mathematics at the University of Jena.* ***John Napier*** *(1550–1617, England) a Scottish nobleman and amateur mathematician invented logarithms, but also warlike machines for the defense of Britain against Spain. He promoted astrology and published a book in which he argued that the Pope was the Anti-Christ.* ***Jobst Bürgi****, a Swiss clockmaker working in Prague, invented the logarithms independently.*

It should be kept in mind that during 1494–1559, *Italy served as a victim of rivalries of the strong monarchies which were rising in Western Europe, all of which coveted the wealth of the peninsula. Spain was engaged in a series of wars against France to control Italy, which was politically divided into four major city-states (Venice, Milan, Florence, and Naples) and the Papal states. These units maintained a precarious balance among themselves and were constantly endeavoring to victimize each other, and ultimately reached the point of inviting in foreigners, with the result that Italy became the prey of French, German and Spanish ambitions.*

Nevertheless, the said period marked the apogee of the Renaissance and the intellectual and artistic primacy of Italy. It seemed as if all the armies of Europe could not stop the ideas whose time had come.

In the field of political science ***Niccolo Machiavelli*** *(1469–1527) was outstanding.* ***Ludovico Ariosto*** *(1474–1533) was a great epic poet. In the field of music* ***Giovanni Palestrina*** *(1525–1594) and* ***Orlando di Lassus*** *(1532–1594) were men of the first rank. Architects and painters of eminence were too numerous to be listed, and it will suffice to recall names like* ***Raffaelo Santi*** *(1483–1520),* ***Michelangelo Buonaroti*** *(1475–1564),* ***Titian*** *(Tiziono Vecelli, 1477–1576),* ***Tintoretto*** *(1512–1594),* ***Paolo Veronese*** *(1528–1588) etc.*

By 1559, *almost all of Italy was under the influence of Spain and so it remained until the early* 1700*'s.*

The most spectacular mathematical achievement of the 16^{th} *century was the discovery, by Italian mathematicians, of the algebraic solution of the cubic and quartic equations. The story of this discovery rivals any page ever written by Benevenuto Cellini:*

About 1515, ***Scipione del Ferro*** *(1465–1526), a professor of mathematics at the University of Bologna, algebraically solved the cubic equation* $x^3 + px = q$, *probably basing his work on earlier Arabic sources. [Bologna* *was at the time one of the oldest of the medieval universities, with a strong mathematical tradition.]*

Consider the identity $(a-b)^3+3ab(a-b) \equiv a^3-b^3$ *and denote* $a-b=x$. *The identity then reads* $x^3+3abx = a^3-b^3$. *Comparing this with the original equation* $x^3+px=q$, *we choose* $3ab=p$, $a^3-b^3=q$, *and solve this pair for* $\{a,b\}$ *in terms of* $\{p,q\}$. *The problem has thus been reduced to the level of the biquadratic equation,* $a^6-a^3q-\left(\frac{p}{3}\right)^3=0$. *A solution of this equation is:* $a^3=\frac{q}{2}+\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}$ *and therefore* $b^3=a^3-q=-\frac{q}{2}+\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}$. *Finally,* $x_1=\left\{\frac{q}{2}+\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}\right\}^{1/3}-\left\{-\frac{q}{2}+\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}\right\}^{1/3}$.

To obtain the other two solutions, one effects the factorization:

$$x^3+3abx+b^3-a^3=[x-a+b][x+\omega b-\omega^2 a][x+\omega^2 b-\omega a]$$

where $\omega=-\frac{1}{2}[1+i\sqrt{3}]$, $\omega^2=-\frac{1}{2}[1-i\sqrt{3}]$, *and finds:* $x_2=\omega^2 a-\omega b$, $x_3=\omega a-\omega^2 b$. *Note that the case of three distinct and real roots occurs only when* $(-a)^3$ *and* b^3 *are complex conjugate. For this "irreducible" case, the trigonometrical solution is adequate, and was first given by* ***Francois Viète*** *in ca* 1579.

Cardano's solution for the general cubic equation

$$a_0x^3+3a_1x^2+3a_2x+a_3=0$$

has the explicit algebraic form

$$x_1=-\frac{a_1}{a_0}-\frac{1}{a_0}\sqrt[3]{\frac{G}{2}+\frac{1}{2}\sqrt{G^2+4H^3}}-\frac{1}{a_0}\sqrt[3]{\frac{G}{2}-\frac{1}{2}\sqrt{G^2+4H^3}},$$

$$x_2=-\frac{a_1}{a_0}-\frac{\omega}{a_0}\sqrt[3]{\frac{G}{2}+\frac{1}{2}\sqrt{G^2+4H^3}}-\frac{\omega^2}{a_0}\sqrt[3]{\frac{G}{2}-\frac{1}{2}\sqrt{G^2+4H^3}},$$

$$x_3=-\frac{a_1}{a_0}-\frac{\omega^2}{a_0}\sqrt[3]{\frac{G}{2}+\frac{1}{2}\sqrt{G^2+4H^3}}-\frac{\omega}{a_0}\sqrt[3]{\frac{G}{2}-\frac{1}{2}\sqrt{G^2+4H^3}},$$

with

$$G=a_0^2a_3-3a_0a_1a_2+2a_1^3; \qquad H=a_0a_2-a_2^2$$

$$\omega = -\frac{1}{2} + \frac{1}{2}i\sqrt{3}$$

For example the equation $x^3 - 7x + 7 = 0$ *is solved by*

$$x_1 = -\sqrt[3]{\frac{7}{2} + \frac{7}{18}\sqrt{-3}} - \sqrt[3]{\frac{7}{2} - \frac{7}{18}\sqrt{-3}},$$

$$x_2 = -\omega\sqrt[3]{\frac{7}{2} + \frac{7}{18}\sqrt{-3}} - \omega^2\sqrt[3]{\frac{7}{2} - \frac{7}{18}\sqrt{-3}},$$

$$x_3 = -\omega^2\sqrt[3]{\frac{7}{2} + \frac{7}{18}\sqrt{-3}} - \omega\sqrt[3]{\frac{7}{2} - \frac{7}{18}\sqrt{-3}}.$$

This is the famous cubic formula which Cardano is reputed to have weaseled out of his fellow mathematician, Tartaglia, under the oath of secrecy.

Consider two examples: the equation $x^3+x = 2$, *has* $x = 1$ *as a solution. The above solution yields* $x = \left[1 + \sqrt{1 + \frac{1}{27}}\right]^{1/3} - \left[-1 + \sqrt{1 + \frac{1}{27}}\right]^{1/3} = 1.263,762,616 - 0.263,762,658$, *which is* $x = 1$ *within* 2×10^{-10}.

We then take the equation $x^3 - 15x = 4$, *which has* $x = 4$ *as a solution. Here, the formula yields:* $x = [2 + \sqrt{-121}]^{1/3} - [-2 + \sqrt{-121}]^{1/3}$, *where* x *is* real *although each of its two members is a complex number. In the year* 1545, *this was a meaningless expression: square roots of negative numbers had no legitimacy and the theory of complex numbers was nonexistent. Only* 255 *years later(!) (when complex numbers were interpreted as points in a coordinate plane) was the mystery lifted.*

Ferro did not publish his solution, but before his death he disclosed it to his pupil ***Antonio Maria Fior****, a mediocre mathematician. Through this intermediary, the solution (or part of it) leaked to Niccolo Fontana, known as* ***Niccolo Tartaglia*** *('the stammerer', 1499–1559) who claimed in* 1535 *to have found the solution of the cubic equation. He divulged his solution to* ***Girolamo (Jerome) Cardano*** *(1501–1576) in return for financial assistance. Although Cardano promised not to publish the solution, he did so in* 1545, *in his book 'Ars magna' which appeared at Nuremberg, Germany. Tartaglia's vehement protests were countered by* ***Lodovico Ferrari****, who argued that Cardano had received his information from* ***del Ferro*** *through a third party and accused Tartaglia of plagiarism from the same source. Since the actors in the above drama seem not always to have had the highest regard for truth, one finds a number of variations in the details of the plot.*

The solution of the cubic and quartic equations was perhaps the greatest contribution to algebra since the Babylonians (almost 4 millennia earlier) had learned how to complete the square for quadratic equations. No other discovery had quite the stimulus to algebraic development as those of Ferrari, Tartaglia and Cardano. It was natural that further study should take aim at the quintic and polynomial equations of higher order. Here mathematicians of the next 200 years would be faced with unsolvable problems, and their negative conclusions would yield much good mathematics.

The explicit solutions of the quadratic, cubic and quartic equations use only the four basic arithmetical operations and the extraction of roots. In the attempts to find general formulae for equations of higher degrees, using only these operations, mathematician after Ferrari have greatly overestimated the 'power of radicals' and were mislead by the fact one can very well find special equations whose solutions can be expressed by radicals (such equations are called 'soluble by radicals', where a radical is a solution of the equation of the n^{th} *degree* $x^n - a = 0$ *and is denoted*[168] *by* $\sqrt[n]{a}$*).*

Indeed, ***Abel*** *(1824) and* ***Galois*** *(1831) put an end to traditional algebra, Italian style, where one provides a neat formula in terms of coefficients, which is applicable to all polynomial equations of a certain degree.*

In the year named, Abel showed for the quintic equation, that even though ***Gauss*** *(1799) guaranteed a solution when coefficients are real or complex, it is impossible to express this solution using only a finite number of rational operations and root extractions on the coefficients. Galois gave a more elegant proof of the same fact, and then established a general theorem indicating the impossibility of finite algebraic formulation of solutions for polynomial equations of all degrees greater than four.*

1507 CE First outbreak of *smallpox* in the New World. By 1520 it spread into Mexico. Several million died.

[168] The n roots of the equation $x^n - 1 = 0$, where n is a positive integer, are given by $x_k = \cos\frac{2\pi k}{n} + i\sin\frac{2\pi k}{n}$, $k = 0, 1, 2, \dots, n-1$. They are known as the *roots of unity*. Clearly, $x_k = \alpha^k$ where $\alpha = \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n}$ (the n^{th} roots of unity) lie on the circumference of the unit circle and divide it into n equal parts.
The more general equation $x^n - \lambda = 0$, $\lambda > 0$ has the solutions $x_k = \lambda^{1/n}, \alpha\lambda^{1/n}, \alpha^2\lambda^{1/n}, \dots, \alpha^{n-1}\lambda^{1/n}$.
Since $x^n - 1$ contains the factor $x - 1$, it follows that $\alpha, \alpha^2, \dots, \alpha^{n-1}$ are the $n-1$ roots of $x^{n-1} + x^{n-2} + \cdots + x + 1 = 0$.

1510 CE Beginning of systematic importation of African black slaves into the West Indies.

1510–1528 CE **Albrecht Dürer** (1471–1528, Germany). Artist (painter, engraver) and mathematician. Laid the foundations of *descriptive geometry*[169]. It is expounded in Dürer's treatise on human proportion [*De symetria Partium in Rectis Formis Humanorum Corporum Libri*], published in Nuremberg after his death in 1528. His engraving *Melancholic* contains the first *magic square* to be seen in Europe. It contains the date 1514 as two entries in the middle of the bottom row. Dürer's geometry was the first printed work to consider the subject of *higher plane curves*, and the first to discuss scientifically the question of such approximate construction as that of the regular *heptagon*.

Dürer was born in the Imperial Free City of Nüremberg and became an apprentice painter and woodcut designer (1486). He traveled to Colmar (1490) and went to Strasbourg and Basel. He returned to Nüremberg (1494) and married Agnes Frey. Dürer visited Venice in 1494–1495 and again between 1505 and 1507. These visits strongly influenced his art. He was appointed court painter to Maximilian I, the Holy Roman emperor (1512) and again under Charles V (1520).

Dürer (and Rembrandt) are credited with the invention of the technique of *dry-point engraving*[170].

1513–1519 CE **Juan Ponce de Léon** (1474–1521, Spain). Explorer. First to describe the *Gulf Stream* (specifically, the Florida Current) in 1513. Sailing from Puerto Rico, he crossed the stream north of Cape Canaveral and then sailed south to Tortugas. The current was so swift that his three ships were frequently unable to stem it. (The island of Cuba was first circumnavigated in 1508, but the current was not reported until 1513.)

In 1515, **Peter Martyr of Anghiera** reported various conjectures about the Gulf Stream. He believed, however, that the North Equatorial Current is deflected by the mainland so as to flow back into the ocean. The westward flow of the Equatorial Current itself was attributed to the general westward motion of the celestial bodies across the sky that allegedly drew the water and air of the equatorial regions along with it (!)

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[169] Later given a sound mathematical basis by **Monge** (1768).

[170] Made by cutting a design into a flat metal plate. The engraved plate is then used to print the design or picture. The artists uses a needle with a diamond or hard steel point. As the needle cuts into the plate (copper or zinc), it throws up a soft ridge of metal called a *burr*. This burr holds the ink and thus forms the lines that appear on the picture.

By 1519 the Gulf Stream was so well known that Spanish ships bound for America by way of the Equatorial Current, passed on their return through the Florida Straits, and followed the Gulf Stream to about the latitude of Cape Hatteras, and then sailed eastward to Spain. In this way they secured favorable winds and avoided contrary currents over the entire voyage.

Ponce de Léon went to America with **Columbus** on the second voyage (1493). He led the settlement of Spaniards in Puerto Rico (1508), and encountered the Gulf Stream while searching for *Bimini*, a fabled island on which was said to be located the Fountain of Youth. This search brought him to a new land at Easter time of 1513. Ponce de Léon claimed the region for Spain and named it *Florida* because of the many flowers he saw there. He returned to Florida in 1521 to start a colony, but was wounded and died on his return to Cuba. Ponce, Puerto Rico, is named in his honor.

1513–1525 CE **Niccolo Machiavelli** (1469–1527, Italy). Political philosopher and historian. A leading literary figure of the Renaissance.

Best known for his book *The Prince* (*Il Principe*, 1513, published 1532), which established Machiavelli as the father of modern political science. He also wrote *History of Florence* (1525), *The Art of War* (1520) and *Discourses upon the First Ten Books of Livy* (1517).

Machiavelli was born in Florence, the son of a jurist, and a member of an old Tuscan family. He became a leading figure in the Republic of Florence after the Medici family was driven out (1498). For 14 years, he served as first secretary of the council of the republic. His duties brought him in contact with the notorious Cesare Borgia. When the Medici regained power (1512), Machiavelli was dismissed from office, tortured and imprisoned, but finally released on order of Pope Leo X. He spent the next 14 years (1512–1526) in retirement near Florence. There he wrote his books on history and politics.

As an historian, he raised the study of history from mere chronicle to an evaluation of motives and causes. He was the first to come along and attack the Church and preach rebellion against the dictatorship of the clergy. Machiavelli's reputation as a diabolical apostle of intrigue, duplicity and power politics is a travesty of his actual work:

The Prince, apparently cynical exposition of a creed of treachery and tyranny, must be read in the context of his other political works, and of the circumstances of the time. The *Discourses* give his long-term views, and in this unconstrained context of security founded on law, he proclaims the superiority of government by people over government by princes. Thus, the precepts of *The Prince* must be taken to be applicable in times of crisis or in desperate circumstances that are comparatively rare.

Much of what he says in *The Prince* is a sardonic description of the political practices of his own day and not a recommendation of such practices. Machiavelli, in his own practice as a diplomat and minister, was hardly '*Machiavellian*': emphasizing power and how it is held, not what should be done to fulfill a providential scheme.

Machiavelli had serious doubts about the chances of long-term success. He maintained that humans tire even of stability and success, and crave novelty. They have a weakness, almost a flair, for corruption; and by the time it becomes visible, it has already taken hold in defiance of the old laws and measures. To restore the healthy political structure then requires extraordinary measures which very few are equal to carrying out.

The collapse of the Soviet Union (1989), the discrediting of Marxism and the defeat of reformist parliamentary socialism are striking examples of the political theory of Machiavelli.[171]

1516 CE **Franz von Taxis** (**Francesco Tassi**, 1460–1517, Italy and Germany). Established the first successful *regular public postal system* in the Habsburg Empire, which soon spread all over Europe. This organization of the Taxis family was in operation for 350 years. In 1867 it was incorporated into the Prussian postal system.

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171 According to the political philosopher **Leo Strauss** (1899–1973), in his book *Thoughts on Machiavelli* (1958), Machiavelli is the key turning point that leads to modern political philosophy, and Machiavelli's sin was to speak esoteric truths openly. He told all within hearing that there is no certain God who punishes wrongdoing; the essence of Machiavellianism is that one can get away with things. Because of this, he turned his back on the Christian virtue that the belief in a retributive God had upheld. Pre-Machiavellian philosophy, be it Greco-Roman of Christian, had taught that the good political order must be based upon human virtues. Machiavelli believed that sufficient virtue was not attainable and therefore taught that the good political order must be based on men as they are, i.e. upon their mediocrity and vices. This is not just realism, or mere cynicism. It amounts to a deliberate choice as to how society should be organized and a decided de-emphasis on personal virtue. It leads to the new discipline of political science, which is concerned with coldly describing men as they actually are. It leads ultimately to Immanuel Kant's statement that, "We could devise a constitution for a race of devils, if only they were intelligent." The ancient view is that this will get you nowhere, because only men with civic virtue will obey a constitution. The modern view leads naturally to value-free social science and social policies that seek to solve social problems through technocratic manipulation that refrains from "imposing value judgements" on the objects of its concern.

1516 CE The first official *ghetto*[172], the street or quarter of a city in which Jews were compelled to live, established in *Venice*, Italy. Ghettos were enclosed by walls and gates which were locked each night. Haphazardly, without a master plan, the isolation of the Jews in the ghettos was achieved within a century of their expulsion from Spain. Out of sight, out of mind, out of influence, they would be expelled from the mainstream of Christianity (and later from Islam too), so as not to contaminate with heresy the minds of Jesus-loving Christians. Placing the Jews into *cordons sanitaires* (antiseptic enclaves) seemed an excellent solution.

The Jew, who for a millennium had been an integral part of the economic, social and intellectual history of Europe, was now relieved of all his rights and privileges. He was now neither essential for Christian salvation nor necessary for national economic survival – he has become the superfluous man in Europe.

During the Middle Ages the Jews were forbidden to leave the ghetto after sunset when the gates were locked, and they were also imprisoned on Sundays and all Christian holy days. By the middle of the 19^{th} century the ghetto system was moribund. It was reinstituted by the Germans during 1939–1945 as a transitory stage to the Jews on their trek to the gas-chambers.

1517–1546 CE **Elia Levita; Eliyahu ben Asher ha'levi Ashkenazi**[173] (1468–1549, Italy). Hebrew grammarian, philologist and lexicographer. Laid the foundation of the lexicography and etymology of the Yiddish language. Published many books on Hebrew Masorah and grammar, and composed the first comprehensive biblical concordance. Claimed that punctuation was established by the Masoretes in the 4^{th} century CE. Composed the first Hebrew–Yiddish dictionary.

Levita was born in the village of Ipsheim, near Nüremberg in Germany and moved to Italy at about 1490, settling in Padua. He lost all his property during the French conquest of Padua (1509) and eventually moved to Rome (1514), where he befriended Cardinal Edidio de Viterbo who later became his patron (1517–1527). In return, Levita taught the Cardinal (and other noted Christian, among them Martin Luther) the Hebrew language and its grammar[174].

172 Abbreviation of Italian *borghetto*, diminutive of borgo = a borough. According to another view it came from *gietto nuovo* = the new foundry, a location near an iron-foundry in the city of Venice. Temporary ghettos existed previously in London (1276), Capua (1375) and Bolonia (1417). The earliest regular ghettos were established in Italy in the 11^{th} century, though Prague is said to have had one already in the 10^{th} century.

173 Known also as the 'Bachur'.

174 The *Renaissance* made the study of Hebrew more popular, and this resulted in

In this sense Levita fulfilled the historic mission of endearing the Hebrew language to enlightened Christians and humanists.

When the Spanish and German mercenaries sacked Rome on May 06, 1527, Rome pre-eminence in the Renaissance ended. All of Levita's property (including some of his finished manuscripts) was lost and he removed to Venice, where he continued to publish his works, and spent the last years of his life.

1517–1546 CE **Girolamo Fracastoro** (**Hieronymus Fracastorius**) (1478–1553, Italy). Physician, astronomer and poet. Universal man of the Renaissance. Explained *fossils* as the remains of actual organisms (1517); Gave name and described symptoms and treatment of *Syphilis*[175] (1530); Prefigured a Copernican model of the solar system[176] in *Homocentrica sive de stellis liber* (1538); made first scientific statement on the nature of *contagion and transmission of diseases by germs* (1546).

Born at Verona and studied at Padua. Became (1502) professor of philosophy and colleague of Copernicus at the University of Padua. He was skilled not only in medicine and literature, but in most arts and sciences. Maintained private medical practice in Verona; studied epidemic diseases; medical consultant of Pope Paul III at the Council of Trent (1545 ff). Intimately acquainted with Julius Scaliger and most of the great man of his time.

1518–1536 CE **Hernando Cortés** (1485–1547, Spain). Spanish conqueror of Mexico. Discovered the peninsula of Lower California.

Cortés was born at Medellin, a small town of Estermadura. He belonged to a noble family of decayed fortune. Being destined for the law, he was sent (1499) to the University of Salamanca, but returned home (1501) resolved to enter upon a life of adventure. He set out (1504) as a soldier for San Domingo and remained there until 1511, when he accompanied **Diego Velasquez** in his expedition to the island of Cuba. Soon after the discovery of Mexico by **Juan Grijalva**, Velasquez entrusted the conquest of the newly discovered

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an increasingly sympathetic attitude on the part of the cultured Europeans toward the literary treasures of the Jews. Italy became the Jewish printing center and Christian printers employed learned Jews to find the best manuscripts and to arrange them in the best manner. This 'honeymoon', however, did not last for long. With the rise of Pope Paulus IV, during the *Counter-Reformation*, the Inquisition in Italy burned Jews and their books (1553–1572) in great bonfires and locked the Jews up in ghettos all over Italy for the next two centuries. Interestingly enough, Levita's sons converted to Christianity and his renegade grandson Vitorio Aliano, urged the Inquisition to burn the Talmud.

[175] The name of a mythical young shepherd who developed the disease.

[176] His planetary system is close to Eudoxos' model; it contains 79 spheres.

country to Cortés. He landed there on the *4th* of March 1519. Coasting along Yucatan and Mexico to San Juan de Uhia (1519), he founded Veracruz and afterwards destroyed his fleet, showing his soldiers that they must either conquer or perish[177]. On march inland, he drew to his side some 6000 natives, hostile to Montezuma, the Aztec monarch.

Believing Cortés to be the god Quetzalcoatl, Montezuma sent him conciliatory gifts. Cortés entered the Aztec capital Tenochtitlan (Nov 8, 1519) as Montezuma's guest, but surrounded and outnumbered by hostile Aztecs he executed a most daring project of taking the monarch hostage in his own capital. The Aztec revolted but, after heavy losses and a long siege, Cortés took the capital and ended Aztec power (13^{th} Aug 1521). These successes were entirely owing to the military genius, valor and profound but unscrupulous policy of Cortés; and the account of them which he transmitted to Spain excited the admiration of his countrymen.

This, however, created for him powerful enemies in Spain and Mexico. Eventually, the King of Spain[178] alarmed by Cortés ambition, ousted him[179].

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177 This moment was immortalized in John Keat's poem "On First Looking into Chapman's Homer" (Oct 1816)

"... Then felt I like some watcher of the skies
When a new planet swims into his ken;
Or like stout Cortez when with eagle eyes
He stared at the Pacific – and all his men
Looked at each other with a wild surmise –
Silent, upon a peak in Darien."

178 Emperor **Charles V** (1500–1558) was the King of Spain as **Charles I** (1516–1556) and also Holy Roman emperor (1519–1556). He was son of **Philip I** ("The Handsome") and the insane Joan who was the daughter of Ferdinand and Isabela. Charles ruled over the Netherlands, Austria, Milan, Naples, Sardinia, Sicily and all the Spanish possessions in America. He fought **Francis I** of France over their rival claims in Italy. He also fought the Turks who threatened central Europe and tried unsuccessfully to put down the Protestants. His troops sacked Rome (1527) and captured **Pope Clement VII**. The acquisition of Mexico and Peru by his intrepid conquistadors, afforded him the means of prosecuting his ambitious and most expensive enterprises; the stream of gold and silver from the New World surpassed all European dreams of wealth. As a result, Spain was to become the leading power in Europe in the 16^{th} century.

179 After the disastrous expedition to Algiers (1541), Cortés fell into neglect, and could scarcely obtain an audience. The story goes that, having forced his way through the crowd which surrounded the emperor's carriage, and mounted on the door-step. Charles, astonished at an act of such audacity, demanded to know who he was. "I am a man" replied the conqueror of Mexico proudly, "Who has

Nevertheless, in 1536, Cortés discovered the peninsula of Lower California, and surveyed part of the gulf which separated it from Mexico.

In 1541, Cortés withdrew from King's court and passed the remainder of his days in solitude on his estate near Seville.

1522–1555 CE **Joseph Caro** (1488–1575, Spain and Turkey). Codifier of Jewish law and Kabbalist. He began writing *Beit Yosef* (*House of Joseph*) in 1522 in Adrianpole and completed it in Safed, Israel (1542). Caro investigated the source of every practical law from its Talmudic origins, traced its development, mentioned divergent views, and recommended the appropriate practice. His *Shulhan Arukh* (*The Set Table*), a digest of *Beit Yosef*, was completed in 1555. It became the pillar of Orthodox Jewish observance.

Caro was born in Toledo, Spain. He was expelled with his family to Portugal (1492) and settled in Lisbon. Expelled again (1496) and reached Constantinople. He finally settled in Safed, Israel (1536) and died there.

Science Progress Report No. 3

The Conquistadors (1521–1548)

As the Spaniards on the Caribbeans began to hear exciting tales of wealthy, half-civilized empires they turned from exploration to conquest. From 1520 *to* 1550*, the Conquistadors of Spain carved out an empire in the West.*

The ***Aztecs of Mexico*** *began their history in* 1168*, when migratory tribes entered the valley of Anahuac. Later in* 1325 *they founded their capital Tenochtitlan (now Mexico City). They were fine architects, evolved a system of picture-writing, but the major thrust of their bizarre culture went into an extraordinary preoccupation with time and the calendar [it was similar in many ways to the Maya Calendar, and thus much more advanced than in either Egypt or Greece].*

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given you more provinces than your ancestors left you cities."

For further reading, see:

- Von Hagen, V.W., *The Aztec*: *Man and Tribe*, Mentor Books: New York, 1962, 224 pp.

Their priest-astronomers had put great efforts into the elaboration of the calendar, which had a 52 *year cycle and "5 empty days" at the end of each year. They believed that during these critical periods, the very future of the world was at balance and might be destroyed. They responded to this issue with mass human sacrifice rituals, where the sacrifices had to be precisely timed so that it would benefit the particular god to whom they were appealing and thus propitiate the right god at the right time. So sacrifice was not mere butchery but rather a parade of elaborately conceived ritual with only one object in view: to preserve human existence.*

During the 'five empty days', fires were extinguished, fasting was general, sexual intercourse ceased, artists left their work, business lay idle. On the dawn of the fifth day, when the priest-astronomers observed the Pleiades rising in the heavens and knew that the world would not end, they slashed open the chest of the sacrificial victim, pulled out his heart, and in the freshly weeping wound kindled a new fire. From it the fires in the temples were rekindled, and people all over the kingdom gathered the new fire for the coming year[180].

Into this formidable empire, entered one ***Hernando Cortés*** *(1485–1547) in* 1519 *with* 600 *men,* 16 *horses, a few cannons and a secret weapon of which he was not aware – the smallpox virus.*[181]

"Thus[182]*, on Saint Hippolytus' Day, Aug.* 31, 1521, *amidst the acrid smoke of a thousand fires, the Spanish conquest was complete and Aztec civilization passed into cultural limbo."*

The Inca culture in and about the Peruvian Cuzco Valley started at about 1250. *By* 1500, *the Inca ruled an empire in the Andes of perhaps as many as* 7 *million people. They were fine architects, potters and metal workers with a well-organized government and superb roads.*

Nevertheless, ***Francisco Pizarro*** *(1471–1541), with* 180 *men and* 27 *horses subjugated the entire empire within three years (1532–1535)*[183]*. So passed the glory of the Incas, whose kingdom extended from Ecuador to as*

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[180] In this connection, read the beautiful story of **D.H. Lawrence**: "The Woman who Rode Away".

[181] It was recently discovered that the epidemics of 1545 and 1576, which exterminated about 8 million Aztecs, were caused by the *Ebola* virus.

[182] Victor W. Von Hagen: "The Aztec: Man and Tribe" Mentor Books New York 1958

[183] At first, the Inca's King, Atahualpa, thought that the Spaniards were the gods themselves, coming to conduct him to Cuzco to his rightful heritage as Inca. But after he had reports that they raped his Virgins of the Sun at the village of Caxas, he knew by then that he dealt with no gods.

far as latitude 35°S in present day Chile, a distance of 3200 *km on a north-south line. With them vanished their great skill in engineering and architectural works, which they accomplished without the use of iron tools, mortar, the wheel, the keystone arch or the written word to help them in their plans and calculations [instead they used the 'quipo', a cord made up of different colored threads, which acted as a superior abacus].*

So vanished too their skill in working precious metals, and their system of education which was based on their credo that "science was not intended for the people, but for those of generous blood. Persons of low degree are only puffed up by it, and rendered vain and arrogant".

The discovery of the New World had disastrous consequences for the natives of South and Central America; during the century that followed the voyages of Columbus, about 40 *million* Indians *were methodically exterminated.*

Amerindians had spent millennia adapting to their microbial environment. The Americas were as untouched by Old World crowd diseases as any remote Pacific island. Therefore one person with even a mild case of smallpox, measles, or mumps could set of an epidemic that could destroy entire cities and that is what indeed happened.

Smallpox was probably introduced to Europe in Roman times, and perhaps reintroduced by returning Crusaders. By 1500, it had become endemic, chiefly a children's disease to which most adults were immune. When, however, the virus reached the New World with the Spaniards, it acted as epidemics had when they killed Romans by the hundred of thousands and drove Huns from the walls of Rome.

In 1520, reinforcement arrived to Cortés from smallpox – stricken Cuba. Among them was an African slave with a mild case of the disease. While the Spanish approached Tenochtitlan, smallpox spread among the natives, first outside the city and then within. In 1521, Cortés attacked with 300 Spaniards and some native allies. Three months later, when Tenochtitlan fell, Cortés learned that half of its people had died, including Montezuma and his successor. The canals were choked with corpses.

The surviving Aztecs were stunned and apathetic, and awed by the white men who went untouched among the dead and dying. Native social and political structures shattered. The terror of smallpox sent people fleeing. Because smallpox incubates for 10–14 days, an apparently healthy refugee could carry it hundreds of kilometers before showing symptoms. By 1530, smallpox had rushed ahead of the conquistadors, covering the Americas from the pampas to the Great Lakes.

South of the Aztec empire, lay the realm of the Mayas. Smallpox reached them before the Spaniards did, and it run through them with searing devas-

tation. It continued south toward the Inca empire. It outran Pizarro, killing hundreds of thousands, among them the chief Inca, his son and heir, and many nobles and generals. In 1533, when Pizarro finally entered Cuzco, the Incas were incapable of serious resistance.

The disease preceded the Spaniards as they ventured northward to the Mississippi valley. In 1539–1542, when ***Hernardo de Soto*** *made his way through the land of the Mound Builders, he found uninhabited towns where corpses were stacked in large houses – the pestilence had done the fighting for him.*

Measles followed smallpox, spread by troops, sailors, missionaries, colonists, messengers and fleeing natives. In 1529, a measles epidemic in Cuba killed two-thirds of the natives who survived smallpox. Two years later it had killed half of the people of Honduras, ravaged Mexico, spread through the Central Americas and attacked the Incas; whipping out whole cities and tribes, cultures and languages lost.

Throughout the 16^{th} *century and beyond, the "great fire" kept raking native peoples from Canada to Chile. Other Old World diseases followed – mumps, typhoid, typhus, influenza, diphtheria, and scarlet fever.*

However daring and resourceful Cortés and Pizarro were, it beggars imagination that each defeated an empire of millions with mere hundreds of soldiers. Their strongest ally was the Fourth Horseman – the Old World virus; It is estimated that the New World population of perhaps 100 million was reduced by about 90 percent. It was a bigger disaster than the Black Death, but spread over 300 years.

History of Geographical Theory (585 BCE–1524 CE)

Geography is the exact and organized knowledge of the distribution of phenomena on the surface of the earth. The fundamental conception of geography is form, including the figure of the earth and the varieties of crustal *relief. Hence* mathematical geography *comes first. It merges into* physical geography, *which takes account of the outer forms of the lithosphere (*geomorphology*), and also of the distribution of the hydrosphere and the rearrangements resulting from the working of solar energy throughout the hydrosphere and atmosphere (*oceanography *and* climatology*). Next follows the distribution of plants and animals (*biogeography*), and finally the distribution of mankind and the various artificial boundaries and redistributions (*anthropogeography*).*

The applications of this latter discipline to human uses, give rise to political *and* commercial geography*, in the elucidation of which all the earlier stages have to be considered, together with* historical *and other purely human conditions.*

The concepts of Darwinian evolution and the more recent Wegenerian plate-motions has revolutionized and unified geography as it did biology, breaking down the old partitions between the various departments and integrating it into one holistic science. The earliest conceptions of the earth were usually expressed in symbolic language, manifested in the old mythological cosmogonies.

The first definite geographical theories to affect the Western world were those first expressed by the Greeks*. The earliest theoretical problem of geography was the* form of the earth: *the natural supposition that the earth is a flat disk, circular or elliptical in outline, had in the time of Homer (ca 800 BCE) acquired a special definiteness by the introduction of the idea of the* ocean river *bounding the whole, an application of imperfectly understood observations.*

Thales of Miletos *(fl. 575 BCE) is claimed as the first exponent of the idea of a* spherical earth. *Although this idea was carried on by* ***Anaximander*** *(fl. 560 BCE), the Pythagorean school (535 BCE)* and ***Parmenides*** *(fl. 470 BCE), the Ionian philosophers (who preferred to deal with facts demonstrable by travel rather than with speculations) did not advance beyond the primitive conception of a circular disk.*

Thus ***Hecataeos of Miletos*** *[fl. 500 BCE, Greek traveler and historian] in his book* Periodos*, modeled the habitable world in the form of a land disk within the ring of ocean. His land mass consisted of* Europe *to the north, and* Asia *to the south, divided by a midland sea.*

Herodotos *(fl. 450 BCE), equally oblivious of the sphere, criticized and ridiculed the* circular *outline of the oceanic ring (which he knew to be longer in the east-west direction than in the north-south direction). He accepted a division of land into* three *continents Europe, Asia and Africa. Beyond the limits of his personal travels, Herodotos applied the Greek theory of* symmetry *to complete in the unknown, outlines of lands and rivers analogous to those* *which had been explored*[184].

Scientific geography was founded by **Aristotle** *(fl. 350 BCE). He demonstrated the sphericity of the earth by three arguments, two of which could be tested by observation:*

- *The tendency of matter to fall together towards a common center.*
- *Only a sphere could always throw a circular shadow on the moon during eclipse.*
- *The shifting of the horizon and the appearance of new constellations or the disappearance of familiar stars, as one traveled from north to south, could only be explained on the hypotheses that the earth was a sphere.*

The first approximately accurate measurement of the globe was made by **Eratosthenes** *(ca 235 BCE) who also was the first to introduce the word geography.*

Aristotle was certainly conversant with many facts, such as the formation of deltas, coast-erosion, and to a certain extent the dependence of plants and animals on their physical environment. He formed a comprehensive theory of the variations of climate with latitude and season, and was convinced of the necessity of a circulation of water between the sea and the rivers[185]*, though, like Plato, he held that this took place by water rising from the sea through crevices in the rocks, losing its dissolved salts in the process.*

Ptolemy *(fl. 110 CE) summarized in his writings all Greek geographical learning and passed it across the gulf of the Middle Ages by the hands of the Arabs, to form the starting-point of the science in modern times. His main sources were* **Marinos of Tyre** *(fl. 150 BCE), and his work was mainly cartographical in its aim, and theory was, as far as possible, excluded. It was the ambition of Ptolemy to describe and represent accurately the surface of the world ocean ("oekumene"), for which purpose he took immense trouble to collect all existing determinations of the latitude of places, all estimates of*

[184] *Symmetry* was in fact the first geographical theory, and the effect of Herodotos' hypothesis that the *Nile* must flow from west to east before turning north in order to balance the *Danube* running from west to east before turning south, lingered in maps of Africa down to the time of **Mungo Park** (1805)!

[185] If we believe that he discovered it himself, then this may put a bound (ca 350 BCE) on the date of composition of the book of *Ecclesiastes* ["*All the rivers run into the sea; yet the sea is not full; unto the place from whence the rivers come, thither they return again*", **1**, 7]. This however, will not be true if both Aristotle and the biblical author had an unknown common source.

longitude, and to make every possible rectification in the estimates of distances by land or sea.

The symmetrically placed hypothetical islands in the great continuous ocean disappeared and the oekumene *acquired a new form of the representation of the* Indian Ocean *as a larger Mediterranean completely cut off by land from the Atlantic. The* terra incognita *uniting* Africa *and* Farther Asia *was an unfortunate hypothesis which helped to retard exploration. In contradistinction to geography, Ptolemy introduced the word* topography *to signify the very detailed description of a small locality.*

The Arab astronomers, studying the Arab translation of Ptolemy's astronomical work (the Almagest*) measured a degree on the plains of Mesopotamia, thereby deducing a fair approximation to the size of the earth (ca 815 CE; in the time of the Caliph* ***al-Mamun****).*

The Middle Ages saw geographical knowledge die out in Christendom, although it retained, through the Arabic translations of Ptolemy, a certain vitality in Islam. The verbal interpretation of Scriptures led ***Lactantius*** *(ca 320 CE) and other ecclesiastics to denounce the spherical theory of the earth as heretical. The wretched subterfuge of* ***Cosmas*** *(ca 550 CE) to explain the phenomena of the apparent movements of the sun by means of a planar rectangular earth reverted geography to the primitive ignorance of the times of Homer.*

The journey of ***Marco Polo*** *(1274 CE), the increasing trade to the East and the voyages of the Arabs in the Indian Ocean prepared the way for the reacceptance of Ptolemy's ideas when the sealed books of the Greek original were translated into Latin by* ***Angelus*** *(1410 CE). The old arguments of Aristotle and the old measurements of Ptolemy were used by* ***Toscanelli*** *and* ***Columbus*** *in urging a westward voyage to India (1492 CE). But not until the voyage of* ***Magellan*** *shook the scales from the eyes of Europe (1522 CE) did modern geography begin to advance. Discovery had outrun theory; the rush of new facts made Ptolemy practically obsolete in a generation, after having been the fount and origin of all geography for a millennium.*

The earliest evidence of the reincarnation of a sound theoretical geography is to be found in a text-book by ***Peter Apian*** *(1524 CE). He based the whole science on mathematics and measurement, following Ptolemy closely.*

1524–1540 CE **Peter Apian** (**Peter Bennewitz**, **Peter Bienewitz**, **Peterus Apianus**) (1495–1552, Germany). Astronomer, mathematician and geographer. First to base the science of geography on mathematics and measurement. Born at Leising, Germany. Professor at Ingolstadt (1527–1552), where he died. *Pascal's triangle* appeared for the first time in one of his books (1527). In his *Cosmographicus liber* (1524) [subsequently edited and added to by **Gemma Frisius** under the title of *Cosmographia*] he created a map projection appropriate for large areas, with latitude circles appearing as parallel straight lines, a prime meridian as a straight line, and the other meridians of longitude as circles with curvature increasing away from the prime meridian. This book includes some of the earliest maps of America and is considered the first text-book on theoretical geography. In his *Instrumentum sinuum sive primi mobilis* (1534) he published a table of sines for every minute of arc. In *Astronomicon Caesareum* (1540), he included description of five comets [including the comet later known as *Comet Halley*], and stated that the tails of comets are pointed away from the sun[186].

1525 CE **Christoff Rudolff** (1500–1545, Germany). Mathematician. Introduced the radical sign $\sqrt{}$, because it resembles a small r, for radix. His book on algebra entitled *Die Coss* was very influential in Germany, and an improved edition was brought out by Michael Stifel (1553).

1528–1530 CE Typhus epidemic in Italy. Hundreds of thousands of lives were lost.

1528–1550 CE **Georg Bauer** (**Georgius Agricola**, 1490–1555, Germany). Mineralogist, Physician and scholar. The father of mineralogy and physical geology , and one of the first to base writings on observation and inquiry rather than received opinion.

He was born at Glauchau in Saxony. Studied philology, philosophy, physics, chemistry and medicine at Leipzig and Italy, and settled at Joachimstal (a center of mining and smelting works) as practicing physician (1527–1533), his object being to test what had been written about mineralogy by careful observation of ores and the method of their treatment. His thorough grounding in philology and philosophy accustomed him to systematic thinking, and this enabled him to construct out of his studies and observations of minerals a logical system which he began to publish in 1528. His *Bermannus, sive de re metallica dialogus*, was the first attempt to reduce to scientific order the knowledge won by practical work. In 1530 Prince Maurice of Saxony appointed him historiographer with an annual allowance, and he moved to Chemnitz (1530 to 1555), a center of mining industry, in order to widen the range of his observations.

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[186] A fact known to the Chinese as early as 635 CE and perhaps earlier, but not known in Europe.

The citizens showed their appreciation of his learning by appointing him town physician and electing him mayor (bürgermeister) in 1546. His popularity was, however, short-lived. Chemnitz was a violent center of the Protestant movement, while Agricola never wavered in his allegiance to the old religion; he was forced to resign his office.

In 1544 he published the *De ortu et causis subterraneorum*, in which he laid the first foundations to a physical geology[187], and criticized the theories of the ancients. His most famous work, *De re metallica*, was published in 1556, though apparently completed several years before. It is a complete and systematic treatise on mining and metallurgy[188].

While all of Chemnitz had gone over to the Lutheran creed, Agricola remained to the end a staunch Catholic. His life ended by a fit of apoplexy brought on by a heated discussion with a Protestant priest. So violent was the theological feeling against him that his body was carried, amidst violent demonstrations, to Zeitz, 10 km from Chemnitz, and buried there.

1528–1554 CE **Jean Francois Fernel** (1497–1558, France). Astronomer and physician. The first man to improve on Eratosthenes' determination of the earth's circumference.

He measured a distance in the direction of the meridian near Paris, by counting the revolutions of his carriage wheels on a journey between Paris and Amiens. His astronomical observations were made with a triangle used as a quadrant. The resulting length of a degree were very near the true value.

Fernel was born at Clermont, and educated at Paris. At first he devoted himself to mathematical and astronomical studies. But from 1534 on he

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[187] In his books, Agricola describes and classifies minerals according to the physical properties of color, luster, odor, shape, state, texture, transparency and weight. He also defines and explains mineral form, hardness, friability, smoothness, solubility, fusibility, brittleness, cleavage and combustibility.
He considered the origin of mountains, hypothesizing that mountains are produced by water erosion, atmospheric winds, earthquakes, and fire from the earth's interiors. He suggested that the subterranean heat apparent in volcanic eruptions is localized under the volcanic centers, and derives from combustion of beds of coal, bitumen, or sulfur, ignited by intensely heated vapors.

[188] This treatise was preceded by *Pirotechnia* (1540) by **Vannoccio Biringuccio of Siena** (1480–1539, Italy). The latter book describes a brass-foundry in Milan where 1200 small objects could be made in one mould – an interesting early example of *mass production*. It also informs us that 5 percent of brass and 8 percent of tin were added to copper to make bronze. Bronze has long been used for casting statues, bells and later also cannons.

gave himself up entirely to medicine, in which he graduated in 1530. His extraordinary general erudition, and the skill and success with which he sought to revive the study of the old Greek physicians, gained him great reputation, and ultimately the office of physician to the court.

In his book *Medicina* (1554) he included descriptions of appendicitis, endocarditis, peristalsis, the systole and diastole of the heart, and anatomical details such as the spinal canal. Fernel introduced the terms '*physiology*' and '*pathology*' into medicine. First to describe '*appendicitis*' and '*peristalsis*' (the waves of contraction in the digestive system that moves food through the alimentary canal).

Fernel was born in Montdidier, Somme, France.

1530 CE, Nov. 01 Dikes burst in Holland and flood the country; 400,000 people perish. Similar catastrophes occurred on: Dec. 4, 1287; Apr. 17, 1421; Nov. 1, 1570; Oct. 1–2, 1574; 1646; 1916; 1953.

1530 CE **Philippus Aureolus Paracelsus** (1493–1541, Switzerland). Alchemist, pioneering chemist and physician. One of the first to apply chemistry to medicine and introduce the use of prescription drugs. Described the properties of many substances. He was the first man who arrived at the conclusion that the processes in the body were of a chemical character, and that when disordered, they were to be put right by counter operations of like kind.

His real name was Theophrastus Bombast von Hohenheim. At the age of 16 he entered the University of Basel, but soon abandoned his chemistry studies in favor of practical medicine and the cure of diseases by means of natural substances made of minerals, ores and metals. He went to Tirol to study the mines and the health problems of the miners, believing that the positive knowledge of nature was not to be got in schools and universities, but only by going to nature herself, and to those who are constantly engaged with her. For ten years (1516–1526) he wandered over a part of Europe and acquired stores of facts, which it was impossible for him to have reduced to order, but which gave him an unquestionable superiority over his contemporaries.

In 1526 he returned to Basel and was appointed town physician and university lecturer. Thus, without a medical degree and against the accepted doctrines of Galen and Avicenna, he had great success in curing or mitigating diseases with his drugs (including the use of opium to deaden pain), for which the regular physicians could do nothing. However, using and advocating a pharmaceutical system of his own aroused the enmity and jealousy of his competitors, causing him to leave Basel in 1529 and wander around in destitution.

He finally settled in Salzburg (1541). Although he proclaimed that he had found the *philosopher's stone*[189] and would live forever, he died before he was 50 – in a fall which some attribute to drunkenness.

Paracelsus was a successful physician and scientist despite his belief in alchemy. His personality is typical of the transition period from the Middle Ages to modern times.

1530–1546 CE **Pedro Nuñes** (**Petrus Nonius**, 1492–1577, Portugal). Jewish mathematician, geographer and cartographer[190]. The father of modern cartography and a peak figure in Portuguese nautical science. He invented the '*nonius*' [an auxiliary movable ruler device for reading fine subdivisions on the scale of astronomical and other instruments which permits greater accuracy in length or arc measurements. The instrument is also called '*vernier*', after the Frenchman **Pierre Vernier** (1580–1637), who reinvented it in 1631]. It is described in his *De crepusculis* (1542). He discovered the *loxodromic curve* in his treatise *De arte atque ratione navigandi* (1546).

Nuñes (pronounced nü-nesh) was born in Portugal at Alcacer do Sal (southeast of Lisbon). He was professor of mathematics at Coimbra University, and in 1529 was appointed cosmographer to the crown, when Spain was disputing the position of the Spice Islands and maps did not agree in their longitude. He then devoted himself to such problems as well as to maps and map projections. This led him to discover (1533) the so-called Mercator-projection and the loxodorme course ahead of Mercator (1568). He published studies of the sphere and of the oceans and a copiously annotated translation of portions of Ptolemy. His clear statements on the scientific equipment of the early Portuguese explorers has become famous.

As the position of the Jews in Portugal became untenable[191] he left the country for Spain (1538). He returned in 1544 to become a leading authority

[189] He also claimed that he was successful in creating a *homunculus*, a living diminutive man *devoid of a soul*. He thus became a model for the *Faust* legend.

[190] For further reading, see:
- Bagrow, Leo, *History of Cartography*, Harvard University Press: Cambridge MA, 1966, 312 pp.

[191] **Nuñes** (also Nuñez) is the family name of Portuguese *Marranos*, which lost many members as martyrs to the Inquisition. Those who were fortunate enough to escape, became prominent physicians, scholars, poets, scientists and financiers during the past 400 years in England, Italy, Holland, South and Central America, and the United States. In fact, the fate of this family reflects, in a microcosmos, the history of the Jews in the Diaspora since the end of the Middle Ages.

The bonfires of the autos-da-fe were operating in Portugal from 1540 to 1791 on the new discoveries of Spain and Portugal. He died at Coimbra in 1577. A complete edition of all his writings appeared at Basel in 1592.

After Nuñes, little of importance for either fundamental or applied science appeared from the Iberian Peninsula.

1530–1576 CE *Botany breaks away from medicine*; plants which adorn the globe in most countries must necessarily have attracted the attention of mankind from the earliest times. The science that treats them dates back to the days of King Solomon (d. 927 BCE), who according to *I Kings* **5**, 13 "*spake of trees, from the cedar of Lebanon to the hyssop on the wall*". The Babylonians, Egyptians and Greeks were the early cultivators of science, and botany was not neglected, although its study was mixed up with crude speculations as to vegetable life, and as to the change of plants into animals.

Aristotle's student **Theophrastos of Eresos** (372–290 BCE, Athens) described over 500 plants in his *Natural history of plants* (ca 300 BCE), including cane sugar and the coconut palm, used for treatment of diseases. **Pliny the Elder** (23–79 CE, Rome) described about a 1000 plants, many of them famous for their medical virtues. Asiatic and Arabic writers also took up this subject.

Little, however, was done in the science of botany[192] until the 16^{th} century. Botany was then an essential part of medical teaching. While the ancient botanists had been satisfied mostly with names and enumeration of qualities and virtues there was now a growing desire to see and handle the plants themselves. Since 1545 (Padua, Italy) *botanical gardens* were attached to medical schools and dried plants were collected in *herbaria*.

One aspect of the Renaissance has often been described and emphasized: the publication of the Latin and Greek classics, many of which had been lost because they were represented by single manuscripts, which were buried and forgotten in the corners of neglected libraries. The discovery of such manuscripts was as thrilling as the discovery today of papyri or clay tablets. There were incunabula editions of the great botanical books of antiquity, those of Theophrastos and Discorides, but those early editions were not illustrated. The descriptions of plants, even when correct, were confusing, because they referred to another flora than that of Western Europe. In this case, the classics have to be rejected and the work of botanical description had to be done over. The pioneers, "the fathers of botany", were Germans:

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and claimed the lives of about 1200 Jews. Among them were Pedro's descendants, Beatrice Nuñes, Fracesco Nuñes, Isabel Nuñes and Clara Nuñes – all burned at the stake on July 04, 1632.

[192] From the Greek $\beta o \tau \alpha \nu \eta$ = *plant*; $\beta o \sigma \kappa \epsilon \iota \nu$ = to *graze*. The science which includes everything relating to the plant kingdom.

- **Otto Brunfels** (1488–1534, Germany). Father of modern botany, a physician in Bern, has been looked upon as the restorer of the science of botany in Europe. In his *Herbarium* (1530–1536) he gave descriptions of a large number of plants, chiefly those of central Europe, accurately illustrated by beautiful woodcuts. He was followed by:

- **Leonhard Fuchs** (1501–1566, Germany), whose *Historia Stirpium* (Basel, 1542) summarized the accepted knowledge regarding some 500 plants. (He introduced European readers to pumpkin and Indian Corn of the Americas.)

- **Valerius Cordus** (1515–1544, Germany) wrote *Dispensatorium* (1535), one of the first pharmacopeias, describing drugs, chemicals and medical preparations. His *Historia Planetarum* (1544) includes description of 500 newly identified plant species. His is the first definite mention of ether and description of its preparation (1540).

- The physician **Hieronymus Bock** (1498–1554, Germany) is also regarded as one of the founders of the science of botany. Author of *Neu Kreutterbuch* (1539), pioneering classic in descriptive botany.

The descriptions in these early works were encumbered with much medical detail, including methods of preparing and administering extracts for medical purposes.

- **Charles de l'Ecluse** (1526–1609, France). Botanist. Introduced *potato* into Europe. Published *Rariorum Planetarum* (1601) on European and American plants.

- **John Ray** (1627–1705, England) did much to advance the sciences of botany and zoology. He promulgated a system of *classification* of plants into *orders* in his *Methodus Planetarum nova* (1682).

1531–1548 CE **The Pizarro brothers**: **Francisco** (1471–1541) and his three half-brothers (one father, three mothers): **Gonzalo** (1506–1548); **Hernando** (1501–1580); **Juan** (1505–1536). Explorers and adventurers. Led the conquest of Peru and its exploitation.

Francisco overcame Atahualpa[193] (1532) and executed him (1533) at Cajamarca for refusal to accept the Chritian faith; Captured Cuzco (1533)

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[193] In the very same year, the great "boss" of Pizarro and Cortés, namely, emperor **Charles V**, held in his hands the fate of two men who offered him a Kingdom in the East:

David Reuveni (ca 1483–1538) arrived in Venice (1523) and later in Rome and secured immense amount of gold. Founded the new capital Lima (1535). Waged a civil was with Almagro (1537–1538) who was defeated and killed. He was slain by followers of Almagro in revenge (1541).

Gonzalo went with Francisco to Peru (1531) and became governor of Quito (1541–1546). After the Spanish government abridged their rights, the conquistadors revolted but were defeated (1546). The age of the conquistadors ended in Cuzco on 10 April 1548, when Gonzalo Pizarro was executed.

Hernando went with Francisco to Peru (1531) and returned to Spain with fifth of the Royal ransom of Atahualpa (1534). On his return to Peru (1537) he helped Francisco overcome Almagro. He was imprisoned in Spain (1540–1560) by Charles V.

Juan went with Francisco to Peru (1530) and became governor of Cuzco (1535), killed in fighting at Cuzco against the Inca Manco Capac.

Gonzalo Pizarro also explored much of northwestern South America. During 1539–1541 he made a perilous journey from Cuzco to Quito, a march of some 1600 km in hostile Indian territory.[194]

1533–1545 CE **Rainer Gemma Frisius** (1508–1555, The Netherlands). Geographer and astronomer. Invented the modern *triangulation* technique for surveying, replacing the pace-out of distances; After measuring a single *baseline*, other distances are calculated by means of *trigonometry*. In *De principis astronomiae et cosmographie* (1533) he pointed out that knowing the correct time according to a mechanical clock and comparing it with sun time, can be used to calculate *longitude*.

Gemma Frisius used Maurolycus' *camera obscura* for his observation of the solar eclipse of Jan. 1544 at Louvain, and fully described the methods he

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(1525), claiming to be of the Lost Tribe of Reuben; for a while he was supported by Pope **Clement VII** and King **John III** of Portugal in a plan to lead a Jewish army against Turks in the Holy Land.

Shlomo Molcho (1500–1532), a Portuguese Marrano from Lisbon by the name Diego Pires, openly proclaimed Judaism. Fleeing the Inquisition, he traveled and preached in Turkey, Israel and Italy.

Reuveni and Molcho eventually joined forces and tried to persuade Charles V that they can help defeat the Turks (1532). The emperor, however turned them over to the Inquisition, who burned Molcho at the Stake in Mantua (1532) and imprisoned Reuveni in Spain where he probably died a few years later.

194 For further reading, see:

- Von Hagen, V.W., *Realm of the Incas*, Mentor Books: New York, 1961, 223 pp.

adopted for making measurements and provided drawings of the eclipsed sun in his *De Radio Astronomico et Geometrico* (1545). He recommended that these methods be used for observation of the moon and stars and also for longitude determination.

Gemma Frisius was born at Dokkum, Friesland, Holland. He was a pupil of **Maurolycus**; died at Louvain (Belgium).

1534–1563 CE **Garcia da Orta** (1500–1560, Goa). Botanist and physician. Made major pioneering contribution to medicine and botany by acquainting Europeans with Oriental medical plants and drugs. His monumental scientific work on which he spent some 30 years of his life in Goa appeared under the title "*Coloquios dos simples e drogos medicinais*" (1563).

Da Orta was born in Elvas, Portugal. Studied at Salamanca and Alcala (1515–1525). Practiced medicine for a while and then became a professor of logic at Lisbon. Being a *Marrano*, he endured great affliction and tribulation: various members of his family, among them a sister, were tried by the Inquisition and sentenced to be burned at the Stake. In 1534 he sailed to India on a voyage of 6 months and later established himself in Goa as a medical practitioner. There he carried out the studies and researches which enabled him to publish his treatise.

Since he lived in Goa as a Jew, he was convicted postmortem on charges of heresy (1580) and his body was exhumed and burned.

1535–1557 CE **Niccolò Fontana (Tartaglia)** (1499–1559, Italy). Amateur mathematician, self-taught engineer, surveyor and bookkeeper, who wrote on mathematics and mechanics. In 1535 he claimed to have discovered a general method of solving the cubic equation $x^3 + ax = b$, independently of **Scipione del Ferro** (1515). In 1537 and 1546 he published works dealing with military tactics, munitions, and ballistics, in which he stated that the impetus of projection and the force of gravity act together on a projectile throughout the course of its flight. Thus the path of the projectile is curved throughout its course. Tartaglia also found an empirical rule connecting the *range* of a cannon with its *angle of inclination*. The range is a maximum, he said, when the cannon is inclined at an angle of 45°, although he did not discover the parabolic shape of the trajectory, later discovered by **Galileo Galilei** (1638). Tartaglia was much concerned with the promotion of mathematics and mechanics. He made the first Italian translation of Euclid's geometry, and published the first edition of Archimedes' mechanics in 1543.

Tartaglia was born at Brecia. His childhood was passed in dire poverty. During the sack of Brecia in 1512 he was horribly mutilated by French soldiers. He slowly recovered from these injuries, but he long continued to stammer in his speech, whence the nickname, adopted by himself, of 'Tartaglia'. Save for the barest rudiments of reading and writing, he tells us that he had no master; yet we find him at Verona in 1521, an esteemed teacher of mathematics. In 1534 he went to Venice. In 1548 Tartaglia accepted the position of professor of Euclid at Brecia, but returned to Venice at the end of 18 months. He died at Venice.

1536–1559 CE **John Calvin (Jean Cauvin)** (1509–1564, France and Switzerland). Theologian and reformer. One of the chief leaders of the *Protestant Reformation*. Calvin's incisive mind, powerful preaching, many books and voluminous correspondence, and capacity for organization and administration made him a dominant figure of the Reformation. He was especially influential in Switzerland, England, Scotland and Colonial *North America*.

Calvin was born in Noyon, France and educated in Paris. He was never ordained as a priest and his education reflected the influence of the liberal and humanistic Renaissance. He adopted Protestantism (1533) and left Paris (1534) to settle in Basel where he published his *Institutes of the Christian Religion* (1536). From 1541 on, Calvin became the dominant personality in Geneva, though he held only the position of pastor.

By 1546, many Protestants in Germany, Switzerland and France were insisting that the people – not just kings and bishops – should share in political and religious policy–making. This idea influenced Calvin and his followers in France, England, Scotland and The Netherlands. Calvin's French followers were called *Huguenots*. The English Protestants whom he influenced were called *Puritans*.

Calvin developed political theories that supported *constitutional government*, *representative government*, the right of the people to change their government, and the separation of civil government from church government. (Calvinists of the 1500's intended these ideas to apply only to the aristocracy, but during the 1600's, further democratic concepts arose, especially in England and later in Colonial America).

Calvin set the *Bible* as the basis to all Christian teachings, and expanded the idea that Christianity was intended to reform all society. Indeed, no other reformer did so much to force people to think about *Christian social ethics*. From this ethical concern, Calvin developed the pattern of church government that today is called *Presbyterian*. He organized the church government distinct from civil government, so that an organized body of churchmen would work for social reform.

The syncretism of the Jewish and Hellenistic cultures that exist at the basis of Christianity, disintegrated at the time of the Reformation. This process enabled **Luther** and **Calvin** to adopt for themselves anew different elements from the various components of Christianity. Luther chose the mystic, Paulinian element, while Calvin chose the nomistic part, emphasizing those elements expressed in the Bible.

Calvin based his doctrine on the Scriptures, returning to the Jewish idea of *God's providence*. His dominant figure is that of God and not the Son. He believed in a world that is ordered, rational and manageable by man, while Luther believed that the visible world was dominated by the *devil*, and that laws were a superfluous devilish phenomena[195].

Calvin maintained, on the other hand, that even God's providence does not relieve man of *responsibility*. Paul and Luther held that man is too weak to save himself by his deeds and therefore not accountable for his sins. While Calvin returned to Jesus' original Christianity, Luther continued to negate the world leaving it under the Devil's control. For this reason, Germany lagged behind England and the United States both politically and spiritually. The devilishness of which Luther spoke so much, finally was realized in the national-socialist movement and the genocide of the Jews in Europe during 1940–1945 by the Germans.

1536–1561 CE **Amatus Lusitanus** (1511–1568, Portugal and Italy). Physician and medical scientist. Among the greatest researchers in medicine during the first half of the 16^{th} century.

Published *Index Dioscoridis* (1536) on medical botany and seven volumes (1549) including 700 medical case histories, which established his name as an original researcher in the fields of internal medicine, anatomy, surgery, skin diseases, etc. Although Lusitanus was associated with the traditions of Hippocrates, Galen and the medieval Arab writers, his accurate observations, examinations, and diagnoses led him to go beyond these authorities and make new discoveries. For example, he was first to describe the *venous valves* (1547).

Lusitanus was born in Castel Branco, Portugal, to a family of marranos under the name Joannus Rodericus. His Jewish name is unknown. He studied medicine, mathematics, Greek and logic at the University of Salamanca, Spain. He fled the Inquisition (1533) and moved to Antwerp where he returned openly to his Jewish faith. He then removed to Ferrara, Italy (1544) and later to Ancona (1547), where he became the physician of the papal court. When pope Paul IV began to prosecute the Jews of Ancona, Lusitanus fled to

[195] Through this, Luther returned to *Paulinian* Christianity, maintaining that since man is devoid of free will, he is incapable of fulfilling laws of any kind. This fact had disastrous consequences for the development of Germany; The Germans never developed their own constitution and that is the reason why Nazi ideology could and did flourish in Germany and not in a Catholic country, let alone in a country under Calvinist influence.

Ragusa and from there to Saloniki (1558), where he died in a local outbreak of the plague.

1537–1555 CE **Andreas Vesalius** (1514–1564, Brussels). One of the foremost anatomists of all times. His book *Fabrica* (1543) contains the first complete description of the human body. For this he is called the *father of anatomy*. He was first to break with the extravagant admiration of antiquity (with the excessive confidence in the writings of **Galen**), and the general practice of principally dissecting bodies of lower animals.

A native of Brussels, he began to study anatomy at the age of 14. The difficulties with which the practical pursuit of human anatomy was beset in France made him look for Italy as a suitable field for the cultivation of this science; in 1536 we find him at Venice, and later at Padua, where he became a professor at the age of 23. Because he dared to correct many of Galen's errors (based on animal dissection), the followers of Galen bitterly attacked him. Discouraged, he burned most of his writings and resigned from Padua in 1544. After teaching at the Universities of Bologna and Pisa he became physician to Philip II of Spain, and to the Holy Roman Emperor Charles V.

Vesalius departed from tradition by performing dissections himself while instructing his medical classes. Traditionally, the dissections were performed by barber-surgeons while the instructor read from the works of earlier writers.

Jewish Scholasticism – Contributions to the Scientific Method

According to the scientific method, science begins from systematic observation and measurement, but it does not stop there – it is not the mere collection of information about nature. The creative act is to generalize from the data, to hypothesize *a possible physical process and to describe the process in* mathematical form. *Mathematics describes a relationship observed in nature, rather than claiming to* be *the underlying reality (as in Platonism). Finally, the hypothesis is judged not on its intrinsic logic or by debate, but solely by its ability to* predict *further measurements.*

In spite of their ancient weakness in mathematics (which can be traced directly to their use of alphabetic notations), the Jewish scholars of the Talmudic era (70–470 CE), and the post Talmudic era (ca 500–1500 CE) must be *given credit for inventing a completely new way of thinking. In their concern to apply the scriptures to the minutest details of everyday life, they supplied not only the definitive method of interpreting the sacred text, but also a logical basis for the scientific method and the logical principles basic to the theory of probability, modern statistics and the inductive method in mathematics. Their achievements are summarized as follows:*

- *First to declare the* principle of economy *(later known as 'Ockham's Razor').*
- *Established the method of reasoning known as 'binyan av'. It is similar to* 'the method of agreement' *used in scientific reasoning to identify causes and effects. This method states that, when there is an invariable association between certain events which happen before the phenomenon, and an absence of the phenomenon when these events do not occur, then the events are related in a causal manner to the phenomenon.*
- *Developed the mode of proof known as 'en la-davar sof'. This is the* reductio ad absurdum *agreement, also attributed to such Greek thinkers as Theaetetos, Hippocrates, and Zeno. It tests some declared truth or opinion by drawing some conclusion from it which is known to be false. If this can be done, the opinion itself must be false.*
- *Introduced the mathematical method of* proof by induction.
- *Formulated the* statistical law of large numbers *some* 250 *years ahead of Jakob Bernoulli (1713). It was first declared by the philosopher* **Itzhak Aramah** *(1420–1494, Spain).*
- *Anticipated many of the laws of* probability *and of* statistical reasoning. *Extensive use of the method of casting lots and constant scrutiny of the system of fairness (because of the tendency of individual priests to manipulate the system for personal gain) was a strong motivation to study the operations of chance. Expressing the idea of the random principle led to the recognition that a number of repetitions of a process of choosing should result in an equal number of choices of the alternatives.*

  The fact that, in the short run, there might be a bias in the results of the draw but that this bias should disappear in the long run of repeated samplings, led them to an intuitive awareness of the law of large numbers (that equal probabilities yield equal frequencies as the draw is repeated more and more times). The recognition that 'counted majorities' yield numbers equal to the actual probabilities which operate in cases of unbiased sampling, was also established.

The factor that ensured the primacy of Jewish savants in pioneering modes of thinking about chance in practical situations, was that Orthodox Jews were forbidden to gamble. The Greek might have discovered probability, for they were addicted to dice-throwing. But the dice they used were made from the astragalus bones of sheep. These were very irregular in shape, so that dice were biased and there was no equal likelihood of each side falling uppermost. Because of the astragalus' imbalance, it was impossible for the statistical law of large numbers to manifest itself in any tractable way. By contrast, the elaborate system of casting lots used by Jewish priests did exactly what it was devised to do; that is, gave everyone an equal chance of being chosen[196].

The theory is virtually the same as was developed later in mathematical form by ***Cardano****,* ***de Moivre****,* ***Laplace****,* ***Gauss****, and* ***Pearson****. But in the religious context in which the rabbis worked, they naturally avoided abstract speculations as an end in itself, and concentrated their analysis on empirical relations. They worked on problems by means of an inductive logic quite foreign to the speculative urges of the Greek philosophers. In doing so, they helped (in the same way as Christian medieval theologians did with their analytic methods and dialectical disputation) to lay the theoretical foundations for the scientific revolution in Western Europe. They set out, albeit in rhetorical and not scientific terminology, the logical principles basic to the modern science of statistics.*

Solved problems pertaining to external geometrical areas and volumes. For example – finding a box of the shape of a cube (or a circular cylinder) with open top and fixed surface area, such that its volume be maximized. A problem of this kind arose, for example, in connection with Jewish dietary laws *which traditionally divide food into pure and impure [Talmud; Kelim]. The addition of a very small proportion of impure food does* not *make a pure food impure. Thus, a cooking vessel whose walls might have in the past absorbed some impurity, can be used safely, provided the ratio of its volume to the total area of its walls is sufficiently large.*

Consequently, many problems of relative area and volume were analyzed by the Talmudists[197].

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[196] The historical origin of the lottery system is the annual choice of one of two goats to be the '*scapegoat*', carrying the heavy load of sin into the wilderness as the *Goat of Azazel*. The concept of an equal chance of being chosen in a 'fair' lottery would be the prime motivation of the whole system. In turn, the notion of equal probabilities, and the fact that such chances would be shown by equal numbers of votes for each participant over a period, could naturally emerge.

[197] **Shlomo ben–Moshe Ashkenazi–Rappoport** (1717–1781, Poland). Talmudic scholar and amateur mathematician, solved this problem in an ingenious way, bypassing the Newtonian calculus (unknown to him). In his book *Mirkevet–Mishne* (1751) he considered a vessel in the form of a box with square

1540 CE **Vannoccio Biringuccio** (1480–1539, Italy). Metallurgist. His *De pirotechnia* (1540), covers the technology of metallurgy of his time: practical aspects of mining, smelting of metals, casting of bells and cannons, and the preparation of chemicals for treating *ores*.

1542–1565 CE **Conrad Gesner** (1516–1565, Switzerland). Universalist and naturalist: physician, encyclopedist, botanist, zoologist, linguist and mountaineer. His compendium *Historiae animalium* (1551–1587) is the starting point of modern zoology. In his *Catalogus plantarum* (1542) he classified the plant genera on the basis of reproductive organs. Distinguished *genus* from *species* and *order* from *class* in his classification of plants. He also classified *fossils* into 15 different types.

Gesner was born in Zurich. He studied (1532–1541) at the Universities of Strasbourg, Bourges, Montpellier and Basel, where he took his degree in medicine (1541). He practiced medicine in Zurich and stayed at the city until his death from the plague.

In 1545 he published his remarkable *Bibliotheca universalis*, a catalogue (in Latin, Greek and Hebrew) of all writers who had ever lived, with the title of their works, etc. A second part, under the title *Pandeclarium sive partitionum universalium Conradi Gesneri Lingurini* appeared in 1548. In 1555 he put forth *Mithridates de differentiis linguis*, an account of about 130 known languages.

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base of side a, and hight h. Then

$$S = \text{ total area } = 4ah + a^2 = 2ah + 2ah + a^2$$

$$V = \text{ volume } = ah^2 = \frac{1}{2}\sqrt{2ah \cdot 2ah \cdot a^2}$$

If we take the total area as constant, then $4V^2 = 2ah \cdot 2ah \cdot a^2$ is the product of three quantities whose sum is constant, and is therefore a maximum when they are all equal, which is when $h = \frac{1}{2}a$. The same argument holds for a cylindric box with a circular base of diameter d, namely $h = \frac{1}{2}d$. This result can be checked against the provable statement that of all *closed* rectangular boxes, the cube encloses the greatest volume for its surface area.

The Emergence of Modern Science

"In the year 1500 *Europe knew less than Archimedes who died in the year* 212 *BCE"*

Alfred North Whitehead, 1953 (1861–1947)

The growth of science was strongly affected by the historical events that shaped Western civilization. During the 130 *years between* 1559 *and* 1689*, Europe passed through a tumultuous and anarchic period of civil wars and rebellions. Each upheaval had its own distinct character, each had multiple causes. The one common denominator, which constantly recurred, was the Protestant-Catholic religious strife.*

Yet, amidst all bloodsheds and turmoil, an intellectual revolution in astronomy, physics and mathematics was steadily and quietly being accomplished. In no other period of history was there such a dense galaxy of brilliant thinkers as ***Copernicus****,* ***Francis Bacon****,* ***Galileo****,* ***Kepler****,* ***Descartes****,* ***Fermat****,* ***Pascal****,* ***Spinoza*** *and* ***Newton****. [This was also the age of* ***Cervantes*** *(1547–1616),* ***Shakespeare*** *(1564–1616),* ***Bernini*** *(1598–1680),* ***Velásquez*** *(1599–1660),* ***Rembrandt*** *(1606–1669),* ***Hobbes*** *(1588–1679),* ***Locke*** *(1632–1704),* ***Montaigne*** *(1533–1592)* and ***Moliere*** *(1622–1673)] – all men of creative genius, whose work still lives today.*

No 16^{th} *or* 17^{th} *century achievement is better remembered today than the accomplishments of this international brotherhood of fellow scientists. They obliterated the traditional view of nature and established scientific practice on an impressive new footing. This great breakthrough equipped the physical scientist with new methods and new standards which worked exceedingly well throughout the* 18^{th} *and* 19^{th} *centuries. So total was the victory, that it requires an effort of imagination to understand how any intelligent man could have taken the pre-Copernican view of nature seriously. Yet this traditional view was scientific within its own terms. For many centuries, thinking men had found it logical and empirical, as well as emotionally convincing.*

We must try to reconstitute this obliterated system in order to appreciate the magnitude of the scientific discoveries which swept it away.

The traditional view of the cosmos, accepted by almost every educated man until well into the 17^{th} *century, was an amalgam of Aristotelian mechanics, Ptolemaic astronomy, and Christian theology. It fitted all the preexisting views of a moral universe in which man occupied a middle place between*

Heaven above and Hell in the earth's core. Everyday experience would seem to indicate that Aristotle's theory of motion did indeed represent the way God operated the universe: All heavenly bodies, said Aristotle, naturally fell toward the center of the universe and rested there, unless propelled by a mover in some other direction. It followed that the round earth, obviously solid and weighty, stood motionless in the center of the universe.

Such teaching, accepted by all *Christians, harmonized nicely with Ptolemaic astronomy in which a concentric series of transparent crystalline spheres revolved around the earth: the moon, the sun, the planets, the fixed stars and the outermost sphere, which drives the entire system, all wheeled in perfect circles, going around the earth once every 24 hours. Beyond the outermost sphere lay God's heavenly abode. This theory coincided, as well as anyone could see, with the crude astronomical observations which stargazers were able to collect.*

There were of course major mysteries that were not written in Aristotle's philosophy, such as plagues, storms, floods, earthquakes, comet apparitions and other natural catastrophes. The fickleness of life seemed to show that God had delegated a role in nature to fate, fortune, or chance. This was the twilight zone in which astrologers and alchemists could thrive. It may also explain the need to believe in small-scale supernatural beings such as witches, fairies, pixies, evil spirits (fallen angels) or even the devil himself, who meddled in human affairs.

In the 14^{th}, 15^{th}, *and* 16^{th} *centuries, however, certain forces in European society were preparing the way for a change in the general view of Nature. Artisans and craftsmen were becoming more skilled in their techniques. The invention of the lens and the development of the glass industry, to take but one example, contained the promise of vastly extending man's power of observing natural processes. New techniques in shipbuilding led to voyages of discovery, which in turn stimulated interest in nature and turned men's attention to problems of navigation. The Renaissance, with its emphasis on literature and art, was in some ways anti-scientific. But humanism stimulated a passionate interest in man. Furthermore, humanistic study revealed conflicting opinions among the ancients on matters of science, just at the moment when the authority of Ptolemy was becoming shaky for other reasons: growing skill in mathematics exposed the clumsiness of Ptolemy's explanations. In the opening years of the* 16^{th} *century, conditions were ripe for change.*

The first thinker to challenge the traditional viewpoint was ***Nicolaus Copernicus****, as early as* 1514*. He was a quiet, conservative Polish cleric, who lived in an obscure East Prussian cathedral town. He had no quarrel with the Ptolemaic vision of concentric crystalline spheres wheeling in perfect circles around the central point in the universe. What troubled him was the fact that Ptolemy's spheres were not perfectly circular, and he therefore published* *his revision to the geocentric theory, hoping to bring Ptolemaic astronomy up to date. He then discovered that he could account for the irregularities in celestial motion by making a single elemental adjustment, namely, to put the sun, rather than the earth in the center of the universe. He could then simplify the Ptolemaic intricate network of 80 interlocking circles by replacing them with only 34 circles. It would then be superfluous to posit that the distant fixed stars wheeled daily around the earth.*

What we call the Copernican Revolution was not made by Canon Koppernigk. His book was not intended to cause a revolution. He knew that much of it was unsound, contrary to evidence, and its basic assumption unprovable. He only half believed in it, in the split-minded manner of the Middle Ages[198]*. Besides, he was denied the essential qualities of a prophet: awareness of mission, originality of vision, and the courage of conviction.*

The scientific revolution began at the medieval universities of ***Oxford*** *and* ***Paris*** *(Roger Bacon, William of Ockham),* ***Padua*** *(Copernicus, Galilei),* ***Tübingen*** *(Kepler) and* ***Cambridge*** *(Newton).*

Other universities such as Bologna, Prague, Vienna, Heidelberg, Leipzig, Uppsala, Königsberg, Jena, Salamanca, Edinburgh and Leyden (established 1158–1575) also participated in the West European movement of intellectual emancipation.

Europe Under the Reformation (1517–1648)

Reformation is the name given to a religious movement that gave birth to Protestantism. It had great impact on man's social, political and economic life

[198] A strange situation formed in astronomy after the Copernican Revolution: Ptolemy's epicycles could still fit the data. In fact, in some respect they did so better than Copernicus' calculations. Yet their forced complexity came to make them seem unconvincing when compared with the eventual attractions of the heliocentric system: There is a deep feeling among those who practice fundamental science, a feeling that has so far proved reliable, that the way to true understanding is the one that satisfies the canons of economy and elegance – the way which is *mathematically beautiful*.

and on the development of science. The Reformation began in 1517 *when the German monk* ***Martin Luther*** *(1483–1546) protested certain practices of the Roman Catholic church. About* 40 *years later, Protestantism was established in nearly half of Europe.*

Before the Reformation, Europe had been held together by the universalism of the Catholic church and by the claim of the Holy Roman Emperor to be the supreme secular *ruler. After the Reformation, Europe had several large Protestant churches and some smaller Protestant religious groups. All of them competed with the Catholic church – and with each other – for the faith and allegiance of men.*

The word Protestant ('one who protests') dates from the diet of Speyer, Germany, in 1529*. In* 1530*, the Lutherans presented the 'Augsburg Confession' to the diet of Augsburg, Germany, and it became the basic statement of Lutheran doctrine. In the 'peace of Augsburg', signed in* 1555*, the Holy Roman Empire officially recognized the Lutheran churches.*

One of the chief leaders of the Protestant Reformation was ***John Calvin*** *(1509–1564). From his center in Geneva he directed efforts to convert the people of France and other West European countries. His followers in France were called the Huguenots. In* 1571*, a moderate form of Protestantism, known as* Anglicanism*, was established in England.*

As a result of the Reformation, Europe was divided between Catholic countries in the south and Protestant in the north. The Protestant ethic encouraged industriousness and careful management of material things. It contributed to the development of industry and commerce during the 18^{th} *and* 19^{th} *centuries.*

Protestant leaders also emphasized education, promoted literacy, an educational curriculum based on ancient Greek and Roman literature and a high respect for teaching and learning. After 1640*, advances in science were promoted more energetically in Protestant lands.*

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The Religious Wars (1562–1689)

Persecution compelled the French Protestants (Huguenots) to take up arms. At the same time they formed a political party. The ensuing struggles, therefore, did not constitute a purely religious war, but also civil war in which the leaders of both parties endeavored to exploit the weakness of the crown and secure control of the government. The Huguenots were recruited primarily from the nobility and from the new capitalist-artisan class. Paris and the northeast in general remained Catholic throughout. After the first three wars (1562–1569), the Huguenots, despite defeat, were given conditional freedom of worship.

On Aug. 24, 1572 (known as "Massacre of Saint Bartholomew"), the pro-Catholic party, with the approval of Queen Catherine dé Medici, murdered some 5000 *Huguenots in Paris and in the French provinces. Killing continued until October* 03. *Catholic Europe congratulated the Queen on her success and the Pope celebrated the event in Rome. The massacre led to the fourth of the religious wars (1562–1598) in which the Huguenots won.*

The wars ended with the Edict of Nant (1598) which gave the Huguenots equal political rights but did not secure them entire freedom of religious worship. The persecutions were resumed in 1620, *and especially after* 1685, *when the Nant Edict was canceled. Many of them (ca 250,000) fled to Switzerland, Netherlands, England, Germany and the U.S.A., where they contributed greatly to the cultures of these countries. Only in* 1789 *did they regain equal religious and social rights.*

The last of the great religious wars in Europe was the Thirty Year War (1618–1648). It involved most of the nations in Europe in a struggle for territory and political power. The underlying cause of the war was the old deep-seated hostility between the German Protestants and the German Catholics.

The war had three stages: The Bohemian period (1618–1620), The Danish period (1625–1629), and finally The Swedish-French period (1635–1648). During the entire duration of the war, the people of Germany suffered misery and hardships. In 1648, *the war ended with the peace of Westphalia. By this treaty, Calvinism was put on an equal footing with Catholicism and Lutheranism.*

By the time the war ended, Germany was in a pitiable condition. Third of its people were killed. Whole cities, villages and farms disappeared, and much property had been destroyed. Art, Science, trade and industry declined. It *took almost* 200 *years for Germany to recover from the effects of the Thirty Years War. Thousands of persons left Europe, especially Germany, and went to America to build a new life.*

The Golden Age of The Netherlands

The Reformation spread through the Netherlands during the early 1500*'s. However, in* 1516 *Charles of Burgundy also became King of Spain. In this way, the Netherlands came under Spanish control. In* 1519*, Charles became emperor of the Holy Roman Empire and tightened his control over the Low Countries. He tried to stop the Protestant threat to Roman Catholicism by persecuting them. His son Phillip II inherited the Netherlands in* 1555 *and tried to gain complete power over the country. In* 1568*, the Nobles there revolted under the leadership of William of Orange. In* 1581*, the northern provinces declared their independence from Spain. The revolt of the Netherlands lasted until 1648 (except for temporary peace from 1609–1621), when Spain finally recognized Dutch independence.*

The first half of the 17^{th} *century, during which the Dutch provinces were still at war with Spain to secure their independence, was nevertheless a period of unexampled flowering in art, science and literature. This was primarily due to an unprecedented expansion of the Dutch commerce which resulted from the closing of Lisbon to Dutch trade and the annexation of Portugal to Spain. The Dutch were obliged to find their own way to the East, and within a remarkably short time they were disputing the command of the Indies with the Portuguese, whom they soon displaced.*

The Dutch 'East India Company' (founded 1602) was given extensive political and military authority and became one of the chief organs of Dutch imperialism. In 1652 *the Dutch established themselves at the Cape of Good Hope, and in* 1667 *they took Sumatra. The 'Dutch West India Company'* *(founded 1621) had the same extensive control over the American and African coast trade.*

In 1623 *the Dutch began extensive conquests in Brazil, until* 1661. *They took the islands of St. Eustace and Curacao (1634–5), Saba (1644) and St. Martin (1648). With this far-flung colonial empire, the Dutch provinces became the commercial center of Europe, Amsterdam easily holding the lead as the financial center.*

Thus, a million and a half people fought, against great odds, the whole might of the Spanish Empire. At the end the Dutchmen not only were able to hold their own, but actually turned the tables on their enemies to such an extent that Spain never recovered from the blow.

The prodigious energy necessary to accomplish such a stupendous victory could not suddenly be subdued the moment peace was signed. Carried forward by its own momentum, this desire to achieve manifested itself in almost every other phase of life. Almost overnight Holland was turned into an economical, intellectual, and artistic beehive.

Never before or since has Holland been the world power it was then – a small country, forced to live by its wits. Because of its tolerance for unorthodox opinions and its tradition of encouraging freedom of thought, it was a haven for intellectuals, who were refugees from censorship and thought-control elsewhere in Europe.

It is therefore not by sheer chance that ***Descartes*** *created most of his philosophy and mathematical writings (1628–1649) in Holland. It was the home of the great Jewish philosopher* ***Baruch Spinoza*** *(1632–1677) and the political scientist* ***John Locke*** *(1632–1704). It was the cradle of the developments of the microscope [****Leeuwenhoek*** *(1632–1723)] and the telescope [****Huygens*** *(1629–1695)], the extensions of human visions to the realms of the very small and the very large, respectively.*

The fundamental studies of ***Snellius*** *(1580–1626) on the refraction of light and* ***Huygens****' wave theory of light are among the remarkable achievements of this golden age.*

This was also the time of ***Rembrandt*** *(1606–1669),* ***Vermeer*** *(1632–1675) and* ***Hals*** *(1580–1666).*

Is there a common denominator to Descartes' 'frame of reference', and the obsession with light and vision of Vermeer, Rembrandt, Huygens, Snellius and Leeuwenhoek? ***Hendrik van Loon*** *(1882–1944, U.S.A.) replied to that in the affirmative:*

"The Low Countries are one of the few parts of the world where every window becomes the frame for a very definite and exceedingly paintable little

landscape, while inside the house that strange light that sweeps across a sky washed clean by everlasting rainstorms, has a clarity and harsh brightness which turns even the most ordinary article of daily usage into mysterious objects that lose their commonplace character and begin to vibrate with all the colors of the rainbow."

ca 1543 CE **Nicolaus Copernicus** (Nicolas Koppernigk, 1473–1543, Poland). Astronomer, physician and Canon of Law. First to revive the heliocentric theory of Aristarchos. Broke away from the Ptolemaic doctrine and put the center of the universe near the sun, with the earth and all the planets revolving about it (1514). This heliocentric model was revolutionary in that it challenged the previous dogma of scientific authority of Aristotle, and caused a complete scientific and philosophical upheaval.

The Ptolemaic system formed the basis of all the writings of the Arab and Jewish astronomers of the Middle Ages, including Maimonides, Avraham bar Hiyya, Levi ben Gershon and others.

Martin Luther (1483–1546) ridiculed the Copernican hypothesis, that the earth revolves around the sun, and appealed to a literal interpretation of scripture to support his arguments. He pointed out that Joshua had made the sun – not the earth – stand still! For half a century after Copernicus' death, his heliocentric theory gained few adherents. It was taught at only one university (Salamanca, Spain) and the general public was unaware of it. **Montaigne** (1533–1592) lightly dismissed it and **Tycho Brahe** (1546–1601) did not accept it either. All this was however quite irrelevant to science; the important thing was that **Kepler** was a Copernican.

Nicolas Koppernigk was born on the 19^{th} of February 1473, at Torun on the Vistula in Prussian Poland, a trading post between East and West. There his father, a native of Cracow, had settled as a dealer in copper (the family business, from which the Koppernigks derived their name). His mother, Barbara Watzelrode, belonged to a family of high mercantile and civic standing. After the death of his father in 1483, Nicolas was virtually adopted by his uncle Lucas Watzelrode, later Bishop of Ermeland.

Placed at the University of Cracow in 1491, he devoted himself during the next four years to mathematical science, but did not take any degree. In 1495 he came to the University of Bologna, where he studied Canon law and astronomy. In 1497 he was nominated a canon of the cathedral chapter of Frauenburg[199] (some 85 km east of Danzig, on the Baltic Sea), but the chapter gave him permission to continue his education in Italy. In 1501, Nicolas entered the medical school of Padua, where he remained until 1505, having taken meanwhile a doctor's degree in canon law at Ferrara in 1503. After his return to his native country in 1506 he resided at the episcopal palace of Heilsberg as his uncle's physician until the latter's death in 1512. He then retired to Frauenburg and attended to his capitular duties. His work was mainly administrative, though he had occasionally to practice medicine. In his spare time he pursued astronomical researches.

The main lines of his work were laid down at Heilsberg and at Frauenburg from 1513 until its completion in 1530. In it he sought, with scant instrumental means, to test by observations the truth of the heliocentric doctrine. In 1530 he circulated a brief popular account, the *Commentariolus*. His disciple **George Joachim Rheticus** (1514–1576) printed in 1540, in Danzig, a preliminary account of the Copernican theory, and simultaneously sent to the press at Nuremberg his master's complete exposition of it in a treatise entitled *De Revolutionibus Orbium Coelestium* (concerning Revolutions of Celestial Spheres). But the final printed copy reached Frauenburg barely in time to be laid on the writer's death-bed. The book was marred by an anonymous preface, addressed to the reader, slipt in by **Andreas Osiander** (1498–1552). It insisted upon the purely hypothetical character of the reasoning it introduced, and explained that the ideas of the book need not be taken too seriously. The trigonometrical section of the book had been issued as a separate book (1542) under the care of Rheticus.

The first edition, Nuremberg 1543, numbered ca 1000 copies, which were never sold out. It had altogether 4 reprints in 400 years: Basle 1566, Amsterdam 1617, Warsaw 1854, and Torun 1873. It is a remarkable negative record, and quite unique among books which made history.

The Copernican system met with very determined opposition, and the Christian Church as well as the Jewish Rabbis denounced it as heretical because the Bible said that the earth was fixed and the sun was moving (*Eccles* **1**, 3–4).

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[199] His uncle was anxious to secure him this post because the economic future of the Baltic Sea ports of the Hanseatic league became uncertain due to the alleged opening of new sea routes to the Orient by Columbus. He drew his prebend, but he neither took holy orders nor was his physical presence at Frauenburg required for the next 15 years. However, the income from this position supported him for the rest of his life. From the age of 22 to 32, the young canon spent his time in Italy, at the Universities of Bologna, Padua and Ferrara on a series of extended leaves of absence. Toward the end of his Italian studies, the heliocentric system began to take shape in Nicolas' mind (ca 1505).

The Copernican Revolution (1543)

In 1543 CE, **Nicolas Koppernigk** *(1473–1543) [now known as* **Nicolas Copernicus***], astronomer, physician and Canon of Law, first revived the heliocentric theory of* **Aristarchos***. He broke away from the Ptolemaic doctrine and put the center of the universe near the sun, with the earth and all the planets revolving about it (1514). This heliocentric model was revolutionary in that it challenged the previous dogma of scientific authority of Aristotle, and caused a complete scientific and philosophical upheaval.*

In his book 'De Revolutionibus', published as late as 1543, he put forward the kinematic explanation to the precession of the equinoxes. Imagining himself as an observer outside *the solar system, he stated that the earth's axis, although it always keeps the same inclination ($66\frac{1}{2}^\circ$) to the plane of the earth's orbit (ecliptic), nevertheless traces out a cone of semiangle $23\frac{1}{2}^\circ$ with respect to the normal to this plane. Copernicus also concluded that the average precessional rate is 50.2″ per year, corresponding to a complete precessional period of about 26,000 years.*

The cause *of this precession remained a mystery until Newton [Principia, Book III, Proposition 39, 1687] gave a quantitative dynamical explanation: the wandering of the earth's axis round the pole of the ecliptic depends upon the same principle as the corresponding motion of a spinning top under the action of terrestrial gravitation. The equatorial bulge of the earth is attracted towards the plane of the ecliptic by the Moon and the Sun. Their turning moment acts upon the earth's angular momentum vector, causing it to precess.*

Although Copernicus is hailed as the harbinger of a new era in natural science, we encounter in his writings Pythagorean-Platonic mysticism and scientific reasoning in a combination which, from a modern standpoint, appear strange to us: he regarded the cosmos as spherical, a 'divine body' endowed with the perfection of its creator, and rejoices in the regularity and order of the world. Circular motion was supposed to be proper to all complete objects, and the state of rest to be more noble than that of motion.

The question arises as to how did Copernicus figured out the sidereal periods of the planets and the sizes of their orbit?

The observed retrograde *motion of the planets inspired* **Aristarchos** *(ca 270 BCE) to suggest a heliocentric cosmology. In Aristarchos' day, however, the idea of a moving earth seemed incompatible with explanation of other phenomena at the earth's surface. About 1800 years elapsed before someone had the insight and the determination to work out the details of a sun-centered cosmology.*

Table 2.7: The synodic and sidereal periods of the planets (in earth time units)

Planet	Synodic Period	Sidereal Period
Mercury	116 days	88.0 days
Venus	584 days	224,7 days
Earth	–	365.2 days
Mars	780 days	687.0 days
Jupiter	399 days	11.9 years
Saturn	378 days	28.6 years

Copernicus realized that, using a heliocentric perspective, he could determine which planets are closer to the sun than the earth and which are further away. Because Mercury and Venus are always observed fairly near the sun in the sky, he concluded that their orbits are smaller than earth's (inferior planets). The other visible planets: Mars, Jupiter and Saturn can be seen in the middle of the night, when the sun is far below the horizon, which can occur only if the earth comes between the sun and the planet. Thus Copernicus concluded that the orbits of Mars, Jupiter and Saturn are larger than that of the earth[200] *(superior planets).*

Since the earth, sun and other planets are all in relative motion w.r.t the distant fixed stars, Copernicus was careful to distinguish between two characteristic time-intervals, or periods, of each planet. Whereas the synodic period[201] *of a planet can be determined by observing the sky, the* sidereal period[202] *must be calculated.*

He derived the relation

$$\frac{1}{P} = \frac{1}{E} \pm \frac{1}{S},$$

where

[200] Other, dimmer superior planets - *Uranus*, *Neptune* and *Plato* - were discovered after the telescope came into use.

[201] Time that elapses between successive identical configurations (as seen from earth from one *opposition* to the next, for example, or from one *conjunction* to the next).

[202] The time it takes the planet to complete one orbit of the sun - the true orbital period.

$$\begin{array}{lcl} P & = & \textit{sidereal period of the planet (in earth years)} \\ E & = & \textit{sidereal period of the earth, which Copernicus} \\ & & \textit{knew to be equal to } 365\tfrac{1}{4} \textit{ days} = 1 \textit{ earth year} \\ S & = & \textit{synodic period of the planet (in earth years)} \end{array}$$

and $(+/-)$ *refers to inferior/superior planets, respectively.*

For example, taking the synodic period of Jupiter to be 398.88 *days* = 1.092 *earth years, and* $E = 1$, *we set:*

$$\frac{1}{P} = 1 - \frac{1}{1.092} = 0.084 = \frac{1}{11.9}$$

It thus takes Jupiter to complete one full orbit of the sun in 11.9 *earth years. Table 2.7 renders the values obtained by the method of Copernicus.*

To determine the size of an inferior planet's orbits, Copernicus measured the angle α *between the sun and the planet when this angle (known as elongation) is maximal. This occurs when the triangle formed by earth, the inferior planet, and the sun is a right angle. The hypotenuse of the triangle has a length of one astronomical unit (1 AU), and hence the radius of the inferior planet's orbit is equal to* $\sin\alpha$, *also measured in AU.*

Determining the size of the superior planet is somewhat more complicated. Here Copernicus used a triangle formed by the sun, earth and the planet at quadrature[203]*, which again contains a right angle at the earth. Simple trigonometry shows that in this configuration, the radius of the superior planet's orbit is equal to* $\frac{1}{\cos(\beta-\gamma)}$ *(measured in AU), where* β *is the angle the earth has spanned from its initial fiducial position at an approximate rate of* 1° *per day. The angle* γ *is that spanned by the planet from the earth's initial fiducial position and can be calculated knowing the sidereal period of the planet.*

Copernicus compiled his ideas and calculations into his book De Revolutionibus Orbium Celestium ("On the Revolutions of the Celestial Spheres") that was published in 1543, the year of his death. Although he assumed that the earth travels around the sun along the circular path, he found that perfectly circular orbits cannot accurately describe the paths of the other planets. He therefore had to add an epicycle to each planet *to account for the slight variation in speed along the orbit. Thus, according to Copernicus, each planet revolves around a small epicycle, which in turn orbits the sun along a circular path. Table 2.8 compares the average distances of the planets from the sun calculated by Copernicus with modern values.*

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[203] In this position, the planet's elongation is 90°.

Table 2.8: Average distances of the planets from the sun (in AU)

Planet	Copernicus	Modern
Mercury	0.38	0.39
Venus	0.72	0.72
Earth	1.00	1.00
Mars	1.52	1.52
Jupiter	5.22	5.20
Saturn	9.07	9.54

Tycho Brahe *tried to test Copernicus' ideas with detailed observations of the sky. We know that, when we walk from one place to another, nearby objects appear to shift their position against the background of more distant objects. Tycho argued that, if Copernicus was correct, nearby stars should shift against the background stars as the earth orbits the sun.*

Tycho spent his lifetime making accurate observations of the positions of the stars and planets. Yet he could not detect any shifting of star positions. He therefore concluded that Copernicus was wrong. Actually, the stars are so far away that naked-eye observations could not possibly detect any tiny shifting of star positions. That has now been confirmed with telescoping observations.

Nevertheless, Tycho Brahe's astronomical records were destined to play an important role in the development of heliocentric cosmology. Upon his death (1601), many of Tycho's charts and books fell into the hands of his gifted assistant, ***Johannes Kepler****.*

Could you prove today that Copernicus was right?

Any school-boy knows that the earth revolves about the sun and not the other way round. But most people, among them, no doubt, some professors of physics would be hard pressed to explain why they are so sure of this.

The solar system is much smaller then the distance between the sun and the nearest star. That the universe is such a big place was only understood once it was realized that the parallax (as viewed from earth in, say, winter *and summer) of even the nearest star is minuscule, even with a base-line of* 3×10^8 *km (2 AU). Nonetheless, such parallaxes have been measured*[204].

Together with optical measurements of stellar aberrations, Doppler shifts and inertial effects caused by the earth's revolution, this provides evidence for the earth's relative *orbital motion w.r.t the sun and fixed stars.*

While none of these arguments can distinguish between the Copernican theory and, say, the Tycho Brahe's approach[205]*, the immense stellar distances revealed by parallax measurements to the nearest stars has rendered the Tycho interpretation unlikely in the extreme. It is simply unreasonable to believe that such huge numerous and distant objects conspire to rotate in unison around the earth.*

Another powerful argument for the correctness of the pure Copernican interpretation follows from a judicious combination of empirical facts and theoretical principles. Once one is armed with Newton's Second law of motion *and his* Law of Universal gravitation *can the issue be resolved in several steps as follows:*

- *Use* Kepler's third law *to compare the lunar orbital elements with those of the earth's revolution in order to deduce the mass ratio of earth and sun. This requires at least two parallax measurements from two different points on the earth's surface in order to relate one AU and the lunar-earth distance to the earth's radius.*
- *Use Newton's Law of Universal gravitation and his second law of motion to show that the earth and the sun must both revolve around a common center of mass which is inside the sun.*

The problem with this "proof" is that it is somewhat indirect. A better way is to directly measure the centrifugal acceleration *due to earth's circumsolar revolution. This can, in fact, be done with sensitive Eotvös-type experiment.*

Additionally, if the center of the earth's mass were really stationary relative to an inertial frame, there would be no way of understanding why there are

[204] The vastness of the universe, as revealed by conjoining the Copernican hypothesis with the minute value of stellar parallaxes, intimidated the minds of scholars and tended to prejudice many against the Copernican theory. Little did they know that the discovery of the extragalactic nature of many nebulae in the 1920's would once again vastly expand the scale of man's universe by many order of magnitudes.

[205] Namely, the compromise view that non-terrestrial planets revolve around the sun, which in turn revolves around the earth!

solar and/or lunar tides *when the respective heavenly bodies were on the other side of the earth. Incidentally, the earth's rotation about an axis is also easy to demonstrate: it partly accounts for, via the centrifugal force, for the reduced weight of objects (and the acceleration of gravity) on the equator relative to higher latitudes.*

The following four effects demonstrate that the earth rotates relative to the sun and the fixed stars:

1. Triangulation *of nearest stars.*

2. Doppler-shifts *of all stars with period of one earth-year.*

3. Stellar aberration *of light from all stars, with the same period and 90° out of phase with the Doppler-shift oscillation.*

4. Inertial forces *observed in Eotvös-type experiments that are due to orbital acceleration.*

None of these four effects can settle the question of whether the earth acceleration relative to sun and fixed stars, or the other way round. Even effects no. 4 cannot resolve this because of the Lenz-Thirring-Brill-Cohen-Mach effect. However, since effect no. 1 shows the immense distance of even the nearest stars, the Tycho interpretation appears unlikely in the extreme.

On Copernicus

"It is necessary to alter the motion of the stars a little because of the (annual) motion of the earth"

Aristarchos, ca 280 BCE (310–230 BCE)

"The Ptolemaic astronomy is nothing so far as existence is concerned; but it is convenient for computing the non-existent"

Muhammad Ibn Rushd, ca 1170 (1126–1198)

"Since, then, the earth cannot be the center, it cannot be entirely devoid of motion... It is clear to us that the earth is really in motion though this may not be apparent to us, since we do not perceive motion except by comparison with something fixed... Moreover, neither the sun, nor the moon, though to us it seems otherwise, can in its motion describe a true circle, because they do not move around a fixed base...".

Nicolas of Cusa, 1440 (1401–1464)

"... I cannot get over my amazement at the mental inertia of our astronomers in general who, like credulous women, believe what they read in the books, tables, and commentaries as if it were the divine and unalterable truth; they believe the authors and neglect the truth".

Regiomontanus, 1464 (1436–1476)

"There is talk of a new astrologer who wants to prove that the earth moves and goes round instead of the sky, the sun, and moon... The fool wants to turn the whole art of astronomy upside-down...".

Martin Luther, 1533 (1483–1546)

"... I have been informed that you have... created a new theory of the Universe according to which the Earth moves and the Sun occupies central position...; moreover, that you have written a treatise on this entirely new theory of astronomy, and also computed the movement of the planets and set them out in tables, to the greatest admiration of all.
Therefore, learned man, I beg you most emphatically to communicate your discovery to the learned world...".

From a letter of Nicolaus Schoenberg, Cardinal of Capua, written to Copernicus on Nov. 1, 1536, at the initiative of Pope Paul III.

"Copernicus had opened the eyes of the most intelligent to the fact that the best way to get a clear grasp of the apparent movements of the planets in the heavens was to regard them as movements round the sun conceived as stationary. If the planets moved uniformly in a circle round the sun, it would have been comparatively easy to discover how these movements must look from the earth".

Albert Einstein, 1930 (1879–1955)

"In philosophy proper, the 15^{th} *and* 16^{th} *centuries are on the whole not very spectacular. On the other hand, the spread of the new learning, the dissemination of books, and, above all, the renewed vigor of the ancient traditions of Pythagoras and Plato, paved the way for the great philosophic systems of the* 17^{th} *century.*
It was in the wake of this revival of ancient modes of thought that the great scientific revolution began. Starting from a more or less orthodox Pythagoreanism, it gradually overthrew the established notions of Aristotelian physics and astronomy, to finish by going right behind the appearances and discovering an immensely general and powerful hypothesis. In all this, the men who furthered such inquiries knew that they stood directly in the Platonic tradition...
The theory as Copernicus propounded it was not free from difficulties, and in some ways was dictated by preconceived notions going back to Pythagoras. That the planets must move steadily in circles seemed to Copernicus a forgone conclusion, because the circle is a symbol of perfection, and uniform motion is the only kind becoming to a heavenly body. Within the scope of the observations available, the heliocentric view with circular orbits was, however, much superior to the epicycles of Ptolemy. For here, at last, was a simple hypothesis that by itself alone saved all appearances".

Bertrand Russell, '*Wisdom of the West*' (1892–1970)

"It is a strange paradox that at every time when man was beginning to conquer nature he was obliged to drive himself away from the center of things; in proportion as he grew wiser he had to make himself smaller."

George Sarton

"Copernicus was a poor observer and it had been easier for him (as it was for Aristarchos) to formulate his new theory, because he was not embarrassed by good observations."

George Sarton

"The notion of limitlessness or infinity, which the Copernican system implied, was bound to devour the space reserved for God on the medieval astronomer's charts. They had taken it for granted that the realms of astronomy and theology were contiguous, separated only by the thickness of the ninth crystal sphere. Henceforth, the space-and-spirit continuum would be replaced by a space-time continuum. This meant, among other things the end of intimacy between man and God. Homo sapiens *had dwelt in a universe enveloped by divinity as by a womb; now he was being expelled from the womb.*

During the remainder of the sixteenth century, the new system of the universe went, like an infectious disease, through a period of incubation. Only at the beginning of seventeenth did it burst into the open and caused the greatest revolution in human thought since the heroic age of Greece.

1600 CE is probably the most important turning point in human destiny after 600 BCE. Astride that milestone, born almost exactly a hundred years after Copernicus, with one foot in the sixteenth, the other in the seventeenth century, stands the founder of modern astronomy, a tortured genius in whom all the contradictions of his age seem to have become incarnate: Johannes Kepler".

Arthur Koestler, 'The sleepwalkers', 1959

1543 CE The works of Archimedes appeared in Latin translation. **Vesalius** published *Des Humani Corporis Fabrica*.

1543–1555 CE **Petrus Ramus** (**Pierre de La Rameé**, 1515–1572, France). Humanist, philosopher, mathematician and logician[206]. Published (1543) *Aristotelicae Animadversiones* – an attack on Aristotelian logic and physics. It represented a break with the authority of medieval tradition.

Ramus defended the audacious thesis that "*Everything Aristotle taught was false*". This iconoclastic attitude was not so much an attack on Aristotle as on the medieval Peripatetic tradition which had become embedded in Scholasticism. One can therefore justly choose the year 1543 to mark the separation of medieval from modern times.

In his second book *Dialecticae partitiones* (1543), Ramus criticized university curriculum and argued for return to teaching of the seven liberal arts:

His works were suppressed and he was forbidden to teach logic (1544–1547). However, the ban was lifted by Henry II (1547) through the influence of the Cardinal of Lorraine. He was then appointed professor of philosophy at the College de France (1551) and embraced Calvinism (1561). Persecuted thereafter by academic and ecclesiastic enemies, he was assassinated in the Massacre of St. Bartholomew.

His system of logic, known as *Ramism*, emphasized logic as method of disputation and had influence in the 16^{th} and 17^{th} centuries.

Ramus was born at the village of Cuth in Picardy, a member of a noble but impoverished family. His father was a charcoal–burner. He gained admission to the college of Navare, working by day and carrying his studies at night. On taking his degree (1536) he actually took as his thesis: "Everything Aristotle taught was false". At the College de France he lectured, for a considerable time, before audiences numbering as many as 2000. He published 50 works in his lifetime and nine appeared after his death.

1544 CE **Lodovico Ferrari** (1522–1565, Italy). Mathematician. A pupil of **Cardano**. Solved the quartic equation by reducing it to a cubic

[206] Ramus was obsessed by the *Tree of Porphyry* and applied the binary tree to so many topics that it was thereafter known as the *Tree of Ramus*. Ramus coined the term *radius* (1569) saying: "*Radius est recta a centro ad perimetrum*".

equation[207]. Ferrari was taken into Cardan's house as a servant at the age

[207] His method, summarized in modern notation, is as follows: First, the general quartic is reduced to a canonical form

$$x^4 + px^2 + qx + r = 0,$$

which is recast into:

$$(x^2 + p)^2 = px^2 - qx + (p^2 - r).$$

Using this, one obtains for *arbitrary* y:

$$(x^2 + p + y)^2 = (p + 2y)x^2 - qx + (p^2 - r + 2py + y^2).$$

Then, y is chosen such that the r.h.s. of the last equation is a square. This occurs when a *cubic in* y is satisfied, namely

$$4(p + 2y)(p^2 - r + 2py + y^2) - q^2 = 0.$$

Once this is solved and y is known in terms of (p, q, r), we are left with a quadratic equation in x.
An alternative procedure consists of expressing the original quartic

$$x^4 + ax^3 + 3x^2 + cx + d = 0 \quad (1)$$

as a product of two quadratic factors:

$$(x^2 + p_1x + q_1)(x^2 + p_2x + q_2) = 0 \quad (2)$$

To find the coefficients (p_1, q_1, p_2, q_2), one carries out the multiplication of the two quadratic factors and equates its coefficients with (1), obtaining the simultaneous equations

$$p_1 + p_2 = a; \quad p_1p_2 + q_1 + q_2 = b; \quad p_1q_2 + p_2q_1 = c; \quad q_1q_2 = d \quad (3)$$

Introducing the notation

$$m = q_1 + q_2 \quad (4)$$

we eliminate the four unknown (p_1, q_1, p_2, q_2) from the five relations (3) and (4), obtaining the eliminant

$$m^3 - bm^2 + (ac - 4d)m + (4bd - c^2 - a^2d) = 0 \quad (5)$$

Assuming m to be solvable explicitly, the equations $q_1q_2 = d$ and $q_1 + q_2 = m$ yield a quadratic equation for the roots q_1 and q_2

$$q^2 - mq + d = 0 \quad (6)$$

of 14. Cardan taught him Latin, Greek and mathematics. Ferrari became

After this, we find p_1 and p_2 from (3) via

$$p_1 = \frac{aq_1 - c}{q_1 - q_2}; \qquad p_2 = a - p_1 \tag{7}$$

Thus, the solution of a 4^{th} -order equation to a solution of a cubic equation (5) and of three quadratic equations (6) and (2).
The explicit algebraic solution of the *general quartic equation*

$$a_0x^4 + 4a_1x^3 + 6a_2x^2 + 4a_3x + a_4 = 0,$$

then hinges on the solution of the *resolvent cubic*

$$u^3 - g_2u + 2g_3 = 0,$$

where

$$g_2 = a_0a_4 - 4a_1a_3 + 3a_2^2$$
$$g_3 = a_0a_2a_4 - a_0a_3^2 + 2a_1a_2a_3 - a_1^2a_4 - a_2^3$$

Denoting the roots of the cubic by $u_1, \quad u_2, \quad u_3$ (using Cardan formula), the roots of the cubic are

$$-\frac{a_1}{a_2}+\frac{1}{a_0}\sqrt{\frac{1}{2}a_0u_1 - a_0a_2 + a_1^2}+\frac{1}{a_0}\sqrt{\frac{1}{2}a_0u_2 - a_0a_2 + a_1^2}+\frac{1}{a_0}\sqrt{\frac{1}{2}a_0u_3 - a_0u_2 + a_1^2}$$

The four possible combination of the signs of the square roots are being chosen so that their product equals $-\frac{1}{2}(a_0^2a_3 - 3a_0a_1a_2 + 2a_1^3)$.

For example, the equation $x^4 + 4x^2 - 24x - 24 = 0$ is solved by

$$x_1 = -1 + \sqrt{1 + \sqrt[3]{3}} + \sqrt{1 + \omega\sqrt[3]{3}} + \sqrt{1 + \omega^2\sqrt[3]{3}},$$
$$x_1 = -1 + \sqrt{1 + \sqrt[3]{3}} - \sqrt{1 + \omega\sqrt[3]{3}} - \sqrt{1 + \omega^2\sqrt[3]{3}},$$
$$x_1 = -1 - \sqrt{1 + \sqrt[3]{3}} + \sqrt{1 + \omega\sqrt[3]{3}} - \sqrt{1 + \omega^2\sqrt[3]{3}},$$
$$x_1 = -1 - \sqrt{1 + \sqrt[3]{3}} + \sqrt{1 + \omega\sqrt[3]{3}} + \sqrt{1 + \omega^2\sqrt[3]{3}},$$

where

$$w = -\frac{1}{2} + \frac{1}{2}i\sqrt{3}$$
$$w^2 = -\frac{1}{2} - \frac{1}{2}i\sqrt{3} = w^*.$$

Cardan's personal secretary and succeeded him (1540) as public lecturer on mathematics in Milan. On Aug 10, 1548 Ferarri defended Cardan in a debate with Nicolo Tartaglia in Milan before a large and distinguished gathering. In 1565, Ferrari became professor of mathematics at the University of Bologna. It is claimed that he died of arsenic poisoning, administered by his own sister!

1544 CE **Michael Stifel** (1486–1567, Germany). The greatest German algebraist of the 16^{th} century, was born in Esslingen and died in Jena. He was educated in the monastery of his native town, and afterwards became a Protestant minister. The study of the significance of mystic numbers in the book of Daniel drew him to mathematics. He then studied German and Italian works and published (1544) in Latin, a book entitled *Arithmetica integra*. Its three parts treat respectively rational numbers, irrational numbers, and algebra. Stifel gave a table containing the numerical values of the binomial coefficients for powers below the 18^{th}. He made early steps towards logarithmic computations, and used alphabet letters to designate unknowns.

Stifel was one of the oddest personalities in the history of mathematics. Converted by Martin Luther, he became a fanatical reformer. His erratic mind led him to indulge in number mysticism. From an analysis of Biblical writings, he prophesied the end of the world on October 3, 1533 and was forced to take refuge in a prison after ruining the lives of many believing peasants who had abandoned work and property to accompany him to heaven.

An extreme example of Stifel's mystical reasoning is his proof, by arithmography, that Pope Leo X was the "beast" mentioned in the *Book of Revelation*[208] (**13**, 18).

1545 CE **Girolamo Cardano** (1501–1576, Italy), also known as Jerome Cardan. The most celebrated mathematician of the age. Physician, astrologer and a Professor of mathematics at the universities of Bologna and Milan. Was first to appreciate complex numbers and use them in computations. In his influential treatise 'Ars Magna', he put the complete solution of a general cubic equation in terms of square roots of numbers that may be positive *or* negative. In his book he admits quite candidly that he was not the original discoverer of the solution. Cardano initiated the study of probability theory in his gambler's manual "Liber de Ludo Aleae" (Book on Games of Chance) published in 1663. He discovered the p^n law for the probability of n successes in n independent repetitions of an event with probability p.

Cardano was one of the most extraordinary characters in the history of mathematics: a physician by trade, an inveterate gambler, heretic and a caster

[208] Others throughout history had claimed the "number of the beast" (616) must be interpreted as Nero, the Pope of Rome (by Napier), Martin Luther, Kaiser Wilhelm, and finally the Nazi Führer.

of horoscopes [he was imprisoned for a time for heresy because he published a horoscope of Christ's life], and undoubtedly an eccentric talent.

His contemporary fame was very largely medical and philosophical. He studied medicine in Pavia and Padua and obtained his doctorate in 1526, but was not admitted into the College of Physicians of Milan because of his illegitimate birth. He settled down as a country doctor in Sacco (near Padua) and married. According to his own recollections, the years he spent there (1526–1532) were the happiest of his life. In 1532 he was able to return to Milan as a lecturer in mathematics; but his first published book was medical (1536). In 1517 he was appointed professor of medicine in Pavia.

The celebrity which Cardan had acquired led in 1551 to his journey to Scotland as a medical advisor of Archbishop Hamilton of St. Andrews. At this point Cardan attained the summit of his prosperity, and the rest of his life was little but a series of disasters: one of his sons was a scoundrel and the other was executed for poisoning his wife. Cardan's reputation and practice waned, and crushed by the blows he addicted himself to gambling.

He was ultimately banished from Milan on some unspecified accusation and he found it advisable to accept a professorship at Bologna (1562). There he was suddenly arrested on a charge of heresy. Through the intervention of some influential cardinals Cardan was released, but was deprived of his professorship, prohibited from teaching and publishing any further. Resigning his chair in Bologna, he moved to Rome and became a distinguished astrologer, receiving a pension as astrologer to the Papal court.

1545–1563 CE **Ambroise Paré** (1510–1590, France). Surgeon. Introduced many medical innovations in the field of surgery: use of artificial limbs and tying severed arteries to stop bleeding after amputation. Instead of treating gunshot wounds with hot oil, according to the practice of the day, he had the temerity to trust a simple bandage. In 1545 he published at Paris *La Méthode de traicter les playes*. His next book was *Anatomy* (1550) and *Cinq livres de chirurgie* (1563).

He began life as apprentice to a barber-surgeon in Paris. His earliest opportunities were in military surgery during the campaign of Francis I in Piedmont. Paré was adored by the army and greatly esteemed by successive French kings; but his innovations were opposed, as usual, by the faculty, and he had to justify it as well as he could by quotations from **Galen** and other ancients.

1547–1570 CE **Andrea Palladio** (di Pietro della Gondola, 1508–1580, Italy). Architect. One of the most influential figures in Western architecture. His palaces and villas were imitated for 400 years all over the Western world; He was the first architect to systematize the plan of a house and consistently to use the ancient Greco-Roman temple front as a portico (roofed perch supported by columns). The influence of Palladio's buildings and publications reached its climax in the architecture of the 18^{th} century, particularly in England, Ireland, the United States and Italy, creating a style known as Palladianism, which is turn spread to all quarters of the world.

Palladio was born in Padua. As a youth he was apprenticed to a sculptor in Padua and at the age of 16 enrolled in Vicenza in the guild of the bricklayers and stonemasons. In 1538, he was discovered by the humanist and scholar Gian Giorgio Trissino. The name Palladio was then added by Trissino to Andrea as an allusion to the mythological figure Pallas Athena, the Greek goddess of wisdom. Palladio then turned to architecture (the ancient literature on architectural engineering and military science) and was influenced by Alberti and Bramante. He adopted principles of Roman architecture, revolting against ornamentalism. He built large palaces, churches and numerous country villas (most of them are near Vicenza, Rome, Venice and various parts of the countryside of Northern Italy). In Venice he built the facade of the churches of *San Giorgio Maggiore* (1566) and *Ill Redentore* (1576).

1550–1700 CE Leading European poets and novelists during the periods of Late Renaissance-Reformation-Baroque:

- Torquato Tasso 1544–1595
- Miguel de Cervantes Saavedra 1547–1616
- Christopher Marlowe 1564–1593
- William Shakespeare 1564–1616
- John Donne 1572–1631
- John Milton 1608–1674

1551–1555 CE **Pierre Belon** (1517–1564, France). Author of *Histoire naturelle des éstranges poisson marins* (1551) and *L'Histoire de la nature de oysseaux* (1555), containing pioneering work in *comparative anatomy* and *embryology*. He also wrote *Les Observations de plusieurs singularitez et choses memorables* (1553) on his tour of the Eastern Mediterranean.

Belon was born near Le Mans. He studied medicine at Paris, where he took the degree of a doctor, and then became a pupil of the botanist Valerius Cordus (1515–1544) at Wittenberg, with whom he traveled in Germany. During 1546–1549 he traveled through Greece, Asia Minor, Egypt, Arabia and Israel.

Belon was assassinated at Paris one evening when coming through the Bois de Boulogne.

1551–1563 CE **George Joachim** (**von Lauchen**) **Rheticus** (1514–1567, Germany). Astronomer, mathematician and physician. Latinized his name into Rheticus since his birthplace, Feldkirch in Tyrol, was anciently the territory of Rhaetica. As a young man he studied at the Universities of Zürich, Nuremberg and Göttingen. At the age of 22, he was appointed professor of mathematics and astronomy at the young University of Wittenberg, center and glory of Protestant learning.

Being greatly attracted by the new Copernican theory, he resigned the professorship in 1539 and went to Frauenburg with a determined purpose to set in motion the Copernican Revolution which Copernicus himself tried to suppress. Soon, a most peculiar situation arose in which Rheticus was pressing for publication of 'De Revolutionibus', while Copernicus maintaining his stubborn opposition. Eventually, he prepared his master's work for publication in 1542, but afterwards lost all interest in the subject – because Copernicus failed to mention his name anywhere in the text.

Rheticus survived his master by more than 30 years, leading a restless and hectic life in Leipzig, Italy, Cracow and Cassovia in Hungary. During 1551–1563 he completed a monumental work on trigonometry which secured him an honorable place in the history of mathematics: he was first to define the trigonometric functions in a right triangle and prepare tables of these functions to seven decimal places at intervals of 10 arcseconds. These were published posthumously by his pupil **Valentinus Otho** (ca 1550–1605).

1552–1553 CE **Thomas Gresham** (1519–1579, England). The financial adviser to Queen Elizabeth I. Developed methods to raise the value of the pound sterling by operations that involved actuarial problems in probability, however rudimentary. Bequeathed his property for the foundation of the first chair of geometry in London's Gresham College in 1596, founded by him. **Henry Briggs** was the first incumbent, to be followed later by **John Wallis**, **Edmund Halley** (1656–1742) and **Christopher Wren** (1632–1723).

Established the economic law (known as: *Gresham's Law*): bad coin drives out good coin.

1553 CE *Burning of the Talmud* by **Pope Julius III** on September 9 (Jewish New-Year day) at Plazza Campo de Fiori in Rome [at the same place they burned Giordano Bruno in 1600]. On September 12, the head of the Inquisition, **Giovanni Pietro Caraffe** ordered the Talmud to be burned throughout Italy. When this man was elected **Pope Paul IV**[209] (1555), he issued a decree which, for ferocious anti-Jewishness, was not equaled until the coming of Hitler in modern Germany. During his terrible reign, Jews and their books kept burning. When he died (18^{th} Aug. 1559) Romans vented their hatred by demolishing his statue, liberating the prisoners of the Inquisition and scattering its papers.

209 Paul's want of political wisdom and ignorance of human nature were fatal to the Christian Church; he joined with France (1555) in order to drive the Spaniards out of Italy. But the victory of Philip II at St. Quentin (1557) and the threatening advance of Alva upon Rome forced him to abandon his French alliance. He denounced the peace of Augsburg as a pact of heresy; nor would he recognize the abduction of Charles V and the election of Ferdinand. By insisting upon the restitution of the confiscated church-lands, regarding England as a papal fief, requiring Elizabeth (whose legitimacy he aspersed) to submit her claims to him, he brought about the final break of England with the Church of Rome.

Science Progress Report No. 5

The "Marranos" – Or how Portugal and Spain sank back into the Middle Ages (1498–1615)

The banishment or forcible conversion of the Jews deprived Portugal of its middle class and of its most scientific traders and financiers. Though the Jews had always been compelled to reside in Ghettos, they had been protected by the earlier Portuguese kings. Before 1223 *their courts had received autonomy in civil and criminal jurisdiction; their chief rabbi was appointed by the king and entitled to use the royal arms on his seal.*

Alphonso V even permitted his Jewish subjects to live outside the Ghetto, relieved them from the obligation to wear a distinctive costume (enforced 1325), and nominated a Jew, ***Itzhak Abravanel*** *(1437–1508) as his minister of finance. In all cultural disciplines, the Portuguese Jews surpassed their rulers. Many of them were well versed in Aristotelian and Arabic philosophy, astronomy, mathematics, cartography, nautical science, and especially in medicine. Three Jewish printing presses were established between* 1487 *and* 1495*; both John II, and Emanuel I, employed Jewish physicians. It was a Jew –* ***Avraham ben Shmuel Zacuto*** *(1450–1515) – who supplied Vasco da Gama with nautical instruments; and it was another Jew,* ***Pedro Nuñes*** *(1492–1577) who invented the* nonius*, the Mercator projection and as a peak figure in Portuguese science became the cartographer Royal in* 1529*. Moreover, the Jews were employed in the overland journeys by which the Portuguese court first endeavored to obtain information on Far Eastern affairs.*

However, while protected by the kings and tolerated by the lower classes, the other orders – the ecclesiastics and nobles resented their religious exclusiveness or envied their wealth, and gradually fostered the growth of popular prejudice against them. In 1449 *the Lisbon Ghettos were stormed and sacked, and between* 1450 *and* 1481 *the Cortes four times petitioned the Crown to enforce the anti-Jewish provisions of the canon law. John II gave asylum to* 90,000 *Jewish refugees from Castile* (1492)*, in return for heavy poll-tax and on condition that they leave the country within* 8 *months, in ships furnished by himself. These ships were not provided in time, and the Jews who were thus unable to depart were enslaved, while their children were deported to the island of St. Thomas and there perished.*

In 1496 *Emanuel I desired to wed Isabella*[210]*, daughter of Ferdinand and Isabella, but found that he was first required to purify his kingdom of the*

[210] She died, anyway, one year after the wedding, leaving Emanuel with neither wife nor Jews.

Jews, who were accordingly commanded to leave Portugal before the end of October 1497*. But in order to avoid the economic dangers threatened by such an exodus, every Jew and Jewess between the ages of* 4 *and* 24 *was seized and forcibly baptized (March 19, 1497): "Christians" were not required to emigrate.*

In October 1497*,* 20,000 *adults were treated in the same way. These "New Christians" or "Marranos"*[211] *as they were called, were forbidden to leave the country between* 1497 *and* 1507*. In April* 1506*, most of the Marranos who resided in Lisbon were massacred during a riot. The rest were permitted to emigrate – an opportunity of which the majority took advantage. Large numbers settled in Holland where their scientific, commercial, and financial skills greatly assisted the Dutch in their rivalry with the Portuguese.*

After the main body of Jews had been banished from Spain and had fled from Portugal, the Inquisition was turned against converted Moors and eventually upon the Christians themselves.

The Reformation never reached Portugal, but even here the critical tendencies which elsewhere preceded Reform, were already at work. Their origin is to be sought not so much in the Revival of Learning as in the fact that the Portuguese had learned, on their voyages of discovery, to see and think for themselves. This interest in the physical world and the true scientific spirit was seen by orthodox churchmen as a threat to religious doctrines previously regarded as beyond criticism.

To check this "dangerous" trend, the Holly Office *was established in Lisbon in* 1536*, where the first* auto-da-fe *was held in* 1540*. The worst vices of the Inquisition were the widespread system of delation it encouraged by paying informers out of the property of the condemned. Quite as serious, in their effects upon national life, were the severe censorship to which all printed matter was liable before publication and the control of education by the Jesuits.*

Portuguese education centered in the national University of Coimbra*, which had long shown itself ready to assimilate new ideas. By* 1555 *The Jesuits had secured control over Coimbra – a control which lasted for two centuries and extended to the whole educational system of the country. The effects of this change upon the national character were serious and permanent. Portugal sank back into the Middle Ages. The old initiative and self-reliance of the nation, already shaken by the brain-drain of the Jews and years of disaster, were now completely undermined, and the people submitted without show of resistance to a theocracy disguised as absolute monarchy.*

In Spain, the harassment of the Marranos became unbearable as time went on. Those who possessed the means sought to escape from the country.

[211] Meaning *pigs* in Spanish.

Unfortunately, the great majority of Marranos had become desperately poor. They saw little hope of cutting themselves loose from the fatal net of their double loyalty: as Christians openly, and as Jews secretly. Many resigned themselves to the hopelessness of their lot, and went over completely to the Church.

But eventually the power of Spain started to crumble. First came the successes of the Dutch in their struggle to gain independence from Spanish rule, which was marked by the Union of Utrecht *in* 1579*. This was followed by the crushing defeat the Great Armada by the English navy* (1588) *– the prison walls of Spain and Portugal finally fell down. In* 1593 *the Iberian* Marranos *began to move to the free Protestant Netherlands. Official permission was granted to them to settle in Amsterdam in* 1615*. The Dutch never had occasion to regret their settlement in their country; from their activities prosperity and energy flowed to Amsterdam. With their help the city became one of the principal maritime centers of Europe. By and large, the* Marranos *were enterprising merchants and traders. They were fluent in many tongues and were thus ideally suited for carrying on international commerce.*

Moreover, they had world-wide connections with the dispersed communities of Spanish and Portuguese Jews and Marranos*, especially in the countries of North Africa, in Arab lands, Turkey, Greece, Persia and India. They also established important shipping branches in such strategic trade ports as Livorno, Genoa, Venice and Naples.*

Finding a ready and unrestricted field for their energies, the ex-Marranos of Amsterdam founded new factories and industries. They also did an effective banking business; at one time they controlled more than a quarter of the stock of the East India Company which played such a decisive role in the history of New York. The prosperity and the relative freedom they enjoyed in Amsterdam drew more and more Marranos *from Spain and Portugal to the city.*

The exceptional vitality and capability of the freedom-seeking Marranos exemplified itself in the story of The House of Nasi, The Duke of Naxos*. Operating from Muhammedan Turkey, the family played an important role in European politics during 1553–1569. The main characters in the drama was* ***Donna Gracia (Hanna) Mendez-Nasi*** *(ca 1510–1569) and her nephew* ***Yosef Nasi*** *(ca 1524–1579).*

The compulsory baptism in Portugal (1497) left the Mendez family [Francisco and Diego Mendez and their wives Beatrics and Reyna] residing in Lisbon. They were bankers with wide connections, with branch offices in Holland and debts due them from the kings of France. After the death of Francisco (1536) and Diego (1543) the clever and gracious Donna Beatrics took over, with the Inquisition always at her heels, she managed to move her business *through Antwerp and Venice to Turkey (1553), where, under the protection of the powerful sultans Suleimen and Selim, her business flourished to such a degree that she had her own private navy.*

In Turkey she openly returned to Judaism and resumed her original name of Donna Gracia Nasi. Her nephew, Joã Migues followed her there, married her daughter and adopted his Jewish name, Yosef Nasi. He rose high in the favor of the Sultan, who created him duke of Naxos. Yosef exerted great influence on the foreign affairs of the Turkish empire at its height and for a while was among the most powerful statesmen in Europe; he had conquered Cyprus for the Sultans and negotiated with the Emperor of Germany, Maximillian II, William of Orange and Sigismund August II, King of Poland. His career was one of the tokens of the new era that was to dawn for the Jews as trusted public officials and as members of the state.

Everywhere they went, the Marranos *brought with them to Amsterdam their superior culture. Even under the oppression of the Holy Office they had functioned in Spain and Portugal as doctors, lawyers, scholars, writers, university professors, army officers, and even as statesmen, diplomats and landed* hidalgos*. In* New Jerusalem *(as Amsterdam was called by the Jews) they picked up the threads of their past callings and many achieved great distinction in them. They helped advance both Jewish and Dutch cultures in all branches.*

Fore and foremost in eminence is the philosopher ***Baruch Spinoza*** *(1632–1677) whose grandfather came from Portugal in* 1579*. Among the physicians,* ***Avraham Zacuto II*** *(1575–1641) [*known as ***Zacutus Lusitanus****] was the foremost Jewish doctor in Holland in the days of Rembrandt and a pioneer in medical history. He came to Amsterdam from Lisbon* (1625)*, where his great grandfather was a famous astronomer. Zacuto II published important books in which he foreshadowed later medical discoveries. In France,* ***Eliyahu Montalto*** *(1550–1616) was a member of the large group of Marranos who distinguished themselves in medicine.*

Itzhak Cardoso *(1610–1685), studied medicine at Salamanca and settled (1632) in Vallabolid. Became chief physician to the Court in Madrid, but was forced to flee the Inquisition and settled in Verona, Italy. There he reverted to Judaism, taking the name Itzhak. Published a number of books on medicine, the natural sciences and philosophy. His brother* ***Michael Avraham Cardoso*** *(1627–1706), also a physician, became one of the ardent followers of Shabetai Zvi.*

Immanuel Rosales *(1593–1668) was a mathematician, astronomer and a famous physician. His family originated from the town Castallvi de Rosanes near Barcelona. Upon the expulsion of the Jews from Spain (1492), the family fled to Lisbon, where they became Marranos. Rosales studied mathematics* *and medicine at Montpelier and became a famous physician. Fleeing from the Inquisition he reached Rome (1625) and then practiced medicine in Amsterdam, where he reverted openly to Judaism. The Rosales (Rosanes) family spread from Spain to North Africa, Europe and the Near East.*

Ludovico Mercato *(1525–1611) from Vallaboild, Spain, became physician to Kings Phillip II and Phillip III.*

The Marrano family **de Castro** *issued a number of famous physicians who escaped the jaws of the Inquisition;* ***Roderigo*** *(1546–1627) was one of the foremost physicians to establish gynecology in the Renaissance period and a prolific medical writer. He received his education in Pisa and became a professor of medicine. Died in Hamburg*[212]*;* ***Itzhak*** *(1620–1687) was also a prolific medical writer. Died in Amsterdam.*

The physician ***Garcia da Orta*** *(1500–1560) fled Portugal as far as Goa and during 1534–1563, prepared singlehandedly a unique encyclopedia of medical plants. But the long hand of the Inquisition reached sentenced and burned his "heretic" body, twenty years after his death.*

The Portuguese Marrano family **Teixeira** *(also known by the name Teixeira de Mattos or Teixeira de Sampayo) became noted in Western Europe and Brazil during the past four centuries for their philanthropy as well as their financial, diplomatic and scientific achievements:*

Benito Teixeira *(ca 1545–1600) was an author and martyr. He lived in Brazil for 30 years and described his travels in books. The Inquisition arrested him in Bahia, sent him to Lisbon where he was burned at the Stake.*

Pedro Teixeira *(1570–1650) became one of the greatest explorers of the* 17^{th} *century, and reverted to Judaism in Antwerp.*

Diego Teixeira Sampayo *(Avraham Senior Teixeira, 1581–1666) became the diplomatic representative of Queen Christina (of Sweden) in Hamburg. He left Portugal (1643) and openly acknowledged Judaism (1647).*

His son ***Manuel Teixeira*** *(Itzhak Hayyim Senior Teixeira, 1625–1705) continued the diplomatic and financial career of his father and became the leader of the Spanish–Portuguese community in Amsterdam. In the* 18^{th} *century we find members of his family as educators, writers, statesmen and scientists in Brazil, Italy, Holland, England, Austria and Germany. Among them:* ***Gomes Teixeira*** *(1860–1941) was the president of Portugal (1923–1925); the poet and writer* ***Teixeira de Pascoais***[213] *(1877–1952); the mathematician* ***F.G. Teixeira*** *who extended Bürmann's theorem (1900);* ***Anisio Spinola***

[212] His son **Benedict** (1597–1684), also a physician at Hamburg, became the personal physician of Queen **Christina** of Sweden.

[213] Pseud. of Joaquim Pereira de Vasconcelos

Teixeira, *a Brazilian educator;* ***Mario Teixeira de Carvalho*** *(b. 1906), a Brazilian physician and writer.*

Among the Marranos lost to Judaism were the Spanish writer ***Fernando de Rojas*** *(1465–1541, Spain), the French philosopher* ***Michel de Montaigne***[214] *(1533–1592) and* ***Mario Soares***, *former prime minister of Portugal.*

1553–1572 CE **Moshe Isserles** ("REMA"[215], ca 1525–1572, Poland). Jewish scholar, Talmudist, codifier and philosopher. One of the great halakhic authorities and one of the founders of rabbinic learning in Poland and Germany. He was recognized as the authority not only in rabbinic law but also in philosophy, Kabbalah, astronomy and the secular sciences. Wrote ten books on halakhic, philosophical, exegetical and scientific subjects[216]. These works contain Biblical exegesis and commentaries, codification of religious laws, responsa and philosophical matters. In his philosophical system he followed Maimonides to which he can be compared in his universal outlook, manner of study, character and his attachment to both Talmudic and secular knowledge. His philosophy of Judaism is expounded in his book *Torat ha-Olah* (Prague, 1570).

Isserles was born in Cracow, Poland, a son of a wealthy and influential Talmudic scholar. He studied in Lublin and in 1553 built a synagogue in Cracow (called 'the synagogue of the Rema'), which still exists today. He died in Cracow and was buried next to his synagogue. Until WW II, thousands of Jews from all over Poland made a pilgrimage to his grave every year on the anniversary of his death.

1555 CE The Age of the Jewish *Ghetto* began officially when compulsory segregation was imposed by Pope Paul IV. By the end of the 16^{th} century the ghetto had become an accepted institution in Italy, from Rome to the Alps. All ghettos were locked at night. Jews who went outside the ghetto were required to wear a distinguished badge on their garments. They could not enter a profession except (with severe restrictions) that of medicine. To

[214] His mother Antoinette de Louppes (Lopez) was of Spanish Marrano origin.

[215] Acronym of Rabbi Moshe Isserles. His full name Israel–Eliezer was shortened to Isserles

[216] E.g. he wrote a commentary on the *Theorica of George Peurbach*.

travel out of town they required special permits. Almost everywhere they were compelled to attend conversionist sermons. The police gave adequate protection to the ghetto from concerted attacks, but only reluctantly in cases of individual molestation. There were about 30,000 Jews living in Italy in the 17^{th} and 18^{th} centuries.

When the French armies entered Italy (1796–1798), the new revolutionary spirit momentarily triumphed: the walls of the ghetto were demolished and the Jews received equal rights. However, in 1815, the restoration resulted in a complete and almost general renewal of the old conditions.

1556 CE, Jan. 23 An *earthquake* in Shansi Province, China caused the death of ca 830,000 persons. No higher death toll from a natural disaster has ever been recorded (ca 700,000 people perished on July 27, 1976 during a major earthquake in the Hopeh Province, China).

1557 CE **Robert Recorde** (ca 1510–1558, England). Welsh physician and mathematician. Lived at a time of social change, economic expansion and religious strife in England. An active participant in the turbulent life of his times, Recorde rose to a position of great trust and responsibility, becoming a physician to Edward VI and Queen Mary. A courageous man with the rare gift of loyalty and compassion, the span of his life was not without its tragedy.

He entered the University of Oxford in 1525. He went later to Cambridge to study medicine and received his M.D. there in 1545. Recorde introduced the equality sign (=) into algebra and systematized its notation. He justified his adoption of a pair of equal parallel line segments for the symbol of equality, as follows: "*bicause noe 2 thynges can be moare equalle*".

Recorde virtually established the English school of mathematics and was first to introduce algebra in England.

Recorde died in King's Bench prison, South Wark, where he was confined for debt.

1558 CE **Luigi Cornaro** (1475–1566, Italy). Dietician. Established theories on the relationship between food and health. A member of the powerful Cornaro family of Venice, he spent the first 40 years of his life indulging his passion for food and drink. After a period of serious ill health, he was threatened by his physician with death if he continued to indulge himself. He then resolved to restrict his diet drastically, eventually reducing it to a single egg a day. His *Discorsi sulla vita sobria* (1558) includes some of the first systematic accounts of diet. It enjoyed great popularity and was widely translated, largely because Cornaro himself lived to the age of 91.

1558–1565 CE **Federigo Commandino** (1509–1575, Italy). Mathematician. Rendered a new translation (1558) of the works of Archimedes into

Latin, thus stimulating subsequent interest in integration techniques. In his work *De centro gravitatis solidorum* (1565) he established a theorem (may have been known to the ancient Greeks), bearing his name and stating that: *The four medians of a tetrahedron* (lines joining vertices to centroid of opposite faces) *are concurrent in a point that quadrisects each median*. In this book he also applied integration techniques of Archimedes to the determination of *centers of gravity*.

1559 CE **Matteo Realdo Colombo** (1516–1559, Italy). Anatomist. Demonstrated pulmonary circulation, the process of blood circulating from the heart to the lungs and back. This showed that Galen's teachings were wrong[217], and was of help to William Harvey (1628).

Colombo was born at Cremona. He was a pupil of **Andreas Vesalius** and became his successor at the University of Padua. Colombo is also remembered for his 'discovery' of the *clitoris*.

1560 CE *Smallpox* epidemic sweeped Brazil; millions died.

1560–1667 CE '**Accademia Secretorum Naturae**' (Academy of the Secrets of Nature) established in Naples. It was the oldest scientific society in modern times. This organization was dissolved after several years following charges of witchcraft practice. Another society, in Rome, '**Accademia dei Lincei**', was established in 1601 and included Galileo. It was closed in 1630 due to pressure from the Church. A third society was established in Florence in 1650, named '**Accademia dei Cimento**' (Academy of the Experiments) and included Torricelli. It dispersed in 1667 when its patron, Leopold Medici was appointed cardinal. Its last member fell into the hands of the Inquisition and committed suicide.

1560–1572 CE **Rafael Bombelli** (1526–1572, Italy). Mathematician, engineer and architect. Inventor of *complex numbers*. In a work entitled 'L'Algebra', he extended Cardano's work and initiated the actual algebra of complex numbers. Improved algebraic notation, including the earliest approach to *index notation*. The first person to write down the rules for addition

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[217] His *De re anatomica* (1559) claims that blood circulates from the right chamber of the heart to the lungs and then to the left chamber. **Galen** thought that the blood passes directly between the two chambers.

Michael Servetus' anonymously published book on theology contains (1553) his view (to be demonstrated by Colombo in 1559), that blood circulates from the heart to the lungs and back. When his authorship was discovered, his unorthodox theological views resulted in Servetus being burned at the Stake (1553) in Geneva by **John Calvin**.

and multiplication of complex numbers. He showed that using his methods, correct real solutions could be obtained from the Cardan–Tartaglia formula for the solution of the cubic equation even when the formula gave an expression involving the square roots of negative numbers.

Presented the first explicit use of imaginary numbers in solving the cubic equation $x^3 = 15x + 4$. Employed continued fractions to approximate square roots. In our modern symbolism he showed that $\sqrt{a^2+b} = a + \frac{b}{2a+}\frac{b}{2a+}\cdots$. A similar result appeared in 1613 in a treatise published by **Pietro Antonio Cataldi** (1548–1626, Italy).

Rafael Bombelli was born in January 1526 in Bologna, Italy, the eldest son of Antonio Mazzioli, a wool merchant, and Diamante Scudieri, the daughter of a tailor. Sometime early in the sixteenth century, the Mazzioli family changed its surname to Bombelli, perhaps due to political difficulties surrounding the family's support of a failed coup. The young Rafael always went by the Bombelli name.

Little is known about Bombelli's early life. He was a student of the engineer-architect, Pier Francesco Clementi of Corinaldo, and it was thus under Clementi that Rafael learned this trade. Throughout most of his working life, Bombelli worked in the employ of Monsignor Alessandro Ruffini, a Roman nobleman and later bishop of Melfi. Most notably, Bombelli helped engineer the reclamation of the marshes of the Val di Chiana which began, under Ruffini's patronage, some time prior to 1549. This project was suspended for some years between 1555 and 1560, and it was during part of this hiatus (1557–1560) that Bombelli did much of the algebraic work that ultimately became his book, *Algebra*, part of which was published in 1572.

Bombelli traveled repeatedly to Rome, working as a consultant to Pope Pius IV on the proposed reclamation of the Pontine marshes. It was during one of Bombelli's Roman sojourns in the 1560s that he met and began working with Antonio Maria Pazzi on the newly found manuscript of Diophantus' *Arithmetica*.

Bombelli died in 1572. At the time of his death, only the algebraic books, Books I–III, of the *Algebra* had been published. The geometrical part, Books IV–V, were discovered in manuscript form in 1923 and were published for the first time only in 1929.

Evolution of Algebraic Notation

Some of the symbols encountered by a student of mathematics, such as $>$, $<$, $=$, $+$, $-$, $\times$, $:$, $\{\ \}$, $[\ ]$, $(\ )$, $::$, $!$, $\sqrt{\ }$, $\sqrt[3]{\ }$, ∞, Σ, π, e, i, a^x, a^n, log, sin, cos, tan, a_n, $a, b, c, d, e, \ldots, m, n, \ldots, x, y, z$, $f(x)$, $\Gamma(x)$, *are little more than* 400 *years old. Yet we know that already the ancient mathematicians of Babylonia, India and Greece, communicated via comprehensible algebraic statements. How did they do it?*

We can discern three stages in the historical evolution of algebraic symbolism:

(1) Rhetorical algebra. Prior to Diophantos of Alexandria (ca 250 CE), solutions to problems were written, without any abbreviations or symbolism, as pure prose statements.

(2) Syncopated algebra. A method by which stenographic abbreviations were adopted for some of the frequently recurring entities, relations and operations.
One of Diophantos' significant contributions to algebraic development was his syncopation of Greek algebra. In the rest of the world (with the exception of India) rhetorical algebra persisted for many hundreds of years. Specifically, in Western Europe algebra remained essentially rhetorical until the 15^{th} *century.*

(3) Symbolic algebra. Solutions to problems appear largely in a mathematical shorthand made up of symbols having little apparent connections with entities and ideas they represent. Symbolic algebra made its first appearance in Western Europe in the 16^{th} *century, but developed so slowly that it did not become widespread until about 1650.*

In his book Arithmetica [arithmos = number, techne = science], ***Diophantos*** *used the following method of syncopation: Greek letters were given numerical values:*

$$\alpha(1), \beta(2), \ldots, \iota(10), \kappa(20), \ldots, \rho(100), \sigma(200), \ldots, \omega(800)$$

[thus $13 = \iota\gamma$, $31 = \lambda\alpha$, $742 = \psi\mu\beta$*]. Unknowns were denoted by a final sigma* (ς)*, unknown square by* Δ^{Υ} *[first two letters of the Greek word dunamis (ΔΥΝΑΜΙΣ)], unknown cube by* K^{Υ} *[first two letters of the word kubos (ΚΥΒΟΣ),* 4^{th} *power by* $\Delta^{\Upsilon}\Delta$, 5^{th} *power by* ΔK^{Υ} *and* 6^{th} *power by* $K^{\Upsilon}K$*. The symol* $\wedge$ *stood for the minus sign. Thus*

$$x^3 - 8x^2 + 2x - 3$$

would appear as

$$K^{\Upsilon}\alpha\varsigma\beta \wedge \Delta^{\Upsilon}\epsilon\mathring{M}\gamma$$

and read literally as (unknown cubed 1*, unknown 2) minus (unknown squared 8, units 3). Here,* $\mathring{M}$ *is an abbreviation of the Greek word monades (MONAΔΕΣ) for 'units'.*

The Hindus had their own syncopation. Addition was indicated by juxtaposition, subtraction by placing a dot over the subtrahend, multiplication by writing bha (the first syllable of the word bhavita, "the product"), division by writing the divisor beneath the dividend, square root by writing ka (from the work karana, "irrational") before the quantity. ***Brahmagupta*** *(628) indicated the unknown by* $y\bar{a}$*, known integers by* $r\bar{u}$ *and a second unknown by* $k\bar{a}$*. In this notation*

$$5xy + \sqrt{13} - 4$$

appears as

$$y\bar{a}\ k\bar{a}\ 5\ bha\ ka\ 13\ r\bar{u}\ \dot{4}.$$

The symbolic stage may have began in 1486*, when* ***Johannes Widman*** *introduced the* $+$ *and* $-$ *signs. It was followed with the radical sign* ($\sqrt{\ }$) *in* 1525 *by* ***Christoff Rudolff*** *and the equality sign* (=) *of* ***Robert Recorde*** *in 1557.* ***Rafael Bombelli*** *(1572) denoted square root by Rq and cubic root by Rc. The compound expression* $\sqrt{7 + \sqrt{14}}$ *would have been written by him as* $R[7pR14]$*, whereas the expression* $\sqrt[3]{4 + \sqrt{-11}} + \sqrt[3]{4 - \sqrt{-11}}$ *would be cast as* $Rc[4p\ dim\ Rq\ 11]p\ Rc[4m\ dim\ Rq\ 11]$.

The following table demonstrates the notation of Bombelli:

Modern notation	*Bombelli printed*	*Bombelli wrote*
$5x$	$\overset{\overset{1}{\smile}}{5}$	$\overset{\overset{1}{\smile}}{5}$
$5x^2$	$\overset{\overset{2}{\smile}}{5}$	$\overset{\overset{2}{\smile}}{5}$
$\sqrt{4 + \sqrt{6}}$	$Rq\lfloor 4pRq6\rfloor$	$R\lvert\underline{4pR6}\rvert$
$\sqrt[3]{2 + \sqrt{0 - 121}}$	$Rc\lfloor 2pRq\lfloor 0m121\rfloor\rfloor$	$R^3\lvert\underline{2pR\lvert\underline{0m121}}\rvert\rvert$

Although authors such as ***Pacioli*** *had made limited use of notation, others such as* ***Cardano*** *had used no symbols at all. Bombelli, however, used quite* *sophisticated notation. It is worth remarking that the printed version of his book uses a slightly different notation from his manuscript, and this is not really surprising for there were problems associated with the printing of mathematical notation which to some extent limited the type of notation which could be used in print.*

***Francois Viète** (1579), the greatest French mathematician of the* 16^{th} *century, did much for the development of algebraic symbolism. He used vowels to represent unknown quantities and consonants to represent known ones, but had no symbol for equality. He would have written*

$$5BA^2 - 2CA + A^3 = 0$$

as:

$B5$ *in* A *quad* $- C$ *plano* 2 *in* $A + A$ *cub aequatur* D *solido.*

In 1586, ***Simon Stevin** introduced a notation of encircled numerals to denote mere exponents, e.g.*

$$9 \textcircled{4} -14 \textcircled{3} +6 \textcircled{1} -5$$

meant

$$9x^4 - 14x^3 + 6x - 5.$$

Through this notation he did not avoid fractional exponents, and was ignorant only of negative exponents.

***René Descartes** (1637) introduced our present custom of using the latter letters of the alphabet for unknowns and the early letters for knowns. He also introduced the present system of indices* x, x^2, x^3, *etc.* ***Thomas Harriot** (1588) gave us our present inequality signs* $>$ *and* $<$ *and* ***William Oughtred** (1631) left us the cross* ($\times$) *for multiplication, and the four dots* (::) *used in a proportion.* ***John Wallis** (1655) introduced negative and fractional exponents and the symbol* ∞ *for infinity.* ***William Jones** was first to use* π *for the ratio (perimeter/diameter) of a circle, in* 1706.

*We owe to **Euler** the symbols* $f(x)$ *for functional dependence,* Σ *for the summation sign,* $\iota = \sqrt{-1}$, e *and the adoption of* π *in* 1737. *The factorial symbol* $n!$ *was introduced in* 1808 *by **Christian Kramp** of Strasbourg.*

Table 2.9 exhibits milestones in the history of symbolic algebra. Once introduced, the symbols did not become popular overnight. On the contrary, often fifty years or more elapsed before anything resembling a unanimous adoption of the symbol was achieved.[218]

[218] Additional material on this subject can be found in:
- Cajori, F., *A History of Mathematical Notation*, Dover publications, Inc: New York, 1993, volume I (451 pp.) and volume II (365 pp.).

Table 2.9: Milestones in the History of Symbolic Algebra

Symbol	Meaning or Name	Date Introduced	Inventor
$+$	Plus	1486	Johannes Wideman
$-$	Minus	1486	Johannes Wideman
$\sqrt{\ }$	Square root	1525	Christoff Rudolff
$()$	Parentheses	1556	Nicolo Fontana (Tartaglia)
$=$	Equals	1557	Robert Recorde
.	Decimal point	1617	John Napier
$a > b$	a greater than b	1631	Thomas Harriot
$a < b$	a less than b	1631	Thomas Harriot
$\times$	Multiplication	1631	William Oughtred
$\cdot$	Multiplication	1631p	Thomas Harriot
AB	Multiplication by juxtaposition	1637	René Descartes
x, y, z	Letters near the end of the alphabet for unknown quantities	1637	René Descartes
a, b, c	Letters near the beginning of the alphabet for known quantities	1637	René Descartes
$a^1, a^2, a^3 \ldots$	positive integer powers	1637	René Descartes
$a^{-1}, a^{\frac{1}{2}}, \ldots$	negative integer or a fractional powers	1656	John Wallis
∞	Infinity	1656	John Wallis
$\div$	Division	1659	John Rahn
a^n	n any real number	1676	Isaac Newton
π	The ratio of circumference to diameter in a circle	1706	William Jones
$a \leqq b$	a less than or equal to b	1734	Pierre Bouguer
$a \geqq b$	a greater than or equal to b	1734	Pierre Bouguer
$\neq$	Not equal	1740	Leonhard Euler

p = posthumously

1565 CE **Bernardino Telesio** (1509–1588, Italy). Natural philosopher. Proposed the first system of physics to rival that of Aristotle. In it he sowed the seeds from which sprang the scientific methods of Bacon and Descartes, with their rich and manifold results. He therefore abandoned the purely intellectual sphere and proposed an inquiry into the data gathered by the senses, from which he held that all true knowledge really comes.

Telesio was born of noble parentage near Naples and was educated at Milan by his uncle, himself a scholar of eminence. His studies included a wide range of subjects: classics, science and philosophy, which constituted the curriculum of the Renaissance savants. He lectured at Naples and finally founded the Academy of Cosenza. His ideas were expounded in his work *De Rerum Natura*. The heterodox views which he maintained aroused the anger of the Church, and a short time after his death his books were placed on the Index.

1566 CE **Aldus Manutius (Manuzio) the Younger** (1547–1597, Venice). Publisher and erudite; defined the *full stop sign* as a dot at the end of the sentence, in his punctuation handbook "*Interpungendi ratio*". Here he described, for the first time, its ultimate role and aspect. He thought he was offering a manual for typographers; he could not have known that he was granting all future writers and readers, the gifts of sense and music in all the literature to come.

The need to indicate the end of a written phrase is probably as old as writing itself, but the solution, brief and wonderful, was not set down until the Italian Renaissance.

"No iron", **Isaac Babel** wrote, "can stab the heart with such force as a full stop put just at the right place".

Aldus was a member of an Italian family , famous in the history of printing as organizers of the *Aldine Press*. He became head of the Aldine press (1574). Appointed professor of literature to the Cancellaria at Venice; Occupied the chair of eloquence at the University of Bologna (1585).

1568 CE **Gerhardus Mercator** (**Gerhard Kremer, Gerard de Cremere**, 1512–1594, Netherlands). Cartographer. Published a projection for world maps ('Mercator Projection')[219] suited for navigation, with a grid of

[219] **Pedro Nuñez** employed the same projection already in 1533 and described it in his treatise *De arte atque ratione navigandi* (1546). He also discovered the *loxodromic curves* before Mercator, in 1533.

Mercator's projection – in which a spherical surface (representing the earth) is straight parallel lines representing longitudes and latitudes. It was one of the first attempts to base cartography upon solid mathematical principles. His chart facilitated both dead reckoning and the problem of rhumb-line sailing.

Mercator was born at Rupelmonde, in Flanders (now Belgium). He studied at Louvain and independently under the mathematician **Gemma Frisius** from whom he derived much of his inclination to cartography and scientific geography. He started to produce maps in 1537. In 1538 appeared Mercator's map of the world in hemispheres (north and south). In 1541 he issued the celebrated *terrestrial globe*. In 1551 a *celestial globe* followed.

Mercator was inclined toward Protestantism; in 1533 he had retired for a time from Louvain to Antwerp, partly to avoid inquiry into his religious beliefs; in 1544 he was arrested and prosecuted for heresy, but escaped serious consequences (two of the 42 arrested with him were burnt, one beheaded, two buried alive). Consequently, he was released and emigrated to Germany to become cosmographer to the Duke of Cleve (1564). He spent the rest of his life issuing regional maps of Europe, using his own projection. He did excellent service in helping to free the 16^{th} century geography from the tyranny of Ptolemy.

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projected from the sphere's center onto the circumscribing cylinder tangent to it at the equator – is a particular case of the *conical orthomorphic* group. The introduction of this type of projection is due to the fact that for navigation it is very desirable to possess charts which yield correct *local* outlines (orthomorphic) and at the same time represent by a straight line any line which cuts the meridians at a constant angle (conformal). The latter condition clearly necessitates parallel meridians, and the former – a continuous increase of scale as the equator is departed from, i.e., the scale at any point must be equal to the scale at the equator times a function of the latitude angle θ – which turns out to be $\sec\theta$ for the Mercator projection.

In early days the calculations were made by assuming that for a small increase of latitude, say $1'$, the scale was constant, then summing up the small lengths so obtained. Nowadays (assuming for simplicity that the earth is spherical) we say that an infinitesimal length of a meridian is represented in this projection as $a\, sec\theta d\theta$, and the length of the meridian in the projection between the equator and latitude θ is $\int_0^\theta a\, sec\theta d\theta = a \log_e \tan\left(\frac{\theta}{2} + \frac{\pi}{4}\right)$. The projection mapping is therefore: $x = \varphi$; $y = \log_e \tan\left(\frac{\theta}{2} + \frac{\pi}{4}\right)$. This is a conformal representation of the sphere on the plane, in which $\varphi = 0$, $\theta = 0$ corresponds to $x = 0$, $y = 0$; the equator $\theta = 0$ is mapped onto the x-axis and equally spaced parallels $\theta =$ constant correspond to lines parallel to the x-axis at ever increasing distance when θ increases, until North and South poles are mapped at infinity. Mercator's projection, although indispensable at sea, is of little value for land maps.

The early Greeks had mostly imagined the earth to be a flat disc, surrounded by an ocean and vaulted by the heavens. However, ***Pythagoras*** *(ca 540 BCE) had taught that the earth was spherical in shape and* ***Aristotle*** *(ca 354 BCE) and* ***Archimedes*** *(ca 250 BCE) tried to prove this fact.* ***Eratosthenes*** *(ca 200 BCE) derived a fairly accurate value for the circumference of our planet. The spherical shape of the earth was certainly common knowledge to the educated in ancient Greece; yet, in late Roman times, this geographical knowledge was rarely remembered. Most of the Church Fathers inferred from the Bible that the earth must be a flat disc. Accordingly it was described as a wheel having Jerusalem as its center. During the Renaissance, the old Greek writings were rediscovered, and science revived. In 1492, when Columbus discovered America,* ***Martin Behaim*** *(1459–1507, Germany) constructed the first globe since antiquity. With the increase of trade and navigation, the preparation of accurate maps became a major scientific, technological and commercial task.*

Portuguese cartographers and theoretical writers on navigation seem to have been the first to recognize the errors of plane charts, which ignored the earth's curvature and the convergence of meridians. From about 1520, *graduation in longitude began to appear, usually along the equator. Latitude graduation has been introduced somewhat earlier than this* (1500). *The earliest surviving printed sea-chart intended for use on board a ship was the map of the eastern Mediterranean, published in Venice in* 1539.

1569–1572 CE **Itzhak Luria Ashkenazi** (**ha-Ari**, 1534–1572, Israel). Philosopher and Kabbalist. Gave rise to a new and influential form of Jewish mysticism.

Luria, a man of German Jewish ancestry, was born in Jerusalem and raised and educated in Egypt. He settled in Safed, Israel, and lived the life of an ascetic. Although he wrote nothing down and merely taught, the Kabbalistic teachings by Luria's disciples and apostles made a tremendous impact not only on Jewish life throughout the communities in Europe but also on Christian thought.

Lurianic Cosmology (1572) ***and modern physics*** (1979)

Western philosophy and science, which had died with the Greco-Roman Hellenistic culture in the first several centuries CE, was reborn in the 16^{th} *and* 17^{th} *centuries. A 1400 year philosophical and scientific dark age lies between Epictetos and Marcus Aurelius on the one hand and Bacon, Descartes, Leibnitz, Copernicus, Kepler, Galileo, and Newton on the other. Something must have sparked this rebirth, but what? Did perhaps the Kabbalistic*[220] *metaphysical speculations of such Jewish and Christian scholars as* ***Avraham Ibn Latif*** *(1220–1290),* ***Raymond Lully*** *(1235–1316),* ***Pico della Mirandola*** *(1470–1533),* ***Johannes Reuchlin*** *(1455–1522), and the contributions of such Jewish scientists as* ***Avraham bar Hiyya*** *(1065–1136),* ***Levi ben Gershon*** *(1288–1344) and* ***Immanuel Bonfils*** *(fl. 1340–1377) have something to do with laying the intellectual foundations for the* 17^{th} *century rebirth of philosophy and the establishment of scientific methodology in Western Europe?*

Indeed, the burst of Christian scientific and philosophical activity did not take place in the centuries between 1100 and 1500, nor did it take place in Eastern Europe[221]*. It took place in the* 17^{th} *century, in Western Europe, in the area where Jewish Kabbalists and scientists had flourished for 400 years. There is no reason to doubt that Copernicus, Kepler, Galileo, Newton, Bacon, Descartes, Locke, Leibnitz and others were familiar both with Kabbalistic thought and the scientific writings of the Jews. In the* 17^{th} *century all these writings were available in Latin and widely distributed in the libraries and universities of Europe.*

One outstanding fact about the Scientific Revolution is that its initial (and in a sense most important) stages were carried through before *the invention of the new measuring instruments – the telescope, microscope, thermometer and accurate clock which were later to become indispensable for setting scientific answers to the questions at the forefront of science. In its initial stages, in fact, the Scientific Revolution came about rather by a* systematic change in the intellectual outlook*, in the type of questions asked, than by an increase in technical equipment. Such a revolution in methods of thoughts was stimulated and inspired by the preexisting* ideas *prevailing in Jewish occult philosophy.*

220 The word *Kabbalah* comes from the Hebrew verb *Kabeil*, meaning 'to receive' - hence 'tradition' or 'revelation'. It was the name given to Jewish mystic philosophy.

221 In Eastern Europe, Kabbalah was put in the service of alleviating the misery of the Jewish people; with its doctrine of the imminence of the Messiah, the Kabbalah held out hope for the Jews.

Throughout the centuries, this mystical undercurrent was present alongside the Torah and the Talmud. It fed on noncanonized prophecy, Zoroastrian resurrection mythology, Greek science, numerology, gnostic heresies. This was the material Jewish scholars worked on for centuries, distilling it, shaping it, blowing life into it.

Not until the 8^{th} century CE, did the first of these undercurrents of mysticism break through the surface with the publication of the Book of Formation, *compiled in southern Italy. In the thirteenth century, the second undercurrent emerged into medieval Jewish civilization with the appearance of the* Zohar *("Glow" in Hebrew), written and compiled in Spain. The* Book of Formation *is concerned mainly with the ecstatic experience of God. The* Zohar *can best be described as an encyclopedia of occultism and metaphysical speculations on God, universe, and science. These two books combined, constitute the Kabbalah, a body of mystic and occult thought, a distinctly Jewish metaphysical philosophy.*

With the appearance of the Zohar, *Kabbalism did not continue for long to course through Jewish life as a unified current, but branched out into two streams. One stream sought out the rational and the scientific and became metaphysical in its orientation. This current led to* **Spinoza** *and the rationalist school of Western philosophers and scientists, finding adherents among both Jewish and Christian scholars. The other stream had its source in Germany and coursed for centuries through Eastern Europe. It began with mysticism and degenerated into superstition as its central theme.*

Both the Zohar *and the* Book of Formation *were translated into Latin and other Western tongues, and the writings of Jewish and Christian scholars, humanists, and scientists, based on or inspired by the Kabbalah, were widely disseminated throughout the universities. This body of Kabbalistic work may even have had a large share in the sudden efflorescence of science in the seventeenth century. This was the century when Kabbalism reached the height of its influence and also saw the beginnings of its demise, perhaps because it was no longer needed after science was reborn.*

Because logic alone could not explain their doctrine of the "exalted experience of God", the Kabbalists introduced symbolic thinking and symbolic language into their speculations. They abandoned the ordinary meanings of words, gave numerical values to letters, and attributed mystical properties to both letters and numbers. This symbolic language consisted of the first ten numbers and all the letters in the Hebrew alphabet, and together they formed the Kabbalistic thirty-two avenues to wisdom. With this abstract shorthand the Kabbalists developed a fantastic metaphysical world where one element was transformed into another, where numbers stood for properties possessed by objects, and the world revolved around its own axis. These Kabbalists also *had an ear for language and a flair for style. They wrote great poetry which survived in Hebrew liturgy and literature.*

A scholar who coupled Kabbalism and science was the Spanish Jew ***Avraham ibn Latif*** *(1220–1290). He wove Kabbalism, Aristotelianism, mathematics, and natural science into a unified system. His works were translated into Latin and caught the attention of* ***Raymond Lully****, a Christian scholar, and the outstanding scientist of thirteenth-century Spain. Lully, searching for a way to break through the stranglehold which Scholasticism had upon science, used the Kabbalah and the works of ibn Latif as the basis for his book on logic, Ars Magna, which was widely used in the medieval European universities. The Muhammedans stoned him to death for preaching the gospel in North Africa.*

It was in the fifteenth and sixteenth centuries, however, that Kabbalism received its greatest dispersal in the Christian world. In the late fifteenth century, for instance, ***Pico della Mirandola****, a Renaissance humanist and philosopher, translated the Zohar into Latin. But the Christian scholar who did the most to popularize Kabbalism was, of course,* ***Johannes Reuchlin****, who, early in the sixteenth century, freely asserted that his theological philosophy was based on the Kabbalah.*

A new metaphysical philosophy was injected into Kabbalism in the sixteenth century by one of the greatest Kabbalistic scholars, ***Itzhak Luria****.*

A strange parallel exist between his Kabbalistic cosmology and the recent reformulation of the 'big bang' theory via so-called 'inflationary models' of cosmic creation and expansion.

Luria stated that all cosmic energy passed through a cosmic drama of three stages:

Stage 1 *'Tzimtzum' (= 'contraction'). God ushered in all the world's dissident elements into a tiny region, while at the same time withdrew Himself from that which he has created.*

Stage 2 *'Shevirat ha-Keilim' (= 'breaking of the vessels'). Everything that has been brought together in the first stage was expanded and shattered.*

Stage 3 *'Tikkun' (= 'restoration'). All that had been shattered is unified into a new final totality.*

In this process, the Ein-Sof (= infinity) contracted unto itself, forming a 'tehiru' (= false vacuum), while the Ein-Sof then expanded outwards, and out of it matter and light were later to emerge. Luria's cosmos is not a static one, but the world for him emanated out of a dynamic interplay of archetypical forces, unlike the cold rigidity of earlier Greek models of the universe.

According to the Lurianic cosmology, the cosmos was created ex-nihilo (out of nothing) which parallels a central idea of modern cosmology — that the universe emerged out of the vacuum state.

Present day cosmologists picture the universe beginning around 14 billion years ago in the "Big Bang". The energy densities involved in this explosion of space-time, matter and energy out of nothing were enormous. However, in the past several decades particle accelerators have allowed scientists to explore some of these energy densities in their laboratories (corresponding to the state of the universe on trillionth of a second after the Big Bang), and the theories that have emerged about the Big Bang are to some extent supported by experimental evidence and not merely upon speculation.

As the universe emerged out of the initial singularity, the causally connected space it occupied rapidly expanded until it engulfed the vast expanses explored by astronomers. During epochs close to the Big Bang, all the energy and matter of the universe must therefore have been packed into a much smaller space, and therefore the universe had a much larger energy density, pressure and temperature. The earlier we go back in time, closer to the event of creation – to 1 second after the Big Bang, to 10^{-3} *sec, to* 10^{-9} *sec, and so on – the smaller a volume of space the currently-observable universe occupies.*

The simplest of Big Bang models thus assume that at the instant of creation the universe had infinite density and temperature. The idea was that the universe emerged out of a naked space-time singularity, a kind of a knot or foam of space-time, like a black hole in reverse. This model of an explosive expansion from a point of nothingness (which had infinite density), raised more questions than it answered. In particular it is still far from clear why the various physical constants and relationships between different particles have the values and patterns they do; for example, the ratio of particles of matter to photons of light, or the relative strength of the four fundamental forces of nature – gravity, electromagnetism, weak interactions, and the strong nuclear force. If the value of some of these constants had been different by a minuscule amount, the universe would have taken a radically different course.

On the macro scale, stars and planets would not have come into being, or would have lasted a much shorter time, or been too hot or cold, etc., while on the smaller scale, the long-chained carbon-based molecules that are the building blocks of living cells could not have come about unless the physical constants which constrain the nature of chemical bonding had adopted the values they have, or unless the stars had the right temperatures and lifetimes, or unless the nuclear and electromagnetic forces would have strengths whose ratios are fine-tuned to enable carbon nuclei to form from three helium nuclei.

Some philosophers and theologians saw the possibility of invoking the hand of God acting to adjust these various values to create the particular special conditions that gave rise to the universe we know today.

This period of theorizing about the Big Bang, which began in the 60's and 70's of the 20th *century, is to some extent akin to the earlier Kabbalistic cosmology, in which God had to play an active formative role in structuring the chain of events. There arose the further problem of what existed before the Big Bang singularity, and what caused it to happen. God could again be called upon for assistance.*

***Luria** realized that if God played a formative role in the structuring of the cosmos, then the cosmos would be a direct manifestation of Him. God would not have been able to separate Himself from his creation, and therefore our created world would in fact be part of God's body.*

In a similar way present day cosmologists did not feel inwardly happy with creation theories in which some factor, outside the equations and mechanics of creation, set the critical values of the constants of nature that determined the form and contents of our universe as we know it.

*In 1980 **Allan Guth**, an American physicist, devised a theory which seems to have solved many of the technical problems inherent in the simplistic Big Bang theory.*

He considered a very early stage in the development of the universe; At around 10^{-43} *seconds after the Big Bang (the 'Planck era'), when the strength of the gravitation forces comes to equal that of other fundamental forces — quantum gravitational events dominated the emerging universe, its dense bubble of space-time being subject to quantum fluctuation in its very geometry. The universe itself could indeed be described at that early epoch as a quantum fluctuation in the vacuum. The energy that the vacuum contained was bound up in various fields and thus (highly symmetric) vacuum configuration could have been unstable (a so-called* false vacuum*) even when the universe was much older than* 10^{-43} *sec (say,* 10^{-30} *or* 10^{-20} *sec old), and its space-time manifold already quite smooth.*

Once the temperature fell sufficiently, Guth's "inflationary" scenario became possible. In inflationary models, the field energy of the initial vacuum is released *by a phase transition triggered by the cooling. The released energy influences space-time geometry via GTR, causing a* 10^{50}*-fold,* faster-than-light expansion *of the universe within a fraction of a second; part of the released energy is dissipated into heat, which becomes the matter and radiation observed in the present epoch by astronomers. The new, "true" vacuum is* stable*, but is* less symmetric *in its field-configuration than the original* false vacuum *— much as a* ferromagnetic domain *below the Curie temperature is less symmetrical, yet more stable, than a microscopically-disordered magnetic phase. Such a breaking of symmetries is called* spontaneous symmetry breaking.

We can see a parallel here between the inflation-precipitating fields and the vessels (Keilim) of the sephiroth, which were unable to hold the light energy that poured through them. The matter and light in the universe arose out of the breaking of the symmetries of the fields to which the Lurianic Kabbalah parallel would be the "breaking of the vessels", and the falling down through the worlds of the husks or shells (Kelipoth).

The inflationary model resolves various problems with the naive Big Bang models, – such as the problem of the large scale uniformity and near-flatness of the universe; the non-observation of arcane particles called magnetic monopoles; and other difficult and paradoxical aspects of the earlier theory.

As indicated above, cosmologists have been speculating about even earlier periods in the life of the universe, before the inflationary period, in which the universe was a foam of space-time emerging out of quantum fluctuations in an even earlier false vacuum state.

One speculation which has received some credence is that the universe began as a quantum fluctuation in an eleven dimensional space. This resulted in four of the dimensions expanding (these being the three dimensions of the space and one of the time), while the other seven became wrapped up into a seventh dimensional sphere of extremely small size. These seven "compact" dimensions remain hidden from our universe on the macro scale (or even currently-observable subnuclear scales) which only know the four outer space-time dimensions, though the compact dimensions do participate in the inner structure of particles of matter.

This idea is strangely paralleled in the Lurianic doctrine of the Ein-Sof contracting unto itself and forming a tehiru or vacuum while its Ein-Sof expands outwards. The Tsimtsum of the Kabbalists and folding up of seven of the eleven dimensions of space-time seem similar. Both of these cosmologies place this contraction before the formation of the false vacuum out of which the matter and light particles of the universe were later to emerge.

Nobody would claim, of course, that Luria foresaw the problems of 20^{th} *century physics, or that cosmologists and particle physicists are secretly adept in obscure areas of Kabbalah.*

It seems, though, that both disciplines were addressing the same cosmological problem, though using different set of ideas. What these parallels do reveal is the way in which the human mind formulates and pictures an event as vast and awesome as the creation of the cosmos.

The simplistic archetype of the cosmos emerging from a single source or event, in a straightforward way, does not satisfy the patterning of our minds; and both these cosmologies found ways of introducing a 'falling into matter' *which harmoniously touches some archetype within our being. So, although the formulations of these two cosmologies was separated by some 400 years, we can recognize that they addressed the same problem - that of the emanation of our cosmos out of nothing.*

In a strange way the physicists of today have come to retrace the philosophical and theosophical steps taken by Kabbalists 400 years ago.

Incidentally, the sources of the Lurianic cosmology are to be found in the works of the great Jewish philosopher ***Shlomo Ibn-Gabirol*** *(1021–1058, Spain), who described what might be construed as the "Big Bang" and the subsequent inflation in these words:*

> *"Calling unto the void and it was cleft,*
> *And unto existence and it was urged,*
> *And to the universe and it was spread out".*

1570 CE, Nov. 2 *Flood in the Netherlands*[222]: the great cities flooded, possibly as many as 400,000 drowned. Flood extended from Northern France to Northwestern Germany.

1570–1587 CE **Avraham Ortelius**, **Ortels**, **Wortels** (1527–1598, Antwerp). Scientific geographer, cartographer and publisher of maps. One of the founders of historical geography. Issued the first modern systematic atlas, *Theatrum Orbis Terrarum*[223] (Theatre of the World, 1570) which consists of 53 maps. Published 20 historical maps and a geographic dictionary (*Thesaurus Geographicus*, 1587) in which he laid the basis for critical treatment of ancient geography.

[222] This event marks the beginning of the general advance of glaciers in what is known as the "Little Ice-Age" in Europe. It was preceded by the floods of April 17, 1421 (100,000 victims) and Nov. 1, 1530 (50,000 victims) and followed by the flood of 1646 (110,000 victims). Similar events, though of lesser magnitude, occurred in the 20^{th} century: Jan. 14, 1916 (10,000 victims) and Jan. 31, 1953 (2000 victims).

[223] Errors, of course, abound, both in general conceptions and in detail. Thus, South America is very faulty in outline, but taken as a whole, this atlas, with its accompanying text was a monument of rare erudition and industry.

Ortelius came from Augsburg, Germany. He traveled extensively in Western Europe and was a friend of Gerhard Kremer (Mercator). He was appointed (1575) geographer to Philip II of Spain.

Antwerp and Amsterdam became great centers of cartographic activity, and they maintained their pre-eminence until the beginning of the 18^{th} century.

1571 CE, Oct. 7 *Battle of Lepanto* (off Greece) for the domination of the Mediterranean; combined papal and Venetial fleet under Don John of Austria defeated the Ottoman Turks under Ali Pasha. Lepanto was the end to the Turkish threat to Europe from the sea. (**Cervantes** lost his arm in this battle.) It marks the first stage of the second unsuccessful drive of Islam to dominate Western civilization.

1571–1578 CE **Azariah** (**Bonainto**) **dei Rossi** (1511–1578, Italy). Physician and scholar. One of the greatest Hebrew savants of the Renaissance. Published *Me'or Einayim* (Light of the Eyes), a series of historical essays. Using classical Greek, Latin and Jewish sources, dei Rossi's work is the first since antiquity to deal with the Hellenistic-Jewish cultural encounter and to subject the Jewish calendar to historical scrutiny. On account of his critical method and refusal to accept rabbinic legend as literal truth, the work was banned in many Jewish communities. Die Rossi described in detail the great earthquake of 1561.

1571–1588 CE **Michel-Eyquem de Montaigne** (1533–1592, France). Influential skeptic and humanist philosopher. In his main work, the "*Essais*" (1588), he used the self as a subject to study the basic features of human nature. It is "the dialogue of the mind with itself."

The Renaissance was a period of expanding horizons, and one in which there was a vast increase in knowledge of the world and its inhabitants. At the same time Europeans were recovering Latin culture and a much more complete grasp of Greek literature. Science was developing. New horizons made previous truths seem wrong or parochial. These discoveries provided Montaigne and other skeptics with a treasure chest of new facts which they used to increase our sense of relativity of all man's beliefs about himself and the world in which he lives.

The practical and self-centered world-view of the Renaissance was manifested in the autobiographical writings of **Cellini** and **Montaigne**, the historical analyses of **Machiavelli**, and **Leonardo**'s drawing of the Vitruvian man. Montaigne was the first to use the term "essay" to describe the literary form to which he had devoted himself.

Montaigne's *Essais* had great influence not only in France, but also in England, where his works were quoted by **William Shakespeare** and imitated by **Francis Bacon**. No real models existed for Montaigne's essays. His literary apprenticeship had been slight: his only early noteworthy publication had been a work of translation. Montaigne's purpose in his essays was self-knowledge: "The greatest thing in the world is to know how to be oneself." But the self one finds in his writings is not narcissistic, although he admitted: "Painting myself for others, I represent myself in a better coloring than my own natural complexion." Montaigne gives room for dialogue, addressing his thoughts to the potential reader, and combining the form of a letter with the form of a dialogue with an ideal friend. Later the French philosopher **René Descartes** (1596–1650) developed Montaigne's unsystematic thoughts into their logical conclusion in his famous "Cogito; ergo sum" (Je pense, donc je suis; I think, therefore I am).

Montaigne was born at his family estate in Château de Montaigne, near Bordeaux, in southwest France. His grandfather, Ramon Eyquem, had bought the estate of Montaigne in 1477, and thus gained the right to its name. Montaigne's father, a lawyer, had served as a soldier in Italy and adopted advanced views about education, which benefited his son. He had married (1533) Antoniette de Lopez (or Louppes) of a wealthy Spanish-Portuguese Jewish family from Toulouse. Montaigne was sent to a small cottage with a peasant family and a tutor until he was six, and while he lived there he spoke exclusively in *Latin*, the language of the educated class.

He received his early education at the Collège de Guyenne in Bordeaux, and then studied law at Bordeaux and Toulouse (1546). He was a counselor of the Court des Aides of Périgueaux, in 1557 he was appointed councilor of the Bordeaux Parliament, and from 1561 to 1563 he was at the court of Charles IX. When his friend Etienne de la Boëtie died in 1563 at thirty-two, Montaigne suffered the most severe emotional experience of his life. Thereafter he never had a close relationship.

In 1564 he married Francoise de la Chassaigne and had five daughters, but only one survived childhood. In 1570 at the age of 37 he sold his post of counselor, and in the following year retired to the Château de Montaigne. There, from 1571 to 1580 he wrote his "Essais."

The first edition of this work contained only two books. He then set out on a journey which lasted a year and a half, of which he has written in his "Journal." He went to Lorraine and Alsace, started for Switzerland, crossed Bavaria and came down to the Tyrol, saw Venice and reached Rome, the end of his journey, where he received letters of citizenship. During his absence he had been made mayor of *Bordeaux*, which office he held for four years (1581–85), his duties coming to an end when the pest broke out. Montaigne being absent from the town did not feel obliged to return to it. In 1588 he published a new edition of his "Essays," corrected and augmented by a third book. He continued to revise his work until his death.

The "Essais" reflected his wide interests and erudition. It is the result of his personal experience and very extensive reading. He writes about his disgust with the religious conflicts of his time, his belief that humans are not able to attain true certainty (*skepticism*), and even alludes to *cultural relativism*, all rather modern notions.

Montaigne considered *marriage* necessary for the raising of children, but disliked the strong feelings of *romantic love* as being detrimental to true freedom. In education, he favored concrete examples and experience over the teaching of abstract knowledge that has to be accepted uncritically.

He argued that the beliefs of different cultures should be respected, and covered in his texts a huge range of subjects, including how to converse properly, how to endure pain, how to prepare for death, how to read well, how to bring up children, and how to deal with the sexual urge. Even his cat did not escape his watchful attention: "When I play with my cat, who knows whether she isn't amusing herself with me more than I am with her?" Montaigne's voice is skeptical and sincere; "I am myself the subject of my book; it is not reasonable to expect you to waste your leisure on a matter so frivolous and empty."

Worldview VI: Michel de Montaigne

* * *

Let us permit nature to have her way. She understands her business better than we do.

* * *

Love to his soul gave eyes; he knew things are not as they seem. The dream is his real life; the world around him is the dream.

* * *

Make your educational laws strict and your criminal ones can be gentle; but if you leave youth its liberty you will have to dig dungeons for ages.

* * *

Marriage is like a cage; one sees the birds outside desperate to get in, and those inside equally desperate to get out.

* * *

My life has been full of terrible misfortunes most of which never happened.

* * *

My trade and art is to live.

* * *

No man is exempt from saying silly things; the mischief is to say them deliberately.

* * *

No wind serves him who addresses his voyage to no certain port.

* * *

Not being able to govern events, I govern myself.

* * *

Nothing fixes a thing so intensely in the memory as the wish to forget it.

* * *

Nothing is so firmly believed as that which we least know.

* * *

Lend yourself to others, but give yourself to yourself.

* * *

I prefer the company of peasants because they have not been educated sufficiently to reason incorrectly.

* * *

He who establishes his argument by noise and command shows that his reason is weak.

* * *

Rejoice in the things that are present; all else is beyond thee.

* * *

The world is all a carcass and vanity, The shadow of a shadow, a play. And in one word, just nothing.

* * *

If you don't know how to die, don't worry; Nature will tell you what to do on the spot, fully and adequately. She will do this job perfectly for you; don't bother your head about it.

* * *

It should be noted that children at play are not playing about; their games should be seen as their most serious-minded activity.

* * *

If you press me to say why I loved him, I can say no more than because he was he, and I was I.

* * *

Fashion is the science of appearances, and it inspires one with the desire to seem rather than to be.

* * *

It is not death, it is dying that alarms me.

* * *

Fame and tranquility can never be bedfellows.

* * *

A man of understanding has lost nothing, if he has himself.

* * *

The way of the world is to make laws, but follow custom.

* * *

It is good to rub and polish our brain against that of others.

* * *

I have no more made my book than my book has made me.

* * *

We can be Knowledgeable with other men's knowledge, but we cannot be wise with other men's wisdom.

* * *

There is no conversation more boring than the one where everybody agrees.

* * *

There are some defeats more triumphant than victories.

* * *

Death, they say, acquits us of all obligations.

* * *

An untempted woman cannot boast of her chastity.

* * *

When I am attacked by gloomy thoughts, nothing helps me so much as running to my books. They quickly absorb me and banish the clouds from my mind.

* * *

When I play with my cat, who knows whether she is not amusing herself with me more than I with her.

* * *

Even on the most exalted throne in the world we are only sitting on our own bottom.

* * *

The most profound joy has more of gravity than of gaiety in it.

* * *

A wise man sees as much as he ought, not as much as he can.

* * *

1572–1601 CE **Tycho Brahe** (1546–1601, Denmark). The first modern astronomical observer. His series of planetary observations made possible Kepler's study of the laws of planetary motion, the basis of our modern view of the solar system.

He constructed the most accurate instruments that were possible at his time and took great precautions in making masses of astronomical observations that reached the limits of naked-eye accuracy, being thoroughly modern in his attempts to avoid errors. Brahe did *not* believe in the Copernican heliocentric theory and clung to the geocentric theory. For, he argued, if the earth rotated and revolved, a stone dropped from a high tower would fall to the *west*, which in fact did *not* happen.

In science-fiction parlance, Tycho Brahe was something like a mad scientist. On a Danish island he built and operated the fantastic castle of Uraniborg, equipped with outside instruments, observatories, and laboratories where he also conducted astrological and alchemical studies.

1572–1604 CE New stars (actually supernova explosions) were seen in the sky and interpreted as portents of imminent destruction. When astronomers observed sunspots through the new telescopes, it was taken as evidence that the sun was decaying too. Previously, the constellations had been thought to be changeless, but when the Protestant Reformation got under way, belief in Biblical chronology strengthened.

1573 CE **Valentinus Otho** (ca 1550–1605, Germany). Mathematician. A pupil of the early table maker **Rheticus**. Rediscovered the ancient Chinese ratio $\pi = 355/113 = 3.1415929$. In 1585 **Adrian Anthoniszoon** (1527–1607, Netherlands) rediscovered it independently.

1574–1608 CE **Christoph Clavius** (1537–1612, Germany and Italy). Mathematician and astronomer. Contributed to algebraic notation (1608) and developed the proposal adopted as the Gregorian calendar reform (1582). Author of an extended commentary on Euclid's *Elements* (1574). In his book *Astrolabium* (1593), he used the identity[224]

$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

to shorten calculations by replacing products by sums ("*prosthaphaeresis*"). Clavius was born in Bamberg, Germany, and entered the Jesuit order (1555); professor at the Collegio Romano, Rome (1565–1612).

[224] Proved by **Ibn Yunus** around 975 CE in Cairo and first used by the German astronomer **Johann Werner** (1468–1522). as an aid to calculations. This was used by **Rheticus** and **Brahe** up to the invention of logarithms.

1575–1580 CE **Bernard Palissy** (1514–1589, France). Potter, ceramist and naturalist. Made important contributions to both science and art. His views are set forth in his work (1580): *Discours Admirables de la Nature des Eaux at Fonteines, tant naturelles qu'artificielles, de metaux, de sels et salines, des pierces, des terres, du feu et des emaux*. In it he covered a wide range of chemical and geological ideas. He was first to recognize that rain and melting snow were the source from which springs and rivers derived their waters. Recognized for the first time that *fossils* are remains of *living* organisms. Rediscovered the enameling of pottery (1575).

Palissy, a true son of the Renaissance, was born of humble parents in Aquitaine. He first studied drawing and modeling. He served as an apprentice to a glass-painter and later practiced the art of glass-painting. He endeavored unsuccessfully to discover the secret of Chinese porcelain. He had recourse to the alchemists and apothecaries in order to learn the nature and properties of materials (in an age where chemistry had not yet developed into a science).

While gaining from them useful information, he found in their teachings a large element of imposture and fraud. He then broke away completely from the speculative attitude of the medieval writers and based his whole treatment of the subject on the facts of nature which he had *observed*. His opponents pointed a finger of scorn at him because he knew neither Latin nor Greek, a fact which he did not attempt to deny.

After a long period of poverty and disappointment he achieved success: his pseudo-chemical knowledge gained from the alchemists, aided him in the preparation of fine glazes for the adornment of pottery.

Later, when in danger of his life because he had embraced the reformed religion, he was taken under the protection of the King and Court. He lectured to large audiences in Paris on natural history, illustrating his remarks by specimens from collections which he had made and insisting that the *direct study of nature and experiment* was the true path to knowledge.

Palissy was eventually thrown into the Bastille because he was a Calvinist, and died in one of its dungeons.

1576–1578 CE **Martin Frobisher** (1535–1594, England). Navigator and explorer. Commanded an expedition (1576) in search of the *Northwest Passage* to India and China and discovered Frobisher Bay in the Eastern Arctic. The discovery of such a route was the motive of most of the Arctic voyages undertaken at that period and for long after, but Frobisher's special merit was in being the first to give to this enterprise a national character. For fifteen years he solicited in vain the necessary means to carry his project into execution. Finally, with the help of the earl of Warwick, he was put in command of an expedition consisting of two tiny ships, the 'Gabriel' and 'Michael' of about 20 tons each with a crew of 35.

He returned to the same region in search for gold, which was not there (1577, 1578). He then became vice admiral under Drake in the West Indian expedition (1586); Commanded the *Triumph* against the Spanish Armada (1588); Vice admiral under Hawkins (1590). Died fighting a Spanish force off French Coast

Frobisher was one of the greatest seamen in the reign of Queen Elizabeth I. His three attempts to reach Asia by sailing west extended geographical knowledge. On the first voyage (1576), he rounded the Southern end of Greenland, visited Labrador, and became the first European to sail into the bay in Baffin Island.

Frobisher took back to England a rock that some persons thought to be gold ore. On his next two voyages (1577, 1578) he brought back some 1200 tons of the rock. But it proved valueless. Yet, in spite of this and his failure to find the Northwest Passage, Frobisher changed the coarse of English imperialism and world's history by making possible 300 years of British colonialism in North America.

1576–1589 CE **Thomas Digges** (ca 1546–1595, England). Mathematician and astronomer. The first Copernican to claim that space is *unbounded* and that stars were scattered through this infinite space. He became the first to popularize Copernicus' ideas to a broad audience, and wrote a book about it in English, instead of scholarly Latin (1576). Already in 1572, Digges and other astronomers had studied the supernova of that year, showing that the heavens do in fact change, contrary to tradition – a sight visible to all.

Digges synthesized the works of **Copernicus** and **Nicholas of Cusa**, proclaiming the universe to be infinite, populated with innumerable suns and worlds.

Thomas Digges was a Member of Parliament (1582, 1585) and served as mustermaster general of English forces in the Netherlands (1586–1594). His father, **Leonard Digges** (d. 1571), also a mathematician, experimented with magnifying effects from combination of lenses, and was said to have anticipated the invention of the *telescope*.

1576–1600 CE **Robert Norman**. English pioneer in accurate magnetic work. Published in 1581 his book 'The New Attractive' in which he stated the fundamental law that unlike poles attract while like poles repel. Found that the magnetic needle dips with the vertical and a freely suspended needle in a horizontal plane orients itself in the north-south direction. The downward tendency of the north-pole magnet, pivoted on a horizontal axis, had been observed by **G. Hartmann** of Nüremberg in 1544, but his observations were not published till much later.

Up to that time, only two magnetic phenomena of importance, besides that of attraction, had been observed: (1) the north seeking property of a horizontal magnet; (2) the magnetization of a piece of iron, when brought into contact with a magnet. The first known magnets were hard black stones, known as 'lodestones'. No one knows when or by whom they were discovered, but the ancient Greek knew their power to attract iron. Throughout the middle ages, many people believed that the 'lodestone' had medical powers.

1577 CE, Oct. 27–Nov. 10 An extraordinary apparition of a *comet*, known thereafter as "*The Great Comet of* 1577". Contemporary descriptions noted that it was seen through the clouds like the moon and rivaled Venus is brightness. It was first recorded on November 01, 1577 in Peru and last recorded January 26, 1578 by **Tycho Brahe** (1546–1601). The comet reached perihelion on Oct. 27, 1577 when it was 0.18 AU from the sun, well inside the orbit of Mercury. The comet's nearly *parabolic* motion around the sun was opposite to that of the earth and planets (retrograde), and it approached closest to earth, at 0.63 AU, on November 10, 1577.

The works published on this comet by the German astronomer **Michael Mästlin** (1550–1631) and Tycho Brahe form a turning point in the history of astronomy because precise observations were used to demonstrate that Aristotle's view that comets occur in the earth's atmosphere (like rainbow), were wrong[225]. Thus, belief in Aristotle's perfect celestial system was shaken by the appearance of the Great Comet.

[225] The comet was shown to be well above the moon: Tycho and his colleagues observed the comet from two different locations on the earth's surface. If the comet was below the moon and close to the earth, each observer would see it appearing against an entirely different stellar background. Since each observer noted the comet appearing against nearly the *same* background stars, the comet must have been quite distant and beyond the lunar sphere. Tycho found however that the comet was much closer to earth than the supernova of 1572, since its motion (relative to distant stars) across the sky was greater.

The placement of the comet of 1577 above the moon by Tycho Brahe and his colleagues was about one step in the eventual discarding of the Aristotelian cosmology. Since Tycho still believed in the geocentric system, he faced a serious problem: on one hand he had the earth at the center of the universe with the moon orbiting around it; on the other hand he had the sun circling the earth, with the interior planets Mercury, Venus, Mars and the Great Comet, all circling the moving sun. There was thus an imminent danger that the crystalline sphere of the comet would smash into that of the moon or that of Mars. By 1583, Tycho accepted as possible the intersection of the planetary and lunar orbits, thus casting aside the notion of solid crystalline spheres for the planetary orbs.

1577–1580 CE *The second circumnavigation of the world* accomplished by the English explorer **Francis Drake** (1540–1596). On Dec. 13, 1577 Drake sailed from Plymouth aboard the *Gololen Hind* (a 100 ton gun-boat, 33 meters long). After passing the Straits of Magellan, Drake cruised along the western coasts of South and Central America and then sailed north to what is now San Francisco Bay. He then crossed the Pacific Ocean, stopping for water at the Philippine Islands. After crossing the Indian Ocean, he sailed around the Cape of Good Hope and reached Plymouth on Sept. 26, 1580.

Drake's voyage broadened English knowledge about the world and paved the way for later explorations.

1579 CE **Francois Viète** (**Franciscus Vieta**), Seigneur de la Bigotière (1540–1603, France). French mathematician. A lawyer by profession, in Poitiers. Being a Huguenot, persecution forced him to flee his native town for several years. He spent this period (1567–1580) largely on mathematics. With the accession of Henry of Navarre to the throne of France, Viète filled in 1589 the position of Royal Privy Counselor, and remained in that post till his death. He endeared himself to the King by breaking the Spanish code, consisting of more than 500 characters, thus enabling the French to read all secret enemy dispatches. His fame, however, rests entirely upon his achievements in mathematics.

Viète introduced the algebraic notation in which letters represented unknowns. To be sure, **Regiomontanus** and **Stifel** in Germany and **Cardano** in Italy, had used letters before him, but Viète extended the idea and was first to make it an essential part of algebra. The new algebra was called by him *logistica speciosa* in contradistinction to the old *logistica numerosa*.

In 1579 Viète published his *Canon mathematicus seu ad triangula cum appendicibus* in which he systematically applied algebra to trigonometry and discovered new trigonometric identities. It gave first systematic elaboration in the Occident of the methods of computing plane and spherical triangles with the aid of the six trigonometric functions. He paid special attention also to trigonometry, developing such relations as

$$\sin\alpha = \sin(60+\alpha) - \sin(60-\alpha),$$

and

$$\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha},$$

with the aid of which he could compute the functions of angles below 30° or 45°. Letting $x = 2\cos\alpha$ he expressed $\cos n\alpha$ as a function of x for all integers $n < 11$.

An ambassador from the Netherlands once told Henry IV that France did not possesses a single geometer capable of solving a problem by a Belgian mathematician, **Adrianus Romanus**. It was the solution of the equation of the 45^{th} degree:

$$y^{45} - 45y^{43} + 945y^{41} + \cdots + 9563y^5 - 3795y^3 + 45y = C.$$

Henry IV called Viète, who saw at once that this awe-inspiring problem was simply the equation by which $C = 2\sin\phi$ was expressed in terms of $y = 2\sin\{\frac{\phi}{45}\}$. Since $45 = 3 \cdot 3 \cdot 5$, it was necessary only to divide an angle once into 5 equal parts, and then twice into 3, a division which could be effected by corresponding equations of the 5^{th} and 3^{rd} degrees. Thus Viète brilliantly discovered that the above equation has 23 roots (the remaining ones involve negative sines, which were unitelligible to him!)

Detailed investigations on the famous old problem of the section of an angle into an odd number of equal parts, led Viète to the discovery of a trigonometrical solution of Cardano's irreducible case (3 distinct real roots) of the cubic equation. To this end he applied the identity

$$\left(2\cos\frac{1}{3}\phi\right)^3 - 3\left(2\cos\frac{1}{3}\phi\right) = 2\cos\phi$$

to the solution of the equation $x^3 - 3a^2x = a^2b$ when $a > \frac{1}{2}b$, by placing[226] $x = 2a\cos\left(\frac{1}{3}\phi\right)$ and determining ϕ from $b = 2a\cos\phi$.

In 1593 he found the representation of π as an infinite irrational product:

$$\frac{2}{\pi} = \frac{\sqrt{2}}{2}\frac{\sqrt{2+\sqrt{2}}}{2}\frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2}\cdots.$$

The value of π itself was calculated by him to 9 significant figures, using the Archimedean method by taking a polygon of 393,216 sides obtained by 16 successive doubling of the original hexagon. This enabled him to improve the Archimedean bounds to $3.1415926535 < \pi < 3.1415926537$.

[226] The solutions of the equation

$$x^3 + a_1x^2 + a_2x + a_3 = 0$$

then assume the forms given in his book *De Emendatione*: $x_1 = -2\sqrt{Q}\cos\frac{\phi}{3} - \frac{a_1}{3}$; $x_2 = -2\sqrt{Q}\cos\left(\frac{\phi+2\pi}{3}\right) - \frac{a_1}{3}$; $x_3 = -2\sqrt{Q}\cos\left(\frac{\phi+4\pi}{3}\right) - \frac{a_1}{3}$ where $Q = \frac{1}{9}(a_1^2 - 3a_2)$,

$$R = \frac{1}{54}(2a_1^3 - 9a_1a_2 + 27a_3); \quad Q^3 - R^2 \geq 0; \quad \phi = \arccos\left\{\frac{R}{\sqrt{Q^3}}\right\}.$$

Table 2.10: PROMINENT EUROPEAN MATHEMATICIANS OF THE 16^{th} CENTURY

NAME	N	LIFE-SPAN	MAJOR CONTRIBUTION
Scipione del Ferro	I	1465–1526	Algebraic solution of the cubic equation
Albrecht Dürer	G	1471–1528	Descriptive Geometry (1514)
Michael Stifel	G	1486–1567	Arithmetic and Geometric series; Algebraic notation
Nicolo Fontana (Tartaglia)	I	1499–1559	Algebraic solution of the cubic equation; The idea of logarithms (1544)
Christoff Rudolff	G	1500–1545	Algebraic notation
Girolamo Cardano	I	1501–1576	The cubic equation: probability calculations
Pedro Nuñes	J	1502–1577	The 'Nonius' calculator
R.Gemma Frisius	D	1508–1555	Trigonometric surveying (1533): Triangulation; Finding longitude at sea (1534); First terrestrial and celestial globe
Robert Recorde	E	1510–1558	Algebraic notation and terminology
Gerhardus Mercator	G	1512–1594	Map projection
George Joachim (Rheticus)	G	1514–1576	Trigonometric functions as ratios of sides of a right triangle; Sine tables
Jacques Peletier	F	1517–1582	Geometry
Lodovico Ferrari	I	1522-1565	Quartic equations; Binomial coefficients
Rafael Bombelli	I	1526–1572	Algebra of complex numbers; Algebraic notation
Adrian Anthoniszoon	D	1527–1607	$\pi \sim \frac{355}{113} = 3.141592\ldots$ (1573)
Christopher Clavius	G	1537–1626	Continued fractions; Gregorian Calendar

Table 2.10: (Cont.)

Name	N	Life-Span	Major Contribution
Francois Viète	F	1540–1603	Application of algebra to trigonometry; New trigonometric identities; π as an infinite product; Algebraic symbolism and notation
Ludolph van Ceulen (1596)	G	1540–1610	$\pi = 3.14159\ 26535\ 89793\ 23846\ 26433\ 8327950$
Simon Stevin	D	1546–1620	Vector decomposition; Use of decimal fractions
Thomas Digges	E	1546–1595	Mathematical Astronomy
Antonio Cataldi	I	1548–1626	Continued fractions
Thomas Finck(e)	S	1561–1656	Trigonometry: Law of tangents
Bartholomew Pitiscus	G	1561–1613	Calendar Reform; Trigonometric tables
Galileo Galilei	I	1564–1620	Cycloid (1599); Idea of equivalence of infinite classes (future set theory)
Paul H. Guldin	SW	1577–1643	Center of gravity

N = Nationality

(I = Italian; G = German; D = Dutch; E = English; F = French;

J = Jewish; S = Swiss; SW = Swedish)

1580 CE **Gaspare Tagliacozzi** (1546–1599, Italy). Surgeon. Pioneered in *plastic surgery*. He was the first to repair noses lost in duels or through syphilis. He also repaired ears. His method involved taking flaps of skin from the arm and grafting them into place.

ca 1580 CE **Itzhak ben Avraham ha-Rofeh of Troky** (1533–1594, Lithuania). Karaite philosopher and physician; Wrote *Hizuk Emunah* (Fortification of Faith) – a polemic in defense of Judaism against the repeated attempts of the Christian Church to establish its faith via a slanted and tendentious interpretation of the Old Testament. The author refutes the linkage of the Christian Biblical references and aims a thorough discussion at vulnerable points of the Christian tradition by exposing the internal contradictions and inconsistencies of the books of the New Testament and misinterpretation of texts of the Old Treatment by Paul and the other authors of the Evangelions. The book was translated into many languages and was highly praised by anticlerical authors of the 18^{th} century, such as **Voltaire**.

1580–1600 CE **Yehudah Liwa ben Bezaleel**, MAHARAL[227] OF PRAGUE (1512–1609, Posen and Prague). Humanist, scholar, Talmudist and philosopher. The greatest Jewish savant of the Renaissance era. A philosopher of history[228] whose original ideas on the evolution of national cultures stand out in their uniqueness on the background of the teachings of his contemporaries **Machiavelli** (1469–1527), **Calvin** (1509–1564) and **Grotius** (1583–1645). Foreshadowed the systems of **Vico** (1668–1743) and **Hegel** (1770–1831).

The Maharal lived in a time of great changes in European history, at the cross-roads of the Middle Ages and the Modern Era. The violent collision of Judaism and Christianity in the 13^{th} and 14^{th} centuries with all its tragical consequences[229] on one hand, and the collapse of the Medieval World (as a result of the Copernican revolution, geographic explorations, Gutenberg's invention and the wars of the Reformation) on the other – place him forcibly at the center of all ideological currents of his age.

Equipped with the scholastic tools of a Talmudic erudite, a deep knowledge in the history of Judaism and Christianity, and the keen understanding of the philosophical and sociological problems of the age of humanism, the Maharal

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227 Hebrew acronym: MORENU HARAV LIWA.

228 B. Gross, "*L'Eternite d'Israel*" *du Maharal de Prague*, Editions Klincksieck, Paris, 1969.

229 Burning of the Talmud (1242); expulsions from England (1290), France (1306), Germany (1336), Spain (1492); Black Death (1348); Disputations (1263, 1240, 1413–14); Inquisition etc.

laid out a new outlook of the significance of the history of Israel and its fate on the *universal level*, and addressed the confrontation of Judaism with the humanistic trends in Europe.

In his trilogy *Gevurot Hashem* (1582), *Tifeeret Israel* (1599) and *Netzah Israel* (1600), the Maharal rendered, for the first time, a daring synthesis that was aimed to bridge the gap between the dualistic *inevitable opposition* and the *necessary tie* between Israel and the rest of the world. Reading correctly the internal dynamics of Jewish history and its overall message and mission in the annals of Western civilization, he proposed a wide Jewish *educational reform* aimed at the peaceful coexistence of all different cultures, each keeping its own uniqueness, and all striving together toward a universal messianic revelation.

Being deeply versed in the new scientific trends of his age, he was entirely in favor of *scientific research* in so far as the latter did not contradict divine revelation. His message, however, fell on deaf ears; during the 350 years that followed Jews were busy seeking other solutions: prior to the *emancipation* of Jews (after the French Revolution), the horizon of most Jews did not venture beyond the world of the Talmud. Their interaction with their environment were limited on account of their being locked inside the ghetto and their special vernacular. The constant humiliation and brutality to which they were subjected, encouraged conservative trends and led to derisive indifference regarding the cultural achievements of the outside world. All their spiritual energy was introverted. When the emancipation finally arrived, Jews were blinded by the European culture, and their eagerness to conform led them on the sure road to *assimilation*, and the voice of the Maharal was lost in the void.

The Maharal was born in *Posen*, Poland, whither his family had fled from persecution in *Worms*, Germany, toward the end of the 15^{th} century. Following his rabbinical studies in his home town and his marriage (1544), he was appointed rabbi at Nikolsburg, Moravia (1553–1573). He then moved to Prague, where he established a private academy (1573–1584). On account of his unconventional ideas his bid for the chief rabbinate of Prague was rejected and he returned to Nikolsburg. His repeated effort (1592) to gain the rabbinate of Prague met again with failure and he took the rabbinate of Posen (1592–1598). Finally, at the age of 86, he became the chief rabbi of Prague, and there his life-work ended.

The Maharal was greatly influenced by the teachings of **Yehudah Halevi** in the latter's book *Ha'Kuzari* (1139). Both rejected the application of the Aristotelian rationalistic system in matters that concern religion, as taught by **Maimonides**. According to Maharal, the laws of nature were created by God *arbitrarily* (in "reply" to **Einstein**'s famous question – God indeed *had* a choice!) and all deviations from the apparent laws of physics (miracles) have their origin in another universe, which he called the "upper universe"[230].

1581 CE *The Dutch Republic was begun.* The history of The Netherlands began with Caesar's conquest (58 BCE) of much of the Low Countries, including what is now *The Netherlands*[231].

During ca 400–800 CE, the Franks controlled the region. In 870 CE The Netherlands became part of the East Frankish Kingdom (now Germany). During the 1300's – 1400's the French dukes of Burgundy united most of the Low Countries. In 1516 Duke Charles of Burgundy, ruler of the Low Countries, also became the King of Spain.

In 1648, Spain recognized Dutch independence. France controlled The Netherlands during 1795–1813. In 1815 The Netherlands became an independent kingdom united with Belgium (Belgium was an independent country during 1598 – 1621), but in 1830 Belgium declared its independence from The Netherlands.

1581 CE, Mar. 30 The fruitful epoch of Jews in medicine, which had lasted uninterruptedly for some 800 years, came ingloriously to an end with the severe repression that was initiated by the Church in Rome during the Counter-Reformation.

Contra Medicos Hebreos, a Bull issued by Pope Gregory XIII, which forbade Christians to receive medical treatments from Jewish doctors, on the ground that this involved danger to the patient's *soul*. This, however, did not stop the Christians from calling on Jewish medical experts. In the late Middle Ages the popes themselves used Jewish physicians. The very fact that

230 Paradoxically, while the Maharal became a legendary figure already in his lifetime, and Jewish folk-legend ascribed to him the fashioning of the *Golem* in the attic of the Gothic-styled Altneuschul – his philosophy remained totally unknown and was, in fact, ignored by philosophers and historians alike until the second half of the 20^{th} century. The actuality of his teachings became very acute after the genocide of the Jews in WWII and the consequential establishment of the State of Israel.

231 The Netherlands is often called *Holland*, though this name actually refers to only one part of the country. The people of The Netherlands call themselves *Hollanders* or *Netherlanders*, but in English speaking countries they are known as *Dutch*. "God created the world, but the Dutch created Holland", according to an old Dutch saying. More than $\frac{3}{5}$ of the country's land was covered by the sea, or by lakes, or by swamps. The Dutch "crated" this land by pumping out the water.

the Jewish physicians were so skillful and popular made their Christian competitors the more anti-Jewish. Indeed, there was no group, with the possible exceptions of monks, throughout the history of the Jews in Europe, more bitterly anti-Jewish than the Christian members of the medical profession. Eventually they succeeded in reducing the number of Jewish physician to insignificance. Nevertheless, when a Jewish physician was available, he always found Christian patients, especially among the higher nobility and the upper ranks of the clergy.

Challenging the Pope's Bull, the Jewish physician **David de Pomis**[232] (1525–1595) from Spoleta, bravely defended the integrity of the Jewish doctors in his publication *De Medico Hebraeo enarratio apologetica* (1581).

Jews and Medicine

"... therefore choose life..."

(Deutronomy *30*, 19)

* * *

"Two of the greatest hygienic thoughts of mankind owe their origins to the Hebrews... the weekly day of rest and the direct prophylaxis of disease. Had Judaism given nothing more to mankind than the establishment of a weekly day of rest, we should still be forced to proclaim her one of the greatest benefactors of humanity.
It is most interesting fact that, despite its theory of natural causation, Greek medicine was blind to the fact of contagion, or direct transmission of disease. But in the Old Testament we have a methodical inspection of a leper by the priest who, according to the diagnosis, isolated the patient temporarily or permanently, and admitted him again to free intercourse only after indubitable convalescence or cure."

Karl Sudhoff

[232] One of the greatest physicians in Italy at that time. Descended from the families exiled by the Roman Emperor Titus from Jerusalem to Rome ('de Pomis' = from the apples). Received his M.D. in 1552 and settled in Venice.

The science of healing has an intensely revered tradition among Jews, with an unbroken continuity of more than 2000 years. To be a doctor was generally considered as the most exalted moral and worldly calling a Jew could aspire to. The doctor could claim great humanitarian and social usefulness – all the more important to a Jew because of the religious belief that the alleviation of human suffering is one of the most meritorious pursuits of righteousness. In any case, the medical profession always had enormous appeal for Jews. Centuries of social idealization have fixed for them its tradition and pattern to this day. They have been producing more doctors in proportion to their number than any other ethnic or cultural group. For a thousand years Jews were among the most honored healers in Europe.

Throughout the ages the Jew has shown a remarkable predilection for the healthy art and in all epochs his skill has been recognized. Indeed there are significant points of encounter between the history of the Jewish people and the history of medicine. Jewish physicians always had special interest in medical ethics and the Hebrew language played an important role in the history of medical writing.

The Hebrews' medical concepts and practices were influenced by the surrounding nations, whose knowledge was highly developed. However, the uniqueness of Biblical "medicine" lies in its prophylactic nature, regulations of hygiene and the weekly day of rest.

Talmudic writings emphasize the sanctity of human life, the importance of health and the saving of life (Piku'ah Nefesh). Patients, are required, not merely permitted, to seek medical help and all restrictions should be set aside.

The Karaites, opponents of Rabbinical Judaism, held that "God alone should be sought as a physician and no human medicine should be resorted to". But the Talmud does not regard calling upon a physician for medical aid as a failure to rely upon God: "Whoever is in pain, let him go to a physician", (Baba Kamma 46b). A physician had to receive adequate fees, since "a physician who takes nothing is worth nothing" (Baba Kamma 85a). At the same time, Jewish physicians had to show special consideration for the poor and the needy - a tradition that was maintained throughout the centuries.

Beginning with the 9^{th} century they helped found the finest medical colleges. Those in Tarentum, Palermo, Salerno and Bari; a little later, those in Rome and Montpellier, furnished Europe for centuries with its best doctors:

***Zedakiah** (fl. ca 1000 CE) was court physician to the Carolingian Kings, Louis 'the Meek' and Charles 'the Bold'*

*The three great figures in 10^{th} century medicine were **Haroun of Cordova**, **Yehudah Hayyuj of Fez** and **Amram of Toledo**.*

The study of medicine was introduced as part of the regular curriculum in rabbinic academies about the year 1000 CE. A long and distinguished line of rabbi-physicians were graduated from them and served caliphs, emperors, popes, kings, bishops and princes.

In the Middle Ages, half of the best known Jewish scientists, scholars, philosophers and litterateurs - men like ***Maimonides***, ***Yehudah ha-Levi***, ***Immanuel of Rome****, and so on - were physicians by profession; a striking illustration of the respect in which that calling was generally held. The bibliographer* ***Moritz Steinschneider***[233] *(1816–1907), who devoted much of his life to a study of Jewish contributions to medieval science and mathematics was able to enumerate no less than 2168 Jewish physicians who were of sufficient eminence to be recorded and who flourished between the Dark Ages and the* 18^{th} *century.*

The contribution of Jewish doctors in the Middle Ages lay mainly in their work as translators. They constituted an important link in the transmission of Arab medicine to Europe, thus enhancing the emergence of modern science. This period is also marked by a revival of Hebrew as the language of scientific writing. Hebrew medical works were important in preserving and spreading knowledge until the beginning of the 16^{th} *century.*

Knowledge of Hebrew was considered extremely important in the study of medicine. In 1518 the rector of the University of Leipzig, ***Mosellanus****, said in his inauguration speech: "In the libraries of the Jews a treasure of medical science lies hidden, a treasure as scarce as is to be found in any other language. Nobody ... will be able to get access to this treasure without intimate knowledge of Hebrew grammar".*

One of the outstanding features of this period was the constant emphasis on ethical and social behavior. Medical aphorisms concerning ethical conduct were composed by every major medical writer from ***Asaf*** *to* ***Maimonides****. An interesting glimpse into this world is given in the ethical will of* ***Yehudah ibn Tibbon*** *(ca 1120–1190), an eminent physician and translator, to his son:*

"My son, let thy countenance shine upon the sins of man: visit the sick, and let thy tongue be a cure to them; if thou receive payment from the rich, attend gratuitously on the poor".

The most important aspect of the physician's activity, however, was the care of the sick. In the Middle Ages and until the late 18^{th} *century, sick people were usually treated at home. And the commandment of visiting the sick and of strengthening family ties was kept religiously. This explains why the necessity for a hospital was not felt in Jewish communities. The Hekdesh*

[233] The father of modern Jewish bibliography. Melvina, the daughter of his cousin Sigmund, married the philosopher **Edmund Husserl** (1859–1938).

was a hospice of sorts and served as a shelter for wandering peddlers and for poor sick Jews. It was administered by the Hevra Kaddisha, a brotherhood that cared for the sick and the dead. Servants had a right to special status as a kind of social insurance.

As merchants and travelers, the Jews met the best scholars of their time and became acquainted with drugs, plants and remedies from many parts of the world. The large number of Jewish physicians during the Middle Ages may be explained by the fact that scholars could turn to the practice of medicine to earn their living, and indeed many Jewish physicians were also rabbis, authors or poets. As men of wide general knowledge, they attained high positions in the countries in which they lived. In spite of renewed ecclesiastical opposition, bishops and popes, kings and sultans - all summoned Jewish physicians to their courts.

With the possible exception of ***Itzhak Israeli****, the most renowned Jewish physician in the Middle Ages, who was also a distinguished rabbi and philosopher, was* ***Moshe Maimonides*** *(the RAMBAM 1137–1204). Born in Cordova, he fled to Fez and later moved to Israel and then to Cairo, where he became court physician of Saladin and his sons. Maimonides had a prodigious literary output, including extensive writing on medical matters. These were written at the end of his life, after his monumental Halakhic and philosophical works.*

Maimonides strongly believed in prophylactic medicine. He wrote:

"Among a thousand persons only one dies a natural death: the rest succumb early in life owing to ignorant or irregular behavior".

Guide To The Perplexed

"Medicine teaches man to restrict his boundless lust which undermines his health and to choose the manner of living. It helps to maintain the fitness of the body and enables him to purify and raise his strength to an uplifted ethical plane."

Mishne Tora

The ***ibn Alfakhar*** *family of Christian Spain (12^{th}, 13^{th} and 14^{th} centuries) is known for its distinguished scholars, diplomats and physicians.* ***Yosef ibn Alfakhar*** *was court physician to Alfonso VIII and his son* ***Yehudah ibn Alfakhar*** *served as physician to Ferdinand III of Castile during the first part of the 13^{th} century.*

*The **ibn Shoshan** family was very influential in Toledo during the* 12^{th} *and* 13^{th} *centuries. The family included physicians, scholars, Kabbalists, poets, grammarians, philosophers, diplomats and rabbis. Through their political work the family became wealthy and powerful.*

The 16^{th} *century was a period of immense exploration, discovery and progress - the century of **Paracelsus**, **Servetus** and **Fallopius**. In 1543 **Andreas Vesalius**, the 29 year old Flemish professor of anatomy in Padua, published De Humani Corporis Fabrica (The fabric of the human body). His great work showed for the first time how nerves penetrated muscles, the relationship between the abdominal organs and the structure of the brain. Vesalius gave Hebrew names together with Greek and Latin equivalents for the anatomical structures.*

The beginning of the medical renaissance had tragic consequences for the Jews of Europe. At the Church Council of Basel (1437–1447) a catalogue of restrictions was drawn, including a decree prohibiting Jews to receive a university degree. Those Jews that were admitted to universities, were subject to special rules, charged special fees and forced to listen to conversional lectures. After graduation they were generally forbidden to treat Christian patients.

At the end of the 15^{th} *century, Spain and Portugal compelled the Jews to embrace Christianity. Many became Marranos, and other emigrated. As a result, during the* 16^{th}, 17^{th} *and* 18^{th} *centuries a very high proportion of noted physicians, scientists and scholars in Europe were of Spanish and Portuguese origin. The Marranos continued to practice medicine in Spain and Portugal until the* 18^{th} *century despite their precarious position and the continual persecution by the Inquisition.*

*Ranking high on the seemingly endless list of eminent Jewish doctors and medical scientists who flourished in Europe as the modern era approached were **Amatus Lusitanus** (1511–1568), **Eliyahu Montalto** (1550–1616) and **Roderigo Lopes** (1525–1594). Lopes was physician to Queen Elizabeth I of England since 1586 and one of about 80 Marranos to live in London during her reign. He was executed on a false charge of attempting to poison the Queen. His fate caused the Marrano community to dwindle away.*

*A particularly horrifying example is that of **Garcia da Orta**, a physician who had left Portugal for Goa, India (then a Portuguese colony) in 1534. There he completed a pioneer scientific work on Oriental medicinal plants, Coloquios dos simples e drogas e cousas medicinaes da India (1563). Da Orta died in 1568 but his work had already attracted great attention. But da Orta was a Marrano and various members of his family, among them his sister, were tried by the Inquisition and sentenced to be burned at the stake. In 1580, following da Orta's posthumous conviction on the charge of having lived as a Jew, his body was exhumed and burned.*

In spite of the limitations and continuous persecution, many Jewish physicians who distinguished themselves in medical practice and literature were also concerned with the moral responsibility which their profession required.

In Renaissance Italy there was evidence of a more liberal spirit. Some Italian universities, mainly those of Padua and Perugia were among the few that allowed Jews to enter the medical faculties. Relative tolerance was only one aspect of their life in that period. In the extensive anti-Semitic literature published at the time, physicians were not spared. Successful Jewish physicians aroused the jealousy of their Christian counterparts, and thus became subject to bitter attacks and innumerable calumnies. It is not surprising therefore, that Jewish physicians published several books containing scholarly defenses. Among them were ***David de Pomis****,* ***Benedict (Baruch) de Castro (1597–1684)*** *and* ***Itzhak Cardoso*** *(1604–1681).*

Cardoso, at the age of 29, after studying medicine and philosophy, became court physician to Philip IV in Madrid, and published several medical works. Persecuted by the Inquisition, he fled to Italy and worked as a physician of the poor Sephardic community. There he published his apologia, Las Excelencias Y Caluminias De Los Hebreos. In ten chapters he emphasized the distinguishing features of the Jews, their selection by God, their separation from all other peoples by special laws, and their compassion for the suffering of others; on ten other chapters he refuted the calumnies brought against them.

The famous Jewish physician in the 17^{th} *century were:*

- ***Benjamin Mussafia*** *(1606–1675), an ex-marrano who practiced in Hamburg (1634), Denmark (1635–1647; private physician of Christian, King of Denmark) and Amsterdam (from 1648).*
- ***Tuvia ha-Rofeh*** *(1652–1729). Physician and Talmudic scholar. Born in Metz, Germany and died in Jerusalem. Descended from* ***Ezra the Scribe*** *through the* ***Maniscriba*** *family. His family flew Poland following the 1648 persecutions of the Jews. Studied medicine in Padua, and became a chief physician to the Turkish Sultan.*

The 18^{th} *century marked the beginning of the Haskalah (Enlightenment), whose aspirations matched those of educated gentiles, for the moral and social betterment of Jews and the abolishment of all social and legal discrimination. This period marked the beginning of the movement of Jews into German universities and the increase in the number of Jewish physicians.*

Marcus Herz *represents the change which took place in the status of Jewish physicians in the Modern Era. Born in Berlin, he studied in Halle* *and was a favorite pupil of Kant. After graduation, Herz lectured on Kantian philosophy and physics. At the same time he worked at the Jewish Hospital in Berlin and was reputed to be one of the best doctors of his time. As his illustrious predecessors, Herz was concerned with the ethical aspects of his profession and in 1783 published "The Physicians' Prayer". It was similar to those which had come before, but Herz added a section about the thirst for knowledge, which reflected the spirit of his times:*

"May I be moderate in everything except in the pursuit of the knowledge of science. Grant me the strength and opportunity always to correct what I have learned... for knowledge is boundless".

The first half of the 19^{th} *century was characterized by a progressively increasing interest in the natural sciences. This period coincided with the Emancipation and the opening of universities, hospitals and scientific research institutes to Central European Jews. Jewish physicians were thus able to take an active part in the development, and there is scarcely an area in the broad domain of medicine in which they were not prominent as pioneers.*

Nevertheless, for many years Jewish physicians were not accepted as university professors unless they converted. They were not welcomed in the fields of surgery and internal medicine. Consequently, they worked primarily in private clinics and hospitals, where they did their research. They developed new fields of medicine that did not attract their non-Jewish colleagues, such as dermatology, venereology, hematology and psychiatry. Freud who was not only a physician but a gifted writer, formulated the reason for this trend with great perceptiveness:

"Soon there was the insight that I had only my Jewish nature to thank for two of the qualities that had become indispensable of my difficult path through life. Because I was a Jew, I found myself free from my prejudices that limit others in the use of their intellect; as a Jew I was prepared to go into opposition and forgo the acceptance of the 'compact majority'".

Emancipation brought with it many changes, among them the pattern of sick care within the community. With the improvement of the general hospital, the Jewish hospital too changed its character from a social institution to a medical one. The acceleration in the creation of modern Jewish hospitals continued in the 20^{th} *century, and by 1933 there were in Poland alone 48 such institutions. Some of them continued their activity in the ghettos even after the German occupation.*

The racial laws introduced into Germany at the beginning of the 1930s and, subsequently, into all of occupied Europe, severed the careers of Jewish physicians. Many emigrated, mainly to the US, where a center of Jewish medical activity was emerging, coinciding with the rise of American medicine.

*The motto taken from Deuteronomy, "*Therefore choose life...*" establishes the basis of the Jewish attitude towards medicine,– yet there has never been a "Jewish Medicine" per se. Jews have always adopted the medical teaching of cultures in which they lived, and enhanced them through their own contribution.*

History of Biology and Medicine, II – The Middle Ages and the Renaissance

The history of biology traces man's understanding of the living world from the earliest recorded history to modern times. Though the concept of biology as a single coherent field of knowledge only arose in the 19th century, the biological sciences emerged from traditions of medicine and natural history reaching back to the ancient Greeks (particularly ***Galen*** *and* ***Aristotle****, respectively).*

Whereas evolutionary ideas more or less died out in Europe after the fall of the Roman Empire, they continued to be propounded in the Islamic world. For example ***Al-Jahiz*** *considered the effects of the environment on the likelihood of an animal to survive, and* ***Ibn al-Haitham*** *went even further, writing a book in which he argued explicitly for evolutionism (although not, of course, natural selection), and numerous other Islamic scholars and scientists (including* ***Nasir al-Din Tusi****) discussed these ideas. Translated into Latin, these works began to appear in the West after the Renaissance and probably had a large (though subterranean) impact on Western science.*

The decline of the Roman Empire led to the disappearance or destruction of much knowledge. However, some people who dealt with medical issues still studied plants and animals as well. In Byzantium and the Islamic world, natural philosophy was kept alive. Many of the Greek works were translated into Arabic and many of the works of Aristotle were preserved. Of the Arab biologists, al-Jahiz, who died about 868, is particularly noteworthy. He wrote Kitab *al Hayawan (Book of animals). In the 1200's the German scholar named* ***Albertus Magnus*** *wrote De vegetabilibus, seven books, and De animalibus, 26 books. He was particularly interested in plant propagation and reproduction and discussed in some detail the sexuality of plants and animals.*

Persia and other Islamic areas became important in the development of science. Based on Greeks and Indian science and connected to Europe they were in a good position to help science develop. There were also Arab and Turkish scientists but the most important ones were Persians. ***Avicenna*** *(commemorated in the genus Avicenna) recorded many findings. He is sometimes regarded among the fathers of modern medicine.*

Interestingly, as many visual artists were interested in the bodies of animals and humans, they studied the physiology in detail. Such comparisons as that between a horse leg and a human leg were made. ***Otto Brunfels****,* ***Hieronymus Bock*** *and* ***Leonhard Fuchs*** *were three men who wrote books about wild plants; they have been referred to as the fathers of German botany. Books about animals were also made, such as those by* ***Conrad Gesner****, illustrated by, among others,* ***Albrecht Dürer****.*

Medieval medicine was an evolving mixture of the scientific and the spiritual. In the early Middle Ages, following the fall of the Roman Empire, standard medical knowledge was based chiefly upon surviving Greek and Roman texts, preserved in monasteries and elsewhere. Ideas about the origin and cure of disease were not, however, purely secular, but were also based on a spiritual world view, in which factors such as destiny, sin, and astral influences played as great a part as any physical cause.

In this era, there was no clear tradition of scientific medicine, and accurate observations went hand-in-hand with spiritual beliefs as part of the practice of medicine.

This idea of personalized medicine was challenged in Europe by the rise of experimental investigation, principally in dissection, examining bodies in a manner alien to other cultures. The work of ***Andreas Vesalius*** *and* ***William Harvey*** *challenged accepted folklore with scientific evidence. Understanding and diagnosis improved but with little direct benefit to health. Few effective drugs existed, beyond opium and quinine, folklore cures and almost or actually poisonous metal-based compounds were popular, if useless, treatments.*

Table 2.11 lists the most influential contributors to the Life-Sciences and Medicine throughout the Middle Ages and the European Renaissance.

Table 2.11: Leading Biologists and Men of Medicine
(550 CE–1550 CE)

Key:

B = Biology A = Anatomy M = Medicine
BO = Botany P = Physiology S = Surgery
ZO = Zoology EB = Evolutionary Biology

Name	*fl.*	*Specialization*
Asaf ha-Rofeh	*550 CE*	*(M)*
Alexander of Tralles	*550–580*	*(M)*
Paul of Aegina	*660-690*	*(M)*
Al-Jahiz	*830–860*	*(M), (EB), (ZO)*
Itzhak Israeli	*900–930*	*(M)*
Al-Razi	*900–925*	*(M)*
Shabbethai Donnolo	*940–980*	*(M)*
Albucasis (El-Zahrawi)	*960–990*	*(M), (S)*
Alhazen (Ibn al-Haitham)	*1000–1030*	*(M), (P), (EB)*
Avicenna (Ibn Sina)	*1010–1040*	*(M)*
Ibn Zuhr	*1120–1160*	*(M)*
Ibn Rushd	*1160–1195*	*(M)*
Maimonides	*1160–1200*	*(M)*
Ibn al-Baitar	*1220–1240*	*(BO)*
Albertus Magnus	*1240–1280*	*(ZO), (BO)*
Nasir al-Din al-Tusi	*1250–1270*	*(M)*
Ibn-Nafis	*1250–1285*	*(M)*
Mondino dei Liucci	*1300-1326*	*(A)*
Guy de Chauliac	*1330–1368*	*(M), (S)*
Leonardo da Vinci	*1482–1519*	*(A), (P)*
Alesandro Achillini	*1495–1512*	*(A), (S)*
Shmuel Zarfati	*1500–1519*	*(M)*
Albrecht Dürer	*1510–1528*	*(A)*
Otto Brunfels	*1510–1530*	*(BO)*
Girolano Fracastoro	*1517–1546*	*(M), (EB)*
Paracelsus	*1531–1541*	*(M)*
Hieronymus Bock	*1530–1554*	*(BO)*

Table 2.11: (Cont.)

Name	*fl.*	*Specialization*
Garcia da Orta	*1534–1568*	*(M), (BO)*
Amatus Lusitanus	*1536–1561*	*(BO), (P)*
Andreas Vesalius	*1537–1555*	*(A)*
Valerius Cordus	*1540–1544*	*(BO)*
Conrad Heresbach	*1540-1570*	*(ZO)*
Gabriele Fallopio	*1540–1560*	*(A)*
Michael Servetus	*1540–1553*	*(P)*
William Turner	*1540–1568*	*(BO)*
Leonhard Fuchs	*1542–1566*	*(BO)*
Conrad Gesner	*1542–1565*	*(M), (BO), (ZO)*
Realdo Colombo	*1544–1559*	*(M), (A), (S)*
Ambroise Paré	*1545–1563*	*(M)*
Pierre Belon	*1546–1564*	*(BO)*
Bartolomeo Eustachi	*1550–1565*	*(A)*
Andrea Cesalpino	*1555–1609*	*(P), (BO)*
Luigi Cornaro	*1558–1566*	*(M)*
Charles l'Ecluse (Carolus Clusins)	*1580–1609*	*(BO)*

The Gregorian Calendar[234]

In 1582 CE, Pope Gregory XIII (1502–1585, Italy) ordered a calendar reform. The Julian year, with average length $365\frac{1}{4}$ *days, is* 11^m14^s *longer than*

[234] For further reading, see:

- Reingold E.M. and H. Dershowitz, *Calendrical Calculations*, Cambridge University Press, 2001, 422 pp.

the tropical year (length $365^d5^h48^m46^s$*). This slight discrepancy accumulates at a rate of one day in 128 years. Thus, between 46* BCE *and 325 CE, the date of the vernal equinox had slipped back from March 25 to March 21. In the year 730, the medieval scholar* ***Bede*** *(672–735, England; known as the 'Venerable Bede'), showed that the deviation was more than 3 days.* ***Roger Bacon****, in 1200, found an error of 7 or 8 days.* ***Dante****, soon after 1300, was well aware of the need of calendar reform. In 1474 Pope Sixtus IV invited the astronomer* ***Regiomontanus****, to revise the calendar and bring it into line. The premature death of the scientist interrupted the plan. In the following century, numerous memoirs appeared on the subject by mathematicians of note.*

By 1582, the 11 minutes and 14 seconds per year had added up such that the first day of spring was occurring on March 11. If the trend were allowed to continue, eventually Easter and the related days of observance would be occurring in early winter. Therefore, Pope Gregory XIII undertook the long-desired reformation. The author of the adopted system was ***Luigi Lillio Ghiraldi****, a learned astronomer and physician of Naples, who died, however, before the introduction of the reform. The calculations were continued by* ***Christoph Clavius*** *(1537–1612, Germany).*

The Gregorian calendar reform consisted of two steps. First, 10 days were dropped out of the calendar in October to bring the vernal equinox back to March 21, where it was at the time of the Council of Nicaea. This step was expeditiously accomplished. By proclamation, the day following October 4, 1582, became October 15. The second feature of the new Gregorian calendar was that the rule for leap year was changed so that the average length of the year would more closely approximate the tropical year.

In the Julian calendar, every year divisible by 4 was a leap year, so that the average year was 365.250000 *mean solar days in length. The error between this and the tropical year of* 365.242199 *mean solar days accumulated to a full day every 128 years. Ideally, therefore, one leap year should be made a common year, thus dropping one day, every 128 years. Such a rule, however, is cumbersome. Instead, Gregory decreed that in 3 out of every 4 century-years, all leap years under the Julian calendar, would be common years henceforth. The rule was that only century-years divisible by 400 should be leap years. Thus 1700, 1800 and 1900, all divisible by 4 (and thus leap years in the old Julian calendar), were* not *leap years in the Gregorian calendar. On the other hand, the years 1600, and 2000, both divisible by 400, are leap years under both systems.*

Among those who objected to this reform was ***Viète****. The reform was devised so that religious ceremonies could be performed on the correct dates. The Julian calendar was promptly adopted throughout the Roman Empire,* *which comprised the whole civilized Western world. But the Gregorian calendar was formulated after the Reformation, and the Pope's decision was not immediately accepted everywhere.*

The Catholic nations adopted it at once, but the Protestant nations fell in line only gradually: Germany in 1700, England in 1751. By this time, the two calendars were out of step by 11 days, and there was rioting in England with the slogan: "Give us back our eleven days". Notwithstanding, England and its colonies (North America included) jumped directly from 02 Sept. 1752 into 14 Sept. 1752. Russia adopted the Gregorian calendar only on 31 Jan. 1918, which was succeeded by 14 Feb. 1919.

As the Gregorian method of intercalation has been adopted in all Christian countries, it is of interest to examine to what degree of accuracy it reconciles the civil year (365^d) *with the tropical year* ($365.2422^d = 365^d : 5^h : 48^m : 46^s$)*. The Gregorian rule gives 97 intercalations in 400 years; Thus, 400 years contain* $365 \times 400 + 97$, *that is 146,097 days. Consequently, an average year contains 365.2425 days or* $365^d : 5^h : 49^m : 12^s$. *This exceeds the true solar year by 26 seconds, which amount to a day in 3323 years. It is perhaps unnecessary to make any formal provision against an error which will only happen after so long a period of time; but as 3323 differs little from 4000, it has been proposed to correct the Gregorian rule by making the year 4000 and all its multiples common years. With this correction, the commencement of the year would not vary more than a day from its present place in 200 centuries.*

One would prefer, however, a different method of intercalation by which the coincidence of the civil and solar year could be restored in shorter periods (since people in the year 3322 would presumably feel uncomfortable with a discrepancy of 24 hours!). To find this method, the decimal fraction 0.2422 is converted into the continued fraction

$$\cfrac{1}{4+\cfrac{1}{7+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{4+\frac{1}{1+\cdots}}}}}}$$

which gives the series of approximating fractions $\frac{1}{4}, \frac{7}{29}, \frac{8}{33}, \frac{31}{128}, \frac{132}{545}, \frac{163}{673}$, *etc. The first of these,* $\frac{1}{4}$, *gives the Julian intercalation of one day in 4 years, and is significantly too big. It supposes the year to contain* $365^d : 6^h$.

The second, $\frac{7}{29}$, *gives 7 intercalary days in 29 years, and errs in defect, as it supposes a year of* $365^d : 5^h : 47^m : 35^s$.

The third, $\frac{8}{33}$, *gives 8 intercalations in 33 years, or 7 successive intercalations at the end of 4 years, and the* 8^{th} *at the end of 5 years. This supposes* *the year to contain* $365^d : 5^h : 49^m : 5.45^s$. *It implies a year exceeding the true year by* 19.45^s, *while the Gregorian year is too long by 26 sec.*

Thus, the above method produces a much nearer coincidence between the civil and solar years than does the Gregorian method; and by reason of its shortness of period, confines the evagations of the mean from the true equinox within much narrower limits. The discovery of the period of 33 years is ascribed to ***Omar Khayyam*** *(1079* CE*).*

Better yet is the fourth convergent

$$\frac{31}{128} = \frac{3 \times 8 + 7}{3 \times 33 + 29},$$

which combines 3 periods of 33 years with one of 29 and would be very convenient in application. It supposes the year to consist of $365^d : 5^h : 48^m : 45^s$, *and is practically exact.*

To determine the day of the week corresponding to the day of the month in any year, one uses the formula

$$S = Y + D + \frac{Y-1}{4} - \frac{Y-1}{100} + \frac{Y-1}{400}.$$

Here Y *is the year of the Gregorian calendar, and* D *is the day of the year (after February 28 of a leap year,* $D \to D+1$ *relative to a non-leap year). Also, in dividing* $Y-1$ *by 4, by 100 and by 400, the remainder gives the day of the week, 0 indicating Saturday, 1 Sunday etc. The corresponding formula for the Julian calendar is*

$$S = Y + D + \frac{Y-1}{4} - 2.$$

Thus, Columbus discovered America on Oct. 12, 1492 which was a Friday.

The Gregorian calendar, for all its sophistication, is not quite as accurate as that devised by the Maya priests of Central America. The Gregorian year is slightly too long, the error amounting to three days in 10,000 years. The length of the year according to the Maya astronomers was too short, but the defect amounted only to two days in 10,000 years.

In the same year (1582), the scholar ***Joseph Justus Scaliger*** *(1540–1609, Netherlands) suggested that all dates be referred to an arbitrary fiducial date, Jan. 1, 4713* BCE *(he may have believed this to be the day of creation). The date thus reckoned is known as the Julian day (JD), in honor of his father* ***Julius Caesar Scaliger*** *(1484–1558) (no connection to the Julian calendar). Julian days are now used for expressing the times of most astronomical ob-*

servations. They are reckoned from noon, and parts of a day are expressed in decimals to the necessary degree of precision[235].

The basis of modern Western science

Modern science started at the end of the Renaissance (ca 1550) with ***Copernicus***, ***Kepler*** *and* ***Galileo***. *All efforts prior to this date did not produce any significant corpus of scientific and technological knowledge and thus did not lead to the uncovering of the great code of natural laws. Neither the Greeks, nor the Chinese or the Hebrews succeeded in revolutionizing human thought in spite of their great respective advances in astronomy, geometry, logic, philosophy, technology and ethics. Why?*

The ancient Hebrews fostered a belief that nature is governed by the decrees of an omnipotent and divine law-giver. This created an advantageous environment for the emergence of a firm belief in laws of nature, and in the rationality and ordered character of the world: a firm belief that there exists something worth investigating. It emphasizes a common factor behind nature, and establishes nature's universality. Moreover, it establishes the invariance of some elements of nature in the face of the flux of events.

This biblical view exerted a powerful influence during much of the period when science grew into its modern form[236]. *The Old Testament picture of God fashioning the World out of a formless void and ordering it in special ways was quite different to beliefs elsewhere in the ancient world, and to the*

[235] The system has since been modified slightly. Modified Julian Date (MJD) is simply $JD - 2,400,000$ days and 12 hours, putting the zero hour at midnight on November 17, 1858.

[236] The majority of leading British scientists until the beginning of the 20^{th} century were devout Christians, and this had a particular influence upon their scientific work.

earlier magical *view of nature. However, this philosophy, while being* necessary *for the development of scientific thought, was* not sufficient*; The instruction against graven images (Exodus 30, 4-5) forbids the representation of the one true God and therein is ingrained a perpetual contentment with an* abstract *view of things, and a distrust of having useful or symbolic representation of things. Thus, nature was seen primarily as a sign and symbol of its creator, rather then as a puzzle to be solved or a source of power to be harnessed.*

Moreover, although the natural world was acknowledged to be the handiwork of the Creator, it was not held that a knowledge of the works of creation led necessarily to a deeper understanding of God. Deep understanding, or 'wisdom', was to be found in the moral world, and came not through the earthquake, wind or fire, but through the 'still small voice' of the Spirit of God. Science, therefore, could not develop directly within such a world-view. Indeed, the ancient Hebrews never developed any seafaring tradition (avoiding the need to study the heavens for the purpose of navigation) and had motivation for neither architecture nor industry - life remained tied to agriculture and tradition.

In the blundering progress of scientific and technical achievements in the Middle Ages, the Jews of Europe did not fail to contribute their part. Throughout the period one finds mention of them - now translating a fundamental scientific work, now introducing a new process from one country to another, now referred to as authorities, now making their original contribution to mathematics, logic, astronomy, philosophy, philology, physics, chemistry and engineering.

An impressive lot of Jewish inventors and scientists of the Middle Ages could be compiled. Many other instances may be adduced, serving to indicate that, in the gradual, anonymous, development of technical progress before the 19^{th} *century, Jews collaborated with other sections of the European population. This same intellectual alertness was given a fresh outlet after the breakdown of the Ghetto.*

In the course of the 19^{th} *century, the energy and inventiveness which had previously been almost confined to Talmudical studies (with their philosophical and mathematical corollaries), or to the difficult task of earning a living through menial occupations (under adverse conditions of hardships and persecutions), began to be turned to science in its wider sense. Jews now not only earned distinction but produced a few scholars who are the unchallenged leaders in their particular branches of research.*

The idea of a single supreme Deity was foreign to the early Chinese *and as a consequence the fate of natural science in that culture was a curious stillbirth. For the Chinese there existed no concept of a divine being whose decrees formed inviolate 'laws of nature', and who underwrote the scientific enterprise.*

Despite sophisticated technological development in rocketry, printing, and the widespread use of magnetic compasses in their sailing ships, these inventions provoked no urge to explore natural regularities or the geography of the globe.

A central idea of Chinese thought from earliest times until the mid 20th century appears to be that of spontaneous development of order in the world. This notion could have had its roots in observations of the natural world, for example the organized collective behavior of insect colonies, where there arises a mysterious harmony between many separate parts without external human interference.

Alternatively, we might find its roots in the gradual appearance of social order within small peasant groups who found themselves evolving a stable and organized way of life within their communities, without the imposition of rules by some external central government. Rules arose by negotiation and compromise rather than dictatorial decree.

This view eventually evolved in the 6th *century BCE into Confucianism - a world view that did not seek rules for the behavior of nature in logical analysis or through systematic observational studies, but looked instead to the analogy with the harmonious social customs that evolve out of* collective human activity. *Thus, order in nature was sought in social behavior, known as Li. Li operated across the entire spectrum of life; it was the reason for the motions of the moon and the stars, for the successful exercise of self-control in human dealings, and for the social divisions of rich and poor.*

Another Chinese philosophy, known as the Tao (the 'way') opposed this search for order of nature in social behavior. They believed that only by being one with nature could the order within be understood. This holistic view also denies a notion of an external world of physical reality. Thus, despite their technology, the Taoist philosophers never framed any statement that we might call laws of nature. They had no confidence in the ability of reason to unravel the universe. *Their theology of pantheistic naturalism, ran counter to the entire concept of God controlling the universe. They believed that everything had the ability to bring itself into being, and so there was no psychological desire to introduce a personal Creator.*

Thus, whether through Li or Tao, the early Chinese had no reason to believe in an underlying rationality in nature that might be uncovered and understood by detailed observation and codification. The Universe was believed to be far too complex for such an enterprise to be even considered.

The idea of order never carried with it any implication that there must inevitably exist constraining laws and an ordering law-giver. Despite their early technological superiority, the scientific altitude faded and died unfulfilled in ancient China.

It remains to ask why the intellectual atmosphere of Greek *culture, with its stable government and highly developed systems of civil law, did not lead to uncovering of the great code of natural laws.*

At first sight it appears that many trends in Greek thought were taking the perfect coarse toward a scientific revolution: a faith in the intelligibility and rationality of the world, a desire to seek out and comprehend the truth for its own sake and a desire to amass a vast panoply of facts. None the less, the following trends created a barrier through which their thinking about the natural world could not pass:

- *The Greek philosophical schools begun with theories about the nature of* everything *but were too vague and diluted to tell us very much about particular things. Science only began to make dramatic progress during the late Renaissance when it limited its objectives and started to address the particular as the necessary prerequisite to any understanding of the general. Greek philosophers preferred to argue about the meaning of the idea rather then observe what happened in the world. Even the Greek materialist philosophers applied their logic to problems beyond the reach of observation (origin of life, origin of all things, etc.).*
- *Greeks were impeded by their very reverence for inexorable logic; but logic alone cannot reveal to us the existence of new types of entity, and it has little use for experiment and observation. A reverence for geometry fosters belief that the most important properties of the world are* static, *and prevents one focusing upon the* dynamic *aspects of its structure.*
- *Greeks had a low view of manual labor and slavery was a key element in their economy, To make devices and experiments was beneath their aristocratic dignity. It was an activity to be pursued by those who could not think. This resulted in a mentality that all concern with nature was dominated by theory. There was no experiment. Furthermore, it hindered the development of a large pool of independent skilled craftsmen within a society. From the workshops of such individuals, and by their motivation to manipulate nature and build bigger and better artifacts and tools, did the practice of applied science receive its stimulus during the Renaissance.*
- *Many of the physical theories and cosmologies of the Greeks read like rational revisions of the early myths. They were exercises in deductive ingenuity in which the question of observational consequence in the* future *never arose.*

In conclusion: science arose in Western Europe as a direct consequence of the Judeo-Christian heritage. The new scientific enterprise evolved most *successfully in an environment in which there existed a strong belief in the role of law and order in the widest sense: it was a grand merger of two systems - a strong monotheistic religious belief coupled to a strong civil legal system and central government. A culture displaying both of these attributes is an especially advantageous environment for the emergence of a firm belief in laws of nature and in the rationality and ordered character of the World.*

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3. *The Clockwork Universe*

1583 CE–1819 CE

Beyond the Greeks

Infinitesimals and Infinities

Rise of Mechanics

The New Astronomy

Enlightenment

Social Revolutions and Industrialization

Algebraization of Geometry and exploitation of the Calculus

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Stars' orbits you will know; and bold,
You learn what nature has to teach;
Your soul is freed, and you behold
The spirits' words, the spirits' speech.
Though dry reflection might expound
These holy symbols, it is dreary:
You float, oh spirits, all around;
Respond to me, if you can hear me.

What jubilation bursts out of this sight
Into my senses – now I feel it flowing,
Youthful, a sacred fountain of delight,
Through every nerve, my veins are glowing.
Was it a god that made these symbols be
That soothe my feverish unrest,
Filling with joy my anxious breast,
And with mysterious potency
Make nature's hidden powers around me,
manifest?

Am I a god? Light grows this page –
In these pure lines my eye can see
Creative nature spread in front of me.
But now I grasp the meaning of the sage:
"The realm of spirits is not far away;
Your mind is closed, your heart is dead.
Rise, student, bathe without dismay
In heaven's dawn your mortal head."

All weaves itself into the whole,
Each living in the other's soul.
How heaven's powers climb up and descend.
Passing the golden pails from hand to hand!
Bliss-scented, they are winging
Through sky and earth – their singing
Is ringing through the world.

What play! Yet but a play, however vast!
Where, boundless nature, can I hold you fast?
And where you breasts? Wells that sustain
All life – the heaven and the earth are nursed.
The wilted breast craves you in thirst –
You well, you still – and I languish in vain?

From Goethe's '*Faust*'.

Translated by Walter Kaufmann, 1961.

1583–1600 CE **Giordano Bruno** (1548–1600, Italy). Philosopher of the Renaissance. Adopted the view that the universe is infinite with innumerable stars and planetary systems. In his publication "Dell' infinito universo e mondi" ("Of infinity, the universe and the world") he criticized the doctrines of Aristotle and Ptolemy that there was an absolutely fixed center in the universe.

Bruno was christened Filippo. In his 15^{th} year he entered the order of the Dominicans at Naples. But from an early age he was on the move through the cities of Europe: Rome (1576), Geneve (1579), Paris (1581), Oxford (1582), Wittenberg (1587), Prague (1588), Frankfurt (1591), Zürich (1592), and Venice (1593).

He was burned at the stake in Rome by the Inquisition on the charge of believing in the nonexistence of the absolute truth. His last cry from the burning stake was "Eppur si muove!" ("and nonetheless it moves!").

Beyond the Greeks – The Emergence of Modern Science

* * *

"Until the Scientific Revolution of the 17^{th} century, meaning flowed from ourselves into the world; afterwards, meaning flowed from the world to us".

(Chet Raymo, 1999)

In some branches of science, notably astronomy, the ancients made substantial contributions. In physics, the most fundamental of the sciences, the record is scant. Simple engineering tools like the lever, the wheel, and the inclined plane were known before recorded history; with them, by 3000 BCE, the Egyptians had built such magnificent structures as the pyramids. The functioning of these tools, of course, depended on unsuspected physical principles.

The first faint stirring of the science itself — the rigorous examination of physical principles — was apparently a product of Greek civilization. From the contemporary physicist's viewpoint, its most interesting legacy was one of the first recorded scientific controversies, having to do with the fundamental nature of matter. ***Democritos****, who lived about* 400 *BCE, was the leader of an "atomistic" school which held that all matter was composed, in varying combinations, of four different kinds of particles, tiny and indivisible. He believed that their existence was literally a fact.* **Plato** *the philosopher, his foremost opponent, conceived of fundamental matter in terms of mathematical patterns, forms, and "ideas". This ancient controversy between materialism and idealism, has, oddly enough, been revived recently in a quite specific way by modern atomic physics, and especially by the quantum theory.*

The activity of Greek physicists was not limited to theoretical and philosophical problems. Among the earliest experimental physicists was ***Pythagoras****, the* 6^{th}*-century philosopher and mathematician. He and his school attempted to formulate a theory of musical harmony by experimenting with strings of different lengths, thicknesses and tensions. It was indeed the first instance of the application of mathematics to a basic physical phenomenon.* ***Euclid*** *the geometer, who flourished at Alexandria about* 300 *BCE, made studies in the laws of perspective and reflection, and is said to have written on music and mechanics.* ***Hero of Alexandria****, who lived probably about* 150 CE*, made pulleys, gears, siphons, and an engine which used steam to rotate a hollow sphere — the first known utilization of the law of action and reaction.* ***Ptolemy****, an Alexandrian of about the same period, whose cosmology was accepted for many centuries, wrote on reflection and refraction. Unquestionably the greatest ancient figure in both physics and mathematics was* ***Archimedes of Syracuse****, killed by a Roman soldier in 212 BCE. He was famous for his engineering and military inventions. More important, he founded the sciences of statics and hydrostatics.*

About the second half of the first century BCE, the Roman ***Vitruvius*** *wrote De Architectura, an encyclopedia of useful knowledge in the fields of architecture, engineering and construction. He investigated such matters as the measurement of time and acoustics, comparing the waves of sound to those caused by a stone thrown into a pond. Like his fellow countrymen, he laid emphasis on practical applications rather than on theoretical scientific knowledge.*

The above names and a few others make up the meager roster of ancient physicists. Their influence on the main stream of scientific history was slight. The monasteries, the cultural centers of the Middle Ages, were concerned primarily with questions of philosophy and religion — for example, whether God could create a stone so heavy that the Himself could not lift it, a problem which does not lend itself to experimental verification. Over all science lay *the shadow of* ***Aristotle****, the Greek scientist of the* 4^{th} *century BCE. His interests were universal, embracing logic, philosophy, history, politics and the biological and natural sciences. He was the apostle par excellence of rationalism, the belief in logical rather than experimental explanations. From a purely philosophical viewpoint,* ***Francis Bacon****, Galileo's contemporary, did much to overthrow this doctrine. But in actual practice, it was* ***Galileo*** *who who sounded its death knell.*

1583–1637 CE **Galileo Galilei** (1564–1642, Italy). Pioneer of modern applied mathematics, physics and astronomy. The founder of modern physics on account of his willingness to replace old assumptions in favor of new scientifically deduced theories.

Supported the Copernican Revolution and paved the road for Newton's laws of motion. Introduced the method of mathematical analysis for the solution of physical problems.

His major achievements are these:

(1) Originated (1583) modern accurate time-keeping through the discovery of a natural periodic process that can be repeated indefinitely and counted — the swinging pendulum. He found that each simple pendulum has its own period, depending on its length. [The actual step of applying the pendulum to clockwork, so as to record mechanically the number of swings, was taken by **Christiaan Huygens** in 1656.]

(2) First pointed his self-made telescope at the sky in 1609. With this instrument he extended our knowledge by observing many stars which are too faint to be seen directly. Discovered the satellites of Jupiter[1], and phases of Venus[2]. This small-scale model of the solar system convinced him of the truth of the Copernican theory.

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[1] **Simon Mayr** (Mair, Meyer, Marius; 1573-1624, Germany). Astronomer. Assistant to Tycho Brahe (1601). Claimed to have discovered (ca 1610) largest moons of Jupiter and named them: *Io*, *Europa*, *Ganymede* and *Callisto* (a discovery, generally credited to Galileo). Made first telescopic observations of Andromeda spiral nebula (1611).

[2] It was these observations, rather then the ideas of Copernicus, that dealt a deathblow to the Ptolemaic geocentric system: *Both* the Ptolemaic and the Copernican views described the motions of the planets. The heliocentric view of Copernicus was a simpler hypothesis (*Ockham's razor!*). This in itself, however, is not a

(3) While investigating the motion of objects in *free fall*[3] he was first to realize that it is not the velocity of a body, but its acceleration which signifies that there are forces acting on it (1604–1609).

(4) Showed by experiments (or thought-experiments as some claim) that bodies of different constitution and weight (mass) are equally accelerated by gravity (1609), i.e. fall with the same terminal velocity. Formulated correctly the basic *kinematic* laws of falling bodies.

(5) Recognized the concepts of parallelograms of forces and velocities, and with it the separation of projectile motion into horizontal and vertical *components*.

(6) Formulated the restricted *mechanical* '*principle of relativity*', stating that no mechanical experiment will reveal whether a system is at rest, or is moving uniformly in a straight line. In other words — the laws of mechanics are invariant under a 'Galilean transformation'. [In modern notation, $\mathbf{r}' = \mathbf{r} - \mathbf{v}_0 t,\ t' = t$: a coordinate transformation that connects observers in different frames.]

(7) Initiated the modern attitude toward the actual *infinite* in mathematics (1638). Asserted that infinite numbers obey a different "arithmetic" from finite numbers: If using the ordinary notions of "equal" and "less than" on infinite sets leads to contradictions, this is not a sign that infinite sets cannot exist, but rather, that these notions do not apply without modifications to infinite sets. Galileo himself did not see how to carry

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demonstration of its validity. Nature may, after all, be complex.

However, both systems differed in another aspect: According to Ptolemy, the sun circled about the earth, and inside of the sphere of the sun lay the spheres of Venus and Mercury. With such a geometry, it would be impossible for us ever to see the entire bright side of Venus! According to Copernicus however, both Venus and earth circled the sun. Since Venus was sometimes beyond the earth and the sun, it would be possible for us to see its bright side. Thus, when Galileo turned his telescope to Venus, and *saw* that its disk underwent phases from a '*full Venus*' (similar to our full moon), to a *new Venus* (the dark side of Venus, corresponding to our new moon), it was clear that the Copernican hypothesis stood vindicated. *Sunspots* were reported independently in 1611 by Galileo and **Christoph Scheiner** (1573–1650, Germany). However, Galileo and the other "discoverers" of sunspots were well aware of the existence of sunspots, and naked-eye reports of them, *before* they looked at the sun through telescopes.

[3] **Giovanni Battista Benedetti** (1530–1590, Italy) was an important forerunner of Galileo. Studied under Tartaglia. He worked on the *free fall* of bodies and proposed (ca 1560) a theory almost identical to that which Galileo published in *De motu* (1590).

out such a modification of these notions (this was left to **Georg Cantor** some 250 years later).

(8) Invented the *thermoscope* (1556), a prototype of the later thermometer, that could only indicate relative *changes* in temperature: an inverted flask with a narrow neck was lowered into a shallow bowl containing liquid. The liquid would go part way up the flask's neck; changes in the surrounding temperature would either raise or lower the liquid. There was no way to tell what the temperature actually was. It was later improved by his friend **Sanctorius** (1611). Only in 1709 did **Daniel Fahrenheit** invent the calibrated mercury thermometer.

Although Galileo[4] did not define *inertia*[5], he came close to it by understanding that one must exert a force on a body in order to accelerate or decelerate it. It remained for Newton to generalize Galileo's results to forces, in general, and to define mass and inertia.

Galileo's experiments with falling bodies[6] were a crucial landmark in physics in the sense that they marked the *demise of Aristotelian physics*. In 1907, Einstein elevated Galileo's *experiment* into a *principle*, just as earlier in 1905 he had generalized Galileo's principle of relativity to incorporate all laws of physics.

After Galileo's work became known, philosophers and scientists slowly began to realize that the behavior of physical objects could be described in mathematical terms. This led to the idea that there existed laws that had been established by God to regulate his creation. The new idea that nature itself was subject to laws, depended upon the implicit assumption that such laws did not change with time, and that their divine origin guaranteed their eternal endurance.

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[4] Often just called by his first name.

[5] **Nicole Oresme** and **Johannes Buridanus** (ca 1350), in Paris, criticized Aristotle's doctrine of motion. They in turn were influenced by the idea of *impetus* introduced ca 530 by **John Philoponus** (John the Grammarian). Philoponus was a Greek philosopher of Alexandria who speculated that a projectile would gain momentum from the mechanism that fired it, thus arriving at a crude idea of inertia. Galileo amalgamated these notions into his theory of motion.

[6] In 1328, **Thomas Bradwardine** (1290–1349, England) discussed the issue of the hypothetical free fall of bodies in void and concluded that two bodies of the same material but different size, will fall with the same terminal velocity, contradicting the Aristotelian view that the heavier body falls faster. Bradwardine was a theologician and mathematician who became the Archbishop of Canterbury.

Like Copernicus and Kepler, and in contradistinction to the Greek doctrine, Galileo realized that the 'mysteries' of nature could be illuminated only with the assistance of accurate observations and experiments. It was just this approach that was responsible for the epochal success of these scientists. In fact, as soon as Galileo and others began to apply the scientific method in physics and astronomy, a chain of discoveries resulted.

These discoveries fired the imagination and enthusiasm of European thinkers. Scientific societies were organized, scientific journals began to appear. Science, hitherto the pursuit of occasional lone individuals, became a social enterprise and has continued to be so to the present. Furthermore, it became fashionable. Newton's work, for example, made a profound impression on every writer in Europe.

Galileo was born at Pisa. His father Vincenzio was an impoverished descendant of a noble Florentine house, which had exchanged the surname of Bonajuti for that of Galilei. Vincenzio was a competent mathematician and a musician. By his wife, Giulia Ammannati of Pescia, he had 4 daughters and 3 sons, the eldest of which was Galileo. From his earliest childhood, Galileo was remarkable for intellectual aptitude as well as for mechanical invention. His education was principally conducted in the monastery of Vallombrosa, near Florence. There he acquired a fair command of Latin, Greek and logic.

He was at this time attracted toward a religious life, but his father withdrew him permanently from the care of the monks and placed him in 1581, at the University of Pisa on a course of medical studies. In that year, while watching a lamp set swinging in the cathedral of Pisa he observed that, whatever the range of its oscillations, they were invariably executed in equal times. The experimental verification of this fact led him to the important discovery of the isochronism of the pendulum. (More than 50 years later he turned it to account in the construction of an astronomical clock.)

Up to this time he was entirely ignorant of mathematics, his father having carefully held him aloof from a study which he rightly apprehended would lead him to forsake medicine. Listening one day to an accidental lesson in geometry, his attention was riveted and he threw all his energies into the new pursuit. He rapidly mastered the elements of the science, and eventually extracted his father's reluctant permission to exchange **Hippocrates of Cos** and **Galen** for **Euclid** and **Archimedes**. For lack of means, he withdrew from the university in 1585 before obtaining a degree, and returned to Florence. Shortly afterward he invented the *hydrostatic balance*, which he used to find the specific gravity of objects by immersing them in water.

In 1588 he wrote a treatise on the center of gravity in solids, which together with his former invention, made his name known throughout Italy and secured him, at age 24, the post of professor of mathematics at the University of

Pisa. During the ensuing two years (1589–1591) he carried out a series of experiments by which he established the fundamental principles of dynamics. His new views put him on a collision course with Aristotelian physics and he was forced to leave the university. Through the death of his father in July 1591, family cares and responsibilities devolved upon him, and thus his nomination to the chair of mathematics at the University of Padua, was welcome both for the relief it offered from pecuniary need, and as opening the road to scientific distinction.

His residence at Padua (1592–1610) was a course of uninterrupted prosperity. His appointment was renewed three times, on each occasion with the expressions of the highest esteem. His lectures were attended by persons of the highest distinction from all parts of Europe, and such was the charm of his demonstrations, that a hall capable of containing 2000 people had eventually to be assigned for the accommodation of the overflowing audiences which they attracted.

In 1593 he constructed the first *thermoscope*, consisting of a bulb filled with air and water and terminating in a vessel of water.

In spite of his adherence to the Copernican theory, he continued to conform, in his public teaching, to Ptolemaic principles, waiting for the proper opportunity to make an open onslaught upon the Aristotelian axioms[7]. The discovery of the telescope by the obscure Middleburg optician, **Jan (Hans) Lippershey** (1608), provided him with that opportunity. Galileo's direction of his new instrument [after one night of profound meditation on the principles of refraction in June 1609 in Venice, he was able to produce a telescope with a magnifying power of 32] to the heavens opened an era in the history of astronomy.

Discoveries followed upon it with astounding rapidity and in bewildering variety: During 1609–1613 his discoveries indicated: that the *moon* (contrary to the teachings of Aristotle) was *not* a smooth sphere shining by its own light, that the *Milky way* was a mass of numerous stars, that *Jupiter* has 4 bright satellites (which he named after the Medici family, who ruled the province of Tuscany where he was born), the peculiar form of *Saturn*, the phases of *Venus* and finally the *sunspots*[8].

[7] Nevertheless, he was not *completely* free of the Pythagorean and Neo-Platonic doctrines, which had been disseminated during the medieval period and early renaissance: his work is premised on the deep-seated conviction of a *simple*, *ordered world*, free from arbitrariness and disclosing geometrical regularity.

[8] It was discovered earlier (1611) by the German astronomer **Johannes Fabricius** (1587–1615). He concluded from his observations that the spots were integral parts of the sun and that their movement was caused by the *sun's rotation about*

In September 1610 Galileo finally abandoned Padua for Florence. His researches with the telescope had been rewarded by the Venetian senate with the appointment for life to his professorship, at an unprecedented high salary. His discovery of the 'Medicean Stars' earned him the title of 'philosopher and mathematician extraordinary' to the grand duke of Tuscany.

When Galileo firmly upheld the Copernican theory that the earth moves around the sun, Church officials warned him to abandon this 'heretical' system. At the same time (1616), the Church placed the work of Copernicus on the *Index* of prohibited books, where it remained for 200 years.

In 1632, Galileo published his masterpiece, *A dialogue on the Two Systems of the World*[9]. After a long trial, Church officials forced him to say that he gave up his belief in the Copernican theory, and sentenced him to an indefinite prison term, which he spent in his villa near Florence.

Domestic afflictions combined with numerous and painful infirmities to embitter his old age. His sister in law and her whole family, who came to live with him on his return from Rome, perished shortly thereafter in the plague. In 1634, his eldest and best-beloved daughter, a nun in a convent, died.

Galileo was never married, but by a Venetian woman named Marina Gamba he had three children — a son who married and left descendants, and two daughters who took the veil at an early age.

His prodigious mental activity continued undiminished to the last. In 1636 he completed his *Discorsi a due nuove scienze* (Discourses on the Two New Sciences) in which he recapitulated the results of his early experiments and presented mature meditations on the principles of mechanics. It summed up

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its axis. In 1612 Galileo in Italy, **Thomas Harriot** in England, and the German Jesuit **Christoph Scheiner** published their own observations. Galileo came forth with the same explanation of the movement of the spots as did Fabricius, whereas Scheiner said they were small planets revolving around the sun. In his 1613 publication *Istoria e dimostrazioni intorno alle macchie solarie e loro accidenti*, Galileo disproved Scheiner's reasoning and, for the first time, publicly supported the heliocentric theory of Copernicus. Scheiner, who eventually conceded that Galileo was right, went on to make much more accurate observations than Galileo had, and he found that the sun completes a full rotation in 27 days.

[9] For further reading, see:

- Galilei, G., *Dialogues Concerning Two New Sciences* (*1638*), Dover Publications: New York, 1954, 300 pp.
- Drake, S., *Galileo at Work, His Scientific Biography*, University of Chicago Press: Chicago, 1978, 536 pp.

his life's work on motion, acceleration and gravity, and furnished a basis for the three laws of motion laid down by Newton in 1687. His last telescopic discovery — that of the moon's diurnal and monthly librations — was made in 1637, only a few months before he became blind. It was in this condition that **Milton** found him when he visited him in Arcetri in 1638. He continued his scientific correspondence and thought out the application of the pendulum to the regulation of clockwork, which Huygens successfully realized 15 years later.

He was also engaged in dictating to his disciples, **Viviani** and **Torricelli**, his latest ideas on the theory of impact, when he was seized with the slow fever which, within two months, brought him to the grave. He was buried in the Church of Santa Croce in Florence. Fifty years after his death, the city erected a monument at the church in his honor.

Science Progress Report No. 6

"Eppur si muove" (1600–1633)

The Catholic hierarchy recognized, as had the Protestants earlier, that the new cosmology was subversive — incompatible with the traditional, authoritarian society. One of the first victims of the Counter-Reformation was ***Giordano Bruno****, a former monk. Bruno traveled to England and befriended its leading political and scientific figures; and when he returned, he popularized Copernican theory on the continent. Bruno took Digges' version of the infinite, Copernical universe and purged it of remaining Ptolemaic elements, such as the perfect spheres that carried the planet's orbits. He made this infinite universe, with its infinity of inhabited worlds, the basis of his philosophy, incorporating Nicholas of Cusa's thinking and going beyond it. Bruno explicitly challenged the idea of creation* ex nihilo*, arguing that the universe must be unlimited in both space and* time*, without beginning or end.*

Bruno was a philosopher, not a scientist, and he used the tradition of logical argument to support the Copernican worldview. Above all, he considered himself a loyal Catholic bent on reforming, not rejecting, the church. Yet on his return to Catholic territory in 1592*, he was promptly arrested. Cardinal*

Bellarmino, a prominent leader of the Counter-Reformation and the pope's own theologian, saw in Bruno's writing an effort to subvert the church from within. The idea of an infinite number of worlds not only undermined the primacy of the church hierarchy, it contradicted as well sources of authority — the idea was found neither in the Bible nor in Aristotle or Plato. Moreover, it very obviously destroyed the Catholic vision of a material, subterranean hell and an ethereal heaven beyond the cosmic spheres: it portrayed a cosmos in which these threats and enticements would have no place, and would be comprehensible to only a few — certainly not to the ill-educated peasants, as the simple picture of a heaven above and a hell below certainly was.

Over seven years of imprisonment Bellarmino labored to get Bruno to recant the doctrine of the infinite plurality of worlds. Bruno refused, and in 1600 *he was burned at the stake.*

Since the charges against Bruno were never made public, other Catholic scientists, including Galileo, did not take his execution as a sign of Catholic hostility to Copernicus. But this hostility was confirmed even as the new theory triumphed.

The astronomical discoveries of Galileo added support to the Copernican system and brought him more fame. But the new views of the solar system promulgated by Galileo's formidable dialectic zeal alerted the Church, which saw in his scientific teaching a danger to religion. The new astronomy was publicly denounced by the Church, and on February 1615 *the matter was brought before the Inquisition. Consequently, Galileo received a semi-official warning to avoid theology and limit himself to physical reasoning.*

However, Galileo had already committed himself to dangerous grounds. In December 1615 *he lectured before the entire pontifical court, full of confidence that the weight of his arguments and the vivacity of his eloquence could not fail to convert them to his views. He was cordially received, and eagerly listened to, but his imprudent ardor served but to injure his cause.*

On the 24^{th} *of February* 1616, *the consulting theologians of the Holy Office characterized the proposition that the sun is immovable in the center of the world as "absurd philosophy and formally heretical, because expressly contrary to Holy Scripture". The proposition that earth has diurnal rotation was described by the Church as "open to the same censure in philosophy, and at least erroneous to faith". Two days later Galileo was summoned to the palace of Cardinal Bellarmino (1542–1621), and there officially admonished not thenceforward to "hold, teach or defend" the condemned doctrine. This injunction he promised to obey. However, he trusted his dialectical ingenuity to find the means of presenting his scientific convictions under the transparent veil of an hypothesis.*

During 1616–1623, Galileo led a life of studious retirement in the Villa Segni at Bellosguardo, near Florence, and maintained an almost unbroken silence. When a new pope was elected in 1623, Galileo visited Rome with the hope to obtain the revocation of the decree of 1616, through personal influence. Although he failed to achieve this, he expected that the decree would at least be interpreted in a liberal spirit. On his return to Florence, he therefore set himself to complete his work "Dialogo dei due massimi sistemi del mondo". It emerged from the press in 1632. A tumult of applause from every part of Europe followed its publication.

It was at once evident that the whole tenor of this work was in flagrant contradiction with the edict passed 16 years before its publication, as well as with the author's personal pledge of conformity to it. Toward the end of August 1632 the book's sale was prohibited, and on the 1^{st} *of October the author was summoned to Rome by the Inquisition. He pleaded his age, now close to 70 years, his infirm health, and the obstacles to travel caused by quarantine regulations, but the pope was indignant at what he held to be his ingratitude and insubordination, and no excuse was admitted. He arrived on 13 February 1633 and was detained until* 21^{st} *of June, when he was finally examined under threat of torture.*

On the 22^{nd} *of June 1633, in the church of Santa Maria Sopra Minerva, Galileo read his recantation and received his sentence: He was condemned, as "vehemently suspected of heresy", to incarceration at the pleasure of the tribunal, and by way of penance was enjoined to recite once a week for three years the seven penitential psalms. This sentence was signed by 7 cardinals, but did not receive the customary papal ratification. He was held by the Inquisition until December 1633, when he was allowed to return to his villa. There he spent the remaining 8 years of his life — practically under house arrest, in strict seclusion, constantly watched by the Inquisition.*

Galileo did not make the trek to the stake, because he was sensible enough not to die for his beliefs but to live for them — he recanted in public and went on with his studies in private.

He died in 1642, the year Newton was born, surrounded by friends and pupils. His epitaph was written for him by posterity: eppur si muove *— the famous words which he never uttered at his trial. When his friends wanted to erect a monument over his grave, Urban told the Tuscan Ambassador that this would be a bad example for the world, since the dead man 'had altogether given rise to the greatest scandal throughout Christendom'.*

In the year 1992, the Roman Catholic Church did in its acknowledgment that Galileo was right after all, that the earth does revolve around the sun[10].

1583–1592 CE **Simon Stevin** (Stevinus, 1548–1620, Netherlands). Versatile mathematician, scientist and engineer. The greatest mechanician of the long period extending from Archimedes to Galileo. A leading figure in the Dutch school of mathematics and science, and an outstanding representative of the great scholars of the closing years of the Late Renaissance; he combined a capability for theoretical investigation with practical skill and inventiveness.

Stevinus originated the study of modern statics and distinguished stable from unstable equilibrium. He demonstrated (1586) how to *resolve* a force according to the parallelogram law[11] (vector *decomposition*; their composition was known to Galileo). He discovered the hydrostatic 'paradox' that the downward pressure of a liquid is independent of the shape of the vessel, and depends only on its height and base. He also gave the measure of the pressure on any given portion of the side of the vessel. He had the idea of explaining the tides by the attraction of the moon.

In 1586, Stevinus performed experiments concerning the effect of gravity on falling bodies. He made a noteworthy contribution to trigonometry, using the *unit circle*. His greatest success, however, was a small pamphlet, first published in Dutch in 1586 (under the name *De Thiende*, i.e. the tithe), and not exceeding seven pages in the French translation: *La Disme enseignant facilement expédier par Nombres Entiers sans rompuz tous Comptes se recontrans aux Affaires des Hommes*. It presented first systematic account of *decimal fractions* and strongly advocated their usage.

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[10] *Modern* Roman Catholicism has no quarrel with the Big Bang, with a universe 15 billion or so years old, with the first living things arising from prebiological molecules, or with humans evolving from apelike ancestors, although it has special opinions on "ensoulment"

[11] This was rediscovered by **Bernard Lamy** (1640–1715, France) in 1676 and again by **Pierre Varignon** (1654–1722, France) at about the same time. The availability of the works of Archimedes was a boon to the study of *statics*, but it is regrettable that the popularity of these so heavily overshadowed medieval steps toward *dynamics* had to await the genius of Newton a century later (1687).

Decimal fractions had been employed for the extraction of square roots since some five centuries before his time, but nobody before Stevinus established their daily use. So well aware was he of the importance of his innovation that he declared the universal introduction of decimal coinage, measures and weights to be only a question of time. Not until the *French Revolution*, more than a two centuries later, did the large scale use of decimals come into vogue[12]. During the last century it spread all over the world, except, strangely enough in the Anglo-Saxon countries, where it met — and still meets — with resistance, which is the stronger in that it is irrational.

His *notation* (1585) was rather unwieldy: he printed little circles round the exponents of the different powers of one-tenth. For instance $237\frac{578}{1000}$ was printed 237 ⓪ 5 ① 7 ② 8 ③ which was a regression from the early use of 237/578 of **Christoff Rudolff**[13] (1500–1545, Austria) in 1530.

Stevinus was born in Bruges and died at the Hague. He began life as a merchant's clerk in Antwerp and traveled in Poland, Denmark and other parts of Northern Europe. He was an adviser to Prince Maurice of Orange (son of William the Silent and great uncle of William of Orange), who made him a Quartermaster General.

Late in life (at an age of 64) he married a young woman who bore him four children. She remarried in 1623 and died half a century later (1673). In July 1846 a modest monument was erected to Stevin's memory in his native city, Bruges.

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[12] The *decimal Dollar* became the basis unit of money in the United States through the Coinage Act of 1792.

[13] The introduction of the *decimal point* to mark the gap between the integral and fractional part is attributed to **Pelazzi of Nice**, about 1492.
The decimal point separatrix was reinstituted by **G.A. Magini** (1555–1617, Italy) in 1592 and by **Christoph Calvius** (1537–1612, Germany) in 1593, both friends of Kepler.
Finally, it reappeared in the trigonometric tables of **Bartholomaeus Pitiscus** (1612) and was accepted by **John Napier** in his logarithmic papers (1614 and 1619).

History of Number Representations

Our decimal system of numerals involves three distinct ideas:

- *There are only* 10 *symbols to write any number. The choice of the base* 10 *is due to the fact that our ancestors made their family accounts on their fingers or on their toes [the Babylonians, however, used the base* 60 *and the Mayas — the base 20].*
- *The representation of numbers employs the principle of local value, i.e., the position of any digit in the number determines its value.*
- *There is a special symbol (zero) representing a vacant position (no number). It seems that the Maya knew the use of it, but they did not think of the decimal system.*

All sizable calculations in the ancient world were performed with the aid of some kind of abacus. A written number representation was needed for record purposes only.

The earliest method of recording numbers — either by writing or by notches on a tally stick — was simply to make the requisite number of strokes. This procedure sufficed for small numbers. It was supplemented as early as the first Egyptian Dynasty (ca 3400 BCE) by the use of an additional symbol for ten. Further symbols were introduced for 100 *and* 1000 *and the method of grouping by tens was a feature of most of the early civilizations of the Mediterranean. In some cases (Etruscan, early Greek and Roman), additional symbols for* 5 *and* 50 *were incorporated for brevity.*

The Greeks used two number representations. Some time in the 3^{rd} *century BCE, they abandoned their Roman-type notation in favor of another — known as the Alexandrian; the numbers* 1 *to* 9 *were represented by the first nine letters of the Greek alphabet, the numbers* $10, 20, \dots, 90$ *by the next nine letters, and the numbers* $100, 200, \dots, 900$ *by the next nine letters. (The Greek alphabet contained* 24 *letters, so* 3 *additional symbols were borrowed from other alphabets). The notation was extended by various artifices to enable numbers greater than* 999 *to be represented. This system was in fact used for business purposes in the Byzantine Empire until its collapse in* 1453. *All such number representations are non-positional; the position of any symbol in the group is without numerical significance. Thus* 183, *for example, is represented in the Roman system as CLXXXIII, but the order of the symbols* *is irrelevant*[14]*. The notation merely expresses the fact that* 183 *is the sum of hundred, on fifty, three tens and three units.*

Apparently, the first people to use a positional system for writing numbers were the Babylonians. They employed the rather odd radix of 60*, which we still retain in our method of expressing* angles *and* time*. The Babylonians had separate symbols for* 1 *and* 10*, and also one for* 100 *which was seldom used. The symbol* $\vee$ *for* 1 *served also for* 60*, for* $60 \times 60 = 3600$*, and in general for any power of* 60*; while the symbol* $<$ *for* 10 *also served for 10 multiplied by any power of* 60*. It seems that the number of powers of* 60 *in any particular case had to be deduced from the context.*

The commercially minded Babylonians were the great computers of antiquity, and modern research enables us to appreciate the extent of their achievements. For example, they extended the positional notation to deal with fractional numbers*, and some later Babylonian records even contain a symbol for zero. So far there is no evidence that this symbol was used in computation.*

The Mayan civilization of Central America, with its highly developed observational astronomy and its preoccupation with the calendar, also used a positional notation. It was more highly developed than the Babylonian, although it was encumbered with a clumsy mixture of radices: 5, 20, and 360*. The Mayas even had a symbol, resembling a half closed eye, for denoting zero.*

Although the first steps towards the use of a radix notation were taken by the Babylonians in the 3^{rd} *millennium* BCE*, the logical culmination of this approach was not reached for another* 2000 *years. If we leave aside the Mayas on the other side of the world, the credit for this achievement (which cannot be precisely dated) must be given to the Hindus.*

The earliest preserved examples of our present number symbols are found on some stone columns erected in India about 250 BCE *by King Aśoka. Other early examples in India, are found among records carved about* 100 BCE *on the walls of a cave in a hill near Poona, and in some inscriptions of about* 200 CE*, carved in the caves at Nasik. These early specimens contain no zero and do not employ positional notation.*

Probably about 600 CE*, the Hindus found a way of eliminating place names. They invented a symbol* sunya *(meaning* empty*), which we call* zero*. With this symbol, they could write "105" instead of "1 sata, 5". This revolution must have been effected prior to* 800 CE*, for the Persian mathematician*

[14] The late Roman use of the *subtractive form* (e.g. IV instead of IIII) provides an exception to this statement. The absolute position of the pair of symbols I and V is not important, but the relative position is.

Al-Khowarizmi describes such a complete Hindu system in a book of 835 CE*. Thus, the Hindu mathematicians took the two concepts (both known much earlier) of the positional representation of numbers and the decimal scale, and added their own contribution — the concept of* zero *as one of the basic digits.*

This recognition of the need to provide a special symbol to represent an empty column in the abacus *— was a crucial step. It provided the world with a flexible and convenient notation whereby any number, however large, could be represented uniquely by an ordered sequence of symbols drawn from a set of ten. It set the stage for the development of arithmetic during the next few centuries.*

By the 7^{th} *century* CE*, the focus of our interest shifts to the Arabs, who by then had established a vast empire with its capital at Baghdad. The Arabs had substantial commercial dealings with India: they found the Hindu merchants using the decimal notation and soon adopted it themselves. We know, for instance, that some Indian astronomical tables in which decimal digits are employed, were brought to Baghdad and translated into Arabic in the year* 773*. By the end of the* 8^{th} *century, the Arabs had absorbed the main body of Indian mathematics; during the following century they became acquainted with the works of the Greek masters.*

The transfer of the mathematical lore from India to Baghdad was effected by ***Al-Khowarizmi****, who visited India in 830* CE *and then based his algebra treatise on the work of* ***Brahmagupta****. His book was the main source whereby the decimal notation was introduced, some 300 years later, into the West. At that time no clear distinction was made between the disciplines now known as arithmetic and algebra. The new arithmetic — that is to say, the arithmetic based on the* Hindu-Arabic notation *instead of the Roman — was indeed known for several centuries as* algorithm*, or the art of Al-Khowarizmi.*

The Hindu-Arabic mathematics, and with it Greek mathematics as well, diffused slowly to Western Europe via Spain. The Moorish rule in Spain attained its Zenith in the 10^{th} *and* 11^{th} *centuries, but Islamic culture was carefully guarded from Christians and few breaches were made before the* 12^{th} *century.*

One of the first Christians to penetrate the Muslim curtain *was the monk* ***Adelard of Bath****, who disguised himself as a Muslim and studied at the University of Cordova. In about* 1120 CE *he translated some of the works of Al-Khowarizmi and Euclid from the Arabic into Latin.*

The earliest coin bearing the Hindu numerals is one with an Arabic legend struck in 1138 *to commemorate the reign of Roger of Sicily. But the conditions prevailing in Sicily, where Byzantines, Latins and Moslems met on an equal footing, were too exceptional to be representative of Western Europe.*

However, by the end of the 12^{th} *century a small élite was apparently familiar with the new system.*

A pioneer in spreading the new knowledge in Europe was the mathematician ***Leonardo Fibonacci****. His father was sent by his fellows merchants to control a custom house in Barbary, and Leonardo grew up in an Arab cultural environment and became acquainted with the work of Al-Khowarizmi. He returned to Italy as a young man, and in* 1202 *he published his* liber Abaci *in which he explained the Arabic system "in order that the Latin race might no longer be deficient in that knowledge".*

Leonardo was a vigorous propagandist for the use of Arabic numerals in commercial affairs. By the middle of the 13^{th} *century, a large proportion of Italian merchants were employing the new system alongside the old. The changeover was, of course, not achieved without some opposition. In* 1299*, for example, an edict was issued at Florence forbidding the bankers to use the infidel symbols!*

Outside Italy the new notation gained ground more slowly, and merchants throughout most of Europe continued to keep their accounts in Roman numerals until the middle of the 16^{th} *century. The Arabic system was, however, in general use for* scientific *purposes throughout Europe by about the year* 1400*. While no Arabic numerals are to be found in English parish registers or Manor Court rolls before the* 16^{th} *century, a popular account of the new algoristic arithmetic entitled* The Craft of Nombrynge *appeared as early as about* 1300 CE *— one of the first books to be written in the English language.*

Although the new decimal system was a time- and labor-saving invention of the first magnitude, more than 1000 *years elapsed between the discovery and its general acceptance: not until the beginning of the* 17^{th} *century was it finally established in civilized Europe. Even then there were still learned doctors and professors who claimed that the Roman letters were much clearer than the Hindu numerals. Was it not much simpler to write* CCCXLVIII *than 348?*

The Hindus had made to mankind a gift of inestimable value. No strings of any kind were attached to it, nor was the suggested improvement entangled with any sort of religious or philosophic ideas. Those proposing to use the new numerals were not expected to make any disavowal or concession; nor could their feelings be hurt in any way. They were asked simply to exchange a bad tool for a good one. The history of our numerals is but one example, among so many others, of the difficulty of overcoming the enormous inertia of rested traditions.

The advantages of the new system were so great that its universal adoption was only a matter of time. The invention of printing hastened the process.

The first manual on arithmetic to come off the press of Renaissance Italy was printed in Treviso, Venice, in 1478.

The 16th and 17th centuries saw a number of important advances in the technique of practical calculation; mathematical rigor came later. Arithmetical procedures were simplified, additional signs were introduced and the decimal notation was extended to represent fractions. The introduction of the decimal point was finalized at the turn of the 17th century, just before the appearance of logarithms.

Such is the story of the representation of numbers. One of its most arresting features is the length of time that elapsed (at least 3000 years) between the coming into use of the abacus, a concrete embodiment of the positional decimal notation, and the introduction of the same system for the representation of numbers in writing. The whole sequence of events provides a striking illustration of the importance of notation in mathematics.

Even the Greeks, with their unrivaled intellectual prowess, could make little progress in arithmetic because of the unsuitable number representations with which they were burdened. Why did the Greek miss the crucial idea which appears so simple to us now?

A partial answer may be attempted in terms of the social and economic climate of classical Greece, which emphasized the gulf between theory and practice, between the intellectual and the artisan. The point, however, is a wider one, and can be applied to all the ancient civilizations of the Mediterranean and Near East. The very efficiency of the abacus as a computing tool weakened the practical need for an efficient written number representation which would facilitate arithmetical calculations. The calculations of everyday life could be carried on quite satisfactorily with the aid of the abacus. The written symbols were used merely as labels for recording the results. If the records were somewhat cumbersome, no great harm was done. The Greek philosopher with an interest in mathematics could happily devote himself to geometry, with its superior aesthetic and intellectual fascination.

So the Greeks missed their opportunity and it was left to the Hindus to take the crucial step. It is interesting to note that the Hindus and the Arabs made comparatively little use of the abacus; so much more acute was therefore their need for an effective written *number representation.*

The utilitarian motive appears, indeed, to dominate the situation throughout. The main stimulus to the spread of the new notation throughout Europe came from the merchants and traders, with the 'establishment', both lay and clerical, usually fighting a rearguard action against the forces of change.

1584–1603 CE **Walter Raleigh** (1552–1618, England). Poet, historian, scholar, soldier, navigator and explorer. One of the most flamboyant characters in the colorful reign (1558–1603) of Elizabeth I. Raleigh sent an expedition which explored the North American coast from Florida to North Carolina (1584) and named the coast north of Florida "Virginia". He succeeded in introducing potatoes and tobacco into England and Ireland.

He fitted out an expedition to seek the fabulous wealth of Guiana (gold mines), explored the coasts of Trinidad and sailed up the Orinoco river (1595).

Raleigh's daring expeditions to the New World, along with his quick wit, handsome face and ostentatious gallantry, made him a favorite with the Queen, but after the accession of James I (1603) Raleigh fortunes changed. He was accused of treason, committed to the Tower of London and executed in 1618.

1585–1641 CE **Pedro Teixeira** (ca 1570–1650, Portugal). Explorer and author. One of the greatest travelers of his age; circumnavigated the globe during 1585–1601 and commanded an expedition that made the first documented round trip voyage up the *Amazon* (1637–1638) from Pará to Quito and back.

Born in Lisbon of *Marrano parents*. A man of education and a close observer, he traveled on his first journey for 18 months (1585–1586) through the Philippines, China, the Americas and finally back to Lisbon (1601). His second journey took him to India, Persia and other parts of the Orient (1603–1607). He then settled in Antwerp and published a detailed account of these travels, *Relaciones de Pedro Teixeira* ... (Antwerp, 1610), containing data long considered authoritative. If was translated into French in 1681 and the first English version appeared in 1708-10. A complete English translation, *The Travels of Pedro Teixeira*, was published in 1902. The book is still held to be one of the most important sources of information about the Orient at the beginning of the 17^{th} century.

Teixeira arrived in Brazil in the early 1620s and led successful forays against the English and the Dutch. In July 1637, at the request of Philip III of Portugal (Philip IV of Spain), he undertook a journey of exploration in the country. In what was to be his last expedition, Teixeira set our from Pará (Belém) with a party of 2,000 men and *made the first*[15] *documented continuous voyage* up the *Amazon*, finally reaching Quito after an adventurous trip

[15] The first European to see the Amazon (1500) was the Spanish explorer **Vicente Yanẽz Pinzon** 1460–1523), who during 1487–1500 explored the coast of Brazil. Another Spaniard, **Francisco de Orellana** (c. 1490–c. 1546) let the first exploration of the river by a European. His expedition followed the Amazon from

lasting ten months. In the course of this journey he extended the boundaries of Brazil and established a line of demarcation between the Spanish and Portuguese possessions in South America.

Teixeira returned to Europe (1640) and settled in Antwerp, where he reverted to Judaism. A description of his expedition to the source of the Amazon is found in the *Nuevo descubrimiento del Gran Rio de la Amazonas* (1641).

1588 CE **Thomas Harriot** (1560–1621, England). Mathematician, astronomer and geographer. One of Britain's greatest mathematical scientists before Newton. He remained comparatively obscure, because he did not publish his work during his lifetime. His achievements are summarized as follows:

- First European to consider the idea of a binary number system.
- In algebra, he introduced the signs for greater than (>), less than (<), and the raised dot (·) to signify multiplication.
- In optics, he discovered the sine law of refraction, ahead of the Dutch mathematician Willebrod van Roijen Snell.
- Harriot made telescopes in the same year Galileo did, and he used them to observe the moon, sunspots, comets and the satellites of Jupiter.
- Investigated the ballistic trajectory of a projectile under the influence of gravity, a decade before Galileo did. He concluded that the path was a parabola.
- Was interested in the atomic theory of substances. He believed in the hypothesis that substances consist of atoms was plausible, and capable of explaining some of the properties of matter.

His writings contain the following propositions (in his own Elizabethan style and spelling):

(1) "*The more solid bodies have Atoms touching on all Sydes*".

(2) "*Homogeneall bodies consist of Atoms of like figure, and quantitie*".

(3) "*The waight may increase by interposition of lesse Atoms in the vacuities betwine the greater*".

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the mouth of the Napo River in Peru to the Atlantic Ocean (1541–1542).

Orellana took part in the conquest of Peru under Pizarro, and Pinzon Commanded the Ninã on Columbus' first voyage. His brother **Martin Alonso Pinzon** (1441–1493) commanded the Pinta on that voyage, and a third brother **Francisco Martin Pinzon** (1440–1493) was master of the Pinta under Martin Alonso.

(4) "*By the differences of regular touches* (*in bodies more solid*), *we find that the lightest are such, where euery Atom is touched with six others about it, the greatest* (*if not intermingled*) *where twelve others do touch euery Atom*".[16]

Harriot was born in Oxford. He studied at St. Mary Hall, Oxford and received his bachelor of arts there in 1580. He then became tutor and scientific adviser to Sir Walter Rayleigh, who appointed him in 1585 to the office of geographer to the second expedition to the newly founded colony of Roan Island in what is now North Carolina (Harriot published an account of this expedition in 1588). On his return to England in 1587, he resumed his mathematical studies and secured the patronage of Henry Percy, Earl of Northumbria, which yielded him a yearly pension of £300, on which he lived.

But Henry was suspected of complicity in the gunpowder plot and in 1606 was jailed. Harriot remained with his patron, and illness and political turmoil prevented him from completing the promising projects he has undertaken. Given more favorable circumstances, he might have become known as the inventor of analytic geometry or as one who solved the rainbow problem.

Harriot was one of the first algebraists who occasionally placed a purely negative quantity by itself on one side of an equation. **Viète** (1600) discarded negative roots of equations. Indeed we find few algebraists before and during the Renaissance who understood the significance of negative quantities. **Fibonacci** (1202) seldom used them. **L. Pacioli** (1494) stated the rule that "minus times minus gives plus", but applied it only to the development of the product $(a-b)(c-d)$.

The first use of + and − as symbols of algebraic notation was due to the Dutch mathematician **van der Hoecke** (1514). **Stifel** (1544) spoke of

[16] The structure in which every atom is in contact with six others about it that Harriot had in mind, is probably the simple cubic arrangement; in this arrangement of atoms the unit of structure is a *cube* that contains one atom, which can be assigned the coordinates $(0,0,0)$. Each atom is then in contact with six other atoms, which are at the distance d from it. The volume of the unit cube is accordingly d^3. If the mass of the atom is M, the density for this arrangement is $\frac{M}{d^3}$.
The denser structure referred by Harriot, where twelve atoms are in contact with each atom, is the *cubic closest-packed arrangement*. The cubic unit of structure for this arrangement contains four atoms. Its edge a is equal to $d\sqrt{2}$ and its volume to $2\sqrt{2}d^3$. The mass contained in the unit cube is $4M$ and the density is accordingly $\sqrt{2}\frac{M}{d^3}$, that is 41 percent denser then the simple cubic packing. Harriot had apparently discovered that there is no way of packing *equal* hard spheres in space that gives a greater density.

numbers which are "*absurd*" or "*fictitious below zero*". However, these ideas remained sparse, and until the beginning of the 17^{th} century, mathematicians dealt exclusively with absolute positive quantities.

As regards the recognition of negative roots, **Cardano** (1545) and **Bombelli** (1572) were far in advance of all writers of the Renaissance, including **Viète**. Yet even they mentioned these so-called false or fictitious roots only in passing, and without grasping their real significance and importance. On this subject Cardano and Bombelli had advanced to about the same point as had the Hindu **Bhaskara** (1150), who saw negative roots, but did not approve of them.

The generalization of the concept of algebraic quantity, so as to include the negative, was an exceedingly slow and difficult process in the development of algebra.

1588 CE The Spanish *Armada* was defeated by the English fleet under Howard of Effingham, Francis Drake and John Hawkins.

How Thomas Digges defeated the Spanish Armada

It was in England that the two streams of Nicolas of Cusa's influence – scientific method and the new infinite cosmology – first merged. England had nurtured its own scientific tradition from the time of Bacon, and English scholars and politicians kept abreast of the latest developments in Italian philosophy. The practical impetus for astronomical and general scientific research was stronger in England than anywhere else.

After the feudal nobility had killed themselves off in the War of the Roses, a collateral royal line, previously involved in trade rather than landholding, came to power with Henry VII. By the time Elizabeth became Queen in 1558, English navigation was in a state of fevered expansion, attempting to wrest control of trade from Catholic Spain.

Elizabethan England, recently freed from the intolerance of Mary's rule, welcomed that antihierarchical and anti-authoritarian teaching of the Copernican system. ***Thomas Digges****, a leading English astronomer, became the* *first to popularize Copernicus' ideas to a broad audience, writing a book about it in English, not scholarly Latin, in 1576.*

Already in 1572, Digges and other astronomers had studied the supernova of that year, showing that the heavens do in fact change, contrary to tradition – a sight visible to all. Indeed, Copernicus' ideas, backed by Digges' prestige as a leading scientist, became the property of the common man.

Digges synthesized Copernicus' and Nicholas of Cusa's work, proclaiming the universe to be infinite, populated with innumerable suns and worlds. But above all he explicitly criticized the ancients' method:

"I have perceived that the ancients progressed in reversed order from theories, to seek after true observations, when they ought rather to have proceeded from observation and then to have examined theories."

In a country where free labor was increasingly drawn into manufacture, and the need for both technological advances and an educated work force became acute, Digges championed the idea that scientific and technological advances are welded together, and that scientific knowledge must become common to all.

Since technological advance would be most rapid when the common workers had combined scientific knowledge with practical experience, Digges vowed to write all his work in English. Digges and others began a series of practical scientific manuals aimed at the widest audience. By 1589 publicly sponsored scientific lectures drew crowds of artisans, soldiers, and sailors eager for knowledge.

The conflict between the old and the new cosmologies was not settled by scholarly argument, but by the battles of the old and new societies – embodied in the struggles of nations. Protestants, in manufacturing Holland, revolted against its Catholic imperial ruler, Spain; and in 1584 the main Protestant power, England, allied with Holland. The Spanish empire was based on forced labor – serfs at home and serfs and slaves in the huge empire of the New World. The English and Dutch relied mainly of free labor.

The Copernican scientific worldview gave not only ideological justification to the Protestant side, but also decisive technological advantage. By synthesizing theoretical science with craft skill, English industry moved ahead of Spain in critical areas, such as the casting of naval artillery, producing lighter guns with greater range and accuracy.

The Copernican revolution had also meant throwing out Aristotelian physics – based on the idea that moving objects sought their "proper" place in the hierarchy. This had significant application in the science of ballistics. Aristotle had taught, and the medieval scholars accepted, that a projectile flew upward in a straight line, then fell vertically to earth. Leonardo and his *successor in engineering, Tartaglia, showed by experiment that the trajectory is a curve, and compiled a gunnery table linking the elevation of the gun to the range of the shot.*

Digges and other English scientists systematized their results, producing widely read manuals of naval gunnery. English ships, manned by draftees drawn from the artisan and working classes, had by 1588 both seamen and officers on board trained in the basics of the new ballistics. Spain, by contrast, had no use or interest in the new sciences. Nor could their uneducated sailors use them[17].

The related differences in social structure, technology, and training proved decisive when the Spanish Armada sailed to invade England. The English ships mounted mostly small guns, called culvetines, whose effective range was one thousand meters. The Spaniards had crude cannons, effective only at point-blank range – that is, before the shot began to fall significantly, perhaps three hundred meters. With this and other advantages the English battered the Spaniards at long range, while the Spaniards' ammunition fell far short of the targets. For one hundred thousand cannonballs fired, the Spaniards killed one English officer and two dozen seamen, sinking no vessels. The English, with about half as many shots and lighter guns, sank or disabled seventeen Spanish ships and inflicted thousands of causalities. When the Spanish ran out of ammunition, the English chased the shattered Armada out of the channel.

Thus, in a very practical way, the superiority of the empirical worldview was demonstrated – with cannon, not with debate. In fact, the defeat of the Armada determined which worldview would triumph, since it determined which society would survive.

The revolutionary changes of the last quarter of the eighteenth century were not universally hailed, and neither were the new scientific theories. The capitalists who ruled Great Britain owed their power to the social revolutions of the seventeenth century and the industrial revolution of the eighteenth, but they had no desire to lose that power in further social upheavals. Great Britain became the major foe of all social change, fearing the development of rival industrial powers abroad and a continual evolution of social structure at home. From Britain, religious and philosophical replies were launched against the ideas of human and natural progress.

***Thomas Malthus**, rebutting the **Marquis de Condorcet**, the French theorist of progress, argued that population growth will always outstrip agricultural production, condemning most people to hunger and blocking material progress. Geologist **John Williams** blasted **Hutton**'s theories on theological grounds. Hutton's "wild and unnatural notion of the eternity of the earth*

[17] Had they not expelled the *Jews* in 1492, they could have fared better in 1588!

leads first to skepticism and at last to downright infidelity and atheism. If we once entertain a firm persuasion that the world is eternal, and can go on itself in the reproduction and progressive vicissitudes of things, we may then suppose that there is no use of the interposition of a Governing Power," he wrote, concluding that "all rebellions soon end in anarchy, confusion and misery and so does our intellectual rebellion."

But these efforts proved generally unsuccessful: in the course of the first half of the nineteenth century, Europe continued to be rocked by repeated popular revolutions, and the industrial revolution transformed British society as well.

While many of the social gains at the height of the revolution were subsequently rolled back, the fundamental outlook and goals of society had been irreversibly transformed. The English government's sponsorship of scientific research put English science far ahead of that of any other country; this, together with England's swift economic development, propelled it a century later into the industrial revolution.

The scientific revolution was thus not an inevitable process, a natural outgrowth of human intellectual development. It was the result of a fierce social conflict, in which cosmological questions were matters of life or death for individuals and whole societies.

Certainly the people of the time did not think that the defeat of Spain, the victory of England and Holland, and later the victory of the English revolution were at all inevitable. Yet without those victories, the scientific revolution would certainly have not occurred. Only the open society born in the sixteenth and seventeenth centuries could have nurtured the infinite unlimited cosmos of modern science. And only such a worldview could have given the new society the moral and material strength to prevail.

1588–1613 CE **Pietro Antonio Cataldi** (1548–1626, Italy). Mathematician and astronomer. Took the first steps in the theory of *continued fractions* and made contributions to the early theory of numbers, especially Mersenne primes and perfect numbers. Wrote a number of mathematical works.

Cataldi was born in Bologna and taught mathematics and astronomy in Florence, Perugia and Bologna, where he died.

1588–1623 CE **Gaspard Bauhin** (1560–1624, Switzerland). Physician, botanist and anatomist. Introduced (1623) a binomial system of nomenclature for botany. One of the first to describe *ileocecal valve* (1588), known as the *Bauhin valve*.

His book *Pinax Theatri Botanici* (1623) contains classification and description of over 6000 plants and was much used by *Linnaeus*.

Bauhin was born in Basel. Studied at Padua, Montpellier, Paris and Tübingen. Professor at Basel University.

1589–1606 CE **Giovanni Battista, della Porta** (1535–1615, Italy). Natural philosopher and inventor. Made the first distinct step from Hero's *aeolipile* toward the steam engine, by using steam instead of air as the displacing fluid (1601). In his *Magia Naturalis* (1589) he describes a number of optical experiments, including a description of the *camera obscura* to which he proposed to add a convex lens. He claimed to be the inventor of the telescope although he does not appear to have constructed one before Galileo. He was however first to recognize the heating affects of light rays (1589).

The *Inquisition* banned the publication of his works for a number of years. Although Porta made important physical observations, much of his work was from point of view of magic and alchemy.

Della Porta founded in Naples the *Accademia Secretorum Naturae* and in 1610 became a member of the *Accademia dei Lincei* at Rome.

1592–1613 CE **David (ben Shlomo, Seligman) Gans** (1541–1631, Germany and Prague). Chronologist. In his book *Zemach David* compiled a chronology of ancient and medieval events up to 1592. Author of textbooks on astronomy, mathematics, geography and cosmography.

Studied under the **Maharal of Prague** and interacted with **Kepler**, **Regiomontanus** and **Tycho Brahe**.

Gans was born in Lippstadt (Westphalia) and died in Prague.

1591–1639 CE **Tommaso Campanella** (1568–1639, Italy). Italian Renaissance philosopher. A precursor of modern empirical science. His work was a source of inspiration for **Descartes**, **Spinoza** and **Leibniz**. Many of his ideas are similar to those of modern-day existentialists. Though neither an original nor a systematic thinker, he stands in the uncertain half-light which preceded the dawn of modern philosophy and science.

Campanella was born in Stilo, Calabria. Before he was 13 years of age he had mastered nearly all the Latin authors presented to him. He entered the Dominican Order (1582), but in 1599 was sentenced to life imprisonment during the Spanish rule for political plotting and heresy. During his stay in prison he wrote a valiant vindication of Galileo. After 30 years of incarceration, Campanella succeeded in escaping to France, where he remained for the remainder of his life under the aegis of Cardinal Richelieu.

His philosophy is a blend of medieval thought combined with the methods of modern science: he rejected Aristotelian scholasticism and insisted that knowledge should be based on close observation of the natural world. His views were strongly influenced by **Bernardo Telesio** (1509–1588, Italy) and also by those of **Nicolas of Cusa** (1401–1464). Telesio founded and directed the *Academia Telesiana*, a school in Naples that propagated the scientific approach to knowledge and advanced the scientific movement in the Renaissance.

1591–1603 CE **Prospero Alpini** (1553–1616, Italy). Physician and botanist. Studied plants for their therapeutic medicinal use. Introduced (1591) the first European descriptions of the coffee bush and the banana tree. First to establish the sexual difference of plants.

Alpini was born at Marostica, in the Republic of Venice. In his youth he served in the Milanese army and in 1574 went to study medicine at Padua, taking his doctor's degree in 1578. To extend his knowledge of exotic plants he traveled to Egypt (1580) as physician of the Venetian consul in Cairo. On his return (1583) he resided in Genoa and then (1593) was appointed professor of botany at Padua. Published *De Plantis Agyptic Liber* (1592).

1595–1620 CE **Francis Bacon** (1561–1626, England). A forerunner of the scientific revolution. Lawyer, essayist, statesman and philosopher. The first union in English literature of the man of letters and the man of science (there have been only a few striking examples since).

Bacon was not himself an active scientist, yet he can be likened to a signpost which shows the way. He had an enduring influence on an entire generation of great scientists. His chief works include: *Essays* (1597) — concise expressions of practical wisdom and shrewd observations; *Advancement of Learning* (1605) — a survey in English of the state of knowledge (incomplete project); *Novum Organum* (1620), in Latin, key to his system for the new systematic analysis of knowledge, intended to replace the deductive logic of Aristotle with inductive methods in interpreting nature. In his utopian tale *New Atlantis*, published posthumously (1627), he predicted robots, telephones, tape recorders and electric motors.

Until the time of Bacon, man had more or less 'drifted' in the natural world. His culture had grown up without conscious self-examination or attention to the fact that man might improve his own society through science. People decided all questions not by investigating the observable facts, but by appealing to a priori reasoning, received folk wisdom, religious dogmas, and the teachings of infallible authorities, living and dead – for instance in medieval Europe, the Church fathers and Aristotle. Education in Bacon's day was largely confined to metaphysical arguments along with the readings of Greek and Roman classics. At Cambridge, learning was largely pretense that all was of the past[18]. Men endlessly wove and rewove a gossamer webs of ideas derived from Greek and Roman sources. The world of Bacon and Shakespeare was only semiliterate, steeped in religious contentions, with its gaze turned backwards in wonder upon the Greco-Roman past.

Bacon waged a vigorous battle against the deductive method of scholasticism. He went much further beyond that to outline, with unique prophetic vision, the future of science and its role in the affairs of man. Bacon:

- Recognized that the triumph of the experimental method demands the thorough institutionalization of science *at many levels of activity and ability*. He eliminated reliance upon the rare elusive genius as a safe road into the future. It involved of too much risk and chance to rely upon such men alone.
- Grasped the cumulative nature of culture and the fact that inventions multiply in a favorable social environment. Science and its traditions had to be transmitted through the universities, and its efforts had to be *publicly supported*. He studied ways by which Cambridge and Oxford might be encouraged toward fostering laboratories and other educational tools.
- Recognized the value of the history of science.
- Observed that the lower organisms might reveal secrets of life which in the higher organisms lay more hidden. (This biological observation, and others in the social sciences, were made too early. By the time these subjects had emerged as recognized disciplines, his far-reaching, anticipatory insights were submerged in a welter of new books and newer phrasing.)
- Foresaw the necessity of using mathematics in the examination of nature.
- Entertained the idea of the universe as a problem to be solved.

For all his interest in scientific inquiry and the proper pursuit of science, Bacon missed practically all the most important developments of his own

[18] Even toward the close of the 19^{th} century, the greatest universities in England were still primarily devoted to the classical education of gentlemen!

time. He was unaware of the work of Kepler; and, though he was a patient of Harvey, did not know of the doctor's researches on the circulation of the blood. The rejection of the centrality of the syllogism in what still passed for natural philosophy in his day, led him to underestimate the function of *deduction* in scientific inquiry. In particular, he had little appreciation of the mathematical methods that were actually developing in his time. The role of *induction* (in itself a notion that was not new — Aristotle had already used it) in the framing of hypotheses is but one facet of the scientific method. Without the mathematical and logical deductions which lead from the hypotheses to concrete, testable predictions, there would be no knowing what to test against experiment.

Bacon was born in London, the son of an important government official. He attended Trinity College, Cambridge, from 1573 to 1575. In 1576, he joined the staff of England's ambassador to France. Bacon was elected to Parliament in 1584 and knighted in 1603. He held several high government positions[19] until 1621, when he was framed[20] and convicted of taking bribes and briefly imprisoned.

Bacon was never free of financial insecurity. In a mercenary age he lacked the means to buy advancement. Although the prestige of his final offices [Attorney General (1613); Lord Keeper (1617); Lord Chancellor (1618)] gave greater weight to his literary pronouncements and financed his publications, he was nonetheless a stranger in his own age — a civilized man out of his time and place. Even when one has measured the three sides of his triangular life (as man of letters, man of science and public servant), one is still at a loss to understand all the motives governing him in his contradictory actions.

Rumors persist that he did not die in the year 1626 but rather escaped to Holland[21]; that he was the real author of Shakespeare's plays; and that he was the unacknowledged son of Queen Elizabeth. These rumors are a measure of

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[19] In 1605, Bacon devised a code for sending secret diplomatic messages. Each letter of the alphabet was represented by a five-letter group of a's and b's. For example A=aaaaa, B=aaaab, C=aaaba, D=aaabb, ..., X=babab, Y=babba, Z=babbb.

[20] He became a victim of the conflict between King James I and his Parliament. Bacon's enemies, in frustration at their inability to vent their rage on the King, set to destroy the one man who had sought to temper the royal excess and preserve the state. Traditional homage was deliberately redescribed as bribery. King James advised him to avow his guilt and trust his protection to the Crown.

[21] He went to a farmhouse in a snowstorm to get a chicken to test his idea that snow could be used as a preservative instead of salt. The exposure to which the experiment subjected him caused his death soon after.

his power to captivate the curiosity of men — a power that has grown century by century since his birth in 1561.

In spite of certain mystifying aspects of his life, there is no evidence sufficient to justify these speculations, though a vast literature betokens their fascination and appeal.[22]

[22] For further reading, see:

- Eiseley, L., *The Man Who Saw Through Time*, Charles Scribner Sons: Ne York, 1973, 125 pp.
- Bacon, Francis, *The Essays*, Penguin Books, 1985, 285 pp.

Worldview VII: Francis Bacon

* * *

"Man can only conquer nature by obeying her".

* * *

"We are not to imagine or suppose, but to discover*, what nature does or may be made to do".*

* * *

"For the history that I require and design, special care is to be taken that it be of wide range and made to the measure of the universe. For the world is not to be narrowed till it will go into the understanding (which has been done hitherto), but the understanding is to be expanded and opened till it can take in the image of the world".

* * *

"I say without any imposture, that I... frail in health, involved in civil studies, coming to the obscurest of all subjects without guide or light, have done enough, if I have constructed the machine itself and the fabric, though I may not have employed or moved it".

* * *

"Science is not a belief to be held but a work to be done".

* * *

"Make the time to come the disciple of the time past and not its servant".

* * *

“Many parts of nature can neither be observed with sufficient subtlety, nor demonstrated with sufficient perspicuity without the aid and intervening of mathematics”.

* * *

“This third period of time will far surpass that of the Grecian and Roman learning only if men will employ wit and magnificence to things of worth, not to things vulgar”.

* * *

“Many things are reserved which kings with their treasures cannot buy, nor with their force command, their spies and intelligencers can give no news of them, their seamen and discoverers cannot sail where they grow”.

* * *

“Every act of discovery, advances the art of discovery”.

* * *

“Mere power and mere knowledge exalt human nature but do not bless it; we must gather from the whole store of things such as make most for the uses of life”.

* * *

“Books must follow sciences, and not sciences books”.

* * *

“If a man will begin with certainties, he shall end in doubts; but if he will be content to begin with doubts, he shall end in certainties”.

* * *

"The men of experiment are like the ant, they only collect and use; the reasoners resemble spiders, who make cobwebs out of their own substance. But the bee takes the middle course: it gathers its materials from the flowers of the garden and field, but transforms and digests it by a power of its own.

Not unlike this is the true business of philosophy [science]; for it neither relies solely or chiefly on the powers of the mind, nor does it take the matter which it gathers from natural history and mechanical experiments and lay up in the memory whole, as it finds it, but lays it up in the understanding altered and digested.

Therefore, from a closer and purer league between these two faculties, the experimental and the rational (such as has never been made), much may be hoped".

* * *

"That all things are changed, and that nothing really perishes, and that the sum of matter remains exactly the same, is sufficiently certain".

* * *

"Great discoveries appear simple once they are made".

* * *

"I take it, that all those things are to be held possible and performable, which may be done by some persons, though not by everyone; and which may be done by many together, though not by one alone; and which may be done in the succession of ages, though not in one man's life; and lastly, which may be done by public designation and expense, though not by private means and endeavor".

* * *

"It is not the pleasure of curiosity nor the raising of the spirit, nor victory of wit, nor lucre of profession, nor ambition of honor or fame, nor opportunity

for business, that are the true ends of knowledge. It is a restitution and reinvesting of man to the sovereignty and power which he had in the first state of creation".

* * *

"The technological arts have an ambiguous or double use, and serve as well to promote as to prevent mischief and destruction, so that their virtue almost destroys or unwinds itself. All natural bodies have really two faces, a superior and inferior. He who will not attend to things like these can neither win the knowledge of nature nor govern it".

* * *

"There is no excellent beauty that hath not some strangeness in the proportion".

* * *

"Truth is more likely to emerge from error than from confusion".

* * *

"Crafty men condemn studies, simple men admire them, and wise men use them".

* * *

"Knowledge is power".

* * *

History of Theories of Light I

The science of optics and optical devices embraces a vast body of knowledge accumulated over roughly 5000 years of the human scene. To view modern optics in full perspective one must trace the road that led us there. The complete story has myriad subplots and characters, heroes, quasi-heroes and an occasional villain or two. Yet from our vantage in time, we can discern 4 epochs in the history of optics:

A. The Beginning — Optics of Reflection and Refraction *(3000 BCE–1589 CE)*

The ancients were more familiar with optics than with any other branch of physics due to the fact that for the knowledge of external things man is indebted to the sense of vision in a far greater degree than to other senses. That light travels in straight lines (i.e. that an object is seen in the direction in which it really lies) must have been realized in very remote times. The antiquity of mirrors points to some acquaintance with the phenomena of reflection. The lens, as an instrument of magnifying object or for concentrating rays to effect combustion, was also known.

The cuneiforms of Sumer describe a highly sophisticated society more than 5000 years ago. These cuneiforms were written by pressing a stylus of bone or hard reed into a tablet of soft river mud. Some of the cuneiform letters are of the order of a millimeter in size and cannot be read (and written!) without a magnifying glass. **Henry Austin Layard** *(1885) excavated, amongst the ruins of the palace of King Sennacherib (705–681 BCE) of Assyria, a quartz magnifier: It seemed to be cut and polished to the shape of a plano-convex lens with $f \approx 10$ cm and could have been used as a magnifying glass, for both making the inscriptions and reading them.*

Another significant development in optics during this period is the use of metallic hand-held mirrors. Early mirrors were made of polished copper, bronze and later of speculum, a copper alloy rich in tin. Specimens have survived from ancient Egypt: a mirror in perfect condition was found along with some tools nearby the Pyramid of Sesostris II (ca 2000 BCE) in the Nile Valley. Exodus ***38****: 8 (ca 1200 BCE) recounts how* **Bezalel***, while preparing the ark and tabernacles, recast "the looking-glasses of the women" into a brass laver (a ceremonial basin).*

*The Bible also tells us that **Joshua** (**8**, 18) used a mirror to send a light signal (sun's reflected rays) to his ambush force, to rise and to take the city of Ai.*

Chinese philosophers (Mohists, Ca 479–381 BCE), studied the reflection of light from plane, concave and convex mirrors, and obtained empirical rules connecting the size and position of objects and images with the curvature of the mirror used.

*In the first systematic writings, of which we have any definite knowledge, the Greek philosophers: **Pythagoras**, **Empedocles**, **Democritos**, **Plato** and **Aristotle** speculated (550–350 BCE) over the nature of light and vision, evolving several theories. The aversion of Greek thinkers to detailed experimental inquiry stultified the progress of the science; instead of acquiring facts necessary for formulating scientific laws and correcting hypotheses, the Greeks devoted their intellectual energies to philosophizing on the nature of light.*

In their search for theory the Greeks were mainly concerned with vision: they sought to determine how an object was seen, and to what its color was due. Emission theories, involving the concept that light was a stream of particles, were formulated.

Their hypothesis was that the eyes emanate vision rays *and the returned rays create vision, a principle similar to that of modern day radar or sonar. As a result of the concept of the eye-ray, direction of arrows and the designation of the incident and reflected angles were reversed from what these are today*[23].

The Pythagoreans *assumed that vision and color was caused by the bombardment of the eye by minute particles projected from the surface of the object seen. The* Platonists *subsequently introduced three elements – a stream of particles emitted by the eye which united with the solar rays, and, after the combination had met the stream from the object, returned to the eye and excited vision.*

***Democritos** maintained that extremely small particles chip off from the object and go into the viewer's eye and imprinted there by the moisture in the eyes*[24]. ***Aristotle** argued against this theory because it could not explain the inability to see in the dark.*

The left-right reversal of the image of a vertical mirror, or the upside-down image of a horizontal mirror, aroused the curiosity of the Greek philosophers, but even Plato could not provide a satisfactory explanation. From a treatise an optics, the catoptrics, *assigned to **Euclid** (ca 300 BCE) by **Proclos***

[23] It took 1400 years before the direction of the arrows was reversed by **Alhazen** (1026).

[24] This is perhaps the origin of the corpuscular theory of light.

*and **Marinos**, we learn that the rectilinear propagation of light, the law of reflection (viz. the equality of the angles of incidence and reflection) were known to the Greeks. **Hero** of Alexandria attempted to explain both these phenomena by asserting that light traverses the shortest allowed path between two points*[25] *(150 BCE).*

The Greeks were also acquainted with the production of images by plane, cylindrical and concave and convex spherical mirrors. Reflections, or catoptrics, was the key-note of the Greeks explanations of most optical phenomena: it is to the reflection of solar rays by the air that Aristotle ascribed twilight, and from his observations of the colors formed by light on spray, he attributed the rainbow to reflections from drops of rain.

*A burning-glass (a positive lens) was alluded to by **Aristophanes** in his comic play The Clouds (424 BCE). The apparent bending of objects partly immersed in water is mentioned in Plato's Republic. **Archimedes** (250 BCE) used concave mirrors as burning-glasses. Certain elementary phenomena of refraction were studied by **Cleomedes** (50* CE*) and later by **Claudios Ptolemy** of Alexandria (150* CE*) who attempted to explain the 'coin-in-a-cup' experiment*[26] *of **Ctesibios** (50 BCE).*

Ptolemy measured the refractive effects of water and discussed refraction in the atmosphere. He tabulated fairly precise measurements of angles of incidence and refraction for several media and obtained the small-angle approximation to Snell's law, concluding that the ratio of the angles of incidence and refraction were constant. He also discussed the refraction of starlight by the atmosphere but held to the theory that vision is due to rays emitted from the eye. The quantitative law of refraction was unknown (in fact it was not formulated until the beginning of the 17th century).

*It is clear from the accounts of **Pliny the Elder** (23–79* CE*) that the Romans also possessed burning-glasses. Several glass and crystal spheres, which were probably used to start fires, have been found amongst Roman ruins, and a planar convex lens was recovered in Pompeii. The Roman philosopher **Seneca** (4 BCE–65* CE*) pointed out in his book Naturalium Quaestionum that a glass globe filled with water could be used for magnifying purposes. It is certainly possible that Roman artisans may have used magnifying glasses to facilitate very fine detailed work.*

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[25] A forerunner of Fermat's principle of least *time* (ca 1638).

[26] The apparent *elevation* of a coin in a basin, by filling the basin with water. Similarly, the Greeks sought to explain the apparent *bending* of the oar at the point where it met the water.

Seneca also observed the analysis of white light into the continuous spectrum of rainbow colors by transmission through prism. *His friend, the Emperor Nero (37–68 CE), may have been the first to use a* monocle, *employing an emerald lens to view bloody gladiator combats in the Coliseum. In Rome, during the first century CE mirrors were made of polished glass, behind which a sheet of silver was placed.*

Aristotle (ca 330 BCE) describes image projection in terms of the camera obscura[27]. *His concept involves a '*darkened box or chamber*' with a small hole in one side though which light is admitted. An inverted image of the scene is projected onto an interior wall, where it can be viewed and traced by an artist. From the opening passage of Euclid's Optics (ca 300 BCE), it would appear that the above phenomena of the simple darkened room were used by him to demonstrate the rectilinear propagation of light by the passage of sunbeams or the projection of the images of objects through small openings in windows.*

The first reference to persistence of vision *appears in De Rerum Natura (Book **4**, lines 771–810) by the Roman poet and natural philosopher **Titus Lucretius Carus** (98–55 BCE):*

"... when the first image perishes and a second is then produced in another position, the former seems to have altered its pose. Of course this must be supposed to take place very swiftly: so great is their velocity, so great the store of particles in any single moment of sensation, *to enable the supply to come up."*

Here Lucretius describes frame sequential animation almost 2000 years before the advent of motion pictures.

*All through the Dark Ages, optics lay dormant in Europe, but the center of scholarship shifted to the Arab world (where the scientific and philosophical treasures of the past were translated and preserved) and eventually extended at the hands of **Alhazen** (ca 1010–1030* CE*). He elaborated on the law of reflection, (putting the angles of incidence and reflection in the same plane normal to the interface), studied spherical and parabolic mirrors and gave a detailed description of the human eye as an optical instrument.*

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[27] The invention of this instrument has generally been ascribed to **Giovanni Battista della Porta** (ca 1558). However, all he seems really to have done was to popularize it. In southern climes, where during the summer heat it is usual to close the rooms from the glare of the sunshine outside, we may often see depicted on the walls vivid inverted images of outside objects formed by the light reflected from them passing through chinks or small apertures in doors or window-shutters.

By the latter part of the 13^{th} *century, Europe was only beginning to rouse from its intellectual stupor. Alhazen's work was translated into Latin and had a great effect on the writings of* ***Robert Grosseteste****, the Polish mathematician* ***Vitelo*** *(ca 1230–1275, Silesia), and the textbook of* ***John Peckham*** *(ca 1230–1292), the archbishop of Canterbury. All of these were influential in rekindling the study of optics.*

Their works were known to ***Roger Bacon*** *(1215–1294), who initiated the idea of using lenses for correcting vision and even hinted at the possibility of combining lenses to form a telescope. After his death optics again languished. Even so, by ca 1350, European paintings were depicting monks wearing eyeglasses, and alchemists had come up with a liquid amalgam of tin and mercury that was rubbed onto the back of glass to make mirrors.*

The great Italian artist, architect and scientist, ***Leonardo da Vinci*** *(1452–1519) followed up Alhazen's experiments and developed the pinhole camera. He indulged in the study of color, made analogy between sound and light waves and believed that light is a wave and color is determined by its frequency. In 1589, the Italian* ***Giovanni Battista della Porta*** *(1535–1615) published his treatise Magiae Naturalis in which he discussed multiple mirrors and combinations of positive and negative lenses. This work can be viewed as contributing to the theoretical preparation for the invention of the telescope in 1608.*

1596 CE **Ludolph van Ceulen** (1540–1610, Netherlands). A 'digit-hunter', at the University of Leyden, who calculated π to 32 decimal places. The value of π was therefore often named "*Ludolph's number*". His performance was considered so extraordinary, that the numbers were carved on his tombstone (now lost) in St. Peter's churchyard, at Leyden [he used the Archimedean method of in- and circumscribed polygons].

1596–1616 CE **Eliyahu de Luna Montalto** (1560–1616, Italy and France). Distinguished physician and medical researcher. Author of extensive medical writings dealing especially with the mind and the nervous system. Physician at the court of Maria de Medicis and Louis XIII, France.

Montalto was born in Castel Branco, Portugal in a Marrano family under the name Philippe Rodrigues. Studied medicine at the University of Salamanca and moved to Livorno, Italy (1596). He was summoned to the French Court in Paris at a period when Jews had been exiled from France for two centuries. On his return to Italy he was appointed to the chair of medicine at the University of Pisa, where he published his research in the fields of optics and medicine (1606). He returned formally to Judaism in Venice. In 1611 he was invited back to Paris to serve as the personal physician of the Queen with a special permission from the Pope to practice his own religion. Died of the plague in Tour, France and buried in Amsterdam.

1597–1613 CE **Andreas Libau** (Libavius, 1540–1616, Germany). Physician, alchemist and chemist. Wrote the first important textbooks in chemistry (*Alchemia*, 1597; *Syntagma*, 1611), in which he described a wide range of chemical methods and preparations such as: hydrochloric acid (HCl), sulfuric acid (H_2SO_4), tin tetrachloride ($SnCl_4$, 1605), ammonium sulphate [$(NH_4)_2SO_4$], and others. Wrote medical texts emphasizing the importance of chemistry for medicine (1606). He pointed out in 1597, before Steno, that the salts present in mineral waters could be ascertained by an examination of the shapes of the *crystals* left upon evaporation of the water.

Libau studied medicine at the University of Jena (1586–1591) and became a professor of history and literature there. He then practiced medicine at Rotenburg, serving also as superintendent of schools until 1607. He was among the first to introduce the study of science into the school curriculum.

He was a follower of **Paracelsus**, and as such belongs to the transition period from alchemy to chemistry. He is counted among the pioneers of the independent science of chemistry.

1599–1603 CE **Ulisse Aldrovandi** (1522–1605, Italy). Physician and naturalist. One of the founders of modern zoology. The results of his various researches were embodied in a *magnum opus*, which was designed to include everything that was known about natural history. The first three volumes, comprising his *ornithology*, were published in 1599, and a fourth, treating of insects, appeared in 1602. After his death a number of other volumes were compiled from his manuscript materials, under the editorship of several of his pupils, to whom the task was entrusted by the senate of Bologna.

1600 CE Pestilence and famine stroke Russia. Ca 500,000 people perished.

From Alchemy to Chemistry[28] (1530–1789)*, or – the Alchemists died poor*

Alchemy was one of the earliest forms of chemistry. This ancient practice originated amongst the followers of Lao Tsu in China and Pythagoras in Greece (6^{th} century BCE), and combined science, religion, philosophy and magic. Alchemy developed as men applied theories about nature to metalworking, medicine, and other crafts. As the practice of alchemy developed and moved Westward, Taoist ideas about chemicals were combined with Pythagorean number mysticism. Another strand of alchemical tradition came from the Egyptian embalmers.

In China, the early alchemists were searching for the elixir of life[29] *(a substance that would provide long or never-ending life and health). Chinese alchemy was passed on to the Hindus, who were more interested in using alchemical ideas to cure diseases. About 300–400 CE the Alexandrians supposedly used sorcery to convert base metals to gold.*

Eventually the Arabs put together the ideas from the East with the Alexandrian traditions of alchemy that had descended from the Pythagoreans. In this form of alchemy, astrological influences were important; chemical reactions were believed to be determined by the influences of the planets, the shapes of the vessels and numerology, and the elixir of life became mingled with the concept of a philosopher's stone *(an object whose presence would enable to transmute other metals into gold).*

Jabir Ibn Hayyan *(721–815, Baghdad) claimed that all base metals consisted only of brimstone (sulfur) and mercury. To make the metal less coarse, the sulfur had to be driven out. According to the alchemists, gold contained almost no sulfur. Arabian alchemists developed a theory in which different metals could be formed by combining mercury and sulfur in various proportions.*

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[28] The word *chemistry* probably originated in 400 BCE from the Greek word *chemeia*, which designated the art of metal working. At a later time, the Arabs added the prefix *al*. The new word *alchemy* signified the art of chemistry in general.

[29] Some of their accomplishments were remarkable: a woman (known as the *Lady of Tai*) was buried about 185 BCE in a double coffin filled with a brown liquid containing mercuric sulphide (HgS) and pressurized methane. There was no observable deterioration of her flesh when she was exhumed after more than 2000 years.

The alchemists also thought that bodies were made up of 'matter' and 'spirit', and they supposed that in some cases they could isolate the spirit by heating the body and condensing the exuded vapor. Thus they obtained alcohol, or 'spirit of wine', and hydrochloric acid, or 'spirit of salt'. In this way, the alchemists managed to obtain various practical results, including the first strong acids and the distillation of alcohol.

It has been estimated that in the past 2000 years over 100,000 tomes have been written by Western Alchemists. Who were the Alchemists? We know that ***Geber*** *(fl. 1350; Spain) and* ***Avicenna*** *(fl. 1020) were physicians and alchemists. In the Middle Ages,* ***Albertus Magnus*** *(fl. 1250),* ***Thomas Aquinas*** *(fl. 1260) and* ***Raymond Lully*** *(fl. 1280) were adept alchemists.*

Arabs and Moors invaded and conquered most of Spain during the 700's. Spanish scholars did not, however, translate Arabic alchemy books into Latin until the 1100's. These translations introduced alchemy to England and the rest of Europe.

The English philosopher and alchemist ***Roger Bacon*** *(1214–1294) laid the foundation for the experimental method of chemical research. Unlike the early alchemists, Bacon planned his experiments and carefully interpreted his laboratory work.*

During the Renaissance, the West absorbed Arabic alchemy along with more substantial Arabic science. By the 16^{th} century, alchemy was being practiced mainly in Europe; some alchemists and physicians began to apply their knowledge of chemistry to the treatment of disease.

Since ancient times, man had known how to prepare and use various drugs. He did so, however, without understanding how the drugs worked. The medical chemistry of the 15^{th} and 16^{th} centuries is called iatrochemistry (from the Greek iatros = physician).

*Iatrochemists were the first to study the chemical effects of medicines on the body [**Paracelsus**, 1530; **Libau**, 1597; **Helmont**, 1620]. As scientists learned more about medicine, they gradually lost interest in the impractical theories of alchemy.*

However, the tradition of alchemy persisted well into the 18^{th} century: ***Newton*** *(1642–1727) spent much of his later life trying to find the philosopher's stone, and may have gone mad from mercury poisoning caused during his experiments. Other most distinguished 17^{th} century scientists,*

G.W. Leibniz *(1646–1716) and* ***Robert Boyle*** *(1627–1691), "the father of modern chemistry", clearly accepted the theory of alchemical transmutation.*[30]

Finally, ***Lavoisier*** *(1743–1794) put together a scientific view of chemistry that effectively abolished the alchemical tradition that had persisted for over two millennia.*

The ancient dream of the alchemists was realized in 1941 CE through the artificial production of several isotopes of gold (Atomic number = 79) from Mercury (Atomic number = 80; Atomic mass = 201) by ***Sherr****,* ***Bainbridge*** *and* ***Anderson****, via a nuclear reaction initiated by fast-neutron bombardment of mercury.*

The 17th century Often is known as the 'age of genius' – and this for at least two reasons. The century *effectively invented far more than its share of scientific instruments: the thermoscope, the telescope, the microscope, the pendulum clock*, are but a few of these. But, more than this, the 'age of genius' also produced more than its just measure of ideas: among them, the *circulation of the blood*, the *wave theory of light*, and the *law of gravitation*. To some extent, it is true, the instruments and ideas had been adumbrated by earlier periods; but it probably is safe to say that in no century, with the possible exception of the 20th, was *the interplay of instruments and ideas more effective that during the 'age of genius'*.

1600–1750 CE The European *Baroque Period* in music. The leading composers are:

- Heinrich Schütz 1585–1672
- Dietrich Buxtehude 1637–1707
- Alessandro Stradella 1642–1682
- Arcangelo Corelli 1653–1713
- Johann Pachelbel 1653–1706

[30] For further reading, see:

- Leicester, H.M., *The Historical Background of Chemistry*, Dover: New York, 1971, 260 pp.
- Partington, J.R., *A Short History of Chemistry*, Dover: New York, 1989, 415 pp.

• Giuseppe Torelli	1658–1709
• Henry Purcell	1659–1695
• Tommaso Vitali	1663–1745
• Francois Couperin	1668–1733
• Thomasso Albinoni	1671–1751
• **Antonio Vivaldi**	**1678–1741**
• Francesco Manfredini	1680–1748
• Jean-Baptiste Loeillet	1680–1730
• **Georg Telemann**	**1681–1767**
• Jean-Philippe Rameau	1683–1764
• Domenico Scarlatti	1685–1757
• **Johann Sebastian Bach**	**1685–1750**
• **Georg Friedrich Handel**	**1685–1759**
• Francesco Geminiani	1687–1762
• Johann Friedrich Fasch	1688–1758
• Carlo Tessarini	1690–1765
• Giuseppe Tartini	1692–1770
• Pietro Locatelli	1695–1764
• Giovanni Batista Sammartini	1701–1775
• Giovanni Batista Pergolesi	1710–1736
• Christoph Willibald von Gluck	1714–1787
• Pietro Nardini	1722–1793

1600 CE **William Gilbert** (1544–1603, England). Physician and scientist. The father of the science of magnetism[31]. Asserted that the earth is a giant magnet, thus explaining for the first time why the compass needle

[31] Gilbert must have been aware of the contributions of **William Borough** (1536–1599, England) and **Robert Norman**. Borough published *A Discourse of the Variation of the Compass, or Magnetical Needle* (1581), based on his observations during several marine expeditions. Norman (1581) described his discovery made some years before (1570) of the *inclination* or *dip*. He devised a form of a dip-circle, and found the value for the inclination in London.

Another fundamental discovery, that of the *secular change of declination*, was made in England by **Henry Gellibrand** (1597–1636), a mathematician and astronomer, professor of mathematics at Gresham College, who described it in his *Discourse Mathematical on the Variation of the Magnetical Needle together with its Admirable Diminution lately discovered* (1635).

Gellibrand also noticed *diurnal* changes in the declination, which he attributed to instrumental uncertainties. However, the reality of this phenomenon was first emphasized by **George Graham** (1675–1751, England), a London instrument maker, in 1724.

seeks the poles. His findings were published in his book: "*De magnete magneticisque corporibus, et de magno magnete tellure physiologia nova*". In his book (1600) Gilbert convincingly demonstrated, with the aid of an enormous body of experimental material, that the magnetic field of the earth is like the field of a uniformly magnetized sphere made of magnetic iron ore[32]. Gilbert's book laid the foundations for the scientific approach to magnetism *in general*, and to terrestrial magnetism in particular. For two centuries following his discovery, nothing of substance that was not either a repetition or a development of what Gilbert had already done, was added to the subject.

Gilbert's work, which embodied the results of many years of research, was distinguished by its strict adherence to the scientific method of investigation by experiment, and by the originality of its material. He explained not only the north-south alignment of the magnetic needle, but also the variation in the dipping (inclination) of the needle. Gilbert's is therefore the first systematic contribution to the science of magnetism.

Gilbert was born at Colchester of an ancient Suffolk family. He entered St. John's College, Cambridge in 1558, and graduated M.D. in 1569. After spending three years in Italy and other parts of Europe, he settled in London, where he practiced as a physician with great success. In 1599 he became president of the college of physicians, and in 1601, court physician to Queen Elizabeth I. On the death of the queen in 1603 he was reappointed by her successor, but died soon thereafter of the plague.

1603–1644 CE **Theodore Turquet de Mayerne** (1573–1655, France and England). Physician, Physiologist and Chemist. One of the great physicians of the Baroque era: added chemical principles to humoral medicine, a greater empiricism to a more rational approach to medicine, and an interventionist therapeutics to a more cautious view of therapy. Thus he was influential in the introduction and support of chemical therapy in medicine, endorsing the use of chemical remedies in his practice.

Turquet was born in Mayerne, near Geneva, the son of a noted *Huguenot* historian and political theorist, **Louis Turquet de Mayerne**. He completed his early schooling in Geneva and took his undergraduate degree at the University of Heidelberg. He received his M.D. in 1597 at the University of Montpellier. For 50 years he served as a royal physician to three kings in France and England (Henri IV, James I, Charles II).

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[32] In explaining terrestrial magnetism Gilbert suggested that the earth was made of magnetized iron, which created the magnetic field; but his proposition was not correct. He himself discovered that iron, at the high temperatures that we now know to exist at the center of the earth, completely loses its magnetic qualities.

Turquet was one of the 17^{th} century most renowned authority on the technical aspects of painting and art: he prepared instructions for varnishes, painting mediums, coating canvases, enamels and pigments for **Peter Paul Rubens**, **Anthony van Dyck** and a host of other well known painters and craftsmen of the Baroque[33].

[33] Many of the practices used by Renaissance and Baroque painters were often kept secret.

History of Magnetism I (1100 BCE–1600 CE)

Certain naturally occurring substances (e.g. magnetite Fe_3O_4*, magnetic pyrites* $6FeS \cdot Fe_2S_3$*) posses the property of attracting neighboring particles of iron over considerable distances. Such bodies are called* magnets. *If a steel rod be stroked with such a natural magnet, it also assumes the property of attracting particles of iron. A splinter of magnetite, hanging by a thread, takes up a definite position, resulted in being called* loadstone *or* lodestone.

These curious facts were known to the ancient Greeks *at least as early as 800 BCE. Apart from these two magnetic phenomena, no additional knowledge about magnetism was gained up to the end of the* 15^{th} *century. Upon one of these is based the principle of the mariner's* compass[34]*, which is said to have been known to the* Chinese[35] *as early as 1100 BCE, thought it was not introduced into Europe until more than 2000 years later.*

A passage in De Rerum Natura (VI, 910–915) by the Roman poet ***Lucretius*** *(ca 60 BCE) indicates that in his time the phenomenon of magnetization by induction has been observed. The property of orientation, in virtue of which a freely suspended magnet points approximately to the geographical north and south, is not referred to by any European writer before the* 12^{th} *century (****A. Neckham*** *of Great Britain in 1187 CE).*

The needles of primitive compasses, being made of iron, would require frequent re-magnetization, and a "stone" for the purpose of "touching the needle" was therefore generally included in the navigator's outfit. With the constant practice of this operation, it is hardly possible that the repulsion acting between like poles should have entirely escaped recognition; but though it appears to have been noticed that the loadstone sometimes repelled *iron instead of attracting it, no clear statement of the fundamental law that unlike poles attract while like poles repel was recorded before the publication (1581) of the New Attractive by* ***Robert Norman****.*

The foundations of the modern science of magnetism were laid by ***William Gilbert*** *(1600) in his book De Magnete. It contains many references to the exposition of earlier writers from Plato to the author's own age. He admitted therein that the north seeking property of magnetite was brought to Europe from China by* ***Marco Polo****. Gilbert showed that the earth's magnetic field was equivalent to that of a permanent magnet, lying in a general north-south direction, near the earth's rotational axis.*

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[34] From the Latin *cum* = with, *passus* = a step; compass = a measuring instrument.

[35] First mentioned by **Shen Kua** of China in 1088 CE.

No material advance upon the knowledge recorded in Gilbert's book was made until the establishment by ***Coulomb*** *(1785) of the law of magnetic action.*

1603–1614 CE **Santorio Santorio or Sanctorius Sanctorius** (1561–1636, Italy). Physician and physiologist. Pioneer of quantitative experimental medicine. His experimental studies established quantitative metabolic phenomena of body weight (1614). Introduced measurements and quantification into physiology and medicine.

Santorio was born in Justinopolis, Venetian Republic (now Koper, former Yugoslavia) to a noble Venetian family. He studied philosophy and medicine at Padua (1579), where he received his M.D. (1582). Served as a personal physician of a Croatian nobleman (1587–1599) and then set up a medical practice in Venice (1599). Here he became part of the circle of learned men, befriending **Galileo** and other leading figures of the *Scientific Revolution*. Appointed to the chair of theoretical medicine at the University of Padua (1611), where he taught until his retirement (1624).

Santorio is best known for his investigations into *metabolism*: over a period of 30 years he carried out an elaborate series of measurements, described in his *De Statica Medicina*. He placed himself on a platform suspended from an arm of an enormous balance, and weighted both himself and his food, drink, and waste products. He determined that over half of normal weight loss is due to 'insensible perspiration'. He invented instruments to measure humidity, temperature (1611) and pulse rate (1603).

Although in treating his patients Santorio did not stray far from Hippocratic and Galenic practice (based on the notion of a balance of fluids), he differed from the classical authors a great deal in his theory and method of investigation. Rather than relying on authority in the first instance, he argued that one should first rely on sense experience, then on reasoning, and only lastly on authority.

Rather than describing the body and its functions in terms of Aristotelian and Galenic elements and qualities, Santorio argued that the fundamental properties were mathematical ones, such as number, position, and form. The body was like a clock, the working of which was determined by the shapes and positions of its interlocking parts. This was a radical break with traditional medical theory and natural philosophy, in which the discourse was about qualities and essences.

1603 CE **Johann Bayer** (1572–1625, Germany). Amateur astronomer and lawyer. Introduced the method of describing the locations of stars and of naming them with Greek letters and by the constellation they are in; this system continues to be used today. His *Uranometria* (1603) is the first attempt at a complete celestial atlas.

Bayer was born in Rain, Germany.

1604–1619 CE **Hieronymus Fabricius** (Geronimo Fabricio; Girolamo Fabrici, 1537–1619, Italy). Surgeon, anatomist and embryologist. Founder of comparative anatomy.

He was born at Aquapendente. Student of **Gabriele Fallopio** and his successor at Padua (1562–1613). Conducted studies in embryology of various animals and man, published in his *De formato foetu* (1604) and *De formatione ovi et pulli* (1621).

1605–1638 CE **Willem Janszoon Blaeu** (1571–1638, Holland). Map maker and astronomer. One of the leading map makers of the early 17^{th} century. His works include a world map issued in 1605, a three-volume sea atlas [*The Light of Navigation* (1608–1621)], and a series of atlases.

Blaeu was born in Alkmaar and developed his geographical and astronomical skills under the guidance of Tycho Brahe in Denmark. He founded a publishing house (1599), specializing in cartography. His instruments and globes featured unprecedented precision.

1608–1609 CE **Hans Lippershey** (1587–1619) and **Zacharias Jansen** (1588–1630), Dutch spectacle makers from Middleburg, and **James Metius of Alkmaar** invented both the compound microscope and the telescope. **Anton van Leeuwenhoek** (1632–1723, Netherlands, 1668) first used microscopes for scientific research.

1609 CE First regularly published *newspaper* in Germany.

1609–1621 CE **Johannes Kepler** (1571–1630, Germany). Court astrologer and astronomer. The founding father of modern astronomy. By careful observations and years of painstaking calculations, was able to derive the laws of elliptical planetary motion, thus providing evidence for the Copernican system.

With his resolution to submit every physical and astronomical law to the test of experiment and observation, he contributed much to the inauguration of the present scientific age. Kepler dissented from the Aristotelian metaphysics of his day and maintained that the Copernican system was not merely a convenient hypothesis but a true image of nature, and that it was amenable to verification through quantitative measurements.

Born in Weil-der-Stadt, Württemberg (near Stuttgart), he attended the University of Tübingen and studied for a theological career. There he learned the principles of the Copernican system. In 1594 he was offered a position of teaching mathematics and astronomy at the Lutheran school in Graz. As part of his duties, he prepared astronomical almanacs and furnished astrological "data". But he left Graz rather then undergo compulsory conversion to Roman Catholicism. While he was seeking another post, he formed an association with **Tycho Brahe** which shaped the rest of his life.

Tycho set Kepler to work trying to find a satisfactory theory of planetary motion — one that was compatible with the long series of observations that he had made. Brahe, however, was reluctant to supply Kepler with enough data to enable him to make substantial progress, perhaps because he was afraid of being "scooped" by the young mathematician.

After Tycho's death in 1601, Kepler succeeded him as mathematician to Rudolph II, the Holy Roman Emperor, and obtained possession of the majority of Tycho's records: Their study occupied most of Kepler's time for more than 20 years. In 1604 Kepler observed what is today known as a *supernova explosion*. [In the same year he also suggested that the opposite ends of a straight line meet at infinity and that two parallel lines intersect at infinity!]

Kepler made his most significant discoveries when he tried to find an orbit to fit all Brahe's observations of the planet Mars. Earlier astronomers thought that a planetary orbit was a circle or a combination of circles. But Kepler could not find a circular orbit that would agree with Brahe's observations. He spent *several years* on this problem. At one point he found a combination of circular arcs that agreed with the observations to within 8 arcminutes (quarter of a diameter of a full moon), but he believed that Tycho's observations could not have been in error by even this small amount, and so, with characteristic integrity, he discarded the hypothesis. He then took the bold step of assuming that the orbit of Mars *cannot be circular*, and tried to represent it with an *oval* instead. He soon discovered that the orbit could be fitted well by an *ellipse* (Kepler's First Law).

Kepler found that the eccentricity of the orbit of Mars is only 0.1: the orbit, drawn to scale, would be practically *indistinguishable from a circle*. It is a tribute to Tycho's observations and to Kepler's perseverance, that he was able to determine that the orbit was an ellipse at all. Kepler's achievement in dislodging the 2000 year old belief in circular orbits, is all the more remarkable since he himself was quite partial to perfect heavenly spheres.

In the year 1609, Kepler published his new results in a book '*Astronomia Nova*', on which he worked altogether for six years. Before he saw that the orbit of Mars could be represented accurately by an ellipse, Kepler had already investigated the manner in which the planet's orbital speed varied. After some calculations, he found that Mars speeds up as it comes closer to the sun and slows down as it pulls away from the sun. Kepler expressed this relation by imagining that the sun and Mars are connected by a straight, elastic line. As Mars travels in its elliptical orbit around the sun, the areas swept out in space by this imaginary line in equal intervals of time are always equal (Kepler's Second Law).

At the time of publication of his book in 1609 Kepler appeared to have demonstrated the validity of his two laws for the case of Mars alone. However, he expressed the opinion that they hold also for the other planets.

Kepler believed in an underlying harmony in nature, and he constantly searched for numerological relations in the celestial realm. This belief was triumphantly vindicated when he found a simple algebraic relation between the length of the semi-major axis of a planet's orbit and its sidereal period: namely that the squares of the sidereal periods of the planets are in direct proportion to the cubes of the semi-major axes of their orbits (Kepler's Third Law[36]).

Kepler published this third law in a second book, "*De Harmonice Mundi*" in 1619.[37] To arrive at this law it was not necessary for him to know the *actual* distances of the planets from the sun, only the distance in units of the earth's distance. [There were very slight discrepancies when the third law was applied to the orbits of Jupiter and Saturn. Decades later, Newton gave an explanation for them, but within the limits of accuracy of the observational data available in 1619, Kepler was justified in considering his formula to be exact.]

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[36] Kepler himself never realized the real importance of his three laws. Indeed, without differential calculus and analytical geometry, these laws show no apparent connection with each other – they are disjointed bits of information which do not make much sense. Once you know the inverse square law of gravitation and Newton's mathematical equations, all this become beautifully self-evident. Thus, Kepler's laws seem to have no particular raison dêtre: of the First he was almost ashamed – it was a departure from the circle sacred to the ancients and there was nothing to recommend it in the eyes of God. The Second Law he regarded as a mere calculating device. The Third he saw as necessary link in the system of harmonies, and nothing more.

[37] For further reading, see:

- Adler, M.J. (ed), *Great Books of the Western World. No. 16. Ptolemy, Copernicus, Kepler*, William Benton, Publisher, The University of Chicago, 1952, 1085 pp.

Much of the rest of "*De Harmonice Mundi*" deals with Kepler's attempts to associate numerical relations in the solar system with the regular Platonic Solids[38] and with music. He tried to derive notes of music played by the planets as they move harmoniously in their orbits (!). The earth, for example, plays the notes mi, fa, mi, which he took to symbolize the "miseria" (misery), "fames" (famine), "miseria" of our planet.

Buried amongst the musical notes was a curious little relationship: "It seems that the squares of the periods of revolution (T) of any two planets are as the cubes of their mean distance from the Sun (r)."

	Year (T)	T squared	Orbit (r)	r cubed
Mercury	0.2408	0.0580	0.388	0.0584
Venus	0.6152	0.3785	0.724	0.3795
Earth	1.0000	1.0000	1.000	1.0000
Mars	1.881	3.5378	1.524	3.5396
Jupiter	11.862	140.71	5.200	140.61
Saturn	29.457	867.72	9.510	860.09

This was to become – even though he didn't know it himself – Kepler's Third Law. It is the key to the orderliness of the solar system, for it indicates in what way the motions of the five planets are mathematically interdependent.

The book containing it was universally ignored. Three days after the completion of *The Harmony of the Worlds*, the Thirty Years War broke out.

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38 Kepler attempted here to bare the ultimate secret of the universe in an all-embracing synthesis of geometry, musics, astrology, astronomy and epistemology. It was the first attempt of this kind since Plato, and it is the last to our day. After Kepler, science is divorced from religion, religion from art and matter from mind.

According to Kepler, the existence of just six planets (with the five intervals between them) matching the five Platonic Solids, was not by chance – but a divine arrangement: into the orbit (or sphere) of *Saturn* he inscribed a *cube*; and into the *cube* another sphere, which was that of *Jupiter*. Inscribed in that was the *tetrahedron* and inscribed in it was the sphere of *Mars*. Between the spheres of *Mars* and *Earth* came the *dodecahedron*; between *Earth* and *Venus* the *icosahedron*; between *Venus* and *Mercury* the *octahedron*.

In the Third Law, Kepler saw the pinnacle of all his achievements: here at last was the connection between characteristic *distances and times* associated with the solar system – the ultimate harmony of the spatial architecture of the Platonic Solids and the temporal musical scale of the planetary spheres.

It is indeed surprising to perceive in his work copious signs of superstition and a keen devotion to astrology. Neo-Platonic and religious conceptions are even more evident than in Copernicus. Still under the spell of apriorism, he was anxious to interpret the universe as motivated by mathematico-aesthetic numerical harmony and exhibiting a surpassing simplicity and unity.

In 1618, 1620, and 1621, Kepler published his text "*Epitome Astronomiae Copernicanae*". Here, he stated that his first two laws had been tested and found valid for the other planets besides Mars, and for the moon. Also, he reported that the third law applies to the motions of the four newly discovered satellites of Jupiter as well as to the motions of the planets about the sun.

In 1623, Kepler concluded work on his last book, the "*Tabulae Rudolphinae*", which consisted of tables and rules for determining the positions of the planets and a catalogue of star positions, mostly based on the data of Brahe. This book ranked for a century as the best aid to astronomy. The printing of this book was delayed [by the *30 year war* (1618–1648) which raged at that time in Europe] and was finalized only in 1627.

Kepler also studied optics and designed a telescope that he probably built but never used. He discovered the inverse-square law of the decrease in the brightness of a source of light, for he saw instinctively that light from a faint source spreads out spherically and that the brightness of the source therefore varies inversely as the square of the observer's distance from it. Kepler also investigated the *refraction of light* and showed that Ptolemy's approximate law of refraction (i.e. the proportionality of the angles of refraction and incidence) holds only for small angles of incidence. However, he did not discover the correct law of refraction[39].

At that time, the insolvent imperial exchequer owed Kepler some 12,000 florins, for which **Wallenstein** assumed full responsibility. But Wallenstein's promises to Kepler were not kept. In lieu of the sums due, he offered him a professorship at Rostock, which Kepler declined. An expedition to Ratisbon, undertaken for the purpose of presenting his case to the diet, terminated his life: shaken by the journey, which he had performed across Europe entirely on horseback in the autumn of 1630, he came down with fever and died at Ratisbon, on the 15^{th} of November 1630 in the 59^{th} year of his life. By his first wife (ca 1611) he had five children, and by his second wife — seven children. Of these only two, a son and a daughter, reached maturity. In 1615 his mother was charged with witchcraft; it was only due to his indefatigable

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[39] Kepler used his own approximation $i = \frac{kr \cos r}{k \cos r - (k-1)}$, where i is the *angle of incidence* (w.r.t. the normal) and r is the corresponding *angle of refraction*; k is a fixed number for any pair of media

efforts that she was acquitted, after having suffered 13 month's imprisonment under imminent threat of torture.

Kepler was buried in a cemetery outside the town of Ratisbon. The cemetery was destroyed during the 30 years war and his bones were scattered. There remained, however, the epitaph that he had prepared for himself:

"*Mensus eram coelos, nunc terrae metior umbras,*
Mens coelestis erat, corporis umbra iacet".

Worldview VIII: Johannes Kepler

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"Ubi materia, ibi geometria"

* * *

"Expectet ille suum lectoremper annos centum; si Deus ipse perannorum sena millia contemplatorem praestolatus est."

"(It may well wait a century for a reader, as God has waited six thousand years for an observer.)"

(Harmonice Mundi, 1619)

* * *

"I measured the skies, now the shadows I measured. Sky-bound was my mind, earth-bound the body rests."

* * *

"When the storm rages and the state is threatened by shipwreck, we can do nothing more noble than to lower the anchor of our peaceful studies into the ground of eternity."

(1629)

* * *

"I have the answer, the orbit of the planet is a perfect ellipse."

(1609)

* * *

"God always geometrizes"

* * *

"The universe was stamped with the adornment of harmonic proportions"

* * *

"I undertake to prove that God, in creating the universe and regulating the order of the cosmos, had in view the five regular bodies *of geometry as known since the days of Pythagoras and Plato, and that he has fixed according to those dimensions, the number of heavens, their proportions, and the relations of their movements."*

* * *

Others on Kepler

* * *

"Johannes Kepler set out to discover India and found America. It is an event repeated over and again in the quest for knowledge. But the result is indifferent to the motive. A fact, once discovered, leads an existence of its own, and enters into relations with other facts of which their discoverers have never dreamed. Apollonios of Perga discovered the laws of the useless curves which emerge when a plane intersects a cone at various angles: these curves proved, centuries later, to represent the paths followed by planets, comets, rockets, and satellites."

(Arthur Koestler "The Watershed", 1960)

* * *

How did Kepler derive his three 'laws'?[40] (1609–1619)

The manner in which Kepler used the empirical astronomical data available to him (consisting of antiquity's accumulated lore plus Tycho Brahe's observations) is an instructive case study of how Science's knowledge of the laws of nature is actually abstracted from observations and experience. It was done through a sequence of interactive, iterative *and* convergent *interplays between empirical investigations on the one hand, and theoretical speculation*

[40] This article was written by Dr. Shahar Ben-Menahem.

and modeling on the other. It led to Kepler's three laws of planetary motion, and then to Newton's theories of mechanics and universal gravitation.

In order to deduce his 'laws', Kepler had to determine the distances of the planets from the sun and show that the orbits are not circles, but ellipses. Before we see how he accomplished this feat, let us regress momentarily to ***Copernicus*** *(1543) who went back to the doctrine of* **Aristarchos** *(270 BCE) and put the sun at the center of the whole planetary system, including the earth as a planet. Having no telescopic information, he stuck to the idealistic belief that each planet moves in the most perfect plane.*

The hypotheses which Copernicus adopted may be summarized under four headings:

1. *The apparent diurnal rotation of the celestial sphere is due to the complete rotation of the earth about its polar axis in a period of 24 hours.*
2. *The moon revolves around the earth in a period of* $27\frac{1}{3}$ *days.*
3. *The earth and the planets revolve in circular orbits about the sun in the same direction as the earth's diurnal motion.*
4. *The orbits of Mercury and Venus lie between the sun and that of the earth, while the orbits of Mars, Jupiter, and Saturn, lie beyond the earth's orbit.*

The tracks of the planets lie close to the ecliptic plane. So it is better to calculate their positions in celestial longitude and latitude as Copernicus did. For the purpose of grasping the principles employed in tracing out their orbits it will be sufficient to employ right ascension to measure their angular displacement. This is equivalent to projecting their movements onto the plane of the celestial equator.

Once one accepts the Copernican system for the solar system, the simplest set of assumptions is that each planet (earth included) describes a closed orbit around the sun — and furthermore, that these orbits are circular (centered at the sun), and that each planet moves around its orbit at a uniform angular speed. These assumptions are 'theoretical' in the sense that the only data that were actually 'measured' from any single observatory on earth (that is to say, excluding 'triangulation' measurements using some terrestrial distance as baseline) were angular position (right-ascensions and declinations) at which the planets and sun appear, in reference to the celestial sphere anchored to the fixed stars. Note that all the above-mentioned theoretical assumptions turned out in the end to be not quite accurate — but they did play an initial role in interpreting the "pure", apparent-angular-position data.

Kepler's first task was to calculate the planets' sidereal periods. True, Copernicus and his predecessors calculated it from the synodic period (known to the ancients); the latter is the time elapsed between two successive occasions when Mars (or any other planet), the sun and the earth occupy the same relative positions. It is done by noting when Mars is in opposition, i.e., when it is on the meridian at midnight, and counting the number of days which intervene before its next midnight meridional crossing (780 earth days on the average).

But the Copernican formula was based on the assumption of constant angular speed (circular orbit). Kepler, however, found very soon from Tycho's observations that the assumption of a circular Martian orbit is in conflict with the data. He therefore realized that strictly speaking, there is no such thing as a precisely-defined synodic period. In other words, because of the non-uniform planetary speeds, the times of conjunction and opposition do not occur with exact regularity and one can speak only of a mean synodic period.

So Kepler was led to determine the planets sidereal periods in a better way, one that does not make use of the (now discredited) 'theory' of uniform angular speeds.

The new, better method involved scanning the Tables for a planets' apparent angular positions at a sequence of dates separated by an integral number of earth years; one might term this the "strobing out" method — the well-known periodicity of the earth's own sidereal motion (namely, one earth-year) is removed from the compounded motion by viewing the accumulated data through a "stroboscope", as it were, having a period of one earth year. This method, when applied to the planetary data that had accumulated over the centuries, gave Kepler accurate values for each planet's sidereal period.

Having eliminated his dependence upon the uniform-revolutions assumption, Kepler found that the assumption that each planet's orbit is a sun-centered perfect circle, was also wanting. By laborious fitting of Tycho's data[41] *, he found that models using circles simply would not work — even if he shifted the sun away from their centers. Thus, even when a given such model seemed to work for Mars (the planet boasting the most eccentric of the planetary orbits) based on R.A. (Right Ascension) data, it failed when "declinations" (due to the differing orbital planes of Earth and Mars) were taken into consideration. In other words, neither circular orbits nor uniform*

[41] Kepler's various fitting efforts were rendered all the more tedious by his lack of proper, statistics-based best-fit procedures — which were only developed later, starting with the work of Gauss

angular speeds yielded theoretical models which could be reconciled with Tycho's data (to the level of accuracy to which Kepler believed Tycho's purely empirical results held true).

Note that in the case of *inferior* planets (i.e. Mercury and Venus), the assumption of sun-centered circular orbits immediately allows the extraction from Tycho's (or even the ancients') data of a reasonable value for these two planets' orbital radii – in units of earth's radius. This can be done by measuring the elongation angle (maximal apparent angular separation between the planet and the sun) for each inferior planet, and then utilizing simple trigonometry to compute the ratio of the respective planets' orbital radii to that of earth. But the *superior* planets (Mars and outwards) have no elongations as viewed from earth; and in any case Kepler could no longer rely on the sun-centered-circles model – as explained above.

Once Kepler was convinced that a circular Martian orbit about the sun would not do, he had to obtain a real picture, based on Brahe's data which he trusted. However, this was not easy since he only had observation of the *apparent path of Mars from a moving earth.* The true distances were unknown, only angles were measured, and those gave a foreshortened compound of Mars' orbital motion and the earth's. So Kepler decided to attack the earth's orbit first by a method that had the hallmark of genius.

To use Tycho's data to extract the correct shapes and sizes of the planetary orbits and the rates at which the planets move along these orbits, Kepler applied the *'strobing out' method in reverse!!* Namely, by picking out of Tycho's tables the apparent angles of a given planet at many observation times *spaced by integral numbers of that planet's sidereal period* — and assuming, as for the earth, that the given planet's orbit is a closed curve — Kepler was able to use apparent angular positions of a *single position* along the planet's orbit, as viewed by many earth positions, to determine the *distance* of that particular position of the planet from the sun in units of the average earth-sun distance (the so-called "Astronomical Unit" – A.U. for short) — via a simple geometrical construction.

In fact, if one believes that the earth's orbit itself *is* a sun-centered circle, then it suffices to employ *two* earth positions along its year-long orbit to properly triangulate each planet; Kepler then could (and did) repeat this procedure for many different positions of each planet along its orbit, thereby determining the detailed shapes and sizes of all planetary orbits.

However, the earth's orbit – although fairly close to being a sun-centered circle – does have *some* eccentricity; which is why the "strobed triangulation" procedure just described, needs to be over-determined. With enough different earth positions per given position of the planet being investigated, one can *compensate for the earth's orbital eccentricity. As a result of this investigation, Kepler found that the planets, earth included, moved in ellipses with the sun at one focus (Kepler's First Law).*

After he had worked out the geometry of planetary orbits, Kepler proceeded to investigate the detailed motion *of each planet around its orbit — finding his* second law, *governing the way in which a planet's varying distance from the sun modulates its (non-uniform!) angular speed along its orbit, as subtended at the sun.*

Finally, Kepler asked himself whether there is any systematic relationship between the sizes *of these orbits and their respective* sidereal periods; *after various attempts, he found such a simple rule – the celebrated Kepler's Third Law.*

Note that there are many other effects, not mentioned above, which "contaminate" Kepler's interpretation of the "pure data" with unwarranted assumptions: to mention just two, there is the earth's precession (caused by tidal torques upon the earth's equatorial bulge, and resulting in the famed "precession of the equinoxes" thanks to which we are said to be entering "the Age of Aquarius"), and perturbation of planetary orbits due to inter-planetary gravitational attractions.

The tidal-precession effect causes an additional apparent motion of the fixed stars, over and above the familiar diurnal motion — and this additional rotation is about an axis perpendicular to the ecliptic plane, and thus at an angle to the rotation axis of the earth. This effect certainly introduced complication in principle *for Kepler's program – but fortunately, the precession is very slow.*

As for the inter-planetary perturbations – those, too, are small; and once the Keplerian picture (as completed by Newton's new physics) clarified the basic *dynamics of the solar system, these perturbations were used by subsequent scientists to work out such details as three-body dynamics (Lagrange points, etc.) and to successfully predict new, previously unobserved, planets from purely theoretical calculations (the planets Neptune and Pluto were discovered with the aid of successive application of this technique). We thus see that the grand "iterative, interactive interplay of theory and experiment" continues to spin and converge long after Kepler, and each new "twist in the plot" demonstrates anew the fundamental robustness of this never-ending iteration.*

Kepler's three laws of planetary motion – augmented by Galileo's observation of the systematics of the Jovian moons' motion about Jupiter – allowed ***Newton*** *to arrive at his laws of mechanics and universal gravitation.*

Late-17th-century triangulation measurements (via a terrestrial baseline) of distances from earth to the nearest planets (Mars and Venus) by Cassini et al., fixed the absolute distance scale *of Kepler's solar system model – thus allowing the determination of its basic scale, the A.U. (Astronomical Unit), in terms of terrestrial units such as kilometers (although the metric system was yet to be invented).*

The Kepler Problem

The motion of an isolated system of two masses, moving under the sole influence of their mutual gravitation, is known as the two body problem *or the* Kepler problem. *The motion is governed by a single ODE equation of the second order:*

$$\frac{d^2\mathbf{r}_{12}}{dt^2} = -\big[G(m_1+m_2)/r_{12}^3\big]\mathbf{r}_{12}, \tag{1}$$

where $\mathbf{r}_{12}(t)$ *is the* relative *vectorial distance between the mass* m_1 *and the mass* m_2 *at time* t. *In this form the problem is represented in terms of the separation* $\mathbf{r}_{12}$, *which can be determined* directly. *The* force *between the masses is*

$$\mathbf{F}_{12} = -[GM\mu/r_{12}^3]\mathbf{r}_{12},$$

where $M = m_1 + m_2$ *is the total mass and* $\mu = \frac{m_1 m_2}{m_1+m_2}$ *is the* reduced mass. *Thus the orbit of each mass about the other is equivalent to the orbit of a mass* μ *about a mass* M *that is fixed in an unaccelerated, unrotating (inertial) frame.*

The exact solution of the above equation of motion can be written as a time-eliminated polar equation of the conic section curve:

$$r_{12}(\theta) = \Big[\frac{MG}{h^2} + \frac{1}{h}\Big\{2E + \frac{M^2G^2}{h^2}\Big\}^{1/2}\cos\theta\Big]^{-1},$$

where θ *is the angle at the focus of the conic between the radius vector* $\mathbf{r}_{12}$ *and the major axis ('*true anomaly*'), and* (E, h) *are the two constants of motion,* *namely the total energy and the orbital angular momentum, both per unit reduced mass (h is also twice the area swept out by the radius vector per unit time).*

Explicitly

$$E = -\frac{MG}{2a}, \quad 1 - e^2 = \frac{h^2}{MGa}, \quad P = 2\pi a\sqrt{\frac{a}{MG}},$$

where a is the semi-major axis, e is the eccentricity of the orbit and P is the orbital period.

If (a, P) for a binary system can be evaluated by direct astronomical observations, and if the motion of one of the two masses (which could be a star, planet, comet, moon, etc.) w.r.t. the common center of mass is known, the individual masses (m_1, m_2) of the pair can also be determined.

The equation of motion can be further exploited to obtain useful relations: from the area-rate constant

$$r^2 \frac{d\theta}{dt} = h = \sqrt{GMa(1-e^2)}$$

and the energy constant

$$\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right),$$

one obtains (by eliminating $\frac{d\theta}{dt}$)

$$\frac{dr}{dt} = \frac{na}{r}\sqrt{a^2e^2 - (a-r)^2},$$

where $n = \frac{2\pi}{P}$. Defining the eccentric anomaly E via

$$a - r = ae\cos E,$$

a straightforward integration of the above first-order differential equation for $r(t)$ yields the Kepler equation

$$n(t - T) = E - e\sin E,$$

where T is an integration constant.

The geometric interpretation of E is clear from its defining equation:

$$r = a(1 - e\cos E).$$

Construct an auxiliary circle in the orbital plane such that its diameter coincides with the major axis of the orbital ellipse, and their centers coincide.

From a point $S(r,\theta)$ on the ellipse draw a normal to the major axis and extend it until it meets the circle at S'. The angle subtended at the circles' center between the major axis and S' is E.

Then $M = n(t-T)$ is the angle which would have been described by a fictitious point moving on the auxiliary circle with mean angular velocity such that it revolves along the circle (and the ellipse) with period P. The angle M is known as the mean anomaly. *The entity $(t-T)$ is the* epoch *relative to T, the time of the perihelian passage.*

Kepler noticed that given M, one must solve a transcendental equation *for E, namely*

$$E - e\sin E - M = 0.$$

Once $E(e;M)$ is known, the orbit is calculated from $r = a(1-e\cos E)$ and $\tan\frac{\theta}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}$, yielding $R(t,\theta(t))$. During 1609–1819, more than 100 methods of solving Kepler's equation had been proposed, the most elegant being that of ***Bessel*** *(1819).*

Eq. (1) is equivalent to a system of 6 scalar ODE's of the first order in 6 unknown functions, namely the components of the relative separation of the masses and of their relative velocity vector at any given time. This is then a well-posed problem which needs for its complete solution 6 *constants of integration*[42]*. These constants are, for example, the relative positions and velocities of the masses at any given fiducial time.*

A difficulty arises from the fact that the observations which are made from the moving earth give only the direction of the line of sight to the object seen by the observer and furnish no direct information regarding its distance or line-of-sight velocity component. The position of the body in space is therefore not given and of course, its components of velocity are not determined. It thus becomes necessary to secure additional observations *at other times (or use Dopler measurements to extract line-of-sight speeds via spectral means). It*

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[42] The six arbitrary constants of integration can be represented by six independent functions (orbital elements) of these constants, which are direct and indirect observable parameters of the orbit:

a = major semi-axis, which defines the size of the orbit, its energy and its period;

e = the eccentricity, which defines the shape of the orbit;

Ω, i = two angles which together define the position of the plane of the orbit relative to the plane of the ecliptic (longitude of ascending node and inclination to the plane of the ecliptic);

ω (or π) = an angle defining orientation of orbit in the plane of the orbit;

T = time of perihelion passage, defining, with the other elements, the position of the body in its orbit at any time.

is clear that the problem of finding the position of the body and the elements of its orbit from such data present some difficulties.

Clearly, six independent entities must be found by observations in order that the elements could be determined. A single complete geometrical observation gives two quantities: the angular coordinates of the body. Therefore 3 *complete observations are just sufficient to determine the orbit.*

It required the combined genius of **Euler** *(1744),* **Lagrange** *(1778–1783),* **Laplace** *(1780) and* **Gauss** *(1809) to perfect precise and elegant computational tools for determining the orbital elements of planets and comets from observations by earthbound spectators.*

The Advent of Optical Instruments

The growth of maritime commerce was reinforced by the introduction of new technical inventions which emerged in a different context from the world's everyday work.

One of these was the invention of spectacles. *Although devices of one kind or another for magnifying objects are of considerable antiquity, there does not seem to have been any general use of them in everyday life till the close of the Middle Ages.*

The introduction of spectacles at about 1300 *in Florence involved no theoretical discovery about phenomena of which the Alexandrian and Arab astronomers were not fully conversant [Ptolemy, 150; Alhazen, 1026]. It is therefore more reasonable to suppose that introduction of paper, the invention of printing and* the use of books *in the* 15^{th} *century, stimulated the demand for eye glasses. The trade increased during the* 16^{th} *century, especially in Italy and in Southern Germany. By* 1600 *opticians were to be found in most of the larger towns on the continent.*

Two other inventions, which are signposts in the history of science, came as quite fortuitous by-products of the new industry: the telescope (1608) and the microscope (1609).

On Galileo's visit to Venice in May 1609, *he heard that an instrument for making objects appear nearer and larger had been invented. Returning to Padua, he made his first telescope by fixing a convex lens in one end of a leaden tube and a concave lens in the other end. Then he made a better one, went to Venice, and presented the instrument to the Doge Leonardo Donato. His first telescope magnified* 3 *diameters. He soon made others which magnified* 8 *diameters and finally one that magnified* 33 *diameters. Kepler devised an alternative form using a convex eyepiece.*

The three years which followed the invention of the telescope by Lippershey, Jansen and Metius, were eventful. Kepler's account of the motion of Mars appeared in 1609. *His telescope was constructed in* 1611. *Eight years later he was able to announce his complete vindication of the fundamental doctrine of Copernicus and his epoch-making laws of the solar system.*

Meanwhile, Galileo had observed the motion of the sun's spots and had seen the moons of Jupiter. Galileo's discoveries were important partly because it deprived the geocentric view of the universe of the inherent plausibility it enjoyed before people realized that there were other worlds with satellites circling about them.

The Inquisition rightly judged the psychological effect of the new realization that our own small world is not a unique one. Thus the tract on the moons of Jupiter became one of the most decisive battle fields between science and the priestly superstition.

The telescope had a threefold significance for the age of the Great Navigators. The determination of longitude for westerly sailing had become a technical issue of cardinal importance, and on this account astronomy retained its place as the queen of the sciences till the end of the 18^{th} *century. At a time when the only method of determining longitude was based on the use of celestial signals (eclipses and conjunctions), such signals were events of vital significance for the world's work, and the discovery of Jupiter's moons brought a new battery of celestial signals to the aid of seafaring and scientific geography. More directly, the telescope was of value to the mariner as a "spy glass".*

A less obvious use is related to one of the pivotal inventions in the history of mankind. The age of the Great Navigators was a period of revolutionary and imperialist wars in which success depended on exploiting the new technique of artillery. The demands of marksmanship called for accurate devices for surveying and sighting distant objects. Galileo was not slow to recognize the *possibilities of the telescope for navigation. Indeed, he offered his invention consecutively to the Catholic Emperor and to the opposing Protestants in letters adapted to the convictions of either parties.*

The design of better telescopes immediately created two needs: high magnification led inevitably to a more precise statement of the law of refraction by ***Kepler****,* ***Snell*** *(1618) and* ***Descartes*** *(1637). The need to eliminate the colored fringe which blurs the outline of the image obtained with simple lenses, led* ***Newton*** *to the study of the spectrum (1665).*

The invention of the telescope is the culmination of a chain of events that spread over a period of 2000 *years from* ***Euclid*** *to* ***Galileo*** *[Euclid composed a work on the geometrical principles of reflection and* ***Archimedes*** *is credited with constructing concave mirrors for use as burning-glasses].*

1611 CE King James Version of the Protestant Bible: In 1604, King James I of England authorized a committee of 54 scholars to prepare a revision of earlier English translations of the Bible. The new version appeared in 1611 and became known as the *King James*, or *Authorized* Version. The beauty and grace of the translation established it as one of the great treasures of the English language and Western Culture in general. A revised version by the Church of England (1870) failed to compete with the King James Version.

In the Middle Ages the Bible was brought to the people indirectly through the miracle plays and directly through the translations supervised by **John Wyclif** (c. 1330–1384). In the sixteenth century came **William Tyndale** (c. 1494–1536), whose ambition was thus expressed to a well-known divine of his day: "If God spare me life, I will cause the boy that driveth the plow to know more of the Scriptures than you do." Tyndale suffered martyrdom for his work, but his translation of the New Testament enabled his successor, **Miles Coverdale** (ca 1488–1569), to complete it. By 1540 religious dissensions were somewhat quieted down, and this "Great Bible", as it was called, was established in all the churches.

The scholars of the King James Version made considerable use of Tyndale's vigorous phrases, and we owe more to Tyndale than to any other one man.

1611 CE **Marco Antonio de Dominis** (1560–1624, Italy). Natural philosopher, mathematician and theologian. First to put forward an explanation of the rainbow which, with all its faults, was superior to any other published theory over 300 years before him.

Dominis was born of a noble Venetian family in the Island of Arbe, off the coast of Dalmatia. For some time he was employed as professor of mathematics at Padua and professor of rhetoric and philosophy at Brescia. He rose to the rank of archbishop of Spalato (1600). In his endeavors to reform the Church he got involved in quarrel between the papacy and Venice. He crossed to England (1616) and converted to Anglicanism, becoming the dean of Windsor (1619). His attacks on the papacy (1617–1618) aggravated the Church and he was enticed back to Rome by the promise of pardon and rich preferment. But he was doomed to bitter disappointment: he was thrown into the Inquisition's prison and died there. Even this did not end his miseries. By order of the inquisition his body was taken from the coffin, dragged through the streets of Rome, and publicly burnt in the Campo di Fiore.

1614–1617 CE **John Napier** (1550–1617, Scotland). Mathematician, inventor of *logarithms* and the man who first *used* the decimal point in the arithmetic of decimal fractions. In the absence of any exponential notation or concept of bases (let alone any knowledge about '*e*') this self-taught man labored 20 years to develop a geometrical scheme that simulated natural logarithms. In 1624, **Henry Briggs** (1561–1637, England) published tables [as did **Johannes Kepler**] of logarithms to base 10. Briggs introduced the word 'mantissa', which is a late Latin term of Etruscan origin meaning an "addition" or "appendix". The Swiss **Jobst Bürgi** (1552–1632), using an algebraic approach, conceived and constructed a table of logarithms independently of Napier in 1620.

One of the anomalies in the history of mathematics is the fact that logarithms were discovered before exponents were in use (1637). Another fact which stands out in connection with this invention is the well known motto, that necessity is the mother of invention. Indeed, the rapid development of astronomy, trade, navigation, engineering and warfare made ever increasing demands on the speed and accuracy of computations. These demands were met successively by the *adoption* of three remarkable inventions: The Hindu-Arabic notation (ca 1500), decimal fractions (1592) and logarithms (1614).

The nations of antiquity experimented for thousands of years with numerical notations before they developed the so-called '*Arabic notation*'. In the simple expedient of the zero which was introduced by the Hindus, mathematics received one of its most powerful stimuli. One would suppose that once the 'Arabic notation' was thoroughly understood, decimal fractions would occur as an obvious extension of it. But simple as decimal fractions may appear to us, the invention of them is not the result of one mind or even of one age. They came into use by a slow and imperceptible process. The first mathematicians associated with their history did not perceive their true nature and importance, and failed to invent a suitable notation.

The idea of decimal fractions made its first appearance in methods for approximating the square roots of numbers, but the first systematic treatment of decimal fractions is due to **Simon Stevin**, who in his *La Disme* (1585) described the advantages of decimal fractions and decimal division in systems of weights and measures. Stevin applied the new fractions to all operations of ordinary arithmetic, but he lacked a suitable notation. In place of our decimal point, he used a zero.

It has not been agreed yet to whom the first introduction of the *decimal point* or comma should be ascribed. However, if a requirement is made that the point or comma was with the candidates not merely a general symbol to indicate separation, but that the symbol has actually been used in operations including multiplication or division of decimal fractions, then it would seem that the honor falls to **John Napier**, who exhibited such use in his *Rabdologiae* (1617). Napier's decimal point did not meet with immediate adoption. It was only in the first quarter of the 18^{th} century that the decimal point achieved a complete and final victory.

By the beginning of the 17^{th} century the victory of the Arabic system of numeration — for both calculation and recording — was complete in most of Europe. As a result the abacus went out of use in the countries west of Russia. It was a long time, however, before even the basic processes of calculation became either commonly understood or widely practiced[43]. The blockage was cleared by two inventions (one quite minor and the other of the very first importance) which effectively reduced all arithmetical calculations to addition and subtraction. Both were due to the same man — John Napier.

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[43] On 4 July 1662, Samuel Pepys, then in charge of the Contract Division of the Admiralty, wrote in his diary:

"*Up by five o'clock, and after my journal put in order to my office about my business... By and by comes Mr. Cooper, of whom I intend to learn mathematiques, and do begin with him today, he being a very able man. After an hour being with him at arithmetique* (*my first attempt being to learn the multiplication table*)*; then we parted till tomorrow*".

Pepys was one of the best educated men of his time. He was a senior Civil Servant, he had been to Cambridge, and later in life he became president of the Royal Society and a friend of such men as **Isaac Newton** and **Christopher Wren**. Yet the poor man had to struggle with multiplication tables at an early hour in the morning! (He probably could add and subtract reasonably well; it was *multiplication*, and still more *division*, of large numbers, that required the skill of a professional mathematician in his day.)

John Napier[44] was born at the family estate of Merchiston Castle near Edinburgh and was the 8^{th} Baron of Merchiston. His father was only 16 years of age when John was born. In 1563, the year his mother died, he matriculated at St. Salvator's College, St. Andrews. After that, his stay at the university was short, and only the groundwork of his education was laid there. To complete his education he studied at the University of Paris, and visited Italy and Germany. He returned home in 1571 and a year later married Elizabeth Stirling, who died in 1579, leaving him a son and a daughter. A few years afterwards he married again, having by his second wife five sons and five daughters.

During 1588–1614 Napier expended much of his energies in the political and religious controversies of his day. He was violently anti-Catholic and championed the causes of John Knox and James I. In 1593, he published a bitter and widely-read attack on the Church of Rome entitled *A Plaine Discovery of the whole Revelation of Saint John*, in which he endeavored to prove that the Pope was the Antichrist and that the Creator proposed to end the world in the years between 1688–1700. The book run through 21 editions, at least ten of them during the author's lifetime, and Napier sincerely believed that his reputation with posterity would rest upon this book. He also wrote prophetically of various infernal war engines and of "*devices of slaying under water*", accompanying his writings with plans and diagrams. Some of his war chariots are remarkably like a modern tank. It is no wonder that Napier's ingenuity and imagination led some to believe he was mentally unbalanced and other to regard him as a dealer in the black art.

As a relaxation from his political and religious polemics, Napier amused himself with the study of mathematics and science. In 1614 appeared the work which in the history of British science can be placed as second only to Newton's *Principia*: *Mirifici Logarithmorum*[45] *Cannonis Descriptio* ("A Description of the Marvelous Rule of Logarithms"), containing 57 pages of explanatory text and 90 pages of tables. It introduced logarithms and simplified the representation of decimal fractions.

The fundamental idea of relating an arithmetic and a geometric series is physically represented by Napier through the motion of two points on separate parallel straight lines, one point moving with uniform velocity and the other

[44] The family name was originally Lennox. When one of its members distinguished himself in battle the King of Scotland changed his name to Napier, to honor his valor, saying: "*You have Na-Peer*" (i.e. no equal).

[45] The compound of two Greek words: *Logos* (ratio) and *Arithmos* (number).

with accelerated velocity[46]. [The idea originated with him in 1594: John Craig, physician to James VI of Scotland, called on him and told him that on his visit to the astronomical observatory of **Tycho Brahe** in 1590, the latter showed him a marvelous mathematical device through which a product of two numbers is converted into a sum[47].]

The publication in 1614 of the system of logarithms was greeted with prompt recognition, and among the most enthusiastic admirers was Henry Briggs, the first Savilian professor of geometry at Oxford. In 1615, he left his studies in London to do homage to Napier at his home in Scotland. There they discussed possible modifications in the method of logarithms. They agreed that powers of ten should be used, that the logarithm of one should be zero and that the logarithm of ten should be one.

Previous to Napier's publication of his *Cannonis Descriptio* England had taken a minor part in the advance of science, and there is no British author of the time except Napier whose name can be placed in the same rank as those of **Copernicus**, **Tycho Brahe**, **Kepler**, **Galileo**, or **Stevinus**. Scotland had produced nothing, and was perhaps the last country in Europe from which a great mathematical discovery would have been expected.

Napier lived not only in a wild country, which was lawless and unsettled during most of his life, but also in a credulous and superstitious age. Like Kepler and all his contemporaries, he believed in astrology. Such was the state of society in the midst of which logarithms had their birth.

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46 We can arrive at the definition of the Napierian logarithm with the aid of the Newtonian calculus (which was unknown to Napier): A point C moves on a segment $AB = a$ from A to B such that its velocity is always proportional to $x = CB$, i.e. $\frac{dx}{dt}|_C = -x$, $x(0) = a$. A second point F moves uniformly on a segment DE from D to E such that $y = DF$, $\frac{dy}{dt} = a$. The two points start at the same time $t = 0$. Since $y = at$ and $t = \log_e \frac{a}{x}$, we have the Napierian logarithm $= y = a \log_{\frac{1}{e}} \left(\frac{x}{a}\right)$. Napier chose $a = 10^7$ and called $y = DF$ the logarithm of $x = CB$. It is evident from this formula that Napier's logarithms are *not* the same as natural logarithms. The notion of a *base* never suggested itself to him because it is not applicable to his system.

47 Such as $\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$.

Calculating Devices

Calculating with the aid of machines or devices began well before the use of electricity. The most important 'hardware' inventions in this category were:

ca 500 BCE *The bead-and-wire* abacus*, used for adding and subtracting large numbers. Invented probably in ancient Egypt.*

ca 1500 CE *The* quadrant*, an astronomical calculation tool, widely used in Europe.*

1502 CE *The first* watch *– an analogue time-keeper.*

Many of the applied fields in which numerical calculations are important, such as astronomy, navigation, trade, engineering, and war, made ever increasing demands that computations be performed more quickly and accurately. These increasing demands were met successfully by three remarkable 'software' inventions:

- *Hindu-Arabic notation (including 'zero')*
- *Decimal fractions (Stevin, 1585)*
- *Logarithms*

Logarithms were invented independently by ***Napier*** *(1594, 1614) and* ***Bürgi*** *(1600) and developed further by* ***Briggs*** *(1615, 1624). Their big advantage is the replacement of multiplication with simple addition, thus saving calculator's time by a large margin. It enabled European mathematics to break away from slow ancient calculating systems and procedures.*

To multiply a number a by another number b we write

$$a = \epsilon^x, \quad b = \epsilon^y$$

where ϵ is an arbitrary base *(usually chosen as $\epsilon = 10$ or $\epsilon = e$). Then*

$$ab = \epsilon^{x+y}.$$

The number x is the logarithm of a to base ϵ and the number y is the logarithm of b to base ϵ. We write

$$x = \log_\epsilon a; \qquad y = \log_\epsilon b$$

It then follows that

$$\log ab = x + y$$

and multiplication is reduced to addition.

One of the anomalies in the history of mathematics is the fact that logarithms were discovered before exponents were in use (1637).

The basic idea of logarithms was noted by ***Stifel*** *(1544). He observed that the terms of the* geometric progression *$\{1, r, r^2, r^3, \ldots\}$ correspond to the terms in the* arithmetic progression *$\{0, 1, 2, 3, \ldots\}$ formed by the exponents.* Multiplication *of two terms in the geometric progression yield a term whose exponent is the* sum *of the corresponding terms in the arithmetic progression. This observation had already been made earlier by* ***Chuquet*** *(1484). Stifel extended this connection between the two progressions to negative and fractional exponents. Thus the* division *of r^2 by r^3 yield r^{-1}, which corresponds to the term -1 in the extended arithmetic progression.*

Though the definition of logarithms as exponents of the powers that represent the numbers in a fixed base became the common approach, they were not *defined as exponents in the early 17^{th} century because fractional and irrational exponents were not in use.* By the end of the century *a number of mathematicians recognized that logarithms could be so defined, but the first systematic exposition of this approach was made by* ***Euler*** *(1728).*

During 1617–1674 mathematicians in England, France and Germany invented and developed mechanical devices and machines to speed up the execution of arithmetic processes; It all started with Napier himself who invented a mechanical numbering device called "Napier Bones". It was made of horn, bone, or ivory. This device evolved into the logarithmic slide-rule *by* ***Edmund Gunter*** *(1620) and* ***William Oughtred*** *(1622).*

Wilhelm Schickard *(1592–1685) of Tuebingen, Germany, made a 'calculating clock' (1623). This mechanical machine was capable of adding and subtracting up to 6 digit numbers, and warned of an overflow by ringing a bell. The machine and its plans were lost and forgotten in the war that was going on, at that time, then rediscovered in 1935, only to be lost in war again, and then finally re-rediscovered in 1956 by the same man! The machine was reconstructed in 1960, and found to be workable. (Schickard was a friend of the astronomer Kepler.)*

Pascal *(1642) built one of the first calculating machines that handled addition by carrying from units to tens, tens to hundreds, etc.* ***Samuel***

Moreland *(1625–1695, England) produced both adding and multiplication machines (1668–72).* ***Leibniz*** *saw the Pascal machine in Paris and then invented (1671–4) a machine which could carry out multiplication and division. It could multiply numbers up to 5 and 12 digits to give a 16 digit operand. The machine was later lost in an attic until 1879.*

Until 1940 machines of this kind were simply mechanical devices that performed only arithmetic and had no influence on the course of mathematics.

1615–1616 CE **Willem Corneliszoon Schouten** (1580–1625, The Netherlands). Navigator and explorer. First to traverse the *Drake Passage* (1615); discovered *Cape Horn* (1616).

1616–1629 CE **Joseph Solomon Delmedigo** (1591–1655), known as the *Yashar of Candia*. Mathematician, astronomer, philosopher, linguist, and physician. The greatest secular Jewish savant of the late Renaissance era, who exerted great influence on the thinking of **Baruch Spinoza**. He was a keen critic of medieval philosophy of nature and carried the ideas of the scientific revolution into the Near East and Central and Eastern Europe.

Delmedigo peregrinated from his native town of Candia, Crete, to Padua, Italy, where in 1606 he became a pupil and disciple of **Galileo Galilei**. Following his studies of mathematics, astronomy and philosophy, he took on medicine under **Hieronymus Fabricius** (1537–1619). After graduating in 1613, he spent most of his life in travels and short sojourns in Egypt, Turkey, Poland, Lithuania, Bohemia, Hamburg, Amsterdam, Frankfurt, Worms and Prague, where he died. He earned his living as a physician and served during 1620–1623 as court physician to Prince Radzivil of Poland.

He wrote some 50 books on mathematics, mechanics, astronomy, medicine and philosophy, but published only a few — since he had to be careful lest the ecclesiastical and secular authorities be offended by his ideas. He stressed the need for experiments in aviation and for the construction of aircraft to collect data for *weather prediction*(!).

1618–1648 CE *The Thirty Years War*; Germany, the most populous province of the Holy Roman Empire became a playground for the invading armies of Spain, Denmark, Sweden, and France. Seven to eight million people (about one third of the total population) were killed. At the time of the *Peace of Westphalia* (1648) the empire remained politically fragmented, divided into 300 autonomous, sovereign states, most of them very small and weak. Historians have summed up their feelings about this conflict as follows:

"Morally subversive, economically destructive, socially degrading, confused in its causes, devious in its cause, futile in its results — an outstanding example in European history of meaningless conflicts".

1620 CE **Pierre Gassendi** (1592–1655, France). Mathematician, philosopher and scientist. A Catholic priest who taught mathematics at College Royal in Paris. He rejected the dogmatic teaching of Aristotelian science and proclaimed his adherence to the Epicurean belief in the atomistic structure of matter. Like Bacon he urged the importance of experimental research, and formulated correctly the law of inertia in 1636. Helped his friend **Mersenne** to measure the speed of sound in air. Although he added little to the stock of human knowledge, he holds an honorable place in the history of science.

1620–1624 CE **Cornelius Jacobszoon van Drebbel** (1572–1633, Netherlands). Engraver, alchemist, instrument-maker and an inventor far ahead of this time. Built the first navigable submarine that could carry a number of people. It cruised 5 m below the surface of the Thames in London, on several occasions.

Drebbel was born in Alkmaar, the son of a well-to-do farmer. He had no university education and as a young man apprenticed to the famous engraver and alchemist **Hendrik Goltzius** (1558–1617), who taught him some chemical ideas and processes. Drebbel then devoted himself to engraving but soon turned to mechanical inventions and instrument making. About 1604 he went to the court of King James I in England, who became his patron. In 1610 Drebbel visited the court of Emperor Rudolf II in Prague, at the Emperor's invitation.

He lingered a decade and instructed the son of Archduke Ferdinand of Bohemia who would later become Holy Roman Emperor. At the beginning of the Thirty Years' War, Ferdinand V's forces imprisoned Drebbel and took all his possessions, for he was affluent at this time. Through the intervention of Prince Henry, Drebbel was set free to return to England in 1613.

During the next several years he lived mostly in London. About 1620 he began to devote himself to the manufacture of microscopes and to the construction of a submarine. For the next several years he was employed by the British Navy, partly in connection with the submarine, but mostly to make explosive devices with which to attack other ships. During 1626 to 1628, he advised the military on how to relieve the French Huguenots under siege at La Rochelle. From 1629 until his death in 1633 he was extremely poor and earned his living by keeping an alehouse.

His most phenomenal work was definitely the submarine. In 1620, he made the first "rudimentary" submarine. Drebbel constructed his vessel while working for the British Navy. They never used it, but tested it many times. He had a wooden row boat; it had a wooden hull wrapped tightly in waterproofed leather. His row boat was the first to answer the question of air replenishment underwater. Air tubes with floats went to the surface to provide the craft with oxygen. Oars went through the hull at leather gaskets. Twelve oarsmen and some other passengers were on board. The trip at the Thames River took three hours. The secret of the craft was probably the production of oxygen from saltpeter by a process discovered by Drebbel already in 1608.

Drebbel also invented thermostats, a thermoscope and a microscope with two sets of convex lenses. He made compound microscopes as early as 1619. He also made telescopes and constructed a camera obscura with a lens in the aperture. A lunar crater is named after him.

1620–1644 CE **Johann Baptista van Helmont** (1579–1644, The Netherlands). Chemist and physician. The first to understand that there are gases other then atmospheric air, and one of the first to apply chemical principles to physiological processes. He was a forerunner of the iatrochemical school, and rendered an important service to the art of medicine by applying chemical methods to the preparation of drugs. He invented the word *gas*[48] to describe substances that are like air (1620). He even isolated several such gases, including oxides of carbon (CO_2, CO) nitrogen and sulfur (he studied gases released by burning charcoal and fermenting wine). Helmont maintained that gases were substances differing fundamentally one from the other, and from air and condensible vapors.

Helmont was born at Brussels, a member of a noble family. He was educated at Louvain, and after ranging restlessly from one science to another and finding satisfaction in none, turned to medicine, taking his doctor's degree in 1599. The next few years he spent in traveling through Switzerland, Italy, France and England. He settled in 1609 at Vilvorde, where he occupied himself with chemical experiments and medical practice until his death.

Helmont presents curious contradictions, characteristic of chemists of his age: On the one hand he was a disciple of **Paracelsus** (a mystic with strong leanings to the supernatural, an alchemist who believed that with a small piece of the philosopher's stone he could transmute 2000 times as much mercury into gold); on the other hand he was touched with the new learning that was

[48] This he derived from the Greek *chaos*, meaning space. In this way, the word described the ability of a gas to fill any amount of space. Before his time, and indeed for some time after, gases were thought to be different forms of the *element* of air, or air mingled with some impurities.

producing men like Harvey, Galileo and Bacon — a careful observer of nature and an exact experimenter, who in some cases realized that matter can neither be created nor destroyed.

The Rise of the New World, I

The Pilgrims[49] *(1620)*

In 1002, *the Viking Leif Ericsson led an expedition across the north Atlantic to the shores of North America, where colonies persisted for many years.*

Intrepid mariners from half a dozen European nations had explored America's coastal waters before England planted its first colonies on the eastern shore. There were earlier visits by Portuguese, Italian and Spanish seamen: armored Spanish Conquistadors filed north from Mexico to explore the southwest in the mid 1500*'s, and tonsured Franciscan friars from Spain established missions in Florida, Georgia and California late in the* 16^{th} *century.*

But it was England that founded the first permanent *colonies in the early* 1600*'s, and the British, with their* staying power*, who outlasted all their colonial rivals and built the thriving North American empire that eventually became the United States.*

The motives that brought millions of Europeans to America were mixed, but most of the immigrants hoped to find wealth and a new start in life, or religious and political sanctuary.

On April 26, 1607, *three shiploads of* 140 *English adventurers, lead by* ***John Smith*** *(1580–1631), anchored in the James River near the mouth of Chesapeake Bay. In the words of Sir Walter Rayleigh, they came "to seek new worlds for gold, for praise, for glory". They found far more hardship than gold or glory, and many of them died of disease and malnutrition, but they did establish Jamestown, the first permanent English settlement in the New World. They began growing Tobacco (1612), an Indian staple, which*

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[49] To dig deeper, see:

- Johnson, Paul, *A History of the American People*, Harper Collins, 1997, 1088 pp.

came into vogue in Europe and became the economic mainstay of Virginia and much of the Colonial South.

*In 1620 the small ship '**Mayflower**' sailed from Plymouth in England with 102 passengers bound for religious freedom in the New World. Most of them were Puritans who had run afoul of the religious laws of Britain. Some had been in exile in the Netherlands.*

The expedition reached Cape Cod Bay in Massachusetts after a 65-day voyage, and finally landed on part of the rocky shore which had been given the name Plymouth a few years earlier. These early settlers, forerunners of the colonists who were to form the independent United States 150 years later, are generally known as the pilgrim fathers, a term first used in 1799. They settled the first permanent colony of Europeans in New England.

As Jamestown, Plymouth, and other British settlements that soon lined the Atlantic coast, grew and prospered, their restless inhabitants gradually worked their way inland to establish new outposts. England began almost two centuries of struggle with her Colonial rivals.

1618 CE **Willebrod van Roijen Snell** (or **Snel**) (1591–1626, Netherlands). Dutch astronomer and mathematician. Rediscovered the law of refraction of light[50]. In 1637 **Descartes** derived this law from more basic principles. In 1657 **Fermat** derived Snell's law from his principle of least time.

When a ray of light passes from one homogeneous medium into another it undergoes a change of direction, and is said to be *refracted*. The acute angles made by the two parts of the ray with the normal to the surface of separation of the two media at the point of incidence are called the angles of incidence and refraction. The complete relation between the two directions is given by the following laws:

[50] The design of better telescopes created a need for a precise statement of the law of refraction. This explains why the best physicists and mathematicians of the 17^{th} century were preoccupied with this problem. Indeed, **Kepler** (1611), **Snell** (1618), Descartes (1638), **Fermat** (1657), **Newton** (1665) and **Huygens** (1678), contributed to the physics of light propagation through inhomogeneous media.

- The two parts of the ray are in the same plane with the normal and on opposite sides of it.
- The ratio of the sine of the angle of incidence i to the sine of the angle of refraction r is a constant, K, depending on the media, and on the nature of light.

Thus, Snell found empirically (1618) that $\sin i = K \sin r$, where K is a fixed number for any pair of media. That the ratio varies with the nature of light was proved by Newton. The ratio K is known as the *relative index of refraction* of the two media[51]. Clearly, for a ray going in the reverse direction $n_{ir} = \frac{1}{n_{ri}}$.

The first attempts to deduce a law of refraction go back to **Claudius Ptolemy** of Alexandria (150 BCE). His measurements of the angles of refraction of water and glass have come down to us and probably represent the most ancient physical experiments recorded historically.

Clearly, the relation $n_{ri} = \frac{v_i}{v_r}$ could be verified experimentally only in the 19th century, when suitable techniques for measuring velocities of light in material media were developed. In contradiction with Snell law $v_r \sin i = v_i \sin r$, particle kinematics yields, via the law of conservation of linear momentum $v_i \sin i = v_r \sin r$, where i is the angle of incidence and r is the angle of refraction. Thus, in the hands of Huygens, Snell's law provided evidence for the wave theory of light.

In 1617 Snell developed a method of determining distances by trigonometric triangulation. He also devised an efficient method for the evaluation of π to 35 decimal places.

Snell was born in Leyden. In 1613 he succeeded his father **Rudolph Snell** (1546–1613) as a professor of mathematics at the University of Leyden. It is not known just how Snell discovered the law of refraction. When the author died he left his manuscript unedited. This manuscript, which may have been available to Descartes, apparently was last seen by Huygens, and it now appears to be lost.

1622–1632 CE **William Oughtred** (1575–1660, England). Mathematician. Invented the rectilinear slide rule in 1622 soon after the invention

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[51] If we write $K = n_{ri}$, it is shown that

$$\frac{\sin i}{\sin r} = n_{ri} = \frac{v_i}{v_r} = \frac{n_r}{n_i}, \quad \text{or} \quad n_i \sin i = n_r \sin r,$$

where v_i is the velocity of light in the medium of incidence and v_r is the velocity of light in the second medium. Here n_r and n_i are known as the *absolute indices of refraction* of each of the media (i.e. relative to vacuum).

of logarithms (1614). Introduced the multiplication sign (×) into algebra in 1631. In 1632, he invented the circular slide rule.

Oughtred was born at Eton, and educated there and at King's College, Cambridge, of which he became a fellow. He left the University about 1603 to become a priest, and at about 1628 he was appointed by the earl of Arundel to instruct his son in mathematics. He corresponded with some of the most eminent scholars of his time on mathematical subjects. In 1631 he wrote a short compact treatise on arithmetic and algebra, *Calvis mathematicae*, in which he employed new mathematical symbols.

1623 CE **Wilhelm Schickard** (1592–1635, Germany). Scholar and inventor. Built a practical calculating machine which was used by **Kepler**. He invented many machines like one to calculate astronomical dates and one for the Hebrew grammar.

Schickard was professor for biblical languages at Tübingen University. His research was broad and included astronomy, mathematics and surveying. He died of the plague.

1625–1640 CE **Hugo Grotius** (**Huig de Groot**, 1583–1645; The Netherlands). Political philosopher, humanist and statesman. Considered a founder of international law. He wrote '*The Law of War and Peace*' (*De Jure Belli ac Pacis*, 1625), which had influenced Spinoza's political philosophy.

Grotius recognized that the corruption and decline of Papal jurisdiction, and the birth of the modern state, together give rise to an urgent need to a form of legality that would transcend the writ of any particular sovereign: all law must stem either from free association of people, or from the higher law – the law of nature – which applies to all men, and all nations, in every circumstance of life. The law of nature is eternal and immutable; to discern it we have but to employ our reason, which leads us to the perception of right and wrong, as it leads us to the truths of logic and mathematics.

Grotius was by no means an isolated thinker; the ideas which he expressed were current in The Netherlands and were to elicit an equal interest in England.

Grotius was born in Delft and graduated from the University of Leiden at 15. He became chief magistrate of Roterdam (1613). Condemned to life imprisonment (1619) for opposing strict Calvinism, but with the aid of his wife, escaped prison in a trunk of books (1621); Lived in Paris (1621–1631); Swedish ambassador in Paris (1634–1644).

1625–1642 CE **Zacutus Lusitanus** (Avraham Zacuto II, 1576–1642). Physician and medical writer in Amsterdam. One of the most celebrated physicians in Europe during the first half of the 17^{th} century. Studied medicine and philosophy at the Universities of Salamanca (Spain) and Coimbra (Portugal). Fled the Inquisition (1625) to Amsterdam, where he returned openly to his Jewish faith. Published a 12 volumes encyclopedia on the history of medicine: *De medicorum principium historia* (1642).

1626 CE **Peter Minuit** (1580–1638). Paid the equivalent of 24 dollars for the Manhattan Island, currently worth more than 100 billion dollars.

1628–1651 CE **William Harvey** (1578–1657, England). Physician. Discovered how blood circulates in the human body (1628), and established the foundations of modern embryology (1651).

Harvey's book *An Anatomical Treatise on the Motion of the Heart and Blood in Animals*, is considered the most important single volume in the history of physiology. In it Harvey showed that the heart, by repeated contractions, produces a continuous stream of blood throughout the body which continually returns to its source. It is amazing how such a fundamental fact escaped all the savants of antiquity[52] and had to await discovery until the 17^{th} century. Even so, Harvey's theory was severely attacked by followers of Galen[53] in spite of the fact that he based his ideas on firsthand observation

52 The Greek physician **Erasistratos** came very close to recognizing the circulation of the blood (ca 280 BCE). A Cairo physician, **Ibn al-Nafis** (1210–1288), who came from Damascus, pointed out that the dividing wall of the heart, the *septum*, was solid, and quite devoid of pores permitting the passage of blood, which Galen has postulated. Thus, he argued, the blood must flow from the right to the left ventricle of the heart through the lungs. In this way Ibn al-Nafis arrived at the theory of the lesser circulation of blood. His discovery, however, did not pass into the mainstream of science as his work did not come to light until 1924.

53 The first idea of this discovery occurred to him not later than 1616 but he did not publish it until 1628 in a little book dealing with the motion of the heart and blood. One is rather surprised to find that this book did not make more stir; neither did it arouse much opposition, at least in England. In France the opposition to the new theory was considerable, but even there, and bitter as it was, it did not last long. More happy in this than many other forerunners, Harvey was granted a taste of victory before his death in 1657. By 1673 his cause was definitely won, even in France, and the people who had been his contemporaries could witness the complete supremacy of the new doctrine.

Until the time of Harvey, the prevalent conception was that promulgated by Galen. According to him, the blood was produced in the liver from the materials furnished by our food and was then transported to the right half of the heart. Some of it passed into the left half, where it was imbued with new properties, and became fit to nourish the whole body. To use Galenic language, the blood of the and experiment. Harvey lived to see his discovery widely accepted, although full credit only came after his death.

Harvey was born at Folkestone. He studied medicine at Padua under **Fabricius** and became a doctor of medicine in 1602. He returned to London and practiced medicine. Harvey became a member of the Royal College of Physicians in 1607, and later served as physician to James I and Charles I.

In one point only was his demonstration of the circulation incomplete: Harvey could not discover the capillary channels by which the blood passes from the arteries to the veins. This gap in the circulation was bridged in 1661 by the physiologist **Marcello Malpighi** (1628–1694, Italy), who saw in the lungs of a frog, by the newly invented *microscope*, how the blood passes from the one set of vessels to the other. Harvey saw all that could be seen by the unaided eye in his observations on living animals.

Harvey speculated that humans and others mammals must *reproduce through the joining of an egg and sperm*. No other theory made sense It was 200 years before a mammalian egg was finally observed, but Harvey's theory was so compelling and so well thought out that the world assumed he was right long before the discovery was finally made.

Harvey remained a physician at St. Bartholomew's until 1643. He maintained his college lectureship until 1656, the year before his death, missing by a moment the dismantling under Cromwell of the monarchy that had supported his research throughout his life.

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right heart was endowed with "natural spirits", that of the left heart with "vital spirits". The latter blood was thus essentially different from the former. They did not circulate in the body, but both moved in a ceaseless ebb and flow, each in its own domain. But how did the blood pass from the right to the left ventricle? To explain the impossible, Galen had been obliged to assume that it passed through innumerable *invisible* pores in the solid wall which divides the right heart from the left. Nobody ever detected these pores for they are not simply invisible but nonexistent. Yet Galen, supreme pontiff of Greek medicine, and nine centuries later Avicenna, the infallible medical pope of the Middle Ages, had spoken *ex cathedra* with such indisputable authority that this gratuitous assumption was generally taken for gospel.

Even a man like Leonardo da Vinci, endowed with so much genius and originality, and who had himself dissected a large number of bodies and examined very minutely many a heart, even he was subjugated by this intangible dogma. This is the more pathetic in that Leonardo was certainly on the scent of the true explanation, but the invisible holes were too sacred to be touched, and nothing but this prejudice caused his failure to discover and to proclaim the circulation of the blood.

The Cult of the Virtuoso

Throughout the 17th century, the majority of university mathematicians continued in the restricted tradition of scholasticism, and the main impetus for mathematical advance came from the Renaissance humanist reaction against the universities.

The most fruitful and original research was carried out by gifted amateurs, who were sometimes called virtuosi, *as being endowed with a special, individual genius. This tendency to single people out as intellectual heroes fostered a spirit of* competitive individualism, *rather than of co-operative research — an attitude which probably encouraged the development of new ideas, but which tended to recede as mathematics became more and more technical.*

The competitive spirit gave rise to considerable jealousies as to priority over discovery of new theorems and methods. One manifestation of this was the custom of setting challenge-problems. Often the challenger had already solved the problem himself, and wanted to publicize his individual achievement. The emphasis on inventive genius encouraged greater interest in ideas themselves rather than in their detailed elaboration.

With the advent of navigation maps and the Renaissance of algebra, the time was ripe for the algebraization of geometry. *It began with the concept of* coordinate system *in the framework of 'analytic geometry'. It was invented, nearly simultaneously and independently, by Fermat and Descartes.*

1629–1654 CE **Pierre de Fermat**[54] (1601–1665, France). One of the greatest mathematicians of all times. Accomplished Toulouse Jurist and a universalist, who cultivated poetry, Greek philosophy, law and philology, and devoted to mathematics only the leisure of a laborious life. His father was a prosperous leather merchant and his mother came from a family of high social standing. He obtained his law degree from the University of Orleans in 1631,

[54] For further reading, see:

- Mahoney, M.S., *The Mathematical Career of Pierre de Fermat*, Princeton University Press: Princeton, NJ, 1973.

and in that year was appointed to a position in the high court of Toulouse and became entitled to include the honorific ‘de’ in his name.

He began his serious mathematical studies in 1629 when he discovered independently, and ahead of **Descartes**, ‘analytic geometry’.[55] This included: general equations of the straight line, the circle (centered at the origin), the ellipse, the parabola and the rectangular hyperbola. Fermat’s analytic geometry appears to be as general as that of Descartes, but is more complete and systematic, and corresponds much more closely to modern day analytic geometry.

In 1638 he communicated to Descartes his method of drawing tangents to plane curves. Fermat made numerous contributions to the development of differential and integral calculus: in particular he introduced the notion of “difference quotient” which he used to define the *derivative*, and used it in the study of problems of minima and maxima. The French, including **Lagrange**, claim Fermat as the true originator of the calculus.

Along with **Pascal** he is regarded as the founder of the theory of probability (1654). In physics, Fermat discovered in 1657 the ‘*principle of least time*’, valid for the propagation of light in material media. It is also known as the principle of shortest optical path. [The optical path is determined by the integral $\int_{r_2}^{r_1} n\,ds$, where n is the refraction index, which may change from point to point.]

However, the greatness of Fermat rests mainly in his contribution to number theory, and for that he is known as the “father of modern number theory”. Some of his discoveries are:

(1) *Fermat’s little theorem* (1640) [if p is a prime number and if a is an integer, then $a^p \equiv a(\text{mod } p)$. In particular, if p does not divide a then $a^{p-1} \equiv 1(\text{mod } p)$. This was known to the Chinese for $a = 2$.]

(2) Fermat’s method of factorization.

(3) Fermat’s method of infinite descent.

(4) Structure of perfect numbers.

(5) Every prime of the form $4m + 1$ is the sum of two squares in a unique way.

(6) Every positive integer is expressible as a sum of 4 squares of integers.

[55] **Marino Ghetaldi** [1566–1626, Dalmatia (now Croatia)] made early applications of algebra to geometry (1603).

(7) *Fermat's conjecture* ('Last Theorem'[56]): The equation $a^n + b^n = c^n$ has no solution in positive integers if $n > 2$ (1637)[57].

A general proof[58] has been attempted by Euler, Legendre, Gauss, Abel, Dirichlet, Cauchy, Kummer and many others over the past four centuries. **Fermat** himself proved it for $n = 3$ and 4 (1659), **Euler** for $n = 3$ and 4 (independently, 1738), **Legendre** and **Dirichlet** for $n = 5$ (1828–1830), **Lamé** for $n = 7$ (1839) and **Kummer** for $n < 100$ except for $n = 37, 59, 67$ (1859). By 1978, the conjecture was known to be true for all integer exponents up to 150,000 and by 1993 for all exponents less or equal to 4,000,000. A large part of *algebraic number theory* originated through attempts to prove Fermat's conjecture. Thus, in spite of the great frustration that this problem caused 15 generations of mathematicians, it turned out to be a blessing in disguise.

David Hilbert (1862–1943), when asked once why he did not attempt to prove Fermat's conjecture, replied: "*Why should I kill the goose that lays the golden egg*?"

Fermat firmly believed that $f(n) = 2^{2^n} + 1$ would yield primes for all values of n, but he was very wrong. Only 5 primes have been discovered which conform to this formula: $f(0) = 3$, $f(1) = 5$, $f(2) = 17$, $f(3) = 257$ and $f(4) = 65,537$, but already $f(5) = 4,294,967,297 = 641 \times 6,700,417$. The compositeness of some *Fermat numbers* has been established, but no further primes have been discovered among them.

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[56] To dig deeper, see:

- Stewart, I. and D. Tall, *Algebraic Number Theory and Fermat's Last Theorem*, A.K. Peters, 2002, 313 pp.
- Van der Poorten, A., *Fermat's Last Theorem*, Wiley, 1996, 222 pp.

[57] For $n = 2$, the solution of the Diophantine equation $x^2 + y^2 = z^2$ proceeds through the factorization $(x + y\sqrt{-1})(x - y\sqrt{-1}) = z^2$. Putting $(x + y\sqrt{-1}) = (u + v\sqrt{-1})^2 \equiv (u^2 - v^2) + 2uv\sqrt{-1}$, we find $x = u^2 - v^2$, $y = 2uv$, $z = u^2 + v^2$, which indeed yields the Pythagorean triplets.

[58] In 1909 **A. Wieferich** proved that $a^n + b^n = c^n$ is impossible for n an odd prime not dividing abc with n^2 not dividing $2^{n-1} - 1$ (the second condition holds for all $n < 3 \times 10^9$ except 1093 and 3511). In 1922 **L. Mordell** showed that the Fermat conjecture holds with finitely many exceptions for any $n \geq 3$ provided the *Mordell conjecture* is true. In 1983 **G. Faltings** proved the *Mordell conjecture*. In 1987 **D.R. Heath-Brown** proved the impossibility of $a^n + b^n = c^n$ for "*almost all*" n. Finally, **Andrew John Wiles** (b. 1953, England) proved Fermat's Last Theorem in *Modular elliptic curves and Fermat's Last Theorem* which appeared in the *Annals of Mathematics* in 1995.

To sum up: Fermat and Pascal share the invention (1654) of the mathematical theory of probability, Fermat alone founded the theory of numbers, Fermat and Descartes share the invention of analytic geometry and Fermat is a harbinger of the differential and variational calculus.

The influence of most of his works upon his contemporaries seems to have been slight. The impact of his discoveries in number theory were just about non-existent. It might have been greater had he agreed to publish his findings, but he shunned this aspect of communication. He began to be appreciated only after his death. His influence on later generations led to the Renaissance of modern number theory.[59]

Fermat and the Theory of Numbers

I. Fermat Numbers and their Associates

In search of an algebraic expression that would yield primes only, Fermat conjectured (1640) that $F_n = 2^{2^n} + 1$ *is prime for all values of* n*. This is true for* $n = 0, 1, 2, 3, 4$*, yielding the series of primes 3, 5, 17, 257, 65537 respectively. But in 1732* ***Euler***[60] *showed that already* F_5 *in composite, and*

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[59] For further reading, see:

- Mahoney, M.S., *The Mathematical Career of Pierre de Fermat*, Princeton, 1994.
- Singh, S.L., *Fermat's last Theorem*, London, 1997.
- Bell, E.T., *Men of Mathematics*, Simon and Schuster: New York, 1937, 592 pp.

[60] It is suspected that **Fermat** was led to his conjecture that all numbers $F_n = 2^{2^n}+1$, $(n = 0, 1, 2, 3, ...)$ are primes by the *Chinese theorem*, since he could prove that F_n divides $2^{F_n} - 2$, by induction. During Fermat's time it was thought that the Chinese theorem is true, for it was not known then that it breaks down for $n = 341$.

We must not rush to condemn Fermat for his blunder. Since F_5 has 10 digits, in *during the 276 years that followed, no one was able to find even one additional prime number in the series beyond F_4. It is perhaps more probable that the number of primes F_n is finite.*[61]

All numbers of the form $2^{2^n}+1$, whether prime or composite, are called Fermat numbers. They obey the simple recursion $F_{n+1}-2 = F_n(F_n-2)$ which leads to the interesting product

$$F_n - 2 = F_0 F_1 F_2 \cdots F_{n-1}.$$

In other words, $F_n - 2$ is divisible by all lower Fermat numbers:

$$F_{n-k}|(F_n-2), \quad 1 < k \le n.$$

On March 30, 1796, the Fermat numbers, until then largely a numerical curiosity, were raised from dormancy and took on a new beauty, linking number theory with a classical problem of Greek geometry.

*On that day, the young **Gauss** showed that a circle can be divided into n equal parts using ruler and compass alone, if n was a Fermat number. In other words: if F_n is prime, then a regular polygon of n sides can be inscribed in a circle by Euclidean methods. The Greek themselves knew how to construct regular n-sided polygon for n = 3, 4, 5, 6, 8, 10, 12, 15, 16 but progress in this problem had eluded mathematician ever since.*[62]

The most important properties of the Fermat numbers are:

---

order to test its primality, it would be necessary to have tables of primes up to 100,000, which was unavailable to him. He could, of course, derive and use some criterion for a number to be a factor of a Fermat number, but this he failed to do. Euler, on the other hand, *knew* that $5\cdot 2^7+1 = 641$ was a *potential* factor of F_5 and he could do the necessary calculations *in his head* (!) without the need of table or calculators.

61 **Hardy** (1938) suggested, by considerations of probability, that since the corresponding number of primes from 1 through x $\pi(x) \sim \frac{x}{\ln x}$, the probability, that a number is prime is $\frac{1}{\ln n}$. Therefore, the total a priori expectation of Fermat primes is at most

$$\sum\left\{\frac{1}{\ln(2^{2^n}+1)}\right\} < \frac{2}{\ln 2}\sum 2^{-n} < \frac{2}{\ln 2}.$$

This argument assumes that there are no special reasons why a Fermat number should be likely to be a prime. But the fact that no two Fermat numbers have a common divisor greater than 1 and the fact that 2^n+1 is composite if n is not a power of 2, suggest that such special reasons may exist.

62 It is easy to construct a regular 85-gon, using constructions for the 5-gon and 17-gon, and since angles can be bisected, one can construct regular 170-gons, 340-gon and more generally regular polygons for which the number of sides is

- *No two Fermat numbers have a common divisor greater than 1*

  For suppose that F_n and F_{n+k}, where k >0, are two Fermat numbers, and that
$$m|F_n, \quad m|F_{n+k}.$$
  If $x = 2^{2^n}$, we have
$$\frac{F_{n+k}-2}{F_n} = \frac{2^{2^{n+k}}-1}{2^{2^n}+1} = \frac{x^{2^k}-1}{x+1} = x^{2^k-1} - x^{2^k-2} + ... - 1,$$
  and so $F_n|F_{n+k}-2$. Hence
$$m|F_{n+k}, \quad m|(F_{n+k}-2);$$
  and therefore $m|2$. Since F_n is odd, $m = 1$, which proves the theorem.

- ***Fermat** (1640) showed that for 2^n+1 to be prime, n must be a power of 2, i.e. $n = 2^m$. Equivalently n has no odd factors, for if n has an odd factor t, then 2^n+1 has $(2^{n/t}+1)$ as a factor. Therefore 2^n+1 is composite, if n is not a power of 2. The inverse statement is false, as we know that F_m is composite for many values of m >4. In general, for any a^n+1 to be prime, a must be even and $n = 2^m$.*

- ***Euler** (1739) showed that for $n \geq 2$ a prime divisor of F_n is necessarily of the form $p = k2^{n+2}+1$. For assume $p|2^{2^n}+1$. Then $2^{2^n} = -1(mod\ p)$, and upon squaring each side, $2^{2^{n+1}} = 1(mod\ p)$. On the other hand, by Fermat's Little Theorem we know that $2^{p-1} = 1(mod\ p)$. The two relations are compatible iff $p-1 = r2^{n+1}$ for some r. Further investigation shows that r must be even and so $p = k2^{n+2}+1$. Indeed, Euler found that $p = 5 \times 2^7 + 1 = 641$ divides F_5. Note that for n = 2, 3, 4*

$n = 2^k \times F_l \times F_m \times ...$, where k = 0,1,2,... and the Fermat numbers are distinct primes.

Let us take $k = 0$ l, m = 0, 1, 2, 3, 4. Then, these polygons with an odd number of sides are built from the first five Fermat numbers 2^1+1, 2^2+1, 2^4+1, 2^8+1, $2^{16}+1$. If we multiply $1 = 2^1-1$ into the cumulative products we obtain 2^2-1, 2^4-1, 2^8-1, $2^{16}-1$, $2^{32}-1$, the latter being the product of the first five Fermat numbers
$$2^{32}-1 = 3 \times 5 \times 17 \times 257 \times 65357 = 4\ 294\ 967\ 295$$
It's quite probable that there are no more such odd polygons, because it seems likely that
$$3, 5, 17, 257 \text{ and } 65537$$
are the only prime Fermat numbers.

the values of p must be identical with the Fermat number themselves, implying k = 1, 2^3, 2^{10} respectively.

Thus every divisor of F_n occurs in the arithmetic progression

$$1, \quad 2^{n+2}+1, \quad 2\cdot 2^{n+2}+1, \quad 3\cdot 2^{n+2}+1, \ldots$$

For given n, then, we can work out terms of this progression and check to see if any is a divisor of F_n. For $n = 5$ we obtain the sequence 1, 129, 257, 385, 513, 641, 769, ... A great time-saver is provided by the observation that, for any number, the least divisor greater than 1 must be a prime number.

Consequently, in the investigation of F_5, we need not even bother with the composite 129. Since 257 is prime it needs to be tried, but it does not divide. Again, 385 and 513 are composite, so they can be passed over. This brings us to the prime 641, which actually divides F_5.

This procedure is based upon the work of Edward Lucas , who published it in 1877. However, Euler knew almost as much a century and a half earlier. In 1739 one of his publications contained the result that every prime divisor of F_n is of the form $2^{n+1}k+1$. (Lucas' improvement amounts only to showing that k must be even.) Presumably he knew this in 1732 and used it to find the divisor 641. For F_5 we have $k\cdot 2^{n+1} = 2^6k+1 = 64k+1$, and for k = 10 we obtain the factor 641.

Hence with the help of only two divisions we can ascertain that 641 is the smallest prime divisor of the number F_5.

- *F_n divides $2^{F_n}-2$; this is demonstrated in two steps: first it is shown by induction that, for positive integers, $2^n \geq n+1$. This implies that 2^{n+1} divides 2^{2^n}, i.e. for some k, we have $2^{2^n} = k\cdot 2^{n+1}$. Consequently*

$$\begin{aligned} 2^{F_n}-2 &= 2^{2^{2^n}+1}-2 = 2[2^{2^{2^n}}-1] = 2[2^{(2^{n+1}k)}-1] \\ &= 2[(2^{(2^{n+1})})^k - 1^k] = 2[(2^{(2^{n+1})}-1)(\cdots)] \\ &= 2[((2^{2^n})^2-1^2)(\cdots)] = 2[(2^{(2^n)}+1)(2^{(2^n)}-1)(\cdots)] \\ &= 2[(F_n)(2^{(2^n)}-1)(\cdots)]. \end{aligned}$$

It is suspected that this relation led Fermat to his conjecture that all numbers F_n (n = 1, 2, ...) are primes. During Fermat's times it was thought that the so-called Chinese theorem is true, namely the theorem asserting that if an integer $m > 1$ satisfies the relation $m|2^m-2$, then m is a prime (it was checked for first several hundred integers). This breaks down, however, for $m = 341 = 11\cdot 31$, which was not then known.

Table 3.1: PRIME FACTORS OF $2^n + 1, \quad n \leq 128$

	n	
F_0	**1**	**3**
F_1	**2**	**5**
	3	3 · 3
F_2	**4**	**17**
	5	3 · 11
	6	5 · 13
	7	3 · 43
F_3	**8**	**257**
	9	3 · 3 · 3 · 19
	10	5 · 5 · 41
	11	3 · 683
	12	17 · 241
	13	3 · 2731
	14	5 · 29 · 113
	15	3 · 3 · 11 · 331
F_4	**16**	**65537**
	17	3 · 43691
	18	5 · 13 · 37 · 109
	19	3 · 174763
	20	17 · 61681
	21	3 · 3 · 43 · 5419
	22	5 · 397 · 2113
	23	3 ·2796203
	24	97 · 257 · 673
	25	3 · 11 · 251·4051
	26	5 · 53 · 157 · 1613
	27	3 · 3 · 3 · 3 · 19 · 87211
	28	17 · 15790321
	29	3 · 59 · 3033169
	30	5 · 5 · 13 · 41 · 61 · 1321
	31	3 · 715827883
F_5	32	641 · 6700417
	33	3 · 3 · 67 · 683 · 20857
	34	5 · 137 · 953 · 26317
	35	3 · 11 · 43 · 281 · 86171
	36	17 · 241 · 433 · 38737
	37	3 · 1777 · 25781083
	38	5 · 229 · 457 · 525313
	39	3 · 3 · 2731 · 22366891
	40	257 · 4278255361
	41	3 · 83 · 8831418697
	42	5 · 13 · 29 · 113 · 1429 ·14449
	43	3 · 2932031007403
	44	17 · 353 · 2931542417
	45	3 · 3 · 3 · 11 · 19 · 331 ·18837001
	46	5 · 277 · 1013 · 1657 · 30269
	47	3 · 283 · 165768537521
	48	193 · 65537 · 22253377
	49	3 · 43 · 4363953127297
	50	5 · 5 · 5 · 41 · 101 · 8101 ·268501
	51	3 · 3 · 307 · 2857 · 6529 · 43691
	52	17 · 858001 · 308761441
	53	3 · 107 · 28059810762433
	54	5 · 13 · 37 · 109 · 246241 · 279073
	55	3 · 11 · 11 · 683 · 2971 · 48912491
	56	257 · 5153 ·54410972897
	57	3 · 3 · 571 · 174763 ·160465489
	58	5 · 107367629 · 536903681
	59	3 · 2833 · 37171 · 1824726041

Table 3.1: (Cont.)

	n		n	
	60	*17 · 241 · 61681 · 4562284561*	80	*65537 · 414721 · 44479210368001*
	61	*3 · 768614336404564651*	81	*3 · 3 · 3 · 3 · 3 · 19 · 163 · 87211 · 135433 · 272010961*
	62	*5 · 5581 · 8681 · 49477 · 384773*	82	*5 · 10169 · 181549 · 12112549 · 43249589*
	63	*3 · 3 · 3 · 19 · 43 · 5419 · 77158673929*	83	*3 · 499 · 1163 · 2657 · 155377 · 13455809771*
F_6	64	*274177 · 67280421310721*	84	*17 · 241 · 3361 · 15790321 · 88959882481*
	65	*3 · 11 · 131 · 2731 · 409891 · 7623851*	85	*3 · 11 · 43691 · 26831423036065352611*
	66	*5 · 13 · 397 · 2113 · 312709 · 4327489*	86	*5 · 173 · 101653 · 500177 · 1759217765581*
	67	*3 · 7327657 ·6713103182899*	87	*3 · 3 · 59 · 3033169 · 96076791871613611*
	68	*17 · 17 · 354689 ·2879347902817*	88	*257 · 229153 · 119782433 · 43872038849*
	69	*3 · 3 · 139 · 2796203 · 168749965921*	89	*3 · 179 · 62020897 · 18584774046020617*
	70	*5 · 5 · 29 · 41 · 113 · 7416361 · 47392381*	90	*5 · 5 · 13 · 37 · 41 · 61 · 109 · 181 · 1321 · 54001 · 29247661*
	71	*3 · 5640964 3 · 13952598148481*	91	*3 · 43 · 2731 · 224771 · 1210483 · 25829691707*
	72	*97 · 257 · 577 · 673 · 487824887233*	92	*17 · 291280009243618888211558641*
	73	*3 · 1753 · 1795918038741070627*	93	*3 · 3 · 529510939 · 715827883 · 2903110321*
	74	*5 · 149 · 593 · 184481113 · 231769777*	94	*5 · 3761 · 7484047069 · 140737471578113*
	75	*3 · 3 · 11 · 251 · 331 · 4051 · 1133836730401*	95	*3 · 11 · 2281 · 174763 · 3011347479614249131*
	76	*17 · 1217 · 148961 · 24517014940753*	96	*641 · 6700417 · 18446744069414584321*
	77	*3 · 43 · 617 · 683 · 78233 · 35532364099*	97	*3 · 971 · 1553 · 31817 · 1100876018364883721*
	78	*5 · 13 · 13 · 53 · 157 · 313 · 1249 · 1613 · 3121 · 21841*	98	*5 · 29 · 113 · 197 · 19707683773 · 4981857697937*
	79	*3 · 201487636602438195784363*	99	*3 · 3 · 3 · 19 · 67 · 683 · 5347 · 20857 · 242099935645987*

Table 3.1: (Cont.)

n			n	
100	17 · 401 · 61681 · 340801 · 2787601 · 3173389601		115	3 · 11 · 691 · 2796203 · 1884103651 · 345767385170491
101	3 · 845100400152152934331135470251		116	17 · 59393 · 82280195167144119832390568177
102	5 · 13 · 137 · 409 · 953 · 3061 · 13669 · 26317 · 1326700741		117	3 · 3 · 3 · 19 · 2731 · 22366891 · 5302306226370307681801
103	3 · 415141630193 · 8142767081771726171		118	5 · 1181 · 3541 · 157649 · 174877 · 5521693 · 104399276341
104	257 · 78919881726271091143763623681		119	3 · 43 · 43691 · 823679683 · 143162553165560959297
105	3 · 3 · 11 · 43 · 211 · 281 · 331 · 5419 · 86171 · 664441 · 1564921		120	97 · 257 · 673 · 394783681 · 4278255361 · 46908728641
106	5 · 15358129 · 586477649 · 1801439824104653		121	3 · 683 · 117371 · 11054184582797800455736061107
107	3 · 643 · 84115747449047881488635567801		122	5 · 733 · 1709 · 3456749 · 368140581013 · 667055378149
108	17 · 241 · 433 · 38737 · 33975937 · 138991501037953		123	3 · 3 · 83 · 739 · 165313 · 8831418697 · 13194317913029593
109	3 · 104124649 · 2077756847362348863128179		124	17 · 290657 · 3770202641 · 1141629180401976895873
110	5 · 5 · 41 · 397 · 2113 · 415878438361 · 3630105520141		125	3 · 11 · 251 · 4051 · 229668251 · 5519485418336288303251
111	3 · 3 · 1777 · 3331 · 17539 · 25781083 · 107775231312019		126	5 · 13 · 29 · 37 · 109 · 113 · 1429 · 14449 · 40388473189 · 118750098349
112	449 · 2689 · 65537 · 183076097 · 358429848460993		127	3 · 56713727820156410577229101238628035243
113	3 · 227 · 48817 · 636190001 · 491003369344660409	F_7	128	59649589127497217 · 5704689200685129054721
114	5 · 13 · 229 · 457 · 131101 · 160969 · 525313 · 275415303169			

Table 3.2: PRIME FACTORS OF FERMAT NUMBERS $F_n = 2^{2^n} + 1,\ n \leq 19$

		F_n			
n	2^n	Digits	Factors in integer form	Discoverer	Factors in power form
0	1	1	3	Fermat, 1640	
1	2	1	5		
2	4	2	17		
3	8	3	257		
4	16	5	65537		
5	32	10	$641 \cdot 6,700,417 = 4,294,976,297 = 62,264^2 + 20,449^2 = (143)^4$	Euler, 1732	$(a_5 \cdot 2^7 + 1)(b_5 \cdot 2^7 + 1)$
6	64	20	$274,177 \cdot 67,280,421,310,721$	Landry, Lasseur, 1880	$(a_6 \cdot 2^8 + 1)(b_6 \cdot 2^8 + 1)$
7	128	39	59,649,589,127,497,217 · 5,704,689,200,685,129,054,721 = 340,282,366,920,938,463,463,374,607, 431,768,211,457	Morrison, Brillhart, 1974	$(a_7 \cdot 2^9 + 1)(b_7 \cdot 2^9 + 1)$
8	256	78	1,238,926,361,552,897 · 93,461,639, 715,357,977,769,163,558,199,606,896, 584,051,237,541,638,188,580,280,321	Brent, Pollard, 1981	$(a_8 \cdot 2^{11} + 1)(b_8 \cdot 2^{11} + 1)$

$a_5 = 5$ $\qquad b_5 = 3 \cdot 17,449$

$a_6 = 3^2 \cdot 7 \cdot 17$ $\qquad b_6 = 5 \cdot 52,562,829,149$

$a_7 = 116,503,103,764,643$ $\qquad b_7 = 5 \cdot 228,394,219,017,628,537$

$a_8 = 604,944,512,477$ $\qquad b_8 =$ prime number with 59 digits

Table 3.2: (Cont.)

$$
\begin{array}{lcl}
F_9 & = & \mathit{2424833} \cdot \\
 & & \mathit{7455602825647884208337395736200454918783366342657} \\
 & & \cdot\, p_{99} \\
F_{10} & = & \mathit{45592577} \cdot \\
 & & \mathit{6487031809.4659775785220018543264560743076778192897} \\
 & & \cdot p_{252} \\
F_{11} & = & \mathit{319489} \cdot \mathit{974849} \cdot \mathit{167988556341760475137} \cdot \\
 & & \mathit{3560841906445833920513} \cdot p_{564} \\
F_{12} & = & \mathit{.114689} \cdot \mathit{26017793} \cdot \mathit{63766529} \cdot \mathit{190274191361} \cdot \\
 & & \mathit{1256132134125569} \cdot c_{1187} \\
F_{13} & = & \mathit{2710954639361} \cdot \mathit{2663848877152141313} \cdot \\
 & & \mathit{3603109844542291969} \cdot \\
 & & \mathit{319546020820551643220672513} \cdot c_{2391} \\
F_{14} & = & c_{4933} \\
F_{15} & = & \mathit{1214251009232704250386841 7} \cdot \\
 & & \mathit{168768817029516972383024127016961} \cdot c_{9808} \\
F_{16} & = & \mathit{825753601} \cdot c_{19720}, \quad F_{17} = \mathit{31065037602817} \cdot c_{39444} \\
F_{18} & = & \mathit{13631489} \cdot c_{78906} \\
F_{19} & = & \mathit{70525124609} \cdot \mathit{646730219521} \cdot c_{157804}
\end{array}
$$

where the numbers written out in full are primes, and p_N or c_N denotes an N-digit prime or composite number.

II. Fermat's Little Theorem

In a letter to **Bernard Frenicle de Bessy** *dated Oct. 18 1640, Fermat stated without proof one of the most important theorems in the theory of numbers*[63]*:*

"If p is prime and $p \neq a$, then $a^p \equiv a \ (mod\ p)$". This can also be written as $p \mid a(a^{p-1} - 1)$. So if we add the condition $p \not| a$, p must divide $a^{p-1} - 1$:

$$a^{p-1} = 1 \ (mod\ p).$$

63 It was not until 1736 that Euler made public a proof of the theorem though it is known that a similar proof was contained in a manuscript of **Leibniz** (1683), unpublished at the time. **De Bessy** (1605–1675) was an official at the French mint and amateur mathematician, well-known for his unusual ability in numerical computations.

A proof by induction in a is immediate: The theorem is certainly true for $a = 1$ since 1=1 (mod p). Now, suppose it is true that $a^p - a$ is divisible by p for some $a = b$; then it follows that it is true for $a = b + 1$. Indeed, by the binomial expansion:

$$(b+1)^p - (b+1) = \{b^p + 1 + \text{terms divisible by } p\} - (b+1) \tag{1}$$
$$= (b^p - b) + Np, \text{ say.} \tag{2}$$

*But $p|(b^p - b)$ on the strength of the induction assumption, and so $(b+1)^p = (b+1)(\text{mod } p)$ proves the theorem. Another variant of the same proof is due to **Leibniz**: for two arbitrary integers A, and B we have*

$$(A+B)^p = A^p + \binom{p}{1} A^{p-1}B + \cdots + B^p,$$

so

$$(A+B)^p \equiv (A^p + B^p) \ (\text{mod } p);$$

Again

$$(A+B+C)^p \equiv (A+B)^p + C^p \equiv (A^p + B^p + C^p) \ (\text{mod } p),$$

and so in general

$$(A+B+C+\cdots+K)^p \equiv (A^p + B^p + \cdots + K^p) \ (\text{mod } p).$$

It suffices to take $A = B = \cdots = K = 1$ and denote their number by a to get again $a^p \equiv a$ (mod p).

The theorem may be described as "little" in comparison with Fermat's more famous theorems, but his "small" result is truly remarkable because there is nothing analogous to it in the classic theory of polynomial equations. A similar, modern proof of this theorem uses group theory.

Applications

- *Prove that if n is prime, then n divides*

$$S = 1^{n-1} + 2^{n-1} + \cdots + (n-1)^{n-1} + 1.$$

By FLT for $p = n$, $a^{n-1} \equiv 1$ (mod n) for $1 \le a < p$. Thus $S = 0$ (mod n) and the theorem is proved.

We do not know any composite *number satisfying this relation. It has been conjectured that there is no such composite number $n < 10^{1000}$.*

- *Prove that* $S = 1^n + 2^n + 3^n + 4^n$ *is divisible by 5 iff* n *is not divisible by 4.*

  By FLT $a^4 \equiv 1 \ (mod\ 5)$ *for* $a = 1, 2, 3, 4$. *Therefore* $a^{4k} = 1 \ (mod\ 5)$, *where* k *is an integer. Let* $n = 4k + r$, *where* $r = 0, 1, 2,$ *or* 3. *Thus* $a^n = a^{4k}a^r \equiv a^r \ (mod\ 5)$. *Consequently*

$$S = 1^n + 2^n + 3^n + 4^n \equiv (1^r + 2^r + 3^r + 4^r) \ (mod\ 5).$$

  It follows that

$$\begin{aligned} S &\equiv 0 \ (mod\ 5) \quad \text{if } r = 1,2,3 \\ S &\equiv 4 \ (mod\ 5) \quad \text{if } r = 0 \end{aligned}$$

- *Verify that* $97^{104} - 1$ *is divisible by* $105 = 3 \cdot 5 \cdot 7$.

$$\begin{aligned} &97 \equiv 1 \ (mod\ 3) \therefore 97^{104} \equiv 1^{104} \equiv 1 (mod\ 3) \\ &97 \equiv 2 \ (mod\ 5) \therefore 97^2 \equiv 4 \ (mod\ 5) \equiv -1 (mod\ 5) \\ &97^4 \equiv 1 \ (mod\ 5) \therefore (97^4)^{26} = 97^{104} \equiv 1 \ (mod\ 5) \\ &97 \equiv -1 \ (mod\ 7) \therefore 97^{104} \equiv 1 \ (mod\ 7) \end{aligned}$$

  Since 3, 5, 7 have no factor in common we have $97^{104} \equiv 1 \ (mod\ 105)$.

- *By FLT* $10^{p-1} - 1$ *is divisible by* p *if* p *is not a factor of 10, i.e. if* $p \neq 2$ *and* $p \neq 5$. *Thus* $10^6 - 1 = 7k$ *or* $\frac{1}{7} = \frac{k}{10^6 - 1}$, *where* $k = 142857$. *This implies:*

$$\frac{1}{7} = \frac{k}{10^6}\frac{1}{1 - 10^{-6}} = \frac{k}{10^6} + \frac{k}{10^{12}} + \cdots + \frac{k}{10^{6m}} + \cdots$$

  This is the basis for decimal expansion of fractions, *suggesting that any rational number is always periodic. Note that the period-length may be less than* $(p - 1)$ *as for example in* $\frac{1}{3} = 0.\overline{3}$ *or as in* $\frac{1}{13} = .\overline{076923}$, *because in these cases* p *divides* $(10^{\frac{p-1}{2}} - 1)$.

- *Show that* $n^{13} - n$ *is always divisible by 2730:*

$$\begin{aligned} f(n) = n^{13} - n \quad &= n(n^{12} - 1) = n(n^6 + 1)(n^6 - 1) = (n^6 + 1)(n^7 - n) \\ &= n[(n^3)^4 - 1] \\ &= n(n + 1)(n - 1)g(n) \end{aligned}$$

  But

$$\begin{aligned} n^{13} - n \quad &\text{is divisible by 13} \\ n^7 - n \quad &\text{is divisible by 7} \\ n(n + 1)(n - 1) \quad &\text{is divisible by 6} \\ n \text{ or } (n^3)^4 - 1 \quad &\text{is divisible by 5} \quad \therefore 5 \cdot 6 \cdot 7 \cdot 13 = 2730 \text{ divides } n^{13} - n \end{aligned}$$

- *If x, y, z are integers such that $x^2 + y^2 = z^2$, then $xyz = 0$ (mod 60).*

  The general integer Pythagorean triplet is: $x = 2kab$, $y = k(a^2 - b^2)$, $z = k(a^2 + b^2)$ so $xyz = 2k^3ab(a^4 - b^4)$. Either a, b or $a^2 - b^2$ is even $\therefore xyz = 0 \ (mod\ 4)$. Either a, b or $a^2 - b^2$ is a multiple of 3, since by Fermat's theorem $3 \nmid a$, $3 \nmid b$ imply $a^2 - b^2 = (1-1) \ (mod\ 3) = 0 \ (mod\ 3)$ $\therefore xyz = 0 \ (mod\ 3)$. Similarly by Fermat's theorem $a^4 - b^4 = 0 \ (mod\ 5)$ if neither a nor b are divisible by 5. Thus $xyz = 0 \ (mod\ 3 \cdot 4 \cdot 5) = 0 \ (mod\ 60)$.

- *Prove that 19 divides $2^{2^{6k+2}} + 3$ for $k = 0, 1, 2$.*

  We have $2^6 = 64 \equiv 1 \ (mod\ 9)$, hence for $k = 0, 1, 2, \ldots$ we also have $2^{6k} \equiv 1 \ (mod\ 9)$. Therefore $2^{6k+2} \equiv 2^2 \ (mod\ 9)$, and since both sides are even, we get $2^{6k+2} \equiv 2^2 \ (mod\ 18)$. It follows that $2^{6k+2} = 18t + 2^2$, where t is an integer ≥ 0. However, by Fermat's theorem, $2^{18} \equiv 1 \ (mod\ 19)$, and therefore $2^{18t} \equiv 1 \ (mod\ 19)$ for $t = 0, 1, 2, \ldots$. Thus $2^{2^{6k+2}} = 2^{18t+4} \equiv 2^4 \ (mod\ 19)$; it follows that $2^{2^{6k+2}} + 3 \equiv 2^4 + 3 \equiv 0 \ (mod\ 19)$, which was to be proved.

- *Prove that 13 divides $2^{70} + 3^{70}$*

  By Fermat's Theorem we have $2^{12} \equiv 1 \ (mod\ 13)$; hence $2^{60} \equiv 1 \ (mod\ 13)$, and since $2^5 \equiv 6 \ (mod\ 13)$, which implies $2^{10} \equiv -3 \ (mod\ 13)$, we get $2^{70} \equiv -3 \ (mod\ 13)$. On the other hand, $3^3 \equiv 1 \ (mod\ 13)$, hence $3^{69} \equiv 1 \ (mod\ 13)$ and $3^{70} \equiv 3 \ (mod\ 13)$. Therefore $2^{70} + 3^{70} \equiv 0 \ (mod\ 13)$, or $13 | 2^{70} + 3^{70}$, which was to be proved.

- *Prove that $11 \cdot 31 \cdot 61$ divides $20^{15} - 1$*

  Obviously, it suffices to show that each of the primes 11, 31, and 61 divides $20^{15} - 1$. We have $2^5 \equiv -1 \ (mod\ 11)$, and $10 \equiv -1 \ (mod\ 11)$, hence $10^5 \equiv -1 \ (mod\ 11)$, which implies $20^5 \equiv 1 \ (mod\ 11)$, and $20^{15} \equiv 1 \ (mod\ 11)$. Thus $11 | 20^{15} - 1$. Next, we have $20 \equiv -11 \ (mod\ 31)$, hence $20^2 \equiv 121 \equiv -3 \ (mod\ 31)$. Therefore $20^3 \equiv (-11)(-3) \equiv 33 \equiv 2 \ (mod\ 31)$, which implies $20^{15} \equiv 2^5 \equiv 1 \ (mod\ 31)$. Thus, $31 | 20^{15} - 1$. Finally, we have $3^4 \equiv 20 \ (mod\ 61)$, which implies $20^{15} \equiv 3^{60} \equiv 1 \ (mod\ 61)$ (by Fermat's theorem); thus $61 | 20^{15} - 1$ as well.

The Old Chinese Theorem

As early as 500 BCE the Chinese were aware of one divisibility fact included in Fermat's Theorem, for their manuscripts asserted that $2^p - 2$ is divisible by p when p is prime. Thus $2^{11} - 2 = 2046$ is divisible by 11, which *can readily be checked, and* $2^{9941}-2$ *is divisible by the prime* 9941*, a fact which no one would care to verify "by hand".*

But Fermat's theorem implies an infinite number of other divisibility statements. For example, $3^{9941}-3,\quad 4^{9941}-4,\quad 5^{9941}-5,\ldots,$ $9940^{9941}-9940$ *must all be divisible by* 9941*, and* $2^{65537}-2,\quad 3^{65537}-3,\ldots,$ $65536^{65537}-65536$ *are all divisible by the Fermat's prime* 65537*.*

Although 2^n-2 *must be divisible by* n *if* n *is a prime number, the early Chinese (and even, much later, Leibniz himself) erred in conjecturing that the converse statement would be true. They believed that if* 2^n-n *is divisible by* n*, then* n *would, of necessity, be prime, so that the divisibility property could then be used as a test of primality.*

The conjecture was discovered to be false only in 1819, when it was shown that $2^{341}-2$ *is exactly divisible by* $341=11\cdot 31$*, a* composite *number. (Subsequently it was found that* 2^n-2 *is divisible by* n *for an infinite number of other composite values of* n*.)*

To see this we just use the binomial theorem through which it is shown that $(a-b)$ *divides* a^k-b^k*. Since* $(2^{10}-1)=1023=3\cdot 341$ *we can write*

$$\begin{aligned}
2^{341}-2 \qquad &= [(2^{31})^{11}-2^{11}]+[2^{11}-2]\\
&= (2^{31}-2)M_1+(2^{11}-2)\\
&= 2\{(2^{10})^3-1\}M_1+2(2^{10}-1)\\
&= 2(2^{10}-1)M_1M_2+2(2^{10}-1)\\
&= (2^{10}-1)J=341Q
\end{aligned}$$

M_1, M_2, J, Q *integers.*

Another way of showing this is that $2^{340}-1\equiv 0\ (mod\ 341)$*. Indeed,*

$$2^{10}\equiv 1\ (mod\ 11);\qquad 2^{10}\equiv 1\ (mod\ 31)$$

$$\therefore (2^{10})^{34}\equiv 1\ (mod\ 11);\qquad (2^{10})^{34}\equiv 1\ (mod\ 31)$$

This means that 11 and 31 each divide $2^{340}-1$*. But then, since* $(11,31)=1$ *so does their product 341.*

A composite number n *which divides* 2^n-2 *is a* pseudoprime: pseudo*primes can also be* even*;* **D.H. Lehmer** *discovered (1950) the pseudoprime* $161,038=2\cdot 73\cdot 1103$ *yielding*

$$2^{161,038}-2=2(2^{161,037}-1);\qquad 161,037=3^2\cdot 29\cdot 617$$

$$2^{161,037}-1=(2^9)^{29\cdot 617}-1^{29\cdot 617}=(2^9-1)(\cdots)=7\cdot 73(\cdots)$$

Similarly,

$$2^{161,037} - 1 = (2^{29})^{9 \cdot 617} - 1^{9 \cdot 617} = (2^{29} - 1)(\cdots) = 233 \cdot 1103 \cdot 2089(\cdots)$$

Since 73 and 1103 are both primes, dividing $2^{161,037} - 1$, it follows that 161038 is an even pseudoprime.

A composite number n which divides $3^n - 3$, or $4^n - 4$, or etc ..., strikes us as sharing in the property of pseudoprimality. A composite number n which divides $2^n - 2$, and $3^n - 3$, and $4^n - 4$, and ..., and $a^n - a$, and ..., for every integer a, even the negative integers, is certainly the ultimate in this regard, and is called an absolute pseudoprime.

The smallest one is 561. That is to say, 561 is a composite number and $a^{561} - a$ is divisible by 561 no matter what integer is substituted for a. This follows directly from Fermat's Little Theorem: the prime decomposition of 561 is $3 \cdot 11 \cdot 17$. We need to show that $a^{561} - a$ is divisible by each of these primes. We have

$$a^{561} - a = a(a^{560} - 1) = a[(a^{10})^{56} - 1^{56}] = a[(a^{10} - 1)(\cdots)]$$
$$= (a^{11} - a)(\cdots).$$

But $a^{11} - a$ is divisible by 11, by Fermat's theorem, because 11 is a prime number. Thus 11 divides $a^{561} - a$. Similarly 3 and 17 are also shown to be divisors.

A few other absolute pseudoprimes are

$2821 = 7 \cdot 13 \cdot 31$ $4991 = 7 \cdot 23 \cdot 31$ $10585 = 5 \cdot 29 \cdot 73$ $15841 = 7 \cdot 31 \cdot 73$

$29341 = 13 \cdot 37 \cdot 61;$ $5 \cdot 17 \cdot 29 \cdot 113 \cdot 337 \cdot 673 \cdot 2689.$

It is unknown whether or not there exists an infinity of absolute pseudoprimes.

The Isogonic Center

In 1643 ***Fermat*** *posed the following problem to the Italian mathematicians* ***Evangelista Torricelli*** *(a pupil of Galileo, 1608–1647) and* ***Francesco Cavalieri*** *(1598–1647):*

To find a point P of the plane, the sum of whose distances from the vertices of a given triangle ABC is the smallest possible. Torricelli's solution was published posthumously in 1659 by his pupil ***Viviani***[64] *(1622–1703). A simple non-calculus solution, published in 1929, (for the case where each angle in the triangle is less than $120°$) is this: Let P be a point inside the triangle ABC. Rotate the triangle APC by $60°$ about A in a direction away from the opposite vertex B and denote its new position in the plane by $AP'C'$. Clearly, the sum of distances $\overline{AP}+\overline{BP}+\overline{CP}$ is now equal to the sum of the segments $\overline{C'P'}+\overline{P'P}+\overline{PB}$, which in general will constitute a continuous broken line. Since the end points of this line are fixed (the position of C' is independent of P!), its length will be minimal if P and P' are on $\overline{C'B}$.*

This implies that the sought point P is such that the sides of the triangle are seen from P at equal angles of $120°$. The construction of P is simple: build on each side of the triangle a new equilateral triangle and connect the new vertices to the corresponding opposite vertices of the original triangle. The three lines will meet at P. This solution is undoubtedly one of the most beautiful ones in the entire Euclidean geometry. The point P is known as the Fermat point.

It is of interest to mention that the above solution is the amalgam of the contributions of four mathematicians during 1643–1846: The first, ***Torricelli****, knew that the circumcircles of the outward, equilateral triangles on the sides*

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[64] **Vincenzo Viviani** was an assistant to both Galileo and Torricelli. His primary interests lay in geometry, hydraulics and mechanics. He discovered the geometrical theorem (named after him): For a point P inside an equilateral triangle ABC, the sum of the perpendiculars a, b, c from P to the sides is equal to the altitude h.

Viviani studied with the Jesuits in Florence. His years with Galileo took the place of a university education, and he was Galileo's companion and pupil during the final two years of his master's life.

In 1660, together with Borelli, Viviani measured the velocity of sound by timing the difference between the flash and the sound of a cannon. They obtained a value of 350 $\frac{\text{m}}{\text{sec}}$, which is considerably better than the previous value of 478 $\frac{\text{m}}{\text{sec}}$ obtained by Gassendi (the currently accepted value is 331.29 $\frac{\text{m}}{\text{sec}}$ at $0\,°\text{C}$).

of the triangle ABC intersect at P. The second, **Cavalieri**, *found that each side of the triangle ABC is seen from P at an angle of $120°$. The third,* **Thomas Simpson** *(1710–1761), realized in 1750, that the lines joining the outer vertices of the triangle ABC intersect at P. In 1834,* **Heinen** *noted that if one of the interior angles (say B) is greater or equal to $120°$, then the shortest pathway linking A, B, and C consists of the segments AB and BC.*

In 1846, **Eduard Fasbender** *discovered the following maximum property of P associated with its minimum property, if P lies in the interior of ABC: The least value of the sum of distances $\overline{AP} + \overline{BP} + \overline{CP}$ in the triangle ABC is equal to the maximum of the altitudes of all equilateral triangles circumscribing the triangle ABC.*

The point P is now known as the isogonic center of the triangle. It was the first notable point of the triangle to be discovered in times more recent than that of Greek mathematics. A thorough analysis of the problem and its generalization to an arbitrary number of point in any number of dimensions, was given (1843) by the geometer **Jacob Steiner** *(1796–1863) and is known therefore as the Steiner problem.*

The Steiner figure can be obtained in a soap-film experiment: To this end one takes two glass plates kept parallel by three perpendicular pins of equal length. If the configuration is immersed in a soap bath and taken out again, one obtains a system of three soap films perpendicular to each of the plates. These soap laminae touch each plate in three segments that yield the shortest pathway linking the three pins at either plate.

As noted, for two or three points the minimal pathway is uniquely determined. For four or more points, however, we must generally expect more than one minimal pathway. We must even distinguish between stationary and stable pathways. The stable pathways yield either absolute, or merely relative, minima.

The generalization of the Steiner problem to n points in a plane does not lead to interesting results. To find a really significant extension we must abandon the search for a single point P. Instead we look for the "street network" of shortest total length! Thus, if we choose four points that are vertices of a square, then we obtain two different but congruent minimal pathways. If we stretch the square into a rectangle, then we obtain two minimal pathways of different length, one of which is an absolute and the other a relative minimum.

Mathematically expressed, the problem is: Given n points $A_1, A_2, \ldots, A_n$, to find a connected system of straight line segments of shortest total length such that any two of the given points can be joined by a polygon consisting of segments of the system. This problem is known today as the Steiner *problem*[65]*, and its solution has eluded the fastest computers and the sharpest mathematical minds.*

The Steiner problem cannot be solved by simply drawing lines between the given points, but it can be solved by adding new ones called Steiner points*, that serve as junctions in a shortest network.*

To determine the location and number of Steiner points, mathematicians and computer scientists have developed algorithms. Yet, even the best of these procedures running on the fastest computers cannot provide a solution for a large set of given points because the time it would take to solve such a problem is impractically long. Furthermore, the Steiner problem belongs to a class of problems for which many computer scientists now believe an efficient algorithm may never be found.

However, approximate *solutions to the shortest-network problem are computed routinely for numerous applications, among them designing integrated circuits, determining the evolution tree of a group of organisms and minimizing materials used for networks of telephone lines, pipelines and roadways.*

About 200 years after Fermat, when calculus was well established, an analytic solution for the triangle was given: Let (a_1, b_1), (a_2, b_2), (a_3, b_3) *be respectively the coordinates of the vertices* A, B, C, *referred to a system of rectangular coordinates. The function whose minimum is sought is*

$$z(x,y) = \left[(x-a_1)^2 + (y-b_1)^2\right]^{1/2} + \left[(x-a_2)^2 + (y-b_2)^2\right]^{1/2}$$
$$+\left[(x-a_3)^2 + (y-b_3)^2\right]^{1/2}.$$

From the relations $\frac{\partial z}{\partial x} = 0$, $\frac{\partial z}{\partial y} = 0$, *one obtains two algebraic equations*

$$\frac{x-a_1}{a} + \frac{x-a_2}{b} = -\frac{x-a_3}{c}, \quad \frac{y-b_1}{a} + \frac{y-b_2}{b} = -\frac{y-b_3}{c},$$

where $PA = a$, $PB = b$, $PC = c$.

Then, squaring and adding, we find the condition

$$1 + 2\left[\frac{(x-a_1)}{a}\frac{(x-a_2)}{b} + \frac{(y-b_1)}{a}\frac{(y-b_2)}{b}\right] = 0.$$

The geometrical interpretation of this result is straightforward: denoting by α *and* β *the cosines of the angles which the direction* PA *makes with the*

[65] *The Shortest-Network Problem*, M.W. Bern and R.L. Graham, Scientific American, January 1985.

axes x and y, respectively, and by α' and β' the cosines of the angles which PB makes with the same axes, we may write this last condition in the form

$$1 + 2(\alpha\alpha' + \beta\beta') = 0,$$

or by denoting the angle APB by ω,

$$2\cos\omega + 1 = 0.$$

This condition expresses the fact that the segment AB subtends an angle of 120° at the point P. For the same reason, each of the angles BPC and CPA must be 120°.

The sum $PA + PB + PC$ is less than the sum of any two sides of the triangle:

$$AB + AC = \sqrt{a^2 + b^2 + ab} + \sqrt{a^2 + c^2 + ac} > \left(b + \frac{a}{2}\right) + \left(c + \frac{a}{2}\right).$$

Hence $AB + AC > a + b + c$ and P therefore actually corresponds to a minimum. When one of the angles of the triangle is equal or greater to 120°, the minimum must be given by the vertex of the obtuse angle.

1626–1629 CE **Albert Girard** (1595–1632, Netherlands). Mathematician. First to accept and use negative roots of equations in the solutions of geometrical problems. Conjectured that an algebraic equation of degree n has n roots, some of which may be non-real (the *fundamental theorem of algebra*).

First to show how to express the sums of the powers of the roots in terms of the equation's coefficients. First to publish (1629) the equation

$$A = \pi r^2\left(\frac{s}{180} - 1\right),$$

relating the area A of a geodesic triangle on a sphere of radius r to the sum of angles s (in degrees) of that triangle.

Published a treatise on trigonometry (1626) containing the first use of the abbreviations *sin, cos, tan*.

1630–1668 CE **Jan Amos Komensky (Comenius)** (1592–1670, Moravia, Poland and Holland). Pioneer of modern education; educational reformer and philosopher, promoter of scientific societies. His ultimate aim was universal peace.

He recognized that the necessary steps preliminary to the attainment of this goal involved the unification of rival Christian denominations, fundamental reforms in education and new approach to natural science.

It was largely the result of his initiative that scientific societies promoting research were founded throughout Europe during the 17^{th} century. He insisted that education should be free, universally available, and compulsory for every child, that automatic memorization should be replaced by teaching words with perceptual objects, and that the sensual faculties of school children be taken into consideration.

Comenius stands on a transitional figure in the area of science – half-way between the medieval Aristotelianism and modern empiricism. He believed that independent study and observation offered greater intellectual rewards than did constant reliance upon Aristotle or Pliny. His textbooks, translated into 17 languages were used in the early years of Harvard University, and throughout the 17^{th} century schools of Europe, Asia and the New England.

His principal works were: *Gate of Languages Unlocked* (1631); *The Way of Light* (1642); *Patterns of Universal Knowledge* (1651); *The Great Didactic* (1657); *Visible World* (1658); The last was the first textbook in which pictures were as important as text.

Central to his philosophy is the proliferation of *truth*, which, being one and universal, carries a chance for world's peace. Men should be educated trilaterally to spiritual life, secular moral life and faithful religious life. Hence the three aims of education: enlightenment, virtues and God-fearing.

Comenius developed a new philosophy of education. He favored broad general education, rather than the narrow training of his day. His curriculum consisted of: singing, languages, economy, politics, world history, science, geography, arts and handicrafts.

He suggested four stages of education, each of 6 years:

(1) 0–6 "mother school" in the family;

(2) grammar school 6–12, emphasizing the development of imagination, memory and the basic skills;

(3) *Latin school*, 12–18, for the development of the intellect;

(4) *Universities and traveling*, 18–24, to consolidate the will and endeavor to harmonize the various domains of education.

First to advocate teaching of science in schools. Urged the establishment of more schools and universities. Developed new method of teaching languages and issued the first children picture book.

He was born in Comna in Moravia of poor parents and studied at Heidelberg. Fled the Thirty Years' War to Poland (1621), settling with a group of Bohemian Brethren at Leszno. Invited to England (1641–1642) and Sweden (1642–1648) to advise on school educational reforms.

Twice during his lifetime, Komensky lost all his property and manuscripts: in 1621, during the Spanish invasion and the prosecution of the Protestants in Moravia, and again in 1655 when the Poles burned Lissa during the Swedish-Polish War.

1630–1632 CE **John Rey** (1582–1645, France). Metallurgist. One of the earliest scientists to put forward a mechanical theory of chemical change. It has been known for some time that metals increased in weight when they were heated in air and formed a calx. To explain the phenomenon, Rey suggested that air had weight, and that it was taken up by metals on heating. He did not think of the process as a chemical combination of air with the metal but as a *mechanical mixing*, like dry sand taking up water and becoming heavier.

In 1632 Rey improved the thermoscope of **Galilei** (1596) and **Sanctorius** (1611) when he used liquid instead of air to measure temperature changes, that is, the thermoscope had fluid at the bottom and air at the top, more closely resembling modern-day thermometers.

1630 CE Venice and surrounding Italy devastated by plague. 500,000 died. By 1632, the disease reached France, killing ca 100,000 more.

1634–1643 CE **Gilles Personier de Roberval** (1602–1675, France). Geometer and physicist whose extensive correspondence served as a medium for the intercommunication of mathematical ideas. Developed some pre-calculus methods of integration of some trigonometric functions and drawing tangents to plane curves. Asserted that *gravitation* is an inherent property of matter throughout the universe and that the counter balancing force allowing bodies to remain separated is the resistance of the intervening *ether*.

He was consistently tardy in disclosing his discoveries. This has been explained by the fact that for 40 years he held the professorial chair of Ramus at the Collège Royale. This chair automatically became vacant every three years, to be filled by open competition in mathematical contests in which the questions were set by the outgoing incumbent.

1634–1647 CE **Adam Olearius** (b. Oehlschlaeger; 1599–1671, Germany). Geographer, traveler, mathematician and scholar. His travels in Persia and Russia[66] (1634–1639) were described in his book (1647): "*Voyages of the Ambassadors Sent by Frederic, Duke of Holstein, to the Great Duke of Muscovy and the King of Persia.*"

Olearius' accounts of his travels became one of the major early descriptions of Russia by a European. He was the first to introduce Western Europe to Persian culture.

1635 CE **Francesco Bonaventura Cavalieri** (1598–1647, Italy). Mathematician. Advanced certain rules that constituted valuable tools in the computation of areas and volumes. Also produced explicit formulae which showed how to integrate a class of functions. These methods are essentially those of the definite integral and anticipated the development of the calculus later in the century. Cavalieri used his method to evaluate correctly the area of the ellipse and the volume of the sphere. The methods of Cavalieri were later extended by **Torricelli** (1645), **Fermat** (1654), **Pascal** (1654), **Barrow** (1662) and others.

Cavalieri was born in Milan, studied under **Galileo**, and served as a professor of mathematics at the University of Bologna from 1629 until his premature death at the age of 49. His treatise *Geometria indivisibilibus* (1635) is devoted to the pre-calculus *method of indivisibles* that can be traced back to **Democritos** (ca 410 BCE) and **Archimedes**. It is likely that the attempts at integration made by **Kepler** directly motivated Cavalieri.

1636–1641 CE **Jeremiah Horrocks** (1619–1641, England). Astronomer. First to apply Kepler's laws to the actual motion of the moon. This was later used by **Newton** to forge his synthesis of Kepler laws of the motion of heavenly bodies and Galileo's laws of falling bodies and projectiles. Horrocks clearly perceived the significant analogy between terrestrial gravity and the force exerted in the solar system.

Horrocks was born at Toxteth Park, near Liverpool. His family was poor and he pursued his self-education amidst innumerable difficulties. He entered Emmanuel College, Cambridge, in 1632 and his university career lasted three

[66] During the early 17^{th} century, northern European merchants saw Russia as a land through which secure trade routes might be opened to Persia and points east — without danger from or taxation by the Turks, and unknown to Italy, Spain and Portugal. Adam Olearius was appointed secretary to an embassy from the Duke of Holstein to Muscovy and Persia which sought to make that Duchy an entrepot for overland silk trade.

years. On its termination he became a tutor at Toxteth, devoting to astronomical observations his brief intervals of leisure.

In 1639 he applied himself to the revision of the Rudolphine Tables (published by **Kepler** in 1627), and in the progress of this task became convinced that a transit of Venus[67], overlooked by Kepler, would nevertheless occur on the 24^{th} of November 1639.

He indeed observed it, while a curate at Hoole, near Preston. This transit of Venus is remarkable as the first ever observed (that of 1631 predicted by Kepler, having been invisible in Western Europe). Through this observation he was able to introduce some important corrections into the elements of the planet's orbit and obtain a good estimate of its apparent diameter.

Before he was twenty, Horrocks made an important contribution to lunar theory, by showing that the moon's apparent irregularities could be completely accounted for by supposing it to move in an ellipse with a variable eccentricity and a rotating major axis of which the earth occupies one focus. These precise conditions were afterwards demonstrated by Newton to follow necessarily from the law of gravitation.

Jeremiah Horrocks died when only in his twenty-second year.

1636–1644 CE **Girard Desargues** (1593–1662, France). Mathematician, engineer and architect. The most original contributor to projective geometry in the 17^{th} century. A geometer of profoundly original ideas, sustained at the same time by a good spatial intuition, precise knowledge of the great classic works and exceptional familiarity with the whole range of contemporary techniques.

In 1639 he distributed in Paris a twelve-page booklet under the heading (translated): "Proposed Draft on an Attempt to Deal with the Cases of Meeting of a Cone with a Plane". After presenting his rules of practical perspective, Desargues outlines a program dominated by two basic themes: the concern to rationalize and unify the diverse preexisting graphical techniques and the purely geometric study of perspective.

In this book he developed topics to be found in modern courses in projective geometry, such as: harmonic ranges, homology, poles and polars, perspectives and involution. The book included Desargues' well-known (today) 'two-triangle theorem'. In spite of this, the treatise was ignored, forgotten and lost until 1845, when **Michel Charles** (1793–1880) found a manuscript

[67] Transits of Venus (when the planet is passing between the earth and the sun) are among the rarest of astronomical phenomena; many astronomers cannot possibly see one during their lifetimes. Since 1639 there have been transits in 1761, 1874 and 1882. The next pair will occur in 2004 and 2012.

copy, and since that time the work has been regarded as one of the classics in synthetic projective geometry.

Desargues was born in Lyons, one of the nine children of a collector of the tithes on ecclesiastical revenues in the diocese of Lyons. He apparently was educated as an engineer (and architect), for there is evidence for his presence in Paris in 1626 in connection with a certain engineering project. In 1630 he evidently became friendly with several of the leading mathematicians in Paris: **Mersenne**, **Gassendi** and **Roberval**.

After the publication of his booklet in 1636 he won the esteem and respect of **Descartes**, and young **Pascal**, both members of Mersenne's*, Academie Parisienne*. Throughout the period 1636–1644, many attacks were launched against Desargues' work by second-rate mathematicians, which may have caused his scientific and polemic activity to decline; he then embarked on his new career as an architect (1645–1657).

He returned to Paris from Lyons in 1657. In Paris, the authors of the period attribute to Desargues, besides a few houses and mansions, several staircases whose complex structure and spectacular character attest to the exactitude of his graphical stonecutting procedures. It also seems that he collaborated, for the realization of certain effects of architectural perspective, with the famous painter **Phillipe de Champagne** (1602–1674). In the region of Lyons, Desargues' architectural creations were likewise quite numerous.

Desargues' main accomplishment as an engineer, was a system for raising water that he installed near Paris, at the Château of Beaulieu, based on the use (until then unknown) of epicycloidal wheels (described and drawn by **Huygens** in 1671).

In 1660 Desargues was again active in the scientific life of Paris, attending meetings at Montmor's Academy. He was heard of last on the meeting of 9 November 1660, at which Huygens heard him present a report on a geometrical problem.

Descartes was probably the source of both the inspiration and demise of his book, since geometers at that time were totally absorbed in Cartesian geometry to the exclusion of any new idea in the field. However, in the early 19^{th} century, the mathematical community was once again willing and capable to digest novel, nonorthodox geometrical ideas.

1637 CE **René du Perron Descartes** (1596–1650, France). Distinguished mathematician, scientist and philosopher. Published his work "*Discours de la méthode pour bien conduire sa raison et chercher la vérité dans les sciences*", the third and last appendix of which, "La Géométrie" contains a sufficiently complete (although somewhat confusing) presentation of the mathematical theory that since then has been called analytic geometry. This discovery of the utility of coordinate systems in relating geometry and algebra opened up an entire new science by enabling the investigation of geometric objects by algebraic means.

Descartes revised exponential notation for integral exponents almost to its 20^{th} century form. He was first to allow powers higher than the third.

He formalized the classical law of inertia, in the form given in Newton's '*Principia*' 40 years later, but he went further to suggest the conservation of linear momentum. Unlike Newton, who used his theory of gravitation to explain how the orbital motions of the planets and satellites can be maintained, but *not* how they have *originated*, Descartes assumed that originally the world has been filled with matter distributed uniformly. He then sketched out a qualitative theory of successive formation of the sun and the planets.

In his book *La Dioptrique* (1637) he was first to publish the now familiar law of refraction in terms of sines. Although the result was given already by **W. Snell**[68] in 1621, Descartes went beyond Snell and derived the law from a new assumption on the nature of light. He considered it to be essentially a pressure transmitted through a perfectly elastic medium (the '*aether*'), which fills all space. In his view, light was *a stream of tiny particles* and the laws of reflection and refraction were explained by using particle kinematics[69]. [In contradistinction, **Fermat** rederived the law of reflection from his own *principle of least time*, which departed from **Hero**'s shortest-path statement.]

In an appendix to *Discours de la Méthode* (1637), Descartes discovered (using the law of refraction) the key to the rainbow problem — the reason for the clustering of rays about the angle $42°$ in the primary bow. He discovered the effective ray through patient observations and laborious calculations (the Newtonian calculus arrived only in 1671).

While Francis Bacon's empiricism influenced science and philosophy in England, Descartes left a profound mark on the thinking of scientists in Europe for the past 300 years, due to two of his ideas: the first was his conviction that the universe (including man's body but excluding his mind) is a mathematically intelligible machine, that could be deduced from a few simple principles, and eventually even by a single overreaching mathematical theorem. This view was the basis of the later cosmological theories of **Kant**, and **Laplace**.

The second is his program of *total geometrization of physics* via the concept of 'dimension'. This idea began to be realized in the new physics of the 20^{th} century, especially in Einstein's GTR, and in quantum mechanics. Thus,

[68] **Huygens** believed that Descartes had seen Snell's manuscript on refraction.

[69] See (51) p. 1006.

although his generalizations in astronomy, physics and anatomy were often premature and his passion for system-building went beyond his capacity to check by experiment, he remains one of the founders of modern scientific thinking.

Descartes rejected Aristotelian teleology which stated that all natural events are purposeful. He emphasized the use of reason and abstract deductive logic as the chief tool of philosophical inquiry. He greatly influenced later natural philosophers, especially **Berkeley**.

Descartes was the first to term the mathematical rules that others had discovered *"the Laws of Nature"*. God rules the universe through these eternal and unchangeable laws, he maintained. These laws were not mere descriptions of nature, but the very 'legislation' of nature: Descartes' God was the great Lawgiver. Experiment was to be used, as with the Platonists, to illustrate laws that were mathematically deduced from first principles.

Descartes was born at La Haye, in Touraine, midway between Tours and Poitiers. From 1604 to 1612 he studied at a Jesuit school. During the winter of 1612 he took lessons in horsemanship and fencing; and then started, as his own master, to taste the pleasures of Parisian life. Here he renewed an early friendship with **Marin Mersenne**. In 1614, however, he abandoned social life and shut himself up for nearly two years in a secluded house of the Faubourg St. Germain in order to study mathematics.

In may 1617 Descartes set out for The Netherlands and took service in the army of Prince Maurice of Orange. After spending two years in Holland as a soldier in a period of peace, he volunteered in 1619 into the Bavarian service. In 1621 he quit the imperial service and returned to France. Money from an inheritance and from patrons enabled him to devote most of his life to study.

He visited Switzerland and Italy, and lived in Paris before settling in Holland in 1628. Except for short visits to France to settle family affairs, a visit to England in 1630 and an excursion to Denmark (1634), he led a quiet, scholarly life in The Netherlands until 1649, and there most of his philosophical works were written.

During his residence in Holland he lived at 13 different places, and changed his abode 24 times. In the choice of these spots, two motives seem to have influenced him — the neighborhood of a university or college, and the amenities of the situation. His residence in the Netherlands fell in the most prosperous and brilliant days of the Dutch state. Abroad, its navigators monopolized world commerce and explored unknown seas; at home the Dutch school of painting reached its pinnacle in Rembrandt (1607–1669).

In 1649 he accepted an invitation from Queen Christina to visit Sweden. The young queen wanted Descartes to draw up a code for a proposed academy of sciences, and to give her an hour of philosophic instruction every morning at five in her draughty chambers. However, he fell victim to the inflammation of the lungs, and died soon thereafter in Stockholm.

Descartes' new ideas were slow to gain the recognition they deserved. In his appendices of 1637, he hit upon three capital advances yet not one of them was integrated into scientific thought for several decades. The fault lay in part with Descartes himself. In the case of each of the appendices — *La Dioptrique* and *La Geometrie*, as well as *Les Meteores* — the author was primarily boasting of the efficacy of his methodology. He was not explaining, with a meticulous care required in new situations, the value of these contributions to science. He did not explore them further, nor did he determine their implications and their relationship to other phenomena. He did not surround them with an aura of proselytizing enthusiasm. In fact he promptly lost interest in analytic geometry, the law of refraction, and the rainbow.

Descartes never married. In person he was small, having a large head, protruding brow, prominent nose, and eyes wide apart, with black hair coming down almost to his eyebrows. His voice was feeble. He usually dressed in black, with unobtrusive propriety.

In all his travels he only studied the phenomena of nature and human life. He was a spectator, rather than an actor, on the world stage. He entered into the army, merely because the position gave a vantage-ground from which to make his observations. He took no part in the political interests which these contests involved.

The contempt of aesthetics and erudition is characteristic of the Cartesian system; to him all the heritage of the past seemed but elegant trivia. The science of Descartes was physics in all its branches, but especially as applied to physiology. Science, he said, may be compared to a tree; metaphysics in the root, physics in the trunk, and the three chief branches are mechanics, medicine and morals — the three applications of our knowledge to the outside world, to the human body, and to the conduct of life.

Who Invented Analytic Geometry?

"Everything has been thought of already. The problem is — thinking of it again".

Johann Wolfgang von Goethe (1749–1832)

By definition, analytical geometry is concerned with the representation of geometrical figures and their relations by algebraic *equations. This essentially means that a problem in geometry is transformed into a corresponding one in algebra, the algebraic problem solved, and finally the algebraic solution is interpreted in geometrical terms. It follows that, before analytic geometry could assume its highly practical form, it had to await the development of algebraic procedures and symbolism. These decisive contributions were only made in the* 17^{th} *century by* ***René Descartes*** *(1596–1650) and* ***Pierre Fermat*** *(1601–1665). Not until after the impetus given to the subject by these two men, do we find analytic geometry in a form with which we are familiar.*

Nevertheless, one of the basic ingredients of analytic geometry, namely the concept of fixing the position of a point by means of suitable reference frame, was employed in the ancient world by the Egyptian and the Roman surveyors and by the Greek map-makers. And, if analytic geometry implies not only the use of coordinates but also the geometric interpretation of relations among coordinates, then Greek priority is favored by the fact that ***Apollonios*** *derived the bulk of his geometry of* conic sections *from the geometrical equivalents of certain algebraic equations of these curves, an idea that seems to have originated with* ***Menaechmos*** *about 350 BCE!*

All these results must have been known to Fermat and Descartes, who were both deeply versed in the classical literature of mathematics. At any rate, they certainly could not have escaped reading ***Oresme****.*

1640–1662 CE **Blaise Pascal** (1623–1662, France). Mathematician, theologian, physicist and philosopher who made great contributions to science through his studies in hydrostatics and the mathematical theory of probability.

During the 16^{th} and 17^{th} centuries a great deal of the leisure of the European aristocracy was occupied with games of chance and gambling in general.

This class did not include among their number any mathematicians capable of handling the problems that naturally suggested themselves. Thus it happened that from time to time problems of chance were passed on to the mathematicians of the period. We know for example that **Galileo** (1564–1642) had his attention directed by an Italian nobleman to a problem in dice.

Pascal was drawn into probability theory as a result of problems that arose in gambling houses. At the time, a gambling die game was in vogue which had been played for at least a hundred years and which persists to the present day: the "house" offers to bet even money that a player will throw at least one six in four throws of a single die. [This game is mildly favorable to the "house" since, on the average it wins $\left[1-\left(\frac{5}{6}\right)^4\right]$ to $\left(\frac{5}{6}\right)^4$, i.e., in a ratio $\frac{671}{625}$].

A distinguished Frenchman, **Antoine Gombauld Chevalier de Méré, Sieur de Baussay** (1610–1685) was bothered by a number of practical problems concerning the game, one of which is called 'the problem of points' (the division problem): "How should the prize money be divided among the contestants if for some reason it proved necessary to call off the game before it is completed and when the contestants have only partial scores?"

A second problem was: "If the player roles a *pair* of dice, will it be favorable to the 'house' to bet that the player will throw at least one double six in 24 throws of the pair?". Méré consulted Pascal, whom he knew, and Pascal proved that the odds were slightly *against* the house if it wagered on 24 throws, but were slightly *favorable* for 25 throws.

For the solution of the first problem, Pascal introduced the important idea that the amount of the prize any contestant deserved, in a partial game, should depend on the *probability* that this particular player would win the game, were it carried through to its conclusion. And Pascal worked out in detail, for several examples, how the probability of winning could be calculated from a knowledge of the nature of the game and the partial score of each contestant.

Pascal wrote about these matters to Fermat, who had a great reputation as a mathematician and who was in addition a distinguished justice at Toulouse. The resulting exchange of letters went further in working out the mathematics of some games of chance, and became known in the learned society of the day. This episode can properly be regarded as the advent of a new branch of mathematics.

At the time when the theory of probability started at the hands of Pascal and Fermat, they were the most distinguished mathematicians in Europe. [**Descartes** died in 1650. **Newton** (b. 1642) and **Leibniz** (b. 1646) were as yet unknown. **Huygens** (b. 1629) could not, at this time, be placed on the level of Pascal and Fermat.]

It might have been anticipated that a subject of such interest in itself and discussed by the two most distinguished mathematicians of the time, would have attracted rapid and general attention; but such does not appear to have been the case. The two great men themselves seem to have been indifferent to any extensive publication of their investigations. *It was sufficient for each of them to gain the approbation of the other.*

The invention of the calculus by Newton and Leibniz soon offered mathematicians a subject of absorbing interest, and the theory of probability advanced but little during the half century which followed the dates of the correspondence between Pascal and Fermat (1654). In 1658, Pascal published several treatises which established his work as a forerunner of both differential and integral calculus.

In 1648 Pascal formulated the basic laws of equilibrium for fluids (published posthumously in 1663), stating that pressure in a fluid is transmitted equally in all directions, and that the height of the mercury column in a barometer is balanced by the pressure of air. He suggested that the barometer be used to determine altitudes, and further used measurements of the barometric pressure made at the summit of *Mount Puy de Dôme* to estimate the total weight of the atmosphere (his value $= 3.7 \times 10^{18}$ kg).

Pascal was a son of a nobleman. A prodigy of sorts, he had already published an essay on conic sections by the age of 16 in which he discovered and proved '*Pascal's Theorem*'[70]. He also invented one of the early calculating machines that could add and subtract (1642).

In his *Traité du triangle arithmétique* (1654), Pascal united the algebraic and combinatorial theories by showing that the elements of the arithmetic triangle (known as the "*Pascal Triangle*"[71] could be interpreted in two ways: as the coefficient of $a^{n-k}b^k$ in $(a+b)^n$ and as the numbers of combinations of n things taken k at a time. In effect, he showed that $(a+b)^n$ is the *generating function* for the numbers of combinations. As an application, he founded the mathematical theory of probability by solving the problem of division of

[70] If a hexagon is inscribed in a conic, then the points of intersection of the three pairs of opposite sides are collinear, and conversely (1640).

[71] In this triangle, the k^{th} element $\binom{n}{k}$ of the n^{th} row is the sum $\binom{n-1}{k-1} + \binom{n-1}{k}$ of the two elements above it in the $(n-1)^{th}$ row, as follows from the formula $(a+b)^n = (a+b)^{n-1}a + (a+b)^{n-1}b$. The triangle appeared to the depth of six in **Yang Hui** (1261) and to a depth of eight in **Zhu Shijie** (1303). Yang Hui attributes the triangle to **Jia Xian**, who lived in the 11^{th} century.
The numbers $\binom{n}{k}$ appear as the number of combinations of n things taken k at a time in the writing of **Levi ben Gershon** (1321), who gave the formula $\binom{n}{k} = \frac{n!}{(n-k)!k!}$.

stakes[72], and as a method of proof he used mathematical induction for the first time in a really conscious and unequivocal way.

Late in 1654, he became dissatisfied with experimentation and withdraw from science and the world for a life of religious meditations. He turned to the study of man and his spiritual problems and produced a religion-oriented philosophy that concerned itself primarily with the relation of man to God through faith.[73]

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[72] Suppose that a game between players I and II has to be called off with n players remaining, k of which I has to win in order to carry off the stakes. Assuming that I has an even chance of winning each play, the ratio of his chance of winning the stakes to that of II's winning is

$$\left[\binom{n}{n} + \binom{n}{n-1} + \cdots + \binom{n}{k}\right] \Big/ \left[\binom{n}{k-1} + \binom{n}{k-2} + \cdots + \binom{n}{0}\right].$$

[73] For further reading, see:

- Steinmann, J., *Pascal*, Harcourt, Brace and World: New York, 1966, 304 pp.

Worldview IX: Blaise Pascal

* * *

"If God does not exist, one will lose nothing by believing in him, while if he does exist, one will lose everything by not believing. Hesitate not, then, to wager that he exists".

* * *

"We arrive at truth, not by reason only, but also by the heart".

* * *

"Nature is an infinite sphere of which the center is everywhere and the circumference nowhere".

* * *

"Contradiction is not a sign of falsity, nor the lack of contradiction a sign of truth".

* * *

"Man is equally incapable of seeing the nothingness from which he emerges and the infinity in which he is engulfed".

* * *

"What is a man in nature? Nothing in relation to the infinite, all in relation to nothing, a mean between nothing and everything".

* * *

"The more intelligent one is, the more men of originality one finds. Ordinary people find no difference between men".

* * *

"[I feel] engulfed in the infinite immensity of spaces whereof I know nothing, and which know nothing of me. The eternal silence of these infinite spaces alarms me".

* * *

"Reason is the slow and tortuous method by which these who do not know the truth discover it. The heart has its own reason which reason does not know".

* * *

"One can have three principal objects in the study of truth: to discover it when one searches for it, to prove it when one possesses it and to distinguish it from falsity when one examines it".

* * *

"By space, the universe encompasses and swallows me up like an atom; by thought I comprehend the world".

(1657)

* * *

Choice and Chance[74] — *The Mathematics of Counting*[75] *and Gambling* (1654–1855)

[74] For further reading, see:

- Levy, H. and L. Roth, *Elements of Probability*, Oxford University Press: London, 1951, 200 pp.
- Parzen, E., *Modern Probability Theory and its Applications*, John Wiley & Sons: New York, 1960, 464 pp.
- Aczel, A.D., *Chance*, Thunder's Mouth Press: New York, 2004, 161 pp.
- Freund, J.E., *Introduction to Probability*, Dover: New York, 1993, 247 pp.
- Mosteller, F., *Fifty Challenging Probability Problems*, Dover Publications: New York, 1965, 88 pp.
- Rozanov, Y.A., *Probability Theory*, Dover Publications: New York, 1969, 148 pp.
- Bates, G.E., *Probability*, Addison-Wesley, 1965, 58 pp.
- Withworth, W.A., *Choice and Chance, I-II*, G.E. Strechert and Co.: New York, 1945.
- Vilenkin, N.Ya., *Combinatorics*, Academic Press: New York, 1971, 296 pp.
- Ball, W.W.R., *Mathematical Recreations and Essays*, Macmillan and Company: London, 1944, 418 pp.

[75] Anthropologists have found that tribes with limited number vocabularies ("one", "two", and "many") had elaborate ways of counting on their fingers, toes, and other parts of their anatomy in a specified order and entirely in their heads. Most primitive counting systems were based on 5, 10, or 20 (*vigesimal*) for the reason that the human animal has 5 fingers on one hand, 10 on both, and 20 fingers and toes. The ancient Chinese, Egyptians, Greeks and Romans used a base of 10. Babylonians, however, used the *sexagesimal* (base 60) which they adopted from the Sumerians, and with that they achieved a remarkably advanced mathematics. The 20–base system was used by the Mayans (together with zero and positional notation) in one of the most advanced of the ancient number systems.

The ancient Greeks and the Romans had an elaborate hand symbolism which they used for counting from one to numbers in the thousands. So did the ancient Chinese and other Oriental cultures. ***Luca Pacioli*** (1494) illustrated the Italian finger symbolism common in the Medieval and Renaissance periods. Moreover, counting symbolism soon developed into *finger arithmetic* for multiplication. This was called for since few people in the Middle Ages and the Renaissance learned the multiplication table beyond 5×5 or had access to an abacus. A variety of simple methods were in use for obtaining the products of numbers from 6 through 10 using both hands.

The Greeks and the Romans were familiar with some mathematics associated with the game of dice. ***Plato*** *in his Laws (Book 12) cited* 3 *and* 18 *as the most difficult sums to roll with three dice. Indeed, they are the only sums that can be made in only one way (1-1-1 and 6-6-6). Since there are* $6^3 = 216$ *equally probable ways of rolling three dice, the probability of making either a* 3 *or an* 18 *is* $\frac{1}{216}$. *The Greeks called 6-6-6 "Aphrodite" and 1-1-1 "the dog".*

There are many references to these and other dicing terms in Greek and Latin literature. The Roman Emperor ***Claudius*** *even wrote a book called How to Win at Dice indicating the great popularity of the game among the upper classes (in Greece too). Apart from this, there is no evidence of any theory of combinations among the ancients.*

The Latin writers, having little interest in any phase of mathematics except the practical, paid almost no attention to the theory of combinations. The leading exception was ***Anicus Boethius*** *(475–534, Italy) who gave* $\frac{1}{2}n(n-1)$ *as the number of combinations of* n *things taken two at a time.*

The Hindus seem to have given the matter no attention until ***Bhaskara*** *(c. 1150) gave the rules for the permutations of* n *objects taken* k *at a time, with and without repetition, and the number of combinations of* n *objects taken* k *at a time without repetition.*

At about the same time similar results were obtained independently in China and South-Western Europe: in Spain, the great Jewish savant ***Abraham Ibn-Ezra*** *seemed to have been aware (c. 1140) of the rule for finding the combination of* n *objects taken* k *at a time [he knew that* $\binom{7}{2} = \binom{7}{5}$; $\binom{7}{3} = \binom{7}{4}$; $\binom{7}{6} = \binom{7}{1}$*].*

Levi ben Gershon *(1321) in his Maasei Choscheb (Work of the Computer), carried the subject considerably farther. He gave the formula* $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ *for the number of combinations of* n *objects, taken* k *at a time, together with the fact that there are* $n!$ *permutations of* n *elements*[76].

[76] In this treatment of permutations and combinations Levi ben Gershon comes very close to using *mathematical induction*, if not actually inventing it [Rabinovitch, N.L., *Arch. Hist. Ex. Sci.* **6** (1969) 237–248].

Early in the Christian Era there developed a close relation between mathematics and the mystic philosophy of the Hebrews known as *Kabbalah*. This led to the belief in the mysticism of arrangements and hence to the study of permutations and combinations. The movement seems to have begun in the anonymous *Sefer Yetzira* (Book of Creation), composed probably between the 3^{rd} and 6^{th} centuries CE in Israel. It seemed to have attracted the attention of the Arabic and Hebrew writers of the Middle Ages in connection with astronomy. They considered it w.r.t. the conjunction of planets, seeking to find the number of ways in which Saturn could be combined with each of the other planets in particular, and, in

The subject of permutations had a feeble beginning in China in the 12^{th} *century, but most of the relevant literature was lost.*

Oresme *(ca 1360) wrote a work in which he gave the sum of numbers representing the combinations of* 6 *objects taken* $1, 2, 3, 4$ *and* 5 *at a time. He also gave* $\binom{6}{2} = 15$, $\binom{6}{3} = 20$, *etc., in rhetorical form.*

First evidence of permutations in print is found in ***Pacioli****'s Suma (1494), where he showed how to find the number of permutations of any number of persons sitting at a table.* ***Tartaglia*** *(1523) seems first to have applied the theory of the throwing of a dice. In a book Pardes Rimmonim (Orchard of Pomegranates, 1548) the Jewish Kabbalist* ***Moshe Cordovero*** *(1522–1570, Israel) gave an interesting treatment of permutations and combinations and showed some knowledge of the general laws governing them.*

At about the same time ***Joannes Buteo*** *(1492–1572, France) discussed (1559) the question of the number of possible throws with* 4 *dice. The first writer to publish the general rule that* $\binom{n}{k} = \frac{n(n-1)(n-2)\cdots(n-k+1)}{k!}$ *was* ***Pierre Herigone*** *(1634).*

Pascal *(1654) united the algebraic and combinatorial theorems by showing that* $(a + b)^n$ *is a generating function for the number of combinations of* n *objects, taken* k *at a time*[77]*. As an application, he founded the mathematical theory of probability, and as a method of proof he used mathematical induction for the first time in a conscious and unequivocal way.*

An interest in logic led ***Leibniz*** *to write the essay Dissertatio de arte combinatoria (1666). His aim was "a general method in which all truths of reason would be reduced to a kind of calculation". Leibniz foresaw that permutations and combinations would be involved, but he did not make enough progress to interest* 17^{th} *century mathematicians in the project.*

The true pioneer of combinatorial analysis was ***Abraham de Moivre****, who first published in Phil. Trans. (1697) the form of the general coefficient in the expansion of* $(a + bx + cx^2 + dx^3 + \cdots)$ *raised to any power. His work on probability would naturally lead him to consider questions of this nature.*

In 1730 *he introduced the powerful method of generating functions (for the Fibonacci numbers). This method has been of great importance in combinatorics, probability and number theory.*

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general, the number of combinations of the known planets taken two at a time, three at a time, and so on.

[77] The credit for discovering the *Pascal triangle* goes to the Chinese mathematicians **Yang Hui** (1261) and **Zhu Shijie** (1303). This is not the only instance of a mathematical concept being named after a rediscoverer rather than a discoverer, but Pascal deserves here credit for more than just rediscovery.

Mathematicians of the 18^{th} century applied the algebra of permutations and combinations to solve a host of arithmetical and geometrical problems, some of which had immediate application to probability theory. Although it is generally agreed that the doctrine of probability has been founded by Pascal and Fermat, the need for the theory arose already with regard to throwing of dice and other gambling questions. In the mathematical work Suma (1494) by ***Pacioli****, two gamblers are playing for a stake which is to go to the one who first wins n points, but the play is interrupted when the first has p points and the second q points. It is required to know how to divide the stakes. The general problem also appears in the works of* ***Cardan*** *(1539) and of* ***Tartaglia*** *(1556).*

The first printed work on the subject was a tract of ***Huygens*** *(1657). There also appeared (1708) an essay on the subject by* ***Pierre-Rémond de Montmort*** *(1678–1719). However, the first book devoted entirely to the theory of probability was Ars Conjectandi (1713) by* ***Jakob Bernoulli****. The second book on the subject was* ***De Moivre****'s Doctrine of Chances (1718). One of the best known works on the theory of probability is* **Laplace***'s Théorie analytique des probabilites (1812). In this is given his proof of the method of least squares.*

The application of the theory to mortality tables started with ***John Graunt****'s book Natural and Political Observations (London, 1662). The first tables of great importance, however, were those of* ***Edmund Halley*** *(1663) in his memoir on Degrees of Mortality of Mankind, in which he made a careful study of annuities. Although a life-insurance policy is known to have been underwritten in London in* 1583*, it was not until* 1699 *that a well-organized company was established for this purpose.*

A few typical examples of problems of historical and aesthetical value are given below:

- The Bernoulli-Euler problem of misaddressed letters

Someone writes n letters and writes the corresponding addresses on n envelopes. How many different ways are there of placing all the letters in the wrong envelopes?

This problem was first considered by ***Nicholas Bernoulli*** *(1687–1759), the nephew of Jakob and Johann Bernoulli. Later* ***Euler*** *became interested in the problem, which he solved independently of Bernoulli. This problem is particularly interesting because of its ingenious solution:*

Let u_n be the sought number of ways. Pick a certain letter; by definition it is in a wrong envelope. Then there are two possibilities:

(I) The letter that matches the wrong envelope was placed in an envelope that matches the originally selected letter. This cross-derangement can occur in $(n-1)$ *ways. The remaining* $(n-2)$ *letters can be misaddressed in* u_{n-2} *ways. The total number of derangements of this type is therefore* $(n-1)u_{n-2}$.

(II) While the originally selected letter is placed in a wrong envelope, the matching envelope for that letter does not *host the letter matching the wrong envelope.* Pretending *that the latter letter-envelope pair is matched, the number of configurations of type II is found to be* u_{n-1} *per choice of wrong envelope, i.e.* $(n-1)u_{n-1}$. *Altogether, adding the counts for cases I and II:*

$$u_n = (n-1)(u_{n-1} + u_{n-2}).$$

This difference equation is solved by

$$u_n = n!\left[\frac{1}{2!} - \frac{1}{3!} + \cdots + \frac{(-1)^n}{n!}\right].$$

Montfort[78] *(1713) solved this problem (*le problème de recountres*) by effectively using what is known today as* The principles of inclusion and exclusion, *which may have been known to the Bernoullis. It states:*

If of N objects, $N(a)$ *have a property a,* $N(b)$ *property b, ...,* $N(ab)$ *both a and b, ...,* $N(abc)$ *a, b, and c, and so on, the number* $N(a'b'c')$ *with* none *of these properties is given by*

$$\begin{aligned} N(a'b'c'\ldots) = N - N(a) - N(b) - \cdots \\ +N(ab) + N(ac) + \cdots \\ -N(abc) - \cdots \\ + \cdots \end{aligned}$$

The proof by mathematical induction is simple once it is noted that the formula $N(a') = N - N(a)$ *can be applied to any collection of properties which is suitably defined.*

The principle of inclusion and exclusion is an important combinatorial tool.

[78] **Pierre de Montfort** (1678–1715, France). Mathematician. Made a systematic study of games of chance and contributed to combinatorics.

• Euler's problem of polygon division

In how many ways can a plane convex polygon of n sides be divided into triangles by non-intersecting diagonals inside the polygon?

Euler *posed this problem (1751) to* ***Christian Goldbach****. He then communicated it to* ***Johann Andreas von Segner*** *(1704–1777, Germany), disclosing the first seven division numbers E_n*

n	3	4	5	6	7	8	9
E_n	1	2	5	14	42	132	429

Segner was able to derive a recursion relation for E_n

$$E_n = E_2 E_{n-1} + E_3 E_{n-2} + \cdots + E_{n-1} E_2$$

where $E_2 \equiv 1$. His solution matched Euler's own result

$$E_n = \frac{2^{n-2}(2n-5)!!}{(n-1)!}.$$

• Steiner's problem *(1826)*

n lines are drawn in the Euclidean plane in such a way that no 3 are concurrent and no 2 are parallel. What is the maximal number of regions formed?

Let P_n denote this number. An additional line will cut all previous lines, creating $(n+1)$ new regions. Therefore $P_{n+1} = P_n + (n+1)$. This equation, augmented by the initial condition $P_1 = 2$, is uniquely solved by $P_n = 1 + \frac{1}{2}n(n+1)$.

- JOSEPHUS[79] PROBLEM *(Tartaglia, 1546)*

Arrange the numbers $1, 2, \ldots, n$ *consecutively (say, clockwise) about the circumference of a circle and proceed clockwise to remove every* q^{th} *number. Let* $J_q(n)$ *denote the final number which remains for a given pair* (q, n)*, i.e. the last survivor.*

For $q = 2$ *(removing number 2 and then every other number) the survivor's-table has the form*

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$J_2(n)$	1	1	3	1	3	5	7	1	3	5	7	9	11	13	15	1

Suppose that we have $2n$ *people originally. After the first go-around, all even numbers will be eliminated and* 3 *will be the next to go. This is just like starting out with* n *people, except that each person's number has been doubled and decreased by* 1*. That is* $J_2(2n) = 2J_2(n) - 1$, *for* $n \geq 1$*. If we start with* $(2n + 1)$ *people, it turns out that person number* 1 *is wiped out just after person number* $2n$*. Again, we almost have the original situation as with* n *people, but this time their numbers are doubled and increased by* 1*. Thus* $J_2(2n + 1) = 2J_2(n) + 1$ *for* $n \geq 1$*.*

Altogether we have

$$\begin{aligned} J_2(1) &= 1 \\ J_2(2n) &= 2J_2(n) - 1, \qquad \text{for } n \geq 1, \\ J_2(2n+1) &= 2J_2(n) + 1, \qquad \text{for } n \geq 1. \end{aligned}$$

[79] The Latin writer **Hegesippus** (340–397 CE) tells us that the Jewish historian **Josephus** saved his life by knowing the solution to this problem for $q = 3$, $n = 41$. According to his account, after the Romans had captured Yodfat, Josephus and 40 other Judean freedom fighters took refuge in a cave. His companions were resolved to die rather than fall into the hands of the Romans. Josephus and one friend, not wishing to die yet not daring to dissent openly, feigned to agree. Josephus even proposed an arrangement by which the deaths might take place in an orderly manner: The men were to arrange themselves in a circle; then every third man was to be killed until but one was left, and he must commit suicide. Josephus and his friend placed themselves in places 16 and 31. This kind of 'lottery', which Josephus adopted, was similar to that used by the priests of the Second Temple (515 BCE–70 CE) in Jerusalem to win their various daily service-jobs [*Yoma* **2**, 2 (Mishna); *Yoma* **2**, 1 (Yerushalmi); *Yoma* **22**, 2 (Bavli)]. The priests stood in a circle, each one pointing one or two fingers toward the man in charge at the center. This man would then announce a number (usually 100 or 60) which was larger than the total number of participating priests, and then *count fingers* in a specified direction from a certain fiducial person, ending the count of the preassigned number at the winner.

It follows from these recursion relations that $J_2(2) = 2J_2(1) - 1 = 1,\ J_2(4) = 2J_2(2) - 1 = 1,$ *etc. and in general, for all* $m,\ J_2(2^m) = 1$. *Hence we know that the first person will survive whenever* n *is a power of* 2. *In the general case* $n = 2^m + \ell,$ *where* 2^m *is the largest power of* 2 *not exceeding* $n,$ *the number of people is reduced to a power of* 2 *after there have been* ℓ *"executions". The eventual survivor is number* $2\ell + 1$ *in the original ordering, i.e.*

$$J_2(2^m + \ell) = 2\ell + 1, \quad \text{for } m \geq 0 \quad \text{and} \quad 0 \leq \ell < 2^m.$$

This can be proved by induction in two steps, depending on whether ℓ *is even or odd: If* $m > 0$ *and* $2^m + \ell = 2n,$ *then* ℓ *is even and*

$$J_2(2^m + \ell) = 2J_2(2^{m-1} + \ell/2) - 1 = 2(2\ell/2 + 1) - 1 = 2\ell + 1,$$

by the induction hypothesis. A similar proof works in the odd case, when $2^m + \ell = 2n + 1$. *We might also note that*

$$J_2(2n + 1) - J_2(2n) = 2.$$

Either way, the induction is complete and the closed-form solution is established.

To illustrate the solution we compute $J_2(100)$. *In this case we have* $100 = 2^6 + 36,$ *so* $J_2(100) = 2 \cdot 36 + 1 = 73$.

There is no closed-form solution to the Josephus problem for $q > 2,$ *not even a recurrence relation. There is however a computer recipe*

$$J_q(n) = qn + 1 - D_k^{(q)},$$

where $D_0^{(q)} = 1, \quad D_n^{(q)} = \frac{q}{q-1} D_{n-1}^{(q)}$ *for* $n > 0$, *and* k *is as small as possible such that* $D_k^{(q)} > (q-1)n$.

- Rook problems

In how many ways can n *rooks be placed on an* $n \times n$ *chessboard so that no rook can attack another?*

If the rooks are unnumbered the answer is $n!$ *since there is exactly* 1 *rook in each row and each column and thus each configuration of the* n *rooks is a different permutation of* n *objects (numbers). If the rooks are numbered from*

1 to n, the answer is $(n!)^2$, since there are $n!$ ways of placing the numbered rooks (the result of permuting the latter).

Now, if the rooks are restricted to avoid the main diagonal, every position of the non-attacking rooks is a derangement of n objects, and we fall back on the Bernoulli-Euler problem of misaddressed letters, with the result (for un-numbered rooks)

$$n!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\cdots+(-1)^n\frac{1}{n!}\right].$$

Another interesting problem is to find the number of ways of arranging n un-numbered rooks on an $n \times n$ chessboard such that every square of the board is controlled by at least one of them.

The number of ways of arranging n rooks, one in each column, is n^n (the first rook can be placed on any of the n squares of the first column; no matter which square it is put on, the second rook can be put on any of the n squares of the second column, etc.).

The same argument applies to the rows, and it would seem at first glance that the number of arrangements of n rooks for which the rooks controlled all squares of the board would equal to $2n^n$. But in this enumeration we have counted twice each arrangement of the rooks for which there is one rook in each column and simultaneously one rook in each row. Since the total number of such arrangement is $n!$, the correct answer is $2n^n - n!$.

In particular, for an ordinary chessboard $(n = 8)$, we obtain $2 \cdot 8^8 - 8! = 33,514,312$ different arrangements.

- Problems of occupancy

Starting with the basic permutations and combinations of N objects, there is a large class of more complicated cases, some of which require very involved and tricky reasoning that leads to fancy mathematical formulations. We shall look at a few just to appreciate how powerful such methods can be.

Let there be a set of N objects (e.g. balls) which should be placed in a set of n compartments (e.g. urns, boxes, etc.). The number of ways in which the distribution can be effected will depend upon two factors:

(1) Whether the order of the urns, even including empty ones, is taken into account, i.e. whether the urns are distinguishable (alias different, alias labeled, distinct) or indistinguishable (alias identical).

(2) Whether the order of the balls within the urns is taken into account, i.e. whether the balls are distinguishable or not.

The simplest case is when both balls and urns are different. There are n choices for each ball as to which urn it will be placed in. The total number of independent choices is therefore n^N (empty urns are allowed).

If in the previous problem the balls are identical and they are placed in n distinct urns or fewer (i.e. empty urns are allowed) the counting proceeds as follows:

Due to the indistinguishability of the balls, any single distribution can be symbolically represented by short vertical bars (walls of urns) and circles (balls), e.g. $|00|000|0|\cdots$. We must begin and end with walls, and we must have in each distribution N balls and $n-1$ internal walls. So we merely have to count the number of ways to line up N balls and $n-1$ internal walls. There are $N+(n-1)$ positions in a line-up and N of them must be balls. Therefore the answer is $\binom{N+n-1}{N} \equiv \binom{N+n-1}{n-1}$. When no cell is empty, the result turns out to be $\binom{N-1}{n-1}$.

Note that we are actually asking here how many solutions are there, in non-negative (or alternatively positive) integers, to the equation $x_1 + x_2 + \cdots + x_n = N$, where $(x_1, \ldots, x_n)$ is an ordered n-tuple.

The next problem is to count the number of ways in which N different balls can be arranged in exactly n different urns (no empty urn).

The number of arrangements in which empty urns are admissible is n^N; the number of arrangements in which one assigned empty urn is admissible is $(n-1)^N$, and so on. Hence, by the principle of inclusion and exclusion, the sought number[80] is $T(N,n) = \sum_{k=0}^{n-1}(-1)^k\binom{n}{k}(n-k)^N$.

Using this we can find the number of ways $B(N,n)$ to put N different balls into n identical urns, with no empty urns. This amounts to $B(N,n) = \sum_{j=0}^{n} \frac{1}{j!}T(N,j)$.

Finally, in how many ways can N identical balls be put in n identical urns? There is no nice, closed-form solution to this problem.

- Rolling dice

Count the number of ways of obtaining the sum N with n dice?

The number of arrangements in which the partition $x_1+x_2+\cdots+x_n = N$ is effected, with no zero values for any x_i, is $\binom{N-1}{n-1}$. Then, since each x_i is

[80] This is shown to equal the coefficient of x^N in the expansion of $N!(e^x-1)^n$.

limited by the set $\{1,2,3,4,5,6\}$, *the principle of inclusion and exclusion yields the result*

$$\binom{N-1}{n-1} - \binom{n}{1}\binom{N-7}{n-1} + \binom{n}{2}\binom{N-13}{n-1}\cdots$$

- Sum of divisors

Let $p_1,\ldots,p_n$ *be distinct primes. What is the number of divisors of the number* $q=(p_1)^{\alpha_1}(p_2)^{\alpha_2},\ldots,(p_n)^{\alpha_n}$ *where* $\alpha_1,\ldots,\alpha_n$ *are natural numbers (including the divisors* 1 *and* q*) and what is the sum of these divisors?*

Each prime p_k *can enter a divisor of* q *with one of* (α_k+1) *exponents* $0,1,\ldots,\alpha_k$. *By the rule of product, the number of divisors is*

$$(\alpha_1+1)\cdots(\alpha_n+1).$$

To compute their sum, we consider the expression

$$(1+p_1+\cdots+p_1^{\alpha_1})\cdots(1+p_n+\cdots+p_n^{\alpha_n}).$$

In performing the product, we obtain a sum in which each divisor of q *appears exactly once. Using the formula for the sum of a geometric progression, the above product — and therefore also the required sum of the divisors of* q *— is seen to have the value*

$$\frac{p_1^{\alpha_1+1}-1}{p_1-1}\cdots\frac{p_n^{\alpha_n+1}-1}{p_n-1}$$

- The birthday problem

If you know more than 23 people's birthdays, it is more likely than not that two of them occur on the same day.

Consider the probability that n *people's birthdays are all different, i.e. that in a random selection of* n *days out of* 365 *there shall be no day counted more than once. The total number of possible selection is* $(365)^n$, *and the number of selection in which no day is counted more than once is* $365\cdot364\cdot\cdots\cdot(365-n+1)$. *The probability is therefore*

$$p(n)=\left(1-\frac{1}{365}\right)\left(1-\frac{2}{365}\right)\cdots\left(1-\frac{n-1}{365}\right).$$

The occurrence is as likely as not if $p(n)=\frac{1}{2}$. *By taking logarithms, we obtain approximately* $\frac{1}{365}+\frac{2}{365}+\cdots+\frac{n-1}{365}=\log_e 2$ *or* $n(n-1)\cong 506$, *whence*

$n = 23$. The probability that at least two persons in a room containing n persons will have the same birthday is $q(n) = 1 - p(n)$. For $n = 64$, $q(n) = 0.997$.

- *n people are seated around a table, $n > 2$. What is the probability that two persons will be neighbors?*

There are $\frac{n!}{n} = (n-1)!$ arrangements in toto. Of these, $2(n-2)!$ are favorable. Therefore, the sought probability is $p(n) = \frac{2}{n-1}$.

- *Given a convex planar irregular (no three diagonals intersect) n-gon. Enumerate:*

(a) the total number of diagonals;

(b) the number of interior intersection points generated by the diagonals;

(c) the number of interior regions generated by the intersecting diagonals.

A diagonal corresponds to a 2-subset of vertices, of which there are $\binom{n}{2}$. However, not every 2-subset gives a diagonal: the n-pairs of adjacent vertices give sides. *Thus, the first requested number is $\binom{n}{2} - n = \frac{n(n-3)}{2}$.*

Likewise, there is a one-to-one correspondence between interior intersection points and combinations of vertices taken 4 at a time. Hence the second sought number is $\binom{n}{4}$.

Finally, using mathematical induction, the number of regions is found to be $\binom{n}{4} + \binom{n-1}{2} = \frac{1}{24}(n-1)(n-2)(n^2 - 3n + 12)$.

For a given n-gon, all these results are independent of the specific shape of the polygon.

- KIRKMAN'S[81] SCHOOL-GIRLS PROBLEM *(1850)*

In a boarding school there are 15 schoolgirls who always take their daily walks in 5 rows of threes. How can it be arranged so that each schoolgirl walks in the same row with every other schoolgirl exactly once a week?

[81] **Thomas Kirkman** (1806–1895, England). Mathematician. Contributed to combinatorial mathematics. Showed the existence of *Steiner systems* seven years before Steiner's article on the subject.

Let the girls be labeled $X_1, \ldots, X_7, Y_1, \ldots, Y_7, Z$. *The solution is*

Day 1	X_1Y_1Z	$X_2X_6Y_4$	$X_3X_4Y_7$	$X_5X_7Y_6$	$Y_2Y_3Y_5$
Day 2	X_2Y_2Z	$X_3X_7Y_5$	$X_4X_5Y_1$	$X_6X_1Y_7$	$Y_3Y_4Y_6$
Day 3					
Day 4					
Day 5					
Day 6					
Day 7	X_7Y_7Z	$X_1X_5Y_3$	$X_2X_3Y_6$	$X_4X_6Y_5$	$Y_1Y_2Y_4$

The solution has the nice property that the triplets for each day can be obtained from those of the previous day by replacing X_i *by* X_{i+1}, Y_i *by* Y_{i+1} $(i \leq 6)$, X_7 *by* X_1, Y_7 *by* Y_1.

• CHANGING A DOLLAR

In how many ways can a dollar be changed into pennies, nickels, dimes, quarters and half-dollars? A painstaking naive counting yields at length the number 292. Sylvester (1855) developed a general theory *which enables one to derive results like these through a systematic fast algorithm. But even with this tool, the following consideration is useful: In general, if the change adds up to N cents and consists of coins of denomination* $n_1, n_2, \ldots, n_k$ *(no restriction on the number of coins of different denominations), one easily derives the recursion relation*

$$D(N; n_1, n_2, \ldots, n_k) = D(N; n_1, n_2, \ldots, n_{k-1}) + D(N - n_k; n_1, n_2, \ldots, n_k)$$

where the left hand side is the sought number of ways to change N *cents with denominations* $\{n_1, n_2, \ldots, n_k\}$. *The above relation shows that if no* n_k*-cent coin is included in the change, then the full sum* N *is made up of coins of lesser denominations* $\{n_1, n_2, \ldots, n_{k-1}\}$, *and if* at least *one* n_k*-cent coin is used, then the remainder* $(N - n_k)$ *may include coins of denomination* $\{n_1, n_2, \ldots, n_k\}$. *Applying this to our problem we get*

$$D(100; 1, 5, 10, 25, 50) = D(100; 1, 5, 10, 25) + D(50; 1, 5, 10, 25, 50).$$

A repeated application of this relation, leads to the above result after a few steps.

It is clear from the above examples that combinatorial mathematics *is first of all concerned with counting the number of ways of arranging given objects* *in a prescribed way (i.e. satisfying certain conditions). Generally speaking, both the theoretical analysis and the actual construction of discrete sets are much more difficult than those problems in analysis concerning infinite sets.*

The main emphasis and the name of this field have changed from time to time and from person to person. Other names such as combinatorial analysis, combinatorial theory and recently discrete mathematics have also been used to describe the same field[82].

- Combinatorial analysis *(1846–1898)*

Up to the middle of the 19th *century, problems of combination were generally undertaken as they became necessary for the advancement of some particular part of mathematical science; it was not recognized that the theory of combinations is in reality a science in and of itself, well worth studying for its own sake irrespective of applications to other parts of analysis. There was a total absence of orderly development, and until* 1846*, Euler's classical paper remained the only method of combinatorial analysis*[83]*. [Other writers who have contributed to the solution of special problems are* ***James Bernoulli***, ***Ruggiero Boscovich***, ***Karl Friedrich Hindenburg*** *(1741–1808),* ***William Emerson*** *(1701–1782),* ***Robert Woodhouse*** *(1733–1827),* ***Thomas Simpson*** *and* ***Peter Barlow****.]*

In 1846 ***Carl G.J. Jacobi*** *studied the partitions of numbers by means of certain identities involving infinite series that are met in the theory of elliptic functions. Further advance was made by* ***Arthur Cayley*** *and* ***Joseph Sylvester*** *(1855) and during 1888–1898 by* ***Pery Alexander MacMahon*** *(1854–1929).*

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[82] In the 20th century, the subject has come a long way since Kirkman's time and the days are past when the calculus was thought to be the undisputed queen of applied mathematics.

[83] *De Partitione Numerorum* (1748), in which the consideration of the reciprocal of the product $(1-ax)(1-ax^2)(1-ax^3)\cdots$ establishes a fundamental connection between arithmetic and algebra through the identity

$$\frac{1}{(1-ax)(1-ax^2)\cdots(1-ax^n)} = 1+ax\frac{1-x^n}{1-x}+a^2x^2\frac{(1-x^n)(1-x^{n+1})}{(1-x)(1-x^2)}+\cdots .$$

Here **Euler** showed that he could convert arithmetical addition into algebraic multiplication and by that he gave the complete formal solution of the main problem of the *partition of numbers*.

On Chance

* * *

"*I returned and saw under the sun, that the race is not to the swift, nor the battle to the strong, neither yet bread to the wise, nor yet riches to men of understanding, nor yet favour to men of skill: but time and chance happeneth them all*".

Ecclesiastes **9** 11

* * *

"*Everything existing in the Universe is the fruit of chance and necessity*".

Democritos of Abdera (ca 460–370 BCE)

* * *

"*The probable is what usually happens*".

Aristotle (384–322 BCE)

* * *

"*Probability is the very guide of life*".

Marcus Tullius Cicero (ca 50 BCE)

* * *

"*The only certainty is that there is nothing certain*".

Pliny the Elder (23–79 *CE*)

* * *

"*It is a truth very certain that when it is not in our power to determine what is true we ought to follow what is most probable*".

René du Perron Descartes (1596–1650)

* * *

"*It is remarkable that a science which began with the consideration of games of chance should have become the most important object of human knowledge*".

Pierre Simon de Laplace (1749–1827)

* * *

"*The most important questions of life are, for most part, really only problems of probability*".

Pierre Simon de Laplace

* * *

"*Fate, time, occasion, chance, and change — to these all things are subject*".

Percy Bysshe Shelley (1792–1822)

* * *

"*In the field of observation, chance favours the prepared mind*".

Louis Pasteur (1822–1895)

* * *

"*No victor believes in chance*" (Kein sieger glaubt an den zufall).

Friedrich Wilhelm Nietzsche (1844–1900)

* * *

"*Chance is the pseudonym of God when he did not want to sign*".

Anatole France (1844–1924)

* * *

"*I can believe anything, provided it is incredible*".

Oscar Wilde (1854–1900)

* * *

"*The record of a month's roulette playing at Monte Carlo can afford us material for discussing the foundations of knowledge*".

Karl Pearson (1857–1936)

* * *

"*The conception of chance enters into the very first steps of scientific activity in virtue of the fact that no observation is absolutely correct. I think chance is more fundamental concept than causality; for whether in a concrete case, a cause-effect relation holds or not can only be judged by applying the laws of chance to the observation*".

Max Born (1882–1970)

* * *

"*The luck of having talent is not enough; one must also have a talent for luck*".

Hector Berlioz

* * *

"*The harder you work, the luckier you get*".

Anon

* * *

"*What is good luck for the early bird is bad luck for the early worm*".

Anon

* * *

"*Depend on the rabbit foot if you will, but remember — it didn't work for the rabbit*!"

Anon

* * *

1641–1672 CE **Franciscus (Franz) Sylvius de la Boë** (1614–1672, The Netherlands). German born physician, anatomist and chemist. Taught that the functions of the living organism were mainly determined by chemical activities ("effervescences"), particularly by acidic or alkaline characters of body fluids (a precursor of the modern cult of pH). He represents the culmination of chemical medicine (*Iatrochemistry*).

Professor of medicine at Leyden (1658–1672), where he built the first university chemical laboratory. Discovered (1614) *Sylvian fissure* in brain.

1642–1644 CE **Abel Janszoon Tasman** (1603–1659, The Netherlands). The greatest of Dutch navigators and explorers. Discovered *Tasmania* and *New Zealand* (1642), and *Tonga* and the *Fiji* Islands (1643); On his second voyage (1644) he discovered the *Gulf of Carpentaria*.

Tasman entered the service of Dutch East India Co. (1633); sent by Anthony van Diemen, governor general of Dutch East Indies, on expeditions to Indian and Australian waters (Aug. 1642) in quest of "islands of gold and silver".

Tasman was born at Lutjegast at Groningen. Although Tasman contributed to the extension of the Dutch colonial empire, his achievements were coldly received by the Dutch colonial authorities.

The first people to live in New Zealand were the *Maoris*. They arrived around 750 *CE* from the Cook, Marquesas, or Society Islands (NE of New Zealand) by canoes.

Tasman tried (1642) to send a group of men ashore, but the Maori attacked their two small landing crafts and killed several of the men. Tasman made no further attempt to land. No other European came to New Zealand until 1769, when Captain James Cook landed on the North Island, made friends with the Maoris, and explored and charted both the North Island and the South Island. In 1840 the Maoris signed the *Treaty of Waitangi*, which gave Great Britain the sovereignty over New Zealand.

1642–1655 CE **Cyrano de Bergerac** (**Savinien de**) (1619–1655, France). Playwright, soldier and writer of science-fiction and philosophical fiction. He was acquainted with all the philosophical trends of his period (Scholasticism, which he attacked; Skepticism, Epicureanism as revived by Gassendi, Cartesianism, and the Italian philosophers of the Renaissance), and was aware of all the recent discoveries in astronomy and physics since Copernicus, Kepler and Galileo, and in medicine since Harvey. His novels show him as a keen and talented popularizer, and contain amazing forecasts of many later developments in science and technology such as: *the unity of matter, its atomic structure, phagocytes, animal intelligence, aviation, the gramophone, and X-rays*. Known for his sword-fighting and for his long nose [Edmond's Rostand's famous play *Cyrano de Bergerac* (1897) contains a somewhat fanciful account of Cyrano's colorful life].

Cyrano was born in Paris. He received his first education from a country priest. At the age of 19 he entered the corps of the guards, serving in the campaigns of 1639 and 1640, and began the series of exploits that were to make him a veritable hero of romance. After 2 years of this life Cyrano left the service and returned to Paris to pursue literature and science studies, becoming a pupil of **Gassendi**.

Among his writings are two fantastic voyages: *L'autre monde ou les états et empires de la lune* and *Des états et empires du soleil* (1654) (published posthumously 1657 and 1662, after being purged of many religious and philosophical audacities; tr. *Voyages to the Moon and the Sun*, 1923).

Only after 20^{th} century scholarship made the complete text of his novels available, did his talent and originality receive full recognition.

Cyrano's ingenious mixture of science and romance has furnished a model for many subsequent writers, among them **Jonathan Swift** (1667–1745, England) and **Edgar Allan Poe** (1809–1849, U.S.A.). He adopted his fanciful style both for safely conveying ideas that might be regarded as unorthodox, and to relax from the serious study of physics.

Cyrano spent a stormy existence in Paris and was involved in many duels. He entered the household of duke d'Arpajon as secretary in 1653, and died two years later as a result of injuries following an accident.

1642–1656 CE **Thomas Hobbes** (1588–1679, England). Philosopher. Best known for his political philosophy, based on the idea of social contract, for purpose of security of each individual, and absolute authority of a sovereign[84] (*Leviathan*, 1651).

In his travels on the Continent he met **Galileo**, **Gassendi** and **Mersenne**. In England he was friendly with **Bacon** and **Harvey**.

Hobbes was influenced by two developments of his time: the new system of *physics* that Galileo and others were working on and the English Civil War. Men, he concluded, are selfish. They are moved chiefly by desire for power

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[84] According to Hobbes, man creates social laws autonomically, with no dependence on God. This is clearly an antithesis to the teaching of Luther, who claimed that the death of Christ relieves us from all moral obligations to each other.

and by fear of others. Therefore, without an all powerful sovereign to rule them, men's lives would be 'poor, nasty, brutish and short'.

Though modern physics is not so materialistic as it seemed to be in the days of Hobbes and though men's motives are more complex than he supposed, Hobbes influence continues. He raised fundamental and challenging questions about the relationship between science and religion, between thought and physiological processes on which it is based, and the nature and limitations of political power. These are questions that men still struggle to answer. He is probably more important for the questions he asked than for the answers he gave.

Hobbes was born in Westport, England. He was educated at Oxford University, and served as secretary to **Francis Bacon**. During the Civil War in England he fled to the European continent, returning to England while Cromwell's protectorate was still in power.

1642–1680 CE **Johannes Hevelius** (Hevel, Hovels or Höwelcke, 1611–1680, Danzig). German astronomer. Founder of lunar topography. Discoverer of comets and the moon's libration in longitude.

He was born in Danzig (now Gdansk, Poland). Studied law at Leyden in 1630; traveled in England and France, and in 1634 settled in his native town as a brewer and town councilor. From 1639 his chief interest became centered on astronomy, though he took, throughout his life, a leading part in municipal affairs. In 1641 he built an observatory in his house, provided with a telescope (46 m focal length) which he constructed by himself.

Hevelius made observations of *sunspots* (1642–1645), devoted four years to charting the lunar surface, and published his results in *Selenographia* (1647). It is the first map of the side of the moon observable from earth. He discovered 4 comets (1652, 1661, 1661, 1672), and suggested the motion of such bodies in parabolic orbits round the sun.

On 26 September 1679, his observatory, instruments and books were destroyed by arson. He promptly repaired the damage, so far as to enable him to observe the great comet of December 1680, but his health suffered from the shock.

His catalogue of 1564 stars appeared posthumously (1690).

1643 CE Typhoid fever first identified or described with accuracy.

1643 CE **Evangelista Torricelli** (1608–1647, Italy). Geometer and physicist. A pupil of Galileo. Engaged in pre-calculus calculations of areas, arc-lengths and extremum. Applied Galileo's laws of motion to fluids, and invented the first barometer to measure air pressure, using mercury as fluid in a 185 cm glass column sealed at the top. When the tube is upended in a dish, the mercury sinks to about 76 cm, leaving a partial vacuum at the top. Torricelli was motivated by his desire to understand why lift pumps were unable to raise water more than 10.37 m. He then correctly concluded that the 760 mm of mercury balanced the air pressure in the dish.

Mining engineers were long aware of the fact that a suction-pump could not draw water from depths greater than some 10 meters and there was no explanation of why such limit should exist. When the engineers of Cosimo de'Medici II failed in an attempt to build a suction-pump capable of lifting water from a depth of 17 meters, the problem was referred to Galileo, and finally solved by his brilliant pupil Torricelli (1644), who then announced that the pressure of the atmosphere was equivalent to a column of water over 10 meter in height. He predicted that the pressure of the atmosphere would fall with increasing altitude, a truth which he confirmed experimentally in 1647, when a barometer was carried to the top of a 1450 m high mountain in the Auvergne; the height in the mercury in it fell by 7.5 cm during the ascent.

Torricelli created the first man-made vacuum known to science, thus refuting the 2000 year old Aristotelian view that vacuum was impossible.

Aristotle was Exactly Wrong

The ancient Greeks believed that the air through which birds fly extended to the moon (Daedalus failed because human arms were insufficiently strong, but that was their only objection). As ***Aristotle*** *said: "Nature abhors a vacuum". Even* ***Kepler****, in his Somnium (1634) never considered air as a problem.*

In the mid 17th century, the Catholic Church had its own "theory" of why there was no vacuum: "vacuum is nothing; since God is everywhere and in everything he could not be nowhere and in nothing". So the pope decreed that the vacuum did not exist and to talk about vacuum was considered heresy. And that was exactly the reason why the air pressure became a Protestant program.

Torricelli *not only created the first artificial vacuum and demonstrated that air had weight — he essentially* discovered outer space! *for if air did have* *weight, and if one assumes its density was uniform throughout the atmosphere, its weight implied that the atmosphere was only* 8 *km high. Even if one assumed it thinned as it rose, the atmosphere certainly could not be more than some* 150 *km high. From that instant onward, man realized that he inhabited not a unique land in the universe filled with possible places of habitation, but a speck-sized island of life in a vast cosmos of life-inimical emptiness.*

Science has suddenly isolated man, and showed how precarious was his grip on nature. Nature, in fact, prefers a vacuum, and Aristotle was shown to be exactly wrong. How puny man suddenly became, and how horrific his universe. This was one of the first dreadful fears created by science, fears confirmed again and again when the Scottish astronomer ***Thomas Henderson*** *(1798–1844) was able to show (1831) that the Sun's nearest stellar neighbor was an ungodly* 40 *trillion km away; when* ***Charles Darwin*** *showed (1859) that man was just another animal; and when* ***Einstein*** *(1905) turned common sense regarding the most fundamental categories upside-down; and when (1945) science coupled with technology showed that they could destroy men's sense of the world together with the world itself.*

1644 CE **Marin Mersenne** (1588–1648, France). Mathematician and natural philosopher. A Franciscan friar who lived in one of the critical periods of scientific history, overlapping the lives of **Galileo** (1564–1642), **Fermat** (1601–1665) and **Descartes** (1596–1650). Contributing little himself, Mersenne's unique historical importance was his gift for stirring up profitable controversies among his friends. His main accomplishment was his correspondence with many of the intellectuals of his time and the meetings held in his quarters in the Minim convent in Paris. At one such gathering (1647), Pascal first met Descartes. Some 18 years after his death, this group of acquaintances had become the French Academy of Sciences. Thus we can see Mersenne as a catalyst, speeding up the exchange of ideas between others.

Mersenne was born at Oise, France. In 1604 he entered the Jesuit school of La Flèche, where he met Descartes. In 1609 he went to the Sorbonne in Paris. In 1611 he joined the order the Minims and moved (1619) to a cloister at the Place Royale, where he remained most of his life. In 1647 he traveled to meet with Fermat. The hot journey over bad roads wore him out and he died soon afterwards. All his mathematical works, except his bad guess about perfect numbers, quickly became obsolete.

Since the days of Greek science, philosophers were concerned with the mystical significance of natural numbers. The 6 days of creation and the 28 days of the lunar month drew attention to the so-called perfect numbers. Euclid had already proved in his '*Elements*' that if $2^k - 1$ is prime, then $2^{k-1}(2^k - 1)$ is a perfect number. Thus, interest aroused early in primes of the form $2^k - 1$. Clearly, if this number is prime, k itself must be a prime (the converse is however not true). In 1644, Mersenne made the incorrect conjecture that for $p \leq 257$, $2^p - 1$ is prime when and only when $p = 2$, 3, 5, 7, 13, 17, 19, 31, 67, 127 and 257. He made 2 sins of commission (67, 257) and 3 sins of omission (61, 89, 107).

Mersenne, believing in inquiry by experiments, measured in 1636 the speed of sound in air. He also observed during 1623–1647 that the intensity of sound is inversely proportional to the square of the source-observer distance. He made a distinction between sound frequency and sound intensity and found that sound velocity is independent of pitch and loudness. He discovered, before Galileo, that the frequency of swing of a pendulum is inversely proportional to the square root of its length. Although relationships between the frequencies of vibrating strings had been known since the time of Pythagoras (ca 540 BCE), the *absolute* frequency of a musical note was first measured by Mersenne, who published his results in "*Harmonie Universelle*" in 1636.

The Mersenne Primes[85] (1644–2003)

It took **Frank Nelson Cole** *(1861–1926) about 1000 hours to calculate, by 1903, that*

$$2^{67} - 1 = 193,707,721 \times 761,838,257,287.$$

(Nowadays, a fairly standard computer will render the result in 0.1 sec.)

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85 To dig deeper, see:

- Hardy, G.H. and E.M. Wright, *An Introduction to the Theory of Numbers*, Oxford University Press: Oxford, 1989, 426 pp.
- Ore, O., *Number Theory and its History*, McGraw-Hill Book Co., 1948, 370 pp.

In 1911, ***R.E. Powers*** *showed that Mersenne had also missed the primes* $2^{89}-1$ *and* $2^{107}-1$ *and in 1947 it was found that* $2^{257}-1$ *(having 78 digits) is composite, having three prime divisors*

$$231,584,178,474,632,390,847,141,970,017,375,815,706,539,969,\\ 331,281,128,078,915,168,015,826,259,279,871\\ =[535,006,138,814,359]\\ \times[1,155,685,395,246,619,182,673,033]\\ \times[374,550,598,501,810,936,581,776,630,096,313,181,393]$$

Table 3.3: Known Mersenne Primes

	p	Digits	Year	Discoverer	$M_p = 2^p - 1$
1	2	1	—	—	3
2	3	1	—	—	7
3	5	2	—	—	31
4	7	3	—	—	127
5	13	4	1456	anonymous	8,192
6	17	6	1588	Cataldi	131,071
7	19	6	1588	Cataldi	524,287
8	31	10	1772	Euler	2,147,483,647
9	61	19	1883	Pervushin	2,305,843,009,213,693,951
10	89	27	1911	Powers	
11	107	33	1914	Powers	
12	127	39	1876	Lucas	
13	521	157	1952	computers	
14	607	183	1952	computers	
15	1279	386	1952	computers	
16	2203	664	1952	computers	
17	2281	687	1952	computers	
18	3217	969	1957	computers	
19	4253	1281	1961	computers	
20	4423	1332	1961	computers	
21	9689	2917	1963	computers	
22	9941	2993	1963	computers	
23	11213	3376	1963	computers	
24	19937	6002	1971	computers	
25	21701	6533	1978	computers	
26	23209	6987	1979	computers	

Table 3.3: (Cont.)

	p	DIGITS	YEAR	DISCOVERER	$M_p = 2^p - 1$
27	*44497*	*13395*	*1979*	*computers*	
28	*86243*	*25962*	*1982*	*computers*	
29	*110503*	*33265*	*1988*	*computers*	
30	*132049*	*39751*	*1983*	*computers*	
31	*216091*	*65050*	*1985*	*computers*	
32	*756839*	*227832*	*1992*	*computers*	
33	*859433*	*258716*	*1994*	*computers*	
34	*1257787*	*378632*	*1996*	*computers*	
35	*1398269*	*420921*	*1996*	*computers*	
36	*2976221*	*895932*	*1997*	*computers*	
37	*3021377*	*909526*	*1998*	*computers*	
38	*6972593*	*2098960*	*1999*	*computers*	
39	*13466917*	*4053946*	*2001*	*computers*	

Table 3.4: PRIME FACTORS OF $2^n - 1$, $n \leq 128$

n			
2	**3**	30	$3 \cdot 3 \cdot 7 \cdot 11 \cdot 31 \cdot 151 \cdot 331$
3	**7**	**31**	**2147483647**
4	$3 \cdot 5$	32	$3 \cdot 5 \cdot 17 \cdot 257 \cdot 65537$
5	**31**	33	$7 \cdot 23 \cdot 89 \cdot 599479$
6	$3 \cdot 3 \cdot 7$	34	$3 \cdot 43691 \cdot 131071$
7	**127**	35	$31 \cdot 71 \cdot 127 \cdot 122921$
8	$3 \cdot 5 \cdot 17$	36	$3 \cdot 3 \cdot 3 \cdot 5 \cdot 7 \cdot 13 \cdot 19 \cdot 37 \cdot 73 \cdot 109$
9	$7 \cdot 73$	37	$223 \cdot 616318177$
10	$3 \cdot 11 \cdot 31$	38	$3 \cdot 174763 \cdot 524287$
11	$23 \cdot 89$	39	$7 \cdot 79 \cdot 8191 \cdot 121369$
12	$3 \cdot 3 \cdot 5 \cdot 7 \cdot 13$	40	$3 \cdot 5 \cdot 5 \cdot 11 \cdot 17 \cdot 31 \cdot 41 \cdot 61681$
13	**8191**	41	$13367 \cdot 164511353$
14	$3 \cdot 43 \cdot 127$	42	$3 \cdot 3 \cdot 7 \cdot 7 \cdot 43 \cdot 127 \cdot 337 \cdot 5419$
15	$7 \cdot 31 \cdot 151$	43	$431 \cdot 9719 \cdot 2099863$
16	$3 \cdot 5 \cdot 17 \cdot 257$	44	$3 \cdot 5 \cdot 23 \cdot 89 \cdot 397 \cdot 683 \cdot 2113$
17	**131071**	45	$7 \cdot 31 \cdot 73 \cdot 151 \cdot 631 \cdot 23311$
18	$3 \cdot 3 \cdot 3 \cdot 7 \cdot 19 \cdot 73$	46	$3 \cdot 47 \cdot 178481 \cdot 2796203$
19	**524287**	47	$2351 \cdot 4513 \cdot 13264529$
20	$3 \cdot 5 \cdot 5 \cdot 11 \cdot 31 \cdot 41$	48	$3 \cdot 3 \cdot 5 \cdot 7 \cdot 13 \cdot 17 \cdot 97 \cdot 241 \cdot 257 \cdot 673$
21	$7 \cdot 7 \cdot 127 \cdot 337$	49	$127 \cdot 4432676798593$
22	$3 \cdot 23 \cdot 89 \cdot 683$	50	$3 \cdot 11 \cdot 31 \cdot 251 \cdot 601 \cdot 1801 \cdot 4051$
23	$47 \cdot 178481$	51	$7 \cdot 103 \cdot 2143 \cdot 11119 \cdot 131071$
24	$3 \cdot 3 \cdot 5 \cdot 7 \cdot 13 \cdot 17 \cdot 241$	52	$3 \cdot 5 \cdot 53 \cdot 157 \cdot 1613 \cdot 2731 \cdot 8191$
25	$31 \cdot 601 \cdot 1801$	53	$6361 \cdot 69431 \cdot 20394401$
26	$3 \cdot 2731 \cdot 8191$	54	$3 \cdot 3 \cdot 3 \cdot 3 \cdot 7 \cdot 19 \cdot 73 \cdot 87211 \cdot 262657$
27	$7 \cdot 73 \cdot 262657$	55	$23 \cdot 31 \cdot 89 \cdot 881 \cdot 3191 \cdot 201961$
28	$3 \cdot 5 \cdot 29 \cdot 43 \cdot 113 \cdot 127$	56	$3 \cdot 5 \cdot 17 \cdot 29 \cdot 43 \cdot 113 \cdot 127 \cdot 15790321$
29	$233 \cdot 1103 \cdot 2089$	57	$7 \cdot 32377 \cdot 524287 \cdot 1212847$
		58	$3 \cdot 59 \cdot 233 \cdot 1103 \cdot 2089 \cdot 3033169$
		59	$179951 \cdot 3203431780337$

Table 3.4: (Cont.)

60	3 · 3 · 5 · 5 · 7 · 11 · 13 · 31 · 41 · 61 · 151 · 331 · 1321
61	**2305843009213693951**
62	3 · 715827883 · 2147483647
63	7 · 7 · 73 · 127 · 337 · 92737 · 649657
64	3 · 5 · 17 · 257 · 641 · 65537 · 6700417
65	31 · 8191 · 145295143558111
66	3 · 3 · 7 · 23 · 67 · 89 · 683 · 20857 · 599 · 79
67	193707721 · 761838257287
68	3 · 5 · 137 · 953 · 26317 · 43691 · 131071
69	7 · 47 · 178481 · 10052678938039
70	3 · 11 · 31 · 43 · 71 · 127 · 281 · 86171 · 122921
71	228479 · 48544121 · 212885833
72	3 · 3 · 3 · 5 · 7 · 13 · 17 · 19 · 37 · 73 · 109 · 241 · 433 · 38737
73	439 · 2298041 · 9361973132609
74	3 · 223 · 1777 · 25781083 · 616318177
75	7 · 31 · 151 · 601 · 1801 · 100801 · 10567201
76	3 · 5 · 229 · 457 · 174763 · 524287 · 525313
77	23 · 89 · 127 · 581283643249112959
78	3 · 3 · 7 · 79 · 2731 · 8191 · 121369 · 22366891
79	2687 · 202029703 · 1113491139767
80	3 · 5 · 5 · 11 · 17 · 31 · 41 · 257 · 61681 · 4278255361
81	7 · 73 · 2593 · 71119 · 262657 · 97685839
82	3 · 83 · 13367 · 164511353 · 8831418697
83	167 · 57912614113275649087721
84	3 · 3 · 5 · 7 · 7 · 13 · 29 · 43 · 113 · 127 · 337 · 1429 · 5419 · 14449
85	31 · 131071 · 9520972806333758431
86	3 · 431 · 9719 · 2099863 · 2932031007403
87	7 · 233 · 1103 · 2089 · 4177 · 9857737155463
88	3 · 5 · 17 · 23 · 89 · 353 · 397 · 683 · 2113 · 2931542417
89	**618970019642690137449562111**
90	3 · 3 · 3 · 7 · 11 · 19 · 31 · 73 · 151 · 331 · 631 · 23311 · 1883700191127 · 911 · 8191 · 112901153 · 23140471537
92	3 · 5 · 47 · 277 · 1013 · 1657 · 30269 · 178481 · 2796203
93	7 · 2147483647 · 658812288653553079
94	3 · 283 · 2351 · 4513 · 13264529 · 165768537521

Table 3.4: (Cont.)

95	31 · 191 · 524287 · 420778751 · 30327152671
96	3 · 3 · 5 · 7 · 13 · 17 · 97 · 193 · 241 · 257 · 673 · 65537 · 22253377
97	11447 · 13842607235828485645766393
98	3 · 43 · 127 · 4363953127297 · 4432676798593
99	7 · 23 · 73 · 89 · 199 · 153649 · 599479 · 33057806959
100	3 · 5 · 5 · 5 · 11 · 31 · 41 · 101 · 251 · 601 · 1801 · 4051 · 8101 · 268501
101	7432339208719 · 341117531003194129
102	3 · 3 · 7 · 103 · 307 · 2143 · 2857 · 6529 · 11119 · 43691 · 131071
103	2550183799 · 3976656429941438590393
104	3 · 5 · 17 · 53 · 157 · 1613 · 2731 · 8191 · 858001 · 308761441
105	7 · 7 · 31 · 71 · 127 · 151 · 337 · 29191 · 106681 · 122921 · 152041
106	3 · 107 · 6361 · 69431 · 20394401 · 28059810762433
107	**162259276829213363391578010288127**
108	3 · 3 · 3 · 3 · 5 · 7 · 13 · 19 · 37 · 73 · 109 · 87211 · 246241 · 262657 · 279073
109	745988807 · 870035986098720987332873
110	3 · 11 · 11 · 23 · 31 · 89 · 683 · 881 · 2971 · 3191 · 201961 · 48912491
111	7 · 223 · 321679 · 26295457 · 319020217 · 616318177
112	3 · 5 · 17 · 29 · 43 · 113 · 127 · 257 · 5153 · 15790321 · 54410972897
113	3391 · 23279 · 65993 · 1868569 · 1066818132868207
114	3 · 3 · 7 · 571 · 32377 · 174763 · 524287 · 1212847 · 160465489
115	31 · 47 · 14951 · 178481 · 4036961 · 2646507710984041
116	3 · 5 · 59 · 233 · 1103 · 2089 · 3033169 · 107367629 · 536903681
117	7 · 73 · 79 · 937 · 6553 · 8191 · 86113 · 121369 · 7830118297
118	3 · 2833 · 37171 · 179951 · 1824726041 · 3203431780337
119	127 · 239 · 20231 · 131071 · 62983048367 · 131105292137
120	3 · 3 · 5 · 5 · 7 · 11 · 13 · 17 · 31 · 41 · 61 · 151 · 241 · 331 · 1321 · 61681 · 45622845
121	23 · 89 · 727 · 1786393878363164227858270210279
122	3 · 768614336404564651 · 2305843009213693951
123	7 · 13367 · 3887047 · 164511353 · 177722253954175633
124	3 · 5 · 5581 · 8681 · 49477 · 384773 · 715827883 · 2147483647
125	31 · 601 · 1801 · 269089806001 · 4710883168879506001
126	3 · 3 · 3 · 7 · 7 · 19 · 43 · 73 · 127 · 337 · 5419 · 92737 · 649657 · 77158673929
127	**170141183460469231731687303715884105727**
128	3 · 5 · 17 · 257 · 641 · 65537 · 274177 · 6700417 · 67280421310721

Table 3.4: (Cont.)

$$
\begin{array}{lcl}
M_{131} & = & 263 \cdot ... \\
M_{137} & = & 32,032,215,596,496,435,569 \cdot \\
 & & 5,439,042,183,600,204,290,159 \\
M_{139} & = & 5,625,767,248,687 \cdot ... \\
M_{149} & = & 86,656,268,566,282,183,151 \cdot ... \\
M_{151} & = & 18,121 \cdot 55,871 \cdot 165,799 \cdot 2,332,951 \cdot ... \\
M_{157} & = & 852,133,201 \cdot 60,726,444,167 \cdot 1,654,058,017,289 \cdot ... \\
M_{163} & = & 150,287 \cdot 704,161 \cdot 110,211,473 \cdot 27,669,118,297 \cdot ... \\
M_{167} & = & 2,349,023 \cdot ... \\
M_{173} & = & 730,753 \cdot 1,505,447 \cdot 70,084,436,712,553,223 \cdot ... \\
M_{179} & = & 359 \cdot 1,433 \cdot ... \\
M_{181} & = & 43,441 \cdot 1,164,193 \cdot 7,648,337 \cdot ... \\
M_{191} & = & 383 \cdot 7,068,569,257 \cdot 39,940,132,241 \cdot \\
 & & 332,584,516,519,201 \cdot \\
 & & 147322265321145317331353282383 \\
M_{193} & = & 13,821,503 \cdot 61,654,440,233,248,340,616,559 \cdot ... \\
M_{197} & = & 7,487 \cdot ... \\
M_{199} & = & 164,504,919,713 \cdot ... \\
M_{211} & = & 15,193 \cdot 60,272,956,433,838,849,161 \cdot ... \\
M_{223} & = & 18,287 \cdot 196,687 \cdot 1,466,449 \cdot 2,916,841 \cdot \\
 & & 1,469,495,262,398,780,123,809 \cdot ... \\
M_{227} & = & 26,986,333,437,777,017 \cdot ... \\
M_{229} & = & 1,504,073 \cdot 20,492,753 \cdot 59,833,457,464,970,183 \cdot ... \\
M_{233} & = & 1,399 \cdot 135,607 \cdot 622,577 \cdot ... \\
M_{239} & = & 479 \cdot 1,913 \cdot 5,737 \cdot 176,383 \cdot 134,000,609 \cdot \\
 & & 7,110,008,717,824,458,123,105,014,279,253,754,096, \\
 & & 863,768,062,879 \\
M_{241} & = & 22,000,409 \cdot ... \\
M_{251} & = & 503 \cdot 54217 \cdot 178,230,287,214,063,289,511 \cdot \\
 & & 61,676,882,198,695,257,501,367 \cdot ... \\
M_{257} & = & 535,006,138,814,359 \cdot \\
 & & 1,155,685,395,246,619,182,673,033 \cdot ... \\
M_{263} & = & 23,671 \cdot 13,572,264,529,177 \cdot \\
 & & 120,226,360,536,848,498,024,035,943 \cdot ...
\end{array}
$$

*The number M_{59} was factored by **Landry** (1869), M_{67} by **Cole** (1903), M_{73} by **Poulet** (1923) and M_{113} by **Lehmer** (1947).*

It is obvious that Mersenne could not have tested the correct results for $p = 19, 31, 127$. *Some have believed that Fermat had communicated to him an as yet undiscovered theorem, since empirical methods could hardly have been used in Mersenne's time.*

So by 1947, Mersenne's range $n \leq 258$, *had been completely checked and it was determined that the correct list is:*

$$n = 2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107 \text{ and } 127.$$

It thus, took 304 years to set Mersenne right!

Apart from the challenge of calculating Mersenne numbers for higher values of n, *number theorists since Fermat endeavored ceaselessly to produce aids for recognizing if a Mersenne number is a prime, and if not, to determine its factors (a daunting task in light of the limited calculating devices at their disposal up to the middle of the* 20^{th} *century). Most theoretical results were found by Fermat, Euler and Lucas inside of a period of 250 years.*

The Mersenne primes $M_n = 2^n - 1$ *have the following remarkable properties:*

- *If* n *is composite, so is* $2^n - 1$. *For let* $n = rs, \quad r > 1, \quad s > 1$. *Then*

  $$2^n - 1 = (2^r)^s - 1 = (a-1)(a^{s-1} + a^{s-2} + \cdots + a + 1); \quad a = 2^r,$$

  and so $2^n - 1$ *is divisible by* $2^r - 1 > 1$ *and (since* $r > 1$*) cannot be prime.*

  This means that when $2^n - 1$ *is prime,* n *cannot be composite and must be prime. However,* $2^n - 1$ *is frequently composite when* n *is prime (e.g.* $2^{11} - 1 = 23 \cdot 89, \qquad 2^{23} - 1 = 47 \cdot 178,481$*).*

  Thus

  $$\begin{aligned} n &= \text{composite} &\longrightarrow\quad 2^n - 1 &= \text{composite} \\ n &= \text{prime} &\longrightarrow\quad 2^n - 1 &= \text{composite or prime} \\ 2^n - 1 &= \text{prime} &\longrightarrow\quad n &= \text{prime} \end{aligned}$$

  In general, if $n > 1$ *and* $a^n - 1$ *is prime, then* $a = 2$ *and* n *is prime, for if* $a > 2$, *then* $a^n - 1$ *is divisible by* $a - 1$, *so* $a^n - 1$ *cannot be prime. Note that on account of the above result, the problem of the primality of* $2^n - 1$ *is reduced to that of* $2^p - 1$ *where* p *is prime.*

- *If $r > 2$ is a prime, each prime factor p of $M_r = 2^r - 1$ must be of the form $p = 1 + 2kr$. For example*

$$\begin{aligned} M_{43} &= 431 \cdot 9719 \cdot 2,099,863 = 2^{43} - 1 \\ p_1 &= 431 = 1 + 2 \cdot 5 \cdot 43 \\ p_2 &= 9719 = 1 + 2 \cdot 113 \cdot 43 \\ p_3 &= 2,099,863 = 1 + 2 \cdot 3 \cdot 3 \cdot 2713 \cdot 43 \end{aligned}$$

Fermat (1640) proved this statement in the following way: Let $r > 2$ be a prime and p a prime divisor of $M_r = 2^r - 1$. So

$$2^r \equiv 1 (mod\ p); \quad 2^{p-1} \equiv 1 (mod\ p) \quad \text{by FLT}$$

Let d = highest common factor of r and $p - 1$. Then by Euclid's algorithm $d = \alpha r + \beta(p - 1)$ for suitable integers α, β. It follows that

$$2^d = (2^r)^\alpha \left(2^{p-1}\right)^\beta \equiv 1^\alpha \cdot 1^\beta \equiv 1 (mod\ p).$$

Since M_r is odd we see that $p > 2$; since p divides $(2^d - 1)$ we infer that $d > 1$. Because r is a prime and $d > 1$, $d = r$ and $(p - 1)$ is divisible by r. Consequently $p - 1 = sr$ for some s. Finally, $p - 1$ is even and r is odd. Hence s is even, $s = 2k$, say, as claimed.

- *Euler (1750) found a simple criterion for the factorizability of M_p: If both $p = 4k + 3 > 3$ and $(2p + 1)$ are prime then $(2p + 1)$ divides M_p. Thus if p = 11, 23, 83, 131, 179, 191, 239, 251, then M_p has the factors 23, 47, 167, 359, 383, 479, 503 respectively. The theorem was proved by* ***Lagrange*** *(1775) and again by* ***Lucas*** *(1878).*

- *Every $M_p = 2^p - 1$ is prime to every other Mersenne number.*

- *If M_p is a Mersenne prime, then $M_{M_p} = 2^{M_p} - 1$ is not necessarily a prime number. For example*

$M_{13} = 2^{13} - 1 = 8151$ *is a prime*

$M_{8151} = 2^{8151} - 1$ *is composite and has the prime factor*

$$2 \cdot (20,644,229) M_{13} + 1 = 338,193,759,479$$

In this connection one may consider the sequence of numbers

$$C_1 = 2^2 - 1 = 3 = M_2,$$
$$C_2 = 2^{C_1} - 1 = 7 = M_3,$$
$$C_3 = 2^{C_2} - 1 = 2^7 - 1 = 127 = M_7,$$
$$C_4 = 2^{C_3} - 1 = 2^{127} - 1 = M_{127}, \dots, C_{n+1} = 2^{C_n} - 1, \dots.$$

It is not known whether all number C_n are primes and even if there exist infinitely many which are primes. It is impossible yet (2004 CE) to test C_5, which has more than 10^{38} digits!

1645–1667 CE **Ismael Boulliau** (1605–1694, France). Astronomer and classical scholar. An early Copernican, Keplerian and defender of Galileo. First to suggest (without proof) in *Astronomia Philolaïca* (1645) that the central force keeping the planets in their Keplerian elliptical orbits, must be proportional to their inverse-square distance from the sun[86]. This work is arguably the most important book in astronomy between Kepler and Newton.

Newton (1684) *proved* that planets moving under such law will obey the three laws of Kepler. Among other astronomers who preceded Newton in astronomical inquiries and contributed some ideas to the establishment of the true laws that govern motion of planets in their courses, are: **Giovanni Borelli** (1664, Pisa), **Huygens** (1673), **Hooke** (1674) and **Halley** (1684).

Boulliau established (1667) the brightness periodicity of the first known long period variable star, *Mira Ceti*.

Born to Calvinist parents in London, Boulliau converted to Catholicism and moved to Paris in the early 1630s. During the next thirty years he enjoyed the patronage of the family de Thou and assisted the Brothers Dupuy at the Bibliotheque du roi. Boulliau was a friend of **Pascal** and **Gassendi** and a close associate of Huygens.

Newton, in his *Principia*, praised *Astronomia Philolaïca*, particularly for the inverse-square hypothesis and its accurate tables.

1645–1675 CE **Jean Picard** (1620–1682, France). Astronomer and a founding member of the *French Academie Royale des Sciences* (1666). First to

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86 Kepler had claimed proportionality to the inverse distance.

apply the telescope to measurements of angles. Known especially for accurate measurements of a degree of a meridian, from which he computed the size of the earth (1668–1670). Credited with first use of telescopic sights and of pendulum clocks in astronomical observations. Determined latitude and longitude of Tycho Brahe's observatory in Uraniborg so Tycho's observations could be directly compared with others. His measurements of the earth's size were used by Newton in his gravitational theory.

Picard became professor of astronomy (1655) at the College de France in Paris. In 1673 he moved to the Paris Observatory where he collaborated with **Cassini**, **Römer** and **La Hire**.

1646 CE **Athanasius Kircher** (1601–1680, Germany). Jesuit and scholar. Credited with the invention of the *magic lantern*[87] (Laterna Magica), the first early projection device and a forerunner of the modern slide and motion picture projectors.

The device, in its simpler forms, consisted of: (1) the lantern body, (2) a source of light, (3) an optical system for projecting the images. It projected on a white wall or screen largely magnified images of transparent pictures painted (or later, photographed) on glass, or of objects (crystals, animals, etc.) carried on glass slides. The projection was made by means of a concave mirror (acting as condenser) and a projection lens, using sunlight, oil or candle light.

Laterna Magica was used during 1726–7 at the Opera in Hamburg.

Kircher was a professor of mathematics in Rome (1650). He was one of the first to experiment with *moving images*.

1646–1658 CE **Johann Rudolph Glauber** (1604–1668, Germany). Chemist. First to distill coal and obtain benzene and phenol; investigated decomposition of common salt through action of acids and bases. *Glauber's salt* [$Na_2SO_4 \cdot 10H_2O$ (1658)] is named after him. Glauber had a fairly clear

[87] Although Kircher was first to set up various types of the apparatus, is difficult to identify the original inventor. In about 1653 the mathematician **Andreas Tacquet** displayed at Löwen a journey from China to the Netherlands by means of Kircher's lantern — the first lantern lecture ever delivered. **Thomas Walgenstein** (a Danish student at Leyden) got the idea from Tacquet, and demonstrated (1658) in a number of cities a magic lantern with interchangeable painted glass slides and a two-lens objective, creating a great sensation. **Huygens**, who had dealings with Walgenstein, also possessed a proper magic lantern at about that time. He even used a lens as a condenser, whereas Walgenstein still retained the use of a concave mirror.

idea that salts consist of acid and base, and a correct notion on *affinity*. Resided in Amsterdam from 1655.

1648–1656 CE *Fire and sword over Eastern Europe*; the uprising of the serfs and the Greek orthodox Ukrainian Cossacks against the weak regime of the Roman Catholic Polish gentry. The chieftain of the Cossacks, Bogdan Chmialnicki, placed himself under the protection of Russia, thus precipitating a prolonged conflict. Over 200,000 Jews, about one half of the total Jewish population of the Ukraine and Galicia, perished in the decade of this revolution and over 700 of their communities were destroyed. Many more fled to Holland, Germany, Bohemia and the Balkans.

This tragic event brought forth a new spiritual movement generated by **Israel Ba'al Shem Tov** (1700–1760) and known as *Chassidism*. It was a new interpretation of Judaism based not upon reason but faith, not upon intellect but emotion; man could literally escape his unbearable miseries by immersing himself in a mystical-esoteric kindling of the soul with God. To the masses who hungered for a direct, simple, stimulating religion which they could follow without any philosophical sophistications, the doctrine of salvation through prayer and humility rather than study was appealing. The unsuppressed emotions and optimistic Chassidic spirit served as a buffer against the depressing environment of dissolution and terror.

1650 CE **Bernhard Varen** (**Bernhardus Varenius**, 1622–1650, Germany). Geographer and physician. In *Geographia generalis* (1650) he endeavored to lay down the general principles of environmental science on a wide scientific basis, according to the knowledge of his day.

His work long held its position as the best treatise in existence on scientific and comparative geography. The work is divided into:

- *Absolute geography*: investigates mathematical facts relating to the earth as a whole, its figure, dimensions, motions, etc.
- *Relative geography*: considers the earth as affected by the sun and the stars, climates, seasons, the difference of apparent time at different places, variations in the length of the day, etc.
- *Comparative geography*: treats the actual divisions of the surface of the earth, their relative positions, globe and map construction, longitude, navigation, etc.

Geography is viewed as encompassing all aspects of the surface of the earth, including its geologic and oceanographic features, climate, plant and animal life. Varen explains the global wind system by a physical process through which air in the equatorial regions is thinned by the sun's heat and in response the cold, heavier air of the polar regions flows equatorward.

Isaac Newton thought so highly of this book[88] that he prepared an annotated edition which was published in Cambridge (1672), with the addition of the plates which had been planned by Varen, but not produced by the original publishers.

Varen was born at Hitzacker on the Elbe, in the Lüneburg district of Hanover. His early years (from 1627) were spent at Uelzen, where his father was court preacher to the Duke of Brunswick. He studied at the Universities of Königsberg (1643–1645) and Leiden (1645–1649), where he devoted himself to mathematics and medicine, taking his medical degree at Leiden (1649). He then settled at Amsterdam, intending to practice medicine. But the recent discoveries of **Tasman** (1642–1644), and **Schouten** (1615–1616), attracted him to geography. He died only 28 years of age, a victim to the privations and miseries of a poor scholar's life.

ca 1650 CE The intensity of the earth's magnetic field began to decline; it diminished 15 percent during the next 350 years[89]. If the trend continues at the same rate, a reversal of the earth's magnetic field may occur around 4000 CE.

1650–1654 CE **James Ussher** (1581–1656, Ireland). Prelate and scholar. Calculated that God created the world at 9:00 a.m., Sunday, 23 October, 4004 BCE. The calculations of this archbishop were somewhat less precise than the result would seem to indicate. The year 4004 BCE was arrived at by taking **Luther**'s estimate[90] of 4000 BCE [obtained by rounding off various arithmetical calculations of *Biblical chronology*], and then correcting it by four years to allow for Kepler's dating of the birth of Christ in 4 BCE

[88] The reason why Newton took so much care in correcting and publishing Varen was, because he thought him necessary to be read by his audience while he was delivering lectures on the same subject from the Lucasian Chair.
The book was still recommended for students at Cambridge in 1910!

[89] The fact that the magnetic compass was a key factor in navigation led governments to subsidize the science of geomagnetism. As a result magnetic observations have been made since the 16^{th} century.

[90] The literal interpretation of the biblical book of *Genesis* gained increasing devotion by Churchmen, not in the early Middle Ages, when the teaching of the Greeks were still largely accepted in the secular world and the New Testament was still new, but in the later Middle Ages and the beginning of modern times, in a reaction to the scientific explorations of the Renaissance. Early Christian scholars, such as St. Augustine, continued in all essentials the tradition of the Greek philosophers, but the thread of that kind of thinking was lost in late medieval outgrowths of Christian scholasticism and theological idealism.

[based on discrepancy between solar eclipses and New Testament dating of the crucification].

1650–1671 CE **Nicolaus Mercator-Kaufmann** (1619–1687, England). Independently of James Gregory introduced and summed infinite series[91], in connection with the calculations of areas under plane curves.

Mercator [*not* to be confused with **Gerhardus Mercator** (1512–1594), who is known for the *Mercator projection*] was born in Holstein (then a part of Denmark) but spent most of his life in England and was one of the first members of the Royal Society of London. He died in Paris.

1652 CE **Thomas Bartholinus**[92] (1616–1680, Denmark). Physiologist, physician and mathematician. Discovered the *lymphatic system* (1652) and determined its relationship to the circulatory system. Professor of mathematics (1646–1648) and of anatomy (1648–1680) at Copenhagen University; physician to King Christian V (1670–1680). Defended Harvey's doctrine of the circulation of the blood.

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91 The series

$$\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}\cdots$$

is sometimes referred to as the *Mercator series*. It was independently discovered by **G. Saint-Vincent** (1584–1667). In the early days of the calculus, this series was probably derived through a term by term integration of the geometric series expansion

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots, \qquad -1 < x \le 1.$$

Since

$$\log_e\left(\frac{1+x}{1-x}\right) = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \cdots\right),$$

the substitution $x = \frac{1}{2N+1}$ yields the formula

$$\log_e(N+1) = \log_e N + 2\left[\frac{1}{2N+1} + \frac{1}{3(2N+1)^3} + \frac{1}{5(2N+1)^5} + \cdots\right].$$

This series converges rather rapidly for all positive N and was used in the calculations of logarithms.

92 His father **Caspar Berthelsen** (Lat. Bartholinus) (1585–1629), physician, was first to describe the olfactory nerve as first cranial nerve. His brother **Erasmus Bartholinus** (1625–1698), was a physician, mathematician and physicist; discovered the phenomenon of *double refraction of light* in Icelandic feldspar. Both were professors of medicine at the University of Copenhagen.

1653 CE The construction of *Taj Mahal* in Agra, India was completed. About 20,000 workmen built it over 21 years. The Indian ruler Shah Jahan ordered it built in memory of his wife Mumtaz-i-Mahal.

The Taj Mahal is made of white marble. It rests on an eight-sided platform of red sandstone. Each side is about 40 m long. At each corner of the platform stands a slender white minaret. The central structure has four smaller domes surrounding the huge, bulbous central dome. The tombs of the Shah and his wife are in a basement room. Above them in the main chamber are false tombs. Light is admitted into the central chamber by finely cut marble screens. The Taj is amazingly graceful from almost any angle of view, and the precision and care which went into its design and construction are impressive. It is one of the most beautiful and costly tombs in the world.

Scientists fear that after centuries of undiminished glory, industrial pollution could cause irreparable damage to the marble.

1654–1672 CE **Otto von Guericke** (1602–1686, Germany). Soldier, engineer and natural philosopher. Believed in a finite starry cosmos surrounded by an infinite void, as in the Stoic system. Performed spectacular public demonstrations in Magdeburg in which two teams of horses were unable to break apart two large evacuated brass hemispheres, held together by external atmospheric pressure. The vacuum was achieved by means of an air-pump which he developed in 1650. He was also able to show that sound could not travel, flames could not burn and animals could not live in vacuum.

Von Guericke was born at Magdeburg, in Prussian Saxony. He studied law at Leipzig, Helmstadt and Jena, and mathematics and mechanics at Leyden. He then visited France and England and in 1636 became engineer-in-chief at Erfurt. Toward the end of the Thirty Years War he returned to Magdeburg and helped rebuild it. He became mayor (1646–1676) and a magistrate at Brandenburg. His leisure was devoted to scientific pursuits, especially in pneumatics. Enticed by the discoveries of **Galileo**, **Pascal** and **Torricelli**, he attempted to create a vacuum. He also experimented with static electricity and made successful researches in astronomy, predicting the periodicity of the return of comets. In 1672, at the age of 70, he published his ideas and experimental results in *The New Magdeburg Experiments on Void Space*.

Only God and space can be infinite, Guericke said, and though the starry cosmos may be immense, it is nonetheless finite in size. Guericke believed in a *finite* Stoic cosmos, and thought that the gaps between stars reveal to us the emptiness and darkness of an extracosmic void, i.e. the sky is dark at night because we look between the stars and see the starless void beyond.

1655–1663 CE **Francesco Maria Grimaldi** (1618–1663, Italy). Physicist. First to suggest the *wave theory of light* in a book entitled "*Physico-Mathesis de lummine coloribus et iride*", published after his death in 1665. Grimaldi's major contribution to the optics was the discovery of *diffraction* (1660).

Grimaldi found that light did not travel exactly in straight lines, for he discovered that shadows were a little larger than they should be on the supposition that the propagation of light was rectilinear. Moreover, he found that the edges of shadows were often colored, and so he suggested that light was a fluid capable of wave-like motions, *different frequencies being different colors*(!) If the motions of the light-fluid were wave-like, then the edges of shadows should be blurred and colored, he said, for water waves can easily go round an obstacle they encounter. He supposed further that his light-fluid moved with great speeds, undulating all the time.

Grimaldi developed experiments to study phenomena associated with diffraction, interference, reflection and the color of light. His experiments were wide in scope and subtle in arrangement. He succeeded in detecting interference fringes even with such a quasi-coherent source as the pinhole source. He illuminated two closely spaced pinholes with a pinhole source and, in the pattern projected onto a screen, he discovered that some areas of the projected patterns were even darker than when one of the holes were plugged. His observations are basically the same as those of Thomas Young (1773–1829) in an experiment performed 150 years later.

His book planted many seeds which were later cultivated to full bloom by **Huygens**, **Newton**, **Young** and **Fresnel**.

Grimaldi was born in Bologna, son of a wealthy silk merchant. At the age of 14 he joined the *Society of Jesus*, and was educated at his Order's houses at Parma, Ferrar and Bologna, where he became Professor of Mathematics at the Jesuit College (1648).

1655–1678 CE **Christiaan Huygens**[93] (1629–1695, The Netherlands). An eminent Dutch mathematician, mechanician, physicist and astronomer.

He was born at the Hague, the second son of Sir Constantin Huygens, poet and diplomat (1596–1687). From his father he received the rudiments of his

[93] For further reading, see:

- Wolf, E., The life and work of Christiaan Huygens, in Blok, H., H.A. Ferwerda and H.K. Kuiken (Editors), *Huygens Principle*: *1690–1990*: *Theory and Applications*, Elsevier Science Publishers, 1992.

education, which he continued at Leyden (1645 to 1647) and Breda, where he studied law and mathematics.

In 1655 he discovered Titan, satellite of Saturn, with a telescope that he built himself, and suggested that the appendages of Saturn seen earlier by Galileo (1610) are edges of a flat disk surrounding the planet. In 1656, Huygens was the first effective observer of the Orion nebula.

In November 1659 Huygens made the first reliable record of surface features of *Mars*, using a refracting telescope of his own design. After observing a prominent, dark, triangular feature (now called *Syrtis Major*) for several weeks, Huygens concluded that the rotation period of Mars is approximately 24 hours (modern value $= 24^h\,37^m\,23^s$). This was the first in a series of observations that would soon lead to speculations about *life on Mars*.

In 1663, on his visit to England, he was elected a fellow of the Royal Society. During the period 1666–1681 he resided in Paris as a guest of King Louis XIV. He returned to Holland to conclude his studies on physical optics.

In 1656 he built the first reliable isochronous pendulum clock with an accuracy of 10 sec/day. Isochronism was achieved by forcing the bob of the pendulum to mark a *cycloidal arc*[94]. Huygens' clock incorporated the verge

[94] *The cycloidal pendulum*: A vertical pendulum having a bob of mass m suspended from the fixed point O.

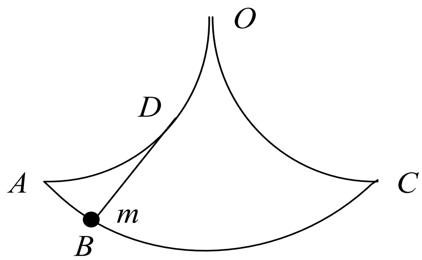

Fig. 3.1: The cycloidal pendulum

As it oscillates, the string winds up on the constant curve ODA (or OC) as indicated in the Fig. 3.1. If the curve ABC is a *cycloid*, the curves ODA and OC are the *evolutes* of the cycloid, and in fact are themselves two halves of an equal cycloid.
Consider the motion of a *free particle* of mass m under gravity on a smooth cycloid ABC whose axis is vertical and vertex lowest, as in the figure.
Let the cycloid be generated by a circle of radius a and let s be the arclength from its lowest point to a general point B on it. Then, it follows from the geometry of the curve that $s = 4a \sin\vartheta$, where ϑ is the angle made by the tangent at B with the horizontal tangent at the vertex (x − axis).

type escapement [In 1676 a much improved *anchor* type was invented, that interfered less with the pendulum's free motion; this device allowed, for the first time, the uniform division of a given time interval] and a spiral balanced spring of his invention. Huygens used continued fractions for the purpose of approximating the correct design for the toothed wheels of a planetarium [rational approximation for irrational gear ratio]. In 1657 Huygens came to Paris and became interested in the new theory of probability. He introduced the concept of *mathematical expectation*.

As an outgrowth of his experimentation with the pendulum and with circular motion, Huygens was able to derive in 1673 the law of *centripetal acceleration* of a mass which moves uniformly in a circle of radius r with velocity v; its acceleration $a = v^2/r$ is directed toward the center of the circle. In his studies of mechanics Huygens gave a clear and concise account of the laws governing the collision of *elastic bodies*[95] (1669) and introduced the important concept of the *moment of inertia*. His studies of pendulum motion made it possible for him to make the first accurate determination of the value of the acceleration of gravity and to show that it varied with latitude[96].

Although his contribution to the calculus was somewhat indirect, he was the first person since Archimedes to calculate areas of portions of surfaces of revolution, such as the paraboloid and hyperboloid.

Huygens used his self-made telescope to make important astronomical discoveries throughout the solar system: the existence of Titan, explanation of

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The force along the tangent at B is given by

$$m\frac{d^2s}{dt^2} = -mg\sin\vartheta \equiv -\frac{mg}{4a}s,$$

so that the motion is *simple harmonic* with the time to the lowest point being $\pi\sqrt{\frac{a}{g}}$, independent of the initial position of the particle. This property will still be true if, instead of the material curve, we substitute a string tied to the particle in such a way that the particle describes a cycloid and the string is always normal to the curve. This will be the case if the string *unwraps and wraps itself on the evolute of the cycloid*. Hence, if a string of length $4a$ is allowed to wind and unwind itself upon fixed metal cheeks in the form of half of the original cycloid each, a particle of mass m attached to its end will have its time of oscillation always *isochronous*, whatever the angle through which the string oscillates. In order words: the period of oscillation will be the same regardless of the amplitude of the oscillations.

[95] These conclusions were arrived at independently by **Christopher Wren** in 1668.

[96] Emil Wolf: The life and work of Christian Huygens, in *Huygens' Principle 1690–1990*, Elsevier Science Publications, pp. 3–17, 1992.

Saturn's rings, Martian rotation period, cloud cover of Venus. In addition he contributed to the design of the first microscopes.

In 1678, Huygens made an explicit development of the wave theory of light, and stated his celebrated principle[97] (in its extended formulation): each surface element of a wavefront at time t_0 is regarded as a source of secondary spherical waves. The wavefront at later time t, is the envelope of all the interfering secondary spherical waves with radius $c(t-t_0)$.

In the same year, Huygens made the fundamental discovery of *polarization*: each of the two rays arising from refraction by *Iceland spar* may be extinguished by passing it through a second crystal of the same material if the latter crystal be rotated by 90° about the direction of the ray[98].

Huygens never married. He died at the Hague, bequeathing his manuscripts to the University of Leyden and his considerable property to the sons of his brother.

1655–1660 CE **William Brouncker** (1620–1684, England). Mathematician. Among the founders of the Royal Society of London and its first president. Worked on continued fractions and calculating logarithms by infinite series. Discovered the expansion (1655)

$$\frac{4}{\pi} = 1 + \cfrac{1^2}{2 + \cfrac{3^2}{2 + \cfrac{5^2}{2 + \cdots}}}.$$

Gave a method of solution to the Diophantine equation $x^2 - ay^2 = 1$ (1657), which first appeared in Fermat's work (1640), to be known later as the *Pell equation*.

[97] Huygens' Principle continued to play an important role in the development of physics: In 1926, **Schrödinger** invoked the Principle to elucidate the transition from classical to quantum mechanics. In 1938, **Zernike** used Huygens' Principle to show how certain statistical features of light, known as its *coherence properties*, are transmitted on propagation.

Feynman (1948) made use of Huygens' Principle, in the so-called path integral formulation of quantum mechanics.

[98] It was however left to **Newton** (1717) to interpret these phenomena. He assumed that rays have "sides"; and indeed this "transversality" seemed to him an insuperable objection to the acceptance of the wave theory, since at that time scientists were familiar only with longitudinal waves. Later (1808), the *polarization of light by reflection* was discovered by **Etrienne-Louis Malus** (1775–1812, France). But Malus did not attempt the interpretation of this phenomenon. Only in 1818 did **Fresnel** establish the transversality of light waves.

Continued Fractions

The origin of continued fractions is hard to pinpoint: we can find examples of these fractions throughout mathematics in the last 2000 years, but its true foundations were not laid until the late 1600's, early 1700's.

The origin of continued fractions is traditionally placed at the time of the creation of Euclid's Algorithm, used to find the greatest common divisor (`gcd`) of two numbers. However, by algebraically manipulating the algorithm, one can derive the simple continued fraction of the rational p/q as opposed to the GCD of p and q. It is doubtful whether Euclid or his predecessors actually used this algorithm in such a manner. But due to their close relationship, the creation of Euclid's Algorithm signifies the initial development of continued fractions.

For more than a thousand years, any work that used continued fractions was restricted to specific examples. The Indian mathematician ***Aryabhata*** *(d. 550 CE) used a continued fraction to solve a linear indeterminate equation. Rather than generalizing this method, he used continued fractions solely in specific examples.*

Throughout Greek and Arab mathematical writing, we can find examples and traces of continued fractions. But again, its use is limited to specifics. More examples were provided during the Late Renaissance by ***Bombelli*** *(ca 1570 CE) and* ***Cataldi*** *(ca 1600 CE), who expressed the square roots of* 13 *and* 18*, respectively, as repeated continued fractions. However, neither of them investigated the properties of the continued fractions.*

Christiaan Huygens *was first to demonstrate practical applications of continued fractions (1695). He used it for the purpose of approximating the correct design for the toothed wheels of a planetarium.*

William Brouncker *discovered in 1655 a continual fraction expansion for $\frac{4}{\pi}$. He then discovered a method to solve the Diophantine equation*

$$x^2 - Ny^2 = 1$$

by a continued fraction. The theory shows that a particular solution is

$$x = p_n, \qquad y = q_n$$

where $\frac{p_n}{q_n}$ is a certain convergent of $\sqrt{N}$. Moreover, from one solution, an infinite number of solutions may be found. Thus from the least-values solution

$$x = 3, \qquad y = 2 \qquad \text{of} \qquad x^2 - 2y^2 = 1,$$

one derives

$$x_n = \frac{1}{2}\left[(3+2\sqrt{2})^n + (3-2\sqrt{2})^n\right]$$

$$y_n = \frac{1}{2\sqrt{2}}\left[(3+2\sqrt{2})^n - (3-2\sqrt{2})^n\right].$$

The field began to flourish when **Leonhard Euler** *(1707–1783),* **Johann Heinrich Lambert** *(1728–1777), and* **Joseph Louis Lagrange** *(1736–1813) embraced the topic. Euler laid down much of the modern theory in his work De Fractionibus Continius (1737). He showed that every rational can be expressed as a terminating simple continued fraction. He also provided an expression for e in a continued fraction form*

$$e-1 = 1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\ddots}}}}}}}}$$

He used this expression to show that e and e^2 are irrational. He also demonstrated how to pass from a series to a continued fraction representation of the series, and conversely.

Lambert generalized Euler's work on e to show that both e^x and $\tan x$ are irrational if x is rational. Lagrange used continued fractions to find the value of irrational roots. He also proved that a real root of a quadratic irrational[99] *is a periodic continued fraction.*

The 19^{th} century can probably be described as the golden age of continued fractions. The subject was known to every mathematician and, as a result, there was an explosion of growth within this field. Some of the more prominent mathematicians to make contributions to this field include **Gauss**, **Cauchy**, **Jacobi** *and* **Hermite**.

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[99] Any number of the form $\frac{P\pm\sqrt{D}}{Q}$, where $P,\ D,\ Q$ are integers and D is a positive integer which is *not* a square.

1655–1695 CE **John Wallis** (1616–1703, England). Mathematician. Contributed substantially to the origins of calculus. Wrote a book, '*Arithmetica infinitorum*' (1655), in which he introduced the concept of *limit*, negative and fractional exponents, and the symbol ∞ for infinity. The whole thrust of his work was to replace geometrical with algebraic concepts and procedures wherever possible. Newton's study of this book was a major influence in his discovery of the general binomial theorem.

Wallis prime objective, however, was to 'square the circle', i.e., to effect the quadrature of the curve $y = (1-x^2)^{1/2}$, by expanding y in power series of x^2 of the form $a_0 + a_1x^2 + a_2x^4 + \cdots$. In this he was unsuccessful; it was left to the young **Newton** to achieve success here. In 1685 Wallis presented a graphical representation of complex numbers in his book: "*Treatise of Algebra*". He also made pioneering contributions to mechanics: In 1668 he suggested *the law of conservation of momentum*. In 1655 he found the infinite rational product $\pi = 2\frac{2\cdot2\cdot4\cdot4\cdot6\cdot6\cdots}{1\cdot3\cdot3\cdot5\cdot5\cdot7\cdots}$.

In his book *Opera Mathematica* (1695) Wallis laid some of the basic groundwork for *continued fractions*. He explained how to compute the n^{th} *convergent* and discovered some of the now familiar properties of convergents. It was also in this work that the term "*continued fraction*" was first used.

Wallis has been described thus: "*One of those youthful prodigies who never quite pay off. But for the mishap of living in the age of Newton, Leibniz, Descartes and Fermat, he might have been a leading mathematician of the 17th century...*"

We also find in history books that he testified against Laud, the Archbishop of Canterbury, and was partly responsible for his execution. For his distinguished services to the crown, he was appointed Savillian professor of geometry at the University of Oxford.

1655–1698 CE **Georg Eberhard Rumpf van Hanau** (**Rumphius**, 1627–1702, Amboina, the Moluccas, East Indies). One of the great naturalists of the 17^{th} century. In his *Herbarium Amboinense* he described the first large herbal of the flora of the Eastern and tropical world (published posthumously 1741–1750, Amsterdam). Earlier (1705), his manuscript *Amboinsche Rariteitkamer*, describing Moluccan shells, was published in Delft.

Rumphius was born near Hanau (on the river Main), the son of a prosperous architect. He was educated in the local gymnasium and soon developed a very adventurous spirit; he enlisted in the Venetian army, but the ship on which he sailed to Brazil was captured by the Portuguese, and the boy was drafted into the Portuguese army. In 1649 he was allowed to return to Hanau. In 1653 he took service as a warrant officer in the Dutch East India Company and sailed to Java and from there to Amboina. This was to become the arena of his scientific activities for the next half-century. In spite of many catastrophes which befell him [loss of eyesight, 1670; loss of his wife and youngest daughter in the earthquake of Feb. 17, 1674; loss of all his manuscript drawings in the Amboina fire of 1687; loss of the text and illustrations of half of the *Herbarium* at sea, when the carrying ship was sunk by the French in 1692], he continued his work. Four more years of labor amidst all the discomforts of tropical life — far away from every library and university, the new manuscript of the whole work (12 books) was ready. In 1696, the Amsterdam officials of the Dutch East India Co. had it in their hands — one of the masterpieces of botanical literature — but they did not considered it worth publishing. It was to remain 40 years hidden in their archives.

After a long period of oblivion, Rumphius' *magnum opus*, the *Herbarium amboinense* was rescued from the Zeventienen archives by the Amsterdam professor, **Johannes Burman** (1707–1779), who decided to edit it, to translate it into Latin, to add various notes, and to publish it with the original illustrations. The work was so enormous and so expensive to produce that no single Dutch firm would assume the whole risk. It was finally issued by a consortium of eight Dutch publishers in six folio parts appearing in Amsterdam from 1741 to 1750.

1656 CE Plague spread from Sardinia to Naples. Ca 400,000 perished. The disease returned by 1672, killing ca 400,000 more.

1656 CE **Thomas Wharton** (1614–1673, England). Physician and anatomist. Gave first thorough account of the *glands* of the human body, which he classified as excretory, reductive and nutrient. Wharton differentiated the viscera from the glands and explained their relationships, describing the *spleen* and the *pancreas*.

Wharton was born in Winston-on-Tees, Durham county. He studied at Cambridge and Oxford, obtaining his M.D at Oxford (1647). He had medical practice in London and was elected a fellow of the Royal College of Physicians (1650).

In 1456 he published his Latin treatise "*Adenographia – a description of the glands of the entire body*".

Wharton discovered the duct of the *submaxillary salivary gland* and the jelly of the umbilical cord, both of which are named for him; he also provided the first adequate account of the *thyroid*, and gave it that name. He explained the role of saliva in mastication and digestion.

1657 CE *Accademia del Cimento*, the first scientific research institute, was founded in Florence with the encouragement and support of Ferdinand II, grand duke of Tuscany (1610–1670).

1657 CE **Johann van Waveren Hudde** (1628–1704, The Netherlands). Mathematician. Introduced letter coefficients which stand for negative as well as positive number. Until this date, negative values *were not allowed*.

1657–1685 CE **Robert Hooke** (1635–1703, England). Inventor and experimental scientist. Because of his varied interests he abandoned many successful but slow-moving experiments without finishing them, originating much but perfecting little. Others profited from his findings.

In 1660, Hooke laid the foundations to the currently accepted theory of elasticity in his motto "*Ut tensio sic vis*". It states the one-dimensional stress-strain relation of linear elasticity. This he discovered while applying spiral springs to the balances of watches.

His other scientific activities were:

- Improved the air pump and used the improved version to confirm Galilei's hypothesis (with a feather and a coin) that in a vacuum all objects fall at the same rate (1657).
- Constructed the first reflecting telescope (1664).
- Invented the *anchor escapement* for clocks and was first to use a *spiral spring* to regulate watches (1658).
- Discovered plant cells (1665).
- Stated the *inverse-square-law* of gravitation prior to Newton's publication (he insisted that Newton mention this fact in his *Principia*!) and approached to a remarkable degree the discovery of universal gravitation (1679).
- His optical investigations led him to adopt, in an imperfect form, the wave theory of light, to anticipate the concept of interference and to observe, independently of **F.M. Grimaldi** (1618–1663), the phenomenon of diffraction.

In personal appearance Hooke made a sorry show: his figure was crooked, his limbs shrunken, his hair hung in disheveled locks over his haggard countenance. His temper was irritable, his habits penurious and solitary.

Many circumstances concurred to embitter the latter years of his life, and the repeated anticipation of his discoveries by others filled him with morbid jealousy. In 1691, the Royal Society made him a grant to enable him to complete his inventions. While engaged in this task he died, worn out with disease, in London.

1658–1671 CE **Johann de Witt** (1625–1672, Netherlands). Dutch statesman and amateur mathematician. He conceived a new and ingenious way of generating conics, essentially the same as that by projective pencils of rays in modern synthetic geometry, but which he treated by means of Cartesian analytic geometry. Using the known theory of probability of his day [mainly via the works of **Fermat**, **Pascal** and **Huygens**], he gave a careful and adequate discussion of the theory of *life-annuities*. This represents his most important contribution to mathematics, and is a remarkable performance for a man deeply involved in the affairs of state.

Witt was born at Dort. He was educated at Leyden and displayed early on remarkable talents in mathematics and jurisprudence. As a student he lived in the house of **Franciscus van Schooten** (1615–1660) [a professor of mathematics at Leyden, remembered for his recommendation of the use of Cartesian coordinates in 3-dimensional geometric problems].

He led a hectic life while leading of the United Provinces through periods of war, in which he opposed the designs of Louis XIV. When in 1672 the French invaded The Netherlands, de Witt was dismissed from office by the Orange party and lynched by an infuriated mob.

1659 CE **Johann Heinrich Rahn** (1622–1676, Switzerland). Mathematician. In his book *Teutsche Algebra* (1659) he introduced the logical symbol $\therefore$ (therefore) and the operational symbol $\div$ for division.

1658–1673 CE **Jan Swammerdam** (1637–1680, The Netherlands). Naturalist. Founder of both comparative anatomy and entomology.

Conducted microscopic examination of aspects of human anatomy. First to observe and record *red blood cells* (1658). Discovered the *valves of the lymph vessels* (1664), which now bear his name.

Performed (1667) series of experiments on *animal respiration*: by compressing and expanding an air bellows attached to the windpipe and lungs of various animals, he was able to examine the effects of inflating and deflating the lungs.

Swammerdam described ovarian follicles of mammals independently of **de Graaf** (1672). Through his investigation of the human reproductive system he was first to show that female mammals produce *eggs*, analogous to birds' eggs.

Studied the anatomy of *insects*, which he classified on the basis of development. His chief works are *Historia Insectorium Generalis* (1669) and *Biblia Naturae* in which he used a simple microscope to make observations of a great range of biological phenomena.

Swammerdam was born in Amsterdam. He studied medicine at Leiden but never practiced.

1659–1661 CE *Cheques and Banknotes*: Messrs Clayton and Morris, bankers in London, handled the first known *cheque* (1659). The first European *banknotes* were issued in Stockholm, Sweden. They were originally receipts issued by bankers for gold deposited with them, promising to repay the deposition (1661).

1660 CE The Royal Society founded in London by **Jonh Wilkins** (1614–1672) and **William Brouncker** (1620–1684).

1660 CE **Robert Boyle** (1627–1691, England). Irish chemist and physicist. Laid the foundations of modern chemistry[100]. In his book *The Sceptical Chymist* (1661) he disputed and refuted the ideas of Aristotle (the 4 Greek "elements": air, earth, fire, water) and Paracelsus (the fundamental nature of sulfur, salt, and mercury; 1530) on the composition of matter. Introduced the modern concepts of *elements*, *alkali*, *acid* and defined *chemical reaction.* Although he argued against the existence of elements, he was first to make attempts to classify all substances into elements, compounds and mixtures. Experimented with his improved air-pump ('Boyle's engine') and showed that sound cannot diffuse in vacuum, whereas light can pass through it unattenuated.

Derived 'Boyle's law' for ideal gases, stating that at fixed temperature the gas volume is inversely proportional to its pressure. Theorized that gas is made of small indivisible spherical particles, in random motion.

Robert Boyle was born at Lismore Castle, in the province of Munster, Ireland, the 14^{th} child of Richard Boyle, the great earl of Cork. While still a child he learned to speak Latin and French, and was only 8 years old when he was sent to Eaton. During 1638–1642 he traveled with a tutor in Europe and being a heir to a great fortune, he decided to dedicate his life to study and scientific research. He settled in Oxford (1654) and set himself, with the

[100] The Late Latin *alchimista* stemmed from *al kimiya*, the prefix *al* being the Arabic article. The remainder of the word may be from the Greek *chimia* ($\chi\upsilon\mu\epsilon\iota\alpha$) [meaning: pouring, infusion and used in connection with the study of juices of plants. Also *cheimeia* = transmutation of metals]. This derivation accounts for the old-fashioned spellings: *chymist* and *chymistry*.

Another view traces it to the Egyptian *kym*, god of the Nile or *khem*, which denotes black earth and occurs in the *Bible* (*Gen* **9**, 24; *Ps* **78**, 51; *Ps* **105**, 23, 37; *Ps* **106**, 22) and **Plutarch**. On this derivation alchemy is explained as meaning the *Egyptian art*. The first occurrence of the word is said to be in a treatise of **Julius Firmicus** (ca 346 CE), an astrological writer.

The prefix *al* was added by a later copyist, and dropped about the middle of the 16^{th} century.

assistance of Robert Hooke, to improve on the air-pump of Otto von Guericke (1657–1660). It was in relation to this work that he discovered in 1661 his famous law. In 1680 he was elected president of the *Royal Society of London for improving natural knowledge* (established 1660). In 1668 he left Oxford for London, where his failing health caused him to withdraw from all his public engagements. He was buried in the churchyard of St. Martin's in the Fields.

1660–1677 CE **Baruch Spinoza** (1632–1677, Netherlands). One of the greatest philosophers of modern times. His philosophical system has come to impregnate the prevailing modern scientific, social and moralistic theories. Spinoza merged in his doctrine the best gems of reason he could extract from Greek philosophy, the Talmud and the Kabbalah, Maimonides and the Christian scholars Hobbes and Descartes.

His philosophy is composed of four elements, subsequently personified by four great Jewish thinkers:

- The need for *piety* (**Leopold Zunz**, 1832)
- The passion for *freedom* and *justice* (**Karl Marx**, 1848)
- The *rational* ordering of all thought (**Sigmund Freud**, 1904)
- The conception of all-embracing *science* of the *universe* (**Albert Einstein**, 1915)

It was mainly the spread and influence of science in its more dogmatic aspects that, toward the end of the 19^{th} century, caused especial interest to be taken in Spinoza's thought. By a sort of instinct Spinoza seems to have anticipated, by deductions from first principles, many of the most fundamental principles of modern science; e.g., the conservation of energy (in his belief that the total quantity of motion in the universe is constant); the non-existence of a vacuum; and the existence of nothing real in the universe but configurations and motions.

Anticipating the methods and fundamental ideas of modern science, he examined the concepts of space, time, causality, free will and natural laws in an holistic attempt to comprehend the entire universe in all its manifestations. Thus, his system advances rational claims for a definite beginning in cosmic time and for a cosmic evolution ["*There was no time or duration before the creation*"; "*We are aware of external things only in relation to each other. All sense experience and all deductions based on them are inadequate*".]

His ideas are present in the writings of Berkeley and Mach, and in Einstein's General Relativity.[101]

Spinoza's teachings also echo in Quantum Mechanics, since he argued that *no system can be understood in isolation*, and we are forced to treat the observer as part of the physical system he is observing. Indeed, according to modern interpretation, both *free will* and *causality* are only rendered meaningful in the presence of external, and thus indeterminate, perturbations. Thus an observer, who himself should ultimately be considered part of the system, must perturb the subsystem that he is focusing on, and measure the result, in order to imbue *causality* with meaning. Likewise, Spinoza would argue that our observer's self-perceived "free will" is an illusion, stemming from his ignorance of external influences acting on *him*.

Only two of Spinoza's writings were published during his life time: *Renati des Cartes Principiorum* (1663), and the *Tractatus Theologico-Politicus* (1670). Three additional works appeared in the year of his death (1677): the *Ethics*, which brought him universal fame in the annals of philosophy; his treatise *On the Improvement of the Understanding*, and his *Political Treatise*.

Baruch Spinoza, or, as he later called himself, Benedict de Spinoza, was born in Amsterdam on the 24^{th} of November, 1632. His forefathers fled from Spain to Portugal in 1492, but in 1498 were forced to convert by the Inquisition (yet remained Jews in spirit). After the establishment of the Union of Utrecht in 1579, his grandfather's family sought refuge in the emancipated Netherlands (1593), and returned there to Jewish orthodoxy. The name, variously written Espinoza, de Spinoza, and Despinoza, probably is derived from the city of *Espinoza de los Monteros* in Leon, not far from the city of Burgos. Baruch's father, Michael de Spinoza, a respectful merchant, was one of the leaders of the Sephardic community of Amsterdam. He married thrice and Baruch was the third of the four children born to him from his second wife (this wife and her sister, which became his third wife, were also from the Espinoza family).

Spinoza was six years old when his mother died (1638), and his father died in 1654. He was trained at the communal school and at the Pereira Yeshivah, over which Manasseh ben Israel and Saul Morteira presided. There he studied Hebrew, Bible, Talmudic literature, and, toward the end of his course, some of the Jewish philosophers: Maimonides, Gersonides, Hisdai

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[101] For further reading, see:

- *Reflections and Maxims* by B.Spinoza (with *Introduction* by Albert Einstein), Philosophical Library, New York, 1965, 92 pp.

- Nadler, S., *Spinoza, a Life*, Cambridge University Press, 2001, 407 pp.

Crescas, Avraham Ibn Ezra (bible commentaries) and other representatives of Jewish medieval thought, who aimed at combining the traditional theology with ideas gotten from Aristotle and his Neoplatonic commentators. The amount of his Kabbalistic knowledge is somewhat doubtful, but both of his teachers were adepts in Kabbala.

During his studies Spinoza had shown early promise of becoming an excellent rabbinic scholar and the Amsterdam Sephardic community had high hopes for him. However, the study of Jewish philosophers of former days led him to turn to the study of philosophy in general. At that time, philosophy was abandoning its interest in theology (which had been its main concern in the Middle Ages), and was turning to the study of natural sciences and the human mind.

Spinoza was attracted by the atmosphere of free thought characteristic of the Dutch Capital. He associated himself with a number of freethinking friends and teachers, both Jews and Christians. It is also likely that his heretical views developed out of heterodox controversies *within* the Amsterdam Jewish community.

Latin, still the universal language of learning, formed no part of Jewish education, and Spinoza, after learning the elements of the language from a German master, resorted to further instruction from Franz van der Ende, an adventurer and polyhistor, under whom he also studied mathematics, physics, mechanics, astronomy, chemistry, and the medicine of the day. The mastery of Latin opened up to him the whole world of modern philosophy and science, both represented at that time by the writings of **Descartes**. Spinoza likewise acquired a knowledge of the scholasticism developed in the school of Thomas Aquinas.

His acquaintance with the works of Descartes (who led Europe in an attempt to establish a philosophy based upon reason, not tradition), accelerated his estrangement from the tradition of the synagogue and finally led to his break with Jewish orthodoxy.

Shortly after leaving the *yeshiva* (Jewish academy), rumors became persistent that young Spinoza had given utterance to heretical views. There was danger in this for the newly established Jewish community, whose enemies might now point out that Judaism was fostering irreligion and disbelief in God.

Desirous to avoid public scandal, the chiefs of the community offered him a yearly pension if he would outwardly conform and appear now and then in the synagogue. His refusal put him on a direct collision course with the congregation, and on the 27^{th} of July 1656 Spinoza was solemnly cut off from the commonwealth of Israel. While negotiations were still pending, he had been set upon one evening by a fanatical ruffian, who thought to expedite matters with the dagger.

Spinoza was thus cast out at the age of 23 from all communion with men of his own faith and race, and there is no evidence of his coming into communication with a single Jewish soul from that time to his death.

Spinoza, however, did not mind. He was an individualist who could find no place in *any* organized religion. In the free environment of Holland he could live peacefully without being a member of any religious group. Socially, he was not alone: he had already formed a circle of friends and disciples, mainly of the *Mennonite* sect known as *Collegiants*, whose doctrines were similar to those of the Quakers; and he attended a philosophical club, with membership drawn mainly from this sect.

During 1656–1661 Spinoza took his abode with a Collegiant friend near Amsterdam, and started his research in optics through the grinding and polishing of lenses for the newly invented microscope and telescope (in which his mathematical knowledge was valuable). He also took pupils in philosophy, Latin and Hebrew.

The five years which followed the excommunication, were devoted to concentrated thought and study. Before their conclusion Spinoza had parted company from Descartes, and the main tenets of his own system were already clearly determined in his mind. He wrote what was later extended into the "*Tractatus Theologico-Politicus*", and a short tractate on "*God, Man and his Well-Being*" (afterwards developed into his *Ethics*).

In 1661 Spinoza removed to Rhijnsburg, near Leyden, then the center of the Collegiants activity. Here he spent the two most fruitful years of his life. In 1663 he removed to Voorburg (a suburb of The Hague), to be near the de Witt brothers[102], then at the height of their power. From Voorburg Spinoza used to send portions of his *Ethics*, written in Dutch, to his band of disciples in Amsterdam, who translated them into Latin. The "*Tractatus Theologico-Politicus*" was published in 1670, without the author's name, and it brought such a storm of opprobrium that it was formally proscribed by the Synod of Dort and by the States General of Holland, Zealand and West Friesland.

Spinoza's reputation as a thinker, however, had by this time been fully established by his two published works, and he was consulted both in person and by letter by many important scientific men of the day, including **Oldenburg**

[102] Spinoza's income was supplemented by a small pension given to him by **John de Witt**. This arrangement assured his independence, and left him sufficient time to pursue his philosophical writings and correspond with the leading scientists of the day.

(secretary of the London Royal Society), **Huygens**[103], **von Tschirnhausen** and **Leibniz**[104]. In 1670 Spinoza settled at the Hague itself where he spent the remaining years of his life in the state of frugal independence which he prized. In 1673 he received an invitation to become a professor of philosophy in the University of Heidelberg, but he declined because it required from him "*that he will not misuse his freedom of speech to disturb the established religion*".

Early in 1677 Spinoza became seriously ill. He had a hereditary tendency to consumption derived from his mother, and this was aggravated by the inhalation of particles of crystal incidental to his work as lens grinder. He died on the 21^{th} of February 1677, with his friend Dr. Meyer as the only witness of his last moments. He was little more than 44 years of age.

Many mourned him; for the simple folk had loved him as much for his gentleness as the learned had honored him for his wisdom. Philosophers and magistrates joined the people in following him to his final rest, and men of varied faiths met at his grave[105]. In 1678, the Dutch government confiscated

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[103] It was as an optician that he first came into contact with Huygens. In fact, Huygens and his brother tried to spy on Spinoza's own techniques of grinding lenses. An optical "*Treatise on the Rainbow*" written by Spinoza was discovered in 1862.

[104] Those of Leibniz's works that have been published give little evidence of any connection with Spinoza other than in the latter's calling as optician, and his public utterances on Spinozism were in every case hostile and derogatory; but more recent evidence shows that during the critical period of his development, from 1676 to 1686, he took a more favorable attitude toward both Spinoza and Spinozism, and this has been traced to an intimate personal association of the two philosophers during a whole month in 1676, not long before Spinoza's death. It was during this period that Leibniz developed from a pure Cartesian into an opponent of Descartes, chiefly as regards the definition of body and the principles of motion; it is known that Leibniz discussed both subjects with Spinoza. When, however, a strong outcry broke out against Spinoza's "atheism", Leibniz devoted himself to finding an escape from Spinozism, and it took him nearly ten years before he arrived at his theory of the monads, which he declared to be the only solution of the difficulty. Bertrand Russell's analysis of the philosophy of Leibniz proved that in his views on soul and body, on God and ethics, he "*tends with slight alterations of phraseology to adopt* (*without acknowledgment*) *the views of the derided Spinoza*".

[105] He was buried in the yard of the New Church, the Hague, in a grave that his friends rented for 20 years. In 1697, upon the termination of the grave-contract, his remains were removed to an unknown location. In 1953, a tombstone was erected, in that yard, by **David Ben-Gurion**, then Israel's prime minister. The

all of Spinoza's writings and his name became a symbol of heresy.

Spinoza lived in the Netherlands at the time when scientific discovery, religious division, and profound political changes has revolutionized the nature and application of philosophy. Philosophy, for Spinoza, was not a weapon, but a way of life – the adoption of truth as one's master and one's goal. But every orders requires a sacrifice, and that demanded by philosophy is neither easily undertaken nor readily understood by those who refuse it. To the mass of mankind, therefore, the philosopher may appear as a spiritual saboteur and a subverter of things lawfully established. So Spinoza appeared to his contemporaries, and for many years after his death he was regarded as the greatest heretic of the 17^{th} century.

Indeed, for more than a century after Spinoza's works were published, their author was bitterly denounced by Catholics, Jews, Protestants and free-thinkers alike. Even **David Hume**, in general a man of kindly disposition, branded him as 'infame', and **Moses Mendelssohn**, the affable advocate of tolerance, was scandalized when he heard that his friend **Lessing** had adopted Spinoza's doctrine[106].

However, a radical change took place in Germany and England: Spinoza was rediscovered by of all people, the poets! This was possibly due to the influence of **Johann Gottfried von Herder** (1744–1803) and **Goethe** (1749–1832), who had both given utterance to great admiration for Spinoza's life and thought. The wide influence of Goethe, whose philosophical views were entirely Spinozian and were expressed in some of the profoundest of his poems, was perhaps the chief influence which drew to Spinoza the attention of such men as **Samuel Taylor Coleridge** (1772–1834), **Matthew Arnold** (1822–1888) and **Joseph Ernest Renan** (1823–1892).

Post-Kantian philosophers and romantic poets in Germany were deeply influenced by Spinoza's conception of nature. In modern times, Spinoza is universally recognized as a philosopher of unsurpassed sublimity and profundity. Even his critics agree that Spinoza had a most lovable personality, one

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stone is adorned by the inscription of a *rose surrounded by thorns*. Two words are inscribed on it too: the Hebrew word 'AMCHA' (meaning: the common folk) and the Latin word *caute* (beware), which was engraved on Spinoza's ring. The philosopher himself, however, does not rest under that stone.

106 Mendelssohn may have realized that Spinoza had first shown how a critique of Judaism could be used to reach radical conclusions about the world. His example has been indeed followed by the French enlightenment, though their treatment of Judaism was far more hostile, and racial, in tone. Two centuries later, the personal anti-semitism of **Karl Marx** would play a similar role in his socio-economical theory

of the purest characters in the history of mankind. His delicate feelings, his benevolence and fondness of plain people never hampered the boldness of his thoughts and the sternness of his will to draw conclusions logically and without any deference to personal inclinations. Philosophical thinking was, to Spinoza, self-education and improvement of the mind of the thinker. His aim was to obtain, by means of reason and science, the same trust in the rules of human behavior that religious traditions claimed to grant their believers.

Spinoza affirmed that God does not exist in the way religion preaches, only as an impersonal and spiritual 'principle', as a substance which constitutes the *reality* of the universe. Nothing exists save the one substance – the self-contained, self-sustaining, and self-explanatory system which constitutes the world. This system may be understood in many ways: as God or Nature; as mind or matter; as creator or created; as eternal or temporal. And to understand it in its totality, is also to know that everything in the world exists by necessity, and that it could not be other than it is[107]. A single stuff, obedient to a single set of laws gives rise to all that we observe. The task of science is to provide the complete description of that substance and the laws which apply to it.

Contrary to Descartes, he denied the possibility of harmonizing reason with Biblical revelation, and in that, Spinoza, not Descartes, became the symbol of the end of medieval philosophy. The *scientific method* offered to Spinoza not only the measure of moral evaluation but a means of gaining

[107] The God of Spinoza is not a *personal* deity with whom man can communicate. In this sense man is *devoid of free will*, unable to change the course of his life. The deity of the Bible activates Nature and as such is *beyond* Nature. God of the Bible negotiates with man (Abraham, Moses, Job, the prophets) and sometimes changes his will in accordance with man's actions.

The conflict between the teachings of Spinoza and Judaism is that of *monotheism* and *pantheism*. i.e. the idea of *oneness* against that of *unity*: Judaism is based upon the notion that the creator (God) is above both nature and man, each authority being subjected to its own set of laws which 'he' made. But in contradiction to inanimate nature, man has a free will.

According to Spinoza, creator and creation are one and the same entity and cannot be separated; there is but one authority and one set of laws. Man is an integral part of nature with no privileges. Man is not a transcendence of nature but an immanence of it, subjected to nature's laws with no free choice whatsoever. Furthermore, while Judaism believes that God is purposeful and created man without the ability to comprehend his purposefulness, Spinoza maintains that God = nature is totally unpurposeful. Hence man is unable to comprehend the purpose of nature not because he is not capable of *knowing* it, but because it *did not exist* ab initio.

eternal bliss. To win supreme happiness or ‘unceasing joy’, Spinoza said, man has to attain knowledge of his union with the whole of nature.

Worldview X: Baruch Spinoza

* * *

Sed omnia praeclara tam difficilia quam rara sunt.
All excellent things are as difficult as they are rare.

* * *

Nothing in Nature is random... A thing appears random only through the incompleteness of our knowledge.

* * *

He who loves God cannot endeavor that God should love him in return.

* * *

In the language of philosophy, it cannot be said that God desires anything of any man, or that anything is displeasing or pleasing him: all those are human qualities and have no place in God.

* * *

The bees, in all their work and the orderly discipline, which they maintain among themselves, have no other end in view than to make certain provisions for themselves for the winter, still, man who is above them, has an entirely different end in view when he maintains and tends them, namely to obtain honey for himself. So also [is it with] man, in so far as he is an individual thing and looks no further than his finite character can reach; but, in so far as he is also a part and tool of the whole of Nature*, because she is infinite, and must make use of him, together also with all other things, as an instrument.*

* * *

Whenever anything in nature seems to us ridiculous, absurd, or evil, it is because we have but a partial knowledge of things, and are in the main ignorant of the order and coherence of nature as a whole, and because we want everything to be arranged according to the dictate of our own reason; although, in fact, what our reason pronounces bad, is not bad as regards the order and laws of universal nature, but only as regards the laws of our own nature taken separately.

* * *

Whatsoever is contrary to nature is also contrary to reason, and whatsoever is contrary to reason is absurd, and, ipso facto, to be rejected.

* * *

Be not astonished at new ideas; for it is very well known to you that a thing does not therefore cease to be true because it is not accepted by many.

* * *

The highest endeavor of the mind, and the highest virtue is to understand things by the intuitive kind of knowledge.

* * *

All bodies are surrounded by others, and are mutually determined to exist and operate in a fixed and definite proportion, while the relations between motion and rest in the sum total of them, that is, in the whole universe, remain unchanged. Hence it follows that each body, in so far as it exists as modified in a particular manner, must be considered as a part of the whole universe, as agreeing with the whole, and associated with the remaining parts.

* * *

If the foundations of their religion have not deserted their minds they may even, if occasion offers, *so changeable are human affairs, raise up their empire afresh, and that God may a second time elect them.*

* * *

Do not weep; do not wax indignant. Understand.

* * *

On Spinoza

* * *

"There is no other philosophy then that of Spinoza"

Ephraim Gotthold Lessing (1729–1781)

* * *

"I feel a deep spiritual affinity between me and Spinoza, albeit his soul is more profound than mine. His doctrine inspires tranquility and calm; it brings the tranquility of God or the tranquility of nature upon me. Yet, I do not dare to claim a thorough apprehension of the ideas of one who ascended to the pinnacle of reason".

Johann Wolfgang von Goethe (1749–1832)

* * *

"To be a philosopher one must first be a Spinozist".

Georg Wilhelm Friedrich Hegel (1770–1831)

* * *

"Whose dwelling is the light of setting suns,
And the round ocean, and the living air,
And the blue sky, and in the mind of man —
A motion and a spirit, which impels

All thinking things, all objects of all thought,
And rolls through all things".

William Wordsworth (1770–1850)

* * *

"I can hardly imagine how one can be a poet, and yet not admire Spinoza".

Karl Wilhelm Friedrich von Schlegel (1772–1829)

* * *

"All the contemporary philosophers, perhaps unknowingly, observe the world through lenses grinded by Baruch Spinoza".

"When we read Spinoza, we have the feeling that we are looking at all-powerful Nature in liveliest repose — a forest of thoughts, high as heaven, with green tops ever in motion — while below the immovable trunks are deeply rooted in the eternal earth. It may be that the spirit of the Hebrew prophets hovered over their remote descendant.

Benedict Spinoza teaches that there is but one substance, God. This one substance is infinite and absolute. All finite substances are derived from it, are contained in it, emerge from it or sink into it; they have only relative, transitory, accidental existence. Absolute substance manifests itself to us in the form of infinite thought as well as infinite extension. We know only these two attributes. But God, absolute substance, may posses other attributes which we do not know.

Only stupidity and malice could term this doctrine 'atheism'. No one has ever expressed himself in more sublime terms regarding the Deity. Instead of saying that he denies God, we should rather say that he denies Man. All finite things are contained in God; the human intellect is only a ray of infinite thought; the human body only an atom of infinite extension; God is the infinite cause of souls and bodies — natura naturans*".*

Heinrich Heine (1797–1856)

* * *

"In Spinoza is contained the fullness of modern science".

Ernest Belfort Bax (1854–1926)

* * *

"No modern writer is altogether a philosopher in my eyes, except Spinoza".

George Santayana (1863–1952)

* * *

"The system of Spinoza remains one of the outstanding monuments of Western philosophy. Though the severity of its tone has a certain Old Testament flavor, it is one of the great attempts, in the grand manner of the Greeks, to present the world as an intelligible whole".

Bertrand Russell (1872–1970)

* * *

"I believe in Spinoza's God who reveals himself in the orderly harmony of all that exists, not in a God who concerns himself with the fate and actions of men".

"I would not think that philosophy and reason itself will be man's guide in the foreseeable future; however, they will remain the most beautiful sanctuary they have always been for the select".

Albert Einstein

* * *

"This man, from his granite pedestal, will point out to all men the way of blessedness which he found; and ages hence, the cultivated traveler, passing

by this spot, will say in his heart: 'The truest vision ever had of God came perhaps, here'".

Ernest Renan, at the unveiling of Spinoza's statue, The Hague (1882)

* * *

"Original ideas are exceedingly rare and the most that philosophers have done in the course of time is to erect a new combination of them".

George Alfred Léon Sarton (1884–1956)

* * *

1660 CE End of the *English Civil War*(also known as the *Puritan Revolution*, 1642–1646) and the beginning of the *Restoration* (1660). The biggest revolution in English history; the last and grandest episode in Europe's age of religious wars. It was mainly a contest over religious and political principles — Puritanism versus Anglicanism and parliamentary self-government versus royal absolutism.

After Cromwell beheaded Charles I (Jan. 30, 1649), England was a *Puritan republic* (1649–1660); Cromwell conquered Ireland (1649) and Scotland (1651) and compelled them to accept union with England, welding Great Britain into a single political unit. During the reign of Cromwell and the Puritans, Anglicans and Catholics were persecuted, but after Cromwell's death (1658) the Puritan Republic collapsed and Charles II was invited to return (1660).

With the restoration of the Stuart monarchy, the Church of England became again the state Church and the persecution of the Puritans was resumed: Puritans were not allowed into government, army, teaching positions, or parliament, which made it difficult for many of them to get a job. The only two places open for them were trade and industry, where most of them did indeed go. As it turned out, all of the great scientists, technologists and innovators of the next hundred years in England were Church of England rejects! (e.g. **James Watt**). Thus, *religious intolerance* kicked off the *industrial revolution* and brought the chemistry it needed.

Marranos, Huguenots and Puritans — three versions of the same history.

1660–1679 CE **Marcello Malpighi** (1628–1694, Italy). Physiologist and anatomist. Founder of microscopic anatomy; the first to apply the microscope to the study of animal and vegetable structure. Malpighi was born near Bologna. He studied medicine there (1649–1653). Professor of medicine at Pisa (1656–1659), at Messina (1662-1666) and again at Bologna (1666–1691). He then moved to Rome to become the private physician to Pope Innocent 12^{th} (1691).

Malpighi studied structure of secreting glands; discovered capillary circulation in the lung of the frog (1660); the deeper portion of the epidermis is known as the *Malpighi layer*; loops of capillaries in the kidney are known as *Malpighi tufts*; masses of adenoid tissue in the spleen are called *Malpighi corpuscles*. He described structure of human lung, development of the chick (1673), structure of the brain and spinal cord, and the metamorphosis of the silkworm (1669). He published *Anatome plantarium* (1675–1679).

1662 CE **John Graunt** (1620–1674, England). Published a book, "*Natural and Political Observations made upon the Bills of Mortality*", in which he laid the foundation to the science of statistics. 43 years later, the first successful life insurance company was established in England.

1662 CE **Lorenzo Bellini** (1643–1704, Italy). Physician and anatomist. Discovered the complex of tubules composing the *kidney* (*Bellini's tubules*) and described the mechanical theory of excretion (1662). Investigated the *sense of taste*.

1662–1687 CE **William Petty** (1623–1687, England). Physician, political economist and statistician. A founder of the Royal Society and a pioneer in the field of *vital statistics*[108]

His "*Treatise of Taxes and Contributions*" contains the first clear statement of the doctrine that price depends on the labor necessary for production. Petty was a professor of anatomy at Oxford (1651).

Evolution of the Calculus[109]

It took over 2500 *years for the calculus to progress from the early notions on the subject to the form we study today. For most of this period the concepts of differential and integral calculus were considered distinct.*

It was not until the latter part of the 17^{th} *century that mathematicians, led by* **Isaac Newton** *in England and* **Gottfried Wilhelm von Leibniz** *in Europe, discovered the connection between these fundamental ideas.*

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[108] Records of the most basic human events – birth, marriage, divorce, sickness and death. This data is essential for legislators, health authorities, sociologists, school administrators, insurance statisticians and market researchers. State bureaus of vital statistics and state health departments maintain files of vital records and compile statistics.

[109] *Calculus* means "pebble" in Latin (hence the word *calculation*). Indeed, in the civilizations of Egypt and the Asian river valleys, numbers were represented by means of pebbles arranged in heaps of ten. This in turn led to the development of the *abacus*, or counting frame in which a number is represented by pebbles put in grooves.

Over the past 300 years, calculus has been put on a firm mathematical foundation and refined to the point that it now follows logically from a few basic notions and principles.

The underlying concept of integral calculus was used by Greek mathematicians at least as early as the time of ***Antiphon the Sophist*** *(fl. 450 BCE),* ***Eudoxos of Cnidos*** *(408–355 BCE) and* ***Archimedes of Syracuse*** *(287–212 BCE), employing the so-called* method of exhaustion *to approximate areas and volumes. Archimedes determined the area of a circle by computing the area of the inscribed and circumscribed polygons of increasing numbers of sides. In this and similar applications he adumbrated the concept behind the Riemann integral. Archimedes also derived laws for determining the tangent lines to certain curves, including parabolas and the curve that bears his name, the 'spiral of Archimedes'. In some sense, Archimedes could be considered the founder of calculus. He did not, however, have a notion of a unified theory that could be applied to more than a few specific cases, nor did he recognize a connection between the differential and integral concepts of calculus*[110].

Little progress was made toward the discovery of the unified theory of calculus until the beginning of the 17^{th} *century. Then, in the course of a mere 64 years, a formidable group of precursors, pioneers, inventors and co-inventors succeeded in creating the basic mathematical framework of the calculus familiar to us today (****Kepler****, 1615;* ***Galileo****, 1619;* ***Fermat****, 1629;* ***Roberval****, 1634;* ***Cavalieri****, 1635;* ***Wallis****, 1656;* ***Barrow****, 1669;* ***Newton****, 1671;* ***Leibniz****, 1673;* ***Huygens****, 1679, 1695).*

The first method of differentiation was introduced by ***Fermat*** *(1629) based on an earlier idea of* ***Kepler*** *(1615).*

Kepler had observed that the increment of a function becomes vanishingly small in the neighborhood of an ordinary maximum or minimum value. Fermat translated this fact into a process for determining such extrema. His method is equivalent to setting

$$\lim_{h\to 0}\frac{1}{h}[f(x+h)-f(x)]=0,$$

i.e setting the derivative of $f(x)$ *equal to zero. Fermat also devised a general procedure for finding the tangent at a point of a curve whose Cartesian equation is given.*

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110 In beginners college courses, it is customary to begin with *differentiation* and later consider *integration*. Historically, however, the idea of integration, in connection with finding certain areas and volumes, was created earlier then that of differentiation, which was associated with problems of tangents to curves and with the question of finding maxima and minima of functions.

Fermat, however, did not *know that the vanishing of the derivative of* $f(x)$ *is only a necessary, but not a sufficient, condition for an ordinary extremum. He also did* not *distinguish between a maximum and a minimum value.*

It was again ***Kepler*** *(1615) who applied crude integration procedures to evaluation of areas which he needed in connection with his Second Law of planetary motion and volumes of solids of revolution.*

Thus, Kepler regarded the circumference of a circle as a regular polygon with an infinite number of sides. If each of these sides is taken as a base of an isosceles triangle whose vertex is at the center of the circle, then the area of the circle is divided into an infinite number of thin triangles, all having an altitude equal to the radius if the circle. Since the area of each thin triangle is equal to half the product of its base (Δl) *and the radius* (r)*, and since* $\sum(\Delta l) = 2\pi r$*, the total area is* πr^2*. Similarly, the volume of a sphere can be imagined to consist of infinitude of small cones, each of volume* $\frac{1}{3}r(\Delta s)$ *where* $\sum(\Delta s) = 4\pi r^2$*.*

Cavalieri *(1635) was influenced by this work by Kepler when he carried the refinement of the infinitesimal calculus a stage further in his method of* indivisibles*.*

The problem of constructing tangents to curves was also taken up by ***Roberval*** *(1634),* ***Descartes*** *(1637) and* ***Barrow*** *(1669). With such active research, it was only a matter of time until the discovery of the notion that differentiation and integration are inverse operations.*

The first published statement concerning this Fundamental Theorem of Calculus appears in Lectiones geometricae*, a treatise published by Barrow in* 1670*. The theorem, however, is believed to have been recognized intuitively by* ***Galileo*** 50 *years earlier in connection with his study of motion.*

This brings us to the time of ***Newton****, a young student at Cambridge in the* 1660*'s, and to* ***Leibniz****, who was born in Leipzig and was self-trained in mathematics. These two men systematically unified and codified the known results of calculus, giving, in essence, algorithmic procedures for the use of these results. Each gave a proof of the Fundamental Theorem of Calculus and each clearly demonstrated the importance of this new theory.*

Newton developed most of his calculus, called the "method of fluxions and fluents", during a period 1664 through 1671, and compiled his results in the tract De analysi per equationes numero terminorum infinitas *in* 1669*. Although this manuscript was circulated and studied by a number of his English contemporaries, it did not appear in print until* 1711*, over* 40 *years later. In fact, Newton probably used his calculus to develop many important* *discoveries regarding gravitation and the motion of objects*[111]*, but his treatise on this subject, Philosophiae naturalis principia mathematica (1687), contains only classical geometric demonstrations.*

It is difficult to determine precisely when Leibniz first became interested in the calculus, but it was probably shortly before he traveled to France and England in 1673 *as a political envoy. While visiting the London home of* ***John Collins*** *(1625–1683), he saw Newton's* 1669 *tract. He probably did not have a sufficient mathematical background to follow Newton's arguments at this time, but he was nevertheless excited by the result, particularly those dealing with series. After studying Descartes' fundamental work La géométrie, he communicated with Newton regarding the discoveries the latter had made. The two exchanged several letters during* 1676–1677*, by which time Leibniz had developed his own theory of calculus. The letters generally describe the extent of their work, but often omit crucial details necessary for the methods of discovery.*

Leibniz understandably expected that Newton would soon publish a treatise on calculus. When it became obvious that this work was not forthcoming, Leibniz began in 1682 *to publish his own discoveries in a series of papers in the Acta eruditorum, a journal published in Berlin with a wide circulation in Europe. In* 1684 *Leibniz published the first work on differential calculus and in* 1686*, the first on integral calculus. His articles are often vague and sketchy and were never collected in a definitive treatise.*

Because of Leibniz's prior publication, his calculus became the version known to the mathematical public of the time, particularly the European scientific community. We use his differential notation, dy/dx*, for differentiation and his elongated S symbol,* $\int$*, to represent integration. He called his integral calculus* calculus summatorius*; the term* integral *was introduced by* ***Jakob Bernoulli*** *in* 1690*. Newton's notation was generally more cumbersome, although his symbol* $\dot{y}$ *to denote the derivative of* y *is still commonly used to indicate differentiation with respect to time.*

Many reasons have been suggested for Newton's failure to capitalize on his discovery of calculus: his reticence, his preoccupation with other research, and his lack of interest in publishing. Certainly he had a complex personality

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[111] The Newtonian calculus enabled mathematicians and physicists, for the first time, to solve more complex problems of motion, which up to his time seemed insoluble. This modern branch of mathematics, having achieved the art of dealing with infinitely small entities (infinitesimals), was unknown to the ancients. It corrected the inevitable errors which the human mind cannot avoid (such as Zenon's paradoxes) when dealing with *discrete* elements of motion instead of *continuous* motion.

and a sensitivity to criticism. Nevertheless, spurred on by their friends and colleagues, Newton and Leibniz were locked in a bitter controversy for nearly 20 *years over who deserved the credit for discovering the differential calculus. This is one of the saddest chapters in the history of mathematics: Leibniz complained that Newton's attitude was the malicious interpretation of a man who was looking for a quarrel, while Newton said that second inventors count for nothing! Of course, there was more than enough honor to go around, and the effect of the quarrel has been only to tarnish the images of both these mathematical giants.*

It should be kept in mind that neither Newton nor Leibniz established their results with anything resembling modern mathematical rigor. An example is the limit concept, so basic to the study of both the differential and the integral calculus. Although this concept is intuitively clear, its definition is quite sophisticated. It was not until 1870 *that* **Eduard Heine** *(1821–1881, Germany) published the definition for the limit of a function that we use today. Heine's work was strongly influenced by that of* **Karl Weierstrass** *(1815–1897, Germany), who was one of the leaders in the movement to place function theory on a firm and rigorous basis.*

Another calculus timeline involves the calculation of arc-lengths and area of surfaces: mathematicians of the early 18^{th} *century became interested in the problem of finding paths of shortest length on a surface using the methods of the calculus. The brilliant and prolific mathematician* **Leonhard Euler** *(1707–1783) presented the first fundamental work on the theory of surfaces in* 1760 *with "Recherches sur la courbure des surfaces", and it may have been in this work that a surface was first defined as a thee-dimensional graph* $z = f(x, y)$. *In* 1771 *Euler introduced the notion of parametric representation of surfaces.*

After the rapid development of calculus in the earth 18^{th} *century, formulas for the lengths of curves and areas of surfaces were developed. The underlying concepts of the length of a curve and the area of a surface were understood intuitively before this time, and the use of the formulas from calculus to compute areas were considered a great achievement.*

The subject of the calculus has played a special role in the history of modern science: Most of physics and engineering, and important parts of astronomy, chemistry and biology, would be impossible without it.[112]

[112] We list below a number of excellent calculus textbooks for self-study, on a number of levels:

- Kline, Morris, *Calculus – An Intuitive and Physical Approach*, Dover: New York, 1998, 943 pp.

- Zeldovich, Ya.B., *Higher Mathematics for Beginners*, Mir Publications:

1662–1677 CE **Isaac Barrow** (1630–1677, England). Versatile scholar, classicist and mathematician. Developed a method of determining tangents that closely approach the methods of calculus. Prolific writer on theology, mathematics and poetry. Translated Euclid, Archimedes and Apollonios into English and Latin. His book *Lectiones Geometricae* (1669) contains the foundations of the calculus in geometrical form. It presents, for the first time, differentiation and integration as inverse processes, integration as a summation, and nomenclature and methods which were direct forerunners of the algorithmic procedures of the calculus. His presentation of the differential triangle, clearly indicates *the mutual influence of Barrow and Newton upon each other*!

Barrow's influence upon **Leibniz**, too, may be inferred from the fact that Leibniz is known to have purchased a copy of Barrow's *Lectiones Geometricae* in 1673.

Barrow was born in London. He entered Trinity College, Cambridge in 1644 and received his B.A. degree in 1648. The next four years were spent in travel, at times highly adventurous, over Eastern Europe. He returned to England in 1659. In 1662 he was elected professor of geometry in Gresham College and in 1663, he became the first Lucasian professor of mathematics at Cambridge. In 1669 he resigned this chair to his great pupil and friend Isaac Newton. In 1675 he was chosen vice-chancellor of the university. He died suddenly of a fever and was interred in Westminster Abbey.

1663–1665 CE First scientific newspapers:

- *Erbauliche Monaths Unterredungen* ('Monthly edifying discussions'). Issued (1663) in Germany.

Moscow, 1972, 494 pp.

- Granville, W.A. et al., *Elements of the Differential and Integral Calculus*, Gin and Company, 1941, 556 pp.
- Piskunov, N., *Differential and Integral Calculus*, P.Noordhoff: Groningen, 1962, 895 pp.
- Fikhtengol'ts, G.M., *The Fundamentals of Mathematical Analysis*, 2 Volumes, Pergamon Press: Oxford, 1965, I, 491 pp; II, 516 pp.
- Smirnov, V.I., *A Course of Higher Mathematics*, Addison-Wesley, 1964, Vols I+II (543 pp.+630 pp.)
- Khinchin, A., *A Course of Mathematical Analysis*, Hindustan Publishing Corporation: Delhi, India, 1960, 668 pp.

- *Journal des Savants* ('Scientists newspaper'). Issued (1665) in France, by **Denis de Sallo**.

1663–1671 CE **James Gregory** (1638–1675, Scotland). Mathematician and astronomer. One of the first to distinguish between convergent and divergent series. Expanded (1667) the infinite series $\tan^{-1} x$, $\tan x$ and $\sec^{-1} x$ and showed in 1671 that

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$$

yields for $x = 1$,

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots .$$

The $\tan^{-1}$ series was used in 1699, with $x = \sqrt{\frac{1}{3}}$, to evaluate π to 71 correct decimal places. In 1671, Gregory preceded **Brook Taylor** (1712) in series expansion of a function about a point.

Gregory, a Reverend's son, was born and educated at Aberdeen. In 1665 he went to the University of Padua where he studied for some years. In 1674 he became a professor of mathematics at the University of Edinburgh.

Many members of the Gregory family attained eminence in various departments of science. During 1650–1850, *fourteen* of them held professorships in mathematics or medicine.

The Reflecting Telescope

Gregory, a Reverands son, was born at Aberdeen. In 1665 he went to the University of Padua, Italy, where he studied for some years. He became a professor of mathematics at the University of St. Andrews(1669) but left that school in 1674 (his salary was not paid!) and was appointed a professor at the University of Edinburgh. In Oct. 1675, while showing the satellites of the *planet Jupiter to some of his students through one of his telescopes, he was suddenly struck with blindness, and died of the stroke a few days afterwards.*

In 1663 Gregory published his famous treatise Optica promota in which he made known his invention (1661) of the Gregorian reflecting telescope.

In the Gregorian arrangement, a concave ellipsoidal secondary mirror *reinverts the image, returning the beam through a hole in the primary image. It was first successfully constructed (with some modification) by* ***Newton*** *(1668), and only became important research tool in the hands of* ***Frederick William Herschel*** *a century later.*

The Gregorian system was incorporated into the design of 1997 Arecibo reflecting radio telescope, *having a spherical mirror with a diameter of 10 meters.*

Gregory also discovered a diffraction grating *(using a birds 'feather'). In a letter to a friend dated May 12, 1673 Gregory pointed out that sunlight passing through a feather would produce a colored pattern and he asked that his observations be conveyed to Mr. Newton .*

Cassegrain *(1625–1650, France), physician and inventor, improved the Gregory–Newton reflecting telescope by utilizing a* convex hyperboloidal secondary mirror *to further increase the angular magnification. A century later it was noted by* ***Ramsden*** *(1735–1800) that this system also partly eliminated* spherical aberration.

The system gives an inverted image of any distant object and is superior to Gregory's in two points: first, the spherical aberrations of the two mirrors tend to correct instead of reinforcing each other, thus promoting good definition of the image; secondly, the necessary radius of aperture of the convex mirror is less, so that the proportion of light stopped is less in this instrument than in Gregory's telescope.

Cassegrain's system of mirrors is used today within many modern reflecting and large refraction telescopes. Nothing is known for certain about Cassegrain's life – not even his first mane. Believed to have been a professor at the College Chartres.

1664 CE, Mar. 06 Appearance of the first issue of the *Philosophical Transactions of the Royal Society*. By 1750, 46 volumes had been published[113].

In 1887 the Phil. Trans. was divided into two series, labeled A and B respectively, the former containing papers of a mathematical or physical character, and the latter papers of a biological character.

In 1832 appeared the first volume of *Abstract of papers printed in the Phil. Trans. from the year* 1800. This publication developed in the course of a few years into a *Proceedings of the Royal Society*.

1664–1672 CE **Thomas Willis** (1621–1676, English). Physician, anatomist and physiologist. Through his studies of the anatomy of the central nervous system and the circulation of the blood he extended the concepts proposed by the Roman physician **Galen**. In his *Cerebri Anatome,* (1664) the most complete and accurate account of the nervous system to that time, he rendered the first description of the hexagonal continuity of arteries (the "circle of Willis") located at the base of the brain and ensuring that organ a maximum blood supply, and of the 11^{th} cranial nerve (spinal accessory nerve) responsible for the motor stimulation of major neck muscles. Willis attempted to correlate the knowledge of anatomy, physiology and biochemistry with chemical findings in neuropathology.

He was a member of the iatrochemistry school, which believed that chemistry was the basis of human function, rather than mechanics, as was the main belief of the time.

Willis was born in Great Bedwyn, Wiltshire. An Oxford professor of natural philosophy (1660–1675). Opened a London practice that became the most profitable and fashionable of the period. Died in London.

1664 CE The Great Plague in London. Ca 100,000 perished.

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[113] 15 million scientific papers were written since modern science began. It was written by 4 million authors, most of them are alive today. Papers are being published *now* (2000) at a rate of one million per year in 40,000 journals. The mean 'life' of most of the journals is 25 years. But half of the reading is confined to only 200 journals.

The number of scientific journals $N(t)$, has doubled every 15 years for the past 200 years. Approximately $N(t) = 4e^{\lambda t}$, where $\lambda \approx 0.0461$, and $t = 0$ corresponds to the year 1794.

1665–1687 CE **Isaac Newton** (1642–1727, England). Physicist, mathematician and astronomer. One of the greatest names in the history of human thought. Discovered the calculus, established the fundamental laws of mechanics and stated the universal law of gravitational attraction, unifying terrestrial and celestial mechanics.

In 1664 Newton began to work on his "Calculus of fluxions", the principles and methods of which were developed by him in three tracts entitled: *De analysi per aequationes numero terminorum infinitas* (1666); *Methodus fluxionum et serierum infinitarum* (1671); and *De quadratura curvarum* (1676). None of these was published until long after they were written (printed 1711, 1736, 1704, respectively).

The infinitesimal calculus was 'almost' discovered by **Fermat** (1629) and **Isaac Barrow** (1630–1677, England). Newton was Barrow's pupil, and he knew to start with, in 1664, all that Barrow knew and that was practically all that was known about the subject at that time.

The discovery of the infinitesimal calculus seems to consist of three parts:

(1) The recognition that differentiation, known to be a useful process, could always be performed, at least for the functions then known. Thus, the problem of tangents could be solved once and for all.

(2) The recognition that the operation of integration is the inverse of differentiation and could be rendered systematic.

(3) The introduction of a suitable notation through which the discovery could be rendered accessible to mathematicians in general.

During the years 1664–1666 Newton started to wonder whether the earth's gravity could account for the motion of the moon and whether the sun's gravitation could account for Kepler's laws. On the second issue, when Kepler's third law is substituted into the expression for the centripetal acceleration of a planet in its orbit about the sun, there results the dependence of the centripetal acceleration on the inverse square average distance from the sun[114]. When he used this law for the earth-moon system, the moon's acceleration

[114] It is possible that this was independently deduced by **R. Hooke**, **C. Wren** and **E. Halley** working together in 1679, using **Huygens**' 1673 law of centripetal force, and Kepler's third law. The idea of inverse-square-law was "*in the air*" when Newton made his calculations. Other scientists were speculating on a cause for Kepler's laws and asking whether planetary motions could be explained by an attraction spreading from the sun. Newton rescued the question from mere speculation and extended the guess to universal gravitation.

toward the earth was found to be equal to $g\left(\frac{r}{R}\right)^2 \simeq \frac{g}{3600}$, where g is the acceleration of gravity on the earth's surface, r the earth's radius, and R the earth-moon distance[115].

Newton then calculated the moon's centripetal acceleration $\frac{v^2}{R}$ in its orbit around the earth. Finding that it was actually equal to $\frac{1}{3600}$ of the value of gravity on earth, he knew that universal gravitation could indeed supply the force needed to maintain the moon in its orbit around the earth. All these preliminary calculations were done by Newton without invoking his own calculus. Later in 1687, however, he used the calculus to justify the assumption that the earth and the moon can be treated as point masses located at their respective centers[116].

During 1665–1672, Newton laid the foundation for the science of optical spectrum analysis. He passed a beam of light through a glass prism and studied the resulting separation of sunlight into its various color components. He was then led in 1668 to construct the first reflecting telescope, in which a reflecting mirror is used instead of a system of lenses to avoid chromatic aberration. Newton's telescope was 15 cm long with a magnification of 38. Through it he saw the satellites of Jupiter.

He believed that light behaves as if it were a stream of tiny particles, such that red light was composed of the largest particles and violet of the smallest. He then showed that Snell's law can be derived from his own principles of mechanics. **Huygens**, on the other hand, argued that light has the nature of a wave propagating in a vacuum. [Modern quantum physics has shown that *both* were right.]

In 1671 Newton introduced new coordinate systems, such as polar coordinates and bipolar coordinates.

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115 Virtually, the moon falls radially toward earth with acceleration $g' = \frac{GM}{R^2}$. On the other hand, for the earth itself $g = \frac{GM}{r^2}$; Therefore $g' = g\left(\frac{r}{R}\right)^2$, where $g' = \frac{v^2}{R} = \frac{GM}{R^2}$. Newton knew that the radius of the moon's orbit was about 60 times the radius of the earth itself, as the ancient Greeks had first shown [**Hipparchos**, ca 130 BCE].

116 Newton could have, and probably did, deduce the inverse-square distance dependence of the law of universal gravitation by amalgamating *Kepler's third law* $[R^3/T^2 = K]$, the expression for *centripetal acceleration* $[a = \omega^2 R]$ and his own *second law* $(F = ma)$.
Indeed, the force F that accelerates a mass m in a circular orbit of radius r with angular velocity ω, is explicitly expressible in the form

$$F = ma = m(\omega^2 R) = mR\frac{4\pi^2}{T^2} = (4\pi^2 K)\frac{m}{R^2}.$$

In 1687, there appeared his book: "*Principia mathematica philosophiae naturalis* ("Mathematical principles of natural philosophy").[117] In this treatise he defined the concepts of *mass* and *force*, stated the three basic laws of mechanics and postulated the universal law of gravitation through the force F between any two point masses m and M at a distance r apart: $F = G\frac{mM}{r^2}$, where G is the universal constant of gravitation. He then showed how the empirical Kepler's laws follow from his laws (1680), but made no hypothesis as to how the gravitational force is transmitted[118].

Newton then applied his theory to explain the ocean *tides* as resulting from the combined attraction of the moon and sun. He also showed that the *precession of the equinoxes* resulted from the earth's equatorial bulge and the attractions of the sun and the moon. In addition he obtained detailed corrections to Kepler's elliptical orbits. In order to make the proper calculations related to all these phenomena, Newton had to develop many mathematical techniques *in addition* to the differential calculus. The phenomenal success of

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[117] For further reading, see:

- Newton, Isaac, *Mathematical Principles of Natural Philosophy*, University of California Press: Berkeley, CA, 1960, 680 pp.
- Maury, J.P., *Newton — The Father of Modern Astronomy*, Harry N. Abrams: New York, 1992, 143 pp.
- Westfall, R.S., *Never at Rest: A Biography of Isaac Newton*, Cambridge University Press: Cambridge, 1980, 908 pp.
- Rankin, W., *Newton for Beginners*, Icon Books, 1993, 176 pp.

[118] With a stroke of genius, Newton created here, without knowing it, one of the fundamental concepts of modern physics — the *field*.
His *action at a distance* force acts with no apparent physical contact between interacting objects, yet this action is ubiquitous, pervading the entire space surrounding the masses.
The enormity of this step can be vividly illustrated by the fact that a steel cable of radius $d = 50$ km would not be strong enough to hold the earth in its orbit. Yet the gravitational force which hold the earth in its orbit is transmitted from the sun across a hundred and fifty million kilometers of space without any material medium to carry that force! Indeed, equating the gravitational force between the two point-mass models of these bodies, we obtain per unit cross-section of the cable: $\{GM_{\oplus}M_{\odot}R^{-2}\}/\pi d^2 = 2.1 \times 10^{12}$ dyn/cm^2, which is above the value of Young's modulus for steel. [In this calculation we took $G = 6.67 \times 10^{-11}\ \frac{\text{Newton}\times\text{meter}^2}{\text{kg}^2}$; $R = 1.5 \times 10^{11}$ meters; $M_{\odot} = 2 \times 10^{30}$ kg; $M_{\oplus} = 6 \times 10^{24}$ kg; 1 Newton $= 10^5$ dyn.]

his efforts owed much to his unusual mathematical skills and superb physical insights.

In his *Principia* (1687) Newton devised a simple way to estimate the distance of the stars nearest to the sun: Assuming that the sun is a typical fixed star, one may estimate the distance to a star by comparing its apparent brightness with that of the sun — in the same manner that a distance to a candle may be judged by comparing its brightness with that of an identical candle nearby. Newton then calculated that our nearest stars are about a million times further than the sun, in good agreement with later measurements.

One of Newton's great achievements was the formulation of the fundamental laws of mechanics (1687). These constitute a codification of observation, experience and theory into 3 propositions:

1. *Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus illud a viribus impressis cogitur statum suum mutare*.
   [Every body persists in its state of rest or uniform motion straight ahead, except in so far as it be compelled to change that state by forces impressed upon it.]

2. *Mutationem motus proportionalem esse vi motrici impressae, et fieri secundum lineam rectam qua vis illa imprimitur*.
   [The change of motion is proportional to the motive force impressed and it takes place along the right line in which that force is impressed.]

3. *Actioni contrariam semper et aequalem esse reactionem*: *sive corporum duorum actiones in se mutuo semper esse aequales et in partes contrarias dirigi*.
   [To an action there is always an equal and contrary reaction: or, the mutual actions of two bodies upon each other are always equal and directed to contrary parts.]

These propositions, while somewhat vague, have been the foundation of much of the science and technology developed to the present time. They are stated in terms of undefined *primitive* concepts such as *mass* (implicitly) and *force* (explicitly) which need to be separately quantified. Notwithstanding, they provide the infrastructure for a precise analysis of a wide range of complex and seemingly unrelated systems and phenomena at a vast range of spatiotemporal scales.

In the 18^{th} and 19^{th} centuries, **L. Euler** (1758–1765), **J. d'Alembert** (1742), **J.L. Lagrange** (1788), **S.D. Poisson** (1813), **C.G.J. Jacobi** (1837), **W.R. Hamilton** (1828) and **J.H. Poincaré** (1889) have put Newton's propositions on a much firmer analytical basis, which led to the birth and development of special branches of continuum mechanics such as hydrodynamics, aerodynamics, gas dynamics, theory of elasticity etc. The spirit of mechanics in all of its manifestations, however, is still easily traceable to Newton.

The first law is an important special case of the 2^{nd} law, since when $\mathbf{F}^{(e)} = 0$ (vanishing of total external force) the acceleration (of a point-particle or of the center of mass of a composite system) vanishes and thus the relevant velocity vector is fixed, both in direction and magnitude.

One may ask, then, why include the first law at all? The answer is that it helps to give *meaning* to the concept of force, by *defining* the case where it vanishes in a suitable class of reference frames! Of course, $\mathbf{F}^{(e)}$ must be better defined than that in order to solve for the motion in terms of initial positions and velocities (which is the aim of classical physics). Thus, it is necessary to use some theoretical form or semi-empirical ansatz (exemplified by the Law of Universal Gravitation on the one hand, and Hooke's law on the other).

Finally, we note that the third law may be replaced by *momentum conservation*. Thus, for a closed system of two interacting masses, let $\mathbf{F}_{12}(\mathbf{F}_{21})$ be the force exerted by 1 on 2 (2 on 1), respectively; we have by the second law

$$\mathbf{F}_{12} = \frac{d}{dt}(m_2\mathbf{V}_2), \qquad \mathbf{F}_{21} = \frac{d}{dt}(m_1\mathbf{V}_1),$$

so the third law $\mathbf{F}_{12} = -\mathbf{F}_{21}$ is equivalent to $m_1\mathbf{V}_1 + m_2\mathbf{V}_2 = \text{const.}$, which is just the statement that the momentum of the overall closed system is conserved. This readily generalizes to a system of n masses, or even a continuous medium, such as an elastic solid or a liquid.

Yet *another* way of expressing this law is that the *center of mass* (COM) position vector of the closed system,

$$\mathbf{r}_{COM} \equiv \frac{1}{\sum_{i=1}^{n} m_i}\Big(\sum_{i=1}^{n} m_i\mathbf{r}_i\Big),$$

moves in a straight line at constant speed. Consulting the first law again, we see that we can think of the n-mass system as a single mass located at its *COM*. When the system is *not* closed, it obeys the second law as if it were a mass acted upon by the total force $\sum_{i=1}^{n}\mathbf{F}_i^{(e)}$, where $\mathbf{F}_i^{(e)}$ is the total external force acting on the i^{th} mass.

The important thing to remember is that the main difference between Newton's second and third laws concerns *the definition of the system under study*: If one wishes to examine the motion of a single object (however defined) that is being acted by forces that originate *outside* the object, one uses

$$\mathbf{F}^{(e)} = \frac{d(m\mathbf{V})}{dt},$$

where $m\mathbf{V}$ is termed the momentum vector of the object.

On the other hand, if we consider a whole closed system composed of parts that can *interact with each other*, then Newton's third law tells us that the sum of the changes of the individual momenta will be equal to zero and that, therefore, the *internal forces must act in pairs* such that each pair is equal and opposite.[119]

An extended closed system (and after all any real-life mass is such a system, made up of myriad of atoms!) obeys another vectorial conservation law — that of *angular momentum*:

$$\mathbf{L}_{\text{total}} \equiv \sum_{i=1}^{n} m_i \mathbf{r}_i \times \dot{\mathbf{r}}_i = \text{const.}$$

Arriving at this law from Newton's laws is more circuitous, and in fact an extra ingredient is required to derive it. It follows from the second law, applied to the individual masses, that

$$\frac{d}{dt}\mathbf{L}_{\text{total}} = \sum_{i=1}^{n} \mathbf{r}_i \times \mathbf{F}_i \equiv \mathbf{T}_{\text{total}},$$

where $\mathbf{F}_i$ is the overall force acting on m_i, by the other masses and by external influences. Here $\mathbf{T}_{\text{total}}$ is the total *torque* acting on the system, and is origin-dependent.

One way to derive

$$\frac{d}{dt}\mathbf{L}_{\text{total}} = 0$$

for a closed system, turns out to require an action principle with rotational symmetry, from which the second law must be derivable as Euler-Lagrange equations; this is true in particular for a closed self-gravitating system of masses, and indeed one can check the angular momentum conservation directly in this case:

$$\mathbf{T}_{\text{total}} = \sum_{i=1}^{n} \mathbf{r}_i \times \sum_{j\neq i} \frac{G m_i m_j (\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^3} = 0.$$

[119] If a subsystem or object is defined in such a way that no mass may enter or leave it, we have $\frac{dm}{dt} = 0$ and Newton's 2^{nd} law assumes the form $\mathbf{F}^{(e)} = m\frac{d\mathbf{V}}{dt} = m\mathbf{a}$, with $\mathbf{a}$ the instantaneous acceleration of the object's COM.

In addition to such so-called ‘conservative’ forces[120], one can add any types of contact forces, for which there is no net torque. Thus, angular momentum is conserved also in the presence of non-conservative forces such as *friction*[121].

Newton’s laws of motion embody what is now known as *Galileo’s principle of relativity*, according to which uniform motion has meaning only when referred to some other body or system and can be detected only by reference to something external to the body in motion. Hence all laws of mechanics are the same in all systems (frames) of reference that move uniformly relative to each other (two such systems are related by a *Galilean transformation*).

To explain the forces that act on bodies in non-uniform motion, Newton invented the concept of ‘absolute space’, regarding it as a *substance within which bodies move*, which reacts but cannot be acted upon. The reaction of absolute space back on the body produces the ‘inertial force’. Thus, the fictitious centrifugal and Coriolis forces are the reaction of absolute space on the rotating body[122]. **Berkeley** (1734) and **Mach** (1872) later rejected

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120 Besides gravity, other examples are: elastic, intermolecular and electrostatic or magnetostatic forces.

121 In *classical mechanics* the law of conservation of angular momentum does not carry the strength of a universal principle; it applies mainly to two cases — particles interacting via *central forces*, and in continuum mechanics with only contact forces.

In a (classical or quantum) system derived from an action principle of fields, however, it *is* a universal law — of no less importance than the law of conservation of linear momentum.

In *special relativistic physics* we must raise the number of dimensions from 3 to 4; consequently energy and momentum become components of a single 4-vector, while the extension of the angular-momentum 3-vector into a skew-symmetric 4-tensor of rank 2 leads one to the uniform rectilinear motion of the center of mass.

When dealing with *fields* or other continuous matter-energy distributions, we are led to consider densities of these conserved quantities, since in each volume element one has energy, momentum and angular momentum w.r.t. some given point. Moreover, it makes little sense to consider energy density by itself, because what is energy density in one reference frame will be some combination of energy density, energy flux density, and momentum flux density as seen from another reference frame (even in Newtonian mechanics, a shift in the spatial origin mixes angular momentum with (linear) momentum, while a Galilean transformation between inertial frames admixes components of momentum into energy). Hence, all these quantities are best considered together.

122 They are ‘fictitious’ only in the sense of arising in non-inertial frames — frames in which Newton’s 1^{st} law is violated. These reference frames are accelerated relative to *inertial* frames.

this simplistic 'solution', but it was not until the advent of Einstein's general relativity (1915) that a more satisfactory interpretation emerged.

Newton also concerned himself with the equivalence of inertial and gravitational mass. The roots of this problem are to be sought in Galileo's result that *in a given gravitational field, all nearby pointlike particles fall with the same acceleration*. This statement implies that to some extent gravitational forces behave in the same way as inertial forces. Galileo's statement can be called *Galileo's principle of equivalence*.

Newton sharpened this principle by combining it with his own second law of motion. This fusion then yielded the statement that the gravitational force is proportional to the mass on which it acts, ergo: the inertial mass and the gravitational mass are proportional. Newton then assumed that these two measures of mass were *equal* and he set forth to determine the precision of this determination. *Newton's principle of equivalence* then states that *gravitational mass* (m_g) *and inertial mass* m_F (resistance to change of motion under action of forces) *are equal*.

Comparing periods of pendula of fixed length but with different masses and composition, he found[123] that

$$\frac{|m_I - m_g|}{m_I} \ll 10^{-3}.$$

Clearly, Newton's principle implies equal accelerations only for bodies of *sufficiently small size* placed in a *sufficiently homogeneous gravitational field*.

Newton's laws of motion have the inherent property that they are covariant under the Galilean transformation, i.e., they retain their form when viewed by different inertial-frame observers — those attached to frames of reference in which no inertial forces are observed; any two such frames move with fixed velocity with respect to each other ($\mathbf{r}' = \mathbf{r} - \mathbf{v}t,\ t' = t$). [A physical law which retains its form under a particular transformation is said to be covariant w.r.t. that transformation.] Frames of reference in which a test particle moves with constant velocity unless acted on by a force are known as *inertial frames*. [One may define such a frame in a picturesque way as one in which it is possible to play three-dimensional billiards.]

Newton's laws of motion do not distinguish between past and future, in the sense that they are *symmetrical w.r.t. reversal of motion*. Indeed, with

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[123] In the period 1891–1908 **Roland von Eötvös** (1848–1919, Hungary) used a torsion balance, which he had developed, to lower Newton's equivalence bound to ca 10^{-9}. This number was further reduced to 10^{-12} in experiments performed at Princeton and Moscow in the 1960's.

the exception of a rare kind of decay among a certain type of elementary particles, all experimental facts known to date about the actual world (even at the quantum level and even in GTR) are consistent with the following symmetry postulate: *to any state or process encountered in the actual world there corresponds a time reversed state or process that is again a possible state or process in the actual world*.

Newton's views on time were clearly stated in his 'Principia' of 1687:

"*Absolute, true and mathematical time, of itself and from its own nature, flows equably without relation to anything external*".

Thus according to Newton, time would continue just the same even if the universe were completely empty.

The work of Newton cannot be properly understood without a knowledge of the science of antiquity. Newton did not create in a void. Without the stupendous work of **Ptolemy** (which completed and closed ancient astronomy) and **Kepler**'s '*Astronomia Nova*', the mechanics of Newton would have been impossible. Without the conic sections of **Apollonios**, which Newton knew thoroughly, his development of the law of gravitation is equally unthinkable. And Newton's integral calculus can be understood only as a continuation of **Archimedes**' determination of areas and volumes. The history of mechanics as an exact science begins with the laws of the lever, the laws of hydrostatics and the determination of mass centers by Archimedes. In short, all the developments of mathematics, mechanics and astronomy which converge in the work of Newton, began in Greece.

And one must not forget **Galileo Galilei**; There is an appropriate symbolism in the fact that the death of Galileo and the birth of Isaac Newton occurred in the same year, 1642. Galileo and Newton are to be considered as the parents of modern science. In the words of Alfred North Whitehead:

"*There would have been no Newton without Galileo; and it is hardly a paradox to say, that there would have been no Galileo without Newton*".

Newton was born at Woolsthorpe, a hamlet in the parish of Colsterworth, Lincolnshire. His father (also Isaac Newton) who farmed a small property of his own, died before his son's birth, a few months after his marriage to Hannah Ayscough. When Newton was two years old his mother remarried and had three more children, to the descendants of which Newton eventually left most of his property. At the age of 12 he was sent to a grammar school at Grantham. At the age of 14 his stepfather died and his mother took him away from school, since she intended him to be a farmer. He was sent back to school, however, and admitted to Trinity College, Cambridge, in 1661.

During the next 3 years he studied Euclid's *Elements*, Descartes' *Geometry* and Wallis' *Arithmetic Infinities*. In 1665 Newton took the B.A. degree. During his college career, Newton showed no exceptional ability and was graduated without any particular distinction. In the years 1665 and 1666, Trinity College was closed on account of the Great Plague in London (ca 70,000 people died). Newton retired to the countryside at his mother's home in Lincolnshire, where he made his preliminary discoveries of the binomial theorem, the method of fluxions, universal gravitation and the light spectrum. He returned to his college in 1667, and took his M.A. degree early in 1668. In 1669 (at age 26), his teacher, Isaac Barrow, resigned the Lucasian chair in favor of Newton, who thus became a professor of mathematics. In 1672 he was elected fellow of the Royal Society and during 1689–1690 represented Cambridge University in Parliament. To afford him a substantial salary, his friends secured for him the vacated mastership of the mint in 1699, where he later drew up the English monetary reform.

In 1701 he resigned his Cambridge professorship and moved to live in London. He was knighted in 1705, and was a very popular visitor at the court of George I. Newton added very little to his achievements in physics and mathematics after 1687 and spent most of the next 40 years of his unmarried life on public activities, experiments in *alchemy* and problems of *theology* and *Biblical chronology*. He died in 1727 and was buried in Westminster Abbey.

Newton was a man of deep religious convictions, bordering on mysticism. From an early period of his life he paid great attention to theological studies. [In fact, he spent *little* of his time studying mathematics, physics and astronomy.] The preoccupation with these matters served as the driving force behind his scientific work, which he considered as the deciphering of nature's code. Most of his life was spent on investigations of the Scriptures and the writings of the Christian saints, where he hoped to find hints to the secrets of the creation. He was a Unitarian and kept it a secret.

We know that Newton adhered to the philosophical view that time is cyclic and was convinced that the world was coming to an end. He believed that the comet of 1680 had just missed hitting the earth, and in his commentaries on '*Revelations*' and the '*Book of Daniel*', unpublished in his lifetime, he indicated that the end of the world could not be long delayed. A particularly striking example of his cyclical philosophy occurs in a letter that he wrote to **Henry Oldenberg**, secretary of the Royal Society, in December 1675:

"*For nature, is a perpetual circulatory worker, ... so perhaps may the Sun imbibe this Spirit copiously to conserve his shining, and keep the Planets from receding further from him*".

Newton's discoveries had been so impressive for nearly two hundred years that they had the hallmark of being the last word. No refinement of his laws had been suggested. His law of gravitation had successfully explained every astronomical observation (with the tiny exception of a wobble in the orbit of the planet Mercury around the sun).

In fact, during his own lifetime the success of his mechanics had led to speculations that his approach might provide a panacea for the investigation of all questions. The impressive completeness of Newton's *Principia* (1687) and the deductive power of his mathematics led to a bandwagon effect with thinkers of all shades aping the Newtonian method. There were books on Newtonian models of governments and social etiquette, and Newtonian methods for children and 'ladies'.

Nothing was imagined to be beyond the scope of the Newtonian approach. Nor was Newton himself entirely divorced from this enthusiasm. His later work on alchemy and biblical criticism reveals a deep-rooted belief in his ability to unveil all mysteries for the human race. Having first revealed the truth about God's design of the physical world, he seems to have seen himself as having a similar commission to fulfill in the realm of the spiritual and the mystical.

Newton is a deeply paradoxical figure when viewed through the lens of modern scientific attitudes. A mathematical genius who possessed the most penetrating physical intuition of any recorded scientist, he nevertheless had one foot in the Middle Ages and displayed a magician's belief in his ability to solve all problems and overcome all barriers. His achievements must have made his contemporaries believe that the end of the seventeenth century was indeed the completion of science.

Newton was basically a very religious person, deeply influenced by the Bible and the religious philosophy of the ancient Hebrews. He believed in the unity of nature and the universality of natural laws — hence the motive for his discovery of the law of universal gravitation which applies to *all* stars in the universe. This was *not* a Greek heritage. This he received directly from the Bible.

Many years of Newton's life were embittered by the professional controversies which his *Principia* evoked. A long time elapsed before his ideas became part of the equipment of the ordinary educated man. It has been said that there were comparatively few scientists in the 20^{s} and 30^{s} of the 20^{th} century who comprehended Einstein's GTR. But there have been far fewer in Newton's day who could appreciate the reasoning of the Principia.

In 1692 and 1693 Newton seems to have a serious illness characterized by insomnia, withdrawal from close friends, headaches, nightmares, loss of hair.

It is now believed that there were symptoms of *mercury poisoning*, caused by his preoccupation with *alchemy* (transmutation of mercury into silver and gold).

Newton maintained that the corpuscles of light associated with various colors excited the ether into characteristic vibrations, where the sensation of red corresponds to the longest vibration of the ether and violet to the shortest. Perhaps the main reason for rejecting the wave theory as it stood then was the blatant problem of explaining rectilinear propagation in terms of waves which spread out in all directions.

Worldview XI: Isaac Newton

* * *

"If I have seen further it is by standing on the shoulders of Giants."

In a letter to Robert Hooke, February 5, 1675

* * *

"I know not what I may appear to the world, but to myself I seem to have been only like a boy playing on the sea-shore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me."

* * *

"The light of the fixed stars is of the same nature (as) the light of the sun."

Mathematical Principles of Natural Philosophy, 1687

* * *

"Physics, beware of metaphysics."

* * *

"Nature is pleased with simplicity, and affects not the pomp of superfluous causes."

* * *

"Absolute space, in its own nature, without relation to anything external, remains always similar and immovable. ... Absolute motion is the translation of a body from one absolute place into another."

Mathematical Principles of Natural Philosophy, 1687

* * *

"Are not gross bodies and light not convertible in to one another?"

* * *

"Amicus Plato, amicus Aristoteles, magis amica veritas.*"*
(Plato is my friend, Aristotle is my friend, but my best friend is truth*)*

* * *

"Absolute, true and mathematical time*, of itself, and from its own nature, flows equally without relation to anything external."*

* * *

"Truth is ever *to be found in* simplicity*, and not in the multiplicity and confusion of things ... He is the God of order and not of confusion."*

* * *

"Whence it is that nature *does nothing in vain; and whence arises all that* order *and* beauty *which we see in the world."*

* * *

On Newton

* * *

"Newton, forgive me. You found the only way that was possible for the man of the highest powers of intellect and creativity. The concepts that you created still dominate the way we think in physics."

"Let no one suppose, however, that the mighty work of Newton can easily be superseded by reality on any other theory. His great and lucid ideas will retain their unique significance for all the time as the foundation of our whole modern conceptual structure in the sphere of natural philosophy."

Albert Einstein

* * *

"Nature, and nature's laws lay hid in night.
God said, Let Newton be! and all was light."

Alexander Pope

* * *

"Nearer to the Gods no mortal may approach."

* * *

"Newton, with his prism, and silent face: The marble index of a mind for ever voyaging through strange seas of thought alone."

William Wordsworth
(on seeing Newton's statue in the chapel at Trinity College by moonlight)

* * *

"One had to be a Newton to notice that the moon is falling when everyone sees that it does not fall."

Paul Valery (1871–1945)

* * *

Time and Tides

The periodic rise and fall of the ocean level, termed simply the Tide, was suggested by ancient peoples to be in some way related to the celestial bodies, long before a word for gravity existed. But the theories advanced were fantastic. It is natural that the writings of the classical authors of antiquity should contain but few references to the tides, for the Greeks and Romans lived on the shores of an almost tideless sea.

Pytheas of Massilia *(fl. ca 310 BCE) was familiar with the tides in the region of the British Isles and North Sea and is said to be the first to have actually* measured *the rise and fall of the tide. The Greek geographer and historian* ***Strabo*** *(ca 64 BCE–20* CE*) quotes from* ***Poseidonios*** *(135–51 BCE) a clear account of the tides on the Atlantic coast of Spain. He also gives the law of* diurnal inequality[124] *of the tide in the Indian Ocean as observed by* **Seleucus the Babylonian***. Seleucus was the first known commentator to offer a rational (thought incorrect) mechanism of tide-generation.*

The Roman historian **Pliny the Elder** *(23-79* CE*) described the variation in the tidal range accompanying the moon's phases and changes in declination. In his* Historia Naturalis *he writes:*

"Much has been said about the nature of waters; but the most wonderful circumstance is the alternate flowing and ebbing of the tides, which exist, indeed, under various forms, but is caused by the sun and the moon*. The tide flows twice and ebbs twice between each two risings of the moon, always in the space of* 24 *hours. First, the moon rising with the stars swells out the tide, and after some time, having gained the summit of the heavens, she declines from the meridian and sets, and the tide subsides. Again, after she has set, and moves the heavens under the earth, as she approaches the meridian on the opposite side, the tide flows in; after which it recedes until she again rises to us. But the tide of the next day is never at the same time with that of the preceding".*

Julius Caesar *and his officers were totally ignorant of the connection between moon and tide: in the year* 55 *BCE, his first assault-landing on the Kentish coast of Britain, near Dover, failed as a direct result of a devastating spring tide; high water came about an hour before midnight, and driven by*

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[124] The difference in the amplitudes (at locations in the middle latitudes of either hemisphere) of the two diurnal tides due to the periodicity in the moon's declination.

the gale, the beach galleys were swept by the breakers, all sustaining heavy damage. As a result, Caesar and his army returned to France.

The next 1500 *years did not advance the tidal lore beyond the observations of* ***Pytheas*** *and* ***Pliny****. The occurrence, at many places, of high tide at about the time of the moon's passage across the meridian may have prompted the idea that the moon exerts some* attraction *on the water, but the occurrence of a second high tide when the moon is on or near the opposite meridian was a great puzzle to the few* philosophers *who thought about it.*

We know about two Englishmen who were occupied with this problem: ***Alexander Neckam*** *(1157–1217), a learned monk, who was for a time a professor at the University of Paris, wrote a book of general knowledge,* De Naturis Rerum *in which he remarked that he was unable to resolve the vexed question as to the cause of the tides, but that the common belief was that they are due to the moon.* **Wallingford** *(d. 1213) made recordings of tidal observations for the purpose of prediction and tabulated the occurrences of floods at London Bridge in ca* 1210.

The Franciscan friar ***Roger Bacon*** *of Oxford attempted, at about* 1250 CE, *a rational (though wholly false) solution to the problem of the 'second high tide', based on the Ptolemaic conception of the universe.*

Johann Kepler *had recognized the tendency of the waters of the ocean to move towards the centers of the moon and the sun, and he wrote of some* attraction *between the moon and the earth's waters.* ***Galileo*** *then expressed regret that so acute a man as Kepler should produced a theory with occult qualities of the ancient philosophers (!!) His own explanation referred the phenomenon to the rotation and orbital motion of the earth, and he considered that it afforded a principal proof of the Copernican system.*

It was not until **Newton** *published the consequences of his law of universal gravitation in the* Principia *(1687) that the basic mechanics of tidal behavior could be understood.*[125] *It is a fact of the history of science that during the entire period from Pliny to Newton, nobody had the conviction that ocean tides are caused directly by the moon or the sun.*

But even if someone could believe it, and even if that someone knew about gravitational attraction between masses, he still could not have accounted quantitatively for the tide generating forces without stating clearly the three fundamental laws of dynamics discovered by Newton! It is the association of these two categories that made Newton see the light. The essence of his reasoning is this: When an astronomical body is moving under the action of

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[125] For further reading, see:

- Cartwright, D.E., *Tides*, Cambridge University Press, 2000, 292 pp.

the gravitational forces of other astronomical bodies, the orbit of the moving body is that of its center of mass. The finite extent *of the body introduces two new effects:*

(1) A precession *of the axis of* rotation *in case of a rotating body (provided the body is not a sphere with homogeneous or concentric distribution of mass).*

(2) Appearance of tidal forces: Under the assumption that the body is rigid, the forces of inertia *due to the orbital motion (revolution without rotation) are uniform for each mass element of the body. They have the same magnitude and opposite direction to that of the* gravitational *force acting on the body's center of mass. The gravitational forces, however, are not uniform throughout the body so that outside the center of mass, differences appear between gravitational forces and the inertial forces. These differences appear as tidal forces: Over the earth's hemisphere nearer to the moon, the gravitational attraction is* greater *and the* centrifugal[126] *acceleration is* less *so that over this hemisphere there is a distribution of unbalanced upward force directed moonward and acting* against *the earth's own gravitation force.*

Over the opposite hemisphere the gravitational force of the moon is less *and the centrifugal force is* greater *than at C, so that there, too, a distribution of unbalanced force results, directed obliquely upward. Particles free to move, like those of the sea and air, will do so under the actions of these unbalanced forces. The mathematical formulation of this concept, on the basis of the law of gravitational attraction, does not require more than elementary algebra.*

A tide-producing force (on the surface of a hypothetical yielding, spherical earth) can be resolved into a tangential (horizontal) component and a normal (vertical) component. Assuming this sphere to be covered with an oceanic layer, the motion *of the water on its surface will be governed by the horizontal component of the tidal force. This motion will lead toward an accumulation, or a heaping up, of water at the sublunar centers (points nearest and farthest from the moon), with an attendant rise in sea level.*

On the other hand, a withdrawal of water will tend to take place along the great-circle zone on a plane normal to the earth-moon line, where the sea level must fall. In general if the whole spherical earth were made of a yielding material and made to respond to the tidal forces by deforming freely, it would assume the shape of an ellipsoid with its major axis coinciding with the earth-moon direction. An equilibrium shape would be reached when the inequalities of the earth's own gravitational attraction (resulting from the development

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[126] An earthbound observer co-revolving with the earth prefers to see it as a *centrifugal* force, while an outside observer will see it as a *centripetal* force. Both views are equivalent.

of its prolated form), exactly counterbalance the tide-producing forces at all points.

Thus, if the earth were not rotating about its own axis, the heaping up of water at the two tidal centers, and the lowering of the sea level along the great-circle zone would quickly reach a state of equilibrium, becoming permanently fixed in geographic coordinates. We would then have no tidal fluctuations due to the moon's attraction. As the earth turns on its axis, however, the direction and magnitude of the tidal force acting at any given place on the earth's surface changes periodically. This causes the two tidal centers (bulges) to move westward as a tidal wave around the earth. At any given geographical point, the period of the lunar tide is 12 *moon-hours, i.e.* 12^h24^m *(the time*[127] *elapsed between successive meridian passages of the moon).*

When Newton developed his theory of tides he assumed, for the sake of simplicity, that the earth was covered by an ocean of uniform depth and that the flow of water to the two centers of tidal rise would quickly bring about an equilibrium form of the sea surface in which pressure differences would exactly balance the horizontal forces. Thus, the water are devoid of inertia in this approximation, whereas its gravitational properties are kept. This is Newton's theory of the equilibrium tide. The observed tides are generally much greater than those derived from the equilibrium theory. The oceans are unable to respond instantly and completely to the rapidly moving system of horizontal forces. Nevertheless, the equilibrium theory is valid as a fundamental explanation of the tide.

Newton was well aware of this discrepancy between theory and fact, but pursued it no further. The quasi-static theory was completed in 1741 by ***Daniel Bernoulli****,* ***L. Euler*** and ***C. Maclaurin****. In* 1774*,* ***P.S. Laplace*** *presented his dynamic theory of tides. He considered tides as waves induced in a uniform ocean layer by periodic forces, taking into account Coriolis forces and friction. But even this theory could not account for local observations. The nature of actual tides is complicated due to the presence of land masses stopping the flow of water, the unknown friction in the oceans and between oceans and the ocean floors, the rotation of the earth, the variable depth of the ocean, winds and other factors*[128].

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[127] During the semi-diurnal period of 12^h, the moon advances in its orbit about the earth. Since its speed is 30 times slower than that of the earth, the earth must spend $\frac{12^h}{30} = 24^m$ more to overtake the moon.

[128] The numerical integration of the *Laplace tidal equation* (1774) for realistic models of the world oceans was undertaken by **C.L. Pekeris** (1908–1993, Israel) during 1969–1978.

Both the times and the heights of high tide vary considerably from place to place on the earth. The earth's rapid rotation causes the tide-raising forces within a given mass of water to vary too rapidly for the water to adjust completely to them. These forces however, recurring periodically, set up forced oscillations *in the ocean surfaces, so that the water over a large area rises and falls in step. Consequently, the highest water does not necessarily occur when the moon is highest in the sky (or lowest below the horizon).*

Sometimes shallow coastal seas have such shapes and sizes that the natural frequency of the basin waters is very nearly the same as that of the tidal period in the adjacent ocean. Then the ocean tide can set up resonance oscillations in the basin (e.g. the Bay of Fundy between New Brunswick and Nova Scotia and the Gulf of Maine; under favorable conditions, the tidal range at the head of the Bay of Fundy can exceed 15 *meters).*

*Tides also occur in the atmosphere (**Laplace**, 1825) and the solid earth (**Lord Kelvin**, 1863).*

The sun too produces tides on the earth, although it is less than half as effective a tide-raising agent as the moon. Actually, the gravitational attraction between the sun and the earth is about 180 *times as great as that between the earth and the moon, but the earth-sun distance is about* 390 *times larger than the earth-moon distance. The moon's tides, therefore, dominate. On the other hand, when the sun and the moon are lined up at new or full moon, both tides reinforce each other* (spring-tides)*. In contrast, when the moon is at first quarter or last quarter, the tides produced by the sun partially cancel out the tides of the moon, and the tides are lower than usual* (neap-tides)[129].

Consider first the lunar effect, and neglect the rotation of the earth about its axis. The centripetal acceleration of the earth and the moon, required for revolution of the earth-moon system about their common mass-center, is provided by their mutual attraction*, but only at the mass center of each body is the gravitational force precisely equal to the centripetal force.*

Put the centers of the earth and the moon at points O and M respectively (a distance d apart on a z-axis). At an arbitrary point P in the earth with coordinates $\{r, \beta\}$ relative to the z-axis, the centripetal acceleration is also in the z-direction. The difference between this acceleration and the acceleration due to the moon's attraction, gives the tidal acceleration at P.

To see this quantitatively we perform a preliminary calculation for points on the earth nearest and farthest from the moon. The moon's attraction

[129] The unit of acceleration in the cgs system is 1 cm s^{-2} and this unit is called 1 *gal*, in honor of **Galileo Galilei** (1564–1642) who was a pioneer in the study of motion of bodies under gravity. One thousandth of a *gal* is called 1 milligal (mgal). One millionth of a *gal* is called 1 microgal (μgal).

on unit masses located at these points, and at the earth's mass center, are $f_1 = GM(d-r)^{-2}$, $f_2 = GM(d+r)^{-2}$ *and* $f_0 = GMd^{-2}$ *respectively, with* M *the lunar mass. Hence*

$$f_1 - f_0 = GM\big[\frac{1}{(d-r)^2} - \frac{1}{d^2}\big] = GM\frac{(2d-r)r}{d^2(d-r)^2};$$

$$f_2 - f_0 = GM\big[\frac{1}{(d+r)^2} - \frac{1}{d^2}\big] = -GM\frac{(2d+r)r}{d^2(d+r)^2}.$$

Since $d \gg r$, *we have to a good accuracy*

$$f_1 - f_0 \cong \frac{2GMr}{d^3}, \qquad f_2 - f_0 = -\frac{2GMr}{d^3}.$$

For points which are not necessarily on the mass-center line, the above calculation involves vector subtraction. It is then more convenient to write the gravitational potential Φ *of the moon, acting on a unit mass at* P *at distance*

$$R = \sqrt{d^2 - 2dr\cos\beta + r^2} = d\sqrt{1 - 2\frac{r}{d}\cos\beta + \frac{r^2}{d^2}}$$

from the moon's center: $\Phi = -\frac{GM}{R} + C$. *Clearly,* C *is a constant that must be assigned the value* $\frac{GM}{d}$ *in order to secure* $\Phi(r=0) = 0$.

Thus

$$\Phi = \frac{GM}{d}\big[1 - \big(1 - 2\frac{r}{d}\cos\beta + \frac{r^2}{d^2}\big)^{-1/2}\big].$$

Expanding the inverse square root in a series of Legendre polynomials we find

$$\Phi = -\frac{GM}{d^2}z - \frac{GMr^2}{d^3}\big[P_2(\cos\beta) + \frac{r}{d}P_3(\cos\beta) + \cdots\big],$$

where $z = r\cos\beta$. *The first term on the r.h.s. is just the uniform centripetal acceleration. The second term dominates the moon's tide generating potential. Since* $\frac{r}{d}$ *is about* $\frac{1}{60}$, *this term is very often sufficient. The force associated with this potential, namely* $\mathbf{f} = -\operatorname{grad}\Phi$, *has the radial (vertical) component*

$$f_v = -\frac{\partial\Phi}{\partial r} = \frac{3GMr}{d^3}\big(\cos^2\beta - \frac{1}{3}\big)$$

and the azimuthal (horizontal) component

$$f_h = -\frac{1}{r}\frac{\partial\Phi}{\partial\beta} = -\frac{3GMr}{d^3}\sin\beta\cos\beta.$$

These expressions show that the amplitudes of the radial and the azimuthal tidal accelerations on the earth's surface are $2g\big(\frac{M}{E}\big)\big(\frac{r}{d}\big)^3$ *and* $\frac{3}{2}g\big(\frac{M}{E}\big)\big(\frac{r}{d}\big)^3$

respectively, where $g = G\frac{E}{r^2}$ *is the surface gravity acceleration and* E *is the mass of the earth. Taking the values* $\frac{M}{E} = \frac{1}{81.5}$, $\frac{r}{d} = \frac{1}{60}$, *one obtains a vertical acceleration of* 0.112 *mgal against a centripetal acceleration of*

$$\frac{GM}{d^2} = g\left(\frac{M}{E}\right)\left(\frac{r}{d}\right)^2 = 3.38 \ mgal.$$

Thus, the vertical tidal acceleration acting on a mass of 1 *kg at sea-level is only* 0.11 *milligram, a mere* 10^{-7} *g! [This will make the weight of the "Queen Elizabeth" ocean liner (83,673 tons) lighter by ca* 9 *kg as she passes under the zenith of the moon, compared to a location where* f_v *vanishes.] However, being an unbalanced force, it may nevertheless cause large displacements.*

The tide-generating potential which incorporates the rotation of the earth about its axis, is obtained directly from the above expression for Φ *if we go over from the intrinsic earth-moon coordinate system to the celestial sphere coordinate system. Applying the trigonometric identity*

$$\frac{1}{2}(3\cos^2\beta - 1) \equiv \frac{1}{2}(3\sin^2\delta - 1)\frac{1}{2}(3\cos^2\theta - 1)$$

$$+\frac{3}{4}\sin 2\delta \sin 2\theta \cos H + \frac{3}{4}\cos^2\delta \sin^2\theta \cos 2H$$

(δ *= moon's declination;* (φ, θ) *= spherical coordinates of observatory;* $H = \varphi + t$; $t = -\varphi_{_M}$ *= hour angle of moon at Greenwich], the three terms on the r.h.s. of this identity represent respectively the lunar fortnightly tide* M_f, *the principal lunar diurnal component* O_1 (25.82^h) *and the principal lunar* M_2 (12.42^h). *The explicit expression for the latter potential is*

$$\Phi_2 = \frac{3}{4}gr\left(\frac{M}{E}\right)\left(\frac{r}{d}\right)^3 \cos^2\delta \sin^2\theta \cos 2(\varphi + t).$$

For $\theta = 90^\circ$, r *= mean equatorial radius, and* $\langle\cos^2\delta\rangle = 0.722$, *the height of the tide at the equator follows the expression*

$$\eta = \frac{1}{g}\Phi_2 = 25.6\cos 2(\varphi + t) \ cm.$$

The corresponding expression for the sun's tide is $11.8\cos 2(\varphi + t)$ *cm. Since* δ *and* d *both depend on time, the tide at any given location, even in the framework of equilibrium tidal theory, is a combination of a great number of Fourier components, each with its own period, amplitude and phase.*

Of special interest in the earth sciences is a rare event, occurring approximately every 1600 *years, when perigee (moon closest to earth) coincides with* *syzygy (centers of sun, earth and moon are collinear) as well as with perihelion (sun closest to earth); the moon's nodes are on the line connecting the earth and the moon (moon on the ecliptic); and the declination between the moon and the sun is zero. These conditions, which give the greatest possible tide-raising force, have the following schedule of occurrence: 3500 BCE, 1900 BCE, 250 BCE, 1433* CE, *3300* CE.

Newton's Calculus of Fluxions (1664–1671)

When Newton received his B.A., at the age of 23, in June 1665, his examiner, Professor Barrow was of the opinion that Newton did not even know his basic Euclid. Newton had indeed sorely neglected the syllabus. What Barrow did not realize was that Newton was already advancing beyond Descartes, who in his turn had already advanced beyond Euclid. Newton was entirely self-taught — in the sense that he worked largely alone, from books. All his work was confined to his notebooks — which nobody else had seen. However, despite the gaps in his knowledge, he was allowed to continue studying for an M.A. degree.

Newton seemed to thrive in isolation, and events now conspired to make sure this continued. By August 1665[130] *Cambridge University had effectively closed down on account of the Great Plague in London, causing Newton to return to Woolsthorpe, where he remained for about a year.*

Newton's first major breakthrough was the development of the differential calculus – a method for finding the tangent to a point on a curve[131]*. This*

130 This was to result in an *annus mirabilis*, the like of which was **Einstein**'s 1905, 240 years later.

131 **Pierre de Fermat** (1638) beat him to that and was considered by **Lagrange** to be the true originator of the differential calculus.

method, which he named the method of fluxions, was communicated to Barrow in 1669, written in 1671, but was not published until 1736.

In this work, Newton considers a curve as generated by the continuous motion of a point. Under this conception the abscissa and the ordinate of the generating point are, in general, changing quantities. A changing quantity is called a fluent (a flowing quantity), and its rate of change is called the fluxion of the fluent. If a fluent, such as the ordinate of the point generating a curve, be represented by y, then the fluxion of this fluent is represented by $\dot{y}$. In another standard notation we see that this is equivalent to $\frac{dy}{dt}$, where t represents time.

In spite of this introduction of time into geometry, the idea of time can be evaded by supposing that some quantity, say the abscissa of the moving point, increases constantly. This constant rate of increase of some fluent is called the principal fluxion, and the fluxion of any other fluent can be compared with this principal fluxion. The fluxion of $\dot{y}$ is denoted by $\ddot{y}$, and so on for higher ordered fluxions.

On the other hand, the fluent of y is denoted by the symbol y with a small square drawn about it, or sometimes by $\acute{y}$ (these notations are no longer used).

Newton also introduces another concept, which he called the moment of a fluent; it is an infinitely small amount by which a fluent such as x increases in an infinitely small interval of time o. Thus, the moment of the fluent x is given by the product $\dot{x}o$.

Newton remarks that we may, in any problem, neglect all terms that are multiplied by the second or higher power of o, and thus obtain an equation between the coordinates x and y of the generating point of a curve and their fluxions $\dot{x}$ and $\dot{y}$.

As an example he considers the cubic curve

$$x^3 - ax^2 + axy - y^3 = 0.$$

Replacing x by $x + \dot{x}o$ and y by $y + \dot{y}o$, we get

$$\begin{aligned} x^3 &+ 3x^2(\dot{x}o) + 3x(\dot{x}o)^2 + (\dot{x}o)^3 \\ &- ax^2 - 2ax(\dot{x}o) - a(\dot{x}o)^2 \\ &+ axy + ay(\dot{x}o) + a(\dot{x}o)(\dot{y}o) + ax(\dot{y}o) \\ &- y^3 - 3y^2(\dot{y}o) - 3y(\dot{y}o)^2 - (\dot{y}o)^3 = 0. \end{aligned}$$

Now, using the fact that $x^3 - ax^2 + axy - y^3 = 0$, dividing the remaining terms by o, and then rejecting all terms containing the second or higher power of o, we find

$$3x^2\dot{x} - 2ax\dot{x} + ay\dot{x} + ax\dot{y} - 3y^2\dot{y} = 0.$$

Newton considered two types of problems. In the first type, we are given a relation connecting some fluents, and we are asked to find a relation connecting these fluents and their fluxions. This is what we did above, and is, of course, equivalent to differentiation. In the second type, we are given a relation connecting some fluents and their fluxions, and we are asked to find a relation connecting the fluents alone. This is the inverse problem and is equivalent to solving a differential equation. The idea of discarding terms containing the second and higher powers of o was later justified by Newton by the use of primitive limit notions.

Newton made numerous and remarkable applications of his method of fluxions. He determined maxima and minima, tangents to curves, curvature of curves, points of intersection, convexity and concavity of curves, and he applied his theory to numerous quadratures and to the rectification of curves.

Newton's awkward notation led him into long and complex calculations. But he eventually derived simple rules for differentiation of the elementary polynomial, algebraic, trigonometric and exponential functions.[132]

This process of the differential calculus provided the new mathematics with one of its most powerful tools — allowing the calculation of all kinds of rates of change. This included, for instance, the determination of maximum and minimum points in any curve – which occur when the rate of change, $\frac{dy}{dx}$, *is equal to zero.*

Problems of maximum and minimum were solved by great mathematicians long before Newton; in Euclid's Elements VI, 27 we read:

> *"Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar and similarly situated to that described on the half of the straight line, that parallelogram is greatest which is applied to half of the straight line and is similar to the defect"*

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132 The rules of thumb for differentiation, in modern notation, are put in very simple terms, and can be mastered even by high-school kids who know what to do, but don't really know what they are doing. Thus, for example, who was not amazed to learn that the function $y = e^x$, like a phoenix rising again from its own ashes, is its own derivative. Indeed, by Newton's own method:

$$\dot{y} = \lim_{h\to 0} \frac{e^{x+h} - e^x}{h} = e^x \lim_{h\to 0} \left\{ \frac{e^h - 1}{h} \right\} \to e^x.$$

After a lapse of 1900 years, problems of maxima and minima were taken up again by ***Fermat*** *(1638). Fermat's method was made more general by Newton, who called it 'the method of fluxions'.*

Early Applications of the Calculus

(1) Numerical Approximations

Today, a student of mathematics pushes a little key on his or her pocket calculator, and there appears π*, correct to* 8 *decimal places, ready to be used. Whenever higher accuracy is required, a simple computer subroutine can produce* π *to hundreds decimal places in a matter of seconds. In the pre-calculus era,* π *was still being calculated through the old Archimedean method of regular polygons. The peak of this endeavor was reached in* 1596*, when* ***Ludolph van Ceulen****, after devoting years of effort to the task, calculated* π *to* 35 *significant figures, using polygons with* 2^{62} *sides!*

This inefficient classical method of approximating π *had carried mathematicians far. But in the* 17^{th} *century came a mathematical explosion of epic proportions, one of whose advances at last supplanted Archimedes' approach and pushed the search for* π *into a new phase. In* 1665*, young* ***Isaac Newton*** *applied his generalized binomial theorem and his newly invented method of fluxions — that is, calculus — to get a very accurate estimate of* π *with relative ease.*

He considered a circle having its center at $C\left(\frac{1}{2},0\right)$ *and radius* $r=\frac{1}{2}$*. Since its equation is* $y=\sqrt{x-x^2}$*, the area of the circular segment* ABD*, with a base* $A(0,0)$ *to* $B\left(\frac{1}{4},0\right)$*, is*

$$S=\int_0^{1/4}\sqrt{x}\,\sqrt{1-x}\,dx,$$

where D *is a point on the circle at* $x=\frac{1}{4},\ y=\frac{1}{4}\sqrt{3}$*. Replacing* $\sqrt{1-x}$ *by its binomial expansion and integrating term by term, Newton obtained*

$$S=\frac{2}{3\cdot 2^3}-\frac{1}{5\cdot 2^5}-\frac{1}{28\cdot 2^7}-\frac{1}{72\cdot 2^9}-\cdots$$

On the other hand, the segment ABD equals the sector ACD less the triangle BCD, and since $CD = \frac{1}{2}$, $BD = \frac{\sqrt{3}}{4}$, *Newton found* $S = \frac{\pi}{24} - \frac{\sqrt{3}}{32}$. *On comparing the two expressions for S, he obtained*

$$\pi = \frac{3\sqrt{3}}{4} + 24\left[\frac{1}{12} - \frac{1}{5\cdot 2^5} - \frac{1}{28\cdot 2^7} - \frac{1}{72\cdot 2^9} - \cdots\right].$$

Here, 22 terms were sufficient to give him 16 decimal places (the last term was incorrect because of the inevitable round-off error). Since $a_n/a_{n+1} \to 4$, *the method is not suitable for the calculation of* many *significant figures.*

Newton also devised and ingenious algorithm for approximating the roots of a numerical equation, known today as the Newton-Raphson method. To find a root x of an algebraic or transcendental equation $f(x) = 0$, *one starts with a given approximation* x_n *and seeks an improved approximation* x_{n+1}.

Let e_n, e_{n+1} *be the respective errors in* x_n, x_{n+1} *so that*

$$x_n = x + e_n, \qquad x_{n+1} = x + e_{n+1}.$$

Expanding by Taylor's series we get

$$0 = f(x) = f(x_n - e_n) = f(x_n) - e_n f'(x_n) + \frac{1}{2}e_n^2 f''(x_n) - \cdots.$$

If $f'(x_n) \neq 0$ *and if we ignore* e_n^2 *and higher powers we get*

$$e_n \approx \frac{f(x_n)}{f'(x_n)}; \qquad x \approx x_n - \frac{f(x_n)}{f'(x_n)}.$$

It follows that

$$x_{n+1} \approx \left[x_n - \frac{f(x_n)}{f'(x_n)}\right] + e_{n+1}.$$

Discarding e_{n+1} *and setting* x_{n+1} equal *to the first r.h.s. term, one can easily show that* $e_{n+1} = ke_n^2$ *where* $k = \frac{1}{2}\frac{f''(x)}{f'(x)}$, *and therefore negligible, within the limits of the claimed accuracy. All told, an improved approximation to x is then*

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)},$$

which is the Newton-Raphson formula.

It has a simple geometrical *interpretation: draw* $y = f(x)$ *and construct a tangent to the curve at* $x = x_1$. *This tangent intersects the x-axis at* $x = x_2$. *At* $x = x_2$ *erect a line normal to the x-axis and let the line intersect the curve. Draw a new tangent to* $f(x)$ *at this point. It intersects the x-axis at* $x = x_3$, *etc. The process can be repeated and the root of $f(x) = 0$ is approached with great rapidity.*

Joseph Raphson *(1648–1715) published (1690) a tract, Analysis aequatorium universalis which essentially describes the method. Newton's earliest printed account appeared in Wallis' Algebra (1685).*

The chief contribution of ***Wallis*** *to the development of the calculus lay in the theory of integration. The first to realize in full generality that differentiation and integration are reverse operations was* ***Isaac Barrow*** *(1670). He developed a method of determining tangents that closely approached the methods of calculus.*

At this stage of the development of differential and integral calculus many integrations had been performed, many cubatures, quadratures, and rectifications effected, a process of differentiation had been evolved and tangents to many curves constructed, the idea of limits had been conceived, and the fundamental theorem recognized.

What more remained to be done? There still remained the creation of a general symbolism with a systematic set of formal analytical rules, and also a consistent and rigorous redevelopment of the fundamentals of the subject. It is precisely the first of these, the creation of a suitable and workable calculus, that was furnished by Newton and Leibniz, working independently of each other. The redevelopment of the fundamental concepts on an acceptably rigorous basis had to outwait the period of energetic application of the subject, and was the work of the French analyst ***Augustin-Louis Cauchy*** *(1789–1857) and his nineteenth-century successors.*

With the invention of calculus, the history of elementary mathematics had essentially terminated. There remained, however, one special preoccupation which still kept haunting the spirit of mathematicians – the mystique of π!

Ever since the ancient Greek circle-squarer's cult, this number held mathematicians in wonder and awe and each generation devised new representation for it. The calculus served as a novel tool to this end:

John Wallis *started from the equation*

$$\int_0^1 \sqrt{1-x^2}\,dx = \frac{\pi}{4}$$

to obtain

$$\frac{\pi}{2} = \frac{2\cdot 2\cdot 4\cdot 4\cdot 6\cdot 6\cdots}{3\cdot 3\cdot 5\cdot 5\cdot 7\cdot 7\cdots} \qquad (1655)$$

William Brouncker *then followed with a novel expression of his own*

$$\frac{4}{\pi} = 1 + \cfrac{1^2}{2 + \cfrac{3^2}{2 + \cfrac{5^2}{2 + \cfrac{7^2}{2 + \cdots}}}}. \qquad (1660)$$

Newton *used*

$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x$$

to obtain

$$\frac{\pi}{3} = 1 + \frac{1}{(3 \cdot 2^3)} + \frac{1 \cdot 3}{4} \frac{1}{(5 \cdot 2^5)} + \frac{1 \cdot 3 \cdot 5}{4 \cdot 6} \frac{1}{(7 \cdot 2^7)} + \frac{1 \cdot 3 \cdot 5 \cdot 7}{4 \cdot 6 \cdot 8} \frac{1}{(9 \cdot 2^9)} + \cdots \qquad (1665)$$

James Gregory *(1638–1675) appeared on the scene in 1663. A Scott mathematician and astronomer, he was one of the first to distinguish between convergent and divergent series. He expanded (1671) the infinite series*

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots ,$$

which for $x = 1$ *yields*

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

In 1699, the $\tan^{-1} x$ *series was used by him, with* $x = \sqrt{\frac{1}{3}}$*, to evaluate* π *to 71 correct decimal places. Thus Gregory preceded* ***Brook Taylor*** *(1712) in series expansion of a function about a point. Note that the* $\tan^{-1} x$ *series can be alternatively written with* $x = \tan\theta$*, as*

$$\theta = \tan\theta - \frac{1}{3}\tan^3\theta + \frac{1}{5}\tan^5\theta + \cdots$$

and as such, has some computational advantages.

Another important series was discovered by ***Nicolaus Mercator-Kaufmann*** *(1650)*

$$\log_e(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \cdots$$

It is sometimes referred to as the Mercator series. It was independently discovered by ***G. Saint-Vincent*** *(1584–1667).*

The arithmetic, algebra, geometry, and trigonometry ordinarily taught in the schools today, along with college algebra, analytic geometry, and the calculus usually taught during the freshman or sophomore year in college, constitute what is generally called "elementary mathematics".

At this point, then, we have virtually concluded the historical treatment of elementary mathematics in the form that we have it today. It is interesting to note, without carrying the generalization too far, that the sequence of mathematics courses studied in the classroom follows quite closely the evolutionary trend of the subject.

(2) Mathematical Astronomy: orbits and gravitation

Orbits were discussed already by the Greeks, and their method of epicycles is, in fact, an early application of Fourier series. ***Copernicus*** *(1543) proposed openly that the planets and the earth were in circular orbit round the sun. However, astronomical observations soon began to show that his proposal was not strictly accurate.*

In 1609, Kepler showed that a planet moved around the sun in an elliptical orbit which has the sun in one of its focii (First Law). He also showed that a line joining the planet to the sun sweeps out equal areas in equal times (Second Law). These laws were not accepted with enthusiasm by Kepler's peers: the first was given a cool reception and was thought to require further work to confirm it. The second was ignored (!) by scientists for about 80 years. Kepler's Third Law, that the squares of the periods are proportional to the cubes of the mean radii of their orbits (1619), was, however, widely accepted from the time of its publication.

Newton *(1665–6; 'Principia' 1687) suggested, for the first time, that planetary motion is a result of a central force, proportional to the inverse-square distance from the centers of the sun and the planet. He then associated this force with a* universal *law of gravitation. It is possible that* ***Hooke*** *(1679) independently deduced the inverse-square-law, using Huygens' law of centripetal acceleration (1673) and Kepler's Third Law.*

In his Principia *(1687), the problem of two attracting bodies is completely solved, showing that an inverse-square law must produce elliptical, parabolic* *and hyperbolic orbits. The observed parabolic orbit of a bright comet (visible 14 Nov 1680 – 05 Dec 1680) confirmed Newton's theory.*

For more than two point masses, only approximations to the motion of a body could be found and this line of research led to large efforts by mathematicians to develop methods to attack this 3-body problem. *But even if the earth-moon system were considered as a 2-body problem, the orbits could not be simple ellipses; neither the earth nor the moon is a perfect sphere and so does not behave as a point-mass. This was to lead to the development of mechanics of rigid bodies, but even this would not give a complete accurate picture of the 2-body problem since* tidal forces *mean that neither the earth nor the moon are rigid.*

Halley *(1682) calculated the perturbations of Jupiter and Saturn on the orbit of a comet which appeared in 1378, 1456, 1537, 1607 and 1682, and used the Newtonian theory to predict its return on 13 April 1759, giving an error of one month on either side of this date. The comet was indeed observed, reaching perihelion on 12 March 1759.*

Note that the notion that bodies fell to earth owing to some form of attraction exerted by the earth did not originate with Newton. His genius, however, showed itself in extending this idea to the whole universe, formulating his result in a single law, and verifying it by examination of the motion of the planets, comets, the earth and the moon.

Newton was worried that his model of the solar system could become gravitationally unstable in the long run. (He was correct, but this was not proven until 1989). To compensate for the instability, he suggested a cyclic process whereby the planets would be assisted by God when they were periodically perturbed from their orbits by their mutual gravitational action.

The great mathematical physicists of the 18^{th} *century, such as* ***Euler****,* ***Laplace****, and* ***Lagrange****, showed that the solar system was in fact stable to first order, the perturbations which worried Newton leading merely to a* cyclic *oscillation of the planetary orbits. The periods of the oscillations were of the order of a few thousand years, and the astronomers of the* 19^{th} *century concluded that the solar system was stable for at least this length of time.*

More than 300 years after the publication of the Principia, *the full implications of Newton's deceptively simple law of gravity, with its surprisingly complicated consequences, still elude us. One has only to look at the strangeness of a chaotically tumbling satellite like* Hyperion *or at the intrinsic difficulties of calculating the moon's itinerary or delving into the solar system's origin, to sense the dynamical mysteries that confront us. Apparently, even in the classical world God, after all, 'plays dice'. Yet, recent studies of the dynamics of the solar system assure us that its past history certainly suggests that it probably remains stable for geologically significant periods.*

Newton, Shakespeare and the Law of Gravitation

*A sign of **Shakespeare's** (1564–1616) many-sided genius is his anticipation of a scientific vision of later times: **Kepler's** Third Law was discovered in 1618 and Newton's law of universal gravitation was stated by him in 1687. Yet, in Troilus and Cressida (1609) the heroine thus expressed herself (iv.2):*

"Time, force, and death,
Do to this body what extremes you can,
But the strong base and building of my love
Is as the very centre of the earth,
Drawing all things to it."

*Indeed, **Newton** cannot rightly be said to have discovered the law of gravitation; he only applied it to the movements of members of the solar system. Even **Aristotle** had defined weight as "the striving of heavy bodies towards the centre of the earth". Among men of classical culture in England in Shakespeare's time, the knowledge that the centre point of the earth attracts everything to it was quite common. It seems that several of the men whose society Shakespeare frequented were among the most highly-developed intellects of the period. That his astronomical knowledge was not, on the whole, in advance of his times is proved by the expression, "the glorious planet Sol" (Troilus and Cressida i,3). He never got beyond the Ptolemaic system.*

*Another example of this kind concerns the field of geology: **Steno** (1669) first systematized geological conceptions; but he was by no means the first to hold that the earth has been formed little by little, and that it was therefore possible to trace in the record of the rocks the course of the earth's evolution. His chief service lay in directing attention to* stratification, *as affording the best evidence of the processes which have fashioned the crust of the earth.*

In the second part of Henry IV (iii, 1), composed in 1597, King Henry says: –

"O God! that one might read the book of fate,
And see the revolution of the times
Make mountains level, and the continent,
Weary of solid firmness, melt itself
Into the sea! and, other times, to see

The beachy girdle of the ocean
Too wide for Neptune's hips; how chances mock,
And changes fill the cup of alteration
With divers liquors!"

The purport of this passage is simply to show that in nature, as in human life, the law of transformation reigns; but no doubt it is implied that the history of the earth can be read in the earth itself, and that changes occur through upheavals and depressions.

There is nothing in these lines that presupposes any special or technical knowledge; Shakespeare's knowledge was not of a scientific cast. He learned from men and from books with the rapidity of genius. Not, we may be sure, without energetic effort, for nothing can be had for nothing; but the effort of acquisition must have come easy to him, and must have escaped the observation of all around him. There was no time in his life for patient research; he had to devote the best part of his days to the theater, to uneducated and unconsidered players, to entertainments, to the tavern. We may fancy that he must have had himself in mind when, in the introductory scene to Henry V (1598) he makes the Archbishop of Canterbury thus describe his hero, the young king: –

"Hear him but reason in divinity,
And, all-admiring, with an inward wish
You would desire the king were made a prelate:
Hear him debate of commonwealth affairs,
You would say, it hath been all-in-all his study:
List his discourse of war, and you shall hear
A fearful battle render'd you in music:
Turn him to any cause of policy,
The Gordian knot of it he will unloose,
Familiar as his garter; that, when he speaks,
The air, a charter'd libertine, is still,
And the mute wonder lurketh in men's ears,
To steal his sweet and honey'd sentences;
So that the art and practice part of life
Must be the mistress to this theoric:
Which is a wonder, how his grace should glean it,
Since his addiction was to courses vain;
His companies unletter'd, rude, and shallow,
His hours fill'd up with riots, banquets, sports;
And never noted in him any study,
Any retirement, any sequestration
From open haunts and popularity."

To this the Bishop of Ely answers very sagely, "The strawberry grows underneath the nettle." We cannot but conceive, however, that, by a beneficent provision of destiny, Shakespeare's genius found in the highest culture of his day precisely the nourishment it required.

1665 CE First mathematical journal appeared.

1665–1684 CE The Italian family of Cassini produced four generations of astronomers who succeeded each other in official charge of the observatory of Paris. The first was **Giovanni Domenico Cassini** (1625–1712), who first determined the rotation periods of Jupiter, Mars and Venus (1665–1667). During 1671–1684 he discovered 4 Saturnian satellites and in 1675 he found the division in Saturn's ring named after him. Made the earliest sustained observations of the zodiacal light.

Cassini was also the first person to see the Martian polar caps, which bear a striking resemblance to the arctic and antarctic polar caps on earth. (More than a century elapsed, however, before **William Herschel** first suggested that the Martian polar caps are made of ice.)

G.D. Cassini was born near Nice. Educated by Jesuits at Genoa, he was nominated in 1650 professor of astronomy at the University of Bologna. In 1657 he was appointed director of waterways in the papal states by Pope Alexander VII. Louis XIV of France applied for his services in 1669. He died at the Paris Observatory. A partial autobiography was published by his great-grandson, Count Cassini, in 1810.

As the quality of telescopes improved, details of the ring and of Saturn's cloud cover (Huygens, 1655) became visible. In 1675 G.D. Cassini discovered a dark division in the ring that looks like a gap about 5000 km wide. Afterwards, astronomers began to view the ring as a system of rings, known today as the *Cassini division*.

Cassini was also the first to make an indirect measurement of the *solar parallax* by measuring Mars' distance from us, at its nearest approach to earth. This he achieved by obtaining measures of the *parallax of Mars* at the same time from two stations (Paris and Cayenne, South America), widely separated on the earth's surface. A value of $9.5''$ was obtained for the solar parallax.

The *mean distance* of the earth from the sun is the average of major and minor axes of the earth's orbit. This distance is defined by the *solar parallax* which is the *angular size* of the earth's radius as seen from the sun. Since the earth is not quite spherical, the *equatorial radius* is the one used, and because its distance from the sun varies, the *mean radius*, corresponding to the mean distance, is employed. The measured quantity is called the sun's *mean equatorial horizontal parallax* (*horizontal*, because it is the angle between the direction of the sun on the horizon and the direction it would have if viewed from the earth's center), or simply a *geocentric parallax*, on the baseline of the earth's radius.

Unfortunately, this parallax is hard to measure directly on account of the great distance to the sun, which makes it less than $9''$. **Aristarchos**, **Kepler** and **Huygens** tried to measure it directly, but obtained very inaccurate results.

Cassini's geometrical method of triangulation was an ingenious way to circumvent a frontal attack: measure first the parallax of Mars at a smaller distance from the sun: every 15 or 17 years, Mars comes to a point where it is nearest to us. At its nearest, Mars' distance from us is little more than $\frac{1}{3}$ of our distance from the sun, and its geocentric parallax is $23''$.

Minor planets are even better subjects than Mars; they have smaller images, and some of them approach nearer than Mars. From all geometrical measures of the solar parallax, the mean value is calculated to be $8''.803 \pm 0.001$. In combination with the best value for the earth's equatorial radius, this gives for our mean distance from the sun the value 1 AU = 149,459,000 $\pm$ 17,000 km. Other methods, based on dynamical and spectroscopic determinations of the solar parallax are in close agreement.

His son **Jacques Cassini** (1677–1756) was born at the Paris Observatory, as was Jacques' son, **César Francois Cassini** (1714–1784), as well as the fourth Cassini, **Jacques Dominique Cassini** (1748–1845). He succeeded in 1784 to the directorate of the observatory, but his plans for its restoration and re-equipment were obstructed in 1793 by the animosity of the National Assembly. He resigned in that year and was thrown into prison in 1794, but released after several months. He then withdrew to his estate at Thury and died there at the age of 97.

1666 CE *Foundation of the French Academy of Sciences in Paris*. The academy arose from an association of a group which used to meet at the cell of **Mersenne** (1588–1648), a man active in spreading the teaching of Galileo. The original members included **Descartes**, **Pascal**, **Fermat** and **Gassendi** (whose commentaries on Epicuros revived the atomistic speculations of the early Greek materialists).

The Paris Academy, like the English Royal Society, was actively interested in all problems related to navigation, then the cornerstone of mercantile supremacy. Under its auspices, the Paris observatory was inaugurated and completed 3 years before the one at Greenwich. A rich harvest of discoveries followed immediately. To Paris came Cassini from Italy and **Römer** from Denmark. Cassini undertook the calculation of tables forecasting eclipses of Jupiter's satellites for use in determining longitude at sea (the project was undertaken in accordance with a suggestion made by Galileo himself).

The determination of longitudes by eclipses of Jupiter's satellites merely depend on the known fact that the same event does not occur at the same solar time in two places on different meridians of longitude. The tables that Cassini prepared for calculating longitude by observations of the satellites of Jupiter, were used by the French Navy during the first half of the 18^{th} century.

The academy sponsored several expeditions, notably one to French Guiana with a view to simultaneous observations on the parallax of Mars from the Paris observatory and Cayenne (Lat. $4°46'$N). This expedition, which gave the first relatively satisfactory scale of the solar system, ushered in a new era in clock technology.

1666–1686 CE **Thomas Sydenham** (1624–1689, England). Physician. A founder of clinical medicine and epidemiology. Often called "the English Hippocrates". Believed and taught that medicine could be learned only at the bedside of the patient. He was a keen observer and gave excellent descriptions of gout, scarlet fever, measles, influenza, smallpox, malaria and hysteria. He had great faith in the healing power of nature, and he felt that fever was nature's way of fighting the injurious matter that caused disease.

Sydenham introduced *opium* into medical practice and adopted *quinine* for the treatment of fevers at the time when many doctors opposed this new drug. He was one of the first to use iron in treating anemia. Studied epidemics in relation to different seasons, years, and ages. Insisted on clinical observations instead of theory.

Sydenham was born at Wynford Eagle, Dorset, and studied medicine (1642–1663) at the universities of Oxford, Cambridge and Montpellier. Served in parliamentary forces in the Civil War. Success came slowly to him, but eventually he gained recognition as one of the great doctors of his time.

1667 CE, July 21 *Treaty of Breda* to end the 'Musk-Seed War' between England, Holland, France and Denmark. It ended a long war over Far-East spice routes between the East-India companies of the respective countries. England retained New Amsterdam (later, *New York*) and Holland got Surinam.

1667–1704 CE **Francis Willughby** (1635–1622, England) and **John Ray** (1627–1705, England). Naturalists. Their plant and animal classification were the first significant attempts since **Aristotle** (334 BCE) to produce systematic taxonomy based on a variety of structural characteristics, including internal anatomy.

From 1663 to 1666 they toured Europe to study flora and fauna and collect specimens.

After the death of Willughby (1672), Ray completed the three-volume *Historica Generalis Planetarium* (1704) in which he attempted to produce an extensive botanical classification based on a scheme of Aristotle but incorporating many of the new plant forms discovered on the 16 th and 17 th century voyagers of discovery. Altogether, 18,600 European species were covered.

Although it was not possible to devise a natural classification system until **Charles Darwin** and **Alfred Wallace** formulated evolutionary theory (1859), Ray's system approached that ideal more closely than those of any of his contemporaries and remained the best attempt at classification until superseded by **Linneaus**' taxonomic work (1735).

Ray was born in Black Notley, near Braintree, Essex. He was educated at Cambridge and was appointed lecturer in Greek (1651), mathematics (1653) and humanity (1655). He was elected FRS in 1667.

1668 CE **John Pell** (1611–1685, England). A scholar whose contributions to mathematics were worthless, but who had the good fortune to propagate his name through the "*Pell* (*or Pellian*) *equation*" erroneously named after him by **Euler** (1759) [some claim that Pell never saw his equation].

Pell was a professor of mathematics at the University of Amsterdam (1643–1646) and a fellow of the Royal Society (1663) [his output is still carefully preserved in the form of 40 folio volumes in the British Museum]. Both **Newton** and **Leibniz** were happy to discuss their latest researches with him, and Oliver Cromwell made him his political emissary to the Protestant cantons of Switzerland. It is not clear today how he earned his reputation as a mathematician. It is known however that for a time he was confined as a debtor in the king's bench prison. Pell died in abject poverty at the College of Physicians in London.

The so-called *Pellian* is the non-linear Diophantine equation[133]

$$x^2 - Ny^2 = 1,$$

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[133] For further reading, see:

- Beiler, A.H., *Recreations in the Theory of Numbers* (The Queen of Mathematics Entertains), Dover Publications: New York, 1964, 349 pp.

where N is not a perfect square natural number and (x, y) are integers. References to individual cases of this equation occur scattered throughout the history of mathematics. The Greeks and the Hindus of ca 400 BCE realized that a/b was a good approximation to $\sqrt{2}$ when $a^2 - 2b^2 = \pm 1$, an equation which, unlike $a^2 - 2b^2 = 0$, is solvable by integers.

The most curious of these occurrences is the so-called Cattle-Problem (Problema Bovinum) of **Archimedes** (ca 250 BCE). It contains eight unknowns (numbers of cattle of various kinds) which satisfy 7 linear equations together with 2 conditions which assert that certain numbers are perfect squares. After some elementary algebra, the problem reduces to that of solving the equation $x^2 - (4,729,494)y^2 = 1$, the least solution of which is a number y of 41 digits. The least solution of the original problem, deduced from this, consist of numbers with hundreds of thousands of digits[134].

By 130 *CE*, **Theon of Smyrna** had shown how to find infinitely many approximations to $x^2 - 2y^2 = 1$. The Hindu mathematicians of about 800 *CE* also claimed to have known how to solve equations of this type. Later, ca 1150, a completely general method was given by the Hindu mathematician **Bháskara**. In modern times, **Fermat** seems to have been the first to state categorically that there are infinitely many solutions to the Pellian. In fact, in 1657 he challenged all European mathematicians to solve $x^2 - 109y^2 = 1$ [there was a slim chance that anybody at that time could have found even the least solution to that equation, since we know now that it is $y = 15, 140, 424, 455, 100$].

Nevertheless, **William Brouncker** (1620–1684, Ireland) discovered a method to solve $x^2 - 313y^2 = 1$, which is essentially the continued fraction method. **Euler** showed in 1759 how to obtain infinitely many solutions of the *general* Pellian by using the continued fraction[135] expansion of $\sqrt{N}$,

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[134] A sphere with the radius of the Milky way could not contain all the cattle even if they were of the size of electrons.

[135] The theory of continued fractions shows that a particular solution of

$$x^2 - Ny^2 = 1$$

is $x = p_n$, $y = q_n$, where p_n/q_n is a certain convergent of $\sqrt{N}$. Moreover, from one solution an infinite number of solutions may be found. Thus from the least-values $x = 3$, $y = 2$ of $x^2 - 2y^2 = 1$ one derives

$$x_n = \frac{1}{2}[(3 + 2\sqrt{2})^n + (3 - 2\sqrt{2})^n];$$

$$y_n = \frac{1}{2}[(3 + 2\sqrt{2})^n - (3 - 2\sqrt{2})^n]\frac{1}{\sqrt{2}}.$$

without giving a proof. This was left to **Lagrange** in 1768.

1668 CE **Francesco Redi** (1626–1697, Italy). Physician, naturalist and poet. One of the first to test scientifically *the theory of spontaneous generation*; showed that no maggots developed in meat protected so that flies could not lay their eggs on it.

1668–1674 CE **John Mayow** (1641–1679, England). Chemist, physician and physiologist. His studies on *respiration* show him to have been an investigator much ahead of his time. In *Tractatus quineue medicophysici* he noted the similarities between combustion and respiration in particular that both use only a small proportion of the available air. He suggested that in respiration the volume of air is reduced, but air must consist of two different gases.

Mayow was born in London and studied Medicine and Law at Oxford, graduating in 1670. He practiced medicine in the City of Bath and was chosen a fellow of the Royal Society (1678). The following year, after his marriage, he died in London.

Mayow, who also gave a remarkably correct anatomical description of the mechanism of respiration, preceded **Priestley** and **Lavoisier** by a century in recognizing the existence of *oxygen* (under the guise of his *spiritus nitro-ereus*) as a separate entity distinct from the general mass of the air; he perceived the part it played in combustion and in increasing the weight of the calces of metals as compared with metals themselves.

Rejecting the common notion of his time that the use of breathing is to cool the heart, or assist the passage of the blood from the right to the left side of the heart, he saw in inspiration a mechanism for introducing oxygen into the body, where it is consumed for the production of heat and muscular activity. His extent of influence on **Lavoisier** is the subject of debate.

1668–1692 CE **Anton van Leeuwenhoek** (1632–1723, Holland). Amateur scientist who, in retrospect, deserves to be called the father of *microbiology*. He was the first in his field to leave written records of his findings. Through tedious hand-grinding techniques, Leeuwenhoek prepared hundreds of lenses with magnifying powers up to 300 and resolutions of about 10^{-3} millimeters. With these lenses he studied such diverse materials as stagnant water (protozoa, 1676), blood cells (he discovered and described the *human*[136]

[136] Red blood cells in *frogs* were observed (via microscope) and described in 1658 by **Jan Swammerdam** (1637–1680, Holland). He also described the *metamorphosis of insects* (1669).

red blood corpuscles[137] in 1673), muscle fibers, and spermatozoa. His *animalcules* (tiny animals) were described as being 1000 times smaller than the eye of a louse (which he used as a standard measurement because its size is remarkably constant). He opposed the popular notion that living things can arise from dead matter.

Leeuwenhoek[138] was among the first to estimate the maximal number of people that the earth could support: he argued that if the whole planet were inhabited with the same population density as that of Holland, the number would reach about 13.4×10^9 individuals. This coincides with one of the present estimates.

Revival of the Medical Sciences

Many of the medical works of the ancient Greeks and **Galen** *reached Western Europe by a roundabout route that took centuries to complete.*

In 431 CE*, the Church banished Nestorius, the heretical patriarch of Constantinople, with his followers. Their descendants fled to Persia where, at Jundi Shapur, they made the university and its medical school and hospital a leading intellectual center. There the Nestorians translated into Syriac all*

[137] **Leeuwenhoek** had a very good idea of the size of a human blood corpuscle (ca 7.5×10^{-6} m). He also was aware of the fact that the blood corpuscle's size was not growing with the size of the animal. It was later found that it is related to the animal's *activity*, namely to its oxygen intake. We know today that the shape of the platelets evolved to meet three criteria: maximum volume, maximum diffusion-rate and maximal flexibility; hence the special disk-like shape which looks like a cross between pancake and doughnut.

[138] There is a fascinating biography of Leeuwenhoek by **Clifford Dobell**. As a young bacteriologist, Dobell was especially interested in studying the microbial flora of the mouth. However, each time he presented his professor with what he thought was the discovery of a new type of microbe, his professor would shake his head and respond, "No, no, Leeuwenhoek already discovered that one." Finally, motivated by a mixture of curiosity and skepticism, he decided to find out more about this man Leeuwenhoek. After 25 years of painstaking research, Dobell published a truly inspiring biography of Leeuwenhoek in 1932.

the Greek books they could find, including the Hippocratic Corpus and the works of ***Galen****.*

With the rise of Islam in the 7^{th} *century, medical schools spread. In the Eastern Caliphate of Baghdad, Muslim scholars and physicians continued to translate Greek works, adding their own commentaries.*

Islam spread through North Africa and into Spain and south-west France, until it was stopped by the Christians at the Battle of Tours (732 CE*). The Western Caliphate was centered on the Spanish city of Cordova, which had* 50 *hospitals,* 70 *public libraries and the most renowned university of the* 10^{th} *century.*

Abu al-Qasim, Khalaf *(Abul Kasim,* ***Albucasis*** *(936–1013), Spanish-Arab physician, was one of the greatest surgeons of the Middle Ages. Born in Cordova. Wrote al-Tasrif, a medical compendium partly based on earlier authors, but containing new material including remarkable illustrations and surgical instructions; This work greatly influenced European surgery for* 500 *years.*

Ibn Zuhr *(****Avenzoar****, 1090–1162), Muslim physician and greatest clinician of Western Caliphate. Born in Seville; his at-Taysir was influential throughout Europe in Latin and Hebrew translations.*

Maimonides *(1135–1204) became physician to the Saracen sultan, Saladin, whose crusader foe, Richard Coeur de Lion of England, tried in vain to secure the Jewish doctor's services. Maimonides studied the patient, not the disease; he also rejected astrology and attempted to separate medicine from religion.*

The works of these men and the works of ***Galen*** *were first translated from Arabic into Latin already in the* 11^{th} *century. The translation was continued in the* 12^{th} *century, when Galen was translated from the Greek. It marked the beginning of the Western rediscovery of the original ancient texts, to be later continued by the humanists of the Renaissance.*

In the Middle Ages, medical scholars were again carrying out human dissection (with ecclesiastical permission), but only rarely — and in circumstances hardly conductive to learning. The professor, in long robes, sat on high in a great chair reading his anatomy lecture, with the cadaver on a table below him. A junior colleague pointed out the line of incision, and a third — the menial demonstrator — did the actual cutting.

The new teaching methods of ***Mondino dei Liucci*** *(1270–1326, Italy), who taught at the University of Bologna, were a major advance and soon spread to other medical schools. His anatomy textbook, the Anathomia (1316), although full of inaccuracies, passed down from Galen and Avicenna,* *is considered the first modern work on the subject and remained authoritative until the appearance of Vesalius' anatomical work (1543).*

In Italy, ***Leonardo da Vinci*** *(1452–1519), among others, spearheaded the new interest in anatomy. Dissecting in the secret of the night, he reproduced exactly what he saw, in 750 anatomical drawings. Unfortunately, his pioneering work remained hidden for more than 300 years, and others had to forge on without knowledge of his discoveries.*

In 1543, ***Andreas Vesalius****, a 29-year-old Flemish professor of anatomy at Padua, published* De Humani Corporis Fabrica *(The Fabric of the Human Body). Like Mondino, he dissected personally, and his work showed, for the first time, how nerves penetrated muscles, the nutrition of bones, the true relationship of the abdominal organs and the structure of the brain.*

Two of Vesalius' assistants also made major findings and are among the founders of modern anatomy: ***Gabriele Fallopio*** *(1523–1562) described the internal working of the ear, the anatomy of bones and muscles, and the sex organs: the tubes leading from the ovaries to the uterus are named after him.*

Bartolomeo Eustachi *(1520–1574) studied the kidneys and the head, describing the anatomy of the teeth and, in particular, the 'Eustachian tubes' from the throat to the middle ear and the 'Eustachian valve' in the heart.*

The true circulation of the blood continued to elude these pioneers. However, ***Miguel Serveto*** *(known as Michael Servetus; also used pseudonyms Michael de Villeneuve and Villanovanus, 1511–1553), a Spanish theologician and physician. Lectured on geography and astronomy; practiced medicine at Charlieu and Vienna (1538–1553). He included the first description of the circulation of the blood in the lungs in a theological work entitled* Christianismi restituto *(1553). For that he was arrested and brought to trial before the Inquisition at Lyons; he escaped, but was apprehended at Geneva; imprisoned at Calvin's request and burned at the stake as heretic.*

Andrea Cesalpino *(1519–1603, Italy), physiologist and botanist, stumbled across not only the pulmonary circulation but the systemic circulation as well (1583). Cesalpino was a professor of materia medica and director of the Pisa botanical gardens, physician to Pope Clement VIII and professor at Rome. [He wrote the first true textbook of botany and created first coherent system of taxonomy, to which Linnaeus acknowledged indebtedness.]*

During the Renaissance, surgery made great progress, while medicine remained the province of book-oriented physicians. Many fine surgeons of the Middle Ages had gained experience and knowledge on the battlefields of Europe; for example, the British surgeon ***John Arderne*** *(1307–1390), who served in the* Hundred Year's War *— and dealt not only with slashes and* *punctures from sword and lance, but also with gaping, dirt-filled wounds caused by bullets from the newly invented guns and artillery.*

As the Black Death swept through Italy in 1347 *and* 1348*, taking its terrible toll, the pretensions of the physicians and barber-surgeons were stripped bare*[139]*. But within* 75 *years or so, there was a new air of inquiry in medicine, as the Renaissance began.*

To bolster up their status, physicians created professional structures for themselves, in order to prevent anyone not properly trained (and, in effect, women and Jews) from practicing. Thus, in 1518*, six prominent physicians in London were granted a charter by the King to form the Royal College of Physicians; it could license doctors, and prosecute, fine and imprison, unlicensed practitioners.*

***Philipp Aurelus Theophrastus Bombastus von Hohenheim** (1493–1541) was a German alchemist and physician who had styled himself **Paracelsus** — implying that he was greater than the great Roman encyclopedist **Celsus**. He had earned his niche in medical history as standard bearer for freedom of scientific inquiry, the central position of the patient in medicine and above all a forerunner of pharmaceutical chemistry. His opinion of the state of contemporary medicine is reflected in his conclusions:*

- *"When I saw that nothing resulted from doctor's practice but killing and laming, that they deemed most complaints incurable... I determined to abandon such a miserable art and seek truth elsewhere".*
- *"The best of our popular physicians are the ones who do the least harm. But unfortunately some poison their patients with mercury, and others purge or bleed them to death. There are some who have learned so much that their learning has driven out all their common sense".*

To emphasize his point, he pitched the books of Galen, Avicenna and other masters of medieval medicine on to a bonfire in a public square.

Paracelsus, in many ways ahead of his time, believed in the power of nature and the imagination to cure the body and the mind. The patient had to be treated as a whole: diet, surroundings, the behavior of doctor and carers — all these and more could have a profound effect on recovery. In his own words:

- *"Medicine does not consist of compounding pills and drugs of all kinds, but it deals with the processes of life, which must be understood before they can be guided".*

[139] Small wonder that Shakespeare wrote in *Timons of Athens* (1607) — "*Trust not the physician, His antidotes are poison*".

The 15^{th}- and 16^{th}-centuries 'Rebirth', or Renaissance, in medicine centered on the rediscovery of the ancient Greek and Roman works in their original form and the discovery of the fabric of the body.

On this more solid basis, medicine grew into the age of enlightenment — two centuries that saw Britain's Glorious Revolution (1688), America's Declaration of Independence (1776) and the French Revolution (1789). But these events did not occur in a vacuum. They were reflections of an attitude of mind: a rejection of social and religious constraints; a belief that progress in science and technology would lead to a utopian existence; and, as far as medicine is concerned, a determination that one day all diseases would be conquered — or so they were convinced.

Scientific and technological progress certainly played its part in medicine, as Galen's erroneous theories were finally overthrown, the circulation of the blood was understood, microbes were revealed by the microscope, and smallpox vaccination was introduced.

Galen believed that the body daily manufactures and eliminates large quantities of blood. In 1628 *this theory was overthrown by the British doctor* ***William Harvey*** *(1578–1657). Actually, Harvey made his discovery already in* 1603*, but delayed the publication of his results because he was not sure about the reaction of the medical establishment and needed more time to design a striking experimental proof.*

In 1661, ***Robert Boyle*** *(1627–1691) rejected Aristotle's* 4*-elements and instead proposed an experimental theory of the elements, thus transforming alchemy into scientific chemistry. In addition he revealed that air was necessary for life. In* 1667*, Boyle's former assistant* ***Robert Hooke*** *(1635–1703) demonstrated that the key to respiration was the alteration of blood in the lungs.*

At the end of the 16^{th} century, the compound microscope was discovered by the Dutch spectacle makers ***Hans*** *and* ***Zacharias Jansen****. Using one, Robert Hooke first described cells, and in* 1660*, an Italian,* ***Marcello Malpighi*** *(1628–1694), discovered the missing link in Harvey's theory: the tiny capillaries that connect the arteries and veins. However, it was a Delft draper,* ***Anton van Leeuwenhoek*** *(1632–1723), who popularized the medical use of the microscope, describing spermatozoa, red corpuscles and stripped voluntary muscles, as well as protozoa and bacteria.*

In 1709, ***Gabriel Daniel Fahrenheit*** *(1686–1736), a German physicist, invented the alcohol thermometer and, five years later, the mercury thermometer and a temperature scale that stood medicine in good stead for almost three centuries.*

Gradually, scientific and medical research ceased to be an activity of isolated men of genius, but more organized in academies — such as the Royal Society in London (1660) [which grew out of informal tavern meetings of a group Boyle called the 'Invisible College'; the Accademia dei Lincei in Rome (1661) and the Académie Royale des Sciences in Paris (1666).

1669 CE **Erasmus Bartholinus** (1625–1698, Denmark). Physician, mathematician and physicist. Discovered the phenomenon of *double refraction* of light in an anisotropic crystalline substance. Bartholinus obtained some beautiful crystals from a sailor who collected them in Iceland, and when he viewed small objects through them, he found that the objects appeared double.

Bartholinus discovered the origin of this phenomenon[140]; if one sends a narrow beam of light — a light ray — into an ordinary transparent medium such as a piece of glass, it is refracted and then proceeds as a single beam. However, when it is refracted at the face of the Iceland spar (calcite, $CaCO_3$; an *anisotropic* trigonal system) a second beam is generated, and this is the reason for the appearance of a second image. Bartholinus suggested that one of the rays, which resembles the usual one in some ways, be called the *ordinary* ray and the other one, which behaves in a somewhat unusual fashion, be called the *extraordinary* ray.

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[140] **Huygens** contributed to the understanding of double refraction. In his book *Traité de la Lumière* (1690) he assumed that when light is incident on the Iceland spar, each element of it produces secondary waves surfaces which are no longer spherical but rather consist of two geometrical surfaces (sheets); one of the sheets is again spherical and is associated with the ordinary rays. The other sheet has the form of an ellipsoid and is associated with the extraordinary rays. Huygens' treatment is rather incomplete, and while of appealing form, is deceptively simple. It presupposes that a diverging bundle of rays which originates from a point source behaves in the same way as a system of mutually independent plane waves. **Lamé** (1852) was the first to recognize that this presents a mathematical problem of wave propagation in an *anisotropic* medium, which is by no means simple. In addition, the existence of double refraction posed difficulties for contemporary theories of light, and was not explained until the early 1800's.

Bartholinus noted that rotating the crystal will cause one image to remain stationary while the other appears to move in a circle about it, following the motion of the crystal.

He was born at Roskilde. His father Gaspard (1585–1629) was a well-known physician and a professor of medicine at Copenhagen. Bartholinus spent 10 years visiting England, Holland, Germany and Italy, and later filled the chairs of mathematics and medicine at Copenhagen. He was **Römer**'s father-in-law.

1669 CE **Nicolaus Steno** (Niels Stenson, 1638–1686, Denmark, Italy). Danish-born naturalist and physician. The first scientist to notice that the horizontal stratification of rocks holds the key to their history. Made the first clear statement that layered rocks show sequential changes and thus laid the foundation to the time-stratigraphic record.

His fame rests on *De solido intra solidum naturaliter contento*, published at Florence in 1669. From his work on the mountains of Western Italy, Steno realized that the principle of *superposition* in the stratified rocks was the essential key. Steno also realized the importance of another principle — *original horizontality* — namely, that strata are always initially deposited nearly horizontally although they may be found dipping steeply. In his book, Steno described various gems, minerals and *fossils* enclosed within solid rocks. He found that the angles between the faces of quartz crystals were the same even though the crystals had different shapes.

Steno was born in Copenhagen and studied medicine and anatomy there and in Paris. After a period of travel he settled in Italy (1666), at first as professor of anatomy at Padua, and then in Florence as house-physician to grand-duke Ferdinand II of Tuscany. He returned to his native city in 1672, but left again for Florence and was ultimately made apostolic vicar of Lower Saxony. He died at Schwerin in Mecklenburg.

1670 CE **Gabriel Mouton** (1618–1694, France). Mathematician. First to suggest the metric system, the decimal system, and the treatment of series by the method of finite differences ahead of Leibniz (1673).

Mouton was born in Lyon, took the holy orders and spent his whole career as the vicar of the Church of St. Paul in Lyon. His most famous work *Observationes* (1670) studied *interpolation*. His methods of interpolation were similar to those used by Briggs in the construction of his log tables. He produced 10 place tables of logarithmic sines and cosines and an astronomical pendulum of remarkable precision. He suggested (1670) a standard unit of length based on the length of the arc of one minute of longitude on the earth's surface and divided decimally.

1671 CE **Jean Richer** (1630–1696, France) and **Giovanni Domenico Cassini** (1625–1712, France-Italy) measured the scale of the solar system (earth-sun distance) from the parallax of Mars at Cayenne and Paris. Their result was about 10 million km short of the actual figure.

Richer's second important work was to examine the periods of pendulums at different point on the earth. He examined the period of a pendulum at Cayenne and found that it beat more slowly than in Paris. From this he deduced that gravity was weaker at Cayenne, so it was further from the center of the earth than was Paris.

History of measurements of absolute distances on earth and inside the Solar System (ca 585 BCE–1671 CE)

Thales *(fl. 585 BCE) measured the height of the Pyramid of Cheops (146 m) by measuring the length of its shadow at a time when the height of a nearby stick of known length was equal to the length of its shadow.*

Eratosthenes *(ca 235 BCE) estimated the length of the earth's circumference to be 160 km in excess of the present accepted value: he found that when the sun was overhead at Syene (Aswan), it was about 7° from the vertical in Alexandria, about 800 km away. Assuming the sun rays to arrive almost parallel to both places, the circumference is* $\approx \frac{360}{7} \times 800 = 40,000$ *km. From the circumference, the diameter of the earth can be calculated by the familiar formula* $D = \frac{40,000}{\pi}$. *Taking Euclid's value* $\pi = 3\frac{1}{7}$, *one finds* $D = 12,700$ *km.*

There is a beautiful simplicity about the method which Eratosthenes used. It invokes no mathematical principles which had not been current in the Greek-speaking world two centuries before his time; and its importance to posterity lies less in any direct impetus to theoretical inquiry than to the fact that it provided an indispensable basis for any successful attempt to measure the distance of the earth from the sun or the moon, as attempted by his contemporary ***Aristarchos*** *(ca 280–240 BCE) with inadequate information.*

At about 150 BCE, Alexandrian astronomy and geography received an enormous impetus from the work of three contemporaries: ***Hypiscles***, ***Hipparchos*** *and* ***Marinos***. *The first introduced the Babylonian system of angular measurement (360° to a full circle) and the sexagesimal fractions.*

Hipparchos introduced a new system of mapping the position of stars by guide lines comparable to our familiar system of terrestrial latitude and longitude. To this end he needed, and had indeed constructed, a table of trigonometrical ratios (probably by the half-angle formulae $\cos\frac{A}{2} = \sqrt{\frac{1+\cos A}{2}}$*).*

Marinos is notable as the first to introduce the use of circles of latitude and longitude to map the habitable globe as then known.

Now, in 130 BCE, Hipparchos made an observation at Rhodes, from which he obtained a remarkable accurate estimate of the earth-moon distance. His method has been suggested by ***Aristarchos****, about 150 years later.*

The method involves a clear understanding of the positional relationship of sun, earth, and moon. First, he knew that sun and moon subtended almost exactly the same angle α *at the earth. Hipparchos measured this angle to be* $0.553°$ *(* $\approx \frac{1}{103.5}$ *radian); he also knew what Aristarchos before him had found — that the sun is far more distant than the moon. Hipparchos used this knowledge in an analysis of an* eclipse of the moon by the earth: *Assume that centers of the sun, earth and moon (in this order) are collinear, and that the rays coming from the extreme edges of the sun and tangent to the earth, cut the moon's circular orbit at two points* A *and* B*. Let the angle subtended between these two boundary rays be* α*. The moon passes through the shadow from* A *to* B*, and from the measured time that passage took, Hipparchos deduced that the angle subtended at the earth's center by the arc* BA *was* $2.5\ \alpha$*. The rest is simple geometry: if the distance from the earth's center to the moon is* D*, the length of the arc* AB *is* $\overline{AB} = 2R_E - \alpha D$ *(*$R_E =$ *earth's radius). Also* $\overline{AB}/D = 2.5\alpha$*. With* $\alpha = \frac{1}{103.5}$*, Hipparchos found* $D/R_E \approx 59$*.*

Nothing new happened in the field of distance measurements in the solar system until 1671 CE. In that year, ***Domenico Cassini*** *and* ***Jean Richer*** *measured for the first time an absolute earth-sun distance from the* parallax of Mars *at Cayenne and Paris. Using a* Galilean telescope *and the theory of* ***Kepler****, their result was 10 million km short of the actual figure of about 150 million km.*

1672–1715 CE **Gottfried Wilhelm von Leibniz** (1646–1716, Germany). Mathematician, logician, scientist, philosopher, theologian, jurist and diplomat — a Universal man with a wide range of interests, who deliberately ignored boundaries between different disciplines and believed in cross-fertilization of ideas, which he saw as essential to the advance both of knowledge and wisdom. Leibniz's most immediate influence was as a mathematician. His philosophical influence was rather less direct. As a logician he was far ahead of his time.

His contributions to the various disciplines are:

Mathematics (1672–1700)

- Discovered the equations for the curves known as *catenary*, *isochrone* and *brachistochrone*. Laid (1694) the foundations of the theory of *envelopes*[141].
- Discovered the basic principles of topology, for which he coined the Latin name: *analysis situs*. He saw it as complementing the analytic geometry of Fermat and Descartes. His ideas in this field remained dormant until the 19^{th} century.

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141 Previously, **Huygens** (1673) originated the idea of *evolutes* of plane curves (envelope of normals to a given curve). However, the concept may be traced to **Apollonios** (ca 200 BCE) where it appears in the fifth book of his *Conic Sections*.

While **Leibniz** (1694) and **B. Taylor** (1715) were first to encounter singular solutions of differential equations, the geometrical significance of envelopes was first indicated by **Lagrange** (1774). Particular studies were made by **A. Cayley** (1872) and **G.W. Hill** (1888).

The envelopes of a family of plane curves $f(x, y; \alpha) = 0$ are determined by the elimination of α between the simultaneous equations $f(x, y; \alpha) = 0$ and $\frac{\partial}{\partial\alpha} f(x, y; \alpha) = 0$.

If a family of curves is given in terms of *two* parameters by the equations $f(x, y; \alpha, \beta) = 0 \quad g(\alpha, \beta) = 0$, the envelopes of this system are determined with the aid of the third equation $\frac{\partial f}{\partial\alpha}\frac{\partial g}{\partial\beta} - \frac{\partial g}{\partial\alpha}\frac{\partial f}{\partial\beta} = 0$.

Examples:

- The family of straight lines $x\cos\alpha + y\sin\alpha - p = 0$. Differentiation of this equation w.r.t. α yields $-x\sin\alpha + y\cos\alpha = 0$. The elimination of α between them reveals that the envelope is the circle $x^2 + y^2 = p^2$.
- The trajectory of shells fired from a gun at velocity v_0 at angular elevation α with the horizon, is given by the parabola $y = x\tan\alpha - \big(\frac{g}{2v_0^2\cos^2\alpha}\big)x^2$. The envelope of all trajectories is the *safety parabola* $y = \frac{1}{4a} - ax^2$, where $a = \frac{g}{2v_0^2}$. No point outside it is within reach of shells fired from a gun with velocity v_0.
- Consider the envelope of a line of constant length moving with its ends upon the two coordinate axes: $\frac{x}{a} + \frac{y}{b} = 1$ where $a^2 + b^2 = 1$. The resulting envelope is the *astroid* $x^{2/3} + y^{2/3} = 1$.

- Improving on Pascal's calculating machine, he devised one which performed the four fundamental operations and also extracted roots (1673).
- Invented *binary arithmetic*[142] (1700). He failed, however, to generalize it into a theory of modular arithmetic with its own special theorems; nor did he try to design a calculating machine which used it[143]. [Leibniz was not the first to discover the binary system. It has already been thought of by **Thomas Harriot** early in the century.]
- Discovered the infinite series representation for π:

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots .$$

  In his efforts to sum these series (squaring the circle!), he discovered the 'Leibniz test' for convergence.
- Discovered the infinitesimal calculus independent of Newton[144] (1673). His discovery arose from the concept of an infinite series converging to a

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142 After thinking that he had invented binary numerals, Leibniz was astonished to find that an ancient Chinese book, the *I Ching*, contained a set of numbered figures, called *hexagrams*. Each *hexagram* consists of 6 lines, each of which is either solid or broken. The *hexagrams* are related to the binary sequence in a simple way.

143 It may seem odd to us, in the age of the computer, that someone who invented both a calculator *and* binary arithmetic should not have put the two together, and come up with something closer in principle to the modern computer. But in the context of the technology of that time, a binary machine would only increase Leibniz's difficulties. There would have been more wheels, more friction, more carrying, and there would have had to be an extra mechanism for translating between binary and decimal, in order to make the calculator usable by ordinary people. The binary system came into its own only with the advent of electronics. As far as Leibniz was concerned, the greatest significance of his discovery was metaphysical, or indeed mystical, as showing how the whole universe could be seen as constructed out of number.

144 Newton beat him to it by 9 years, though Leibniz was first to publish the discovery in 1684. If matters had rested with Newton and Leibniz, there would have been no quarrel between them. But early in the 1700's, their supporters on opposite sides of the channel started squabbling about the respective merits of the two systems and about the priority of their discovery. Newton and Leibniz were soon drawn into the dispute, which became unpleasantly acrimonious. In particular, Leibniz had to defend himself against charges of plagiarizing Newton's letters during the early 70's, and of subsequently tampering with the evidence. It is beyond reasonable doubt that Leibniz's discovery was in fact independent, but the nationalistic fervor aroused by the dispute, and the incontrovertible

limit: the differential calculus was a technique for determining the *limit* of such a series, and the integral calculus for finding its *sum*. But he never thought of the *derivative* as a limit.

Newton's approach was basically geometrical and his notation clumsy and difficult to work with. Leibniz's approach was algebraic, introducing such new notions as: *differential* and *function*. His notation, which we still use today, was clear and elegant. it was based on the letter for 'difference' (as in $\frac{dy}{dx}$), and the contemporary long s ($\int$) for 'sum', or integral[145].

- Made important contributions to the theory of determinants, the calculus of finite differences, and the theory of numbers[146].
- In a letter to Huygens (first published in 1833) he discussed the possibility of creating a system which would serve as a direct method of space analysis. It can be ranked as the first conceptual forerunner of vector analysis.

Mechanics (1671–1695)

While Newton proposed to measure motion by momentum, Leibniz argued for another quantity, the "*vis viva*", which — except for the factor $\frac{1}{2}$ — is identical with our "*kinetic energy*". Leibniz replaced the Newtonian equation by the equation that "the change of kinetic energy is equal to the work done by the force". The ideas of Leibniz were in harmony with later developments in analytical mechanics. Both the kinetic energy and the work of the acting forces could easily be generalized from one single particle to an arbitrary system of particles. The work of the forces could be replaced by another more fundamental quantity, the negative of the "*potential energy*" (a term coined by **W.J.M. Rankine** in 1853). Both kinetic and potential energy were quantities which could characterize a system *as a whole*, and later became essential in the variational formulation of the laws of mechanics.

Symbolic logic (1666–1696)

At the age of 20, he earned the right to teach at the University of Leipzig with his paper entitled: "*Dissertatio de Arte Combinatoria*", his first thoughts on the subject of symbolic logic. He later returned to his fundamental idea of

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evidence in favor of Newton's priority, had disastrous consequences for English mathematics. While the Continental mathematicians of the 18^{th} century made great strides in the theory of the calculus, and in its applications to Newtonian physics, the English stuck loyally to Newton's own much less suitable method of fluxions, and remained in a backwater for over a century!

145 Their 'official' use started only in 1812 due to the reform of **George Peacock**.

146 There is evidence that *Wilson's theorem* [For any prime p, $(p-1)! \equiv -1 (\mathrm{mod}\ p)$] was known to Leibniz long before 1770.

'Language of Concepts' again and again, clarifying, emending and implementing it. Two fragments found among his papers contain Leibniz's introduction to symbolic logic. They clearly establish him as one of the founders of the science. It was not until the work of **George Boole** (who probably did not know of Leibniz's paper) that an algebra came into existence which can be called a realization of Leibniz's ideas.

Philosophy

His philosophical system stands at the interface between the holistic and vitalist world-view of the Renaissance, and the atomistic and mechanistic materialism that was to dominate the 18^{th} and 19^{th} centuries.

Leibniz grasped that space and time were merely phenomenal things (appearances) and not genuine realities. He called these entities '*monads*' (Greek for unity).

Leibniz was born in Leipzig. His father, Friedrich Leibnütz (1597–1652) was a professor of philosophy at Leipzig University. His mother (1621–1664) was Friedrich's third wife. Though the name Leibniz, or Lubeniecz, was originally Slavonic, his ancestors were German, and for 3 generations had been in the employment of the Saxon government.

At an early age he mastered Latin, Greek and scholastic philosophy, which formed the basis of his later massive erudition in the classics. At the age of 14 he enrolled in the University of Leipzig, following the standard 2-year arts course which included philosophy, rhetoric, mathematics, Latin, Greek and Hebrew.

He devoted the next 3 years to legal studies, and in 1666 applied for the degree of doctor of law. Refused on the ground of his youth, he left his native town forever. The doctor's degree refused him there was conferred on him at once (1666) at Altdorf — the university town of the free city of Nüremberg — where his brilliant dissertation procured him the immediate offer of a professor's chair. But by that time Leibniz had changed his mind about an academic career, and decided instead to become more involved in the outside world. It is possible that already at that stage of his mental development, he became hostile to universities as institutions because their rigid faculty structure was bent on intellectual and scientific specialization.

During 1667–1672, Leibniz stayed for some time in Nüremberg where he was associated with a secret brotherhood of alchemists[147]. He soon left them

[147] To his dying day, Leibniz retained a close interest in *alchemy*. Unlike Newton, he never did actual laboratory work. His declared motives were scientific, but in fact he hoped to make his fortune from it. Thus, in 1676 he entered into a formal profit-sharing agreement with two practicing alchemists, his side of the bargain being to provide capital and technical advice.

to become an Assessor in the Court of Appeal of the Elector of Mainz, where he spend the next five years. In the course of his work there he also applied his mind to literary[148] and political activities. Thus he devised a plan to distract Louis XIV away from Northern Europe with an enticing scheme for a French conquest of Egypt (the strategy he suggested was almost identical to the one actually carried out by Napoleon a century and a half later). He was then sent to Paris to try and lay it before the French government.

Strongly attracted to the society of the leading scientists and mathematicians in Paris, Leibniz renewed his mathematical studies under the guidance of **Huygens**. He attacked the current problems in mathematics and science[149] with characteristic gusto (1672–1676) and by the time he left Paris he had already made most of the discoveries that were to earn him his place in the history of mathematics.

In 1673 he went to London and made personal contacts with members of the Royal Society. He showed them his mechanical calculator, which impressed then considerably (at the time even educated people rarely understood multiplication, let alone division!!). His trip to London was cut short by the news of the sudden death of his patron, the Elector of Mainz. He sought a research post attached to the Paris Academy, but it was denied him. He then accepted the post of Court Councilor at the service of the Duke of Hannover. Leibniz remained in the service of the Brunswick family for 40 years to the day of his death (1676–1717).

On his way back to Germany he had 4 days of intense discussions with **Baruch Spinoza** at The Hague (1676). His work in the service of the Duke of Hannover can be divided into 3 periods: During 1676–1686 he was the chief librarian of the great Hanover Library, where his duties were onerous but fairly mundane: general administration, purchase of new books and cataloging[150].

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[148] All his life he prided himself on his Latin poetry and boasted that he could recite the bulk of Virgil's *Aeneid* by heart. In 1676 he translated Plato's *Phaedo* and *Theaeteus* into Latin.

[149] While in Paris, Leibniz was full of technological ideas: a device for calculating a ship's position without using a compass or observing the stars, a compressed-air engine for propelling vehicles or projectiles, a ship which could go under water to escape enemy detection and various improvements to the design of lenses.

[150] **Leibniz** supported himself as a librarian (1676–1717), helping the dukes of Brunswick-Lüneburg in Hanover arrange their collection of 3000 volumes. Then he went on to organize the 30,000-volume ducal Library of Wolfenbüttel, for which he provided one of the first comprehensive alphabetical author catalogue. Leibniz signaled the transition from the royal and ecclesiastical collection for

In the second period (1687–1697) he became the historian and archivist of the House of Brunswick: his genealogical researches in Italy and elsewhere in Europe established the Hanoverian claim for a succession to the throne of Great Britain. He spent much time traveling. Although he had his own coach, it is nevertheless remarkable that he managed to write letters while on the move[151]. In this phase of his life he interacted strongly with the **Bernoulli** brothers, Jakob and Johann, exchanging mathematical challenges with them;

The ubiquitous practice of issuing challenge problems was actually inaugurated at this time by him. They were at first intended merely as exercises in the new calculus.

Thus, in 1687, Leibniz proposed the problem of the *isochrone curve*[152]: it was solved by 'the brothers', Huygens, and himself. Jakob Bernoulli returned the challenge with the *Catenary problem*[153] (1690), which was readily solved by Huygens, 'the brothers' and himself (quite an exclusive club!). During 1698–1714, Leibniz was engaged in diplomatic tasks for Hannover in Vienna, London, Berlin and Paris. He also promoted scientific societies and academies.

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the privileged few to the library serving everyone.

In 1856, **Anthony Panizzi** (1797–1879) became the Principal Librarian of the *British Museum*, establishing an egalitarian Reading Room there. **Thomas Carlyle** (1795–1881) established the *London Library* (1841), where books could be *checked out* by subscribers – the first circulating library in the world. But the *public library* "in every town", which Carlyle demanded, was yet to come. Panizzi still required users to present letters of introduction to enter the Reading Room and his books did not circulate. **Andrew Carnegie** (1835–1919) would spread public libraries across the United States of America.

[151] Leibniz was an avid letter-writer. He was corresponding with intellectuals from all over Europe, sometimes with hundreds of people at a time, on almost every subject under the sun — science, mathematics, law, politics, religion, philosophy, literature, history, linguistics, numismatics. He was obsessive about preserving his letters, and over 15,000 survived.

[152] A particle descends a smooth curve under the action of gravity, describing *equal vertical distances* in equal times, and starting in a vertical direction. Taking x as the horizontal axis and y in the vertical (downward) direction with the initial condition $\dot{x}(0) = 0$, $\dot{y}(0) = V$, the energy equation yields $\frac{1}{2}\left(\frac{ds}{dt}\right)^2 = gy + \frac{1}{2}V^2$ or $\dot{x}^2 + \dot{y}^2 = 2gy + V^2$, while the constraint is $y = Vt$. Eliminating $\dot{y} = V$ and integrating leads to $y^3 = \left(\frac{9V^2}{8g}\right)x^2$, a *semi-cubical parabola*.

[153] To find a curve formed by a chain of uniform weight suspended freely from its ends $\left[y = a\cosh\left(\frac{x}{a}\right)\right]$.

During his last years Leibniz was rather miserable and lonely. He was getting too infirm either to travel or to start a new life elsewhere. He died peacefully in the presence of his secretary and coachman.

1672–1682 CE **Nehemiah Grew** (1641–1712, England). Physician and botanist. A founder of *plant anatomy*. First to hypothesize in print on *sex in plants*.

Through microscopic observations he discovered sexual reproduction in plants and identified the stamen and pistil as the male and female organs respectively, as well as representing detailed drawings of plant anatomy.

Author of *Anatomy of Plants* (1682). It was the first complete account of the subject and remained the most authoritative work in this field for over 150 years.

Grew was born in Atherstone and educated at Cambridge and Leiden.

1672 CE **Regnier de Graaf** (1641–1673, The Netherlands). Physician and anatomist. One of the founders of experimental physiology. Discovered the Graafian vesicles of the *mammalian female gonad*, coining the term '*ovary*' for the organ.

He was born in Schoohoven and studied at Utrecht and Leiden.

1673–1685 CE **Phillipe de la Hire** (1640–1718, France). Mathematician, astronomer, physicist, naturalist, architect and painter. In his book *Sectiones conicae* he argued for the power and potential of *projective geometry*[154] and thus secured its place in mathematics. His work had influenced Newton, yet during de la Hire's lifetime, the mathematical community was not convinced that the synthetic methods of Desargues (1639) can match in power the analytic methods of Descartes (1637). The father of de la Hire, a well-known painter, was a student of Desargues.

[154] *De la Hire's theorem*: On a line L outside a conic (e.g., ellipse) and coplanar to it, we choose three points and draw from each of these points two tangents to the conic; connect the opposite points of tangency by lines; then these three lines meet at a point Q, dual to L.

Table 3.5: GREATEST MATHEMATICIANS OF THE 17^{th} CENTURY

NAME	NAT.	LIFE-SPAN	MAJOR CONTRIBUTION
Joost Bürgi	SW	1552–1632	Logarithms (1603); Decimal and exponential notations
John Napier	E	1550–1617	Logarithms (1614)
Thomas Harriot	E	1560–1621	Mathematical symbols
Henry Briggs	E	1561–1630	Decimal logarithms; Logarithmic tables (1624)
Johannes Kepler	G	1571–1630	Forerunner of calculus (areas and volumes, 1615); New polyhedra; Applied conic sections.
William Oughtred	E	1575–1660	Mathematical symbols; Logarithmic slide-rule (1622)
Edmund Gunter	SW	1581–1626	Cosine, cotangents (1620), slide-rule (1620)
Girard Desargues	F	1591–1661	Early development of synthetic projective geometry
Albert Girard	D	1595–1632	General algebraic equations: roots and coefficients; Fundamental theorem of algebra (conjecture).
Rene Descartes	F	1596–1650	Coordinate geometry; Topology ($v - e + f = 2$); Algebraic notation.
F.B. Cavalieri	I	1598–1647	Precursor of the integral calculus
Pierre de Fermat	F	1601–1665	Modern number theory; Modern analytic geometry; Differential calculus; Probability theory.

Table 3.5: (Cont.)

Name	Nat.	Life-span	Major Contribution
G.P de Roberval	F	1602–1675	Pre-calculus method for tangents and areas.
E. Toricelli	I	1608–1647	First notion that differentiation is the inverse of integration; envelopes of families of curves.
John Wallis	E	1616–1703	Pre-calculus integration; Concept of limit; First geometric representation of complex numbers.
Nicolaus Mercator	E	1620–1687	Mercator series.
William Brouncker	E	1620–1684	Continued fractions; 'Pells' equation; Infinite series.
Johann H. Rahn	SW	1622–1676	Mathematical symbols
Vincenco Viviani	I	1622–1703	Geometer, physicist and inventor of instruments.
Blaise Pascal	F	1623–1662	Calculating machine; Mathematical theory of probability; Projective geometry; Binomial triangle; Cycloid.
Pietro Mengoli	I	1626–1686	Infinite series; Divergence of harmonic series; Infinite product for π.
Christiaan Huygens	D	1629–1695	Probability; Rational approximation for gear-ratio by continued fractions.
John Hudde	D	1628–1704	Algebraic equations with literal coefficients standing for negative and positive numbers.

Table 3.5: (Cont.)

Name	Nat.	Life-span	Major Contribution
James Gregory	E	1638–1675	Convergent and divergent infinite series ($\tan^{-1} x$)
Isaak Barrow	E	1630–1677	Pre-calculus differentiation: tangents and rectification of curves; maxima and minima.
P. de la Hire	F	1640–1718	Projective geometry; pole and polars.
Seki Kowa	N	1642–1708	General determinant theory
Isaac Newton	E	1642–1727	Creation of workable calculus (1671)
G.W. Leibniz	G	1646–1716	Infinitesimal calculus (1673–1676); Calculus of finite differences; Binary arithmetics; Early topology; Theory of determinants (1698); Symbolic logic; Theory of envelopes; Theory of numbers.
Giovanni Ceva	I	1647–1734	Synthetic geometry (Ceva theorem)
Walter von Tschirnhausen	G	1651–1708	Theory of equations; “The T. Transformation”
Michel Rolle	F	1652–1719	Rolle’s theorem (1691)

E = England; I = Italy; G = Germany; F = France;
D = Holland; N = Japan; SW = Switzerland.

1675 CE Scarlet fever first identified or described with accuracy.

1675 CE **Olaus Römer** (1644–1710, Denmark). Astronomer. Made the first measurement of the velocity of light — 309,000 km/sec.

Previously, **Kepler** and **Descartes** believed it to be infinite. **Galileo** failed to devise a successful method of measurement. Römer used observations based on the times of eclipses and occultations of the four large satellites of Jupiter. The four moons could be easily seen with the telescopes of that day. Römer compiled a table of their periods of revolution around the planet. He could predict the times at which each was eclipsed as it moved into Jupiter's shadow or occulted when it passes behind the planet's limb. He found that these phenomena occur sooner than expected during part of the year and later then expected during the other part, and correctly inferred that the advance or delay of the occurrence was due to the finite velocity of light.

Römer's observations showed that there is a difference of 1000 seconds at two dates roughly 6 months apart. In 1675, the diameter of the earth's orbit was known to be 309 million km; hence Römer's value for the speed of light in free space.

In 1849 **Armand Hippolyte Louis Fizeau** (1819–1896, France) made a first laboratory measurement of the velocity of light, by synchronizing the rate of a rapidly rotating toothed wheel with the reflection of a light beam so as to allow the beam to *enter* and *leave* through two adjacent slits. Again the result was close to 300,000 km/sec. Later measurements by **Michelson** (1879, 1887, 1926) improved it gradually to within an error of less than 10^{-5} percent. The meter is nowadays *defined* by means of a value of c, agreed upon by international committee to be $c = 299,792.458$ km/sec.

1675 CE Foundation of the Royal Observatory at Greenwich. This date indicates the beginning of the precise standardization of time measurements, needed primarily for navigation.

1678–1692 CE **Giovanni Ceva**[155] (1647–1734, Italy). Mathematician. Discovered one of the most important results on the synthetic geometry of the triangle between Greek times and the 19^{th} century. His geometrical treatise *De lineis rectis* (1678) contains a theorem now known by his name: "*If three concurrent lines* (*known as Cevians*), *one from each vertex of a triangle, are drawn, they divide the opposite sides into six segments such that the products*

[155] A town in Piedmont, Italy, in the province of Cuneo. In the Middle Ages it was a strong fortress defending the confines of Piedmont towards Liguria, but the fortifications on the rock above the town were demolished (1800) by the French.

of three segments having no common end is equal to the product of the three other segments".

The converse of *Ceva's theorem* is also true.

Ceva's theorem is a direct generalization of the corresponding theorems in elementary plane geometry and marks a point of departure of the new European geometry away from the classical Greek tradition. Its applications lead immediately to some important properties of the triangle[156].

Giovanni was in the service of the Duke of Mantua; known also for his calculations of centers of gravity, areas and volumes of geometric figures. His brother **Tomasso Ceva** (1648–1737) was a teacher of mathematics in the Jesuit College at Milan, and wrote on the cycloid and mathematics in general.

1678–1718 CE **Edmund Halley** (1656–1742, England). Astronomer-royal. A close friend of Isaac Newton and active in many areas of astronomy. Halley is best known for his pioneering study of comets.

After what he termed a 'prodigious deal of calculations', Halley (1705) published parabolic orbital elements for 24 well-observed comets. He noted the similarities in the orbits for the comets of 1682, 1607, and 1531 and published his first correct prediction for the return of a comet. It did return on time (16 years after his death) and has been called "Comet Halley" ever since.

Other discoveries of Halley are:

(1) The proper motions of the stars on the celestial sphere (1718). Halley shattered man's ancient belief in 'fixed stars' by charting the motions of Sirius, Arcturus and Aldebaran, listed as bright stars in Ptolemy's *Almagest*.

(2) *Secular acceleration of the moon's mean motion* (1693). Halley found from a comparison of ancient and modern eclipses that the mean velocity of the moon in its orbit is gradually increasing. Nearly 100 years later (1787), Laplace showed that it is caused partly by the gradual average decrease of the eccentricity of the earth's orbit which has been going on for many thousands of years.

Indeed, due to planetary perturbations *and* the slowing down of the earth's rotation (10^{-3} second per century. First suggested by **William Ferrel** in

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[156] E.g., if each side of the triangle is divided into n equal parts and the Cevian lines are drawn to the first point from each vertex in a clockwise (or anticlockwise) direction around the triangle, the central triangle has an area of $\frac{(n-2)^2}{n^2-n+1}$ of the original triangle.

1856) caused by tidal braking, the moon is speeding up at a rate that is proportional to *square* of elapsed time. This apparent change in the moon's rate is manifested in the appreciable *shift* in tracks of eclipses. When actual eclipse tracks are calculated for ancient eclipses on the basis of *current* motions of sun and moon, they are found to deviate slightly *eastward* from the observations.

(3) Determination of the solar parallax by means of the transit of Venus (1677).

(4) Explanation of the trade-winds and monsoons (1686): published a world map indicating the prevailing winds over the tropical oceans. He explained the equatorward[157] flow of the trades as resulting from a combination of the rising of air near the equator due to solar heating and the resulting surface inward flow of air toward the updraft region. [An improved explanation based on the rotation of the earth and atmosphere was given later by George Hadley (1735).]

Halley went to St. Helena island during 1676–1678 to catalog stars not visible from Northern observatories. His resulting star catalog started the systematic study of the Southern sky. It was the first study based on telescopic, rather then naked eye observations.

Before Halley made his study on comets, most people believed that comet apparitions were random. But Halley argued that comets belonged to the solar system and that their orbits are governed by Newton's law of universal gravitation.

In 1981 Tao Kiang et al. numerically integrated the orbital motion of comet Halley back to 1404 BCE, using the Newtonian equations of motion. They took into account the perturbations by the nine major planets over the past 3500 years and non-gravitational forces due to 'rocket effects' of an outgasing water ice-nucleus. The dynamic model used to compute the long-term motion of the comet successfully reproduced the ancient Chinese observations over nearly two millennia.

If Halley could comment on the wonders of the computer era and its astronomers, he would not be likely to change even one word of what he already said in his paper of 1705:

"*You see therefore an agreement of all the elements in these three, which would be next to a miracle if they were three different comets... Wherefore, if according to what we have already said it should return again about the*

[157] This basic pattern was known already to the ancient Hebrews, for we read in the Bible [*Eccl.* **1**, 6]: "*The winds blow to the south, and turn to the north; round and round it goes, ever returning on its course*".

year 1758, *candid posterity will not refuse to acknowledge that this was first discovered by an Englishman*".

1679–1709 CE **Denis Papin** (1647–1712, France). Physicist and inventor. One of the inventors of the steam-engine (1690). Papin was born in Blois. He studied medicine at the University of Angers (1662–1669). Assisted **Huygens** in Paris in his experiments with the air-pump (1674–1675) and afterwards assisted **Boyle** in his experiments in London. At this time he experimented with hydraulic and pneumatic transmission of power, improving the air-pump, inventing the condensing pump, and the "steam-digester" [a pressure cooker with which he showed that boiling point is raised or lowered as the pressure exceeds or falls below atmospheric pressure]. He also invented the safety valve and is credited with being the first (1690) to apply steam to raise a piston.

In 1687 Papin was appointed to the chair of mathematics in the University of Marburg, and there he remained until 1696. In 1707 he sailed with his family to London in an ingeniously constructed boat, propelled by paddle-wheels. He died in London in poverty and total obscurity.

1680 CE **Edme Mariotte** (1620–1684, France). Physicist. Independently discovered Boyle's law.

1682 CE A Russian Physician reportedly repaired the skull of a wounded nobleman *using bone from a dog*. The surgery was said to be successful, but the Russian Church threatened the nobleman with excommunication, prompting him to have the graft removed.

This is the first recorded case in medical history of animal tissue transplantation.

In the late 1800's *frog skin* was often grafted onto patient's skin in an attempt to heal burns or skin ulcer. Good results were reported.

1682–1683 CE **Ehrenfried Walther von Tschirnhausen** (1651–1708, Germany). Physicist and mathematician. Discovered the caustic of reflection[158] (1682). Endeavored to solve equations of any degree by removing

[158] Parallel light rays from the sun fall onto a nearly full cup of coffee. Each ray is reflected from the circular surface of the cup and these reflected rays form an *envelope* known as a *caustic*. At the caustic, the intensity of the light is theoretically infinite (according to geometrical optics) since the cross-section of the ray pencil at each point on the envelope has zero area. In fact this is not quite true, as is obvious on physical grounds and as a more accurate analysis confirms, but the intensity can indeed be very great: sufficient to burn a piece

all the terms except the first and the last (this procedure has been tried before him by the Frenchman **Francois Dulaurens** and the Scot **James Gregory**).

In 1683 Tschirnhausen published *Method of Eliminating All Intermediate Terms from a Given Equation*. Although the title exaggerated, the paper was the most important idea for the solution of algebraic equations in about 200 years. It showed that a polynomial of degree $n > 2$ can be reduced by his transformations to a form in which the coefficients of the terms of degrees $(n-1)$ and $(n-2)$ are zero.[159]

Tschirnhausen studied at Leyden, and for a while served in the Dutch army. Later he spent some time in England. He visited Paris several times, and was elected (1682) to the French Academy of Sciences. He also set up a glasswork in Italy to further his experiments on light.

Tschirnhausen was a man of wide acquaintance and interests. Everywhere he went he sought contact with leading scientists, collected observations and reported interesting discoveries to **Leibniz**. He thus met with **Spinoza**, **Huygens**, and **Wallis** and corresponded with **Newton**, **Jakob Bernoulli** and

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of paper, for example (hence the Greek name *caustic*).

Let the inner surface of the cup be represented by the unit circle, and let the incident rays be parallel to the (horizontal) x-axis. If a ray is incident on the cup at the point Q, whose coordinates we may take to be $(\cos\theta, \ \sin\theta)$, then since the angle of reflection is equal to the angle of incidence we may easily find the equation of the reflected ray from Q: $(y-\sin\theta)\cos 2\theta = (x-\cos\theta)\sin 2\theta$. Considered as a family of equations with θ as parameter, this represents *all* the reflected rays.

The caustic is the envelope of this family, i.e. the curve which is tangent to every member of the family. Now, the equation of the envelope of a one-parameter family of curves $f(x, y; \theta) = 0$ is found by eliminating the parameter θ from the equation $f = 0$ and $\frac{\partial f}{\partial \theta} = 0$. It is shown that the parametric equations of the envelope are

$$x = \cos\theta - \frac{1}{2}\cos 2\theta\cos\theta; \qquad y = \sin\theta - \frac{1}{2}\sin 2\theta\cos\theta.$$

These are the equations of a curve known as the *nephroid*. Note that throughout this calculation we have been suppressing the z-coordinate; the equation we have derived is really that of a cylinder with the nephroid as cross section, and what we observe in the intersection of this cylinder with a plane $z =$ constant, i.e. with the surface of the coffee.

159 To dig deeper, see:

- Panton, A.W., *The Theory of Equations*, Dover: New York, 1960, Vol I (286 pp.); Vol II (318 pp).

Johann Bernoulli. He also examined unpublished and posthumous papers of **Descartes** and **Pascal**.

However, he exhausted his mathematical talents in searching for algorithms and lacked insight into the more profound mathematical ideas of his age (e.g., he considered infinitesimal symbolism to be of limited applicability). He could have achieved more, but being essentially an autodidact he lacked the guidance of experienced and strict teachers, who might have instilled in him a greater measure of self-criticism.

Algebra and the Theory of Equations

Descartes *(1637) rejected* complex roots *and termed them imaginary. Even* ***Newton*** *did not regard complex roots as significant, most likely because in his day they lacked physical meaning.* ***Leibniz*** *worked with complex numbers formally, but possessed no understanding of their nature.*

Despite the lack of any clear understanding during the 16^{th} *and* 17^{th} *centuries, the operational procedures with real and complex numbers were improved and extended.* ***John Wallis*** *(1673) was first to show how to represent* geometrically *the complex roots of a quadratic equation with real coefficients, as point in a plane. His work was ignored because mathematicians were not receptive to the use of complex numbers.*

It is remarkable that the free use of algebra provoked a host of protests. The philosopher ***Thomas Hobbes*** *(1588–1679), though only a minor figure in mathematics, nevertheless spoke for many mathematicians when he objected to the application of algebra to geometry. He characterized John Wallis' book on the algebraic treatment of conics as a scurry book and as a "scab of symbols". Many mathematicians, including* ***Pascal*** *and* ***Barrow****, objected to the use of algebra because it had no logical foundation; they insisted on* geometric *methods and proofs.*

Unlike ***Descartes****, who still regarded algebra as the servant of geometry,* ***John Wallis*** *and* ***Newton*** *recognized the full power of algebra.* ***Leibniz****, too, noted the growing dominance of algebra and fully appreciated its effectiveness.*

Albert Girard *(1629) conjectured that an equation of degree n always has n solutions in the domain of* complex numbers*. Attempts to prove this were made by* ***Descartes****, d'Alembert and others, but it was only* ***Gauss*** *(1799) who succeeded in giving a rigorous proof without gaps.*

Girard *(1629) was also the first to engage in* nonalgebraic *solutions of algebraic equations, involving an* infinite *number of arithmetic steps, such as infinite series or products. He had shown that trigonometric functions (which are nonalgebraic, or transcendental functions) are effective in obtaining solutions when the* cubic *formula yields irreducible results (3 distinct real roots). Therefore, mathematicians after Galois' day conceived the idea that the* elliptic functions*, which generalize ordinary trigonometric functions, might offer a means of expressing solutions of some higher-degree equations that are not solvable algebraically. The ideas of Girard were picked by* ***Lambert*** *(1757), who suggested a solution base on series.*

The search for meaning of negative and imaginary roots of equations that started with ***Cardano*** *(1545), continued in the 17th century:* ***Albert Girard*** *(1629) interpreted negative numbers as a kind of a relative orientation, which eventually paved the way toward the idea of the* number-line*. Girard retained all imaginary roots of equations because they show the general principles in the formation of equation from its roots. He stated clearly the* relation between roots and coefficients*, allowing of negative and imaginary roots of equations.*

Descartes *(1637) coined the term* '*imaginary*' *for expressions involving square roots of negative numbers, and took their occurrence as a sign that the problem was insoluble.* ***Leibniz*** *(1670) was confused and perplexed by expressions such as*

$$\sqrt[3]{6+\sqrt{-\frac{1225}{7}}}+\sqrt[3]{6-\sqrt{-\frac{1225}{7}}}=4$$

which results from Cardano's solution of $x^3 - 13x - 12 = 0$. Today such relations are considered trivial by a good high-school algebra student.

Apart from the problem of the existence *of solutions of algebraic equations, there is also the problem of* determining *them. After the solution formulae for the cubic and quartic equations had been found during the Renaissance, the mathematicians of the 17th and the 18th centuries searched with great tenacity for the corresponding solution formulae of degree 5 and higher.*

In their quest for a solution of the quintic by radicals, mathematicians were first concerned with the reduction of the general quintic into the simplest possible canonical form. The feasibility of this procedure is based on the pioneering discovery of ***Tschirnhausen*** *(1683) that a special transformation can eliminate the terms of degree $(n-1)$ and $(n-2)$ from any polynomial of* *degree $n > 3$. He started from the observation that the transformation of the equation*

$$x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 = 0$$

by the linear substitution $y = x + h$ eliminates the $(n-1)^{th}$ power by choosing $h = \frac{1}{n}a_{n-1}$. He then noticed that it is possible to remove both the $(n-1)^{th}$ and the $(n-2)^{th}$ powers by effecting a quadratic transformation $y = x^2 + ax + b$. To see this one first assumes the "existence of an equation in y of the same degree as the original in x, i.e.

$$y^n + b_{n-1}y^{n-1} + \cdots + b_1 y + b_0 = 0.$$

This is always possible since x^{n+k} can be expressed as a polynomial of degree not higher than n by a repeated use of the original equation. Thus $\{y, y^2, y^3, \ldots, y^n\}$ will be each a polynomial of degree lower or equal to n. When the equation in y is formed, the free parameters $\{p, q\}$ are chosen such that $b_{n-1} = b_{n-2} \equiv 0$.

Thus he found that by a transformation of the form $y = x^2 + ax + b$, a general cubic is reduced to the form $y^3 = K$. Another such transformation reduces a general quartic to $y^4 + py + q = 0$, and a general quintic to the form $y^5 + \alpha y^2 + \beta y + \gamma = 0$. In general, a Tschirnhausen transformation of a polynomial equation $f(x) = 0$ is one of the form $y = g(x)/h(x)$, where g and h are polynomials and h does not vanish for a root of $f(x) = 0$. The transformation by which Cardano and Viète solved the cubic were special cases of such transformations.

1683 CE **Seki Kowa** (1642–1708, Japan). Mathematician. Considered by the Japanese the greatest mathematician that their country has produced. His most original and important work is the invention of *determinants*, at least 10 years ahead of Leibniz (1693). Also, Leibniz treated only 3 equations with 3 unknowns, whereas Seki considered n equations and gave a more general treatment. Seki knew that a determinant of the n^{th} order, when expanded, has $n!$ terms and that rows and columns are interchangeable. Discovered *Bernoulli Numbers* before Jacob Bernoulli. Credited with the independent invention of the differential calculus. Seki was a great teacher who attracted gifted pupils. He discouraged divulgence of mathematical discoveries made by himself and his school.For that reason it is difficult to determine with certainty the exact origin and nature of some of the discoveries attributed to him.

1683–1699 CE *Islam vs. Christendom*; Since 1656 the Ottoman Empire had been undergoing a revival, and in the 1660's began a new thrust up the Danube Valley directed at their old enemies, the Habsburgs. Louis XIV, also an inveterate enemy of the Habsburgs, allied himself with the Turks and Hungarian rebels against his Austrian foes. This resulted in two of the greatest military confrontations of the second half of the 17^{th} century[160]: the Habsburg's successful resistance to the advance of Turkey into Europe and their subsequent counter-offensive; and the great coalition which halted Louis XIV's attempt to dominate the Continent.

The crisis came in July 1683, when a Turkish army of ca 200,000 *laid siege to Vienna*. For two months the fate of Christendom seemed to hang in the balance. Then volunteers began to flow in from all over the continent to help the emperor in his extremity: Pope Innocent XI contributed moral and material aid, and King Jan Sobieski of Poland arrived with an army that helped rout the Turks by September. This marked the beginning of the decline of the Ottoman Empire, and the end of a thousand year military conflict between Islam and Christianity. Thus failed the last (perhaps) attempt of Islam to subdue Christendom and conquer Western Europe [1^{st}, 732 at Tour, 2^{nd}, 1571 at Lepanto].

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160 *The Jewish connection*: The financier **Samuel Oppenheimer** [1630–1703; a distant relative of **Joseph Oppenheimer**, the so-called "Jud Süss" (1698–1738)] was the Imperial War Purveyor to the Austrian monarchy during 1673–1702 and played a decisive role in the above wars of the Habsburgs. He was running the finances of two-front war, marshaling the resources. Some historians believe that he was indeed the man who saved Vienna during the siege of 1683 when the emperor fled.

No one ever rendered greater services to the Habsburgs. But the Austrian Treasury never payed him back! Moreover, in 1701 his house in Vienna was "accidentally" burned down, destroying most of the financial business records. At that time the Crown owed him more than ten million florins. But Oppenheimer could no longer produce proofs of the debts due to him from the State. So the State produced its own records "proving" that he had been overpaid! All his services were then forgotten and the Jew was rewarded by being thrown into prison while his family was left penniless.

His nephew **David Oppenheimer** (1664–1736), rabbi of Prague, managed to gather a large library of rare Jewish books and manuscripts. These he kept in Hamburg, away from the reach of *Inquisition* in Catholic Bohemia. His collection was purchased by Oxford University early in the 19^{th} century, and now forms the basis of the *Bodleian hebraica*, encompassing over 7,000 books and 1,000 manuscripts.

Historians have explored the question of why the West (nearly) always wins? For 2,500 years, from ancient Greece to the present day, Western armies vanquished their non-Western adversaries in almost every war, with rare exceptions when the West was caught completely by surprise or was exceptionally outnumbered.

A comprehensive examination of important battles, from the Battle of Salamis in which the Athenians defeated the Persian fleet to the Battle of Midway in World War II in which the Americans defeated Japan refutes widespread assumptions that Western military superiority is explained by greater valor, military-technology advantage or greater economic strength. Hanson[161] argues that the secret is that Western military forces are more effective killers. This results from the "citizens' army" model created in a Western "open society," which was born in the ancient Greek tradition of storming the enemy.

While Ancient, Eastern monarchs considered war a sport, a game of balance between forces, ancient Greek democracy gave rise to an utterly unsportsmanlike perception of war. It viewed war as a fight for liberty and freedom of community and citizen, an existential fight to the death. Its primary objective was not to defend city and homeland but, to the greatest extent possible, to prevent the enemy from recovering in time for another round. While non-Western forces strive to gain points, their Western adversaries strive for a knockout.

1685 CE, Oct. 18 *Revocation of the edict of Nantes* (1598) by Louis 14^{th} of France; all religions except Roman Catholicism became forbidden by law. About 400,000 Protestants (Huguenot) fled France and emigrated to the neighboring countries and North America.

There were about a million Huguenots out a total population of perhaps 18 million in France in 1650. After Richelieu deprived them of their military and political privileges, they had become good citizens and remained loyal to the crown. Many were successful in industry and the professions. The French Catholic clergy had long tried to persuade Louis 14^{th} that the continued exercise of the Protestant religion in France was an insult to his dignity and authority, and as Louis became more concerned about his salvation, the idea of atoning for his sins of the flesh by crushing heresy became more attractive to him. The edict was "interpreted" more and more strictly. Protestant children were declared of age at seven and converted to Catholicism, and any attempt of their parents to win them back was punished by imprisonment. Money was offered to converts. Protestant chapels were destroyed,

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[161] The American military historian **Victor Hanson** in his book "*Carnage and Culture*" (Doubleday, 2002).

and troops were quartered on prominent Huguenots to make life miserable for them.

Finally Louis aided and abetted by his mistress and his Jesuit advisers, announced that since all the heretics had finally been converted to Catholicism, there was no further need for the Edict of Nantes and it was therefore revoked; Protestant churches and schools were closed, and all Protestant children were baptized as Catholics. The Revocation was savagely enforced by imprisonment, torture and condemnation to the gallows.

The Huguenots escaped to England, The Netherlands, Brandenburg, and the New World, where their industry and skill contributed appreciably to the economic, technological and cultural life of their new homes. To France the Revocation 'brain-drain' brought both economic and cultural loss.

Thus, under the influence of the clergy, was committed one of the most flagrant political blunders in the history of France.[162]

The Protestant Huguenots experienced in 1685 what the Jews had already gone through during 1492–1498 in the Iberian Peninsula. In this sense the fate resemblance of the *Marranos* and the *Huguenots* is a striking example for the way in which history repeats itself from time to time.

1686 CE **Bernard (Le Bovier) de Fontenelle** (1657–1757, France). Man of letters, thinker and science-fiction writer. Made the first major attempt to present scientific knowledge to the layman in an attractive literary form.

Born in Rouen, a nephew of Corneille. Studied law, but having lost the first case which was entrusted to him, he soon abandoned law and gave himself completely to literary pursuits.

In his *Entretiens sur la pluralite des mondes habites* (1686), he popularized among his countrymen the astronomical theories of **Descartes**. It is a lucid exposition of astronomy according to Copernicus and Descartes, enlivened by speculations concerning *life on other planets*. In 1691 he was received into the French Academy, and in 1697 became its perpetual secretary.

1687–1789 CE Leading European Poets and Novelists in the Enlightenment (Age of Reason):

- Daniel Defoe 1660–1731
- Jonathan Swift 1667–1745

[162] In fact, the oppression of the Huguenots in France and the Puritans in England, during 1640–1685, accelerated the outbreak of the *Industrial Revolution* in England in the 1760's.

•	Alexander Pope	1688–1744
•	Voltaire	1694–1778
•	Johann Wolfgang von Goethe	1749–1832
•	William Blake	1757–1827
•	Friedrich von Schiller	1759–1805

1689–1695 CE **John Locke** (1632–1704, England). Empirical philosopher. Rejected the notion of the 'divine rights' of kings, as well as the infallibility (absolute truth) of religion and the dogma of the Church. Opposed the authority of the Bible and the Church in temporal affairs. Maintained that political sovereignty rests upon the consent of the governed.

His political philosophy is strongly felt in the *American Constitution* and *Declaration of Independence*. In his own words:

"No man has the right to more than others, because we are all equal, of the same species and condition, equal amongst ourselves, with equal rights to enjoy the fruits of nature".

Held that men were free to think of God in their own way, not as any religion told them to. His major works: *Two Treaties on Government* (1689); And *Essay on Human Understanding* (1690).

Locke was born in Wrington in Somerset County. He attended Oxford University. When his friend Anthony Cooper became involved in plots against the King, the suspicion also fall on Locke and he fled to Holland (1684), but returned (1689) as favorite of the court of Prince William of Orange.

1691 CE **Michel Rolle** (1652–1719, France). Mathematician; author of a theorem named after him[163], found in his '*Methode pour résoudre les egalitez*' (1691). The name *Rolle's theorem* was first used in 1834 in Germany and in 1846 in Italy (**G. Bellavitis**).

1690 CE **Jakob (Jacques, James) Bernoulli** (1654–1705, Switzerland). Among the principal contributors to mathematics in the 17^{th} century. Jakob and his brother Johann gave up earlier vocational interests and became mathematicians when Leibniz's papers began to appear in the *Acta eruditorum*. They were among the first mathematicians to realize the surprising

[163] *Rolle's Theorem*: If $f(x)$ is continuous in the interval $a \le x \le b$, $f'(x)$ exists in the open interval $a < x < b$ and $f(a) = f(b) = k$, then there is a point c, such that $a < c < b$, at which $f'(c) = 0$ [one may assume, without loss of generality, that $f(b) = f(a) = 0$ since one may apply the theorem to the new function, $f(x) - k$, instead of $f(x)$]. For polynomials, Rolle's theorem takes the form: between any pair of roots of $P(x) = 0$ lies a root of $P'(x) = 0$.

power of the calculus, and to apply the tool to a plethora of problems, and first to use the term *'integral'* (1690).

Jakob Bernoulli invented polar coordinates (1691). [Newton may have discovered them earlier in 1671, but this is not clear from his writings.] He wrote on infinite series, studied many special functions and introduced the *Bernoulli numbers* that appear in the power series expansion of the function $z(e^z - 1)^{-1}$ and the *Bernoulli polynomials*[164] of number theory. In 1700 he developed further the theory of probability (*Bernoulli distribution*) and rediscovered the *law of large numbers*, a theorem named after him. Was first to apply calculus to probability theory.

The solution of the *Brachistochrone* problem by him and his younger brother Johann, started an acrimonious quarrel between them that dragged on for several years. Jakob is also known for the early use of *radius of curvature* of a plane curve, discovery of the *isoperimetric figures*, the *Bernoulli equation* in the theory of ODE, and his pioneering work in the calculus of variations. In his 1690 solution to the problem of the *isochrone* [curve along which a body will fall with uniform vertical velocity], we encounter for the first time the word "*integral*" in a calculus sense. Leibniz had called the integral calculus *calculus summatorius*, but in 1696, **Leibniz** and **Johann Bernoulli** agreed to call it *calculus integralis*.

Jakob Bernoulli was struck by the way the equiangular spiral reproduces itself under a variety of transformations and asked, in imitation of **Archimedes**, that such a spiral be engraved on his tombstone along with the inscription "*Eadem mutate resurgo*" ("I shall arise the same, though changed").

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164 *Bernoulli numbers* B_n were introduced in his *Ars Conjectandi* (published posthumously, 1713) through the definition:

$$\frac{z}{2}\cot\frac{z}{2} = 1 - B_1\frac{z^2}{2!} - B_2\frac{z^4}{4!} - \cdots - B_n\frac{z^{2n}}{(2n)!} - \cdots, \quad |z| < 2\pi$$

with $B_1 = \frac{1}{6}$, $B_2 = \frac{1}{30}$, $B_3 = \frac{1}{42}$, $B_4 = \frac{1}{30}$, $B_5 = \frac{5}{66}$.
He also introduced the polynomials $\Phi_n(z)$ as the coefficients of $\frac{t^n}{n!}$ in the expansion of

$$t\frac{e^{zt}-1}{e^t-1} = \sum_{n=1}^{\infty}\Phi_n(z)\frac{t^n}{n!}, \quad |t| < 2\pi$$

Explicitly,

$$\Phi_n(z) = z^n - \frac{n}{2}z^{n-1} + C_2^n B_1 z^{n-2} - C_4^n B_2 z^{n-4} + C_6^n B_3 z^{n-6} - \cdots$$

the last term being z or z^2 and C_2^n, C_4^n, $\ldots$ the binomial coefficients.

The Bernoulli family is one of the most illustrious families in the annals of science. They originally came from Antwerp. Driven from Holland during the oppressive government of Spain for their attachment to the Reformed religion, the Bernoullis sought asylum first in Frankfurt (1583) and afterwards in Basel, where they ultimately rose to the highest distinctions.

Jakob was born in Basel. He was educated at the city's public school. Upon the conclusion of his philosophical studies at the university, some geometrical figures which he chanced to see excited in him a passion for mathematics, and in spite of the opposition of his father, who wished him to be a clergyman, he applied himself in secret to his favorite science. In 1676 he visited Geneva on his way to France, and subsequently traveled to England and Holland. While in Geneva he taught a blind girl several branches of science, and also how to write; and this led him to publish *A Method of Teaching Mathematics to the Blind*. In London he was admitted to the meetings of **Robert Boyle**, **Robert Hooke** and other learned men. On his return to Basel in 1682 he devoted himself to physical and mathematical investigations, and opened a public seminary for experimental physics.

In the same year he published his essay on comets, *Conamen Novi Systematis Cometarum*, which was occasioned by the appearance of the comet of 1680. In 1687 the mathematical chair of the University of Basel was conferred upon him, and he was later made rector of his university. In 1684 he had been offered a professorship at Heidelberg; but his marriage to a lady of his native city led him to decline the invitation. He wrote elegant verses in Latin, German and French; but although these were held in high esteem in his own time, it is on his mathematical works that his fame now rests.

1694 CE **Rudolph Jakob Camerarius** (**Camerer**) (1665–1721, Germany). Physician and botanist. In *De Sexu Plantarum Epistola*, presented a conclusive demonstration of the sexuality of plants. Professor at the University of Tübingen (from 1688).

1694–1718 CE **Johann (Jean, John) Bernoulli** (1667–1748, Switzerland). One of the leading mathematicians of the 18^{th} century. A member of a remarkable Swiss family that produced 8 mathematicians — three of them outstanding — who in turn had a swarm of descendants who distinguished themselves in many fields.

In 1694 he took a doctor's degree in medicine but became fascinated by calculus and applied it to many problems in geometry, differential equations and mechanics. In 1695 he was appointed professor of mathematics and physics at Groningen in Holland, and on his brother Jakob's death, succeeded him in the professorship at Basel. In 1696 he proposed the famous '*Brachistochrone*

Problem' as a challenge to the mathematicians of Europe[165] [curve of shortest descent-time between two fixed points in a homogeneous gravitational field — the cycloid]. It was solved by **Newton**, **Leibniz**, his brother Jakob and himself [solved earlier (1673) by **Huygens** and applied by him in the construction of a pendulum clock]. Most of the calculus integration techniques were systematically worked out by the Bernoullis and **Euler**. Johann, however, pioneered the use of *substitutions*.

The so-called *L'Hopital Rule* was actually obtained by him in 1696; but L'Hopital and Bernoulli had an agreement (1692) whereby Johann sent L'Hopital some of his mathematical discoveries, to be used as L'Hopital chose, in exchange for regular salary. It was only after the death of **Guillaume Francois Antoine L'Hopital**, **Marquis de St. Mesme** (1661–1704) that Bernoulli accused him of plagiarism, an accusation that at the time was generally dismissed but now seems to be well founded[166]. The Marquis introduced this method in the *first* calculus textbook "*Analyse des infiniments petits*", published in Paris in 1696. This book had a wide circulation, and brought the differential notation into general usage in France as well as making it known throughout Europe.

Johann Bernoulli was the first to recognize the *principle of virtual work* as a general principle of statics with which all problems of equilibrium could be solved. He introduced the product of the force and the virtual velocity in the direction of the force, taken with a positive or negative sign according to the acute or obtuse angle between force and velocity (scalar product of vectors!). In 1717 he announced the general principle that *for all possible infinitesimal displacements, the sum of all these products must vanish if the forces balance each other*.

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[165] He was the first to denote the acceleration of gravity by the symbol g, and first to write the relation $v^2 = 2gh$. The notation ϕx to indicate a function of x was introduced by him in 1718.

[166] In 1921, Johann's manuscript on the differential calculus was discovered. Together with the correspondence of L'Hopital and Bernoulli, it proved that the Swiss mathematician was the true author of L'Hopital's calculus book.
One of Johann's contributions, made jointly with Leibniz, was the technique of *partial differentiation* The two kept this discovery secret(!) for 20 years in order to use it as a "secret weapon" in problems about a family of curves. Another startling result of Johann (1697) was

$$\int_0^1 x^x dx = 1 - \frac{1}{2^2} + \frac{1}{3^3} - \frac{1}{4^4} + \cdots$$

Remarkably ingenious, too, is another observation of Bernoulli; he compared the motion of a particle in a given field of force with the propagation of *light* in an optically heterogeneous medium, and tried to give a mechanical theory of the refractive index on this basis. Bernoulli is thus the forerunner of the Hamilton-Jacobi theory, that links optical and mechanical systems and presages *quantum mechanics*.

1696–1727 CE **Stephen Gray** (1666–1736, England). Physicist and chemist. A pupil of Newton. One of the first experimenters in static electricity, using frictional methods to prove conduction (1727).

Discovered the *conduction* of electrical charges and made the distinction between conductors and insulators. Gray showed that electricity can be transmitted from one object to another and over distances through conductors and that static electrical charges reside on the surfaces of objects, not in the interiors (1729). Transmitted electrical charges, generated by electric generator, over brass wires 100 meters long. Demonstrated that anything can be charged with static electricity if it is isolated by nonconducting materials (1731). His work had a great influence on the electrical theory of **Du Fay** (1733–1740).

Gray was born in Canterbury and followed his father's trade as a dyer, but the thirst of education led him to Cambridge University. His first scientific paper (1696) described a microscope made of a water droplet, similar to the glass bead microscope made so famous by **Leeuwenhoek** (1703).

1697–1733 CE **Abraham de Moivre** (1667–1754, England). An outstanding mathematician of the 18^{th} century. Extended the pioneering ideas of **Fermat**, **Pascal**, **Huygens** and **Jakob Bernoulli** in probability theory, and originated other 'simple discoveries' which are found today in school textbooks. His achievements are:

(1) Wrote a systematic treatise on probability: "*Doctrine of chances*" (1714). In 1733 he showed the manner in which the normal distribution function arises in probability, as means of approximately evaluating probabilities associated with the binomial law[167]. He proved the central limit theorem for a special case. He is credited with the first treatment of the probability integral $\int_0^\infty e^{-x^2}dx = \frac{1}{2}\sqrt{\pi}$ and of the normal frequency curve $y = ae^{-\lambda^2x^2}$ in statistics theory. Moivre is also noted for his work

[167] De Moivre's observation, in modern notations, is as follows: The binomial law states that

$$(py+q)^n = \sum_{k=0}^{n} P_n(k)y^k,$$

where $P_n(k) = \frac{n!}{k!(n-k)!}p^kq^{n-k}$, $(q = 1-p)$ is the probability of k successes in n independent trials. Simple manipulation via differentiation and the subsequent substitution $y=1$, yield the identities:

$$\sum_{k=0}^{n} P_n(k) = 1, \qquad \bar{k} = \sum_{k=0}^{n} kP_n(k) = np; \qquad \bar{k^2} = \sum_{k=0}^{n} k^2 P_n(t) = n(n-1)p^2 + np$$

It thus follows that the first and second order parameters of the binomial distribution are: mean $= k = np$; dispersion $= D = \bar{k^2} - (\bar{k})^2 = npq$; root mean square deviation$=\sigma = \sqrt{npq}$.

Introducing a new variable $x = k - \bar{k} = k - np$, the binomial probability

$$P_n(k) = \frac{n!}{(np+x)!(nq-x)!} p^{np+x} q^{nq-x}$$

is approximated by means of Stirling's formula $n! \approx n^n e^{-n}\sqrt{2\pi n}$ and becomes

$$\begin{aligned} P_n(k) &= (1+\frac{x}{np})^{-x-np}(1-\frac{x}{nq})^{x-nq}\left[2\pi n(p+\frac{x}{n})(q-\frac{x}{n})\right]^{-1/2} \\ &= \frac{\exp\left[-(np+x)\log_e(1+\frac{x}{np}) + (nq-x)\log_e(1-\frac{x}{nq})\right]}{\sqrt{2\pi n(pq - \frac{x}{n}(p-q) - \frac{x^2}{n^2})}} \end{aligned}$$

Using the Taylor expansions for small $\frac{x}{np}$ and $\frac{x}{nq}$, namely

$$(np+x)\log_e(1+\frac{x}{np}) = (np+x)\left[\frac{x}{np} - \frac{1}{2}\frac{x^2}{n^2p^2} + \frac{1}{3}\frac{x^3}{n^3p^3} - \cdots\right],$$

$$(nq-x)\log_e(1-\frac{x}{nq}) = -(nq-x)\left[\frac{x}{nq} + \frac{1}{2}\frac{x^2}{n^2q^2} + \frac{1}{3}\frac{x^3}{n^3q^3} + \cdots\right].$$

De Moivre obtained the asymptotic expressions

$$P_n(k) \approx \frac{1}{\sqrt{2\pi pqn}} e^{-\frac{1}{2}\frac{x^2}{npq}} \qquad \text{or} \qquad P_n(k) \approx \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(k-\bar{k})^2}{2\sigma^2}}.$$

With $\xi = \frac{k-np}{\sqrt{2npq}} = \frac{k-\bar{k}}{\sigma\sqrt{2}}$, the (approximate) probability of ξ lying between ξ and $\xi + d\xi$ is $p(\xi)d\xi = \frac{1}{\sqrt{\pi}}e^{-\xi^2}d\xi$. Therefore, the probability for k lying between the two values k_1 and k_2 is

$$\frac{1}{\sqrt{\pi}}\int_{\xi_1}^{\xi_2} e^{-\xi^2} d\xi$$

with

$$\xi_1 = \frac{k_1 - np}{\sqrt{2npq}}; \qquad \xi_2 = \frac{k_2 - np}{\sqrt{2npq}}.$$

"*Annuities upon Lives*", which played an important role in the history of actuarial mathematics.

(2) Was first to derive the factorial approximation $n! \simeq (2\pi n)^{1/2}(n/e)^n$, misnamed *Stirling's formula*.

(3) Announced in 1707 the keystone formula of analytic trigonometry[168]:

$$(\cos x + i \sin x)^n = \cos nx + i \sin nx$$

for positive integer n. It was explicitly stated and proved inductively by Euler in 1748.

(4) Introduced (1730) the powerful method of *generating functions* which proved to be of great importance in combinatorics, probability and number theory. He used it to obtain a closed-form expression for the general term of the Fibonacci sequence, namely

$$F_n = \frac{1}{2\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right] + \frac{1}{2}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

(It was rediscovered by Binet in 1843).

Moivre was born in Vitry, Champagne, to a French Huguenot family. His father was a surgeon. He was compelled to take refuge in England at the revocation of the edict of Nantes in 1685. He started his mathematical education in France and furthered it in London. There, he eked out a living by giving private lessons in mathematics and games of chance. He never married.

He was greatly influenced by the "*Principia Mathematica*" of Newton and was among Newton's personal friends. He became distinguished among first-rate mathematicians and was admitted in 1697 to the Royal Society of London. His merits were so well known and acknowledged that the society found him fit to decide the famous contest between Newton and Leibniz. In spite of all this he never secured a university position, perhaps because he was not British by birth.

His old age was spent in obscure poverty. A bizarre story is associated with his death: de Moivre noticed, so the story goes, that each day he required a quarter of an hour more sleep than on the preceding day. When this

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[168] Using this formula he proved that $y = \cos n\theta$ is a polynomial in $x = \cos\theta$. Indeed this is a direct consequence of the relation

$$2y = \left(x + \sqrt{x^2-1}\right)^n + \left(x - \sqrt{x^2-1}\right)^n, \qquad |x| \le 1.$$

He thus anticipated the Chebyshev polynomials $T_n(x) = \cos n(\cos^{-1} x)$.

progression reached 24 hours, de Moivre passed away, almost blind, at the age of 87.

1699 CE **William Dampier** (1652–1715, England). Buccaneer, navigator and oceanographer. In the publication "*Discourse of Winds, Breezes, Storms, Tides and Currents*", he suggested that major ocean currents might be caused by winds (they were previously explained as resulting from ocean height differences produced by evaporation and rain). Distinguished the steady low-latitude trade winds from the mid-latitude westerlies.

Dampier spent 38 years of his life at sea in various capacities: he went to sea as a boy, and joined the British navy in 1672. During 1679–1711 he was engaged in Pirating (1679–1680) and privateering[169] in the South Seas (against the Spaniards). In 1688 he sailed to Australia (then called New Holland) on a pirate ship. In 1699 he reached Australia again in a voyage financed by the British Admiralty. Dampier also reached New Britain and New Ireland, islands off the coast of New-Guinea.

In 1703–1711 Dampier commanded two privateers on an expedition to the South Pacific. Alexander Selkirk[170], the original Defoe's hero, Robinson Crusoe, was the master of one of his vessels.

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169 *Privateer*: armed vessel owned and officered by private person holding a government commission, authorized to use it against hostile nations, especially in capture of merchant shipping. Dampier visited Jamaica (1679) and joined a party of pirates with whom he spent the year 1680 on the Peruvian Coast, sacking, plundering and burning Spanish ships.

170 **Alexander Selkirk** (1676–1721). Scottish sailor. Ran away to sea (1695) and joined Dampier in a privateering expedition to the South Seas, going with the *"Cinque Ports"* galley (96 tons, 16 guns) as sailing master. In September 1704, he quarreled with his captain, and at his own request was put ashore on Mas Afuera, one of Juan Fernandez islets (33.45° S; 80.45° W, some 750 km west of Valparaiso, Chile). There he was marooned in complete solitude for four years and four months (Sept. 1704–Jan. 1709), until taken off by one of Dampier's ships.

He was later given command of one of the privateering vessels. Selkirk met with **Defoe** in Bristol (1711) and handed over his papers to him. Defoe then wrote his novel: *The Life and Strange Surprising Adventures of Robinson Crusoe* (1719). Defoe's narrative is an amalgamation of Selkirk's story and background material from Woodes Rogers' *"Cruising Voyage round the World"* (1712), and Edwards Cooke's "Voyage in the South sea and round the World" (1712) (both, the earliest descriptions of Selkirk's adventures). Nevertheless, most of the incidents in Defoe's masterpiece are fairly independent of his sources; thus the decidedly tropical description of Crusoes island and the whole narrative of the Cannibals' visits etc. agree rather with one of the West Indies than with

Dampier accounts of his voyages [“*New Voyage Round the World*” (1697); “*Voyages and Descriptions*” (1699); “*Voyage to New Holland*” (1699)] are famous. He had great talent for observation, especially of the scientific phenomena affecting the seaman’s life. His style is easy, clear and manly. His knowledge of natural history, though not scientific, appears surprisingly accurate and trustworthy.

1701 CE **Giacomo Pylarini of Smyrna**. A Greek physician. The first immunologist. Inoculated children with smallpox in Constantinople.

1701 CE **Joseph Sauveur** (1653–1716, France). Physicist. Coined the terms ‘*acoustics*’ (study of sound) and ‘*harmonics*’ (multiples of a fundamental frequency) while experimenting with vibrating strings.

1701–1731 CE **Jethro Tull** (1674–1741, England). Gentleman farmer and agriculturist. Technical innovator of the agrarian revolution[171]. Introduced new agricultural machinery (1701) and new farming methods (1731) through the use of manure, pulverization of the soil, growing crops in rows and hoeing to remove the weeds.

Tull was born in Berkshire, and was educated at St. John’s College, Oxford University. He traveled in France and Italy to observe farming methods. In his days, farmers sowed the seed by throwing it by hand. Tull regarded the practice both wasteful and uncertain. So he invented a drill for boring straight rows of holes into which he dropped the seed. His ideas were adopted slowly.

1702 CE First daily newspaper in England.

1703 CE French chemists **Nicolas Lémery** (1645–1715) and **Martin Lister** (1638–1712) promoted a theory that the source of an earthquake was an *explosion* produced by mixing minerals inside the earth composed of the same chemicals used for explosives (iron, sulfur, salt, water). This theory became very popular and **Isaac Newton** in his book *Optiks* (1704) adopted this idea of the *mineral explosion* in subterranean cavities.

Juan Fernandez. The best biography is the “Life and Adventure of Alexander Selkirk” by Jonh Howell (1829).

Selkirk returned to sea, and died as master’s mate of H.H.S. “Weymouth” on Dec. 12^{th} 1721.

171 The English statesman **Charles Townshend** (1674–1738) introduced into England the four-course system of crop rotation (1731), which he first practiced at his Raynham estate.

Robert Bakewell (1725–1795, England) was first to introduce stock-breeding improvements and grassland amelioration by systematic irrigation.

The 'explosion theory' reigned supreme for more than 150 years. It was finally discarded[172] in the wake of the great Napolitan earthquake (1857) under the impact of the new ideas of **Robert Mallet** (1860).

The 'Little Ice Age' (LIA 1560–1850)

Current research on global climate change, drawn from tree rings and Greenland Ice cores, provides much detailed information on weather and climate history[173]. This new information can be correlated with historical accounts on major weather events and their influence on the human condition.

Indeed, this new knowledge provides an engaging history of Western Europe; it reveals that Europe experienced a prolonged warm period known as the Medieval Warm Period[174] (600–1150), cooling of the climate (1150–1460), a brief warming (1460–1560), followed by dramatic cooling known as the Little Ice Age[175] (1560–1850).

For people living near subsistence levels, as most European did before 1800, abrupt changes in weather could mean the difference between prosperity and

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172 As often happen in science, the theory was recently resurrected and incarnated: it has been claimed that under increasing strain, minerals undergo a phase transformation and become metastable polymorphs of higher free energy density. This instability leads to an explosion creating a shock wave with supersonic velocity, which then causes fluidization of the fault core; the fault is thus unlocked, releasing the stored elastic energy, *ergo*, an earthquake.

173 Brian Fagan: "*Little Ice Age*" Basic Books, N.Y. 2001, 246 pp.

174 The *Viking expansion* from Scandinavia through Europe and the North Atlantic (800–1050), occurred through this period.

175 It seems that the LIA affected the entire globe; Temperature data obtained from a *Peruvian* ice core whose layers date from 1600 CE to the present agree well with independent temperature histories derived for the Northern hemisphere, confirming that the LIA of 1400 to 1650 and the climatic impact of the 1815 Tambora volcanic eruption in Indonesia, were global in extent and demonstrating that some mid-latitude glacial records, too, can play important roles in studies of historical climate.

pauperhood, or even between life and death – especially if these changes lasted more than one season. Events such as the French Revolution *and the Irish Potato-famine, are now seen through the lens of weather and its effects on harvests: the colder weather impacted agriculture, health, economics, social strife, emigration and even art and literature. Increased glaciation and storms also had a devastating affect on those that lived near glaciers and the sea.*

The climate of a region is typically defined by its monthly mean temperature *(Fig. 3.2) and annual total* precipitation*. However, direct observations of these variables began only after the invention of the* barometer *(**Torricelli**) and the thermometer (**Galileo**) in the first half of the 17^{th} century.*

To determine earlier climate, investigators infer the climate record from physical and biological fossil data including, among others: oxygen isotope ratios detected in ice cores, tree-rings dating, ice flow and glacier data, and archaeological discoveries, and also from records intended for other purposes such as: weather diaries, shipping logs, tax records, crop production and pricing records, allusions to climate in art and literature, etc.

- *Oxygen Isotope Record: Measurements of the isotopes ratio $^{18}O/^{16}O$ in ice indicates the temperature of the snow at the time if was formed. Higher ratios of the heavier ^{18}O oxygen isotope indicate the snow formed at a higher temperature while lower ratios indicate the snow formed at a cooler temperature.*

- *Tree-Ring Data: individual rings represent individual years while the width of each ring shows the growth-rate during that year. The width of rings from trees found at higher altitudes and higher latitudes in generally a function of temperature, where wide rings indicate warm years and narrow rings indicate cool years. Because the pattern of rings is similar to a fingerprint, dendrochronologists are able to construct a chronology by matching similar ring patterns found in living trees, construction timbers, and fossil trees.*

- *North Atlantic Drift Ice: Drift ice is carried from the Arctic ice pack and the waters north of Iceland by ocean currents. In colder times, arctic waters carry the ice southward while in warmer times the Gulf Stream dominates the Iceland area, keeping drift ice away. Drift ice was carefully observed by Icelanders both from shore and from ships because it threatened ships and therefore affected commerce. Thus, drift ice can be considered a thermometer of the North Atlantic.*

- *Glacier Waxing and Waning: Mountain glaciers in Scandinavia and the Alps can be used to record climatic changes. Because glaciers are*

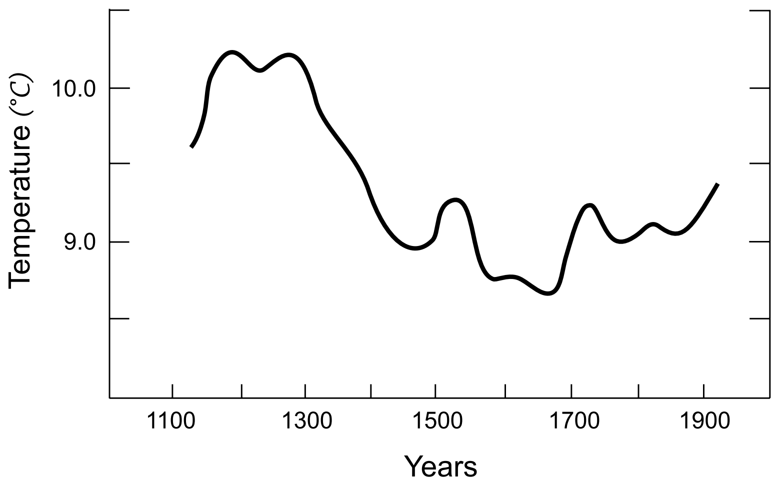

Fig. 3.2: Estimated mean yearly temperatures based on a variety of climatic, political and social indicators. (Redrawn from *Climate, History and the Modern World* **H.H. Lamb** Methuen, London, 1982.)

massive, they respond to long-term temperature and precipitation variations on a time scale of decades and centuries. Glaciers grow during winters by accumulating snowfall and glaciers decline during summers due to above-freezing temperatures. For a glacier to maintain its position, snowfall must equal snowmelt. Cooler summers result in less snowmelt and longer winters increase the number of days of potential snowfall (clearly, cooler winters may also bring drier air which could decrease snowfall, but this factor is much smaller than the former).

There are four main possible causes (forcing mechanisms) of the climatic changes experienced during the LIA. These include: sunspot variations, volcanic eruptions, changes in the large-scale ocean current conveyor belt and changes in the earth's albedo. None of these factors on their own offers conclusive evidence; it is likely that each has played a role.

- *Sunspot Variations: Because the sun is earth's greatest source of energy and is the driving force behind its atmospheric circulation, any variation in solar output will influence the weather. Scientists have observed that the number of sunspots on the surface of the sun has been determined to correspond to solar output variability. More sunspots correspond to a higher solar energy output while fewer sunspots correspond to a lower solar output. A record of sunspot numbers has been recorded through time by various indicators including naked eye observations, auroral reports, and* ^{14}C *isotope concentrations in tree rings.*

  Thus, during the Medieval Warm Period (600–1150 CE) there was a high number of sunspots referred to as the Medieval Maximum, while during the Little Ice Age (1560–1850 CE) there were two periods of very low sunspots numbers called the Spörer Minimum and the Maunder Minimum (1645–1715 CE).

- *Volcanic Eruptions: Ash and other small particulate matter injected into the stratosphere can effectively reduce incoming solar radiation received at the earth's surface. Sulfur compounds from eruptions condense into very tiny sulfuric acid droplets that form clouds which may stay suspended in the stratosphere for years, further reducing incoming sunlight.*

  Large eruptions at low latitudes can cause the greatest global climate change. Weaker eruptions only send their eruptive materials into the troposphere where weather processes quickly remove them and high latitude eruptions only send their materials into one hemisphere. The explosion of Mt. Tambora in 1815 led to the year 1816 being called "the year without a summer" across much of Europe. The eruption of Mt. Pinatubo in 1991 provided a good example of how a large low-latitude eruption can quickly influence global climate: in nine days the sulfur dioxide plume had spread into both hemispheres and around half the planet.

  The result was an estimated $1\,^{\circ}C$ *global cooling that lasted two years. It is unlikely that a single eruption can cause long-term cooling over hundreds of years such as during the LIA. But evidence has shown that there was an increase in the frequency of large eruptions during the LIA that corresponds quite well with the coolest years during this time period.*

- *Large-Scale Ocean Current Conveyor Belt: Warm waters in the upper 1500 meters flow northward to the vicinity of Iceland. Winter cooling increases the density of the water permitting it to sink to great depths. Once at depth, the water flows the length of the Atlantic and becomes mixed into the deep southern hemisphere current. Because the ocean* *and atmosphere are a coupled system, any changes in this large-scale ocean circulation could cause large-scale atmospheric changes on the order of hundreds of years. The ocean is both a heat source for the atmosphere by releasing carbon dioxide, a greenhouse gas, and a heat sink by conducting heat away from the air that rests upon it. Surface water that comes into contact with air is referred to as ventilated water. Scientists have demonstrated that very high rates of deep water ventilation occurred curing the LIA, which means the oceans were removing heat from the atmosphere at a greater rate than normal during that period. That could explain the dramatic cooling observed during the LIA.*

- *Earth Albedo: Albedo is a measure of the reflectivity of a surface. Snow and ice have a high albedo because their properties allow them to reflect up to 90% of incoming sunlight. After a global cooling event has begun, it can become self-perpetuating. With increased snow cover and glaciation, the planet's surface will have a higher albedo, which in turn will cause more incoming sunlight to be reflected. With less sunlight being absorbed at the earth's surface there will be a subsequent cooling effect. This cooling effect may cause even more snow cover and glaciation that would increase the planet's albedo even more. As the climate cooled during the LIA, earth's albedo increased due to more snow and greater glaciation. The process can last for many years; however, it eventually does subside because cooler oceans experience less evaporation which leads to a decrease in cloud cover. Reduced cloud cover allows more sunlight to reach the surface which results in higher global air temperatures.*

The Historical Record

The cooling of 1.5–$2.0\,^{\circ}C$*, synchronous over broad regional areas for a span of several hundred years caused a wide gamut of accompanying phenomena, documented in Europe and North America:*

(i) Glacier movements

Glaciers in many parts of Europe began to advance about the mid-13th *century, influencing almost every aspect of life for those unfortunate enough to be living in their path. It destroyed farmland and caused massive flooding. Glaciers in the Swiss Alps advanced, gradually engulfing farms and crushing entire villages. On many occasions bishops* *and priests were called to bless the fields and to pray that the ice stopped grinding forward. Various tax records show glaciers over the years destroying whole towns caught in their path. A few major advances are:*

***1595 CE**: Gietroz (Switzerland) glacier advances*, dammed Dranse *River, and caused flooding of Bagne with 70 deaths.*

***1600–1610 CE**: Advances by Chamonix (France) glaciers cause massive floods which destroyed three villages and severely damaged a fourth.*

***1670–1680's CE**: Maximum historical advances by glaciers in Eastern Alps. Noticeable decline of human population by this time in areas close to glaciers, whereas population elsewhere in Europe had risen.*

***1695–1709 CE**: Iceland glaciers advance dramatically, destroying farms.*

***1710–1735 CE**: A glacier in Norway was advancing at a rate of 100 m per year for 25 years.*

***1748–1750 CE**: Norwegian glaciers achieved their historical maximum LIA positions.*

In general, habitual structures which were once at high altitude in the Alps were destroyed by glacier activities: a glacier blocked the Saas valley, including its river (1589) and eventually formed a lake. Ice sheets advanced over farms, villages and valleys in Greenland. Once productive Icelandic farms were covered by advancing glaciers. So serious was the climatic change experienced by Icelanders that Denmark, the parent country, considered evacuating all the Icelanders and re-settling them in Europe.

Glaciers advances in North America occurred from 1711–1724 and 1835–1849.

(ii) Storms

During the LIA, there was a high frequency of storms. As the cooler air began to move southward, the polar jet stream strengthened and followed, which directed a higher number of storms into the region. At least four sea floods of the Dutch and German coasts in the thirteenth century were reported to have caused the loss of around 100,000 lives. Sea level was likely increased by the long-term ice melt during the MWP which compounded the flooding. Storms that caused greater than 100,000 deaths were also reported in 1421, 1446, and 1570. Additionally, large hailstorms that wiped out farmland and killed great numbers of livestock *occurred over much of Europe due to the very cold air aloft during the warmer months. Due to severe erosion of coastline and high winds, great sand storms developed which destroyed farmlands and reshaped coastal land regions.*

Two great storms in the North Sea occurred in 1362 and in 1703. The first destroyed the Island of Strand and the city of Ronghold. The second (Nov 26) killed ca 8,000 people on the eastern coast of the British Isles.

(iii) Freezing

The Baltic Sea and rivers such as the Thames in England and the Tagus in Spain, currently ice-free the year around, were regularly frozen several inches thick. Winter Landscape, painted by ***Peter Brueghel the Younger*** *(1601) exhibit the frozen canals of Holland, now regularly ice-free the year around. A generation earlier,* ***Peter Breughel the Elder*** *recorded the merrymaking of Flemish peasantry in their daily lives. His artworks-started off with fairly warm sunny summer weather, but In the 1560s he suddenly switched to cold snow-swept landscapes. This change began with* Hunters in the Snow*, depicting a group of men returning from a hunt, set against a frozen lake. It was at this time that the winter of 1564–1565 struck – the longest and most severe for well over a century.*

In the winter of 1780, New York Harbor froze, allowing people to walk from Manhattan to Staten Island. Sea ice surrounding Iceland extended for miles in every direction, closing the island's nation's harbors to shipping.

(iv) Volcanic Activity

Throughout the Little Ice Age the world also experienced heightened volcanic activity. When a volcano *erupts, its ash reaches high into the atmosphere and can spread to cover the whole earth. This ash cloud blocks out some of the incoming solar radiation, leading to world-wide cooling that can last up to two years after an eruption. Also emitted by eruptions is* sulfur *in the form of SO_2 gas. When this gas reaches the* stratosphere *it turns into sulfuric acid particles, which reflect the sun's rays, further reducing the amount of radiation reaching the earth's surface. The 1815 eruption of* Tambora *in* Indonesia *blanketed the atmosphere with ash; the following year, 1816, came to be known as the* Year Without A Summer*, when frost and snow were reported in June and July in both* New England *and Northern* Europe*.*

There's probably no better example of the artistic weather record than ***Joseph Turner****. Because he was obsessed with the light of the sky, clouds and sea, Turner has given us a stunning insight into the climate of the early nineteenth century. His glorious red skies were a particular sign of strong atmospheric powers at work, because this was a time when volcanic eruptions in the Azores in 1811 and Tambora in 1815 had shot clouds of dust across the globe. That dust cooled the earth and scattered the light, filtering out the blues in the low sun and giving sumptuous red sunrises and sunsets.*

(v) *Agriculture, economics and health*

Crop-failures, poor harvests, increasing grain prices, lower wine-production and severe diseases marred the lives of people in Western Europe during the LIA, especially throughout the Maunder minimum.

These ere some of the many disasters impacted by the dramatic cooling of the climate. Due to the famine, storms, and growth of glaciers, many farmsteads were destroyed, which resulted in less tax revenues collected due to decreased value of the properties.

The change in climate during these years greatly affected crop production and animal husbandry. Famine became more frequent and death from diseases increased.

Each grain crop requires several conditions before a successful growing season and harvest is possible. Minimum temperatures are necessary for seed germination. Higher altitudes are more susceptible to adverse climatic cooling. Frost will occur later in the spring and earlier in the fall causing a shortened growing season. Increase cloud cover and cool weather retard the growing process and prolong the ripening of the grain. In addition, if the summer remains wetter than usual, grain crops may not be able to mature by drying out. If an early frost comes, the still-moist grain will suffer damage. A cooling trend can affect the growing plant in several ways, compounding the possibility of crop failure.

Thus, in the years of the LIA the price of grain increased over five times, imposing an obvious hardship on the poor.

It is estimated that in the coldest decades of the Little Ice Age the growing season was shortened by 3-4 weeks. This may represent an approximate reduction of 20% of the total growing season which would range from May to September in the Northern latitudes.

Exceptionally grim reports of mass deaths are frequent in the literature of this time. There were population decreases in large portions of Europe. While diseases such as bubonic plague (Black Death) definitely *had their effect, the generally weakened health of the people in years of poor harvest must certainly be considered. In fact, population declines attributed to low food levels began 40 years before the plague arrived.*

In conclusion, the cooler climate during the LIA had a huge impact on the health of Europeans, dearth and famine killed millions. Cool, wet summers led to outbreaks of an illness called St. Anthony's Fire. Whole villages would suffer convulsions, hallucinations, gangrenous rotting of the extremities, and even death. Grain, if stored in cool, damp conditions, may develop a fungus known as ergot blight and also may ferment just enough to produce a drug similar to LSD. (In fact, some historians claim that the Salem, Massachusetts, witch hysteria was the result of ergot blight.)

Malnutrition led to a weakened immunity to a variety of illnesses. In England, malnutrition aggravated an influenza epidemic of 1557–8 in which whole families died. In fact, during most of the 1550's deaths outnumbered births The Black Death (Bubonic Plague) was hastened by malnutrition all over Europe.

(vi) *Social Unrest*

Conditions during the LIA led to many cases of social unrest. The winter of 1709 killed many people in France. Conditions were so bad, a priest in Angers, in west-central France, wrote:

"The cold began on January 6, 1709, and lasted in all its rigor until the twenty-fourth. The crops that had been sewn were all completely destroyed... Most of the hens had died of cold, as had the beasts in the stables. When any poultry did survive the cold, their combs were seen to freeze and fall off. Many birds, ducks, partridges, woodcock, and blackbirds died and were found on the roads and on the thick ice and frequent snow. Oaks, ashes, and other valley trees split with cold. Two thirds of the vines died... No grape harvest was gathered at all in Anjou... I myself did not get enough wine from my vineyard to fill a nutshell."

In March the poor rioted in several cities to keep the merchants from selling what little wheat they had left.

The winter of 1739–1740 was also a bad one. After that there was no spring and only a damp, cool summer which spoiled the wheat harvest. The poor rebelled and the governor of Li told the rich to "fire into the middle of them. That's the only way to disperse this riffraff".

One of history's most notorious quotes might have been due in part to a rare extremely warm period during the LIA. In Northern France in 1788, *after an unusually bad winter, May, June, and July were excessively hot, which caused the grain to shrivel. On July 13, just at harvest time, a severe hailstorm (which typically occurs when there is very cold air aloft) destroyed what little crops were left. From that bad harvest of 1788 came the bread riots of 1789 which led to Marie Antoinette's alleged remark "Let them eat cake," and the storming of the Bastille.*

(vii) *Vampires and Violins*

Writers were also influenced by the great change in climate. In 1816, "the year without a summer," many Europeans spent their summers around the fire. ***Mary Shelley*** *(1797–1851) was inspired to write 'Frankenstein', and* ***John Polidori*** *(1795–1821), 'The Vampyre'. Both authors, together with* ***Byron*** *and* ***Percy Shelley****, were in Switzerland, near Lake Geneva where Byron said "We will each write a ghost story." Percy Shelley also referred to a glacier in his poem "Mont Blanc" when he wrote "and wall impregnable of beaming ice. The race of man flies far in dread; his work and dwelling vanish".*

The less intense solar radiation and activity coincided with a sharp decline in temperature, causing a very cold weather in Western Europe. It is clearly seen in tree-ring records from high-elevation forest stands in the European Alps. The long winters and cool summers produced wood that has slow, even growth – desirable properties for producing quality sounding boards.

Antonio Stradivari *of Cremona, Italy, perhaps the most famous of violin makers*[176]*, was born one year before the beginning of the Maunder minimum (1645–1715). He and other violin makers of the area used the only wood available to them from the trees that grew during the Maunder minimum. It was suggested*[177] *that the narrow tree-rings of*

176 The violin first emerged in Northern Italy in the mid 1500's. Many of the most distinguished violins ever created were produced by famous local families. The most famous makers were: **Andrea Amati** (1520–1578; Italy); **Jacob Stainer** (ca 1617–1683; Austria); **Antonio Stradivari** (1644–1737; Italy); **Francesco Stradivari** (1671–1743; Italy); **Andrea Guarneri** (ca 1626–1698; Italy); **Giuseppe Guarneri** (1666–ca 1739); **Giuseppe del Gesu Guarneri** (1687–1745). **Stradivari**, the most famous of these craftsmen, produced over 1100 violas, guitars, cellos and violins. Around 600 of his instruments are extant today. Narrow tree-rings would not only strengthen the violin but would increase the wood's density. Dense wood with narrow growth-rings may help instill a superior tone and brilliance in violins.

177 **H. Grissino-Mayer** and **L. Burckle**: *Dendrochronologia*, **21**, 41–45, Lamont-Doherty Earth Observatory, Dec 2003.

these trees played a role in the enhanced sound quality of instruments produced at this time.

1703 CE, Nov. 26 An ocean tempest killed ca 8000 people on the eastern coast of the British isles. Probably the greatest British storm of the last two millennia. Fifteen warships and hundreds of merchant vessels (with about 1500 seamen on board) were lost. Thousands of trees were laid low throughout the country, including ca 4000 large oaks in the New Forest, Cranbourne Chase, and the Forest of Dean. Houses were blown to pieces; church steeples toppled like skittles; floods were widespread, and Bristol's streets ran into water. Eddystone lighthouse, together with its crew, was swept like a heap of rubble into the sea. When it was all over, the Commons presented an address to Queen Anne and she issued a proclamation of general fast.

1704–1709 CE **Abraham Darby** (1678–1717, England). Iron and brass manufacturer who developed a process for smelting iron using *coke*[178]

[178] *Coke* is prepared by carbonizing coal in *coke-ovens*. The old oven consisted of a covered mound of brickwork, in which coal was partly burnt in a limited supply of air, as in charcoal burning. The high temperature produced carbonizes the rest of the coal, and all the volatile products are lost.

The *blast-furnace* consisted of an outer shell of steel plates, lined with refractory bricks. It was a 15–30 m hight, the greatest width being about 8 m at the "boshes". The mouth was closed with a *cap-and-cone* (above) through which the charge of ore, limestone and fuel was fed intermittently, whilst the gases (carbon monoxide and nitrogen) pass away through a pipe to a *dust-catcher*, and are utilized in heating the blast. The furnace below the boshes narrows gradually to a *hearth* at the base, about 3 m in diameter and the same height. This was pierced with holes for the water – jacketed iron blowing-pipes or *tuyeres*, through which air was forced from an annular pipe by means of powerful blowing engines. About 3–5 tons of air were passed through the furnace per ton of iron made, the power for working the blowing-engines being supplied by coke-oven gas obtained in producing the coke for the blast furnace.

The first extensive use of cast-iron was in England (1544). Formerly, charcoal was used as fuel; coal was used by **D. Dudley** (1599–1684) in England (1619).

The chemical reactions in the blast-furnace are as follows:

- Oxygen unites with carbon at a very high temperature in the *hearth* to produce carbon monoxide: $2C + O_2 = 2CO$.

instead of the more expensive charcoal. Darby produced coke to use in *blast-furnaces*. It was made by partly burning coal in a closed chamber. The heating drives out the volatile material from the coal, including most of the sulfur. This produced a high-carbon fuel which was clearer and hotter than coal. Formerly wood-based charcoal was used to smelt iron.

The discovery greatly increased the market for coal and improved iron production. With the invention of the *Newcomen engine*, this breakthrough marked one of the starting points of the Industrial Revolution in England. (Darby employed the cheaper iron to cast thin pots for domestic use, and after his death it was used for the huge cylinders required by the new steam pumping-engines.)

His grandson, Abraham Darby III (1750–1791) constructed the world's first *iron bridge*, over the river Severn at Coalbrookdale, Shropshire.

Darby was born near Dudely, Worcestershire to a Quaker family and trained in engineering, setting up his own business (1698). He visited Holland (1704) and brought back with him some Dutch brass[179] founders, establishing them in Bristol, later moving to Coalbrookdale. They experimented with substituting cast iron for brass in some products and in 1708 Darby took out a patent for a new way of casting iron pots and other ironware in sand only,

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- Above the boshes, at a dull red-heat (500 °C–900 °C) the ferric oxide is reduced by the carbon monoxide to spongy iron: $Fe_2O_3 + 3CO \leftrightharpoons 3Fe + 3CO_2$.

  The reaction is reversible and the escaping gases contain CO and CO_2. Another reaction also occurs which limits the completeness of the reduction: $2Fe + 3CO \leftrightharpoons Fe_2O_3 + 3C$. In this upper zone the limestone is decomposed: $CaCO_3 \leftrightharpoons CaO + CO_2$, and some carbon dioxide is reduced to monoxide: $CO_2 + C \leftrightharpoons 2CO$. The spongy iron absorbs sulphur from the fuel.

- Near the center of the furnace, at bright red heat, finely-divided carbon is deposited by the reaction: $2CO \leftrightharpoons CO_2 + C$. This and the carbon of the charge complete the reduction:

  $$Fe_2O_3 + 3C = 2Fe + 3CO.$$

- At the white zone, in the lowest part of the furnace, the spongy iron containing carbon, sulphur, phosphorus and silicon, fuses to molten *cast-iron* which is tapped-off from time to time into sand moulds to form *pig-iron*, or is sent in the fused state to the steel furnaces.

[179] After the restoration of the monarchy England's economy had surged, and the demand for household brass rose rapidly.

without loam or clay. This process cheapened utensils much used by poorer people.

Table 3.6: Iron metallurgy – signposts of progress, 1650–1950

In England, the age of iron marched from triumph to triumph; New techniques were developed for its production and new users were found for it. Vast new smelting houses were built. Such was the demand, that Britain had to import some 50,000 tons a year.

1665 CE Smelting iron with *charcoal* in England.

1709 CE **Darby** (England) developed a process using *coke* in blast furnaces.

1740 CE **Huntsman** (England) rediscovered the crucible process of making *cast steel*. The small ingots produced by this process could not be used yet to build bridges or railways.

1781 CE Darby's grandson constructs in England the world's first *cast-iron bridge* (over the Severn River); the bridge is still used by pedestrians. The 378-ton bridge spans 30 m.

1783–1784 CE The English ironmaster **Henry Cort** (1740–1800) invented a process for purifying iron by *puddling* and a method of producing iron bars by means of grooved rollers. He produced *wrought iron*. It used a '*reverberatory furnace*' where raw coal and low-carbon iron were kept separate to reduce impurities in the finished product (Cort himself was ruined by a prosecution for debt and died poor).

1790 CE *Stainless steel* is produced in England.

1801 CE The engineer **James Finlay** (USA) completed the first modern suspension bridge in Pennsylvania, USA. It used iron chains for support.

1822 CE The engineer **George Stephenson** (1781–1848) built the first *iron railroad bridge* in the world. The first *iron steamship* to cross the Channel was assembled on the Thames from parts fabricated in England.

1845 CE The engineer **William Fairbairn** (1789–1874, Scotland) built the first *steel bridge*.

1851 CE Architect **Joseph Paxton** (1801–1865, England) built the *Crystal Palace* of glass and iron for the London exhibition.

1855 CE **Henry Bessemer** (England) developed the *Bessemer process* of converting *pig iron* into steel: cold air was forced through holes in the base of the furnace and through the molten iron, burning up the carbon. The device produced large amounts of steel cheaply.

1861 CE **William Siemens** (1823–1883, Germany and England) and **Pierre Emile Martin** (1824–1915, France) streamlined steelmaking with their independent invention of the *open-hearth process*: air and hot gas pass over the molten pig iron. The gases from the molten metal are then used to heat the air, to save fuel.

1885 CE The first skyscraper is erected in Chicago, USA, using presaged technique introduced by Paxton (1851).

1902 CE **P. Héroult** (France) began producing steel in a *electric-arc furnace*. This gave very high temperatures, producing much purer steel.

1904 CE **Leon Guillet** (France) developed the first stainless steel that resist corrosion.

1913 CE **H. Brearley** (England) first made stainless steel by adding *chromium* to steel. This prevents rusting.

1947 CE **H. Hartley** (England) added *titanium* to iron to produce much stronger iron.

1948 CE The *basic oxygen process* was introduced in Austria. This is the main method of making steel today: A jet of oxygen is blown on the molten iron, quickly burning up the carbon and producing steel. It is ten times faster than the open-hearth process.

NOMENCLATURE:

Ore A natural deposit of a solid containing an insoluble compound of a metal. Ores contain *minerals* (comparatively pure compounds of the metals of interest) and mixed with relatively large amounts of *gangue* (sand, soil, clay, rock and other materials). *Native ores* is the free state of the less active metals: Au, Ag, Pt, Os, Ir, Ru, Rh, Pd, As, Si, Bi, Cu.

Common classes of ores are:

- **Oxide** Fe_2O_3 (hematite); Fe_3O_4 (magnetite); Al_2O_3 (bauxite); SnO_2 (cassiterite); MgO (periclase); SiO_2 (silica).
- **Sulfide** $CuFeS_2$ (chalcopyrite); Cu_2S (chalcocite); ZnS (sphalerite); PbS (galena); FeS_2 (iron pyrites); HgS (cinnabar).
- **Chloride** NaCl (rock salt); KCl (sylvite); $KCl \cdot MgCl_2$ (carnallite).
- **Carbonate** $CaCO_3$ (limestone); $MgCO_3$ (magnesite); $MgCO_3 \cdot CaCO_3$ (dolomite).
- **Sulfate** $CaSO_4 \cdot 2H_2O$ (gypsum); $MgSO_4 \cdot 7H_2O$ (epsom salt); $BaSO_4$ (barite).
- **Silicate** $Be_3Al_2Si_6O_{18}$ (beryl); $Al_2(Si_2O_8)(OH)_4$ (kalinite); $LiAl(SiO_3)_2$ (spodumence).

Alloy Mixing of a metal with another substance (usually other metals) to modify its properties. Among these are: *bronze* (copper+tin), *brass* (copper+zinc), *pewter* (tin+antimony+copper), *German silver* (copper+nickel+zinc), *yellow gold* (gold+copper), *white gold* (gold+palladium+silver), *sterling silver* (silver+copper), *wrought iron* (iron+small percentage of carbon), *cast iron* (iron+2 percent or more of carbon), *steel* (many different alloys of iron containing carbon and one or more metals such as: manganese, nickel, tungsten, molybdenum, cobalt, vanadium and chromium), *stellite* (cobalt+chromium+tungsten), *carboloy* (tungsten+carbon+cobalt), *woods metal* (bismuth+tin+lead+cadmium).

Metallurgy The overall process by which metals are extracted from ores.

Roasting Heating a compound below its melting point in the presence of air. It removes sulfur, CO_2, moisture and other impurities from the ore. The remaining solid material contains a metallic oxide.

Smelting Chemical reduction of a substance at hight temperature in metallurgy. Basically it is a process of melting the ore in such a way as to remove impurities.

In the case of iron, for example, the ore is placed in a huge, brick-lined furnace called a blast furnace and subjected to high heat by blasting hot air into the bottom half of a furnace, producing temperatures up to 1000 °C. Quantities of coke and limestone are also placed in the furnace. As the heat of the furnace is raised, the coke begins to burn and gives off carbon monoxide. This gas takes oxygen from the iron oxide, helping to purify the metal.

Many of the other impurities of the ore melt and combine with the limestone to form a liquid collection of waste materials (*refuse*), which is lighter than iron. This refuse rises to the top of the molten metal, and is taken from the furnace as *slag*.

Pig iron The iron as it comes from the blast furnace. It usually contains 95 percent iron, 3 to 4 percent carbon, and smaller amounts of manganese, phosphorus, sulfur, and other elements.

Coke A substance produced from coal by heating it in the absence of air; the heating drives out volatile material from the coal, including most of the sulfur. It also consolidates the carbon into strong lumps, stronger from either coal or charcoal.

Cast iron Reprocessed pig iron: it is remelted in a coke-burning furnace and cooled. It is brittle because it contains much iron carbide Fe_3C, but cheaper to make.

Wrought iron Pig iron is melted and most of the impurities are removed. The molten iron is then poured over a glassy mass of melted sand, or *silica slag*. The iron separates into droplets which quickly start to harden. Gases are trapped inside each droplet. The gases build up pressure and cause the drops to explode. The iron and silicate form spongelike balls of iron. These sponge balls are placed in presses to squeeze out the excess slag and form the wrought iron into blocks. The tiny threads of iron silicate make wrought iron more *malleable* (easier to hammer) and more resistant to corrosion than other kind of iron.

Wrought iron, with no carbon, was stronger but expensive because of the extra work involved in making it. What everyone was looking for was an effective compromise: cheap iron with just a little carbon, what is now called – *wild steel*. But the only kind of steel then available was unsuitable. From about 1000 BCE onwards bars of iron could only be steeled by hand labor with hammer and anvil or by *roasting* with charcoal in a furnace. The secret of melting steel, though practiced in India before the Christian era, was unknown in the West until 1740 CE, when a Yorkshire clock maker devised a process which made Sheffield steel world famous.

Steel An alloy of iron and small definite amounts of other metals. There are many types of steel, containing alloyed metals and other elements in various controlled proportions. Stainless steel show high tensile strength and excellent resistance to corrosion. The most common kind contains 14 to 18% chromium and 7 to 9% nickel.

Pig iron can be converted into steel by burning most of the carbon with O_2 in an oxygen furnace: Oxygen is blown through a heat-resistance tube inserted below the surface of the molten iron. Carbon burns to CO, which subsequently escapes and burns to CO_2.

1704–1711 CE **Luigi Ferdinando Marsigli** (1658–1730, Italy). Naturalist, adventurer, soldier, writer, and student of the sea, whose life was stranger than fiction: A general in the Austrian army, a slave in Turkey, a pensioner of the Queen of Sweden, and a fellow of the French and London Royal Societies. He wrote one of the first textbooks of oceanography, published in Venice in 1711.

While in the Bosporus, Marsigli observed the currents flowing between the Black Sea and the Mediterranean. He found that the surface water flows out of the Black Sea, but the deep water flows in the opposite direction. The local fishermen had been making good use of this fact. To travel from the Black Sea to the Mediterranean, a fisherman merely drifted in the surface current. To proceed in the opposite direction, he lowered his net into the bottom current. The large net acted as a sea anchor, dragging the boat toward the Black Sea against the surface current.

When Marsigli studied the deposits brought up from the sea bottom by fishermen, he became interested in the depth of the sea. He investigated the variation of temperature in the Mediterranean and found that it does not change significantly with depth. He measured the density of seawater with a hydrometer and found that the density increases with depth.

Marsigli was born at Bologna. After a course of scientific studies in his native city he traveled through Turkey, collecting data on the military organization of that empire, as well as on its natural history. On his return he entered the services of the emperor Leopold (1682) and fought with distinction against the Turks, by whom he was wounded and captured in a battle on the River Raab, and sold to a pasha whom he accompanied to the siege of Vienna. His release was purchased in 1684, and he afterwards took part in the war of the Spanish succession. In 1703 he was appointed second in command under Count Arco in the defense of Alt-Breisach. The fortress surrendered to the Duke of Burgundy, and Marsigli was court martialed and forced to give up soldiering. He then devoted the rest of his life to scientific investigations, in the pursuit of which he made many journeys through Europe, spending a considerable time at Marseilles to study the nature of the sea.

1705–1733 CE **Stephen Hales** (1677–1761, England). Clergyman, physiologist, chemist and inventor. Inaugurated the science of plant physiology and is one of the originators of experimental physiology. First to investigate the role of gases in *plant metabolism* and measure *blood pressure*.

The first volume of his book *Vegetable Staticks* (1727) contains an account of numerous experiments on the exchange of gases in plants, flow of fluids in plants and plant respiration, root pressure, leaf growth and the rise of sap under varying plant conditions, weather conditions and time of day. He concluded that plants drew through their *leaves* some part of their nourishment from the air. In the second volume (1733) on *Haemostaticks* he reported methods of determining *blood pressure* in man and animals and also the rate of flow, and the capacity of different vessels. He first reported observing *elasticity in arteries*[180].

Hales was born in Bekesbourne in Kent, grandson of Sir Robert Hales, who was created a baronet by Charles II, in 1670. Studied divinity, anatomy and chemistry at Cambridge and received the degree of doctor of divinity from Oxford (1733). Elected Fellow of the Royal Society (1717) and foreign associate of the French Academy of Sciences (1753).

Hales invented artificial *ventilator* for ships, prisons and hospitals, and devised forceps to aid remove kidney and bladder stones.

1706 CE **William Jones** (1675–1749, England). Mathematician. Introduced the symbol π, adopted by Euler in 1739.

1706 CE **John Machin** (1680–1752, England). Mathematician. Professor of astronomy in London. Calculated π to 100 decimal places[181], using

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180 As the heart *pumps blood* into the arteries during ventricular *systole*, a greater volume of blood enters the arteries from the heart than leaves them to flow into smaller vessels down-stream, because the smaller vessels have a greater resistance to flow. The arteries' elasticity enables them to expand to temporarily hold this excess of volume of ejected blood, storing some of the energy imparted by cardiac contraction, in their *stretched* walls.
When the heart *relaxes* and ceases pumping blood into the arteries, the stretched arterial walls passively *recoil*. This recoil pushes the excess blood contained in the arteries into the vessels down-stream, ensuring continued blood flow to the tissues when the heart is relaxing and not pumping blood into the system.

181 $\tan 4\beta = 4\frac{t(1-t^2)}{1-6t^2+t^4}$; $\tan\left(4\beta - \frac{\pi}{4}\right) = \frac{\tan 4\beta - 1}{\tan 4\beta + 1}$; $t = \tan\beta$.
Taking $\tan\beta = \frac{1}{5}$ we find $\tan 4\beta = \frac{120}{119}$ and $\tan\left(4\beta - \frac{\pi}{4}\right) = \frac{1}{239}$. Consequently:

$$\tan^{-1}\frac{1}{239} = 4\beta - \frac{\pi}{4} = 4\tan^{-1}\frac{1}{5} - \frac{\pi}{4},$$

which is Machin's formula. Substituting Gregory's series

$$\tan^{-1} x = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \cdots,$$

the arctangent series

$$\frac{\pi}{4} = 4\arctan\left(\frac{1}{5}\right) - \arctan\frac{1}{239}.$$

1706–1716 CE **Roger Cotes** (1682–1716, England). Mathematician. A contemporary of Newton who undertook the publication of the second edition of Newton's *Principia*.

He was the first to develop, in 1714, the important relation

$$i\theta = \log_e(\cos\theta + i\sin\theta),$$

which is usually attributed to Euler. To Cotes we also owe a geometric theorem (1714) which depends on the factorization of a trigonometric function[182], and 'theorem of the *harmonic mean*'.[183]

Cotes was born at Burbage, Leicestershire, the son of a rector. He was educated in Trinity College, Cambridge (1699–1705) and in 1706 was appointed Plumian professor of astronomy and experimental philosophy. He took orders in 1713. After his death, his papers were collected and published under the title *Harmonia Mensurarum* (1722). His meteoric career earned him Newton's exclamation, "*If Cotes had lived, we might have known something*".

1706–1761 CE **Giovanni Battista Morgagni** (1682–1771, Italy). Physician and anatomist. Founder of the science of *pathological anatomy*.

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one finds

$$\frac{\pi}{4} = 4\left[\frac{1}{5} - \frac{1}{3\cdot 5^3} + \frac{1}{5\cdot 5^5} - \cdots\right] - \left[\frac{1}{239} - \frac{1}{3\cdot 239^3} + \frac{1}{5\cdot 239^5} - \cdots\right].$$

182 *The factorization*:

$$x^{2n} + 2x^n y^n \cos n\alpha + y^{2n} = \prod_{0}^{n-1}\left\{x^2 + 2xy\cos\left(\alpha + \frac{2k\pi}{n}\right) + y^2\right\}$$

is proven by factoring the complex bivariate polynomial $\left\{x^n + \exp(in\alpha)\times y^n\right\}$, making use of the n-th root of unity,and taking the modulus assuming x, y to be real.

183 "If, through a fixed point O, a variable line is drawn, cutting an algebraic curve at points $(P_1, P_2, P_3, \ldots, P_n)$, and if a point H is taken on the line such that OH is the harmonic mean of $OP_1, OP_2, \ldots, OP_n$, then the locus of H is a straight line."

His object was to relate the illness to the lesions established at autopsy. His explorations of the female genitals, of the glands of the trachea and of the male urethra, broke new grounds. His work (1761) *De sedibus et Causis Morborum per Anatomen Indagatis* (Caused of Diseases) was grounded on over 600 postmortems. Morgangi was born in Forli. Graduated from the University of Bologna and practiced medicine there. Professor at Padua University 1711–1771.

1706 CE **Francis Hauksbee** (Hawksbee) (c. 1666–1713, England). Physicist. Studied *surface tension* and *capillary* cation in fluids. Invented (1706) an *electrostatic generator*. Constructed a two-cylinder vacuum pump. Experimented with *electroluminescence*. He is called 'the elder' to distinguish his from his nephew of the same name (1688–1763) and the similar scientific interests.

1707–1732 CE **Hermann Boerhaave** (1668–1738, The Netherlands). Physician, chemist and botanist. Founded modern system of clinic instruction. A man with immense academic knowledge who dominated and influenced various branches of science in Europe.

Boerhaave was born in Voorhout, near Leiden. Went to the University of Leiden (1684) where he studied philosophy, botany, languages, chemistry and medicine (graduated 1693). Professor al Leiden (from 1709).

1709–1714 CE **Gabriel Daniel Fahrenheit** (1686–1736, Germany). Physicist. Proposed a temperature scale that bears his name. He also made the thermometer more accurate by using mercury instead of alcohol in the thermometer tube. He determined three fixed temperatures: $0\,^{\circ}$F for the freezing point of ice + salt + water; $32\,^{\circ}$F for the freezing point of pure water and $96\,^{\circ}$F for the normal temperature of the human body. These three temperatures correspond respectively to $-17.77\,^{\circ}$, $0\,^{\circ}$, and $35.55\,^{\circ}$ on the Celsius temperature scale. Later experiments proved the body normal temperature to be $98.6\,^{\circ}$F, or $37\,^{\circ}$C.

Fahrenheit was born in Danzig. For the most part he lived in England and Holland, devoting himself to the study of physics and making a living by the manufacture of meteorological instruments. He also invented an improved form of a hygrometer, of which he published an account in the Phil. Trans. of 1724. He died in Holland.

His temperature scale is still extensively used in the United States and Great Britain.

1710–1744 CE **Giambattista (Giovanni Battista) Vico** (1668–1744, Italy). Historical philosopher and jurist. Criticized radically the aims of science as outlined through the Cartesian system. Claimed that mathematics does not enable us to promote a knowledge of nature as much as the rationalists thought and therefore tried to discover a 'new science' that was both perfectly knowable and about the real world. Advanced the basic principle that we can know only what we can do or make[184].

Vico's work contains the terms of many developments in the philosophy of the 19^{th} century. His ideas echoed through the *Sturm and Drang* movement in Germany (**Goethe**, **Herder**, 1770) and he extended great influence upon **Karl Marx** (1860), **Benedetto Croce** (1902), **Georges Sorrel** (1908) and **Oswald Spengler** (1918).

In his own time, however, and for fifty years after his death, Vico remained practically unknown. He was born in Naples, son of a small bookseller and lived there or in its environs until his death. Educated by priests, he became, at the age of thirty-one, a minor professor of rhetoric at the university of his native city. This somewhat subordinate position he held until his retirement in 1741. Most of his life he was poor. To keep himself and his family he had to eke out his modest salary by giving private tuition and composing inscriptions, Latin eulogies and laudatory biographies for the nobility. In the last years of his life he was rewarded by being appointed official historiographer to the Austrian Viceroy of Naples.

Vico is known mainly for his *Principi d'una Scienza Nuova* (1725). He founded no school and his philosophy seemed to die with him; his name was soon obscured, especially as the Kantian system dominated the world of thought. His reinstatement was completed by **Michelet** (1827)[185], who translated his books.

According to Vico, mathematics, being an arbitrary construction of the human mind is divorced from nature. It is not as Descartes supposed, a discovery of an objective structure, the eternal and most general characteristic of

[184] His approach was to establish a clear distinction between the world as it really is and the world which we create and cognize through the use of mathematical models and physical experiments. He realized that the understanding one has of something created by oneself is of a different nature to that understanding gleaned from simple observations. This distinction means we can never be free from subjectivism. Vico saw that mathematical models appear intelligible and coherent to our minds because our minds alone have made them. All our inquiry is necessarily anthropocentric because we employ man-made tools and human reasons in its pursuit. Vico believed the 'real' world of nature, which obeyed inaccessible rules, differed in kind from our do-it-yourself model of intelligible but man made laws.

[185] **Jules Michelet** (1798–1874, France). Historian. Professor, College de France (1838–1851).

the real world, but rather an *invention*: invention of a symbolic system which men can logically guarantee only because men have made it themselves; but men cannot make the physical world. Nature herself was made by God and therefore only he can fully understand her. As far as man goes, he can learn something about nature by adopting an empirical approach through experiment and observation and not so much through a mathematical procedure[186]. Nature is not completely knowable. His concept of knowledge led Vico to argue further that man can fully know only what he himself invented, created or participated in, i.e., such provinces as participated in, i.e., such provinces as mathematics, mythology, language, symbolism and its own history.

Faced with the choice between a perfect understanding of a philosophical system divorced from reality and an imperfect understanding of the reality of life, Vico chose the latter and developed his concept that, since men could only fully apprehend the reality of their own creations, the task of philosophy should be the study of the universal principles underlying the history of nations. He pleaded that history should be written by philosophers.

Vico was the first thinker who asked, why have we a science of nature, but no science of history? Because our glance can easily be turned outwards and survey the exterior world; but it is far harder to turn the minds eye inwards and contemplate the world of the spirit.

Vico advanced the *cyclical theory of history* which maintains, counter to the Christian concept of time, that humanity advances not in a straight line but along an upward *spiral* staircase, with each spiral bringing man closer to freedom and nearer to God.

He declared that there were three great doors that led into the past: language, myths, and rites (institutional behavior). The task before those who wish to grasp what kind of lives have in the past been led in societies different from their own, is to understand their worlds through each of the above categories. Poetry, for example, is a direct form of self-expression of our remote ancestors, collective and communal. Myths are far-reaching images of past social conflicts out of which many diverse cultures grew.

Vico maintained that the Homeric poems were the sublime expression of a society dominated by the ambition, avarice and cruelty of its ruling class; for only a society of this kind could have produced this vision of life. Later ages may have perfected other aids to existence, but they cannot create the

[186] Vico failed to see the role it plays in scientific research. At the same time one might allow that there was here a *warning* against unbridled mathematical speculation, which sometimes tries to pass for empirical work. The proper approach lies somewhere between these two extremes.

Iliad, which embodies the modes of thought and expression and emotion of one particular kind of way of life; these men literally saw what we do not see.

The scientific method is adequate for establishing bare facts. However, the task of historians is not merely to establish facts and give causal explanation for them. The knowledge that they need is not knowledge of facts or of logical truths, provided by observation or the science of deductive reasoning. They must possess imaginative power of a higher degree. Without this power of entering into minds and situations, the past will remain a dead collection of objects in a museum for us. Without some ability to get into the skin of others, the human condition, history cannot be understood. This use of informed imagination about, and insight into, systems of value, conceptions of life of entire societies, is not required in mathematics or physics.

The ideas of Vico provided a natural prologue to the more critical analysis which were to be developed by **David Hume** and **Immanuel Kant**.

1711 CE Austria and Germany devastated by plague. About 500,000 died.

1712 CE **Thomas Newcomen** (1663–1729, England). One of the inventors of the early steam engine. His 'fire engine' (1712) was used for pumping water from mines until **James Watt** invented one with a separate condenser. Newcomen's engine was the first to use a piston and a cylinder. Earlier models of the machine were constructed by him during 1705–1712 with the aid of the military engineer **Thomas Savery** (1650–1715). The whole situation is confused by a patent granted to Savery and in later years Newcomen paid royalties to Savery. It is also known that Newcomen corresponded with **Robert Hooke** about the previous investigations of **Denis Papin**.

Newcomen was born in Dartmouth, Devon, and set up a blacksmith's shop there, assisted by a plumber called **John Calley**. The Newcomen's engine consumed an enormous amount of coal, because fresh hot steam had to be raised for each piston stroke. The early engines were very expensive, because the cylinder was made by brass; later, iron cylinders were produced but they were thick-walled and consequently even less efficient in terms of coal consumed. However, they were mostly used in coal mines. It was with the Newcomen's engine that the age of steam began.

As late as the French revolution (1793 CE), it has been estimated, Europe drew energy from about 14 million horses and 24 million oxen. All these societies exploited energy sources that where *renewable*: nature could eventually replenished the forests they cut, the wind that filled their sails, the rivers that turned their puddle wheels. Even animals and people were replenished "energy slaves".

A revolutionary shift began after Newcomen's engine. Societies, by contrast, drew their energy from coal, gas and oil – from *irreplaceable* fossil funds.

It meant that for the first time a civilization was eating into natures capital rather than merely living off the interest it provided.

This dipping into the earth's energy reserves provided a hidden subsidy for industrial civilization – a vastly accelerated economic growth. And from that day to this, nations built towering technologies and economic structures on the assumption that cheap fossil fuels would be endlessly available.

1712–1715 CE **Brook Taylor** (1685–1731, England). Mathematician. Discovered the polynomial approximation of analytic functions near a given point[187]. Also, contributed to the general development of the calculus. In

[187] Taylor expansion of a function $f(x)$ $[f^{(n)}(x)$ continuous] about a point $x = a$ is

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n-1)}(a)}{(n-1)!}(x-a)^{n-1} + R(x),$$

with a *remainder*

$$R(x) = \frac{1}{(n-1)!}\int_a^x (x-s)^{n-1} f(s)ds = \frac{f^{(n)}(\xi)}{n!}(x-a)^n, \qquad a < \xi < x.$$

Newton's method of finding an approximate local solution to an equation of the form $f(x) = 0$ follows from Taylor's expansion in the following way: Suppose c denotes the solution to the above equation and $f''(x)$ exists on an interval containing both c and the initial value x_0. Expanding $f(x)$ in Taylor series about x_0 we have

$$0 = f(c) = f(x_0) + (c - x_0)f'(x_0) + \frac{1}{2}f''(\xi)(c - x_0)^2,$$

or

$$c - x_0 + \frac{f(x_0)}{f'(x_0)} = -\frac{1}{2}\frac{f''(\xi)}{f'(x_0)}(c - x_0)^2,$$

where $f'(x_0) \neq 0$. Denoting $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ (Newton's first approximation), Taylor's expansion implies that

$$c - x_1 = -\frac{1}{2}\frac{f''(\xi)}{f'(x_0)}(c - x_0)^2.$$

If a bound M is known for the second derivative of f on an interval about c and x_0 is within the interval, then $|c - x_1| \leq \frac{M}{|2f'(x_0)|}|c - x_0|^2$. This inequality implies that Newton's method has the tendency to approximately double the number of digits of accuracy with each successful approximation.
Succeeding approximations are generated by applying the formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

1715 he introduced the idea of "*integration by parts*"[188]. During 1715–1717, Taylor invented the concept of *finite difference*, thus initiating the calculus of finite differences. Devised the basis principle of perspective in his *Linear Perspective* (1715).

Taylor was born in Edmonton, Middlesex. He was elected a Fellow of the Royal Society (1712) and was appointed in that year to the committee for adjudicating the claims of Newton and Leibniz to have invented the calculus.

'Taylor's Expansion' on a Sumerian Cuneiform Tablet?

The ancient Sumerians in Mesopotamia gave some interesting approximations to the square root of nonsquare numbers, like $\frac{17}{12}$ *for* $\sqrt{2}$ *and* $\frac{17}{24}$ *for* $\frac{1}{\sqrt{2}}$. *A remarkable approximation for* $\sqrt{2}$ *is*

$$1 + \frac{24}{60} + \frac{51}{(60)^2} + \frac{10}{(60)^3} = \mathbf{1.414215}5$$

Choosing $f(x) = x^n - k$ and taking x_0 to be an approximation to $\sqrt[n]{k}$, we find $x_1 = \frac{n-1}{n}x_0 + \frac{k}{nx_0^{n-1}}$.

A useful generalization of the Taylor expansion in which one function is expanded in terms of *another given function*, was discovered by **Heinrich Bürmann** (1799):

$$f(x) = f(a) + \sum_{k=1}^{n-1} \frac{\alpha_k(a)}{k!}\big[g(x) - g(a)\big]^k + R(x),$$

where $\alpha_k(x) = \frac{\alpha'_{k-1}(x)}{g'(x)}$, $k = 1, 2, \ldots$, $\alpha_0(x) = f(x)$, $R(x) = \frac{\alpha_n(\xi)}{n!}\big[g(x) - g(a)\big]^n$, $a < \xi < x$, $f^{(n)}(x)$ and $g^{(n)}(x)$ continuous; $g'(x) \neq 0$. The function $\alpha_k(x)$ is given explicitly by the expression

$$\alpha_k(x) = \Big[\frac{1}{g'(x)}\frac{d}{dx}\Big]^k f(x).$$

188 The name "*integration by parts*" first appeared in 1797 in a book by **Sylvestre Francois Lacroix** (1765–1843, France).

(correct to 5 decimals), found on the Yale tablet YBC 7289 dated about 1600 BCE.

When searching for $\sqrt{x}$, the Sumerian would start with some approximation a and then generate a sequence of increasingly better approximations. In modern notation they calculated

$$a_1 = \frac{1}{2}\left(a + \frac{x}{a}\right), \quad a_2 = \frac{1}{2}\left(a_1 + \frac{x}{a_1}\right), \quad \cdots, \quad a_{n+1} = \frac{1}{2}\left(a_n + \frac{x}{a_n}\right).$$

This iterative algorithm by successive approximation was known to the Greeks, as is evident from the writings of ***Heron*** *(ca 50 CE).*

Let us apply their technique to evaluate $\sqrt{2}$ and take $a = 1$ as our initial guess. Then

$$\begin{aligned} a_1 &= \frac{1}{2}\left(1 + \frac{2}{1}\right) = \frac{3}{2} \equiv 1 + \frac{30}{60}, \\ a_2 &= \frac{1}{2}\left(\frac{3}{2} + \frac{2}{3/2}\right) = \frac{17}{12} \equiv 1 + \frac{25}{60}, \\ a_3 &= \frac{1}{2}\left(\frac{17}{12} + \frac{2}{17/12}\right) = \frac{577}{408} \equiv 1 + \frac{24}{60} + \frac{51}{(60)^2} + \frac{10}{(60)^3}, \end{aligned}$$

which leads to the result inscribed on the Yale tablet!

Al-Khowarizmi *(ca 825 CE) spoke of rational numbers as* audible *and surds as* inaudible*, and it is the latter that gave rise to the word* surd *(deaf, mute in Arabic). The European use of this word begins with* ***Gerhardo of Cremona*** *(ca 1150 CE).*

The Arab mathematicians (e.g. ***Al-Karkhi****, 1020 CE) and medieval writers used the approximation*

$$a + \frac{h}{2a+1} < \sqrt{a^2 + h} < a + \frac{h}{2a}, \qquad 0 < h \le a.$$

Now, on the r.h.s. we recognize the old Sumerian first approximation. Indeed, take $x = a^2 + h$ and then

$$a_1 = \frac{1}{2}\left(a + \frac{a^2 + h}{a}\right) = a + \frac{h}{2a}.$$

The European mathematicians before Newton generalized the surd approximation to the case of cube roots. Thus, ***Joannes Buteo*** *(1492–1572, France) derived $\sqrt[3]{a^3 + h} \approx a + \frac{h}{3a(a+1)}$ (1559) and* ***Stevin*** *followed suit (1634) with $\sqrt[3]{a^3 + h} \approx a + \frac{h}{3a(a+1)+1}$. With the development of the Newtonian calculus* *and the expansions of Taylor and Maclaurin that followed in its wake, it was recognized that the approximation*

$$\sqrt[n]{a^n + h} \approx a + \frac{h}{na^{n-1}} \qquad (h < a)$$

corresponds to the first approximation used by Heron for $n = 2$ *(the relevant Sumerian analogs were discovered only in 1943).*

Moreover, Taylor's polynomial expansion shed some light on the entire Sumerian method of approximating square roots. for, if we apply their technique to $\sqrt{1+x}$, *starting with the approximate value* $a = 1$, *we find*

$$\begin{aligned} a_1 &= \frac{1}{2}\left[1 + \frac{1+x}{1}\right] = 1 + \frac{x}{2} \\ a_2 &= \frac{1}{2}\left[1 + \frac{x}{2} + \frac{1+x}{1+x/2}\right]. \end{aligned}$$

But for $|x| < 1$, $\frac{1}{1+x/2} = 1 - \frac{x}{2} + \frac{x^2}{4} - \cdots$ *renders*

$$a_2 = 1 + \frac{x}{2} - \frac{x^2}{8} + \cdots .$$

Continuing this process will lead to the Taylor polynomial of higher and higher degrees. The Sumerian approximation, however, had the clear advantage of being valid for all values of x *and not just* $|x| < 1$.

1715–1750 CE **Giulio Carlo Fagnano dei Toschi** (1682–1766, Italy). Mathematician. Discovered the formula $\pi = 2i \log_e \frac{1-i}{1+i}$, in which he anticipated **L. Euler** in the use of imaginary exponents and logarithms. His studies on the rectification of the ellipse, the hyperbola and the lemniscate are the starting-points of the *theory of elliptic functions*. Suggested new methods in solving equations of degree 3 and 4. He gave expert advice to Pope Benedict XIV regarding the safety of the cupola of St. Peter's at Rome. In return the Pope promised to publish his mathematical investigations. For some reason, the promise was not fulfilled and they were not published until 1750.

1716–1720 CE **Jacob Hermann** (1678–1733, Switzerland). Mathematician. Worked in mechanics and first to study the '*inverse problem*', where one has to determine the orbit from the knowledge of the law of force. One of the pioneers of '*theoretical mechanics*'. Hermann was a pupil of **Jakob Bernoulli** and was a professor of mathematics in the University of Padua (1707–1713), at Frankfurt a.d.O. (1713–1724), at St. Petersburg (1724–1731) and at Basel.

The Evolution of Trigonometry (280 BCE–1720)

Trigonometry, in its essential form of showing how to deduce the values of the angles and sides of a triangle when other angles and sides are given, is an invention of the Greeks, although the basic trigonometry of the right-angled triangle was known to the Babylonians and the Egyptians.

Thus, the history of trigonometry stretches over a period of some 2000 *years from* ***Aristarchos of Samos*** *to* ***Euler****.*

Trigonometry found its origin in the computations demanded for the reduction of astronomical observations and in other problems connected with astronomical science: After the 3^{rd} *century BCE, mathematical research shifted increasingly away from the pure forms of constructive geometry toward areas related to applied disciplines, in particular to astronomy. Also, in the* 2^{nd} *century BCE, the Greeks first came into contact with the fully developed Mesopotamian astronomical systems and took from them many of their observations and parameters.*

While retaining their own commitment to geometric models rather than adopting the arithmetic schemes of the Mesopotamians, the Greeks nevertheless followed the Mesopotamians' lead in seeking a predictive astronomy based on a combination of mathematical theory and observational parameters. They thus made it their goal not merely to describe but to calculate *the angular positions of the planets on the basis of the numerical and geometric content of the theory. This major restructuring of Greek astronomy, in both its theoretical and practical aspects was primarily due to* ***Hipparchos*** *(ca 150 BCE), whose work was consolidated further by* ***Ptolemy****.*

To facilitate their astronomical researches, the Greeks developed techniques for numerical measurements of angles, a precursor of trigonometry, and produced tables for practical computations. Early efforts to measure numerical ratios in triangles were made by ***Archimedes*** *and* ***Aristarchos****. Their* *results were soon extended, and comprehensive treatises on the measurement of chords (effectively tables of values of the sine function) were produced by* ***Hipparchos*** *and by* ***Menelaos of Alexandria*** *(ca 98* CE*). These works are now lost, but the essential theorems and tables are preserved in Ptolemy's Almagest. For computing with angles the Greeks adopted the Mesopotamian sexagesimal method in arithmetic, whence it survived in the standard units for angles and time employed to this day.*

It so happened that spherical trigonometry was developed before the simpler plane trigonometry.

In place of sine, cosine and tangent, the Greek astronomers Hipparchos and Ptolemy (150 CE) always used chords of arcs of circles. In fact it makes little difference whether one operates with cords or with sines, since what we now call the sine of an angle is the quotient by the radius of one half of the chord of twice the intercepted arc i.e.

$$\sin\alpha = \frac{1}{2R} chord(2\alpha)$$

[it is only since around 1800 CE that we divide by the radius and regard the l.h.s. as more fundamental than chord (2α)].

As early as the 5^{th} century CE, the Hindu astronomers changed from the chords to the sines.

The Hindus, who were much more adept calculators than the Greek, availed themselves of the Greek geometry which came from Alexandria, and made it the basis of trigonometrical calculations. The principal improvement which they introduced consists in the formation of tables of half-cords (or sines) instead of chords.

Although the Hindus could calculate sines and cosines of one degree [$\sin 1^\circ = \frac{10}{573}$, $\cos 1^\circ = \frac{6568}{6569}$] with greater accuracy than Ptolemy, they did not apply their trigonometrical knowledge to the solution of triangles. For astronomical purposes they solved right-angled plane and spherical triangles by geometry.

The Arabs were acquainted with Ptolemy's almagest and they probably learned from the Hindus the use of the sine. The Arab astronomer ***Albategnius*** *(850–929) employed the sine regularly, and was fully aware of the advantage of the sine over the chord. He was also acquainted with the formula*

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

for a spherical triangle ABC.

Abu al-Wafa of Baghdad *(940–998) was the first to introduce the tangent as an independent function. This improvement was forgotten, however, and the tangent was reinvented in the* 15^{th} *century.*

Ibn Yunus of Cairo *(d. 1008), Alhazen's contemporary and countryman (they both lived in Egypt), introduced the formula*

$$2\cos x \cos y = \cos(x+y) + \cos(x-y).$$

[This formula, and 3 similar ones, were used in 16^{th} *century Europe to convert products to sums before the invention of the logarithm!] He showed even more skill then Albategnius in the solution of problems in spherical trigonometry, and gave improved approximate formulae for the calculation of sines.*

The Western Muslim astronomer, ***Jabir Ibn Aflah of Seville*** *(frequently called* ***Jabir*** *or* ***Geber****), who flourished ca* 1130*, discovered the relation* $\cos B = \cos b \sin A$, *valid for a spherical triangle* ABC *with a right angle at* C*. This formula escaped the notice of* ***Ptolemy****. Strangely enough, Jabir made no progress in plane trigonometry.*

George Peurbach *(**Purbach**) (1423–1461), professor of mathematics at Vienna, wrote a work entitled Tractatus super propositiones Ptolemaei de sinubus et chordis (Nuremberg, 1541). This treatise consists of a development of a method of interpolation for the calculation of tables of sines, and was published posthumously by Regiomontanus at the end of his works.*

Johannes Müller *(1436–1476), known as Regiomontanus, was a pupil of Purbach and taught astronomy at Padua; he wrote an exposition of the Almagest, and a more important work, De triangulis planis et sphericis cum tabulis sinuum, which was published in* 1533*, a later edition appearing in* 1561*. He reinvented the tangent and calculated a table of tangents for each degree, but did not make any practical applications of this table, and did not use formulae involving the tangent. His work was the first complete European treatise on trigonometry, and contains a number of interesting problems; but his methods were in some respects behind those of the Arabs.*

Copernicus *(1473–1543) gave the first simple demonstration of the fundamental formulae of spherical trigonometry; the Trigonometria Copernici was published by* ***George Joachim*** *(1514–1576), known as Rheticus (1542). He wrote Opus palatinum de triangulis, which contains tables of sines, tangents and secants of angles at intervals of* $10''$ *from* 0° *to* 90°*. His method of calculation depends upon the formulae which give* $\sin(n\alpha)$ *and* $\cos(n\alpha)$ *in terms of the sines and cosines of* $(n-1)\alpha$ *and* $(n-2)\alpha$*; thus these formulae may be regarded as due to him. Rheticus found the formulae for the sines of the half and third of an angle in terms of the sine of the whole angle.*

In 1595 *there appeared an important work by* ***Bartholomaeus Pitiscus*** *(1561–1613), entitled Trigonometriae seu De dimensione triangulorum, in which the word 'trigonometry' was first coined; this contained several important theorems on the trigonometrical functions of two angles, some of which had been given before by* ***Thomas Fincke***, ***Landsberg*** *(or* Lansberghe de *Meuleblecke) and* ***Adrian van Roomen***.

Francois Viète *or* ***Vieta*** *(1540–1603) employed the equation*

$$\left(2\cos\frac{1}{3}\phi\right)^3 - 3\left(2\cos\frac{1}{3}\phi\right) = 2\cos\phi$$

to solve the cubic $x^3 - 3a^2x = a^2b \ \left(a > \frac{1}{2}b\right)$; *he obtained, however, only one root of the cubic. In* 1593 *Van Roomen proposed, as a problem for all mathematicians, to solve the equation*

$$45y - 3795y^3 + 95634y^5 - \cdots + 945y^{41} - 45y^{43} + y^{45} = C.$$

Vieta gave $y = 2\sin\frac{1}{45}\phi$, *where* $C = 2\sin\phi$, *as a solution, and also twenty-two of the other solutions, but he failed to obtain the negative roots. In his work Ad angulares sectiones* Vieta *gave formulae for the chords of multiples of a given angle in terms of the chord of the angle itself.*

A new stage in the development of the science was commenced after ***John Napier****'s invention of logarithms in* 1614. *Napier also simplified the solution of spherical triangles by his rules for the solution of right-angled triangles. The first tables of logarithmic sines and tangents were constructed by* ***Edmund Gunter*** *(1581–1626), professor of astronomy at Gresham College, London; he was also the first to employ the expressions cosine, cotangent and cosecant for the sine, tangent and secant of the complement of an angle.*

A treatise by ***Albert Girard*** *(1590–1634), published at the Hague in* 1629, *contains the theorems which give areas of spherical triangles and polygons, and applications of the properties of the supplementary triangles to the reduction of the number of different cases in the solution of spherical triangles. He used the notation sin, tan, sec for the sine, tangent and secant of an angle.*

In the second half of the 17^{th} *century the theory of infinite series was developed by* ***John Wallis***, ***Gregory***, ***Mercator***, *and afterwards by* ***Newton*** and ***Leibniz***. *In the Analysis per aequationes numero terminorum infinitas, which was written before* 1669, *Newton gave the series for the angle in powers of its sine; from this he obtained the series for the sine and cosine in powers of the angle; but these series were given in such a form that the law of the formation of the coefficients was hidden.*

James Gregory discovered in 1670 *the series for the angle in powers of the tangent and for the tangent and secant in powers of the angle. The* *first of these series was also discovered independently by Leibniz in* 1673*, and published without proof in the Acta eruditorum* for 1682*. The series for the sine in powers of the angle he published in* 1693*; this he obtained by differentiation of a series with undetermined coefficients.*

In the 18^{th} *century the science began to take a more analytical form; evidence of this is given in the works of* ***Jakub Kresa*** *(1648–1715) in* 1720 *and* ***Mayer*** *in* 1727*.* ***Friedrich Wilhelm von Oppel****'s Analysis triangulorum (1746) was the first complete work on analytical trigonometry. None of these mathematicians used the notation sin, cos, tan, which is the more surprising in the case of Oppel, since* ***Leonhard Euler*** *had, in* 1744*, employed it in a memoir in the Acta eruditorum.* ***Johann Bernoulli*** *was the first to obtain real results by the use of the symbol* $\sqrt{-1}$*; he published in* 1712 *the general formula for* $\tan(n\phi)$ *in terms of* $\tan\phi$*, which he obtained by means of transformation of the angle into imaginary logarithms.*

Further advance was made by ***Euler****, who brought the science in all essential respects into the state in which it is at present. He introduced the present notation into general use; until his time the trigonometrical functions had been, except by Girard, indicated by special letters, and had been regarded as certain straight lines, the absolute lengths of which depended on the radius of the circle in which they were drawn.*

Euler's great improvement consisted in his regarding the sine, cosine, &c., as functions of the angle only, thereby giving to equations connecting these functions a purely analytical interpretation, instead of a geometrical one as before. The exponential values of the sine and cosine, ***de Moivre****'s theorem, and a great number of other analytical properties of the trigonometric functions, are due to Euler, most of whose writings are to be found in the Memoirs of the St. Petersburg Academy.*

1722 CE **Johann Sebastian Bach** (1685–1750, Germany). Composed *The Well-Tempered Clavier*, written in the tempered scale with 12 notes per octave having a *fixed frequency ratio* of $2^{1/12} = 1.0595$. Although there is controversy as to whether Bach ever played on an instrument tuned according to equal temperament, his *Well-Tempered Clavier* had considerable influence on the use of the system.

This scale may have originated in China long before the time of **Pythagoras** (ca 540 BCE). **Michael Stifel** (1544) introduced the scale to Europe. **Mersenne** (1636), however, was the first to give the correct frequency ratios for equal temperament. One should note that Bach was able to employ the tempered scale only because **Napier** had invented the logarithms before him, shortly after 1600.

1724 CE **Jacopo Francesco Riccati** (1676–1754, Italy). An Italian savant who wrote on mathematics, physics and philosophy. He was chiefly responsible for introducing the ideas of Newton to Italy. At one point he was offered the presidency of the St. Petersburg Academy of Sciences, but preferred the leisure and comfort of his aristocratic life in Italy. Though widely known in scientific circles of his time, he did very little original work and his name now survives only through the differential equation $y' = p(x) + q(x)y + r(x)y^2$, bearing his name. Even this was an accident of history, for Riccati only discussed special cases of this equation without offering any solutions, and even most of these were treated by various members of the Bernoulli family. The term '*Riccati's equation*' was given by **d'Alembert** (1763).

1725–1741 CE **Vitus Bering** (1680–1741, Denmark). Navigator and explorer in Russian service. Was dispatched by Peter the Great to explore the waters off north-eastern Siberia. The Bering Island, sea and strait take their name from him. In a series of voyages he discovered the Bering strait, crossed to Kamchatka and explored the Aleutian Islands.

1728–1748 CE **James Bradley** (1693–1762, England). Astronomer. Discovered the phenomenon of *starlight aberration* (1728): because of the earth's orbital motion, if starlight is to pass through the length of the telescope[189] the telescope must be slightly tilted forward in the direction of the earth's motion relative to the *actual* line of sight to the star's position; i.e. the apparent direction of the star is displaced slightly from its geometrical direction, and the displacement is in the direction of the earth's orbital motion.

Since the speed of light is about 10,000 times that of the earth in its orbit, the angle through which a telescope must be tilted forward can be as large as ten-thousandth of a radian, or about $20.5''$. The effect is greatest when the earth is moving at right angles to the star's direction.

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[189] *Analogy*: A man stands still, holding a straight drain pipe in a vertical upright position. If it is raining, and if raindrops fall vertically (no wind), they will fall through the length of the pipe. But if the man walks forward with a fixed speed, v, he must tilt the pipe forward so that drops entering the top will fall out at the bottom without being swept up by the approaching inside of the wall of the pipe. If the raindrops fall with speed V in the earth's frame, the pipe must be tilted at an angle α to the vertical such that $\tan\alpha = \frac{v}{V}$.

Bradley found that if a telescope is pointed in a certain direction to observe a particular star on one night, then 6 month later the telescope must be pointed in a slightly different direction to observe the same star. Let a star be located on a line from the sun that is perpendicular to the earth's orbital plane. Because of the earth's motion with orbital velocity V, the tilt angle α of the star's rays is given by $\tan\alpha = V/c$, where c is the speed of light in vacuum. Bradley measured the difference in sighting angles 6 months apart, obtaining $2\alpha = 40.4''$ (arc seconds). Combined with $V = 30$ km/sec (known independently from celestial mechanics), this value for 2α radians gives

$$c = \frac{V}{\tan\alpha} \approx \frac{V}{\alpha} = \frac{3.0\times 10^4 \frac{m}{sec}}{20.2\times\pi/(3600\times 180)} = 3.06\times 10^8\ m/sec.$$

The *relativistic* expression is

$$\tan\alpha = \beta(1-\beta^2)^{-1/2},$$

$\beta = V/c$. When $\beta \ll 1$, this expression reduces, as in the present case, to $\tan\alpha \approx \beta$.

A star that is on the ecliptic plane appears to shift back and forth in a straight line during the year. A star in a direction perpendicular to the earth's orbit appears to describe a small circle in the sky. [In 1862, **J. Foucault** reversed the logic of the above calculation by using his measurement of the speed of light to calculate the earth's speed, and hence verify its distance to the sun.][190]

In 1737 Bradley discovered the *nutation* of the earth's axis, which is a motion caused by the temporal irregularity of the forces that cause the precession: for instance when the sun or moon is in the plane of the earth's bulge, no tidal torque is applied by the respective body. The sun crosses the celestial equator twice a year and the moon crosses it twice a month, and at these moments of crossing there will be no torque effecting precession due to one or the other. In addition, there are variations in the orientation of the moon's orbit with respect to the ecliptic. All these factors affect precession by causing slight fluctuations in its rate and in the tilt angle of the earth's axis to the ecliptic[191]. These are the nutations (Latin for "nodding"), which

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[190] The distance is approximately equal to $\frac{VT}{2\pi}$ where V is the earth's orbital speed and T is its orbital period (1 year). Knowing this, the mass of the sun can then be estimated from Kepler's third law. This idealized calculation assumes a circular orbit.

[191] However, even a simple spinning top on a flat table, generally undergoes nutation, since precession competes with the tendency of the tilted top to fall.

result in an overall periodic motion of the earth's pole relative to the "fixed stars" much faster than its precession (period: 18.6 years).

Bradley was born in Sherborne, Gloucestershire. He graduated from Oxford University in 1717 and was trained in astronomical observations by his uncle, a skilled astronomer. He became a professor of astronomy at Oxford in 1721 and served until 1742. He then became the director of Greenwich Observatory, succeeding **Edmund Halley** as astronomer royal.

1728–1749 CE **Pierre Bouguer** (1698–1758, France). Mathematician and physicist. Invented the *photometer* (1748) and the *heliophotometer*. Considered as the father of *photometry*. Devised a method to relate gravity anomalies to deficiency of mass in the earth's crust. Participated in the French astro-geodetic expedition[192] to Peru (1735–1744), and made his measurements in the high Andes.

Bouguer was appointed in 1723 to succeed his father as professor of hydrography. In 1730 he was made professor of hydrography at Havre, and succeeded Maupertuis as member of the Académie des Sciences.

1728–1755 CE **Daniel Bernoulli** (1700–1782, Switzerland). A distinguished mathematician of the 18^{th} century. Son of Johann Bernoulli. Studied medicine like his father, and like him gave it up to become a professor of mathematics, at St. Petersburg. In 1733 he returned to Basel and was successively a professor of botany, anatomy and physics. He won 10 prizes from the French Academy, for one of which his father was among the competitors. In a fit of jealous rage Johann threw his son out of the house for winning the prize that he coveted for himself.

His famous book '*Hydrodynamica*' (1738) includes the earliest treatment of the *kinetic theory of gases* and the famous *Bernoulli principle*[193] [obtained

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[192] This expedition, and another to Lapland (1736–1737), established that the earth is flattened at the poles. The flattening, predicted theoretically by **Newton** (1687), was confirmed by the expeditions' measurements of 110,600 m for the length of a degree of latitude in Peru and 111,900 m for the corresponding length in Lapland.

[193] *The Bernoulli principle*: If an incompressible frictionless homogeneous fluid with density ρ moves without friction, the sum of pressure p and kinetic energy per unit volume remains constant (along a streamline): $p + \frac{1}{2}\rho v^2 =$ constant. Thus an increase in the flow velocity v is accompanied by a decrease in the pressure exerted by the fluid on the walls of the container.

Many aerodynamic effects are consequences of Bernoulli's principle. For example, subsonic aircraft obtain most of their lift from the pressure difference between underside and top of the wing, whose profile makes the air flow faster *before* the discovery of the Euler equation, by considerations similar to the modern principle of conservation of energy]. He also used the Fourier series expansion long before Fourier (1828).

He is considered by many to have been the first genuine mathematical physicist.

Apart from Jakob, Johann and Daniel, the Bernoulli family produced another 6 mathematicians of distinction:

- **Nicolas** (**Nicolaus**, 1687–1759). Professor of mathematics at Padua. Contributed to probability theory and the theory of infinite series. Nephew of Jakob and Johann.
- **Nicolas II** (1695–1726). Professor of mathematics at St. Petersburg. Empress Catherine ordered him a state funeral upon his premature death.
- **Johann II** (1710–1790). Professor of mathematics at Basel. Contributed to the theory of heat diffusion and light propagation.
- **Johann III** (1744–1807). Astronomer at the Academy of Berlin. Son of Johann II.
- **Jakob II** (1759–1789). Professor of mathematics at St. Petersburg. Tragically drowned while bathing in the Neva, a few months after his marriage to the granddaughter of Leonhard Euler. Son of Johann II.
- **Daniel II** (1751–1834). Professor of mathematics at Basel. Son of Johann II.

The Bernoulli family, with all its mathematical talent, also had more than its share of arrogance and jealousy, which turned brother against brother and father against son. In three successive generations, fathers tried to steer their sons into nonmathematical careers, only to see them gravitate back to mathematics. The fiercest conflict occurred between James, John, and Daniel.

During his teens Daniel was tutored by his older brother Nicholas II; his father wanted him to go into business, but when that career failed Daniel was permitted to study medicine. During his years at St. Petersburg Academy (1725–1733) he conceived his ideas on modes of vibrations and produced the

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over the top than along the underside. Similarly, the "curve ball" familiar to baseball aficionados is the Bernoulli effect on a moving sphere whose spin will cause a difference in air flow velocity and thus a pressure difference on opposite sides of the sphere.

first draft of his *Hydrodynamica*. Although he missed the basic partial differential equations of hydrodynamics, his book used systematically the principle of conservation of energy. Unfortunately, publication of *Hydrodynamica* was delayed until 1738. His father, John then published a book on hydrodynamics in 1743, dating his book to 1732! - one of the most blatant priority theft in the history of mathematics. Daniel complained to Leonhard Euler (1743) with the result that John's reputation was so tarnished by the episode that he did not even receive credit for parts of his work that *were* original.

1728 CE **Pierre Fauchard** (1678–1761, France). Dentist. Founder of modern dentistry. He practiced in Paris from 1715 and was influential in raising dentistry from a trade into a profession. He advocated the sharing of dental knowledge and wrote the two volume '*La chirurgien Dentiste ou traité des dents*' (1728). It includes detailed discussions on the treatment of caries, the making and using of removable dentures, and a variety of dental instruments. After removal of the carious material, with pain reduced through application of oil of cinnamon, the cavity is filled with small pieces of thin foil of tin, or gold.

After the publication of Fauchard's work the practice of dentistry became more specialized and distinctly separated from medical practice, the best exponents of the art being trained as apprentices by practitioners of ability, who had acquired their training in the same way from their predecessors.

Fauchard suggested porcelain as an improvement upon bone and ivory for the manufacture of artificial teeth.

1729–1753 CE **Jean Astruc** (1684–1766, France). Physician, medical researcher and historian, and a pioneer biblical critique and exegetic. One of the most prolific medical authors of the 18^{th} century. Physician to August II, King of Poland; consultant to Louis XV of France. Descendant from a Jewish Marrano family. Wrote extensively on venereal and skin diseases. Considered as the progenitor of the modern scholarly and textual investigation of sources of Pentateuch (1753).

1730–1746 CE Identification, study and industrial working of the metallic element *zinc*[194] by **Isaac Lawson** (1730, England), **John Champion** (1743,

[194] **Plato** (ca. 400 BCE) refers to brass, an alloy of zinc and copper. An alloy containing 23 percent of zinc and 10 percent of tin was found at *Gezer* (ancient Israel), already in 1500 BCE. The name zinc derived from *tusku* (mentioned in Assyrian tablets of 650 BCE), probably zinc carbonate, $ZnCO_3$, used also by the alchemists. Deposits of *calamine* (native zinc carbonate) occur in the old Greek silver mines of Laurion. The extraction of zinc from its ores was in

England) and **Andreas Sigismund Marggraf** (1746, Germany). Marggraf made the earliest complete study of zinc.

1730 CE **James Stirling** (1692–1770, Scotland). Mathematician. Presented the approximation of the factorial function for large argument $n! \approx \sqrt{2\pi n}(n/e)^n$, or more generally[195]: $\Gamma(x) \sim \sqrt{\frac{2\pi}{x}}(x/e)^x$. He also contributed to the calculus of finite differences [*Stirling's interpolation formula*, *Stirling numbers*[196], and *Stirling factorial series*[197]].

Stirling, a descendant of a noble Scottish family, was educated at Glasgow and Oxford. He was expelled from Oxford for supporting the Jacobite cause, and lived in Venice during 1716–1724. His return to Britain is supposed to have been hastened because he had learned some secrets of the glass industry, and may have feared for his life. Newton, whose friendship he enjoyed, helped him secure fellowship in the Royal Society. Stirling's book *Methodus differentialis*, which appeared in 1730, included most of his mathematical discoveries.

In 1735 he was asked to reorganize the work of the Scottish Mining Company in the lead mines at Leadhills, Lenarkshire. Stirling was a successful administrator and spent most of his time after 1735 in that remote village. In 1748, he was elected to the Berlin Academy of Sciences, even though his mathematical activities had ceased. Stirling's own political principles prevented him from succeeding to the Edinburgh chair left vacant at **Maclaurin**'s death.

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operation on an extensive scale in Bristol (1743), the roasted ore ZnO being distilled with carbon at high temperatures in a crucible.

195 The asymptotic expansion

$$\Gamma(z) = e^{-z}z^{z-1/2}\sqrt{2\pi}\left[1 + \frac{1}{12z} + \frac{1}{288z^2} - \frac{139}{51840z^3} - \frac{571}{2488320z^4} + O(z^{-5})\right]$$

is known today as *Stirling's expansion*. This is a misnomer, however, since it was discovered earlier by **de Moivre**.

196 *Stirling numbers* of the first kind $S_k^{(n)}$ are the coefficients of x^{n-k} in the factorial polynomial of degree n, i.e.

$$x(x-1)\cdots(x-n+1) = S_0^{(n)}x^n + S_1^{(n)}x^{n-1} + \cdots + S_{n-1}^{(n)}x.$$

197 The *Stirling series* $\sum_{s=0}^{\infty} a_s\left\{\frac{s!}{x(x+1)\cdots(x+s)}\right\}$ are of importance in the theory of linear difference equations, where they play a part analogous to that of power series in the theory of differential equations.

1730–1735 CE Advances in navigational instruments: **John Hadley** (1682–1744, England). A country gentleman (b. Hertfordshire) of independent means and instrument-maker of East Barnet, London, and independently **Thomas Godfrey** (1704–1749), a poor glazier in Philadelphia, invented in 1730 the *reflecting sextant*[198] to measure the angle between a star and the horizon. The frame of the sextant supports the graduated arc of a sixth part of a circle, a moveable arm which represents the radius of the circle, two mirrors and a small telescope. One of the mirrors is fixed (known as the *horizon glass*). The second mirror is screwed to the moveable arm, and is called the *index glass*, both mirrors being perpendicular to the plane of the sextant.

Light from a star is reflected from the index glass to the silvered half of the horizon glass and thence through the telescope to the observer's eye. If the moveable arm has been moved so as to make the *image* of the star coincide with that of the horizon, it is seen that the altitude of the star is equal twice the angle which the moveable arm reads on the graduated arc.

In 1731 John Hadley invented the *bubble-sextant*, or *artificial horizon* sextant (it made long-range air navigation possible 200 years later).

In 1735 **John Harrison** made the first accurate chronometer in England.

[198] It was introduced by **Tycho Brahe** to measure angular distances between any two points on the celestial sphere. Originally it was equipped with two sights: one on a fixed radius, the other on a moveable radius, which the observer pointed to the two objects of which the angular distance was to be measured.

Geodesy[199] *and the Gyrostatic Equilibrium of Liquid-like Bodies*

Man has been concerned about the earth on which he lives for many centuries. During very early times this concern was naturally limited to his foraging range or to the immediate vicinity of his dwelling place; later it expanded to encompass a village, a region of land or sea, a country; and finally, with the development of advanced means of transportation man became interested in his whole world. Much of this early "world interest" was evidenced by speculation concerning the size, shape, and composition of the earth.

The early Greeks, in their speculation and theorizing, ranged from the flat disc advocated by ***Homer*** *to Pythagoras' spherical figure — an idea supported later by* ***Aristotle*** *(ca 350 BCE).* ***Pythagoras*** *(ca 530 BCE) was a mathematician, and to him the most perfect figure was a sphere. He reasoned that the gods would create a perfect figure and therefore the earth was created to be spherical in shape.* ***Anaximenes*** *(ca 540 BCE) held that the earth was rectangular in shape.*

The early astronomers, however, had no doubts: reasoning from the uniform level appearance of the horizon, the variations in latitude of the circumpolar stars as one travels toward the north or south, the disappearance of a ship sailing out to sea, and perhaps other phenomena — they came to regard the earth as a sphere.

Since the spherical shape was the most widely supported during the Greek era, efforts to determine its size followed. ***Plato*** *(ca 380 BCE) determined the circumference of the earth to be* 64, 000 *km, while* ***Archimedes*** *(ca 250 BCE)*

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199 *Geodesy* determines by observation and measurement the exact position of points and the figures and areas of large portions of the earth's surface, the shape and size of the earth, and the variation of terrestrial gravity.

Geophysics deals with the physical phenomena and properties of the whole earth (or of its more extensive regions). One of the branches of geophysics is *gravimetry*, which is the science of the earth's gravity field [from the Latin *gravis* (= heavy) and the Greek *meterin* (= to measure)]. The gravimetrician measures gravity, studies the figure and dimensions of the earth's body and relates these to the internal structure and composition of the earth's interior.

The determination of the *figure of the earth* is a problem of the highest importance in astronomy, inasmuch as the diameter of the earth is the unit to which all celestial distances must be referred.

estimated it to be $50,000$ *km. Plato's figure was a guess and Archimedes' a more conservative approximation. Meanwhile, in Egypt,* ***Eratosthenes*** *(ca 250 BCE) set out to make more explicit measurements.*

He had observed that on the day of the summer solstice, the midday sun shone to the bottom of a well in the town of Syene (Aswan). At the same time, he observed that the sun was not directly overhead at Alexandria; instead, a vertical pole cast a shadow with subtended angle equal to $(1/50)^{th}$ *of a circle* $(7^\circ 12')$*. To these observations, Eratosthenes added certain "known" facts: (1) that on the day of the summer solstice, the midday sun was directly over the line of the summer Tropic Zone (Tropic of Cancer) — Syene was therefore concluded to be on this line; (2) the linear distance between Alexandria and Syene was (in today's units)* 804.5 *km; (3) Alexandria and Syene lay on a direct north-south line.*

From these observations and "known" facts, Eratosthenes concluded that, since the angular deviation of the sun from the vertical at Alexandria was also the angle of the subtended arc, the linear distance between Alexandria and Syene on the earth's surface was $\frac{1}{50}$ *of the circumference of the earth; the latter thus came out* $50 \times 804.5 = 40,225$ *km. [A currently accepted value for the earth's circumference at the Equator is* $40,065$ *km, based upon the equatorial radius of the World Geodetic System.] The actual unit of measure used by Eratosthenes was called the "stadia". No one knows for certain what the stadium that he used is in today's units. The measurements given above in km were derived assuming one stadia to be* 160 *meters.*

It is remarkable that such accuracy was achieved in view of the fact that most of the "known" facts, and his observations too, were incorrect: (1) although it is true that the sun at noon is directly overhead at the Tropic of Cancer on the day of the summer solstice, it was erroneously concluded that Syene lay on that line. Actually, Syene is 60 *km to the north; (2) the true distance between Alexandria and Syene is* 729 *km and not* 804.5 *km; (3) Syene lies* $3^\circ 30'$ *east of the meridian of Alexandria; (4) the difference of latitude between Alexandria and Syene is* $7^\circ 5'$ *rather than* $7^\circ 12'$ *as Eratosthenes had concluded.*

Nevertheless, Eratosthenes appears to have seen the first who entertained an accurate idea of the principles on which determination of the figure of the earth really depends, and attempted to reduce them to practice. His method, the comparison of a line measured on earth with the corresponding arc of the heavens, is still valid.

Another ancient measurement of the size of the earth was made by the Greek ***Poseidonios*** *(ca 100 BCE). He noted that a certain star was hidden from view in most parts of Greece but that it just grazed the horizon at Rhodes. Poseidonios measured the elevation of the same star at Alexandria* *and determined that the angle was* $\frac{1}{48}^{th}$ *of circle. Assuming the distance from Alexandria to Rhodes to be* 800 *km, he computed the circumference of the earth as* 38,600 *km. While both his measurements were approximations, when combined, one error compensated for another and he achieved a fairly accurate result.*

Revising the figures of Poseidonios, another Greek philosopher determined 29,000 *km as the earth's circumference. This last figure was promulgated by* ***Ptolemy*** *(ca 150* CE*) through his world maps. The maps of Ptolemy strongly influenced the cartographers of the Middle Ages. It is probable that Columbus, using such maps, was led to believe that Asia was* 5000–6500 *km west of Europe. It was not until the* 16^{th} *century that this concept of the earth's size was revised.*

No improvement on the Greek methods was forthcoming until 1528*, when* ***Jean Francois Fernel*** *repeated the Eratosthenes procedure with greater accuracy.*

G. Mercator *(1568* CE*) made successive reductions in the size of the Mediterranean Sea and all of Europe which had the effect of modifying the size of the earth. The telescope, logarithmic tables, and the method of triangulation were contributed to the science of geodesy during the* 17^{th} *century. Indeed, during* 1617–1669*, measurements of this type (employing a spherical earth model) were redone by* ***Snell*** *(1617),* ***Richard Norwood*** *(1637), and* ***Jean Picard*** *(1669) who was the first to apply the telescope to angular measurements. He performed an arc measurement that is modern in some respects; he measured a base line with the aid of wooden rods, used a telescope in his angle measurements, and computed with logarithms. Earth models departing from spherical symmetry date from* 1672*, when* ***Jean Richer*** *discovered that the magnitude of the force of gravity depends on latitude.*

The first, rather inaccurate measurements of the acceleration of gravity were made by ***Galilei*** *(1564–1642). In ca* 1590 *he discovered that the distance traversed by a falling body in the first second is equal to half the value of the acceleration of gravity at the point of observation. The possibility of determining the shape of the earth from measurements of gravity on its surface, occurred to both* ***Newton*** *(1642–1727) and* ***Huygens*** *(1629–1695) who became interested in the observation of* ***Richer*** *(1630–1696) that the period of a pendulum depends on the latitude of its location via the dependence of its period on* g*. Newton, assuming the earth to be a gravitating rotating ellipsoid of revolution of uniform fluid in hydrostatic equilibrium, demonstrated that a slowly rotating liquid body must necessarily be flattened at the poles. He found its ellipticity to be* $\epsilon = \frac{a-c}{a} = \frac{g_p - g_e}{g_e} \approx \frac{1}{231}$*, where* $\{g_p, g_e\}$ *are the respective values of gravity at the pole and the equator.*

Huygens, on the other hand, assumed a uniform earth with its total mass concentrated at its center. Using the condition of the Geoid[200] *as an equipotential surface, he obtained* $\epsilon = \frac{1}{578}$.

The disagreements between the theoretical considerations of Newton and Huygens were explained by ***Clairaut*** *(1713–1765), who also showed how the flattening of the earth could be computed from gravimetric observations.*

The prediction of Newton (1687) that the earth was oblate *at the poles was contrary to the best astronomical evidence available at the time, and for years after Newton's death, the Parisian school of* ***Cassini*** *(1625–1712) vigorously supported the view that the earth was actually* prolate[201]. *To settle the controversy once and for all, the French Academy of Sciences sent a geodetic expedition to Peru in* 1735 *to measure the length of a meridian degree close to the equator, and another to Lapland to make a similar measurement near the Arctic Circle.*

When the leader of the Arctic party, ***Maupertuis****, returned to Paris, after suffering hunger and shipwreck, with proof that the earth is* oblate *(as Newton had forecast), Voltaire congratulated him on having "flattened the poles and Cassini".*

We now know that the actual ellipticity of the earth is $\sim \frac{1}{294}$, *substantially smaller than Newton's predicted value of* $\sim \frac{1}{230}$. *This discrepancy is interpreted in terms of the inhomogeneity of the earth.*

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200 The *Geoid* is a theoretical smooth surface whose normal at each point is in the direction of gravity at that point, i.e., a surface of constant gravity potential. The shape of the Geoid is that which the surface of water would take, were it to cover the whole surface of the earth. [Sea-level, undisturbed by winds or tides, is an equipotential surface of the earth's gravitation.] The *niveau spheroid* is a mathematical approximation of the Geoid, where all local irregularities caused by lateral density variations were removed. This spheroid is very close to an *ellipsoid of revolution*.

201 **G.D. Cassini** continued Picard's arc northward to Dunkirk and southward to the Spanish boundary, dividing the measured arc into two parts, one northward from Paris, another southward. When he computed the length of a degree from both chains, he found that the length of one degree in the northern part of the chain was shorter than that in the southern part. This unexpected result could have been caused only by an egg-shaped earth or by observational errors.
The results started an intense controversy between French and English scientists. The English claimed that the earth must be flattened, as Newton and Huygens had shown theoretically, while the Frenchmen defended their own measurement and were inclined to keep the earth egg-shaped.

Newton's model of the earth as a rotating homogeneous incompressible fluid body was valid only for small rotation speeds. However, in 1742 ***Maclaurin*** *(1698–1746) generalized Newton's result to the case where the ellipticity caused by the rotation cannot be considered small. He found a class of exact solutions for the equilibrium of a rotating body. In these solutions, known as* Maclaurin spheroids *(the fluid surface is an oblate ellipsoid of revolution) the eccentricity is a function of the angular velocity. Moreover,* ***Simpson*** *and* ***d'Alembert*** *(1717–1783) have shown (1743) that Maclaurin's solution implies: (1) for slow rotation there are two possibilities, one nearly spherical and the other very much flattened; (2) above a critical rotation rate* no *spheroid is a figure of equilibrium.*

In 1834 ***Jacobi*** *proved that when the rate of rotation is not too great there is an ellipsoid of* 3 unequal *axes which is a figure of equilibrium. For a certain rate of rotation it coincides with the more nearly spherical shape of the Maclaurin spheroids. These figures are known as* Jacobi ellipsoids*. In* 1860, ***Riemann*** *went one step further than Jacobi by showing that even the Jacobi ellipsoids are only special members of a much larger family of ellipsoidal equilibrium configurations, the* Riemann ellipsoids.

In 1885, ***Poincaré*** *showed that the Jacobi ellipsoids are actually the preferred configurations of rapidly rotating fluid bodies because they have lower energy for fixed angular momentum and mass.*

A more detailed account of the development of these ideas is as follows: ***Maclaurin*** *showed that for every volume V and for each angular velocity $\omega \leq \omega_L = 1.188\sqrt{\rho G}$ (G = Newton's gravitational constant, ρ = density of liquid), there exist two different rotating oblate spheroids that are in gyrostatic equilibrium. As ω approaches ω_L, the shapes of both spheroids approach that of the same rotating spheroid, rotating with the angular velocity ω_L. As ω approaches a value of zero (that is, as the rotation slows down to zero), one branch of the Maclaurin spheroids will increasingly resemble a ball of volume V, the well-known equilibrium configuration at absolute rest ($\omega = 0$), whereas the other branch will grow into a disc of "infinite diameter".*

For nearly a century it was believed that Maclaurin's spheroids were the only shapes possible for uniformly rotating bodies of homogeneous fluids in gyrostatic equilibrium. ***Lagrange*** *claimed that there could not be any other equilibrium configurations; yet this was not true. In 1834* ***Jacobi*** *discovered that, for every volume V and every value ω of the angular velocity which is neither zero nor too large, there exists an equilibrium configuration in the shape of an asymmetric ellipsoid ($a > b > c$) that rotates about the axis of the smallest principal radius c. Jacobi showed that ω should stay below $\omega_J = 1.084\sqrt{\rho G} < \omega_L$.*

If ω approaches the value of ω_J, the Jacobi ellipsoid will eventually resemble one of the Maclaurin spheroids (which rotate with the angular velocity ω_J) and, if ω approaches a value of zero, the Jacobi ellipsoid will come to resemble a needle of infinite length. [What would life be like on a planet that was very thin and very long, and that rotated very, very slowly?].

***Poincaré** found that a new branch of pear-shaped equilibrium configurations bifurcates from the family of Jacobi ellipsoids, much as the Jacobi ellipsoids branch off one class of the Maclaurin spheroids. Poincaré conjectures "that the bifurcation of the pear-shaped body leads onward stably and continuously to a planet attended by a satellite". He furthermore proclaimed that along the Jacobi sequence there must be other points of bifurcation that give rise to other stable branches that would eventually develop into planets with two, three, or more satellites. In this way Poincaré envisioned a grand scheme that could explain the birth of our solar system by an evolutionary process rather than by sudden catastrophes.*

If one follows the cosmogonic hypotheses of Kant and Laplace, our solar system was at first a huge and slowly rotating gas ball of very low density. Self-gravitation would then lead to a contraction of the gas, thereby increasing density and angular velocity, with the matter then changing from a gaseous into a liquid state. As density and speed increased, the originally sphere-shaped matter would become a more and more oblate Maclaurin spheroid, until the bifurcation point at which the Jacobi ellipsoids became stable configurations. The liquid body would change into a Jacobi ellipsoid and then, with even stronger contraction, into a pear-shaped body, which eventually would fission into a main body and a satellite.

*Poincaré never made the detailed calculations necessary to substantiate such a scenario. Such calculations were instead carried out by **George Darwin** (1898) who claimed that he had proved the stability of the pear forms. Unfortunately, **Lyapunov** (1903) was able to refute Darwin's calculations, and other scientists reached the same conclusion. Thus Poincaré's wonderful model collapsed. Nevertheless, the theory of equilibrium configurations developed by Poincaré and Lyapunov was the beginning of bifurcation theory in nonlinear dynamics. This important theory is a principal tool in such diverse areas as fluid mechanics, mathematical biology, and elasticity theory.*

What happens for large values of ω? it was known that a figure cannot be in gyrostatic equilibrium if its angular velocity ω is too large. There is, in fact, no possible gyrostatic equilibrium if ω^2 is greater than $2\pi G\rho$. The only way out of this dilemma is to consider rotating liquid bodies in which the liquid is in internal motion, but whose shape does not alter. This is a much weaker type of equilibrium, but very likely a more realistic one. It was studied by

Dirichlet and ***Riemann*** *(1858–1860) and completed by* ***Chandrasekhar*** *in the* 1960*s.*

The importance of the Jacobi ellipsoids for galactic dynamics is that their very existence suggests that a rapidly rotating galaxy may not remain axisymmetric.

The equilibrium shape of rotating liquid masses is a special case of a more general problem: the rotation of a homogeneous fluid [either simply connected (like a ball) or multiply connected (like a handle, ring etc.)] under forces generated in the fluid itself such as self-gravitation, surface tension, electrostatic Coulomb attraction and centrifugal forces. In gyrostatic equilibrium, the above four forces balance each other: the contractive forces of surface tension and self gravitation counterbalance the dispersive electrostatic (for charged fluid) and centrifugal forces. The problem is then to find the possible shapes of liquid bodies in stable gyrostatic equilibrium.

According to ***Johann Bernoulli****'s principle of virtual work (1717), the equilibria are stationary states of the potential energy, and the stable equilibrium correspond to the minima of potential energy. The total potential energy of a liquid body is the sum of four terms: total energy = surface energy (proportional to the surface area) + electrostatic energy + gravitational energy + rotational energy (potential energy of the centrifugal forces).*

We have seen that the earliest example was that of rotating bodies of liquids, which served as models of the planets and, later on, of the stars and galaxies. Here the forces of self-attraction caused by gravitation are so large that the influence of surface tension can be neglected. If charged celestial bodies are excluded, the potential energy reduces to the sum of gravitational and rotational energies.

Another special case of interest in physics is where surface tension is the dominant force, whereas self-attraction is virtually nill. This situation was realized by ***Plateau*** *(1873) who derived an apparatus for rotating small uncharged drops of oil, immersed in another liquid of the same density; with increasing angular velocity, the drop decomposes first into a drop + ring, then into a drop with droplet satellites of different sizes. This circumstance brings to mind a solar system with a large central body circled by small satellites. The ring resembles the rings of Saturn or Jupiter*[202].

[202] Recently, Plateau's experiments were repeated and improved on by scientists at the Jet Propulsion Laboratory in Pasadena, CA. Besides the axisymmetric and ring-shaped figures of Plateau, they discovered two-, three- and four-lobed equilibrium shapes. With increasing angular speed, all figures were seen to decay into a one-lobed shape. No satisfactory method of explaining all this is currently available, because, in fact, the friction between the host liquid and the

The more complicated case of a charged drop found application in the nuclear drop model of ***G. Gamow*** *(1929). Here, theory predicts various hourglass figures corresponding to different values of the physical parameters. If energy is introduced into the nucleus — say, by bombarding it with a neutron — the nucleus can be excited into a state of free oscillations; if these vibrations take the nucleus above a certain energy barrier, it will then be split into two parts (fission).*

Can this physical model be applied to unicellular organisms, which are drops of protoplasm, a very viscous fluid, suspended in water? It is believed that tension forces at the surface of a cell can only partly explain its shape, and that internal structures are to a large extent responsible for cell shape.

1731–1743 CE **Alexis Claude Clairaut** (1713–1765, France). Mathematician. Born in Paris and spent most of his life in his native city.

Under his father's tutelage he made such rapid progress that at age 13 he read before the French Academy an account of four curves which he had then discovered. He was first to give analytic expressions to non-planar space curves and study their differential geometry (1729). This procured him admission into the Paris Academy of Sciences in 1731 although he was then below the legal age. He became the youngest person ever elected to the Academy.

During 1736–1738 he participated in the Lapland expedition of Maupertuis. Their geodetic measurements of length of meridian arcs at different latitudes, afforded data which showed conclusively the flattening of the earth at the poles.

Later in 1743, he deduced a theoretical relation between the variation of gravity from equator to poles and the ellipticity of a spheroidal earth model[203]

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oil drop cannot be neglected. This friction causes internal flows, which become rather significant as soon as lobes form.

[203] $g(\varphi) = g(0)[1+\beta\sin^2\varphi]$, where $g(\varphi)$, $g(0)$ are the respective values of gravity at latitude φ and the equator. The ellipticity is then given by $\epsilon = \frac{5}{2}\frac{\omega^2 a}{g(0)} - \beta$, where β is known from observations. For further reading, see:

- Webster, A.G., *Dynamics – Lectures on Mathematical Physics*, Hafner Publishing Co., 1949, 588 pp.
- Jeffreys, H., *The Earth*, Cambridge University Press, 1976, 574 pp.

of mean radius a [his development is valid only to terms of the first order in the flattening]. His results showed for the first time how the oblateness of the earth could be computed from gravimetric observations.

In 1750 Clairaut gained the prize of the St. Petersburg Academy for his essay *Théorie de la lune*, and in 1759 he calculated the perihelion of Halley's comet. The first-order ODE, $y = xy' + f(y')$, bears his name. He studied the 3-body problem and wrote several important memoirs on the calculus.

1733 CE The *Industrial Revolution* is launched in England with the invention of the *Flying Shuttle* by **John Kay**.

1733–1740 CE **Charles Francois de Cisternai Du Fay** (1698–1739, France). Physicist. Demonstrated that there are two different kinds of electric charges, one of which was to be found in rubbed amber and one in rubbed glass. In 1750 they were named '*negative*' and '*positive*' electrical fluids, respectively, by **Benjamin Franklin** (1706–1790, U.S.A.). He noticed that if an object carrying one kind of electricity touched an object carrying an equal quantity of the other kind, the two kinds neutralized each other, leaving both objects electrically 'uncharged'. He then coined the names, 'positive charge' and 'negative charge'.

1733 CE **Girolimo Saccheri** (1667–1733, Italy). Mathematician. Composed '*Euclides ab omni naevo vindicatus*' (Euclid vindicated from all fault), where he inadvertently laid the foundation of non-Euclidean geometry.

1733–1773 CE **(Francois Marie Arouet de) Voltaire** (1694–1778, France). Writer. A universal man of the 18^{th} century. *In his writings* he was at once dramatist, poet, philosopher, scientist, novelist, moralist, satirist, polemicist, letter-writer, and historian. *In his life* he was imprisoned in the Bastille, spent years abroad in England and Prussia, was courtier at Versailles and a wealthy landowner at Ferney.

His long life and his voluminous writings, which show a strong sense of engagement in the world around him, made him a man who bestrided the Age of Enlightenment, and in many ways is the epitome of it. Helped spread Newtonian science through the European intellectual community (*Lettres Philosophiques sur les Anglais*, 1773; *Elements de la Philosophie de Newton*, 1738), and presented new ideas in the field of optics, most particularly regarding the psychology of perception. During 1751–1772 Voltaire participated in the composition of the *French Encyclopedia of the Sciences, Arts and Trades*. In 1772, he encouraged the serious study of *probability theory* in his *Essay on Probabilities Applied to the Law*.

Voltaire (pseud, 1718) was born at Paris, son of a notary who belonged to a class of yeoman-tradesman. He was educated by the Jesuits, and soon began to appear in Paris society, particularly in free-thinking and neo-Epicurean circles. His satiric writings incurred him nearly a year in the Bastille (1717–1718). A quarrel with an influential aristocrat led to exile in England (1726), where he encountered a society living in a state of relative justice and freedom.

The publication of his *Lettres Philosophiques* (1773) occasioned another scandal and forced him to retreat at Cirey; he remained there for a decade, apart from brief visits to Paris, Prussia and the Low Countries [an inheritance from his father in 1721 made him economically independent]. There he continued writing, conducting experiments in physics in his own well-equipped laboratory.

In 1744 he was recalled to Versailles and given official positions at Court. During 1750–1753 he visited the court of Frederick II in Berlin. Eventually he settled (1755) at Les Delices on the outskirts of Geneva, and later moved to Ferney (purchased in 1758), a few miles away inside France. He remained there until 1778, when he went back to Paris and died there.

Voltaire was, in general, an ardent defender of victims of religious persecution with, however, one exception:

In his entry '*Juifs*' [*Dictionnaire Philosophique*, 1764], Voltaire writes:

"We find in the *Jews* only an ignorant and barbarous people, who have long united the most sordid avarice with the most detestable superstition and the most invincible hatred for every people by whom they are tolerated and enriched".

Thus, Voltaire echoes the familiar litany of insults drawn from classical pagan antisemitism, which he, no doubt, owed to his Jesuit upbringing. Not only did he repeat the pagan canard that Jews were the '*enemies of the mankind*', but he even justified the long history of persecution and massacres to which they had been subjected. These diatribes, shared by other prominent thinkers of the French Enlightenment like **Diderot**, **Baron d'Holbach**, and **Rousseau**, should be seen as philosophical expression of a crisis of a religious belief, in which war was conducted against the very roots of the Christian faith led logically to an assault on its Jewish origins.

The achromatic lens story (1733–1758)

Chester Moor Hall *(1703–1717, England) and* ***John Dollond*** *(1706–1761, England). Opticians. Independently invented the achromatic telescope, using an objective lens, composed of two kinds of glass so that the chromatic aberration in one kind of glass is compensated for by the other kind of glass.*

Hall, an amateur scientist from Essex, made achromatic lenses for his own use (1733) to little notice. Dollond developed the achromatic telescope (1758) with his son Peter (1738–1820). Earlier (1754) Dollond, a London optician of Huguenot descent, invented the heliometer, a telescope that produces two images that can be manipulated to determine angular distances accurately, for finding the diameter of the sun or distances between stars. Hall brought an action against the Dollonds on the ground of the priority of his earlier work, but the action was dismissed by the courts. In 1761 Dollond was appointed optician to King George III, only a few month before his death.

Isaac Newton *(1704) gave up trying to remove chromatic aberration from refracting telescope lenses, erroneously concluding that it could not be done: he falsely maintained that two lenses of different refraction indices require an infinite focal distance. Consequently he turned to the design of reflectors.*

Euler *proposed that the undesirable color effect seen in a lens were absent in the eye (which is an erroneous assumption) because the different media present negated dispersion. He suggested that achromatic lenses might be constructed in a similar way. Enthused by this work,* ***Samuel Klingenstjerna*** *(1698–1765, Sweden), professor at Uppsala, reperformed Newton's experiments on achromatism and determined them to be in error. Klingenstjerna was in communication with John Dollond, who was observing similar results. Dollond finally (1758), combined two elements, one of crown glass and the other of flint glass, to form a single achromatic lens. This was an accomplishment of great importance. The full mathematical theory of combinations of thin lenses and its application for correction of spherical and chromatic aberration was given in 1840 by Hungarian mathematician* ***Joseph Max Petzval*** *(1807–1891).*

1734 CE **George Berkeley** (1685–1753, England). Philosopher, economist, mathematician, physicist and bishop. Argued that 'absolute space' does not exist by itself, since it is not a fundamental thing but an attribute, like color or harmony. Hence motion is relative and must be measured against some fiducial. He claimed that "Esse is percipi" — the existence of a thing is our perceiving it and since absolute space cannot be perceived, it cannot do as a reference frame. This led him to the notion that inertia is not intrinsic to a body but produced by motion relative to the fixed stars, i.e. all motion, including acceleration and rotation, should be regarded relative to the fixed stars, not space itself.

In his book "*The Analyst*: *A Discourse Addressed to an Infidel Mathematician*" (1734), he ridiculed infinitesimals as "the ghosts of departed quantities". His criticism was a much needed breath of fresh air; in the early days of calculus, it was practiced by a handful of fanatics, and so it had to be, for the theories of *fluxions* and *fluents*[204] were virtually devoid of rigor. Berkeley's attack forced mathematicians to re-examine the foundations of analysis. There followed 200 years of intense efforts by the best minds in Europe. The result was the rigorous calculus we know it today.

He was some 150 years ahead of his time and his arguments were lost in Newton's shadow.

[204] Berkeley's criticism was meant indirectly against Newton. Newton, however, knew exactly what he was doing; he just could not find a precise way to express it.

Do the infinitesimals really exist?

The concept of the infinitesimal, a number that is infinitely small yet greater than zero, has roots stretching back into antiquity. In spite of its importance as a tool in mechanics and geometry since the golden age of Greece, a never ending war between the finite and the infinite has been going on for the past 24 centuries.

In the 19^{th} century infinitesimal were driven out of mathematics once and for all, or so it seemed. To meet the demands of logic, the infinitesimal calculus of ***Isaac Newton*** *and* ***Gottfried Whilhelm von Leibniz*** *was reformulated by* ***Karl Weierstrass*** *(1872) without infinitesimals. Let us briefly survey the evolution of this idea:*

In ***Euclid's*** *geometry, both the infinite and the infinitesimal are deliberately excluded; we read in Euclid that a point is that which has a position but no magnitude. Certainly this meaningless definition is just a pledge not to use infinitesimal arguments. This was a rejection of earlier concepts in Greek thought: the atomism of* ***Democritos*** *had been meant to refer not only to matter but also to time and space. But then the arguments of* ***Zeno*** *had made untenable the motion of time as a row of successive instants, or the line as a row of successive "indivisibles".* ***Aristotle****, the father of systematic logic, banished the infinitely large or small from geometry.*

One of the first thinkers who came forth in defense of infinitesimals was ***Nicolas of Cusa*** *(ca. 1450). It behooved him to do just that because, as a cardinal of the Church, he believed that the infinite was the "source and means, and at the same time the unattainable goal, of all knowledge". Nicolas was followed in his mysticism by* ***Johannes Kepler*** *who in 1612 used infinitesimals to find the best proportions for a wine cask! He was not troubled by the self contradictions in his method; he relied on divine inspiration, and he wrote that "nature teaches geometry by instinct alone even without ratiocination". Moreover, his formulas for the volumes of wine casks are correct.*

The most famous mathematical mystic was no doubt ***Blaise Pascal****. In answering those of his contemporaries who objected to his reasoning with infinitely small quantities, Pascal said that "the heart intervenes to make the work clear" (1656). Pascal looked on the infinitely large and infinitely small as mysteries proposed by nature to man for him to admire but not to understand.*

The full flower of infinitesimal reasoning came with ***Newton*** *(1664),* ***Leibniz*** *(1673),* ***Jakob Bernoulli*** *(ca. 1700),* ***Johann Bernoulli*** *(ca. 1700) and* ***Leonhard Euler*** *(ca. 1740). The first textbook on the calculus was written* *in 1696 by* ***G. F. A. de L'Hospital****, a pupil of Leibniz and Johann Bernoulli. In it are found two axioms that Aristotle outlawed 2000 years earlier:*

- *Two quantities differing by an infinitesimal can be considered equal (i.e. equal and unequal at the same time).*
- *A curve is the totality of an infinite number of straight segments.*

Curiously enough Newton and Leibniz did not endorse these views: Leibniz did not claim that infinitesimal really existed[205]*, only that one could reason without error as if they did exist. Newton, on the other hand, tried to avoid the infinitesimal: in his Principia Mathematica (as in Archimedes' 'On the Quadrature of the Parabola'), results originally found by infinitesimal methods are presented in a purely finite Euclidean fashion.*

The first critique of the infinitesimal method appeared by ***George Berkeley*** *in his book The Analyst (1734). It was addressed to "an infidel mathematician" [He meant Edmund Halley who financed the publication of the* Principia *and helped prepare it for the press]. In this book he accused both Newton and Leibniz of false reasoning, calling infinitesimals "the ghosts of departed quantities" and naming Newton's fluxions*[206] *"obscure, repugnant and precarious".*

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[205] This syndrome repeats itself at every revolutionary stage in the history of science: e.g. **Einstein** did not believe in the inherent probabilistic interpretation of quantum mechanics and **Schrödinger** did not believe in the reality of his own wave mechanics!

[206] Newton called "*fluent*" what we call today the instantaneous position function (of time) of a moving body. By "*fluxion*" he meant the instantaneous velocity of the same body. In the case of a falling stone, the fluent is given by the formula $s(t) = 16t^2$ with distances measured in feet and time in seconds. To evaluate the velocity of the falling stone at $t = 1$, we let dt stand for the infinitesimal increment of time. The corresponding increment in position is $ds = 16(1 + dt)^2 - 16 = 32dt + 16dt^2$. The ratio $\frac{ds}{dt}$ which yields the average velocity over ds at time dt is equal to $32 + 16dt$. To compute instantaneous velocity one must drop the infinitesimal $16dt$, i.e. assume that $32 + 16dt$ is the same as 32. That is precisely what Bishop Berkeley would not let them do. He said: "dt is either equal to zero or not equal to zero. If dt is *not* zero then $32 + 16dt$ is not the same is 32. If dt *is* zero, then the increment in distance ds is also zero, and the fraction $\frac{ds}{dt}$ is not $32 + 16dt$ but a meaningless expression $\frac{0}{0}$".

Weierstrass, 138 years later, gave the following answer (1872): to find an instantaneous velocity we abandon any attempt to compute the speed as a *ratio*. Instead we define the speed as a *limit*, which is approximated by ratios of *finite*

Although Berkeley's logic could not be refuted at the time, mathematicians went on using infinitesimals for another century, and with great success. Indeed, physicists and engineers have never stopped using them. The 18th century, the great age of the infinitesimals, was the time when no barrier between mathematics and physics was recognized. The leading physicists and the leading mathematicians were the same people. When pure mathematics reappeared as a separate discipline, mathematicians again made sure that the foundations of their work contained no obvious contradictions. "Just go on, and faith will soon return", said ***Jean Le Rond d'Alembert*** *to hesitate mathematical friend, who lacked experience and intuition in handling infinitesimals.*

By the beginning of the 19th century a clear distinction had been established between analysis (the study of infinite processes) and algebra (the study of operations of discrete entities such as natural numbers). A major objective of much of 19th-century mathematical effort was to unify (or at any rate to bridge) these two branches of mathematics. This endeavor was termed 'the arithmetization of analysis'. It was realized that the prime task was to construct a sound logical foundation for the real number system. Although the basic concepts of analysis - function, continuity, limit, convergence, infinity, were progressively clarified and refined during the first half of the 19th century, notably by ***Cauchy****, much remained to be done. This task was left to* ***Weierstrass****,* ***Dedekind*** *and* ***Cantor****, who eventually restored Greek standard of rigor: Modern analysis secured its foundation by doing what the Greeks had done: outlawing infinitesimals.*

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increments. Let Δt be a variable finite time-increment and Δs be the corresponding variable space-increment. Then $\frac{\Delta s}{\Delta t}$ is the variable quantity $32+16\Delta t$. By choosing Δt sufficiently small we can make $\frac{\Delta s}{\Delta t}$ take on values as close as we like to the value 32, and so, by definition, the speed at $t=1$ is exactly 32.

This approach succeeds in removing any reference to numbers that are not finite. It also avoids any attempt to set directly $\Delta t = 0$ in the fraction $\frac{\Delta s}{\Delta t}$. There is, however, a price to pay: the intuitively clear and physically measurable quantity, the instantaneous velocity, becomes subject to the surprisingly subtle notion of '*limit*'. In the mathematical terminology it means that:
"The velocity is v if, for any positive number ϵ, $\left|\frac{\Delta s}{\Delta t} - v\right| < \epsilon$ for all values of $|\Delta t| < \delta$ for some $\delta = \delta(\epsilon, t) > 0$".
We have defined v by means of a relation between two new quantities, ϵ and δ, which are irrelevant to v itself. Ignorance of ϵ and δ never prevented Euler or Bernoulli from finding a velocity. The truth is that in a real sense we already knew what instantaneous velocity was before we learned this definition; for the sake of logical consistency we accept a definition that is much harder to understand than the concept being defined.

The reconstruction of the calculus on the basis of the limit concept and it's 'epsilon-delta' definitions amounted to a reduction of the calculus to the arithmetic of real numbers. The momentum gathered by these foundational clarifications led naturally to an assault on the logical foundation of the real-number system. This was a return, after 2500 years, to the problem of irrational numbers, which the Greeks had abandoned as hopeless after Pythagoras (One of the tools of these efforts was the newly developed field of symbolic logic). For rigorous certainty one had to resort to the cumbersome Archimedean method of exhaustion in it's modern version: the Weierstrass epsilon-delta method.

When we say that infinitesimals do exist, we do mean this in the sense it would be understood by Euclid and Berkeley. Until 100 years ago it was tacitly assumed by all philosophers and mathematicians that the subject matter of mathematics was objectively real in the sense close to the sense in which the subject matter of physics is real. Whether infinitesimals did or did not exist was a question of fact not too different from the question of whether material atoms do or do not exist.

Today, most mathematicians have no such conviction of the objective existence of the objects and structures they study. What mathematicians want from infinitesimals is not material existence *but rather the right to use them in proofs. This, of course, is of no concern to applied mathematicians and physicists since it is quite true that whatever can be done with infinitesimals can in principal be done without them.*

1734 CE **Emanuel Swedenborg** (1688–1772, Sweden). Scientist, philosopher and mystic[207]. Believed that the world evolved from a material point-source and the solar system originated from a sudden explosion of material from the sun.

1734–1742 CE **Colin Maclaurin** (1698–1746, Scotland). One of the ablest mathematicians of the 18^{th} century. He is best known today for his

[207] He wrote a treatise *A new system of reckoning which turns* 8 *instead of the usual turn at* 10 (1718) in which he defended the number 8 as a base. In his own words: "*Should the practice of the use and the use of the practice give its approval, I suppose that the learned world will gain incredible benefits from this octonary reckoning*". Modern computers have long been using a base-8 arithmetic. [It has been recently discovered that crows are capable of counting up to 7.]

exact solution for the spheroidal figure of equilibrium of a uniformly rotating homogeneous fluid mass (1742).

He did not discover the 'Maclaurin expansion' $f(x) = \sum_{n=0}^{\infty} f^{(n)}(0)\frac{x^n}{n!}$, since it is a special case of the '*Taylor expansion*' (1715) and was also given by **James Stirling** a quarter of a century before Maclaurin used it (as acknowledged by Maclaurin himself). He did, however, devise in 1729 a means of finding solutions to systems of linear equations, long before Cramer published it in 1750.

Maclaurin did notable work in geometry, particularly in the study of higher plane curves, and he showed great prowess in applying classical geometry to physical problems. Among his many papers in applied mathematics is a prize-winning memoir on the mathematical theory of tides (1740). In his '*Treatise on Fluxions*' (1742) he undertook the defense of the calculus techniques of Newton, which came under attack in 1734 by a nonmathematician, Bishop **George Berkeley**. In his book Maclaurin treated calculus on the basis of Greek geometry and thus answered all objections to its method as being founded on false reasoning and full of mystery. It is in this work that he expounded his discovery of the '*Maclaurin spheroids*'.

Maclaurin was a mathematical prodigy. At age 11 he entered the University of Glasgow. In 1717 he was elected professor of mathematics in Marischal College, Aberdeen. Two years later he was admitted as Fellow of the Royal Society and made acquaintance with Isaac Newton.

In 1722 Maclaurin traveled as tutor and companion to the eldest son of Lord Polward. In 1725 he was elected professor of mathematics in the University of Edinburgh on the recommendation of Newton. There was some difficulty in obtaining a grant to cover his salary, and Newton offered to bear the cost personally.

In 1745, when the rebels of Charles the pretender were marching on Edinburgh, Maclaurin took part in preparing trenches and barricades for its defense. The hardships to which he was thus exposed, caused a disease to which he later succumbed. He died at Edinburgh.

1735 CE **George Hadley** (1685–1768, England). One of the first contributors to the classical model of the general circulation in the atmosphere. In his paper "*Concerning the cause of the general trade winds*"[208], he revised

[208] *Trade winds*: steady winds, with speed between 5–$7\frac{1}{2}$ m/sec, occupy belts between latitudes 25° and 5° on either side of the equator. North of the equator they blow from the northeast; in the Southern Hemisphere their direction is generally from the southeast. Along the equator, the atmospheric pressure tends to be low and the winds weak. This is the region of the *doldrums*, where sailing vessels can make but very little headway.

and improved Edmund Halley's earlier explanation of the trade winds (1686).

Hadley was well aware of the fact that solar energy drives the winds. He proposed that the large temperature contrast between the poles and the equator would create a thermal circulation very similar to that of sea breeze. As long as the earth's surface is heated unequally, air will move in an attempt to balance the inequalities.

Hadley suggested that on nonrotating earth the air movement would take the form of one large *convection cell* in each hemisphere. The more intensely heated equatorial air would rise and move poleward. Eventually, this upper-level flow would reach the poles where it would sink and spread out at the surface and return to the equator. As the cold polar air approached the equator, it would be reheated and rise again. If the earth were not rotating, this would produce winds blowing from the poles to the equator along the earth's surface. Because of the rotation of the earth, the air moving towards the equator is deflected to blow from east to west (easterly wind), while the flow aloft will be deflected from west to east (westerly wind).

Hadley's paper remained unnoticed for many years. His ideas were based on a *single* thermally direct cell and required high pressure over the poles and low pressure over the equator, with uniform pressure gradient between them. In the 19^{th} century, new observations of surface pressures contradicted this, for belts of high pressure were observed in the subtropics as well as at the poles, with low pressure in middle latitudes as well as at the equator; such distribution required the existence of *three cells* (**Ferrel**, 1856), not one[209]. In the 1920's, the three-cell model (in each Hemisphere) was definitely accepted as correct (and sufficient to accomplish the task of maintaining the earth's heat balance).

In the zone between the equator and roughly 30 degrees latitude, the circulation closely resembles the convective model used by Hadley for the whole earth; hence, the name *Hadley cell* is generally applied to it.

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[209] Hadley's model does not take into account the thermal conditions in the upper atmosphere. Clearly, heating and cooling are not restricted to the earth's surface. However, there is yet another reason why the single-cell model is not acceptable: because of gravity, the atmosphere must rotate with the earth. In the Hadley model the surface winds blow toward the west and would, because of *friction*, oppose the earth's rotation, which is toward the east. Since the atmosphere is attached to the earth, however, its average motion relative to the solid earth's surface must vanish. Thus, easterly flow at one latitude must be balanced by westerly flow at another.

It is of historical interest to note that Hadley realized the effect of the earth's rotation on winds 100 years ahead of **Coriolis**(!), basing his theory upon the law of conservation of angular momentum.

1735–1743 CE **Charles Marie de la Condamine** (1701–1774, France). Naturalist and mathematician. Member of an expedition to Peru (with **Bouguer**) to measure the length of a degree of a meridian arc at the equator (1735). Made first scientific exploration and account of the Amazon river in a 4-month raft journey (1743). It was published in 1751.

1736–1740 CE **Claudius Aymand** (1660–1740, France). Surgeon. Performed the first recorded successful appendectomy (1736).

1736–1760 CE **Israel ben Eliezer, Ba'al Shem Tov** (Master of The Name; Acr. BESHT, ca. 1700–1760, Poland). Religious leader and philosopher. Founder (1736) of the Jewish *Hasidic* ("The Pious") movement in Eastern Europe that influenced the course of Jewish life for over two centuries. The new cult was directly in the line of the traditional Jewish mystics of the *Kabbalah*, but the BESHT endeared it to the masses with a poetic earthiness and love of life and people which the old ascetic Kabbalah lacked.

Upon the rapid decline of Jewish life in Slavic countries following the great devastation and massacres by the Cossack hordes (1648–9), religious worship had become even more formalistic and the great majority of the Jews sank into the most abject poverty and ignorance. In his teachings, the BESHT revived the cult of the *Tzadik*[210], the righteous saintly person who mediates between the Upper and the Lower worlds. He invented a revolutionary form of popular prayer, through which man breaks down the barriers of his natural existence and reaches into the divine world. Thus Hasidism emphasized *joyful* worship of God in prayer and in *all* of one's actions.

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210 The Talmudists placed Ba'al Shem and his followers under the ban, but to no avail. Since eventually *Hasidism* became orthodoxy it could not be excommunicated. One of the dedicated enemy of Hasidism was the great Talmudic scholar **Eliahu ben Shlomo Zalman** (Known as HAGA'ON MI-VILNA; 1720–1797, Lithuania). He was a man of awesome secular and religious knowledge, probably one of the greatest Hebrew scholars ever. He purchased a small house outside Vilna and concentrated entirely on study. He never slept more than two hours a day, and not more than a half an hour at a time. To eliminate distractions he kept his shutters closed even in daytime and studied by candlelight. To stop himself from falling asleep, he cut off the heating and put his feet in a bowl of cold water. He expressed interest in *secular science* as an aid to understanding the Torah. "all knowledge is necessary for understanding the Holy Torah and is included in it".

Nothing of what the BESHT taught was new to Judaism; he merely gave added emphasis to ideas which had been current for millennia [e.g. *Psalms* **47**, 2; **100**, 2]. His techniques, however, offered the Jewish masses an escape from their troubled life and the oppressive authority of the rabbis. Consequently his movement spread with remarkable speed trough Southeastern Europe.

The Ba'al Shem Tov was born in Okop in backward Podolia to very old parents. Orphaned at an early age, his early manhood was spent in the wilderness, in utter poverty, performing miracles, faith-healing, and exorcizing evil spirits (unlike Jesus, he was twice married).

In his fortieth year, (1740), Ba'al Shem threw off his cloak of boorishness and revealed himself in the splendor of a messenger of God. He established himself in Medzibezh, Podolia, where he remained until his death. Henceforth, he led a life of saintliness and piety. The BESHT left no writings. His homilies were put down by his disciples.

1736–1761 CE **Thomas Bayes** (1702–1761, England). Mathematician. Introduced the '*Principle of inverse probability*'[211]: "*The posterior probabilities of the hypotheses are proportional to the products of the prior probabilities and the likelihoods*". In other words: if there is no ground to believe one of a set of alternative hypotheses rather than another, their prior probabilities are equal. When, in addition, posterior evidence is available, then in retrospect the most probable hypothesis is the one that would have been most likely to lead to that evidence. Thus, if the data were equally likely to occur on any of the hypotheses, the former tell us nothing new with respect to the credibility of the latter, and we shall retain our previous opinion, whatever it was. This principle provides a formal rule, in general accordance with common sense, that enables a decision between hypotheses on the basis of available evidence.

Bayes initiated the "Bayesian" school of 'inductive probability' (probability of causes) with his extension of the definition of conditional probability. It has been promoted by **Harold Jeffreys** (1891–1989, England) and applied with considerable success to diagnosis of medical conditions and many other applications of statistical inference and fuzzy logic.

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211 The essence of Bayes' theorem is found already in the Talmud (*Yevamoth*, **4**). The Jewish rabbis solved the problem very much earlier (ca. 100 CE), but expressed the argument in words, not numbers. They also thought of the analysis as a way to solve moral and legal problems, not as an end in itself. The Talmud, though, lacks the clarity of the Bayes' analysis and is content with *relative*, not absolute probabilities. Yet, the rabbis understood the logic underlying this analysis, and their relative probabilities are the same as obtained by the Bayes' formula.

Bayes was born in London. He was privately educated and his mature life were spent as an Anglican minister at the chapel of Turnbridge Wells.

In 1761 he attempted to use the theory of probability to prove the existence of God. To this end he started from a statement expressing the relationship between the conditional probability and its inverse ("The other way around"). The conditional probability of event B, given A is

$$prob(B|A) = prob(A|B)\frac{prob(B)}{prob(A)}$$

Now, let a set of mutually exclusive events $B_1, B_2, \dots, B_n$, be given, and let us assume that the occurrence of one or another of them is a necessary condition for the occurrence of an event A. Since

$$prob(A) \equiv P(A) = P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + \cdots P(B_n)P(A|B_n),$$

there follows *Bayes' theorem*

$$P(B_i|A) = \frac{P(B_i)P(A|B_i)}{P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + \cdots + P(B_n)P(A|B_n)}$$

Two years later (1763) Bayes died and his "*Essay towards solving a problem in the doctrine of chance*" was discovered and published. It caused a great stir in the mathematical community, establishing an entire new field of science, now called *Bayesian Statistics*, and having far reaching implications about scientific inference.[212]

1737, Oct 11 CE A cyclonic storm in the Bay of Bengal killed about 300,000 people in Calcutta, India.

1737 CE **Georg Brandt** (1694–1768, Sweden). Chemist. Discovered and isolated the element *cobalt*. The coper-miners of the Harz Mountains frequently obtained ores looking like copper-ore; these gave an unpleasant smell of garlic or roasting and furnished no copper. The miners attributed their occurrence to the pranks of a mischievous spirit, *Kobold* (from the Greek *kobalos*), and the material was called "false-ore", or *cobalt*. The use of cobalt as a constituent of some blue glazes and blue glass, made in imitation of *lapis*

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212 Using Bayes' theorem in cases where the prior probabilities of B_i cannot be measured directly, may lead to controversial results: misrepresentation of data. if one is not careful enough.

In the 20^{th} century, a leading proponent of Bayesian probability theory was Bruno de Finetti (1906–1985, Italy), who expounded the mathematical relationships between independence and *exchangeability*.

lazuli, has been established for ancient Egyptian (1735 BCE) and Babylonian (1450 BCE) specimens by analysis.

Cobalt chloride solutions were introduced as *invisible ink* in 1705. The pale pink, almost colorless complex $2[Co(H_2O)_6]Cl_2$ in dilute aqueous solution is all but invisible when used to write on white paper. Gentle warming of the paper shifts the equilibrium to $Co(CoCl_4)+12H_2O$ by driving out the complexed water molecules, forming the easily identified blue cobalt chloride complex. However, if the paper is allowed to sit about at room temperature for a while, all is soon invisible again as the anhydrous complex picks up moisture from the atmosphere.

With the advent of nuclear physics in the 20^{th} century, a great variety of radioactive isotopes for medical and industrial use were produced by neutron irradiation of various elements. The most widely used of these is cobalt-60, which is prepared from the normal metal cobalt-59. A moveable cobalt-60 "gun" produces a beam of gamma rays for the irradiation of a selected spot on the patients body.

Since cobalt-60 has a half life of 5 years, the "gun" does not have to be loaded very often. Cobalt-60 is also an exceptionally effective material for making a *dirty bomb*. It can be wrapped around a large hydrogen bomb in almost unlimited amounts to absorb the superfluous neutrons and produce fall-out enormously more potent than that from an ordinary atomic bomb. Cobalt bombs have been mentioned as possibly the main ingredient of a *doomsday machine*.

1737–1753 CE **Bernard Forest de Belidor** (1698–1761, France). Civil engineer. First to apply the Newtonian calculus to practical architectural problem. This he expounded in his influential 4-volume treatise *Architecture hydraulique* (1737–1753). He wrote numerous books dealing with hydraulic, civil and military engineering.

Bedidor was a professor at the Ecole de Artilleric.

1737–1776 CE **David Hume** (1711–1776, Scotland). Agnostic philosopher, historian and political economist. A major figure of the *enlightenment* (1715–1789). Pioneered in the sciences of political and cultural history, economics, comparative history of religions, sociology and psychology. Scandalized Britain with his anti-religious ideas.

After his graduation from Edinburgh University and after fruitless attempts to make a place for himself in law and in business (1734), David

Hume went to France and there wrote *Treatise on Human Nature*[213] (1735–1740). Settling in his family estate of Ninewells (1740) he wrote an *Enquiry Concerning Human Understanding* (1748). His reputation made by this essay, was solidified in England and on the continent with *An Enquiry Concerning the Principles of Morals* (1752).

Hume's reputation as a skeptic led to his failure to obtain the Chair of Ethics and Philosophy at Edinburgh University (1744). Later on he was a soldier (1745–1746), librarian (1751–1757), diplomat (1763–1765) and Under-Secretary of State (1767–1768). Thereafter he settled down in Edinburgh living among his friends with Epicurean ease, and dedicating his time to the writing of a history of England and his posthumously published *Dialogues Concerning Natural Religion* (1779).

In his *Dialogues*, Hume mounted a skeptical attack on the logical structure of many naive features of the Newtonian clockwork-universe ("God wound up the Universe and set it going") and indeed also upon the rational basis of any form of scientific inquiry. Hume calls the *Design Argument* 'the religious hypothesis', and proceeds to attack its foundation from a variety of directions. Hume's approach was entirely negative; whereas most of his contemporaries accepted the rationality and ordered structure of the world without question, Hume did not. A commonsense view of the world, along with the metaphysical trimmings that had been added to the Newtonian world model, Hume rejected.

His objections are threefold: Firstly, the *Design Argument* is unscientific; there can be no causal explanation for the order of nature because the uniqueness of the world removes all grounds for comparative reference. Secondly, analogical reasoning is so weak and subjective that it could not even provide us with a reasonable conjecture, let along a definite proof. And finally: all negative evidence has been conveniently neglected.

Hume maintains that a dispassionate approach could argue as well for a disorderly cause if it were to concentrate upon the disorderly aspects of the world structure.

Humean tirade against the simple design arguments of the English physicists fell upon deaf ears, and must have seemed rather naive when held up against the staggering quantitative achievements of the Newtonian system. He became an isolated and ignored figure in literary circles even during his lifetime, and appeared a 'crank' to the Newtonians.

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[213] In Germany, the philosopher, statistician and scientist **Johannes Nicolaus Tetens** (1736–1807) expounded a similar theory in his treatise (1777): "*Philosophische Versuche über die Menschliche Natur und ihre Entwicklungen*." He is therefore known as the 'German Hume'.

Hume's theory of knowledge can be summarized as follows:

- *Science* and *religion* are mutually exclusive. Religion is not a form of *knowledge*; it is not even a form of *knowing*; it is rather a complex kind of *feeling*, without recourse to science and reasons. Religion simply postulated unknown causes: reason is limited to the realm of human experience, and therefore it cannot decide ultimate questions such as the origin of the cosmos.
- Except for abstract reasoning concerning quantity and numbers (mathematics) reasoning involves belief rather than knowledge and is referable to human feelings, instincts and emotions (passions).
- The problem of *truth* in questions of fact and existence is referable to psychology rather than to logic. Hume stressed that empirical facts must be given due weight against the testimony of men. He gives a number of examples showing that testimony of otherwise reputable men cannot be trusted. *Mass delusion*[214] can occur: given enough time for people to talk to each other, the delusion can develop consistency. Delusions are especially likely in cases where people are trying to interpret an extraordinary event, e.g. the natural law (confirmed billions of times by all humans everywhere) that the dead do not rise must outweigh testimony to the contrary.
- The idea that bases the existence of God on the majestic and wondrous design of the Universe (i.e., natural theology) is rejected. The world could have existed throughout eternity, requiring no first cause.
- There is no observable 'soul' behind the process of thought; what we call 'mind' is only an abstract name for perceptions, memories and feelings. (By dissipating the concept of soul, Hume destroyed orthodox religion.)
- Man cannot know ultimate reality or achieve any knowledge beyond a mere awareness of phenomenal sensory images. The only knowledge we can possess consists of a mere sequence of ideas (perceptions, or assumptions) none of which can be proved to be true; all knowledge is therefore restricted to mental states or experiences; of those only we can be certain. (He thus challenged all alleged truths except those of mathematics and the immediate intuitive awareness of our sense experience.) This made him reject the idea of *scientific law* as objective reality, i.e., science must

[214] We have now vastly more experience with mass delusions than Hume had, experience primarily obtained in the process of investigating UFOs. The modern world does not believe in messengers from God (angels), but it does believe in extraterrestrial intelligent life: imaginary super-beings from other stars that play the same psychological role in modern society as angels did in the past.

limit itself strictly to mathematics and direct experiment; it cannot trust to unverified deduction from "laws".
There is no such thing as 'natural law' or 'necessity' in the sequence of effect upon cause; we never perceive causes, or laws: we perceive events and sequences, and *infer* causation and necessity. 'Law' is an observed *custom* in the sequence of events; but there is no 'necessity' in custom.

- An event C and a subsequent event E are related as *cause* and *effect*, if the occurrence of C (or a situation *similar* to it[215]) is always followed by E, and if E never occurs unless C has occurred previously. But the fact that we have become aware of a particular cause sequence (like C to E) even a very large number of times, is no proof that C will be followed by E on all future occasions. He concluded that our belief in causality is no more than a *habit* which is not an adequate basis for belief.

Causality according to this definition[216] cannot be gained from material given by the senses. To connect one occurrence with some other by the notion of cause and effect is not the result of rational knowledge but of a habit of expecting the perception of the second after having perceived the first; because that sequence has previously taken place in innumerable cases. This *habit* is founded upon a belief which can be explained psychologically but cannot be derived by abstraction from the ideas of the two events (objects) or the impressions of the senses. Hume did not deny that causality works. He only denied that reason is capable of understanding it.

There can be no causal explanation for the order of nature: perhaps the development of the world is *random* but has had an infinite amount of time available to it so all possible configurations arise until eventually a stable self-perpetuating form is found or — matter may posses some intrinsic *self-ordering property* (1748).

Hume has influenced the development of the best philosophers who came after him (**Kant**, **James**, **Russel**, **Santayana**) and gave speculative philosophy a new direction[217].

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[215] Hume included it in his definition because he wanted to make causality *experimentally verifiable*.

[216] Modern physics, whose causal laws are elaborated inferences from the observed course of nature, have supported Hume's challenge to the traditional causal connection.

[217] **Immanuel Kant** read Hume's Dialogues in 1780 and subsequently acknowledged his debt to him for awakening him 'from his dogmatic slumbers'.
Hume's idea that the world might have been gradually *evolved* from very small beginning, increasing by the activity of its inherent principles rather than by a sudden decree of God was taken up by the zoologist **Erasmus Darwin**

(1731–1802), who was Charles Darwin grandfather. Erasmus was starting to take up early steps (1794) toward an evolutionary theory of animal biology, maintaining that the components of an animal or plant were not designed for the use to which they are currently applied, but rather, have grown to fit that use by a process of gradual improvement.

Hume had also influenced young **Albert Einstein**, who said of him: "*One is amazed that many, sometimes highly esteemed, philosophers after him have been able to write so much obscure stuff and even find grateful readers for it*".

Worldview XII: David Hume

* * *

"Reason is limited to the realm of human experience, and therefore it cannot decide ultimate questions such as the origin of the cosmos".

* * *

"While Newton seemed to draw off the veil from some of the mysteries of nature, he showed at the same time the imperfections of the mechanical philosophy; and thereby restored her ultimate secrets to that obscurity in which they ever did and will remain".

* * *

"If we take in our hand any volume, of divinity or school metaphysics, for instance; let us ask 'Does it contain any abstract reasoning concerning quantity or number?' No. 'Does it contain any experimental reasoning concerning matter of fact and existence?' No. Commit it then to the flames: for it can contain nothing but sophistry and illusion".

* * *

"Look around this world: Contemplate the whole and every part of it. You will find it to be nothing but one great machine, subdivided into an infinite number of lesser machines... All these various machines and even their most minute parts, are adjusted to each other with an accuracy, which ravishes into admiration all men who have ever contemplated them. The curious adapting of means to ends, throughout all nature, resembles exactly, though it much exceeds, the productions of human contrivance; of human design, thought, wisdom and intelligence...".

* * *

"... We are guilty of the grossest, and most narrow partiality, and make ourselves the model of the Universe... What peculiar privilege has this little agitation of brain which we call thought, that we must thus make it the model of the whole Universe".

* * *

"A very small part of this great system, during a very short time is very imperfectly discovered to us: And do we thence pronounce decisively concerning the origin of the whole?... Let us remember the story of the Indian philosopher and his elephant. It was never more applicable than to the present subject. If the material world rests upon a similar ideal world, this ideal world must rest upon some other; and so on, without end. It were better, therefore never to look beyond the present material world".

* * *

"But were this world ever so perfect a production, it must still remain uncertain whether all the excellences of the work can justly be ascribed to the workman... Many worlds might have been botched and bungled, throughout an eternity, ere this system was struck out; much labour lost, many fruitless trials made; and a slow but continued improvement carried on during infinite ages of world-making".

* * *

1737–1783 CE **Leonhard Euler**[218] (1707–1783, Switzerland). The greatest mathematician of the 18^{th} century. Freed the analytical calculus from all geometric bounds, and thus established analysis as an independent science, which from his time on has maintained an unchallenged leadership in the field of mathematics.

Euler was born in Basel, the son of a preacher. The father's wish was that his son should follow in his footsteps, but on entering the University of Basel in 1720, Euler met **Johann Bernoulli** and learned much mathematics from him. Nevertheless he emerged with a master of philosophy degree and joined the department of theology, with ample knowledge in Greek and Hebrew and a strong religious conviction that stayed with him all his life.

At 19 he won a prize from the French Academy of Sciences for a paper on the masting of ships, and was consequently invited to join the Academy of Sciences in St. Petersburg. In 1740 he lost sight in one eye, and at the request of Frederick the Great joined the scientific community in Berlin, where he stayed for the next 25 years. In 1766 he returned to St. Petersburg, and by 1771 he had become totally blind.

Euler's output, range and energy were phenomenal: he published hundreds of papers in almost every field in the pure and applied mathematics of his day, plus several books on a wide range of topics. While in Berlin he supervised the observatory, the botanic gardens and the publication of maps and calendars. He also advised on financial matters, including lotteries and pensions. In addition he was required to work on canal improvements and to translate a military book into German. He continued to work in his blindness for twelve years, producing an 800-page book on lunar motion and 50 research papers totaling 1000 pages. He fathered 13 children, and enjoyed playing with them whilst simultaneously contemplating mathematics.

Euler contributed new essential ideas in number theory, algebra, calculus, calculus of variations, functions of complex variables, differential geometry, difference and differential equations, special functions, acoustics, optics[219], mechanics, fluid dynamics, astronomy, artillery, navigation, statistics, finance and philosophy of science. In addition, he was also a prodigious calculator.

In his memoir '*De Fractionibus Continius*' he laid the foundations for the modern theory of continued fractions which play an important role in present day mathematics. They constitute a most important tool for new

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[218] For further reading, see:

- Wittle, T., *Leonhard Euler 1707–1783*: *Beiträge zu Leben und Werke*, Gedenkband des Kantons Basel-Stadt, Birkhäuser: Basel, 1983.

[219] E.g., suggested (1766) a design for *achromatic lenses*.

discoveries in number theory and in the field of Diophantine approximations. In numerical analysis, continued fractions are used to give rapid numerical approximation.

Euler (1728) introduced the notation e for the base of the natural logarithms[220]. In 1739 he adopted the symbol π (**Jones**, 1706). Later (1750) he introduced the functional notation $f(x)$, the summation symbol $\sum$; and in 1777, the symbol $i = \sqrt{-1}$. In 1755 he discovered the differential equations of motion of non-viscous fluids. During 1758–1765, he proved that the instantaneous displacement of a rigid body can be expressed as a sum of an axial rotation and a translation. To describe the rotation he introduced the '*Euler angles*', thus establishing the basis for the algebra of finite rotations. He then derived the equations of motion of a rigid body about a point, thus laying the mathematical foundation for the analysis of gyroscopic behavior. His formulation emphasizes the crucial role of the components of the *inertia tensor*, the first tensor entity to enter physics.

In 1765 he suggested that the earth might undergo a free *nutation* with a period of $A/(C-A) = 305$ sidereal days. Euler started the systematic investigation of variational problems, a subset of which is known as *isoperimetric problems*. These maximum-minimum problems attracted the interest of the best minds — such as **Newton**, **Leibniz**, **Jakob** and **Johann Bernoulli** — from the very start of the infinitesimal calculus. Euler found a differential equation which gave the implicit solution of an extended class of such problems.

In 1768, Euler published his three-volume treatise '*Institutiones calculi integralis*' in which he presented exhaustive methods for evaluating definite and indefinite integrals in terms of elementary functions. He also developed the theory of ordinary and partial differential equations[221].

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220 Seeking a function $f(x)$ whose derivative is equal to itself, **Newton** (1665) had shown that $f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$. From this he deduced that $f(1) = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!} + \cdots$.
Euler (1728) proved that $f(1) = \lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n$ and gave this number the symbol e.
Lambert (1776) proved that e is irrational and **Hermite** (1873) showed that e is also transcendental.
Euler discovered the beautiful relationship $e^{\pi i} = -1$ and, more generally that e is related to the trigonometric functions by $e^{i\theta} = \cos\theta + i\sin\theta$.

221 *Example*: Euler derived a closed-form solution to the general linear ODE of the second order in the form of a *continued fraction*. Starting from $y(x) = P_1(x)\frac{d^2y}{dx^2} + Q_0(x)\frac{dy}{dx}$ this equation is differentiated and becomes $y' = Q_1 y'' + P_2 y'''$, where $Q_1 = \frac{Q_0 + P_1'}{1 - Q_0'}$, $P_2 = \frac{P_1}{1 - Q_0'}$. This procedure is repeated indefinitely, and a set of relations $y^{(n)} = Q_n y^{(n+1)} + P_{n+1} y^{(n+2)}$ is obtained, where

The legacy of Leonhard Euler in unsurpassed in the long history of mathematics. In body, quantity and quality his achievements are overwhelming. Euler's collected works fill over 70 volumes, a testament to the genius of this unassuming man who changed the face of mathematics so profoundly.

Throughout his career, Euler was blessed with a phenomenal memory. His number-theoretic investigations were aided by the fact that he had memorized not only the first 100 prime numbers, but all their first six powers as well. While others were digging through tables or pulling out pencil and paper, Euler could simply recite from memory such quantities as 241^4 or 337^6. He was able to do mental calculations requiring him to retain in his head up to 50 significant figures, and that without apparent effort, "*just as men breath, as eagles sustain themselves in the air*" — in the words of Francois Arago. Yet this extraordinary mind still had room for the entire text of Virgil's *Aeneid*, which Euler had committed to memory as a boy, and still could recite flawlessly half a century later. No writer of fiction would dare provide a character with a memory of this caliber[222].

$n = 1, 2, 3, \ldots,$ and $Q_n = \frac{Q_{n-1}+P'_n}{1-Q'_{n-1}}$, $P_{n+1} = \frac{P_n}{1-Q'_{n-1}}$. Then

$$\frac{y}{y'} = Q_0 + \frac{P_1}{(y'/y'')} = Q_0 + \frac{P_1}{Q_1 + \frac{P_2}{(y''/y''')}}.$$

Let

$$\lambda(x) = Q_0 + \cfrac{P_1}{Q_1 + \cfrac{P_2}{Q_2 + \cfrac{P_3}{Q_3 + \cfrac{}{\ddots + \frac{P_n}{Q_n+R_n}}}}}$$

where $R_n = P_{n+1}\frac{y^{(n+2)}}{y^{(n+1)}}$.

If the fraction terminates, $y(x) = e^{\int \frac{dx}{\lambda(x)}}$; if it does not terminate, the problem of its convergence arises. To this end the following fundamental theorem is available: $\{\lambda(x)\}^{-1}$ converges and has the value y'/y if $y \neq 0$ and (1) $P_n \to P$, $Q_n \to Q$ as $n \to \infty$, (2) the roots ρ_1 and ρ_2 of the equation $\rho^2 = Q\rho + P$ are of unequal modulus; if further $|\rho_2| < |\rho_1|$ then $\lim |y^{(n)}|^{1/n} < |\rho_2|^{-1}$ provided that $|\rho_2| \neq 0$. When $|\rho_2| = 0$ the last condition is replaced by the condition that the limit is finite.

222 However, even the great Euler was not always right. His conjecture (1769) that $x^n + y^n + z^n = c^n$ has no solution if $n \geq 4$ was proven wrong:

Noam Elkies found (1988) the counterexample

$$2,682,440^4 + 15,365,639^4 + 18,756,760^4 = 20,615,673^4$$

Part of Euler's well-deserved reputation rests upon the textbook he authored. In all his texts, Euler's exposition was quite lucid and his mathematical notation was chosen so as to clarify, not obscure, the underlying ideas.

Euler's *Opera Omnia*, consists of 73 volumes. It contains 886 books and articles — written variously in Latin, French and German. His output was so huge and the pace of its production so rapid — even in the darkness of his later life — that a publication backlog is reported to have lasted 47 years after his death. It has been estimated that if one were to collect *all* publications in the mathematical sciences produced during 1725–1800, roughly 1/3 of these were from the pen of Leonhard Euler.

Virtually every branch of mathematics has theorems of major significance that are attributed to Euler.

One can get a feel for Euler's profound insight through the following examples:

To prove that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ Euler began with the key equation

$$\frac{\sin x}{x} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots = \left[1 - \frac{x^2}{\pi^2}\right]\left[1 - \frac{x^2}{4\pi^2}\right]\left[1 - \frac{x^2}{9\pi^2}\right]\cdots .$$

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and **Roger Frye** followed with the simpler result

$$95,800^4 + 217,519^4 + 414,560^4 = 422,481^4.$$

Moreover, Elkies showed that there are *infinitely many solutions* of

$$x^4 + y^4 + z^4 = c^4$$

in coprime natural numbers x, y, z, and c. He also provided a second solution in four astronomical numbers, each width 70 digits:

$$\begin{aligned}
x &= 1439965710\ 6489544922\ 6850677183\ 3175267850\ 2014266153\ 0044221829\ 2336336633,\\
y &= 4417264698\ 9945384969\ 4359748975\ 4952845854\ 6722971790\ 4789886412\ 4209346920,\\
z &= 9033964577\ 4825324980\ 5948245939\ 8457291004\ 9479250057\ 4302814746\ 5732645880,\\
c &= 9161781830\ 0354368478\ 3245239826\ 7266038227\ 0029622572\ 4366207037\ 0888722169.
\end{aligned}$$

Euler further conjectured (1769) that the general Diophantine equation

$$x_1^n + x_2^n + \cdots + x_{n-1}^n = x_n^n \quad (n \geq 4)$$

has no solutions in positive integers. Yet, **L. J. Lander** and **T. R. Parkin** were able to furnish the first counterexample (1966) for $n = 5$

$$27^5 + 84^5 + 110^5 + 133^5 = 144^5.$$

Performing the infinite multiplication on the right hand side, Euler obtained

$$1-\left[\frac{1}{\pi^2}+\frac{1}{4\pi^2}+\frac{1}{9\pi^2}+\frac{1}{16\pi^2}+\cdots\right]x^2+(\cdots)x^4-\cdots.$$

Equating the coefficient of x^2 on both sides, the required result follows. Thus, Euler found the answer that had escaped mathematicians for decades[223].

From the above key equation Euler deduced, for $x=\frac{\pi}{2}$, the known Wallis' infinite-product representation

$$\frac{2}{\pi}=\left(1-\frac{1}{4}\right)\left(1-\frac{1}{16}\right)\left(1-\frac{1}{36}\right)\cdots$$

or

$$\frac{\pi}{2}=\frac{2\cdot 2\cdot 4\cdot 4\cdot 6\cdot 6\cdots}{1\cdot 3\cdot 3\cdot 5\cdot 5\cdot 7\cdot 7\cdots}.$$

Moreover, by equating the coefficients of x^4 on both sides of the above key equation, he could announce that $\sum_{n=1}^{\infty}\frac{1}{n^4}=\frac{\pi^4}{90}$.

Another of Euler's beautiful results is the relation (1737) known as '*Euler's product formula*' $\sum_{n=0}^{\infty}\frac{1}{n^s}=\prod_p\left(1-\frac{1}{p^s}\right)^{-1}$, where s is a real number greater than 1 and the expression on the right denotes an infinite product in which p runs over all primes.

In the field of Diophantine equations, Euler found that $p=a^3-9ab^2$; $q=3a^2b-b^3$; $r=a^2+3b^2$ solve the equation $p^2+3q^2=r^3$

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223 Historically,

$$1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots$$

was the first series that mathematicians were unable to sum using elementary algebraic methods. After the Bernoulli family had tried and failed, Euler finally cracked the problem (1734) by means of a brilliant unorthodox argument. Today, such results can be derived in a systematic way using *residues*:
Consider the function $g(z)=\frac{cot(\pi z)}{z^2}$. With N a positive integer, let S be the origin – centered square with vertices $(N+\frac{1}{2})(\pm 1\pm i)$. Adding up the residue inside S,

$$\frac{1}{2\pi i}\oint_s g(z)=Res\,[g(z),0]+\sum_{n=-N}^{-1}Res[g(z),n]+\sum_{n=1}^{N}Res\,[g(z),n]=$$

$$=-\frac{\pi}{3}+\frac{2}{\pi}\sum_{n=1}^{N}\frac{1}{n^2}$$

As N tends to infinity, the integral on the LHS tends to zero and from this fact we immediately deduce Euler's result.

It is incredible how Euler could have discovered the identity:

$$x^4 + y^4 = z^4 + t^4$$

where

$$\begin{aligned}
x &= a^7 + a^5b^2 - 2a^3b^4 + 3a^2b^5 + ab^6,\\
y &= a^6b - 3a^5b^2 - 2a^4b^3 + a^2b^5 + b^7,\\
z &= a^7 + a^5b^2 - 2a^3b^4 - 3a^2b^5 + ab^6,\\
t &= a^6b + 3a^5b^2 - 2a^4b^3 + a^2b^5 + b^7.
\end{aligned}$$

One of Euler's achievements (1748) was the discovery of the four-square analogue of $(a^2 + b^2)(c^2 + d^2) = (ac \pm bd)^2 + (ad \mp bc)^2$, namely:

$$\begin{aligned}
(x_1^2 + x_2^2 + x_3^2 + x_4^2)(y_1^2 + y_2^2 + y_3^2 + y_4^2) = \quad & (x_1y_1 + x_2y_2 + x_3y_3 + x_4y_4)^2\\
&+(x_1y_2 - x_2y_1 - x_3y_4 + x_4y_3)^2\\
&+(x_1y_3 + x_2y_4 - x_3y_1 - x_4y_2)^2\\
&+(x_1y_4 - x_2y_3 + x_3y_2 - x_4y_1)^2.
\end{aligned}$$

Euler also showed (1737) that

$$\tanh(1) = \frac{e^2 - 1}{e^2 + 1} = \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{7 + \cfrac{1}{\ddots}}}}} = 0.76159415\ldots$$

where e is the basis of the natural logarithms.

The Saga of $i = \sqrt{-1}$

During the 16^{th} century, mathematicians encountered square roots of negative numbers through the general solutions of quadratic and cubic equations.

Since the square of every real number is either positive or zero, the equation $x^2 + 1 = 0$ *cannot be solved in the field of real numbers.*

A book on algebra by ***Rafael Bombelli****, which dates from* 1572, *contains a consistent theory of roots of negative numbers. These numbers were used by mathematicians since the middle of the* 17^{th} *century and were since known as imaginary numbers. Later, the theory of numbers of the form* $a + b\sqrt{-1}$ *(complex numbers) was advanced by* ***Johann Bernoulli****. However, the symbol* $\sqrt{-1}$*, was not satisfactory as it lead to paradoxes such as:* $-1 = (\sqrt{-1})^2 = \sqrt{-1}\sqrt{-1} = \sqrt{(-1)(-1)} = \sqrt{1} = 1$. *To avoid this,* ***Leonhard Euler***[224] *introduced in* 1777 *the notation* i *with the basic property* $i^2 = -1$. *The two roots of the equation* $x^2 = -1$ *are now* $\pm i$. *The symbol* i *is called the imaginary unit. The choice of the word imaginary is unfortunate, but it indicates the distrust with which complex numbers were viewed. These suspicions slowly vanished at the end of the* 18^{th} *century, when* ***Caspar Wessel*** *in* 1797, ***Carl Friedrich Gauss*** *in 1799 (doctoral thesis) and* ***Jean Robert Argand*** *in* 1806, *gave simple geometric representation to complex numbers* $a + ib$ *as vectors (points) in the Cartesian plane.*

This simple interpretation of complex numbers made mathematicians feel much more comfortable with them, and their existence was slowly accepted.

With Euler began the study of functions and power series in a complex variable. He observed that the formal substitution of x *by* ix *in the exponential function*

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

leads to

$$e^{ix} = \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots\right) + i\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right),$$

that is, $e^{ix} = \cos x + i \sin x$. *However, all these formal results were lacking in mathematical rigor and often led to paradoxes*[225].

It was not until the 19^{th} *century that this naive approach to complex analysis was replaced by a rigorous treatment. In* 1833, ***William Rowan Hamilton*** *presented a paper before the Royal Irish Academy in which he*

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[224] For further reading, see:

- Nahin, P.J., *The Imaginary Tale*, Princeton University Press, 1998, 257 pp.

[225] For example, we know that the real-valued function $y = \tan x$ for $-\pi/2 < x < \pi/2$ takes on all real values. Suppose that this function could be generalized so as to take on all complex values while retaining the ordinary law of the tangent of sums. There should then exist a complex number x_0 such that $\tan x_0 = -i$. Thus for any complex number x with $\tan x \neq \pm i$, we should have $\tan(x + x_0) = \frac{\tan x + \tan x_0}{1 - \tan x \tan x_0} = \frac{\tan x - i}{1 + i \tan x} = -i$, which is absurd.

introduced a formal algebra of ordered pairs of real numbers, the rules of combination being precisely those given today for the system of complex numbers.

The founders of the theory of functions of complex variable (and of all analysis), were ***Augustin Louis Cauchy****, professor at the École Polytechnique in Paris (1848),* ***Karl Weierstrass****, professor at the University of Berlin (1864), and* **Bernhard Riemann***, professor at Göttingen (1859). Cauchy introduced the concept of the complex line integral in* 1814 *and published his basic theorems on functions of complex variable in* 1825*. During the second half of the* 19^{th} *century, Riemann developed the theory of complex functions from a physico-geometrical standpoint, and Weierstrass developed it from a logically rigorous standpoint.*

The invention of set theory by ***Georg Cantor*** *at the end of the* 19^{th} *century helped enormously in the development of the foundations of complex analysis.*

Euler versus Bernoulli versus Leibniz

Nothing sheds more light on the state of knowledge in any given era than the issues debated among the scientists of that time. One of the investigations continued from the 17^{th} *century was the solution of polynomial equations. In this context, the question was raised whether an arbitrary polynomial with real coefficients can be decomposed into a product of linear factors (or a product of linear and quadratic factors with real coefficients, to avoid the use of complex numbers).* ***Leibniz*** *(1702) thought this was not possible and gave the example*

$$x^4 + a^4 = (x^2 - ia^2)(x^2 + ia^2) = (x + a\sqrt{i})(x - a\sqrt{i})(x + a\sqrt{-i})(x - a\sqrt{-i}),$$

claiming that no two of these four factors render a quadratic factor with real coefficients upon multiplication. Had he been able to express the square root *of* i *and* $-i$ *as ordinary complex numbers, he would have seen his error. Indeed,* ***Nicholas Bernoulli*** *(1687–1759), a nephew of James and John, pointed out in* 1719 *that*

$$x^4 + a^4 = (a^2 + x^2)^2 - 2a^2x^2 = (a^2 + x^2 + ax\sqrt{2})(a^2 + x^2 - ax\sqrt{2})$$

[which meant that the function $(x^4 + a^4)^{-1}$ *could be integrated in terms of trigonometric and logarithmic functions!]*

Notwithstanding this result, Nicholas did not believe that his decomposition can be effected for every *polynomial with real coefficients. Euler, however, took the correct stand: In a letter to Nicholas of October* 1, 1742*, Euler affirmed (without proof) that a polynomial of* arbitrary *degree with real coefficients could be decomposed into linear and quadratic factors with real coefficients.*

Nicholas replied on December 15, 1742 *with an example of his own, which he said, contradicts Euler's assertion:*

$$f(x) = x^4 - 4x^3 + 2x^2 + 4x + 4 = (x - z_1)(x - z_2)(x - z_3)(x - z_4)$$

where

$$z_1 = 1 + \sqrt{2 + i\sqrt{3}}; \qquad z_2 = 1 + \sqrt{2 - i\sqrt{3}};$$
$$z_3 = 1 - \sqrt{2 + i\sqrt{3}}; \qquad z_4 = 1 - \sqrt{2 - i\sqrt{3}}.$$

Euler then showed that since $z_1 = \overline{z}_2$, $z_3 = \overline{z_4}$ *(conjugate pairs), the factorization yields*

$$f(x) = \left[x^2 - 2(1+p)x + (1+p)^2 + q^2\right]\left[x^2 - 2(1-p)x + (1-p)^2 + q^2\right],$$

where p, q *are real and*

$$z_1 = 1 + p + iq; \qquad z_2 = 1 + p - iq;$$
$$z_3 = (1 - p) - iq; \qquad z_4 = (1 - p) + iq.$$

Euler thus proved Nicholas wrong on this count, but he still lacked a general proof. The kernel of the problem of factoring a real polynomial into linear and quadratic factors with real coefficients was to prove that every such polynomial had at least one real or complex root. The proof of this fact, called the fundamental theorem of algebra*, became a major goal. Proofs afforded by* **d'Alembert** *and* **Euler** *were incomplete. The first substantial proof was given by* ***Gauss****, in his doctoral thesis (1799).*

Worldview XIII: Leonhard Euler

* * *

"If a nonnegative quantity was so small that it is smaller than any given one, then it certainly could not be anything but zero. To those who ask what the infinitely small quantity in mathematics is, we answer that it is actually zero. Hence there are not so many mysteries hidden in this concept as they are usually believed to be. These supposed mysteries have rendered the calculus of the infinitely small quite suspect to many people. Those doubts that remain we shall thoroughly remove in the following pages, where we shall explain this calculus".

* * *

"Mathematicians have tried in vain to this day to discover some order in the sequence of prime numbers, and we have reason to believe that it is a mystery into which the human mind will never penetrate".

* * *

[Upon losing the use of his right eye] "Now I will have less distraction".

* * *

On Euler

* * *

The whole form of modern mathematical thinking was created by Euler. It is only with the greatest difficulty that one is able to follow the writings of any author preceding Euler, because it was not yet known how to let the formulae speak for themselves. This art Euler was the first to teach.

F. Rudio

* * *

The Earth's Rotation (350 BCE–1765)

The revolution of the earth about the sun and its rotation about its own axis were known to ***Heracleides*** *(ca 355 BCE). During the next two centuries, the Greeks accumulated sufficient astronomical data which enabled them to discover the forced precession of the earth's axis of rotation. This astronomical lore was put in 'deep freeze' for some* 1700 *years and resurged in Europe after the Copernican revolution (1543).*

By the time of ***Euler*** *(1758) the following facts about the earth's rotation were known:*

(1) The inertial behavior of the earth is compatible with a figure that closely resembles an ellipsoid of revolution[226] *(spheroid), slightly flattened at the poles with ellipticity (flattening) of* $\frac{1}{297}$*, i.e. possessing an equatorial diameter greater by about* 43 km *than its polar diameter.*

The figure has its principal moment of inertia C*, in the direction of the symmetry axis. The points at which the axis of symmetry pierces the surface of the earth is the geometric North Pole.*

(2) The earth is not rotating at present about a principal axis. Therefore the axis of rotation is not fixed relative to itself. The points at which the angular velocity vector cuts through the earth's surface are the celestial poles of rotation. Vertically above these points stars would have no diurnal motion. (The star Polaris is near the celestial North Pole). This axis of spin[227] *is called the polar axis. The polar axis is inclined to the plane of ecliptic by* $23°27'$*.*

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[226] This is known as the 'reference ellipsoid'. It is generated by the rotation of the ellipse $R_1 = b(1 - \epsilon^2 \cos^2 \varphi_1)^{-1/2}$, where b is the semi-minor axis, R_1 is the geocentric radius, φ_1 is the geocentric latitude and $\epsilon^2 = (a^2 - b^2)/a^2$, where a is the semi-major axis $[a = 6378.388 \text{ km}, \ b = 6356.912 \text{ km}]$ and the flattening is $(a - b)/a$.

[227] To an earthbound observer, the principal axes of inertia of the earth are fixed. The angular velocity vector $\boldsymbol{\omega}$ is fixed in magnitude and rotates about the major axis of inertia $\mathbf{e}_3$ (*free Eulerian precession*). In the *forced precession*, the inertia axis $\mathbf{e}_3$ rotates about the normal to the ecliptic, as seen to an observer outside the earth.

Spin is the component of the angular velocity vector in the direction of the greatest moment of inertia. Its value is $\omega_3 = 7.29211 \times 10^{-5}$ rad/sec.

(3) *According to the Eulerian theory of free precession, the tip of the angular velocity vector (relative to the body axes) describes a circle (the 'polohods') about the axis of symmetry. In the case of the earth, the shape of this curve is somewhat irregular, but its diameter never exceeds* 15 *meters, with a mean radius of* 4 *m. The period of revolution about the figure axis is about* 305 *days.*

(4) *Because of the earth's oblateness ('equatorial bulge') and the inclination of the polar axis just mentioned, the resultant attraction of the sun or moon does not pass through the earth's center of mass. The resultant attraction is therefore equivalent to a force acting through the center of mass and a couple which tends to bring the earth's equator into coincidence with the ecliptic. The earth, being an enormous gyroscope, reacts to this couple by a precession of its spin axis (roughly directed along its axis of symmetry) around a normal to the ecliptic (Newton, 1687). The precession due to the moon is more than twice as great as that due to the sun. This precession manifests itself through two equivalent motions:*

(a) *a slow conical motion of the earth's polar axis in space (about a normal to the ecliptic);*

(b) *a continual revolution of the line of nodes (line of equinoxes in astronomical usage, i.e. the intersection of the plane of the earth's equator with the plane of the ecliptic) in the plane of the ecliptic.*

Both motions have a period of about $25,800$ *years and amount to a rotation of* $50.4''$ *per year*[228]*. The diameter of the apparent circular motion at the earth's poles is about* 52 *cm which is about* 10 *times smaller than the corresponding amplitude of the Eulerian free precession.*

(5) *Superposed on the precessional motion of the principal inertial axis, there is an additional periodic irregularity, called forced nutation [from Latin nutare, to nod]. This appears as a small elliptical motion*[229] *about the mean pole, depending on the moon's inclination, with a period of about* 18.6 *years and amplitude of about* $9''$ *(**Bradley**, 1748).*

228 The average precession rate per year due to the combined action of the sun and the moon is $-\frac{6\pi^2}{\Omega}\left(\frac{C-A}{C}\right)\cos\theta\left[1+\frac{m_L}{m+m_L}\frac{1}{\tau_1^2}\right]$ where Ω = angular velocity of the earth's rotation relative to the inertial frame of the fixed stars $= 2\pi \times 365\frac{1}{4}$ radians per year; $(C-A)/C = 0.0032$; $\theta = 23°27'$; $\frac{m_L}{m+m_L} = \frac{1}{82.5}$ (m = mass of earth; m_L = mass of moon); τ_1 = period of revolution of the moon around the earth (27.32 days) in units of solar years $= \frac{27.32}{365\frac{1}{4}}$. The total result is $-15.9'' - 34.5'' = -50.4''$ per year.

229 The major axis of this ellipse points towards the Pole of the ecliptic and is only $18''$ long, and the minor axis is $14''$ long. These are about the angular dimensions of a lemon seen from a distance of a kilometer.

The Enlightenment (Age of Reason 1687–1789)

Central Europe emerged from the calamity of the Thirty Years' War greatly diminished in population, badly disorganized economically and more than ever broken up into tiny political states. The largest of this states was Austria, whose king was usually also emperor of the so-called Holy Roman Empire*, though his powers as emperor were purely nominal. The next in size was Prussia, whose ruler (1700) changed his title of elector to that of King. Then there where hundreds of almost independent principalities, cities, bishoprics. The head of each of these, or its group of ruling patricians, exercised authority with few limitations, the common people having no say whatever in the government. Few of the rulers, moreover, had any idea how to improve the economic situation of their people. Their interests were confined to taxing, conducting war and living in as grand a style as possible.*

The burgers, on the other hand, were bound to the old ways of doing things and feared the slightest change in the methods of commerce and industry. Their guilds regulated every thing to a degree that it was impossible for trade to make any progress. England and Holland had broken with these old-fashioned methods and were rising rapidly in wealth and power, while the rest of Europe was standing still. Alone among the princes of the Continent, Frederick William, elector of Prussia (reigned 1640 – 1688), saw the need for reforming the economic conditions of his country by breaking the stranglehold of the guilds, thus starting Prussia on the road to military and political power. This power derived from the changes he forced in the economic order of his lands.

The one hundred years that preceded the French revolution witnessed the rise of kings to unmatched power and influence in European affairs. These years also encompassed the birth, maturation, and waning of the Enlightenment*. In the latter half of the period, during the rule of* "philosopher kings" *(the enlightened absolutists), the monarchical tradition and the new intellectual development was reflected. This period was brought to an abrupt end by the movement toward representative government and the stirrings of political and social revolts.*

The overwhelming success of the Newtonian physics and world-view induced the thinkers of the 18^{th} century to apply the methods and principles *of* 17^{th} *century mathematics and physics to heal the economic, social, political and ecclesiastical elements of society. The Paris-centered enlightenment movement dates from the visit of* ***Voltaire*** *(1694–1778) to England (1726–1729), and the subsequent dissemination of the ideas of* ***Newton*** *and* ***Locke*** *(1632–1704) across the channel. Other leaders of the enlightenment were* ***Montesquieu*** *(1689–1755, France) [who tried to apply methods of the natural sciences to the study of governmental forms], and* ***Denis Diderot*** *(1713–1784, France).*

The period produced many important advances in the fields of astronomy, chemistry, mathematics and physics. Philosophers of the Age of Reason organized knowledge in encyclopedias and founded scientific institutes. They explored issues in education, law, philosophy and politics, and attacked tyranny, social injustice, superstition and ignorance. Many of their ideas contributed directly to the outbreak of the American and French revolutions[230].

The principal publication of the Age of Reason is the 'Encyclopédie'[231]*, edited by* ***Diderot*** *and* ***d'Alembert*** *in* 17 *volumes of text and* 11 *volumes of illustrations during* 1751–1772. *This monumental endeavor became important in the democratization of scientific knowledge. Technology was given equal importance to that of pure science or philosophy. For the first time in the history of science a group of scholars and savants addressed their ideas in writing to a broad public.*

While science, literature and philosophy flourished in France, there was little evidence for such cultural activity in Germany; here, the setback caused by the Thirty Year's War (1618–1648) lasted until the second half of the 18^{th} *century. The like was true of Jewish culture: the ghetto imprisonment, the impoverishment and the terrors of war had not only destroyed schools, but also crushed the independence of spirit necessary for cultural progress*[232].

230 In the Middle Ages, opposite forces were held together by the pressure of the Church. As this pressure has diminished, the opposite forces rebelled against each other, leading to revolutions.

231 It was by no means the first endeavor of its kind. The Chinese encyclopedia 'Yung lo ta tien' was written in the 15^{th} century.

232 As odd as it may seen, the Enlightenment in France marks the beginning of modern secular anti-judaism. The torchbearers of this new age, namely **Voltaire**, **Diderot** and to a lesser degree **Jean-Jacques Rousseau**, carried over into the mainstream of Enlightenment thinking the medieval Christian stereotypes of the Jew.

Instead of disappearing with Enlightenment, antisemitism simply found new guise, one which no longer blamed the Jews for crucifixion of Christ but held them responsible for all the crimes and perversities committed in the name of

Other major figures of the enlightenment are: ***Swift*** *(1667–1745),* ***Berkeley*** *(1685–1753),* ***Buffon*** *(1707–1788),* ***Frederick the Great*** *(1712–1786),* ***Sterne*** *(1718–1768),* ***Helvetius*** *(1715–1771),* ***Winckelmann*** *(1717–1768),* ***Adam Smith*** *(1723–1790),* ***Lessing*** *(1729–1781),* ***Moses Mendelssohn*** *(1729–1786),* ***Burke*** *(1729–1797),* ***Priestley*** *(1733–1804),* ***Wieland*** *(1733–1813),* ***Coulomb*** *(1736–1806),* ***Gibbon*** *(1737–1794),* ***Galvani*** *(1737–1798),* ***Lavoisier*** *(1743–1794).*

The late enlightenment created [toward the end of the 18^{th} *century] a reaction to its materialism*[233] *and rationalism. It is generally called 'Romanticism' in France and England. In art, music and literature this reaction emphasized the great elemental motions and denied the supremacy of reason. Romanticism did not come to fruition in most countries until the first half of the* 19^{th} *century.*

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the monotheistic religions: The Jews were judged to be inherently perverse, and their 'fossilized' religion to be *an obstacle to human progress*!

Interestingly enough it was *Jews*, and non other, that helped Germany recover from the destructive effects of the Thirty Years' War by strengthening its shaky economy and catalyzing the diffusion of the spirit of the age of reason into Germany. Jews were being called out of the ghetto-prisons to help rebuild the lands ravaged by war. However the *Christian* population continued in its accustomed hostility, despite the new ideas which were slowly spreading among the new cultured classes.

As one of the means to achieve his ends, Frederick William of Prussia made use of the Jews. To overcome the opposition to the burgers (who, on the slightest suspicion of competition by Jews bestirred themselves to force the Jews out of the occupation involved) He granted the Jews certain trading privileges, and then used these privileges to bargain with the burgers, either to gain greater authority for himself or to make some change in general economic situation, which enhanced interstate and international commerce for the benefit of all concerned. Thus there emerged the figure of the Court-Jews who served the various princelings as financial agents and as civilian quartermasters for their armies. The two most noted of these were Samuel Oppenheimer of Vienna (1630–1703) and his distant relative Joseph Süss Oppenheimer ("*Jud Süss*") (1692–1738, Feb 04).

[233] A philosophical doctrine which examined both nature and social life from a mechanistic point of view. Basing themselves on *mechanics*, which in those days was the height of science, materialist philosophers imagined that the same mechanical laws can be applied automatically to life and nature. Moreover, since these laws are immutable, society changes very little except for repeating itself mechanically via wars, hunger, government etc. Consequently, mankind can do nothing to change things.

The romantic era marked an interlude between the more disciplined, rationalist 'Weltanschauung' of the 18^{th} *century, which reflected the order and regularity of the Newtonian universe, and the science-oriented outlook that was to triumph in the second half of the* 19^{th} *century.*

At any rate, what remained of the romantic mood was shattered by the mid-century revolutions of 1848. The abortive uprisings of that year seemed to prove that ideals were not enough, that in the last analysis physical force, material resources, and power were what counted in human relations

1738 CE, Feb 04 **Joseph Süss Oppenheimer** (1692–1738). The Jewish Financial Minister of Karl Alexander, the Duke of Württemberg. He has been envied and hated for his role in planning and implementing radical economics reforms. Arrested after the Duke's death and placed in an iron cage suspended from a high beam over the Württemberg city square, for all to see and mock. When the mob tired of the spectacle, Oppenheimer was strangled. In 1939, the German Nazis made an anti-semitic movie named *Jüde Süss*, which was very successful all over Germany. The public murder of Oppenheimer took place in the middle of European *Enlightenment* (Age of Reason) in the days of Voltaire, Montesquieu, Diderot, Rousseau and Lessing.

1739 CE, Oct **Jose Antonio da Silva** (1705–1739, Brazil and Portugal). A Converso writer, was garroted and burnt at a Lisbon auto-da-fe (on charges of Sabbath observance) by the Inquisition – *Enlightenment* Portuguese style. His wife, who witnessed his death, did not long survive him. Da Silva's tragic story has inspired several modern writers, including the Portuguese Camilo Castelo Branco (author of the novel *O Judeu*), who was himself of Converso origin.

1740 CE **Benjamin Huntsman** (1704–1776, England) inventor and steel-manufacturer. Produced a satisfactory *cast steel*, purer and harder than any steel then in use[234]. Born to German parents in Lincolnshire. He started business as a clock, lock and tool maker at Doncaster, and attended a considerable local reputation for scientific knowledge and skilled workmanship.

[234] Steel had been made in small quantities even before Christian era. However, in 1722 **René Antoine de Réaumur**, a French physicist, learned how to make larger quantities by placing malleable iron in a bath of cast iron.

Finding that the bad quality of the steel then available for his products seriously hampered him, he began to experiment in steel manufacture, first at Doncaster and subsequently in Handsworth, near Sheffield, to where he moved in 1740 to secure cheaper fuel for his furnaces.

After several year's trials he at last produced satisfactory cast steel[235]. The Sheffield cutlery manufactures, however, refused to buy it, on the ground that it was too hard, and for a long time, Huntsman exported his whole output to France. The growing competition of imported French cutlery made from Huntsman's cast-steel at length alarmed the Sheffield cutlers (who vainly endeavored to get the exportation of steel prohibited by the British government) and compelled them in self-defense to use it.

Huntsman had not patented his process and its secret was discovered by a Sheffield ironfounder. Huntsman's business was subsequently greatly developed by his son **William Huntsman** (1733–1809).

1740–1747 CE **Moshe Hayyim Luzzatto**[236] (1707–1747, Italy Amsterdam and Israel; acronym RaMHaL). Philosopher, mystic moralist, accomplished linguist, poet and the progenitor of a Hebrew revival. His philosophy

[235] *Steel* is a purified alloy of iron, carbon, and other elements that is manufactured in the liquid sate. Most steels are almost freed from phosphorus, sulfur, and silicon, and contain between 0.15 to 1.5 percent of carbon. High-carbon steels (0.70–1.5 percent) are used for making razors, surgical instruments, drills and other tools.

The *Crucible* is the oldest method of making steel, it is a small pot made of clay and graphite. A number of crucibles are placed on the hearth of a furnace, which is heated by gas. Carefully selected scrap is melted in these crucibles. Huntsman melted together pieces of iron and charcoal (Ca 20 kg) in a covered crucible for a few hours. The resulting steel, with a relatively high but evenly distributed carbon content, was exceptionally hard. Because he cast it in molds, Huntsman called it *cast steel*. However, small ingots of this size could not yet be used to build bridges and railways.

[236] Luzzatto (Luzzatti) is the name of Italian scholars that is derived from the province of *Lausitz* in Eastern Germany (Lat. *Lussatia*). According to tradition the family emigrated into Italy in ca 1450, settling in the Venetian territories. The earliest member of the family of whom there is a record is Abraham Luzzatto (1586); one of his sons settled in Safed, Israel.

During 1500–1900 CE, the Luzzatto family has provided an uninterrupted lineage of some 14 generations of creative scholars in many fields of human intellectual endeavor: philosophers, scientists, physicians, historians, statesmen, authors and religious leaders. Among them: (i) **Shmuel David Luzzatto** (acronym *SHaDaL*; 1800–1865), philosopher, philologist, translator, Bible commentator; (ii) **Luigi Luzzatti** (1841–1927); Prime Minister of Italy 1910–1911.

inspired millions of people in Europe and continues to be a living tradition in Judaism.

His treatise *The Path of the Upright* (*Mesillat Yesharim*, 1740) stands on it's owns as one of the most influential and inspirational ethical works of Judaism [alongside with the *Bible book of Genesis* (ca. 750 BCE); *Book of Job* (ca. 600 BCE); *Book of Ecclesiastes* (ca. 250 BCE); *Book of the Khazar* (Yehuda Halevi, 1139) and *The Guide for the perplexed* (Maimonides, 1190)][237]. Luzzatto was born in Padua, son of a wealthy merchant. Since his childhood prodigy, he became thoroughly knowledgeable in Judaic literature, classical and modern languages, contemporary Italian culture and the secular sciences. In his early poetry and dramas (1724–1727) he created a new school of Hebrew literature. But through 1727–1734 he began to lean toward Kabbalistic mysticism, becoming leader of a group of religious thinkers. This brought him to a direct conflict with the Venetian Rabbinate who, fearing a new messianic pretender, put Luzzatto under the ban (1734) for "*practicing sorcery and pronouncing incantations*".

Subjected thus to persecution and excommunication, Luzatto went to Amsterdam (1736) where he could freely teach and write on diverse topics, such as ethics, philosophy, poetry and Kabbalah. Like Spinoza before him, he earned

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[237] The spiritual giants of the Jewish people can be divided into two groups: One is called *"Ma'atikei Shmu'a"* – those who faithfully record what has been passed down to them for posterity. The other is composed of those who have tried to rewrite the tradition that passed down to them.

The members of the first group earned admiration, trust, and love during their lifetimes. The members of the second group were not trusted and were considered to be controversial; after their death, however, they where given unlimited admiration, to the point of becoming legends. It is to this second mysterious group that Maimonides and Rabbi Moshe Hayyim Luzzatto belong. An intellectual and tragic common denominator connects them: it was after their death that both earned total admiration, which has not diminished since. Based upon the following famous Talmudic passage Luzatto wrote *Mesillat Yesharim* in order to blaze a trail that man must follow to attain ethical perfection: Rabbi Pinchas ben-Yair says: (Mishna; *Sota*, **9**)

"Watchfullness leads to alertness,
Alertness leads to cleanliness,
Cleanliness leads to abstinence,
Abstinence leads to purity,
Purity leads to saintliness,
Saintliness leads to humility,
Humility leads to fear of sin,
Fear of sin leads to holiness."

his living by grinding optical lenses, but unlike him he remained ardently devoted to the cause of Judaism. However, he did not turn his thoughts from the mysticism that not only incited his loftiest aspirations but also inspired him to the conception of high ethical principle.

Indeed, it is in Amsterdam that he wrote his important philosophical treatise *Mesillat Yesharim* (The path of the Upright) on the path man must follow to attain ethical perfection[238]. This ethical work, written in Hebrew, became one of the most influential books read by Eastern European Jewry in the late 18^{th} and 19^{th} centuries.

In other ethical and theosophical works Luzzatto studied some basic theological questions: the ways of divine justice, the ways to overcome evil desires, prayer, the Commandments, relationship between the just and the sinner, original sin, the aim of creation, the next world and the world of redemption. All his works in this field were widely read and accepted, and contributed to his metamorphosis to sage and saint.

Luzatto visited London, but finally he was determined to escape from the prohibition to teach Kabbalah. Filled with longing for the Holy Land, and after many hardships he moved with his wife (m. 1731) and son to Safed, the Kabbalistic center in Israel at that time. He died of the plague on May 06, 1747 in Kfar Yassif near Acre and was buried at Tiberias beside Rabbi Akiva

Luzatto, though persecuted when alive, was accepted by the three main 19^{th} century Jewish movements, which were fighting bitterly among themselves: the *Hassidim* saw him as a saintly mystic and used some of his Kabbalistic ideas. Their opponents, the *Mitnaggedim* regarded his ethical works as the clearest pointers toward a Jewish ethical way of life; and the enlightenment (*Haskalah*) writers saw Luzzatto as a progenitor of their own movement, and his works as the beginning of Hebrew aesthetic writings. Every facet of

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238 This treatise has been compared to John Bunyan's *The Pilgrim's Progress* (1675), though it was not influenced by the latter. Though written in the 18^{th} century, *Messilat Yeshrim* is essentially a medieval book, for Jewish medievalism outlasted European medievalism by almost 400 years. The work was printed many times, and translated into many languages.

A common groundless accusation against Judaism, which is repeated *ad nauseam*, is that Judaism was nothing but a formal system of practices which exacted outward conformity regardless of inner meaning of mind and heart. This misleading disinformation, which is all too opt to be accepted uncritically, is shuttered in the face of the vast ethical literature which the Jews have produced. *Messilat Yesharim* cultivates the inwardness of the laws and duties which the Jew has to live up.

Luzatto's work, therefore, remained alive and creative in the divided and confused Jewish culture of 19^{th} century.

1740–1744 CE **Pierre Louis Moreau de Maupertuis** (1698–1759, France). Mathematician and astronomer.

Was born in St. Malo. At 20 he entered the army, becoming lieutenant in a regiment of cavalry and spending his leisure on mathematical studies. In 1723 he quit the army and was admitted as a member of the Academy of Sciences. In 1728 he visited London and was elected a fellow of the Royal Society. In 1736 he was the head of an expedition sent by Louis XV into Lapland to measure the length of a degree of the meridian for the sake of determining the oblateness of the earth. On the basis of these measurements he found that the earth is flattened at the poles and oblate at the equator, as predicted by Newton. His findings corrected earlier results of Cassini.

In 1740 Maupertuis went to Berlin on the invitation of the King of Prussia, and took part in the battle of Mollwitz where he was taken prisoner by the Austrians. Returning to Berlin in 1744 at the request of Frederick II, he was chosen president of the Royal Academy of Sciences.

In 1744, he stated his "*principle of least action*" and applied it to optics and mechanics. He believed that it is a mathematical principle through which nature acts in the grand scheme of the universe to secure greatest economy.

Maupertuis was a man of considerable ability, but his restless, gloomy disposition involved him in constant quarrels, of which his controversy with Voltaire during the latter part of his life furnishes an example.

1741–1765 CE **Johann Peter Süssmilch** (1707–1767, Germany). Prussian regimental pastor and a pioneer in the field of population statistics. In his book (1761): "*Die göttliche Ordnung in den Veränderungen des menschlichen Geschlechts aus der Geburt, dem Tode, und der Fortpflanzung desselben erweisen*" he made a systematic attempt to make use of a class of facts which up to that time had been regarded as belonging to "political arithmetic" (today — "vital statistics").

In his book, Süssmilch investigated whether war and plague were part of God's plan[239] for the decimation of human surplus on earth. To this end he

[239] In this he was influenced by a paper (1710) of **John Arbuthnot** (1667–1735) called "*An argument for Divine Providence, taken from the constant regularity observed in the birth of both sexes*". In this note Arbuthnot (Physician to Queen Anne during 1709–1714, satirical writer and collaborator of Jonathan Swift) claimed to demonstrate that divine providence, not chance, governed sex-ratio of birth.

estimated that the earth had then a billion people (an overestimate), and calculated that the population could grow for centuries before it reached the maximum number of people the earth could support. This number he estimated at 13.9 billion. Süsmilch then piously concluded that war and pestilence were *not* part of the divine plan for reducing human population.

Süssmilch had arrived at a perception of what has been later termed the "*laws of large numbers*". He endeavored to form a general theory of society, based upon quantitative aggregate observation. Although he did not enter his investigation with an open mind, his work was nevertheless a most valuable one since it *pointed out the road* which other unbiased researchers were not slow to follow. Thus, Süssmilch's success was the origin of a mathematical school of statisticians[240].

How Many People Have Ever Lived on Earth?

Leeuwenhoek *(1769) published the first quantitative estimate of 13.4 billion people. Since then, 65 different estimates have been made, ranging from 1 billion to 1000 billion, depending upon initial assumptions made.* ***Süssmilch*** *(1761) gave an estimate of 13.9 billion.*

[240] The word *statistics* is derived from the Latin *status*, which in the Middle Ages came to mean a *state* in the political sense, denoting inquiries into the condition of a polity.

As human societies became more and more organized, a considerable body of official statistics came into existence, and intended to aid administration. The Romans were careful to obtain accurate information regarding the resources of the state, and they appear to have taken the census with a regularity which has hardly been surpassed in modern times. The material for statistics therefore existed at a very early period, but it was not until within the last four centuries that systematic use of the information available began to be made for purposes other than mere administration.

Statistics in the modern sense of the word, did not really come into existence until Süssmilch's publication.

The total number of people who have ever lived on earth can be estimated in a reasonable way as follows:

One begins by determining the mean population size for a birth-death stochastic process (i.e., the average behavior of a population whose size varies stochastically, growing over time due to random occurrence of births and deaths). One then assumes a starting population of two persons 1.5 millions years ago and divides the total time span into a number of smaller subintervals by using times for which estimates of world population have been made (e.g., N (8000 BCE) $= 5 \times 10^6$*; N (0 BCE)* $= 250 \times 10^6$*; N (1750 CE)* $= 800 \times 10^6$*; N (1825 CE)* $= 10^9$*; N (1930)* $= 2 \times 10^9$*; N (1960)* $= 3 \times 10^9$*; N (1980)* $= 4.4 \times 10^9$*). The total number of people who ever lived since 1.5 million years before present is then found to range from 50 to 100 billion* (10^{11}*).*

One can also show that the number of people living today (2008 CE) is almost equal to the number of offsprings from a single pair of parents (The primordial Adam and Eve of, say, the Homo Sapiens branch) 140,000 years ago.

1742 CE **Christian Goldbach** (1690–1764, Germany and Russia). Mathematician. Made notable contributions to the theory of infinite series and the integration of differential equations, but is mainly known on account of the *Goldbach conjecture*[241]: in a letter to **L. Euler** (1742) he claimed: (1) Each *even* positive integer $n > 2$ is expressible as a sum of two primes. (2) Each positive integer greater than 2 can be represented as a sum of three primes.

Goldbach was born in Königsberg, Prussia. He became a professor of mathematics at St. Petersburg (1725). In 1728 he became a tutor to Tzar Peter II, and from 1742 on served as a staff member of the Russian Ministry of Foreign Affairs.

[241] The first conjecture was found valid up to $n = 100,000$, but no definitive proof has been found. In 1937, **Ivan Matveyevich Vinogradov** (1891–1983, Russia) gave a partial proof of the second conjecture, restricting n to be *a sufficiently large odd number* ($\geq 3^{3^{15}}$).

1742 CE **Benjamin Robins** (1707–1751, England). Military engineer. Laid the groundwork for modern ballistic theory. Invented the *ballistic pendulum*[242], first described in his *New Principles of Gunnery* (1742).

Artillery men of 18^{th} century endeavored to improve their cannon-firing with little success. Robins noted that one of the causes of imperfection was the deflection of the bullet's path due to *friction against the bore of the gun*. He suggested remedying this by *scoring the bore longitudinally*.

Euler rejected both Robin's *observations* and his solution. It was more than a century before they were recognized as fully justified[243].

1742 CE **Anders Celsius** (1701–1744, Sweden). Astronomer. Described the centigrade thermometer in a paper read before the Swedish Academy of Sciences. Born in Uppsala. Occupied the chair of astronomy in the university of his native town (1730–1744), but traveled during 1732 and some subsequent years in Germany, Italy and France. In Paris he advocated the measurement

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[242] *Ballistic pendulum*: A device used to measure the velocity of such projectiles as bullets, arrows, and darts by applying the *momentum principle*. It consists of a rather massive block of wood that is suspended by parallel cords and is initially hanging at rest. A test projectile (e.g., a bullet) is fired horizontally into the block, which is thick enough to bring the bullet to rest, embedded inside it (inelastic collision). The block and embedded bullet swing up to a maximum deviation h. From the known masses and h, the final velocity of the bullet is $v = \frac{m+M}{m}\sqrt{2gh}$. Because h is generally small and difficult to measure, this result is expressed in terms of x_m (the maximum horizontal displacement) and L (the length of the pendulum chord). Thus, for $x_m^2 >> h^2$ we find $h \cong \frac{x_m^2}{2L}$, $v = \frac{m+M}{m}x_m\sqrt{\frac{g}{L}}$. Note that during the stopping time of the bullet, momentum is conserved, but kinetic energy is *not*, whereas later, as the pendulum begins to swing, energy is conserved, but the momentum of the pendulum of the block changes due to the unbalanced forces that then begin to act.
If $m = 10$ g, $M = 3$ kg, $x_m = 25$ cm, $L = 1$ m, calculations yield $v = 235$ m/sec.
Momentum conservation allows us to obtain a result for the velocity of the bullet even though the *force* exerted on the bullet by the block during the stopping time is extremely complicated (even unknowable). To obtain the result from the time-developed equations of motion, using Newton's second law, would be exceedingly difficult (even impossible).

[243] **Euler**, like most of his contemporaries, adopted the wrong philosophy of explaining science rather then observing it. He thus made an impressive blunder which halted the progress of ballistics for a hundred years. There are innumerable examples of this type throughout the history of science.

of an arc of the meridian in Lapland, and took part, in 1736, in the expedition organized for that purpose by the French Academy.

1742–1747 CE **Jean Le Rond d'Alembert** (1717–1783, France). Mathematician, physicist and man of letters. Pioneer in the study of partial differential equations and their application in physics. Studied the equilibrium and motion of fluids, hydrodynamics, mechanics of rigid bodies, the 3-body problem in astronomy and atmospheric circulation. In science he is remembered for his four contributions:

(1) *d'Alembert's principle* (1742): by introducing the concept of 'force of inertia', which is created by the body's own motion, it is possible to reduce problems of motion to problems of equilibrium in the body's co-moving frame of reference. d'Alembert went further to generalize the *principle of virtual work* to all mechanical systems, thus furnishing a bridge between the Newtonian formulation of the laws of mechanics and the later Lagrangian formulation.

The first veiled formulation of the *principle of virtual work* is contained in the *Physics* of **Aristotle** (384–322 BCE). However, he used *virtual velocities* rather than *virtual displacements*, and this is the form in which the principle was used up to the 19^{th} century. Aristotle derived the law of the lever from his principle and **Stevinus** (1548–1620) used it to deduce the equilibrium of pulleys. **Galileo** (1564–1642) improved the formulation of Aristotle's principle by recognizing that it is not the velocity, but rather the velocity in the direction of the force which counts.

His method amounts to the recognition of the "*work*" as the "product of the force and the displacement in the direction of the force". He applied the principle of virtual work to the equilibrium of a body on an inclined plane, and showed how his principle gives the same result that Stevinus found on the basis of the energy principle. **Johann Bernoulli** (1667–1748) was first to formulate the principle of virtual work as a general principle of statics with which problems of equilibrium could be solved (1717).

The principle states: "If a system of n material points $A_1, \dots, A_n$ is without friction, then the necessary and sufficient condition for the equilibrium of the acting forces $\mathbf{F}_1, \dots, \mathbf{F}_n$, is that to every virtual displacement $\delta\mathbf{r}_1, \dots, \delta\mathbf{r}_n$ the inequality $\sum_{j=1}^{n} \mathbf{F}_j \cdot \delta\mathbf{r}_j \leq 0$ holds for the virtual work of the acting forces" [virtual displacements = infinitesimal displacements *possible* at a point A]. The importance of the principle consists in the fact that it gives a condition of equilibrium of the acting forces without the aid of reactions. In the d'Alembertian formulation the principle of virtual work assumes the form: $\sum_{j=1}^{n}(\mathbf{F}_j - m_j\ddot{\mathbf{r}}_j) \cdot \delta\mathbf{r}_j = 0$.

(2) Discovered the scalar wave equation of a vibrating string and found its general solution (*d'Alembert's solution*) (1747).

(3) Calculated the perturbation of the planets on the orbits of the moon and the earth. This theory was previously developed by Newton from a geometrical standpoint. d'Alembert and Clairaut, each in his own way, formulated the result in the form of series solutions of differential equations (1747–1754).

In 1754 d'Alembert developed the mathematical theory of the perturbing effects of the planets (mainly Jupiter) on the motions of earth. He showed that because of these perturbations, the luni-solar precessional period of 26,000 years [known to **Hipparchos** (120 BCE), and shown by Newton (1687) to be caused by the gravitational pull that the sun and the moon exert on the earth's equatorial bulge], must be modified to include *precession of the earth's perihelion* ($47''$ per century) to yield a *general precession of the equinoxes* with a period of 22,000 years.

This must be understood as follows: In the absence of planetary perturbations, the plane of the earth's equator turns in a retrograde direction about the normal to the ecliptic, with the latter regarded as *fixed* relative to the fixed stars. The rate of this nearly uniform rotation of the earth's spin axis is about $50''$ per year, making a full revolution in $\frac{60\times 60\times 360}{50} = 26,000$ years. During this motion, the obliquity of the earth's axis w.r.t. the ecliptic is unchanged, only the orientation of the polar axis relative to the fixed stars varies [however, the lunar forced *nutation* (Bradley, 1737) must, of course, be considered as well].

When planetary perturbations are introduced, the elliptic orbit, as a whole, executes a slow rotation, known as the *precession of the perihelion*. Consequently, as seen from earth, the curve described by the North Pole, is not quite a perfect circle, and it does not close back on itself during 26,000 years but rather sooner, in 22,000 years.

(4) Was first to notice (1754) the unsatisfactory state of the foundation of analysis and see that a theory of *limits* is needed. The actual process of banishing intuitionism and formalism from analysis started in 1797 with **Lagrange**. This led in the 19^{th} century to the *arithmetization of analysis*.

His main lifework was his collaboration with Diderot in preparing the famous *Encyclopédie*, which played a major role in the French enlightenment by emphasizing science and literature and attacking the forces of reaction in church and state.

d'Alembert was a foundling: having been abandoned near the church of St. Jean le Rond, Paris, he was discovered on the 17^{th} of November, 1917.

It afterwards became known that he was the illegitimate son of a Parisian notable. He was called Jean le Rond after the church near which he was found; the surname d'Alembert was added by himself at a later period. In 1730 he entered the Mazarin College, where his exceptional talents were soon noted. His knowledge of higher mathematics was acquired by his own unaided efforts after he had left college.

On leaving college he returned to the house of his foster mother, where he continued to live for 30 years. He studied law and medicine but in 1740 resolved to fully devote his time to mathematics. His association with Diderot in the preparation of the *Dictionaire Encyclopédique* led him to take a somewhat wider range than that to which he had previously confined himself (1754). Apart from contributing mathematical articles to the Encyclopedia, he wrote literary and philosophical works which extended his reputation but also exposed him to criticism and controversy. d'Alembert was interested in music both as a science and as an art. His fame spread rapidly throughout Europe: Frederick the Great, Catherine of Russia, and Pope Benedict XIV each invited him to live in their respective country on lucrative salaries, but he preferred to stick to the quiet and frugal life dictated by his simple tastes.

His latter years were saddened by circumstances connected with a romantic attachment to a noted consort of literary men and savants. On her part there seems to have been nothing more than a warm friendship, but his feelings toward her were of a stronger kind and her death in 1776 deeply affected him.

The chief features of d'Alembert's character were benevolence, simplicity and independence. Though his income was never large, and during the greater part of his life was very meager, he continued to find means to support his foster mother in her old age, to educate the children of his first teacher and to help various deserving students during their college career. His conversation was a singular mixture of feigned malice, goodness of heart and delicacy of wit.

1743–1750 CE **Jean Antoine Nollet** (1700–1770, France). Physicist. Invented the first *electroscope* (1747–1750). Discovered and described the phenomenon of *osmosis* and *osmotic pressure* (1748).

Osmosis is derived from a Greek word, meaning to *push*. A membrane partition separates pure water, say, from a weak solution of a substance (solute) in water (solvent). The membrane is such that the molecules of the water can pass through it, but not those of the solute (a semi-permeable membrane). After some time the level of the pure solvent (water) becomes *lower* than the level of the solution. The process of penetration of a solvent through a semi-permeable membrane is called *osmosis*. The pressure difference created between the two sides of the membrane is called *osmotic pressure*. The new state of equilibrium can be understood either as due to the entropy gain attendant to the further dilution of the solute (*thermodynamic* argument), or, alternatively, as an impairment of the rate of effusion of water molecules from the *solution* to the *pure-water* side, caused by the presence of solute molecules in the former.

The exact microscopic mechanism of this impairment is not completely understood. Apparently, a complex interaction between the molecules and the semi-permeable membrane is at work. The magnitude of the osmotic pressure depends on the concentration of the solute molecules; the greater the concentration, the higher the osmotic pressure difference. On the other hand, for a given *weight* of solute, the lower the osmotic pressure, the higher the *molecular weight*. This enables one to determine molecular weights, of proteins say, through osmotic pressure measurements.

Nollet was born near Noyon (Oise) to a peasant family. His parents destined him for the clergy, but after finishing his theological studies in Paris, he came under the influence of **Réaumur**, and began the study of the exact sciences.

In the Church he ultimately attained the rank of abbé, but his tastes lay in the direction of experimental physics. In 1734 he was admitted as member of the London Royal Society, and in 1739 he entered the Academy of Sciences at Paris. In 1753 he was appointed to the newly instituted chair of experimental physics in the College de Navarre. He discovered osmosis while experimenting with water diffusing into sugar solution from which it is separated by an animal membrane.

1743–1750 CE **Thomas Simpson** (1710–1761, England). An able self-taught mathematician. Was active in perfecting trigonometry as a science. His name is preserved in the so-called *Simpson's rule* published in his *Mathematical Dissertations on a Variety of Physical and Analytical Subjects* (1743) — a rule for approximate quadrature using parabolic arcs (this result appeared in somewhat different form in 1668 in the *Exeritationes Geometricae* of **James Gregory**).

Simpson's father was a weaver, and, intending to bring his son up into his own business, took little care of the boy's education. Young Simpson was so eager for knowledge that he neglected his weaving, and in consequence of a quarrel was forced to leave his father's house. Until 1743 his life was rather turbulent; he managed to sustain himself through a gamut of odd jobs as fortune-teller, oracle, astrologer and private teacher. After publishing 5 books on mathematics he was finally appointed professor of mathematics in the Royal Military Academy at Woolwich, and in 1745 he was admitted as fellow of the Royal Society of London.

1744 CE **Jean Philippe Löys de Chéseaux** (1718–1751, Switzerland). Astronomer. 'Solved' the riddle of the dark night sky by assuming that starlight is slowly *absorbed* while traveling across the immense gulfs of interstellar space in a boundless universe. In his essay "*On the intensity of light, its propagation through the ether, and the distance of the fixed stars*" (1744) he wrote:

"*The enormous difference between this conclusion and experience demonstrates either that the sphere of fixed stars is not infinite but actually incomparably smaller than the finite extension I have supposed for it, or that the intensity of light decreases faster than the inverse square of distance. This latter supposition is quite plausible, it requires only that starry space is filled with a fluid capable of intercepting light very slightly*".

Chéseaux' calculation, in the updated form given by **Lord Kelvin** (1901), is as follows: we assume that all stars are sun-like, of radius a, and uniformly distributed with density n per unit volume. The number of stars in a shell of radius $q \gg a$ and thickness dq approximately equals $4\pi nq^2dq$, and the *sum of their uneclipsed projected areas* is this number multiplied by πa^2, thus giving $4\pi^2na^2q^2dq$; If we divide this area of stellar disks by the area $4\pi q^2$ of the shell, we find that *the fraction of the sky covered by the stars* is of the shell $n\pi a^2dq = n\sigma dq$, where $\sigma = \pi a^2$ denote the geometric cross-section of a star.

We now integrate out to distance r and find that the fraction of the sky covered is $\alpha = n\sigma r = \frac{r}{\lambda}$, where $\lambda = \frac{1}{n\sigma}$ is the *mean free path* of a light ray traced backwards from a point on earth, terminating on the surface of the star which emitted it. (The mean free path — a term commonly used in the kinetic theory of gases — is the average distance a particle travels between collisions.) When $r = \lambda = \frac{1}{n\sigma}$ (or $\alpha = 1$), the whole sky is covered with stars. The corresponding distance is known as the *background limit*. If $V = \frac{1}{n}$ is the average volume occupied by one star, the background limit in a star-filled universe is $\lambda = \frac{V}{\sigma}$.

Let $N = 4\pi n\frac{r^3}{3}$ stand for the number of stars out to distance r. The number of uniformly scattered stars needed to cover the entire sky is obtained by inserting $r = \lambda = \frac{V}{\sigma}$ in the above equation. Then $N = \frac{4\pi V^2}{3\sigma^3}$. Even in an infinite universe containing an infinite number of stars, we see only out to a *finite distance* and a *finite number of stars*.

Assuming then for simplicity that all stars are similar to the sun in size and luminosity, we take $\sigma = 1.5 \times 10^{12}$ km^2. Noticing that there are about 10 stars within a distance of 10 light-years from the sun, we obtain $V \approx 100$ (light-years)^3. Consequently[244] $\lambda \approx 6 \times 10^{15}$ light-years, and $N \sim 10^{46}$.

It was the immensity of the background limit that prompted Chéseaux to think that absorption in space, even the slightest, would veil the most distant stars and create the observed dark night sky. Indeed, the above toy model can easily be made to incorporate the effects of both *geometric overlap* by stars of intermediate shells and *absorption*. To this end we multiply our former expression for the fraction of the sky covered by stars in the shell first by $e^{-q/\lambda}$, where $\lambda = \frac{1}{\pi n a^2}$, and then by $e^{-q/\mu}$, where μ is the absorption mean free path. Hence, the fraction of the sky covered by stars in the shell is $\left\{\frac{dq}{\lambda}\right\}e^{-q(\frac{1}{\lambda}+\frac{1}{\mu})}$. Integrating from $q = 0$ to $q = r$, we find $\alpha = \frac{\mu}{\lambda+\mu}\left[1 - e^{(\frac{1}{\lambda}+\frac{1}{\mu})}\right]$, where α is the fraction of the sky covered by unobscured stellar surface of effectively-solar apparent brightness. As $r \to \infty$ in an unlimited and uniform universe, the fraction of the sky covered by stars becomes $\alpha = \frac{\mu}{\lambda+\mu}$. If the *absorption limit* μ is much less than the overlap limit λ, $\alpha \approx \frac{\mu}{\lambda} \ll 1$, and most stars are obscured from view.

Because a system of concentric shells may be constructed about *any* point in space, always yielding the *same* result, we conclude that observers at all places will perceive the sky as consisting of one continuous, though dimmed, stellar surface.

Chéseaux was born in the Swiss village of Chéseaux near Lausanne, the son of a landowner of modest wealth. Educated by his grandfather, the mathematician **Jean-Pierre de Crousaz**, he developed an interest in astronomy while a youth and constructed his own observatory. At age 17 he wrote papers on the physics of collisions, retardation of cannonballs by air resistance, and sound propagation. Never very robust, he died while on a visit to Paris, at the age of 33.

Chéseaux' name is associated with the magnificent comet of 1744, one of the finest of the 18^{th} century. Though not discovered by him, this comet is often referred to as Chéseaux' because he computed its orbit and ephemeris and described its impressive, multiple tails.

ca 1745 CE During the *Silesian wars* between Prussia (Frederick the Great) and Austria (Maria Theresa), the latter saw a disgrace in her loss of Silesia. Deeply frustrated, the Austrian empress blamed her military defeat on the *Jews of Prague*(!) and on Dec. 18, 1744 she ordered their expulsion

[244] Note, however, that in this estimate Chéseaux uses the average star density in our galaxy — which is far higher than the average for the Universe as a whole.

from the country within six weeks. Thus, in the blistering cold of a February day, thousands of people of all ages were clogging the highways and the roadside was lined with the dead and the sick. Eight centuries of continuous Jewish community life and of dynamic intellectual striving were wiped out. It is indeed odd that this most dramatic expulsion should have taken place not during the Dark Ages but in the year 1745 when the Industrial Age and the modern spirit had already made their appearance in Europe.

Three years later the Imperial Treasury in Vienna began to feel acutely the financial loss resulting from the expulsion. This made the empress regret her excessive resentment against the Jews. Consequently, the Imperial Military Council denounced their own previous charges against the Jews and ordered their immediate return to Prague.

ca 1745 CE **Hugh Jones** (1692–1760, North America[245]). Mathematician. An ardent advocate of the octary (radix eight) system (which is used today in connection with certain electronic computers). Jones, a professor of mathematics at the College of William and Mary, was a reformer who asserted that the base eight makes fractional work simpler because octary fractions are just a matter of repeated halving. Moreover, computation is facilitated because the radix eight is a perfect cube, and four, which is one-half of the radix, is a perfect square.

1745–1746 CE **Ewald Georg von Kleist** (1700–1748, Germany) and **Pieter van Musschenbrock** (1692–1761, Holland) independently invented the *Leyden Jar*, an early version of an *electrical capacitor*.

Von Kleist was a German ecclesiastic and scientist. Dean of the Cathedral of Kamin, Pomerania. He discovered it on 04 Nov, 1745. Van Musschenbrock was a Dutch mathematician, physician and physicist. Von Kleist was member of a notable Leiden family[246] of instrument makers (air pumps, microscopes, and telescopes). He was a professor at Duisburg (1719–1723), Utrecht (1723–1740), Leiden (1740–1761). The jar device accumulates electrical charges produced by a static machine. When voltage reaches a critical value, there occurs a discharge through the air-gap. Theirs was the first working model of an electrical storage device.

1745–1785 CE **George Louis Leclerc de Buffon** (1707–1788, France). Naturalist and mathematician. Although not a profound original investigator,

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[245] At this time, a British colony.

[246] To this illustrious family of scientists, soldiers and poets belong also: **Ewald Christian von Kleist** (1715–1759), poet and soldier; **Heinrich Wilhelm von Kleist** (1777–1811), a great dramatist, poet and prose writer; **Paul Ludwig Ewald von Kleist** (1881–1954), army general in WWII.

he is remembered today due to his initiation of some geometrical aspects of probability[247]: he showed how to get *experimental* estimates of π by tossing a needle across a grid a large number of times (1777).

Previously, in 1745, he advanced the first of the so-called *catastrophic theories* which envision a solitary sun disrupted by some singular cataclysmic event. Buffon suggested that a massive body (comet) passed so close to the sun that its gravitational pull drew material out of it, which then condensed to form the planets (1785). He also tried, for the first time, to determine the age of the earth (his result: 74,832 years!).

He regarded (1749) spermatozoa as "living organic molecules" which multiply in the semen.

Buffon was born at Montbard (Côte d'Or). He studied law and mathematics at the Jesuit College at Dijon. Being a rich nobleman he led a life of a scientist-at-large, occupying himself with whatever he liked. His son, an army officer, died by the guillotine at the age of thirty in 1793.

Buffon was a member of all the learned societies of Europe. He was known during his time chiefly due to his great work (44 quarto volumes) on natural history, the publication of which extended over 50 years.

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247 *Buffon's Needle problem*: A table of infinite expanse has inscribed on it a set of parallel lines spaced a units apart. A needle of length $l < a$ is twirled and tossed on the table. What is the probability that when it comes to rest it crosses a line?

What matters is the needle's *angle* θ with the horizontal, and the distance x of the needle's-center from its nearest parallel. Since the needle's-center is equally likely to fall anywhere between the parallels, then for a fixed θ, the chance that the line crosses on of the parallels is $\frac{2x}{2a}$, because the line crosses a parallel if the center falls within x units of either parallel. On account of the twirling, the angle θ might be thought of as uniformly distributed from 0 to $\frac{\pi}{2}$ radians, because crossing that happens for angle θ also happens for angle $(\pi - \theta)$. All we need then is the mean value of $\frac{x}{a}$, or, since $x = l\cos\theta$, the mean value of $\left(\frac{l}{a}\right)\cos\theta$, which is equal to $\frac{l/a}{\pi/2}\int_0^{\pi/2}\cos\theta d\theta = \frac{2l}{\pi a}$. This is indeed the desired result. In the particular case in which $2l = a$, the probability of intersection is $1/\pi$. When the needle is tossed N times onto the ruled plane and on n of these occasions the needle intersects on of the lines, the *Law of Large Numbers* dictates $\frac{n}{N} \approx \frac{1}{\pi}$.

From Divination through the Buffon Needle to Monte Carlo Methods

*Throughout history simple games of chance have been used to communicate with, or seek guidance from, the supernatural. In some primitive societies a person's innocence or guilt was determined by drawing or casting lots [Joshua **7**, 14–18; **18**, 10; Jonah **1**, 7; I.Chronicles **26**, 13–14]. In ancient Greece and Rome oracles based predictions on casts of* astragali *(forerunners of modern dice), and in the Bible there is a reference to an occasion where the direction in which the army was to proceed was determined by shaking arrows in a quiver and observing the direction in which the first one fell [Ezekiel **21**, 26–28: "For the king of Babylon stood at the parting of the way, at the head of the two ways, to use divination"; also in the Talmud; Gittin **5**, 56 p. 1].*

*These early crude attempts to generate random (and sometimes 'rigged'!) events, were later replaced by rolling dice or flipping coins. Yet the harnessing of a gambling device in solving a problem of pure mathematics had to await the year 1777 CE, when the French naturalist **George de Buffon** showed that if a very fine needle of length l is thrown at random on a board ruled with equidistant parallel lines, the probability w that the needle intersect one of the lines is $\frac{2l}{\pi a}$, where $a > l$ is the distance between the parallel lines.*

*The problem and its solution were largely forgotten for the next 35 years, until the great French mathematician **Pierre Simon de Laplace** (1812) called attention to it and gave it a new twist. Writing Buffon's result in the form $\pi = \frac{2l}{aw}$, Laplace realized that it provides a new method of calculating π!*

Indeed, the remarkable thing about this result is that it involves the constant $\pi = 3.1415926\ldots$, which can be thus estimated *by actually tossing a needle on a board suitably ruled with parallel lines. Early experiments of this kind (1850) gave the probability $w = 0.5064$, based on 5000 throws with a needle 36 mm long and a distance of 45 mm between the parallels. This yielded $\pi \approx 3.151496$. Note that a* probabilistic approach *has been used here to solve a* non-probabilistic *problem, very far from the ancient divinations, where other non-probabilistic problems were solved by interpreting chance happenings as divine intent.*

It is not difficult to calculate the probability of obtaining π correct to K decimals in N throws. The result of such calculation show that this method is very inefficient as far as the numerical computation of π is concerned. Yet, Laplace had discovered a powerful method of computation that did not come into its own until the advent of electronic computers. The method that

Laplace proposed consists of finding a numerical value by realizing a random event many times and observing the outcomes experimentally.

A similar procedure, though non-geometrical, was devised for estimating e. It is based on the observation that if $2K$ uniformly distributed numbers x_i are independently drawn in a sequence from a random source, and assuming each draw value to be uniformly distributed, then the probability that they are all in ascending order $x_1, x_2, x_3, \dots, x_{2K}$ is $\frac{1}{(2K)!}$. This is so because $2K$ numbers can have $(2K)!$ possible orderings and only one ordering will be an ascending one. The probability that a sequence of trials yielding an increasing sequence of x_i's will fail on the odd trial $2K+1$ is the difference $\frac{1}{(2K)!} - \frac{1}{(2K+1)!}$.

Thus, the total probability that a sequence of drawings of random numbers from an equilikely source will produce a rising sequence that ends with an even number of numbers is

$$\sum_{k=1}^{\infty}\left[\frac{1}{(2k)!} - \frac{1}{(2k+1)!}\right] = \sum_{k=0}^{\infty}\left[\frac{1}{(2k)!} - \frac{1}{(2k+1)!}\right] = \frac{1}{e}$$

An experiment using 252 runs gave $\frac{1}{e} = 0.381$, a 3.5 percent error.

This result serves as a basis for a winning strategy of a well-known game in which N cards, assigned with random numbers, are uncovered one at a time by a player. The player must announce his decision of whether or not an uncovered card bears the largest number of the lot. The optimal strategy is to delay decision until after $\left(\frac{100}{e}\right)$ percent of the cards have been uncovered.

In recent decades, the practice of chance happenings [namely, statistical experiments] has become quite respectable through the use of the so-called Monte Carlo methods[248]*; we should add, though, that there is no longer any question of divine intervention. Monte Carlo methods are essentially simulation techniques, and they enable us to study in the classroom or in the laboratory random processes which would otherwise be difficult to observe.*

They have been used, for example, to study the effect of changes in an assembly procedure without actually having to put the changes into operation, the effects of pollution without having to induce them in our environment, as well as complex series of chemical and nuclear reactions (before an engine or reactor is actually built, or to check its design and performance). Very often, the use of Monte Carlo methods eliminates the cost of building and operating experiments; it is thus used in the study of collisions of photons with electrons, the scattering of neutrons, evolution of biological populations, and other complicated phenomena. The chance factors in all these processes

[248] The term "Monte Carlo" was coined by **N. Metropolis** and **S. Ulam** in 1949 [J. Amer. Statistical Assoc. **44**, 241–335].

are simulated by means of appropriate gambling devices, which, most of the time, are themselves simulated by means of electronic computers.

Monte Carlo methods solve certain types of problems through the use of random or pseudo-random numbers[249]*, whose values depend on the outcome of a random or pseudo-random event. The former may be generated by using the outcomes of random physical processes such as throwing of dice, spinning a roulette wheel, scintillation in a Geiger-Müller counter, noise generated by electrical transmission systems, etc.; the latter can be generated via deterministic numerical algorithms.*

Monte Carlo methods offer two types of applications:

- *Sampling: deducing properties of a large set of elements by studying only a small, random subset. Thus an average value of $f(x)$ over an interval may be estimated from its average over a finite, random subset of points in the interval. Since the average of $f(x)$ is actually an integral, this amounts to a Monte Carlo method for approximate integration.*
- *Simulation: providing arithmetical imitations of "real" phenomena. In a broad sense this describes the general idea of applied mathematics. The classical example is the simulation of neutron's motions and absorptions in a nuclear reactor, its zigzag path being imitated by a type of arithmetical random walk.*

Of the mathematical problems to which the Monte Carlo method has been applied, one may mention: solving systems of linear equations, matrix inversion, evaluating multiple integrals, solving the Dirichlet problem, and solving functional equations of a variety of types.

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[249] *Random numbers*, in the context of computations and communication *pseudo-random* and *not* numbers generated by a random, analog physical process (such as the flip of a coin or the spin of a wheel). Instead they are numbers generated by a completely deterministic arithmetical process, the resulting set of numbers having various statistical properties which together approximate *randomness*. A typical algorithm for generating pseudorandom numbers is

$$x_{n+1} = rx_n (\text{mod } N).$$

An initial element x_0 is repeatedly multiplied by r, each product being reduced modulo N. With decimal computers $x_{n+1} = 7^9 x_n (\text{mod } 10^5)$, $x_0 = 1$ is quite satisfactory, while with binary computers a good choice is $x_{n+1} = (8t - 3)x_n(\text{mod } 2^5)$, $x_0 = 1$, with t being some large number. *Truly* random computer bits can be produced by means of e.g. amplified and discretized electronic noise, or a Geiger counter monitoring nuclear decays.

Suppose, for example, that it is required to evaluate $I = \int_0^1 f(x)dx$, *where* $f(x)$ *is assumed to be bounded above and below so that it can be transformed to satisfy the condition* $0 \leq f(x) \leq 1$. *Monte Carlo integration then proceeds as follows: random points are chosen within the unit square. The integral* I *is then estimated as the fraction of random points that fall below the curve* $f(x)$. *The number of points must be sufficiently large and their uniform distribution must be truly random in both the* x- *and* y-*dimensions, so that one deals with* n *mutually independent trials. Uniformly distributed random numbers make it possible to break off the procedure at a value* n *for which the successive estimates differ by less than a prescribed limit of accuracy. If* $(k-1)$ *is the number of counting steps executed so far and* I_{k-1} *the resulting estimate of* I, *then the recursive counting scheme*

$$I_k = I_{k-1} + (\xi_k - I_{k-1})/k = [(k-1)I_{k-1} + \xi_k]/k$$

has proved useful, where $\xi_k = 1$ *if the* k^{th} *point falls in the region of* $f(x)$, *and* $\xi_k = 0$ *otherwise.*

If only uniformly distributed random numbers x_i *in the one-dimensional interval* $[0,1]$ *are chosen for the argument, and* $f(x_i)$ *is calculated for each, then the statistical mean* $M[f(x)]$, *multiplied by the width of the interval 1, is an estimate for the required integral. Because the arithmetic mean is an effective estimate for* $M[f(x)]$, *one obtains*

$$\frac{1}{n}\sum f(x_i) \approx \int_0^1 f(x)dx.$$

Here the deterministic recursive formula

$$I_k = I_{k-1} + [f(x_k) - I_{k-1}]/k = [I_{k-1}(k-1) + f(x_k)]/k$$

has proved suitable (i.e. ξ_k *was replaced by its expectation* $f(x_k)$*).*

This method can, in principle, be extended to multidimensional volumes V. *One picks* N *points, uniformly randomly distributed in* V. *Call them* $x_1, x_2, \ldots, x_N$. *Then the basic theorem of a Monte Carlo integration estimates the integral of a function* f *over the multidimensional volume,*

$$\int f dV \approx V\langle f\rangle \pm V\sqrt{\frac{\langle f^2\rangle - \langle f\rangle^2}{N}}\ .$$

Here $\langle\ \rangle$ *denote taking the arithmetic mean over the* N *sample points,*

$$\langle f\rangle = \frac{1}{N}\sum_{i=1}^{N} f(x_i) \quad ; \qquad \langle f^2\rangle = \frac{1}{N}\sum_{i=1}^{N} f^2(x_i)\ .$$

The "plus-or-minus" form is a one standard deviation error estimate for the integral, which is a rough indication of the probable error range.

1747–1753 CE **James Lind** (1716–1794, Scotland). Surgeon in the Royal Navy. Pioneer of preventive medicine and nutrition. Founder of naval hygiene and promoter of the use of citrus fruits and fresh vegetables to prevent and cure *scurvy*. The discovery was ignored until forty years after he discovered the cure.

1747–1760 CE **Johann Tobias Mayer** (1723–1762, Germany). Mathematician, physicist and astronomer. Made a careful investigation of the *libration of the moon*[250] (1747–1748), and published tables on the positions of the moon which allow determinations of *longitude at sea* (1753). With the tables in hand, a mariner could obtain his longitude through tedious calculations with an accuracy of half a degree. The method was that suggested by **Peter Apian** (1524).

Mayer was born at Marbach, in Würtemburg, and brought up at Esslingen in poor circumstances. A self-taught mathematician, he entered (1746) to work in a cartographic establishment at Nuremberg. Here he introduced many improvements in map-making and gained a scientific reputation which led (1751) to his election to the chair of mathematics in the University of Göttingen. In 1754 he became superintendent of the astronomical observatory of that university. He left behind him an essay on color, in which 3 primary colors are recognized; a memoir on the proper motions of 80 stars; papers on atmospheric refraction (1755), on the motion of Mars as affected by the perturbations of Jupiter and the earth (1756), and on terrestrial magnetism (1760), where he made the first definite attempt to establish a mathematical theory of magnetic action.

The British Government paid his widow a grant of 3000 Sterling for the lunar tables which he submitted to them in 1755.

1747–1784 CE **Benjamin Franklin** (1706–1790, U.S.A.). Scientist, inventor, statesman and diplomat. One of the first men to experiment with electricity (1747–1752). Invented the *lightning rod* (1749) and created such

[250] The moon slightly *wobbles* as it moves along its orbit. This wobbling, called *libration*, permits us to view 59 percent of its surface.

electrical terms as *armature*, *condenser*, and *battery*. Invented *bifocal eyeglasses* (1784) which allowed both reading and distant lenses to be set in a single frame. Published the first chart of the North Atlantic *Gulf Stream* (1770), based on his own observations.

He was the first to relate the severe Northern Hemisphere winter of 1783/4 to the eruption of the volcano Laki in Iceland in the summer of 1783, speculating that solar heating of the earth is reduced due to the ash and other particles injected by the volcano into the atmosphere. Franklin was instrumental in establishing the *American Philosophical Society*, the first scientific society in the United States (proposed in 1743, established in 1769 at Philadelphia).

Franklin was born in Boston, the 15^{th} child and youngest son in a family of 17 children. His formal schooling ended at the age of 10, but he continued to educate himself throughout his life. From 1723 to 1730 Franklin worked for various printers in Philadelphia and London, England. He became the owner of a print shop in 1730, and began publishing *The Pennsylvania Gazette*, writing much of the material for this newspaper himself. His name gradually became known throughout the colonies. Seeking to improve the poor colonial postal service he became Philadelphia's postmaster in 1737, and in 1753 he became deputy postmaster general for all the colonies.

He started his electrical experiments in 1747, with the discovery that a pointed conductor can draw off electric charge from a charged body. In 1751 he described electricity as a single fluid and distinguished between positive (excess) and negative (deficiency) electricity. He also showed that electricity can magnetize and demagnetize iron needles. In June 1752, Franklin performed his famous kite experiment, showing that lighting is a form of electricity, similar to the discharge from a Leyden jar. This was the first recorded experiment on atmospheric electricity[251], and the first human endeavor to harness this natural source of power.

Franklin became the first scientist to study the movement of the Gulf-Stream. He spent much time charting its course, and recording its temperature, speed, and depth. He hoped that use of his chart would help ships to avoid the current and to speed the mail in crossing from Great Britain to America[252].

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[251] In 1912 **Victor Franz Hess** (1883–1964, Austria) discovered, through manned balloon flights, the agent of this phenomenon — cosmic radiation. The bombardment of the earth's surface by massive particles from outer space was named '*cosmic rays*' by **Millikan** in 1925.

[252] *Gulf Stream*: a current of warm waters that flows from the Straits of Florida in a north-easterly direction across the Atlantic toward Europe. It flows as fast as 220 km/day and its rate of flow, measured in volume per second, is about 1000

Franklin was a fierce supporter of America's struggle for independence: he played an important part in drafting the declaration of independence (1776) and the United States constitution (1787). During 1776–1785, he served as ambassador to France.

1748–1750 CE **Charles-Louis de Secondat**, **Baron de la Brède et de Montesquieu** (1689–1755, France). Political and social philosopher. His *L'Esprit des lois* (1748) [*The Spirit of the Laws*] is a seminal contribution to political theory which profoundly influenced political thought in Europe and America.

His family belonged to the lesser nobility of Guyenne, with a distinguished tradition of legal service in Bordeaux. He accordingly trained for the law at Bordeaux and Paris, where he was in touch with some of the most emancipated minds. A literary career (1721–1726) led to his election (1727) to the French Academy and in 1728 he began a 3-year European tour which took him to Italy and England and greatly nourished his interest in political and social institutions. He lived in England (1729–1731) and came to admire the English political system.

In his magnum opus he analyzed human institutions and the laws which embody them in terms of their dependence upon forms of government, upon the external relations of the state, upon national temperament, climatic and economic factors. The most influential part of the work, however, has been his analysis of the conditions which create political liberty, and his advocacy (based upon his readings of the English constitution) of a system of equilibrium based upon *a separation of the legislative, executive and judicial powers of the state.*

Montesquieu believed that laws underline all things – human, natural, and divine. One of philosophy's major tasks was to discover theses laws. Man was

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times greater than that of the Mississippi River. It is the second largest ocean current.

The Gulf Stream is partly responsible for the warm southwesterly winds that make the climate of Great Britain and Northwestern Europe much warmer than parts of North America that lie equally far north. These winds pick up heat and moisture from the Sargasso Sea and the Gulf Stream. The Gulf Stream is also an aid to shipping. Many large oil tankers and ore carriers, traveling from South America to Atlantic coastal harbors, attempt to "ride" the current on their northbound journey.

The stream is about 80 km wide and 910 meters deep. It is formed in the Caribbean from the union of the North and South Equatorial currents. These currents, in turn, are generated by trade winds, as suggested by **Benjamin Franklin** already in 1770.

difficult to study because the laws governing his nature were highly complex. Yet Montesquieu believed that these laws could be discovered empirically. Knowledge of the laws would ease the ills of society and improve human life. He maintained that liberty and respect for properly constituted law could exist together.

1748–1768 CE **Johann Joachim Winckelmann** (1717–1768, Germany). One of the fathers of modern archeology and art historian who set the foundation of our modern views on the arts. His writings reawakened the taste for classical art and was responsible for generating the neoclassical movement in the arts.

Born at Stendal in Brandenburg, the son of a poor shoemaker. As a child, Johann was influenced by the ancient Greek culture, especially Homer. He studied theology and medicine at Halle and Jena Universities. In 1748 he discovered the world of ancient Greek art while serving as a librarian near Dresden. There he wrote the essay "Reflections on the Painting and Sculpture of the Greeks" (1755). This was recognized as a manifesto of the Greek ideal in education and art. His other works include "Geschichte der Kunst des Altertums" (1764, "History of the Art of Antiquity").

In 1763 he became superintendent of Roman antiquities, but soon he rose to the position of librarian at the Vatican and later became the secretary to Cardinal Albani, who had an extensive collection of classical art.

In his work, Winckelmann sets forth both the *history of Greek art* and *the principles on which it seemed to him to be based*. He also presents a glowing picture of the political, social and intellectual conditions which he believed tended to foster creative activity in ancient Greece. The fundamental idea of his theories is that the end of art is beauty, and that this end can be attained only when individual and characteristic features are strictly subordinated to the artist's general scheme.

The true artist, selecting from nature the phenomena fitted for his purpose, and combining them through the imagination, creates an ideal type marked in action by "Edle Einfalt und stille Größe" ("*noble simplicity and quiet grandeur*") — an ideal type in which normal proportions are maintained, particular parts, such as muscles and veins, not being permitted to break the harmony of the general outlines.

In the historical portion he used not only the works of art he himself had studied but the scattered notices on the subject to be found in ancient writers; and his wide knowledge and active imagination enabled him to offer many fruitful suggestions as to periods about which he had little direct information.

Many of his conclusions, based on inadequate evidence of Roman copies, have been modified or reversed by subsequent research, but the fine enthusiasm of his work, its strong and yet graceful style, and its vivid descriptions of works of art give it enduring value and interest. It marked an epoch by indicating the spirit in which the study of Greek art should be approached, and the methods by which investigators might hope to attain solid results. To Winckelmann's contemporaries it came as a revelation, *and exercised a profound influence on the best minds of the age.*

On June 8, 1768 on his way back to Rome from Germany and Austria, he was murdered *by a chance acquaintance in Trieste*, Italy, which was where he was buried.

1749–1752 CE **Frederik Hasselquist** (1722–1752, Sweden). Traveler and naturalist; The first modern researcher of the fauna and flora of the Holy Land.

Born at Törnevalla, East Gothland and studied at Uppsala under Linnaeus. On account of the frequently expressed regrets of the latter regarding the natural history of the Holy Land, Hasselquist resolved to undertake a journey to that country. He visited parts of Asia Minor, Egypt, Cyprus and the Land of Israel, making large natural history collections. But his constitution, weakened by chronic consumption, gave way under fatigues of travel, and he died near Smyrna on his way home.

His collections reached home in safety, and five years after his death his notes were published by Linnaeus under the title *Resa till Heliga Landet, 1749–1752.* It was translated into French (1762) and English (1766). Among his discoveries: the fig-wasp (*Blastophaga psenes*), St. Peter's fish (*Thilapia galilaeae*), the common jerboa (*Jaculus jaculus*). His herbal collection *Flora Palestina* (1763) includes 600 species.

The Honeycomb — or, How to Hold the Most Honey for the Least Wax

One of the most beautiful hexagonal arrays is the honeycomb constructed by bees. The walls of the main body of connected cells form regular hexagonal prisms. The bottom of each cell is shaped like a concave triangular pyramid and constructed from three equilateral rhombs. The cell walls are slightly tilted toward the rim, which prevents honey from running out before the cells are closed.

The first question that comes to mind is, why the hexagonal cross-section? After all, the bees might have built their cells with rounded walls as the bumblebees do or as they themselves build for the cradles of their queens. Or they could base their architectural style on some other geometrical configuration. However, if the cell were round or, say, octagonal or pentagonal, there would be empty spaces between them. This would not only mean a poor utilization of space; it would also compel the bees to build separate walls for all or part of each cell, and entail a great waste of material.

These difficulties are avoided by the use of triangles, squares, and hexagons. But of those three geometrical figures with equal area (and for equal-depth cells — also with equal volume) the hexagon has the smallest circumference. This means that the amount of building material required for cells of the same capacity is the least in the hexagonal construction. The geometry of the cell-bottoms and the manner in which they dovetail into each other, contributes to the stability of the comb. A comb measuring 37 *by* 22.5 *centimeters can hold two kilogram of honey. Yet in the manufacture of such a comb, the bees use only* 40 *grams of wax.*

This natural architectural marvel must have attracted the attention and excited the admiration of mathematicians from time immemorial.

The writings of **Pappos of Alexandria** *(ca 300* CE*) inform us that the ancient Greeks had already tried to explain the regularity of beehive cells by means of an optimum principle. He has left us an account of its hexagonal plan, and drew from it the conclusion that the bees were endowed with "a certain geometrical forethought... There being, then, three figures which of themselves can fill up the space around a point, viz. the triangle, the square and the hexagon, the bees have wisely selected for their structure that which contains most angles, suspecting indeed that it could hold more honey than either of the other two".*

Erasmus Bartholinus *(1669) was the first to suggest that the hypothesis of 'economy' was* not *warranted, and that the hexagonal cell was no more than* *the necessary result of equal pressures, each bee striving to make its own little circle as large as possible.*

The understanding of the particular shape of the bottom of the cell was a more difficult matter than that of its sides, and came later. ***Kepler*** *was first to deduce from the space-filling symmetry of the honeycomb that its angles must be those of the* rhombic dodecahedron; and **Swammerdam** *(1673) also recognized the same geometrical figure in the base of the cell. But Kepler's discovery passed unnoticed, and to the Italian astronomer* ***Giacomo Filippo Malardi*** *[(1665–1729), a nephew of D. Cassini; lived in Paris] goes the credit of ascertaining the shape of the rhombs and the solid angle which they bound, while watching the bees in the garden of the Paris Observatory (1712).*

He found the angles of the rhomb to be $110°$ *and* $70°$*. He later observed that the angles of the three rhombs at the base of the cell depend on the basal angles of the* 6 *trapezia which form its sides. It then occurred to him to ask what must these angles be, if those on the floor and those of the sides are equal to one another. The solution to this geometrical problem yielded the theoretical values of* $70°32'$ *and* $109°28'$*. Thus, invoking the two* principles of simplicity and mathematical beauty, *Malardi obtained a theoretical result very close to the observed values!*

The next step, taken by the French physicist and naturalist ***René-Antoine Ferchault de Réaumur*** *(1734), had been foreshadowed long before by Pappos. Though Euler had not yet published his famous discussion on curves,* maximi minimive proprietate gaudentes, *the idea of* maxima *and* minima *was in the air as a guiding postulate, a heuristic method, to be used as Malardi used his principle of simplicity.*

So it occurred to Réaumur that the hexagonal structure of the bee's honeycomb should follow from a minimum principle: the bee would build its cells with the greatest economy in order to use as little wax as possible; and that, just as the closed-packed hexagons gave the minimal extent of boundary in a plane, so the figure determined by Malardi, namely the rhombic dodecahedron, might be that which employs the minimum of surface for a given volume; or which, in other words, should hold the most honey for the least wax[253].

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[253] Consider a right prism of height h, having regular hexagonal base *abcdef*, top $ABCDEF$, both with side s (volume $= \frac{1}{2}3\sqrt{3}s^2h$). At the top, we cut off the corners B, D, F by planes through the lines AC, CE, EA. Using these lines as 'hinges', we rotate the so-formed three tetrahedrons such that they all meet at a common vertex V. The new body with top faces $AXCV$, $CYEV$, $EZAV$ (rhombuses), is the bee's cell and has the same volume as the original prism. The hexagonal base at the opposite end is the open end. One parameter

Réaumur posed his conjecture to **Samuel Koenig**, *a young Swiss mathematician: Given a hexagonal cell terminating with three similar and equal rhombs, what is the configuration which requires the least quantity of material for its construction? Koenig (1739) found that the angle* $109°24'$ *followed from the minimum principle proposed by Réaumur [Koenig's own paper, sent to Réaumur, remained unpublished and was lost and his method of solution is unknown]. Thereupon* **Bernard Le Bovier de Fontenelle** *(1657–1757), the perpetual secretary of the French Academy, declared that bees had no intelligence; yet they were "blindly using the highest mathematics by divine guidance and command".*

In spite of the striking success of the calculus in explaining the cell's geometry in terms of 'wax economy', a line of mathematicians since Bartholinus doubted the philosophical implications of this theory. **Glaisher** *(1873) summed up the matter as follows:*

"As the result of a tolerably careful examination of the whole question, I may be permitted to say that the economy of wax has played a very subordinate part in the determination of the form of the cell. I should not be surprised if it were found that the form of the cell had been determined by other considerations, into which saving wax did not enter, although I would not go as far as to say that the amount of wax required was a matter of absolute indifference to the bees".

D'Arcy Thompson *(1860–1948) commended in the same spirit that it makes more sense to suppose:*

"that the beautiful regularity of the bee's architecture is due to some automatic play of the physical forces" than to suppose "that the bee intentionally seeks for a method of economizing wax".

But all this assumes that the bees have somehow hit upon the optimal honeycomb. Have they? This question was investigated by the Hungarian

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is, however, left to our choice: the angle through which we cut-off the three tetrahedrons, or alternatively, the angle θ which the vertical at V makes with the line VX. The bees form the faces by using wax. When the volume is given, it is economic to spare wax and, therefore, to choose the angle of inclination θ in such a way that the *surface area S of the bee's cell is minimized*.
Simple geometrical considerations reveal that $S(\theta) = 6hs + \frac{3}{2}s^2\left(\frac{\sqrt{3}}{\sin\theta} - \cot\theta\right)$. The derivative $S'(\theta)$ vanishes if, and only if, $\cos\theta_0 = \frac{1}{\sqrt{3}}$, yielding $\theta_0 = 54.7°$ or $2\theta_0 = 109°24'$, independent of the choice of h and s. It is worth comparing the result with the actual angle chosen by the bees. It is difficult to measure this angle. However, the average of all measurements does not differ significantly from the theoretical value of $2\theta_0 = 109°24'$. Thus, the bees strongly prefer the optimal angle. It is rather unlikely that the result is due to chance.

mathematician **Fejes Tóth** *(1964). In his paper "What the bees know and what they don't know", he considered honeycombs, which he defined as a set of congruent convex polyhedra called cells, filling the space between two parallel planes without overlapping and without interstices in such a way that:*

(1) each cell has a face (called a base or opening) on one and only one of the two planes; and

(2) every pair of cells is congruent in such a way that their bases correspond to each other.

The cells built by the bees are prismatic vessels, the openings (and cross sections) of which are regular hexagons, whereas their bottoms consist of three equal rhombi.

The bees construct their honeycomb in such a way that the hexagonal openings of the cells are attached to one of the two planes. Is the zigzagged bottom surface constructed by the bees the most economical one? (It is certainly more advantageous than a plane.)

In order to state the problem precisely, we formulate (following Tóth) the isoperimetric problem for honeycombs: Given any two numbers V and W, find a honeycomb of width W whose cells have smallest surface area and yet enclose the volume V. (The width W is the distance between the two parallel planes that bound the honeycomb.)

We don't know yet what the solution is, but definitely it cannot be the bee cell, because Fejes Tóth found another cell that yields a slightly better result. The bottom of this cell consists of two hexagons and two rhombi. The advantage of Tóth's cell amounts to less than 0.35% of the area of an opening (and a much smaller percentage of the surface area of a cell). Hence we can state that the bees do a pretty good but not a perfect job, although their practical result, taking the margin of error into account, might still be optimal.

Evolution of Minimum and Variational Principles[254]

Many fundamental ideas in science were conceived in antiquity, and our present way of thinking owes a great deal to our predecessors. One essential idea that modern science has inherited from the classical world is the concept of a fundamental order and harmony to the universe, a harmony that could be reflected in the beauty of mathematical structures. Because Greek mathematics was mainly restricted to geometry, the ancient scientists used geometric models to describe nature.

Thus, since the nascence of the Milesian school (ca 600 BCE), Greek philosophers and scientists sought to reduce the manifold phenomena of nature to a basic set of unifying laws. This quest for simplicity was continued by ***Pythagoras*** *(ca 540 BCE), but whereas the Ionian physicists postulated a single* substance *from which all substances comprising the cosmos were derived, Pythagoras put the emphasis on mathematical reasoning and believed that the concepts of* harmony *and* number *(positive integers) embrace the whole structure of the universe (mathematics and physics were in his time indistinguishable).*

Plato *(427–347 BCE) continued the Pythagorean legacy and upheld the view that number rules the universe. Although he rejected the experimental*

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[254] For further reading, see:

- Elsgolts, L., *Differential Equations and the Calculus of Variations*, Mir Publishers: Moscow, 1980, 440 pp.
- Yourgrau, W. and S. Mandelstam, *Variational Principles in Dynamics and Quantum Mechanics*, Dover: New York, 1968, 201 pp.
- Weinstock, R., *Calculus of Variations*, Dover: New York, 1974, 326 pp.
- Gelfand, I.M. and S.V. Fomin, *The Calculus of Variations*, Prentice Hall, 1965, 232 pp.
- Lanczos, C., *The Variational Principles of Mechanics*, University of Toronto Press, 1964, 367 pp.
- Woodhouse, R., *A Treatise on Isoperimetrical Problems, and the Calculus of Variations*, Chelsea Publications: New York.
- Todhunter, I., *A History of the Progress of the Calculus of Variations in the Nineteenth Century*, Chelsea Publications: New York.

method, he still adhered to the conceptual representation of the phenomenal world through ideas of simplicity, uniformity, order and perfection.

Aristotle *(384–322 BCE) mentioned the fact that of all curves enclosing a given area, the circle possesses the shortest perimeter. This marks the transition from the belief in simplicity to a* minimum *principle, explicitly stated for the first time. This minimum hypothesis was not dictated by any appeal to quantitative measurement and was not subject to rigorous scrutiny.* ***Hero of Alexandria*** *(ca 150 BCE) gave a geometrical demonstration of the principle of shortest optical path (distance) for light rays reflected from a plane mirror. No further development of this idea was made until the advent of Fermat's least-time principle in the* 17^{th} *century.*

In the interim period, however, the claim for the simplicity of nature was strongly advocated by **William of Ockham** *(1285–1349). His 'razor' principle, while indicating a viewpoint similar to the simplicity hypothesis of Aristotle, differs from it in the sense that while the Greek philosopher held that nature possesses an immanent tendency to simplicity, Ockham demanded that in* describing *nature one should avoid unnecessary complications. Both doctrines appear simultaneously in the writings of* **Copernicus** *(1473–1543),* ***Galileo*** *(1564–1642) and* **Kepler** *(1571–1630) in the form of Pythagorean-Platonic mysticism and deep-rooted convictions of a simple, harmonious and ordered universe.*

Both **Newton** *(1642–1727) and* **Leibniz** *(1646–1716) reformulated the principle of simplicity: "We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances" (Newton); "The perfectly acting being... can be compared to a clever engineer who obtains his effect in the simplest manner one can choose" (Leibniz). The French philosopher* **Nicolas de Malebranche** *(1638–1715) replaced the word* simplicity *by* economy *and arrived at a similar view which he called the 'Economy of Nature'*[255].

In 1621, ***Snell*** *derived empirically the law of light refraction across a boundary between two homogeneous media in which the velocity of light has the values* c_1 *and* c_2*, respectively. If the ray passes from one region to the other, then it must consist of two straight-line-segments which satisfy the*

[255] There were, however, other more prosaic motives for the preoccupation of scientists with extremum problems; the question of shortest and quickest connections became especially important to the European powers during the 15^{th} and 16^{th} centuries, when they were searching for the best routes to the Far East and to the New World. Faster sailing routes promised greater profits. The well-known expeditions of Vasco da Gama and Christopher Columbus must be seen mainly in economic terms.

law of refraction, $\sin\alpha_1/\sin\alpha_2 = c_1/c_2$, *where* α_1 *and* α_2 *are the angles between the normal to the boundary and the two line-segments at the point of intersection.*

In 1657, ***Fermat*** *succeeded in deriving Snell's law from a new principle — the principle of least time. It stated that a light ray requires less time along its actual path between two points than it would require along any other conceivable ('virtual') path satisfying the given condition.*

It is remarkable that Fermat demonstrated the principle using elementary algebra only (no derivatives!). He later generalized his result to curved surfaces separating the two media and also for inhomogeneous media. He thus arrived at the general Fermat principle of geometrical optics:

"In an inhomogeneous medium, a light ray traveling between two points follows a path along which the time taken is minimum w.r.t. paths joining the two points"[256].

Minimizing the travel-time t *between two points* P *and* Q *means minimizing the integral* $I = \int_P^Q \frac{ds}{v}$, *where* s *is the arc-length along the ray and* $v = v(s) = v(\mathbf{r}(s))$ *is the velocity at a general point* $\mathbf{r}(s)$ *on the ray. The principle then states that* $\delta\int_P^Q \frac{ds}{v} = 0$, *meaning that the variation between the time taken to travel along the actual path and that needed to cover an infinitesimally adjacent virtual path is zero.*

Thus, 1800 years had to pass before Hero's observation could be improved upon and generalized. The ideas unfolded by Fermat have had a tremendous influence on the development of physical thought in and beyond the study of classical optics; including its analogues in classical and quantum mechanics as well as both classical and quantum field theories. Fermat's principle provided science with an insightful and highly useful way of anticipating the behavior of light, matter and energy. Note that Fermat's principle is not so much a computational device as it is a concise way of thinking about the propagation of light. It is a statement about the grand scheme of things, without reference to any underlying causal mechanisms.

The first real justification of Fermat's principle was given by ***Huygens*** *who, in 1678, deduced the laws of reflection and refraction on the basis of the wave theory of light (Huygens' principle). Furthermore, he demonstrated that*

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[256] Fermat's principle is a *true minimum principle* (and not merely a stationary value principle) if we make comparisons in the *local* sense. However, it is required that all along the trajectory the wave surfaces shall be well defined, single-valued with definite normals (no intersection of ray trajectories!). The mathematical machinery needed to derive the equations of the rays for a given $v(\mathbf{r})$ was not known to Fermat and had to await for another 100 years.

the travel-time of light upon refraction was a real minimum. Fermat's achievement, with Huygens' support, stimulated a great deal of effort to supersede Newton's laws of mechanics with a similar variational formulation.

In 1740, *the French mathematician* ***Maupertuis*** *(1698–1759) announced le principe de la moindre quantité d'action — the famous principle of least action.*

According to this principle all events in nature take place such that a certain quantity, called "action", is rendered minimum. He postulated that the action must depend on the mass and the histories of the velocities and displacements; he therefore defined action as an integral of the product of these three factors (dimensionally it is also equal to the product of energy and time or to angular momentum). The bold universality of this assumption is admirable and well in line with the spirit of the 18^{th} *century. It conforms with the spirit of the Platonic-Pythagorean cosmology, as well as with the natural philosophy of Leibniz, and follows in the footsteps of Hero and Fermat.*

However, Maupertuis' original definition of action (as a product of mass, velocity and distance without integration) was very obscure, owing to the fact that the distance covered by the moving body varies with time, and his failure to specify the time interval for which the product is to be computed. For these reasons Maupertuis could not establish satisfactorily the quantity to be minimized. He applied his principle to the derivation of the laws of elastic collision. This phenomenon is very intricate if treated as a minimum problem and requires great skill in handling (the mathematical powers of Maupertuis were far behind the high standards of his period); he obtained the correct result by an incorrect method. More satisfactory was his treatment of the law of refraction, in which he showed how Fermat's principle of least time can be replaced by the principle of least action (this result was earlier recognized by ***Johann Bernoulli****).*

Thus, the original statement by Maupertuis was vaguely theological and could hardly pass muster today. The integral formulation which today bears his name is actually due to ***Euler*** *who discovered the principle in* 1743 *in an entirely correct form (he may have been inspired, in part at least, by Maupertuis'* 1740 *paper). In particular, Euler knew that both actual and virtual motions have to satisfy the law of conservation of energy. Without this auxiliary condition, the action quantity of Maupertuis, even if corrected from a sum (the form in which he used it) to an integral, loses all significance.*

Euler, who confined himself to a single particle moving on a plane curve, asserted:

"When a particle travels between two fixed points, it takes the path for which $\int vds$ *is a minimum",* v being the velocity of the particle and ds the

corresponding element of the curve. Euler also gave an alternative formulation through which the actual path can be mathematically evaluated:

"A particle travels between two fixed points in such a way that the difference between the integral $\int v ds$ taken along the actual path and that taken along any neighboring virtual path between the two points, is an infinitesimal quantity of second order; the particle is supposed to travel along the virtual path with the velocity for which the energy is equal to the given energy" (virtual path is one along which the particle may be imagined to move *without* satisfying Newton's laws of motion).

The condition is thus

$$\delta \int_P^Q v ds = 0$$

where P and Q are the initial and final points and δ denotes the variation *of the integral under the aforementioned restrictions.*

The nascence of the calculus of variations was in **Euler***'s work (1744) "Methodus Inveniendi lineas Curvas Maximi Minimive proprietate gaudentes" (a method to find curved lines that enjoy a maximum or minimum property).*

He was seeking an admissible function $y(x)$ that extremalizes the functional given by the integral $I = \int_{x_1}^{x_2} f(x, y, y')dx$. He showed that if $y(x)$ exist, it must obey the differential equation

$$\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) - \frac{\partial f}{\partial y} = 0.$$

Euler applied the calculus of variations to the study of elasticity, examining the bending, buckling and vibrations of bands and plates. Note that since in Euler's equation $f = f\big(x, y(x), y'(x)\big)$, $\frac{\partial f}{\partial y'}$ is in general an explicit *function of x as well as an* implicit *function of x via $y(x)$ and $y'(x)$. Therefore*

$$\frac{d}{dx} = \frac{\partial}{\partial x} + \frac{dy}{dx}\frac{\partial}{\partial y} + \frac{d^2y}{dx^2}\frac{\partial}{\partial y'}.$$

Consequently the second order ODE *for $y(x)$ is found to be*

$$\left(\frac{\partial^2 f}{\partial y'^2}\right)\frac{d^2y}{dx^2} + \left(\frac{\partial^2 f}{\partial y \partial y'}\right)\frac{dy}{dx} + \left(\frac{\partial^2 f}{\partial x \partial y'} - \frac{\partial f}{\partial y}\right) = 0.$$

The solution of this equation constitute a two-parameter family of curves, and among these, the stationary functions are those in which the two parameters are chosen to fit the given boundary conditions.

In the case of three independent variables, the integral in question in

$$I[u] = \iiint\limits_R F\left(x, y, z; u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}\right) dxdydz,$$

where $u = u(x, y, z)$*, the local stationary point of the functional* $I[u]$ *obeys the Euler equation*

$$\frac{\partial I}{\partial u} - \frac{\partial}{\partial x}\left(\frac{\partial I}{\partial u_x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial I}{\partial u_y}\right) - \frac{\partial}{\partial z}\left(\frac{\partial I}{\partial u_z}\right) = 0.$$

Suppose we wish to find $u(x, y, z)$ *which has a minimum average value of the square of the gradient in a certain region in space, i.e.*

$$F = (\nabla u)^2 = u_x^2 + u_y^2 + u_z^2.$$

The resulting Euler equation is simply the Laplace equation $\nabla^2 u = 0$ *which must be satisfied, for instance, by the electric potential in free space.*

The Laplace equation is therefore the necessary condition that the average electrostatic field energy be minimized in a given volume. If the same quantity is to be made stationary, but with the additional requirement that $\int u^2 dxdydz$ *shall have a fixed value, another interesting equation results. In that case we define*

$$F = (u_x)^2 + (u_y)^2 + (u_z)^2, \qquad F_1 = u^2.$$

If we extremalize the integral $\iiint (F - \lambda^2 F_1) dxdydz$*, with* λ^2 *the Lagrange multiplier of the constraint* $\int u^2 dxdydz = \text{constant}$*, Euler's equation then reads*

$$\nabla^2 u + \lambda^2 u = 0$$

which is the Helmholtz wave equation for monochromatic waves. Such a wave may therefore be characterized as a disturbance in which the displacement u *has a fixed mean square value and at the same time a minimum mean square gradient.*

Another example of the power of the variational calculus is the propagation of light in an inhomogeneous medium: Let $v(x, y, z)$ *be the velocity of light at each point of the medium. The square element of distance between two points on a light ray* (x, y, z) *and* $(x + dx, y + dy, z + dz)$ *is* $ds^2 = dx^2\left[1 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{dz}{dx}\right)^2\right]$*, where* $y = y(x), z = z(x)$ *constitute the equations describing the ray.*

Therefore, the travel time along a path between two fixed points P *and* Q *is* $t = \int_P^Q \frac{\sqrt{1+y'^2+z'^2}}{v(x,y,z)} dx$*. It is required to simultaneously find two functions* $y = y(x)$*,* $z = z(x)$ *such that the functional* t *is the smallest.*

Writing the system of Euler equations for this functional, i.e.

$$\frac{\partial v}{\partial y}\frac{\sqrt{1+y'^2+z'^2}}{v^2}+\frac{d}{dx}\frac{y'}{v\sqrt{1+y'^2+z'^2}}=0;$$

$$\frac{\partial v}{\partial z}\frac{\sqrt{1+y'^2+z'^2}}{v^2}+\frac{d}{dx}\frac{z'}{v\sqrt{1+y'^2+z'^2}}=0,$$

we obtain the two coupled ordinary differential equations for the curve along which light propagates. Once $y(x)$, $z(x)$ *are known, the path of light from* P *to* Q *is given in the parametric form* $\{x, y(x), z(x)\}$.

Another important application of Euler's equations is the determination of the path of shortest distance between two points on a surface $\mathbf{r}=\mathbf{r}(u,v)$. *If such a path exists, we call it a geodesic. The line-element on the surface is*

$$ds^2 = d\mathbf{r}\cdot d\mathbf{r} = \Big(\frac{\partial \mathbf{r}}{\partial u}du+\frac{\partial \mathbf{r}}{\partial v}dv\Big)^2 = Edu^2+2Fdudv+Gdv^2$$

where

$$E=\frac{\partial \mathbf{r}}{\partial u}\cdot\frac{\partial \mathbf{r}}{\partial u}, \qquad F=\frac{\partial \mathbf{r}}{\partial u}\cdot\frac{\partial \mathbf{r}}{\partial v}, \qquad G=\frac{\partial \mathbf{r}}{\partial v}\cdot\frac{\partial \mathbf{r}}{\partial v}.$$

A curve on the surface has the parametric representation $u=u(t)$, $v=v(t)$. *Hence, the distance between* $P(t_1)$ *and* $Q(t_2)$ *on the surface and along the curve is given by the line integral*

$$J[u,v]=\int_{t_1}^{t_2}\sqrt{Eu'^2+2Fu'v'+Gv'^2}\;dt,$$

where $u'=\frac{du}{dt}$, $v'=\frac{dv}{dt}$. *Writing Euler equations for the functional* J, *we obtain the simultaneous differential equations for the parametric functions* u *and* v:

$$\frac{A_u}{\Delta}-\frac{d}{dt}\frac{C}{\Delta}=0; \qquad \frac{A_v}{\Delta}-\frac{d}{dt}\frac{D}{\Delta}=0,$$

where

$$\Delta=\sqrt{Eu'^2+2Fu'v'+Gv'^2};$$

$$A_u=\frac{\partial E}{\partial u}u'^2+2\frac{\partial F}{\partial u}u'v'+\frac{\partial G}{\partial u}v'^2;$$

$$A_v=\frac{\partial E}{\partial v}u'^2+2\frac{\partial F}{\partial v}u'v'+\frac{\partial G}{\partial v}v'^2;$$

$$C=2(Eu'+Fv'); \qquad D=2(Fu'+Gv').$$

Although Euler was first to implement Maupertuis' conjecture, the credit for having given the correct formulation of the principle of least action goes to **Lagrange** *(1736–1813). It is true that Euler was first to introduce the concept of variation and stationarity (minimum or maximum) instead of an exclusive minimum, but he still held to the conviction that some sort of maximum or minimum law prevails throughout nature. Lagrange, on the other hand, showed that the principle of least action together with the law of conservation of energy is fully equivalent to Newton's law of motion and may, indeed, be employed as an alternative formulation of the principles of dynamics.*

Lagrange himself remarked, in conformity with his general outlook on natural philosophy, that the principle of least action was to be considered not as a metaphysical postulate, but as a simple and general consequence of the laws of mechanics (1788).

For the next half century, the principle of least action was thought of as interesting rather than important, and no use at all was made of it. Indeed, as late as 1837, *it was described as "only a useless rule" by Poisson, who failed to read* **Hamilton***'s* 1835 *paper. There, Hamilton gave the first exact formulation of the principle of least action for systems which are not necessarily conservative (showing it to be equivalent to the Lagrange equations of motion) and stated his principle*

$$\delta \int L dt = 0.$$

This effort culminated in the celebrated Hamilton-Jacobi equation (1828–1837) which brought about the geometrization of dynamics and the mathematical analogy between optical rays and mechanical paths of point-masses. A bridge had finally been established between Fermat's principle of least time and Hamilton's principle of least action. Thus, a long line of thinkers from **Hero** *through* **Fermat***, and even* **Euler***, believed in an underlying metaphysical optimum law of one kind or another. But such postulates were transformed by* **Lagrange***,* **Hamilton** *and* **Jacobi** *into exact analytic instruments capable of solving concrete problems in physics, mathematics and astronomy.*

The striking similarity between the principles of Fermat and Hamilton played an important role in **Schrödinger***'s development of quantum mechanics. In* 1942 **R.P. Feynman** *showed that quantum mechanics can be formulated in an alternative way using a variational approach*[257]*. And so, the*

[257] Feynman used a continuous sum (*path integral*) over all virtual trajectories between two given particle positions at two given times: the *classical* path, and its neighboring trajectories, determine the path integral in the classical limit. In this way, the Huygens principle is seen to apply to quantum mechanics.

continuing evolution of variational principles take us back to optics via the modern formalism of the matter waves of quantum mechanics.

So far we have concentrated mainly on dynamical problems. In the geometrical vein matters had a history of their own, and in order to see it in the right perspective we must return to the period 1690–1701, namely to the emergence of the calculus of variations due to the efforts of the brothers ***Jakob and Johann Bernoulli****.*

The pivotal year is 1696, in which two problems were proposed: In December of that year Johann Bernoulli challenged the mathematicians of his age in the journal Acta Eruditorum to solve the problem of the brachistochrone by Easter 1697: He asked to determine a curve of the quickest descent of a massive particle moving between two given points in a homogeneous gravitational field. In time, three mathematicians solved the problem: Johann and Jakob Bernoulli, and Leibniz. The path, which happens to be a cycloid, is known as the brachistochrone[258].

[258] Let the particle start from rest at the origin; the terminal point of the motion is (x_2, y_2). It is convenient to extend the y-axis to the right and to measure x downwards. From the energy equation

$$mgx = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{ds}{dt}\right)^2 = \frac{1}{2}m\left[\frac{\sqrt{dx^2+dy^2}}{dt}\right]^2,$$

we find $dt = \left[1 + (y')^2\right]^{1/2}(2gx)^{-1/2}dx$. The integral to be minimized is therefore

$$\sqrt{2g}\; t = \int_0^{x_2}\left(\frac{1+{y'}^2}{x}\right)^{1/2} dx.$$

Euler's equation reads

$$\frac{d}{dx}\frac{y'}{[x(1+{y'}^2)]^{1/2}} = 0;$$

hence $y' = x\left(\frac{x}{c} - x^2\right)^{-1/2}$, with c a constant. If we introduce $2a = \frac{1}{c}$, integration leads to

$$y = a\cos^{-1}\left(1 - \frac{x}{a}\right) - \sqrt{2ax - x^2}.$$

It represents an inverted cycloid with its base along the y-axis and the cusp at the origin. The constant a must be so adjusted that the cycloid passes through the point (x_2, y_2).

This problem must not be confused with another problem, the *tautachrone*, proposed by **Jakob Bernoulli** in 1690: to determine a curve on a vertical plane, along which a massive particle will arrive at a given point of the curve in the same time interval, no matter from what initial point of the curve it

The other problem to which attention was called in 1696 by Jakob Bernoulli is the isoperimetric problem (iso = equal, perimetron = circumference), of which the Greek mathematicians were well aware. It has been one of the most stimulating and influential problems in the history of mathematics. Its origins lay in the ancient legend associated with the founding of the city of Carthage (ca 900 BCE). Dido, a Pheonician princess, fled from the city-state of Tyre when her ruthless brother Pygmalion murdered her husband to usurp her possessions. She bought a parcel of land from the King of Numidia under the condition that she would obtain only as much land as she could enclosed by the skin of an ox. To maximize the land she cut the hide in thin strips and tied them together to form a cord of some 1500 meters, and then formed with it a semi circle with the Mediterranean coast as its diameter.

The Greek thus knew that among all closed lines of the same perimeter, the circle has the maximal area[259]. An incomplete proof of the isoperimetric property of the circle was given by ***Zenodoros*** *(ca 180 BCE).* ***Jakob***

started (This curve, too, is a cycloid!). It was solved earlier by Huygens (1673) and Newton (1687) and applied by Huygens in the construction of pendulum clocks. The isochronous property of the cycloid is this: a pendulum constrained to swing between two successive arches of an inverted cycloid must oscillate such that the time to the lowest point is $\pi\sqrt{\frac{a}{g}}$, where a is the radius of the circle that generates the cycloid, and irrespective of the oscillation's angular amplitude.

259 The Greeks, who held some rather impressive notions of beauty and perfection, came to the conclusion that the circle was the most beautiful curve. After all, the sun and moon were round, the horizon was round, and the planets (so they thought) orbited in circles. The circle must be the perfect figure, for the architect of the universe would certainly not deal with imperfect creations. Some views about the circle were even more sweeping. The philosopher **Empedocles** held that the nature of God is a circle whose center is everywhere and whose circumference is nowhere.

But this theory received a blow even at ancient times. When astronomers were able to measure the paths of the planets accurately, they were found not to travel in true circles. Ptolemy invented an ingenious but ad hoc system of epicycles — that is, of circles and circles upon circles — to generate the paths of the planets. In the 17^{th} century, when **Kepler** found that the planets moved in ellipses around the sun, the circle was dethroned from its position as the most perfect curve. Yet, no other curve shares the following list of characteristics:

- Every point on the circle is at the same distance from the center.
- Every diameter of the circle is an axis of symmetry.
- A circle is a figure with constant width.
- Every tangent to a circle is perpendicular to the radius drawn from the center to the point of tangency.

Bernoulli *gave a proof in* 1701. *A complete proof, however, was first given by* **Weierstrass** *in* 1865.

Denoting the given perimeter of the curve by L, *all permissible areas* A *formed by* L *obey the isoperimetric inequality* $A \leq \frac{L^2}{4\pi}$. *This inequality has the following theorem as a consequence:*

Among all figures of equal area, the circle has minimal perimeter.

This can easily be seen, because the area A *of a circle of perimeter* $\overline{L}$ *equals* $\overline{L}^2/4\pi$. *If there were a plane figure with the same area but a smaller perimeter* $L < \overline{L}$, *we would have* $A > \frac{L^2}{4\pi}$, *which contradicts the isoperimetric inequality.*

This explains the circular shape of oil slicks: the molecular forces generate a figure of least potential energy (least surface area) for a given amount of oil. There are several other optimum properties of the circle. For instance: among all plane domains of a given area, the disc can support the largest sand pile; among all cross-sections of a perfectly elastic column of equal area, the disc can withstand the largest torsional moment; out of all drums with a given cross-sectional area, the circular membrane has the lowest tone [this last result was conjectures by **Rayleigh** *(1877) on the basis of experiments, but it was proved by* **Faber** and **Krahn** *(1923–1924)].*

The term isoperimetric problem is usually extended beyond its classical content to include the general case of finding extremals for one integral (subject to constraints) requiring a second integral to take on a prescribed value[260].

- The curvature of a circle is constant at every point.
- Of all curves that enclose the same area, the circle has the least perimeter, and of all curves with the same length of perimeter, the circle encloses the greatest area. Thus, given any plane figure of area A and perimeter L, then $\frac{4\pi A}{L^2} \leq 1$ (equality holds only for the circle). For a semicircle, this ratio is about 0.75, for a square 0.79 and for a perfect hexagon 0.91.

  The 3-*dimensional* isoperimetric property can be expressed by the inequality $\frac{36\pi V^2}{A^3} \leq 1$ between the surface area A and the volume V, the equality holding only for the sphere.

260 If the curve is expressed parametrically by $x(t)$ and $y(t)$ and it is traversed once counterclockwise as t increases from t_1 to t_2, then the enclosed *area* is known to be $A = \frac{1}{2}\int_{t_1}^{t_2}(x\dot{y} - y\dot{x})dt$, [where $\cdot = \frac{d}{dt}$], which is an integral depending on two unknown functions [this integral expression for A is a special case of Green's theorem]. Since the length of the curve is $L = \int_{t_1}^{t_2}\sqrt{\dot{x}^2 + \dot{y}^2}\,dt$, the problem is to maximize A subject to the constraint that L must have a constant value. Using the method of *Lagrange multipliers*, the problem reduces to maximizing the unconstrained functional $I = \int_{t_1}^{t_2} F(x, y; \dot{x}, \dot{y})dt$ where $F = \frac{1}{2}(x\dot{y} - y\dot{x}) + \lambda\sqrt{\dot{x}^2 + \dot{y}^2}$.

In August 1697, Johann Bernoulli again publicly posed the problem of finding the shortest line between two given points on a convex surface. This was meant as a challenge to his brother Jakob, with whom he was publicly feuding. The unfortunate rivalry of the two brothers eventually became so intense, and their polemics so ugly, that the scientific journals of the time declined to publish them. Anyway, the challenge was met by Jakob who solved the problem in 1698 for all surfaces of revolution.

Johann then announced that he had found a solution of the shortest connection problem for an arbitrary *surface (1698). His unpublished solution appeared in the form of a geometric theorem:*

"at each point P of a shortest line C, the corresponding osculating plane *of C intersects the tangent plane to the surface in a right angle (the osculating plane includes the tangent to C at P and the principal normal at P)".*

Thirty years later, in December 1727, Johann again posed the problem to his student Euler! Euler published his solution in 1728 under the title Da linea brevissima in superfice quacunque duo quaelibet puncta jungente *("On the shortest line on an arbitrary surface connecting any two points whatsoever").*

In contradistinction to the geometric solution of Johann Bernoulli, Euler reduced the problem to the solution of a differential equation. Euler stated that we can easily solve the problem of shortest connection between two points on a convex surface by a simple mechanical artifice: we fix a string at one of the points and pull it taut in the direction of the other. The string then yields the shortest connection between the two points.

The Euler equations for this case then read:

$$\frac{d}{dt}\left(\frac{\partial F}{\partial \dot{x}}\right) - \frac{\partial F}{\partial x} = 0$$

and

$$\frac{d}{dt}\left(\frac{\partial F}{\partial \dot{y}}\right) - \frac{\partial F}{\partial y} = 0.$$

The integration of these equations yields the *circle*:

$$(x - c_1)^2 + (y - c_2)^2 = \lambda^2.$$

Early Theories of Cosmic Evolution

1750–1785 CE *Descartes' idea of a universe evolving by natural processes of separation and combination was the source of a succession of theories of cosmic evolution by* ***Swedenborg*** *(1688–1772, Sweden; 1734),* ***Thomas Wright*** *(1711–1786 , England; 1750),* ***Immanuel Kant*** *(1724–1804, Germany; 1755),* ***Johann Heinrich Lambert*** *(1728–1771, Germany; 1761),* ***Georges Louis Leclerc de Buffon*** *(1707–1788, France; 1785) and others.*

Their theories related to the formation of the solar system and the phenomenon of the 'Milky Way' [galaxy is the Greek word for 'milk']. Kant held to the idea that in the beginning all matter was in a gaseous state and was spread more or less uniformly throughout the universe (his interpretation of Genesis ***I****, 1–2). He assumed that we live in an evolutionary universe in the sense that the past was essentially simpler than the present. Subsequently a giant cloud of gas, contracting under its own gravitation, began to rotate and shed matter from its center, to in turn form the planets by further gravitational contraction.*

The phenomenon of the 'Milky Way' was interpreted by similar speculations of ***Wright*** *and* ***Lambert*** *who came very close to the truth. Wright suggested that the Milky Way consisted of a flattened distributions of stars forming a disc, which rotates about its center on an axis normal to the disc plane. He also suggested that what appear to be nebula are actually galaxies and that the solar system comprises a small portion of one of the universe's endless galactic structures.*

These ideas remained speculative until 1785, *when they were confirmed by the observations of* ***Frederick William Herschel*** *(1738–1822, England).* ***Kant*** *suggested that the universe was hundreds of millions of years old, and that it was in a state of* continuous dynamical evolution *that is manifested through motion, creation and disintegration. Kant's theory liberates time from its earthly confinement and links it with cosmic processes.*

1750 CE **Gabriel Cramer** (1704–1752, Switzerland). Mathematician. Widely known among students of mathematics for his rule for solving a system of linear equations by determinants[261].

Cramer was born in Geneva. He belonged to an ancient Holstein family known first in Strasbourg, and then in Geneva, where his father and grandfather were physicians. Cramer was educated at the University of Geneva, and in 1724 was given an appointment there as a professor of mathematics. In 1727 he took a two-year leave for travel, during which time he made the acquaintance of **Jean Bernoulli** in Basel. He died in Bagnols near Nimes in the south of France, where he sought to restore his failing health.

1750 CE **Maria Gaetana Agnesi** (1718–1799, Italy). Mathematician and philosopher. Became the first woman to occupy a chair of mathematics in modern times. It happened at the University of Bologna, Italy. The plane curve $y(x^2 + ya^2) = 8a^3$, now known as the *witch of Agnesi*[262], is named after her.

ca 1750 CE **Eugene Aram** (1704–1759, England). Self-taught philologist. Recognized in advance of scholars the Indo-European affinities of *Celtic* and disputed the derivation of Latin from Greek. But he was not destined to live in history as a pioneer of philology, as he should; In 1759 he was convicted of murdering his wife's lover (1745) and executed. This was the subject of a romance by Bulwer Lytton *Eugene Aram* (1832).

1750–1784 CE **John Michell** (1724–1793, England). Geologist, astronomer and the founder of the science of seismology. Expounded novel and farsighted ideas on a wide range of subjects:

- Made accurate magnetic observations, described a method of *magnetization* and gave a lucid exposition of the nature of *magnetic induction* (1750).
- Invented the *torsion balance* (1784) independently of **Coulomb** (1777). Michell described it in his proposal of a method for obtaining the mean density of the earth. He did not live to put his method into practice;

[261] This rule was discovered independently by **Colin Maclaurin** (1698–1746, Scotland) in 1742.

[262] The name is a misnomer; It seems that Agnesi confused the old Italian word "*versorio*" [given earlier (1703) to the curve by **Guido Grandi** (1671–1742, Italy)] which means 'free to move in any direction' with *versiera* which means 'Devil's wife' or 'goblin'. This curve was treated earlier by **Fermat** (1663). A similar curve was studied by **James Gregory** (1658) and used by **Leibniz** (1674) in deriving the series $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$.

this was done by **Henry Cavendish**, who made, by means of Michell's apparatus, the celebrated determination that now goes by the name of *Cavendish's experiment* (*Phil. Trans.*, 1798).

- In his geological essay entitled *Conjectures concerning the cause and observations upon the phenomena of earthquakes* (*Phil. Trans.* **60**, 1760), he recognized that earthquakes originate *within* the earth and send out elastic waves through the earth's interior.
- Originated the concept of a *black hole*[263] in his essay *On the means of discovering the distance, magnitude etc. of the fixed stars* (*Phil. Trans.*, 1784), 12 years ahead of **Laplace**. Reasoning, a la Newton, that light is composed of particles, he calculating that a star with the same density as the sun but with a radius 500 times larger could, due to its gravitation alone, prevent the escape of light and consequently be invisible to the rest of the universe.

Michell was educated at Queens' College, Cambridge. He became M.A. in 1752, received his doctor's degree in 1761, and taught mathematics, theology, Greek, Hebrew and philosophy there. Appointed Woodwardian professor of geology in 1762, and in 1767 became rector of Thornhill in Yorkshire. He was elected a fellow of the Royal Society in the same year as Henry Cavendish (1760). Michell had a wide circle of scientific friends, among them **Joseph Priestley**, **John Smeaton** and **William Herschel**.

1750–1820 CE The *Classical Period* in music. Its leading composers are:

• Johann Wilhelm Hertel	1727–1789
• **Joseph Haydn**	**1732–1809**
• Luigi Boccherini	1743–1805
• Domenico Cimarosa	1749–1801
• Carl Stamitz	1745–1801
• Giovanni Batista Viotti	1755–1824
• **Wolfgang Amadeus Mozart**	**1756–1791**

263 The velocity of escape v from the gravitational influence of a massive star of mass M and radius R is given by $v^2 = \frac{2GM}{R} = \frac{8\pi}{3}G\rho R^2$, where ρ is the star's density and G is the universal gravitational constant. If we require $v > c$ (velocity of light), light will be trapped inside the star; this happens whenever $\rho R^2 \geq \frac{3c^2}{8\pi G}$. Inserting the numerical values: $c = 3 \times 10^{10}\ \frac{\text{cm}}{\text{sec}}$, $G = 6.672 \times 10^{-8}$ cgs, ρ (sun's mean density) $= 1.41\ \frac{\text{g}}{\text{cm}^3}$, we find: $R \geq 500R_\odot$, where $R_\odot$ (sun) $= 6.96 \times 10^{10}$ cm. The entity $R = \frac{2GM}{c^2}$ (obtainable from the equality $v = c$, i.e. $\frac{GM^2}{R} = \frac{1}{2}Mc^2$) is known today as the radius of the *event horizon* of a black hole of mass M. It is remarkable that already **Newton** (1687) hypothesized that light is attracted by massive bodies.

- Luigi Cherubini 1760–1842
- Johann Hoffmann 1760–1820
- **Ludwig van Beethoven** **1770–1827**
- Johann Nepomuk Hummel 1778–1837

1751 CE **Johann Andreas von Segner** (1704–1777, Germany). Physicist and mathematician. Introduced the concept of *surface tension* of liquids.

Segner was a professor at Jena (1732–1735), Göttingen (1735–1755), and Halle (1755–1777). Invented (1750) a simple reaction waterwheel, later developed by Leonhard Euler into a crude turbine.

1751–1753 CE **Joseph-Jérome Le Francais de Lalande** (1732–1807, France) and **Nicolas-Louis de Lacaille** (1713–1762, France) obtained for the earth-moon distance the figure of 60 earth radii. The two French astronomers arrived at this result from measurement of the moon's parallax at the Cape of Good Hope and Berlin. Their result provided a more exact value than the estimates known since antiquity.

Lacaille was born at Rumigny, in the Ardennes. He studied theology at the Collège de Lisieux in Paris, and after taking his deacon's orders, he devoted himself exclusively to science. Through the patronage of **G.D. Cassini** he was employed in remeasuring the French arc of the meridian (1739). Subsequently, he was appointed a professor of mathematics in Mazarin College. His desire to observe the Southern heavens led him to propose (1750), an astronomical expedition to the Cape of Good Hope.

Lalande was born at Bourg. He studied law in Paris but was accidentally drawn to astronomy. On the completion of his legal studies he was about to return to Bourg to practice there as a lawyer, when **Lemonnier** sent him to Berlin to make observations on the lunar parallax in concert with those of Lacaille at the Cape of Good Hope.

1751–1772 CE **Jean-Jacques Rousseau**[264] (1712–1778, France). Philosopher of history and social reformer, whose ideas had great influence on Western civilization. He was the first to diagnose, from secular aspects, the symptoms of the crisis of modern civilization, that has not yet come to an end in the age of two World Wars. Both modern civilization and the entire history that shaped its features were condemned by Rousseau as deviations from nature.

[264] For further reading, see:

- De Beer, G., *Rousseau*, G.P. Putnam's sons: New York, 1972, 117 pp.

He asserted that *progress in arts and sciences was disastrous for mankind*; cultural life is degenerating more and more because vital needs of the human heart are neglected. He demanded a radical reform that does not mean return to primitive barbarism, but rather a restitution of the natural order in which reason and sentiment become harmonized. Free and equal men with inalienable wills have a right to institute a State through mutual agreement, by engaging in a social contract.

Rousseau became the precursor of the French and American Revolutions and caused a literary turmoil that started soon after the publication of his principal works [*Discours sur les arts et sciences* (1750); *Du contract social* (1762); *Emile* (1762)]. This religious creed is a deism that relies more on feelings than on reason, without excluding rational principles. In the field of education, his ideas were adopted by **Pestalozzi** (1746–1827).

Among the philosophers, his teachings are reflected in the works of **Kant** (1724–1804), **Fichte** (1762–1814), **Hegel** (1770–1831), and **Karl Marx** (1818–1883). His literary influence remained strong from the times of **Goethe** (1749–1832) and **Byron** (1788–1824) to the days of **R.L. Stevenson** (1850–1894) and **D.H. Lawrence** (1855–1930). Notwithstanding the excesses of the French Revolution [**Maximilien Robespierre** (1758–1794) was one of Rousseau's most devoted followers!], Rousseau continued to be regarded the apostle of democracy, although it it was discovered that some aspects of his philosophy favor totalitarian dictatorship.

The tormented soul of Jean-Jacques Rousseau was born to a **Huguenot** family of watchmakers in Geneva. Since the age of 10 he lived as an orphan. His unstable temperament, insatiable need for love, and deep sense of guilt explain his restless, wandering existence. An engraver's apprentice from 1727, he fled Geneva (1728), and in Annecy encountered Mme de Warens, to whom he would return at intervals in the following years. She sent him to Turin where he became a Catholic convert. Thereafter he was a footman, a seminarist, a music master (with little knowledge of music, he undertook to revolutionarize musical notation!), a tutor.

During 1732–1740 he settled near Mme Warens at Chambéry and made up for many gaps in his education by voracious readings. But from 1840 he was rootless again. He went to Paris (1741) and entered on a career in society, which included sojourn abroad as a secretary to the French ambassador in Venice (1743–1744), but otherwise he lived in Paris. He became attached (1746) to an illiterate inn-servant by whom he had 5 children, all placed in a foundling hospital.

At the same time he collaborated with Diderot in the *Encyclopedia*, almost exclusively on musical subjects. The reputation which he enjoys today as a writer is entirely founded on work written from 1750 onwards. The 1750's saw quarrels with philosophers like **Voltaire** and **Diderot**, who had formerly befriended him, and the pattern was to be repeated until his death.

With tireless energy he wrote operas, plays, novels, essays, political tracts, autobiography and social discourses, enough to fill 47 volumes. Nearly everything that came from his pen was controversial and combative enough to make for him many distinguished enemies, which he constantly fled from. These miseries fed his persecution complex.

Several years of his life were spent in exile. It was in England (1766–1767) that he wrote his *confessions* and quarreled with **Hume**. However, some respite was granted in 1770, when he found humble but quiet lodging in Paris, where he wrote his latest works. He was undoubtedly partly insane during the 10–15 last years of his life.

Rousseau's *social contract* was not only an influence on his time and his country, but also on the revolutionary founders of democracy in America.

1751–1776 CE **Denis Diderot** (1713–1784, France). Encyclopedist. One of the first *evolutionary* thinkers. He was the editor of the great *Encyclopedia of the Sciences and Crafts*, whose publication in 35 volumes, 1751–1776, impeded by government censorship, was the culminating event of the Enlightenment. Diderot himself wrote many articles on industrial and technical processes, studying them first hand for this purpose. He gained thereby a genuine appreciation for practical and experimental knowledge that led him to urge *the founding of scientific laboratories*. Diderot followed **Montesquieu** (1689–1755, France) in breaking the chains of Biblical chronology of nature. He thought that the age of the universe was a matter of several millions of years.

1752–1756 CE **James Dodson** (1705–1757, England). Mathematician, actuary[265], and innovator in the insurance industry. Published (1756) "First Lectures on Insurance".

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[265] An actuary is a business professional who deals with the financial impact of risk and uncertainty.

Actuaries have a deep understanding of financial security systems, their reasons for being, their complexity, their mathematics, and the way they work. They evaluate the likelihood of events and quantify the contingent outcomes in order to minimize losses, both emotional and financial, associated with uncertain undesirable events. Since many events, such as death, cannot be totally avoided, it is helpful to take measures to minimize their financial impact when they occur. These risks can affect both sides of the balance sheet, and require asset management, liability management, and valuation skills. Analytical skills, business knowledge and understanding of human behavior and the vagaries of information systems are required to design and manage programs that control risk.

Dodson's pioneering work on the level premium system led to the formation of the 'Society for Equitable Assurances on Lives and Survivorship' (1762) which used the actuarial principles that Dodson had developed over the previous decade. This was the first life insurance company to use premium rates which were calculated scientifically for long-term life policies.

Actuaries' insurance disciplines may be classified as life; health; pensions, annuities, and asset management; social welfare programs; property; casualty; general insurance; and reinsurance. Life, health, and pension actuaries deal with mortality risk, morbidity, and consumer choice regarding the ongoing utilization of drugs and medical services risk, and investment risk. Products prominent in their work include life insurance, annuities, pensions, mortgage and credit insurance, short and long term disability, and medical, dental, health savings accounts and long term care insurance. In addition to these risks, social insurance programs are greatly influenced by public opinion, politics, budget constraints, changing demographics and other factors such as medical technology, inflation and cost of living considerations.

Casualty actuaries, also known as non-life or general insurance actuaries, deal with catastrophic, unnatural risks that can occur to people or property. Products prominent in their work include auto insurance, homeowners insurance, commercial property insurance, workers compensation, title insurance, malpractice insurance, products liability insurance, directors and officers liability insurance, environmental and marine insurance, terrorism insurance and other types of liability insurance. Reinsurance products have to accommodate all of the previously mentioned products, and in addition have to properly reflect the increasing long term risks associated with climate change, cultural litigiousness, acts of war, terrorism and politics.

Actuaries use skills in mathematics, economics, finance, probability and statistics, and business to help businesses assess the risk of certain events occurring, and to formulate policies that minimize the cost of that risk. For this reason, actuaries are essential to the insurance and reinsurance industry, either as staff employees or as consultants, as well as to government agencies such as the Government Actuary's Department in the UK or the Social Security Administration in the US. Actuaries assemble and analyze data to estimate the probability and likely cost of the occurrence of an event such as death, sickness, injury, disability, or loss of property. Actuaries also address financial questions, including those involving the level of pension contributions required to produce a certain retirement income and the way in which a company should invest resources to maximize its return on investments in light of potential risk. Using their broad knowledge, actuaries help design and price insurance policies, pension plans, and other financial strategies in a manner which will help ensure that the plans are maintained on a sound financial basis.

In ancient Rome, the title of *actuarius* was given to the secretary of the senate, responsible for compiling the *Acta Senatus*. Prior to 1762, the use of the term had been restricted to an official who recorded the decisions (or 'acts') of ecclesiastical courts.

The 17^{th} century was a period of extraordinary advances in mathematics in Germany, France and England. At the same time there was a rapidly growing desire and need to place the valuation of personal risk on a more scientific basis. Independently from each other, compound interest was studied and probability theory emerged as a well understood mathematical discipline. Another important advance came in 1662 from a London draper named **John Graunt**, who showed that there were predictable patterns of longevity and death in a defined group, or cohort, of people, despite the uncertainty about the future longevity or mortality of any one individual person. This study became the basis for the original life table. It was now possible to set up an insurance scheme to provide life insurance or pensions for a group of people, and to calculate with some degree of accuracy, how much each person in the group should contribute to a common fund assumed to earn a fixed rate of interest. The first person to demonstrate publicly how this could be done was Edmond Halley. In addition to constructing his own life table, Halley demonstrated a method of using his life table to calculate the premium someone of a given age should pay to purchase a life-annuity (Halley 1693). Dodson built on these statistical mortality tables.

In the eighteenth and nineteenth centuries, computational complexity was limited to manual calculations. The actual calculations required to compute fair insurance premiums are rather complex. The actuaries of that time developed methods to construct easily-used tables, using sophisticated approximations to facilitate timely, accurate, manual calculations of premiums. In the 1930s and 1940s, however, rigorous mathematical foundations for stochastic processes were developed. Actuaries could now begin to forecast losses using models of random events instead of the deterministic methods. Computers further revolutionized the actuarial profession. From pencil-and-paper to punchcards to microcomputers, the modeling and forecasting ability of the actuary has grown exponentially.

1752–1773 CE **Victor Albrecht von Haller** (1708–1777, Switzerland). Physician, naturalist, anatomist, physiologist, botanist, historian of science and poet.

One of the founders of experimental physiology. Elucidated the mechanism of respiration. Discovered the function of *bile*. First to distinguish and relate muscle irritability and nerve sensibility and show transmission of nerve impulse (1752). His book *Elementa physiologiae* was the demarcation line between modern physiology and whatever preceded it (9 volumes, 1759–1776).

He was first to show (1766) that nerves stimulate muscles to contract and that all nerves lead to the spinal cord and brain.

Haller was born in Bern. He was known as a child prodigy and at age 10 already mastered Latin, Greek and Hebrew. He studied at the Universities of Tübingen (1723), Leiden (1725–1727), Paris (1728) and Basel (1728). He earned his medical degree in 1727 and studied mathematics under **John Bernoulli** in Basel. He served as a professor of medicine and botany in Göttingen (1736–1753) and practiced medicine at Bern (1753–1777). Haller published 650 articles on almost every branch of human knowledge and wrote books, treatises and bibliographies on physiology, medicine, history of science, botany, philosophy and poetry. He topped this prolific literary and research activity with three marriages, having 8 children.

In 1773 the state of his health rendered necessary his entire withdrawal from public business; for some time he supported his failing strength by means of opium; it is believed that the excessive use of the drug hastened his death.

1753–1763 CE **Carolus Linnaeus** (Carl von Linné, 1707–1778, Sweden). Naturalist and botanist. Established the modern scientific method of classification and naming of plants, animals, minerals and diseases. In this system, each living thing has a name with two parts; the first part is the *genus* (group), and the second part is for the *species* (kind). Linnaeus' book *Species Plantarum* (1753) forms the basis for plant classification. His *Systema Naturae* (1758) covers animal classification, while his *Genera Morborum* (1763) classifies diseases.

Linné was born in Råshult, in the province of Småland, Sweden. In 1726 his father destined him to be an apprentice to a shoemaker. He was, however, saved from this fate through his town physician, who expressed his belief that he would yet distinguish himself in medicine, and who further instructed him in physiology.

In 1728 he entered the University of Uppsala and studied botany. In 1732 he undertook to explore Lapland; with the equivalent of 50 dollars given to him by the Royal Society of Science, he spent 5 months collecting plants while walking nearly 1600 kilometers. Linnaeus then went to The Netherlands, where he earned his medical degree in 1735. He returned to Stockholm in 1738 to practice medicine as a naval physician. In 1741 he was appointed to the chair of medicine at Uppsala, but soon changed it for that of botany (1742). In 1761 he was granted a title of nobility with the name **Carl von Linné**.

When Linné appeared upon the scene, new plants and animals in increasing numbers were daily discovered thanks to the increase in trade. To him belongs the honor of having first enunciated the principles for defining genera and species. No naturalist has impressed his own character with greater force upon his pupils than did Linné. He imbued them with his own intensive acquisitiveness, taught them in an atmosphere of enthusiasm, trained them to close and accurate observation, and then dispatched them to various parts of the globe.

1754–1761 CE **Jean Etienne Montucla** (1725–1799, France). Historian of the mathematical sciences. His book *Histoire des mathématiques* (2 volumes, 1758; second edition, 4 volumes, 1795–1802) is essentially a history of science from a mathematical viewpoint. It is the first comprehensive modern evaluation of the evolution of mathematical thought, especially with reference to the 17^{th} and 18^{th} centuries.

Montucla was born in Lyon. He received his first education in the Jesuit College of Lyon. It included a thorough training in mathematics, Greek and Latin. He later picked up sufficient understanding of Italian, English, German and Dutch. In 1745 he studied law in Toulouse and a few years later established himself in Paris. There he came under the influence of **Diderot**, **d'Alembert**, **Lalande** and others and began his investigations on the history of mathematics. His first publication concerns the history of the attempts to square the circle (*Histoire des recherches sur la quadrature du cercle*, 1754).

After an ill-fated trip to French Guiana (1764–1765), where he was appointed royal astronomer of that colony, he lived for the rest of his life in Versailles, where he was superintendent of royal buildings, gardens, manufactures, and academies. During that period of peaceful activity, Montucla devoted his leisure to historical studies. In spite of the fact that he has been a clerk in the royal administration, he had good friends among the revolutionaries who kept him unharmed and unaffected by the revolution.

1754–1798 CE **Immanuel Kant** (1724–1804, Germany). Idealist philosopher and speculative scientist. Established a system of thought that dominated the philosophy of the 19^{th} century. No other philosopher of modern times has been throughout his work so imbued with the fundamental conceptions of physical science; no other has been able to hold with such firmness the balance between empirical and speculative ideas.

The early writings of Kant are almost without exception on questions of physical science. It was only by degrees that philosophical problems began to engage his attention, and that the main thrust of his literary activity turned toward them. The following are the most important of his works which bear directly on physical science:

- *The Nebular Hypothesis* (1755) was motivated by the faint patches of light which telescopes revealed in large numbers. A particularly troublesome item was a cloudy patch of light in the constellation *Andromeda*[266]. In his book *Universal Natural History and Theory of the Heavens*[267], Kant hypothesized a primeval, slowly rotating cloud of gas (nebula) which in some unspecified fashion condenses into a number of discrete globular bodies. The rotation of the parent nebula is preserved in the rotation of the sun, the revolution of the planets about the sun, and the rotation of the planet about their axes — all in the same direction.

According to Kant and Laplace, the original mass of gas cooled and began to contract. As it did, the rotational speed increased until successive rings of gaseous material spun off from the central mass by centrifugal forces. In the final stages the rings condensed into planets. While Laplace considered the Andromeda Nebula to represent a planetary system in the process of formation, Kant did not accept the Andromeda as a visible support of his own theory.

Instead, he suggested that Andromeda and similar bodies, might represent immensely large conglomerations of stars, which appeared as small, fuzzy

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[266] Visible to the naked eye as a small object of the 4^{th} magnitude that looks like a fuzzy star. Some Arab astronomers had noted it in their maps, but the first to describe it in modern times was the astronomer **Simon Martin** (1570–1624, Germany) in 1612.

[267] Published anonymously. The publisher went bankrupt and the stock was seized by the creditors, so that very few copies reached the public.

Laplace proposed essentially the same theory in 1796, without the mathematical formulation which he was incapable of providing. Had he been able to provide it, he might have discovered some serious flaws. Indeed, **Maxwell** and **Jeans** showed about 100 years later that there was not enough mass in the rings to provide the gravitational attraction for condensation into individual planets. The coup de grâce was delivered in 1906, when **Forest Ray Moulton** (1872–1952, U.S.A.) showed that the nebular hypothesis violated the observation that the planets carry 99 percent of the angular momentum (the sun, which collected 99.9 percent of the mass should have gathered most of the angular momentum of the system). Nevertheless, *recent* theories tend to be neo-Kantian in the sense that they revive the idea of primordial, rotating cloud of gas and dust whose shape and internal motions were determined by gravitational and rotational forces. At some moment, gravitational attraction became the dominant factor, contraction began, and the rotation speeded up. The cloud tended to flatten into a *disk*; matter began to drift toward the center, accumulating into the *proto-sun*. The proto-sun collapsed due to its own gravitation, ending with the known scenario of sustained thermonuclear reactions. The formation of the planets and how they picked up the necessary angular momentum is, however, still poorly understood.

patches only because they were immensely far away. He felt they might represent "*island universes*", each one a separate galaxy, so to speak. However, this suggestion of Kant's was not based upon any observational data available to the astronomers of the time. It made very few converts, and was dismissed as a kind of a science fiction[268].

- *Secular retardation of the earth's rotation* (1754). Pointed out that the tide-generating forces of the moon might act through the oceans to produce a breaking effect on the earth's rotation. (The attendant acceleration in the *orbital* motion of the moon had been suggested by Halley in 1695.) First to suggest that tidal friction would cause a lengthening of the day.
- Calculated (1754) that if the sun's light came from ordinary combustion, it would have burned out in 1000 years.
- Conjectured (1786) that the major forces of nature are manifestation of a single force, and can be converted one into the other.
- *Theory of winds* (1756). Independently of Hadley (1735), pointed out how the varying velocity of rotation of the successive zones of the earth's surface furnishes a key to the phenomena of periodic winds.

Consideration of these works is sufficient to show that Kant's mastery of the science of his time was complete and thorough, and that his philosophy is to be dealt with as having throughout a reference to general scientific conceptions.

Trained in the philosophy of **Leibniz**, he was influenced by the mathematical theories of **Newton**, by the psychological theories of **John Locke** (1632–1704, England), and especially by the philosophy of **David Hume** (1711–1776).

His own system was rooted in a rationalistic outlook, but sought to implant a comprehensive method and doctrine of experience that would improve upon mere intellectual idealism. His *Critique of Pure Reason* (*Kritik der Reiner Vernunft*) (1781) cost him fifteen years of critical analysis of human thought. Like other philosophers before him he maintained that only part of our knowledge is based on experience. The world we observe is only a part of a reality that we are able to conceive. Another part is *not* inferred inductively from our experience but is acquired by our senses, then filtered and elaborated by

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268 The nebular hypothesis was given observational support in 1983, when an orbiting telescope in space, the *Infrared Astronomical Satellite*, found the first evidence for disks of particles orbiting stars.

To date, such disks of gas and dust, which may be proto-solar-systems and/or debris left over from the formation of the planets, have been found around as many as a quarter of all nearby stars.

our reason, thinking and intelligence. Here, reason brings laws, order and regularity into the observed phenomena.

Thus, the laws conceived are the result of the process of reason, which does *not* derive from nature. We understand these phenomena because we approach them with certain notions and concepts for which Kant uses the term *a priori*. Among such necessary notions, or categories, are space and time. They are prerequisite and basic structures into which we must fit all our perceptions. We cannot imagine that there could be no space, even if we can imagine that there should be nothing in the space. The same reflections apply to time. Without these two *a priori* notions we should be unable to perceive a well-ordered universe.[269]

The law of *causality* is another *a priori* notion: When an event is observed, it must be determined by a preceding event. For Kant the *a priori* category of the law of causality forms an absolute necessity of all science; it is not an empirical assertion that can be proved or disproved by experiment. Rather it forms the *basis* of all experience.

These *a priori* categories are based on Newtonian mechanics, which strongly influenced not only Kant's philosophy but that of the 19th century. These laws of physics had absolute validity for Kant and were not subject to any question.[270]

Kant argued that no description of the World can free itself from the reference to human experience. Although the world that we know is not of

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[269] Kant's proposition that the human mind inevitably imposes order on the world so as to make sense of it ceased to impress scientists of the 20th century. Kant knew nothing of atomic or nuclear structure, yet the study of the atom revealed the same sort of mathematical regularities — many more of them in fact — that occur in the organization of the solar system. This fact has nothing to do with the way we choose to perceive the world. Moreover, it is difficult to be convinced that the deep and complex mathematical symmetries evinced in the operation of the fundamental forces is of no significance except as a tribute to the tidy nature of the human mind.

[270] Their limited applicability only became apparent through the results of modern physics. Moreover, in Newtonian physics, the geometry that formed the essential basis in his concepts was that of Euclid. Not till the 19th century was a new geometry developed, particularly by the pioneer work of **Gauss** (1777–1855), which then greatly influenced thinking in physics and philosophy. Kant obviously could not have foreseen the startling developments of modern physics, neither the theory of relativity nor quantum theory. The former forced the change of the *a priori* concepts of time and space. The latter demonstrated that the law of causality is not strictly applicable to events in the atom.

our creation, it cannot be known except from the point of view that is ours. All attempts to break through the limits imposed by experience, and to know the world '*as it is in itself*', from the absolute perspective of 'Pure Reason' – end in contradiction. 'Ideas' of reason can never be coherently applied, and although we may have intimations of an 'absolute' or 'transcendental' knowledge, that knowledge can never be ours: to be sure, we know only appearances, colors, sounds and the like, never the *thing-in-itself* ("Ding an Sich").

Thus Kant maintained that true knowledge cannot transcend experience. The temptation of Pure Reason, Kant argued, can never be overcome. It is part of our nature as rational beings that we should aspire towards the 'transcendental' perspective. This yearning of reason toward the eternal is at the root of morality. Transformed into practical imperative, the Ideas of Reason provide a moral law which guides us. Kant was certain that there cannot be morality without some belief in God or immortality. This obliged one to presuppose the existence of God as a necessity.

Kant's '*Idealist*' philosophy was the exact opposite to 17^{th} century *materialist* philosophy. The materialists wanted to reach an absolute truth through *science*. Kant claimed that this truth is subordinate to our senses and for this reason science is unable to discern the *essence* of things independently of the process of understanding. Reason enables man to conceive the universe but his senses prevent him from doing so. To establish universal laws one must go beyond all possible experience. The objective of knowledge is just a myth, although Kant still accepts the objective *existence* of things, which he called 'Being'. 'Being' is the very essence of things, and independent of the way in which things appear to us.

Kant was born at Königsberg. His grandfather was an emigrant from Scotland, and the name Cant is not uncommon in the north of Scotland. In his youth he studied theology and his inclination at this time was towards the classics. During his university course, which began in 1740, Kant was principally attracted towards mathematics and physics. During 1746–1755 he was much disturbed by poverty and was compelled to earn his own living as a private tutor. But with the aid of friends he was able to resume his studies, and during 1755–1770 he slowly and patiently worked his way from the rank of privatdocent to that of professor of logic and metaphysics at Königsberg.

In the course of 1781–1793, the Kantian philosophy made rapid progress in Germany, and the *Critique of Pure Reason* was expounded in all the leading universities, and even penetrated the schools of the Church of Rome. Young men flocked to Königsberg as to a shrine of philosophy.

In 1792, Kant was involved in a dispute with the government on the question of his religious doctrines, since his *moral rationalism* could not be reconciled to the literal doctrines of the Lutheran Church. The government, influenced by hatred and fear of the French Revolution, banned his writings in Berlin, and exacted from him a pledge not to lecture or write at all on religious subjects in the future. Consequently, in 1794, he withdrew from society and in 1797 he ceased altogether his public affairs, after an academic career of 42 years.

His stature was small, and his appearance feeble. He was little more than 5 feet high; his breast was almost concave, and he had a deformed right shoulder. His senses were quick and delicate, and though of weak constitution, he stayed healthy through a strict regimen. His life was arranged with mechanical regularity; and, as he never married, he kept the habits of his studious youth to old age. His man-servant, who woke him summer and winter at 5 o'clock, testified that he had not once failed in 30 years to respond to the call.

After rising, he studied for 2 hours, then lectured for another two, and spent the rest of the morning, till one, at his desk. He then dined at a restaurant (which was his only regular meal), and often held prolonged conversation until late in the afternoon. He then walked out for at least one hour in any weather, always at the same time (the burghers used to set their watches when he passed under their windows!). The evenings were spent in lighter reading[271], except for an hour or two devoted to the preparation of his next day's lectures, after which he retired between 9 and 10.

His acquaintance with books of science, general history and travels was boundless. He was fond of newspapers and works on politics. As a lecturer, Kant avoided altogether that rigid style in which his books were written. He sat behind a low desk, with a few jottings on slips of paper or book margins, and delivered an extemporaneous address, opening up the subject by partial glimpses and many anecdotes or familiar illustrations, until a complete idea of it was conveyed. His voice was extremely weak, but sometimes rose into eloquence, and always commanded perfect silence. Though kind to his students, he refused to remit their fees, as this, he thought, would discourage independence. Another of his principles was that his chief exertions should

[271] At 70 he wrote an essay "*On the Power of the Mind to Master the Feeling of Illness by Force of Resolution*". One of his favorite principles was to breathe only through his nose, especially when outdoors; hence, in autumn, winter and spring he would permit no one to talk to him on his daily walks; better silence than a cold. He applied philosophy even to holding up his stockings — by bands running up his trousers to the pockets, where they ended in springs contained in small boxes. He remained a lifelong bachelor; he felt that marriage would hamper him in the honest pursuit of truth.

be bestowed on the intermediate class of talent, as the geniuses would help themselves and the dunces were beyond remedy.

Truthful, kind-hearted and high-minded as Kant was in all moral respects, he was somewhat deficient in sentiment. He held little enthusiasm for the beauties of nature, and indeed never sailed into the Baltic, or traveled more than 60 km from Königsberg; shunned music and poetry, and held the female sex in low esteem. Though faithful in a high degree to the duties of friendship, he could not bear to visit his friends in sickness, and after their death he repressed all allusion to their memory. His engrossing intellectual efforts no doubt tended to harden his character, and in his zeal for rectitude of purpose he forgot the essential part which affection and sentiment play in human affairs.

Yet, the influence of Kant on Europe was enormous: the entire philosophic thought of the 19th century revolved about his speculations. After Kant, all Germany began to talk metaphysics: **Schiller** and **Goethe** studied him; **Beethoven** quoted with admiration his famous words; and **Fichte**, **Schelling**, **Hegel** and **Schopenhauer** produced in succession systems of thought reared upon the idealism of Kant. His criticism of reason, and his exaltation of feeling, prepared for the teachings of Schopenhauer, **Nietzsche**, **Spencer**, **Bergson** and **William James**. His identification of the laws of thought with the laws of reality gave to **Hegel** a whole system of philosophy.

Immanuel Kant made disparaging statements about Jews and non-whites. However, because of the magnitude of his achievement, scholars have tended to downplay his unwholesome writings on Jews and non-white people.

Clearly, Kant did not generate his anti-Semitism out of thin air: As with other figures of the Enlightenment (e.g. **Voltaire** and **Thomas Paine**), his mind was furnished with the medieval thinking he intended to refute. Going back to at least the 12th century, European culture had developed a distorted image of the Jews as grasping materialists and as slaves to pedantic legality. These perceived traits (encapsulated in the Shakespearean figure of Shylock) were contrasted with an idealized revision of Christianity committed to otherworld values and spiritual freedom – providing the structure for Kant's world view[272].

[272] The historian **Michael Mack** in his study "*German Idealism and the Jews*" (University of Chicago Press) argues for a deep affinity between modern anti-Semitism and the philosophy of Immanuel Kant. By Mack's account, Kant's contempt for the Jew is intimately related to the central themes of his world view, and sheds light on the limits of Enlightenment thinking. According to Mack, all the positive traits of Kantian philosophy (freedom, autonomy, reason) are formed by being contrasted with a negative image of unenlightened

Worldview XIV: Immanuel Kant

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"Human reason is burdened by questions which, as prescribed by the very nature of reason itself, it is not able to ignore, but which, as transcending all its power, it is also not able to answer."

* * *

"It is impossible to prove the existence of God through any normal means."

* * *

"Every intent, whether scientific or religious, to define reality is nothing other than pure hypothesis."

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humanity. *He saw Judaism as an inherently materialist religion, based upon a quid pro quo between God and His chosen people.*

"In order to fully define the formal structures of his philosophy (autonomy, reason, morality and freedom), Kant almost unconsciously fantasized about the Jews as its opposite," Mack notes. "*He posited Judaism as an abstract principle that does nothing else but, paradoxically, desire the consumption of material goods.*"

As portrayed in Mack's book, Kant is a pivotal figure in Western thought because he *took this earlier religious hostility toward Jews and reformulated it in philosophic language.* By showing that the traditional critique of the Jews could be made by an Enlightenment philosopher, *Kant set the stage for modern secular anti-Semitism.* In the central chapters of his book, Mack argues that what he believes is Kant's fundamental antinomy (free enlightened humanity versus *Jews enslaved to materialism*) provided the framework for future anti-Semites, notably the philosopher **G.W.F. Hegel** and the musician **Richard Wagner**. Since Wagner in particular was a cultural hero for Adolf Hitler, Kant's own anti-Semitism can be seen as having a far-reaching effect.

"Every attempt to apprehend transcendental knowledge is vain, since for every thesis the mind produces, one can oppose an equally valid anti-thesis."

* * *

"Give me matter and I will construct a world out of it."

* * *

"I call it the 'thing in itself'. I differentiate it from phenomena, that is, the world as it appears to us."

* * *

"Seek not the favor of the multitude; it is seldom got by honest and lawful means. But seek the testimony of the few, and number not voices but weigh them."

* * *

"Two things fill the mind with ever new increasing admiration and awe: the starry heavens above me, and the moral law within me."

* * *

"Concepts without factual content are empty; sense data without concepts are blind: the understanding cannot see, the senses cannon think. By their union only can knowledge be produced."

* * *

"One must understand that the greatest evil that can oppress civilized peoples derives from wars, not, indeed, so much from actual present or past wars, as form the never-ending arming for future war. To this end all the nation's powers are devoted, as are all those fruits of its culture that could be used to build a still greater culture."

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"*Understanding* is the knowledge of the general.
Judgment is the application of the general to the particular.
Reason is the power of understanding the connection between the general and the particular."*

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"Intelligence divorced from judgment produces nothing but foolishness."

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*

"The mark of a mature man is to live for a cause, that of an immature man to die for a cause."

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"Memory should only be occupied with such things as are important to be retained, and which will be of service to us in real life."

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"Science is organized knowledge. Wisdom is organized life."

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1755 CE, Nov. 01 A major earthquake destroyed two-thirds of the city of Lisbon and killed more than 60,000 people in Portugal.

1755 CE **Samuel Johnson** (1709–1784, England). Writer, moralist and scholar. Composed the first comprehensive authoritative *Dictionary of English Language*[273], including definitions for some 114,000 words. In the preface he declared that

"The Dictionary was written *with little assistance of the learned, and without any patronage of the great; not in the soft obscurities of retirement, or under the shelter of academic bowers, but amidst inconvenience and distraction, in sickness and in sorrow*".

Though plagued by ill health and stricken by the death of his wife, he produced the work just $8\frac{1}{2}$ years after he had begun. He legislated standard English into existence – by the power of a printed dictionary.

In his preface Johnson explained that language was inevitably changed by conquests, migration, and commerce, and by the progress of thought and knowledge; "*No dictionary of a living tongue ever can be perfect, since while it is hastening for publication, some words are budding and some falling away*".

1755–1783 CE **Ruggiero Giuseppe Boscovich** (Rudjer Josip Bošcović; 1711–1787, Croatia and Italy). Mathematician, astronomer and physicist. One of the earliest continental savants to adopt Newton's gravitation theory and apply it to the calculation of orbits and rotations of celestial bodies, and to the figure of the earth. Advanced an atomistic theory of matter (1758) in which atoms possess inertia and mutual interaction. He considered that chemical elements result from combination of point atoms, and chemical compounds from combination of chemical elements. These ideas influenced both **Humphry Davy** and **Michael Faraday**.

He published many remarkable memoirs, among them solutions of the problem to determine the orbit of a comet from three observations, and the *achromatic telescope* (1778).

[273] The first English dictionary appeared in 1604, authored by **Robert and Thomas Cawdrey**, schoolmaster father and his son, and entitled: "*A Table Alphabeticall, conteyning and teaching the true writing and understanding of hard usuall English wordes, borrowed from the Hebrew, Greeke, Latine, or French, etc.*" Next came *The New World of English Words* (1658) by **Edward Phillips** (1630–1696). It was followed by *A New English Dictionary* (1702) by **John Kersey**.

Two other famous dictionaries of the English language are **Noah Webster**'s *American Dictionary of the English Language* (1828) and **James A.H. Murrey**'s (1837–1915) *Oxford English Dictionary* (1925).

Boscovich was born at Ragusa in Dalmatia. Joined the Jesuits (1725), and on completing his noviciate at Rome, studied mathematics and physics at the Collegium Romanum. Taught in Rome (1740), Pavia (1764) and Milan (1770), and became director of optics for the French navy (1773–1883). He took part in the Portuguese expedition for the survey of Brazil, and the measurement of a degree of the meridian (1743). He also measured an arc of two degrees between Rome and Rimini.

In 1783 he returned to Italy. But his health was failing, his reputation was on the wane, and his works did not sell. He fell into melancholy, and finally madness, with lucid intervals, and died in Milan.

1755–1788 CE **Joseph Louis Lagrange** (1736–1813, France). One of the greatest mathematicians of the 18^{th} century. He belongs to that brilliant group of mathematicians whose magnanimous rivalries helped to accomplish the task of generalization and deduction reserved for the post-Newtonian era. Indeed, it is by no means easy to distinguish and apportion the respective merits of the competitors. This is especially the case between Lagrange and **Euler** on the one side and between Lagrange and Laplace on the other. Lagrange's mathematical career can, however, be viewed as a natural extension of the work of his older and greater contemporary **Euler**, which in many respects he furthered and refined.

In 1755 Lagrange communicated to Euler his method of multipliers for solving isoperimetric problems. He is justly regarded as the inventor of the *calculus of variations* (the name given by Euler in 1766).

During 1773–1784, Lagrange undertook the demanding task of verifying Newton's universal gravitation via the observed motions of the planets and comets of the solar system. Using the method of planetary perturbations and transferring the origin of coordinates from the center of the sun to the center of gravity of the sun-planet system, he was able to achieve great simplification.

With **Alexandre Vandermonde** he introduced in 1770 the notion of a 'group' (though not the term). In 1773 he originated the idea of scalar gravitational potential.

Lagrange took conspicuous part in the advancement of almost every branch of pure mathematics. In the theory of numbers he furnished proofs of many of Fermat's theorems, and added some of his own. In algebra he discovered the method of approximating the real roots of cubic and quartic equations by means of continued fractions. [*Traité de la résolution des équations numerique de tous degrés* (1767).] To the calculus of finite differences he contributed the beautiful formula of *interpolation* which bears his name (although substantially the same result seems to have been previously obtained by Euler) and the *Lagrange expansion* (1770).

Lagrange's contributions to the theory of equations were doubtless the most potent anticipations of Galois' later breakthrough (1831). In a 1770–1771 memoir, Lagrange attempted to find a uniform procedure for solving equations of all degrees. He analyzed the methods that had yielded general solutions for degrees 2, 3, 4, and found that in each case the technique involved the use of a *resolvent* equation. Although the latter was of lower degree than the original for $n = 2, 3, 4$, Lagrange discovered that application of the previously successful pattern to the quintic $(n = 5)$, led to an irreducible sextic $(n = 6)$, and the problem became more difficult instead of being resolved. He then hinted at the impossibility of solution by radicals, and let the matter drop.

His greatest achievement was the transformation of mechanics [defined by him as a "*Geometry of four dimensions*"] into a branch of analysis, by exhibiting mechanical principles as simple results of the calculus: instead of following the motion of each individual mass, he determined their collective configuration by a sufficient number of dynamical variables, whose number is that of the scalar motional degrees of freedom, there being as many equations as the system has degrees of freedom. The kinetic and potential energy of the system can then be expressed in terms of these, and the differential equations of motion follow by simple differentiations.

Lagrange gave the solution of isoperimetric problems quite independently of Euler, and with entirely new methods. He developed for this purpose the new *calculus of variations*.

His work had deep influence on later mathematical research, for he was the earliest first-rank mathematician to attempt a rigorization of the calculus. His cardinal idea was the representation of a function $f(x)$ by a Taylor's series. The notation $f'(x)$, $f''(x)$ is due to Lagrange. But he failed to give sufficient attention to matters of convergence and divergence, which were later taken up by his pupil **Cauchy**.

Lagrange[274] was born at Turin of mixed French-Italian ancestry. His interest in mathematics was aroused through the reading of a paper by Halley on the uses of algebra in optics. An intensive self-study for two years placed him on a level with the greatest of his contemporaries and at the age of 19 he was appointed professor of geometry in the Royal Artillery School in Turin (1754). At the age of 26, Lagrange found himself at the summit of European fame (1762). In 1764 he carried off the prize offered by the Paris Academy of Sciences for the best essay on the *librations of the moon*. He won four more such prizes: *theory of Jovian systems* (1766), *restricted 3 body problem* (1772),

[274] He was born with the name *Lagrangia*.

secular equation of the moon (1774) and *the theory of cometary perturbations* (1778).

In 1776, when Euler left Berlin for St. Petersburg, he suggested to Frederick the Great that Lagrange be invited to take his place. The invitation conveying the wish of the "greatest king in Europe" to have the "greatest mathematician" at his court, was sent to Turin. Lagrange accepted and lived in Berlin for twenty years (1766–1786) until the death of Frederick. There he had ample leisure for scientific research, and royal favor sufficient to secure him respect without exciting envy. During this period he introduced the concept of *velocity potential*, and made the first use of the *stream function* in the analysis of fluid motion (1781). In 1788 he wrote the treatise *Mécanique Analytique* in which he unified and developed analytical mechanics, introducing the '*Lagrangian*'[275] and the '*Lagrange equation*'[276]. In this book Lagrange created a new and powerful tool which could solve any mechanical problem on

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[275] For further reading, see:

- Doughty, N.A., *Lagrangian Interaction*, Addison-Wesley, 1990, 569 pp.

[276] Consider the dynamical system composed of n mass points located at vector positions $\mathbf{r}_j$ $(j = 1, 2, \ldots, n)$ w.r.t. some origin. The resultant external force on the j^{th} mass is denoted $\mathbf{F}_j$. We shall designate by q_k $(k = 1, 2, \ldots, m)$ the generalized coordinates necessary to describe the system; in general $m \leq 3n$ due to possible constraints. Since $\mathbf{r}_j = \mathbf{r}_j(q_1, q_2, \ldots, q_m)$, we have: $\delta\mathbf{r}_j = \sum_{k=1}^m \frac{\partial \mathbf{r}_j}{\partial q_k}\delta q_k$; $\dot{\mathbf{r}}_j = \sum_{k=1}^m \frac{\partial \mathbf{r}_j}{\partial q_k}\dot{q}_k$ $\left(\text{dot} = \frac{d}{dt}\right)$. We confine our attention to *holonomic* systems, that is, systems in which the δq_k and $\delta\dot{q}_k$ are independent. The virtual work is then

$$\delta w = \sum_{j=1}^{n} \mathbf{F}_j \cdot \delta\mathbf{r}_j = \sum_{k=1}^{m} Q_k \delta q_k,$$

where

$$Q_k = \sum_{j=1}^{n} F_j \cdot \frac{\partial \mathbf{r}_j}{\partial q_k}$$

are defined as *generalized forces*. The d'Alembertian principle of Virtual Work then reads:

$$\sum_{k,j} (\mathbf{F}_j - m_j \ddot{\mathbf{r}}_j) \cdot \frac{\partial \mathbf{r}_j}{\partial q_k}\delta q_k = \sum_{k=1}^{m} \left[Q_k - \sum_{j=1}^{n} m_j \ddot{\mathbf{r}}_j \cdot \frac{\partial \mathbf{r}_j}{\partial q_k} \right] \delta q_k = 0.$$

In this expression

$$m_j \ddot{\mathbf{r}}_j \cdot \frac{\partial \mathbf{r}_j}{\partial q_k} = \frac{d}{dt}\left[m_j \dot{\mathbf{r}}_j \cdot \frac{\partial \mathbf{r}_j}{\partial q_k} \right] - m_j \dot{\mathbf{r}}_j \cdot \frac{\partial \dot{\mathbf{r}}}{\partial q_k}. \tag{1}$$

the basis of pure calculation, without any reference to physical or geometrical considerations, provided that the kinetic and potential energies of the system were given in analytical form.

He returned to Paris in 1787 and accepted a professorship at the newly established École Polytechnique[277]. Marie Antoinette warmly patronized him, he was lodged at the Louvre and received a generous pension. He emerged unscathed from the turmoil of the French Revolution, since he was respected and held in affection by all political parties: the revolutionary tribunals overlooked his association with the aristocracy, and even his pension was continued by the National Assembly. Lagrange, however, was revolted by the cruelties of the Terror. When the great chemist **Lavoisier** went to the guillotine, Lagrange expressed his indignation at the stupidity of the execution: "*It took the mob only a moment to remove his head; a century will not suffice to reproduce it*".

Later in life Lagrange was subject to fits of loneliness and despondency. He was rescued from these, when he was 56, by a young and beautiful girl nearly forty years his junior — the daughter of his friend, the astronomer

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The last term in (1) is equivalent to

$$-\frac{\partial}{\partial q_k}\left[\frac{1}{2}m_j\dot{\mathbf{r}}_j\cdot\dot{\mathbf{r}}_j\right]=-\frac{\partial}{\partial q_k}\left(\frac{1}{2}m_jv_j^2\right).$$

The term preceding this takes a similar form when $\frac{\partial \mathbf{r}_j}{\partial q_k}$ is replaced by $\frac{\partial \dot{\mathbf{r}}_j}{\partial \dot{q}_k}$, allowed by $\frac{\partial \dot{\mathbf{r}}_j}{\partial \dot{q}_k}=\frac{\partial \mathbf{r}_j}{\partial q_k}$. Putting these forms into (1) and summing over the particles (j) yields:

$$\sum_k\left[Q_k-\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_k}\right)+\frac{\partial T}{\partial q_k}\right]\delta q_k=0.$$

With all the δq_k arbitrary, this can vanish if and only if, for each degree of freedom $k=1,2,\ldots,m$,

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_k}\right)-\frac{\partial T}{\partial q_k}=Q_k,$$

where T is the total kinetic energy. In cases of forces derivable from a potential $V(q_1\cdots q_k;t)$ such that $Q_k=-\frac{\partial V}{\partial q_k}$, there follow *Lagrange's equation* of motion

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right)-\frac{\partial L}{\partial q_k}=0,$$

for $L=T(q,\dot{q},t)-V(q,t)$ and $k=1,2,\ldots,m$.

[277] A famous school in the history of mathematics, where many of the great mathematicians of modern France were trained and held professorships.

Lemonnier (1715–1799). She was so touched by Lagrange's unhappiness that she insisted on marrying him (1792). He had no children by this or his former marriage.

Lagrange was nominated president of the Academic Commission for the reform of weights and measures and for the establishment of the metric system. Napoleon loaded him with personal favors and official distinctions. He became a senator, a count of the Empire, and a grand officer of the Legion of Honor.

Toward the end of his life, Lagrange felt that mathematics had reached a dead end and that the physical and biological sciences would attract the ablest minds of the future. His pessimism might have been relieved if he had been able to foresee the coming of Gauss and his successors, who made the 19^{th} century the richest in the long history of mathematics.

He was buried in the Pantheon on April 10, 1813. The funeral oration was pronounced by Laplace. Hamilton called Lagrange the "Shakespeare of Mathematics" on account the extraordinary beauty, elegance and depth of his methods.

The astronomy historian **Agnes Mary Clerke** (1842–1907) succinctly summarized his life work in the statement:

"*His treatises are not only storehouses of ingenious methods, but models of symmetrical form. The Clearness, elegance and originality of his mode of presentation give lucidity to what is obscure, novelty to what is familiar, and simplicity to what is abstruse. His genius was one of generalization and abstraction, and the aspirations of the time towards unity and perfection received, by his serene labors, an embodiment denied to them in the troubled world of politics*".

Lagrange and the '3-Body Problem' (1772)

Euler *(1760) seems to be the first to have studied the general problem of three bodies' motion under their mutual gravitation, although at first he only considered the restricted three bodies problem when one of the bodies has a negligible mass.*

It is then assumed that the motions of the other two can be solved as a two body problem, the body of negligible mass having no effect on the other *two. Then the problem is to determine the motion of the third body attracted to the other two bodies which orbit each other.*

Even in this form the problem does not lead to exact solutions. ***Euler****, however, found a particular solution with all three bodies in a straight line.*

The Paris Academy Prize of 1772 for work on the orbit of the Moon was jointly won by ***Lagrange*** *and* ***Euler****.* ***Lagrange*** *submitted Essai sur le problème trois corps in which he found another solution where three bodies were at the vertices of an equilateral triangle.*

The motion of an isolated system of two attracting point masses[278] *is solvable exactly in the framework of Newtonian dynamics. Lagrange considered*

[278] "Point mass": a model for a spherically-symmetric mass distribution, which for purposes of Newtonian attraction is considered to be concentrated at the sphere's center. It is tacitly assumed in astronomical applications that the mass' dimensions are small compared to the inter-mass distance (no other restriction on size of mass).
While this is only a very good first approximation for real masses in the universe, it nevertheless underlines the mathematical theory of nearly all problems in celestial mechanics. In the following, the word 'mass' or 'body' will mean 'point-mass', unless otherwise stated.
The attraction (external gravitational potential) of a finite body that is *not necessarily symmetric*, at a point P a distance r from the body's mass-center O, is given by the expression

$$\phi(P) = -G\left[\frac{M}{r} + \frac{\mathfrak{Q}\colon \mathbf{e}_r\mathbf{e}_r}{2r^3} + O\left(\frac{1}{r^4}\right)\right].$$

Here $M = \int \rho(\mathbf{r}')dr'$ is the total mass $(r' \ll r)$, $\rho(r)$ the density, $\mathfrak{Q} = \int \rho(\mathbf{r}')(3\mathbf{r}'\mathbf{r}' - r'^2 I)dr'$ is the *mass quadrupole moment tensor*, and $\mathbf{r} = r\mathbf{e}_r$ is the position vector of the field point relative to the mass center.
The potential ϕ can be represented in a number of alternative forms:

(I) *Maccullagh's formula*: same as the above with $\mathfrak{Q}\colon \mathbf{e}_r\mathbf{e}_r = A + B + C - 3J$, where $\{A, B, C\}$ are the principal moments of inertia of the body relative to the *mass center*, and J is the body's moment of inertia about the *axis* OP.
(II) *Multipole expansion*:

$$\phi(r > r') = -G\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\frac{4\pi}{2\ell+1}\left[\frac{\widehat{Y}_{\ell m}(\theta,\varphi)}{r^{\ell+1}}\right]Q_{\ell m}$$

where

$$Q_{\ell m} = \int \rho(\mathbf{r}')r'^{\ell}\widehat{Y}^*_{\ell m}(\theta',\varphi')d^3\mathbf{r}';$$

$$\widehat{Y}_{\ell m}(\theta,\varphi) = \sqrt{\frac{2\ell+1}{4\pi}\frac{(\ell-|m|)!}{(\ell+|m|)!}}\;P_\ell^{|m|}(\cos\theta)e^{im\varphi}\delta_m;$$

the motion of an isolated system of three masses, free to move under their mutual Newtonian attractions. It is governed by 3 2^{nd} order vector ODE's, or equivalently by a system of 18 first-order ODE's. An explicit analytic solution does not exist[279] *for the general case. This is so because only* 10 *out of the* 18 *constants of integration (i.e. coordinates and velocities of each mass at a common fiducial time) are expressible as conservation laws. They are: the system's total mechanical energy content,* 6 *components of the mass-center position and velocity vectors, and* 3 *components of the total angular momentum vector at any chosen time.*

To see this, we write the equations of motion in the form:

$$\ddot{\mathbf{r}}_1 = G\left[\frac{m_2}{r_{12}^3}\mathbf{r}_{12} - \frac{m_3}{r_{31}^3}\mathbf{r}_{31}\right];$$

$$\ddot{\mathbf{r}}_2 = G\left[\frac{m_3}{r_{23}^3}\mathbf{r}_{23} - \frac{m_1}{r_{12}^3}\mathbf{r}_{12}\right];$$

$$\ddot{r}_3 = G\left[\frac{m_1}{r_{31}^3}\mathbf{r}_{31} - \frac{m_2}{r_{23}^3}\mathbf{r}_{23}\right],$$

where (m_1, m_2, m_3) *are the three masses in question,* G *is the universal constant of gravitation,* $\{\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3\}$ *are the time-dependent respective position-vectors of the masses w.r.t. an inertial frame of reference and* $\mathbf{r}_{jk}$ *is a vector with origin at* m_j *and terminus at* m_k*. Simple algebraic and differential manipulations of the above equations yield the following results:*

$$\delta_m = \begin{Bmatrix} 1 & m \geq 0 \\ (-)^m & m < 0 \end{Bmatrix}.$$

When this expansion is applied to a nonspherical earth (bulged at the equator, flattened at the poles but azimuthally symmetric), it reduces to

$$\phi = -\frac{GM}{r}\left[1 - 10^{-6}\sum_{\ell=2}^{\infty} J_\ell \left(\frac{r_e}{r}\right)^\ell P_\ell(\sin\lambda)\right],$$

where $r_e =$ equatorial radius, $\lambda =$ geocentric latitude $= \sin^{-1}\frac{z}{r}$, and $J_2 = 1082.64 \pm 0.03$; $J_3 = -2.5 \pm 0.1$; $J_4 = -1.6 \pm 0.5$; $J_5 = -0.15 \pm 0.1$; $J_6 = 0.57 \pm 0.1$; $J_7 = -0.44 \pm 0.1$.

279 Although there is no closed-form analytical solution, the problem is still solvable numerically by the use of computers, since the number of equations and unknowns is compatible. However, the computer cannot give answers to questions about the behavior of the system which require an *infinite time* to answer, such as whether some member of the system will escape from it or eventually collide with another member.

(1) *The center of mass of the system either remains at rest or moves uniformly in space on a straight line.*

(2) *The total angular momentum of the system is fixed in magnitude and direction at all times.*

(3) *The sum of potential and kinetic energies of the system is constant.*

Result (1) is mathematically expressed as

$$\mathbf{R} \equiv \frac{1}{M}(m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + m_3\mathbf{v}_3) = \frac{\mathbf{a}_1}{M}t + \frac{\mathbf{a}_2}{M}$$

where $M = \sum_{i=1}^{3} m_i$ *and* $\{\mathbf{a}_1, \mathbf{a}_2\}$ *are two constant vectors. This equation therefore yields* 6 *scalar equations satisfied by the mass coordinates.*

Result (2) has the mathematical form

$$\frac{d}{dt}\mathbf{H} = 0, \qquad \mathbf{H} = (\mathbf{r}_1 \times m_1\mathbf{v}_1) + (\mathbf{r}_2 \times m_2\mathbf{v}_2) + (\mathbf{r}_3 \times m_3\mathbf{v}_3).$$

It consists of 3 *additional scalar relations linking the positions and velocities of the masses.*

Result (3) reads

$$\frac{1}{2}\sum m_i\dot{\mathbf{r}}_i^2 - G\left(\frac{m_1m_2}{r_{12}^2} + \frac{m_2m_3}{r_{23}} + \frac{m_3m_1}{r_{31}}\right) = E = const.$$

It yields one additional relation (integral of motion). Thus, 10 *of the* 18 *integrals which are necessary for complete solutions are known, and they are all algebraic functions when expressed in rectangular coordinates. In 1887,* ***H. Bruns*** *showed that when rectangular coordinates are chosen as the independent variables in the* 3*-body problem, the* 10 *integrals (constants) described above are the only integrals to be expected, and no more such algebraic integrals exist.* ***Poincaré*** *showed, in the same year, that there are no new transcendental integrals, even when the masses of all bodies except one are small.*

Knowing all this (although lacking Brun's proof), ***Lagrange*** *sought particular exact solutions to the* 3*-body problem which do not require more than* 10 *constants of integration. These particular solutions were discovered by him in a prize memoir in* 1772.

Lagrange considered two separate problems. The first, known as the restricted three-body problem, is of particular importance in discussions of space probes moving in the gravitational fields of the earth and the moon. In this *case one of the three masses, referred to as the 'particle', is so small in comparison with the other two that its gravitational effects on these two masses can be neglected. The particle is thus considered as a* test mass *whose motion is the object of calculation. [The earth-moon system together with a small artificial satellite or spacecraft constitutes such a system, if we ignore the presence of the sun, the lack of sphericity of the earth and the eccentricity of the moon's orbit.]*

It is then advantageous to set up the following model: two massive bodies (masses m_1 and m_2 respectively) move in circular *orbits about their center of mass, which is taken as the origin of a coordinate system that revolves around its z-axis (such that the xy axes are in the plane of motion of the two finite masses) with an angular velocity that is equal to their orbital velocity, $\omega = \frac{G\sqrt{m_1+m_2}}{a^{3/2}}$, $(m_1 \geq m_2)$ by Kepler's third law (a = radius of circular orbit).*

In this special rotating system, the equation of motion of the particle is

$$\mathbf{a} + 2(\boldsymbol{\omega} \times \mathbf{v}) + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) = -G\left[\frac{m_1}{\rho_1^3}\boldsymbol{\rho}_1 + \frac{m_2}{\rho_2^3}\boldsymbol{\rho}_2\right],$$

where $\boldsymbol{\omega} = \omega\mathbf{e}_z$; $\mathbf{a} = \ddot{x}\mathbf{e}_x + \ddot{y}\mathbf{e}_y + \ddot{z}\mathbf{e}_z$, $\mathbf{r} = x\mathbf{e}_x + y\mathbf{e}_y + z\mathbf{e}_z$; $\{\boldsymbol{\rho}_1, \boldsymbol{\rho}_2\}$ are the vector distances of the particle from the masses m_1 and m_2 respectively and $\{\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z\}$ are the three unit vectors along the axes of the rotating system.

The general problem of determining the motion of the test particle requires six integrals for its complete solution. Simple manipulations of the above equation of motion yield the first integral in the form which resembles the energy integral in a two-body system. Choosing the unit of mass such that $m_1 + m_2 = 1$ [i.e. $m_1 = 1 - m$, $m_2 = m$], and furthermore choosing the units of distance and time such that $G = 1$, $a = 1$ $(\omega = 1)$, the first integral reads

$$\dot{x}^2 + \dot{y}^2 = (x^2 + y^2) + \frac{2(1-m)}{\rho_1} + \frac{m}{\rho_2} - c.$$

There remain five integrals to be found. However, by restricting the motion of the particle to the xy plane, it is possible to reduce the total number of the needed constants to a total of two[280]*. One new integral is therefore needed, but cannot in principle be found, on the strength of the above-mentioned Brun's theorem. Nevertheless, the first integral is sufficient to deduce the*

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280 This was shown by **C.G.J. Jacobi**, in 1844. The first 'energy-like' integral is also due to him and known as *Jacobi's integral*. Lagrange arrived at his results in another way.

salient features of the motion of the particle, as was done by Lagrange: he derived the contours of zero velocity, by examining the equipotential contours

$$x^2 + y^2 + \frac{2(1-m)}{\sqrt{(x-x_1)^2+y^2}} + \frac{2m}{\sqrt{(x-x_2)^2+y^2}} = c.$$

It turns out that there are five points in the xy plane, known as the Lagrangian points[281]*, which are of special significance in the three body problem. At each of these 5 positions (relative to the two masses in mutual circular revolution) the particle, once placed, will also move on a circular orbit, always maintaining a fixed orientation w.r.t. to the other two masses. Three of these points, called the Lagrangian points L_1, L_2 and L_3, along the line joining the two masses, are* unstable, *in the sense that if the particle is displaced slightly from one of them, it will leave its circular orbit.*

Because small perturbations are always likely to occur, we would not expect to find many examples in nature in which three bodies revolve exactly in those configurations.

The two remaining points, known as L_4, and L_5 ("L" in honor of Lagrange) are, however, stable. *A particle at one of those positions* cannot *be*

[281] The equipotential contours display the combined gravitational fields of the two massive objects. The field has a constant strength at each point (x, y) along the curve. Thus, we can think the equipotential contour map as a sort of topographical map, showing 'hills' and 'valleys' in the gravitational field. A small object like an asteroid can be permanently trapped at one of the stable Lagrange points.

In 1776, *asteroids* had not yet been discovered, and Lagrange knew no actual case that would demonstrate the existence of the $\{L_1, L_2, L_3\}$ points. However, 90 years after Lagrange's theoretical work, **Daniel Kirkwood** (1866) showed that it applied perfectly to Jupiter and the asteroids. Those places between Mars and Jupiter where no asteroid would be found have been known as *Kirkwood's gaps* ever since.

The *Trojan asteroids* at L_4 and L_5 along Jupiter's orbit have been known since 1906 and provided the first proof of Lagrange's theoretical ideas about these points. Since then, stable Lagrange points have been found to exist at many places in the solar system; while passing Saturn, the *Voyager* spacecraft (1980) discovered tiny satellites at the L_4 and L_5 points of the Saturn-Tethys and the Saturn-Dione systems. A group called the L_5 *society* argues that the L_5 point on the Earth-Moon system would be an ideal location for a huge space station with a permanent human population. Despite careful searches, no asteroids have been found at the stable Lagrange points of the Earth-Sun and Saturn-Sun systems.

forced away by slight perturbations. It can be shown that in these configurations, the particle and the two masses are at the corners of an equilateral triangle*. We do, in fact, find natural examples of this kind of motion: The best known is the equilateral configuration defined by the sun, the planet Jupiter and the two groups of* Trojan asteroids *(the sun and Jupiter move in nearly circular paths around their mutual COM, and the minor planets have negligible mass in comparison).*

Lagrange's solution to the restricted 3-body problem also specifies the regions of space within which the particle can *move relative to the two larger ones. In recent years his theory found another application in the theory of evolution of massive stars: there are many* binary star systems *in which the two stars revolve about each other in nearly circular orbits. If the two stars are relatively close together and if one evolves to a large enough size, the atoms of its outer distended layers, having negligible mass, move about (in the role of particles) in a manner predicted by Lagrange.*

We thus find that during the evolution of stars in binary systems, matter can flow from one star to another, or can flow in an orbit around one or both stars, or can even flow into space, escaping the two stars altogether (from the inner Lagrangian points). This mass exchange*, believed to occur between many stars in closed binary systems, can have profound effects of the evolution of the stars in a system, possibly accounting for such phenomena as* novae *and* supernovae*. It can also lead to the formation of a large circumstellar disk or ring of matter around the binary system, and even be involved in the creation of* neutron stars *and* black holes*.*

The second problem considered by Lagrange was that of special stationary solutions of the three-body problem for arbitrary *masses. By a stationary solution we mean one in which the geometric configuration of the three masses remains self similar w.r.t. time. If the motion of the masses is such that their mutual distances from each other remains unchanged, the configuration simply* rotates in its own plane *around the center of mass. On the other hand, an expansion or contraction may take place which does not alter the* shape *of the patterns of points.*

Lagrange showed that there are only two such configurations: one in which the three masses lie on a straight line*, and the other in which the masses form an* equilateral triangle *whose base is the segment* a *between two masses. In this latter case the motion is such that the plane through the three masses is fixed in space while the plane rotates with fixed angular velocity* $\omega = \left\{\frac{G(m_1+m_2+m_3)}{a^3}\right\}^{1/2}$ *onto itself. The resultant Newtonian forces on each of the three masses passes through their common mass center. Finally, the three points describe conic sections similar to each other, with the common mass-center lying at the focus. In a coordinate system rotating with angular velocity*

ω in the plane of the masses, the Lagrangian points (masses) are fixed: at these points, the gravitational and the centrifugal forces just balance each other.

In the straight-line solution, the masses will be located at Lagrangian points (also known as libration points) if they are arranged on a line (the x-axis, say) with coordinates $\{x_1, x_2, x_3\}$ such that if $x_2 - x_1 = 1$, then $x_3 - x_2 = p$, where p is the only positive root of the quintic equation:

$$(m_1 + m_2)p^5 + (3m_1 + 2m_2)p^4 + (3m_1 + m_2)p^3$$

$$-(m_2 + 3m_3)p^2 - (2m_2 + 3m_3)p - (m_2 + m_3) = 0.$$

The angular velocity for this case[282] *is*

$$\Omega = \omega \left[\frac{m_1 p^2 - m_3}{m_1 p^2 - m_3 p^3}\right]^{1/2}.$$

[282] To translate this into a mathematical language, we write the total coplanar acceleration of any one of the mass points $\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\mathbf{e}_\theta$, where $\mathbf{r} = r\mathbf{e}_r$ is its position vector (drawn from the system's mass-center) and $\{\mathbf{e}_r, \mathbf{e}_\theta\}$ are unit vectors in radial and transverse directions respectively. Since we have assumed $\dot{r} = \ddot{r} = 0$, $\dot{\theta} = \omega =$ the constant angular speed of revolution about the mass-center, the accelerations become $\mathbf{a}_i = -r_i\omega^2\mathbf{e}_i$ where $(\mathbf{e}_r)_i = \mathbf{e}_i$ $(i = 1, 2, 3)$. Using these values in the general equations of motion given at the beginning of this section, we obtain the *differential equations*

$$\ddot{r}_i = -\omega^2 r_i \qquad (i = 1, 2, 3),$$

where general solutions are conic sections. If the orbits are ellipses, where ε_s is the *eccentricity* and E_s is the *eccentric anomaly*, then $r_i = a_i(1 - \varepsilon_s \cos E_s)$. Thus the various conics described by the three bodies are all similar and the masses occupy corresponding positions in their orbits at any given instant. To derive the dependence on the angular velocity ω on the constants of the system we must solve the three simultaneous *algebraic vector equations*

$$-\omega^2\mathbf{r}_1 = \frac{m_2}{r_{12}^3}\mathbf{r}_{12} - \frac{m_3}{r_{31}^3}\mathbf{r}_{31};$$

$$-\omega^2\mathbf{r}_2 = \frac{m_3}{r_{23}^3}\mathbf{r}_{23} - \frac{m_1}{r_{12}^3}\mathbf{r}_{12};$$

$$-\omega^2\mathbf{r}_3 = \frac{m_1}{r_{31}^3}\mathbf{r}_{31} - \frac{m_2}{r_{23}^3}\mathbf{r}_{23}$$

with the additional center of mass condition

$$m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + m_3\mathbf{r}_3 = 0,$$

where $\mathbf{r}_{12} = \mathbf{r}_2 - \mathbf{r}_1$; $\mathbf{r}_{23} = \mathbf{r}_3 - \mathbf{r}_2$; $\mathbf{r}_{31} = \mathbf{r}_1 - \mathbf{r}_3$.

In conclusion, the n-body problem $(n > 2)$ can be solved in general only by laborious numerical calculations. There are, however, some special circumstances in which there exist solutions, or partial solutions, in the form of algebraic equations. Usually, these solutions apply only when the mass-system has a very particular and most unlikely configuration. Nevertheless, the Lagrange's theory can serve as an approximate model for certain important and interesting astrophysical phenomena.

There is yet another aspect of the three-body problem that one should not overlook: consider the case of two spherical bodies that move under the influence of their mutual attractions, each describing a conic section w.r.t. their center of mass as a focus. If there is a third body attracting the other two under consideration, their orbits will cease to be exact conic sections. The difference between the coordinates and components of velocity in the actual orbits *and those which the bodies would have had if the motion had been undisturbed are the* perturbations.

For example: in the solar system, to first approximation, each planet moves as though it and the sun constituted a two-body system. This suggests, therefore, that we first study the motion of a planet as a part of a two-body system. Then we determine the deviations from a purely two-body motion that will result from the presence of other disturbing bodies, namely — the perturbations. This approach supplements the above results of Lagrange, who was interested in the motion of the test particle and ignored its perturbing effect on the large masses.

It is of interest to note that Lagrange considered his solutions for the 3-body problem as inapplicable to the solar system. We now know, however, that both earth and Jupiter have asteroids sharing their orbits in the equilateral triangle solution configuration discovered by Lagrange. For Jupiter, these bodies are called Trojan planets, the first to be discovered being Achilles (1908).

The first comet to have a calculated elliptical orbit which was far from a parabola was observed (1769) by **Charles Messier** *(1730–1817). The elliptical orbit was computed by* **Lexell** *(1740–1784) who correctly realized that the small elliptical orbit had been produced by the perturbation of Jupiter. The comet made no reappearance and Lexell correctly deduced that Jupiter had changed the orbit so much that it was thrown far away from the sun.*

The Lagrange theory found interesting and important applications in recent times. It was shown that the forces at the Lagrangian balance-points could capture objects and keep them orbiting. The European Space Agency has taken advantage of one balance point by launching a sun-observatory called SOHO that currently orbits at L_1. The orbits of objects at these *points are exotic, often tadpole-shaped and rarely horseshoe-like. The horseshoe orbit involves movement around L_3, L_4 and L_5 points.*

In 1986 astronomers discovered a new Near-Earth asteroid, named 3753 Cruithne[283] *(also known as 1986 TO). At that time no one had tracked its path thoroughly enough to detect its rare orbit. Then, in 1997, it was found that Asteroid 3753 follows a spectacular horseshoe orbit and has characteristics never before seen or even anticipated, either in theory or in computer simulations.*

Cruithne is co-orbital with the earth (meaning that it shares the earth's orbit). In a co-rotating frame with the earth (in which earth is stationary) Cruithne is on an spiraling horseshoe orbit: every year, the asteroid traces out a kidney bean. Over time, this kidney bean drifts along the earth's orbit, tracing out a spiral which, when complete (after 385 years) fills in an overlapping horseshoe.

Cruithne avoids collision with earth: at its closest approach it only gets to within 15 million km. Each year, it is at its closest in the autumn, and at this point it will pass almost directly beneath the earth's South Pole.

The asteroid is about 5 km wide and takes 770 years to complete its horseshoe-shaped orbit around the earth. It is believed that it is a temporary companion, remaining in a suspended state around the earth for at least 5000 years.

The Lagrange theory for this 3-body problem (earth-Cruithne-sun) provides for new dynamical channel through which free asteroids become temporarily moons of the earth and stay there for periods ranging from a few thousand years to several tens of thousands of years. Thus, some asteroids that cross the earth's orbit may be trapped in orbits caused by the gravitational dance between earth and sun.

It is believed that the laws of nature would make it very difficult for an asteroid to have entered into this orbit recently. The asteroid may be as old as the solar system itself, and it might have found its way into this orbit when the solar system was forming. On the other hand, if it joined us more recently, the mechanics and physics that would have been needed to get this asteroid into orbit in recent times are akin to threading a needle.

Asteroid 3753 is following the most complicated horseshoe orbit ever seen, and it is unique in our solar system. It has unique characteristics, including: a spiraling motion; a big inclination (titled path) and an overlap at the end of the horseshoe.

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[283] The *Cruithne* (=croo-een-ya) were the first Celtic tribal group to come to the British Isles between about 800 to 500 BCE.

1754–1778 CE **Joseph Black** (1728–1799, Scotland). Physician, chemist and physicist. Rediscovered *carbon dioxide* (1754). First to introduce the concepts and theories of *heat capacity* and *latent heat* (1760). These theories contributed substantially to Watt's development of the steam engine. He visualized heat as a certain imponderable fluid (called "*calor*"), which can penetrate all material bodies and thus increase their temperature. Mixing a gallon of boiling water with a gallon of ice cold water, he noticed that one finds the temperature of the mixture just halfway between two initial temperatures, and he interpreted this fact by saying that, after the mixing, the excess of "calor" in hot water is equally distributed between the two portions.

He defined the unit of heat as the amount necessary to raise the temperature of 1 lb of water by 1 °F (in the modern metric system we speak of *calorie*, which is the amount of heat it takes to raise the temperature of 1 gm of water by 1 °C). He concluded that equal weights of different materials heated to the same temperature contain different amounts of "calor" since, indeed, by mixing equal weights of hot water and cold mercury, one gets a temperature which is much closer to the original temperature of water than of mercury. Therefore, he argued, cooling a certain amount of water by 1 ° liberates more heat than is necessary to heat an equal weight of mercury by 1 °.

This led him to the notion of the *heat capacity* of different materials, characterized by the amount of heat needed to raise their temperature by 1 °. Another important notion introduced by Black was that of *latent heat*, which is the amount of heat needed for a change of phase, e.g. to turn ice into ice water (both at 0 °C), or to burn boiling water into water vapor (both at 100 °C). He thought that adding a given amount of the imponderable heat fluid to a piece of ice loosens up its structure, making it liquid, and that, in a similar way, adding more heat to the hot water further loosens its structure, turning it into vapor.

Black was born at Bordeaux, where his father, a native of Belfast but of Scottish descent, was engaged in the wine trade. He studied medicine in Glasgow. James Watt, at the University of Glasgow, later absorbed the thermal theories of Joseph Black (by then professor of medicine at the university) and applied them in his invention of the improved condensing steam-engine (1765).

1756–1763 CE *Seven Years' War*. Prussia and Austria fought for control of Germany in a war that involved nearly every nation in Europe. It pitted Prussia (Frederick the Great) against Austria, Sweden and France. The war ended exactly where it began, with no territorial changes in Europe. In North

America, however, France gave up Canada to Britain and also yielded its colonies in North America.

1756–1774 CE **John Smeaton** (1724–1792, England). Civil engineer and inventor. Founder of the civil engineering profession during the early days of the Industrial Revolution in England. Improved instruments used in navigation and astronomy. His major achievements were

- Rediscovered (1756) *hydraulic cement*, unknown since the fall of Rome.
- Made improvements on windmills and watermills (1759).
- Design large pumping engines; improved diving bell; rebuilt *Eddystone* lighthouse (1759).
- Constructed Ramsgate harbor (1774), Forth and Clyde Canal, and Perth, Banff, and Coldstream bridges.
- Improved the steam-engine of James Watt (1775).

Smeaton was born at Austhorpe Lodge, near Leeds. Left the grammar school of Leeds in his 16 th year to become apprentice to an instrument maker and in 1750 set up his own business. In 1759 he read a paper before Royal Society entitled '*An Experimental Inquiry concerning the Native Powers of Water and Wind to turn Mills and other Machines depending on a Circular Motion*' for which he received the Copley medal.

In 1754 he made a tour of the Low Countries to study the great canal works there. He died at Austhorpe and was buried in the old parish church of Whitkirk.

The Watch[284] ***and Modern Time-Culture*** (1502–1760)

The invention of portable timepieces dates from the end of the 15^{th} *century, and the earliest manufacture of them was in Germany. It is known that* **Peter Henlein** *(1480–1542), a locksmith of Nuremberg, built, during* 1502–1510, *a small round clock with steel mainspring enclosed in a box. It was known as the Nuremberg Egg. Being too large for the pocket it were frequently hung from the girdle. It was the first pocket watch ever made. Before Henlein invented the watch, time was told by clocks that used heavy weights. The mainspring supplied the power to turn the wheels. The manufacture of watches by hand soon spread throughout Europe. The difficulty with these early watches was the inequality of action of the mainspring.*

An attempt to remedy this was provided through 1525–1540. *In early watches, the escapement was the same as in early clocks, namely, a crown wheel and pallets with a balance ending in small weights. Such an escapement was, of course, very imperfect; since the force moment acting on the balance does not vary with the displacement, the time of oscillation varies with the arc, and this in turn varies with every variation of the driving force. An immense improvement was therefore effected when the hair-spring was added to the balance, which was replaced by a wheel. This was done about the end of the* 17^{th} *century.*

During the 18^{th} *century a series of escapements were invented to replace the old crown wheel, ending in the chronometer escapement. Though great improvements in detail have since been made, the modern mechanical watch may nevertheless be called an* 18^{th}*-century invention.*

Early watches had only an hour hand. The minute hand was developed in 1687. *In the* 1800*'s, new machinery made it possible to produce accurate watches cheaply.*

The invention of the clock in the 14^{th} *century and its technical improvements during the* 15^{th} *and* 16^{th} *centuries, rendered a useful means for the fulfillment of religious and social functions: it was regularly fixed on the front walls of churches and town halls, or placed in city squares, where its chimes*

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[284] From the Old English word *waecce* = a keeping guard or watching, from *wacian* = to guard, watch, *wacan* = to wake. Hence watch = that which keeps watchful or wakeful observation or attention over anything. The term was used for persons who patrolled the streets, called the hours, and performed the duties of modern police. The term was later applied to a period of time marked by the change of sentries or ship crews.

served to assemble the burghers to prayer and public meetings. Only in the 16th *century, when clocks became part of the household, and more so in the* 17th *century, when portable watches were in the private use of individuals, did the* modern time-culture *begin. This was the time when man's concept of time underwent a* revolutionary *change.*

Prior to the invasion of the metric time keeper into the household, the day- and night-cycle and the annual cycle of the seasons dominated the conduct of human life. In the agrarian pre-industrial society, all activities were predetermined by the calendar*, by the constant march of generations and the ages of man, and the periodic change of the seasons. Beyond that was the consciousness of the existence of an eternity beyond life, granted by faith. This routine was totally disrupted when life according to the calendar changed to life according to the watch; Western civilization has come to be* dominated *by the clock and the timetable, and Westerners have had little sympathy with people who have escaped this domination.*

Hours, days and periods that were of unique value to societies and cultures, and previously sanctioned by their calendars, were absent from the indifferent faces of the new timekeepers, which from now on became the only measure of homogeneous, universal and objective time. During the next 300 *years, there occurred a gradual but perpetual subjugation of all norms, concepts and values to clock time; Westerners adopted a new puritan world outlook, known by its motto:* TIME IS MONEY *(**Benjamin Franklin**, 1733).*

The message was clear: no more spontaneous prayers, easygoing work, communal togetherness and mutual aid; life became more mechanized and more personal. Cooperation gave way to mere synchronization. People became more punctual, more pedantic, more purposeful. Even basic functions such as eating and sleeping became mundane; Europeans dined not out of hunger but when the clock said so, and they turned in to sleep not when tired but when the time came. Puritans turned from a life of meditation and abstinence to a life of creativity and labor. The harnessing of inanimate physical forces in the Industrial revolution made it possible for work to be carried on for 24 *hours a day throughout the year — under cover, by artificial light, and at a controlled temperature.*

But that was not all: clock-time intensified man's consciousness of the fleeting moment, the discretized unit of time, and increased his fear of death[285]*. Time became less abstract and more real, symbolized by the per-*

[285] Great poets and artists, as always, are first to feel the deep implications of social changes.

The clock as a harbinger of death is clearly portrayed in the sonnets of **Shakespeare** (1609) and the woodcuts of **Hans Holbein**, "*The Dance of Death*" (1538).

petual moving hands of the clock. It became the idol of a new mercantile-industrial society. The value of goods was measured by the time needed to produce it and vice versa: the value of time was measured by the amount of goods produced. The clock thus became a machine that produced time!, and obviously, like any other material object whose value is measured by it usefulness, the collective time became redundant after its use; it lost its moral value as soon as it passed and there was no motivation whatsoever in its keeping. The clock turned time into a one dimensional disposable entity that is not accumulated in any cultural collective consciousness or tradition. It did not turn anymore into a significant past — it became a historic time.

1757–1776 CE **Johann Heinrich Lambert** (1728–1777, Germany). Physicist, mathematician and astronomer. Came close to being the founder of non-Euclidean geometry. His mathematical discoveries were extended and overshadowed by the work of his contemporaries.

In 1770 he derived the continued fraction representation

$$\pi = 3 + \cfrac{1}{7 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{292 + \cdots}}}}.$$

It yields as partial fractions the historical approximations $\frac{3}{1}$, $\frac{22}{7}$, $\frac{333}{106}$, $\frac{355}{113}$, $\frac{103993}{33102}$, $\frac{104348}{33215}$, $\cdots$ where the 4^{th} fraction yields π with an error of at most 3 units in the 7^{th} decimal place[286]. Later (1776), Lambert proved that π and e are *irrational*[287].

286 This expansion does not seem to have any regularity. Apparently, it was obtained by transforming the decimal fraction for π into a continued fraction. The first 23 terms are: $[3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1, 84, 2, \ldots]$.

287 Lambert's proof is rather hard. The following simple proof, of unknown origin, appeared in the *Mathematics Preliminary Examination* at Cambridge in 1945: Consider the integral $I_n = \int_{-1}^{1}(1-x^2)^n \cos\left(\frac{\pi}{2}x\right)dx$. Two integrations by parts yield $I_n = \left(\frac{2}{\pi}\right)^{2n+1} n! P_n$, where P_n is a polynomial in $\left(\frac{\pi}{2}\right)$ of degree $\leq 2n$ and with integral coefficients depending on n. Assuming $\frac{1}{2}\pi = \frac{b}{a}$, where a and b are integers, it follows that $\frac{b^{2n+1}}{n!} I_n = P_n a^{2n+1}$. The right side is an integer. But $0 < I_n < 2$, and as $n \to \infty$, $b^{2n+1}/n! \to 0$. Hence for some m and

In 1761 he loosely speculated that various solar systems might revolve about a common center, that such systems might in turn revolve about another system[288] and "where shall we stop?" In 1776 he argued in favor of developing a non-Euclidean geometry by building a logically consistent system through the explicit rejection of the parallel postulate, while keeping all other postulates intact.

Among his other contributions is his series solutions of the equation $x^m + px = q$ (1757), which was extended by **Euler** and **Lagrange**, and the first systematic development of the theory of hyperbolic functions. He also contributed to the mathematics of descriptive geometry, the determination of cometary orbits and the theory of *map projections*, some of which bear his name.

Lambert was born at Mulhausen, Alsace (then part of Swiss territory), to a poor family. He was self-educated and worked his way up patiently. In 1764 he removed to Berlin, where he received many favors at the hand of Frederick the Great and was elected a member of the Royal Academy of Sciences. He died of consumption.

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all $n > m$, $\frac{b^{2n+1}}{n!} I_n < 1$, and therefore $= 0$ since it is a non-negative integer. Hence $I_n = 0$ for $n > m$.
But for $-1 < x < 1$, $\cos \frac{1}{2}\pi x$ is positive, and $1 - x^2$ is positive. Hence $I_n > 0$ for all n. This contradiction shows that $\frac{1}{2}\pi$ cannot be of the form b/a.

The irrationality of e is even easier to demonstrate; let $e = \frac{p}{q}$ be rational (p and q integers). We may write $e = e_n + R_n$, where $e_n = \frac{p_n}{q_n} = 1 + \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{n!}$ or $n!\frac{p_n}{q_n} = 1 + nf(n)$. Since $R_n < \frac{1}{n!}$ we can replace $\frac{p_n}{q_n}$ by $\frac{p}{q}$ for a large enough n. Then $n!\frac{p}{q} = 1 + nf(n)$. But we can choose $n > q$ in which case the l.h.s. is divisible by n whereas the r.h.s. is not. The irrationality of e^π was proved by **Gelfond** (1929).

288 This idea is borne out by our current knowledge of the *hierarchical structure of the universe: stars clusters, spiral arms, galaxies, galaxy clusters and superclusters, etc.*

The 1758 return of Comet Halley

In 1705, **Halley** *predicted his comet return in late 1758 or early 1759. The prediction was confirmed by* **Cheseaux** *(1744) and* **Euler** *(1746). In 1757,* **Lalande** *suggested that the comet would be most easily seen during the month of November. What was needed was an improvement in knowing the time of the comet's perihelion passage. This task was entrusted in the hands of the French mathematician* **Alexis-Claude Clairaut***.*

Applying the first approximation to the 3-body problem, Clairaut began his computations in June 1757. Since the return was imminent, he was racing against time. Initially the plan was to compute the comet's motions around the sun over the 1607 to 1759 interval, taking into account perturbative effects of both Jupiter and Saturn. To assist him in the lengthy computations Clairaut enlisted the aid of his young colleague **Lalande***, who in turn enlisted the aid of Madame Lepaute, wife of the clock maker to King Louis XV. The three of them made calculations from morning to night over six months. The discrepancy of 33 days is only a modest error considering the uncertainty in the planetary masses, the perturbations from neglected or undiscovered planets, and the approximations that had to be made in the method itself.*

Clairaut's first paper on the predicted return was read to the Academy of Sciences in Paris on Nov 14, 1758, thus winning the race between himself and the comet. Had he waited with his announcement after the paper was published and the comet recovered prior to the announced result, their work might have been perceived as a mere footnote in astronomical history rather than the classic work it turned to be. Indeed, the published version of his prediction did not appear until January 1759 – well after the first sighting of the comet by **Palitzsch** *on December 25, 1758.*

In 1760, after the comet was recovered, Clairaut corrected some errors in the earlier work, made more comprehensive perturbation calculations for Saturn, and suggested a perihelion passage of April 4, 1759. His essay two years later moved this date back further, to March 31, 1759, which Clairaut considered to be within 19 days of the observed perihelion passage. A competition made in 1985 between Clairaut's work and computer results based on modern astronomical data, showed that 6 days of this remaining error as due to the planets Uranus and Neptune, which had not yet been discovered; another 6 days due to neglected effects of Mercury, Venus, Earth, and Mars; and 4 days from errors in the masses of Jupiter and Saturn that Clairaut adopted.

The arduous work left Clairaut with an unspecified malady that changed his temperament for the rest of his life.

For mid-European observers, the comet's apparition was broken into 3 phases:

- *The first phase:* From December 24, 1758, through February 14, 1759. It ended when the comet disappeared into the evening twilight.
- *The second phase:* Rounding perihelion on March 13, 1759, the comet again became visible in early April, before it sank below the local horizon.
- *The third phase* of visibility was from early May to when it was last seen on June 22. The comet passed within 0.12 AU of the earth on April 26, 1759, and became a rather impressive naked-eye object.

The bold prediction and successful recovery of comet Halley in 1758 and 1759 was the most visible confirmation of Newtonian dynamics in the 18th century.

Apart from the man vs. nature aspect of the 1758 apparition of comet Halley, there is another side to the story: ***Johann Georg Palitzsch*** *(1723–1788) was a German farmer and amateur astronomer. Palitzsch lived in Prohlis, a small town near Dresden in Saxony. On the nights of 24–26 Dec 1758 he observed the comet with his eight-foot telescope but did not identify it with Halley's. His observations were however published in a Dresden newspaper. To their chagrin, members of the Paris Academy (Clairaut included) learned about the comet recovery more than 3 months later (April 1, 1759), unable to understand how a German farmer beat them to it.*

Palitzsch also observed the June 6, 1761 transit of Venus. *He observed a black band linking Venus and the sun near the beginning and end of the transit and correctly interpreted this as evidence that* Venus *possessed an atmosphere. He also found that the brightness of Algol varied with a period of 2 days, 20 hours, 53 min.*

1759 CE **Franz Aepinus** (**Franz Maria Ulrich Theodor Hoch** 1724–1802, Germany). Physicist. Professor at St. Petersburg (1757–1798).

Discoverer of the *pyroelectric effect* in the gemstone tourmaline[289]. He also noticed that when tourmaline is subjected to a mechanical stress it can generate electric charges. Conversely, it can change its shape when voltage is applied to it.

Aepinus was first to apply mathematics systematically to phenomena of electricity and magnetism. He constructed the first condenser with parallel plates.

1759–1772 CE **James Brindley** (1716–1772, England). Canal engineer of remarkable mechanical ingenuity. Pioneered in construction of canals and aqueducts at the dawn of the industrial revolution.

Brindley was born at Thornsett, Derbyshire. His innate ingenuity more than compensated for his lack of training.

1759–1787 CE **Jean-Sylvain Bailly** (1736–1793, France). Astronomer, historian of science and statesman. Computed the orbit of Comet Halley (1759); studied major satellites of Jupiter (1766). Author of histories of ancient and modern astronomy (1775–1787).

Bailly was born in Paris; originally intended for the profession of a painter, he preferred writing tragedies until attracted to science by the astronomer **Nicolas de Lacaille**. Gained high literary reputation by his writings on **Moliere**, **Corneille** and **Leibniz** (1770); admitted to all three French Academies (1784–5).

The cataclysm of the French Revolution interrupted his studies. He became president of the National Assembly (1789) and first mayor of Paris (1789). Imposed martial law and called out the National Guard to keep order, leading to massacre of Champ de Mars (1791). Late in 1793, Bailly quited his Nantes home to join his friend Pierre Simon de Laplace at Melun;

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[289] Sodium aluminum borosilicate

$$Na(Mg, Fe, Mn, Li, Al)_3Al_6(Si_6O_{18})(BO_3)_3 \cdot (OH, F)_4.$$

Commonly used as a gemstone. It crystallizes in rhombohedral, *hexagonal* system as prisms. It consists of six-membered silica rings and 3-membered borate rings held together by sodium, aluminum, and other positive ions.

Transported to Europe from Sri Lanka by Dutchmen (1703). Its name comes from the Sinhalese 'tormalli'. The columns of tourmaline are charged when heated, positively at one end, and negatively at the other.

The Sri Lanka variety is reddish to vividly red (*rubellite*). The less conspicuous black tourmaline (*schorl*) was known in Europe long before, but its dull coloring did not arouse their curiosity.

but was there recognized, arrested and brought before the Revolutionary Tribunal at Paris, where he was guillotined amid the insults of a howling mob. He met his death with patient dignity.

1760 CE **John Harrison** (1693–1776, England). Horologist. Solved the greatest scientific problem of his time: built the first mechanical marine chronometer, accurate to within 0.1 sec/day, leading to longitude determinations accurate to within ca 1.3 minutes of arc. Harrison's instrument was tested on a voyage to Jamaica, and on its return to Portsmouth in 1762 it was found to have lost just under 2 min. The function of the clock was to keep Greenwich time, needed in celestial navigation to determine the *longitude* at sea from the time-dependent star position. [*Latitude* could be determined by the stars alone, through the identification of new constellations as the ships moved south, as well as the elevation of Polaris.]

Harrison, the son of a carpenter, was born at Faulby in Yorkshire. At first he learned his father's trade and worked at it for several years, but later became interested in mechanical devices, and during 1715–1726 made ingenious clocks. In 1714 Queen Ann authorized a public reward of $ 20,000 to any person who should construct chronometers that would determine a ship's longitude in the open sea within 30 minutes of arc[290].

In this connection **Isaac Newton** said: "*That, for determining the longitude at sea, there have been several projects, true to the theory, but difficult to execute: one is, by a watch to keep time exactly; but, my reason of the motion of a ship, the variation of heat and cold, wet and dry, and the difference in gravity in different latitudes, such a watch hath not yet been made*".

Harrison applied himself vigorously to the task, and in 1735 went to the *Board of Longitude* with a watch which he also showed to **Edmund Halley** and others. Through their influence he was allowed to proceed in a king's ship to Lisbon to test it. The result was so satisfactory that he was paid $500 to carry out further improvements. Harrison continued to work on the subject with the utmost perseverance for the next 25 years. In 1762, Harrison claimed the full reward of $ 20,000, but it was not until 1773 that he was paid in full. He was never able to express his ideas clearly in writing for lack of formal education, although in conversation he could give a very precise and exact account of his many intricate mechanical contrivances.

In Harrison's watches, compensation for changes in temperature was applied for the first time by means of a "*compensation-curb*", designed to alter

[290] Since the earth rotates once every 24 hours, the time at noon changes by one hour every $\frac{360^\circ}{24} = 15^\circ$ of longitude. To be accurate to $\frac{1}{2}^\circ$, a clock must not vary by more than 2 minutes at the end of the voyage.

the effective length of the balance-spring in proportion to the expansion or contraction caused by variations in temperature. Harrison's timekeeper was used on Cook's last voyage of 1776, and Cook had nothing but praise for it.

Further improvements were made in the 18^{th} century, especially in the development of the escapements. The best solution to the temperature-compensation problem was ultimately proposed by **Pierre le Roy** (1717–1785, France) in 1765, and perfected in 1785 by **Thomas Earnshaw** (1749–1829, England). Their idea was to diminish the inertia of the balance-wheel in proportion to the increase of temperature, by means of the unequal expansion of the metals composing the rim. Earnshaw's chronometer made the voyage of the *Bounty* with Captain William Bligh in 1791.

The Longitude Problem[291]*, or John Harrison against the Admiralty* (1714–1760)

"For lack of a practical method of determining longitude, every great captain in the Age of Exploration became lost at sea despite the best available charts and compasses. From Vasco da Gama to Vasco Núñez de Balboa, from Ferdinand Magellan to Sir Francis Drake — they all got where they were going willy-nilly, by forces attributed to good luck or the grace of God.

As more and more sailing vessels set out to conquer or explore new territories, to wage war, or to ferry gold and commodities between foreign lands, the wealth of nations floated upon the oceans. And still no ship owned a reliable means for establishing her whereabouts. In consequence, untold numbers of sailors died when their destinations suddenly loomed out of the sea and took them by surprise. In a single such accident, on October 22, 1707, at the Scilly Isles near the southwestern tip of England, four homebound British warships ran aground and nearly two thousand men lost their lives".

"The active quest for a solution to the problem of longitude persisted over four centuries and across the whole continent of Europe. Most crowned heads

291 Includes quotations from: '*Longitude*' by D. Sobel, Penguin Books, 1996, New York, 184 pp.

of state eventually played a part in the longitude story, notably King George III of England, and King Louis XIV of France. Seafaring men such as Captain William Bligh of the Bounty *and the great circumnavigator Captain James Cook, who made three long voyages of exploration and experimentation before his violent death in Hawaii, took the more promising methods to sea to test their accuracy and practicability.*

Renowned astronomers approached the longitude challenge by appealing to the clockwork universe: ***Galileo Galilei***, ***Giovanni Domenico Cassini***, ***Christiaan Huygens***, ***Isaac Newton***, *and* ***Edmund Halley***, *all entreated the moon and stars for help. Palatial observations were founded in Paris, London, and Berlin for the express purpose of determining longitude by the heavens. Meanwhile, lesser minds devised schemes that depended on the yelps of wounded dogs, or the cannon blasts of signal ships strategically anchored – somehow – on the open ocean.*

In the course of their struggle to find longitude, scientists struck upon other discoveries that changed their view of the universe. These include the first accurate determinations of the mass of the earth, the distance to the stars, and the speed of light".

As time passed and no method proved successful, the search for a solution to the longitude problem assumed legendary proportions. The governments of the great maritime nations — including Spain, the Netherlands, and certain city-states of Italy — periodically roiled the fervor by offering jackpot purses for a workable method. Finally, in 1714 Queen Ann (through the famed Longitude Act of Parliament) authorized a public reward of £20,000 to any person who should construct chronometers that would determine a ship's longitude in the open sea within 30 minutes of arc.

There lay the problem; and as it often happens in the history of science, or history in general for that matter, the nation that produced the problem, also produced the individual that was equal to the challenge, one ***John Harrison***, *clockmaker, a mechanical genius who pioneered the science of portable precision timekeeping, who devoted his life to the quest. He accomplished what Newton had feared was impossible: he invented a clock that would carry the true time from the home port to any remote corner of the world. He would build the first mechanical marine chronometer, accurate to within 0.1 sec/day, leading to longitude determination accurate to within 1.3 minutes of arc, thus solving the greatest scientific problem of his time.*

There is yet another side to this story which must be told. As in a Shakespearean play, heroes go with villains, and the archvillain in our epic is ***Nevil Maskelyne*** *(1732–1811). Reverend and the* 5^{th} *astronomer royal, who contested his claim to the coveted prize money, and whose tactics at certain junctures can only be described as foul play. As a member of the longitude-prize* *board, he made all possible obstructions to prevent Harrison from getting the prize. In addition, he took all Harrison chronometers from him (1766) and placed them in a damp cellar at Greenwich, untouched until 1920. Moreover, the commissioners charged with awarding the longitude prize (orchestrated by Maskelyne) changed the contest rules whenever they saw fit, so as to favor the chances of professional astronomers over the likes of Harrison and his fellow 'mechanics'.*

Thus, in 1767, Maskelyne published a nautical almanac *(1767) which gave the positions of each heavenly body for exact time and dates. By observing the direction of several stars and measuring their angles above the horizon, the navigator could* roughly *estimate his longitude at sea. The method was much inferior to Harrison's clocktime determination and could not meet the prize conditions.*

"With no formal education or apprenticeship to any watchmaker, Harrison nevertheless constructed a series of virtually friction-free clocks that required no lubrication and no cleaning, that were made from materials impervious to rust, and that kept their moving parts perfectly balanced in relation to one another, regardless of how the world pitched or tossed about them. He did away with the pendulum, and he combined different metals inside his works in such a way that when one component expanded or contracted with changes in temperature, the other counteracted the change and kept the clock's rate constant.

But the utility and accuracy of Harrison's approach triumphed in the end. His followers shepherded Harrison's intricate, exquisite invention through the design modifications that enabled it to be mass produced and enjoy wide use.

An aged, exhausted Harrison, taken under the wing of King George III, ultimately claimed his rightful monetary reward in 1773 – after forty struggling years of political intrigue, international warfare, academic backbiting, scientific revolution, and economic upheaval".

1760–1797 CE **Eliahu ben Shlomo Zalman** (1720–1797, Lithuania; known as THE 'VILNA GAON'; acronym: HA'GRA). Scholar, teacher and leader. One of the greatest Jewish scholars of the 2^d millennium CE. Sought to lead the Jews out of their mental ghetto into the wide world of general culture without doing harm to their specifically Jewish culture. He felt that Jewish learning had excluded too much of the secular knowledge which could be helpful in understanding the world as well as Judaism.

Born in Vilna to a family of distinguished rabbis, he turned out to be a prodigy: at the age of six he had completed the study of the Bible and was deep in the Talmud[292]. By thirteen he had mastered most rabbinic and mystical literature. He steadfastly refused to undertake the responsibilities of active rabbinate, but preferred to live exemplary life on a meager stipend left him in a relative will, so that he might have more time for study. His reputations, however, grew despite his seclusion and before long he was recognized as the unofficial spiritual head of all the communities of Eastern Europe.

The Vilna Gaon was the last of the great Jewish scholars of Talmudism, revered by the orthodox but ignored by the moderns. Through his interest in science, he had shown the Talmudic students the way to Western Enlightenment. The seeds for the coming massive Jewish involvement in modern science were sown when the Vilna Gaon had encouraged not only his but other Talmud students to study and translate scientific works into the language of the prophets.[293]

History repeats itself: as in the Greco-Roman and Islamic days, when these 18^{th} century Jewish youth came in contact with new ideas, they also became imbued with them.

The Vilna Gaon left no written works, but over 40 volumes of his textural notes and his student's notes have been published.

1761–1766 CE **Joseph Gottlieb Kölreuter** (1733–1806, Germany). Botanist. Pioneer of hybridization experiments with plants. Recognized the importance of insects and the wind in pollinating flowers: Published reports describing 136 quantitative experiments in artificial hybridization, foreshadowing the work of Mendel. Professor of natural history and curator of Botanical Gardens at Karlsruhe (1764–1786).

1764–1770 CE **James Hargreaves** (c. 1722–1778, England). Inventor, weaver and carpenter. Invented the '*spinning jenny*', the first machine to spin many threads at a time. He turned the spindles of several spinning wheels upright and placed them in a row. He then added a frame which alternately held and pulled the *rovings* (crude twists of cotton) from which threads were made. He patented the 'jenny' in 1770. Earlier, (1733) **John Kay** invented

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[292] At the age of ten he wanted to become a scientist, but his horrified father turned him from science to Talmud. He never forgot his early interest in science. Had he been born in the 12^{th} century he would have been a great philosopher. At the 18^{th} century he was an anachronistic man.

[293] The Gaon urged one of his pupils, who knew German, to translate Euclid's *Geometry*, for example, which he felt ought to be studied by Jews.

the *flying shuttle loom* which doubled the amount of cloth that weavers could make, but Hargreaves' invention supplied the weavers with more thread.

Hargreaves was a weaver in Standhill, England, and first used the '*jenny*' at home. He then sold some machines. The sales made his patent invalid, and he was never rewarded for his invention. Local spinners worried that the increased amount of yarn the '*jenny*' spun might cost them their jobs. They burned Hargreaves' machine and drove him from the town.[294] He moved to Nottingham (1768) and helped found a prosperous spinning mill. His machine was used in the mill. Other manufacturers used the '*jenny*' without paying him. No one really knows the origin of the term *jenny*.

1765 CE *The Lunar Society* of Birmingham, an informal club of technologists, was founded in England. The society included men such as **Erasmus Darwin** (1731–1802) and **James Watt**. Its members, consisting of Midland scientists and manufacturers, met once a month on the occasion of the new moon to discuss technology and other subjects of shared interest, such as chemistry of clays and glazes, surveying, geology and the developing science of climate and weather. They projected plans for new canals, and devices for harnessing the power of wind and steam. The Lunar Society was the intellectual seedbed for the industrial revolution.

1765–1774 CE **James Watt** (1736–1819, England). A Scottish engineer whose improved engine design first made steam power practicable.

Crude steam engines were used before Watt's time but burned large amounts of coal and produced little power. Their lateral motion restricted their use to operating pumps. Watt's invention of a separate condenser made steam engines more efficient, and his further development of crank movement enabled them to turn wheels and made possible their wider application (patented, 1769). The first primitive steam-engine to convert heat into mechanical energy (used to drain mines) was invented by **Thomas Newcomen** (1663–1729, England) in 1712. His machine was improved by **John Smeaton** (1724–1792, England).

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[294] Wool weavers afraid of loosing their job destroyed **John Kay**'s loom in 1733 and sent him packing to France. Nevertheless, about 1750, cotton workers started using the flying shuttle.

Table 3.7: The evolution of surface transportation (1769–1997) (Trains, Automobiles, Airplanes, Ships, Submarines)

1769 CE	**James Watt** (Scotland) patented his improved *steam engine*.
1770 CE	**Nicolas Cugnot** (France) built first *steam-powered wagon*.
1787 CE	**John Fitch** (USA) built first successful *steamboat*.
1802 CE	**John Stevens** (USA) constructed steamboat that uses *screw propeller*. **Richard Trevithick** (England) built the first *steam railway locomotive*.
1807 CE	**Robert Fulton** (USA) directed the building of the 'Clermont', the first steamboat to become a practical and financially successful (20 HP engine).
1814–1829 CE	**George Stephenson** (England) built the first *reliable railway locomotive*. Completed the adaptation of the steam engine to the railroad.
1815 CE	**John McAdam** (England) introduced a new method of road-building, using crushed rocks.
1830 CE	**Robert L. Stevens** (USA) invented the *railroad rail* (inverted "T").
1834 CE	**Thomas Davenport** (USA) built the *electric streetcar*.
1836 CE	The *screw propeller* for ships was patented.
1838 CE	The *Sirius* (England), first *steamship* to cross the Atlantic Ocean without sails. It made crossing in 18 days.
1839 CE	**Charles Goodyear** (USA) invented the process of *rubber vulcanization*.
1845 CE	**Robert W. Thomson** (England) invented the *pneumatic rubber tire*.
1860 CE	**Etienne Lenoir** (France) invented first practical gas engine for a road vehicle.
1863 CE	First successful subway built in London.

1865 CE **Pierre Lallement** and **Ernest Michaux** (France) constructed the pedal-powered *bicycle*.

1869 CE The *Suez Canal* opened.
First *transcontinental railway* completed in the U.S.

1874 CE Ocean liners cross the Atlantic in only 7 days.

1875 CE **Siegfried Marcus** (Germany) built the first successful 4-cycle petrol driven engine and carriage.
Nickolaus Otto achieved this feat a year later.

1879 CE First *electric locomotive* demonstrated in Berlin.

1881 CE First *electric streetcar* built in Berlin.

1885 CE **Karl Benz** (Germany) built a *gasoline-powered automobile*.

1887 CE **J.B. Dunlop** (Scotland) invented the *air-inflated rubber tire*.

1896 CE **Rudolph Diesel** (Germany) built the first successful *diesel engine*.

1896 CE **Samuel P. Longly** (USA) made first successful powered flight of an unmanned heavier then-air-plane. The craft, weighting 12 kg, is powered by a small steam engine.
Otto Lilienthal (Germany) was killed while flying one of his experimental gliders after making hundreds of successful flights. His pioneering work heavily influenced the Wright brothers' airplane design.

1900 CE *Electrical ignition* system invented for internal combustion engine.

1903 CE The **Wright brothers** (USA) made the first successful *airplane flight*.

1907 CE **Louis Bréquet** and **Paul Cornu** (France) made the first successful *helicopter flight*.

1908 CE *Gyroscopic compass* was invented.
Henry Ford introduced the Model T car.

1914 CE The *Panama Canal* opened.
Red and green traffic lights utilized for the first time in Cleveland, Ohio.

1927 CE **Charles A. Lindbergh** (USA) completed first nonstop solo transatlantic flight: flew 5180 km from New York to Paris in $33\frac{1}{2}$ hours.

1930 CE **Frank Whittle** (England) patented the first *jet engine*.

1932 CE First successful *synthetic rubber* became available commercially.
Diesel-electric trains were introduced.

1935 CE The French ocean liner 'Normandie' crossed the Atlantic in only 4 days.
Gas-turbine engines patented in England and Germany contributed to the development of the *jet aircraft engine*.

1939 CE First flight of jet-powered aircraft in Germany.

1947 CE An experimental rocket-plane broke the *sound barrier* in the United States.

1949–1952 CE First commercial *turbo-jet airliner* (the De Havilland 'Comet') was unveiled in Great Britain. Went into *regular service* in 1952.

1950 CE Jet aircraft made its first transatlantic flight.

1954 CE U.S. launched '*Nautilus*', world's first nuclear submarine.

1955 CE First practical *hovercraft* was built.

1958 CE U.S. launched '*Savannah*', world's first nuclear-powered cargo ship.

Boeing 707, first American jet air-liner, begun regular commercial service.

1964 CE Boeing 727 commercial airliner was introduced.

1969–1970 CE 'Concorde' – *supersonic jet-liner* (French-British) and the Soviet Tu-144, fly at supersonic speeds for the first time.
Pan-American World Airlines began commercial flights of the *362-passenger* Boeing 747 jet.

1986 CE Lightweight airplane 'Voyager' completed record *round-the-world flight* without refueling.

1988 CE Largest suspension bridge constructed in Japan; it has a span of 2 km.

1997 CE (Oct 15) **Andy Greene** (England) broke the sound barrier (1220 km/sec) with his *supersonic car* in the Nevada desert. The car was driven by two turbo-jet engines.

1765–1784 CE **Carl Wilhelm Scheele** (1742–1786, Sweden). Apothecary and chemist. First discoverer of *oxygen* (1772, ahead of Priestley), *chlorine*, *manganese*, and *barium* (1774). Claimed that air consists of oxygen and nitrogen (1777). Discovered and isolated various organic acids: [*prussic* (1765), *tartaric* (1770), *oxalic* and *uric* (1776), *lactic* (1780), *citric* (1784), *malic* (1785), *gallic* (1786)] and also *glycerine* (1783). Discovered action of light on silver salts (1777). Formed HCN (hydrocyanic acid) by the action of ammonia on a mixture of charcoal or graphite and potassium carbonate (1782).

Scheele was born at Stralsund, then the chief town of Swedish Pomerania. He was apprenticed in 1757 to an apothecary in Gothenburg, where he began to study chemistry. He occupied positions in pharmacies in Malmö, Stockholm, Uppsala and Köping, where he died at an early age.

Scheele was a man of great modesty and his circumstances were often poor. He worked with very simple apparatus and in periods of scanty leisure, in a cold and uncomfortable laboratory, yet he made a great number of discoveries of the very first rank.

1765–1785 CE **Lazzaro Spallanzani** (1729–1759, Italy). Physiologist, naturalist and 'microbe-hunter'. Known for his experiments in digestion, circulation of the blood, fertilization and regeneration of animals. Disproved the theory of spontaneous generation; pioneered in volcanology. The first to watch isolated bacterial cells divide. His main achievements:

- Suggested preserving the quality of food by sealing it in airtight containers (1765).
- Demonstrated (1767–8) that the experiments of John Needham (1713–1781, England), allegedly 'proving' spontaneous generation of microorganisms, were invalid since they derived from germs transported in the air.

- Discovered (1773) digestive action of saliva.
- Established importance of semen for fertilization.
- Showed that *digestion* was clearly a *chemical* process rather than a mechanical grinding of food (1780).
- Performed artificial semination of a dog (1785).
- Lay the foundations of modern *volcanology* and *meteorology*.

Born in Pavia. First educated by his father, who was a lawyer. At the age of 15 was sent to a Jesuit college at Reggio de Modena and took orders of the Roman Catholic Church. Studied natural history, languages and mathematics at the University of Bologna. Professor at the Universities of Reggio (1754–1760), Modena (1760–1769) and Pavia (1769–77). Made many journeys along the shores of the Mediterranean.

1766–1794 CE **Peter Simon Pallas** (1741–1811, Germany). Zoologist and botanist. Influenced the development of evolutionary theory.

Pallas was born in Berlin and attended the Universities of Halle, Göttingen and Leiden, where he earned his doctor's degree at the age of 19. His books *Miscellania Zoologica* (1766) and *Spicilegia Zoologica* included a new system of animal classification as well as a discovery of several vertebrates new to science.

In 1767 he was invited by Catherine II of Russia[295] to become a professor at the St. Petersburg Academy of Sciences, and during 1769–1774 he led an expedition to *Siberia* collecting natural history specimens on their behalf. He explored the upper *Amur*, the *Caspian Sea*, and the *Ural* and *Altai mountains*, reaching as far eastward as *Lake Baikal*. Between 1793 and 1794 he led a second expedition to southern Russia, visiting the *Crimea* and the *Black Sea*.

The first expedition resulted in his book: "*Journey through various provinces of the Russian Empire*" (1776–1778).

The stony-iron meteorite of *Krasnoyarsk* as well as a number of animals are named after him. His work provided great amounts of data on a variety of subjects, including botany, zoology, geology, geography, ethnography, philology, and medicine. Employing the comparative method, he thus laid the foundations of a new natural history that was influential in the development of evolutionary theory.

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[295] During the reign of this empress, Russia became increasingly receptive to Western science, technology and culture. The German-born monarch invited scores of foreign scholars to take up residence in Russia in the hope of developing the material resources and intellectual life of her empire.

1766–1798 CE **Henry Cavendish** (1731–1810, England). Physicist and chemist. In 1766 he discovered the properties of hydrogen and identified it as an element. Later he showed that water is composed of hydrogen and oxygen.

In 1798, Cavendish performed a novel laboratory experiment to measure Newton's universal gravitation constant G [the apparatus he employed was devised by **John Michell** in 1784]. This constant remained unknown for over half a century after Newton. A rough estimate of G from guesses like Newton's of the average density of the earth, showed that the attractions between small objects in a laboratory must be almost hopelessly small. The common forces of gravity seem strong; but they are due to the huge mass of the earth[296]. The sun, with enormously greater mass still, controls the whole planetary system with its gravitational pull. But the gravitational tugs between human-sized objects are so small that we never notice them compared with earth-pulls and the forces between objects in "contact". It was clear that to measure G, very delicate and difficult experiments would be needed.

In a desperate attempt, several scientists at the end of the 18^{th} century tried to use a measured mountain as the attracting body. They estimated G by the pull of the mountain on a pendulum hung near it. They had to measure *astronomically*, the tiny deflection of the pendulum from the vertical caused by the sideways attraction of the mountain. They then had to *geologically* estimate the mass of the mountain and its "average distance" from the pendulum. Substituting these measurements in $F = G\frac{Mm}{d^2}$ gave the estimated value[297] $G \approx 7.5 \times 10^{-8}$ cgs and consequently $\rho = 4.5$ gm/cm^3 (1774).

Cavendish placed a pair of small metal balls on a light trapeze suspended by a long thin fiber. He brought large lead balls near the small ones in such positions that their attractions on the small balls pulled the trapeze about the fiber axis, twisting the fiber until its elastic forces balanced the effects of the tiny attractions.

He measured the masses and the distances between the small balls and the large attracting balls, but to calculate the value of G he also needed to know the attracting forces. Since the fiber was far too thin and delicate

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[296] For a mass m on the surface of a homogeneous spherical earth of radius a, mass $M = \frac{4\pi}{3}\rho a^3$ and average density ρ, Newton's law yields: $G\frac{Mm}{a^2} = mg$, where g is the earth's surface gravity. The two equations render the relation $g = \frac{4\pi}{3}G\rho a$. For known g and a, this relation enables one to calculate the mean density if G is measured independently.

[297] In 1887 **Thomas Preston** (1860–1900) obtained, in a similar experiment, $G = 6.6 \times 10^{-8}$ cgs.

for any direct measurement, Cavendish let the trapeze and its small balls twist to and fro freely with simple harmonic motion and timed the *period* of that isochronous motion. From that, with measurements of mass and dimensions of the trapeze, he could calculate the twisting strength of the fiber. He then proceeded to obtain a good estimate of G. To avoid convection currents, Cavendish placed his apparatus in a closed room and observed it with a telescope from outside the room. Cavendish's value for G was 6.75×10^{-8} cgs. The ensuing average density of the earth was 5.48 gm/cm^3 (1798).

Cavendish was born in Nice, France, the elder son of Lord Charles Cavendish [brother of the 3^{rd} duke of Devonshire] and Lady Anne Gray, daughter of the duke of Kent. During 1749–1753 he studied in Cambridge without taking a degree. In the latter part of his life he inherited a fortune which made him one of the richest men of his time. He owned a huge private library, where he used to attend on appointed hours to lend the books to men who where properly vouched for. So methodical was he that he never took down a volume for his own use without entering it in the loan-book. He never married.

1768–1774 CE **William Hewson** (1739–1774, England). Surgeon, anatomist and physiologist. Sometimes referred to as the 'father of haematology.' Isolated *fibrin*, a key protein in the blood coagulation process. He also contributed work on the *lymphatic system* by showing the existence of lymph vessels in animals and explaining their function. Demonstrated that *red blood cells* were flat rather than spherical (as had been previously supposed by Leeuwenhoek).

In 1773 he produced evidence for the concept of a *cell membrane* in red blood cells — however, this last work was largely ignored.

Hewson was born in Hexham. He studied at Newcastle upon Tyne and Edinburgh, being the assistant of **William Hunter** (1761–1762). He died, at the age of 35, as a result of sepsis contracted whilst dissecting a cadaver.

An article "William Hewson (1739–74): the father of haematology" was published in May 2006 in the *British Journal of Haematology.*

Evolution of the Steam Engine[298]

A crude prototype of the first engine, in the form of an apparatus which employed the kinetic energy of jets of steam, is mentioned amongst the writings of **Hero of Alexandria** *(ca 150 BCE). In his book Pneumatica, he describes a primitive steam reaction turbine (Hero's engine was considered in his time to be mainly an interesting toy). Another apparatus described by Hero is a mechanism to close or open temple doors by a hidden mechanism: A hollow altar containing air is heated by a fire kindled on it. The air, in expanding, drives some of the water contained in a spherical vessel beneath the altar into a bucket. The descending bucket pulls ropes that are entwined on a pair of vertical posts, to which the doors are fixed, causing them to open. When the fire is extinguished, the air cools, the water leaves the bucket, the ascent of which closes the doors. In another device, a jet of water driven out by expanding air is turned to account as a fountain.*

Today, not only do jet propulsion and rocket motors run airplanes — there are also gasoline engines for cars and planes; diesel engines for trucks, boats and trains; steam turbines to generate electricity and propel boats; and steam engines to run boats and locomotives.

But all of these engines make use of the same basic principle which operated Hero's toy: a hot flame imparts increased motion to molecules and causes expansion of gases. When a substance is heated, its molecules move at great speeds but in random, haphazard motion. As many molecules go one way as another. The problem is to organize this chaos of movement so that the molecules act together, applying their energy in one direction. In all heat engines this collimation is accomplished by permitting the hot gas to

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298 A steam engine is a machine for the conversion of heat into mechanical work, in which the working substance is water and water vapor. The working substance may be regarded from two points of view: *Thermodynamically* it is the vehicle by which heat is conveyed to and through the engine from the hot source (the furnace and boiler). Part of this heat undergoes a transformation into work as it passes through, and the remainder is emitted, still in the form of heat.

Mechanically, the working substance is a medium capable of exerting pressure, which effects this transformation in doing work by means of a change of volume which it undergoes in the operation of the machine.

Regarded as a thermodynamic device, the function of the engine is to extract as much work as possible from a given quantity of heat (or from the combustion of a given quantity of fuel). Accordingly, a question of primary importance is what is called the *efficiency* of the engine.

create pressure in a chamber which is completely enclosed except on one side. Sometimes there is a movable piston on this side and the bombardment of the molecules cause it to move. In a jet propulsion engine this side is left open; then the hot expanding gases rush out the back, at the same time reacting on the engine to push it forward. In the steam turbine, the motion of the gas rushing out of the open end pushes a wheel and makes it turn.

From the time of Hero to the 17^{th} *century no progress was recorded, though here and there we find evidence that appliances like those described by Hero were used for trivial purposes, such as organ blowing and the turning of spits.*

However, in 1601 ***Giovanni Battista della Porta*** *described in his treatise on pneumatics, an apparatus similar to Hero's fountain but with steam instead of air: Steam generated in a separate vessel passes into a closed chamber containing water, from which a pipe (open under the water) leads out. He also pointed out that the condensation of steam in the closed chamber may be used to produce a* vacuum *and suck up water from a lower level. In fact, his suggestions anticipated the machine which a century later became the steam engine.*

In 1629, ***Giovanni Branca*** *designed an engine shaped like a water-wheel, to be driven by the impact of a jet of steam on its vanes, and in its turn to drive another mechanism for various useful purposes.*

To **Edward Somerset**, 2^{nd} *marquis of Worcester, appears to be due the credit of proposing (1663), if not making, the first useful steam engine: Its object was to raise water, and it probably worked like della Porta's model — but with a pair of displacement-chambers, from which water was alternatively forced by steam from an independent boiler, while the other vessel was allowed to refill.*

The steam engine first became commercially successful in the hands of ***Thomas Savery*** *(1650–1715, England), who in* 1698 *obtained a patent for a water-raising engine. In the use of artificial means to condense the steam, and in the application of the vacuum so formed to raise water by suction from a level lower than that of the engine, Savery's engine was probably an improvement on Somerset's.*

Earlier, in 1678, *the use of piston and cylinder (long before known as applied to pumps) in a steam engine had been suggested by* ***Jean de Hautefeuille*** *(France), who proposed to use the explosion of gun-powder to raise a piston. In* 1680, ***Christiaan Huygens*** *described an engine in which the explosion of gun-powder in a cylinder expelled part of the gaseous contents, after which the cooling of the remainder caused a piston to descend under atmospheric pressure, doing work in the process by raising a weight.*

In 1690, ***Denis Papin*** *suggested that the condensation of steam should be employed to make a vacuum under a piston, previously raised by the expansion of the steam. Papin's was the earliest cylinder and piston engine.*

The first usable engine which made use of heat obtained by burning coal was invented by ***Thomas Newcomen*** *in* 1705. *In this engine, the pressure of the steam moving through a pipe controlled by a valve made the piston rise. Then the valve was shut manually and another valve on the opposite side was opened to let the steam out, condense it and thus make a vacuum under the piston. Air pressure above the piston then pushed it back to repeat the cycle (boys were hired at very low pay to turn the valves for* 14 *hours a day!).*

About half a century after Newcomen's engine first appeared, it was greatly improved by ***James Watt*** *(1769). He gets the credit for inventing the steam engine because he made the valve operation automatic and thus creating a practical engine. The non-condensing, high-pressure engine was the invention of* ***Richard Trevithick*** *(1800) and* ***Oliver Evans*** *(1755–1819, U.S.A.) in* 1805.

A steam engine is called an external-combustion engine because the fuel is burned outside the cylinder of the engine. There is a furnace to burn the coal and a boiler for the production of steam. In most steam engines, the furnace and the boiler are much larger than the cylinder itself. Watt's improvements and other improvements added since Watt's time, have made the modern steam engine 10 *times as efficient as Newcomen's original. Nevertheless, on the average, less than* 15 *percent of the total heat energy that is put into the engine is converted into mechanical energy. With expensive equipment and the greatest care of operation, this efficiency figure can be raised to* 27 *percent.*

1768–1777 CE **Jesse Ramsden** (1735–1800, England). Precision instrument-maker. One of the most skillful designers of mathematical, astronomical, surveying and navigational instruments in the 18^{th} century. Introduced the first satisfactory *screw-cutting lath*[299] (1770), which had far reaching consequences.

[299] The availability of accurately cut screws, engaging with equally accurately cut gears, made it possible to effect the controlled movement of the scriber that cut the graduations of the scale. In this field **Duc de Chaulnes** (1714–1769, France) did much pioneer work, introducing the use of *microscopes*, with cross-

Using the ideas of Duc de Chaulnes (1768), he built the first dividing-engine suitable for work on an industrial scale. His machines excited great interest, and early in the 18^{th} century many of a similar type were built.

Ramsden was first to carry out *in practice* a method of reading off angles by measuring the distance of the index from the nearest division line by means of a *micrometer screw* which moves one or two fine threads placed in the focus of a *microscope*. His specialty was divided circles, which began to supersede *quadrants* in observatories toward the end of the 18^{th} century. He took out patents for improvements in the *sextant*, *theodolite*, *barometer* and *micrometer*. He also invented the *electrostatic machine* with glass plates (1768).

He was elected Fellow of the Royal Society (1786) and received the Copley Medal (1795).

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hair in the field of view, for the precise location of the graduations of the master plate; he also used the *tangent-screw drive*.

The development of the steam-engine by **Watt** (1769) and **Trevithick** (1800) made possible the creation of a civilization based on *power-driven machinery* but did not of itself create such a civilization. In fact, the steam-engine took almost 50 years to establish itself as the principal source of power for industry. One reason was the poor trade conditions existing during and after the great French wars; another reason was the purely technical difficulty of constructing steam-engines and the machinery they were to drive: the making by hand of parts of machinery to *precise standards* could prove not merely prohibitively expensive but even a practical impossibility. To this end, accurately threaded screws were important for a variety of purposes in the making of both precision machines and machine-tools [as, for example, the moving of the tool-holder — since each turn of the screw must correspond to a precisely determined linear movement forward].

The rate of which standards changed is illustrated in the fact that in 1776 the error in boring a meter-long cylinder was about 2 millimeters, whereas by 1856 workshop machines were capable of measuring 250 parts per million of a millimeter!

Ship of Doom

Principal causes of mortality among Royal Navy warship crews during the late 18^{th} century were: Enemy action 8.3%; Fire, sinking, wreck 10.2%; Accident 31.5%; Disease 50%.

Eye-patch, hook-hand and peg-leg, equipment of the pantomime pirate, are theatrical details with origins of historical accuracy. Throughout its history, the sailing ship was a death trap to the men who sailed in it, and mutilation often the lot of those who survived a lifetime at sea. Accidents could be of different origins:

- *Falls* were an everyday hazard to man racing aloft to take in sail at the onset of a squall. They meant either broken bodies on deck or "man lost overboard". A fall into a billowing sail would catapult a sailor far into the sea.
- *Electrocution* (a storm hazard), if lightning struck a mast.
- *Rupture*, caused by hauling on ropes, or reefing heavy sails, was common among crew members.
- *Snapping cables* whiplashed, severing the limbs of bystanders if not killing them.
- On deck, a *gun breaking loose* from its mount would crush anyone in its path and had to be tipped on its side to stop it.

After months at sea, the air below decks was rank and fetid and the bilges fouled. Respiratory diseases — tuberculosis, pneumonia — were common; stomach disorders, caused by bad food and bad water, were things with which every sailor had to live.

Headroom below decks was so limited that cracked heads were common. If a man found his way to the surgeon, he was in the hands of a man who supplied his own drugs and instruments and knew as little about his job as the man he treated.

To these everyday hazards, a passage in the tropics added many more. Yellow jack, or the black vomits, and malaria reduced crews to a point where, unless more hands could be pressed, a ship sailed undermanned and was more prone to foundering and running aground. Finally there was scurvy, *a horrifying deficiency disease leading to disfigurement and death; it remained common until steam came along to shorten passage times and reduce the crew's reliance on stored food. During the late 18^{th} century, when British sea power was approaching its peak, press-gangs roamed the seaports, clubbing* *unwary passers-by senseless and carrying them off to lives of hardship and likely death — not in action, but from accident or disease.*

1768–1779 CE **James Cook** (1728–1779, England). Navigator, surveyor and explorer of the Pacific Ocean. Commanded three scientific expeditions around the globe.

The first (1768–1771), was launched under the joint sponsorship of the English Admiralty and the Royal Society, to observe the transit of Venus (June 03, 1769, Tahiti), produce a detailed survey of the coastline of the '*South Continent*'[300], and observe its flora, fauna and inhabitants.

In the *H.M.S. Endeavor* (368 tons, 31 m long, crew of 97 men), he circumnavigated the North and South islands of New Zealand and mapped its coasts. During this voyage, the naturalist **Joseph Banks**[301] (1743–1820) and his assistants collected specimens of 1400 previously unidentified species of plants.

Soon afterwards, Cook embarked on his 2^{nd} voyage (1772–1775) with the *Resolution* (462 tons), the *Adventure* (330 tons), and a crew of 193 men. He became the first man to sail across the Antarctic circle. His pioneering work led him to conclude that a frozen continent lay further south in the Antarctic (later explorers proved him right).

Cook's 3^{rd} and last voyage (1776–1779) was primarily to settle the question of the *northwest passage*. He proved that there was no direct water route from the Pacific Ocean to Hudson Bay. He was killed by the Hawaii islanders in a trivial incident.

"*In ten years, he explored more of the earth's surface than any other man in history*". This tribute was to James Cook, who by sheer competence as a navigator, sailor and leader of men rose in his life from obscure origins to

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[300] Geographers have speculated for hundreds of years the existence of a continent that extended from the South Pacific to the South Pole. Like many explorers before him, he looked for it in vain.

[301] A wealthy young naturalist and member of the Society, who put up £10,000 to help the expedition and supplied some of the telescopes and other scientific instruments. With him were **Carl Solander**, a Swedish botanist and pupil of **Linnaeus**. There were also two artists: **Alexander Buchan**, a landscape painter, and **Sydney Parkinson**, who specialized in natural history.

a permanent place in history. In his youth and early twenties he served in collier brigs working out of Whitby (a busy port in his native Yorkshire), before joining the Royal Navy in 1755 as a seaman. In two years he was master — warrant officer in charge of handling a warship — aboard the 64-gun *Pembroke*, and by his navigational skills was instrumental in the successful assault on Quebec by General Wolfe in 1759. The talents that had gained him promotion made him a giant among explorers.

1769 CE **Richard Arkwright** (1732–1792, England). Inventor and manufacturer. Improved on earlier versions of *spinning machines* by adding mechanical details that made them work. The machine was powered by water. Sets of rollers turning at different speeds drew cotton from the carding machine, which straightened out the fibers. Spindles then twisted the cotton into thread.

Arkwright was born in Preston in Lancashire, the youngest of 13 children. After serving his apprenticeship in his native town, he established himself as a barber about 1750, and later amassed a little property from dealing in human hair, and dyeing it by a process of his own. He worked 16 hours a day and studied at night to make up for his lack of schooling. He was knighted in 1786.

1769–1781 CE **Pierre Sonnerat** (1748–1814, France). Naturalist and explorer. Made several voyages to southeast Asia, visiting the *Philippines and Molucces* (1769–1772), *India and China* (1774–1781) and *New Guinea* (1776). He was the first person to give a scientific description of the south Chinese fruit tree *lychee.*

In the latter half of the 18^{th} century, France made serious attempts to break the monopoly in the spice trade which the Dutch had long enjoyed. Having annexed the Seychelles islands in the Indian Ocean (1743), they built permanent settlements (1768) and spice plantations, later dispatching expeditions to India, the Malay archipelago, and elsewhere. Sonnerat was a naturalist accompanying one such voyage. He made extensive observations of primitive societies and exotic wildlife, which he subsequently reported.

1770 CE **Edward Waring** (1734–1798, England). Physician and mathematician. In his *Meditationes algebraicae* asserted without proof:

- Every positive integer is the sum of nine or fewer cubes (known as the '*Waring Conjecture*'; yet unproven).

- Every positive integer is a sum of a fixed number s of non-negative k^{th} powers (known as the '*Waring Problem*').

Clearly, the 'conjecture' is a special case of the 'problem' for which $s(3) = 9$. In general $s = s(k)$, such that for a given non-negative integer N

$$N = u_1^k + u_2^k + \cdots + u_{s(k)}^k, \qquad k \geq 2.$$

For each s there are two problems:

(1) Prove that $s(k)$ exists (done by **Hilbert** in 1908).

(2) Find the minimum value $s(k)$ for a given N.

Lagrange proved (1770) that $s(2) = 4$ and Waring himself claimed that $s(3) = 9, \qquad s(4) = 19$.

G.H. Hardy had pointed out (1938) that the most fundamental and most difficult aspect of the problem is that of deciding not how many cubes are required for the representation of *all* numbers, but how many are required for the representation of *all large* numbers, i.e. of all numbers with some finite number of exceptions.

In 1986, **Ramachandran Balasubramanian**, **Jean-Marc Deshouillers** and **Fancois Dress** proved that $s(4) = 19$.

Waring also classified quartic curves and was first to set forth a method of approximating values of imaginary roots of polynomial equations.

Waring practiced medicine in various London Hospitals. From 1760 he was Lucasian professor of mathematics at Cambridge, although he did not give up practicing medicine until 1770.

1770 CE **John Wilson** (1741–1793, England). Discovered a theorem that bears his name:

$$(p-1)! + 1 \equiv 0 (\text{mod } p)$$

for p prime [or: $\frac{1\cdot 2\cdot 3\cdots(p-1)+1}{p}$ is an integer]. This theorem was presumably discovered on numerical evidence alone, and reported without proof in Waring's book: '*Meditationes Arithmeticae*' (1770). Among the posthumous papers of Leibniz there were later found similar calculations on the remainders of $n!$, and he seems to have made, already in 1682, the same conjecture. The first proof of the theorem was given by **Lagrange** in 1770.

Wilson was a senior Wrangler at Cambridge and left the field of mathematics quite early to study law. Later he became a judge and was knighted.

1770 CE **Nicolas Joseph Cugnot**. French army captain. Operated successfully a three-wheeled steam-powered vehicle. It was used as a tractor for hauling cannon. It could travel 5 km/hour and had to stop every 10 or 15 minutes to build up steam.

1770–1789 CE **Antoine Laurent Lavoisier** (1743–1794, France). French chemist who gave the first accurate scientific explanation of the mystery of fire. In 1777, after a series of careful experiments, he stated that burning is the result of rapid union of the burning material with oxygen and that *respiration* is a form of combustion (1780). In 1789 he wrote the first modern textbook of chemistry[302] in which he formulated the principle of conservation of matter[303]. These ideas led him to write the first chemical equation.

Originated the modern concept of the chemical *element* through the definition: "*an element is a substance that cannot be decomposed into simpler substances*". On the basis of this definition, he drew up a list of 30 or so elements, most of which are still recognized as such.

Lavoisier was born in Paris of well-to-do parents, and attended the College Mazarin, where he studied mathematics, astronomy, chemistry, and botany. In 1768 he became a member of the Academy of Sciences. He established an agricultural experiments station, and tried to improve farming methods in France. Among his other varied interests were the increase of production of salt, improved manufacture of gunpowder, plans for improving the social and economic conditions of the community by means of saving banks, insurance societies, canals, and workhouses. He was further associated with committees on hygiene, coinage, the casting of cannon, etc., and was secretary of the treasurer of the commission appointed in 1790 to secure uniformity of weights and measures.

In 1787, Lavoisier, **Berthollet** and their associates introduced the first method of chemical nomenclature based on scientific principles ("*Method d'une nomenclature chimique*").

He was executed on the guillotine for his membership in a financial company that collected taxes for the government[304].

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302 *Traité élémentaire de chimie* (1789), a work sometimes described as marking the start of chemistry as a science, and classed with Darwin's *Origin of the Species* (1859) in biology and Newton's *Principia mathematica* (1687) in physics. In his book, Lavoisier describes *fermentation* as the splitting of sugar into alcohol and CO_2. He characterized the reaction as an oxidation-reduction process. He also made the first measurements on human metabolic rate.

303 He arrived at this conclusion after realizing that the *total weight* of all the products of a chemical reaction must be exactly equal to the total weight of the reacting substances. He was able to draw correct inferences from his weighings because, unlike many of the *phlogistonists*, he looked upon heat as imponderable.

304 A petition in his favor addressed to Coffinhal, the president of the tribunal, is said to have been met with the reply: "*La Republique n'a pas besoin de*

The name Lavoisier is indissolubly associated with the overthrow of the phlogistic doctrine that had dominated the development of chemistry for over a century, and with the establishment of the foundations upon which the modern science rests. Justus von Liebig said of him:

"*He discovered no new body, no new property, no natural phenomenon previously unknown; but all the facts established by him were the necessary consequences of the labors of those who preceded him. His merit, his immortal glory, consists of this — that he infused into the body of science a new spirit; but all the members of that body were already in existence, and rightly joined together*".

Founders of Modern Chemistry – from Lavoisier to Mendeleev (1778–1889)

If a specific date is to be set for the science of chemistry, it may be said to have begun with ***Robert Boyle****'s clear definition of a chemical element in his* Sceptical Chymist *of 1661. Nevertheless, it would be quite wrong to suppose that there was a sharply defined transition from empiricism to science. Chemical theory was built on a strong foundation of knowledge laboriously built up over the centuries by practical men and, on a level more detached from reality, by the alchemists with their fruitless preoccupation with the transmutation of base metals into gold and the preparation of an elixir of life. More realistic than the alchemists, though more limited in ambition, were the so-called iatrochemists of the* 16^{th} *century Paracelsian school, who looked on chemistry as primarily the handmaiden of medicine.*

The discovery of gases[305] *during* 1620–1774*, and the investigation of their properties slowly undermined the* phlogiston *theory*[306]*, and after* 1785 *it*

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savants".

[305] *Hydrogen* (Boyle, 1670; **Cavendish**, 1766); *Ammonia* (**Kunckel**, 1677; **Berthollet**, 1787); *Oxygen* (**Scheele**, 1772; **Priestley**, 1774); *Nitrogen* (**Cavendish** and others, 1772); *Chlorine* (Scheele, 1772; Priestley, 1774); *Carbon dioxide* (**Van Helmont**, 1620; **Black**, 1754); *Carbon monoxide* (**Daniel Rutherford**, 1772).

[306] The name *phlogiston* was coined (1703) by **Georg Ernst Stahl** (1660–1734), a professor of medicine and chemistry at Halle.

rapidly disappeared except among a few very conservative chemists. [Although the theory had the advantage of coordinating a large number of facts into a system, it retarded the progress of chemistry, and prevented a number of the best investigators from seeing the correct explanation of the facts they brought to light.]

With the publication in 1789 *of the* Elements of chemistry *by* Lavoisier, *the science of chemistry severed its remaining connections with the alchemical past and assumed a modem form. Lavoisier stressed the importance of quantitative methods of investigation in chemistry, and in this connection, he introduced* the principle of conservation of matter. *Lavoisier's new viewpoint led to the elaboration of several empirical laws. The first was the* law of equivalent proportions *(1791), formulated by* ***Jeremias Benjamin Richter*** *(1762–1807, Germany). After this discovery, tables of* equivalent weights *were drawn up, showing the relative amounts of chemical elements that would combine with each other. Richter also introduced the name* stoichiometry.

A second law, that of constant proportions, *was put forward (1797) by* ***Joseph Louis Proust*** *(1754–1826, France). Finally, the revival of the atomic theory (1803) by* ***John Dalton*** *(1766–1844, England), opened the road to the quantitative analysis and synthesis of compounds and put chemistry on solid foundations upon which the scientific method could rest.*

Gay-Lussac *(1778–1850, France) then established the* law of combining gaseous volumes *(1807), followed by an hypothesis (1811) of* ***Amadeo Avogadro*** *(1776–1856, Italy) which reconciled Dalton's atomic theory with Gay-Lussac's law. But Avogadro's hypothesis was not accepted until the* 1860*'s, and chemists long continued to base atomic weights on arbitrary rules*[307].

William Prout *(1785–1850, England), a London physician, suggested (1815) that the atoms of all the elements were composed of a discrete number of hydrogen atoms, but* ***Jöns Jacob Berzelius*** *(1779–1848, Sweden), who*

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307 Dalton himself denied to the end the validity of Avogadro's hypothesis(!) because Avogadro pointed out that the molecules of elementary gases are not necessarily the atoms themselves, but usually consist of *groups* of atoms. Both kind of particles, atoms and molecules, had been called "atoms" by Dalton, but they are really different. Dalton held that like atoms must repel one another and could not combine. With his logic, the fact that one volume of oxygen combined with one volume of nitrogen to produce two volumes of nitric oxide meant that nitric oxide should contain only half as many particles in a given volume as nitrogen or oxygen. But the true reaction is $N_2+O_2=2NO$ in full accord with Avogadro's hypothesis. The hypothesis was also rejected by **Gay-Lussac** and **Berzelius**.

devised the modern chemical symbols (1813) and introduced the name Halogen (1825), showed that the atomic weights of the elements were not *exact multiplies of the weight of the atom of hydrogen.*

From about 1820 *to* 1860 *the atomic theory did not play a prominent role in chemistry. For the most part chemist preferred to use the directly determined equivalent weights of the elements, rather than the atomic weights which involved uncertain estimates as to the combining numbers of the atoms. The rejection of Avogadro's hypothesis left chemists without a general method of ascertaining the combining numbers of the elementary atoms.*

As early as 1824*, chemists discovered* isomers *(compounds with the same chemical formulas but different molecular structure). It all began with* ***Eilhard Mitscherlich*** *(1794–1863,* Germany*), one of Berzelius' pupils, who noticed (1819) that compounds with similar chemical formulae had the same crystalline form. This was the advent of* isomorphism*, through which Berzelius could determine the formulae of many salts and the atomic weight of their constituent elements.*

In the same year ***Pierre Louis Dulong*** *(1785–1838,* France*) and* ***Alexis Thérèse Petit*** *(1791–1820,* France*) found in Paris that in the case of a number of metals, the product of their atomic weight and specific heats was constant. This law enabled rough values of the atomic weights of the metals to be determined.*

The discoveries of ***Galvani*** *(1771) and* ***Volta*** *(1775) and the availability of the* Voltaic cell*, soon led to the development of the new branch of* electrochemistry*. It appeared that electricity could bring about chemical action;* ***William Nicholson*** *(1753–1815,* England*) and* ***Anthony Carlisle*** *(1768–1840,* England*) performed (1800) the first electrolysis of water. The experiments of* ***Humphry Davy*** *(1778–1829,* England*) during* 1801–1806 *on the electrolysis of salt solutions led him to the theory that the chemical reaction between the elements, was essentially of an electric character and paved the road to the electrical theory of chemical affinity developed further by Berzelius. The laws of electrolysis were introduced (1833) by* ***Michael Faraday*** *(1791–1867,* England*).*

Inorganic chemistry developed rapidly in the period 1790–1830*, as geologists discovered numerous minerals for the chemists to analyze. Berzelius himself described the preparation, purification, and analysis of over* 2000 *inorganic compounds in the decade* 1810–1820*. Hundreds of chemists, mainly in German, French, English and Swedish universities were discovering each year new elements and compounds and determining their properties:* Uranium *(1789) was discovered by* ***Martin Heinrich Klaproth*** *(1743–1817,* Germany*);* Chromium *(1798) by* ***Louis Nicolas Vauquelin*** *(1763–1829,* France*);*

Bromine (1826) by **Antoine Jérome Ballard** *(1802–1876,* France*); Palladium and Rhodium (1803) by* **William Hyde Whollaston** *(1766–1828); hydrogen peroxide by* **Louis Jacques Thenard** *(1777–1857,* France*).*

Early recognitions of the law of mass action were made by **Carl Friedrich Wenzel** *(1740–1793,* Germany*) in* 1777 *and by* **Claude Louis Berthollet** *(1748–1822) in 1799.* **Jean Antoine Chaptal** *(1756–1832,* France*) proposed the name nitrogen (1790) and* **William Whewell** *(1794–1866,* England*) coined the names electrolysis, electrolyte, anode, cathode, anion and cation at the request of* Michael Faraday *(1833).* **Robert Wilhelm Bunsen** *(1811–1899) studies the chemical action of light (1857) and applied the spectroscope to chemistry (1859).*

In 1807, **Berzelius** *named the class of solid substances that melt upon heating — inorganic and those that burn — organic. It was soon discovered that while minerals could be characterized by the relative amounts of the elements which were contained in them, organic compounds from the start were seen to be complex arrangement of few elements, notably of carbon (C), hydrogen (H), oxygen (O), and nitrogen (N); quantitative analysis did not go far toward the characterization of such compounds.*

Isolated studies of carbon compounds go back to the Middle Ages (e.g., alcohol, ether, acetone). The investigations of **Scheele** *during* 1770–1784*, resulted in the discovery of many organic acids in plants and fruits, glycerin, HCN and esters.*

The first satisfactory method of organic analysis was worked out by **Gay-Lussac** *and* **Thenard** *(1810).* **Michel Eugène Chevreul** *(1786–1889) investigated the composition of oils and fats (from 1813), explained the reaction of saponification and worked on the analysis of organic compounds. The first amino acid was isolated and studied by* **Henri Braconnot** *(1781–1855, France) in* 1820.

The development of organic chemistry was boosted considerably by the works of the German chemists **Friedrich Wöhler** *(1800–1882) and* **Justus von Liebig** *(1803–1873). Liebig went to Paris to study under Gay-Lussac at the Ecole Polytechnique, and Wöhler went to Stockholm to study under Berzelius. The later, as late as* 1819*, had thought that organic compounds did not obey the law of constant proportions and did not belong to chemistry proper, as they were the products of "vital forces". But in* 1828*, Wöhler broke down the hypothetical barrier dividing inorganic substances from organic substances by heating some ammonium cyanate, classed as inorganic, and got urea, an organic chemical*[308].

[308] He had mixed solutions of *silver cyanate* (AgCNO) and *ammonium chloride* (NH_4Cl), producing the isomer *ammonium cyanate* ($NH_4.CNO$) which, when

Now organic and inorganic chemistry were brought close together. Nevertheless, certain fundamental difference were emerging. It was found already in 1815 *by* ***Jean Baptiste Biot*** *(1774–1862, France) that tartaric acid produced by grapes [HOOC−(CHOH)$_2$−COOH] polarizes light, while seemingly the same acid produced in the laboratory, did not polarize light — both acids having the* same *chemical formula. Liebig and Wöhler found other similar situations in* 1824. *In* 1830, *Berzelius named such pairs of compounds* isomers.

Louis Pasteur *(1822–1895, France) investigated the optical activity of organic compounds (1848) and had worked out the mechanism by which two otherwise identical isomers behave differently in living organisms. He suggested that the shape of the molecule might be different between the isomers. He also provided experimental proofs for the vitalistic theory of* fermentation *(1857) and later carried out fundamental research in bacteriology, disproving 'spontaneous generation'.*

Jean Baptiste Dumas *(1800–1884, France), chemist and politician, suggested that the chemical properties of organic compounds were due to their particular structural arrangements, or* type, *and not to the electrical character of the elements which compose them (1838). His theory had a direct influence on the revival of the atomic theory, which meanwhile had receded into the background of chemical theory.*

The theory of types was further developed by ***August Laurent*** *(1808–1853) [who also discovered* anthracene *(1832)] and* ***Charles Frederic Gerhardt*** *(1816–1856, France) who revived the theory of acid radicals and contributed to the developing concept of atomic weights.* ***Charles Adolphe Wurz*** *(1817–1884, France) developed the method of synthesizing* long-chain hydrocarbons *using hydrocarbon iodides and sodium (1855).*

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warmed, gave crystals of urea identical to those obtained as a waste product in urine. He wrote: "*I must tell you that I can make urea without requiring a kidney or an animal, either man or dog*".

Previously, urea had been obtained synthetically by **John Davy** in 1811 by the action of ammonia gas on carbonyl chloride (the poisonous gas *phosgene*, obtained when a mixture of equal volumes of chlorine and carbon monoxide is exposed to bright sunlight). The reactions are: $Cl_2+CO \Rightarrow COCl_2$, $4NH_3+COCl_2 \Rightarrow CO(NH_2)_2+2NH_4Cl$. Urea is then separated from ammonium chloride by warming with alcohol in which urea is soluble.

Davy, however, was not aware that urea was formed in the reaction.

In 1859, ***Marcel Morren****, the dean of the science faculty of Marseille, discovered acetylene* ($H-C \equiv C-H$) *by activating an electric spark in a glass containing carbon electrodes and hydrogen*[309] ($2C+H_2 \rightleftharpoons C_2H_2$).

Pierre-Eugéne-Marcellin Berthelot *(1827–1907) succeeded in synthesizing many organic compounds (1854–1868), among them alcohol (1857) and the first that do not occur naturally (1860). Through this he seriously undermined the remaining support for the theory of vitalism. Berthelot also carried out important work in physical chemistry on reaction velocity, thermochemistry and detonation waves.*

Christian Friedrich Schönbein *(1799–1868, Switzerland) discovered ozone (1840) and synthesized (1846) the new organic compound nitrocellulose*[310] *which heralded the age of high explosives.* **Thomas Graham** *(1805–1869, Scotland), one of the founders of physical chemistry, discovered his law of diffusion of gases (1829).* **Hermann Kolbe** *(1818–1884, Germany) foreshadowed modern structural formula and valence.*

However, it was **Edward Frankland** *(1825–1899, England) who introduced the concept of valence of elements into chemistry (1852) [valenc = the definite capacity of each atom to combine with other atoms] and recognized that the valency of an element could vary. He noted that the elements fell into groups which had the same valency.*

Friedrich August Kekulé *(1829–1896, Germany) recognized the 4-valency of carbon (1857) and began to use structural diagrams based on bonding in organic chemistry (1861). His diagrams showed that Pasteur was correct in assuming that the shape of the organic molecule determines its properties. Kekulé also put forward the hexagon formula for benzen (1865).*

309 **Morren** reported that he obtained 'carbonized hydrogen', the nature of which he has not yet established. Three years later (1862) **Marcelin Berthelot** repeated the same feat. However, he *knew* that he had synthesized acetylene. There was one difference between the two experiments. Morren had made a *discovery* and Berthelot an *invention*. **Jean Baptiste Dumas**, who was president of the Academy, drew Berthelot's attention to what Morren had done before him. Berthelot replied that Morren had not been able to verify his production of a carbonized hydrogen. In fact, Morren had verified it by a *spectral analysis* of the gas obtained, but he was not up to contesting the matter with Berthelot; he stepped down and history has forgotten his name.

310 It was discovered when Schönbein's wife's apron, which he had used to wipe up a spilled mixture of acids, exploded and vanished in a puff of smoke. When others tried to manufacture guncotton in quantity, many were killed by premature explosions.

At the international Karlsruhe conference (1860) ***Stanislao Cannizzero*** *(1826–1910, Italy) revived the work of Amadeo Avogadro, particularly, Avogadro's hypothesis and the important distinction between atoms and molecules (1811). Cannizzero employed the Avogadro hypothesis in the straightforward determination of molecular weights of gaseous compounds by comparing the weight of the volume of the gas to that of an equal volume of hydrogen at the same pressure and temperature. From molecular weights he proceeded to atomic weights. The valency could then be obtained by dividing the atomic weight by the equivalent weight of the element.*

With settled values for the valencies of the elements, structural models of their compounds were constructed. The reaction of those compounds provided tests for the validity of such structures, while, in turn, the proposed structures indicated possible new reactions.

The final addition to the classical theory of molecular structure came in 1874 *when* ***Joseph Achille Le Bell*** *(1847–1930, France), and* ***Jacobus Hendricus Van't Hoff*** *(1852–1911, Holland), suggested independently that the* 4 *valencies of carbon were directed in space toward the apices of a regular tetrahedron, in order to account for the two isomeric forms of tartaric acid isolated by Pasteur (1848), and other cases of optical isomerism discovered later.*

The acceptance of Avogadro's hypothesis, followed by the establishment of the definitive valencies and atomic weights of the elements, had its influence upon inorganic as well as organic chemistry. The works of ***Johann Dobereiner*** *(1780–1849, Germany) and* ***Antoine Jérome Ballard*** *(1802–1876, France) helped to classify the elements into equivalency groups.* ***Alexandre Émile Beguyer de Chancourtois*** *(1820–1886, France) was first to publish a list of elements in the order of their atomic weights (1862), but since he failed to furnish an accompanying diagram, the periodicity of the elements was far from clear.*

Finally, ***Dimitri Ivanovich Mendeleev*** *(1834–1907, Russia) in* 1869 *and* ***Julius Lothar Meyer*** *(1830–1895, Germany) in* 1870, *formulated the periodic law, stating that the properties of the elements varied in a periodic manner with their atomic weights. Both emphasized that there were gaps in the periodic table which elements as yet unknown should occupy. Mendeleev specified three gaps, all of which were filled by discoveries between* 1875 *and* 1885. *As a consequence, he got most of the credit for the periodic table.*

The periodic classification provided the first theoretical guide to the search for new elements: 23 elements known to Lavoisier had been discovered by the trial and error study of their specific chemical relations. Practical chemical analysis became more systematized, and, applied to the mineral specimens *provided by the geologists, it led to the discovery of* 31 *new elements in the period* 1790–1830.

Between 1830 *and* 1860 *little was accomplished in regard to the isolation and identification of new elements, save the rare-earths lanthanum and erbium by* **Carl Gustaf Mosander** *(1797–1858,* Sweden) *in* 1839–1841. *With his new spectroscope,* **Bunsen** *discovered the new alkali metals cesium and rubidium in* 1860–1861.

In London, **William Crookes** *(1861–1919) found thallium (1861) and in the Freiberg School of Mines,* **Ferdinand Reich** *(1799–1882,* Germany) *discovered indium (1863). Then came the discoveries of gallium (1874), ytterbium (1878), scandium (1879), gadolinium (1880) and germanium (1885) by the respective chemists* **Paul Emile Lecoq de Boisbaudran** *(1838–1912, France; Ga),* **Jean Charles Galissard de Marignac** *(1817–1894,* Switzerland; Gd), **Lars Fredrik Nilson** *(1840–1859,* Sweden; Sc) *and* **Clemens Alexander Winkler** *(1838–1904,* Germany; Ge).

The development of chemistry in the second half of the 19^{th} *century was mainly due to the rapid growth of synthetic organic chemistry: attempts were made to prepare in the laboratory those compounds which built up the plant and animal organisms. In addition, numerous drugs and dyestuffs have been prepared which are not found in the storehouse of nature.*

Three of the leading organic chemists who contributed most of this trend were **Adolf Johann Friedrich Wilhelm von Baeyer** *(1835–1917),* **Emil Hermann Fischer** *(1852–1919), and* **Victor Meyer** *(1848–1898) — all from Germany. Baeyer synthesized the dye indigo blue (1878) Fischer won the Nobel Prize (1902) for sugar and parine synthesis (1891–1898), and Meyer discovered thiophene (1882).*

In the 19^{th} *century chemistry held sway as the leading science. Yet it was running ahead of its theories, emerging with little understanding of why certain rules worked; for example, the concept of valence was introduced in* 1852 *and the periodic table in* 1869, *but it was not until the discovery of the Pauli exclusion principle in* 1925 *that either of these could be understood from first principles. Likewise, with no understanding of how electron behave (indeed, without any suspicion of electrons!) the wonders of spectroscopy emerged without a theoretical basis.*

1771 CE Encyclopedia Britannica published.

1771–1772 CE **Alexandre-Théophile Vandermonde** (1735–1796, France). Mathematician and musician. Founded the general theory of determinants (1772).[311]

His role in this field is similar to the one played by Cayley with regard to matrices in 1857. In his "*Mémoire sur la resolution des équations*" (1771), he approached the general problem of solubility of algebraic equations through a study of functions invariant under *permutations* of the roots of the equation.

Kronecker (1888) claimed that the study of modern algebra began with this paper of Vandermonde. **Cauchy** stated that Vandermonde had priority over **Lagrange** (1771) for this remarkable idea which eventually led to the study of *group theory*. He thus discovered the first truly group-theoretic properties of permutations and the key to understanding of the solution of equations by radicals.

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311 His name is best known today for the *Vandermonde determinant*. Yet nowhere in his four mathematical papers (1771–1772) does this determinant appear! Let

$$P(x) = x^n - s_1x^{n-1} + \cdots + (-)^n s_n = (x - x_1)(x - x_2)\cdots(x - x_n),$$

let $\sigma_i = \sum_{j=1}^{n} x_j^i$ for $i = 1, 2, ...$ and let A and B be the $n \times n$ matrices

$$A = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \\ x_1^2 & x_2^2 & \cdots & x_n^2 \\ \vdots & \vdots & \cdots & \vdots \\ x_1^{n-1} & x_2^{n-1} & \cdots & x_n^{n-1} \end{bmatrix},$$

$$B = \begin{bmatrix} n & \sigma_1 & \sigma_2 & \cdots & \sigma_{n-1} \\ \sigma_1 & \sigma_2 & \sigma_3 & \cdots & \sigma_n \\ \sigma_2 & \sigma_3 & \sigma_4 & \cdots & \sigma_{n+1} \\ \vdots & \vdots & \cdots & \vdots & \\ \sigma_{n-1} & \sigma_n & \sigma_{n+1} & \cdots & \sigma_{2n-2} \end{bmatrix}.$$

It can be then shown that

- $\det A = \prod_{i>j}(x_i - x_j) =$ *Vandermonde's determinant*
- $B = AA^T$ $\quad (A^T =$ transpose of $A)$
- $D(s_1, s_2, ..., s_n) =$ discriminant of $P(x) = \det B$
- Discriminant of $P(x) = x^n + px + q$ is $(-)^{\frac{n(n-1)}{2}}\left[(-)^{n-1}(n-1)^{n-1}p^n + n^n q^{n-1}\right]$

Solved (1771) the irreducible cyclotomic equation

$$(z^{11}-1)/(z-1) = z^{10}+z^9+z^8+z^7+z^6+z^5+z^4+z^3+z^2+z+1 = 0$$

in radicals.

1772 CE *Nitrogen* was discovered independently by **Daniel Rutherford** (1749–1819, Scotland), **Carl Scheele** (1742–1786, Sweden), **Joseph Priestley** (1733–1804, England), and **Henry Cavendish** (1731–1810, England).

1772 CE **Johann Bode** (1747–1826, Germany) stated an empirical relation that gives the approximate mean distance of the known planets from the sun. It is known as *Bode's law* although it was stated earlier (1766) by **Johann Titius** (1729–1796, Germany) and it is not a law in the strict physical sense. If 4 is added to 0, 3, 6, 12, 24, 48, 96, and each sum is divided by 10, a sequence of number results, each of which is the approximate distance of a planet in astronomical units[312] [Mercury 0.4 (0.39); Venus 0.7 (0.72); Earth 1.0 (1.00); Mars 1.6 (1.52); Ceres 2.8 (2.77); Jupiter 5.2 (5.20); Saturn 10.0 (9.54); Uranus 19.6 (19.18); Neptune 38.8 (30.06); Pluto 77.2 (39.44).]

When Bode's law was stated, there was an apparent gap in the series between the distances of Mars and Jupiter and it was thought that there was a planet in the gap.

On Jan. 1, 1801, **Guiseppe Piazzi**[313] (1746–1826, Italy) discovered the first known asteroid. This "star" was found to move and was observed for about a month before it became "lost" owing to the illness of its discoverer.

Based on complicated calculations with meager evidence (41 days of data), **Carl Friedrich Gauss** (1771–1855, Germany), then only 23 years old, determined its orbit using a new method developed by him ad hoc. When its distance from the sun was found to fit almost exactly the mean distance computed by Bode's law, it was assumed to be the missing planet and named Ceres (diameter $\sim$ 1000 km), for the goddess of agriculture and protector of Sicily.

It is not clear today whether Bode's law is just a coincidence of numbers, or whether it describes some deeper interrelation among the planet's orbits. If this progression of numbers indeed has any meaning, it could provide some insight into the *early history* of the solar system. But it could also just be an

[312] The number in brackets is the actual value.

[313] The astronomer **Piazzi** was born in Ponte di Valtellina, Italy. He became a Theatine monk, professor of theology in Rome (1779), and professor of mathematics at the Academy of Palermo (1780). He set up an observatory at Palermo (1789) and published a catalogue of fixed stars (1813), listing 7646 entries.

arrangement that would hold for *any* system of bodies orbiting about a center of mass, given enough time for those bodies to reach some state of dynamic equilibrium.

Gauss' method depends on accurate positions of the body on 3 dates, preferably separated by a few weeks. It is still used in modified form. Ceres was found again in the position predicted by Gauss. Other asteroids were discovered (often called minor planets): **Pallas** (1802), **Juno** (1804), **Vesta** (1807), **Astraea** (1845). Presently, nearly 2000 asteroids are known.

The fame earned by **Gauss** through his efforts on the problem eventually led, in 1807, to his appointment as director of the Göttingen Observatory, where he remained for the rest of his life.

1773–1778 CE **Otto Frederik Müller** (1730–1784, Denmark). Naturalist. Taxonomically separated bacteria[314] from protozoa and was able to distinguish two morphological types of bacteria: *bacillum* and *spirillum*. Bacteria were first *stained* (with indigo and carmine) by **Wilhelm Friedrich von Gleichen-Russworm** (1778).

1773–1825 CE **Pierre Simon de Laplace** (1749–1827, France). An eminent mathematician and astronomer. His most outstanding work was done in the fields of celestial mechanics, probability, differential equations and geodesy.

He was first to examine the conditions of stability of the system formed by Saturn's rings, pointed out the necessity for their rotation and assigned to it a period ($10^h 33^m$) virtually identical with that established by the observations of **Herschel**. In 1773 he began his studies of the figure of equilibrium of a mass of rotating fluid. The related subject of the attraction of spheroids was also promoted by him, and in 1784 he generalized the results of **Legendre** and **Maclaurin** and treated exhaustively the general problem of the attraction of any spheroid form upon a particle situated outside or on its surface.

Laplace was born of poor parents at Beaumont-en-Auge in Normandy. His early mathematical ability won him a teaching post in the military school of Beaumont. He came to Paris and with the support of d'Alembert, became a professor of mathematics in the Ecole Militaire of Paris.[315]

At the age of 24 he rose to fame following his discovery (1773) concerning the mutual gravitational interactions of the constituents of the solar system

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[314] First discovered by **Leeuwenhoek** (1683).

[315] **Napoleon Bonaparte** was a cadet at this school from October 1784 to October 1785.

(sun, planets and their satellites). He showed that while perturbations[316] introduced small changes into planetary orbits, these changes were *periodic*: that is, the orbit would alter its properties in one direction, then back in the other, and so on indefinitely. Over the long run, the *average* shape of the orbit would remain constant[317]. This is Laplace's celebrated conclusion of the invariability of the planetary mean motions, carrying the proof as far as the cubes of the eccentricities and inclinations. This was the first and most important step in the establishment of the stability of the solar system. It meant that the solar system was in dynamic equilibrium, and could continue indefinitely into the future and might already have existed through an indefinite past [assuming of course that there is no appreciable overriding influence by stars outside the system].

Laplace's results were followed by a series of investigations, in which **Lagrange** and Laplace alternatively surpassed and supplemented each other in assigning limits of variation to the several elements of the planetary orbits.

In his monumental five-volume treatise: "*Traité de Mécanique Céleste*" (1799–1825), Laplace summed up the work on gravitation of several generations of illustrious mathematicians [giving credit only to himself (!) and suppressing references to discoveries of his predecessors and contemporaries, including Lagrange]. The principal legacy of *Mécanique Céleste* to later generations lay in Laplace's wholesale development of *potential theory*[318], with its

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316 To dig deeper, see:

- Nayfeh, A., *Perturbation Methods*, Wiley, 1973, 425 pp.
- Bellman, R., *Perturbation Techniques in Mathematics, Physics and Engineering*, Holt, Rinehart and Winston, 1964, 118 pp.
- Bush, A.W., *Perturbation Methods for Engineers and Scientists*, CRC Press, 1992, 303 pp.
- Hinch, E.J., *Perturbation Methods*, Cambridge University Press, 1991, 160 pp.
- Bender, C.M. and S.A. Orszag, *Advanced Mathematical Methods for Scientists and Engineers*, McGraw-Hill, 1978, 593 pp.

317 **Newton** himself was bewildered by the complexity of the solar system and was of the opinion that divine intervention would occasionally be needed to prevent this complex mechanism from degenerating into chaos. Laplace, apparently, decided to seek reassurance elsewhere.

318 For further reading, see:

- Kellogg, O.D., *Potential Theory*, Dover, 1953, 384 pp.
- MacMillan, W.D., *The Theory of the Potential*, Dover, 1958, 469 pp.

far-reaching implications for different branches of physics ranging from gravitation and fluid dynamics to electromagnetism and atomic physics. Even though he lifted the idea of the potential from Lagrange without acknowledgements, he exploited it so extensively that ever since his time the fundamental partial differential equation of potential theory has been known as the *Laplace equation*[319].

The overall aim of Laplace in his treatise was to "offer a complete solution of the great mechanical problem presented by the solar system, and to bring the theory to coincide so closely with observation that empirical equations should no longer find a place in astronomical tables".

The first part of the work (2 volumes, 1799) contains methods for calculating the movements of translation and rotation of the heavenly bodies, for determining their figures and resolving *tidal* problems.

In his book '*Exposition du Systeme du Monde*', published in 1796, Laplace speculated on the subject of planetary origin in his famous *nebular hypothesis*.

Already in 1644, **Descartes** advanced the idea that the sun and its solar system formed from a gigantic whirlpool, or vortex, in a universal fluid, with the planets and their satellites forming from smaller eddies. This crude theory did not include any clearly specified idea of the nature of the cosmic substance from which the sun and planets arose, but it did account for the fact that all orbital motions are in the same direction.

The hypothesis of Descartes[320] was the first of a general type known as *evolutionary theories*, in which the formation of the solar system is posited to

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- Webster, A.G., *Partial Differential Equations of Mathematical Physics*, Dover, 1956, 440 pp.
- Webster, A.G., *Dynamics – Lectures on Mathematical Physics*, Hafner, 1949, 588 pp.
- Bateman, H., *Partial Differential Equations of Mathematical Physics*, Cambridge University Press, 1959, 522 pp.

[319] It is the equation $\nabla^2\phi = 0$ of the gravitational potential ϕ, outside the source region. It had actually been discovered by Euler in 1752, in connection with his studies of hydrodynamics.

[320] The Cartesian idea of a set of universal laws which control natural occurrences exercised a powerful appeal in the succeeding centuries. Laplace, even as he developed his theory of a naturally evolving cosmos, took the motion to its logical conclusion by endorsing the idea that, given the laws of gravitation and other forces, Newtonian mechanics, and the "initial conditions" of the universe, every subsequent event not only can be accurately predicted, but is *predetermined*. The whole history of the universe, and of earth, is but the inevitable unfolding

have occurred as a natural by-product of the sequence of events that produced the sun.

Immanuel Kant in 1755 further elaborated Descartes' idea by applying the recently discovered Newtonian mechanics to show that a rotating interstellar gas cloud would flatten into a disc as it contracted.

Laplace added to this model the notion that as the spinning cloud flattened into a disc, rotational inertial forces broke off concentric rings of matter, so that at one point the early solar system would have resembled the planet Saturn with its rings. Each ring was supposed to have condensed into a planet.

In the same book, Laplace raised another speculation about an object which he called "*corps obscurs*". He noted (1796) that a consequence of Newtonian gravity and Newtonian corpuscular theory of light was that light would not be able to escape from a sufficiently massive object, but would be bent around and stay trapped near the object[321]. In spite of this deduction, the idea that a '*black hole*' could actually exist in nature did not occur to astronomers for almost 2 centuries. Laplace's "*Corps obscurs*" were taken up in the mid 1960's by modern physicists, armed with the new General Relativity Theory.

His other masterpiece was the treatise "*Théorie Analytique des Probabilités*" (1812). Nowhere did Laplace display his genius more conspicuously than in the theory of probabilities. The science which **Pascal** and **Fermat** had initiated, was brought by him to near perfection. In this book he amalgamated his own discoveries with many ideas of others (unacknowledged!),

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of the consequences of a set of eternal laws. Laplace believed that mathematical physics is capable of explaining *everything*, although in practice he ignored physical phenomena not governed by the basic mathematical laws known in his day. For example, he did not take into account (because he could not) the electrical and magnetic interactions of bodies, their chemical reactions, their nuclear transformations, the processes by which they are heated and cooled — in short, all the phenomena now known to science but unknown to him. Nevertheless, the Laplacian idea of a deterministic, natural evolving universe – suitably modified by modern insight of quantum physics and chaos theory – is nowadays taken for granted by science – even in the realms of *biology* and *cosmology* (the study of the universe as a whole, including its very *creation*).

[321] This was noted before (1784) by **John Michell**. Laplace calculated that no light could escape from a body with the *earth's density* and a radius 250 times that of the sun. Did Laplace read Michell's paper, published in the Philosophical Transactions of the Royal Society of London?

but even discounting this, his book is considered to be the greatest contribution to this branch of mathematics by any one man. Here he harnessed, for the first time, the powerful machine of the infinitesimal calculus to discrete mathematics.

To this end he invented the *Laplace Transform*, *generating functions* and many other highly nontrivial tools. He showed how the *Laplace transform* can be used to reduce the solutions of linear differential equations to definite integrals, and furnished an elegant method by which a linear partial differential equation of the second order might be solved.

Laplace died exactly 100 years after the death of **Isaac Newton**. His last words were: "*That which we know is a trifle — that which we are ignorant of is immense*".

Worldview XV: Pierre-Simon de Laplace

* * *

Nature laughs at the difficulties of integration.

* * *

Read Euler: he is our master in everything.

* * *

Such is the advantage of a well constructed language that its simplified notation often becomes the source of profound theories.

* * *

"An intelligence, which at a given moment knew all of the forces that animate nature, and the respective positions of the beings that compose it, and further possessing the scope to analyze these data, could condense into a single formula for the movement of the greater bodies of the universe and that of the least atom: for such an intelligence nothing could be uncertain, and past and future alike would be before its eyes."

(1812)

* * *

Napoleon and his scientists

(**Lagrange** 1736–1813; **Monge** 1746–1818; **Laplace** 1749–1827; **Fourier** 1768–1830)

*In September 1785, **Laplace** examined and passed a cadet by the name of Napoleon Bonaparte in a military school at Beaumont. With this education, Napoleon qualified himself for the artillery.*

***Napoleon Bonaparte** (1769–1821) Emperor of the French, was an enthusiastic amateur mathematician, particularly fascinated by geometry, which of course had great military value. He was also a man with unbounded admiration for the creative French mathematicians of his day. Whatever Napoleon's ability as a geometer may have been, it is to his credit that he so revolutionarized the teaching of French mathematics that, according to several historians of mathematics, his reforms were responsible for the great upsurge of creative mathematics in the 19th century France.*

*It is said that in 1797, while discussing geometry with **Lagrange** and **Laplace**, Bonaparte surprised them by explaining some of **Mascheroni**'s solutions that were completely new to them. "General", Laplace reportedly remarked, "we expect everything of you, except lessons in geometry".*

Yet, a theorem named after Napoleon exists. It states that the centers of equilateral triangles constructed on the sides of an arbitrary triangle form another equilateral triangle (Napoleon's theorem). [The theorem provides a generalization in which the word 'equilateral' is replaced by 'similar'.] It is very doubtful that Napoleon was well enough versed in geometry to have discovered and proved it himself.

***Monge** gained the close friendship and admiration of Napoleon and accompanied the latter, along with **J. Fourier**, on the Egyptian Expedition (1798).*

*The publication of Mécanique Céleste gained **Laplace** world-wide celebrity. Asked by Napoleon why in the entire work he had not once mentioned God, Laplace replied: 'Sire, je n'avais pas besoin de cette hypothèse' (Sir, I had no need for that hypothesis).*

But scientific distinctions by no means satisfied his ambition, and after the French revolution Laplace's political talents and greed for position came into full play. He set a memorable example of a genius degraded to servility for the sake of a riband and a title. He smoothly adapted himself by changing *his principles – back and forth between fervent republicanism and fawning royalism – and each time emerged with a better job and grander titles. The ardor of his republican principles gave place to devotion towards Napoleon, a sentiment promptly rewarded with the post of minister of the interior. His incapacity for affairs was however so flagrant that it became necessary to supersede him at the end of 6 weeks. "He brought into the administration", said Napoleon, "the spirit of the infinitesimals".*

His failure was consoled by elevation to the senate, of which body he became chancellor in 1803. The title of Count he had acquired on the creation of the Empire. Nevertheless, he cheerfully gave his voice in 1814 for the dethronement of his patron, and his compliance merited a seat in the chamber of peers and, in 1817, the dignity of a marquisate.

Table 3.8: Greatest Mathematicians of the 18^{th} century

Name	Nat.	Life-span	Major Contribution
Jakob Bernoulli (Jacque, James)	SW	1654–1705	Theory of probability; isoperimetric problems (early calculus of variation); Bernoulli numbers and polynomials.
Antoine Parent	F	1666–1716	Solid analytical geometry (1700).
G. Saccheri	I	1667–1733	Forerunner of non-Euclidean geometry.
Johann Bernoulli (Jean, John)	SW	1667–1748	Principle of virtual work; L'Hospitale rule; partial differentiation; Brachistochrone; Applied calculus.
Abraham de Moivre	E	1667–1754	Normal and binomial distributions; Probability theory; De-Moivre formula; generating functions; Approximation for $n!$.
J.F. Riccati	I	1676–1754	Differential equations.
Roger Cotes	E	1682–1716	Algebraic equations; Trigonometry.
Fagnano dei Toschi	I	1682–1766	Addition theorems for elliptic integrals; Rectification of curves; $\pi = 2i \ln \frac{1-i}{1+i}$.
Brook Taylor	E	1685–1731	Polynomial approximation to analytic functions; Integration by parts; Calculus of finite differences.

Table 3.8: (Cont.)

Name	Nat.	Life-span	Major Contribution
Christian Goldbach	G	1690–1764	Goldbach conjecture (1742).
Colin Maclaurin	E	1698–1746	Applied calculus; Attraction of ellipsoids; determinants.
P.L.M. de Maupertuis	F	1698–1759	Principle of least action (optics and mechanics).
Daniel Bernoulli	SW	1700–1782	Hydrodynamic theory; First 'Fourier expansions'.
Thomas Bayes	F	1702–1761	Principle of Inverse probability (conditional probability).
Gabriel Cramer	SW	1704–1752	Determinants.
Leonhard Euler	SW	1707–1783	One of the last, and one of the greatest mathematical universalists. Contributed to all fields of pure and applied mathematics. Established analysis as an independent science.
Comte de Buffon	F	1707–1788	Geometrical probability. Forerunner of Monte-Carlo methods.
Alexis-Claude Clairaut	F	1713–1765	Differential geometry. Differential equations.
Jean Le Rond d'Alembert	F	1717–1783	Scalar wave-equation; Mathematical theory of gravitational perturbation; Foundations of calculus.

Table 3.8: (Cont.)

Name	Nat.	Life-span	Major Contribution
John Landen	E	1719–1790	Elliptic Integrals and functions.
Johann H. Lambert	G	1728–1777	Irrationality of π and e; Non-Euclidean geometry; map projections; Infinite Series; descriptive geometry.
Etienne Bezout	F	1730–1783	Algebraic equations; (Bezout eliminant).
A.T. Vandermonde	F	1735–1796	Theory of determinants. Notion of 'group'.
Erland S. Bring	S	1736–1798	The quintic equation ($x^5 + px + q = 0$, 1786).
Joseph Louis Lagrange	F	1736–1813	Calculus of variations; Theory of numbers; Interpolation formulae; Theory of equations; Continued fractions; 3-body problem; Theoretical dynamics.
John Wilson	E	1741–1793	Theory of numbers (1770).
Caspar Wessel	S	1745–1818	Geometry of complex numbers.
Gaspard Monge	F	1746–1818	Descriptive geometry; differential geometry of space curves and surfaces.
Jean-Baptiste-Joseph Delambre	F	1749–1822	Spherical trigonometry.

Table 3.8: (Cont.)

Name	Nat.	Life-span	Major Contribution
Pierre Simon de Laplace	F	1749–1827	Celestial mechanics; Theory of analytic probability; Partial differential equations.
Lorenzo Mascheroni	I	1750–1800	Geometry; infinite series.
Marie Adrien Legendre	F	1752–1833	Number theory; Elliptic Integrals; method of least squares; Law of quadratic reciprocity.
Lazare Carnot	F	1753–1823	Synthetic geometry.
J.B.M.C. Meusnier	F	1754–1793	Differential geometry; Minimal surface (1785).
Aimé Argand	SW	1755–1803	Geometry of complex numbers.
Marc-Antoine Parseval	F	1755–1836	Parseval Equality.
Paolo Ruffini	I	1765–1822	The quintic equation (1799).

Key: SW = Switzerland; F = France; I = Italy; E = England; G = Germany; S = Scandinavia.

1774–1784 CE **Marcus (Mordecai) Herz** (1747–1803, Germany). Physician, physicist and philosopher. One of the best physicians of his time. Was concerned with the ethical aspects of his profession and published (1783) "The Physician's Prayer".

Born to a poor Jewish copyist of scriptures. At the age of 15 left his parent's home and moved to Königsberg, where he began his philosophy studies, under **Kant**. Befriended **Moses Mendelssohn** in Berlin (1770) and studied medicine (1770–1774) at the University of Halle. Published books on philosophy and medicine. King Frederick William III (1744–1797) of Prussia appointed him Professor for life. His wife Henriette (1764–1847) was a famous beauty and society leader who conducted a brilliant salon frequented by **Boerne**, **Humboldt**, **Fichte** and **Schleiermacher**. She married Herz at age of 15 and after his death adopted the Christian faith (1817).

1774–1786 CE **Joseph Priestley** (1733–1804, England). Chemist. Shares the credit for the discovery of oxygen[322] with **Carl Wilhelm Scheele** of Sweden. His experiments are described in his 6 volume treatise "*Experiments and Observations of Different Kinds of Air*". Priestley prepared and examined oxygen, nitrous oxide, nitric oxide, nitrogen dioxide, hydrogen chloride, ammonia, silicon fluoride and sulphur dioxide. His work firmly established the fact that different gaseous forms of matter exist, each with definite properties.

Priestley was born near Leeds. He studied for the ministry and became a *dissenting* (nonconformist) minister in Leeds and Birmingham. Through his friendship with **Benjamin Franklin** he became interested in electricity, on which he performed many brilliant experiments. He turned to chemistry in 1772.

Priestley's sympathies for the cause of the French Revolution made him unpopular in England. In 1791 an angry mob burned his home and chapel in Birmingham. He then left England and moved to the United States in 1794.

1774–1800 CE **Alessandro Giuseppe Antonio Volta** (1745–1827, Italy). Physicist. Pioneer of electrical science. Invented the electric battery, the first electrochemical source of electric current. His discovery of the decomposition of water by electrical current laid the foundation of electrochemistry. He also invented the electric condenser.

Volta was born in Como, Italy, a member of a noble family. By 1774 he had established a reputation by his research work in electricity. In 1779, a

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[322] Priestley called the gas "*dephlogisticated air*". The French chemist **Antoine Lavoisier** named it *oxygen*. At that time gases were called "*airs*".

chair of physics was founded in Pavia, and Volta was chosen to occupy it. In 1782 he journeyed through France, Germany, Holland and England, and became acquainted with many scientific celebrities. In 1801 Napoleon called him to Paris, to show his experiments on contact electricity, and a medal was struck in his honor. He was made a senator of the kingdom of Lombardy. In 1815, the emperor of Austria made him a director of the philosophical faculty of Pavia. The *volt*, a unit of electric potential, is named for him.

1774–1804 CE **Johann Heinrich Pestalozzi** (1746–1827, Switzerland). Educational reformer. Influenced strongly methods of instruction in elementary schools throughout Europe and America.

Pestalozzi believed a pupil learned best by using his own senses and by discovering things for himself. His emphasis, therefore, lay upon concrete approach in education, with objects used to develop powers of observation and reasoning.

He was born in Zurich. He first studied for the ministry, but later changed to law. Poor health forced him to abandon law, and Pestalozzi settled on his farm near Zurich. In 1774 he established a school for poor children on his estate and endeavored to put in practice educational theories of **Jean-Jacques Rousseau**; although the school failed (1780), he derived from his experience a knowledge of certain principles for effective education, which he explained in his influential book *Lienhard und Gertrude* (1787). His most famous educational experiments were carried on at an institute for training teachers which he established at Yverdon (1805–1825). His theories are also expounded in *Abendstunde eines Einsiedlers* (1780) and *Wie Gertrude ihre Kinder lehrt* (1801).

1775–1785 CE **William Withering** (1741–1799, England). Physician and botanist. Discovered the use of *digitalis*, the most important drug in the treatment of heart disease[323].

Withering was born in Wellington, England. In 1766 he received his medical degree from the University of Edinburgh and established a general practice in the town of Stafford in Shropshire. He listened with interest (1775) to the country folk in his native district as they described the benefit of foxglove-tea

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[323] *Digitalis* was a medical herb for centuries. **Dioscorides** (ca 80 AD) praised it as a plant whose leaves, applied to the skin, could cure many diseases. Rural people made hot water infusions of leaves and drank *foxglove tea* to experience inexpensive but dangerous intoxication.

for 'dropsy'[324]. For several gold sovereigns he purchased the recipe from a local 'witch' and for the next ten years studied digitalis therapy in dropsy. Chemistry was not sufficiently advanced to permit the isolation of the active ingredient; biology in general and human physiology in particular were just in their infancy.

It was thus left to Withering to answer questions such as: which part of the plant was most active, could the leaves be dried; what was the best solvent for the active material (cold water); should one pick leaves in early or late summer and, most importantly, what was the optimum dose and how frequently should it be administered. In 1785, he published *An Account of the Foxglove and Some of its Medical Uses*, a clinical study so detailed and so accurate, so as to make the use of digitalis effective and safe.

The report was something of a bombshell in England where the standard treatment of dropsy was to puncture the water-logged tissues with an unsterilized scalpel and stretch the patient over bedsprings to allow the fluid to drip into buckets.

William Withering died of tuberculosis (1799) and was buried in a vault on which a *Digitalis* plant was engraved.

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[324] A cardiac-stimulating chemical, occurring naturally in the dried leaves of the common garden flower purple foxglove (*Digitalis purpurea*). It is a mixture of several cardiac *glucosides*. Doctors use digitalis in small doses to stabilize heartbeat when the action of the heart muscle is too weak to force blood out of the heart normally.

Dropsy (edema) is a condition in which watery fluid gathers in the body cavities and tissues caused by disorders in blood circulation in anemia, heart disease, kidney failure, etc. The word *Digitalis* was coined (1539) from the Latin *digitis* = little finger, and is directly derived from the German name for the plant *fingerhut*.

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"Behold, I make all things new"

(Revelations 21:5)

* * *

"The Revolution was effected before the War commenced."

(John Adams)

* * *

1775–1783 CE *American War of Independence*. Thirteen of Britain's North American Colonies broke away from rule by the mother-country and formed the *United States of America*; they were: Connecticut, Delaware, Georgia, Massachusetts, Maryland, North Carolina, New Hampshire, New Jersey, New York, Pennsylvania, Rhode Island, South Carolina, and Virginia. The colonies already made their own local laws, but the Britain Parliament kept control of financial matters, and particularly trade. The colonies had to use their own or British ships and to trade mainly with Britain or British colonies.

The Revolution could not have taken place without the religious background afforded by the *Great Awakening* — an American revivalist movement (1719–1775)[325]. It arose in response to the growing formalism of early 18^{th} century American Christianity, but was also influenced by the European

[325] Just as in France, rather late in the century, the combination of Voltairean rationalism and Rousseauesque emotionalism was to create a revolutionary explosion, so in America, but in a characteristically religious context. The thinking elements, and the fervid, personal elements were to combine to make Americans see the world with new eyes.

The essential difference between the *American Revolution* and the *French Revolution* is that the American Revolution, in its origin, was a religious event, whereas the French Revolution was an anti-religious event. That fact was to shape the American Revolution from start to finish and determine the nature of the independent state it brought into being. Indeed, in his *Farewell Address*

Enlightenment and the economic boom of middle-class people in Colonial America.

Revival began in New Jersey in 1719; Key figures were: **William Tennent** (1673–1745), a Presbyterian preacher, **Jonathan Edwards** (1703–1758), a puritan scholar, and **George Whitefield** (1714–1770), evangelist. Finally, Presbyterians, Baptists, Calvinists and Methodists all over America embraced the new movement. By questioning established authority, founding new colleges, and revivifying evangelical zeal, it helped to prepare the revolutionary generation in America.

The historian **Paul Johnson** in his book '*A History of the American People*' (1997) summarized the impact of the *Great Awakening*, in these words:

> *"The Revolution was in the mind and hearts of the people... It was the marriage between the rationalism of the American elites touched by the Enlightenment, with the spirit the Great Awakening among the masses which enabled the popular enthusiasm thus aroused to be channeled into the political aims of the Revolution — itself soon identified the coming eschatological event. Neither force could have succeeded without the other."*

After the *Seven Years' War* which brought Britain the French possessions in North America (1755–1763), Britain felt it necessary to keep a standing army there and taxed the colonies to pay for it. The colonists objected to 'taxation without representation' in Parliament. The British tried imposing taxes on newspapers, tea, paper, lead, and paint, but had to repeal all but the tea tax when the colonists refused to buy British goods as protest.

On December 16, 1773, a band of colonists disguised as Indians boarded British ships in Boston harbor and threw cargos of tea overboard. To this '*Boston tea party*' the British Parliament retorted with the so-called '*Intolerable Acts*', which included closing the port of Boston.

The *First Continental Congress* at Philadelphia (Sept. 1774) protested at the Acts, and the colonies decided not to buy British goods. British troops were sent from Boston to destroy an arms cache held by the colonists in

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(1796), **Washington** dispelled for good any notion that America was a secular state. He insisted: "*Religion and Morality are indispensable supports... There can be no security for property, for reputation, for life, if the sense of religious obligation desert the oaths which are the instruments of investigation in the Courts of Justice.*"

In fact, Washington was saying that America, being a free republic, dependent for its order on the good behavior of its citizens, cannot survive without religion.

nearby Concord. Just after dawn on April 19, 1775, at Lexington on the road to Concord, the troops were confronted by armed colonists and the war began.

The British retreated from Concord to Boston, and in June won the *Battle of Bunker Hill*, near Boston, despite heavy losses. The *Second Continental Congress* assembled in May 1775 and on July 4, 1776 issued the *Declaration of Independence*, largely drafted by Thomas Jefferson, claiming complete freedom from the British rule.

In 1777, the British army gained an important victory at Brandywine Creek (Pennsylvania), but a few weeks later, under General John Burgoyne, were forced to surrender at Saratoga (New York). France entered the war on the American side, followed later by Spain.

The end came when the British under Cornwallis surrendered to the American commander-in-chief, **George Washington** at Yorktown (Virginia) on October 19, 1781. The *Treaty of Paris* (September 03, 1783) formally recognized the independence of the United States. Washington was elected first president in 1789.

1776 CE The submarine is first used in combat, during the American Revolution. This 2-meter vessel, called the *Connecticut Turtle*, was designed (1775) by **David Bushnell** (1742–1824) of wood, iron and pitch. Driven by a hand-cranked propeller, it attempted unsuccessfully to sink British warships in New York harbor.

1776 CE **Adam Smith** (1723–1790, Scotland). Economist. Regarded the founder of modern economics. Worked out a theory of division of labor, money, prices, and wages in his book "Inquiry into the Nature and Causes of the Wealth of Nations". It laid foundations of the science of *political economy* and is the most influential economic treatise ever written, founding the classical school of economy. It contains the germ of nearly all economic ideas which have since appeared, even in rival systems.

The book dealt with the basic problem of how social order and human progress can be possible in a society where individuals follow their own self-interests, free from any government interference. This is the policy of '*laissez faire*' – leaving things done. Smith argued that this individualism led to order and progress. In order to make money, people produce things that other people are willing to buy. Buyers spend money for those things that they need or want most. When buyers and sellers meet in a market, a pattern of production develops that results in social harmony provided that all this would happen without any conscious control or direction. Smith also believed that:

- *Labor* (not land or money) was both the source and the final measure of value; *wages* developed on the basic needs of the workers and *rent* on the productivity of the land. *Profits* were the difference between selling prices and the cost of labor and rent.

- *Profits* should be used to expand *production*; this expansion would in turn create more jobs, and the *national income* would grow.

- *Free trade* and a self-regulating economy would result in *social progress*. Government need only preserve law and order, enforce justice, defend the nation, and provide for a few social needs that could not be met through the market.

Smith attacked the British mercantile system's limit on free trade and criticized the British government's tariffs and other limits on individual freedom in trade.

Smith was born in Kirkcaldy, Scotland and studied at the University of Glasgow and Oxford University. He became professor at Glasgow (1751). Appointed tutor of the young duke of Buccleuch (1764) and later received a regular income from that family. This enabled him to retire from teaching and devote the years 1766–1776 to the writing of his book.

1776–1784 CE **Jean Baptiste** (**Marie Charles**) **Meusnier** (de la Place) (1754–1793, France). Mathematician, engineer and army general. A pupil of Monge at the school in Mézières. Derived the *Meusnier theorem* (1776) on curvature at a point on a surface[326]. In 1783–1784, after Montgolfier's ascent in a balloon, he did fundamental research on aerostatics and designed (1784) a dirigible balloon. In this period he collaborated with **Lavoisier** in his work on decomposition of water into its elements. Meusnier joined the Jacobins

326 *Euler's theorem* (1760): $k_n = k_1 \cos^2 \alpha + k_2 \sin^2 \alpha$. It expresses the normal curvature (l.h.s.) to a surface in an arbitrary direction α in terms of the principal curvatures k_1, k_2.

Meusnier's theorem (1776): If a set of planes be drawn through a tangent to a surface in a nonasymptotic direction, then the osculating circles of the intersections with the surface (*normal sections*) lie upon a sphere ($k_n = k \cos \gamma$, where k_n = normal curvature, k = curvature of normal section, γ = angle between the principal normal to the curve and the normal to the surface).

The theorem was published in his *Mémoire sur la courbure des surfaces*, which he wrote after Monge had shown him Euler's paper. Together with Euler's theorem it gives full information concerning the curvature of any curve through a point on the surface.

(1790), became a field-marshal (1792) and was killed in defense of the fortress of Kassel, Mainz.

1777–1789 CE **Charles Augustin de Coulomb** (1736–1806, France). Scientist, inventor and military engineer. Made fundamental contributions in the fields of friction, electricity, and magnetism. In 1777 he invented the *torsion-balance* for measuring torsional elasticity, which he used to derive the laws of *torsion* of metal wires[327], strands of hair and thin silk (1784). In his memoir on the theory of simple machines (1779–1781) he discovered the fundamental law of *friction*; In 1785 he made precise measurements of the forces of attraction and repulsion between charged bodies and between magnetic poles, using his torsion balance. He then demonstrated conclusively that electric charges and magnetic poles obey the *inverse-square laws* like that of static gravity.

Coulomb was born at Angouleme. He chose the profession of military engineer. After spending nine years at Martinique in the West Indies he was stationed, in 1781, permanently at Paris. Upon the outbreak of the Revolution in 1789, he resigned his position and retired to a small estate at Blois. But he was recalled to Paris, to take part in the new determination of weights and measures, decreed by the revolutionary government. He was appointed inspector of public instruction in 1802 and died in Paris a few years later.

The practical unit of quantity of electricity, the *coulomb*, is named after him.

1778–1802 CE **Joseph Bramah** (1748–1814, England). Engineer and prolific inventor whose inventions introduced practical techniques that founded the engineering industry. Suggested the possibility of screw propulsion for ships (1785), and the hydraulic transmission of power (1802). Invented the hydraulic press (1795), a numerical printing machine (1806) and the ball-drive siphon system for a water closet (1778).

Bramah was born at Stainborogh in Yorkshire, the son of a farmer. He worked as a cabinet-maker in London, where he subsequently started a business of his own.

1779 CE **Jan Ingenhousz** (1730–1799, Holland). Physician and scientist. Discovered *photosynthesis*[328], and the *carbon cycle* in the earth-atmosphere system.

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[327] The torque τ in a thin cylinder with diameter d and length ℓ is $\tau = \mu d^4\theta/\ell$, where μ is the rigidity and θ is the angle of twist.

[328] The overall process by which plants absorb, store and use radiant energy: red light is absorbed by certain plant pigments (mainly *chlorophyll*), and converted into potential chemical energy. This energy is used to break the water molecule

He showed that in the presence of sunlight plants absorb water and carbon dioxide and give off oxygen through their green portions. In the dark the roots, flowers, and fruits give off carbon dioxide. In this process, plants obtain carbon *from the atmosphere* and not from the soil. On the other hand, he maintained, animals, by eating plants and breathing oxygen, recombine plant tissue and oxygen and re-form carbon dioxide and water. Thus, he concluded, plant and animal life on earth formed a balance.

Moreover — nothing material is used up; carbon, hydrogen, and oxygen shuttle between plants and animals, and from land to sea in a process that is most commonly called "*the carbon cycle*". Other elements, too, are engaged in cyclic processes: nitrogen, sulfur, phosphorus, and so on are absorbed *from the soil* by the plants and incorporated into their tissue. Animals eat plants and make use of the various elements, then finally restore them to the soil in their droppings, and in the form of their own bodies when death is followed by bacterial decomposition. Only one thing is permanently used up in these cyclical chemical changes, and that is the energy of the solar radiation.

Ingenhousz was born in Breda, Holland. During 1772–1779 he was court physician to empress Maria Theresa of Austria. He died in England.

Ingenhousz' discovery was not accidental, but rather a natural consequence of the Industrial Revolution: Following the ascent of the first hydrogen balloons, and the introduction of coal, gas and metallurgical innovations associated with the use of steam power, chemical science grew rapidly.

Black's work on CO_2 (1756), **Rutherford**'s isolation of nitrogen (1772), the researches of **Watt** (1776), **Cavendish** (1766–1781), and **Charles** (1787) on hydrogen, those of **Priestley** (1770), **Lavoisier** (1783–1789), and **Scheele** (1771) on oxygen — distinguished the principal constituents of

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into H and OH, thus reducing atmospheric CO_2 to glucose, according to the general scheme: $683\ \mathrm{Kcal} + 6CO_2 + 12H_2O \rightarrow C_6H_{12}O_6 + 6O_2 + 6H_2O$. The above equation glosses over the complexity of what is perhaps a 100-step molecular sequence(!), each step probably requires the action of specific catalytic enzymes, with monosaccharides, proteins, and fats (lipids) also being produced along the way. None of this, of course, was known to Ingenhousz in 1779.

The *reverse process*, that which occurs in the dark, is none other than *animal respiration*, through which energy is formed by oxidation of the stored glucose (or any other carbohydrate) and giving off CO_2. It thus became evident that CO_2 and H_2O are *essential for life*: both take an active part in carrying out the various life functions of living organisms, and in storing energy from the sun in the form of sugars and starches — through a process by which the chlorophyll of green plants catalyzes the formation of carbohydrates from carbon dioxide and water, through the action of sunlight.

air and the 4 elements which make up the bulk of plant tissues. They reawakened interest in the problem of breathing and made it possible to analyze the elementary constituents of the atmosphere and of plant body.

While prosecuting the researches which led to the modern view of metallurgical processes, Scheele, Priestley, and Lavoisier also devoted their efforts to the analogous problems of combustion, breathing, and animal heat. An important by-product of these subsidiary inquiries were two contradictory observations: One was made by Priestley who claimed that plants remove CO_2, and the other by Scheele who maintained that they produce it.

Lazzaro Spallanzani (1729–1799) then noticed (1768) that aquatic plants give off bubbles of oxygen in sunlight and do not do so in darkness. At this point, Ingenhousz took up the clues and showed that while plants remove carbon dioxide from the air and give up oxygen to it in sunlight, they evolve carbon dioxide and take up oxygen in the dark. He thus showed that two kinds of gaseous exchange between the green plant and the air occur: one is comparable to respiration in animals, the other is essentially different from it.

1780–1794 CE **Luigi Galvani** (1737–1798, Italy). Physician and Physicist. Made pioneering researches in electrophysiology, as causing muscular contraction in a frog's legs by application of static electricity. Galvani was professor of obstetrics at Instituto delle Scienze, Bologna (1782 to 1798).

1781 CE **Charles Messier** (1730–1817, France). Astronomer. First to compile a systematic catalog of nebulae, galaxies and star clusters. There are about 110 objects in the Messier catalog. Messier's objects (known by the prefix M and their catalog numbers) include the emission nebulae M8 (Lagoon) and M16 (Eagle) – two of the most famous hot clouds of interstellar matter (emission nebulae).

Massier was born in Badonville, Lorraine. He began professional career as an assistant to **Delisle** in Paris (1751). He observed the return of the Halley Comet (1759) and from that time onwards was an arid searcher of comets, discovering independently a total of 13 of them. His purpose in compiling his catalog was to make comet hunting easier by taking careful note of permanent deep-sky objects that might be mistaken for comets.

1781 CE **Johann Carl Wilcke** (1732–1796, Sweden). Physicist. Introduced the idea of *specific heat*[329], quantity of heat required to raise the temperature of a given substance by a given amount (usually 1 °C).

[329] The Irish physician and chemist **Adair Crawford** (1747–1795) may also be credited with an independent introduction of the concept of specific heat in 1779 in connection with his treatment of heat generation in animals.

Wilcke was born in Wismar, Germany. Moved to Sweden with his parents (1739). Entered Uppsala university (1750) to study theology but concentrated instead on physics and mathematics. Received his doctorate at Rostock (1757). Prepared a comprehensive map of the earth's magnetic inclination (1768). Drew up a list of specific heats for different substances (1781), independently of **Joseph Black** (1728–1799).

Wilcke also discovered that in changing phase (e.g. from solid to liquid), a body absorbs (or releases) heat without changing temperature (*latent heat*). These were the first important discoveries about heat in modern times.

1781–1786 CE **Moses Mendelssohn of Dessau** (1729–1786, Germany). Distinguished scholar. The apostle of Jewish enlightenment in Germany. Catalyzed the merger of German and Jewish cultures, thus creating the proper climate and opportunity for the involvement of Jews in modern European science.

However, by asserting the pragmatic principle of the possible plurality of truths and his unfailing efforts to emancipate the Jews at the price of weakening the firm adhesion to their traditional values – he opened the floodgates of apostasy, secularization and assimilation on a scale that the Jewish people had never known before. Mendelssohn own descendants – a brilliant circle, of which the composer Felix[330] [331] was the most noted – left the synagogue for the Church.

Moses' father, Mendel Heyman was a poor scribe – a writer of Torah scrolls. The maternal grandfather, however, was a direct descendant from a

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330 His father, Abraham Mendelssohn-Bartholdy (1776–1835) reflected on his status: "Before, I was known as the son of my father, and now I am the father of my son."

331 Moses had 5 sons and 5 daughters. Of these: 4 died young, 4 were baptized [Dorothea 1764–1839; married the philosopher **Friedrich Schlegel** in 1804; Henriete 1775–1831; Avraham 1776–1835; Nathan] and 2 remained Jews [Rachel 1767–1831; Joseph 1770–1848]. The converts were baptized during 1814–1815. Abraham had 3 daughters and one son:

- Fanny 1805–1847, grandmother of the mathematician **Kurt Hensel**.
- Felix 1809–1847 (composer).
- Rebeca 1811–1858, married the mathematician **Dirichlet**.
- Ottilie 1819–1848, married the mathematician **Kummer**. Their daughter Marie-Elisabeth married the mathematician **Herman Amandus Schwartz**.

most illustrious rabbinic lineage[332]. His early education was cared for by his father and by the local rabbi. The latter, besides teaching him the Bible and Talmud, introduced him to the philosophy of Maimonides. When the rabbi received a call to Berlin (1743) the lad followed him there.

For the next seven years he embarked on a program of self-education, learning German, English, French, Latin, mathematics, philosophy and general history. His life at this period was a struggle against crushing poverty, but his scholarly ambition never relaxed. In 1750 he was appointed by a wealthy silk-merchant as teacher to his children. Mendelssohn soon won the confidence of his benefactor, who made the young student successively his book-keeper and his partner.

No stage director would have dared select an ugly ghetto hunchback as the central character in this drama. *But history dared.* It selected Moses Mendelssohn from the ghetto of Dessau, to reintroduce a knowledge of Judaism to the Christians and sell Christians cultural values to the ghetto dwellers. In a matter of a few years he befriended **Lessing** (then Germany's foremost dramatist and the great liberator of the German mind) and **Immanuel Kant**. His subsequent philosophical works earned him the sobriquet "German Socrates"; his reviews on literature made him the leading German stylist, while his critical essays on art made him the founder of modern aesthetic criticism.

He was soon challenged publicly to quit straddling the religions issue and either refute Christianity or be baptized (1781). In wrestling with his conscience, Mendelssohn became reinfected with the spirit of Judaism. From then on he dedicated the rest of his life to the emancipation of the Jews. To this end he saw his task as twofold: first, to give the Jews a tool for their own emancipation; second, to prepare a new basis for Judaic values once the old religious norms were rejected.

The German language was to be the tool whereby the Jews would lift themselves out of the ghetto. It was with this in mind that Mendelssohn translated the Pentateuch into lucid German, written in Hebrew letters (1783). His book *Jerusalem* (1793) was a forcible plea for freedom of conscience and noninterference of the state with the religion of its citizens.

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332 Moses' grandfather, Saul Whal of Dessau, was the 6^{th} generation of Rabbi Meir Katzenellenbogen, known as the MAHARAM OF PADUA (1482–1565). **Karl Marx** was the 12^{th} generation of this very ancestor along another route.

Mendelssohn had 10 children: 4 died young, 4 were baptized and 2 assimilated. All his grandchildren but one were apostates, and the last Jewish Mendelssohn died in 1871, thus bringing this rabbinic line into final extinction.

Whether his influence was for good or evil in the next generation has been a subject of much debate. But the real difficulty was not at all of his doing. It was due to the fact that the Jews caught his spirit of eagerness to re-enter European society much more quickly than the Christians were willing to permit them to enter.

1781–1800 CE **Frederick William (Friedrich Wilhelm) Herschel** (1738–1822, England). One of the greatest observational astronomers in history and founder of the present day system of stellar astronomy. Made a series of astronomical discoveries that established the universality of the law of gravitation — its not being confined to the solar system alone.

In 1781 he discovered the planet *Uranus* and the phenomenon of *binary stars*. In 1782 he discovered that *the entire solar system is moving* relative to the fixed stars. Finally in 1783 cosmology received an enormous boost when Herschel observed diffuse patches of light, or *nebulae*, through his telescope. He considered them to be 'island universes'.

Thomas Wright and **Immanuel Kant** had previously speculated about such nebulae, but Herschel's observations established extragalactic astronomy as an independent branch of astronomy. He realized that the 'Milky Way' might be similar in structure and scale to other faint nebulae. He was also able to resolve the globular star clusters in our own galaxy into stars.

In so doing Herschel took a major step toward placing the earth in its proper perspective with respect to the rest of the universe.

In 1800, he discovered infrared radiation by moving a thermometer along the color spectrum produced by a prism.[333]

Herschel was born in Hanover. His father, Isaac, was a Jewish musician employed in the Hanover guard. His grandfather's family had left Moravia for Saxony in the early part of the 17^{th} century on account of religious persecutions. He started his career as an oboist in the Prussian army, but the hardships of the Seven Years War caused his parents to send him to England, where he became organist and teacher of music.

During 1766–1772 he was director of all public musical entertainment at Bath, and in his free time taught himself mathematics and astronomy. Moreover, in 1772 he brought his sister **Caroline Lucretia** (1750–1848) to England and both built, after toiling at it for few years, a 7 ft. Newtonian reflecting

[333] Although William Herschel is best remembered for his hand-built telescopes and his discovery of Uranus (1781), the simple experiments he performed with glass prisms and thermometer (1800), with which he detected what is now known as *infrared light*, were far more momentous: They gave science its first evidence that an entire world lay hidden beyond the limit of our visual perceptions.

telescope — having an aperture of $6\frac{1}{2}$ inch (16.25 cm). His observations were communicated by him to the Royal Society in a series of memoirs from 1781 to 1797. In 1782 he accepted the offer of King George III to become his private astronomer.

In a series of papers from 1784 up until 1818 (when he was 80 years of age) he demonstrated that our sun is an ordinary star of the Milky Way, and that all the stars visible to us lie more or less in clusters scattered throughout a comparatively thin, but immensely extended disc. In 1789 he finished the construction of his large 4 ft (122 cm) aperture reflecting telescope, through which he could observe the Saturnian system with its 7 satellites, two of which he discovered himself (*Enceladus* and *Mimas*). The 8^{th}, *Hyperion*, escaped his notice.

His son **John Frederick William** (1792–1871) continued his father's studies on double stars and nebulae and contributed further to the knowledge of the *Milky Way*. His sons: **Alexander Stewart** (1836–1907) and **John** (1837–1921) were also astronomers.

1781–1828 CE **Caroline Lucretia Herschel** (1750–1848, England). Astronomer. She was born at Hanover, Germany, while this territory was still part of the British crown. Her father, Isaac Herschel, a musician in the Hanoverian guard, encouraged the development of her musical talents and she learned to play the violin competently enough to perform in concerts.

After her father's death, she was brought by her brother William to England to keep his house for him. During her stay at Bath she established herself as a popular vocalist and also an as assistant to her brother in his astronomical observations. When William became the Astronomer Royal, Caroline became his official assistant at a stipend of 50 pounds annually. Never before or since has any government purchased such a dedicated servant for such a relatively low cost of hire.

To this end she taught herself mathematics and took care of all the laborious numerical calculations and reductions, all the record keeping and the other tedious minutiae. She also did her own observations and during 1783–1797 she discovered 8 comets, 3 nebulae and issued a comprehensive star catalogue. In 1828 she completed the cataloging of 1500 nebulae. For this immense and valuable labor, the Royal Astronomical Society presented her with a gold medal and in 1835 elected her an honorary member of the society.

Synthetic vs. Analytic Geometry

Projective geometry investigates those properties of geometrical figures that are unaltered by projection. The impetus for these investigations was provided by the study of perspective in painting and architecture.

The first beginnings of this synthetic approach *(in contradistinction to the* analytic *geometry of Fermat and Descartes) are to be found in the work of* ***Pappos*** *(ca 300* CE*) who introduced the cross-ratio, referring to the lost work of* ***Apollonios of Perga*** *(262–200* BCE*).*

The first projective geometer of modern times is ***Girard Desargues*** *(1593–1662). In a highly original treatise on conic sections (1639), he went beyond the Greek geometers and presented a systematic foundation to projective geometry, and in addition — a number of beautiful theorems unknown to Apollonios.*

Following the development of descriptive geometry, principally by ***Gaspard Monge*** *(1746–1818), the first outline of projective geometry was given by* ***Victor Poncelet*** *(1788–1867). Analytical methods in projective geometry were introduced mainly by* ***August Ferdinand Möbius*** *(1790–1868) and* ***Julius Plücker*** *(1801–1868), while* ***Jacob Steiner*** *(1796–1863) and* ***Christian Von Staudt*** *(1798–1867) perfected a development of projective geometry without these methods.*

The connection between projective and Euclidean geometry was clarified by ***Felix Klein*** *(1849–1925). He also introduced the idea of a geometry as the invariant theory of certain groups of mappings.*

The essence of analytical geometry of space consists in setting up a correspondence between the points of the space and real numbers: Curves (1-dimensional manifolds) and surfaces (2-dimensional manifolds) then correspond to solution of sets of equations, and geometrical constructions can be replaced by algebraic and analytic methods. Since these methods form the basis of analytic geometry, the subject did not arise until progress was made in algebra and analysis.

1781 CE **Gaspard Monge** (1746–1818, France). Mathematician. The inventor of descriptive geometry and the father of differential geometry of space curves and surfaces.

Monge was born at Beaune. He started his career as a teacher at the military school in Nézieres (on the Meuse in N. France), where he discovered a clever representation of 3-dimensional objects by appropriate projections on a 2-dimensional plane. His method was adopted by the military and classified as top-secret(!) It later became widely taught as *descriptive geometry*.

In 1778 Monge married Mme Horbon, a young widow whom he had previously defended in a very spirited manner from an unfounded charge, and in 1780 he was appointed to a chair of hydraulics at the Lyceum in Paris. Unlike the three L's (**Lagrange**, **Laplace**, **Legendre**) who remained aloof from the French Revolution, Monge was an active Jacobine and occupied leading scientific positions.

As temporary head of the government on the day of the King's execution, he incurred lasting royalist resentment as the *chief regicide*. He served as a Minister of Marine and engaged in the manufacture of arms and gunpowder for the army.

After 1795 he was the principal organizer of the Polytechnical School in Paris [the prototype of all technical institutes in Europe and the U.S., even West Point] and became a professor of mathematics there.

Monge gained the close friendship and admiration of Napoleon and accompanied the latter, along with **J. Fourier** on the Egyptian expedition in 1798. Monge was a great teacher and his lectures in algebraic and differential geometry inspired many young men. Among them were: **E.L. Malus** (1775–1812), **J. Dupin** (1784–1873, geometry of surfaces), **J.V. Poncelet** (1788–1867, projective geometry), **A.L. Cauchy** (1789–1857), **O. Rodrigues** (1794–1851), **A.J.C. Barré de Saint-Venant** (1796–1886, theory of curves through his work in elasticity), **M.A. Lancret** and **J.B. Meusnier**, all of whom have theorems in differential geometry named after them. Others, whose principal papers in differential geometry were written in the period 1840–1850, are: **F. Frenet** (1816–1888), **J.A. Seret** (1819–1885), **V. Puiseax** (1820–1883) and **J. Bertrand** (1822–1900).

The school of Monge contributed greatly to the geometry of surfaces, introducing the concept of *developable surfaces*[334].

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[334] A surface that may be unrolled or developed *into a plane* without stretching or tearing, e.g.: cylinder, cone. In order to find geodesics on such surfaces, we "unwrap" the surface, flattening it to a plane, draw the relevant straight lines in the plane, and then wrap the plane up again. Using this idea, it is not too

Monge himself contributed to differential geometry in the topics of space evolutes and lines of curvature on 3-dimensional surfaces.

In 1816 he was discharged as director of the Polytechnical School after the fall of the Emperor. He was also purged with his friend, the geometer **Lazare Carnot** (1753–1823) from the Académia, while **Cauchy** was assigned to it by royal decree — although he had not been elected by the members of the Académia. This created an enormous scandal in scientific circles and Cauchy became very unpopular. [The tables were turned in 1830: Louis-Philippe came to power and Cauchy, as a loyal Burbon, refused to swear allegiance to the new government. He then went into voluntary exile.] Monge died soon afterwards.

1781–1793 CE **Jean Pierre Francois Blanchard** (1753–1809, France). Aviation pioneer and inventor. Proposed heavier-than-air machines in 1781. But as soon as the Montgolfiers made successful balloon flights, Blanchard became an ardent balloonist; he made his first balloon ascent in England (1784) and on Jan. 07, 1785 made the first aerial crossing of the English channel with Dr. **John Jeffries** (1745–1819, U.S.A.), an American physician. Invented the *parachute* and survived the first jump (1784). In 1793 he made the first balloon ascent over North America (Philadelphia, 1793). He was born in Les Audelys, France.

1783 CE The brothers **Jacque Étienne** (1745–1799) and **Joseph Michel** (1740–1810) **Montgolfier** (France) invented and built the first balloon to carry men into the air. The balloon was made of cloth and paper, filled with *hot air*. On its first public trial (June 05, at Annonay, France), their balloon (unmanned) rose about 1800 meters. Five months later (Nov 21), two men: **Francois Pilatre de Rozier** and **Marquis d'Arlandes** rose to height of 24 meters and flew across Paris for 25 minutes in a Montgolfier balloon – the first human beings to fly. Ten days later, the physicist **Jacques Charles** and a member of his team made the first *hydrogen balloon* flight.

1783 CE, June 08 The eruption of the *Laki volcano* in Iceland, started. Fluid basalt lavas flooded out of the fissure for a period of 2 months, spreading out over tens of square kilometers. Large volumes of sulphurous fumes were emitted throughout the eruption, forming a ground-hugging layer extending many kilometers down-wind from the fissure. A heavy fall of ash also rained

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difficult to show that *geodesics* on a cylinder or a cone are curves that make constant angle with the elements of the cylinder or cone. In the case of the cylinder, the geodesics are *helices*.

A *sphere*, for example, is not developable. It can be shown that the *Gaussian curvature* of a developable surface is zero.

on the countryside. 10,000 people, about $\frac{1}{5}$ of Iceland's population in the 18^{th} century, died of the eruption. Most died a lingering death of the resulting environmental damage: ash-fall destroyed growing crops and carpeted grazing lands, so that cattle either died of starvation, or were forced to eat ash-covered grass. The loss of livestock produced a severe famine which resulted in starvation. The Laki event was the largest eruption of historic times in terms of the volume of material erupted.

Benjamin Franklin, while serving in Paris in 1783 as the first diplomatic representative of the newly-formed United States of America, related the severe Northern Hemisphere winter of 1783–1784 to the Laki eruption and speculated that the injection of ash, dust and gases from the volcano into the atmosphere could result in lower temperatures by screening out some of the solar radiation.

1783–1790 CE Advent of the *steam boat*; Thomas Newcomen's first practical *steam-engine*, employing piston and cylinder, was first used to power a 45 m paddle-boat by the **Marquis de Jouffroy** (France) in 1783. Two years earlier, **James Watt** patented a way to change the power produced by a steam engine from a back-and-forth motion to rotary motion. This was used by **John Fitch** (England) in 1787 to successfully test his steam boat on the Delaware river. By 1790, one of Fitch's steamboats was in regular service for several weeks during the summer, but it was a commercial failure.

1783–1801 CE **Jacques-Alexandre-César Charles** (1746–1823, France). Physicist and mathematician. First, in 1783, to employ hydrogen for the inflation of balloons (he made the first ascent in that year to an altitude of 3.2 km). In 1787 he discovered (ahead of **Joseph Gay-Lussac**, 1802) that a gas expands, under constant pressure, such that its volume is proportional to the absolute temperature (*Charles law*)[335].

Charles was born at Beaugency, Loiret. After spending some years as a clerk in the ministry of finance, he turned to scientific pursuits, and attracted

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[335] Charles did not publish his findings, but explained his experiments to the French chemist **Joseph Louis Gay-Lussac** (1778–1850). The latter performed similar experiments and published his results in 1802.

As a result, Charles' law is sometimes called Gay-Lussac's law. The *ideal gas law* $PV = nRT$ combines Boyle's law $[(PV)_T = \text{constant}]$, Charles' law $[(V/T)_P = \text{constant}]$, and Avogadro's law into a single statement. [P = pressure; V = volume; T = absolute temperature in degrees Kelvin (K); n = number of moles of gas; R = universal gas constant.] For one mole ($n = 1$), $R = \frac{PV}{T}$.

Taking a gas at $p = 1$ atm, $T = 0°\text{C} = 273.15°\text{K}$ and molar volume of 22.4 liter, one obtains $R = 8.3144 \frac{\text{J}}{\text{mole}\times°\text{K}}$. In general $PV = nRT = NkT$, $N = nN_A$ = total number of molecules in the sample, N_A = Avogadro's considerable attention by his skillful and elaborate demonstrations of physical experiments. In 1785 he was elected to the Academy of Sciences, and subsequently became professor of physics at the Conservatoire des Arts et Metiers.

1784 CE **George Atwood** (1746–1807, England). Applied mathematician. Graduated from Trinity College, Cambridge in 1769 and remained there until 1784 as a fellow and tutor. In 1776 he was elected a fellow of the Royal Society of London. In 1784 he was appointed to the office of a patent searcher of the customs. In the same year he published his work "*Treatise on the Rectilinear Motion and Rotation of Bodies*" in which he invented a machine for the demonstration of the laws of free fall (*Atwood machine*)[336]. With this machine he was able to improve the accuracy of the measurement of the acceleration of a body in free fall.

1784–1794 CE **William Jones** (1746–1794, England). Orientalist, linguist and jurist. Recognized and demonstrated that six groups of kindered languages (known today as Indo-European) — Sanskrit, Greek, Latin, Gothic, Celtic and Persian originated from a *common source* (proto Indian-European) which no longer exists[337].

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number, k = Boltzmann's constant. Therefore

$$k = \frac{R}{N_A} = \frac{8.3144\ \mathrm{J/mole} \times {}^\circ\mathrm{K}}{6.02205 \times 10^{23}/\mathrm{mole}} = 1.38066 \times 10^{-23}\ \frac{\mathrm{J}}{{}^\circ\mathrm{K}}.$$

336 The apparatus "dilutes" the effect of gravity so that acceleration could be accurately measured to determine the value of g. It consists of a light frictionless pulley on which two nearly equal masses $m_2 > m_1$, are hung vertically. Neglecting the mass and rotational effect of the pulley, the energy conservation can be expressed as:

$$(m_2 - m_1)gs = \frac{1}{2}(m_1 + m_2)v^2,$$

where s is the vertical displacement of the blocks and v their linear velocity. But since $v^2 = 2as$, the acceleration a of the system is

$$a = \frac{m_2 - m_1}{m_2 + m_1}g.$$

337 Two hundred years earlier, an Italian, **Filippo Sassetti**, had already noticed the similarity between Sanskrit and Italian. Sassetti lived in India (1581–1588).

Jones was born in London and distinguished himself at Harrow and Oxford in the study of Oriental languages. By 1766 he mastered Arabic, Hebrew, Persian, Chinese, French, Italian, Spanish and Portuguese. During 1768–1774 he occupied himself with translations from Asiatic to European languages. To enhance his income he studied law and gained high reputation in this field both in England and America. In 1783 he was appointed judge of the supreme court at Calcutta. In this capacity (1783–1794) he compiled a digest of Hindu and Muhammadan law and translated from the ancient Hindu literature into English.

An extraordinary linguist knowing 13 languages well, and having a moderate acquaintance with 28 others, his range of knowledge was enormous. As a pioneer in Sanskrit learning he rendered the language and literature of the ancient Hindus accessible to European scholars, and thus became the indirect cause of later achievements in the field of Sanskrit and comparative philology.

1784–1809 CE **René Just Haüy** (1743–1822, France). Mineralogist and the founder of the science of crystallography. Elucidated geometrical properties of various crystals and laid theoretical basis for further work in his *Traité de mineralogie* (1801) and *Traité de cristallographie* (1822). Also studied pyroelectricity.

Haüy was born at St. Just, Oise. He studied at the college of Navarre and afterwards at that of Lemoine. Becoming one of the teachers (Abbé Haüy) at the latter (1770 to 1784), he began to devote his leisure hours to the study of botany; but an accident directed his attention to another field of natural history: while looking at a particular collection of minerals, he supposedly dropped a group of calcite crystals that crystallized as hexagonal prisms. As he went down to examine the shattered fragments, he found they were all perfect rhombohedra, in every detail the identical shape of Iceland spar, a different crystalline form of calcite (= calcium carbonate, or limestone in one form).

Thus he found that all crystals of calcite, whatever their external form, could be reduced by cleavage to a rhombohedron with interfacial angle of 75°. Further, by stacking together a number of small rhombohedra of uniform size he was able to reconstruct the various forms of calcite crystals[338].

[338] In the same manner a regular octahedron is built of cubic elements, such as given by the cleavage of rock-salt. By making the steps one, two or three bricks in width and one, two or three bricks in height the various secondary faces on the crystal are related to the primitive form or "cleavage nucleus" by the law of whole numbers, and the angles between them can be arrived at by mathematical calculation. By measuring, with a goniometer, the inclination of the secondary faces to those of the primitive form, Haüy found that the secondary forms are

When the revolution broke out he was thrown into prison (1792), and his life was even in danger, when he was saved by the intercession of the naturalist **Étienne Geoffray Saint-Hilaire**[339] (1772–1844, France). He later taught at École des Mines (1795 to 1802). In 1802, under Napoleon, he became a professor of mineralogy at the museum of natural history and the Sorbonne (1809), but after 1814 he was deprived of his appointment by the government of the Restoration. His latter days were consequently clouded by poverty, though he lived cheerful and respected till his death in Paris.

1784–1809 CE **Adrien Marie Legendre** (1752–1833, France). An outstanding mathematician whose works have placed him at the forefront of achievement in widely distinct fields of pure and applied mathematics. He had the misfortune of seeing most of his best work — in elliptic integrals, number theory and the method of least squares — superseded by the achievements of younger and abler men.

For 40 years he slaved over *elliptic integrals* (his 2-volume treatise appeared in 1827) without noticing what both **Abel** and **Jacobi** saw almost at once (1828) that by considering the *inverse functions* the whole subject drastically simplified.

The readiness with which Legendre, who was then 76 years of age, welcomed these important researches, that quite overshadowed his own, and included them in successive supplements to his work, eloquently testify to his integrity.

Legendre was born in Paris. In 1775 he was appointed professor of mathematics in the École Normale. In 1783 he was elected a member of the French Academy in succession of **J. Le Rond d'Alembert**. During the revolution, he was one of the three members of the council established to introduce the metric system.

In 1767–1769 Legendre used continued fractions to find approximations to the irrational roots of algebraic equations, and approximate solutions of ordinary differential equations.

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always related to the primitive form (on crystals of numerous substances) in the manner indicated, and that the width and the height of a step are always in a simple ratio, rarely exceeding that of 1: 6. This laid the foundation of the important *law of rational indices* of the faces of crystals.

339 Geoffray was a student of medicine in Paris when Haüy, his former teacher, and other professors of the colleges of Lemoine and Navarre were arrested by the revolutionists as priests. Through the influence of Daubenton, Geoffray obtained an order for the release of Haüy in the name of the Academy.

In 1784, Legendre encountered his polynomials in his research on the gravitational attraction of ellipsoids. He introduced the celebrated expressions, which though frequently called 'Laplace coefficients', are more correctly named after Legendre. The definition of the coefficients is that if $(1 - 2h\cos\phi + h^2)^{-1/2}$ be expanded in ascending powers of h, and if the general term be denoted by $P_n h^n$, then P_n is the Legendre coefficient of the n^{th} order.

In 1805 Legendre issued the first published account of the *method of least squares* in connection with his work on the orbits of comets. It had, however, been applied earlier (1795) by **Gauss**, and was independently used by **Laplace** (1810).

To Legendre is due the theorem known as *law of quadratic reciprocity* in the theory of numbers, the most important general result in the science of numbers which has been discovered since the time of **Fermat** and which was called by Gauss "the gem of arithmetic". The symbol $\left(\frac{a}{p}\right)$ for odd prime p is known as *Legendre's symbol*[340]. *Legendre's formula* $\{x/(\log_e x - 1.08366)\}$ for the approximate number of primes less or equal to a number x, was first given by him in 1801 in his famous treatise '*Théorie des nombres*'.

In 1825 Legendre provided a complete proof, for the case $n = 5$, of the Fermat conjecture.

In 1794 he published a popular textbook on Euclidean geometry called '*Eléments de géométrie*', in which he attempted a pedagogical improvement of Euclid's '*Elements*' by rearranging and simplifying many of the propositions. This book was translated by **Thomas Carlyle** (1795–1881), who early in his life was a teacher of mathematics, and ran through 33 American editions for 100 years.

In one respect, Legendre's life resemble that of Lagrange: in 1792, at the age of 40, he married a girl 22 years his junior. His young wife helped him put his affairs in order and also brought the tranquility to his life which greatly aided him in his work.

[340] Equal to zero when p divides a; equal to $+1$, when p is prime to a but p divides $N^2 - a$, for some N; equal to -1, when p is prime to a but there is no N such that p divides $N^2 - a$.

The Remarkable Legendre

Legendre' works have placed him at the very forefront in the widely distinct subjects of elliptic functions, theory of numbers, potential theory, geodesy and analysis. His versatility is demonstrated in the following relations discovered by him.

I. Elliptic Integrals (1811)

The relation

$$EK' + E'K - KK' = \frac{\pi}{2}$$

relates the complete elliptic integrals

$$\begin{aligned}
K(k) &= \int_0^{\pi/2} d\theta (1 - k^2 \sin^2\theta)^{-1/2}, \qquad\qquad k^2 + {k'}^2 = 1 \\
K'(k) &= \int_0^{\pi/2} d\theta (1 - {k'}^2 \sin^2\theta)^{-1/2} \equiv K(k') \\
E(k) &= \int_0^{\pi/2} d\theta (1 - k^2 \sin^2\theta)^{1/2} \\
E'(k) &= \int_0^{\pi/2} d\theta (1 - {k'}^2 \sin^2\theta)^{1/2}.
\end{aligned}$$

It is known as the Legendre relation. *To prove it, one must show that*

$$\frac{dE}{dk} = \frac{1}{k}(E - K), \qquad \frac{dK}{dk} = \frac{1}{k{k'}^2}(E - {k'}^2 K),$$

$$\frac{dE'}{dk} = \frac{k}{{k'}^2}(K' - E'), \qquad \frac{dK'}{dk} = \frac{1}{k{k'}^2}(k^2 K' - E').$$

It then follows that $\frac{d}{dk}(EK' + E'K - KK') = 0$. *To find the value of the constant, we let* $k \to 0$.

With the aid of the Legendre relation and Gauss' arithmetic-geometric mean, **Eugene Salamin** *discovered (1976) a new formula that is currently used as a computer algorithm for the fast computation of* π *with the property of doubling the number of digits at each step.*
Thus, **Y. Tamura** *and* **Y. Kanada** *computed* π *in 1982 to 4,194,293 digits with a* CPU *time of 2 hours and 53 minutes!!.*

II. Prime Factorization of $m!$

Legendre enriched mathematics, both pure and applied, with many important and beautiful results. But one can recognize the signature of the master even in one of his lesser known theorems.

Suppose we wish to find how many zeros there are at the end of the number

$$1000! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot 999 \cdot 1000$$

Clearly, the number of terminal zeros of a number depends on how often the factor $10 = 2 \cdot 5$ *occurs in its factorization. We must therefore find the exponents of the factors* 2 *and* 5 *in the prime factorization of* 1000!. *The smaller of these exponents will yield the largest exponents, say* α, *for which* $10^\alpha = (2 \cdot 5)^\alpha$ *divides* 1000!, *and* α *will be the number of terminal zeros in* 1000!.

Now, every fifth one of the numbers

$$1,\ 2,\ 3,\ \ldots,\ 1000$$

is a multiple of 5. *Since*

$$1000 = 5 \cdot 200 + 0,$$

there are 200 *factors in* 1000! *which are divisible by* 5. *Of these* 200, *i.e., of*

$$5,\ 10,\ 15,\ 20,\ 25,\ \ldots,\ 1000,$$

every fifth is a multiple of 5^2. *Since*

$$200 = 5 \cdot 40 + 0,$$

there are 40 factors divisible by 5^2. *Moreover, since*

$$40 = 5 \cdot 8 + 0, \qquad 8 = 5 \cdot 1 + 3, \qquad 1 = 5 \cdot 0 + 1,$$

there are 8 *numbers divisible by* 5^3, *one by* 5^4, *and none by any higher power of* 5.

Thus, the prime factorization of 1000! *contains* $5^{200} \cdot 5^{40} \cdot 5^8 \cdot 5^1$; *the exponent of* 5 *in the prime factorization is*

$$200 + 40 + 8 + 1 = 249.$$

On the other hand, the prime 2 *occurs to a higher power in the prime factorization of* 1000!, *since* 500 *of the factors are even,* 250 *are divisible by* 2^2, *etc. Hence there are* 249 *zeros at the end of* 1000!.

Legendre attacked the general problem of the exponents in the prime factorization of $m!$. *He asked: What is the highest power, say* p^α, *of a prime* p *such that* p^α *divides* $m!$, *where* m *is any given natural number? In order to answer this question he generalized the procedure used in the above solution for the case* $m = 1000,\ p = 5$.

First, divide the given number m *by the prime* p:

$$m = pq_1 + r_1 \qquad (0 \le r_1 < p);$$

Next, divide the quotient q_1 *by* p:

$$q_1 = pq_2 + r_2 \qquad (0 \le r_2 < p).$$

One continues this process until one obtains a quotient which is zero, i.e.,

$$q_{k-1} < p, \qquad q_k = 0.$$

By examining the solution of the problem, it is seen that the exponent α *(which was* 249 *in the case* $m = 1000,\ p = 5$*) is the sum of the quotients obtained in the above algorithm:*

$$\alpha = q_1 + q_2 + \cdots + q_{k-1}.$$

It is now claimed that the remainders $r_1, r_2, \ldots, r_k$ *obtained in this algorithm are the digits in the representation of the number* m *to the base* p. *To see this, just substitute repeatedly for the quotients; thus,*

$$m = pq_1 + r_1 = p(pq_2 + r_2) + r_1 = \cdots = p^{k-1}r_k + p^{k-2}r_{k-1} + \cdots + pr_2 + r_1.$$

The last expression proves the claim. In the solution of the problem, the remainders in the divisions were $0, 0, 0, 3, 1$ *and the representation of* 1000 *to the base* 5 *is, indeed,* 13000.

Next it is shown how to express the exponent α *in terms of the remainders. One adds the expressions*

$$\begin{array}{rcl} m & = & p\,q_1 + r_1 \\ q_1 & = & p\,q_2 + r_2 \\ \vdots & \vdots & \vdots \\ q_{k-1} & = & p \cdot 0 + r_k \end{array}$$

and obtains

$$m + q_1 + q_2 + \cdots + q_{k-1} = p\,(q_1 + q_2 + \cdots + q_{k-1}) + r_1 + r_2 + \cdots + r_k$$

or

$$m + \alpha = p\,\alpha + s,$$

where $s = r_1 + r_2 + \cdots + r_k$. *Solved for* α, *this gives*

$$\alpha = \frac{m-s}{p-1}\ .$$

This proves the following theorem of Legendre:

> *If* m *is a positive number and* p *a prime, then the exponent of* p *in the prime factorization of* $m!$ *is*
>
> $$\frac{m-s}{p-1}$$
>
> *where* s *is the sum of the digits of the representation of* m *to the base* p.

The prime power in a factorial, i.e. the highest power of p *that divides* $m!$ *is alternatively given by the expression*

$$\left[\frac{m}{p}\right] + \left[\frac{m}{p^2}\right] + \left[\frac{m}{p^3}\right] + \cdots$$

which include only finitely many non-zero terms. Here $[A]$ *indicates the greatest integer not exceeding* A.

To see this we write

$$m! = 1 \cdot 2 \cdots (p-1) \cdot p \cdot (p+1) \cdots (2p) \cdots (p-1)p \cdots p^2 \cdots\ .$$

It is obvious that there are $\left[\frac{m}{p}\right]$ *multiples of* p, $\left[\frac{m}{p^2}\right]$ *multiples of* p^2, *and so on. Similarly, the number*

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

is an integer because the power of p *in* $\binom{n}{r}$ *is*

$$\sum_m \left(\left[\frac{n}{p^m}\right] - \left[\frac{r}{p^m}\right] - \left[\frac{n-r}{p^m}\right] \right)\ .$$

Since $[A] - [B]$ *is either* $[A-B]$ *or* $[A-B]+1$, *the sum is a non-negative integer.*

Examples:

- *If* $m = 1000$, $p = 3$ *then*

$$\left[\tfrac{1000}{3}\right] = 333; \qquad \left[\tfrac{1000}{3^2}\right] = 111; \qquad \left[\tfrac{1000}{3^3}\right] = 37; \qquad \left[\tfrac{1000}{3^4}\right] = 12;$$

$$\left[\tfrac{1000}{3^5}\right] = 4; \qquad \left[\tfrac{1000}{3^6}\right] = 1$$

  Therefore the exact power of 3 *which divides* 1000! *is* 498.

- *Defining* $3!!! = [(3!)!]! = 720!$, *prove that it has more than* 1000 *digits, and find the number of zeros at the end of the expansion.*

  Since $(720)! > 99!100^{621} > 10^{1242}$, 3!!! *has more than* 1000 *digits. The largest power of* 5 *which divides* $3!!! = 720!$ *is*

$$\left[\frac{720}{5}\right] + \left[\frac{720}{25}\right] + \left[\frac{720}{125}\right] + \left[\frac{720}{625}\right] = 144 + 28 + 5 + 1 = 178,$$

  while the largest power of 2 *dividing* 720! *is still greater (since already* $\left[\frac{720}{2}\right] = 360$*). It follows that the number* 3!!! *has* 178 *zeros at the end of its decimal expansion.*

1784–1810 CE **Johann Wolfgang von Goethe** (1749–1832). Poet-philosopher and scientist. The last of the great universal men. In the course of a long life he engaged in a wealth of activities: poet, lawyer, politician, civil servant, physicist, botanist, zoologist, painter, theater manager and literary critic. Yet there is nothing fragmentary about him, and his mature writings are the expression of the harmony he created by conscious effort out of the manifold experiences of a richly varied life.

Goethe made important contributions in *anatomy*, *botany* and *optics*. He tried to introduce an *evolutionary perspective* into every one of these disciplines. He advocated a holistic approach toward science, emphasizing intuition and a concern for the whole rather than a separation into parts (1791).

As a student he studied in Leipzig and Strasbourg where his thinking was strongly influenced by the works of **Bacon**, **Spinoza** and **Kant**.

Goethe discovered the intermaxilary bone, a feature of the human upper jaw that is missing in most other mammals.

In the two-volume *Zur Farbenlehre* (1810), he attacked Newton's theory of light (1704) and presented a psychologically-oriented examination of color. However, Goethe would not recognize the distinction between *physical* and *physiological*[341] optics; this was the reason for his fruitless fight against Newton: reviving the old Aristotelian view, Goethe abhorred the theory that white light is a mixture of the seven colors of the rainbow.

He was certainly correct in regard to the white[342]-*sensation* which he had primarily in mind, but the rainbow should have convinced him that white light is decomposed into colors by a spectral apparatus (in this case, water droplets).

In attempting to explain the metamorphosis of plants (1789) he claimed incorrectly that all plant structures are modified leaves, but clearly espoused *evolution*.

[341] Today we understand without difficulty that the *sensation yellow* which is caused by the *D*-lines of sodium is a phenomenon which is entirely different from the wavelengths $\lambda = 5890\text{Å}$ and $\lambda = 5896\text{Å}$ by which we must describe these lines physically. For we know that the psychological response to an event is something entirely different from the physical event itself; the two are different in nature, though related.

[342] We perceive the sun's natural light as white (i.e., as lacking all spectral colors) because the eye is *adapted* to see sun; that is to say, because our eye and the associated physiological-psychological vision apparatus has in its evolutionary development adapted itself to the spectrum of the sun. If we lived in the vicinity of a *red giant*, we would presumably perceive its red color as the normal white.

Worldview XVI: Johann Wolfgang von Goethe

* * *

"In the inside of Nature — Nature has neither kernel nor shell".

* * *

"The history of science is science itself; the history of the individual, the individual".

* * *

"As for what I may have done as a poet, I take no pride in it whatever... Excellent poets have lived at the same time with me, poets more excellent lived before me, and others will come after me. But that in my country I am the only person who knows the truth in the difficult science of colors — of that, I say, I am rather proud, and here I have a consciousness of superiority to many".

* * *

"In the eternal silence within a crystal they may see the happenings of the world outside".

* * *

"Gray and ashen, my friend, is every science. And only the golden tree of life is green".

* * *

"Duration is change".

* * *

"Man is naturally disposed to consider himself as the center and end of creation, and to regard all the beings that surround him as bound to subserve his personal profit... He cannot imagine that the least blade of grass is not there for him".

* * *

1785 CE *The Times* (London) was founded.

1785–1788 CE **James Hutton** (1726–1797, Scotland). Geologist, chemist, physician and farmer. The father of modern geology. Concluded (1785), on the basis of geological observations, that vast periods of time were required for the earth to have reached its present state and form. He saw the earth as a living machine, immensely old, continuously changing and powerful[343].

Noticing how little the *Hadrian wall* was affected by erosion during the 1600 years of its existence, he concluded that there was simply no time for mountains and valleys to be carved during the meager 6000 years allocated by **Ussher** (1650)[344].

Hutton was born in Edinburgh and educated at the high school and university of his native city. He completed his medical education in Paris and Leyden (1749) but later abandoned the medical profession, and after extensive travels in the Low Countries and France (1750–1754) he settled on his own farm in Berwickshire. In 1768 he established himself in Edinburgh for the rest of his life, living unmarried with his 3 sisters. Surrounded by congenial literary and scientific friends he devoted himself to research.

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[343] Hutton observed the layering of rock in Scotland's provinces, and deduced that the layers of limestone, sandstone, and shales had been laid down in distant times as soft sediments that settled to the bottom of the sea. He then conjectured that these sediments had been compacted and slowly turned to stone by the pressure of sediments settling above them; at last the sea had withdrawn, or the sea bed had risen, exposing some of the rock to the open air. The wearing action of wind and weather (erosion) had broken up the topmost layers of rock into fine bits, helping to make the rich mix we call soil. Rains *continually* wash the soils into streams and rivers and hence to the sea, where the sediment is compacted once again into solid rock. Heat within the earth heaves this bedrock up above the sea to form new mountains. This cycle, Hutton argued, from rock to soil to rock, from sea to air and again to sea, had endured for an extraordinary length of time, and will go on indefinitely. In his own words: "*We find no vestige of beginning — no prospect of an end*". Hutton did not venture to estimate the time-scale involved in these processes. His ideas were further elaborated by **Wegener** (1912).

[344] Ussher's age for the earth proves much too small for a totally different reason: we know the Sumerians invented writing in ca 3000 BCE. We cannot expect that in 900 years, a civilization complex enough to require a writing system can be developed.

Edinburgh was at that time the center of Scottish Enlightenment, which included **James Watt**, **Adam Smith**, **David Hume**, **John Clerk** (1728–1812) and **Joseph Black**. They met one evening a week at the *Oyster Club*, which Hutton helped to found. These men brought into the world the engines of the *Industrial Revolution* and some of the ideas and attitudes that made the revolution possible.

Hutton communicated his views in 1785 to the recently established *Royal Society of Edinburgh*, in a paper entitled *Theory of the Earth*.

1785–1787 CE **Edmund Cartwright** (1743–1823, England). Inventor and clergyman. Developed a steam-powered loom for wearing cotton that led to the invention of more effective power looms and to the development of modern wearing industry.

Cartwright was born at Marnham, in Nottinghamshire. He studied literature at Oxford, but had no scientific education. He became pastor of a rural parish in Goadby Marwood in Leicestershire. In 1784 Cartwright learned of the need for a wearing machine that could make cloth faster than the hand loom. Even though he had never seen a loom in operation, he hired a carpenter and a blacksmith to help him build a power loom. In 1787 he used it in a spinning and wearing factory he opened at Doncaster.

In 1791, a mill at Manchester ordered 400 of Cartwright's looms. But the factory was burned down by workmen who feared the new power machinery would eliminate their jobs. Although Cartwright's looms were never fully practical, Parliament recognized his pioneering work (1809) by awarding him the equivalent of $ 50,000.

1785–1803 CE **Claude Louis Berthollet** (1748–1822, France). Chemist. Made the first systematic attempt to grapple with the problems of *chemical physics* (the physics of chemistry), such as chemical affinity and rate of chemical reactions.

Berthollet was born at Talloire, near Annecy in Savoy. He graduated in medicine at Turin. In 1722 he moved to Paris and became the private physician of Phillip, duke of Orleans. In 1785 he declared himself an adherent of the Lavoisierian school, and in 1787 participated in the revision of chemical nomenclature with **Lavoisier** himself. He determined the composition of ammonia (NH_3) and hydrogen sulphide (H_2S), and introduced the process of chemical *bleaching*[345] (1785). After 1794 he became a professor of chemistry at the École Polythechnique. He accompanied Napoleon to Egypt in 1798.

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[345] Berthollet found that a solution of chlorine in water, when exposed to *light*, gave of bubbles of *oxygen*, which causes the bleaching action

$$Cl_2 + H_2O \rightleftarrows HCl + HOCl \rightleftarrows H + Cl + HOCl;$$

During 1799–1803 he determined the factors influencing chemical reaction: *affinity* (tendency to combine) and *concentration*. In 1803 Berthollet anticipated the *law of mass-action*[346] which was formulated in a general form by **Guldberg** and **Waage** (1864 to 1867).

Under the empire Berthollet became a Count, and after the restoration of the Bourbons he was made a peer. In the later years of his life he had established, at Arcueil, a well-equipped laboratory which became a center frequented by some of the most distinguished scientific men of the time.

1785–1802 CE **William Paley** (1743–1805, England). Philosopher and churchman. Author of "*The Principles of Moral and Political Philosophy*" (1785) and "*Natural Theology*" (1802).

Argued that living things are far too complicated to have arisen by chance, and that the existence of creatures as beautifully fitted for their way of life as ourselves, reveal the presence of a designer. Paley's arguments come both from biology and astronomy: he realized, for example, that the inverse-square law of the gravitational attraction force is unique in giving rise to stable orbits. If the law of gravity had, for example, been an inverse *cube*, then planetary orbits would be unstable, and a planet that moved a little closer to the sun would immediately begin to fall inwards permanently, while one that moved slightly outwards in its orbit would continue receding forevermore. Tiny changes, such as those caused by the impact of a meteorite, would be disastrous. In our universe, if the earth's orbit, say, shifts slightly inwards or outwards because it is hit by a piece of rock from space, the natural tendency is for the planet to return close to its old, regular path. Paley saw this

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$$HOCl \rightleftarrows HCl + O.$$

The photon energy decomposes the chlorine molecule into two chlorine atoms, and the oxygen, through the oxidizing OH group, helps to convert pigments and strains from cotton goods.

346 A *chemical reaction*, e.g. $A + BC \rightarrow AB + C$, is countered by the inverse reaction: $AB + C \rightarrow A + BC$ which can often take place under the *same* conditions and simultaneously with the direct reaction. A state of *equilibrium* is then reached when the two opposing reactions balance each other, i.e., proceed with equal rates. This is denoted $A + BC \rightleftarrows AB + C$ [example: reducing steam by heated iron $3Fe + 4H_2O \rightleftarrows Fe_3O_4 + 4H_2$]. Such reactions are called *reversible reactions*.

According to Berthollet, the *activity* of a substance is proportional to the affinity and the concentration. The product of these he called active *mass*. His *law of mass-action* then states: In the equilibrium state, the extent of chemical change is proportional to the active masses of the interacting substances.

“choice” of the universe square law of gravity as another example of the work of intelligent design in a universe suitable for human life. He did not elaborate, however, on the fact that the inverse square law is a byproduct of the fact that the universe has three spatial dimensions — although this had been noticed by Immanuel Kant earlier in the eighteenth century.

Paley’s arguments go back to authors such as **John Ray**, and have had a long intellectual history, surviving to the present day in the critique of Darwinian evolution. Yet Charles Darwin, while himself a student at Christ’s College of Cambridge University, not only had to read Paley, but was deeply impressed with Paley’s arguments. Even though Paley’s concept of God as a designer is very different from Darwin’s theory of natural selection, Darwin took from his reading of Paley a belief in adaptation — that organisms are somehow fit for the environments in which they live, that their structure reflects the functions they perform throughout their lives.

Where natural theology ran into trouble was in explaining the many cases of apparent pain, waste, and cruelty in the living world: why would a benevolent Designer have made cats play with mice before killing them, or parasites that eat their hosts from the inside?

Paley struggled to reconcile the apparent cruelty and indifference of nature with his belief in a good God, and finally concluded that the joys of life simply outweighed its sorrows. Where Darwin departed from Paley was in his concept of natural selection as a process that could produce adaptation and design without the all-encompassing intervention of a benevolent Designer.

Paley was born in Peterborough and graduated from Christ’s College, Cambridge (1763). In 1782 he became Archdeacon of Carlisle.

1785–1794 CE **Antoine-Nicolas Caritat Marquis de Condorcet** (1743–1794, France). Mathematician. Tried to apply probability theory to situations of human judgment, such as the probability of election of a candidate by a given number of voters or the probability of a tribunal arriving at a true verdict in a trial if to each juror a number can be assigned that measures the chances he will speak or understand the truth. This *probabilité des judgments*, with its overtones of the Enlightenment philosophy, was prominent in the work of Condorcet[347]. Though an advocate of the French Revolution (and a believer in the necessary progress of the human race towards happiness and perfection), he himself became ironically, a victim of the revolutionary tribunal.

[347] His *Voteŕs Paradox* (1785) plays a central role in both the theory of group choice (voting) and the theory of group preference (utility aggregation), finding application in modern *game theory*

1787–1794 CE **Nicolas Leblanc** (1742–1806, France). Chemist and Surgeon. Father of modern industrial chemistry. Invented the first industrial chemical process[348] to be worked on a really large scale.

The great expansion to textile manufacturers in Britain which began with the industrial revolution, together with the expansion of glass-making and soap manufacture, greatly increased the demand for alkali, and so great a strain was put on natural resources, that before long the synthesis of alkali became essential. In France the general shortage of alkali has been made acute by the difficulties arising from the wars, and in 1775 the Academy offered a prize of 2400 livres for the satisfactory method of making soda from salt.

Leblanc, then a physician to the Duke of Orleans, took the challenge and established his process in a works by means of a loan from the Duke (1791). He opened factories at St. Denis, Rouen, and Lille, but reaped no lasting benefit from what was to remain for a century one of the most fundamentally important of all industrial processes. In 1793 the Duke was guillotined by the friends of liberty and fraternity, and Leblanc's factory was confiscated and dismantled. Moreover, Leblanc was forced to reveal the secrets of his process.

By way of compensation for the loss of his rights, the works were handed back to him in 1800, but all his efforts to obtain money enough to restore them and resume manufacturing were vain. Worn out with disappointment, the unfortunate inventor died by his own hand in a shelter for the poor in St. Denis, near Paris and buried in an unknown pauper's grave.

Four years after his death, Michel Jean Jacques Dizê published a paper in the *Journal de Physique* claiming that it was he himself who had first suggested the addition of chalk; but a committee of the French Academy came to the conclusion that the merit was entirely Leblanc's (1856).

Although Leblanc's process was announced in 1787, it was not worked in Britain until nearly 40 years later, an important factor in the delay being the excise duty on salt (1702–1823).

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[348] *The* '*Lablanc process*': the preparation of Soda (Na_2CO_3) from common salt. In this process, sulphuric acid made from pyrites is heated with salt: $2NaCl+H_2SO_4 = Na_2SO_4+2HCl$, the hydrochlorin acid being absorbed and converted into chlorine, used in the manufacturing of bleaching powder. The 'salt-cake' (Na_2SO_4) is now heated with carbon in the presence of limestone (chalk, $CaCO_3$). The reaction occurs in two stages, sodium sulphate being first reduced to sulphide: $Na_2SO_4+2C = Na_2S+2CO_2$, and the sulphide then reacting with chalk to form a mixture of sodium carbonate and calcium sulphide, together with unchanged carbon and impurities, called *black ash*: $Na_2S+CaCO_3 = Na_2CO_3+CaS$.

1788 CE **Charles Bladgen** (1748–1820, England). Scientist and physician. Discovered that the lowering of the freezing point of a solvent by a substance in dilute solution is proportional to the concentration of the solute.

Blagden was born in Gloucestershire. Assistant of **Cavendish**. Visited **Lavoisier** (1783) and told him of Cavendishe's experiments. Established the importance of sweating in maintaining constant body temperature of animals.

1789 CE **Jeremy Bentham**[349] (1748–1832, England). Social philosopher. Remembered as a legal and political critic and reformer.

Founded the philosophy known as *Utilitarianism*. It defines virtue in terms of utility (the enhancement of the happiness of many, expressed in the formula, "*the greatest happiness of the greatest number*", as the proper goal of society, and the function of a good government. Utilitarians advocated the intellectual and social independence of individuals, defended civil liberties, and declared their belief in democratic ideals. All such values, however, were regarded merely as steps toward the fundamental goal of universal happiness, *not as absolute truths*.

Bentham avowed his faith in the democratic rights of man, but he considered them to be only fictions necessary for the successful conduct of life. He proposed to organize a country's laws and institutions so that they placed the general good above each person's pleasure, effecting harmony between public and private interests. His criticisms brought about many needed *reforms*.

Bentham dabbled in various scientific and intellectual pursuits; for example, he propounded schemes for cutting canals through the isthmus of *Suez* and the isthmus of *Panama*.

As a teacher of the principles of legislation, Bentham inquires of all institutions whether their utility justifies their existence. His writings have been and remain a storehouse of instruction for statesmen and legal reformers, and the great legal revolution (1873) which in England accomplished the fusion of law and equity can be traced to him.

Bentham identified the useful and the good. This outlook influenced English thought, which took its spirit from a life of industry and trade, and looked up to *matters of fact* with a certain reverence.

The Baconian tradition had turned *thought* in the direction of *things*, *mind* is the direction of *matter*; The *materialism* of Hobbes, the *sensationalism*

[349] The stuffed and clothed skeleton of Jeremy Bentham are preserved according to his instructions in his will by the University of London. He left his entire estate to the university with the provision that his remains be present at all meetings of the board.

of Locke, the *skepticism* of Hume, and the *utilitarianism* of Bentham were variations on the theme of a practical and busy life.

1786–1789 CE **Abraham Bennet** (1750–1799, England). Physicist. Early pioneer of electricity. Tried to relate atmospheric electricity to the weather. In the course of these investigation invented (1786) the *gold-leaf electroscope* [based on the "portable electrometer" of **Tiberius Cavallo** (1740–1809), an Italian, who settled in London]. He also invented a simple *electric induction machine* (1789).

Bennet was born Taxal, Cheshire, the son of the schoolmaster.He was ordained in London (1785) and appointed to curacy in Wirksworth, Derbyshire. He held several other posts at the same time, including librarian to the Duke of Bedford.

1789 CE Bad weather and disastrous harvests infected with *ergot* resulted in mass hallucinations in Brittany – leading to widespread panic and irrational fears of food being stolen; it is not known how many people died of ergot outbreaks, but it had an impact on events leading up to the French Revolution.

1789–1795 CE *The French revolution*[350] transformed the government of France, shook the Establishment throughout Europe and led to many changes in ideas of government.

By 1789 France was deeply in debt because of expensive wars, and badly governed by an elite of the nobility, who lived in luxury while many poor people starved.

Faced with national bankruptcy, the King, Louis 16^{st}, decided to summon the *Estates General*, a national parliament which has not met since 1614. It consisted of 300 noblemen, 300 clergy and 600 commoners. Each estate had one vote, which meant that the nobility and clergy could outvote the commoners. So the commoners formed a national *Constituent Assembly*, pledged to make a new constitution for France.

Louis planned to dismiss the Assembly. This aroused the fury of the Paris mob, which stormed the fortress-prison of the Bastille on July 14, 1789. Louis had to give way, and the Assembly proceeded to bring many reforms. Louis then conspired with his allies in Austria and Prussia, and in June 1791 tried to flee the country. He was captured and taken back to Paris. War with Austria and Prussia followed in April 1792.

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350 For further details, see:

- Hobsbawm, E.J., *The Age of Revolution 1789–1848*, Mentor Books: New York, 1962, 416 pp.

In August the Paris mob attacked the King in the Palace of the Tuileries, butchering his guards and imprisoning him. French victory against the Prussians in the *Battle of Valmy* (1792) encouraged the revolutionaries. A new assembly, the *National Assembly*, declared the monarchy abolished and set up a republic on Sept. 21, 1792. Power in the Convention passed to a political group called the *Girondists*, who had Louis tried for treason and executed (1793).

During 1793, a more extremist group, the *Jacobians*, gained power. The Girondists were executed, and a *Committee of Public Safety* ruled the country, headed by Maximilien Robespierre. Under his influence anyone suspected of opposing the new regime was executed, in a blood-bath known as the '*Reign of Terror*'. In July 1794, Robespierre himself was accused and guillotined, and the Terror gradually died away. In 1795, a new two-chamber assembly was elected, and order returned to France.

The French Revolution marked a turning point in European history. It unleashed forces that altered not only the political and social structures of states but also the map of Europe. Europe entered a world of class conflict, middle-class ascendancy, acute national consciousness, and popular democracy. Together with industrialization, the Revolution reshaped the institutions, the societies, and even the mentality of European men[351].

Industrial Chemistry I

No beginning can be set to chemical industry, for at the earliest times for which we have either archaeological evidence or written records, a considerable number of both chemicals and of chemical processes were in use.

[351] The revolution did not, however, bring immediate equality for *women*; the French took the last word of the revolutionary slogan *liberté*, *égalité*, *fraternité* so literally that French women were not given the right to vote until 1945.
In the United States, women had to struggle for more than half a century before the 19^{th} Amendment to the constitution gave them full voting rights (1928); Great Britain reluctantly made the same concession in 1928.

One of the first chemicals in demand was salt[352]*, an early consequence of cooking food on fire; many cooking processes remove salt from the raw food. In the hot climate of the early civilizations much salt was also lost in sweat, and the need of replenishment was correspondingly great. For these reasons salt trade was one of the most ancient in the world. In addition of its use for seasoning food, salt was also used at an early date to preserve both meat and fish.*

Natron (an impure form of soda) was preferred for the preservation of the body after death. The production of natron, derived from three main natural sources in Egypt, and especially from the Wadi Natrun, was a state monopoly in Ptolemaic times.

While the roasting and grilling of food became possible as soon as mastery of fire had been won, boiling had to await the availability of vessels that would withstand the heat of the fire. Thus, cooking begat pottery vessels, the glazing of which demanded chemical skill; Egyptian potters used naturally occurring iron oxide to form red and black glazes.

Fermentation processes, too, had their origin in the preparation of alcoholic beverages by the conversion of sugar by yeast. Although Alexandrian alchemists were familiar with the processes of distillation, it is doubtful whether apparatus was sufficiently advanced for pure alcohol to have been available before the 12^{th} *century.*

In ancient Egypt it was known that the fermentation could proceed further, resulting in the formation of vinegar: chemically, this involves oxidation of alcohol to acetic acid.

The art of painting produced a need for natural pigments: blacks were produced with manganese dioxide, red with iron oxide and yellow with iron carbonate. By the time of the ancient empires, the paint for the decoration of houses, temples and tombs was made viscous by addition of such substances as egg-white, gum, or honey. Pigments used included red lead, yellow lead oxide, malachite and green copper silicate — the preparation of which needed considerable chemical skill.

The plastering of walls with lime, made by roasting limestone or chalk in kilns to expel carbon dioxide was introduced already around 2500 BCE. Likewise, roasted gypsum (hydrated calcium sulphate) was used for decoration of walls.

[352] The Bible is abundant with such evidence (e.g., *Job* **6**, 6; *Lev* **2**, 13; *Zep* **2**, 9). Wars were fought for control over salt and asphalt sources (*Gen* **14**; *Chron II* **25**, 11; *Kings II* **4**, 7).

With increasing sophistication there came the increasing demand for artificial illumination; lamps of metal and pottery were modeled on sea-shells, using oil mixed with little salt to give a yellower and more luminous flame. Oil was made either from olives or the seed of the sesame plant.

The demands of clothing provided the most powerful stimulus for the development of chemical processes: To this day, the chemical and the textile industries are very closely related. The origin of soap (in the chemical sense of saponified fats and oils) is probably in the 4^{th} century CE. Long before that, however, various cleansing agents of a different chemical character were in use. The basic process in soap-making is to boil fats or vegetables oils with strong alkali. From the 12^{th} century on, soft soap for the use of the textile industry was prepared using caustic alkali.

The practice of dyeing goes back to remote times, and the earliest records show that it was already a complex craft relying heavily on chemical processes. Until the 19^{th} century, virtually all dyes were of vegetable or animal origin. From very early times it was known that cloth would take up colors much more intensely and permanently if it was first treated with what we now know to be salts of aluminum (alums). The Greeks and Romans used potassium alum, obtained from certain volcanic regions, but by the 13^{th} century a method of purifying natural aluminum sulphate was described by Arabic writers.

Of the origins of the three principal acids of modern chemical industry[353]

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[353] By far the most important industrial chemical is sulfuric acid. It is used as a solvent and reactant in the preparation of a large number of other chemicals. Thus, it is involved in the manufacture of phosphate fertilizers, of inorganic pigments, of iron and steel, of ammonium sulfate [$(NH_4)_2SO_4$] and aluminum sulfate [$Al_2(SO_4)_3$], and of *rayon*. Sulfuric acid is also involved in the processing of nonferrous metals and in the manufacture of a variety of petroleum products. After H_2SO_4, ammonia (NH_3) is the industrial chemical produced in the greatest quantity. Its largest use is in the preparation of fertilizers. It is also used in the manufacture of soda ash, nylon, dyes, rubber and various plastics. HCl is used in the petroleum, food and metal industries. HNO_3 has its largest use in the manufacture of nitrates, explosives and as an oxidizing agent.

Some idea of the variety of compounds and uses can be obtained from the following list of the uses of the salts of sodium:

$NaCl$ (sodium chloride): table salt, manufacture of NaOH, Cl_2 and in the paper industry.

Na_2SO_4 (sodium sulfate): preparation of paper pulp and in the manufacture of glass.

$NaHSO_4$ (sodium bisulfate): dye industry.

$NaHSO_3$ (sodium bisulfite): tanning and bleaching of textiles.

$Na_2S_2O_3$ (sodium hyposulfate): photography.

— sulfuric (H_2SO_4), hydrochloric (HCl), and nitric (HNO_3), sulfuric acid seems to have been unknown until the early 16^{th} century, when it was made in Germany by dry distillation of green or blue vitriol (iron or copper sulfate). It was of virtually no industrial importance until the 17^{th} century, which is also when HCl was first clearly distinguished. Nitric acid, commonly obtained by distilling nitre (potassium nitrate) with vitriol, was described by the 8^{th} century Arabic alchemist, ***Jabir*** *(**Geber**). It was industrially important for separating large quantities of silver, which dissolves in it, from gold.*

Far more important than nitric acid was nitre (KNO_3), which with sulfur and charcoal, is an essential ingredient of gunpowder. By 1300 this mixture was prepared for use in artillery and, later, in small arms. The common source of nitre was from stables, pig-sites etc., in which it resulted from bacterial action on manure. At first, mixing of the three ingredients was done by artillerymen in the filed, but power-mills were soon established; the earliest were manually operated, but water-power had been introduced by the 17^{th} century.

Up to the 18^{th} century, the main specifically chemical trades were those of the apothecary, who prepared compounds on a small scale for use in medicine, and of the alum makers, who prepared alum on a comparatively large scale for the treatment and coloring of skins, paper, and textiles.

The new spinning and weaving machines introduced during the 18^{th} century increased the output of textiles to such a degree that the chemical problems of bleaching, and later of dyeing cloth, became considerable. Traditionally, textiles had been bleached by dipping them alternately in acid solutions of sour milk and alkaline solutions of plant ashes, and exposing them to the sun on 'bleach fields', a process that occupied all of the summer months in a given year. A shortage was soon experienced in sour milk and then also in natural alkali.

At the end of the 18^{th} century, the discoveries of ***Antoine Lavoisier*** *(1789) and* ***Nicolas Leblanc*** *(1791) in France had propelled a small chemical industry. But it was in Germany, which became the leading country in theoretical chemistry, that chemical research had the biggest impact on industry. By the end of the 19^{th} century, the country had developed into the*

$NaBO_3$ (sodium perborate): oxidizing and bleaching agent.

Na_2CO_3 (sodium carbonate; soda ash): manufacture of glass, soap and detergents, paper.

$NaOCl$ (sodium hypochlorite): disinfectants, deodorants, bleaches.

$NaClO_3$ (sodium chlorate): manufacture of rocket propellant and explosives.

Na_2S (sodium sulfide): preparation of dyes.

largest manufacturer of such chemicals as dyes, fertilizers, and acids used in industrial processes[354].

In England, **William Henry Perkin** *(1838–1907) was trying to synthesize quinine when he accidentally produced the first synthetic dye, which we know as mauve. Perkin got rich on mauve and then went on (1875) to create the first synthetic perfume ingredient coumarin.*

Perkin's chemistry teacher, **August Wilhelm von Hoffmann** *(1818–1892, Germany) was a German chemist teaching in England. He synthesized his first dye, magenta in* 1858. *After he returned to Germany he discovered many chemicals and aniline dyes. Other chemists in Germany worked on producing natural dyes from easily available chemicals, obtaining a red called alzarin (1869) and indigo (1880). All these dyes became the basis of an immense German chemical industry. They also had an impact on biology, for biologists discovered that coloring bacteria or cells with dyes made previously invisible structures apparent.*

1789–1842 CE **Martin Klaproth** (1743–1817, Germany). Chemist. Discovered an unknown metal in *pitchblende* (1789). Although he was unable to isolate the new metal, he named it *uranium* after the recently discovered plant *Uranus* (1781, William Herschel). In 1842 the chemist **Eugène Melchior Peligot** (1811–1890, France) isolated the element.

1790–1800 CE **Salomon ben Joshua Maimon** (1753–1800, Germany). Philosopher, historian of philosophy and logician. Attempted to expound an algebraic *symbolic system of logic* (1794). Developed a form of *monism* (1797) (i.e. there is but one fundamental reality) that pervaded not only philosophy, but all sciences, and by which **Fichte**, **Schelling** and **Hegel** were influenced. **Goethe**, **Schiller**, **Kant** and **Mendelssohn** payed him tributes of praise.

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354 The relationship between scientific education and technological progress became fully understood during the 19^{th} century. Following the example of the *Ecole Polytechnique* in France, Germany (and later the United States) also founded technical schools with the idea of applying science to technology. At the end of the century, these technical universities played an essential role in the rapid expansion of Germany's industry. They developed the various kind of engineers who used science to solve technological problems rather than to advance knowledge.

Key works:

- *Versuch über die Transzendentalphilosophie* (Essay on the Transcendental Philosophy, 1790). Criticism of Kantian philosophy. Kant acknowledged Maimon as the most acute of his critics.
- *Versuch einer Neuen Logik* (Essay on the New logic, 1794)
- *Kritische Untersuchungen über den Menschlichen Geist* (Critical elaborations on the human spirit, 1797).
- *Lebengeschichte* (Autobiography, 1793). An important source for the study of Judaism and Hasidim in Eastern Europe in that period.

In his later writings he achieved synthesis of rationalism and Judaism. In his *Kritische Untersuchungen*, the great question at issue is Kant's question: *"Has man any ideas which are absolutely and objectively true?"*. The answer to this question depends on another question: *"Has man any ideas independent of experience?"*, for if all ideas depend on experience, there can be no question of objective ideas, experience being essentially subjective.

Kant answered the second question in the affirmative, and the first in the negative. He showed that in consciousness certain elements are given which are not derived from experience, but which are necessarily true. However these given elements or "things in themselves" man knows only as they appear to him, but not as they are "per se". This concept of "things in themselves" is rejected by Maimon, who holds that the matter of exterior objects which produce impression on man's sensibility is absolutely intelligible.[355] He also

[355] Maimon seized upon the fundamental incompatibility of a consciousness which can apprehend, yet is separated from, the "thing-in-itself". That which is object of thought cannot be outside consciousness; just as in mathematics $\sqrt{-1}$ is an unreal quantity, so things-in-themselves are *ex-hypothesis* outside consciousness, that is to say, unthinkable.

The Kantian paradox he explains as the result of an attempt to explain the origin of the "given" in consciousness. The form of things is admittedly subjective; the mind endeavors to explain the material of the given in the same terms, an attempt which is not only impossible but involves a denial of the elementary laws of thought. Knowledge of the given is, therefore, essentially incomplete. Complete or perfect knowledge is confined to the domain of pure thought, to logic or mathematics. Thus the problem of the thing-in-itself is dismissed form the inquiry, and philosophy is limited to the sphere of pure thought. The Kantian categories are, indeed, demonstrable and true, but their application to the given is meaningless and unthinkable.

contested the Kantian distinction between sensibility and understanding as well as the subjectivity of the intuitions of time and space. For him, sense is imperfect understanding, and time and space are sensuous impressions of diversity, or diversity presented as externality.

In practical philosophy he criticized Kant for having substituted an unpractical principle for the only motive for action – pleasure. The highest pleasure is in knowing, not in physical sensation, and because it recognizes this fact the "Ethics" of Aristotle is much more useful than the Kantian.

Maimon's autobiography was published by K. Ph. Moritz (Berlin, 1793). In this work he gives a résumé of his views on the Kabbalah, which he had expounded in a work written while he was still in Lithuania. According to him the Kabbalah is practically a modified Spinozism, in which not only is the world in general explained as having proceeded from the concentration of the divine essence, but every species of being is derived from a special divine attribute. God, being the ultimate substance and the ultimate cause, is called "En Sof," (infinity) because He can not be predicated by Himself. However, in relation to the infinite beings, positive attributes were applied to Him, and these attributes were reduced by the Kabbalists to ten – the *ten sefirot*. The ten "circles" correspond to the ten Aristotelian categories, without which nothing can be conceived.

In the same work Maimon expresses his views on Judaism. He divides Jewish history into five main periods:

(1) the period of natural religion, extending from the Patriarchs to Moses;
(2) the period of revealed or positive religion, from Moses to the Great Sanhedrin;
(3) the Mishnaic period;
(4) the Talmudic period;
(5) the post-Talmudic period.

Maimon censures the Rabbis for having burdened the people with minute prescriptions and ceremonies, but praises their hight moral standard.

Maimon was born at Nieswiez, Polish Lithuania. He was a child prodigy in the study of rabbinic literature. Married off by his father at the age of 11, he became a father himself at 14. He supported his family by working as a tutor

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By this critical skepticism Maimon takes up a position intermediate between Kant and Hume. Hume's attitude to the empirical is entirely supported by Maimon. The causal concept, as given by experience, expresses not a necessary objective order of things, but an ordered scheme of perception; it is subjective and cannot be postulated as a concrete law apart form consciousness.

in neighboring towns. In his spare time he educated himself in philosophy, secular sciences and foreign languages. He adopted the name **Maimon** in honor of Maimonides, as a token of reverence for that great master.

Harassed both by his implacable mother-in-law and by his correligionists (who regarded him as a heretic) he left home and family (1770) to begin a life of material insecurity and wandering over Northern Europe which terminated only in 1790; he was then offered a retreat on the estate of Count Adolph Kalkereuth of Nieder Siegersdorf (Silesia), where he died.

1790–1820 CE **John Rennie** (1761–1821, England). Civil engineer. Constructed many canals, bridges, docks, breakwaters and harbors in Scotland and England. The most conspicuous are: Waterloo bridge, Southwark bridge and London bridge over the Thames.

Born at Phantassie, Haddingtonshire, and educated (1780–1784) at Edinburgh University.

A feature of his work was the use of iron for many portions of the machines which had formerly been made of wood.

1790–1850 CE Leading Western poets and novelists in the *Age of Romanticism* an *Naturalism.*

- Friedrich Hölderlin 1770–1843
- William Wordsworth 1770–1850
- Samuel Taylor Coleridge 1772–1834
- Heinrich von Kleist 1777–1811
- Adelbert von Chamisso 1781–1838
- Stendhal 1783–1842
- Jakob Grimm 1785–1863
- Wilhelm Grimm 1786–1859
- Lord Byron 1788–1824
- Percy Bysshe Shelley 1792–1822
- John Keats 1795–1821
- Heinrich Heine 1797–1856
- Adam Mickiewicz 1798–1855
- Honore de Balzac 1799–1850
- Alexander Pushkin 1799–1837
- Victor Hugo 1802–1885
- Prosper Merimeé 1803–1870
- Hans Christian Andersen 1805–1875
- Henry W. Longfellow 1807–1882
- Edgar Allan Poe 1809–1849
- Nikolai Gogol 1809–1852

•	Alfred Tennyson	1809–1892
•	Berthold Auerbach	1812–1882
•	Ivan Goncharov	1812–1891
•	Mikhail Lermontov	1814–1841
•	Herman Melville	1819–1892

1791 CE **Jeremias Benjamin Richter** (1762–1807, Germany). Chemist. Formulated the *Law of Equivalent Proportions* which states that if an amount x of substance A combines chemically with amount y of substance B and also with amount z of substance C, then amount y of substance B will combine with amount z of substance C. After this discovery, tables of equivalent weights were drawn up, showing the relative amounts of the chemical elements that would combine with one another.

Richter was born at Hirschberg in Silesia, and became a chemist at Breslau mines and the Berlin porcelain factory. Richter was a pupil of the philosopher **Immanuel Kant**, and he held, with his master, that the physical sciences were all branches of applied mathematics.

Emancipation – The second Exodus (1791–1917)

In antiquity the Jews were the great innovators in religion and morals. In the Dark Ages and early medieval Europe they were still an advanced people transmitting scarce knowledge and technology. Gradually they were pushed from the van and left behind until, by the end of the 18^{th} century, they were seen as bedraggled and obscurantist rearguard in the march of civilized humanity.

But then came an astonishing second burst of creativity. Breaking out of their ghettos, they once more transformed human thinking, this time on the secular scientific sphere.

With the decline of religious faith in post-medieval European society, the traditional theological hostility towards the 'deicide' people became less relevant, especially to intellectuals, who identified with the skeptical temper of the Age of Enlightenment. The rise of rational thinking in the 17^{th} and 18^{th}

centuries appeared to be positive development for the Jew, for it attacked the foundations of Christian religion and the unified Christian state which had excluded or oppressed Jews for reasons of creed.

*It was partly from rationalist assumptions of the German Enlightenment that the Habsburg **Emperor Joseph II** derived his Toleration edicts of the 1780s, that **Moses Mendelssohn** felt empowered to build a bridge between traditional Jewish and modern German cultures and that his friend **Gotthold Lessing** immortalized a more positive image of the Jew in his famous play, Nathan the Wise. Without the philosophy of the Enlightenment, the Prussian bureaucrat **Christian Wilhelm Döhm** would never have written his tract "Concerning the Civic Amelioration of the Jews" (1781), an indictment of the responsibility of the Christian world for the degradation of the Jews.*

Other forces were also in action; with the disappearance of the last vestiges of feudalism, the Court Jews went also. Their financial manipulations, which often saved European monarchs from bankruptcy, made way for the public banking system which we know today. Their financial services and counsel to sovereigns, states and private enterprises were vastly important in the commercial development and industrial growth of Europe. Thus, at the turn of the 19^{th} *century, the rulers of Europe were not blind to the economical potential of the Jews.*

*The Emperor, as well as his neighbor, **Frederick the Great** of Prussia, have certainly not overlooked the substantial revenue the Jew brought into their realms by stimulating industry. Indeed, the industrial revolution in Germany found its most enterprising pioneers among Jews. The first iron industry (1840), coke industry, and railroad industry were all founded and built by Jewish investors and entrepreneurs. The electrical, chemical, shipping and dye industries also owe much to Jews.*

With those factors combined, the ghetto dwellers faced, at the end of the 19^{th} *century, the most momentous political event in Jewish history since the loss of their state in 70 CE – the Emancipation:*

- *Sept. 28, 1791: Jews were declared to be equal (on paper) with all men and free citizens of the Republic of France.*
- *1798: The ghetto gates in Bonn, Germany, were broken down by Christians*

Consequently, the Jews of Germany and Austria emerged from the mental and physical isolation of ghetto life and rushed with burning enthusiasm into *the arts and sciences*[356]*. They may have been newcomers to the German universities, but it seemed as though they had been preparing for the entrance examination for a thousand years.*

Their bibliophile tradition as "people of the Book" took them almost as a matter of course, into medicine, biology and mathematics as well as literature and music. Their grandfathers had studied the Talmud. They, no less attentively, read ***Kant****,* ***Goethe*** *and* ***Hegel****.* ***Karl Marx*** *(grandson of two orthodox rabbis) unconsciously reverted to an old Kabbalist technique when he "turned Hegel upside down" in order to formulate his own dialectical materialism. In the sciences, though most professorships still remained closed to them*[357]*, new doors were constantly opening.*

There was, however, a price to pay for joining the modern world; the "ticket of admission to European culture" was the baptismal certificate[358]*, common mostly in the first half of the 19*th *century. After 1848, apostasy declined while other forms of assimilation (intermarriage and renegades) were more fashionable.*

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356 **Moses Mendelssohn** translated the Pentateuch into German (1783): **Leopold Zunz** and his friends established (1819) the *Society for promotion of Jewish Culture and Science*

357 From 1818, Jews in Germany were excluded from state academic posts, by decree of King Frederick William III. Jews were also dismissed from state positions and conversion to Christianity was actively encouraged.

358 During the 19th century, at least 250,000 Jews converted to Christianity in Europe alone. [Germany 22,500; Britain 23,500; Russia 84,500; Poland 21,500; Austro-Hungary 45,000]. This amounted to about 5 percent of the total Jewish population during that century. Most apostates belonged to the wealthy and intellectual circles in major cities, who constituted about 15 percent of the Jewish population. The rest were poor religious people who would not change their old living style, thus being immune to assimilation of any form. Total Jewish population in Europe reached 1,430,000 (1800) [Russia and Poland 800,000; Austria, Hungary and Galicia 300,000; Germany 200,000; France 80,000; Holland 50,000] and 8,690,500 (1900); 9,462,000 (1939). In *Berlin*, the Jewish population never exceeded 4 percent [2000 (1743); 3300 (1812); 12,000 (1852); 24,280 (1864); 45,500 (1876); 106,000 (1900); 172,600 (1925); 160,500 (1933); 82,780 (1939); In *Vienna*: 178,000 (1933); 91,500 (1939)]. The number of Jews in Germany grew to 420,500 (1871) and 564,400 (1925), where the 1925 figure included some 80,000 immigrants from the east.

Intermarriages during 1906–1930 amounted to 27% of total Jewish marriages. In general, Jewry lost 80% of descendants via intermarriages. The Jewish intellectual elite amounted to about 3 percent of their total number at any given time during 1830–1930.

Apostasy was usually Protestant in Northern Germany (for ***Heine****,* ***Marx****,* ***Felix Mendelssohn*** *and many others) but Catholic in Bavaria and the Habsburg empire. Conversion tended to be a matter of good form rather than an act of faith. Most converted Jews blended entirely into the social background. A generation or two later, no one remembered that* ***Johann Strauss, Sr.*** *("the demon of Viennás innate musical spirit", as Richard Wagner described him), was the son of a baptized Jewish tavern-keeper from Budapest. Many German Jews bore the names of localities which, with the addition of a* von *or* zu*, furnished a pure Aryan flavor.*

Even those who stopped short of conversion, however, tended to abandon the more visible aspects of the Jewish faith. Since German life was becoming increasingly secular, baptism as such ceased to be the touchstone of social assimilation. Most of the emancipated German Jews did their best to become outwardly indistinguishable from their neighbors.

But despite all the efforts of the liberal elements and the strenuous fight of the Jews themselves to remove the barriers to their full acceptance by assimilation, anti-Semitic forces remained powerful, especially in the lower middle class and nobility. They frequently brought about violent outbreaks and riots differing in strength and extent in the cities and states where they took place. Whenever adverse events (economic, political or social) occurred, these feelings of hostility erupted.

The attitude of Bismarck to this problem during his reign (1862–1890) was ambiguous; on one hand he had great respect for the high qualities, great talents and competence of many Jews and had not the slightest hesitation to use their services for Germany or for his own interests.[359] *But Bismarck also used antisemitism as a convenient political weapon in his fight against liberals and Social Democrats, many of whose leaders were Jewish. Nevertheless, he would not tolerate any violent actions against Jews and would not permit their civil rights violated, fully recognizing their great value for the strength of Germany.*

The attitude of Kaiser Wilhelm II toward Jews was more complex and sometimes more emotional. During his reign (1888–1914), Jews increasingly

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359 An example is his close association to the Jewish banker **Gerson von Bleichröder** (1822–1893) who helped to finance two of Bismarck's wars and was instrumental in the building of the empire. He was also Bismarck's personal financial adviser, as well as that of Benjamin Disraeli. Bleichröder's contributions to the greatness of Germany earned him only envy scorn and hatred, becoming the target of strong antisemitic reactions, especially from the economically declining ruling Junker class.

penetrated academic professions and many became recognized leaders in science and many other fields. But, owing to the newly accumulated wealth, Jews began to play a prominent role in society, and many Junkers were unable to compete with them. These Junkers, whose wealth was essentially based on their large estates were impoverished by the rise of capitalism and industrialization. Attributing their misfortunes to the Jews and not to the economic trends prevailing in Western Europe, they succeeded (with the full support of Wilhelm) to block the Jews from both government and the army.

Unfortunately, the Kaiser fell under the influence of the antisemitic ideology of Cosima Wagner[360] *and her son-in-law, Houston Stewart Chamberlain. Consequently he began to consider Jews in general as the deadly enemy of the "Aryan" Germans. This did not prevent many of his entourage from having Jews as their personal physicians, bankers, or advisers, despite their gross antisemitism. As for Wilhelm himself, he was fully aware of the major contribution of German Jews to German science and industry and like Bismarck would not tolerate any violent eruption of antisemitism; Law and order were untouchable!*

What were the factors permitting the Jews in Germany to become instrumental in the rapid rise of science in the Wilhelmian era (1888–1914) and later in the Weimar Republic until Hitler (1919–1933)?

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[360] In December 1914, **Lord Balfour** (Britain's foreign secretary 1916–1919) told the scientist and Zionist leader, **Chaim Weizmann** that on his previous visit to Cosima Wagner in Bayreuth she had expressed the opinion that "...the Jews in Germany have captured Stage, Press, Commerce and the Universities. They are putting in their pockets, after only a hundred years of emancipation, everything for which the Germans have worked for centuries. We resent very much having to receive all the moral and material culture at the hands of the Jews ..."

To this, Weizmann had commented to Balfour: "The essential point which most non-Jews overlook and which forms the crux of the Jewish tragedy is that those Jews who are giving their energies and their brains to the Germans are doing it in their capacity as Germans and are enriching Germany and not Jewry, which they are abandoning .... The tragedy of it all is that whereas *we do not recognize them as Jews, Madame Wagner does not recognize them as Germans*, and so we stand there as the most exploited and misunderstood people."

At the turn of the century these extreme views of Cosima Wagner were shared by a small number of people, but they became widespread in the era of the Weimar Republic. It is remarkable how unaware Jews, as well as many non-Jews, were of these deep-rooted feelings, almost until the era when the Nazis came to power.

One factor was the rapid growth of cities which offered talented, intelligent Jews, with their willpower and drive, an unexpected chance.

A second factor was the rapid development of the capitalistic and industrial economy around 1850. New elite and new ideologies were formed; a new upper middle class began to emerge at the expense of the previously dominant nobility, the craftsmen, the lower middle class, the peasants, and the landowners, who lost many of their privileges. Jews seized upon these new opportunities and played an important role in the new economy.

The third factor was the vast expansion of the universities and the technical institutions. About 10% of the students were Jews while Jews constituted not quite 1% of the total population! In 1907, they amounted to 6% of all German physicians and dentists, 14% of all lawyers, and 8% of private scholars, journalists and writers.[361]

The acceptance of Jews in the universities was greatly facilitated by their assimilation to German civilization. At the turn of the century many Jews had lost almost all connections with Jewish tradition and the Jewish community. Many of them considered themselves as German citizens of Jewish faith. But the Jewish religion actually had little meaning for many of them. Even in the relatively liberal Wilhelmian era, a period of great prosperity and relative affluence, the freedom that the Jews enjoyed in the academic and free professions was not extended to all fields. Even baptized Jews were admitted to public office and civil service in very limited and insignificant numbers. They were virtually excluded from the government and the army. In these fields, the situation began to change only in WWI, when many Jews fought in the army.

Full emancipation (the right of religious freedom and the right to be chosen to governing bodies) was declared in The Netherlands (1796); Italy (1798); Belgium (1831); Canada (1832); Germany (1871); England (1878); U.S.A. (1785–1877); Russia[362] *(1917).*

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361 When the Nazis came to power (1933) almost half of the 6000 physicians in Berlin were Jews.

362 There was no other state on the European continent which officially pursued such repressive anti-Jewish policies in the 19^{th} century as the Tzarist Russian Empire. By 1897, more than 5 million Jews (about half of the world Jewry) lived under the totalitarian rule of the Tzars in poverty and deprivation, subjected to endless humiliating decrees. What the Russians did was to engage in the first modern exercise of *social engineering*, treating Jews as earth or concrete, to be shoveled around. Firstly they confined the Jews to what was called the "Pale of the Settlement" (1812) which consisted of one million square kilometers stretching from the Baltic to the Black Sea. A series of statutes (1804) forbade

The Jewish intellectual tradition, nurtured continuously for so many centuries by Talmudic studies and enriched at various periods by fusion with other cultures, received a new impetus with the emancipation.

Although severely handicapped by the numerus clausus *(an anti-Semitic device for limiting Jewish students in universities on a percentage quota), Jews nonetheless entered into all fields of study. They distinguished themselves especially in the sciences. While science does not know any racial or national boundaries, since it aims to serve all mankind, in Germany, however, sharp distinctions were often made between Jewish and non-Jewish scientists — to the detriment of science and of Germany.*

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them to live and work in the villages, thus destroying the livelihood of a third of the Jewish population, without allowing them to do any labor on the land. The real aim was to drive Jews into accepting baptism. The next turn of the screw came in 1827, when Nikolai I issued the 'Cantonist Decrees' which conscripted all male Jews, from 12 to 25, to 25 years of military service, the object again being to promote baptism. During 1827–1856, some 60,000 kids were forcibly kidnapped and conscripted, half of which were eventually baptized. The government destroyed Jewish education. Jewish books were censored or burned. Movements outside the pale were banned, and inside it – restricted.

Russian antisemitism was in its origins a combination of simple primitive hatred for the Jews as 'aliens' and of Christian orthodox religious prejudice which regarded Jewish people as deicides. Such prejudice remained alive and virulent both at the state level and among the millions of superstitious and illiterate Russian peasants. In fact, Russia was the only country in Europe, at this time, where antisemitism was the official policy of the government. It took innumerable forms, from organizing massacres (pogroms, 1871–1906) to forging and publishing the *Protocols of the Elders of Zion*.

History of Theories of Light II

B. Waves Versus Corpuscles, A *(1608–1800)*

The 17^{th} *century opened with a shower of new inventions and ideas. It seemed as if all the latent optical lore of* 2000 *years, suddenly burst the floodgates of human consciousness and materialized in the form of telescopes, microscopes, prisms, the new phenomena of dispersion, polarization, diffraction and aberration and the principles of least-time and Huygens'.*

Above all, however, hovered the great controversy on the nature of light: was it a stream of particles as maintained by ***Democritos*** *(420 BCE),* ***Descartes*** *(1637) and* ***Newton*** *(1672) or a rapid undulation of ethereal matter as argued by* ***da Vinci*** *(1490),* ***Grimaldi*** *(1665),* ***Hooke*** *(1665) and* ***Huygens*** *(1678)?*

Until about the middle of the 17^{th} *century, it was generally believed that light consisted of a stream of corpuscles, emitted by light sources, such as the sun or a candle flame, and traveled outwards from the source in straight lines. This theory provided simple explanations to the simple laws of reflection and refraction from smooth surfaces. With the discovery of the phenomenon of light diffraction by* ***Grimaldi*** *(1665) and* ***Römer's*** *proof (1676) that light travels with a definite velocity,* ***Huygens*** *(1690) showed that the laws of reflection and refraction could be explained on the basis of a wave theory (through the wavelets and secondary wavefronts) and that such a theory could furnish a simple explanation to the recently discovered phenomenon of double refraction by* ***Erasmus Bartholinus*** *(1670).*

*The great moments of this epoch were undoubtedly the invention of the telescope (1608), the mathematical statement of the law of refraction (****Snell****, 1621),* ***Fermat****'s principle of least-time (1657), the advent of wave theory [****Grimaldi****,* ***Hooke****, 1665;* ***Huygens****, 1678], the determination of the velocity of light (****Römer****, 1676) and* ***Newton****'s theory of corpuscles, dispersion and color (1672). The authority of Newton led to the rejection of the wave-theory and the abeyance of optics for nearly a century.*[363] *But it still found an occasional supporter, such as the great mathematician* ***Leonhard Euler***

363 For one thing, it was objected that if light were a wave motion one should be able to see around corners, since waves can bend around obstacles in their paths. We know now that the wavelengths of light are so short that the bending, while it does actually take place, is so small that it is not ordinarily observed. The significance of Grimaldi's results was not realized at the time.

(1746). It was not until the beginning of the 19^{th} *century that the decisive discoveries were made which led to general acceptance of the wave theory.*

1791–1819 CE **William Smith** (1769–1839, England). Geologist, engineer and surveyor. Founder of stratigraphical geology. Discovered a method to assign relative ages to individual rock strata (formations) by means of their fossilized content and thus was first to point out the relationship between fossils and geologic data (1791).

Units of similar lithology are not continuous even in one region and the stratigraphic sequences differ significantly among widespread localities. The need for detailed mapping of rock formations required a new unifying principle — a new tool by which units could be categorized and recognized widely. [Detailed geologic mapping in a humid region like Europe is difficult. Almost everywhere the rocks are superficially covered by soil, vegetation, or alluvium.]

Smith found fossils to furnish just such a tool. His investigations of roads, quarries, mines, and canals acquainted him intimately with much of England's countryside. During his travels he recognized and traced out numerous sedimentary rocks, and he soon noticed that each successive unit contained its own diagnostic assemblage of fossils by which it could be distinguished from other units of different ages. Utilizing this principle he produced, in 1815, the first geological map of England, and correlation between distant localities now became feasible. The way was prepared to erect a stratigraphic classification based on *time relations* of strata rather then on rock type.

1792–1808 CE **Jean-Baptiste-Joseph Delambre** (1749–1822, France). Astronomer, erudite and historian of astronomy. Discovered new formulas in *spherical trigonometry* (1808). Published tables of the location of planets and their satellites (1792); with **Méchain**, measured an arc of the meridian between Dunkirk and Barcelona (1792–1799); wrote histories of ancient, medieval and modern astronomy. A large crater is named for him on the moon.

Delambre was born at Amiens. Despite extreme penury he studied indefatigably ancient and modern languages, history and literature, and it was not until he was 36 years of age that he begun a serious study of astronomy and mathematics. He was 40 before he published anything on the subject, and it was some years later that he was awarded a prize by the Academy for his tables of Uranus. He succeeded **Lalande** (1807) as professor of astronomy at the College de France.

1792 CE Ca 800,000 died of the plague in Egypt. By 1799 the disease reached North Africa with ca 300,000 additional casualties.

1784–1830 CE **William Murdock** (1754–1839, England). Inventor. Invented *coal-gas lighting* (1792). First to construct a model of steam powered carriage (1784). Made important improvements of the steam engine.

Murdock was born near the village of Auchinleck in Ayrshire. He was first to realize that coal gas might be used for light. In 1807, London streets began to be illuminated by coal-gas lighting.

At the celebration of the centenary of gas lighting (1892), the bust of Murdock was unveiled by Lord Kelvin.

1793 CE **Christian Konrad Sprengel** (1750–1816, Germany). Botanist. Discovered the part played by nectaries, insects and the wind in the *pollination* of flowers (plant fertilization).

1793–1798 CE **Eli Whitney** (1765–1825, U.S.A.). Inventor. The father of *mass production*. His *cotton gin* (1793) made cotton-growing profitable, and helped make the United States the largest cotton producer in the world. His method of making guns by machinery (1798) marked the beginning of *mass production* in the world's industry.

Whitney was born in Westborough, Mass., the son of a farmer. Times were hard after the Revolutionary War, and Whitney did not have the money to go to college. He taught school for five years and with his saving financed his studied at Yale during 1788–1792. By 1793 he had built the cotton gin, which could clean cotton as fast as 50 men working by hand.

In 1798, he built a factory near New Haven and began to make muskets by a new method. Until then, each gun had been handmade by a skilled craftsman, and no two guns were alike. Whitney invented tools and machines that enabled unskilled workmen to turn out absolutely uniform parts.

1793–1814 CE **Thomas Young** (1773–1829, England). Distinguished physicist, physician and philologist. Discovered and explained the phenomenon of light interference and consolidated the wave theory of light on a firm experimental basis. Opposed Newton's particle theory of light in favor of light as a wave in the cosmic aether (1801). In 1807 he anticipated the nature of infrared radiation from hot bodies, claiming that heat, like light, is a wave vibration rather than a material substance. In 1809 he applied the wave theory of light to the phenomena of light refraction and dispersion, although he believed light vibration to be mainly longitudinal.

Young is also the founder of physiological optics: in 1793 he explained the mode in which the eye accommodates itself to vision at various distances depending on the change of the curvature of the lens. In 1801 he described the defect known as *astigmatism*. In 1802 he put forward the theory that color perception depends on the presence in the retina of 3 kinds of nerve fibers which respond respectively to red, green and violet light.

In another field of research, he was one of the first successful workers at the decipherment of Egyptian hieroglyphic inscriptions: by 1814 he had completely translated the enchorial (demotic) text of the *Rosetta stone*.

Young, like Leonardo da Vinci, was a remarkably versatile scholar. His epitaph reports that "he first penetrated the obscurity which had veiled for ages the hieroglyphics of Egypt"; but his work shows only one facet of a brilliant career. His work in medicine and science later led the physiologist and physicist **Helmholtz** to say of him: "*He was one of the most profound minds that the world has ever seen*".

Young was born to a Quaker family in Somerset, England, the youngest of 10 children. At age 14 he was acquainted with Latin, Greek, French, Italian, Hebrew, Persian and Arabic. In 1796 he obtained his M.D. degree at Göttingen, Germany. Upon the death of his grand-uncle in 1797, he became financially independent and in 1799 established himself as a physician in London. In 1801 he was appointed professor of physics, but resigned his professorship in 1803, fearing that its duties would interfere with his medical practice.

1793–1828 CE **Thomas Telford** (1757–1834, Scotland). Civil engineer. Devised and improved methods of road construction. The Telford method of using large flat stones for road foundations is named after him. Telford engineered roads, bridges, harbors, docks, canals and waterways. He built the Menai Strait suspension bridge in Wales, the Ellesmere Canal in England, the Caledonian Canal in Scotland and the Göta Canal in Sweden.

Telford was born in Eskdale, Scotland and died in London (buried in Westminster Abbey). He was a son of a shepherd. From early childhood he was employed as a herd, occasionally attending the parish school of Westerkirk. He was mostly self-educated, learning architectural drawing in his spare time. He never married, living most of his adult life in hotels. He was a fellow of the Royal Societies of London and of Edinburgh.

Science Progress Report No. 7

"The Revolution has No Need for Savants"

With the rise to power of the Jacobines in 1793, *the French revolution took a more radical turn, and many of the old institutions were closed down, including the Paris Academy of Sciences; scientists associated with the regime of the Girondists were executed, notably Lavoisier. The vice-president of the tribunal that tried Lavoisier declared that France "Already had too many scholars".*

However, after 40,000 *people were killed by the government and its agents, the National Convention was sobered by the terror and with the fall of the Jacobines in the summer of* 1794, *the revolution fell back into the hands of the bourgeoisie, which was the class that in the end gained the most from it.*

1794–1835 CE **Carl Friedrich Gauss**[364] (1777–1855) Germany)[365]. Physicist, astronomer, and one of the greatest mathematicians of all times. Published the treatise '*Disquisitiones Arithmeticae*', which includes his discoveries in number theory and is one of the most important works in the history of mathematics.

Gauss was born in Brunswick, Germany, into a poor family. He was a child prodigy. At the age of 14 he discovered the prime number theorem, and completed his first original work at 19, when he showed how to construct a regular 17-sided polygon with a ruler and compass, the first 'new' n-gon for

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[364] For further reading, see:

- Bühler, W.K., *Gauss*, Springer-Verlag, 1981, 208 pp.
- Hall, T., *Carl Friedrich Gauss*, Massachusetts Institute of Technology Press: Cambridge, 1970, 176 pp.
- Dunnington, G.W., *C.F. Gauss Titan of Science*, New York, 1955.
- Gauss, C.F., *Disquisitiones Arithmeticae* (*1801*), Yale University Press: New Haven, CT, 1966, 472 pp.

[365] During 1777–1783, *both* Euler and Gauss were alive.

2000 years[366]. His doctoral thesis (1799) contained the first acceptable proof of the fundamental theorem of algebra, a result whose proof had defeated such giants as **Euler** and **Lagrange**. He did fundamental work in probability, geodesy, mechanics, optics, actuarial science and electromagnetism [with **W.E. Weber**, he build in 1833 the first operating electric telegraph]. Like Euler, he was a prodigious calculator.

In 1795, Gauss developed the method of least squares[367], thus founding the field of *mathematical statistics*. He applied it to problems as diverse as astronomy and prime number counting.

In 1811 Gauss opened the modern period of research on infinite series with his memoir on the *hypergeometric* series [name given by **Johann Friedrich Pfaff** (1765–1825, Germany)]. **Euler** had studied it and introduced its defining differential equation, but Gauss was the first to master it. He made the first adequate study of its convergence and associated functional relations. The hypergeometric function played a central role in Gauss' thinking, be-

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366 In 1801 Gauss took up the ancient problem of finding *all* regular polygons that can be constructed by means of compass and ruler. The construction of regular polygons of 2^n, $3\cdot 2^n$, $5\cdot 2^n$ and $15\cdot 2^n$ have been known since the time of the Greeks, but no one suspected before Gauss that polygons of any other number of sides could be constructed by ruler and compass. The way had to be paved by numerous theorems in algebra. This Gauss did, showing eventually that a circle can be divided by ruler and compass into n equal parts if and only if n is of the form

$$2^{\alpha_0}\left[2^{(2^{\alpha_1})}+1\right]\left[2^{(2^{\alpha_2})}+1\right]\cdots\left[2^{2^{\alpha_n})}+1\right],$$

where each of the quantities in parenthesis is a prime and where $\alpha_1, \alpha_2, \dots, \alpha_n$ are all different positive integers. The only known primes of the form $F_n = 2^{2^n}+1$ (*Fermat numbers*) are 3, 5, 17, 257 and 65537 corresponding to n values of 0, 1, 2, 3 and 4. Euler showed (1732) that F_5 is *not* prime. No prime F_n has yet been found for $n \geq 4$. In fact, F_n is known to be composite for all n such that $5 \leq n \leq 21$, as well as for some larger n.

Gauss was so proud of his discovery showing the relation between prime Fermat numbers and inscribed polygons that he wished to have a 17–gon inscribed on his tombstone (emulating the tombstone of Archimedes, which was decorated by a figure of a sphere and circumscribed cylinder, suggesting his formula for the area of a sphere).

For some reason his request was not granted and on Gauss' grave in Göttingen there is no such polygon. It does, however, appear on the side of a monument in his native town of Brunswick.

367 *Formal* priority belongs to Legendre (1805). Gauss *published* his results only in 1821.

cause he encountered many special cases of its defining series in the theory of elliptic integrals.

In 1819 Gauss obtained an explicit formula for the combination of two finite rotations (never published). This led him to the discovery of the quaternions 24 years in advance of Hamilton.

In 1827 Gauss made the first systematic study of quadratic differential forms in his *Disquisitiones generales circa superficies curves*, thus laying the foundation of differential geometry. He was led to this by his geodetic work, which concerned the precise measurement of large triangles on the earth's surface. This provided the stimulus that led him to found the intrinsic differential geometry of general curved surfaces. For this work he introduced curvilinear coordinates u and v on a surface. He obtained the fundamental quadratic differential form $ds^2 = Edu^2 + 2Fdudv + Gdv^2$ for the element of arclength ds, which makes it possible to determine geodesic curves.

He formulated the concepts of *metric coefficients*, *Gaussian curvature* and *total curvature*.

His main specific results were the famous *theorema egregium*, which states that the Gaussian curvature depends only on E, F, G, and the *Gauss-Bonnet theorem* on total curvature for the case of a *geodesic triangle*[368], which in its general form is the central fact of modern differential geometry. It is

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[368] Gauss was able to find a formula for the sum of the angles of a *geodetic triangle* on any surface. If s is the sum of the angles, measured in degrees, then $s = 180\big[1 + \frac{1}{\pi}\int KdA\big]$, where K is the Gauss' curvature and the integral is taken over the interior of the triangle. In the plane, $K = 0$ and we have $s = 180$ for any triangle. On a sphere of radius r, we have $K = \frac{1}{r^2}$ and $\int KdA = \frac{A}{r^2}$, where A is the area of the triangle. Then $s = 180\big(1 + \frac{A}{\pi r^2}\big)$ or $A = \pi r^2\big(\frac{s}{180} - 1\big)$. (This formula was first published by the Flemish mathematician **Albert Girard** (1629).) By measuring A and the sum of angles s, this equation can be used to determine the radius r of a sphere. In general, on a surface with positive curvature we have $K > 0$, $\int KdA > 0$, and the sum of the angles s is greater than 180 degrees. Negative curvature implies $K < 0$ and s is then less than 180 degrees.

Thus, an inhabitant of a 2-dimensional spherical surface can discover the radius of his spherical world by simply measuring the area and the sum of angles of an arbitrary spherical triangle, using the formula

$$r = \sqrt{\frac{(A/\pi)}{\frac{s}{180^\circ} - 1}}$$

e.g. a quarter-hemisphere spherical triangle has: $s = 270^\circ$, $A = \frac{1}{2}\pi r^2$.

the generalization of these concepts that opened the door to Riemannian geometry, tensor analysis and the ideas of Einstein.

In 1831, Gauss turned his attention again to number theory, where he broadened the ideas of number into the complex domain. He defined complex integers (now called '*Gaussian*' *integers*) as complex numbers $a + ib$ with a, b as ordinary integers. This led him to introduce a new concept of prime numbers in which 3 remains prime but $5 = (1 + 2i)(1 - 2i)$ does not.

He then proved the unique factorization theorem for these integers and primes. The ideas of this paper inaugurated algebraic number theory. [He used these concepts to prove Fermat's conjecture for $n = 3$.]

In 1839 Gauss published his fundamental paper on the general theory of inverse square forces, which established *potential theory* as a coherent branch of mathematics. Among his discoveries were the *divergence theorem*[369], the basic mean value theorems for harmonic functions and the very powerful statement which later became known as "*Dirichlet's Principle*" and was finally proved by **Hilbert** in 1899.

Unlike **Euler**, he restricted the amount of his research that he made public and in his publications, obliterated any description of how his ideas had been generated. He stuck to his motto "Few, but ripe". Like **Newton** before him, he ascribed his success in solving problems where others failed to 'always thinking about them'.

In his unpublished notes it was discovered, after his death, that Gauss had considered non-Euclidean geometry before **Lobachevsky**, quaternions before **Hamilton**, elliptic functions[370] before **Abel** and **Jacobi** as well as much of **Cauchy**'s complex variable theory. In a letter written to his friend **Bessel** in 1811, Gauss explicitly stated *Cauchy's integral theorem* (1827) and

369 *Gauss' divergence theorem* (1839): The flux of a vector field out of a closed oriented surface equals the integral of the divergence of that vector field over the *volume* enclosed by the surface. The results parallel Stokes' theorem in that it relates an integral over a closed geometrical object (curve or surface) to an integral over a contained region (surface or volume).
Let $\mathbf{F}$ be a smooth vector field defined on Ω. Then

$$\int_{\Omega} \operatorname{div} \mathbf{F}\, dV = \int_{\partial\Omega} \mathbf{F} \cdot d\mathbf{S} = \int_{\partial\Omega} (\mathbf{F} \cdot \mathbf{n}) dS,$$

where $\partial\Omega$ is an oriented closed surface that bounds Ω and $\mathbf{n}$ is the outward unit normal to Ω.
This theorem arose in connection with electrostatic problems.

370 In 1799, Gauss defined the *sinus lemniscaticus* function (sin lem) $x = \text{sl}\, u$ via the inverse relation $u = \int_0^x \frac{dt}{\sqrt{1-t^4}}$. He then defined the lemniscate cosine by

remarked that he had found a fairly simple proof, and also that he knew how to make series expansion of functions of complex variable [**Laurent**, 1843]. For some reason, however, the suitable occasion for the publication of these theorems did not arise.

By 1820 he was in full possession of the main theorems of *non-Euclidean geometry* (the name is due to him), but he did not reveal his conclusions. The reason for this may be sought in the dominance of the ideas of **Kant** at that time in Germany, namely, the idea that Euclidean geometry is the only possible way of thinking about space. Gauss knew that this idea was totally false and that the Kantian system was a structure built on sand. However, he valued his privacy and quiet life and held his peace in order to avoid wasting his time on disputes with the philosophers.

The same thing happened again in the theory of elliptic functions, a very rich field of analysis that was launched primarily by **Abel** in 1827 and by **Jacobi** in 1828–1829. Gauss had published nothing on this subject and claimed nothing, so the mathematical world was filled with astonishment when it gradually became known that he had found many of the results of Abel and Jacobi before these men were born![371] Abel was spared this devastating knowledge

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$\text{cl}\ u = \text{sl}\left(\frac{\varpi}{2} - u\right)$, where $\varpi = 2\int_0^1 \frac{dt}{\sqrt{1-t^4}}$, and showed that

$$\text{sl}^2 u + \text{cl}^2 u + (\text{sl}^2 u)(\text{cl}^2 u) = 1.$$

The equation of the John Bernoulli lemniscate (1694) is

$$(x^2+y^2)^2 - 2a^2xy = 0, \qquad \text{or} \qquad r^2 = a^2\sin 2\theta.$$

Clearly, $\text{sl}\ 0 = 0$, $\text{cl}\ 0 = 1$, $\text{sl}\left(\frac{\varpi}{2}\right) = 1$, $\text{sl}(\varpi - u) = \text{sl}\ u$, $\text{cl}(\varpi - u) = -\text{cl}\ u$. The graphs of sl u and cl u resemble in their appearance those of the circular functions $\sin u$ and $\cos u$. The lemniscate is the locus of points P, the product of whose distances from *two* fixed points F_1, F_2 (the foci) $2a$ units apart is constant and equals a^2.

371 **Gauss** (1818) devised a remarkable method for the numerical calculation of elliptic integrals: To begin with, Gauss (and independently, **Lagrange**) introduced (ca 1785) the concept of the *arithmetico-geometric mean* in the following way: Let two numbers $\{a_0, b_0\}$ be given. Then the arithmetic mean a_1 is defined by $a_1 = \frac{1}{2}(a_0+b_0)$ and the geometric mean by $b_1 = \sqrt{a_0b_0}$. He then formed the new means $a_2 = \frac{1}{2}(a_1+b_1)$, $b_2 = \sqrt{a_1b_1}$. By continuing this process, one obtains two series of numbers obeying the coupled recursions

$$a_{n+1} = \frac{1}{2}(a_n+b_n), \qquad b_{n+1} = \sqrt{a_nb_n}.$$

As $n \to \infty$, a_n steadily decreases and b_n steadily increases toward the *same* limit $M(a_0, b_0)$ known as the *arithmetico-geometric* mean. This follows from by his early death in 1829, at the age of 26, but Jacobi was compelled to swallow his disappointment and go on with his work. His attention was caught by a cryptic passage in the *Disquisitiones*, whose meaning can only be understood if one knows something about elliptic functions. He visited Gauss on several occasions to verify his suspicion and tell him about his own most recent discoveries, and each time Gauss pulled a 30-year-old manuscript out of his desk and showed Jacobi what Jacobi has just shown him. The depth of Jacobi's chagrin can readily be imagined. At this point in his life Gauss was indifferent to fame, and was actually pleased to be relieved of the burden of preparing a treatise on the subject which he had long planned.

In 1832 Gauss established with **Wilhelm Eduard Weber** (1804–1891, Germany) the now-standard CGS system of units.

In 1835 he discovered that a moving charge exerts a different electric force from a charge at rest. His (unpublished) result was rediscovered by Weber in 1846. Earlier, in 1833, Gauss and his friend Weber built the first experimental electromagnetic telegraph which transmitted signals across a wire, 2000 meters long, connecting Gauss' house with his observatory.

Gauss married twice (1805, 1811) and had altogether 2 daughters and 3 sons, two of which emigrated to the U.S.A. after an extended conflict with

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the inequalities:

$$a_{n+1} - b_{n+1} = \frac{1}{2}(\sqrt{a_n} - \sqrt{b_n})^2 > 0$$

$$a_n - a_{n+1} = \frac{1}{2}(a_n - b_n) > 0$$

$$b_{n+1} - b_n = \sqrt{b_n}(\sqrt{a_n} - \sqrt{b_n}) > 0.$$

Next, Gauss established the relation

$$\int_0^{\pi/2} \frac{d\theta}{\sqrt{a_0^2\cos^2\theta + b_0^2\sin^2\theta}} = \int_0^{\pi/2} \frac{d\theta}{\sqrt{a_1^2\cos^2\theta + b_1^2\sin^2\theta}}.$$

By applying this transformation repeatedly, he obtained

$$\lim_{n\to\infty} \int_0^{\pi/2} \frac{d\theta}{\sqrt{a_n^2\cos^2\theta + b_n^2\sin^2\theta}} = \int_0^{\pi/2} \frac{d\theta}{\sqrt{M^2\cos^2\theta + M^2\sin^2\theta}} = \frac{\pi}{2M}.$$

Choosing $a_0 = 1,\ b_0 = k' = \sqrt{1-k^2}$, Gauss clinched the final beautiful result

$$K(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{\cos^2\theta + k'^2\sin^2\theta}} = \frac{\pi}{2M(1,k')}$$

which provides a powerful means of constructing a table of values of $K(k)$.

him. His second wife died in 1831 and his youngest daughter kept the house for him until his death in 1855. Gauss did not seem to relish travel: In the last 27 years of his life he slept away from his observatory only once.

Gauss lived in a period of extraordinary political and social upheavals, even when measured by the standards of our fast-moving and eventful age. He was 12 years old when the French Revolution broke out (1789), 29 when the 1000-year old Holy Roman Empire was dissolved (1806), 38 when Napoleon was defeated (1815), and over 70 when Germany had its own Liberal Revolution (1848). During the same period, the so-called first Industrial Revolution took place, with its lasting and incisive effects on everyday life and on the political and social order. All this affected Gauss' life in an explicit and tangible way.

Gauss' contemporaries in Germany were: **Ludwig van Beethoven** (1770–1827), **Franz Schubert** (1797–1828), **Arthur Schopenhauer** (1788–1860), **Georg Wilhelm Friedrich Hegel** (1770–1831), **Johann Wolfgang von Goethe** (1749–1832), **Heinrich von Kleist**[372] (1777–1811) and **Caspar David Friedrich** (1774–1840).

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[372] His father's uncle, **Ewald Georg Christian Johann von Kleist** (1700–1748, Germany) invented the *Leyden jar* (1745). It is a glass jar, partially filled with water and containing a nail projecting from its cork stopper, basically an early version of an electrical capacitor.

Worldview XVII: Carl F. Gauss

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"Few, but ripe" (his motto)

* * *

If others would but reflect on mathematical truths as deeply and as continuously as I have,, they would make my discoveries.

* * *

I confess that Fermat's Theorem as an isolated proposition has very little interest for me, because I could easily lay down a multitude of such propositions, which one could neither prove nor dispose of.

[A reply to Olbers' attempt in 1816 to entice him to work on Fermat's Theorem.]

* * *

There are problems to whose solution I would attach an infinitely greater importance than to those of mathematics, for example touching ethics, or our relation to God, or concerning our destiny and our future; but their solution lies wholly beyond us and completely outside the province of science.

* * *

I have had my results for a long time: but I do not yet know how I am to arrive at them.

* * *

You know that I write slowly. This is chiefly because I am never satisfied until I have said as much as possible in a few words, and writing briefly takes far more time than writing at length.

* * *

God does arithmetic.

* * *

We must admit that, while number is purely a product of our minds, space has a reality outside our minds, so that we cannot completely prescribe its properties a priori.

Letter to Bessel, 1830

* * *

I mean the word proof not in the sense of the lawyers, who set two half proofs equal to a whole one, but in the sense of a mathematician, where half proof = 0, and it is demanded for proof that every doubt becomes impossible.

* * *

Gauss' Class-Number Conjecture[373] (1801–1983)

Consider Gaussian integers of the form $a + b\sqrt{-N}$, *where* N *is some positive integer other than* 1 *and* (a, b) *are integers or half-integers. The question may arise as which values of* N *result in unique factorization. For* $N = 1, 2, 3$ *we do get it but for* $N = 5$ *we do not, since, for example* $6 = 2 \cdot 3 = (1 + \sqrt{-5})(1 - \sqrt{-5})$. *In Gauss' time,* 9 *values of* N *were known for which the system of numbers* $a + b\sqrt{-N}$ *has a unique factorization. They are* $N_1 = 1, 2, 3, 7, 11, 19, 43, 67, 163$.

Despite considerable efforts by Gauss (1801) and others in the decades that followed, no one was able to find higher values of N_1. *However, in* 1952, *the Swiss mathematician* **Kurt Heegner** *proved that the special number 163 is the largest value of* N *for which the number system* $a + b\sqrt{-N}$ *allows unique factorization! [An independent and different proof was given in* 1967 *by* **Harold Stark** *(U.S.A.) and* **Alan Baker** *(England).]*

We turn to number systems that fail the unique factorization. One can still group them into different classes according to the number of ways there are for factoring numbers in the system into primes in that system. We assign to each such class a figure of merit, called the class number (Gauss, 1801), and denote it by $h(N)$. *Thus* $h(N_1) = 1$ *is given to the above class of values* N_1 *for which unique factorization holds.*

The class number $h(N_2) = 2$ *is assigned to the class where unique factorization just fails:* $N_2 = 5, 6, 10, 13, \ldots$. *To* $h(N_3) = 3$ *corresponds the series* $N_3 = 23, 31, 59, \ldots$ *and to* $h(N_4) = 4$ *holds for* $N_4 = 14, 17, 41, \ldots$ *and so on.*

In Article 303 *of his Disquisitiones Arithmeticae, Gauss described some extensive computations of class numbers, and observed that for each class number* k, *there seemed to be a largest value for* N_k. *Thus,* $N_1 = 163$, $N_2 = 427$, $N_3 = 907$ *are maximal in their respective classes. But Gauss was able neither to confirm that any of these values really was maximal, nor to prove that there always was a largest* N, *though he conjectured that this was, nevertheless, the case.*

The class number problem, which assumes the truth of Gauss' conjecture, is to determine for each class number k *the largest* N *for which* $h(N) = k$. *In*

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[373] To dig deeper, see:

- Stark, N.M., *An Introduction to Number Theory*, The MIT Press, 1987, 344 pp.

1983, ***Don Zaiger*** *(U.S.A.) and* ***Benedict Gross*** *(U.S.A.) announced that they had proven Gauss' conjecture, and the ' class-number conjecture' was finally put to rest. Yet this subject still has a number of interesting sidelines.*

Euler *discovered in* 1772 *that the formula* $f(n) = n^2 + n + 41$ *yields primes for all values of* n *from zero to* 39. *No other quadratic formula has been discovered which produces as many prime numbers. Of the first* 10 *million values, the proportion of primes is about one in three — far greater than for any other quadratic formula.*

It turns out that the roots of $f(n) = 0$ *are* $-\frac{1}{2} \pm \frac{1}{2}\sqrt{-163}$. *Is it a coincidence that the number* 163 *arises again in connection with primes? Yet this is not all: It was found that the number* $e^{\pi\sqrt{163}}$ *differs from an integer by less than* 10^{-12}*, namely*

$$e^{\pi\sqrt{163}} = 262\ 537\ 412\ 640\ 768\ 743\ .\ 999\ 999\ 999\ 999\ 250\ \cdots .$$

Incidentally[374]*, the very simple* $2n^2 + 29$ *found by* ***Legendre*** *in* 1798 *generates* 29 *primes for* $n = 0, 1, 2, \ldots, 28$.

Statistics Comes of Age – The Normal Distribution and the Method of Least Squares (1795–1827)

Statistics is the system of computation that deals with the collection, classification, analysis and interpretation of numerical data. By using the theory of probability it aims at discovering laws that govern complex physical

[374] In 1752, **Christian Goldbach** (1690–1764, Germany) proved that no polynomial with integer coefficients can yield a prime for all integer values of x. The proof of this statement is elementary: Let $g(x) = a_0 + a_1x + a_2x^2 + \cdots + a_kx^k$, with a_i integers and $g(m)$ prime for any integer m. Then, if p is a prime derived from this formula when $x = m$, we have $p = g(m)$. Likewise, let q be another prime such that $q = g(m + np)$. Clearly $q = p + pf(m, n)$ with f integer, and thus q is composite for some integer m. This shows that no such polynomial $g(x)$ can exist.

as well as biological and social systems. As a science, statistics began in Germany in the 18^{th} and 19^{th} centuries when governments used statistics to count their citizens and to collect taxes. Today, statistics helps all sciences to deal with masses of facts and it makes vital contributions to business and industry; advertising, finance, insurance, manufacturing, retailing, and many other fields depend on statistics. It helps politicians plan their campaigns, and the use of statistics forms the basis of public-opinion polls.

I. History[375]

Over the two centuries, from 1700 to 1900, statistics underwent a simultaneous horizontal and vertical development: horizontal in that the method spread among disciplines, from astronomy and geodesy, to psychology, to biology, and to social sciences, being transformed in the process; vertical in that the understanding of the role of probability advanced as the analogy of games of chance *gave way to probability models for* measurements*, leading finally to statistical inference.*

The roots of modern statistics, since 1650, encompassed the following disciplines and scientists:

- *The works of mathematicians on* probability*:* ***P. Fermat*** *(1601–1665),* ***B. Pascal*** *(1623–1662),* ***C. Huygens*** *(1629–1695),* ***Jacob Bernoulli*** *(1654–1705),* ***A. de Moivre*** *(1667–1754),* ***Daniel Bernoulli*** *(1700–1782),* ***T. Bayes*** *(1701–1761),* ***P.S. Laplace*** *(1749–1827),* ***S.D. Poisson*** *(1781–1840).*

- *The works of* astronomers and geodesists *on the solutions of overdetermined set of equations:* ***L. Euler*** *(1707–1783),* ***Tobias Mayer*** *(1723–1762),* ***Ruggiero Boscovich*** *(1711–1787).*

- *The ideas of mathematicians concerned with the* errors of measurement *and* combination of observations*:* ***C.F. Gauss*** *(1777–1855),* ***Legendre*** *(1752–1833).*

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[375] For further reading, see:

- Stigler, S.M., *The History of Statistics*, Harvard University Press, 1986, 410 pp.

- Larsen, R.J. and M.L. Marx, *An Introduction to Mathematical Statistics and its Applications*, Prentice-Hall: Englewood Cliffs, NJ, 1981, 596 pp.

- *The labors of social scientists to extend a calculus of probability to the social sciences:* ***John Graunt*** *(1620–1674),* ***A. Quetelet*** *(1796–1874),* ***A.A. Cournot*** *(1801–1877),* ***Francis Galton*** *(1822–1911),* ***Wilhelm Lexis*** *(1837–1814),* ***F.Y. Edgeworth*** *(1845–1926),* ***Karl Pearson*** *(1857–1936),* ***G.U. Yule*** *(1871–1951),* ***W.S. Gosset*** *(1876–1937),* ***Ronald Fisher*** *(1890–1962).*

Early work in mathematical probability was motivated by problems in the social sciences, annuities, insurance, meteorology, and medicine, but the paradigm for the mathematical development of the field was the analysis of games of chance. Concepts applied there were applied in astronomy. Thus, the consideration of games of chance led to the first mathematical treatment of the quantification of uncertainty.

By the end of the 17^{th} century the mathematics of many simple games of chance was well understood and widely known. ***Fermat***, ***Pascal***, ***Huygens***, ***Leibniz*** and ***Jacob Bernoulli*** *— all had examined the ways in which the mathematics of permutations and combinations could be employed in the enumeration of favorable cases in a variety of games of known properties. These works had been concerned with a priori computations: given an urn known to contain Q white balls and P black balls, the chance of a white ball being drawn is $\frac{Q}{P+Q}$.* ***Jacob Bernoulli*** *(1713) was the first to consider the inverse problem or the a posteriori question: to determine P and Q from observations of the outcomes of a game.*

In his search for a solution to this problem, Bernoulli developed (1713) the theory of the binomial distribution (alias 'Bernoulli's Formula'). The physical picture was that of a sequence of experiments satisfying the following conditions (known as 'Bernoullian trials'):

- *For each experiment, the possible results are classified as either success or failure.*
- *The probability of success is the same for every experiment.*
- *Each trial is independent of all the others.*

Under these conditions, Bernoulli showed (on the basis of the Newtonian binomial theorem) that the probability of exactly k successes in n Bernoullian trials is $C_k^n p^k q^{n-k}$ where $q = 1 - p$ and $C_k^n = \binom{n}{k}$. It then followed that the probability of at least k successes in n trials is $\sum_{s=k}^{n} \binom{n}{s} p^s q^{n-s}$, while the probability of at most k successes in n trials is $\sum_{s=0}^{k} \binom{n}{s} p^s q^{n-s}$.

The next major step was made by **de Moivre**. *In a work that started in 1721 and culminated in 1733, he succeeded in approximating the terms of a binomial expansion and derive what we now call the* normal approximation *to the binomial distribution. In achieving this goal he used his own approximation to* $n!$ *(factorial* n*) ahead of* **Stirling** *(1730). He also recognized that the root mean square deviation is proportional to* $\sqrt{n}$*, and calculated values of the normal integral*[376] $\int_{-a}^{a} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$ *as approximations to the binomial probabilities (e.g. in the case* $p = q = \frac{1}{2}$*)* $P(\frac{1}{2}n - \frac{1}{2}a\sqrt{n} \leq X \leq \frac{1}{2}n + \frac{1}{2}a\sqrt{n})$.

Looking at de Moivre's work from a perspective of 270 years it is easy to appreciate the profound influence it had upon later mathematical developments and the solution of a wide variety of scientific problems. Yet his contemporaries greeted these advances with indifference and missed the potential in this masterful work. No application or extension of these ideas occurred before 1774, the year that **Laplace** *revisited the inverse probability problem of Jacob Bernoulli.*

Here, the analytic superiority of Laplace enabled him to apply probability to statistical inference *for the first time and succeed where Bernoulli and Thomas Bayes had failed.*

In another vein, astronomers and geodesists were struggling with the solution of large sets of overdetermined equations. The outcome of this endeavor was a novel idea that had a profound effect on the theory of statistics.

The method of least squares *was the dominant theme of* 19^{th} *century mathematical statistics. It was to statistics what the calculus had been to mathematics a century earlier. Indeed, disputes on the priority of its discovery signaled the intellectual community's recognition of the method's value. This "calculus of observations", like the calculus of mathematics, did not spring into existence without antecedents, and the exploration of its subtleties and potential took over a century. Throughout much of this time statistical methods were commonly referred to as "the combination of observations".*

The genesis of the method of least squares is anchored in the three major physico-astronomical problems of the 18^{th} *century:*

- *To determine and represent mathematically the motions of the moon.*
- *To account for an apparently nonperiodic (secular) perturbations that had been observed in the motions of the planets Jupiter and Saturn.*

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376 $P(a) = \int_{-a}^{a} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$ is the probability for a measurement to occur in an interval within $a\sigma$ of the *mean*, where σ is the *variance*. In particular $P(0.6745) = 1/2$ is the median deviation from the mean (probable error).

- *To determine the shape (figure) of the earth.*

These problems involved astronomical observations and the theory of gravitational attraction, and they all presented intellectual challenges that engaged the attention of many of the ablest mathematical scientists of the period.

The chain of development that led to the method of least squares began with ***Johann Tobias Mayer*** *(1723–1762, Germany), who in 1747 undertook the study of the complex irregular minor perturbations of the moon's motion.*[377] *The specific work of Mayer that most influenced statistical practice was his study (1750) of the librations of the moon. He made numerous observations of the position of several prominent lunar features and in his memoir he showed how these data could be used to determine various characteristics*

[377] In the 18^{th} century the problem of accurately accounting for these minor perturbations in the moon's movement, either by a mathematical formula or by an empirically determined table describing *future lunar positions*, was of great scientific, commercial and even military significance. Its *scientific* importance lay in the general desire to show that Newtonian gravitational theory can account for the movement of our nearest celestial neighbor if allowance is made for the attraction of other bodies (such as the sun), for periodic changes in the earth's and the moon's orbits, and for the departure from sphericity of the shapes of the earth and the moon. But it was the potential *commercial* and *military* usefulness of a successful accounting of the moon (as an aid to navigation) that prompted the widespread attention the problem received. Over the previous 19 centuries, from **Hipparchos** and **Ptolemy** to **Newton** and **Flamsteed**, the linked development of theoretical and practical astronomy had played a key role in freeing ship's navigation from a dependence upon land sightings as a way of determining the ship's position. The developments of better nautical instruments (including the sextant, 1731) and a more accurate understanding of astronomical theory, increasingly enabled navigators to map their ships' courses across previously trackless seas.

By 1700, it had become possible to determine ship's *latitude* at sea with relative precision by the *fixed stars*, simply by measuring the angular elevation of the celestial pole above the horizon. The determination of *longitude*, however, was not so simple. Indeed, in 1714 England established the "commissioners for the discovery of longitude at sea", a group that by 1815 had disbursed £101,000 in prizes and grants to achieve its goal. The two most promising methods of ascertaining longitude at sea were the development of an *accurate clock* (so that Greenwich time could be maintained on shipboard and longitude determined by the comparison of the fixed stars' positions and Greenwich time) and the creation of *lunar tables* that permitted the determination of Greenwich time (and thus longitude) by *comparison of the moons position and the fixed stars.*

of the moon's orbit: all in all he ended up with an overdetermined *system of 27 equations for the calculations of 3 unknown parameters.*

Mayer divided his equations into three groups of nine equations each, added each of the three groups separately, and solved the resulting three linear equations for their unknowns. He then made a numerical estimate of the accuracy of his empirical determination. This way of combining observations and making an error assessment was remarkable for this time.

Mayer's story of statistical success showed that there was a potential gain to be achieved through the combination of observations. Yet the discovery of the method of least squares was not possible in the intellectual climate of 1750 and certain conceptual barriers had to be crossed before this climate became sufficiently supportive for the later advances.

In a widely read treatise, Astronomie *(1771),* ***Joseph Jerome Lalande*** *presented an extensive discussion of Mayer's work for the specific purpose of explaining how large number of observational equations could be combined to determine unknown quantities. It was this exposition that called the method to the attention of contemporary astronomers.: Boscovich, Laplace and Legendre. After it was decided that the earth was oblate (1735) it remained only to establish the size of the oblateness (ellipticity) because different pairs of arcs gave different results.*

In 1760, ***Ruggiero Giuseppe Boscovich*** *introduced a statistical procedure for resolving measurements of the length of a meridian arc. Boscovich was aware, as others before him had been, that to obtain an accurate determination of the figure of the earth it would be necessary to compose measurements widely separated in latitude, as even small errors made in proximate arc measurements would be greatly exaggerated in any pairwise combination of them.*

He thus focused his attention on only five determinations that were made at well-separated locations and were likely to be accurate. He then calculated the inverse ellipticity *and* polar excess *(the amount by which a degree at the pole exceeds a degree at the equator) for all possible ten pairs. He next focused upon the discrepancy between his* average value *of polar excess and the ten components that made up the average*[378]*, coming forth with two conditions that would lead to best choice of the results:*

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[378] As his average yielded the value of $\frac{1}{155}$ for the ellipticity, Boscovich tried to improve this value by rejecting pairs which looked "different from the others". It finally occurred to him that he must subject this arbitrary rejection of data to certain principles.

(i) Since positive and negative error are equally likely, the sum of positive corrections should be equal to the sum of negative corrections.

(ii) The sum of the corrections, taken without regard of sign, was to be a minimum.

In 1770, Boscovich appended the French translation of his 1760 paper with a geometric description of an algorithm which was based on the above two principles. Through this he calculated the ellipticity as $\frac{1}{230}$*, in close proximity to Newton's own value. Boscovich gave no further development of the method, no analytic formulation, and no application to problems other than the figure of the earth. The method might thus have faded into obscurity had not a brief reference to its existence, in a 1772 review of the 1770 translation, caught the eye of* ***Pierre Simon Laplace****.*

In the course of a memoir on the perturbation of the motions of Saturn and Jupiter (1787), Laplace proposed and extension of Mayer's method of reconciling inconsistent linear equations. In his epochal work, Laplace finally laid to rest what was then a century-old problem by showing that the perturbations were in fact periodic with a very long period.

In 1810, Laplace produced a major result in probability theory, known today as the Central Limit Theorem (CLT)*. Roughly speaking, it states that whenever a random variable* X *may be expressed as a sum of a very large number of independently varying random variables, then the probability density of* X *is approximately normal. Combined with his unrivaled ability to derive asymptotic approximations to integrals, the CLT enabled Laplace to show that quite general sums or averages had distributions well approximated by the* normal curve*.*

Thus, two major avenues of attack on statistical problems were at the disposal of mathematicians in 1810:

- *Legendre's method of least squares (1805) and Laplace's way of combining observations in complex situations (1787).*
- *The probability apparatus developed by de Moivre and Laplace for the analysis of binomial distributions and its limiting case of the normal distribution.*

What was missing was any connection between these two lines of work. In 1809 ***Gauss*** *provided the key, and within two years a remarkable synthesis was achieved.*

Laplace must have encountered Gauss' work soon after April 1810, and it struck him like a bolt. Before seeing Gauss' work Laplace had not seen *any connection between his limit theorem and the method of least squares, but almost immediately afterward he could see how it all fit together: If the errors of Gauss' formulation were themselves random variables, then the limit theorem implied they should be approximately distributed as what would later be called normal, or Gaussian curve. And once Gauss' choice of curve was given a rational basis, the entire development of least squares fell into place, just as Gauss had showed.*

One of the Gauss' most efficient tools in his research was the method of least squares. When he first developed it (1795), he did not consider it very important. Although formal priority belongs to **Legendre** *(1805), it seems that the motivation, deduction, and systematic application of the method of least squares is more interesting than the problem of deciding who happened to discover, use, and publish it first.*

Suppose we try to measure some quantity x *and make* M *measurements* x_i. *We do not see* x *but only measurements* x_i *with errors of measurement* ε_i; *that is, we observe* $x_i = x + \varepsilon_i, \quad i = 1, 2, ..., M$. *We will regard the residuals* ε_i *as "noise" and call* x *the true value, whatever that may mean in a situation in which it cannot be measured directly.*

The principle of least squares states that the best estimate $\hat{x}$ *of* x *is that number which minimizes the sum of the squares of the deviations of the data from their estimate,*

$$f(\hat{x}) = \sum_{i=1}^{M} \hat{\varepsilon}_i^2 = \sum_{i=1}^{M} (x_i - \hat{x})^2. \tag{1}$$

In the final analysis, the usefulness of this principle rests on how useful the results turn out to be in practice and how easy it is to use.

This principle is equivalent to the assumption that the average (sample mean)

$$\bar{x} = \frac{1}{M} \sum_{i=1}^{M} x_i \tag{2}$$

is the best estimate. To prove this equivalence, we first show that the least-squares principle leads to the average. We regard

$$f(\hat{x}) = \sum_{i=1}^{M} (x_i - \hat{x})^2 \tag{3}$$

as a function of $\hat{x}$ *to be minimized. Applying the usual calculus rule* $\frac{df}{d\hat{x}} = 0$, *(3) yields at once* $\hat{x} = \bar{x}$ *as given in (2). Thus* $\bar{x} = \hat{x}$ *minimizes the sum of* *squares of the residuals. We note also that* $\frac{d^2f}{dx^2} = 2M > 0$ *and hence we have an absolute minimum.*

We have now proved that the principle of the least squares and the choice of the average as the best value are equivalent.

The maximum-likelihood estimator *of an unknown parameter is motivated by the Bayes-type notion that we should select the value that maximizes the likelihood of observing it. If the sample is from some probability density* $\varphi(x;\theta)$*, then the likelihood of the value* θ *for* x *is the product of the individual (independent) observations*

$$L(\theta) = \varphi(x_1;\theta)\varphi(x_2;\theta)\cdots\varphi(x_m;\theta). \tag{4}$$

In the case that the errors come from a normal distribution

$$\frac{k}{\sqrt{\pi}}e^{-k^2(x-\theta)^2},$$

then this leads to

$$L(\theta) = \frac{k^M}{\pi^{M/2}}e^{-k^2\sum(x_i-\theta)^2}.$$

When this is maximized θ *clearly is the solution to a least-squares problem, and is the maximum likelihood estimator,*

$$\theta = \frac{1}{M}\sum_{i=1}^{M} x_i = \overline{x}. \tag{5}$$

Thus lest squares can be derived from the normal distribution via the (assumed) maximum-likelihood estimator.

In his decisive papers (1821, 1823)[379] *Gauss defines the function* $\varphi(x)$ *as the relative frequency of errors in the observations* X*. Then* $\varphi(x)dx$ *expresses the probability of the error lying between* x *and* $x + dx$*. The function* φ *is required to fulfill the two conditions:*

$$\int_{-\infty}^{\infty} \varphi(x)dx = 1\ ; \quad \int x^2\varphi(x)dx \quad \text{attains a minimum.}$$

These conditions expresses the idea that the squares of the error *is its most suitable weight. This is where Gauss' approach differs from that of Laplace,*

379 In his several publications Gauss derived the method in substantially different ways. His most mature approach was developed in the two papers "*Theoria combinationis observationum erroribus minimis obnoxiae*", *I and II.*

who earlier tried to use the absolute value of an error *for its weight. This is why Gauss' method is called the method of least squares; computationally, it is clearly superior to Laplace's original method.*

After developing the theoretical basis of his method, a suitable distribution function $\varphi(x)$ had to be found. In general, the distribution of errors will not be known in advance. After some heuristic preparations, Gauss introduced the experimental density[380] $\varphi(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}$ *as a particularly natural law according to which errors of observation occur*[381]*. This law was found to represent the errors of observations in astronomy and other physical sciences remarkably well. Hence its name "Law of Errors". This law occupies a central position in statistical theory.*

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380 This probability density is often called the *Laplacian* density by the French and the *Gaussian* density by the Germans. *Normal* is British usage. Probability theorists and statisticians use 'normal', while physicists and engineers often use 'Gaussian'. The least-squares criterion is widely used, and often believed to be the "right one" to use.

There is a saying that mathematicians believed that it is a physical principle while physicists believe that it is a mathematical principle.

Either we can assume the principle or we can assume some other principles and deduce that of least squares – something must be assumed in any case. There appears to be a widespread belief that the principle of least squares implies the normal law of errors. This belief is false. Another belief sometimes encountered is that the normal law is "a law of nature". Certainly, the normal law has been found in practice to be a useful model in many applications. Deviations from it usually occur from having more values in the "tail" of the distribution (when x is large) than the model indicates there should be. The reason for this is that often there is a small effect which has a wide variability. In such cases, a mixture of two normal distributions with different parameters sometimes is useful. The theory of *quality control* is , in part, based on the observed excesses in the tails.

381 It can be easily shown that if (4) and (5) are *assumed*, then the Gaussian Law follows. To see this, one differentiates (4) logarithmically, uses (5) and solves the ensuing differential equation $\frac{\varphi'(x)}{\varphi(x)} = \lambda x$, where λ is a constant.

Gauss' 'Clock-arithmetic' [382]

Consider a clock, numbered (in unorthodox fashion) with the hours $0, 1, 2, \ldots, 11$. *Such a clock has its own peculiar arithmetic. For example, since three hours after 5 o'clock is 8 o'clock, we could say that* $3 + 5 = 8$, *as usual. But 3 hours after 10 o'clock is 1 o'clock, and 3 hours after 11 o'clock is 2 o'clock; so by the same token,* $3 + 10 = 1$ *and* $3 + 11 = 2$. *Not so standard!*

Nevertheless, this 'clock arithmetic' has a great deal going for it, including almost all of the usual laws of algebra. Following Gauss, we describe it as arithmetic to the modulus *12, and replace '$=$' by the symbol '$\equiv$' as a reminder that some monkey-business is going on. The relation '$\equiv$' is called a* congruence. *In arithmetic* modulo *(that is, to the modulus)* 12, *all multiples of* 12 *are ignored. So* $10 + 3 = 13 \equiv 1$ *since* $13 = 12 + 1$ *and we may set* $12 \equiv 0$.

If a scientist is performing an experiment in which it is necessary for him to keep track of the total number of hours that have elapsed since the start of the experiment, he may label the hours sequentially $1, 2, 3$, *etc. When 41 hours have elapsed, it is 41 o'clock "experimental time." How does he reduce experiment time (e.t.) to ordinary time? If zero hours e.t. corresponded to midnight, his task is easy: he simply divides by* 12 *and the* remainder *is the time of day.* 41 *e.t. is thus 5 o'clock, because* 12 *goes into* 41 *with a remainder of* 5; 41 *is congruent to* 5 *modulo* 12*:*

$$41 \equiv 5 (\text{mod } 12).$$

For the purpose of telling the time of day it is not necessary to know how many *times* 12 *is contained in* 41, *but only the remainder,* 5. *Of course if one wants to distinguish between a.m. and p.m., then it would be better to divide by* 24. *We then find that* 41 *is congruent to* 17 *modulo* 24. *This means that* 41 *e.t. is 17 hours (military time), namely 5 p.m.*

Any other number n may be used as the modulus: now multiples of n are neglected. The resulting arithmetical system consists only of the numbers

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[382] For further reading, see:

- Deskins, W.E., *Abstract Algebra*, Dover: New York, 1955, 624 pp.
- Childs, L.N., *A Concrete Introduction to Higher Algebra*, Springer-Verlag, 1995, 522 pp.
- Littlewood, D.E., *A University Algebra*, Dover Publications: New York, 1970, 324 pp.

$0, 1, 2, \ldots, n-1$, and has its own multiplication and subtraction rules, as well as addition. Division is less pleasant: but if n is a prime then it is possible to divide by any non-zero number. For example, modulo 7 we find that $3/5 = 2$, since $2 \cdot 5 = 10 \equiv 3$. This bizarre arithmetic was introduced by Gauss because it is ideal for handling questions of divisibility. Number theory has used it for this purpose ever since.

Every integer may be expressed as the multiple of a lower integer plus a remainder. The number n can accordingly be expressed as

$$qm + r$$

where r is the remainder when n is divided by m; qm begin a multiple of m. Where n is expressed as a multiple of m, and m is a factor of n, the remainder will obviously be equal to zero.

The number n will not, however, be the only number which gives the remainder r when divided by m, and this is the basic fact upon which the theory of congruences was founded by Gauss.

The numbers 40 and 64, for instance, give the same remainder of 4 when divides by 12. This is another way of saying that both numbers are equal to multiples of 12 added to 4. They both, therefore have something in common in their relationship to the number 12. In mathematical parlance the latter number is called the 'modulus' with respect to which the two other numbers have similar properties, and these two numbers are said to be 'congruent' to each other for that modulus.

Thus, if

$$a = pm + r$$

and

$$b = qm + r$$

then a is said to be congruent to b for the modulus m, and the relationship is expressed in either of the equivalent forms:

$$\left.\begin{array}{c} a \equiv b \pmod{m}\ ; \quad a - b = km\ ; \quad a - b \equiv 0 \pmod{m} \\ a \equiv b \end{array}\right\} \tag{1}$$

where k may be positive, zero, or negative.

For example $10 \equiv 3 \pmod{7}$, and $13 \equiv 28 \pmod{5}$. Also $a \equiv b \pmod{p}$ if $a = b + kp$.

Sometimes the notation is used in a wider sense: the general solution, in radian units, of $\tan\theta = \tan\alpha$ may be written $\theta \equiv \alpha \pmod{\pi}$.

It will be seen that the algebra of congruences is so like that of ordinary integer arithmetic that no inconvenience arises from using the same notation *for both. The explicit statement of the modulus at each stage of the work is not necessary when the same modulus is used throughout.*

The congruence $a \equiv r \pmod{m}, \quad 0 \le r < m$ *means that*

$$a = r + km\ , \quad r = 0, 1, 2, \dots, m-1.$$

The set of all possible m *remainders (including zero) constitute a complete residue system (mod* m*). On the other hand, since* k *is an arbitrary integer (positive or negative), all integers will group into* m *classes, known as residue classes (mod* m*), where each class is characterized by one of the remainders of the residue system* $r = 0, 1, \dots, m-1$*. For a given* r *and* m*, these classes will be*

$$a = km,\ 1 + km,\ 2 + km, \dots, (m-1) + km.$$

No number from one class is congruent to any number from another class.

For example for $m = 2, \quad r = 0, 1$ *there are two classes:*

1st class:	$a = 2k$	$\dots,\ -4,\ -2,\ 0,\ 2,\ 4, \dots$	*(even)*
2nd class:	$a = 1 + 2k$	$\dots,\ -3,\ -1,\ 1,\ 3,\ 5, \dots$	*(odd)*

For $m = 3,\ r = 0, 1, 2$

1st class:	$a = 3k$	$\dots,\ -6,\ -3,\ 0,\ 3,\ 6, \dots$
2nd class:	$a = 1 + 3k$	$\dots,\ -5,\ -2,\ 1,\ 4,\ 7, \dots$
3rd class:	$a = 2 + 3k$	$\dots,\ -4,\ -1,\ 2,\ 5,\ 8, \dots$

Clearly, all numbers in a residue class have the same greatest common divisor with the modulus m*.*

Consider next the case $m = 5, \quad r = 0, 1, 2, 3, 4$ *with the corresponding residue classes*

class	*residue class*
$a = 5k$	$\dots, -10,\ -5,\ \boxed{0},\ 5,\ 10, \dots$
$a = 5k + 1$	$\dots, -9,\ -4,\ \boxed{1},\ 6,\ 11, \dots$
$a = 5k + 2$	$\dots, -8,\ -3,\ \boxed{2},\ 7,\ 12, \dots$
$a = 5k + 3$	$\dots, -7,\ \boxed{-2},\ 3,\ 8,\ 13, \dots$
$a = 5k + 4$	$\dots, -6,\ \boxed{-1},\ 4,\ 9,\ 14, \dots$

Where the squares indicate a particular choice of a set of 5 numbers, no two of which are congruent to each other. This can be done in an infinite number of ways by picking just one number from each class. Such a set will be referred to as a complete residue system. Thus, instead of the canonical system $r = 0, 1, 2, 3, 4$*, we may select* $r = 5, 6, 7, 8, 9$*, or (as above)* $r = 0, 1, 2, -2, -1$*. In general, we shall say that the* m *numbers* $\{a_1, a_2, \dots, a_m\}$ *form a complete* *system of residues if every number is congruent to* a_i *(only one); each residue* a_i *then represents its class in mod-m arithmetic.*

The word 'congruence' can be used synonymously with 'residue class'.

Returning to the basic logic of clock arithmetic, we next consider the operations of addition and multiplication of congruences. One can easily verify the following addition and multiplication tables for modulo 5 *arithmetic.*

Addition mod 5

	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

Multiplication mod 5

	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

The following properties of congruences are obvious from their definition:

- *Symmetry:* when $a \equiv b \pmod{m}$ then $b \equiv a \pmod{m}$.
- *Transitivity:* $a \equiv b \pmod{m}$ and $b \equiv c \pmod{m}$ imply $a \equiv c \pmod{m}$.
- *Residue classes of the same modulus may be added, subtracted, multiplied by an arbitrary integer and multiplied together unambiguously. Namely, if* $a \equiv b \pmod{m}$ *and* $a' \equiv b' \pmod{m}$ *and if* (q, r) *are integers, then*

$$qa + ra' \equiv qb + rb' \pmod{m}$$
$$aa' \equiv bb' \pmod{m}$$
$$a^n \equiv b^n \pmod{m} \qquad [(a^n - b^n) \text{ is always divisible by } a - b]$$

- *If* $a \equiv b \pmod{m}$ $f(a) \equiv f(b) \pmod{m}$ *for polynomials of integer coefficients* $f(x)$.
- *If any polynomial* $f(x)$ *and* $g(x)$ *have congruent coefficients of corresponding powers modulo* m, *and* $a \equiv b \pmod{m}$, *then* $f(a) \equiv g(b) \pmod{m}$.

The notion of congruence began with **Euler** *(1783). In his treatise Disquisitiones Arithmeticae,* **Gauss** *(1801) developed the systematic algebra of congruences, treating congruence polynomials of the n-th degree with a prime modulus. He showed that congruences w.r.t. the same modulus can be treated like ordinary Diophantine equations: they can be added, subtracted and multiplied, and one can ask for a solution of congruences involving unknowns, e.g.*

$$Ax^n + Bx^{n-1} + \cdots Mx + N = 0 \pmod{p},$$

where p is a prime not dividing A. Gauss proved that this equation cannot have more than n noncongruent roots. A famous example is the Fermat's Little Theorem

$$a^{p-1} - 1 \equiv 0 \pmod{p},$$

where p is prime and a is not a multiple of p. Gauss' book was indeed a new way of looking at old things, introducing the concept of residue classes. In his own words:

> *"If a number A divides the difference of two numbers B and C, B and C are called congruent with respect to A, and if not, incongruent. A is called the modulus; each of the numbers B and C are residues of each other in the first case, and non-residues in the second."*

Does it seem strange that Gauss should write a whole book about the implication of $A \mid (B - C)$? It surely is not clear a priori why this group of symbols should be worthy of such protracted attention. In fact, these opening sentences are completely unmotivated and hardly understandable, except in the light of historical perspective. But in that light, the time was ripe for such an investigation. Gauss may not have been aware of the underlying structure of the works of Fermat and Euler that evolved from their preoccupation with perfect and Mersenne numbers. But his interest in periodic decimals called for a new notation and new notions to handle an algebra of ambiguity and an arithmetic of remainders.

The power of the congruence algebra is manifested in the following simple examples:

I. *Since $(2n+1)^2 = 4n(n+1) + 1$, every even power of any odd number is congruent to $1 \pmod{8}$. This modulus partitions the integers into 8 residue classes, all the odd numbers being congruent to one of $1, 3, 5, 7$. If we square these four possible congruences, we find that every odd square is congruent to one of the numbers $1, 9, 25,$ or $49 \pmod{8}$. But these all happen to be $\equiv 1 \pmod{8}$. Raising the original residue to an arbitrary even power will not change this congruence.*

II. *A number is divisible by* 3 *or* 9 *if and only if the sum of its digits is divisible by* 3 *or* 9. *For let*

$$N = a_0 + a_1 \cdot 10 + a_2 \cdot 10^2 + \cdots + a_n \cdot 10^n.$$

Multiplying the congruence $10 \equiv 1 \pmod{3 \text{ or } 9}$ *repeatedly, one obtains* $10^k \equiv 1 \pmod{3 \text{ or } 9}$ *and therefore* $a_k \cdot 10^k \equiv a_k$ *for* $k = 1, 2, \ldots, n$. *Then* $N \equiv a_0 + a_1 + a_2 + \cdots + a_n \pmod{3 \text{ or } 9}$. *This means that* N *is divisible by* 3 *or* 9 *iff the sum of its digits is so divisible.*

In a similar way the congruences $10 \equiv -1 \pmod{11}$, $10^k \equiv (-1)^k \pmod{11}$ *yield* $N \equiv a_0 - a_1 + a_2 + \cdots \pmod{11}$. *Hence an integer is divisible by* 11 *iff the sum of its digits with alternating signs is divisible by* 11.

III. *Prove that* 999, 999 *is divisible by* 7*:*

$$\begin{gathered} 999,999 = 10^6 - 1 \\ 10 \equiv 3 \pmod{7} \\ 10^6 \equiv 3^6 \pmod{7} \equiv (3^2)^3 \pmod{7} \equiv 9^3 \pmod{7} \\ 9 \equiv 2 \pmod{7} \quad \therefore 9^3 \equiv 2^3 \pmod{7} \equiv 1 \pmod{7} \\ \therefore 10^6 \equiv 1 \pmod{7} \quad \therefore 10^6 - 1 \equiv 0 \pmod{7}. \end{gathered}$$

IV. *Prove that* $2^{11} - 1$ *has* 23 *as one of its factors; The steps are:*

$$\begin{gathered} 2^5 = 32 \equiv 9 \pmod{23} \\ 2^{10} \equiv 81 \pmod{23} \equiv 12 \pmod{23} \\ 2 \equiv 2 \pmod{23} \\ 2^{11} = 2^{10} \times 2 \equiv 12 \times 2 \pmod{23} \equiv 1 \pmod{23} \\ \textit{Therefore} \quad 2^{11} - 1 \equiv 0 \pmod{23}. \end{gathered}$$

V. *Prove that* $3^{4n+2} + 5^{2n+1}$ *is divisible by* 14*:*

$$\begin{gathered} 3^{4n+2} = 9 \cdot 81^n \equiv 9 \cdot [11 \pmod{14}]^n \equiv 9 \cdot 11^n \pmod{14} \\ 5^{2n+1} = 5 \cdot 25^n \equiv 5 \cdot [11 \pmod{14}]^n \equiv 5 \cdot 11^n \pmod{14} \\ \therefore 3^{4n+2} + 5^{2n+1} \equiv 14 \cdot 11^n \equiv 0 \pmod{14}. \end{gathered}$$

VI. *Find the remainder when* 2^{1000} *is divided by* 13.

$$\begin{gathered} 2^3 = 8; \quad 2^6 = 64 \equiv -1 \pmod{13}; \quad \textit{But since } 1000 = 6 \cdot 166 + 4 \\ \textit{and} \quad 2^{996} = (2^6)^{166} \equiv (-1)^{166} \pmod{13} \equiv +1 \pmod{13}, \ \textit{we have:} \\ 2^{1000} \equiv 2^4 \pmod{13} \equiv 16 \pmod{13} = 3 \pmod{13}. \end{gathered}$$

The remainder is therefore 3.

VII. *What are the last two digits of* 3^{1234}? *In* mod 100 *arithmetic*

$$\begin{aligned}
&3^2 \equiv 9\ ;\ 3^4 \equiv 81\\
&3^8 \equiv 81^2 \equiv 61\\
&3^{10} \equiv 9 \cdot 61 \equiv 49\\
&3^{20} \equiv 49^2 \equiv 1\\
&1234 = 20 \times 61 + 4 + 10\\
&3^{1234} \equiv (3^{20})^{61} 3^4 3^{10} \equiv 81 \cdot 49 \equiv 69 \pmod{100}
\end{aligned}$$

The last two digits are seen to be 69.

VIII. *Show that* $A = 2903^n - 803^n - 464^n + 261^n$ *is divisible by* 1897 *for any natural number* n.

Write

$$A = (2903^n - 464^n) - (803^n - 261^n).$$

The first group is divisible by $2903 - 464 = 9 \cdot 271$ *and the second by* $803 - 261 = 2 \cdot 271$, *so* A *is divisible by* 271 *But we can also write*

$$A = (2903^n - 803^n) - (464^n - 261^n),$$

where the first group is divisible by $2903 - 803 = 7 \cdot 300$ *and the second by* $464 - 261 = 7 \cdot 29$, *so that* A *is also divisible by* 7. *Since* 271 *is not divisible by the prime* 7, A *is divisible by the product* $271 \cdot 7 = 1897$.

IX. *Prove that the* 5^{th} *Fermat number* $F_5 = 2^{2^5} + 1 = 2^{32} + 1$ *is divisible by* 641 *(Euler's claim, 1732):*

$$\begin{gathered}
640 = 5 \cdot 128 = 5 \cdot 2^7 \equiv -1 \pmod{641}\\
5^4 \cdot 2^{28} = (5 \cdot 2^7)^4 \equiv (-1)^4 \equiv 1 \pmod{641}\\
5^4 = 625 \equiv -16 \pmod{641} \equiv (-2^4) \pmod{641}\\
\therefore \quad -(2^4) \cdot 2^{28} \equiv 1 \pmod{641}\\
-(2^{32}) \equiv 1 \pmod{641}\\
2^{32} \equiv -1 \pmod{641}
\end{gathered}$$

$$2^{32} + 1 \equiv 0 \pmod{641}$$

Alternatively,

$$F_5 = 2^{2^5} + 1 = 2^{32} + 1 \equiv (5^4 \cdot 2^{28} + 2^{32}) - (5^4 \cdot 2^{28} - 1)$$

But

$$\begin{aligned} 5^4 \cdot 2^{28} + 2^{32} &= 2^{28}(5^4 + 2^4) = 641 \cdot 2^{28} \\ 5^4 \cdot 2^{28} - 1 &= (5^2 \cdot 2^{14} + 1)(5^2 \cdot 2^{14} - 1) \\ &= (5^2 \cdot 2^{14} + 1)(5 \cdot 2^7 - 1)(5 \cdot 2^7 + 1) \\ &= 641 \cdot (5^2 \cdot 2^{14} + 1)(5 \cdot 2^7 - 1) \end{aligned}$$

Application - Calendar problems

How does one find the relation between dates and days of the week in the Gregorian calendar?

According to this calendar, the common year consists of 365 days and each leap year of 366 days. Leap years are the years for which the number is divisible by 4*, except the centurial years, which are leap years only if divisible by* 400*. Thus, the first centurial leap year after the reformation of the calendar, which occurred in the catholic countries in 1582, was 1600, but 1700, 1800, 1900 were common years; the next centurial leap year was 2000, and so on.*

It is easy to determine the number of leap years between 1600 exclusive and a given year N inclusive. The number of years divisible by 4 *in the assumed interval is the same as the number of integers x such that*

$$400 < x \leq \frac{N}{4};$$

that is,

$$\left[\frac{N}{4}\right] - 400.$$

But from this we must exclude the number of centurial years not divisible by 400*. The number of all centurial years between 1600 exclusive and N inclusive is*

$$\left[\frac{N}{100}\right] - 16,$$

and among them there are

$$\left[\frac{N}{400}\right] - 4$$

divisible by 400*. Consequently the number of centurial years which are not leap years is*

$$\left[\frac{N}{100}\right] - \left[\frac{N}{400}\right] - 12$$

and the requested number of all leap years between 1600 exclusive and N inclusive is thus

$$T = \left[\frac{N}{4}\right] - \left[\frac{N}{100}\right] + \left[\frac{N}{400}\right] - 388.$$

This expression can be put into more convenient form by setting

$$N = 100C + D$$

where C is the century number and $D < 100$. Then

$$\left[\frac{N}{4}\right] = 25C + \left[\frac{D}{4}\right]; \qquad \left[\frac{N}{100}\right] = C; \qquad \left[\frac{N}{400}\right] = \left[\frac{C}{4}\right]$$

and

$$T = \left[\frac{D}{4}\right] + \left[\frac{C}{4}\right] + 24C - 388.$$

Since in a leap year an additional day is added at the end of February, it is convenient to proceed as if the years begin in March. Then March, April, May, ... will be counted as the first, second, third, ... months of the year N, while January and February of the same year will be considered as the eleventh and twelfth months of the year $N - 1$. It will also be convenient to denote days of the week beginning with Sunday by 0, 1, 2, ..., 6.

Now suppose that the first of March of the year 1600 had the weakday number a. Since the next year 1601 was a common year, 365 days elapsed between March 1, 1600, and March 1, 1601. But 365 days consist of 52 full weeks and 1 day; hence March 1, 1601, had the number $a + 1$ or this number diminished by 7.

Again, since the years 1602 and 1603 were common years, March 1, 1602, and March 1, 1603, had the numbers $a + 2$ and $a + 3$ or these numbers diminished by a proper multiple of 7. *Between March 1, 1603, and March 1, 1604, since 1604 was a leap year, 366 days or 52 weeks and 2 days elapsed; hence the number of March 1, 1604, was $a + 5$ or the least positive residue of it modulo* 7.

It is now clear that every common year elapsing augments the number of March 1 modulo 7 *by one unit and every leap year by two units. Hence, to find the number of March 1 in the year N, we have to add to a the number of all years between 1600 exclusive and N inclusive and also the number of leap years in the same interval, and to reduce the sum to its least positive residue* mod 7. *Thus March 1 of the year N will have the number a' determined by the congruence*

$$a' \equiv a + 100C + D - 1600 + \left[\frac{D}{4}\right] + \left[\frac{C}{4}\right] + 24C - 388 \pmod 7$$

or

$$a' \equiv a + D + \left[\frac{D}{4}\right] + \left[\frac{C}{4}\right] - 2C \pmod 7.$$

For the year 1938, March 1 was on Tuesday, so $a' = 2$; again for the same year

$$D = 38, \quad C = 19; \quad D + \left[\frac{D}{4}\right] + \left[\frac{C}{4}\right] - 2C \equiv 6 \pmod 7,$$

whence

$$2 \equiv a + 6 \pmod 7, \quad a = 3.$$

That is, March 1, 1600, was a Wednesday, and the preceding expression for a' becomes

$$a' \equiv 3 + D + \left[\frac{D}{4}\right] + \left[\frac{C}{4}\right] - 2C \pmod 7.$$

This congruence determines the day of the week on which March 1 falls in every year after the Gregorian reform.

Origins of the Vector Concept (1589–1831)

The foundations of vector analysis were laid in the 1840*'s. The deep roots of this concept were latent in various forms, in the works of* 10 *men:*

*(1) Parallelograms of velocities and forces (**Galileo**, 1589).*

*(2) 'Geometry of situations' (**Leibniz**, 1693).*

*(3) Geometrical representation of complex numbers (1797–1831). **Caspar Wessel** (1745–1818, Norwegian Surveyor, 1797), **Carl Friedrich Gauss** (1777–1855, Germany, ca* 1797*, first published in 1831), **Abbé Buée** (1805, France), **Jean Robert Argand** (1768–1822, Switzerland, 1806), **John Warren** (1828, England), **C.V. Mourey** (1828, France).*

(4) 'Sensed magnitudes' first to be employed systematically in projective geometry by ***Lazare Carnot*** *(1803) and later by* ***August Ferdinand Möbius*** *(1827).*

Wessel *was the first to render a clear exposition on the subject in his paper 'On the Analytical Representation of Directions', which he read before the Royal Academy of Science and Letters of Denmark. It contained a complete development of laws governing operations with directed line segments as representation of numbers in the form* $a + b\sqrt{-1}$ *and their applications, as well as a partial theory of rotation.*

Wessel was born in Jonsrud, Norway, to a family blessed with 13 *children. In* 1763 *he went to Copenhagen, and in the following year he was engaged by the Danish Academy of Sciences as an assistant in the preparation of a map of Denmark. He remained in the employment of the Academy until* 1805.

It speaks well for the Academy that they received Wessel's paper sympathetically, since he was neither a member nor was he considered a mathematician. Written in Danish (in Volume 5 *of the Memoirs of the Academy, 1799), it failed to achieve wide accessibility to the mathematicians of other countries — with the result that this excellent and significant work did not become generally known until a French translation of it was published in 1897!*

1795–1805 CE **Mungo Park** (1771–1806, Scotland). African explorer. Explored the course of the Niger River. Born on a farm in Selkirkshire, Scotland, the seventh son in a family of thirteen. In 1791 he obtained a surgical diploma at the University of Edinburgh. Through his connections with **Joseph Banks**, president of the Royal Society, he was sent in 1795 by the African Association with a small expedition to ascertain the course of the River Niger. He ascended the Gambia River, crossed Senegal, followed the course of Niger (1795 to 1796), was captured by an Arab chief, and escaped after four months of imprisonment. In 1797 he reached England, where he had been given up for dead, by way of America. An account of his adventures by his own pen appeared in 1799 (*Travels in the Interior of Africa*). He then married (1799) and settled in Peebles, where he worked as a country doctor. The hardness and monotony of life at Peebles impelled him to accept the government's offer to lead another expedition to the Niger (1805). Park and his European colleagues were either killed or drowned in an encounter with forces of the King of Haoussa, 800 km south of the Niger delta (1806).

1796 CE **Edward Jenner** (1749–1823, England). Physician and discoverer of *vaccination*[383]. Laid the foundation of modern immunology. It was introduced by him as a preventive measure against smallpox. The success of the smallpox vaccine led to the search for vaccines to prevent other serious diseases. Before his time, no mother counted her children safe until all had passed through smallpox.

Smallpox (*Variola major*) replaced the plague as the foremost epidemic disease. The first reasonably effective method of control in the early 18^{th} century was *variolation* — inserting pus from a smallpox pustule into a scratch on someone unaffected. In 1768, the '*inoculator*' **Thomas Dimsdale** (1712–1800) treated the Russian Empress Catherine the Great, her son and her court, and was rewarded with a considerable fee, pension and the rank of a baron. Unfortunately, variolation could sometimes lead to a fatal attack or fail to give protection, and all who underwent it became infectious and had the potential to spread the disease.

It was common knowledge in Jenner's time that a person could catch smallpox only once. Many people tried to inoculate themselves with matter

[383] From the Latin *vacca* = cow, since it referred to the injection of cowpox virus to prevent smallpox. In general, *vaccination* is the introduction of dead or weakened viruses or bacteria, or their *toxins* (poisons) into the body to develop resistance to disease. The material introduced is called a *vaccine*. The vaccine causes the body to manufacture substances called *antibodies* which fight the effects of bacteria, toxins and viruses. Vaccines must be strong enough to excite resistance, but too weak to cause serious illness.

from smallpox sores. They hoped to catch a light case of the disease and then be immune for it for the rest of their lives.

Mary Wortley Montagu (1689–1762), an English author, had introduced the practice of inoculation in 1717 from Turkey. She had her own children inoculated, but encountered a vast amount of prejudice against this procedure. The method was, indeed, unsafe.

Jenner was born at Berkeley, Gloucestershire. In 1770 he went to London to study medicine and in 1792 he obtained the degree of doctor of medicine from St. Andrews College. In 1796, Jenner took matter from the hand of Sarah Nelms, a Berkeley dairymaid[384] who caught the cowpox disease while milking the cows. Jenner made two cuts on the arm of James Phipps, a healthy 8-years-old boy, and infected the matter from one of Sarah's sores. The boy then caught cowpox. Six weeks later, Jenner risked his medical reputation by introducing variolous matter into the boy's arm. Ordinarily fatal, the smallpox matter had no effect.

Subsequently the method proved routinely successful[385], and honors began to shower on him from abroad[386]: He was elected a member of almost all the chief scientific societies on the continent of Europe. In his own country his merits were less recognized; In 1813 the University of Oxford conferred on him the degree of M.D. but the college of physicians would not admit him until he had undergone an examination in classics! To which Jenner replied: "*To brush up my classics — I would not do it for a diadem*". He continued to vaccinate gratuitously all the poor who applied to him, so that he sometimes had as many as 300 persons waiting at his door. Only in 1858 was a statue of him erected by public subscription in London.

With Jenner's vaccination, smallpox could be controlled, and by 1975, it had been eradicated.

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[384] Cowpox is a minor disease in humans, that causes a few sores on the hands, but carries little danger of disfiguration and death. People believed that dairymaids who had caught cowpox could not catch smallpox.

[385] An early advocate of vaccination was the physician **Jacob Ezekiel Aronsson** (1759–1845) of Alsace-Lorraine.

[386] In 1796, **Catherine the Great**, the Empress of Russia, caused the first child operated upon to receive the name **Vaccinov**, and to be educated at the public expense. On one occasion, when Jenner was endeavoring to obtain a release of some of the unfortunate Englishmen who had been detained in France on the sudden termination of the Peace of Amiens (1803), **Napoleon** was about to reject the petition, when Josephine uttered the name of Jenner. The Emperor paused and exclaimed: "*Ah, we can refuse nothing to that name*".

1796–1815 CE *The Napoleonic Wars*:

- 1796–1797 *The Italian campaign*: Napoleon defeated the Austrians and the Piedmontese in a series of 8 battles (Millesimo, Mondovi, Ladi, Castiglione, Roverdo, Bassano, Arcola and Rivoli).
- 1798–1799 *The Egyptian expedition*: Napoleon wins the land-battles of the Pyramides (against the Mamluks) and Abukir (against the British and the Turks), but looses the sea-battle of the Nile.
- 1798–1799 *War of the second coalition* (Britain, Russia, Austria, Naples, Portugal and Ottoman Empire): The French are driven out of Italy in a series of five battles (Magnano, Cassano, Zürich, Trebbia and Novi).
- 1800 *Battles of Marengo and Hohenlinden*: France defeats Austria.
- 1805 *Battle of Ulm*: France defeats Austria.
  Battle of Trafalgar: British navy under Nelson defeats the combined French and Spanish fleets.
  Battle of Austerlitz: France defeats the combined armies of Austria and Russia.
- 1806 *Battles of Jena and Auerstädt*: France defeats Prussia.
- 1807 *Battle of Friedland*: France defeats Russia.
- 1808–1814 *The Peninsular War* of the British against the French in Portugal and Spain. A series of 4 battles (Vimerio, Corunna, Talavera and Ocana).
- 1809 *Battle of Aspern and Essling* and *Battle of Wagram* in which Napoleon crushed the Austrians.
- 1812 *Battle of Borodino*: Russian retreated and abandoned Moscow. The retreating French army fought at *Jaroslavetz* and *Viazma* against Kutuzov.
- 1813 *Battle of Dresden*: Napoleon defeated the allied army of Prussia, Russia and Austria.
  Battle of Leipzig: Napoleon is driven out of Germany.
- 1813–1814 Napoleon is driven out of Spain. The allies enter Paris and Napoleon is exiled to Elba.
- 1815 *Battle of Waterloo*.

Impact of Social Revolutions (1775–1814)

From 1775 *to* 1783 *Britain's North American colonies, with a population of well over 2 million, broke away from rule by the mother country. The thirteen ex-colonies formed the United States of America* [Connecticut, Delaware, Georgia, Massachusetts, Maryland, North Carolina, New Hampshire, New Jersey, New York, Pennsylvania, Rhode Island, South Carolina, Virginia]. *In 1778 France entered the American War of Independence in support of the colonists.*

The success of the American Revolution had profound effects on Europe, and eventually on other parts of the world. It passed on to the western world the ideas of constitutional government and popular sovereignty.

Encouraged by the American example, the French Revolution (1789–1799) not only transformed the government of France, but also shook the establishment throughout Europe and led to new ideas that remained influential throughout the next century.

France was the hive of advanced ideas. A widespread rebellion had broken out against the tyranny of the clergy and the monarchy which finally culminated in the French Revolution. It did more to spread political ideas than philosophical ones. Following the U.S. and French example, other parts of America fought to free themselves from Europe and Europe from the Pope. These ideas liberated the world from the yokes of religion and with this liberation came the flowering of new sciences.

Indeed, for the next two centuries, nobody was able to think of the era of the sciences without referring to the French Revolution, when the scientists quite plainly took power. An astronomer was Mayor of Paris, the inventor of topology was at the head of the Committee for Public Health, the scholars occupied the institutions before the people did and in their place, and a geometrician, although a minor one, gained the title of Emperor. The nobility and the clergy collapsed, society no longer lived according to the same divisions or the same offices and scientists at last formed a class, replacing the clerics and forming a new "Church".

Under the influence of the social ferment, the movement of ideas and beliefs which had been dominant until this time – the Enlightenment, *with its emphasis on reason and natural law – gave way to the Romantic movement in the arts, which favored emotion before reason and espoused* free individual expression.

The French revolution marked the coming of a modern world — a world of class conflict, middle-class ascendancy, acute national conflicts and popular democracy. Together with industrialization, *the revolution reshaped the institutions, the societies and even the mentalities of the European peoples.*

In France itself, the revolution stimulated the rapid growth of science at the turn of the 18*th century. The scientists of France found their activities directed towards practical ends, which appears to have given them a greater taste for experimentation than they previously had. Simultaneously, scientific institutions were established which trained the French talent that was to dominate the cutting edge of science during the early years of the* 19*th century.*

The first practical problem which the revolutionaries posed to the scientists of France was the standardization of weights and measures throughout the country. During the 18*th century weights and measures varied in France from region to region. The meter, for example, measured* 100 *centimeters in Paris, was* 98 *cm at Marseilles,* 102 *cm at Lille and* 96 *cm at Bordeaux. By* 1799 *'the astounding and scandalous diversity in measures' was brought to an end.*

In 1794 *the National Convention founded the École Polytechnique and the École Normale Supériere, which were important institutions devoted to scientific education and research in France throughout the* 19*th century. The Supériere was closed down after* 4 *months, and did not become important until* 1808*, when it was reopened by Napoleon Bonaparte (1769–1821). The Polytechnique, however, flourished from the start. It opened in* 1794 *with* 400 *pupils and a staff composed of the leading scientists of the time: mathematical physics were taught by* ***Laplace*** *and* ***Lagrange****, geometry by* ***Monge*** *and chemistry by* ***Berthollet****. Amongst their students were* ***Poncelet****,* ***Poisson****,* ***Cauchy****,* ***Carnot****,* ***Gay-Lussac****,* ***Dulong*** *(1785–1838) and* ***Petit*** *(1791–1820).*

Napoleon himself encouraged the practical side of science by offering prizes for useful discoveries. He also discouraged the speculative thinkers who continued the tradition of the earlier materialist philosophers. Thus French science became more practical and experimental during the Napoleonic period.

A marked anti-scientific movement arose in the official and fashionable circles in France with the restoration of the Bourbons in 1815*. The movement was particularly opposed to the mathematical tradition of French science. But the scientific institutions set up by the National Convention in* 1794 *had the effect of concentrating the scientific activity of France in the capital, at the Paris schools. During the* 19*th century the Polytechnique and the Supériere became the Mecca of young French scientists from the provinces as well as from the metropolis.*

1796 CE At the age of 19, **Carl Friedrich Gauss** conceived the first proof that a 17-sided regular polygon is constructible by means of a compass and a ruler: The first mathematician to thus go beyond the ancient Greeks.

Cyclotomic Equations and the Roots of Unity – de Moivre to Gauss (1730–1801)

During the 36 centuries that elapsed from he Old Babylonian period to the end of the 19^{th} century, algebra was the science of solution of equations.

The solution of algebraic equations occupied the minds of the finest mathematicians of Europe in the 17^{th}, 18^{th} and 19^{th} centuries and demanded the combined efforts of men like **de Moivre** *(1730),* **Euler** *(1749),* **Lagrange** *(1770),* **Vandermonde** *(1771),* **Gauss** *(1801),* **Ruffini** *(1810),* **Abel** *(1824) and* **Galois** *(1831).*

1. Historical overview

Archaeological research in the 20^{th} century has revealed that the people of Mesopotamia around 1700 BCE had an advanced mathematical culture, including a knowledge of the Pythagorean theorem *(a millennium before* Pythagoras*), a* sexagesimal system of arithmetic *and a method of solution of* quadratic equations.

There was only modest progress in algebra in the 3000 years that followed; During the Middle Ages, Europe had learned about algebra from the Arabs and had begun to improve it by devising new symbols and notations. Then, in the 16^{th} century, the algebraic solution of cubic equations was discovered (1515), and closely thereafter the solution of quartic equations (1544).

It was not until almost 300 years later that it was shown – first by ***Abel*** *(1824), then by* **Galois** *(1831) – that it is impossible to solve the quintic equation in the same manner that the quadratic, cubic and quartic were solved;*

specifically, by using a finite number of additions, subtractions, multiplications, divisions, and the extractions of roots.

However, certain classes of quintic (and higher order) equations can *be solved in this manner. Thus, any algebraic equation can be associated with a Galois group, which may be the* symmetric (permutation) group S_n, metacyclic group M_n, dihedral group D_n, alternating group A_n, *or the* cyclic group C_n. *Solvability of a quintic is then predicated on its corresponding group being a* solvable group. *An example of a quintic equation with a solvable cyclic group is*

$$x^5 - x^4 - 4x^3 + 3x^2 + 3x - 1 = 0$$

which arises in the computation of $\sin\left(\frac{2\pi}{11}\right)$.

2. Roots of unity

In the 18th *century, the problem of solving the* n^{th} *degree equation centered on the special case* $x^n = 1$, *called the* binomial equation. **Roger Cotes** *(1714) and* **de Moivre** *(1707, 1730) showed, through the use of complex numbers, that the solution of this problem amounts to the division of the circle into* n *equal parts; hence the alternative name* cyclotomic equation. *To obtain the roots of this equation by radicals (trigonometric solutions are not necessarily thus expressible) it is sufficient to solve the case of* n *an odd prime* p. *Indeed, assume this: Then if* $n = pm$, *let* $y = x^m$. *But* $y^p - 1$ *is solvable. By assumption, and for each such* $y = y_j$, $x^m = y_j$ *can be solved if* m *is either prime or, if not,* m *can be decomposed in the same manner that* n *is.*

To solve $x^n = 1$ *we write*

$$x^n = 1 = \cos 2k\pi + i \sin 2k\pi, \qquad k = 0, 1, \ldots, n-1 \tag{1}$$

Then, using de Moivre's theorem, we have

$$x_k = (\cos 2k\pi + i \sin 2k\pi)^{\frac{1}{n}} = \cos\left(\frac{2k\pi}{n}\right) + i \sin\left(\frac{2k\pi}{n}\right), \tag{2}$$

which renders all the n^{th} *roots of unity when* k *sweeps the range* $k = 0, 1, \ldots, n-1$. *Denoting*

$$R = \cos\frac{2\pi}{n} + i \sin\frac{2\pi}{n}, \tag{3}$$

we use again de Moivres' theorem to obtain

$$R^k = \cos\frac{2k\pi}{n} + i \sin\frac{2k\pi}{n} \qquad (k \text{ an integer}). \tag{4}$$

Comparing with (2) we see that the n^{th} roots of unity are powers of R. The n distinct roots of unity are

$$R, R^2, R^3, \ldots, R^{n-1}, R^n = 1. \tag{5}$$

Since the absolute value (modulus) of R^k is 1, the points representing the nth roots of unity are equally spaced on the circumference of the unit circle. Joining these points by straight line segments, a regular polygon of n sides is formed. The possibility of construction of such regular polygons with the use of straightedge and compass alone is discussed next.

An nth root of unity which is not also a pth root (for some prime $p < n$) is called a primitive root*. The number R defined by (3) is a primitive nth root of unity.*

Of the numbers (5) the primitive nth roots are those whose exponents are prime to n. To see this we consider the root R^s in (5) $(s < n)$, namely

$$R^s = \cos\frac{2s\pi}{n} + i\sin\frac{2s\pi}{n}.$$

Suppose s and n are not relatively prime. Let k be the g.c.d. of s and n; Then $n = ka$, $s = kb$; and $1 < k < n$ (all lower case Latin letters but i represent natural numbers here, unless stated otherwise). We have

$$(R^s)^a = (R^n)^b = 1^b = 1.$$

Since $a < n$, then R^s is not a primitive n^{th} root of unity by virtue of the definition of a primitive n^{th} root.

But if s and n have no common factor other than unity, then $(R^s)^r \neq 1$ for r a positive integer less than n, since for

$$(R^s)^r = \cos\frac{2rs\pi}{n} + i\sin\frac{2rs\pi}{n}$$

to be unity, $\frac{rs}{n}$ must be an integer. However s is by hypothesis prime to n. Therefore $\frac{r}{n}$ must be an integer; yet this is impossible, since $r < n$.

The primitive n^{th} roots of unity are

$n = 3$	ω, ω^2	$\omega = -\frac{1}{2} + \frac{1}{2}i\sqrt{3}$
$n = 4$	$i, -i$	
$n = 5$	R, R^2, R^3, R^4	$R = \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}$
$n = 6$	R, R^5	$R = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$
$n = 8$	R, R^3, R^5, R^7	$R = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$

If n is a prime number, all the complex roots are primitive and given by (5). In general, when one primitive nth root is known, all the other are obtainable.

The properties of the roots of the binomial equation are summarized in the following six theorems:

- *If α is a complex root of $x^n - 1 = 0$, then α^m, m integer, is also a root.*
  Proof: $\alpha^n = 1 \therefore (\alpha^n)^m = 1$ *or* $(\alpha^m)^n = 1$; *Therefore* α^m *is a root of* $x^n - 1 = 0$.

- *If m and n are prime to each other, the equations $x^m - 1 = 0$, $x^n - 1 = 0$ have no common root except unity.*
  Proof: Let α be a common root $\alpha^m = 1$, $\alpha^n = 1 \therefore \alpha^{(mb-na)} = 1$ for integers a and b. But since $(m, n) = 1$ there exist integers a, b such that $mb - na = 1$, so $\alpha = 1$ and 1 is the only common root of the given equations.

- *If k is the greatest common divisor of two integers m and n, the roots common to the equations $x^m - 1 = 0$ and $x^n - 1 = 0$ are roots of the equation $x^k - 1 = 0$.*
  Proof: $m = km'$, $n = kn'$, $(m', n') = 1$. So integers a, b can be found such that $m'b - n'a = 1 \therefore mb - na = k$. Thus if α is a common root then $\alpha^{mb-na} = 1$ and also $\alpha^k = 1$. This proves that α is a root of $x^k = 1$.

- *When n is a prime number, and α any complex root of $x^n - 1 = 0$, all the roots are included in the series,*

$$1, \alpha, \alpha^2, \ldots, \alpha^{n-1} .$$

  Proof: By our first theorem, these entities are all roots of $x^n - 1$, and by the second they are all different.

- *When n is a composite number formed of the factors $p, q, r, \ldots$, the roots of the equations $x^p - 1 = 0$, $x^q - 1 = 0$, $x^r - 1 = 0$ etc., all satisfy the equation $x^n - 1 = 0$.*
  Proof: Let α be a root of $x^p - 1 = 0$; then $\alpha^p = 1$. Then

$$(\alpha^p)^{qr\cdots} = 1 \quad \text{or} \quad \alpha^n - 1 = 0.$$

3. Solution by radicals

(a) $n = 3, 4, 5, 6, 8, 10, 12$

The most important result of the previous section is that the primitive n^{th} roots of unity are

$$e^{2k\pi i/n} = \cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n} \tag{6}$$

where k runs over the positive integers which are less than n and relatively prime to n. In particular, when n is prime, then every n^{th} root of unity except 1 is primitive.

Now, Eq. (6) renders the roots of unity in a trigonometric form. Yet, one of the basic problems in the theory of algebraic solution of equations is to give algebraic solutions of the cyclometric equation $x^n = 1$.

In the case $n = 3$, **Lagrange** *(1771) showed how this can be elegantly achieved: starting from*

$$x^3 + qx + p = 0 \tag{7}$$

he substituted $x = y - \frac{q}{3y}$, obtaining the 6^{th} degree equation

$$y^6 + py^3 - \frac{q^3}{27} = 0.$$

Putting $r = y^3$, one derives the quadratic equation $r^2 + pr - \frac{q^3}{27} = 0$ with the explicit solutions $r_{1,2} = \frac{1}{2}\left[-p \pm \sqrt{p^2 + \frac{4}{27}q^3}\right]$. *It then remains to solve*

$$y^3 - r = 0;$$

Clearly

$$y = \sqrt[3]{r_j}; \qquad \omega\sqrt[3]{r_j}; \qquad \omega^2\sqrt[3]{r_j}; \qquad j = 1, 2; \qquad r_1 r_2 = -\frac{q^3}{27},$$

where

$$\omega^3 = 1, \qquad 1 + \omega + \omega^2 = 0; \qquad \omega = \frac{1}{2}(-1 + \sqrt{-3}),$$

and therefore

$$\begin{aligned} x_0 &= \sqrt[3]{r_1} + \sqrt[3]{r_2} \\ x_1 &= \omega\sqrt[3]{r_1} + \omega^2\sqrt[3]{r_2} \\ x_2 &= \omega^2\sqrt[3]{r_1} + \omega\sqrt[3]{r_2} \end{aligned} \tag{8}$$

This solution exhibits the link between the solutions of the general cubic and the cyclotomic equation of degree 3, $\omega^3 = 1$. *The solution of the latter is an immediate result of the factorization*

$$\omega^3 - 1 = (\omega - 1)(\omega^2 + \omega + 1) = 0 \tag{9}$$

and so

$$\begin{aligned} x_1 &= \cos\tfrac{2\pi}{3} + i\sin\tfrac{2\pi}{3} = -\tfrac{1}{2} + \tfrac{i}{2}\sqrt{3} = \omega \\ x_2 &= \cos\tfrac{2\pi}{3} + i\sin\tfrac{2\pi}{3} = -\tfrac{1}{2} - \tfrac{i}{2}\sqrt{3} = \omega^2 \end{aligned} \tag{10}$$

In the case $n = 4$, *the algebraic solution is*

$$x^4 - 1 = (x^2 - 1)(x^2 + 1) = 0,$$

resulting in

$$x_1 = 1, \quad x_2 = -1; \quad x_2 = i, \quad x_4 = -i.$$

The points $(1, 0)$; $(-1, 0)$; $(0, i)$; $(0, -i)$ *then divide the unit circle into four equal parts.*

In the case $n = 5$

$$x^5 - 1 = (x - 1)(x^4 + x^3 + x^2 + x + 1) = 0 \tag{11}$$

To solve Eq. (11), *one uses the trick discovered by* **de Moivre** *(1707) to solve general reciprocal equations,* namely the substitution $x + \frac{1}{x} = u$. Then, since

$$\begin{aligned} x^2 + \frac{1}{x^2} &= u^2 - 2, \\ x^3 + \frac{1}{x^3} &= u^3 - 3u, \\ x^4 + \frac{1}{x^4} &= u^4 - 4u^2 + 2, \\ x^5 + \frac{1}{x^5} &= u^5 - 5u^3 + 5u, \quad \textit{etc.}, \end{aligned} \tag{12}$$

the quartic reciprocal equation yields,

$$\begin{aligned} x^4 + x^3 + x^2 + x + 1 \quad &= x^2\left[(x^2 + \frac{1}{x^2}) + (x + \frac{1}{x}) + 1\right] \\ &= x^2(u^2 + u - 1) = 0. \end{aligned}$$

It then remains to solve the pair of quadratic equations:

$$u^2 + u - 1 = 0; \qquad x^2 - ux + 1 = 0, \tag{13}$$

yielding

$$\begin{aligned}
x_1 &= \tfrac{1}{4}[(\sqrt{5}-1)+i\sqrt{10+2\sqrt{5}}] = \cos\tfrac{2\pi}{5}+i\sin\tfrac{2\pi}{5} = e^{\frac{2\pi i}{5}}\\
x_2 &= \tfrac{1}{4}[-(\sqrt{5}+1)+i\sqrt{10-2\sqrt{5}}] = \cos\tfrac{4\pi}{5}+i\sin\tfrac{4\pi}{5} = e^{\frac{4\pi i}{5}}\\
x_3 &= \tfrac{1}{4}[-(\sqrt{5}+1)-i\sqrt{10-2\sqrt{5}}] = \cos\tfrac{6\pi}{5}+i\sin\tfrac{6\pi}{5} = e^{\frac{6\pi i}{5}}\\
x_4 &= \tfrac{1}{4}[(\sqrt{5}-1)-i\sqrt{10+2\sqrt{5}}] = \cos\tfrac{8\pi}{5}+i\sin\tfrac{8\pi}{5} = e^{\frac{8\pi i}{5}}
\end{aligned} \tag{14}$$

The very same result could have been obtained through the factorization

$$x^4+x^3+x^2+x+1=(x^2+Ax+1)(x^2-\frac{1}{A}x+1)=0 \tag{15}$$

where

$$A=\frac{\sqrt{5}+1}{2}, \qquad \frac{1}{A}=\frac{\sqrt{5}-1}{2}.$$

Note that since $x_1=\cos\frac{2\pi}{5}+i\sin\frac{2\pi}{5}$, *it follows that*

$$u_1=x_1+\frac{1}{x_1}=e^{\frac{2\pi i}{5}}+e^{\frac{-2\pi i}{5}}=2\cos\frac{2\pi}{5}=\frac{1}{2}(\sqrt{5}-1).$$

The geometrical interpretation of this result renders the key to the construction of a regular pentagon *inscribed in a unit circle: one draws a unit circle centered at* O *and two perpendicular diameters* AA' *and* BB'. *Let the midpoint of* OA' *be* C. *Draw an arc with* C *as center and* CB *as a radius, cutting* OA *at* D. *Then, if* S_n *represents a side of a regular polygon of* n *sides,* $S_{10}=OD$ *and* $S_5=BD$.

Apart from $x^5=1$, *there is a whole class of quintic equations solvable by radicals. Furthermore, certain quintics have solutions expressible in terms of the fifth roots of unity. Thus the equation*

$$x^5-5ax^3+5a^2x-2b=0, \tag{16}$$

sometimes known as de Moivre's quintic, *has the explicit simple solutions*

$$x_k=\epsilon^k u_1+\epsilon^{4k}u_2 \qquad k=0,1,2,3,4, \qquad \epsilon=e^{\frac{2\pi i}{5}}. \tag{17}$$

To solve for u_j, *set* $k=0$ *and substitute* $x=u_1+u_2$ *in (16) and obtain*

$$u_1^5+u_2^5+5(u_1+u_2)(u_1^2+u_1u_2+u_2^2-a)(u_1u_2-a)-2b=0.$$

Letting $u_1^5+u_2^5=2b$ *and* $u_1^5u_2^5=a^5$, *the solution is*

$$u_1=\sqrt[5]{b+\sqrt{b^2-a^5}}\,; \qquad u_2=\sqrt[5]{b-\sqrt{b^2-a^5}}\,. \tag{18}$$

It can be shown[387] *that if a and b are rational numbers such that the quintic trinomial x^5+ax+b is irreducible over the rationals, then the equation $x^5+ax+b=0$ is solvable by radicals iff there exist a sign $\theta(=\pm 1)$, and reals $c(\geq 0)$ and $e(\neq 0)$ such that $a=\frac{5e^4(3-4\theta c)}{c^2+1}$, $b=\frac{-4e^5(11\theta+2c)}{c^2+1}$, in which case the roots of $x^5+ax+b=0$ are*

$$\begin{gathered}
x_k = e\left[\epsilon^k u_1+\epsilon^{2k}u_2+\epsilon^{3k}u_3+\epsilon^{4k}u_4\right], \qquad k=0,1,2,3,4 \\
u_1=\left(\frac{v_1^2 v_3}{D^2}\right)^{1/5}, \quad u_2=\left(\frac{v_3^2 v_4}{D^2}\right)^{1/5}, \\
u_3=\left(\frac{v_2^2 v_1}{D^2}\right)^{1/5}, \quad u_4=\left(\frac{v_4^2 v_2}{D^2}\right)^{1/5} \\
v_1=\sqrt{D}+\sqrt{D-\theta\sqrt{D}} \qquad v_2=-\sqrt{D}-\sqrt{D+\theta\sqrt{D}} \\
v_3=-\sqrt{D}+\sqrt{D+\theta\sqrt{D}} \qquad v_4=\sqrt{D}-\sqrt{D-\theta\sqrt{D}} \\
D=1+c^2
\end{gathered} \tag{19}$$

For $a=0$, $b=-1$, we fall back on (14).

In the case $n=6$

$$x^6-1=(x^3-1)(x^3+1)=(x-1)(x+1)(x^2+x+1)(x^2-x+1)=0. \tag{20}$$

So, the only roots of $x^6-1=0$ which are not roots of lower order equations are those of $x^2-x+1=0$, namely $\alpha=\frac{1+i\sqrt{3}}{2}$, $\beta=\frac{1-i\sqrt{3}}{2}$ with the additional provisions $\alpha\beta=1=\alpha^6$ or $\beta=\alpha^5$. Therefore, the primitive roots of $x^6-1=0$ are

$$\alpha,\alpha^5 \qquad \text{or} \qquad \beta^5,\beta \quad \text{or} \quad \alpha,\frac{1}{\alpha}.$$

In the case $n=8$

$$x^8-1=(x^4-1)(x^4+1)=(x^2-1)(x^2+1)(x^2-x\sqrt{2}+1)(x^2+x\sqrt{2}+1)=0. \tag{21}$$

[387] B.K. Spearman and K.S. Williams Am. Math. Monthly **101**, 986–992, 1994.

The eight roots are

$$x_k = \cos\frac{k\pi}{4} + \sin\frac{k\pi}{4} \qquad k = 0, 1, \ldots, 7,$$

which correspond explicitly to the series

$$1, \quad -1, \quad i, \quad -i, \quad \frac{1+i}{\sqrt{2}}, \quad \frac{1-i}{\sqrt{2}}, \quad \frac{-1+i}{\sqrt{2}}, \quad \frac{-1-i}{\sqrt{2}}.$$

The case $n = 10$, yields

$$\begin{aligned} x^{10} - 1 \quad &= (x^5 - 1)(x^5 + 1) = (x^5 - 1)(x + 1)(x^4 - x^3 + x^2 - x + 1) \\ &= (x^5 - 1)(x + 1)(x^2 + ax + 1)(x^2 + bx + 1) \end{aligned} \tag{22}$$

with

$$a = \frac{\sqrt{5} - 1}{2}, \qquad b = -\frac{\sqrt{5} + 1}{2}.$$

There are four primitive roots, corresponding to the roots of the quadratic equations $x^2 + ax + 1 = 0$ *and* $x^2 + bx + 1 = 0$. *The division of the circle into* 10 *equal parts is then feasible with a compass and a ruler.*

The case $n = 12$, likewise, lends itself to the factorization

$$\begin{aligned} x^{12} - 1 = \quad & (x^6 - 1)(x^6 + 1) \\ & x^6 - 1 = (x - 1)(x + 1)(x^2 + x + 1)(x^2 - x + 1) \\ & x^6 + 1 = (x^4 - x^2 + 1)(x^2 + 1) \end{aligned} \tag{23}$$

Clearly, there are only 4 *primitive roots corresponding to the roots of*

$$x^4 - x^2 + 1 = 0 \qquad \therefore x^2 = \frac{1}{2}(1 \pm \sqrt{-3}),$$

namely

$$x_{1,2,3,4} = \pm\sqrt{\frac{1}{2}\left(1 \pm \sqrt{-3}\right)}.$$

But since

$$\frac{1 \pm \sqrt{-3}}{2} = \left(\frac{\sqrt{3} \pm i}{2}\right)^2, \qquad x_{1,2,3,4} = \pm\frac{\sqrt{3} \pm i}{2},$$

(the two signs are independent), the 4 primitive roots are:

$$\alpha = \frac{\sqrt{3}+i}{2}\,, \quad \frac{1}{\alpha} = \frac{\sqrt{3}-i}{2}\,; \quad \alpha_1 = \frac{-\sqrt{3}+i}{2}\,, \quad \frac{1}{\alpha_1} = \frac{-\sqrt{3}-i}{2}\,.$$

Division of the circle into 12 equal parts is enabled simply because $12 = 2 \cdot 2 \cdot 3$. For the same reason, the division of the circle into $16 = 2^4$ equal parts is possible, and also into $15 = 3 \cdot 5$ parts, because $\frac{2\pi}{15} = \frac{\pi}{3} - \frac{\pi}{5}$.

(b) $n = 7, 9, 11$

The Greeks, as well as succeeding mathematicians, tried in vain to construct a regular heptagon. It was especially frustrating since regular polygons of $3, 4, 5, 6, 8, 10$ sides could be constructed with a compass and a ruler.

The source of the difficulty becomes obvious once we try to solve

$$x^7 - 1 = (x-1)(x^6 + x^5 + x^4 + x^3 + x^2 + x + 1) = 0. \tag{24}$$

Applying the substitution $y = x + \frac{1}{x}$ and using (12), we end up solving a pair of equations:

$$y^3 + y^2 - 2y - 1 = 0\,, \qquad x^2 - xy + 1 = 0\,. \tag{25}$$

The three solutions of the cubic are obtained via the irreducible Cardano solution

$$\begin{aligned} y_1 &= 2\cos\tfrac{2\pi}{7} = u + v - \tfrac{1}{3} \\ y_2 &= 2\cos\tfrac{4\pi}{7} = \omega u + \omega^2 v - \tfrac{1}{3} \\ y_3 &= 2\cos\tfrac{6\pi}{7} = \omega^2 u + \omega v - \tfrac{1}{3} \end{aligned} \tag{26}$$

$$u = \tfrac{1}{3}\sqrt[3]{\tfrac{7}{2}}\,\left[1 + 3i\sqrt{3}\right]^{1/3}, \qquad v = \tfrac{1}{3}\sqrt[3]{\tfrac{7}{2}}\,\left[1 - 3i\sqrt{3}\right]^{1/3}$$
$$\omega = -\tfrac{1}{2} + \tfrac{\sqrt{3}}{2}i, \qquad \omega^2 = -\tfrac{1}{2} - \tfrac{\sqrt{3}}{2}i$$

It follows from (26) that

$$2\cos\frac{2\pi}{7} = \frac{1}{3}\sqrt[3]{\frac{7}{2}}\left[(1+3i\sqrt{3})^{1/3} + (1-3i\sqrt{3})^{1/3}\right] - \frac{1}{3}\,. \tag{27}$$

Since the separation of the real and imaginary parts of $(1 \pm 3i\sqrt{3})^{1/3}$ *leads back to the cubic in (25), it is impossible to express* $\cos\frac{2\pi}{7}$ *by an algebraic expression involving real radicals of any kind. Indeed, it can be rigorously proved that an irreducible cubic equation with rational coefficients is not solvable by real roots. This explains why it is not possible to construct a regular polygon with seven sides with a compass and a ruler.*[388]

The same fate befalls the case $n = 9$*, leading to the cyclotomic equation*

$$x^9 - 1 = (x^3 - 1)(x^6 + x^3 + 1) = 0. \tag{28}$$

The trigonometric solution is

$$x_k = \cos\frac{2k\pi}{9} + i\sin\frac{2k\pi}{9}\,, \qquad k = 0, 1, \ldots, 8\,,$$

and yields $x_1 = \cos 40^\circ + i\sin 40^\circ$*. The corresponding algebraic solution is* $x_1 = \frac{1}{\sqrt[3]{2}}(-1 + i\sqrt{3})^{1/3}$*. This implies that*

$$\cos 40^\circ = \frac{1}{2\sqrt[3]{2}}\left\{(-1+i\sqrt{3})^{1/3} + (-1-i\sqrt{3})^{1/3}\right\}. \tag{29}$$

However, the complex–algebra process indicated on the r.h.s. of this equation leads again to a cubic equation, the solution of which is $2\cos 40^\circ$*. It is thus impossible to express* $2\cos 40^\circ$ *by an expression involving real radicals of any kind.*

Note that here again $x^6 + x^3 + 1 = x^3(y^3 - 3y + 1)$ *with* $y = x + \frac{1}{x}$*, where* $y^3 - 4y + 1 = 0$ *is an irreducible cubic. Ergo – a regular polygon of* 9 *sides cannot be constructed with a compass and a ruler.*

Until 1771, no one knew how to solve by radicals the equation $x^n - 1 = 0$ *for* $n > 10$*. A paper by* ***Alexandre-Théophile Vandermonde*** *(1735–1796), in which he solved (1775) the case* $n = 11$ *was regarded as an important advance. In his paper, Vandermonde had pinpointed (without himself being aware of that!*[389]*) the very basic idea of Galois theory (1831), namely, that in*

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[388] **Archimedes**, however, devised a method for trisecting an angle using *a pair* of compasses and a ruler with two marks on it which enables a most ingenious method for constructing a regular heptagon using the same instruments.

[389] On this missed opportunity **Lebesgue** said:

order to determine the 'structure' of an equation, deciding eventually whether is solvable by radicals, one has to look at the permutations of the roots; but one needs only consider those permutations which preserve the relations between the roots.

Vandermonde started from the factorization

$$x^{11} - 1 = (x-1)(x^{10} + x^9 + \cdots + x + 1) = 0. \qquad (30)$$

Substituting $u = -(x + \frac{1}{x})$ and using (12), the solution of (30) lead him to the irreducible quintic (solvable!)

$$u^5 - u^4 - 4u^3 + 3u^2 + 3u - 1 = 0, \qquad (31)$$

which he set to solve by radicals over the field of complex numbers. His novel idea was to generalize the solutions of the cubic and the quartic equations [see (8)] by the introduction of the auxiliary entity, known today as the Lagrange resolvent:

$$t_2 = a + \alpha b + \alpha^2 c + \alpha^3 d + \alpha^4 e. \qquad (32)$$

Here $(1, \alpha, \alpha^2, \alpha^3, \alpha^4)$ are the primitive roots of $x^5 = 1$, [given explicitly in Eq. (14)] and (a, b, c, d, e) are the solutions of (31), yet undetermined.

Now, in (32) we effect the permutation

$$a \to b \to d \to c \to e \to a \qquad (33)$$

and consequently define

$$\left.\begin{aligned} t &= a + b + c + d + e \\ t_1 &= a + \alpha b + \alpha^2 c + \alpha^3 d + \alpha^4 e \\ t_2 &= b + \alpha d + \alpha^2 e + \alpha^3 c + \alpha^4 a \\ t_3 &= d + \alpha c + \alpha^2 a + \alpha^3 e + \alpha^4 b \\ t_4 &= c + \alpha e + \alpha^2 b + \alpha^3 a + \alpha^4 d \end{aligned}\right\} \qquad (34)$$

"Surely, any man who discovers something truly important is left behind by his own discovery; he himself hardly understands it, and only by pondering over it for a long time. But Vandermonde never came back to his algebraic investigations because he did not realize their importance in the first place, and if he did not understand them afterwards, it is precisely because he did not reflect deeply on them; he was interested in everything, he was busy with everything; he was not able to go slowly to the bottom of anything. To assess exactly what Vandermonde saw, understood and what he did not catch, one would have to reconstruct not only the mind of a man from the eighteenth century, but Vandermonde's mind, and at the moment when he had a glimpse of genius and went ahead of his age."

Since

$$x_k = \cos\frac{2\pi k}{11} + i\sin\frac{2\pi k}{11} = e^{\frac{2\pi k i}{11}}\ , \qquad k = 1, 2, \dots, 10$$

we have

$$\begin{aligned} u_1 &\equiv a = -2\cos\tfrac{2\pi}{11} \\ u_2 &\equiv b = -2\cos\tfrac{4\pi}{11} \\ u_3 &\equiv c = -2\cos\tfrac{6\pi}{11} \\ u_4 &\equiv d = -2\cos\tfrac{8\pi}{11} \\ u_5 &\equiv e = -2\cos\tfrac{10\pi}{11} \end{aligned} \tag{35}$$

Vandermonde then used the trigonometric formula

$$2\cos\theta_1\cos\theta_2 = \cos(\theta_1+\theta_2) + \cos(\theta_1-\theta_2) \tag{36}$$

to obtain relations between a, b, c, d *and* e*. For instance, substituting* $\frac{2\pi}{11}$ *for* θ_1 *and* θ_2*, one obtains* $2\cos^2\frac{2\pi}{11} = \cos\frac{4\pi}{11} + \cos 0$*, leading to the relation* $a^2 = -b+2$*. Likewise, substituting* $\frac{2\pi}{11}$ *for* θ_1 *and* $\frac{4\pi}{11}$ *for* θ_2 *one finds that* $ab = -c-a$*. Altogether one can easily verify the relations*

$$\begin{array}{llll} a^2 = -b+2 & ab = -a-c & bc = -a-e & cd = -a-d \\ b^2 = -d+2 & ac = -b-d & bd = -b-e & ce = -b-c \\ c^2 = -e+2 & ad = -c-e & be = -c-d & \\ d^2 = -c+2 & ae = -d-e & & de = -a-b \\ e^2 = -a+2 & & & \end{array} \tag{37}$$

Note that the relations in (37) are preserved under the permutation (33).

Next, Vandermonde created the algebraic expressions for the fifth powers $t^5, t_1^5, t_2^5, t_3^5, t_4^5$*. Using (37) repeatedly and the known relations between the roots of (31) and its coefficients, namely*

$$\begin{aligned} &a+b+c+d+e = 1 \\ &abcde = 1 \\ &abcd + bcde + cdea + deab + eabc = 3 \\ &abc + abd + cda + cdb + eab + ecd + eac + ead + ebc + ebd = -3 \\ &ab + bc + cd + de + ea + ac + ce + eb + ad + bd = -4, \end{aligned} \tag{38}$$

it is possible to greatly simplify the algebraic expressions for the fifth powers [in fact $t = 1$*, by (31)]. It is thus finally shown that fifth root of* t_j^5*,* $j = 1, 2, 3, 4$*, are linear combinations in the roots* a, b, c, d, e*. It then follows, after some algebra, that*

$$a = \frac{1}{5}\left[1 + \sqrt[5]{t_1^5} + \sqrt[5]{t_2^5} + \sqrt[5]{t_3^5} + \sqrt[5]{t_4^5}\right] \tag{39}$$

Using the above simplifications for t_j^5, we can express them in terms of the coefficients of the quintic in (31). We then find

$$\begin{aligned}
(t_1)^5 &= \tfrac{11}{4}\left[(89 + 25\sqrt{5}) + i(45\sqrt{5-2\sqrt{5}} - 5\sqrt{5+2\sqrt{5}})\right]\\
(t_2)^5 &= \tfrac{11}{4}\left[(89 + 25\sqrt{5}) - i(45\sqrt{5-2\sqrt{5}} - 5\sqrt{5+2\sqrt{5}})\right]\\
(t_3)^5 &= \tfrac{11}{4}\left[(89 - 25\sqrt{5}) - i(45\sqrt{5+2\sqrt{5}} + 5\sqrt{5-2\sqrt{5}})\right]\\
(t_4)^5 &= \tfrac{11}{4}\left[(89 - 25\sqrt{5}) + i(45\sqrt{5+2\sqrt{5}} + 5\sqrt{5-2\sqrt{5}})\right]
\end{aligned} \tag{40}$$

Eq. (39) for the first root is augmented with similar ones for the roots b, c, d, e. Note that since (t_1^5, t_2^5) and (t_3^5, t_4^5) are complex conjugate in pairs, the expression for a in (39) is real, as it should be.

Note also that to obtain the solutions x_k for $x^{11} - 1 = 0$, one must yet solve the quadratic equation

$$x_k^2 + x_k u + 1 = 0$$

for $u = a, b, c, d, e$. Thus for $k = 1$:

$$x_1 = -\frac{a}{2} + i\sqrt{1 - \left(\frac{a}{2}\right)^2} = \cos\frac{2\pi}{11} + i\sin\frac{2\pi}{11} \tag{41}$$

etc.

(c) Gauss and $n = 17$ (heptadecagon)

Apart from the three classical outstanding ancient Greece non-construction problems (squaring the circle, trisection of an angle and the doubling of the cube), Greek geometers also focused their interest on several other problems, among them the construction of regular polygons *and platonic bodies (regular polyhedra).*

The number of regular polygons which can be constructed in 2-dimensional space is unlimited. The number of regular convex polyhedra in a space of 3 dimensions is five. The Pythagoreans, who were interested in such matters, regarded the dodecahedron *as begin worthy of special respect. By extending the sides of one of its pentagonal faces to form a star, they arrived of the* pentagram, *or* triple triangle, *which they used as a symbol and badge of the Society* *of Pythagoras. By this sign they recognized a fellow member. The construction of the pentagon*[390] *and the pentagram discovered by Pythagoreans (and given by Euclid), is directly based on the Golden Section ratio* $\frac{\sqrt{5}+1}{2}$ *and on the formulas:*

$$p_r = \textit{side of the regular pentagon} = \frac{R}{2}\sqrt{10-2\sqrt{5}}$$

$$p_s = \textit{side of the star-pentagon} = \frac{R}{2}\sqrt{10+2\sqrt{5}}$$

$$\frac{p_s}{p_r} = \frac{\sqrt{5}+1}{2} = \frac{\textit{diagonal of regular pentagon}}{\textit{side of a regular pentagon}}$$

where R is the radius of the circumscribed circle.

The construction of the regular n-gons with $n = 3, 4, 5, 6, 8, 10, 12, 15, 16$ *were known to the mathematicians of ancient Greece.*[391]

It was not until 1796 that any further constructions of regular polygons were discovered. In that year, Gauss, a student of mathematics at Göttingen who had just turned 19, proved that it is possible to construct the regular 17-gon with ruler and compass.

In the 7^{th} and last section of Disquisitiones arithmeticae (1801), Gauss turns to the general problem of constructing regular polygons by compass and ruler[392]*. The geometric entities that are constructible from known data by means of compass and ruler correspond algebraically to those expressions that*

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390 No pentagon or decagon is to be found in *Egyptian* monuments, although it is easy enough to divide a circle into 5 equal parts *without* geometrical consciousness of any kind. Pentagonal ornaments occur in *Mycenaean* art, prisms of heptagonal shape were found in Babylon, and regular dodecahedron of *Etruscan* and *Celtic* origins were discovered. Thus, elaborate geometrical ornaments can be drawn without explicit geometry.

391 The case $n = 15$ is interesting: knowing how to construct geometrically the angle for the equilateral triangle (120°) and the regular pentagon (72°), the angle for the regular 15-gon (24°) is half the difference angle $\left[\frac{120^\circ - 72^\circ}{2} = 24^\circ\right]$. It can also be constructed as the sum $\frac{60^\circ}{4} + \frac{72^\circ}{8} = 15^\circ + 9^\circ = 24^\circ$.

392 It is assumed that each of these two instruments be used only for a single, specific operation: with the compass, circles with *given* center and circumference-point can be drawn; with the ruler, a straight line can be drawn through two *given* points. Thus, marking on the ruler cannot be utilized. Any construction that can be performed by compass and ruler can be made by compass alone, and, also, if a fixed circle has been drawn, the construction may be achieved by ruler alone!

Since the algebraic calculation of the intersection points of a circle and a straight *may be deduced from given numbers by repeated use of the four rational operation and square root extraction. Thus, in principle, construction problems are transliterated into questions in the theory of equations.*

Therefore, to decide on the possibility of solving a construction problem, one must examine first whether the quantity to be found satisfies an algebraic equation that is rational in the given quantities, and second whether this equation has a constructible solution, i.e. whether it is solvable by square roots.

There may be several such equations, but among them there is one of minimal degree, which cannot be factored further with rational coefficient (known as irreducible), and it divides all other equations of the same kind. For this minimal equation to be solvable by square roots it must have very special properties. One of these is that its degree must be a power of 2. Indeed, the unsolvable problem of the duplication of the cube leads to the cubic equation $x^3 - 2 = 0$. *Similarly, the trisection of any angle* α *leads to the cubic equation* $4x^3 - 3x - \cos\alpha = 0$ *and, in general, one cannot decompose this equation further into factors whose coefficients depend rationally on* $\cos\alpha$.

A regular polygon with n *sides has its vertices equidistant on a circle (say of unit radius, without loss of generality). Since each side of the polygon corresponds to a central angle of* $\frac{360^\circ}{n}$, *the problem is to divide a full angle of* 360° *into* n *equal parts. Now, any angle can be bisected, so when a regular polygon with* n *sides has been obtained, one can successively construct a polygon with* $2^m n$ *sides. On the other hand, from a polygon with* $2n$ *sides, one can draw one with* n *sides by joining every second vertex by a side. Consequently, one can limit the considerations only to regular polygons with an odd number of sides. From the fact that regular polygons with* $3, 4, 5$ *sides can be easily constructed, it follows that all polygons with* 2^m, $3 \cdot 2^m$, $5 \cdot 2^m$ *sides are constructible. Furthermore, given the constructability of polygons with sides* a *and* b, *where* a *and* b *are relatively prime, a polygon with* ab *sides is obtainable. Clearly, the side of a regular* n*-polygon inscribed in unit circle with* n *sides is* $s_n = \sqrt{2 - 2\cos\frac{2\pi}{n}}$.

All this was known before Gauss. But instead of dealing with these quantities directly, Gauss took a step that at the time was innovative: he used

line, or of a circle with another circle leads to a second degree equation, the coordinates are obtained as the sum of a rational expression and the *square root* of such an expression. The distance between two points is also expressible as a square root. Since all other allowed constructions can be composed of a series of these simple operations, those magnitudes that can be constructed from given ones may be computed algebraically by repeated operations of the four arithmetic operations and by extracting square roots (and vice versa).

the unit circle in the complex plane. In the circle he inscribed a regular polygon with n sides such that one vertex lies on the positive real axis at the point $x = 1$. The next vertex will correspond to the complex number $R = R_1 = \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n}$ and the subsequent ones to $R_k = \cos\frac{2\pi k}{n} + i\sin\frac{2\pi k}{n}$, $k = 1, 2, 3, \dots, n-1$ with $R_k = R^k$ by the theorem of de Moivre. This means that R as well as its powers are roots of the algebraic equation $x^n - 1 = 0$.

Gauss then proved that a necessary and sufficient condition that a regular polygon with n sides could be inscribed in a circle (constructed) by compass and ruler is that

$$n = 2^m p_1 p_2 \dots p_n,$$

where the prime factors are also Fermat numbers $p_k = 2^{2^k} + 1$.

Since $17 = 2^{2^2} + 1$, a regular polygon of 17 sides can be inscribed in a circle by ruler and compass. The possibility of this construction is proved if we show that $\cos\frac{2\pi}{17}$ can be constructed.

Starting from the equation $x^{17} - 1 = 0$, we arrive at

$$x^{16} + x^{15} + x^{14} + \cdots + x^2 + x + 1 = 0,$$

with

$$R = \cos\frac{2\pi}{17} + i\sin\frac{2\pi}{17},$$

Gauss arranged the roots in the order

$$R, R^3, R^9, R^{10}, R^{13}, R^5, R^{15}, R^{11}, R^{16}, R^{14}, R^8, R^7, R^4, R^{12}, R^2, R^6,$$

each of which is the cube of the preceding, the first being the cube of the last.

Set

$$y_1 = R + R^9 + R^{13} + R^{15} + R^{16} + R^8 + R^4 + R^2$$
$$y_2 = R^3 + R^{10} + R^5 + R^{11} + R^{14} + R^7 + R^{12} + R^6.$$

Then $y_1 + y_2 = -1$, $y_1 y_2 = -4$, and each y satisfies the equation $y^2 + y - 4 = 0$ whose roots are

$$y = \frac{\pm\sqrt{17} - 1}{2}.$$

But

$$y_1 = (R + R^{16}) + (R^2 + R^{15}) + (R^4 + R^{13}) + (R^8 + R^9)$$
$$= 2\cos\frac{2\pi}{17} + 2\cos\frac{4\pi}{17} + 2\cos\frac{8\pi}{17} + 2\cos\frac{16\pi}{17} > 0$$

$$\therefore \quad y_1 = \frac{\sqrt{17}-1}{2}, \qquad y_2 = \frac{-\sqrt{17}-1}{2}\,. \tag{42}$$

Now set

$$z_1 = R + R^{13} + R^{16} + R^4 = 2\cos\frac{2\pi}{17} + 2\cos\frac{8\pi}{17} > 0$$
$$z_2 = R^9 + R^{15} + R^8 + R^2 = 2\cos\frac{4\pi}{17} + 2\cos\frac{16\pi}{17} < 0.$$

Then $z_1 + z_2 = y_1, \quad z_1 z_2 = -1,$ *and each* z *satisfies the equation* $z^2 - y_1 z - 1 = 0,$ *whose roots are*

$$z_1 = \frac{\sqrt{17}-1}{4} + \frac{\sqrt{34-2\sqrt{17}}}{4};$$
$$z_2 = \frac{\sqrt{17}-1}{4} - \frac{\sqrt{34-2\sqrt{17}}}{4}. \tag{43}$$

Now set

$$w_1 = R^3 + R^5 + R^{14} + R^{12} = 2\cos\frac{6\pi}{17} + 2\cos\frac{10\pi}{17} > 0$$
$$w_2 = R^{10} + R^{11} + R^7 + R^6 = 2\cos\frac{12\pi}{17} + 2\cos\frac{14\pi}{17} < 0.$$

Then $w_1 + w_2 = y_2, \quad w_1 w_2 = -1,$ *and each* w *satisfies the equation*

$$w^2 - y_2 w - 1 = 0,$$

whose roots are

$$w_1 = \frac{-\sqrt{17}-1}{4} + \frac{\sqrt{34+2\sqrt{17}}}{4};$$
$$w_2 = \frac{-\sqrt{17}-1}{4} - \frac{\sqrt{34+2\sqrt{17}}}{4}. \tag{44}$$

Now set

$$u_1 = R + R^{16} = 2\cos\frac{2\pi}{17}, \qquad u_2 = R^4 + R^{13} = 2\cos\frac{8\pi}{17}.$$

Then

$$u_1 > u_2 > 0, \qquad u_1 + u_2 = z_1, \qquad u_1 u_2 = w_1.$$

Whence each u *satisfies the equation* $u^2 - z_1 u + w_1 = 0$ *whose roots are*

$$u_1 = \frac{z_1 + \sqrt{z_1^2 - 4w_1}}{2} = 2\cos\frac{2\pi}{17}, \quad u_2 = \frac{z_1 - \sqrt{z_1^2 - 4w_1}}{2}, \tag{45}$$

and we see that $u_1/2 = \cos\frac{2\pi}{17}$ *can be obtained by a finite number of rational operations and extraction of square roots of real numbers. Hence the regular polygon of* 17 *sides is constructible.*

Gauss' final algebraic expression, as obtained by the substitution of (43) and (44) into (45) is

$$\begin{aligned}\cos\tfrac{2\pi}{17} = -\tfrac{1}{16} + \tfrac{1}{16}\sqrt{17} + \tfrac{1}{16}\sqrt{34 - 2\sqrt{17}} \\ + \tfrac{1}{8}\sqrt{17 + 3\sqrt{17} - \sqrt{34 - 2\sqrt{17}} - 2\sqrt{34 + 2\sqrt{17}}};\end{aligned} \tag{46}$$

Note that the problem is not entirely *algebraic because the* sign *of the radicals in Gauss' formula (46) must be determined by* nonalgebraic *means.*

The construction of regular polygons had interested Gauss since 1796 when he conceived the first proof that the 17-sided polygon is constructible. There is a story about this discovery. One day Gauss approached his professor A.G. Kästner at the University of Göttingen with the proof that this polygon is constructible. Kästner was incredulous and sought to dismiss Gauss, much as university teachers today dismiss angle-trisectors. Rather than take the time to examine Gauss' proof and find the supposed error in it, Kästner told Gauss the construction was unimportant because practical constructions were known. Of course Kästner knew that the existence of practical or approximate constructions was irrelevant for the theoretical problem. To interest Kästner in his proof Gauss pointed out that he had solved a seventeenth degree algebraic equation. Kästner replied that the solution was impossible. But Gauss rejoined that he had reduced the problem to solving an equation of lower degree. "Oh well," scoffed Kästner, "I have already done this."

Gauss also proved the following important theorems:

- *For every prime* p, *the cyclotomic polynomial*

$$f_p(x) = x^{p-1} + x^{p-2} + \cdots + x + 1 \tag{47}$$

  is irreducible over the field of rational numbers.

- *For* every *integer* n, *the* n^{th} *roots of unity have expressions by radicals.*

4. Circulant matrices and the roots of unity

There is an interesting and useful connection between matrix algebra and *roots of polynomials through a special kind of matrices known as circulants*[393]. *An $n \times n$ circulant matrix is defined by*

$$C_n = \begin{bmatrix} c_1 & c_2 & c_3 & \cdots & c_n \\ c_n & c_1 & c_2 & \cdots & c_{n-1} \\ c_{n-1} & c_n & c_1 & \cdots & c_{n-2} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ c_2 & c_3 & c_4 & \cdots & c_1 \end{bmatrix}, \tag{48}$$

where the c's are real or complex. Each row in (48) consists of the elements of the preceding row shifted one position to the right, with the 'overflow' element begin moved to the first position. The matrix is entirely determined by its first row. Three properties can be seen immediately by inspection of (48):

- *The elements along each diagonal line parallel to the principal diagonal (including the principle diagonal itself) are equal.*
- *Transpose of a circulant is also a circulant.*
- *C_n is symmetric w.r.t. its secondary diagonal (the line from top right corner to bottom left corner.*

Using the notation $C_n = \text{circ}(c_1, c_2, \ldots, c_n)$, an important special circulant matrix is

$$W_n = \text{circ}(0, 1, 0, \ldots, 0)_n = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix}_n, \tag{49}$$

known as the permutation matrix of order n or the shift matrix, because the postmultiplying of any matrix by W_n shifts its columns one place to the right (a similar shift is applied to rows on premultiplying by W). Clearly

$$C_n = c_1 I + c_2 W + c_3 W^2 + \cdots + c_n W^{n-1} = \sum_{k=1}^{n} c_k W^{k-1} \tag{50}$$

[393] *Circulants* were introduced by **Catalan** (1846) and further investigated by **Bertrand** (1850), **Sylvester** (1855), **Cremona** (1856), **Bellavitis** (1857) and **Souillart** (1858).

and

$$C_nW_n = W_nC_n \tag{51}$$

For example, with $n = 3$*:*

$$C_3 = \begin{bmatrix} a & b & c \\ c & a & b \\ b & c & a \end{bmatrix} = a\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + b\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} + c\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \quad aI_3 \quad + \quad bW_3 \quad + \quad cW_3^2$$

Using (50), it can be shown that the inverse of C_n *is also a circulant matrix. Note that if we denote* $C = [c_{ij}]$*, then* $c_{ij} = c_{j-i+1}$ *with* $c_{-k} = c_{n-k}$ *for* $k \geq 0$*. With this notation, one easily shows that the matrix product of any two circulant matrices is also a circulant. In other words, circulants form a group under multiplication. It also follows directly from (50) that any two circulants of the same order commute*

$$C_1C_2 = C_2C_1 \tag{52}$$

Eigenvalues of a circulant

It is known from the theory of matrices that if λ *is an eigenvalue of an* $n \times n$ *matrix* A *i.e.* $A\overrightarrow{x} = \lambda\overrightarrow{x}$ *or* $|A - \lambda I| = 0$*, then the Cayley-Hamilton theorem guarantees that* $P(A)\overrightarrow{x} = P(\lambda)\overrightarrow{x}$*, where* $\overrightarrow{x}$ *is an eigenvector and* P *is any polynomial. Let us first calculate the eigenvalues of* W_n*, through the equation*

$$|W_n - \lambda I_n| = \begin{vmatrix} -\lambda & 1 & 0 & \cdots & 0 \\ 0 & -\lambda & 1 & \cdots & 0 \\ \vdots & & & & \vdots \\ & & & & 1 \\ 1 & & \cdots & & -\lambda \end{vmatrix}_n \tag{53}$$

Clearly, the characteristic polynomial of W_n *is* $\lambda^n = 1$ *and therefore its eigenvalues are the* n^{th} *roots of unity. Because of (50) and on the strength of the Cayley-Hamilton theorem, the* k^{th} *eigenvalues of a circulant* C_n *is* $(\lambda_1 = 1)$

$$r_k = c_1 + c_2\lambda_k + c_3\lambda_k^2 + \cdots + c_n\lambda_k^{n-1}, \qquad k = 1, 2, \ldots, n \tag{54}$$

This implies that the determinant of C_n *can be expressed in the compact form*

$$|C_n| = \prod_{k=1}^{n}(c_1 + c_2\lambda_k + c_3\lambda_k^2 + \cdots + c_n\lambda_k^{n-1}). \tag{55}$$

For example

$$\begin{vmatrix} x_1 & x_2 & x_3 \\ x_3 & x_1 & x_2 \\ x_2 & x_3 & x_1 \end{vmatrix} = (x_1+x_2+x_3)(x_1+\omega x_2+\omega^2 x_3)(x_1+\omega^2 x_2+\omega x_3),$$

where ω and ω^2 *are the complex cube roots of unity.*

We note that (54) can be recast in the matrix relation

$$\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \lambda & \lambda^2 & \cdots & \lambda^{n-1} \\ 1 & \lambda^2 & \lambda^4 & \cdots & \lambda^{2(n-1)} \\ \vdots & & & \cdots & \\ 1 & \lambda^{n-1} & \lambda^{2(n-1)} & \cdots & \lambda^{(n-1)^2} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \\ c_n \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ \vdots \\ r_n \end{bmatrix}. \tag{56}$$

The $n \times n$ *matrix on the l.h.s. is a Vandermonde type matrix* $V(1,\lambda,\lambda^2,\ldots,\lambda^{n-1})$. *When multiplied by* $\frac{1}{\sqrt{n}}$ *it is known as the Fourier-Transform matrix* $F_n = \frac{1}{\sqrt{n}}V$. *It is a unitary matrix, its inverse being equal to its conjugate transpose*

$$F^{-1} = F^*.$$

Let

$$D_n = \begin{bmatrix} 1 & & & & 0 \\ & \lambda & & & \\ & & \lambda^2 & & \\ & & & \ddots & \\ 0 & & & & \lambda^{n-1} \end{bmatrix}, \qquad \lambda = e^{-\frac{2\pi i}{n}}\,. \tag{57}$$

Using (50), (56) (or 54), one readily proves the important relations

$$\begin{aligned} W_n &= F_n^{-1} D_n F_n \qquad & D_n &= F_n W_n F_n^{-1} \\ C_n &= F_n^{-1} \Delta_n F_n \qquad & \Delta_n &= F_n C_n F_n^{-1} \end{aligned} \tag{58}$$

where

$$\Delta_n = c_1 I + c_2 D + c_3 D^2 + \cdots c_n D^n\,. \tag{59}$$

This means that all elements of a circulant C_n are simultaneously diagonalized by the same unitary matrix.

A special type of circulant matrix is defined as

$$C_n = \begin{bmatrix} 1 & \binom{n}{1} & \binom{n}{2} & \cdots & \binom{n}{n-1} \\ \binom{n}{n-1} & 1 & \binom{n}{1} & \cdots & \binom{n}{n-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \end{bmatrix}, \tag{60}$$

where $\binom{n}{k}$ is a binomial coefficient. The determinant of C_n is given by the formula

$$C_n = \prod_{j=0}^{n-1} \left[(1+\lambda_j)^n - 1 \right] . \tag{61}$$

Thus, the computation of a circulant's eigenvalues is actually quite trivial: simply generate a polynomial from the first row

$$q(t) = c_1 + c_2 t + \cdots + c_n t^{n-1}$$

and then evaluate the polynomial at $t = \lambda_k$. For example, if

$$C_4 = \begin{bmatrix} 1 & 2 & 1 & 3 \\ 3 & 1 & 2 & 1 \\ 1 & 3 & 1 & 2 \\ 2 & 1 & 3 & 1 \end{bmatrix}$$

Since $q(t) = 1 + 2t + t^2 + 3t^3$ and $\lambda_1 = 1$; $\lambda_2 = -1$; $\lambda_3 = i$; $\lambda_4 = -i$, we have for the eigenvalues of C:

$$q(1) = 7; \qquad q(-1) = -3; \qquad q(i) = -i; \qquad q(-i) = i \, .$$

Suppose that $\mu_1, \mu_2, \ldots, \mu_{r-1}$ are the roots of the polynomial

$$q(z) = c_1 + c_2 z + \cdots + c_n z^{n-1} \tag{62}$$

known as the representer *of the circulant (to be distinguished from the eigenvalues of C_n). Then*

$$C_n = q(W) = a_r(W - \mu_1 I)(W - \mu_2 I) \cdots (W - \mu_{r-1} I) \, . \tag{63}$$

This gives us a factorization of any circulant into a product of circulants $(W - \mu_k I)$ that are of a particular elementary type.

Let C_n be nonsingular. i.e.: non of the eigenvalues of C_n is zero, namely, $\lambda_j = q(W^{j-1}) \neq 0, \quad j = 1, 2, \ldots, n$. This will be true iff $\mu_k^n \neq 1, \quad lk = 1, 2, \ldots, r-1$. Then, from (63) one has

$$C_n^{-1} = a_r^{-1}(W - \mu_1 I)^{-1}(W - \mu_2 I)^{-1} \cdots (W - \mu_{r-1} I)^{-1} . \tag{64}$$

It can be shown that for $\mu^n \neq 1$

$$(W - \mu I)^{-1} = \frac{1}{1 - \mu^n}\left[\mu^{n-1} I + \mu^{n-2} W + \mu^{n-3} W^2 + \cdots + W^{n-1}\right] . \tag{65}$$

The characteristic polynomial of C_n

Given a $n \times n$ circulant matrix C_n, the polynomial

$$P_n(x) = \det(xI - C_n) = x^n + p_{n-1}x^{n-1} + \cdots + a_1 x + a_0 \tag{66}$$

is the characteristic polynomial *of C_n. Its roots r_k, as determined in (54), are the eigenvalues of C_n. Clearly*

$$p_{n-1} = -(r_1 + r_2 + r_3 + \cdots + r_n) = \text{trace of } C_n = -nc_1 . \tag{67}$$

If we effect the transformation $\quad y = x - \frac{1}{n}p_{n-1} \quad$ in (66), the term of degree $n-1$ is eliminated and this operation corresponds to making the trace of C_n vanish. Thus, the circulant matrix for the modified polynomial *has vanishing diagonal and trace; such a matrix is called a* traceless circulant.

Suppose we wish to obtain expressions for the roots of $p_3(x) = x^3 + \beta x + \gamma$ as the eigenvalues of a circulant matrix

$$C_3 = \begin{bmatrix} 0 & b & c \\ c & 0 & b \\ b & c & 0 \end{bmatrix} \tag{68}$$

The characteristic polynomial of C_3 is $\quad x^3 - 3bcx - (b^3 + c^3)$. This equals $p_3(x)$ if $\quad b^3 + c^3 = -\gamma; \quad 3bc = -\beta$. To complete the solution of the original equation, we must solve this system for b and c, and then apply $q(x) = bx + cx^2 \quad$ to the cube roots of unity. That is, for any a and b satisfying $\quad b^3 + c^3 = -\gamma \quad$ and $3bc = -\beta$, we obtain the roots of p as

$$\begin{aligned} q(1) &= b + c \\ q(\omega) &= b\omega + c\omega^2 \\ q(\bar{\omega}) &= b\bar{\omega} + c\bar{\omega}^2 \end{aligned} \tag{69}$$

Solving, then, the above equations for the unknowns b^3 *and* c^3 *we obtain*

$$b = \left[\frac{-\gamma + \sqrt{\gamma^2 + 4\beta^3/27}}{2}\right]^{1/3}, \qquad c = -\frac{\beta}{3b}, \qquad (70)$$

which is essentially the Cardano solution. What distinguishes this approach is the role of the roots of unity and the immediate extendability of the circulant approach to the quadric equation and solvable polynomial equations of higher degree.

Circulant matrices have important applications to diverse disciplines including physics, image processing, probability and statistics, numerical analysis, number theory and geometry. The built-in periodicity also means that circulants are closely related to Fourier analysis and group theory.

1796–1825 CE **Georges (Léopold Chrétien Frédéric Dagobert) Cuvier** (1769–1832, France). Geologist and paleontologist. Author of the geological-historical concept of world revolutions in nature. Founder of comparative anatomy and paleontology (1805). Attributed fossil succession to extinction caused by a series of *natural catastrophes*[394] rather than to evolution (1812–1825). Developed a method of classifying mammals (1796) and gave an account of the whole animal kingdom, dividing it into four groups (1817).

[394] *Crisis*: an event that occurs in the history of a system, when stress is sufficient to cause the imminent alteration of the system's principal structures, but, through the absorption of this stress into its subsystems, the system survives.

Catastrophe: an event that occurs in the history of a system, when stress is sufficient to cause the imminent alteration of the system's principal structures; and the subsystems fail to absorb all of the stress but survive, although the system fails. In such cases, a new and modified system is then formed to take the place of the failed system.

Cataclysm: an event that occurs in the history of a system, when stress is sufficient to cause the imminent alteration of the system's principal structures, and both the system and its subsystems fail.

In each of the three events just described, the source of the stress is left undefined; but, for the present, it can be inferred to be external. Crises occur often, catastrophes happen less often, and cataclysms rarely occur on a grand scale.

Cuvier was born at Montbéliard, in Wurthenburg (now a part of Burgundy) of a poor Lutheran military family of Huguenot stock. After spending four years at the Caroline University near Stuttgart, he accepted the position of tutor in the family of the Comte d'Hericy. Like **Laplace**, he was appointed (1795) assistant at the Muséum d'Histoire Naturelle. He later became professor of natural history in the Collège de France (1799), and titular professor at the Jardin des Plantes (1802).

During the early years of the 19 th century Cuvier was a man of considerable influence, earning for himself in the sciences the title of 'the dictator of biology.' In 1808 he was placed by Napoleon upon the council of the Imperial University, assisting the latter in the reorganization of higher education. In 1831 he was raised by Louis-Philippe to the rank of peer of France.

Cuvier lived through turbulent times: the fall of the nobility, the French Revolution, the reign of Napoleon, the return of the nobility, the fall of the Church, and the resurgence of its influence. Like **Laplace**, he was a "survivor", who died rich, famous and powerful. His vanity was boundless, as was his hunger for honors and praise. He was said to have had an exceptional memory and to have known the contents of all 19,000 books in his library.

His life story and character may explain why he chose the catastrophic point of view.

1796–1826 CE **Aloys Senefelder** (1771–1834, Germany). Inventor of *lithography*. In 1826 he invented a process of lithographing in color. Born in Prague. Director of the royal printing office in Münich (1809).

1796–1833 CE **Samuel Hahnemann** (1755–1843, Germany). Physician. Founded *homeopathic medicine*. This medical system is based on simple remedies (exercise, a nourishing diet, and pure air), and on two fundamental principles:

- diseases are cured by drugs which produce in healthy persons the symptoms found in those who are ill;
- the smaller the dose, the more efficacious the medicine.

Hahnemann was born in Meissen, Saxony. He studied medicine at Leipzig and Vienna and received his M.D. at the University of Erlangen (1779). Through his practice he quickly discovered that the medicine of his day (purgatives, emetics, blistering, cupping, sweating, bloodletting and huge doses of calomel and other mineral drugs) did as much harm as good.

He then gave up his practice and made his living as a writer and translator. While translating William Cullen's *A Treatise on the Materia Medica*, Hahnemann encountered the claim that Cinchona, the bark of a Peruvian tree, was effective in treating malaria because of its astringency. Hahnemann realized that other astringent substances are not effective against malaria and began to research cinchona's effect on the human organism very directly: by self-application. He discovered that the drug evoked malaria-like symptoms in himself, and concluded that it would do so in any healthy individual. This led him to postulate a healing principle: "*that which can produce a set of symptoms in a healthy individual, can treat a sick individual who is manifesting a similar set of symptoms.*" This principle, '*like cures like*', became the first of a new medicinal approach to which he gave the name *homeopathy*.

Hahnemann began systematically testing substances for the effect they produced on a healthy individual and trying to deduce from this the ills they would heal. He quickly discovered that ingesting substances to produce noticeable changes in the organism resulted in toxic effects. His next task was to solve this problem, which he did through exploring dilutions of the compounds he was testing. He discovered that these dilutions, when done according to his technique of succussion (systematic mixing through vigorous shaking) and potentization, were still effective in producing symptoms.

Hahnemann began practicing medicine again using his new technique, which soon attracted other doctors. He first published an article about the homeopathic approach to medicine in a German medical journal in 1796; in 1810, he wrote his *Organon of the Medical Art*, the first systematic treatise on the subject.

Hahnemann continued practicing medicine, researching new medicines, writing and lecturing to the end of a long life. He died in 1843 in Paris, 88 years of age, and is entombed in a mausoleum at Paris' Pére Lachaise cemetery.

1797 CE **Lorenzo Mascheroni** (1750–1800, Italy). Mathematician and poet. Published a variety of mathematical works. He shares with Euler the name of the number $\gamma = \lim_{n\to\infty}\left[\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}-\log_e n\right]$, known as the *Mascheroni-Euler number*.

$\gamma = 0.577\ 215\ 664\ 901\ 532\ 860\ 606\ 512\ 090\ 082\ 402\ 431\ 042\ 159\ 335\ 939\ 92\ldots$

is one of the most mysterious of all arithmetic constants. It appears unexpectedly in several places in number theory.

In *Geometria del compasson* (1797) he showed that every compass-and-straightedge (unmarked ruler) construction can be done with a compass alone[395] (it is assumed that two points, obtained by arc intersections, define a straight edge). Among the problems solved by Mascheroni, using the compass alone, were:

- Locating the center of a given circle.
- Finding a point midway between two given points A and B.
- Dividing a circle, its center given, into four equal arcs (known as "Napoleon problem"[396]).

Mascheroni was ordained as a priest at the age of 17. At first he taught rhetoric, then, from 1778, he taught physics and mathematics at the seminary at Bergamo. In 1786 he became professor of algebra and geometry at the university of Paris. He later became rector of that University.

1797–1808 CE **Joseph Louis Proust** (1754–1826, France). Chemist. Discovered the quantitative nature of chemical combination through the *Law of Definite Proportions*. He also was first to distinguish a chemical *compound* from a simple *mixture* of elements. Identified the sugars: *glucose*, *fructose* and *sucrose* in plant juices (1808).

1798 CE **Benjamin Thompson** (1753–1814); **Count Rumford**. British-American scientist, adventurer and political figure. In "*An Inquiry*

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395 A Danish geometer **George Mohr** with no other claim to fame, published this surprising result already in 1672 in a 24-page booklet. It was issued in a Danish edition under the name *Euclides Danicus* and a Dutch edition (1673) under *Compendium Euclidis Curiosi*. The Dutch edition, published anonymously, was translated (1674) into English, but the Danish book was discovered only in 1928 in Copenhagen.

Jean Victor Poncelet suggested a proof (1822) that *all* compass-and-straightedge constructions are possible with a straightedge and a *fixed compass*. But again, a little-known geometer, **Servais**, published this result earlier (1805). Incidentally, the first systematic effort to go beyond the Greek by imposing more severe restrictions on instruments used in construction problems, is ascribed to the Persian mathematician Abu al-Wafa (ca 970 AD). In his work he described constructions possible with a straightedge and a fixed compass.

396 Young Mascheroni was an ardent admirer of Napoleon and the French Revolution. His book *Problems for Surveyors* (1793) was dedicated in verse to Napoleon. The two men met and became friends in 1796, when Napoleon invaded Northern Italy. A year later, when Mascheroni published his book on constructions with the compass alone, he again honored Napoleon with a dedication in a lengthy ode.

Concerning the Source of the Heat Which is Excited by Friction", he reported his experiments which discredited the caloric theory of heat and established heat as a form of energy rather than a substance. Since heat was being introduced by motion, he suggested that heat is a form of motion itself. In 1797, Rumford conjectured the existence of large-scale convection currents in the world oceans. In his essay "*On the propagation of heat in fluids*", he concluded that the existence of cold water at depth in the tropics implies a meridional circulation, transporting deep water from the polar regions toward the equator.

Thompson was born in Woburn, Massachusetts, to a family of wealthy farmers that had settled in New England around 1650. His father died when he was very young and his mother speedily remarried. At the age of 14 he was already versed in algebra, geometry, astronomy and higher mathematics. In 1768 he was apprenticed to a storekeeper at Salem, and occupied himself in chemical and mechanical experiments.

He began his checkered career when at the age of 19 he married a wealthy widow (from the township of Rumford), 14 years his senior. He was allegedly engaged in spying for the British during the American Revolution and had to leave America when the British troops left Boston in 1776. On his arrival in London he entered the civil service and within 4 years rose to a rank of under-secretary of state. His official duties, however, did not interfere with the prosecution of his scientific pursuits, and in 1779 he was elected a fellow of the Royal Society. He then left the civil service and joined the cavalry, which he quit in 1783 at the rank of lieutenant-colonel. He then joined the Austrian army, for the purpose of campaigning against the Turks. At Strasbourg he was introduced to Prince Maximilian, afterwards elector of Bavaria, and was invited by him to enter the civil and military service of that state.

During 1787–1798 he remained in Münich as a minister of war, minister of police and grand chamberlain to the elector. His work to improve the living conditions of the poor in Münich gained him the title of Count of the Holy Roman Empire in 1791. His political and courtly employments, however, did not absorb all his time, and during his stay in Bavaria he contributed a number of papers to the *Philosophical Transactions*. In 1798 he made his greatest contribution to science while supervising the boring of cannons.

The death of the elector Karl Theodor, the rise of Napoleon and the fact that the Bavarians were beginning to find him tiresome, led Rumford to return to England in 1799. In 1800 he helped found the British Royal Institution. In 1804 he established himself in Paris, where he was married briefly and unhappily to the widow of **Lavoisier**. He died at Auteuil, near Paris, at the age of 61.

1798–1801 CE *Napoleon's Egyptian campaign*. A French expedition of 151 scientists, engineers, medical men and scholars created the first modern vision of Egyptian antiquity and its natural history. It resulted in a monumental encyclopedia *Le Description de L'Égypte*, printed between 1809 and 1828 in ten folio volumes of plates (50 cm by 65 cm), three atlases (65 cm by 100 cm) and nine accompanying volumes of text comprising approximately 7000 pages of memoirs, description and commentary.

Serving as permanent secretary of the project was **Jean Baptist Fourier**, who had yet to invent the analysis that bears his name. Among the scientists in the expedition were **Gaspard Monge** (exact sciences), **Claude Louis Berthollet** (physical chemistry), **Étienne Geoffray Saint-Hilaire** (vertebrate zoology), **Jules César Lelorgne de Savigny** (invertebrate zoology, ornithology), **Francois-Michel de Rozière** (mineralogy), **Dominique Jean Larrey** (medicine) and **Jean Francois Champollion** (archaeology).

On the first of July 1798, an armada of 400 ships appeared of the coast of Alexandria. By the end of the day, an army of 36,000 men, under the command of Napoleon Bonaparte landed ashore. On July 21, this army defeated the Mamelukes in the *Battle of the Pyramids*. Ten days later, Admiral Horatio Nelson destroyed the French fleet, marooning the expeditionary force for the next three years[397].

The most famous discovery of the expedition remains the *Rosetta stone*; it was only in 1822 that Champollion succeeded in matching the name Ptolemy in the three scripts — hieroglyphic, demotic and Greek — inscribed on the Rosetta stone, and not until the 1850's were scholars able to construe whole texts.

Another archaeological feat was the excavation of the route of the canal that had linked the Red Sea to the Mediterranean in ancient times.

As for science is the ordinary sense, the Egyptian environment created exceptional opportunities. Some of these achievements are:

(1) **Monge**'s explanation of *mirages* as the effect of light rays from beyond the horizon, reflected from the surface of a layer of air superheated at

[397] In 1798, when all of England's allies in the war against France had been defeated and when the Spaniards had changed sides, the Royal Navy was obliged to withdraw from the Mediterranean. Bonaparte then set out from Toulon with a powerful fleet with the intension of conquering Egypt and attacking the British in India. He had captured Malta, stormed Alexandria and taken Cairo. On Aug. 01, Nelson discovered the French fleet in the Bay of Aboukir and although outmanned and outgunned, he attacked without delay. His victory shattered Napoleon's scheme in Egypt and India and had great political influence in Europe.

ground level by the sun-soaked sand. Although modern optics attributes the effect to dual refraction within the surface layer, Monge had the underlying physics right.

(2) Motivated by the *natural* occurrence of the reaction

$$CaCo_3 + 2NaCl \xrightarrow[\text{humidity}]{\text{heat}} Na_2Co_3 + CaCl_2$$

in limestone formations surrounding saline lakes, **Berthollet** advanced (1803) his theory that the course of reactions is determined not only by the relative concentration of reagents but also by exterior physical factors such as pressure, heat and light. This, in retrospect, is considered as the point of departure for *physical chemistry*.

(3) The zoological studies of the naturalists **Geoffray** and **Savigny** moved beyond *taxonomy* (classification) to *morphology* (form and structure); the former had been the main preoccupation of the natural history of the 18^{th} century; the latter became important subdiscipline of the emerging science of biology in the 19^{th} century.

(4) In a monograph '*On the Physical Constitution of Egypt and its Relation with the Ancient Institutions of the Country*' **Rozière** observed that in no other country has a highly developed society such as that of ancient Egypt, ever exhibited such dependence on a single set of physical factors: everything in the laws of the land and the customs of the people derived from the behavior of the Nile. The rise and fall of the river not only shaped the civilization of Egypt but also accounted for the influence of its culture on the theogonies, the sciences, and the arts and crafts of all antiquity.

(5) **Larrey** gave clinical descriptions of trachoma, bubonic plague, tetanus, yellow fever, leprosy, elephantiasis and gigantism. In his view, the etiology of some of these diseases involved a specific external agent, for which he sometimes used the word *virus* and sometimes *germ*.

With the French conquest of Egypt, began the spread of European science and its appurtenances to African and Asian societies under the aegis of military conquest and political power.

1798–1816 CE *Extinction of German universities*. The political storms which marked the turn of the 19^{th} century dealt a death-blow to some 15 of the old universities in Germany. Among them: **Mainz** (1476–1798), **Cologne** (1388–1798), **Bamberg** (1648–1804), **Altdorf** (1580–1807), **Frankfurt a.o.** (1506–1809), **Wittenberg** (1502–1815) and **Erfurt** (1379–1816).

Rumford and Caloric

Many earlier experimenters believed that heat was a weightless, highly elastic, self-repellent fluid, indestructible and uncreatable. This fluid they called caloric[398]*. The caloric theory offered an explanation of the facts then known: bodies emitted heat because the particles of caloric repelled one another strongly. Differences in specific heats were due to the different attracting powers of different substances for the fluid. Expansion occurred because the self-repellent fluid tended to increase the volume of any body in which it was lodged. Latent heat was supposed to enter into combination with the particles of the material: thus water = ice + latent heat. If a simple theory of this kind served to explain all the observed facts, it was quite reasonable to look no further. But the generation of heat by percussion and by friction presented difficulties.*

The theory stated that the rise of temperature of a block of lead, when hammered, was due to the extrusion of caloric under pressure, much as water issues from a sponge when it is squeezed. The rise in temperature of two bodies when rubbed together was due to the diminution of the bodies themselves: as the small particles rubbed off, the bodies' overall power of attracting caloric, decreased, and some of it was thereby freed — that is, the specific heat of a finely powdered substance was less than that of the same substance in one solid mass. No attempt seems to have been made to detect this diminution of specific heat, and this argument could not possibly explain the generation of heat by the churning of a liquid.

Rumford performed a series of experiments (1798) at the Münich military arsenal, Germany, in which heat was generated by rotating a blunt cannon borer in a large mass of gun-metal. He observed that a large quantity of heat (sufficient to raise nearly 12 kg of water from the freezing point to the boiling point in one experiment) was released from the abrasion of a very small quantity of metallic dust, and he found that the specific heat of this dust was not appreciably different from that of the solid material, which showed that the caloric theory was false, on this point at least. Further, the supply of heat appeared to be inexhaustible, and it was clear that no closed system could supply unlimited amounts of any material substance.

In a paper published in 1799, ***Humphry Davy*** *(1778–1829, England) reported that rubbing two blocks of ice together in a vacuum, using a clockwork*

[398] The term "phlogiston" was used before "caloric", in a similar but not identical manner.

mechanism, caused the ice to melt at the surface in contact. As it was well known that considerable latent heat is required to turn ice into water, and as no other possible source of heat was available, this experiment was claimed to demonstrate that heat was evolved here by mechanical action only. Also, the specific heat of the product (water) is approximately double that of the solid used. Although this work has been accepted for many years, it seems very doubtful if Davy, then 19 years of age, could have carried out such an experiment, which would tax the ingenuity of any trained physicist.

In spite of the works of Rumford and Davy, the caloric theory remained in favor for some 50 *more years, until finally wiped out by the theories and experiments of* ***Mayer*** *(1840),* ***Joule*** *(1847),* ***Helmholtz*** *(1847),* ***Kelvin*** *(1852) and* ***Rankine*** *(1853).*

1798–1820 CE **Thomas Robert Malthus** (1766–1834, England) Economist. Aroused controversy by his *Essay of the Principles of Population Theory*, based on the premise that population, when unchecked, tends to increase in a geometrical progression (doubling every 25 years), whereas the means of subsistence tend to increase only in an arithmetic progression. From this Malthus concluded that:

- population always increases as the means of subsistence increase.
- population is limited by the means of subsistence.
- population is kept from overgrowing the means of subsistence by two kinds of checks: *positive checks* (disease, war, famine etc.) and *preventive checks* (voluntary abstinence from sex indulgence).

The Malthusian population theory went hand in hand with *Wage Theory*: If laborers receive wages affording them more than mere subsistence, they will raise more children. The number of people will thus increase until there are more than can be fed and the population will reduce to numbers that can just be supported by the available means of subsistence. It is thus useless to attempt to relieve the laboring classes of their misery: in the struggle for survival the fittest come out on top, the unfit perish. This is better than to keep the unfit alive through charity and to let the fit die instead. **Darwin** built his theory of evolution on this Malthusian idea.

Although it contains much that is true, the theory has been criticized in several counts: (1) The ratios of increase of population and the means of subsistence are hypothetical, not actual; (2) Population does not always increase to the full extent of its biological capacity[399]; (3) Means of subsistence may increase faster than population due to *technological improvements*; the experience of the last 150 years bears this out.

The doctrine of Malthus was a corrective reaction against the superficial optimism diffused by the school of Rousseau and its blindness to the real conditions that circumscribe human life.

Malthus was born near Guilford, Surrey. He was educated by private tutors and went to Cambridge (1784–1797). He then became a curate at a small parish in Albury, Surrey (1797); During 1805–1834 he was professor of history and political economy at the East India Company's college at Haileybury.

1798–1805 CE **Johann Wilhelm Ritter** (1776–1810, Germany). Physicist. His suggestion that the galvanic current was due to a *chemical* interaction between the metals (1798), was the first electrochemical explanation of this phenomenon. Discovered the process of *electroplating* (1800); discovered existence of ultraviolet radiation through its effect of darkening a silver chloride film (1801); observed thermoelectric currents (1801); invented the *dry voltaic cell* (1802) and the electrical storage battery (1803).

Ritter was born in Samitz, Silesia (now Poland) and began his career as an apothecary. He then studied at the University of Jena (1796). The basic concept of electrolysis and electroplating was discovered by Ritter at the same time or in some cases earlier than the experiments of **Carlisle**, **Nicholson** and **Davy**.

In 1801 he observed thermoelectric currents, anticipating the discovery of *thermoelectricity* by **Seebeck** (1821). In 1805, Ritter moved to Munich to take a position at the Bavarian Academy of Science. He died at the young age of 33, as a direct result of exposing his body to very high voltages in his experiments on the electrical excitation of muscle and sensory organs.

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[399] On the eve of the *Agricultural Revolution* (10,000 BCE), the human species numbered about 4 million people. On the eve of the *Industrial Revolution* (ca 1750 CE), the total world population was estimated at 800 million people. In 1950 world population reached 2485 ($\pm$ 5%) millions, and in 1990 it was about 5300 million. Projections to the years 2000 and 2050 CE yield the respective estimates of 6200 and 10,000 million.

1797–1815 CE **Johann Friedrich Pfaff** (1765–1825, Germany). Mathematician. Presented the theory of *Pfaffian forms* and *equations*[400] (1797). His work constituted the starting point of a basic theory of integration of PDE which, through later work of **Jacobi**, **Lie** and others, has developed into the modern **Cartan**'s exterior calculus of differential forms.

Pfaff was born in Stuttgart to a distinguished family of Württemberg civil servants. At age of 9 he went to the Hohe Karlsschule in Stuttgart, a school with harsh military discipline, serving chiefly to train servile government officials. Pfaff completed his legal studies there in 1785 and then spent a few years in travel and study at the Universities of Göttingen, Berlin, Vienna, Halle, Jena and Prague.

He finally settled down as a professor of mathematics at the University of Helmstadt. Gauss, after completing his studies at Göttingen (1795–1798), lived in Pfaff's house. Pfaff recommended Gauss' doctoral dissertation and, when necessary, greatly assisted him. Gauss always retained a friendly memory of Pfaff both as a teacher and as a man.

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[400] The expression $\sum_{i=1}^{n} F_i(x_1, x_2, \ldots, x_n)dx_i$ in which the F_i $(i = 1, 2, \ldots, n)$ are functions of the n independent variables $x_1, x_2, \ldots, x_n$, is called a *Pfaffian differential form* in n variables. Similarly, the relation $\sum_{i=1}^{n} F_i dx_i = 0$ is called a *Pfaffian differential equation*.

In the case of 2 variables, the form $P(x, y)dx + Q(x, y)dy = 0$ is equivalent to $\frac{dy}{dx} = f(x, y) = -\frac{P}{Q}$. If $\{P, Q\}$ are single-valued functions, then $\frac{dy}{dx}$ is single-valued, and the solution to the above ODE which satisfies the boundary condition $y_0 = y(x_0)$ consists of a curve which passes through this point and whose tangent at each point is defined by the DE. Thus the original Pfaffian equation defines a one-parameter family of curves in the xy-plane. It can be shown that a Pfaffian DE in 2 variables always possesses an *integrating factor*, i.e. there exist $\mu(x, y)$, $\phi(x, y)$ such that $0 = \mu(Pdx + Qdy) = d\phi$ or $\frac{1}{P}\frac{\partial\phi}{\partial x} = \frac{1}{Q}\frac{\partial\phi}{\partial y} = \mu$.

When there are 3 variables, the Pfaffian DE is of the form $Pdx+Qdy+Rdz = 0$. If we introduce the vectors $\mathbf{x} = (P, Q, R)$ and $d\mathbf{r} = (dx, dy, dz)$, we may write this equation in vector notation as $\mathbf{x} \cdot d\mathbf{r} = 0$. A necessary and sufficient condition that the Pfaffian DE $\mathbf{x} \cdot d\mathbf{r} = 0$ should be integrable is that $\mathbf{x} \cdot \operatorname{curl} \mathbf{x} = 0$ [this theorem figures prominently in theoretical thermodynamics. Moreover, the exterior calculus of **Cartan** can be used to extend it to criteria for solvability of systems of ODE's].

The representation of a given vector-field $\overrightarrow{f}$ in the form $\overrightarrow{f} = \nabla w + u\nabla v$, where (u, v, w) are scalar functions of the coordinates, is known as the *Pfaff Problem*.

In 1810, the University of Helmstadt was closed and Pfaff went to Halle, where he stayed to the end of his life.

1798–1817 CE **Louis Nicolas Vauquelin** (1763–1829, France). Chemist. Discovered *chromium* (1798), *beryllium*[401] (1798) and [with Pierre Jean Robiquet (1780–1840)] first isolated an amino acid, *asparagine*[402], from asparagus (1806).

1799 CE **Marc Antoine Parseval des Chênes** (1755–1836, France). Mathematician. His reputation rests on a single formula for summing special cases of series of products. Since its appearance in print in 1800, dozens of equations have been called Parseval equations, theorems or identities both in the theory of Fourier series and the theory of the Fourier integral; most of them only remotely resemble the original. In his memoirs, which were presented at the Academy of Sciences, Parseval applied his theorem[403] to

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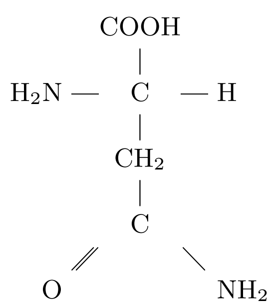
401 He discovered it in the gems *beryl* and *emerald*, but did not isolate the element. Beryllium was finally isolated by **Friedrich Wöhler** (1828).

402 *Asparagine* was synthesized by **Wilhelm Körner** (1839–1925) in 1887. It has the structural formula



Asparagine is important in the metabolism of nitrogen and the anabolism of nitrogen-containing compounds.

403 In modern notation: If in the series

$$M = A_0 + A_1 s + A_2 s^2 + \cdots$$

and

$$m = a_0 + a_1 s + a_2 s^2 + \cdots ,$$

s is replaced by e^{iu} and the real and imaginary parts are separated so that $M = P + iQ$ and $m = p + iq$, then

$$\frac{2}{\pi}\int_0^{\pi} Ppdu = 2A_0 a_0 + A_1 a_1 + A_2 a_2 + \cdots .$$

Today, *Parseval's theorem* (or *identity*) for Fourier series states: Let $f(x)$

the solution of certain differential equations suggested by **Lagrange** and **d'Alembert**.

Little is known of Parseval's life or work; he was a member of a distinguished French family. An ardent Royalist, he was imprisoned in 1792 and later fled the country when Napoleon ordered his arrest for publishing poetry against the regime.

1799–1813 CE **Paolo Ruffini** (1765–1822, Italy). Physician and mathematician. Taught mathematics as well as clinical medicine at the University of Modena. In his book *Teoria generale dell equazioni* (1799), and later in 1813, he continued the thread of thought of **Lagrange** (1770), giving an incomplete proof that virtually established the unsolvability of the quintic equation by means of algebraic functions of the coefficients involving radicals.

Previously, **Euler**'s attempts (1750) to reduce the solution of the quintic equation to that of a quartic equation met with total failure. Ruffini's proof was later improved by Abel (1824).

1799–1831 CE **Aimé Jacques Alexandre Bonpland** (1773–1858, France and South America). Naturalist, botanist, horticulturist, agricultural experimenter and physician. As a botanist of the Humboldt expedition to the Spanish territories of South America (1799–1804), he garnered and described some 60,000 plant specimens, which he personally managed to collect in the equatorial swamps and rain forests. Later, during his stay in Argentine, Brazil, Uruguay and Paraguay (1816–1858) he continued to enrich European science with new floral specimens. He collected thousands of specimens of new plants and diagnosed them. Being a physician he had a special interest in plants which might have medicinal virtues and he sent many of them to the Paris Muséum for chemical analysis. He was first to investigate the culture of certain herbs and try to improve them in a scientific manner. He was one

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and $g(x)$ be bounded and integrable in $(-\pi, \pi)$ such that

$$f(x) = A_0 + \sum_{n=1}^{\infty} [A_n \cos nx + B_n \sin nx],$$

$$g(x) = a_0 + \sum_{m=1}^{\infty} [a_m \cos mx + b_m \sin mx].$$

Then

$$\frac{1}{\pi} \int_{-\pi}^{\pi} f(x)g(x)dx = 2A_0B_0 + \sum_{k=1}^{\infty} (A_k a_k + B_k b_k).$$

of the first botanists to observe one of the marvels of the floral world — the giant water lily (1819).

Bonpland was born in the parish of St. Bartholomew of La Rochelle. He received a medical education in Paris, and became a surgeon in the French navy. Under the influence of **Lamarck**, he had developed a deep interest in natural history, chiefly botany, which needed only a little encouragement to flare out. Having completed his naval service (1795), he returned to Paris to continue his medical studies. He then acquainted **Alexander von Humboldt**, and the two friends embarked together on their famous expedition to South America.

After their triumphal return to Paris (1804), Humboldt secured for him an appointment as botanist to the empress Joséphine[404] (1763–1814) who had a deep interest in flowers and was an ambitious horticulturist. After the empress' death he emigrated to Buenos Aires (1816), where he established a plantation on the Paraná River. There he undertook agricultural experiments on a large scale. However, in 1821 his plantation was sacked by troops of the Paraguayan dictator Francia, and he himself was imprisoned in Paraguay for more than seven years. Finally, thanks to the intervention of Humboldt, he was freed and settled (1831) in San Borja, on the eastern shore of the Uruguay River. He established there a large plantation, where he continued to conduct horticultural experiments, especially with regard to citrus fruits.

1799–1839 CE **Augustin-Pyrame de Candolle** (1778–1841, Switzerland). Botanist. Laid the foundation for modern studies on plant evolution and classification in his *Théorie Elementaire de la Botanique* (1813); *Regni Vegetabilis Systema Naturale* (1817–1821), and *Prodromus Systematis Naturalis Regni Vegetabilis* (1824–1839).

De Candolle was born in Geneva. Studied in Paris (1796); professor, Montpellier (1808–1817), Geneva (1817–1841). Coined the term "taxonomy" as a method of classifying plants by structure (1818). His son **Alphonse-Louis-Pierre-Pyrame de Candolle** (1806–1893) succeeded him as professor in Geneva (1842–1893) and continued the *Prodromus* to 17 volumes; author of *Geographic botanique raisonnée* (1855).

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404 Born at Martinique, she accompanied her father to France (1779) and married there the viscount Alexandre de Beauharnais; he was beheaded in July 1794. In 1796, she married general Bonaparte and was crowned with him on Dec. 2, 1804. Napoleon divorced her in 1809. However, she was richly endowed and was able to keep a royal establishment at Malmaison near Paris, which she had bought during Napoleon's absence in Egypt. She died there in 1814.

1799–1858 CE **Alexander von Humboldt** (1769–1859, Germany). Naturalist, geographer and explorer. The first modern geographer to become a great traveler, and thus to acquire an extensive stock of first-hand information on which an improved system of geography might be founded. Laid the foundation to physical geography and meteorology, and pioneered in plant geography and climatology. The theory of geography was advanced by Humboldt mainly by his insistence on the great principle of the *unity of nature*. He brought all the "observable things" which the eager collectors of the previous century had been heaping together regardless of order or system, into relation with the vertical relief and the horizontal forms of the earth's surface. Thus he demonstrated that the forms of the land exercise a directive and determining influence on climate, plant life, animal life and on man himself[405].

He traveled extensively in the Spanish territories of America (1799–1804), and the results of this voyage were published by him in 23 volumes during 1804–1823. Among his novel contributions to science throughout this expedition one may mention his delineation of *isothermal lines*, through which he devised the means of comparing the climatic conditions of various countries.

He first investigated the rate of decrease in mean temperature with increase of elevation above the sea-level, and afforded, by his inquiries into the origin of *tropical storms*, the earliest clue to the detection of the more complicated law governing atmospheric disturbances at higher latitudes. Studied meteorite showers, volcanoes, the earth's magnetic field and communication between the water-systems of the Orinoco and Amazon rivers, and introduced the fertilizing properties of the guano into Europe.

Humboldt was born in Berlin. During 1788–1792 he studied geology, biology, and political science at the University of *Göttingen*, mining and metallurgy at the School of Mines in *Freiburg*, commerce and foreign languages at *Hamburg*, and anatomy and astronomy at *Jena*. His studies were directed, with extraordinary insight and perseverance, to the purpose of preparing himself for his distinctive calling as a scientific explorer. Through the years 1799–1804 he was engaged in the scientific exploration of Central and South America.

In 1808 he settled in Paris, then a center of geographical learning, and lived there for the next 20 years. In 1811 he speculated about the mechanism that

[405] This in itself was no new idea; it had been familiar for centuries in a less definite form, deduced from a prior consideration, and so far as regards the influence of surrounding circumstances upon man, **Kant** had already given it full expression (1765). Humboldt's concrete illustrations and the remarkable power of his personality enabled him to enforce these principles in a way that produced an immediate and lasting effect.

drives the current flowing along the coast of Peru, later named the *Humboldt current* in his honor. In 1827 he settled permanently in Berlin. In 1829 he was the first scientist ever to organize an all-European research program in earth-magnetism and meteorology in which Germany, France, Britain and Russia participated. In 1845 he began the publication of his five-volume treatise *Kosmos*, in which he tried to unify all of the physical science of his day.

In 1829 he made a voyage for the Russian czar, who sent him to the Ural Mount and Central Asia to report on mineral resources. Between May and November of that year he traversed, with his associates, the wide expanse of the Russian Empire from Neva to the Yenesei, accomplishing in 25 weeks a distance of some 16,000 km. One of the most important fruits of this journey were the correction of the prevalent exaggerated estimate of the height of the Central Asian plateau.

The last decade of his life was devoted to the continuation of his *Kosmos*. The scope of this remarkable work may be briefly described as the representation of the unity amid the complexity of nature. In it the large and vague ideals of the 18^{th} century are sought to be combined with the exact scientific requirements of the 19^{th} century. And, in spite of inevitable shortcomings, the attempt was quite successful. The science historian **Agnes Mary Clerke** summed up his personality and lifework in the statement:

"*After every deduction has been made, he yet stands before us as a colossal figure, not unworthy to take his place beside Goethe as the representative of the scientific side of the culture of his country*".

1800 CE **Louis-Francois-Antoine Arbogast**[406] (1759–1803, France). Mathematician. Introduced discontinuous functions and conceived the calculus as algebra of operational symbols. In his book *Calcul des Derivations* (1800) introduced integer powers of $D = \frac{d}{dx}$ as operational symbols for differentiation and integration. In this respect he was far ahead of his time.

1800–1802 CE **Karl Friedrich Burdach** (1776–1847, Germany). Physiologist. Introduced the term *biology* (1800), using it in a restricted sense to denote the combined morphological, physiological, and psychological study of human beings (1800). A broader definition was given in 1802 by **Gottfried Treviranus** (1776–1837, Germany) and **J.B. Lamarck** (1744–1829, France) to signify the study of life in general.[407]

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406 Arbogast was a name of a Frankish general in the Roman army (c. 334–394 CE), one of the greatest soldiers of the late empire, and one of the most interesting personalities of the 4^{th} century.

407 The word biology is formed by combining the Greek $\beta\iota o\varsigma$ (bios), meaning "life", and the suffix '-logy', meaning "science of", "knowledge of", "study of", based

1800–1804 CE **Richard Trevithick** (1771–1833, England). Engineer and inventor. A great rival of **James Watt** in improvement on the steam engine. His earliest invention of importance was his improved plunger pole pump (1797) for deep mining, and in 1798 he applied the principle of the plunger pole pump to the construction of a water-pressure engine, which he subsequently improved in many ways. In 1800 he built a *high-pressure non-condensing steam engine*, which became a successful rival of the low-pressure steam-vacuum engine of Watt. He was a precursor of **George Stephenson** in the construction of *locomotive engines* and introduced *rails* into steam transportation.

In February 1804 he invented and constructed the first *steam locomotive* (railway) in South Wales which was able to haul twenty tons of iron and 70 men. It traveled at 8 km/h on the 16 km track. He was the first to recognize the importance of iron in the construction of large ships, and in various ways his ideas also influenced the construction of steamboats.

Trevithick was born in the parish of Illogan, Cornwall, where his father was manager of important Cornish mines. He had little formal education and was a big man of exceptional physical strength. At the age of 18 he began to assist

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on the Greek verb $\lambda\varepsilon\gamma\varepsilon\iota\nu$, 'legein' = "to select", "to gather" (cf. the noun $\lambda o\gamma o\varsigma$, 'logos' = "word"). The term "biology" in its modern sense appears to have been introduced independently by :

- **Karl Friedrich Burdach** in 1800,
- **Gottfried Reinhold Treviranus** (*Biologie oder Philosophie der lebenden Natur*, 1802) and
- **Jean-Baptiste Lamarck** (*Hydrogéologie*, 1802).

The word itself appears in the title of Volume 3 of **Michael Christoph Hanov**'s *Philosophiae naturalis sive physicae dogmaticae: Geologia, biologia, phytologia generalis et dendrologia*, published in 1766.
Before biology, there were several terms used for study of animals and plants. *Natural history* referred to the descriptive aspects of biology, though it also included mineralogy and other non-biological fields; from the Middle Ages through the Renaissance, the unifying framework of natural history was the *scala naturae* or *Great Chain of Being*. *Natural philosophy* and *natural theology* encompassed the conceptual basis of plant and animal life, dealing with problems of why organisms exist and behave the way they do, though these subjects also included what is now geology, physics, chemistry, and astronomy. Physiology and (botanical) pharmacology were the province of medicine. *Botany*, *zoology*, and (in the case of fossils) *geology* replaced *natural history* and *natural philosophy* in the 18^{th} and 19^{th} century before *biology* was widely adopted.

his father and soon showed considerable aptitude for mechanical invention. He went to work in Peru and Costa Rica (1814–1826), but was financially ruined in the Peruvian revolution of the 1820's. He returned to England in 1827, and in 1828 petitioned parliament for a reward for his inventions, but without success. He died penniless, at Dartford. A *Live of Richard Trevithick, with an account of his Inventions* was published in 1872 by his third son, Francis Trevithick (1812–1877).

1801 CE **Johann Georg von Soldner** (1776–1833, Germany). Mathematician. Used classical Newtonian gravitation theory to calculate the bending of starlight rays in the sun's gravitational field, based on the assumption that light consists of particles moving with velocity c, scattered by the sun. The correct answer[408] was given by Einstein in 1915 in the framework of General Relativity. Einstein was not aware of this work and it was rediscovered only in 1921!

1801–1808 CE **John Dalton** (1766–1844, England). Chemist and physicist. Introduced atomic theory into chemistry. Revived, sharpened and

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408 Soldner's derivation is as follows: By Newtonian mechanics, a *hyperbolic orbit* of a small mass about a massive star, is subjected to the relation $\sin(\frac{\delta}{2}) = \frac{1}{e}$, where $e > 1$ is the eccentricity of the hyperbola, and δ is the angle between the asymptotes, which in turn represents the *angle of deflection* of the orbiting mass. The eccentricity, however, can be shown to be expressible in the form $e = 1 + 2r_{\min}E/GM$, where G is the universal gravitational constant, M is the star's mass, $r_{\min}$ the distance of closest approach and $E = \frac{1}{2}v^2$ the energy per unit mass of the orbiting body, whose orbital velocity is v. For a light particle, $v = c$ and hence $e = 1 + c^2 r_{\min}/GM \approx c^2 r_{\min}/GM$, since $c^2 r_{\min}/GM \gg 1$ in all practical cases. Thus e is very large and δ is very small, leading to the final result

$$\delta \approx 2GM/c^2 r_{\min}.$$

For light grazing the surface of the sun, $r_{\min} = R_\odot = 6.960 \times 10^{10}$ cm, $G = 6.672 \times 10^{-8}$ cgs, $M = M_\odot = 1.989 \times 10^{33}$ g, yielding the "*classical*" value $\delta = 0.87$ seconds of arc. Note that, although the mass of the photon cancels out in the derivation, the *mass must be finite* (i.e., $m \neq 0$); and this was *not* established in prerelativity physics. Moreover, if we had used the *special theory of relativity* instead of Newtonian mechanics, we would have found $\delta = 0$, because light tracks are null geodetics, and in flat spacetime these are straight lines! Shortly after developing the general theory Einstein calculated the deflection and got the same answer. However, he later further developed the theory and found that GTR actually predicts a value twice as large, or 1.75 seconds of arc. The experiment was first performed in 1919 and the result was 1.7 seconds of arc.

quantified the atomic theory of matter, expounded by Leucippos and Democritos 23 centuries before. Prepared the first list of *atomic weights* (1803).

In 1801 Dalton formulated his *law of partial pressures* for gases. [It states that the pressure exerted by a mixture of gases in a closed vessel held at a fixed temperature, is the sum of the pressures which each gas alone would exert if separately confined in the whole volume occupied by the mixture.]

Dalton's atomic theory (1803) supposes that:

(1) all matter is made up of minute particles, called *atoms*;

(2) all atoms of the same element are identical in all respects, particularly in weight or mass. Different elements have atoms differing in weight, and each element is characterized by the weight of its atom[409];

(3) in chemical compounds, a *whole* number of atoms of one element is associated with a *whole* number of atoms of another element, to form a *molecule* of the compound;

(4) each kind of atom has a definite small weight or mass.

Realizing that the absolute weights of atoms are very small, Dalton directed his attention to the determination of the *relative weights*, taking the weight of the lightest atom, that of hydrogen, as unity[410]. With his simple

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[409] Dalton's second assumption has been considerably modified by the discovery of *isotopes*, and can no longer be maintained. One element, e.g. chlorine, may have atoms differing in mass, and the atoms of such an element are not necessarily all the same, since an ordinary element may be a mixture of isotopes. It is the *atomic number* (net positive charge on the nucleus of the atom) rather then the *atomic weight*, which characterizes an element. Distinct isotopes of the same elements differ in atomic weight and have minutely different chemistries (quantitatively as well as qualitatively). Only at temperatures close to absolute zero (0° K $\approx -273.15^\circ$ C) can isotopic differences be (sometimes) very significant (as in e.g. superfluid Helium).

[410] Since the relative average *masses* of atoms are in the ratio of nearly integral numbers in most cases, it is convenient to define an *atomic weight* scale that specifies the weights of all atoms relative to an arbitrary standard [the *relative weights* of two objects are always the same at any given common altitude, and are thus always equivalent to the *relative masses*].

We could choose this standard as the (standard isotope) hydrogen atom H, and we could arbitrarily choose the atomic weight of this species to be a dimensionless number 1 (exactly). Suppose we now express this atomic weight in units of gram, which is the most convenient mass unit for most chemical purposes. Since the mass of the hydrogen atom has been found to be 1.67×10^{-24} g, the number of H atoms in 1.00 g of hydrogen is approximately $\frac{1.00 \text{ g}}{1.67\times 10^{-24} \text{ g/atom}} = 5.99 \times 10^{23}$ atoms. [Therefore, for a pure sample of *any* assumptions Dalton could explain the basic laws of stoichiometry, such as the law of constant proportions (**Proust**, 1799), the law of multiple proportions (**Dalton**, 1802), and the law of equivalent proportions (**Cavendish**, 1788).

John Dalton was born at Eaglesfield, a village near Cockermouth in Cumberland. His father was a poor weaver and a Quaker. As a boy he earned a living partly by teaching rustic youth, and partly as a farm laborer. He had received some instruction in mathematics from a distant relative, and in 1781 left his native village to become an assistant to his cousin who kept a school at Kendal. There he passed the next 12 years, becoming in 1785 a joint manager of the school with his brother Jonathan. In 1793 he moved to Manchester, where he spent the rest of his life. Mainly through **John Gough** (1757–1825), a blind philosopher (to whose aid he owed much of his scientific knowledge), he was appointed teacher of mathematics and natural science at Manchester's New College.

Apart from his work on atomic theory, he published papers on meteorology and *color vision*[411]. Altogether Dalton contributed 116 memoirs to the Manchester Literary and Philosophical society. In 1822 he was elected to the fellowship of the Royal Society and in 1830 he became a corresponding foreign member of the French Academy of Sciences. He never married, but there is evidence that he delighted in the society of women of education and refinement.

1801–1814 CE **William Hyde Wollaston** (1766–1828, England). Physician, chemist and physicist. Discovered the elements *palladium* and *rhodium* (1804) and isolated the second amino acid, *cystine*, from a bladderstone (1810). Invented the total reflection refractometer (1802). First to

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element, the mass corresponding to the number of grams given by the atomic weight of the element, would contain about 6×10^{23} atoms (*Avogadro's number*, 1811).] The international atomic weight scale presently in use is based on the exact number 12 for the atomic weight of the dominant carbon-12 isotope, ^{12}C (the atomic weight of *natural carbon* on earth is 12.01115 due to the presence of stable isotope ^{13}C (1.11% in abundance) as well as the unstable (but constantly replenished) radioactive ^{14}C isotope.

411 Dalton suffered from red-green color-blindness. He published (1794) the earliest scientific description of the condition in a paper "*Extraordinary Facts Relating to the Vision of Colors*" [before him, **Joseph Huddart** (1741–1811, England) described the condition in a letter to the chemist **Joseph Priestley** (1777)]. Dalton believed that color blindness is caused by coloration of the eye's vitreous humor. He bequeathed his eyes for science, but upon a post mortem examination it was found to be normal. In 1995, his eye's DNA were examined and the cause of his color-blindness was finally discovered — a genetic defect.

observe (1802) Fraunhofer lines in the spectrum. First to fix a meniscus lens to a Camera Obscura (1812). Secretary of the Royal Society (1804–1816).

Before the invention of *photography*, a *Camera Obscura* was a blackened box-like apparatus which was used by painters for the reproduction of complex images (the image was inverted on the base and it simply had to be sketched by hand). Initially, simple biconvex lenses had been used in these dark chambers, but the problem with them was that the images they produced were clear in the middle and blurred around the edges. Wollaston therefore introduced the *meniscus* which was convex on one side and concave on the other, and to which he adopted a diaphragm on the concave side. With its equal focal distances the meniscus gave much better definition[412].

Wollaston also proved the elemental nature of *niobium* and *titanium*. He developed a method of making platinum malleable. His consideration of geometrical arrangements of atoms led him into crystallography and the invention of the reflecting goniometer to measure angles of crystal faces. The mineral *Wollastonite* was named in his honor.

Though he was formally educated as a physician, his great curiosity led him into study and research in the fields of chemistry, physics, astronomy, botany, physiology, pathology, and crystallography. He was one of the most influential scientists of his time.

Wollaston was born in East Derham, Norfolk and died in London.

1802 CE **William Symington** (1763–1831, Scotland). Engineer. Built the tug *Charlotte Dundas*, the first practical *steamboat* equipped with stern paddle.

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[412] It still had appreciable *lateral chromatic aberration* which produced colored fringes on the outer parts of the field. Then there was the *astigmatic deformation* beyond the half-field of 20^0 and the optical distortions that made straight lines appear curved. Finally, the last fault, which appeared when the Wollaston device was used in photographic instruments, was that the aperture was limited to $f/11$ due to *spherical aberrations*. In 1821 the Frenchman **Charles Chevalier** made a positive lens using crown-glass which has a low refractive index and a weak dispersion. He stuck it on to a negative lens made of lead glass, (flint glass), which has a high refractive index and high dispersion. This was the first *double lens objective* which eliminated the *chromatic aberration*, but did not eliminate astigmatism, which remained in the edges. The field distance also remained limited to $f/11$. Finally, **Joseph Petzval** (1841) added a modified telescope lens to the Chevalier lens and thus noticeably eliminated their spherical distortions. It allowed an aperture of $f/3.6$, a previously unknown photographic speed. Even at apertures as large as $f/1.6$ this objective maintained an excellent definition up to 5^0 off the axis

1802 CE **Charles-Francois Brisseau de Mirbel** (1776–1854, France). Botanist. Founder of plant-cytology and physiology (1802).

Mirbel concluded from his numerous observations of plant structure that "*the plant is wholly formed of a continuous cellular membranous tissue. Plants are made up of cells, all parts of which are in continuity and form one and the same membranous tissue*".

Mirbel worked at the Musée d'Histoire Naturelle (1798–1803) and was director of gardens at Malmaison from 1803.

1802–1816 CE **Lorenz Oken (Ockenfuss)** (1779–1851, Germany). Naturalist and philosopher. Sought to unify the natural sciences. Contended that natural sciences can only offer partial knowledge and demand completion by a metaphysical – idealistic interpretation of nature. [His book: *Philosophy of Nature* (1802).] In his speculations, he foreshadowed theories of the cellular structure of organisms, the protoplasmic basis of life (1805), and that light is a state of stress of the ether (1808).

Okenfuss was born at Bohlsbach, Swabia. He changed his name to Oken upon his appointment to privatdocent at Göttingen (1801). His reputation at Göttingen has reached the ear of Goethe, and in 1807, Oken was appointed an associated professor of medical sciences in the University of Jena. In 1808 he advanced the preposition that "light could be nothing but a polar tension of the ether". Founded the influential periodical *Isis* (1816). He then became a professor at Munich (1829), and Zurich (1833).

1802–1826 CE **Heinrich Wilhelm Matthias Olbers** (1758–1840, Germany). Astronomer and physician. Pointed out that in an infinite, homogeneous, static Newtonian universe the mean radiation density would be as high as on the surface of a star.[413]

Born at Arbergen near Bremen, where his father was a minister. He studied medicine and mathematics at Göttingen during 1777–1780. In 1779 he devised a method of calculating cometary orbits.

Olbers settled as a physician in Bremen (1781) and practiced medicine actively for about 40 years — during day-time only. The greater part of each night (he never slept more than 4 hours) was devoted to astronomy, the upper portion of his house being fitted up as an observatory. He paid special attention to comets, and that of 1815 (period = 74 years) bears his name.

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[413] For further reading, see:

- Harrison, E., *Darkness at Night*, Harvard University Press, 1987, 293 pp.

On 28 March 1802 he discovered the asteroid *Pallas* and later the minor planet *Vesta*.

While watching the sky for many years, it occurred to him to ask the naive question (1826): "*Why is the night sky dark, away from the Milky way*?" This question is known as '*Olbers' Paradox*', since according to the classical Newtonian cosmology [eternal-infinite-Euclidean-static-homogeneous universe] there are enough stars to fill the sky, and their light should be sufficiently intense to set fire to the earth[414].

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414 In the absence of absorption, the apparent luminosity of a star of absolute luminosity L at a distance r in a Newtonian universe model will be $\frac{L}{4\pi r^2}$. If the density of such stars is a constant N, then the number of stars at distances between r and $r + dr$ is $4\pi N r^2 dr$, so that the total radiant energy density due to all stars at a given locus in the universe, should be proportional to $\int_0^\infty \left(\frac{L}{4\pi r^2}\right) 4\pi N r^2 dr = LN \int_0^\infty dr = \infty$. *In words*: doubling the distance of a star reduces the light received from it to one quarter. At the same time, doubling the radius of the shell increases the number of stars fourfold; therefore, we should receive from each concentric equi-thickness celestial shell-volume about us the same amount of starlight. A distant shell of many faint stars gives as much starlight as a nearer shell of fewer and brighter stars.

Olbers attempted to resolve this paradox by suggesting that space was filled with a tenuous absorbing medium. This explanation is invalid, however, as the intervening gas would be heated by the radiation it absorbs until it attained a temperature such that it radiates as much energy as it received, and so no reduction in the average radiation intensity would result.

The stars themselves are of course opaque, and block out the light from sufficiently distant sources. This however does *not* resolve the Olbers' Paradox since every line of sight must terminate at the surface of a star, so the intensity would tend to the average surface brightness of the stars — that is to say, comparable to the surface brightness of the solar disk.

Modern cosmological models avoid the Olbers' Paradox. In such models the mean total energy density of starlight anywhere at epoch t_0 is proportional to $\int_{-\infty}^{t_0} \mathcal{L}(t_1)\left[\frac{R(t_1)}{R(t_0)}\right]^4 dt_1$, $\mathcal{L}(t_1) \equiv \int n(t_1, L) L dL$, where: L is the absolute luminosity of a star as reckoned in a comoving coordinate system, t_0 is the time (epoch) the star is *observed*, t_1 is the time the light is *emitted*, $n(t_1, L)dL$ is the number density of stars of luminosity between L and $L + dL$ at time t_1, and $R(t)$ is the *radial scale factor* of the spacetime metric. In a "big-bang" cosmology there is obviously no paradox, since the integral is cut off at a lower limit $t_1 = 0$, and the integrand vanishes at $t_1 = 0$ roughly like $R(t_1)$. The question of an Olbers' Paradox arises only in models such as the (now defunct) *steady state cosmology*, in which the universe is supposed to have existed for an infinitely long time. In such models, a necessary condition for avoidance of the Olbers' Paradox is that $t_1 R_1^4(t_1)\mathcal{L}(t_1) \to 0$ for $t_1 \to -\infty$. In the case of the

The same question was already asked in 1744 by the Swiss astronomer **Jean Philippe Löys de Chéseaux**. The astronomer **Edmund Halley** stumbled on the paradox even earlier (1721) and **Johannes Kepler** wrote about it in 1610. The first to worry about the problem seems to have been **Thomas Digges** in 1576. Olbers had a copy of Chéseaux's book in his library but apparently never read it, and most scholars credit him with having arrived at the idea on his own.

This paradox, which is based on a superficial observation, has a deep cosmological significance and suggests that the naive Newtonian cosmology is wrong.

Modern cosmological models, based on GTR, avoid the Olbers' Paradox, and the threatened fiery furnace is transmuted into the tepid 2.7 °K microwave background. To see this we note that the divergence of the total energy integral can be avoided by assuming that the stars had not been shining forever but had turned on at some finite time in the past. In that case, the absorbing matter might not have heated up yet or the light from distant stars might have not reached us yet. Even in an infinitely old *expanding*, steady-state universe model with a constant average luminosity per unit volume, Olbers' Paradox is avoided on account of the *red-shift*, which weakens the contribution of distant galaxies over and above the inverse square law, such that the integrated radiant energy remains finite and even negligible. In the favored Big-Bang model, both mechanisms are operative, with the finite age of stars being quantitatively more important in avoiding Olbers' Paradox[415].

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oscillating model, absorption is needed during the highly contracted state and *redshift* during the expansion stage, to save the phenomenon (as the universe expands and galaxies drift apart owing to the expansion of intergalactic space, white light emitted by stars in galaxies far away and long ago arrives feeble and red, and the feeblest starlight arriving from the farthest galaxies is red-shifted into invisibility).

415 The idea that the universe had a finite lifetime also existed in the mid-19th century, although only on the popular fringes of science. The first suggestion that the universe originated in a creative explosion — the first Big Bang — actually came from the pen of **Edgar Allan Poe** (1809–1849, U.S.A.). Poe was not only a well-known writer and poet, but also a scientific popularizer who kept abreast on the latest in astronomical research. In the book-length essay *Eureka* (1848) Poe rejected the idea of an infinite universe, citing Olbers' objections. He reasoned that a universe governed by gravitation would collapse in a heap if not kept apart by some form of repulsion. He postulated that God had, in an enormous explosion at the creation, thrust all the stars apart. Like a rocket racing into the sky, the stars and galaxies would first expand, and then contract into a final catastrophe, the end of the world.

One may use Olbers' Paradox to provide significant constraints on the luminosity that can be attained by very remote galaxies. In this way astronomers were compelled to conclude that most of the luminosity from these young remote galaxies must have been greatly red-shifted.

In *Eureka* Poe offers a solution to *Olbers' Paradox* in which he proposes that the light from very distant stars has not yet reached us. This solution requires only slight amendments to fit in with the standard modern cosmology. In his own words:
"*Were the succession of stars is endless, then the background of the sky would present us a uniform luminosity, like that displayed by the Galaxy — since there could be absolutely no point, in all that background, at which would not exist a star. The only mode, therefore, in which, under such state of affairs, we could comprehend the voids which our telescopes find in innumerable directions, would be by supposing the distance of the invisible background so immense that no ray from it has yet been able to reach us at all*".
It is remarkable that the same man that solved Olbers' Paradox with such brilliant intellectual aplomb, had the capability to match it with an equally grand poetic soul:

"*Deep into the darkness peering, long I stood*
there, wondering, fearing,
Doubting, dreaming dreams no mortal ever
dared to dream before"

Poe was born in Boston. his father deserted the family and his mother died before Poe was three years old. He was raised as a foster child and lived with his new family in London (1815–1820). In 1826 he entered the University of Virginia, but soon left to pursue a literary career. Served in the US Army (1827–1829) and attained the rank of sergeant major. Published his first poetry volume (1827), and married his cousin Virginia Clemm (1836), who at the time was only 14 years old. His most productive years as a fiction writer (1837–1845) were spent in New York City and Philadelphia. His wife died (1847) of tuberculosis and he was engaged (1849) to marry his childhood sweetheart when he suddenly died in Baltimore, apparently of hydrophobia caused by a cat's bite a year earlier.

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Meteorites[416]

"And it came to pass that the Lord cast down great stones from heaven upon them".

Joshua **10**, *11*

It was once thought that the world was small and flat. ***Columbus*** *sailed across the Atlantic (1492) and enlarged it,* ***Magellan*** *(1522) sailed around the world and made it round. It was thought that the earth was at the center of the universe until* ***Copernicus****(1543) restored the sun to its proper place and set the earth in orbit around it. We thought that the earth and its moon were unique until* ***Galileo*** *(1609) showed us that there were similar planets and moons elsewhere in the heavens. And we thought, until the turn of the* 19^{th} *century, that not even a stone could fall from the sky, that nothing could shatter our isolation from the cosmos.*

Fallen meteorites have been recovered throughout history and description of meteorites appear in ancient Hebrew, Chinese, Greek and Roman literature. There are numerous examples of meteorite veneration, such as the black stone of Kaaba enshrined at Mecca.

Yet, the extraterrestrial origin of meteorites was rejected by most people, scientists included, prior to 1800. *Upon hearing a lecture by two Yale professors, President Thomas Jefferson (1743–1826) is said to have remarked: "I could more easily believe that two Yankee professors could lie than that stones could fall from Heaven". The mere fact that meteorite falls had been widely witnessed and specimens had been collected (as, for example, in the* 1751 *fall*

416 *Meteor*: Greek $\mu\epsilon\tau\epsilon\omega\rho\alpha$, literally: "things in the air", from $\mu\epsilon\tau\alpha$ = beyond and $\alpha\epsilon\iota\rho\epsilon\iota\nu$ = to lift up. Hence *meteorology* — the study of the atmosphere and the weather. A chunk of matter in space is called a *meteoroid*. A *meteor* (shooting star) is a brief flash of light that is visible at night when a meteoroid strikes the earth's atmosphere. When it reaches the ground it becomes a *meteorite*. This happens with extreme rarity at any one locality, but over the entire earth probably about 500 meteorites fall each year. The total mass of the meteor-producing meteorites that enter the atmosphere each day is estimated to be from 10 to 100 tons. Of these, only about 10 tons reach the ground each year. For further reading, see:

- Heide, F., *Meteorites*, The University of Chicago Press, 1957, 144 pp.

near Zagreb, Yugoslavia) did not tip the scales in favor of the extraterrestrial theory of meteorite origin.

In 1772, *the distinguished French chemist* ***Antoine Lavoisier*** *wrote a memorandum in which he concluded that stories of stones falling from the sky are mere fabrications — since meteorites could not possibly come from outside the earth. He was later beheaded by the revolutionaries' guillotine, although not for that reason.*

In that year (1794), ***Ernst Florens Friedrich Chladni*** *(1756–1827, Germany) openly suggested that meteorites are not terrestrial but cosmic in origin. Conclusive evidence came on April* 26, 1803, *when many witnesses observed the explosion of a bolide that pelted the French village of l'Aigle (Orne). Afterwards, many meteorite stones were found, reportedly still warm, on the ground. The austere French Academy, whose members were among the last diehards, sent the noted physicist* ***Jean Baptiste Biot*** *to investigate the matter and collect evidence to refute the rumors about stones falling from the sky. In his exhaustive report he stated that he believed the witnesses and finally convinced the scientific community of the extraterrestrial nature of meteorites.*

One of the barriers to accepting the existence of meteors and meteorites was the absence of a theory which predicted they exist. About the time ***Biot*** *made his observations,* ***Laplace*** *put forward his* Nebular Hypothesis *for the formation of the solar system. In the* Nebular Hypothesis, *one would expect debris in the form of stones in interplanetary space remaining to this day. Thus, the science of astronomy was transformed from a bastion of powerful empirical arguments against their existence into a strong supporter.*

Occasionally, scientists are wrong, and the illiterate peasants reporting observations of exotic events are correct. But in most cases it is the other way round. An interesting experiment was conducted in 1962 by the astronomer ***Frank D. Drake****; in 1962 two very bright fireballs (a type of meteor) burst over West Virginia, USA, at about 10 P.M. about a month apart. Astronomers were sent to collect meteorite bits and interview people about what they saw. We know what they* should have seen, *since fireballs are well-studied physical phenomena, so the interviews were a* test *of observation by inexpert witnesses who suddenly were exposed to unfamiliar phenomena.*

It turned out that 14 % of witnesses reported hearing a loud noise at the same time they saw the fireball, despite the fact that the witnesses had no contact with each other. A few simple calculation show that the visual stimulus could not have been accompanied by any sound whatsoever. Drake *suggested this auditory delusion to be due to a crossover in the brain when strong uninterpreted stimuli occur*[417].

Diogenes and the French Academy, or – History Lessons

During the first five centuries BCE, Greek philosophers established the foundations of much of modern science. ***Aristarchos*** *of Samos proposed that the earth moves around the sun;* ***Democritos*** *and* ***Leucippos*** *described atomic structure;* ***Herophilos*** *of Thrace described the brain as the organ of thought;* ***Empedocles*** *proposed the idea of primal elements and forces;* ***Pythagoras*** *and* ***Euclid*** *developed geometry;* ***Diogenes*** *proclaimed that meteors move in space and frequently fall to earth; and* ***Archimedes*** *founded the subjects of mechanisms and hydrostatics.*

Many of these enduring ideas were not easily accepted; Pythagoras, for example, was forced to flee Magna Graecia because of his bizarre suggestion that numbers constitute the true nature of things.

By the 6^{th} *century CE, the Greek and the Roman civilizations had declined. Much of Greek science had been recast into theological scripture, which rapidly crystallized into dogma. Deviations from the official view were not tolerated.*

During the "Dark Ages", not much happened. With the exception of a few enlightened individuals such as ***Leonardo da Vinci*** *in the* 15^{th} *century, scientific and scholarly development in the Western world was rather quiet*

[417] Drake's findings also revealed that a witness's memory of exotic events fades very quickly: after one day, about half of the reports are clearly erroneous; after two days, about 3/4 are clearly erroneous; after four days, only 1/10 are good; after five days, people report more imagination than truth. Later they were *reconstructing* in their imagination an event based on some dim memory of what happened. We know today that *collective delusions* are extremely common in UFO 'observations'.

for about a thousand years. Because mirrors were not readily available in the dark Ages, da Vinci survived persecution for his dangerously heretical ideas by writing his notes backward. By contrast, the Eastern world continued to develop a thriving civilization and enjoy many ingenious fireworks displays.

By the beginning of the 16^{th} *century, as astronomical measurements improved, astrologers and calendar makers had grown increasingly dissatisfied with the inaccuracy of Ptolemy's geocentric (earth-centered) cosmology.* ***Copernicus*** *and others proposed a heliocentric (sun-centered) cosmology, which directly challenged scriptural doctrine.*

Despite attacks by theological authorities, the heretical Copernican model proved to be more accurate than the Ptolemaic and was eventually adopted, but not without causing a profound shock to Western's society's metaphysical assumptions. The new cosmology dethroned the earth, and by implication, human beings, from the center of the universe.

By the end of the 17^{th} *century, Renaissance luminaries such as* ***Newton****,* ***Galileo****, and* ***Descartes*** *had changed the course of Western civilization by splitting the world into two distinct realms. Theology became the authoritative voice for spirituality, morality, and the mental world, and science became the authority for the material world. Although the effects of their revolution would not directly affect most people for a century or more, the Church's reaction to the growing scientific revolution was brutal and persistent.*

Galileo was persecuted for his audacious proposal that moons orbit the planet Jupiter. ***Kepler*** *was accused of blasphemy in suggesting that the moon controlled the motion of the tides. Many Renaissance scientists were charged with heresy; some survived the wrath of the orthodox, many did not.*

While not fully appreciated for several centuries, Newton's famous paper on the nature of light, published in 1671 in the Philosophical Transactions of the Royal Society*, described an experiment that could not have worked the way he said it did. He was well aware that his experiment, involving the refraction of white light through a glass prism, was an idealization, but he did not acknowledge this until he was challenged by a contemporary who tried and failed to repeat his experiment. Newton was apparently so convinced that his theory about light was correct that he fabricated an experiment to confirm it. Fortunately for Newton, his intuition was correct.*

By the middle of the 18^{th} *century, there were as many theories about the nature of electricity as there were experimenters. Most of the theories had something in common – the Newtonian-Cartesian concept of a mechanical-corpuscular world – but although all the experiments involved electricity and the experimenters read each other's works, their theories bore no more that a meager resemblance.*

Meanwhile, the Royal Society *in the United Kingdom was suppressing evidence that supported the existence of phenomena they interpreted as "witchcraft". Centuries later, those phenomena would be largely understood in terms the psychological concepts of suggestion, hypnosis, and hysteria.*

By the end of the 18^{th} *century, the Newtonian-Cartesian worldview, with its underlying principles of* positivism *(what is real is measurable),* reductionism *(complex systems can be understood by reducing them to their individual parts) and* materialism *(everything real is made of matter), had created an outstanding success of modern science. But the new emergence of successful scientific theories carried a severe price – systematic exclusion and denial of natural phenomena that did not fit the prevailing theories.*

By paying homage to prevailing theories, the French Academy *(of Science) soundly denounced public reports of "hot stones" falling from the sky, because both common sense and science agreed that there were obviously no stones in the sky, thus there was nothing to fall. The reported phenomena were declared as delusions, and therefore the witnesses of such phenomena were officially pronounced mentally deranged. A few radical scientists suggested that possibly a few stones might be cast into the sky by distant volcanic eruptions, but the prestige of the French Academy was so great that museums all over Western Europe threw away their specimens of rocks that fell from the sky. As a result, there are very few preserved meteorite specimens in France that date prior to 1790.*

In 1879, Thomas Edison developed the first successful electric light bulb. He was already famous for many other successful inventions. But when Edison announced his new invention, scientists worldwide were incredulous. In response to the critics, Edison wired up the streets of Menlo Park, New Jersey, the location of his famous laboratory, and artificially illuminated the night sky for the first time in history. A professor, Henry Morton, who lived nearby and personally knew Edison, did not bother to view the evening exhibition, which went on night after night. Instead, he was so confident that the claimed invention was impossible that he offered the sober opinion that Edison's experiments were a "conspicuous failure, trumpeted as a wonderful success. A fraud upon the public".

Of course, Edison had already been denounced as a fraud for his invention of the phonograph years earlier, so one can imagine his amusement upon reading the opinion of Edwin Weston, a respected specialist in arc lighting, who asserted that Edison's claims were "so manifestly absurd as to indicate a positive want of knowledge of the electric circuit and the principles governing the construction and operation of electrical machines".

Meanwhile, back in Britain, after the lime was proposed as a cure for scurvy (a serious disease caused by malnutrition due to depletion of vitamin

C) the British medical establishment declared the proposal laughable and refused to put limes aboard ships. It took another 50 years to convince an entirely new generation of physicians that the cure actually worked, and over that sad half-century, thousands of sailors needlessly lost their lives.

About the same time, prior to the radical "germ theory" of disease, a Viennese physician named ***Semmelweiss*** *reported that washing one's hands before obstetrical assistance could prevent what was then a widespread disease threatening newborns: childbed fever. He was viciously scorned and rejected by his contemporaries and died a broken man several years later in an insane asylum.*

By the middle of the 19^{th} *century, Scottish physicist* ***James Clerk Maxwell*** *had brilliantly synthesized 150 years of unorganized empirical observations about electricity and magnetism, and biologist* ***Charles Darwin*** *had described his theory of evolution. These and other significant developments aroused great hostility among mainstream scientists of the day. Maxwell's theory was called "scandalous"; Darwin's theory was condemned as absurd by both scientists and theologians;* ***von Helmholtz****'s idea that physical experimentations could teach us how the human body worked was severely denounced.*

By the end of the 19^{th} *century, physics professors were so confident in the highly accurate results of Newtonian physics that they began to discourage their best students from pursuing careers in physics because most of the difficult problems had already been solved. Most of the rest of physics was expected to be little more than a "mopping up" operation – adding a few more decimal places to the known physical constants and resolving a few minor questions about puzzles known as the "ultraviolet catastrophe" and the "photoelectric effect".*

Year before the Wright brothers flew their airplane at Kitty Hawk, Rear-Admiral George Melville, chief engineer of the US Navy, declared that attempting to fly a heavier-than-air aircraft was simply "absurd". A few weeks before the airplane flew, ***Simon Newcomb****, a distinguished professor of mathematics and astronomy at Johns Hopkins University, stated that heavier than air powered human flight was, in scientific terms, "utterly impossible". According to Newcomb, any form of powered flight would require the discovery of entirely new force. With such eminence behind these statements, the mainstream media of the day meekly followed the lead of the authorities, and sneered at the ridiculous notion of the powered flight.*

To add injury to insult, more than two years after *the Wright brothers had first flown their aircraft, and in spite of the fact that dozens of eyewitnesses had actually seen them fly, the popular* Scientific American *magazine continued to ridicule the "alleged" flights.*

Many years later, when the editor of the Wright brothers' hometown newspaper was asked why he had refused to publish anything about their amazing accomplishment, he replied "We just didn't believe it. Of course, you remember that the Wrights at that time were terribly secretive." The interviewer responded incredulously, "You mean they were secretive about the fact that they were flying over an open field?" The editor considered the question and replied sheepishly, "I guess the truth is we were just plain dumb."

*At about the same time as the Wright brothers were flying their impossible machine, **Einstein**, **Bohr**, **Heisenberg** and others had begun to revolutionize physics with the quantum theory. Einstein's theories were vigorously attacked on the basis that their acceptance would throw back science to the Dark Ages.*

*The inventor **Lee De Forest**, was prosecuted for fraud in 1913 for claiming that if was possible to transmit the human voice across the Atlantic by radio.*

The Atoms of Leucippos and Dalton (460 BCE–1803 CE)

The empirical laws of chemical combination, particularly the law of multiple proportions, suggest that the chemical elements react together as though the matter of which they are composed is parceled out into exceedingly minute portions. Each such particle is incapable of further subdivision, so that when two elements combine they do so in masses which are whole multiples of the masses of these individual portions.

Two possible guesses as to the ultimate structure of matter present themselves. The first saw matter as a continuous *structure, completely filling the space occupied by bodies in the same way as jelly fills a mould. The second saw matter filling space discontinuously, with interstitial gaps, much as small pellets fills a barrel. The first view is associated with the Elea school of Greek philosophy, founded by **Xenophanes** (ca 530 BCE); the second is the* atomic *hypothesis, due to* **Leucippos** *(ca 460 BCE), but particularly developed by* **Democritos of Abdera** *(ca 420 BCE). This assumes the division of matter into exceedingly small particles, or atoms, incapable of further division by physical means. It is uncertain whether it was independently proposed in India, but it certainly appears there in a novel form in later Buddhist and Jainist treatises. The idea of divisible atoms (i.e., molecules) was put forward by* **Asclepiades of Bithynia** *(100 BCE).*

Epicuros *(310 BCE) and* **Lucretius** *(57 BCE) adopted the theory, and we possess a long Latin poem of the latter, De Rerum Natura, dealing principally with the atomic theory.* **van Helmont**, **Nicolas Lémery** *(1645–1715, France, 1675),* **Hermann Boerhaave** *(1668–1738, Holland, 1724),* **Boyle** *and* **Newton**, *made use of the hypothesis. Indeed, the latter gave a mathematical demonstration of Boyle's law on the basis of the hypothesis that gases consist of atoms repelling one another with forces inversely proportional to the distances.* **Ruggiero Boscovich** *(1711–1787, 1758) also made extensive application of a similar theory, but considered the atoms as mere points, centers of attractive and repulsive forces, and endowed with mass.*

Bryan Higgins *and* **William Higgins**[418] *(1762–1825, Ireland), in 1777 and 1789 respectively, made some applications of Newton's atomic theory to chemistry, but the merit of having independently elaborated a chemical atomic theory capable of coordinating all the known facts, and of being modified and extended with the progress of the science, belongs unquestionably to* **John Dalton** *(1776–1844).*

Dalton's theory provided no means of determining even the relative weights of atoms. Although 7.94 parts of oxygen combine with 1 part of hydrogen, we do not know how many atoms of each element the molecule of water contains. If it contains one atom of each element (as Dalton supposed), the atomic weight of oxygen is 7.94, but if it contains 2 atoms of hydrogen to 1 atom of oxygen, as **Davy** *supposed from the combining of volume ratio, the atomic weight of oxygen is* $2 \times 7.94 = 15.88$. *Thus, Dalton's assumption that the particles of elements in the free state are single atoms was the main source of the difficulties of the earlier theory.*

[418] William Higgins' book '*Comparative View of the Phlogistic and Anti-Phlogistic Theories*' anticipated the atomic theory later developed by **Dalton** (1803); however, the book was not widely distributed, and Dalton probably never saw it. Higgins also anticipated later chemical symbolism developed by **Berzelius** when he used first letter abbreviations for many elements.

1802–1815 CE **Joseph Louis Gay-Lussac** (1778–1850, France). Chemist and physicist. Among the distinguished chemists of the early 19^{th} century. Discovered (independently of **J. Charles** and **J. Dalton**) that a volume of gas at constant pressure is proportional to the absolute temperature (1802; known as *Charles-Gay Lussac law*). Deduced the equation for alcoholic *fermentation*. Made balloon ascents to investigate the effects of terrestrial magnetism, composition, temperature and moisture of air at altitudes as high as 7016 meters (1804).

Enunciated the law of volumes (or *Gay-Lussac law*), stating that two gases combine chemically such that the volumes involved are in ratio of small numbers (1808). Investigated the composition of water (1805; with **A. von Humboldt**). Established the properties of potassium (1808; with **L.J. Thenard**). Isolated *boron* (in the same year that **Humphry Davy** did), and devised improved methods for analyzing organic compounds (1809). Demonstrated that *sulfur* is an element (1810). Conducted studies of fermentation and improved processes for manufacturing of sulfuric acid, oxalic acid, etc. (1811–1815). Showed that *iodine* is an element and was first to predict the existence of *isomers* (1814). Discovered the gas *cyanogen* and was first to recognize the importance of radicals in chemical reactions (1815).

Gay-Lussac was born at St. Léonard-le-Noblat. In 1797 he was admitted to the École Polytechnique, and in 1800 became an assistant to **C.L. Berthollet** at the École de Ponts at Chaussées. He was a professor of physics at the Sorbonne (1808 to 1832) and subsequently a professor of chemistry at the Jardin des Planets.

Gay-Lussac will be remembered as a bold and energetic scientist. His early adventures heralded the fearless aeronauts: on Sept. 16, 1804 with the thermometer marking $9\frac{1}{2}$ degrees below freezing, he ascended in a balloon, unaccompanied to the altitude of 7 km above sea-level. He remained at this dizzying height, for a considerable time. A year later he investigated at close range the volcanic eruption of Vesuvius. He exhibited great fortitude of spirit and will power throughout a health crisis and under the laboratory accidents that befell him. Only at the very end, when the disease from which he was suffering left him no hope, did he complain with some bitterness of the hardship of leaving his world while the many discoveries being made pointed to yet greater discoveries to come.

1803–1806 CE **Lazare Nicolas Marguerite Carnot**[419] (1753–1823, France). French army general and geometer who took a leading part in the revolutionary changes at the end of the 18th century and the revival of projective geometry. He was also one of the harbingers of the vector concept.

Carnot was born at Nolay in Burgundy. Following the custom of many of the sons of well-to-do French families, he prepared himself for the army and was thus led to the military school at Mézières, where he studied under Monge, becoming a Captain of engineers in 1782. In 1783 he published his first work '*Essai sur les machines en genéral*' in which he proved that kinetic energy is lost in the collision of imperfectly elastic bodies.

The Revolution drew him into political life. As a republican member of the Assembly he voted in 1793 for the execution of Louis XVI as a traitor. In the same year he undertook the organization of the French army to oppose the million-man force that Europe launched against France. In this capacity he was technically responsible for the acts of the Reign of Terror. In 1796 he opposed Napoleon's coup d'etat, and had to flee to Geneva, where he wrote a semi-philosophical work on the metaphysics of the calculus.

In 1800 he became Minister of War, but opposed the increasing monarchism of Napoleon who, however, gave him a pension and commissioned him to write a book on fortification for the military school at Metz. It is during these years that he published *Géometrie de position* (1803) and *Essai sur la théorie des transversals* (1806). In these works the influence of **Desargues** (1593–1662) and **Pascal** is evident, but Carnot went beyond them in the extension of well-known theorems of geometry as well as the development of various coordinate system that are independent of any particular choice of axes.

In 1814, when France was once more in danger, Carnot offered his services and was made a general of a division. He joined Napoleon during the Hundred Days and was made minister of the interior. On the second restoration he was exiled and lived in Magdeburg, occupying himself with science.

His son Sadi became a celebrated physicist. Of his other son Hippolyte he had one grandson, Sadi, who became the 4th president of the 3rd French Republic (1837–1894) and a second grandson, Adolpe, who became an eminent chemist.

1803–1808 CE **William Henry** (1774–1836, England). Chemist. Formulated *Henry's law*: the weight of a gas dissolved by a liquid is proportional

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419 For further reading, see:

- Gillispie, C.C., *Lazare Carnot – Savant*, Princeton University Press: New Jersey, 1971, 359 pp.

to the pressure of the gas over the liquid. It contributed directly to the atomic theory of **John Dalton**, who extended the law to mixtures of gases, in conjunction of his own *Law of Partial Pressures*.

1803–1838 CE **Adelbert von Chamisso** [(Louis Charles Adelaide de Chamisso de Boncourt), 1781–1838, Germany]. Botanist, explorer, novelist and poet of French origin. Some of his poems are known in their musical settings, such as the touching song cycle "Frauenliebe und Leben" ("A Woman's Love and Life"), memorably set by Schumann. Chamisso is also known for the novella "Peter Schlemihl's Remarkable Story," the tale of a man who sells his shadow to the devil for a bottomless purse.

Indeed, this trade brings wealth to Peter Schlemihl, but also exclusion from society and he ends in despair. To end his ordeal, the demon offers him a second deal: his shadow against his soul. But Peter declines, although it means that he looses the woman he loves. With the aid of a pair of magic boots he is wandering the world searching for peace of mind; he finds it as a naturalist, and not in endless wealth. But his magic boots cannot bring him everywhere, he regrets not being able to visit the Pacific islands, notably the coral islands. So even his magic tools, do not enable him to get the ultimate knowledge.

This admirable story of Peter Schlemihl certainly reflects the life and struggles of the naturalist Adelbert Chamisso.

The secret of his popularity lies in his poised blend of the conservative and the progressive, the intellectual and the emotional. Though disguised as a simple tale, it has a profound psychological significance that has kept it all as classic, until the present day.

Chamisso was born at the Chateau of Boncourt in Champagne, France, the ancestral seat of the family. Driven out by the French Revolution, his parents settled in Berlin. During 1798–1808 Chamisso served in the Prussian infantry regiment as lieutenant.

He lived in Berlin (1808–1810) and Switzerland (1810–1812), studying botany, natural science and medicine. He continued his botanical research in Berlin and during 1815–1818 served as a botanist aboard the Russian ship "Rurick" which Otto von Kotzebue commanded on a scientific voyage round the world. His diary of the expedition (1821) is a fascinating account of the expedition to the Pacific Ocean and the Bering Sea[420] During this trip Chamisso

[420] For further reading, see:

- Von Kotzebue, Otto, *A New Voyage Round the World, in the Years 1823, 24, 25, and 26*, Henry Colburn & R. Bently: London, 1830, Two volumes. vol.1:(6), 341 pp. plus three maps (two folding); vol.2(2), 362 pp.

described a number of new species found in what is now the San Francisco Bay Area. Several of these, including the California poppy, Eschscholzia californica, were named after his friend Johann Friedrich von Eschscholtz, the Rurik's entomologist. In return, Eschscholtz named a variety of plants, including the genus Camissonia, after Chamisso. On his return in 1818 he was made custodian of the botanical gardens in Berlin, and was elected a member of the Academy of Sciences, and in 1820 he married.

1803–1843 CE **Marc Isambard Brunel** (1769–1849, England). Inventor and civil engineer. Best known for the construction of the *Thames tunnel* (1825–1843).

Brunel was born in France. Arrived in New York (1793) as a refugee of the French Revolution and practiced there as architect and civil engineer. Sailed to England (1799) in order to submit to the British government his plans for the mechanical production of ships' blocks, instead of the manual processes then employed. His proposals were adopted and the machinery was installed at the Portsmouth dockyard (1803–1806).

He erected many sawmills, experimented with steam navigation, invented a knitting machine (1816), a timber bending machine, etc. He also invented a tunneling shield (1818) and with it bore a tunnel under the Thames river between Wapping and Rotherhithe. He used Portland-cement concrete (1828) for filling in the river bed over this tunnel. (It came into large-scale use a generation later, when 70,000 tons of it went into the making of the London main drainage system.)

- Von Chamisso, Adelbert, *A Sojourn at San Francisco Bay 1816*, Book Club of California: San Francisco, 1836, Folio, v+16 pp+8 plates.

- Von Chamisso, Adelbert, *A voyage around the world with the Romanzov exploring expedition in the years 1815–1818 in the Brig Rurick, Captain Otto von Kotzebue*, translated by H. Kratz University of Hawaii Press: Honolulu, 1986.

- Von Chamisso, Adelbert, *The Alaska Diary 1815–1818*, translated by Robert Fortuine, Cook Inlet Historical Society, 1986.

- Fischer, Robert, *Adelbert of Chamisso: Citizen of the World, Natural Scientist and Poet*, Klopp, 1990.

Brunel got into financial difficulties and was thrown into prison (1821), from which he was freed by his friends, who secured for him a grant of £5000 from the government.

Many difficulties were encountered during the Thames tunnel construction; the river broke through the roof of the tunnel (1827), and after a second irruption (1828), work was discontinued for lack of funds. Seven years later it was resumed with the aid of money advanced by the government, and after three more irruptions the tunnel was completed and opened in 1843. Aided by his son, Brunel displayed immense determination and extraordinary skill and resourcefulness in the various emergencies with which he had to deal, but the anxiety broke down his health.

The second phase of the industrial revolution was affected almost continuously by the economy of the Great French Wars (1792–1815). Even while Napoleon was banished to Elba, Britain was still at war with America. This war regime reached a climax in the years 1806–1811, when Napoleon attempted to exclude British trade from the Continent and Britain counteracted his so-called "continental system" blockade by depriving the Continent of all trade which did not pass through British ports. Although this situation created a strain felt by the munitions industries, including the building, arming and servicing of war ships (on which the safety of the island nation was seen to depend), these wars served to boost technological progress[421] — as the block-making machinery introduced by **Brunel** at Portsmouth dockyard clearly illustrates. In general, the increased use of steam-power and machinery enabled Britain to press home the advantage which she derived from her superiority in iron production. The wars also encouraged their use in the textile industries, amplifying (for instance) the insatiable demand for American cotton which, in turn, bolstered the slavery-based culture of the American South and made inevitable the U.S. Civil War.

1804 CE **Nicolas Théodore de Saussure**[422] (1767–1845, Switzerland). Chemist and naturalist. Laid the foundations of *plant chemistry* (phy-

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421 There has always been a strong underlying relationship between man's general history and the history of his technological progress. The Roman empire, for example, rested upon the achievements of its engineers, including the great road-makers, as truly as it did upon its more abstract concepts of law and duty. The expansion of Europe in the 16^{th} century depended upon the existence of new means of crossing the oceans. In the same way, the bewilderingly rapid and numerous political changes during 1750–1900 influenced, and were in turn influenced by the technological revolutions of the time.

422 Name of a distinguished Swiss family including **Nicolas** (1709–1790),agriculturist). His son **Horace Benedict** (1740–1799), was a professor of philosophy and physics at Geneva who developed the first *electrometer* (1766), invented the

tochemistry); made pioneering work on nutrition and respiration in plants, on fermentation, germination, composition of alcohol, and transformation of starch into sugar. Asserted the importance of CO_2 and soil nitrogen to green plants, and demonstrated that plants absorb water. He showed that plants receive their carbon from atmospheric CO_2, not from the soil as earlier theorists had supposed. Moreover, his experiments represent the first treatment of the subject of *photosynthesis*, using quantitative methods and modern chemical terminology.

1804–1806 CE *Lewis and Clark Expedition*: A group of US frontiermen sent by president **Thomas Jefferson** to reconnoiter the vast new territories west of the Mississippi River acquired by the United States in the Louisiana Purchase. The Journals of the expedition provided valuable scientific information about Indian tribes and the natural wealth of the Western lands.

The expedition included about 5000 frontiermen under the leadership of **Meriwether Lewis** (1774–1809) and **William Clark** (1770–1838). The expedition set out on May 14, 1804 from St.Louis on a round trip of 13,000 km to the Pacific Ocean. They went up the Missouri River across the Rocky Mountains, then down the Columbia River to the Ocean in canoes. On the return trip Clark went down the Yellowstone River, reaching St. Louis on Sept. 23, 1806.

1804–1812 CE **Nicolas-Francois Appert** (1750–1841, France). Chef and inventor. Produced a process for preserving food in hermetically sealed containers: he put precooked foods in sealed glass bottles and heated them in boiling water. Appert published his reports in 1810 – a pioneering enterprise in heat sterilization of food. He later established the first commercial cannery (1812).

1804–1820 CE **Jean-Baptiste Biot** (1774–1862, France). Physicist, mathematician and astronomer. Discovered with **Felix Savart** that the intensity of the magnetic field set up by a current flowing through a wire varies inversely with the distance from the wire (known as *Biot-Savart Law*). This law is fundamental to modern electromagnetic theory. Made a balloon ascension with **Gay-Lussac** (1804) to study the upper atmosphere and terrestrial magnetism. Collaborated with **Arago** to study refractive properties of gases.

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world's first *Solar Collector* (1767), built first *hygrometer* utilizing a human hair (1783) and introduced the term *geology* (1799). Nicholas Théodore was the eldest son of Horace. Horace's grandson, **Henri** (1829–1905) was an entomologist. **Ferdinand** (1857–1913), son of Henri, was a linguist, regarded as the father of modern linguistics.

Investigated polarization of light passing through chemical solutions (1815). Became a professor of Mathematical Physics at the College de France (1800).

1804–1834 CE **Louis Poinsot** (1777–1859, France). Mathematician. Contributed significantly to *analytical* and *geometrical mechanics*. Introduced the concept of a *force-couple*, and proved that every system of forces is equivalent to a system consisting of a sum (resultant) acting at an arbitrary point O and a couple whose moment is equal to the moment of the system about O. In 1834, Poinsot gave a geometrical description of the force-free motion of a rigid body about a fixed point, known as *Poinsot's construction*[423].

Poinsot (1809) wrote an important book on polygons and polyhedra [*Mémoire sur les polygons et les polyèdres*], discovering four new regular polyhedra[424]. Two of these appear in **Kepler**'s work *Harmonice Mundi* (1619), but Poinsot was unaware of this. On the subject of polygons, he determined the number of n-pointed regular (noncompound) polygons that can be drawn around the circumference of a circle[425]. Today, this enumeration has become important in electrical network theory, statistical mechanics and numerical analysis.

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[423] Describes the motion of the *inertia-ellipsoid* relative to a plane perpendicular to the angular momentum vector of the body about the fixed point (*invariable plane*). According to Poinsot, the motion of the body is equivalent to the rolling the momental ellipsoid on the fixed tangent plane.

[424] A simple polyhedron is a closed shape enclosed by faces, all of which are plane polygons (e.g. pyramid, prism, frustum). A convex polyhedron is said to be regular if its faces are regular and equal (e.g. tetrahedron, cube etc.). Non-simple polyhedra can have holes. **Kepler** described the small and the great *stellated dodecahedra*, which do not fit Euler's relationship $(F = 12,\ V = 12,\ E = 30)$, since their faces intersect themselves. **Cauchy** (1810) proved that any regular polyhedron must have the same face planes as the 5 platonic solids and thus deduced that no further regular polyhedra can exist.

[425] n points are drawn at evenly spaced intervals on the circumference of a circle. These points are then joined with line segments to form a polygon. The points can be joined consecutively or skipped over any fixed interval $(d-1)$. There are three possible classes of outcomes:
(a) regular convex polygons $(d = 1)$ for any n;
(b) $d > 1$ but prime to n, resulting in non-convex regular polygon (sides have equal length and consecutive sides form equal angles; e.g. the 5-pointed star or the 6-pointed star of David);
(c) $d > 1$, but n and d have common factors. The degenerate form is known as a *compound polygon*. Cases (a) and (b) together yield non-compound *polygons*. Poinsot asked: how many *non-compound* polygons are generated by n points.

Poinsot was born in Paris. He studied at the École Polytechnique during 1794–1797, and left in order to enter the École des Ponts et Chaussées. He eventually gave up the idea of becoming an engineer. From 1809 until 1826 he was both inspector general of the Université de France and teacher and examiner at the École Polytechnique. From 1839 until his death he worked at the Bureau des Longitudes. He showed no interest in algebra and was one of the principal leaders of the revival of geometry in France during the first half of the 19^{th} century.

1805 CE, Oct. 21 *Battle of Trafalgar* (sandy cape on Spain's southern coast, at the western entrance to the Strait of Gibraltar). British navy under Horatio Nelson defeated a combined French and Spanish fleets in one of the greatest naval battles in history. The victory ended the invasion threat of Napoleon and gave England undisputed domination of the seas throughout the 19^{th} century.

1805 CE The German pharmacist **Friedrich Sertürner** (b. 1783) extracted *morphine* from opium (1805) and used it to relieve pain. [It was not until 1925 that the chemical structure of morphine and other alkaloids were fully known.]

Sertürner named the new crystalline substance *morphium*, but the name soon changed to morphine. In 1817 he determined the alkaloid nature of morphine, thus marking the beginning of alkaloid chemistry. His isolation is the first of an alkali with a vegetable origin.

In 1905, the physician **Carl Gauss** of Freiburg, brought down the wrath of the Lutheran Church when he used morphine to induce *dammerschlaf* (the twilight sleep treatment) in women experiencing difficult births. The Church fathers — non of them mothers — declared that Dr. Gauss has fallen from

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His answer was

$$N = \begin{cases} \frac{1}{2}(n-1) & \text{if } n \text{ is prime} \\ \frac{n}{2}\left(1-\frac{1}{m_1}\right)\left(1-\frac{1}{m_2}\right)\left(1-\frac{1}{m_3}\right)\cdots\left(1-\frac{1}{m_k}\right) & \text{if the \textit{different} prime} \\ \quad \text{factors of } n \text{ are } m_1, m_2, m_3, \ldots, m_k, & \text{excluding } n \text{ and unity.} \end{cases}$$

Thus for $n = 7, 8, 9, 10$ we have respectively $N = 3, 2, 3, 2$.
The preceding relationship between the *geometry* of star polygons and the *theory of numbers* (1809) followed the earlier discovery by **Gauss** (1801) that polygons with a prime number of sides could be constructed (using only a compass and a straightedge) if and only if the number of sides was a *prime* of a special form. Both examples hint to a close relationship between geometry and the theory of numbers.

grace and was near unto heresy because the Bible says (*Gen* **4**, 16) that women were to bring forth in pain.

1805 CE **Joseph Marie Jacquard** (1752–1834, France). Inventor. A silk weaver who perfected the loom with an attachment that made the loom weave patterns *automatically*. The attachment automated the weaving of fabric through use of a series of cards with punched holes, forerunners of the punched cards used later for input to early computers.

1805–1831 CE **Sophie Germain** (1776–1831, France). A mathematician, contemporary of **Gauss**, **Cauchy** and **Legendre**, with whom she corresponded. Contributed to number theory, acoustics and elasticity. Proved a *restricted* form of Fermat's conjecture: The equation $a^p + b^p = c^p$ has no solution in integers prime to p, if p is an odd prime and $2p + 1$ is also a prime.

It follows from this theorem that the equation $a^p + b^p = c^p$ has no solution in integers not divisible by p. The proof of the theorem is quite simple and shows how far one can go with very elementary arguments. In 1831 Germain introduced the notion of *mean curvature* of a surface, $M = \frac{1}{2}(k_1 + k_2)$.

Germain took correspondence courses from the Ecole Polytechnique in Paris since women were not allowed in the building of the school. During 1811–1816 she presented memoirs on the theory of vibrating plates based on the experimental work of **Chladni**. **Gauss** was so impressed by her work that he recommended her for honorary degree from the University of Göttingen. Unfortunately Germain died before the degree could be awarded.

1805–1814 CE **Francois Joseph Servois** (1767–1847, France). Mathematician. Published ideas on 3-dimensional vectorial systems, and developed the first elements of what became known as the *operational calculus* (1814). **Hamilton** later attributed to him the nearest approach to an anticipation of vectors and quaternions. Developed the notion of a mathematical '*operator*'. Introduced the term '*pole*' in projective geometry, and was one of the chief precursors of the English school of symbolic algebra.

Servois was born at Mont-de-Laval, Doubs, France, a son of a merchant. He was ordained a priest at Besancon at the beginning of the Revolution, but in 1793 gave up his ecclesiastical duties in order to join the army. In 1794, after a brief stay at the artillery school of Chalons-sur-Marne, he was made a lieutenant. With the support of **Legendre**, he was appointed professor of mathematics at the artillery school of Besancon (1801). He later served in this capacity at Metz (1802–1808), and finally was appointed curator of the artillery museum at Paris (1816–1827).

Although Servois did not produce a major body of work, he made a number of original contributions to various branches of mathematics and paved the road for important later developments in vector theory, operational calculus, projective geometry and symbolic algebra. Thus, his memoirs directly inspired the work of **George Boole** (1847).

1805–1814 CE **William Congreve** (1772–1828, England). Inventor and pioneer of military rocketry. Developed a war rocket that could carry explosives and was driven by gun-powder. It was used by British troops in the Napoleonic wars [the shelling of and burning of Boulogne (1806), Copenhagen (1807) and Leipzig (1813)] and in their war against the United States Army (1812–1814)[426]. The Congreve rocket was in use by the British army until 1860, when cannons became more accurate.

Congreve was a versatile man of science: he invented a process of color printing (1821), water-marks on banknotes, and was first to suggest the armoring of battleships.

He was educated at Trinity College, Cambridge.

1806–1820 CE **Charles Julien Brianchon** (1785–1864, France). Mathematician. Contributed mainly to projective geometry. Discovered jointly with **J.V. Poncelet** (1820) the *nine-point circle*[427].

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[426] After watching the rocket attack of British troops on Fort McHenry in Maryland (1812), **Francis Scott Key** described *the rocket's red glare* in "*The Star-Spangled Banner*".

[427] *Brianchon's Theorem*: if all the sides of a hexagon are tangent to a conic, then the diagonals joining opposite vertices are concurrent.

Brianchon discovered it when he was a 21-year-old student and published it in 1806 in the Journal of l'École Polytechnique.

The nine-point circle: the mid-points of the sides, the feet of the altitudes, and the mid-points of the lines joining the orthocenter to the vertices of the triangle are concyclic.

The theorem concerning this circle is named for neither **Brianchon** nor **Poncelet** (joint paper in Gergonne's *Annales* for 1820–1821), but for yet a third mathematician **Karl Wilhelm Feuerbach** (1800–1834) who in 1822 published this and some related theorems. Feuerbach showed that the center of the circle lies on the *Euler line* and is midway between the orthocenter and the circumcenter. *Feuerbach's Theorem* then states that the *nine-point circle* of any triangle is tangent internally to the *inscribed circle* and tangent externally to the three *escribed circles*, possibly one of the most beautiful theorem in elementary geometry that has been discovered since Euclid.

In the 19^{th} century the *geometry of the triangle* made noteworthy progress by

Brianchon was born at Sevres. During 1804–1808 he studied at the École Polytechnique under **Monge**. He then became a lieutenant of artillery in the armies of Napoleon (1808–1813). In 1818 he was appointed professor at the Artillery School of the Royal Guard.

The 'Elastic Skin' of Liquids

*If a thin glass tube (a fraction of a millimeter in diameter) is lowered into water, then in violation of the law of communicating vessels, the water in it will begin to rise rapidly, and its level will become considerably higher than that of the large vessel. The discovery of this phenomenon, known as capillarity[428], is attributed to **Leonardo da Vinci** (ca 1500).*

*What forces are supporting the weight of the column of liquid that has risen up? The answer was given some 300 years later by **John Leslie**[429] (1802): the rise is accomplished by the forces of adhesion between the water*

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the above authors and **Steiner**. But it was many years before the subject attracted much attention. **Lemoine** (1840–1912) was the first (1873) to take up the subject in a systematic way and to contribute extensively to its development. **Henri Brocard** (1845–1922; France) discovered certain critical points of the triangle that bear his name.

428 From the Latin *capilla* = thin as a hair.
Plants and trees have an entire system of long ducts and pores. The diameters of these ducts are less than a hundredth of a millimeter. Because of this, capillarity forces (aided by negative osmotic pressures) raise soil moisture to a considerable height and distribute water through the plant.
If a sheet of blotting paper is observed through a microscope, it is seen to consists of a sparse network of paper fibers, forming thin and long ducts that play the role of capillary tubes. Capillarity causes kerosene to rise through the wick of a lamp, and in the technology of the dyeing industry, frequent use is made of a fabric's ability to draw in a liquid through the thin pores formed by its threads.

429 **John Leslie** (1766–1832, Scotland). Mathematician and physicist. In 1805 he was elected to succeed **John Playfair** to the chair of mathematics at Edinburgh, and in 1819 was promoted to the chair of natural philosophy.

and the glass (for a diameter of 0.01 *mm, the height of the rise is about* 15 *cm).*

The physicist ***Francis Hauksbee*** *(1666–1713, England) made first accurate observations of the capillary action of tubes and glass plates, and ascribed the action to an* attraction *between the glass and the liquid (1709). He concluded that only those particles of the glass which are very near the surface have any influence on the phenomenon.* ***James Jurin*** *(1718) showed that the height to which the same liquid rises in tubes, is inversely proportional to their radii. The concept of* surface-tension[430] *was first introduced in* 1751 *by* ***Johann Andreas von Segner*** *(1704–1777). He ascribed it to short range attractive forces. Segner also attempted to calculate the effect of surface tension in determining the form of a drop of liquid. His results had a most important effect on the subsequent development of the theory: first, they showed that* macroscopically, *the surface of a liquid is in a state of tension similar to that of a two-dimensional elastic membrane, stretched equally in all direction. Second, it gave hope for deducing this surface tension from a* microscopic *molecular theory.*

Indeed, the works of ***Thomas Young*** *(1804) and* ***P.S. Laplace*** *(1806) provided the natural quantitative aspect to Segner's ideas. Thomas Young founded the theory of capillary phenomena on the principle of surface tension. He also observed the constancy of the angle of contact of a liquid surface with a solid, and showed how to deduce from these two principles the phenomena of capillary action. He supposed particles to act on one another with two different kind of forces: one of which, the attractive force of cohesion, extends to particles at a greater distance than those to which a repulsive force is confined. The attractive force is constant throughout the small distance to*

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[430] In liquids (and solids) the average distance between molecules is about the same as a molecular diameter, so molecules are essentially in contact with their nearest neighbors.

The molecules of a liquid experience strong attractive (*cohesive*) forces that resist attempts to separate molecules. These forces have short ranges. Consequently, the molecules in a liquid interact only with their nearest neighbors. But surface molecules have less neighbors than bulk ones. So the surface effectively has a positive potential energy proportional to the number of molecules, i.e. to the surface area. Any physical system will tend spontaneously toward a condition of minimum potential energy (apart from entropy effects). In a liquid, this entails a tendency toward minimizing the surface area. Because the surface area in equilibrium is minimal, work is required to increase the surface area. The amount of this work per unit area is called the *surface tension*. A fixed volume of free liquid will therefore assume the shape of a sphere, because this shape has the least surface area for a given volume.

which it extends, but the repulsive force increases rapidly as the distance diminishes.

The subject was taken up by Laplace, who furnished us with seminal quantitative results that have never been surpassed.

Consider an experimental set up in which air can be pumped into a spherical soap bubble. To increase the radius of the bubble by an amount dr*, a certain amount* dW *of external work must be done to overcome the resistance of the surface to an increase of its area. Clearly* $dW = pdV = p4\pi r^2 dr$*, where* p *is the excess pressure inside the bubble above the outside pressure and* dV *is the change of volume due to expansion.*

In a state of equilibrium, the bubble is held together by a surface tension T $\left(\frac{\text{force}}{\text{length}}\right)$*. The work done by this force is* TdS*, where* $dS = d(4\pi r^2)$ *is the change of surface area due to the expansion. Equating the two expressions we arrive at Laplace's law:* $p = \frac{2T}{r}$*. When the same logic is applied to a cylinder of radius* r*, we obtain* $p = \frac{T}{r}$*.*

Laplace (1806) generalized these results to arbitrary curvature radii R_1 *and* R_2 *in two perpendicular directions:* $p = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$*. This equation equates the outward excess pressure* p *to the* inward *pressure due to the surface tension. At equilibrium,* p *and* T *are fixed over the entire surface, that is*

$$\frac{1}{R_1} + \frac{1}{R_2} = \textit{mean curvature} = \textit{const},$$

at all points. This equation then represents a surface of minimal area *for the volume enclosed, otherwise known as a* minimal surface*. Planes, cylinders and spheres belong to the family of such surfaces.*

Note that for fixed T*,* p *varies like* $\frac{1}{r}$*. This explains why very small spheres with only thin walls can withstand enormous internal pressures (plant cells often have internal pressures of* 10 *atmospheres with walls only a few microns thick!)*

The rise, h*, of a wetting liquid of density* ρ *in a capillary of radius* r *can be calculated by equating the total upward surface-tension force* $2\pi r(T\cos\beta)$ *[*β *= angle between tube wall and tangent to the liquid surface at the wall] to the weight of the liquid column* $(\pi r^2 h)\rho g$*, yielding*

$$h = \frac{2T\cos\beta}{\rho g r}.$$

The same result can be obtained from energy considerations: the total energy is $E(h) = \frac{1}{2}\cdot(\pi r^2 h^2 \cdot \rho g) - (2\pi r h)T\cos\beta + c$*. Solving the equation* $\frac{\partial E}{\partial h} = 0$ *for* h*, yields the same result.*

*The next step was taken by **C.F. Gauss** (1830). Instead of calculating the direction and magnitude of the resulting force on each particle, arising from the action of neighboring particles, he formed a single expression for the sum of the potential energies of the system constituents: the first depending on the action of gravity, the second on the mutual action between the particles of the fluid, and the third on the action between particles of the fluid and the particles of a solid or fluid in contact with it. The condition of equilibrium is that this expression shall be a minimum. The condition, when worked out, gives not only the equation of the free surface in the form already established by Laplace, but the conditions of the angle of contact of this surface with the surface of a solid.*

During 1830–1869, ***J.A.F. Plateau** made an elaborate study of the phenomena of surface tension. **Lord Kelvin** (1887) calculated the effect of surface tension on the propagation of surface waves of a liquid.*[431]

1807 CE **Robert Fulton** (1765–1815, U.S.A.). An American inventor. Designed and built the first commercially successful *steamboat*. Also made important contributions to the development of the *submarine*.

Fulton was born on a farm in Lancaster County, Pennsylvania, and showed inventive talent at an early age. He went to Philadelphia at the age of 17 and was apprenticed to a jeweler. At the age of 21 he went to England and made a moderate living in London as an artist. After 1793 he gave his full attention to developments in science and engineering, and painted only for amusement. He began to travel, studied science and higher mathematics and learned French, Italian and German.

About 1797 Fulton turned his attention to submarines, a project which claimed his energies until 1806. The problem of submarine navigation received his practical attention during the time that he was making his experiments on steam propulsion. He constructed two submarine boats in France, and one in America. One of the former, the "*Nautilus*"[432] was built with the direct

[431] In the 1970's a Minkowski-space version of the Plateau minimal-surface-area problem was applied to studying the relativistic, quantum dynamics of *fundamental strings*, leading to major advances in elementary particle theory.

[432] It seems that this name was adopted by **Jules Verne** (1828–1905, France) in his book "*Twenty Thousand Leagues Under the Sea*" (1870). It is a story about Captain Nemo, a mad sea captain who cruises beneath the oceans in an internationally-manned submarine.

encouragement of Napoleon in 1801. It was 6.4 m long, and supplied with compressed air for respiration. He descended in it to a depth of 8 m, remaining under water for fully 4 hours. Although Fulton's submarine ideas interested both Napoleon and the British Admirality, neither nation showed much interest in the craft, even though it sank several ships in demonstrations[433].

Fulton's submarine was propelled by manual power; two horizontal screws were employed for propulsion and two vertical screws for descending and ascending. It was built of wood with iron ribs, and was sheathed with copper.

In 1802 Fulton became interested in the steamboat. An experimental boat, launched on the Seine River in Paris in 1803, sank because the engine was too heavy. But a second boat, which was built in the same year, operated successfully[434].

Fulton returned to the United States in 1806, and in 1807 he directed the building of a steamboat in New York, which he named the *Clermont*. On Aug. 17, 1807, this vessel began its first successful trip up to Hudson River to Albany.

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[433] Fulton failed to convince either the English, French or the United States governments of the adequacy of his submarine boats. Thus, in Brest harbor, he was able (1801) to blow up a small vessel with a torpedo sent from his 6.5 m long Paris-built *Nautilus*, before the watchful eyes of a commission appointed by Napoleon. Although it was still propelled manually, the *Nautilus* was equipped with a *reduction gearing system*; it could maintain 4 men under water for 4 hours. His *Nautilus II*, was launched at Brest harbor heading for the British fleet, but was unable to get close to any of the ships because the English (informed by spies) had rowing boats on constant patrol around their vessels. *Nautilus II* returned to port without having attached its explosive charge to anything, and France lost interest in the so-called *fish-boat*. Turning to the enemy (1805), Fulton tried to persuade England to adopt his submarine. Despite having the support of the Prime Minister Pitt, he came up against the opposition of the first Lord of the Admiralty, John Jervis, who saw in it a serious treat to the British supremacy of the seas. When Fulton succeeded, experimentally, in blowing up the schooner *Dorothy* by attaching to it an explosive charge which was detonated from a distance using an electric cable, the success of his demonstration only served to reinforce Jervis' hostility. Queen Victoria had her own peculiar reservation to "submarine" warfare on the ground that it was an unBritish (i.e. ungentlemanly!) way to win a war!

[434] Allegedly, Fulton offered Napoleon his services in building for him a fleet of steam battleships for the invasion of Britain. Napoleon, however, rejected the idea. One wonders whether or not history could have changed its course, had the French warlord accepted the challenge.

1807–1816 CE **Humphry Davy** (1778–1829, England). Chemist. Isolated potassium and sodium through electrolysis (1807). Invented the arc lamp in 1809. Proposed *Chlorine*[435] as an element (1810) [Until then it was commonly believed that it was a *compound* which contained Oxygen and known by the name *Oxymuriatic acid*]. **Davy** (1816) was also able to finally prove that diamond is actually carbon.

Diamond (16–1955 CE)

Diamond is one of the most valuable precious stones. Its unequaled physical properties and intrinsic beauty place it in a unique position among other minerals. The discovery of diamonds dates back to ancient times[436]*, but the first undoubted application of the name to diamond is found in* ***Manilius*** *(16 CE) and* ***Pliny the Elder*** *(ca 75 CE), though Romans only knew diamonds of small dimensions. Most of the known large, valuable stones were*

[435] In 1811 **J.S.C. Schweigger** proposed the name *halogen* for Chlorine. **Berzelius** (1823) accepted Davy's theory that Chlorine is an element. Then (1825), Berzelius used the name *halogen* for the elements Fluorine, Chlorine and Iodine. Bromine was not discovered and added until 1826.

[436] The name '$\alpha\delta\alpha\mu\alpha\varsigma$' ("the invincible") was probably applied by the Greek to hard metals, and thence to corundum [(Al_2O_3), has some valuable transparent colored varieties, including *ruby* and *sapphire*]. The "diamond" (*Yahalom*) mentioned in the Old Testament [Ex **39**, 11], used in the breastplate of the high priest, was certainly some other stone, for it bore the name of a tribe, and methods of engraving the true diamond cannot have been known so early. The stone became familiar to the Romans only after being introduced from India, where it was probably mined at a very early period. Later Roman authors mentioned various rivers in India as yielding the *Adamas* among their sands. The name *Adamas* became corrupted into the forms *adamant*, *diamaunt*, *diamant*, *diamond*.

found only at the beginning of the 17^{th} century. However, the nature of the diamond still remained a mystery.

Isaac Newton *(1675) conjectured that the diamond was combustible on account of its high refractive power; this was first established experimentally by the Italian Academicians,* ***Averani***[437] *and* ***Targioni***[438].

Smithson Tennant *(1797) and* **Humphry Davy** *(1816) finally proved that diamond actually is carbon.*

Of all the solid elements (at S.T.P.), only sulphur, gold and diamond (carbon) are found in nature in their pure state. Since ancient times, diamonds have been found in the form of grains, or small octahedrons, in alluvial deposits, whence they had been carried from dark, igneous rocks. Efforts to find diamonds in the original rock hardly ever met with success.

There are only four known instances of diamonds being found in their parent rock. The oldest one is the mine in the Kimberley region of South Africa, where they occur in a decomposed olivine rock (Kimberlite). Another substantial deposit of diamonds in parent rock is in the Vilyuy river basin in Yakutsk (Russia). In 1961, diamonds in Kimberlite were discovered in Sierra Leone. The most well-known diamond alluvial deposits are in Zaïre, Minas Gerais (Brazil), Angola, Tanzania, Ghana and on the West African coast (Guinea, Ivory Coast, Liberia). In North America they are found in Arkansas, Ohio, Indiana and Wisconsin.

Diamonds are formed in nature at great depths (80 km or more) at $1100-1300\,^{\circ}C$ *under great pressure by eruption in the volcanic pipes. It is carried with the rising Kimberlite to the surface of the earth. Diamonds, together with other minerals and rocks, are carried away by water, along rivers, some of them into the sea. Therefore, secondary deposits are found in river beds, on old valley terraces and along the Atlantic coast on old beaches. Small diamonds were also discovered in certain meteorites, both stones and irons; [e.g. Novo-Urei, Penza, Russia 1886; Carcote, Chile; Canyon Diablo, Arizona].*

The history of many of the diamond discoveries is interesting and the fate which pursued the prospectors and the stone themselves is vivid and

[437] **Giuseppe Averani** (1662–1738, Italy). Professor at the University of Pisa. Performed various experiments in physics and botany. Member of the London Royal Society.

[438] **Giovanni Targioni-Tozzetti** (1712–1783, Italy). Naturalist and physician. Curator of the Botanic Garden and professor of botany at the University of Florence. Director of the Magliabechiana Library (1739).

dramatic[439]*. Many large diamonds of rare quality are now the property of royalty or of governments. The largest stone ever discovered was the Cullinan (Kimberly, 1905, original weight* $= 3106$ *carats;* 1 *carat* $= 0.2054$ *gram.*

The commercial manufacture of diamonds was begun in 1955 by the General Electric Company in the US, after technique for obtaining very high pressure (over $70,000$ *atm.) at high temperature (*$2000\,°C$*) had been developed. The crystallization of artificial diamonds is favored by the addition of a small amount of a metal such as nickel. Artificial diamonds therefore contain some nickel atoms replacing pairs of carbon atoms.*

Physical structure and properties:

Density $= 3.510$*; refractive index* $= 2.417$ *for the D line; high dispersive power.*

Diamond has a cubic unit of structure with side $a = 3.56\text{Å}$*; there are*

8 *atoms in the unit cell at coordinates:*

$(0,0,0)$; $(\frac{1}{2},\frac{1}{2},\frac{1}{2})$; $(\frac{1}{2},0,\frac{1}{2})$; $(\frac{1}{2},\frac{1}{2},0)$; $(\frac{1}{4},\frac{1}{4},\frac{1}{4})$; $(\frac{1}{4},\frac{3}{4},\frac{3}{4})$; $(\frac{3}{4},\frac{1}{4},\frac{3}{4})$; $(\frac{3}{4},\frac{3}{4},\frac{1}{4})$.

Carbon, with 4 *electrons missing from a complete octet, can form* 4 *covalent bonds. In the diamond, each atom is bonded strongly to* 4 *neighboring atoms which are held about it at the corners of a regular tetrahedron. These covalent bonds bind all of the atoms into a single giant molecule. Since the C-C bonds are very strong, this* network solid *crystal is very hard, in fact the hardest substance known.*

Recently (1999) it was conjectured that Saturn's atmosphere rains diamonds (!), the free fall of which converts gravitational energy into heat, thus driving that planet's weather system. The same may be true for Uranus and Neptune.

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[439] Perhaps the most dramatic of them all is *the affair of the diamond necklace* (1784–1786) — a mysterious incident at the court of Louis 16^{th} of France, which involved the queen Marie Antoinette and contributed to render her very unpopular, probably sealing her fate. It involved: cardinal Louis de Rohan (formerly ambassador to Vienna), Comte and Comtesse de Lamotte, Boehmer and Bassenge (Paris jewelers), Marie Lejay, Reteaux de Villete (small time crook) and Cagliostro (famous charlatan) — a complex story of greed, passion and doom.

1807–1820 CE Gas lights first introduced in London (1807). By 1820 much of the cities of London and Paris were lighted.

1807–1821 CE **Georg (Wilhelm Friedrich) Hegel** (1770–1831, Germany). Philosopher. Perhaps the most abstruse of the Teutonic thinkers. Built a huge edifice to contain the whole of human knowledge. In his effort to reveal the implications of reality and reason, he employed the method of thesis, antithesis and synthesis, with analysis as the starting point. The examination of contradictions as the second step, and finally the arrival at unity by means of reason in a summation of ultimate truths. Hegel's system is applied to the whole experience, beginning with logic, going on to the philosophy of nature and then to the philosophy of mind and spirit. Within these categories, anthropology, philosophy, metaphysics, law, ethics, morality, government, property, the family, emotions, customs, art, religion, history and many other facets of thought and life were examined analytically, in their opposites and finally in synthesis.

His point of view, that *everything* is a logical process of thought obeying the laws of evolution from the simple to the more complex, held sway in Germany and influenced other countries until the middle of 19^{th} century; even though it lost some of its popularity after that date, it continued to influence world thought for many years thereafter to such a degree that it had profound effects on the ideas and *political events* of 20^{th} century.

His theories echo through the writings of **Marx**, **Kierkegaard**, **A. N. Whitehead**, **John Dewey** and a group of British and American thinkers known as *Neo-Hegelians*.

Among his important work are: *Phenomenology of mind* (1807); *Science and Logic* (1812–1816), *Encyclopedia of Philosophy* (1817); *Philosophy of Law* (1821).

Hegel developed the most systematic and comprehensive philosophy of modern times. He sought to synthesize the ontology of the ancient Greeks (Particularly the theories of **Aristotle**) with Kantianism[440]. Some of his views may be traced to the influence of Heraclitos, **Spinoza**, **Schelling** and **Fichte**. In fact, notwithstanding its revolutionary emphasis, *the Hegel system represents largely a synthesis of other philosophers*.

Kant had set a precedent for such synthesis when he combined the conceptual (rational) world of ideas with the phenomenal world of perception as the basis for valid knowledge. Similarly, Hegel set out to synthesize all opposites

[440] Kant argued that one could *suppose* God's existence, but no system could *prove* it; Hegel instead seeks to justify the *idea* of God. Kant separated science from religion, Hegel wanted to make religion into a new science.

to arrive at truth which is considered as an organic unity of applied parts; according to his grammar of logical thinking, every condition of thought or of things, every idea or every situation in the world (*thesis*), leads irresistibly to its opposite (*anti-thesis*), and then unites with it from a higher or more complex whole (synthesis)[441]. This scheme constitute the formula and secret

[441] This '*dialectic movement*' runs through everything that Hegel wrote. It is an old thought, foreshadowed by **Empedocles**, and embodied in the 'golden mean' of **Aristotle**, who wrote that '*the knowledge of opposites is one*'. Moreover it comprises one of the 13 logical principles on which the Hebrew Talmudic exegetics are based; ["*Shnei ketuvim ha'makishim ze et ze...*"; Barai'ta d'Rabi Ishmael; ca 115 AD].

Hegel generalized this principle to embrace all things and thoughts; thus, a social system with free economy stimulating individualism is required in a period of economic adolescence and unexploited resources, but in a later age a cooperative commonwealth is preferable; the future will see neither the present reality nor the envisioned ideas, but a synthesis in which something of both will come together to beget a higher life. For Hegel, history too is a dialectical movement, almost a series of revolutions, in which people after people, and genius after genius, become the instrument of the spirit of the Age (*Zeitgeist*). What actually happens to a state or people represents the final judgment as to the worth of a national policy or course of action. For Hegel world history constitutes the world's court of justice. Reason is constantly evolving in history toward an absolute goal. God exists only as a 'world-spirit' which is real because it is rational. HISTORY ADVANCES AND PROGRESSES ONLY BECAUSE OF CONFLICTS, WARS, REVOLUTIONS, I.E. THROUGH RELIGIOUS STRUGGLES. PEACE AND HARMONY DO NOT MAKE FOR PROGRESS. HEGEL'S LOGIC LEADS TO THE CONCLUSION THAT WAR IS JUSTIFIED BECAUSE IT IS THE MEANS BY WHICH PROGRESS IS MADE. Moreover, it carries the unfortunate implication that whatever has been successful is thereby also somehow 'right' and superior to what had been unsuccessful. Whatever vanished from the memory of history (because it was destroyed or unsuccessful) was to Hegel '*unjustified existence*'. The Hegelian system was adopted by the Prussian state and many Prussian thinkers held that the Prussian state was destined to carry forward the realization of universal reason through its eventual conquest of the world.

The dialectical process makes change in the cardinal principle of life. No condition is permanent; in every stage of things there is a contradiction which only the strife of opposition can resolve.

After Hegel died, German philosophers gravitated around him, some approved him, and some other supported his theories. His followers '*Young Hegelians*' eventually splitted into '*right*', '*left*' and '*center*' over questions of theology: 'Right Hegelians' defended traditional Christianity; 'Center' sought to reinterpret religious dogma in Hegelian terms to give it a new, more scientific language;

of all development and all reality[442].

For not only do thoughts develop and evolve according to this '*dialectical movement*'[443], but things do equally; every condition of affairs contains a contradiction which evolution must resolve by a reconciling unity. The higher stage, if reached, too will divide into a productive contradiction, and rise still to loftier levels of organization, complexity and unity. The movement of thought, then, is the same as the movement of things; in each there is a dialectical progression from unity trough diversity to diversity in unity. Thought and being follow the same law; and logic and metaphysics are one.

Hegel, the native of Stuttgart, studied philosophy and theology at the national university of Tübingen (1788–1793). He took part in the walks,

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'Left Hegelians' criticized Christianity and developed Hegel's ideas toward radical conclusions, not only in theology. **Mosses Hess** (1812–1875), **Ludwig Feuerbach** (1804–1872) and **Karl Marx** (1818–1883) turned Hegel's philosophy of history into a theory of *class struggles* leading by Hegelian necessity to inevitable socialism. In place of the Absolute as determining history through the *Zeitgeist*, Marx offered mass movements and *economic forces* as the basic causes of every fundamental change, whether in the world of things or in the life of thoughts. He has argued a la Hegel, that since change is a road to better thing, a society based on private property would give way to one in which socialism was supreme via a synthesis of opposites. The collapse of Soviet Communism in 1989 demonstrated that at least Marx's extrapolation was wrong.

Since the second half of the 19^{th} century, *positivists* and *existentialists* questioned the role of Hegel's philosophical reasoning and sought to replace it [**Comte** (1798–1857); **Kierkegaard** (1813–1855); **Husserl** (1859–1938)].

442 In modern physics one could view the electron through 'Hegelian eyes' according to a triadic structure: thesis (particle), antithesis (wave) => synthesis (non-classical quantum-mechanical entity). Likewise, in modern biology, protein synthesis is achieved through the combination of two single ladders into a double-stranded DNA molecule. In general, the movement of evolution is a continuous development of oppositions, and their merging and reconciliation. **Darwin** (1809–1882), like Hegel, also starts from what has been empirically successful and argues back to the supposed necessity of it's appearance. In Darwin, however, there is no longer a rational dialectic of nature, but instead a principle of '*natural selection*'. Both Hegel and Darwin can be miss-construed to support a belief in the '*survival of the fittest*'. Seen in the light of such a 'Darwinian Hegelienism', world history presents a very ugly spectacle – at its most grotesque in the rise and fall of Nazism and Soviet communism.

443 Way back in ancient times, Greek philosophers applied this strategy to arrive at truth; a system of arguments, which bring out the *contradiction* in ones opponent's reasoning.

beer-drinks and love-making of his fellows and gained most from intellectual intercourse with his contemporaries **Hölderlin** and **Schelling**.

After leaving the university he became a private tutor at Bern and lived there in intellectual isolation (1793–1796). Schelling recommended his appointment to the faculty of Jena University (1797–1808). But in 1807, the wife of Hegel's landlord gave birth to Hegel's illegitimate son and the philosopher moved to Nuremberg, where he assumed the post of a teacher of philosophy in a classical high school for boys (1808–1816). During that period he married (1811) Marie von Tucker (the daughter of a respected Nuremberg family) scarcely half his age. When, Christiana Burckhardt, the mother of Hegel's illegitimate son, Ludwig, heard of the marriage and tried create a stir, Hegel had been paying money to support his son and appeared to have placated her.

At the age of 46 (1816) Hegel went to Heidelberg to take up his first secure full-time academic post. Finally, in 1818 he succeeded **Fichte** at the University of Berlin. From this point on, in accordance with his new status and public role in Berlin, Hegel's philosophizing took on the form of *lectures*.

In his last years, Hegel denounced the radicals and aligned himself with the Prussian Government and basked in the sun of its academic favors. His enemies called him '*the official philosopher*'[444]. He began to think of the Hegelian system as part of the natural laws of the world; he forgot that his own dialectic condemned his thought to impermanence and decay. Never did philosophy assume such a lofty tone, and never were its royal honors so fully recognized and secured as in 1830 in Berlin.

When the cholera epidemic came to Berlin in 1830, his weakened body was one of the first to succumb to the contagion. Just as the space of a year had seen the birth of Napoleon Beethoven and Hegel, so in the years from 1827 to 1832 Germany lost Goethe, Hegel and Beethoven. It was the end of an epoch, the last fine effort of Germany's greatest age.

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[444] E.g.: Hegel did not believe in the immorality of the soul. But being a respectable civil servant of the Prussian State he was forced to give in a bit and not let his ideas to be spread among people.

Worldview XVIII: Georg W.F. Hegel

* * *

"Philosophy always comes an the scene too late to give instructions to what the world ought to be. As a thought of the world, it appears only when it is already there, cut and dried, after the process of formation has been completed."

* * *

"What exists is reason; ... Reason is the substance of the universe, ... The design of the world is absolutely rational[445]".

* * *

"In a true tragedy, both parties must be right".

* * *

"The people are that part of the state which does not know what it wants".

* * *

"We learn from the history that we do not learn from history".

* * *

445 In the light of the experience of the Holocaust and Stalin's totalitarianism, reason itself appears insane as the world acquires systematic totality.

"My task is to turn Kantian criticism into a true system, in other words, to overcome the divisions it still contains by deriving all its elements from a single fundamental principle".

* * *

"What is rational is actual, and what is actual is rational".

* * *

"As far as history goes, we must rather deal with what had been and what is. In philosophy, on the other hand, with what is *and is eternally".*

* * *

"Beauty is the mediation between the sensible (or sensuous) and the rational (or intellectual). My definition of beauty as 'pure appearance of the idea to sense' is true of beauty throughout the history of its embodiment in art. But art has the particular task of showing within the realm of the human, the essence of the divine".

* * *

"... It is science which had led you into this labyrinth of the soul, and science alone is capable of leading you out again and healing you".

* * *

"I believe that philosophy, like geometry, is teachable and must no less than geometry have a regular structure".

* * *

"Nothing great in the world has been accomplished without passion".

* * *

"Life is not made for happiness, but for achievement".

* * *

"The history of the world is not a theater of happiness; periods of happiness are blank pages in it, for they are periods of harmony".

* * *

"Great men had no consciousness of the general idea they were unfolding, ... but they possessed insight into the requirement of the time – what was ripe for development. This was the very Truth for their age, for their world; they merely placed another stone on the pile, as other have done; somehow he has the good fortune to come last, and when he placed his stone the arch stood self-supported".

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Philosophers on Hegel

* * *

"Philosophers are doomed to find Hegel waiting patiently at the end of whatever road we travel".

Richard Rorty

* * *

"All the great philosophical ideas of the past century – the philosophies of Marx and Nietzsche, phenomenology, German existentialism, and psychoanalysis – had their beginnings in Hegel".

Maurice Merieau-Ponty (1908–1961)

* * *

"Whether through logic or epistemology, whether through Marx or Nietzsche, our entire epoch struggles to disentangle itself from Hegel".

Michel Foucault (1926–1984)

* * *

"It may well be that the future of the world, and thus the sense of the present and the significance of the past, will depend in the last analysis on contemporary interpretation of Hegel's work".

Alexandre Kojère (1900–1968)

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"The highest of audacity in serving up pure nonsense, in stringing together senseless and extravagant mazes of words, such as had previously been known only in madhouses, was finally reached by Hegel, and become the instrument of the most bare-faced general mystification that has ever taken place, with a result which will appear fabulous to posterity, and will remain as a monument to German stupidity."

Arthur Schopenhauer (1788–1860)

* * *

"We surrealists, recognize Hegel as one of the first of our own mad company, willing to explore the furthest reaches of Unreason in order to win a new, expanded and higher form of Reason".

Andre Breton (1892–1966)

* * *

"One could write intellectual history of our century without mentioning Hegel. The 19th century thinkers whose spirits have dominated the 20th century have been Marx, Kirkegaard and Nietzsche. At the beginning of this century Sigmund Freud brought to light the unconscious and Ferdinand de Saussure the structure of Language. Meanwhile science had made explosive progress, more or less obvious to the continuing debates among philosophers of science. It is possible to leave Hegel out of the picture".

Lloyd Spencer (1996)

* * *

1807–1822 CE **Jean Baptiste Joseph Fourier** (1768–1830, France). Mathematical physicist and politician who exerted great influence on his field. Was first to assert (1807) that an *arbitrary* function (such as were understood in his time), given in the interval $(-\pi, \pi)$, could be expanded in a trigonometric series

$$a_0 + \frac{1}{\pi}\sum_{n=1}^{\infty}[a_n \cos nx + b_n \sin nx],$$

if

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx;$$

$$a_n = \int_{-\pi}^{\pi} f(x)\cos nx dx;$$

$$b_n = \int_{-\pi}^{\pi} f(n)\sin nx dx, \qquad n > 1.$$

First to establish a mathematical theory of heat conduction in isotropic solids (1811).

He proved that the above expansion, known today as the *Fourier expansion*, or *Fourier series*, holds for certain simple functions which he needed in the problems of heat conduction. Since then, these series have been used extensively in the solution of the differential equations of mathematical physics[446].

Although **Euler** (1748), **d'Alembert** (1749, 1754), **Clairaut** (1757), **Lagrange** (1759) and again **Euler** in 1777, used the above coefficients (Fourier made no claim to its discovery!) the credit goes to Fourier[447] because he was the first to apply these coefficients to the representation of an entirely arbitrary function. He was also the first to allow that the arbitrary function might be given by different analytical expressions in different parts of the interval.

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446 The Fourier coefficients a_n and b_n have the remarkable property that they give the best least-squares fit among all possible approximations when a function $f(x)$ is expanded in terms of an orthonormal set of functions. This implies that the *mean square error* $\frac{1}{2\pi}\int_{-\pi}^{\pi}\left[f(x) - F_n(x)\right]^2 dx$ is minimized when $F_n(x)$ coincides with the partial n-th sum of the Fourier-series expansion for $f(x)$.

447 Why did great mathematicians like **Euler** and **Lagrange** miss the final crucial step?

It was, no doubt, partially because **Fourier**'s disregard for rigor, that he was able to take *conceptual* steps which were inherently impossible to men of more critical genius. However, once the floodgate of the new idea opened, mathematicians hurried to exploit it and make it into one of the most efficient tools of modern linear mathematics.

Indeed, the theory of Fourier Series was further developed by **Poisson** (1820), **Cauchy** (1826), **Dirichlet** (1837), **Stokes** (1847), **Riemann** (1854), **Lipschitz** (1864), **H.E. Heine** (1870), **Cantor** (1872), **Du Bios-Reymond** (1875), **U. Dini** (1880), **Jordan** (1881), **Lebesgue** (1902), **Fejer** (1904), **Riesz** (1907) and **Fisher** (1907). The origin of the theory of the *Fourier Integrals* is found in Fourier's "Analytical Theory of Heat" (1822). *Fourier transforms* are due to **Cauchy** who pointed out the reciprocity of the Fourier integrals (1826).

Fourier was born at Auxerre. He was the son of a tailor, and was orphaned in his 8^{th} year. His admission into the military school of his native town was secured through the kindness of a friend. He soon distinguished himself in mathematics. Barred from entering the army on account of his poverty and low birth, he was appointed teacher of mathematics at the same school. In 1787 he became a novice at the abbey of St. Benoit-sur-Loire, but he left in 1789 and returned to his college. From 1789 to 1794 Fourier taught in secondary schools and also became actively involved in the French Revolution. As a result of this latter activity, Fourier spent some time in the prison of Auxerre in 1794. In 1795 he was on the faculty of the École Normale in Paris and thereafter occupied the chair of analysis at the École Polytechnique.

Fourier was one of the savants who accompanied Bonaparte to Egypt in 1798. During his expedition he was called on to discharge important political duties in addition to his scientific ones. He was for a time virtually governor of half of Egypt. He returned to France in 1801 and during the rest of his life combined scientific and political activities. As a politician Fourier achieved uncommon success, but his fame rests chiefly on his strikingly original contributions to science and mathematics.

The theory of heat conduction in solids engaged his attention quite early, and in 1812 he obtained a prize offered by the Academy of Sciences for his memoir: *Theorie des mouvements de la chaleur dans les corps solides* — an epochal paper in the history of mathematical and physical science. The works of Fourier have been collected and edited by **Gaston Darboux** in 1889–1890.

Scientists on Fourier Analysis

* * *

"Fourier's book was of paramount importance in the history of mathematics and pure analysis perhaps owed it even more than applied mathematics."

(Poincaré, 1895)

* * *

"Looking back, we can see Fourier's memoir as heralding the surge of new mathematical methods and results which were to mark the new century. His ideas are built into the commonsense of our society."

(T.W. Körner, 1988)

* * *

"It is difficult to say which of Fourier results is most to be praised: their uniquely original quality, their transcendently intense mathematical interest, or their perennially important instructiveness for physical science."

(Lord Kelvin, 1880)

* * *

"Fourier's book is a great mathematical poem."

(J.C. Maxwell)

* * *

Fourier and his Integral [448]

When Joseph Fourier published his book "Téorie analytique de la chaleur" (1822), he certainly could not foresee that he had provided scientists of the 20^{th} century with one of their most powerful research tools. Indeed, one can hardly find a better example for the metamorphosis of a successful idea, than the story of the Fourier integral and its applications.

Historical Perspectives

In the 17^{th} century, ***Isaac Newton*** *showed that the way to understand the natural world is to use differential equations that govern the motion of objects under given forces.* ***Albert Einstein*** *merited it as "the greatest intellectual stride that has ever been granted to any man to make". Predictive science became possible, prompting* ***Laplace*** *to imagine a single formula that would describe the motion of every object, for all time.*

Some 150 years after Newton, ***Joseph Fourier*** *provided a practical way to extract the truth from a whole class of such equations: linear partial differential equations. He asserted that virtually any 2π-periodic function $f(x)$ can be represented as the infinite sum of sines and cosines, now known as a Fourier series:*

$$f(x) = a_0 + \sum_{k=1}^{\infty}(a_k \cos kx + b_k \sin kx), \tag{1}$$

where the Fourier coefficients $\{a_0, a_k, b_k\}$ are calculated by

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(x)dx;$$

[448] For further reading, see:

- Wiener, N., *The Fourier Integral and Certain of its Applications*, Dover Publications: New York, 1958, 201 pp.
- Titchmarsh, E.C., *Introduction to the Theory of the Fourier Integrals*, Oxford University Press: Oxford, 1948, 391 pp.
- Sneddon, I.N., *Fourier Transforms*, McGraw-Hill Book Company: New York, 1951, 542 pp.

$$a_k = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos kx dx; \tag{2}$$

$$b_k = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin kx dx.$$

Roughly, what this means is that any curve that periodically repeats itself, no matter how jagged or irregular, can be expressed as the sum of perfectly smooth oscillations. Knowing $\{a_0, a_k, b_k\}$*, one may reconstruct the original function from its Fourier coefficients.*

The Fourier coefficients a_k *and* b_k *have the remarkable property that they give the best least-squares fit among all possible approximations when a function* $f(x)$ *is expanded in terms of an orthogonal set of functions. This implies that the mean square error* $\frac{1}{2\pi} \int_{-\pi}^{\pi} [f(x) - F_n(x)]^2 dx$ *is minimized when* $F_n(x)$ *coincides with the partial n-th sum of the Fourier-series expansion for* $f(x)$*.*

To prove this we want to minimize, for an arbitrary expansion of $f(x)$*, the expression:*

$$S(a_k, b_k) = \int_{-\pi}^{\pi} [f(x) - a_0 - \sum_{k=1}^{n} (a_k \cos kx + b_k \sin kx)]^2 dx.$$

This requirement is met when the partial derivatives of S *w.r.t.* a_0, a_r*, and* b_r *are set to zero, namely*

$$\int_{-\pi}^{\pi} [f(x) - a_0 - \sum_{k=1}^{n} (a_k \cos kx + b_k \sin kx)] dx = 0$$

$$\int_{-\pi}^{\pi} \cos rx [f(x) - a_0 - \sum_{k=1}^{n} (a_k \cos kx + b_k \sin kx)] dx = 0, \qquad r = 1, 2, ..., n$$

$$\int_{-\pi}^{\pi} \sin rx [f(x) - a_0 - \sum_{k=1}^{n} (a_k \cos kx + b_k \sin kx)] dx = 0$$

Using the orthogonality relations

$$\int_{-\pi}^{\pi} \cos rx \cos kx dx = \pi \delta_{kr} = \int_{-\pi}^{\pi} \sin rx \sin kx dx,$$

$$\int_{-\pi}^{\pi} \sin rx \cos kx dx = \int_{-\pi}^{\pi} \cos rx \sin kx dx = 0,$$

we regain the Fourier coefficients:

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx;$$

$$a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos kx dx; \qquad k \neq 0$$

$$b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin kx dx.$$

***Dirichlet** (1829, 1837) showed that when the function $f(x)$ is bounded in the interval $(-\pi, \pi)$, and this interval can be broken up into a finite number of partial intervals in each of which $f(x)$ is monotonic, the Fourier series converge at every point within the interval to $\frac{1}{2}[f(x+0) + f(x-0)]$, and at the end-points to $\frac{1}{2}[f(-\pi+0) + f(\pi-0)]$. These sufficient conditions (and their extensions to unbounded function) cover most of the cases that are likely to be required in the applications of Fourier series to the solution of the differential equations of mathematical physics and engineering.*

*During 1850–1905, we pass into the domain of pure mathematics. **Riemann** aimed at finding a necessary and sufficient conditions which an arbitrary function must satisfy so that, at a point x in the interval, the corresponding Fourier series shall converge to $f(x)$. The question Riemann set himself to answer has not yet been solved. But in the consideration of the problem he realized that the concept of the definite integral should be widened. And thus it transpires that we owe the Riemann Integral to the study of Fourier series.*

Riemann showed (1854) that for any bounded and integrable function $f(x)$, the integral

$$\frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \left[\begin{array}{c} \sin nx \\ \cos nx \end{array} \right] dx$$

tends to zero as n tends to infinity. This theorem implies that, if $f(x)$ is bounded and integrable in $(-\pi, \pi)$, the convergence of its Fourier series at a point in $(-\pi, \pi)$ depends only on the behavior of $f(x)$ in the neighborhood of that point.

*The nature of the convergence of Fourier series received attention, especially after the introduction of the concept of uniform convergence (**Stokes***

1847, ***Seidel*** *1848).* ***Jordan*** *(1881) simplified the treatment of Fourier series by introducing his functions of bounded variation. His criterion states that the Fourier series for the integrable function $f(x)$ converges to $\frac{1}{2}[f(x+0)+f(x-0)]$ at every point of which $f(x)$ is of bounded variation.*

If Fourier's Series for $f(x)$ is not convergent, it may converge when one or the other of the methods of 'summation' applied to divergent series is adopted. ***Fejer*** *(1904) proved that when the series is summed by the method of arithmetical means, its sum is $\frac{1}{2}[f(x+0)+f(x-0)]$ at every point in $(-\pi,\pi)$ at which $f(x\pm 0)$ exist. The condition attached to $f(x)$ is:*

- *If bounded, it shall be integrable in $(-\pi,\pi)$.*
- *If unbounded, $\int_{-\pi}^{\pi} f(x)dx$ shall be absolutely convergent.*

Although applied mathematicians were quite satisfied with the new limiting processes placed in their hand by ***Dirichlet***, ***Riemann***, ***Cantor*** *and* ***Jordan***, *pure mathematician were still unsatisfied because of the lack of unity, symmetry and completeness of the overall theory. Some advancement made during 1905–1920 greatly improved this situation. The most important contribution were made by* ***Lebesgue*** *(1902–1905),* ***Fejer*** *(1904),* ***de la Vallée Poussin*** *(1893) and* ***W.H. Young*** *(1912). One of the advantages of the Lebesgue integral is that a function which is integrable-L (Lebesque) need not be continuous 'almost everywhere' in the interval of integration, as is the case of a function integrable-R (Riemann). The Riemann-Lebesgue Lemma now guarantees that if $f(x)$ is integrable-L in (a,b), then*

$$\lim_{n\to\infty}\int_a^b f(x)\left[\begin{array}{c}\sin nx\\ \cos nx\end{array}\right]dx=0, \tag{3}$$

whether $f(x)$ is bounded or not.

One of the most remarkable results which follow from the use of the Lebesgue integral in the theory of Fourier Series is the converse of Parseval's Theorem, known as the Riesz-Fisher Theorem: Any trigonometric series for which $\sum_1^\infty(a_n^2+b_n^2)$ converges is the Fourier Series of a function whose square is integrable-L^2 in $(-\pi,\pi)$.

From Fourier Series to the Fourier Integral

When applying the theory of Fourier Series to time signals it is convenient to change the name of the variable x to t and f to g. Thus, if $g(t)$ is a *continuous function with a finite number of bounded discontinuities, defined in the interval* $T_1 \le t \le T_2, \quad T = T_2 - T_1$*, one defines the Fourier expansion of* $g(t)$ *over* T *by*

$$g(t) = \sum_{n=-\infty}^{\infty} c_n e^{in\omega_0 t}, \qquad c_m = \frac{1}{T}\int_{T_1}^{T_2} g(s) e^{-im\omega_0 s} ds, \qquad \omega_0 = \frac{2\pi}{T}. \tag{4}$$

Clearly, the Fourier series representation of $g(t)$ *is* periodic *with period* T*.*

If $g(t)$ *is also periodic with period* T*, the Fourier series will render* $g(t)$ *for all values of* t*; If, however,* $f(t)$ *is not periodic, the sum will be equal to* $g(t)$ *only for* $T_1 \le t \le T_2$*. If* $g(t)$ *is real* $c_m = c^*_{-m}$*. At points of discontinuity of* $g(t)$ *the sum will converge to the arithmetic mean of the values of the function on both sides of the discontinuity.*

For real $g(t)$ *we may write*

$$g(t) = \frac{a_0}{2} + \sum_1^{\infty}[a_n \cos n\omega_0 t + b_n \sin n\omega_0 t] = \frac{a_0}{2} + \sum_1^{\infty} B_n \cos(\omega_0 nt - \varphi_n), \tag{5}$$

where

$$\begin{aligned}
a_n &= \frac{2}{T}\int_{T_1}^{T_2} g(s)\cos(n\omega_0 s)ds; \qquad b_n = \frac{2}{T}\int_{T_1}^{T_2} g(s)\sin(n\omega_0 s)ds;\\
\varphi_n &= \tan^{-1}\frac{b_n}{a_n}; \qquad B_n = \sqrt{a_n^2 + b_n^2}\\
a_0 &= \frac{2}{T}\int_{T_1}^{T_2} g(s)ds; \qquad b_0 = 0; \quad c_0 = \frac{a_0}{2};\\
c_n &= \frac{a_n - ib_n}{2}; \qquad c_{-n} = \frac{a_n + ib_n}{2};\\
B_n &= 2\sqrt{c_n c_{-n}} \quad .
\end{aligned} \tag{6}$$

Inserting the integral expression of c_m *from (4) into the infinite-sum representation of* $g(t)$ *with the provisions*

$$T_1 = -\frac{T}{2}, \quad T_2 = \frac{T}{2}, \quad c_m = c^*_{-m} \quad [\text{real } g(t)] \tag{7}$$

one obtains

$$g(t) = \sum_{n=-\infty}^{\infty} \left[\frac{1}{2\pi} \int_{-\frac{T}{2}}^{\frac{T}{2}} g(s) e^{-in\omega_0 s} ds \right] \omega_0 e^{in\omega_0 t}. \qquad (8)$$

In the limit $T \to \infty$

$$\omega_0 = \Delta\omega \ (say) \to 0; \qquad n\omega_0 = n\Delta\omega \to \omega \quad as \quad n \to \infty, \qquad (9)$$

where any 'harmonic' $n\omega_0$ *must now correspond to the general frequency variable which describes a continuous spectrum.*

Consequently the summation in (8) becomes an integration over ω*, and the function* $g(t)$ *has the representation*

$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} g(s) e^{-i\omega s} ds \right] e^{i\omega t} d\omega. \qquad (10)$$

If we define

$$G(\omega) = \int_{-\infty}^{\infty} g(t) e^{-i\omega t} dt, \qquad (11)$$

then (10) becomes

$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) e^{i\omega t} d\omega. \qquad (12)$$

Eqs. (11) and (12) comprise the Fourier representation of the nonperiodic function, and are known together as the Fourier Integral Theorem.

Note the fundamental difference between the representation of a $g(t)$ through a Fourier Series and a Fourier integral; the Fourier Series of a *periodic function* concerns only those sines and cosines whose frequencies are *integer multiples* of the base frequency. If a function is *not periodic* but decreases sufficiently fast at infinity, it is still possible to describe it as a superposition of sines and cosines i.e. to analyze it in term of its frequencies. But now we must compute the coefficients for *all* possible frequencies. To this end, one may recast (11) and (12) in the *symmetric form* (for a variable x which does not necessarily represent time)

$$f(x) = \int_{-\infty}^{\infty} F(\xi) e^{2\pi i \xi x} d\xi; \qquad F(\xi) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i \xi x} dx \qquad (13)$$

and call $F(\xi)$ the Fourier transform of $f(x)$. The Fourier transform is essentially a mathematical prism, breaking up the function $f(x)$ into the frequency components that compose it, as a prism disperses white light into colors.

With the substitution $\omega = 2\pi\xi$, (13) can be recast in the less symmetric form

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{i\omega x}d\omega; \qquad F(\omega) = \int_{-\infty}^{\infty} f(x)e^{-i\omega x}dx, \tag{14}$$

which agrees with (11)–(12).

Mathematicians spent much of the 19th century coming to terms with the ideas of Fourier. This process yielded a surge of new mathematical methods and results. Thus, questions of convergence and divergence have provided a great deal of work for mathematicians[449]. The need to formulate conditions on Fourier- decomposeable or -transformable functions, sum-shorted (together with advances in DE) the discipline of function spaces (topological and vector); measure theory; functional analysis, as well as special and generalized functions.

The Fourier Integral Theorem and its immediate consequences

Michaël Plancherel *(1885–1967, Switzerland) formulated (1910) the Fourier Integral Theorem in a form which is completely symmetrical with the aid of a new concept known as mean convergence. If $f_n(x)$, $\quad n = 1, 2, 3...$ are absolutely square integrable functions over (a, b) and if*

$$\lim_{n\to\infty}\int_a^b |f_n(x) - f(x)|^2\, dx = 0, \tag{15}$$

where $f(x)$ is square integrable over (a, b), then we say that $f_n(x)$ converges in the mean to $f(x)$ with index 2, and write it as

$$f(x) = \underset{n\to\infty}{\text{l.i.m}} f_n(x). \tag{16}$$

(limit in the mean)

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449 Virtually every periodic function can be represented as a series, or sum, of sines and cosines, but not every series of sines and cosines represent a function. If a series can be proved to *converge*, one can work with a *finite number of terms*, confident that adding more terms will not significantly change the results. If, however, the coefficients of the series do not become small fast enough, the series diverges and does not represent the function.

A similar notation holds if the parameter n tending to infinity is replaced by a variable tending continuously or discretely to some other limit.

Note that pointwise convergence does not imply convergence in the mean [e.g. $f_n(x) = n^{3/2}xe^{-n^2x^2} \to 0$ for every x while $\int_{-1}^{1} |f_n(x)|^2 dx \to \frac{1}{2}\sqrt{\pi}$ and $f_n(x)$ does not converge in the mean]. However, if can be shown that if $f_n(x)$ converges to a limit $f(x)$ almost everywhere on (a, b) and at the same time converges in the mean to a limit $g(x)$, then $f(x) = g(x)$ almost everywhere.

Plancherel's theorem: If the complex function $f(t)$ is absolutely square integrable on $(-\infty, \infty)$ than

$$F(\omega) = \underset{n\to\infty}{\mathrm{l.i.m}} \int_{-\lambda}^{\lambda} f(t)e^{-i\omega t}dt, \tag{17}$$

known as the Fourier transform of $f(t)$, exists and is absolutely square integrable, and we have

$$f(t) = \underset{\lambda\to\infty}{\mathrm{l.i.m}} \frac{1}{2\pi} \int_{-\lambda}^{\lambda} F(\omega)e^{i\omega t}d\omega \tag{18}$$

where

$$\int_{-\infty}^{\infty} |F(\omega)|^2 \, d\omega = 2\pi \int_{-\infty}^{\infty} |f(t)|^2 \, dt. \tag{19}$$

The conditions of this theorem are only sufficient. This means that the Fourier Integral Theorem may be valid for many functions which do not obey these conditions.

It is easy to prove the Fourier Integral Theorem for the more restrictive class of piecewise smooth functions $f(t)$ for $-\infty < t < \infty$ where t is a real variable, and $\int_{-\infty}^{\infty} |f(t)|dt$ converges. This is done with the aid of the δ-function concept. We wish to show that

$$f(t) = \lim_{\lambda\to\infty} \frac{1}{2\pi} \int_{-\lambda}^{\lambda} e^{i\omega t}d\omega \int_{-\infty}^{\infty} f(\xi)e^{-i\omega\xi}d\xi. \tag{20}$$

To this end define the Fourier transform of $f(t)$

$$F(\omega) = \int_{-\infty}^{\infty} f(\xi)e^{-i\omega\xi}d\xi, \tag{21}$$

multiply both sides by $e^{i\omega t}$ and integrate over the ω-range $(-\lambda, \lambda)$, to obtain

$$\int_{-\lambda}^{\lambda} F(\omega)e^{i\omega t}d\omega = \int_{-\infty}^{\infty} f(\xi)d\xi \int_{-\lambda}^{\lambda} e^{i\omega(t-\xi)}d\omega. \tag{22}$$

However

$$\lim_{\lambda\to\infty} \int_{-\lambda}^{\lambda} e^{i\omega(t-\xi)}d\omega = 2\pi\delta(t-\xi), \tag{23}$$

where δ is the 'Dirac delta function'. Eqs. (22)–(23) then yields $f(t) = \int_{-\infty}^{\infty} F(\omega)e^{i\omega t}d\omega$, by use of the "sifting" property of $\delta(\cdot)$.

In the 20th century the concept of l.i.m. has been extended to 'stochastic processes' in which all functions are random variables and the l.h.s. of (15) is replaced by bits 'expectation'.

A number of important special cases and consequences can be drawn from the Fourier Integral Theorem:

Sine and cosine transforms

If $f(t)$ is real and $F(\omega) = R(\omega) + iX(\omega)$, then

$$R(\omega) = Re\ F(\omega) = \int_{-\infty}^{\infty} f(t)\cos(\omega t)dt \equiv \int_{0}^{\infty} [f(t) + f(-t)]\cos\omega t dt$$

$$X(\omega) = Im\ F(\omega) = -\int_{-\infty}^{\infty} f(t)\sin\omega t dt \equiv -\int_{0}^{\infty} [f(t) - f(-t)]\sin\omega t dt$$

So, if $f(t)$ is real and even

$$R(\omega) = R(-\omega) = 2\int_{0}^{\infty} f(t)\cos\omega t dt, \qquad X(\omega) = 0$$

and hence (by the Fourier Integral Theorem)

$$f(t) = \frac{1}{\pi}\int_{0}^{\infty} R(\omega)\cos(\omega t)d\omega. \tag{24}$$

On the other hand, if $f(t)$ is real and odd

$$R(\omega) = 0, \qquad X(\omega) = -X(-\omega) = -2\int_0^\infty f(t)\sin(\omega t)dt,$$

$$f(t) = -\frac{1}{\pi}\int_0^\infty X(\omega)\sin(\omega t)d\omega. \tag{25}$$

Since an arbitrary function $f(t)$ can always be decomposed into a sum of an even and an odd function

$$f(t) = \frac{f(t)+f(-t)}{2} + \frac{f(t)-f(-t)}{2} = f_{even} + f_{odd},$$

we can write,

$$f_{even}(t) = \frac{1}{\pi}\int_0^\infty R(\omega)\cos(\omega t)d\omega, \qquad f_{odd}(t) = -\frac{1}{\pi}\int_0^\infty X(\omega)\sin(\omega t)d\omega. \tag{26}$$

Next, consider the so-called causal function *$f(t) = 0$ for $t < 0$. Then since for this case*

$$f(t) = 2f_{even}(t) = 2f_{odd}(t), \qquad t > 0 \tag{27}$$

we find that

$$\begin{gathered} f(t) = \frac{2}{\pi}\int_0^\infty R(\omega)\cos\omega t\omega = -\frac{2}{\pi}\int_9^\infty X(\omega)\sin\omega t d\omega, \quad t > 0 \\ f(0) = \frac{1}{\pi}\int_0^\infty R(\omega)d\omega = \frac{1}{2}f(0^+). \end{gathered} \tag{28}$$

In this case, the functions $R(\omega)$ and $X(\omega)$ are not independent. In fact they are Hilbert transforms of each other.

Cauchy *(1826) pointed out that the sine and cosine transforms lead to reciprocal relations between pairs of functions. If we write*

$$F_c(u) = \sqrt{\frac{2}{\pi}}\int_0^\infty f(t)\cos utdt,$$

then,

$$f(t) = \sqrt{\frac{2}{\pi}}\int_0^\infty F_c(u)\cos tudu, \tag{29}$$

and the relation between $f(x)$ and $F_c(x)$ is thus reciprocal. For example e^{-x}, $\sqrt{\frac{2}{\pi}}\frac{1}{1+x^2}$ are a pair of Fourier cosine transforms. Likewise, from Fourier's sine formula, we obtain

$$F_s(u) = \sqrt{\frac{2}{\pi}}\int_0^\infty f(t)\sin utdt,$$

$$f(t) = \sqrt{\frac{2}{\pi}}\int_0^\infty F_s(u)\sin tudu, \tag{30}$$

which are sine transforms of each other. Thus e^{-x}, $\sqrt{\frac{2}{\pi}}\frac{x}{1+x^2}$ belong to this family.

The Poisson Summation formula

Consider the function

$$S(x) = \sum_{n=-\infty}^{\infty} f(x+nx_0), \tag{31}$$

where f is some function. $S(x)$ is periodic *with period x_0 because with $n+1 = m$*

$$S(x+x_0) = \sum_{n=-\infty}^{\infty} f[x+(n+1)x_0] = \sum_{m=-\infty}^{\infty} f(x+mx_0) = S(x), \tag{32}$$

and so it may be expressed as a Fourier Series

$$S(x) = \sum_{l=-\infty}^{\infty} C_l e^{\frac{2\pi i l x}{x_0}}; \qquad C_l = \frac{1}{x_0}\int_{-\frac{x_0}{2}}^{\frac{x_0}{2}} S(x)e^{-\frac{2\pi i l x}{x_0}}\,dx. \tag{33}$$

Substituting $S(x)$ from (31) into (33) and changing variables via $y = x+nx_0$, we have

$$C_l = \frac{1}{x_0}\sum_{n=-\infty}^{\infty}\int_{(n-1/2)x_0}^{(n+1/2)x_0} f(y)e^{-\frac{2\pi i l y}{x_0}}\,dy =$$

$$= \frac{1}{x_0}\int_{-\infty}^{\infty} f(y)e^{-\frac{2\pi i l y}{x_0}}\,dy = \frac{1}{x_0}F\left(\frac{2\pi l}{x_0}\right)$$

where $F(k)$ is the Fourier transform of $f(x)$ (Eq. (21)). Therefore we arrive at the Poisson sum formula:

$$\boxed{S(x) = \sum_{n=-\infty}^{\infty} f(x + nx_0) = \frac{1}{x_0} \sum_{l=-\infty}^{\infty} F\left(\frac{2\pi l}{x_0}\right) e^{\frac{2\pi i l x}{x_0}}.} \tag{34}$$

In many cases the summation over Fourier-transformed functions is easier then the original sum. For the special case $x = 0$, $x_0 = 1$

$$\sum_{n=-\infty}^{\infty} f(n) = \sum_{l=-\infty}^{\infty} F(2\pi l) \equiv \sum_{l=-\infty}^{\infty} \int_{-\infty}^{\infty} f(u) e^{-2\pi i l u} du. \tag{35}$$

Example 1

To sum the series

$$S = \sum_{n=0}^{\infty} \frac{1}{a^2 + n^2} = \frac{1}{2a^2} + \frac{1}{2} \sum_{n=-\infty}^{\infty} \frac{1}{a^2 + n^2}, \tag{36}$$

we put $f(u) = \frac{1}{a^2+u^2}$, $v = 2\pi l$. It remains to calculate the integral

$$\int_{-\infty}^{\infty} \frac{e^{-iuv}}{a^2 + u^2} du = \int_{-\infty}^{\infty} \frac{\cos uv}{a^2 + u^2} du + i \int_{-\infty}^{\infty} \frac{\sin uv}{a^2 + u^2} du = \int_{-\infty}^{\infty} \frac{\cos uv}{a^2 + u^2} du.$$

For $v > 0$, the residue theorem is applied to a closed contour completed in the lower v half-plane, yielding $\frac{\pi}{a} e^{-av}$ for the integral. For $v < 0$, the contour is deformed around the upper half-plane and the result is similarly $\frac{\pi}{a} e^{av}$. Combining the two results

$$S = \frac{1}{2a^2} - \frac{\pi}{2a} + \frac{\pi}{a} \frac{1}{1 - e^{-2\pi a}} \equiv \frac{\pi}{2a} \left[\coth(\pi a) + \frac{1}{\pi a}\right]. \tag{37}$$

When $a \to 0$, $S(a) - \frac{1}{a^2} \to \frac{\pi^2}{6}$, so $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. By the same method one can prove the more general result

$$\sum_{n=0}^{\infty} \frac{\cos nx}{n^2 + a^2} = \frac{1}{2a^2} + \frac{\pi \cosh[a(\pi - x)]}{2a \sinh(\pi a)}$$

Example 2

Let

$$f(u) = e^{-2u^2+2\pi i a u\lambda}.$$

Then Eq. (34) yields

$$1 + 2\sum_{m=1}^{\infty} e^{-m^2\lambda^2}\cos(2m\pi\lambda a) =$$

$$\frac{\sqrt{\pi}}{\lambda}e^{-\pi^2 a^2}\left[1 + 2\sum_{n=1}^{\infty} e^{-\frac{\pi^2 n^2}{\lambda^2}}\cosh\frac{2\pi^2 na}{\lambda}\right]. \tag{38}$$

Taking $\lambda = \pi t$, $v = \lambda a$, we obtain

$$1 + 2\sum_{m=1}^{\infty} e^{-m^2\pi^2 t^2}\cos(2m\pi v) = \frac{1}{t\sqrt{\pi}}\sum_{n=-\infty}^{\infty} e^{-\left[\frac{(n+v)}{t}\right]^2}. \tag{39}$$

If $f(x)$ is an even function of x which can be expanded in a Fourier Series of cosines, in the open interval $(-a, a)$, (34) takes the form ($x_0 = 2a$)

$$\sum_{n=-\infty}^{\infty} f(x + 2na) = \frac{1}{a}\int_0^{\infty} f(x')dx' + \frac{2}{a}\sum_{n=1}^{\infty}\cos\frac{n\pi x}{a}\int_0^{\infty} f(x')\cos\frac{\pi n x'}{a}dx'. \tag{40}$$

If in (34) we set $x = 0$, $x_0 = \beta$, $\alpha\beta = 2\pi$, $n = l$ and define the symmetric cosine transforms

$$f(x) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} F(t)\cos xt dt, \qquad F(t) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x)\cos xt dx \tag{41}$$

then, for $\alpha > 0$, $\beta > 0$, the summation formula for even $f(x)$ takes the form

$$\boxed{\sqrt{\beta}\sum_{n=-\infty}^{\infty} f(n\beta) = \sqrt{\alpha}\sum_{n=-\infty}^{\infty} F(n\alpha)}. \tag{42}$$

*This formula may have been known earlier to **Gauss**. When it was first observed by **A.L. Cauchy**, his admiration was very great. He said that it was a discovery worthy of the genius of **Laplace**. It has much in common with the Euler-Maclaurin summation formula and in some applications is superior to it.*

The techniques that Fourier invented have had an impact well beyond studies of heat, or even solutions of differential equations. Real data tend to be very irregular. Consider electrocardiogram arabesques – tantalizing curves that contain all the information of the signal but hide it from our comprehension. Fourier analysis translates these signals into a form that makes sense, transforming a signal that varies with time (or in some cases, with spatial dimensions) into a new function, the Fourier transform *of the signal, which tells how much of each frequency the signal contains.*

In many cases these frequencies correspond to the frequencies of the actual physical waves making up the signal. This applies to sound waves (e.g. speech, music) and all kinds of electromagnetic waves (radio waves, microwaves, infrared, visible light and x-rays), not even known in Fourier's time.

Being able to break down such waves into frequencies has myriad uses, including tuning your radio to your favorite station, interpreting radiation from distant galaxies, using ultrasound for medical diagnosis, and making cheap long-distance telephone calls.

With the discovery of quantum mechanics, it became clear that Fourier analysis is directly relevant to any dynamical system. On the "position space" side of the Fourier transform, one can talk about an elementary particle's position; on the other side, in "Fourier space," one can talk about its momentum or think of it as a wave; and similarly for time *vs.* energy*; electric vs. magnetic field;* intensity *vs.* phase *of an EM wave; and other such "conjugate pairs" of dynamical variables. The modern realization that matter and energy at very small scales behave differently from matter and energy on a human scale – that (for example) an elementary particle does not simultaneously have a precise position and a precise momentum – is naturally expressed in the language of Fourier analysis and transform.*

While irregular functions, defined on compact domain, can be expressed as sums of sines and cosines, usually those sums are infinite. Why translate a complex signal into an endless arithmetic problem in which one must calculate an infinite number of coefficients and sum an infinite number of waves?

We seem to be jumping from the pot into the frying pan. Fortunately a small number of coefficients is often adequate. In the case of the heat diffusion equation, for example, Fourier showed that the coefficients of high-frequency sines and cosines rapidly approach zero, so all but the first few frequencies can safely be ignored. In other cases engineers may assume that a limited number of calculations gives a sufficient approximation, until proved otherwise.

In addition, engineers and scientists using Fourier analysis often don't bother to add up the sines and cosines to reconstruct the signal; instead they "read" Fourier coefficients (or at least the amplitudes; phases are more *difficult) to get the information they want, the way some musicians can hear music silently by reading the notes.*

They may spend hours on end working happily in this "Fourier space," rarely emerging into "physical space." (For one-dimensional signals, "physical space" generally corresponds to time, but Fourier analysis can also be applied to pictures. In this case, "physical space" corresponds to position.)

Fourier decomposition and transforms have been extended, in both classical and quantum physics and engineering, to other sets of base functions besides sinusoids. These may be orthonormal or not. Examples are: spatial vibration eigenmodes in non-rectangular geometries; wavelets and Gabor transforms in space an/or time; and coherent states in quantum optics.

But the time it takes to calculate Fourier coefficients is a problem: without computers and fast algorithms, Fourier analysis would have remained a theoretical tool, and digital signal-processing technology would not pervade modern life.

1808 CE **Christian Kramp** (1760–1826, France). Mathematician at Strasbourg. Introduced the factorial symbol $n!$

1808–1810 CE **Etienne-Louis Malus** (1775–1812, France). Engineer and physicist. Discovered polarization of light by reflection and presented a theory explaining double refraction of light in crystals (1810).

Malus was educated at the Ecole Polytechnique and remained associated with it all his life as an examiner, but his main career was in the army. As an engineer he accompanied Napoleon's expedition to Egypt and Syria (1798–1801).

1808–1837 CE **Simeon Dennis Poisson** (1781–1840, France). Notable mathematician. A principal successor to **Laplace**, both in interests and position. There are few branches of mathematics to which he did not contribute something, but it was in the application of mathematics to physical subjects that his greatest services to science were performed. He considered such matters as physical astronomy, stability of planetary orbits (1808), heat conduction (1811), analytical mechanics (1833), the attraction of ellipsoids, probability theory, definite integrals, Fourier series and theory of elasticity. One encounters *Poisson Brackets*, *Poisson's Constant*, *Poisson Integral*, *Poisson*

Equation, *Poisson Summation Formula* (1827) and the *Poisson Distribution* (1837).

He was first to predict the existence of longitudinal and transverse elastic waves[450] (1828) and deduced, in an alternative way to that of Navier, the basic equations of a viscous fluid. He also studied the propagation of waves in anisotropic media (crystals).

He derived the equation satisfied by the gravitational potential *within* a distribution of matter, which now bears his name ($\nabla^2\psi = 4\pi G\rho$, 1813).

Poisson was born in Pithiviers. He was educated by his father who had served as private soldier in the Hanoverian wars but deserted, disgusted by the ill-treatment he received from his patrician officers. Poisson entered the École Polytechnique to study mathematics (1798) and immediately began to attract the notice of Lagrange and Laplace, the latter regarding him almost as his son. In 1806 he became a full professor, in succession to **J. Fourier**. In 1808 he became astronomer to the Bureau des Longitudes and in 1809, when the Faculté des Sciences was instituted, he was appointed professor of rational mechanics.

His father, whose early experience let him to hate aristocrats, bred him in the stern creed of the First Republic. Throughout the Empire period, Poisson faithfully adhered to the family principles and refused to worship Napoleon. After the Second Restoration, his fidelity was recognized by his elevation to the dignity of Baron in 1825, but he never used the title. The revolution of 1830 threatened him with the loss of all his honors, but his disgrace was averted with the help of his friend **Francois Arago** (1786–1853) and in 1837 he was made a peer of France — not for political reasons but as a representative of French science.

In all his work, his role was that of an insightful extender rather than that of a bold originator. As a scientist, however, his activity has rarely, if ever, been equaled. Notwithstanding his many official duties, he found time to publish more than 300 works, several of them extensive treatises, and many of them memoirs dealing with the most abstruse branches of pure and applied mathematics. There is a remark of his that explains how he accomplished so much: "*La vie c'est le travail*".

1809–1810 CE **George Cayley** (1773–1857, England). Father of modern aeronautics. Contributed many ideas to early aviation. Clearly defined for the first time the idea that sustentation can be accomplished by moving

[450] His findings, however, created at that time a new difficulty in the wave theory of light: for if the luminiferous ether behaved like an elastic solid, his analysis showed that *two* waves, instead of one, should be visible!

an inclined surface in the flight direction, provided one has mechanical power to compensate for the air resistance which hinders this motion.

He belonged to a group of enthusiasts who tried to empirically solve the problem of flight by building models and studying bird flight. In his papers, published in 1809–1810, he clearly defined and separated the problem of sustentation, or in modern scientific language — the problem of lift, from that of drag i.e. the component of total resistance that works against the flight direction, and must be compensated by propulsion in order to maintain level flight.

Cayley understood the effect of streamlining on drag, and advocated borrowing from nature in the design of low-drag cross sections (e.g. spindles of the trout and woodcock). The shape of his profiles almost exactly coincided with certain modern airfoil sections. Cayley wrote about helicopters and parachutes. He conceived the biplane and built a glider that carried a coachman for 270 meters. Cayley was born in Brompton, England.

1809–1822 CE **Jean-Baptiste (Pierre Antoine de Monet, le Chevalier) de Lamarck** (1744–1829, France). Naturalist. A thinker who played an important part in preparing the way for universal acceptance of the doctrine of evolution . He also propounded a theory, known by his name as *Lamarckism* that evolutionary change might have occurred by the inheritance of 'acquired characteristics', i.e. he believed that changes that came about during an organism's lifetime, as a result of active adaptation to circumstances, would become impressed upon its genome, or chromosomes, and thus be reproduced in succeeding generations. His ideas were outlined in his book *Philosophie Zoologique* (1809).

Lamarck was born in Bazentin, Picardy. He studied medicine, meteorology and botany, and traveled across Europe as botanist to King Louis XVI from 1781. In 1793 he was made professor of zoology at the Museum of Natural History in Paris.

Lamarck was the first to distinguish vertebrate from invertebrate animals by the presence of a bony spinal column. He was also the first to establish the *crustaceans*, *arachnids*, and *annelids* among the invertebrates[451]. It was Lamarck who coined the word '*biology*'. His studies of both living and fossil invertebrates were described in his book *Natural History of Invertebrate Animals* (1815–1822).

Unfortunately, all attempts to demonstrate a Lamarckian scenario in real-life heredity have failed, either because the demonstration itself has been

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[451] So little was known about invertebrates at this time that some scientists grouped snakes and crocodiles with insects.

unconvincing or because the phenomenon said to have been observed were open to an alternative, Darwinian interpretation. The incentive to support Lamarck lay partly in the fact that his theory seems only fair: the skills that human beings obtain by their own endeavors and exertions should surely become part of their children's heritage. This, after all, is what happens regularly in the kind of inheritance that takes place extragenetically through culture[452].

1810–1820 CE **Franz Joseph Gall** (1758–1828, Germany and France). Physician and anatomist. Pioneer in ascribing cerebral functions to various areas of the brain; first to identify grey matter of brain with neurons and white matter with ganglia; sought to establish relationship between faculties and shape of skull (*phrenology*)[453]. Wrote *Anatomy and Physiology of the Nervous System* (1810–1820).

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[452] The Lamarckian idea was not new: the first theory of evolution came from the Book of *Genesis* (**25** – **30**), where the story about breeding suggests that environmental influences can affect heredity.

According to the story, Jacob came to his father-in-law, Laban, and to Laban's daughters to claim his reward for 20 years of service to them. After some discussion it was agreed that his reward should be to take for his own from Laban's flocks all the brown sheep and all the spotted, speckled and banded goats.

Jacob accordingly set about increasing the proportion of such goats in the flock. He did this by causing them, as the famous depiction by the seventeenth-century Spanish painter Bartolomé Murillo shows, to mate in the presence of rods, or wands, of green poplar, hazel and chest-nut stripped of bark in such a way that they were emblazoned with alternating bands of white and dark wood. This is alleged to have done the trick. The offspring of goats influenced by the stripes now had among them, according to the tale, an increased proportion of "ring-straked (banded) speckled, and spotted" goats!

The story of Jacob and the sheep of Laban is by no means uplifting, but it does illustrate the great antiquity of Lamarckian ideas and helps to explain why those ideas enjoy a popular revival every few decades.

[453] A pseudo scientific theory based on the idea that certain mental faculties and character traits are indicated by the configuration of a person's skull, i.e. that *mental* qualities are associated with *physical* characteristics. The sciences of physiology and psychology have shown that different portions of the brain do have certain functions, but these usually merely receive sensory stimuli and relate them to action. However, the knowledge we have accumulated so far tends to disprove phrenology.

Gall was born in Tiefenbrunn, Germany; Physician in Vienna (1785); took up residence in Paris (1807). His lectures on phrenology were popular, but suppressed (1802) as being subversive to religion.

1810–1836 CE **Jöns Jacob Berzelius** (1779–1848, Sweden). Distinguished chemist. Made valuable contributions to the development of atomic theory. Discovered the elements selenium and thorium (1829). He was the first to isolate the elements *calcium*, *silicon* (1823) and *tantalum*, and to note and describe *allotropy*, *isomorphism* and *chemical catalysis* (1835). He originated the system of writing chemical symbols[454] and formulae (1813), and undertook an extensive program to determine the relative atomic weights of the known elements (1818–1826).

Berzelius was born at Väfversunda Sorgard, near Linköping, Sweden. He went to Uppsala University, where he studied chemistry and medicine, and graduated as M.D. in 1802. He served as a professor of chemistry at Stockholm from 1807 until 1832. About 1807 he began to devote himself to what he made the chief object of his life — the elucidation of the composition of chemical compounds through atomic theory. During 1810–1820 he analyzed over 2000 inorganic compounds to determine the weight ratios of the various constituent elements, using *oxygen* as the basis of reference. This resulted in the first table of atomic weights which he published in 1818 and 1826: most elements are presented with atomic weights very close to those accepted in the 20^{th} century.

Another service of the utmost importance which he rendered to the study of chemistry was in continuing and extending the efforts of **Lavoisier** and his associates to establish a convenient system of chemical nomenclature, later

[454] Some of the elements retained their old names: *copper* = the metal from Cyprus (301 AD); *gold* = gelb = yellow (teutonic; the Latin *aurum* is akin to Aurora, goddess of the dawn).

Many elements have names derived from Greek roots: *chlorine*, from its color, chlorus = yellowish green (**Davy**, 1811); *chromium*, from chroma = color. Other elements have been named after mythological deities or personages: *vanadium* from Vanadis, one of the names of the Norse goddess Freya; *thorium* from Thor, the Scandinavian war-god; *tantalum* and *niobium* from Tantalus and Niobe, of Greek mythology. Names of places where compounds of elements were first discovered have sometimes formed the bases of other names: *strontium*, from Strontian, in Scotland; *ruthenium*, from Ruthenia (Russia); *ytterbium*, from Ytterby (Sweden); *hafnium*, named after Copenhagen, formerly called Hafnia; *masurium*, after a lake in East Prussia; *rhenium*, from the Rhine; *palladium* and *uranium* after Pallas and Uranus, discovered about the same time; *selenium* and *tellurium* are named after the Moon (selene) and the Earth (tellus).

to become accepted in the scientific literature: an element is generally represented by the first letter of its Latin name, or, in the event of elements with the same first letter, by the first two letters. Compounds are symbolized by juxtaposing the element symbols, superscribed[455] with the number of atoms involved if greater than one; e.g., carbon dioxide is symbolized as CO^2. Berzelius was first to classify minerals on a chemical basis.

1811 CE **Amadeo Avogadro** (1776–1856, Italy). Physicist. Postulated the *Avogadro hypothesis* which states that for a given temperature and pressure, equal volumes of gas have the same number of molecules (moles)[456]. This provides an explanation for the law of integral volume ratio [asserts that when two gases combine chemically, they do so such that the two volumes involved are in the ratio of whole numbers]. It was discovered by **Joseph Louis Gay-Lussac** (1778–1850, France) in 1809.

Avogadro's hypothesis was ignored until 1865, when **Joseph Loschmidt** (1821–1895, Austria) used the new kinetic theory of gases to obtain the number of molecules of an ideal gas in a cubic centimeter as 2.69×10^{19} under standard conditions. [*Loschmidt-Avogadro number*. Also given as 6.022×10^{23} molecules/mole, since $\frac{6.022}{2.69} 10^4 \cong 22400$, the number of cm^3 in 22.4 liters.] This number is one of the fundamental constants of nature. Unlike the dimensionless constants, this one belongs to the category of constants whose numerical values depend on conventions and system units. Here specifically,

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455 In 1834, **Justus von Liebig** revised this by replacing superscript by subscript, e.g., $CO^2 \to CO_2$.

456 The ideal gas equation [which combines *Boyle's law* (1660) with *Charles law* (1787)], is: $PV = \alpha T$, where $\alpha = \frac{P_0 m}{\rho_0 T_0}$ is a constant which depends on the selected mass m of a particular gas. There are two options for making this a useful law: either to agree on a *fixed mass* of gas (say, one gram) or choose a *variable mass*, but one that always has the same number of molecules (molecular mass, mole). In the first case, α will differ from one gas to another. In the second case, α (denoted by R) will have the same value for 1 mole of all gases. It is known as the *gas constant*. Thus, the ideal gas equation, referring to one mole of any gas, is $PV = RT$, and $PV = nRT$ for n moles. Here $n = \frac{m}{M}$, m being the mass in grams and M is the molecular weight (mass). It has been found experimentally that one mole of any gas under standard pressure and temperature occupies approximately 22.4 liters.

Avogadro's hypothesis (which later became a *law* in the framework of kinetic theory of gases) removed a serious obstruction to progress in chemistry since it provided a simple way of comparing masses of molecules by weighing equal volumes of two gases. The results agreed with other evidence, leading chemists to trust the hypothesis.

on the centimeter unit and the values of standard temperature and pressure (for Loschmidt's number), and of the numerical scale of atomic and molecular weights (for Avogadro's number[457]). Once these definitions are made, the value of the constants is immutable. All that remains is to measure it as accurately as possible.

Avogadro was born at Turin. He was for many years professor of physics at Turin University. He published numerous physical memoirs but is chiefly remembered for his "*Essai d'une maniére de déterminer le masses relatives des molécules elémentaires des corps, et les proportions selon lesquelles elles entrent dans les combinaisons*", in which he enunciated his hypothesis. He coined the term *molecule*.

1811 CE Birth of the Siamese twins **Eng and Chang** (1811–1874), identical twins joined together at the hip. They ended up as American citizens, taking the name Bunker, and before the Civil War they were shareholders in North Carolina. Eng and Chang had seven daughters and three sons for Chang, seven sons and five daughters for Eng. (The birth of Siamese twins is very rare, about 1:50,000).

1811 CE **Charles Bell** (1774–1842, Scotland). Surgeon and anatomist. Discovered distinct functions of sensory and motor nerves and the dual nature of spinal nerves (1811).

Asserted in his *Idea of a New Anatomy of the Brain* that different parts of the brain undertake different functions and that the specific functions of each of the various divisions of the peripheral nerves derive from the part of the brain connected to that division.

1812 CE Volcanic eruption in the Azores (1811) led to a bitter winter in 1812, and was a major factor in the defeat of Napoleon's army in Russia.

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457 The precise value of Avogadro's number for the ^{12}C atomic weight scale is 6.022169×10^{23} atoms per gram atomic weight. One gram atomic weight of any element (i.e., the atomic weight of the element, expressed in grams) is called a *mole*. One mole of *any* pure substance — whether it is composed of atoms, molecules, ions, electrons, or any other kind of particle — contains (by definition) Avogadro's number of particles. For this reason, Avogadro's number is given by $N_A = 6.022169 \times 10^{23}$ mole^{-1} with the numerator "particle" being understood. Thus, 1 mole of H atoms weighs precisely 1.00797 g; 1 mole of N atoms weighs 14.0067 g. Similarly, 1 mole of water molecules H_2O weighs 15.9994 g+2(1.00797 g) = 18.0153 g. Avogadro's number of *photons* (a mole of photons) is called 1 *einstein*. The energy of one einstein at wavelength λ is $\frac{E_A hc}{\lambda}$.

1811–1816 CE Organized bands of English rioters called *Luddites*[458] destroyed labor-saving machines as a protest against their low wages and terrible working conditions, and because of the widespread prejudice that its use produced unemployment.

The riots arose out of severe distress caused by the war with France. Apart from this prejudice, it was inevitable that the economic and social revolution implied in a change from manual labor to work by machinery should give rise to great misery. The riots began (1811) with the destruction of stocking and lace frames in Nottingham. Continuing through the winter and the following spring, it spread into Yorkshire, Lancashire, Derbyshire and Leicestshire. They were met with severe repressive legislation. In 1816 the rioting was resumed (caused by depression which followed the peace of 1815 and aggravated by one of the worst recorded harvests) and extended over the whole kingdom. Vigorous repressive measures, and, especially, reviving prosperity, brought the movement to an end.

1811–1848 CE **Dominique-Francois-Jean Arago** (1786–1853, France). Physicist and statesman. Contributed to the discovery of laws of *light polarization*, ruling out the previously assumed longitudinal nature of light. He thus lead **Young** (1817) to the correct transverse nature of light's vibrations.

Arago devised an experiment (1816) by which the nature of light was demonstrated via its reduced speed through dense media. Working with Biot, he made measurements of arc length on the earth which led to the standardization of the metric system of lengths. Encouraged his student Le Verrier to investigate irregularities in *Uranus'* orbit, which led to the discovery of *Neptune*.

Arago was educated at the Ecole Polytechnique in Paris and became professor of geometry there at the age of 23. Later (from 1830) he became director of the Paris Observatory. He was minister of war and marine in the provisional government (1848); responsible for the abolition of slavery in the colonies.

1812 CE, June Napoleon invaded Russia with a grand army of ca 600,000 men. Of these, only some 90,000 reached Moscow. The rest succumbed to the common campaign diseases of *dysentery* and *typhus*. Typhus had been endemic in Poland and Russia for many years. Lack of water and insufficient changes of clothing made bodily cleanliness impossible. Fear of Russian attack and Polish reprisals caused the men to sleep close together in large groups.

[458] Named after **Ned Ludd**, who in 1779 destroyed frames used in stocking machines in a village in Leicestershire.

The lice of infested hovels crept everywhere, clung to the seams of clothing, to the hair and bore with them the bacteria of typhus. Disease alone had rubbed Napoleon's central force of some 265,000 men of its effective strength by the end of the first month!

By June 1813, less than 3000 of his grand army were alive. By the autumn of 1813 some 470,000 new troops were mobilized for the final battle.

1812–1823 CE **Jacque-Philippe-Marie Binet** (1786–1856, France). Mathematician and astronomer. Discovered the rule for *matrix multiplication*. He continued to investigate the foundation of matrix theory, thus setting the scene for later work by Cayley and others. Derived the laws of motion of a particle in a field of a central force (*Binet's formulas*[459]).

Binet was a student at the Ecole Polytechnique in Paris and after graduating worked for the Department of Bridges and Roads of the French government. Appointed to the chair of astronomy at the College de France (1823).

1813 CE **Simon-Antoine-Jean Lhuilier** (1750–1840, Switzerland). Mathematician. Noticed that Euler's formula $v-e+f=2$ was wrong for solids with holes in them and derived instead the more general formula $v-e+f=2-2g$ where g is the number of holes. This was the first known result on what we call today *topological invariant*.

1813–1822 CE **Pierre-Charles-Francois Dupin** (1784–1873, France). Differential geometer. Invented the *Dupin indicatrix*[460] which gives an indication of the local behavior of a surface. Dupin was a pupil of Monge at the Paris Ecole Polytechnique. Entering the Napoleonic Navy as an engineer, Dupin lived to be a promoter of science and industry, a peer of France and a senator under Napoleon III. He was a professor at Conservatoire des Artes (1819–1864).

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[459] *Binet's formulas*: Given the equation of the particle's path $r=r(\theta)$ under a central force in the xy-plane, the first formula is $v^2 = h^2\left[\frac{1}{r^2}+\left(\frac{d\,\frac{1}{r}}{d\,\theta}\right)^2\right]$ where v is the particles's velocity; the second formula is $F=-\frac{mh^2}{r^2}\left[\frac{1}{r}+\frac{d^2}{d\,\theta^2}\left(\frac{1}{r}\right)\right]$, where F is the centripetal force acting in the central motion, m is the particle's mass, and h is twice the areal velocity.

[460] *Dupin's indicatrix* is the ellipse $\kappa_1 x^2+\kappa_2 y^2 = 1$ with principal semi-axes $\left\{\frac{1}{\sqrt{\kappa_1}}, \frac{1}{\sqrt{\kappa_2}}\right\}$ where $\{\kappa_1,\kappa_2\}$ are the principal curvatures of a surface. The *normal curvature* $\kappa=\kappa_1\cos^2\alpha+\kappa_2\sin^2\alpha$ is obtained graphically by intersecting the ellipse with a line $y = x\tan\alpha$ through the center; the distance intercepted by the ellipse in the α direction is then equal to $\frac{1}{\sqrt{\kappa}}$.

1814–1823 CE **Joseph von Fraunhofer** (1787–1826, Germany). Optician and physicist. The true founder of astrophysics; laid the foundations of solar and stellar spectroscopy. Born at Straubing in Bavaria to a very poor family, he was apprenticed to an optician at the age of 11 and lived in a half-ruined house in Munich. One day this slum collapsed, killing all its occupants except the young boy, who was pulled out of the ruins seriously injured. The Elector of Bavaria showed his compassion to the survivor by granting him the sum of 18 ducats, which Joseph spent on books and optical instruments. Through solitary and dogged labor he became an expert optician, and at 19 he went to work for a large glassware and scientific instrument factory. Three years later he became one of its directors!

In an attempt to improve telescope objectives, he embarked on a study of prisms and refraction. He repeated Newton's experiments, but added to the prism a small telescope which received the colored beams and gave a particularly clear image of the spectrum. Having far better prisms at his disposal than had Newton, he discovered (1814) that the continuous spectrum of the sun is interrupted by a large number of dark lines: certain wave-lengths were lacking, or at least were greatly suppressed, in the light of the sun.

These dark lines were subsequently called *Fraunhofer lines*[461]. He not only discovered them, but undertook the measurement of their relative positions, making up a map of the spectrum. By a series of ingenious experiments, he proved that these dark lines really are a property of sunlight and are not due to instrumental shortcomings.

Fraunhofer then turned to the spectra of other celestial bodies. In the spectrum of Sirius he could see lines which were completely different from those he found in sunlight. He noticed that different stars had different spectra, thus showing the way to *stellar spectroscopy*. At the same time this tireless optician was studying artificial light sources; he noticed in particular that the same bright yellow line, or rather a pair of lines, was to be found in nearly all flames. This double line is in exactly the same position as a very strong line in the solar spectrum, called the *D line* by Fraunhofer.

He invented the *diffraction grating* and established the fundamental law which makes it possible to find the wavelength of monochromatic radiation from the position of the corresponding line[462] in the grating spectrum of order

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461 **Whollaston** had detected 4 of the strongest lines in 1802, but he thought that they were the *natural* separations between the colors!

462 A diffracted wave, obtained by a grating from a plane wave at normal incidence, has local maxima in directions making angles θ with that of the white central image such that $\sin\theta = n\frac{\lambda}{d}$, where d is the distance between 2 successive slits in the grating and n are integers. Thus, the grating disperses light composed of

n. With the aid of his gratings Fraunhofer determined the first accurate value of the wavelength of the sodium[463] D lines in 1822.

In his studies he used a beam of light coming practically from infinity (plane-wave), which is the one used in observing the stars through a telescope. This kind of diffraction has since been named *Fraunhofer diffraction* in his honor [in contradistinction to the *Fresnel diffraction* from *close* point-sources]. In 1823 he was appointed conservator of the physical cabinet at Munich, and in the following year he received from the elector of Bavaria the civil order of merit.

All the scientific achievements of Joseph von Fraunhofer were carried out in his spare time, and one sometimes wonders how such an enormous amount of work could have been done in so short a life. He died at Munich and was buried near Reichenbach. On his tomb is the inscription "*Approximativ sidera*".

Fraunhofer started his apprenticeship at a tender age, produced a large number of inventions, both large and small, and died before he was 40. One might say that he was, in a way, the Mozart of physics.

1814–1825 CE **George Stephenson** (1781–1848, England). Engineer and inventor. Known as the '*Founder of Railways*'. Completed the adaptation of the steam engine to the railroad.

He was born in Wylam, near Newcastle, the son of a coal-ship fireman. In boyhood he was employed as a cowherd, in his 14^{th} year he became assistant fireman to his father at a shilling a day, and in his 17^{th} year was yet unable to read. In his 18^{th} year he began to attend a night school and made remarkably rapid progress. In 1804 he moved to Killingworth and there devised his miner's safety lamp (1815), independently of **Humphrey Davy** who was producing his lamp at about the same time.

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various wavelengths more, the closer the spacing of the slits. Hence the efforts of this ingenious physicist to engrave gratings with closer and closer lines. He constructed a machine that could engrave 3000 lines in one centimeter of glass, with a diamond point.

463 Sodium (Na) was first isolated by **Humphry Davy** (1807) through *electrolysis* of Na_2CO_3 (caustic soda), using a *voltaic battery* (1800). Previous to this discovery, caustic alkalies were regarded as elements, although **Lavoisier** (1789) hinted that alkaline earths might be oxides of unknown metals.
Fraunhofer did not know in 1822 that the lines that he tabbed "D" were due to sodium, but **Kirchhoff** (1859–1861) showed that Fraunhofer's D-lines were produced by the cool *sodium vapor* of the solar atmosphere.

In 1814 he completed his construction of a traveling engine for the tramroads between the coal-ship and the shipping port, 15 km distant. The engine, which he named '*My Lord*', ran a successful trial on 25^{th} of July, 1814 at a speed of 10 km/h. His second engine, '*Puffing Billy*', embodied his invention, the steam blast. This device increased the draft in the boiler. In turn, the fire became hotter and made steam of higher pressure.

He was instrumental in opening the world's first railway, the *Stockton and Darlington Railroad* on Sept. 27, 1825. His locomotive *The Rocket* (1829) traveled at the then unheard of speed of 48 km/h, and became a model for later locomotives.

With the wealth he amassed from his inventions he became a philanthropist for the miners cause, establishing night schools for miners and educational and recreational facilities for their children.

1815 CE, April 5–10 *The greatest volcanic explosion of recent centuries*. The eruption of *Mount Tambora* (8°15′S, 118°00′E) on the Island of Sumbawa in Indonesia killed ca 100,000 persons. About 150 km^3 of tephra $(1.7 \times 10^6$ tons) were ejected into the atmosphere, giving rise to remarkable sunsets and luminous twilights in England for 6 months after the eruption[464]. The total energy released in the two series of eruptions (April 5 and April 10), is estimated at 8.4×10^{26} erg, 80 times bigger than that of *Krakatoa* (1883).

The year that followed has sometimes been called the *year without a summer*, there being only 3 or 4 days without rain between May and October 1816 in Wales with subsequent poor harvests and food shortage.

The explosion affected climate on a world-wide scale: temperatures dropped by about 2°–4°C in Paris, Geneva, Milan, and some North American locations[465], resulting in considerable famine and extremely cold winters in many parts of the world. This lasted for about 3 years. It was not until 1847 that the first scientific expedition went to Sumbawa to study Tambora.

1815 CE, June 16-18 *Battle of Waterloo* (Belgium): An allied army under the command of Wellington (mixed British-Dutch-German-Belgian force of 100,000 men) and Blücher (Prussian force, ca 120,000 strong) defeated Napoleon's French army (ca 124,000 men); Austrian and Prussian monarchies

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[464] These may have inspired some of the best works of the English painter J.M.W. Turner and the novel *Frankenstein* by **Mary Shelley** who lived at that time in Geneve with **Shelley** and **Byron**.

[465] A freezing cold was reported in New England on the night of June 10, 1816, and on July 04, 1816 during daytime!

restored; German confederation replaced Confederation of the Rhine; Kingdom of Netherlands formally united Belgium and Holland. France's boundaries restored to those of 1790.

Napoleon very nearly defeated Wellington at Waterloo; Napoleon's ill-health may have provided the necessary weight to tilt the balance (migraine, hemorrhoids, gall stone colic, peptic ulcer and thyroid deficiency).

1815–1820 CE **John Loudon McAdam** (1756–1836, Scotland). Inventor. Originated the paving of roads with crushed rock, known as the *macadam* type of road surface. He was the first man to recognize that dry soil supports the weight of traffic, and that pavement is useful only for forming a smooth surface and keeping the soil dry. His macadam pavements consist of crushed rock packed into thin layers. McAdam methods of road building spread to all nations.

He was born at Ayr, Scotland, being descended on his father side from the McGregors. In 1770 he went to New York and returned with a considerable fortune (1783). The highways of Great Britain were at this time in a very bad condition and McAdam at once began to consider how to effect reforms. In pursuing his investigations he had traveled over 50,000 km of roads. In 1819 he published a *Practical Essay on the Scientific Repair and Preservation of Roads*, followed, in 1820, by the *Present State of Road Making*. As a result of a parliamentary inquiry in 1823 into the whole question of road-making, his views were adopted, and in 1827 he was appointed general surveyor of roads.

History of Roads[466] *and Highways*

Early roads were built in the Near East soon after the wheel was invented (ca 3500 BCE). As travel developed between villages, towns, and cities, trade routes were made. One such early system of roads was the Old Silk Trade

[466] The word *road* came from the Middle English word *rode*, meaning a *mounted journey*.This, in turn, was derived from the Old English *rad*, from the word *ridan*, meaning *to ride*.

Route which extended over 10,000 km connecting China and Rome and pre-Christian Europe across Turkestan, India, and Persia. The first road markers were piles of stones at intervals. Trails through forests were marked by blazing trees.

The Egyptians, Carthaginians and Etruscans all built roads. But the first really great road-builders were the Romans. They knew how to lay a solid base, paved with flat stones and recognized that the road must slope slightly from the center toward both sides to drain off water. They also dug ditches along the sides of the road to carry water away. Roman roads were intended mainly to transport soldiers across their empire. These roads ran in almost straight lines and passed over hills instead of cutting around them. The Roman built more than 80,000 km of roads and some are still in use.

In the Middle Ages there was little reason to build good roads, because most of the travel was on horseback. In South America from the 1200's to the 1500's the Inca Indians built a network of 16,000 km of roads connecting their cities. In England, certain main roads were higher than the surrounding ground because earth was thrown from the side ditches toward the center. Hence they were called highways. *These roads were under the protection of the king's men and were open to all travelers. In North America, early roads were surfaced with hand-broken stone and gravel. Some roads were covered with logs or planks, laid crosswise, and were therefore very bumpy.*

When the steam locomotive arrived in 1830, the rapid development of railroads began and people became convinced that the railroad was the best means of travel over long distances. From 1830 to 1900, there was little change in the surfacing materials for roads and highways. Even in the cities, only wood blocks, brick, and cobblestones were used. By 1900, because of the rapid development of the United States, there was a growing demand for good roads. It was mainly for roads extending a short distance from the railroad so farmers could get their produce to the rails. But with the ever growing use of the automobile after 1900, the demand arose for good roads to all places.

The first concrete road was laid in Detroit in 1908.

1815–1827 CE **William Prout** (1785–1850, England). Chemist and physician. Practiced in London. Suggested that *hydrogen* is the fundamental unit from which all elements are built (1815–1816). Among the first to classify food components into fats, carbohydrates, and proteins (1827). Made significant determinations of the density of air (1822–1823).

1815–1840 CE **Olinde Rodrigues** (1794–1851, France). Mathematician, economist and reformer with a brief career in mathematics. Born to Jewish parents of Portuguese ancestry. A student of **Monge** at the École Polytechnique. In 1815 he contributed to the differential geometry of surfaces (*Rodrigues formula* and *Rodrigues theorem*) and in 1816 his name became attached to a theorem in the theory of Legendre functions (also *Rodrigues formula*). Soon thereafter he became interested in the scientific organization of society, but made his living off the family banking business.

In 1840 he found some spare time to prove that every displacement of a rigid body is the resultant of a rotation and a translation. Described a rotation by *four* parameters, the first three determining the direction of the axis. He then developed explicit formulae for the resultant of two rotations and stressed the fact that the product is *not* commutative. He came to the aid of *Saint-Simon*[467] (founder of Socialism) in his destitute old age, supported him during the last years of his life and became one of his earliest adherents.

1816 CE **Renè Theophile Hyacinth Laënnec** (1781–1826, France). Physician. Invented the *stethoscope*: a device physicians use to hear the sounds produced by certain organs of the body, such as the heart, lungs, veins, and arteries.

Laënnec was a pupil of Napoleon's personal physician, Corvisart. He made the first stethoscope from a hollow wooden tube.

1816–1822 CE **Francois Magendie** (1783–1855, France). Physiologist. Showed for the first time that *nitrogenous* foods were needed for life. Professor at College de France (from 1831).

He fed dogs on diets composed of distilled water and one specific food, such as sugar, olive oil, or butter. The dogs in every case died after about a month.

Extended the work of Charles Bell (1811) on the functions of the dorsal and ventral roots of spinal nerves (1822). Formulated and demonstrated the *Bell-Magendie Law* that the anterior roots of the spinal cord control movements while the posterior roots control sensation.

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467 **Claude-Henri de Rouvroy**, **Comte de Saint-Simon** (1760–1825, France). Volunteer with the French troops fighting with Americans in the American Revolution (1777–1783); on his return to France (1783) made a fortune in land speculation but lost it (by 1805), and lived thereafter in poverty. Founded a 'religion of socialism', combining the teaching of Jesus with ideas of science and industrialism. His disciples spread his system, known as *Saint-Simonianism* throughout Europe.

History of Biology and Medicine, III – The 'Age of Reason'

During the Renaissance and Age of Discovery, renewed interest in empiricism as well as the rapidly increasing number of known organisms led to significant developments in biological thought; ***Vesalius*** *inaugurated the rise of experimentation and careful observation in physiology, and a series of naturalists culminating with* ***Linnaeus*** *and* ***Buffon*** *began to create a conceptual framework for analyzing the diversity of life and the fossil record, as well as the development and behavior of plants and animals. The growing importance of natural theology — partly a response to the rise of mechanical philosophy was also an important impetus for the growth of natural history (though it also further entrenched the argument from design).*

In the 18^{th} century many fields of science — including botany, zoology, and geology — began to professionalize, forming the precursors of scientific disciplines in the modern sense (though the process would not be complete until the late 1800s). ***Lavoisier*** *and other physical scientists began to connect the animate and inanimate worlds through the techniques and theory of physics and chemistry.*

In 1665, using an early microscope, ***Robert Hooke*** *discovered cells in cork, and a short time later in living plant tissue. The German* ***Leonhart Fuchs****, the Swiss* ***Conrad von Gesner****, and the British authors* ***Nicholas Culpeper*** *and* ***John Gerard*** *published herbals that gave information on the medicinal uses of plants.*

In 1628 ***William Harvey*** *explained that blood circulates throughout the body, and is pumped by the heart.* ***Antony van Leeuwenhoek****'s use and improvement of the microscope in about 1650 opened up the micro-world of biology. The History of Plants was greatly extended, almost into an encyclopedia, by* ***Giovanni Bodeo da Stapel*** *in 1644 CE.* ***Jan Swammerdam*** *(1658) and* ***Marcello Malpighi*** *(1660) were the first to observe and describe red blood cells, while Leeuwenhoek was the first to describe spermatozoa, bacteria and infusoria in the 1670's and 1680's. By the 1690's plants were, like animals, known to be sexual, having stamens and pistils.*

Systematizing, naming and classifying dominated biology throughout much of the 17th and 18th centuries. ***Carolus Linnaeus*** *published a basic taxonomy for the natural world in 1735, and in the 1750's introduced scientific names for all his species. The discovery and description of new species, and collecting specimens became a widespread passion of biologists.*

This work of classification was led by the Frenchmen ***Antoine de Jussieu*** *(1789),* ***Geoffray St. Hilaire*** *(1796),* ***Georges Cuvier*** *(1812) and* ***August de Condolle*** *(1819).*

One of the major evolutional trends during 1530–1750 CE was the passage from alchemy to medical chemistry. The ancient Greek biologists and medical writers had never considered the physiology of the human body in specifically chemical terms. Since ancient the Greek philosophers and the medieval scholars were not greatly interested in chemical substances and their properties, most medicines were not derived from mineral sources. However, some alchemists became interested in the application of alchemy to medicine.

Such a movement culminated in the work of ***Paracelsus*** *(1531), who endeavored to bring into being a new science of medical chemistry (iatrochemistry), by uniting medicine with alchemy.*

He put forward a theory that the human body was essentially a chemical system composed of mercury, sulphur and salt. Illness, according to Paracelsus, could arise from a lack of balance between these three elements, and the balance was to be restored by mineral medicines, not organic remedies. Iatrochemistry was developed further by ***van Helmont*** *(1648).*

One of the earliest chemists to put forward a mechanical theory of chemical change was ***John Ray*** *(1630). His line of thinking was extended by* ***Robert Boyle*** *(1684). Boyle was interested in the work of iatrochemists, particularly in their empirical observations, but he was of the view that those observations should be explained in terms of the mechanical philosophy, namely — that matter consists of particles of corpuscles in motion.*

The English school of medical chemists of the 17th century, including ***Robert Hooke*** *(1635–1703),* ***Richard Lower*** *(1631–1691), and* ***John Mayow*** *(1645–1679) — did not survive, and modern chemistry was founded in France at the end of the 18th century. Boyle had arrived at a reasonable definition of a chemical element and at a promising conception of method in chemistry.*

The discovery of the circulation of the blood by ***Harvey*** *(1628) established its primacy and many 18th century authors attributed to it alone all the properties formerly associated with the other humours. From the end of the 18th century physiological investigations concentrated more upon its constituent parts and assigned properties to them and, although modern science reliance on blood tests and transfusions has emphasized its role in diagnosis and therapy, it is now viewed primarily as a carrier and transmitter of other, more important, chemical substances round the body (e.g. hormones).*

The concept of the brain *as an anatomical entity emerged quite early in Western thought. The brain is first mentioned in Egyptian papyri and much of its detailed gross anatomy was described by the Greeks especially after the establishment of dissection as a valid method of inquiry.* ***Galen****, for example, was able to produce a classification of the* cranial nerves.

With the revival of anatomy during the Renaissance, more features of the brain were described, notably its arterial supply by ***Thomas Willis*** *(1664).* ***Marcello Malpighi*** *(1660) first investigated cerebral structure microscopically. By 1800, this anatomical tradition has elucidated most of the major visible features of the organ except for the regularity of the cerebral convulsions.*

Harvey's discovery of the blood's circulation (1628) also helped to elucidate the mechanics of breathing (respiration). First, ***Robert Boyle****,* ***John Mayow*** *and others (1645–1679) showed air necessary for life, Mayow recognizing a component of air indispensable for combustion, respiration and converting venous into arterial blood.*

Then ***Malpighi*** *(1679) microscopically identified the pulmonary capillaries and* ***Albrecht von Haller*** *(1752) expounded the mechanics of breathing. The identification of Mayow's* aerial nitre *with oxygen by* ***A. Lavoisier*** *(1780) and others, sealed the analogy between combustion and respiration.*

Finally, ***E.F.W. Pflüger*** *showed that the essential chemical changes of respirations occur in the tissues and cells rather than the lungs (metabolism).*

Table 3.9: Leading Biologists and Men of Medicine (1600–1820)

Key:

B = Biology	ZO = Zoology	CL = Chemistry of Life
A = Anatomy	M = Medicine	EP = Epidemics
P = Physiology	EM = Embriology	T = Taxonomy
BO = Botany	MB = Microbiology	IM = Immunology
S = Surgery	MR = Medical Research	PA = Pathology
		EB = Evolutionary Biology

Name	*fl.*	*Specialization*
Gaspard Bauhin	*1588–1623*	*(BO), (A)*
Eliyahu de Luna Montalto	*1596–1616*	*(M), (MR)*
John Gerard	*1597–1607*	*(BO)*
Andreas Libau	*1597–1613*	*(P), (CL)*
John Tradescant	*1600–1638*	*(BO)*
Adrian van der Spiegel (Spigelius)	*1603–1625*	*(BO), (A)*
Santorio Santorio	*1603–1614*	*(P), (M)*
Theodore Turquet de Mayerne	*1603–1644*	*(P), (CL), (MR)*
Hieronimus Fabricius	*1604–1619*	*(EM), (A), (M)*
Joseph Solomon Delmedigo	*1616–1629*	*(M)*
Johann Baptista van Helmont	*1620–1648*	*(BO), (M), (CL)*
Zacutus Lusitanus	*1625–1642*	*(M), (MR)*
William Harvey	*1628–1651*	*(P), (M)*
Franciscus de la Boë	*1641–1672*	*(M), (A), (CL)*
Thomas Browne	*1645–1680*	*(BO), (M)*
Georg Rumpf van Hanau	*1655–1698*	*(BO)*
Thomas Wharton	*1656–1673*	*(M), (A)*
Jan Swammerdam	*1658–1673*	*(P), (A), (EM)*
Marcello Malpighi	*1660–1679*	*(P), (A)*
Lorenzo Bellini	*1664–1704*	*(M), (A)*
William Petty	*1664–1687*	*(A), (M)*
Thomas Willis	*1664–1672*	*(A), (P)*
Richard Lower	*1665–1691*	*(P)*
Robert Hooke	*1665–1703*	*(P)*
Thomas Sydenham	*1666–1686*	*(EP), (M)*

Table 3.9: (Cont.)

Name	*fl.*	*Specialization*
Francis Willughby	*1667–1704*	*(T)*
John Ray	*1667–1705*	*(T), (CL)*
Francesco Redi	*1668–1697*	*(B)*
Anton van Leeuwenhoek	*1668–1692*	*(MB)*
Regnier de Graaf	*1668–1673*	*(M), (A)*
Nehemiah Grew	*1672–1682*	*(M), (BO)*
John Mayow	*1674–1679*	*(P), (M)*
Robert Boyle	*1684–1691*	*(P)*
Rudolph Camerarius	*1694–1721*	*(M), (BO)*
Giacomo Pylarini of Smyrna	*1701*	*(M), (IM)*
Stephen Hales	*1705–1730*	*(P)*
Hermann Boerhaave	*1707–1732*	*(M), (BO)*
Antoine de Jussieu	*1719–1758*	*(BO)*
Pierre Fauchard	*1728–1761*	*(M)*
Jean Astruc	*1729–1753*	*(M)*
Carolus Linnaeus	*1735–1763*	*(BO), (T)*
Georges Louis Leclerc (de Buffon)	*1739–1788*	*(BO)*
Julien de la Mettrie	*1740–1751*	*(M), (S)*
Percival Pott	*1740–1780*	*(M), (S)*
Giovanni Battista Morgagni	*1740–1760*	*(M), (A)*
James Lind	*1747–1794*	*(S), (M)*
Frederik Hasselquist	*1749–1752*	*(BO)*
Victor Albrecht von Haller	*1752–1773*	*(M), (A), (P), (BO)*
John Hunter	*1760–1790*	*(M), (P)*
Joseph Gottlieb Kölreuter	*1761–1766*	*(BO)*
Lazzaro Spallanzani	*1765–1785*	*(P), (B)*
Peter Simon Pallas	*1766–1794*	*(BO), (ZO)*
William Hewson	*1769–1774*	*(P), (S), (A)*
Otto Frederik Müller	*1773–1778*	*(B)*
Jan Ingenhousz	*1779–1799*	*(M), (CL)*
Jiri Prochaska	*1784–1820*	*(A), (P), (MB)*
Christian Sprengel	*1793–1816*	*(BO)*
Edward Jenner	*1796–1823*	*(IM)*

Table 3.9: (Cont.)

Name	*fl.*	*Specialization*
Georg Cuvier	*1796–1825*	*(B)*
Étienne Saint Hilaire	*1798–1801*	*(ZO)*
Samuel Hahnemann	*1793–1843*	*(M)*
Jules de Savigny	*1798–1801*	*(ZO)*
Aimé Jacques Bonpland	*1799–1813*	*(M), (BO)*
Augustin de Candolle	*1799–1839*	*(BO)*
Karl Friedrich Burdach	*1800–1802*	*(B), (P)*
Gottfried R. Treviranus	*1802–1837*	*(B), (EB)*
Charles-Francois de Mirbel	*1802–1854*	*(BO)*
Nicolas Théodore de Saussure	*1804–1845*	*(CL), (BO)*
Friedrich Sertürner	*1805–1841*	*(CL)*
Jean-Baptiste de Lamarck	*1801–1822*	*(B), (T)*
Franz Joseph Gall	*1810–1820*	*(A), (M)*
Charles Bell	*1811–1842*	*(A), (M)*
William Prout	*1815–1827*	*(P)*
Renè Theophile Laënnec	*1816–1826*	*(M)*
Francois Magendie	*1816–1822*	*(P)*
Pierre Joseph Pelletier	*1817–1842*	*(CL)*
Joseph-Bienaime Caventou	*1817–1877*	*(CL)*
Alfred Donné	*1829–1878*	*(P), (BM), (M)*
Thomas Addison	*1837–1860*	*(P), (M)*
J.H. Bennett	*1845*	*(P)*
E.F.W. Pflüger	*1868*	*(P), (EB)*
Claudius Aymand		*(S), (M)*

1816 CE **John Farey** (1766–1826, England). Geologist and surveyor. In an article '*On a curious property of vulgar functions*' published in the *Philosophical Magazine* (1816) he constructed a sequence of common fractions (now called the 'Farey Sequence') defined as follows: For a fixed number n, one observes all rationals between 0 and 1 which, when expressed in their lowest terms, have denominator not exceeding n. The sequence is then written in ascending order of magnitude beginning with the smallest. For example[468]

$$F_5 = \left\{ \frac{0}{1}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{1}{1} \right\} .$$

It is then found that this sequence has the following 'curious property': each member of the sequence is equal to the rational whose numerator is the sum of the numerators of the fractions on either side, and whose denominator is the sum of the denominators of the fractions on either side. Thus

$$\frac{2}{5} = \frac{1+1}{3+2}; \quad \frac{1}{3} = \frac{1+2}{4+5}; \quad \frac{2}{3} = \frac{3+3}{5+4} .$$

In the final paragraph of his article, Farey wrote:

> *I am not acquainted, whether this curious property of vulgar fractions has been before pointed out?; or whether it may admit of some easy or general demonstration?; which are points on which I should be glad to learn the sentiments of some of your mathematical readers ...*

One mathematical reader (at least of a French translation) was **Cauchy**, and he gave the necessary proof in his *Exercises de mathématique* which was published in the same year as Farey's article. This might have been the end of the story but there is more to tell.

Farey was not the first to notice the property. **Haros**[469], in 1802, wrote a paper on the approximation of decimal fractions by common fractions. He

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468 The question may arise as to how long is the Farey Sequence? It can be shown that the n^{th} sequence has the length

$$L(n) = 1 + \Phi(1) + \Phi(2) + \cdots + \Phi(n-1) + \Phi(n)$$

where $\Phi(n)$ is the Euler totient function, equal to the number of numbers smaller and relatively prime to n [e.g. $\Phi(1) = 1$, $\Phi(2) = 1$, $\Phi(3) = \Phi(4) = \Phi(6) = 2$, $\Phi(5) = \Phi(8) = \Phi(10) = 4$, $\Phi(7) = \Phi(9) = 6$, $\Phi(100) = 40$]. There is no simple formula for the above sum of totient numbers, but it is known that *asymptotically* for large n the sum is $L' = \left(\frac{3}{\pi^2} n^2\right)$. For example $L'(10) = 30.4$ compared to $L(10) = 30$.

469 Haros, C.: "Tables pour evaluer une fraction ordinaire avec autant de decimales qu'on voudra; et pour trouver la fraction ordinaire la plus simple, et qui approche sensiblement d'une fraction decimale", in Journal de L'Ecole Royale Polytechnique, Tome IV (cahier 11), pp. 364–368, Paris, 1802.

explains how to construct what is in fact the Farey sequence for $n = 99$ and Farey's "curious property" is built into his construction. However, this is certainly not a proof, nor for that matter a general statements of the "curious property".

Farey himself gave no proof, and it is unlikely that he had found one, since he seems to have been at the best an indifferent mathematician. As a geologist he is forgotten. However, the one thing in his life which survives is just his sequence.

The Farey sequence F_n for the first few values of n are

$$F_1 = \left\{\tfrac{0}{1}\,,\ \tfrac{1}{1}\right\}$$

$$F_2 = \left\{\tfrac{0}{1}\,,\ \tfrac{1}{2}\,,\ \tfrac{1}{1}\right\}$$

$$F_3 = \left\{\tfrac{0}{1}\,,\ \tfrac{1}{3}\,,\ \tfrac{1}{2}\,,\ \tfrac{2}{3}\,,\ \tfrac{1}{1}\right\}$$

$$F_4 = \left\{\tfrac{0}{1}\,,\ \tfrac{1}{4}\,,\ \tfrac{1}{3}\,,\ \tfrac{1}{2}\,,\ \tfrac{2}{3}\,,\ \tfrac{3}{4}\,,\ \tfrac{1}{1}\right\}$$

Except for F_1, each F_n has an *odd* number of terms and the middle term is always $\frac{1}{2}$. Let $\frac{p}{q}$, $\frac{p'}{q'}$ and $\frac{p''}{q''}$ be three successive terms in a Farey Sequence. Then

$$qp' - pq' = 1; \qquad \frac{p'}{q'} = \frac{p+p''}{q+q''}\,.$$

These two statements are actually equivalent.

A method of computing a Farey Sequence of order $n+1$ from a sequence of order n is as follows: Let $\frac{a}{c}$ be directly followed by $\frac{b}{d}$ in the n sequence. Then, the fraction $\frac{a+b}{c+d}$, $\quad c+d \geq n+1$ is the *mediant fraction* between $\frac{a}{c}$ and $\frac{b}{d}$ in the $(n+1)^{th}$ sequence. Interpolating the Farey Sequence of order n with such mediant $\frac{a+b}{c+d}$, satisfying $c+d \geq n+1$, we obtain the Farey Sequence of order $n+1$.

1816–1823 CE **Francis Ronalds** (1788–1873, England). Pioneer of telegraphy whose ideas were largely ignored. Experimented with sending messages of words and numbers over few hundred meters by single wires.

On the ground of his estate in Hammersmith, London, he erected an experimental telegraph system. It used a clockwork-driven rotating dials, engraved with letters of the alphabet and numbers, synchronized with each other, at both ends of the circuit. They were connected with an iron wire hung on two strong wooden frames.

Ronalds successfully transmitted and received letters. The British Admiralty was informed of his success but rejected his invention; they felt that the telegraph was not needed in peacetime and that the existing semaphore system was adequate. In their own words: "*Telegraphs of any kind are wholly unnecessary*" (1823).

1816–1844 CE **Friedrich Wilhelm August Froebel** (1782–1852, Germany). Educator and founder of the *Kindergarten* system.[470] Created (1837) an environment for young children that nurtured self-education, spontaneous play, and intimacy with nature – the kindergarten.

Born in Oberweissbach, Turingia, Froebel was the fifth son of the village Lutheran pastor. His mother died when he was an infant and the boy would spend most of his time in the gardens and forests surrounding his home. Perhaps because of its botanical heritage, the village became the place where Froebel would begin to feel a deep, mystical connection with nature, which later influenced his ideas on education.

Having taken up the education of the sons of his deceased brother, Friedrich marveled at the auto-didactic nature of their play. Each was different, yet each led himself to new understandings and discoveries through individual role playing and adventures. Perhaps the prevailing idea that children should be "seen and not heard" was not correct. Perhaps children carried in them the seed of self-development, which should be encouraged, guided and nurtured by adults. "Kommt, lasst uns unsern Kindern leben" **Come, let us live with our children** became the cornerstone for a new approach to early childhood education. Children are like the tiny flowers; they are varied and need care, but each is beautiful alone and glorious when seen in the community of peers. "My school shall be called Kindergarten – the garden of children," he reasoned.

Froebel labeled his approach to education as "self-activity." This idea allows the children to be led by their own interests and to freely explore them. The teacher's role, therefore, is to be a guide rather than lecturer.

Froebel studied and worked under **Pestalozzi** at Yverdon, Switzerland (1808–10); served during the anti-French campaign (1813–14); assistant in mineralogical museum, Berlin (1814–16). Founded school at Griesheim (1816); moved to Keilhau (1817); founded a kindergarten at Blankenburg, Thuringia (1837); established training courses for kindergarten teachers and introduced kindergartens throughout Germany. Author of *Die Menschenerziehung* (1826), *Mutter- und Koselieder* (1844), etc.

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[470] Prior to Froebel's kindergarten, children under the age of 7 did not attend school, since it was held that young children did not have the ability to focus or to develop cognitive emotional skills before this age. Froebel's ideas seem correct enough to us today, yet were radical in his day.

Froebel's ideas were promoted in Germany through the intervention of Baroness Bertha von Marenholtz-Buelow. Through her connections to the more liberal Weimar court and Thuringian nobility, as well as liberal urban educators and intellectuals in Dresden, Leipzig, Frankfurt and Berlin, Madame von Marenholtz-Buelow convinced skeptics and adherents alike that there was worth in his ideas. She took Froebel's philosophy to Switzerland, Holland, Belgium and England. In London, **Charles Dickens** attended her lectures and wrote that he was favorably impressed. Other liberal educators and followers of Froebel transplanted the educational system to the United States, Canada, and even Japan.

The influence of Froebel's system was not to end in the 19^{th} century. Although modified, child-centered kindergartens are now found throughout the world. There is a Froebel College on Roehampton Lane in London and another in Dublin, Ireland, which together with the Pestalozzi Froebelhaus in Berlin to this day further the child-friendly ideas started by the Thuringian educator over 165 years ago.[471]

1817 CE **David Ricardo** (1772–1823, England). Economist. Founder of the *classical school*[472] of economics. One of the leading economists of the 19^{th} century. In "*On the Principles of Political Economy and Taxation*" (1817) developed his theory of rent, profit and wages, and presented clear statements on the quantitative theory of money.

Ricardo was born in London to a religious Jewish family. His father (descended from Portuguese marano's) emigrated from Holland and became a successful member of the London Stock Exchange. At the age of 14, Ricardo

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471 Norman Brosterman, in his recent book *Inventing Kindergarten* (New York: Harry Abrams, 1997), theorizes that Froebel was the impetus for the creations of a number of renowned modern architects and artists, all who had attended Froebelian kindergartens where abstraction of natural forms through geometric shapes was explored. Hence, one finds commonalities in the work of such figures as **Georges Braque**, **Piet Mondrian**, **Paul Klee**, **Wassily Kandinsky**, **Frank Lloyd Wright**, and **Le Corbusier** amongst others.

472 *Classical economy* is based on the assumption that people behave *rationally*; they desire to maximize gain and have freedom of choice (of goods, occupations, etc.). The theory then *deduces* from these premises how people act individually and how their actions collectively determine prices and quantities of goods in the market. Deductive theory based on the assumption of *rational action* to maximize gain is still adhered to by many economists who see no possibility of any other theory.

The basic elements of classical economics were already contained in the writings of **Adam Smith** (1776) and **Robert Malthus** (1798).

entered his father's office, where he showed much aptitude for business. But in 1793 he married physician's daughter of a Quaker family and converted to the Anglican Church, severing his ties with his family and his faith-sakes altogether. He then entered a successful career in the profession to which he had been brought up and at the age of 25 was already rich. Ricardo retired from business (1819), became a land proprietor, and entered parliament (1819–1823).

Ricardo's work was the real first textbook on economics; he defined the conditions that would enable a nation's economy to reach its greatest potential. He believed that the accumulation of capital was the key to rapid economic growth[473], and argued that allowing businessmen to seek high profits would bring about a rapid accumulation of capital. He considered labor to be the most important source of wealth.

Labor, to Ricardo, was very much like any other commodity. When it was plentiful, it was cheap; when it was scarce, it was expensive. As long as there is an ample supply of workers, wages will inevitably sink to the lowest possible level of subsistence, just above starvation. To try to remedy this situation by lowering profits and raising wages would be futile, since it would merely increase the number of worker's children and, by limiting the supply of capital, cut down production. He therefore advocated that wages should be left to the fair and free competition of the market, and should never be controlled by the interference of the legislation.

Ricardo's theories influenced other thinkers; his theory of comparative advantage is still the basis for the modern theory of *international trade*. **Karl Marx** was influenced by Ricardo's *labor theory of value*, which held that the value of a commodity is determined by the amount of labor needed in its production. **John Stuart Mill** used Ricardo's ideas as the basis for a philosophy of social reform. In general, the tenets of Ricardo's theory were enthusiastically adopted by a rising manufacturing class which sought low wages and freedom from governmental interference.

1817 CE **Johann Wolfgang Döbereiner** (1780–1849, Germany). Chemist. Recognized (52 years ahead of Mendeleev) relationships between the properties of the chemical elements and their *atomic weights*, upon which the periodic table of the elements is based; classed closely related elements in group of three (known as *Döbereiner's triads*).

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[473] The entire economic activity of Ricardo took place in the shadow of the *Napoleon Wars* and the following periods of Restoration. His theory offered solutions to concrete problems that bugged the economy of Britain at that time. Ricardo's economics enjoyed an immense practical success, culminating in the adoption of *free trade* in England (1846).

He was a professor at Jena from 1810.

1817–1848 CE **Richard Roberts** (1789–1864, England). Engineer and mechanical inventor. Machine-tools pioneer. Made a long series of inventions of machines in the cotton, railway and steam-engine industries. His inventions include a screw-cutting lathe and a planing machine. In 1848 he invented a machine for punching holes in steel plates. Incorporating the Jacquard method, he devised a machine for punching holes of any pitch in bridge plates and boiler plates. He later invented a machine for simultaneously shearing iron and punching both webs of angle iron to any pitch.

Roberts was born at Carreghova, Montomeryshire, Wales, — a son of a shoemaker. He had very little formal education. Starting as a toolmaker at Manchester he became one of the greatest mechanical engineers of the 19^{th} century. But with all his inventive genius, his lack of business acumen led him eventually to die in poverty.

1817–1820 CE **Bernhard (Bernhardus Placidus Johann Nepomuk) Bolzano** (1781–1848, Prague). Czech priest, mathematician and philosopher. Made many important contributions to mathematics in the first half of the 19^{th} century. Freed calculus from the concept of the infinitesimal. Was one of the first to recognize that many "obvious" statements about continuous functions[474] require proof. His observations concerning continuity were published posthumously in 1850.

In 1834 Bolzano devised a function which is continuous throughout an interval but has no derivative at any point on that interval. This work was overlooked for almost 30 years and credit for this function is given to **Weierstrass**, who rediscovered it in 1861. In 1840, Bolzano introduced the concept of denumerable and nondenumerable sets, 32 years ahead of **Cantor** (1872).

In 1865 Karl Weierstrass proved that if S is a bounded infinite set of points, then there exists a point P such that every neighborhood of P contains points of S. This is known as the *Bolzano-Weierstrass Theorem* in recognition of the earlier contribution of Bolzano.

Bolzano was born in Prague. His father, an Italian emigrant, was an art dealer and his mother was the daughter of a hardware tradesman. Bolzano studied philosophy, physics, mathematics and theology at the University of

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[474] *Bolzano's theorem*: Let $f(x)$ be continuous at each point of a closed interval $[a, b]$ and assume that $f(a)$ and $f(b)$ have opposite signs. Then there is at least one point c in the open interval (a, b) such that $f(c) = 0$.

Bolzano's definition of continuity: $f(x)$ is continuous for $x = \xi$ if, given $\delta > 0$, we can choose $\epsilon(\delta) > 0$ so that $|f(x) - f(\xi)| < \delta$ if $0 < |x - \xi| \leq \epsilon(\delta)$.

Prague (1796–1804). He distinguished himself at an early age, and after his ordination to the priesthood was appointed professor of the philosophy of religion at Prague University (1807). In 1816 he was accused of being connected with some of the student's revolutionary societies. He was compelled to resign and was also suspended from his priestly functions, spending the rest of his life in literary work. He was influenced by **Leibniz** and **Kant**.

1818–1827 CE **Augustin Jean Fresnel** (1788–1827, France). Prominent physicist. Derived the equation of wave-surfaces of purely transverse plane waves in anisotropic media (crystals). He discovered that the structure of anisotropic media permits two plane waves with different linear polarizations and distinct velocities of propagation, in any given direction.

In 1818 he was first to give a correct explanation to the phenomena of *diffraction* of light as the mutual interference of secondary waves from an aperture. In a series of calculations he demonstrated the ability of a transverse wave theory of light to account for the details of the observed phenomena of reflection, refraction, interference, polarization, and diffraction patterns that appear as light spreads around objects. His theory led to such fundamental concepts[475] as *Fresnel-diffraction*, *Fresnel-Huygens principle*, *Fresnel-zones*, *Fresnel integrals*, and *Fresnel equations*.

Fresnel, the son of an architect, was born at Broglie (Eure). His early progress in learning was slow, and when 8 years old he was still unable to read. At the age of 13 he entered the École Centrale in Caen, and at 16 he entered the École Polytechnique. Then he went to the École des Ponts et Chaussées and started his career as a civil engineer, engaged in the construction of roads in Southern France. Fresnel spoke openly against Napoleon, and as a consequence he had to resign his government position. This freed him to

[475] *Diffraction* — the deviation of light from rectilinear propagation. It occurs whenever the waves encounter an obstacle (either transparent or opaque), and results in alteration of the amplitude and phase of the incident radiation. The various segments of the wavefront propagate beyond the obstacle and *interfere* to cause the particular energy-density distribution referred to as a *diffraction pattern*. The hypothesis that each point on a wavefront is a source of secondary waves [Huygens' principle] was supplemented by Fresnel with the statement that these secondary waves are mutually coherent, and the waves emitted by them interfere. Thus, while analyzing the propagation of waves we must take into consideration their amplitudes *and* their phases. If we use *plane waves* to begin with and look at the interference pattern *far away* from the obstacle, the phenomenon is called *Fraunhofer diffraction.* If the original wavefront is not plane and if we study the interference pattern just past the obstacle, the phenomenon is called *Fresnel diffraction.*

devote himself entirely to optics. He was elected a member of the Académie des Sciences at Paris in 1823, and in 1825 he became a member of the Royal Society of London. In 1819 he was nominated a commissioner of lighthouses. He died of consumption at Ville-d'Avray, near Paris.

His work in optical science received only scant public recognition during his lifetime, and some of his papers were not printed by the Academie till many years after his death. But, as he wrote to **Young** in 1824: "*In me, that sensibility, or that vanity, which people call love of glory, had been blunted. All the compliments that I have received from **Arago**, **Laplace** and **Biot**, never gave me so much pleasure as the discovery of a theoretic truth, or the confirmation of a calculation by experiment*".

The Diffraction of Light – Fresnel vs. Poisson

*In the year 1678 **Christiaan Huygens** expressed the intuitive conviction that if each point on the wavefront of light signal were considered to be the source of a 'secondary' spherical disturbance, then the wave front of any later instant could be found by constructing the* 'envelope' *of these secondary wavelets. With this construction he could explain the wave phenomena of reflection. However, the phenomenon of* diffraction*, observed by **F.M. Grimaldi** (1660), could not be accounted for, neither by him nor by any of his contemporaries.*

*In 1801, **Thomas Young** discovered the* interference *of light waves and thus paved the way for **Augustin Jean Fresnel** (1818) to establish the real cause of diffraction: by making some rather arbitrary assumptions about the effective amplitudes and phases of Huygens' secondary sources, and by allowing the various wavelets to mutually interfere, Fresnel was able to calculate the distribution of light in diffraction patterns with excellent accuracy.*

The transversally *of light motion was recognized by Young (1817) and the* polarization *of light was discovered by **Malus** (1809). Thus, in a span of little more than one decade, all major difficulties in the wave theory of light were resolved. The centuries – old question of the nature of light was answered by stating that light was a transverse motion of waves in the elastic ether.*

In the meantime, the corpuscular theory had been developed further by ***P.S. de Laplace*** *and* ***J.B. Biot****, and under their influence the Paris Academy proposed the subject of diffraction for the prize question of 1818, in the expectation that a treatment of this subject would lead to the crowning triumph of the corpuscular theory.*

To their dismay, and in spite of strong opposition, the prize was awarded to ***A.J. Fresnel****, whose treatment was based on the wave theory. His work was the first of a succession of investigations, which, in the course of a few years, were to discredit the corpuscular theory completely.*

In his memoir Fresnel effected a synthesis of Huygens' envelope construction with Young's principle of interference. This was sufficient to explain diffraction phenomena. Fresnel calculated the diffraction caused by straight edges, small apertures, and screens. [He was advised by ***Francois Jean Arago*** *(1786–1853, France) to read the publications of Grimaldi and Young, but could not follow this advice because he could read neither English nor Latin.]*

Fresnel's theory predicted that in the center of the shadow of a small aperture there should appear a bright spot. This counter-intuitive fact caused ***S.D. Poisson*** *to refute the theory. Fresnel was saved by Arago, who performed the experiment by himself and verified that Fresnel's theory was indeed correct. Poisson acquired his share of fame in the event: the spot became known as Poisson's spot!*

The ideas of Huygens and Fresnel were put on a firmer mathematical foundation by ***Helmholtz*** *(1860),* ***Rayleigh*** *(1871) and* ***Kirchhoff*** *(1882). Helmholtz developed the mathematical theory of the Huygens principle for monochromatic steady-state scalar waves and Kirchhoff generalized the results of Helmholtz for a source with an arbitrary time-dependence. Both employed a mathematical vehicle formulated earlier by* ***Green*** *(1828), but had went unnoticed until 1845, when it was publicized by* ***Kelvin****. Both succeeded in showing that the amplitudes and phases ascribed to the secondary Huygens sources by Fresnel were indeed logical consequences of the wave nature of light.*

Kirchhoff based his mathematical formulation on two assumption about the boundary values of light incident on the surface of an obstacle placed in the way of the propagating light. These assumption were later proved inconsistent by ***Poincaré*** *(1892).*

As a consequence of these criticism, Kirchhoff's formulation of the so-called Huygens-Fresnel principle must be regarded as a first approximation although under most conditions it yield results that agree amazingly well with experiments.

*The advent of the Maxwell electromagnetic theory [***Maxwell** *(1864),* **Hertz** *(1888)], left no doubt that light was not an elastic wave, and in 1881* **Rayleigh** *analyzed the* scattering of light *by small particles based on the electromagnetic theory of light. It is interesting to note that in the beginning Kirchhoff and Rayleigh based their diffraction theories upon the elastic theory of light,* ignoring Maxwell's equations. *However, as far as their approximations were concerned, light could be treated either way.*

The first truly rigorous solution of a diffraction problem was given in 1896 by **Arnold Sommerfeld** *and in 1897 by* **Rayleigh** *and is known as the* Rayleigh-Sommerfeld diffraction theory. *In it, Sommerfeld treated the 2-dimensional case of a plane-wave incident on an infinitesimally thin, perfectly conducting half-plane.*

In general, rigorous diffraction theory involves solving the Maxwell equations subject to boundary conditions assumed for the aperture screen. The geometrical limitations inherent in the Fresnel-Kirchhoff theory do not exist in the rigorous theory, which therefore renders a complete description of the field in the vicinity of the aperture boundary, as well as at great distances. Unfortunately, the number of cases that can be treated rigorously is very limited, and even the simplest case, (that treated by Sommerfeld) involves complicated mathematical analysis.

However, even so, the Rayleigh-Sommerfeld theory employs certain simplifications and approximations ab initio. Central to these is the treatment of light as a scalar *phenomenon. i.e., only the scalar amplitude of a single transverse component of either the electric or the magnetic field vector is considered, it being assumed that any other component of interest can be treated* independently *in a similar fashion. Such on approach entirely neglects the fact that the various component of the electric and magnetic field vectors are* coupled through Maxwell's equations *and cannot be treated independently.*

Experiments have shown that the scalar theory yields accurate results if two conditions are met[476]*:*

- *The diffracting aperture must be large compared with the wave-length.*
- *The diffracted field must not be observed too close to the aperture.*

476 There exist important problems for which these conditions are *not* met, e.g., in the theory of high-resolution *diffraction gratings*. There, the vectorial nature of the fields *must* be taken into account if reasonably accurate result are to be obtained.

We next give a succinct account on scalar diffraction theory, encompassing the main results of **Fresnel**, **Helmholtz**, **Kirchhoff** *and* **Rayleigh**. *For the sake of both clarity and brevity we do not follow the historical sequence of evolution, and use modern notation.*

Scalar diffraction theory[477]

Although from a wave point of view, we find it quite reasonable that waves can "bend around corners", it is still to be seen how to handle the phenomenon quantitatively. Historically, most of the development of the theory of diffraction has been in the context of visible light, but it must be recognized that the theory is applicable to any physical process that can be described by the ordinary wave equation in two or three spatial dimensions (e.g. electromagnetic, acoustic, seismic, etc.)

A point-source at P_s emits an outgoing monochromatic spherical wave which interacts with an opaque screen S, having a small aperture ΔS. According to **Huygens**' *construction, every point of the wavefront may be considered as a center and source of a secondary disturbance which gives rise to spherical wavelets, and the wavefront at any later instant may be regarded as an envelope of these wavelets.* **Fresnel** *supplemented the Huygens construction with the postulate that the secondary wavelets mutually interfere. This combination of Huygens' construction with the principle of interference is known as the* Huygens-Fresnel principle. *Let us invoke it in the above setup: let the wave reach an aperture point Q centered on an infinitesimal area element ds at distance r_s from P_s. There it excites secondary spherical wavelets which reach an observation point P_0 at distance r_0 from Q.*

Now, the wave-amplitude reaching Q from P_s is a solution of the wave equation

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\Psi = 0,$$

namely $\Psi = \frac{A}{r}e^{i(kr-\omega t)}$, where A is the amplitude at unit distance from the source, λ is the wavelength, $k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$ is the wave-number, ω is angular frequency and c is the wave velocity. A unit wavelet with zero phase, emerging

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[477] To dig deeper, see:

- Born, M. and E. Wolf, *Principles of Optics*, Macmillan Co.: New York, 1964, 808 pp.
- Stone, J.M., *Radiation and Optics*, McGraw-Hill, New York, 1963, 544 pp.

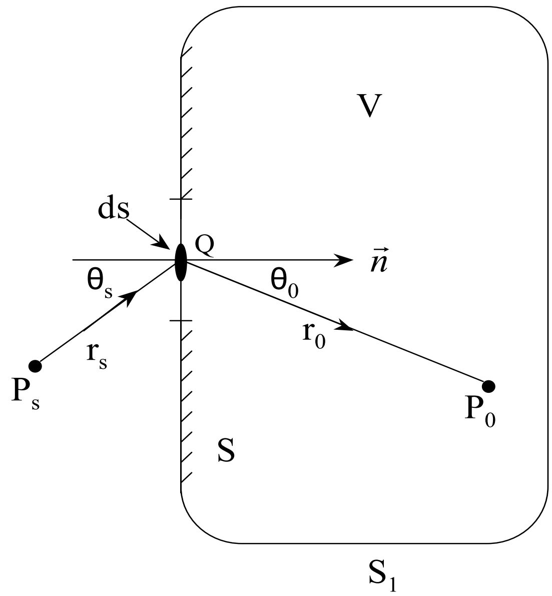

Fig. 3.3: Scalar diffraction by a small aperture

from Q, reaches P_0 with amplitude $\frac{1}{r_0}e^{ikr_0}$. Accordingly, the wavelet that arrives at P_0 from the secondary source element at Q may be written as

$$d\Psi = \Psi_0 f(\theta_s, \theta_0)\left[\frac{A}{r_s}e^{i(kr_s-\omega t)}\right]\left[\frac{1}{r_0}e^{ikr_0}\right],$$

where Ψ_0 is a constant complex source-amplitude and $f(\theta_s, \theta_0)$ represents the dependence of the amplitude of the secondary wavelet on its angular position in the aperture relative to P and P_0 (the angle θ_s is between the normal $\mathbf{n}$ at Q and the vector $\mathbf{r_s}$, and the angle θ_0 is between the same normal at Q and the vector $\mathbf{r_0}$. The entire disturbance at P_0 is found by summing up all contributions across the aperture

$$\Psi(P_0) = \Psi_0 A e^{-i\omega t}\int_{\Delta S} f(\theta_s, \theta_0)\frac{e^{ik(r_s+r_0)}}{r_s r_0}ds. \quad (1)$$

The results is known as the Fresnel-Kirchhoff formula. **Fresnel** *rendered a convenient contrivance to calculate the above integral, known as the Fresnel zone construction.*

Helmholtz *and* ***Kirchhoff*** *realized that in order to obtain explicit values for* Ψ_0 *and* $f(\theta_s, \theta_0)$ *in (1), the above heuristic derivation must be properly represented as a boundary value problem. To this end they used the 1824 Green's theorem which states that for surface* Σ *enclosing a volume* V

$$\int_V \left(\Phi\nabla^2\Psi - \Psi\nabla^2\Phi\right) d^3x = \int_\Sigma \left[\Phi\frac{\partial\Psi}{\partial n} - \Psi\frac{\partial\Phi}{\partial n}\right] d\Sigma. \tag{2}$$

Let the field Ψ *be assumed to satisfy the homogeneous scalar Helmholtz wave equation*

$$(\nabla^2 + k^2)\Psi(\mathbf{x}) = 0$$

and let $\Phi = G$ *be the Green's function for the Helmholtz wave equation*

$$(\nabla^2 + k^2)G(\mathbf{x}, \mathbf{x}') = -\delta(\mathbf{x} - \mathbf{x}').$$

Eq. (2) then yields for points $\mathbf{x}$ *inside* V *and points* $\mathbf{x}'$ *on* Σ

$$\Psi(\mathbf{x}) = \int_\Sigma \left[\Psi(\mathbf{x}')\mathbf{n}' \cdot \nabla' G(\mathbf{x}, \mathbf{x}') - G(\mathbf{x}, \mathbf{x}')\mathbf{n}' \cdot \nabla'\Psi(\mathbf{x})\right] d\Sigma, \tag{3}$$

where $\mathbf{n}'$ *is the inward directed normal to* Σ.

Choosing $G(\mathbf{x}, \mathbf{x}') = \frac{e^{ikr_0}}{4\pi r_0}$, $r_0 = |\mathbf{x} - \mathbf{x}'|$, *we divide the integral over* Σ *into two parts, one over the screen and its aperture* (ΔS)*, the other over a surface* S_∞ *which is made to recede to infinity. It can be shown that the contribution from* S_∞ *vanishes under a requirement on the behavior of* Ψ *at infinity, known as the Sommerfeld radiation condition. Then (3) becomes*

$$\Psi(\mathbf{x}) = -\frac{1}{4\pi} \int_{\Delta S + Screen} \frac{e^{ikr_0}}{r_0}\mathbf{n}' \cdot \left[\nabla'\Psi + ik\left(1 + \frac{i}{kr_0}\right)\mathbf{e}_{r_0}\Psi\right] ds \tag{4}$$

where $\mathbf{e}_{r_0}$ *is a unit vector in the direction of* $\mathbf{r}_0$*. In order to apply (4), it is necessary to know the values of* Ψ *and* $\frac{\partial\Psi}{\partial n}$ *on* ΔS *and the screen. But these values are not known, unless the problem has been solved. Kirchhoff's approach was to approximate the values of* Ψ *and* $\frac{\partial\Psi}{\partial n}$ *over the aperture and the screen by assuming:*

- Ψ *and* $\frac{\partial\Psi}{\partial n}$ *vanish everywhere on the screen.*

- *The values of* Ψ *and* $\frac{\partial \Psi}{\partial n}$ *on the aperture are equal to the values of the incident wave in the absence of the obstacle, ergo* $\Psi \simeq A\frac{e^{ikr_s}}{r_s}\, e^{-i\omega t}$.
- $kr_0 >> 1$ *('far-field' approximation).*

With these assumptions, the Sommerfeld-Kirchhoff integral becomes

$$\Psi(P_0,t) = -\frac{iA}{\lambda}e^{-i\omega t}\int\limits_{\Delta S}\left[\frac{\cos\theta_s + \cos\theta_0}{2}\right]\frac{e^{ik(r_s+r_0)}}{r_s r_0}ds, \tag{5}$$

where we have used the ancillary relations $(\mathbf{n}' \cdot \mathbf{e}_{r_s}) = \cos\theta_s$ *and* $(\mathbf{n}' \cdot \mathbf{e}_{r_0}) = \cos\theta_0$. *Upon comparison with the Fresnel-Kirchhoff formula we find*

$$\Psi_0 = -\frac{i}{\lambda}, \qquad f(\theta_s,\theta_0) = \frac{1}{2}(\cos\theta_s + \cos\theta_0).$$

The factor $(-i)$ *signifies that Huygens wavelet is radiated with a phase advance of* $90\,^\circ$, *a feature not anticipated in the phenomenological treatment of Huygens and Fresnel. The factor* $f(\theta_s,\theta_0)$ *has a maximum value of unity in the forward direction and goes to zero for the portion of the Huygens wavelet returning toward the source.*

It can be shown that there are serious mathematical inconsistencies in the first two assumptions of Kirchhoff. **Rayleigh** *showed that these can be removed by an alternative choice of the Green's function. Indeed, choosing*

$$G_1(\mathbf{x},\mathbf{x}') = \frac{1}{4\pi}\left[\frac{e^{ikR}}{R} - \frac{e^{ikR'}}{R'}\right], \tag{6}$$

where $R' = |\mathbf{x} - \mathbf{x}''|$, $\mathbf{x}''$ *being the mirror-image of* $\mathbf{x}'$ *[i.e.*

$$R^2 = (x-x')^2 + (y-y')^2 + (z-z')^2;$$
$$R'^2 = (x+x')^2 + (y+y')^2 + (z+z')^2.$$

With this choice G_1 *will vanish on* Σ *and consequently*

$$\Psi(\mathbf{x}) = \int\limits_{\Sigma}\Psi(\mathbf{x}')\mathbf{n}'\cdot\nabla' G_1(\mathbf{x},\mathbf{x}')d\Sigma,$$

leading to the Rayleigh integral

$$\Psi(P_0,t) = -\frac{iA}{\lambda}e^{-i\omega t}\int\limits_{\Delta S}\cos\theta_0\frac{e^{ik(r_s+r_0)}}{r_0 r_s}. \tag{7}$$

Similarly, the choice

$$G_2(\mathbf{x}, \mathbf{x}') = \frac{1}{4\pi}\left[\frac{e^{ikR}}{R} + \frac{e^{ikR'}}{R}\right]$$

will end with

$$\Psi(P_0, t) = -\frac{iA}{\lambda}e^{-i\omega t}\int_{\Delta S}\cos\theta_s\frac{e^{ik(r_s+r_0)}}{r_0 r_s}. \tag{8}$$

Note that in (7), Ψ is approximated on ΔS, while in (8) it is $\frac{\partial\Psi}{\partial n}$ which is approximated on ΔS. Hence the difference. But how can we have three different approximation to the Helmholtz-Kirchhoff integral? The answer is simple: if the source point P_s and the observation point P_0 are far from the screen in terms of aperture dimensions, the function $f(\theta_s, \theta_0)$ can be treated as constant. For normal incidence all values of f are approximately unity.

Rayleigh diffraction formulas (1897)

In the analysis of diffraction by an aperture in a plane screen one must determine the solution of Helmholtz wave equation $(\nabla^2 + k^2)\Psi(\mathbf{r}) = 0$ for the wavefield $\Psi(x, y, z)$ valid throughout the Half-space $z > 0$ from (approximate) knowledge of the boundary values $\Psi(x, y, z)$ on the half-space boundary $z = 0$.

An outgoing monochromatic wave in the half-space has the Fourier-integral representation

$$\Psi(x, y, z; t) = e^{-i\omega t}\iint_{-\infty}^{\infty} a(p, q)e^{ik(px+qy+mz)}dpdq \tag{9}$$

where the support of $a(p, q)$ is assumed contained in the disk $p^2 + q^2 \leq 1$, $m^2 = 1 - p^2 - q^2$, and

$$a(p, q) = \left(\frac{k}{2\pi}\right)^2\iint_{-\infty}^{\infty}\Psi(x', y', 0; 0)e^{-ik(px'+qy')}dx'dy'. \tag{10}$$

Substituting (10) into (9) and interchanging the orders of integrations, one obtains the expression of the field in terms of the boundary values

$$\Psi(x, y, z; t) = e^{-i\omega t}\iint_{-\infty}^{\infty}\Psi(x', y', 0; 0)G(x - x', y - y', z)dx'dy' \tag{11}$$

with

$$G(x-x', y-y', z) = \left(\frac{k}{2\pi}\right)^2 \iint_{-\infty}^{\infty} e^{ik[p(x-x')+q(y-y')+mz]} dpdq$$

where $p^2 + q^2 + m^2 = 1$.

We next use Weyl's integral (Weyl, 1919) with $\mathbf{r} = (x, y, z > 0)$, $\mathbf{r} = (x_0, y_0, 0)$ *which represents an outgoing spherical wave as a superposition of plane waves*

$$\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} = \frac{ik}{2\pi} \iint_{-\infty}^{\infty} \frac{1}{m} e^{ik[p(x-x')+q(y-y')+mz]} dpdq, \tag{12}$$

from which we deduce that

$$G(x-x', y-y', z) = -\frac{1}{2\pi}\frac{\partial}{\partial z}\left[\frac{e^{ikR}}{R}\right], \qquad R^2 = (x-x')^2+(y-y')^2+z^2. \tag{13}$$

Inserting this in (11), we arrive at the first Rayleigh formula

$$\Psi(x, y, z; t) = -\frac{e^{-i\omega t}}{2\pi} \iint_{-\infty}^{\infty} \Psi(x', y'; 0) \frac{\partial}{\partial z}\left[\frac{e^{ikR}}{R}\right] dx'dy', \tag{14}$$

since

$$\frac{\partial}{\partial z}\left[\frac{e^{ikR}}{R}\right] \simeq \frac{ikz}{R}\frac{e^{ikR}}{R}$$

for $kR \to \infty$ *(the 'far field').*

Then since $\frac{z}{R} = \cos\theta_0$, *one can write*

$$\Psi(x, y, z; t) = -\frac{i}{\lambda} e^{-i\omega t} \iint_{-\infty}^{\infty} \cos\theta_0 \frac{e^{ikR}}{R} \Psi(x', y', 0) dx'dy', \tag{15}$$

which a more general form of the Rayleigh integral in (7).

Summary

The Fresnel-Kirchhoff diffraction theory is intrinsically a high-frequency approximation; it gives incorrect results when the aperture dimensions are much smaller than a wavelength. Furthermore, even if such dimensions are *large and one uses theory to predict fields at only those distances which are large compared with a wavelength, the predictions may be in substantial disagreement at large angular deviations from the direction* **n**.

Nevertheless, the theory is satisfactory for explaining small-angle, high-frequency diffraction phenomena and has an advantage in simplicity compared with rigorous theories of diffraction.

It is extensively used in optics; applications to acoustics are limited (except for ultrasonics*) because many of the diffraction phenomena of interest either involve dimensions small compared with a wavelength or require on understanding of diffraction through large angles.*

The Fresnel-zone construction

According to ***Huygens****' construction, every point of a wavefront may be considered as a center of a* secondary *disturbance which gives rise to spherical wavelets and the wavefront at any later instant may be regarded as the* envelope *of these wavelets.* ***Fresnel*** *was able to account for diffraction by supplementing Huygens' construction with the postulate that the secondary wavelets mutually interfere. This combination of Huygens' construction with the principle of interference is called the* Huygens-Fresnel principle.

Before applying it to the study of diffraction effects one is tempted to verify that, with certain simple additional assumptions, the principle correctly describes the propagation of light in free space.

To see this we consider the instantaneous *position of a spherical monochromatic wavefront of radius r_s which proceeds from a point source P_s to another point P_0 where the light disturbance is to be determined.*

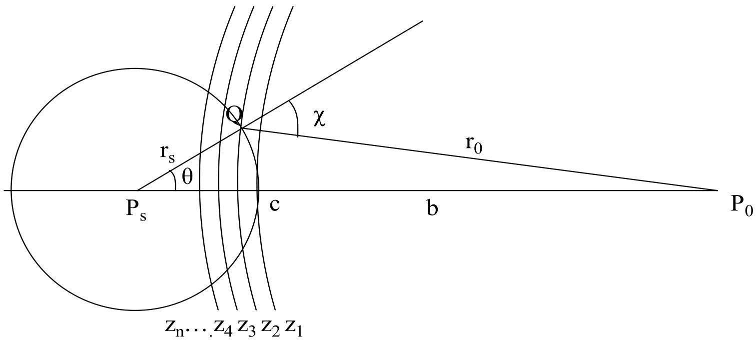

Fig. 3.4: Fresnel's zone construction

Let the time factor be $e^{-i\omega t}$ *(omitted) and let the disturbance at* Q *on the wavefront be* Ae^{ikr_s}/r_s*, where* A *is the amplitude at unit distance from the source. In accordance with the Huygens-Fresnel principle we regard each element of the wavefront as the center of a secondary disturbance which is propagated in the form of spherical wavelets, and obtain for the contribution* $d\Psi(P_0)$ *due to the element* dS *at* Q *the expression*

$$d\Psi = k(\chi)A\frac{e^{ikr_s}}{r_s}\frac{e^{ikr_0}}{r_0}dS,$$

where $r_0 = QP_0$ *and* $k(\chi)$ *is the inclination factor, describing the variation with direction of the amplitude of the secondary waves,* χ *being the angle between the normal at* Q *and the direction* QP_0*.*

We assume that k *is maximum for* $\chi = 0$ *and zero for* $\chi = \pi/2$*. Hence the total disturbance at* P_0 *is given by*

$$\Psi(P_0) = A\frac{e^{ikr_s}}{r_s}\iint_{S'}\frac{e^{ikr_0}}{r_0}k(\chi)dS \tag{16}$$

where S' *is that part of* S *which is not obstructed by obstacles situated between* P_s *and* P_0*.*

To evaluate (16) we draw spheres from P_0 *of radii*

$$r_j = b + j\left(\frac{\lambda}{2}\right) \quad j = 0, 1, 2, ..., n, \qquad r_n^2 = (b + r_s)^2 - r_s^2$$

(P_0Q tangent to the primary wavefront at $r = r_n$).

These spheres divide S into a number of zones $z_1,\ z_2,\ z_3, ..., z_n$. Assuming $r_s >> \lambda, \quad r_0 >> \lambda$, then k may be assumed to have the same value K_j, for points on one and the same zone.

Now by the law of cosines,

$$r_0^2 = r_s^2 + (r_s + b)^2 - 2r_s(r_s + b)\cos\theta$$

so that

$$r_0 dr_0 = r_s(r_s + b)\sin\theta d\theta \qquad b, r_s \text{ fixed}$$

and therefore

$$dS = r_s^2 \sin\theta d\theta d\phi = \frac{r_s}{r_s + b} r_0 dr_0 d\phi \qquad \phi = \text{azimuth angle}$$

Hence, the contribution of the j^{th} zone (spherical ring) to $\Psi(P_0)$ is:

$$\begin{aligned} \Psi_j(P_0) &= 2\pi \frac{Ae^{ikr_s}}{r_s + b} K_j \int_{b+(j-1)\frac{\lambda}{2}}^{b+j\frac{\lambda}{2}} e^{ikr_0} dr_0 \\ &= 2i\lambda(-1)^{j+1} K_j \frac{Ae^{ik(r_s+b)}}{r_s + b} \end{aligned}$$

The total wave at P_0 is

$$\Psi(P_0) = 2i\lambda A \frac{e^{ik(r_s+b)}}{r_s + b} \sum_{j=1}^{n} (-1)^{j+1} K_j. \tag{17}$$

The contributions of the successive zones are alternately positive and negative

$$\Sigma = K_1 - K_2 + K_3 - ... + (-1)^{n+1} K_n.$$

It can be shown that (approximately)

$$\Sigma \approx \frac{K_1}{2} + \frac{K_n}{2} \qquad (n \text{ odd})$$

$$\Sigma \approx \frac{K_1}{2} - \frac{K_n}{2} \qquad (n \text{ even}).$$

Since $K_n = 0 \quad [\chi = \frac{\pi}{2}]$ *for the last zone,* $\Sigma \approx \frac{K_1}{2}$ *and we have approximately:*

$$\Psi(P_0) = i\lambda K_1 A \frac{e^{ik(r_s+b)}}{r_s + b} = \frac{1}{2}\Psi_1(P_0), \tag{18}$$

showing that the total disturbance at P_0 *is approximately equal to half of the disturbance due to the first zone. This last result is in agreement with the field at* P_0 *obtained by simply assuming a spherical wave beginning at* P_s *and ending at* P_0. *i.e*

$$\Psi(P_0) \approx A\frac{e^{ik(r_s+b)}}{r_s + b}$$

if $i\lambda K_1 = 1$, *namely*

$$K_1 = -\frac{i}{\lambda} = \frac{1}{\lambda}e^{-\frac{\pi i}{2}}.$$

The factor $e^{-\frac{\pi i}{2}}$ *may be accounted for by assuming that the secondary waves have an initial phase retardation of a quarter of a period, relative to the primary wave. There is also the amplitude factor of* $\frac{1}{\lambda}$.

By means of the above method, Fresnel was able to "calibrate" his approximate integrating scheme and thus test the validity of his principle. To enable additional experimental tests he introduced a plane screen with circular opening, perpendicular to the optical axis P_sP_0, *with its center on this line. The total disturbance at* P_0 *must now be regarded as due to wavelets from only those zones that are not obstructed by the screen. Four experiments could be performed:*

- *The screen covers all but half the first zone; according to Eq. (18) with* $j = 1$

$$\Psi(P_0) = \frac{1}{2}\Psi_1(P_0) = i\lambda K_1 \frac{Ae^{ik(r_s+b)}}{r_s + b} = \frac{Ae^{ik(r_s+b)}}{r_s + b}.$$

  This is the same disturbance as would be obtained if no screen were present.

- *All zones are covered except the first one; then (17)–(18) yields*

$$\Psi(P_0) = 2i\lambda K_1 \frac{Ae^{ik(r_s+b)}}{r_s + b} = 2\frac{Ae^{ik(r_s+b)}}{r_s + b}.$$

  The intensity $I(P_0) = |\Psi(P_0)|^2$ *is four times larger than if the screen were absent. [Conservation of energy clearly demands that there be other points where the intensity has decreased.]*

- *Only the first two zones are left open; since K_1 and K_2 are nearly equal (which can be shown), there will be almost complete darkness. In general, when the size of the opening is varied, there is a periodic fluctuation in intensity at P_0. Similar results are obtained when the size of the opening and the source's position are fixed but P_0 gradually moved along the axis.*

- *When only the first zone is obstructed by a small circular disk placed at right angles to P_sP_0, the field is*

$$\Psi(P_0) = 2i\lambda \frac{Ae^{ik(r_s+b)}}{r_s+b} \underbrace{[-K_2 + K_3 - K_4 + \cdots]}_{\approx -\frac{K_2}{2}},$$

but since $K_1 \approx K_2$ it follows that there is light in the geometrical shadow of the disk! and (even more remarkably) the intensity there is the same as if no disk were present.

This prediction[478] of Fresnel's theory made a strong impression on his contemporaries, and was one of the decisive factors which temporarily ended the long battle between the corpuscular and the wave theories of light in favor of the latter.

We next integrate the expression

$$dS = \frac{r_s}{r_s+b} r_0 dr_0 d\phi$$

over the j-th zone to get the area of that zone:

$$A_j = \frac{2\pi r_s}{r_s+b} \int_{b+(j-1)\frac{\lambda}{2}}^{b+j\frac{\lambda}{2}} r_0 dr_0 = \frac{\lambda\pi r_s}{r_s+b}\left[b + \frac{(2j-1)\lambda}{4}\right]$$

$$= \frac{\lambda\pi r_s b}{r_s+b}\left[1 + \frac{(2j-1)}{4}\frac{\lambda}{b}\right].$$

[478] That a bright spot should appear at the center of the shadow of a small disk was deduced from Fresnel's theory by **S.D. Poisson** (1818). Poisson, who was a member of the committee of the French Academy which reviewed Fresnel's prize memoir, considered this conclusion contrary to experiment and so rejected Fresnel's theory. However, **Arago**, another member of the committee, performed the experiment and found that the surprising prediction was correct. A similar observation was made in 1723 by **Jacques Philippe Maraldi** (1665–1729), a nephew of G.D. Cassini, but was forgotten.

It is also found that the mean distance from the field point P_0 to the j-th zone is $r_j = b + \left(\frac{2j-1}{4}\right)\lambda$ *so that A_j/r_j is constant.*

When $\lambda << b$ and for small j values, we have approximately

$$A \approx \frac{r_s}{r_s+b}\pi b\lambda \qquad \text{(independent of } j\text{)}$$

If the aperture has a radius R, a good approximation for the number of zones within it is thus simply

$$\frac{\pi R^2}{A} = \frac{(r_s+b)R^2}{r_s b\lambda}.$$

If the point source has been moved so far from the aperture (diffraction screen) that the incoming wave can be regarded as a plane wave ($r_s \to \infty$), two facts emerge:

- $A \simeq \pi b\lambda$ *independent of j*
- *Since* $r_j = b + j\frac{\lambda}{2}$, *we have* $\pi R_j^2 \approx \pi(b + j\frac{\lambda}{2})^2 - \pi b^2$ *or*

  $$R_j^2 \approx jb\lambda + j^2\frac{\lambda^2}{4} \simeq jb\lambda \qquad \therefore \qquad R_j \cong \sqrt{jb\lambda}$$

  as long as j is not extremely large. So the radii are proportional to the square roots of the integers.

It should be borne in mind that, the sensor at P_0 merely records the light amplitude (or intensity), the zones having no reality. It is just a convenient contrivance for the evaluation of the field.

1818–1844 CE **Arthur Schopenhauer** (1788–1860, Germany). Philosopher. One of the first Western thinkers to concern himself with the dilemmas and tragedies of real modern life, not just with abstract philosophical problems. Espoused pessimism that saw life as being essentially evil and futile. Under influence of Eastern thought, he saw hope in aesthetics, sympathy for others and ascetic living. His ideas influenced the fields of music, psychology, literature and physics (through **Einstein and Schrödinger**). Accorded the arts a more important place in the overall scheme of things than any other major philosopher.

While still in his twenties he wrote his masterpiece: "*The World as Will and Representation*" (1818), and then "*On the Will as Nature*" (1836), showing that the ongoing progress of science was supporting the arguments of his main work. Finally he produced two books on ethics: "*The Freedom of the Will*" (1841), and "*The Foundations of Morality*" (1841).

Schopenhauer was born near Danzig, the son of a rich Hanseatic merchant of Dutch heritage. As his parents had strong feeling against any kind of nationalism, the name Arthur was selected especially because it was the same in English, German and French.

After the city fell to Prussia during the second partition of Poland (1793), the family fled to Hamburg. In 1805 Schopenhauer's father died (possibly by suicide) and his mother, Johanna, moved to Weimar, where she kept a literary salon at which she entertained such figures as Goethe and the Brothers Grimm.[479]

Schopenhauer studied at the University of Göttingen and was awarded a PhD from the University of Jena. As a youth, he traveled widely, becoming fluent in English and French, so that his prose style acquired a lightness and clarity quite unlikely the murky philosophic German of his times. His first education was that of a man of the world, only later did he obtain the usual academic credentials.

He became a friend of Goethe, and in 1816 wrote a small book on color theory[480] inspired by the ideas of the older man.

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479 She herself achieved fame as a romantic novelist, and one of her poems was set to music by **Schubert**. When **Goethe** told her that he thought her son was destined for great things, Johanna objected: she had never heard there could be *two* geniuses in a single family.

480 Schopenhauer's philosophy of science has its embarrassing aspects: Schopenhauer did not understand the new physics of light and electricity that had been developed by **Thomas Young** (1773–1829) and **Michael Faraday** (1791–1867). He disparaged the wave theory of light, which Young had definitively established, as a "crude materialism", and "mechanical, Democritean, ponderous, and truly clumsy." Unfortunately, Schopenhauer does not seem to have understood the evidence for Young's discoveries about light, or even for Newton's — he still clung to Goethe's clever but clueless theory of colors. Nevertheless, Schopenhauer would have been happy to learn how his beloved *qualitates occultae* would return in force with quantum mechanics: Things like strangeness, charm, baryon number, lepton number, etc., are exactly the kinds of irreducible types he demanded.

Because of a large inheritance from his father, Schopenhauer was able to retire early, and, as a private scholar, was able to devote his life to the study of philosophy.

Schopenhauer was influenced by **Friedrich Schelling**, regarded himself as the true spiritual descendant of **Kant**, and despised **Hegel**. He thought that Hegel's belief in a happy ending to human history was the ramblings of a "stupid and clumsy charlatan." He maintained that Hegel, and other university philosophers had perverted the Kantian gospels.

The first edition of his major work, *The World as Will and Representation* appeared in 1818. He followed Kant in the belief that the mind is not merely a passive recipient of sense impressions, but takes an active role in fitting the phenomena into the categories of space and time, the principle of causality being the necessary method for creating this representation of the world. Kant taught that the real world, the *noumenon*, the thing-in-itself [*Ding an sich*], can never be accessible to human thought or experience. Schopenhauer did not agree: he believed that the thing-in-itself can be identified as *will*. Every person experiences himself in two different ways, as an object like any other, and through self-consciousness as a *will*. The will is neither a phenomenon nor a representation, it is a directly experienced reality.

What is true of the microcosm of man, is also true of the world: its thing-in-itself is will. On the foundation of this primary intuition, which of course can be neither proved nor disproved, Schopenhauer constructed a philosophy that has continued to fascinate and influence thinkers of all kinds: **Nietzsche**, **Tolstoy**, **Chekhov**, **Zola**, **Maupassant**, **Bernard Shaw**, **R.M. Rilke**, **Thomas Mann**, **Freud**, **Klimt**, and **Schrödinger**, to mention a few. The reason for this wide range of influence must be sought in a combination of an unparalleled depth of insight into the human condition with a literary style of exceptional quality.

This use of the term "will" has led to much misunderstanding, because people find it difficult to think of a will that has no personality, no kind of mind or intelligence, and no aims or goals: but this is what Schopenhauer says quite clearly that he means. He would have regarded the discovery by physics in the 20^{th} century that the entire contents of the empirical world, including all material objects, are reducible to energy and fields of force, operating in a space-time framework, as fitting in perfectly with his philosophy.

Schopenhauer agreed with Kant that human beings can only ever live in the phenomenal world. But for Schopenhauer, the phenomenal world is an illusory one, always controlled by the *Will*. The Will directs every living being, including humans.

Human beings like to believe that their own individual lives have some kind of higher meaning, but there is no more to their lives than the urge to satisfy their desires. Different individual wills then inevitably come into conflict, and this is what produces human suffering.

According to Schopenhauer, the *Will to Live* (Wille zum Leben), is defined as an inherent drive within human beings, and indeed all creatures, to stay alive and to *reproduce*. He refused to conceive lore as either trifling or accidental, but rather understood it to be an immensely powerful force lying unseen within man's *psyche* and dramatically shaping the world. He saw '*falling in love*' as the process whereby the noumenon, Kant's 'thing in itself,' enters the world of phenomena.

He had more to say about sexual love than any previous philosopher, since 'the sexual relation in the world of mankind ... is really the invisible central point of all action and conduct ... The ultimate aim of all love affairs, whether played in sock or in buskin, is actually more important than all other aims in man's life; and therefore it is quite worthy of the profound seriousness with which everyone pursues it. What is decided by it is nothing less than the *composition of the next generation*.' The new individual who will arise from the love affair is like a new Platonic idea, and 'just as all the Ideas strive to enter into the phenomenal with the greatest vehemence, avidly seizing for this purpose the matter which the law of causality divides among them all, so does this particular Idea of a human individuality strive with the greatest eagerness and vehemence for its realization in the phenomenon. This eagerness and vehemence is identical with the passion for each other of the two future parents.'

Schopenhauer's view of the human condition is that of a world of violence and injustice ending in death. He took the blackest view of our existence that is possible to take and still remain sane.

He believed, however, that a momentary release from our imprisonment in the dark dungeon of this world could be achieved through the *arts*: painting, sculpture, poetry, drama, and above all – music.

Through these media we are in touch with something *outside* the empirical realm. We are taken out of time and space and also out of ourselves (body included). In particular, he regarded music as a sort of super-art transcending all the others in metaphysical significance.

Worldview XIX: Arthur Schopenhauer

* * *

All truth passes through three stages. First, it is ridiculed. Second, it is violently opposed. Third, it is accepted as being self-evident.

* * *

Each day is a little life; every waking and rising a little birth; every fresh morning a little youth; every going to rest and sleep a little death.

* * *

After your death you will be what you were before your birth.

* * *

Every possession and every happiness is but lent by chance for an uncertain time, and may therefore be demanded back the next hour.

* * *

Human life must be some form of mistake.

* * *

If a man sets out to hate all the miserable creatures he meets, he will not have much energy left for anything else; whereas he can despise them, one and all, with the greatest ease.

* * *

The discovery of truth is prevented more effectively, not by the false appearance things present and which mislead into error, not directly by weakness of the reasoning powers, but by preconceived opinion, by prejudice.

* * *

The first forty years of life give us the text; the next thirty supply the commentary on it.

* * *

The fundament upon which all our knowledge and learning rests is the inexplicable.

* * *

Physics is unable to stand on its own feet, but needs a metaphysics on which to support itself.

* * *

The closing years of life are like the end of a masquerade party, when the masks are dropped.

* * *

A man must have grown old and lived long in order to see how short life is.

* * *

Fate gives us the hand, and we play the cards.

* * *

Wealth is like sea-water; the more we drink, the thirstier we become; and the same is true of fame.

* * *

The alchemists in their search for gold discovered many other things of greater value.

* * *

A man can do what he wants, but not want what he wants.

* * *

The wise have always said the same things, and fools, who are the majority have always done just the opposite.

* * *

Money is human happiness in the abstract; he, then, who is no longer capable of enjoying human happiness in the concrete devotes himself utterly to money.

* * *

The present is the only reality and the only certainty.

* * *

Martyrdom is the only way a man can become famous without ability.

* * *

Sleep is the interest we have to pay on the capital which is called in at death; and the higher the rate of interest and the more regularly it is paid, the further the date of redemption is postponed.

* * *

The deep pain that is felt at the death of every friendly soul arise from the feeling that there is in every individual something which is inexpressible, peculiar to him alone, and is, therefore, absolutely and irretrievably lost.

* * *

The highest, most varied and lasting pleasures are those of the mind.

* * *

Life swings like a pendulum backward and forward between pain and boredom.

* * *

Natural ability can almost compensate for the want of every kind of cultivation; but no cultivation of the mind can make up for the want of natural ability.

* * *

To go to the theater is like making one's toilet with a mirror.

* * *

Will power is to the mind like a strong blind man who carries on his shoulders a lame man who can see.

* * *

The difficulty is to try and teach the multitude that something can be true and untrue at the same time.

* * *

The more unintelligent a man is, the less mysterious existence seems to him.

* * *

Without books the development of civilization would have been impossible. They are the engines of change, windows on the world, "Lighthouses" as the poet said "erected in the sea of time." They are companions, teachers, magicians, bankers of the treasures of the mind, Books are humanity in print.

* * *

Books are like a mirror. If an ass looks in, you can't expect an angel to look out.

* * *

Authors may be divided into falling stars, planets, and fixed stars: the first have a momentary effect; the second have a much longer duration; but the third are unchangeable, possess their own light, and work for all time.

* * *

Any book, which is at all important, should be reread immediately.

* * *

Almost all of our sorrows spring out of our relations with other people.

* * *

With people of limited ability modesty is merely honesty. But with those who possess great talent it is hypocrisy.

* * *

Every man takes the limits of his own field of vision for the limits of the world.

* * *

Boredom is just the reverse side of fascination: both depend on being outside rather than inside a situation, and one leads to the other.

* * *

Do not shorten the morning by getting up late; look upon it as the quintessence of life, as to a certain extent sacred.

* * *

Journalists are like dogs, whenever anything moves they begin to bark.

* * *

Exaggeration of every kind is as essential to journalism as it is to dramatic art, for the object of journalism is to make events go as far as possible.

* * *

To find out your real opinion of someone, judge the impression you have when you first see a letter from them.

* * *

Religion is the masterpiece of the art of animal training, for it trains people as to how they shall think.

* * *

Buying books would be a good thing if one could also buy the time to read them in: but as a rule the purchase of books is mistaken for the appropriation of their contents.

* * *

Great men are like eagles, and build their nest on some lofty solitude.

* * *

A man's face as a rule says more, and more interesting things, than his mouth, for it is a compendium of everything his mouth will ever say, in that it is the monogram of all this man's thoughts and aspirations.

* * *

Change alone is eternal, perpetual, immortal.

* * *

A word too much always defeats its purpose.

* * *

We forfeit three-fourths of ourselves to be like other people.

* * *

Honor has not to be won; it must only not be lost.

* * *

It is a clear gain to sacrifice pleasure in order to avoid pain.

* * *

Ignorance is degrading only when found in company with great riches.

* * *

In action a great heart is the chief qualification. In work, a great head.

* * *

Compassion is the basis of morality.

* * *

So long as we are given up to the throng of desires with its constant hopes and fears, we never obtain lasting happiness or peace.

* * *

Actions are transitory while works remain: The most noble action *still has only a temporary effect; the work of genius, on the other hand, lives and has beneficial and uplifting effect through all times.*

* * *

1819–1851 CE **Leopold (Yom-Tov Lipmann) Zunz** (1794–1886, Germany). The first modern Jewish thinker. Devoted his long life to the refurbishment of the old-style Jewish learning and its presentation in a modern scientific spirit.

Zunz and his friends of the immediate post-Napoleonic period, called their work the *Wissenschaft des Judentums* (The science of Judaism). In 1819, immediately after the *Hep-Hep riots*, they realized the fragility of the acceptance of the Jews even in modern-minded Germany, and set up the *Society for the promotion of Jewish Culture and Science*[481]. Its object was to investigate the nature of Judaism by modern scientific methods and demonstrate the universal value of Jewish knowledge.

Zunz then embarked on a grand project: an encyclopedia of Jewish intellectual history: He translated an enormous amount of Jewish literature and elaborated a philosophy of Jewish history. He visited the great libraries in search of material, in an overall effort to emancipate Jewish writing from the theologians and rise to the historical viewpoint. In practice it involved accepting that the history of the Jews was merely an element of world history. In that Zunz was influenced by Hegelian ideas of progression from lower to higher forms, and inevitably applied this dialectic to Judaism.

There had been only one period in Jewish history, he said, when their inner spirit and their external form had matched, and they had become the center of world history, and that was under the ancient commonwealth. Thereafter they were delivered into the hands of other nations. Their internal history became a history of ideas, their external history a long tale of suffering. A day will come, he believed, that the distilled legacy of Jewish ideas became part of the common property of enlightened mankind.

While Zunz' interpretation of Jewish history and learning as a contribution to the world stock made some impression on gentile society, it involved almost by definition a severance from a great part of Judaism.[482]

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[481] The society soon dissolved (1824), because most of its members and officers converted to Christianity in the most opportunistic manner. Nonetheless, under the guidance of Zunz it had successfully initiated the scientific method in the study of Jewish religion, history and culture. Among the initiators was **Heinrich Heine**, then still a fledgling poet.

[482] The Jewish orthodoxy in his time, rejected this dualism. To them it was not Judaism: there are *no* two kinds of knowledge, sacred and secular; there was only *one*. Moreover, there was only one legitimate purpose in acquiring it: to discover the exact will of God, in order to obey it. Hence the 'Science of Judaism', as a dislocated academic discipline, was contrary to Jewish belief. They believed that, without Israel, there would have been no world and therefore no history.

Zunz was born in Detmold. Studied at Berlin University (1815–1821) and received his Ph.D. at the University at Halle. Eventually he became the principal of a teachers' seminary established by the Jews in Berlin (1840). His foremost work is "*The Religious Discourses of the Jews*". It proved that Judaism never stood still, but underwent changes in accordance with requirements of time and place; it changed itself even when there was no reformers advocating change.

1819 CE **William George Horner** (1786–1837, England). Mathematician. Rediscovered the ancient Chinese computational scheme for the evaluation of a polynomial and hence solving for the real roots of algebraic equations.

Horner was born in Bristol. He began his humble career as an assistant schoolmaster at Kingston (1802), worked his way to become a Headmaster there (1806), and founded his own school at Bath (1809). Although not a man of great ability as a mathematician, he succeeded in making for himself a name that is well known to students of Algebra.

While a schoolteacher at Bath, he came independently upon a method known to Chu Shih-Chieh (ca 1300 CE) for the approximation of the real roots of a numerical algebraic equation. This method, which has been practically forgotten in China, was made known in a paper read by Horner before the London Royal Society (1819), and since that time has become familiar in all parts of the English speaking world.

It was recently discovered that **Ruffini** (1765–1822) had described a similar method a few years earlier (1813).

An Age of Transition[483]

"Greek civilization depended essentially on slave-labor but could not progress without the harnessing of natural forces to labor-saving machines. Only the free man, not a slave, has a disposition and interest to improve implements or to invent them. Accordingly, in the devising of a complicated machine, the workmen employed upon it are generally co-inventors. The eccentric *and the* governor, *most important part of the steam-engine, were devised by laborers. The improvement of established industrial methods by slaves, themselves industrial machines, is out of question.*

Justus von Liebig (1803–1873)

"Prior to 1890, *the steam engine did more for science than science for the steam engine".*

L.J. Henderson

The industrial age started in England at about 1740, *continued to France in ca* 1810, *and arrived in Germany and the U.S.A. in about* 1830. *It became pronounced after* 1815. *By* 1850 *industrialization had become widespread in Western Europe as well as the northeastern United States. This process eventually took manufacturing out of the home and workshop. Power-driven machines replaced handwork, and factories developed as the best way of bringing together the machines and the workers to operate them.*

As industrialization grew, private investors and financial institutions were needed to provide money for its further expansion. Financiers and banks thus became as important as industrialists and factories in the growth of the industry. For the first time in European history, wealthy businessmen called capitalists *took over the control and organization of manufacturing.*

This was a great turning point in the history of mankind. It changed the Western world from a rural and agricultural society to a basically urban

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[483] For further reading, see:

- Craig, G.A., *Europe Since 1815*, The Dryden Press: Hinsdale, IL, 1974, 620 pp.
- Talmon, J.L., *Romanticism and Revolt* (*Europe 1815–1848*), Harcourt, Brace and World: New York, 1970.

society. Industrialization brought many material benefits, but also created a large number of problems that still remain critical in the modern world. Some of the most acute of these problems today are air and water pollution, the depletion of natural resources and other man-made alterations of the biosphere.

The transformation of Europe's economy from agriculture to industry affected science in both direct and indirect ways: the invention of the steam engine stimulated interest in thermodynamics and the concepts of power, work and energy began to be formalized. The advent of chemical industry based on chemical processes accelerated the renaissance of the atomistic theory of matter. The kinetic theory of gases and the discovery of electromagnetism are also associated with the industrial age.

In addition, industrialization stimulated the rise of mechanical inventions, especially those associated with transportation on land and sea.

*The advent of the efficient steam engine (**Watt**, 1769), the steam locomotive (**Trevithick**, 1804), the steamboat (**Fulton**, 1807) and magnetic telegraphy (**Morse**, 1838) revolutionized transport, travel and communication.*

The first commercial steam railroad was opened between Liverpool and Manchester in 1830. *By* 1840 *the first transatlantic steamer line was established. In* 1844 *the first telegraph line was connected between Washington and Baltimore and in* 1850, *the first submarine cable was laid under the English Channel between Dover and Calais.*

*The negative reaction to Newtonian science and mathematics, and the industrial revolution that followed in its wake, found its powerful expression in the best-known lyric of **William Blake** (1810):*

And did those feet in ancient time
Walk upon England's mountains green?
And was the holy Lamb of God
On England's pleasant pastures seen?

And did the Countenance Divine
Shine forth upon our clouded hills?
And was Jerusalem builded here
Among these dark Satanic mills?

Bring me my Bow of burning gold:
Bring me my Arrows of desire:
Bring me my Spear: O clouds unfold!
Bring me my Chariot of fire.

I will not cease from Mental Fight,
Nor shall my Sword sleep in my hand

Till we have built Jerusalem
In England's green & pleasant Land.

Set to music, and usually misnamed Jerusalem, it is frequently sung as a hymn today, even though its allusions are obscure. Blake believed in the legend that England had once been part of Atlantis, the home of mankind during the Golden Age. To Blake mills are both the ugly factories and symbols of the chains of Newtonian science. The weapons of gold and fire are weapons of the imagination to be used in the Mental Fight to restore the harmony of reason and vision in man's thinking.

The Agricultural revolution of the 10^{th} millennium BCE and the Industrial revolution of the 18^{th} century AD, on the other hand, created deep breaches in the continuity of the historical process. With each one of these two Revolutions, a new story begins, dramatically and completely alien to the previous one. Continuity is broken between the cave-man and builders of the pyramids, just as continuity is broken between the ancient ploughman and the modern operator of a nuclear power station. Clearly, each Revolution had its roots in the past, but each created a deep break with the very same past; the first "Revolution" transformed hunters and food-gatherers into farmers and shepherds, while the second "Revolution" transformed farmers and shepherds into operators of machines fed with inanimate energy.

The ten millennia or so that separate the two "Revolutions" witnessed a great number of discoveries and innovations that increased man's control over energy sources, but until the Industrial Revolution man continued to rely mainly on plants, animals and other men for energy – plants for food and fuel, animals for food and mechanical energy, other men for mechanical energy. The use of other available sources – mainly wind and water power – remained limited.

If the Agricultural Revolution is the process whereby man came to control and increase the supply of biological converters (plants and animals), the Industrial Revolution can be regarded as the process whereby the large scale exploitation of new sources of energy by means of inanimate converters was set on foot.

Looking at things from this point of view, one easily understands the key role played by the cultural revolution of the 16^{th} and 17^{th} centuries is the shaping of the destiny of mankind. It was in fact the cultural revolution that gave to man the conceptual tools which enabled him to master new sources of energy. The conscious systematic investigation of phenomena revealed in man's environment became a fundamental cultural trait of early modern Europe since the days of the Renaissance.

In the north-west part of Europe the 16^{th} and 17^{th} centuries witnessed also a most remarkable mercantile development which favored the accumulation of physical wealth and of entrepreneurial skills. In England these cultural and economic developments happened to coincide with a shortage of a traditional form of energy (timber) and the presence of large supplies of coal. It was the union of certain happy mental qualities with material resources of an altogether peculiar character that provided the explosive formula.

In the second half of the 18^{th} century, ***James Watt*** *perfected previous discoveries and constructed a steam engine (1765), the commercial use of which mounted through 1800–1820. Steam engines were used in metallurgical and textile activities as well as in mining coal and in surface transportation. As more machine power made it possible to produce more coal and to transport it at an enormously accelerated rate, more coal in its turn meant more machine power. Coal became a strategic element in the emergence and diffusion of the industrial civilization. It meant a rapidly expanding supply of energy that could be used for heating and lighting and for power in sea and land transportation and in almost all the various forms of industry*[484].

A cumulative interaction was soon set in motion; the extraordinary growth in the supply of energy stimulated economic growth, *which in turn stimulated* education and scientific research *leading to the discovery of new sources of energy*[485]*! Under the impact of these discoveries, the process quickened: the more energy was produced, the more energy was sought. Man turned to the sun, the tides, earth-heat, tropical waters, and atmospheric electricity. Then, toward the middle of the 20^{th} century, man discovered that energy could be obtained from atoms through the process of fusion or fission.*

Man needs capital to trap energy, and still more capital to exploit this energy for productive purposes. Capital accumulation is a necessary condition for any society's survival and progress. There is a definite correlation between capital and output. In a hunting economy, *the capital needs are very limited: a few bones (used as tools or weapons), and in more developed cultures: bows, arrows and stone implements. In an* agricultural economy *the capital needed is: stocks of seeds, fertilizers, ploughs, draught animals, silos, mills,*

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484 Around 1800 the world production of coal amounted to $15 \times 10^6 \frac{ton}{year}$. By 1860 it rose to $132 \times 10^6 \frac{ton}{year}$ (= 1057×10^6*megawatt-hours*) and by 1900 to $702 \times 10^6 \frac{ton}{year}$. By 1950 the corresponding figures were $1454 \times 10^6 \frac{ton}{year}$.

485 The year 1860 marks the advent of the American oil-well industry. The *gas-engine* was patented in the same year by **Lenoir**. The *electric* industry was born with Faraday's discoveries (1822–1831). By 1870 practical types of generators were already available to produce either direct or alternating current. The great consumption of electricity followed the evolution of the incandescent lamp.

boats, wagons, etc. In an industrial economy capital needs are still more complex and much larger: machinery, railways, chemical and atomic plants, dams, research laboratories, etc. The greater the production, the greater the volume of capital needed.

Capital is made possible by saving. If resources are consumed they are obviously not available for capital accumulation (if you eat your cow today, you cannot hope to have your milk tomorrow!). Only by forgoing present consumption can a society cumulate capital. In any agricultural society, given the low per capita income, saving per capita is rather low. Temples, pyramids, mansions, jewelery, warfare etc absorb a large quota of resources squeezed out of current income.

Furthermore, pre-industrial societies are typically characterized by inadequate transport facilities. Mass transportation was generally non-existent and communications were costly and insecure. Consequently, any pre-industrial society must have kept inventories of all commodities in much larger proportion to current production than any industrial society does.

To accomplish the transition from agricultural to industrial society the active population must acquire new skills and adopt new pattern of living to change the patterns of capital formation; further capital is needed for investment in education.

In all agricultural societies of our past we find that, mainly because of limitations of energy sources known and exploited, the great mass of people could hardly afford to satisfy anything but the more elementary needs: food, clothing and housing, and even these at rather unsatisfactory levels. Correspondingly, the most of the available resources were employed in agriculture, textile manufacture, and building. On the fringe, there was always some trade.

All historical records seem to show that where trade flourished, demographic and economic levels were the highest attainable within the range of agricultural possibilities. Actually, almost all the great agricultural civilizations of the pre-industrial past were founded on the expansion of the mercantile sector. Indeed, it was an exaggerated expansion of this sector in the 17^{th} *– and* 18^{th}*–century England that created the preconditions of the Industrial Revolution.*

Under this regime, new sources of energy, larger amounts of capital, and more efficient use of factors of production increased the per capita real income and improved the diet, clothing and housing of the masses. While expenditure on food decreased as a percentage of total private expenditure, expenditure on transportation, medical care, education, amusement etc, increased more than proportionally.

In conjunction with the exploitation of new kinds of energy and new prevailing consumption patterns, one observes a general decline in the relative importance of agriculture, which also suffers from the fact that the other productive sectors tend to lose their dependence on it. The building industry substitutes steel and cement for timber. *The textile industry substitutes artificial fibers (rayon, dacron, etc.) for natural ones. The pharmaceutical industry substitutes chemical products for* spices *and* herbs. *Even the food industry follows the trend: vitamin pills replace natural* fruits, *and Coca Cola replaces* wine.

Correspondingly, both the percentage of total active population employed in agriculture and the proportion of income produced by the agricultural sector shrink markedly while a great expansion is experienced in the new key sections: the chemical, the metallurgical, and the mechanical.

In an industrial society, the contribution of science *and scientific methods to production is obviously great. Consequently, the rate of growth of an industrial society is largely influenced by the amount of resources devoted to research and education and by the efficiency at which these resources are used. In an industrial society a good deal of economic growth is due to* technological change, *better education and the training and retraining of the labor force. The growth of inputs (labor and capital) and their progressive more efficient utilization brought forward an extraordinary expansion of production. Production increased faster than population, and thus* per capita *income grew over the long run.*

The passage of a society from one type of economic organization to another also implies drastic cultural and social changes. Four generation ago more than 2/3 of the people living on earth were peasants. In ca 2050, less than 1/3 will live in the fields. The Industrial revolution is spreading all over the world. We witness changes that are not merely industrial but also social and intellectual. A new style of life is emerging, as another disappears for ever. We know what is disappearing but we do not know what to expect.

This is an age of transition as well as an age of uncertainty and anguish. Every aspect of life has to be geared to the new models of production. Family ties are not on the wane and give way to broader perspectives for larger social groups. Individual savings *gives way to collective social services, undistributed profits, and taxes. The rounded* philosophical education of the few *is set aside for the technical training of the masses.* Artistic intuition *must give way to technical precision. New juridical institutions, new types of ownership and* *management, different distribution of income, new tastes, new values, new ideals have to emerge as an essential part of the industrialization process*[486].

Economic activity depends on the earth's capacity to supply raw materials, to produce food, and to absorb waste. While it took 100,000 years for the world's human population to reach 6000 million, it will now take merely 50 years to add another 6000 million. However, improvement in quality of the human species is not necessarily alternative to a growth in quantity. A larger population may mean greater possibilities in the division of labor and economies of scale. These possibilities may contribute to the growth of per capita *income, to better levels of living, and to better education. But beyond certain points,* quantity and quality may become competitive[487].

It is inevitable that, as humans beings become over-abundant in relation to other resources, their marginal value diminishes and the dignity of human life deteriorates correspondingly. 100,000 years may seem a very long span of time, but from the point of view of the whole history of the earth and mankind, this time interval is a brief fragment. It is however remarkable that during this time span homo sapiens *has turned himself from savage into the conqueror of the earth: considering that the Neolithic Revolution diffused into Europe between 5000 BCE and 2000 BCE, slightly more than 150 generation separate Europeans from their ancestors.*

Thus, within a relatively small number of generations, man has come to control his environment and to master the powerful forces of Nature. However, the selective process that favored the success and the multiplication of the

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486 When 'industrialization' occurs *gradually*, these socio-cultural changes take place in a balanced process with economic changes. But when 'industrialization' is speeded up *artificially*, the socio-cultural environment may show much greater degree of resistance to change than the economic structure and *a socio-political revolution* may emerge (e.g. as in Africa, Latin America, Iran, Turkey and the Soviet Union); All the miseries and the hardships that follow then become part of the price of industrialization.

487 In 1794, **John Barrow** (1764–1848), during a journey through China, witnessed a peculiar scene:

"Of the number of persons who had crowded down the banks of the grand canal (to Canton), several had posted themselves upon the high projecting stern of an old vessel which, unfortunately, breaking down with the weight, the whole group tumbled with the wreck into the canal. Although numbers of boats were sailing about the place, non were perceived to go to the assistance of those that were struggling in the water; one fellow was observed very busily employed in picking up, with his boat-hook, the hat of a drowning man."

This happened because men were over-abundant and hats were scarce.

aggressive type was certainly not *interrupted by the Neolithic Revolution. It continued to operate well into 'civilized' times and to a large extent* still operates today, *when man can commend immensely powerful forces, and his efficiency – for good and evil – has increased in spectacular fashion.*

Single man, like Stalin and Hitler as recent history has dramatically demonstrated, can today bring about unspeakable catastrophes that affect the entire world and the entire human species[488].

[488] The naturalist and ethologist K. Lorenz, (1903–1989) himself a Nazi supporter during 1933–1945, wrote (1966): "An unprejudiced observer from another planet, looking upon man as he is today, in his hand the atom bomb, the product of his intelligence, in his heart the aggression drive inherited from his anthropoid ancestors, which this same intelligence cannot control, would not prophesy long life for the species."

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* * *

"I am the daughter of earth and water,
And the nursling of the sky;
I pass through the pores of the ocean and shores;
I change, but I cannot die.
For after the rain, when with never a stain
The pavilion of heaven is bare,
And the winds and sunbeams, with their convex gleams,
Build up the blue dome of air,
I silently laugh at my own cenotaph,
And out of the caverns of rain,
Like a child from the womb, like a ghost from the tomb,
I arise and unbuild it again."

P.B. Shelley ("The Cloud", 1820)

1820 CE **Hans Christian Oersted** (1777–1851, Denmark). Physicist and chemist. Discovered electromagnetism (the magnetic effects of currents) and concluded that there exists a magnetic field surrounding a current. This phenomenon was soon quantified by **Jean Baptiste Biot** (1774–1862, France) and the physician and physicist **Felix Savart** (1791–1841, France).

Oersted was born at the town of Rudkobing, on the Island of Langeland, Denmark.

The Law of Biot-Savart

In 1819 Oersted observed that wires carrying electric currents produced deflections of permanent magnetic dipoles placed in their neighborhood. Thus the currents were sources of magnetic-flux density. Biot and Savart (1820), and later Ampère (1820–1825), in much more elaborate and thorough experiments, established the basic experimental laws relating magnetic induction $\boldsymbol{B}$ *to the source currents, and established the* law of force *between one current and another. The final analytic results were derived by Ampère (1826). In a form later written by* **Maxwell**, *a time-independent magnetic field due to a static current-density* $\boldsymbol{J}(\boldsymbol{r})$ *is given by*

$$\mathit{curl}\ \boldsymbol{H} = \frac{4\pi}{c}\boldsymbol{J}(\boldsymbol{r}); \quad \boldsymbol{B} = \mathit{curl}\ \boldsymbol{A}, \quad \mathit{div}\ \boldsymbol{A} = 0, \quad \nabla^2\boldsymbol{A} = -\frac{4\pi\mu}{c}\boldsymbol{J}(\boldsymbol{r}).$$

Integration of the vector Poisson equation gives $\boldsymbol{A} = \frac{\mu}{c}\int \frac{\boldsymbol{J}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}\mathrm{d}\boldsymbol{r}'$, *and consequently* $\boldsymbol{B} = \frac{\mu}{c}\int \frac{\boldsymbol{J}(\boldsymbol{r}')\times(\boldsymbol{r}-\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^3}\mathrm{d}\boldsymbol{r}'$. *If the conductor in which the current flows is sufficiently thin (thin wire), and if we are interested only in the field in the surrounding space, the thickness of the wire may be neglected. The integration over the volume of the conductor is then replaced by an integration along its length, i.e. we put* $\boldsymbol{J}\,\mathrm{d}\boldsymbol{r}' \to J\,\mathrm{d}\boldsymbol{l}$, *where* J *is the total current in the conductor. Hence*

$$\boldsymbol{A} = \frac{\mu J}{c}\oint_{\text{wire}}\frac{\mathrm{d}\boldsymbol{l}}{R}; \qquad \boldsymbol{H} = \frac{J}{c}\oint_{\text{wire}}\frac{\mathrm{d}\boldsymbol{l}\times\boldsymbol{R}}{R^3}; \qquad \boldsymbol{R} = \boldsymbol{r}-\boldsymbol{r}',$$

which is Biot and Savart's law. Note that the field $\boldsymbol{H}$ *is independent of the magnetic susceptibility of the medium. Above,* $\mathrm{d}\boldsymbol{l}$ *is an element of length* *(pointing in the direction of current flow) of a filamentary wire carrying a current J, and $\boldsymbol{R}$ is the coordinate vector from the element of length to an observation point.*

Ampère's experiments did not deal directly with the determination of the relation between currents and magnetic induction, but were concerned rather with the force that one current-carrying wire experiences in the presence of another; the force experienced by a current element $J_1 \mathrm{d}\boldsymbol{l}_1$ in the presence of a magnetic induction $\boldsymbol{H}$ is $\mathrm{d}\boldsymbol{F} = \frac{J_1}{c}(\mathrm{d}\boldsymbol{l}_1 \times \boldsymbol{H})$. If the external field $\boldsymbol{H}$ is due to a closed current loop with current J_2, then the total force which another closed current loop with current J_1 experiences is

$$F_{12} = \frac{J_1 J_2}{c^2} \oint \oint \frac{\mathrm{d}\boldsymbol{l}_1 \times (\mathrm{d}\boldsymbol{l}_2 \times \boldsymbol{R}_{12})}{|\boldsymbol{R}_{12}|^3},$$

*where the line integrals are taken around the two loops and $\boldsymbol{R}_{12}$ is the vector distance from line element $\mathrm{d}\boldsymbol{l}_2$ to $\mathrm{d}\boldsymbol{l}_1$. This is the mathematical statement (in modern notation) of Ampère's observations about forces between current-carrying loops, as obtained by **Grassman** (1809–1877) in 1844. By manipulating the integrand it can be recast in a form which is symmetric in $\mathrm{d}\boldsymbol{l}_1$ and $\mathrm{d}\boldsymbol{l}_2$ and which explicitly satisfies Newton's third law. Thus*

$$\frac{\mathrm{d}\boldsymbol{l}_1 \times (\mathrm{d}\boldsymbol{l}_2 \times \boldsymbol{R})}{|\boldsymbol{R}_{12}|^3} = -(\mathrm{d}\boldsymbol{l}_1 \cdot \mathrm{d}\boldsymbol{l}_2)\frac{\boldsymbol{R}_{12}}{|\boldsymbol{R}_{12}|^3} + \mathrm{d}\boldsymbol{l}_2 \left[\frac{\mathrm{d}\boldsymbol{l}_1 \cdot \boldsymbol{R}_{12}}{|\boldsymbol{R}_{12}|^3}\right].$$

The second form involves a perfect differential in the integral over $\mathrm{d}\boldsymbol{l}_1$. Consequently, it gives no contribution to the integral in $\boldsymbol{F}_{12}$, provided the paths are closed or extend to infinity. Ampère's law of force between current loops then becomes

$$\boldsymbol{F}_{12} = -\frac{J_1 J_2}{c^2} \oint \oint \frac{(\mathrm{d}\boldsymbol{l}_1 \cdot \mathrm{d}\boldsymbol{l}_2)\boldsymbol{R}_{12}}{|\boldsymbol{R}_{12}|^3}.$$

The line integral *for the field $\boldsymbol{H}$ can be transformed into an equivalent integral over a* surface *S bounded by the line, obtaining*

$$\boldsymbol{H} = \mathrm{grad}\ \Phi; \qquad \Phi = \frac{J}{c}\int_S \frac{\mathrm{d}\boldsymbol{s} \cdot \boldsymbol{R}}{|\boldsymbol{R}|^3},$$

where $\mathrm{d}\boldsymbol{s}$ is the vectorial area element, and Φ is a harmonic potential. The surface integral is, geometrically, the solid angle *Ω subtended by the closed contour at the point considered. As this point describes a closed path round the wire, the angle Ω changes suddenly from 2π to -2π, rendering Φ a* multi-valued *potential.*

The law of Biot-Savart is applicable, mutatis mutandis, *in the theories of hydrodynamics and elasticity.*

Clearly, the work done by moving a unit magnetic pole once around the wire or a closed curve C is $\oint_C \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l} = \oint_C d\Phi = 4\pi J/c$. *This is known as Ampère's circuital theorem.*

1820 CE **Robert Gibbon Johnson** (USA). Consumed two tomatoes on the steps of the courthouse in Salem MA, thus refuting the then widely circulating belief that the tomato was poisonous[1]. This changed forever the human-tomato relationship.

1820–1827 CE **André Marie Ampère** (1775–1836, France). Physicist. Attempted to render a combined theory of electricity and magnetism. Stimulated by the discovery of the *phenomenon* of electromagnetism by Oersted (1820), he soon followed with his own discovery of the basic *laws of electrodynamics* (Ampère coined this term).

He showed that parallel electric currents attract each other if the currents move in the same direction, and repel each other if their directions are opposite. In 1820 he showed that an electric current flowing through a coiled wire acts like a magnet. This led him in 1824 to the invention of the *galvanometer*, an instrument for detecting and measuring electric currents[2].

From his experiments, **Ampère** derived a number of quantitative empirical laws concerning the interaction of circuits carrying *direct* electric currents. Among the laws stated, is the inverse square law of force between two current elements (analogous to Newton's law of gravitation, 1687; and Coulomb's law

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[1] *Tomato* (*Lycopersicum esculentum*; order — *Solanaceae*). Annual plant, native of South America, probably Peru. The family includes: potato, eggplant, bell peppers, hot chili peppers, pimentos, paprika. Within this family are 4 genera which have been involved in most foul murders, in witchcraft and demonology, in military campaigns, in sly seduction, and in sexual orgies. These are: jimsonweed (*Datura*), deadly nightshades (*Atropa*), henbanes (*Hyoscyamus*), and mandrakes (*Mandragora*). These genera contain three *alkaloids*: atropine, hyoscyamine, and scopolamine which are representative of the tropane series of alkaloids, similar in structure to cocaine.

[2] **Johann Salomo Christoph Schweigger** (1779–1857, Germany) made an independent, similar experiment: he built a galvanometer (1820) consisting of a wire wound around a magnetic needle and measured the angle of deflection of the magnetic needle by the current. He named his instrument in honor of Luigi Galvani.

of the static force between point charges, 1785). Another is the Ampère circuital law relating the current flowing in a closed circuit to the magnetic field produced by the current[3]. Consequently, he treated magnetism by postulating small closed circuits inside the magnetized substance. This approach became fundamental in the 19^{th} century. Later, **Maxwell** modified this law for the case of time-varying electric fields.

Ampère was born at Polémieux, near Lyons. At an early age he learned Latin, which enabled him to read the works of Euler and the Bernoullis, but his reading also embraced history, travels, poetry, philosophy and all natural sciences. His father, an anti-revolutionary justice of peace, perished at the scaffold. This event produced a profound impression on the youth's susceptible mind. He was married in 1799 and moved to Bourg in 1801 to become a professor of physics and chemistry. His wife died in 1804 and he never recovered from the blow. In 1809 he was elected a professor of mathematics at the polytechnique school in Paris.

He died at Marseilles. The great amiability and childlike simplicity of Ampère's character are well brought out in his '*Journal et Correspondance*' (Paris, 1872).

1820–1836 CE **John Frederic Daniell** (1790–1845, England). Chemist and meteorologist. Invented (1820) a *dew-point hygrometer* (a device that indicates atmospheric humidity) which came into widespread use. Daniell (1823) revealed his findings on the behaviour of the atmosphere and on trade winds, in addition to giving details of improved meteorological equipment. Improved the voltaic cell by devising a cell giving steady current (1836). This 'Daniell-cell' was used as a source of energy in the electromagnetic telegraphy built by **W.F. Cooke** and **C. Wheatstone**. It gave new impetus to electric research and found many commercial applications.

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[3] Ampère discovered the following experimental facts concerning the magnetic fields produced by an electric current:

- A small coil (or loop) of current $\boldsymbol{J}$ behaves like a magnetic dipole of moment $\boldsymbol{m}$.
- $\boldsymbol{m}$ is perpendicular to the plane of the coil.
- $\boldsymbol{m}$ forms a right-handed screw with the flow of the current round the coil.
- $|\boldsymbol{m}|$ is proportional to the current J flowing in the coil.

With a proper choice of the unit of current (e.m.u), one can write $\boldsymbol{m} = \boldsymbol{J}\,\mathrm{d}\boldsymbol{s}$, where $d\boldsymbol{s}$ is the vectorial area element of the *magnetic shell*.

Daniell was born in London. In 1831 he became the first professor of chemistry in Kings College.

1820–1847 CE **John Herapath** (1790–1868, England). Natural philosopher. One of the first to discover that heat was not a substance but a form of motion. His *kinetic theory* was presented in an ambitious *Mathematical Inquiry into the Causes, Laws and Principal Phenomena of Heat, Gases, Gravitation etc.*, submitted to the Royal Society (1820) but rejected as being too speculative. It was however studied by **James Joule**, who in his own work on heat (1843–7), almost certainly followed Herapath.

Herapath was born in Bristol, and entered his father's profession as a maltster, but left business (1815) to open a mathematical academy in Bristol. He gave up teaching (1832), moved to Kensington, and began to write about the growing British railway network. This in turn aroused his interest in heat engines and hence modeling steam-gas physics in terms of elastic collisions between self-repulsive particles.

1820–1875 CE **Christian Gottfried Ehrenberg** (1795–1876, Germany). Naturalist. One of the first explorers of marine life. He was born at Delitzsch in Saxony, and studied at Leipzig and Berlin, where he took the degree of doctor of medicine in 1818. He was appointed professor of medicine at the University of Berlin in 1827.

Ehrenberg traveled widely, exploring the natural history of Egypt, Abyssinia, Arabia, and Russia, all the way to the frontier of China. After returning from these voyages, he examined his collections under the microscope. He discovered that many of the rock samples he had brought back were not inorganic products, as he had thought, but consisted of the remains of countless microscopic animals. In 1836 he showed that many silicate rocks were similarly composed of the remains of diatoms, sponges, and radiolaria.

Next he showed that living organisms similar to the ones that make up rocks still inhabit the sea. He reasoned that these rocks are continually forming as a result of the constant rain of dead organisms to the sea bottom. Ehrenberg also showed that the phosphorescence of the sea is due to the presence of microscopic organisms. Thus life in the sea extends from the largest living animal, the whale, to microscopic organisms which are so numerous that their accumulated remains make up thick layers of rock.

1820–1910 CE The European *Period of Romanticism* in music; Its leading composers are:

- Mauro Giulliani 1781–1828
- Nicolo Paganini 1782–1840

- Carl Maria von Weber 1786–1826
- Gioachino Rossini 1792–1868
- **Franz Schubert 1797–1828**
- Abraham Niedermeyer 1802–1861
- Hector Berlioz 1803–1869
- Mikhail Glinka 1804–1857
- Felix Mendelssohn 1809–1847
- Robert Schumann 1810–1856
- Frederic Chopin 1810–1849
- Franz Liszt 1811–1886
- **Giuseppe Verdi 1813–1901**
- Cesar Franck 1822–1890
- Edouard Lalo 1823–1892
- Bedrich Smetana 1824–1884
- **Johannes Brahms 1833–1897**
- Alexander Borodin 1834–1887
- Camille Saint-Saëns 1835–1921
- Georges Bizet 1838–1875
- Max Bruch 1838–1920
- Modest Mussorgsky 1839–1881
- Peter Ilytch Tchaikovsky 1840–1893
- Antonin Dvorak 1841–1904
- Edvard Grieg 1843–1907
- Nicolai Rimsky-Korsakov 1844–1908
- Gabriel Fauré 1845– 1924
- Francesco Paolo Tosti 1846–1916
- Charles Hubert Parry 1848–1918
- **Giacomo Puccini 1858–1924**
- Mikhail Ippolitov-Ivanov 1859–1935
- Gustav Mahler 1860–1911
- Claude Debussy 1862–1918
- Alexander Glazunov 1865–1936
- **Jean Sibelius 1865–1957**
- Sergei Rachmaninoff 1873–1943
- Maurice Ravel 1875–1937

1821 CE **Jean Francois Champollion** (1790–1832, France). Egyptologist. The first to decipher hieroglyphic writing. The Rosetta stone, discovered in 1799 during Napoleon's campaign in Egypt, provided the key to the language of ancient Egypt.

1821 CE **Joseph Maria Wronski** (1776–1853, Poland and France). Originated the '*Wronskian determinant*' in the theory of linear ODE. This was his only contribution to mathematics. He is the sole Polish mathematician of the 19^{th} century, remembered today. Wronski, who had an impecunious and erratic personality, spent most of his life in France.

The Emergence of Dynamic Elasticity Theory[4]

In 1787 *the German physicist* **Ernst Florens Friedrich Chladni** *(1756–1827) studied vibration of plates by means of sand figures (1827). In his experiments he poured fine sand on top of a glass plate. He then rubbed a bow against the plate, causing a vibration. The sand bounced away from regions that vibrated and collected at* nodes *(places that remained still). Within a second the plate was covered with a series of sandy curves. The patterns were symmetric and spectacular: circles, stars and other geometric figures. The character of the pattern depended on the shape of the plate, the position of the supports and the frequency of vibration.*

Since this phenomenon could not be explained by contemporary German and Swiss mathematicians, Chladni visited Paris in 1808 *and presented his experiments before mathematicians and physicists of the Institute of France, a section of the French Academy of Science. Chladni's experiments so astounded the scientists that they asked him to repeat his demonstration before Napoleon. The Emperor was impressed, and he agreed that the Academy should award a medal of one kilogram of gold to anyone who could devise a theory that explained Chladni's experiments. The contest was announced in* 1809 *and a deadline of two years was set for all entrees.*

In 1811, **Sophie Germain** *(1776–1831, France) was the only entrant in the contest: she tried to explain the behavior of elastic plates by applying the methods that* **Euler** *(1751) had used: Euler had suggested that a force applied*

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4 For further reading, see:

- Love, A.E.H., *A Treatise on the Mathematical Theory of Elasticity*, Dover Publications: New York, 1944, 643 pp.
- Todhunter, I. and K. Pearson, *A History of the Theory of Elasticity from Galileo to Lord Kelvin*, 2 Volumes, Dover Publications: New York, 1960.

to a rod is counteracted by an internal force of elasticity that is proportional at any point along the rod to the curvature of the rod. Similarly, she proposed that at any point on the surface, the force of elasticity is proportional to the sum of the major curvatures of the surface at that point. (The major curvatures are the maximum and minimum values of curvature out of all the curves formed when planes cut through the surface perpendicularly).

Germain's work did not win the award; she had not derived her hypothesis from principles of physics nor could she have done so at the time, because she lacked knowledge of analysis and the calculus of variations.

But her work did spark new insights: ***Lagrange****, who was one of the judges of the contest, corrected the error's in Germain's calculations and came up with the equation*

$$\frac{\partial^2 z}{\partial t^2} + K^2\left(\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} + 2\frac{\partial^4 z}{\partial x^2 \partial y^2}\right) = 0,$$

where $z(x, y)$ is the amplitude of the vibration for small z, t is time and (x, y) represent a point on the surface.

In 1811, the Academy extended the contest deadline by two years, and again Germain submitted the only entry. She demonstrated that Lagrange's equation did yield Chladni's patterns in several simple cases. But still, she could not devise a satisfactory derivation of Lagrange's equation from physical principles. For this work, she received honorable mention from the Academy.

At about the same time ***Simeon Dennis Poisson*** *approached the subject of elasticity with all the resources available to a 19^{th} century mathematician and physicist. In 1812, he was elected to the Academy and was therefore ineligible to compete for the prize. In 1814 Poisson published an article on elastic plates which sought to arrive at Lagrange's equation by applying a Newtonian model in which the plate consists of molecules that mutually repel and attract each other. By modern standards, Poisson's assumptions seem absurd.*

In her third entry in the contest (1815) Germain proposed to regard the work done in bending as proportional to the integral of the square of the sum of the principal curvatures taken over the surface. From this assumption and the principle of virtual work she deduced the equation of flexural vibration in the form now generally admitted. Later investigations have shown, however, that the formula assumed for the work done in bending was incorrect.

The judges in the contest were, at that time, ***Legendre****,* ***Laplace*** *and* ***Poisson****. They could not accept all of her assumptions, and with this reservation she won the prize. Germain did not attend the award ceremony.*

At the end of 1820, *no one knew yet how to combine the Newtonian conception of the constitution of bodies with Hooke's law. In the words of A.E.H. Love in the* 1892 *edition of his* Treatise on the Mathematical Theory of Elasticity:

"the fruit of all the ingenuity expanded on elastic problems might be summed up as — an inadequate theory of flexure, an erroneous theory of torsion, an unproved theory of the vibrations of bars and plates, ...".

*Yet, very little was needed to combine the older researches: the recognition of the distinction between shear and extension (*strain*) was there. So was the recognition of forces across the elements of a section of beam (*stress*). Also, there was an awareness that deflection of a bent beam and vibrations of rods and plates are expressible in terms of* differential equations*. Finally, the generalization of the principle of virtual work in the* Mécanique Analytique *of* Lagrange *threw open a broad path in this as in every other branch of mathematical physics. Physical science had emerged from its incipient stages with definite methods of hypothesis and induction and of observation and deduction, with the clear aim to discover the laws by which phenomena are connected with each other, and with a fund of analytical processes of investigation.*

This was the hour for production of general theories, and the men were not wanting: in a span of seven years, 1821–1828, ***Navier****,* ***Cauchy*** *and* ***Poisson*** *established the modern theory of elasticity and applied the general theory to special problems. It was further applied by* ***Lamé*** *and* ***Clapeyron*** *(1831–1833) to numerous problems of vibrations and of static elasticity, and thus means were provided for testing its consequences experimentally.*

After the equations of elasticity had been formulated, little advance seems to have been made in the treatment of problems of shells and plates. Only in 1860 *did* ***Kirchhoff*** *succeed formulating the equation of small motion of a plate in a correct way. He deduced it from the principle of virtual work and applied it to the problem of the flexural vibrations of a circular plate. Chladni's experiments were finally explained.*

1821–1823 CE **Claud Loui Marie Henri Navier** (1785–1836, France). Engineer and applied mathematician. Worked on topics such as engineering, elasticity and fluid mechanics. He was first to develop a theory of suspension bridges which hitherto had been built on empirical principles. Presented a molecular theory of an elastic body, giving the equation of motion for the displacement of a particle in an elastic solid[5]. Led by formal analogy with the theory of elasticity, he succeeded in setting up the differential equation of motion of a viscous fluid[6] (*Navier-Stokes equations*, 1823).

Navier was born in Dijon and educated at the École Polytechnique. In 1819 he became professor of applied mechanics at the École des Ponts et Chaussées in Paris, and in 1831 a professor of analytical mechanics at the École Polytechnique. He was a disciple and a friend of **Fourier**.

1821–1831 CE **Augustin Louis Cauchy** (1789–1857, France). The most outstanding analyst of the first half of the 19^{th} century. Pioneered the study

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[5] The field equations for linear elastic media are derived from three fundamental physical principles applied to the elastic continuum: conservation of mass, linear momentum and angular momentum. The conservation of linear momentum leads to the *Cauchy equation of motion*

$$\operatorname{div}\mathfrak{T} + \rho\boldsymbol{F} = \rho\frac{\partial^2\boldsymbol{u}}{\partial t^2},$$

where $\mathfrak{T}$ is the *stress tensor*, $\boldsymbol{F}$ is a *body force* per unit mass, ρ is the density, and $\boldsymbol{u}(\boldsymbol{r}, t)$ is the displacement field. [The conservation of mass manifests itself in the relation $\delta\rho = -\rho\operatorname{div}\boldsymbol{u}$, reflecting changes of density due to wave motions, which are usually negligible in many applications. The conservation of angular momentum leads to the *symmetry* of the stress tensor.] For linear elastic solids, the stress-strain relation (*Hooke's Law*) is of the form

$$\mathfrak{T} = \overset{4}{C} : \frac{1}{2}(\nabla\boldsymbol{u} + \boldsymbol{u}\nabla),$$

where $\overset{4}{C}$ is the fourth-order *tensor of elastic moduli*. The special case of an *isotropic homogeneous solid* renders

$$\mathfrak{T} = \lambda I\operatorname{div}\boldsymbol{u} + \mu(\nabla\boldsymbol{u} + \boldsymbol{u}\nabla).$$

The substitution of this relation into the Cauchy equation yield the *Navier elastodynamic equation* for the unknown displacement field,

$$(\lambda + 2\mu)\operatorname{grad}\operatorname{div}\boldsymbol{u} - \mu\operatorname{curl}\operatorname{curl}\boldsymbol{u} + \rho\boldsymbol{F} = \rho\frac{\partial^2\boldsymbol{u}}{\partial t^2},$$

where (λ, μ) are the *Lamé parameters* of the elastic solid.

[6] **Navier** derived the proper form of these equations although he had no conception of shear stress in a fluid and despite not fully understanding the physics of the situation which he was modeling. He rather based his work on modifying **Euler**'s equation to take into account forces between molecules in the fluid.

of analysis and the theory of permutation groups. Researched the convergence and divergence of infinite series, differential equations, determinants, probability and mathematical physics. He wrote extensively and profoundly on both pure and applied mathematics and can be ranked next to Euler in volume of output[7] [27 large quarto volumes of collected works, including 789 papers]. His work, however, is of uneven quality.

His greatest contributions to the mathematical sciences are couched in the rigorous methodology which he introduced since 1821, banishing formal manipulations and intuition from analysis. These are mainly embodied in his 3 great treatises: *Cours d'analyse de l'Ecole Polytechnique* (1821), *Le Calcul infinitésimal* (1823), *Lecons sur les applications du calcul infinitesimal a la geometrie* (1826–1828). In these volumes he developed the theory of convergence and divergence of infinite series [root test, ratio test, integral test, product of series, absolute convergence] and of infinitesimal calculus [limits, continuity, differentiability, definite integral as a limit of a sum, etc.].

He was the first to prove Taylor's theorem rigorously, establishing his well-known form of the remainder. He was also first to properly define length of arcs and surface areas by integrals [the question of defining surface areas independent of integrals was taken up somewhat later, but this posed many difficult problems that were not properly solved until the 20^{th} century].

It is true that the first known appearance of infinite series occurred in the work of Archimedes and that infinite series were freely used in the late 17^{th} century by **Newton**, **Leibniz** and others, but little or no attention was given to the general question of establishing rigorous tests for their convergence or divergence.

During 1823–1828 Cauchy laid the foundations of today's mathematical theory of elasticity. He introduced the notions of stress and strain tensors, derived the 3D stress-strain relations and also correctly established the number of elastic constants: two for an isotropic solid and 21 for a crystal.

In 1831 he developed his complex function theory where one encounters *Cauchy's inequality*[8], *Cauchy's integral theorem*, *Cauchy's integral formula*,

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[7] In 1835 the Academy of Sciences began publishing its *Comptes Rendus*. So rapidly did Cauchy supply this journal with articles that the Academy became alarmed over the mounting printing bill, and accordingly passed a rule, still in force today, limiting all published papers to a maximum length of 4 pages.

[8] *Cauchy inequality*: Let $f(x)$ and $g(x)$ be two functions, not identically equal to zero, given on the interval (a, b). We choose arbitrary numbers λ and μ and form the expression

$$\int_a^b |\lambda f(x) - \mu g(x)|^2 dx \geq 0, \quad \text{i.e.} \quad 2\lambda\mu C \leq \lambda^2 A + \mu^2 B,$$

Cauchy-Riemann equations and *Cauchy's power series expansion* of an analytic function.[9]

Cauchy was born in Paris. He received his early education from his father Louis Francois Cauchy (1760–1848), who held several minor public appointments and counted Lagrange and Laplace among his friends.

Later, at the École du Panthéon, he excelled in ancient classical studies. In 1805 he entered the École Polytechnique and won the admiration of Lagrange and Laplace. In 1807 he enrolled at the École des Ponts et Chaussées, where he prepared for a career as a civil engineer. In 1810 he left Paris for Cherbourg and returned in 1813 on account of his health, whereupon Lagrange and Laplace persuaded him to renounce engineering and devote himself to mathematics.

He obtained an appointment at the École Polytechnique but was forced to give up his professorship after the revolution of 1830, and was excluded from

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where

$$\int_a^b f(x)g(x)dx = C, \qquad \int_a^b f^2(x)dx = A, \qquad \int_a^b g^2(x)dx = B.$$

Since the above inequality is valid for arbitrary values of λ and μ, we may choose $\lambda = \sqrt{\frac{C}{A}}$, $\mu = \sqrt{\frac{C}{B}}$. Substituting these values of λ and μ in the inequality, we obtain $\frac{C}{\sqrt{AB}} \leq 1$, or the Cauchy inequality:

$$\int_a^b f(x)g(x)dx \leq \left\{ \int_a^b f^2(x)dx \int_a^b g^2(x)dx \right\}^{1/2}.$$

[9] To dig deeper, see:

- Titchmarsh, E.C., *The Theory of Functions*, Oxford University Press: London, 1939, 452 pp.
- Lavrentjev, M.A. and B.W. Shabat, *Methoden Der Komplexen Funktionentheorie*, Deutscher Verlag der Wissenschaften: Berlin, 1967.
- Needham, T., *Visual Complex Analysis*, Oxford University Press, 2000, 592 pp.
- Dettman, J.W., *Applied Complex Variables*, Dover Publications, 1984, 481 pp.
- Moretti, G., *Functions of a Complex Variable*, Prentice-Hall of India, New Delhi, 1968, 456 pp.
- Fisher, S.D., *Complex Variables*, Dover, 1999, 424 pp.

public employment for 18 years. A short sojourn at Freiburg in Switzerland was followed by his appointment, in 1831, to the newly-created chair of mathematics and physics at the University of Turin. In 1833 the deposed King Charles X, living in exile in Prague, summoned Cauchy to tutor his grandson, the 13-year-old Duke of Bordeaux. For five years the great analyst served as a baby-sitter of sorts to the pampered youth, and Charles made him a baron for this martyrdom.

Finally, in desperation, Cauchy escaped to Paris, saying he had to attend the celebration of his parents' golden anniversary. Once back in France, he was permitted to resume his post at the Académie. In 1848, after teaching in some church schools in Paris, he was allowed to return to the École Polytechnique without having to take the oath of allegiance to the new government. Cauchy was disliked by most of his colleagues. He displayed self-righteous obstinacy and an aggressive religious bigotry. His last words were: "*Men pass away, but their deeds abide*".

The New Mathematics

A tidal wave of mathematical inventiveness and novelty began to sweep the European continent at the dawn of the 19^{th} *century. This movement began with a quintet of brilliant minds:* ***N.H. Abel*** *(1824),* ***W.R. Hamilton*** *(1818),* ***C.G.J. Jacobi*** *(1829),* ***E. Galois*** *(1829) and* ***P.G.L. Dirichlet*** *(1829).*

With them the centroid of mathematical activity shifted from France to Germany. During the post-Newtonian era (ca 1740–1819)[10]*, the leading French school of mathematics was strongly tied to problems of Newtonian mechanics and astronomy. The French revolution, with its ideological break from the past and its many sweeping changes, created favorable conditions for the growth of mathematics. Thus in the* 19^{th} *century mathematics underwent a great forward surge, first in France and later, as the motivating forces spread over Northern Europe, in Germany, and still later in Britain.*

The new mathematics began to free itself from its narrow interest in mechanical problems, and a more general and abstract outlook evolved. The great mathematicians of the 19^{th} *century seem to be almost of a different species from their predecessors. They were not content with special problems, but attacked and solved general problems whose solutions yielded those of a multitude of problems which, in the* 18^{th} *century, would have been considered one by one.*

The 60 *years starting with the pivotal* 1829*, witnessed a most extraordinary phenomenon in the history of human thought: mathematicians prepared for the physicists of the* 20^{th} *century most of the mathematical tool needed to model the world of* 20^{th} *century physics: groups, matrices, vectors, quaternions, sets, non-Euclidean geometry, integral transforms, integral equations, partial differential equations, special functions and symbolic algebras.*

Indeed, during 1828–1893*, these weapons were forged by armies of enthusiastic workers that wheeled into the front ranks of analysis, geometry, algebra, theory of functions, theory of numbers and applied mathematics, under the leadership of* ***G. Green*** *(1828)*[11]*,* ***N.I. Lobachevsky*** *(1832),* ***J. Bolyai*** *(1832),* ***A.F. Möbius*** *(1832),* ***J.J. Sylvester*** *(1834),* ***J. Liouville*** *(1837),* ***H.G. Grassmann*** *(1844),* ***L. Kronecker*** *(1845),* ***J.B. Listing***

[10] The discovery of electromagnetism (1820), which marked a new era in *physics*, is contemporary with the rise of the new *mathematics* whose harbingers were **Poncelet** (1822) in geometry and **Abel** (1824) in *analysis*.

[11] Year in parenthesis indicates *beginning* of activity.

(1847), ***K.T.W. Weierstrass*** *(1848),* ***E.E. Kummer*** *(1850),* ***G.F.B. Riemann*** *(1851),* ***A. Cayley*** *(1854),* ***G. Boole*** *(1854),* ***S. Lie*** *(1870),* ***J.W.R. Dedekind*** *(1872),* ***G.F.L.P. Cantor*** *(1872),* ***W.K. Clifford*** *(1873),* ***J.W. Gibbs*** *(1876),* ***O. Heaviside*** *(1881),* ***G. Darboux*** *(1887),* ***V. Volterra*** *(1887),* ***G. Ricci-Curbastro*** *(1887),* ***A.M. Lyapunov*** *(1892) and* ***K. Pearson*** *(1893).*

The very foundations of mathematics were re-examined, and fundamental principles were worked out anew. The terms number, function, limit, continuity, infinity and integral were given more precise meaning. One of the phases of the quest for rigor was the re-defining of mathematics. An old idea which goes back to Aristotle defined mathematics as the science of quantity.

Auguste Comte *(1798–1857, France), philosopher and mathematician, defined mathematics as "the science of indirect measurement". These definitions had to be abandoned, however, since the modern branches of mathematics, such as the theory of groups, topology, projective geometry, theory of numbers and the algebra of logic, have no a priori relation to quantity and measurement. Moreover, the continuum, the central supporting pillar of modern analysis, as constructed by K. Weierstrass, R. Dedekind and G. Cantor, did not bear any reference whatsoever to quantity. In this light we understand the definition of* ***Benjamin Peirce*** *(1870) that "mathematics is the science which draws necessary conclusions".*

Thus, reasoning which seemed absolutely conclusive to one generation, no longer satisfied the next.

1822 CE The first meeting of the association of German speaking scientists and doctors (*Deutscher Naturforscher Versammlung*), was held at Leipzig. Some 20 scientists who had published work attended, together with 60 guests. First viewed with suspicion[12], these congresses later became instruments for increasing national unity in Germany; subsequent meetings, which were held

[12] Being both liberal and nationalistic in tone, the first congresses drew the suspicion of the rulers of the German states. Members attending the 1^{st} meeting refused to allow their names to be recorded, for fear their governments should find out. **Metternich** is Austria suggested to Viennese scientists applying for passports that it would be contrary to their own interests to go, with the result that the Austrian scientists were not represented until 1832, when the annual meeting was held at Vienna.

annually in one or other of the main German cities, gradually grew larger: some 600 attending the 1828 meeting at Berlin, and a 1000 the 1842 meeting at Mainz. Quite early on, the Prussian government saw that the national science congresses could become a controlled force for German unity, and it extended patronage to the meetings from 1828 on. Thereafter the congresses came more under the control of the German governments, with the state acting as host for a particular annual congress, appointing the president of the congress for that year and the secretary who organized and ran the meetings.

The idea of the association was conceived and realized by **Lorentz Oken** (Ockenfuss, 1779–1851), a Swabia-born German naturalist (who in 1806 elaborated on Goethe's theory that the skull in vertebrates evolved from enlargement and fusion of Vertebrae; this theory was disproved in 1858 by **Thomas Huxley**).

1822–1826 CE **Jean Victor Poncelet** (1788–1867, France). A mathematician and engineer. One of the founders of modern *projective geometry*. Born at Metz. From 1808–1810 he attended the École Polytechnique and was a pupil of **Monge**. In 1812 he became lieutenant of engineers and took part in the Russian campaign, during which he was taken prisoner and confined at Saratov on the Volga. It was during his imprisonment that he began his researches on projective geometry, which led to his great treatise on that subject: "*Traité des propriétés projectives des figures*" (1822), which is a study of those properties which remain invariant under projection. It contains fundamental ideas such as the *cross-ratio*, *perspective*, *involution*, and circular points at infinity. In 1826, Poncelet discovered the *principle of duality*. It was applied in 1826 by **Joseph Diaz Gergonne** (1771–1859) to the theorem of **Desargues**, and proved by **Plücker** in 1829.

From 1815 to 1825 Poncelet was occupied with military engineering at Metz and from 1825 to 1835 he was professor of mechanics at the École d'Application there. From 1838 to 1848 he was professor at the faculty of science at Paris and from 1848 to 1850, director of the École Polytechnique.

1822 CE Chemist **Georges-Simon Serulas** discovered iodoform and its antibacterial action.

1822 CE **Thomas Johann Seeback** (1770–1831, Germany). Physicist. Invented the *thermocouple* and discovered *thermoelectricity*; he showed, in his Berlin laboratory, that a temperature difference between the junctions of two different metals in a closed circuit can create an electric current. But, because of a mistaken interpretation of what was involved, he did not do any practical application for it. Thermoelectricity lay undisturbed for over a hundred years like a Sleeping Beauty. The Prince that awoke her was the semiconductor.

1822 CE **Joseph Nicèphore Niépce** (1765–1833, France). Physicist. Was the first person to make a permanent photographic image[13]. He exposed a light-sensitive metal plate in a camera, and then used an engraving process to "fix" the image to obtain what could be called a "photograph". A photograph he made in 1826 still exists today.

In 1839, **Louis Jacques Mandé Daguerre** (1787–1851, France) produced the first popular form of photography, the *Daguerreotype*. He based his process on Niépce's work, exposing a light-sensitive metal plate and developing the image with mercury vapor. He then 'fixed' the image with common salt. Also in 1839, **William Henry Fox Talbot** (1800–1877, England) invented a light-sensitive paper. This paper, coated with salt and silver-nitrate, produced a negative image from which positive prints could be made. This was the first negative-positive system of making photographs. The astronomer **John Frederick William Herschel** (1792–1871, England) named the invention *photography* and suggested the use of sodium thiosulphate (hypo) as a fixing agent.

1822–1842 CE **Eilhard Mitscherlich** (1794–1863, Germany). Physical and organic chemist. Discovered the phenomenon of *isomorphism* ("the same shape"), namely — that substances with similar chemical composition may have the same shape of crystal (1822). He demonstrated it with crystals of potassium arsenate and potassium phosphate and with some of the sulphates. He further noticed that sulphur forms either rhombic or monoclinic crystals, and this led him to the discovery of *dimorphism*, the capacity of some elements to occur in two distinct forms. He synthesized nitrobenzene in the laboratory (1832), which he termed *benzine* (1834). He also synthesized artificial minerals by fusing silica with various metallic oxides. He showed that yeast (which in 1842 he identified as a microorganism) can invert sugar in solution.

Mitscherlich was born in Jever, Lower Saxony, and entered Heidelberg University to study Oriental languages, but had to abandon it with the fall of Napoleon. He instead studied science at Göttingen and then worked with the Swedish chemist **Jöns Berzelius** in Stockholm for two years. He became professor at the Friedrich Wilhelm Institute in Berlin (1825).

1823 CE **Michel Eugène Chevreul** (1786–1889, France). Chemist. One of the founders of modern organic chemistry. Elucidated the true nature of

[13] Niépce "almost" discovered *radioactivity*! He knew that uranium salt may darken a metal plate but failed to understand what he was doing.

soap and explained clearly for the first time the reaction of *saponification*[14] (soap formation) in a classical paper (1823): "*Recherches chimiques sur les corps gras d'origine animale*". In this work he established the fact that soap is formed by a combination of alkali with an acid constituent of the fat[15], the other constituent (glycerol) being set free. It was one of the first works ad-

[14] *Saponification* is the reaction between an ester (fat; fatty acid) and a base (such as NaOH or KOH) in aqueous solution to form an alcohol and a salt (soap), e.g.,

$$H_3CCH_2{-}O{-}\overset{\overset{\displaystyle O}{\|}}{C}{-}CH_3 + (Na^+)_{aq} + (OH^-)_{aq} \rightarrow H_3CCH_2OH + CH_3{-}C\begin{matrix} /\!\!/ O \\ \backslash O^-Na^+ \end{matrix}$$

ethyl acetate — sodium hydroxide — ethyl alcohol (ethanol) — sodium acetate (soap)

Another typical soap molecule is *sodium laurate* $CH_3(CH_2)_{10}COO^-Na^+$. The long hydrocarbon chain in this molecule is *nonpolar* and does not dissolve well in water. This end (tail) of the molecule is called *hydrophobic* ("water hating"). On the other hand, the polar end of the molecule (head) $-COO^-Na^+$ is a salt, and dissolves well in water. It is *hydrophilic* ("water loving") and therefore gives soap its stability.

The hydrocarbon tails of the molecules can mingle with the grease, which is typically a mixture of fats and oils. As a result, the surface of a grease droplet becomes surrounded by a sheath of head groups, which do not mix with the grease. The head groups form hydrogen bonds with water, which lifts them and the droplet and washes them away from the object. Thus, soap acts as an enveloping coating over the grease and dirt particles.

Detergent is the name used for synthetic substances that are not soaps but have similar properties. In the most common type, the polar carboxylate end of the soap molecule is replaced by a sulfate group. A typical detergent molecule is sodium dodecylsulfate

$$CH_3(CH_2)_{10}CH_2{-}O{-}\underset{\underset{\displaystyle O}{\|}}{\overset{\overset{\displaystyle O}{\|}}{S}}{-}O^-Na^+$$

Whereas soaps tend to precipitate in hard water (i.e., water containing bicarbonates of such divalent metals as Mg or Ca), leaving a ring of Mg soap or Ca soap around a bathtub, most detergents do not, and are therefore superior for use in hard water. Most detergents are not *biodegradable*, however, so in recent years their environmental impact has become the source of increasing concern.

[15] This fact was asserted already by the apothecary and medical chemist **Otto**

dressing the issue of the fundamental structure of a large class of compounds. In 1825, Chevreul and Gay-Lussac patented a method of making candles from fatty acids; these candles were a great improvement on tallow (fats of cattle and sheep) candles, then commonly in use.

Chevreul was born at Avigers, a son of a physician. He came to Paris (1803) and studied under **Vauquelin**. He later occupied technical positions in Paris, including the directorship of the famous Goblin tapestry works. Chevreul became (1826) member of the Academy of Sciences and foreign member of the Royal Society of London. He subsequently became professor of organic chemistry in the Natural History Museum (1830). As a result of the researches of Gay-Lussac, Vauquelin and Chevreul, Paris became a center of work and research in the new science of organic chemistry. His completion of his 100^{th} year was celebrated with public rejoicing and after his death he was honored with a public funeral.

Soap-making is one of the oldest chemical syntheses: *soap* both as medicinal and as a cleansing agent was known to **Pliny**, who mentions it as originally a Gallic invention for giving a bright hue to the hair. Thus, there is reason to believe that soap came to the Romans from Germany. *Detergents*, however, were in use in earlier times and mentioned as soap in the Bible (*Jer* **2**, 22; *Mal* **3**, 2; etc.). These refer to the alkali-rich ashes of plants and other such purifying agents.

Phosphates are added to detergents to provide the optimal acidity for the functioning of the *surfactant* (surface-active agents like soap) molecules, to remove calcium and magnesium ions by wrapping around them and hiding them away from other ions with which they precipitate and form a scum, and to attache to dirt particles that have been washed off the fabric to prevent them from redepositing. Unfortunately, since phosphates are fertilizers, the waste water from a load of wash is highly nutritious and can promote the growth of microorganisms in rivers and lakes. This can lead to *eutrophication*, or overnourishment, which leads to clogging by organic growth, perhaps to the point of transforming a lake into a swamp.

1823–1828 CE **Niels Henrik Abel** (1802–1829, Norway). A great mathematician of the 19^{th} century with a meteoric career. Abel was born in Findo, Norway. His father was a country minister of considerable culture. His mother's outstanding characteristic seems to have been her beauty. Burdened with the support of his mother and five brothers and sisters when he was only 18, Abel struggled to take care of them and pursue his mathematical studies at the University of Christiania.

Tachenius (1620–1699, Germany) in his book *Hippocrates Chimicus* (1666). In this book he also stated that every salt is composed of acid and alkali.

The first explicit appearance of *integral equations* in the history of mathematics[16] was in Abel's thesis (1823) on tautochrones. His first notable work was a proof of the impossibility of solving the *quintic equation* by radicals (1824–1826).

State aid enabled him to visit Germany and France in 1825. In Freiburg he made his brilliant researches in the *theory of elliptic, hyperelliptic and Abelian functions*. He came to Paris in 1826, and during a ten months' stay he met the leading French mathematicians, but he was little appreciated, for his work was scarcely known. Pecuniary embarrassments, from which he had never been free, finally compelled him to return to Norway. When he realized that he was dying of tuberculosis, he praised the good qualities of his fiance Crelly to his friend Kielhan, and indeed Kielhan did marry Crelly after Abel's death.

In 1829, August Crelle (1780–1855) was able to secure for him an appointment as professor of mathematics at the University of Berlin, but the offer did not reach Norway until after his death. His premature death at the age of 26 cut short a career of extraordinary brilliance and promise.

1823–1839 CE **Jan Evangelista Purkyne** (**Purkinje**) (1787–1869, Bohemia). Physiologist. Known for observations and discoveries in physiology and microscopic anatomy. Professor in Breslau (1823–1850) and Prague (1850–1869). Director (1839) of the first institute of physiology in Breslau (Wroclaw, Poland).

His main contributions:

- Developed first system for classifying *fingerprints* (1823) and recognized fingerprints as means of identification.
- Discovered sweat glands (1833).
- Noted that animal tissues are made from cells (1835).
- Outlined the key features of the *cell* theory (to be more fully propounded by **Schwann** in 1839). Discovered ciliary movements in vertebrates; a class of pear-shaped cells in the middle layer of the cerebellar cortex are known as *Purkyne's cells* (1837).
- Observed *cell division* under the microscope (1838).

[16] There is an opinion that the first appearance of an integral equation was marked by **Laplace** (1782), when he introduced the *Laplace transform*.

- Discovered networks of fibers made up of large muscle cells in cardiac muscles, known as *Purkyne's network*, system or tissue (1839). Promoted the word *protoplasm* in the modern sense.
- Discovered ganglionic bodies in the brain.

1823–1855 CE **Justus von Liebig** (1803–1873, Germany). Chemist. Discovered (1823) the concept of chemical *isomers* (compounds with the same chemical composition but very different properties). The name *isomer* (Greek for *equal parts*) was coined in 1830 by **Jöns Jacob Berzelius** (1779–1848, Sweden), who also discovered *silicon* in 1823.

Liebig discovered the composition of many organic compounds. Made a systematic organization of organic chemistry, based on the radicals (radicals in organic chemistry act analogously to the elements in mineral chemistry, with the same general principles of combination and reaction). Discovered *chloroform* ($CHCl_3$) in 1831, and explained the theory of exchange of carbon and nitrogen in plants and animals (1840). Founded agricultural chemistry (chemistry of fertilization).

Liebig was born in Darmstadt. Studied at the Universities of Bonn and Erlangen and graduated as Ph.D. in 1822. He then went to Paris and practiced chemistry under **Gay-Lussac**. In 1826 he became a professor of chemistry at Giessen. In this small town his most important work was accomplished. There he established a new chemical laboratory, unique of its kind at the time, that soon rendered Giessen the most famous school in the world. It gave a great impetus to the progress of chemical education throughout Germany. In 1852, Liebig accepted the chair of chemistry at Munich University.

1824–1844 CE **Friedrich Wilhelm Bessel** (1784–1846, Germany). Astronomer. As director of the Königsberg observatory (1810–1846) he inaugurated the modern era of precision astronomy. Determined the positions and proper motions of over 50,000 stars and discovered the parallax of 61 Cygni. In 1838 Bessel made the first authentical measurement of a star's distance from the sun, using stellar parallaxes. This was achieved by employing an instrumental technique that enabled measurements of angles to a fraction of an arc second. In 1831–1832 Bessel measured an arc of the meridian in East Prussia and deduced for the earth's figure, in 1841, an ellipticity of $\frac{1}{299}$.

In 1844 he discovered the companion of Sirius[17] (called Sirius B), a star of the type known later as a *white dwarf*. Bessel *deduced* the binary nature of

[17] In 1914, **Walter Sydney Adams** (1876–1956, U.S.A.) succeeded in taking the spectrum of Sirius B, from which he inferred that it was a 'white' star, not very

Sirius by noticing that the star was moving back and forth slightly (wobbling), as if orbited by an unseen object. Only in 1863 was the companion first seen.

Bessel (1835) used dates of disappearances of the *Saturn Rings*[18] (as viewed from earth) in an effort to determine the orientation of Saturn's pole (w.r.t. to its own orbital plane about the sun). To this end he surveyed the astronomical

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different from its companion. He found that it had a surface temperature of ca 10,000 °C, which according to its luminosity, would have been quite *small*. Recent satellite observations at ultraviolet wave lengths showed that the surface temperature of Sirius B is about 30,000 °K.

[18] *Saturn*, whose equatorial diameter is 9.44 earth diameter, orbits at a mean distance of 9.539 AU from the sun with a mean orbital velocity of 9.6 $\frac{\text{km}}{\text{sec}}$ (compared to 29.8 $\frac{\text{km}}{\text{sec}}$ for earth). The Saturnian year is 29.46 earth years and its equatorial rotation period is $10^h13^n59^s$. The inclination of its orbit to the ecliptic plane is $2°29'$.

Earth-based views of the Saturnian ring-system (first identified by Huygens in 1655) change dramatically as Saturn orbits slowly about the sun. This change is observed because the rings (which are in the plane of Saturn's equator) are tilted $26°44'$ from the plane of Saturn's orbit. Thus, over the course of Saturn's year, the rings are viewed from various angles by earth-based observers. At one time, the observers look "down" on the rings; one-half of a Saturnian year later, the "underside" of the rings is exposed to view from earth. At intermediate times, the rings are seen *edge-on*, and they then disappear entirely from the view of the earth-based observer.

Saturn turns ringless as the earth passes the plane of the planet's razor-thin rings. Most of the time, however, the ring-plane does not cut the earth's orbit. It does so with a period of 14.7 years, and on that year, there are 3 ring-plane crossings (i.e., 3 disappearances of the rings at unequal intervals) due to the approximate ratio 3: 1 of the orbital velocities of earth and Saturn. Thus, the four last ring-plane crossings were {May 22, 1855; Aug. 10, 1985; Feb. 12, 1986; May 22, 1995}. Not until 2009 will the rings be aligned directly toward us once more. The ring-plane takes about 12 to 28 minutes to sweep across the earth.

Prior to the mid-19th century, observers often imagined the rings to be a thin, solid, opaque disc, divided into two concentric portions by the dark gap of the *Cassini Division*. This was despite **Laplace**'s demonstration (1785) that such broad rings, if truly solid (and thus in a state of uniform rotation) would be torn apart by Saturn's tidal forces. A series of many narrow solid rings could evade Laplace's objection, but **Maxwell** showed (1857) that even such narrow rings would be unstable. Only then did most astronomers reluctantly accept that the rings must be composed of myriads of independently orbiting moonlets. Arguments about *fluid* rings persisted until **Harold Jeffreys**' definitive work (1946) showed that the moonlets must be separate bodies.

literature to collect all reliable ring-plane crossing observations between 1714 and 1833. He even attempted to determine the precession period of the pole, obtaining a value of 340,000 years (the modern estimate is 1.7 million years).

Bessel was born at Minden. Early work on Comet Halley in 1804, which he communicated to **Olbers**, and an investigation of the comet of 1807, enhanced his reputation and he was summoned by the King of Prussia in 1810 to help establish a royal observatory.

In 1819 Bessel introduced into an investigation of *Kepler's problem* the solutions of the differential equation

$$y'' + \frac{1}{x}y' + \left(1 - \frac{m^2}{x^2}\right) y = 0,$$

which now bear his name: '*Bessel functions*'[19].

These functions were known earlier to **Daniel Bernoulli** (1732) [obtained the Bessel function of order zero as a solution to the problem of oscillations of

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[19] **Bessel** (1824) discovered the intriguing continued-fraction expansion of a ratio of the modified functions

$$\frac{I_1(2)}{I_0(2)} = \cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{5+\cdots}}}}} = \frac{\sum_{k=0}^{\infty} \frac{1}{k+1}\frac{1}{(k!)^2}}{\sum_{k=0}^{\infty} \frac{1}{(k!)^2}}$$

$$= \frac{1.59063685\ldots}{2.27958530\ldots} = 0.697\ 774\ 658\ldots$$

It is interesting that the continued fraction converges, thus differing from the harmonic series in this regard.
To prove the above relationship we write the n^{th} convergent for the continued fraction as $\frac{P_n}{Q_n}$ where

$$P_n = nP_{n-1} + P_{n-2}, \qquad Q_n = nQ_{n-1} + Q_{n-2}, \qquad n \geq 3$$
$$P_1 = 1, \qquad Q_1 = 1, \qquad P_2 = 2, \qquad Q_2 = 3.$$

Now, the modified Bessel functions K_n and I_n obey the recurrence relations

$$\begin{aligned} K_{n+1}(x) &= \tfrac{2n}{x}K_n(x) + K_{n-1}(x) \\ I_{n+1}(x) &= -\tfrac{2n}{x}I_n(x) + I_{n-1}(x). \end{aligned}$$

a chain[20] suspended at one end] and **Leonhard Euler** (1764), who employed

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Hence $A_n = K_{n+1}(2)$ and $A_n = (-)^{n+1} I_{n+1}(2)$ are independent solutions of $A_n = nA_{n-1} + A_{n-2}$. Consequently

$$\begin{aligned} P_n &= \alpha K_{n+1}(2) + \beta(-)^{n+1} I_{n+1}(2) \\ Q_n &= \gamma K_{n+1}(2) + \delta(-)^{n+1} I_{n+1}(2). \end{aligned}$$

The values of $(\alpha, \beta, \gamma, \delta)$ are found from the initial conditions and the Wronskian relation $K_{n+1}(x)I_n(x) + K_n(x)I_{n+1}(x) = \frac{1}{x}$ namely,

$$\alpha = 2I_1(2), \qquad \beta = 2K_1(2), \qquad \gamma = 2I_0(2), \qquad \delta = 2K_0(2).$$

But $I_n(2)$ tends to zero as n tends to infinity, while $K_{n+1}(2) \sim \frac{1}{2}n!$ as n tends to ∞. This yields $\frac{P_\infty}{Q_\infty} = \frac{I_1(2)}{I_0(2)}$.

20 In 1781, the problem was taken up by **Euler**, who formalized it as follows (modern notation): A chain (or a massive thread, devoid of flexural rigidity), with line density ρ and total length L, is suspended at one end. Take the origin o at that point, with the x-axis pointing downward along the undisturbed chain, and the y-axis pointing to the right.
Let the chain execute a motion with *small transverse* (y) *amplitude*. At a general point A, the tension makes an angle ψ with ox. The y-component of this tension is $T \sin\psi \approx T\frac{\partial y}{\partial x}$. An adjacent point B, at $x + dx$, experiences a tension differing by $\frac{\partial}{\partial x}\left(T\frac{\partial y}{\partial x}\right) dx$. The mass of the element AB is $\rho\, dx$ and its acceleration is $\frac{\partial^2 y}{\partial t^2}$. Moreover, since the oscillations are small, it is sufficiently accurate to take the tension T as the weight of the chain below A; hence $T = \rho g(L - x)$. This renders the equation of motion

$$\rho \frac{\partial^2 y}{\partial t^2} = \frac{\partial}{\partial x}\left[\rho g(L - x)\frac{\partial y}{\partial x}\right].$$

Assuming a harmonic motion $y = ue^{i\omega t}$ and denoting $L - x = z$, the equation for the amplitude u becomes

$$\frac{d^2 u}{dz^2} + \frac{1}{z}\frac{du}{dz} + k^2\frac{u}{z} = 0, \qquad k^2 = \omega^2/g.$$

Its general solution is

$$u(z) = aJ_0(2k\sqrt{z}) + bY_0(2k\sqrt{z}).$$

The free-end conditions implies $b \equiv 0$. At the fixed end, $u(x = 0) = 0$ yields $J_0(2k\sqrt{L}) = 0$, which furnishes an equation for the eigenfrequencies $\omega_n = \omega_n(L, g)$. By an extremely ingenious analysis Euler proceeded to obtain the values 1.445795, 7.6658 and 18.63 for the three smallest roots of the period equation [more accurate values are 1.4457965; 7.6178156; 18.7217517].

And here is how he did it: first, he *assumed* that all zeros are distinct and real

$$0 < \alpha_1 < \alpha_2 < \alpha_3 \ldots$$

and that it is possible to express it as the infinite product

$$J_0\left(2\sqrt{x}\right) = \prod_{n=1}^{\infty}\left(1 - \frac{x}{\alpha_n}\right).$$

If it is differentiated logarithmically, then

$$-\frac{d}{dx}\log J_0\left(2\sqrt{x}\right) = \sum_{n=1}^{\infty}\frac{1}{\alpha_n - x} = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty}\frac{x^m}{\alpha_n^{m+1}}$$

provided that $|x| < \alpha_1$, and the last series then converge absolutely.

Put

$$\sum_{n=1}^{\infty}\frac{1}{\alpha_n^{m+1}} = \sigma_{m+1}$$

and change the order of summations; then

$$-\frac{d}{dx}J_0\left(2\sqrt{x}\right) \equiv J_0\left(2\sqrt{x}\right)\sum_{m=0}^{\infty}\sigma_{m+1}x^m.$$

Replace $J_0\left(2\sqrt{x}\right)$ on each side by its series expansion

$$1 - \frac{x}{1^2} + \frac{x^2}{1^2\cdot 2^2} - \frac{x^3}{1^2\cdot 2^2\cdot 3^2} + \cdots,$$

multiply out the product on the right, and equate coefficients of the various powers of x in the identity; we thus obtain the system of equations

$$\begin{aligned}
1 &= \sigma_1,\\
-\frac{1}{2} &= \sigma_2 - \sigma_1,\\
\frac{1}{12} &= \sigma_3 - \sigma_2 + \frac{1}{4}\sigma_1,\\
-\frac{1}{144} &= \sigma_4 - \sigma_3 + \frac{1}{4}\sigma_2 - \frac{1}{36}\sigma_1\\
\frac{1}{2880} &= \sigma_5 - \sigma_4 + \frac{1}{4}\sigma_3 - \frac{1}{36}\sigma_2 + \frac{1}{576}\sigma_1\\
-\frac{1}{86400} &= \sigma_6 - \sigma_5 + \frac{1}{4}\sigma_4 - \frac{1}{36}\sigma_3 + \frac{1}{576}\sigma_2 - \frac{1}{14400}\sigma_1,
\end{aligned}$$

whence

$$\sigma_1 = 1, \quad \sigma_2 = \frac{1}{2}, \quad \sigma_3 = \frac{1}{3}, \quad \sigma_4 = \frac{11}{48}, \quad \sigma_5 = \frac{19}{120}, \quad \sigma_6 = \frac{473}{4320}, \ldots$$

Since $0 < \alpha_1 < \alpha_2 < \alpha_3 \ldots$, it is evident that

$$\frac{1}{\alpha_1^m} < \sigma_m, \qquad \sigma_{m+1} < \frac{\sigma_m}{\alpha_1},$$

and so

$$\sigma_m^{-1/m} < \alpha_1 < \frac{\sigma_m}{\sigma_{m+1}}.$$

By extrapolating from the following table:

m	$\sigma_m^{-1/m}$	σ_m/σ_{m+1}
1	1.000 000	2.000 000
2	1.414 213	1.500 000
3	1.442 250	1.454 545
4	1.445 314	1.447 368
5	1.445 724	1.446 089
6	1.445 785	—

Euler inferred that $\alpha_1 = 1.445795$, whence

$$\frac{1}{\alpha_1} = 0.691661, \qquad 2\sqrt{\alpha_1} = 2.404824.$$

By adopting this value for α_1, writing

$$\sum_{n=2}^{\infty} \frac{1}{\alpha_n^m} = \sigma'_m,$$

and then using the inequalities

$$\frac{1}{\alpha_2^m} < \sigma'_m, \qquad \sigma'_{m+1} < \frac{\sigma'_m}{\alpha_2},$$

Euler deduced that $\alpha_2 = 7.6658$, and hence that $\alpha_3 = 18.63$, by carrying the process a stage further.

Poisson (1833) calculated α_1 by solving the quadric equation obtained by equating to zero the first five terms of the series for $J_0(2\sqrt{x})$:

$$1 - \frac{x}{1^2} + \frac{x^2}{2^2} - \frac{x^3}{6^2} + \frac{x^4}{24^2} = 0,$$

obtaining $\alpha_1 = 1.446796491$. **Rayleigh** (1874) used the method of Euler to calculate the smallest zero of $J_\nu(x)$.

the functions of both zero and integral orders in an analysis of the vibrations of a stretched membrane. Later, **Schlömilch** (1857) defined these functions as the coefficients of the power of t in the expansion of $\exp\left\{\frac{1}{2}z(t - t^{-1})\right\}$, namely:[21]

$$e^{\frac{1}{2}z\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} J_n(z)t^n.$$

History of Thermometry (1593–1848)

In Aristotelian philosophy, hot and cold were associated with the terrestrial elements, earth, water, air and fire, which conveyed the sensations of coldness (earth-water), wetness (water-air), heat (air-fire), and dryness (fire-earth). Therefore, it was natural for Greek scientists, although not too keen on the experimental approach, to investigate the qualitative differences between these elements.

*The first thermoscope was developed by **Philo of Byzantium** in 250 BCE. This rudimentary device was able to distinguish between a balloon filled with cool air and the same balloon exposed to the heat of the sun. **Heron of Alexandria** later constructed a more refined thermoscope, but in the social conditions of the Hellenistic world and the Middle Ages that followed, these devices remained solely objects of amusements for more than 1500 years.*

*The first step in the development of the science of heat was of necessity the invention of the thermometer — an instrument for indicating temperature and measuring its changes[22]. The first requisite for such an instrument is that it should always give, at least approximately, the same indication at the same temperature. The invention of such a device is generally attributed to **Galileo***

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[21] To dig deeper, see:

- Watson, G.N., *A Treatise on the Theory of Bessel Functions*, Cambridge University Press: Cambridge, 1966, 804 pp.

[22] The invention of the thermometer preceded by some 260 years the notion that temperature is linearly related by the *mean molecular kinetic energy* of a certain kind of system in thermodynamic equilibrium.

at about 1593. An improved version was made by him in 1612. It consisted of alcohol hermetically sealed in a glass bulb, with an attached graduated fine tube. In order to render the readings of such instruments consistent with each other, it was necessary to select a fixed point (standard temperature) at the zero starting-point of the graduations.

It was soon found preferable to take two fixed points *and to divide the interval between them into the same number of degrees. It was natural in the first instance to take the temperature of the human body as one of the fixed points.*

In 1701, ***Newton*** *proposed a scale in which the freezing-point of water was taken to be zero, and the temperature of the human body as* $12°$.

In 1714, ***G.D. Fahrenheit*** *proposed to take as zero the lowest temperature obtainable with a freezing mixture of ice and salt, and to divide the interval between this temperature and that of the human body into* $12°$. *To obtained finer graduations, the number was subsequently increased to* $96° = 8 \times 12°$. *The freezing point of water was at that time supposed to be somewhat variable, because as a matter of fact, it is possible to cool water several degrees below its freezing-point, in the absence of ice.*

Fahrenheit showed, however, that as soon as ice began to form, the temperature always rose to the same point, and that the mixture of ice or snow with pure water, always gave the same temperature. At a latter date he also showed that the temperature of boiling water varied with the barometric pressure, but that it was always the same at the same pressure, and might therefore be used as a second fixed point — provided that a definite pressure, such as the average atmospheric pressure, were specified. The freezing and boiling points on one of his thermometers came out in the neighborhood of $32°$ *and* $212°$ *respectively, giving an interval of* $180°$ *between the points. Shortly after his death (1736), the freezing and boiling points of water were generally recognized as the most convenient fixed points to adopt, but different systems of subdivision were employed.*

Fahrenheit's scale, with its small degrees and its zero below the freezing-point, possesses undeniable advantages for meteorological work, and is still retained in most English-speaking countries. But for general scientific purposes, the centigrade system, in which the freezing-point is marked at $0°$ *and the boiling-point at* $100°$, *is now almost universally employed, on account of its greater simplicity from the arithmetical point of view. For work of precision the fixed points have been more exactly defined, but no change has been made in the fundamental principle of graduation.*

In general, any property of a suitable substance which varies as the temperature is changed, can be used to compare temperature differences with *the fundamental interval. For example: the volume of a liquid enclosed in a vessel, the volume of a fixed mass of gas maintained at constant pressure, the pressure of a fixed mass of gas maintained at constant volume, the electrical resistance of a piece of metal, the saturated vapor pressure of a liquid, are among the many measurable physical properties which alter reproducibly as the temperature changes. Any one of these can be made the basis of a temperature scale*[23].

It was soon observed that thermometers constructed with different liquids (such as oil, alcohol and mercury) did not agree precisely in their indications at points of the scale intermediate between the fixed points, and diverged even more widely outside these limits.

In 1802 *the research of* ***Gay-Lussac*** *showed that the laws of expansion of gases are much simpler than those of liquids, and that almost all gases expand nearly equally such that the differences between them cannot be detected without the most refined observations. This affords a strong a priori argument for selecting the scale given by the expansion of gas as the standard scale of temperature. Among liquids, mercury is found to agree most nearly with the gas scale, and is therefore used as a secondary standard to replace the gas thermometer within certain limits.*

In 1848, ***Lord Kelvin*** *proposed to take advantage of the fact that the efficiency*[24] *of a reversible Carnot ideal engine is independent of the nature of the working substance and dependent on temperature alone. This, he argued, could serve as a basis for an* absolute temperature scale. *The defining equation* $Q_1/Q_2 = T_1/T_2$ *does not, however, prescribe the numbering of the scale, nor indicate how the scale is to be realized in practice.*

If, however, T_0 *is the temperature of the ice-point and* $T_0 + 100$ *that of the steam-point, then if* Q_0 *and* Q_{100} *are the quantities of heat absorbed and ejected by the Carnot engine between the ice-point and the steam-point*

$$(T_0 + 100)/T_0 = Q_{100}/Q_0.$$

[23] Consider any one quantity, the magnitude x of which changes linearly with temperature. Then $T = 100\frac{x_T - x_0}{x_{100} - x_0}$, where x_0, x_{100}, x_T are the respective values of x at the ice-point, the steam-point and the unknown temperature T. In a mercury thermometer, for example $x = \ell$, where ℓ is the length of the mercury column above the bulb. For an ideal gas $x = PV$ (P = pressure, V = volume). This leads to the *ideal gas temperature scale.*

[24] If a Carnot engine takes in a quantity of heat Q_1 at temperature T_1, and ejects a quantity of heat Q_2 at a lower temperature T_2, its efficiency is $\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}$ and hence $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$. This defining equation is the basis upon which the Kelvin (°K) scale of temperature is founded.

Assuming that both Q_{100} and Q_0 can be measured, this equation enables us to find T_0, the thermodynamic absolute temperature of the ice point, and the grading of the scale is thus settled.

To realize the absolute scale in practice, the working substance of the Carnot engine is chosen as a unit mass of an approximately ideal gas. It is then shown that the Kelvin Absolute Thermodynamic Scale of temperature and the ideal gas scale of temperature are identical — both giving the one truly absolute scale of temperature.

1824 CE **Henri J. Paixhans** (1783–1854, France). Artillerist. Introduced the *shell gun*, a revolutionary invention in the history of warfare.

1824 CE **Sadi Nicolas Léonhard Carnot** (1796–1832, France). A French engineer. The founder of the science of thermodynamics. An original and profound thinker of the foremost rank, whose full stature was not recognized until pointed out by **Lord Kelvin** in 1848.

The only work he published was *Réflection sur la puissance motrice du feu et sur les machines propres a developer cette puissance* (Paris, 1824) [Reflections on the Motive Power of Heat and on Proposed Machines to Develop that Power].

In this manuscript Carnot described a working cycle, now called a *Carnot cycle*, that is of great importance from both a practical and a theoretical viewpoint. Paving the way to the forthcoming Second Law of Thermodynamics, he gave the first substantial theory of heat engines.[25]

The steam engine was well known to Carnot[26]. He knew that it had been made increasingly efficient over the years, and he wondered whether there

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[25] For further reading, see:

- Noakes, G.R., *A Text-Book of Heat*, Macmillan, 1945, 469 pp.
- Rocard, Y., *Thermodynamics*, Pitman and Sons: London, 1961, 681 pp.
- Sommerfeld, A., *Thermodynamics and Statistical Mechanics*, Academic Press: New York, 1955, 401 pp.
- Bruhat, G., *Thermodynamique*, Masson and C^{ie}, 1947, 428 pp.

[26] The first real steam *engine* was invented by **Newcomen** in 1705. It was an inefficient contraption until **James Watt** introduced, during 1769–1774, a number of innovations which made the steam engine a practical device. **Fulton**'s steamboat was driven by one of Watt's engines in 1807. Another heat engine that

was some limit to its improvement. He appreciated that real steam engines leaked steam and that friction reduced their efficiency. So he imagined the *ideal engine*, one that we call reversible, on the basis of which he formulated the problem in exactly the right way.

Carnot showed that such a heat engine operating in an ideal, reversible cycle between two heat reservoirs, would be the most efficient engine possible. Such an ideal engine, called a *Carnot engine*, establishes an upper limit on the efficiencies of all engines. That is, the net work done by the working substance taken through the Carnot cycle is the largest possible for a given amount of heat supplied to the working substance. This efficiency is found to be $\eta = 1 - T_l/T_h$, where $\{T_l, T_h\}$ are the low and high temperature limits of the working substance [Carnot's theorem].

In addition to the frustration encountered by any inventor who tries to contravene the law of conservation of energy, another fundamental limitation governs all engines designed to convert heat into mechanical work or other forms of useful energy: it is impossible to construct an engine that will do work by extracting heat from a *single* heat reservoir.

For example, universal frustration is met by inventors who want to construct a device consisting of a box which, when immersed into the ocean, will via some mechanism inside the box convert the heat content of the ocean into some other form of energy. Such a hypothetical device, which is no way violates the law of conservation of energy, is called a perpetual motion engine "of the second kind", to distinguish it from perpetual motion engines "of the first kind" (which do not conserve energy).

The origin of this impossibility can be traced to the fact that to transfer heat from a reservoir at lower temperature to a reservoir at higher temperature requires *additional* work. Indeed, if the perpetual engine were feasible one could, in principle, use the work drawn from the box to boil some water taken from the ocean. The net effect would resemble an experiment in which the water in a kettle placed on a stove would freeze by transfer of heat to the stove, which would become hotter. This sort of thing does not happen in nature.

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influenced Sadi Carnot was due to the French engineer, physicist and inventor **Charles Cagniard de la Tour** (1777–1859) who reported (via Lazare Carnot) to the Academy of Sciences in 1809 on his novel invention: his "buoyancy engine" relied on air expanding in a liquid to produce work. Cagniard was born in Paris and studied at the École Polytechnique and the École du Genie Geographie. Between 1809 and 1838 he made several inventions (the *cagniardelle*, a forced-draft blowing machine and a *siren* for acoustical studies) and worked on crystallization and fermentation.

The most efficient conversion of heat into work is effected by a *reversible* engine, operating between *two* reservoirs of temperatures T_1 and T_2. Such an engine can convert at best only a part W of the amount of heat Q_1 drawn from the reservoir at T_1 into work; the balance $Q_2 = Q_1 - W$ must be transferred as heat to the other reservoir at T_2. An engine is "reversible" if none of its moving parts generates any heat by friction, and if all heat transfers between different parts of the engine take place "isothermally", i.e., only between parts that do not differ in temperature by more than an infinitesimal amount.

The classic example of such an ideal device is provided by Carnot's cyclic engine, which does work in four strokes of a frictionless piston with an ideal gas as working substance. Carnot noticed that the efficiency of such an engine depends on the two temperatures *only*, and is independent of the working substance. This observation is now generally known as "Carnot's theorem". It enables one to *define* an absolute temperature scale by writing $W = \frac{T_1 - T_2}{T_1} Q_1$ or $Q_1 = \frac{T_1}{T_1 - T_2} W$, where all temperatures are measured in the absolute scale. The factor $(T_1 - T_2)/T_1$ is called the ideal efficiency and is, obviously, always ≤ 1. By measuring this efficiency for an engine operating between various heat reservoirs one can, in principle, establish the absolute temperature scale experimentally. Indeed, the efficiency of a reversible engine approaches 1 as the temperature of the colder reservoir approaches the absolute zero of temperature ($0\,^\circ\mathrm{K} = -273\,^\circ\mathrm{C}$).

Another instructive way of looking at the above definition of the absolute temperature scale is obtained if one draws from engineering experience the following inference: the amount of heat Q_1 extracted by a given reversible engine from the hotter reservoir does not depend on what happens to the heat later, and is thus the same for given T_1 independent of the value of T_2. Similarly, the amount Q_2 of heat delivered by a given engine to the reservoir at T_2 does not depend on the value of T_1.

If this is accepted as empirically obvious, then one can arrive at an absolute temperature scale by first introducing as standard $T_S = 1\,^\circ$, the temperature of a reservoir that accepts from the given engine a standard amount of heat $Q_S = S$ (say), and then defining T_0 as the temperature of a reservoir from which the engine draws T times the standard amount S, namely, $Q = TS$. Since, by definition, this is true for all temperatures, one has, specifically, $\frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \frac{Q_S}{T_S} = S$.

The law of conservation of energy as applied to a reversible engine, $Q_1 = Q_2 + W$, allows one to write the above equation in the form $\frac{T_2}{T_1} Q_1 = Q_1 - W$ or $W = \frac{T_1 - T_2}{T_1} Q_1$. One then obtains an expression for Q_2 in terms of W, $Q_2 = \frac{T_2}{T_1 - T_2} W$ or $W = \frac{T_1 - T_2}{T_2} Q_2$.

Since these relations must hold even if the engine is run in reverse, so that it acts as a heat pump, the above equation can be used to define the factor $(T_1 - T_2)/T_2$ as the ideal inefficiency (sometimes called "performance" coefficient) of a heat pump. The lower the temperature T_2 of the colder reservoir, the larger the unavoidable inefficiency of the pump; it will take more and more work W to transfer the amount Q_2 out of T_2 into T_1 as T_2 is made lower and lower.

For example, a steam engine operating between a reservoir at $T_1 = 127\,^\circ\text{C} = 400\,^\circ\text{K}$ and outside air at $T_2 = 27\,^\circ\text{C} = 300\,^\circ\text{K}$ cannot exceed the efficiency $[(400 - 300)/400] = \frac{1}{4}$. A refrigerator operating between an ice box at $T_2 = -3\,^\circ\text{C} = +270\,^\circ\text{K}$ and the kitchen at $T_1 = 27\,^\circ\text{C} = 300\,^\circ\text{K}$ cannot have an inefficiency less than $[(300 - 270)/270] = \frac{1}{9}$.

The beauty of Carnot's result is that it does not depend on the particular design of the reversible engine. All reversible engines must have the same efficiency, independent of the working substance, which may be a gas as in the case of the steam engine, or consist of electrons as in the case of a thermoelectric device. To see this, suppose the assertion were not true. Then there should be an engine 1 giving work W and another engine 2 giving work $\widetilde{W} > W$. By running engine 1 backward with the work W delivered by engine 2, one would have as net result an engine that delivers the work $\widetilde{W} - W$ by drawing this amount of heat from a single reservoir T_2, contrary to the observed impossibility of perpetual motion of the second kind. To avoid this inconsistency one must conclude $\widetilde{W} \leq W$, which proves the assertion[27].

Thus all heat engines convert only part of their heat intake into work, and discard the remainder into the surrounding medium. This limitation is not contained within the First Law of Thermodynamics, nor does it result from imperfections in the engines. This suggested that there must be a Second Law of Thermodynamics which imposes limits not expressed by the First Law. Indeed, the Second Law of Thermodynamics is *implied* here in the assumption that energy cannot flow spontaneously of its own accord from a colder to a hotter body [such a flow *happens* in refrigerators, but at the expense of additional *external* electrical energy and is thus *not* spontaneous]. This law was stated explicitly by **Rudolf Clausius**, 25 years later.

[27] **Maxwell** found his kinetic theory of gases to be in conflict with the ideas of Carnot and he postulated a hypothetical situation where *intelligence* could contradict Carnot's principle (Maxwell's demon). Maxwell then correctly concluded that the second law is of *statistical nature*. In the twentieth century, it has been realized (**Szilard** and **Brillouin**) that the doings of the hypothetical sorting demon involve information processing, itself requiring some energy input.

Sadi Carnot was born in Paris. He entered the École Polytechnique in 1812. Later he served as officer in the French army, but was unemployed in his profession because of his father's political activity. He then devoted himself to mathematics, chemistry, natural history, technology, music, art and athletic sports. He became Captain in the Engineers in 1827, but left the service altogether in 1828.

His naturally feeble constitution, further weakened by excessive study, finally broke down in 1832. An attack of scarlatina led to brain fever, and he had scarcely recovered when he fell victim to cholera, of which he died in Paris at the mere age of 36.

A quotation from his memoir is appropriate:

"*The steam engine works our mines, impels our ships, excavates our ports and our rivers, forges iron... Notwithstanding the work of all kinds done by steam engines, notwithstanding the satisfactory condition to which they have been brought today — their theory is very little understood*".

1824–1859 CE **Johann Franz Encke** (1791–1865, Germany). Astronomer. Used the data from a Venus transit to deduce a sun-earth distance of 153,200,000 km[28] (1824). Encke studied the comets of 1680 and 1812, the orbit of a comet that now bears his name (1818) and the motion of asteroids. He expounded a method of determining an elliptic orbit from 3 observations (1849).

Encke was born at Hamburg. Graduating from the University of Göttingen in 1811, he devoted himself to astronomy under **Carl Friedrich Gauss**. He enlisted in the Hanseatic Legion for the campaign of 1813–1814 and became lieutenant of artillery in the Prussian service in 1815. He returned to Göttingen in 1816 to start his astronomical observations in the Seeberg Observatory near Gotha. He visited England in 1840.

1825 CE **Thomas Drummond** (1797–1840, England). Engineer and administrator. Invented the limelight lamp: an intense beam of light focused by a parabolic mirror and produced by burning lime in an alcohol flame enriched by addition of oxygen.

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[28] This was too high by a little over 3.2 million km. In 1931 the asteroid *Eros* was scheduled to approach earth to within a distance of only about $\frac{2}{3}$ that of Venus. Since Eros held no atmosphere to fuzz its outlines, its position could be determined with great accuracy. An international project was set up to determine the position and parallax of Eros and it was found that the *average sun-earth distance* is just a bit less than 149,079,000 km [at perihelion 146,514,000 km and at aphelion 151,644,000 km].

1825–1836 CE **William Sturgeon** (1783–1850, England). Electrical engineer and inventor. Built the first *electromagnet* capable of supporting more than its own weight (1825). This device led to the invention of the telegraph, the electric motor, and numerous other devices basic to modern technology. He built an electric motor (1832) and invented the *commutator*, an integral part of most modern electric motors. He invented the first *suspension coil galvanometer* (1836), a device for measuring current.

Sturgeon was born in Whittington, Lancashire, England.

1826 CE **August Leopold Crelle** (1780–1855, Germany). Civil engineer and mathematical enthusiast who made various discoveries in the geometry of the triangle (1816). A unique figure in the annals of mathematics. Founded (1826) a new periodical devoted exclusively to mathematics, the *Crelle's Journal der Mathematik*. He started it off by publishing a whole series of papers by **Abel**, including the great one that proved the unsolvability of the general 5^{th}-degree equation. Thus Crelle was able to give an international circulation to Abel's first important results, while Abel could supply papers of a quality that ensured the success of the new journal.

Crelle constructed most of the Prussian highroads (1816–1826) and planned the Berlin-Potsdam railway. He published a German translation of **Legendre**'s geometry (1822) and **Lagrange**'s mathematical work (1823–1824).

1826 CE The last recorded *auto-da-fé* of the *Spanish Inquisition* took place in Valencia, Spain, where a Jew and a Quaker were tortured to death. Between 1481 to 1826, there were 2000 autos-da-fé with about 30,000 persons (mostly *Jews*) burned alive at the stakes.

1826–1837 CE Repeated outbreaks of *cholera* ravaged Europe; millions perished; Ca 900,000 in 1831 alone.

1826–1837 CE **Henri** (**René Joachim**) **Dutrochet** (1776–1847, France). Physician and physiologist. First to discover the quantitative dependence of the *osmotic pressure* on the difference of *concentrations* over the two sides of the membrane (1826). In 1837 he discovered that carbon dioxide is absorbed only by those plant cells that contain green pigment and only in the presence of light.

He was born at Chateau de Néon (Indre). In 1802 he began the study of medicine at Paris, and was subsequently appointed chief physician to the hospital at Burgos. He returned to France in 1809 and dedicated himself to the natural sciences.

1826–1842 CE **Peter Gustav Lejeune Dirichlet** (1805–1859, Germany). One of the eminent German mathematicians of the 19^{th} century. At **Gauss**' death in 1855 he was appointed his successor at Göttingen, a fitting honor for a mathematician who was Gauss' former student and a lifelong admirer of his mentor.

Dirichlet was born in Duren, Germany. As a young man he attended a Jesuit college in Cologne, where one of his teachers was **Georg Simon Ohm** (1787–1854). In 1822 he went to Paris to learn from the great French masters Laplace, Legendre, Fourier and Cauchy. In particular, he found the work of Fourier appealing. In 1828 Dirichlet moved to Berlin to teach mathematics at the military academy. In 1831 he was made a member of the Berlin Academy and married Rebecca Mendelssohn, the sister of the composer. While at Göttingen, he hoped to finish Gauss' incomplete works, but his early death in 1859 prevented this.

At the time when the hegemony in mathematics moved from France to Germany, Dirichlet, being fluent in both German and French, served as liason between mathematicians of the two nationalities.

In 1829, Dirichlet found a sufficient condition for the convergence of Fourier series which suffices for practical purposes and covers a wide class of functions, including functions with discontinuities. This theorem is of paramount importance in harmonic analysis of physical signals. The undertaking led him to generalize the classical function concept [through what we call today the *Dirichlet function*[29] $\{\sin \lambda t/\pi t\}$] and derive the *Dirichlet conditions*.

The name of Dirichlet is associated with a number of other topics: *Dirichlet test* for uniform convergence, *Dirichlet theorem* (1826) on primes [every arithmetic progression in which the first term and the common difference are primes contains an infinite number of primes] and the *Dirichlet boundary value problem* for the Laplace equation [solve $\nabla^2\psi = 0$ inside (outside) V such that ψ takes prescribed values f on the boundary of V]. The Dirichlet problem is related to the calculus of variations because the solution of the Dirichlet interior problem minimizes the integral $I = \int_V |\nabla\psi|^2 d\tau$. This is known as the *Dirichlet principle*.

Weierstrass later disagreed with **Riemann** about the automatic existence of a function which makes this integral minimum, but **Hilbert** later showed that provided certain conditions on f are satisfied, Dirichlet's variational problem always possesses a solution. The value of the method lies in the fact that in certain cases "direct methods", (i.e. methods which do not reduce the variational problem to one in differential equations), may produce

[29] A hundred years later, the limit of this function as λ tends to infinity, became known as one of the useful representations of the 'delta-function'.

a solution of the variational problem more easily than the classical methods could produce a solution of the corresponding interior Dirichlet problem.

The variational method is also of great value in providing approximate solutions, especially in certain physical problems in which the minimum value of I is the object of some interest[30].

Dirichlet contributed notably to number theory. In 1832 he provided a proof for the special case $n = 14$ of Fermat's conjecture. He studied the *Dirichlet series* [$\sum a_n n^{-s}$, including the Riemann zeta-function as a special case] which are of great importance in applications of analysis to the theory of numbers.

1827 CE **Felix Savary** (1797–1841, France). Astronomer. Showed that the motion of binary stars is in full accord with Newton's theory of universal gravitation.

1827–1865 CE **August Ferdinand Möbius** (1790–1868, Germany). Astronomer and mathematician. He is known and appreciated for his work in five fields:

(1) In his book '*Barycentrische Calcül*', Möbius presented, 20 years ahead of **Grassmann** and **Hamilton**, the ideas of vectors and quaternions (1827).

(2) Introduced the '*Möbius function*' and the '*Möbius transform*' (1832) into the theory of numbers. After Euler's totient function, these are among the most important tools of number theory[31].

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[30] e.g. in electrostatic problems, I is closely related to the *capacity* of the system. In this connection, one notices that $\frac{1}{8\pi}\int_V |\nabla\psi|^2 d\tau$ is the potential energy of the field.

[31] The Möbius function: $\mu(n) = 1$ for $n = 1$, $\mu(n) = 0$ if n is divisible by a square and $\mu(n) = (-)^k$ if n is a product of k distinct primes. The Möbius transform of $f(x)$ is defined as

$$F(x) = \sum_{n=1}^{\infty} f(nx)n^{-s}$$

and its inverse is

$$f(x) = \sum_{n=1}^{\infty} \mu(n)F(nx)n^{-s}.$$

For $x = 1$ (Dirichlet series), $f(n) = 1$ yields for $F(1)$ the Riemann zeta function $\zeta(s)$ and therefore

$$[\zeta(s)]^{-1} = \sum_{n=1}^{\infty} \frac{\mu(n)}{n^s}.$$

(3) Discovered the 'Möbius bilinear mapping' in the complex plane $\left[w = \frac{az+b}{cz+d}\right]$. It is linked to projective geometry via the cross-ratio $\frac{(w-w_1)/(w-w_3)}{(w_2-w_1)/(w_2-w_3)}$, which is invariant under a projective transformation (1840).

(4) Originated the concept of *homogeneous coordinates* (1827).

(5) Constructed a one-sided, one-edged surface, the '*Möbius strip*' (1865). This appeared in a paper in which a polyhedral surface was viewed as a collection of joint polygons, which in turn introduced the concept of 2-complexes into topology.

Möbius was born near Naumberg, Germany. Through his father, a dance teacher, he was a descendant of **Luther**. In 1809 he entered the University of Leipzig to study law, but ultimately devoted himself to mathematics and astronomy and was a student of **Gauss** at Göttingen. In 1816 he became an associate professor at Leipzig University, and later was chosen as the director of the university observatory. He waited 28 years to become a full professor (1844).

The Dawning of Topology (1735–1914)

"A Geometry is defined by a group of transformations, and investigates everything that is invariant under the transformations of this given group".

Felix Klein (1849–1925)

The Möbius function is also tied up with the *prime zeta-function*

$$\mathcal{P}(s) = \sum_p p^{-s},$$

p prime, since

$$\mathcal{P}(s) = \sum_{k=1}^{\infty} \frac{\mu(k)}{k} \log_e \zeta(ks).$$

"It has been said that geometry is the art of applying good reasoning to bad diagrams. This is not a joke but a truth worthy of serious thought. What do we mean by a poorly drawn figure? It is one where proportions are changed slightly or even markedly, where straight lines become zigzag, circle acquire incredible humps. But none of this matters.

An inept artist, however, must not *represent a closed curve as if it were open, three concurrent lines as if they intersected in pairs, nor must he draw an unbroken surface when the original contains holes".*

Henri Jules Poincaré, 1895

In the first half of the 19^{th} *century there began a completely new development in geometry that was soon to become one of the great forces in modern mathematics. The new subject, called* analysis situs *or topology, has as its object the study of the properties of geometrical figures that persist even when the figures are subjected to deformations so drastic that all their metric and projective properties are lost.*

The major actors in this drama were **L. Euler** *(1707–1783),* **Lhuilier** *(1750–1840),* **Gauss** *(1777–1855),* **Möbius** *(1790–1868),* **Listing** *(1808–1882),* **Betti** *(1823–1892),* **Kirchhoff** *(1824–1887),* **Riemann** *(1826–1866),* **C. Jordan** *(1838–1922),* **F. Klein** *(1849–1925)* **H. Poincaré** *(1854–1912),* **Hausdorff** *(1868–1942),* **Lebesgue** *(1875–1941),* **Frechet** *(1878–1973),* **F. Riesz** *(1880–1956),* **Veblen** *(1880–1960),* **Brouwer** *(1881–1966),* **Lefschetz** *(1884–1972) and* **Alexander** *(1888–1971).*

In Euclidean geometry, the allowed movements are rigid motions (translations, rotations, reflections), in which the distance between any two points of the figure are not changed. Thus the geometric properties are those which are invariant under the group of rigid motions. Since the transformations are the rigid motions, two figures are considered equivalent if they are congruent.

In projective geometry, *two figures are considered equivalent if one may be projected into the other. As the projections include parallel projection, planar, projective geometry includes Euclidean geometry as a special case. Here, not only are similar figures equivalent, but* any *two triangles are equivalent. This means that all triangles may be placed in the same equivalence class and may therefore be treated alike as being congruent.*

Although distances are no longer invariant under projection, collineation is preserved. [i.e. if three points lie on a line in one configuration, their transformed images will likewise be on a line; if three lines go through a single point, their images go also through a single point.]

Whereas Euclidean transformations may be expressed by polynomials of the first degree, projective transformation may be expressed as the ratio of polynomials of degree one.

In algebraic geometry *(which includes projective geometry as a subset) the class of transformations is widened to that of algebraic transformations. All such transformations also constitute a group. Thus we have extension upon extension in geometry, each extension comprising all the other lower in the hierarchy as special cases. This systematized categorization of the geometries in due to* ***Felix Klein*** *(1849–1925) and is known as his Erlangen Program.*

In topology, the allowed movements are continuous invertible deformations that might be called elastic motions. *We imagine that our figures are made of perfectly elastic rubber and, in moving the figure, we can stretch, twist, pull and bend it at pleasure. We are even allowed to* cut such a rubber figure and tie it in a knot, *provided that we later sew up the cut exactly as it was before, so that points which were close together before we cut the figure, are close together after the cut is sewed up. We are not allowed to force two different points to coalesce into just one point. Two figures are* topologically equivalent *iff one figure can be made to coincide with the other by such an elastic motion.*

The topological properties of a figure are those which are invariant under all such continuous transformation. *So, topology is not concerned with the issues of Euclidean geometry — the measurements of lengths, areas, volumes, angles, the making of scale drawings or enlargements. Moreover, topology is not limited to the rules of* projective geometry, *where straight lines may change in position but may never be distorted into curves, and circles may be transformed into ellipses and vice versa, but may not acquire humps or altogether arbitrary closed contours.*

Thus, topology might be described as the general study of continuity, *that concept whose various aspects have challenged philosophers and mathematicians from Pythagoras' day.*

One of the first topological observations is due to ***Descartes****, who as early as* 1640 *deduced an equation relating the numbers of vertices, edges and faces of simple polyhedra. This formula was rediscovered by* ***Euler*** *in 1752. [The typical character of this relation as a topological theorem, became apparent much later, after* ***Poincaré*** *has recognized 'Euler's formula' and its generalizations as one of the central theorems of topology.]*

Euler's name is linked to the subject through another problem which deserves to be considered as the beginning of topology: In 1736 *he presented a memoir to the St. Petersburg academy, in which he solved the problem of the Königsberg bridges:*

The river Pregelarme has two islands linked by a bridge. One island has one bridge crossing from it to each bank; the other has two bridges to each bank. Can the citizens of Königsberg cross all seven bridges (each being traversed only once) in a continuous walk? Euler showed that to be impossible[32]*, and he solved the most general problem of the same type.*

His method is based on the observation that it is the way the bridges connect, not their precise positions or sizes, that matters. It is evident that the question will not be affected if we suppose the islands to diminish to points and the bridges to lengthen out. In this way one ultimately obtains a geometrical figure of a network*. Euler's problem therefore consisted in finding whether a given network can be described by a point moving so as to traverse every line in it once and only once.*

Euler then proved the rule $V - E + F = 1$ *for planar networks in contradistinction to* $V - E + F = 2$ *for polyhedra. [This problem marks the origin of today's* graph theory *which in itself is part of* combinatorial topology*. It is applied to electrical networks, perturbative Feynman graphs, industrial management science, linear programming, game theory, statistical mechanics, social psychology and other behavioral sciences.]*

Not much happened in topology during the next 100 years[33]*. However, in the middle of the* 19^{th} *century, topology began to assert itself as a separate branch of geometry (known then as '*analysis situs*'), soon to become one of the main themes in modern mathematics. About 1850* ***Francis Guthrie*** *adduced a conjecture concerning the 4-color problem: that any map on a plane or on a sphere can be colored with at most 4 colors. The problem was later taken up by* ***Augustus de Morgan****,* ***Arthur Cayley*** *and others.*

One of the great geometers of the time was ***A.F. Möbius****, a man whose lack of self-assertion destined him to the career of an insignificant astronomer in a second-rate German observatory. Möbius probably did not think of himself as a topologist, because at that time there was no general subject called* topology*; nevertheless his ideas have had a profound influence on the development of the subject. At the age of* 68 *he submitted to the Paris Academy*

[32] In 1875, an eighth bridge was built. The addition of this bridge made it possible to solve the problem.

[33] **Gauss** made several contributions to topology. Of the several proofs that he furnished of the fundamental theorem of algebra, two are explicitly topological. His first proof of this theorem employs topological techniques and was given in his doctoral dissertation in 1799 when he was 22 years old. Later, Gauss briefly considered the theory of knots, which today is an important subject in topology. Although he added little beyond these few abstractions, much has been achieved by his students: **Möbius**, **Listing**, **Kirchhoff** and **Riemann**.

a memoir on 'one-sided' non-orientable surfaces that contained some of the most surprising facts of this new kind of geometry. Like other important contributions before it, his paper lay buried for years in the files of the Academy until it was eventually made public.

Independently of Möbius, the astronomer ***J.B. Listing*** *(one of Gauss's students) in Göttingen had made similar discoveries, and at the suggestion of Gauss, had published in* 1847 *a little book,* Vorstudien zur Topologie. *In this book, the first devoted to the subject, Listing introduced the term* topology. *The theory of Euler's networks is included as a particular case among the propositions proved by Listing in his book. [In* 1857, ***W.R. Hamilton****, using combinatorial analysis and group theory, solved some special problems in network theory.]*

G.R. Kirchhoff*, another of Gauss' students, employed (1847) the topology of linear graphs in his study of electrical networks.*

But of all of Gauss's students, the one who contributed by far the most to topology was ***Bernhard Riemann****, who, in his doctoral thesis of 1851, introduced topological concepts into the study of complex-function theory.*

When Bernhard Riemann came to Göttingen in 1847 *as a student, he found the mathematical atmosphere of that university town filled with keen interest in these strange new geometrical ideas. Soon he realized that therein was the key to the understanding of some deep properties of analytic functions of a complex variable. His was the first major contribution to* analysis situs *since Möbius, namely the so-called* Riemann surfaces *or* Riemann sheets *which enable to set up a one-to-two, or more, correspondence between the function* $w(z)$ *and its argument* z *[e.g. two* z*-sheets for* $w^2 = z$ *and an infinite number of* z*-sheets for* $w = \log z$*].*

He pictured the sheets as attached at certain special points ("branch points"), and also envisioned abstract "bridges" enabling passage from one sheet to another. In this way he established the homeomorphism *or* topological equivalence *of the* w*-plane to the many-sheeted* z*-plane (Riemannian surface[34]). Nothing, perhaps, has given more impetus to the later developments of topology than the structure of Riemann's theory of functions, in which topological concepts are absolutely fundamental.*

J.C. Maxwell *(1873) used the topological theory of connectivity in his study of electromagnetic fields. Others, such as* ***H. Helmholtz*** *and* ***Lord***

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[34] Such a surface is topologically equivalent to a sphere to which several *handles* have been added or, to put it another way, to a plane with several *holes*.

Kelvin, *can be added to the list of physicists who applied topological ideas with success*[35].

The next mathematician in chronological order, as far as combinatorial topology is concerned, was ***H. Poincaré***, *who developed many of his topological methods while studying ordinary differential equations which arise in the study of certain astronomy problems. Indeed he was led to topology through his efforts to solve the n-body problem for the case* $n = 3$. *This involves the determination of all-time orbits for sun, earth and moon, for example. [The 3-body problem involves the solution of a system of* 9 *differential equations.]*

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[35] **Helmholtz** (1858), building on the ideas of **Riemann** (1851, 1857), introduced topological considerations into hydrodynamic theory. He defined *vortex lines* as lines integrating the local directions of the axes of rotation of the fluid, and *vortex tubes* as bundles of vortex lines through infinitesimal elements of area. Helmholtz showed that the vortex tubes had to close up and also that the particles in a vortex tube at any given instant would remain in the tube indefinitely. So, no matter how much the tube was distorted, it would retain its topological shape. Helmholtz was aware of the topological ideas in his paper, particularly of the fact that the region *outside* a vortex tube was multiply connected, which led him to consider many-valued potential functions.

Tait (1867) verified Helmholtz's theoretical claims regarding two circular vortex rings via experiments with smoke rings. Curiously enough, Helmholtz's topology, driven by physical ideas of fluid-flow, impacted the Scottish mathematical physicists, **Kelvin** (Thomson), **Maxwell** and **Tait**, each in a different way: Kelvin concocted an elaborate theory according to which atoms, viewed as knotted vortex tubes in the ether, interact (chemically) as fluid vortices do. Tait and Maxwell wished to model the interaction of linked current circuits after the dynamics of vortex rings in a fluid, using the integral formula counting the linking number of two closed curves which **Gauss** had discovered (1833). Thus, these physicists became involved in topological concepts, in particular *knot theory*, because it entered their physical considerations in a natural way.

Of the three, **Maxwell** was ahead of his time by some 50 years. Although his approach lacked mathematical rigor, he defined (1868) the "*Reidemeister moves*" which, later (1920's), would be shown to be the fundamental moves in modifying. Moreover, Maxwell considered a region of 3D space bounded by one external surface of genus n, and m internal surfaces of genera $n_1, n_2, \ldots n_m$ and showed that the region possessed $N = n + n_1 + n_2 + \cdots + n_m$ cycles. Now in modern terminology, Maxwell was claiming that the first **Betti** number of the region was N. Again we should note that **Maxwell** did not give precise mathematical definitions of the concepts he was dealing with, so no rigorous proof was possible. It is reasonable to ask how, then, did he find the correct answer. The answer is that he reached his correct results using correct physical understanding, rather than mathematical intuition.

Poincaré showed that if two of the bodies have masses that are small compared to the third, periodic *solutions exist. In* 1912 *he proved that certain orbits could be periodic, provided that a simple geometric theorem, topological in nature, is true [the problem being to prove that when a certain topological transformation of the annular area between* 2 *concentric circles, is carried out,* 2 *of its points must remain fixed].*

It is hard to believe that a serious issue of dynamical astronomy can depend on such an apparently simple question that sound like an exercise in high school geometry.

Poincaré also put on a completely rigorous basis (1895) the concept of connectivity, *elaborated earlier by* ***Listing***, ***C. Jordan*** *and* ***Betti***. *He introduced the concepts of* homology *and* homotopy, *gave a more precise definition of the* Betti numbers, *and generalized Euler's convex polyhedra formula to* p*-dimensional space.*

At the same time, topology developed along another route through the generalization of the ideas of convergence. *This process began already in 1817 when* ***B. Bolzano*** *removed the association of convergence with a sequence of numbers and associated convergence with any bounded infinite set of real numbers.* ***Cantor*** *(1872) introduced the concept of a* set of limit points, *defined closed subsets of the real line as subsets containing their set of limit points, and introduced the idea of an* open set *— a fundamental concept in point set topology.*

Weierstrass *(1877) introduced the concept of a* neighborhood *of a point.* ***Hilbert*** *(1902) used this concept when he stated that a continuous transformation group is differentiable.* ***Frechet*** *(1906) extended the concept of convergence from Euclidean space by defining* metric spaces, *and showed that Cantor's ideas of open and closed subsets extended naturally to metric spaces.*

Riesz *(1909) disposed of the metric altogether, and proposed a new axiomatic approach to topology based on the definition of a set of limit points,* with no concept of distance. ***Hausdorff*** *(1914) followed suit by defining neighborhoods via four axioms devoid of metric considerations. These contributions allow the definition of* abstract topological spaces.

Topological concepts also entered mathematics via functional analysis, *pioneered by* ***Volterra*** *(1887). This topic arose from mathematical physics and astronomy because the methods of classical analysis were somewhat inadequate in tackling certain types of problems in the calculus of variations. Further advances in the theory of functionals were made by* ***Hadamard*** *(1903),* ***Frechet*** *(1904), and* ***Schmidt*** *(1907).*

However, the pioneers, like Poincaré, were forced to rely largely upon geometrical intuition. Recent work has brought topology within the framework *of rigorous mathematics, where intuition remains the source but not the final validation of truth. During this process, starting by* ***L.E.J. Brouwer*** *(1881–1966), the significance of topology has steadily increased, and the collection of methods developed by Poincaré was built into a complete topological theory.*

Today, topology, together with abstract algebra, is at the root of almost all of modern pure mathematics. In fact, topology has penetrated into other mathematical subjects and pervades current activity. The subject of topology itself consists of several different branches such as point-set topology, algebraic topology, differential topology, analytic topology *and* combinatorial topology[36]*. We know today that some very fundamental features of our physical reality are topological.*

1827 CE **Georg Simon Ohm** (1787–1854, Germany). Physicist. Born in Erlangen and educated at the university there. In 1817 he became a professor of mathematics in the Jesuits' college at Cologne. In a pamphlet published in Berlin in 1827 with the title "*Die galvanische kette mathematisch bearbeitet*" he stated that a current flowing in a wire is proportional at each point to the gradient of the potential $(E = RI)$. This he modeled after the flow of heat over a temperature gradient, mutatis mutandis. This phenomenological law, albeit approximate, had a most notable influence on the whole development of the theory and applications of dynamical electricity.

However, when the law was announced it seems too good to be true, and was not believed(!). Ohm was considered unreliable because of this, and was so badly treated that he resigned his professorship at Cologne and lived for several years in obscurity and poverty before it was recognized that he was right. So, in 1833, Ohm returned to become a professor at the polytechnic school in Nuremberg, and in 1852 he was appointed a professor of experimental physics at the University of Munich. He died soon thereafter, in 1854.

Peter Dirichlet was one of Ohm's pupils at Cologne.

1827–1828 CE **Robert Brown** (1773–1858, England). Scottish botanist. Discovered the erratic microscopic movement of small inorganic particles suspended in fluids. While investigating the pollen of several different plants, he

[36] In many cases a problem originally conceived as number-theoretic, algebraic, analytic, or geometric has eventually turned out to be *combinatorial*. Recent progress with electronic computers is playing an important role in the solution of various combinatorial problems arising in large systems, and the study of combinatorial topology is now quite active in terms of both theoretical development and applications.

observed that pollen dispersed in water in a great number of tiny particles exhibit uninterrupted irregular zig-zag motion. This phenomenon, which can also be observed in gases, is referred to as *Brownian motion.* Although it soon became clear that Brownian motion is an outward manifestation of the molecular motion postulated by the atomic theory of matter, it was not until 1905 that **Albert Einstein** first advanced a satisfactory theory.

Brown was one of England's greatest botanists. He is best known for his discovery of the nuclei of plant cells, as well as for classifying a large number of unfamiliar plants which he brought back from an Australian expedition in 1801–1805. In 1810 he became librarian to the Royal Society. Though offered a university chair he preferred to retain this job, where he had the use of valuable collections. In 1828 Brown wrote a pamphlet entitles "*A brief account of microscopical observations made in the months of June, July and August, 1827, on the particles contained in the pollen of plants and on the general existence of active molecules in organic and inorganic bodies*".

Brown was born at Montrose, a son of a Scottish Episcopalian clergyman. He studied medicine at the Universities of Aberdeen and Edinburgh and spent 5 years (1795–1800) in the British army as an assistant surgeon. He gained international reputation and was elected member in many learned foreign societies. He died in London.

The motions of microscopic particles in fluids were observed by biologists long before Brown, but were considered to be of *organic character*. [**John Turberville Needham** (1713–1781, in 1767); **Lazzaro Spallanzani** (1729–1799, in 1767) and others before 1800.] What Brown showed was that this phenomenon was not biological but *physical* in nature, thus removing the subject from the realm of biology to the realm of physics.

Brown had other claims to fame, and Brownian motion is not mentioned in his biography in the *Encyclopaedia Britannica*'s 9^{th} edition, 1878. In the 13^{th} edition of 1925, it merited a few words in passing.

1827–1861 CE **Anyos Istvan Jedlik** (1800–1895, Hungary). Physicist and inventor. Invented the first prototype of a *dynamo*.

Jedlik was born in Zemna (the Kingdom of Hungary, now Slovenia). Became a Catholic priest (1817). Lectured on physics (1839–1879) at the Budapest University of Science.

In 1827 he started experimenting with electromagnetic rotating devices. Discovered the principle of the *self-excited generator* (1856–1861), at least six years ahead of Ernst Werner von Siemens. His idea was that the performance of current generators[37] could be perfected by using the current produced by the machine to feed their magnets. In fact, he observed that the very slight remnant magnetization present in the iron core of the electromagnets was sufficient to start the process of induction.

This principle of self-excitation (i.e. the dynamo) was financially exploited a few years later by Siemens and Wheatstone, to whom Jedlik had not been known.

1828–1832 CE **Friedrich Wöhler** (1800–1882, Germany). Chemist. Synthesized the first artificial product which is created in nature within a living being[38], thus shattering the popular belief that some mysterious *vital* principle was at work in organic chemicals (1828).

Wöhler was born at Eschersheim, near Frankfurt-on-Main. He took his degree in medicine and surgery at Heidelberg in 1823, but was persuaded to devote himself to chemistry. He studied in Berzelius' laboratory at Stockholm. He later taught chemistry in Berlin (1825–1831), Cassel (1832–1836) and Göttingen (1836–1882), where he held the position of professor of chemistry in the medical faculty.

Wöhler maintained lifelong friendships with both **Berzelius** and **Liebig**. With the latter he carried out a number of joint researches.

1828–1835 CE **Lambert Adolph Jacques Quetelet** (1796–1874, Belgium). Statistician, astronomer and meteorologist. The father of statistics.

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[37] **Faraday**'s discovery (1831) of *electromagnetic induction* opened up the possibility of generating *electric currents* by the mechanical movement of a conductor in a magnetic field: The reversal of the process makes it possible to obtain *mechanical* work by the action of a magnetic field upon and electric current. Accordingly, we may classify electrical machines into:

- *Current generators*, by which mechanical work is transformed into electrical energy.
- *Motors*, by which electrical energy is transformed into mechanical work.

In 1867 **W. von Siemens** introduced the self-excited generator (dynamo) in which the magnetic field is established not by permanent magnets but by the generator itself. (It is the residual magnetization of the iron that makes it possible for a machine to excite itself once set running.)

[38] Prepared urea, $CO(NH_2)_2$, by evaporating the inorganic isomer ammonium cyanate NH_4OCN. People were astounded in his time because they thought that organic compounds can be produced only by living organisms.

Demonstrated the use of probability models in describing social and biological phenomena. Conducted statistical research on the development of the physical and intellectual qualities of man, formulating a theory of the "average man" as a basic type. First to use the *normal curve* other than as an error law.

Quetelet was born in Ghent. Became professor of mathematics at the University of Brussels (1819). Was the founder and director of the new Royal Observatory at Brussels. Conducted (1829) the first statistical breakdown of a national census, examining for the Belgian census the correlations of death with age, sex, occupation and economic status.

Quetelet studied briefly with **Laplace** and the latter's influence on him was unmistakable. He traveled through Europe in the ensuing years, spreading with fervor the statistical "gospel". (By 1870, "modern" mathematical statistics was poised and ready for its debut.)

His book *Sur l'homme et le développement de ses facultés, ou Essai de physique sociale* (1835) was the first attempt to apply mathematical analysis to the study of man — not only of his body but of his behaviour and morality, his mind and soul. In this respect he may be considered the first modern sociologist[39]. The rest of his life was devoted to the consolidation of his initial effort. He laid a great stress on the universal applicability of the binomial distribution.

The roots of Quetelet's thought must be sought outside the statistical literature. There are two main sources:

- The calculus of probabilities which originated (1654) in a correspondence between **Pascal** and **Fermat** and reached its climax in **Laplace**'s *Théorie analytique des probabilités* (1812). (Quetelet was introduced to him in Paris, in 1823.)
- The "*political arithmetic*" which was developed in England during 1662–1683 by **John Graunt** (1620–1674), **William Petty** (1623–1687) and **Edmund Halley** (1656–1742) who estimated mortality rates drawn from tables of births and funerals with an attempt to ascertain the price of annuities upon lives.

[39] **Auguste Comte** (1798–1857) was probably the first to speak of social physics (as early as 1822) and of sociology (1839). But Comte wrote on these matters with prolixity and conceit whereas Quetelet was not only saying what need be done, but actually doing it, and much better than Comte could imagine. For the real difficulties and crucial points only appear when one is tackling concrete problems.

1828–1837 CE **George Green** (1793–1841, England). One of the most brilliant and original mathematical physicists of the 19^{th} century; created a number of ideas that were far ahead of his time. Made major contributions to *potential theory.*

In 1828 he printed privately in Nottingham the tract "*An essay on the application of mathematical analysis to the theories of electricity and magnetism*". In this manuscript he presented a theorem, named after him, which concerns the relationship between a line integral over a simple closed curve in the plane and a double integral over a region bounded by the curve.

Green's theorem in the plane (1828): Arose in connection with gravitational and electric potential theory. States the following: Let D be a region in the xy-plane bounded by a simple closed curve C which consists of a finite number of smooth arcs. Then, if $P(x,y)$, $Q(x,y)$ are continuous functions with continuous first partial derivatives we have

$$\int\int_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dxdy = \oint_C (Pdx + Qdy),$$

where the circuit integral is taken in the *positive sense*: (a person making the circuit will always have the region D on his left). Green's theorem has the *vector form*

$$\int_C \boldsymbol{F} \cdot d\boldsymbol{s} = \int_D (\nabla \times \boldsymbol{F}) \cdot \boldsymbol{e}_z dA$$

where

$$\boldsymbol{F} = (P, Q), \qquad d\boldsymbol{s} = (dx, dy) = \text{tangent vector line element},$$

$\boldsymbol{e}_z$ is a unit vector normal to D, and $dA = dxdy$. If we choose $\boldsymbol{F} = (-Q, P)$, with $\boldsymbol{n}ds = (dy, -dx) =$ normal vector line element to the curve C, the above vector form changes into

$$\int_C \boldsymbol{F} \cdot \boldsymbol{n}ds = \int_D \operatorname{div} \boldsymbol{F} dA.$$

Green's theorem is very useful because it relates a line integral around a boundary of a region to an area integral over the interior of that region and in many cases it is easier to evaluate the line integral than the area integral, e.g. if we know that $P(x,y)$ *vanishes* on the boundary, we can conclude that $\int_D \frac{\partial P}{\partial y} dxdy = 0$ even though $\frac{\partial P}{\partial y}$ need not vanish in the interior.

The latter form of Green's theorem generalizes – for the case of three-dimensional curves, surfaces and vector fields – to *Stokes' theorem* (1850). It relates the line-integral of a vector-field around a simple curve C to an integral over a surface S for which C is a boundary. If $f(\boldsymbol{r})$ is a continuously differentiable vector function over a two-sided piecewise smooth oriented surface S, spanning the closed curve C, then

$$\int_S \operatorname{curl} \boldsymbol{f} \cdot d\boldsymbol{S} = \oint_C \boldsymbol{F} \cdot d\boldsymbol{s}, \quad d\boldsymbol{S} = \boldsymbol{n} dS, \quad d\boldsymbol{s} = \boldsymbol{t} ds,$$

where $\boldsymbol{n}$ is the positive normal to S and $\boldsymbol{t}$ is the tangent to C in the positive sense.

The theorem states that the *circulation* of a vector field around the contour of some surface is equal to the *flux* of the curl of the vector field through this surface. Otherwise stated: the integral of the *normal* component of the curl of a vector-field over a surface is equal to the integral of the *tangential* component of the same vector-field around the boundary of that surface.

In 1837, Green defined the *elastic strain-energy density*, derived the elastodynamic equation of motion in anisotropic media from the principle of virtual work, and correctly established the boundary conditions at the surface.

Green was primarily self-taught, and the tract was published with the aid of a patron who later helped him enter Cambridge as an undergraduate in 1833. The essay contained not only his theorem, but many other important results. Green graduated from Cambridge in 1837 and although he continued his research, none of his subsequent work had the depth or importance of his essay.

Green's essay went largely unnoticed until it was discovered by **Lord Kelvin** (1845), who arranged to have it printed. It is now regarded as one of the great classics of mathematical physics.

1828–1843 CE **William Rowan Hamilton** (1805–1865, Ireland). A great mathematician of the 19^{th} century, and Ireland's greatest claim to fame in the field of mathematics.

In 1828 he transformed the Lagrange 2^{nd} order equations of a dynamical system to a set of canonical first order equations, with twice as many variables, considering the position coordinates and the momenta as independent variables. These $2n$ first order differential equations are called *Hamilton's equations* for the system, and can replace those of Lagrange in giving the solution of a given problem.

Hamilton gave the first exact formulation of the *principle of least action*.

He also realized that problems in mechanics and geometrical optics can be tackled from a united viewpoint, where the *characteristic function* satisfies the same partial differential equation. He was first to grasp the concept of *group velocity* (1839).

With the rapid development of Newtonian dynamics and the geometric representation of complex numbers, the vector concept and its applications were coming of age. In 1843, Hamilton originated *quaternions*[40]and coined the words: *scalar*, *vector* and *tensor*. [His quaternions consist of four terms, three of which correspond to *vector* components and the fourth being a *scalar*. The term *tensor* was used by Hamilton to define the root of the sum of squares of the four elements of the quaternion. This definition has nothing whatsoever to do with the later use of this word.] During 1843–1850, Hamilton developed for the first time the underlying concepts of vector analysis (e.g. scalar and vector products, ∇ operator[41]) within the framework of his quaternion theory.

It is told of him that on the evening of Oct. 16, 1843, while walking along the Royal Canal in Dublin, the algebra of Quaternions dawned upon him and he carved on a stone on Brougham Bridge the formulas $i^2 = k^2 = j^2 = ijk = -1$.

Hamilton was born in Dublin of a branch of a Scottish family which had settled in the north of Ireland in the times of James I. He was early orphaned and his upbringing was entrusted to an uncle who gave the boy a strenuous but lopsided education with strong emphasis on languages. William proved to be a prodigy and when he reached the age of 13 he acquired, besides modern European languages, Persian, Hebrew, Arabic, Hindustani, Sanskrit and Malay. At 16 he mastered a great part of Newton's *Principia* and at 17 he read Laplace's *Mécanique céleste*. Hamilton's career at Trinity College, Dublin, was unexampled, for in 1828, when he was still a 23 year old graduate student, the university electors unanimously appointed him Royal Astronomer of Ireland and a professor of astronomy.

Two unhappy love affairs (he attempted to drown himself after the first one), a hypochondriac wife and alcoholism marred the personal life of this great Irish mathematician. Although it is thought by many that these difficulties lowered the quality of his mathematical thought, Hamilton's output continued unabated to the end of his life.

Hamilton was inspired by the necessity for appropriate mathematical tools to enable the application of Newtonian mechanics to various aspects of astronomy and physics. The theory of quaternions was too complicated in structure and was not able to survive in its original form as a tool of classical mathematical physics. However, quaternions can be used to represent complex variables, vectors and rotations in 3-space and 4-space. Moreover, the algebra of quaternions is of deep significance and its importance has been emphasized in recent

[40] The origin of the name comes from the New Testament *Acts 12*, 4.

[41] In his quaternion theory, he introduced (1853) the symbolic operator $\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$, which Heaviside later named "*nabla*".

decades by applications in group representation theory, quantum mechanics of spinors and special and general relativity.

Fifty years later, **Oliver Heaviside** said of Hamilton: "*Vector analysis without quaternions could have been found by any mathematician, but to find out quaternions required a genius*".

The highest tribute to Hamilton was perhaps paid by **Erwin Schrödinger**:
"*I dare say not a day passes — and seldom an hour — without somebody, somewhere on this globe, pronouncing or reading or writing or printing Hamilton's name. His famous analogy between mechanics and optics virtually anticipated wave mechanics, which did not have to add much to his ideas, only had to take them seriously — a little more seriously than he was able to take them, with the experimental knowledge of a century ago. The central conception of all modern theory in physics is 'the Hamiltonian'*".

The Principle of Least Action

Consider n particles of masses m_j, located at points $\boldsymbol{r}_j(t)$, and acted upon by resultant external and internal forces $\boldsymbol{F}_j$. By d'Alembert's principle, we write $\sum_{j=1}^{n}(m_j\ddot{\boldsymbol{r}}_j - \boldsymbol{F}_j)\cdot\delta\boldsymbol{r}_j = 0$ for arbitrary variations $\delta\boldsymbol{r}_j$. Call $\delta W = \sum_{j=1}^{n}\boldsymbol{F}_j\cdot\delta\boldsymbol{r}_j$ the virtual work done by all the forces. We have $\frac{d}{dt}(\dot{\boldsymbol{r}}_j\cdot\delta\boldsymbol{r}_j) \equiv \dot{\boldsymbol{r}}_j\cdot\frac{d}{dt}(\delta\boldsymbol{r}_j) + \ddot{\boldsymbol{r}}_j\cdot\delta\boldsymbol{r}_j$, which together with $\frac{d}{dt}(\delta\boldsymbol{r}_j) = \delta\dot{\boldsymbol{r}}_j$ permits us to conclude that for the j^{th} particle

$$m_j\ddot{\boldsymbol{r}}_j\cdot\delta\boldsymbol{r}_j = \frac{d}{dt}(m_j\dot{\boldsymbol{r}}_j\cdot\delta\boldsymbol{r}_j) - \delta T_j,$$

where T_j = kinetic energy of j^{th} particle $= \frac{1}{2}m_j\dot{\boldsymbol{r}}_j^2 = \frac{1}{2}m_jv_j^2$. Summing over all particles, we finally have

$$\frac{d}{dt}\sum_{j=1}^{n}m_j\dot{\boldsymbol{r}}_j\cdot\delta\boldsymbol{r}_j = \delta T + \delta W.$$

Consider two times t_0 and t_1 at which we assume $\delta\boldsymbol{r}_j = 0$, i.e., two times at which actual and virtual paths coincide. Integrating the last equation between

t_0 and t_1, we have $\sum_{j=1}^{n} m_j \dot{\boldsymbol{r}}_j \cdot \delta \boldsymbol{r}_j |_{t_0}^{t_1} = \int_{t_0}^{t_1} (\delta T + \delta W) dt$. *But the l.h.s. is zero by virtue of our restriction on* $\delta \boldsymbol{r}_j$; *hence* $\int_{t_0}^{t_1} (\delta T + \delta W) dt = 0$. *If we assume that W is a work function arising from a potential energy such that* $W = -V(\boldsymbol{r_1}, \ldots, \boldsymbol{r_n})$, *then* $\delta W = -\delta V$ *leads to*

$$\delta \int_{t_0}^{t_1} (T - V) dt = \delta \int_{t_0}^{t_1} L dt = 0,$$

which is Hamilton's principle for conservative dynamical systems. *L is known as the Lagrangian function of the system.*[42]

We may enunciate the principle as follows: a system moves from one configuration to another in such a way that the variation of the integral $\int_{t_0}^{t_1} L dt$ *between the actual path taken and a neighboring virtual path, co-terminous in space and time with the actual path, is zero. In other words,* $\int_{t_0}^{t_1} L dt$, *called the action – a functional of the virtual trajectory in the system's configuration space — is stationary at the actual trajectory, if all virtual trajectories are constrained to evolve between the same two spatial points at the end-times t_0 and t_1.*

This principle is equivalent to the Lagrange equations. Indeed: for a single particle in one dimension x $\left(\dot{x} = \frac{dx}{dt}\right)$,

$$\begin{aligned}
\delta \int_{t_0}^{t_1} L\left(x, \frac{dx}{dt}; t\right) dt &= \int_{t_0}^{t_1} \left\{ \frac{\partial L}{\partial x} \delta x + \frac{\partial L}{\partial \dot{x}} \delta(\dot{x}) \right\} dt \\
&= \int_{t_0}^{t_1} \left\{ \frac{\partial L}{\partial x} \delta x + \frac{\partial L}{\partial \dot{x}} \frac{d}{dt} (\delta x) \right\} dt \\
&= \int_{t_0}^{t_1} \left\{ \frac{\partial L}{\partial x} \delta x - \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) \delta x + \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \delta x \right) \right\} dt \\
&= \int_{t_0}^{t} \left\{ \frac{\partial L}{\partial x} - \frac{d}{dt} \frac{\partial L}{\partial \dot{x}} \right\} \delta x \delta t + \left[\frac{\partial L}{\partial \dot{x}} \delta x \right]_{t_0}^{t_1}
\end{aligned}$$

The last term vanishes (δx being zero at t_0 and t_1), and, since δx may otherwise vary arbitrarily in the time interval (t_1, t_2), we deduce the Euler-Lagrange equation:

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0$$

as a necessary and sufficient condition for the functional $\int_{t_0}^{t_1} L dt$ *to be stationary at the trajectory* $x(t)$, $t_0 \leq t \leq t_1$. *When there are several dependent*

[42] For further reading, see:

- *The Feynman Lectures on Physics*, Volume II, Addison-Wesley, 1964.

variables (i.e. the physical system is multi-dimensional or has multiple degrees of freedom for another reason), there results a system of (generally coupled) Euler-Lagrange equations, one for each dependent variable. Note that the Lagrangian of a closed system does not depend explicitly on time.

Denoting the system's *generalized coordinates* by q_i, $i = 1, \dots, n$, the Lagrangian is a function of $q_i(t)$, their first time derivatives, and (possibly) also depends explicitly on time. Thus, the total differential of L is

$$dL = \sum_i \frac{\partial L}{\partial q_i} dq_i + \sum_i \frac{\partial L}{\partial \dot{q}_i} d\dot{q}_i + \frac{\partial L}{\partial t} dt = \sum_i \dot{p}_i dq_i + \sum p_i d\dot{q}_i + \frac{\partial L}{\partial t} dt$$

with $\frac{\partial L}{\partial \dot{q}_i} = p_i$ by definition (generalized canonical momenta) and $\frac{\partial L}{\partial q_i} = \dot{p}_i$ by the Euler-Lagrange equations. The above equation then yields

$$d\left[\sum_i p_i \dot{q}_i - L\right] = -\sum_i \dot{p}_i dq_i + \sum_i \dot{q}_i dp_i - \frac{\partial L}{\partial t} dt.$$

The argument of the differential is called the *Hamiltonian* of the system: $H(p, q; t) = \sum_i p_i \dot{q}_i - L$, and $dH = -\sum_i \dot{p}_i dq_i + \sum_i \dot{q}_i dp_i - \frac{\partial L}{\partial t} dt$, with the first two sums numerically canceling each other since $dq_i = \dot{q}_i dt$, $dp_i = \dot{p}_i dt$. Note that it is tacitly assumed here that the relations $\dot{p}_i = \frac{\partial L(q, \dot{q})}{\partial q_i}$ can be uniquely inverted to yield $\dot{q}_i = \dot{q}_i(p, p)$[43]. From this follows that[44] $-\frac{\partial L}{\partial t} = \frac{\partial H}{\partial t}$ and:

$$\dot{q}_i = \frac{\partial H}{\partial p_i}, \qquad \dot{p}_i = -\frac{\partial H}{\partial q_i},$$

which are known as *Hamilton's equations* (1835). These constitute a system of first order ODE's in (q_i, p_i). They represent the simplest and most desirable form into which the differential equations of the variational problem can be brought. Hence the name *canonical equations* by which **Jacobi** designated them.

The total time derivative of the Hamiltonian is

$$\frac{dH}{dt} = \frac{\partial H}{\partial t} + \sum \frac{\partial H}{\partial q_i} \dot{q}_i + \sum \frac{\partial H}{\partial p_i} \dot{p}_i = \frac{\partial H}{\partial t},$$

---

[43] The cases where the inversion is *not* unique — termed *constrained dynamical system* — encompass some of the most important applications of the least-action principle to quantum field theory, including *non-abelian gauge theories*, *quantum gravity*, and *string theory*.

[44] $\frac{\partial H}{\partial t}$ is $\frac{\partial}{\partial t} H(p, q; t)$ with $\{p_j, q_j\}_{j=1}^n$ held fixed, while $\frac{\partial L}{\partial t}$ means $\frac{\partial}{\partial t} L(q, \dot{q}; t)$ with $\{\dot{q}_j, q_j\}_{j=1}^n$ held fixed.

on account of Hamilton's equations. For a conservative closed system, neither $L(q, \dot{q})$ nor $H(p, q)$ depend explicitly upon time and the Hamiltonian – equal to $T + V$ – is, numerically, the conserved system energy.

The Hamilton-Jacobi equation (1828–1837) is the most important first-order PDE *that occurs in mathematical physics. It is derived as follows: the action integral $S = \int_{t_1}^{t_2} L dt$ is taken along a path between two given positions which the system occupies at given instants t_1 and t_2. In varying the action, we compare the values of this integral for neighboring paths sharing the same values of the vectors $q^{(1)} = q(t_1)$ and $q^{(2)} = q(t_2)$ and find that generally only one of these paths corresponds to the actual motion, namely the path for which the integral has its minimum (or, more generally, extremum) value.*

Consider now the action integral S on the true path as a function of the value of the vector $q(t_2)$ at the upper limit of integration t_2 $[q(t_1) = q^{(1)}$ is held fixed]. In general

$$\delta S = \left[\sum_{i=1}^{n} \frac{\partial L}{\partial \dot{q}_i} \delta q_i \right]_{t_1}^{t_2} + \int_{t_1}^{t_2} \sum_{i=1}^{n} \left(\frac{\partial L}{\partial q_i} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} \right) \delta q_i dt,$$

but since paths of actual motion satisfy Lagrange's equations, the integral in δS is zero. Since $\delta q(t_1) = 0$ and $\frac{\partial L}{\partial \dot{q}_i} = p_i$, we have for $\delta S = \sum_i p_i \delta q_i$, where we have denoted $\delta q(t_2)$ by δq. From this relation it follows that $\frac{\partial S}{\partial q_i} = p_i$.

Now the action may similarly be regarded as an explicit function of time (even for a closed system) by considering paths starting at a given instant t_1 and at a given point $q^{(1)}$ and ending at a given point $q^{(2)}$ at various times $t_2 = t$. Clearly if $q^{(2)} = q(t_2)$ is allowed to evolve with $t = t_2$ in accordance with actual motion,

$$\frac{dS}{dt} \equiv L = \frac{\partial S}{\partial t} + \sum_i \frac{\partial S}{\partial q_i} \dot{q}_i = \frac{\partial S}{\partial t} + \sum_i p_i \dot{q}_i.$$

Hence $\frac{\partial S}{\partial t} = L - \sum p_i \dot{q}_i$ or $\frac{\partial S}{\partial t} = -H$. Combining this result with $\frac{\partial S}{\partial q_i} = p_i$ we have

$$dS = \sum_i p_i dq_i - H dt$$

for the total differential of the action as a function of independently-varied coordinates and time at the upper limit of integration.

The action $S(q, t)$ thus obeys the equation

$$\frac{\partial S}{\partial t} + H(p, q, t) = 0.$$

Accordingly, replacing the generalized momenta p_i in the Hamiltonian by the partial derivatives $\frac{\partial S}{\partial q_i}$, we have the equation which must be obeyed by the function $S(q,t)$. This first order nonlinear PDE *is called the Hamilton-Jacobi equation:*

$$\frac{\partial S}{\partial t} + H\left(q_1, \dots, q_n, \frac{\partial S}{\partial q_1}, \dots, \frac{\partial S}{\partial q_n}; t\right) = 0.$$

Like Lagrange's equations and the canonical equations, the Hamilton-Jacobi equation is the basis of a general method of integrating the equations of motion.

If the system is closed and conservative, H does not depend upon the time explicitly and its numerical value is time-independent along actual paths. Thus, the integration of $\frac{\partial S}{\partial t} = -H$ yields $S = S_0(q) - Et$, where E is the conserved total system energy and also equals the constant value of H. The Hamilton-Jacobi equation then assumes the somewhat simpler from[45]

$$H\left(q_1, \dots, q_n; \frac{\partial S_0}{\partial q_1}, \dots, \frac{\partial S_0}{\partial q_n}\right) = E.$$

A particular solution to the latter energy equation can be obtained for the motion of a point particle in a field of potential energy V in rectangular coordinates.

The energy-equation then takes the form

$$\frac{1}{2m}(p_1^2 + p_2^2 + p_3^2) + V(x,y,z) = E,$$

or, since $p_i = \frac{\partial S}{\partial q_i} = \frac{\partial S_0}{\partial q_i}$,

$$(\nabla S_0)^2 = 2m(E - V),$$

i.e.

$$\left(\frac{\partial S_0}{\partial x}\right)^2 + \left(\frac{\partial S_0}{\partial y}\right)^2 + \left(\frac{\partial S_0}{\partial z}\right)^2 = 2m(E - V).$$

Given constants m, E and the function V, it is then required to find at any given field point $P(x,y,z)$ a surface $S_0(x,y,z) = const.$, such that the modulus of the normal ∇S_0 is the scalar function $\sqrt{2m(E-V)}$; the Hamilton-Jacobi theory then guarantees that the geometric path of the moving point-mass is normal to this family of surfaces at every point. We obtain a family of possible paths by constructing the orthogonal trajectories to the

[45] This simplification procedure is equivalent to applying the *separation of variables* technique to the full Hamilton-Jacobi PDE.

surface $S_0 = const.$ *These mechanical paths have the ray property, because they behave exactly like light rays in optics, the latter being orthogonal to the wave surfaces*[46].

The mechanical-optical analogy that follows from the Hamilton-Jacobi equations draws parallels between Fermat's principle of least time and the Hamilton principle of least action; between surfaces of equal time in optics to surfaces of equal action in mechanics. Moreover, the basic differential equation of geometrical optics:

$$\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2 + \left(\frac{\partial\phi}{\partial z}\right)^2 = \frac{n^2}{c^2}$$

$n =$ *inhomogeneous refractive index,* $c =$ *light speed in vacuum,*

$\phi =$ *wavefront-phase* $= \omega T,$ $\omega =$ *monochromatic ray angular frequency,*

$T =$ *ray travel time;* *Wavefronts are perpendicular to the rays,*

which expresses Huygens' principle in infinitesimal form (known as the eikonal equation), has the same form as the Hamilton-Jacobi equation with the correspondence

$$\phi = \alpha S_0, \qquad \frac{n}{c} = \alpha\sqrt{2m(E-V)},$$

α *being an arbitrary constant.*

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[46] This orthogonality does not always involve orthogonality in the ordinary Euclidean sense, although it does in our above point-mass mechanical example; e.g. an electron moving in a magnetic field, does *not* cross the surfaces $S_0 =$ const. perpendicularly; nor do light rays in crystals, in general. This is because $p_i = \frac{\partial S}{\partial q_i}$ is not always parallel to the vector $\dot{q}_i$

Non-commutative systems: Quaternions and Polyadics

The efforts of ***Gauss*** *(1819) to build a 3-dimensional complex-number system within the framework of common algebra have failed. The isomorphism of complex numbers and two-dimensional vectors in a plane prompted* ***Hamilton*** *(1843) to extend 3-dimensional vector algebra to include both multiplication and division. But he soon noticed that if one tries to define vector-division by seeking a vector* $\boldsymbol{C}$ *such that* $\boldsymbol{B}\times\boldsymbol{C}=\boldsymbol{A}$ *(or* $\boldsymbol{C}\times\boldsymbol{B}=\boldsymbol{A}$*) for two* given *vectors* $\boldsymbol{A}$ *and* $\boldsymbol{B}$*, then this operation is well-defined only when* $\boldsymbol{A}\cdot\boldsymbol{B}=0$*. It is however* non-unique *on account of the identity*

$$\boldsymbol{B}\times\boldsymbol{C}=\boldsymbol{B}\times(\boldsymbol{C}-\lambda\boldsymbol{B}).$$

Thus, Hamilton was led to invent a new division algebra for quadruples *of numbers (an analogue of 2-D complex numbers) at the price of* relinquishing the commutative law of multiplication.

Hamilton considered a 4-dimensional vector-space with abstract unit base elements $\{e_0,e_1,e_2,e_3\}$*. A general vector in this space, known a* quaternion *("four-fold" numbers) is written in the form*

$$q=q_0e_0+(q_1e_1+q_2e_2+q_3e_3)=q_0e_0+\boldsymbol{q}.$$

Quaternions obey the rules of common algebra w.r.t. addition and multiplication by a scalar. The number q_0 *is the* scalar *of the quaternion and* $\boldsymbol{q}$ *is its* vector.

Multiplication is defined by the table:

	e_0	e_1	e_2	e_3
e_0	e_0	e_1	e_2	e_3
e_1	e_1	$-e_0$	e_3	$-e_2$
e_2	e_2	$-e_3$	$-e_0$	e_1
e_3	e_3	e_2	$-e_1$	$-e_0$

Clearly, the product of two unit quaternions with different indices is in general non-commutative, since $e_re_s=-e_se_r$ $(r,s=1,2,3,\ r\neq s)$*. Using the table and the distributive law, one verifies that the product of two general quaternions is*

$$pq=(p_0q_0-\boldsymbol{p}\cdot\boldsymbol{q})e_0+p_0\boldsymbol{q}+q_0\boldsymbol{p}+(\boldsymbol{p}\times\boldsymbol{q})\neq qp$$

where

$$\boldsymbol{p}\cdot\boldsymbol{q} = p_1q_1 + p_2q_2 + p_3q_3$$
$$\boldsymbol{p}\times\boldsymbol{q} = (p_2q_3 - p_3q_2)e_1 + (p_3q_1 - p_1q_3)e_2 + (p_1q_2 - p_2q_1)e_3.$$

The multiplication table shows that $e_0^2 = e_0$ *and* $e_0e_r = e_re_0$*; therefore we may choose* $e_0 = 1$*. For* $e_0 = 1$, $\{\pm e_1, \pm e_2, \pm e_3\}$ *are the six square roots of* -1*. Two limiting cases are of interest:*

- $p_2 = q_2 = p_3 = q_3 = 0,\quad e_0 = 1,\quad e_1 = \sqrt{-1},\quad p = p_0 + ip_1,$ $q = q_0 + iq_1$*. Quaternions reduce to ordinary 2-D complex numbers with the corresponding algebra.*
- $p_0 = q_0 = 0$*:* $pq = -(\boldsymbol{p}\cdot\boldsymbol{q})e_0 + (\boldsymbol{p}\times\boldsymbol{q})$*. Neglecting the scalar piece, one may view the elements* $(\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3)$ *as unit vectors in 3D. Then* pq *is the ordinary vector cross product which is covariant under coordinate rotation.*

Just as a complex number can be viewed as an ordered pair of numbers, a general quaternion can be viewed as an ordered pair of complex numbers. Indeed, since $e_1 = e_2e_3$*, we may write, identifying* $e_0 = 1$ *and* $e_3 = i$*:*

$$q = (q_0 + q_3e_3) + (q_2 - q_1e_3)e_2 = x + ye_2 \Rightarrow (x, y),$$

with $x = q_0 + iq_3,\ y = q_2 - iq_1$*. Addition and multiplication are then defined as*

$$(x, y) + (u, v) = (x + u, y + v)$$
$$(x, y)(u, v) = (xu - yv^*, xv + yu^*) \qquad {}^* = \text{complex conjugation}$$

In analogy to the algebra of complex numbers, one defines the quaternion conjugate

$$q^t = (x^*, -y) = q_0 - q_1e_1 - q_2e_2 - q_3e_3,$$

the square of the norm

$$\|q\|^2 = qq^t = q^tq = q_0^2 + q_1^2 + q_2^2 + q_3^2,$$

and the inverse

$$q^{-1} = \frac{q^t}{\|q\|^2}.$$

One then finds a one-to-one correspondence between the complex matrices $(e_1 \Rightarrow i)$

$$q \rightarrow \begin{bmatrix} x & y \\ -y^* & x^* \end{bmatrix} = \begin{bmatrix} q_0 + q_3i & q_2 - q_1i \\ -q_2 - q_1i & q_0 - q_3i \end{bmatrix}$$

and the quaternions $q_0 + q_1e_1 + q_2e_2 + q_3e_3$, *which preserves multiplication.*

So far we have not specified the nature of the abstract base elements $\{e_0, e_1, e_2, e_3\}$, *except for some special cases. But, although obeying the same multiplication table, different representations of these elements may exist. To stress this important point we first recast the above* 2×2 *matrix representation of* q *in the form*

$$q \to q_0 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + q_1 \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} + q_2 \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + q_3 \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$$

$$= q_0E + q_1I + q_2J + q_3K.$$

We then 'discover' that the 2×2 *matrices* $\{E, I, J, K\}$ *have the same properties as the basis quaternions* $\{e_0, e_1, e_2, e_3\}$. *In fact*

$$E^2 = E, \quad I^2 = J^2 = K^2 = -E$$

$$IJ = K = -JI; \; KI = J = -IK; \; JK = I = -KJ.$$

The 2×2 *matrix representations of the basis quaternions are intimately connected with the Pauli spin matrices (1925)*

$$\sigma_1 = iI; \quad \sigma_2 = -iJ; \quad \sigma_3 = -iK$$

which were found to be of central significance in quantum mechanics!

It is easy to show that $\{e_0, e_1, e_2, e_3\}$ *have yet another representation, as* 4×4 *matrices. To see this we begin with*

$$q \to \begin{bmatrix} x & y \\ -y^* & x^* \end{bmatrix}$$

and then represent each of the four complex numbers by its own matrix representation with 1 *and* i *represented as* $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ *and* $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, *respectively. This leads to a* 4×4 *real matrix for each quaternion*

$$q = q_0e_0 + q_1e_1 + q_2e_2 + q_3e_3 \to \begin{bmatrix} q_0 & q_3 & q_2 & -q_1 \\ -q_3 & q_0 & q_1 & q_2 \\ -q_2 & -q_1 & q_0 & -q_3 \\ q_1 & -q_2 & q_3 & q_0 \end{bmatrix},$$

such that the one-to-one correspondence again preserves multiplication.

For example:

$$(2 - 4e_1 + e_2 + 3e_3)(1 + 2e_1 + 3e_2 - e_3) = 10 - 10e_1 + 9e_2 + 13e_3$$

and

$$\begin{bmatrix} 2 & 3 & 1 & 4 \\ -3 & 2 & -4 & 1 \\ -1 & 4 & 2 & -3 \\ -4 & -1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 & 3 & -2 \\ 1 & 1 & 2 & 3 \\ -3 & -2 & 1 & 1 \\ 2 & -3 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 10 & -13 & 9 & 10 \\ 13 & 10 & -10 & 9 \\ -9 & 10 & 10 & 13 \\ -10 & -9 & -13 & 10 \end{bmatrix}$$

Again, if we write $q = q_0E_4 + q_1I_4 + q_2J_4 + q_3K_4$, *where*

$$E_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad K_4 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$J_4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, \qquad I_4 = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

we find that the 4×4 *basis matrices* E_4, I_4, J_4, K_4 *obey the same algebra as* $\{e_0, e_1, e_2, e_3\}$.

Quaternions and finite rotations

Suppose that a rigid body is first rotated by a certain angle ϕ *in a given sense around the axis* OA *passing through a given point* O, *and that it is then rotated by an angle* ϕ_1 *around another axis* OB *passing through the same point. The question is: Around what axis and by what angle must the body be rotated in order to bring it from its first position directly to the third? This is the well-known problem of addition of finite rotations. True, it can be solved by means of the ordinary analytic geometry, as was done already by* ***Euler*** *in the 18th century. However, its solution assumes a far more lucid form by means of quaternions.*

Define

$$\boldsymbol{n} = \frac{1}{h}(q_1e_1 + q_2e_2 + q_3e_3), \quad \boldsymbol{n}^2 = -e_0,$$

$$N = \sqrt{q_0^2 + q_1^2 + q_2^2 + q_3^2}, \quad h = \sqrt{q_1^2 + q_2^2 + q_3^2} = N \sin\frac{\phi}{2},$$

$$q_0 = N\cos\frac{\phi}{2}$$

Under this definition, every quaternion is reduced to the standard form

$$q = N(e_0 \cos\frac{\phi}{2} + \boldsymbol{n}\sin\frac{\phi}{2}); \quad q^{-1} = \frac{1}{N}(e_0 \cos\frac{\phi}{2} - \boldsymbol{n}\sin\frac{\phi}{2})$$

It then follows that with $\boldsymbol{n} = x\boldsymbol{e}_1 + y\boldsymbol{e}_2 + z\boldsymbol{e}_3$

$$\begin{aligned}\boldsymbol{r}' &= q\boldsymbol{r}q^{-1} = \boldsymbol{r}\cos\phi + (1-\cos\phi)\boldsymbol{n}(\boldsymbol{n}\cdot\boldsymbol{r}) + \sin\phi(\boldsymbol{n}\times\boldsymbol{r}) \\ &= [I\cos\phi + (1-\cos\phi)\boldsymbol{n}\boldsymbol{n} + \sin\phi(\boldsymbol{n}\times I)]\cdot\boldsymbol{r} = \mathfrak{R}\cdot\boldsymbol{r},\end{aligned}$$

where $\mathfrak{R}$ *describes an active rotation of space about an axis given by the vector* $\boldsymbol{n}$, *relative to the fixed axes* $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3\}$, *by an angle* $\phi = 2\tan^{-1}\{\frac{1}{q_0}\sqrt{q_1^2 + q_2^2 + q_3^2}\}$.

Applying a second rotation represented by a quaternion p, *the combined action is given by the expression*

$$p(q\boldsymbol{r}q^{-1})p^{-1} = (pq)r(pq)^{-1}$$

since $q^{-1}p^{-1} = (pq)^{-1}$ *by the* associative *law of multiplication. This means that the result of two successive rotations, characterized by the quaternions* q *and* p, *is the rotation characterized by the product quaternion* pq. *In other words the* addition *(or more precisely, the* composition*) of the rotations corresponds the* product *of the respective quaternions.*

The Pauli spin matrices, *mentioned earlier, tie in with the subject of finite rotation in the following way: we adopt the representation (*I *is the* 2×2 *unit matrix)*

$$e_0 = I, \quad e_1 = -i\sigma_1, \quad e_2 = -i\sigma_2, \quad e_3 = -i\sigma_3, \quad \boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$$

which leads us to the unit quaternion

$$q(\boldsymbol{n}, \phi) = I\cos\frac{\phi}{2} - i\sin\frac{\phi}{2}(\boldsymbol{\sigma}\cdot\boldsymbol{n}),$$

where

$$\boldsymbol{\sigma}\cdot\boldsymbol{n} = \begin{bmatrix} n_3 & n_1 - in_2 \\ n_1 + in_2 & -n_3 \end{bmatrix}.$$

The matrices σ_k *are* Hermitian *(transpose = complex conjugate) and* traceless. *Moreover, they obey appropriate laws of multiplication.*

Since any such rotation can be decomposed into 3 successive rotations with Euler angles (α, β, γ) *about the respective* fixed space axes $\{\boldsymbol{e}_z, \boldsymbol{e}_y, \boldsymbol{e}_z\}$, *we find that*

$$
\begin{aligned}
q(\boldsymbol{n},\phi) &= q(\boldsymbol{e}_z,\alpha)q(\boldsymbol{e}_y,\beta)q(\boldsymbol{e}_z,\gamma) \\
&= \begin{bmatrix} e^{-i\frac{\alpha}{2}} & 0 \\ 0 & e^{+i\frac{\alpha}{2}} \end{bmatrix} \begin{bmatrix} \cos\frac{\beta}{2} & -\sin\frac{\beta}{2} \\ \sin\frac{\beta}{2} & \cos\frac{\beta}{2} \end{bmatrix} \begin{bmatrix} e^{-i\frac{\gamma}{2}} & 0 \\ 0 & e^{+i\frac{\gamma}{2}} \end{bmatrix} \\
&= \begin{bmatrix} \cos\frac{\beta}{2}e^{-\frac{i}{2}(\gamma+\alpha)} & -\sin\frac{\beta}{2}e^{\frac{i}{2}(\gamma-\alpha)} \\ \sin\frac{\beta}{2}e^{-\frac{i}{2}(\gamma-\alpha)} & \cos\frac{\beta}{2}e^{\frac{i}{2}(\gamma+\alpha)} \end{bmatrix} = \begin{bmatrix} q_0 - iq_3 & -(q_2+iq_1) \\ q_2 - iq_1 & q_0 + iq_3 \end{bmatrix}
\end{aligned}
$$

which is a unimodular unitary matrix. If we shift ϕ to $\phi + 2\pi$ (or by any odd multiple of 2π), $\Re(\boldsymbol{n},\phi)$ remains the same while $q(\boldsymbol{q},\phi)$ changes its sign. Thus, both $\pm q$ represent the same rotation and the correspondence between unimodular quaternions and 3D rotations is indeed established, though it is multi-valued.

Hamilton introduced the vector quaternion differential operator ($e_0 - 1$, $e_1 = i$, $e_2 = j$, $e_3 = k$)

$$\boldsymbol{\nabla} = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

(known today as the gradient operator*). He then showed that when ∇ operates on the vector quaternion $\boldsymbol{v} = v_1 i + v_2 j + v_3 k$, their formal "product" yields the vector quaternion*

$$\nabla\boldsymbol{v} = -div\ \boldsymbol{v} + curl\ \boldsymbol{v}.$$

Thus, $\nabla\boldsymbol{v}$ is a quaternion with a scalar $\{-div\ \boldsymbol{v}\}$ and a vector curl $\boldsymbol{v}$, where

$$div\ \boldsymbol{v} = \frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z};$$

$$curl\ \boldsymbol{v} = (\frac{\partial v_3}{\partial y} - \frac{\partial v_2}{\partial z})i + (\frac{\partial v_1}{\partial z} - \frac{\partial v_3}{\partial x})j + (\frac{\partial v_2}{\partial x} - \frac{\partial v_1}{\partial y})k.$$

Quaternions form a 4 dimensional non-commutative associative algebra over the reals (in fact a division algebra*) and contain complex numbers, but do not form an algebra over the complex numbers. The quaternions, along with the complex numbers and real numbers, are the* only *finite dimensional skew fields over the field of real numbers.*

Hermann Grassmann[47] *(1844) [and independently* ***Saint-Venant*** *(1832)*[48]*] invented the noncommutative algebra of polyadics in n-dimensional*

[47] Grassman, H.D., *Die Linear Ausdehnungslehre*, Leipzig, 1844.

[48] In 1832 the French engineer **Adhémar, Comte de Saint-Venant** (1797–1866) exposed mathematical ideas similar to those which are present in the Grassmanian system. Among other things he defined the dyadic product of two vectors.

Euclidean space. Grassmann's work remained neglected until its resurrection by ***James Clerk Maxwell*** *(1871),* ***Josiah Willard Gibbs*** *(1881) and* ***Oliver Heaviside*** *(1893) who built upon its foundation the modern algebra and analysis of vectors and dyadics in 3-dimensional Euclidean space.*

Grassmann's ideas in Gibbs' notation are as follows: Let $\boldsymbol{a} = a_1\boldsymbol{e}_1 + a_2\boldsymbol{e}_2 + a_3\boldsymbol{e}_3$; $\boldsymbol{b} = b_1\boldsymbol{e}_1 + b_2\boldsymbol{e}_2 + b_3\boldsymbol{e}_3$ *represent two vectors with the respective components* $\{a_1, a_2, a_3\}$ *and* $\{b_1, b_2, b_3\}$ *in an orthogonal Cartesian coordinate system with unit vectors* $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3\}$. *One then defines three types of products between the two vectors:*

- *The scalar (inner) product* $\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{b} \cdot \boldsymbol{a} = a_1b_1 + a_2b_2 + a_3b_3$
- *The vector (outer) product* $(\boldsymbol{a} \times \boldsymbol{b}) = -(\boldsymbol{b} \times \boldsymbol{a}) = \lambda_1\boldsymbol{e}_1 + \lambda_2\boldsymbol{e}_2 + \lambda_3\boldsymbol{e}_3$

$$\lambda_1 = a_2b_3 - a_3b_2;\ \lambda_2 = a_3b_1 - a_1b_3;\ \lambda_3 = a_1b_2 - a_2b_1$$

$$(\boldsymbol{e}_i \cdot \boldsymbol{e}_j) = \delta_{ij}; \qquad (\boldsymbol{e}_i \times \boldsymbol{e}_j) = \sum_k \boldsymbol{e}_k \epsilon_{ijk},$$

  with ϵ_{ijk} *the totally antisymmetric Levi-Civita symbol.*

- *The dyadic (indeterminate) product*

$$\boldsymbol{ab} = \sum_{i,j} a_i b_j \boldsymbol{e}_i \boldsymbol{e}_j \neq \boldsymbol{ba} \qquad i, j = 1, 2, 3$$

  The dyadic product is a tensor of the second rank with 9 components.

An important scalar is the triple-product of the vectors $\{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}$:

$$(\boldsymbol{a} \times \boldsymbol{b}) \cdot \boldsymbol{c} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

equal in absolute value to the volume of a parallelepiped constructed by the vectors $\boldsymbol{a}$, $\boldsymbol{b}$ *and* $\boldsymbol{c}$.

The connection between the Hamilton and Grassmann algebras is the following: the Grassmann inner product of two vectors is equivalent to negative of the scalar of Hamilton quaternion product of two vectors; the Grassmann outer product is precisely Hamilton's vector of the quaternion product of two vectors. However, in a theory of quaternions, the vector appears as a subsidiary part of the quaternion, whereas in the Grassmann algebra the vector is a basic quantity.

While physicists ignored quaternions (up to 1928), Hamilton's work led mathematicians to the theory of linear associative algebras and beyond.

Grassmann's work, however, was redeemed sooner and became a powerful tool in exploiting the theory of the electromagnetic field. **Maxwell** and **Clifford** *separated Hamilton's* $\nabla \boldsymbol{v}$ *into a scalar divergence* and *the curl* vector, *establishing the identities:*

$$\text{divcurl } \boldsymbol{v} \equiv 0, \qquad \text{curlgrad } \boldsymbol{v} \equiv 0$$

and defining the Laplacian operator

$$\text{divgrad } = \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

Maxwell's work made clear that vectors were real tool for physical thinking and not just an abbreviated notation. Thus, by Maxwell's time a great deal of vector analysis was created by treating the scalar and vector parts of quaternions separately. The formal break with quaternions and the inauguration of a new independent subject, 3-dimensional vector analysis, was made independently by **J. W. Gibbs** and **Oliver Heaviside** *in the early 1880's. By the beginning of* 20^{th} *century, the physicists were quite convinced that vector analysis was what they wanted. The mathematicians finally followed suit and introduced vector methods in analytic and differential geometry. With the rise of quantum mechanics in the 1920's, physicists would return to embrace quaternions as representing a new physical reality.*

From the purely algebraic standpoint quaternions were exciting because they furnished an example of an algebra that had the properties of real numbers and complex numbers except for commutativity of multiplication. During the second half of the 19^{th} *century mathematicians explored other varieties of noncommutative algebras.* **Clifford** *(1873),* **B. Peirce** *(1881),* **F. G. Frobenius** *(1878),* **C. S. Peirce** *(1881)* and **A. Hurwitz** *(1898), made important contributions to the field of linear associative algebra.*

1828–1868 CE **Julius Plücker** (1801–1868, Germany). Distinguished mathematician and physicist. In a unique double career as geometer and experimental physicist, he was both the founder of *line geometry* (1830) and one of the first promoters of gas spectroscopy.

Plücker established Poncelet's *principle of duality* as a fundamental conceptual tool in projective geometry, and extended it to three dimensions where the duality is between points and planes, lines being unchanged (1828–1831).

Plücker introduced *line-geometry*, where straight lines are used as elements in 3-D space, rather than points (1830).

In 1839, Plücker established the field of algebraic geometry. He discovered six equations connecting the number of singularities of algebraic curves. He discovered *homogeneous coordinates* independently of **Möbius**, **E. Bobillier** and **Feuerbach**, defined *Plücker's coordinates* and *Plücker's equations*.

In 1846 he switched to experimental physics, and his research centered on spectroscopy of rare gases. After 1855, improved vacuum techniques enabled **Plücker** and **Crookes** to investigate the properties of the so-called '*Cathode-rays*', which led in 1897 to their identification with electron-streams by **J.J. Thomson**.

It is believed that Plücker was the first to identify three lines of hydrogen in the spectrum of the Solar Corona in 1858, and also first to invent the cathode-ray tube (1859).

Plücker was born at Elberfeld and was educated at the universities of Bonn, Heidelberg and Berlin. In 1823 he went to Paris and came under the influence of the school of French geometers established by **Monge**. He returned to Bonn in 1825 and stayed there until 1833, moving thereafter to Berlin and Halle. In 1836 Plücker returned to Bonn as a professor of mathematics, becoming in 1847 also a professor of physics.

1829 CE The word *technology* coined.

1829–1851 CE **Carl Gustav** (**Jacob Shimon**) **Jacobi** (1804–1851, Germany). One of the leading mathematicians of the 19^{th} century and the greatest mathematician in Germany after Gauss.

True to the spirit of his time, a spirit compounded of equal parts of faith and nearly incredible ingenuity, he derived in his magnum opus, *Fundamenta Nova Theoriae Functionum Ellipticum*, many elegant and intricate results by means of algebraic manipulations that surpassed even Euler and Gauss. He uncovered a treasure-house of results whose variety, aesthetic appeal and capacity for arousing our astonishment have not been equaled by research in any other area.

Jacobi was born of Jewish parents in Potsdam, Prussia and later converted to Christianity, without which he could not have pursued an academic career in the Germany of those days. He was introduced to mathematics at an early age by his maternal uncle Lehmann, who prepared him to enter the Potsdam Gymnasium in 1816. His unusual talents were recognized already at school and he left in 1821 to enter the University of Berlin. He taught himself algebra, calculus and number theory through the direct reading of the works of Euler and Lagrange. This earliest self-instruction was to give Jacobi's first outstanding work — in elliptic functions — its definite direction, for Euler, the master of ingenious devices, found in Jacobi his brilliant successor. For sheer manipulative ability in tangled algebra, Euler and Jacobi have no rival, except perhaps Srinivasa Ramanujan in the 20^{th} century.

Jacobi's student days at Berlin lasted from 1821 to 1825. During the first two years, he divided his time about equally between philosophy, philology and mathematics. Mathematics, however, finally won him over, and in 1825 he obtained his degree and moved to the University of Königsberg, where he joined, amongst others, Friedrich Bessel. He soon rose to the rank of associate professor, due to his great talents as an inspiring teacher and his work on cubic reciprocity in number theory. The latter excited Gauss' admiration and with his recommendation, the Ministry of Education promoted Jacobi over the heads of his colleagues.

In 1829 he published his first masterpiece, *Fundamenta Nova* on the theory of elliptic functions and modular equations[49] and obtained his full professorship at the age of 25. This work is one of the greatest mathematical classics that has ever been written — a book perhaps never equaled in the annals of mathematics in the shear number of new and important results first given in it. He continued to work incessantly, with Gauss watching his phenomenal activity with more than a mere scientific interest — as many of Jacobi's discoveries overlapped some of his own youth, which he had never published. The two met in September 1839, when Jacobi, collapsing from 12 years of overwork, returned from a vacation in Marienbad.

In 1842 Jacobi met **Hamilton** at Manchester. It was one of Jacobi's greatest glories to extend Hamilton's work in dynamics, which Hamilton forsook in favor of his quaternions. Later in the year Jacobi became seriously ill with diabetes. Through the efforts of Dirichlet and von Humboldt, he was granted financial support to enable him to visit Italy for a few months and restore his health. On his return he moved to Berlin, where he lived as a royal pensioner. In February 1851 his health deteriorated again. He first contracted influenza and then, on the point of recovery, caught smallpox and died within a week.

Jacobi was the greatest university mathematical teacher of his generation, stimulating and influencing an unprecedented number of able students. He rejected the notion that before doing research, students should first master what has already been accomplished and held that young mathematicians "ought to be pitched into the icy water to learn to swim or drown by themselves, or else they never acquire the knack of independent work".

[49] Jacobi studied *modular equations* for elliptic functions. The equation $u^6 + v^6 + 5u^2v^2(u^2 - v^2) + 4uv(1 - u^4v^4) = 0$ is fundamental for Hermite's 1858 solution of quintics.

His investigations of *elliptic functions*, the theory of which he established upon quite a new basis, and more particularly his development of the *theta functions*, constitute his greatest analytical discovery.[50]

He contributed to complex variable theory and was one of the early founders of the theory of *determinants*. He developed extensively the properties of the functional determinant formed by the n^2 partial derivatives of n given functions w.r.t. their n independent variables, which now bears his name — the *Jacobian* (1829) (although it was known to Cauchy already in 1815). This function plays an important role in differential geometry. The Jacobian of two functions $u(x, y)$, $v(x, y)$ is defined as the determinant

$$\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial v}{\partial x} \equiv (\nabla u \times \nabla v)_z.$$

It is represented by the symbol $\frac{\partial(u,v)}{\partial(x,y)}$ or $J(\frac{u,v}{x,y})$. A necessary and sufficient condition that two continuously differentiable functions $u(x, y)$ and $v(x, y)$ in a region R satisfy the relation $F(u, v) \equiv 0$ for some function F is that their Jacobian vanish in R.

Similarly, the Jacobian of three functions $u(x, y, z)$, $v(x, y, z)$ and $w(x, y, z)$ is defined by the determinant

$$J\left(\frac{u, v, w}{x, y, z}\right) = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix} = (\nabla u \times \nabla v) \cdot \nabla w.$$

A necessary and sufficient condition that three continuously differentiable functions u, v, w satisfy an equation $F(u, v, w) \equiv 0$ in R is that their Jacobian vanish in this region.

A general differentiable transformation of coordinates $\bar{x}_i = f_i(x_1, x_2, x_3)$ in which the functions f_i are single-valued for all points in R can be solved to render $x_i = g(\bar{x}_1, \bar{x}_2, \bar{x}_3)$ iff $J = \left|\frac{\partial \bar{x}}{\partial x}\right| = \det \frac{\partial \bar{x}_i}{\partial x_j} \neq 0$ everywhere in R. The *volume element* is altered by the transformation according to the relation

$$d\bar{V} = \left| J(\frac{\bar{x}_1, \bar{x}_2, \bar{x}_3}{x_1, x_2, x_3}) \right| dV.$$

[50] On a higher order of originality is his discovery, of *Abelian functions*. Such functions arise in the inversion of an *Abelian integral*, in the same way that the elliptic functions arise from the inversion of an elliptic integral. Here he had nothing to guide him, and for long he wandered lost in a maze that yielded no clue. The appropriate inverse functions in the simplest case are functions of *two* variables having *four* periods; in the general case, the functions have n variables and $2n$ periods; the elliptic functions correspond to $n = 1$.

There are also the *Jacobi identity* for associative algebras, *Jacobi elliptic functions*, *Jacobi polynomials*, *Jacobi zeta function*, *Jacobi epsilon function*, and *Jacobi identity* for a triple infinite product.

Jacobi extended Hamilton's equations of motion via the *canonical transformations*, to what is known as the *Hamilton's-Jacobi equation*. In his formalism, geometrical optics, mechanics and wave mechanics [**Louis Victor de Broglie** (1892–1987, France, 1924) and **Erwin Schrödinger** (1887–1961, Germany, 1925)] meet on common ground: the geometrization of physical phenomena.

Jacobi's contributions to number theory were extensive. In 1827 he stated the law of cubic reciprocity. He applied elliptic functions to the theory of numbers, obtaining the Fermat-Lagrange four-square theorem. Furthermore, Jacobi's theory could determine the number of distinct ways in which each number can be represented.

Jacobi contributed to the theory of differential equations and to the calculus of variations. He introduced (1837) the concept (though not the term) of a *self-adjoint* differential equation: Using modern notation, we consider the general linear second-order PDE

$$\sum_{i,j=1}^{n} A_{ij}\frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{i=1}^{n} b_i \frac{\partial u}{\partial x_i} + cu + F = 0,$$

or in its index-free vector form,

$$\mathcal{L}[u] + F = \mathfrak{A}\colon \nabla\nabla u + \boldsymbol{b}\cdot\nabla u + cu + F = 0,$$

where $\mathfrak{A}$, $\boldsymbol{b}$, c, F are functions of the coordinates $(x_1, \ldots, x_n)$, one of which can be time.

Define the *adjoint operator* as

$$\overline{\mathcal{L}}[u] = \operatorname{div}\operatorname{div}(\mathfrak{A}u) - \operatorname{div}(\boldsymbol{b}u) + cu.$$

Using certain vector identities it is shown that if $\mathfrak{A}$ is a symmetric tensor and $\boldsymbol{b} = \operatorname{div}\mathfrak{A}$, the operators $\mathcal{L}$ and $\overline{\mathcal{L}}$ are identical. In that case we say that the original PDE is *self-adjoint*, and write it in the compact form

$$\operatorname{div}[\mathfrak{A}\cdot\nabla u] + cu + F = 0.$$

As an example, set $u = u(x_1, x_2, x_3, t)$, $\nabla u = \left\{\frac{\partial u}{\partial x_1}, \frac{\partial u}{\partial x_2}, \frac{\partial u}{\partial x_3}, \frac{\partial u}{\partial t}\right\}$ and choose $\mathfrak{A} = \begin{bmatrix}\lambda & & & \\ & \lambda & & \\ & & \lambda & \\ & & & -\rho\end{bmatrix}$ with λ, ρ function of *position* only. The self-adjoint equation then becomes

$$\rho \frac{\partial^2 u}{\partial t^2} = \operatorname{div}(\lambda \operatorname{grad} u) + cu + F,$$

which is recognized as the *wave-equation*, with div and grad as the usual operators in 3-dimensional space. On the other hand, the *diffusion equation* is *not* self-adjoint, and can be derived from the original equation, with the aid of the identity $\operatorname{div}[\mathfrak{A} \cdot \nabla u] = \operatorname{div} \mathfrak{A} \cdot \nabla u + \mathfrak{A} \colon \nabla\nabla u$. The equation then becomes

$$\operatorname{div}[\mathfrak{A} \cdot \nabla u] + (\boldsymbol{b} - \operatorname{div} \mathfrak{A}) \cdot \nabla u + cu + F = 0.$$

Choosing

$$\mathfrak{A} = \begin{bmatrix} \lambda & & & \\ & \lambda & & \\ & & \lambda & \\ & & & 0 \end{bmatrix}, \qquad \boldsymbol{b} - \operatorname{div} \mathfrak{A} = (0, 0, 0, -\kappa),$$

we obtain

$$\kappa \frac{\partial u}{\partial t} = \operatorname{div}(\lambda \operatorname{grad} u) + cu + F,$$

which renders the diffusion equation.

Lagrange (1762 to 1765) was probably the first to present examples of adjoint differential equations. **Liouville** (1838) gave a special pair of adjoint differential systems. The term *adjoint* is due to **Fuchs** (1873). The theory of self-adjoint differential equations was further developed by **Frobenius** (1873 to 1878).

We note that $u\mathcal{L}[v] - v\overline{\mathcal{L}}[u] = \operatorname{div} \boldsymbol{P}$, where

$$\boldsymbol{P} = uv\boldsymbol{b} + u\mathfrak{A} \cdot \nabla v - v \operatorname{div}(\mathfrak{A}u).$$

When $\mathcal{L}$ is self-adjoint, $\boldsymbol{P} = (u\boldsymbol{\nabla} v - v\boldsymbol{\nabla} u) \cdot \mathfrak{A}$.

For $\mathfrak{A} = \mathfrak{I}$, the special case of *Green's identity*, $u\nabla^2 v - v\nabla^2 u = \operatorname{div}(u\boldsymbol{\nabla} u - v\nabla u)$, is obtained.

In a single spatial dimension, the self-adjoint PDE degenerates into the self-adjoint ODE

$$\frac{d}{dx}\left[p_0(x)\frac{du}{dx}\right] + p_2(x)u + F = 0,$$

where $p_0 \neq 0$ in the interval $a \leq x \leq b$.

One arrives at this result with the explicit requirement that

$$\mathcal{L}[u] = \left[p_0(x)\frac{d^2}{dx^2} + p_1(x)\frac{d}{dx} + p_2(x)\right]u(x) + F$$

be made equal to the *adjoint operator*

$$\overline{\mathcal{L}}[u] = \frac{d^2}{dx^2}(p_0 u) - \frac{d}{dx}(p_1 u) + p_2 u + F$$

$$\equiv \left[p_0\frac{d^2}{dx^2} + (2p_0' - p_1)\frac{d}{dx} + (p_0'' - p_1' + p_2)\right]u + F,$$

which happens whenever $p_0' = p_1$.

In this case, an operator can always be made self-adjoint upon its multiplication with

$$\frac{1}{p_0}\exp\left\{\int^x \left(\frac{p_1}{p_0}\right)dx\right\} = \mu(x),$$

leading to the self-adjoint

$$\mu(x)\mathcal{L}[u] = \frac{d}{dx}\left[p_0\mu\frac{du}{dx}\right] + p_2\mu u + F.$$

Also, for $F = 0$

$$u\mathcal{L}[v] - v\overline{\mathcal{L}}[u] = \frac{\partial}{\partial x}\left[p_0 uv' - v(p_0 u)' + p_1 uv\right].$$

To the Newton-Laplace-Lagrange theory of attraction Jacobi made substantial contributions, by his investigations on the functions which recur repeatedly in that theory and by the application of elliptic and Abelian functions to the attraction of ellipsoids.

Jacobi (1841) introduced the notation d and ∂ for total and partial derivatives (*differentialia partialia*), respectively, i.e. he was first to write $df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$. This he generalized to a function of n variables $f(x_1, x_2, \ldots, x_n)$. The notation $\frac{\partial f}{\partial x}$ advocated by Jacobi did not meet with immediate adoption. It took half a century for it to secure a generally recognized place in mathematical writing.[51]By 1898, Jacobi's notation was accepted

[51] When **Cayley** (1857) abstracted Jacobi's paper, he paid no heed to the new notation and wrote all derivatives in the form $\frac{df}{dx}$, etc.

Partial derivatives appear in the writing of **Newton**, **Leibniz**, and the in England, where Hamilton's gradient operator was written for the first time as $\nabla = \boldsymbol{i}\frac{\partial}{\partial x} + \boldsymbol{j}\frac{\partial}{\partial y} + \boldsymbol{k}\frac{\partial}{\partial z}$.

The Elliptic Wonderland of Jacobi[52]

"God ever geometrizes"

Plato (427–347 BCE)

"God ever arithmetizes"

C.G.J. Jacobi (1829 CE)

A large number of important properties of elliptic integrals were observed by **Euler** *and* **Legendre** *before it was realized that the inverses of certain*

Bernoullis, but as a rule without any special symbolism. **Euler** (1776) used $\frac{\partial^\lambda}{p} \cdot V$ to indicate the λth derivative, partial w.r.t. the variable p, operating upon V. The use of the rounded letter ∂ in the notation for partial differentiation occurs again (1786) in an article by **Legendre**, but he himself soon abandoned his own notation in later papers.

52 For further reading, see:

- Lawden, D.F., *Elliptic Functions and Applications*, Springer-Verlag: New York, 1989, 334 pp.
- Dutta, M. and L. Debnath, *Elements of the Theory of Elliptic and Associated Functions* (With Applications), The World Press Private, 1965, 290 pp.
- Eagle, A., *The Elliptic Functions as They Should Be*, Gallaway and Porter: Cambridge, England, 1958, 508 pp.
- Oberhettinger, F. and W. Magnus, *Anwendung Der Elliptischen Functionen in Physik und Technik*, Springer-Verlag: Berlin, 1949, 126 pp.

standard types of elliptic integrals, rather that the integrals themselves, should be regarded as fundamental functions of analysis. This idea is due to ***Gauss****,* ***Abel*** *and* ***Jacobi****.*

Gauss inverted the lemniscate integral (1797)

$$u = \int_0^x \frac{dt}{\sqrt{1-t^4}}$$

and defined through it the "lemniscate sine function"

$$x = \mathrm{sl}(u).$$

He found that the function was periodic, like the sine, with period

$$2\widetilde{\omega} = 4\int_0^1 \frac{dt}{\sqrt{1-t^4}}.$$

From the relation

$$\frac{d(it)}{\sqrt{1-(it)^4}} = i\frac{dt}{\sqrt{1-t^4}}$$

he deduced $\mathrm{sl}(iu) = i\mathrm{sl}(u)$ *and hence that the lemniscate sine has a second period* $2i\widetilde{\omega}$*. Thus Gauss discovered double periodicity, one of the key properties of elliptic functions, though at first he did not realize its universality. However, the importance of elliptic functions became clear to him when he independently discovered (1799) an ingenious method to calculate numerically the values of complete and incomplete elliptic integrals by using the arithmetic-geometric mean.*

To grasp the revolutionary idea of inversion, consider, for example, the fundamental elliptic integral of the first kind

$$u(x) = \int_0^x \frac{dt}{\sqrt{(1-t^2)(1-k^2t^2)}}$$

$$= \int_0^\phi \frac{d\theta}{\sqrt{1-k^2\sin^2\theta}}, \qquad t = \sin\theta, \qquad x = \sin\phi,$$

where the parameter k *is known as the modulus and* $k' = \sqrt{1-k^2}$ *is the complementary modulus. In the trivial case* $k^2 = 0$ *we have*

$$u(x) = \phi = \sin^{-1} x = \int^x \frac{dt}{\sqrt{1-t^2}}$$

where u is a multivalued function of x. The inverse relation $x = \sin u$ is simple and represents a single-valued periodic function of period 2π. A similar situation occurs with $u = \log x = \int^x \frac{dt}{t}$ and $x = e^u$.

In light of this analogy, ***Jacobi*** *defined for all $k \leq 1$*

$$u(x) = \int_0^x \frac{dt}{\sqrt{(1-t^2)(1-k^2t^2)}} = \mathrm{sn}^{-1}x.$$

The inverse functions, single-valued and analytic, are then defined through the new symbols

$$x = \mathrm{sn}(u,k), \quad \phi = \mathrm{am}(u,k).$$

The fundamental new functions are related through the equations:

$$x = \mathrm{sn}(u,k) = \sin\phi = \sin[\mathrm{am}(u,k)]$$

$$\sqrt{1-x^2} = \mathrm{cn}(u,k) = \cos\phi = \cos[\mathrm{am}(u,k)]$$

$$\sqrt{1-k^2x^2} = \mathrm{dn}(u,k) = [1-k^2\sin^2(\mathrm{am}(u,k))]^{1/2}$$

$$\mathrm{am}(u,0) = u, \quad \mathrm{sn}(u,0) = \sin u, \quad \mathrm{cn}(u,0) = \cos u, \quad \mathrm{dn}(u,0) = 1$$

$$\mathrm{sn}^2u + \mathrm{cn}^2u = 1; \quad \mathrm{dn}^2u + k^2\mathrm{sn}^2u = 1$$

$$u = \mathrm{sn}^{-1}x = \mathrm{cn}^{-1}\sqrt{1-x^2} = \mathrm{dn}^{-1}\sqrt{1-k^2x^2}$$

Euler *(1761) has shown that if $R(\xi)$ is a rational polynomial in ξ of the 4^{th} order, then there exists an algebraic function $W(x,y)$ such that*

$$\int_0^x \frac{d\xi}{\sqrt{R(\xi)}} + \int_0^y \frac{d\xi}{\sqrt{R(\xi)}} = \int_0^{W(x,y)} \frac{d\xi}{\sqrt{R(\xi)}}.$$

Thus, for $R(\xi) = (1-\xi^2)(1-k^2\xi^2)$, Euler found

$$W(x,y) = \frac{x\sqrt{1-y^2}\sqrt{1-k^2y^2} - y\sqrt{1-x^2}\sqrt{1-k^2x^2}}{1-k^2x^2y^2}.$$

*Using the notation of **Jacobi***

$$u = \int_0^x \frac{d\xi}{\sqrt{R(\xi)}}, \quad x = \mathrm{sn} u, \quad v = \int_0^y \frac{d\xi}{\sqrt{R(\xi)}}, \quad y = \mathrm{sn} v$$

$$W = \mathrm{sn}(u+v);$$

one can recast Euler's result in the form of an addition theorem

$$\mathrm{sn}(u+v) = \frac{\mathrm{sn}\ u\ \mathrm{cn}\ v\ \mathrm{dn}\ v + \mathrm{sn}\ v\ \mathrm{cn}\ u\ \mathrm{dn}\ u}{1 - k^2 \mathrm{sn}^2 u \mathrm{sn}^2 v}.$$

In inverse-function notation this reads

$$\mathrm{sn}^{-1} x + \mathrm{sn}^{-1} y = \mathrm{sn}^{-1} W(x,y).$$

In the limit $k = 0$, the last two relations degenerate into the familiar trigonometric formulas

$$\sin(u+v) = \sin u \cos v + \cos u \sin v,$$

$$\sin^{-1} x + \sin^{-1} y = \sin^{-1}[x\sqrt{1-y^2} + y\sqrt{1-x^2}]$$

The theory of elliptic functions includes two fundamental parameters: the modulus k and the complete elliptic integral of the first kind

$$K(k) = \mathrm{sn}^{-1}(1) = \int_0^1 [(1-t^2)(1-k^2t^2)]^{-1/2}\, dt = \frac{\pi}{2} {}_2F_1(\frac{1}{2}, \frac{1}{2}, 1, k^2),$$

$$\mathrm{am}\ K = \frac{\pi}{2}, \qquad {k'}^2 + k^2 = 1.$$

Associated with K is the quantity

$$K' = \int_0^1 [(1-t^2)(1-{k'}^2 t^2)]^{-1/2}\, dt = K(k').$$

The elliptic functions reduce to circular functions with

$$k = 0, \quad k' = 1, \quad K = \frac{\pi}{2}, \quad K' = \infty$$

and to hyperbolic functions with

$$k = 1, \quad k' = 0, \quad K = \infty, \quad K' = \frac{\pi}{2}$$

in which case:

$$\mathrm{sn} x = \mathrm{th}\ x, \quad \mathrm{cn}\ x = \mathrm{dn}\ x = \frac{1}{\mathrm{ch}\, x}.$$

The elliptic functions are doubly periodic in the complex u-plane. To see this important feature, one effects the substitution

$$x = \frac{iy}{\sqrt{1-y^2}} = \sin\phi = i\tan\psi,$$

implying

$$\cos\phi = \frac{1}{\cos\psi}, \quad -\sin\phi\, d\phi = \sec\psi\tan\psi\, d\psi,$$

$$\sqrt{1-k^2\sin^2\phi} = \frac{1}{\cos\psi}\sqrt{1-k'^2\sin^2\psi}.$$

Then

$$u = \int_0^x \frac{dt}{\sqrt{(1-t^2)(1-k^2t^2)}} = \int_0^\phi \frac{d\theta}{\sqrt{1-k^2\sin^2\theta}}$$
$$= i\int_0^\psi \frac{d\alpha}{\sqrt{1-k'^2\sin^2\alpha}} = i\int_0^y \frac{d\xi}{\sqrt{(1-\xi^2)(1-k'^2\xi^2)}} \equiv iW.$$

This further implies $y = \sin\psi = \mathrm{sn}(W, k')$ and

$$\mathrm{sn}(u,k) = \mathrm{sn}(iW,k) = \sin\phi$$
$$\mathrm{cn}(u,k) = \mathrm{cn}(iW,k) = \cos\phi$$
$$\mathrm{dn}(u,k) = \mathrm{dn}(iW,k) = \sqrt{1-k^2\sin^2\phi}.$$

For $k = 0$ (trigonometric functions) $\int_0^1 \frac{dt}{\sqrt{1-t^2}} = \frac{\pi}{2} = \frac{1}{4}T$ *where $T = 2\pi$ is the period. It is therefore natural to expect that K will assume the role of the quarter-period of the elliptic function. To see this we calculate*

$$\int_0^{\pi n+\beta} \frac{d\phi}{\sqrt{1-k^2\sin^2\phi}} = \int_0^{\pi n} \frac{d\phi}{\sqrt{1-k^2\sin^2\phi}} + \int_{\pi n}^{\pi n+\beta} \frac{d\phi}{\sqrt{1-k^2\sin^2\phi}}$$

$$= 2nK + \int_0^{\beta} \frac{d\phi}{\sqrt{1-k^2\sin^2\phi}} = 2nK + u.$$

The above relation can be translated into $\sin\beta = \mathrm{sn}u$, $\sin(\beta + \pi n) = \mathrm{sn}(2nK + u)$, *or* $\mathrm{sn}(u \pm 4K) = \mathrm{sn}u$. *In fact, $4K$ is the period of all three elliptic functions* $\mathrm{sn}u$, $\mathrm{cn}u$ *and* $\mathrm{dn}u$.

Similar manipulation involving k' show that

$$\mathrm{sn}(u + 4K) = \mathrm{sn}(u + 2iK') = \mathrm{sn}\ u$$

$$\mathrm{cn}(u + 4K) = \mathrm{cn}(u + 2K + 2iK') = \mathrm{cn}\ u$$

$$\mathrm{dn}(u + 2K) = \mathrm{dn}(u + 4iK') = \mathrm{dn}\ u,$$

exhibiting the double periodicity *of the elliptic functions in the complex x plane.*

It can be shown that a function of complex variable cannot have two incommensurate periods in the same direction, *but if one of the periods is in a* different direction *in the complex plane (i.e. the ratio of the periods is not a real number), this is possible. Thus, instead of a* one-dimensional *sequence of periods (as in the case for the trigonometric functions) there will be a* two-dimensional *lattice of parallelograms, with the function repeating, in each parallelogram, its behavior in every other parallelogram. The smallest unit within which the function goes through all its behavior is called the* unit cell *for the function; each side of the unit cell is one of the fundamental periods for the function.*

Thus, for real W, one has

$$\{\mathrm{sn}\ iW,\ \mathrm{cn}\ iW,\ \mathrm{dn}\ iW\}$$

defined for purely imaginary argument $u = iW$ in terms of the real Jacobi functions with real argument W and complementary modulus k'.

Furthermore, the same reasoning that permitted us to define $\operatorname{sn} u$ *and hence* $\operatorname{cn} u$ *and* $\operatorname{dn} u$ *as periodic functions with real period* $4K$ *shows that we can take* $\operatorname{sn}(W,k')$, $\operatorname{cn}(W,k')$ *and* $\operatorname{dn}(W,k')$ *as real periodic functions with real period*

$$4K' = 4\int_0^1 \frac{dx}{\sqrt{(1-x^2)(1-k'^2x^2)}}.$$

Clearly, $4iK'$ *is a second period, purely imaginary, for* $\operatorname{sn} u$, $\operatorname{cn} u$, *and* $\operatorname{dn} u$, *that is*

$$\operatorname{sn}(u+4iK') = \operatorname{sn} u$$

etc.

Jacobi elliptic functions are therefore doubly periodic functions of the complex variable u. *However, all told, the Jacobi elliptic function, although defined for real* u *and purely imaginary* u, *were never defined for complex* $u = \sigma + i\tau$. *Abel and Jacobi got around this by using the addition formula for* $\operatorname{sn} u$, *etc.*

This procedure, unfortunately, breaks down when one wishes to use complex values of k.

Thus, if one wishes to define Jacobi elliptic functions as functions of a complex variable, by using the idea of inverting the elliptic integral of the first kind, [i.e., considering one limit of integration as a complex variable, and the value of the integral as a complex line integral over some curve], then one must use a thoroughly complex variable technique which takes into account all the difficulties of integrating a multi-valued function, with branch cuts in complex plane. The correct technique was discovered by ***Riemann****, who introduced the notion of Riemann surface precisely to handle such problems.*

There are no functions of complex variable z *which have more than two independent periods.*

The elliptic function $y = \operatorname{sn}(u,k)$ *has simple zeros at* $u = 2mK + 2nK'i$ $(m,n = 0,\pm1,\pm2,\dots)$ *and has simple poles at* $u = 2mK + i(2n+1)K'$ *(the first pole at* $u = iK'$ *on the imaginary axis);*

It thus has a row of zeros along the real axis, spaced at distance $2K$ *apart and a row of poles along the line* $y = K'$, *each vertically above a zero on the real axis, and so on. [The residue at the pole* $u = iK'$ *is* $\frac{1}{K}$, *and the residue at* $u = 2K + iK'$ *is* $(-\frac{1}{K})$.*]*

This property makes the elliptic function useful in the solution of certain potential problem in electrostatics.

The 5-Fold Way

The number 5 has the following remarkable traits:

- *The geometry of art, aesthetics and life is associated with the pentagon, the pentagram and the Golden Section, that is inherent in both. Five is also the 4^{th} Fibonacci number. In Phyllotaxis (regular arrangement of leaves of a stems or petals in flowers), a pattern with 5 units occurs very frequent [whimsical: 5 fingers on human limbs]. Five is also the hypotenuse of the smallest Pythagorean triangle.*

- *There are 5 Platonic solids: the regular tetrahedron, cube, octahedron, dodecahedron and icosahedron (all but the cube were named after the Greek word for their number of faces). They were all known to the Greeks.* ***Euclid*** *showed that there are no more than 5.*

  Kepler *used them, with typical confidence in their mystical properties, to explain the relative sizes of the orbits of the planets.*

- *The 'worst' close-regular-packing of spheres*[53] *in any dimension is at dimension 5.*

- *The smallest integer n for which $F_n = 2^{2^n} + 1$ (Fermat number) is composite: $F_5 = 2^{32} + 1 = 641 \cdot 6,700,417 = 4,294,967,297$.*

- *The n^{th} Fibonacci number is given by the formula*

$$F_n = \frac{1}{\sqrt{5}}[(\frac{1+\sqrt{5}}{2})^n - (\frac{1-\sqrt{5}}{2})^n].$$

- *The smallest Z_n group that cannot be a symmetry of a periodic lattice is $Z_5 = 5$-fold symmetry.* ***D. Schechtman*** *(1984, Israel) has demonstrated that this symmetry is nevertheless realized in non-periodic quasi-crystals.*

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[53] The volume of a *unit sphere* in n-dimensions is $V_n = \pi^{n/2}/\Gamma\left(\frac{n}{2}+1\right)$. It is largest at $n = 5$ ($V_1 = 2$, $V_2 = 3.14\ldots$, $V_3 = 4.19\ldots$, $V_4 = 4.93$, $V_5 = 5.26$, $V_6 = 5.17\ldots$, $V_\infty = 0$). In a 5-dimensional space the *density of packing* has a minimal value of $\frac{\sqrt{2}}{60}\pi^2 = 0.2325\ldots$.

- *The general algebraic equation of the* 5^{th} *or higher degree cannot be solved in terms of the coefficients by using only a finite sequence of arithmetical operations and radicals. This was first proved by Abel during* 1824–1826. *By* 1831, ***Galois*** *established the theory of algebraic solution of equations in its most complete form, associating it with subgroups of the group of permutation of the roots. The results of Galois are much deeper and more general than those of Abel. Moreover, Galois found that algebraic equations of orders* 5, 7 *and* 11 *are related to the modular equations in the theory of elliptic functions.*

  Thus, by shutting the door to the possibility of algebraic *solution of a class of polynomial equations, he simultaneously opened another door to* nonalgebraic *solutions of the same class, requiring an* infinite *number of arithmetical operations on the coefficients. Indeed in* 1858, ***Hermite*** *used this method in a very elegant manner to obtain all* 5 *solutions of the quintic equation in terms of elliptic functions [analogously to the trigonometrical solution of the cubic equation]. Finally, it was shown by* ***C. Jordan*** *in* 1870 *that the solutions of the general algebraic equation of degree higher than* 5 *are not expressible in terms of* elliptic functions *alone.*

- *Plays an unexpected part in the Rogers-Ramanujan identities (1894)*

$$1 + \sum_{1}^{\infty} \frac{x^{m^2}}{(1-x)(1-x^2)\dots(1-x^m)} = \prod_{0}^{\infty} \frac{1}{(1-x^{5m+1})(1-x^{5m+4})}$$

$$1 + \sum_{1}^{\infty} \frac{x^{m(m+1)}}{(1-x)(1-x^2)\dots(1-x^m)} = \prod_{0}^{\infty} \frac{1}{(1-x^{5m+2})(1-x^{5m+3})}$$

It is also reflected in Ramanujan's most bizarre result, obtained with the aid of the above identities (1913)

$$u = \cfrac{x}{1 + \cfrac{x^5}{1 + \cfrac{x^{10}}{1 + \cfrac{x^{15}}{1 + \cfrac{x^{20}}{1 + \dots}}}}}; \qquad v = \cfrac{x^{1/5}}{1 + \cfrac{x}{1 + \cfrac{x^2}{1 + \cfrac{x^3}{1 + \cfrac{x^4}{1 + \dots}}}}}$$

$$v^5 = u \frac{1 - 2u + 4u^2 - 3u^3 + u^4}{1 + 3u + 4u^2 + 2u^3 + u^4}$$

As in the previous item, the number 5 seems to emerge out of the theory of elliptic functions. Yet, there is here a hidden connection between seemingly unrelated formulas: even after we understand what has been done, the feeling of bewilderment and wonder is not lifted.

After x is eliminated, one is left with a quadratic equation in v^5, namely

$$\frac{(u^{-1}-1-u)^6}{u^{-5}-11-u^5}=\frac{1}{v^5}-11-v^5,$$

which upon simplification yields the desired result.

1829–1832 CE **Évariste Galois** (1811–1832, France).

"Down, down, down into the darkness of the grave
Gently they go, the beautiful, the tender, the kind;
Quietly they go, the intelligent, the witty, the brave.
I know. But I do not approve. And I am not resigned".

Edna St. Vincent Millay, '*Dirge Without Music*'

A most brilliant mathematician who, in a brief meteoric career, laid the foundations to the theory of groups and the theory of algebraic equations. His theory of equations is based upon concepts of group theory and supplies criteria for the possibility of solving an algebraic equation by radicals.

Galois resolved the deeper issues of solvability. His *group-theoretic* approach superseded the *algebraic* theories of Lagrange (1770), Ruffini (1799) and Abel (1824).

In 1815 **Gauss** gave an algebraic proof of the fundamental theory of algebra. The problem with this theorem is, however, that it does not tell us what the roots are.

After **Abel's** work (1826), the situation was as follows: Although the general equation of degree higher then four was known to not be solvable by radicals, there were special equations (e.g. $x^p = a$, p *prime*) that were solvable by radicals. It remained to determine *which equations are solvable by radicals.* This task was successfully undertaken by **Evariste Galois**.

Galois' idea was to associate to any polynomial equation a group in such a way that the properties of the group and the nature of the solutions of the equations are closely related. In particular, he devised groups that reflect the symmetry properties of the roots of general polynomial equations.

To this end he introduced the concept of the *Galois group of an equation*: If $f(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$ is a polynomial over the rationals, then there are certain rational functions with rational-valued coefficients $H(\alpha_1, \dots, \alpha_n) = 0$, among the solutions $\alpha_1, \dots, \alpha_n$ of $f(x) = 0$. The group of all permutations that leave all the relations $H(\alpha_1, \dots, \alpha_n) = 0$ invariant, is called the *Galois group*[54] of the equation[55]. It can then be shown that *any* rational relation (in the above sense) left invariant by all permutations in the Galois group, is rational-valued (in either of the senses explained in the previous footnote).

The central theorem in Galois' theory then states that a polynomial equation is soluble by radicals if and only if its group is 'solvable'. When the Galois group for any equation has been found, a criterion devised by Galois will indicate whether or not the group is 'solvable'.

The association of the group concept with solutions of algebraic equations can be illustrated with aid of the following example:

The equation $x^3 - 2 = 0$ has the three roots: $x_1 = \sqrt[3]{2}$, $x_2 = \omega\sqrt[3]{2}$, $x_3 = \omega^2\sqrt[3]{2}$, where $\omega = -\frac{1}{2} + \frac{1}{2}i\sqrt{3}$ is a primitive cube root of unity. The three roots can be pictured in the plane of complex numbers as 3 points, equally spaced on a circle of radius $\sqrt[3]{2}$. There are 6 operations of permuting the roots, such that $x^3 - 2 = (x - x_1)(x - x_2)(x - x_3)$ remains invariant[56]. They are:

(1) Rotation of each root-vector by 120° counterclockwise:

$$\sqrt[3]{2} \to \omega\sqrt[3]{2} \to \omega^2\sqrt[3]{2} \to \sqrt[3]{2}.$$

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[54] For further reading, see:
- Maxfield, J.E. and M.W. Maxfield, *Abstract Algebra and Solutions by Radicals*, Dover Publications, 1992, 209 pp.

[55] Here "rational-valued" could mean either rational numbers, *or* rational functions of the coefficients $\{\alpha_j\}$.

[56] The coefficients of this polynomial, or of any polynomial, are totally symmetric polynomials of the roots, and furthermore, any rational function of the roots that is totally symmetric under root permutations, can be shown to be a rational function of these symmetric polynomials.

The corresponding permutation is

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}.$$

(2) Rotation of each root-vector by 240° counterclockwise:

$$\sqrt[3]{2} \to \omega^2 \sqrt[3]{2} \to \omega \sqrt[3]{2} \to \sqrt[3]{2}.$$

The permutation is

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}.$$

(3) Reflection about the horizontal (real) axis:

$$\sqrt[3]{2} \to \sqrt[3]{2}, \quad \omega \sqrt[3]{2} \to \omega^2 \sqrt[3]{2}, \quad \omega^2 \sqrt[3]{2} \to \omega \sqrt[3]{2},$$

corresponding to the permutation

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}.$$

(4) The identity

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}.$$

(5) Reflection about an axis going through the origin and $\omega \sqrt[3]{2}$, corresponding to the permutation

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}.$$

(6) Reflection about an axis going through the origin and $\omega^2 \sqrt[3]{2}$, the corresponding permutation being

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}.$$

These 6 permutations constitute the group of symmetries of the equation.

Consider next the particular equation

$$x^4 + px^2 + q = 0, \qquad (p, q \quad \text{rational numbers})$$

having the explicit roots

$$x_1 = \sqrt{\frac{-p + \sqrt{p^2 - 4q}}{2}}; \qquad x_2 = -\sqrt{\frac{-p + \sqrt{p^2 - 4q}}{2}}$$

$$x_3 = \sqrt{\frac{-p - \sqrt{p^2 - 4q}}{2}}; \qquad x_4 = -\sqrt{\frac{-p - \sqrt{p^2 - 4q}}{2}}$$

Let $\mathbb{Q}$ be the field of the rationals. Clearly

$$x_1 + x_2 = 0, \qquad x_3 + x_4 = 0$$

holds. Of the 24 possible permutations of the above 4 roots, the following 8 substitutions (permutations)

$$\begin{aligned}
E &= \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ x_1 & x_2 & x_3 & x_4 \end{pmatrix} & E_1 &= \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ x_2 & x_1 & x_3 & x_4 \end{pmatrix} \\
E_2 &= \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ x_1 & x_2 & x_4 & x_3 \end{pmatrix} & E_3 &= \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ x_2 & x_1 & x_4 & x_3 \end{pmatrix} \\
E_4 &= \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ x_3 & x_4 & x_1 & x_2 \end{pmatrix} & E_5 &= \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ x_4 & x_3 & x_1 & x_2 \end{pmatrix} \\
E_6 &= \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ x_3 & x_4 & x_2 & x_1 \end{pmatrix} & E_7 &= \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ x_4 & x_3 & x_2 & x_1 \end{pmatrix}
\end{aligned} \tag{1}$$

leave the two relations true in $\mathbb{Q}$. One could show that these 8 are the *only* substitutions of the 24=4! which leave invariant *all* rational relations in $\mathbb{Q}$ among the roots. These 8 are *the Galois group of the equation in* $\mathbb{Q}$ and constitute a subgroup of the full group. That is, the group of an equation w.r.t. a field $\mathbb{Q}$ is the group of substitutions on the roots which leave invariant the rational relations with coefficients in $\mathbb{Q}$ (among the roots of the given equation).

One can say that the number of substitutions that leave all rational relations in $\mathbb{Q}$ invariant is a measure of our ignorance of the root because we cannot distinguish them under these 8 substitutions.

Now consider

$$x_1^2 - x_3^2 \equiv \sqrt{p^2 - 4q}. \tag{2}$$

We adjoin this radical to $\mathbb{Q}$, and form the field $\mathbb{Q}'$, which is the smallest fields containing $\mathbb{Q}$ and $\sqrt{p^2 - 4q}$. Since $x_1 + x_2 = 0$ and $x_3 + x_4 = 0$ we also have

$$x_1^2 = x_2^2, \quad x_3^2 = x_4^2.$$

We then notice that of the 8 substations listed in (1) only $E,\ E_1,\ E_2,\ E_3$ leave the $\mathbb{Q}'$-valued relation (2) invariant. Then these four substitutions, since they leave *every* true $\mathbb{Q}'$-valued rational relation among the roots invariant, are the Galois group of the original quartic equation over $\mathbb{Q}'$. These four comprise a subgroup of the eight-member Galois group of the equation.

Suppose next that we adjoin to $\mathbb{Q}''$ the quantity $\sqrt{\frac{1}{2}\left(-p - \sqrt{p^2 - 4q}\right)}$, thereby forming the field $\mathbb{Q}''$. Then

$$x_3 - x_4 = 2\sqrt{\frac{1}{2}\left(-p - \sqrt{p^2 - 4q}\right)} \tag{3}$$

is a rational relation in $\mathbb{Q}''$. This relation remains invariant only under the substitution E and E_1, but not under the rest of the eight. Thus, the group of the equation in $\mathbb{Q}''$ consists of these two substitutions, because every rational relation in $\mathbb{Q}''$ among the roots remains invariant under theses two substitutions. The two comprise the subgroup of the previous four-substitutions subgroup.

If we finally adjoint to $\mathbb{Q}''$ the quantity $\sqrt{\frac{1}{2}\left(-p + \sqrt{p^2 - 4q}\right)}$ we get $\mathbb{Q}'''$. In which we have

$$x_1 - x_2 = 2\sqrt{\frac{\left(-p + \sqrt{p^2 - 4q}\right)}{2}}$$

It is found that the only substitution leaving all the rational relations over $\mathbb{Q}'''$ invariant is just E (the trivial subgroup) – and this is the group of the equation over $\mathbb{Q}'''$.

Now Galois showed that when the group of an equation w.r.t. a given field is just E, then the roots of the equation are members of that field.

There is next a straightforward process for finding the roots by rational operations in $\mathbb{Q}'''$. Galois pointed out, however, that his work was not intended as an efficient practical method of solving equations. Yet the Galois theory shows that the general n^{th} - degree equation for $n > 4$ is not solvable by radicals whereas for $n \leq 4$ they are. (see the following essay).

Galois was born in the village Bourg-la-Reine near Paris, the son of the village mayor. Throughout his school years, Galois was hampered by teachers who discouraged his interest in mathematics.

Galois discovered mathematics with the reading of Legendre's textbook of Euclidean geometry at the age of 13. Finding his school algebra textbook boring, he started at the age of 14 to read the original memoirs of Lagrange and Abel, whose algebraic analyses were addressed to professional mathematicians. Galois tried twice (1827, 1829) to enter the École Polytechnique, but was refused admission for inability to meet the formal requirements of his examiners, who completely failed to recognize his genius. This failure drove him in upon himself and embittered him for the remainder of his short life.

In 1828, at the age of 17, Galois was already making discoveries of epochal significance in the theory of equations, discoveries whose consequences are not yet exhausted after almost two centuries. In 1829, he published his first paper, on continued fractions, and entered the École Normale to prepare himself to teach. At about this time he presented an abstract of his fundamental discoveries to **Cauchy** for presentation to the Academy of Sciences. Cauchy promised to present this, but forgot — and also lost the manuscript. Embittered and frustrated, Galois was drawn by democratic sympathy into the turmoil of the 1830 revolution. He was expelled from school and spent several months in prison. Shortly after his release, he was killed in a pistol duel with a friend: both men, having fallen in love with the same girl, decided the outcome by a gruesome version of Russian roulette. The night before, he wrote his scientific testament in the form of a letter to one of his friends. He was buried in the common ditch of the South Cemetery, so that today there remains no trace of the grave of Évariste Galois. His enduring monument is his collected works, 60 pages in all.

Hermann Weyl, a leading 20^{th} century mathematician, had this to say (1952):

"*If judged by the novelty and profundity of ideas it contains, it is perhaps the most substantial piece of writing in the whole literature of mankind*".

In 1846, **Joseph Liouville** published several of Galois' memoirs and manuscripts in his *Journal de mathématique.* The importance of Galois' ideas became apparent only after they were applied in 1870 by **Camille Jordan**, **Felix Klein** and **Sophus Lie**.

Galois and the Dawn of Abstract Algebra

*The first stirrings of modern abstract algebra began with investigations of the theory of equations, and studies of n-object permutations that arose in this theory. This line of work began with successive attempts to algebraically prove the fundamental theorem of algebra (**Euler**, **Lagrange**, **Laplace**).*

Lagrange's work, in particular (1771–3), introduced the use of symmetric functions of the roots of a general polynomial, and proved what later became known as Lagrange's theorem in group theory[57]*. Those early proofs of the Fundamental Theorem*[58] *suffered from the common flaw of assuming that the roots of any polynomial exist, in some sense.*

***Gauss** gave the first (almost) algebraic proof which escaped this apparent tautology, by means of the so-called "principle of continuation of identities"*[59]*. In the course of this proof (1815), Gauss introduced a congruence of polynomials modulo a given polynomial, thus paving the way to such abstract-algebra concepts as ideals, quotient rings, field extensions and splitting fields.*

***Cauchy** independently co-discovered a subset of these new concepts and methods (1815); and his own studies of symmetry permutation groups of*

[57] *Lagrange Theorem*: The order (# of elements) of a subgroup divides the order of the larger group.

[58] *Fundamental Theorem of Algebra*: every n-th order polynomial with real coefficients is factorizable into linear and quadratic factors.

[59] It states that if a function F of the n fundamental symmetric polynomials

$$\sigma_1 = \alpha_1 + \cdots + \alpha_n, \ldots, \qquad \sigma_n = \alpha_1 \cdot \alpha_2 \cdot \cdots \cdot \alpha_n$$

is identically zero, $F(\sigma_1, \ldots, \sigma_n) \equiv 0$, then this also holds for *any* n reals $\sigma_j = s_j$. Here α_j are the n putative roots.

algebraic functions led to what later came to be known as Cauchy's theorem of group theory[60]. *Additionally, Gauss' earlier work on the roots of unity and cyclotomic fields, together with* ***Abel****'s extensions, pioneered many related concepts, including what were later recognized as Galois groups (the Abelian, or commutative case).*

Galois*, having read the relevant works by* ***Legendre*** *and* ***Gauss****, made a complete study of finite fields, and introduced, for the first time, explicit definitions of groups, normal subgroups, field extensions and related concepts (albeit with different names, in some cases, from later nomenclature in what came to be known as "Galois' theory") His efforts to characterize the hidden structure of polynomials in terms of groups – especially with regard to the solvability-by-radicals of polynomials – were so complete and successful that the old "theory of equations" ended, in effect, with him, while giving birth to modern abstract algebra.*

Later (1843), ***Hamilton*** *managed to finally extend the field of complex numbers into the non-commutative (yet still associative) field of quaternions. Hamilton's approach – developed with physical applications in mind – led him to do what Galois and his followers did: namely to generalize ordinary addition and multiplication (and sometimes division) into abstract operations among a priori-undefined elements, thus breaking ground for modern axiomatics. Galois' work was continued by* ***Cayley****,* ***Jordan****,* ***Serret*** *and others*[61]*; Hamilton's quaternions were subsumed by Gibbs' vectors, but remained a cornerstone of modern algebra and re-entered physics in the guise of the Pauli matrices – describing quantum-mechanical spin – and was generalized to Clifford algebras in the context of quantum field theories and GTR. The abstract algebra pioneered bay Galois, Hamilton and their predecessors & followers, exercises a unifying effect within modern mathematics itself, and has also led to many important applications in modern physics.*

Here, we shall present a simplified modern synopsis of that part of Galois' theory dealing with the solvability by radicals of algebraic equations[62].

Let $f(x)$ be any n-th order polynomial over the field of rational numbers[63]*, with the coefficient of the highest power normalized to unity:*

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[60] *Cauchy's theorem*: a group of order $n = p \cdot m$, p prime, has a subgroup of order p

[61] Including **F. Klein**, **E. Moore**, **Hölder**, **L. Kronecker**, and **S. Lie**.

[62] Another branch of his work – that dealing with finite fields – has found modern applications in digital logic design and cryptography

[63] Galois theory applies to polynomials over *any* algebraic field, not just $\mathbb{Q}$.

$$f(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0, \qquad a_j \in \mathbb{Q}$$

where $\mathbb{Q}$ denotes the field of (real) rational numbers. By the fundamental theorem of algebra, $f(x)$ has n complex roots (not all of which need be distinct):

$$f(x) = (x - d_1) \cdot (x - d_2) \cdot \cdots \cdot (x - d_n), \qquad d_j \in \mathbb{C}$$

with $\mathbb{C}$ the field of complex numbers. Expanding this product, we find that the coefficients a_j are symmetric polynomials *in the roots $\{d_k\}$:*

$$a_0 = (-1)^n d_1 \cdot d_2 \cdot \cdots \cdot d_n, \ \ldots \qquad a_{n-1} = -d_1 - d_2 - \cdots - d_n.$$

These n-variable *polynomials are 'symmetric' in the sense that they remain invariant under arbitrary* permutations *of the n roots of $f(x)$. Some of these roots might be rational (i.e. lie in $\mathbb{Q}$).*

Denote a maximal, linearly-independent (over $\mathbb{Q}$) subset of the irrational *roots, if any, by $\{c_1, \ldots, c_m\}$ (we only include* distinct *irrational roots in this set). Note that if* all *roots of $f(x)$ are in $\mathbb{Q}$, $m = 0$ and the set is empty.*

It can be shown that any *symmetric rational function of the roots $\{d_j\}$ can be expressed as a rational function of the coefficients $\{a_j\}$ (which are themselves symmetric polynomials, as seen above.) Since a_j are in $\mathbb{Q}$, it follows that any symmetric rational $R(d_1, \ldots, d_n)$ has a numerical value in $\mathbb{Q}$. Galois posed the following question: are there any rational functions of $\{d_j\}$ which are* not *fully symmetric, yet nevertheless assume rational numerical values for a given polynomial $f(x)$? The answer is clearly in the affirmative; for example, if $f(x)$ has even a single rational root (say d_1), the rational function $R(d_1, \ldots, d_n) \equiv d_1$ is not invariant under all permutations, yet assumes a rational numerical value.*

Galois was thus led to associate to each polynomial a group: *The group*[64] *$\mathbb{G}(f)$ of all permutations of the n roots such that* a generic rational function *$R(d_1, \ldots, d_n)$ assumes a rational numerical value if, and only if, it is invariant under the permutation belonging to $\mathbb{G}(f)$. Clearly, $\mathbb{G}(f)$ is a subgroup of the group of* all *possible permutations of the n roots; the latter group is known as the* symmetric group *of n objects, and denoted S_n. It can be shown that, in fact, $\mathbb{G}(f) = S_n$ for a* generic *n-th order polynomial over $\mathbb{Q}$; but $\mathbb{G}(f)$ is a proper subgroup (i.e. smaller than S_n) for special choices of the coefficients a_j, or of algebraic relations among them. If all the d_j are rational, $R(d_1, ..., d_n)$*

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[64] To be more precise, this group should be denoted $\mathbb{G}_{\mathbb{Q}}(f)$ and referred to as the Galois group of $f(x)$ over $\mathbb{Q}$. However, we shall employ the simpler notation.

is a rational number for any rational function R, so clearly $\mathbb{G}(f)$ *is the trivial group*[65] $\{\mathbf{1}\}$, *consisting only of the trivial (identity) permutation*

$$\begin{pmatrix} 1 & 2 & \dots & n \\ 1 & 2 & \dots & n \end{pmatrix} \equiv \mathbf{1}.$$

There is another, equivalent way of determining the Galois group – an abstract-algebraic way, as follows. One defines an extension E *of the field* $\mathbb{Q}$, *such that* E *is also a field and all roots* d_j *belong to* E. *The basis irrational roots* $\{c_1, \dots, c_m\}$ *and their powers generate* E *over* $\mathbb{Q}$ *(if* $m = 0$, *we simply have* $E = \mathbb{Q}$*). This minimal extension field, denoted as* $E = \mathbb{Q}(c_1, \dots, c_m)$, *is a finite-dimensional vector space, because* $(c_j)^n$ *and higher powers can be expressed as linear combinations of lower powers by using* $f(c_j) = 0$.

An automorphism of the field E *is a one-to-one mapping* σ *of* E *onto*[66] *itself,* $x \to \sigma(x)$, *which preserves addition and multiplication:*

$$\sigma(x + y) = \sigma(x) + \sigma(y), \quad \sigma(xy) = \sigma(x)\sigma(y), \quad x \in E,\ y \in E.$$

It can be proven that the subfield $\mathbb{Q}$ *is invariant under any automorphism* σ: $\sigma(x) = x$ *when* $x \in \mathbb{Q}$. *Clearly,* σ *maps any root* d_j *of* $f(x)$ *into another such root*[67], d_j. *And since* σ *is one-to-one, it acts on the set of roots* $\{d_j\}$ *by permuting them. The set of all automorphisms of* E *is readily seen to be group, called* $Aut(E)$, *with the map* $x \to x$ *being the identity element* ($\sigma = \mathbf{1}$) *and group multiplication being defined by map composition:*

$$(\sigma_1 \cdot \sigma_2)x \equiv \sigma_1(\sigma_2(x)), \quad x \in E, \quad \sigma_1 \in Aut(E), \quad \sigma_2 \in Aut(E)$$

Since any $x \in E$ *is a linear combination of powers of roots, the permutation induced by any mapping* $\sigma \in Aut(E)$ *completely determines the action* $\sigma(x)$ *on all elements of* E. *It is thus possible to identify the group of automorphisms of the extension field with a subgroup of the permutations group* S_n. *It can be shown that this subgroup is exactly the Galois group:*

$$Aut(E) = \mathbb{G}(f)$$

This, then, is the abstract-algebraic way of defining the Galois group. It is very useful, because abstract entities involved in this approach – field extensions, automorphisms and the riches of finite-group theory – obey numerous,

[65] In this case the set $\{c_1, \dots, c_m\}$ is an empty set.

[66] That σ is "onto" means that for any $y \in E$, there exits an $x \in E$ that maps into it: $\sigma(x) = y$.

[67] Since $0 = \sigma(f(d_i)) = f(\sigma(d_i)) = 0$.

quite powerful theorems. Using these tools – which he pioneered himself – Galois was able to answer questions concerning the solvability of a polynomial equation $f(x) = 0$ by precisely mapping these questions into corresponding ones about the group $\mathbb{G}(f)$ and its subgroups.

For a general finite group G, let $G^{(1)}$ denote the set of commutators

$$xyx^{-1}y^{-1}, \quad x \in G, \quad y \in G.$$

Clearly the unit element $\mathbf{1}$ of G is also a member of $G^{(1)}$,since we may choose $x = y$ and then $xyx^{-1}y^{-1} = \mathbf{1}$. It is immediately seen that $G^{(1)}$ is a subgroup of G. One can repeat the procedure to form the subgroup $G^{(2)}$ of $G^{(1)}$ consisting of all commutators of element pair $x \in G^{(1)}, \quad y \in G^{(1)}$. If the repeated application of this procedure eventually yields the trivial, subgroup $G^{(r)} = \{\mathbf{1}\}$ after finite number of steps r, the original group G is said to be solvable.

Galois' two main theorems concerning polynomial solvability (specialized to the case of rational coefficients) can now be stated[68]*:*

(i) *A non-constant, n-th order polynomial $f(x)$ over $\mathbb{Q}$ is solvable by radicals if and only if, its Galois group $\mathbb{G}(f)$ is a solvable group;*

(ii) *For $n \geq 5$, a generic*[69] *n-th order polynomial $f(x)$ over $\mathbb{Q}$ has the Galois group $\mathbb{G}(f) = S_n$, i.e the full n-object permutation group.*

From the above definition of group solvability, it is a mere mechanical task to check whether any given finite group is solvable. The only catch is that it is often quite quite difficult to actually determine the group $\mathbb{G}(f)$ for a given polynomial $f(x)$. Once this is done, however, theorem (i) enables a straightforward determination as to whether $f(x)$ is solvable by radicals or not.

The correspondence between the solvability of $f(x)$ by radicals and the solvability of $\mathbb{G}(f)$ as a group is, in fact, more intimate than indicated by Theorem (i). If the finite sequence of nested subgroups of a solvable $\mathbb{G}(f) \equiv G^{(0)}$ is $G^{(0)}, G^{(1)}, ..., G^{(r)} = \{\mathbf{1}\}$, then the number of distinct sets $xG^{(j+1)} = \{xy_1, xy_2, ...\}$ where $x \in G^{(j)}$, and $y_1, y_2, \ldots$ range over all elements of $G^{(j+1)}$, is an integer, n_{j+1}; theses sets are called congruences or

[68] The first theorem exposes the reason for naming the just-described property of some groups "solvability"!

[69] By a "generic", or "general", polynomial is meant : any $f(x)$ except some special classes definable via algebraic relations among the coefficients.

cosets, and they form (for each j) an Abelian group.[70]*In Galois theory, it can then be shown that solving $f(x)=0$ by radicals (if possible at all) can be done by solving successive algebraic equations of orders $n_1, n_2, ..., n_r$. Furthermore, for n_j prime, the j-th of these algebraic equations can be written in the form $y^{n_j}=g$, where g is a function (of the roots of f(x)) whose group of symmetries is $G^{(j-1)}$. The procedure of extracting the n_j-th root of g is then a step in a sequence of field extensions that iteratively build up the field E introduced above.*

The combinations of (i) and (ii), plus the easily demonstrated fact[71] *that S_n is solvable for $n \leq 4$ and insolvable for $n \geq 5$, leads us immediately to Galois' celebrated result – that a general 5^{th} or higher-order polynomial is not solvable by radicals*[72]*.*

We conclude with several examples.

(1) *$n=1$: any 1^{st}-order (linear) polynomial is, of course, solvable via simple subtraction – even radicals are not needed! Thus if $x+a_0=0$, the solution is simply $x=-a_0$. And, indeed, $\mathbb{G}(f)$ is in this case the trivial group $\{\mathbf{1}\}$ (since $S_1=\{\mathbf{1}\}$ and $\mathbb{G}(f)$ is a subgroup of S_1). This group is (trivially) solvable.*

(2) *$n=2$: The quadratic equation $x^2+a_1x+a_0=0$ is always solvable by radicals: $d_{1,2}=\frac{1}{2}(-a_1 \pm \sqrt{(a_1)^2-4a_0})$. If $(a_1)^2-4a_0$ happens to be a perfect square in $\mathbb{Q}$, then both roots are rational numbers and, as explained above, $\mathbb{G}(f)=\{\mathbf{1}\}$ in this case - a solvable group (as in example (1)).*

But if $(a_1)^2-4a_0$ is not a perfect square, $\{d_j\}$ are both irrational, and it is readily seen that the only rational functions of (d_1, d_2) over $\mathbb{Q}$ that are rational numbers are symmetric rational functions – which, for $n=2$, means rational functions of $(d_1 \cdot d_2, \quad d_1+d_2)$. Thus, by the first definition of the Galois group, $\mathbb{G}(f)=S_2$ in this case. S_2 consists of $2!=2$ permutations:

$$S_2=\left\{\mathbf{1}, \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}\right\},$$

[70] The set of cosets $xG^{(j+1)}$, $x \in G^j$, is a group by virtue of the fact that $G^{(j+1)}$ is a *normal* subgroup of $G^{(j)}$, i.e. $x^{-1}yx \in G^{(j+1)}$ for any $x \in G^{(j)}$, $y \in G^{(j+1)}$.

[71] In the examples below it will be shown that S_1, S_2, S_3 and S_4 are *solvable* groups, and that S_5 is insolvable

[72] For rational coefficients. However, this also holds for a general $n \geq 5$ polynomial over $\mathbb{R}$. This can be seen by either noting that $\mathbb{Q} \subset \mathbb{R}$, or by applying Galois' theory but replacing $\mathbb{Q}$ with the field of rational functions of the coefficients $\{a_0, a_1, \ldots, a_{n-1}\}$ (of $f(x)$) over $\mathbb{Q}$.

where $\mathbf{1}$ *is again the trivial permutation:*

$$\mathbf{1} = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}.$$

S_2 *is clearly an abelian group, so its commutators are all* $\mathbf{1}$*;* $\mathbb{G}(f) = S_2$ *is, once again, a solvable group. Thus* $\mathbb{G}(f)$ *is always solvable for* $n = 2$ *– confirming Galois' result.*

(3) $n = 3$*: Theorem (ii) states that for a general* 3^{rd} *order polynomial,* $\mathbb{G}(f) = S_3$*. In particular, this holds for* $f(x) = x^3 - 2$*. This polynomial is easily solved by radicals, with three complex roots* $d_1 = \sqrt[3]{2}$, $d_2 = \omega(\sqrt[3]{2})$, $d_3 = \omega^2(\sqrt[3]{2})$*, where* $\omega = \frac{-1 \pm i\sqrt{3}}{2}$*. We now use the second (abstract algebra) definition of the Galois group: any permutation of these roots is an automorphism of the extension field* E*. For instance, the permutation*

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$$

maps ω *into its complex-conjugate* $\omega^2 = \omega^*$*, and vice versa, leaving* $\sqrt[3]{2}$ *invariant; it thus amounts to redefining* $i = \sqrt{-1}$ *as* $-i$*, which clearly preserves addition and multiplication in the field* $E = \mathbb{Q}(\omega, \omega^*, \sqrt[3]{2})$*. Likewise, the cyclic permutation* $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ *merely multiplies each root by* $e^{2\pi i/3}$*, and is equivalent to redefining the "canonical" root,* $d_1 = \sqrt[3]{2}$*, to be* $\omega\sqrt[3]{2}$*; and similarly with the other* $3! - 3 = 3$ *nontrivial permutations. Thus indeed* $\mathbb{G}(f) = S_3$ *for the particular polynomial* $x^3 - 2$*. By working out all two-element commutators in*

$$S_3 = \left\{ \mathbf{1} = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \right\}$$

we easily find:

$$S_3^{(1)} = \left\{ \mathbf{1}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \right\}$$

This is the subgroup of cyclic permutations, and is abelian; hence $S_3^{(2)} = \{\mathbf{1}\}$*, and* $S_3 = \mathbb{G}(f)$ *is therefore solvable – in agreement with Galois result.*

In this example – and in fact for the general cubic $f(x) \equiv 0$, except special cases – the sequence of nested normal subgroups is:

$$G^{(0)} = S_3 \quad \text{(a group of order 6, i.e. having 6 elements)}$$

$$G^{(1)} = A_3 = \text{“alternating group”} = \left\{\mathbf{1}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}\right\} = \text{subgroup of order 3;}$$

$$G^{(2)} = \{\mathbf{1}\}.$$

The order ratios are nothing but the indices $\{n_j\}$ mentioned above: $n_1 = 6/3 = 2, \quad n_2 = 3/1 = 3$. And as these indices are both prime, Galois theory tells us that the general cubic may be solved by taking a square root and then a cubic root (with rational operations before, after and between these radical operations corresponding to the standard Cardano-Tartaglia solution.

A more cumbersome algorithm having the same radical sequence – an algorithm, in fact, directly related to Galois theory – is as follows.

Let $\alpha,\ \beta,\ \gamma$ denote the roots of a general cubic. The three fundamental symmetric functions:

$$\sigma_1 = \alpha + \beta + \gamma$$

$$\sigma_2 = \alpha\beta + \beta\gamma + \alpha\gamma$$

$$\sigma_3 = \alpha\beta\gamma$$

are just the (rational) non-leading coefficients of $f(x)$ (up to signs). Each σ_j has as its group the full S_3, and any *function with this invariance group can be expressed as a rational function of $\sigma_1,\ \sigma_2,\ \sigma_3$ over $\mathbb{Q}$.*

An example of a function of $\alpha,\ \beta,\ \gamma$ which corresponds to the subgroup $G^{(1)} = A_3$, is:

$$\tau \equiv \alpha^2\beta + \beta^2\gamma + \gamma^2\alpha$$

Galois theory predicts that τ satisfies a quadratic equation over the rationals – and indeed, some algebra shows that

$$\tau = \frac{1}{2}\left(A \pm \sqrt{B}\right), \quad \text{where}$$

$$A = \alpha^2\beta + \beta^2\gamma + \gamma^2\alpha + \alpha\beta^2 + \beta\gamma^2 + \gamma\alpha^2 =$$

$$= \frac{1}{3}(\alpha+\beta+\gamma)^3 - \frac{1}{3}(\alpha^3+\beta^3+\gamma^3) - 2\alpha\beta\gamma,$$

$$B = (\alpha-\beta)^2(\alpha-\gamma)^2(\beta-\gamma)^2.$$

A and B are fully symmetric, i.e. their group is S_3, and thus are rational functions of $\sigma_1,\ \sigma_2,\ \sigma_3$; therefore A, B are rational numbers and thus τ solves a rational quadratic equation – as predicted by Galois theory. It can likewise be shown that $\alpha,\ \beta,\ \gamma$ can be obtained from τ by extracting a third root of a rational function of τ over the extension field $\mathbb{Q}(\sqrt{B})$.

(4) *$n = 4$: The generic 4^{th}-order polynomial has $\mathbb{G}(f) = S_4$, and it has been known for centuries that a general quartic is solvable by radicals. This is in agreement with Galois theory, because S_4 is a solvable group: this is proven by constructing the sequence of groups $S_4^{(1)},\ S_4^{(2)},\ldots$ (as done above for S_3).*

For a general quartic $f(x)$, the sequence of commutator subgroups over the extension field $\mathbb{Q}(\sqrt{B})$ is as follows:

$G^{(0)} = S_4$, of order $4! = 24$; $G^{(1)} = A_4$ = set of even *permutations of order (2).*

$$G^{(2)} = \left\{\mathbf{1},\ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix},\ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix},\ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}\right\},$$

of order 4; and then,

$$G^{(3)} = \left\{\mathbf{1},\ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}\right\},$$

is of order 2.

The indices are:

$$n_1 = 24/12 = 2; \quad n_2 = 12/4 = 3; \quad n_3 = 4/2 = 2.$$

Thus the general quartic could *be solved by solving a* quadratic*, then a cubic, and finally another quadratic.*

But the well-known Ferrari Solution of quartic proceeds by first *solving an auxiliary cubic and* then *two successive quadratics. This corresponds to a different sequence of groups, not invoking successive commutator; one of the groups is not a normal subgroup of the preceding one. This illustrates the fact* *the solutions by radicals suggested by Galois theory, are quite cumbersome – one can usually do better!*

(5) $n = 5$: S_5 *is not a solvable group. Indeed, by computing all commutators one can verify that* $S_5^{(1)} = A_5$, *the subgroup of all even permutations; and that* $S_5^{(2)} = A_5^{(1)} = A_5$, *so that* $S_5^{(r)} \neq \{\mathbf{1}\}$ *for any natural number* r.

Hence S_5 *is insolvable, and the general quintic equation is not solvable by radicals.*

A concrete example of a quintic not solvable by radicals is

$$f(x) = x^5 - 4x + 2,$$

for which the abstract "machinery" of Galois theory can be used to prove that, indeed, $\mathbb{G}(f) = S_5$. *But for*

$$f(x) = x^5 - 2, \qquad d_j = \sqrt[5]{2}e^{2\pi ij/5} \qquad (0 \leq j \leq 4)$$

In this case $f(x)$ *is solvable by radicals, and indeed* $\mathbb{G}(f)$ *is a 20 - element, solvable subgroup of the 120-element* S_5 *permutation group.*

This order-20 subgroup M_{20} *of* S_5 *is the so-called metacyclic group for the case* $n = 5$; *i.e. it is the subgroup of substitutions*

$$j \to rj + g \mod 5, \qquad r \in \{1\ ,2\ ,3\ ,4\},\ g \in \{0, 1, 2, 3, 4\}.$$

This is not a normal subgroup, so unless $\mathbb{G}(f)$ *is this metacyclic group – it cannot be used as part of a Galois group-sequence. However, even for an* $f(x)$ *for which* $\mathbb{G}(f) = S_5$, *it can be proven that a function of the roots having* M_{20} *as its group satisfies an algebraic equation (with rational coefficients) of* 6^{th} *order (since* $120/20 = 6$ *is the index of* M_{20} *in* S_5*).*

In some special cases this sextic is solvable by radicals – in which case, so is the original quintic. The $\mathbb{G}(f)$ *is again* M_{20}, *as occurred in the case* $f(x) = x^5 - 2$.

1829–1832 CE **Nicolai Ivanovitch Lobachevsky** (1793–1856, Russia). A geometer of great originality. Pioneer of modern geometries which deal with spaces other than Euclidean.[73] Published (1829) the first account of non-euclidean geometry to appear in print.

This revolutionary development marked the liberation of geometry from its traditional mold established by the Greeks. A deep-rooted and centuries-old conviction that there could be only one possible geometry was shattered, and the way opened for the creation of many different systems of geometry. Moreover, it became apparent that geometry is not necessarily tied to actual physical space as long as its postulates are self-consistent. And as in other instances, it turned out, less than a century later, that these "artificial geometries" are not less physical than the Euclidean geometry. **Clifford** (1845–1879) called Lobachevsky "the Copernicus of geometry".

Lobachevsky was born in Makariev, Nizhniy Novgorod. His father died around 1800 and his mother, who was left in poor circumstances, removed to Kazan with her three sons. In 1807 Nicolai entered the University of Kazan, then recently established. In 1823 he rose to a rank of a full professor of mathematics and retained the chair until 1846. His first contribution to non-Euclidean geometry is believed to have been given in a lecture at Kazan in 1826, but the subject is also treated in many of his memoirs.

Gauss (1777–1855) and **Janos Bolyai** (1802–1860, Hungary) share with Lobachevsky the credit for the discovery of non-Euclidean geometry. Although Gauss failed to publish anything on the matter throughout his life, there is ample evidence to show that he was first to reach penetrating conclusions concerning the parallel postulate.

Bolyai published his findings in 1832 in an appendix to a mathematical work of his father. Because of language barriers and the slowness with which information on new discoveries traveled in those days, Lobachevsky's work did not become known in Western Europe for some years[74].

1829–1841 CE **Jacques Charles Francois Sturm** (1803–1855, Switzerland and France). Mathematician and physicist. Made major contributions to

[73] For further reading, see:

- Brannan, D.A. et.al., *Geometry*, Cambridge University Press, 1998, 497 pp.

[74] In 1824, **F.A. Taurinius** (1794–1874, Germany) communicated to Gauss two monographs on non-Euclidean geometry. Earlier, in 1817, **F.K. Schweikert** (1780–1859, Germany), discussed his ideas with Gauss and is also known to have developed a non-Euclidean geometry.

the theory of algebraic and differential equations [Sturm's theorem[75], Sturm-Liouville equation]. Made the first accurate determination of the velocity of sound in water (1826).

Sturm was born in Geneva. After completing his studies at the Geneva Academy, he became (1823) a tutor to the youngest son of Mme de Staël at the Château of Coppet near Geneva. There he met the Duke Victor de Broglie[76]. He then accompanied the Duke to Paris and through him was able to enter the capital's scientific circles. He became a French citizen (1833). In Paris he met Arago, Ampère, Gay-Lussac, Dulong and Fourier. Upon the death of Ampère, he was elected to the vacant seat in the Académie des Sciences (1836), and in 1838 he became a professor of analysis and mechanics at the École Polytechnique, and succeeded Poisson to the chair of mechanics there (1840).

Around 1851 Sturm's deteriorating health obliged him to arrange for a substitute at the Sorbonne and at the École Polytechnique. Four years later he died in Paris. Sturm also made contributions to experimental and mathematical physics in the fields of analytical mechanics, optics, heat conduction and the study of vision.

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[75] *Sturm's Theorem* shows how to find for any equation, by rational methods, the exact number of real roots which lie within a given range of values. (**Descartes**, **Newton**, **Lagrange**, **Fourier** and **Cauchy** had tried to find suitable criteria to decide whether a root of a polynomial lies in a given interval of the domain of definition.) Given a polynomial of degree n, Sturm defined a chain of $n+1$ functions $f(x), f'(x), f_2(x), \dots, f_n(x)$ where $f'(x)$ is the derivative, $f_2(x)$ is the remainder of the division of $f(x)$ by $f'(x)$, $f_3(x)$ is the remainder of the division of $f'(x)$ by $f_2(x)$ etc. Substituting for x a particular value a in the polynomials of Sturm's chain gives a sequence of real numbers: $f(a), f'(a), f_2(a), \dots, f_n(a)$. If two consecutive numbers $f_i(a)$ and $f_{i+1}(a)$ in this sequence have different signs, one speaks of a *sign change*. Let $W(a)$ denote the number of sign changes in the Sturm's chain for a value $x = a$. Sturm's theorem then states: "Let $f(x)$ be a polynomial with only simple zeros, where $a < b$ and $f(a) \neq 0$, $f(b) \neq 0$; then $\{W(a) - W(b)\}$ is equal to the number of zeros of the polynomial in the closed interval $[a, b]$".

[76] Victor Claude, Prince de Broglie was executed at Paris in June 1794. His son, the Duke Achille Charles Léonce Victor de Broglie (1785–1870) escaped with his mother to Switzerland, where they remained until the fall of Robespierre. In 1816 he was married to the daughter of **Madame de Staël**. In 1832, he took office as minister for foreign affairs. His son, Jacques Victor Albert (1821–1901), was a prime minister of France in 1877. Jacques' grandson was the physicist **Louis Victor de Broglie** (1892–1987).

1829–1855 CE **Thomas Graham** (1805–1869, Scotland). Chemist. A founder of *physical chemistry*. Conducted research on gases and solutions. In 1829 he formulated *Graham's law of diffusion*[77], which explains how two gases mix with each other. He also did pioneering work with colloids[78], founding the science of *colloidal chemistry* (1850).

Graham was born in Glasgow. In 1819 he entered the University of Glasgow with the intention of becoming a minister of the Church, but 'converted' to experimental science and concentrated his studies on molecular physics, a subject which formed the main preoccupation throughout his life. He graduated in 1824, and in 1837 was appointed to the chair of chemistry in University College, London. In 1855 he became Master of the Mint.

1829–1858 CE **Isambard Kingdom Brunel** (1806–1859, England). Railway and bridge engineer, and naval architect. One of the greatest English engineers of the 19^{th} century. Son of **Marc Isambard Brunel** (1769–1849). Took a leading part in the systematic development of ocean steam navigation. Designer and builder of railroads, bridges, tunnels, steamships and docks.

Brunel studied in Paris (1820–1823) and during 1823–1828 assisted his father in the Thames-tunnel project. First designed the Clifton suspension bridge over the Avon (1829; completed 1864). In 1833 he became chief engineer of Great Western Railway and constructed all its viaducts, bridges and tunnels, including the Royal Albert bridge across the River Tamar into Cornwall. During 1838–1845 he designed two highly successful steamships for regular transatlantic service: the *Great Western* (1838) was a wooden steamship, measuring 72 m long and 11 m wide with two huge side wheels that drove it at a speed of 9 knots. The *Great Britain* (1845) was the first large iron-hulled screw-driven steamship. Then in 1853 he began the construction of the *Great Eastern*, the largest steamship of its time[79] (1858).

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[77] The ratio of speeds at which two different gases diffuse is inverse to the ratio of the square roots of the gas densities. The same law applies to the flow of gas through a small aperture (*effusion*).

[78] Colloids: tiny particles of one material evenly distributed in another.

[79] It measured 211 m long, 26 m wide with a total tonnage of 18,918 tons, accommodating 4000 passengers. The *Great Eastern* was intended to show the full potentialities of the iron steamship by carrying enough fuel for a voyage to Australia and back, out of the Cape and home via the Horn. This was a bold attempt to overcome the great obstacle to the development of steamships, namely the fact that coal took up so much space and there was no room for the other commodities less profitable than passengers and mail. As an advertisement of the structural possibilities of iron, the ship was a great success; as a demonstration of the economic use of coal it was a dismal failure – it did not attract enough

1830 CE **George Peacock** (1791–1858, England). Mathematician. Was first to study the fundamental principles of algebra and its structure and pass from 'symbolized arithmetic' to 'symbolic algebra'. In 1830 he published *Treatise on Algebra* which attempted to give algebra a logical treatment comparable to Euclid's *Elements*. First to define and introduce *symbolic algebra* as the science which treats the combinations of arbitrary signs and symbols by means defined through arbitrary and consistent laws. As an undergraduate at Cambridge he made friends with John Herschel and Charles Babbage and together they formed the Analytical Society whose aims were to bring the advanced continental methods to Cambridge. In 1836 he was appointed professor of geometry and astronomy at Cambridge.

Peacock was followed by **Duncan Farquharson Gregory** (1813–1844, England, 1840), **Augustus de Morgan** (1806–1871, England, 1860), **George Boole** (1815–1864) and finally **Hermann Hankel** (1839–1873, Germany, 1867). These studies led to the liberation of algebra (as in geometry) and opened the floodgates of modern abstract algebra. Thus, it seemed inconceivable in the early 19^{th} century that there could exist *non-commutative* algebras. However, **Hamilton** and **A. Cayley** applied it soon enough to quaternions and matrices.

The founders of *quantum mechanics* showed in the 1920's that the atoms and electrons must live by the rules of a non-commutative algebra. Later on, non-associative algebras, such as Jordan algebras and the Lie algebras, were introduced.

Abstract Algebraic Structures

The modern abstract point of view requires a pure science to be founded on postulates (assumptions) about undefined elements, which are not necessarily numbers or points but abstractions (elements), potentially capable of varied

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passengers to pay the enormous operation costs; it was used (1866) to lay the first transatlantic telegraph cable. In 1881 it was sold for scraps.

During the late 1800's, steel began to replace iron for ships. Steel ships were stronger and lighter than iron ones. In 1881, the *Servia*, a British vessel, became the first all-steel passenger liner to cross the Atlantic.

interpretations consistent with the basic assumptions. With this in mind, one might begin with systems in which a set of undefined elements is given, as well as two operators between these elements, $\oplus$ and $\otimes$. These symbols are used to suggest some kinship with ordinary addition and multiplication, although in different concrete realizations and interpretations might be considerably different from the usual ones. Then our postulate set can include those 'laws' or 'properties' such as closure, commutativity, and associativity for both $\oplus$ and $\otimes$, and also distributivity of $\otimes$ w.r.t. $\oplus$.

One of the simplest sets of abstract elements is a modulus: *a set S of numbers such that the sum and difference of any two members of S are themselves members of S, i.e. $m \in S$, $n \in S \Rightarrow (m \pm n) \in S$. The elements of a modulus need not necessarily be integers or even rational: they may be complex numbers or quaternions.*

The single number 0 *forms a modulus (the* null modulus*). For any modulus S and $a \in S$ we have $a - a = 0 \in S$, and also: $a + a = 2a \in S$. Repeating this argument we see that $na \in S$ for any integer n. More generally, $a \in S$, $b \in S$ implies $xa + yb \in S$ for any integer x, y. Thus the set of values of $xa + yb$ also forms a modulus.*

It can be shown that $xa + yb$ is the set of multiples of $d = (a, b)$, the greatest common divisor *of a and b. But a number representable as $ax + by$ is, per definition,* linearly dependent on a and b. *Clearly, the property of linear dependence on a and b is preserved by addition, subtraction and multiplication by a number and is not affected by interchanging a and b. Indeed,*

$$(ax_1 + by_1) \pm (ax_2 + by_2) = a(x_1 \pm x_2) + b(y_1 \pm y_2)$$

and

$$\lambda(ax + by) = (\lambda x)a + (\lambda y)b.$$

We shall next exhibit five fundamental algebraic structures which have a wide application in the physical world.

Before we discuss algebraic structures, let us consider first the subject of complex numbers.

When operating with ordinary real numbers, it is noticed that the square root of negative numbers has no meaning, because the square of every real number is positive or zero. However, the solution of quadratic and cubic equations compelled mathematicians to regard expressions of the form $a + b\sqrt{-1}$. If it is assumed that these 'imaginary' numbers are subject to the same laws (axioms) of common arithmetical operations as the ordinary numbers, then *all square roots of negative numbers can be expressed in terms of the quantity* $i = \sqrt{-1}$, *and the result of arithmetical operations performed any finite number of times on real or imaginary numbers can always be expressed in the form* $a + bi$, *where* a *and* b *are real numbers.*

Clearly, this definition of imaginary numbers runs counter to common sense: First it was stated that expression $\sqrt{-1}$, $\sqrt{-2}$, *and so forth, have no meaning, and then it was proposed that these meaningless expressions be called imaginary numbers. This circumstance caused many mathematicians of the 17*th *and 18*th *century to doubt the validity of the use of complex numbers. However, these doubts were dispelled at the beginning of the 19*th *century, when a geometrical interpretation was found for the complex numbers by points in a plane.*

Another purely arithmetical foundation of the theory of complex numbers was discovered by ***Hamilton*** *(1833) who noted that the complex number* $a+bi$ *can be viewed simply as an ordered pair of real numbers, subject to the addition and multiplication rules*

$$(a, b) + (c, d) = (a + c, b + d);$$

$$(a, b)(c, d) = (ac - bd, ad + bc).$$

For example, we have

$$(2, 3) + (1, -2) = (3, 1) \qquad (2, 3)(1, -2) = (8, -1)$$

$$(3, 0) + (2, 0) = (5, 0) \qquad (3, 0)(2, 0) = (6, 0)$$

These examples show, in particular, that the arithmetical operations on pairs with a zero in the second place reduce to the same operations on their first terms, so that the arithmetic of real numbers is just a special case of the arithmetics of complex numbers. Indeed, if we introduce the notation i *for the pair* $(0, 1)$ *then we have*

$$(a, b) = a(1, 0) + b(0, 1) = a + bi$$

$$i^2 = (0, 1)(0, 1) = (-1, 0) = -1$$

i.e., we have the usual notation for complex numbers.

One then defines the conjugate *of a complex number by* $(a, b)^* = (a, -b)$, *the square of the* norm *of* (a, b):

$$(a, b)(a, -b) = (a^2 + b^2, 0) = ||a, b||^2$$

and the multiplicative inverse *of* (a, b) *as*

$$\frac{(a, b)^*}{||a, b||^2},$$

whenever (a, b) *is nonzero.*

Sylvester (1852) noted that complex numbers could alternatively be represented by matrices[80]

$$\begin{pmatrix} a & b \\ -b & a \end{pmatrix} = a\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + b\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

Since

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix},$$

the matrix

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

plays the 'role' of $i = \sqrt{-1}$.

Thus it would be improper to state that complex numbers were invented so that negative numbers would have square roots, or, equivalently, so that all quadratic equations would have solutions. It is certainly true that they do provide these, as well as many other interesting and useful properties.

Before negative numbers were invented, mathematicians would say that the equation $x + 1 = 0$ *has no solution. Similarly, before complex numbers were introduced, mathematicians could state that* $x^2 + 1 = 0$ *has no solution.*

The real reason that complex numbers gained acceptance in mathematical circles, has to do with cubic equations. It was recognized that all cubic equations have at least one real root. However, when the cubic formula was discovered, it was found that sometimes complex numbers were needed as an intermediate step in finding that one real root. One could not just dismiss a cubic as having no solutions; but at the same time maintain that real numbers were insufficient to solve it.

Operations on the complex numbers can be used to describe geometrical operations on the plane. For instance, multiplication by a real number corresponds to scaling of the plane. Multiplication by complex numbers with

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[80] The matrices

$$\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$$

form a group w.r.t. matrix multiplication. This group is *isomorphic* to the multiplicative group of non-zero complex numbers

$$\begin{pmatrix} a & b \\ -b & a \end{pmatrix} \leftrightarrow a + bi.$$

a modulus ("length") of unity corresponds to rotation *of the plane. Adding complex numbers corresponds to* translation *of the plane. Thus,* transformation *of the plane is easily modeled with complex numbers.*

By 1830, it was well established that complex numbers behave algebraically like vectors in a plane.[81]

Let us investigate this interesting observation in more detailed: consider the complex numbers

$$z = x + iy = re^{i\theta}$$
$$iz = -y + ix = re^{i(\theta+\pi/2)}.$$

Multiplication by i then rotates the 'vector' z by 90° counterclockwise. Two consecutive operations of this kind rotate the vector by 180°, yielding a vector that is anti-parallel to the original vector z.

Now suppose that we start from the plane vector $\boldsymbol{r} = x\boldsymbol{e}_x + y\boldsymbol{e}_y$ and cross it from the left by $\boldsymbol{e}_z$

$$\boldsymbol{e}_z \times \boldsymbol{r} = -y\boldsymbol{e}_x + x\boldsymbol{e}_y.$$

This operation is again rotating the vector by 90° clockwise. Both cases can be represented by the coordinate transformation

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ x \end{bmatrix}.$$

We may then write the symbolic equation in the xy plane

$$i \Leftrightarrow \boldsymbol{e}_z \times .$$

Moreover, the product of two complex numbers $\overline{f} = a - ib, \quad g = c + id$ can be written as

$$\overline{f}g = (ac + bd) + i(ad - cb) \Leftrightarrow (\boldsymbol{f} \cdot \boldsymbol{g}) + i\{\boldsymbol{f} \times \boldsymbol{g}\},$$

where

$$\boldsymbol{f} = a\boldsymbol{e}_x + b\boldsymbol{e}_y,$$
$$\boldsymbol{g} = c\boldsymbol{e}_x + d\boldsymbol{e}_y,$$
$$\boldsymbol{f} \times \boldsymbol{g} = \boldsymbol{e}_z\{\boldsymbol{f} \times \boldsymbol{g}\}.$$

When the two vectors are perpendicular, the real part of their product vanishes. If, on the other hand they are parallel, the imaginary part of their product vanishes.

[81] **Aristotle** knew that forces can be represented as vectors and that the combined action of two forces can be obtained by the 'parallelogram law'. **Simon Stevin** employed this law in problems of statics, and **Galileo** stated the law explicitly.

This notion can be further extended into the realm of the calculus: Let

$$\nabla = \frac{\partial}{\partial x} + i\frac{\partial}{\partial y}$$

operate on $S = u + iv$. *Then* $\overline{\nabla}$ *gives the divergence and the rotation of a vector* $\boldsymbol{S} = u\boldsymbol{e}_x + v\boldsymbol{e}_y$

$$\begin{aligned} \overline{\nabla}S = \left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)(u+iv) &= \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + i\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \\ &= div\ \boldsymbol{S} + i\{curl\ \boldsymbol{S}\}_z \end{aligned}$$

If S *is analytic, the Cauchy-Riemann relations*

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}, \qquad \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

enable us to write

$$\overline{\nabla S} = \left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)(u - iv) = 0$$

$$\nabla S = \left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)(u + iv) = 0$$

The complex representation of plane vectors can be extended to plane tensors. Consider the symmetric plane dyadic

$$\overleftrightarrow{A} = a_{11}\boldsymbol{e}_x\boldsymbol{e}_x + a_{12}(\boldsymbol{e}_x\boldsymbol{e}_y + \boldsymbol{e}_y\boldsymbol{e}_x) + a_{22}\boldsymbol{e}_y\boldsymbol{e}_y,$$

having the scalar invariant $A_1 = a_{11} + a_{22}$. *Let us set our former correspondence* $\boldsymbol{e}_y = (\boldsymbol{e}_z\times)\boldsymbol{e}_x = i\boldsymbol{e}_x$. *We may then recast* $\overleftrightarrow{A}$ *in the symbolic form*

$$\overleftrightarrow{A} = (a_{11} - a_{22})\boldsymbol{e}_x\boldsymbol{e}_x + 2ia_{12}\boldsymbol{e}_x\boldsymbol{e}_x = A_i\boldsymbol{e}_x\boldsymbol{e}_x,$$

where $A_i = (a_{11} - a_{22}) + 2ia_{12}$ *is called the complex invariant of* $\overleftrightarrow{A}$. *We can then use it as a complex representation of the dyadic* $\overleftrightarrow{A}$.

1. Group

A group G is a set of elements a, b, c, etc. (objects, symbols, quantities) for which a composition law $*$ between any two elements (ordered pair) has been uniquely defined and for which the following four conditions are fulfilled.

(i) *Closure:* If a belongs to G and b belongs to G, then $a * b$ also belongs to G.

(ii) *Associativity:* For any three elements a, b, c in G

$$(a * b) * c = a * (b * c).$$

(iii) *Existence of a unit element (the identity):* There exists an element e in G such that operating with e has no effect on a, namely $e * a = a * e = a$ for every a of G. [Actually the slightly weaker condition of the *right identity* $a * e = a$ would also suffice.]

(iv) *Existence of the inverse element:* Corresponding to each a of G there exists an element denoted by a^{-1} such that $a^{-1} * a = a * a^{-1} = e$ for every a in G. [Again, the existence of a *right inverse* only could suffice.]

Of course, $a * b \neq b * a$ in general. A group is said to be *Abelian* or *commutative* if in addition to the group axioms (i)–(iv) we also have $a * b = b * a$ for any pair of elements in G. It is usual in this case to call the composition law *addition* and write $a + b$ for $a * b$ which is then called the *sum* of the element a and b. The identity is then denoted by 0 and is called the *zero* element while the inverse of a is called the *negative* of a and denoted by $-a$. Thus, for an Abelian group, the axioms (i)–(iv) take the form:

(i) $a \in G, \quad b \in G \Rightarrow a + b = c \in G; \quad a + b = b + a$

(ii) $a + (b + c) = (a + b) + c$

(iii) $a + 0 = a \quad$ for every $a \in G$

(iv) $a + (-a) = 0 \quad$ for every $a \in G$

Examples of groups:

(1) The set of all integers with ordinary addition as the composition law.

(2) *The set of all $n \times m$ matrices A with complex elements a_{ij} under the addition law*

$$(A+B)_{ij} = a_{ij} + b_{ij} = (A)_{ij} + (B)_{ij}.$$

The negative of A is defined by $(-A)_{ij} = -a_{ij} = -(A)_{ij}$, so that $A + (-A) = (a_{ij} + (-a_{ij})) = (0) = 0$, where $0 = (0)$, the null matrix, all of whose elements are zero, is the zero element of the group.

(3) *The set of permutation of n objects. Such a permutation may be written*

$$S = \begin{pmatrix} 1 & 2 & 3 & \dots & n \\ s_1 & s_2 & s_3 & \dots & s_n \end{pmatrix} = \begin{pmatrix} k \\ s_k \end{pmatrix},$$

$k = 1, 2, \dots n$: this means that the object in cell 1 was sent to cell s_1, the object in cell 2 was sent to cell s_2 etc. under the permutation S. Observe that S remains the same in permuting its columns in any way. Consider the permutation

$$T = \begin{pmatrix} k \\ t_k \end{pmatrix} \equiv \begin{pmatrix} s_k \\ t_{s_k} \end{pmatrix}.$$

The product

$$TS = \begin{pmatrix} k \\ t_k \end{pmatrix} \begin{pmatrix} k \\ s_k \end{pmatrix}$$

means that we first carry out the permutation S that sends the object in cell k to cell s_k, and then from s_k to t_{s_k}, namely

$$TS = \begin{pmatrix} k \\ t_{s_k} \end{pmatrix}, \quad k = 1, 2, \dots n.$$

In general, $TS \neq ST$. The permutation

$$E = \begin{pmatrix} k \\ k \end{pmatrix}$$

which leaves the objects where they are is the unit element, and

$$S^{-1} = \begin{pmatrix} s_1 & s_2 & s_3 & \dots & s_n \\ 1 & 2 & 3 & \dots & n \end{pmatrix}.$$

The algebra of permutations can alternatively be executed with the aid of matrices proper: A permutation on n objects is represented by *an $n \times n$ orthogonal matrix (of determinant ± 1) in which the column (k, a_k) in*

$$\begin{pmatrix} 1 & 2 & 3 & \dots & k & \dots & n \\ a_1 & a_2 & a_3 & \dots & a_k & \dots & a_n \end{pmatrix}$$

is represented by placing unity in the k^{th} row and the a_k^{th} column of the matrix, and zero elsewhere in that row. Thus, for example

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 2 & 5 & 3 \end{pmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

(4) *The set of all non-singular complex square matrices of order n under matrix multiplication.*

(5) *The set of all non-singular complex quaternions under quaternion multiplication.*

2. Ring

A ring R is a set of elements a, b, c etc. which is closed under two distinct composition laws between any two elements and for which the following four conditions are fulfilled:

(i) *R is an additive abelian (commutative) group*

(ii) $a \in R$; $\quad b \in R \Rightarrow ab \in R$ $\quad$ *for any a, b (Closure under 'multiplication')*

(iii) $a(bc) = (ab)c$ $\quad$ *for any a, b, c of R (Associativity for 'multiplication')*

(iv) $a(b + c) = ab + ac$ $\quad$ *and $(b + c)a = ba + ca$ (Distributivity)*

Note again that the commutative operation $(+)$ and the operation of multiplication may be only abstractly connected with the usual connotations of these expressions. In general ab need not be equal to ba and the corresponding ring is then called a non-commutative ring.

An immediate consequence of (iv) is $a \cdot 0 = 0$ for any $a \in R$, where 0 is the zero element of R. In some rings the product of two non-zero elements *may itself be zero, i.e. $ab = 0$ while $a \neq 0$, $b \neq 0$. In that case, these elements are called zero divisors. Furthermore, a ring need not contain either a unit element e in the sense that $ae = e$, or a multiplicative inverse a^{-1} of a given a. A commutative ring which contains no zero divisors is called an integral domain. In this domain $ab = ac$ implies $b = c$ if $a \neq$ zero (cancellation law).*

Examples of rings:

(1) *The set of all integers, $0, \pm 1, \pm 2, \ldots$ under ordinary addition and multiplication. The integer 1 serves as a unit element for multiplication but there does not exist an integral inverse n^{-1} for any n except $n = \pm 1$. Note that the subset of all even integers $0, \pm 2, \pm 4, \ldots$ is itself a ring. This ring does not contain even a unit element.*

(2) *The set of all polynomials $A(x) = a_0 + a_1 x + \cdots + a_n x^n$ of arbitrary degrees under addition and multiplication[82]: we define a zero polynomial as the one with all its coefficients zero and it is simply denoted by 0. The negative $-A(x)$ of $A(x)$ is evidently the polynomial whose coefficient of x^k is $-a_k$ ($k = 0, 1, \ldots, n$).*

In this manner, the set of all polynomials becomes an additive abelian group. We make it into a ring by defining the product polynomial of $A(x)$ and $B(x)$ (of orders m, n respectively) as one of degree $n + m$,

$$A(x)B(x) = c_0 + c_1 x + \ldots c_k x^k \cdots + c_{n+m} x^{n+m}$$

$$c_k = \sum_{r+s=k} a_r b_s; \qquad (k = 0, 1, \ldots n + m)$$

[82] The exact significance of the symbol x varies subtly with the context. In the early stages of algebra x denoted some *unknown number*, to be discovered eventually at the outcome of the analysis. Later, with the introduction of the concept of a function the unknown x is replaced by the *variable* x, which can range over all numbers of a given set. But unknown and variables obey exactly the same algebraic laws and it is not always necessary to have a clear idea which one is being used. Sometimes one passes from unknown to variable without noticing. Thus it may be required to solve the equation $x^3 + 3x + 2 = 0$ for the unknown x. But a convenient method of solving this equation is by finding the intersection of the curve $y = x^3$ with the line $y + 3x + 2 = 0$. Here, the symbol x ceased to be an unknown and has become a variable.

The laws of algebra are not concerned, however, with whether x represents an unknown, a variable or a constant. A further important generalization is introduced here: the letter x can denote an *indefinable*, i.e. just a symbol about which nothing is assumed, except that it obeys a certain arbitrary system of formal algebraic operations, that may or may not be commutative.

One can similarly define rings of polynomials in several unknowns.

(3) *The set of all square matrices under the composition laws of addition and multiplication of matrices. The ring is non-commutative, contains zero divisors* [e.g. $\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} a & b \\ -a & -b \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$] *and contains both singular and non-singular matrices* [*i.e., some elements have no inverse*].

(4) *The set of complex quaternions with the product definition*

$$AB = (a_0 b_0 - \boldsymbol{a} \cdot \boldsymbol{b}, \qquad a_0 \boldsymbol{b} + b_0 \boldsymbol{a} + \boldsymbol{a} \times \boldsymbol{b}),$$

where $A = (a_0, \boldsymbol{a})$; $B = (b_0, \boldsymbol{b})$.

Here again, the ring consists of all quaternions, both singular $[|A| = a_0^2 + a_1^2 + a_2^2 + a_3^2 = 0]$ *and non-singular, and must be distinguished from the group of nonsingular complex quaternions introduced earlier. This ring too, contains zero divisors* [*e.g.* $(1 + ie_1)(1 - ie_1) = 0$ *where* e_1 *is a quaternion unit* $e_1^2 = -1$] *and is non-commutative.*

3. Division ring (skew-field)

We observe that a general ring is a group w.r.t. addition and only a semigroup *w.r.t. the multiplication since only closure and associativity are assumed to hold relative to this second composition law. If on the other hand, the elements of the ring (with exception of its zero element), form a multiplicative group as well, it is called a* division ring.

More explicitly, a division ring is a ring that contains a unit element e *and also the inverse* a^{-1} *of every* $a \neq 0$ *such that* $ae = a$ *and* $aa^{-1} = e$. *Here in addition to the unique solution* $x = -a + b$ *of an equation* $a + x = b$, *an equation* $ay = b$ *also possesses the unique solution* $y = a^{-1}b$ *and hence, the name* division ring *for this structure.*

In general, a division ring is not commutative w.r.t. multiplication[83].

[83] However **J.H.M. Wedderburn** (1882–1948) proved (1905) that any finite division ring must be commutative (and thus a *field*).

Example of a non-commutative division ring:

The set of all real quaternions under the definition of the inverse to $A(a_0, \boldsymbol{a})$

$$A^{-1} = \frac{\bar{A}}{|A|^2}$$

where $\bar{A} = (a_0, -\boldsymbol{a})$; $|A|^2 = a_0^2 + a_1^2 + a_2^2 + a_3^2$. *We also have a unit element* $(1, \mathbf{0})$.

4. Field

A commutative division ring is a field. In other words: a field is a set of elements forming a ring w.r.t. two binary operations, addition and multiplication, for which the set of all elements except the unit element w.r.t. addition forms an abelian group w.r.t. multiplication. In yet simpler language one could say that a field is a set of entities which is closed w.r.t. the four rational arithmetical operations of addition, subtraction, multiplication and division by any non-zero member of the set.

Obvious instances of fields are the set of all rational numbers, the set of all real numbers and the set of all complex numbers. We notice that each of these examples is a subfield of the one that succeeds it. Note that $x^2 = 2$ *cannot be solved over the field of rationals while* $x^2 + 1 = 0$ *cannot be solved over the field of real numbers.*

Consider the field of rational numbers $\mathbb{Q}$. *Each rational number can be represented uniquely as a point on a straight line, the 'number axis'. Each such point is a rational point. The rational number* A *is said to be less than the rational number* B $(A < B)$ *if* A *lies left of* B *on the axis. Equivalent statements are that* B *is greater than* A $(B > A)$ *if, or that* $B - A$ *is positive.*

It then follows that, if $A < B$, *the points (numbers) between* A *and* B *are those which are both* $> A$ *and* $< B$. *Any such pair of distinct points, together with the points in between, is called a segment, or interval,* (A, B). *The distance of a point* A *from the origin, considered as positive, is called the absolute value of* A *and is indicated by the symbol* $|A|$. *By definition*

$$\begin{aligned} |A| &= A, \text{ if } A > 0 \\ &= -A, \text{ if } A < 0 \end{aligned}$$

It is clear that if A and B have the same sign, the equality $|A+B| = |A| + |B|$ holds, while if A and B have different signs, we have $|A+B| < |A| + |B|$. Hence, combining these two statements we have the general inequality

$$|A+B| \leq |A| + |B|,$$

which is valid irrespective of the signs of A and B.

The absolute value $|x|$ of $x \in \mathbb{Q}$ therefore satisfies the three properties

(i) $|x| \geq 0, \qquad |x| = 0 \Longleftrightarrow x = 0$

(ii) $|xy| = |x||y|$

(iii) $|x+y| \leq |x| + |y|$

Any function on $\mathbb{Q}$ with properties (i)–(iii) is called a norm. [The absolute value $|x|$ is not the only norm possible over the rationals]

Once the norm is defined, one can go further and define a metric D over the rationals

$$d(x,y) = |x-y|$$

which renders the distance between any two rational points on the number axis. It has the following properties

(i) $d(x,y) = 0$ *iff* $x = y$

(ii) $d(x,y) = d(y,x)$

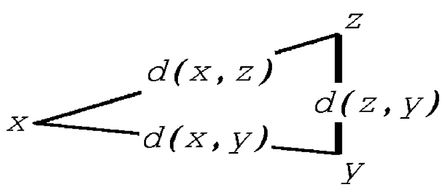
(iii) $d(x,y) \leq d(x,z) + d(z,y)$

for all $z \in \mathbb{Q}, \quad x \in \mathbb{Q} \quad$ and $\quad y \in \mathbb{Q}$.

In the field of complex numbers $\mathbb{C}$, with the metric

$$d(a+bi, c+di) = \sqrt{(a-c)^2 + (b-d)^2}$$

the above condition (iii) is known as the triangle inequality. Indeed, in the complex plane, with the above metric, (iii) states that the sum of two sides of a triangle is greater than the third side:



The integral domain of polynomials over a field can be extended to form a field by the use of infinite series. If a_0 differs from zero, the inverse of the polynomial $(a_0 + a_1x + a_2x^2 + \cdots + a_nx^n)$ can formally be generated by assuming the existence of an infinite series of ascending powers of x such that

$$(a_0 + a_1x + a_2x^2 + \cdots + a_nx^n)(b_0 + b_1x + \cdots + b_nx^n + \dots) = 1.$$

Solving successively for the coefficients b_k we obtain

$$b_0 = \frac{1}{a_0}; \quad b_1 = -\frac{a_1}{a_0^2}; \quad b_2 = \frac{a_1^2 - a_0a_2}{a_0^3}; \quad b_3 = -\frac{a_1^3 - 2a_0a_1a_2 + a_3a_0^2}{a_0^4}$$

etc. No question of convergence arises; the system of polynomials is merely extended to include infinite series. The system is not yet a field, as the element x has no reciprocal. But the system of polynomials and series of the form $x^p(a_0 + a_1x + a_2x^2 + \dots)$ where p is a positive or negative integer does form a field.

5. Linear Vector Space[84]

One considers an additive abelian group V with elements $\mathbf{0}, \boldsymbol{x}, \boldsymbol{y} \dots$ whose general element is denoted by $\boldsymbol{v}$. One then considers a field F with elements

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[84] For further reading, see:

- Deskins, W.E., *Abstract Algebra*, Dover: New York, 1995, 624 pp.
- Littlewood, D.E., *University Algebra*, Dover, 1970, 324 pp.
- Childs, L.M., *A Concrete Introduction to Higher Algebra*, Springer-Verlag, 2000, 522 pp.

$0, \lambda, \mu, a, b \ldots$ *We call* V *a linear Vector Space over the ground field* F *if we can define an operation called* scalar multiplication *which associates with each* $\lambda \in F$ *and each* $\boldsymbol{v} \in V$ *one unique element of* V *denoted* $\lambda\boldsymbol{v}$ *and satisfying the following axioms in addition to those of the abelian group for* $\boldsymbol{v}$ *and the field* F*:*

$$\begin{aligned}
&\lambda(\boldsymbol{x}+\boldsymbol{y}) = \lambda\boldsymbol{x}+\lambda\boldsymbol{y}\\
&(\lambda+\mu)\boldsymbol{x} = \lambda\boldsymbol{x}+\mu\boldsymbol{x}\\
&\lambda(\mu\boldsymbol{x}) = (\lambda\mu)\boldsymbol{x} \equiv \lambda\mu\boldsymbol{x}\\
&1\boldsymbol{x} = \boldsymbol{x}; \qquad 1 \text{ is the unit element of } F.
\end{aligned}$$

The elements of V *are called* vectors *while the elements of* F *are called* scalars. *If the ground field* F *is the real number field,* V *is a* real linear vector space; *if* F *is the complex number field,* V *is called a* complex vector space. *An obvious consequence of the above axioms is*

$$\boldsymbol{x} \in V, \boldsymbol{y} \in V \Rightarrow \lambda\boldsymbol{x}+\mu\boldsymbol{y} \in V \qquad \text{for arbitrary } \lambda, \mu \in F.$$

If each element $\boldsymbol{v}$ *of* V *is equivalent to a* finite *sum of* n *scalar-multiplied fixed vectors of* V

$$\boldsymbol{v} = \lambda_1\boldsymbol{v}_1+\lambda_2\boldsymbol{v}_2+\ldots\lambda_n\boldsymbol{v}_n,$$

the vector space is said to be finite-dimensional *over* F*. If, in addition,* $\boldsymbol{v}_1, \ldots \boldsymbol{v}_n$ *are linearly independent*[85]*,* V *is called an* n*-dimensional vector space over* F*, usually designated* V_n*.*

Let the ground field F *possess a norm as defined above. Then a vector space* V *over* F *is called* normed *if to every element* $\boldsymbol{v}$ *of* V *there corresponds a real, non-negative number* $\|\boldsymbol{v}\|$ *called the* norm *of* $\boldsymbol{v}$ *such that*

(i) $\|\lambda\boldsymbol{v}\| = |\lambda|\|\boldsymbol{v}\|, \quad \lambda \in F, \quad \boldsymbol{v} \in V$

(ii) $\|\boldsymbol{v}_1+\boldsymbol{v}_2\| \le \|\boldsymbol{v}_1\|+\|\boldsymbol{v}_2\| \qquad$ for $\boldsymbol{v}_1, \boldsymbol{v}_2 \in V$

(iii) $\|\boldsymbol{v}\| > 0 \qquad$ for $\boldsymbol{v} \neq 0$

It is easily proven that $\|\mathbf{0}\| = 0$.

An example of a norm is the magnitude of the real vector $\boldsymbol{v}$*, namely* $|\boldsymbol{v}|$*, its euclidean distance from a fiducial origin.*

A vector space is called metric *if for each* pair *of elements* $\boldsymbol{v}_1, \boldsymbol{v}_2$ *in the space there is a real, non-negative number* $d(\boldsymbol{v}_1, \boldsymbol{v}_2)$ *such that*

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[85] A set of vectors $\boldsymbol{v}_1, \boldsymbol{v}_2 \ldots \boldsymbol{v}_n$ is said to be *linearly dependent* if scalars $\lambda_1, \lambda_2 \ldots \lambda_n$, not all of them zero, exist such that $\lambda_1\boldsymbol{v}_1+\lambda\boldsymbol{v}_2+\ldots\lambda_n\boldsymbol{v}_n = \mathbf{0}$.

(i) $d(\boldsymbol{v}_1, \boldsymbol{v}_2) = 0 \qquad$ *iff* $\qquad \boldsymbol{v}_1 = \boldsymbol{v}_2$

(ii) $d(\boldsymbol{v}_1, \boldsymbol{v}_2) \leq d(\boldsymbol{v}_3, \boldsymbol{v}_1) + d(\boldsymbol{v}_3, \boldsymbol{v}_2) \qquad$ *for all* $\boldsymbol{v}_3$ *in* V

These properties imply that $d(\boldsymbol{v}_1, \boldsymbol{v}_2) = d(\boldsymbol{v}_2, \boldsymbol{v}_1)$. *Clearly a normed vector space can be made metric by taking*

$$d(\boldsymbol{v}_1, \boldsymbol{v}_2) = \|\boldsymbol{v}_1 - \boldsymbol{v}_2\|,$$

which is the 'distance' between the vectors $\boldsymbol{v}_1$ *and* $\boldsymbol{v}_2$.

Examples of linear vector spaces:

(1) The set of all 3-dimensional real vectors as oriented line segments with the triangle law of vector addition and with the real number field as the ground field.

(2) The set of all n*-tuples of the form*

$$x = (x_k) = (x_1, x_2, \dots x_n)$$

where x_k *are elements of a ground field, and*

$$x + y = (x_k) + (y_k) = (x_k + y_k) = (x_1 + y_1, x_2 + y_2, \dots x_k + y_k)$$

The zero element is clearly $0 \equiv (0, 0, \dots 0)$ *and the negative of* x *is* $-x = (-x_k)$; *also* $\lambda x = (\lambda x_k) = (\lambda x_1, \dots, \lambda x_n)$. *The* n*-tuple* x *when written as a row is called a* row vector *and when written as a column is called a* column vector.

(3) The set of all matrices A *of order* n*, with complex elements, on defining* $(\lambda A)_{ij} = \lambda A_{ij}$, *where* λ *is a complex number.*

(4) The set of all polynomials $A(x)$ *of degree less than or equal to some number* n *over a field* F*, on defining*

$$\lambda A(x) = (\lambda a_0) + (\lambda a_1)x + \dots (\lambda a_n)x^n, \quad \lambda \in F.$$

It may be noted that in the above abstract structures no mention has been made of a vector product; *this is because that structure is in general not a vector at all from the viewpoint of linear transformation theory; i.e. its components do not transform to different coordinate systems as do those of an ordinary vector, unless* $n = 3$.

Table 4.1 overviews the development of abstract algebra during the 19^{th} *century.*

Table 4.1: MAJOR EVENTS AND TURNING POINTS IN THE EVOLUTION OF ABSTRACT ALGEBRA, 1771–1880

MATHEMATICIAN	YEAR(S)	ACHIEVEMENT
J.L. Lagrange	1771–1774	Employment of symmetric and similar functions in the solution of algebraic equations by radicals.
A.T. Vandermonde	1771	First group-theoretic theorem. In his memories "*Memoire sur la resolution des équatious*" he approached the general problem of solvability of algebraic equations through the study of functions invariant under permutations of the roots of the equations. **Kronecker** (1888) claimed that the study of modern algebra began with this paper by Vandermonde.
R. Ruffini	1799–1813	Introduction of subgroup of substitutions; notions of transitivity and primitivity of groups. First to prove (with groups) the *Abel-Ruffini Theorem*, using permutation groups. May have come up first with some of the ideas of Galois.
C.F. Gauss	1801	Showed that equation $x^p - 1 = 0$ can be reduced to solving a series of quadratic equations, whenever p is *Fermat prime**.
	1815	Algebraic proof of the *fundamental theorem of algebra*. Pioneered early concepts of: group; special (abelian) case of Galois group; *field*; *splitting field*; *quotient ring*; Extensions of cyclotomic fields; primitive roots.

Table 4.1: (Cont.)

Mathematician	Year(s)	Achievement
N.H. Abel	1824–6	Complete independent proof of *Abel-Ruffini theorem* of the impossibility of the algebraic solution of general algebraic equation of degree higher than the forth.
E. Galois	1823–1829	Complete theory of *finite fields*. Theory of *field extensions*; Solvability conditions of algebraic equations by radicals. Completion of the theory of equations. *Group* concept; *Normal subgroups*. A turning point in the rise of group theory. Advent of abstract algebra. Decisive paper published by J. Liouville only in 1846.
G. Peackock **D.E. Gregory** **A. de Morgan** **G. Boole**	1834–1841	*Symbolic algebra* and *logic*
C.G. Jacobi	1834	Theories of *determinants*, *quadratic forms* and *invariants*.
W.R. Hamilton	1843	Advent of *hypercomplex numbers*; *Quaternions*.
H.G. Grassmann	1844	*Polyadic algebra* (n-dimensional 'exterior algebra'); harbinger of tensor algebra.
A. Cauchy	1844–1846	*Permutation subgroups*, *splitting fields* (1815); *Cauchy theorem* in group theory: ('every group whose order is divisible by a prime number p must contain one or more subgroups of order p').

Table 4.1: (Cont.)

Mathematician	Year(s)	Achievement
A. Cayley	1849–1859	Theories of *matrices* (1858); abstract *finite groups* [*Cayley theorem: every finite group is isomorphic to a subgroup of* S_n]. Theory of algebraic invariants. With **Hamilton** and **Grassmann** opened abstract algebra to a variety of structures.
S.H. Aronhold **J. Silvester** **R.F.A. Clebsch** **L.O. Hesse** **P. Gordan**	1850–1872	Theory of invariants (later to become essential in *tensor algebra*)
J.A. Serret	1866	Gave the first exposition of Galois' ideas in his book '*Cours d'algebre superiere*'.
C. Jordan	1870	Consolidation of group theory (*normal subgroups*, *simple groups*, *homomorphism*, *matrix groups*). First full and clear presentation of Galois theory.
L. Kronecker	1870	Generalized Gauss' work (1815) and solved the general problem of polynomial splitting field.
L. Sylov	1872	Extended Cauchy's theorem
J.W Dedekind **B. Peirce**	1872–1880	Creation of structural theory of semisimple algebras.
W.K. Clifford	1878–80	'Clifford algebras'; biquaternions.

Table 4.1: (Cont.)

(*) *Example*, since $x^5 - 1 = (x-1)(x^4 + x^3 + x^2 + x + 1)$, the equation to be solved is $x^4 + x^3 + x^2 + x + 1 = 0$. We put $z = x + \frac{1}{x}$. Since $x^5 = 1$, we have

$$\frac{1}{x} = x^4, \quad z = x^4 + x, \quad z^2 = x^2 + 2 + \frac{1}{x^2} = x^2 + 2 + x^3.$$

This yields the equation $z^2 + z - 1 = 0$ for z and the equation $z^2 - zx + 1 = 0$ for x. Solving these equations we obtain

$$z_{1,2} = \frac{-1 \pm \sqrt{5}}{2}, \quad x_{1,2} = \frac{z \pm i\sqrt{z+3}}{2}.$$

Thus,

$$x_{1,2} = \frac{-1 \pm \sqrt{5}}{2} + i\sqrt{\frac{5 \pm \sqrt{5}}{8}}; \quad x_{3,4} = \frac{-1 \pm \sqrt{5}}{4} - i\sqrt{\frac{5 \pm \sqrt{5}}{8}}.$$

Vandermonde (1771) produced an *algebraic solution* in radicals for the binomial equation $x^{11} - 1 = 0$

1830 CE **Joseph Jackson Lister**[86] (1786–1869, England). Optician. Showed how microscope lenses could be made to correct for *chromatic and spherical aberrations*[87] (seventy years after the first achromatic *telescope* lenses were made). His invention improved the resolving power of compound microscopes. This, in turn, enabled physiologists to advance cell theory on a solid basis[88].

1830–1833 CE **Marshall Hall** (1790–1857, England). Physician and physiologist. Described the mechanism by which a stimulus can produce a response independently of sensation or volition and coined the term "*reflex*". Maintained his theory in face of denunciation by colleagues. Denounced *bloodletting*[89] as a treatment for disease (1830).

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86 Not to be confused with the English surgeon **Joseph Lister** (1827–1912), founder of *antiseptic surgery* (1865–1877) who first demonstrated that the use of carbolic acid as an antiseptic reduced the danger of surgery.

87 When a bundle of rays originates from an axial point, the image distances are not the same for all rays but depend on the original slope angle at the object point. This means that the *rays do not converge to a single focus*. This common feature of spherical reflecting and refracting surfaces (such as lenses, mirrors, prisms etc.) is known as *spherical aberration*. Due to *dispersion*, the focal length of a simple lens also varies with the wavelength. This variation, called *chromatic aberration*, can be reduced substantially by means of a lens combination in which the component lenses are made of glasses having different dispersions. An *achromatic* combination of focal length f for two thin lenses in contact is obtained if the focal lengths of the component lenses are

$$f_1 = f\left(1 - \frac{\delta_1}{\delta_2}\right), \qquad f_2 = f\left(1 - \frac{\delta_2}{\delta_1}\right),$$

where

$$\delta_1 = \frac{1}{n_1 - 1}\frac{dn_1}{d\lambda}, \qquad \delta_2 = \frac{1}{n_2 - 1}\frac{dn_2}{d\lambda},$$

n_1, n_2 being the indices of refraction and λ the wavelength. Since $\frac{dn}{d\lambda}$ also varies with wavelength, a lens can be achromatized over a limited wavelength interval only. Spherical aberration, on the other hand, can be minimized by appropriate choice of *lens curvatures* and *separations*.

88 **Pierre Turpin** (1826) reported his observations of *cell division* in algae; **Franz Meyen** observed (1830) that each cell is an independent unit which nourishes itself, build itself up and incorporates raw nutrients that are taken up into different substances and structures. **Robert Brown** (1831) discovered the widespread occurrence of nuclei in cells. **Hugo von Moll** (1805–1872, Germany) showed that plant-cells alone possessed walls (1835–1839).

89 An ancient practice, found in virtually all periods and cultures, based on magic

1830–1833 CE **Charles Lyell** (1797–1875, England). Founder of modern historical geology. Building on Hutton's concept of *gradual* change through existing physical causes, Lyell marshaled all the observations he could collect in support of the doctrine that the earth has changed slowly and gradually through the ages by means of processes that are still going on.

Between 1830 and 1833 he published the three volumes of his *Principles of Geology*, which organized existing information about that science with lucidity and clarity. Until then many persons believed that changes in the earth occurred in sudden *worldwide* upheavals. Almost singlehandedly, Lyell established *uniformitarianism* at the expense of *catastrophism*, as the accepted philosophy for interpreting the history of the earth. In so doing he introduced, with profound impact, the concept of *unlimited time*. Geological problems now could be solved by reference to natural laws *still active* and available for study in the real world about us instead of by reference to former, shadowy, mythical, or supernatural events; the present became a key to the past.

Lyell's wide influence prepared the ground for the succeeding accomplishments of the 19^{th} century, including those of **Charles Darwin**, whose ideas on the gradual development of living things could not have flourished without the intellectual framework of vast time. Hence, the uniformitarian doctrine was eminently successful in nourishing scientific progress.

In retrospect, however, it appears that the pendulum swung a bit too far[90]. Not only did Lyell strictly reject any process that could not be shown to accord with constant and presently verifiable laws of nature, but he would not even entertain the thought that rates of change, or the *relative importance* of geological agents, ever differed from what they have been within human experience. In short, strict uniformitarianism possessed its own rigid and stifling aspects, brought on by allowing, for all the geological past, only the present rates of natural processes.

Lyell was born in Kinnordy, Scotland. In 1816 he entered Exeter College, Oxford, studying law and geology. During 1821–1826 he was simultaneously practicing law and active in geological research, becoming a member of the Royal Society in 1826. In 1827 he finally abandoned the legal profession and

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and other supernatural explanations. Although various exuberant Renaissance *phlebotomists* were attacked by Paracelsus (1493–1541) and J.B. van Helmont (1579–1644) as upholders of outmoded traditions, bloodletting was still widely used into the 19^{th} century and died out only gradually toward 1900.

90 e.g. Lyell's doctrine would preclude catastrophic episodes in the earth's history such as the one posited in the contemporary theory of the extinction of the dinosaurs, or nonlinear processes such as the reversal of the polarity of the geomagnetic field.

devoted himself to geology. In 1841 he spent a year in traveling through the United States and Canada; he returned the America in 1845. During these journeys he studied the annual average accumulation of alluvial matter in the Mississippi delta and the coal-formations in Nova Scotia.

Among his characteristics were great thirst for knowledge, perfect fairness and sound judgment. He was buried in Westminster Abbey.

1830–1842 CE **Auguste Comte** (1798–1857, France). Philosopher and social thinker. Founder of the history of science (1832). Founded a philosophical system concerned with the impact of modern science on society, known as *positivism*. In it he tried to arrange the entire field of scientific study in a comprehensive and logical order. Each science in the hierarchy contributes to the entries that follow it, but not to those who precede it; the list is headed by mathematics and followed by astronomy, physics, chemistry, biology and sociology[91].

Comte adopted the view there are three phases in the historical development of human society: (1) initial *theological* phase, where man's speculations were dominated by superstition and prejudices; (2) *metaphysical* phase, where man's search for reality took the form of rational speculation unsupportable by facts; (3) final *positive* phase, where dogmatic assumptions began to be replaced by factual and rational scientific knowledge. This phase brings the historical process to its ultimate state of perfection[92].

One may consider positivism as a type of social physics describing a society rigidly governed by natural laws, with reason playing a key role in social evolution. This evolution goes through three stages: a military-theological stage, a critical-metaphysical stage, and a scientific-industrial stage. In this last stage man no longer concerns himself with ultimate causes, as he did during the metaphysical stage, but is satisfied with the material world and with whatever he might learn from observing it. Here was a philosophy that

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[91] The idea of such an order is extremely old, going back as far as Aristotle, and was later adopted by the philosophical movement of the *encyclopedists*.
Comte himself coined sociology for the *science of man*, the last and most complex study in the hierarchy. He considered himself as its founder.

[92] Such a view reflects the "rampant optimism" of the 19^{th} century, shared by Hegel, Spencer and Marx. **Herbert Spencer** (1820–1903, England) sought to interpret society in terms of principles derived from *mechanics* (within 50 years of Comte and Spencer a positivist account of mechanics came to be given by **Mach**). In his *First Principles* he amassed an enormous amount of data, systematically arranged and accompanied by consistent body of theory. He was the chief exponent of the philosophy of evolution.

accepted science as its only guide and authority and which for that reason was eminently suited to the late 19^{th} century.

Comte maintained that it is necessary to study the evolution of the different sciences to understand the development of the human mind and the history of mankind. It is not sufficient to study the history of one or of many particular sciences; one must study the history of *all* sciences, taken together. Indeed, as early as 1832, Auguste Comte made an application to the minister Guizot for the creation of a chair, devoted to the general history of the sciences[93] (*histoire générale des sciences*).

Comte further maintained that Positive humanity will be ruled by the moral authority of a scientific élite, while the executive power will be entrusted to technical experts, an arrangement that is similar to the ideal state of Plato's *Republic*.

Comte was born in the ancient university town of *Montpelier*, at a time when social and political conditions were highly unstable. He was the son of a respectable and conventional family of government clerks. His father was a monarchist and a rigid Catholic. When at the École Polytechnique in Paris (1816), he was expelled for taking part in a student rebellion against one of their professors. This later prevented him from gaining university employment.

Throughout his life Comte was frail in health, and suffered from recurrent mental depression which drove him to the verge of suicide. He made a living by giving private lessons in mathematics and by gifts from friends and admirers. He was twice committed to an insane asylum; the first time, as a result of his unhappy marriage (1825–1842); the second, after the death of his friend Clotilde de Vaux (the wife of a man imprisoned for life) in 1846.

1831–1843 CE **James Clark Ross** (1800–1862, England). Polar explorer. On June 1, 1831, Ross located the *north magnetic pole*[94] in Boothia Peninsula. Commanded Antarctic expedition (1839–1843), discovering Ross Sea,

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[93] It was sixty years before this wish of his was granted; the course entrusted to Pierre Laffitte was inaugurated at the Collège de France in 1892, thirty-five years after Comte's death. The real heir to Comte's thought was **Paul Tannery** (1887).

[94] At the *magnetic poles*, on the earth's surface, the horizontal component of the magnetic field vanishes and a completely free magnetic needle sets itself vertically. The line joining the poles is the magnetic axis of the earth. In 1963, the poles were approximately at 75 °N, 101 °W and 67 °S, 143 °E. In contradistinction, the *geomagnetic poles* (the best dipole approximation to the earth's true field) are at $78\frac{1}{2}$ °N, 69 °W and $78\frac{1}{2}$ °S, 111 °E.

the Ross Ice Shelf, Victoria Land, and Mount Erebus, an active volcano. His uncle **John Ross** and **William Edward Parry** trained him during six arctic voyages in search of the *Northwest Passage* (1818–1834).

Tracking the North Magnetic Pole (1831–2000)

The discovery of the directive property of a magnetic needle in the earth's field and the invention of the mariners compass is obscure. The earliest mention in European literature is ascribed to the monk ***Alexander Neckham*** *(1157–1217). Using a model of the earth made from loadstone (a naturally occurring magnetic rock),* ***William Gilbert*** *came to the conclusion (1600 AD) that the earth behaved substantially as a* uniform magnetized sphere, *its magnetic field being due to causes within the earth, and not from any external agency, as was supposed at that time. The field of a uniformly magnetized sphere can be represented by a dipole at its center. Gilbert showed that there should be two points on the earth where a magnetized needle should stand vertically: at the North and South magnetic poles.*

This is basically the same definition used today. At the magnetic poles, the earth's magnetic field is perpendicular to the earth's surface. Consequently, the magnetic dip, or inclination (the angle between the horizontal and the direction to the earth's magnetic field), is 90°. *And since the magnetic field is vertical, there is no force in a horizontal direction. Therefore, the* magnetic declination, *the angle between true geographic north and magnetic north, cannot be determined at the magnetic poles.*

Gilbert believed that the North Magnetic Pole coincided with the north geographic pole. Magnetic observations made by explorers in subsequent decades showed that this was not true, and by the early nineteenth century, the accumulated observations proved that the pole must be somewhere in Arctic Canada.

In 1829, ***John Ross*** *set out on a voyage to discover the Northwest Passage. His ship became trapped in ice off the northwest coast of Boothia Peninsula, where it was to remain for the next four years. John's nephew,* ***James Clark Ross****, used the time to make magnetic observations along the Boothia coast. These convinced him that the pole was not far away, and in* *the spring of 1831 he set out to reach it. On June 1, 1831, at Cape Adelaide on the west coast of Boothia Peninsula, he measured a dip of* 89°59′. *For all practical purposes, he had reached the North Magnetic Pole.*

In 1839, **Gauss**, *by spherical harmonic analysis, showed that the field of uniformly magnetized sphere was an excellent first approximation to the earth's magnetic field. Gauss further analyzed the irregular part of the earth's field, i.e. the difference between the actual observed field and that due to a uniformly magnetized sphere. With the data then available, he showed that both the regular and irregular components of the earth's field were of internal origin.*

The next attempt to reach the North Magnetic Pole was made some 70 *years later by the Norwegian explorer* **Roald Amundsen**. *In 1903 he left Norway on his famous voyage through the Northwest Passage, which, in fact, was his secondary objective. His primary goal was to set up a temporary magnetic observatory in the Arctic and to re-locate the North Magnetic Pole.*

A pole position was next determined by scientists shortly after World War II. **Paul Serson** *and* **Jack Clark** *measured (1947) a dip of* 89°56′ *at Allen Lake on Prince of Wales Island (*73.9° *N,* 100.9° *W). This, in conjunction with other observations made in the vicinity, showed that the pole had moved some* 250 *km northwest since the time of Amundsen's observations. Subsequent observations by scientists in 1962, 1973, 1984, and in 1994, showed that the general northwesterly motion of the pole is continuing, and that during the* 20^{th} *century it has moved an average of 10 km per year*[95].

If, as Gilbert believed, the earth acts as a large magnet, the pole would not move, at least not so rapidly as it does. We now know that the cause of the earth's magnetic field is much more complex. We believe that it is produced by electrical currents that originate in the hot, liquid, outer core of the earth.

In nature, processes are seldom simple. The flow of electric currents in the core is continually changing, so the magnetic field produced by those currents also changes. This means that at the surface of the earth, both the strength and direction of the magnetic field will vary over the years. This gradual change is called the secular variation of the magnetic field.

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[95] These measurements were:

1962	Loomer and Dawson	75.1° N	100.8° W
1973	Niblett and Chairboneau	76.0° N	100.6° W
1984	Newitt and Niblett	77.0° N	102.3° W
1994	Newitt and Barton	78.3° N	104.0° W
1999, 2000	Newitt, McKee, Mandea and Orgeval	81.3° N	110.8° W

The position of the North Magnetic Pole is strongly influenced by the secular variation in its vicinity. For example, if the dip is $90°$ at a given point this year, that point will be the North Magnetic Pole, by definition. However, because of secular variation, the dip at that point will change to $89°58'$ in about two years, so it will no longer be the pole. However, at some nearby point, the dip will have increased to $90°$, and that point will have become the pole. In this manner, the pole slowly moves across the Arctic.

The more rapid daily motion of the pole around its average position has an entirely different cause. If we measure the earth's magnetic field continually, such as is done at a magnetic observatory, *we will see that it changes during the course of a day, sometimes slowly, sometimes rapidly. The ultimate cause of these fluctuations is the sun. The sun constantly emits charged particles that, on encountering the earth's magnetic field, cause electric currents to be produced in the upper atmosphere. These electric currents disturb the magnetic field, resulting in a temporary shift in the pole's position. The distance and speed of these displacements will, of course, depend on the nature of the disturbances in the magnetic field, but they are occurring constantly. When scientists try to determine the current average position of the pole, they must average out all of these transient wanderings.*

In April and May of 1994, Larry Newitt, of the Geological Survey of Canada, and Charles Barton, of the Australian Geological Survey Organization, *conducted a survey to determine the average position of the North Magnetic Pole at that time. Working out of Resolute Bay, N.W.T., they established a temporary magnetic observatory on Longheed Island, close to the predicted position of the pole. This allowed them to monitor the short-term fluctuations of the magnetic field that result in the daily motion of the pole.*

The strength and direction of the magnetic field were measured at this site, and at seven additional sites in the region. From these observations, the point at which the average dip was $90°$ could be determined.

They determined that the average position of the North Magnetic Pole in 1994 was located on the Noice Peninsula, southwest Ellef Ringnes Island, at $78.3°N$, $104.0°W$.

Note that the magnetic North and South Poles are not *diametrically opposite, each being about 2300 km from the point antipodal to the other. The magnetic poles must not be confused with the geomagnetic poles, which are the points where the axis of the* Gaussian geocentric dipole *(which best approximates the earth's field) meets the surface of the earth.*

The geomagnetic poles are situated approximately at $78\frac{1}{2}°N$, $69°W$ and $78\frac{1}{2}°S$, $111°E$ and the geomagnetic axis is thus inclined at $11\frac{1}{2}°$ to the earth's geographical axis. If the geocentric dipole field *were the total field, the dipoles* *and geomagnetic poles would of course coincide. A better approximation to the earth's field can be obtained by displacing the center of the equivalent dipole by about 300 km towards Indonesia.* **Vestine** *(1953) has determined the position of the eccentric dipole from 1830 to 1945 and found a change in longitude of about* $0 \cdot 30°$ *per year.*

1831–1839 CE In 1831 the *cell nucleus*[96] was discovered by **Robert Brown** (1773–1858).

During 1838–1839 botanist **Jacob Matthias Schleiden** (1804–1881, Germany) and physiologist **Theodor Ambrose Hubert Schwann** (1810–1882, Germany) originated what we now call *cell biology*.

1831–1846 CE **Michael Faraday** (1791–1867, England). Distinguished experimental physicist and chemist. Discovered (1831) *electromagnetic induction* and introduced the concepts of *lines of force* and a physical *field* (1845). As a chemist he discovered and isolated *benzene* (C_6H_6) in 1825.

His experiments started in 1821, when he showed that a current carrying wire is surrounded by circular lines of magnetic field which he called 'lines of force'. Electromagnetic induction was independently discovered by **Joseph Henry** in 1832. On the other hand, **Oersted** preceded Faraday in discovering the magnetic field of a current (1820).

In 1831 Faraday discovered that electric current is generated by changes in the magnetic field — a phenomenon complementary to Oersted's discovery of the magnetic effects of currents. In 1845 he discovered *diamagnetism* and *paramagnetism*. He also showed that a magnetic field affects the polarization of light in a medium. In the same year he conjectured that light is essentially electromagnetic waves. In 1846 he suggested that electromagnetic energy is transmitted by a transverse vibrations of the lines of force, and no fluid agent, such as the 'ether', is needed for the transmission of light.[97]

1831–1848 CE **Macedonio Melloni** (1798–1854, Italy). Physicist. First to claim that radiant heat and light were different modes of the same process.

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[96] *Nucleus* = little nut (Latin: Diminutive of nux, nuc).

[97] For further reading, see:

- Williams, L.P., *Michael Faraday, A Biography*, Basic Books: New York, 1965, 531 pp.

Measured the heating effect (infrared radiation) from the sun's light scattered from the moon and reaching the earth during the night.

Melloni was born in Parma, and became a professor of Physics there (1824–1831). Had to flee to France on account of political activities. Returning to Naples (1839), he became the director of the Vesuvius Observatory. He died of cholera.

From Thales to Faraday and beyond, or — What is Electricity?

As early as 600 BCE, ***Thales of Miletos****, is supposed to have made the first observation on this mysterious entity, by noting that amber rubbed with another substance attracted certain light objects. Since then its exact nature has been a matter of dispute. The ancients considered it a kind of soul or spirit inhabiting otherwise lifeless objects.* ***Cardano*** *(1557) described it as a material substance, a fluid that flows from object to object.* ***Galvani*** *(1791) held that it was a "vital force", the element essential to life, for which philosophers had searched for centuries.*

Du Fay *(1733) offered evidence that there were not one but two different types electricity, vitreous and resinous.* ***Benjamin Franklin****, in a dangerous experiment, showed (1752) that lightning and electricity were akin.* ***Oersted*** *(1820) proved that there was a relationship between electricity and magnetism.*

Faraday*, a consummately skillful experimentalist, went even further. He knew of Oersted's observation; he also knew that heat and chemical reactions could generate electricity and vice versa. With an insight which characterizes a great scientist, he stated: "I believe that the various forms under which the forces of matter are made manifest, have one common origin; or, in other words, are so directly related and mutually dependent, that they are convertible, as it were, into one another".*

Before Faraday, electricity was a plaything of natural philosophers, a source of entertainment for the fashionable audiences who attended their lectures. No one had the slightest intimation of its practical possibilities.

Faraday had little interest in such possibilities. He was concerned with fundamental research — with the establishment of principles linking seemingly diverse phenomena. He was, however, more farsighted than his predecessors, and in response to a query by W. Gladstone, he is said to have replied, "Some day you will tax it".

Faraday's discoveries should have led immediately to a major electrical industry, but it took a very long time to develop. The reason was mainly economic: although it would have been possible to have produced electricity in a big way in 1830, electricity could not be sold because there were no buyers!

The first call for electricity was through fashion: people were becoming moderately rich, not rich enough to afford silver spoons, but what about having electroplated spoons? For that a good source of current was required and the magneto machine of Faraday, slightly improved, worked very well for this purpose. Then it was used for where really bright lights were needed, arc lights, and for lighthouses. Gradually the uses increased and as the demand for it increased, so did production.

1832 CE Following the *Reform Act*, elementary education in England became the concern of the state rather then that of the Church. The government stepped in with grants, and toward 1850, elementary education became universal in *public schools*, for the first time in history.

1832 CE **Joseph Henry** (1797–1878, U.S.A.). Physicist and inventor. Discovered the principles of electromagnetic self-inductance. Proposed a single wire telegraph (1816).

1832 CE A Chicago carpenter, **George W. Snow(e)**, reinvented[98] the *balloon frame* which revolutionized home construction. This simple method,

[98] The idea was not original. Carpenters in 17^{th} century Virginia employed a similar method when confronted with pressures to build rapidly. But no matter the type of frame, carpenters could not reduce substantially the handwork necessary for building a house until the 1880's. Then, Chicago carpenters replaced mortized-and-tenoned sills with box sills that used only dimensional lumber joined by nails. By this time, factories produced most windows, doors, and trim, as well as kiln-dried dimensional lumber with tighter tolerances. Carpenters on the site merely fit and installed these products.

The balloon frame evolved slowly over the course of the 19^{th} century. Companies utilizing standard size boards and machine-cut nails allowed even unskilled workers to build houses, quickly, cheaply and easily.

Traditional building methods used heavy, hand-hewn timbers and hand carved mortise-and-tenon joints held by hand-cut dowels or hand-made nails. An entire wall was assembled on the ground by skilled craftsmen and then lifted into place by a crew of about twenty laborers.

The balloon frame was built with much lighter pre-cut 2×4 studs and held together by factory-made nails. This reduced the cost and made affordable building materials available to middle-income and low-income families. These houses could be built quickly and easily by only two workers using basic carpentry techniques.

The method was first used by Augustine Taylor on St. Mary's Church in Fort Dearborn, near Chicago. His crew framed the church with 2×4s and 2×6s — using studs, joists, and cross members, all nailed together. There were no mortises, no tenons, no dovetail joints to be carved. It was built with just boards and nails.

Professional carpenters said the church would blow away in a stiff wind and labeled the technique "balloon construction." It was a derisive term which stuck. The style also stuck as most homes today still used this method with some modifications. In early construction, the studs ran from the foundation to the roof. In case of fire, this long open space created a chimney effect and allowed fires to spread rapidly. Today, the studs are broken by the floors and the spaces between the studs are filled with insulation reducing the chimney effect.

Two architectural practices, the balloon frame and Chicago Construction, made Chicago the world's first vertical city. Builders using the balloon frame method created a skeleton of two-by-four covered by wooden siding. First widely used in 19^{th}-century Chicago and still employed today, the balloon frame not only sped up the building process; It also made construction less costly.

The new balloon frame helps to explain the astonishingly rapid expansion of Chicago. By 1848, it became an important port equipped with facilities for handling the biggest inland ships in the world with 100 trains a day arriving on eleven different railroads. From a small village (1830), with a population less than 200, it grew (1887) into a city of 800,000 people.

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in Chicago then produced ready-made houses with balloon frames that were sold to various Western cities attempting to meet the needs of rapidly expanding populations.

The balloon frame was a precursor to a great Chicago innovation: the practice of attaching a facade onto a strong yet light steel frame. Though skyscrapers were born in New York, the method called Chicago Construction, developed by Chicago architects and engineers between 1880 and 1883, provided the basic structural system for building modern steel-and-glass office towers.

Balloon frame construction helped to make possible the incredible growth of Western U.S., where trees were scarce. Wood from the Midwest, cut into standard-size boards, was shipped by rail to the West. Most wooden buildings erected today still use the method of construction derived from this system.

1832–1846 CE **Joseph Liouville** (1809–1892, France). A remarkable mathematician of the 19^{th} century.

He was born into a distinguished family in St. Omer, France. He studied at the École Polytechnique and was appointed professor there in 1833. In 1836 he founded the *Journal des Mathématique Pures et Appliquées*, which upheld the high standard of French mathematics throughout the 19^{th} century. He was appointed professor at the Sorbonne and the Collège de France in 1839.

Liouville contributed significantly to many fields of mathematics, especially to boundary-value problems for 2^{nd} order linear differential equations (*Stürm-Liouville theory*). He was also interested in number theory, differential geometry and Hamiltonian dynamics. Today he is also remembered for having published the works of **Galois**, after the latter's untimely death.

In 1838 he discovered an important theorem which found applications both in classical and quantum mechanics. It states that the volume of any region in phase-space remains invariant under any Hamiltonian time evolution. Otherwise stated it means that the 'phase-fluid' moves like an incompressible fluid[99]. Of historical importance is his general method of solution of integral equations by successive substitution (1837).

A number of important theorems bear his name: *Liouville theorem* in the theory of functions states that if $f(z)$ is analytic for all values of z and $|f(z)| < k$, where k is constant, then $f(z)$ is constant. Another *Liouville theorem* states that an elliptic function $E(z)$ with no poles in a cell is merely a constant. Then there are a number of theorems that concern the

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[99] **Boltzmann** (1867) applied the theorem in the context of his statistical-mechanics theory. An explicit *equation* was first derived by **Gibbs** (1884), who recognized its potential usefulness in astronomy.

solvability of second order differential equations[100], and in particular of the Riccati equation.

Liouville proved the existence of *transcendental numbers* (1844) and constructed an infinite class of such numbers. He wrote over 400 papers in total, many of them of major importance in mathematics.

1832–1863 CE **Jacob Steiner** (1796–1863, Switzerland). An outstanding geometer. Laid the foundations of modern synthetic geometry. His mathematical works are confined to geometry, which he treated synthetically, to the total exclusion of analysis. In his own field he surpassed all his contemporaries. His investigations are distinguished by their great generality, rigor and profound intuition.

Steiner clarified many of the concepts of projective geometry and stressed the fundamental importance of the *principle of duality*. Using exclusively synthetic methods he was able to prove theorems that belong to the realm of analysis. Among his contributions: *the Steiner-Poncelet theorem*, which states that second order problems (Euclidean constructions) can be solved with the aid of a straight-edge and a circle with a given center.

Steiner was born in the village of Utzendorf (canton Bern). He learned to read and write at the age of 14. At 18 he became a pupil of **Heinrich Pestalozzi** and afterward studied mathematics at Heidelberg and Berlin, where he made his living by giving private lessons. He was helped by **A.L. Crelle**, and due to the influence of **G.C.J. Jacobi** and the brothers **Alexander** and **Wilhelm von Humboldt** he was appointed professor of geometry at the University of Berlin (1834), where he taught for the rest of his life.

1832–1873 CE **Joseph Antoine Ferdinand Plateau** (1801–1883, Belgium). Physicist. Devised an experimental method of visualizing minimal surfaces, and described it in his 1873 treatise on molecular forces in liquids. The essence of the matter is that if a piece of wire is bent into a closed curve and dipped in a soap solution, then the resulting soap film spanning the wire will assume the shape of a minimal surface in order to minimize the potential energy due to surface tension. During 1830–1869 Plateau performed many striking experiments on surface tension and capillary phenomena, and

[100] There is a partial nonlinear differential equation, which bears his name: $u_{xy} = e^u$. It has the *exact* solution $e^u = 2\frac{\alpha'(x)\beta'(y)}{[\alpha(x)+\beta(y)]^2}$, where $\alpha(x)$, $\beta(y)$ are arbitrary functions of x and y respectively.

since his time the problem of minimal surfaces has been known as *Plateau's problem*.[101]

Plateau was born in Brussels. From 1835 and on he was a professor of physics at Ghent. He did most of his work in the fields of physiological optics and molecular forces. We owe to him the *stroboscopic*[102]method of studying the motion of a vibrating body, by looking at it through equidistant radial slits in a revolving disc.

In 1829 he imprudently gazed at the midday sun for 20 seconds, with the view of studying after effects. It caused him to lose his eyesight in 1843. But this calamity did not interrupt his scientific activity. Aided by his wife, son and son-in-law, he continued to the end of his life his researches on vision, although he did not see many of his own experiments.

In 1832 he developed the *phenakistoscope*[103], the first device that gave pictures the illusion of movement: Plateau placed two discs on a rod. He painted pictures of an object or a person along the edge of one disc. Each picture slightly advanced the subject's position. Slots were cut in the other

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[101] For further reading, see:

- Hildebrandt, S. and A. Tromba, *The Parsimonious Universe*, Springer-Verlag, 1995, 330 pp.
- Courant, R., *Dirichlet's Principle, Conformal Mapping and Minimal Surfaces*, Interscience-Wiley: New York, 1950, 330 pp.

[102] Plateau and **Simon von Stampfer** (1792–1864, Austria) invented the *stroboscope* independently around 1823. Stampfer was a professor at the Technical College of Vienna. The stroboscope and the principle of persistence of vision were at the base of all early attempts to produce moving pictures, culminating with the *cinematograph* of the Lumiére brothers (1885).

[103] The *phenakistoscope* was constructed with **Simon Ritter**. It was then developed in a number of directions in an attempt to produce moving pictures. **Franz von Uchatius** (1811–1881, Germany) was the first person to project visible moving images on the screen: he scanned a series of painted slides (places around a disc) through slits cut in a second disc. As the discs were rotated, apparently moving images were projected by light onto a screen (1853). **Ottomar Anshutz** (1846–1907, Prussia) made the first noteworthy attempt (1892, two years before Edison's peepshow *kinetoscope*) to project moving sequences of *photographs* using his *projecting Electrotachyscope* which was in principle just an elaborated stroboscope.
The *commercial history* of the moving pictures began with **Edison's** kinetoscope (1854) and the invention of the Kodak celluloid film by **George Eastman** (1888).

disc. When both discs were rotated at the correct speed, the pictures seemed to move as they appeared in the slots.

1833 CE **Charles Babbage** (1792–1871, England). Mathematician. The great ancestral figure in the history of computers. The first man to put forward detailed proposals for an all purpose automatic calculating machine. He designed and tried to build a complicated machine, dubbed the *Analytical Engine*. The design for his vast mechanical calculators rank among the most startling intellectual achievements of the 19^{th} century. Yet Babbage failed in his efforts to realize those plans in physical form, because the demands of his devices lay beyond the capabilities of Victorian mechanical engineering.

Contemporary computers are based on many of the principles used in Babbage's machine: It was designed so that it would perform mathematical operations from a set of instructions ('program'). The machine would 'read' the program from 'punched cards', an idea derived partly from the punched cards of the *Jacquard loom* (1805). The computer was equipped with a memory and a central processor. A long sequence of different operations could be performed with no human intervention after the punched cards were fed in.

The first machine conceived by Babbage, already in 1812, was the *Difference Engine* which he intended as a device for computing and printing tables of mathematical functions. He noticed that tables of polynomials can be easily constructed to any desired accuracy if one employs their first, second etc. *differences*, using the addition operation only. Since most functions can be represented to sufficient accuracy (at least over a limited range) by means of polynomials, their values can be constructed in a similar way. It was this process that Babbage proposed to mechanize with his *Difference Engine*.

Babbage constructed a small machine with 3 registers which would tabulate quadratic functions. This he demonstrated in 1822, to such effect that he secured the support of the Royal Society for the construction of a full size machine to compute and check tables of 6^{th} degree polynomials to no less than 20 decimal places. The machine was never constructed. A part of it is now in the London Science Museum. [In 1853, **Pehr Georg Scheutz** (1785–1873, Sweden), stimulated by some published accounts of Babbage's ideas and funds from the Swedish Academy, completed an improved version of the Difference Engine that would tabulate 4^{th} degree polynomials to 14 decimal places.] In 1832 Babbage lost interest in the *Difference Engine*.

Babbage was born in Teignmouth, Devonshire. He was educated at a private school, and afterwards entered St. Peter's College, Cambridge, where he graduated in 1814. Though he did not compete in the mathematical tripos, he acquired a great reputation at the university. In the years 1815–1817, he contributed three papers to the *Philosophical Transactions* and in 1816 was made a fellow of the Royal Society. Babbage's attention seems to have been drawn at an early stage to the number and importance of errors introduced into astronomical and other tables.

From 1828 to 1839 Babbage held the post of Lucasian Professor of Mathematics at Cambridge — but without delivering a single lecture at the university. He was busy enough in other directions, however. Not only did he attempt to reform the Royal Society, Greenwich Observatory, and the teaching of mathematics at Cambridge, but he also found time to analyze the operation and economics of the Post Office, the pin-making industry and the printing trade, to publish one of the first reliable actuarial 'life tables', and to make some of the earliest dynamometer measurements on the railway, running a special train on Sundays for the purpose.

The essential constituents of the 'Analytical Engine' are:

- a *store* (sometimes called a *memory*) for holding numbers — both those forming the data of the problem and those generated in the course of the calculation;
- an *arithmetic unit* — a device for performing arithmetic operations on those numbers (Babbage called it the *mill*);
- a *control unit* — a device for causing the machine to perform the desired operation in the correct sequence;
- *input devices* whereby numbers and operating instructions can be supplied to the machine;
- *output devices* for displaying the results of the calculation.

For storage Babbage proposed to use columns of wheels, each wheel being capable of resting in any one of ten positions and so of storing one decimal digit. Transfer of numbers between the store and the mill was to be accomplished by means of elaborate mechanisms of gears, rods, and linkages. The store itself was to accommodate 1000 numbers, each number being represented by no less than 50 decimal places. It seems that Babbage intended that numbers would normally be set on the storage wheels or on the mill by hand, but he also envisaged supplying mathematical tables to the machine in punched-card form. Several alternative kinds of output were envisaged: direct printing, the production of moulds from which printer's blocks could be cast, and punched cards.

Babbage's ideas were sound, but there was no technology in the mid 19^{th} century to implement them. For that reason, the Analytical Engine was never completed, although Babbage continued to work on it until his death, spending much of his own money. His schemes were simply too ambitious (consider, for instance, his idea of working with numbers to 50 decimal places!) at a time when simple desk calculators were far from reliable mechanically and could not be made in any quantity.

As time went on the frustrated inventor became increasingly embittered by a sense of failure[104]. He quarreled with many of his contemporaries, from his own craftsmen to the astronomer royal; he became ever more intolerant of criticism, more caustic in his judgments, more out of sync with his time. He suffered the unhappy fate of a misunderstood genius who is too far ahead of his time.

It took the world a century to catch up with him. Today, when his germinal ideas are bearing so rich a fruit, we can appreciate the magnitude of his achievements and the depth of his prophetic insight.

In 1831, **Babbage**, **John Frederick William Herschel** (1792–1871) and **David Brewster** (1781–1868) created the 'British Association for the Advancement of Science' to promote British science, which was on the decline. The reason was that the mathematics taught in Britain during the early years of the 19^{th} century did not go much beyond the level of Newton's time. In the calculus, Newton's somewhat clumsy notation[105] was adhered to, whilst the more elegant symbolism introduced by Leibniz, and the advances made by the French, were largely ignored. Moreover, scientific research in England was still largely an amateur activity, and a reform in the universities was needed.

The idea of the association had originated from a national congress of German scientists, which Babbage had attended in Berlin in 1828. Other founders appear to have been stimulated by the writings of **Francis Bacon**, who in 1626 had suggested the formation of a national academy for the advancement of the sciences and crafts.

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[104] Babbage's vain attempts to make a universal digital adding machine, are partly the result of the obtuse avarice and shortsightedness of Prime Minister **Robert Peel** (1788–1850); just when the 'analytical engine' was near completion and the inventor desired to adopt a new principle – Peel declined government grants, unwilling to accept further risk.

[105] **Herschel** introduced the new mathematical notation:
- $\sin^{-1}$, $\cos^{-1}$ etc., to indicate the *inverse* trigonometric functions (1813);
- single-line representation of a continued fraction (1820); e.g.,

$$\sqrt{13} = 3 + \frac{1}{\underset{*}{1+}}\frac{1}{1+}\frac{1}{1+}\frac{1}{1+}\frac{1}{\underset{*}{6+}}\frac{1}{1+}\cdots$$

1833 CE **Jean Marie Constant Duhamel** (1797–1872, France). Applied mathematician. Known for his resolution of boundary value problems for the diffusion equation (*Duhamel's Theorem*[106]), and the *Duhamel Superposition Integral*[107] in the theory of linear systems. In acoustics, Duhamel studied the vibration of strings and suggested, independently of *Ohm* (1843), that one perceives a complex sound signal as a linear superposition of elementary sinusoidal components.

Duhamel entered the École Polytechnique in Paris in 1814. The political events of 1816, which caused reorganization of the school, obliged him to return to Rennes, where he studied law. He taught at the École Polytechnique from 1830 to 1869, becoming a professor of analysis and mechanics in 1836. The commission of 1850 demanded his removal because he resisted a program for change, but he returned as professor of analysis in 1851, replacing Liouville.

1833–1845 CE **Urbain Jean Joseph LeVerrier** (1811–1877, France). Astronomer. His main work was in *celestial mechanics*. His discovery of a discrepancy in the motion of the perihelion of Mercury was important as early evidence for Einstein's GTR.

During 1833–1843 he developed formulas for calculating past changes in the earth's orbit, and reconstructed the orbital history of the past 100,000 years. Published in 1843, these calculations were based on the orbits and masses of the seven planets known at the time. He was then led in 1845 to postulate the existence of planet *Neptune* [co-predicted by **John Couch**

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[106] *Duhamel's Theorem*: If $T = F(\boldsymbol{r}, t_0, t)$ is the temperature at point $\boldsymbol{r}$ in a heat conducting solid at time t due to zero initial temperature, a *fixed* heat source $s(\boldsymbol{r}, t_0)$ and fixed boundary temperature $\phi(\boldsymbol{r}, t_0)$ for some t_0 such that $0 < t_0 < t$, then the temperature in the same body due to zero initial temperature and *variable* heat source $s(\boldsymbol{r}, t)$ and boundary temperature $\phi(\boldsymbol{r}, t)$, will be given by the *Duhamel Integral*

$$T(\boldsymbol{r}, t) = \frac{\partial}{\partial t} \int_0^t F(\boldsymbol{r}, \tau, t - \tau) d\tau.$$

Note that the theorem does *not* tell us how to find F, but only how to reduce a problem with time-dependent source and boundary conditions to time-independent source and boundary conditions.

[107] *Duhamel's Superposition Integral*: Given the step-response $h(t)$ of a causal linear system, its response to arbitrary excitation $f(t)$ is given by

$$g(t) = f(0^+)h(t) + \int_0^t f'(\tau)h(t - \tau) d\tau.$$

Adams[108] (1819–1892, England)] on the basis of calculations of its perturbation of the orbit of Uranus. He encouraged astronomer **Johann Gottfried Galle** (1812–1910, Germany) to search for it. The latter indeed found it on Sept. 23, 1846, only 52 seconds of arc from LeVerrier's predicted position.

In 1859 LeVerrier postulated the existence of a planet ('Vulcan') between Mercury and the sun, as the cause of an *anomalous* precession of Mercury's perihelion (that part of the precession which remains after the perturbations due to known planets are subtracted). However, no such planet was ever found. In 1916, Einstein explained the anomalous precession of Mercury's orbit as a consequence of small non-Newtonian spacetime curvature effects in his theory of General Relativity. LeVerrier worked at the Paris observatory for the most of his life.

1833–1855 CE **Wilhelm Eduard Weber** (1804–1891, Germany). Physicist. With Gauss, investigated terrestrial magnetism, and devised the *electromagnetic telegraph* (1833). Introduced the absolute system of electrical units after Gauss' system of magnetic units. His ratio between the electrodynamic and electrostatic units of charge (1855) was crucial to Maxwell in his electromagnetic theory of light. Weber found the ratio was 3.1074×10^8 m/sec but failed to take any notice of the fact that this was close to the speed of light. The Weber, a magnetic flux unit, is named in his honor.

Weber entered the University of Halle (1822) and wrote his doctoral dissertation there (1826). He became a professor at Göttingen from 1831. When Victoria became Queen of Britain (1837) her uncle became ruler of Hanover and revoked the liberal constitution. Weber was one of seven professors at Göttingen to sign a protest and all were dismissed. He remained in Göttingen without a position until 1843, when he became a professor of physics at Leipzig. In 1848 he returned to his old position at Göttingen. He and Dirichlet became temporary directors of the astronomical observatory there.

1833–1861 CE **William Whewell** (1794–1866, England). Philosopher, historian of science and mathematician. Suggested to the British Association for the Advancement of Science in Cambridge that their members be called

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[108] The idea that an unknown planet is causing the observed perturbation of the orbit of Uranus occurred to Adams already in 1841, when he was still an undergraduate at Cambridge University, following an earlier conjecture made by **Mary Fairfax Sommerville** (1780–1872), a writer on mathematics and physical science. After 4 years of work Adams obtained a solution, calculated the position of the unknown planet and sent his results to **G.B. Airy**, then the Astronomer Royal of England. Unfortunately, Airy was not interested in the prediction and made no search for the perturbing body.

scientists. The word gradually caught on and began to displace *natural philosopher*. (Today, we would call few of those BAAS members scientists, since most were amateurs or supporters of science.)

In his book *The Philosophy of Inductive Sciences* (1849) he analyzed the method exemplified in the formation of ideas, in the new induction of science, and in the applications and systematizations of these inductions. Whewell articulated that the success of western science is due to the broad theoretical consistency of *physics*, that with its astonishing congruity with mathematics, came to undergird *chemistry*, which in turn proved foundational for *biology*[109].

Whewell was born at Lancaster. His father, a carpenter, wished him to follow his trade, but his success in mathematics in local grammar-school enabled him to proceed to Cambridge (1812). He was a professor of mineralogy (1828–1832) and philosophy (1838–1855) at Cambridge. He died from the effects of a fall from his horse.

1834 CE **Emile** (**Benoit Pierre**) **Clapeyron** (1799–1864, France). Engineer and physicist. Born in Paris and educated at the École Polytechnique. He went to Russia in 1820 with **G. Lamé** at the invitation of the czar Alexander I to supervise a program of public works. Upon his return in 1830 he was employed in the Paris-Versailles railroad project. In 1834 he revived the forgotten theory of Carnot by applying it to practical steam engine problems. By considering a *Carnot engine* operating between two reservoirs differing infinitesimally in temperature, and by letting the working substance undergo a change in phase, he derived an important relation, giving the slope of the equilibrium lines in a pressure-temperature diagram. This was later generalized by Clausius, and is known today as the *Clausius-Clapeyron equation*[110].

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109 He introduced the concept of *consilience* as literally a "jumping together" of facts and theory to form a common network of explanation across the scientific disciplines. He said: "The Consilience of Inductions takes place when an Induction, obtained from one class of facts, coincides with an Induction, obtained from another different class. This Consilience is a test of the truth of the Theory in which it occurs."

Western scientists succeeded because they believed that the abstract laws of the various disciplines in some manner interlock and interconnect. *Consilience* proved to be the way of the natural sciences.

110 For a pure crystalline solid the change of state from solid to liquid (the process of *melting*) takes place at a single definite temperature under fixed pressure. This temperature is called the melting point for that pressure. The melting point at a given pressure is the temperature at which the solid and the liquid are in equilibrium under that pressure. Melting is accompanied by a change of volume, which may be either an *increase* or a *decrease* (there is a decrease in

1834 CE Foundation of the Statistical Society of London. Though it has contributed little to the theory of statistics, it has had a considerable influence on the practical work of carrying out statistical investigations in the United Kingdom and elsewhere.

1834–1837 CE **Charles Wheatstone** (1802–1875, England). Experimental physicist and practical founder of modern telegraphy. In 1834 he measured the velocity of current electricity by examining sparks produced at the ends of a lengthy electric circuit with a revolving mirror. He estimated that electricity traveled at a speed which was one half the speed of light. The great velocity of electrical transmission suggested to him the possibility of utilizing it for sending messages, and after many experiments and business collaboration with **William Fothergill Cooke** (1806–1879), a patent for an electric telegraph was taken out in their joint name in 1837. Wheatstone is also known for his "bridge", a circuit for comparison of resistors.

Wheatstone was born in Gloucester. He was educated at several private schools. In 1823 he and his brother inherited their father's business. In 1829 he retired to devote himself to experimental research in sound physics. By 1834 he was appointed professor of experimental philosophy at Kings College, London. In 1868, after completion of his automatic telegraph, he was knighted. Wheatstone was the uncle of **Oliver Heaviside**.

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volume when ice melts, but increase in volume when wax melts).

The effect of a change of pressure on the melting point is such that dp/dT has always the same sign as $V_2 - V_1$ [$V_{2,1}$ = volume of a unit mass of liquid (solid)]. The thermodynamic equation governing this phenomenon is known as the *Clausius-Clapeyron* equation:

$$\frac{dT}{dp} = \frac{T(V_2 - V_1)}{LJ}.$$

Here dT is the change in the absolute temperature T of the melting point caused by the change in pressure dp, L is the latent heat of melting in cal. per gram, and the conversion factor J is the mechanical equivalent of heat (1 calorie $\equiv$ 4.2 joules).

If $V_2 > V_1$, the substance expands on melting and dT/dp is positive, whence increasing the pressure *raises* the melting point. If $V_2 < V_1$, the substance contracts on melting, dT/dp is negative, whence increasing the pressure *lowers* the melting point.

The *Clausius-Clapeyron equation* applies to changes of state in general, e.g. change of vapor-pressure with temperature, and enables dp to be calculated if dT and other quantities are provided. As it stands, the equation cannot be integrated unless $L(T)$, $V_1(T)$ and $V_2(T)$ are explicitly known.

1834–1840 CE **Jean Charles Athanase Peltier** (1785–1845, France). Experimental physicist and meteorologist. Discovered experimentally that a junction between two dissimilar metals tends to absorb heat when an electric current is passed across it in one direction, but tends to lose heat when the current is passed in the opposite direction. The thermoelectric cooling or heating of the junction was later termed the *Peltier effect*, and is now commonly used e.g. to cool semiconductor chips. Introduced the concept of *electrostatic induction* (1840).

Peltier was born at Ham (Somme). He was originally a watchmaker, but retired from business about the age of 30 and devoted himself to experimental and observational science.

1834–1856 CE **James Nasmyth** (1808–1890, England). Engineer and inventor. Developed the *self-acting principle* in machine tool design, by which a mechanical hand moving along a slide holds a tool. Using this principle, Nasmyth invented the *planning mill* and a *nut-shaped machine*.

Nasmyth was born in Edinburgh, the son of a noted artist. He became assistant to **Henry Maudslay**, tool designer and manufacturer. In 1834, Nasmyth started the Bridgewater Foundation at Manchester, which became famous for machine tool and steam-engine construction. He then invented the *shaper* (1834) and the *steam-hammer* (1839).

When **James Watt** began his experiments with the steam engine (1763) he could not find anyone who could drill a perfect hole! As a result, his engines leaked steam until the Englishman **John Wilkinson** (1728–1808) invented the *boring machine*. The *planner* was developed (1800–1825) by **Matthew Murray**, **Joseph Clements**, and **Richard Murray**. The principle of the *lathe* had been known since ancient times, and probably originated with the *potter's wheel*. Until 1800, lathes were crude machines that could not be used to cut screw threads accurately. In that year, **Henry Maudslay** (1771–1831, England) invented the first good screw-cutting lathe.

Nasmyth did much for the improvements of machine-tools, and his inventive talent devised many new appliances.

1834–1884 CE **James Joseph Sylvester** (1814–1897, England). One of the foremost mathematicians of his time. Developed the theories of matrices, algebraic invariants and quadratic forms, partition of numbers[111], algebraic

[111] Sylvester addressed the problem of "*denumeration*", i.e. the number of partitions of a number N into m parts $n_1, n_2, \dots, n_m$, repeated or not. This is the same thing as finding the number of solutions in integers of $n_1x_1 + n_2x_2 + \cdots + n_mx_m = N$. Sylvester (1855) introduced the name "*denumerant*" for this number of partitions and denoted it by the symbol

elimination and substitution, determinants, theory of equations, mechanics, optics and astronomy. Sylvester coined the terms *matrix*, *latent roots* (*eigenvalues*) and *Jacobian*.

Sylvester was born of orthodox Jewish parents in London as James Joseph. His eldest brother emigrated to the United States, where he took the name of Sylvester, an example followed by the rest of the family. In 1831 Sylvester entered St. Johns College, Cambridge. Being a Jew he was ineligible for fellowship and could not even take a degree. This last, however, he obtained at Trinity College, Dublin, were religious restrictions were no longer in force. After leaving Cambridge he was appointed to the chair of natural philosophy at University College, London, where his friend **A. de Morgan** was one of his colleagues, but he resigned in 1840 in order to become professor of mathematics in the University of Virginia, U.S.A. There, however, he remained only a few months, for a certain event entailed unpleasant consequences and caused his return to England[112].

$D(N; n_1, n_2, \ldots, n_m)$. He then proved that the denumerants are the coefficients in the expansion of the *generating function*

$$\frac{1}{(1-t^{n_1})(1-t^{n_2})\cdots(1-t^{n_m})} = \sum_n D(n; n_1, n_2, \ldots, n_m)t^n.$$

Multiplying this equation by $(1-t^{n_m})$ and equating coefficients of t^n of two equivalent sums, we get the relation

$$\begin{aligned} D(N; n_1, n_2, \ldots, n_m) =& D(N-n_m; n_1, n_2, \ldots, n_m) \\ &+ D(N; n_1, n_2; \ldots, n_{m-1}) \end{aligned}$$

which upon repetition, enables one to evaluate the denumerant.

112 Sylvester was America's first Jewish professor. He arrived in Charlottesville late in November 1841 and left suddenly in March 1842. Being both a Jew and an Englishman he attracted the hatred of the local protestant community even before his arrival. The *watchman of the South*, organ of the Presbyterian Church, the most influential denomination in Virginia, led a venomous racist attack on his appointment, driven by the fear that "his powerful ascendancy over the young minds may contaminate their pure Christian morality". This crusade provoked some of his students to abuse him to such a point that he had no choice but leave the hornet's nest.

The virtual ouster of Sylvester did great damage not only to his career, but especially to the advancement of science itself. His most creative years were lost to humanity. Disgraced, outcast from the mathematical community, unable to secure any teaching post, unemployed for more than a year in New York City, Sylvester sought his livelihood for 10 years as an actuary and at the legal bar.

He then proceeded to spend almost ten years in business and then turned to the study of law, in connection with which, in 1850, he first met A. Cayley. The two men were to remain lifelong friends, and ultimately both left the law. In 1855 Sylvester took a position at the Royal Military Academy at Woolwick. In 1877 he was appointed professor of mathematics in the Johns Hopkins University, Baltimore, where he stayed until 1883. His stay there gave an enormous impetus to the study of higher mathematics in America, and during that time he contributed to the *American Journal of Mathematics*, of which he was the first editor. In 1883 he was appointed to the Savillian chair of geometry at Oxford[113], from which he retired in 1893 due to failing health.

Sylvester was a good linguist and a diligent composer of verse, in English, Latin and Greek.

1835 CE *The Roman Catholic Church* finally takes the books of Copernicus, Galileo and Kepler off its *Index of Prohibited Books* (the decision to lift the ban was made in 1822). Thus, heliocentricity is officially restored 13 centuries after Justinian and 21 centuries after Aristarchos of Samos.

1835 CE **Samuel Colt** (1814–1863, US). Designer and manufacturer of the first successful repeating pistol – the *'colt revolver'*. It had a cylinder of several chambers that could be discharged in succession by the same locking and firing mechanism. The idea for a revolver dates back to the early 1500's, but Colt was the first person to make it simple and rugged enough for long use.

Samuel Colt was born in Hartford, Conn. He established a factory there, where he also produced arms used during the Mexican War and the Civil War.

1835 CE **Augostino Maria Bassi** (1773–1856, Italy). Bacteriologist. Anticipated **Pasteur** and **Koch** in formulating *germ* theory.

Demonstrated that a disease of silkworms was caused by a parasitic *fungus* (1835). Theorized that many diseases are caused by parasites. This discovery gave impetus to the germ theory of infectious diseases.

At the beginning of the 19th century Bassi studied the silkworm disease (*muscardine*). He discovered (1807) that it was caused by a minute parasitic

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He also took a few private pupils. One of them was **Florence Nightingale**, then six years younger than her teacher.

[113] After the abolition of the *religious tests* (1871), this appointment could go through.

fungus (the fungus was later named *Botrytis bassiana* after its first discoverer) that was transmitted by infected food and from animal to animal by contact. He went on to describe methods for treating fungally infected worms, which was of considerable interest at the time, as *muscardine* was causing financial losses to those working in the European silk trade.

Bassi was born in a village near Lodi in what was then part of the Austrian Empire but is now a part of Italy. He graduated in law and worked as a civil servant in Italy while devoting much of his spare time to the study of living organisms using an early version of the microscope.

Although **Anton van Leuwenhoek** first discovered and described such minute microorganisms as bacteria (1676), the link between these tiny organisms and the induction of infectious diseases was not recognized for another two hundred years. Bassi was the first to understand this link.

1835 CE **Gaspard Gustave de Coriolis** (1792–1843, France). Physicist. Presented an analysis of a body's motion in a rotating frame in his paper: "*Memoire sur les équations due mouvement relatif des systèmes de corps*". He applied his study to fluid motions on a rotating earth[114].

[114] Nevertheless, problems associated with the dynamics of the earth's rotation continue to challenge scientists even today. Consider, for example, the phenomenon of the "*bathtub vortex*", i.e. the rotation developed when water drains out through a hole at the bottom of a vertical tank. In a carefully controlled experiment, water in the tank is allowed to settle for some 24 hours before opening the drain to begin the experiment, so that the residual vorticity is reduced to a value less than that corresponding to the earth's angular velocity; a perceptible *counterclockwise* (looking down on the tank) rotation appears in the Northern Hemisphere after 10–15 min of drain, indicating that vorticity can be developed from the earth's rotation. Under similar controlled conditions, a *clockwise* rotation (looking down on the tank) is developed in the Southern Hemisphere.
The equation of motion for a homogeneous, inviscid and incompressible fluid of density ρ relative to a reference frame having angular velocity $\boldsymbol{\omega}$ (relative to an inertial frame), is

$$\frac{D\boldsymbol{v}}{Dt} \equiv \frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v}\cdot\nabla\boldsymbol{v} = \boldsymbol{g} - 2(\boldsymbol{\omega}\times\boldsymbol{v}) - \mathrm{grad}\,\frac{P}{\rho},$$

where P, $\boldsymbol{v}$ are the pressure and velocity fields and $\boldsymbol{g}$ is the body force per unit mass, assumed to be conservative. This equation assumes that centripetal acceleration terms involving the square of the earth's angular velocity are negligible. Taking the curl of the above equation $(\boldsymbol{\Omega} = \mathrm{curl}\,\boldsymbol{v})$, one obtains the evolution equation for the vorticity $\boldsymbol{\Omega}$:

$$\frac{\partial \boldsymbol{\Omega}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{\Omega} = (\boldsymbol{\Omega} + 2\boldsymbol{\omega}) \cdot \nabla \boldsymbol{v}.$$

The flow for the situation to be considered (that of vorticity generation as water flows out of an exit in the center of the bottom of a vertical cylindrical tank), is *symmetric*. We accordingly employ the cylindrical coordinate system (r, θ, z), with the z direction being downward. In these coordinates (neglecting the term $\boldsymbol{v} \cdot \nabla \boldsymbol{\Omega}$ for the small velocities under consideration), the equations for the components of the vorticity vector become

$$\frac{\partial \Omega_z}{\partial t} = \Omega_r \frac{\partial v_z}{\partial r} + (\Omega_z + 2\omega_z) \frac{\partial v_z}{\partial z};$$

$$\Omega_r = -\frac{\partial v_\theta}{\partial z}; \qquad \Omega_z = \frac{1}{r} \frac{\partial}{\partial r}(r v_\theta).$$

They give the growth of the vertical vorticity component Ω_z in a frame having angular velocity ω_z in terms of the prescribed velocity gradients $\frac{\partial v_z}{\partial r}$, $\frac{\partial v_z}{\partial z}$ and $\frac{\partial v_\theta}{\partial z}$.

Additional relations are obtained from the equation of continuity

$$\operatorname{div} \boldsymbol{v} = \frac{1}{r} \frac{\partial}{\partial r}(r v_r) + \frac{\partial v_z}{\partial z} = 0.$$

We take v_θ to be independent of θ on account of axial symmetry. The temporal growth of the circulatory velocity v_θ associated with the growth of the vertical vorticity Ω_z is taken as the primary effect, and we assume that only the vertical component ω_z of the angular velocity of the frame of reference is operative in generating vorticity. Then, a solution satisfying $\operatorname{div} \boldsymbol{v} = 0$ is obtained with $\Omega_r = 0$, $\Omega_\theta = 0$, $v_z = az$, $v_r = -\frac{1}{2} ar$, $\omega_z = -\omega_E \sin\lambda$, where ω_E is the angular velocity of the earth's rotation and λ is the latitude of the location of the experiment (assumed to be in the Northern Hemisphere). The explicit form of the solution for Ω_z is

$$\Omega_z = -2\omega_E \sin\lambda (e^{at} - 1) + (\Omega_z)_0 e^{at},$$

where $(\Omega_z)_0$ is the *residual vorticity* of the water in the tank on beginning the discharge at time $t = 0$. If this residual vorticity is absolutely removed by stilling, then vorticity will be generated and grow (initially) exponentially with time, according to

$$\Omega_z = -2\omega_E \sin\lambda (e^{at} - 1),$$

being negative in the Northern Hemisphere (counterclockwise when viewed from above). Taking $\omega_E = 7.3 \times 10^{-5}$ radians per sec, we find that at latitude 50° North, a counterclockwise vorticity of about one revolution in 6 sec would be generated after 15 min with a value of $a = 0.01 \text{ sec}^{-1}$. The actual value of a depends upon the dimensions of the tank. Note, though, that the assumed velocity profile in the no-rotation limit $(v_z = az, \ v_r = -\frac{1}{2} ar)$ does not hold for any actual draining container; furthermore, eventually Ω_z is large enough to invalidate the approximates made, so Ω_z does not *continue* to grow exponentially.

Coriolis was assistant professor of mathematics at the Ecole Polytechnique, Paris (1816–1838). He introduced the terms *'work'* and *'kinetic energy'* with their present scientific meaning. In 1835 he wrote a mathematical theory of *billiards.*

Let $\boldsymbol{\omega}$ be the angular velocity vector of the earth, with the vector pointing in the direction of advancement of a right-hand screw (that is, northward along the earth's axis). Since the earth turns once in 24 hours, the magnitude of $\boldsymbol{\omega}$ is $\frac{2\pi}{24\times 3600} = 0.729 \times 10^{-4}$ rad/sec.[115] If $\boldsymbol{v}$ is a body's velocity relative to the earth's frame, then the Coriolis acceleration $= -2\boldsymbol{\omega} \times \boldsymbol{v}$. If u and v are the east and north components of the velocity $\boldsymbol{v}$, the corresponding components of the Coriolis acceleration are: $C_u = 1.46 \times 10^{-4}(v \sin\lambda)$; $C_v = -1.46 \times 10^{-4}(u \sin\lambda)$, where λ is the latitude.

If a car is driven at 90 mph (40 m/sec) in any direction, the Coriolis acceleration at latitude $\lambda = 45°$ will tend to push it to the right with an acceleration of 0.4 cm/sec^2. Meanwhile, gravity will be acting downward with an acceleration of 980 cm/sec^2. As a result, suspended objects in the car will tend to lean to the right by 4 parts in 10,000, or 1.4 minutes of arc. There is no need to bank the freeways for the Coriolis effect. The effect is tiny, but sufficient to cause lateral wear on railway tracks, except near the equator. For example, suppose a train of mass 500 tons $= 5 \times 10^5$ kg moves with a speed of $v = 40$ m/sec toward the north at latitude $\lambda = 30\,°$, so that the component of its speed perpendicular to the earth's axis is $v_V = v \sin\lambda = 20$ m/sec. The Coriolis acceleration it experiences is about 3×10^{-3} m/sec^2, and the Coriolis force it experiences on the track through the flanges of its wheels is thus about 1.5×10^3 Newton.

In the Northern Hemisphere winds will circle around a low-pressure area in a counterclockwise direction, as recorded on a weather map. As a low-pressure area is developing, air will be drawn into its center, and as the wind gathers speed it will be deflected, by the Coriolis effect, toward the right. The net result is a circulation of air around the low-pressure area in a counterclockwise direction. In the southern hemisphere this sense of circulation is reversed, as borne out by meteorological observation.

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[115] Other measures of rotation-rate are:

- RPM – rotations per minute;
- deg/s – degrees per second.

The relations between them are:

1 RPM = 360 deg / 60 sec = 6 deg/sec
1 rad/sec = 180 deg / π sec = 57.3 deg/sec

Objects dropped from a tower will be deflected, except at the poles, toward the east by the amount $d = \frac{2}{3}\omega_0 \cos\lambda\sqrt{\frac{2h^3}{g}}$, where h is the height of the tower, g the local acceleration of gravity, λ the local latitude angle and ω_0 the angular speed of the earth's spin. For $\lambda = 40^\circ$ and $h = 100$ m, $d = 1.6$ cm. This effect can be made plausible by the following line of reasoning: consider a tower of height h located at the equator. The velocity of the tower's base is $v_{_R} = 2\pi R/T$ and the velocity of the tower's top is $v_{_{R+h}} = 2\pi(R + h)/T$, directed from west to east because of the earth's rotation. Accordingly, an object at rest at the top will have an eastward horizontal component of its velocity with respect to ground of amount $v_{_E} = v_{_{R+h}} - v_{_R} = \frac{2\pi h}{T} = \omega_0 h$. Neglecting the fact that the local vertical and horizontal are slowly rotating with the earth, one estimates the deflection toward the east, if the falling time is t, to be of amount $d \cong v_{_E} t$ with $t = \sqrt{\frac{2h}{g}}$. Combining these result, we obtain the above formula except for the factor $\frac{2}{3}$.

In 1735, exactly 100 years before Coriolis published his theory, **George Hadley** (1685–1768, England) proposed a theory based on the *conservation of angular momentum* to explain the existence of *trade winds.*

In 1775 **Laplace** included the horizontal components of the 'Coriolis acceleration' in his hydrodynamic tidal equations, antedating the work of Coriolis.

1835–1837 AD **Edward Blyth** (1810–1873, England and India). Chemist and naturalist. Presented a precursor of Darwin's work on evolution. In a number of papers, published in the *Magazine of Natural History*, heralded elements of the theory of evolution by natural selection, some twenty years ahead of **Charles Darwin** (1859)[116]. He stated therein:

> 'A variety of important considerations here crowd upon the mind, foremost of which is the inquiry that, as *man*, by removing species from

[116] In his book *Darwin and the Mysterious Mr. X* (1979), **Loren Eiseley** vigorously promoted the thesis that Darwin read Blyth's papers and quite likely had derived a major inspiration from it without ever mentioning this in his writings. Eiseley argues that Darwin was to use many of Blyth's ideas years later when writing his "Origin", yet he had given Blyth little or no acknowledgment. Darwin, however, having been influenced by Blyth's ideas, changed natural selection around to mean evolutionary descent of all beings from a common ancestor. Loren Eisely wrote: "But let the world not forget that Edward Blyth, a man of poverty and bad fortune, shaped a key that dropped half-used from his hands when he set forth hastily on his own ill-fated voyage. That key, which was picked up and forged by a far greater and more cunning hand, was no less then *natural selection.*"

> their appropriate haunts, superinduces changes on their physical constitution and adaptations, to which extent may not the same take place in wild *nature*, so that, in a few generations, distinctive characters may be acquired, such as are recognized as indicative of *specific diversity*. May not then, a large proportion of what are considered species have descended from a common *heritage*?'

Blyth was an ardent *creationist*, and his papers flowed with his sense of awe and reverence for the God of creation who had so wonderfully and wisely made all of his creatures.

Unlike Darwin, Blyth was not born into wealth. His father died when he was ten, leaving his widowed mother to raise four children. She managed to send Edward, her eldest son, to school where he excelled in chemistry and natural history. He went to India (1841) and was eventually appointed a curator of the Museum of the Royal Society of Bengal. He lived there for many years on a meager stipend. Plagued by continuing poor health, and afflicted by a personal tragedy, he returned to England (1862), living on a small pension.

1835–1839 CE **Theodor Ambrose Hubert Schwann** (1810–1882, Germany). Physician and physiologist. Laid the foundation of *cell-biology*. Discovered and isolated *pepsin* (1835), a digestive catalyst *enzyme*[117] (which he called *ferment*), the first known animal enzyme. Discovered (1837) that yeast is made of small living organisms[118].

The theory of fermentation was immediately attacked by the leading chemists of the time: **Berzelius** (1839) concluded that microscopic evidence was of no value and that nothing was living in yeast! In the same year **Justus von Liebig** and **Friedrich Wöhler** added sarcasm to scorn by ridiculing the Schwann-de la Tour discovery. It took a man of the caliber of **Pasteur** to settle the problem once and for all.

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[117] The latter term was coined (1876) by **Wilhelm Künhe** (1837–1900, Germany) from Greek words meaning *in yeast*, because they acted outside cells as ferments did inside cells such as yeast. Künhe, a physiologist, discovered the enzyme *trypsin* in the pancreatic juice. Born in Hamburg, died at Heidelberg.
Schwann coined (1839) the word *metabolism*, taken from the Greek and meaning literally "throw into different position" (and therefore implying 'change').

[118] This was independently discovered in the same year by the French inventor **Charles Cagniard de la Tour** (1777–1859).

Schwann was born at Neuss in Prussia. Educated at Bonn and Würzburg, where he graduated M.D. in 1834. During 1838–1847 he lectured at the Catholic University of Louvain, and in 1847 he was appointed professor at Liége, where he remained.

1835–1840 CE **Nachman (Kohen) Krochmal**[119] (1785–1840, Ukraine). Philosopher of history. The first thinker to view Jewish history not as a distinct and independent entity, but as a part of the whole of civilization in the framework of world history. In his *Moreh Nevuchei ha-Zman* (*Guide to the Perplexed of the Age*) he set forth his ideas on reconciling essential Judaism with modern thought; he showed that while the history of every nation undergoes the inevitable stages of growth, blossoming and decay, that of Israel is cyclic, i.e., always rises again to begin a new cycle.

Drawing from **Maimonides**, **Avraham Ibn Ezra**, **Yehuda Halevi**, **Maharal**, **Kant**, **Hegel** and **Schelling**, Krochmal's philosophy of Jewish history is based on the concept of '*national spirit*' that consists of its religious greatness and spiritual gifts; this spirituality permeates all the people's intellectual achievements, explains the ability of the Jews to overcome the forces of decline and is the secret of their self-rejuvenation and national revival.

Krochmal was born in Brody (Poland), lived most of his life in Zalkieve and died in Tarnopol. He was a merchant, and later a bookkeeper at Nestrov, near Lvov. He ordered his disciples to send the manuscript of his book to **Yom-Tov Lippmann Zunz** (1794–1886, Germany) in Berlin, who published it posthumously (1851).

1835–1865 CE **Panfuty Lvovich Chebyshev** (1821–1894, Russia). An outstanding, versatile mathematician with rare talent for solving difficult problems by elementary methods.

Conjectured at the age of 14 that $\left\{\frac{x}{\log x}\right\}$ is a good approximation to the number of primes less or equal to x. In probability theory, Chebyshev introduced the concepts of *variance* and *arithmetic mean* of random variables. Known also for his *inequality*[120], *set of polynomials* and *problem*[121]. He was the

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[119] Known by his acronym: RANAK.

[120] If $a_1 \geq a_2 \geq \cdots \geq a_n \geq 0$ and $b_1 \geq b_2 \geq b_3 \geq \cdots \geq b_n \geq 0$ then

$$\frac{1}{n}\sum_1^n a_k b_k \geq \left(\frac{1}{n}\sum_1^n a_k\right)\left(\frac{1}{n}\sum_1^n b_k\right),$$

$n = 1, 2, \ldots$.

[121] To find the probability that two integers, chosen at random, are prime to one another (the answer is $\frac{6}{\pi^2}$).

principal founder of the theory of approximations. Proved (1850) Bertrand's conjecture (1845) that there is always at least one prime between n and $2n$ for $n > 3$.

Chebyshev was born at Borovsk. He was educated at the University of Moscow, and in 1859 became a professor of mathematics in the University of St. Petersburg, a position from which he retired at 1880.

Approximations — Minimax vs. Least-Squares

"After having spent years trying to be accurate, we must spend as many more in discovering when and how to be inaccurate".

Ambrose Gwinett Bierce (1842–1914)

Mathematical models of natural processes inevitably contain some inherent *errors. These errors result from incomplete understanding of natural phenomena, the stochastic or random nature of many processes, and uncertainties in experimental measurements. Often, a model includes only the most pertinent features of the physical process and is deliberately stripped of superfluous detail related to second-level effects. Therefore, we approximate because we must!*

Even if an error-free mathematical model could be developed, it could not, in general, be solved exactly *on a digital computer. A digital computer can only perform a limited number of simple* arithmetico-logical *operations on finite, rational numbers. Fundamentally important* mathematical *operations such as differentiation, integration, and evaluation of infinite series cannot, in general, be implemented directly on a digital computer. All computers have finite memories and computational registers; only a discrete subset of the real, irrational numbers may be generated, manipulated, and stored. Thus, it is impossible to represent infinitesimally small or infinitely large quantities, or even a continuum of real numbers on a finite interval.*

Algorithms that use only finitistic arithmetic operations and certain logical operations (such as a branching based upon algebraic comparison or logical) are called numerical methods. *The error introduced in approximating the solution of a mathematical problem by a numerical method is usually termed the* truncation error *of the method. When a numerical method is actually run on a digital computer after transcription to computer program form, another kind of error, termed* round-off error, *is introduced. These are caused by the rounding of results from individual arithmetic operations because only a finite number of digits can be retained after each operation, and will differ from computer to computer, even when the same numerical method is being used.*

The art *of approximations is as old as mathematics itself; the greatest mathematicians since* ***Archimedes***, *including* ***Newton***, ***Euler***, ***Lagrange***, ***Gauss***, ***Legendre*** *and* ***Ramanujan*** *devised ingenious and even beautiful ad-hoc techniques to approximate π, e, arclengths, areas, sums of series, roots of equations, solutions of differential equations and other entities.*

However, the rapid growth of applied mathematics in the wake of the Industrial Revolution called for the establishment of a discipline of approximations, through which algorithms could be systematized and developed methodically to answer the growing needs of the exact sciences. The banner of this new trend in mathematics was carried by ***Panfuty L. Chebyshev***.

To understand the ideas of Chebyshev, a brief survey of polynomial approximation is needed: It is sometimes useful to approximate one function $f(x)$, by a sum of 'suitable', simpler function. Such simpler functions are: monomials $\{x^k\}$, $k = 0, 1, \ldots, n$, trigonometric functions $\{\sin kx, \cos kx\}$ or exponential functions $\{e^{\lambda_k x}\}$. A linear combination of monomials leads to an algebraic polynomial *of degree n, $p_n(x) = \sum_{k=0}^{n} a_k x^k$. Polynomials are easy to evaluate, and their sums, products, differences, derivatives and integrals — are also polynomials. In addition, they remain polynomials under the transformations of scaling and of origin translation[122]. These favorable properties are possessed by the trigonometric functions as well. The Weierstrass*

[122] A natural generalization of polynomial approximation consists in approximation by *ratios* of polynomials, that is, by rational functions. Such approximations are expressed conveniently in terms of *continued fractions*. As an example, consider the continued-fraction expansion of **Thorvald Nicolai Thiele** (1909)

$$f(x) = a_0 + \cfrac{x - x_0}{a_1 + \cfrac{x - x_0}{a_2 + \cfrac{x - x_0}{a_3 + \cdots}}}$$

approximation theorem[123] *(1885) then provides the analytical justification for*

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with coefficients

$$a_0 = f(x_0), \qquad a_k = \frac{k}{\left[\dfrac{d\rho_{k-1}(x)}{dx}\right]_{x_0}}$$

for $k = 1, 2, \dots$ where the function $\rho_k(x)$ follows from the recursion relation

$$\rho_k = \rho_{k-2}(x) + \frac{k}{\rho'_{k-1}(x)}$$

for $k = 1, 2, \dots$ with $\rho_{-1}(x) = 0$, $\rho_0(x) = f(x)$. Also

$$a_k = \rho_k(x_0) - \rho_{k-2}(x_0).$$

For small values of $x - x_0$, the Thiele expansion can be seen as an alternative to the Taylor expansion of $f(x)$ about $x = x_0$.
As an example, take $f(x) = e^x$ at $x_0 = 0$, to obtain

$$a_0 = 1, \quad a_{2n} = 2(-1)^n, \quad a_{2n+1} = (-)^n(2n+1).$$

A generalization of the above expansion leads to an analog of the *Bürmann-series* expansion in the form:

$$F(x) = A_0 + \cfrac{G(x) - G(x_0)}{A_1 + \cfrac{G(x) - G(x_0)}{A_2 + \cfrac{G(x) - G(x_0)}{A_3 + \cdots}}}$$

where

$$A_k = \Phi_k(x_0), \qquad \Phi_k(x) = P_k(x) - P_{k-2}(x), \qquad \Phi_{k+1}(x) = (k+1)\frac{G'(x)}{P'_k(x)},$$

$$P_{-2}(x) = P_{-1}(x) = 0, \qquad \Phi_0(x) = F(x).$$

The first few Φ's are readily found to be governed by the equations $\Phi_0 = F$, $\Phi_1 = \frac{G'}{F'}$, $\Phi_2 = 2\frac{G'}{\Phi'_1}$, $\Phi_3 = 3\frac{G'}{F' + \Phi'_2}$. Thus, for example, if we take $F(x) = e^x$, $G(x) = \sin x$, $x_0 = 0$, we obtain, near $x = 0$,

$$e^x = 1 + \cfrac{\sin x}{1 + \cfrac{\sin x}{-2 + \cdots}}.$$

[123] If $f(x)$ is continuous in the closed interval $[a, b]$ then, given any $\varepsilon > 0$, there is some polynomial $p_n(x)$ of degree $n(\varepsilon)$ such that $|f(x) - p_n(x)| < \varepsilon$, $a \le x \le b$.

believing that polynomials can yield good approximations for a given function. Weierstrass' theorem is of little value in cases where $f(x)$ is unknown, except for a few sampled values. But even if $f(x)$ is known, the theorem does not tell us how *the polynomial $p_n(x)$ can be produced.*

If the data is given in the form of $n+1$ paired values $\{x_i, f(x_i)\}$, $i = 0, 1, \ldots, n$, the determination of the approximating polynomial

$$p_n(x) = \sum_{i=0}^{n} a_i x^i$$

boils down to the determination of the coefficients a_i from the set of $n+1$ equations $p_n(x_i) = f(x_i)$, $i = 0, 1, \ldots, n$. The result is known as the interpolating polynomial *of the n^{th} degree. It does not guarantee accurate approximation of $f(x)$ for $x \neq x_i$, unless $f(x)$ itself is a polynomial of degree n or less.*

There are situations which render the above procedure inefficient. This is especially true when the degree of reliability of the discrete data is not well established. There is no sense then in attempting to determine a polynomial of high degree which fits the vagaries of such data exactly and hence, in all probability, is represented by a curve which oscillates violently about the true function. In this case it is preferable to apply a postulate that is often known as the principle of least squares *(**Gauss**, 1795; **Legendre**, 1806).*

The basic idea behind this principle is the requirement that $f(x)$ and its approximant $p_n(x)$ (or some other function) agree as closely as possible *in a specific sense. Of the many meanings which might be ascribed to "as closely as possible", the principle assumes that the* best approximation *is that for which the integral (or sum) of the* squared error *is least.*

More generally, if $W(x_i)$ is a measure of the relative precision of the value assigned to $f(x)$ when $x = x_i$, the criterion is modified by requiring that the squared error at x_i be multiplied by the weight *$W(x_i)$ before the sum is calculated.*

For a given $f(x)$ and basis functions $\phi_k(x)$, one minimizes the integral

$$I = \int_a^b W(x) \left[f(x) - \sum_{k=0}^{\infty} A_k \phi_k \right]^2 dx.$$

Setting $\frac{\partial I}{\partial A_k} = 0$ yields at once

$$A_k = \int_a^b W(x) f(x) \phi_k dx,$$

provided ϕ_k *are orthonormal with weight* $W(x)$ *in* $[a, b]$. *One can show that the truncated Fourier series*

$$T_M(x) = \frac{1}{2}A_0 + \sum_{k=1}^{M}(A_k \cos kx + B_k \sin kx)$$

minimizes the integral

$$I = \int_0^{2\pi} [f(t) - T_M(t)]^2 \, dt.$$

In other words, to minimize I *we should choose*

$$A_k = \frac{1}{\pi}\int_0^{2\pi} f(t)\cos kt\, dt, \qquad B_k = \frac{1}{\pi}\int_0^{2\pi} f(t)\sin kt\, dt.$$

The minimum value of I *then assumes the form*

$$I_{\min} = \pi \sum_{k=M+1}^{\infty}(A_k^2 + B_k^2).$$

If $f(x)$ *is discontinuous, the truncated Fourier-series does not give a good approximation to* $f(x)$ *in the vicinity of the discontinuity points, no matter how large* M *is chosen (Gibbs' phenomenon)*[124].

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[124] In 1904, **Lipót Fejer** (1880–1959, Hungary) has shown that a better approximation to $f(x)$ is obtained if one replaces $T_M(x)$ by the arithmetic mean of the partial sums

$$s_N(x) = \frac{1}{N}\{T_1(x) + T_2(x) + \cdots + T_{N-1}(x)\}.$$

As N increases, this series tends to a limit that is equal to the mean discontinuity, i.e.

$$\frac{1}{2}[f(x+0) + f(x-0)].$$

Since trigonometric series can in turn be represented by power series, $s_N(x)$ can be approximated by a polynomial in x.

Fejer Theorem: Let $f(x)$ be a function of the real variable x, $-\pi \le x \le \pi$, and defined by the equation $f(x + 2\pi) = f(x)$ for all real values of x; and let $\int_{-\pi}^{\pi} f(x)dx$ exist and (if it is an improper integral) let it be absolutely convergent. Then the Fourier series associated with the two limits $f(x \pm 0)$ exist and its average is

$$s = \lim_{N\to\infty} s_N(x) = \frac{1}{2}[f(x+0) + f(x-0)].$$

Another popular criterion for how close is "as closely as possible", termed the minimax principle, requires that the coefficients of the approximating polynomial $p_m(x)$ *be chosen so that the maximum magnitude of the difference* $f(x_i) - p_m(x_i)$, $i = 0, 1, \ldots, n$ $(m < n)$ *be as small as possible. Then the minimax polynomial of degree* m *must satisfy the condition* $\max_i |f(x_i) - p_m(x_i)| =$ *minimum, that is,* $p_n(x)$ *must minimize the maximum error. In more general form* $\max_{a \le x \le b} |f(x) - p_m(x)| =$ *minimum. The principle was created by Chebyshev (1859), and the minimax polynomials are closely related to the Chebyshev polynomials of the first kind* $T_n(x)$.

These polynomials are defined by $T_n(x) = \cos(n \cos^{-1} x)$. *This may look trigonometric at first glance, but it is indeed algebraic [the symbol* T_n *comes from the French spelling used for his name in French, Tchebychef*[125]*]. To see this, we recall de Moivre's theorem*

$$(\cos\theta + i \sin\theta)^n = \cos n\theta + i \sin n\theta.$$

Expanding the binomial and taking the real part, we get, with $x = \cos\theta$:

$$T_n(x) = \cos n\theta = \frac{n}{2} \sum_{k=0}^{[n/2]} (-)^k \frac{\Gamma(n-k)}{k!(n-2k)!} (2x)^{n-2k}.$$

With the aid of the recurrence formula $T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$ *one can generate* $T_n(x)$ *for any* n. *In particular* $T_0(x) = 1$; $T_1(x) = x$; $T_2(x) = 2x^2 - 1$; $T_3(x) = 4x^3 - 3x$.

The Chebyshev polynomials have a number of interesting and useful properties:

Fejer has shown that

$$s_N(x) = \int_0^\pi [f(x+t) + f(x-t)] \left\{ \frac{\sin^2 \frac{N}{2} t}{\sin^2 \frac{t}{2}} \right\} \frac{dt}{2\pi N}$$

where the function in the curly braces is the *Fejer kernel*. Since

$$\lim_{N\to\infty} \left\{ \frac{\sin^2(\frac{N}{2}t)}{\frac{1}{2}\pi N t^2} \right\} = \delta(t),$$

the above result is then obvious.

[125] **Abram S. Besicovitch** (1891–1970) once said, in his thick Russian accent: "Zey are called T-polynomials because T is the first letter of ze name Chebyshev".

(1) $T_n(x)$ *is a polynomial of degree* n*. If* n *is even,* $T_n(x)$ *is an even polynomial; if* n *is odd,* $T_n(x)$ *is an odd polynomial. The coefficient of* x^n *in* $T_n(x)$ *is* 2^{n-1}.

(2) $T_n(x)$ *has exactly* n *real zeros on the interval* $[-1,1]$*. These zeros are located at* $x_j = \cos\frac{2j+1}{n}\frac{\pi}{2}$, $j = 0,1,2,\ldots,n-1$.

(3) $|T_n(x)| \le 1, \quad -1 \le x < 1$ *for all* n*. For* $n > 0$, $T_n(x)$ *attains its bounds* ± 1*, alternately at the points* $x_j = \cos\frac{\pi j}{n}$, $j = 0,1,\ldots,n$; $T_n(x_j) = (-1)^j$.

(4) The Chebyshev polynomials are orthogonal in the interval $[-1,1]$ *over weight* $W(x) = (1-x^2)^{-1/2}$*, namely*

$$\frac{2}{\pi}\int_{-1}^{1}\frac{T_i(x)T_j(x)}{\sqrt{1-x^2}}dx = \left\{\begin{matrix} 0 & i \ne j \\ 1 & i = j \ne 0 \\ 2 & i = j = 0 \end{matrix}\right\}.$$

Given a function $f(x)$*, the least-squares approximation to* $f(x)$ *in terms of the Chebyshev polynomials yield the series* $\sum_{k=0}^{\infty} A_k T_k(x)$ *with* $A_k = \frac{\varepsilon_k}{\pi}\int_{-1}^{1}\frac{f(x)T_n(x)}{\sqrt{1-x^2}}dx$ *where* $\varepsilon_0 = 1$, $\varepsilon_k = 2\ (k \ne 0)$.

The polynomials $T_n(x)$ *also satisfy the discrete orthogonality relation*

$$\frac{2}{n}\sum_{k=1}^{n} T_i(x_k)T_j(x_k) = \left\{\begin{matrix} 0 & i \ne j \\ 1 & i = j \ne 0 \\ 2 & i = j = 0 \end{matrix}\right\}$$

with $0 \le i < n$, $0 \le j < n$*. Here* $x_k\ (k = 1,\ \ldots,\ n)$ *are the* n *zeros of* $T_n(x)$*. If* $f(x)$ *is an arbitrary function in the interval* $[-1,1]$*, and if* N *coefficients* c_j $(j = 0,1,\ldots,N-1)$ *are defined by*

$$c_j = \frac{2}{N}\sum_{k=1}^{N} f(x_k)T_j(x_k),$$

then the approximation formula

$$f(x) \approx \left[\sum_{k=0}^{N-1} c_k T_k(x)\right] - \frac{1}{2}c_0$$

is exact for x *equal to all* N *zeros of* $T_N(x)$.

The importance of this should be appreciated because it means that we can approximate the continuous case in a natural way by simply doing the discrete *case. This property is not enjoyed by the other classical set of orthogonal polynomials.*

(5) *Since* $T_n(x)$ *belongs to a class of functions of the special form* $T_n(x) = f\left[nf^{-1}(x)\right]$, *one derives the unique relation*

$$T_m\left[T_n(x)\right] = f\left[mf^{-1}T_n(x)\right] = f\left[mf^{-1}fnf^{-1}(x)\right] = f\left[mnf^{-1}(x)\right] \\ = T_{mn}(x) = T_{nm}(x) = T_n\left[T_m(x)\right].$$

(6) *Minimax Property. Let* $p_n(x)$ *be any polynomial of degree* n *with leading coefficient unity. Then*

$$\max_{-1\le x\le 1} |2^{1-n}T_n(x)| \le \max_{-1\le x\le 1} |p_n(x)|,$$

i.e. $\left\{T_n(x)/2^{n-1}\right\}$ *has the smallest maximum magnitude on the interval* $[-1,1]$ *of all polynomials in this class.*

This property is of great interest in numerical computations, since any error that can be expressed as an n^{th} *degree polynomial, can be minimized by equating it with* $T_n(x)/2^{n-1}$, *provided there is freedom of choice in selecting the base points* x_i *[which one chooses as the roots of* $T_n(x)$*]. It has been shown that if the function* $f(x)$ *can be expanded in terms of Chebyshev polynomials* $f(x) = \sum_{k=0}^{\infty} a_k T_k(x)$, *then the partial sum* $p_M(x) = \left\{\sum_{k=0}^{M} a_k T_k(x) - \frac{1}{2}a_0\right\}$ *will usually be a very good approximation to the minimax polynomial, that is,* $p_M(x)$ *will be near-minimax*[126], *but whereas the minimax polynomial is very difficult to find, the Chebyshev approximating polynomial is very easy to compute.*

The polynomial $p_M(x)$ *is known as the minimax polynomial approximation to* $f(x)$*. If* $f(x)$ *is given by a polynomial* $p_n(x)$, *it is possible in many cases to obtain a minimax polynomials with* $M < n-1$. *The procedure for replacing a polynomial of a given degree by one of lower degree is known as economization.*

126 *Note that* $p_M(x)$ *is just the Fourier cosine series expansion of the function* $f(\cos\theta)$.

1835 CE **Giusto Bellavitis** (1803–1880, Padua, Italy). Mathematician. Created a two dimensional vectorial system (calculus of 'equipollences'), thereby describing geometrical entities that are in all ways equivalent in behavior to complex numbers. He gave numerous and ingenious applications of his method to mathematical and physical problems. Made significant contributions to algebraic geometry and descriptive geometry.

Bellavitis was born in Bassano. He was an autodidact who did not pursue regular studies. During 1822–1843 he worked for the municipal government of Bassano, occupying his free time with mathematical studies and research. In 1845, he became a professor of descriptive geometry at the University of Padua (through competitive examination). In 1866 he was elected a senator of the Kingdom of Italy.

1835–1846 CE **Jean Léonard (Louis) Marie Poiseulle** (1799–1869, France). Physician and physiologist. Discovered experimental laws for viscous laminar flow in straight circular pipes, known as *Hagen-Poiseulle flow*[127]. He wanted to understand the flow of blood through capillaries and determined the relevant laws in painstaking detail.

If a pressure difference Δp drives the viscous fluid (of *shear viscosity* η) in a cylinder of length L and radius R, the velocity profile is given by the parabola

$$V(r) = \frac{\Delta p}{4\eta L}(R^2 - r^2),$$

where $0 \leq r \leq R$ (*Poiseulle law*).

This *Hagen-Poiseulle flow* is a steady unidirectional axisymmetric flow in a circular cylinder. The law of flow is derivable theoretically through a straightforward integration of the Navier-Stokes equations for steady, axially-homogeneous axially-directed incompressible flow,

$$\eta \nabla^2 \boldsymbol{V} = -\frac{\Delta p}{L}$$

where $\boldsymbol{V}(\boldsymbol{r}, t) = V(r)\boldsymbol{e}_z$, $\boldsymbol{e}_z$ being a unit vector along the axis. We find

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dV}{dr}\right) = -\frac{\Delta p}{\eta L}.$$

[127] **Gotthilf Heinrich Ludwig Hagen** (1797–1884, Germany). Hydraulic engineer in Prussian state service. Known for his studies of laminar and turbulent flow, and for the independent discovery of the law of laminar flow in circular pipes (1839). A *laminar flow* is a an orderly non-turbulent flow in which the fluid particles move in smooth layers without mixing.

Integrating twice w.r.t. r and imposing the boundary condition $V(R)=0$, we arrive at the Poiseulle law.

The volume velocity (volume flow per unit time) through the tube is given by $Q_c = \frac{\pi R^4}{8\eta}\frac{\Delta p}{L}$ (*Poiseulle equation*) and serves to determine η when all other entities are known. The sensitive dependence of Q_c on R explains why small changes in diameter can cause large changes in flow[128]. The application of this law to flow in blood vessels must be modified by the *elastic* properties of the capillary wall and the presence of *erythrocytes*. (It is convenient to define the resistance to flow via the relation $Q_c = \frac{F}{\Omega}$, where $F=\pi R^2(\Delta p)$ is the driving force and $\Omega = \frac{8\eta L}{R^2}$ is the resistance.)

The above relation is sometimes recast in the form $\Delta p = Q_c \cdot r$ where Δp =mean arterial pressure, Q_c =cardial output and r =total peripheral resistance.

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[128] If the pipe has an elliptic cross section

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,$$

the solution of the above problem becomes

$$V(x,y)=\frac{\Delta p}{2\eta L}\left(\frac{a^2b^2}{a^2+b^2}\right)\left(1-\frac{x^2}{a^2}-\frac{y^2}{b^2}\right),$$

which describes the flow of a fluid of viscosity η through an elliptic pipe. The flux through the pipe is

$$Q=\iint\limits_{\frac{x^2}{a^2}+\frac{y^2}{b^2}\le 1} V(x,y)dxdy=\frac{\pi\Delta p a^3b^3}{4\eta L(a^2+b^2)}.$$

Setting $a=b=R$, we deduce that the flux through a circular pipe of radius R is given by $Q_c=\frac{\pi\Delta p R^4}{8\eta L}$. Since the area of an ellipse is πab, a circular pipe with the same cross-sectional area as the ellipse must have radius $\sqrt{ab}$. Hence the fractional flux reduction caused by deforming the circle into an ellipse with the same area is $1-\frac{Q}{Q_c}=\frac{(a-b)^2}{a^2+b^2}$. This is always non-negative and is clearly minimized by taking $a=b$. Thus for a given cross-sectional area and driving pressure drop, a circular pipe carries a greater quantity of fluid per unit time than any elliptical one, and this is why pipes are circles! This optimality property is intuitively clear: because the circle, being the curve of minimum length for a given enclosed area, minimizes the area of viscous friction between fluid and pipe per unit pipe length.

Blood pressure, the force per unit area exerted by the blood against a vessel wall and normal to it, depends on the volume of blood contained within the vessel and the compliance of the vessel wall (how easily they can be stretched). If the volume of blood entering the arteries were equal to the volume of blood leaving the arteries during the same period, *arterial blood pressure* would remain fixed. This is not the case, however. During ventricular *systole*, a stroke volume of blood enters the arteries from the ventricle while only about 1/3 as much blood leaves the arteries to enter the arterioles. During *diastole*, no blood enters the arteries, while blood continues to leave, driven by elastic recoil (**Hale**, 1733).

The maximum pressure exerted in the arteries when blood is ejected into them during systole, the *systolic pressure*, averages 120 mm Hg. The minimum pressure within the arteries when blood is draining off into the remainder of the vessels during diastole, the *diastolic pressure*, averages 80 mm Hg. The arterial pressure does not fall to 0 mm Hg because the next cardiac contraction occurs and refills the arteries before all the blood drains off.

In computing the average pressure (or *mean arterial pressure*) responsible for driving blood forward into the tissues throughout the cardiac cycle, it must be taken into account that arterial pressure remains closer to diastolic than to systolic pressure for a longer portion of each cardiac cycle; numerically, mean arterial pressure $=$ diastolic pressure $+\frac{1}{3}[$ systolic $-$ diastolic$]$.

Poiseulle's main interest was the flow of blood through the vessels of the circulatory system, but he actually worked with water because of the difficulty at that time of preventing blood from clotting on exposure to air. Poiseulle's law is so well established experimentally that it is often used in order to determine the viscosity coefficient of viscous fluids. When blood is examined in this manner[129], its viscosity coefficient is found to be about 5 times the value for water, if the diameter of the tube is relatively large ($\eta_B = 0.035$ poise[130], where 1 poise = 1 dyn sec/cm^2).

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[129] *Estimate of total number of capillaries in the body*: The cardiac output is about $K = 5.5$ liter/minute. The mean blood flux, Q_c, through a typical capillary of radius 3.5 micron (a mean value for the entire body) is calculated from Poiseulle's equation to be 0.13×10^{-6} milliliter/sec. The total number of capillaries in the body, N, is then calculated from $K = fNQ_c$ ($f \simeq 0.7$ is the fraction of capillaries that are open), yielding $N \approx 1.0 \times 10^9$. This estimate agrees with other estimates in order of magnitude. The mean velocity through a capillary is 2.5 mm/sec.

[130] A unit of viscosity named after Poiseulle. In 1975 the unit was changed to {Pascal·sec}, where Pascal (Pa) is a unit of pressure ($= 1$ N/m^2) and 1 Pa·sec $= 10$ poise.

1836 CE Introduction of the marine *screw propeller*, developed by **John Stevens** (1749–1838, U.S.A.) and later applied by **John Ericsson** (1803–1889, Sweden, U.S.A.) and **Francis Pettit Smith** (1808–1874, England).

The Rainbow —
From Noah to Airy (1836) and Beyond

"The triumphal arch through which I march,
With hurricane, fire, and snow,
When the powers of air are chained to my chair,
Is the million-colored bow;
The sphere-fire above its soft colors wove,
While the moist earth was laughing below".

Percy Bysshe Shelley, *'The Cloud'*

I. Phenomenology

The rainbow (formerly known as iris) is best seen in the sky after a rain storm when the sun is low but is shining brightly through a section of the sky that is clear. To see the rainbow, one must turn one's back to the sun and look toward a region that still has rain clouds. Under good conditions one sees a colored arc consisting of concentric circular bands having their common center on the line joining the eye of the observer to the sun. Each band within the bow has its own color, with blue-violet on the inside (lower boundary) and red on the outside (upper boundary). This is known as the primary rainbow; it has an angular radius of about 41°, and exhibits a fine display of the colors of the spectrum.

Sometimes an outer bow, the secondary rainbow, is observed; this is much fainter than the primary bow, and it exhibits the same play of colors, with the important distinction that the order of colors is reversed — the red being inside and the violet outside. Its angular radius is about 57°. It is also to be noticed that the space between the two bows is considerably darker than the rest of the sky. The third or tertiary bow, having about the same radius as that of the primary and colors in the same order, lies between the observer and the sun, but is so faint that it is rarely seen in nature. In addition to these prominent features there are sometimes to be seen a number of colored bands, situated at or near the summits of the bows, close to the inner edge of the primary and the outer edge of the secondary bow; these are known as the spurious, supernumerary or complementary rainbows.

The higher the sun, the lower the bow; if the sun is on the horizon, the observer will see a full 180° of the rainbow (and an observer on a high mountain might see the whole circle of the bow). If the sun is higher in the sky, only a small section of the upper arc can be seen. If the sun is higher than 42° above the horizon, the rainbow disappears completely. However, if one is looking down into a canyon where there is a waterfall or even down the mist produced by a lawn sprinkler, one can see a miniature rainbow though the sun is high in the sky. Under the right conditions, one may be able to see the whole 360° of the bow from an airplane!

(Occasionally, the light from the moon forms a feeble lunar rainbow. *But this phenomenon is rarely seen except about great waterfalls and along certain showery coasts.)*

Seven colors are discernible in each rainbow: violet, indigo, blue, green, yellow, orange, and red. But these colors blend into each other so that the observer really perceives only four or five of them. The angular width of each color band varies, and depends chiefly on the size *of the raindrops in which the rainbow forms.*

Like other optical phenomena, rainbows have been used by people as a means of predicting the weather. A well-known weather proverb says:

Rainbow in the morning, sailors take warning
Rainbow at night, sailors delight.

This bit of weather lore relies on the fact that weather systems in the mid-latitudes usually move from west to east. Remember that an observer must be positioned with his back to the sun and facing the rain in order to see the rainbow. When a rainbow is seen in the morning, the sun is located to the east of the observer and the raindrops that are responsible for its formation must therefore be located to the west. In the early evening, the opposite situation exists — the rain clouds are located to the east of the observer.

Thus, we predict the advance of foul weather when the rainbow is seen in the morning because the rain is located to the west of the observer and is traveling toward him. On the other hand, when the rainbow is seen late in the day, the rain has already passed. Although this proverb does have a scientific basis, a small break in the clouds, which lets the sun shine through, can generate a late-afternoon rainbow. In this situation, a rainbow may certainly be followed shortly by more rainfall.

Rainbows differ among themselves[131]*, as one snowflake from another. Close observations of rainbows show that not even the* colors *are always the*

[131] Furthermore, each observer sees his "own" rainbow, generated by a different set of droplets and different sunlight from that which produces another person's rainbow. In this sense each observer would find "his" rainbow responding to

same; neither is the band of any color of constant angular width; nor is the total breadth of the several colors at all uniform; similarly, the purity and brightness of the different colors are subject to large variations. All these differences depend essentially upon the size of the drops.

II. Optics

The rainbow is produced by the combined effects of refraction, reflection, dispersion, scattering and *diffraction* of sunlight by drops of rain[132].

Consider first the formation of the primary bow in terms of geometrical optics[133] *(ray theory). Consider first a monochromatic (fixed wavelength, λ) plane wave of sunlight falling on a spherical drop of water. According to ray*

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his motion in the same sense as his shadow. Rainbow, like beauty, is in the eye of the beholder.

[132] If we look at a very bright rainbow through a monochromatic red glass we see a succession of circular arcs, alternately bright and dark, similar to the *diffraction rings* which are formed when the light from a point source (sunlight or a distant arc lamp) falls through a small circular stop on to a white screen.

[133] The laws of *geometrical optics* are asymptotic laws of propagation of electromagnetic waves (light), valid in the limit of wavelengths small relative to typical spatial dimensions of the problem. In this regime one assumes that the wave-fronts near any point are sufficiently characterized by their normals and by their local radii of curvature. This approximation breaks down *near* the rainbow, where the *Airy theory* applies. The visible spectrum stretches from $\lambda = 0.7\mu m$ (red) to $\lambda = 0.4\mu m$ (violet), while the diameters of a rain drop range from 200μ to 2000μ. Visible *fog* may have characteristic particle sizes as small as 5–20μ.

The general explanation for rainbows is that of all the parallel rays of light which fall on a drop of water and emerge after one or more internal reflections, those which emerge in appreciably the same direction reinforce each other, and therefore produce a definite sensation on the eye. The order of the colors is explained by the fact that the direction of these “accumulated” rays depends on the index of refraction of the drop, and is therefore different for different colors.

This explanation is not entirely satisfactory, nor are the results it predicts absolutely consistent with the facts. The rays that leave the drop in the same direction take slightly different paths in the drop, and may therefore be in a condition to produce *interference* effects in accordance with the principles of *theory, this wave is represented by its normal (ray). Because of the drop's spherical symmetry, it is sufficient to determine the effects of this interaction in the plane of a great circle containing the ray, rendering the problem 2-dimensional*[134].

Let the ray enter with incidence angle i (w.r.t. the local normal of the sphere) and angle of refraction r (w.r.t. the same normal), be internally reflected n times, and finally be refracted into air again. The path of the ray will lie throughout in the initial plane of incidence. Simple geometric considerations show that the angle by which the incident ray will be bent from its original direction (known as total deviation) is given by

$$D = 2(i - r) + n(\pi - 2r).$$

Since $\sin i = \mu \sin r$ *(Snell's law of refraction; $\mu =$ index of refraction of water relative to air), the deviation becomes a function of* $\{i, \mu, n\}$, *having the explicit form*

$$D = \pi n + 2i - 2(n + 1)\sin^{-1}\left\{\frac{1}{\mu}\sin i\right\}.$$

Now, according to ray theory, the intensity of the emergent ray at large distance R from the sphere is

$$I_n = \frac{a^2}{R^2} I_0 \epsilon_n^2 G,$$

where a is the radius of the drop, I_0 the incident intensity, ϵ_n the fraction that yields the refracted part of the total energy, and

$$G = \frac{\sin i \cos i}{\sin\theta\left|\frac{dD}{di}\right|}$$

is the divergence coefficient ($\theta =$ azimuth angle of emergent ray at the sphere's center relative to incident direction in the plane of incidence).

Clearly, the intensity is very sensitive to $\left|\frac{dD}{di}\right|$, *and since* $\frac{a^2}{r^2}$ *is usually very small, it is just this amplification due to G that makes the rainbow visible! But, alas, the rays which render* $D(i)$ *extremal [i.e.* $\frac{dD}{di} \equiv 0$*] are precisely*

Physical Optics. In the purely ray-theory calculation, one must determine the general form of the *caustic surface* enveloped by a system of rays, originally parallel and emerging after any number of reflections within a drop of water.

134 To obtain the 3-dimensional picture, one then *rotates* this plane about a line in the plane that bisects the angle between the incident and emerged paths of any given ray in the same plane.

those which invalidate the geometrical optics approximation. Simple calculus shows that the angle of incidence of such rays is given by $\cos i_c = \sqrt{\frac{\mu^2-1}{n^2+2n}}$, *and that the total deviation there is a minimum.*

If the rays suffer one internal reflection (primary bow), the deviation is a minimum when the ray is incident at an angle $\cos^{-1}\left\{\sqrt{\frac{\mu^2-1}{3}}\right\}$. *If we take* $\mu = 1.3311$ *for red rays* ($\lambda = 6562.9$ Å), *we find that the corresponding angles are* $(n = 1)$ $i_c = 59°31'$, $r_c = 40°21'$, $D = \pi - 42°22'$. *With* $\mu = 1.3435$ *for violet* ($\lambda = 3968.5$ Å) *we obtain:* $i_c = 58°48'$, $r_c = 39°33'$, $D = \pi - 40°36'$.

Now, if a line be drawn through the eye parallel to the direction of sun's rays, all drops which lie on a cone of semi-opening angle $\{\pi - D\}$ *with this line as axis, will be in a position to allow the emergent parallel rays to enter the eye*[135]*. The apparent arc is therefore composed of arcs of different colors; and the angular radii will be* $42°22'$ *for the red arc and* $40°36'$ *for the violet arc. However, as each point of the sun's disc sends rays giving rise to a bow, the apparent breadth of the bow exceeds the difference of these radii by the sun's angular diameter (ca* $32'$*), yielding altogether* $2°18''$.

Thus, geometrical optics allows us only to explain the overall shape and color-geometry of the primary (and secondary) bow. However, intensities at or near the zones of minimum deviation for each color, must be evaluated from the reconstruction of the caustic formed by the physical-optics process of interference of the refracted and reflected wave-fronts inside the drop.

To this end, ***Airy*** *(1836) first derived the equation of the emergent wave-surface, which he showed to be the involute of the caustic surface. His lengthy analysis boils down at the end to the rather simple equation* $y = \frac{h}{3a^2}x^3$ *where*

$$h = \frac{(n^2+2n)^2}{(n+1)^2(\mu^2-1)}\sqrt{\frac{(n+1)^2-\mu^2}{\mu^2-1}}.$$

Here $y(x)$ *is the curve in the plane of incidence which results from the intersection of the involute with the said plane. It represents the distortion of the straight-line segment of the incident plane wave due to its interaction with the drop, i.e. the initial plane wave surface becomes curved on emerging from*

[135] Since the rainbow may be regarded as consisting of coaxial, hollow conical beams of light of different colors seen edgewise from the vertex, they may have great depth, or extent, in the line of sight. The drops that produce the bow may be nearby or far away, and the question "what is the rainbow's distance" is therefore meaningless.

the drop; it is bent in opposite directions on either side of the least deflected ray. Only those rays that lie close to the inflection point $(x = 0,\ y = 0)$ *reach the eye and give rise to the rainbow image on the retina*[136].

Airy then calculated the amplitude at a distant point in the direction θ *from the ray of minimum deviation. Taking the origin of the coordinates at the point of inflection of the emitted wave-front near the drop, the spatial part of the amplitude is the real part of the Airy "rainbow integral"*

$$A = A_0 \int_{-\infty}^{\infty} e^{-ik[y(x)\cos\theta - x\sin\theta]}\, dx$$

where $k = \frac{2\pi}{\lambda}$ *is the light wavenumber,* $y(x) = \frac{h}{3a^2}x^3$, *and* A_0 *is the amplitude per unit length of the front.*

The above integral can be transformed into a form which yields the luminous image intensity produced in the eye by this active part of the wave surface: $A^2 = M^2 f^2(z)$. *Here*

$$f(z) = \int_0^{\infty} \cos\frac{\pi}{2}(u^3 - zu)\, du, \qquad z = 2\sqrt[3]{6}\frac{\sin\theta}{(\cos\theta)^{1/3}}a^{2/3}\lambda^{-2/3}h^{-1/3},$$

$$h = \frac{(n^2 + 2n)\sin i_c}{(n+1)^2\cos^3 i_c}, \qquad M = 2A_0\left[\frac{3a^2\lambda}{4h\cos\theta}\right]^{1/3}.$$

Airy's theory predicts periodic changes of intensity *of monochromatic light via the function* $f(z)$; *the first maxima does* not *coincide with* $z = 0$, *nor, therefore, with* $\theta = 0$, *the direction of the ray of minimum deviation.*

When the source of light simultaneously emits radiations of various wavelengths, as does the sun, a corresponding sequence of bows, each consisting of a sequence of maxima and minima, are partially superimposed on each other. In this way different colors are mixed, and thus the familiar polychromatic rainbow produced. The mixing of colors is governed by two causes: First, the angular intervals between two successive maxima increase with $\lambda^{2/3}$, *and consequently, coincident distribution of the intensities of any two colors is impossible. Second, since the direction of the ray of minimum deviation varies*

[136] This equation, then, represents a curve *very nearly* coincident with that portion of the wave-front to which the rainbow phenomena are due, and, since the effects computed from it substantially agree with those observed when the drops are not too small, the approximation is sufficient for most practical use. Indeed the approximation that raindrops are perfectly spherical involves, perhaps, a greater error (the *undersides* of a falling drop depart most from a spherical shape — the largest drops look like hamburger buns with concave undersides).

with the refractive index, the direction of the zero ($\theta = 0$) *point on the intensity curve, near which the first maximum lies, correspondingly varies. These two causes, together, produce all sorts of colors mixings that in turn give rise to widely varied sensations.*

The actual intensity is proportional to $\left\{\frac{a^{7/3}}{\lambda^{1/3}}\right\}$. *The breadth of the line is proportional to* $\left\{\frac{\lambda^{2/3}}{a^{2/3}}\right\}$. *Thus, the rainbow bands produced by very small droplets (fog) are not only broad, but also feeble; as their colors necessarily are faint they frequently are not distinguished — the bow appearing as a mere white band.*

Note that in the above analysis certain contributions to the incident radiation and scattering by the drop were totally neglected, since they contribute little to the rainbow. (Thus, for example, radiation backscattered to the sun through reflection off the back surface was ignored.)

III. History

The rainbow affords a means of bridging the gap between the sciences and the humanities.

Mankind has been thinking, talking, and writing about the rainbow for thousands of years. Virtually every volume on mythology contains legends connected with the rainbow, and practically all modern textbooks of physics include some exposition of the optical principles which account for the bow.

Man's story of the rainbow, like other aspects of the history of science, has no inescapable origin and no discernible end. There is no record, oral or written, of the precise date at which a rainbow was first noticed; and even now, at the dawn of the 21^{th} *century, it is not possible to boast that the formation of the bow is accounted for in all its details. The primeval theories of the rainbow must have arisen from man's sense of wonder; and now, thousands of years later, the theory has become enmeshed with the intricacies of advanced mathematics. To trace the gradual development of this theory from early primitive conjectures to sophisticated contemporary formulations is to tread the pathway of human knowledge.*

The earliest documented mentions of the rainbow are in Homer's Iliad, a Chaldean story of the flood, an early Sumerian hymn and above all in the beautiful Biblical passage on God's covenant with ***Noah*** *(Genesis* ***9****, 13–16):*

> *"I do set my bow in the cloud, and it shall be for a token of a covenant between me and the earth".*
>
> *"And it shall come to pass, when I bring a cloud over the earth, that the bow shall be seen in the cloud".*
>
> *"And I will remember my covenant, which is between me and you and every living creature of all flesh; and the waters shall no more become a flood to destroy all flesh"*[137].
>
> *"And the bow shall be in the cloud; and I will look upon it, that I may remember the everlasting covenant between God and every living creature of all flesh that is upon the earth".*

Most exegetes interpreted the passage broadly as indicating that God here gave to the already familiar beauty of the rainbow a new significance, causing it to be a symbol of divine promise. Rain, sunlight, and cloud formations would appear to have been sufficiently similar (throughout temperate regions and during the period of man's existence) to those familiar today to justify the assumption that rainbows were observed by our most primitive ancestors. The circular-arc form of the rainbow may have been perceived by the earliest forms of life endowed with a sense of color-vision. At any rate, the rainbow probably existed more than a billion years ago, independently of the observer, as soon as suitable atmospheric conditions for its formation came into being.

Primitive peoples viewed the rainbow with fear and misgiving, as is evident from the various myths and legends. There is no precise date at which mythology gave way to science in the theory of the rainbow, nor did the transition take place at the same time or at the same rate in all cultures. With the four potamic[138] *civilizations (Tigris-Euphrates, Nile, Indus, Yangtze) flourishing several thousand years ago, one might expect to find some theory of the rainbow; yet there is no evidence of an attempt at a scientific explanation in those cultures.*

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[137] God indeed kept his promise, for He used the *fire* next time to wipe out Sodom and Gomorrah (*Genesis* **19**, 24–25).

[138] In a very broad and over-simplified sense, one can recognize, in the development of civilizations, three general stages which may be designated respectively as *potamic*, *thalassic*, and *oceanic*, according as the dominant cultures centered about rivers, seas or oceans. The first of the stages left nothing scientific on the rainbow. With the advent of the thalassic civilizations, which thrived throughout the whole Mediterranean area during the first millennium BCE, the situation changed.

Among the peoples pressing down from the north were the Hellenes, who occupied the peninsula between the Adriatic and Aegean Seas and then spread east and west to colonize the shores of Asia Minor and the tip of Italy (Magna Graecia). They acquired with amazing alacrity all the knowledge that the potamic civilizations had accumulated, and then they looked about for new intellectual fields to conquer. Unencumbered by hoary traditions and relatively unhampered by political and cultural authoritarianism, Greek scholars investigated nature with an exhilarating freedom and ingenuity. With them the scientific point of view became a dominant characteristic, for they sought to coordinate observations of natural phenomena into a consistent theoretical structure.

***Anaximenes** (ca 575 BCE), a member of the Ionian school led by Thales of Miletos, was first to issue a naturalistic statement on the rainbow. First, he pointed out the obvious relation of the rainbow to the appearance of the sun. Then, he explained the colors as resulting from the admixture sunlight with the blackness of the cloud. Finally, he claimed that the cloud is bending the rays of the sun toward the eye.*

No documents of the period have survived the ravages of time, and the little that is known of the Milesian school is reported by others who lived long afterwards.

***Anaxagoras** (ca 460 BCE) declared that the rainbow is but a reflection of the sun from a spherical cloud, as from a mirror. His theory that the rainbow is caused by* reflection *persisted, in variously elaborated forms, for about* 2000 *years. One cannot, however, determine whether Anaxagoras, and later **Democritos** (to whom Albertus Magnus ascribed the idea that the colors of the rainbow are due to positions from which it is viewed), were aware of the optical law of reflection, or if it had been applied in those days to a geometrical demonstration of the formation of the rainbow.*

*The law may have been discovered shortly after the Periclean age, for **Plato** (in the* Timaeus, *ca 380 BCE) seems to have been aware of some uniformity in the angles in optical reflection.*

***Aristotle** (ca 340 BCE) did* not *contribute significantly to the physics of the rainbow, and his "theory" is today untenable. The most serious deficiency is the ascription of the bow to reflection alone, with no role accorded the essential phenomenon of refraction. A characteristic of his explanation which was perhaps even more obstructive was the macroscopic approach — the concentration of attention on the cloud and the meteorological sphere, rather than on the "little mirrors" of the cloud, where the key to the problem was, in the end, to be found. Finally, one misses in his account any mensurational element, although his geometry, even without measurement, was more sophisticated than that of any successor for well over a millennium. Moreover,* *his work includes the idea that the size of the rainbow could be explained geometrically in terms of the relative positions of the sun, the rain cloud, and the eye of the observer.*

Whatever may be one's judgment on the place of Aristotle in the history of science, criticism must be in terms of the status of knowledge at that time. The rainbow concerns one of the most elusive portions of science; and when one compares the idiosyncrasy of the atmosphere with the regularity of the heavens, it is easier to appreciate why the rainbow appeared so enigmatical to the ancients. In view of the fact that Aristotle placed the explanation for rainbow not in optics, but in meteorology, along with hydrology, seismology, geology, and other portions of natural philosophy, it is greatly to his credit that he gave a thoroughly mathematical treatment to the bow.

Aristotle was undoubtedly acquainted with the colors formed when sunlight passes through a glass prism, but he seems not to have associated these with the rainbow. He was in fact, primarily a philosopher and biologist; and hence it is all the most surprising that the first mathematical theory of the rainbow *should have come from him. The surprise deepens into admiration when one realizes that no superior explanation was proposed for a period of more than* 1500 *years.* ***Archimedes*** *(ca 250 BCE), the greatest mathematical scientist of antiquity, was especially interested in optical phenomena; yet, so far as one knows, he left the problem of the rainbow quite untouched*[139]*.*

Seneca *added little of permanent value in the theory of the rainbow. His chief contribution is his emphasis upon the role of the* individual raindrops *or "mirrors". The practical Romans were ever poor mathematicians, and one looks to them in vain for any improvement over the Aristotelian geometrical theory of the rainbow.*

Ptolemy*, who left us in his* Optics *the earliest surviving tables of angles of refraction from air to water, could have attributed the bow to refraction, but this is not mentioned in that part of* Optics *which came down to us.*

The first man to refute the old idea that the rainbow is due to reflection of the sun's rays by the surface of a cloud (as from a concave or convex mirror) was ***Robert Grosseteste*** *(ca* 1217 *to 1235 CE) in his book De Iride Seu de Iride et Speculo. He hinted vaguely to the role of refraction in the formation*

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[139] Here one sees a sharp difference in approach of the two outstanding scientists of ancient times. **Aristotle** gave answers — often times rough-and-ready, occasionally more sophisticated — to *all* questions that turned up; and hence many of his answers have not stood the test of time. **Archimedes** concentrated his attention upon a few aspects of mechanics and optics, and his treatises are as impeccable today as when they were written.

of the rainbow but gave no specific explanation and made no attempt at quantitative treatment.

Albertus Magnus *(ca 1260 CE) reiterated the part played by the individual drops, and in that sense he was the initiator of the microscopic doctrine. In his* Opus Majus *(1266–1267 CE),* ***Roger Bacon*** *followed the lead of Grosseteste and Albertus and stated that the rainbow must be produced by many reflections in numberless drops of water. Nevertheless, he utterly failed to clear up the problem of the rainbow, and the seed planted by Grosseteste sprouted elsewhere, in far-away Poland:* ***Witelo*** *(b. 1230 CE) was brought up in the neighborhood of Cracow, but he had been educated at Paris, as well as at Padua and Viterbo, and hence may have been acquainted with the work of Grosseteste. He was however mostly influenced by* ***Alhazen****'s* Treasury of Optics. *He wrote, sometimes between* 1270 *and* 1278, *a treatise on* Optics.

In his theory of the rainbow, some rays were reflected directly from the convex surfaces of drops, others were refracted through drops *before being reflected at the other surfaces of other drops lying further within the medium. Refraction served primarily to condense the light; the drops served as spherical lenses, to enhance the lights impression upon the eye. He mistakenly believed that the reflections, as well as refractions, participated in the formation of the colors.*

Witelo also furnished tables of refraction from water (or glass) to air. In so doing he used some of Ptolemy's values from the reciprocal law, *(i.e. independence of the refracted ray on the sense in which the path is traversed). Witelo tried, unsuccessfully, to find general mathematical relations between angles of incidence and refraction, but on the other hand he anticipated Newton's discovery of dispersion, believing that the refraction of different rays through different angles produced the various colors. He did* not *succeed, however, to render an overall picture of the rainbow.*

The next significant advance in the theory of the rainbow was made by ***Dietrich of Freiberg*** *in his book* De Iride Radialibus Impressionibus *(1304–1310). Possessed of experimental skill and persistence as well as theoretical imagination, and deeply versed in the optical learning available at the time, he was admirably equipped to exploit to the full the accumulated wisdom of the rainbow, and draw from it correct and clear physical conclusions.*

Consequently, his explanation is an essentially correct *(though incomplete)* description of the mechanism producing the rainbow, *and vastly superior to that of any one of the eminent scholars before him who had sought unsuccessfully to explain the bow. The merits of his contribution are summarized as follows:*

- *Provided for the first time a clear-cut and unambiguous qualitative theory of the formation of the primary and secondary bows in terms of total reflections and refractions in a raindrop.*
- *Discovered that each drop is responsible but for one color in the bow.*

His theory was nevertheless wrong, because he did not discard the Aristotelian macroscopic circle of altitude, and in his microscopic raindrop model he did not use the essential geometric angle between the incident and emergent rays (deviation). Consequently he failed to account for the radius of the rainbow and the tertiary bow. In linking the orthodox macroscopic geometric explanation to a new microscopic consideration of the geometry of the raindrop, he proposed the only quasi-quantitative theory of the rainbow to appear in the long interval from Aristotle to Copernicus. His work, with all its faults, represents one of the greatest scientific triumphs of the Middle Ages.

Similar ideas were presented simultaneously and independently by the Persian scholar ***Kamal al-Din al-Farisi***[140] *between the years* 1302 *and* 1311*. This amazing case of simultaneous discovery can be understood as due to the common intellectual heritage available to them.*

The first clear-cut break with the Aristotelian tradition is due to the Sicilian mathematician **Francisus Maurolycus** *(1494–1575) of Messina, Abbot of Castronuovo. In his book Diaphaneon* (*written* 1553–1567*; published 1611) he abandoned the meteorological sphere and focused attention for the first time on the basic question to which Aristotelian writers had given only fleeting consideration: How can one account for the apparent size of the rainbow? Why is the angle between the incident and reflected ray close to* 45°*? Of course, Dietrich knew from experience that there was a particular path through the drop designated by nature as appropriate for the production of the primary bow, and he successfully traced this path even though he could not explain it in terms of number or measure.*

Maurolycus seems to have felt that the geometrical basis was discoverable without recourse to experimental observation. However, his suggested scheme through which rays are sufficiently reinforced to reach the eye was physically impossible, inasmuch as it was based on reflection without refraction[141]*.*

[140] Kamal says that he was greatly assisted by his teacher **Qutb al-Din al-Shirazi** (1236–1311), a distinguished Persian scientist. Hence the discovery of the theory presumably belongs to al-Shirazi, its elaboration to al-Farisi. Both Dietrich and al-Shirazi derived their inspiration from the *Meteorologica* of **Aristotle** and *Kitab al-Manazir* (Treasury of Optics) of **Alhazen**.

[141] In another book, *Theoremata de Lumine Umbra* (1521), Maurolycus investigated the optical problems connected with the passage of rays of light through

***Kepler** came close to solving the problem of the rainbow (1608) through the study of refraction in a spherical globe of water. He recognized the fact that colors arise only at places where the refraction is maximum, but lacking the mathematical expression for the law of refraction, he could not make the final step and became discouraged. Yet to Kepler one owes the clear recognition that "to measure is to know"; and to him physics is indebted for the earliest* quantitative *theory of the rainbow based upon refraction in raindrops. Had he but measured more accurately, he might have anticipated the theory that Descartes gave seven years after Kepler's depth.*

*In 1611, **Marco Antonio de Dominis**[142] (1560–1624, Italy), theologician, natural philosopher and mathematician, issued the publication De Radiis Visus et Lucis in Vitris Perspectivis et Iride Tractatus. His explanation of the rainbow, with all its faults, is superior to any other published in the interval of three centuries from* 1311 *to* 1611.

His theory was not derived from any one source, but was rather a mosaic of notions borrowed from the philosophical and optical traditions, *verified or modified perhaps by direct experimental evidence. Dietrich's work was clearly of a higher order in precision and correctness of thought as far as what takes place within the raindrop; but at least Dominis correctly followed Maurolycus in abandoning the old incubus, the Aristotelian meteorological sphere.*

Nevertheless, in several respects Dominis' views are quite inferior to the unpublished opinions of Kepler. In the first place, Kepler's explanation was consistent with the elementary principles of geometrical optics, for he recognized the inevitability of the second refraction. Then, too, Dominis made no

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small apertures with and without lenses (*Camera Obscura*). He applied it to solar observations in a darkened room (1535).

[142] Born of a noble Venetian family in the island of Arbe, off the coast of Dalmatia. He was educated by the Jesuits in their colleges at Loreto and Padua. For some time he was employed as professor of mathematics at Padua, and professor of philosophy at Brescia. He was appointed bishop of Segnia (1596), archbishop of Spalato (1598), and primate of Croatia and Dalmatia (1600). His endeavors to reform the church involved him in a quarrel between the papacy and Venice, and made his position intolerable. He crossed to England (1616), where he became convert to Anglicanism and dean of Windsor (1619). He attacked the papacy in a number of publications (1616 to 1619). He was enticed back to Rome by the promise of pardon and the prospect of a cardinal's hat, only to be doomed to bitter disappointment. Upon his return (1623) he was thrown in prison and died soon thereafter in a dungeon of the Inquisition in St. Angelo. Later the Inquisition tried him posthumously and found him guilty. His corpse was exhumed, dragged through the streets of Rome and publicly burnt in the Campo di Fiore.

*attempt to account for the size of the bow, a problem which Kepler essayed, albeit unsuccessfully. Yet, the explanation of Kepler has been universally overlooked and in many an authoritative treatises on physics one can read that "the elementary theory of the rainbow was first given by de Dominis". (**Newton**, **Leibniz**, **Goethe** and others virtually accused Descartes of plagiarism from de Dominis!)*

The abortive efforts to solve the problem of the rainbow came to an end 326 *years after the first scientific theory was propounded by Dietrich. The man who reaped what others have sowed over more than three centuries was non other than **René Descartes**.*

In the third appendix to the Discours de la Méthode, *one to which he gave the title* Les Météores, *Descartes solved for the first time the fundamental problem of the size of the* rainbow. *Following many experiments and calculations he concluded (1637): "I took my pen and made an accurate calculation of the paths of the rays which fall on the different points of a globe of water to determine at what angles, after two refractions and one or two reflections they will come to the eye, and then I found that after one reflection and two refractions there are many more rays which can be seen at an angle of from* 41 *to* 42 *degrees than at any smaller angle; and that there are none which can be seen at a larger angle. I found also that, after two reflections and two refractions there are many more rays which come to the eye at an angle from* 51 *to* 52 *degrees than at any larger angle, and none which come at a smaller angle".*

Thus Descartes gave the 14^{th} *century theory true scientific status by showing the quantitative agreement of theoretical calculations with the results of observation. He discovered the key to the rainbow problem — the reason for the clustering of rays about the angle* $42°$ *in the primary bow. This he achieved through patient observations and laborious calculations (the* calculus *arrived only in 1671). Yet, he had not really answered* all *the problems connected with the rainbow, as future generations were to find out.*

Descartes' work was not exempt from the rule that new ideas do not meet with immediate acceptance; in fact, it was not integrated into scientific thought for several decades, and consequently, the Aristotelian theory of the rainbow had not suddenly been overthrown. Philosophical disagreement was not the only impediment to the spread of the Cartesian explanation. In 1637 *there were no scientific periodicals, and news traveled slowly. Thus, his work, even though published again in Latin (1656), was slow to achieve the recognition it deserved.*

*The next character in the rainbow drama is **Huygens**, who played a role as a transition figure between the age of Descartes and that of Newton. He held Descartes' explanation of the rainbow in high regard. The chief contribution* *of Huygens to the theory of the rainbow was indirect, and its influence was not felt until well over a century later. In the Cartesian geometrical theory it matters little what light is, or how it is transmitted, so long as propagation is rectilinear and the laws of reflection and refraction are satisfied. But rainbow developments of the* 19^{th} *century were to hinge closely on the nature of light, and here Huygens introduced a major change (1679) — the wave theory of light and a new derivation of the law of refraction by means of the "Huygens Principle".*

This led him to conclude that light travels faster in air than in water, contrary to the conclusions of ***Descartes*** *and* ***Hooke****. But Huygens was unable to verify this inference, nor was he able to make use of his principle to explain the colors of the rainbow. The reason for this lay in the fact that he disregarded the oscillatory and dispersive characteristic of waves. Huygens never really accepted the challenge which the problem of the colors presented and felt that, except for the question of color formation, the work of Descartes was definitive. He probably never dreamed that his theory of light some day would revolutionize the explanation of the rainbow.*

It is of interest to note that whereas Descartes had laboriously calculated the paths of innumerable rays, one by one, Huygens expressed the deviation of the emergent ray as a function of the angle of incidence and then calculated, by the method of ***Fermat****, the values for which this deviation is a maximum or a minimum (a procedure equivalent to the use of the calculus).*

Huygens may have been the original inspiration for a little-known treatise on the rainbow Stelkonstige Reeckening Van Den Reegenboog, composed by ***Baruch Spinoza*** *and published posthumously (1687), the year of Newton's Principia. In this manuscript the author combined the use of a variant of the method of Fermat and Cartesian analytic geometry to arrive at the radii* $40°57'$ *and* $54°25'$ *for the two bows.*

Then came ***Newton****. For thousands of years men had looked at colored spectra produced by light passing through spheres and prisms of water and glass; but Newton looked at the spectrum more carefully than had any one of his predecessors. He saw that rays of differing color were refracted by differing amounts. Ever since antiquity it had been realized that the amount by which light was refracted depended on the angle of incidence, as well as upon the media in question; but Newton first showed (1666 to 1672) that it depends also on the color of the light involved, each color having its own characteristic index of refraction. Thus, for the first time color was reduced to an orderly quantitative basis, and, also, for the first time, an adequate explanation was possible for the width of the rainbow.*

Traditionally (since Aristotle) it had been understood that white light was pure and homogeneous, and that color, such as that of the rainbow, was *the result of a loss in strength or purity. Newton's experiments indicated, however, that the reverse is true — only colored light is pure and homogeneous, and it is not a result of weakening. Newton realized that such a drastic departure from previously accepted views would not be accepted by his contemporaries without strong supporting evidence. Consequently, he went out of his way to get an audience for his ideas. In* 1672 *he presented to the Royal Society a paper describing his discovery of the composite nature of white light. But the reception accorded Newton's great discovery was a great disillusionment to the young author. Half a dozen scientists, including Hooke and Huygens, criticized his work. From that time on Newton was most prudent indeed. He withheld from publication anything further on optics until* 1704, *the year after Hooke, his sharpest critic, had died. Meanwhile his ideas went pretty much unnoticed, with credit sometimes ascribed to others who did similar work [e.g.* **Edme Mariotte** *(1679)]. Newton made two other contributions to the theory of the rainbow; he was first to render calculations concerning rainbows of order higher than two (1669–1671, published 1704), and he derived for the first time an explicit formula from which the radii of bows of all orders (and for any index of refraction) can be deduced*[143].

The theory of the rainbow during the Newtonian age had reached the point where no one untrained in advanced mathematics could hope to follow it. In the field of poetry the change in attitude toward the rainbow was variously received. In England, some times later, on December 28, 1817, *in a dinner gathering,* **Charles Lamb** *(1775–1834) and* **John Keats** *(1795–1821) agreed that Newton had destroyed all the poetry of the rainbow by reducing it to its prismatic colors, and all the guests drank a toast: "Newton's health, and confusion to mathematics". Not long afterwards Keats composed the familiar lines of Lamia:*

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143 [$\cos i_c = \sqrt{\frac{\mu^2-1}{(n+1)^2-1}}$; $\mu =$ water index of refraction; $n = 1$ for the primary rainbow, etc.; $i_c =$ critical angle of incidence which makes the deviation D extremal;

$$D(i_c) = \pi n + 2i_c - 2(n+1)\sin^{-1}\left\{\frac{1}{\mu}\sqrt{\frac{(n+1)^2-\mu^2}{(n+1)^2-1}}\right\}].$$

> *"Do not all charms fly*
> *At the mere touch of cold philosophy?*
> *There was an aweful rainbow once in heaven:*
> *We know her woof, her texture; she is given*
> *In the dull catalogue of common things.*
> *Philosophy will clip an Angel's wings,*
> *Conquer all mysteries by rule and line,*
> *Empty the haunted air, and gnomed mine –*
> *Unweave a rainbow".*

For 99 years after the Opticks *appeared (1704) there was nothing of comparable significance in the story of the rainbow. It was generally assumed that the last word has been written; the theory appeared to be in such satisfactory shape that little refinement seemed to be necessary.*

The first substantial studies in the physiology of color, as well as the first credible explanation of the supernumerary rainbows*, came in the work of* ***Thomas Young*** *(1803). His discovery of optical interference unlocked one of natures best-kept secrets — the cause of supernumerary rainbows: Young saw that for each angle of incidence upon a raindrop greater than that of the Cartesian effective ray, i_c, there is another of smaller angle such that the two rays emerge from the drop in parallel, or nearly parallel, paths.*

It can be shown that these two rays are reflected at the same point on the rear surface of the drop. These two rays, being deviated more than $D(i_c)$, will appear inside *the primary bow (for the secondary rainbow they appear* outside *the bow). It is clear that the two rays, on traversing the drop, will have followed paths which are not quite equal in distance, and so they will arrive at the eye's retina with a certain phase-difference and interfere. If the difference in the lengths of the paths is an integral multiple of the wavelength of a given color, the rays will be reinforced; if it is an odd multiple of half a wavelength, the rays will extinguish each other. Several positions are expected where reinforcement takes place, and also other intermediate positions where the rays annihilate each other — the familiar phenomenon of* Newton rings*, namely, the formation of a whole series of bows.*

Thus, there are potentially infinitely many bands of each hue in the primary bow*, the bands becoming fainter and narrower as the radii diminish. The spacing of the bands depends on the variations in the length of the path, and these are determined by the size of the drops. Ordinarily there is considerable overlapping in the bands of various colors, especially when the drops are not of uniform size; and this accounts for the fact that most people see only a single primary rainbow.*

If the drops are unusually minute (as in a fine mist), the interference bands may become so intermingled that the result is a superposition of all colors, that is, a white bow*.*

For large drops of rain, only one brightly colored primary rainbow is usually seen; but Young found that supernumerary bows are clearly visible when the raindrops are uniformly sufficiently small, and noted that there was a regularity in their spacing which corresponds to that of Newton's rings. Young actually advanced this phenomenon as an argument supporting his doctrine of interference[144].

Young's theory, however, was unable to explain 18^{th} *century observations that the radius of the bow is not constant, but rather varies considerably; and the explanation had to wait for another* 35 *years. In the meantime, Young's work did not receive the recognition it deserved, partly due to the discovery of the phenomenon of polarization of light [**Malus** (1808); **Biot** (1812); **Brewster** (1815)].*

Indeed, the observation that light from the two rainbow arcs is almost entirely polarized in the planes which pass through the eye and the radii of the arcs, could not be fitted into Young's interference theory, since the latter was based (prior to 1816) on the wrong concept of light as a sound-like longitudinal motion. But in 1816 *both **Fresnel** and **Young**, independently, finally saw that polarization made it necessary to abandon this preconception to conceive instead of light as a transverse vibration, in which the displacements take place at right angles to the direction of propagation. They assumed that light is a bundle of transverse waves in planes variously oriented, and that in reflection and refraction some of the planes of vibration are screened out to leave a beam which is wholly or partially polarized. This idea saved the wave theory from the incubus presented by the non-interference of polarized light, for transverse vibrations in different planes could scarcely be expected to affect each other.*

Since transverse vibrations were regarded as incompatible with the fluid state, Fresnel was forced to assume that the luminiferous ether behaves like an elastic solid; with an elasticity greater than that of steel. But scientists found it difficult to believe that the heavenly bodies are moving resistlessly through such a solid, and it was not until 1838 *that the Newtonian emission theory gave way to the wave theory of Young and Fresnel.*

Neither Young nor Fresnel gave the adequate mathematical exposition which was needed for the formation of the rainbow. The definitive explanation of the rainbow was to a large extent the work of three Cambridge men, not

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[144] The interference theory of the rainbow made clear why the bow is brighter near the earth and why the supernumerary arcs seem to appear near the highest part of the bow; raindrops tend to increase in size as they fall, and the results of Young showed that where the drops are uniformly larger, there will the bow be brighter, but unaccompanied by supernumerary arcs.

one of whom was primarily a mathematician: ***George Biddell Airy*** *(1801–1892) was an astronomer,* ***William Hallowes Miller*** *(1801–1880) was a mineralogist, and* ***Richard Potter*** *(1799–1886) was a chemist and physicist with medical training. Their theory was further refined by other Cambridge scholars and professors, notably* ***Stokes***, ***Larmor*** *and* ***Rayleigh***.

The final major assault on the rainbow problem was started by ***Potter*** *(1835): He integrated all the former concepts of* ***Descartes*** *(limiting ray),* ***Huygens*** *(wave front),* ***Newton*** *(dispersion), and* ***Young*** *(interference) into a single mathematical theory. To this he added a central idea which escaped the notice of his predecessors — the caustic*[145] *wave front formed in the raindrop: following the refraction at the concave surface of the drop, the wave-front is no longer rectilinear, but curvilinear. In fact, some of the rays intersect others even before they strike the rear surface. Descartes had traced the path through the drop of one ray at a time, and so he failed to call attention to this intersection. These rays, after the first refraction, form a caustic by refraction.*

Potter then found that the orthogonal trajectory of the rays reflected from the rear concave surface of the drop is an s-shaped curve, with an equation approximately of the form $y = kx^3$. *Finally, the wave front which emerges from the drop after the second refraction consists of two convex portions, mutually tangent but with unequal radii of curvature, which form a cusp at a point slightly below the Cartesian limiting ray. The Cartesian rainbow band can therefore be thought of as a caustic, and the rays of each color have their own caustic, each one corresponding to a colored band in the rainbow as explained by Newton in different terms.*

Potter called attention to the fact that close to the caustic, the nearly parallel rays will exhibit the interference phenomenon of which Young had pointed out, creating the Newton-rings pattern. Consequently, the intensity of illumination does not fall of monotonically as one departs from the effective ray, as Descartes believed; the decline in intensity is oscillatory.

The precise analytical expression for the intensity of illumination at each and every point of the region brightened by the bow (as a function of the angular deviation of the ray from the least-deviated ray) was given by ***Airy***

[145] *Caustic surface* — the envelope of a family of reflected or refracted rays. An example of a *caustic curve*, which is a plane section of a caustic surface, can easily be seen by noting the bright arcs formed by *reflected* light rays on the bottom of a teacup. If the equations of the tangent lines (rays) forming the caustic are known, the equations of the orthogonal trajectories (wave-fronts) and envelopes (caustics) can be found by the methods of advanced calculus. Caustics can be formed either by reflection or by *refraction*, usually by *intersecting rays*.

(1836) in his diffraction theory of the rainbow. He found that the intensity of light is given by the square of an integral which since has come to be known as "Airy's rainbow integral". His calculations showed that the region of greatest brightness (as viewed by any particular observer) lie appreciably within the radius computed on the basis of geometrical theory.

Airy also showed that the radius of the primary bow (and not only the colors and spacing of the arcs) varies with the size of the drops. Moreover, whereas Descartes, Young and Potter maintained that there should be no light whatever returned to the eye at an angle greater than that of the least deviated ray, Airy's calculations show that this assumption is erroneous[146] *and that diffraction must be taken into account in any complete theory of the rainbow.*

***Miller** (1841) extended Airy's analysis to include the secondary bow and performed experiments which verified the results of Airy. **Stokes** (1850) derived a more expeditious device for calculating the values of Airy's rainbow integrals, and calculated intensities of illumination sufficient to place the first fifty maxima.*

*The theory of Potter and Airy ignored the finite size of the sun's disc; throughout the computations of the rainbow integral the light was assumed to come from a point source. **Keiichi Aichi** (b. 1880, Japan) and **Aikischi Tanakadate** (b. 1856) extended the Airy theory for a circular source of light (1904). They had been struck by the fact that, according to the theory of Airy, one should anticipate numerous supernumerary arcs, whereas in nature the bow generally is accompanied by only a very limited number. They suspected, from some approximations, that this discrepancy might be accounted for by fact that the sun is not a point source of light. Their elaborate analysis showed that, for a finite source, the supernumerary arcs of the natural rainbow lose most of their color, especially for large drops. They demonstrated that the degree of luminous intensity depends on the breadth of the source, as well as on the size of the drops.*

Since 1945 *rainbows have been "used" for the first time as a means of calculating how large the drops of water in a cloud are. The results were used in aircraft icing investigations, where the free-water content and the size of drops becomes a matter of immediate concern; a camera "rainbow recorder" was used, both in natural clouds and in experimentally-controlled*

[146] Diffraction occurs for negative values of the Airy parameter $z \approx 2\theta \left\{ \frac{a^2}{h\lambda^2} \right\}^{1/3}$. Airy also showed that, as far as the rainbow is concerned, there is no need to integrate over the cusped wave-front that emerges from the drop, but over the *simpler s-shaped caustic-curve* that results from the reflection at the back of the drop.

fog chambers, to find the difference in viewing angle between the principal bow and the first supernumerary arc: the rainbow-calculated drop diameters differed by as little as 2 to 5 percent from those computed by other means. Airy's theory proved, however, inadequate in the range of drop sizes from 10 to 15 microns, and a new approximation for the equation of the generating caustic wave-front was derived.

The story of the rainbow had passed from Iris to Mathesis through a mythological *state, a* reflection *stage, a* refraction *state, a* geometrical *stage, a* dispersion *state, an* interference *stage, and a* diffraction *stage. But although much is known about the production of the rainbow, little has been learned about its perception; our knowledge of what goes on between the eye and the brain when one sees a rainbow is pretty much in a state of flux. Which part of what we see is due to physical factors, and which is due to purely entoptic reasons — is still unknown. As long as man by nature desires to know and yearn for beauty, just so long will Iris continue to inspire both exact science and romantic literature. For poets the rainbow had served as a ubiquitous source of inspiration, but mathematics has also given the bow a beauty which only the deeply initiated can fully appreciate.*

1836–1855 CE **Nicholas Joseph Callan** (1799–1864, Ireland). Priest, scientist and inventor. Created the first *induction-coil* (1836), which led to the modern transformer [ahead of **Ruhmkorff** (1851)].

Callan was influenced by the work of **William Sturgeon**, who invented (1825) the first electromagnet and by the discoveries of **Michael Faraday** (1831) and **Joseph Henry** (1832) concerning electromagnetic induction. Working from 1834 on, Callan employed a horseshoe-shaped iron-bar and wound it with thin insulated wire (primary coil) and then wound thick insulated wire over the winding of the thinner wire (secondary coil). He discovered that, when a DC current (sent by a battery) through the primary coil was interrupted, a high voltage was produced in the *open* secondary coil. In doing so he constructed what is today known as *autotransformer*. Callan's induction-coil also used a "breaker", consisting of a rocking wire that repeatedly dipped into a small cup of mercury. A clock mechanism was used to interrupt the current in the primary coil 20 times a second. It generated a 40-cm spark in the open secondary coil over an open-circuit voltage of some $600,000$ Volt.

Like Cavendish before him, Callan made an independent discovery of Ohm's law (Ohm, 1827). In applied science he discovered several types of galvanic battery and influenced the study of high-voltage electricity. He also constructed one of the first DC electric motors.

In 1838, Callan stumbled on the principle of the self-excited dynamo; moving his electromagnet in the earth's magnetic field, he found he could produce electricity without a battery. In his words, he found that "by moving with the hand some of the electromagnets, sparks are obtained from the wires coiled around them, even when the engine is in no way connected to the voltaic battery". The effect was feeble so he never pursued it, and the discovery is generally credited to **Werner Siemens** (1866).

Callan was born at Darver, Ireland. After ordination as priest (1823) he went to Rome where he obtained a doctorate in divinity (1826) at the Sapienza University. While at Rome he became acquainted with the experiments of Galvani and Volta. In 1826 he was appointed to the chair of Natural Philosophy in Maynooth University (near Dublin) and remained in that post until his death.

Unfortunately, his name was forgotten and his inventions were attributed to other scientists: Maynooth was a theological university where science was marginal in the curricula. Callan's colleagues often told him that he was wasting his time. In such an atmosphere, Callan's pioneering work was soon forgotten after his death, and **Ruhmkorff** (who like all instrument makers, put his name on every instrument he made) got into the textbooks and thus received the pioneering credit for the induction-coil. It was never challenged until Callan's publications were rediscovered in 1936 and first put into physics textbooks in 1953.

1836–1858 CE **Robert Remak** (1815–1865, Germany). Neurologist and biologist. Made important discoveries in nerve and muscle diseases (1859). Developed new cell theory for animals, emphasizing protoplasm as cell substance and that cells are formed by division of existing cells. Showed (1845) that there are only three layers present in the early development of the embryo which he named: ectoderm, mesoderm and endoderm.[147]

Remak was born in Posen (Poznan), the oldest of the five children of Salomon Meyer Remak, who ran a tobacco shop and lottery office, and Friedrike

[147] In this he revised earlier theory of **Christian Pander** (1820) and **Karl von Baer** (1826) who first maintained that an embryo has heterogeneous structural layers, called *germ layers*, which always give rise to the same physiologically differentiated adult tissues. Remak emphasized that the formation of the germ layers occurs by *cell division*.

Caro[148]. The family were Orthodox Jews and in 1815 Poznan had returned from Polish to Prussian sovereignty by the Congress of Vienna. Remak received his earliest education at home and enrolled at the University of Berlin (1833) to study medicine. His pioneering studies on the nerve tissue (1836) gained him the M.D. (1838).

Although Remak wished to make a career in teaching, the way was barred to him, since in Prussia at that time Jews were not admitted to that profession. He therefore continued his laboratory research, and in 1839 discovered ganglion cells in the human heart. This finding seemed to him to explain the relatively autonomous action of the heartbeat. During 1843–1856, Remak applied many times for a teaching position, but in spite of his growing fame and the intervention of **Alexander von Humboldt** and other eminent friends on his behalf, his repeating requests were refused. Finally (1859) he was appointed assistant professor at the University of Berlin, but this belated and meager recognition had no effect upon his subsequent career.

His son **Ernst Julius** became a professor of medicine at the Berlin University (1902).

His grandson **Robert** became an important researcher in number theory. The name REMAK is an acronym for **R**abbi **M**oshe **K**ordovero[149][1512–1570, Safed, Israel], and the Remak family probably stemmed from the same Spanish-Italian ancestry.

1837–1838 CE Based on the discoveries of **Oersted** (electromagnetism, 1820), **Sturgeon** (electromagnet, 1825) and **John Daniell** (steady current cell, 1836), three men developed successful *wire-telegraphy*: in England, working together, **William Fothergill Cooke** (1806–1889) and **Charles Wheatstone** (1802–1875), and in the U.S.A. the painter **Samuel Finley Breese Morse** (1791–1872). The *Morse code*, patented by Morse in 1840, uses patterns of dots and dashes to represent letters, numerals, punctuation and other signs.

1837–1844 CE **Samuel Finley Breese Morse** (1791–1872, USA). Portrait painter and inventor. Developed the first successful electric

148 Kisch, B., "Forgotten Leaders in Modern Medicine", *Trans. Amer. Phil. Soc. 44*, 227–296, 1954; Pagel, J., *Allgemeine deutsche Bibliographie 28*, 191–192, Leipzig, 1889

149 Provided the first complete and systematic theory of the *Kabbalah*.

telegraph[150] in the United States and invented the *Morse Code*, still used occasionally to send telegrams.

Morse was born in Charlestown MA, the eldest child of the Reverand Jedidiah Morse and his wife, Elizabeth Ann Breese. He graduated from Yale College (1810), went to London (1811) and studied two years at the Royal Academy of Arts. He returned home in 1815 and within the next ten years became a well-known portrait painter. His interest in telegraphy began in 1832 and after working at it for five years he demonstrated his equipment in 1837. His symbolic alphabet, known as the Morse code was invented in 1840. A line was constructed between Baltimore and Washington and the first message, sent in May 24, 1844, was "What hath God wrought!". Morse and his telegraph were known within 12 years throughout North America and Europe. In 1861 the United States were linked by telegraph from coast to coast. Electromagnetic waves were not yet discovered at that time. Morse was not the first to invent the telegraph, but he is known as the "father" of the telegraph because he *created a new industry.*

1837–1853 CE **Heinrich Gustav Magnus** (1802–1870, Germany). Physicist and chemist. Investigated the motion of spinning spherically or cylindrically-shaped solids in a fluid (liquid or gas) and discovered an effect named after him ("*Magnus effect*")[151]. It is responsible for the "*curve*" of a served tennis ball or a driven golf ball, and affects the path of a spinning artillery shell. Analyzed (1837) gases in the blood and showed that a higher

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[150] Independently, **William O'Shaughnessy** set up a 22 km demonstrator telegraph system in India, near Calcutta (1839) and later (1854) completed a 1300 km telegraph line in India, between Calcutta and Agra.

[151] In ideal-fluid aerodynamics (neglecting friction), the force exerted by the fluid on a finite rigid body moving with a constant velocity through it is zero, if the fluid closes behind the body (*d'Alembert's paradox*). The result implies that the so-called *drag-force* on the body due to fluid resistance is zero. [It also predicts a *zero lift force* for lifting bodies such as wings of an airplane!]
If however, a circulatory flow is superposed, such as occurs when the body is spinning, *Bernoulli's theorem* predicts the existence of a force that tends to divert the body from its straight trajectory. In the case of a cylinder (e.g. spinning artillery shell) with a clockwise rotation and moving to the right, the fluid velocity above the cylinder increases, whereas the velocity below it decreases. Consequently there is a low pressure ("*suction*") above it and a high pressure below it. The result is a lift on the cylinder [Rayleigh, 1876]. The lift on an airplane wing does not require a rotating body; the shape of the wing creates a velocity distribution with circulation, but no vortices. The lift is then caused by a Bernoullian pressure-gradient.

concentration of oxygen exists in the blood flowing in arteries than in that flowing in veins. This suggests that respiration takes place in the tissues[152].

Magnus was born in Berlin to parents of Jewish origin. He studied for a while under **Gay-Lussac** in Paris. In 1831 he returned to Berlin as a lecturer on technology and physics at the university, and in 1845 he became a full professor there.

1837–1859 CE **Gabriel Lamé** (1795–1870, France). Engineer and mathematician. Invented curvilinear coordinates. Made the following forecast of the scientific significance of coordinate systems:

"*Should anyone find it singular that we have been able to found a Course of Mathematics on the sole concept of a system of coordinates, he may be reminded that it is precisely these systems which characterize the phases and stages of science. Without the invention of rectangular coordinates, algebra might still be where Diophantos and his commentators left it, and we should lack both the infinitesimal calculus and analytic mechanics. Without the introduction of spherical coordinates, celestial mechanics would be absolutely impossible; and without elliptic coordinates, illustrious mathematicians would have been unable to solve several important problems of this theory... Subsequently the reign of general curvilinear coordinates supervened, and these alone are capable of attacking the new problems* [*of mathematical physics*] *in all their generality. Yes, this definitive epoch will arrive, but tardily: those who first recognized these new implements will have ceased to exist and will be completely forgotten — unless some archaeological mathematician revives their names. Well, what of it, provided science has advanced?*"

Lamé insistence on the importance of coordinates has been justified in modern physics. His early work (1839) on the conduction of heat in ellipsoids led him to discover the functions which bear his name.

Lamé's investigation in curvilinear coordinates led him into the field of number theory. In 1840 he was able to prove Fermat's Last Theorem for the case $n = 7$. In 1847 he developed a solution, in complex numbers, of the form $A^5 + B^5 + C^5 = 0$ and in 1851 a complete solution, in complex numbers, of the form $A^n + B^n + C^n = 0$.

[152] This was later confirmed by the physiologist **Eduard Friedrich Wilhelm Pflüger** (1829–1910, Germany), who showed that the essential *chemical* changes of respiration occur in the tissues and cells rather than in the lungs. Finally **John Scott Haldane** (1860–1936, England) and **Joseph Bacroft** (1872–1947, England) elucidated the fine physical mechanism of respiration.

He also proved the following theorem: The number of divisions required to find the greatest common divisor of two numbers is never greater than 5 times the number of digits in the smaller of the numbers.

Lamé was born in Tours. He attended the École Polytechnique during 1813–1817. He then continued at the École des Mines from which he graduated in 1820. In the same year he accompanied **E. Clapeyron** to Russia. He was appointed director of the School of Highways and Transportation in St. Petersburg, where he taught the exact sciences. He was also busy planning roads, highways, and bridges that were built around that city.

In 1832 he returned to Paris and accepted the chair of physics at the École Polytechnique. In 1836 he was appointed chief engineer of mines. He also helped plan to build the first two railroads from Paris to Versailles and to St. Germain. In 1851 he became professor of physics and mathematics at the University of Paris.

It is difficult to characterize Lamé and his work. Gauss considered him the foremost French mathematician of his generation. French mathematicians, however, considered him too practical, while French scientists viewed him as too theoretical. Yet the work he began was generalized almost as soon as it appeared by such mathematicians as **Klein** and **Hermite**.

1837–1861 CE *Origins of the telephone*. In 1837, **C.G. Page** of Salem, Massachusetts, drew attention to the sound given of by an electromagnet at an instant when the electric current is closed or broken. He later discussed the musical note produced by rapidly revolving the armature of an electromagnet in front of the poles. In 1854, **Charles Bourseul** (Paris) recommended the use of a flexible plate which would vibrate in response to the varying pressure of the air, and thus open or close an electric circuit. A similar plate at the receiving station would be acted on electromagnetically, and thus produce as many pulsations as there are breaks in the current.

In 1861, **Johann Philipp Reis** (1834–1874, Germany) succeeded in transmitting speech and music electrically down a wire using a device he called 'das Telephon' in a lecture delivered before the physical society of Frankfurt. He described an apparatus in which he caused a membrane to open and close an electric circuit at each undulation, thus transmitting as many electric pulses through the circuit as there were periodic amplitude vibrations in the sound. These electric pulses were made to act on an electromagnet at the receiving station, which gave out a sound corresponding to the number of times it was magnetized or demagnetized per second. Reis could not, however, reproduce human speech with sufficient clarity. The suggestion of Bourseul and the experiments of Reis are founded on the idea that a succession of currents, corresponding in number to the successive undulations of the pressure on the membrane of the transmitter, could reproduce at the receiving station sounds of the same character as those produced at the sending station. The professors to whom this invention was presented were not very impressed and his version of the "telephone" never received any financial support and no patent ensued.

Reis was born in the town of Gelnhausen, in Hesse-Cassel, Germany, where his father was a master baker and a petty farmer. Orphaned at an early age, he interrupted his high school education to become merchant but in 1855 he became a schoolteacher of mathematics and science.

1838 CE **Antoine-Augustin Cournot** (1801–1877, France). Economist and mathematician. Attempted to apply mathematics to solution of economic problems; pioneer in mathematical economics. In *Recherches sur les principles mathematiques de la theorie des richess* (1838) discussed supply and demand functions and introduced the concept of '*elasticity of demand*'. He also considered conditions for equilibrium with monopoly, duopoly and perfect competition. He considered the effect of taxes, treated as changes in production costs, and discussed problems of international trade. Conducted research in the theory of probability.

Cournot was professor at Lyons (1834), rector of academies at Grenoble (1835–1838) and Dijon (1848–1862).

1838 CE **Gerhardus Johannes Mülder** (1802–1880, Holland). Chemist. Coined the name *protein* from the Greek word for "first". He studied the chemical structure of the albuminous substances and concluded that they were built up of a basic building block to which various amounts of modifying structures were added. Mülder's speculation turned out to be not quite right, but the name remained.

1838–1839 CE **Matthias Jacob Schleiden** (1804–1881, Germany). Botanist. Recognized the importance of the cellular element of plants and stated that the *cell*[153] was the basic unit of life: an individual living and reproductive organ. The next year, the physiologist **Theodor Schwann** (1810–1882, Germany) advanced the same idea. Neither of them originated this concept. A number of other scientists had already come to believe that all organisms were made of cells. But from that time on, all biologists regarded the cell as the building block of life.

[153] In 1665, **Robert Hooke** (1635–1702) coined the word *cell* for the infrastructural unit of a piece of cork which he saw through his microscope.

The Cell (1831–1925)

The word cell *owes its existence to* ***Robert Hooke****, who first noticed the cellular structure of cork (1665). What Hooke really saw were dead cell walls in the bark of the cork oak. Other early microscopists soon observed cells in all kind of plants. Animals contained similar units, but these were harder to see because animal cells lack the thick walls that surround plant cells. Observers also reported the existence of many tiny* unicellular *organisms, each consisting of only one cell. Thus,* **Antoine van Leeuwenhoek** *observed (1674) bacteria 2 micron long, as well as blood cells and spermatozoa. More than 150 years later, in 1831, the appreciation of the cell as the basic unit of the organism was finally manifested through the works of* **Robert Brown***, who coined the term 'cell nucleus'.* ***Schleiden*** *and* **Schwann** *soon followed (1838–1839) with the first theory on nucleus and cell formation. This theory states that:*

- *cells are the fundamental unit of life – the smallest entities that can be called "living"*
- *all organisms are made up of one or more cells.*

Robert Remak *(1845) was first to demonstrate that cells are formed by division of existing cells.* ***Rudolf Virchow*** *(1855) generalized and popularized Remak's discovery, using the aphorism:* Omnis cellula e cellula *- all cells from cells. He added a third statement:*

- *cells arise only by division of other cells. In other words, cells are the fundamental structural, functional and reproductive units of life.*

Why are there cells? The metabolism of a living organism requires a chemical environment different from any found in the nonliving world. A cell organizes an "environment in miniature" by maintaining strict control of the chemical composition within its boundaries. In the controlled environment of a cell, all the activities of life occur: acquiring energy; using this energy to maintain the chemical environment, to build organic molecules, to grow, and reproduce by division into two new cells.

Many organisms are unicellular, but most plants and fungi, and all animals, are multicellular*, composed of many cells. All cells must carry out certain basic activities; in addition, each cell of a multicellular organism makes a specialized contribution to the economy of the body as a whole. For example, a muscle cell in the heart is specialized to contract and help pump blood.*

Since it is deep within the body, it cannot capture its own food or obtain oxygen from the air, but must rely on other specialized cells, such as those of the digestive tract, lungs, and blood, to provide its food and oxygen.

Thus there is division of labor among the cells of a multicellular organism. Unicellular organisms may also be highly specialized for their own ways of life, with features much more complex than those in the cells of most plants and animals. A specialized cell is usually distinguished by the exaggeration or modification of one or more features common to most types of cells, rather than by possession of structures or chemicals that other calls lack. We can think of specialized cells as variations on the basic theme of cell structure and function.

An idealized "basic cell" has three main parts:

1. *The plasma membrane, covering the outside of the cell. (in plants, this lies just inside the nonliving cell wall.)*
2. *The cytoplasm (cyto = cell), containing water, various salts, and organic molecules. The cytoplasm also contains a variety of larger structures, collectively called organelles, which are the working parts of the cell. Many of these "little organs" are surrounded by membranes very similar to the plasma membrane. The cytoplasm fluid is crisscrossed by a barely visible network of microtubules involved in transporting molecules to and from the membrane.*
3. *The cell nucleus (in bacteria, the nuclear area), containing the cell's genetic material.*

The Plasma Membrane

Molecules and ions are in constant, random motion. Left to themselves, these substances would diffuse down their concentration gradients and eventually become uniformly distributed. However, the chemical composition and the physical environment is usually not completely appropriate for the biochemical reactions of life. Cells require higher concentrations of some substances and lower concentrations of others.

To remain alive, a cell must maintain chemical homeostasis ("same-standing"); that is, it must keep its internal chemical composition constant *within the narrow limits suitable for life. However, a cell cannot create a suitable internal environment and then seal itself off from the world to avoid gaining or losing substances by diffusion. The biochemical reactions of metabolism require raw materials from outside the cell and generate waste products that must be expelled. Hence the cell must maintain homeostatis while continuously exchanging substances with its environment. Control of what substances enter and leave the cell is the task of the* plasma membrane*, also called the* cell membrane *or* plasmalemma*.*

The idea of homeostatis is due to **Claude Bernard** *(1857) who coined the aphorism: "la fixité du milieu interieur c'est la condition de la vie libre" (free life depends on the constancy of the internal environment). Indeed, the plasma membrane plays an important role in homeostatis, the constancy of the internal environment at the cellular level.*

The plasma membrane is selectively permeable[154]*; that is, it permits some substances to pass more freely than others, and even prevents the passage of certain kinds of molecules, to which it is* impermeable*. Hence the concentration of ions, nutrients, and other substances inside the cell differ from those in the cell's environment.*

For biological systems to operate, some parts of organisms must be separated from other parts. On a cellular level, the outside of the cell has to be separated from the inside of the cell. "Greasy" lipid membranes *serve as the barrier. In addition to isolating the contents of the cell, membranes allow the selective transport of ions and organic molecules into the cell.*

All biological membranes consist of lipids[155]*and* proteins*. The actual kinds*

[154] Discovered by **Charles E. Overton** (ca 1895) and established further by the experiments of **I. Langmuir** (1905).

[155] *Lipids* are a class of biochemical compounds composed mainly of fats and oils. They are soluble in fat solvents such as benzene, ether, and chloroform, but insoluble in water. Lipids do not contain a common chemical unit, and their composition as well as their structure varies widely. Most lipids, however, are ethers of fatty acids. Lipids are essential constituents of almost all living cells. The more complex lipids are concentrated in brain and nerve tissues.
Lipids may be classified into the following categories:

I. *Simple lipids* that contain C, H, O, N, and P

(a) Phospholipids (phosphatides)

(1) Lecithins
(2) Cephalins
(3) Sphingosides

and proportions of molecules present depend upon the membrane, its location and function.

The plasma membrane is very thin, only about 7.5 *to* 10 *nanometers (nm) thick* ($1\ nm = 10^{-9}\ m = 10^{-6}\ mm$)*. Therefore, it cannot be seen with a regular microscope. However, its existence was deduced from the behavior of cells long before the much more powerful electron microscope showed that it really is there. Electron micrographs show the membrane as a continuous double line surrounding the cell.*

The classic studies of membrane structure used the red cells of mammals (warm-blooded animals with fur or hair, including humans). These cells were chosen because of their simple structure. Mammalian red blood cells lack most of the internal components of other cells; they consist of a single, sac-like plasma membrane containing little other than the red, oxygen-carrying hemoglobin.

An experiment done in 1925, by **E. Gorter** and **F. Grendel***, established that plasma membranes contain a double layer of lipid molecules. These researchers broke open blood cells and separated the membranes from the*

(b) Glycolipids

II. *Derived lipids*

(a) Sex hormones

(b) Fat-soluble vitamins A, D, E, and K

Fats are esters of three molecules of fatty acid and one molecule of glycerol. Fats are called *triglycerides* because they are triesters of glycerol and three fatty acids. They are the most abundant as well as the most important class of lipids in the average diet (they comprise 10 percent of the body weight of mammals). Triglycerides are used to store energy, and they provide protection against heat loss and mechanical shock.

Many internal organs, such as the kidneys, are enveloped in a thick layer of fat to protect them from the effects of violent shock.

Among lipids found in biological membranes is *cholesterol* (a derived lipid) and *phospholipids.* The latter are complex esters composed of an alcohol, fatty acids, phosphoric acid, and a nitrogenous base. They are present in every tissue of the body, but especially in the nervous system.

In the energy economy of the cell, glucose reserves are like ready cash, whereas lipid reserves are like a fat savings account. The stored energy of lipids resides in the fatty acid chains of triacylglycerols. When there are excess calories, fatty acids are *synthesized* and stored in fat cells. When energy demands are great, fatty acids are *catabolized* to liberate energy.

hemoglobin. They dissolved the membrane lipids in acetone (the main component of nail polish remover). When the acetone-lipid preparation was mixed with water, the lipids rose to the top and spread over the water surface in a monolayer *only one lipid molecule thick. This lipid monolayer occupied two times the surface area calculated for the plasma membranes of the original red blood cells. Hence the researchers concluded that the plasma membrane's lipid must exist in a double layer, or* bilayer*. We now know that all biological membranes have double layers of lipid molecules. In fact, lipids found in biological membranes may form bilayers spontaneously even when they are removed from the cell.*

Membranes have two main functions: they form a physical barrier around a cell or organelle, and they control the passage of substances into or out of the enclosed area. Some aspects of the membrane permeability have been understood for a long time, but not until the 1970s did it become clear that substances cross biological membranes in only three distinct ways: by dissolving through the lipid layers, by being moved through the lipid layers by way of the membrane proteins, or by moving within a sac formed from part of the membrane. We shall consider each of these in turn.

Some molecules cross biological membranes by virtue of their interactions with the lipid molecules in the membrane. In essence, these substances dissolve in the lipid on one side of the membrane and emerge at the opposite face. These substances move by diffusion, each entering or leaving the cell according to its own concentration gradient. To study these interactions without interference from membrane proteins, researchers work with artificial lipid bilayers. The lipids in these bilayers behave essentially like lipids in intact biological membranes. A lipid bilayer's hydrophobic interior makes it relatively impermeable to ions and to many polar molecules. As a result, the plasma membrane prevents most of the water-soluble cell contents from escaping. However, small uncharged molecules can slip between the hydrophilic heads of the membrane phospholipids and will diffuse across the bilayer. The rate at which such a substance can diffuse through the lipid bilayer depends on its solubility in lipids and its molecular size.

Hence small, nonpolar molecules such as oxygen (O_2)*, nitrogen* (N_2)*, benzene, ethylene, and ether cross membranes rapidly. Uncharged polar molecules also cross the lipid bilayer rapidly if they are small enough. For example, ethanol and urea cross rapidly; glycerol, which is also uncharged but larger, crosses much more slowly, and glucose, twice the size of glycerol, can hardly cross an artificial lipid bilayer at all.*

Because water is relatively insoluble in lipids, it is somewhat surprising that water molecules cross lipid bilayers quite rapidly. This is partly because of the water molecule's small size, but it may also be that the molecules' unique *bipolar structure somehow permits it to pass the membrane's hydrophilic outer layers especially easily.*

Artificial lipid bilayers are about 10^9 *(one billion) times more permeable to water molecules than to charged ions, even such small ones as sodium* (Na^+) *and potassium* (K^+)*. The inability of ions to penetrate the plasma membrane is partly due to their electric charge. In addition, ions in solution are* hydrated*, that is, surrounded by a layer of water molecules, which in effect makes them much larger.*

Many (particularly polar) molecules move rapidly across biological membranes even though they cross artificial lipid bilayers very slowly. Examples include various small ions, glucose, and amino acids. These substances are transported by membrane transport proteins *or* carriers*. Each transport protein is specific in that it transport only one or a few chemically similar substances.*

Some transport proteins merely move one type of solute across the membrane. In others, transfer of one solute depends upon the simultaneous transfer of a second kind of solute in the same or the opposite direction. Some proteins move their solutes in only one direction, while others work in both directions. Here we consider some of the more important types of protein transport systems.

In passive transport*, the membrane protein provides a means for an ion or molecule to cross the membrane, moving down its electrochemical gradient. It is well-known that molecules move according to their concentration gradients, going from areas where their concentration is greater to areas where it is less. However, a second factor also influences the movement of ions: the electrical environment.*

In most plasma membranes, transport proteins move ions in such a way that electrical charge is unequally distributed on the two sides of the membrane.

Therefore we say that an electrical potential difference*, or* membrane potential*, exists across the membrane. For instance, if the interior of a particular cell has an electrical potential of* -50 *millivolts (mV) compared with the exterior, its membrane has a membrane potential of* -50 *mV. So the membrane potential has this effect: positively charged ions tend to enter the cell readily, attracted by the excess negative charges there, but negatively charged ions tend to remain outside the cell, attracted by the external positive charge and repelled by the negatively charged interior.*

Now we can see that the diffusion of a given ion across a membrane depends on two factors: (1) the ion's own concentration gradient and (2) the overall electrical potential gradient across the membrane, which is the gradient of the *amalgamated concentrations of all electrically charged species present. Both these gradients together constitute the* electrochemical gradient *for the ion, and this determines how rapidly the ion diffuses across the membrane.*

Perhaps the simplest case of passive transport occurs where membrane proteins form channels *through the lipid membrane. These channels contain an aqueous solution and permit small solutes to cross the bilayer by simple diffusion down their electrochemical gradients, avoiding the membrane's hydrophobic interior.*

While some of the channels are open all the time, others, called gated channels, *behave as if they have gates that open and close. Some gated channels open when a specific substance binds to a receptor on the plasma membrane. Others open in response to changes in the membrane potential. Still others open when the concentration of a particular ion inside the cell increases. Many "gates" close again automatically even if the stimulus that caused them to open is still present. Gated channels permit the membrane's permeability to change from time to time. This feature is vital, among other things, to the working of nerves and muscles.*

In facilitated diffusion, *a carrier protein combines with a specific solute and moves it from one side of the membrane to the other, down its electrochemical gradient. This in effect increases the membrane's permeability to the substance and so allows the substance to cross membranes faster than it otherwise would.*

An example is the system that facilities the diffusion of glucose into the cells of some tissues of vertebrates (animals with backbones). In the liver, the lens of the eye, and red blood cells, facilitated diffusion moves glucose across the plasma membrane in both directions by means of a carrier molecule. The carrier molecule is more likely to encounter and pick up a glucose molecule on that side of the membrane where glucose is more plentiful. When a cell is breaking down glucose quickly during respiration, the glucose concentration inside the cell falls; glucose is then more plentiful outside the cell, and it is moved into the cell more rapidly than it is moved out.

Facilitated diffusion is just as important in increasing the rate at which glucose leaves a cell. Cells in the liver, for instance, not only remove glucose from the bloodstream when the blood glucose level is high but also replenish the blood glucose when its level drops.

In active transport, *substances are moved either with or against their electrochemical gradients; this process requires the expenditure of energy. The source of energy for active transport may be the high-energy ATP molecule or the electrochemical gradient of an ion across a membrane. Common sources of such electrochemical energy are steep differences in sodium ion* (Na^+) *or* *hydrogen ion* (H^+) *concentration on the two sides of a membrane. Since there is a strong tendency for ions to move down a steep gradient, such a gradient represents a source of energy.*

Among the many active systems are those that take up amino acids, peptides and potassium in the bacterium Escherichia coli. These substances move into the bacterium only in the presence of the appropriate carriers and of a source of energy.

Another example is a calcium pump, found in many cells, which pumps Ca^{2+} *out of cells and so keeps their internal* Ca^{2+} *concentration much lower than external levels. A spectacular pump found in the stomach wall is responsible for secreting "stomach acid": using the energy of ATP, it exports* H^+ *against a* pH *gradient of about a million to one! But one of the most widespread and best examples is the sodium-potassium pump.*

In order for a cell to maintain homeostatis, it must have strict control over its chemical content, which includes not only the absolute amounts of solutes but also their concentration. Thus the content of the solvent, water, in a cell must also be precisely regulated. Vital as water is to living cells, cells have known carriers or other direct means of transporting water in or out. Water seems to travel through the plasma membrane quite freely – faster, in fact, than any other substance.

Osmosis*, the process by which water moves through a selectively permeable membrane, is a special case of diffusion: it involves the diffusion of a solvent, such as water, rather than the diffusion of substances dissolved in the solvent. In osmosis in living cells, water moves across a membrane from a weak, or dilute, solution into a strong, or concentrated, solution (this process is* spontaneous *since it increases the overall solute entropy).*

A simple way to demonstrate osmosis is to separate distilled (pure) water from an aqueous solution by a membrane that is permeable to water but not to the solute. As time passes, the volume of the solution increases and that of the distilled water decreases. Therefore water must be moving by osmosis from the water, across the membrane, and into the solution.

What is the molecular mechanism for this entropy-increasing process? Water molecules can cross the membrane in either direction. However, the water molecules in the higher-concentration solution bump into the solutes and also experience forces attracting them to solute particles; this retards the movement of the water molecules in the solution, and so water moves into that side of the membrane solution faster than it moves out.

In a laboratory osmotic system, the net movement of water into the solution increases the height of the solution in the tube, and the weight of the column of solution exerts hydrostatic pressure*. As water enters the solution,* *the hydrostatic pressure builds up until it is pushing water molecules out as fast as they enter. The solution will remain at this level.*

The extent of movement of water across a membrane can be predicted by knowing the osmotic potentials of the two solutions separated by the membrane. The osmotic potential *of a solution is its tendency to gain water when separated from pure water by an ideal selectively permeable membrane. A stricter definition of osmotic potential is that it is the negative of the* osmotic pressure*, which is the minimum pressure that must be applied to a solution to prevent it from gaining water when it is separated from pure water by an ideal selectively permeable membrane.*

The osmotic potential is expressed in negative terms: the more concentrated the solution, the lower (more negative) *its osmotic potential, and the greater its tendency to gain water from a solution with a higher* (less negative) *osmotic potential. The osmotic potential is the driving force of osmosis, since water tends to move in a downhill direction in terms of free energy, that is, in the direction of the lower osmotic potential,*

The osmotic potential in a system depends on the concentration of particles in the solution and on the attraction of water molecules to the particles, which slows the movement of the water molecules. There may be only one type of molecule dissolved in a solution, or there may be many, as in a living cell. Each molecule of a strong-electrolyte ionic substance dissociates into more than one particle in aqueous solution; $NaCl$ *dissociates into two particles,* $MgCl_2$ *into three, and so forth. The more particles there are in a solution, the lower the osmotic potential. If the solute particles are able to pass through the membrane, then the osmotic potential of the solution will gradually change as solute particles enter or leave it.*

We can now see how a cell can control its water content, and hence its volume. The cell can create a difference in osmotic potential across its membrane by the active transport of solutes, especially by means of the sodium-potassium pump. Water will then move by osmosis toward the side of the membrane where the osmotic potential is lower.

Cells behave as an osmotic system. A living cell has a selectively permeable plasma membrane, which encloses the cell's internal solution of various particles dissolved in water. To remain alive, the cell must be covered by at least a thin layer of water, which also has solutes dissolved in it. If this extracellular (extra = outside) solution is in osmotic balance with the intracellular (intra = within) solution, no net exchange of water occurs between them, and the cell is said to be living in an isotonic *solution.*

If the external solution is made more concentrated, so that the cell loses water to its environment, such an extracellular solution is said to be hypertonic to the cell contents. And if the cell is placed in a solution dilute enough for the cell to gain water from outside, this environment is said to be *hypotonic* to the intracellular solution.

Some animal cells in dilute (hypotonic) solutions may take in so much water that their internal pressure ruptures the plasma membrane, allowing the cell contents to escape. This process is known as *lysis* (bursting) of a cell. In the same situation, the rigid wall of a plant cell produces a *wall pressure* which opposes the outward pressure of swelling and makes most plant cells more resistant to swelling in a hypotonic solution.

Many animals live in freshwater environment which are hypotonic to their cell cytoplasm. Why don't these animals take up so much water by osmosis that they swell up and burst? Most of a freshwater animal's body surface is covered by a layer of rather impermeable material, which retards water uptake. Such layers include the mucus of fish and worms or the wax-impregnated chitinous armor of aquatic insects and spiders. In addition, the excretory structures of such organisms have well-developed active transport mechanisms, which allow them to rid their bodies of water while conserving precious salts.

Freshwater protozoa (unicellular organisms that lack cell walls, such as *Amoeba* and *Paramecium*) gain a great deal of water by osmosis. These organisms void excess water by way of specialized structures called *contractile vacuoles*, which accumulate water and then contract, squeezing the water back into the environment. Like all other freshwater organisms, protozoans face a scarcity of available salts. So, before the contractile vacuole expels its contents, salts are removed by active transport. Every 4 to 8 minutes the contractile vacuoles of a paramecium eject a volume of water equivalent to the volume of the entire cell!

The cells of a multicellular organism communicate with their neighbors by the exchange of substances in the cytoplasm. Such transfers can be accomplished most effectively by direct cytoplasm-to-cytoplasm connections. In many cases we find plasma membranes arranged to permit such connections.

In plants, the cytoplasm of neighboring cells is often connected by strands of cytoplasm called *plasmodesmata* (singular: *plasmodesma*). These cytoplasmic bridges pass through interstices in the cell walls between the two cells, and both the cytoplasm and the plasma membranes of these cells are essentially continuous with each other.

Gap junctions occur between many kinds of animal cells and are thought to permit passage of ions and small molecules from cell to cell. This cell-to-cell connection can be shown by placing microelectrodes inside two adjacent cells that are linked by gap junctions. The electrical resistance between the electrodes is very low, indicating that electrically charged substances can move *unimpeded between the cells. In contrast, if the electrodes are placed so that one is inside and the other outside a cell, the electrical resistance measured is high. This shows that the flow of ions through the membrane is highly controlled.*

The main electrically charged particle in biological systems are proteins and nucleic acids, which are too big to leave the cell (even through gap junctions), and small ions such as Na^+, K^+, *and* Cl^-. *The low electrical resistance between adjacent cells suggests that ions can move from one cell to another, probably through the intercellular channels of the gap junction. It is sometimes possible to confirm such a finding by tracing the movement of fluorescent or radioactive substances from one cell to another.*

In some tissues, the function of electrical coupling between cells is clear. For instance, coupling helps to coordinate the contractions of adjacent heart muscle cells. However, the function of this coupling is not yet understood in other cases, such as the many gap junctions in early animal embryos.

Electron micrographs of gap junctions show an array of proteins channels linking the cytoplasm of two neighboring cells. Each channel consists of two short, pipe-like sections, one in each plasma membrane, lined up so that they meet in the intercellular space to form a continuous passageway. One end of each "pipe" juts its counterpart on the opposite membrane. The walls of each "pipe" consists of six rather cylindrical membrane proteins arranged in a circle surrounding a channel about 1.5 *to* 2.0 *mm in diameter.*

The complex process by which a cell nucleus gave rise to two daughter nuclei was worked out during the 1870's and the 1880's, largely by German investigators who gave the first clear description of the appearance of chromosomes *in* mitosis. *These phenomena gained importance (1890) when* ***August Weismann*** *took up the theories of* hereditary *particles developed by* ***Hugo de Vries*** *and others, combined them with studies of cytology and embriogenesis, and developed the idea that nuclei, and particularly chromosomes, contained determinants which directed the development of the cells and hence controlled the characteristics of the whole animal or plant. Hence the nucleus, previously seen as a part of the active protoplasm of the cell, came to be viewed as a store of information.*

Summary

Cells must maintain the internal concentrations of all substances at appropriate levels. At the same time, cells must maintain a lively commerce *with their environments, taking in new raw materials for their metabolism and expelling waste products.*

The plasma membrane regulates what enters or leaves the cell. It is permeable to many types of small molecules and ions, yet sufficiently impermeable to prevent the loss of such materials as nucleic acids, proteins, and polysaccharides.

A biological membrane consists of a fluid lipid bilayer with various proteins floating in it, some mobile in the bilayer and some anchored to stable cellular structures. Oligosaccharides are attached to some protein and lipid molecules, forming glucoproteins and glycolipids.

This basic structure has two properties crucial to membrane function. First, lipid bilayers spontaneously form closed compartments, thereby keeping the solutions inside and outside the membrane separate. Second, membranes are asymmetrical, with different lipid and protein components in each of the two layers, and with molecules oriented so that they consistently face one membrane surface or the other. For example, active transport molecules are oriented so that they move substances in only one direction.

A membrane's lipid bilayer is freely permeable to water. It also admits small, lipid-soluble molecules, which diffuse through the lipid layers according to their concentration gradients.

Most ions and polar molecules can cross the membrane only with the aid of protein transport molecules. Each protein is specific for a particular solute or a few closely related solutes. Channel proteins form aqueous channels through the membrane; some are gated so that they open in response to specific polar molecules and ions down their electrochemical gradients. Other proteins mediate active transport, which can move a solute either with or against its electrochemical gradient (used e.g. to transport ions such as Na^+ *or* H^+*). The sodium-potassium pump, powered by ATP, pumps* Na^+ *out of a cell and* K^+ *in. This pump is largely responsible for the membrane potential of a cell, and the electrochemical gradient of sodium that it creates also provides energy for the active transport of solutes such as glucose.*

When the cell acquires macromolecules or larger particles, the membrane surrounds them and pinches off to become a vesicle or vacuole inside the cell, by the process of endocytosis. Substances can be discharged from many cells by the opposite process of exocytosis.

Cells gain or lose water by osmosis. The membrane does not control the movement of water molecules directly; rather, it performs active transport of solutes and so creates an osmotic potential difference that will induce osmosis. The cell wall of a plant cell exerts a pressure that limits the cell's water *content. Many protozoans void excess water taken in by osmosis by means of a contractile vacuole.*

The plasma membrane may be expanded to provide additional surface area for exchange of substances with the environment. The plasma membranes of adjacent animal cells may interact. Tight junctions seal membranes together and prevent seepage of substances between cells. Intermediary junctions and desmosomes provide mechanical strength by attaching the membranes of adjacent cells. Gap junctions act as "pipes" through animal cell membranes, providing for direct transfer of ions from cell to cell. In plants, direct transfer between cytoplasm of adjacent cells occurs by way of plasmodesmata.

1838–1840 CE **Ferdinand Minding** (1806–1885, Germany). Mathematician. A forerunner of hyperbolic non-Euclidean geometry. Contributed to the differential geometry of surfaces of constant curvature and was first to introduce the concept of the pseudosphere[156] and show that the hyperbolic

[156] *Pseudosphere*: A surface of revolution formed from the plane pursuit curve called *tractrix*. This curve was first studied by **Newton** (1676) and later by **Huygens** (1693), **Leibniz** (1693), **Johann Bernoulli** (1728), and **Liouville** (1850). Also called *Tractory* and *Equitangential curve*. It is the path of a particle pulled by an inextensible string whose other end moves along a line (x-axis, say). The simplest example is that of a toy-boat pulled by a boy with a string: he starts walking in a direction perpendicular to the string, always keeping the string taut. The tractrix is the boats' path in the water (ignoring the boy's height). Since the thread is stretched in the direction *tangent* to the curve, we can write $\frac{dy}{dx} = -\frac{y}{\sqrt{p^2-y^2}}$ where p is the length of the string. Integration yields the equation

$$x = p\,\mathrm{ch}^{-1}\left(\frac{p}{y}\right) - \sqrt{p^2-y^2}\,,$$

or in parametric form:

$$y = p\sin\phi, \qquad x = p\left[\log\tan\frac{\phi}{2} - \cos\phi\right], \qquad -\frac{\pi}{2} < \phi < \frac{\pi}{2},$$

where ϕ is the angle between the string and the x-axis (the asymptote).

Consider the surface of revolution formed by rotating the curve about the entire formulas for triangles hold on the pseudosphere (1840). Minding was a student of **Gauss** and later became a professor at the University of Dorpat (now Tartu, Estonia).

Hyperbolic non-Euclidean geometry (nEg) can briefly be characterized by the fact that there are (at least) two straight lines passing through a point and parallel to the given line, and the sum of the angles of a triangle is less than π. In *Elliptic* nEg there does not exist a straight line parallel to a given one, and the sum of the angles of a triangle is more than π. The intrinsic geometry of surfaces with positive constant Gaussian curvature is, at least locally, identical with elliptic nEg.

Minding established the theorem (1839) that all surfaces with the same constant curvature are isometric; in particular, every surface of constant negative Gaussian curvature $K = -\frac{1}{\lambda^2}$ can be isometrically mapped onto a pseudosphere of pseudoradius λ.

Minding started from the standard relation for a spherical triangle of sides $\{a, b, c\}$ and angle A opposite the side a:

$$\cos\frac{a}{R} = \cos\frac{b}{R}\cos\frac{c}{R} + \sin\frac{b}{R}\sin\frac{c}{R}\cos A,$$

where R is the sphere's radius and $\frac{a}{R}$ is the angle subtended at the sphere center by side a, in radians, and similarly for $\frac{b}{R}$, $\frac{c}{R}$. In the plane limit $R \to \infty$ it reduces to the Euclidean law of cosines $a^2 = b^2 + c^2 - 2bc\cos A$. On a surface with constant curvature K, the above relation is generalized into

$$\cos(a\sqrt{K}) = \cos(b\sqrt{K})\cos(c\sqrt{K}) + \sin(b\sqrt{K})\sin(c\sqrt{K})\cos A.$$

Applying this to surfaces with *negative* curvature $K = -1$, one finds $\mathrm{ch}\, a = \mathrm{ch}\, b\ \mathrm{ch}\, c - \mathrm{sh}\, b\ \mathrm{sh}\, c\ \cos A$, which holds for a geodetic triangle on a *pseudosphere.* This very equation was obtained earlier (1837) by **Lobachevsky** who did not realize that it holds on a pseudosphere. It is one of the great examples of noncommunication in mathematical history. Neither Minding nor Lobachevsky seem to have read the other's paper. *Nobody* seem to have read them both and realize that Lobachevsky's "imaginary geometry"

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asymptote. It has an area of $4\pi p^2$ and a volume of $\frac{2}{3}\pi p^3$ (Huygens showed in 1693 that they are finite). The *mean curvature* of the said surface (arithmetic mean of maximum and minimum curvatures at a point) is a *negative* constant $\left(-\frac{1}{p}\right)$. It is for this reason that the surface is called the *pseudosphere* with *pseudoradius p*. Because of this property the *pseudosphere* serves as a model for non-Euclidean hyperbolic geometry, just as a *sphere* does for non-Euclidean elliptic geometry.

was nothing more than the very real geometry of a particular surface. Hence, the importance of this result for hyperbolic geometry was totally missed. Perhaps it was clear that the pseudosphere cannot serve as a "plane", because it is infinite in only one direction. It was not until 1868, when **Beltrami** extended the pseudosphere to a true *hyperbolic plane*[157] — a surface *locally* like the pseudosphere but infinite in all directions — that hyperbolic geometry was finally placed on a firm foundation.

1838–1852 CE **Ferdinand Gottfried Max Eisenstein** (1823–1852, Germany). Mathematician. One of the most gifted mathematicians in Germany, of the two generations after Gauss.

Contributed to the fields of elliptic functions, algebra and number theory. In the latter he created the theory of cubic forms. His work led to several theorems for quadratic and biquadratic residues, cyclotomy and quadratic partition of prime numbers[158] and reciprocity laws. He was Gauss' favorite student and Gauss wrote of him to von Humboldt: "*Eisenstein belongs to those talents who are born but once in a hundred years*".

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[157] *Hyperbolic plane*: consider a circle of radius 2 in the x–y plane. Define a metric tensor w.r.t. a polar coordinate system (r, θ) at the origin:

$$g_{11} = \left(1 - \frac{r^2}{4}\right)^{-2}, \qquad g_{12} = g_{21} = 0, \qquad g_{22} = r^2\left(1 - \frac{r^2}{4}\right)^{-2}.$$

The *Gaussian curvature*

$$K = -\frac{1}{\sqrt{g_{11}g_{22}}}\left[\frac{\partial}{\partial r}\left(\frac{1}{\sqrt{g_{11}}}\frac{\partial\sqrt{g_{22}}}{\partial r}\right) + \frac{\partial}{\partial\theta}\left(\frac{1}{\sqrt{g_{22}}}\frac{\partial\sqrt{g_{11}}}{\partial\theta}\right)\right]$$

is found to be equal to -1, while the *geodesics* consist of Euclidean straight lines through the origin and circles whose center lies *outside* the boundary $r = 2$ and intersect $r = 2$ orthogonally. Thus, through a point not on a given geodesic circle, there exist infinitely many "parallel lines" (circles) to the given one.

[158] Prime numbers can be defined in fields other than integers. In the complex field $\mathbb{C}$ we have the *Gauss integers* $n + im$ (n, m integers) and the *Gauss primes* [primes of the form $4k - 1$ over the integers are still primes in $\mathbb{C}$, but 2 and primes of the form $4k + 1$ can be factored in $\mathbb{C}$, e.g. $2 = (1 + i)(1 - i)$, $5 = (2 + i)(2 - i)$, $13 = (2 + 3i)(3 - 3i)$, $19 = (4 + i)(4 - i)$, etc.].
Eisenstein defined the *Eisenstein integers* $n + \omega m$, where ω is the complex cube root of unity $[\omega = \frac{1}{2}(1 - \sqrt{-3});\ 1 + \omega + \omega^2 = 0]$. The prime 2 and primes of the form $6k - 1$ are also *Eisenstein primes*, but 3 and primes of the form $6k + 1$ can be factored, e.g. $3 = (1 - \omega)(1 - \omega^2)$, $7 = (2 - \omega)(2 - \omega^2)$, $13 = (3 - \omega)(3 - \omega^2)$, $9 = (3 - 2\omega)(3 - 2\omega^2)$, etc.

Eisenstein was born in Berlin to Jewish parents. He started his university studies while still in high school and on his second year of study he received his Ph.D. degree, honoris causa, from the University of Breslau.

Eisenstein suffered all his life from bad health. After leaving school he traveled with his parents to England where they were looking for a better life. While in Ireland Eisenstein met **Hamilton**, who gave him a copy of a paper that he had written on Abel's work on the impossibility of solving quintic equations. This stimulated Einstein to do research in mathematics and on his return to Germany he enrolled at the University of Berlin. In 1844 Eisenstein went to Göttingen for a short time and met **Gauss**.

Throughout this year he published no fewer than 25 papers in German mathematical journals. In 1852 he became a professor of mathematics at the University of Berlin and was elected to the Prussian Academy of Science. He was an influential teacher of **Riemann** in Berlin.

But his health worsened after 1847. Soon he spent most of his time in bed. Eisenstein spent a year in Sicily in an attempt to improve his health but after his return to Germany he died of pulmonary tuberculosis at the age of 29. The collected works of Eisenstein were published by Gauss already during his lifetime (1847) [his *autobiography* was published in Volume **40** of *Zeitschr. f. Math. u. Physik*, pp. 143–200, 1895].

1839, Aug 19 *Official date for the start of photography*

The Academy of Science and the Academy of Fine Arts met jointly in Paris to debate one last time the process invented by **Daguerre** (1839) and (after the French state purchased the inventor's patent at the price of a life annuity) made the world a gift of that process.

Evolution of early photography (1826–1851)

The discovery of photography during the first third of the 19th century was the response of the technological era to the new tastes of a middle class of considerable economic means. Indeed, manual graphic techniques such as drawing, etching and lithography must have seem outmoded to people living in an age when steam machine energy was being pressed into the service of *capitalist production, when mechanical looms were gradually replacing labor force, and when railroads were on the verge of bringing distant regions within the reach of one another, thus endowing humanity with new mobility.*

Above all, such manual and therefore largely subjective forms of representation hardly corresponded with the objective vision of the world and the environment to which the rational positivism of the times aspired.

Strictly speaking, the basic principles of photography had been known for some time. They only had to be combined and the final missing link supplied. The working of the camera obscura, for example, had been known since ancient times and had also served as an aid to artists interested in achieving more true-to-life perspective in their work since the Renaissance. Similarly, the sensitivity to light of silver salts was a known fact by, at the latest, the time of ***Johann Heinrich Schulz*** *(1727). What remained lacking up until the end of the 18th century was a socially rooted interest in obtaining pictures by mechanical means and a way of making permanent the camera obscura "sun pictures".*

Various separate developments took place during the first half of the 19th century: in 1826, the Frenchman ***Nicéphore Niepce*** *had succeeded in taking a photograph of the view from his workroom window by using bitumen coatings on a copper plate; since 1834, England's* ***Henry Fox Talbot*** *had been successfully experimenting with sensitized paper negatives; and by 1837,* ***Louis Jacques Mondé Daguerre*** *had succeeded in producing the type of photograph which was named after him: the daguerreotype.*

The international reaction to the publication of the technique by the French Academies (1839) was as immediate as it was tremendous – a proof that Western industrial society had been waiting for just such a technical means of producing pictures. People were enchanted with the way the process brought out every detail of a picture equally, and with the method's speed. Although lack of color was disappointing, what fascinated and finally convinced most people was the mechanical, almost automatic nature of the process.

The potential range of applications of the new medium was soon recognized: architectural and landscape photography, art reproductions, portraiture, and a tool of astronomy and photometry.

The widely traveled ***Alexander von Humboldt*** *reflected on the usefulness of a camera on journeys*[159]*.*

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[159] The long exposure times required in 1839 left the achievements of portraits in the realm of the utopia. For example, after a first visit to Daguerre (Feb 1839), Humboldt mentions exposure times taking 10 minutes! Even in 1842, after

From a technical point of view, it was the invention of the negative by ***Talbot*** *which laid the cornerstone for the popularization of photography. Both in theory and in practice, negatives affording countless positives was a winning formula. At the start, however, Talbot's brown-spotted "catlotypes" could hardly measure up to the finely lined, steel-gray and sharply defined daguerreotypes of the day. Even stiffer competition was afforded by the colored motifs in the stereoscopic daguerreotypes which so enthused the public and even now remain astonishing. But steady improvement in the technique from approximately 1840 onwards soon made its basic superiority apparent.*

One of the first commercial photographers was ***Antoine-Jean-Francois Claudet*** *(1797–1867, France and England), who was a French glass merchant, living in High Holborn, England. He learned the details of the daguerreotype process from its inventor, and bought from him a license to operate in England. In 1841 he set up a studio on the roof of the Adelaide Gallery (near the Nuffield Centre), behind St. Martins in the Fields Church, London, and later on in two other sites in London.*

In 1847 glass was used for the first time as an emulsion support, while albumen paper, long appreciated for its sheen, was put on the market in 1850. Finally, the arrival of the colodion wet-plate process (1851) at last made available photographic material combing a high level of light-sensitivity with sharp definition. By the middle of the 19th century, the main steps had been taken w.r.t. both the practical aspects of the medium and the over-widening scope of its subject matter.

In addition to the various chemical-technical improvements, there were also new discoveries and refinements of optical features and the cameras themselves.

1839 CE **Charles Goodyear** (1800–1860, U.S.A.). A New-England hardware merchant; discovered a method of vulcanizing rubber.

The rubber in use at that time became hard in the winter and sticky in the summer. Without having any idea of what he was doing (he was no chemist), Goodyear began a series of experiments to try to improve the properties of rubber. He worked with rubber-sulfur mixtures, because he had heard of other similar experiments. When he accidentally spilled one of his

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the introduction of improved lenses and more sensitive photographic plates, exposure times were rarely less than 30 seconds.

concoctions on a hot stove, he found that he had created a superior type of rubber whose properties did not alter with heat or cold. Thus was the process of *vulcanization* born.

Unfortunately, Goodyear himself reaped no benefit from it. Before his discovery, his life had been marked by debtor's prison and bankruptcy, after it by patient litigation and the borrowing of huge sums to promote his invention. He died not quite 60 years old and hundreds of thousands of dollars in debt.

1839 CE **James Maccullagh** (1809–1847, Ireland). Irish mathematician and physicist. Produced an elastic model of the ether as a solid, the potential energy of which depends only on the rotation of a volume element, thus explaining the single wave velocity $c = (\mu/\rho)^{1/2}$. In terms of the later Maxwell theory, his displacement vector $\boldsymbol{u}$ corresponds to the magnetic field vector, and $\mu\,\mathrm{curl}\,\boldsymbol{u}$ to the electric field vector [μ = rigidity, ρ = density]. He also derived the gravitational potential of a finite body at an external point in terms of its principal moments of inertia.

Maccullagh was born near Strabane, Ireland. After a brilliant career at Trinity College, Dublin, he held the chair of mathematics during 1832–1843 and in 1843 was transferred to the chair of natural philosophy. Overwork induced mental disease, and he died at his own hands in 1847.

The Principle of Conservation of Energy

The failure to comprehend the distinction between velocity and acceleration retarded the study of dynamics for centuries. The study of heat was retarded by a somewhat similar confusion between temperature and heat, and by the further misapprehension that heat was a substance. Discovery of the true relationships involved some of the most illustrious names in theoretical and experimental physics.

The principle of conservation of kinetic and potential energy for rigid bodies was intuitively recognized already by ***Galileo Galilei*** *(1564–1642), following his experiments with bodies in free fall. The generalization of this principle to the entire field of mechanics is due to* ***C. Huygens*** *(1629–1695),* ***G.W. Von Leibniz*** *(1646–1716),* ***J. Bernoulli*** *(1667–1748) and* ***L. Euler*** *(1707–1783).*

*The concept of work originated with **J.V. Poncelet** (1788–1867). The word energy occurs already in **Aristotle**'s writings. It was introduced into the language of science by **T. Young** and **William Rankine** (1853). **Robert Mayer** (1840), **James Prescott Joule** (1843), **William Robert Grove** (1846), **H.L.F. Helmholtz** (1847) and **Lord Kelvin** (1852) reformulated the energy principle to include thermal, electrical and chemical phenomena and found the proper numerical conversion factors.*

1839–1842 CE *The Chinese Opium War* with Great Britain. The Chinese government had long been alarmed by the flourishing trade in opium and had vainly tried to stop it. In 1839 it moved to confiscate and destroy the vast quantities of the drug stored in Canton. This provided Britain the rationale for taking over China, something that it had long desired. A punitive force, assisted by the fleet of the East India Company, invaded China. The Chinese were no match for the experienced Westerners. Britain lost 500 troops; the Chinese lost 30,000. After three years of intermittent fighting, the Chinese were forced to agree to Britain's terms as laid down in the Treaty of Nanking (1842).

Opium had been used in China for medical purposes for centuries, but there was only a small amount of addiction among the people. China had little use for Western goods and ideas; its society was stable, and the vast country supplied all its own needs.

Europe, however, was extremely interested in the goods of China. Its *tea*, *silks*, *spices*, and *porcelain* commanded high prices on a market fascinated by the Orient.

The conquest of Bengal (1773) by Britain and the development of the World's finest merchant fleet allowed England's East India Company to obtain a foothold in the China trade and, by 1800, to monopolize it; after delicate negotiations with the Chinese government, a small island off the shores of Canton was established as a basis for trade. But the Chinese insisted that all goods be paid for in *silver*. By 1810–1815, China had acquired a good share of the silver of Europe which, because of scarcity, was rising rapidly in price, thus reducing the profits of British merchants.

Moreover, the Napoleonic wars have dwindled the gold reserves of Britain to a degree that it could not pay for the imported tea from China. Obviously, there had to be something that the Chinese wanted (except silver) and, with unerring intelligence, the British decided to sell Indian-made *opium* to the Chinese.

This required two preliminary steps: first, making the Chinese masses addicted to the drug, and second, securing a safe ocean route for trafficking the drug from India to China. These two objects were simultaneously achieved: on the one hand a British military expedition, occupied *Java* and controlled the *Straits of Malacca* (1811). In 1819 **Thomas Stamford Raffles** (1781–1826) founded *Singapore*.

Simultaneously opium was given away, and as addiction spread, prices rose accordingly. Because of its absolute control over India, the East India Company subverted the agriculture of Benares, Baher, and other areas of India to the growing of the poppy and the production of opium. Poppy cultivation was compulsory, and since the production of food crops was limited, the people of these India provinces were reduced near to starvation.

As addiction in China rose to astronomical proportions, silver began to move out of the country back into Europe where its price fell and the profits of the East Indian Company soared. Since silver was the currency of China, taxes went unpaid and internal business was disrupted. Based on Confucian and Tao ideals, Chinese society was grounded in the philosophy of self-discipline and a hierarchical arrangement of duties to family and emperor. The addict to his habit sacrificed family, duty, and self-discipline, and the fabric of Chinese society and government began to collapse. Nevertheless, the British government refused to order the East India Company to stop the opium trade; tax revenues from India, tea duties, and opium sales were simply too profitable.

1839–1846 CE **Christian Friedrich Schönbein** (1799–1868, Germany). Chemist. Discovered *ozone* (1839) and *guncotton* (1846), a powerful explosive, which he prepared and applied as a propellant in fire-arms.

Schönbein was born at Metzingen, Swabia. After studying at Tübingen and Erlangen, he taught chemistry and physics at Germany and England, but most of his life he spent at Basel, where he was appointed full professor (1835–1868). He was a prolific writer and carried on a large correspondence with other men of science such as **Berzelius**, **Faraday**, **Liebig** and **Wöhler**.

1839–1846 CE **William Robert Grove** (1811–1896, England). Jurist, physicist and electrochemist-inventor. "*Father of the Fuel Cell*".

A *fuel cell* is a device that produces electricity by combining hydrogen and oxygen — the reverse process of *electrolysis*. In his classic "*On the Correlation of Physical Forces*" (1846), he enunciated the *principle of conservation of energy* a year before the German physicist **Hermann von Helmholtz**.

Grove invented two cells of special significance. His first cell consisted of zinc, in dilute sulfuric acid and platinum in concentrated nitric acid, separated by a porous pot (*Grove Cell*).

Grove's nitric acid cell was the favorite battery of the early American telegraph (1840–1860), because it offered strong current output. This cell had nearly double the voltage of the first Daniell cell. As telegraph traffic increased, it was found that the Grove cell discharged poisonous nitric dioxide gas. Large telegraph offices were filled with gas from rows of hissing Grove batteries. As telegraphs became more complex, the need for constant voltage became critical and the Grove device was necessarily limited (as the cell discharged, nitric acid was depleted and voltage was reduced). By the time of the American War, Grove's battery was I replaced by Daniell battery.

His second cell, a "gas voltaic battery" was the forerunner of modern *fuel cells*. He produced the first fuel cell in 1839 basing his experiment on the fact that sending an electric current through water splits the water into its component parts of hydrogen and oxygen. So, Grove tried reversing the reaction — combining hydrogen and oxygen to produce electricity and water. This is the basis of a simple fuel cell. The term "fuel cell" was coined later in 1889 by **Ludwig Mond** and **Charles Langer**, who attempted to build the first practical device using air and industrial coal gas.

Grove was born at Swansea, South Wales. He was educated by private tutors and then at Brasenose College, Oxford, and also studied law at Lincoln's Inn and was called to the bar in 1835. His scientific career flourished while he was a professor of physics at the London Institution (1839–1864). At that period he also invented the earliest form of a filament lamp intended for use in mines.

He was elected FRS in 1840 and was one of the leaders of the reform movement. His law career was resumed in 1879, when he became a Judge at the Court of Common Pleas. He moved to the High Court of Justice in 1880 and became a Privy Councilor in 1887. Grove was knighted in 1872.

In 1846 he published his book on *The Correlation of Physical Forces*, the leading ideas of which he had already put forward in his lectures: its fundamental conception was that each of the manifested energy-forms of nature — light, heat, electricity, etc — is definitely and equivalently convertible into each other, and that where experiment does not give full equivalent, it is because the initial energy has been dissipated, not lost, by conversion into heat.

1839–1876 CE **John William Draper** (1811–1882, U.S.A.). Pioneer scientific photographer, photochemist, historian and author. His contributions:

- Made portrait photography possible by his improvements (1839) on Daguerre's process. Made first telescopic daguerreotype of the moon (1840) and the sun's diffraction spectrum (1844). Took photographs of the solar spectrum (1876) and anticipated development of *spectrum analysis.*
- Showed that plants grown in solution of sodium bicarbonate can liberate oxygen in light (1844).
- Investigated the dependence of the color of a heated substance upon its temperature.

Draper was born at St. Helen near Liverpool, England. He studied at the University of London. Went to the U.S.A. (1831) and continued his studies at the medical school of the University of Pennsylvania (1835–1836). Professor of chemistry at the University of the City of New York (1838–1882).

His son **Henry Draper** (1837–1882, U.S.A.), astronomer, was professor at the University of the City of New York (1860–1882). Built and mounted a 28-inch reflector (1869) with which he did pioneering work in stellar spectroscopy: obtained the first photograph of the stellar spectrum of *Vega* (1872), and the *Orion Nebula* (1880).

1840 CE **Germain Henri Hess** (1802–1850, Switzerland and Russia). Chemist, Formulated *Hess's Law*[160], which states that the net heat evolved or absorbed in any chemical reaction depends only upon the initial and final stages, It was a forerunner of the more complete law of the conservation of energy.

Hess was born in Geneva, Switzerland and became a professor of chemistry at St.Petersburg, Russia (1830–1850).

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160 *Hess's Law*: If a reaction is carried out in stages, the algebraic sum of the amounts of heat evolved in separate stages (heat absorbed being reckoned negative) is equal to the total evolution of heat when the reaction occurs directly.

e.g.: the *heat of combustion* of carbon to carbon dioxide: $C+O_2 = CO_2 + 94$ kilocalories; the *heat of combustion* of carbon monoxide to dioxide: $CO+\frac{1}{2}O_2 = CO_2 + 67.8$ kilocalories. By subtracting the second of these equations from the first, we find the *heat of formation* of carbon monoxide: $C+\frac{1}{2}O_2 = CO+26.2$ kilocalories.

Table 4.2: MAIL SERVICES AND NEWSPAPERS THROUGHOUT HISTORY

900 BCE	China has an organized postal service for government use
500 BCE	Persia has a form of pony express
59 BCE	Julius Caesar ordered posting of *Acta Diurna*
100 CE	Roman couriers carry government mail across the empire
1200 CE	European monasteries communicate by letter system
1300 CE	Incas and Aztecs employ courier runners to carry messages over their kingdoms roads at top speed of ca 400 km/day
1305 CE	The Taxis family began a private postal service in Europe
1450 CE	First newsletters began circulating in Europe
1533 CE	A postmaster in England
1609 CE	First regularly published *newspaper* appeared in Germany
1627 CE	France introduced registered mail
1650 CE	A daily newspaper in Leipzig, Germany
1840 CE	First postage stamps sold in Britain

1840 CE **Joseph Max Petzval** (1807–1891, Hungary). Mathematician and optician. Contributed to the design of precision optical systems through his work on lenses and aberrations. These had great impact in the design of modern cameras; Petzval produced an achromatic portrait lens that was vastly superior to the simple meniscus lens then in use. *Petzval curvature*, *Petzval surface*, *Petzval theorem. Petzval condition* and *Petzval lens* are named after him.[161]

[161] To dig deeper, see:

- Born, M. and E. Wolf, *Principles of Optics*, Macmillan: New York, 1964, 808 pp.

The discovery of photography (1839) by Daguerre was chiefly responsible for early attempts to extend the Gaussian theory. Practical optics, which until then was mainly concerned with the constructions of *telescope objectives*, was confronted with the task of producing objectives with *large apertures and large fields*. Petzval attacked with considerable success the related problem of supplementing the Gaussian formulae by terms involving *higher powers of the angular inclination of rays to the axis*. Unfortunately, Petzval's extensive manuscript on the subject was destroyed by thieves; what is known about his work comes mainly from semi-popular reports.

He was professor at the University of Vienna and worked for much of his life on the *Laplace transform*, being influenced by the work of **Liouville**. He pioneered the application of the Laplace transform to the solution of linear differential equations although he did not use contour integration to invert the transform.

1840 CE **Friedrich Gustav Jacob Henle** (1809–1885, Germany). Anatomist and pathologist. First to argue that infectious diseases are transmitted by living organisms which can reproduce. In his work *Pathologische Untersuchungen* (Pathological investigations) he presented an early version of germ theory of disease in which parasite living matter can be transmitted through contact or through the atmosphere. His contention was proved by his pupil Robert Koch 40 years later.

Henle was born at Fürth, a grandson of a rabbi, and was baptized at the age of 11. He took his doctors degree in medicine at Bonn (1832) and latter became a professor at Heidelberg (1844).

1840 CE Polio first identified or described with accuracy.

1840–1862 CE *Cholera* spread worldwide; fatalities were in the millions.

1840–1865 CE **Karl Friedrich Schimper** (1803–1867, Germany). Naturalist and poet. Proposed many of his scientific ideas in poetic form, including botany, geology, and the formation of the Alps during the Ice Age. A pioneer of modern plant morphology. Originated the modern concepts of the Ice Age and the climatic cycles. Formulated the theory of phyllotaxis.

Schimper was born in Mannheim. He studied theology at Heidelberg (1822) and medicine at Munich (1829) but failed to secure an academic post nor any other regular appointment. He never married despite two engagements and eventually moved to Schwetzingen where he died of dropsy.

His cousin, **Wilhelm Philipp Schimper** (1808–1890) was a botanist. Studied at the University of Strasbourg, where he became Director of the local Natural History Museum (1835). Identified the *Paleocena Period* in earth's history (1874). His son **Andreas Franz Wilhelm Schimper** (1856–1901) was also a botanist and a professor at Basel (1898–1901). Proved (1880) that starch is a source of stored energy for plants and a product of photosynthesis.

1840–1870 CE **Joseph Whitworth** (1803–1887, England). Mechanical engineer and inventor; a leader in tool design and manufacture. Invented measuring machines and found a method of milling and testing plane surfaces. Introduced a *system of standard measures*, gauges, and screw threads.

Whitworth was born in Stockport, England, and died at Monte Carlo.

1840–1895 CE Leading Western poets and novelists in the Age of Naturalism and Realism:

- Charles Dickens 1812–1870
- Ivan Turgenev 1818–1883
- Walt Whitman 1819–1892
- Charles Baudelaire 1821–1867
- Gustav Flaubert 1821–1880
- Feodor Dostoevsky 1821–1881
- Charles de Coster 1827–1879
- Henrik Ibsen 1828–1906
- Lev N. Tolstoi 1828–1910
- Bjornstjerne Bjornson 1832–1910
- Mark Twain 1835–1910
- Emil Zola 1840–1902
- Edmondo de Amicis 1846–1908
- Henryk Sienkiewicz 1846–1916
- Jens Peter Jacobsen 1847–1885
- August Strindberg 1849–1912
- Guy de Maupassant 1850–1893
- Robert Louis Stevenson 1850–1894
- Anton Chekhov 1860–1904

1841–1847 CE **Edward Forbes** (1815–1854, England). Naturalist and oceanographer. One of the first men to take a scientific interest in the ocean *depths*. As a naturalist on board the surveying ship *H.M.S. Beacon*, he did some dredging in the Aegean Sea, studying the distribution of flora and fauna and their relation to depth, temperature and other factors. His pioneering work led the way to the *Challenger* expedition.

Forbes was born at Douglas, in the Isle of Man. In 1854 he became professor of natural history in the University of Edinburgh, but died soon afterwards.

1841–1852 CE **James Prescott Joule** (1818–1889, England). Physicist. With **J.R. Mayer** and **H.L.F. Helmholtz** established the First Law of Thermodynamics and the mechanical equivalent of heat.

Joule began his work with the discovery of the rate of heat production by an electric current in a conductor and showed it to be proportional to the square of the current strength and the wire resistance (*Joule's Law*, 1841). In 1843, Joule read before the British Association at Cork his paper, entitled: "*The Caloric Effects of Magneto-Electricity and the Mechanical Value of Heat*".

In 1847, he generalized his former results and asserted equivalence and convertibility of heat, mechanical, electrical and chemical forms of energy, rendering some numerical conversion factors. In 1852 he established, with **W. Thomson**, the Joule-Thomson effect.

Joule was born at Salford, near Manchester. Although he received some instruction from **John Dalton** in chemistry, most of his scientific knowledge was self-taught.

1841–1852 CE **David Gruby** (1811–1898, France). Distinguished physician and pioneer in the fields of modern microbiology, veterinary protozoology, and parasitology. Created the field of pathological mycology of humans and pets. First to show that fungi can cause diseases in humans. His decisive and pioneering contribution to the development of microbiology and parasitology has been underestimated in the history of medicine and biology.

Gruby was born in a small village, near Novi-Sad, Hungary, the son of a poor Jewish farmer. He left his home at an early age and went to Budapest and from there to Vienna, where he studied medicine. In Paris (1835) he distinguished himself as a lecturer in the Museum of Natural History. He ceased his research activity in 1852 and dedicated all his time to his medical practice. He was the personal physician of Heinrich Heine and and Alexandre Dumas.

1841–1852 CE **Alexander Bain**[162] (1811–1877, Scotland). A mechanical genius of the first order who came before his time. Clockmaker, inventor, telegraphy pioneer and the 'father of the *facsimile' (fax)*, which can be said to be a primitive forerunner of television. Proposed facsimile

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[162] Not to be confused with his namesake and country sake **Alexander Bain** (1818–1908), philosopher and psychologist

telegraph[163] transmission system (1843). Made the first *electric clock* (1841). Invented the first *chemical telegraph* (1846). Details of his inventions are:

- *Electrical clock*: Electromagnetic pendulum is kept going by an electric current instead of springs and weights. He improved on this idea in following patents, and also proposed to derive the motive electricity from an 'earth battery', by burying plates of zinc and copper in the ground.

- *Facsimile*: His method for sending a facsimile image cleverly explored the transmission of electrical signals over telegraph wires. The telegraph was a relatively new device in Bain's day but was rapidly gaining on popularity. Both amateur and professional inventors were trying their utmost to find new ways to use it.

  Bain's operated as follows: the sender write a message on a sheet of conducting tin in non-conducting ink. The sheet was then fixed to a curved metal plate and scanned by a needle controlled by a swinging pendulum. This 'scanner' read the text line by line, point by point at a rate of three lines per millimeter. It emitted an electrical signal, which registered at one strength as it passes through the images' black points (ink) and at another as it passed through the images' white points (absence of ink, i.e. metal). The two distinct signals traveled over the telegraph wire to the receiver where a *synchronized pendulum* controlled a stylus that marked out with Prussian blue ink on a paper soaked in potassium ferrocyanide – leaving behind images of black and white dots that had defined the original text.

  To ensure that both needles scanned at exactly the same rate (so that they would begin and end the scan lines at the same point) two extremely accurate clocks should be used. This, however, could not have been achieved in Bain's time. Other improved on Bain's invention in the years to come.

- *Chemical telegraph*: Bain recognized that the Morse telegraph was comparably slow in speed, owing to the mechanical inertia of the parts; and he saw that if signal currents were made to pass through a ribbon of paper soaked on a solution of iodide of potassium, a brown mark could be made at the point of contact due to liberation of iodine and consequently a very high speed could be obtained.

[163] Note that the first patent on a working fax machine had been filed and granted 33 years *before* Alexander Graham Bell patented his telephone, and even before Bell was even born!

> When the chemical telegraph was tried between Paris and Lille, the speed of signaling attained was 325 words/min as compared to 40 words/min on the Morse electro-magnetic instrument. Others later improved on the neglected method of Bain and reached a recording rate of 113 words/min.

Bain was born of humble parents in the little town of Thurso at the extreme north of Scotland. Learning the art of clock-making, he went to Edinburgh, and subsequently removed to London. By 1870, his royalties from patents for electric telegraph and clocks were exhausted, and he sank into poverty. Moved by this unhappy circumstances, **Lord Kelvin** and **William Siemens** obtained for him from the Prime Minister W. Gladstone (1873) a pension of 80 pound a year; but the beneficiary lived in such obscurity that it was a considerable time before his lodging could be discovered. The Royal Society had previously made him a gift of 150 pounds.

In his later years his health failed and he was removed to the Home for Incurables at Broomhill, Kirkintilloch, where he died. He was a widower, and had two children, but they were said to be abroad at the time, the son in America and a daughter on the continent.

Several of Bain's earlier patents were taken out in two names; owing to his poverty he was compelled to take a partner to share the patent fees. Considering the early date of his achievement, and his lack of education or pecuniary resource, we cannot but wonder at the strength, fecundity, perseverance and prescience of his creative faculty. Beyond a few facts in a little pamphlet (published by himself) there is little to be gathered about his life; a veil of silence had fallen alike upon his triumphs, his errors and his miseries.

1841–1873 CE **David Livingstone** (1813–1873, Scotland and Africa). Missionary, physician, geographer and explorer in Africa. No single African explorer has ever done so much for African geography as Livingstone during his thirty years' work: his travels covered one-third of the continent, extending from the Cape to the equator, and from the Atlantic to the Indian Ocean. He did his journeying leisurely, carefully observing, mapping and recording all that was worthy of note, with rare geographical instinct and with the eye of a trained scientific observer, studying the ways of the people, eating their food, living in their huts, and sympathizing with their joys and sorrows.

In all the countries through which he traveled his memory was cherished by the native tribes as a superior being. Indeed, in the annals of exploration of Africa, he stands preeminent above all. His example and death raised in Europe a powerful feeling against the slaver trade that through him it received its death-blow. The motto of his life was advice he gave some school children in Scotland: "Fear God, and work hard".

Livingstone was born at the village of Blantyre Works, in Lanarkshire, Scotland. Received a degree in medicine from Glasgow University (1840) and then became connected with the London Missionary Society. He went to South Africa (1841) to begin his missionary work. His aims were to convert African natives to Christianity, to put a stop to the slave trade, and to explore the mysterious African continent. He arrived at *Lake Ngami* (1849), *Zambezi River* (1851), and sighted *Victoria Falls* (1855). He explored *Lake Nyasa* region, the *Shire River* (1858), *Lake Shirwa* (1858), reached the southern end of *Lake Tanganyika* (1867) and *Lake Bangweulu* (1868).

Concern over his safety led to the expedition of **Henry Morton Stanley** and the two met near Lake Tanganyika (1871). Livingstone refused to return to the coast with Stanley (1872) and continued his travels for another year. Weakened by illness, he arrived at *Lake Bangweulu* where he died on April 30, 1873. He was later buried at Westminster Abbey in London.

1842 CE **Johann Christian Doppler** (1803–1853, Austria). Physicist. Discovered the law that determines the change of frequency of a moving source of mechanical radiation (*Doppler Effect*)[164]. Since its inception, this law became a major tool in determining translational and angular velocities in all branches of physics and astronomy where the moving bodies might be electrons, satellites, stars or galaxies, in the framework of classical physics, quantum physics and general relativity.

Doppler was born in Salzburg. He was educated at the Polytechnisches Institut in Vienna and became professor of experimental physics at the University of Vienna in 1850.

1842 CE **Julius Robert Mayer** (1814–1878, Germany). Physicist and physician. First to recognize that the law of conservation of energy goes beyond the framework of classical mechanics, without giving this idea a precise mathematical formulation. It is remarkable that in spite of inaccurate reasoning and data of limited quality, he was able to obtain a correct numerical result for the mechanical equivalent of heat. Thus, on account of his boldness, insight and intuition, it can be claimed that he was the father of the First Law of Thermodynamics[165].

Mayer, the son of the owner of an apothecary shop, was born at Heilbronn. He studied medicine at Tübingen, Münich and Paris, and after a journey to

[164] Later found to apply to electromagnetic radiation as well, with the appropriate relativistic correction.

[165] On this point there was no agreement between **Sommerfeld** on the one hand and **Lord Kelvin**, **Maxwell** and **G.G. Stokes** on the other. The British physicists claimed that distinction for their countryman **J.P. Joule**.

Java in 1840 as surgeon of a Dutch vessel, in the East Indies, obtained a medical post in his native town. It was here, by a curious route, that he was led to the idea of the conservation of energy. That route involved medicine, not physics.

Letting out blood was a common medical cure of the time, and while letting the blood of sailors arriving at Java, Mayer noted that their blood was unusually red. He reasoned that the heat of the tropics reduced the metabolic rate needed to keep the body warm and therefore reduced the amount of oxygen that needed to be extracted from the blood. Accordingly, the sailors had a surplus of oxygen in their blood, causing its extra redness. This hypothesis, and its apparent validation, were taken by Mayer to support the link between heat and chemical energy, the energy released by the combustion of oxygen.

After deciding that there must be a balance between the input of chemical energy and the output of heat in the body, Mayer made a conceptual leap. Friction in the body, from muscular exertion, also produced heat, and the energy associated with this heat also had to be strictly accounted for by the intake of food and its content of chemical energy.

Mayer, being a physician and not a physicist, was at first not familiar with the principles of mechanics, and his first paper on energy had errors. It was rejected. Although disappointed, Mayer immediately took up the study of physics and mathematics, learned about kinetic energy, and submitted a new paper a year later. In 1842 he published a little paper "*Bemerkungen über die Kräfte der unbelebten Natur*" in which he expounded his ideas concerning conversion and conservation of energy. This paper did not receive much attention, but within the next few years other physicists, mainly **J.P. Joule** (1843–1849) and **H.L.F. Helmholtz** (1847) put the First Law on a much firmer foundation.

Despondent over his lack of recognition, Mayer attempted suicide in 1850. He suffered episodes of insanity in the early 1850s and was confined in asylums on several occasions.

After 1860, Mayer was finally given the recognition he deserved. Many of his articles were translated into English, and such well-known scientists as **Rudolph Clausius** in Germany and **John Tyndall** in England began to champion Mayer as the founder of the law of the conservation of energy.

From his marriage Mayer had 7 children, 5 of whom died in childbirth. He died of tuberculosis in 1878.

1842 CE **Joseph Alphonse Adhémar** (1797–1862, France). Mathematician. First to propose an astronomical theory of the ice ages based on the precession of the equinoxes.

Adhémar theorized that glacial climates occur whenever a hemisphere enters winter while at its farthest distance from the sun. Thus, every 11,000 years[166] (every half cycle) an ice age would occur, alternately in one hemisphere and then in the other; a series of abnormally cold winters would allow snow and ice to build up and would pitch the globe into an ice age.

In 1852, **Alexander von Humboldt** (1769–1859, Germany) pointed out that Adhémar's basic idea was incorrect: the average temperature of either hemisphere is controlled not by the number of *hours* of daylight and darkness, but the total number of calories of solar energy received each year. And, as **d'Alembert**'s calculations had demonstrated many years before, any decrease in solar heating that occurs during one season (earth farther from the sun), is exactly balanced by an increase during the opposite season, when the earth is closer to the sun. Therefore, the total amount of heat received by one hemisphere during the year is always the same as that received by the other.

Although Adhémar's theory was proved wrong, it was nevertheless an important step toward understanding the ice age mystery. The idea that *astronomical* phenomena such as the precession of the equinoxes might have a significant effect on the earth's climate was not forgotten, and would set the stage for further discoveries.

1842 CE **Samuel Earnshaw** (1805–1888, England). Mathematician. Showed that a set of physical point-objects (charges, masses, magnetic poles), governed by the classical long-range inverse-square law (electrostatic, magnetostatic, gravitational), cannot be maintained in a stationary stable equilibrium configuration. This is known as 'Earnshaw's Theorem'.

Earnshaw was born in Sheffield and graduated Senior Wrangler and Smith's Prizeman (1831) in Cambridge University. He worked there as tripos coach (1831–1847) and was also appointed to the parish church at St. Michael, Cambridge (1846).

He published several articles and books on classical physics and mathematics, but is best known for his article: "On the Nature of the Molecular Forces which Regulate the Constitution of the Luminiferous Ether" (Trans. Camb. Phil. Soc. 7, 97–112, 1842). Earnshaw's Theorem has consequences for levitation by means of electromagnetic fields, as it shows the impossibility of stable levitating permanent magnets without active control. Note that the

[166] Previously (1754) it was shown by **d'Alembert** that the simultaneous precession of the equinoxes (due to the combined pull of the sun and the moon), and the precession of the earth's perihelion (due to the perturbation of the planets) cause the earth to undergo a *general* precession of the equinoxes with a period of 22,000 years.

case of a point charge in arbitrary static electric field is a simple consequence of Gauss' law: $\text{div}\boldsymbol{E} = -\Delta U = 0$ at field points, where $\boldsymbol{E}(\boldsymbol{r})$ is the electric field and $U(\boldsymbol{r})$ is the potential and $\text{curl}\,\boldsymbol{E} = 0$ in free space. Thus, at any equilibrium point, there must be some direction along which the equilibrium is unstable.

The theorem also means that even dynamic system of charges are unstable in the long term due to EM radiation. This lead, for some time, to the puzzling question of how matter is held together electromagnetically. The answer came via the quantum-mechanical structure of the atom. It was then discovered that the Pauli exclusion principle and the uncertainty principle are responsible for holding bulk matter in a rigid form.

1842–1843 CE (**Augusta**) **Ada Byron**, Countess of Lovelace; 1815–1852, England. An amateur mathematician[167] who wrote the first computer program for Babbage's *analytical machine*. It was a set of instructions for the machine to compute the Bernoulli numbers (the analytical engine never reached the stage of allowing the program to be run).

Ada was the only child of Lord and Lady Byron. She had considerable mathematical talent (in this she took after her mother, who was described by the poet as the 'Princess of Parallelograms'), and frequently visited Babbage while he was working on his engine. In 1840, Babbage gave a series of lectures in Turin. Among his audience was **L.F. Menabrea**[168], a young engineer officer on the staff of the Military Academy in that city. Menabrea wrote an account of Babbage's ideas and published it in a Geneva Journal in 1842. The paper was translated into English by Lady Lovelace, who added extensive notes of her own, and was published in Taylor's *Scientific Memoirs* in 1843. She had a remarkable grasp of Babbage's ideas and her lucid notes make fascinating reading even today.

Her notes are mainly concerned with the formulation of a schedule of instructions (the program) which will enable the machine to carry out a desired calculation automatically. Lady Lovelace went into the subject in considerable detail, and illustrated her points by describing several programs for performing

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[167] Ada lost much of her fortune by using her computations to predict horse races.

[168] **Luigi Federico Menabrea** (1809–1896, Italy). Became professor of mechanics at the military academy and at the University of Turin (1842). Embarked in a political career which led him to become Italian Premier and Foreign Minister (1867). During this period of politics he still continued his scientific work, giving the first precise formulation of methods of structural analysis based on the principle of virtual work. Published, jointly with **J.L.F. Bertrand**, the first correct proof of the principle of least work (1870). This was later called the *Castigliano principle*.

advanced mathematical calculations, some of them of considerable sophistication. The two basic features of her programs are: the use of repetitive cycles, whereby the same set of instructions is executed over and over again, and the use of a jump (branching) instruction to enable the calculation to take one or the other of two alternative paths.

In one of her more elaborate programming examples she introduced a number in a certain register for the specific purpose of counting the number of repetitions of a group of instructions. This number is arranged to change sign when the desired number of cycles has been executed, and a jump instruction is inserted to cause the machine to move out of the loop at this point to the next part of the calculation.

Since the punched cards in the **Jacquard** loom (1805) pass through the mechanism in a fixed order which cannot be varied once the loom is set up, Lady Lovelace suggested the provision of an additional *hardware* facility to enable the *backing* of the cards of the analytical engine: the drum over which the train of cards passes must be able to *rotate in the reverse direction* in an amount determined by the program. She wrote:

"*The object of this extension is to secure the possibility of bringing any particular card or set of cards into use any number of times successively in the solution of one problem. The power of repeating the cards reduces to an immense extent the number of cards required. It is obvious that the mechanical improvement is especially applicable wherever* cycles occur in the mathematical operation, *and that, in preparing data for calculations by the Engine, it is desirable to arrange the order and combination of the processes with a view to obtain them as much as possible* symmetrically and in cycles, *in order that the mechanical advantages of the* backing *system may be applied to the utmost*".

Already in 1842, this remarkable pioneer of modern programming had a full grasp of the 'soul of the computer', as she put it in her unusual clarity: "*The Analytical Engine has no pretensions whatever to originate anything. It can do whatever we* know how to order it *to perform*".

The computer language "Ada" was named by the designers after her.

Ada was the first wife of Baron King, who in 1838 was made earl of Lovelace. They had two sons and a daughter.

1842–1845 CE Physicians in the United States first used *anesthesia* to ease pains during treatment of patients. **Crawford Williamson Long** (1815–1878), surgeon, was first to use *ether* vapor (1842), in Jefferson Ga, to knock a patience unconscious during operation. In 1845, he used ether for the first time in delivering a child; **William Thomas Green Morton** (1819–1868), dentist, administered ether (1846) during a surgical operation at Mass. General

Hospital (Boston). He did this at the suggestion of **Charles Thomas Jackson**[169] (1805–1880), physician, chemist and geologist at the Harvard Medical School. **James Young Simpson** (1811–1870, Scotland), physician, was first to use *chloroform* (1847) to reduce pain at childbirth. This was quicker and more effective than ether; Queen Victoria was one of the first women to be anesthetized during childbirth.

1842–1855 CE **Ludwig Otto Hesse** (1811–1874, Germany). Mathematician. Introduced the '*Hessian normal form*'[170] and also the '*Hessian function*' and the '*Hessian matrix*'[171] that appear in extremalization of real-valued functions and the theory of algebraic invariants.

Hesse was born in Königsberg. His studies at the University of Königsberg (1832–1840) were interrupted for an educational journey throughout Germany and Italy and his subsequent high-school teaching career. Hesse studied

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[169] Practiced medicine in Boston (1832–1836) but abandoned medicine to work in chemistry and mineralogy (1836). Claimed to have pointed out to S.F.B. Morse the basic principles of the electric telegraph; also claimed the priority in discovery of guncotton (explosive). In 1852, the French Academy awarded a prize of 5000 francs jointly to Jackson and Morton. Both men claimed sole credit for the discovery, and Morton refused to share the prize with Jackson. A bitter quarrel and lawsuits followed, and Morton was ruined financially. Long did not get any credit since he published the account of his early discovery only in 1849.

[170] The *vector form* of a plane in space is $\boldsymbol{x}\cdot\boldsymbol{x}_1 + d = 0$, where $\boldsymbol{x} = (x, y, z)$ and $\boldsymbol{x}_1 = (a, b, c)$, i.e. $ax + by + cz + d = 0$. If this equation is divided by the normalization factor $\pm(a^2 + b^2 + c^2)^{1/2}$, one arrives the *Hessian normal form*

$$x\cos\alpha + y\cos\beta + z\cos\gamma - p = 0,$$

where (α, β, γ) are the direction cosines of the normal $\boldsymbol{x}_1$, and p is the distance of the plane from the origin.

[171] Let P be some particular point chosen as the origin of a coordinate system with coordinates $\boldsymbol{x}$. Then, any function f can be approximated by its Taylor series

$$f(\boldsymbol{x}) = f(P) + \boldsymbol{x}\cdot \operatorname{grad} f|_P + \frac{1}{2}\boldsymbol{x}\boldsymbol{x}\colon \operatorname{grad}\operatorname{grad} f|_P + \cdots$$

The matrix

$$(\operatorname{grad}\operatorname{grad} f)_{ij} \equiv \frac{\partial^2 f}{\partial x_i \partial x_j}|_P,$$

whose components are the second partial derivative matrices of the function, is called the *Hessian matrix* of the function at P. The determinant functional $H[f] = \|\nabla\nabla f\|$, known as the *Hessian determinant*, has found many applications in algebraic geometry.

For example, if in the bilinear form

mainly under **C.G.J. Jacobi**, who greatly stimulated his mathematical investigations.

He then taught at Königsberg (1845–1855), Heidelberg (1856–1868) and Münich (1868–1874). He became an ordinary professor in 1855 and among his students were **G. Kirchhoff**, **S.H. Aronhold**, **C. Neumann**, **A. Clebsch** and **R.O.S. Lipschitz**.

1842–1866 CE **Siemens**: Name of German brothers: inventors, electrical engineers and industrialists.

- **Ernst Werner von Siemens** (1816–1892); invented the *electroplating process* (1842); invented the *dial telegraph* (1846); laid an underground electric telegraph for the army (1847). Founded the Siemens firm for manufacture of telegraphic equipment and electrical apparatus (1847). Laid cables across the Mediterranean and from Europe to India; invented the self-excited generator (1866).
- **Karl Wilhelm (Charles William) Siemens** (1823–1893). Naturalized British citizen (1859). Invented the regenerative steam engine

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$$f(x_1, x_2) = a_0x_1^2 + 2a_1x_1x_2 + a_2x_1^2$$

we effect the linear transformation

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \xi_1 & \eta_1 \\ \xi_2 & \eta_2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix},$$

we obtain the new bilinear form

$$F(X_1, X_2) = A_0X_1^2 + 2A_1X_1X_2 + A_2X_2^2$$

where

$$A_0A_2 - A_1^2 = (a_0a_2 - a_1^2)D^2,$$

and

$$D = \det \begin{bmatrix} \xi_1 & \eta_1 \\ \xi_2 & \eta_2 \end{bmatrix} = \xi_1\eta_2 - \xi_2\eta_1.$$

It is also true that $H[F] = D^2H[f]$, where under an orthogonal transformation $D^2 = 1$; the Hessian functional is

$$H[f] = \frac{\partial^2 f}{\partial x_1^2}\frac{\partial^2 f}{\partial x_2^2} - \left(\frac{\partial^2 f}{\partial x_1 \partial x_2}\right)^2 = \text{Hessian of } f = \det \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix}.$$

(1847) and Siemens process for making steel (1861). Laid the first cable between Britain and the USA (1875).

- **Friedrich Siemens** (1826–1904). Invented (1856) a regenerative smelting oven widely used in the glass and steel making industries.
- **Karl von Siemens** (1829–1906). Organized and directed the Russian branch of the firm.

1843 CE **Pierre Alphonse Laurent** (1813–1854, France). Engineer and mathematician. Discovered the relationship between power series and analytic functions in a domain bounded by two concentric circles[172].

Laurent was born in Paris. After studying for two years at the École Polytechnique, he joined the army engineering corps and took part in the expeditions to Algeria. He then returned to France and spent about 6 years

[172] *Laurent series*: Let $f(z)$ be a function analytic in the ring-shaped region between two concentric circles C and C', of radii $R' < R$, and center a, and on the circles themselves. Then $f(z)$ can be expanded in a series of positive and negative powers of $z - a$, convergent at all points of the ring-shaped region:

$$f(z) = \sum_{n=-\infty}^{\infty} a_n (z-a)^n;$$

$$a_n = \frac{1}{2\pi i}\int \frac{f(\omega)}{(\omega - a)^{n+1}} d\omega,$$

for all values of n.
The integral is taken round any simple closed contour within the region. In the particular case where $f(z)$ is analytic inside C', all the $n < 0$ coefficients are zero (by Cauchy's theorem), and the series reduces to a Taylor series.
In the neighborhood of a pole of the n^{th} order, the Laurent series are truncated at the negative power with exponent n. In a similar way, a function defined by a Laurent series has a pole of the n^{th} order at infinity when the terms with positive powers ends at the term with power n.
If $f(z)$ is regular in an arbitrary narrow annulus

$$1 - \varepsilon < |z| < 1 + \varepsilon \qquad (0 < \varepsilon < 1),$$

it can be represented there by the series $\sum_{n=-\infty}^{\infty} c_n z^n$, where

$$c_n = \frac{1}{2\pi i}\oint_{|z|=1} \frac{f(\zeta)d\zeta}{\zeta^{n+1}} = \frac{1}{2\pi}\int_0^{2\pi} f(e^{i\theta})e^{-in\theta}d\theta \qquad (n = 0, \pm 1, \pm 2, \ldots).$$

In particular, at the points $z = e^{it}$ on the *unit circle*, $f(z)$ is represented by in the port of Le Havre, directing hydraulic construction projects. In the midst of these technical operations he submitted two memoirs to the Academy of Sciences, one of which dealt with the 'Laurent series expansion'. It was communicated by **Cauchy**, but due the negligence of the latter was never published by the Academy. It did not appear until 1863, when it was published in the Journal de l'École Polytechnique.

1843 CE **Heinrich Samuel Schwabe** (1789–1875, Germany). Apothecary and amateur astronomer. Discovered the 11-year cycle of solar activity, the sunspot cycle; also made (1831) the first known detailed drawing of the Great Red Spot on Jupiter.

In 1825, Schwabe began to study the sun and its sunspots. He spent 17 years looking at it (with the proper precautions to avoid blindness) and discovered that the number of spots rose and fell in what seemed a cycle of 10 years (more like 11, according to continuing studies by others). This contributed to the beginning of the science of *astrophysics*, the study of physical phenomena in stars and other objects in the universe.

1843 CE **John Stuart Mill** (1806–1873, England). Philosopher and logician. Delineated the foundations of *inductive logic* and the scientific method in his book *A System of Logic* (1843). He applied principles of Empiricism to the scientific method, interpreted as a system of inductive logic. Rounding out and perfecting Francis Bacon's inductive technique, he advocated induction as a new approach of problem-solving that would supersede the Aristotelian method of deductive logic.

Mill was born in London and educated completely by his father. He began to study Greek at the age of 3, and at 14, had mastered Latin, classical literature, logic, history, political economy and mathematics. He entered the East India Company at the age of 17. Like his father, he became director of the company. He retired after 33 years of service and was elected member of Parliament (1865).

$$F(t) = f(e^{it}) = \sum_{n=-\infty}^{\infty} c_n e^{int},$$

where

$$c_n = \frac{1}{2\pi}\int_0^{2\pi} F(\theta)e^{-in\theta}d\theta \qquad (n = 0, \pm 1, \pm 2, \ldots).$$

This is none other then the complex form of the Fourier series of $F(t)$. Thus, on the unit circle $z = e^{it}$, the Laurent series, considered as a function of the real variable t, is the *Fourier series* of the function $F(t) = F(e^{it})$.

The main task of the *System of Logic*[173] is the analysis of inductive proof. His canons of inductive methods for comprehending the causal relations between phenomena are valid under the assumption of the validity of the *law of causality*, which cannot be accepted except on the basis of induction — making the whole argument circular.

1843–1849 CE **Søren Aabye Kierkegaard** (1813–1855, Denmark). Philosopher. Attacked social and religious complacency. His assault on institutional Christianity and on traditional Western philosophy generated a crisis that produced a radically new way of philosophizing and made him the founder of a school that would later be called *Existentialism* – centered on the obsession with the particularity of human existence. To Kierkegaard, reality was personal, subjective – it began and ended with the individual. To him, **Hegel**'s system[174] was an immense fraud which, by its verbose techniques of reconcilia-

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173 Mill presented five rules (canons) of inductive reasoning:

(1) *Rule of Agreement*: If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree is the cause (or effect) of the given phenomenon.

(2) *Rule of Difference*: If an instance in which the phenomenon under investigation occurs, and an instance in which it does not occur, have every circumstance save one in common, that one occurring only in the former, the circumstance in which alone the two instances differ is the effect, or cause or a necessary part of the cause, of the phenomenon.

(3) *Joint Rule of Agreement and Difference*: If two or more instances in which the phenomenon occurs have only one circumstance in common, while two or more instances in which it does not occur have nothing in common save the absence of that circumstance, the circumstance in which alone the two sets of instances differ, is the effect, or cause, or a necessary part of the cause, of the phenomenon.

(4) *Rule of Residues*: Subduct (subtract) from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents.

(5) *Rule of Concomitant Variations*: Whatever phenomenon varies in any manner whenever another phenomenon varies in some particular, is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation.

His rule (1) is almost identical to the hermeneutic Talmudic rule *Binyan-Av* of **Hillel** (ca 32 BCE). It was used by the latter as a tool of logical deduction from the juxtaposition of two legal sections.

174 **Hegel** had treated the human individual as an insubstantial being of secondary importance to social institutions and the state. He saw God as a poetic myth anticipating in a primitive way the higher philosophical truth embodied in his own doctrine of the Absolute.

tion and rationalization, refused to confront the actual circumstances of man in the world, and in particular the fact of deaths and the remoteness and inscrutability of God.

Kierkegaard argued that actual existence, as we experience it in life, cannot be rationalized in Hegel's way: men and the world they inhibit cannot be tidily explained. Belief in God is not the solution to a theoretical problem but a free act of faith. In this framework he saw the Protestant Church as a means of perverting its original messages, and the very symbol of self-satisfied bourgeois snugness that stands between the individual and the truth. *Knowledge*, as Kierkegaard construes it, is always abstract but *existence* cannot be thought, because it is always concrete. Existence must, at its very core, be experienced as anguish or dread of the possibility of death at any moment.

Kierkegaard took the position that religion was a personal experience. He divided experience into three categories: *aesthetic*, *ethical*, and *religious*. The child is an example of an individual who lives almost exclusively at the aesthetic level: all choices are made in terms of pleasure and pain, and experience is ephemeral, having no continuity, no meaning, but being merely a connection of isolated, non-related moments. The *ethical* level of experiences involves choices, whenever conscious choice is made, one lives at the ethical level. At the *religious* level, one experiences a commitment to oneself, and an awareness of one's uniqueness and singleness. To live in the religious level means to make any sacrifice, any antisocial gesture that is required by being true to oneself.

These levels are not mutually exclusive but may coexist. He concluded that only when man experiences the suffering of firm commitment to the religious level of experience can he be considered truly religious. If religion then is a purely personal matter, truth is clearly subjective, quite separate from the "truth" of religious doctrine. Objective truth, such as that of geometry, is acquired by the *intellect*; subjective truth must be experienced by the total individual. One may *have* objective truth, but one must *be* religious truth.

His main works: *Fear and Trembling* (1843); *Either/Or* (1843); *The Concept of Dread* (1844);*The Sickness unto Death* (1849).

Kierkegaard lived most of his life in Copenhagen. In 1840 he became engaged to a 17-year old girl, but he broke off the engagement after about a year. Their affair continued to haunt him throughout his life. For two years he traveled in Germany (1840–1842) where he studied Hegelian philosophy at the Berlin university under Schelling with his classmates **Friedrich Engels**, **Ludwig Feuerbach** and **Michael Bakunin**. Kierkegaard died in the middle his violent battle against the Lutheran Church establishment in Denmark; he died a lonely man with hardly a follower.

There were riots at his funeral, caused by theology student's outrage at the way the Church tried to take over in death the man who had opposed it so bitterly with his last breath. He had wanted to have written on his tombstone simply "THE INDIVIDUAL".

In his short life, Kierkegaard wrote more than twenty-five books. After his death, his works slipped into obscurity.

He has been 'discovered' only in the 20th century, and has influenced both modern Protestant theology and the Existentialist philosophy (e.g. of **Heidegger**). **Jean-Paul Sartre** said of Kierkegaard: "I want to catch hold of him, and it is myself I catch".

Worldview XX: Kierkegaard

* * *

"To seek objectivity is to be in error."[175]

* * *

"Existing is a form of doing, not a form of thinking."

* * *

"People demand freedom of speech to make up for the freedom of thought which they avoid."

* * *

"Wherever there is a crowd there is untruth."

* * *

"Life can only be understood backwards; but it must be lived forwards."

* * *

"The tyrant dies and his rule is over; the martyr dies and his rule begins."

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"It is not true that the scientist goes after truth. It goes after him."

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[175] Compare with the cynical wit of **Oscar Wilde** (1856–1900): "It is only about things that do not interest one that one can give a really unbiased opinion, which is no doubt the reason why an unbiased opinion is always absolutely valueless."

Kierkegaard probably got this skeptical view from reading **Hume** (1711–1776) who held that no *moral* claim can ever be grounded in objective fact.

1843–1857 CE **Ernst Eduard Kummer** (1810–1893, Germany). Mathematician. Showed that the Fermat conjecture was true for all prime powers smaller than 100 except 37, 59 and 67. [First three '*irregular primes*'.] In his efforts to prove the conjecture, Kummer extended (1846) the notion of 'prime' numbers in the integers to general algebraic domains, preserving the existence of a unique factorization into prime factors. The prime factors in the more general domains are called 'ideal numbers'. In 1849 he extended the theory of Gaussian complex numbers.

1843–1857 CE **John James Waterstone** (1811–1883, Scotland). Physicist and engineer. First formulated the essential features of the kinetic theory of gases. Submitted to the Royal Society (1845) a speculative memoir on gases linking heat with molecular motion. In it he included a calculation of the ratio of specific heats at constant temperature and constant volume. The memoir was dismissed by the referees as '*nothing but nonsense*'. In 1892 it was reproduced in complete form by **Rayleigh** (1842–1919). However, many of Waterstone's key ideas had by then been published by **Clausius** and **Maxwell**. Waterstone's misfortune[176] resulted from the fact that the idea of *energy conservation* became accepted only in 1858. Thus the rejection of his work delayed progress by about 15 years.

Waterstone was born and educated in Edinburgh. He moved to London (1833) to do surveying for the railways, then took a job in the Hydrographers' Department of the Admiralty. Went to India (1839) as teacher of the East India Company's cadets in Bombay. He returned to Edinburgh (1857) to devote all his efforts to research. His work, however, was repeatedly rejected or ignored, causing him to withdraw from the scientific community.

Waterstone wrote other papers on gravitation, sound, capillarity, physiology, latent heat and various aspects of astronomy. He also estimated the temperature of the sun (1857).

1843–1858 CE **Haim Zelig Slonimsky** (1810–1904, Poland). Mathematician, astronomer and inventor. Made important contribution to the study of the Hebrew calendar (1852). Invented a novel calculating machine (1843) and developed a method of delivering simultaneously four messages via a telegraph wire. Befriended the German astronomers and mathematicians **Bessel**, **Crelle**, **Encke** and **Jacobi** and especially **Alexander von Humboldt** who remained his lifelong friend.

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[176] He did not even merit a mention in the *Britannica's* 11th edition (1910)!

Slonimsky was trained as a rabbinical Talmudic scholar up to his 18th year. After his marriage (1828), he self-educated for six years in the home of his father-in-law, publishing a number of books on various mathematical and astronomical subjects, and also mathematical papers in the Crelle Journal. His *Yesodei ha-Ibur* (Foundations of the Calendar, 1844) is still the seminal work in this field.

Slonimsky was born in Bialystok and later lived in Warsaw. The Russian government appointed him supervisor of the rabbinical academies (1862). In the same year he began publishing the Hebrew scientific weekly, *Hazefira*, which turned (1886) into daily newspaper. Slonimsky's grandchildren became prominent figures in the Polish and Russian literature.

1843–1865 CE **Claude Bernard** (1813–1878, France). Physiologist. Investigated chemical phenomena of *digestion*, discovering role of pancreas in digestion of fat and the glycogenic function of the liver; discovered regulation of blood supply by vasomotor nerves.

In an attempt to prove that animals could *synthesize* food materials in their bodies instead of having to obtain all nutrients from plant life, Bernard discovered that the liver could serve as a source of blood sugar (1843); in 1857 he indeed announced the isolation of *glycogen* from the liver[177]. Thus, the foundations for an understanding of *carbohydrate metabolism* had been laid, though the real comprehension of the reaction involved had to wait until the structure of the sugars had been worked out.

Bernard was a professor at the Sorbonne (1854), College de France (1855–1868), and the Musée d'Histoire Naturelle (1868–1878).

1843–1873 CE **Charles Hermite** (1822–1901, France). One of the eminent French mathematicians of the 19^{th} century. A professor at the Sorbonne (1869) and the teacher of **Picard** (1856–1911), **Borel** (1871–1956) and **Poincaré** (1854–1912).

Abel had proven in 1824 that the quintic equation cannot be solved by functions involving only rational operations and root extractions. One of Hermite's most surprising achievements (1858) was to show that this equation can be solved by *elliptic functions*. In 1873 he proved that e is transcendental[178].

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[177] *Glycogen* was independently discovered (1857) by **Viktor Hensen** (1835–1924, Germany), a medical student who worked under **Rudolf Virchow** (1821–1902, Germany).

[178] In a sense this is paradigmatic of all the discoveries of Hermite. By a slight adaptation of Hermite's proof, **Felix Lindemann** (1882) obtained the much more exciting transcendence of π. Thus, Lindemann, a mediocre mathemati-

Hermite was born in Dieuze, Lorraine, the sixth of seven children. His father, Ferdinand, a man of strong artistic inclination who had studied engineering, entrusted his draper's trade to his wife, Madeleine, in order to give full rein to his artistic bent. Around 1829 Charles' parents transferred their business to Nancy. They were not much interested in the education of their children, but Charles continued his studies in Paris; his mathematics professor was the same Richard who 15 years earlier had taught Évariste Galois. So, instead of seriously preparing for his examination Hermite, at the age of 17, read Euler, Gauss and Lagrange. He was thus admitted to the École Polytechnique in 1842 with the poor rank of 68. After a year's study, he was refused further study, because of a congenital defect of his right foot, which obliged him to use a cane.

At this time, Hermite resembled a Galois resurrected: Owing to the intervention of influential people the decision was reversed, but under conditions to which Hermite was reluctant to submit and he declined the paramount honor of graduating from the École Polytechnique, contenting himself with the career of a high school teacher. In 1847 he became acquainted with Jacobi's work on elliptic and hyperelliptic functions, and already in 1843, at the age of 20, he was able to generalize some of Abel's results, thus placing himself in the ranks of the first analysts. He communicated his discovery to **Jacobi**, who did not conceal his delight.

The association of Hermite with the École Polytechnique was resumed in 1848 and through the influence of **Pasteur** (1822–1895), a special position was created there for him. In 1869 he took over **J.M.C. Duhamel**'s chair as professor of analysis both at the École Polytechnique and the Sorbonne positions which he kept until 1876 and 1897 respectively. He was an honorary member of a great many academies and learned societies, and was awarded many decorations. His 70^{th} birthday gave scientific Europe the opportunity to pay homage in a way accorded very few mathematicians.

Hermite married the sister of **Joseph Bertrand** (1822–1900); one of his daughters married **Émile Picard** (1856–1941). Hermite was seriously ill with smallpox in 1856, and under **Cauchy**'s (1789–1857) influence became a devout Catholic. He studied Sanskrit and ancient Persian.

Throughout his life Hermite exerted a great scientific influence by his correspondence with other prominent mathematicians. If Hermite's work were

cian, became even more famous than Hermite — for a discovery for which Hermite had laid all the groundwork and that he had come within a gnat's eye of making. [The irrationality of π and e had previously been demonstrated by **Lambert** (1776).] Hermite also produced an 'artificial' new transcendental number $\sum_{n=0}^{\infty} 2^{-n!}$.

scrutinized more closely, one might find more instances of Hermitian preludes to important discoveries by others, since it was his habit to disseminate his knowledge lavishly in correspondence, in his courses, and in short notes. His correspondence with T.J. **Stieltjes**, for instance, consisted of at least 432 letters written by both between 1882 and 1894. Hermite's most important results have been so solidly incorporated into more general structures that they are rarely attributed to him.

Several of his purely mathematical discoveries had unexpected applications many years later in mathematical physics: *Hermitian forms* and *matrices* which he invented in connection with certain problems of number theory turned out to be crucial for **Heisenberg**'s 1925 formulation of quantum mechanics, and *Hermite polynomials* and *functions* appear in the solution of **Schrödinger**'s wave equation for a harmonic oscillator, as well as in solutions of the *classical* wave equation representing narrow beams.

1843–1876 CE **George Gabriel Stokes** (1819–1903, Ireland and England). A British mathematician and physicist with an extraordinary combination of mathematical prowess and experimental skill. His contributions range from optics, acoustics, and hydrodynamics to viscous fluid problems (a unit of viscosity is named for him).

Stokes was born in Skreen, Ireland. He entered Bristol College at 16 and matriculated at Pembroke College, Cambridge, in 1837. He became a Fellow of Pembroke College in 1841 and in 1849 received the Lucasian Professorship of mathematics at Cambridge, held by **Airy** from 1826. Baron since 1889, member of parliament (1887–1892) and president of the Royal Society (1885–1890).

In 1843 he gave a new deduction of the general equation of viscous flow (discovered by **Navier** in 1823; *Navier-Stokes equation*[179]). Anticipated the instability of laminar flow patterns. In 1847 he created the concept of *uniform convergence* of series. In 1849 he conceived the first mathematical model of a point source in an elastic solid ('luminiferous ether'), treating light as a transverse wave in the elastic ether.

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179 The *Navier-Stokes equation* governs the motion of *Newtonian fluids* (viscous fluids for which the shearing stress in linearly related to its rate of deformation).

The laws of conservation of mass, linear momentum, and angular momentum lead directly to the two basic field equations:

Stokes' theorem was discovered by William Thomson (Lord Kelvin) and communicated to his friend Stokes in a postscript to a letter of July 2, 1850. Stokes replied that the result was very elegant and new to him and that he had constructed his own proof. He never claimed it as his own or published a proof, but he did include a question on it in the Smith's Prize Examination for 1854 [a competitive examination given to the best mathematics students at Cambridge University]. One of the students who took the 1854 examination, and who tied for first place on it, was **James Clerk Maxwell** (1831–1879). Stokes' theorem is of critical importance in electromagnetic theory and in the formulation of Maxwell's equations.

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \boldsymbol{V}) = 0$$

(*equation of continuity*: $\rho(\boldsymbol{r}, t) =$ density, $\boldsymbol{V}(\boldsymbol{r}, t) =$ particle velocity),

$$\operatorname{div} \mathfrak{T} + \rho \boldsymbol{F} = \rho \frac{D\boldsymbol{V}}{Dt}$$

(*Euler's equation of motion* relative to an inertial frame,

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \boldsymbol{V} \cdot \boldsymbol{\nabla};$$

$\boldsymbol{F} =$ force per unit mass, $\mathfrak{T} =$ stress tensor).
The adequate stress tensor for isotropic linear viscous fluid (*Navier-Poisson law*) is derived on the basis of experimental evidence:

$$\mathfrak{T} = -p\mathfrak{I} + \left(\overline{\lambda} - \frac{2}{3}\eta\right) \mathfrak{I} \operatorname{div} \boldsymbol{V} + \eta(\nabla \boldsymbol{V} + \boldsymbol{V}\nabla),$$

where I is the unit dyadic, $\overline{\lambda}$ is the *bulk viscosity* and η the *shear viscosity*. Assuming uniform $\overline{\lambda}$, η, *a* substitution of the explicit form of $\mathfrak{T}$ in Euler's equation yields the *Navier-Stokes equation* (non-linear in $\boldsymbol{V}$):

$$\rho \frac{D\boldsymbol{V}}{Dt} = \rho \boldsymbol{F} - \operatorname{grad} p + \eta \nabla^2 \boldsymbol{V} + \left(\overline{\lambda} + \frac{1}{3}\eta\right) \operatorname{grad} \operatorname{div} \boldsymbol{V}.$$

We thus have 4 scalar equations in the 5 unknown functions $(\boldsymbol{V}, \rho, p)$. The *equation of state* $p = p(\rho)$ supplies the missing relation.

Introduced the *Stokes parameters* (1852), useful in the experimental determination of the state of polarization of a light beam. Derived the expression for the *drag force* on a sphere moving slowly (small 'Reynolds numbers') in a viscous fluid [$F = 6\pi a\eta u$; a = sphere's radius[180], u its velocity, η = dynamic viscosity. It was used by **Millikan** in his famous experiment to determine the charge of the electron (1910) and by **Einstein** in analyzing Brownian motion in external fields].

Stokes named and explained the phenomenon of *fluorescence* (1852).

Discovered the *Stokes phenomenon* (1857), namely — the discontinuity of the constants in the asymptotic expansion of integral functions. Stokes illustrated the change with the aid of Bessel functions whose orders are 0 and $\frac{1}{3}$, the latter being those associated with the **Airy** integral. On this discovery, **George Neville Watson** (1886–1965) remarked that "*the discovery was apparently one of those which are made at three o'clock in the morning*".

Stokes was first to derive an analytical expression for *group velocity*[181] (1876).

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[180] This law is still valid in the form $F = A\eta u$ for non-solid and non-spherical objects, where the parameter A depends on the shape of the body, and upon its physical state. For example, $A = 4\pi a$ (air bubble), $16a$ (disk, moving face-on), $\frac{32}{3}a$ (disk, moving edge-on), $12a$ [disk, moving at random. The 'addition law' is

$$\left(\frac{1}{A}\right)_{\text{random}} = \frac{1}{3}\left(\frac{1}{A_x} + \frac{1}{A_y} + \frac{1}{A_z}\right)].$$

Thus we have:

$$A = \frac{4\pi a}{\log_e \frac{2a}{b} - \frac{1}{2}}$$

(Ellipsoid, $a \gg b$, moving lengthwise), and

$$A = \frac{8\pi a}{\log_e \frac{2a}{b} + \frac{1}{2}}$$

(Ellipsoid, $a \gg b$, moving sidewise).

[181] Subsequently developed by **Rayleigh** (1877). It appears however that as early as 1839 **Hamilton** had made investigations into the velocity of advance of a finite train of waves in a dispersive medium, but his researches were only published in short abstracts and have been entirely overlooked until recently. Also in 1839, **George Green** derived the formula for the *phase velocity* of water waves in terms of wavelength ["*Note on the Motion of Waves in Canals*",

1843–1889 CE **Joseph-Louis-Francois Bertrand** (1822–1900, France). Mathematician. Known for his contributions to differential geometry, number theory and probability theory.

Conjectured (1845) that there is at least one prime number between n and $2n-2$ for $n>3$. This was proved by **Chebyshev** (1850). His book *Calcul des probabilités* (1889) contains what Poincaré later called, *Bertrand's paradox*[182] and *Bertrand's coin problem*[183].

1844 CE **Johann Martin Zacharias Dase** (1824–1861, Germany). A calculating prodigy who calculated π correctly to 200 decimal places in less than two months; using the formula

$$\frac{\pi}{4} = \arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{5}\right) + \arctan\left(\frac{1}{8}\right),$$

with a series expansion for each arctangent [the '*Gregory-Leibniz*' formula is not suitable for practical calculation of π, since one would need 100,000 terms to calculate π to 5 decimal places].

Dase gave exhibitions of his extraordinary calculating prowess in Germany, Austria and England. During an exhibition in Vienna in 1840 he made acquaintance with **Schultz von Strassnitzky** (1803–1852, Austria), who urged him to make use of his powers for the calculation of mathematical tables.

Dase calculated the natural logarithms of the first 1,005,000 numbers, each to 7 decimal places, in his spare time in 1844–1847, when employed by the Prussian Survey. On the recommendation of **Gauss**, the Hamburg Academy of Sciences agreed to assist him financially, for a preparation of table of factors

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Trans. Camb. Phil. Soc. **7**, 87–95]. Later extensions have originated in **Kelvin**'s *method of stationary-phase* (1887).

182 *Bertrand's paradox*: A chord is chosen randomly in a circle of radius r. What is the probability that the length X of the chord will be less than the radius r? The answer depends on the method for randomly choosing points to determine the chord. In this manner one is able to obtain *distinct answers* for the probability $P(X<r)$. The importance of the paradox lies in that it serves as a warning to all persons who adopt practical policies on the basis of theoretical solutions, without first establishing that the *assumptions* underlying the solutions are in good accord with the experimentally observed facts.

183 Of 3 identical boxes, one contains 2 gold coins, one contains a gold and a silver coin, and the third contains 2 silver coins; a box is selected at random and a coin taken from it. Given that the chosen coin is gold, what is the probability that the other coin in the selected box is also gold?

of all numbers from 7 to 10 million. He died in 1861, after he had finished about half of it.

Calculating Prodigies of the 18^{th} and 19^{th} centuries

Among the self-taught calculators who showed their power in youth[184] *were:* ***Jedediah Buxton*** *(1707–1772, England). With the exception of his power of dealing with large numbers, his mental faculties were of low order and he remained throughout his life a farm laborer. He could calculate* 2^{140}. *When asked later to square this number, he gave the answer* $2\frac{1}{2}$ *months later, and he said he had carried on the calculations at intervals during that period (he could not read or write). In 1754 he reached London and was examined by various members of the Royal Society. It was suggested that he counted by multiples of 60 and of 15 and thus reduced the multiplication to addition.*

Of billions, trillions, etc. he had never heard, and in order to represent the high numbers required in some of the questions posed to him, he invented a notation of his own, calling 10^{18} *a* tribe, *and* 10^{36} *a* cramp. *He could stop in the middle of a piece of mental calculation, take up other subjects, and after an interval of weeks, could resume the consideration of the problem.*

Zerah Colburn *(1804–1840, U.S.A.) showed extraordinary powers of mental calculation while still less than 6 years old, which were displayed in a tour of America and later in London (1812). He was born at Cabut, Vermont, the son of a small farmer. At the age of 8 he could calculate* 8^{16} *in a few seconds. He gave the answer to such questions so rapidly that the gentleman who was taking them down was obliged to ask him to repeat them more slowly. His power of factorizing numbers less than a million was exceptional. In 1814, his English and American friends raised money for his education. With education, his calculating powers fell off.*

George Parker Bidder *(1806–1878, England) had mental capabilities similar to those of Colburn. He could calculate the square root of* 119,550,669,121 *in 30 seconds. Bidder later graduated from the University*

[184] Excluding *educated* prodigies who channeled their energy into rational mathematics, such as **Wallis**, **Ampère**, **Gauss**, **Ramanujan** and others.

of Edinburgh as a civil engineer and rose to high distinction. He retained his power of rapid mental calculation to the end of his life. Other members of his family have also shown exceptional powers of a similar kind as well as an extraordinary memory.

Jacques Inaudi *(1867–1939, Italy) was employed in his early years as a shepherd and was ignorant of reading and writing even in his teens. He could find integral roots of equations and could represent numbers less than* 10^5 *as a sum of four squares in a minute or two. He could mentally reproduce the sound of the declamation of the numbers' digits in his own voice, and was confused, rather than helped, if the numbers were shown him in writing. A number of 24 digits, having been read to him in 59 seconds, was memorized by its sound. His memory was excellent for numbers, but normal or subnormal for other things.*

Most of these calculating prodigies found it difficult or impossible to explain their methods. There are a few analyses by competent observers of the processes used, notably of Bidder on his own work and that of ***Darboux*** *of Inaudi.*

[Bidder performed multiplication, say of 397×173, *by forming the product* $(100 + 73 + 3)$ *and* $(300 + 90 + 7)$ *and adding up all the partial products. This method he used even when multiplying a 9 digit number by another 9 digit number.]*

Dase visualized recorded numbers, working in much the same way as with pencil and paper, while Bidder made no use of symbols and recorded successive results verbally in a sort of cinematographic way.

In multiplication of a number of n *digits, the strain on the mind varied approximately as* n^x *(measuring it by the time taken in answering the question) where* $x \sim 5$ *for Bidder and* $x \sim 3$ *for Dase.*

1844 CE **Hermann Günther Grassmann** (1809–1877, Germany). Mathematician. The harbinger of modern abstract algebra, especially vector and polyadic algebra.

In his book '*Ausdehnungslehre*' (1844) he developed a mathematical system involving a theoretical algebraic structure (calculus of linear extensions) on which geometry of any number of dimensions in affine and metric spaces could be based. He used invariant symbolism in which we now recognize *vector* and *tensor* (dyadic) notation. His "gap" products correspond to Gibbs' later 'indeterminate products'. Vector addition and subtraction, the two major kinds of vectorial products, vector differentiation and the elements of the linear vector function were all presented in forms either equivalent or nearly equivalent to their modern counterparts.

His '*Ausdehnungslehre*' includes the concept of hypercomplex numbers and their algebras, and Hamilton's algebra and matrix algebra are just special cases of his broader concepts — which embraces even the tensor algebra of general relativity.

Grassmann never attended a university mathematical lecture, and the great mathematicians of his day such as **Gauss**, **Kummer**, **Möbius**, **Hamilton** and others, failed to appreciate the greatness of his achievement. Thus, his ideas were overlooked in the main during his lifetime, and their importance was not recognized until the twentieth century. A later generation utilized parts of Grassmann's structure to build up vector and dyadic analysis for affine and metric spaces. All in all, the geometrical tradition of Hamilton and Grassmann led to the extremely useful vector algebras of classical mechanics and mathematical physics and eventually to tensor algebra and calculus.

Furthermore, Grassmann's non-commutative algebra was implemented in the matrix mechanics of quantum theory by **Werner Heisenberg** (1901–1976, Germany, 1925). It seems probable that Grassmann did not anticipate any such outcome for his extremely general 'geometric algebra'[185].

Grassmann was a high-school teacher in Stettin, Germany. His father, Justus Günther Grassmann once said: "*I would be happy if Hermann became a gardener or a craftsman*".

1844 CE **Gabriel Gustav Valentin** (1810–1883, Germany). Physiologist and physician. Discovered that pancreatic juice breaks down food in digestion. Contributed to the physiology of metabolism, the digestive tract and the nervous system.

Valentin was born to Jewish parents in Breslau. He became a professor of physiology in the University of Bern (1836).

1844–1859 CE **Carl Friedrich Wilhelm Ludwig** (1816–1895, Germany). Physiologist. One of the founders of physiochemical school of physiology. Helped create an autonomous discipline of physiology, with its research schools, professional societies and specialized journals.

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[185] In 1845 the French engineer **Adhémar, Comte de Saint-Venant** (1797–1866) exposed mathematical ideas similar to those which are present in the Grassmannian system. Among other things he defined the dyadic product of two vectors.

Ludwig was born in Witzenhausen, Hesse and studied at Marburg (though temporarily compelled to leave the university as a result of his political activities[186]), Erlangen, and the surgical school in Bamberg. He wasprofessor at Marburg (1846–1849), Zürich (1849–1855), Vienna (1855–1865), Leipzig (1865–1895).

Ludwig showed (1844) that the epithelium of the kidney tubules serve as a passive filter in urine production. Demonstrated the influence of nerves on the distribution of blood and on the secretion of the glands. Developed (1846) the *kymograph* — first physical device for a continuous recording of *blood pressure*[187] and other physiological or muscular processes. Proved (1854) that blood circulation is purely mechanical, such that no mysterious *vital processes* outside ordinary physics need to be involved. First to keep animal organs alive in vitro outside the body, which he achieved by pumping blood through them (1859). Devised medical instruments, useful especially in diagnostic technology. Energetic and influential teacher. Sought explanation of living processes in the paradigms of physics and chemistry (reductionism).

1844–1871 CE **Pierre-Ossian Bonnet** (1819–1892, France). Mathematician. Contributed to the differential geometry of curves and surfaces.[188] The field was opened by **Euler**, **Monge** and **Gauss**[189], but at the time was lacking a systematic treatment. Between 1840 and 1950, this challenge was taken

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186 He had a stormy student career: dueling left him with a heavily scarred lip.

187 Before the late 19^{th} century, blood pressure studies required sticking a tube directly into the arteries.

188 For further reading, see:

- Struik, D.J., *Lectures on Classical Differential Geometry*, Dover Publications: New York, 1988, 232 pp.
- Weatherburn, C.E., *Differential Geometry of Three Dimensions*, Cambridge University Press: Cambridge, 1939, 268 pp.
- Mishchenko, A. and A. Fomenko, *A Course of Differential Geometry and Topology*, Mir Publications: Moscow, 1988, 455 pp.
- Kreyszig, E., *Differential Geometry*, Dover Publications: New York, 1991, 352 pp.

189 *Gauss-Bonnet theorem* (Bonnet, 1848; known earlier to Gauss): If the Gaussian curvature K of a surface is continuous in a simply connected region A, bounded by a closed curve C composed of k smooth arcs making at the vertices exterior angles $\theta_1, \theta_2, \dots, \theta_k$, then:

$$\int_C K_g ds + \iint_A K dA = 2\pi - \sum_{i=1}^{k} \theta_i,$$

up by Bonnet and a group of younger French mathematicians, among them **Serret**, **Frenet**, **Bertrand** and **Puiseux**[190]. Bonnet demonstrated the invariance of the geodetic curvature under bending of the surface and stressed the usefulness of special coordinate systems, such as isometric and tangential coordinates.

Bonnet was born at Montpelier. He studied at the École Polytechnique and became a teacher there in 1844. He succeeded the astronomer LeVerrier to a Sorbonne chair in 1878.

1844–1890 CE **John Fowler** (1817–1898, England). Civil engineer. Pioneer in the construction of railway systems (including bridges and deep tunneling 'tubes') in England, Italy, Egypt and Sudan.

Fowler was born at Wadsley Hall, near Sheffield and flourished in an era of railway construction initiated by the Stepensons. In 1890 he completed the *Forth bridge* with his partner Benjamin Baker.

1845–1867 CE **Robert William Thomson** (1822–1873, Scotland). Engineer and inventor. Invented the *vulcanized rubber pneumatic tire*[191]. He patented his invention in 1845, and it was successfully tested in London. However, it was abandoned because it was thought too expensive for common use. The tire was re-invented by John Dunlop in 1888.

Thomson also patented the *fountain pen* (1849) and a steam traction engine (1867). He was born in Stonehaven, Scotland.

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where K_g represents the geodetic curvature of the arcs.
This theorem is an application of Green's theorem, known from the theory of line integrals and surface integrals in the plane, to the integral of the geodetic curvature.

[190] **Victor Alexandre Puiseux** (1820–1893, France). Mathematician. Furthered Cauchy's work on functions of complex variable. First to distinguish *poles*, *essential singularities* and *branch points*.

[191] It consisted of inflexible casings around an *inner tube* and was designed for vehicles pulled by *animals*. They were ousted after a few years by *solid tires*. The **Michelin brothers** (France) were the first to fit *motor vehicles* with tires with *inner tubes* (1895).

The Real Number System

The middle of the 19^{th} century saw the main thrust of the program for the arithmetization of analysis, which started with **d'Alembert** *(1754),* **Lagrange** *(1797) and* **Cauchy** *(1821). In the first stage of this process, the foundations of the real number system were rigorized. This was done in several different ways.*

One of the methods starts with the positive integers as undefined concepts, states some axioms concerning them, and then uses them to build a larger system consisting of the positive rational *numbers (quotient of positive numbers). The positive rational numbers, in turn, are used as a basis for constructing the positive* irrational *numbers (such as $\sqrt{3}$, π, etc.). The final step is the introduction of the negative real numbers and zero. The most difficult part of the whole process is the transition from the rational numbers to the irrational numbers.*

Although the need for irrational numbers was apparent to the ancient Greeks from their study of geometry, satisfactory methods for constructing irrational numbers from rational numbers were not introduced until late in the 19^{th} century.

Three different theories were outlined by **Karl Weierstrass** *(1815–1897),* **Georg Cantor** *(1845–1918) and* **Richard Dedekind** *(1831–1916). In 1889, the Italian mathematician* **Giuseppe Peano**[192] *(1858–1932) listed 5 axioms for the non-negative integers that could be used as the starting point of the whole construction:*

(1) *Zero is a number.*

(2) *If a is a number, the successor of a is a number.*

(3) *Zero is not a successor of a number.*

(4) *Two numbers of which the successors are equal are themselves equal.*

(5) *If a set S of numbers contains zero and also the successor of every number in S, then every number is in S (axiom of induction).*

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192 For further reading, see:

- Kline, M., *Mathematical Thought from Ancient to Modern Times*, Oxford University Press, 1990, 1211 pp.

Here, the postulational method attained a new height of precision, with no ambiguity of meaning and no concealed assumptions.

1845–1881 CE **Leopold Kronecker** (1823–1891, Germany). Distinguished mathematician and mathematical philosopher who planted the seed of *intuitionism*[193] in modern mathematics (although his views should not be confused with those of the present-day movement). In general Kronecker adhered to an arithmetical approach to algebra, via a postulational treatment of algebraic structures in terms of various number fields, and insisted that arithmetic and analysis be based on the whole numbers[194].

He categorically rejected the real number construction of his day on the ground that they cannot be achieved through finite processes only, and he

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[193] *Intuitionism* asserts that mathematics is built solely on *finite* constructive methods, employing a *finite* number of steps [e.g. a Galois field having a finite number of elements, as for example the field of integers modulo a prime number]. For the intuitionists, an entity whose existence is to be proved must be shown to be constructible in a finite number of steps.

Intuitionism stresses that mathematics has priority over logic; the objects of mathematics are constructed and operated upon in the mind by the mathematician, and it is impossible to define the properties of mathematical objects simply by establishing a number of axioms.

[194] *Kronecker's theorem in one dimension* (1884): If ν is irrational, α is arbitrary, and N and ϵ are positive, then there are integers n and p such that $n > N$ and $|n\nu - p - \alpha| < \epsilon$.

The theorem implies that there are integers n for which $n\nu$ is as near as we please to any number in $(0, 1)$. Alternatively, if ν is irrational, then the set of points $(n\nu)(\text{mod } 1)$ is *dense* in the interval $(0, 1)$. The theorem has a simple application to a plane geometrical-optics problem: a ray of light leaves a point inside a square, the sides of which are reflecting mirrors. What is the nature of the path? The equivalent geometrical theorem then states: Either the path is closed and periodic or it is dense in the square, passing arbitrarily near to every point in the square. A necessary and sufficient condition for *periodicity* is that the angle between a side of the square and the initial direction of the ray should have a rational tangent.

Kronecker himself proved his theorem for the more general case of a space of K dimensions. Later, **Harald Bohr** (1934) and **Georgii Fedoseevich Voronoi** (1868–1908) extended the theorem to spaces of infinite number of dimensions.

called for an arithmetical revolution that would ban the irrational numbers as nonexistent(!)[195]. In analysis, Kronecker openly criticized his contemporaries (especially **Weierstrass** and **Cantor**) in lectures and conversation. He believed that mathematics should deal only with finite numbers and a finite number of operations.

Kronecker made significant contributions to algebra: with **Kummer** and **Dedekind** he invented the modern theory of algebraic numbers. They did for higher arithmetic and the theory of algebraic equations what **Gauss**, **Lobachevsky** and **Riemann** did for geometry, in emancipating it from the narrow Euclidean dogma. Thus the creators of the theory of algebraic numbers have unified the separate theories of equations, algebraic curves and surfaces, and numbers into one firm supersystem based on a firm background of postulates.

In addition, Kronecker investigated the curvature of hypersurfaces in Euclidean space in n-dimensions (1869) and introduced (1881) his famous δ symbol. [$\delta_{ij} = 1$ if $i = j$; $\delta_{ij} = 0$ if $i \neq j$.]

He used the method of residues and the integral

$$\int \frac{e^{\frac{2\pi i}{m} z^2} dz}{1 - e^{2\pi i z}}$$

to render a simple proof of the *Gauss sum*

$$\sum_{s=0}^{m-1} e^{\frac{2\pi i}{m} s^2} = \frac{i + i^{1-m}}{i+1}\sqrt{m}.$$

Gauss himself devoted several painful years to determine the exact form of this sum. He later deduced from it the law of quadratic reciprocity for real primes.

Kronecker was born at Liegnitz, Prussia, of Jewish parents. At school he excelled in Greek, Latin, Hebrew, philosophy and mathematics. His mathematical talent appeared early under the expert guidance of Kummer. He acquired a broad liberal education in the Greek classics, painting and sculpture, and was an accomplished pianist and vocalist. Upon entering the University of Berlin in 1841 he came in contact with **Dirichlet**, **Jacobi**, **Weierstrass**, **Steiner** and **Eisenstein**. After taking his Ph.D. degree at the age of 22, he spent the years 1845–1853 managing a successful farming business.

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[195] His motto: "*Die ganze Zahl schuf der liebe Gott, alles übrige ist Menschwerk*" (God made the integers, men made the rest.)

Until the last decade of his life, Kronecker was a free man with obligations to no employer. From 1861 to 1883 he conducted regular courses at the University of Berlin, principally on his personal researches. In 1883 Kummer retired, and Kronecker succeeded him as ordinary professor.

Kronecker was of very small stature and extremely self-conscious about his height. In fact he attacked rigorously anyone whose mathematics he disapproved.He believed that the mathematical analysis of Weierstrass, based on his conception of irrationals as defined by infinite sequences of rationals, is all wrong. His finitism obviously embarrassed Weierstrass, but it was **Cantor** whom he wounded most seriously. Not only did Kronecker stand in the way of a position for Cantor in Berlin, but he sought to undermine the branch of mathematics that Cantor was creating. In 1884 Cantor suffered the first of the nervous breakdowns that were to recur throughout the remaining 33 years of his life.

Kronecker died of bronchial illness in Berlin. On his death bed he converted to Christianity.

Analysts at his time regarded his views as excessively metaphysical. After a temporary decline, his views reappeared in a new form in the works of **Poincaré** (1902–1906) and **Brouwer** (1908). This school of intuitionism has gradually strengthened with the passage of time. It won over some eminent present-day mathematicians, and has exerted great influence on all thinking concerning the foundations of mathematics.

1846 CE, Sept 23 **Johann Gottfried Gale** (1812–1910, Germany), astronomer, discovered the planet *Neptune* using predictions of its position by **Urbain LeVerrier** (1811–1877, France) and **John Couch Adams** (1819–1892, England).

The discovery of Neptune was a dramatic and spectacular achievement of mathematical astronomy. The very existence of this new member of the solar system, and its exact location, were demonstrated with pencil and paper; there was left to observers only the routine task of pointing their telescopes at the spot the mathematicians had marked.

1846 CE **Hugo von Mohl** (1805–1872, Germany). Botanist. Pioneer in the field of plant cell structure and physiology. His meticulous observations were the first attempts at *cytochemistry*; he identified a substance he called *protoplasm*. Mohl was the first person (1846) to use the term protoplasm in cell biology. He was the first to clearly explain *osmosis*.

Mohl was born in Stuttgart and studied medicine at Tübingen. Professor of Physiology at Bern (1832–1835) and of Botany at Tübingen (1835–1872).

1846 CE, Aug 10 The *Smithsonian Institution* founded by act of Congress in Washington D.C., with a $100,000 bequest from English chemist and mineralogist **James Smithson** (1765–1829). It is a federal chartered nonprofit corporation of scientific, educational, and cultural interests, established for the "*increase and diffusion of knowledge among men*". The Smithsonian conducts scientific research and publishes the results of studies, explorations, and investigations. It preserves and displays items representing aeronautics and space exploration, science and technology and natural history.

James Smithson (known until 1801 as James Louis Macie) was born in Paris, the illegitimate son of Hugh Smithson Percy, 1^{st} Duke of Northumberland, and Elizabeth Macie. The mineral *smithsonite* (calamine) is named after him.

1846 CE **Ernest Heinrich Weber** (1795–1878, Germany). Anatomist and physiologist. Founded *experimental psychology*, studying the *response* of humans to *physical stimuli*. Professor at Leipzig (1818–1878). Established the empirical law (1846):

"Noticeable differences in sensation occur when the increase of stimulus is a constant percentage of the stimulus itself".

If s is the magnitude of a measurable stimulus and (Δs) the increase just required for discrimination, then the ratio $r = \frac{\Delta s}{s}$ is constant. This applies to sound, light and taste reception[196].

Weber's law is at best a good approximation to reality. It fails when s is either too small or too large.

1846–1885 CE **Louis Pasteur** (1822–1895, France). Distinguished chemist, microbiologist and humanist. Pioneered in the field of modern stereochemistry in proving the existence of *optical isomers* (1846) and explaining the phenomenon.

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[196] *Example*: Assume a person holds a weight of 20 grams in his hand and that he is tested for the ability to distinguish between this weight and a slightly higher weight. Experiments show that a person is not able to discriminate between 20.5 g and 20 g, but that he finds 21 g to be heavier than 20 g most of the time. Now, a person cannot reliably discriminate between 41 g and 40 g. The detectable increase is 2 g instead of 1 g. It is found that, in general, discrimination is possible if s is increased by 5 percent of the original value. The following list of r-values may illustrate the sensitivity of human senses:

visible brightness	1:50	(s =light intensity)
tone	1:10	(s =sound intensity)
smell for rubber	1:8	(s =number of molecules)
taste for saline solution	1:4	(s =concentration of solution)

Discovered that fermentation of wine and beer is caused by *microorganisms* (yeast), not by chemical means, as previously supposed and proved that these organisms do not arise by spontaneous generation (1856–1871). Determined that excess fermentation could be eliminated by boiling the liquid or filtering the microorganisms (1856). Discovered the bacilli causing two distinct diseases of *silkworm* and found a method of preventing spread of the disease (1868), thus saving the silk industry in France. Extended his theory of fermentation to the germ theory of disease (1862–1885) and developed effective inoculation against several specific diseases: *chicken cholera* (1880), *anthrax* (1882) and *rabies* (1885). Identified the bacteria streptococcus (1879). Invented the process of milk '*pasteurization*' (1885).

Pasteur was born at Dôle, Franche-Comté. In 1838 he was sent with a friend to Paris, to a preparatory school for the École Normale. But being a nervous and excitable boy, his health broke down, and he returned home, to Arbois. He then continued his education at the Royal College of Besancon. His admittance to the École Normale was hampered by a low grade in chemistry (1842). This only increased his incentive for a serious study of chemistry. After his brilliant solution of the isomeric problem (1846) which had baffled the greatest chemists and physicists of the time, he was immediately appointed professor of chemistry at the faculty of science at Strasbourg, where he soon married Mlle Laurent.

In 1854 he was appointed professor of chemistry and dean of the Faculty of sciences at Lille. In his inaugural address he used significant words, the truth of which was soon manifested in his case: "*In the field of observation, chance only favors those who are prepared*".

The diseases of beer and wine had from time immemorial baffled all attempts at cure. Pasteur one day visited a brewery containing both sound and unsound beer. He examined the yeast under the microscope, and at once saw that the globules from the sound beer were nearly spherical, while those from the sour beer were elongated; and this led him to a discovery the consequence of which have revolutionized chemical as well as biological science. It was the beginning of a series of experimental researches in which he proved conclusively that the notion of spontaneous generation was a chimera.

Up to this time the phenomenon of fermentation was considered strange and obscure. Explanations had indeed been put forward by men as eminent as **Berzelius** and **Liebig**, but they lacked experimental foundation. This was given in the most complete degree by Pasteur. For he proved that various changes occurring in the several processes of fermentation are invariably due to the presence and growth of minute organisms.

In a series of delicate and intricate experiments Pasteur was able to show that when the atmospheric germs are absolutely excluded, no changes take place. The application of these facts to surgical operations has revolutionized surgical practice in Pasteur's own time.

Pasteur left Lille in 1857 to become the director of the École Normale in Paris (1857–1867). His discoveries on fermentation inaugurated a new era in the brewing and wine-making industries. Empiricism, hitherto the only guide, was replaced by exact scientific knowledge; the connection of each phenomenon with a controllable cause was established. Yet, in spite of rising fame and success, he still had to withstand grave opposition from powerful foes in the academy.

His powers of patient research and exact observation were about to be put to a severe test: An epidemic of a fatal character had ruined the French silk producers. Up to that time he had never seen a silkworm, and hesitated to attempt so difficult a task; but at the reiterated request of his friends he consented, and in June 1865 went to the south of France for the purpose of studying the disease on the spot. In September of the same year he was able to announce results which pointed to the means of securing immunity from the epidemic, thus bringing back prosperity to the silk trade of France.

In 1880 Pasteur attacked the problem of chicken cholera, an epidemic which destroyed 10 percent of the French fouls; after the application of inoculation the death-rate was reduced to below one percent.

Next came the successful attempt to deal with the fatal cattle scourge known as *anthrax*. Many million of sheep and oxen all over the world have been treated by Pasteur's method, and the rate of mortality reduced from 10 to less than one percent. It is estimated that the monetary value of these discoveries was sufficient to cover the whole cost of the war indemnity paid by France to Germany in 1870.

The most spectacular of Pasteur's anti-microbial wars was launched against the dread disease of *hydrophobia* in man and of *rabies* in animals. This was accomplished in spite of the fact that the virus causing the disease had not been identified. Here again, the method of inoculation proved to be successful. On the 14^{th} of November 1888, the 'Institut Pasteur' was founded. Thousands of people suffering from bites from rabid animals, from all lands, have been treated in this institute, and the death-rate from this disease has been reduced to less than one percent[197].

Pasteur brought to microbiology the spirit and logic of the exact methods of physics and chemistry. This enabled him to bring under the domain of scientific laws the phenomenon of disease. Rich in years and honors, but

[197] **Paul Muni** (1895–1967; born Muni Weisenfreund in Lemberg, Austria) played the character of Pasteur in the movie "*The Story of Louis Pasteur*" (1936).

simple and affectionate in his demeanor, this great benefactor of humanity passed quietly away near St. Cloud on the 28^{th} of September 1895.

In 1874 Pasteur said: "Life, as is known to us, is a direct result of the *asymmetry*[198] of the universe or of its indirect consequences. The universe is asymmetric."

Now, at that time, the only known asymmetry pertaining to this comment was that of optical isomers in the field of organic chemistry. From our present vantage point this reads as a prophetic statement because life, physics, matter and even the fabric of the vacuum which we inhabit, are known to stem from spontaneous breaking of a string of symmetries. e.g.: *time-reversal*, *electroweak gauge symmetry* and *chiral symmetry*.

In biology, the fundamental symmetry of the double helix molecule is a case in point.

Worldview XXI: Louis Pasteur

* * *

"Let me tell you the secret that has led me to my goal: my strength lies solely in my tenacity."

* * *

"Travailler, travailler toujours."

* * *

[198] *Symmetry* is the Greek word $\Sigma YM\text{-}METPIA$ = "the same measure".

"Blessed is he who carries within himself a god and an ideal and who obeys it — an ideal of art, of science, of gospel virtues. Therein lie the springs of great thoughts and great actions."

* * *

"In the field of observation, chance favors the prepared mind."

* * *

"Science owns no fatherland."

* * *

"Unfortunate are those scientists who have only clear thoughts in their heads!"

* * *

"There does not exist a category of science to which one can give the name applied science. There are science and the applications of science, bound together as the fruit of the tree which bears it."

* * *

At the inauguration of his institute (1888) he closed his oration with the following words:

"Two opposing laws seem to me now in contest. The one, a law of blood and death, opening out each day new modes of destruction, forces nations to be always ready for the battle. The other, a law of peace, work and health, whose only aim is to deliver man from the calamities that beset him. Which of these two laws will prevail, God only knows. But of this we may be sure, that science, in obeying the law of humanity, will always labor to enlarge the frontiers of life."

* * *

"Où en êtes-vous? Que faites-vous? Il faut travailler" (on his death-bed, to his devoted pupils, watching over him).

The Spontaneous Generation Controversy (340 BCE–1870 CE)

"Omne vivium ex Vivo."

(Latin proverb)

Although the theory of spontaneous generation (abiogenesis) can be traced back at least to the Ionian school (600 B.C.), it was Aristotle (384–322 B.C.) who presented the most complete arguments for and the clearest statement of this theory. In his "On the Origin of Animals", ***Aristotle*** *states not only that animals originate from other similar animals, but also that living things do arise and always have arisen from lifeless matter. Aristotle's theory of spontaneous generation was adopted by the Romans and Neo-Platonic philosophers and, through them, by the early fathers of the Christian Church. With only minor modifications, these philosophers' ideas on the origin of life, supported by the full force of Christian dogma, dominated the mind of mankind for more that 2000 years.*

According to this theory, a great variety of organisms could arise from lifeless matter. For example, worms, fireflies, and other insects arose from morning dew or from decaying slime and manure, and earthworms originated from soil, rainwater, and humus. Even higher forms of life could originate spontaneously according to Aristotle. Eels and other kinds of fish came from the wet ooze, sand, slime, and rotting seaweed; frogs and salamanders came from slime.

Rather than examining the claims of spontaneous generation more closely, Aristotle's followers concerned themselves with the production of even more remarkable recipes. Probably the most famous of these was ***van Helmont's*** *(1577–1644) recipe for mice. By placing a dirty shirt into a bin containing wheat germ and allowing it to stand 21 days, live mice could be obtained. Another example was the slightly more complicated but equally "foolproof" recipe for bees. By killing a young bullock with a knock on the head, burying him in a standing position with his horns sticking out of the ground, and finally sawing off his horns one month later, out will fly a swarm of bees.*

The more exact methods of observation that were developed during the seventeenth century soon led to a realization of the complex nature of the anatomy and life cycles of certain living organisms. Equipped with this better understanding of the complexity of living organisms, it became more difficult for some to accept the theory of spontaneous generation. This skepticism signaled the beginning of three centuries of heated controversy over a theory that had gone unchallenged for the previous 2000 years. What is more significant is that the controversy was to be resolved not by powerful arguments but by ingeniously designed, simple experiments.

The controversy went through four phases:

I. Redi (1688) Vs. Aristotelian school and Church dogma

Redi *was first to use carefully controlled experiments to test the theory of spontaneous generation. He put some meat in two jars. One he left open to air (the control); the other he covered securely with gauze. At that time it was well recognized that white worms would arise from decaying meat or fish. Sure enough, in a few weeks, the meat was infested with the white worms but only in the control jar which was not covered. This experiment was repeated several times, using either meat or fish, with the same result. On closer examination he noted that common houseflies went down into the meat in the open jar, later the white worms appeared, and then new flies. Redi reported that he had observed the flies deposit their eggs on the gauze; however, worms developed in the meat only when the eggs got to the meat. He therefore concluded from his observations that the white worms did not arise from the putrid meat. The worms developed from the eggs that the flies had deposited. The white worm then was the larva of the fly, and the meat served only as food for the developing insect.*

Redi's experiment provided the impetus for testing other well-established recipes. In all cases that were examined carefully, it was demonstrated that the living organism arose not by spontaneous generation, but from a parent. Thus it was shown that the theory of spontaneous generation was based on a combination of the weakness of the human eye and bits and snatches of information gathered by accidental observation. The early biologists had seen earthworms coming out of the soil and frogs emerging from the slime of pond water, but they had not been able to see the tiny eggs from which these organisms arose. Because their observations had not been systematic, they had not seen how the mice invaded the grain bin in search of food, so they thought that the grain produced the mice. Based on the more exact methods of observation, the evidence that supported the theory of spontaneous generation of animals and plants was largely demolished by the end of the seventeenth century.

II. Spallanzani Vs. Needham (1767–1768)

As soon as the discoveries of Leeuwenhoek[199] *became known, the proponents of spontaneous generation turned their attention to these microscopic organisms and suggested that surely they must have formed by spontaneous generation. Finally, experimental "proof" for this notion was published in 1748 by an Irish priest,* ***John Tuberville Needham*** *(1713–1781).*

Needham reported that he had taken mutton gravy fresh from the fire, transferred it to a flask, heated it to boiling, stoppered it tightly with a cork, and then set it aside. Despite boiling, the liquid became turbid in a few days. When examined under a microscope, it was teeming with microorganisms of all types. The experiments were repeated by and gained the support of the famous French naturalist, ***Georges Louis Le-clerc, Comte de Buffon*** *(1707–1788). Needham's demonstration of spontaneous generation was generally accepted as a great scientific achievement, and he was immediately*

[199] The development of *microscopy* started with **Janssen** (1590) and continued with **Hooke** (1660), **Leeuwenhoek** (1676) and **Zeiss** (1883). Just as the theory of the abiogenesis of higher organisms was being refuted, the controversy was reopened, more heated than ever, because of the discovery of microorganisms by Antony van Leeuwenhoek. Leeuwenhoek patiently improve his microscopes and developed his techniques of observation for 20 years before he reported any of his results.

elected into the Royal Society of England and the Academy of Sciences of Paris.

Meanwhile in Italy, Lazzaro Spallanzani (1729–1799) performed a series of brilliantly designed experiments of his own that refuted Needham's conclusions. Spallanzani found that if he boiled the food for one hour and hermetically sealed the flasks (by fusing the glass so that no gas could enter or escape), no microorganisms would appear in the flasks. If, however, he boiled the food for only a few minutes, or if he closed the flask with a cork, he obtained the same results that Needham reported. Thus he wrote that Needham's conclusions were invalid because (1) he had not heated the gravy hot enough or long enough to kill the microorganisms, and (2) he had not closed the flask sufficiently to prevent other microbes from entering.

Count Buffon and Father Needham immediately responded that, of course, Spallanzani did not generate microorganisms in his flasks because his extreme heating procedures destroyed the vegetative force *in the food and the* elasticity *of the air. Regarding Spallanzani's experiments, Needham wrote, "from the way he has treated and tortured his vegetable infusions, it is obvious that he has not only much weakened, and maybe even destroyed, the vegetative force of the infused substances, but also that he has completely degraded ... the small amount of air which was left in his vials. It is not surprising, thus, that his infusions did not show any sign of life."*

Rather than engage in theoretical arguments over the possible existence of these mystical forces, Spallanzani returned to the laboratory and performed another set of ingenious experiments. This time he heated the sealed flasks to boiling not for one hour, but for three hours, and even longer. If Needham was right, this treatment should certainly have destroyed the vegetative force. As Spallanzani had previously observed, nothing grew in these heated, sealed flasks. However, when the seal was broken and replaced with a cork, the broth soon became turbid with microbes. Since even three hours of boiling did not destroy anything in the food necessary for the production of microbes, Needham could no longer argue that he had killed the vegetative force by the heat treatment.

Spallanzani continued to perform experiments that led him to the conclusion that properly heated and hermetically sealed flasks containing broth would remain permanently lifeless. He was, however, unable to answer adequately the criticism that by sealing the flasks he had excluded the "vital forces" in the air that Needham claimed were also necessary ingredients for spontaneous generation. With the discovery of oxygen gas in 1774 and the realization that this gas is essential for the growth of most organisms, the possibility that spontaneous generation could occur, but only in the presence of air (oxygen), gained additional support.

III-1. Schwann Vs. Berzelius, Liebig and Wholer (1836–1839) – the fermentation controversy

The art of brewing was developed by trial and error over a 6000-year period and practiced without any understanding of the underlying principles. From long experience, the brewer learned the conditions, not the reasons, for success. Only with the advent of experimental science in the eighteenth and nineteenth centuries did man attempt to explain the mysteries of fermentation. Let us, then, from our vantage point in time, trace the observations, experiments, and debates from which evolved our present understanding of fermentation and biological catalysis.

For centuries, fermentation had a significance that was almost equivalent to what we would now call a chemical reaction, an error that probably arose from the vigorous bubbling seen during the process. The conviction that fermentation was strictly a chemical event gained further support during the early part of the nineteenth century, when French chemists led by ***Lavoisier*** *and* ***Gay-Lussac*** *determined that the alcoholic fermentation process could be expressed chemically by the following equation:*

$$\underset{\text{glucose}}{C_6H_{12}O_6} \rightarrow \underset{\text{ethyl alchohol}}{2C_2H_5OH} + \underset{\text{carbon dioxide}}{2CO_2}$$

It was, of course, known that yeast must be added to the wort in order to ensure a reproducible and rapid fermentation. The function of the yeast, according to the chemists, was merely to catalyze the process. All chemists agreed that fermentation was in principle no different from other catalyzed chemical reactions.

Then in 1837, the French physicist ***Charles Cagniard-Latour*** *and the German physiologist* ***Theodor Schwann*** *independently published studies that indicated yeast was a living microorganism. Prior to their publications, yeast was considered a proteinaceous chemical substance. The reason the two workers came up with the same observations at approximately the same time is most likely due to the production of better microscopes.*

It should be mentioned that one of the reasons it was difficult to ascertain whether or not yeast is living was because, like most other fungi, yeast is not motile. The organized cellular nature of yeast was discovered only when improved microscopes became available. Schwann and Cagniard-Latour also observed that alcoholic fermentation always began with the first appearance of yeast, progressed only with its multiplication, and ceased as soon as its growth stopped. Both scientists concluded that alcohol is a by-product of the growth process of yeast.

The biological theory of fermentation advanced by Cagniard-Latour and Schwann was immediately attacked by the leading chemists of the time. The eminent Swedish physical chemist ***Jons Jakob Berzelius*** *reviewed the two papers in his Jahresbericht for 1839 and concluded that microscopic evidence was of no value in what was obviously a purely chemical problem. According to Berzelius, nothing was living in yeast.*

This opinion was supported by ***Justus von Liebig*** *and* ***Friedrich Wöhler****. Liebig argued that:*

1. *Certain types of fermentation, such as the lactic acid (souring of milk) and acetic acid (formation of vinegar) fermentations, can occur in the complete absence of yeast.*
2. *Even if yeast is living, it is not necessary to conclude that the alcoholic fermentation is a biological process. The yeast is a remarkably unstable substance which, as a consequence of its own death and decomposition, catalyzes the splitting of sugar. Thus, fermentation is essentially a chemical change catalyzed by breakdown products of the yeast.*

Liebig's views were widely accepted, partly because of his powerful influence in the scientific world and partly because of a desire to avoid seeing an important chemical change relegated to the domain of biology. And so the stage was set – biology against chemistry – for the entrance of Louis Pasteur.

III-2. Pasteur Vs. Liebig and Berzelius (1857–1860)

In 1851, ***Pasteur*** *published his first paper on the topic of fermentation. The publication dealt with lactic acid fermentation, not alcoholic fermentation. Utilizing the finest microscopes of the time, Pasteur discovered that souring of milk was correlated with the growth of a microorganism, but one considerably smaller than the beer yeast. During the next few years, Pasteur extended these studies to other fermentative processes, such as the formation of butyric acid as butter turns rancid. In each case he was able to demonstrate the involvement of a specific and characteristic microorganism; alcoholic fermentation was always accompanied by yeasts, lactic acid fermentation by nonmotile bacteria, and butyric acid fermentation by motile rod-shaped bacteria. Thus, Pasteur not only disposed of one of the opposition's strongest arguments, but also provided powerful circumstantial evidence for the biological theory of fermentation.*

Now Pasteur was ready to attack the crucial problem, alcoholic fermentation. Liebig had argued that this fermentation was the result of the decay of *yeast; the proteinaceous material that is released during this decomposition catalyzes the splitting of sugar. Pasteur countered this argument by developing a protein-free medium for the growth of yeast. He found that yeast could grow in a medium composed of glucose, ammonium salts, and some incinerated yeast. If this medium is kept sterile, neither growth nor fermentation takes place. As soon as the medium is inoculated with even a trace of yeast, growth commences and fermentation ensues. The quantity of alcohol produced parallels the multiplication of the yeast. In this protein-free medium, Pasteur was able to show that fermentation takes place without the decomposition of yeast. In fact, the yeast synthesizes protein at the expense of the sugar and ammonium salts. Thus Pasteur concluded in 1860:*

"Fermentation is a biological process, and it is the subvisible organisms which cause the changes in the fermentation process. What's more, there are different kinds of microbes for each kind of fermentation. I am of the opinion that alcoholic fermentation never occurs without simultaneous organization, development and multiplication of cells, or continued life of the cells already formed. The results expressed in this memoir seem to me to be completely opposed to the opinion of Liebig and Berzelius."

Pasteur argued effectively, and more important, all the data were on his side. Thus the vitalistic theory of fermentation predominated until 1897, when an accidental discovery by **Eduard Buchner** *(1860–1917) demonstrated that the alcoholic fermentation of sugars is due to action of* enzymes *contained in the yeast.*

The controversy was thus finally resolved and the door was thrown open to modern biochemistry.

IV. Pasteur and Tyndall Vs. Pouchet (1859–1885)

The spontaneous generation controversy was brought to a crisis in 1859 when **Felix Archimède Pouchet** *(1800–1872), a distinguished scientist and director of the Museum of Natural History in Rouen, France, reported his experiments on spontaneous generation. Pouchet claimed to have accomplished spontaneous generation using hermetically sealed flasks and pure oxygen gas. These experiments, he argued, demonstrated that "animals and plants could be generated in a medium absolutely free from atmospheric air and in which therefore no germ of organic bodies could have been brought by air."*

The impact of Pouchet's experiments on his contemporaries was so great that the French Academy of Sciences offered the Alhumpert Prize in 1860 for *exact and convincing experiments that would end this controversy once and for all. Pasteur first set out to demonstrate that air could contain numerous microorganisms. From his microscopic observation, Pasteur concluded that there are large numbers of organized bodies suspended in the atmosphere. Furthermore, some of these organized bodies are indistinguishable by shape, size, and structure from microorganisms found in contaminated broths. Later he showed that these organized bodies that collected on the cotton fibers not only looked like microorganisms, but when placed in a sterile broth were capable of growth!*

Pasteur's second series of experiments provided further circumstantial evidence that it was the microbes on floating dust particles and not the so-called vital forces that were responsible for sterilized broth's becoming contaminated. In these experiments, Pasteur carried sterile-sealed flasks to a wide variety of locations in France. At the various sites, he would break the seal, allowing air to enter the flask. The flask was immediately resealed and brought back to Paris for incubation. The conclusion from these numerous experiments was that where considerable dust existed, all the flasks would become turbid. For example, if he opened sterile flasks in the city, even for a brief period, they all became turbid, whereas in the mountainous regions, especially at high altitudes, a large proportion of the flasks remained sterile.

His third and most conclusive experiment utilized the now famous swan-neck flask. As a result of the experiments described, Pasteur hypothesized that the source of contamination was dust. If true, then it should be possible to keep a broth sterile even in the presence of air as long as the dust is kept out. In order to test this hypothesis. Pasteur constructed several bent-neck flasks. After placing broth into the flask, he boiled the liquid for a few minutes, driving the air from the orifice of the flask. As the flask cooled, fresh air entered the flask. Despite the fact that the broth was in contact with the gases of the air, the fluid in the swan-neck flask always remained sterile. Pasteur reasoned correctly that the dust particles that entered the flask were absorbed onto the walls of the neck and never penetrated into the liquid. As an experimental control, Pasteur demonstrated that nothing was wrong with the broth. If he broke the neck off the flask or tipped liquid into the neck (in both cases dust would enter the broth), the fluid soon became turbid with microorganisms.

With these simple, ingenious experiments, Pasteur not only overcame the criticism that air was necessary for spontaneous generation but he was also able to explain satisfactorily many of the sources (dust) of the contradictory findings of other investigators. Although Pasteur's conclusions gained wide support in both the scientific and the lay communities, they did not convince all the proponents of spontaneous generation.

Pouchet and his followers continued to publish reports of spontaneous generation. They claimed their techniques were as rigorous as those of Pasteur. Where Pasteur failed to obtain spontaneous generation they succeeded in every case. For example. they carefully opened 100 flasks at the edge of the Maladetta Glacier in the Pyrenees Mountains at an elevation of 10,850 feet. In this region which Pasteur had found to be almost dust free, all 100 of Pouchet's flasks became turbid after a brief exposure to the air. Even when Pouchet used swan-neck flasks, there was growth.

To Pasteur, this disagreement no longer revolved around the interpretation of experiments; rather, either Pouchet was lying or his techniques were faulty, Pasteur had complete faith in his own procedures and results and had no respect for those of his opponents. Thus he challenged Pouchet to a contest in which both of them would repeat their experiments in front of their esteemed colleagues of the Academy of Science. Pouchet accepted the challenge with the added statement, "If a single one of our flasks remains unaltered, we shall loyally acknowledge our defeat."

A date was set, and the place was to be the laboratory in the Museum of Natural History at the Jardin des Plantes, Paris[200]*. Pasteur arrived early with the necessary apparatus for demonstrating his techniques. Newspaper photographers and reporters were also on hand for this event of great public interest. However, Pouchet did not show up, and Pasteur won by default. It is difficult to ascertain whether Pouchet was intimidated by Pasteur's confidence or, as he later stated, he refused to take part in the "circus" atmosphere that Pasteur had created, and that their scientific findings should instead be reported in the reputable scientific journals. At any rate, in Pouchet's absence, Pasteur repeated his experiments in front of the referees with the same results he had previously obtained. As far as the scientific community was concerned, the matter was settled*[201]*. The law* Omne vivium ex vivo *(All life from life) also applied to microorganisms.*

In retrospect, however, the most ironic aspect of this extraordinary contest was not that Pouchet failed to show up, but rather that if he had appeared, he would have won! *Pouchet's experiments are reproducible. Pouchet performed his experiments in the following manner: He filled swan-neck flasks with a*

[200] **Henri Milne-Edwards** (1800–1885), a French naturalist and zoologist (then a professor at the Museum and from 1864, its director) lent political and scientific support to Pasteur during the Pasteur-Pouchet debate. He wrote important works on crustaceans, mollusks, and corals and wrote a major opus on comparative anatomy and physiology.

[201] Yet, the Pasteur-Pouchet debate had a chilling effect on French evolutionary research for decades.

broth made from hay, boiled them for one hour, and then allowed the flasks to cool. He obtained growth in every flask. Pasteur's experiments differed in only two respects. Pasteur used a mixture of sugar and yeast extract for broth and boiled it for just a few minutes. Pasteur never obtained growth in his swan-neck flasks. The reason for their contradictory results was not understood until 1877, 17 years later.

Mainly because of the careful work of the English physicist ***Tyndall*** *(1820–1893), Pouchet's experiments could be explained without invoking spontaneous generation. Tyndall found that foods vary considerably in the length of boiling time required to sterilize them. For example, the yeast extract and sugar broth of Pasteur could be sterilized with just a few minutes of boiling, whereas the hay medium of Pouchet required heating for several hours to accomplish sterilization. Tyndall postulated that certain microorganisms can exist in heat-resistant forms, which are now referred to as spores. Furthermore, studies by Tyndall and the French bacteriologist* ***Ferdinand Cohen*** *revealed that hay media contain a large number of such spores. Thus the contradictory results of Pasteur and Pouchet were due to differences in the broths they used.*

Tyndall went on to demonstrate that nutrient medium containing spores can be sterilized easily by boiling for one-half hour on three successive days. This procedure of discontinuous heating, now called Tyndallization, *works as follows: The first heating kills all the cells that are not spores and induces the spores to germinate (in the process of germination, the spores lose their heat resistance as they begin to grow); on the second day, the spores have germinated and are thus susceptible to the heating. The third day heating "catches" any late germinating spores. Thus, with the publication of Tyndall's work, all the evidence that supported the theory of spontaneous generation was destroyed. Since that time, there has been no serious attempt to revive this theory.*

It should be pointed out, however, that by its very nature, the theory of spontaneous generation cannot be disproved. *One can always argue that the conditions necessary for spontaneous generation have not yet been discovered. Pasteur was well aware of the difficulty of a negative proof, and in his concluding remarks on the controversy, he merely showed that spontaneous generation had never been demonstrated.*

There is no known circumstance in which it can be affirmed that microscopic beings came into the world without germs, without parents similar to themselves. Those who affirm it have been duped by illusions, by ill-conducted experiments, and by errors that they either did not perceive, or did not know how to avoid.

1847 CE **Augustus De Morgan** (1806[202]–1871, England). Mathematician and logician, a contemporary of **Boole**. Laid the foundation of modern *symbolic logic* and developed new technology for logical expressions. Formulated *De Morgan's laws*. Introduced and vigorously defined the term *mathematical induction*. He endeavored to reconcile mathematics and logic, but compared with Boole, his impact on modern mathematics and its applications is small[203], and he is remembered mainly as a logical reformer. He is most noteworthy as the founder of the *logic of relations* and as a developer of the *algebra of logic* which reconstructed the logic of Aristotle upon a mathematical basis.

De Morgan was born in India, and taught at University College in London during 1836–1866. Although a convinced theist, he never joined a religious congregation. He renounced his professorship in 1866 when a colleague was denied a chair at University College because he was a unitarian.

The Basic Ideas of Topology

I. Polyhedra and surfaces[204]

A simple polyhedron is a body enclosed by faces, all of which are plane polygons (some examples of polyhedra are: pyramid, prism, frustum). It has

[202] De Morgan was always interested in odd numerical facts; thus in 1849, he noticed that he had the distinction of being x years old in the year x^2 ($x = 43$).

[203] Nevertheless, he shall be remembered in mathematics proper due to his discovery of the summation formula:

$$\sum_{n=1}^{N} \frac{x^{2^{n-1}}}{x^{2^n} - 1} = \frac{1}{x-1} - \frac{1}{x^{2^N} - 1} \qquad (x \neq 1).$$

[204] For further reading, see:

- Cundy, H.M., *Mathematical Models*, Oxford University Press, 1961, 286 pp.
- Coxeter, H.S.M., *Regular Polytopes*, Dover, 1973, 321 pp.
- Fauvel, T. et al (eds), *Möbius and his band*, Oxford University Press, 1993, 172 pp.

no holes, and can be continuously deformed into a sphere. A convex *polyhedron*[205] *is said to be* regular *if its faces are regular and congruent polygons (e.g. cube, tetrahedron). The study of polyhedra held a central place in Greek geometry, which already recognized most of their salient geometrical features. Greek geometers correctly concluded that the only polygons that can occur as faces of a regular polyhedron are the* regular polygons *having 3, 4 or 5 sides, bringing the total number of possible regular polyhedra to five.*

Now, all five of these possible forms actually exist. They were well known as early as **Plato** *(ca 390 BCE), and he gave them a very important place in his Theory of Ideas, which is why they are often known as the "Platonic Solids"*[206]*. The most important data on the regular polyhedra are given in Table 4.3 (L = length of edge, R = radius of circumsphere).*

While the sphere encloses the most volume *of all shape having a given surface area, the tetrahedron, of all polyhedra, encloses the* least volume *with a given surface area [this ratio is equal to $(\frac{1}{12}a^3\sqrt{2})/a^2\sqrt{3} = \frac{a}{12}\sqrt{\frac{2}{3}}$, where a is the side length]. Table 4.3 suggests that for* simple *polyhedra $V - E + F = 2$, a fact first stated by* **Descartes** *(1635), proved incompletely by* **Euler** *(1751)*

- Henle, M., *A Combinatorial Introduction to Topology*, Dover: New York, 1994, 310 pp.
- Flegg, H.G., *From Geometry to Topology*, Dover: New York, 2001, 186 pp.

[205] The designation '*convex*' applies to every polyhedron that is entirely on one side of each of its faces, so that it can be set on a flat table top with any face down. Although convexity is *not* a topological property it *implies* a topological property, since every convex polyhedron is necessarily simple.
There is a peculiar difference between the convex and the non-convex polyhedra: whereas every closed convex polyhedron is rigid, there are closed non-convex polyhedra whose faces can be moved relative to each other.

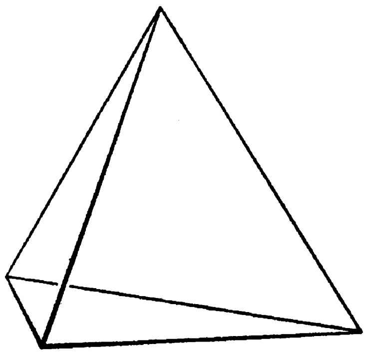
[206] It seems probable that **Pythagoras** (c. 540 BCE) brought the knowledge of the cube, tetrahedron and octahedron from Egypt, but the icosahedron and the dodecahedron have been developed in his own school. He seems to have known that all five polyhedra can be inscribed in a sphere. These solids played an important part in Pythagorean cosmology, symbolizing the five elements: *fire* (tetrahedron), *air* (octahedron), *water* (icosahedron), *earth* (cube), *universe or earth* (dodecahedron). The Pythagoreans passed on the study of these solids to the school of Plato. Euclid discusses them in the 13th book of his *Elements*, where he proves that no other regular bodies are possible, and shows how to inscribe them in a sphere. The latter problem received the attention of the Arabian astronomer Abu al-Wafa (10th century CE), who solved it with a single opening of the compass.

for convex polyhedra, and proved generally by **Cauchy** *(1811). It may have been known to* **Archimedes** *(ca 225 BCE), although the Greeks usually associated geometrical properties with measurements and not with mere counting.*

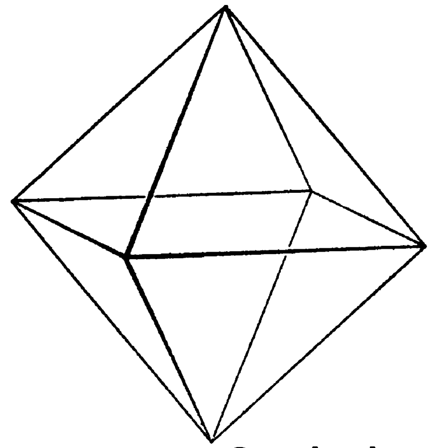
We have extant specimens of icosahedral dice that date from about the Ptolemaic period in Egypt. There are also a number of interesting ancient Celtic bronze models of the regular dodecahedron still extant in various museums. There was probably some mystic or religious significance attached to these forms. Since a stone dodecahedron found in northern Italy dates back to a prehistoric period, it is possible that the Celtic people received their idea from the region south of the Alps, and it is also possible that this form was already known in Italy when the Pythagoreans began to develop their teachings in Crotona.

Table 4.3: Regular polyhedra

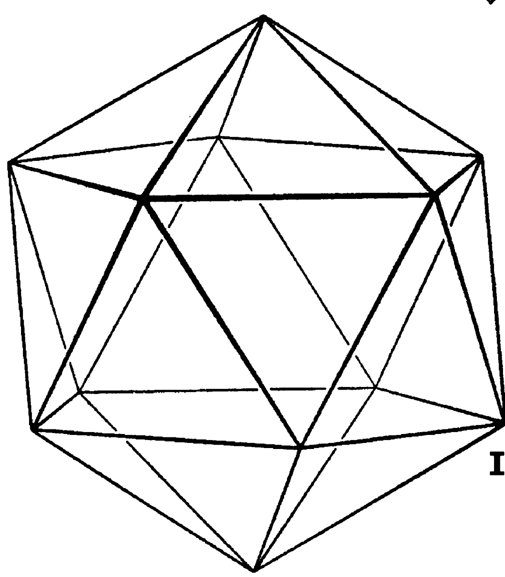
Name of Polyhedron	Polygons forming the faces	Number of Vertices V	Number of Edges E	Number of Faces F	Number of Faces meeting at a vertex	$\frac{R}{L}$
Tetrahedron	*Triangles*	*4*	*6*	*4*	*3*	$\frac{\sqrt{6}}{4}$
Octahedron	*Triangles*	*6*	*12*	*8*	*4*	$\frac{1}{\sqrt{2}}$
Icosahedron	*Triangles*	*12*	*30*	*20*	*5*	$\frac{1}{2}\sqrt{\frac{5+\sqrt{5}}{2}}$
Cube (Hexahedron)	*Squares*	*8*	*12*	*6*	*3*	$\frac{\sqrt{3}}{2}$
Dodecahedron	*Pentagons*	*20*	*30*	*12*	*3*	$\frac{1}{2}\sqrt{\frac{5+3\sqrt{5}}{2}}$



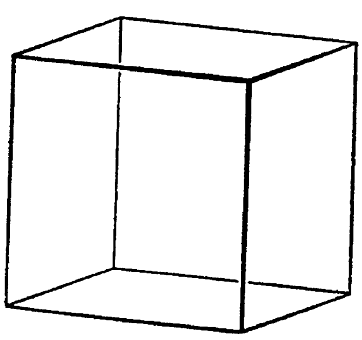
Tetrahedron



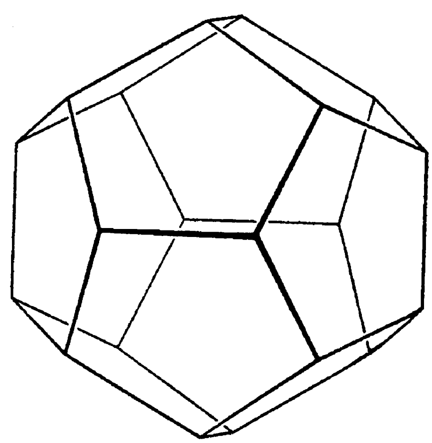
Octahedron



Icosahedron



Cube



Dodecahedron

The five regular polyhedrons attracted attention in the Middle Ages chiefly on the part of astrologers. At the close of this period, however, they were carefully studied by various mathematicians. Prominent among the latter was ***Pietro Franceschi****, whose work* De Corporibus Regularibus *(c. 1475) was the first to treat the subject with any degree of thoroughness. Following the custom of the time* ***Pacolli*** *(1509) made free use of the works of his contemporaries, and as part of his literary plunder he took considerable material from this work and embodied it in his* De Divina Proportione.

Albrecht Dürer*, the Nürnberg artist, showed how to construct the figures from a net in the way commonly set forth in modern works.*

Thus, Platonic and Archimedean polyhedra have sparked the imagination of creative individuals from Euclid to Kepler to Buckminster Fuller[207]*. These polyhedra are rich in connections to the worlds of art, architecture, chemistry, biology, and mathematics. In the realm of life, the Platonic Solids present themselves in the form of microscopic organisms known as* radiolaria.

Three other groups of polyhedra drew the attention of mathematicians throughout the ages:

- Archimedean Solids: *characterized by having all their angles equal and all their faces regular polygons, not necessarily of the same species. Archimedes' own account of them is lost. Thirteen such solids exist mathematically, some realized in crystalline forms: truncated tetrahedron (8 faces); cuboctahedron (14); truncated cube (14); truncated octahedron (14); rhombicuboctahedron (26); icosidodecahedron (32); truncated icosahedron ($V = 60$, $E = 90$, $F = 32$); snub cube (38); rhombicosidodecahedron (62); snub dodecahedron (92). Recently, the truncated icosahedron showed up in chemistry as the molecule C_{60}, known as a* fullerene *(after* **Buckminster Fuller***).*

- Kepler-Poinsot Polyhedra *have as faces congruent regular polygons, and the angles at the vertices all equal, but their center is multiply enwrapped by the faces (convex polyhedra).*

[207] American engineer and inventor (1895–1983); among his numerous inventions is his *geodesic dome* structure (1947), based on 3-dimensional structural principles that were developed to achieve maximum span with a minimum material. His designs find parallels to such natural molecular geometries as the *tetrahedron* and the *truncated icosahedron* (C_{60}, named "Buckeyball" or "Fullerene" in his honor).

Fuller also built the geodetic dome at the American Pavilion in the 1970 World Fair in Montreal.

Four such solids exist: small stellated dodecahedron ($F = 12$, $V = 12$, $E = 30$)*; great dodecahedron* ($F = 12$, $V = 20$, $E = 30$)*; great icosahedron* ($F = 20$, $V = 12$, $E = 30$)*. They were described and studied by* ***Kepler*** *(1619),* ***Poinsot*** *(1810),* ***Cauchy*** *(1813) and* ***Cayley*** *(1859).*

- *Semi-regular Polyhedra: solids which have all their angles, faces, and edges equal, the faces* not *being regular polygons. Two such solids exist: rhombic dodecahedron, a common crystal form; and semi-regular triacontahedron.*

On the basis of Euler's formula it is easy to show that there are no more than five regular polyhedra. For suppose that a regular polyhedron has F faces, each of which is an n-sided regular polygon, and that r edges meet at each vertex. Counting edges by faces, we see that

$$nF = 2E;$$

for each edge belongs to two faces, and hence is counted twice in the product nF; but counting edges by vertices,

$$rV = 2E,$$

since each edge has two vertices. Hence from $V - E + F = 2$ we obtain the equation

$$\frac{2E}{n} + \frac{2E}{r} - E = 2$$

or

$$\frac{1}{n} + \frac{1}{r} = \frac{1}{2} + \frac{1}{E}.$$

We know to begin with that $n \geq 3$ and $r \geq 3$, since a polygon must have at least three sides, and at least three sides must meet at each polyhedral angle. But n and r cannot both be greater *than three, for then the left hand side of the last equation could not exceed $\frac{1}{2}$, which is impossible for any positive value of E. Therefore, let us see what values r may have when $n = 3$, and what values n may have when $r = 3$. The totality of polyhedra given by these two cases yields the number of possible regular polyhedra.*

For $n = 3$ the last equation becomes

$$\frac{1}{r} - \frac{1}{6} = \frac{1}{E};$$

r can thus equal 3, 4, or 5. (6, or any greater number, is obviously excluded, since $1/E$ is always positive.) For these values of r we get $E = 6, 12$, or 30,

corresponding respectively to the tetrahedron, octahedron, and icosahedron. Likewise, for $r = 3$ *we obtain the equation*

$$\frac{1}{n} - \frac{1}{6} = \frac{1}{E},$$

from which it follows that $n = 3, 4,$ *or* 5*, and* $E = 6, 12,$ *or* 30*, respectively. These values correspond respectively to the tetrahedron, cube, and dodecahedron.*

While Euler's formula is valid for simply-connected polyhedra (regular and truncated polyhedra, pyramids, prisms, cuboids, frustums, crystal-lattice unit cells of various kinds) which are all topological spheres*, it fails for solids with holes in them and non-convex star-polyhedra. Thus, Kepler (1619) described the small and great* stellated dodecahedra *with* $V = 12,\ F = 12,\ E = 30,\quad V - E + F = -6$ *and* **Lhuilier** *(1813) noticed that Euler's formula was wrong for certain families of solid bodies. For a solid with* g *holes Lhuilier showed that* $\quad V - E + F = 2 - 2g.$

Consider for example a non-simply-connected polyhedron such as the prismatic block, consisting of a regular parallelepiped with a hole having the form of a smaller parallelepiped with its sides parallel to the outer faces of the block. Introducing just enough extra edges and faces to render all faces simply-connected polygon interiors (rectangles and trapezoids), this polygon is seen to have $V = 16,\quad E = 32$ *and* $\quad F = 16$ *such that* $\quad V - E + F = 0.$ *This corresponds to Lhuilier's formula with* $g = 1$.

To understand the significance of the number g *and its role in the topological classification of surfaces*[208]*, we compare the surface of the sphere with that of a torus. Clearly, these two surfaces differ in a fundamental way: on the sphere, as in the plane, every simple closed curve separates the surface into two disconnected parts. But on the torus there exist closed curves that do not separate the surface into two parts — for example, the two* generator *circles on the torus surface. Furthermore, such a closed curve cannot be continuously shrunk to a point — whereas* any *closed curve on a* sphere *can be so shrunk. This difference between the sphere and the torus marks the two surfaces as belonging to two topologically distinct classes, because this shows that it is impossible to deform one into the other in a continuous way.*

Likewise, on a surface with two holes we can draw four *closed curves each of which does not separate the surface into disjoint components; these can be*

[208] For the time being, we consider only *two-sided* and *closed* surfaces — i.e., we assume the surface has no boundary and that an ant, walking on one of its two sides, can never reach the opposite side without puncturing the surface. A 2-sided surface is also known as an *oriented* surface.

chosen to be the four generator curves (two per hole). Furthermore, one can draw two (non-intersecting) closed curves that, drawn simultaneously, still do not separate the two-hole surface. The torus is always separated into two parts by any two non-intersecting closed curves. On the other hand, three closed non-intersecting curves always separate the surface with two holes.

These facts suggest that we define the genus of a (closed and 2-sided) surface as the largest number of non-intersecting simple closed curves that can be simultaneously drawn on the surface without separating it. The genus of the sphere is 0, that of the torus is 1, while that of a 2-holed doughnut is 2. A similar surface with g holes has the genus g. The genus is a topological property of a surface and thus remains the same if the surface is deformed. Conversely, it may be shown that if two closed 2-sided (oriented) surfaces have the same genus, then one may be continuously deformed into the other, so that the genus $g = 0, 1, 2, \dots$ of such a surface characterizes it completely from the topological point of view.

For example, the two-holed doughnut and the sphere with two "handles" are both closed surfaces of genus 2, and it is clear that either of these surfaces may be continuously deformed into the other. Since the doughnut with g holes, or its equivalent, the sphere with g handles, is of genus g, we may take either of these surfaces as the topological representative of all closed oriented surfaces of genus g.

Suppose that a surface S of genus g is divided into a number of regions (faces) by marking a number of vertices on S and joining them by curved arcs. As stated above, it has been shown that

$$V - E + F = 2 - 2g,$$

where V = number of vertices, E = number of arcs, and F = number of faces or regions[209]*. The topological invariant on the L.H.S. is usually denoted χ and is known as the Euler characteristic of the surface (this invariant admits a generalization to even-dimensional manifolds of dimension higher than two). We have already seen that for the sphere, $V - E + F = 2$, which agrees with the above equation, since $g = 0$ for the sphere.*

Another measure of non-simplicity which is used in the classification of surfaces will emerge from the following example. Consider two plane domains: the first of these, a, consists of all points interior to a circle, while the second, b, consists of all points contained between two concentric circles. Any closed

[209] An outline of the proof: S can be constructed from a particular partitioning of the sphere by *identifying* $2g$ distinct sphere faces pairwise. This reduces E and V by the same integer, and reduces F by $2g$, thus resulting in a reduction of $V - E + F$ by $2g$ from its sphere value (2), as claimed.

curve lying in the domain a can be continuously deformed or "shrunk" down to a single point within the domain. *A domain with this property is said to be* simply connected. *The domain b, however, is not simply connected. For example, a circle concentric with the two boundary circles and midway between them cannot be shrunk to a single point within the domain, since during this process the curve would necessarily sweep through the center of the circles, which is not a point of the domain. A domain which is not simply connected is said to be* multiply connected. *If the multiply connected domain b is cut along a radius, the resulting domain is simply connected.*

More generally, we can construct domains with two "holes". In order to convert this domain into a simply connected domain, two cuts are necessary. If $h-1$ *non-intersecting cuts from boundary to boundary are needed to convert a given multiply connected planar domain D into a simply connected domain, the domain D is said to be h-tuply connected. The degree of connectivity of a domain in the plane is an important topological invariant of the domain. The number h is called the* connectivity number *assigned to every surface. It extends also,* mutatis mutandis, *to 3-dimensional bodies.*

As an example, consider a closed, non-self-intersecting polygon (a chain) *consisting of edges of a polyhedron. If the* surface *of the polyhedron is divided into two separate parts by every such closed chain of edges, we assign the connectivity* $h = 1$ *to the polyhedron. Clearly, all simple polyhedra have connectivity 1, since the surface of the sphere is divided into two parts by every closed curve lying on it. Conversely, it is readily seen that all polyhedra with connectivity 1 can be continuously deformed into a sphere. Hence the simple polyhedra are also called* simply connected.

A polyhedron is said to have connectivity h if $h-1$ *is the greatest possible number of chains that, when simultaneously drawn, do not cut the surface in two. Since* $h-1=2$ *for the prismatic block, its connectivity is* $h=3$.

We thus set $h=1$ *for the sphere and* $h=3$ *for the torus. Surfaces of higher connectivity can be constructed by flattening a sphere made of a plastic material, cutting holes into it, and* identifying *(sewing together) each pair of stacked hole-boundary closed curves.*

We shall call such surfaces pretzels. *It can be proved that a pretzel with g holes (i.e. a g-handle surface) must have connectivity* $h=2g+1$.

On a general surface, the curves can be chosen more freely than on a polyhedra, where we restricted the choice to chains of edges. Various other definitions can be given for the connectivity of surfaces – for example, the following:

On a closed surface of connectivity h*, we can draw* $h - 1$ closed *curves without cutting the surface in two, but every system of* h *closed curves cuts the surface into at least two separate parts. On a closed surface of connectivity* $h = 2g + 1$ *there is at least one set of* g *closed, mutually non-intersecting curves – and no set of more than* g *such curves – having the property that the curves in the set do not cut the surface in two when drawn simultaneously.*

All the polyhedra and closed surfaces we have considered thus far had odd *connectivity numbers* h *and* even *Euler characteristics* ($\chi = 2 - 2g$)*, related by the formula* $\chi = 3 - h$*. If we extend both concepts to surfaces with boundaries (i.e. open) — with* χ *still defined as* $V - E + F$ *and* h *now defined as the maximal number of simultaneous cuts (along closed* or *boundary-to-boundary open curves) leaving the surface connected — the formula becomes*[210] $\chi = 2 - h$*. And for such surfaces,* χ *and* h *may be both even* or *both odd.*

The numbers χ*,* g *and* h *are all* topological invariants*. So is the orientability/non-orientability property, which we explain next.*

The question arises whether there are any closed *(boundary-less) surfaces at all with* even connectivities *or* odd χ values*; or whether there are boundary-less surfaces for which genus and connectivity are* not *related by* $h = 2g + 1$*. Indeed, such surfaces* do *exist and are called* one-sided *(or* non-orientable*) surfaces.*

Hitherto we have been dealing with "ordinary" surface, i.e. those having two sides. This restriction applied to closed surfaces like the sphere or the torus and to surfaces with boundary curves, such as the disc, a sphere with two holes (i.e. with two discs removed) – equivalent to a cylinder – or a torus from which a single disc has been removed.

The two sides of such a surface could be painted with different colors to distinguish them. If the surface is closed, the two colors never meet. If the surface has boundary curves, the two colors meet only along these curves. A bug crawling along such a surface and prevented from puncturing it or crossing boundary curves, if any exist, would always remain on the same side.

Möbius *made the surprising discovery that there exist surfaces with only* one *side. The simplest such surface is the so-called Möbius strip (Figure 2), formed by taking a long rectangular strip of paper and pasting its two ends*

[210] Also, the formula $h = 2g + 1$ does *not* always apply for a non-closed surface. For instance, a cylinder with g handles – equivalent to a g-handle sphere with two discs cut out – has $h = 2g + 2$; for $g = 0$ (a simple cylinder) $h = 2$, since it can be cut once ($1 = h - 1$) while maintaining connectedness — e.g. from boundary to boundary along the cylinder axis.

together after giving one end a half-twist. A bug crawling along this surface, keeping always to the middle of the strip, will return to its original position upside down and on the opposite side of the surface! The surface is thus indeed one-sided when considered globally*; only local portions of it can be said to have two sides. The Möbius strip also has but one edge, for its boundary consists of a single closed curve. The ordinary two-sided surface formed by pasting together the two ends of a rectangle without twisting has two distinct, disconnected closed boundary curves; topologically it is a cylinder (or a sphere missing two discs).*

If this surface is cut along a plane separating the two closed boundary-curves, it falls apart into two *such disjoint cylinder surfaces, each with a new closed-curve component to its boundary. Like the cylinder, the Möbius strip has a continuous family of closed curves in its interior, each having the property of not being continuously deformable to a single point. And, as in the case of the cylinder, all such curves of unit* winding-number *(i.e. consisting of a single component if the surface is cut back into the original rectangle) can be deformed into each other, and are thus topologically equivalent.*

However, unlike the cylinder, if the Möbius strip is cut along one of its non-shrinkable closed curves, we find that it remains in one piece[211]*. It is rare for anyone not familiar with the Möbius strip to predict this behavior, so contrary to one's intuition of what "should" occur. If the surface that results from cutting the Möbius strip along the middle is again cut along its middle, two separate but intertwined strips are formed.*

The connectivity of the Möbius strip is $h = 2$*, just as the untwisted open cylinder. It also may be characterized by means of another important topological concept which can be formulated as follows: Imagine every point of a given surface (with the exception of boundary points, if any) to be enclosed in a small closed curve that lies entirely on the surface. We then try to fix a certain sense (handedness) on each of these closed curves in such a way that any two curves that are sufficiently close together have the same sense. If such a consistent determination of sense of traversal is possible in this way, we call it an* orientation *of the surface and call the surface* orientable.

While all two-sided surfaces are orientable, one-sided surfaces are not. Thus the classification of surfaces into two-sided and one-sided surfaces is identical to the classification into orientable and non-orientable surfaces.

[211] The cut strip is in fact equivalent to a rectangular strip subjected to *two* half-twists before identifying its two (short) opposite sides — both half-twists being in the same sense. This strip is topologically equivalent to a cylinder, yet cannot be deformed into it without self-intersection if embedded in 3-D space ($\mathbb{R}^3$).

It is easy to see that a surface is non-orientable if and only if there exists on the surface some closed curve such that a continuous family of small oriented circles whose center traverses the curve will arrive at its starting point with its orientation reversed.

The Möbius strip is an open *one-sided surface and does not intersect itself. But it can be proven that all one-sided* closed *surfaces embedded in* $\mathbb{R}^3$ *(Euclidean 3-dimensional space) have self-intersections. However, the presence of curves of self-intersection need not represent a topological property in the sense that in some cases it can be transformed away by deformation, or eliminated by defining the surface* intrinsically *(without embedding it in a 3-D* $\mathbb{R}^3$ *space), or else by embedding it in an* $\mathbb{R}^n$ *space with* $n > 3$*. If this is not the case we say that the surface has* singular points *which are a topological property.*

This raises the question of whether there can exist any one-sided closed surface (2-D intrinsic manifold) that has no singular points. Such a surface was first constructed mathematically by Felix Klein, as follows. Consider an open tube (cylinder). A torus[212] *is obtained from it by bending the tube until the ends meet and then gluing (identifying) the boundary circles together. But the ends can be welded in a different way:*

Taking a tube with one end a little thinner than the other, we bend the thin end over and push it through the wall of the tube, molding it into a position where the two circles at the ends of the tube have nearby and concentric positions. We now expand the smaller circle and contract the larger one a little until they meet, and then join them together (Fig. 7). This does not create any singular points and gives us Klein's surface, also known as the Klein bottle. It is clear that the surface is one-sided and, in any $\mathbb{R}^3$ *embedding, intersects itself along a closed curve where the narrow end was pushed through the wall of the tube.*

The connectivity number of the Klein bottle is 3, like that of a torus. It can be shown that any closed, one-sided surface of genus g *is topologically*

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212 *Torus*: a surface (intrinsic or embedded in $\mathbb{R}^3$). The *intrinsic* torus is a rectangle with opposite ends identified without twists (Fig. 9(f)). An $\mathbb{R}^3$-embedded torus is generated by revolving a circle about a line (in its plane) that does not intersect the circle. One of its parametric representations in Gaussian surface coordinates (u, v) is

$$\boldsymbol{r}(u,v) = [(a + b\cos v)\cos u; \quad (a + b\cos v)\sin u; \quad b\sin v],$$
$$a > b > 0; \qquad 0 \le u < 2\pi, \quad 0 \le v < 2\pi.$$

a and b are the two radii of the $\mathbb{R}^3$ torus, while the coordinates u, v are azimuths along two generating circles.

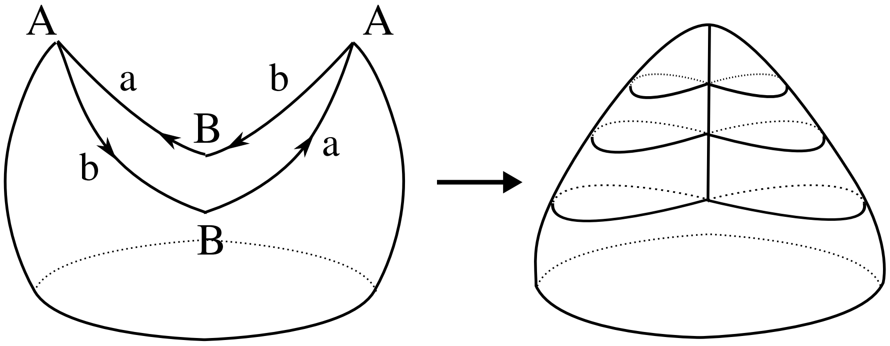


Fig. 1: Sewing up a cylinder to yield a representation of the Möbius strip as a topological sphere with cross-cap. The two copies of point A are identified, and similarly for B and the directed arcs a, b

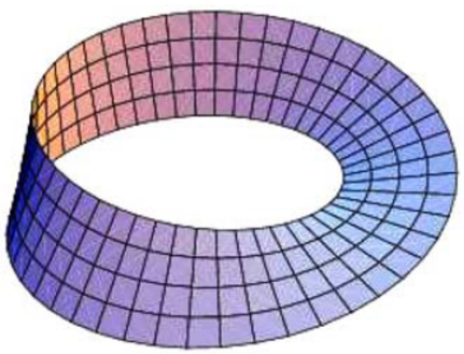


Fig. 2: An embedding of the Möbius strip in $\mathbb{R}^3$

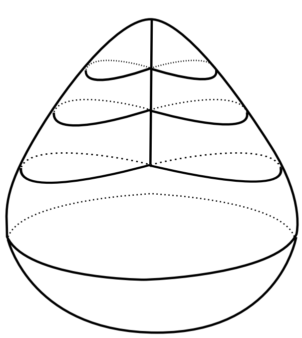


Fig. 3: The Real Projective Plane (sphere with one cross-cap)

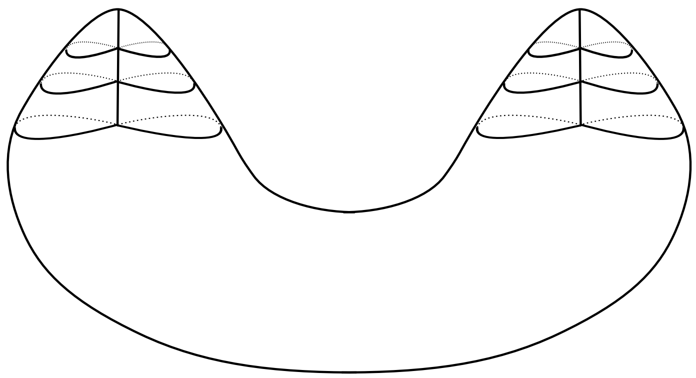


Fig. 4: Klein bottle represented as a topological sphere with two cross-caps

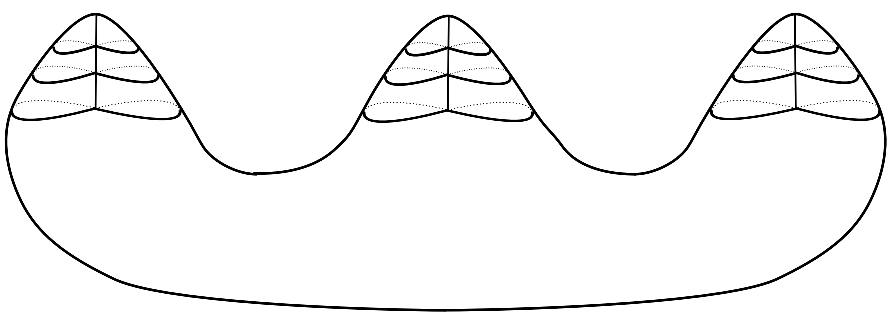


Fig. 5: The triple-crosscap surface (topological sphere with three cross-caps)

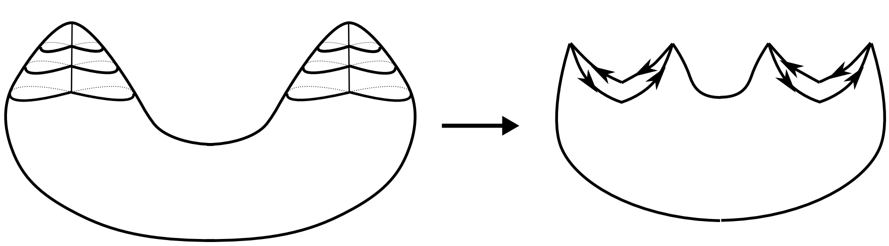


Fig. 6: Cutting the Klein bottle along to closed curves while maintaining connectivity

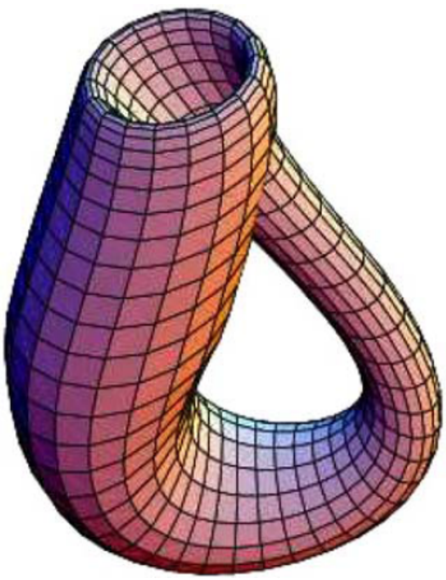
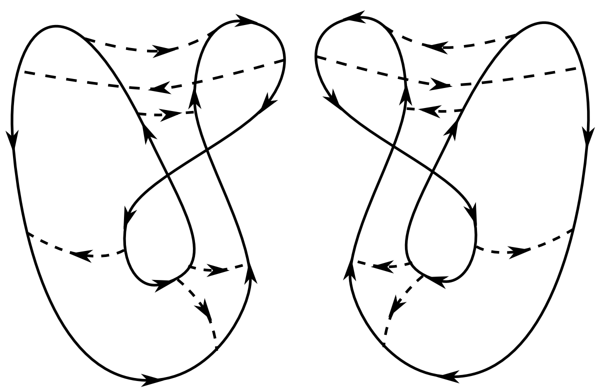
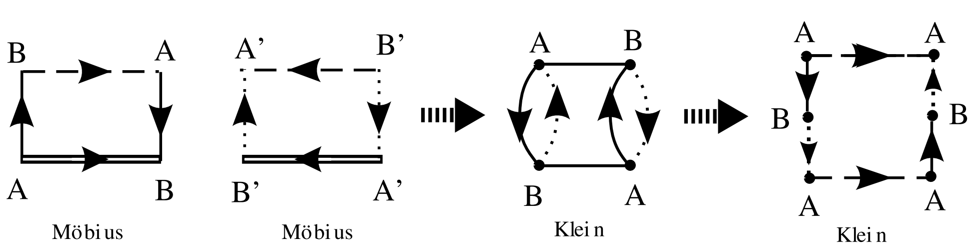


Fig. 7: A Klein bottle represented as a self-intersecting, nonsingular embedding (*immersion*) in $\mathbb{R}^3$



(a) Two Möbius strips with their boundaries sewn (identified) yield a Klein bottle.



(b)

Fig. 8

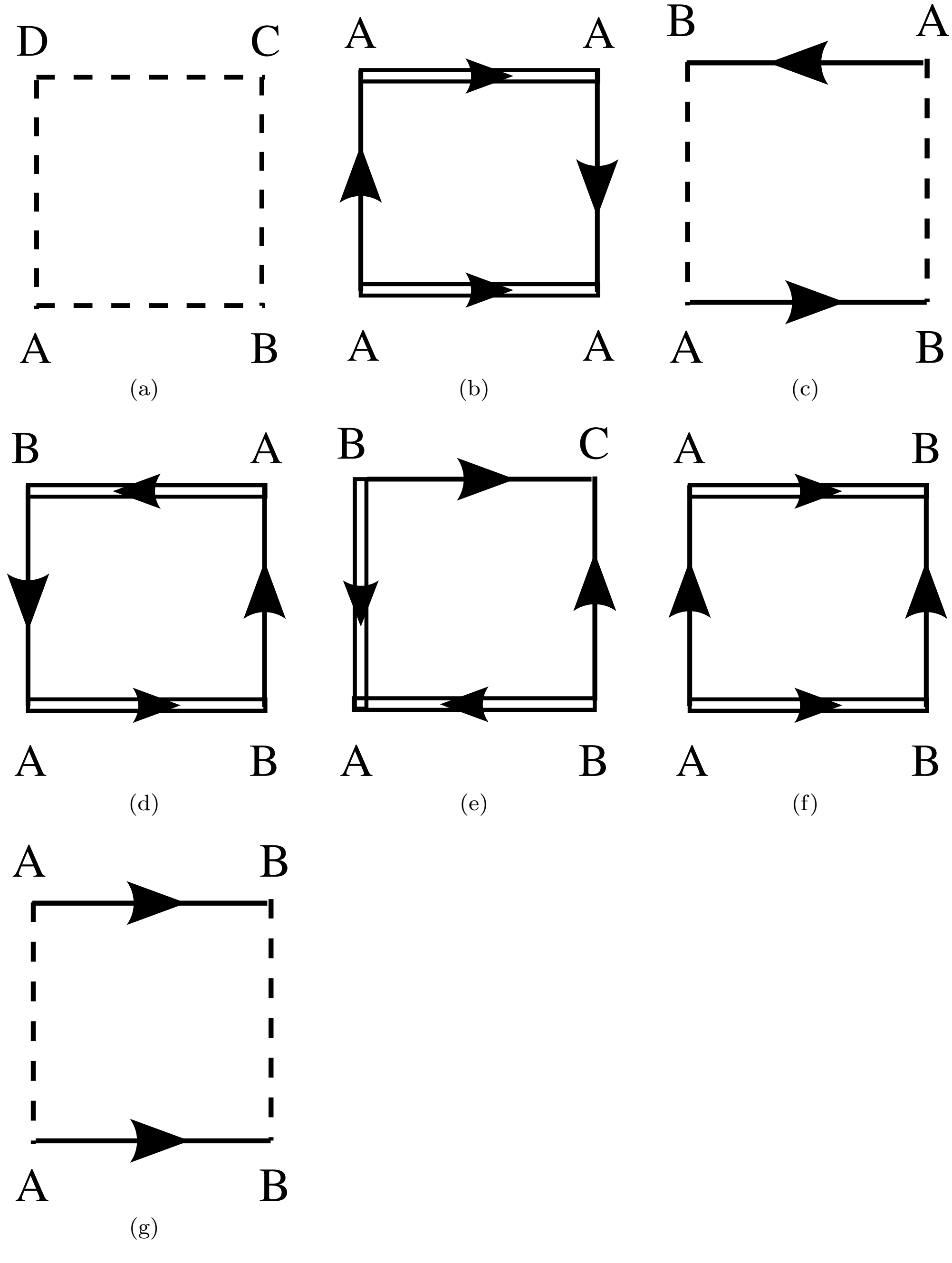


Fig. 9: Surfaces (open and closed, orientable and non-orientable) obtained from a rectangle by identifying (sewing) edges in various ways. Broken-line edges are not identified; arrows (single-line or double lines) are identified with same-type arrows (head with head and tail with tail). (a): topological disc; (b): Klein bottle; (c): Möbius strip; (d): real projective plane; (e): sphere; (f): torus; (g): cylinder (tube)

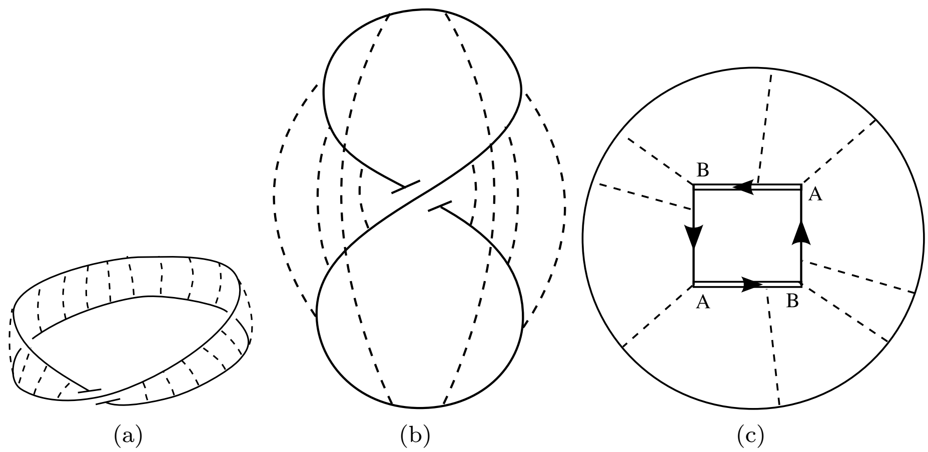


Fig. 10: Deforming the *standard* Möbius-strip representation into a *disk-with-crosscap*

equivalent to a sphere from which g discs have been removed and replaced by local topological constructs called cross-caps.[213]

[213] Cross-Caps: Modular building blocks for non-oriented surfaces.

The Möbius strip is the basic unit of non-oriented surfaces. It can be represented as described above: by half-twisting a rectangle and then identifying a pair of opposite sides (Figs. 2 and 9(c)). In the latter, the two corners labeled "A" are identified – "sewn" together – as are the two corners labeled "B"; and the two solid edges are identified in accordance with the arrow-indicated directions. But in this representation the Möbius strip has a complicated-looking boundary. Figures 1 and 10(a) to 10(c) show how to continuously deform this into the standard disc-with-crosscap representation. Unfolding the solid-line closed boundary of 10(a) yields 10(b) (the dotted lines indicate the Möbius-strip interior). Then, untwisting the boundary of 10(b) into a circle yields Fig. 10(c). In 10(c) it was necessary to cut the surface along some closed curve ABA to avoid intersections among the broken lines. This cut results in the rectangle depicted in 10(c), in which the two single-solid lines are identified along their arrows — as are the two double-solid lines. Re-sewing the cut ABA, as depicted in Fig. 1, results in a disc with one crosscap. This is a convenient representation of the Möbius strip, because its boundary is a simple curve (a circle if we wish), and also because any non-orientable surface (open or closed) is topologically equivalent to a sphere with some number of local discs removed, with some or all of these discs replaced by cross-caps. On the other hand — as clearly seen in Fig. 1 — the cross-cap representation of the Möbius strip makes it self-intersecting in a 3-D embedding (though it has no singular point).

From this it easily follows that the Euler characteristic $V - E + F$ of such a surface is related to g by the equation

$$V - E + F = 2 - g.$$

Thus for the Klein bottle, $g = 2$. For an orientable closed surface we have $\chi = 2 - 2g$, while for a non-orientable closed surface $\chi = 2 - g$.

Figs. 3-5 depict the three simplest classes of *closed* non-orientable surfaces, represented as a sphere with one, two and three local cross-caps, respectively. The class shown in Fig. 3 includes the *real projective plane* ($\mathbb{RP}^2$), obtained from $\mathbb{R}^3$ by identifying all points $(\lambda x, \lambda y, \lambda z)$ with fixed (x, y, z) and all real numbers λ; this class is also a Möbius strip with its boundary sewn to a disc. Fig. 4 — a sphere with two cross-caps — is equivalent to a Klein bottle (Fig. 7). This is because a Klein bottle can be constructed by sewing together the boundaries of two Möbius strips — as shown in two different ways in Figs. 8(a) and 8(b). Fig. 8(a) shows how two standard (twisted-strip) representations of Möbius strips are sewn along their boundaries to yield a single Klein bottle. In Fig. 8(b) it is done another way, by identifying the boundaries of two Möbius strips. Each separate Möbius strip is represented as a rectangle with two of its edges identified, as in Fig. 9(c). The final Klein bottle can be represented as a rectangle with its four edges identified pairwise as in Fig. 9(b). The vertices A, A' are identified with each other, as are B and B'; the two single-broken-line arrows are identified with each other (base with base and arrow-tip with arrow-tip), and the two double-solid-line arrows are similarly identified. The two remaining arrow pairs are separately identified *within* each Möbius strip, as in Fig. 9(c) (single-solid-arrows identified with each other, as are the single-dotted-arrows). Proceeding from left to right, the first solid-whit arrow indicates the sewing together of the boundaries of the two Möbius strips. The second solid-white arrow indicates two further operations: a 180° twisting of the *right* closed curve ABA to align it with the *left* closed ABA curve, followed by a cut along a curve between the two copies of point A. Neither operation changes the Klein bottle's topology.

Fig. 5 shows a sphere with three cross-caps; this can be shown to be topologically equivalent to a torus with a small disc removed and replaced with a cross-cap (*Dyck's theorem*).

Fig. 6 shows how a Klein bottle can be simultaneously cut along two closed curves while remaining a connected surface: the two cuts open the surface's two cross-caps, converting them into two closed-curve boundaries — the Klein bottle is thus converted into a topological cylinder.

Finally, Figs. 9(a)-9(g) show how to obtain various surfaces — open and closed, orientable and one-sided — by sewing (identifying) the vertices and edges of a single rectangle in various ways.

In general, for a closed orientable surface S of genus g, for which the $\mathbb{R}^2 \to \mathbb{R}^3$ mapping $\boldsymbol{r}(u,v)$ is twice continuously differentiable, the integral curvature *is equal to $\iint_S K\,dA = 4\pi(1-g)$, where K is the* Gaussian curvature *(this follows from the Gauss-Bonnet theorem). For the torus ($g=1$) the right-hand side vanishes. And indeed, in terms of the parametric representation of the $\mathbb{R}^3$-embedded torus given above:*

$$K = \frac{\cos v}{b(a+b\cos v)}, \qquad dA = (a+b\cos v)b\,du\,dv$$

and therefore the integral curvature is zero as claimed, as $\int_0^{2\pi} dv\cos v = 0$. Another interesting feature of the torus is that an elliptic function *defines a mapping of a plane into a torus. It arises from the notion that the curve $y^2 = ax^3 + bx^2 + cx + d$ can be parametrized as $x = f(z)$, $y = f'(z)$, where f and f' are elliptic functions (**Jacobi**, 1834).*

II. Topological mappings

Starting with the concept of a set *(such as points in a plane, lines through a point, rotations in 3-D, etc.) one can generalize the idea of a* function *to that of a* map: *A map is a relation between two specified sets that associates a unique element of the second to each element of the first.*

To establish topological equivalence *between sets, one must have a mathematical machinery that is able to transform one of the sets into the other, and this transformation must be a map endowed with various properties.*

There are various methods of mapping one surface (or higher-dimensional manifold space, whether intrinsically defined or embedded) onto another. The most faithful image of a surface is obtained by an isometric, *or* length-preserving, *mapping*[214]*. Here the geodesic distance between any two points is preserved (assuming the surface or space is endowed with a* metric[215]*), all angles remain unchanged, and geodesic lines are mapped into geodesic lines. An isometry also preserves the Gaussian curvature at corresponding points. Hence the only surfaces that can be mapped isometrically into a* part of the plane *are surfaces whose Gaussian curvature is everywhere zero; this excludes, for example, any portion of the sphere. In consequence, no* geographical map *(i.e. map of the earth's surface) can be free of distortions.*

Less accurate, but also less restrictive, are the area-preserving mappings. *They are defined by the condition that the area enclosed by every closed curve be preserved. With the aid of such a mapping portions of the sphere* can *be mapped onto portions of the plane, and this is frequently used in geography. It is achieved in practice by projecting points of the sphere onto the cylinder along the normals of the cylinder. If the cylinder is now slit open along a generator and developed into a plane, the result is an area-preserving image of the sphere in a plane; the distortion increases the further we are from the circle along which the cylinder touches the sphere.*

Another type of mapping, especially useful for navigation, is that of geodesic maps, *where geodesics are preserved. If, for example, a portion of a*

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[214] In the Euclidean plane all isometries can be generated by combining a two-parameter translation, a one-parameter rotation, and a single reflection about some fixed axis. In Euclidean $\mathbb{R}^3$, there are 3 translations parameters and 3 rotation-angle parameters, but still only one independent reflection, which can be implemented with the help of a *mirror*. No more than 3 mirrors (i.e. three reflection planes) are needed to generate any isometry in $\mathbb{R}^3$.

[215] In the case of a 2D surface embedded in $\mathbb{R}^3$, the natural surface metric is the one inherited from the Euclidean metric of the "host" $\mathbb{R}^3$ space.

sphere is projected from its center onto a plane, then the great circles are mapped into straight lines of the plane, and the map is therefore geodesic. At the same time, this gives us a (local) geodesic mapping of all surfaces of constant positive Gaussian curvature *into the plane, because all these surfaces can be mapped isometrically into spheres. All surfaces with* constant negative Gaussian curvature *can also be mapped into the plane by a geodesic mapping.*

Yet another type of mapping is that of the conformal*, or* angle-preserving*, mappings, for which the angle at which two curves intersect is preserved. The simplest examples of conformal mapping, apart from the isometric mapping, are* stereographic projections *and the circle-preserving transformations*[216]*. A stereographic projection map, in which a sphere (with its north-pole removed) is placed atop a plane and projected onto it by drawing straight lines from that pole, is also a circle-preserving map.*

It can be shown that very small figures *suffer hardly any distortion at all under general conformal transformations; not only angles are preserved, but the* ratios of distances *(although not the distances themselves) are approximately preserved.* In the small*, the conformal mappings are thus the nearest thing to isometric mappings among all the types of mappings mentioned earlier, for area-preserving and geodesic mappings may bring about arbitrarily great distortions in arbitrarily small figures.*

The most general mappings that are at all comprehensible to visual intuition are continuous invertible mappings (homeomorphisms)*. The only condition here is that the mapping is* one-to-one *and that neighboring points (and* only *such) go over to neighboring points. Thus a homeomorphic mapping may subject any figure to an arbitrary amount of distortion, but it is not permitted to* tear *connected regions apart or to stick separate regions together. Yet, continuous mappings do not always exist that can map (which we refer to here as "continuous" for simplicity's sake) two given surfaces onto each other (Example: the circular disc and the plane annulus bounded by two concentric circles cannot be mapped continuously into each other, even not their boundaries alone!). Clearly, the class of continuous mappings embraces all the types of mapping mentioned so far. The question of when two surfaces can be mapped onto each other by a continuous mapping is one of the problems of topology.*

The simplest type of topological mapping of a surface consists of a continuous mapping (homeomorphism) which is such as to transform the surface

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[216] An example of a circle-preserving map is the $\mathbb{R}^2 \to \mathbb{R}^2$ *inversion map* w.r.t. a given circle. If the latter is $x^2 + y^2 = a^2$, this inversion is $x \to \frac{a^2 x}{x^2+y^2}$, $y \to \frac{a^2 y}{x^2+y^2}$. It is a special type of conformal transformation that maps *any* circle into another circle.

as a whole onto itself, and *which is arcwise-connected with the trivial (unity) map (in which each surface point is mapped to itself*[217]*). This type of mapping is called a* deformation. *The* reflection *of a plane about a straight line, on the other hand, is an example of a topological mapping which is not a deformation; for a reflection reverses the sense of traversal (orientation) of every circle, whereas deformations cannot reverse the sense of traversal.*

A point that is mapped onto itself under the mapping is called a fixed point *of the mapping. In the applications of topology to other branches of mathematics, "fixed-point" theorems play an important role. The* theorem *of* ***Brouwer*** *states that every continuous deformation of a circular disc (with the points of the circumference included) onto itself has at least one fixed point. On a sphere, any continuous transformation which carries no point into its diametrically opposite points (e.g. any* small *deformation) has a fixed point.*

Fixed point theorems provide a powerful method for the proof of many mathematical "existence theorems" which at first sight may not seem to be of a geometrical character. Also, topological methods have been applied with great success to the study of the qualitative behavior of dynamical systems. A famous example is a fixed point theorem conjectured by **Poincaré** *(1912), which has an immediate consequence: the existence of an infinite number of* periodic orbits *in the restricted problem of three bodies.*

Apart from the choice of the mapping transformation there is yet another problem that must be resolved; in describing a surface or other manifold, there is the freedom of choice of a suitable coordinate system *(CS). In general we cannot restrict ourselves to manifolds which can be covered by a* single *CS such as is suitable for an n-dimensional Euclidean space $\mathbb{R}^n$; simple examples of various kinds of surfaces embedded in E^3 indicate that, in general, no single CS can exist which covers a given surface completely.*[218]

The simplest example is a 2-dimensional spherical surface in E^3 (the latter having Cartesian coordinates (x_1, x_2, x_3)), which we wish to map onto a planar disc. To obtain a one-to-one correspondence in the mapping, one may choose

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217 Two continuous mappings $f : A \to B$, $g : A \to B$ from a set A to a set B are said to be arcwise-connected (or *continuously deformable* into each other) if there exists an arc of continuous functions $h(s) : A \to B$, $s \in [0, 1]$, such that $h(0) = f$, $h(1) = g$, and h is a continuous map from $[0, 1] \times A$ to B.

218 E^n is the space $\mathbb{R}^n$ with a Euclidean metric (norm). A CS (coordinate system) is a homeomorphism between a region (open subset) of the surface (or higher-dimensional manifold) and $\mathbb{R}^m$, m being the manifold's *dimension* ($m = 2$ for a surface). For a manifold requiring more than one CS, it is assumed that the open subsets *cover* the manifold, and that in the intersection of any two subsets, the two CS maps compose to yield a $\mathbb{R}^m \to \mathbb{R}^m$ homeomorphism.

the hemisphere for which $x_1 > 0$, which is then continuously mapped onto a disc in the x_2x_3 plane. Accordingly, this hemisphere is referred to as a coordinate neighborhood.

Similarly, 5 other hemispheres corresponding to the respective restrictions $x_1 < 0$; $x_2 > 0$; $x_2 < 0$; $x_3 > 0$; $x_3 < 0$ can be regarded as coordinate neighborhoods. The totality of these 6 hemispheres covers the sphere completely, and in the overlap of any pair of them, composing the two corresponding maps yields a continuous map of one planar disc onto another. In general, the existence and overlap structure of suitable coordinate neighborhoods depends on the topological properties of the surface taken as a whole. This shows that one must give up on the construction of a unique CS for all points of a space under consideration and use different CS for different parts of the space.

A surface, however curved and complicated, can be thought of as a set of little curved patches glued together; and topologically (though not geometrically) each patch is just like a patch in the ordinary Euclidean plane. It is not this local *patch-like structure that produces things like the hole in a torus: it is the* global *way all the patches are glued together. Once this is clear, the step to n dimensions is easy: one just assembles a space from little patches carved out of n-dimensional space instead of a plane. The resulting space is an n-dimensional manifold. For example: the motion of three bodies under mutual gravitational forces involves an 18-dimensional phase-space manifold, with 3 position coordinates and 3 velocity coordinates per body.*

III. Algebraic topology

Algebraic topology is the study of the global *properties of spaces by means of abstract algebra. One of the earliest examples is Gauss's linkage formula which tells us whether two closed space curves are linked, and – if so – how many times does any one of them wind around the other. The linkage number remains the same even if we continuously deform the space curves. The central idea here is that continuous geometric phenomena can be understood by the use of integer-valued topological invariants.*

One of the strengths of algebraic topology has always been its wide degree of applicability to other fields. Nowadays that includes fields like theoretical physics, differential geometry, algebraic geometry, and number theory. As an example of this applicability, here is a simple topological proof that every non-constant polynomial $p(z)$ has a complex zero (root) — a key component in proving the fundamental theorem of algebra.

Consider a circle of radius R and center at the origin of the complex plane. The polynomial transforms this into another closed curve in the complex plane. If this image curve ever passes through the origin, we have our zero. If not, suppose the radius R is very large. Then the highest power of $p(z)$ dominates and hence $p(z)$ transforms the circle into a curve which winds around the origin the same number of times as the degree of $p(z)$. This is called the winding number *of the curve around the origin. It is always an integer and it is defined for every closed curve which does not pass through the origin. If we deform the curve, the winding number has to vary continuously but, since it is constrained to be an integer, it cannot change and must be a constant unless the curve is deformed through the origin.*

Now deform the image curve by shrinking the radius R to zero and suppose that the image curve never passes through the origin, that is to say, the original circle, in shrinking, never passes through a zero of the polynomial. The image curve gets very small since $p(z)$ is continuous; hence it must have winding number 0 around the origin unless it is shrinking to the origin (which cannot be the case unless $p(0) = 0$). If the image curve is shrinking to the origin, the origin is a zero of $p(z)$. If not, the winding number is 0 which means that the polynomial must have degree 0; in other words, it is a constant.

The winding number of a curve illustrates two important principles of algebraic topology. First, it assigns to a geometric object, the closed curve, a discrete invariant, the winding number which is an integer. Second, when we deform the geometric object, the winding number does not change, hence, it is called an invariant of deformation or, synonymously, an invariant of homotopy*. The field is called* algebraic *topology because an equivalence class of geometric entities possessing the same invariant — e.g. linkage number between curves; winding numbers of curves about points, or of closed surfaces in many-to-one mappings about other closed surfaces; winding numbers of non-shrinkable generator curves on the surface of a surface of nonvanishing genus; et cetera — turn out to form algebraic structures, such as rings and groups, under various geometric operations.*

IV. From curves and knots to manifolds

A simple closed curve (one that does not intersect itself) is drawn in the plane. What property of this figure persists even if the plane is regarded as a sheet of rubber that can be deformed in any way? The length of the curve and the area that it encloses can be changed by a deformation. But there is a *topological property of the configuration which is so simple that it may seem trivial: A simple closed curve C in the plane divides the plane into exactly two domains, an inside and an outside. By this is meant that those points of the plane not on C itself fall into two classes — A, the outside of the curve, and B, the inside — such that any pair of points of the same class can be joined by a curve which does not cross C, while any curve joining a pair of points belonging to different classes must cross C. This statement is obviously true for a circle or an ellipse, but the self-evidence fades a little if one contemplates a complicated curve like the twisted polygon. This problem was first stated by* **Camille Jordan** *(1882) in his Cours d'analyse. It turned out that the proof given by Jordan was invalid.*

The first rigorous proofs of the theorem were quite complicated and hard to understand, even for many well-trained mathematicians. Only recently have comparatively simple proofs been found[219]*. One reason for the difficulty lies in the generality of the concept of "simple closed curve", which is not restricted to the class of polygons or "smooth" curves, but includes all curves which are topological images of a circle. On the other hand, many concepts such as "inside", "outside", etc., which are so clear to the intuition, must be made precise before a rigorous proof is possible.*

It is of the highest theoretical importance to analyze such concepts in their fullest generality, and much of modern topology is devoted to this task. But one should never forget that in the great majority of cases that arise from the study of concrete geometrical phenomena it is quite beside the point to work with concepts whose extreme generality creates unnecessary difficulties. As a matter of fact, the Jordan curve theorem is quite simple to prove for the reasonably well-behaved curves, such as polygons or curves with continuously turning tangents, which occur in most important problems.

A knot is formed by first looping and interlacing a piece of string and then joining the ends together. The resulting closed curve represents a geometrical figure the "knotiness" of which remains essentially the same even if it is deformed by pulling or twisting without breaking the string. But how is it possible to give an intrinsic characterization that will distinguish a knotted closed curve in space from an unknotted curve such as the circle? The answer is by no means simple, and still less so is the complete mathematical analysis of the various kinds of knots and the differences between them. Even for the simplest case this has proved to be a daunting task.

Consider, for example, two knots which are completely symmetric mirror images of one another. The problem arises whether it is possible to deform

[219] A generalization of the *Jordan theorem* to arbitrary surface is used in proving the *surface classification theorem* cited earlier.

one of these knots into the other in a continuous way. The answer is in the negative, but the proof of this fact requires considerable knowledge of the technique of topology and group theory.

Knots are the most immediate topological features of curves in space. Beyond curves come surfaces; beyond surfaces come multidimensional generalizations called manifolds*, introduced by* ***Riemann****.*

*Whereas mathematical analysis and the theory of differential equations deal primarily with "*local*" properties of a function (only infinitesimally adjacent points are considered), geometry studies the "*global*" properties of functions (i.e. their properties are analyzed by considering finitely spaced points). This intuitive idea of globality has given rise to the fundamental concept of* manifold *as a generalization of the concept of* domain *in Euclidean space.*

A coordinate system describing the positions of points in space is an indispensable tool for studying geometrical objects. Using coordinate systems, we can apply the methods of differential and integral calculus to solve various problems. Therefore, an analysis of spaces which admit such concepts as differentiable or smooth functions, differentiation and integration, has emerged as an independent branch of geometry.

Topologists would like to do for manifolds what they have already done for surfaces and knots. Namely:

(1) Decide when two manifolds are or are not topologically equivalent.

(2) Classify all possible manifolds.

(3) Find all the different ways to embed one manifold in another (e.g. a knotted circle in 3-space).

(4) Decide when two such embeddings are, or are not, the same.

The answer to problems (1) and (2) lies in an area called homotopy theory *which is part of* algebraic topology*. It endeavors to associate various algebraic invariants with topological spaces.* ***Poincaré*** *was one of the fathers of this theory. But problems (1) and (2) have not yet been fully resolved. Problem (3) led topologists to some surprising and counter-intuitive results, as the following example shows. It has been asked: when can two 50-dimensional spheres be* linked *[i.e. embedded such that they cannot be separated by a topology-preserving transformation of the surrounding n-dimensional space]. The answer is:*

cannot link	*for*	$n \geq 102$
can link	*for*	$n = 101, 100, 99, 98$
cannot link	*for*	$n = 97, 96$
can link	*for*	$n = 95, \ldots, 52.$

V. Networks

Graph (or Network) theory had its origin in a paper by Euler (1736) including the famous problem of the bridges of Königsberg. ***Euler*** *saw that the problem could more easily be studied reducing island and banks to points and drawing a network (graph) in which two points are connected by an edge whenever there is a bridge connecting the corresponding two land masses. In this way Euler was able to abstract the problem so that only information essential to solving it was highlighted, and he could dispense with all other aspects of the problem. He could thus rephrase the problem as follows: "Given a connected graph, find a path that traverses each edge of the graph without retracing any edge." Such a path is called a Eulerian traversal or Eulerian path*[220]*. Some experimentation and application of logic lead to the conclusion that in order to have a Eulerian path, it is necessary that for any edge along which the path enters a vertex, there must correspond a distinct edge along which the path leaves it — and that all such edge-pairs be distinct for any given vertex. The only exception occurs for the beginning and ending vertices of the path, if these points are different.*

Networks can be used to solve mazes and guarantee that one can find a path through a maze, if such a path exists, even when no map is explicitly given. Other procedures enable people to retrace their steps to the beginning of a labyrinth. Some of these procedures have applications to problems of computer processing, traffic control, electrical engineering, and many other fields.

During the ten generations elapsed since 1736, mathematicians have developed a new branch of geometry — a geometry of dots and lines, otherwise known as graph theory — that preserves geometrical relations only in their most general outlines. Here lines do not have to be straight, nor are there such things as perpendicular or parallel lines, and it does not make sense to talk about bisecting lines or measuring lengths or angles. The power of graph theory (a sub-field of topology) is that it can be used to model many patterns

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[220] *A practical architectural application*: In the hallways of a museum, pictures are hung on one side of each hall. How does one design a tour that will enable a person to see each exhibit exactly once?

in nature — from the branching of rivers to the cracking or brittle of surfaces to subdivision of cellular forms, as well as many abstract concepts. It gives us a way to study spatial structures unencumbered by the details of Euclidean geometry.

A geographical map *shows countries, borders and corners. From such a map we may prepare an abstract* mathematical map *in which countries are* faces *(F), borders are chains of pairs of adjacent* edges *(E) and corners are* vertices *(V). In order to study the topology of a map in the technical language of mathematics, we must forget its geographical significance and treat it as merely a network, or graph, being a set of faces, edges and vertices, $M = \{F, E, V\}$, with certain* incidence relations *among them (e.g. face f_1 has edges (e_2, e_3); edge e_2 is shared by faces $\{f_1, f_3\}$; vertex v_1 is shared by $\{f_1, f_2, f_3\}$ and also by $\{e_2, e_3, e_4\}$; etc.). In this context the face is represented by some polygon and each edge lies in exactly two faces. Copies of a map formed by placing it on a flexible membrane and stretching the membrane without cutting, are considered identical or* homeomorphic*. Edges and faces thus become distorted but the sets E, F and V and their relational structure (incidence relations) maintain their integrity.*

From a mathematical point of view, maps on a plain and maps on the sphere*, with one point removed, are isomorphic. Since all the enclosed areas, including one additional outer one, are now considered to be faces, and maps are always considered to be in one piece (connected), one can show that* Euler's formula $V - E + F = 2$ *holds for connected planar maps on either a plane or a sphere.*

There is a family of maps for which each vertex, edge or face is like every other vertex, edge or face. They are called regular maps *and are said to have perfect symmetry. Upon finding oneself stranded in a mathematical country defined by such a map, one would experience vistas of sameness in all directions and be hopelessly lost.*

There are only five[221] *such regular maps and they correspond to the five 3-dimensional Platonic Solids. In fact, they are obtained by projecting the edges of a Platonic polyhedron onto a plane from a point directly above the center of one of its faces, and counting the infinite area outside the boundary as an additional face. These are known as* Schlegel diagrams.

Visually, this amounts to holding one face of a polyhedron quite close to one's eyes, looking at the structure through the face, and drawing the projection of the structure as seen in this exaggerated perspective. The number of vertices, faces and edges for the Schlegel diagrams then becomes identical to those of the corresponding Platonic Solids.

221 Except for two *trivial* families, one of which consist of all regular polygons.

Just as there are only five regular maps on the sphere (or plane), there are only three classes of regular maps that can be created on a torus.

For a surface homeomorphic to a sphere with g handles Euler's formula becomes $V - E + F = 2 - 2g$.

On a torus, for example, we have (with $g = 1$) $V - E + F = 0$.

G. R. Kirchhoff *enunciated (1845) laws which allow calculations of currents, voltages and resistances of electrical networks. In the framework of these laws he became interested in the mathematical problem of the number of independent circuit equation in a given network. Considering the electrical network as a geometrical object (map) constructed from points (vertices, V) and lines (edges, E), Kirchhoff proved that, in general, the number of independent circuits[222] is equal to $(E - V + 1)$.*

His paper is quite modern in its approach, and he used various constructions which we now think of as standard in graph theory. But he did not have the algebraic techniques that are needed to extend the results to higher dimensions. However, the basic ideas were latent in Kirchhoff's paper, and it was just those ideas which mathematicians were able to develop in the second half of the 19th century, in order to create what we now call 'algebraic topology'. This development did not happen overnight.

The apparatus of vectors, matrices, and what we call now linear algebra, as well as the abstract algebra of groups, rings, homeomorphisms etc., were not available to Kirchhoff, Listing, and the other mathematicians of the 1840's. However, in the course of time all these ingredients developed into a program which turned some very vague and descriptive ideas about the 'holeyness' of solids into an impressive general theory – an algebraic context within which these ideas can be formulated independently of any intuitive notions. There are many famous names associated with this program. One of them was the Italian mathematician **Enrico Betti***, who introduced numbers, knows as Betti numbers, which turn out to be a generalization of the Kirchhoff number $(E - V + 1)$.*

But the person who made the greatest advances, in a series of papers published around 1895, was the French mathematician ***Henri Poincaré****. He formulated everything in terms of multi-dimensional objects (complexes), built out of what he called simplexes, and he showed how the rules by which they are fitted together can be described by means of matrices. He also showed how the 'holeyness' of complexes can be described algebraically in terms of properties of these matrices.* **Veblen** *(1916) gave a modern treatment of Poincaré's theory.*

[222] This is compatible with *Euler's formula* if we equate the number of independent circuits to $(F - 1)$.

1847 CE **Johann Benedict Listing** (1806–1882, Germany). Mathematician. Started the systematic study of topology as a branch of geometry, and coined the word 'topology'. Some topological problems are found in the works of **Euler**, **Möbius** and **Cantor**, but the subject only came into its own in 1895 with the work of **Poincaré**.

1847–1852 CE **Matthew O'Brien** (1814–1855, England). Mathematician. A forerunner of **Gibbs** and **Heaviside**. Introduced the modern symbols for vector multiplication.

History of the Wave Theory of Sound[223]

The speculation that sound is a wave phenomenon grew out of observations of water waves. The rudimentary notion of a wave *is that of an oscillatory* disturbance *that moves away from some source and transports no discernible amount of matter over large distances of* propagation.

The possibility that sound exhibits analogous behavior was emphasized by the Greek philosopher ***Chrysippos*** *(ca 240 BCE), by the Roman architect*

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[223] For further reading, see:

- Crighton, D.G. et all, *Modern Methods in Analytical Acoustics*, Springer Verlag: Berlin, 1992, 738 pp.
- Pierce, A.D., *Acoustics*, American Institute of Physics, 1989, 678 pp.
- Dowling, A.P. and J.E. Ffowcs Williams, *Sound and Sources of Sound*, Ellis Horwood, 1983, 321 pp.
- Lord Rayleigh, *Theory of Sound*, Vols I-II, Dover: New York, 1945.
- Morse, P.M. and K.U. Ingard, *Theoretical Acoustics*, McGraw-Hill, 1968, 927 pp.

*and engineer **Vitruvius** (ca 35 BCE), and by the Roman writer **Boethius**[224] (ca 475–524).*

*The pertinent experimental result that the air motion generated by a vibrating body (sounding a single musical note) is also vibrating at the same frequency as the body[225], was inferred with reasonable conclusiveness in the early 17^{th} century by **Marin Mersenne** (1636) and **Galileo Galilei** (1638).*

Mersenne's description of the first absolute determination of the frequency of an audible tone (at 84 Hz) implies that he had already demonstrated that the frequency ratio of two vibrating strings, radiating a musical note and its octave, is as $1:2$. The perceived harmony (consonance) of two such notes would be explained if the ratio of the air oscillation frequency is also $1:2$, which in turn is consistent with the source-air motion frequency equivalence hypothesis.

*The analogy with water waves was strengthened by the belief that air motion associated with musical sound is oscillatory and by the observation that sound travels with finite speed. Another matter of common knowledge was that sound bends around corners, which suggested diffraction, a phenomenon often observed in water waves. Also, **Robert Boyle**'s (1660) classic experiment on the sound radiation by a ticking watch in a partially evacuated glass vessel provided evidence that air is necessary, both for the production and transmission of sound.*

The apparent conflict between ray and wave theories played a major role in the history of the sister science of optics, but the theory of sound developed almost from the beginning as a wave theory.

*When ray concepts were used to explain acoustic phenomena (as was done by **Reynolds** and **Rayleigh** in the 19^{th} century), they were regarded, either explicitly or implicitly, as mathematical approximations to a well-developed wave theory.*

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[224] Born into an aristocratic Christian family and became a consul (510). He wrote texts on geometry and arithmetic which were of poor quality but used for many centuries during a time when mathematical achievements in Europe were remarkable low. Boethius fell from favor and was imprisoned and later executed for treason and magic.

[225] The history of this is intertwined with the development of the laws of vibrating strings and the physical interpretations of musical consonances, which goes back to **Pythagoras** (ca 550 BCE) and perhaps earlier. Thus, the dual nature of wave-motion in both time and frequency domains goes back all the way to the ancient Greeks.

The successful incorporation of geometrical optics into a more comprehensive wave theory had demonstrated that viable approximate models of complicated wave phenomena could be expressed in terms of ray concepts. This recognition has strongly influenced 20^{th} *century development in architectural acoustics, underwater acoustics, and noise control.*

The mathematical theory of sound propagation began with ***Isaac Newton*** *(1642–1727), whose Principia (1686) included a mechanical interpretation of sound as being* pressure *pulses transmitted through neighboring fluid particles*[226]*. Substantial progress toward the development of a viable theory of sound propagation resting on firmer mathematical and physical concepts was made in* 1759–1816 *by* ***Euler***, ***d'Alembert***, ***Lagrange*** *and* ***Laplace***. *During this era, continuum physics, or field theory, began to receive a definite mathematical structure. The* wave equation *emerged in a number of contexts, including the propagation of sound in air. The theory ultimately proposed for sound in the* 18^{th} *century was incomplete from many standpoints, but the modern theories of today can be regarded for the most part as refinements of that developed by Euler and his contemporaries.*

The linearized equations of the acoustic field are derived directly from the general equations of fluid motion on the basis that the fluid velocity $\boldsymbol{u}$, *the* change *of pressure* p, *and the* change *of density* ρ *— are all small compared to the sound velocity* c, *average ambient density* ρ_0, *and average background pressure* p_0, *respectively, such that products of the small entities can be neglected in the equations.*

There are three fundamental equations relating the above entities:

(1) Newton's equation of motion (conservation of the fluid linear momentum) *relating the pressure gradient to the linear fluid acceleration:*

$$\nabla p = -\rho_0 \frac{\partial \boldsymbol{u}}{\partial t};$$

where $\boldsymbol{u}$ *is the fluid velocity vector.*

(2) the equation of continuity (conservation of mass)*:*

$$\rho_0 \operatorname{div} \boldsymbol{u} + \frac{\partial \rho}{\partial t} = 0;$$

[226] The fundamental relation $\lambda f = c$ [λ = wavelength; f = frequency; c = phase velocity] appeared explicitly for the first time in Newton's *Principia* (1686). The first measurement of the sound speed in air was evidently made by **Mersenne** (1635, 1644). The time was measured from the visual sighting of a firing of a cannon to the reception of the transient sound pulse at a known distance from the source.

(3) the equation of state, specifying the functional dependence $p = p(\rho)$, *subjected to the expansion*

$$p - p_0 = (\rho - \rho_0)\left(\frac{dp}{d\rho}\right)_{\rho_0} + \cdots \approx (\rho - \rho_0)c^2,$$

where $c^2 = \left.\frac{dp}{d\rho}\right|_{\rho_0}$ *and* c *the ambient velocity of sound.*

Newton *(1686), applying Boyle's law* $p = \rho f(T)$ *[isothermal process], obtained* $c = \sqrt{\frac{p_0}{\rho_0}} = 290\ \frac{m}{\sec}$ *at* $T = 293\,^\circ K$, *15% lower than the observed value.*

Laplace *(1816) improved on Newton's result by correctly assuming that sound waves pass too rapidly for a significant exchange of heat to take place. For an adiabatic expansion in a perfect gas, he used* $p\rho^{-\gamma} = p_0\rho_0^{-\gamma}$, *which led him to*

$$c^2 = \frac{dp}{d\rho} = \gamma\frac{p_0}{\rho_0} = \gamma RT$$

with $c = 343\ \frac{m}{\sec}$ *at* $T = 293\,^\circ K$,

$$\gamma = c_p/c_v \ = \ \textit{ratio of specific heats},$$

and

$$R \ = \ \textit{universal gas constant.}$$

[Clearly, the theoretical prediction of the speed of sound in liquids is more difficult than in gases. For example, c *in* sea water *depends on the pressure, salinity, water temperature and the amount of dissolved and suspended gas.]*

The above equations then imply the approximate relations

$$p = \rho c^2 + const.,$$

$$k\,\mathrm{div}\,\boldsymbol{u} + \frac{\partial p}{\partial t} = 0,$$

where $k = \rho_0 c^2$ *is the* incompressibility. *The combination of the conservation laws for mass and momentum leads to the wave equation*

$$\nabla^2 p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

for the acoustic pressure changes. The further assumption $\boldsymbol{u} = \mathrm{grad}\,\psi$, *implies that the fluid velocity* $\boldsymbol{u}$ *also obeys the same wave equation. It then follows that all field entities are expressible in terms of the potential* ψ*:*

$$p - p_0 = -\rho_0\frac{\partial\psi}{\partial t};$$

$$\rho - \rho_0 = -\frac{\rho_0}{c^2}\frac{\partial\psi}{\partial t}; \qquad \boldsymbol{u} = \nabla\psi.$$

The wave equation for ψ is

$$\nabla^2\psi = \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}.$$

For one-dimensional motion,

$$\psi = \psi(x - ct); \qquad u = \psi'$$

implies at once the relations $p = \rho_0 cu$; $\rho = \rho_0 \frac{1}{c} u$. *Certain entities formed of the basic field elements* $\{p, u, c, \rho_0\}$ *are of use in acoustic engineering:*

$$Z = \rho_0 c \equiv \sqrt{\rho_0 k} \qquad \textit{(impedance)};$$

$$W = \frac{1}{2}\rho_0|u|^2 + \frac{1}{2k}|p|^2 = \rho_0|u|^2 = \frac{|p|^2}{\rho_0 c^2}$$

(wave energy density = fluid momentum flux);

$$I = pu = Wc$$

(sound intensity = rate at which acoustic energy crosses a unit area per unit time).

The application of the Fourier transform to the pressure wave equation yields the Helmholtz equation (1860):

$$\nabla^2 p + \frac{\omega^2}{c^2}p = 0,$$

where ω is the angular frequency of the harmonic Fourier component.

It is of interest to note that ***Euler****, in his "Continuation of the Researches on the Propagation of Sound" (1759, 1766), already derived the Helmholtz spectral wave equation for the* particle displacement *(or velocity).*

The solution of the Helmholtz equation for a symmetrical point-source *yields the well-known result that the sound intensity falls of as the square of the distance from the source in a free open space. For sources of large area, the approximation does not hold and the sound intensity may at first fall off proportionally to the first power of the distance. Finally, in enclosed regions the sound intensity may decrease very slowly, or not at all, with distance.*

Pressure is measured in units of Pascal, denoted $Pa = \frac{\text{Newton}}{\text{m}^2} = 10\ \frac{\text{dyn}}{\text{cm}^2}$ *[Newton = 10^5 dyn, Joule = Newton×meter; Watt = Newton×meter/sec].*

Another unit is the $\quad bar = 10^5\ Newton/m^2 = 10^6\ dyn/cm^2 \approx Kg/cm^2$*;* $1\mu bar = 10^{-6}\ bar = dyn/cm^2$*. Atmospheric pressure* $\approx 10^5 Pa \approx Kg/cm^2$*.*

1847–1856 CE **Jean Frederic Frenet** (1816–1888, France). Mathematician. Contributed to differential geometry of curves and surfaces. Introduced the so-called *Frenet-Serret*[227] *formulae* for the moving-trihedral on a space curve. He was a man of wide erudition and a classical scholar.

Frenet was born at Perigueux and graduated from the École Normale Superior (1840). He was a professor at Toulouse and Lyons.

1847–1861 CE **Ignaz Philipp Semmelweis** (1818–1865, Hungary). Obstetrician. Pioneer of antisepsis[228]. Proved (1847–1849) that *puerperal fever* (childbed fever) is brought to the woman in labor by the hands and instruments of examining physicians and can be eliminated through a thorough cleansing, in a solution of water with chloride of lime, of the hands, instruments, and other items brought in contact with the patient. Published (1861) *Die Aetiologie, der Begrift und die Prophylaxis des Kindbettfiebers*.

Semmelweis was born in Buda to Jewish parents and was educated at the Universities of Pest and Vienna, graduating M.D. in 1844. At the time when he was appointed assistant professor in a maternity ward, the mortality rate from puerperal fever stood at about 20 percent. His antiseptic measures caused this rate to drop to 1.2 percent by May 1847. His superior, Johann Klein, apparently blinded by jealousy and vanity, and supported by other reactionary teachers, drove Semmelweis from Vienna (1849).

Fortunately, in the following year Semmelweis was appointed obstetric physician at Pest in the maternity department, then as terribly afflicted as Klein's clinic had been. In the course of his six years of tenure there he succeeded, by antiseptic methods, in reducing the mortality rate to 0.85 percent. However, constant conflicts with his uncooperating superiors brought

[227] **Joseph Alfred Serret** (1819–1885, France). Mathematician. Graduated from the Ecole Polytechnique (1840). Professor of celestial mechanics at College de France (1861); Professor of Mathematics at Sorbonne (1863). Succeeded Poinsot in the Academie des Sciences (1860).

[228] In 1854, **Heinrich Schröder** and **Theodor von Dusch** showed that bacteria could be removed from air by filtering through cotton-wool. In 1867, **Joseph Lister** (1827–1912, England) reported his method of antiseptic surgery [son of Joseph Jackson Lister (1786–1869)].

him within the gates of an asylum (1865). He brought with him into this retreat an infected dissection wound which caused his death — a victim of the very disease for the relief of which he had already sacrificed health and fortune.

1847–1894 CE **Hermann Ludwig Ferdinand von Helmholtz** (1821–1894, Germany). One of the foremost scientists of the 19^{th} century. Surgeon, physiologist, physicist, mathematician, chemist, musical scientist and philosopher. Helmholtz was among the last of the universalists: his research spanned almost the entire gamut of science.

In one of the epoch-making papers of the century, he formulated in 1847 the universal law of conservation of energy. Presented (1858) the first mathematical account of *rotational* fluid flow, introducing the important concepts of *vorticity*, *circulation*, *vortex flow*[229] and *vortex lines*. In 1860, Helmholtz

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[229] Circulatory flow that is irrotational everywhere (except possibly at $r = 0$) is possible and is known as *circulatory flow without rotation*. In this case if the fluid is also incompressible and the flow *stationary* the velocity field has to satisfy both the matter conservation $(\mathrm{div}(\boldsymbol{V}) = 0)$ and irrotationality $(\boldsymbol{\Omega} = \mathrm{curl}\,\boldsymbol{V} = 0)$ conditions. The simplest solution of this class exhibiting circulatory flow about $r = 0$ has

$$\boldsymbol{V} = u_\theta(r)\boldsymbol{e}_\theta,$$

while irrotationality requires

$$\boldsymbol{\Omega} = \frac{\partial(ru_\theta)}{r\partial r} = 0.$$

It therefore follows that

$$ru_\theta = K = \text{constant}$$

(which is the law of conservation of angular momentum in disguise; the fluid angular-momentum density is $J = \rho u_\theta r$]. Thus

$$\boldsymbol{V} = K\frac{\boldsymbol{e}_\theta}{r}$$

representing irrotational motion except at the point $r = 0$, where the vorticity $\boldsymbol{\Omega}$ and the velocity become infinite (this is obviously an idealization of actual such flows). The circulation along a steamline $r = \text{const.}$ is

$$\Gamma = \oint \boldsymbol{V} \cdot d\boldsymbol{\ell} = 2\pi K$$

and the motion is known as *vortex flow*. It plays an important role in aerodynamics. On the basis of experimental evidence and the theory of viscous flow, one can assume that there is a fluid core or nucleus surrounding the center of developed the mathematical theory of Huygens' principle for 'monochromatic' steady-state scalar waves. He also showed that an arbitrary continuously differentiable vector-field can be represented at each point as a superposition of the gradient of a scalar potential and a curl of a vector potential.[230]

Helmholtz made a great contribution to our understanding of thermodynamics; he was first to apply minimum principles to thermodynamics, and showed that for reversible processes, the role of the *action* was played by the "*Helmholtz free energy*", F.

In 1854 Helmholtz seized upon the problem of the sun's luminosity. Previously, **Kant** had calculated that if the sun's light came from ordinary combustion, it would have burned up in only 3000 years. Helmholtz then argued that the tremendous weight of the sun's outer layers, pressing radially inward, should cause the sun to gradually contract: Consequently, its interior gases will become compressed, and heat up. Hence gravitational contraction causes the sun's gases to become hot enough to radiate energy into space. He was thus able to boost the theoretical age of the sun to some 20 million years. This in turn meant that the sun extended beyond the earth's orbit only 20 millions years ago, to which geologists could not agree on the basis of the earth's present surface features. **Kelvin** supported and 'improved' Helmholtz's theory and it is known as *Helmholtz-Kelvin contraction*.

In other fields of science, Helmholtz contributed to the subjects of: fermentation, animal heat and electricity, muscular contraction, velocity of nerve

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the flow and that the core rotates approximately like a solid body. Within the core we have circulatory flow with constant angular velocity and outside the core we have circulatory flow without rotation. Inside the core

$$u_\theta \sim r$$

while outside

$$u_\theta \sim \frac{1}{r}.$$

Such a combination is known as an *eddy* or simply a *vortex*. The central core is called the vortex core. The *tornado* and *water spout* (or even the common *bathtub vortex*) are examples of such a flow. The *stability* of the vortex is determined by its Reynolds' number.

If an eddy occurs in a fluid that is otherwise undisturbed, the spatial location of the eddy remains unaltered. However, if a uniform stream is superposed on it, it will *move* with the stream. Such a vortex is known as a *free vortex*.

230 This theorem is now recognized as a special case of a result from Cartan's exterior calculus in an arbitrary, n-dimensional manifold. The more general result relates to algebraic topology through the *de Rham cohomology*.

impulses[231], invention of the ophthalmometer, physiological optics, color vision, physiological acoustics and meteorological physics.

From 1869 to 1871 Helmholtz involved himself in the verification of Maxwell's predictions concerning electromagnetic waves. He entrusted the subject into the hands of his favorite pupil, **Heinrich Hertz**, and the latter finally gave an experimental verification of their existence and velocity.

Helmholtz was born in Potsdam, near Berlin. His father was a high school teacher and his mother was a lineal descendant of the Quaker **William Penn** (founder of the state of Pennsylvania).

As his parents were poor and could not afford to allow him to pursue a purely scientific career, he became a surgeon in the Prussian army. He lived in Berlin from 1842 to 1849, when he became a professor of physiology in Königsberg. In 1855 he removed to assume the chair of physiology in Bonn. In 1858 he became professor of physiology at Heidelberg, and in 1871 he was called to occupy the chair of physics in Berlin.

Helmholtz married twice and had 4 children. He was a man of simple but refined tastes, noble carriage and somewhat austere manner. His life, from first to last, was one of devotion to science.

1848 CE A year of revolutions in almost every European country. It was the natural climax of a process of reaction and revolt which began after the defeat of Napoleon at Waterloo in 1815. Thereafter, Europe entered a period of instability, characterized by a long series of upheavals. The revolution of 1848 was the culmination of the political, economical and social unrest of the time — of the struggle between the aristocracy and the middle classes, the rapid increase of population from 180 million in 1800 to 266 million in 1850, the fact that more and more people now lived in cities, the conflict between the bourgeoisie and the rising proletariat, and the movements for national liberation and reunion. And it confounded all the protagonists, compelling a reappraisal of ideas and a realignment of forces.

In some sense, the French Revolution and its sequel in Napoleonic imperialism, disrupted the historic continuity of European society and shattered most of its traditions.

All the significant problems of the period arose out of these events. This break in continuity engendered a quest for new patterns of interpretation — nationalism, socialism, vast philosophical systems like those of **Marx** and **Hegel**, new conceptions of historical, scientific, literary and artistic ideas.

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[231] He actually measured the speed of nerve impulses (1852).

Table 4.4: Timeline of the Industrial Revolution, 1770–1848

- ca 1770 — Consolidation of the steam engine by **James Watt**
- 1775–1783 — American Revolution
- 1780 — Industrial Revolution under way
- 1789–1794 — The French Revolution
- 1799–1815 — Reign of Napoleon
- 1800–1850 — Romanticism in literature and the arts
- 1815 — The Congress of Vienna and the congress system of European diplomacy
- 1820 — Revolutions in Greece and Spain
- 1830 — Rise of liberalism and nationalism
- 1830, 1848 — Periods of revolution in Europe
- 1832 — Parliamentary reform in Great Britain
- 1848 — **Karl Marx's** 'Communist Manifesto'.

Europe's search for stability after 1815 was marked by a contest between the forces of the past and the forces of the future. For a while it seemed as though the traditional agencies of power — the monarchs, the landed aristocracy and the Church — might once again resume full control. But potent new forces were ready to oppose relapse into the past. With the quickening of industrialization, there was now not only a middle class of growing size and significance but a wholly new class, the urban proletariat. Each class had its own political and economical philosophy — liberalism and socialism respectively — which stood opposed to each other as well as to the traditional conservatism of the old order.

Nationalism as an awareness of belonging to a particular nationality was nothing new. What was new was the *intensity* that this awareness now assumed: for the mass of the people, nationalism became their most ardent emotion, and national unification or independence their most cherished aim.

The Vienna settlement (1815) ignored the stirrings of nationalism and the hopes for democracy that had been awakened by the French Revolution. It was mainly interested in peace and order and the restoration of conditions as they were before the French Revolution[232]. Indeed, there was no war among the great European powers for 40 years, and no war of world-wide dimensions for a whole century. The Triple Alliance of Austria, Russia and Prussia guaranteed to maintain the territorial *status quo* in Europe and the existing form of government in every European country, i.e. aiding legitimate governments against revolutions.

A first wave of reaction that followed the peace settlements of 1815, manifested itself in the first wave of revolutions (1820–1829) in Spain, Portugal, Italy, Greece and Russia. The second wave of Revolutions swept France, Belgium and Poland during 1830–1833. The third wave (1848–1849) lasted for over a year and affected most of Europe with the exception of England and Russia. In Italy, Germany, Austria, and Hungary, the fundamental grievance was still the lack of national freedom and unity. In Western Europe the chief aim of revolutions was the extension of political power beyond the upper middle class. With the revolutions of 1848, socialism for the first time became an issue of modern politics.

In addition, severe economic crises particularly affected the lower classes: everywhere the small artisan was fighting against the competition of large-scale industry, which threatened to deprive him of his livelihood. At the same time, the industrial workers in the new factories were eking out a miserable existence on a minimum wage. There were also periodic upheavals in agriculture, primarily as a result of crop failures.

The revolutions of 1848 had failed everywhere due to weaknesses in the revolutionary camp (lack of widespread popular support, indecision among their leaders and the lack of well-defined programs) and the continued strength of the forces of reaction. The burden of the revolution fell on the workers whereas the middle class, in most countries, did not really want a revolution. It preferred to achieve its aims through reform, as had been done in England. There was no attempt to coordinate the revolutions in different countries, although the forces of reaction worked together.

Two forces emerged from the revolutions that henceforth were to dominate the history of Europe — nationalism and socialism. These now became, respectively, the main issues in the struggle of nation against nation and class against class.

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[232] In Spain and Naples the returning Bourbons abolished the liberal reforms that had been granted in 1812. In the Papal States, Pope Pius VII got rid of the French legal reforms, re-established the Jesuits, put the Jews back into the ghettos, and forbade vaccination against smallpox!
In Piedmont, Victor Emmanuel I had the French botanical gardens torn up by the roots and the French furniture thrown out of the windows of his palace!

1848–1867 CE **Karl** (**Heinrich**) **Marx** (1818–1883, Germany and England). Political economist. A critic of capitalism. Sought a scientific formula for social justice. The most influential social thinker of modern times. Marx approach was philosophical, Hegelian, and his materialist conception of history is basic to his philosophy of economic determinism (*Historical Materialism*). Marx defined *Communism* as the common ownership of the means of production, an ideal system to be achieved by shifting control over economic resources from the capitalists to the proletariat. This transfer of properly rights would result in the permanent abolition of private property, with public ownership of all means of production, including the farms and factories, raw materials, transportation and communication facilities, and the like.

In his major work *Das Kapital* (*Capital*, 1867, 1885, 1894), Marx systematically developed his theory of *surplus value*, which maintains that the worker is exploited in an inequitable distribution of the products of his labor by the owners of the means of production. The surplus is the difference between what he gets in order to subsist and what is totally derived from what he creates[233].

Marx accepted the *Hegelian dialectic*, which states that every thesis contains its own antithesis, its negation, opposite, or contradiction, and that the

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[233] **Ayn Rand** (1905–1982, USA), social philosopher, maintained that most workers in a capitalistic economy earn far more – both in the value of their wages and through the infrastructure made possible by such an economy – than they could ever bring into existence on their own. Thus, according to Rand, the "surplus value" works the *other* way, and is a de facto gift from enterpreneuring individuals to those whom they employ (or whose employment is made possible by the entrepreneurs' inventions and business acumen).

She was born in St. Petersburg, Russia, as Alissa Rosenbaum, to Jewish parents. Emigrated to the USA (1926). Espoused her philosophy of "rational selfishness" (*Objectivism*) in novels, and in non-fiction books such as "*For the New Intellectual*" (1961); "*The Virtue of Selfishness*" (1965); "*Capitalism: The Unknown Ideal*" (1966); and "The New Left" (1971). Rand held that the source of all happiness, progress and justice lies with the *productive individual*, free to pursue his own agenda by relentlessly applying *reason* and by engaging in *voluntary*, *non-coercive* cooperation/competition with other individuals for mutual benefit and satisfaction. She thus staunchly opposed religion and all other forms of *mysticism*, as well as any social order based upon *altruism*; regarded **Aristotle** as the most important philosopher; and taught that the *United States* – the only country founded upon laissez-faire capitalism and the principles of the enlightenment – was (in the 19^{th} century), and potentially still is, the most moral county in the history of mankind.

two conflicting forces merge to produce a synthesis, a new and greater reality. He applied this logical principle to socio-economic history. The two socio-economic classes, the property-owing class (capitalists) and the workers (proletariat), who must sell their labor in order to survive — are antithetical to each other.

Under the influence of **Ludwig Andreas Feuerbach**[234] (1804–1872, Germany), a pupil of Hegel in Berlin, Marx adopted the economic interpretation of history (*Historical Materialism*). It contends that a particular society's mode of economic production determines the nature of its cultural and social structure. Marx traced the relevant cause-effect relationship from ancient to modern times. He noted that the chief mode of production among the ancient Greeks and Romans was replaced by feudalist methods of production during the medieval period. Feudalism and the institution of serfdom upon which it depended yielded to capitalism in the modern period when the mode of production was changed through wider use of machinery and the factory system. Marx concluded that capitalism, by its very nature, is self-destructive and hence must capitulate to Socialism, that owing to the dialectical character of history, each historical period carries within itself the "*germs of its own destruction*" (Hegel's principle of negativity).

Marx held that the victory of the proletariat in their struggle could be predicted with the certainty of a scientific experiment (hence the term '*Scientific socialism*'). However, the history of the world in the 13 decades that elapsed since the appearance of the *Capital* (culminating with the collapse of Communism in Soviet Russia and Eastern Europe) proved that there were many blind spots in Marx' socialist theories: while the rich were getting richer, the poor did not necessarily get poorer.[235] The general standard of living in the world's industrial nations was to reach heights undreamed of by Marx. Man, furthermore, does not seem to be moved exclusively, or even primarily, by economic concerns. Despite Marx' attack on religion, the established churches have continued to play an important role even in the lives of the lower classes.

Another force that increasingly came to command the allegiance of rich and poor alike was *nationalism*. The great wars of the last century have been fought not between the "oppressed" and their "oppressors" but between workers of different nations for the defense or the greater glory of their own country. Marx' fundamental errors arise from an uncritical extrapolation of

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[234] Brother of the mathematician **Karl Wilhelm Feuerbach** (1800–1834, Germany), after which the '9-point circle' is named.

[235] **Paul Samuelson** (Nobel prize in economics, 1970) said: "one may ignore the entire life-work of Karl Marx for the impoverishment of the working class simply did not happen."

what he observed in capitalist societies[236] to all class societies, and from a disregard of the enormous influence which political, national, and moral forces have exerted on the development of capitalism as an economic system.

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[236] Much of the political tension of Europe during the first half of the 19^{th} century was a manifestation of underlying economic unrest caused by the gradual transformation of Europe's economy from agriculture to industry. In this new scheme of things, the impact of the railroad was overwhelming. Here was an entirely new industry, answering a universal need, employing thousands of people, offering unprecedental opportunities for investment, and introducing greater speed into all industrial and commercial transactions. As the workers grew in number, they became aware that they constitute a new and separate class whose interests conflicted with those of their employers. This conflict prompted the emergence of *Utopian Socialism* that proposed to distribute the profits of human labor in such a way that every member of society receive an equitable share, economically, socially and politically. ("From each according to his capacity, to each according to his need".)

However, the Utopian Socialists [**Henri de Saint-Simon** (1760–1825), **Charles Fourier** (1772–1837); **Louis Blanc** (1811–1882), **Robert Owen** (1771–1858)] failed because they believed in the natural goodness of man and the perfectibility of the world. A more realistic and more militant type of Socialism was needed that would use the worker's potential economic and political power to wrest concessions from the middle class.

The class-struggle, as enunciated by Karl Marx, is the main doctrine of the theory as well as the means of achieving socialism. Because the economic forces in the modern world are in constant conflict, Marx proclaimed that the working classes, out of historic necessity, must make their bid for power by uniting, bringing about social and political changes and achieving dominance in society through the "dictatorship of the proletariat". Marx's basic error was his failure to appreciate the importance of *noneconomic forces* in society: religion, emotion, prestige, genius, stupidity and such factors as climate and geography. His economic theories themselves based on conceptions of *static economy*: the theory of surplus value did not take sufficient account of the importance of capitalist equipment, administrative ability, initiative, and the willingness to take risks. The capitalist-industrial revolution, far from pressing more and more people into proletarian poverty, increased production to such an extent that it improved the general standard of living of all men. Indeed, in the most highly industrialized countries the proletarian class is rapidly disappearing. The great revolutions inspired by Marxist doctrines have taken place not in industrial societies (where Marx expected them to occur), but in societies still overwhelmingly agricultural, beset by very real economic hardships. In all revolutions inspired by Marxism, the *state* has played a dominant role in the reorganization of society, but nowhere is there any sign of its withering away.

Despite errors and shortcomings in his teachings, however, the contribution of Marx to modern thought have been most fruitful. By bridging the gap between politics and economics, he enriched our understanding of the past. Prior to Marx, the division of society into rich and poor, haves and have-nots, was accepted as a natural, unchangeable fact. It was chiefly due to Marx that we came to realize the importance of economic factors, being jolted out of complacent acceptance of the *status quo*. By predicting far reaching changes, he made people aware that changes were possible. The threat of revolutionary changes, conjured up in Marx' writings, did much to hasten the peaceful evolution that has so markedly improved the condition of the lower class in all industrial societies. Almost every social movement in the 20th century has been influenced by Marxist ideology.

Karl Marx was born in Trier (Trèves), Rhenish Prussia. His paternal grandfather, Meir Levi (later surnamed Marx[237]), was the Rabbi of Trèves and his paternal grandmother, Chaya (neé Lwow), was a descendant of an unbroken chain of Ashkenasi rabbis, at least 8 centuries long[238]. His father, Hirschel (Heinrich) Marx (1782–1838), was a lawyer and judge in Trèves. He married Henriette Pressburg, daughter of the Rabbi of Nijmegen, Holland, and had his entire family baptized as Christian Protestants (1824) for business and social reasons.[239]

Marx went (1835) to the universities of Bonn and Berlin. He studied first law, then history and philosophy and in 1841 took the degree of doctor of philosophy. In Berlin he became acquainted with the philosophy of Hegel and interacted with the 'Young Hegelians'[240]. At 24 he became an editor of a

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[237] Marx was the 12^{th} generation of the famous Jewish medieval scholar **Rabbi Meir Katzenellenbogen**, the MAHARAM of Padua (1482–1565), the great ancestor of Europe's leading Rabbis, Talmudists and heads of the Rabbinic Courts in principal cities and towns for over three centuries. The MAHARAM himself was the 17^{th} generation of RASHI (1040–1105).

[238] **Marcus** is an ancient Roman name and means: "belonging to the god Mars". Jews with Hebrew name of **Moshe** or **Mordecai** often selected Marcus or Mark as the non-Hebrew name. This became the family name Marks or Marx.

[239] Although a trained lawyer, he could not practice law as a Jew in Trier, Prussia.

[240] **Karl Marx** was a product of this school of thought. Unlike his fellow renegades **Ludwig Feuerbach**, **Bruno Bauer** [*Die Judenfrage*, 1843], and **G. F. Daumer** who became virulent racist antisemites, Marx himself stopped short of full-fledged antisemitism, but in his own way reinforced the negative stereotype of the Jew as the personification of modern capitalism, which would later be adopted by the Nazis and their imitators. Thus, the implementation of Marx's vision ["As soon as society succeeds in destroying the empirical essence of Judaism, the Jew will become *impossible*... The total emancipation of Jewry, is

paper in Cologne, Germany, but his radical ideas soon got him into censorship trouble and he had to flee to Paris to escape arrest. With him went his young wife, Jenny von Westphalen, whom he had married (1842) in spite of both families misgivings. [She was a most faithful companion to Marx during all the vicissitudes of his career and died in 1881 after bearing him 6 daughters, 3 of whom reached marriageable age and 2 of whom outlived him.]

In Paris (1844) Marx met **Friedrich Engels** (1820–1895), a son of a German textile manufacturer, whose ideas were in complete accord with Marx' and who collaborated with him in writing. This was the beginning of a close friendship and an uninterrupted collaboration and exchange of ideas which lasted for nearly 40 years. He also befriended the poet **Heinrich Heine**, who contributed some of his poems to Marx' radical magazine. Following his expulsion from Paris (1845), Marx lived for a time in Brussels, Belgium. He later returned to Paris (1848) but only to be expelled again (1849).

Marx then went to England and made his home in London for the next 34 years. He lived in wretched poverty (3 of his children died through the lack of medicines). Sometimes Marx could not go out because his clothes were at the pawnbroker. He spent day after day in the British Museum library, his bills being paid by Engels.[241]

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the emancipation of society from Judaism", 1843] of Communism in the USSR in the name of 'human emancipation' would cause untold suffering to Jews and other national or religious minorities. His writings were used in the Soviet Union to justify the most vulgar antisemitic propaganda.

At the same time, the fact that the founder of Communism was himself born a Jew, made him the arch-symbol of Jewish revolutionary subversion for the conservative and radical Right all over the world! Modern antisemitism seized on the prominent role which 'non-Jewish Jews' like Marx played in Socialist, Communist and other radical movements to construct a new myth of the Jew as the 'ruthless cosmopolitan' enemy of all national values, religious traditions, social cohesion, and bourgeois morality.

[241] Marx great intellectual talents could only be matched by his tenacity and perseverance: he would be found at his writing desk from nine in the morning until three o'clock the next morning — with time off only for meals and bedtime stories. When he worked at the British Museum he would arrive when the library opened at nine, and leave only when it closed at seven: to a penniless refugee, the great domed reading room offered the advantages of dependable central heating and comfortable chairs.

He would encourage his disciples to study harder. "Learn, learn," was the categorical imperative which he would shout often at them, though the message was already conveyed in the example he set up and of what they could see of

While in Brussels, Marx and Engels had written the epochal revolutionary pamphlet '*Manifesto of the Communist Party*' (1848). It contains the simplest expression of Marx' beliefs (see Table 4.4). The ideas in it were later developed at length in the three volumes of his major work '*Das Kapital*'.

Some historians claimed that the ideas expounded in the *Manifesto* were directly taken by Karl Marx from the writing of **Adam Weishaupt**[242] (1778), the founder of the *Order of the Illuminati* in Bavaria.

Only eight people were present to hear Engels' funeral oration in Highgate Cemetary on March 15, 1883. Marx' descendants, the Longuet family, live today in France.

Marx never had a steady income. No one knew anything about him outside a small circle of German exiles and a few intellectuals (only in 1917, with the rise of the communists in Russia, the works of Marx became known in Europe). Marx's economic theories made no immediate impact on the debate inside the worker's movement or on other thinkers except after his death (1883). This is true of his theories on *value* and *surplus value*, *accumulation*, *exploitation*, *crisis and appropriation*, *class struggle* and *revolution*. But by the end of the 19th century, several such theories were hotly discussed with the worker's movement.

Marx was an apostate Jew, he had no Jewish education and never sought to acquire any. He tried to shut Judaism showing the smallest concern for any of the injustice inflicted on Jews throughout his lifetime. But his suppressed

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his own tremendous labors.

[242] **Weishaupt** (1748–1811, or 1830) was born in Ingolstadt, Bavaria of orthodox Jewish parents who had converted to Catholicism in 1748. When his father died (1754) young Adam was turned over to be raised by the Jesuits. He graduated from the University of Ingolstadt (1768) and was made a professor of Law (1772), after his conversion to Protestantism. He was initiated as a Freemason (1774) and then founded (1776) the *Order of the Illuminati*, some proto-communist organization dedicated to bringing about a proletarian revolution. In 1784 the Illuminati attempted a coup against the Habsburgs, but the plot was revealed by police spies that had infiltrated the order. This led to the total ban of all secret societies in Bavaria and membership was punished by death. Weishaupt was forced to flee to a neighboring province (1785). The *French Revolution* of 1789 has been widely attributed to the machinations of the Illuminati. Although this statement is an exaggeration, it cannot be denied that several persons who were intensively involved in the revolution were active members, among others the **Comte de Mirabeau** (1749–1791). After the rise of **Napoleon Bonaparte**, the Illuminati were utterly crushed.

self-hatred manifested itself through venomous attacks on friends, benefactors and especially Jews.

However, despite his ignorance of Judaism as such, there can be no doubt about his Jewishness: his notion of progress was profoundly influenced by Hegel, but his sense of history as a positive and dynamic force in human society, governed by iron laws, is profoundly Jewish. His Communist vision is deeply rooted in Jewish apocalyptic thought and messianism. His methodology too, was wholly rabbinical: all his conclusions were derived solely from books, his temperament was religious, and he was quite incapable of conducting objective, empirical research. Marx's theory of how history, class and production operate, and will develop, is not a scientific theory at all but a Kabbalistic theory of the Messianic Age.

The roots of Marx's anti-Semitism went deep. He was not merely a Jewish thinker, but also an anti-Jewish thinker. Therein lies the paradox, which has a tragically important bearing both on the history of Marxist development and on its consummation in the Soviet Union and its progeny. Marx's personal anti-Semitism, however disagreeable in itself, might have played no great part in his lifework had it not been part of a systematic theory in which Marx profoundly believed. In fact it is true to say that Marx's theory of Communism was the end-product of his theoretical anti-Semitism.

Worldview XXII: Marx

* * *

"Die Philosophen haben die Welt nur verschieden interpretiert; es kommt darauf an, sie zu verändern."
"The philosophers have only interpreted *the world in various ways; the point, however, is to* change *it."*

(Epitaph on his tomb in Highgate Cemetery, London)

* * *

"Darwin's book is very important and serves me as a basis in natural science for the class-struggle in history."

* * *

"Natural science will in time include the science of man, as the science of man will include natural science; there will be one science."

* * *

"Lucretius is the truly Roman heroic poet; his heroes are the atoms, indestructible, impenetrable, well-armed, lacking all qualities but these; a war of all against all, the stubborn form of eternal substance. Nature without gods, gods without a world."

* * *

"At the entrance to science, as at the entrance to Hell, there should be posted the demand: 'Here the spirit should be firm. Here the promptings of fear should be heeded'."

* * *

"The more of himself man attributes to God, the less he has left in himself."

* * *

"Religion is the opiate of the people."

* * *

"Social revolution never occur because of the weakness of the strong; for that you need the strength of the weak."

Philosophies of Social Criticism

By the middle of the nineteenth century, the coming of the Industrial Age, with its revolutionary political, social, and economic effects, had made itself felt over most of Europe and it also extended to other parts of the world. For the next two decades, people's attention in Europe and the United States became absorbed by momentous political developments. Once the political situation had become stabilized, however, shortly after 1870, a second wave of economic development swept over Europe and the world, a wave of such magnitude that it is often referred to as a "Second Industrial Revolution."

Marxian socialism, in its ultimate effects on society, turned out to be the most important attack on the capitalist philosophy of laissez faire. There were other critics of this philosophy, however, who tried in various ways to awaken their contemporaries to the social problems created by the industrialization of society.

Humanitarianism

Writers like ***Honoré de Balzac*** *in France and* ***Charles Dickens*** *in England, by dwelling in their novels on the more sordid aspects of the new industrialism, played on human sympathy in the hope of creating a climate favorable to reform. The historian* ***Thomas Carlyle****, in his* Past and Present *(1843), showed deep concern over the growing division between the working classes on the one hand and the wealthy classes on the other. He turned against the "mammonism" and the "mechanism" of his age and admonished the new captains of industry to be aware of their responsibilities as successors to the old aristocracy. Like Carlyle,* ***Benjamin Disraeli****, one of the rising young Tories, in his social novel* Sybil *(1845), deplored the wide gap that industrialization had opened between the rich and the poor. It was, he said, as though England had split into two nations "between whom there was no intercourse and no sympathy."*

Anarchism

One other form of social protest, the effects of which were not felt until later in the 19^{th} *century. It is the belief that every form of regulation or government is immoral, and that restraint of one person by another is an evil which must be destroyed (*Anarchism *comes from Greek word meaning* without *government). Anarchism dates back to ancient times. It also appeared among early Christian groups.*

Anarchism, like socialism, hoped to overthrow capitalism. But while the socialists were ready to use the state as a stepping stone for the realization of their aims, the anarchists were deeply opposed to any kind of governmental authority and organization. One of the earliest theorists of anarchism and the first to use the word anarchy, *was the French social philosopher* ***Pierre-Joseph Proudhon*** *(1809–1865). In his pamphlet First Memoir on Property (1840) he asked the question, "What is property?" and replied with the well-known slogan, "Property is theft!" This seeming opposition to private property appeared to align Proudhon with communism and endeared him to Marx. The latter's admiration cooled, however, when he discovered that Proudhon was less interested in overthrowing the middle class than in raising the worker to the level of that class. Proudhon was against any kind of government, be it by one man, a party, or a democratic majority. "Society," he wrote, "finds its highest perfection in the union of order with anarchy."*

Thus Proudhon, often called the father of anarchism, became the first to make anarchism a mass movement. His philosophical, *or* individualistic, *anarchism urged the willing cooperation of free men without any regulation (law) or government.*

Terroristic *anarchism began under the leadership of* ***Mikhail Bakunin*** *(1814–1876, Russia). A theorist of anarchism, he also practiced what he preached. Bakunin was involved in several revolutions, was three times condemned to death, and spent long years in prison and Siberian exile. Most of the evils of his day Bakunin attributed to two agencies — the state and the Church. His ideal society was a loose federation of local communities, each with a maximum of autonomy. In each of these communities the means of production were to be held in common. The way to achieve this governmentless state of affairs, Bakunin held, was not by waiting patiently for the state to wither away, as Marx had held, but by helping matters along, if necessary by means of terrorism, assassination, and insurrection. The years 1881–1912 witnessed a whole series of assassinations attributed to anarchists (Czar Alexander II of Russia, 1881; President Carnot of France, 1894; Prime Minister of Spain Antonio Canovas del Castilio, 1897; Empress Elizabeth of Austria, 1898; King Humbert of Italy, 1900; President William McKinley of the United States, 1901; Prime Minister of Spain José Canalejas y Mendez, 1912).*

Anarchism under the leadership of the Russian physical geographer and political philosopher Prince ***Peter Alekseievitch Kropotkin*** *(1842–1921), during the late 1800's assumed a more rigid communistic form. Kropotkin* *rejected the terroristic methods of Bakunin, but he also opposed the authoritative type of communism.*

Under such kind of anarchism, the state would be eliminated and the future society would be built on the communes, *or* village communities, *which had existed in feudal Russian society. Each commune would be a self-sufficient group.*

He advanced a theory of mutual aid *(1904) as a rationale for eliminating all forms of government except true self-government. He based this theory on the evidence of various studies of animal behavior, and his main source for inspiration was* **Darwin's** *theory of evolution through the survival of the fittest. Kropotkin insisted that* mutual aid *is as important a principle of nature as* mutual hate; *that* mutual hate, *and the struggle for existence, as Darwin had shown, exists only with respect to different species; that among the same species there is a spirit of mutual* cooperation *for existence; and that since man is a single species, all men should cooperate and help each other to survive. Thus, "the aspirations of man are at one with nature," and "mutual aid, therefore, is the predominant fact of nature."*

Born into the Russian nobility, Kropotkin entered the Imperial army in 1862 and served until 1867. Visiting Switzerland in 1872 he became a convinced anarchist, under the influence of Bakunin's teaching. Back in Russian he began active propaganda for the movement, and in 1874 was arrested and imprisoned. Two years later he escaped and fled from Russia, beginning an exile, mainly spent in London, ended only by the revolution of 1917. He saw anarchist communism very much as the next natural stage of social revolution, and part of the wider evolutionary process. In contrast to Darwin's social disciples, he believed that mutuality and cooperation were features of the animal world and already significant forces in society, however masked by coercive government. With **Marx** *he believed that modern productive techniques opened up possibilities of good living conditions for all; capitalism with its wage system must be replaced by communism. The Soviet state which emerged from the Bolshevik revolution did not have his sympathies. He did not reject the use of force, and supported the Allies against Germany in WWI. His* Mutual Aid: a Factor of Evolution *(London, 1902) is one of the continuing classics of anarchist thought.*

During the 1870's some of the nihilists fell under the influence of Mikhail Bakunin and his philosophy of anarchism. In 1879 this terrorist faction formed a secret society, "The Will of the People," whose aim was to overthrow the government by direct action and assassination.

Frightened by these manifestations of radicalism, Alexander II reverted to a policy of renewed reaction. Yet by reverting to repression, he merely helped to strengthen the revolutionary forces he hoped to combat. This fact was *brought home to him in several attempts on his life, and in 1880 he tried once again to return to his initial policy of reform. But by then it was too late.*

Anarchists tried to mobilize working-class support behind the Russian General Strike which was a central feature of the Russian Revolutions of 1905 and 1917. But anarchism never developed into a well-defined movement, partly because of Bakunin's death in 1876, partly because of the impracticable nature of its doctrine.

The strength and influence of anarchism declined throughout the world in the 1900's after the rise of totalitarian states elsewhere. They were active in the Spanish Civil War of 1936–1939, and in the latter half of the 20th century anarchism attracted urban terrorists.

1848–1851 CE **Armand Hippolyte Louis Fizeau** (1819–1896, France). Outstanding experimental physicist. In an experiment (1851) of great historical value[243], he showed that the Galilean transformation of velocities does not apply to the velocity of light in moving media. This result constituted a strong motivating factor for Einstein in his development of relativity theory.

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[243] Consider a medium in which the velocity of light is $u = \frac{c}{n}$ when the medium is at rest w.r.t. an observer. If the medium itself moves with velocity v w.r.t. that observer, the STR predicts that the combined velocity of light relative to the observer is

$$U = \left(\frac{c}{n} + v\right) / \left(1 + \frac{c}{n}\frac{v}{c^2}\right) = \frac{c}{n} + v\left(1 - \frac{1}{n^2}\right)\left(1 + \frac{v}{nc}\right)^{-1} \approx \frac{c}{n} + v\left(1 - \frac{1}{n^2}\right),$$

keeping only terms of order $\left(\frac{v}{c}\right)$. This is in agreement with Fizeau's experiment, but contrary to the prediction

$$U = \frac{c}{n} + v$$

which is obtained by using the Galilean, rather then the Lorentz transformation. In pre-relativistic times, the extra term was attributed to the dragging along of the ether by the moving body which, in turn, was accounted for by ad-hoc theories of **A.J. Fresnel** (1788–1827) and **G.G. Stokes** (1819–1903). After the ether theory was demised by the Michelson-Morley experiment, Fizeau's experiment remained without a plausible explanation until 1905.

In 1849, Fizeau devised a laboratory experiment to measure the velocity of light in air and in water. In 1850, he measured the velocity of propagation of the electromagnetic field in matter: his values ranged from $\frac{1}{3}c$ in iron wires to $\frac{2}{3}c$ for copper wires. In 1848, he established experimentally the existence of the *Doppler effect* for light waves and discovered the so-called '*red shift*'.

1848–1872 CE **Heinrich Eduard Heine**[244] (1821–1881, Germany). Mathematician. He was born in Berlin, Germany, the eighth of nine children. His father was a banker. Eduard studied at Berlin and Göttingen and from 1848 was a professor at the University of Halle. He was still teaching there when Georg Cantor joined the faculty in 1874.

Influenced by Weierstrass' lectures at Göttingen, Heine introduced the ϵ–δ definition of limits. He published about 50 papers, most of them dealing with special functions, but his name is best known for its association with the so-called Heine-Borel theorem. Borel's name is associated with this result due to his recognition of its importance and his use of it as a separate theorem in 1895.

1848–1878 CE **Karl Theodor Wilhelm Weierstrass** (1815–1897, Germany). One of the greatest mathematical analysts of the 19^{th} century. Sought to separate analysis, especially the calculus, from geometry and reduce the principles of analysis to real number concepts (a program known as '*arithmetization of analysis*').

Weierstrass began his mathematical career with papers on Abelian functions, but his most important contribution to mathematics is his construction of the *theory of complex functions* by means of power series. He showed special interest in entire functions and functions defined by infinite products. He rigorized the concept of *uniform convergence*[245] (1854) and exhibited a class of continuous non-differentiable functions[246].

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244 A member of the Jewish banking family founded by **Solomon Heine**, and therefore a relative of the poet **Heinrich Heine**.

245 Discovered independently at about the same time by **Cauchy** (1853) and **G.G. Stokes** (1847) and by **P.L.V. Seidel** (1821–1896, Germany) in 1848.

246 In 1876 Weierstrass stated that the function

$$\sum_{n=0}^{\infty} b^n \cos(\pi a^n x), \qquad a > 1, \qquad \frac{1}{a} < b < 1,$$

is continuous but nowhere differentiable. Earlier, in 1872, he published the result only for $a =$ odd integer and $ab > 1 + \frac{3\pi}{2}$. Weierstrass was not the first to produce functions of this kind. Riemann had asserted already in 1861

In algebra, he gave a postulational definition of a determinant and contributed to the theory of bilinear and quadratic forms. One of the important contributions of Weierstrass to analysis is known as *analytic continuation.* He has shown that the infinite power series representation of a function $f(z)$, about a point z_1 in the complex plane, converges at all points within a circle C_1 whose center is z_1 and which passes through the nearest singularity. If now, one expands the same function about a second point z_2 within C_1, $z_2 \neq z_1$, this series will be convergent within a circle C_2 having z_2 as center and passing through the singularity nearest to z_2. This circle may include points outside C_1, hence one has expanded the area of the plane within which $f(z)$ is defined analytically by power series. The process can be continued with still other circles. Weierstrass thus defined an analytic function as one power series together with all those that are obtainable from it by analytic continuation. The impact of this idea is felt particularly in mathematical physics, in which solutions of differential equations are rarely found in any form other than as an infinite series.

In his drive to arithmetize the calculus, Weierstrass contributed to the definition of a real number and provided an improved definition of the limit concept. He is the herald of the *age of rigor*, replacing older heuristic devices and intuitive views by critical, logical precision. In todays textbooks the definitions of a limit of a function are in essence those introduced by Weierstrass and Heinrich Eduard Heine (1821–1881, Germany) a century ago, and the so-called delta-and-epsilon proofs, or *epsilontics*, are now part of the mathematicians stock-in-trade.

Weierstrass was born at Ostenfelde in the district of Münster, Germany. His father was a customs officer in the pay of the French (who at the time

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(without proof) that the function

$$f(x) = \sum_{n=1}^{\infty} \frac{\sin(n^2 x)}{n^2}$$

was nowhere differentiable. A deeper insight into this function was, however, only achieved in 1916, when **Hardy** proved that $f(x)$ has no finite derivative at any point $\pi\xi$, where ξ is either irrational or rational of the form $\frac{2A}{4B+1}$ or $\frac{2A+1}{2B}$ (A and B integers). **J. Gerver** (1970) continued Hardy's efforts and showed that for rational ξ of the form $\frac{2A+1}{2B+1}$ the Riemann function has, on the contrary, the finite derivative $\left(-\frac{1}{2}\right)$. Thus Gerver succeeded to show that the Riemann function does not in fact belong to the class of continuous, nowhere differentiable functions. In 1972 **A. Smith** gave a proof that $f(x)$ has no finite derivative *at any point other* than those of the Gerver form.

dominated Europe). His mother died when he was 11 years old, and his stepmother contributed very little, to say the least, to Karl's education.

Weierstrass not only did extremely well at school, but at the age of 15 was able to secure a job as an accountant. After analyzing Karl's qualities, his father concluded that he should prepare for public service: the youngster was sent to study law at the University of Bonn. After four years at Bonn, Karl returned home an expert in drinking and fencing, but without the law degree. He was then sent by his family to embark on a secondary-school teaching career, at the Academy of Münster, for which he was ready in 1841. At Münster Weierstrass became fascinated by the lectures of his mathematics professor, **Christoph Gudermann**[247], who was an enthusiast of elliptic functions. Gudermann's idea was to base everything on the power series representation of a function. This idea was the main tool for the greatest part of Weierstrass' work. Karl spent the next 15 years in the capacity of an unknown school teacher in an obscure village. However, the publication of his memoir on Abelian functions in the Crelle Journal in 1854 brought him at once into the limelights of the mathematical world, and he moved to Berlin in 1856. But only in 1864 was he awarded full professorship at the University of Berlin and could finally devote all his time to advanced mathematics.

Due to occasional spells of vertigo, he never trusted himself to write his own formulae on the blackboard, but dictated to an assistant who wrote them for him. Among the most important of his students were **Hermann Amandus Schwarz** (1843–1921), **Sonja Kovalevsky** (1850–1891), **Georg Cantor** (1845–1918), **Magnus Gösta Mittag-Leffler** (1846–1927) and **David Hilbert** (1862–1943).

1849 CE **Paul Julius von Reuter** (1816–1899, Germany). Journalist. Founded in Aachen (1849) a central telegraphic and pigeon-post bureau for collecting and transmitting news, forerunner of Reuter's News Agency with headquarters in London (from 1851); removed to England (1851) and became naturalized British subject; created baron (1871) by Duke of Saxe-Coburg-Gotha.

Born at Cassel, Germany as **Israel Beer Josaphat**, he was baptized (1844), when he assumed the name Reuter. At the age of 13 he became a

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247 **A. Cayley** called the function $\phi(u) = \sin^{-1}\{\tanh u\}$ the *Gudermannian* of u, and denoted it by $gd\,u$. Then

$$u = gd^{-1}\phi = \log\{\tan\phi + \sec\,\phi\} \equiv \log\tan\left[\frac{\pi}{2} + \frac{\phi}{2}\right]$$

clerk in his uncle's bank at Göttingen, where he chanced to make the acquaintance (1829) of **Carl Friedrich Gauss** (1777–1855), whose experiments in telegraphy were then attracting some attention. Reuter's mind was thus directed to the value of the speedy transmission of information, and in 1849, on the completion of the first telegraph lines in Germany and France he found an opportunity of turning his ideas to account.

There was a gap between the termination of the German line at Aix-la-Chapelle and that of the French and Belgian line at Verviers. Reuter organized a news-collecting agency at each of these places, his wife being in charge of one, himself at the other, and bridged the interval by pigeon-post. On the establishment of through telegraphic communications, Reuter endeavored to start a news agency in Paris, but finding that the French government restrictions would render the scheme unworkable, removed to England (1851).

The first submarine cable (between Dover and Calais) had just been laid, and Reuter opened an office in London for the transmission of intelligence between England and the Continent. In 1859 Reuter extended his sphere of operations all over the world. In 1866 he laid down a special cable from Cork to Crookhaven, which enabled him to circulate news of the American Civil War several hours before the steamer could reach Liverpool.

1849–1859 CE **Arthur Cayley** (1821–1895, England). One of the principal mathematicians of the 19^{th} century and one of the most prolific mathematicians of all times, rivaled in productivity only by Euler and Cauchy [the number of his papers and memoirs exceeded 800]. Cayley wrote upon every subject of pure mathematics and also upon theoretical dynamics, physical astronomy and physical geography. He was primarily an algebraist, and his main achievements are as follows:

(1) Created the theory of matrices[248] and developed it as a pure algebra

[248] Among the key theorems discovered by him is the *Cayley-Hamilton Theorem*: Every matrix satisfies its own characteristic equation, i.e. if $f(\lambda)$ is the *characteristic polynomial* of $\boldsymbol{A}$, then $f(\boldsymbol{A}) = 0$. This theorem was established by **Hamilton** (1853) for a special class of matrices. **Cayley** (1858) enunciated the general result without proof.

The manipulation of matrices is often generally facilitated by the Cayley-Hamilton theorem, which provides an easy method for expressing any polynomial in $\boldsymbol{A}$ as a polynomial of degree not exceeding $n-1$, when n is the degree of $f(\lambda)$. Thus, if $\boldsymbol{A}$ satisfies the equation

$$\boldsymbol{A}^4 - \pi^2\boldsymbol{A}^2 = 0,$$

we have

$$\sin\boldsymbol{A} = \boldsymbol{A} - \pi^{-2}\boldsymbol{A}^3.$$

(1857); showed that quaternions can be represented as matrices of special form. [Matrices, considered as arrays of coefficients in homogeneous linear transformations, were tacitly in existence long before Cayley, but their properties were not studied for their own sake.] The importance of matrix theory in the mathematical machinery of modern physics echo the prophetic statement made by P.G. Tait: "*Cayley is forging the weapons for future generations of physicists*[249]".

(2) Initiated the ordinary analytic geometry of n-dimensional space (1843), using determinants as the essential tool and introducing the modern notation for determinants. [Simultaneous work on the same subject was done by **Grassmann** and **Ludwig Schläfli** (1814–1895, Switzerland, 1852).]

(3) Introduced the notion of an abstract finite group and what we call today '*finite group algebra*'.

(4) Contributed to the theory of algebraic invariants, which later proved to be essential to tensor algebra.

(5) Research on the singularities of curves and surfaces[250].

Similarly, it can be shown that if

$$\boldsymbol{S} = \begin{bmatrix} 0 & \nu & -\mu \\ -\nu & 0 & \lambda \\ \mu & -\lambda & 0 \end{bmatrix},$$

then

$$e^{\boldsymbol{S}} = \boldsymbol{I} + \frac{\sin\omega}{\omega}\boldsymbol{S} + \frac{1-\cos\omega}{\omega^2}\boldsymbol{S}^2,$$

where

$$\omega^2 = \lambda^2 + \mu^2 + \nu^2.$$

The latter result is immediately applicable to the general finite three-dimensional rotation matrix about an arbitrary axis.

249 His work on matrices served as a primary mathematical tool for the theory of quantum mechanics as developed by **Heisenberg** (1925)

250 In a memoir "on Contour and Slope Lines" (*Philosophical Magazine 18* p.264 1859) Cayley introduced the first elements of *physical geography* such as topological *contour-lines* and its application to geological surveying, as well as to mathematical topology. He discovered the relations $S = P + 1$ ($S =$ number of summits, $P =$ number of passes) and $I = B + 1$ ($I =$ number of bottoms, $B =$ number of bars), deducing it from the theory of maxima and minima of continuous functions of two variables.

Unaware of this contribution, **J. C. Maxwell** (*Philosophical Magazine* 1870) rederived most of Cayley's results.

(6) By developing algebras satisfying structural laws different than those obeyed by common algebra, he opened [together with **Hamilton** and **Grassmann**] the floodgates of modern abstract algebra to an enormous variety of systems. Some of these are known as: groupoids, quasigroups, semigroups, monoids, rings, lattices, fields, vector spaces and Boolean algebras[251].

(7) Showed in 1885 that three-dimensional as well as four-dimensional rotations can be represented by quaternions. Similar results were obtained by **Felix Christian Klein** (1849–1925); hence the '*Cayley-Klein rotation parameters*'. These were systematically used by Felix Klein and **Sommerfeld** (1868–1951, Germany) in their classical book "*Über die Theorie des Kreisels*" (1897).

Cayley was born at Richmond in Surrey, of Russian origin on his mother's side. At age 14 he arrived at King's College school, London. He soon showed remarkable mathematical ability and entered Trinity College, Cambridge. In 1842 he graduated Senior Wrangler. In 1846 Cayley decided to adopt the law as a profession and indeed practiced law during 1849–1863. Then he was elected to the new Sadlerian chair of pure mathematics in Cambridge, which he held thereafter.

[251] As early as 1849 Cayley wrote a paper linking his ideas on permutations with Cauchy's. In 1854 Cayley wrote two papers which are remarkable for the insight they have of abstract groups. At that time the only known groups were groups of permutations and even this was a radically new area, yet Cayley defined an abstract group and gave a table to display the group multiplication. He realized that matrices and quaternions formed groups.

Matrix Algebra[252] *– A powerful Mathematical Tool*

It is clear from the historical survey shown in Table 4.5 that the earliest notions of determinants arose 23 centuries ago in connection with the simplest algebraic structures then known to mathematicians, namely the solution of linear systems of equations.

The subject of matrices, too, was well developed before it was 'officially' created by ***A. Cayley*** *(1858). Logically, the idea of a matrix precedes that of a determinant but historically the order was the reverse and this is why the basic properties of matrices were already clear by the time matrices were introduced: just because the use of matrices were well-established, it occurred to Cayley to introduce them as distinct entities.*

Determinants and matrices arose in connection with elimination theory, transformation of coordinates, change of variables in multiple integrals, solution of systems of differential equations arising in planetary motion and reduction of quadratic forms in 3 or more variables (geometrical surfaces) to standard form.

In themselves matrices and determinants say nothing directly that is not already stated in the equations or the transformations themselves. Neither determinants nor matrices have influenced deeply the course of mathematics despite their utility as compact expressions. Nevertheless, both concepts have proved to be highly useful tools and are now part of the apparatus of mathematics.

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252 For further reading, see:

- Mirsky, L., *An Introduction to Linear Algebra*, Oxford University Press: London, 1955, 433 pp.
- Turnbull, H.W. and A.C. Aitken, *An Introduction to the Theory of Canonical Matrices*, Blackie & Son: London, 1952, 200 pp.
- Turnbull, H.W., *The Theory of Determinants, Matrices and Invariants*, Dover Publications: New York, 1960, 374 pp.
- Barnett, S., *Matrices*, Oxford University Press, 1990, 450 pp.
- Heading, J., *Matrix Theory for Physicists*, Longmans, Green and Company: London, 1958, 241 pp.
- Gantmacher, F.R., *The Theory of Matrices*, Vols I-II, Chelsea Publishing Company: New York, 1960, 366+276 pp.
- Lay, D.C., *Linear Algebra and Its Applications*, Addison Wesley, 2003, 492 pp.
- Hohn, Franz E., *Elementary Matrix Algebra*, Dover: New York, 2002, 522 pp.
- Stephenson, G., *An Introduction to Matrices, Sets and Groups for Science Students*, Dover: New York, 1965, 164 pp.

Table 4.5: Milestones in the history of matrices and determinants

Year(s)	Mathematicians and their achievements
ca 300 BCE	*Babylonians studied problems which lead to simultaneous linear equations.*
200–100 BCE	*In the text 'Nine Chapters of the Mathematical Art', written during the Han Dynasty, Chinese mathematicians gave the first known example of matrix methods: the author set up coefficients of a system of 3 linear equations in 3 unknowns as a table on a 'counting board' and then proceeded to solve the system by a method now known as the 'Gaussian elimination' (early 19th century).*
1545 CE	***Cardano** (in his 'Arts Magna') gave a rule for solving a system of two linear equations, now known as 'Cramer's Rule'.*
ca 1658 CE	***De Witt** (in his 'Elements of Curves') showed how a transformation of the axes reduces a given equation of a conic to canonical form. This amounts to diagonalizing a symmetric matrix.*
1683 CE	***Leibniz** (Germany) and **Seki Kowa** (Japan) independently introduced determinants and gave methods for calculating them. Seki knew that a determinant of n^{th} order, when expanded, has $n!$ terms and that rows and columns are interchangeable. Leibniz, on the other hand, knew that the solubility condition for an homogeneous linear system of equations is that the coefficient matrix has determinant zero. He also proved what is essentially Cramer's Rule and that a determinant could be expanded using any column (now called the 'Laplace expansion'). Leibniz also studied quadratic forms which led naturally towards matrix theory.*

Table 4.5: (Cont.)

YEAR(S)	MATHEMATICIANS AND THEIR ACHIEVEMENTS
ca 1735 CE	***Maclaurin*** *(in his Treatise of Algebra) proved Cramer's Rule* for 2×2 and 3×3 *systems of equations.*
1747 CE	**d'Alembert** *introduced the concept of an eigenvalue while studying the motion of a string with masses attached to it at various points.*
1750 CE	***Cramer*** *gave the general rule for a* $n \times n$ *system of equations. It arose out of a desire to find the equation of a plane curve passing through a number of given points.*
1764–1771 CE	**Bezout** and **Vandermonde** *gave methods for calculating determinants.*
1772 CE	***Laplace*** *gave an expansion of a determinant (he called it 'resolvent') which now bears his name. It arose in connection with his studies of the orbits of the inner planets.*
1773 CE	***Lagrange****, solving a problem in mechanics, showed that the volume of a tetrahedron formed by 4 points (0,0,0),* (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) is *expressible by the determinant* $$\frac{1}{6}\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}.$$
1801–1809 CE	***Gauss*** *introduced the term 'determinant' while studying quadratic forms. In this connection he described matrix multiplication and the inverse of a matrix. In his work on the orbit of the asteroid Pallas,*

Table 4.5: (Cont.)

Year(s)	Mathematicians and their achievements
	Gauss obtained a system of 6 linear equations in 6 unknowns. He gave a systematic method for solving such equations which is now known as the Gaussian elimination method.
1812–1841 CE	***Cauchy** expounded the first systematic work on determinants, introducing the concept of minors and adjoints. He proved the multiplication theorem for* $n \times n$ *determinants,* $c_{ij} = \sum_n a_{ik}b_{kj}$, *(1841). In the context of quadratic forms, Cauchy used the term 'tableau' for the matrix of coefficients. He calculated the eigenvalues and gave results on diagonalization of a matrix in the context of converting a form to a sum of squares (1826). He also introduced the idea of similar matrices (1826), showed that if two matrices are similar they have the same characteristic equation and proved (again, in the context of quadratic forms) that every real symmetric matrix can be diagonalized (1826).*
1829 CE	***Sturm** (Switzerland) defined the concept of the eigenvalue in the context of solving an ordinary differential equation. However, neither d'Alembert nor Sturm realized the generality of the idea they were introducing and saw them only in the specific context in which they were working.*
1841 CE	***Jacobi** generalized the determinant concept to include elements that are functions. **Cayley** used two vertical lines on either side of the array to denote a determinant.*
1850–1851 CE	***Sylvester** introduced the term matrix. Defined equivalence of two matrices (1851).*

Table 4.5: (Cont.)

Year(s)	Mathematicians and their achievements
1853 CE	***Hermite*** *introduced Hermitian matrices (matrix equal to its transpose conjugate) and showed that its eigenvalues are real. In 1854 he was first to use orthogonal matrices.*
1858 CE	***Cayley*** *created the theory of matrices, singling out the matrix for its own sake, giving it an abstract definition and establishing matrix algebra (addition, multiplication, scalar multiplication and inverses). He gave an explicit construction of the inverse of a matrix and also proved that a* 2×2 *matrix satisfies its own characteristic equation.*
1870 CE	***Jordan*** *defined the canonical or normal form of a matrix.*
1874 CE	***Kronecker*** *defined the direct matrix product.*
1878–1879 CE	***Frobenius*** *defined the minimum polynomial of a matrix as the polynomial of the lowest degree which the matrix satisfies, formed from the factors of the characteristic polynomial. He also defined the rank of a matrix (1879) as the least* r*-rowed minor whose determinant is not zero. Gave a formal definition to orthogonal matrices(equal to the inverse of its transpose). Defined congruent matrices. Proved the Cayley–Hamilton theorem for* $n \times n$ *matrices.*
1885 CE	***A. Buchheim*** *(1859–1888) proved that the eigenvalues of a real symmetric matrix are real (**Cauchy** proved it for determinants).*
1892 CE	***W.H. Metzler*** *introduced transcendental functions of a matrix, writing each as a power series in a matrix. He established series for* e^A, e^{-A}, lnA, $sinA$, $sin^{-1}A$ *for matrices* A.

Table 4.5: (Cont.)

YEAR(S)	MATHEMATICIANS AND THEIR ACHIEVEMENTS
1904 CE	***K. Hensel*** *proved that the minimal polynomial of a matrix divides any other polynomial satisfied by the matrix.*
1907 CE	*The textbook 'Introduction to Higher Algebra' by* ***M. Bôcher*** *brought matrices into their proper place within mathematics.*
1908 CE	***H. Minkowski*** *gave covariant formulation of relativistic electrodynamics in terms of matrices.*
1925 CE	***W. Heisenberg***[253] *formulated quantum mechanics in terms of matrices, establishing matrix mechanics.*

A. DETERMINANTS

Systematic treatments of determinants began with **Cauchy** *(1812–1840) and continued throughout the 19^{th} century by* **Jacobi** *(1832–1846),* **Catalan** *(1839–1846),* **Bertrand** *(1850),* **Hermite** *(1854–1856),* **Cayley** *(1855),* **Cremona** *(1856),* **Bellavitis** *(1857),* **Souillart** *(1858),* **Weierstrass** *(1858),* **H.J. Smith** *(1861),* **R.F. Scott** *(1878) and* **Hadamard** *(1892).*

The determinant

$$D_3 = \begin{vmatrix} a_{11} & \boxed{a_{12}} & a_{13} \\ \boxed{a_{21}} & a_{22} & a_{23} \\ a_{31} & a_{32} & \boxed{a_{33}} \end{vmatrix} \qquad (1)$$

stands for the number

253 Because matrix algebra was not taught in the curriculum of graduate physics in German universities, neither Heisenberg nor Born knew what to make of the appearance of matrices in the concept of the atom. **David Hilbert** had to tell then to look for differential equations with the same eigenvalues.

$$a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{12}a_{23} - a_{13}a_{22}a_{31} - a_{23}a_{32}a_{11} - a_{12}a_{21}a_{33}. \quad (2)$$

Each term in (2) comprises of a product of three elements of the determinant, such that no two elements are from the same row or the same column; e.g. the circled elements, corresponding to the last term in (2). The first index of the element indicates its row and the second index stands for its column. Note that the indices of each term in (2) can be considered as a permutation *of 1, 2, 3. Thus $a_{11}a_{22}a_{33}$ corresponds to the identity permutation*

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix},$$

while $a_{13}a_{22}a_{31}$ corresponds to the permutation

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$

and $a_{12}a_{21}a_{33}$ to the permutation

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}.$$

In general, the determinant D_n of the array

$$D_n = \begin{vmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{vmatrix}, \quad (3)$$

is the number

$$D_n = \sum \pm a_{1i}a_{2j}...a_{np} = \sum \pm a_{i1}a_{j2}...a_{pn}, \quad (4)$$

where the summation is over all permutations $(i, j, ..., p)$ of $(1, 2, ..., n)$ and the sign accords with the parity of the permutation. In each term of the sum there is one element from each row and one from each column but no two elements have their row or column in common.

***Cauchy** (1826) encountered determinants in his study of the quadratic form*

$$f(x_1, x_2, x_3) = a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + 2a_{12}x_1x_2 + 2a_{13}x_1x_3 + 2a_{23}x_2x_3. \quad (5)$$

This function can be associated with the determinant

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{31} \end{vmatrix}, \qquad a_{ij} = a_{ji},$$

where the roots of the characteristic equation $|a_{ij} - \lambda\delta_{ij}| = 0$ *determine the principal axes.*

Sylvester *(1840) came across a determinant in his studies of the theory of equations. He asked: what is the condition under which the two equations*

$$f(x) = ax^3 + bx^2 + cx + d = 0;$$

$$g(x) = px^2 + qx + r = 0$$

possess a common root? Now , if such a root exist, it is also a common root of the system of the 5 equations

$$xf(x) = 0, \quad f(x) = 0, \quad x^2 g(x) = 0, \quad xg(x) = 0, \quad g(x) = 0,$$

namely

$$\begin{aligned} ax^4 + bx^3 + cx^2 + dx \quad &= 0, \\ ax^3 + bx^2 + cx + d &= 0, \\ px^2 + qx + r &= 0, \\ px^3 + qx^2 + rx \quad &= 0, \\ px^4 + qx^3 + rx^2 \quad &= 0. \end{aligned}$$

If we treat these as 5 linear equations, homogeneous in $\{x^4, x^3, x^2, x, 1\}$*, the condition for their consistency is, by the theory of linear equations,*

$$\begin{vmatrix} a & b & c & d & 0 \\ 0 & a & b & c & d \\ 0 & 0 & p & q & r \\ 0 & p & q & r & 0 \\ p & q & r & 0 & 0 \end{vmatrix} = 0. \tag{6}$$

This determinant is Sylvester's eliminant *and comprises the relation that the 7 parameters* $(a, b, c, d; p, q, r)$ *must obey.*

Certain classes of determinants gained importance in numerical analysis and mathematical physics:

- Continuants

  A continuant is a determinant all of whose elements are zero except those in the main diagonal and in the two adjacent diagonal lines parallel to and on either side of the main diagonal, i.e. $D_{ij} = 0$ *for* $|i - j| > 1$.

A general continuant has the form

$$D_{n,1} = \begin{vmatrix} a_1 & b_1 & \cdots & & & & & \\ c_1 & a_2 & b_2 & & & & & \\ & c_2 & a_3 & b_3 & & & & \\ \vdots & & & \ddots & \ddots & \ddots & & \\ & & & & & c_{n-2} & a_{n-1} & b_{n-1} \\ & & & & & & c_{n-1} & a_n \end{vmatrix}. \tag{7}$$

Expansion in terms of the elements of the first column, leads to the difference equation $D_{n,1} = a_1 D_{n,2} - b_1 c_1 D_{n,3}$, *where* $D_{n,j}$ *means that the first element of the main diagonal is* a_j *and the last is* a_n. ***Sylvester*** *(1855) showed that continuants are linked to continued fractions in the following way:*

$$\frac{D_{n,1}}{D_{n,2}} = a_1 - \frac{b_1 c_1}{D_{n,2}/D_{n,3}}$$

develops into

$$\frac{D_{n,1}}{D_{n,2}} = a_1 - \cfrac{b_1 c_1}{a_2 - \cfrac{b_2 c_2}{a_3 - \cfrac{b_3 c_3}{\ddots \quad a_{n-1} - \cfrac{b_{n-1} c_{n-1}}{a_n}}}}. \tag{8}$$

The case $a_n = a, \quad b_n = b, \quad c_n = c$ *was investigated by* ***R.F. Scott*** *(1878)*

$$D_n = \begin{vmatrix} a & b & & & 0 \\ c & a & b & & \\ & \ddots & \ddots & \ddots & \\ & & c & a & b \\ 0 & & & c & a \end{vmatrix}_n \tag{9}$$

The $n \times n$ *determinant obeys the difference equation*

$$D_{n+2} - aD_{n+1} + bcD_n = 0 \qquad n \geq 1,$$

the solution of which is

$$D_n = \frac{(a+\Delta)^{n+1} - (a-\Delta)^{n+1}}{2^{n+1}\Delta}, \qquad \Delta = \sqrt{a^2 - 4bc}.$$

For $a = 2\cos\theta, \quad c = b = 1, \qquad D_n = \frac{\sin(n+1)\theta}{\sin\theta} = U_n(\cos\theta).$

Considering D_n *as an* $n \times n$ *matrix where* $a,\ b,\ c$ *are real and* $bc > 0$, *the eigenvalues of* D_n *are given by*

$$\lambda_s = a + 2\sqrt{bc}\cos\frac{s\pi}{n+1}, \qquad s = 1,\ 2, ..., n.$$

In the 1950's, investigations of the stability of numerical solutions of the heat conduction equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, \quad u = u(x,t), \quad u(0,t) = u(1,t) = 0, \quad t > 0$$

led to the explicit finite-difference scheme

$$u_{i,j+1} = ru_{i-1,j} + (1-2r)u_{i,j} + ru_{i+1,j}.$$

This, in turn, is manifested through the behavior of the eigenvalues of the above tridiagonal matrix with $a = 1-2r, \quad b = c = r$. *It was found that the scheme is stable for* $r \leq \frac{1}{2}$, *where* $r = \frac{k}{h^2}, \quad h =$ *spatial mesh-size,* $k =$ *temporal mesh size.*

Another case of interest is $\quad b_n = 1, \quad c_n = -1.$ *Here, the continued fraction has the form:*

$$\frac{P_n}{Q_n} = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \ddots + \cfrac{1}{a_{n-1}}}}. \tag{10}$$

where

$$P_n = \begin{vmatrix} a_0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ -1 & a_1 & 1 & \cdots & 0 & 0 & 0 \\ \vdots & & \ddots & \ddots & \ddots & & \vdots \\ 0 & 0 & 0 & \cdots & -1 & a_{n-2} & 1 \\ 0 & 0 & 0 & \cdots & 0 & -1 & a_{n-1} \end{vmatrix}_{n=1,\ 2,...}$$

$$Q_n = \begin{vmatrix} a_1 & 1 & 0 & \cdots & 0 & 0 & 0 \\ -1 & a_2 & 1 & \cdots & 0 & 0 & 0 \\ \vdots & & \ddots & \ddots & \ddots & & \vdots \\ 0 & 0 & 0 & \cdots & -1 & a_{n-2} & 1 \\ 0 & 0 & 0 & \cdots & 0 & -1 & a_{n-1} \end{vmatrix}_{n=2,\ 3,\ldots} \qquad (Q_1 = 1),$$

and where the sequential subdeterminants are

$$\begin{array}{ll} P_1 = a_0 & Q_1 = 1 \quad \textit{(defined)} \\ P_2 = a_0 a_1 + 1 & Q_2 = a_1 \\ P_3 = a_0 a_1 a_2 + a_0 + a_2 & Q_3 = a_1 a_2 + 1 \\ P_4 = a_0 a_1 a_2 a_3 + a_0 a_3 + a_2 a_3 + 1 & Q_4 = a_1 a_2 a_3 + a_1 + a_3 \end{array}$$

An example of a practical application of the above theory was given by ***Muir*** *(1889): a rapidly converging series for the extraction of a square root. It was based on previous work done on continued fractions by* ***Lagrange****, who proved that any quadratic number has a continued fraction expansion which is periodic from some point onward. So if $N > 0$ is an integer which is not a perfect square,*

$$\sqrt{N} = a_1 + \frac{1}{a_2+}\frac{1}{a_3+}\cdots\frac{1}{2a_j+}\frac{1}{a_{j+1}+}\cdots \tag{11}$$

for some $j \geq 1$. For example

$$\sqrt{41} = 6 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{12 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{12 + \cdots}}}}}} \tag{12}$$

In general, it can be shown that

$$\sqrt{N} = a_1 + \frac{Q_n(a_2, \ldots, a_n)}{P_n(a_1, \ldots, a_n)} + \frac{(-)^n}{2P_n(a_1, \ldots, a_n)Q_n(a_2, \ldots, a_n;\ a_1)} - \cdots \tag{13}$$

For the above example, this yields $\sqrt{41} = 6.403\ 124\ 237$, with an error *less than* 5×10^{-9}. *The next approximant improves the accuracy to 25 decimal places.*

Another, more complicated example of a continuant, applicable to the theory of musical scales, is the algorithm

$$\log_{a_0} a_1 = \frac{\log a_1}{\log a_0} = \cfrac{1}{n_1 + \cfrac{1}{n_2 + \cfrac{1}{n_3 + \cdots}}}, \tag{14}$$

where $\{n_1,\ n_2,\ n_3, ...\}$ *is a sequence of positive integers, and* $a_0 > a_1 > 1$. *The* n_i *are determined by the relations* $a_i^{n_i} < a_{i-1} < a_i^{n_i+1}$ *with the sequence* a_i *recursively defined by* $a_{i+1} = a_{i-1}/a_i^{n_i}$, $i = 0,\ 1,\ 2...$ *Thus, for* $a_0 = 3$, $a_1 = 2$ *the* n_i *sequence is* $[1, 1, 1, 2, 2, 3, 1...]$; *already the first 5 terms yield an excellent approximation for the musical fifth:* $\log 2/\log 3 \approx 12/19$, *from which the Greek result,* $\left(\frac{3}{2}\right)^{12} \approx 2^7$, *follows!*

Another result obtained through the above algorithm is $\log 3/\log 5 \approx 13/19$.

Note that $\frac{p_k}{q_k}$, *the* k^{th} *convergent to* $\frac{\log a_1}{\log a_0}$ *is given by* $p_k = n_k p_{k-1} + p_{k-2}$, $q_k = n_k q_{k-1} + q_{k-2}$ $(n_0 = 0,\ p_{-2} = 0,\ p_{-1} = 1,\ p_0 = 0,\ p_1 = 1;\ q_{-2} = 0,\ q_{-1} = 0)$, *where* $\{p_k,\ q_k\}$ *are the respective continuants:*

$$p_k = \begin{vmatrix} n_2 & 1 & & & & \\ -1 & n_3 & 1 & & & \\ & -1 & n_4 & 1 & & \\ & & \cdot & \cdot & \cdot & \\ & & & \cdot & \cdot & \cdot \\ & & & \cdot & -1 & n_{k-1} & 1 \\ & & & & & -1 & n_k \end{vmatrix}, \quad k = 2, 3, ...; \tag{15}$$

$$q_k = \begin{vmatrix} n_1 & 1 & & & & \\ -1 & n_2 & 1 & & & \\ & -1 & n_3 & 1 & & \\ & & \cdot & \cdot & \cdot & \\ & & & \cdot & \cdot & \cdot \\ & & & \cdot & -1 & n_{k-1} & 1 \\ & & & & & -1 & n_k \end{vmatrix}, \quad k = 1, 2, 3, ... \tag{16}$$

*The study of continuants began with **Jacobi** (1850) and **Sylvester** (1853).*

- Alternants

When the elements of the first row of a determinant are all functions of one variable, the elements of the second row are the same respective functions of a second variable, and so on, the determinant is called an alternant (and similarly for columns): for example

$$\begin{vmatrix} \sin x & \cos x & 1 \\ \sin y & \cos y & 1 \\ \sin z & \cos z & 1 \end{vmatrix} \tag{17}$$

*A well-known alternant is due to **A.T. Vandermonde**(1772),*

$$V_n = \begin{vmatrix} 1 & 1 & \cdots & 1 \\ \lambda_1 & \lambda_2 & \cdots & \lambda_n \\ \lambda_1^2 & \lambda_2^2 & \cdots & \lambda_n^2 \\ \vdots & \vdots & & \vdots \\ \lambda_1^{n-1} & \lambda_2^{n-1} & \cdots & \lambda_n^{n-1} \end{vmatrix} \tag{18}$$

It can be shown that

$$V_n = \prod_{n \geq j > i \geq 1} (\lambda_j - \lambda_i).$$

The corresponding Vandermonde matrix is thus non-singular iff all the λ's are different from each other. This matrix finds application in numerical analysis, where the coefficients of an interpolating polynomial are determined from the data: it is required to determine the polynomial

$$y = a_0 + a_1 x + a_2 x^2 + ... + a_{n-1} z^{n-1}$$

so that it passes through n given points $(x_1, y_1),\ (x_2, y_2), ..., (x_n, y_n)$. The coefficients are determined by the matrix equation $aV = y$, where

$a = [a_0, a_1, ..., a_{n-1}]$, $\quad y = [y_1, ..., y_n]$ and V is the Vandermonde matrix with $\lambda_i = x_i$.

*The study of alternants began with **Cauchy** (1812) and **Jacobi** (1841).*

- Recurrents

Determinants associated with polynomials, ratios of polynomials, binary quartics and ratios of infinite power series are known as recurrents. For example

$$a_0x^n + a_1x^{n-1}y + a_2x^{n-2}y^2 + \cdots + a_ny^n = \begin{vmatrix} a_0 & a_1 & a_2 & \cdots & a_{n-1} & a_n \\ y & x & 0 & \cdots & 0 & 0 \\ 0 & y & x & \cdots & 0 & 0 \\ . & . & . & \cdots & . & . \\ . & . & . & \cdots & . & . \\ 0 & 0 & 0 & \cdots & y & x \end{vmatrix}. \tag{19}$$

Another example comes from the algebra of infinite series. If

$$\frac{a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \cdots}{b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \cdots} = c_0 + c_1\frac{x}{1!} + c_2\frac{x^2}{2!} + \cdots, \tag{20}$$

then c_n is given by the determinant

$$c_n = \frac{1}{b_0^{n+1}} \begin{vmatrix} b_0 & 0 & 0 & \cdots & a_0 \\ b_1 & b_0 & 0 & \cdots & a_1 \\ b_2 & 2b_1 & b_0 & \cdots & a_2 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ b_n & C_1^n b_{n-1} & C_2^n b_{n-2} & \cdots & a_n \end{vmatrix}. \tag{21}$$

If however, we take the ratio of two polynomials

$$\phi(x) = c_0x^m + c_1x^{m-1} + \cdots + c_m,$$
$$f(x) = a_0x^n + a_1x^{n-1} + \cdots + a_n,$$

its formal Laurent series is

$$\frac{\phi(x)}{f(x)} = A_0 x^{m-n} + A_1 x^{m-n-1} + A_2 x^{m-n-2} + \cdots,$$

and have, upon equating coefficients on both sides of the equation $f(x)\frac{\phi(x)}{f(x)} = \phi(x)$

$$c_0 = a_0 A_0, \quad c_1 = A_0 a_1 + A_1 a_0, \quad c_2 = A_0 a_2 + A_1 a_1 + A_2 a_0, ...$$

Solving for A_r*, we get (**Hagen**, 1883)*

$$A_r = \frac{(-)^r}{a_0^{r-1}} \begin{vmatrix} c_0 & a_0 & 0 & 0 & \cdots & 0 \\ c_1 & a_1 & a_0 & 0 & \cdots & 0 \\ c_2 & a_2 & a_1 & a_0 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\ c_{r-1} & a_{r-1} & a_{r-2} & a_{r-3} & \cdots & a_0 \\ c_r & a_r & a_{r-1} & a_{r-2} & \cdots & a_1 \end{vmatrix}_{r+1} . \tag{22}$$

*A famous recurrent is associated with **Laplace**, who produced an explicit expression for the values of the Bernoullian numbers (Johann Bernoulli, 1713), defined as the coefficients of the power series expansion*

$$\frac{t}{e^t - t} = \sum_{n=0}^{\infty} B_n \frac{t^n}{n!}, \qquad |t| < 2\pi.$$

Laplace gave the formula

$$B_n = (-)^n n! \begin{vmatrix} \frac{1}{2!} & 1 & 0 & \cdots & 0 \\ \frac{1}{3!} & \frac{1}{2!} & 1 & \cdots & 0 \\ \frac{1}{4!} & \frac{1}{3!} & \frac{1}{2!} & \ddots & 0 \\ \cdot & \cdot & \cdot & \cdot & 1 \\ \frac{1}{(n+1)!} & \frac{1}{n!} & \frac{1}{(n-1)!} & \cdots & \frac{1}{2!} \end{vmatrix} . \tag{23}$$

The numbers B_n figure in the power expansion of the functions $\tan t$, $\tanh t$, $t\cot t$, $t\coth t$, *in the Euler-Maclaurin summation formula, and in the asymptotic form of Euler's gamma function. Some Bernoullian numbers are:*

$$B_0 = 1, \quad B_1 = -\frac{1}{2}, \quad B_2 = \frac{1}{6}, \quad B_4 = -\frac{1}{30}, \quad B_6 = \frac{1}{42}, \quad B_8 = -\frac{1}{30},$$

$$B_{10} = \frac{5}{66}, \quad B_{12} = -\frac{691}{2730}, \quad B_{14} = \frac{7}{6},$$

$$B_{2n+1} = 0, \quad n = 1,\ 2,\ 3, \ldots$$

The first 62 Bernoullian numbers were computed by **Adams** *(1877).*

- Circulants

A determinant such that any row is obtained from the preceding one by passing the last element to the first place, or the first element to the last place, is called a circulant. Thus:

$$c(a_1, a_2, \ldots, a_n) = \begin{vmatrix} a_1 & a_2 & \cdots & a_n \\ a_n & a_1 & \cdots & a_{n-1} \\ \cdot & \cdot & \cdots & \cdot \\ a_2 & a_3 & \cdots & a_1 \end{vmatrix};$$

$$c'(a_1, a_2, \ldots, a_n) = \begin{vmatrix} a_1 & a_2 & \cdots & a_n \\ a_2 & a_3 & \cdots & a_1 \\ \cdot & \cdot & \cdots & \cdot \\ a_n & a_1 & \cdots & a_{n-1} \end{vmatrix} \qquad (24)$$

By transposition of rows it appears that

$$c'(a_1, a_2, \ldots, a_n) = (-)^{\frac{1}{2}(n-1)(n-2)} c(a_1, a_2, \ldots, a_n),$$

where c' belongs to the class of symmetric determinants.

Certain circulants may form a group under determinant multiplication,

e.g.

$$\begin{vmatrix} a & c & b \\ b & a & c \\ c & b & a \end{vmatrix} \begin{vmatrix} x & y & z \\ z & x & y \\ y & z & x \end{vmatrix} = \begin{vmatrix} A & C & B \\ B & A & C \\ C & B & A \end{vmatrix} \qquad (25)$$

Explicitly

$$(a^3+b^3+c^3-3abc)(x^3+y^3+z^3-3xyz)=(A^3+B^3+C^3-3ABC),$$

where

$$A=ax+by+cz,\quad B=bx+az+cy,\quad C=cx+bz+ay$$

or

$$A=ax+bz+cy,\quad B=bx+ay+cz,\quad C=cx+az+by.$$

If $a=x,\ b=y,\ c=z$, *we obtain by induction*

$$\begin{vmatrix} x & y & z \\ z & x & y \\ y & z & x \end{vmatrix}^{2^n} = \begin{vmatrix} A_n & B_n & C_n \\ C_n & A_n & B_n \\ B_n & C_n & A_n \end{vmatrix}. \tag{26}$$

Explicitly:

$$(x^3+y^3+z^3-3xyz)^{2^n}=A_n^3+B_n^3+C_n^3-3A_nB_nC_n,$$

with $A_n=A_{n-1}^2+2B_{n-1}C_{n-1}$ *etc.*

Likewise, for $n=4$

$$\begin{vmatrix} x & y & z & u \\ u & x & y & z \\ z & u & x & y \\ y & z & u & x \end{vmatrix} = (x^2+z^2-2yu)^2-(u^2+y^2-2zx)^2$$

$$\begin{vmatrix} x & y & z & u \\ u & x & y & z \\ z & u & x & y \\ y & z & u & x \end{vmatrix}\begin{vmatrix} X & Y & Z & U \\ U & X & Y & Z \\ Z & U & X & Y \\ Y & Z & U & X \end{vmatrix} = \begin{vmatrix} A & B & C & D \\ D & A & B & C \\ C & D & A & B \\ B & C & D & A \end{vmatrix}$$

where

$$\begin{aligned} A &= xX+yY+zZ+uU \\ B &= uX+xY+yZ+zU \\ C &= zX+uY+xZ+yU \\ D &= yX+zY+uZ+xU \end{aligned}$$

Circulants were introduced by ***Catalan*** *(1846) and further investigated by* ***Bertrand*** *(1850),* ***Sylvester*** *(1855),* ***Bellavitis*** *(1857) and* ***Souillart*** *(1858).*

- Pfaffians

The determinant of a skew-symmetric matrix can always be written as the square of a polynomial in the matrix elements. This polynomial is called the Pfaffian of the matrix. The Pfaffian is nonvanishing only for $2n \times 2n$ skew-symmetric matrices, in which case it is a polynomial of degree n. For example

$$Pf\begin{bmatrix} 0 & a \\ -a & 0 \end{bmatrix} = a; \qquad Pf\begin{bmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{bmatrix} = af - be + dc$$

B. Matrices

Matrices entered mathematics with Cayley in connection with linear transformations of the type

$$\begin{aligned} x' &= ax + by \\ y' &= cx + dy \end{aligned} \tag{27}$$

(where $a,\ b,\ c,\ d$ are real numbers), which may be thought of as mapping the point (x, y) into the point (x', y'). Since the above transformation (or map) is completely determined by the four coefficients $a,\ b,\ c,\ d$ it can be symbolized by the square array

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix},$$

which is called a (square) matrix (of order 2).

If the transformation given above is followed by a second transformation

$$\begin{aligned} x'' &= ex' + fy', \\ y'' &= gx' + hy', \end{aligned} \tag{28}$$

the combined (composition) map can be shown to be the transformation

$$\begin{aligned} x'' &= (ea + fc)x + (eb + fd)y, \\ y'' &= (ga + hc)x + (gb + hd)y. \end{aligned} \tag{29}$$

This leads to the following definition for the product of two matrices,

$$\begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ea + fc & eb + fd \\ ga + hc & gb + hd \end{bmatrix}. \tag{30}$$

For brevity we shall state Cayley's definition for 2 by 2 and 3×3 *matrices though the definitions apply to* $n \times n$ *matrices. Two matrices are equal iff their corresponding elements are equal. Cayley defined the* zero matrix *and the* unit matrix

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} , \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Addition of matrices is defined by

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a + e & b + f \\ c + g & d + h \end{bmatrix}, \tag{31}$$

and if λ *is any real number*

$$\lambda \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \lambda = \begin{bmatrix} \lambda a & \lambda b \\ \lambda c & \lambda d \end{bmatrix}. \tag{32}$$

In the resulting algebra of matrices, it may be easily shown that addition is both commutative and associative and that multiplication is associative and distributive over addition. But multiplication is not commutative, *as is shown by the simple example*

$$\begin{aligned} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \\ \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. \end{aligned} \tag{33}$$

This example also demonstrates that the product of two matrices may be zero without either being zero.

A special class of a linear transformations in two dimensions is that of rotations of a plane through an angle θ_1 *in a counterclockwise sense,while the* *coordinate axes remain fixed (active rotation). If the point (x, y) is carried into position (x', y'), then*

$$x' = x\cos\theta_1 - y\sin\theta_1,$$
$$y' = x\sin\theta_1 + y\cos\theta_1. \tag{34}$$

In matrix notation, this takes the form

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}. \tag{35}$$

Planar rotation through the angle θ_2 followed by a second rotation through θ_1 is written as

$$\begin{bmatrix} x'' \\ y'' \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}. \tag{36}$$

Applying the law of matrix multiplication to this product, we obtain, through the use of certain trigonometric identities,

$$\begin{bmatrix} x'' \\ y'' \end{bmatrix} = \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}. \tag{37}$$

which just states the expected result that two consecutive rotations by angles θ_1 and θ_2 are equivalent to a single rotation by the sum of the angles $(\theta_1+\theta_2)$.

The set of rotation matrices in (37) constitute a multiplicative group *of matrices known as the* orthogonal group *of $\mathbb{R}^2$, or $O(2)$; any matrix in $O(2)$ is called* orthogonal*, defined as having the property*

$$A^T A = AA^T = I, \tag{38}$$

where A^T is the transpose of A and I is the unit matrix. Explicitly, these relations imply

$$\sum_{k=1}^{n} a_{kr}a_{ks} = \delta_{rs}, \qquad (r,\ s = 1, ..., n)$$
$$\sum_{k=1}^{n} a_{rk}a_{sk} = \delta_{rs}, \qquad (r,\ s = 1, ..., n) \tag{39}$$

While either one of the relations (39) implies (38), Eq. (38) is also equivalent to the property that the vectors $A \cdot \begin{bmatrix} x \\ y \end{bmatrix}$, $A \cdot \begin{bmatrix} u \\ v \end{bmatrix}$ are orthogonal iff $\begin{bmatrix} x \\ y \end{bmatrix}$ and $\begin{bmatrix} u \\ v \end{bmatrix}$ are (hence the adjective "orthogonal" for matrices in $O(2)$).

This property, in turn, is equivalent to $\boldsymbol{u}' \cdot \boldsymbol{v}' = \boldsymbol{u} \cdot \boldsymbol{v}$ *for any two vectors* $\boldsymbol{u} = \begin{bmatrix} x \\ y \end{bmatrix}$, $\boldsymbol{v} = \begin{bmatrix} \xi \\ \eta \end{bmatrix}$, *where* $\boldsymbol{u}' = A\boldsymbol{u}$, $\boldsymbol{v}' = A\boldsymbol{v}$, *and* $\boldsymbol{u} \cdot \boldsymbol{v} = \sum_{i=1}^{n} u_i v_i$ *is the scalar product of two vectors.*

Equations (38), (39) and the preservation of scalar products, all generalize to orthogonal matrices in $n = 3$ *and higher dimensions, where the group of orthogonal matrices is denoted* $O(n)$; *but (36), (37) imply* $O(2)$ *is a commutative group, which does not hold for* $O(n)$ *with* $n \geq 3$.

Another interesting outcome of the law of matrix multiplication is the result

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \tag{40}$$

If we denote

$$X = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix},$$

Eq. (40) can be written as

$$X^2 = -I. \tag{41}$$

As I *plays the role for matrices that 1 plays for numbers, this suggests that we should think of the matrix* X, *in some sense, as a square root of minus one. Note that since* X *is obtained from the rotation matrix*

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

by inserting $\theta = \pi/2$, *the interpretation of (41) is that the symbol* $a + ib$ *stands for the matrix* $Ia + ib$, *namely*

$$\begin{bmatrix} a & -b \\ b & a \end{bmatrix} = a \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + b \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \tag{42}$$

Given two $n \times n$ *matrices* A *and* B, *three types of products find applications in linear mathematical physics (linear vector spaces):*

- *The ordinary matrix product* $(A \cdot B)_{ik} = \sum_{j=1}^{n} A_{ij} B_{jk}, \quad (i, k = 1, 2, ...n)$
- *The scalar product* $A : B = \sum_{i,k=1}^{n} A_{ik} B_{ik}$
- *The Kronecker (direct) product* $(A \otimes B)_{ik,jl} = (A)_{ij} (B)_{kl}$

An example of the Kronecker product is

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B \\ a_{21}B & a_{22}B \end{bmatrix} = \left[\begin{array}{cc|cc} a_{11}b_{11} & a_{11}b_{12} & a_{12}b_{11} & a_{12}b_{12} \\ a_{11}b_{21} & a_{11}b_{22} & a_{12}b_{21} & a_{12}b_{22} \\ \hline a_{21}b_{11} & a_{21}b_{12} & a_{22}b_{11} & a_{22}b_{12} \\ a_{21}b_{21} & a_{21}b_{22} & a_{22}b_{21} & a_{22}b_{22} \end{array}\right], \quad (43)$$

where

$$trace(A \otimes B) = (traceA)(traceB) \quad (44)$$

since (using the summation convention for repeated indices) $(A \otimes B)_{ik,ik} = A_{ii}B_{kk}$.

The Kronecker product arises in the following way: Let $\overrightarrow{x}$, $\overrightarrow{y}$ *be two vectors in* n *dimensions,* $\overrightarrow{x} = x_i\overrightarrow{e_i}$, $\overrightarrow{y} = y_k\overrightarrow{e_k}$; *then*

$$\overrightarrow{x} \otimes \overrightarrow{y} = x_i y_k(\overrightarrow{e_i} \otimes \overrightarrow{e_k}).$$

Explicitly $\overrightarrow{x} \otimes \overrightarrow{y}$ *is a column vector with components* $(i = 1, ..., n;$ *k=1,...,n)*

$$\overrightarrow{x} \otimes \overrightarrow{y} = x_i y_k = x_1 y_1, ... \; x_1 y_n ... \; x_n y_1, ... \; x_n y_n.$$

Next apply the linear transformation of the coordinates

$$\overrightarrow{x'} = A\overrightarrow{x} \quad i.e. \quad x'_i = A_{ij}x_j$$

$$\overrightarrow{y'} = B\overrightarrow{y} \quad i.e. \quad y'_k = A_{kl}y_l$$

Then, the Kronecker product transforms according to the law

$$\overrightarrow{x'} \otimes \overrightarrow{y'} = (A \otimes B)(\overrightarrow{x} \otimes \overrightarrow{y})$$

It remains to express $(A \otimes B)$ *explicitly in terms of* A *and* B. *But since*

$$(\overrightarrow{x'} \otimes \overrightarrow{y'}) = x'_i y'_k = A_{ij}x_j B_{kl}y_l = (A \otimes B)_{ik,jl}x_j y_l$$

we have

$$(A \otimes B)_{ik,jl} = (A)_{ij}(B)_{kl}.$$

- Inverse of a matrix

A linear system of n equations in n unknowns can be written in concise matrix notation as a matrix product

$$A\overrightarrow{X} = \overrightarrow{b} \tag{45}$$

where $\overrightarrow{X}$ is the column vector

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

and $\overrightarrow{b}$ is the column vector

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}.$$

Its solution, when A is non-singular ($|A| \neq 0$) is written as

$$\overrightarrow{X} = A^{-1}\overrightarrow{b} \tag{46}$$

Here the matrix A^{-1} is the inverse *of the matrix A, given explicitly by*

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} A_{11} & A_{21} & \cdots & A_{n1} \\ A_{12} & A_{22} & \cdots & A_{n2} \\ \vdots & \vdots & \dots & \vdots \\ A_{1n} & A_{2n} & \cdots & A_{nn} \end{bmatrix}, \tag{47}$$

where A_{jk} is the cofactor *of a_{jk} in $|A|$. We note that in A^{-1} the cofactor A_{jk} occupies the same place as a_{kj} (not a_{jk}) does in A.*

Thus, for example, for

$$A = \begin{bmatrix} a & c \\ b & d \end{bmatrix}, \qquad ad \neq bc,$$

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -c \\ -b & a \end{bmatrix} \tag{48}$$

A Hermitian matrix H, is such that its transpose is equal to its complex conjugate, i.e., $H^T = H^*$. An example is

$$H = \begin{bmatrix} 1 & 2-i & 4i \\ 2+i & 3 & -1-i \\ -4i & -1+i & 4 \end{bmatrix}, \qquad i = \sqrt{-1}. \tag{49}$$

For real matrices the concept of Hermitian matrix reduces to that of a symmetric matrix. A skew Hermitian matrix satisfies $H^T = -H^*$, which for real matrices reduces to skew-symmetry.

A unitary matrix U is such that its inverse equal its conjugate transpose, i.e., $U^{-1} = (U^*)^T$ or $UU^{*T} = U^{*T}U$. Two examples are:

$$U_2 = \frac{1}{2}\begin{bmatrix} 1+i & 1-i \\ 1-i & 1+i \end{bmatrix}; \qquad U_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{bmatrix} \tag{50}$$

A real unitary matrix is simply an orthogonal matrix. Note that U_3 may arise in the following way: consider a vector in a spherical coordinate system

$$\boldsymbol{u} = \boldsymbol{e_r} u_r + \boldsymbol{e_\vartheta} u_\vartheta + \boldsymbol{e_\varphi} u_\varphi.$$

We can also recast this in the form

$$\boldsymbol{u} = \boldsymbol{e_0} u^0 + \boldsymbol{e}_- u^- + \boldsymbol{e}_+ u^+$$

where we use complex basis vectors and complex components:

$$\begin{aligned} \boldsymbol{e}_- &= \tfrac{1}{\sqrt{2}}(\boldsymbol{e_\vartheta} - i\boldsymbol{e_\varphi}) & u^- &= \tfrac{1}{\sqrt{2}}(u_\vartheta + iu_\varphi) \\ \boldsymbol{e_0} &= \boldsymbol{e_r} & u^0 &= u_r \\ \boldsymbol{e}_+ &= \tfrac{1}{\sqrt{2}}(-\boldsymbol{e_\vartheta} - i\boldsymbol{e_\varphi}) & u^+ &= \tfrac{1}{\sqrt{2}}(-u_\vartheta + iu_\varphi). \end{aligned} \tag{51}$$

Then

$$\begin{bmatrix} u^0 \\ u^- \\ u^+ \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} u_r \\ u_\vartheta \\ u_\varphi \end{bmatrix} \tag{52}$$

Just as an orthogonal $n \times n$ matrix preserves the $\mathbb{R}^n$ scalar product of vectors, $\boldsymbol{u} \cdot \boldsymbol{v}$, so a unitary $n \times n$ matrix can easily be shown to preserve the scalar product $\boldsymbol{u}^* \cdot \boldsymbol{v} = u_j^* v_j$, where $\boldsymbol{u}, \boldsymbol{v}$ belong to $\mathbb{C}^n$

(vector space of complex n-tuples). The set of $n \times n$ unitary matrices is again a multiplicative group, denoted $U(n)$ and called the $(n \times n)$ unitary group.

- Transformation of matrices

There are 4 fundamental relations possible between two given square matrices:

- *Equivalence* $B = PAQ$ (**H.J.S. Smith**, *1861*)
- *Similarity* $B = P^{-1}AP$ (**Frobenius**, *1878*)
- *Congruence* $B = P^{T}AP$ (**Frobenius**, *1878*)
- *Hermitian Congruence* $B = P^{*T}AP$ (**Hermite**, *1854*)

If $P^{-1} = P^{T}$ we have orthogonal similarity; if $P^{-1} = P^{*}$ we have unitary similarity, and if $P^{T} = P^{*}$ the transformation is real.

- Eigenvalues and eigenvectors

Let $A = [a_{jk}]$ be a given $n \times n$ matrix and consider the vector equation

$$Ax = \lambda x \tag{53}$$

where λ is a number (scalar) and x is a vector.

It is clear that the zero vector $x = 0$ is a solution of (53) for any value of λ. A value of λ for which (53) has a solution $x \neq 0$ is called an eigenvalue or characteristic value (or latent root) of the matrix A. The corresponding solutions $x \neq 0$ of (53) are called eigenvectors or characteristic vectors of A corresponding to that eigenvalue λ. The set of the eigenvalues is called the spectrum of A. The largest of the absolute values of the eigenvalues of A is called the spectral radius of A.

The problem of determining the eigenvalues and eigenvectors of a matrix is called an eigenvalue problem. Problems of this type occur in connection with physical and technical applications.

Let us consider (53). If x is any vector, then the vectors x and Ax will, in general, be linearly independent. If x is an eigenvector, then x and

Ax are linearly dependent; corresponding components of x and Ax are then proportional, the factor of proportionality being the eigenvalue λ.

Any $n \times n$ square matrix has at least 1 and at most n distinct (real or complex) eigenvalues.

To see this we write (53) out:

$$\begin{aligned} a_{11}x_1 + \cdots + a_{1n}x_n &= \lambda x_1, \\ a_{21}x_1 + \cdots + a_{2n}x_n &= \lambda x_2, \\ &\cdots\cdots\cdots \\ a_{n1}x_1 + \cdots + a_{nn}x_n &= \lambda x_n. \end{aligned}$$

By transferring the terms on the right-hand side to the left-hand side we obtain

$$\begin{array}{ccccccc} (a_{11}-\lambda)x_1 & + & a_{12}x_2 & + \cdots + & a_{1n}x_n & = 0 \\ a_{21}x_1 & + & (a_{22}-\lambda)x_2 & + \cdots + & a_{2n}x_n & = 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1}x_1 & + & a_{n2}x_2 & + \cdots + & (a_{nn}-\lambda)x_n & = 0 \end{array}$$

In matrix notation,

$$(A - \lambda I)x = 0.$$

By Cramer's theorem, this homogeneous system of linear equations has a nontrivial solution if and only if the corresponding determinant of the coefficients is zero:

$$D(\lambda) = \det(A - \lambda I) = \begin{vmatrix} a_{11}-\lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22}-\lambda & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn}-\lambda \end{vmatrix} = 0. \quad (54)$$

$D(\lambda)$ is called the characteristic determinant, and (54) is called the characteristic equation corresponding to the matrix *A. By developing*

$D(\lambda)$ we obtain a polynomial of n^{th} degree in λ. This is called the characteristic polynomial *corresponding to A.*

Once the eigenvalues have been determined, corresponding eigenvectors can be determined. Since the system is homogeneous, it is clear that if x is an eigenvector of A, then kx, where k is any constant, not zero, is also an eigenvector of A corresponding to the same eigenvalue.

Let us, for example, determine the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}.$$

The characteristic equation

$$D(\lambda) = \begin{vmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{vmatrix} = \lambda^2 - 7\lambda + 6 = 0$$

has the roots $\lambda_1 = 6$ and $\lambda_2 = 1$. For $\lambda = \lambda_1$ the system assumes the form

$$-x_1 + 4x_2 = 0$$

$$x_1 - 4x_2 = 0.$$

Thus $x_1 = 4x_2$, and

$$x_1 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

is an eigenvector of A corresponding to the eigenvalue λ_1. In the same way we find that an eigenvector of A corresponding to λ_2 is

$$x_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$

where x_1 and x_2 are linearly independent vectors.

- Application to a simple mechanical system

Matrices find many application to geometry, mechanics, electromagnetic theory, relativistic electrodynamics, quantum mechanics, probability theory and game theory.

Historically, the concept of vectors and matrices were automatically derived from the application of Newton's laws to simple physical configurations governed by a system of linear ordinary differential equations with constant coefficients.

A mechanical system is composed of a linear array of n equal masses m interconnected by linear springs of equal stiffness μ. In addition, each mass is connected to a common support force $F(t)$ by means of a spring of stiffness ν. Taking into account frictional attenuation and 'next-neighbor interaction', the displacement of the i-th mass at distance $x_i(t)$ from equilibrium is given by the differential equation

$$m\frac{d^2x_i}{dt^2} = \mu_i(x_{i+1} - x_i) - \mu_{i-1}(x_i - x_{i-1}) - \nu_i x_i - 2hm\frac{dx_i}{dt} + mF(t)$$

or

$$\ddot{x}_i = \frac{\mu}{m}(x_{i+1} - 2x_i + x_{i-1}) - \frac{\nu}{m}x_i - 2h\dot{x}_i + F(t). \tag{55}$$

In abbreviated notation, (55) takes on the form

$$\ddot{\boldsymbol{X}} + 2h\dot{\boldsymbol{X}} - [K_n]\,\boldsymbol{X} = F(t)\,[I_n] \tag{56}$$

where

$$[K_n] = -\frac{\nu}{m}\,[I_n] + \frac{\mu}{m}\,[T_n]\,, \tag{57}$$

$$\boldsymbol{X} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \qquad [T_n] = \begin{bmatrix} -2 & 1 & & & & \\ 1 & -2 & 1 & & & \\ & 1 & -2 & 1 & & \\ & & \ddots & \ddots & \ddots & \\ & & & 1 & -2 & 1 \\ & & & & 1 & -2 \end{bmatrix} \tag{58}$$

and $[I_n]$ is the n-dimensional unit matrix.

To obtain a formal solution of (56) we fall back for a moment on the one-dimensional equation of motion of a single attenuated harmonic oscillator driven by a time-dependent force

$$\ddot{X} + 2h\dot{X} - \frac{k}{m}X = F(t).$$

The solution of this equation is known to be

$$X(t) = \int_0^t F(\tau)G(t-\tau)d\tau + X(0)G_1(t) + \dot{X}(0)G(t) \tag{59}$$

where

$$G(t) = \frac{e^{\alpha_1 t} - e^{\alpha_2 t}}{\alpha_1 - \alpha_2}; \qquad G_1(t) = \frac{\alpha_2 e^{\alpha_1 t} - \alpha_1 e^{\alpha_2 t}}{\alpha_2 - \alpha_1};$$

$$\alpha_{1,2} = -h \pm \sigma; \qquad \sigma = \sqrt{h^2 + k/m}. \tag{60}$$

Explicitly, for $F(t) = qU(t)$, $X(0) = 0$, $\dot{X}(0) = 0$, *with* $U(t)$ *standing for the Heaviside unit step-function*

$$X(t) = e^{-ht}\left[a_0 \cosh(\sigma t) + b_0 \frac{\sinh(\sigma t)}{\sigma}\right] U(t) - a_0 U(t), \tag{61}$$

$$a_0 = mq/k; \quad b_0 = mhq/k; \quad q = force\ per\ unit\ mass$$

In a similar way, we may write the solution of (56)

$$\boldsymbol{X}(t) = \left\{e^{-ht}\left[\cosh(\sigma t) + h\frac{\sinh(\sigma t)}{\sigma}\right] - I_n\right\} K_n^{-1} \boldsymbol{q} U(t) \tag{62}$$

where $\cosh(\sigma t)$, $\frac{\sinh(\sigma t)}{\sigma}$, K_n *and* K_n^{-1} *are* $(n \times n)$ *matrices,* $\boldsymbol{q}$ *a column* n*-vector and*

$$\sigma^2 = h^2 I + K_n. \tag{63}$$

Since the eigenvalues of σ^2 *are*[254]

$$\lambda_s = h^2 - \nu - 4\mu \sin^2 \frac{\pi s}{2(n+1)}, \qquad s = 1, 2, ..., n, \tag{64}$$

the explicit forms of the matrices participating in (62) are

$$\cosh \sigma t = \sum_{s=1}^{n} B_n^{(s)} \cosh\left(t\sqrt{\lambda_s}\right), \qquad \frac{\sinh \sigma t}{\sigma} = \sum_{s=1}^{n} B_n^{(s)} \frac{\sinh\left(t\sqrt{\lambda_s}\right)}{\sqrt{\lambda_s}}$$

$$\frac{1}{K_n} = \sum_{s=1}^{n} \frac{1}{\hat{\lambda}_s} B_n^{(s)} \qquad \hat{\lambda}_s = -\nu - 4\mu \sin^2 \frac{\pi s}{2(n+1)} \tag{65}$$

$$\left\{B_n^{(s)}\right\}_{ij} = \frac{2}{n+1} M_{is}^{(n)} M_{sj}^{(n)} (\textit{no summation over } s),$$

where M_n *is the symmetric modal matrix whose columns are the eigenvectors of* K_n *and its* $(sk)^{th}$ *term is* $\sin\left(\frac{\pi sk}{n+1}\right)$

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[254] The *eigenvalues* of $[T_N]$ are known to be equal to

$$\left\{-4\sin^2 \frac{\pi n}{2(N+1)}\right\}, \quad n = 1, 2, ...N$$

The corresponding *eigenvectors* are

$$\left[\sin \frac{\pi n}{N+1}, \ \sin \frac{2\pi n}{N+1}, ..., \sin \frac{\pi n N}{N+1}\right].$$

In general, the eigenvalues of the 3-diagonal matrix

$$\begin{bmatrix} a & b & & & & \\ c & a & b & & & \\ & c & a & b & & \\ & & \ddots & \ddots & \ddots & \\ & & & c & a & b \\ & & & & c & a \end{bmatrix}$$

are

$$\lambda_n = a + 2\sqrt{bc}\cos\left(\frac{\pi n}{N+1}\right), \qquad n = 1, 2, ..., N.$$

If $a = 1 - 2r, \quad b = c = r$, then $\lambda_n = 1 - 4r\sin^2\left(\frac{\pi n}{2(N+1)}\right)$.

$$M_n = \begin{bmatrix} \sin\dfrac{\pi}{n+1} & \sin\dfrac{2\pi}{n+1} & \cdots & \sin\dfrac{n\pi}{n+1} \\ \sin\dfrac{2\pi}{n+1} & \sin\dfrac{4\pi}{n+1} & \cdots & \sin\dfrac{2n\pi}{n+1} \\ \vdots & \vdots & \cdots & \vdots \\ \sin\dfrac{n\pi}{n+1} & \sin\dfrac{2n\pi}{n+1} & \cdots & \sin\dfrac{n^2\pi}{n+1} \end{bmatrix}, \qquad M^{-1} = \frac{2}{n+1}M. \tag{66}$$

Defining

$$Q(\lambda_s) = 1 - \left(\cosh t\sqrt{\lambda_s} + h\frac{\sinh t\sqrt{\lambda_s}}{\sqrt{\lambda_s}}\right)e^{-ht}, \qquad s = 1, 2, ..., n$$

we may present the solution as

$$\boldsymbol{X}(t) = \left[\sum_{s=1}^{n} \frac{Q(\lambda_s)}{(-\overset{\wedge}{\lambda}_s)} B_n^{(s)}\right] \boldsymbol{q}U(t). \tag{67}$$

- Triangular Matrices

A square $n \times n$ matrix L is called lower triangular matrix if all elements of L above the principal diagonal are zero

$$L = \begin{bmatrix} l_{1,1} & & & & 0 \\ l_{2,1} & l_{2,2} & & & \\ l_{3,1} & l_{3,2} & \ddots & & \\ \vdots & \vdots & \ddots & \ddots & \\ l_{n,1} & l_{n,2} & \ldots & l_{n,n-1} & l_{n,n} \end{bmatrix}.$$

Analogously, a matrix of the form

$$U = \begin{bmatrix} u_{1,1} & u_{1,2} & u_{1,3} & \cdots & u_{1,n} \\ & u_{2,2} & u_{2,3} & \cdots & u_{2,n} \\ & & \ddots & \ddots & \vdots \\ 0 & & & & u_{n,n} \end{bmatrix}$$

is called upper triangular matrix.

If the entries on the main diagonal are 1*, the matrix is termed normed (or unit) upper/lower triangular. Because matrix equations with triangular matrices are easy to solve they are very important in numerical analysis.*

A special type of a normed triangular matrix is one in which all the off-diagonal entries are zero except for entries in one column. Such a matrix is called a Gauss matrix, and its inverse is again a Gauss matrix.

$$L_i = \begin{bmatrix} 1 & & & & & 0 \\ & \ddots & & & & \\ & & 1 & & & \\ & & l_{i+1,i} & \ddots & & \\ & & \vdots & & \ddots & \\ 0 & & l_{n,i} & & & 1 \end{bmatrix},$$

$$L_i^{-1} = \begin{bmatrix} 1 & & & & & 0 \\ & \ddots & & & & \\ & & 1 & & & \\ & & -l_{i+1,i} & \ddots & & \\ & & \vdots & & \ddots & \\ 0 & & -l_{n,i} & & & 1 \end{bmatrix}.$$

Here the off-diagonal entries are replaced by their opposites.

Note that:

- *A matrix which is simultaneously upper and lower triangular is diagonal. The identity matrix is the only matrix which is both normed upper and lower triangular.*
- *A matrix which is simultaneously triangular and normal, is also diagonal. This can be seen by looking at the diagonal entries of* A^*A *and* AA^**, where* A *is a normal, triangular matrix.*
- *The transpose of an upper triangular matrix is a lower triangular matrix and vice versa. The determinant of a triangular matrix equals the product of the diagonal entries, and the eigenvalues of a triangular matrix are the diagonal entries.*
- *The product of two upper triangular matrices is upper triangular, so the set of upper triangular matrices forms an algebra.*

– *A matrix equation in the form* $Lx = b$ *is very easy to solve. It can be written as a system of linear equations*

$$\begin{array}{ccccccccc} l_{1,1}x_1 & & & & & & & = & b_1 \\ l_{2,1}x_1 & + & l_{2,2}x_2 & & & & & = & b_2 \\ \vdots & & \vdots & & \ddots & & & & \vdots \\ l_{m,1}x_1 & + & l_{m,2}x_2 & + & \cdots & + & l_{m,m}x_m & = & b_m \end{array}$$

which can be solved by the following recursive relation

$$\begin{aligned} x_1 &= \tfrac{b_1}{l_{1,1}}, \\ x_2 &= \tfrac{b_2 - l_{2,1}x_1}{l_{2,2}}, \\ &\vdots \\ x_m &= \tfrac{b_m - \sum_{i=1}^{m-1} l_{m,i}x_i}{l_{m,m}}. \end{aligned}$$

A matrix equation $Ux = b$ *with an upper triangular matrix* U *can be solved in an analogous way.*

Let A *be an invertible square matrix. An 'LU decomposition'*[255] *gives an algorithm to decompose* A *into normed lower triangular matrix* L *and an upper triangular matrix* U *in the form* $A = LU$, *where* L *and* U *are of the same size as* A. *For a* 3×3 *matrix this becomes*

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$= \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & (l_{21}u_{12} + u_{22}) & (l_{21}u_{13} + u_{23}) \\ l_{31}u_{11} & (l_{31}u_{12} + l_{32}u_{22}) & (l_{31}u_{13} + l_{32}u_{23} + u_{33}) \end{bmatrix}$$

It yields 9 equations in 9 unknowns $\{l_{21}, l_{31}, l_{32}, u_{11}, u_{12}, u_{13}, u_{22}, u_{23}, u_{33}\}$.

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[255] For further reading, see:

- Press, W.H. et al, *Numerical Recipes in C*, Cambridge University Press, 1988, 735 pp.
- Lay, D.C., *Linear Algebra and its Applications*, Addison-Wesley, 2003, 492 pp.
- Horn, R.A. and C.R.Johnson, *Matrix Analysis*, Cambridge University Press, 1985.

In general there are n^2 equations in n^2 unknowns. To solve the matrix equation

$$Ax = LUx = L(Ux) = b,$$

we first solve $Ly = b$ *for* y *and then solve* $Ux = y$ *for* x.

Other decompositions by means of triangular matrices are:

LDU decomposition
: $A = LDU$, *where* D *is a diagonal matrix, and* L, U *are normed triangular matrices.*

PLU decomposition
: $A = PLU$, *where* P *is a permutation matrix (i.e., a matrix of zeros and ones that has exactly one entry in each row and column).*

PLUQ decomposition
: $A = PLUQ$, *where* P *and* Q *are permutation matrices.*

It can be shown that:

(1) *An invertible matrix admits an* LU *factorization if and only if all its principle minors are non-zero. The factorization is unique if we require that the diagonal of* L *(or* U*) consists of ones. The matrix has a unique* LDU *factorization under the same conditions.*

(2) *If the matrix is singular, then an* LU *factorization may still exist. In fact, a square matrix of rank* k *has an* LU *factorization if the first* k *principal minors are non-zero.*

(3) *Every invertible matrix admits* PLU *factorization. Finally, every square matrix* A *has a* $PLUQ$ *factorization.*

(4) *The matrices* L *and* U *can be used to calculate the matrix inverse.*

Cholesky[256] Decomposition *(1905)*

Every real symmetric and positive definite matrix A *(i.e.* x^TAx *is positive for every non-zero vector* x*) can be expressed in the Cholesky decomposition*

$$A = LL^T = U^TU,$$

[256] **Andre-Louis Cholesky** (1875–1918, France). Mathematician. His novel method of solving simultaneous algebraic linear equations was discovered by him in 1905, published posthumously in 1924, and became widely known through A.M. Turing in 1948. Cholesky was born near Bordeaux, France. He graduated from the Ecole Polytechnique (1897) under Camille Jordan and the Army Artillery School (1899). He then served in the Army's Geodetic Section in Tunisia, Algeria and Crete (1902–1912). He was killed in action in 1918 in North of France.

where U is an invertible upper triangular matrix whose diagonal entries are positive, and L is a lower triangular matrix with positive diagonal elements. Thus L can be seen as the "square root" of A. To solve $Ax = b$, one solves first $Ly = b$ for y, and then $L^T x = y$ for x. Cholesky decomposition is often used to solve the normal equations in linear least squares *problems; they give $A^T Ax = A^T b$, in which $A^T A$ is symmetric and positive definite.*

If A is Hermitian *and positive definite, then we can arrange matters so that U is the conjugate transpose of L. In this case*

$$A = LL^*.$$

The Cholesky decomposition always exists and is unique.

To derive $A = LL^T$, we simply equate coefficients on both sides of the equation:

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{31} & a_{32} & \cdots & a_{3n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & \cdots & 0 \\ l_{21} & l_{22} & \cdots & 0 \\ l_{31} & l_{32} & \vdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ l_{n1} & l_{n2} & \cdots & l_{nn} \end{bmatrix} \begin{bmatrix} l_{11} & l_{21} & \cdots & l_{n1} \\ 0 & l_{22} & \cdots & l_{n2} \\ 0 & 0 & \vdots & l_{n3} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & l_{nn} \end{bmatrix}$$

to obtain:

$$\begin{array}{lcl} a_{11} = l_{11}^2 & \rightarrow & l_{11} = \sqrt{a_{11}} \\ a_{21} = l_{21} l_{11} & \rightarrow & l_{21} = a_{21}/l_{11} \\ a_{22} = l_{21}^2 + l_{22}^2 & \rightarrow & l_{22} = \sqrt{(a_{22} - l_{21}^2)} \\ a_{32} = l_{31} l_{21} + l_{32} l_{22} & \rightarrow & l_{32} = (a_{32} - l_{31} l_{21})/l_{22}, \text{ etc.} \end{array}$$

In general, for $i = 1, \ldots, n$ and $j = i + 1, \ldots, n$:

$$\begin{aligned} l_{ii} &= \sqrt{\left(a_{ii} - \sum_{k=1}^{i-1} l_{ik}^2\right)} \\ l_{ji} &= \left(a_{ji} - \sum_{k=1}^{i-1} l_{jk} l_{ik}\right) / l_{ii}. \end{aligned}$$

Because A is symmetric and positive definite, the expression under the square root is always positive, and all l_{ij} are real.

- Application to Modern Control Theory

Consider a set of linear differential equations

$$\frac{d\boldsymbol{x}}{dt} = A\boldsymbol{x} + bu, \tag{68}$$

where $\boldsymbol{x} = [x_1, x_2, \dots, x_n]^T$ *is a vector of variables describing the state of a system,* $\frac{d\boldsymbol{x}}{dt} = [\frac{dx_1}{dt}, \frac{dx_2}{dt}, \dots, \frac{dx_n}{dt}]^T$, A *is a given* $n \times n$ *matrix,* b *is a given constant column* n*-vector, and* u *is a scalar control variable which can be manipulated. If there are* m *control variables, the set (68) is replaced by* $\frac{d\boldsymbol{x}}{dt} = A\boldsymbol{x} + Bu$*, where* B *is a* $n \times m$ *matrix and* u *is a* m*-vector.*

It is frequently convenient to seek an approximate solution to a problem governed by a differential equation by first obtaining a difference equation which approximately simulates that equation and then satisfying the new equation at a certain discrete mesh of points by direct algebraic methods, with the expectation that the solution of the simulating problem will indeed simulate the solution of the true problem at these points. In modern digitally controlled systems, state variables measurements and actuator commands, occur, in any case at discretely spaced times.

Thus, if (68) is subjected to the prescribed initial conditions $\boldsymbol{x}(t_0) = \boldsymbol{x}_0$*, one uses a Taylor expansion to approximate the derivative* $\frac{d\boldsymbol{x}}{dt}$ *by the divided difference* $\frac{\Delta\boldsymbol{x}}{h}$*, where* h *is a conveniently chosen spacing, and hence rewrite (68) in the form*

$$\boldsymbol{x}(t+h) = \boldsymbol{x}(t) + h[A\boldsymbol{x}(t) + Bu] + O(h^2). \tag{69}$$

We then denote by $\boldsymbol{y}(t)$ *the solution of the difference equation which results from ignoring the terms of order* h^2*, and require that the resultant equation holds when* $t = t_0, t_1 = t_0 + h, \dots, t_k = t_0 + kh$*. With the usual abbreviation* $\boldsymbol{y}_k = \boldsymbol{y}(t_k)$*, the equation determining the approximation* $\boldsymbol{y}_k$ *then takes the form*

$$\boldsymbol{y}_{k+1} = A\boldsymbol{y}_k + B\boldsymbol{u}_k, \qquad k = 0, 1, 2, \dots \tag{70}$$

where the associated truncation error accordingly is of order h^2 *when* h *is small. Eq. (70) can be treated as a recurrence formula. In general, the controlled plant dynamics (68)–(70) may involve explicit time dependencies in* A *and* B*.*

Eqs (68) and (70) (as well as non linear and other variants) are applicable to control problems, which gained importance in the last three *decades of the 20^{th} century as a discipline for engineers, mathematicians, scientists and other researchers. Examples of control problems include landing a vehicle on the moon, controlling a power-plant car engine or the macroeconomy of a nation, designing robots, and controlling the spread of an epidemic.*

In our example, the time-discretized uncontrolled physical system is governed by the homogeneous difference equation $\boldsymbol{y}_{k+1} = A\boldsymbol{y}_k$, where A is an (assumed known) $n \times n$ matrix. To control this system (i.e. to induce it to behave in a predetermined fashion), we introduce into it a forcing term, or a control, $\boldsymbol{u}_k$. Thus, the controlled system is the inhomogeneous system $\boldsymbol{y}_{k+1} = A\boldsymbol{y}_k + \boldsymbol{u}_k$. In realizing this system, it is assumed that the control can be applied to directly affect each of the state variables $y_{1,k}, y_{2,k}, \ldots, y_{n,k}$ of the system, at each timestep t_k. In most applications, however, this assumption is unrealistic[257]. Thus, a more realistic model for the controlled system is

$$\boldsymbol{y}_{k+1} = A\boldsymbol{y}_k + B\boldsymbol{u}_k, \tag{71}$$

where B is a $(n \times m)$ matrix, and $\boldsymbol{u}_k$ is an $(m+1)$ vector, with m indicating the number of control variables $u_1(k), u_2(k), \ldots, u_m(k)$, where $m \le n$.

In control problems, two basic questions need to be answered in deciding whether or not a control solution exists. These questions may be posed thus:

(i) *Can we transfer the system from any initial state to any other desired state – or make it follow a desired trajectory – to prespecified accuracy and over a given time interval, by application of a suitable control force? (controllability and stability)*

(ii) *Knowing the vector of output (sensed, measured) variables for a finite length of time, can we determine the initial state of the system? (observability)*

*The answers to these questions were conceptualized (1960) by **R.E. Kalman**[258].*

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[257] Thus for example economists do not know how economic growth and rate of inflation can be controlled, but can affect them by altering some or all of the following variables: taxes, the money supply, prime bank lending rate, etc.

[258] **Rudolf Emil Kalman** (b. 1930, USA). Mathematical system theorist and electrical engineer. Co-invented the *Kalman Filter*, a mathematical technique widely used in control systems and avionics to extract a signal from a series of incomplete and noisy measurements by a succession of optimized updates of

More precise definitions of controllability and observability are as follows:

- *A system is said to be completely state controllable if it is possible to transfer the system state from any initial state $\boldsymbol{x}(t_0)$ to any other desired state $\boldsymbol{x}(t_0)$ in specified finite time by a control trajectory $\boldsymbol{u}(t)$.*
- *A system is said to be completely observable, if every state $\boldsymbol{x}(t_0)$ can be completely identified by measurements of the output $\boldsymbol{z}(t)$ over a finite time interval.*

If a system is not completely observable, this implies that some of its state variables are shielded from observation. As an example consider the system governed by the discretized equations

$$\begin{aligned} y_1(k+1) &= a_{11}y_1(k) + a_{12}y_2(k) + bu(k) \\ y_2(k+1) &= a_{22}y_2(k) \end{aligned} \tag{72}$$

Here

$$A = \begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix}, \quad B = \begin{bmatrix} b \\ 0 \end{bmatrix}.$$

Clearly, this system is not completely controllable, since by inspection, $u(k)$ has no influence on $y_2(k)$. Moreover, $y_2(k)$ is entirely determined by the second equation and is given by $y_2(k) = (a_{22})^k y_2(0)$.

In general, Kalman proved[259] *that the system (71) is completely con-*

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feedback gains and controls. Kalman filters were first used during the *Apollo program* of NASA.

Kalman was born in Budapest, Hungary. Obtained his M.Sc. degree from MIT (1954) and his doctorate from Columbia University (1957). Professor of Stanford University (1964–1971), University of Florida (1971–1992) and ETH, Zurich (1973–1992).

[259] The proof hinges on the fact that the explicit solution of the difference equation $y(k+1) = Ay(k) + Bu(k)$ for constant matrices A and B is

$$y(k) = A^n y(0) + W\bar{u}(k),$$

where we define a new $m \times k$ vector:

$$\bar{u}(k) = \begin{bmatrix} u(k-1) \\ u(k-2) \\ \vdots \\ u(0) \end{bmatrix}.$$

trollable if and only if $\operatorname{rank} W = n$*, where*

$$W = [B, AB, A^2B, \ldots, A^{n-1}B] \tag{73}$$

is a matrix of n *rows and* mn *columns.*

Consider the system

$$\begin{aligned} y_1(k+1) &= ay_1(k) + by_2(k) \\ y_2(k+1) &= cy_1(k) + dy_2(k) + u(k), \end{aligned}$$

where $ad - bc \neq 0$*. Here* $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ *and* $u(k)$ *is a scalar control sequence. Now*

$$W = [B, AB] = \begin{bmatrix} 0 & b \\ 1 & d \end{bmatrix}$$

has rank 2 *iff* $b \neq 0$*. Thus the system is completely controllable iff* $b \neq 0$*. If, however, we consider the control system* $\boldsymbol{y}_{k+1} = A\boldsymbol{y}_k + Bu_k$ *with* $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$*, subjected to* $\boldsymbol{y}(0) = \boldsymbol{y}_0 = \begin{bmatrix} y_{01} \\ y_{02} \end{bmatrix}$*, we have* $\boldsymbol{y}(1) = A\boldsymbol{y}_0 + B\boldsymbol{u}(0) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y_{01} \\ y_{02} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u_0 = \begin{bmatrix} y_{02} \\ 0 \end{bmatrix} + \begin{bmatrix} u_0 \\ 0 \end{bmatrix}$*. So, if we pick* $u_0 = -y_{02}$*, then we will have* $y(1) = 0$*. Therefore the system is controllable to zero. But since* $\operatorname{rank}[B, AB] = \operatorname{rank} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = 1 < 2$*, the system is not completely controllable.*

In the previous theory it was assumed that the observed discretized output of the control system is the same as that of the state of the system $\boldsymbol{y}(k)$*. In practice, however, one may not be able to observe the state of the system* $\boldsymbol{y}(k)$ *but rather an output* $\boldsymbol{z}(k)$ *that is related to* $\boldsymbol{y}(k)$ *in a specific manner. The mathematical model of this type of system is given by*

$$\boldsymbol{y}(k+1) = A\boldsymbol{y}(k) + B\boldsymbol{u}(k); \quad \boldsymbol{z}(k) = C\boldsymbol{y}(k) \tag{74}$$

where A *is an* $n \times n$ *matrix,* B *is a* $n \times m$ *matrix,* $\boldsymbol{u}(k)$ *is an* m*-dimensional column vector, and* C *is* $r \times n$ *matrix. The control* $\boldsymbol{u}(k)$ *is the input of the system, and* $\boldsymbol{z}(k)$ *is its output (Fig. 11).*

Roughly speaking, observability *means that it is possible to determine the state of the system* $\boldsymbol{y}(k)$ *by measuring only the output* $\boldsymbol{z}(k)$*. Hence it is useful in solving the problem of reconstructing unmeasurable state*

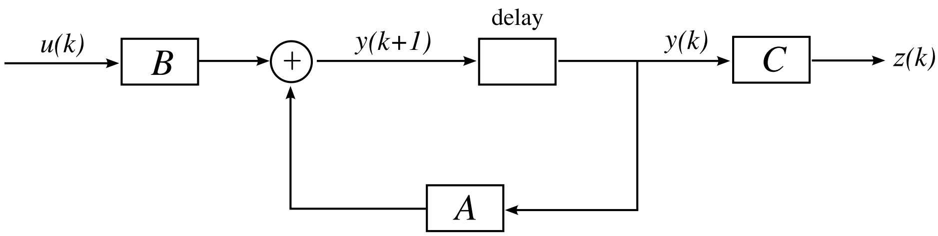

Fig. 11: Flow diagram of a control system

variables from measurable ones. The input-output system (74) is completely observable if for any integer $k_0 \geq 0$*, there exists* $N > k_0$ *such that the knowledge of* $\boldsymbol{u}(k)$ *and* $\boldsymbol{z}(k)$ *for* $k_0 \leq k \leq N$ *suffices to determine* $y(k_0) = y_0$.

For constant matrices A, B, C*, the exact solution of (74) for* $k \geq k_0$ *is*

$$\boldsymbol{z}(k) = CA^{k-k_0}y_0 + \sum_{j=k_0}^{k-1} CA^{k-j-1}B\boldsymbol{u}(j). \tag{75}$$

Since the second term on the r.h.s. of (75) is known, it may be subtracted from the observed value of $\boldsymbol{z}(k)$*. Hence, for investigating a necessary and sufficient condition for complete observability it suffices to consider the case where* $\boldsymbol{u}(k) = 0$.

It can then be shown that the system (74) is completely observable iff $\operatorname{rank} V = n$*, where*

$$V = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \end{bmatrix}.$$

Returning to the continuous time model, let the initial system state be $\boldsymbol{x}(0)$ *and the final state be* $\boldsymbol{x}(t_f)$*. We say that the system (68) is controllable if it is possible to construct a control signal which, in finite time interval* $0 < t \leq t_f$*, will transfer the system state from* $\boldsymbol{x}(0)$ *to* $\boldsymbol{x}(t_f)$.

For simplicity we restrict attention to the case of a single component (control input) variable. Let us first assume that the eigenvalues of the matrix A are all distinct, so that the ODE system (68) can be *transformed into the canonical state variable form*

$$\begin{bmatrix}\dot{z}_1\\ \vdots\\ \dot{z}_n\end{bmatrix} = \begin{bmatrix}\lambda_1 & \cdots & 0\\ & \ddots & \\ 0 & \cdots & \lambda_n\end{bmatrix}\begin{bmatrix}z_1\\ \vdots\\ z_n\end{bmatrix} + \begin{bmatrix}\tilde{b}_1\\ \vdots\\ \tilde{b}_n\end{bmatrix} u.$$

This equation can be written in component form as

$$\dot{z}_i = \lambda_i z_i + \tilde{b}_i u, \quad i = 1, 2, \ldots, n,$$

which has the solution

$$z_i(t) = e^{\lambda_i t} z_i(0) + e^{\lambda_i t}\int_0^t e^{-\lambda_i \tau}\tilde{b}_i u(\tau) d\tau.$$

The system described by Eq. (68) is then completely controllable if the state variable z_i can be transferred from any initial state $z_i(0)$ to any final state $z_i(t_f)$ in a finite time t_f. In other words, the system is controllable if it is possible to construct a control signal $u(t)$ such that the following equation is satisfied

$$\frac{z_i(t_f) - e^{\lambda_i t_f} z_i(0)}{e^{\lambda_i t_f}} = \int_0^{t_f} e^{-\lambda_i \tau}\tilde{b}_i u(\tau) d\tau.$$

This inverse problem is easily solvable. In fact there are an infinity of functions $u(t)$ on the interval $(0, t_f)$, which solve it, provided $\tilde{b}_i \neq 0$, because otherwise the link between input and the corresponding state variable gets broken and hence it is no longer possible to control that particular state variable.

It therefore follows that the necessary condition of complete controllability is simply that the vector $\tilde{\boldsymbol{b}}$ should not have any zero elements. If any element of this vector is zero, then the corresponding state variable is not controllable. It can be further shown that the condition stated here is in fact both necessary and sufficient.

The result just obtained can be extended to the case where the control variable $\boldsymbol{u}$ is an m-dimensional vector. For the system described by

$$\dot{\boldsymbol{z}} = \boldsymbol{\Lambda}\boldsymbol{z} + \tilde{\boldsymbol{B}}\boldsymbol{u}$$

where $\boldsymbol{\Lambda} = diag(\lambda_1, \lambda_2, \ldots, \lambda_n)$, and

$$\tilde{\boldsymbol{B}} = \begin{bmatrix}\tilde{b}_{11} & \tilde{b}_{12} & \ldots & \tilde{b}_{1m}\\ \tilde{b}_{21} & \tilde{b}_{22} & \ldots & \tilde{b}_{2m}\\ \vdots & & & \\ \tilde{b}_{n1} & \tilde{b}_{n2} & \ldots & \tilde{b}_{nm}\end{bmatrix}$$

the necessary and sufficient condition for controllability is that the rank of the matrix $\tilde{\boldsymbol{B}}$ must be n_0. It is observed from the above equation that otherwise, it is not possible to influence (all) state variables by the control forces and hence the system is not fully controllable.

Consider the state model of an n-th order single-input, single-output linear time-invariant system,

$$\begin{aligned}\dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}u \\ y &= \boldsymbol{c}^T\boldsymbol{x}\end{aligned}$$

The state equation may be transformed to the canonical form by the linear transformation $\boldsymbol{x} = \boldsymbol{M}\boldsymbol{z}$. The resulting state and output equations are

$$\dot{\boldsymbol{z}} = \boldsymbol{\Lambda}\boldsymbol{z} + \tilde{\boldsymbol{b}}u \tag{76}$$

$$\begin{aligned}y &= \tilde{\boldsymbol{c}}^T\boldsymbol{z} \\ &= \tilde{c}_1 z_1 + \tilde{c}_2 z_2 + \cdots + \tilde{c}_n z_n\end{aligned} \tag{77}$$

Since diagonalization decouples the state variables, no z-component now contains any information regarding any other component, i.e., each state must be independently observable. It therefore follows that for a state to be observed through the output y, its corresponding coefficient in Eq. (77) should be nonzero. If any particular $\tilde{c}_i$ is zero, the corresponding z_i can have any value without its effect showing up in the output y. Thus the necessary (and also sufficient) condition for complete state observability is that none of the $\tilde{c}_i$'s (i.e., none of the elements of $\boldsymbol{c}^T\boldsymbol{M}$) should be zero.

The result may be extended to the case of multi-input, multi-output systems where the output vector, after canonical transformation, is given by

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_p \end{bmatrix} = \begin{bmatrix} \tilde{c}_{11} & \tilde{c}_{12} & \cdots & \tilde{c}_{1n} \\ \tilde{c}_{21} & \tilde{c}_{22} & \cdots & \tilde{c}_{2n} \\ \vdots & & & \\ \tilde{c}_{p1} & \tilde{c}_{p2} & \cdots & \tilde{c}_{pn} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix}$$

or

$$\boldsymbol{y} = \tilde{\boldsymbol{C}}\boldsymbol{z}.$$

The necessary condition for complete observability is that none of the columns of the matrix $\tilde{\boldsymbol{C}}$ be zero.

Kalman's test of observability is as follows. A general n-th order multi-input, multi-output linear time-invariant system

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}$$
$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x}$$

is completely observable if and only if the rank of the composite matrix

$$\boldsymbol{Q}_0 = [\ \boldsymbol{C}^T \vdots \boldsymbol{A}^T\boldsymbol{C}^T \vdots \dots (\boldsymbol{A}^T)^{n-1}\boldsymbol{C}^T\ ] \tag{78}$$

is n.

Example *Let us examine the observability of the system given below*

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}u \tag{79}$$

$$y = \begin{bmatrix} 3 & 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \boldsymbol{c}^T\boldsymbol{x} \tag{80}$$

The characteristic equation is

$$\boldsymbol{A} - \lambda\boldsymbol{I} = \begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 0 & -2 & -3-\lambda \end{vmatrix} = 0$$

or

$$\lambda(\lambda+1)(\lambda+2) = 0.$$

Therefore the eigenvalues of matrix $\boldsymbol{A}$ are

$$\lambda_1 = 0, \lambda_2 = -1, \lambda_3 = -2.$$

The diagonalized matrix is then

$$M = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & 1 & 4 \end{bmatrix}$$

Under the linear transformation $\boldsymbol{x} = \boldsymbol{M}\boldsymbol{z}$, the output is given by

$$\boldsymbol{y} = \boldsymbol{c}^T\boldsymbol{M}\boldsymbol{z} = \begin{bmatrix} 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

It is found that the system is not completely observable, since the state variable z_2 is hidden from observation.

Let us apply the Kalman's test to the same system. From Eqs. (79) *and* (80)

$$\boldsymbol{A}^T(\boldsymbol{c}^T)^T = \boldsymbol{A}^T\boldsymbol{c} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -2 \\ 0 & 1 & -3 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix};$$

$$(\boldsymbol{A}^T)^2(\boldsymbol{c}^T)^T = (\boldsymbol{A}^T)^2\boldsymbol{c} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -2 \\ 0 & 1 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ -2 \end{bmatrix}.$$

Therefore the composite matrix in Eq. (78) *is given by*

$$\boldsymbol{Q}_0 = [\ \boldsymbol{c} \vdots \boldsymbol{A}^T\boldsymbol{c} \vdots (\boldsymbol{A}^T)^2\ ] = \begin{bmatrix} 3 & 0 & 0 \\ 4 & 1 & -2 \\ 1 & 1 & -2 \end{bmatrix}.$$

Since

$$\begin{vmatrix} 3 & 0 \\ 4 & 1 \end{vmatrix} \neq 0 \textit{ and } \begin{vmatrix} 3 & 0 & 0 \\ 4 & 1 & -2 \\ 1 & 1 & -2 \end{vmatrix} = 0$$

the rank of the matrix $\boldsymbol{Q}_0$ is $r = 2$, while $n = 3$. Hence one of the state variables is unobservable.

The concepts of controllability and observability found important applications in signal processing and control theory[260]*, where it is sometimes desired to "track" (i.e. maintain an estimate of) a time-varying signal in the presence of noise. If the signal is known to be characterized by some number of parameters that vary only slowly, then the formalism of Kalman filtering tells how the incoming, raw measurements of the signal should be processed to produce best parameter estimate as a function of time. For example, if the signal is a frequency-modulated*

[260] For further reading, see:

- Kalman, R.E., *A New Approach to Linear Filtering and Prediction Problems*, Transaction of the ASME **82**, 35–45, 1960.
- LaSalle, J.P., *The Stability and Control of Discrete Processes*, Springer-Verlag, New York, 1986.
- Barnett, S. and R.G. Cameron, *Introduction to Mathematical Control Theory*, Oxford University Press, 1985.
- Franklin, G.F. et al, *Feedback Control of Dynamic Systems*, Prentice-Hall, New York, 2001.

sine wave, then the slowly varying parameter might be the instantaneous frequency. The Kalman filter for this case is called a phase-locked loop *and is implemented in the circuitry of good radio receivers.*

In control theory a system is said to be controllable *if it is possible to manipulate the control variables in such a way that the system starts out from any initial state and finishes up in any desired state — for example, transferring a spacecraft from an orbit round the earth to a specified orbit round the Moon, or to a 'soft landing' on the Moon, by suitably controlling the rocket motors.*

Indeed, when a deep space probe is launched, corrections may be necessary to place the probe on a precisely precalculated trajectory. Radio telemetry provides a stream of discretized-time observed state vectors, $\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_k$*, giving information at different times about how the probe's position compares with its planned trajectory.*

Similarly, space shuttle[261] *control systems are absolutely critical for flight. Because the shuttle is an unstable air-frame, it requires constant computer monitoring during atmospheric flight. The flight control system sends a stream of commands to aerodynamic control surfaces and many small thruster jets.*

Modern control theory involves besides input and output variables also feedback and feedforward loops. These feed information on the sensed and desired trajectories into the input. Adaptive control schemes estimate unknown parameters such as inertia or weights of a neural net estimator of unknown dynamics. These parameters are then involved alongside the normal state vector. A control scheme must guarantee suitable levels of stability and controllability in the presence of nonlinearities, delays and noise.

261 The *Columbia* (12 stories high and weighing 75 tons, launched in April 1981) was the first U.S. space shuttle, a triumph of control systems engineering design, involving many branches of engineering — aeronautical, chemical, electrical, hydraulic, and mechanical.

1849–1857 CE **Antonio Santi Guiseppe Meucci** (1808–1889, Italy and USA). Made pioneering experiments of transmission of the human voice by electricity. Designed and constructed a prototype of an *electromagnetic telephone* many years before Alexander Graham Bell (1876). However, the vagaries of history and the patent office have determined that Meucci will be recognized only in Italy as the true inventor of the telephone.

Meucci was born in Florence, Italy. During 1821–1827 he studied chemistry and mechanics in the '*Accademia di Belle Arti*'. In 1833–1834 he was involved in the conspiracies for the liberation of Italy, being jailed with other patriots. In 1835 he fled the violence of the civil insurrections which raged throughout Italy and reached Havana, Cuba and stayed there as chief engineer of the local Opera House. In 1850, the Meucci's moved to Clifton (Staten Island) NY where he begun to conduct experiments on telephony communications over distances of a few km.

Today in Bensonhurst, New York, there is an iron-fenced triangle of land with fresh sod, some trees, and a small monument that reads: "ANTONIO MEUCCI, 1808–1889, FATHER OF THE TELEPHONE. FIRST US PATENT CAVEAT 3335". He was also memorialized in an Italian postal stamp.

1849–1877 CE **Emil Heinrich Du Bois-Reymond**[262] (1818–1896, Germany). Physiologist. Showed the existence of electrical currents in nerves, correctly arguing that it would be possible to transmit nerve impulses chemically. His experimental techniques proved the basis for almost all future work on electrophysiology.

He was born and educated in Berlin. He studied a wide range of subjects for two years before he finally chose a medical training. Graduating in 1843, he plunged into research on animal electricity and especially on electric fishes. By 1849 he developed a delicate instrument for measuring nerve currents which enabled him to detect an electric current in ordinary muscle tissues, notably contracting muscles. Du Bois-Reymond denounced the *vitalistic* doctrines that were in vague among German scientists and denied that nature contained mystical life forces independent of matter.

He became a professor of physiology at the Berlin university (1858) and was appointed the head of the new Physiological Institute which first opened in Berlin (1877).

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[262] His brothel Paul (1831–1889) was a mathematician, who made contributions to the theory of functions. Their father was a Swiss teacher who settled in Berlin. The family was French-speaking.

1850–1866 CE **James Young** (1811–1883, Scotland). Industrial chemist. Started commercial production of paraffin from crude oil made from heated coal. The crude oil was distilled into its components (or fractions), in containers heated by steam. Thus Young established the basis for *oil refining*[263].

Young was born in Glasgow and studied chemistry under **Thomas Graham** at University College, London. He worked as a chemist in Lancashire (1839) and after 1850 directed his efforts to the establishment of the Scottish mineral-oil industry — for the production of lubricating oils, illuminating oils and paraffin wax.

1850 CE Advent of large sanitary municipal improvement in Western Europe. Before this date the practice of *bathing* was not a general one, and was entirely confined to river and sea baths.

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263 Traditionally, oil was brought to the surface in buckets by workmen who lowered themselves into hand-dug wells.

Science in the Age of Nationalism[264] *(1850–1890)*

The Germans and Italians were the pioneers of modern science, reaching their first peak achievement with the works of **Kepler** *and* **Galileo** *respectively in the early decades of the* 17^{th} *century. But they did not sustain this effort, and almost* 200 *years were to elapse before they produced men of science who were at all comparable.*

The great geographical discoveries opened up opportunities which were the more effectively exploited by England, France and Holland, and these lands became the main centers of European endeavor. In science, as in other fields, England and France retained their leadership right down to the mid or the late 19^{th} *century [their activities were somewhat complementary: the French were inclined toward theoretical interpretation of nature, while the British leaned more to empirical investigation]. In the early decades of the* 19^{th} *century French scholars were the leaders in the world of science, but during* 1850–1870, *the British rose to the forefront once more.*

Meanwhile, the Germans and the Italians adhered to traditions which had been laid down in the 16^{th} *century. Politically they remained divided up into a number of petty principalities (in contrast to the unified states of Britain and France), whilst in science they retained an active interest, but produced little that was novel during most of the* 18^{th} *century.*

It is noteworthy that, of the 90 *or so scientific journals founded before* 1815, 53 *were German,* 8 *were Italian,* 15 *were French and* 11 *were English, whilst America, Sweden and Holland, had one each. For such a number of scientific journals to be founded, there must have been a considerable interest in science amongst the Italians and Germans, but it seems that this interest was not active enough to produce markedly novel advances.*

In general, the second half of the 19^{th} *century is marked with an overriding interest and deep belief in science, to a degree that a veritable 'cult of science' developed. Science inspired a positive alternative to the seemingly futile Idealism and Romanticism of the early* 19^{th} *century.*

Scientific research, formerly the domain of a few scientists and gentleman scholars, now became the concern of large numbers of people, especially as the application of science to industry gave an incentive to new inventions. "Pure"

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264 For further reading, see:

- Rich, N., *The Age of Nationalism and Reform* (*1850–1890*), W.W. Norton and Company: New York, 1977, 270 pp.

science continued to be of fundamental importance, but "applied" science — the fusion of science and technology — now took precedence in the minds of most people. A virtually endless series of scientific inventions seemed to provide tangible evidence of man's ability to unlock the secrets of nature.

By the end of the 19^{th} *century, Germany has outstripped both England and France and held the leading position in the physical sciences and mathematics, which climaxed in the 'second scientific revolution' of* ***Planck*** *and* ***Einstein*** *during 1900–1905.*

1850–1857 CE **Rudolf Julius Emanuel Clausius** (1822–1888, Germany). Theoretical physicist who laid the foundations to thermodynamics and the kinetic theory of gases.

Based on the theoretical results of **James Joule** (1818–1889, England, 1847) and the former theory of heat engines of **Sadi Carnot** (1796–1832, France, 1824), Clausius stated (1850–1865) the first and second laws of thermodynamic[265]and introduced the concept of *entropy*. He formulated (1854–1857) the kinetic theory of gases, defining the concept of *mean free-path*. He assumed different molecular velocities, but the statistical velocity distribution function is due to **Maxwell** (1859). In 1870 Clausius applied to the theory of gases a theorem in mechanics due to the astronomer and mathematician **Yvon Villarceau** (1813–1883, France), known today as the *scalar virial theorem*; the *virial* is the integral of the moments of the molecular forces, partaking in the equation of conservation of mechanical energy. This theorem leads directly to the Van der Waals equation of state.

The scalar virial theorem: although not as important as the conservation of *angular momentum* under central force, on the conservation of *energy* under a conservative force, assumes considerable importance in the kinetic theory of gases, and in its applications to galactic dynamics[266].

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[265] In a paper of 1865 he stated these in the following form:

1. The energy of the universe is constant.

2. The entropy of the universe tends to a maximum.

[266] Consider a general system of mass points with position vectors $\boldsymbol{r}_i$ and applied forces $\boldsymbol{F}_i$ (including any forces of constraint). Starting with the equation of

motion of a single particle

$$\frac{d}{dt}(m_i \boldsymbol{V}_i) = \dot{\boldsymbol{p}}_i = \boldsymbol{F}_i,$$

we derive from it the vector identity

$$m_i \boldsymbol{V}_i^2 + \boldsymbol{F}_i \cdot \boldsymbol{r}_i \equiv \frac{d}{dt}(m_i \boldsymbol{V}_i \cdot \boldsymbol{r}_i).$$

Summing over all particles and time-averaging this equation over a time interval τ, we obtain

$$2\overline{T} + \overline{\sum_i \boldsymbol{F}_i \cdot \boldsymbol{r}_i} = \frac{1}{\tau}\left[G(\tau) - G(0)\right],$$

where

$$G = \sum_i m_i \boldsymbol{V}_i \cdot \boldsymbol{r}_i,$$

$$\overline{T} = \frac{1}{\tau}\int_0^\tau \sum_i \left(\frac{1}{2} m_i V_i^2\right) d\tau.$$

If the motion is periodic, or if coordinates and velocities of all particles remain finite (such that there is an upper bound for G), or if the forces are derived from a potential — then the entity

$$\frac{1}{\tau}\left[G(\tau) - G(0)\right]$$

vanishes or can be made as small as desired. The ensuing result

$$2\overline{T} + \overline{\sum_i \boldsymbol{F}_i \cdot \boldsymbol{r}_i} = 0,$$

is known as the *virial theorem*.

The quantity $-\frac{1}{2}\sum(\boldsymbol{F}_i \cdot \boldsymbol{r}_i)$ is called the *virial* of the system. For a single *particle* moving under a *conservative central force* the theorem reduces to $\overline{T} = \frac{1}{2}\frac{r\partial V(r)}{\partial r}$, where $\boldsymbol{F} = -\boldsymbol{e}_r\frac{\partial V}{\partial r}$. The virial theorem differs in character from mechanical conservation laws in being *statistical* in nature, i.e., it is concerned with time averages of various mechanical quantities.

In general, $\boldsymbol{F}_i$ can be separated into external ($\boldsymbol{f}_i$) and internal ($\boldsymbol{f}_{ij}$) forces. Then,

$$2\overline{T} = -\left[\sum_{\substack{\text{all}\\ \text{particles}}} \boldsymbol{f}_i \cdot \boldsymbol{r}_i + \sum_{\substack{\text{all pairs}\\ \text{of particles}}} \boldsymbol{f}_{ij} \cdot \boldsymbol{r}_{ij}\right], \qquad \boldsymbol{r}_{ij} = \boldsymbol{r}_i - \boldsymbol{r}_j.$$

One of the most interesting applications of the virial theorem is the derivation of the *equation of state of a gas*, which describes the relations between the macroscopic quantities such as pressure, volume, and temperature. **Clausius**

Clausius was born at Köslin, in Pomerania. He studied at Stettin, Berlin and Halle. In 1855 he was appointed professor of physics at Zürich. He then held appointments at the universities of Würzburg (1867), and Bonn (1869).

During the Franco-German war he was at the head of an ambulance corps composed of Bonn students.

1850–1871 CE **Adhemar-Jean-Claude Barre de Saint-Venant** (1797–1886, France). Applied mathematician. Contributed to mechanics, elasticity, hydrostatics and hydrodynamics.

Derived solutions for the torsion of noncircular cylinders (1850). Extended Navier's work on the bending of beams (1864). Derived the equations for non-steady flow in open channels. Also developed a vector calculus similar to that of Grassmann. Stated (1855) the *Saint-Venant principle*[267].

(1870) has shown that for N gas molecules enclosed in a volume V at absolute temperature T and pressure p, the virial theorem leads to the result

$$pV = NkT + \frac{1}{3}\left(\sum_{\substack{\text{all}\\ \text{pairs}}} \boldsymbol{f}_{ij}\cdot\boldsymbol{r}_{ij}\right)_{\text{average}} .$$

For an *ideal gas*, the intermolecular forces are considered zero, and this result reduces to the classical result $pV = NkT$ (k = Boltzmann's constant). In all other cases, it is a good approximation except when the molecules are closely packed or the temperatures are very low. Clausius also generalized Clapeyron's equation expressing the relation between the pressure and temperature at which two phases of a substance are in equilibrium (*Clausius-Clapeyron equation*).

267 *Saint-Venant principle* (as formulated by **Boussinesq** (1889)): In elastostatics, if the boundary tractions on a part S_1 of the boundary S are replaced by a *statically equivalent* traction distribution, the effect on the stress distribution in the body are negligible at points where distance from S_1 is large compared to the maximum distance between points of S_1. The principle has been widely accepted on *empirical grounds*, and a precisely stated version of it was proved by **S. Sternberg** (1954). The principle is of great importance in applied elasticity, where it is frequently invoked to justify solutions in long slender structural members where the end traction boundary conditions are satisfied only in an average sense, so that the correct stress resultant acts on the ends. In such solutions, the *actual* stress distribution near the ends may differ considerably from the *calculated* stress distribution. The *exact* solutions in such cases require elaborate calculations.

The principle is not limited to linear elastic solid or infinitesimal displacements.

Saint-Venant was a student of **Liouville** (1839–1840). He taught mathematics at the Ecole de Ponts et Chausses when he succeeded **Coriolis**.

1850–1871 CE Six European wars established a new balance of power, out of which came the unifications of Italy and Germany and the unprecedented economic growth and scientific development of Europe.

The *Crimean War* (1854–1856) arose out the conflict between Russia and the Western powers over economical interests in the Near East, caused by the slow disintegration of Turkish rule in the Balkans. The defeat of Russia curtailed its influence over the area adjacent to the Ottoman Empire. More than 500,000 people lost their lives in the war. The cost of the war (both sides) have been about $310 million[268] (1903).

In the war of 1859 Austria lost to France and Italian forces, and was consequently driven out of Lombardy. In 1860, Sicily and Southern Italy were liberated from French rule by a small expedition force of ca 1000 men under the leadership of **Giuseppe Garibaldi**, who defeated an army twenty time its size. The kingdom of Italy[269] was proclaimed in 1861.

Although no precise proof is available, **Goodier** (1937) has argued on the basis of energy as follows: Let p be the order of magnitude of the *surface forces*, and a the order of magnitude of the linear dimension representative of the surface ΔS upon which the forces act. Then the components of the *stress tensor* will be of order (pa^2), the components of the *strain tensor* of order $\frac{p}{E}$, the components of the *displacement* of order $\frac{pa}{E}$, the total *work* done by the applied forces of order $\frac{p^2a^3}{E}$ and the *energy density* is of order $\frac{p^2}{E}$. The work done by the applied forces is just sufficient to affect a volume whose magnitude is of order a^3; outside this volume there can be no deformation and one can therefore assume that the region affected will be the immediate vicinity of the surface ΔS upon which the surface forces act.

The concept of Saint-Venant principle does not apply to problems in *elastodynamics*, where the governing equations are hyperbolic, and we know that any fine structure of the surface pressure distribution is propagated all the way to infinity, or at least to the far-field [e.g. the field of a line load traveling at supersonic speeds over the free surface].

268 On the night of November 14, 1854 a violent storm in the Black Sea wrecked the entire British supply fleet; nearly 30 vessels with their cargo were sunk. Nobody bothered to read the *barometer*!

269 The independence and unification of Italy was conducted under the joint efforts of **Giuseppe Mazzini** (1805–1872), **Camillo di Cavour** (1810–1861) and **Giuseppe Garibaldi** (1807–1882). These men were, respectively, the '*soul*', the '*brain*' and the '*sword*' of the independence movement.

In the Austro-Prussian war of 1866 (battle of Sadowa[270]), the Austrians were defeated and surrendered Venice to Italy.

Previously, in 1863, Denmark had suffered a crushing defeat at the hands of Prussia and Austria and had to surrender Schleswig and Holstein to the victors.

Finally in 1870, the battle of *Sedan* decided the outcome of the Franco-German war[271].

Thus, in 1871, Germany emerged as a great power — both militarily and culturally. From 1871 to 1945, the influence of that belatedly unified nation made itself felt in every major international crisis and in the history of every country. Compared to German unification, the unification of Italy seems of minor importance today, though it did not appear so at the time. Of much greater consequence was the tragic fate of the Second French Empire. Its defeat at the hand of Prussia sowed some of the seeds that brought forth the great wars of the 20^{th} century. At the time however, it seemed as though the Continent had at long last found the stability that statesmen before 1850 had tried so hard to achieve. The future was to show the precariousness of the new order in Europe.

1850–1881 CE **Ferdinand Julius Cohn** (1828–1898, Germany). Botanist and bacteriologist. Defined and named the term *bacterium* and founded the study of the *bacteriology.* First to treat bacteria systematically by dividing them into genera and species (1872). Assisted **Robert Koch** in his work on *anthrax* (1876). Helped disprove the notion of spontaneous generation. Showed that plant and animal protoplasm are one and the same substance.

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[270] The battle of *Sadowa.* Austria was deeply divided and poorly prepared. She was further handicapped by having to fight on two fronts. The Prussians were in excellent military form and led by a master-strategist, **Helmut von Moltke**.

[271] The main cause of the Franco-Prussian War of 1870–1871 was French resentment of the growing power of Prussia. Relations between the two countries grew worse when Prussia appeared to support the claim of a German prince to the throne of Spain. The final spark occurred when **Otto von Bismarck** (1815–1898), chief minister of Prussia, made public a telegram from King William which he had altered to appear insulting to the French. Bismarck hoped war with France would unite Germany behind Prussia. France at once declared war, although not ready to fight. In six weeks its main army, with the Emperor Napoleon III, had surrendered, and in January 1871 Paris capitulated after 132-day siege. Under the Treaty of Frankfurt, which ended the war in 1871, France gave Prussia the provinces of Alsace and Lorraine, and paid an indemnity of 5000 million Franc.

Cohn was born in Breslau (then Wroclaw, Poland) to Jewish parents. He was educated at Breslau and Berlin[272] (PhD: 1847, at the age of 19), and became an associate professor at Breslau already at the age of 31. At an early age he exhibited astonishing ability with the microscope, which he did much to improve. Although his early researches were especially on algae, he soon widened the scope of his interest to fungi and bacteria and other lower life-forms. He had also a clear perception of the important bearings of mycology and bacteriology in infective diseases. Cohn founded the first institute of plant physiology (1866), the world's first institute specializing in plant physiology.

Bacterial studies outside medicine remained superficial until Cohn. He distinguished four groups on the basis of external form and specific fermentive activity. He recognized that bacteria take *nitrogen* from simple ammonia compounds, elucidated their life-cycles, identified spores and suggested that bacteria were motile cells devoid of walls. Cohn is regarded as the father of bacteriology in that he was the first to account it a separate science and define bacteria. He observed sexual formation of spores in the fungal genera *Sphaeroplea Pilobolus.*

1851 CE **John Gorrie** (1803–1855, USA). Physician and inventor. Invented the first machine for mechanical refrigeration, based on principles of present-day mechanical refrigerators: a steam-engine driven piston compressed the air in a cylinder. When the piston withdrew, the air expanded, absorbing heat from a bath of brine in which the cylinder was immersed.

Gorrie spent most of his life practicing medicine in Apalachicola, Fla. There he investigated the artificial cooling of sickrooms and hospitals. In 1851, he patented an ice-making machine but he lacked funds to manufacture it.

1851 CE **Heinrich (Henri) Daniel Ruhmkorff** (1803–1877, Germany and France). Mechanic, manufacturer of instruments of physics and electrical researcher. Invented (1851) the *Ruhmkorff induction-coil* which could produce

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[272] He went to the University of Breslau (1842) in order to study philosophy and soon became interested in botany, but because he was a Jew he was unable to obtain a degree there. This prompted his transfer to the University of Berlin. Upon his return to Breslau (1849) he had to wait ten years before he was made associate professor and another thirteen years to become a full (ordinary) professor of botany — the first non-assimilated Jew in Prussia to obtain this rank. Indeed, **Robert Remak** (1815–1865) was not allowed to hold a senior teaching position in any German university. **Julius Sachs** (1832–1897), however, was appointed full professor at Breslau already in 1868, but he had been fully assimilated and relinquished his religion long before.

sparks more than 30 cm in length. He thus improved on the two-winding induction spark-coils of **Callan** (1836), on the basis of the research conducted by **Mason** and **Breguet** (1842).

The Ruhmkorff coils, which produced high-voltage current within a second armature winding, were used for operation of *Geissler and Crooks tubes*, in the first *radio transmitters*, for detonation devices as well as in other primitive electrical and electronic devices.

The induction-coil is built as follows: upon an iron core is wound a primary coil consisting of a relatively small number of turns of thick wire, and over this (generally in several layers insulated from one another) a secondary coil consisting of a large number of turns of thin wire. The necessary variations of the magnetic field of the primary current (supplied by a voltage source in its circuit) are produced by making and breaking this current at a rapid speed. A condenser is usually connected in parallel with the make-and-break. It consists of a large number of sheets of tin-foil insulated from each other by means of paraffined paper or sheets of ebonite. Alternate tin-foil sheets are connected together so that the capacity of the whole is very great.

The action of the induction-coil is as follows: When the primary circuit is "made", the magnetic flux through the coil increases. When the primary circuit is suddenly "broken", the magnetic field disappears rapidly and the corresponding field energy, previously stored up chiefly in the air, becomes available. It cannot give rise to a current in the primary coil, for this is now open. The whole of the field energy therefore goes to produce current in the secondary circuit, provided that this is closed. If the secondary circuit is also open, then in consequence of the rapid decrease of magnetic induction and the large number of turns in the secondary coil, a high voltage is produced between the secondary terminals. This tends to cause a spark or arc to pass, with consequent of *closing* of the secondary circuit and consumption of the greater part of the field energy.

The object of the condenser is to reduce the voltage between the contacts of the make-and-break at the moment of breaking the primary circuit, thus preventing sparking and arcing at this point. Since it is connected in *parallel* with the make-and-break, the condenser acts as a shunt. Hence, the introduction of the condenser not only protects the contacts of the make-and-break against damage by arcing, but also increases the efficiency of the induction coil.

Ruhmkorff was born in Hannover, Germany. After apprenticeship to a German mechanic, he worked in England with **Joseph Brahmah**, inventor of the hydraulic press. In 1855 he opened his own shop in Paris, which became widely known for the production of high-quality electrical apparatus. The induction-coil awarded him (1858) a 50,000-franc prize by the Emperor

Napoleon III as the most important discovery in the application of electricity. Ruhmkorff coil was popular for energizing discharge tubes and in particular for *generating X-rays* (discovered in 1895 by Roentgen). His doubly wound induction-coil later evolved into the *alternating-current transformer*.

1851–1859 CE **Georg Friedrich Bernhard Riemann** (1826–1866, Germany). A profound mathematician who greatly influenced the mathematics of the 20^{th} century. His ideas concerning geometry of space had a major effect on the development of modern mathematical physics and provided the concepts and methods used later in General Relativity Theory. He was an original thinker, and a host of methods, theorems and concepts are named after him [*Riemann surface*; *Riemann integral*; *Riemann hypothesis* etc.]. Obtained his doctoral degree in Göttingen (1851) under **Gauss**. His work can be classified according to the following topics:

(1) *Theory of functions of complex variable*, based upon the Cauchy-Riemann relations. Introduced geometrical representation of multi-valued functions (Riemann surfaces), rendering geometric interpretation to the hidden analytical properties of functions. In this he paved the road to modern topology.

Riemann is considered, with **Cauchy** and **Weierstrass**, as one of the three founders of complex function theory. The *method of steepest descent* (also known as the *saddle-point method*) occurs in a posthumously-published fragment of Riemann (Gesammelte Werke, 1892). It was rediscovered in 1910 by **Peter Debye** (1884–1966, Holland).

(2) *Theory of functions of real variable* (1854). Developed the concept of the *Riemann integral*. Generalized Dirichlet's criteria for the validity of Fourier expansions. This inspired Cantor's theory of sets and then led to the concept of the *Lebesgue integral*.

(3) *Differential equations*. Aimed to characterize all linear differential equations whose solutions are expressible in terms of Gauss' hypergeometric function, and to achieve systematic classification of all linear differential equations with rational coefficients according to the number and nature of their singularities.

(4) *Differential geometry* (1854). Followed up the work of his teacher **Gauss** on curved surfaces and took the final step in a far-reaching generalization of differential geometry.

While Gauss' theory is a direct descendant of cartography and geodesy, Riemann, in one of the most prolific contributions ever made to geometry, passed immediately to the general quadratic differential form in n variables, with variable coefficients. He introduced space as a topological manifold in an arbitrary number of dimensions. A *metric* was defined in such a manifold by means of a quadratic differential form:

$$ds^2 = \sum_{i,j=1}^{n} g_{ij} dx_i dx_j,$$

where the g_{ij}'s are suitable functions of $(x_1, x_2, \ldots, x_n)$; different systems of g_{ij}'s define different Riemannian geometries on the manifold under discussion.

He introduced the *Riemannian curvature tensor*, which reduces to the Gaussian curvature when $n = 2$, and whose vanishing he showed to be necessary and sufficient for the given quadratic metric to be equivalent (isometric) to the Euclidean metric. From this point of view, the curvature tensor measures the deviation of the Riemannian geometry from Euclidean geometry. The physical significance of geodesics appears in its simplest form as a consequence of Hamilton's principle in the calculus of variations.

In a general Riemannian space, g_{ij} is a symmetric non-singular $(\det g_{ij} \neq 0)$ covariant second-rank tensor field. The dependence of g_{ij} on the coordinates x^j is *arbitrary* except that its partial derivatives will be assumed to exist and be continuous to any required order.

A special case of a Riemann space in which a global Cartesian system can be set up is known as a *Euclidean space*. This condition imposes certain restrictions on the metric tensor g_{ij}, namely that the independent components of the *Riemann-Christoffel tensor* $R^r_{\cdot ijk}$ vanish everywhere.[273] There are thus $\frac{n(n-1)}{2}$ conditions on the $\frac{1}{2}n(n+1)$ independent metric coefficients of an Euclidean space, in any coordinate system.

The Cartesian coordinate system has the advantage that the distance ds between two neighboring points $\boldsymbol{x}$ and $\boldsymbol{x} + d\boldsymbol{x}$ is given by the Pythagorean theorem $ds^2 = dx^i dx^i$ and therefore $g_{ij} = \delta_{ij}$. If $(\boldsymbol{x}',\ \boldsymbol{x}' + d\boldsymbol{x}')$ are the coordinate of the *same* point in another Cartesian frame, then $ds^2 = d\overline{x}^i d\overline{x}^i$ and it follows that ds^2 is invariant w.r.t. a transformation of the coordinates from one rectangular frame to another. However, in an Euclidean space it is often convenient to employ a coordinate frame which is not Cartesian, and this is achieved by a curvilinear transformation of the coordinates. Thus, *curvilinear orthogonal coordinate frames* are generated with metric tensors g_{ij} represented by diagonal matrices. (If the curvilinear coordinates are non-orthogonal, off-diagonal elements of g_{ij} will appear.)

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[273] $R^r_{\cdot ijk}$ and g_{lm} together obey, identically, certain algebraic symmetry conditions and differential equations (the latter are the Bianchi identities). This reduces the number of functional conditions on g_{ij} to $\frac{1}{2}n(n-1)$.

In another class of Riemannian spaces, no admissible global transformation exists which reduces $ds^2 = g_{ij}dx^i dx^i$ to the Pythagorean form $ds^2 = dy^i dy^i$, i.e. no global Cartesian coordinate system can be found. These spaces are *non-Euclidean*, e.g. the 2-dimensional surface of a sphere. In this example we can always find a *local* Cartesian frame in which $ds^2 = du^2 + dv^2$ at a single *point* or even along a local *curve*, but never in any local *two dimensional* region, let alone globally (in contradistinction, the surfaces of the right circular cylinder and cone are locally, though not globally, Euclidean.) Clearly, one may *approximate* any sufficiently small region by a *flat* (Euclidean) *space* provided that the region taken is small enough.

Consider a 2-dimensional Riemann surface with metric equation $ds^2 = g_{11}dv^2 + 2g_{12}dvdw + g_{22}dw^2$ where (v, w) are some Gaussian coordinates and $g_{11} > 0$. The coefficients g_{11}, g_{12}, and g_{22} are functions of position and contain all the information about the geometry of the surface. A point P on the surface is selected and local coordinates (x, y) are found for which the metric is *locally Euclidean* at P. A general definition of the new coordinates is

$$dv = A(x,y)dx + B(x,y)dy,$$
$$dw = C(x,y)dx + D(x,y)dy,$$

where

$$A = \frac{\partial v}{\partial x}, \quad B = \frac{\partial v}{\partial y}, \quad C = \frac{\partial w}{\partial x}, \quad D = \frac{\partial w}{\partial y}.$$

Then

$$ds^2 = g'_{11}dx^2 + 2g'_{12}dxdy + g'_{22}dy^2,$$

where

$$g'_{11} = A^2 g_{11} + 2ACg_{12} + C^2 g_{22},$$
$$g'_{12} = ABg_{11} + (AD + BC)g_{12} + CDg_{22}$$

and

$$g'_{22} = B^2 g_{11} + 2BDg_{12} + D^2 g_{22}.$$

We are free to choose not only the values of A, B, C, D at P, but also the values of their first derivatives at P, provided the two compatibility conditions ($\frac{\partial A}{\partial y} = \frac{\partial B}{\partial x}$, $\frac{\partial C}{\partial y} = \frac{\partial D}{\partial x}$) are obeyed. Thus, the values of A, B, C, D, and their 6 independent first derivatives provide 10 free variables, enough flexibility to arrange at P:

$$g'_{12} = 0, \quad g'_{11} = g'_{22} = +1,$$
$$\frac{\partial g'_{11}}{\partial x} = \frac{\partial g'_{12}}{\partial x} = \frac{\partial g'_{22}}{\partial x} = \frac{\partial g'_{11}}{\partial y} = \frac{\partial g'_{12}}{\partial y} = \frac{\partial g'_{22}}{\partial y} = 0.$$

It follows that a Euclidean surface with metric equation $ds^2 = dx^2 + dy^2$ will match the current surface locally at P, up to deviations quadratic in $x - x_p$, $y - y_p$. In other words, a *plane* can always be drawn so as to pass through any arbitrary point on a 2-dimensional Riemann surface so that it is *locally tangential* to the surface. Furthermore, this plane can be deformed such that it remains intrinsically un-curved and its (previously) straight lines match all of the manifold's geodesics at P in both direction and curvature. (Notice that the conditions on the metric components and derivatives only make up 9 equations, whereas there are 10 degrees of freedom. The residual degree of freedom amounts to the choice of *orientation* of the x and y axes on the plane.)

It is even possible to choose A, B, C, D such that g'_{ij}, $\partial g'_{ij}/\partial x$, $\partial g'_{ij}/\partial y$ vanish along a *finite curve* on the manifold which passes through P.

A similar procedure can be followed in higher dimensional spaces; some coordinate transformation can always be found which converts the metric coefficients locally (or in the vicinity of a curve section) to a sum of squares, up to corrections quadratic in the geodetic distance from the given point or curve. Riemann spaces are thus said to be locally flat (or locally Euclidean) in this restricted sense. It is *not* possible, however, to arrange that the 2^{nd} derivatives as well as the first derivatives of the coefficients g_{11}, g_{12}, and g_{22} all simultaneously vanish.

If $g_{11}g_{22} - g_{12}^2 < 0$, the quadratic form obtained in the locally-flat coordinate system is $ds^2 = dx^2 - dy^2$ rather than $x^2 + dy^2$. The space involved is still locally flat (in the above sense). It is referred to as a *pseudo-Riemannian space*. The space-time of STR is pseudo-Euclidean (*Minkowski space*), that is, its metric assumes the form $ds^2 = c^2dt^2 - d\mathbf{r}^2$ *everywhere*.

A work published after Riemann's death contains what is now known as the *Riemann-Christoffel tensor* in the general-relativistic theory of gravitation (GTR). Riemann made the remarkable conjecture that his new metrics would reduce questions concerning the material universe and the "binding forces" holding it together, to problems in *pure geometry.*

His unifying principle enabled him to classify all existing forms of geometry and allowed the creation of any number of new types of abstract spaces, many of which have since found a useful place in geometry and modern physical theories.

Einstein conceived the geometry of spacetime as a pseudo-Riemannian geometry (locally pseudo-Euclidean) in which the curvature and geodesics are determined by the distribution of matter. In this curved space, planets move in their orbits around the sun by simply coasting along geodesics, instead of being pulled into curved paths by a mysterious force of gravity whose nature no one had ever really understood.

(5) *Analytic number theory* was founded by Riemann in his path-breaking paper of 1859, devoted to the *Prime Number Theorem*. It launched a tidal wave in several branches of pure mathematics, and its influence will probably still be felt for many years to come. He generalized Euler's identity to complex values of s. The resulting function is known as the *Riemann zeta-function*:

$$\zeta(s) = 1 + 2^{-s} + 3^{-s} + \cdots; \qquad s = \sigma + iy.$$

He made six conjectures with regard to the nature of this function[274].

[274] This Dirichlet series is convergent for $\sigma > 1$, and uniformly convergent in any finite region in which $\sigma \geq 1 + \delta, \quad \delta > 0$. It therefore defines an analytic function $\zeta(s)$, regular for $\sigma > 1$.

An equivalent definition of the zeta-function is

$$\zeta(s) = \prod_p \left(1 - \frac{1}{p^s}\right)^{-1},$$

where p runs through all the primes. This is known as *Euler's product* and is absolutely convergent for $\sigma > 1$. Euler considered it for particular values of s only, and it was **Riemann** who first considered $\zeta(s)$ as an analytic function of a complex variable. Since a convergent infinite product of non-zero factors is not zero, $\zeta(s)$ *has no zeros for* $\sigma > 1$.
The analytic function $\zeta(s)$ can be *continued* beyond the half-plane $\sigma > 1$. One such extension is through the relation

$$(1 - 2^{1-s})\zeta(s) = 1 - 2^{-s} + 3^{-s} - 4^{-s} + \cdots, \qquad \sigma > 0.$$

Riemann has extended the definition of the zeta-function for *all* complex values of s through a line integral in the complex s plane. He has also shown that the zeta function satisfies the remarkable functional equation:

$$\Gamma\left(\frac{s}{2}\right)\pi^{-\frac{s}{2}}\zeta(s) = \Gamma\left(\frac{1-s}{2}\right)\pi^{-(\frac{1-s}{2})}\zeta(1-s).$$

Some special values and relations involving $\zeta(s)$ are:

$\zeta(0) = -\frac{1}{2}; \quad \zeta(1) = \infty;$

$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} = \lim_{n \to 1}\left[(1 - 2^{1-n})\zeta(n)\right] = \ln 2;$

$\zeta(2) = \frac{\pi^2}{6}; \quad \zeta(4) = \frac{\pi^4}{90};$

> Assuming these six, Riemann then proved the Prime Number Theorem, namely, that the number of primes less then a given number x asymptotically approaches $\{x/\log_e x\}$. Since then, five of his six conjectures have been proven true.

The famous *Riemann Hypothesis*: All complex solutions of the equation: $1 - \frac{1}{2^s} + \frac{1}{3^s} - \frac{1}{4^s} + \cdots = 0$ lie on the line $s = \frac{1}{2} + it$, for some $t \neq 0$, has not yet been proved.

Riemann initiated the study of many more topics, but he died too young to have completed all the projects he started.

Riemann was brought up in a warm family atmosphere, and was of poor health due to poverty at home. Thanks to his father's understanding he did not practice theology, for which he was trained.

$\zeta(2n) = \frac{(2\pi)^{2n}}{2(2n)!}|B_{2n}|, \quad n = 1, 2, \ldots$ (B_n = Bernoulli numbers);

$\zeta(-2m) = 0$;

$\zeta(1-2m) = -\frac{B_{2m}}{2m}$;

$\zeta(-m) = -\frac{B_{m+1}}{m+1}, \quad m = 1, 2, 3, \ldots$;

$\gamma = \sum_{n=2}^{\infty} \frac{(-)^n}{n}\zeta(n)$ (Euler-Mascheroni constant);

$\frac{1}{2} z \coth z = \sum_{n=0}^{\infty} (-)^{n+1}\zeta(2n)\left(\frac{z}{\pi}\right)^{2n}$.

The infinity of primes is a direct consequence of the relation

$$\frac{6}{\pi^2} = \left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\cdots\left(1 - \frac{1}{p_n^2}\right)\cdots,$$

and the fact that the l.h.s. is irrational.

The Riemann Hypothesis (RH) and the Prime Number Theorem[275] *(PNT)*

The functional equation derived by Riemann for the zeta function exhibits a symmetry about $s = \frac{1}{2}$*. Consequently it is expected that the line* $\mathrm{Re}\, s = \frac{1}{2}$ *will play an important role in the theory of the zeta function. On the basis of this functional equation, and some preliminary calculations and asymptotic analyses which he made, Riemann conjectured that all the zeros of* $\Gamma\left(\frac{s}{2}\right)\zeta(s)$ *were on the line* $s = \frac{1}{2} + it$.

Despite its apparent simplicity, this statement has never been confirmed or refuted. As a result of the concerted effort of a number of mathematicians — in the pre-computer era: ***E. Landau*** *(1911–1933),* ***T.H. Gronwall*** *(1913),* ***G.H. Hardy*** *(1914),* ***J.E. Littlewood*** *(1914–1928),* ***S. Ramanujan*** *(1915),* ***C. de la Vallée-Poussin*** *(1916),* ***C.L. Siegel*** *(1922–1948),* ***A.E. Ingham*** *(1926–1933),* ***J. Hadamard*** *(1927),* ***E.C. Titchmarsh*** *(1928–1947),* ***A. Selberg*** *(1942–1946),* ***A.M. Turing*** *(1943) — a great deal has been learned about the distribution of zeros of* $\zeta(s)$.

It has been proven that:

(1) $\zeta(s)$ *has an infinity of zeros in the strip* $0 \le \sigma \le 1$, *all of which are complex (no zeros on the real axis between* 0 *and 1);*

(2) the zeros either lie on $\sigma = \frac{1}{2}$ *or occur in pairs symmetrical about this line;*

(3) there is an infinite number of zeros of $\zeta(s)$ *on* $s = \frac{1}{2} + it$ *[**Hardy**, 1914].*

Nevertheless, the RH remains one of the outstanding challenges of mathematics, a prize which has tantalized and eluded some of the most brilliant mathematicians of 20th *century. In fact, the RH stand today as the most important unsolved problem in mathematics. Not only is it tied up with the prime number theorem, but many other theorems are conditioned on the RH — that is, their proofs assume that the RH is true. Other results are known to be equivalent to the RH.*

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275 For further reading, see:

- du Sautoy, M., *Music of the Primes*, Perennial, 2003, 335 pp.
- Derbyshire, J., *Prime obsession*, Joseph Henry Press: Washington, D.C., 2003, 422 pp.

The search for the zeros is a story in itself: In 1903, ***J.P. Gram*** *published values of* t *of the first* 15 *zeros [14.13, 21.02, 25.01, 30.42, 32.93, 37.58, ... to two decimal places], thus rendering the first solid evidence in support of the RH. There was even some wonderment at how Riemann had arrived at his prediction, since his paper on the zeta-function contained no computations at all. It was generally believed that Riemann had based his hypothesis on aesthetics and intuition — two major driving forces of mathematical research.*

However, in 1932, ***Carl Ludwig Siegel*** *proved that there was more to it than aesthetics and intuition. Searching through Riemann's unpublished papers in the archives of the University Library at Göttingen, Siegel discovered that Riemann had indeed computed several zeros of the zeta-function. Not only that, but he had done so by a method superior to those that Gram and others had used after him!*

Siegel cleaned up Riemann's method, and the so-called Riemann-Siegel formula *became the basis for computing zeros of the zeta-function. The list of zeros has grown with the advent of high-speed computers. In* 1952, ***Alan Turing*** *identified the first* 1054 *zeros ("the first" meaning the zeros closest to the real axis — the purported line of zeros is perpendicular to the real axis at the point* $\frac{1}{2}$*).*

The list grew to $25,000$ *in* 1956, 3.5 *million in* 1968 *and* 81 *million in* 1979. *In* 1985, *it reached a staggering* 1.5 *billion zeros —* every *one of which lies on the predicted line. In* 1990, *the* 10^{20}*-th zero of the Riemann zeta-function was found to be* $\frac{1}{2} + [15,202,440,115,920,747,268.629,029,9\ldots]i$. *Thus, to date, mathematicians have amassed impressive amounts of evidence in favor of the hypothesis.*

But that is not all: statistical studies of the distribution of spacing between the zeros of the zeta-function *have led to another conjecture, namely that the distribution of these spacing is similar to that of eigenvalues of random matrices that are studied in* many-particle systems *in physics. This hypothesis suggests that the zeta-function could be used as a model of* quantum chaos.

The importance of $\zeta(s)$ *in the theory of prime numbers lie in the fact that it connects two expressions, one of which contains the primes explicitly, while the other does not.*

The theory of primes is largely concerned with the function $\pi(x)$, *the number of primes less or equal to* x.

Gauss *(1792, age 14!) was first to notice that* $\pi(x)$ *can be estimated by the function* $\left\{\frac{x}{\log_e x}\right\}$ *or*

$$Li(x) = \int_2^x \frac{dt}{\log_e t} = \frac{x}{\ln x} + \frac{1!x}{(\ln x)^2} + \ldots \frac{(k-1)!x}{(\ln x)^k} + O[\frac{x}{(\ln x)^{k+1}}];$$

He did not publish this result. In 1798, ***Legendre****, independently, suggested that* $\pi(x) \sim \frac{x}{\log_e x - 1.08366}$. *At that time, these relations seemed completely inexplicable, since* $\log_e x$ *arose in differential calculus in connection with problems of* continuous *growth and decay and was not known to be related in any way to* discrete prime numbers. *The approximation, in percentage terms, grows better and better as* x *increases. Gauss, being both a number theorist and the man who founded mathematical statistics, used his "*method of least squares*" to show that, as* x *approaches infinity, the errors are likely to eventually approach zero [for* $x = 10^3$, *the error is 16.0%, for* $x = 10^9$, *it is 5.4%, while for* $x = 10^{14}$, *it is only* 3.2 *percent].*

It took 50 *years before anyone made any progress toward proving the Gauss-Legendre conjecture. The first person to do so was* ***P. Chebyshev*** *in* 1850. *He obtained a partial result and his ideas were then imitated by others. But eventually it turned out that his methods would not go any further, and they were abandoned.*

In 1859, ***Riemann*** *published a small paper entitled: "On the Number of Primes Less Than a Given Magnitude" (in German). Its reasoning contained large gaps, and very little was definitively proven, but nearly everything that has been done in the theory of numbers since then has been influenced by that paper.*

For 40 *years, other mathematicians tried to prove the main result enunciated in Riemann's* 8*-page paper — but to no avail. In* 1896, ***Hadamard*** *and* ***de la Vallée-Poussin****, working independently, finally proved that* $\lim_{n\to\infty} \left\{ \frac{\pi(n)}{n/\log_e n} \right\} = 1$, *the PNT.*

Further research followed: In 1908, ***E. Landau*** *showed that*

$$\pi(x) = Li(x) + O[xe^{-\gamma\sqrt{\ln x}}].$$

In 1914, ***J.E. Littlewood*** *showed that the difference* $\{Li(n) - \pi(n)\}$ *changes from positive to negative infinitely many times as* n *runs up through the positive integers, although the first change of sign occurs for a* very large *n [in* 1986 ***J.J. te Riele*** *showed that this number is* smaller *than* 6.69×10^{370}. *A* computer search *made as far as* 10^9 *failed* *to produce such a number. It may never be possible to discover the* actual number*!]*

An approximation to $\pi(n)$ *which involves the zeta-function explicitly was derived in* 1903 *by* ***Gram*** *from Euler's product formula:*

$$R(n) = 1 + \sum_{k=1}^{\infty} \frac{1}{k\zeta(k+1)} \frac{(\log n)^k}{k!}.$$

Thus, for $n = 10^9$,

$$\pi(n) = 50,847,534; \qquad \left\{\frac{x}{\log_e x}\right\} = 48,254,942;$$
$$\left\{\frac{x}{\log_e x - 1.08366}\right\} = 50,917,519, \qquad R(n) = 50,847,455;$$

$R(n)$ *thus yields best estimate with a percentage error of only* 1.5×10^{-4}*!* ***Ramanujan*** *discovered (1913) the alternative form*

$$F(x) = \int_0^\infty \frac{(\log x)^t dt}{t\Gamma(t+1)\zeta(t+1)}$$

for the sum.

The importance of the RH lies in the fact that the errors of the approximations to $\pi(x)$ *depend on the zeroes of the zeta-function. The connection between* $\pi(x)$ *and RH also lies behind a great deal of other known facts about primes. If the RH does turn to be true, then the connection with the function* $\pi(n)$ *will enable even more information about the prime numbers to be deduced than is at present known.*

Moreover, the prime number theorem is important not only because it makes an elegant and simple statement about primes and has many applications, but also because much new mathematics was created in the attempt to find a proof. This is typical in number theory and topology, where problems which are very simple to state are often extremely difficult to solve. Mathematicians working on these problems often create new areas of mathematics of independent interest[276]*. Two additional examples stand out:*

(1) the creation of Algebraic Number Theory as a result of work on the Fermat Conjecture;

(2) the creation of Graph Theory as a result of the search for the solution to the 4-color problem.

[276] This phenomenon, which happened over and over again in mathematics, brings to mind the well-known tale about a farmer who had three lazy sons that were loafing around without doing any substantial work. On his deathbed, the farmer told them that a treasure was buried somewhere on the farm. Following his death the sons began to dig the farm inside out in search of the fortune. In doing so, they unknowingly cultivated the land and became very prosperous. The conjecture of Riemann was such a hidden ‘treasure’.

1851 CE The first successful *submarine telegraph cable* was laid between Dover and Calais.

1851 CE The *brown rats* (alias *Norway rats*) reach the Pacific coast[277]of the United states after some 50 years of migration from the East coast (average diffusion rate of ca. 300 meter/day). It reached the ports of the New World from Europe as stowaways on ships. Brown rats migrated to Europe from Asia, apparently from *North China*. They are known to have reached Paris in 1753.

1851–1855 CE Tuberculosis ravaged England; Ca 250,000 died.

1851–1897 CE **William Thomson** (**Lord Kelvin**, 1824–1907, England). A distinguished physicist of the 19^{th} century. Kelvin published more than 600 papers on a wide range of scientific subjects, and he patented 70 inventions. Queen Victoria knighted Kelvin for his work as an electrical engineer in charge of laying the first successful transatlantic cable[278] in 1866.

In 1851 he proposed the gas thermometer as the basis of an *absolute temperature scale* (Kelvin scale; 1848), with degree intervals equivalent to those on the centigrade scale but with the fiducial zero point at $-273.7\,°$C [today at $-273.15\,°\text{C}{=}-459.67\,°\text{F}$], called the *Absolute zero*. [According to classical physics, ideal gases at this temperature contract to solids and all molecular motion ceases.] He coined the word *Thermodynamics* (1849).

In 1852 he discovered with **James Prescot Joule** the '*Joule-Thomson Effect*', according to which gases undergo a change of temperature when made to expand freely [the effect was utilized in 1877 by **Carl von Linde** (1842–1934) to design an Ammonia gas refrigerator]. Greatly interested in the improvement of physical instrumentation, he designed and improved many new devices.

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[277] Some 300 years after **Balboa**, the first European to see the Pacific Ocean in 1513. Since the second half of the 19th century brown rats arrived everywhere with the speed of trains, ships and cargo planes, spreading diseases such as plague, typhus, anthrax and trichinosis.

[278] Experimental testing of physiologists in the 1930's provided important evidence confirming the relevance of telegraphic cable theory to *nerve axons*. **Alan Lloyd Hodgkin** (1946–1947) and his co-workers presented derivations of the Kelvin cable equation for nerve cylinders and included transient solutions as well as methods for estimating the values of key parameters. The application of cable theory to *dendritic neurons* began in the late 1950's, when it became necessary to interpret experimental data obtained from individual neurons by means of intercellular microelectrodes located in the neuron soma.

Among Kelvin's inventions are the mirror galvanometer (1867) and the marine compass free of magnetic influence (1873). In 1876 he proposed the principle of the *differential analyzer*[279] (a misnomer for a mechanical computer, destined to solve mainly ordinary differential equations). In 1897 he finalized his estimate of the age of the earth, using the theory of conductive cooling of a semi-infinite half-space model. He hypothesized that the earth was formed at a uniform high temperature and that its surface was subsequently maintained at low temperature. He then assumed that a thin near-surface boundary layer developed as the earth cooled. Since the boundary layer would be thin compared with the radius of the earth, he reasoned that a one-dimensional heat-equation model could be applied. His calculations then yielded the value of ca 20 million years for the age of the earth. The discovery of radioactivity (1896) showed that his basic assumptions were wrong[280].

Kelvin was also wrong on three other counts: he was convinced that the Eulerian period of 10 months for the free precessional motion of the earth's axis of rotation was real, and ignored the effect of the period lengthening by 4 months due to the non-rigidity of the earth.

He was totally blind to the impact of vectors on physical theory. In 1886 he wrote to Hayward: "*Quaternions came from Hamilton after his really good work had been done; and, though beautifully ingenious, have been useless to those who have touched them in any way, including Clerk Maxwell*". In a letter to G.F. Fitzgerald in 1890 he wrote: "*Vector is a useless survival, or off-shoot, from quaternions, and has never been of the slightest use to any creature*". Finally, Kelvin did not accept the atomic theory of matter.

Applied mathematicians and physicists will always be grateful to him for discovering the *method of stationary phase* (1887), for asymptotic evaluation of special integrals[281].

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[279] The proposal was unfortunately neglected. Not until 50 years later was a workable machine constructed by **Vannevar Bush** and his colleagues at the Massachusetts Institute of Technology.

[280] Kelvin also viewed the sun as some sort of a dwindling self-gravitating coal pile that illuminated the earth for only a few tens of millions of years.

Kelvin would not accept geological arguments against his estimate and once, in the heat of a dispute over the earth's age, said that geology was as intellectually respectable as collecting postage stamps.

[281] The method is applicable to integrals of the form $K(\lambda) = \int_{-\infty}^{\infty} g(\omega)e^{i\lambda f(\omega)}d\omega$, where $g(\omega)$, $f(\omega)$ are real functions and λ is a large, positive constant. When λ is large, the exponential function $e^{i\lambda f(\omega)}$ will, in general, oscillate very

Kelvin resolved *Olber's paradox* quantitatively and correctly in the framework of a transparent, uniform, and *static* universe. In a paper entitled "On ether and gravitational matter through infinite space", published in the *Philosophical Magazine* (1901), he was the first to show, on the strength of the Kant-Laplace nebular hypothesis, that if one assumes that we live in a universe of *finite age* (or in a universe of unlimited age in which the stars have been shining for only a limited time), then the observed phenomenon of a dark starlit sky would categorically necessitate a cosmological regime in which the size of the visible universe is less than the background limit[282]. This Kelvin

rapidly. If $g(\omega)$ changes slowly, the rapid phase-change will tend to cause cancellations. In total, the integral will approximately vanish except around points $\omega = \omega_0$ where $f(\omega)$ is stationary, i.e., $f'(\omega_0) = 0$. For one stationary point, the quantitative analysis of this statement yields *Kelvin's formula*:

$$\int_{-\infty}^{\infty} g(\omega)e^{i\lambda f(\omega)}d\omega = \left[\frac{2\pi}{\lambda|f''(\omega_0)|}\right]^{1/2} g(\omega_0)e^{i\lambda f(\omega_0)+i\delta}\left[1 + O\left(\frac{1}{\lambda}\right)\right],$$

where $\delta = \frac{\pi i}{4}\,\mathrm{sgn}\, f''(\omega_0)$.

282 The treatment of Kelvin elucidates what was previously shown by **Halley** (1721), **Cheseaux** (1744), and **Olbers** (1826). Let all stars be sun-like, of radius a, and uniformly distributed (n per unit volume). Let α denote the fraction of the sky covered by the discs of stars out to radius r. Then $\alpha = \frac{r}{\lambda} \geq 1$, where $\lambda = \frac{1}{\pi n a^2}$ is the mean free path of a light ray. We note that $\alpha = 1$ when the radius r of a surrounding sphere of stars equals λ, and hence λ is also the *background limit* of a star-filled universe. (In this case the entire sky is covered in a distribution of stars of infinite extent and the average distance observable from any position is the background limit λ.)
Let each star have *luminosity* L. The contribution to the *radiation density* u at its center of a shell of radius q and thickness dq is $du = \frac{nL}{c}dq$. By integrating from $q = 0$ to $q = r$ we find $u = u^*\frac{r}{\lambda} \equiv u^*\alpha$, where $u^* = \frac{L}{\pi a^2 c}$ is the radiation density at the surface of a star. Therefore, $\alpha = \frac{u}{u^*}$ demonstrates the truth of Kelvin's statement that α is the ratio of the apparent brightness of our starlit sky to the brightness of our sun's disc. Kelvin thus showed that,

$$\frac{\text{brightness of starlit sky}}{\text{brightness of sun's disc}} = \frac{\text{size of visible universe}}{\text{background limit}} = \frac{\text{fraction of sky}}{\text{covered by stars}}.$$

Since the left hand fraction is much less the unity, any viable theory must explain why the size of the visible universe (the part we see) is much smaller than the background limit. In his estimate for the size of the background limit $\lambda = \frac{1}{\pi n a^2}$, he followed the reasoning of Cheseaux and Olbers, and the astronomical data available to him, which gave him a value of ca 3×10^{15} light-years. To ensure that the *visible* universe remains always smaller than λ, his static uniform model forced him to limit the *age* of the universe (or equivalently, the age of stars in a universe otherwise of unlimited age). Kelvin chose (erroneously), a visible was easily able to show on the basis of the then available astronomical information.

Although he solved the riddle of cosmic darkness according to the conditions prescribed in its original conception for a uniformly populated and static universe, it can be shown that all the variants of this primitive standard model that resort to absorption, hierarchy, and redshift (owing to expansion) merely accomplish a state of greater darkness in a universe already dark.

Kelvin disbelieved in paradoxes. In his *Baltimore Lectures* (1884) he more than once declared: "*There are no paradoxes in science*". He took the rationalist attitude that paradoxes are the result of misunderstandings; they lie in ourselves and not the external world.

Kelvin was born in Belfast, Ireland. His father James was a professor of mathematics at Glasgow University. He was educated at the universities of Glasgow, Cambridge, and Paris. Kelvin became a professor of natural history at the University of Glasgow in 1846 and remained there until his retirement in 1899. He was married twice (1852, 1874). However, there was no heir to his title, which became extinct.

Electrostatics and Number Theory

In 1853, ***Lord Kelvin*** *used his method of images to calculate, in a very elegant way, the mutual capacitance* (C_{12}) *of a configuration consisting of* 2 *spheres of radii* a *and* b, *with their centers a distance* $c \geq (a+b)$ *apart. He was able to show that the result can be represented in the form of a converging modified Lambert series* (**Lambert**, *1771)*

$$C_{12} = \frac{EI}{c}\sum_{n=1}^{\infty}\frac{\alpha^n}{1-\alpha^{2n}}; \quad \alpha = \frac{E-I}{E+I} \leq 1.$$

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universe of 14 million light-years, but even with modern values of 14 billion light-years, the darkness at night is fully guaranteed.

Cheseaux and **Olbers** assumed that the stars shine long enough for light to travel from the background limit. Had they questioned this assumption, they might have realized that there was no need to postulate the absorption of starlight.

Here,

$$E = \sqrt{c^2 - (a-b)^2}, \qquad I = \sqrt{c^2 - (a+b)^2}$$

are the lengths of the external and internal tangents, respectively, to the circles obtained by cutting the spheres with a plane through their centers.

*This problem has drawn the attention of mathematicians and physicists, who tried to improve the convergence of the above series [**E.W. Barnes** (1903), **A. Russell** (1911), **J.H. Jeans** (1915), **W. Smythe** (1939)]. One hundred years after its inception (1953), the problem was revisited by **Balthazar van der Pol**, who noticed that*

$$\sum_{n=1}^{\infty} \frac{\alpha^n}{1-\alpha^{2n}} \equiv \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} \alpha^{n(2m+1)}.$$

Therefore,

$$C_{12} = \frac{EI}{c} \sum_{n=1}^{\infty} \left\{ d(n) - d\left(\frac{n}{2}\right) \right\} \alpha^n.$$

The Dirichlet divisor function, $d(n)$, characteristic of number theory, represents the number of divisors of n. [$d\left(\frac{n}{2}\right)$ is to be replaced by zero for any odd n.] This function is related to the Riemann zeta-function $\zeta(n)$, via the Voronoi relation[283]

$$\{\zeta(s)\}^2 = \sum_{n=1}^{\infty} \frac{d(n)}{n^s}.$$

*Also, the above series is closely related to the famous divisor problem of **Dirichlet**[284], which is that of determining the asymptotic behavior as $x \to \infty$ of the sum*

$$D(x) = \sum_{n \le x} d(n) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \zeta^2(z) \frac{x^z}{z} dz$$

(with $c > 1$ and x not integer).

1851–1852 CE **Jean Bernard Léon Foucault** (1819–1868, France). A distinguished experimental physicist. Demonstrated the absolute rotation of

[283] **G. Voronoi** (1904).

[284] Dirichlet proved that $D(x) = x \log x + (2\gamma - 1)x + O(x^{1/2})$, where γ is the Euler-Mascheroni constant.

the earth at the Paris exhibition by means of a pendulum (1851). An iron ball of about 30 kg was suspended from the ceiling of the Pantheon from a wire 60 m long, so as to be free to swing in any direction. To an observer in an inertial frame outside the earth, the plane of motion of a pendulum at either pole remains fixed in space while the earth rotates underneath. To an observer on earth the plane of motion rotates relative to earth, and the pendulum's motion is attributed to the action on it of a *Coriolis force*. (In any non-polar location on earth, however, the pendulum's plane of motion is fixed in *neither* frame.)

In 1852 Foucault constructed a refined gyroscope to demonstrate the rotation of the earth. Because the device exhibited the rotation of the earth before his eyes, Foucault named it the 'gyroscope', which means etymologically 'to see rotation' [from the Greek '*gyros*' = ring, '*skopien*' = to view].

During 1850–1865, he produced many important experiments, discoveries and inventions: he used a revolving mirror to measure the speed of light, and showed that light travels slower in water than in air and that its speed varies inversely with the index of refraction. He also made various improvements in the mirrors of reflecting telescopes and invented the *parabolic mirror telescope* which did away with spherical aberration. He discovered the existence of eddy currents, which are produced in a conductor moving in a magnetic field.

Foucault was the son of a publisher at Paris. After an education received chiefly at home he studied medicine, which he abandoned for physical science.

History of Gyroscopic Phenomena and Technologies, I

A. General theory

The earliest appreciation of gyroscopic phenomena appears to date to the time of **Newton** *(1642–1727). It arose from a study of the motion of our planet, which is itself a massive gyroscope. A description of its motion will, in fact, help in understanding some of the essential characteristics of the general case. The earth approximates closely to a free gyroscope, for its axis remains almost fixed in the direction of the North Star, Polaris, irrespective of its transit around the sun. The direction of the axis, however, has been changing* *slowly throughout the centuries, a phenomenon known as "precession of the Equinoxes". Its polar axis is sweeping out a cone, with an apex angle of* $46\,^\circ 54'16''$ *and a period of* $25,800$ *years.*

Though extremely slow, this motion is similar to the precession of a spinning top. It arises from the gravitational torque to which the earth is subjected by the sun and the moon, as a combined result of its lack of sphericity and the inclination of its axis to the ecliptic plane. A further periodic movement is also present, in which the earth's axis describes a much smaller cone whose diameter at the North Pole is approximately 800 *cm. This movement, known as Eulerian motion, has an observed period of* 428 *days and corresponds to the free oscillation or nutation of a gyroscope. The above phenomena are superposed on the orbital motion of the earth around the sun.*

From the above we may identify three gyroscopic attributes, namely directional stability, precession and nutation. The technological applications of the gyroscope are based upon these properties. For example, directional stability, which may be regarded as the reluctance of a body to change its orientation, provides the basis of modern inertial navigation.[285]

Also, it is found that rate of precession is proportional to applied torque, and as the latter may be produced by the acceleration of a platform upon which the gyro rides, linear acceleration may be measured by angular velocity, and consequently linear velocity by angular displacement. This integrating ability of the gyroscope is made use of in instruments carried by rockets for recording their position in space.

A further characteristic depends on the opposite nature of action and reaction. If forced to precess, a gyroscope exerts a reactive moment proportional to the product of the velocities of spin and precession. Moments of immense magnitude may thus be produced by the precession of high-speed rotors. This feature is utilized in gyroscopic vibration absorbers and in some ship stabilizers.

The mathematical foundation of gyroscopic behavior must undoubtedly be ascribed to ***Euler*** *(1707–1783). His initial work in this field concerned the general motion of a rigid body for which he derived a set of dynamical equations involving relations between applied moment, inertia, angular acceleration and angular velocity. These, known as* Euler's equations*, were stated with reference to axes fixed in the body. Later he established the independence of rotation and translation of a rigid body, and devised the so-called* Eulerian angles *to define its orientation with respect to a system of fixed axes.*

[285] Although it is being replaced with such technologies as *laser gyros* and MEMs (= Micro Electro-Mechanical devices), which detect rotations and accelerations by other, optical and mechanical, means.

From this background came his first direct contribution to gyrodynamics: the general motion of a rigid body, fixed at a point and otherwise free from external force. This problem includes that of the free gyroscope, and was to occupy the attention of mathematicians for many years. The general displacement of the body is, in fact, only expressible in terms of elliptic functions, mainly associated with the name of ***Jacobi*** *(1804–1851). Euler's later contributions included the inertial properties of bodies, which led to the concepts of principal axes and momental ellipsoid.*

There is an important concept which first appears to have been recognized by ***Clairaut*** *(1713–1765) in* 1742*, though credited much later to* ***Coriolis****. This concerns the force to which a particle is subjected when moving on a surface which is itself subjected to rotation. Although this had not been neglected in earlier work, Clairaut indicated its application to a moving frame of reference.*

From the death of Euler in 1783 *until the early part of the* 19^{th} *century, little was added to the theory of the gyroscope. A revival of interest, however, is evident in the work of* ***Poinsot***[286] *(1777–1859) who approached the subject by way of analogy. He demonstrated theoretically that if a free body, fixed at a point, were replaced by its momental ellipsoid, the path of motion of the ellipsoid when rolled on a fixed plane was identical to that of the body. In this representation, the distance of the plane from the fixed point was a function of the energy and momentum of the body. At a later date,* ***Sylvester*** *(1814–1897) showed that if a solid homogeneous ellipsoid were used, not only the path but also the* transit times *at each position would be identical to that of the actual body.*

Many contributions of ***Poisson*** *(1781–1840) are associated with gyrodynamics. In particular, he appears to have been the first to investigate the motion of the spinning top, a much more complex problem than that of the free gyroscope. Because of the torque due to the gravitational force, a top may perform a large variety of complicated motions; and during the latter half of the* 19^{th} *century, much thought was devoted to this subject. Poisson also made a comprehensive study, related to the work of Poinsot, which dealt with the rolling of bodies of various shapes on a plane.*

The following years provided new approaches to gyroscopic problems. ***Peter Guthrie Tait*** *(1831–1901) investigated the motion of the free gyroscope by vector methods, while* ***Edward John Routh*** *(1831–1907) studied the stability of gyroscopic motion and gave a geometrical construction for determining the rise and fall of a spinning top. The contributions of* ***Lord Kelvin*** *(1824–1907) were both practical and theoretical. He made a suggestion for*

[286] **Poinsot** introduced the concept of 'torque' (1804).

using a gyro-compass as early as 1883, and later developed analogies between gyroscopic motion and the motion of electrons in magnetic fields. The work of ***Felix Klein*** *(1849–1925) also deserve mention. He approached the motion of a top by using parameters which later became known as the Cayley-Klein parameters.*

By the turn of the 20^{th} century gyroscopic theory was virtually complete, and since then emphasis has shifted to gyroscopic applications. This has stimulated much theoretical work involving the solution of specific problems, rather than the discovery of new phenomena. The gyroscope has become the province of the inventor rather than of the mathematician.

B. Gyro-technology (1744–1930)

The early history of the gyroscope is obscure. Its modern history begins with the Englishman ***Serson***[287]*, who in 1744 constructed a spinning rotor for indicating the position of the horizon at sea, when the real horizon was obscured. It was supported at its centroid (to avoid precession) so as to be free from disturbance by the motion of the ship, and was the forerunner of the 'gyroscopic horizon', used in modern aircraft.*

The early part of the 19^{th} century saw gyroscopes being used in the teaching of dynamics.

In 1810 ***Bohnenberger*** *(Germany) constructed the earliest type of gyroscope now in use. In 1819 the English instrument maker* ***Edward Troughton*** *(1753–1835) produced an improved 'gyroscopic horizon' in which the center of gravity of the rotor could be adjusted accurately by means of screwed plugs. In 1832* ***Walter Rogers Johnson*** *(Philadelphia, U.S.A.) constructed an improved type and used it to illustrate the dynamic of rotating bodies. He called it a 'rotascope'. In 1852,* ***Foucault*** *constructed a gyroscope with which he successfully demonstrated the earth's rotation.*

In the last decade of the 19th century, the stage was set for the application of the gyroscope to real world problems. These were quick in coming;

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[287] Serson was sent to sea by the Admiralty to test his instrument, but he was lost in the wreck of the "victory" in 1744. His invention was reported, however, in *Phil. Trans. Royal Soc.* **47**, 1752.

three things drove the transformation of the gyroscope from a child's toy, or inventor's curiosity to that of a usable technology. These were:

- *The increasing use of steel in ships made the vessels unstable.*
- *Unreliability of the magnetic compass within a steel ship.*
- *Preparation of the great powers to conduct underwater warfare in steel hull ships.*

In 1883 ***Lord Kelvin*** *made suggestions for using a gyro-compass, which was indeed designed during* 1908–1911 *by* ***Hermann Anschütz-Kaempfe*** *(Germany) and* ***Elmer Ambrose Sperry*** *(1860–1930, U.S.A.), mainly for the use of polar expeditions. Anschutz and Sperry both built on the properties of the gyroscope: stability and precession. If force is exerted on it, it will react at right angles to the applied force. The characteristic of a gyro combined with other elements of precession, pendulousity and damping will allow the gyro to settle toward the true north.*

In 1908 Anschutz patented the first north seeking gyrocompass with the United Kingdom's Patent Office. The same year Elmer Sperry invented and introduced the first ballistic gyrocompass, which included vertical damping. Both of these first devices were of the single pendulum type. Earlier, ***Schlick*** *(Germany) made first attempts to stabilize a ship against rolls by means of a gyroscope. However, the solution to this problem was finally perfected by Sperry in* 1907. *In 1923,* ***Max Schuler*** *(Germany) introduced his finely-tuned gyro-pendulum*[288].

Since that date, the gyroscope has been used in a variety of ways to steer torpedoes, navigate ships, rockets and missiles, to stabilize the rolling of ships, to counter vibration and to operate innumerable control mechanisms. The small directional unit of the gyro-compass operates by the same principles as the massive rotor of the ship stabilizer, weighting up to 100 *tons.*

The conventional gyroscope, however, always consists of a symmetrical rotor spinning rapidly about its axis and free to rotate about one or more perpendicular axes. Freedom of movement about an axis is normally achieved by supporting the rotor in a gimbal, and complete freedom can be approached by using two gimbals. None other than ***Albert Einstein*** *spent much of his valuable time on the improvement of the gyrocompass during 1915–1925, as a*

[288] With its universal *Schuler-period* of $T = 2\pi\sqrt{\frac{R}{g}} = 84.4^m$, where R is the earth's mean radius.

consultant to the Kiel-based firm Hermann-Anschütz-Kaempfe. After WWI, Einstein and Anschutz collaborated intensively on the development of a fundamentally improved gyrocompass. In the 1930's virtually every navy in the world, except the British and the American, was equipped with gyrocompasses by the Anschütz firm. The construction of these gyrocompasses also involved a patent of Albert Einstein.

As of 1925, Einstein's share in the profits of the gyrocompass project was contractual, receiving 3% of the sales price of each instrument, and 3% of any revenue from licenses. The contract was not with the Kiel firm, but with the Dutch firm Giro, a distribution company founded by Anschütz primarily to evade the ban imposed by the treaty of Versailles on exports of military articles. This firm was liquidated in 1938 since the parent firm in Kiel no longer needed a Dutch branch to circumvent armaments controls. By then Einstein had no longer received any payments from the German Reich. He was thus at least spared any disquieting thoughts on the propriety of earning royalties from a device which guided German U-boats and Japanese aircraft carriers.

1852 CE **Francis Guthrie** (1831–1899, England). Mathematician. Formulated the four-color conjecture. This states that any map on a plane or a sphere can be colored with the use of only four colors, in such a way that no two adjacent domains are of the same color.

Chromatic Numbers (1852–1952)

"I know by the color. We're right over Illinois yet... Indiana ain't in sight... Illinois is green, Indiana is Pink. You show me any pink down here, if you can. No, sir; it's green... I've seen it on the map".

"Indiana pink*?... Well, if I was such a numskull as you, Huck Finn, I would jump over. Seen it on a map! Huck Finn, did you reckon the states was the same color out-of-doors as they are on the map?"*

"Tom Sawyer, what's a map for? Ain't it to learn facts? ...there ain't no two states the same color. You get around that*, if you can, Tom Sawyer".*

Mark Twain, '*Tom Sawyer Abroad*' (1835–1910)

There are many topological questions, some of them quite simple in form, to which intuition gives no satisfactory answer. A problem of this kind, known as the 4*-color problem, arose out of the practical needs of map-makers already in the* 16^{th} *century. These men were familiar with the notion that* not more than 4 colors *are necessary in order to color a map of a country (divided into districts) in such a way that no two contiguous districts shall be of the same color*[289].

The problem was mentioned by ***A.F. Möbius*** *in his lectures in* 1840*, but was first stated as a mathematical conjecture in* 1852 *by* ***Francis Guthrie****. Shortly after he had completed his studies at University College, London, he was coloring a map showing counties of England. As he did so, it occurred to him that, in order to color* any *map [subject to the requirement that no two regions sharing a length of a common boundary should be given the same color], the maximum number of colors required seemed likely to be* 4*. Being unable to prove this, he communicated the problem to* ***Augustus de Morgan*** *(1806–1871), one of the major mathematicians of his time, and through him the proposition then became generally known.*

[289] *Contiguous = districts having a common* line *as part of their boundaries. The map is drawn on a simply-connected surface, such as a plane or a sphere. The number of districts is finite and no district consists of two or more disconnected pieces. The map may or may not fill up the whole surface. Some maps can be colored with fewer than* 4 *colors, such as a chess-board, which requires only* 2*, or a hexagonal tessellation, which requires* 3*.*

Like Guthrie, de Morgan had no difficulty proving that at least 4 *colors are necessary (i.e. that there are maps for which* 3 *colors are not sufficient). He also proved that it is* not *possible for* 5 *regions to be in a position such that* each *of them is adjacent to the other* 4 *[this may, at first glance, appear to imply that* 4 *colors are always sufficient, but it does* not *in fact imply this at all. Numerous false 'proofs' of the 4-color conjecture that appeared during* 1852–1976 *were based upon this invalid implication].*

Unable to solve the problem, de Morgan passed it on to his students and to other mathematicians, among them ***W. Hamilton****, giving credit to Guthrie for raising the question. However, the problem did not seem to attract much interest until* 1878*, when* ***Arthur Cayley****, unable to determine the truth or falsity of the conjecture, called attention to it by asking the members of the London mathematical society if they knew a proof of the conjecture. His question was published in the society's proceedings, and this was the first mention of the problem in print.*

Within a year after Cayley's challenge, ***Arthur Bray Kempe*** *(1849–1922), a London barrister and a member of the London Mathematical Society, published a paper that claimed to prove that the conjecture was true. However, in 1890* ***Percy John Heawood*** *(1861–1955, England) pointed out that Kempe's argument was in error. Heawood was, however, able to salvage enough to prove that* 5 *colors are always adequate.*

Heawood, in trying to attack the problem, investigated a generalization of the original conjecture: The maps studied by Guthrie and Kempe were maps in a plane or on a sphere. Heawood also considered maps on more complicated surfaces containing "handles" and "twists". He was able to derive upper bounds *for the numbers of colors required to color maps on these surfaces (the numbers themselves are known today as the* chromatic numbers*). His method, however, was* not *applicable to the plane*[290]*.*

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[290] **Heawood** proved that for a surface of *Euler characteristic* n

$$(= V\ (vertices) - E\ (edges) +\ F\ (faces)),$$

such that $n \leq 1$, the number of colors that suffice to color all maps drawn on the surface is $\frac{1}{2}[7 + \sqrt{49 - 24n}]$, where $[x]$ denotes the largest integer contained in x.
For a sphere $n = 2$; for the torus and the *Klein bottle*, $n = 0$, and for a double-torus, $n = -2$. Though topologically equivalent surfaces *do* have the same n value, topologically different surfaces may or may not.
Thus for the torus, 7 colors are sufficient. For the *Klein bottle*, the formula gives the answer 7, while only 6 colors are needed.

Heawood continued to work on the 4*-color problem for the next* 60 *years (1890–1950). During that time numerous mathematicians and even a greater number of amateurs, investigated the* 4*-color problem, developing in the process a great many mathematical techniques that ultimately proved to have applications elsewhere in mathematics. In fact, much of what is now known as Graph Theory (the geometry of wiring diagrams and airline routes) grew out of the work done in attempting to prove it.*

In 1913*,* ***George D. Birkhoff*** *used the idea of* ***Kempe*** *to develop much of the basis for later progress. Using these results,* ***Philip Franklin*** *(1898–1965, U.S.A.) proved in* 1922 *that every map with* 25 *or fewer zones could be colored with* 4 *colors. In* 1975 *this figure reached* 96*. In* 1950*,* ***H. Heesch****, who had been working on the* 4*-color problem since* 1936*, indicated for the first time that the problem would be solved* only *with the aid of a computer, capable of handling vast amounts of data, and he indeed advocated, and attempted, a computer-aided assault on the problem.*

1853–1869 CE **Johann Wilhelm Hittorf** (1824–1914, Germany). Physicist. Pioneer in electrochemical research. Investigated the migration of ions during electrolysis (1853) and suggested that different ions in a solution impelled with an electric current travel at different rates. He then developed expressions to account for the measured transport numbers. Studied electrical phenomena in rarefied gases, the *Hittorf tube* being named for him. Determined a number of properties of *cathode-rays* (before Crookes) including the deflection of the rays by a magnet (1869).

Hittorf was born in Bonn, Rhenish Prussia. He was a professor of physics at Münster (1852–1890).

1853–1871 CE **William John Macquorn Rankine** (1820–1872, Scotland). Ingenious engineer and physicist. Was among the founders of the science of thermodynamics on the basis laid by **Sadi Carnot** and **J.P. Joule**. His work was extended by **Maxwell**.

Although the word *energy* occurs already in the writing of Aristotle, it was introduced into the language of science by Rankine in 1853. His *manual of the steam engine* (1859) coined most of the modern terms used in the field. Introduced the *Two-Phase Rankine Cycle* in the ideal steam engine and the *Rankine Temperature Scale*. He demonstrated (1865) that the functioning of the propeller is based on the principle of reaction and was first to recognize that the essential point in its action is the acceleration of the air mass passing through the circular area swept by its blades. He also contributed to the theory of *shock waves* (1870). Rankine also wrote on *fatigue* in the metal of railway axles and on *soil mechanics*.

Rankine was born in Edinburgh and completed his education in its university. In 1855 he was appointed to the chair of civil engineering in Glasgow.

1853–1876 **Heinrich (Zvi Hirsch) Graetz** (1817–1891, Germany). The first Jewish historian of modern times. His eleven-volume *History of the Jews* is one of the great monuments of the 19th century historical writings.

Graetz was born in Posen[291]and received his doctorate from the University of Jena (1846). He momentary thought of entering the rabbinate, but he was unsuited to that career. For some years he supported himself as a tutor, but in 1856 the publication of the 3rd volume of his history made him famous. No Jewish book of the 19th century produced such a sensation as this. The work has been translated into many languages; it appeared in English in five volumes in 1891–1895. In 1854 he was appointed on the staff of the new Breslau Seminary and passed the remainder of his life in this office. In 1869 he was created professor by the government of the Breslau University. He kept this post until his death at Munich.

Graetz made use of a vast number of sources in many languages, but his vision of the Jew was rooted in Deutero-Isaiah. The Jews, he argued, had

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[291] *Poznan* (Posen); a city at the confluence of the Cybina and the Warthe rivers. One of the oldest towns in Poland (ca 800 CE), and the residence of some of the early Polish princes. It became the seat of Christian bishopric about the middle of the 10th century. The original settlement was on the right bank of the Warthe, but the new town established on the opposite bank by German settlers (1250), soon became the more important part of the double city. Posen became a great depot for the trade between Germany and Western Europe on the one hand and Poland and Russia on the other. The city attained the climax of its prosperity in the 16th century (p. 80,000). The intolerance shown to Protestants, the troubles of the Thirty Years' war, the plague and other causes, soon conspired to dwindle its population to 12,000 in the 18th century. Since its annexation by Prussia and the 2^{nd} partition of Poland (1793) its growth has been rapid. The German rule (1793–1806, 1815–) ended in 1918 when most of German inhabitants left the city. During WWII the Germans exiled its citizens and filled it with Germans from the Baltic states. It reverted to Poland after the war.

Jews lived in Poznan from the 12th century to the Holocaust [3000 (1519); 76,00 (1840); 26,599 (1910); 5000 (1920); 0 (1940)]. Its orthodox community constantly supplied the German Jewery with religious spiritual leaders as well as distinguished intellectuals that contributed to Judaism and science.

always been powerful and productive in religious and moral truths for the salvation of mankind. Judaism was, by divine providence, self-created. In that respect it was unlike any other great religion. Its 'sparks' had ignited Christianity. Its 'seeds' had brought forth the fruits of Islam. From its insights could be traced the origins both of scholastic philosophy and Protestantism.

Graetz was not interested in the social and economic motivations of human society, and laid the blame for the persecution of the Jews on the narrowness and bigotry which characterized Christianity during the Middle Ages. This, obviously, aroused resentment among many Christians and German nationalists.

Graetz's history was accepted by the Jews with mixed feelings: although the Jews of the world over hailed it with enthusiasm, it carried no real message to the great masses of East European Jews. He disparaged their study of the Talmud, as vain and useless scholasticism. Knowing little about Jewish mysticism, he had nothing but contempt for *Hasidism*, which was so widespread among them. To him it seemed pure superstition.

He failed to see the beautiful piety which prevailed in Eastern Europe despite hostility and poverty. Had he taken the trouble to learn more about the living Jews of Russia, Poland and Romania, Graetz would have found among them that very loyalty to Jewish life which he claimed so much in the Jews of the Middle Ages and which he was trying to revive among the Jews of Germany.

Nevertheless, Graetz had written a *justification of Jewish life* and, in a sense, gave a *pledge of Jewish continuity.* The *History of Jews* appeared at an opportune moment: Forces were already in motion in Germany which were opposed to that spirit of freedom and democracy which had brought emancipation to the Jews.

1854 CE A wire telegraph was established between London and Paris.

1854 CE **George Boole** (1815–1864, England). Irish logician and mathematician. One of the principal founders of symbolic logic.

Boole was born in Lincoln. His extraordinary mathematical talents did not manifest themselves in early life. During 1832–1849 he was a school teacher. In 1849 however, he was appointed a professor of mathematics in the Queen's College at Cork. He published some 50 papers and a few books and pamphlets. His most important work is "*An Investigation of the Laws of Thought, on which are founded the Mathematical Theories of Logic and Probabilities*" (1854) in which he put logic on a mathematical basis.

Boole was first to put laws of human reason in symbolic form. Today, Boolean algebra is considered as a mathematical system used to solve problem in logic, probability and engineering. In the context of this algebra, logical statements are formulated symbolically so that they can be written and proved in a manner similar to that used in ordinary algebra.

Boolean algebra deals with relationships between *sets* (collections of entities). Such sets and operations on them (unary or binary) are represented by letters and symbols of operations [e.g. $A \cap B$ represents the set of those elements that are in both sets A and B].

In 1881 Boole's work was extended by **John Venn** (1834–1923), and later in 1901 by **Giuseppe Peano** (1858–1932).

The Algebra of Switching Circuits (1854–1901)

By switching circuit is meant a connected set of circuit elements which may be opened (thereby interrupting a portion of the circuit) or closed.

Let x represent the condition of an element by taking the value

$$x = \begin{cases} 1 & \text{closed} \\ 0 & \text{open} \end{cases}$$

Let the operation + denote elements in parallel *and "multiplication" represent elements in* series.

Since the switching circuit design is an arrangement of wires and switches where an open *switch prevents the flow of current while a* closed *switch permits the flow, the tables below exhaust all possible configurations of a subcircuit consisting of two distinct switches x and y, through each of which current may flow or not.*

(*A*)

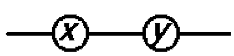
x	y	*parallel circuit*	x	y	$x+y$
on	*on*	*on*	*1*	*1*	*1*
on	*off*	*on*	*1*	*0*	*1*
off	*on*	*on*	*0*	*1*	*1*
off	*off*	*off*	*0*	*0*	*0*

(B)

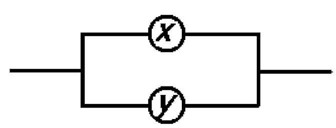
x	y	*series circuit*	x	y	xy
on	*on*	*on*	*1*	*1*	*1*
on	*off*	*off*	*1*	*0*	*0*
off	*on*	*off*	*0*	*1*	*0*
off	*off*	*off*	*0*	*0*	*0*

Note that $x + x = x$. Also $1 + x = 1$, because having one element of a parallel pair closed ensures the pair will 'act' closed. Also, to each element x there is an element $\bar{x}$ (with $x + \bar{x} = 1$ and $x \cdot \bar{x} = 0$) which is open when x is closed and closed when x is open.

The two binary operations introduced above can be represented by the diagrams:

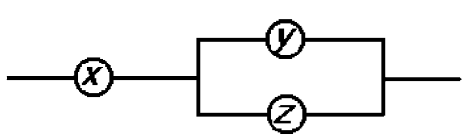


xy



x+y

Each switching circuit has an associated switching function which describes whether it is on or off as a whole, as a function of the states of its individual switches. Thus for the circuit



the switching function would be $f = x(y + z)$ which is unity if and only if $x = 1$ and $(y + z) = 1$ (i.e. y or z or both are closed) and which is zero if and *only if $x = 0$, or if y and z are zero, or if x, y and z are all zero. The overall state of the switching circuit can be specified by the vector (x, y, z).*

If x, y and z are circuit conditions (switches), the following algebraic properties hold [$\bar{x}$ is the complement of x defined by $x + \bar{x} = 1$ and $x \cdot \bar{x} = 0$]:

(i)	$x + 0 = x$	additive identity
(ii)	$x \cdot 1 = x$	multiplicative identity
(iii)	$x + y = y + x$	commutative laws
(iv)	$xy = yx$	
(v)	$(x + y) + z = x + (y + z)$	associative laws
(vi)	$(xy)z = x(yz)$	
(vii)	$x(y + z) = xy + xz$	distributive laws
(viii)	$x + yz = (x + y)(x + z)$	
(ix)	$x + \bar{x} = 1$	
(x)	$x\bar{x} = 0$	
(xi)	$x + 1 = x$	
(xii)	$x \cdot 0 = 0$	
(xiii)	$x^2 = x$	
(xiv)	$\overline{x + y} = \bar{x}\bar{y}$	De Morgan's laws
(xv)	$\overline{xy} = \bar{x} + \bar{y}$	

Each of these identities can be proved using the above tables. Equations (viii) through (xiii) have no analogues in ordinary algebra. In particular, (viii) is a 'weird' fact since ordinary algebra has instead

$$x^2 + xy + xz + yz = (x + y)(x + z)$$

The switching circuit algebra is an example of a Boolean algebra. *It is easy to see that there exists a 1-1 correspondence between a* disjunction *(join, union) and a parallel circuit (+) on one hand, and between a* conjunction *(meet, intersect) and a series circuit $(\cdot)$ on the other. Thus, in general Boolean algebra we replace + by the symbol $\vee$ and $(\cdot)$ by the symbol $\wedge$, to remind us of the corresponding set-theoretic operations.*

Nonassociative Algebraic systems

By developing algebras satisfying structural laws different from those obeyed by common algebra, ***Hamilton***, ***Grassmann***, *and* ***Cayley*** *opened the floodgates of modern abstract algebra. Indeed, by weakening or deleting various postulates of common algebra, or by replacing one or more of the postulates by others, which are consistent with the remaining postulates, an enormous variety of abstract systems can be studied.*

As examples of these systems we mention groupoids, quasi-groups, loops, semigroups, monoids, groups, rings, integral domains, lattices, division rings; Boolean rings, Boolean algebras, fields, vector spaces, Jordan algebras, and Lie algebras, the last two being examples of nonassociative algebras.

It is probably correct to say that mathematicians have, to date, studied well over 200 such algebraic structures. Most of this work belongs to the twentieth century and reflects the spirit of generalization and abstraction so prevalent in mathematics today. Abstract algebra has become the vocabulary of much of present-day mathematics, and has increasingly penetrated even engineering textbooks.

Octonions *were discovered by* ***John T. Graves*** *(1843) and independently by* ***Arthur Cayley*** *(1845). They are sometimes called* Cayley numbers *or* Cayley algebra.

Every octonion is a real linear combination of unit octonions

$$1, e_1, e_2, e_3, e_4, e_5, e_6, e_7$$

which thus form a basis of a vector space of octonions over the field of real numbers $\mathbb{R}$. The multiplication table for this 8-dimensional algebra, shown below, describes the result of multiplying the element in the i-th row by the element in the j-th column. Multiplication of two general non-basis octonions is defined by means of the distributive laws and the general properties of vector spaces:

$$\begin{aligned}(a + \sum_{j=1}^{7} b_j e_j) \cdot (a' + \sum_{j=1}^{7} b'_j e_j) &= aa' - \sum_{j=1}^{7} b_j b'_j \\ &+ \sum_{j=1}^{7} (a' b_j + a b'_j) e_j + \sum_{j,k=1}^{7} b_j b'_k e_j e_k\end{aligned}$$

	1	e_1	e_2	e_3	e_4	e_5	e_6	e_7
1	1	e_1	e_2	e_3	e_4	e_5	e_6	e_7
e_1	e_1	-1	e_4	e_7	$-e_2$	e_6	$-e_5$	$-e_3$
e_2	e_2	$-e_4$	-1	e_5	e_1	$-e_3$	e_7	$-e_6$
e_3	e_3	$-e_7$	$-e_5$	-1	e_6	e_2	$-e_4$	e_1
e_4	e_4	e_2	$-e_1$	$-e_6$	-1	e_7	e_3	$-e_5$
e_5	e_5	$-e_6$	e_3	$-e_2$	$-e_7$	-1	e_1	e_4
e_6	e_6	e_5	$-e_7$	e_4	$-e_3$	$-e_1$	-1	e_2
e_7	e_7	e_3	e_6	$-e_1$	e_5	$-e_4$	$-e_2$	-1

The interesting things that one learns from this table are:

- $e_1, e_2, \ldots, e_7$ *are square roots of* -1: $e_i^2 = -1, \quad i = 1, 2, \ldots, 7$
- e_i *and* e_j *anticommute when* $i \neq j$: $e_i e_j = -e_j e_i$
- *whenever* $e_i e_j = s e_k$ *with* $s = \pm 1$, $e_{i+1} e_{j+1} = s e_{k+1}$ *with index addition understood to be defined so that* 8 *equals* 1 *(index cycling property)*
- *whenever* $e_i e_j = s e_k$ $e_{2i} e_{2j} = s e_{2k}$ *where index doubling is again understood to obey the cycling property "*$8 = 1$*" (index doubling property)*
- $e_i(e_j e_k) \neq (e_i e_j) e_k$ *unless* $i = j$ *of* $j = k$ *(non-associativity)*

  e.g. $e_1(e_2 e_3) = e_1 e_5 = e_6 \qquad e_6(e_7 e_3) = -e_6 e_1 = -e_6$
  $(e_1 e_2) e_3 = e_4 e_3 = -e_6 \qquad (e_6 e_7) e_3 = e_2 e_3 = e_5$

The definitions of the norm, conjugate and inverse are similar to those of quaternions. The norm $N(A)$ *is defined as* $N(A) = A^* A = A A^*$, *where* A^* *is the conjugate octonion, i.e.* e_j *replaced with* $-e_j$. *For a nonzero octonion* A, *the multiplicative inverse* A^{-1} *is also an octonion (division algebra), since* $A^{-1} = \frac{1}{N(A)} A^*$. *For two octonions* A *and* B

- $(AB)^* = B^*A^*$
- $N(AB) = N(A)N(B)$ *(composition algebra* property*)*
- $A(AB) = A^2B$ *(alternativity* property*)*
- $A^*(AB) = (A^*A)B = N(A)B$

Since octonions do not form an associative algebra, they cannot be represented directly by matrices. The following describe a method of representing octonions:

An octonion A is written as an ordered pair of two 4-dimensional quaternions q_1 and q_2 as $A = (q_1; q_2)$. Then the rule of multiplication of two octonions A and B is

$$AB = (q_1;\ q_2)(q_3;\ q_4) = (q_1q_3 + \beta q_4^* q_2; \quad q_2q_3^* + q_4q_1)$$

where β is a field parameter.

Octonions are the largest of the 4 normed division algebras, but were somewhat neglected due to their nonassociativity. Their relevance to geometry was quite obscure until 1925, when **Elie Cartan** *used them to establish symmetry between vectors and spinors in 8-dimensional Euclidean space. Their potential relevance to physics was noticed in 1934 by* **Jordan**, **von Neumann** *and* **Wigner**, *but attempts to apply octonions quantum mechanics to nuclear and particle physics were met with little success.*

In 1985 it was realized that octonions explain some curious features of string theory. Beside their possible role in physics, octonions seem to tie together some mathematical structures. Today they stand at the crossroads of many interesting fields of mathematics: Clifford algebras and spinors, projective and Lorentzian geometry, Jordan algebras, and the exceptional Lie groups. They seem to have applications in quantum logic, special relativity and supersymmetry.

Sedenions

The word sedenion is derived from sexdecim, meaning 16. A sedenion is a hypercomplex number constituted from 16 basal elements obeying the multiplication table:

	1	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}
1	1	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}
e_1	e_1	-1	e_3	$-e_2$	e_5	$-e_4$	$-e_7$	e_6	e_9	$-e_8$	$-e_{11}$	e_{10}	$-e_{13}$	e_{12}	e_{15}	$-e_{14}$
e_2	e_2	$-e_3$	-1	e_1	e_6	e_7	$-e_4$	$-e_5$	e_{10}	e_{11}	$-e_8$	$-e_9$	$-e_{14}$	$-e_{15}$	e_{12}	e_{13}
e_3	e_3	e_2	$-e_1$	-1	e_7	$-e_6$	e_5	$-e_4$	e_{11}	$-e_{10}$	e_9	$-e_8$	$-e_{15}$	e_{14}	$-e_{13}$	e_{12}
e_4	e_4	$-e_5$	$-e_6$	$-e_7$	-1	e_1	e_2	e_3	e_{12}	e_{13}	e_{14}	e_{15}	$-e_8$	$-e_9$	$-e_{10}$	$-e_{11}$
e_5	e_5	e_4	$-e_7$	e_6	$-e_1$	-1	$-e_3$	e_2	e_{13}	$-e_{12}$	e_{15}	$-e_{14}$	e_9	$-e_8$	e_{11}	$-e_{10}$
e_6	e_6	e_7	e_4	$-e_5$	$-e_2$	e_3	-1	$-e_1$	e_{14}	$-e_{15}$	$-e_{12}$	e_{13}	e_{10}	$-e_{11}$	$-e_8$	e_9
e_7	e_7	$-e_6$	e_5	e_4	$-e_3$	$-e_2$	e_1	-1	e_{15}	e_{14}	$-e_{13}$	$-e_{12}$	e_{11}	e_{10}	$-e_9$	$-e_8$
e_8	e_8	$-e_9$	$-e_{10}$	$-e_{11}$	$-e_{12}$	$-e_{13}$	$-e_{14}$	$-e_{15}$	-1	e_1	e_2	e_3	e_4	e_5	e_6	e_7
e_9	e_9	e_8	$-e_{11}$	e_{10}	$-e_{13}$	e_{12}	e_{15}	$-e_{14}$	$-e_1$	-1	$-e_3$	e_2	$-e_5$	e_4	e_7	$-e_6$
e_{10}	e_{10}	e_{11}	e_8	$-e_9$	$-e_{14}$	$-e_{15}$	e_{12}	e_{13}	$-e_2$	e_3	-1	$-e_1$	$-e_6$	$-e_7$	e_4	e_5
e_{11}	e_{11}	$-e_{10}$	e_9	e_8	$-e_{15}$	e_{14}	$-e_{13}$	e_{12}	$-e_3$	$-e_2$	e_1	-1	$-e_7$	e_6	$-e_5$	e_4
e_{12}	e_{12}	e_{13}	e_{14}	e_{15}	e_8	$-e_9$	$-e_{10}$	$-e_{11}$	$-e_4$	e_5	e_6	e_7	-1	$-e_1$	$-e_2$	$-e_3$
e_{13}	e_{13}	$-e_{12}$	e_{15}	$-e_{14}$	e_9	e_8	e_{11}	$-e_{10}$	$-e_5$	$-e_4$	e_7	$-e_6$	e_1	-1	e_3	$-e_2$
e_{14}	e_{14}	$-e_{15}$	$-e_{12}$	e_{13}	e_{10}	$-e_{11}$	e_8	e_9	$-e_6$	$-e_7$	$-e_4$	e_5	e_2	$-e_3$	-1	e_1
e_{15}	e_{15}	e_{14}	$-e_{13}$	$-e_{12}$	e_{11}	e_{10}	$-e_9$	e_8	$-e_7$	e_6	$-e_5$	$-e_4$	e_3	e_2	$-e_1$	-1

The sedenions form a 16-dimensional algebra over the reals. *Like octonions their multiplication is neither commutative nor associative. But in contrast to octonions the sedenions do not even have the property of being* alternative. *They do, however, have the property of being* power-associative: $A^n A^m = A^{n+m}$.

The sedenions have multiplicative inverses, but they are not a division algebra because they have zero divisors.

Every sedenion is a real linear combination of the unit sedenions $1, e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}$ and e_{15}, *which form a basis of the* vector space of sedenions.

A sedenion may be represented as an ordered pair of two octonions. The product of the sedenions S and T is then given as

$$ST = (A;\ B)(C;\ D) = (AC + \gamma D^*B; \quad BC^* + DA)$$

where B^* is the conjugate of B and γ is a field parameter.

Classical Group Theory[292] *(1770–1903)*

A group G is a set of objects, symbols or operations (elements), $a, b, c, \ldots$ for which there exists a certain binary composition law (usually called 'multiplication') that associates with each ordered pair of elements a unique element (called their 'product'), such that the following conditions are satisfied:

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292 To dig deeper, see:

- Hall, G.G., *Applied Group Theory*, American Elsevier Publishing Company: New York, 1967, 128 pp.
- Smirnov, V.I., *Linear Algebra and Group Theory*, McGraw-Hill, 1961, 464 pp.
- Barnard, T. and H. Neill, *Mathematical Groups*, Teach Yourself Books, 1996, 218 pp.
- Lyubarskii, G.Ya., *The Applications of Group Theory in Physics*, Pergamon Press, 1960, 381 pp.
- Bishop, D.M., *Group Theory and Chemistry*, Dover, 1993, 300 pp.
- Kramer, E.E., *The Nature and Growth of Modern Mathematics*, Princeton University Press, 1982, 758 pp.

- *Closure:* If a and b are two elements of G, then the product ab is also an element of G.
- *Associativity:* For any 3 elements a, b, c multiplication is associative, i.e. $(ab)c = a(bc)$.
- *Existence of identity:* There exists an element I called the identity such that $aI = Ia = a$ for every element a of G.
- *Existence of inverse:* Corresponding to each element a of G there exists an element denoted by a^{-1} and called the inverse of a, such that $aa^{-1} = a^{-1}a = I$ for every a of G.

Of course $ab \neq ba$ in general. A group is said to be finite if the number of elements in it is finite. The number of distinct element is called the order *of the group. Otherwise, it is said to be an* infinite group*. A group G is said to be* Abelian *or* commutative *if in addition to the group postulates we also have $ab = ba$ for any pair of elements a, b of G.*

To illustrate the group properties, consider the group of rigid geometrical operations that transform the figure of a square into itself. There are 8 independent operations in the plane*, generated by rotations and reflections, which achieve this goal. Labeling the corners of the squares with numbers 1 through 4, beginning at the upper right and going around clockwise, it is clear that the numbers will be permuted by each of the 8 operators as indicated in the following list:*

Identity	(I)	${}^{4}_{3}\square^{1}_{2}$
90° *Counterclockwise rotation*	(R_1)	${}^{1}_{4}\square^{2}_{3}$
180° *Counterclockwise rotation*	(R_2)	${}^{2}_{1}\square^{3}_{4}$
270° *Counterclockwise rotation*	(R_3)	${}^{3}_{2}\square^{4}_{1}$
Reflection through horizontal axis	(H)	${}^{3}_{4}\square^{2}_{1}$
Reflection through vertical axis	(V)	${}^{1}_{2}\square^{4}_{3}$
Reflection through diagonal joining corners 1 and 3	(D_1)	${}^{2}_{3}\square^{1}_{4}$
Reflection through diagonal joining corners 2 and 4	(D_2)	${}^{4}_{1}\square^{3}_{2}$

These are the 8 basic geometrical operations for transforming a square into itself. If, for example, we effect R_2 after R_1, the combined effect of both operations is exactly the same as would have been obtained from the original square with R_3 alone, so we can write symbolically $R_2R_1 = R_3$, which illustrates closure. Proceeding in this manner, the entire 'multiplication table' for the square operators can be established, with each particular entry being the product of the same-row element in the left column (left factor) with the same-column element of the topmost row (right factor), as is shown in the following table:

	I	R_1	R_2	R_3	H	V	D_1	D_2
I	I	R_1	R_2	R_3	H	V	D_1	D_2
R_1	R_1	R_2	R_3	I	D_1	D_2	V	H
R_2	R_2	R_3	I	R_1	V	H	D_2	D_1
R_3	R_3	I	R_1	R_2	D_2	D_1	H	V
H	H	D_2	V	D_1	I	R_2	R_3	R_1
V	V	D_1	H	D_2	R_2	I	R_1	R_3
D_1	D_1	H	D_2	V	R_1	R_3	I	R_2
D_2	D_2	V	D_1	H	R_3	R_1	R_2	I

The table exhibits that the 8 operators do indeed form a group, but since $D_2R_3 = H$, $R_3D_2 = V$ etc., the group is not Abelian. Any subset of elements in a group (usually a smaller subset) which in themselves satisfy the group postulates is called a subgroup of the initial group. Thus $\{I, R_2\}$ is a subgroup of order 2, while $\{I, R_1, R_2, R_3\}$ is a subgroup of order 4.

A group such as that of the square has a certain formal or abstract structure which does not depend upon geometrical associations for its meaning. Thus, a set of eight matrices could be determined which would satisfy the group postulates just as the geometrical operators did. These are, for example, $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$; $R_1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$; $R_2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$; $R_3 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$;

$H = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$; $V = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$; $D_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$; $D_2 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$, *under* *ordinary matrix multiplication. The elements of this group of matrices and those of the group of operators can be put into one-to-one correspondence with each other which preserves the multiplication table, and the groups are then said to be* isomorphic *to one another. The matrices are said to form a* representation *of the group.*

A second example is the group S_n of rearrangements (permutations) of n objects, known as the symmetric group. *A typical element of S_5 might be written as [24153], which means: put the second object first, the fourth object second, etc. Two elements are "multiplied" by performing first the rearrangement on the right, then the rearrangement on the left. For example, $[24153][51234]abcde = [24153]eabcd = acedb = [13542]abcde$, where letters represent the 5 objects; therefore $[24153][51234] = [13542]$. The order of S_n is obviously $n!$.*

An example of a group with a different kind of binary operation (composition law) is the group of 6 functions: $I(x) = x$; $A(x) = \frac{1}{1-x}$; $B(x) = 1 - \frac{1}{x}$; $C(x) = \frac{1}{x}$; $D(x) = 1 - x$; $E(x) = \frac{x}{x-1}$. The law of composition is the substitution of one function into the other as a function of a function, e.g., $AE = A(E(x)) = A\left(\frac{x}{x-1}\right) = \frac{1}{1-\frac{x}{x-1}} = 1 - x = D(x)$.

The general representation *of a group G is a group of square non-singular matrices, one matrix M for each group element g, with matrix multiplication as the composition law. If the matrices corresponding to different elements of G are themselves different, the two groups are isomorphic. If, however, one matrix M represents more than one group element of G, the group is said to be* homomorphic *to the matrix group. An isomorphism of a group onto itself is an* automorphism.

Consider a particular representation D. The matrix associated with the group element g, will be written as $D(g)$. We can form another representation D' in quite a trivial way by defining $D'(g) = S^{-1}D(g)S$, where S is any *non-singular matrix. Such representations, connected by the* equivalence transformation, *are said to be* equivalent, *and will, in practice, be considered to be the* same *representation. From two representations of the same element, $D^{(1)}(g)$ and $D^{(2)}(g)$, we can form a new representation*

$$D(g) = D^{(1)}(g) \oplus D^{(2)}(g) = \begin{pmatrix} D^{(1)}(g) & 0 \\ 0 & D^{(2)}(g) \end{pmatrix}.$$

If $D^{(1)}$ and $D^{(2)}$ have dimensionalities n_1 and n_2, the dimensionality of $D(g)$ is clearly $n_1 + n_2$. The representation D is said to be reducible *and splits into two smaller representations $D^{(1)}$ and $D^{(2)}$. A representation which is* *not of the above form and cannot be brought into this form by an equivalence transformation, is called an irreducible representation. The irreducible representations of a group are the "building blocks" for the study of group representations, since an arbitrary representation can be decomposed into a linear combination of irreducible representations*[293].

Since the same group can be represented by infinite number of equivalent matrix groups which are the same for most physical purposes, it would seem preferable to identify the particular group by something that is invariant under similarity transformations. One such invariant is the trace of the matrix. We therefore define the character of the i^{th} *representation* $\chi^{(i)}(g)$ *as the trace of* $D^{(i)}(g)$. *In the case of* S_3, *for example, we have the following irreducible representation with its associated characters:*

[293] A *tensor* in n-dimensional space is reducible if there exists a less general class of the same rank which transforms onto itself by any $\mathbb{R}^n$ rotation (the *group* of these rotations is called $SO(n)$, a subgroup of the orthogonal group $O(n)$).

Vectors are always *irreducible* since for any two given vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ it is always possible to find a rotation $\mathfrak{R}$ such that $\mathfrak{R}\cdot\boldsymbol{a}=\boldsymbol{b}$. With *dyadics* (second rank tensors) the situation is different since in general, for any given two dyadics $\mathfrak{A}$ and $\mathfrak{B}$ one *cannot* always find a rotation $\mathfrak{R}$ such that $\mathfrak{R}\cdot\mathfrak{A}\cdot\mathfrak{R}^{-1}=\mathfrak{B}$, because not all matrices are similar. In fact, the group of all dyadics is reducible into three groups: For an arbitrary dyadic,

$$\mathfrak{A}=\frac{1}{3}\lambda\mathfrak{J}+\frac{1}{2}(\mathfrak{A}-\mathfrak{A}^T)+\left[\frac{1}{2}(\mathfrak{A}+\mathfrak{A}^T)-\frac{1}{3}\lambda\mathfrak{J}\right],$$

where $\lambda=\text{trace }\mathfrak{A}$, $\mathfrak{J}$ is the unit dyadic and $\mathfrak{A}^T$ is the transpose of $\mathfrak{A}$. Then, with $\mathfrak{B}=\mathfrak{R}\cdot\mathfrak{A}\cdot\mathfrak{R}^{-1}$, $\lambda=\text{trace }\mathfrak{B}=\text{trace }\mathfrak{A}$:

- $\mathfrak{R}\cdot(\lambda\mathfrak{J})\cdot\mathfrak{R}^{-1}=\lambda\mathfrak{J}$.
- $\mathfrak{R}\cdot(\mathfrak{A}-\mathfrak{A}^T)\cdot\mathfrak{R}^{-1}=\mathfrak{B}-\mathfrak{B}^T$.
- $\mathfrak{R}\cdot\left(\frac{1}{2}\mathfrak{A}+\frac{1}{2}\mathfrak{A}^T-\frac{1}{3}\lambda\mathfrak{J}\right)\cdot\mathfrak{R}^{-1}=\frac{1}{2}\mathfrak{B}+\frac{1}{2}\mathfrak{B}^T-\frac{1}{3}\lambda\mathfrak{J}$.

These three sub-representations (identity, skew-symmetric and symmetric traceless) are irreducible.

g	$D(g)$	*rotations*	$\chi(g)$
[123]	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$	$R_z(0)$	2
[231]	$\frac{1}{2}\begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix} = A$	$R_z(120\,^\circ)$	-1
[312]	$\frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} = B$	$R_z(240\,^\circ)$	-1
[321]	$\frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} = E$	$R_{_E}(180\,^\circ)$	0
[213]	$\frac{1}{2}\begin{pmatrix} 1 & -\sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix} = D$	$R_{_D}(180\,^\circ)$	0
[132]	$\frac{1}{2}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = C$	$R_{_C}(180\,^\circ)$	0

Note that $D(g)$ *serves also to represent the* 6*-element crystallographic dihedral group with three axes of symmetry, when* 3 *identical atoms are fixed at the vertices of an equilateral triangle in the* x*–*y *plane:* $(0, 1)$, $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ *and* $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$. *There are* 3 *in-plane rotations and* 3 *reflections which leave the triangle invariant. These* 6 *symmetry operations are: 3 rotations* R_z *about a* z*-axis perpendicular to the* x*–*y *plane at its origin with the respective angles* 0, $120\,^\circ$, $240\,^\circ$, *a reflection* $R_{_C}$ *about the* y *axis, a reflection* $R_{_D}$ *about an altitude drawn from* $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$, *and finally a reflection* $R_{_E}$ *about an altitude drawn from* $\left(\frac{-\sqrt{3}}{2}, -\frac{1}{2}\right)$.

It is an indication of the versatility of the group concept, that in addition to the groups whose order is finite or denumerably infinite, there are groups the elements of which form a continuum. For example, the real numbers form an Abelian group L *under addition. Also, the* 2×2 *active rotation matrices* $R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ *form an Abelian group* $O(2)$ *under matrix multiplication, since* $R(\theta_1)R(\theta_2) = R(\theta_1 + \theta_2)$ *[clearly* L *and* $O(2)$ *are not isomorphic, but rather homomorphic].*

The group O_2 *is an example of a group whose elements depend on one or more continuous parameters. The functional dependence is not only continuous but also differentiable to any order provided a suitable set of parameters* *is used in each region of the group manifold. Such groups are known as Lie groups. Obviously, we cannot write down a "multiplication table" in the ordinary sense for such a group. The neighborhood of the identity (i.e. of* $\theta = 0$ *in the* $O(2)$ *example) is of special significance, since the identity element is* $I = R(0)$, *and the inverse of* $R(\theta)$ *is* $R(-\theta)$. *In the neighborhood of the unit element the group structure of* L *and* O_2 *is similar. Such a neighborhood is known as a group germ, and both* L *and* O_2 *have isomorphic group germs. Indeed, for small angles* $\delta\theta$, *the infinitesimal rotation operator takes the linear form*

$$R(\delta\theta) = \begin{pmatrix} 1 & \delta\theta \\ -\delta\theta & 1 \end{pmatrix}.$$

Some examples of Lie groups will indicate the range of applications of the ideas. Generally, a Lie group of r *parameters has elements which depend on* r *real parameters in an* r *dimensional space and are so related that if* $A(\boldsymbol{\gamma}) = A(\boldsymbol{\beta})A(\boldsymbol{\alpha})$, *where* $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$ *each have* r *components, then the parameters satisfy* $\boldsymbol{\gamma} = f(\boldsymbol{\beta}, \boldsymbol{\alpha})$ *and each component of* f *is analytic in all components of* $\boldsymbol{\alpha}$ *and* $\boldsymbol{\beta}$[294]. *Some of the more important Lie groups are:*

- $\boldsymbol{GL}(n)$: *The general linear group in* n *dimensions consist of all real non-singular* $n \times n$ *matrices. It has* n^2 *parameters ranging over a non-compact domain.*
- $\boldsymbol{SL}(n)$: *The special linear (or unimodular) group is a subgroup of* $\boldsymbol{GL}(n)$ *which consists of matrices whose determinant is unity. This condition reduces the number of parameters by one.*
- $\boldsymbol{O}(n)$: *The orthogonal group in* n *dimensions consists of all real* $n \times n$ *matrices* A *which satisfy* $AA^T = I$. *There are* $\frac{1}{2}n(n-1)$ *angle parameters that have a compact domain. Only proper rotations can be reached by starting from the identity and applying successive infinitesimal rotations. The subgroup of* $O(n)$ *consisting of proper rotations only (no reflection allowed) is* $SO(n)$, *which can also be defined as a set of matrices* A *of* $O(n)$ *such that* $\det A = 1$.
- $\boldsymbol{U}(n)$: *The unitary group in* n *dimensions consist of all* $n \times n$ *matrices* U *with complex elements satisfying* $UU^+ = I$, *where* $U^+ = (U^*)^T$. *The number of real parameters is* n^2. *[In 2 dimensions* $U = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, *where* $aa^* + bb^* = 1, \quad cc^* + dd^* = 1, \quad ac^* + bd^* = 0.$*]*

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[294] In general, more than one chart of coordinates $\boldsymbol{\alpha}$ may be needed to cover the *group manifold*, with analytic maps $\boldsymbol{\alpha}'(\boldsymbol{\alpha})$ relating the coordinates of any two intersecting chords inside their overlap region

- $\boldsymbol{SU}(n)$: *The special unitary group is a subgroup of* $\boldsymbol{U}(n)$ *whose matrices have a determinant of unity. This reduces the number of real parameters to* n^2-1.

- *The affine group in three dimensions consists of all transformations of the form* $\boldsymbol{r}' = A\boldsymbol{r} + \boldsymbol{a}$, *where* A *is a non-singular matrix and* $\boldsymbol{a}$ *is a vector. This group has 12 parameters. One important subgroup is the group of rigid rotations and translations, in which* A *is restricted to orthogonal matrices.*

- *The fractional linear (or projective) group, under substitution in two variables, consists of all transformations of the type* $\overline{x} = \frac{a_{11}x+a_{12}y+a_{13}}{a_{31}x+a_{32}y+a_{33}}$, $\overline{y} = \frac{a_{21}x+a_{22}y+a_{23}}{a_{31}x+a_{32}y+a_{33}}$ *and has 8 parameters.*

  In one variable, all expressions of the form $w = \frac{az+b}{cz+d}$ *form a group under substitution, where* a, b, c, d *are parameters which satisfy* $ad-bc=1$. *If* w *and* z *are complex variables and* (a, b, c, d) *are complex parameters then this is a group of conformal representations. One particular finite subgroup of this continuous group has already been mentioned earlier. It consists of the 6 functions* $\left\{z, \frac{1}{1-z}, 1-\frac{1}{z}, \frac{1}{z}, 1-z, \frac{z}{z-1}\right\}$.

- *A one-parameter group of general transformations: Consider the transformation of coordinates (not axes) given by* $x' = f(x, y; a)$; $y' = \phi(x, y; a)$, *where* a *is an arbitrary parameter. The transformation carries the point* $M(x, y)$ *in the* x*–*y *plane to another point* $M'(x', y')$ *in the same plane. To each value of the parameter* a *corresponds a definite transformation. Varying this parameter, we obtain an infinite number of different transformations.*

Suppose that we carry out in succession two different transformations of this set, corresponding to two values of a *and* b *of this parameter. The first transformation will carry the pair* (x, y) *into* (x', y') *according to the above equations. The second transformation will carry* (x', y') *into a third pair* (x'', y'') *such that* $x'' = f(x', y'; b)$; $y'' = \phi(x', y'; b)$. *Substituting* (x', y') *from the first transformation, we find* $x'' = F(x, y; a, b)$, $y'' = \Phi(x, y; a, b)$, *which defines a point-transformation depending on two parameters* a *and* b.

We shall say that the set of transformations f *and* ϕ *form a continuous one-parameter group if the new transformation belongs to this set. It is necessary and sufficient for this that* $x'' = f(x, y; c)$; $y'' = \phi(x, y; c)$, *where* $c = \psi(a, b)$ *for some function* ψ.

Examples are: $\{x' = x + a;\quad y' = y + 2a\}$; $\{x' = x\cos\alpha - y\sin\alpha;\ y' = x\sin\alpha + y\cos\alpha\}$; $\{x' = ax;\quad y' = a^2y\}$. *But* $x' = x + a$; $y' = y + a^2$ *do not form a group.*

Such a group contains the identity transformation

$$f(x, y; a_0) = x, \qquad \phi(x, y; a_0) = y$$

for some value $a = a_0$ *of the parameter. If* ϵ *is small, the transformation* $x_1 = f(x, y; a_0 + \epsilon)$; $y_1 = \phi(x, y; a_0 + \epsilon)$ *will be such that* x_1 *differs only infinitesimally from* x, *and* y_1 *from* y. *This transformation therefore differs only infinitesimally from the identity transformation, and is said to be an infinitesimal transformation.*

The idea of a group is one which pervades the whole of mathematics, both pure and applied. In the particular form of the study of symmetry, group theory can claim to have its origin in prehistoric times. Nowadays, group theory is developed in an abstract way so that it can be applied in many different circumstances — but many of these still concern symmetry.

The notion of group *(though not the term) originated (1770) in the works of* ***Lagrange*** *and* ***Vandermonde*** *on* permutation groups. *Lagrange moved toward the definition of the group concept in his attempts to solve the general* quintic equation. *He proved that the number of elements in a group is divisible by the number of elements in any of its subgroups (1770, Lagrange's Theorem).* ***A.L. Cauchy*** *(1815) investigated permutation groups and discovered several basic theorems.* ***Galois*** *(1829 to 1832) laid the foundations to the theory of groups in his group-theoretical approach to the problem of solvability of algebraic equations by radicals.* ***Auguste Bravais***[295] *(1811–1863, France) studied the symmetry of crystalline lattices by means of rotations and translations of their patterns into themselves. He thus advanced both crystallography and group theory (1848 to 1851).*

A. Cayley *(1854) presented the first technical definition of a group, listing postulates and representing a finite group by its multiplication table. He later (1878) asserted the equivalence of isomorphic groups*[296]. ***Ludvig Sylow*** *(1832–1918, Norway), discovered (1872) the fundamental theorem of permutation groups*[297].

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[295] French physicist. Served in the navy (1831–1857), and as a professor at the École Polytechnique (1845 to 1856). Demonstrated (1850) the 14 possible lattice configurations, known as *Bravais lattices*.

[296] *Cayley theorem*: Every group of finite order is isomorphic to a subgroup of the permutation group $S_n = \begin{pmatrix} 1, 2, 3, \dots, n \\ s_1, s_2, s_3, \cdots, s_n \end{pmatrix}$, also known as the symmetry group for some n.

[297] If p is a prime number and α is a positive integer such that p^α divides the order of the group but $p^{\alpha+1}$ does not, then the group has a subgroup of order p^α.

The progress that was made by means of theory of groups in the solution of algebraic equations of higher degree, induced mathematicians of the mid 19^{th} *century to attempt to use the theory of groups in the solution of equations of other forms, in the first instance the solution of differential equations, which play such an important role in the applications of mathematics. This attempt was rewarded with success.*

Although the place occupied by groups in the theory of differential equations is entirely different from their role in the theory of algebraic equations, the applications of the theory of groups to the solution of differential equations led to substantial extension of the very concept of a group and to the creation of a new theory of the so-called continuous group (Lie groups)*, which have proved to be extremely important for the development of many branches of mathematics and physics.*

M.S. Lie[298] *established (1874 to 1891) the theory of* continuous groups *and applied it to the classification and integration of* ODE*.* ***Georg Frobenius*** *(1896 to 1903) expanded the study of group representations to all finite abstract groups. He introduced and developed the concepts of* reducible *and* completely reducible *representations.* ***Eliakim Moore*** *(1862–1932, U.S.A.) developed (1893) the theory of the group of* automorphism *of any finite group.*

1854–1863 CE **Francesco Brioschi** (1824–1897, Italy). Mathematician, engineer and architect. Contributed to the theory of *determinants* (1854) and the application of *elliptic modular functions* to the solution of algebraic equations of the 5^{th} and 6^{th} degree.

Brioschi was born in Milan, and graduated from the University of Pavia (1845). He taught mechanics, architecture and astronomy at Pavia and the Istituto Tecnico Superiore in Milan (1863–1897).

1854–1883 CE **Gustav Robert Kirchhoff** (1824–1887, Germany). Distinguished physicist. Made important contributions to the theory of circuits, using *topology*, and to elasticity.

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[298] **Lie** and **F. Klein** were students together in Berlin in 1869–1870 when they conceived the notion of studying mathematical systems from the perspective of transformation groups which left these systems invariant. Thus, Klein in his famous *Erlangen program*, pursued the role of *finite* groups in the studies of regular bodies and the theory of algebraic equations, while Lie developed his notion of continuous transformation groups and their role in the theory of ODE.

Born and educated at Königsberg, Prussia. A student of **Gauss**. Served as a professor of physics at Breslau (1850), Heidelberg (1854) and Berlin (1875–1887). Made numerous important contributions to mathematical physics. In 1857 he showed that the mechanical forces manifested by static and current electricity were related by a constant which has the dimensions of velocity. By comparing the attractive force of two static charges with the magnetic force produced when they are discharged, he demonstrated that the constant has the same magnitude as the velocity of light. Established the fundamental laws which govern the distribution of currents in a network of conductors (*Kirchhoff circuit laws*).

During 1859–1861, Kirchhoff and his collaborator **Robert Wilhelm Bunsen** (1811–1899) invented the Kirchhoff-Bunsen spectroscope for chemical analysis of metals placed in a flame, whereby the bright lines in the spectra of elements could be accurately recorded. They conjectured that each element produces a characteristic spectrum. By discovering the metal *cesium* they demonstrated how new elements could be discovered via spectroscopic analysis.

Furthermore, Kirchhoff found that for any emitting body in thermal equilibrium, the coefficient of emission and the coefficient of absorption are in a ratio that is a function only of wavelength and temperature. He introduced the concepts of *blackbody* and *emissivity*. He mapped the solar spectrum, and showed that elements such as sodium can be detected in the atmosphere of the sun by means of the dark (absorption) lines they cause in the spectrum[299].

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[299] The heated vapors produced an *emission spectrum*: bright lines against a dark background. The nature of these bright lines depended on the elements present in the vapor. Each element produced *its own pattern of bright lines*, and the same line in precisely the same position was never produced by two different elements. The emission spectrum served as a sort of fingerprint of the elements present in the glowing vapor. In the course of their studies, Kirchhoff and Bunsen detected lines that were not produced by any known element. They suspected the presence of new and hitherto undiscovered elements and were able to verify the fact by chemical analysis. The new elements were named *cesium* and *rubidium* from Latin words meaning *sky blue* and *red* respectively, signifying the colors of the lines that led to the discovery (the first elements to be discovered spectroscopically).

Kirchhoff and Bunsen then worked with light from a glowing solid (which produced white light that consisted of a continuous spectrum) and passed that light through a cool vapor. They found that the vapor absorbed certain wavelengths of light, and that the spectrum that was formed after the light had passed through the vapor was no longer completely continuous, but was crossed by dark lines which marked the position of the absorbed wavelengths. This was

In 1883 he generalized the Helmholtz harmonic time solution of the scalar wave equation (1860) to the case of transient waves.

1855 CE **Heinrich Geissler** (1814–1879, Germany). Physicist. Developed a mercury pump which he uses to produce the first good vacuum tubes. Such tubes were used in 1869 to produce "cathode rays", leading eventually to the discovery of the electron.

1855 CE **Henry Bessemer** (1813–1898, England). Inventor and manufacturer. Developed the *Bessemer process* of converting pig iron to steel. In this process a blast of air burns most impurities out of the molten pig iron. In 1830 he established his own steelworks at Sheffield and financed the experiments that promoted his invention. It was soon adopted throughout the world.

1855 CE **William Parsons** (1800–1867, England and Ireland). Astronomer. First to record observations of the spiral structure of galaxies. The nature of these spiral nebulae remained a source of speculations until 1924.

Using a large telescope of his own design (built 1827–1845) he was able to distinguish spiral structure in what we know today as the *whirlpool galaxy* (M51, in the constellation of Canes Venatici, 15 million light years away from earth).

William Parsons, the 3^{rd} earl of Rosse, was born in York, England. He was rich, liked machines, and was fascinated with astronomy. Accordingly he set about the business of building gigantic telescopes. In February 1845 he completed a 183 cm reflecting telescope at his estate in Birr Castle, Ireland. The contraption was mounted at one end of a 18.3 m tube that was controlled by cables, straps, pulleys, and cranes. For a brief time it was the largest telescope in the world.

In the course of 20 years, Lord Rosse examined many of the nebulae that had been discovered and catalogued by **William Herschel**, and observed that some of these have a distinct spiral structure. Because he did not have any photographic equipment, he had to make *drawings* of what he saw. His drawing of the M51 captured the salient features of modern photographs of the galaxy. Views such as this inspired Lord Rosse to echo **Kant**'s (1755) theory that these objects might be "*island universes*" — vast collections of stars far beyond the confines of the Milky Way.

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an *absorption spectrum*, and it seemed clear that the solar spectrum was an example of this.

The poor weather over Birr Casle usually limited the capabilities of the big telescope. Nevertheless, the whirlpools of light stood clearly against the black background of space.

1855–1859 CE **John Henry Pratt** (1809–1871, England and India). Geophysicist and clergyman. Introduced (with **Airy**) the concept of *isostasy compensation* and calculated the average depth of density compensation to be 100 km. Gave 43 km as the difference between equatorial and polar radii of the earth.

Pratt postulated (1855) density difference in the crust of the earth: lower density under mountains, high density in the lowlands — to explain the too nearly constant values obtained for gravity of a given latitude. In the same year, Airy offered a different explanation (though based on the same principle) of the gravity data. Both proposals have their merits but are oversimplification of the actual situation.

Pratt was archdeacon of Calcutta (1850–1871).

The Hindu Puzzle

Before the middle of the 18th century, geologists and geodesists regarded mountains as composed of matter of much the same density as the rest of the crust, and it was not recognized that their weight would be expected to produce any deformation of the matter below them, nor that the density of the matter below a mountain range might differ systematically from that of the matter at an equal depth below a plain or even an ocean. Now, if a mountain is considered merely as an extra mass superposed on a previously uniform crust, and its deforming effect in the interior is ignored, it is possible to compute all components of its contribution to gravity on bodies in its neighborhood. The attraction can also be found experimentally and the result compared with that calculated.

The experiment was carried out on several mountains during the 18th century with unexpected results. Indeed, the measurements of ***P. Bouguer*** *(1749) of the deflection of gravity due to the mountain Chimborazo (Andes), of* ***Maskelyne*** *(1774) at Schiehallion (England) and* ***Petit*** *(1849) in the Pyrenees, showed that the attraction of mountains was generally nearer to zero than to the values calculated on the supposition that the underlying matter was of normal density.*

The modern development began with the discussions of ***Pratt*** *(1855) and* ***Airy*** *(1855) on the deflections of gravity observed by* Everest *during the great land survey in India (1830–1843): the distance between Kaliana (some 100 km south of the Himalayas range, and Kalianpur (600 km further south), was determined in two precise ways – by measurement over the surface and by reference to astronomical observations – and the results disagreed by some 150 meters over 600 kilometers. This may seem to be a small amount, but it was an intolerable surveying error even by 19th century standards.*

The astronomical method of measuring distances uses the angles of stars w.r.t. the vertical, which are defined by a plumb line (a weight suspended on a string). To account for the difference, it was proposed that the plumb line was tilted toward the Himalayas because of the gravitational attraction of the mountains on the plumb bob, causing an error in the distance measurement. When, however, the calculation was actually made, it was found that the mountains should have introduced an even larger error – one of about 450 meters – thus compounding the puzzle!

Following an earlier suggestion by ***Cavendish*** *(1772),* ***Pratt*** *and* ***Airy*** *(1855), independently, came forward with an explanation for the discrepancy that contained the basis of the* principle of isostasy *(the actual word isostasy* *was coined in 1889 by* ***C.E. Dutton****). Accordingly, the enormously heavy mountains are not supported by a strong rigid crust below, but that they float in a "sea" of rock. Thus, the excess mass of the mountain above sea-level is* compensated *by a deficiency of mass in an underlying root (since the lighter mountain-base material locally displaces the denser "rock-sea" material). This root provides the buoyant support, in the manner of all floating bodies such as a ship with a deep hull or an iceberg. The plumb bob "feels"* both *the excess mass on top and the deficiency of mass below; hence the reduced deflection.*

The principle of isostasy *is thus the Archimedes principle of buoyancy applied to the flotation of continents and mountains, which holds that the relativity light continents float on a more dense mantle. The supportive root must develop as part of a process that provides buoyancy and keeps the load from sinking*[300]*. A simplified quantitative treatment of the above idea develops as follows:*

If the mass of the earth is M, its mean density $\bar{\rho}$, and its radius a, the acceleration of gravity at the surface in the absence if any disturbance is

$$g_0 = G\frac{M}{a^2} = \frac{4}{3}\pi G\bar{\rho}a.$$

At height h above the surface, in the free air, the intensity is

$$g_0\frac{a^2}{(a+h)^2} \simeq g_0(1-\frac{2h}{a}).$$

If instead of the space between sea-level and height h being empty, it is filled by matter of density ρ_m and a shape of width $\sim h$, the theory of Newtonian attraction predicts an additional contribution of $(2\pi G\rho_m h)$ to the intensity of gravity above it. In order to remove *the gravitational attraction of* local topography *the* ***P. Bouguer*** *gravity anomaly* correction*, $\Delta g = -2\pi G\rho_m h$, must be applied to the data.The overall excess of gravity at height h over its value at sea-level should be*

$$-[\frac{2g_0 h}{a}(1-\frac{3}{4}\frac{\rho_m}{\bar{\rho}})].$$

Although the Bouguer gravity formula is effective in removing the gravitational influence of local topography, it is not effective in removing the influence

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[300] One variant should be mentioned: if for some reason (e.g. regional *heating*) a part of the upper mantle becomes less dense than the adjacent mantle, it will also exert a buoyant force that can support elevated topography above it without the need for a crustal root. Here the lower density mantle serves as a root.

of regional topography: a mountain or valley with a small horizontal scale, say 10 km, can be supported by the elastic lithosphere without deflection and consequently does not influence the density distribution at depth. However, the load due to a mountain range with a larger horizontal scale, say 100 km, deflects the lithosphere downwards as well as the so-called Moho discontinuity in which it is embedded. Because crustal rocks are lighter than mantle rocks, this results in a low density root for the mountain range with a large horizontal scale. The mass associated with the topography of the mountains is compensated *at depth by a low-density root. According to* ***Pratt****, the density of the root varies horizontally as a function of the elevation h according to $\rho_p = \rho_0 \frac{w}{w+h}$, where ρ_0 is the surface density corresponding to zero elevation and w is referred to as the depth of compensation. According to* ***Airy*** *(1855), the density of the crust ρ_c and the mantle ρ_M are assumed to be constant. A crust feature with an elevation h has a crustal root of thickness*

$$b = \frac{\rho_c}{\rho_M - \rho_c} h,$$

derived from the principle of hydrostatic equilibrium.

Clearly, compensation in the lithosphere may be a complex combinations of both the Pratt and the Airy models.

The resolution of the 'Indian Puzzle' not only led to the notion of isostasy but also introduces gravity surveying *as a method for detecting mass variations in the interior by their corresponding gravity variations.*

1855–1858 CE **Rudolf Ludwig Carl Virchow** (1821–1902, Germany). Pathologist and political leader. Founded *cellular pathology.* Extended and applied cell theory to problems of pathology[301] and disease and set forth the principle that the outward symptoms of disease are merely the reflections of impairment at the level of cellular organization. He also advanced the notion that all cells arise from pre-existing cells: "*Omnis cellula e cellula*". His book

[301] *Pathology* — the study of disease. Pathology took definite form with **Giovanni Morgagni**'s (1682–1771, Italy) *Seats and Causes of Disease* (1761), correlating disease symptoms with the underlying pathology of the organs.

Xavier Bichat (1771–1802, France) introduced the concept of *tissue* to underlie pathological anatomy. **Karl von Freiherr Rokitansky** (1804–1878, Austria) systematized modern post-mortem protocol and described many specific conditions such as gastric ulcer and acute yellow atrophy of the liver.

Cellular Pathology (1858) established the cell as the fundamental pathological unit and permitted such processes, as *inflammation*, *tumor growth* (cancer) and degeneration, to be understood in cellular terms.

Virchow was born at Schivelbein, in Pomerania. He took a M.D. degree in Berlin (1843). Professor of pathological anatomy, Würzburg (1849) and Berlin (1856). Carried on research on blood, phlebitis, tuberculosis, rickets, tumors, trichinosis, etc. Made sanitary reforms in Berlin; established farms utilizing sewage for fertilizing the land. Founder and leader of the German Liberal party. Member of the German Reichstag (1880–1893); opposed policies of Bismarck.

1855–1868 CE **Jules-Antoine Lissajous** (1822–1880, France). Physicist. Studied the vibrations of bodies under combined excitations of different frequencies phases and amplitudes[302]. Found a simple way visualizing and studying these vibrations by reflecting a light beam from a mirror attached to a vibrating object onto a screen (1855–1857); e.g. by successively reflecting light from mirrors on two tuning forks vibrating at right angles. The resulting *Lissajous-figures* could be seen only because of persistence of vision in the human eye (no oscilloscopes available at that time!).

Lissajous entered the Ecole Normale Superieure (1841) and was awarded a doctorate thesis on vibrating bars using Chladni's sand pattern method to determine nodal positions (1850).

1855–1868 CE **Nathanael Pringsheim** (1823–1894, Germany). Botanist. One of the first to demonstrate sexual reproduction in algae (1855). Observed sperm penetration of the egg of *Oedogonium*.

1855–1876 CE **Alexander Bain** (1818–1903, Scotland). Philosopher and psychologist. Referred to by many as the first real psychologist. Developed psychology as a discipline apart from philosophy and physiology and

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[302] These had been observed earlier (1815) by **Nathaniel Bowdich** (1773–1838, USA), mathematician and astronomer. Consider a particle forced to vibrate harmonically, with two simultaneous motions at right angles

$$x = x_m \cos(\omega_x t + \phi_x); \quad y = y_m \cos(\omega_y t + \phi_y).$$

The path of the particle in the $x - y$ plane is a *Lissajous curve.* If $\frac{\omega_x}{\omega_y}$ is a rational number, the angular frequencies are commensurable and the curve is *closed* i.e. the motion repeats itself at regular intervals. Even for $\omega_x = \omega_y$, $x_m = y_m$ the curves will depend on the phase-difference $\phi_x - \phi_y$. If $\frac{\omega_x}{\omega_y}$ is irrational, then the curve is *open.* Lissajous observed *beats* when his tuning forks had slightly different frequencies; it showed as a rotating ellipse for the case $\omega_x = \omega_y$.

attempted to relate known physiological facts to psychological facts. Extended the associationist approach to all areas of psychological functioning, including habit and learning. Coined the term “trial and error”; wrote the first textbook on psychology in English (1855, 1859); and founded the first psychological journal, *Mind*, in 1876.

Bain was born in Aberdeen. Studied mathematics, physics and philosophy at Marischol College and later came under the influence of **John Stuart Mill** (1842). Appointed professor of mathematics and natural philosophy in the University of Glasgow but in 1846 resigned his position and devoted himself to literary work. Lived in London since 1848 and acquired wide influence as a logician and grammarian. Guided the awakened psychological interest in of British thinkers of the second half of the 19th century.

1855–1878 CE **David Edward Hughes** (1831–1900, England and USA). Inventor. Invented (1855) a keyboard telegraph with rotating type-wheel printer that grew into modern *telex* industry. Invented the *carbon microscope*[303] (1878).

Efforts to improve the telephone transmitter invented by Alexander Graham Bell (1876) led to the development of the microphone. Other microphone inventors included: **Philip Reis** (1861), **Emile Berliner** (1877) and **Thomas Edison** (1877).

Hughes was born and died in London. He emigrated with his parents to the United States (1838). In 1850 he became a professor of music at the College of Bardstown, Kentucky. He abandoned his academic career (1854) and moved to Louisville to manufacture his type-printing telegraph machines. In the succeeding ten years it came into extensive use all over Europe. It used a keyboard in which each key caused the corresponding letter to be printed at a distant receiver. It worked a bit like a ‘golfball’ typewriter and was produced before the typewriter was invented. The modern teleprinter, telex system and the computer keyboard are all direct descendants of his invention. His invention of the loose-contact carbon microphone became vital to *telephony* and later to *broadcasting* and *sound recording.* Hughes refused to patent his inventions and revealed it to the general public.

[303] **Charles Wheatstone** was first to use the word *microscope* (1827). **Hughes** (1878) revived the term in connection with his discovery that a loose contact in a circuit containing a battery and a telephone receiver would give rise to sounds in the receiver corresponding to vibrations impinged upon the diaphragm of the mouthpiece or transmitter.

1856 CE **Norman Robert Pogson** (d. 1891, England). Amateur astronomer. Proposed a quantitative scale of stellar magnitude that is now generally adopted.

1856 CE **Philipp Ludwig von Seidel** (1821–1896, Germany). Mathematician. Presented the earliest systematic treatment of third-order geometrical aberrations[304] which was extremely important in the design and construction of lens system in cameras and other optical instruments. [In earlier optical system such as telescopes, only those points and rays were considered which lie in the immediate neighborhood of the axis. The resulting theory is known as *Gaussian optics*].

Seidel entered the University of Berlin in 1840 and studied under **Dirichlet**. He moved to Königsberg where he studied under **Bessel**, **Jacobi** and **Franz Neumann**. He obtained his doctorate from Munich University (1846) and he went on to become a professor there.

1856 CE **Adolf Eugen Fick** (1829–1901, Germany). Physiologist. Developed fundamental laws of diffusion in living organisms. Professor at Zürich (1852–1868) and Würzburg (1868–1899).

Fick's law consists of the observation that for small concentration gradients, the diffusive flux $\boldsymbol{J}$ is proportional to the gradient of the concentration c [$\boldsymbol{J} = -D\,\mathrm{grad}\,c$, where D is the diffusion coefficient and the minus sign shows that the flow is in the direction from higher to lower concentration]. This law treats diffusion from the *macroscopic* point of view.

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[304] First order geometrical ray theory (Gaussian optics) is based on the assumption that the optical system is restricted to operate in an extremely narrow region about the optical axis — this is known as the *paraxial approximation.* Mathematically, one takes here $\sin\varphi \approx \varphi$. Obviously, if rays from the periphery of a lens are to be included in the formation of the image, the statement that $\sin\varphi \approx \varphi$ is unsatisfactory. Seidel retained the first *two* terms in the expansion

$$\sin\varphi = \varphi - \frac{\varphi^3}{3!} + \frac{\varphi^5}{5!} - \frac{\varphi^7}{7!} + \dots$$

which resulted in a *third* order theory. Departures from first-order theory are then embodied in the five *primary aberrations*: *spherical aberration*, *coma*, *astigmatism*, *field curvature* and *distortion* ('Seidel aberrations'). The difference between the results of exact ray tracing and the computed primary aberrations are the sum of all contributing *higher aberrations.* More refined treatment of aberrations is based on *physical optics* via *diffraction theory.* **K. Schwarzschild** (1905–1906) extended Seidel's analysis to include 5th order terms.

Fick was the first to put diffusion on a quantitative basis by adopting the mathematical equation of heat conduction derived some years earlier by **Fourier** (1822). Moreover, he recognized that the transfer of heat by conduction is due to *random molecular motion*. He was one of the first to actually experiment with *contact lenses* on animals and then, finally, fit contact lenses to human eyes.

1856 CE Discovery of the first Neanderthal remains in Germany.

1856–1901 CE **Pierre Eugène Marcellin Berthelot** (1827–1907, France). Chemist and politician. One of the most distinguished chemists of the 19^{th} century. His interests were extraordinarily wide and his work was highly original and of a fundamental character. First to synthesize organic compounds from inorganic ones[305].

By the synthetic production of numerous hydrocarbons, natural fats and sugars, he showed that organic compounds can be formed by ordinary methods of chemical manipulations, and obey the same laws as inorganic substances. He held that chemical phenomena are not governed by special laws peculiar to themselves, but rather are explicable in terms of the general laws of physics, and are in operation throughout the universe (1860). Discovered the *partition law*[306] for solubility of a substance in a mixture of liquids (1872).

Berthelot was born in Paris, the son of a doctor. After distinguishing himself at school in history and philosophy, he turned to the study of science. In 1851 he became a member of the staff of the College de France, and in 1865 became a professor of organic chemistry there. He was appointed inspector general of higher education in 1876, minister of public instruction (1886–1887), and held the portfolio of foreign affairs in the cabinet of 1895–1896.

1856 CE **Christoph Hendrik Diederik Buys-Ballot** (1817–1850, Holland). Meteorologist. Professor at Utrecht University (1847). Formulated the law determining the swirl direction of large storms and hurricanes[307] [*counterclockwise* in the Northern Hemisphere as viewed from

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[305] When he began his active career it was generally believed that on the whole, organic chemistry must remain an analytical science and could not become a synthetic one, because the formation of organic compounds required the intervention of *vital activity* of some form.

[306] The weights of dissolved substance per unit volume of each liquid are in constant ratio.

[307] Big storms in the atmosphere are usually centered on low-pressure areas. Independently, **William Ferrel**, in the United States (1859), gave a mathematical formulation of atmospheric motions on a rotating earth and applied his theory to the general circulation of both the atmosphere and the oceans.

above the North Pole (direction of earth's spin) and *clockwise* in the Southern Hemisphere – in accord with the *Coriolis effect* (1835)].

1857–1866 CE A *submarine telegraphic cable* was laid across the Atlantic Ocean from Ireland to Newfoundland. **Otway H. Berryman** made a line of soundings from the U.S.S. *Arctic* to verify the existence of a submarine ridge on which it was proposed to lay the telegraph cable (1856) [see Table 4.4]

1857–1880 CE **Joseph Wilson Swan** (1828–1914, England). Inventor. Notable for his achievements in photography, synthetic textiles and *electric lighting*. A rival of Edison.

Swan was born in Sunderland and after leaving school at 13 was apprenticed to a druggist. In 1846 he joined a pharmaceutical business in Newcastle which also manufactured photographic plates, and thus Swan was led to one of the advances in photography with which his name is associated – the production of extremely rapid dry plates based on his observation that heat increases the sensitiveness of a gelatino-bromide of silver emulsion (1857). The patent was bought by **George Eastman**, founder of Kodak, and helped make photography cheaper and thus widely popular.

He was one of the first (since 1848) to undertake the production of electric lamp in which light should be produced by the passage of an electric current through a *carbon filament*, but it was not until **Herman Sprengel** (1834–1906, Germany and England)developed his mechanized *air-pump* (1865) that it became possible to achieve the necessary vacuum in the bulb. A *scientific American* article of July 1879 mentioned how Swan had designed an incandescent lamp using carbon shaped into a form of a cylinder.[308]

In 1880 he established a small factory near Newcastle and within three years he was manufacturing 10,000 lamp bulbs a week. In 1883 he amalgamated his business with Edison to form the 'Ediswan' Electric Light Company.

[308] Edison admitted to have read this article. Swan defenders claim Edison stole this idea from Swan. Edison backers claim Edison read this article after he designed his own carbon filament. Apparently, neither of them was the sole inventor. More than 30 experimenters have been known to work on the perfection of incandescent electric lighting during 1802–1879 and carbon had been an ingredient of experimental light bulbs 50 years before Edison.

Table 4.6: Development of world-wide electromagnetic Telegraphy Communications (1831–1866)

1831	**Joseph Henry** (USA) sent an electric charge over 1.5 km of a single wire, where an electromagnet produces a force on a suspended permanent magnet that rang a bell.
1831	**C.F. Gauss** and **W. Weber** (Germany) built an electromagnetic telegraph that operated over a distance of 2 km. It used a mirror-galvanometer as a receiving device.
1836–1837	**John Daniell** and **Charles Wheatstone** improved the voltaic cell, creating a stable source of current.
1838	**Carl August von Steinheil** (1801–1870, Germany) discovered the possibility of using the *earth* for a return conductor in telegraphy (*grounding*). He also invented a telegraph system in which characters are printed on a paper ribbon.
1839	**William Oshaughnessy** (British) laid a telegraph cable which crossed the Ganges in India.
	Samuel Morse (USA) laid down a telegraph cable in the port of New York (Detected the use of an electromagnet for transmitting signals in 1837).
1844	**Samuel Morse** (USA) set up a 60 km telegraph line between Washington DC and Baltimore.
1846	New York City was linked with Washington DC.
1851, Dec 31	England and France combined efforts to lay the first submarine cable across the English Channell from Dover to Calais (ca 33 km).
1852	England was linked telegraphically to Ireland, Belgium, Holland, and Denmark.
1853	The Rhine was crossed at Worms.
1854	Turkey was linked with Crimea, across the Black Sea.

1855 **David Edward Hughes** (England and USA) invented a keyboard-telegraph with rotating type-wheel printer that grew into modern *telex* industry.

1855 The Mediterranean was crossed from Italy to Corsica, from Corsica to Sardinia and from Sardinia to Bône.

1856 Twelve companies in the USA combined to form the *Western Union Telegraph Company*.

1858, Aug 18 The first cablegram from America to Europe was sent across the Atlantic. It took 35 minutes to arrive. The text consisted of a congratulation of US President Buchanan to Queen Victoria. It read:

"Europe and America are united by telegraphy. Glory to God in the highest, on earth peace, goodwill toward man. It is a triumph more glorious because for more useful to mankind than was ever won by a conqueror on the field of battle."

The line remained in service for 23 days only when the cable broke. Silence fell over the Atlantic for the next 6 years.

1861, Oct 24 Transcontinental coast to coast telegraph line was established in the USA. That day, Stephen J. Field, chief justice of California, sent the first message to President Abraham Lincoln. It declared California's loyalty to the Union. The transcontinental telegraph ended the *pony express*, which had operated only about 19 months.

1865 A telegraph system between India and England. It took, on the average, 6 days to telegraph a message overland between the two countries.

1866, Sept 08 **Cyrus West Field** (1819–1892, USA) and **Lord Kelvin** (1824–1907, England) succeeded in laying a telegraph cable across the Atlantic Ocean. The Old and the New Worlds were joined together again, for "better or for worse".

Searching for a better filament for his bulbs, Swan dissolved cellulose in acetic acid and extruded it through narrow jets into a coagulating fluid. Soon afterwards **Chardonnet** (1839–1924, France) adopted this process to make *rayon*, and it was further developed by **Charles Cros** (1855–1935, England) and **Edward Beran** (1856–1921, England) who, in conjunction with **Samuel Courtaulds** (1893–1881, England) laid the foundation of the *synthetic textile industry*. Swan also made significant improvement to lead-plate batteries by designing cellular lead plates which held the lead oxide more securely.

Swan was a self-taught experimentalist and entrepreneur, much like Edison in the USA. He also elected FRS (1874), knighted (1904), and received many other honors.

1858 CE **Archibald Scott Couper** (1831–1892, Scotland). Chemist. Introduced the idea of the *valence bond* and drew the first structural formulas independently of **Kekulé** in Germany. Proposed the tetravalency of carbon and the ability of carbon atoms to bond with each other. Couper recognized two valencies of carbon, one in CO and one in CO_2. He also assumed that carbon atoms form the basic bone of organic compounds. Couper, who was only 27, had sent his contribution to his former teacher, **Charles Adolphe Würtz** (1817–1884) in Paris, to be presented to the French Academy of Sciences. But Würtz unaccountably neglected to do this. His work first appeared through the intervention of **Jean Baptist André Dumas** (1800–1884) a little less than a month after Kekulé's publication, and consequently Kekulé received most of the credit.

Couper's health, poor since childhood, thereafter failed, and after a nervous breakdown he made no further contributions to chemistry[309]. Kekulé, whose conceptions were not quite as close to modern ideas as Couper's, went on to develop the structure of benzene.

Couper was born in Kirkintillach, Scotland.

1858–1862 CE *Discovery of the sources of the Nile.* For thousands of years the people of Egypt revered the Nile as a sacred river. They did not know where it originated, nor what caused its annual flooding. But they did know that without it their civilization might never have come into being.

With a length of 6650 km from its farthest headstream to the Mediterranean, the Nile is the world's longest river. Its drainage basin, estimated at 3,349,000 km^2, includes parts of 9 countries and encompasses about $\frac{1}{10}$ of Africa's land area.

[309] H.C. Brown, *J. Chem. Educ.* **11** (1934) 331; **36** (1959) 104–110; O.T. Bently, *ibid.*, 319–320, and E.N. Hiebert, *ibid.*, 320–327.

Down through the ages the source of the great river was shrouded in mystery despite many efforts to discover it. The task was complicated, as it turned out, by the fact that the Nile has not one but three major sources, since its northward flow unites the waters of its longest branch, the so-called *White Nile*, with those of the *Blue Nile* and the smaller *Atbara*.

If the ancient Egyptians knew of the Blue Nile and its source, the knowledge was lost. It was not until 1615 that the Jesuit missionary **Pedro Paez** (1564–1622, Spain), working in the service of the Portuguese, visited the source of the Blue Nile in the Ethiopian highlands (h. 1830 m; 11 °N, 37 °E), southeast of Lake Tana. It was rediscovered by **James Bruce** (1730–1794, Scotland) in 1770.

The White Nile proved a more difficult problem. In 150 *CE*, the Greek astronomer Ptolemy placed its headwaters in a range called the *Mountains of the Moon* — a range that has since been identified as the Ruwenzori Mountains on the border between Uganda and Zaire. Although Ptolemy was not far from the truth, attempts to confirm his theory were unsuccessful.

In the early 1800's, European knew little more about Africa than the Phoenicians had known in 500 BCE. However, the impetus given to research and exploration in the prosperous Victorian era, made possible the organization of a series of British expeditions in an effort to unveil the last mysteries of the '*Dark Continent*'. During the second half of the 19^{th} century these expeditions finally discovered the river's ultimate headstream, the *Kagera River*, which rises in the present-day Burundi (h. 2130 m; 2 °20′S, 29 °20′E) and flows northeast 400 km into *Lake Victoria*. The overflow from the northern end of this lake, in turn, is the beginning of the White Nile proper.

Flowing northward through *Lake Kyoga*, the White Nile plunges 37 m over *Murchison Falls* and begins a rapid descent from the lake plateau to the low flat plains of southern Sudan. There the river slows drastically as it spreads out across a broad marshy area, and eventually after a northward journey of 800 km is joined by the Blue Nile at Khartoum. Although much shorter than the White Nile, with a length of 1370 km, the Blue Nile carries 63% of the Nile's total waters (annual average 10^9 m^3), being fed by summer rains on the Ethiopian highlands. It is this sudden influx of water that accounts for the annual flooding of the arid lower Nile Valley.

The discovery of the sources of the White Nile is a most exciting human drama: In January 1858 *Lake Tanganyika* was discovered by **Richard Burton** (1821–1890, England) and **John Hanning Speke** (1827–1864, England). On Aug. 03, 1858, Speke discovered *Lake Victoria Nyanza* (he did not know at that time that he had reached the head reservoir of the White Nile). On the 28^{th} of July 1862 Speke, at the head of a new expedition, stood by the *Ripon*

Falls, where the Nile issues from Lake Victoria. In his journey he discovered the *Kagera River*, now known as the most remote headstream of the Nile.

On March 14, 1864, **Samuel White Baker** (1821–1893, England) reached *Lake Albert Nyanza*. These discoveries virtually solved the Nile problem so far as the source of the main stream was concerned, but there remained much to be done before the hydrography of the whole Nile basin was made known.

The project was achieved in two steps: During 1874–1889 **Henry Morton Stanley** (1841–1904, Wales) filled in the gaps left by Speke and Baker, exploring the *Kagera*, *Lake Kyoga*, *Lake Albert Nyanza*, and the '*Mountains of the Moon*'. Between 1891–1908, British and German teams made accurate surveys of the entire source region.

Stellar Brightness, Magnitudes, Luminosities, and Spectra

Hipparchos *(ca 150 BCE) and* ***Ptolemy*** *(ca 150 CE) used a scale of magnitude to indicate the* apparent brightness *of stars on their charts. The notion of brightness is based on the amounts of light energy, or* luminous flux *(erg per sec per cm^2) received from stars, which are among the most important and fundamental observational data of astronomy. It is used in estimating the distances and the actual output of stellar energy.*

By 1856*, stellar photometry had developed to such a degree that accurate magnitudes could be determined by visual methods. Photography was adapted to astronomy at about the same time.* ***John Frederick William Herschel***[310] *(1792–1871, England) and* ***N.R. Pogson*** *noticed that an average first-magnitude star was about* 100 *times brighter than a star of* 6^{th} *magnitude, i.e., it delivers to earth somewhat more than* 100 *times as much*

[310] His father, **William Herschel**, devised a simple method to measure the relative intensity of starlight (photometry): let two stars appear with different brightness in the field of view of a telescope. They can be made to appear equally bright, when viewed one at a time, by adjusting the aperture size for either of them. The ratio of the relative light *aperture areas* then serves as an approximate measure of the relative light intensities arriving from the two stars.

light as a star that is just barely visible on a dark night. Therefore, a difference of five magnitudes corresponds to a luminous flux ratio of 100: 1.

Since the physiology of sense perception is such that equal differences of brightness correspond to equal ratios *of light flux energies, Pogson suggested that the ratio of light flux corresponding to a step of one magnitude be* $\sqrt[5]{100} = 2.512$. *By assigning a magnitude* 1.0 *to the bright stars Aldebaran and Altair, Pogson's new scale gave magnitudes that agreed roughly with those in current use at the time. Thus, for each difference of ca* 5 *magnitudes, the ratio of brightness increases (or decreases) by a factor of* 100. *In general, if* m_1 *and* m_2 *are the magnitudes corresponding to stars from which we receive visible light flux in the amounts* ℓ_1 *and* ℓ_2, *the difference between* m_1 *and* m_2 *is* $m_1 - m_2 = 2.5 \log_{10} \frac{\ell_2}{\ell_1}$.

The star possessing the highest apparent brightness, Sirius, sends us about ten times as much light as the average star of the first magnitude and so it has the magnitude $1.0 - 2.5 = -1.5$. *Venus, the brightest planet, is of magnitude* -4. *The sun, with a magnitude of* -26.5, *sends us* 10^{10} *as much light energy as Sirius; and we also receive* 10^{10} *times as much light from Sirius as from the faintest star that can be photographed with a* 5 *meter telescope. Magnitudes are determined by eye estimates (*visual*), by blue-sensitive photographic plates (*photographic *magnitude) and with photo cells (*photoelectric *magnitude).*

Since the stars are not all at the same distance from the sun, it is desirable to calculate all magnitudes as if *all were at the same distance. The term* absolute magnitude *means that the magnitude of a star is calculated for a standard distance of* 10 *parsecs (32.6 light-years), assuming the light intensity to vary inversely with the square of the distance [the absolute magnitude of the sun is about* $+5$*]. The extreme range for absolute magnitudes observed for normal stars in* -10 *to* $+19$, *a range of a factor of more than* 10^{11} *in intrinsic light output* $[2.512^{29} = 10^{11.6}]$.

The difference between the star's apparent magnitude m *and its absolute magnitude* M *is a measure of its distance*[311]. *The light energy-flux arriving at the earth is called a star's* brightness *and is usually expressed in erg per* cm^2 *per sec. The eye receives* 2.512 *times more energy per* cm^2 *per sec from a* 3^{rd}*-magnitude star than from a* 4^{th} *magnitude star.*

In determining a star's absolute magnitude, astronomers must make allowance for non-visible radiation and for the absorption and scattering of light in the atmosphere. The apparent magnitude of a star that we see in the sky

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311 From the definition of magnitudes $m - M = 2.5 \log_{10} \frac{\ell(10)}{\ell(r)}$, where r is the *actual* distance in parsecs. Combining this with the inverse-square law $\frac{\ell(10)}{\ell(r)} = \left(\frac{r}{10}\right)^2$, we obtain $m - M = 5 \log_{10} \frac{r}{10}$.

could be misleading if the star happens to emit a significant fraction of its radiation at non-visible wavelengths [e.g., a very luminous and hot star with surface temperature of $35,000$ K appears deceptively dim to our eyes simply because most of the star's radiation is emitted at ultraviolet wavelengths. Furthermore, the earth's atmosphere is opaque to many non-visible wavelengths, and thus a sizable fraction of the radiation from the hottest stars and the coolest stars simply does not penetrate the air to get at our eyes or telescopes].

To cope with this difficulty, astronomers have defined the bolometric magnitude *of a star as the star's apparent magnitude measured* above *the earth's atmosphere and over* all *wavelengths. In recent years, satellites have allowed us to determine the bolometric magnitude of many stars. The absolute magnitude deduced from the bolometric magnitude is called the* absolute bolometric magnitude *(M_{bol}) of a star and is always smaller than the star's absolute visual magnitude (M) deduced from ground-based observations at visible wavelengths alone, per fixed* distance *estimate.*

By comparing satellite and ground-based data, astronomers have figured out how much they must subtract from a star's absolute visual magnitude to get its absolute bolometric magnitude. This correction is called the bolometric correction *(BC). The* luminosity *(L) of a star[312] is its total energy output in units of erg·sec^{-1}. The star's absolute bolometric magnitude is directly related to its luminosity[313], assuming the correct distance was used in computing the former.*

A discovery by ***P.A. Secchi*** *in 1863, opened the field of* stellar spectroscopy*: in addition to its magnitudes and luminosity, a star could be characterized by its* spectrum*, or* spectral type*. The physical interpretation of this phenomenon became available only in the early 1900's, when Niels Bohr explained the structure of the hydrogen atom[314].*

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312 For the *sun*: absolute bolometric magnitude = +4.75; absolute visual magnitude = +4.85; $L_\odot = 3.90 \times 10^{33}$ erg·sec^{-1}; energy flux = $E_\odot = \frac{L_\odot}{4\pi R_\odot^2} = 6.41 \times 10^{10}$ erg·cm^{-2}·sec^{-1}. Using the Stefan-Boltzmann law, we find for the sun's surface temperature $T_\odot = \left\{\frac{E_\odot}{\sigma}\right\}^{1/4} = 5800$ K.

313 $M_{\text{bol}} = 4.75 - 2.5\log_{10}\left(\frac{L}{L_\odot}\right)$. Knowing the star's M_{bol} and using this equation, astronomers can calculate how much energy is released from the stars surface each second.

314 Each dark line in a stellar spectrum is due to the presence of a particular chemical element in the atmosphere of the star observed. The differences in both continuous stellar spectra and their elemental absorption lines are due to the widely differing *temperatures* in the outer layers of the various stars. Hydrogen, for example, is by far the most abundant element in all stars, but hydrogen lines

1858–1866 CE **Max Johann Sigismund Schultze** (1825–1874, Germany). Anatomist, zoologist and cytologist. Altered the conception of the cell (1861); advanced the correct theory of retinal function (1866); demonstrated minute nerve endings in the ear (1858) and nose (1863); introduced new techniques in histology.

Schultze was born at Freiburg (Baden). He studied medicine in Berlin; Professor at Halle (1854–1859) and Bonn (1859–1874).

Schultze recognized the *protoplasm* with its nucleus as the fundamental common substance for all forms of life (emphasizing the living protoplasm and *not* the membrane). Advanced the *duplexity theory* of retinal function by identifying the *retinal cones* as color receptors and the *rods* for night vision.

1858–1872 CE **Siegfried Heinrich Aronhold** (1819–1884, Germany) and **Rudolph Friedrich Alfred Clebsch** (1833–1872, Germany). Mathematicians. Developed independently a consistent symbolism in invariant theory: a method for systematic investigation of algebraic invariants. We now recognize in this symbolism as well as in Hamilton's vectors, Grassmann's 'gap' products and Gibbs' dyadics — special aspects of *tensor algebra*.

Clebsch was born at Königsberg in Prussia, and studied at the university of that town. During 1858–1863 he held the chair of theoretical mechanics at the Polytechnicum in Carlsruhe. In 1863 he accepted a position at the University of Giessen. In 1868 he went to Göttingen, and remained there until his death. He worked successively in the fields of mathematical physics, the calculus of variations, partial differential equations of the first order, the general theory of curves and surfaces, the theory of invariants and Abelian functions. He introduced the topological concept of a *genus* of a curve.

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do not necessarily show up in a star's spectrum: if the star is much hotter than 10,000 K, high energy photons pouring out of the star's interior easily knock electrons out of the hydrogen atoms in the star's outer layers, ionizing the gas. The hydrogen ions have no electrons in their lower energy levels to absorb photons and produce Balmer lines. Conversely, if the star is much cooler than 10,000 K, the majority of photons escaping from the star do not posses enough energy to boost many electrons up from the ground state of the hydrogen atoms. These unexcited atoms also fail to produce Balmer lines. A prominent set of Balmer lines is a clear indication that the star's surface temperature is about 10,000 K. Only then is an appreciable number of atoms excited to the second energy level, from which they can absorb additional photons and rise to still higher levels of excitation. These photons correspond to the wavelengths of the Balmer series, which is the part of the spectrum that is readily observable.

Aronhold was born in Angerburg to Jewish parents, and died in Berlin. He was educated at the University of Königsberg (1841–1845) under **Bessel**, **Jacobi**, **Hesse** and **F. Neumann** and in Berlin under **Dirichlet** and **Steiner**. From 1852 to 1854 Aronhold taught at the Artillery and Engineer's School at Berlin. He also taught at the Royal Academy of Architecture at Berlin from 1851. Aronhold was appointed professor at the Royal Academy of Arts and Crafts. In 1864 he became a professor at the Berlin Royal Academy of Architecture.

1858–1875 CE **Joseph William Bazalgette** (1819–1891, England). Civil Engineer. Planned and constructed London's main drainage system and Thames embankment (1860–1874). It consisted of 130 km of large intersecting sewers, draining more than 250 km^2 of buildings, and calculated to deal with some 1.7 million m^3 a day. The cost was 4.6 million pounds.

As late as 1850, towns and cities were plagued by three problems: dispersing of the *rain-water* which might cause floods, of the miscellaneous *rubbish* which in the course of times would make the streets impassable, and of the decomposing of *organic matter* which was not merely an offensive nuisance but a grave danger to health.

Since the Great Fire of London (1666), dumping places for rubbish has been officially provided in the streets of the City, from which refuse was removed by a paid staff of 'rakers'. The content of the privies were removed by 'night-soil men' at times when the streets were deserted. Much was sold for agricultural uses and the rest was tipped into Thames. By the end of the 18th century a town like London, with more than a million inhabitants, was driven to attempt a number of solutions, all of which proved increasingly inadequate. One of those was the *water-closet*: when flushed, discharged the content directly to a cesspool (in the basement or under the garden) which was emptied at something like annual intervals. But the cesspool constituted a double danger to health, from the effluvia which commonly entered the house and from the leakage which tainted wells, rivers and water-pipes.

By 1840 the situation in London became horrible: evil-smelling mudbanks proclaimed the fact that the river fleet and the other rainwater sewers were discharging vast quantities of household effluent into the Thames, to be carried to and for on the tide even in the heart of the capital. In 1855, after 20,000 Londoners had died in two *cholera epidemics*, a Metropolitan Board of Works was set up, with **Bazalgette** as its chief engineer.

Bazalgette built five main sewers running parallel to the course of the Thames, three on the north bank and two on the south, which would be capable of dealing with all household sewage, together with the normal flow of rainwater. At first all sewage was discharged into the Thames. Later, however, chemical clarification of the river waters was established.

1858–1882 CE **Friedrich August Kekulé** (von Stradonitz, 1829–1896, Germany). Distinguished organic chemist. Made far-reaching contributions to chemical theory, especially in regard to the constitution of *carbon* compounds. Established, simultaneously with **Couper**, the 4-valence[315] of carbon (1858) and the fact that carbon atoms can chemically combine with one another to form *chains*. First to perceive the correct structure of *Benzene*.

Kekulé drew chemical structural formulae in which he represented each atom in the molecule as possessing a number of hooks, or *bonds* equal to its valence, and then wrote those bonds into a formula so that the atoms seemed held together in tinker-toy fashion. For over a century now, chemists have been able to use the Kekulé system as a guide to the possible structure of new compounds and to the number of *isomers* possible in a given case. The system has been greatly refined and made at once more complicated and more flexible, but its main outline still stands.

Kekulé was born in Darmstadt. While studying architecture at Giessen he came under the influence of **Liebig** and was induced to take up chemistry. From Giessen he went to Paris, and then visited England. On his return to Germany he started a small chemical laboratory at Heidelberg and in 1858 was appointed professor of chemistry at Ghent. In 1865 he was called to Bonn to fill a similar position.

1858–1901 CE **Peter Guthrie Tait** (1831–1901, Scotland). Physicist. Creator of new methods in quaternion analysis, many of which were later transferred to vector analysis. Changed the emphasis in quaternion analysis towards its usefulness as a tool for physical science. Extended the ∇ (Nabla) operator to vector fields, and developed it as a fundamental tool in modern vector analysis.

Tait did important work on the 'Four-Color' problem[316], and wrote an analysis of the physics of golf balls in flight!

He refused fellowship in the Royal Society of London, declaring that a fellowship in the Royal Society of Edinburgh was quite good enough for him.

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[315] The name *valence* (from a Latin word meaning *power*) was suggested in 1852 by the chemist **Edward Frankland** (1825–1899, England).

[316] For further reading, see:

- Wilson, R., *Four Colors Suffice*, Princeton University Press, 2005, 262 pp.

History of Algebraic Equations[317]

The solution of polynomial equations continued to occupy the center stage in the algebra of the early 19th century. In the previous century, the efforts of **Euler**, **Vandermonde**, **Lagrange** *and* **Ruffini** *to solve algebraically general equations of degree greater than 4 came to nought. Indeed, the quintic equation boggled the minds of the finest mathematicians of Europe for about 300 years!*

So ended also the efforts of mathematicians to furnish a general algebraic solution to the binomial equation $x^n - 1 = 0$. ***Gauss****, however, opened (1801) the 18th century with the algebraic solvability of the cyclometric equation* $x^p - 1 = 0$ *(where* p *is a prime*[318]*), which is the equation of the division of the circle into* p *equal parts. The latter refers to the fact that the roots of this equation are (by de Moivre's theorem)*

$$x_k = \cos(k\frac{2\pi}{p}) + i\sin(k\frac{2\pi}{p}), \quad k = 1, 2, \dots, p$$

and the complex numbers x_k*, when plotted geometrically, are the vertices of a* p*-sided regular polygon that lie on the unit circle.*

Gauss then showed that the cyclometric equation is solvable in radicals if and only if $p = 1 + 2^{(2^n)}$ *[Fermat's Number], namely*

$$p = 3,\ 5,\ 17,\ 257,\ 65537, \dots.$$

For an arbitrary n*, Gauss proved that* $x^n - 1 = 0$ *is solvable in radicals if and only if* $n = 2^{\alpha} p_1 p_2 p_3 \dots p_n$ *where all prime factors are distinct Fermat's*

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317 For further reading, see:

- Dehn, E., *Algebraic equations*, Dover Publications Inc: New York, 1960, 208 pp.

318 The case of p prime takes care of $x^n - 1 = 0$. For if $n = pq$, let $y = x^q$ and the problem reduces to $y^p - 1 = 0$ which is solvable. Moreover $x^q =$ const. can be solved if q is a prime, and if not, q can be decomposed in the same manner that n was.

Numbers and α is a positive integer or zero. Clearly this implies that a regular polygon with n sides can be constructed with compass and ruler only when n is of the above form.

***Gauss** (1801) proved the Fundamental theorem of Algebra stating that every polynomial equation of degree n,*

$$f(x) = x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_{n-1}x + a_n = 0$$

in which the a_i ($i = 1, 2, \ldots, n$) are real or complex numbers, has at least one solution in the domain of complex numbers. If this solution is denoted by x_1, one finds that $f(x) - f(x_1) = (x - x_1)[x^{n-1} + \cdots + a_{n-1}] = 0$. By the fundamental theorem, the expression in the square brackets also has a solution, say x_2. If this process is iterated, one obtains a product representation of $f(x)$, from which it follows that an equation of degree n has exactly n roots, which need not necessarily be distinct from each other.

*The question of the solution of general equations of degree higher than four was settled by **Abel** (1826). He first proved the theorem:*

"The roots of the equation *solvable by radicals* can be given such a form that each of the radicals occurring in the expression for the roots is expressible as a *rational function* of the roots of the equation and certain roots of unity".

Abel then used this theorem to prove the impossibility of solving by radicals the general equation of degree greater than four.

In 1828, Abel discussed the general solution of the cyclometric equation. One of the reasons why it was so difficult to arrive at an understanding of the solvability of equations of higher degree, was the fact that special equations of higher degree can be solvable by radicals.

In particular, there are two classes of equations which can be solved; The first are those in which the polynomial can be written as a product of polynomials of lower degrees. The second class are those whose polynomial can be decomposed into powers of a polynomial of a lesser degree.

Even for cubic and quadratic equations, the results can be extremely complicated. If the parameters in equations like these are symbolic; there can also be some subtlety in what the solutions mean: the results you get by substituting specific values for the symbolic parameters into the solution may not be the same as what you get by doing the substitutions in the original equation.

An example of an equation solved by radicals is

$$x^6 - 9x^4 - 4x^3 + 27x^2 - 36x - 23 = 0,$$

where one solution is $x = \sqrt[3]{2} + \sqrt{3}$.

In another example, the equation

$$x^5 + 20x + 32 = 0$$

has a root

$$\begin{aligned} x_1 = \frac{1}{5}\Bigg(& -\sqrt[5]{2500\sqrt{5} + 250\sqrt{50 - 10\sqrt{5}} - 750\sqrt{50 + 10\sqrt{5}}} \\ & - \sqrt[5]{-2500\sqrt{5} + 750\sqrt{50 - 10\sqrt{5}} + 250\sqrt{50 + 10\sqrt{5}}} \\ & + \sqrt[5]{2500\sqrt{5} + 750\sqrt{50 - 10\sqrt{5}} + 250\sqrt{50 + 10\sqrt{5}}} \\ & - \sqrt[5]{2500\sqrt{5} - 250\sqrt{50 - 10\sqrt{5}} + 750\sqrt{50 + 10\sqrt{5}}}\Bigg). \end{aligned}$$

Indeed, it was shown (1885) by ***J.S.C. Glashan*** *(1844–1932),* ***G.P. Young*** *(1819–1889), and* ***C. Runge*** *(1856–1927) that all irreducible solvable quintics, with the quadratic, cubic and quartic terms missing, have the following form, with* ν *and* μ *rational*

$$x^5 + \frac{5\mu^4(4\nu + 3)}{\nu^2 + 1}x + \frac{4\mu^5(2\nu + 1)(4\nu + 3)}{\nu^2 + 1} = 0.$$

The previous quintic example is a special case for $\mu = 1$, $\nu = \frac{1}{2}$.

Thus, the next step after Abel's work was to discover general criteria *for the solvability of algebraic equations with* $n > 4$. *This task fell to* ***Galois*** *(1830–1), using a novel approach that revolutionalized mathematics. An important point to be emphasized is that "algebraic" solution requires expression in a* finite *number of arithmetic steps. Solution of general equations of degree higher than 4 is possible if an* infinite *number of steps is permitted. But such solutions are nonalgebraic, and are sometimes expressed in terms of special non-algebraic (transcendental) functions. One may think of such functions as formulated by the* infinite series *so important in analysis, and in this way realize that an infinite number of arithmetic steps is involved.*

Now, ***Girard*** *(1629) had already shown that trigonometric functions (which are nonalgebraic or transcendental functions) are effective in obtaining solutions when the Cardano's formula yields irreducible results. Therefore, mathematicians after Galois' day conceived the idea that the* elliptic functions, *which generalize ordinary trigonometric functions, might offer a means of expressing solutions of some higher-degree equations that are not* *solvable algebraically. Hence,* ***Charles Hermite*** *(1858) succeeded in solving the general quintic equation ($n = 5$) in terms of* elliptic modular functions.

Another part of Galois' ideas is his theory of fields, which is needed to clarify the notion of rational functions. *The idea of a field was introduced by* ***Abel*** *(1826). By a field of numbers he meant (like Galois) a collection of numbers such that the sum, difference, product, and quotient of any two numbers in the collection (except division by zero) are also in the collection. Hence, rational numbers, real numbers and complex numbers form fields. A polynomial is said to be* reducible *in a field (usually the field to which its coefficients belong) if it can be expressed as the product of two lower-degree polynomials over the field.*

After Galois, ***Bravais*** *(1849),* ***Cayley*** *(1849),* ***Jordan*** *(1869),* ***Sylow*** *(1872),* ***Sophus Lie*** *(1874),* ***Frobenius*** *(1887) and* ***Hölder*** *(1889) continued researches in the theory of groups. With them the study of groups assumed its abstract form (independent of the solution of algebraic equations), and developed at a rapid pace. The notion of group came to play a major role in geometry, and in algebra it became an important factor in the 20th century rise of* abstract algebra.

The technique of solving the quintic equation (and higher order equations) by means of transcendental functions (especially elliptic modular functions*) was perfected over a period of some 200 years. We shall next give a brief survey of these efforts since* ***E.S. Bring's*** *reduction of the general quintic (1786) into a canonical form.*

George Birch Jerrard *graduated at Trinity College (1827) and heard about* ***E.S. Bring's*** *Lund publication (1786) only in 1861. Both achieved reduction of the quintic be means of the* ***Tschirnhausen*** *substitution (1683) method, through which the reduction is effected by the extraction of only square and cubic roots. However, Jerrard found a* single *Tschirnhausen transformation that converts an nth degree polynomial equation ($n \geq 5$) into an nth degree polynomial equation in y in which the coefficients of* $y^{n-1}, y^{n-2}, y^{n-3}$ are all zero.

Thus, the general quintic

$$x^5 + a_1x^4 + a_2x^3 + a_3x^2 + a_4x + a_5 = 0$$

can be transformed, via a single Tschirnhausen transformation, into the ***Bring-Jerrard*** *form*[319] $y^5 - A_4y + A_5 = 0.$

[319] This simplification has, however, a 'price': The evaluation of (A_4, A_5) from $\{a_1, a_2, a_3, a_4, a_5\}$ requires the solution of three quadratic equations and one cubic equation. The final result of the general case is therefore quite messy.

Starting from the Bring-Jerrard canonical form, mathematician of the second half of the 19th century concentrated their efforts on infinite series solution of the quintic[320]. ***Chebyshev*** *(1838),* ***Eisenstein*** *(1844) and others considered the function $y(x) = x^5 - x - \rho$ together with its inverse $x = x(y, \rho)$.*

The formal series expansion of $x(y;\rho)$ is then calculated for small y values, yielding at $y = 0$,

$$x(0;\rho) = -\rho - \rho^5 - 5\rho^9 - 35\rho^{13} - 285\rho^{17} - 2530\rho^{21} \\ - 23,751\rho^{25} - 231,880\rho^{29} + O(\rho^{33}).$$

These are the first 8 terms of a series having the general term $\{\frac{-\rho^{4k+1}}{4k+1}\binom{5k}{k}\}$. This series can be summed using hypergeometric functions; one of the roots of the quintic is

$$x_1(\rho) = -\rho\ {}_4F_3\left(\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}; \frac{1}{2}, \frac{3}{4}, \frac{5}{4}; \frac{3125}{256}\rho^4\right). \quad (1)$$

In principle this method will work for any polynomial.

During 1860–1862, ***James Cockle*** *(1819–1895) and* ***Robert Harley*** *(1828–1910) developed a method for solving algebraic equations of the type $x^p + bx^q + \rho = 0$ based on differential equations. They showed that these equations have roots that can be represented in terms of hypergeometric functions of one variable. In particular they solved the quintic equation $x^5 - x - \rho = 0$.*

Their idea was to consider the function $x(\rho)$ and derive for it a linear differential equation based on the algebraic equation. Assuming the form:

$$a_1\frac{d^4x}{d\rho^4} + a_2\frac{d^3x}{d\rho^3} + a_3\frac{dx^2}{d\rho^2} + a_4\frac{dx}{d\rho} + a_5x + a_6 = 0,$$

the repeated differentiation of the original algebraic equation w.r.t. ρ leads to a system of 5 linear equations in the a_j, yielding

$$\begin{aligned} &a_1 = (256 - 3125\rho^4)/1155, && a_2 = -6250\rho^3/231, && a_3 = -4875\rho^2/77, \\ &a_4 = -2125\rho/77, && a_5 = 1, && a_6 = 0. \end{aligned}$$

The general solution of the differential equation is then a linear combination of four independent solutions with as yet unknown four parameters. These

For this reason, mathematician in the second half of the 19th century devised various ingenious 'bypasses' of the Tschirnhausen transformation.

320 **Lambert** (1757) was first to suggest (based on the ideas of Girard in 1629) a solution based on infinite series.

in turn are found by substituting the known general solution back into the quintic equation, and expand it about $\rho = 0$.

Collecting terms in ρ *and setting the coefficients equal to zero, there results a system of 4 linear equations for the four parameters. This completes the procedure of deriving the five roots of the given quintic. The explicit form of one of them is the same as (1).*

Hermite *(1858) produced an elliptic function solution of the quintic equation by combining previous ideas of* ***Galois***[321] *with the* ***Bring-Jerrard*** *solution. Here indeed, in the case* $n = 5$, *the modular equation of order 6 depends on an equation of order 5. Conversely, the general quintic equation may be made to depend upon this modular equation of order 6. Thus, assuming the solution of this modular equation, Hermite was able to solve (not by radicals) the general quintic equation, analogously to the trigonometric solution of the cubic equation.*

Hermite's explicit solution of the Bring-Jerrard reduced equation $x^5 - x - a = 0$ *is given in the form:* $x_j = \frac{1}{\lambda} z_j$, $j = 1, 2, 3, 4, 5$, *where*

$$\lambda = 2 \cdot 5^{3/4} \cdot p^{1/4} \cdot (1 - p^2)^{1/2}, \quad p = \tan \frac{\alpha}{4}, \quad \sin \alpha = \frac{16}{\sqrt{5^5 - a}}$$

$$z_1 = (v_1 - v_2)(v_2 - v_6)(v_4 - v_5)$$

$$z_2 = (v_2 - v_4)(v_3 - v_1)(v_5 - v_6)$$

$$z_3 = (v_3 - v_2)(v_4 - v_3)(v_6 - v_1)$$

$$z_4 = (v_4 - v_2)(v_5 - v_4)(v_1 - v_3)$$

$$z_5 = (v_5 - v_2)(v_1 - v_4)(v_3 - v_4)$$

$$v_m = p^{5/4} \mathrm{sn}(K - 4\omega_m)\, \mathrm{sn}(K - 8\omega_m), \qquad m = 1, 2, 3, 4, 5, 6$$

$\mathrm{sn}\, u =$ Jacobi's elliptic function

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321 Among other results demonstrated and announced by Galois may be mentioned those relating to the modular equations of the theory of elliptic functions (derived by Jacobi in 1829): for the transformations of order 5, 7, 11, the modular equations of orders 6, 8, 12 are reducible to the orders 5, 7, 11 respectively, but for n prime and > 11, the reduction is not possible.

$$K = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - p^2 \sin^2 \theta}}, \qquad K' = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - p_1^2 \sin^2 \theta}}, \qquad p^2 + p_1^2 = 1$$

$$\omega_1 = \frac{K}{5}, \qquad \omega_2 = i\frac{K}{5}, \qquad \omega_3 = \frac{1}{5}(K + iK'),$$
$$\omega_4 = \frac{1}{5}(K + 2iK'), \quad \omega_5 = \frac{1}{5}(K + 3iK'), \quad \omega_6 = \frac{1}{5}(K + 4iK')$$

In 1877, ***Klein*** *published Lectures on the Icosahedron and the Solution of Equation of the Fifth Degree. In this book and a later article he gave a complete solution of the quintic in terms of ratios of hypergeometric functions.*

Klein used a Tschirnhausen transformation to reduce the general quintic to the form

$$z^5 + 5az^2 + 5bz + c = 0.$$

He found the solution of this reduced quintic by first solving the related icosahedral equation

$$z^5(z^{10} + 11z^5 - 1)^5 - \lambda[z^{30} - 10005(z^{20} + z^{10}) + 522(z^{25} - z^5) + 1]^2 = 0,$$

where λ *can be expressed in radicals in terms of* $\{a, b, c\}$*. A solution of the icosahedral equation using hypergeometric function is*

$$z = \frac{\lambda^{-1/60}\ {}_2F_1(-\frac{1}{60}, \frac{29}{60}; \frac{4}{5}; 1728\lambda)}{\lambda^{11/60}\ {}_2F_1(\frac{11}{60}, \frac{41}{60}; \frac{6}{5}; 1728\lambda)}.$$

Gordan *(1878) described an alternative method, avoiding the difficult Tschirnhaus transformation to the quintic form.* ***Brioschi*** *(1858) found a simpler Tschirnhausen transformation that takes the general quintic into the same form*

$$z^5 + 5az^2 + 5bz + c = 0$$

and requires only a single square root. *Another Tschirnhaus transformation of the same kind yields the* ***Brioschi quintic***

$$u^5 - 10\lambda u^3 + 45\lambda^2 u - \lambda^2 = 0$$

which depends on a simple parameter λ*.* ***Kiepert*** *(1878) transformed the Brioschi equation into the Jacobi Sextic equation*

$$s^6 + \frac{10}{\Delta}s^3 - \frac{12g_2}{\Delta^2}s + \frac{5}{\Delta^2} = 0,$$

$$\Delta = -\frac{1}{5}, \qquad g_2 = \frac{1}{12}\sqrt[3]{\frac{1-1728\lambda}{\lambda^2}}.$$

This sextic is then solvable with Weierstrass Elliptic functions.

In 1884 ***F. von Lindemann*** *expressed the roots of an* arbitrary *polynomial in terms of theta functions.* ***Birkeland*** *(1905) showed that the roots of an algebraic equation can be expressed using hypergeometric functions of several variables.* ***Mellin*** *(1915) solved an arbitrary polynomial equation with the aid of Mellin integrals.*

One could assume that the quest for new solutions of the quintic would slow down after the beautiful results given by Hermite and Klein, but this was not to be: a new generation of mathematicians asked for more, and the 300-year old race continued.

Carl Woldemar Heymann *(1855–1910) solved trinomial equations using integrals. In the same vein,* ***Robert Hjalmar Mellin*** *(1854–1933) solved an arbitrary polynomial equation, using his Mellin transform. Finally,* ***Paul Emile Appell*** *(1855–1930) and* ***Joseph Marie Kampe de Fériet*** *(1893–1982) recognized (1926) the hypergeometric functions in the series solution of the quintic.*

Table 4.7: MILESTONES IN THE HISTORY OF ALGEBRAIC EQUATIONS

ca 2000 BCE

• Babylonians solve quadratic in radicals.

ca 300 BCE

• **Euclid** demonstrates a geometrical construction for solving a quadratic.

1515

• **Scipione del Ferro** (University of Bologna, Italy) gave an algebraic closed-form solution of the cubic equation $x^3 + px = q$, probably basing his work on earlier Arabic sources.

1579

• **Francois Viete** (France) gave a trigonometric solution for the 'irreducible' case of the cubic equation.

1669

• **Newton** introduced his iterative method for the numerical approximation of roots.

1757

• **Johann Heinrich Lambert** gave infinite series solutions of the trinomial equation $x^m + x + r = 0$.

1767

• **Lagrange** expressed the real roots of a polynomial equation in terms of a continued fraction. He showed (1770) that algebraic equation of degree five or more cannot be solved by the methods used for quadratics, cubics and quartics.

1799

• **Paolo Ruffini** gave an incomplete proof of the unsolvability of the quintic equation by means of algebraic functions of the coefficients. **Abel** (1826) gave a complete proof.

• **Gauss** proved the fundamental theory of algebra. In 1801 he solved the cyclotomic equation $x^{17} = 1$ in square roots.

1832

• **Galois** discovered the connection between solutions of algebraic equations and group theory, and showed that the general equation of degree $n > 4$ is not solvable in radicals.

1858

• **Hermite**, **Kronecker** and **Brioschi** independently solved a general quintic in terms of elliptic modular functions. Earlier studies of **Jacobi** (1829) of modular equation (for elliptic functions)

$$u^6 + v^6 + 5u^2v^2(u^2 - v^2) + 4uv(1 - u^4v^4) = 0$$

is fundamental for the Hermite solution.

1870
- **Camille Jordan** showed that algebraic equations of *any* degree can be solved in terms of the modular functions.

1877
- **Felix Klein** solved the icosahedral equation in terms of hypergeometric functions, thus rendering a closed-form solution of a principal quintic.

1884–1892
- **Ferdinand von Lindemann** expressed the roots of an arbitrary polynomial in terms of theta functions.

1895
- **Emory McClintock** gave series solutions for all the roots of a polynomial.

1905
- **R. Birkeland** showed that the roots of algebraic equation can be expressed using hypergeometric functions in several variables.

1915
- **Mellin** solved an arbitrary equation with the aid of Mellin integrals.

“*From so simple a beginning, endless forms most beautiful and most wonderful have been, and are being evolved*”.

Charles Darwin, in the final words of ‘*The Origin of the Species*’ (1859)

“*God created men because he was disappointed in the monkey*”.

Mark Twain

1858–1871 CE **Charles Robert Darwin** (1809–1882, England). A British naturalist whose theory of evolution through natural selection caused a revolution in the biological sciences, and had strong impact on natural philosophy and all of the sciences — especially geoscience, astronomy, chemistry, linguistics and anthropology. His book “*On the Origin of the Species*[322] *by Means of Natural Selection, or the Preservation of Favoured Races in the Struggle for Life*” (1859), gave facts on which he based his concept of gradual changes of plants and animals.

According to Darwin, *biological evolution* is the process whereby new species arise from earlier by accumulated changes. As this process of *speciation* proceeds with time, increasing number of species appear, becoming increasingly different.

After many years of careful study, Darwin attempted to show that higher species had come into existence as a result of gradual transformation of lower species and that the process of transformation could be explained through the selective effect of the natural environment upon organisms. He thus concluded that the principles of *natural selection* and *survival of the fittest*[323] govern all life. According to Darwin, nature does not optimize the good of the species, only the good of the individual, upon which natural selection acts. In 1871 Darwin published *The Descent of Man*, outlining his theory that man came from the same group of animals as the chimpanzee and other apes.

Darwin was born in Shrewsbury and was educated at the Universities of Edinburgh and Cambridge. Soon after his graduation at the age of 22 he sailed

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[322] *Species* can be defined as a population which reduces all the individuals that could mate or are likely to mate with one another.

[323] This prong of Darwin’s theory is a tautology: who are the fittest? The ones that survive! So this is a circular statement: Through the process of natural selection, the “fittest” survive.

aboard *H.M.S. Beagle*[324] on a 5-year cruise (1831–1836) around the world's oceans. During an exploratory voyage along the coast of South America, the Galapagos[325] and other Islands of the Pacific, Darwin searched for fossils and studies plants, animals and geology.

Destined for the church, Darwin was happily prepared to champion the Book of Genesis. But everything he encountered on the voyage — from the primitive people of Tierra del Fuego to the famous finches of the Galapagos Islands, from earthquakes and eruptions to fossil seashells gathered at 4000 m elevation in the Andes — conspired to wean the young scientist from the simple faith of the *Beagle's* commander, Captain **Robert FitzRoy** (1805–1865)[326], and force upon him the subversive conclusions of '*The Origin of the Species*'. Darwin was influenced in the formulation of his theory by the writings of the economist **Robert Malthus** (1766–1834) and the geologist **Charles Lyell** (1797–1875).

Lyell, in his "*Principles of Geology*" (1830–1833), had restated the thesis already advanced some 50 years earlier by the Scottish physician, Landowner and agriculturist **James Hutton** (1726–1797), that the earth's physical appearance was the result of the same geological processes that are still active at the present time. This idea of vast changes brought about by natural causes, which Lyell had applied to the inorganic world, Darwin applied to the world of organisms. In searching for an explanation of organic evolution, Darwin was impressed by Malthus' account [*Essay on Population*, 1798] of the intense competition among mankind for the means of subsistence.

Darwin never stopped working. "*When I am obliged to give up observation and experiment*", he said, "*I shall die*". He was working on 17 April, 1882; he died two days later. He was interred at Westminster Abbey, with Huxley, Hooker and Wallace among the pall-bearers.

The popular success of *The Origin of the Species* distinguishes it from most other novel ideas in the history of science. Isaac Newton's *Principia* was, and still is, inaccessible to the general reader: its mathematical argument is so

[324] For further reading, see:

- Moorehead, A., *Darwin and the Beagle*, Penguin Books: England, 1971, 280 pp.

[325] The bishop of Panama reached the Galapagos Ills in 1535 and named them after their giant *tortoises*. These creatures allegedly may reach the age of 200 years. Thus, some of those living there today may have *seen* Darwin!

[326] Later Vice-Admiral, hydrographer and a meteorologist pioneering in weather prediction. Sick in body and mind, he took his own life in a spasm of righteous despair.

obtruse that it took many years of patient analysis before the *scientific* community fully understood its implications. Darwin's book, on the other hand, is amazingly simple for a major scientific book; it is written in such straightforward English that anyone who is capable of following a logical argument can recognize what it means.

Although Darwin made many important observations of his own, the facts which would have supported his theory were already known and had been widely discussed before. Moreover, by 1859 the scientific atmosphere was saturated with the possibility of evolution. It was only a matter of time before someone stumbled on this idea. Why then, had no one thought of it before?

What happened to Darwin's predecessors (and to some of his contemporaries as well) was that their vision was obscured by a strong *preconception* (i.e. not because they were short of facts, but because they had reasons for 'seeing' these facts in a different way). These were: the Biblical notion of special creation (*Creationism*) and the Greek philosophical notion of Ideal Forms (*Essentialism*). Darwin's theory overturned the *catastrophic* history of the world (as promulgated by the world's leading religions), changing it from a series of separate tableaux into a slow-motion picture.

Although Darwin had already formulated the essential outlines of his theory as early as 1839, he delayed its publication for 20 years, waiting for the scientific world to become thoroughly familiar with the issue of evolution. The sources of his scruples were fear of controversy and persecution, his own religious beliefs, and finally his scientific caution; the mechanism which he had invoked was contrary to all of the most dearly held beliefs of Victorian Christianity. In his notebook Darwin had grimly reminded himself of the persecution meted out to other scientists who had flouted traditional belief.

The most important factor was, however, Darwin's doubt about the *scientific credibility* of his own theory. He recognized that evolution could not be observed *directly*. The only way of overcoming this difficulty was to collect such an overwhelming mass of *indirect evidence* that the deduction could be inescapable.

The objections to his theory gave Darwin serious trouble. The first was raised by the zoologist **Jackson St. George Mivrat** (1827–1900, England), who argued that although natural selection might account for the success of well-established adaptations, it could not possibly explain the *initial* stages of their development. The biological usefulness of the eye is self-evident, but how did such an organ get started in the first place? In other words, there must have been a stage at which the incipient organ had no recognizable function, and would therefore have conferred no selective advantage. Therefore, useful organs must have developed *with a view* to a function they would eventually serve!

Darwin's answer was that a random novelty which gains a foothold by conferring one kind of biological advantage might end up conferring a different sort of advantage altogether (e.g., a primitive feather probably served as a heat insulator, and only subsequently developed its aerodynamical advantage. It is a mystical nonsense to suppose the feather emerged in order to realize the remote possibilities of flight!).

The second objection was the *absence of intermediate types*. Darwin was confidence that subsequent research would restore the episodes of gaps in the fossil record, but this has not happened. There is now overwhelming evidence pointing to the conclusion that certain forms remained stable for long periods of time, only to be suddenly succeeded by new forms altogether. Thus, whilst the process of imperceptible change has an all-important part to play in the origin of the species, it is often superseded by *abrupt transformations* which result in the emergence of comprehensively new designs.

The third objection raised against Darwin's theory arose when **Lord Kelvin** calculated from the temperature of the earth's interior that Darwin has grossly overestimated the age of the earth and hence that an evolutionary mechanism which is based on the slow accumulation of small invisible novelties simply does not have such huge lengths of time at its disposal.

Darwin rightly suspected that Kelvin's calculations would turn out to be wrong. If he'd lived longer, he would have been gratified to discover that the earth was even older than he supposed[327].

Darwin's theories, although they partially accounted for the origin of the species, did not at all account for the origin of life. Only since 1953 have we had the scientific basis to make biochemical guesses about that.

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[327] Darwin did indeed project a vista of slow, gradual, steady, progressive change. In his day, it was a temerity to suggest that the earth was older than a few millions of years. Darwin ventured to ascribe many millions of years to the earth's antiquity. There was good circumstantial evidence for it — but, more than that, he *needed* what then seemed to be a vast amount of time for his notions of how evolution works to hold. The fossil record, as it is known today, suggests that some of the specific ideas that Darwin (and many of his successors) had on *how life evolves*, may well be at least partly wrong; rather than a stately progression, the gross history of life shows a mixture of status quo and revolutions. But the *order* is there: more primitive forms antedate more advanced. Some episodes in evolution proceed faster than others, but the fossil record abundantly affirms the general notion that *life has evolved*.

It is sometimes claimed by historians of science that Darwin 'borrowed' material from other writers (e.g. **Lamarck**, **E. Darwin**, **P.L. Maupertuis**) and lifted his central ideas (without giving due credit) from a number of precursors, including earlier evolutionists and formulators of the principle of *natural selection* (e.g. **Blyth**, 1835). On the other hand, his defenders claim that Darwin, like any scientist, had *influences*, but that he was honest in his theoretical developments and was working as a bona fide scientist of his day.

Darwin's theories consist of seven main hypotheses. He was neither original nor claimed to be on *transmutation*, *the struggle for existence*. He extended or modified earlier theories of *common descent*, *biogeographical speciation* and *natural selection*. *Sexual selection* was his own theory, not influenced either by earlier formulation or by Wallaces' independent discoveries. His theory of *heredity* was not original except in his specific and mistaken hypothesis of pangenesis. Table 4.8 summarizes the "evolution of evolution".

Table 4.8: Origin of Evolutional Hypotheses

Hypothesis	**Original to Charles Darwin**	**Influenced**	**First author**
Transmutation of species	No	Possibly, by Lamarck, E Darwin, and Lyell's anti-Lamarckian arguments	Lamarck or Erasmus Darwin in the scientific tradition
Struggle for existence	No	Yes, by numerous scientists, and writers (e.g., Malthus, Tennyson)	Heraclitos
Common descent	No, but first to propose single ancestor of all life	Yes, by numerous scientists, especially von Baer and Owen	Maupertuis
Biogeographical speciation	No	Numerous scientists, esp. Wallace	Gmelin, von Buch
Natural selection	No	Yes, by Blyth (1835)	Patrick Matthew (1831) and William Charles Wells (1813)
Sexual selection	Yes	Possibly by comments by Erasmus Darwin	C. Darwin
Heredity (use and disuse)	No	Yes, possibly by Lamarck	Ancient
Heredity (pangenesis)	Yes	Yes	C. Darwin

Details of the hypotheses are given below:

1. **Transmutationism** (also called by Darwin "Descent with Modification"). This word means in context that species change ("mutate", from the Latin) from one species to another. It is in opposition to the prevailing Aristotelian views that species were natural kinds that were eternal.

2. **Common descent.** This is the view (not held by all evolutionists prior to Darwin or even after) that similar species with similar structures (homologies) were similar because they were descended from a common ancestor. Darwin tended to present the cases for limited common descent – i.e., of mammals or birds – but extended the argument to the view that all life arises from a common ancestor or small set of common ancestors.

3. **Struggle for existence.** This is the view that more organisms are born than can survive. Consequently, most of those zygotes that are fertilized will die, and of those that reach partition (birth) many will either die or not be able to reproduce. The competition here is against the environment, which includes other species (predators and organisms that use the same food and other resources). This is *inter*specific (between species) competition.

4. **Natural selection.** This is a complex view that species naturally have a spread of variations, and that variants that confer an advantage on the bearer organisms, and are hereditable, will reproduce more frequently than competitors, and change the "shape" of the species overall. Notice here that this competition is mostly *intra*specific, i.e., between families of the same species (and indeed of the same population).

5. **Sexual selection.** Many features of organisms are obvious hindrances (such as the tails of birds of paradise), and these often occur in one sex only. Darwin argued that there was competition for mating opportunities and any feature that initially singled a member of one gender out as a good mating opportunity would become exaggerated by the mating choices of the opposite gender. Competition here is between conspecifics of the same gender.[328]

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[328] *The struggle for survival* is above all a struggle for reproduction. Individuals are constantly and automatically tested for their ability to multiply under certain conditions of existence and produce descendants that can live in certain territories. The struggle between males for the possession of females results in the strongest and most wily having the most descendants.
The neo-Darwinists of the beginning of the 20^{th} century argued that the decisive factor in natural selection is not the *struggle for life* (an expression that came from Herbert Spenser and not from Darwin) but the differential rate of reproduction within a given species.

6. **Biogeographic distribution.** Darwin and Wallace were concerned to explain why species were found in the areas they were, and argued that dispersal of similar, but related, species was due to their evolution in one place and migration into other regions.

7. **Heredity.** Darwin knew very little about what we would call the principles of genetics. He accepted the prevailing and old view that the use of features of the organism would change the way those features were inherited.

In conclusion, Darwin and Darwin alone can be seen to be responsible for the theory of *sexual selection*. He was the first person to scientifically posit common descent for *all* life. He and Wallace independently uncovered the causes of biogeographical distribution, though not of the phenomenon itself, and of natural selection in a time of limited resources and change, despite prior sketches. The idea of a transmutation of species was not original to any 19th century scientist, although Darwin and Wallace, along with **Huxley**, **Haeckel**, **Gray**, **Hooker**, **Lyell** and others, were chiefly responsible for its acceptance by the scientific and general community and the success of the view of differentiating and branching evolution.

All biologists until **Weismann** accepted some version of the use and disuse theory of heredity that is known today as "Lamarckism". Even then, the views known as Mendelian genetics were not widely accepted until the turn of the 20th century. Darwin's pangenesis was a heroic but doomed effort.

D'Arcy Thompson said of him (1915):

> "That wise student and pupil of the ant and the bee, who curiously conjoined the wisdom of antiquity with the learning of today; whose Provencal verse seems set to Dorian music; and who, being of the same blood and marrow with Plato and Pythagoras, saw in Number *le comment et le pourquoi des choses*, and found in it *la clef de voûte de l'Univers*".

After Darwin, there were many attempts to extend the idea of evolution into social affairs and explain everything and anything by the same principle of the 'survival of the fittest'. Few of these speculations were well founded but they gave rise to a particular concept of progress and a direction of change.

Evolution did away with the idea that the living world is a finished product. This opened the door to ideas of progress (and regress) and to speculations about what the world might be like in the future. These ideas come more naturally to life scientists. Physical scientists who study the mathematical laws of nature lay much emphasis upon the unchanging character of those laws.

Before the twentieth century, the most successful applications of those laws were to the motions of the moon and the planets. The changes seen in the astronomical realm were slower, simpler, and more predictable than those in the living world. Not until the twentieth century would astronomers have to come to terms with radical new theories about the origin and evolution of stars and galaxies, and the discovery of the expansion of the universe.

There is abundant evidence in ancient history and the geological record for flood and fire catastrophes that can be associated with impacts of objects such as fireballs, comets and asteroids falling from the heavens in significant numbers, and causing widespread damage, extinctions and loss of life. Consequently, a preoccupation with the sky was an integral part of the earliest civilizations; a fear of certain heavenly phenomena was built on an awareness that the sky presented a real threat to one's survival.

In contradistinction, the Newtonian world pictured everything to be under control, ordered on the whole, and unchanging on the average. The ancient view of a sky filled with arbitrary events capable of devastating civilization therefore gave way to one in which the universe acts with unthreatening and clockwork regularity.

The Newtonian approach was seductive because it implied that we live in a more-or-less predictable and hence comfortable world; the random collision with a comet did not fit such a picture.

Darwin's concept of evolution, invoked *gradual* biological change triggered by equally gradual changes in otherwise benign environment.

Worldview XXIII: Darwin

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"The preservation of favorable variations and the rejection of injurious variations, I call Natural Selection, or Survival of the Fittest. Variations neither useful nor injurious would not be affected by natural select and would be left a fluctuating element."

* * *

"I believe there exists, and I feel within me, an instinct for truth, or knowledge or discovery, of something of the same nature as the instinct of virtue, and that our having such an instinct is reason enough for scientific researches without any practical results ever ensuing from them."

* * *

"False facts are highly injurious to the progress of science, for they often endure long; but false views*, if supported by some evidence do little harm, for everyone takes a salutary pleasure in proving their falseness."*

* * *

"As many more individuals of each species are born than can possibly survive; and as consequently there is a frequently recurring struggle for existence, it follows that any being, if it vary ever so slightly in a manner profitable to itself ... will have a better chance of survival, and thus be naturally selected."

* * *

"If insects had not been developed on the face of the earth, our plants would not have been decked out with beautiful flowers but would have produced such poor flowers as we see in our fir, oak, nut and ash trees, as grasses, docks and nettles which are fertilized through the action of the wind."

Science Progress Report No. 9

The Oxford Meeting[329] *(June 19, 1860)*

Darwin's theory was presented to the Linnaean Society of London in 1858. It had rather little impact. The president (a dentist interested in reptiles) claimed that the year had not "been marked by any of those striking discoveries which at once revolutionize, so to speak, the department of science on which they bear; it is only at remote intervals that we can reasonably expect any sound and brilliant innovation that shall produce a marked and permanent impression on the character of any brand of knowledge."

"On the Origin of Species" was put out by the publisher John Murray in 1859*, and the first edition of* 1250 *copies sold out on the day of publication.*

"The interest aroused was intense; but the subject was too novel and heretical and so, most scientists did not take sides, preferring to reserve their judgment. But it was too revolutionary an issue to lie dormant. What Darwin was saying, or at any rate suggesting, was that the world had not been created in a week, and certainly not in the year 4004 *BCE, as revealed through the obscure calculations of* **James Ussher** *in* 1650*. It was inconceivably older than this, it had changed out of recognition, and was still changing: all living creatures had changed as well, and man, far from being made in God's image, may have begun as something much more primitive. The story of Adam and Eve, in brief, was a myth. This was intolerable: people were furious at the idea that they might share a common lineage with animals. They wrongly thought that he was saying that man had descended from an ape; in fact, what he did believe was that modern man and modern apes have diverged in the remote past from a common line of ancestors."*

"Naturally, the church entered the fray: by 1860*, when Darwin's book had run through* 3 *editions, the clergy were throughly aroused and chose to come out and join battle at the meeting of the British Association for the Advancement of Science, set to take place at Oxford on June* 19, 1860*. The clergy arrived at the meeting, led by the formidable figure of* **Samuel Wilberforce***, the Bishop of Oxford, a man whose impassioned eloquence was a little too glib for some people (he was known as 'Soapy Sam'), but whose influence was very great indeed."*

329 Includes quotations from "*Darwin and the Beagle*" by **Alan Moorehead**, Penguin Books, 1971.

"Wilberforce announced beforehand that he was out to 'smash Darwin'. He was supported by the anatomist ***Richard Owen***[330]*, who was a rabid anti-Darwinist, and who probably supplied the Bishop with scientific ammunition. Darwin was ill and could not come, but his old teacher,* ***J.S. Henslow*** *presided, and he had two ardent champions in* ***Thomas Henry Huxley*** *(1825–1895) and the botanist* ***Joseph Hooker****."*

"Wilberforce, with his priestly clothes and his air of confident episcopal authority, accused Darwin of merely expressing sensational opinions that flew in the face of the divine revelations of the Bible. The Bishop, rising to the height of his peroration, then turned to Huxley, who was sitting on the platform, and demanded to know if it was through his grandmother of his grandfather that he claimed to be descended from the apes."

"Huxley was not a man to provoke lightly. When he heard how ignorantly the Bishop presented his case, ending with his 'insolent question', he said in an undertone, "The Lord hath delivered him into my hands". He got up and announced that he would certainly prefer to descend from an ape rather than from a cultivated man who prostituted the gifts of culture and eloquence to the service of prejudice and falsehood."

"Uproar ensued. The undergraduates clapped and shouted, the clergy angrily demanded an apology, and the ladies from their seats under the windows fluttered their handkerchiefs in consternation. One of them even collapsed from shock and had to be carried out. Amid the hubbub, a slightly grey-haired man got to his feet. His thin aristocratic face was clouded with rage, and he waved a Bible aloft like an avenging prophet. Here was the truth, he cried, here and nowhere else. Long ago he had warned Darwin about his dangerous thoughts. Had he but known then that he was carrying in his ship such a . . . He was shouted down and the rest of his words were lost. There were those in the audience who recognized Vice-Admiral **Robert FitzRoy***, and it must have been disturbing to hear him so passionately denounce his old shipmate."*

So ended the overture to the future 'Monkey Trial' of 1925*. Darwin lived on for another* 22 *years after the Oxford meeting, and his health somewhat improved. His reputation grew steadily and he was given an honorary Doctor's degree at Cambridge (but not Oxford!), and when he attended a lecture at*

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[330] **Richard Owen** (1804–1892), a comparative anatomist, knew more than enough biology to recognize the truth. But he was driven by wounded pride to write a long spiteful article in which he deliberately twisted the facts in an effort to discredit the new theory. Darwin wisely disregarded these objections, insisting that he could have written much more damaging criticism himself.

the Royal Institution, the whole assembly rose to their feet and applauded him.

On the Galapagos Islands there is a biological research station maintained by the Charles Darwin Foundation. Charles Darwin is now recognized as the man who, as ***Julian Huxley*** *(1887–1975) said: "provided a foundation for the entire structure of modern biology", but during his lifetime he received no official honor from the state. The Church was strong enough to see to that.*

Evolution — Origins and Impact

The idea of evolution was nothing new. Both in the general sense of gradual development of human society from simple to more complex institutions, and the more narrow biological sense that all organisms had evolved out of more elemental forms, the concept had deep roots in Western thought. Darwin's major contribution was to provide an observational basis for what has previously been a mere hypothesis.

The first evolutionist on record was ***Anaximander*** *(ca 560 BCE) who believed in dynamical natural hierarchy and taught that man evolved from aquatic animals.*

Aristotle *(384–322 BCE) noted that species were characterized by their reproductive isolation. He wrote extensively on the classification and structure of over 500 species of animals from the Mediterranean area. He accepted the idea of the origin of life as a spontaneous event, but was also concerned about the problem of heredity. His classification of life embraced a complete gradation from the lowest to the highest organism — man.*

Spontaneous generation of living creatures from nonliving matter became increasingly suspect in the 17^{th} century. The physician ***Francesco Redi*** *(1621–1697, Italy) became convinced that the maggots found in meat were derived not from the meat itself but from eggs laid by flies.*

The voyages of discovery of the 15^{th} and 16^{th} centuries, and the invention of the microscope revealed a diversity of animal and plant form and function unknown to Aristotle. With these new observations, changes in classification took place. ***John Ray*** *(1627–1705, England), a naturalist, introduced (1686) the present idea of* species *(based on common descent) and higher categories in classification. Ray showed that groups of similar species could be classified into sets, which he called* genera*. (This system is the basis for the international one still in use today.) In 1749* ***George Buffon*** *(1707–1788), in the first volume of his Histoire naturelle, defined species as a group of inbreeding individuals who cannot breed successfully outside the group.*

Carl Linnaeus[331]*(1707–1778, Sweden), a naturalist, developed the present system and method of biological classification (taxonomy).* ***Erasmus Darwin*** *(1731–1802, England), grandfather of Charles Darwin, a physician, poet and naturalist, was impressed by the extent of changes in form — within the lifetime of individual animals (frogs, for example), by influence of selective breeding in horses and dogs, and by differences due to climate. He also noted the close affinities of the mammals, which (he reasoned) implied their common origin.*

The notion of natural hierarchy was further elaborated on by ***Leibniz****, while* ***Julien Offroy de La Mettrie*** *(1709–1751, France) prefigured the conception of progress through a struggle for existence in his book* Man as Mechanism *(1748). The encyclopedist* ***Denis Diderot*** *(1713–1784) suggested (1754) that the hierarchy was not static but resulted from continuous development through time. The simpler organisms came first, the more complex ones evolved from them in progressive stages. Thus, the idea of evolution was there already in the 18^{th} century, although another hundred years were to elapse before its mechanism was explained plausibly.*

Jean Baptiste de Lamarck *(1744–1829, France), soldier and biologist, published (1809) his work 'Zoological Philosophy'. In it he expounded a consistent and well-reasoned theory according to which species descend from other species by gradual change over many generations. He argued that species retain constant characteristics only in unchanging environments, but plants and animals will change their form to adapt to their new environment (he thought, of course, that* individual organisms *change, and knew nothing of mutations and natural selection).*

Lamarck's work clearly anticipated many fundamental ideas that were later to be popularized by others, but it was his misfortune to be ahead of

[331] Yet, Linnaeus still believed in the biblical story (*Gen, 8*, 19) that animals and all other living forms started to migrate over the earth from Noah's Ark after the Flood had subsided. Today, it seems ridiculous even to children.

his time. His theory was strongly criticized by leading naturalists of his day and, as a result, never received the attention or credit it deserved. Man's everyday experience provided little support for species' development: in spite of circumstantial evidence, no one had yet seen one species turn into another. It remained for Darwin, writing 50 *years later, to convince the scientific world of the truth of evolution. To be in this receptive mood biology had to progress to a point where the existence of evolution should seem reasonable and therefore deserve a scientific explanation. This proper climate for evolution theory was created by advances in five independent scientific fields:*

(1) *Embryology: The work of* ***Christian Heinrich Pander*** *(1794–1865, Germany and Russia) and* ***Karl Ernst von Baer*** *(1792–1876, Germany) showed that the early development of the embryo is similar for wide classes of animal species (1828–1837).*

(2) *Paleontology: The emerging understanding of* fossils *as remains of living creatures and the realization that many fossils were of species that no longer existed. The father of* paleontology *was the naturalist* ***Georges Cuvier*** *(1769–1832, France), who studied the fossil vertebrates of the Paris basin and attributed the succession of fossil forms to a series of simultaneous extinctions caused by* natural catastrophes *(1796).*

(3) *Geology: The understanding that the time required for the evolution was available in the earth's history — namely, that the development of geological features required great stretches of time. This step was necessary since evolution at the species level is* not *observable during a human lifetime*[332]*. These concepts were introduced by* ***James Hutton*** *(1727–1797),* ***Abraham Gottlob Werner*** *(1749–1817, Germany, 1774),* ***Charles Lyell*** *(1797–1875, England, 1830) and* ***Robert Chambers*** *(1802–1871, Scotland, 1852).*

(4) *Economy: The "*industrial revolution*" mostly benefited only a minority, the middle class, while it brought utmost misery and destitution to the growing proletariat. A tremendous population growth and large-scale urbanization inflicted great miseries on the working classes.* ***Robert Malthus*** *(1766–1834, England), clergyman and economist, was unconvinced that man is perfect. Disbelieving the universal peace, equality and bounty predicted by the politicians and utilitarian philosophers of the* 18^{th} *century, Malthus wrote an anonymous "Essay on Population"*

[332] Except for organisms of very short lifespan, namely insects and microorganisms. Such creatures often evolve to adapt themselves to artificial agents (pesticides and drugs) devised to eradicate them.

(1798). In it he stated that human population cannot expand indefinitely. Populations tend to expand at a geometric rate of increase with which food supplied can never keep pace: Famine, disease and war, Multhus argued, will limit the increasing size of the human populations. Darwin read Malthus in 1838*, and it struck him at once that in a 'struggle for existence' favorable variations would tend to be preserved and unfavorable ones to be destroyed*[333].

(5) *Philology: The orientalist* ***William Jones*** *(1746–1794, England) had drawn attention to the phonetic similarities between certain key words in Latin, Greek and Sanskrit (1790). By 1816, the philologist* ***Franz Bopp*** *(1791–1867, Germany) suggested that all European languages had descended from the same Indo-European root.*

Alfred Russel Wallace *(1823–1913, England), surveyor and naturalist, independently suggested the theory of natural selection and its relation to the geographical distribution*[334] *of the species. Wallace ventured into the Amazon (1848), expressly for the purpose of solving the problem of the origin of the*

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333 This also occurred to **Karl Marx** (1818–1883), who (1848) proclaimed in his *Communist Manifesto* that civilization is an organism evolving irresistibly by circumstantial selection. In *Das Kapital* (1867) he claimed that "*the relation of the bourgeoisie to society was grossly immoral and disastrous and that it concealed and defended the most infamous of all tyrannies and the basest of all robberies*". Marx thus became an inspired prophet in the mind of every generous soul whom his book reached.

334 The *geographical distribution* of species was immensely important for Darwin's ideas on evolution. The English ornithologist **Philip Lustley Sclater** (1829–1913) studied the geographical distribution of birds (1858) and Wallace, elaborating on his ideas, divided the globe into six major biogeographical areas and eventually published the classic text of 19^{th} century *zoogeography*, *The Geographical Distribution of Animals* (1876).

From earliest times, travelers noted that different kinds of plants and animals are to be found in different parts of the world. Only in the 18^{th} century, however, was special attention diverted to questions concerning the geographical distribution of living things. In the 19^{th} century, the geographic distribution of plants was studied by **Alexander von Humboldt** (1769–1859), **Augustin Pyramus de Candolle** (1778–1841), **Charles Lyell** (1797–1875), and **Edward Forbes** (1815–1854). The latter hypothesized that the existence of land bridges in the past explained the similarities among the faunas and floras of Britain and various continental areas. Recent *continental drift* theory has reintroduced the idea that geographical distribution must be understood in part in terms of *geological history*. It is clear today that Wallace reached independently *species. He could not have gone to a better place for clues: the lowland Amazon basin has an astonishingly high number of species. Overwhelmed with the abundance of life, he soon began to discern patterns in the jumble of life around him. He then decided that the rivers, with their forceful water dynamics, were creators of much of the diversity, affecting and shaping both the landscape and the forest animals as well. He argued that the rivers, so broad and uncrossable, were acting much like fences, keeping the species apart.*

Already convinced of the fact of evolution, he conceived the idea of natural selection while lying sick with fever in the Moluccas (February 1858). He recalled the Essay of Population *by Robert Malthus, which he had read twelve years before, and saw its application to evolution in a flash of intuition. In June* 1858*, Wallace sent Darwin his essay, entitled "On the Tendencies of Varieties to Depart Indefinitely from the Original Type". Faced with this unnerving anticipation of his own hard-won new synthesis, he expedited the publishing of his theory (1859), and simultaneously arranged for a joint paper with Wallace to be presented to the Linnean Society the following month.*

By applying the idea of evolution to all living organisms, including man, Darwin destroyed many of the most cherished beliefs of his contemporaries. Yet, to an age that worshiped science, the thought that man was just as much subject to the logic of science as was everything else in nature also held a great fascination. Underlying much of Darwin's work was the idea of progress*, an idea dear to the* 19^{th} *century. History, the study of man's past, suddenly appeared in a new light — as a march toward some far-off, lofty goal.*

The theory of evolution and the origin of species began to change our sense of human time. The pace of technological change led people to wonder about the shape of the future. The notions of natural and supernatural, which had seemed so firm when science was merely experimenting and measuring, became shaky when science began constructing and destroying. Things that had seemed fantastic became actuality, from planes and rockets to wonder drugs and superbombs. In response to this sense of technological change and fantastic possibilities in a future that became increasingly more real, new fictional forms began to emerge.

In order to appreciate the nature of the first future shock, one must imagine how people of the pre-modern era visualized the future. In ancient times most people saw the future as being simply a continuation of the present — until the end of the world. For many, the myth of a golden age, from which men had fallen and to which they might be restored at the end of time, provided some

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the same explanation for evolution as Charles Darwin did. Unfortunately he is remembered in the history of science as *'The man who was not Darwin'*.

comfort. But the notion that the world would change regularly was simply not a part of human thought until modern time. Plato, who saw as far as anyone, saw only cycles — as tyranny, oligarchy, democracy, anarchy, and, once again, tyranny succeeded one another in time. Others saw history as having involved a steady decay from gods to heroes to men, which could only be renewed by the gods returning to earth, possibly destroying it, and beginning the cycle again. The idea of steady and irreversible growth in human capabilities was unthinkable until a few hundred years ago, and the idea of humanity as the product of an evolution from less highly organized forms of life would have seemed fantastic beyond blasphemy until the last century.

Darwin's scheme depends on the interaction between individuals and their environment: random mutations introduce diversity among individuals and the environment acts as a filter, selecting through differentiated reproductive rates the ones best tuned to their surroundings.

Although this mechanism of natural selection acts upon individuals*, it is* species *that evolve, and through them all higher populational entities, like the set of species in a given ecological system and in the final account, the entire biosphere. The ensuing freedom in the choice of evolutional unit has often led to confusion in the analysis of these phenomena.*

Darwin assumed that geological times are long enough to provide opportunity for major modifications of species, so that they could be transformed into different species. Yet he never attempted to deal with the origin of life.[335]

One of Darwin's opponents was the Swiss naturalist and geologist ***Jean Louis Rodolphe Agassiz*** *(1807–1873). He studied many kinds of animals in Europe and America. As a geologist he showed that glaciers once covered large areas of the earth. He became noted for his work on fossil forms of fishes. Agassiz established a zoological laboratory on an island in Buzzard's*

[335] Since the early 1960s, developments in the field of *molecular biology* demonstrated that all organic life is programmed by the DNA, which is essentially a specific coded statement (like an alphabetic statement), with digital (discrete) but also analog aspects. Simplistic calculations show that if the hemoglobin protein evolved by chance there would be one chance in 10^{650} of it actually arising. Similarly, the specificity of the T4 bacteriophage is represented by the number $10^{78,000}$, with only one chance in $10^{78,000}$ of it actually occurring by random shuffling. When these figures are set against the age of the universe (10^{18} sec), it seems as if there is no possibility of *life* evolving through Darwin's theory of natural selection, operating on chance mutations. This paradox of the apparent statistical impossibility of Darwinism on the molecular level may just reflect our present ignorance of knowing how to calculate the correct probabilities of early life processes.

Bay, off the coast of Massachusetts, to provide a place to study animals in their natural surroundings. He believed that animal species do not *change, and criticized Darwin's theory of evolution.*

Agassiz, the son of a Protestant pastor, was born in Motier, on the shore of the Lake of Morat. Educated at first at home, then spending four years at the gymnasium of Bienne, he completed his elementary studies at the academy of Lausanne. Having adopted medicine as his profession, he studied successively at the Universities of Zürich, Heidelberg and Munich, where he extended his knowledge of natural history. In 1829 *he took a degree of doctor of philosophy at Erlangen, and in* 1830 *that of doctor of medicine at Munich.*

Agassiz came to the United States in 1846*, and in* 1848 *became a professor of zoology and geology at Harvard.*

Another 'heretic' was ***Jean Henri Casimir Fabre*** *(1823–1915,* France*), one of the greatest naturalists of the* 19^{th} *century. He spent his life observing insects and spiders, mostly in the gardens and fields near his home in Sérignan. Fabre was called by many the* Poet of Science *who "thinks as a* philosopher*, sees as an* artist*, and feels and expresses himself like a* poet*".* ***Charles Darwin****, in his* Origin of Species*, called him* 'the incomparable observer'*, and* ***Victor Hugo*** *crowned him as* The Homer of Insects*.*

Fabre was born in the small upland village of Saint-Léons in the Rouergue Mountains of southern France. His parents were so poor that, when Henri was five, they sent him to live with his grandparents on a farm at Malaval; when he was six, he had already an enormous curiosity about nature. Smock-clad and barefoot, the boy keenly studied every new and strange animal and plant — he looked, examined and made mental notes, always driven by his insatiable desire to know. At seven, Fabre returned to his parents' home to begin his schooling. He worked while attending school and in 1842*, at eighteen, he obtained his diploma from the Normal College of Avignon. He then began his teaching career as a primary schoolteacher at Carpentras. Here, his meager salary was often in arrears. While at Ajaccio, Corsica, where he taught science for a few years, he contracted malaria and was forced to return to the mainland. Finally (1852) he became a teacher at the Lycée of Avignon. Here he labored for nearly* 20 *years; when he left, his rank, title and salary were the same as when he began.*

During those years, on his precious Thursday afternoons — the traditional half-holiday of the French school system — and the summer holidays, the schoolmaster became a schoolboy again, devoted to the study of insects. The hours spent along the banks of the Rhone filled his notebooks with entries.

These activities, while doing little to enhance his stature with his school superiors, gave him a local reputation in this strange field of endeavor. ***Louis***

Pasteur[336] *(1822–1895) was sent to see Fabre when he began his study of the silkworm disease. Victor Duruy, energetic minister of public instruction during the reign of Napoleon III, was so impressed by the obscure provincial teacher, that he invited him to Paris, with the hope that he might become a tutor of the imperial family. He conferred upon him the Ribbon of a chevalier of the Legion of Honor, but the simple son of the Rouergue peasant was ill-fitted for life in the royal court. He fled from the great city declaring that he had "never felt such loneliness before".*

Back in Avignon, he outraged his superiors when he admitted girls to his science classes. The clergy denounced him from the pulpit, and in 1870*, when the German armies were overrunning France, Fabre was dismissed from the Lycée and ejected from his house with his wife and five small children. He was saved by his friend* **John Stuart Mill** *(1806–1873, English economist and philosopher), then living at Avignon. Mill loaned Fabre $600 to see him through the crisis. During the next* 9 *years, Fabre found sanctuary in a house at the edge of Orange and supported himself by writing books on popular science. He continued, however, to observe and record the life of the insects. In* 1879 *he was able to buy a small foothold of earth, sun-scorched and thistle-ridden, at the edge of the village of Sérignan. It was inhabited by wasps, and wild bees, and all manner of other creatures — to which he devoted the remaining years of his life. During the next* 3 *decades (1879–1907) he issued his 10-volume saga* Souvenirs Entomologiques*. Oftentimes, Fabre felt that he had reached the end of his strength and that his grand scheme would not be fulfilled. In the final paragraph of Volume III he wrote: "Dear insects, my study of you has sustained me in my heaviest trials. I must take leave of you for today. The ranks are thinning around me and the long hopes have fled. Shall I be able to speak to you again?".*

Fabre never accepted the theory of evolution. He was an empiricist, and opposed to hypotheses: "I observe, I experiment and I let the facts speak for themselves". This attitude, in addition to his isolation and remoteness from centers of research, his narrow knowledge of entomological literature, and his not being a trained entomologist, caused him to ignore the role of instincts in the action of many insects and look upon them as 'programmed' machine-like creatures that stick to their course like a train on its rails. Consequently he was dominated by the general rule and failed to ascribe much significance to exceptions to the rule as modern research workers have found they must do. Yet his harvest of facts is invaluable to students of experimental biology, since he led the study of living *entomology at a time when that science seemed*

[336] Pasteur questioned the theory of evolution, because Darwin did not base his ideas on experimental proof. Louis said: "Do not put forward anything you cannot prove by experimentation."

preempted by those whose horizon of interest was limited to the dead insect and the pinned specimen. Each of his experiments was an adventure — and he was able to transmit his enthusiasm to others, never losing sight of humanity in his writings, which possess a charm that defies definition.

His work was recognized when he was already in his eighties. In 1910 *the President of France came to Sérignan to visit him.*

***Faraday**, **Maxwell** and **Kelvin** also rejected Darwinian evolution: they were religious men who adopted without question the view that nature laws were imposed by Divine decree.*

*Finally, many other 19th century scientists did not accept the theory of evolution. Among them: **C. Babbage**, **J.F.W. Herschel**, **James Joule**, **G. Mendel**, **W. Ramsay**, **Lord Rayleigh**, **B. Riemann**, **G.G. Stokes** and **R. Virchow**.*

1859–1860 CE **William Ferrel** (1817–1891, USA). Meteorologist. Applied the theory of the *Coriolis effect* to the general circulation of atmospheric and oceanic currents. Accepted the theory of **James Pollard Espy** (1785–1860, USA) that the energy of cyclones is largely due to the latent heat of condensation when air ascends (1840), and went on to show that differential heating is the initial cause of both cyclones and the general circulation. He attempted to analyse quantitatively the effect of horizontal temperature gradients on the horizontal pressure field at different levels in the atmosphere. From this work he derived the concept of the *thermal wind*.

1859 CE, Aug 27 *Birth of the Oil Industry.* **Edwin Laurentine Drake** (1819–1880, USA), a retired railroad conductor, *drilled* an oil well at Titusville, PA, USA. He found oil 21 meters bellow the surface. Drake used a wooden rig and a steam-operated drill similar to the cable-tool drills of today. He drove an iron pipe 12 meters long into the ground to solid rock, and drilled inside the pipe. This pipe served as a casing. Drake put a pump on the well, which produced 10 to 35 barrels a day. The company sold the oil for $20 a barrel. Other men drilled wells nearby, after Drake showed them how to do it. As a result, the price of oil dropped to 10 cents a barrel in less than three years. In the early 1860's, over 600 oil companies were incorporated in Pennsylvania.

In early times man used petroleum that seeped to the surface from underground springs. The ancient Egyptians coated mummies with *pitch* (natural asphalt). The Chinese found natural gas while drilling for salt, and used natural gas for fuel as far back as 1000 BCE. About 600 BCE, **King Nebuchadnezzar** used asphalt to build the walls and pave the streets of Babylon. the Assyrians and Persians also used asphalt to build their cities. Boatmen on the Euphrates River made vessels of woven reeds smeared with asphalt. American Indians used petroleum for fuel and medicine hundreds of years before the white man came; remains of their ancient oil wells have been found in the oil regions of Pennsylvania, Kentucky and Ohio.

Some historians believe that the first oil industry began in Romania, which produced about 2000 barrels of oil already in 1857. Workmen used bags and buckets to bring up oil from hand-dug wells. Also in 1857, **James Miller Williams** of Canada dug an oil well and established a refinery near present-day Oil Springs, Ontario. He distilled and sold oil for lamps. But most historians trace *the start of the industry on a large scale* to Drake's well (1859).

James Young (1850, England) started the commercial production of *paraffin* from crude oil by slow distillation, thus creating the paraffin oil-shale industry. The first off-shore oil wells were drilled in 1900.

Drake's pioneering endeavor started a process that would eventually fund much large-scale geological research in search for more oil.

1859–1879 CE **James Clerk Maxwell**[337] (1831–1879, Scotland). The greatest mathematical physicist since Newton, and one of the great theoretical physicists of all time. Made revolutionary investigations in electromagnetism and the kinetic theory of gases, along with substantial contributions in several other theoretical and experimental fields: (1) Color vision, (2) the theory of Saturn's rings, (3) geometrical optics, (4) photoelasticity, (5) thermodynamics, (6) the theory of servomechanisms, (7) viscoelasticity, (8) relaxation processes. He wrote 4 books and about 100 papers.

His greatest achievement was the construction of a unified field theory for electricity and magnetism that integrated the accumulated experimental results known since **Coulomb** (1785), **Oersted** (1820), **Ampère** (1827), **Faraday** (1831) and **Gauss** (1833). He then represented all known electromagnetic phenomena by four partial differential equations, known as *Maxwell's equations* (1873). These represent: absence of magnetic monopoles, electrostatic field of charges (Coulomb's law in Gauss' form), law of induction (Faraday's law) and the magnetic effect of current (Ampère's law in Stokes'

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[337] For further reading, see:

- Everitt, C.W.F., *James Clerk Maxwell, Physicist and Natural Philosopher*, Charles Scribner's Sons: New York, 1975, 205 pp.

form). This last law was modified by Maxwell by adding a term responsible for the *displacement current*, such that the whole system of equations could render *electromagnetic waves*.

Thus, Maxwell established the theory of the electromagnetic fields, putting the field notion of **Faraday** on a solid mathematical footing. He showed that these fields can propagate as *electromagnetic waves*, carry with them a definite amount of energy and move with the velocity of light.

At one time it was thought that gravity, magnetism and electricity are the result of bodies acting on each other via '*action at a distance*'. According to this idea (originated by Newton for the case of gravitation), given any two bodies there is an '*action*' between them, that causes them to attract each other by a force that is proportional to each of their masses and inversely proportional to the square of the distance between them. In the '*field*' picture, a body surrounds itself by a field of force, which exists *whether or not* a second body is present to feel it and be attracted. A field of force is related to a *potential*, whose variation in space determines the field (force per unit test mass; test-charge in the case of electricity).

Maxwell unified the regimes of optics, electricity and magnetism. Moreover, his electromagnetic theory predicted the existence of X-rays, gamma rays, radio waves and ultraviolet and infrared radiation.

These predictions were soon to be verified. In 1883, **George Francis FitzGerald** (1851–1901, Ireland), professor of natural philosophy at Dublin, pointed out that if Maxwell's theory were valid, it should be possible to generate electromagnetic waves purely electrically — by varying an electric current periodically in a circuit. [**Kelvin** had demonstrated in 1853 that the discharges of a Leyden jar, and other electrical condensers, are oscillatory phenomena.] Accordingly, FitzGerald suggested that a discharging condenser would be a good source of the electromagnetic radiation predicted by Maxwell's theory, and he showed that the shorter their wavelength, the greater the amount of energy they would carry, and thus the easier they should be to detect.

During 1886–1889 **Heinrich Hertz** (1857–1894, Germany) confirmed Maxwell's theory by producing, transmitting and receiving electromagnetic waves in the laboratory. They were shown to be transverse and propagate with the velocity of light.

In spite of this, Maxwell's electromagnetic theory was slow to gain general acceptance. In the words of Max Born (1933): "*It seems to be characteristic of the human mind that familiar concepts are abandoned only with the greatest reluctance, especially when a concrete picture of phenomena has to be sacrificed*".

Indeed, Maxwell himself and his followers tried for a long time to describe the electromagnetic field with the aid of mechanical models. It was only gradually, as Maxwell's concepts became more familiar, that the search for an "explanation" of his equations in terms of mechanical models was abandoned.

In 1861 Maxwell created the science of quantitative colorimetry. He proved that all colors may be matched by mixtures of 3 spectral stimuli[338], provided that subtraction as well as addition of stimuli is allowed. He revived **Thomas Young**'s 3-receptor theory of color vision and demonstrated that color blindness is due to the ineffectiveness of one or more receptors. He also projected the first *color photograph* and made other noteworthy contributions to physiological optics. [Helmholtz' paper of 1852 contained useful work, but he overlooked the essential step of putting negative quantities in the color equations and explicitly rejected the 3-color hypothesis.]

In 1859 Maxwell finished his study on the rings of Saturn (**Huygens**, 1655). He proved mathematically that a model assuming broad, rigid, thin sheets of matter would break apart and concluded that Saturn's rings are composed of "*an indefinite number of unconnected particles*". [Supporting observational evidence came 4 decades later when **James Edward Keeler** (1857–1900, U.S.A.), working at Lick Observatory, observed (1895) Doppler shifts in sunlight reflected from the Saturnian rings.]

The problem of determining the motion of large numbers of colliding bodies came to Maxwell's attention while he was still investigating Saturn rings. Then, when he read the new papers by **Rudolf Clausius** (1858 and 1859) on the kinetic theory of gases, he set forth to go a step further and remove the simplifying assumption that all molecules of any one kind have the same speed. This led him (1860) to a statistical formula for the distribution of velocities in a gas at uniform temperature. Maxwell's idea of describing actual physical processes by a *statistical function* was an extraordinary novelty.

He next applied the distribution function to evaluate coefficients of viscosity, diffusion and heat conduction, as well as other properties of gases not studied by Clausius. He interpreted viscosity as the transfer of momentum between successive layers of molecules moving, like Saturn rings, with differential transverse velocities. Finally, Maxwell evaluated the distribution of energy among different modes of motion of the molecules — translational, rotational, etc. [*equipartition law*].

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338 Artists had indeed known centuries before Maxwell and Helmholtz that the 3 so-called primary pigments, red, yellow and blue, yield any desired hue by mixture; but the weight of Newton's claim that the prismatic spectrum contains 7 primary colors clouded interpretation of the phenomenon.

It is important to note that Maxwell was the first to state that the second law of thermodynamics[339] is *statistical in nature* (1868).

Maxwell introduced the concepts of *'curl'*, *'gradient'* and *'convergence'* (negative 'divergence') of vector fields. He also introduced the distinction between axial and polar vectors, and gave a physical treatment of the two classes of tensors later distinguished mathematically as covariant and contravariant. He gave (1871) a simple physical interpretation of the Laplace operator[340].

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[339] **Maxwell** nevertheless did not quite comprehend The Second Law of Thermodynamics (SLT). In his *Theory of Heat* (1871) he posed a way to defy SLT, saying: "The second law is undoubtedly true as long as we can deal with bodies only in mass, and have no power of pressing or handling the separate molecules of which they are made up". But he postulated that a "being" [known as: *"Maxwell's Demon"*] small enough to manipulate molecules, should be able to defy SLT and use *all* the available heat energy without expending any in the process, in effect creating a perpetual motion machine of the 2^d kind.

To prove this, Maxwell described a vessel with two chambers, A and B, which were connected by a tiny hole, which the Demon can quickly open or close so as to allow only the swifter (hotter) molecules to pass from A to B, and only the slower (cooler) molecules to pass from B to A. He will thus, without expenditure of work, raise the temperature of B and lower that of A, in contradiction to SLT [i.e. constructing a virtual air-conditioner that needs no power supply!].

In 1922, within months of submitting his doctoral thesis, **Leo Szilard** (1898–1964; then a student of Max von Laue, Max Planck and Albert Einstein in Berlin) wrote a paper on thermodynamic equilibrium: *"On the Decrease of Entropy in a Thermodynamic System by the Intervention of Intelligent Beings"*. In it he argued convincingly that *thinking generates entropy*, demonstrating that Maxwell's demon could *not* decrease entropy in the system (since his selective opening and closing of the valve must involve what today would be called "data processing", a.k.a. *thinking*)and thus could not violate SLT. It thus turns out that there is a deep connection between SLT and *information theory*.

Modern developments in the fields of *quantum computing*, *reversible computing* and *nanotechnology* continue to provide new twists on the theme of Maxwell's Demon and the relations between information, statistics, and thermodynamics.

[340] The quantity

$$\nabla^2\Phi$$

is a measure of the difference between the value of the scalar function Φ at a given field point and the *average values* of Φ in an infinitesimal neighborhood of that point. Indeed, define the average of $\Phi(\boldsymbol{r})$ at P inside a sphere of radius R centered about $P(\boldsymbol{r})$,

$$\Phi_{\text{av}} = \frac{1}{V}\int_V \Phi(\boldsymbol{r}+\boldsymbol{\eta})d^3\boldsymbol{\eta},$$

In the introduction to his book: *Treatise on Electricity and Magnetism* (1873) Maxwell abandoned Faraday's description of the electric field as a state of elastic stress in the ether, arguing that under this concept, no measurements could be made. He exchanged it for a mere *mathematical law*. This led to a revolutionary change in our attitude toward the physical world.

Up to Maxwell's time, physics had been divided into its various disciplines according to the human senses (the *anthropomorphic* classification), e.g. optics, acoustics, heat etc.; every observed phenomenon would be reduced to the appropriate sensing organ and classified accordingly. But with the discovery of new phenomena which could not be classified by this method it became necessary to divide physical phenomena according to the respective *mathematical laws*, as Maxwell did. This approach bears the advantage that parallels can be drawn between different phenomena that are subjected to the same *mathematical law* (say, the Laplace equation $\nabla^2\phi = 0$ which appears in hydrodynamics, potential theory, and electricity).

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where $V = \frac{4\pi}{3}R^3$, $0 \leq |\boldsymbol{\eta}| \leq R$. Expand Φ in a Taylor series about $\boldsymbol{r}$; for small R,

$$\phi(\boldsymbol{r}+\boldsymbol{\eta}) = \Phi(\boldsymbol{r}) + \boldsymbol{\eta}\cdot\nabla\phi + \frac{1}{2}\boldsymbol{\eta\eta}\colon \nabla\nabla\phi + O(R^3).$$

Since

$$\frac{1}{V}\int_V \phi(\boldsymbol{r})d^3\boldsymbol{\eta} = \phi(\boldsymbol{r}),$$

$$\frac{1}{V}\int_V \boldsymbol{\eta\eta}d^3\boldsymbol{\eta} = \frac{3}{5}R^2\mathfrak{I},$$

$$\frac{1}{V}\int_V \boldsymbol{\eta}\cdot\nabla\phi d^3\boldsymbol{\eta} \equiv 0$$

(symmetry), and

$$\mathfrak{I}\colon \nabla\nabla\Phi = \nabla^2\Phi,$$

it follows that

$$\nabla^2\Phi = \frac{10}{R^2}\left[\{\Phi(\boldsymbol{r})\}_{\text{av}} - \Phi(\boldsymbol{r})\right].$$

If Φ is harmonic, the average of Φ inside a sphere is equal to its value at the sphere's center. Writing

$$\{\phi(\boldsymbol{r})\}_{\text{av}} = \phi(\boldsymbol{r}) + \frac{1}{10}R^2\nabla^2\Phi(\boldsymbol{r}),$$

it appears that the average value of Φ in a small sphere equals its value at the center plus a *correction term due to spatial variation*. This correction term is governed by the Laplacian of Φ. Since $\nabla^2\Phi$ is very important in the differential equations of physics, Maxwell's interpretation enables us to attach a simple physico-geometrical meaning to many field equations in physics.

In the century that followed Maxwell, classical physics was divided into the following main categories:

- *Particle physics*, including the mechanics of a particle and systems of discrete particles and small bodies, kinetic theory of gases, and classical statistical mechanics. The unifying mathematical law is Newton's equation $m_i \ddot{q}_i(t) = F_i$ where m_i, $q_i(t)$, F_i are the respective masses, generalized coordinates and generalized forces. The mathematical vehicle is therefore the theory of systems of *ordinary differential equations* in the time variable.
- *Continuum physics*, including the classical physics of rigid bodies, elasticity, electromagnetism, fluid dynamics etc. Here we have functions, each of a finite number of variables, that vary continuously over a given domain, constituting a *field*. The state of the system is governed by field functions $F_i(x_1, x_2, x_3; t)$ [e.g. the velocity field in a fluid $\boldsymbol{V}(x_1, x_2, x_3; t)$]. The appropriate mathematical theory is that of *partial differential equations*.

Maxwell was born in Edinburgh, Scotland, the son of wealthy parents. His mother died when he was nine. At age 16 he entered the University of Edinburgh, and at 21 Trinity College, Cambridge. His class included such later celebrities as **Thomson** (Kelvin), **A. Cayley**, **Ferrers**, **Tait** and **Routh**.

In 1858 Maxwell married Katherine Mary Dewar, seven year his senior. They had no children. Earlier Maxwell had an emotional involvement with his cousin Elizabeth Cay, a girl of great beauty and intelligence, which they had to terminate because of the perils of consanguinity in a family already inbred. From 1860 to 1865, Maxwell served as a professor of natural history at King's College, London. In 1865 he retired from regular academic life to write his celebrated "*Treatise on Electricity and Magnetism*". In 1871 he was appointed professor of experimental physics at Cambridge, and planned and developed the Cavendish Laboratory. He died of abdominal cancer on 5 November, 1879.

The advance of physics during the two centuries following the publication of Newton's '*Principia*' was made possible largely due to two convictions:

- The elucidation of scientific laws was a prerequisite for the systematic ordering of empirical data, and reflected the fundamental order within the realm of objective reality.
- The basic concepts and processes reflected in these laws are mechanical in nature.

The usefulness of the second of these two pillars of the Newtonian clockwork world view reached its zenith, and the beginning of its end, with Maxwell's equations of the electromagnetic field. The aptitude with which he combined the experimental results of Faraday with his own mathematical intuition, serves as a quintessential example of the scientific method at work.

Maxwell unified electricity, magnetism and light: the electromagnetic spectrum runs the wavelength (and frequency) gamut from gamma rays, through X rays to ultraviolet light, to visible light to infrared light to radio waves, encompassing the technologies of radio, television and radar. His four equations unified the experimental results of **Oersted**, **Ampère** and **Faraday**. Light now appeared to behave as waves and to derive from electric and magnetic fields.

Maxwell has ushered in the age of modern physics – on his own, driven only by curiosity, costing the government almost nothing, himself unaware that he was laying the grounds for the next great revolutions in both science and technology.

Like most other great British scientist (Faraday, Darwin, Dirac and Crick), Maxwell was never knighted. Moreover, the communications media, the instrument of education and entertainment that Maxwell made possible, have never offered even a mini series on the life and thought of their benefactor and founder: he is almost forgotten in popular culture.

On Maxwell

"The greatest change in the axiomatic basis of physics — in other words, of our conception of the structure of reality — since Newton laid the foundation of theoretical physics, was brought about by Faraday's and Maxwell's work on electromagnetic phenomena.

Before Maxwell people conceived of physical reality as material points, whose changes consist exclusively of motions, which are subject to total differential equations. After Maxwell they conceived physical reality as represented by continuous fields, not mechanically explicable, which are subject to partial differential equations. This change in the conception of reality is the most *profound and fruitful one that has come to physics since Newton."*

Albert Einstein[341] (1931)

"From a long view of the history of mankind there can be little doubt that the most significant event of the 19th century will be judged as Maxwell's discovery of the laws of electrodynamics. Even the American Civil War, will pale into provincial insignificance before this more powerful event of the 1860's."

Richard Feynman (1964)

"Immortality of the soul, in its old religious sense, had been thoroughly discredited. But there is another and far nobler sense in which the soul truly was immortal. In living our lives, each of us makes some impression on the world, good or bad, and then dies; this impression goes on to affect future events for all times, so that part of us lives after us, diffused through all humanity, more or less, and all Nature. This is immortality of the soul."

Oliver Heaviside (1886)

Electromagnetic waves — from Oersted to Maxwell (1820–1864)

The course of physics up to about 1820 was a triumph of the Newtonian scientific program. The "forces" of nature – heat, light, electricity, magnetism, chemical action – were being progressively reduced to instantaneous attractions and repulsions between the particles of a series of fluids. Magnetism and static electricity were already known to obey inverse-square laws similar to the law of gravitation. The first 40 years of the 19th century witnessed a growing reaction against such division of phenomena in favor of some kind of "correlation of forces".

[341] Maxwell died in 1879, the year that Einstein was born.

Oersted's *discovery of electromagnetism (1820) was at once the first vindication and the most powerful stimulus of the new tendency, yet at the same time it was oddly disturbing; the action he observed between an electric current and a magnetic field* differed *from previous phenomena in two essential ways:*

- *It was developed by electricity* in motion;
- *The magnet was neither attracted to nor repelled by – but set* transversally *to the wire carrying the current.*

To such a strange phenomenon widely different reactions were possible. ***Faraday*** *took it as a new irreducible fact by which his other ideas were to be shaped. He was first to suggest that the force acting between two separate objects arises because of a* field, *created by the existence of electric charge. Faraday went on to a major discovery: Electric fields were not only created by charges but also by changing magnetic fields. The two heretofore different forces were thus connected in both directions. But the seminal breakthrough was still to be made.*

Ampère *and his followers sought to reconcile electromagnetism with existing views about instantaneous* action at a distance. *Indeed, shortly after Oersted's discovery, Ampère discovered that a force also exists between two electric currents and put forward the brilliant hypothesis that* all magnetism is electrical in origin. *In 1826 he established a formula which reduced the known magnetic and electromagnetic phenomena to an inverse-square force along the line joining two current elements $j\,\boldsymbol{dl}$, $j'\,\boldsymbol{dl'}$ separated by a distance r*

$$F_{jj'} = G\frac{jj'\,dl\,dl'}{r^2}, \tag{1}$$

where G is a geometrical factor involving the angles between r, $\boldsymbol{dl}$ and $\boldsymbol{dl'}$.

In 1845, ***F.E. Neumann*** *derived the potential function corresponding to Ampère's force and extended the theory to electromagnetic induction. Another extension developed by* ***W. Weber*** *was to combine Ampère's law with the law of electrostatics to form a new theory, which also accounted for electromagnetic induction, treating the electric current as the flow of two equal and opposite groups of charged particles, subject to a force whose direction was always along the line joining two particles e and e', but whose magnitude depended upon their relative velocity $\dot{\boldsymbol{r}}$ and relative acceleration $\ddot{\boldsymbol{r}}$ along that line:*

$$F_{ee'} = \frac{ee'}{r^2}[1 - \frac{1}{c^2}(\boldsymbol{r}^2 - 2\boldsymbol{r}\cdot\ddot{\boldsymbol{r}})], \tag{2}$$

c being a constant with dimensions of velocity.

In 1856, Weber and **F.W.G. Kohlrausch** *(1840–1910, Germany) determined c experimentally by measuring the ratio of electrostatic to electrodynamic forces. Its value in the special units of Weber's theory was about 2/3 of the velocity of light! Equations (1) and (2) and Neumann's potential theory provided the starting points for almost all the work done in Europe on electromagnetic theory until the 1870's.*

The determining influences on **Maxwell** *were* **Faraday** *and* **William Thomson**. *He progressively extended their ideas about lines of electric and magnetic force, and merged it with the result of Kohlrausch and Weber.*

By 1863, Maxwell had found a link of a purely phenomenological kind between electromagnetic quantities and the velocity of light. His paper: "A Dynamical Theory of the Electromagnetic Field" (1865), clinched matters. It provided a new theoretical framework for the subject, based on experiment and a few general dynamic principles from which the propagation of electromagnetic waves through space followed without any special assumptions about molecular vortices or the forces between electric particles.

In 1865, Maxwell developed a group of 8 scalar equations describing the electromagnetic field. The principle they embody is that electromagnetic processes are transmitted by the separate and independent action of each charge (or magnetized body) on the surrounding space rather than by direct action at a distance. Formulas for the forces between moving charged bodies may indeed be derived from Maxwell's equations, but the action is not along the line joining them, is not instantaneous, and can be reconciled with dynamical principles only by taking into account the exchange of momentum (and energy and angular momentum) with the field.

Maxwell discovered that the data, formulated and reconciled by his mathematical equations, produced a permanent marriage of the electric and magnetic fields. Not only did changing magnetic fields produce electric fields but changing electric fields produced magnetic fields. This implied the existence of self-sustaining and moving electromagnetic waves. These waves were soon identified with light. The entire spectrum of electromagnetic waves now stretches from the ultra-low frequencies and hence long wavelengths (kilometers or even larger) through the infrared, the visible region, to the ultraviolet, to the ultra-short-wave, X- and γ-rays radiated by excited heavy atoms, nuclei and elementary particle collisions (10^{-9} m down to 10^{-15} m and smaller).

We now had two force fields capable of acting through great distances: gravitation and the electromagnetic field (gravitation was still described as action-at-a-distance until the advent of GTR). The unification by Maxwell of three historically diverse phenomena — electricity, magnetism and optics — led to a deeper understanding of the phenomena and was an inspiring lesson for what would come later (the theories of Relativity, quantum-mechanical *matter waves, quantum field theories, and the quest for further field unifications).*

Maxwell's equations[342] *for stationary material media and slowly moving sources, expressed in Gaussian units ($\boldsymbol{E}$, $\boldsymbol{D}$, ρ in electrostatic units and $\boldsymbol{H}$, $\boldsymbol{B}$, $\boldsymbol{J}$ in electromagnetic units), are as follows:*

(1) $\operatorname{curl}\boldsymbol{E} = -(1/c)\partial\boldsymbol{B}/\partial t - (4\pi/c)\boldsymbol{J}_m,$ *Faraday's law;*

(2) $\operatorname{curl}\boldsymbol{H} = (1/c)\partial\boldsymbol{D}/\partial t + (4\pi/c)\boldsymbol{J}_e,$ *Ampère-Maxwell law;*

(3) $\operatorname{div}\boldsymbol{D} = 4\pi\rho_e,$ *Coulomb's law;*

(4) $\operatorname{div}\boldsymbol{B} = 4\pi\rho_m,$ *Gauss' law.*

Here c is the velocity of electromagnetic waves in vacuum, $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric and magnetic induction vectors respectively, $\boldsymbol{D}$ is the electric displacement vector, and $\boldsymbol{H}$ is the magnetic-field vector. The entities $\{\rho_e, \boldsymbol{J}_e\}$ are the respective free electric charges and free current density, whereas $\{\rho_m, \boldsymbol{J}_m\}$ are the free magnetic charge and free current density.

The current and charge densities are subject to the local conservation laws

(5) $\operatorname{div}\boldsymbol{J}_e = -\partial\rho_e/\partial t, \qquad \operatorname{div}\boldsymbol{J}_m = -\partial\rho_m/\partial t,$

which are easily verified upon taking the respective divergence of (1) and (2), using (3) and (4).

For linear, isotropic and non-conducting media, the constitutive relations

(6) $\boldsymbol{D} = \epsilon(\boldsymbol{r})\boldsymbol{E}, \qquad \boldsymbol{B} = \mu(\boldsymbol{r})\boldsymbol{H}$

are assumed, where the dimensionless point-functions ϵ, μ are respectively the dielectric constant (permittivity) and magnetic permeability.

The elimination of $\boldsymbol{H}$ and $\boldsymbol{D}$ between (1), (2) and (6) leads to a wave equation in the electric field vector, in which the current densities are assumed to be known:

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[342] To dig deeper, consult:

- Schwinger, Julian et al., *Classical Electrodynamics*, Perseus Books, 1998, 569 pp.
- Jackson, J.D., *Classical Electrodynamics*, John Wiley & Sons: New York, 1975, 848 pp.
- Landau, L.D. and E.M. Lifshitz, *Electrodynamics of Continuous Media*, Addison-Wesley, 1960, 417 pp.
- Ben-Menahem, A., "Green's Tensors and Associated Potentials for Electromagnetic Waves in Inhomogeneous Media", *Proc. Roy. Soc. (London) A* **426** 79–106, 1989.

$$(7)\quad \operatorname{curl}\left(\tfrac{1}{\mu}\operatorname{curl}\boldsymbol{E}\right)+\tfrac{\epsilon}{c^2}\tfrac{\partial^2\boldsymbol{E}}{\partial t^2}=-\tfrac{4\pi}{c^2}\tfrac{\partial\boldsymbol{J}_e}{\partial t}-\tfrac{4\pi}{c}\operatorname{curl}\left(\tfrac{1}{\mu}\boldsymbol{J}_m\right).$$

In the special case $\boldsymbol{J}_e=0,\ \boldsymbol{J}_m=0,\ \rho_e=0$ *and* $\mu=const.$ *(non-magnetic insulator with no free electric charges), equation (7) reduces to*

$$(8)\quad \nabla^2\boldsymbol{E}-(\epsilon\mu/c^2)\partial^2\boldsymbol{E}/\partial t^2=-\operatorname{grad}(\epsilon^{-1}\nabla\epsilon\cdot\boldsymbol{E}).$$

The corresponding wave equation for the magnetic vector is obtained in a similar way,

$$(9)\quad \operatorname{curl}(\epsilon^{-1}\operatorname{curl}\boldsymbol{H})+\tfrac{\mu}{c^2}\tfrac{\partial^2\boldsymbol{H}}{\partial t^2}=-\tfrac{4\pi}{c^2}\tfrac{\partial\boldsymbol{J}_m}{\partial t}+\tfrac{4\pi}{c}\operatorname{curl}(\epsilon^{-1}\boldsymbol{J}_e).$$

Although physically realizable electromagnetic sources can be described solely in terms of electric charges and currents, the use of equivalent magnetic current is sometimes a convenient artifice (for example, a magnetic line current element in an isotropic medium is equivalent to a circular electric current flowing around a path of vanishingly small radius in a plane normal to the element).

In the modern formulation of Maxwell's electrodynamics (both classic and quantum), $\rho_m=\boldsymbol{J}_m=0$*; all magnetic fields (whether due to macroscopic electricity flow or single molecules, atoms, ions and electrons) are due to electric currents; and the* full *electric current and charge in bulk media (which determine* $\boldsymbol{E}$ and $\boldsymbol{B}$ *through the* vacuum *Maxwell's equations) are given by*

$$(10)\qquad \rho_{total}=\rho_e-\operatorname{div}\boldsymbol{P}$$

$$(11)\qquad \boldsymbol{J}_{total}=\boldsymbol{J}_e+\tfrac{\partial\boldsymbol{P}}{\partial t}+c\ \operatorname{curl}\boldsymbol{M}$$

where

$$(12)\qquad \boldsymbol{P}=\tfrac{1}{4\pi}(\boldsymbol{D}-\boldsymbol{E})=\tfrac{\epsilon(\boldsymbol{r})-1}{4\pi}\ \boldsymbol{E}$$

is the electric-dipole-moment density, and

$$(13)\qquad \boldsymbol{M}=\tfrac{1}{4\pi}(\boldsymbol{B}-\boldsymbol{H})=\tfrac{\mu(\boldsymbol{r})-1}{4\pi}\ \boldsymbol{H}$$

is the effective magnetic-dipole-moment density, and $\{\rho_{total}-\rho_e,\ \boldsymbol{J}_{total}-\boldsymbol{J}_e\}$ *are the* bound *charge- and current-density, respectively.*

Maxwell's synthesis (1865) of the empirical laws of electricity *and* magnetism, *gathered over the previous 150 years – together with his new "displacement current" term* ($\frac{1}{c}\frac{\partial\boldsymbol{D}}{\partial t}$) *in equation (2), introduced for mathematical and aesthetic reasons – and the consequent explanation of the entire field of* optics *as an electromagnetic phenomenon – was the most far-reaching advance of the 19th century theoretical physics. These were previously considered to* *be unrelated phenomena; magnetism appeared with lodestones; current flowed in wires. Electricity had to do with rubbing amber rods to produce "charged" objects. These phenomena were known since* ***Thales****, and many* optical *phenomena are so ubiquitous that they were known to most persons since the dawn of humanity. But now, mathematical descriptions of experimental results were available. Some of the experiments that provided the data were obtained by* ***Coulomb****,* ***Cavendish****,* ***Orsted****,* ***Ampère*** *and* ***Faraday****.*

The Maxwell theory also explained all the known data in geometrical and wave optics, and predicted all future results in radio wave generation, propagation, and reception. When coupled with STR and quantum mechanics, Maxwellian electrodynamics became 20th-century QED – a theory that explains to a very high accuracy all branches *of* natural science *except gravitation, cosmology and nuclear- and particle physics (in particular, it should be able to explain* life*). In the 1970's, QED was successfully unified with the* weak nuclear force*, and its* gauge principle*, in a generalized form, underlining the* entire *Standard Model of particle physics.*

The Kinetic Theory of Gases, or —
How fared the atoms of Democritos?

The basic ideas of the Greeks were transmitted via ***Epicuros*** *to the Roman philosopher* ***Lucretius****, and through that connection to the Renaissance scientists, including* ***Galileo****.* ***Newton*** *used these hard, massive atoms in his work on chemistry and optics.*

In about 1800, ***Roger Boscovitch*** *wrote presciently about the primary elements of matter as point-like entities, having no extension in space but acting upon one another by forces that he describes in detail: strong repulsion when the particles are close, attraction when they are more distant.*

Experimental techniques for studying matter were improved by chemists, such as ***Lavoisier*** *(1783) who began to clarify the idea of chemical elements.*

***Dalton**, in about 1800, summarized and advanced the idea that an atom is the simplest structure that contains all the properties of an element, and he recognized twenty elements. This number would grow to ninety by the end of the century. A suggestive step in the process of understanding atoms was made by **J.L. Meyer** and **Mendeleev** (1869) who discovered the chemical periodicity of their properties.*

***Prout** (1815), aware of the simple relation of atomic weights, suggested that all elements were made of hydrogen (recalling Thales and his contentious students). That a repetitive pattern in properties of elements would suggest a complex and similarly repetitive internal structure of the atoms corresponding to these elements, came much later. The model of atoms, hard, indivisible, but subject to Newton's laws, led to new ideas about heat and energy.*

*Two main ideas, each advocated by a different group of people, were beginning to emerge. Following the Newtonian track, **Daniel Bernoulli** (1733), **J. Herapath** (1820) and **T.J. Waterston** (1843) advanced the notion that gas molecules interact by elastic collisions between mutually-repulsive particles (atoms, molecules, ions) and that heat is a form of motion.*

They produced early kinetic gas models, but even as late as 1845, these models were persistently rejected by the scientific establishment, namely the London Royal Society.

*In another vein, the universal principle of conservation of energy was established by the experiments of **Rumford** (1798), **Carnot** (1824), **Joule** (1843) and **Helmholtz** (1847). Once this idea became accepted, the treatment of heat as a form of mechanical energy arising from the motion of polyatomic molecules made more sense. It was now up to **A.K. Krönig** (1822–1879) and **Rudolf Clausius** to make the final step in forging the kinetic theory*[343] *of gases as a connection between the physicists' thermodynamics and the chemists' 'atomic theory' through Avogadro's hypothesis. Clausius' partitioning of the total energy of a system between motions of translation, rotation and vibration, encouraged work on specific heats, while his introduction of the concept of average distance traversed by molecules before collision ('mean free path') stimulated the development of a statistical interpretation of thermodynamics and molecular physics, namely – statistical mechanics. Since heat was now viewed as the kinetic energy of restless atoms, and since there are too many atoms to keep track of via their individual Newtonian equations of motion, they must be treated actuarially.*

Here again, with the application of the idea that all matter is made of atoms, a second great synthesis took place. The jiggling and chattering of

[343] The term *'kinetic theory'* was popularized in the 1870's by **O.E. Meyer** (1834–1915) as a title of a textbook.

atoms bombarding vessel walls explained "pressure". The increase in temperature of a gas is simply an increase in average speed of the atoms (faster jiggling). Heated liquids evaporate because their atoms move fast enough to escape. An enormous collection of observations on the properties of solids, liquids and gases became understood by this kinetic theory, much of it developed by ***James Clerk Maxwell*** *(1859) and* ***Ludwig Boltzmann*** *(1866).*

Despite certain difficulties in deriving specific heat capacities (unresolved until quantum mechanics) most Victorian physicists regarded the kinetic theory as a triumphant example of the mathematization of physics.

History of the Theories of Light III

C. Rebirth of the wave theory (1801–1888)

The 19*th century opened with a series of experimental and phenomenological studies which soon put the wave theory of light on a secure foundation, as a transverse propagating undulation of the elastic ether, explicable in mechanical terms.*

The first step toward this was the enunciation by ***T. Young*** *(1801) of the principle of interference, and the explanation of the colors in thin films. His views, however, were expressed largely in a qualitative manner and therefore did not gain general recognition. Young was also first to recognize (1817) that the wave motion of light was transverse. Earlier, in* 1808, *polarization of light by reflection was discovered by* ***Etienne Louis Malus*** *(1775–1812,* France*), who did not attempt an interpretation of this phenomena.*

In the meantime, the corpuscular theory had been developed further by ***P.S. de Laplace*** *and* ***J.B. Biot****, and under their influence the Paris Academy proposed the subject of diffraction for the prize question of* 1818*, in the expectation that a treatment of this subject would lead to the crowning triumph of the corpuscular theory.*

To their dismay, and in spite of strong opposition, the prize was awarded to ***A.J. Fresnel****, whose treatment was based on the wave theory. His work was the first of a succession of investigations, which, in the course of a few years, were to discredit the corpuscular theory completely. In his memoir*

Fresnel effected a synthesis of Huygens' envelope construction with Young's principle of interference. This was sufficient to explain diffraction phenomena. Fresnel calculated the diffraction caused by straight edges, small apertures, and screens. [He was advised by **Francois Jean Arago** *(1786–1853,* France) *to read the publications of Grimaldi and Young, but could not follow this advice because he could read neither English nor Latin.]*

Fresnel's theory predicted that in the center of the shadow of a small disc there should appear a bright spot. This counter-intuitive fact caused ***S.D. Poisson*** *to reject the theory. Fresnel was saved by Arago, who performed the experiment by himself and verified that Fresnel's theory was indeed correct. Poisson acquired his share of fame in the event: the spot became known as Poisson's spot!*

In 1818 *Fresnel developed his theory of the partial convection of the luminiferous ether by matter [a theory apparently confirmed in* 1851 *by the direct experiment carried out by* ***A.H.L. Fizeau****]. In* 1821 *Fresnel gave the first indication of the cause of* dispersion, *by taking into account the molecular structure of matter. The first terrestrial determination of the speed of light was performed by Fizeau in* 1849. *His colleague* ***J.B.L. Foucault*** *then followed suite and measured the speed of light in water (1851), finding it to be* less *than that in air.*

In contrast, the corpuscular theory *explained refraction in terms of attraction of the light-corpuscles at the boundary toward the optical denser medium, and this implies a greater velocity in the denser medium. On the other hand the wave theory demands, according to Huygens' construction, that a smaller velocity is obtained in the optically denser medium. Thus, the direct measurement of the velocity of light in air and water decided unambiguously in favor of the wave theory.*

In another vein, a major effort during 1821–1876 *was aimed at establishing the theory of the elastic ether. This theory persisted, in spite of many difficulties, for a long time and most of the great physicists of the* 19^{th} *century contributed to it. Among these were* ***Lord Kelvin****,* ***Lord Rayleigh*** *and* ***G. Kirchhoff****.*

While all this was happening in optics, the study of electricity and magnetism was also bearing fruits, culminating in the discoveries of ***M. Faraday*** *(1839).* ***J.C. Maxwell*** *(1873) succeeded in synthesizing all previous experiences in this field in a system of equations, the most important consequence of which was to establish the possibility of electromagnetic waves.*

Maxwell was able to show, purely theoretically, that the electromagnetic field could propagate as a transverse wave in the luminiferous ether. Solving for the velocity of the wave, he arrived at an expression in terms of electric *and magnetic properties of the vacuum ('ether'): $c = (\epsilon_0\mu_0)^{-1/2}$. Upon substituting empirically determined values for ϵ_0 and μ_0 [**Rudolph Kohlrausch** (1809–1858) and **Wilhelm Weber** (1804–1891) in 1856], Maxwell obtained a numerical result equal to the measured velocity of light. This led Maxwell to conjecture that light waves are electromagnetic waves.*

However, it soon became apparent that the new electromagnetic theory of light, while capable of explaining all phenomena associated with the propagation of light, failed to elucidate the processes of emission and absorption, in which the finer features of interaction between matter and light-radiation are manifested.

As often happens in the sciences, the limits of applicability of a theory exist in latent form long before the theory itself is demised. Indeed, a body of stubborn spectroscopical data had been accumulating since the early days of the 19^{th} century, which 100 years later turned the tide again in favor of a corpuscular aspect of light:

*In 1802, **William Hyde Whollaston** (1766–1828, England) made the earliest observations of the dark lines in the solar spectrum. Because of the slit-shaped aperture generally used in spectroscopes, the output consisted of narrow colored band of light, the so-called spectral lines. In 1814, **Joseph Fraunhofer** (1787–1826, Germany) independently rediscovered the dark lines in the solar spectrum, since named after him. These were interpreted in 1859 as absorption lines on the basis of experiments by **G. Kirchhoff** and **Robert Wilhelm Bunsen** (1811–1899, Germany), in the following way: The light of the continuous spectrum of the sun, passing through cooler gases of the sun's atmosphere, losses by absorption just those wavelengths which are emitted by the gases.*

This discovery was the beginning of spectrum analysis, which is based on the recognition that every gaseous chemical element has its own signature of a characteristic array of spectral lines. The problem of how light is produced or destroyed in atoms involves the mechanics of the atom itself, and the laws of spectral lines reveal not so much the nature of light as the structure of the emitting particles. But this story began to unfold only in the 20^{th} century.

History of Magnetism II (1600–1894 CE)

(A) Background

Although it was known (since 1581) that unlike magnetic poles attract and like poles repel, it was difficult to establish experimentally the law of magnetic force between poles. Thus, an exact determination of the mutual action could only be made under conditions which were in practice unattainable. The difficulty was finally overcome by ***Coulomb*** *(1785), who [by using very long and thin magnets, so arranged that the action of their distant poles was negligible] succeeded in establishing the law named after him. It stated that the force of attraction or repulsion exerted between two magnetic poles varies inversely as the square of the distance between them.*

Several previous attempts had been made to discover the law of force, with various results,some of which correctly indicated the inverse square law; in particular ***John Michell*** *(1750),* ***J. Tobias Mayer*** *(1760)* and ***Johann Heinrich Lambert*** *(1766) may fairly be credited with having anticipated the law which was afterwards more satisfactorily established by Coulomb*[344]*. The accuracy of this law was confirmed by* ***Gauss*** *(1832), who employed an indirect but more rigorous method than that of Coulomb.*

Gauss continued to work on terrestrial magnetism (1833)and other magnetic phenomena (1838).

H.C. Oersted *discovered (1819) that a magnet placed near a wire carrying an electric current tended to set itself at right angles to the wire, a phenomenon which indicated that the current was surrounded by a circulating magnetic field. The discovery constituted the foundation of electromagnetism*[345]*, and its publication (1820) was immediately followed by*

[344] **Joseph Priestley** showed (1767) that *electric charges* obeyed the Newtonian inverse-square force law. **John Michell** suspended a magnet by a thread and brought up another magnet, measuring the repulsive force between them by means of the twist imparted to the thread. **Coulomb** rediscovered Michell's torsion balance and with it, from 1785 to 1789, demonstrated the inverse-square law for *both* electrical and magnetic attractions and repulsions.

[345] The German natural philosophers, headed notably by **Friedrich Schelling** (1775–1859), believed that there was only one kind of power behind the development of nature, namely, that of the *World Spirit* (*weltgeist*). They thus held that light, electricity, magnetism, and chemical forces, were all interconnected; all where different aspects of the same thing. One of **Schelling's** disciples was

***A.M. Ampère's** experimental and theoretical investigation of the mutual actions of electric currents (1820) and of the equivalence of a closed electric circuit to a polar magnet.*

*In the same year **D.F. Arago** (1820) succeeded in magnetizing a piece of iron by an electric current and in 1825, **W. Sturgeon** (1783–1850) exhibited the first actual electromagnet. The experiments of **Michael Faraday**, which ran from 1831 to 1895, established the phenomena of* electromagnetic induction, paramagnetism, diamagnetism *and* permeability, *describing electric and magnetic field in terms of 'lines of force'.*

*The unification of all the known electric and magnetic phenomena was finally accomplished by **James Clerk Maxwell** (1873), who translated Faraday's ideas into a mathematical form. Maxwell explained electric and magnetic forces, not by the action at a distance assumed by earlier mathematicians, but by stresses in a medium permeating all space, and possessing qualities like those attributed to the old luminiferous ether. In particular, he found that the calculated velocity with which it transmitted electromagnetic disturbance, was equal to the observed velocity of light. Hence he was led to believe, not only that his medium and the ether were one and the same, but, further, that light itself was an electromagnetic phenomenon.*

The hypothesis known as molecular theory of magnetism *originated with **Ampère** who proposed in 1823 that magnetism was due to electric currents circulating within matter. The idea was then developed further by **W.E. Weber** who suggested that the molecules of a ferromagnetic metal are small permanent magnets, randomly oriented under ordinary conditions. These notions were based upon two age old observations:*

- *An unmagnetized bar of steel can be made into a permanent magnet by stroking the bar with a loadstone. Careful investigation of the process reveals that nothing material has been imparted to the bar of steel. The presence of the loadstone apparently had only a* directing effect *on something already present in the steel bar.*

- *If we were to take a steel bar which has been converted into a permanent magnet and cut it in two, we would find two magnets, and, if the process were continued until molecular dimensions were approached, each resulting particle would prove to be a magnet. Thus, the steel bar, even in its unmagnetized condition, possesses magnetic particles of molecular or atomic dimensions distributed throughout the bar in perfectly random manner, so that the gross effect is zero magnetization. The presence of the loadstone with its magnetic*

Hans Christian Oersted (1777–1851), who announced (1807) that he was looking for the connection between magnetism and electricity.

field had the effect of aligning the elementary particles so that their magnetic axes are made parallel.

While the identification of the 'Amperian currents' with the motion of electrons had to await the discovery of the electron at the turn of the 20th century, Maxwell's electromagnetic field theory consolidated the understanding of the bulk (macroscopic) magnetic properties of isotropic matter that had already started with **Poisson**, **Gauss**, **Faraday**, **Weber**, **F.E. Neumann** and **Lord Kelvin**. *By 1873, the concepts of magnetic moment, magnetization, magnetic induction, magnetic susceptibility, paramagnetism, diamagnetism and permeability were in use both in theory and practice.*

(b) The Classical Theory

(i) Magnetostatics — the magnetic force

The expression for the magnetic force $\boldsymbol{F}$ *between two magnetic point-monopoles* m_1 *and* m_2 *at distance* r *apart (force on* m_2 *by* m_1*) is obtained from the Coulomb law*

$$\boldsymbol{F} = \frac{m_1 m_2}{\mu r^2}\boldsymbol{e}_r, \tag{1}$$

where $\boldsymbol{e}_r$ *is a unit vector directed from* m_1 *to* m_2 *and* μ *is a constant of proportionality, known as the permeability of the medium surrounding the magnet. It depends on the units chosen and also upon the medium between the poles. Using electromagnetic units (emu),* $\boldsymbol{F}$ *is in dynes and* r *in cm, and* $\mu = 1$ *for the vacuum.*

The poles themselves are a mathematical fiction, since they cannot exist isolated but only in pairs, and the force between two current loops is not even exactly a magnetic dipole-dipole force, except asymptotically for $r \gg$ *loop sizes. However, if we assume two very long bar magnets with two poles close together and the other two far apart, the situation is fulfilled in practice. The sign convention adopted is that a positive pole is one which is attracted towards the earth's north magnetic pole.*

(ii) Magnetic field-strength and the Lorentz force law

A pole of strength m is positioned at the origin of a coordinate system, embedded in an infinite medium of permeability μ. The force it exerts upon a unit pole at a position P, at position $\boldsymbol{r}$, defines the magnetic field-strength $\boldsymbol{H}$ at that point, namely

$$\boldsymbol{H} = \left(\frac{m}{\mu r^2}\right)\boldsymbol{e}_r, \tag{2}$$

with $\boldsymbol{e}_r = \frac{\boldsymbol{r}}{r}$, $r = |\boldsymbol{r}|$.

The force on a pole of m' units at P, will then be $\boldsymbol{F} = m'\boldsymbol{H}$. It is assumed that m' is not large enough to disturb the field $\boldsymbol{H}$ at the point of measurement, i.e., $m' \ll m$. In this way the notion of the field is decoupled from the source in the sense that it becomes a local property of space *(locally in space and time) divorced from the source that created it. In emu, $\boldsymbol{H}$ is in Oersteds = dynes per unit pole charge. The field $\boldsymbol{H}$ may also be generated by* current *flowing in a wire rather than by a pole or poles of magnetized material.*

According to the law of ***Biot-Savart*** *(1820), the magnetic field at position $\boldsymbol{r}$ due to an element $\boldsymbol{ds}$ of a straight line wire at the origin, through which a current J flows, is $|d\boldsymbol{H}| = \frac{J\,\boldsymbol{ds}\times\boldsymbol{e}_r}{4\pi r^2}$ with a corresponding force on a pole m' at $\boldsymbol{r}$ being $d\boldsymbol{F}' = m'\,d\boldsymbol{H}$. By Newton's law of action and reaction there must be a force on the current equal to*

$$d\boldsymbol{F} = -d\boldsymbol{F}' = J\,\boldsymbol{ds} \times \left(\frac{-m'\boldsymbol{e}_r}{4\pi r^2}\right) = J\,\boldsymbol{ds} \times \boldsymbol{B}(\boldsymbol{o}),$$

where $\boldsymbol{B}(\boldsymbol{o}) = \mu\boldsymbol{H}(\boldsymbol{o})$ and $\boldsymbol{H}(\boldsymbol{o})$ is the field strength at the current element due to m (according to Eq. (2)). This is a derivation of the Lorentz force law. Note that Newton's third law can only be applied in the magnetostatic *case ($\boldsymbol{J}$, $\boldsymbol{ds}$, m, $\boldsymbol{r}$ static), for otherwise the EM field can absorb momentum.*

Note that the molecular Amperian electric-current loops cause magnetic dipole moments, and spatial variations in the density $\mu\boldsymbol{M}$ of these magnetic dipole moments lead to additional atomic currents $\boldsymbol{j}_{\text{mag}} = c\,\text{curl}\,\boldsymbol{M}$. Accordingly we have from Maxwell's equations $4\pi(\boldsymbol{j} + \boldsymbol{j}_{\text{mag}}) = c\,\text{curl}\,\boldsymbol{B} - \frac{\partial \boldsymbol{D}}{\partial t}$, or

$$4\pi\boldsymbol{j} = c\,\text{curl}(\boldsymbol{B} - 4\pi\boldsymbol{M}) - \frac{\partial \boldsymbol{D}}{\partial t} = c\,\text{curl}\,\boldsymbol{H} - \frac{\partial \boldsymbol{D}}{\partial t}$$

with the new field $\boldsymbol{H} = \boldsymbol{B} - 4\pi\boldsymbol{M}$, analogous to the definition $\boldsymbol{D} = \boldsymbol{E} + 4\pi\boldsymbol{P}$, where $\boldsymbol{P}$ is the electric polarization – the density of electric *dipole moments.*

(iii) Magnetic moment, current loop, magnetic shell

Since the (fictional) magnetic poles always exist in pairs, the fundamental magnetic entity is the magnetic dipole; two poles of strength $+m$ *and* $-m$ *separated by distance* h. *Then the magnetic moment is defined as* $\boldsymbol{m} = m\, h\boldsymbol{e}$, *the unit vector* $\boldsymbol{e}$ *extending from the negative pole towards the positive pole.*

An example is a very short magnet in the form of a thin sheet with one face magnetized as an N-pole and the other as an S-pole.

Apart from permanent natural magnets (macroscopic, atomic or scales in between), a magnetic field is produced by an electric field and/or current: A small planar loop of wire with area ds *and unit normal vector* $\boldsymbol{n}$ *carrying a steady current* J *in a counterclockwise direction about* $\boldsymbol{n}$ *behaves like a magnetic dipole of moment* $d\boldsymbol{m}$ *such that (in vacuo)*

$$d\boldsymbol{m} = \lambda(J\, ds)\boldsymbol{n} = \lambda J\, d\boldsymbol{s} \tag{3}$$

Here λ *is a constant of proportionality which can be set to* $\lambda = 1$ *by a proper choice of units*[346].

If a current flows in a circuit C *of finite area* S *(magnetic shell), it can be criss-crossed into a virtual net of tiny meshes, each with a current* J *flowing around it.*

Each such mesh, at position $\boldsymbol{r}'$, *has a magnetic moment* $d\boldsymbol{m}$, *with a magnetic potential* $d\Omega = d\boldsymbol{m}\cdot\boldsymbol{\nabla}'\frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|}$ *where* P *is a point outside the circuit at position* $\boldsymbol{r}$. *The total potential is*

$$\Omega(P) = \int_S d\boldsymbol{m}(\boldsymbol{r}')\cdot\boldsymbol{\nabla}'\frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|} = J\int_S \frac{\cos\theta}{|\boldsymbol{r}-\boldsymbol{r}'|^2}\, ds = J\int_S d\omega = J\omega, \tag{4}$$

where $\cos\theta = (\boldsymbol{r}-\boldsymbol{r}')\cdot\boldsymbol{n}$ *and* ω *is the solid angle suspended at* P *by the loop* C. *The magnetic field of the circuit is given by* $\boldsymbol{H} = -\nabla\Omega$. *It is then shown that this magnetic field is given by the line integral* $\boldsymbol{H}(\boldsymbol{r}) = J\oint_c \frac{d\boldsymbol{l}\times\boldsymbol{r}}{r^3}$, *where* $d\boldsymbol{l}$ *is the vectorial loop element (Biot-Savart law), and the closed curve* C *follows the wire loop in the direction of positive current. The work done*

[346] These are known as *electromagnetic units* e.m.u for short.
It can be shown that

1 e.m.u $= c$ e.s.u $\qquad c = 3\times 10^{10}$ cm/ sec (velocity of light in vacuum)

1 ampere $= \dfrac{1}{10}$ e.m.u of current

by the field $\boldsymbol{H}$ *can only be approximated by a dipole far from the loop, and furthermore* Ω *is not single valued: if a unit magnetic charge is carried in a closed loop* Γ *that links* C once *topologically (in a clockwise sense), the work done by the wire field is*

$$\Delta\Omega = \oint_\Gamma \boldsymbol{H} \cdot d\boldsymbol{l} = 4\pi J$$

(Ampere circuital theorem).

This result is compatible with (2) since $\Delta\omega = 4\pi$ *upon a complete traversal of* Γ*. For a current flowing in a straight wire, Ampere's theorem for* Γ *a circle of radius* r *about the wire, yields* $4\pi J = (2\pi r)H(r)$ – or $H(r) = \frac{2J}{r}$ *for the (circumferential) field at distance* r *from the axis.*

The magnetic moment of a particular volume *containing currents of density* $\boldsymbol{j}_m$ *is defined as*

$$\boldsymbol{m} = \frac{1}{2}\int(\boldsymbol{\xi} \times \boldsymbol{j}_m)\,dv \tag{5}$$

$$\boldsymbol{\xi} = \text{ coordinate vector inside the volume.}$$

If $\boldsymbol{j}_m = \rho\boldsymbol{u}$, $\rho =$ charge density moving with velocity $\boldsymbol{u}$,

$$\boldsymbol{m} = \frac{1}{2}\int\rho(\boldsymbol{\xi} \times \boldsymbol{u})\,dv \tag{6}$$

This is analogous to the expression for mechanical angular momentum $\boldsymbol{s}$ *in term of the velocity of a distribution of mass densities* ρ_m

$$\boldsymbol{s} = \int\rho_m(\boldsymbol{\xi} \times \boldsymbol{u})\,dv \tag{7}$$

It is convenient to define:

$$\Gamma = \frac{|\boldsymbol{m}|}{|\boldsymbol{s}|} = \text{ gyromagnetic ratio } = \frac{e}{2m} \text{ for an electron.}$$

This is the classical *result, and also holds for orbital electron motions inside atoms and molecules. For more complex structures* $\Gamma = g\,\frac{e}{2m}$*. Due to different masses and g-factors for different particles, atoms and molecules,* Γ *for a typical macroscopic bulk medium may be quite complicated to compute.*

For the intrinsic electron spin *contribution to material magnetic moments* $g = 2$*. For atomic nuclei, the* g *values are not well understood, though known empirically.*

(iv) Magnetic induction — flux density

If we imagine an isolated positive magnetic pole of m *units at the center of a sphere of radius* r*, then on the surface* $\boldsymbol{B} = \mu \boldsymbol{H}$ *points radially outwards and* $|\boldsymbol{B}| = \frac{m}{r^2}$*, where* μ *is the permeability of the medium.*

$\boldsymbol{B} \cdot \boldsymbol{n}\, ds$ *is the magnetic flux threading through a vectorial surface element* $d\boldsymbol{s} = \boldsymbol{n}\, ds$*, and* $\boldsymbol{B}(\boldsymbol{r})$ *is called the magnetic induction field. The unit of induction (in the unit system employed here) is called the gauss*[347]*. In air, for which* $\mu \approx 1$*,* B *and* H *are numerically equal.*

(v) Intensity of Magnetization: moment density; susceptibility

A magnetic body placed in an external magnetic field becomes magnetized by induction. The intensity of magnetization is proportional to the strength of the field and its direction in isotropic materials, is in the direction of that field. It is defined as the magnetic moment per unit volume, that is, $\boldsymbol{M} = \frac{\boldsymbol{m}}{v} = M\boldsymbol{e}$*. Practically, this magnetization by induction amounts to lining up the dipoles of the magnetic material: for this reason* $\boldsymbol{M}$ *is often referred to as the magnetic polarization. We write* $\boldsymbol{M} = k\boldsymbol{H}$*, where* k *is the magnetic susceptibility.*

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[347] *International Systems of units* (1974) are based on the following six basic entities:

Length = meter (m); *Mass* = kilogram (kg); *Time* = second (s); *Electric current* = ampere (A); *Temperature* = Kelvin (K); *Amount of substance* = mole (mol).

The derived units are:

Force = Newton (N) = $\frac{\text{kg}\cdot\text{m}}{\text{s}^2}$; *Energy, work* = Joule (J) = N · m;

Power = Watt (W) = J/s; *Electric charge* = Coulomb (C) = A · s;

Magnetic induction vector $\boldsymbol{B}$ whose unit is 1 Tesla = $\frac{\text{N}}{\text{A}\cdot\text{m}}$.

Another unit of this vector is the gauss (G) such that $1T = 10^4 G$;

also $1\gamma = 10^{-5} G = 10^{-9} T$.

Magnetic flux = $\int_S (\boldsymbol{B} \cdot \boldsymbol{n})\, dS$. Its unit is 1 Weber (Wb) = T · m^2.

Hence $\boldsymbol{B}$ has also the meaning of *magnetic flux density* with the unit 1 T = $\frac{1\ \text{Wb}}{\text{m}^2}$. Note that 1 Wb = Volt × sec.

Table 4.9: SUSCEPTIBILITY OF SOME COMMON SUBSTANCES

SUBSTANCE	SUSCEPTIBILITY, $k \times 10^6$
Cerium	100
Manganese	80
Chromium	26
Aluminum	1.7
Magnesium	1.2
Tin	0.2
Oxygen	0.15
Air	0.03
Silicon	-0.20
Water	-0.72
Copper	-0.80
NaCl	-1.0
Zinc	-1.0
Silver	-1.5
Mercury	-2.4
Gold	-3.1
Carbon	-8.0
Bismuth	-14.0

The value of k is positive for paramagnetic *substances and negative for* diamagnetic *substances. Most substances have a permeability which differs only very slightly from unity. Some examples are given in Table 4.9.*

Macroscopic permanent magnets (e.g. ferromagnets) may have a permanent component $\boldsymbol{M}_0$ to $\boldsymbol{M}$ but, unless saturated, also have a small-field susceptibility (which depends on $\boldsymbol{M}_0$):

$$\boldsymbol{M} - \boldsymbol{M}_0 = k\,\boldsymbol{P} + O(H^2).$$

In a non-isotropic medium, k should be replaced by a susceptibility tensor.

(vi) Diamagnetism

In 1778 ***Anton Brugmans*** *(1732–1789, Holland) observed that a small piece of bismuth was repelled when a magnet was brought up to it. When a ball of bismuth is suspended from a thread and brought between the poles of a powerful electromagnet, it is driven out of the field when the current is switched on.* ***Faraday*** *(1845) was the first to carry out systematic investigations of the magnetic properties of a range of substances.*

The behavior of Bismuth is due to its diamagnetic properties.

Diamagnetism is caused by the variation in the frequency of an electron circulating in an atom; it occurs upon a change in the magnetic induction due to the introduction of the atom into a magnetic field or during the ramping-up of a magnetic field (e.g. by an electromagnet). At the classical level, consider an undisturbed orbit in the xy plane and equate the centripetal force to some expression $f(r)$ *which is a function of the radius* r *only,* $m\omega_0^2 r = f(r)$*. If a uniform magnetic field* $\boldsymbol{B}$ *is now applied along the* z*-axis, this field will exert a force* $\boldsymbol{F} = e(\boldsymbol{v} \times \boldsymbol{B})$ *which is directed radially outwards if the electron revolves counterclockwise in the xy plane. The total force on each electron is therefore*

$$m\omega^2 r = f(r) \pm evB, \tag{8}$$

where $v = \omega r$*. For moderate fields* B*, the radius of the orbit will not change while the electron speed in its orbit will increase or decrease.*

Let $\omega = \omega_0 + \Delta\omega$*. Then*

$$mr(\omega^2 - \omega_0^2) = \pm e\omega r B$$

or approximately

$$\Delta\omega = \pm\frac{eB}{2m}, \tag{9}$$

independent of ω_0 *if* $|\omega - \omega_0| \ll \omega_0$*.*

Thus, an electron in a magnetic field acquires an additional angular velocity characterized by the frequency $\omega_L = |e|\frac{B}{2m}$*, known as the Larmor frequency. It can be shown that the angular-momentum vector assigned to any electron orbit precesses about the lines of force of an applied magnetic field where* ω_L *is the angular velocity of precession.*

Since the electron velocity in an atom placed in a magnetic field varies, its kinetic energy varies as well. On the other hand, since r *remains unchanged, the potential energy also does not change. But since the magnetic field is always perpendicular to the electron's velocity, the magnetic field does not* *perform any work. However, according to Faraday's Law of magnetic induction, a sudden change in* $\boldsymbol{B}$ *gives rise to an induced electric field which must accelerate or decelerate the electron in its orbit.*

The angular momentum of the electron is $|\boldsymbol{L}| = |m(\boldsymbol{r} \times \boldsymbol{v})| = m\omega r^2$. *Its magnetic moment is*

$$|\boldsymbol{p}| = \pi r^2 \cdot \frac{e}{T} = \pi r^2 \cdot \frac{e\omega}{2\pi} = \frac{1}{2} e r^2 \omega = \frac{e}{2m} |\boldsymbol{L}|.$$

But the equation of motion of the electron is

$$\frac{d\boldsymbol{L}}{dt} = \boldsymbol{N} = \text{ torque } = -\boldsymbol{p} \times \boldsymbol{B} = -\frac{e}{2m} (\boldsymbol{B} \times \boldsymbol{L}).$$

If we consider the electron's orbit as a perfectly rigid body rotating with angular velocity $\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{r}$, *its motion is given by the equation* $\frac{d\boldsymbol{L}}{dt} = \boldsymbol{\omega}_L \times \boldsymbol{L}$. *Therefore* $\boldsymbol{\omega}_L = -\frac{e}{2m}\boldsymbol{B}$, *and the whole atom precesses in a magnetic field like a gyroscope — the Larmor precession.*

Now, a circular current appearing as a result of the Larmor precession of each electron in an atom causes an additional magnetic induction due to this induction current which is directed against the magnetic vector of the external field. The magnetic moment of the atom, appearing as a result of the precession as well as the magnetization, is directed against the magnetic induction of the external field. This is the basis if diamagnetism. It can be shown that the diamagnetic susceptibility is

$$\chi_d = -\mu_0 \frac{ne^2}{2m} \overline{r^2} \tag{10}$$

where n = *number of electron per unit volume,* $\chi_d = \frac{M}{H}$ *and* $\overline{r^2}$ = *mean square distance between the electrons and the nucleus.*

Typical values are $\overline{r^2} \approx 10^{-20}$ *meter,* $m = 9 \times 10^{-31}$ *kg,* $\mu_0 = 12.5 \times 10^{-7} \frac{\text{Weber}}{\text{Amp·meter}}$, $e = 1.6 \times 10^{-10}$ *Coul,* $n = 10^{28}/\text{meter}^3$, *so that* $\chi_d \sim 10^{-5}$.

The diamagnetic susceptibility is independent of temperature since the thermal motion and collision of atoms are unable to draw them from the state of Larmor Precession for any appreciable time.

(vii) Paramagnetism (Faraday, 1845)

Paramagnetic materials are substances whose molecules have a constant magnetic moment. The energy that the magnetic moment $\boldsymbol{p}$ *has in an external magnetic field is equal to* $W = -\boldsymbol{p} \cdot \boldsymbol{B}$. *The minimum value of energy is attained when* $\boldsymbol{p}$ *aligns with the direction of the magnetic field. In this case, when a paramagnetic material is introduced into a magnetic field, a preferred orientation of magnetic moments of paramagnetic atoms takes place in the direction of the magnetic induction in accordance with the Boltzmann distribution, and the body is accordingly magnetized.*

The additional induced field coincides with the direction of the external field and enhances it. The failure of the magnetic moments of individual atoms to fully align with the field results from collisions and interaction between atoms. The fraction that does line up depends on the ratio of the magnetic energy to the mean thermal energy, $f = \frac{pB}{3kT}$. *Here, the factor* $\frac{1}{3}$ *is geometrical, stemming from an average over dipole orientations. Thus* $M = npf = \frac{np^2B}{3kT} = \chi_p H$ *where* $\chi_p = \mu_0 \frac{np^2}{3kT}$ *(The Curie Law). At room temperature* $\chi_p \sim 10^{-3}$, *which is two orders of magnitude higher than the diamagnetic susceptibility.*

1859 CE **Raymond Gaston Planté** (1834–1889, France). Physicist. Invented the first practical *storage battery* (accumulator). In improved form, his invention has become the most wildly used *rechargeable battery*.

Planté was born in Orthez, France. His academic career began as a lecture assistant in physics at the Conservatory of Arts and Crafts in Paris (1854). He then became a professor of physics at the Association Polytechnique, Paris (1860).

Prior to Planté, primary cell batteries eventually lost all of their electricity when the chemical reactions were spent. Planté overcame this shortcoming by constructing his cell with two thin lead plates separated a rubber sheets. He rolled the combination up and immersed it in a dilute sulfuric acid solution. Initial capacity was extremely limited since the positive plate had little active material available for reaction. Therefore, Planté had positively charged one of his plates, making it lead oxide. The other was simply lead, which has a negative charge; The created flow of electrons from the negative plate was taken out of the battery as electricity and then *fed back into the battery*, thus creating a rechargeable battery.

1859 CE **Jean Joseph Étienne Lenoir** (1822–1900, France). One of the first to build a practical internal-combustion engine. By 1860 he was producing reliable engines equipped with an electric ignition system and using coal gas as a fuel. Many hundreds of these engines were constructed; they were used throughout Paris. About 1863, Lenoir built one of the first *automobiles*[348] to use a gas engine. The engine was a one-cylinder unit of his own design. The efficiency of the engine was, however, very low and it traveled about 5 km per hour.

1859 CE **Richard Christopher Carrington** (1826–1875, England). Astronomer. Discovered the differential rotation of the sun about its axis, with equatorial period of about 25 days. This was measured in 1887 by means of a spectroscopic Doppler effect (light coming from receding and approaching limbs).

Galileo (1613) first demonstrated that the sun rotates on its axis, by recording the apparent motions of sun-spots as the turning sun carried them across its disc. He found that the rotation period was about four weeks. (A typical sunspot group lasts about two months, so it can be followed for two periods.)

Earlier, **Christoph Scheiner** (1573–1650, Germany) announced the discovery of sunspots (1611), although ancient Chinese astronomers recorded such sightings already in ca 1000 BCE.

Very sensitive Doppler shift measurements over the interval 1973–1977 showed an average period of 24.65 days at the equator and 35 days at latitude 80°.

The sun, with over 99.8 percent of the total mass of the solar system, contains only 2 percent of all its angular momentum. It rotates in the same direction as the planets revolve, about an axis inclined at an angle $82^\circ 49.5'$ to the ecliptic.

1859–1871 CE **Zénobe Théophile Gramme** (1826–1901, Belgium). Electrical engineer and inventor. Invented and built the first commercially practical generator for producing *alternating current* (1869) and *direct current* (1859). Opened factory (1871) to produce dynamos, armature-rings, etc.

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[348] Yet, no revolution in transportation occurred until four decades later because the public was content with horses and railroads!

1859–1874 CE **William Stanely Jevons** (1835–1882, England). Economist, logician and statistician. Pioneer of mathematical economics and the application of mathematics to political economy. His main contributions outside economics are in mathematical logic where he developed the 'logical piano', a machine with 21 keys for operations in equational logic. It has many features which were later incorporated into computer design.

Jevons was born in Liverpool and studied at University College, London (1851–1853, 1859–1862). He became a professor of political economy at Manchester (1866–1879) and University College (1876–1880). He drowned whilst bathing near Hastings at the age of 47.

Jevons developed *marginal utility theory of value*[349] (1862) and a *sunspot theory*[350] of business cycles.

1860 CE **Gustav Theodor Fechner** (1801–1887, Germany). Physicist, philosopher, psychologist. A founder of *psychophysics*[351]. Used the *Weber law*[352] of discriminants to *scale* responses to stimuli. The **Weber-Fechner** law states that in order that the intensity of sensation may increase in an arithmetical progression, the stimulus must increase in a geometrical progression. Hence, in general, if M denotes a suitable quantity for scaling sensation, we get

$$M = a \log s + b$$

where s is the magnitude of a measurable stimulus, and (a, b) are contents of a particular phenomenon.

For example, the subjective impression of loudness L (sensation) is related to the physical intensity of sound (stimulus) I, measured in Watt/m^2, via the experimental relation

$$L = a \log I + b.$$

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[349] It was not till after the publication of this work that Jevons became acquainted with the application of mathematics to political economy made by earlier writers, notably **Antoine Augustin Cournot** (1838) and **H.H. Gossen** (1854). The theory of utility was developed independently by **Carl Manger** in Austria and **M.E.L. Walras** in Switzerland (1870).

[350] Sunspots influence the weather, the weather affects the crops, and the crops affect business conditions. Although few economists accept this explanation, some even today devote their efforts to proving the connection between storms in the sky and storms in the business atmosphere.

[351] Usually this term is nowadays avoided because of metaphysical implications.

[352] *Weber's law*: "Noticeable differences in sensation occur when the increase of stimulus is a constant percentage (about 5) of the stimulus itself".

The constants a and b are determined in the following way: At a frequency of 1000 Hz, the threshold of audibility (lowest intensity that can be heard) is nearly $I_0 = 10^{-12}$ Watt/m^2. Then L is made equal to

$$L = 10(\log I - \log I_0) = 10 \log(I/I_0).$$

The unit of L is called decibel[353]and abbreviated by dB. For $I = I_0$, L is zero decibel. Another application of the Weber-Fechner law is used in the dose-response relationship in biological assay, assuming that the *response of a chemical drug* (vitamin, hormone, poison etc) is linearly dependent on the logarithm of the dose.

Fechner discovered yet another important application of his law to astronomy[354]: the experienced *brightness* of a star is by no means proportional to the light energy received by the eye. Again, we have a linear relationship between *brightness* (sensation) and the logarithm of light *intensity* I (stimulus).

A standard formula is

$$m = x - 2.5 \log I.$$

Here c is a constant determined by the unit in which I is measured, and m is called the *apparent magnitude* of a star [the magnitude of *Sirius*, the brightest star, is -1.6, *Vega* has magnitude 0.1, and Betelgeuse 0.9. The clumsiness of negative magnitude might have been avoided if the magnitude zero had been better placed].

Fechner was born at Gross-Särchen, near Muskau, in Lower Lusatia, where his father was a pastor. He was educated at the University of Leipzig, in which city he spent the rest of his life. Appointed professor of physics (1834), but in 1839 contracted an eye disease while studying the phenomena of color

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353 1 decibel= $\frac{1}{10}$Bel in honor of **Alexander Graham Bell** (1847–1922), the inventor of the telephone. For a tone of any frequency other than 1000 Hz the unit dB cannot be used for the human ear.

354 Fechner found that the eye can distinguish two brightnesses if their ratio (not *difference* between them!) amounts to a definite and constant amount (the one at least about 5 percent greater than the other).
This explains the daytime disappearance of the stars: the difference in brightness between a star and its surrounding is always the same, but the ratio of the brightnesses in the *daytime* differs from that at night. As a rule, it may be said that our visual impressions are determined mainly by the brightness ratios. This aspect of our sense of vision is of the utmost importance for our daily life. Thanks to this, the objects around us remain definite, recognizable entities, even in changing conditions of illuminations.

and vision, and, after much suffering, resigned. Subsequently recovering, he turned to the study of the mind and the relations between body and mind. He set out to prove a universal parallelism expressible through a logarithmic function[355].

1860 CE Wet-plate photographs of the *sun's prominences* – the first significant astronomical result achieved by *photography* – taken during a total eclipse by **Warren de la Rue** (1815–1889).

The first astronomical photographs were taken by use of the daguerreotype process, and during the 1840s photographs were obtained of the *sun*, *moon* and *solar spectrum*. In 1850 the first successful *star photograph* was secured at the Harvard College Observatory. With the discovery of the wet collodion process (1851) more sensitive plates were made available though limited to an effective exposure time of ten minutes. During the 1870s and 1880s the wet plate was in turn supplanted by the *dry plate*, ushering in the modern era of astronomical photography since the exposure times of the dry plates could be extended almost indefinitely.

In the late 19th and early 20th centuries *photography* transformed the methods of astronomical investigation because instead of having to rely on visual observations astronomers were now able to record permanently the light from sources, inspecting the photographs at their leisure.

1860 CE **Stanislao Cannizzaro** (1826–1910, Italy). Chemist. Employed Avogadro's hypothesis in the determination of molecular weights of gaseous compounds by comparing the weight of a volume of gas to that of an equal volume of hydrogen. From molecular weights he proceeded to atomic weights, thus establishing the usefulness of atomic weights in determining the formulae for organic compounds.

1860–1861 CE **Johann Philipp Jacob Reis** (1834–1874, Germany). Physicist and teacher. Invented the first *electrical telephone*. It could transmit speech through a wire over 100 meters — a forerunner of Bell's telephone.

Reis was born to a Jewish family in Gelnhausen. He was a physics teacher at a private school near Frankfurt-am-Main, and designed and exhibited the telephone for the entertainment of his pupils. Although Reis lectured on and demonstrated his machine publicly, he was unable to realize its full potential; he died at age 40, after a long illness which eventually robbed him of his voice. Bell (1876) acknowledged that he drew upon Reis' ideas in the construction of his telephone.

[355] Modern psycho-physics favors a *power form* of the law.

1860–1873 CE **Émile Léonard Mathieu** (1835–1890, France). Mathematician. Extended and developed the formulation and solution of PDE's for a wide range of physical problems. Discovered (1860, 1873) 5 transitive permutation groups: M_{12}, M_{11}, M_{24}, M_{23} and M_{22}, known as the *Mathieu groups*. These are simple groups with exceptional properties. In the context of the 20^{th} century classification of all finite groups, these are a subset of the *special groups*. The best-known of his achievements are the *Mathieu functions*, which arise in solving the two-dimensional wave equation for the motion of an elliptic membrane (1868).

Mathieu was born in Metz. He was a student at the École Polytechnique in Paris and took his D.Sc. in 1859. He worked as a private tutor until 1869, when he was appointed to a chair of mathematics at Besancon. He moved to Nancy in 1874, and remained as a professor there until his death.

Mathieu's shy and retiring nature have accounted, to some extent, for the lack of worldly success in his life and career.

1860–1889 CE **Enrico Betti** (1823–1892, Italy). Mathematician. Noted for his contributions to algebra and topology. Derived important theorems in the mathematical theory of elasticity (*Betti's relation*, *Betti's reciprocity theorem*). In 1871 he did pioneering work in topology (*Betti's numbers*)[356] and wrote the first rigorous exposition on the theory of equations developed by **E. Galois**. Betti thus made an important contribution to the transition from classical to modern algebra. He showed (1854) that the quintic equation could be solved in term of integrals resulting in elliptic functions.

Betti was born near Pistoia, Tuscany. He studied at the University of Pisa where he rose to the rank of professor of mathematics in 1857. Under his leadership the Scuola Normale Superiore in Pisa became the leading Italian center for mathematical research education. Along with **Brioschi** and **Casorati** he visited mathematical centers in Europe (Göttingen, Berlin, Paris) making many important mathematical contacts. His work in the theory of elasticity was inspired by **Bernhard Riemann** who had visited Pisa in 1863. In 1874 he served for a short time as undersecretary of state for public education. He died in Pisa.

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[356] The (first) *Betti number* of a surface is the largest number of cross cuts which can be made without dividing the surface into more than one piece. The concept was extended by Poincaré to n-dimensional manifolds, where $n+1$ Betti numbers are defined; b_i is the number of independent, boundary-less i-dimensional submanifolds that are not themselves boundaries of any $(i+1)$-dimensional submanifold. One has $b_0 = b_n = 1$ and $b_i = b_{n-i}$, and the alternating sum of Betti numbers is the Euler characteristic of the whole manifold. Thus for $n = 2$ (a surface), $b_0 = b_2 = 1$, $b_1 = 2h$ (number of handles), and $b_0 - b_1 + b_2 = 2(1-h) = \chi$, the Euler characteristic.

Time's Arrow

The fundamental laws of motion (both classical and *relativistic), optics and electromagnetism that deal with macroscopic physics are governed by second order partial and ordinary differential equations in space and time. These equations remain equally valid when the direction of time is reversed. All phenomena described by these equations are therefore* reversible *in time.*

However, in spite of the fact that these equations permit two symmetrical solutions, natural macroscopic processes governed by these equations are for the most part irreversible.

*Thus, electromagnetic theory (using Maxwell's equations) is as compatible with the outflow of light from stars as with its inflow into them. (*retarded *potential solutions vs.* advanced *potential solutions). Yet we never see light flow* into *a star; nature seems to reject the advanced potential solution. This circumstance creates asymmetry between time and space coordinates, and also between past and future along the time axis itself.*

Physics had, therefore, to devise yet another kind of law *to account for the unidirectional trend of events in the universe – the* arrow of time. *It turned out that the observed irreversibility of natural phenomena emerges almost automatically when we begin to consider the coarse-grained stochastic evolution of large* aggregates *of particles, events or processes — despite of the fact that the underlying microscopic processes (even at the quantum-mechanical level) are* individually reversible[357]. *What Newton's, Maxwell's and Einstein's macroscopic laws and the governing equations of quantum mechanics had completely ignored, is accounted for by the science of* statistical mechanics. *Its laws are* statistical, *the laws of* crowds of events. *A crowd of individually reversible processes becomes irreversible in the bulk.*

By visualizing any macroscopic object as consisting of a very large, but finite, *number of atoms, molecules and/or other fundamental, few-degrees-of-freedom constituents, one can derive the behavior of substances in thermal equilibrium (or even away from equilibrium in some cases) by application of statistics. In particular, the non-occurrence of large fluctuations in the temperature distribution of a system in equilibrium can be understood through considerations of probabilities. Indeed, if one simplifies the description of a gas in thermal equilibrium as consisting of a number of fast molecules and an equal number of slow molecules in random motion within a box, one* expects *to*

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[357] Except for a sub-variety of weak nuclear force, of negligible relevance in most situations where a statistical time asymmetry arises.

find about equal ratios of fast and slow molecules in any given sizable portion of the box.

Even if a very large number of observations were made, it would be very improbable to ever find a state realized in which all fast molecules are accidentally in one half of the box and the slow molecules in the other half. Quite generally, the probability of finding a particular macroscopically-described state of the gas will be proportional to the number of microscopic *states of realizing that coarse-grained state, and the gas tends towards macroscopic states of ever-increasing probabilities. The entropy of a state should be connected in a quantitative way with N, the number of possibilities of realizing that state; this connection would then furnish a statistical-mechanical explanation of the 2^{nd} thermodynamical law (non-decrease in entropy of a closed system).*

To find that connection, consider two separate systems, each in thermal equilibrium, so that the numbers of possibilities of realizing the respective states are N_1 and N_2, and the entropies of the respective systems are S_1 and S_2. If the two systems are completely independent of each other and boundary effects are neglected, the entropies will simply add: $S = S_1 + S_2$. The number of possibilities of realizing the combined state, N, however, is equal to the product $N = N_1 N_2$.

Thus, if there is any connection between S and N at all, S must be proportional to the logarithm of N, because no other function $f(x)$ satisfies the functional relation $f(x_1 x_2) = f(x_1) + f(x_2)$. One thus has the famous relation, due to ***Boltzmann****:*

$$S = k \log N,$$

where k is a positive proportionality factor depending on the units in which S is measured. In the mks (SI) system of units k has the value $k = 1.38 \times 10^{-23}$ J/°K.

Unfortunately, these considerations prove to be of little help when one tries to answer two questions, arising from the existence of irreversible processes in nature:

(1) Why is the part of the actual world in which we find ourselves in a state that appears to be very far from a state of thermal equilibrium? [Indeed, if we were not living in a part of the universe very far from statistical equilibrium we would not be here to speculate about this question, since living organisms and their habitats, including the earth as a whole, are of necessity open, far-from-equilibrium thermodynamical systems.]

(2) *Why does one observe in irreversible bulk processes, starting from initial states that are not produced by extremely unlikely statistical fluctuation, an increase and never a decrease in entropy? [No one has ever observed measurable entropy decrease in a closed macroscopic system.]*

Neither of these two questions would pose a conundrum if the present actual state of our (observable) part of the universe were *the result of a statistical fluctuation.*

Indeed: if the highly ordered state we notice about us – at the terrestrial, astronomical and cosmological scales – is the result of a statistical fluctuation, then there is an overwhelming probability that the parts of the universe we have not yet looked at should be in a state of higher entropy, and appear less ordered, than the part we are seeing now.

Judging by past experience, when the part of the universe accessible to observation was much smaller, we venture to predict, however, that every new advance in the art of telescopy will reveal, as it has time and again in the past, new distant parts of the universe that are at least as far away from thermal equilibrium as we are.

All astronomic experience indicates that the universe as a whole is in a state far away from equilibrium, and has been in such states for a long time. It should be admitted that a satisfactory answer to question (1) has not yet been found.

In some authors' opinion the universe should not be expected to attain a state resembling thermal equilibrium, on the grounds that the universe is not an isolated system in a constant environment — because the all-pervasive gravitational field and the cosmological expansion furnish an environment that cannot, in principle, remain constant in time.

But even if one could resolve the problem posed by question (1), there would remain the puzzle of question (2). Practically all initial states that lead to observation of irreversible processes (for example melting of an ice cube) are states that were prepared *by us or by natural processes and are just* not *picked at random from an ensemble of possible states. Apparently, an initial state (whether of an ice cube in this epoch, or of the entire universe at the time of the Big Bang) carries within itself the template for its development, when left alone, into states of higher entropy.*

This apparent "miracle" of the arising of an arrow of time in macroscopic systems is then ultimately traceable to the assumption that the initial conditions of the universe are microscopically *random. As time wears on, the interactions in the universe (or in a closed subsystem) gradually manifest* *this randomless macroscopically, which we perceive as an increase in disorder (entropy).*

This is despite the fact that the laws of mechanics (even quantum mechanics) allow in principle the existence of prepared initial states which, when left alone, would evolve into states of lesser entropy.

In some authors' opinion the arrow of time observed in laboratory experiments is tied up with the arrow of time of the universe as a whole; in such a view there is a mechanism, perhaps gravitation, which makes it in principle impossible to truly isolate a system, and which imprints the arrow of time of the entire universe on all its parts.

Others, reluctant to accept the implication that what happens here on earth inside an isolated box, containing water and an ice cube, should be tied up with what happens inside another isolated box, with water and ice cube, on some planet in some distant galaxy, feel it ought to be possible to find sufficient reason for the arrow of time by considering only relatively small isolated systems. Thus question (2) is still open.

An individual molecule has no way of distinguishing between the two directions of time, and its behavior (apart from some tiny components of the weak nuclear force) is described by the basic time-symmetric laws of nature. Nothing prevents us from speculating about whether individual particles might be able to travel backwards in time, since there are no laws of physics that forbid motion backward in time[358].

A theory of this sort was proposed in 1949 *by* ***Richard Phillips Feynman*** *(1918–1988, U.S.A.) to explain pair-creation and pair-annihilation (transformation of one or more γ-ray photons into an electron and positron, or conversely the collision of the latter two to produce γ-ray photons and/or other quanta) by regarding a positron as a negative-energy electron traveling backwards in time (time-reversal compensating for charge and energy reversal). Thus, both of the above processes can be described by a single particle performing occasional "reflections" into the past*[359].

Though the arrow of time can disappear on the subatomic level, large fluctuations are extremely unlikely on the macroscopic or even mesoscopic

[358] If we allow such motion in a theory of physics, however, care must be taken to avoid paradoxes, such as a person traveling back in time and killing one of his grandmothers in time to render his own existence paradoxical! Such care must be exercised at the microscopic, as well as the macroscopic, level.

[359] Feynman's theory is causal, i.e., it allows the future (*or* past) quantum states to be determined from an arbitrary initial (or final) state, so it leads to no paradoxes. Recently, however, the possibility that Einstein's theory of gravity might allow time paradoxes, has aroused renewed interest.

level. [Ice cubes are not created by chance fluctuations — they are made in refrigerators; wineglasses are not created by chance — they are manufactured.] This guarantees that the arrow of time will not disappear in the world around us, and although it depends on statistical averages, it is very real.

Moreover, the arrow of time exists also in the universe as a whole, i.e. the direction of time is not a local phenomenon. When astronomers look at the universe, they see low entropy in the past, and they are justified in expecting that there will be higher entropy in the future. The cosmos, unlike the electron, is subject to the arrow of time.

The Rise of the New World,[360] *II*
The immigrants (1860–1924)

The first U.S.A. census in 1790 *recorded a population of almost* 4 *million, of which ca* 700,000 *were Negro slaves.*

Almost $\frac{1}{4}$ *of the white population was of non British ancestry: Swedish, Polish, German, Italian and Dutch, French Huguenots, Jews from Spain and Portugal, Scotch-Irish Presbyterians and others.*

The first great wave of the 19^{th} *century immigration (1830–1860) brought* 2 *million people, half of which were Catholic Irish. Thousands of English, Scottish, Welsh, French, Dutch and Swiss settlers assimilated with relative ease into American communities established by their predecessors. The abortive* 1848 *revolution in Germany brought thousands of its exiled leaders, students, intellectuals and artisans. By* 1900 *some* 5 *million more Germans (mostly peasants, but also tradesmen and craftsmen) followed.*

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360 For further reading, see:

- *The Story of America*, Reader's Digest Association: New York, 1975, 527 pp.

Beginning in 1825, "American fever" also swept Scandinavia and Finland, where political turmoil, overcrowding, feudalism and a series of poor harvests spurred a peasant exodus, that by 1920 *would bring to the United States a million Swedes and another million Norwegians, Danes and Finns.*

The dizzying rush of industrialization following the Civil War, the Homestead Act of 1862 *and the growth of a railroad network that made virtually every region accessible to settlement — all these provided new allures for Europe's poor. Agents for U.S.A. railroads, shipping companies and other industrial concerns fanned out across Europe.*

This contributed to history's most massive immigrant tide: 31 million people arrived in America in the period 1860–1924. *The big wave of the 1880's was made up largely of Germans, English, Scandinavians, and Canadians. During the peak years of* 1900–1920, *more than* 3 *million immigrants came from Italy alone. Another* 3 *million came from the heart of the crumbling Austro-Hungarian Empire. An additional* 3 *million arrived from Russia and Poland, mostly persecuted Jews. [In 1907, the record year for immigration to the United States, the newcomers came primarily from Russia, Central Europe, and Italy.] During the same two decades, well over* 5 *million more people came from Britain, Scandinavia, Germany, France, Portugal, Greece, Armenia, Canada, Mexico and other Latin American countries.*

By 1890 *the population shot up to* 63 *million and by* 1924 *to a total of* 115 *million. Altogether,* 40 *million people immigrated to the U.S.A. since its birth in* 1776.

Most of the immigrants were uneducated, unskilled, poorly clothed, destitute or nearly so. The vast majority settled in the industrial Northeast: most of the rest went to the industrial regions of the North Central States. By 1912, *for example, there were men and women from* 25 *nations, speaking* 45 *tongues, manning the looms of Lawrence, Massachusetts.*

The isolationist fervor that gripped the nation after WWI, *compelled congress to pass a law in* 1921, *that for the first time restricted the number of immigrants.*

1860–1871 CE **Hermann Hankel** (1839–1873, Germany). Mathematician. Contributed to theory of functions, special functions and the history of mathematics.

Hankel was born at Halle. His father was a professor of physics at Halle and Leipzig. Hankel acquired a considerable knowledge in Greek at the Leipzig Gymnasium and improved upon it by reading the ancient mathematicians in the original. He studied in the University of Leipzig under **Möbius** and at Göttingen under **Riemann** (1860). The following year he studied in Berlin with **K. Weierstrass** and **L. Kronecker**, and in 1862 received his doctorate at Leipzig. In 1867 he became a full professor at Erlangen.

In 1869 he was called to Tübingen, where he spent the last four years of his life. His most important contribution to mathematics was his development of the theory of Bessel functions (1869), mainly integral representations and asymptotic expansions. In honor of Hankel, **Nielsen** denoted the linear combinations $J_\nu(z) \pm iY_\nu(z)$ by the symbol H_ν (1904), and they are known today as "Bessel functions of the third kind" or simply *Hankel functions*.

1860–1867 CE **Benjamin Peirce** (1809–1880, U.S.A.). Mathematician. Developed the theory of linear associative algebra, a classification of all complex associative algebras of dimension less than seven. He used the, now familiar, tools of idempotent and nilpotent elements. Defined mathematics as "*the science which draws necessary conclusions*". He worked on a wide range of mathematical topics from celestial mechanics and geodesy on the applied side to algebra and number theory on the pure side.

Peire graduated from Harvard and became a professor there. He established the Harvard Astronomical Observatory and helped determine the orbit of Neptune.

1861 CE *Massachusetts Institute of Technology* (M.I.T.) founded in Boston. It moved to its present location on the Charles River in Cambridge, Mass., in 1916.

1861 CE **Julius Weingarten** 1836–1910, Germany). Mathematician. Worked on the fundamental equations of differential geometry. *Weingarten equations* and *Weingarten surfaces* are named after him. Received his Ph.D. from Halle (1864).

1861 CE **Pierre-Paul Broca** (1824–1880, France). Surgeon and anthropologist. Discovered seat of motor control of speech in the brain, now referred to as 'Broca's area'.

Broca was born in the township of Saint-Foy-La-Grande and studied medicine in Paris.

1861 CE The interpretation of **R.W. Bunsen** and **G. Kirchhoff** of the *Fraunhofer lines* in the solar spectrum marked the beginning of modern spectroscopy, and provided the first observation that led in 1913 to the Bohr model of the atom.

1861–1865 CE *The American Civil War*[361]. A tragic conflict between northern and southern states over the issue of black slavery and also due to economic rivalry between the industrial north and the agricultural south. It took more American lives than any other war in American history. It ended the southern way of life that depended on slave labor in the cotton and tobacco fields and cemented the union of states. It granted freedom to the American black people but not equality.

In 1860, Abraham Lincoln, a Republican whose party wanted to limit slavery, was elected president. Afraid of being outnumbered by non-slave states, 11 southern states separated from the union (23 states) into a new nation, the *Confederate States of America*. Lincoln refused to recognize this secession, and fighting broke in April 1861.

The Civil War was expected to be a brief conflict in which the immense advantages of the North in resources and manpower would prove decisive. But the South, having the advantage of fighting on its own soil and superior commanders, put up a valiant fight, and in the early part of the war won some brilliant victories.

Nobody would have predicted that the war would last four years and would turn into one of the most costly military ventures up to that time. The confederacy finally surrendered to the Union forces in April 1865; ca 618,000 had died (out of a total fighting force of some 2.5 million soldiers).

The total cost of the war is estimated at 15 billion dollars (1976). Many southern cities and towns were destroyed and the economy of the South almost completely collapsed. The victory of the North was achieved mostly due to the military competence of Ulysses S. Grant. It was the first war to apply new warfare technological such as telegraphy, photography, balloon reconnaissance, repeated rifles, trenches, railroad transportation, wire entanglement, submarines and armored vessels.

1861–1879 CE **William Crookes** (1832–1919, England). Physicist, chemist, and inventor. Discovered the element *thallium* (1861), invented the *radiometer* (1873), and investigated passage of electrical discharge through

[361] Johnson, Paul, *A History of the American People*, Harper Collins, 1998, 1088 pp.

highly rarefied gases. He also pioneered investigations of *cathode rays* in high-vacuum tubes of his own design (1878), now known as *Crookes' tubes*. This latter work led directly to the discovery of the electron by **J.J. Thomson**.

Crooks was born in London and studied at the Royal College of Chemistry. He set up (1856) a private laboratory in London and made his living as a chemical consultant and editor of scientific journals. In 1861, while conducting a spectroscopic examination of the residue left in the manufacture of sulphuric acid, he observed a bright green line which has not been noticed previously. He then succeeded in isolating a new element, *thallium*. He served as president of the Royal Society (1913–1915).

Between 1874 and 1876 the scientific world has been stirred by Crookes' experiments with the *radiometer*. This device is composed of partially evacuated chamber containing a paddle wheel with vanes blackened on one side and silvered on the other, which spins rapidly when radiant heat impinges on it.

As the invention of the radiometer came shortly after the publication of Maxwell's *Treatise*, some persons (Maxwell included) though that the motion of the wheels can be ascribed to *light pressure*, but the forces were much greater than predicted from the electromagnetic theory, and in the wrong direction. It soon became evident that the effect is due to the residual gas. The observed rotation occurs because the light heats up the black faces more than the white ones. Molecules in the residual gas that drift up against a black (hotter) side of a vane therefore get a stronger kick than from a white vane, and the corresponding stronger recoil drive the rotation with the black sides receding[362].

1861–1884 CE **Carl von Voit** (1831–1908, Germany). Physiologist. Conducted, with the assistance of **Max von Pettenkofer** (1818–1901, Germany) pioneering experiments in animal and human *metabolism*, making first measurements of *energy* requirements and determinations of oxygen and nutrients utilization. Professor at Münich (1863–1908).

von Voit was influenced by the conceptions of energy that had become dominant in physics and chemistry at that time. He was a trained physician, but studied chemistry under Liebig. He began his physiological work

[362] The detailed quantitative behavior of Crookes' radiometer was much investigated during 1873–1930. Many theoretical papers were written, including important ones by **J.C. Maxwell** (1879), and **A. Einstein** (1924). Many experiments were performed [e.g. **H. Marsh** et al. (1925). A detailed discussion with many references is given in **I.B. Loeb**, *The Kinetic Theory of Gases* (1934, pp 364–388) and in **R.W. Wood** *Physical Optics* (1934, p 794).

by establishing the fact that healthy adult animals are normally in *nitrogen equilibrium* (excepting as much nitrogen as they take in).

In 1865 he showed that combination with oxygen was *not* the first step in energy production, but that a large number of *intermediary substances* were formed from the original food before the final union with oxygen occurred. Not all these intermediates were necessarily oxidized completely. Hence, oxygen did not cause metabolism. Much of modern biochemistry consists of the search for these products of *intermediate metabolism.*

1862 CE Astronomers **Alvan Clark** (1808–1887, USA) and his son **Alvan Graham Clark** (1832–1897, USA) discovered *Sirius B*, a *white dwarf.* Alvan Sr. was also a lens maker; his firm, Alvan Clark and Sons made the 66 cm refraction telescope[363] for U.S. Naval observatory and the 91 cm telescope for the Lick Observatory. His son discovered 16 double stars and made the 102 cm lens for the Yerkes telescope (1897).

Sirius consists of a pair of stars for which individual masses can be determined. The brighter component had been known since 1844, to be moving across the sky in a sinuous path, and it was rightly surmised that this sinuous motion was the result of orbital motion around an unseen companion. Its magnitude is +8.7, some 10,000 times fainter than its bright companion. This makes it very difficult to observe except when the two stars are farthest apart.

1862–1871 CE **Ernst Felix Immanuel Hoppe-Seyler** (1825–1895, Germany). Physiologist and chemist. A founder of *physiological chemistry.* Identified[364] and isolated (1862) *hemoglobin* as the oxygen-carrying substance. Prepared a crystalline form of hemoglobin (1862) and was able to show

[363] A *refracting telescope* consists of a large long-focus-length objective lens and a small, short-focus-length eyepiece that magnifies the image formed by the focus of the objective lens. The magnification, or *magnifying power* of a refracting telescope is equal to the focal length of the objective divided by the focal length of the eyepiece lens. *Chromatic aberration* is the most severe of a host of optical problems that must be solved in designing a high-quality refracting telescope. During the 19th century, master opticians devoted their lives to overcoming those problems.

Modern astronomers in the 20th century lost interest in this type of telescope, since all its technical shortcomings can be avoided by using *mirrors* instead of lenses. There are now 14 refractors around the world with objective lenses larger than 65 cm in diameter.

[364] This was independently done at about the same time by **Otto Funke** (1828–1879, Germany.

that hemoglobin contains a compound called 'heme' as part of its structure. ['Heme' was not an amino acid, but a rather complex atom grouping containing an *iron* atom.] Measured metabolic phenomena in isolated tissues. Discovered (1871) the enzyme *Invertase*, that speeds up conversion of sucrose into glucose and fructose.

Hoppe-Seyler was born in Freiburg-an der-Unstrut. He was a professor at Tübingen (1861–1872) and Strasbourg (1872–1895).

1862–1887 CE **Julius (von) Sachs** (1832–1897, Germany). Botanist and plant physiologist. The creator of experimental botany. Contributed to all branches of botany and his name will always be associated with the great development of plant physiology which marked the latter half of the 19th century. Under his general and enthusiastic leadership, Wuerzburg became an international center of plant physiology where some of Europe's most eminent botanists were trained.

Sachs discovered that:

- *photosynthesis* occurs in the chloroplasts (the structure in the plant cell containing the green pigment *chlorophyll*) and produces oxygen. Specifically: chlorophyll is the key component that forms CO_2+water into starch while releasing oxygen (1832).
- *Starch* in chloroplasts results from absorption of CO_2; a simple iodine test can be used to show the existence of starch in a whole leaf. Starch is then translocated from the leaf in the form of sugar.
- *Light* is necessary for the synthesis of chlorophyll.

Sachs also pioneered in studies of the nutritional requirements of plants: he published the first formula for a standard culture solution, a necessary basis for identifying the mineral elements essential for growth.

Sachs was born to a poor Jewish family in Breslau, Silesia (also Wroclaw, Poland). Left high-school early because of the death of his parents. He managed to find a job as an assistant to the physiologist **Johannes Purkinje** in Prague (1850) and was later able to complete his schooling. He attended Prague University, graduating with PhD (1856). Became a professor of botany at Wuerzburg (1868), remaining there for nearly 30 years.

Science and Musicology

"Music is a secret arithmetical exercise and the person who indulges in it does not realize that he is manipulating numbers".

Gottfried Wilhelm von Leibniz (1646–1716)

Although music played an important role in the cults of many ancient civilizations[365]*, it was the Greeks who first discovered the mathematical basis of music, and thus laid the foundations of the scientific development and evolution of this discipline.*

The Greek $\mu o \upsilon \sigma \iota \kappa' \eta$, from which music *is derived, was used very widely to embrace all those arts over which the Nine Muses ($\mu o \upsilon \sigma \alpha \iota$) were held to preside. Contrasted with gymnastics it included those branches of education concerned with the development of the mind as opposed to the body. Thus, such widely different arts and sciences as* mathematics, astronomy, poetry, literature, *and even reading and writing would all fall under $\mu o \upsilon \sigma \iota \kappa' \eta$, besides the singing and setting of lyric poetry. The philosophers placed special emphasis on the educational value of music in the formation of character, and this attitude affected their aesthetic analysis. $A\rho\mu o \nu \iota \alpha$ (harmony) was the name given by the Greeks to the art of arranging sounds for the purpose of creating a definite aesthetic impression.*

[365] In ancient *Egypt*, priests trained choirs in singing ritual music, and court musicians sang and played reed pipes and string instruments such as lyres, lutes, and harps as early as 4000 BCE. In *Babylonia*, court musicians played ornate instruments in the 2600's BCE. The Bible contains the words of many *Hebrew* songs and chants, such as the Psalms. It mentions harps, drums, trumpets, cymbals, and other instruments. The music in Solomon's Temple at Jerusalem in the 900's BCE included trumpets and choral singing to the accompaniment of stringed instruments [*I Chron* **25**, mentions a total of 228 skilled musicians in the service of the temple]. The ancient kingdoms of the Mediterranean recognized dance and music as integral forms of celebration. The early *Chinese* thought that music reflected the order in the universe (Chinese music used the five-tone scale. It had no half-steps, and sounded somewhat like the 5 black keys of the piano). Musical traditions in *India* go back to the 1200's BCE. Hindus believed that music was directly related to the fundamental processes of human life. They worked out musical theories by about 300 BCE. Their music was not, however, based on a system of whole steps and half steps, like the diatonic scale.

The Greeks were first to use letters of the alphabet to represent musical tones. They grouped the tones in tetrachords *(succession of 4 tones). The first and fourth tones have a relationship somewhat like that between C on the piano and the next F above. By combining these tetrachords in various ways, the Greeks created groups of tones called* modes*. Modes were the forerunners of the more modern* major *and* minor *scales.*

Trepander of Lesbos *(710–670 BCE, Sparta) is known as the Father of Greek music. He founded at Sparta the earliest musical school of Europe. He improved the* cithara *(7-chord), by adding a note at the top of the scale and leaving out the third from the top, so as to attain an octave with one note of the scale omitted.*

Pythagoras *(fl. 532 BCE) founded the mathematical theory of acoustics and music. He established the 7 note* diatonic musical scale *based on the primes* $\{2, 3, 5\}$*; the ancient Greeks, with their abundance of string instruments, discovered that when a string (or a flute) is shortened to half its length, the resulting tone, when played with the original one, resulted in a pleasant musical sound. Similar experiments with length combinations of* $3: 2$ *(fifth),* $4: 3$ *(fourth), and* $5: 4$ *(third) also resulted in luminous sounds. This led Pythagoras and his followers to believe that music and mathematics provide keys to the secrets of the world.*

Hippasos of Metapontium *(ca 500 BCE) developed the theory of the* harmonic mean*. To the three consonant intervals: octave, fifth and fourth he added the double octave and the fifth beyond the octave.* ***Archytas of Tarentum*** *(fl. ca 390 BCE) was a mathematician, mechanician, statesman and Pythagorean philosopher. He was a friend of Plato and founder of theoretical mechanics. Ptolemy called him the most important theoretician of music of the Pythagorean school. He calculated numerical ratios for new musical scales by means of arithmetical harmonic means.*

Plato *(in* Timaeus*, ca 380 BCE) believed that the 7 notes of the musical scale also embodied the intervals between the 7 known planets as viewed from an earth-centered perspective (Mercury, Venus, Mars, Jupiter, Saturn,* *the sun, and the moon), which he later referred to (in the Republic) as the "harmony of the Spheres"*[366].

Aristoxenes of Tarentum *(fl. ca 320 BCE) was a pupil of Aristotle, philosopher and mathematician, and the greatest theoretician of music in ancient times.* **Nicomachos** *(fl. ca 100 CE) wrote a manual on harmony which is our oldest source on Pythagorean music.* **Theon of Smyrna** *(fl. 127–132 CE) was a Platonic philosopher who developed the mathematical theory of music.*

The Italian monk **Guido d'Arezzo***, in ca* 1025*, laid the basis for modern musical notation: the four-line staff, the F clef and the notes ut (do), re, mi, fa, sol, and la.*

The theory of music was considered a part of mathematics almost until modern times. It was one of the main ingredients of medieval education.

The greatest explorer of musical physics in recent times was **Hermann von Helmholtz** *(1821–1894), a German physicist who who wrote The Sensations of Tone (1862), the foundation book on sound as it is made and heard. He was also a talented pianist with a thorough practicing knowledge of his subject from several points of view. In an address given in* 1878*, the great Scottish physicist* **James Clerk Maxwell** *said: "Helmholtz, by a series of daring strides, has effected a passage for himself over that untrodden wild between acoustics and music — that Serbonian*[367] *bog where whole armies of scientific musicians and musical men of science have sunk without filling it up".*

The science of music developed due to the combined efforts of physicists and musicians, who complemented each other. Thus, we can place opposite Helmholtz, the musical scientist, the scientific musician **Theobald Boehm**

366 These connections deeply influenced the *neoplatonists* of the Renaissance who felt that, as a result of this connection, the soul must have some kind of ingrained mathematical structure. Plato's emphasis on the importance of ratio of small integers had the greatest influence on Renaissance architecture.

367 Maxwell chose his words with skill: Herodotos described the engulfing of armies in the mud of Lake Serbonis, a lake in the northern Sinai peninsula, which has since dried up. Many early musical instruments were made of the kind of cane which grows to this day in swampy places around the Mediterranean.

(1794–1881, Germany). The latter was trained as a goldsmith in the family business, but very quickly showed his ability on the flute. He made solo concert tours for several years before beginning to feel that many limitations of the flute of his day could be remedied.

Between 1830 *and* 1850 *he performed an extensive series of carefully chosen experiments, guided by what little acoustic theory was available to him, and in the end produced an instrument which was essentially the same as the modern flute. Not only does this instrument show the excellence of his researches into sound, but it also reveals Boehm as a first-class engineer. Most of the improvements which took place in the construction of other woodwinds during the last half of the* 19^{th} *century are directly descended from his ideas and from the stimulus of his success.*

The effect of a musical sound upon our ears depends first and foremost upon its frequency (pitch), i.e., the number of vibrations per second of the body emitting the sound. To say that we hear middle C*, is to say that our ears register* 256 *vibrations per second. Therefore a number is associated with each sound, and conversely, with each number, is associated a sound. For musical purposes however, we are led to employ only a limited number of sounds (frequencies), in each octave (if a string produces a certain note, half the length will produce the octave). Among the* 300 *discernible sounds in an octave, one must choose a series of monotonically increasing frequencies (scale)*[368].

Once a scale was adopted, it became practically impossible to change it; the continuity of musical life requires the continuing use of the same scale or those with practically negligible differences — which is in fact what has happened in the course of the history of Western music; nearly 2500 *years have passed and the present day scales are in fact only variations of the Greek scale.*

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[368] *Colors*, too, are differentiated by their frequencies. Indeed our eyes and ears are analyzers of frequencies, but while a painter can put colors of any frequency whatever on his canvas, a composer cannot place sounds of arbitrary pitch in his work. Why is this? First, he must write his music and for this he needs a discrete alphabet, or else he would need an infinite number of symbols; deciphering such notation would be very slow in any case. Second, music is made to be played, and the large majority of our instruments can produce only a limited number of sounds. Finally, our ear is incapable of discerning two sounds that are too close, which makes it fruitless to use all frequencies. It is agreed that a practical ear can distinguish about 300 sounds in one octave; this is still too much for musical notation and for the capacity of the instruments (a concert piano of 8 octaves would have 2400 keys and a total length of some 100 meters, exceeding by far the hand span of a human pianist).

The reasons for this consistency are rooted in the nature of the human auditory system (anatomy of the ear and the signal processing in the brain); when two notes are played together or in succession, the resulting sound is generally more harmonious to the ear when the corresponding frequencies are in simple ratios, and much music takes advantage of this fact: particular intervals sounding especially harmonious are those with frequency ratios of $2:1$ *(octave),* $3:2$ *(perfect fifth),* $4:3$ *(perfect fourth), and* $5:4$ *(major third).*

In general, the aesthetic effect of a chord depends almost exclusively on the ratio of frequencies. *The whole question of harmony is therefore a question of the choice of* ratios[369]. *Furthermore, two sounds will be more agreeable to the ear, especially if heard* simultaneously, *to the extent that they offer harmonics in common (thus, unpleasant sound consists of incommensurable frequencies). That is why the A produced by a musical instrument such as a violin sounds richer and more pleasant than the rather mechanical sound of a pure A from a tuning fork. When two or more notes are sounded together from different instruments, the ear is pleased if their various overtones have a harmonic relationship. The more combinations there are of overtones that are the same as each other, or 2 or 4 times the frequency of each other, the better the ear likes it.*

The Greek diatonic *scale consists of 7 tones within its* octave *(Latin, for* 8^{th}*). If we normalize the lowest frequency to unity, the scale will consist of the numbers* $\{1, \frac{9}{8}$ *(major interval),* $\frac{5}{4}$ *(major third),* $\frac{4}{3}$ *(fourth),* $\frac{3}{2}$ *(fifth),* $\frac{5}{3}$ *(sixth),* $\frac{15}{8}$ *(seventh), 2 (octave)*$\}$*. In the* Just C Major *scale, the absolute frequencies corresponding to these notes are* $\{264, 297, 330, 352, 396, 440, 495, 528\ Hz\}$.

These are denoted by the letters $\{C_4, D_4, E_4, F_4, G_4, A_4, B_4, C_5\}$ *respectively, or by the names* $\{$*do-re-mi-fa-sol-la-ti-do*$\}$*. These notes are produced by the* white keys *of the piano keyboard. The sequence of frequencies of the above scale is neither arithmetic nor geometric, but the subset* $\{264, 330, 396\}$ *is geometric (with ratios of* $4:5:6$*) and known as a* major triad. *Since these notes have many harmonic overtones that match each other, tonal clusters which contain them are most pleasant to the ear.*

Note that the subsets $\{352, 440, 528\}$ *and* $\{396, 495, 594\}$ *are also major triads. If it were not for the preference of our ears, it might seen logical to divide the octave into 8 equally spaced tones. However, this is not the way the standard scale works. As one goes up the scale one meets 3 different ratio's of*

369 For that reason, an *interval* between two musical notes is understood as the *ratio* of the frequencies of these two notes.

frequencies: $\frac{9}{8} = 1.125$ *(known as a whole tone or major whole),* $\frac{10}{9} = 1.111$ *(minor whole), and* $\frac{16}{15} = 1.067$ *(half tone or major semitone).*

For instance, the ratio of frequencies between D and C (re and do) is $\frac{9}{8}$*. The next step up to mi is almost as large,* $\frac{10}{9}$*. As one goes on to fa, the step is only half a tone (as re has a frequency about 12% larger than the do, whereas the frequency of the fa is only* $6\frac{1}{2}\%$ *higher than the mi). As we proceed on to sol, la and ti, we go up whole tones, but between ti and do there is only a half tone again*[370].

The tones of the diatonic scale make up a specific pattern of whole steps $(C-D, D-E, F-G, G-A, A-B)$ *and half steps* $(E-F, B-C)$*. The first tone of the scale gives the scale its name. For example, the D major scale will start with* $D = 297$ *Hz and proceeds with the same intervals (ratios) as the C scale over the frequency set* $(297, \overline{334}, \overline{371}, 396, \overline{445}, 495, \overline{557}, 594,$ *where the bars indicate rounding-off to the nearest integer).*

Likewise, the E major scale proceeds over the set

$$\{330, 372, \overline{413}, 440, 495, \overline{550}, \overline{618}, 658\ Hz\}.$$

The Greeks also used the Minor diatonic scale, in which the intervals are the same as in the Major scale, but in a different order. Thus, for the C minor scale, the normalized set is $\left\{1, \frac{9}{8}, \frac{6}{5}, \frac{4}{3}, \frac{3}{2}, \frac{8}{5}, \frac{9}{5}, 2\right\}$*, with the corresponding frequencies* $\{264, 297, \overline{317}, 352, 396, \overline{422}, \overline{475}, 528\ Hz\}$*. This combination of frequencies includes three subsets with frequency ratios* $10\colon 12\colon 15$*, known as minor triads. They are:* $\{264, \overline{317}, 396\}$*,* $\{352, \overline{422}, 528\}$*,* $\{396, \overline{475}, 594\}$*. The matching harmonics of this scale also sound pleasant to the ear, but western culture interprets the result as a sad sound.*

If we wish to play more than one scale (say, the above C Major, D Major, E Major and C Minor) on a single musical instrument, certain complications arise; changing from one key to another introduces new tones slightly different from the corresponding tones of the former key, while preserving the ratios. If a person is playing an instrument with continuous tuning like a slide trombone or a violin, and that person is very skillful, he can make the slight adjustment so that the frequency ratios of the notes are exactly right.

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370 Note also that $D = \frac{1}{2}(C+E)$, $E = \frac{1}{2}(C+G)$, $F = \frac{2C_4C_5}{C_4+C_5}$ (harmonic mean), $G = \frac{1}{2}(C_4+C_5)$. With the piano, the C major scale resides in the *white keys*. If one wishes to test the pleasantness of a scale made exclusively of whole tones, one will have to use some of the *black keys*, which does not sound better!

With valve instruments, however, that is harder to do, and with a piano it is impossible. If we had to have a different set of keys and strings on the piano for every key that might be played, the piano could not fit into a living room and no human would be able to play it. The most common tuning system alleviating this problem is called the equally tempered scale[371]*: the octave is divided into twelve equal intervals* $\frac{1}{12}$ *octave apart.*

An interval with a frequency ratio of $2^{1/12} = 1.0595$ *is called a half step and corresponds approximately to a semitone* $\frac{16}{15} = 1.0666$.

Any two half steps approximate a major interval
$\left(2^{2/12} = 1.1225 \approx \frac{9}{8} = 1.1250\right)$,
any four a major third $\left(2^{4/12} = 1.2599 \approx \frac{5}{4} = 1.2500\right)$,
any five a fourth $\left(2^{5/12} = 1.3348 \approx \frac{4}{3} = 1.333\right)$,
any seven a fifth[372] $\left(2^{7/12} = 1.4893 \approx \frac{3}{2} = 1.500\right)$,

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371 The first mention of temperament is found in 1496, in the treatise *Practica musica* by the Italian theorist **Franchino Gafori**.

372 The relation $2^{7/12} \approx \frac{3}{2}$ was known to the Greeks; the Pythagoreans asked themselves whether an integral numbers of octaves could be constructed from the fifth alone by repeated application of the simple frequency ratio $\frac{3}{2}$.
In mathematical notation, the Greeks sought a solution of the equation $\left(\frac{3}{2}\right)^n = 2^m$ in positive integers n and m. But the equation $3^n = 2^k$ has *no* integer solutions for $n > 0$. However, the Greeks were not discouraged and, by trial and error, found the excellent approximation $\left(\frac{3}{2}\right)^{12} \approx 2^7$, which is based on the near equality of $3^{1/19}$ and $2^{1/12}$. A systematic way of finding such near-coincidences is based on the continued fraction expansion (Daniel Shanks, *A Logarithm Algorithm*, *Mathematical Tables and Other Aids to Computation* **8**, 60–64, 1954)

$$\log_{a_0} a_1 = \frac{\log a_1}{\log a_0} = \cfrac{1}{n_1 + \cfrac{1}{n_2 + \cfrac{1}{n_3 + \ddots}}},$$

where $\{n_1, n_2, n_3, \ldots\}$ is a sequence of positive integers, and $a_0 > a_1 > 1$. The n_i are determined by the relations

$$a_i^{n_i} < a_{i-1} < a_i^{n_i+1} \qquad \text{and} \qquad a_{i+1} = a_{i-1}/a_i^{n_i}, \qquad i = 0, 1, 2, \ldots.$$

Thus, for $a_0 = 3$, $a_1 = 2$ the n_i sequence $[1, 1, 1, 2, 2, 3, 1, \ldots]$ yields an excellent approximation for the *musical fifth*: $\log 2/\log 3 \approx 12/19$, from which the Greek result, $\left(\frac{3}{2}\right)^{12} \approx 2^7$, follows!

any nine a sixth $\left(2^{9/12} = 1.6818 \approx \frac{5}{3} = 1.6667\right)$,
and any eleven a seventh $\left(2^{11/12} = 1.8877 \approx \frac{15}{8} = 1.8750\right)$.

A piano keyboard has 7 white keys and 5 black keys (12 in all) per octave and can be tuned with such a scheme. Insofar as the human ear cannot

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Other results obtained through the above algorithm are

$$\log 3/\log 5 \approx 13/19 \quad \text{or} \quad 3^{1/13} \approx 5^{1/19},$$

$$\log 3/\log 7 \approx \frac{13}{23} \quad \text{or} \quad 3^{1/13} \approx 7^{1/23}.$$

These relations find application in musical scales with frequency ratio 3: 1 (called *tritave*) for which the basic frequency ratio of a tempered scale is $3^{1/13} = 1.088\ldots$.

Note that $\frac{p_k}{q_k}$, the k^{th} convergent to $\frac{\log a_1}{\log a_0}$, is given by

$$p_k = n_k p_{k-1} + p_{k-2}, \quad q_k = n_k q_{k-1} + q_{k-2}$$

$$(n_0 = 0, \quad p_{-2} = 0, \quad p_{-1} = 1, \quad p_0 = 0, \quad p_1 = 1;$$

$$q_{-2} = 0, \quad q_{-1} = 0, \quad q_0 = 1),$$

where $\{p_k, q_k\}$ are the respective *continuants*:

$$p_k = \begin{vmatrix} n_2 & 1 & & & & & \\ -1 & n_3 & 1 & & & & \\ & -1 & n_4 & 1 & & & \\ & & \cdot & \cdot & \cdot & & \\ & & & \cdot & \cdot & \cdot & \\ & & & & & n_{k-1} & 1 \\ & & & & & -1 & n_k \end{vmatrix}, \qquad k = 2, 3, \ldots;$$

$$q_k = \begin{vmatrix} n_1 & 1 & & & & & & \\ -1 & n_2 & 1 & & & & & \\ & -1 & n_3 & 1 & & & & \\ & & -1 & n_4 & 1 & & & \\ & & & \cdot & \cdot & \cdot & & \\ & & & & \cdot & \cdot & \cdot & \\ & & & & & & n_{k-1} & 1 \\ & & & & & & -1 & n_k \end{vmatrix}, \qquad k = 1, 2, 3, \ldots.$$

perceive the discords caused by the deviations of the tempered ratios for fifths and fourths from their ideal values, the scheme is satisfactory, although to some trained listeners the discord in the major third is on the limit of unpleasantness.

Note that since the fundamental interval is an irrational number, the tempered scale does not possess any simple interval (ratio), a fact which would have driven Pythagoras to despair! Moreover, the notes of which it is composed have no harmonics in common, which is very far from the physicist's conception of the affinity of sounds. Thus the tempered scale is clearly based upon a more complicated mathematical conception than the diatonic scale and could not have been conceived before the invention of logarithms, when nobody could calculate $2^{1/2}$*!*

With this system only 12 notes are needed to play all the tones and half tones of a full scale. The frequencies of the 4^{th} *octave are taken as*

$$\{262, 277, 294, 311, 330, 349, 370, 392, 415, 440, 466, 494, 523\ Hz\}$$

with the corresponding notation

$$\{C, C_{\#} = D_b, D, D_{\#} = E_b, E, F, F_{\#} = G_b, G, G_{\#} = A_b, A, A_{\#} = B_b, B, C\}.$$

The compromise in intonation is rarely greater than 1% for all scales, but the tones of the well-tempered scale are harsh when compared with tones corresponding to the ratio of small integers of the Just scale. A close rational approximation to the well-tempered scale is

$$\left\{1, \textcircled{\tfrac{16}{15}}, \frac{8}{9}, \textcircled{\tfrac{6}{5}}, \frac{5}{4}, \frac{4}{3}, \textcircled{\tfrac{45}{32}}, \frac{3}{2}, \textcircled{\tfrac{8}{5}}, \frac{5}{3}, \textcircled{\tfrac{7}{4}}, \frac{15}{8}, 2\right\},$$

where the circles denote black keys.

***Bela Bartok** (1881–1945, Hungary and USA) based the entire structure of his music on the golden mean* $\left[\frac{1+\sqrt{5}}{2} = 1.6180339\ldots\right]$ *and Fibonacci series. His interest in folk music led him to realize that most Eastern European folk music lies outside the traditional major-minor system. He therefore formed his own type of harmonic system, one which could accommodate melodies not based on a major-minor tonality.*

Bartok used the pentatonic scale *which is perhaps the most ancient human sound system and lies at the basis of the oldest folk melodies and the simplest nursery songs. It rests on the Fibonacci sequence* $\{2, 3, 5, 8\}$, *where the numbers are interpreted as the* number of semitone intervals *separating a* *note from the fundamental tone in the 12-tone chromatic (well-tempered) scale. The black keys on the piano make up a pentatonic scale (successions of 2 and 3 halftones are the intervals between the black notes).*

Bartok also used Fibonacci numbers in an another way. Roughly speaking, the fabric of his music may be imagined to be built up of cells 2, 3, 5, 8 and 13 in size, i.e., the minor second (2 halftones), minor third (3 halftones), fourth (5 halftones), minor sixth (8 halftones), and augmented octave (13 halftones).

In musical notation the notes are: D, E flat, F, A flat, C sharp or $\left\{\frac{9}{8}, \frac{6}{5}, \frac{4}{3}, \frac{8}{5}\right\}$. *Bartok contrasted this Fibonacci scale with a scale obtained by subtracting his halftone series from the 12-tone chromatic scale. The result is (with the exception of one term, the major second* $\frac{9}{8}$, *and with accuracy of 2%) the arithmetic series* $\left\{1, \frac{9}{8}, \frac{10}{8}, \frac{11}{8}, \frac{12}{8}, \frac{13}{8}, \frac{14}{8}, \frac{15}{8}, \frac{16}{8}\right\}$. *We may consider this scale as being based on the overtones of the fundamental note. Thus, the chromatic scale can be separated into Fibonacci and overtone scales, each being a part of the whole and neither of which can exist apart from the other.*

The revolution in orchestral textures (which was one of the most characteristic achievements of the Romantic composers) was only possible because of the intense period of mechanical invention in the early decades of the 19^{th} *century, which led to radical changes in the wind section of the orchestra. Indeed, during the relatively short interval 1820–1847, the flute, clarinet, oboe, and bassoon were improved by means of various mechanical and structural innovations*[373].

There appeared new instruments such as the 4-octaves saxophone (1840, ***Adolph Joseph Sax****, 1814–1894, Belgium), the harmonica (1829,* ***Charles Wheatstone****, 1802–1875, England), the modern orchestral xylophone (1840), the accordion (1822,* ***Friedrich L. Buschmann****, Germany), and the celesta (1886,* ***Auguste Mustel****, France). The revolutionary development in the brass instrument was the introduction of the valve by* ***Heinrich Stölzel*** *(1818, Germany).*

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[373] *Clarinet*: Originated ca 2700 BCE in Egypt. Modernized in 1690 CE by **Johann Christoph Denner** (Germany). The orchestral flute was introduced in 1843 by **H. Klose** and **A. Buffet** in France; Range — 2 octaves.

Oboe: Originated ca 4000 BCE in Egypt. Its final orchestral form was shaped in 1876 in France; Range — 3 octaves.

Bassoon: Originated ca 1500 by **Afranio**, canon of Ferrara. The double bassoon was introduced in 1620 by **Hans Schreiber** in Germany. The Contra Bassoon was introduced in 1739 by **Stanesby** (England). Improved for orchestral use by the physicist **G. Weber** (Germany) in 1825; Range — $3\frac{1}{2}$ octaves.

Flute: Orchestral version introduced in 1847 by **Theobald Boehm**, with improved acoustics and mechanical hole control; Range — 3 octaves.

Every musical instrument consists of some source of vibrations and some arrangement for transmitting the energy to the air with reasonable efficiency. The basic vibration is rarely sinusoidal, but instead consists of a series of pulses of various shapes. The vibrating system is part of or is connected to a resonating system with its own pattern of preferred frequencies of oscillation.

The resonating system responds to the fundamental and overtones of the driving pulse and radiates these into the air with varying efficiency (spectral amplitude response) depending on the frequency of each and the shape of the resonator. The initial vibrator may be a stretched string, or vibrating lips, or vocal chords, or turbulent air in a constricted channel. The resonating system may be a carefully shaped wooden box that can vibrate in response to a whole range of frequencies or it may be a column of air enclosed in a pipe that will respond only to certain harmonic frequencies[374].

Thus, an open end[375] *instrument like a* bugle *of effective length* $L = 1.86\ m$ *produces the fundamental mode* $f_0 = 92\ Hz$ *(taking* $c = 343\ \frac{\mathrm{m}}{\mathrm{s}}$ *for the speed of sound), and its harmonics* $f_n = 184, 276, 368, 460, 553$ *corresponding to the five notes* $F_{\#}(185), C_{\#}(278), F_{\#}(370), B_{\#}(476)$ *and* $C_{\#}(555)$*. A bugler can sound these different pitches (the fundamental sounds like a* nonmusical *growl, and is called the* pedal note*). The F sharp is the lowest* musical *pitch, and comprises two pressure pulses at a time in the tube.*

Since most wind instruments act like pipes open at both ends (with the driving vibration occurring at one end), the fundamental wavelength is approximately twice the length of the pipe, and the frequency of the lowest note that can be used is usually twice that of the pedal note. For that note, 2 different pulses are introduced into the instrument during the time for one round trip.

The length of the pipe is changed by pressing down valves in the case of the trumpet*, by sliding out a length of pipe in the case of the* trombone*, or by opening and closing holes in the side of the tube, which effectively changes its length, in the case of the* flute *or* clarinet*. With the violin, the lowest note produced by the G string is about 4 times the length of the case.*

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[374] In the *mechanical siren*, for example, compressed air blows through holes in a rapidly revolving disc. The frequency is strictly determined by the number of holes passing the air blast each second. Invented in 1820 by the physicist **Charles Cagniard de La Tour** (France).

[375] A *closed end pipe* of length L accommodates a series of harmonics with wavelength $\lambda_n = \frac{4L}{2n+1}$ $(n = 0, 1, 2, 3, \ldots)$, where $\lambda = 4L$ for the fundamental mode. An *open end pipe* of length L has $\lambda_n = \frac{2L}{n+1}$ $(n = 0, 1, 2, 3, \ldots)$ with $\lambda = 2L$ for its fundamental mode.

The fundamental frequency corresponding to the lowest frequency for the human singing voice is around 60 *Hz for a low base, while for the highest pitch it is about* 1300 *Hz for a very high soprano voice. The range of any voice, however, rarely exceeds* 2 *octaves, although there are Iranian singers capable of* 3 *or* 4 *octaves and there is at least one opera singer (**Yma Sumac**[376], b. 1924) capable of a possible range of* 5 *octaves.*

The frequency ranges for musical instruments are much wider than for voice. The frequency corresponding to the lowest pitch that the ear recognizes as sound is about 30 *Hz, yet the piano goes down to a frequency of* 27 *Hz, and some organs descend to* 8 *Hz. At the other extreme, the piano can get up to a frequency of* 4186 *Hz* ($7\frac{1}{2}$ *octaves). There are organs with pipes* 18 *mm long that can go to* 8372 *Hz (the frequencies referred to are those of the fundamental tones, not the higher overtones). In orchestral instruments, the lowest frequency is carried by the harp (32 Hz) and bass viola (41 Hz). The piccolo at* 3729 *Hz has the highest fundamental frequency of the orchestral instruments. Overtones that accompany the fundamentals can go beyond the limits of hearing (20,000 Hz).*

The practical invention of the pianoforte (Hammerklavier) *is due to the Italian* **Bartolomeo di Francesco Cristofori** *(1655–1731,* Padua) *who in* 1709 *had the idea of combining in one the qualities of two keyboard instruments then in use: the clavichord and the harpsichord. Of the former he borrowed the action (struck string), but replaced the metal blade which set the strings in vibration by wooden hammers whose heads were covered with leather. Of the second, the harpsichord, he retained a row of jacks which, fitted with cloth, formed the dampers.*

Although all the mechanical elements of the modern pianoforte are found in Cristofori's early specimens, the pianoforte did not have much initial success; its wide dynamic range did not quite compensate for the dullness of its tone in comparison with that of the harpsichord. However, during the 18^{th} *century its mechanism was further perfected and by the* 1770*'s* ***J.S. Bach**,* ***Haydn*** *and* ***Mozart*** *were writing for it.*

The London cabinet-maker ***John Broadwood*** *invented (1783) the sustaining and damper pedals. By* 1802 *the harpsichord no longer appears in the titles of Beethoven's works and the piano was established in the concert hall. The piano soon became the instrument of the Romantic composers and performers like Chopin, Liszt, the Schumanns, Mendelssohn and others.*

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376 Known as the '*Voice of the Incas*'; Allegedly born in a village in the Peruvian High Andes. The combination of her extraordinary voice, exotic looks and stage personality, made her a hit with American audiences.

Vibrations of pianoforte strings: The quantitative laws governing the vibrational frequencies have been known since 1713, *and can be summarized as follows: the frequency of a string rises proportionally with the square root of the tension, varies inversely with its length, and for fixed tension and length, is inversely proportional to the string's diameter.*

If all strings are to have the same diameter and tension, it turns out that a string-length of 5 cm *for the highest C entails a length of ca* 7 *meters for the lowest C! It was also found that one cannot vary the tension parameter much without affecting low tone quality. If, to avoid these difficulties, one tries to increase the diameter of the strings at the lower end of the scale, one finds that the lowest "wires" would have to be little steel* bars *nearly as thick as a pencil, with a resulting horrible tone. In addition, a wire of this thickness, length, and tension vibrates at a frequency considerably higher than the simple formula $f_0 = \frac{1}{Ld}\sqrt{\frac{F}{\pi\rho}}$ (ideal string of length L, diameter d, mass per unit volume ρ, stretching force F;* **Brook Taylor**, *1713) would lead us to expect.*

A real wire, *on the other hand, has some stiffness in an amount that increases with the diameter. Such a wire vibrates under the influence of two sets of forces: one set arising from the string tension, and the other from its stiffness. As a result, the vibrational shapes and frequencies of all the various modes of oscillation are of a sort of intermediate between those of flexible string under tension and those of a stiff but unstretched bar whose ends are clamped. The equation of motion is*

$$\rho\frac{\partial^2 y}{\partial t^2} = \frac{F}{S}\frac{\partial^2 y}{\partial x^2} - Qk^2\frac{\partial^4 y}{\partial x^4}$$

where S is the area cross-section, k its radius of gyration, and ρ and Q are the density and modulus of elasticity of the material, respectively.

The usual boundary conditions, corresponding to a wire clamped at both ends, lead to the approximation for the fundamental mode $f = f_0(1+\epsilon)$, where $\epsilon^2 = 4\frac{QSk^2}{L^2F}$ and $\epsilon \ll 1$. To get a good musical string *(in which harmonics are whole-number multiples of the lowest frequency) the piano maker wants to use as thin and flexible strings as possible that can stand the highest possible tension.*

Most of the difficulties mentioned above were resolved by practical men over the years without help of formal technical knowledge. In general, the piano makers have had to back away from the ideal in order to get an instrument of practical size, and have arranged things so that the heavily used middle of the piano is good, while the tone quality at the high and low ends is gradually spoiled; down to an octave below the middle C, the strings are *lengthened by a factor of* 1.94 *instead of* 2 *per octave, their diameter is increased by 9.3% per octave, and the tension is reduced to the proper amount to bring the string into tune.*

Below this point the wires lengthen very little, and the pitch is lowered by using strings wound closely with copper wire, in such a way as to add the mass without increasing the stiffness unbearably. The diameter, and therefore the stiffness, of the last unwound string is chosen to match the stiffness of the first wound one, so that an even-sounding scale is obtained.

Around middle C the notes of almost any honestly built piano can sound musical and clear, because the string proportions have not been compromised very much. At the bass end of the scale, however, anyone can hear the difference between the noble sounds issuing from a first-class concert grand and the clumping noises from shrunken pianos sold as pieces of furniture. The reason is that the bass strings of a concert grand are about twice as long as they are in a small piano, so that they can be pulled up to four times tension if the thicknesses are the same.

On a concert grand the tension may go as high as 200 *kg per string, and the total pull distributed over the frame is about* 20 *tons! With forces like these to contend with, a piano designer must be a good mechanical engineer as well as a capable vibration physicist.*

A piano string is set into forceful vibration by means of a hammer blow. The place where the hammer strikes, the fact that the hammer may be soft or hard, wide or narrow, and the strength of the blow are among the prime factors that determine the spectral content of the pulse around the center frequency of the struck key.

***Thomas Young** discovered (1800) that no possible mode of vibration can be set up that has a node at the position at which the disturbance is applied. **Helmholtz** (1862) advised to place the hammer strokes* $\frac{1}{7}$ *to* $\frac{1}{9}$ *along the length of the string because that was the distance that would make the* 7^{th} *and* 9^{th} *harmonics weak and since these are less consonant, the tone would be more pleasant without them. Others have found that placing the hammer close to, but not at* $\frac{1}{8}$ *of the length, enhances the fundamental and gives a strong and full tone.*

The quality of the tone is due to the strength and number of the harmonics, and these change with the speed of which the hammer strikes the string. However, when notes are struck simultaneously and in succession, then the time intervals between them may generally determine the quality of the tone produced. In general, the duration of a harmonic depends on its amplitude and because of this, weak harmonics die out sooner than strong ones. The *duration may be extremely short and a fraction of a second difference between the onset of two notes can alter the total picture of the number of harmonics that are present at any instant. It is probably the variation of these extremely small time sequences between succeeding notes that is meant by touch.*

The motion of violin strings: The violin is a peculiarly shaped box on which 4 strings are stretched so that they run over a bridge that couples their vibrations to the box and its enclosed air. Sounds are brought out of this device by rubbing the strings with a bow, and the player chooses different pitches by pressing down with his fingers on one or the other of the bowed strings in such a way as to shorten its vibrating length. The 4 strings of the violin are tuned a fifth apart at G_3*,* D_4*,* A_4 *and* E_5*.*

The highest fundamental that can be played is B_7 *(3951 Hz). The G-string is wound with a spiral of fine aluminum or silver wire to provide it with sufficient mass to have the desired low fundamental. The violin string is excited by the friction of the moving bow*[377]*. The number of bow hairs in*

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[377] The motion of the bow-driven string with friction, moving between two rigid supports, is governed by the equation:

$$\frac{\partial^2 y}{\partial t^2} + 2k\frac{\partial y}{\partial t} - c^2\frac{\partial^2 y}{\partial x^2} = f(x,t),$$

where y is the string's lateral displacement, T the fixed tension of the string (given), $c = \sqrt{\frac{T}{\mu}}$, $k = \frac{R}{2\mu}$, $f(x,t)$ is the applied force per unit mass of the string [with $f(x,t) = 0$ for $t < 0$], R is the frictional force of the medium per unit length per unit velocity, and μ is the string's mass per unit length. The solution can be given in the form

$$y(x,t) = \int_a^b G(x,x_0;t)dx_0,$$

where $G(x,x_0;t)$ is a solution of the above equation for a point force at $x = x_0$, namely

$$\frac{\partial^2 G}{\partial t^2} + 2k\frac{\partial G}{\partial t} - c^2\frac{\partial^2 G}{\partial x^2} = \delta(x-x_0)f(x_0,t),$$

and (a,b) is that part of the string over which the force is applied. It can be shown that if

$$f(x_0,t) = f(t) = \frac{1}{2}P_0 a e^{-a|t|},$$

then for $t > 0$,

$$y(x,x_0;t) = \frac{2P_0 a^2 c^2}{L}\sum_{n=1}^{\infty}\left\{\frac{\sin(\pi n x/L)\sin(\pi n x_0/L)}{\omega_n W_n^2}\right\}e^{-kt}\sin(\omega_n t - 2\phi_n)$$

contact with the string is changed by varying the angle and the plane that the bow makes with the strings. Studies on bowing have shown:

(1) increased bow pressure tends to increase the intensity of the Fourier components above the fundamental;

(2) intensity of tone depends on the speed of bowing and the number of bow hairs in contact with the string;

(3) the closer the bow is to the bridge, the more prominent are the higher Fourier components.

The air in the box is capable of vibrating at any of a large number of resonant frequencies. If the motion of the wooden enclosure is at one or another of these cavity resonance frequencies, the air will radiate strongly into the room by way of the f-holes cut in the belly of the violin. The top of the bridge vibrates back and forth on its slim waist when it is driven at certain frequencies, and thus alters the forces which it passes on to the violin belly in a way strongly depending on frequency. Thus, the sound radiated from the violin's body is controlled by the bow-string-bridge system. Physicists have obtained the response of a violin to a single exciting frequency (not the same as that for a bowed violin):

The results show a peak at about 300 Hz (D_4)*, which is due to the air resonating in the body of the violin, and another due to the vibration of the body of the violin at about 440 Hz* (A_4)*. These peaks may be up or down by at least a semitone in various different violins. A comparison between good and poor violins shows that fine instruments seem to have frequency-response*

$$+\frac{P_0 a e^{-at}}{2k'_a}\begin{cases}\dfrac{\operatorname{sh}(k'_a x)\operatorname{sh}[k'_a(\ell-x_0)]}{\operatorname{sh}(k'_a L)}, & x < x_0\\[2ex] \dfrac{\operatorname{sh}(k'_a x_0)\operatorname{sh}[k'_a(\ell-x)]}{\operatorname{sh}(k'_a L)}, & x > x_0\end{cases}$$

where

$$k'_a = \tfrac{a}{c}\sqrt{1-\tfrac{2k}{a}}, \qquad \omega_n = \tfrac{\pi n c}{L}\sqrt{1-\left(\tfrac{kL}{\pi n c}\right)^2},$$

$$W_n^2 = [\omega_n^2 - k^2 + a^2]^2 + 4k^2\omega_n^2, \qquad 2\phi_n = \operatorname{tg}^{-1}\tfrac{2\omega_n^2}{W_n}.$$

The response consists of a term having the behavior of the forcing function plus a series representing the free vibration of the system caused by the discontinuity of the forcing function at time $t = 0$.

curves that are fairly uniform over the range from 300 *to* 4000 *Hz, and fall almost linearly from* 4000 *to* 6000 *Hz.*

If one compares response curves of a violin made by ***Antonio Stradivarius*** *(1644–1737, Italy; made* 1116 *violins during* 1700–1725 *of which very few genuine specimens remain) with those of modern scientifically constructed violins, one is struck by the similarity of the resonance curves.*

Organ: A keyboard musical instrument. While a piano makes sounds by causing steel strings to vibrate, a pipe organ *creates sound by forcing air through metal or wooden tubes called pipes. It is the largest and most powerful of all musical instruments, and may have more than* 5000 *pipes, each producing a different frequency. The longest pipes, producing the lowest notes, may be more than* 9 *meters long and* 30 *cm in diameter. The smallest pipes, which produce the highest notes, are only* 18 *cm long and less than* 6 *mm in diameter. Some organs have* 6 *keyboards. Organ pipes are designed to play only the* fundamental frequency. *Consequently, there is only one pressure pulse at a time in the pipe (with other wind instruments, however, the driving vibrator can be controlled to send in one, two, three, or more pulses into the pipe before the first one has returned).*

In ca 250 *BCE,* ***Ctesibios of Alexandria****, a Greek engineer and inventor, built an organ that used water power to force air into the pipes. The major features of the modern organ were developed during* 200–1600 *CE. By* 1900*, interest in the organ had declined among composers and performers, and the instrument was played regularly only in churches as part of religious services. In 1934,* ***Laurens Hammond*** *(U.S.A.) patented the first commercially practical electronic organ.*

Leonard Bernstein on Music

"The genius of Johan Sebastian Bach was to balance so delicately and so justly the two forces of chromaticism and diatonicism that were equally powerful and presumably contradictory.

What makes music dramatic? Contrast; duality of two tones, two contrasting ideas or emotions within a single movement. Contrast makes drama — black against white, good against evil, day and night, grief and joy.

Bach represents the last stand against the dualistic concept. Any single movement is always concerned with one single idea. He clung to the older concept of one thing at a time — grief or joy, day or night: once the theme *is stated at the beginning, the main event is over. The rest of the movement will be a constant elaboration, recitation, and discussion of that main event. But if you are expecting any change in mood – say, a sudden yielding to sentimentality or lyricism – you are not going to get it. Contrast is there all right, but it is restricted to laud and soft, or key change, or different instrument grouping, but the dramatic contrast of themes is not there.*

Consider the vast catalogue of Bach's output: songs, dances, suites, partitas, sonatas, tocatas, preludes, fugues, cantatas, oratorios, masses, passions, fantasies, concertos, chorales, variations, motets, passacalias – the creation of fifty years. What is that holds all these pages together, that makes it all inevitably the product of one man — the religious spirit.

For Bach, all music was religion; writing was an act of faith; and performing was an act of worship. Every note was dedicated to God and nothing else. This is the spine of Bach's work: simple faith. Otherwise, how could he have ever turned out all that glorious stuff to order, meeting deadlines, playing the organ, directed a choir, taught school, instructed his army of children, attended board meetings and keeping his eye out for better-paying jobs.

Bach was a man, after all, not a god; but he was a man of God, and his godliness informs his music from first to last."

* * *

"In the opera, the basic human emotions are pinpointed and magnified way beyond life size so that you can't miss them. Each emotion comes at you gigantically, in a clear direct, uncluttered, full-blown way. One of the chief reasons for the direct power of opera is that it is sung: Indeed, among of all the different instruments in the orchestra, there is none that can compete in any way with the expressivity of the human voice. And when such a voice, or several, or many together, carry the weight of a drama, then there is nothing in all the theater to compare with it for shear immediacy of impact. Now these emotions are not merely presented to us; they are hurled at us. You see, music is something very special. It does not have to pass through the censor of the brain before it can reach the heart; it goes directly to the heart."

1862–1889 CE **Eugenio Beltrami** (1835–1899, Italy). Distinguished differential-geometer. Professor of mathematics at the Universities of Pisa, Pavia and Rome. Placed *hyperbolic geometry* on a firm foundation, developed the Riemannian calculus of n-dimensional manifolds, and discovered the so-called 'differential parameters' that bear his name. Contributed to the mathematical theory of elasticity, optics and thermodynamics.

He brought Riemann's work into connection with non-Euclidean geometry. In his work "*An attempt to interpret the non-Euclidean geometry*" (1868), he demonstrated that the plane geometry of Lobachevsky-Bolyai holds on surfaces of constant negative curvature embedded in Euclidean space, straight lines being replaced by geodesics. Such surfaces are capable of a conformal representation on a plane, in which geodesics are represented by straight lines[378]. Interest in hyperbolic geometry was rekindled in the 1860's when unpublished work of **Gauss** (d. 1855), came to light. Learning that Gauss has taken hyperbolic geometry seriously, mathematicians became more receptive to non-Euclidean ideas.

The works of **Lobachevsky**, **Bolyai** and **Minding** were rescued from obscurity and, approaching them from the viewpoint of differential geometry, Beltrami was able to give them the concrete explanation that had eluded all his predecessors. He was interested in the geometry of surfaces and had found the surfaces which could be mapped onto the plane in such a way that their geodesics went to straight lines. They turned out to be just the surfaces of constant curvature. In the case of positive curvature (the sphere), such a mapping is a central projection onto a tangent plane, though this maps only half of the sphere onto the whole plane. The mapping of surfaces of constant negative curvature, on the other hand, take the whole surface onto only part of the plane.

He derived (1892) the so-called *Bertrami-Michell compatibility equation* in linear elasticity theory, serving as *integrability conditions* in terms of the components of the stress tensor and the applied forces.

Beltrami was born in Cremona to an aristocratic family. He was educated at the University of Pavia under Brioschi. During 1856–1861 he was a secretary to a railroad engineer, but in 1862 returned to the academia as a professor of rational mechanics in Bologna (1862–1864), Pisa (1864–1866), Bologna (1866–1873), Rome (1873–1876), Pavia (1876–1891) and again Rome (1891–1899).

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[378] Beltrami's method allows us to map only a *part* of the Lobachevsky plane on a *part* of a surface of negative curvature. **Hilbert** has shown that it is impossible to continue an analytical surface of constant negative curvature indefinitely without meeting singular lines when this surface is embedded in ordinary Euclidean space.

1862–1899 CE **Ernst Heinrich (Philipp August) Haeckel** (1834–1919, Germany). Biologist, physician, eugenicist and philosopher. Popularized Darwinism in Central Europe and applied it to some of the oldest problems of philosophy and religion. Outlined the essential elements of modern zoological classification (1864). Hypothesized that the nucleus of a cell contains *hereditary information* (1866). First to use the term *ecology* (1866) to describe the study of living organisms and their interaction with other organisms and with their environment. Haeckel's attempt to describe human evolution in racial terms later became a part of the pseudo-scientific basis for *Nazism*.

He was born at Potsdam and studied medicine and science at Würzburg, Berlin and Vienna. Graduated at Berlin as M.D. (1857). At the wish of his father he began to practice as a doctor in that city, but his patients were few in number, and after a short time he turned to more congenial pursuits. He came to the University of Jena in 1861 and was later appointed to the chair of zoology (1865–1909). He was on scientific expeditions to the Canary Islands (1866–1867), Red Sea (1873), Ceylon (1881–1882), and Java (1900–1901).

It happened that just when he was beginning his scientific career Darwin's *Origin of the Species* was published (1859), and such was the influence it exercised over him that he became the apostle of Darwinism in Germany. He therefore gave a wholehearted adherence to the doctrine of organic evolution and treated it as the cardinal conception of modern biology. He was first to draw up a genealogical tree relating all the various orders of animals, showing the supposed relationship of the various animal groups.

Haeckel tried to discover (1862) the symmetry of crystallization in Radiolarians (one-celled sea animals). Promoted the theory of *recapitulation* which states that the embryo repeats the evolutionary changes that its ancestors underwent.[379]

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[379] Haeckel believed that the development of the embryo imitated an organism's entire evolution as a species. He supported his theory with embryo drawings that have since been shown to *deliberately faked* to get more support for his ideas (almost every biology book for the past century has included pictures of vertebrate embryos made by him, purportedly demonstrating the amazing similarity of fish, chickens, and humans in the womb).

In the 1990's, British embryologist **Michael Richardson** was looking at vertebrate embryos through a microscope and noticed that they look nothing at all like Haeckel's drawings. Richardson and his team of researchers examined vertebrate embryos and published actual photos of the embryos in the August 1997 issue of the journal *Anatomy and Embryology.* It turned out that Haeckel had used the same woodcuts for some of the embryos and doctored others to make sure that the embryos looked alike. Indeed, Haeckel's drawings turned out to be one of the most famous fakes in biology. It turned out that this has

His integrated views on the philosophical implications of the theory of evolution were published under the title *Die Welträtsel* (1899), which in 1901 appeared in English as the *Riddle of the Universe*. In this book, adopting an uncompromising *monistic* attitude, he proposed that all nature is a unity with life originating in crystals and evolving to man; matter alone is the one fundamental reality (material monism); the mind depends upon the body, and hence does not survive after it; animals with a central nervous system possess consciousness. He believed in the singularity of essence of both the organic and the inorganic, and rejected religions and their ideas of God.

According to his "carbon theory", the chemico-physical properties of carbon in its complex albuminoid compounds are the sole and the mechanical cause of the specific phenomena of movement which distinguishes organic from inorganic substances, and the first development of living protoplasm arose from such nitrogenous carbon compounds by a process of spontaneous generation.

He regarded psychology as merely a branch of physiology, and psychical activity as a group of vital phenomena which depend solely on physiological actions and material changes taking place in the protoplasm of the organism in which it is manifested. Every living cell has psychic properties, and the psychic life of multicellular organisms is the sum-total of the psychic functions of the cells of which they are composed.

Moreover, just as the highest animals have been evolved from the simplest forms of life, so the highest faculties of the human mind have been evolved from the soul of the brute beasts, and more remotely from the simple cell-soul of the unicellular Protozoa. As a consequence of these views Haeckel was led to deny the immortality of the soul, the freedom of will, and the existence of a personal God[380].

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been known for a century!!. **Stephen Jay Gould** responded in the March 2000 issue of *Natural History* magazine, saying he had known all along. But Darwinists kept mum because Haeckel's crackpot theory constituted one of the main pieces of evidence in support of evolution.

Oddly enough, in 2005, the *New York Times* reported that biology textbooks were still running Haeckel's fake drawings. The *Times* specially singled out the third edition of *Molecular Biology of the Cell*, the bedrock text of the field, as one of the culprits.

380 **D'Arcy Wentworth Thompson** reacted to these views in his *On Growth and Form* (1917): "*Many a beautiful protozoan form has lent itself to easy physico-mathematical explanations; others, no less simple and no more beautiful, prove harder to explain. Nature keeps some of her secrets longer than others*".

1862–1921 CE **Josef Popper-Lynkeus** (1838–1921, Austria). Inventor, scientist, social thinker and humanist. Held in high esteem by **Mach**, **Freud** and **Einstein**. Foreshadowed some of the fundamental ideas of aerodynamics, electric power transmission, relativity and quantum physics that were later formulated by others. Attempted to enunciate a general science of energetics. Anticipated Freud's essential characteristics and most significant part of *dream theory* (the reduction of dream-distortion to an inner conflict). Rejected, years ahead of **L.E.J. Brouwer**, the logical *principle of the excluded middle*.

Josef Popper was born in the Jewish ghetto at Kohlin, Bohemia and lived therein up to his 15^{th} year. There he attended the elementary school where he was educated in a devoutly religious environment. In 1854 he was admitted to the Polytechnikum in Prague, where for three years he majored in physics, mathematics and engineering. He continued his studies at the University of Vienna but being a Jew he could not obtain an academic position[381] (although recommended by his teachers!). However, his income from his invention royalties enabled him to live humbly and pursue his writings. His major achievements were:

- Conducted pioneering experiments and proposed ways of transmission of electric power from its natural sources [1862; *Die Physikalischen Grundsätze der elektrischen Kraftübertragung*, 1883); (Physical Principles of Electric Power Transmission)].
- Suggested a connection between the laws of conservation of *mass and energy* (1883).
- Suggested an experiment to establish the existence of an *energy-quantum* through which one could interpret the periodic table of the elements. (In a letter to Mach, 1884.)
- Elaborated on the principles of *heavier-than-air flight* (*Flugtechnik*, 1888; *Der Machinen und Vogelflug*, 1911 = Mechanical Aviation and the Flight of Birds).
- Proposed a social reform plan in which he viewed the state as no *more* than a utilitarian association to assure security of existence for individuals living on a common soil, and to lighten the burden of their lot on this earth. He rejected compulsory military service; no one should be compelled to kill or to be killed. Even the freedom of criminals should be curtailed no *more* than the protection of society requires.

[381] Unlike Karl Marx and many others he was a proud Jew and refused to convert to Christianity for the sake of a university position.

> However, every man should give a decade of his lifetime to work for the state in order to assure (through organized production of all truly necessary means) a secure existence to each individual for the rest of his life. All other economic endeavor should be completely free. [*Das Recht zu leben und die Pflicht zu sterben* (The Right to Live and the Duty to Die), 1878; *Das Individum und die Bewertung menschlicher Existenzen* (The Individual and Evaluation of Human Existences), 1910; *Die allgemeine Nährpflicht als lösung der sozialen Frage* (The Obligation of Securing a Guaranteed Subsistence for all as the Solution to the Social Problem), 1912; *Krieg, Wherpflicht und Staadsverfassung* (War, Military Service and the State Constitution), 1921.]

Sigmund Frued touched upon the hidden background that linked him with Popper (1899):

"*A special feeling of sympathy drew me to him, since he too had clearly had painful experience of the bitterness of the life of a Jew and of the hollowness of the ideals of present-day civilization*".

Albert Einstein elucidated the unique phenomena of a scientist as a humanist and moralist (1954):

"*Popper-Lynkeus was a prophetic and saint person, and at the same time a thoroughly modern man. In love with the natural sciences and modern technology, he remained throughout a long, strenuous, and difficult life steadfastly true and dedicated to the aim he set for himself — that of contributing to the improvement of the lot of mankind and to their moral advancement. He affirmed passionately the technological advancement of our age as a liberator from soul-destroying physical labor and as the originator of cognition and creativeness which he loved for their own sake, for their own beauty*".

1863 CE **Luigi** (**Antonio Gaudenzio Giuseppe**) **Cremona** (1830–1903, Italy). Mathematician. Known for his work in projective geometry. The birational *Cremona Transformation*[382] is named after him.

Cremona was born to Jewish parents. He was a professor in Bologna (1860), Milan (1866) and Rome (1873). He became senator (1879) and Minister of Education (1898).

[382] A transformation of a plane curve in a plane: (x, y) goes into

$$(R_1(x, y), R_2(x, y)),$$

where R_1 and R_2 are rational algebraic functions. He later generalized it to a rational transformation in 3-dimensional space.

1863–1898 CE *Underground railway*: The growing congestion of 19th century urban traffic prompted **Charles Pearson** (1843) to suggest the building of underground tunnels in London through which railway lines could be laid. The project was approved in 1853 and construction began in 1860. The project was successful, for it transported some 10 million passengers in its first year of service (1863). The London network expanded and became electrified (1890). Similar projects followed in Glasgow (1886), Boston, Budapest and Paris (1898).

1863 CE **Pietro Angelo Secchi** (1818–1878, Italy). Astronomer. First to classify stars into four major classes according to the general arrangement of the dark lines in their spectra. Proved that prominences seen during solar eclipses are features of the sun itself. This marks the nascence of *stellar spectroscopy*.

During 1814–1823, **Joseph von Fraunhofer** compared the spectra of the sun and the stars, but the first fairly comprehensive attempt at classification was undertaken by Secchi "to see if the composition of the stars is as varied as the stars are innumerable". He noticed that while the stars are innumerable, their spectra can be grouped in certain distinct groups. He observed the spectra visually by attaching a spectroscope to his telescope and pointing it toward the stars.

In those days, the nature and cause of spectral lines were not well understood, and astronomers classified each star by assigning a letter from A through P, depending on the strength of the hydrogen Balmer lines (**Johann Jakob Balmer**, 1825–1898, Switzerland) in the star's spectrum. The A stars have the strongest Balmer lines and the P stars have the weakest. Secchi was unable to see the fainter lines, which were not observed until the application of photography to this study.

Secchi was born at Reggio in Lombardy and entered the Society of Jesus at an early age. In 1849 he was appointed director of the Vatican Observatory. There he devoted himself with great perseverance to researches in physical astronomy and meteorology. He completed the first spectroscopic survey of stars, cataloging the spectrograms of about 4000 stars (1868).

1863–1864 CE **Arminius Vambery** (Herman Wamberger, 1832–1913, Hungary). Orientalist and explorer. The first European to explore Turkestan, Uzbekistan and Afghanistan.

He was born of poor orthodox Jewish parents at Duna-Szerdahely, a village on the island of Shütt, on the Danube near Pozsony. The name of the family was originally Bamberger. He got an orthodox Jewish education, and studied the Talmud in the village school until the age of 12. His mother destined her son, who was lame, for the trade of a dressmaker. After being for a short time apprentice to a ladies' tailor, he became a tutor to an innkeeper's son. With the aid of friends he was enabled to enter the Gymnasium during the difficult and unsettled time of the revolution of 1848, but lack of resources forced him to quit school and go to Slavonia as a tutor. From then on he educated himself and at 20 gained knowledge in 15 European and ancient languages.

Later he studied at Vienna and Budapest and turned his attention to the study of Turkish and Arabic. In 1854 he migrated to Constantinople, where he worked as a tutor. During his 6-year stay there he acquired some 20 oriental languages and Turco-Tartar dialects, and published a Turkish-German dictionary. He became a secretary to Fuad Pasha and, for all practical purposes, a Muslim.

In 1861, the Hungarian Academy sponsored his journey to Central Asia. Under the name of Reshid Effendi, and in the guise of a Sunnite dervish, he traveled with Muslim pilgrims across the Turkestan desert to Khiva, Bokhara and Samarkand. He experienced hardships rarely sustained by a European before, braving the risk of being detected and put to death by the offended Muslims. He left the pilgrims and continued to travel to Herat in Afghanistan.

In November 1863 Vambery left Herat for Meshed, having joined a caravan of pilgrims and merchants. This was the first journey of its kind undertaken by a European; and since it was necessary to avoid suspicion, Vambery could not take even fragmentary notes except by stealth. He returned to Europe in 1864, and in the next year received the appointment of professor of oriental languages in the University of Budapest, retiring therefrom in 1905.

Earlier he became an adherent of the Protestant faith. On several occasions he carried out diplomatic missions for Great Britain in the Near East. He became a personal friend of the Prince of Wales, later King Edward VII. Sultan Abdul Hamid consulted him on problems of foreign policy, and **Theodor Herzl** (1860–1904) enlisted Vambery's aid in his negotiations with the sultan on behalf of his Zionist movement.

1863–1873 CE **John Tyndall** (1820–1893, Ireland). Experimental physicist, educator, pioneer researcher of the physics of the atmosphere, science writer and alpinist. Discovered[383] (1869) the *Tyndall effect* – the scattering of light by invisibly small colloidal particles in solution, thus making the light

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[383] **Leonardo da Vinci** understood the basic phenomenon around 1500 CE. In particular, his experiments with the scattering of sunlight by wood smoke observed against a dark background [see: *The Notebooks of Leonardo da Vinci*, Dover edition]. The phenomenon was confirmed (1871) by theoretical studies of **Lord Rayleigh** and known as the *Rayleigh scattering*.

It is the incoherent scattering of electromagnetic radiation (light) by gas mole-beam visible when viewed from the side. Suggested that the blue color of the sky is due to greater scattering of the shorter wavelength blue light by the colloidal particles of dust and water vapor in the atmosphere [beyond the atmosphere the sky is black!]

Tyndall was born in Leighlin-Bridge, County Carlow, Ireland. Self educated. Employed as a railway engineer (1844) and college teacher (1847) before studying physics (1848–1851) at the University of Marburg (under **Bunsen**) and Berlin (under **Magnus**), obtaining his PhD in 1851.

He became professor of natural philosophy at the Royal Institution (1854) and the Royal School of Mines in London (1859–1868). Tyndall toured the US 1872–1873. He died from accidental poisoning with chloral. Apart from the *Tyndall effect*, his other investigations and discoveries are:

- confirmed Pasteur's claim of the fallacy of the doctrine of *spontaneous generation* by showing that air contains living organisms (1881);
- carried out experimental work on the absorption and transmission of heat by water vapor and atmospheric gases which was important in the development of *meteorology*;
- the first scientist to describe the *greenhouse effect* for water vapor (1863);
- discovered that the transmission of sound is affected by variation of density in the atmosphere;

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cules or other randomly distributed electric dipole scatterers in accordance with the $\{\sin^4\theta/\lambda^4\}$ distribution of the flux density of an oscillating dipole field [$\theta =$ scattering angle, $\lambda =$ wavelength]. This explains the blueness of the sky, the redness of sunrise and sunset, and the scattering of radar waves by droplets of ice crystals.

The theory correctly describes scattering by *molecules*, as well as tiny spherical particles whose radius is smaller than about $(0.03)\lambda$. When the diameter is comparable to the wavelength of the incident radiation, *Mie scattering* theory takes into account the particle size and the dipole model is inadequate. The Mie theory predicts weak dependence of the scattered field on wavelength. This is significantly different from Rayleigh scattering, and because of this the *clouds* are white and the sky is blue.

Rayleigh scattering is also accompanied by *polarization*. Unpolarized incident light that is scattered through $90°$, is linearly polarized in the direction perpendicular to the plane of incidence. Thus, unpolarized sunlight becomes almost completely polarized upon scattering through an angle of $90°$ by air molecules.

* made the first demonstration (1870) of the guiding of light by internal total reflection. In front of an audience of the Royal Academy of London, he demonstrated that light illuminating the top surface of water in a pail can be guided along a semi-arc of water streaming out through a hole in the side of the pail — a precursor of *fiber optics*.

1863–1875 CE **William Huggins** (1824–1910, England). Astronomer. First to identify some of the lines of stellar spectra with those of known terrestrial elements (1863). Huggins made the first radial-velocity determination of a star in 1868. He observed the Doppler red-shift[384] in one of the hydrogen-lines in the spectrum of Sirius, and found that the star was receding from the sun at a velocity of about 45 km/sec.

Huggins was born in London. He built in 1856 a private observatory in the south of London. Kirchhoff's discoveries in spectrum analysis turned his attention to the problem of the internal constitution of stars. The advent of photographic astronomy led him in 1875 to adopt and adapt the *gelatin dry plate*, enabling him to obtain stellar spectrograms with exposures of any desired length and thus produce permanent accurate pictures of celestial objects so faint as to be completely invisible to the eye, even when aided by a

[384] The radial or "line-of sight" velocity of a star can be determined from the Doppler shift of the lines of its spectrum, using the formula

$$\frac{\Delta\lambda}{\lambda} = \sqrt{\frac{1+v/c}{1-v/c}} - 1 \approx \frac{v}{c}$$

(for small v/c). Here c is the speed of light in vacuum, λ is the wavelength emitted by the *source*, and $\Delta\lambda$ the difference between λ and the wavelength measured by the *observer*. v is the relative line-of-sight velocity of the observer and the source, which is counted as positive if the velocity is one of recession and negative if it is one of approach. If a star approaches (recedes) from us, the wavelengths of light in its continuous spectrum appear shortened (lengthened), as well as those of the dark lines. However, unless its speed is tens of thousands of km/sec, the star does not appear noticeably bluer or redder than normal. The Doppler-shift is thus not easily detected in a continuous spectrum (except for very remote galaxies) and cannot be measured accurately in such a spectrum. On the other hand, the wavelengths of the absorption lines can be measured accurately, and their Doppler-shift is relatively simple to detect. The known wavelengths of the bright lines in the spectrum of a laboratory source (such as the bright lines in the spectrum of the arc lamp) serve as standards against which the wavelengths of the dark lines in the star's spectrum can be accurately measured.

powerful telescope. [His results were, however, affected by serious systematic errors. During 1888–1891, German astronomers at the Potsdam Observatory reduced the average probable error in the radial velocity measurements to only 2.6 km/sec.]

In 1900 Huggins, commenting on the future of radial-velocity determinations, concluded: "*This method of work will doubtless be very prominent in the astronomy of the near future, and to it probably we shall have to look for the more important discoveries in sidereal astronomy which will be made during the coming century*".

1863–1875 CE Global attack of *cholera*. In 1866, ca 300,000 died in Europe. Worldwide deaths were in the millions.

1863–1892 CE **Francis Galton** (1822–1911, England). Scientist. A cousin of Charles Darwin. Pioneer in modern meteorology and statistics. The father of racist 'eugenics', to be later embraced by Hitler in his *Mein Kampf*.

Invented the *teletype printer* (1850). Introduced modern weather-mapping techniques and established the existence and theory of *anticyclones* (high-pressure areas of the atmosphere) in his book *Meteorographica* (1863); it was the first serious attempt to chart the weather on an extensive scale.

Noted the uniqueness of each individual's *fingerprints* and worked out a system of classifying them (1885). His work became important for law enforcement through *fingerprint identification*.

Introduced the concept of *correlation* (the measure of interdependence of two sets of variables) and defined the useful *correlation coefficients* (1888–1889). Those he incorporated in his statistical techniques related to *genetics* and his pioneer use of statistics in psychological measurements[385].

His name, however, is most closely associated with studies in anthropology and especially *heredity*, which he expounded in *Hereditary Genius* (1869), *Human Faculty* (1884) and *Natural Inheritance* (1889).

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[385] He studied, for example, the familial tendency to inherit brilliance, or the related problem of the extinction of family surnames. In this connection he raised the question [known as the *Galton problem* (1873)]: If each male in a population has a family with x sons, where the random variable x has the distribution $F(s) = \sum_{x=0}^{\infty} p(x)s^x, \ \ |s| \ll 1;$ and so on for the next generation, what is the probability $p(x)$ of any particular male-line dying out?

The first solution of this problem was given by H.A. Watson (1874), and the complete solution by Steffensen (1930).

Galton (1883) coined the word '*eugenics*' (from the Greek 'good in birth' or 'noble in heredity'). Eugenics was defined as the science of improving the human stock through systematic selective breeding, by checking the birth-rate of the *unfit* (paupers, insane, physically impaired and feeble-minded) and furthering the production of the *fit* (talented, gifted, healthy, beautiful, etc.). Galton thus endeavored to improve or impair the racial qualities of further generations, either physically or mentally. He thus recommended forced sterilization of "unfit" humans, saying they could not be persuaded to stop breeding on their own. Eugenics, he said, "must be introduced into the national consciousness as a new religion". Hitler's worldview was based on the evolutionary ethic of Galton and his followers.

For Galton, science and progress were almost inseparable. Men could be improved by scientific methods, in the same way that plant and horse breeders improve their stock. Would it not, he wondered, be "*quite practicable to produce a highly gifted race of men by judicious marriages during several consecutive generations*?" The scientific assumptions behind this were explicit: most human attributes are inherited. His program was derived from ideas about natural selection and evolution. Not only was talent perceived of as being inherited, so too were pauperism, insanity and any kind of perceived feeble-mindness.

Galton graduated in medicine from Cambridge (1844). After inheriting ample fortune he was able to abandon his medical career, holding no scientific or teaching posts. Instead, he set out to see the world, traveling in Europe, Asia Minor, the Holy Land and southwest Africa.

Eugenic ideas may be detected as early as **Plato** (427–347 BCE), but eugenic became significant only after the publication of **Charles Darwin**'s (1809–1882) *Origin of the Species* (1859), which implied that man was the outcome of a natural process of evolution. Galton's campaign on behalf of eugenic breeding stimulated a popular social movement (from 1900) and the formation of centers of eugenic study in Britain, America, the Soviet Union and Nazi Germany[386].

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[386] The ideas of Galton, amplified by Karl Pearson from University College London, received support from a variety of sources, which included Fabians such as Bernard Shaw and psychologists like **Havelock Ellis**.

In the United States, **Charles Davenport** (1904) came to believe that certain races were feeble-minded. To this end he favored a selective immigration policy coupled to the prevention of reproduction of the genetically defective. The list of distinguished scientists that initially gave eugenics positive support is impressive: **Ronald Aylmer Fisher** (1890–1962), **J.B.S. Haldane** (1892–1964), **J.S. Huxley** (1887–1975), **W.E. Castle** (1867–1962), **T.H. Morgan** (1866–1945).

1864–1875 CE **James Croll** (1821–1890, Scotland). Geophysicist. Presented an astronomical theory of the ice ages caused by periodical changes in the earth's *orbital eccentricity* (100,000 year cycle), and the *precession of the equinoxes* (22,000 year cycle). Croll noticed the relevance of the periodic variation in the tilt of the earth's axis to the insulation and heating of the polar regions. But since he depended on the calculations of **LeVerrier**[387], he did not follow the consequences of this important line of reasoning, save the qualitative notion that ice ages would be more likely to occur during periods when the axis is closer to vertical, for then the polar regions receive a smaller amount of heat.

Croll plotted orbital changes during the past 3 million years, and found cyclical changes with long intervals of low eccentricity and long intervals of high eccentricity. He then concluded that ice ages occurred during periods of *high eccentricity*, alternating from the Northern to the Southern Hemisphere in response to the 22,000 year precession cycle.

Wrongly believing the crucial factor to be minimum winter solar radiation, Croll postulated that when the eccentricity is high, the hemisphere whose winter occurs at the time of the earth's farthest distance from the sun will

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However, in the 1930's, Huxley, Haldane, **Hogben** and other biologists at last began to react against many of the wilder claims for eugenics. But it was too late, for the ideas had permeated into mainland Europe, and especially into the ideology of the German National Socialists. They claimed that there is a biological basis for the diversity of mankind: what makes a Jew a Jew, a Gypsy a Gypsy is in their blood, that is to say in their genes — all this based on the genetic ideas of the eugenic movement. Thus it is quite easy to see the direct line from the eugenic movement to the statement by the animal behaviorist **Konrad Lorenz** (1935; Nobel prize, 1973): "*It must be the duty of racial hygiene to be attentive to a more severe elimination of morally inferior human beings than is the case today… . This role must be assumed by a human organization; otherwise humanity will be annihilated by the degenerative phenomena that accompany domestication*".

Another metaphor from Lorenz is the '*analogy between bodies and malignant tumors on the one hand, and a nation and individuals within it who have become asocial because of their defective constitution*'.

In 1933, the Nazi Cabinet promulgated a *Eugenic Sterilization Law* which can be considered as leading to the atrocities by doctors and others in the Nazi concentration camps.

387 In 1843 **LeVerrier** used perturbation theory to show that in the past 100,000 years, the earth's orbital eccentricity has varied from a low near zero to a high about 6%. He calculated that the tilt of the earth's axis fluctuates within the range $23\frac{1}{2}^\circ \pm 1\frac{1}{2}^\circ$, but did not determine the period of this motion.

experience an ice age. Nevertheless, he was the first scientist to develop the idea now referred to as *positive feedback*[388].

James Croll was born in a peasant family at Little Whitefield in Perthshire. Lacking any formal education, he drifted from one occupation to another: mechanic, millwright, carpenter, shopkeeper, hotel-keeper, salesman and janitor. Finally in 1864, at the age of 43, he came across **Adhemar**'s book *Revolutions of the Sea* (1842). Although he realized that the French mathematician was wrong in believing that a change in the length of warm and cold seasons could cause an ice age, Croll was convinced that some other astronomical mechanism must lie behind these geological phenomena.

Following the publication of his theory, he received an appointment in the Scottish Geological Survey (1867), and for 13 years he took charge of the Edinburgh office. He has been compelled by ill-health to withdraw from public service in 1880. He was elected Fellow of the Royal Society in 1876.

Croll's theory created an immediate and profound impression on the world of science. Here, at last, was a plausible theory of the ice age that could be tested by comparing its predictions with the known geological record. Over the next 30 years Croll's ideas were widely and hotly debated: scientific expeditions were organized to dig for facts in drift deposits all over the world; articles and scientific journals probed the details of Croll's theory; and arguments pro and con filled many pages in geological textbooks.

As time went on, however, many geologists in Europe and America became more and more dissatisfied with Croll's theory, which maintained that the last Glacial Epich began about 250,000 years ago and ended some 80,000 years ago. The new evidence showed that the last ice age ended 10,000 years ago — at variance with Croll's results. Moreover, theoretical arguments were advanced against the theory by meteorologists who calculated that the variations in solar heating described by Croll were too small to have any noticeable effect on climate. By the end of the 19^{th} century, the tide of scientific opinion had turned against Croll, and his astronomical theory came to be treated as an historical curiosity, interesting but no longer valid. Eventually it was almost forgotten.

Almost, but not quite, for it would be picked up years later by a Yugoslavian astronomer. But in 1890, when James Croll lay on his deathbed in

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[388] Croll reasoned that a decrease in the amount of sunlight received during *winter* favors the accumulation of snow. Any small initial increase in the size of the area covered by snow must result in an additional loss of heat by reflecting more sunlight back into space. Therefore, any astronomically induced change in solar radiation (however small) would be amplified.

Scotland, **Milutin Milankovich** was only 11 years old, quite unaware of the task that the goddess of science had destined for him.

1864–1877 CE **Siegfried Marcus** (1831–1897, Germany). Engineer and inventor. Inventor of the automobile[389]. Built the first horseless carriage (1864) and in 1875 the second, which was the first 4-stroke engine, petrol-driven vehicle to function. This he drove about the streets of Vienna, amid general astonishment.[390]His automobile patents were registered in Germany (1882) and it was not until four years later (1886), that the first Daimler motor-car was built.

Marcus was born in Malchin, Mecklenburg, to Jewish parents and settled in Vienna (1853). He first began to study medicine, but later turned to electrotechnics and chemistry. Apart from his theoretical studies, he did practical work and thus acquired a sound knowledge of mechanics. His research work in chemistry drew his attention to the problem of fuel. In 1864, Marcus built his first model which he improved in 1875.

This vehicle (now kept in the Technical Museum of Vienna) contained all the essential parts of todays motorcar. The engine was driven by petrol supplied and mixed with air by a carburetor. This mixture entered the cylinder through a conic valve operated by a camshaft. The ignition spark was provided by a magneto at the moment when the piston arrived at the top dead center. The exhaust gas escaped through an outlet valve. The engine-power was transmitted by a conical clutch and two belt pulleys to the rear axle. The body, which looked like a horse-carriage was equipped with shock absorbers in the form of rubber buffers placed between the body and the rear axle. Two half-elliptical springs were fitted above the front axle. The steering box was of the worm gear in use today.

In 1975, on the occasion of a festival in honor of the inventor, Siegfried Marcus' car was put to use; the vehicle was ready after minor repairs. Experts were surprised at the ease with which the car ran.

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[389] Whilst history records the German **Otto** as the inventor of the 4-stroke engine (1877), the German **Daimler** as the first to use petrol to drive on engine (1885) and the Frenchman **Levassor** (1887) as the first to utilize a petrol-engine to drive a vehicle — in fact, all these inventions had been made previously by one man, Siegfried Marcus.

[390] The first traffic report to the first driver in the first motorcar was made by a Viennese policeman, when he stopped Siegfried Marcus' car and forbade him to continue his journey because of the noise he was making (1875). In doing so, this policeman had delayed the development of an invention which was destined to change the face of the earth.

1864–1880 CE **Cato Maximilian Guldberg** (1836–1902, Norway), chemist and mathematician, and **Peter Waage** (1833–1900, Norway), chemist, formulated the *law of mass action*[391] (1864–1867), the basic law of chemical kinetics. In it simplest form, it states that in a state of equilibrium, the fraction of molecules of one kind changing into molecules of another kind is a time independent constant[392].

Guldberg became a professor at the University of Christiania (Oslo) in 1869. With **Henrik Mohn** (1835–1916, Norway) he published a book on the circulation of the atmosphere, providing the theoretical foundation for *dynamic meteorology* (1876–1880). In their study, they incorporated the Coriolis deflection and the friction between the earth and the atmosphere.

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[391] **Berthollet** (1803) recognized that the concentrations of the reacting compounds influence the reaction but failed to render a general mathematical formulation.

[392] In general, reactants with concentrations $A, B, C, \ldots$ may combine in various well-defined proportions to give products with concentrations $G, H, \ldots$ according to the stoichiometric equation

$$\nu A + \mu B + \eta C + \cdots = \gamma G + \delta H + \cdots,$$

where the integers ν, μ, η, γ, δ are called *stoichiometric coefficients* (namely, the numbers of molecules that partake in the reaction). The law of mass-action states that at equilibrium

$$\frac{G^{\gamma} H^{\delta} \cdots}{A^{\nu} B^{\mu} C^{\eta} \cdots} = K(T, P),$$

where K is a constant independent of time. Large values of K indicate the formation of large quantities of new products.

The law of mass action results in a natural way from the differential equation governing the reaction kinetics. Consider for example the symbolic reaction equation $A + X \rightleftharpoons B + Y$, which means that whenever a molecule of component A encounters a molecule of X, there is a certain probability a reaction will take place and a molecule of B and a molecule of Y will be produced. Likewise, the collision between molecules of Y and B can set off the opposite reaction.

The total variations in concentrations of the chemicals is given by the balance between the forward and the reverse reaction. Consequently

$$\frac{dX}{dt} = \frac{dA}{dt} = -\frac{dY}{dt} = -\frac{dB}{dt} = -kAX + k'BY.$$

In the state of equilibrium the forward and reverse reactions compensate one another statistically so that there is no longer any overall variation in the concentrations $\left(\frac{dX}{dt} = 0\right)$. This compensation implies that the ratio between equilibrium concentrations is given by $\frac{AX}{YB} = \frac{k'}{k} = K$.

1864–1884 CE **Julius Friedrich Cohnheim** (1839–1884, Germany). Physician. Pioneer of pathological anatomy. Revolutionized medical thought and practice when he advanced the basic theory of *inflammation* (1864–1867) and pus formation. Devised techniques of freezing tissue samples before sectioning. Author of classical textbooks in general pathology (1877–1880).

Cohnheim was born in Demmin, Pommerania, to Jewish parents. Converted to Christianity to advance his career, he studied in Berlin, where he graduated in medicine and studied for a year with the cellular pathologist **Rudolf Virchow**. Professor of Kiel (1868), Breslau (1872), Leipzig (1878).

Cohnheim devised new ways of looking at specimens of human tissue under the microscope and worked out many of the early cellular events that occur in inflammation. He showed by experiments how the blood cells vessels respond in the early stages of inflammation, and proving that the white cells (leukocytes) passed through capillary walls where inflammation was occurring, later degenerating to become pus corpuscles.[393]

Cohnheim worked on a whole range of diseases, including tuberculosis, myocardial infarct and cancer. **Metchnikov** was among many later workers who confirmed and extended his early studies.

1864–1886 CE **Edmond Nicolas Laguerre** (1834–1886, France). Mathematician. Although most of his research efforts were in the field of geometry, this part of his output [foci of algebraic curves (1853), geometric interpretation of homogeneous forms and their invariants, curves mapped onto themselves by inversion, 4^{th} order curves, studies of curvature and geodesies and pioneering investigations of the complex projective plane] has been largely absorbed by later theories or has passed into the general body of geometry without acknowledgment[394].

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[393] The essence of the inflammatory response is the migration of white blood cells to a wound. The inflammatory response results in an increased flow of blood to the site of injury and an increased permeability of the endothelium (the tissue that comprises the walls of capillaries). Both of these effects assist the migration of phagocytic (germ engulfing) white blood cells from the blood to the interstitial fluid. Here the white blood cells can begin engulfing debris and any infecting microorganisms.

[394] e.g. his work on differential invariants is included in the more comprehensive *Lie group* theory. He was also one of the first to point out that a distance function (metric) can be imposed on the coordinate plane of analytic geometry in more than one way.

Laguerre's current reputation rests on his discovery (1879) of the *Laguerre differential equation* and its polynomial solutions (*Laguerre polynomials*). These functions have wide use in mathematical physics and applied mathematics — for example, in the solution of the Schrödinger equation for hydrogen-like atoms and in the study of electrical networks and dynamical systems. The 1879 memoir of Laguerre is significant not only because of the discovery of the Laguerre equations and polynomials, but also because it contains one of the earliest infinite continued fractions which are known to be convergent.[395]

Laguerre was born in Bar-le-Duc. His education was completed at the École Polytechnique in Paris. In 1854 he left school and accepted a commission as an artillery officer (1854–1864), where he published nothing. Upon his return to Paris he became a tutor at the École Polytechnique and in 1874 was appointed *examinateur*. In 1883 he accepted, concurrently, the chair of mathematical physics at the Collège de France. In 1886 his continually poor health broke down and he returned to Bar-le-Duc, where he died.

In evaluating the life-work of Laguerre one encounters a phenomena common to many brilliant and innovative minds: his name is little known and his work so seldom cited because *he was primarily occupied with details and did*

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[395] The polynomials

$$\begin{aligned} L_n(x) &= e^x \frac{d^n}{dx^n}(x^n e^{-x}) \\ &= n!\left[1 - \binom{n}{1} x + \binom{n}{2}\frac{x^2}{2!} - \binom{n}{3}\frac{x^2}{3!} + \cdots + (-)^n \frac{x^n}{n!}\right] \end{aligned}$$

satisfies the differential equation $xy'' + (1-x)y' - ny = 0 \quad (n = 0, 1, 2, \ldots)$. The *associated Laguerre polynomials* $L_n^m(x) = (-)^m \frac{d^m}{dx^m} L_{n+m}(x)$, satisfy the generalized equation $xy'' + (m+1-x)y' + ny = 0$. Laguerre arrived at his equation through an investigation of the exponential-integral function $\int_x^\infty \frac{e^{-u}}{u} du$. for which he obtained the continued-fraction representation

$$\int_x^\infty \frac{e^{-u}}{u} du = \cfrac{e^{-x}}{x + 1 - \cfrac{1}{x + 3 - \cfrac{4}{x + 5 - \cfrac{9}{x + 7 - \cfrac{16}{x + 9 - \cdots}}}}}.$$

Laguerre proved that the m^{th} convergent of the fraction could be written as $e^{-x}\,[\varphi_m(x)/f_m(x)]$, where $f_m(x)$ is the Laguerre polynomial of degree m, $L_m(-x)$, and thus demonstrated that a divergent power series can be converted into a convergent continued fraction.

not step back to draw together various pieces and put them into a single theory. The result is that his work has mostly come down as various interesting special cases of more general theories discovered by others.

1865 CE **Eugene Charles Catalan** (1814–1894, Belgium). Mathematician. Contributed to the theory of continued fractions and number theory. The constant[396] $G = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^2} = 0.915,965,594\ldots$ is named after him.

Catalan was in Liouville's class at Ecole Polytechnique (1833) but was expelled the following year. Allowed to resume his studies in 1835. With Liouville's help he obtained a lectureship in descriptive geometry at the Ecole Polytechnique (1838) but his career was damaged by being politically active with strong left-wing views.

1865 CE The London Mathematical Society founded[397].

1865 CE **Gregor Johann Mendel** (1822–1884, Austria). Botanist. Discovered the mathematical principles of heredity. Observing the contrasting characteristics of different pea plant species, he grew successive generations of such plants and studied how these characteristics were inherited.

Mendel was born in Heinzendorf, Austria. He became interested in plants while a youth on his father's farm. In 1843 he entered the Augustinian

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[396] It is a special case of the *Dirichlet series* $\beta(z) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^z}$. Catalan's constant has many series, integral and continued fraction representations. It is unknown whether $G = \beta(2)$ is irrational. **Ramanujan** showed that $G = \frac{\pi}{4}\ {}_3F_2(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 1, \frac{3}{2})$ and that

$$2G = 2 - \cfrac{1}{3 + \cfrac{2^2}{1 + \cfrac{2^2}{3 + \cfrac{4^2}{1 + \cfrac{4^2}{3 + \ldots}}}}}$$

[397] The following *mathematical societies* were established in the indicated order: *France* (1872), *Edinburgh* (1883), *Palermo* (1884), *New York* (1888), *Germany* (1890), *India* (1907), *Spain* (1911), *U.S.A.* (1915).

The first seven International Congresses were held at: *Paris* (1889), *Chicago* (1893), *Zürich* (1897), *Paris* (1950), *Heidelberg* (1904), *Rome* (1908) and *Cambridge* (1914).

The number of *mathematical periodicals* increased as follows: 1700 (17), 1800 (210), 1900 (950), 1990 (ca 3800). This roughly fit the curve $N(t) = 17(1+t)^{1/4}e^{t/75}$, where $t = 0$ corresponds to the year 1700.

About 200,000 new mathematical theorems are being proved each year, since 1990.

Monastery in Brünn. Except for his education at the University of Vienna and short periods of teaching natural history at nearby schools, Mendel spent his life in the monastery.

In 1865, Gregor Mendel read his paper before the Brünn Society for the Study of Natural Science. The records of the society state that there were neither questions nor a discussion following his presentation. Like a stone dropped down a well, Mendel's work disappeared from view of the scientific community — without causing so much as a ripple.

It was not until 1900, sixteen years after his death, that biologists came to appreciate what he had accomplished. At that year his work was rediscovered by three distinguished biologists: **Hugo de Vries** (1848–1935), **Carl Correns** (1864–1933) and **Erich Tschermak** (1871–1962). Thereafter, Mendel's ideas have steadily gained ground, and came to exert upon biology an influence not less than that associated with the name of Darwin.

1865 CE **Hermann Johann Philipp Sprengel** (1834–1906, England). Chemist, physicist and inventor. Invented the high vacuum pump which had far reaching effects: for example. it made possible **Crooke's** investigations of radiation in a high vacuum, leading eventually to the discovery of the electron by **J.J. Thomson** (1895).

Sprengel was born in Schillerslage, near Hanover, and educated at the universities of Göttingen and Heidelberg. He moved to England (1859) and carried out research at Oxford and in the laboratories of several institutions in London. He mechanized the pump devised by **Heinrich Geissler** (1858), making the action of the pump much swifter and more efficient.

1865 CE **Immanuel Lazarus Fuchs** (1833–1902, Germany). Mathematician. One of the creators of the modern theory of differential equations.

Fuchs was born to Jewish parents in Moschin, near Posen and studied at Berlin with **Kummer** and **Weierstrass**. He became professor at the University of Berlin (1884), after converting to Christianity. In 1865 he combined two methods in the study of linear differential equations with complex functions as coefficients. One, using power series, as elaborated by **A.L. Cauchy**; the other method uses the hypergeometric series as has been done by **G.F.B. Riemann**. A special type of linear ordinary differential equations bear his name.

One of his most able students, **Zvi Hermann Shapira** (1840–1898), became a professor of mathematics at Heidelberg (1887–1898) and contributed to the theory of co-functions. He reissued and annotated (1880, Leipzig) the medieval mathematical treatise of **Avraham bar Hiyya**. Shapira was also active in the Zionist movement and suggested the idea of the Jewish National Fund (1897).

1865–1877 CE **Heinrich Anton de Bary** (1831–1888, Germany). Botanist. Founder of science of mycology and of plant pathology. First to work out life histories, morphology and physiology of many *fungi*, esp. parasitic fungi; first to demonstrate *heteroecism*. Demonstrated symbiotic nature of lichens.

Born at Frankfurt, Germany. Professor at Freiburg (1855–1866), Halle (1867–1872), Strasbourg (1872–1888).

1865–1881 CE **Carl Gottfried Neumann** (1832–1925, Germany). Mathematician and theoretical physicist. Pioneered in boundary value problems of potential theory and contributed to the theory of Bessel functions. He coined the term '*logarithmic potential*' (1870). Neumann was born in Königsberg. His father **Franz Ernst Neumann** (1798–1895) was a known mineralogist and physicist, and his mother was a sister-in-law of the astronomer F.W. Bessel. From 1868 until 1911 he was a professor at the University of Leipzig.

1866–1882 CE **Camille Marie Ennemond Jordan** (1838–1922, France). Mathematician. Known for his important contributions to algebra, topology and group theory. Gave a generalization of the Serret-Frenet formulae for a curve in an R^n space, and also established the existence of principal directions for any subspace of such a manifold. Introduced with **Giuseppe Peano** (1858–1932) the concept of 'Riemann content' in measure theory. Made significant contributions to topology ('*Jordan curve theorem*'), group theory and measure theory. Showed that algebraic equations of *any* degree can be solved in terms of *modular functions*.

Jordan studied mathematics at the Ecole Polytechnique and from 1873 taught there and at the College de France. He introduced important topological concepts (1866) such as *homotopy* and defined a homotopy group of a surface without explicitly using group terminology [he was aware of Riemann's work but not of the work of Möbius]. His introduction of group concepts into geometry (1869) was motivated by studies of crystal structure. Defined the *normal form* for matrices (1870) over a finite field, and brought *permutation groups* to a central role in mathematics. Originated the concept of *functions of bounded variation* and is known especially for his definition of the length of a curve (1882). He also generalized the criteria for convergence of *Fourier series*.

Two of Jordan's students, **Sophus Lie** and **Felix Klein**, drew upon his studies to produce their own theories of continuous and discontinuous groups.

1866–1896 CE **Robert Whitehead** (1828–1905, England). Engineer and inventor. Father of the modern *torpedo.* Designed and built the first unmanned, self-propelled torpedo. It was propelled by a compressed-air engine and carried 9 kg of dynamite. Its most important feature was a self-regulating device which kept it at a constant preset depth. Many of the basic component parts used in his early prototypes were, in fact, still in use during the Second World War and the overall form of the torpedo has been retained to the present day. He was first to use the *gyroscope* in military equipment (1896).

Whitehead was born near Bolton, Lancaster, UK and came from a family of engineers. After a long apprenticeship with a company in Manchester he left in 1840 to seek his fortune abroad. In 1864 he began to work for the Austrian Navy and undertook to build for them an unmanned, self-propelled surface boat packed with explosives which could be directed at blockading ships. In 1870 he brought two of his weapons to England for trial with the Royal Navy. The larger was $4\frac{1}{2}$ m long, diameter 40 cm, charged with 9 kg of dynamite and having a range of ca 1000 m. The Royal Navy were very impressed and bought the manufacturing rights for 15,000 sterling in 1871. In 1896, Whitehead used the gyroscope to steady the motion of his torpedo.

Whitehead was clearly one of the greatest British inventors of the 19th century. The type of torpedo that he invented exerted more influence over the tactics of naval warfare than all the world's top admirals and naval architects put together. Yet although he was honored by many other nations, he received minimal recognition from his country of birth. Even today, apart from current and past members of the Royal Navy, his name remains virtually unknown. (You will not find his name mentioned in any of the Britannica editions prior to 1975!).

1866 CE **Georges Leclanché** (1839–1882, France). Engineer. Invented the first *dry cell* (Zinc-Carbon cell), where the electrolyte is moist. [It is called "dry" in comparison with cells like the Daniell cell, in which the electrolytes are *aqueous solutions.*]

The container of this dry cell, made of zinc, also serves as one of the electrodes. The other electrode is a carbon rod in the center of the cell.

The zinc container is lined with porous paper to separate it from the other materials of the cell. The rest of the cell is filled with a moist mixture (the cell is not really dry) of ammonium chloride (NH_4Cl), manganese oxide ($Mn(IV)O_2$), zinc chloride ($ZnCl_2$), and a porous, inert filler. Dry cells are sealed to keep the moisture from evaporating. As the cell operates (the electrodes must be connected externally), the metallic Zn is oxidized to Zn^{2+}, and the liberated electrons flow along the container to the external circuit. Thus, the zinc electrode is the anode (negative electrode).

$$Zn \rightarrow Zn^{2+} + 2e^- \qquad \text{(oxidation, anode)}$$

The carbon rod is the cathode, at which ammonium ions are reduced.

$$2NH_4^+ + 2e^- \rightarrow 2NH_3 + H_2 \qquad \text{(reduction, cathode)}$$

Addition of the half-reactions gives the overall cell reaction

$$Zn + 2NH_4^+ \rightarrow Zn^{2+} + 2NH_3 + H_2, \qquad E_{\text{cell}} = 1.6V$$

As H_2 is formed, it is oxidized by MnO_2 in the cell, while the Mn is reduced. This prevents collection of H_2 gas on the cathode, which would stop the reaction.

$$H_2 + 2Mn(IV)O_2 \rightarrow 2Mn(III)O(OH)$$

The ammonia produced at the cathode combines with zinc ions and forms a soluble compound containing the complex ions, $[Zn(NH_3)_4]^{2+}$:

$$Zn^{2+} + 4NH_3 \rightarrow [Zn(NH_3)_4]^{2+}$$

Leclanche's invention, which was quite heavy and prone to breakage, was steadily improved over the years. The idea of encapsulating both the negative electrode and porous pot into a zinc cup was first patented by **J.A. Thiebaut** in 1881. But it was **Carl Gassner** of Mainz who is credited as constructing the first commercially successful "dry" cell. Variations followed. By 1889 there were at least six well-known dry batteries in circulation. Later *battery* manufacturing produced smaller, lighter batteries, and the application of the tungsten filament in 1909 created the impetus to develop batteries for use in torches (flashlights).

Leclanché was born in Parmain, France, and educated in England. After completing a technical education in Paris (1860) he began to work as an engineer.

1866 CE **Gabriel Auguste Daubrée** (1814–1896, France). Geologist and mineralogist. Suggested that the center of the earth is a core of iron and nickel.

He was born at Metz and educated at the École Polytechnique in Paris. Qualified as a mining engineer, he was put in charge of the mines in Alsace (1838) and was subsequently a professor of mineralogy and geology at Strasbourg (1852). In 1861 he was appointed professor of geology at the museum of natural history in Paris. The minerals *daubreeite* and *daubreelite* are named for him.

1866–1867 CE **Daniel Kirkwood** (1814–1895, U.S.A.). Astronomer. First drew attention to gaps in the distribution of asteroids' mean distances from the sun, known today as *Kirkwood gaps*. He noticed that very few asteroids have orbits whose orbital periods correspond to simple fractions (such as $\frac{1}{2}$, $\frac{3}{7}$, $\frac{2}{5}$, $\frac{1}{3}$) of Jupiter's orbital period. *Resonant gravitational perturbations* due to the repeated alignments with Jupiter have deflected asteroids away from these orbits and prevented the formation of a planet between Mars and Jupiter[398]. Kirkwood pointed out that the divisions in the ring structure of *Saturn* may have similar origin (the *Cassini divisions*, 1675), created by gravitational perturbations of the Saturnian moons on the icy fragments of the rings.

The Planet that Failed to Form[399]

The giant planet Jupiter orbits the sun in an ellipse that is 5 *times larger and somewhat more eccentric (e* = *0.048) than the orbit of the earth. It comes to perihelion at* 740, 558, 340 *km and recedes to* 815, 602, 000 *km at aphelion, so its distance from the sun varies by some* 75 *million km. Its mean orbital speed of* 13.1 *km/sec carries it once around its orbit (sidereal period) in nearly* 12 *years. Since its synodic period*[400] *is* 399 *days, it comes to opposition every*

[398] The inner planets formed when swarms of planetesimals a few kilometers in size collided at velocities low enough to permit bodies to grow larger by accretion. Numerous resonances from the rapidly growing and massive planet Jupiter probably permeated the region between 2 and 4 AUs. These resonances may have pumped up the orbital eccentricities of the planetesimals there, accelerating the objects to velocities so high that successful accretion on a planetary scale was impossible. Today the asteroids remain in an environment dominated by collisions, encountering one another at about 5 km/sec.

399 For further reading, see:

- Gallant, R.A., *Our Universe*, National Geographic Society, 1994, 284 pp.
- Moore, P., *Atlas of the Universe*, Philips, 2005, 288 pp.
- Caprara, G. (ed.), *The Solar System*, Firefly Books, 2003, 255 pp.

[400] The time taken by the earth to catch up with Jupiter by one lap.

13 months. Its rapid rotation with a period of only 9^h50^m *has produced a noticeable flattening at its poles (0.062).*

The mass of Jupiter (as determined from the motions of its inner satellites and the perturbation it produces on the motion of asteroids) is 318 *times the mass of the earth, and it is nearly* 11 *times the earth in diameter. In both volume and mass, it is larger than all the other planets put together*[401]*. Jupiter's gravitation perturbs the motion of the sun and the other planets, and holds its own satellites in orbit.*

During the 18^{th} *century, when post-Newtonian astronomers began to seek for law and order in the solar system, they noticed that all the ratios of distances of neighboring planets from the sun lie between* 1.3 *and* 2.0 except *the Jupiter-Mars ratio which came to* 3.4*. Thus, the gap between these two planets is twice as great as it might be expected to be. It is almost as though a planet ought to exist between Mars and Jupiter, and doesn't.*

Toward the end of the 1700*'s, astronomers were thinking along these lines and were beginning to plan a telescopic sweep of the sky in order to see if such a missing planet could be spotted. Between* 1800 *and* 1845*,* 5 *minor planets were found.* ***Herschel*** *named them* asteroids*. By* 1866*, enough asteroids had been discovered so that one could see that the average distances were spread out fairly evenly between the orbits of Mars and Jupiter — but not entirely evenly!*

The modern theory of the formation of the solar system has it beginning in a huge cloud of dust and gas. Slowly this cloud turned and came together under its own gravitational pull. As the cloud condensed into a smaller and smaller object, it turned faster and faster. Eventually, the central part of it condensed into the sun, while some of it at its midsection was kept in the outskirts by the centrifugal force, like a large equatorial bulge. The thinner cloud of dust and gas that spread out beyond the sun's midsection formed larger and larger objects that kept colliding until the planets were formed, all circling more or less in the equatorial plane of the sun.

Most of the total angular momentum of the solar system lodges in the orbital motion of the massive outer planets such as Jupiter and Saturn

[401] *The center of gravity* of the solar system shifts in a complicated fashion as the planets circle the sun, but most of the time it is about 45,000 km above the sun's surface in the general direction of Jupiter, causing the sun to wobble slightly — making one complete wobble in about 11.86 years, close to the orbital period of Jupiter.

($J_{\text{Jup}} \approx 16J_{\odot}$)[402]. *The part of the dust cloud lying between the orbits of Mars and Jupiter has collected into small solid bodies of various sizes, but could not take the final step of coalescing into a single large body, because the gravitational influence of Jupiter kept stirring up the asteroids, preventing them from coming together. Attempts to explain the Kirkwood gaps go all the way back to Kirkwood. Chief among them is the so-called gravitational hypothesis; it suggests that asteroids drift away from the commensurable orbits under the influence of Jupiter's gravitational perturbation alone, needing no help from collisions.*

The mechanism of this interaction is as follows: Every time the asteroid wheels into that part of its orbit which happens to be near Jupiter's position at the time, it feels Jupiter's pull particularly strongly. If Jupiter happens to be a little ahead of the asteroid at the time of closest approach, it will pull the asteroid forward. If Jupiter happens to be a little behind, it will pull the asteroid backward. On the average, the forward and backward pulls will cancel each other and, in the long run, the asteroid's orbit will remain unchanged.

If, however, the period of revolution of an asteroid is some simple fraction of the period of revolution of Jupiter, their relative position will be repeated periodically every T years; the perturbations will not balance out but tend to accumulate in a preferred direction, with the consequence that the asteroid will regularly be pushed out of its orbit closer or farther from the sun. A gap in the asteroid belt will form at a series of distances which are simple fractions of the period of revolution of Jupiter.

The asteroids all revolve about the sun in the same direction as the principal planets (from west to east), and most of them have orbits that lie near the plane of the earth's orbit. The main asteroid belt contains minor planets with orbits of semimajor axes in the range 2.2 to 3.3 AU, with corresponding periods of orbital revolution about the sun from 3.3 to 6 years.

Calculations show that perturbations by Jupiter of asteroids near or in the Kirkwood gaps can result in ejecting asteroids to the part of the solar system occupied by the earth.

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402 The orbital angular momentum of a satellite of mass m in a circular orbit of radius r around a gravitating center of mass M is proportional to the square root of r. This follows from Newton's law of gravitation which requires the satellite's centripetal acceleration to have the value $v^2/r = GM/r^2$, so that $vr = \sqrt{GMr}$ and therefore the orbital angular momentum is $J = mvr = m\sqrt{GMr}$. One may thus compare the orbital angular momentum of the planet Jupiter ($m \approx 10^{-3}M$; $M \approx 2 \times 10^{30}$ kg; $r \approx 8 \times 10^{11}$ m) with the spin angular momentum of the sun $J_{\odot} = \frac{2}{5}MR^2\omega$ obtained if one treats the sun approximately as a rigid sphere ($R = 7 \times 10^8$ m) rotating with a period of about 25 days (i.e., $\omega \approx 3 \times 10^{-6}$ sec^{-1}).

In recent years (1985–1992), celestial-mechanics theorists brought to bear a new outlook upon the formation mechanism of the Kirkwood gaps: given enough time, it seems an asteroid with a period commensurable with that of Jupiter will experience a chaotic burst of eccentricities, high enough to put it in an orbit where it is likely, sooner or later, to have a close encounter with Mars. The modern study of chaos deals with the onset of wild and unpredictable fluctuations in a system governed by simple deterministic equations from which one would naively expect nothing but good behavior.

When one speaks of chaos, especially in an essentially conservative, Hamiltonian system like the solar system, one does not mean the unpredictability inherent in intrinsically disorderly phenomena such as thermal noise or innumerable random collisions. What is meant here is a dependence on initial conditions so hypersensitive that it thoroughly destroys predictability, despite the simple, deterministic equations that govern the system. After a few hundred thousand years, two asteroids that were initially traveling together in one of the Kirkwood gaps will become completely uncorrelated.

During 1866–1981, *nobody was able to present an analytically tractable solution that offer a detailed explanation of the Kirkwood gaps. The explicit numerical integration of Newton's equations without radical approximations consume so much computer time that the orbits of the nine planets, including their mutual interactions, have never been explicitly calculated beyond* 5×10^5 *years into the past and future. A thousand fold increase in computing speed is gained by a method developed by* **Boris Chirikov** *(1979) and applied by* **Jack Wisdom** *(1981) to the study of transition to chaos in the solar system.*

Through this method one replaces the full differential equation describing the behavior of the system by an algebraic mapping that carries the system over a sequence of discrete time intervals. Then one looks at the system only at stroboscopic intervals corresponding to the orbital period of Jupiter. If there were no longer-term variations in the problem, the mapping point would remain fixed in phase space from one strobe time to the next. The movement of the map-point describes only variations slower than the annual revolution. The thousandfold increase in computing speed is gained because the mapping algorithm obviates the need to integrate the differential equations over many smaller time intervals within the 12*-year strobe step; and because the mapping rule is algebraic rather than differential, one has better digital accuracy.*

When the mapping was applied to the $3:1$ *Kirkwood gap, the eccentricity variation suddenly shot up chaotically to fluctuations reaching 35% after behaving itself for* $20,000$ *years, sufficient for the asteroid to cross the orbit of Mars.*

The Kirkwood gaps now had a plausible origin. Close encounters (not necessarily collisions) with Mars would eventually perturb these high-eccentricity orbits out of the commensurable band.

There is a class of asteroids whose orbits come close to or cross *that of the earth. They are divided into 3 groups: The* Atens *have orbits that cross the orbit of the earth, but lie wholly within the orbit of Mars. The* Apollos *are objects that cross both the orbits of the earth and Mars. The* Amors *cross the orbit of Mars but do not, at present (1993), quite come as close as the earth's orbit. Some earth-crossing asteroids have been observed at their near-earth passes: of these,* Hermes *passed very close to earth in* 1937. Icarus *missed our planet by only* 6.4 *million km on June* 14, 1968, *and* Geographos *by only* 10 *million km in 1969*[403].

Thus, the missing planet never had a chance to form. What remains today in the gap between the orbits of Jupiter and Mars appears to be simply a remnant of the scattered debris from the original solar nebula that elsewhere accreted into planets. Effects of resonances*, or locations where the orbital period of a body is some exact integer ratio of Jupiter's orbital period — are clearly visible.*

1866–1884 CE **Ludwig Eduard Boltzmann**[404] (1844–1906, Austria). One of the greatest physicists of the 19^{th} century. Opened the door to an

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[403] Asteroid fragments (usually called *meteoroids*) have in the past collided with our planet. It resulted in *impact crater* whose diameter depends on both the mass and the speed of the impinging object. One of the most impressive and best preserved impact craters is the *Barringer Crater* near Winslow, Arizona. It measures 1200 meters across and is 200 meters deep. The crater was formed some 25,000 years ago when an iron-rich object measuring 50 meters across struck the ground with a speed estimated at 11 km/sec. The resulting blast, was mechanically equivalent to the detonation of a 20-megaton hydrogen bomb.

[404] For further reading, see:

- Broda, E., *Ludwig Boltzmann: Man. Physicist. Philosopher*, Ox Bow Press, 1983, 169 pp.
- Harris, S., *An Introduction to the Theory of the Boltzmann Equation*, Holt, Rinehart and Winston, 1971, 221 pp.
- Rumer, Yu.B. and M.S.Ryvkin, *Thermodynamics, Statistical Physics and Kinetics*, Mir Publishers, Moscow, 1980, 600 pp.

understanding of the macroscopic systems in a manner consistent with their reversible microscopic *molecular dynamics*. Among the founders of classical statistical mechanics. The originality of Boltzmann's ideas made them difficult for some of his contemporaries to grasp. His important achievements are:

- Molecular kinetic gas theory: *Boltzmann transport equation*[405] (nonlinear integro-differential equation for the phase-space distribution function), Maxwell-Boltzmann distribution, the *H-theorem*. The probabilistic interpretation of *entropy* (1866).
- Stress-strain relation for a most general linear viscoelastic solid (1876), known as the *Boltzmann superposition principle*[406].
- *Ergodic hypothesis* (1877).

- Reif, F., *Fundamentals of Statistical and Thermal Physics*, McGraw-Hill, 1965, 651 pp.
- Huang, K., *Statistical Mechanics*, John Wiley & Sons: New York, 1963, 470 pp.
- Jackson, E.A., *Equilibrium Statistical Mechanics*, Dover, 2000, 241 pp.
- Schrödinger, E., *Statistical Thermodynamics*, Cambridge University Press: Cambridge, 1962, 95 pp.
- Sommerfeld, A., *Thermodynamics and Statistical Mechanics* (Lectures on Theoretical Physics), Vol. 5, Academic Press: New York, 1964, 401 pp.

[405] Arises in the determination of the phase-space distribution of particles of an ideal gas in an enclosure on which there act external forces.
Another equation, which also bears Boltzmann's name, is an equation for the evolution of a probability density $\phi(x,t)$ over time (furnished with an initial condition),

$$\frac{\partial \phi(x,t)}{\partial t} = -\lambda \phi(x,t) + \lambda \int_0^\infty K(x,s)\phi(s,t)\,ds$$

[406] A generalization of *Hooke's solid*, *Newtonian fluid* and the *viscoelastic models* of **Maxwell**, **Kelvin** and **Voigt** (in retrospect). It has the form of a convolution integral

$$\mathfrak{T}(\boldsymbol{r},t) = \int_{-\infty}^{t} \overset{4}{\Psi}(\boldsymbol{r}, t-\tau) \colon \frac{\partial \mathfrak{E}(\boldsymbol{r},\tau)}{\partial \tau} d\tau,$$

where $\overset{4}{\Psi}$ is a fourth-order *relaxation tensor, $\mathfrak{T}(\boldsymbol{r},\tau)$ is a stress tensor and $\mathfrak{E}$ is a strain tensor*.

- *Stefan-Boltzmann law* for blackbody radiation (1883–1884), through which he connected Maxwell's electrodynamics with thermodynamics.

Boltzmann established the statistical nature of the second law of thermodynamics, which stipulates that heat passes, *in a closed system*, spontaneously from a hot (higher temperature) body to a cold (lower temperature) one and never in the reverse direction. Boltzmann's statistical proof of the second law of thermodynamics addresses only the *average* variation of entropy of an isolated system and *does not rule out* the possibility of an occasional decrease in its value. Fluctuations from the average *must*, in fact, occur, and their frequency and typical magnitudes depend on the *size* of the system. Indeed, **Smoluchowski** (1872–1917) has shown that microscopic phenomena can have no intrinsic arrow of time, as far as internal entropy changes are concerned.

When Boltzmann (1872) first presented a statistical theory purporting to prove that an improbable distribution will *always* proceed, when left alone, to a more probable distribution with higher entropy, his colleague Joseph Loschmidt is said to have questioned the general validity of Boltzmann's theorem by pointing out that, if a skilled experimentalist would at some instant reverse all motions in an equilibrium state that had evolved in the manner envisaged by Boltzmann from a non-equilibrium state, then this reversed equilibrium state would return (without further interference from the outside) to a non-equilibrium state, and the entropy would decrease during that return. This counterexample to Boltzmann's supposed proof, nowadays known under the name "*Loschmidt's paradox*", is clearly based on the validity of symmetry with respect to reversal of motion, as it invokes the existence of the reversed process to every process that may take place.

Boltzmann is reported to have silenced Loschmidt at the time by pointing a finger at him and saying, "*You reverse the momenta*". Ever since, there has existed a body of opinion that dismisses Loschmidt's paradox on the grounds that simultaneously reversing the motions of a huge number of molecules is a task in principle beyond the capability of the most skilled experimenters. If indeed this is what Boltzmann meant by his cryptic remark, then it did not dispose of Loschmidt's objection, because there *are* experiments that precisely realize the kind of reversal of motion Loschmidt had in mind. A Loschmidt type of reversal *is* realized in all so-called *spin echo experiments*, which are conducted daily in many laboratories since their first performance by the American physicist **Erwin Hahn** (1950). Indeed, if one were considering a gas of particles of the degree of dynamic simplicity as that dealt with in spin echo experiments, even Loschmidt might have been able to reverse the momenta.

To deal with Loschmidt's objection satisfactorily one must keep in mind that the precise initial positions and velocities of the molecules remain encoded, through the laws of motion, in the future positions and velocities of the molecules *only if the gas remains completely isolated.*

Definition and measurement of temperature T and/or entropy S, on the other hand, require that the gas be put into thermal contact with a heat bath, thus breaking the condition of isolation. Whenever the gas is in thermal contact with a heat bath, its molecules will be subject, as a matter of principle, to random changes in their positions and velocities *which destroy the encoded memory of the initial state*. In other words, the thermal contact couples the *thermodynamic arrow of time* of the measured sample, to that of its environment.

Thus, if one reverses all motions *after* the equilibrium temperature T and/or entropy S have been measured, the gas will *not* return to a non-equilibrium state of lesser entropy, but rather evolve into other states whose entropy is either equal to or larger than S. Furthermore, as shown by **L. Szilard** (1921) in connection with the *Maxwell's Demon* paradox, any micro-management by a macroscopic (or even microscopic) "Demon" of molecular degrees of freedom, entails *its own* entropy increase due to the attendant data processing. We conclude that sequences of actual measurements of entropy on a real-life, macroscopic sample of matter will never show a decrease, even though reversal experiments of the Loschmidt type are possible in certain special circumstances.

Boltzmann's principle represents the entropy S in terms of the probability W of macroscopic states and expresses it in the formula

$$S = k \log W$$

(carved out on Boltzmann's tombstone in the Central Cemetery in Vienna].

Boltzmann never wrote down the equation in this form. This was done by **Planck** (1906), who also introduced the constant k. Boltzmann only referred to the proportionality between S and the logarithm of the probability of a state. The designation of *Boltzmann's principle* was advocated by **Einstein** for the reverse of this relation, namely:

$$W = \exp\{S/k\}$$

in which S is considered to be known empirically.

The insight that the second law of thermodynamics can be understood only in terms of a connection between entropy and probability, was one of the seminal achievements of 19^{th} century scientific thought.

Boltzmann was born in Vienna and received his doctorate there in 1866. After a few years as assistant to his teacher, **Joseph Stefan**, he taught at Graz and then moved on to Heidelberg and Berlin for further studies with such notables as **Gustav Kirchhoff** and **Hermann von Helmholtz**. He returned to Vienna in 1873 as professor of mathematics, but soon left for Graz again, where he served this time as a professor of experimental physics from 1876 to 1889. From Graz Boltzmann went to Münich as professor of theoretical physics, after refusing an invitation to succeed Kirchhoff in Berlin.

He returned to his native Vienna once more in 1894, this time as professor in his own field, but his wanderings were not over yet. He was to leave Vienna for Leipzig in 1900, and then return to his still vacant chair in Vienna in 1902 for the remaining few years of his life. In beginning his inaugural speech in Vienna in 1902, Boltzmann remarked that he could spare his audience the conventional hymn of praise for his predecessor since he and the speaker were identical!

Boltzmann admired the republic of the United States of America and visited it several times. In 1905 he was invited to give a course of lectures (in English) in the summer session at the University of California in Berkeley, where he arrived on 26 June. During his stay he visited Stanford University and Lick Observatory[407].

By 1906, at age 62, Boltzmann had suffered for years from periods of serious depression, and from the perhaps not unrelated burden of serious asthma. During his later years he was plagued by an anxiety that his own wit and memory would suddenly leave him in the midst of a lecture. The combination of his recurrent depressions and fears became too much for him to bear, and he took his own life on 5 September 1906, while on a summer vacation at Duino, near Trieste.

One of the causes of this tragic event was the intense philosophical opposition to his work, which now forms an integral part of physics. Ironically,

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[407] His recollections of that summer survive in his well-known popular essay: "*Reise eines deutschen Professor ins Eldorado*". Of James Lick, the founder of the observatory, he said: "*I have often asked myself which is a more remarkable fact about America: that millionaires are idealistics, or that idealistics become millionaires. What a fortunate land!...*"

Boltzmann was generally having a wonderful time in California; he smuggled wine into Berkeley and was a weekend house guest at the Hearst estate near Livermore, where he played a Schubert sonata on a Grand Steinway before an audience after dinner. He was astonished to find in the Berkeley bulletin an announcement of a course of lectures, by a female colleague, on the preparation of salads and desserts, alongside the syllabus of his own lectures.

Boltzmann died just a year after the publication of Einstein's first paper on Brownian motion, the harbinger of Boltzmann's ultimate triumph.

1867 CE **Charles-Joseph-Étienne Wolf** (1827–1918, France) and **George-Antoine-Pons Rayet** (1839–1906, France). Astronomers. First to observe visually very broad emission lines in several 8^{th} magnitude stars in Cygnus: the spectra of V1042, MR103 and MR100 was observed by them (1867), before systematic use of photographic plates. It was the first known instance of a *laser* being observed about 100 years before the first artificial one was built. The 'bands' were originally thought to be due to hydrogen molecules.

Stars of this class are called today '*Wolf-Rayet Stars*' (WR). They are very rare (only about 150 in our galaxy of 10^{11} stars) and represent an important phase of stellar evolution.

Wolf-Rayet stars have masses in the range 30–50 solar masses (a solar mass $\approx 2 \times 10^{33}$ gram) and lie near the main sequence of the H-R diagram. A very large percentage of these rare and beautiful stars have been confirmed to be members of close binary systems. Although such stars are few in number, they are important in the generation of the chemical elements, and they play a key role in the life-cycle of stars.

It is believed today that WR stars are at the end of their stellar lives ($\leq 4 \times 10^6$ yr). As these stars age, material which the stars have cooked up in their central nuclear furnaces (like carbon and oxygen) gradually reach the surface of the star. When enough material reaches the surface, it absorbs so much of the intense light from the star that an enormously strong *wind* starts to flow from the star's surface. This wind (which, essentially is an ejected hot gas at a typical velocity of 100 km/sec) becomes so thick that it totally obscures the star. The amount of material which the wind carries away is very large. Typically, WR stars lose mass at a rate of about $10^{-6} - 10^{-5}$ solar masses per year. By comparison, our sun loses about 10^{-14} solar masses per year in its solar wind.

This mass loss is so large that it significantly shortens the stars' life.

Astronomers believe that very massive stars become Wolf-Rayet stars just before they explode as *supernovae*.

WR stars became an important object of research for astronomers and continue to challenge our understanding of massive star evolution and the physics of radiative processes in very dense hot (50,000 to 100,000 degree K) star winds. It has rather broad astrophysical implications; e.g. vigorous stellar winds near the very hot WR star *HD* 56925 (WN4) produces *nebulousity with visible shock fronts.*

Because of their intrinsic brightness and their remarkable spectra, some WR can be observed in distant galaxies up to 60 Mps away (1 parsec=3.2 LY). It makes these stars excellent candidates for measuring distances significantly beyond the Cepheid limit. Because of their remarkable optical spectra, dominated by strong emission lines, WR can be distinguished easily with low-resolution spectroscopy, or with narrow-band photometry. Their spectra is thus useful for determining their atmosphere constituents, radial velocities and their distance from us.

Recently (1995–2003), WR stars were linked to *hypernovae*, which in turn are associated with *gamma-ray bursters*.

Wolf worked in the Paris observatory from 1862 and was a professor of astronomy in Paris during 1875–1901.

1867 CE **Alfred Bernhard Nobel** (1833–1896, Sweden). Invented dynamite (a combination of nitroglycerin with an absorbent substance). Within a few years, he became one of the world's richest men. Nobel set up a fund of about 9 million dollars, the interest from which was to be used for annual award prizes in six different fields [physics, chemistry, physiology (or medicine), literature, peace and economics]. Prizes for the first five categories were first presented in 1901.

Dynamite and Peace

The era of modern explosives began in 1739 with the discovery of glycerin by ***Carl Wilhelm Scheele*** *(1742–1786, Sweden), a struggling apothecary who made many first-class contributions to experimental chemistry. Glycerin — a sweet, syrupy liquid — could be obtained by heating various oils of plant or animal origin. This organic substance, frequently used as a humectant in candy, cosmetics, skin lotion, ink and tobacco, was destined to become a substance of first importance in the manufacturing of modern explosives.*

However, organic chemistry began to take shape as a definite branch of science only about 1830, *and it was not until* 1858 *that its fundamental theory of molecular structure was put forward by* ***Friedrich August Kekulé***

(1829–1896, Germany) and ***Archibald Scott Couper*** *(1831–1892, Scotland). [These dates are significant, because a command of organic chemistry was essential before an organic explosive could be prepared and applied.]*

Thus, in 1846*, an Italian chemist,* ***Sorbero****, first prepared* nitroglycerin *(glycerol trinitrate) by treating glycerin with a mixture of sulphuric and nitric acids at low temperatures. This oily liquid with its sweet burning taste, detonates violently on the slightest touch: there is sufficient oxygen in the molecule to convert all the carbon and hydrogen present into carbon dioxide and water, liberating molecular nitrogen. The reaction instantaneously releases a large amount of gas ($7\frac{1}{2}$ moles) into small volume, (initially occupied by liquid) at a relatively high temperature.*

This physical process, in turn, results in an explosion and shock-wave of enormous proportions. [In marked contrast, nitroglycerin has had an interesting pharmaceutical history in the treatment of angina pectoris, as a coronary vascodilator, when taken in tablet form.] Meanwhile, other developments in organic chemistry were taking place: in 1838 *the French chemists* ***Théophile Jules Pelouze*** *and* ***Henri Braconnot*** *obtained highly inflammable material by treating* cotton *with strong nitric acid, and thereby opened the way to the study of materials which became known as* nitro-celluloses*.*

This process was improved in 1845 *by* ***Christian Friedrich Schönbein*** *(1799–1868, Switzerland), by using a mixture of nitric and sulphuric acids for the nitration of cotton, and resulted (1846) in a new explosive far exceeding gun powder in its power. It was left for Nobel to unite the two lines of research starting from glycerin and cotton, and to show that the explosive properties of nitrated cotton could be tamed for propellant purposes by gelatinizing the fibrous material with nitroglycerin. This discovery, that the two most powerful explosives then known could be blended to furnish a slow burning propellant was so startling that it was received with incredulity, which soon gave place to astonishment. Table 4.10 summarizes the history of explosives.*

1867–1876 CE **Ludwig Schläfli** *(1814–1895, Switzerland). Mathematician. Pioneered in higher-dimensional point geometry and elliptic modular functions (1870). Also made significant contributions to the theory of Bessel functions. The bulk of his work was not published until several years after his death.*

Schläfli was born in Grasswil, Bern. He enrolled in the theological faculty at Bern but, not wishing to pursue an ecclesiastical career, accepted a post as a teacher of mathematics and science at the Burgerschule in Thun. He taught there for ten years, using his few free hours to study higher mathematics. In the autumn of 1843 he accompanied ***Steiner***, ***Jacobi*** *and* ***Dirichlet*** *on their travels in Italy, as an interpreter, and had thus the opportunity to learn* *from the leading mathematicians of his time. It was not until 1868, when he became a full professor at Bern University, that he was free from financial concerns.*

An examination of his posthumous manuscripts reveal that in 1867, ten years ahead of ***Dedekind****, Schläfli discovered the domain of discontinuity of the modular group and used it to make a careful analysis of the Hermite modular function. We have today the* Schläfli modular equation*, as well as Schläfli polynomial, function and hypergeometric series in the theory of Bessel functions.*

Besides his mathematical achievements, Schläfli was an expert on the flora of the canton of Bern and an accomplished student of languages. He possessed a profound knowledge of the Veda*, and his posthumous manuscripts include ninety notebooks of Sanskrit and commentary on the* Rig-Veda*.*

1868 CE **Felice Casorati** *(1835–1890, Italy). Mathematician. Proved the important* Casorati-Weierstrass theorem *which claims that in any neighborhood of an essential singularity of a function, it comes arbitrarily close to any given value.*

Casorati was a student in Pavia and later taught at Pavia and Milan.

Table 4.10 MAJOR EVENTS IN THE HISTORY OF EXPLOSIVE MATERIALS[408]

900–1000	Gunpowder developed in China.
1242	English monk **Roger Bacon** (1220–92) described the preparation of gunpowder (using an anagram).
c.1250	German alchemist **Berthold Schwarz** claimed to have reinvented gunpowder.
1771	French chemist **Pierre Woulfe** discovered picric acid (originally used as a yellow dye).
1807	Scottish cleric **Alexander Forsyth** (1767–1843) discovered mercury fulminate.
1833	French chemist **Henri Braconnot** (1781–1855) nitrated starch, making a highly flammable compound (crude nitrocellulose).
1838	French chemist **Théophile Pelouze** (1807–67) nitrated paper, making crude nitrocellulose.
1845	German chemist **Christian Schönbein** (1799–1868) nitrated cotton, making nitrocellulose.
1846	Italian chemist **Ascania Sobrero** (1812–88) discovered nitroglycerin.
1863	Swedish chemist **J. Wilbrand** discovered trinitrotoluene (TNT).
	Swedish chemist **Alfred Nobel** (1833–96) invented a detonating cap based on mercury fulminate.
1867	Alfred Nobel invented dynamite by mixing nitroglycerin and kieselguhr.
1871	German chemist **Hermann Sprengel** showed that picric acid can be used as an explosive.

[408] For further reading, see:

- Read, John, *Explosives*, Pelican Books, 1942, 159 pp.

1875 Alfred Nobel invented blasting gelatin (nitroglycerin mixed with nitrocellulose).

1885 French chemist **Eugene Turpin** discovered ammonium picrate (Mélinite).

1888 Alfred Nobel invented a propellant from nitroglycerin and nitrocellulose (Ballistite).

1889 British scientists **Frederick Abel** (1826–1902) and **James Dewar** invented a propellant (Cordite) similar to Ballistite.

1891 German chemist **Bernhard Tollens** (1841–1918) discovered pentaerythritol tetranitrate (PETN).

1899 **Henning** discovered cyclotrimethylenetrinitramine (RDX or cyclonite).

1905 US army officer **B.W. Dunn** (1860–1936) invented ammonium picrate explosive (Dunnite).

1915 British scientists invented amatol (TNT + ammonium nitrate).

1955 US scientists developed ammonium nitrate-fuel oil mixtures (ANFO) as industrial explosives.

1868 CE, Jan 30 A bright fireball streaked through the sky over the Polish town of Pultusk (52.42°N; 21.02°E; 30 km north of Warsaw): A small asteroid, with an estimated mass of the order of ten tons ripped through the earth's atmosphere at about 20 km/sec and exploded over the town. It pelted the countryside with a shower of rocks, the fragment of which ranged from the size of peas to chunks weighing about 10 kg. The bombardment occurred when the men of Pultusk were at home instead of working in the fields, so no one was killed or injured. In recent years, scientists have analyzed some of the fragments. It was found that the original asteroid was a piece of the primordial material from which the planets formed $4\frac{1}{2} \times 10^9$ years ago.

1868 CE **Joseph Norman Lockyer** (1836–1920, England). Astronomer. Suggested the existence of a new element, not yet discovered on earth. It was named *Helium* (from the Greek word for the sun), because its characteristic lines were found in the spectrum of solar radiation. In 1895, this sun-element was discovered on earth.

1868–1870 CE **Paul Albert Gordan** (1837–1912, Germany). Mathematician. Contributed to invariant theory and algebraic geometry. Proved that every binary form has an associated finite complete system of invariants and covariants. He also showed that any finite system of binary forms has associated with it such a system of invariants and covariants. Gordan was born in Breslau of Jewish parents. He studied under **Kummer** and **Jacobi** and worked with **Clebsch**. **Emmy Noether** was his only doctoral student.

1868–1893 CE **Karl Hermann Amandus Schwarz** (1843–1921, Germany). Mathematician. A pupil of Weierstrass and his successor as professor of mathematics at Berlin (1897). One of the most distinguished researchers on the calculus of variations in the 19^{th} century. Contributed significantly to many branches of mathematics, including the theory of minimal surfaces, the theory of functions and its applications to potential theory (Dirichlet problem), set theory and conformal mappings.

Schwarz showed that smooth parts of a soap film will intersect a smooth supporting surface perpendicularly: he proved that if a *minimal surface* has a free boundary Σ on a support surface S, then it meets S along the curve Σ at a right angle. **Gergonne** (1816) posed the problem: Divide a cube into two parts by a surface M in such a way that M is attached at two inverse diagonals that lie on opposite faces of the cube, and M is of minimal surface (*Gergonne's surface*). A solution was found by Schwarz in 1872.

H.A. Schwarz was the first to solve the *Plateau problem* for the simplest contour which does not lie in a plane (1865). He also discovered two important *reflection principles* for minimal surfaces and periodic minimal surfaces known as *Schwarz chain*.

Named after him are: *Schwarz' inequality*[409], *Schwarz' theorem*, *Schwarz' lemma*, *Schwarz-Christoffel transformation*, *Schwarzian derivative or differential invariant*[410] and the *Schwarz problem*[411].

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409 $\int_a^b \phi^2(x)dx \int_a^b \psi^2(x)dx \geq \left\{\int_a^b \phi(x)\psi(x)dx\right\}^2$.

410 $\{z,u\} = \frac{z'''(u)}{z'(u)} - \frac{3}{2}\left\{\frac{z''(u)}{z'(u)}\right\}^2$.

411 Schwarz's problem: Given an acute triangle, find an inscribed triangle with the *smallest possible perimeter*. Schwarz discovered that the solution is given by

Schwarz was the son of a Jewish architect. He held the chair of mathematics successively at Halle (1867), Zürich (1869), Göttingen (1875) and Berlin (1892). His research was marked by originality of thought, vivid imagination and a love of detail. His problems were concretely and clearly outlined and usually derived from geometrical or mathematical-physics sources. He exercised a decisive influence on the development of mathematics in the second half of the 19^{th} century.

He was married to Kummer's daughter.

1869 CE, Nov. 17 The *Suez Canal* opened (construction began April 25, 1859). Constructed by a French company under the direction of **Ferdinand Marie de Lesseps** (1805–1894, France). It is a narrow waterway, 160 km long, that connects the Mediterranean and the Red Sea. Lesseps, canal builder and diplomat, was born in Versailles, and during 1825–1849 worked in the French diplomatic service.

1869 CE **John Wesley Hyatt** (1837–1920, U.S.A.). Inventor. A printer in Albany, N.Y., who invented *Celluloid*, the first synthetic plastics material to receive wide commercial use. Hyatt was seeking a substitute for ivory to make billiard balls. Celluloid could be sawed,carved, and made into sheets. As a result, new plastics products appeared on the market, including the first photographic roll film. But celluloid was hard to mold and it caught fire easily.

1869 CE First systematic *color photography*[412] (subtractive method) done simultaneously by the Frenchmen **Emile Hortensius Charles Cros** (1842–1888) and **Louis Ducos de Hauron** (1837–1920). Both based their system

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the *altitude triangle*, whose vertices are obtained by dropping the perpendicular from each vertex of the original triangle to its opposite side. It is easy to see that the altitude triangle must be a *light-ray triangle*; if we think of a triangular room with mirror walls, the inscribed triangle represents a closed path of travel for a ray of light in the room. It can be shown that the perimeter of the inscribed light-triangle is equal to $a\cos A + b\cos B + c\cos C$, where ABC is the original triangle.

[412] The first color photograph was demonstrated by **James Clerk Maxwell**, already in 1861.
The physicist, **Gabriel Jonas Lippman** (1845–1921, France) invented in 1891 the first *color photographic process* (based on the phenomenon of light interference). This earned him the physics Nobel prize for 1908. The relative long time required for a film to develop by this method precluded its commercial success, and it was therefore superseded by the Maxwell 3-color procedure. However, *Lippman's plate* found new applications in *holography* (1962).

on the *Young-Maxwell* color separation and mixing theory, by superposing three color positive pictures on one another.

1869–1871 CE **Dimitri Ivanovich Mendeleev** (1834–1907, Russia). Chemist. Introduced order into inorganic chemistry by devising the Periodic Table, that systematized the properties of the elements known at his time and permitted prediction of the existence of new ones. The later synthesis of new elements has been based on his work. Various chemists[413] had traced numerical sequences among the atomic weights of some of the elements and noted connections between them and the properties of the different substances, but it fell to him to give a full expression to the generalization, and to treat it not merely as a system of classifying the elements according to certain observed facts, but as a *law of nature* — which could be relied upon to predict new facts and to disclose errors in what were supposed to be old facts. Thus in 1871 he was led, by certain gaps in his tables, to assert the existence of 3 new elements, hitherto unknown to chemistry, and to assign them definite properties [*gallium*, discovered 1871; *scandium*, 1879; *germanium*, 1886].

The youngest of a family of 17, Mendeleev was born at Tobolsk, Siberia. After attending the gymnasium of his native town, he went to study natural science at St. Petersburg where he graduated in chemistry (1856), subsequently becoming *privatdocent*. In 1860 he went to Heidelberg where he started a laboratory of his own, but returned to St. Petersburg in 1861. He became professor of chemistry in the technological institute there in 1863, and three years later succeeded to the same chair at the University. In 1890

[413] The periodic law was proposed in 1869 independently by **Julius Lothar Meyer** (1830–1895, Germany), who plotted *atomic volumes* (atomic weight/density) of the elements against their atomic weight. The resulting curve exhibits periodicity in the case of other properties, such as expansion by heat, thermal and electrical conductivities, magnetic susceptibility, melting point, refractive index, boiling point, crystalline form, compressibility, atomic heat at low temperatures, heats of formation of oxides and chlorides, hardness, malleability, volatility, volume change on fusion, viscosity and color of salts in aqueous solution, mobility of ions, electrode potentials of metals, frequency of atomic vibrations in solids, distribution of elements in nature, distribution of spectral lines, and valence. As Mendeleev said, "*these regularities can hardly be the result of chance*".

Lothar Meyer pointed out that gaseous elements, occur at the maxima and on ascending portions of the atomic volume curve.

Lewis Reeve Gibbes (1810–1894, U.S.A.) constructed in 1870 a table of the chemical elements which arranged the elements into families according to valences. This work was not published until 1886, by which time the tables of Mendeleev and Meyer were well established.

he resigned the professorship, and in 1893 he was appointed director of the Bureau of Weights and Measures, a post which he occupied till his death.

Boyle, who laid the foundations of modern chemistry in the 17^{th} century, was familiar with the concept of atoms, which may have assisted him in his attempts to classify all substances into elements, compounds and mixtures. However, the full importance of the atomic theory in chemistry was not realized until **Dalton** used it to expound the laws of chemical combinations in 1803. Since the chemical elements react with each other in *fixed proportions by weight*, it appeared that atoms of different elements are combining to form compound atoms or *molecules*. The success of this idea led to the introduction of chemical formulae for the simpler compounds, each formula indicating the atoms present in a single molecule of the compound.

An important development was the introduction of the concept of the *valency* concept in 1852 by **Edward Frankland** (1825–1899, England), which is, ideally, the number of hydrogen atoms combining with one atom of the element considered. Through the introduction of single, double, and triple bonds of the 'covalent' type, the greater part of organic chemistry could be brought into one scheme. It was found, however, that a single valency could not be assigned for many atoms [e.g. nitrogen and sulphur, which do not form ions and thus exhibit variable valency].

Mendeleev's periodic classification was a major development amid the confusion of chemical ideas which prevailed in the middle years of the 19^{th} century. This table grouped elements with valence properties in vertical columns, numbered from I to VIII. The chemical resemblance of the first two rows (*lithium* to *fluorine* and *sodium* to *chlorine*) had been recognized for some time. Although the scheme was received with skepticism, its essential correctness was demonstrated eventually by the discovery of the inert gases (*helium*, *neon*, *argon*, *krypton*, and *xenon*), which provided a completely new column in the table.

In its modern form (Table 4.11), the periodic table is based on the *atomic number* (Z) of the elements, where Z is the number of electrons accommodated outside the nucleus of the atom. The periodic classification shows immediately that the electrons in an atom possesses some kind of *shell structure*. It was only since 1925, the year of the *Pauli exclusion principle*, that the '7^{th} veil' was finally lifted, and the true infrastructure of the periodic table was understood.

Table 4.11: Periodic Table of the Elements

1 1A	2 2A	3 3B	4 4B	5 5B	6 6B	7 7B	8 8B	9 8B	10 8B	11 1B	12 2B	13 3A	14 4A	15 5A	16 6A	17 7A	18 8A
1 **H** 1.00794																	2 **He** 4.00260
3 **Li** 6.941	4 **Be** 9.01218											5 **B** 10.811	6 **C** 12.011	7 **N** 14.0067	8 **O** 15.9994	9 **F** 18.9984	10 **Ne** 20.1797
11 **Na** 22.9898	12 **Mg** 24.3050											13 **Al** 26.9815	14 **Si** 28.0855	15 **P** 30.9738	16 **S** 32.066	17 **Cl** 35.4527	18 **Ar** 39.948
19 **K** 39.0983	20 **Ca** 40.078	21 **Sc** 44.9559	22 **Ti** 47.88	23 **V** 50.9415	24 **Cr** 51.9961	25 **Mn** 54.9381	26 **Fe** 55.847	27 **Co** 58.9332	28 **Ni** 58.693	29 **Cu** 63.546	30 **Zn** 65.39	31 **Ga** 69.723	32 **Ge** 72.61	33 **As** 74.9216	34 **Se** 78.96	35 **Br** 79.904	36 **Kr** 83.80
37 **Rb** 85.4678	38 **Sr** 87.62	39 **Y** 88.9059	40 **Zr** 91.224	41 **Nb** 92.9064	42 **Mo** 95.94	43 **Tc** (98)	44 **Ru** 101.07	45 **Rh** 102.906	46 **Pd** 106.42	47 **Ag** 107.868	48 **Cd** 112.411	49 **In** 114.818	50 **Sn** 118.710	51 **Sb** 121.757	52 **Te** 127.60	53 **I** 126.904	54 **Xe** 131.29
55 **Cs** 132.905	56 **Ba** 137.327	57 ***La** 138.906	72 **Hf** 178.49	73 **Ta** 180.948	74 **W** 183.84	75 **Re** 186.207	76 **Os** 190.23	77 **Ir** 192.22	78 **Pt** 195.08	79 **Au** 196.967	80 **Hg** 200.59	81 **Tl** 204.383	82 **Pb** 207.2	83 **Bi** 208.980	84 **Po** (209)	85 **At** (210)	86 **Rn** (222)
87 **Fr** (223)	88 **Ra** 226.025	89 **Ac** 227.028	104 **Rf** (261)	105 **Db** (262)	106 **Sg** (263)	107 **Bh** (262)	108 **Hs** (265)	109 **Mt** (266)	110 (269)	111 (272)	112 (272)		114 (287)		116 (289)		118 (293)

*Lanthanide series	58 **Ce** 140.115	59 **Pr** 140.908	60 **Nd** 144.24	61 **Pm** (145)	62 **Sm** 150.36	63 **Eu** 151.965	64 **Gd** 157.25	65 **Tb** 158.925	66 **Dy** 162.50	67 **Ho** 164.930	68 **Er** 167.26	69 **Tm** 168.934	70 **Yb** 173.04	71 **Lu** 174.967
Actinide series	90 **Th** 232.038	91 **Pa** 231.036	92 **U** 238.029	93 **Np** 237.048	94 **Pu** (244)	95 **Am** (243)	96 **Cm** (247)	97 **Bk** (247)	98 **Cf** (251)	99 **Es** (252)	100 **Fm** (257)	101 **Md** (258)	102 **No** (259)	103 **Lr** (260)

Atomic masses are relative to carbon-12. For certain radioactive elements, the numbers listed (in parentheses) are the mass numbers of the most stable isotopes.

The Elements

Name	Symbol	Atomic Number	Relative Atomic Weight
Actinium	Ac	89	227.028
Aluminum	Al	13	26.9815
Americium	Am	95	(243)
Antimony	Sb	51	121.757
Argon	Ar	18	39.948
Arsenic	As	33	74.9216
Astatine	At	85	(210)
Barium	Ba	56	137.327
Berkelium	Bk	97	(247)
Beryllium	Be	4	9.01218
Bismuth	Bi	83	208.980
Bohrium	Bh	107	(262)
Boron	B	5	10.811
Bromine	Br	35	79.904
Cadmium	Cd	48	112.411
Calcium	Ca	20	40.078
Californium	Cf	98	(251)
Carbon	C	6	12.011
Cerium	Ce	58	140.115
Cesium	Cs	55	132.905
Chlorine	Cl	17	35.4527
Chromium	Cr	24	51.9961
Cobalt	Co	27	58.9332
Copper	Cu	29	63.546
Curium	Cm	96	(247)
Dubnium	Db	105	(262)
Dysprosium	Dy	66	162.50
Einsteinium	Es	99	(252)
Erbium	Er	68	167.26
Europium	Eu	63	151.965
Fermium	Fm	100	(257)
Fluorine	F	9	18.9984
Francium	Fr	87	(223)
Gadolinium	Gd	64	157.25
Gallium	Ga	31	69.723
Germanium	Ge	32	72.61
Gold	Au	79	196.967
Hafnium	Hf	72	178.49
Hassium	Hs	108	(265)
Helium	He	2	4.00260
Holmium	Ho	67	164.930
Hydrogen	H	1	1.00794
Indium	In	49	114.818
Iodine	I	53	126.904
Iridium	Ir	77	192.22
Iron	Fe	26	55.847
Krypton	Kr	36	83.80
Lanthanum	La	57	138.906
Lawrencium	Lr	103	(260)
Lead	Pb	82	207.2
Lithium	Li	3	6.941
Lutetium	Lu	71	174.967
Magnesium	Mg	12	24.3050
Manganese	Mn	25	54.9381
Meitnerium	Mt	109	(266)
Mendelevium	Md	101	(258)
Mercury	Hg	80	200.59
Molybdenum	Mo	42	95.94
Neodymium	Nd	60	144.24
Neon	Ne	10	20.1797
Neptunium	Np	93	237.048
Nickel	Ni	28	58.693
Niobium	Nb	41	92.9064
Nitrogen	N	7	14.0067
Nobelium	No	102	(259)
Osmium	Os	76	190.23
Oxygen	O	8	15.9994
Palladium	Pd	46	106.42
Phosphorus	P	15	30.9738
Platinum	Pt	78	195.08
Plutonium	Pu	94	(244)
Polonium	Po	84	(209)
Potassium	K	19	39.0983
Praseodymium	Pr	59	140.908
Promethium	Pm	61	(145)
Protactinium	Pa	91	231.036
Radium	Ra	88	226.025
Radon	Rn	86	(222)
Rhenium	Re	75	186.207
Rhodium	Rh	45	102.906
Rubidium	Rb	37	85.4678
Ruthenium	Ru	44	101.07
Rutherfordium	Rf	104	(261)
Samarium	Sm	62	150.36
Scandium	Sc	21	44.9559
Seaborgium	Sg	106	(263)
Selenium	Se	34	78.96
Silicon	Si	14	28.0855
Silver	Ag	47	107.868
Sodium	Na	11	22.9898
Strontium	Sr	38	87.62
Sulfur	S	16	32.066
Tantalum	Ta	73	180.948
Technetium	Tc	43	(98)
Tellurium	Te	52	127.60
Terbium	Tb	65	158.925
Thallium	Tl	81	204.383
Thorium	Th	90	232.038
Thulium	Tm	69	168.934
Tin	Sn	50	118.710
Titanium	Ti	22	47.88
Tungsten	W	74	183.84
Uranium	U	92	238.029
Vanadium	V	23	50.9415
Xenon	Xe	54	131.29
Ytterbium	Yb	70	173.04
Yttrium	Y	39	88.9059
Zinc	Zn	30	65.39
Zirconium	Zr	40	91.224

Atomic masses in this table are relative to carbon-12 and limited to six significant figures, although some atomic masses are known more precisely. For certain radioactive elements the numbers listed (in parentheses) are the mass numbers of the most stable isotopes.

The Elements[414]

Chemistry started to emerge from its alchemical roots in the 18th century, partly with the discovery of new elements: between 1735 and 1826, no fewer than 40 were added to the 9 known to the ancients (copper, silver, gold, iron, mercury, lead, tin, sulphur and carbon) and the few discovered in the Middle Ages (arsenic, antimony and bismuth). The discovery of these new elements forced certain questions on every chemist: How many elements were there? Was there any limit to their number? Were they all related somehow? And if so, how could they be classified?

Kinships were recognized among some. Chlorine, bromine and iodine — all colored, volatile, hungrily reactive — seemed a natural family, the halogens. Calcium, strontium and barium, the alkaline earth metals, were another family, for they were all light, soft, readily set alight and strongly reactive with water.

In 1817, ***Johann Döbereiner*** *observed that the atomic weights of the alkaline earth metals formed a series, the atomic weight of strontium being just midway between those of calcium and barium. He later discovered other such triads, as well as triads in which the elements had similar properties but almost identical atomic weights.*

Döbereiner's triads convinced many chemists that atomic weight must represent a fundamental characteristic of all elements. But confusion about the basics remained — about the difference between atoms and molecules and about the combining power, or valency, of atoms. As a consequence, many accepted atomic weights were wrong. Dalton himself — the originator of the atomic hypothesis — assumed, for instance, that the formula of water was HO *and not* H_2O*, giving him an atomic weight for oxygen that was only half the correct number.*

In 1860, the first international gathering of chemists was convened at Karlsruhe, Germany, for the expressed purpose of clearing up this confusion. Here, ***Stanislao Cannizzaro*** *proposed a reliable way of calculating atomic weights from vapor density, and his beautifully argued presentation carried the day, leading to a consensus: now, at last, with corrected atomic weights and a*

414 For further reading, see:

- Emsley, John, *Nature's Building Blocks*, Oxford University Press, 2003, 539 pp.

clear idea of valency, the way was open for a comprehensive classification of the elements.

It is a remarkable example of synchronicity that no fewer than six such classifications, all pointing toward the discovery of periodicity, were independently devised in the next decade. Of these, ***Dmitri Ivanovich Mendeleev's*** *system was the most comprehensive, and also the most audacious, for it ventured to make detailed predictions of elements as yet unknown.*

Mendeleev was the author of a chemistry text "The Principles of Chemistry", and he had brooded since 1854 on how the chemical elements might be classified.

With the old, pre-Karlsruhe atomic weights, one could get, as Döbereiner did, a sense of local triads, or groups. But one could not easily see that there was a numerical relationship between the groups themselves, Only when Cannizzaro showed that the proper atomic weights for the alkaline earth metals, calcium, strontium and barium, were 40, 88 and 137 did it become clear how close these were to those of the alkali metals, potassium (39), rubidium (85) and cesium (133). It was this closeness, and the closeness of the atomic weights of the halogens — chlorine, bromine and iodine — that incited Mendeleev in 1868 to make a small, two-dimensional grid juxtaposing the three groups:

Cl	35.5	K	39	Ca	40
Br	80	Rb	85	Sr	88
I	127	Cs	133	Ba	137

And it was at this point, seeing that arranging the three groups of elements in order of atomic weight produced a repetitive pattern — a halogen followed by an alkali metal followed by an alkaline earth metal — that Mendeleev felt this must be a fragment of a larger pattern and leaped to the idea of a periodicity governing all the elements, a periodic law.

Mendeleev's first small table had to be filled in and then extended in all directions, as if filling up a crossword puzzle. Alternating between conscious calculation and hunch, between intuition and analysis, Mendeleev arrived within a few weeks at a tabulation of 30-odd elements in order of ascending atomic weight, a tabulation that suggested that there was a recapitulation of properties with every eighth element.

On the night of Feb. 16, 1869, it is said, Mendeleev had a dream in which he saw almost all of the 65 known elements arrayed in a grand table. The following morning, he committed this to paper.

This first table was to undergo considerable revision over the next few years, but by 1871 it had taken its now familiar form of a chunky rectangle with intersecting groups and periods.

It was this table that was to be found in every textbook, lecture room and museum for a century. One could read the table up and down, going from one group to another (each vertical group was a family of elements with similar reactivity and valency) — this was what Döbereiner and the pre-1860 chemists would have done.

But one could also read it horizontally, getting a feel for each period as it moved through the eight groups. One could see the way in which the properties of the elements changed with each increment of atomic weight, until suddenly the period came to an end and one found oneself on the next row and period, where all the elements echoed the properties of those above. It was this, above all, that gave one a feel for the mysterious periodicity of the table, the reality of the great law it enshrined.

The periodic table did not actually tell one the properties of the elements, but like a family tree, it assigned them places. One could plot the physical and chemical properties of all the elements against their atomic weights and obtain the most tantalizing graphs. If one plotted atomic volume against atomic weight, for example, one would get a many-peaked curve, with summits for the light Group I metals, valleys for the dense Group VIII metals. Every property, it seemed, varied periodically and was somehow linked with atomic weight. But why any of the elements should have the properties they had, and why such properties should recur in periodicity with atomic weight, were complete mysteries.

From 1869 to 1871, Mendeleev expanded the table, going so far as to reposition elements that did not fit, revising their accepted atomic weights to make them fit, a practice that shocked some of his contemporaries. Further challenges were presented by two groups of elements, the transition elements (these included rare metals like vanadium and platinum, as well as common ones like iron and nickel) and the rare-earth elements. Neither of these seemed to fit in the neat "octaves" of the earlier periods. To accommodate them, Mendeleev and others experimented with new forms of the table — helical forms, pyramidal forms, etc. — that, in a sense, gave it extra dimensions.

In an act of supreme confidence, Mendeleev reserved several empty spaces in his table for elements "as yet unknown". He asserted that by extrapolating from the properties of the elements above and below (and also, to some extent, from those to either side), one might make a confident prediction as to what these unknown elements would be like. He did exactly this, predicting in great detail a new element that would follow aluminum in Group III: it would be a silvery metal, he thought, with a density of 6.0 and an atomic weight of 68. Four years later, in 1875, just such an element was found: GALLIUM.

He also predicted with equal precision the existence of SCANDIUM *and* GERMANIUM, *and these too were soon discovered. It was this ability to predict* *elements in such detail that stunned his fellow chemists and convinced many of them that Mendeleev's system was not just an arbitrary ordering of the elements but a profound expression of reality.*

But Mendeleev was astonished, as everyone was, by the discovery in the 1890's of an entire new family of elements, the inert gases. He was at first skeptical of their existence. (He initially thought that ARGON*, the first found, was just a heavier form of nitrogen.) But with the discovery of* HELIUM, NEON, KRYPTON, XENON *and finally* RADON*, it was clear that they formed a perfect periodic group. They were identical in their inability to form compounds; they had a valency, it seemed, of zero. So to the eight groups of the table, Mendeleev now added a final Group 0.*

With the inert gases in place, the number of elements in each period stood out: 2 (hydrogen and helium) in the first period; 8 each in the second and third; 8 typical plus 10 transition elements, or 18 each, in the fourth and fifth periods; 8 plus 10 plus 14 rare-earth elements, or 32, in the sixth period. These were the magical numbers — 2, 8, 8, 18, 18, 32. But what did they mean? And what, in broader terms, was the basis of chemical properties?

Mendeleev constantly returned to these questions. He yearned for a new "chemical mechanics", comparable to the classical mechanics of Newton. And yet one wonders what he might have thought of the actual form of the revolution that took place after his death, a revolution wholly unimaginable in terms of classical mechanics.

The new insight into the internal constitution of atoms came in 1911, four years after Mendeleev's death, when ***Ernest Rutherford*** *(bombarding gold foil with alpha particles and finding that, very occasionally, one was deflected back) inferred that the atom must have a structure like a miniature solar system, with almost all of its mass concentrated in a minute, very dense, positively charged nucleus surrounded at relatively great distances by relatively light electrons.*

But the very essence of atoms was their absolute stability. And such an atom as Rutherford's, if ruled by the laws of classical mechanics, would not be stable; its electrons would lose energy (by EM radiation) as they orbited, eventually diving into the nucleus.

Niels Bohr*, working with Rutherford in 1912, was intensely aware of this, and of the need for a radically new approach. This he found in the quantum theory, which postulated that electromagnetic energy — light, radiation — was not emitted or absorbed continuously, but rather in discrete packets, or "quanta". Bohr, by an astounding leap, connected these concepts with the Rutherford model and with the well-known but previously inexplicable nature of optical spectra — that these were not only characteristic for each element* *but consisted of a multitude of discrete lines or frequencies which obeyed Ritz's combination law.*

All of these considerations came together in the Bohr atom, where electrons were conceived to occupy a series of orbits, or "shells", about the nucleus – of differing radii and energies. Unlike classical orbits, which would decay, these quantum orbits had a stability that allowed them to maintain themselves, potentially, forever. (But if the atom was excited, some of its electrons might leap to higher energy orbits for a while and in returning to their ground state emit a quantum of light energy of a certain frequency; it was this that caused the characteristic absorption and emission lines in their spectra.)

Bohr presented his model of the atom in the spring of 1913. A few months later, ***Henry Moseley*** *found a most intimate relationship between the order of the elements and their X-ray spectra. These spectra could be correlated, Moseley thought, with the number of positive charge units carried by the nucleus, and for this the term "atomic number" was used. With atomic numbers, there were no gaps or fractions or irregularities, as with atomic weights. It was atomic number, not atomic weight, that determined the order of the elements. And Moseley could now say with absolute confidence that there were only 92 elements between hydrogen and uranium, including half a dozen as yet undiscovered. (Three of these had been predicted, though vaguely, by Mendeleev.)*

Bohr's model suggested that every element's chemical properties, its position in the periodic table, depended on the number of its electrons and how these were organized in successive shells. Valency and chemical reactivity, the definers of Mendeleev's groups, were correlated with the number of valence electrons in the outer shell: with the maximum of eight electrons, an atom was chemically inert; with more, or less, than the maximum, it would tend to be more reactive. Thus the halogens, only one electron short in their outermost shells, were avid to pick up an eighth electron, whereas the alkali metals, with only a single electron in their outer shells, were avid to get rid of it, to become stable in their own way.

To this basic "eightness", extra sub-shells were added in the later periods: two 10-electron shells for the transition elements and two 14-electron shells for the rare-earth elements.

Bohr *and* ***Moseley*** *thus provided a spectacular confirmation of the periodic table, grounding it, as* ***Mendeleev*** *had hoped, in "the invisible world of chemical atoms". The periodicity of the elements, it was now clear, emerged from their electronic structure. And the mysterious numbers that governed the periodic table — 2, 8, 8, 18, 18, 32 (and eventually, another "32") — could now be understood as the number of electrons added in each period.*

Such an electronic periodic table is basically identical with Mendeleev's table, posited nearly half a century earlier on purely chemical grounds. Moseley and Bohr worked from the inside, with the invisible world of chemical atoms, and Mendeleev and his contemporaries worked from the outside, with the visible macroscopic and manifest physico-chemical properties of the elements — and yet they arrived at the same point. This is the beauty of the periodic table, indeed, that it looks both ways, uniting classical material science and chemistry and quantum physics in a magical synthesis.

Given Bohr's orbits[415] *of different energy levels, together with the property of* electron spin *and the* Pauli exclusion principle *(both discovered in the 1920's, close on the heels of Quantum Mechanics), one can, in principle, build up the whole periodic table by adding electrons to the atom (and protons to its nucleus) one at a time, climbing the rungs of an atomic ladder from hydrogen to uranium. And it is by such a building-up that we have been able to create new elements absent in nature, like the 20 elements (93–112) that now follow uranium in the periodic table, heavier atoms that do not depart from the regularities of the periodic law.*

In principle, one can work out the periodic table to element 200 and beyond and predict some of the properties of such elements. (These predictions are largely theoretical, because the highly radioactive transuranic elements tend to get more and more unstable. One may only be able to produce an atom at a time, and this may be gone in a few millionths of a second. And there are theoretical reasons for believing that for atomic numbers above about 137, the intense electric fields near the nucleus might locally destabilize the vacuum, *producing pair-created electrons and positrons and destroying such heavy nuclei before they have had a chance to form atoms.)*

The periodic table is still the icon of chemistry, as it has been since 1869. It continues to guide chemical research, to suggest new syntheses, to allow predictions of the properties of never-before-seen materials. It is a marvelous map to the whole geography of the elements and their compounds and alloys.

[415] (later renamed *orbitals* when quantum mechanics – which supplanted the old quantum theory of Planck, Einstein and Bohr – showed them to be fuzzy, variously-shaped, continuous probability distributions rather than Keplerian orbits.)

1869–1874 CE **Johann Friedrich Miescher** (1844–1895, Switzerland). Chemist and physiologist. Professor at Basel (1871–1895). Discovered (1869) *nucleic acids* in cell nuclei. Miescher, isolated (1874) a substance from tissue, that turned out to be neither carbohydrate, lipid, nor protein. Since he had obtained it from cell nuclei, he named it *nuclein*. In time the substance turned out to have acid properties, so it was renamed *nucleic acid*, but it was not connected either to heredity or to chromosomes. This substance was eventually (1944) found to be joined to the protein of chromosomes, and given the name *nucleoprotein*. In Miescher's time, however, no one understood its significance. [Miescher later discovered that salmon sperm are almost entirely nucleic acid plus a simple protein; but he failed to connect this fact with heredity.]

1869–1877 CE **Elwin Bruno Christoffel** (1829–1900, Germany). Mathematician. Discovered the procedure now known as '*covariant differentiation*' (1869) and introduced two symbols, now named after him. Independently of Riemann, he discovered the concepts of space curvature and metric. In 1877 Christoffel derived the cubic equation for the three plane-wave phase velocities in general anisotropic elastic media. He was also one of the first contributors to the theory of shock waves.

Christoffel studied at the University of Berlin, where he was taught by Dirichlet. Obtained his doctorate in 1856 and became eventually a professor of mathematics at the University of Strasbourg (1872–1892).

1869–1882 CE **Rudolf Otto Sigismund Lipschitz** (1832–1903, Germany). Mathematician. Invented independently the process of *covariant differentiation* (1869). Discovered new theorems concerning subspaces of Riemannian and Euclidean manifolds, the mean curvature vector and minimal subspaces.

Lipschitz was born to Jewish parents, on his father's estate near Königsberg. At the age of 15 he began the study of mathematics at the Königsberg University. He received his doctorate from the University of Berlin in 1853. From 1864 onwards he was a full professor at Bonn. Lipschitz was a corresponding member of the academies of Paris, Berlin, Göttingen and Rome.

With Christoffel, Aronhold and Clebsch he laid the foundations to Ricci's absolute differential calculus[416].

[416] If $f(x,y)$ is defined in a region S such that, for *any* two points (x,y) and $(x,\overline{y})$ in S, $|f(x,y)-f(x,\overline{y})| \leq N|y-\overline{y}|^{\alpha}$, where N, α are positive constants, then $f(x,y)$ is said to satisfy the *Lipschitz condition* in S.

1870 CE[417] **Eugène Rouché** (1832–1910, France). Mathematician. Worked on complex functions, descriptive geometry, algebra and probability theory. Was born in Sommiéres, and died in Lunel, France. Known mainly for *Rouché's Theorem*[418].

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Darwinism ad Absurdum

A scientific theory that had the most revolutionary impact on almost every facet of Western thought and society in the second half of the 19^{th} *century was Darwin's theory of evolution. To an age that worshiped science, the thought that man was just as much subject to the logic of science as was everything else in nature, also held great fascination.*

Underlying much of Darwin's work was the idea of progress, an idea dear to the 19^{th} *century. History, the study of man's past, suddenly appeared in a new light — as a march toward some far-off, lofty goal. The concept of life as a struggle for existence in which the fittest would survive had particular appeal to his contemporaries. The philosophy of laissez faire, with its emphasis on competition, had long been hailed as the root of economic success. With the advent of Darwinism, this belief seemed to have been given scientific sanction.*

If $f(x,y)$ has a continuous partial derivative w.r.t. y in S, and if this partial derivative is bounded in S, then $f(x,y)$ satisfies the Lipschitz condition with $\alpha = 1$.

If $f(x,y)$ is continuous and satisfies the Lipschitz condition with $\alpha = 1$ in $S\{0 \le x \le b, |y - y_0| < c\}$, then the initial-value problem $\frac{dy}{dx} = f(x,y)$, $y(0) = y_0$, $M = \max|f(x,y)|$ in S has a unique solution for $0 \le x \le \min\left[b, \frac{c}{M}\right]$.

417 In the absence of additional biographical material, this year was arbitrarily chosen for the inception of the *Rouché theorem*.

418 Let $f(z)$ and $g(z)$ be analytic within and on a simple closed contour C and satisfy the inequality $|g(z)| < |f(z)|$ on C, where $f(z)$ does not vanish. Then $f(z)$ and $f(z) + g(z)$ have the same number of zeros inside C.
This theorem provides a proof that an algebraic equation of degree n has n roots (cannot be proved by *purely* algebraic methods!).

Big business, according to John D. Rockefeller, was "merely a survival of the fittest... the working out of a law of nature and a law of God". But not only the Capitalists derived great comfort from Darwin. His emphasis on the importance of environment for the improvement of man also gave hope to the socialists in their demands for social and economic reform. More than ten years before Darwin published his Origin of the Species, ***Karl Marx****, the "*Darwin of the Social Sciences*", had already sketched the evolution of society through a series of struggles among social classes.*

The application of Darwin's theory to groups and states, rather then individuals, was promoted by **Herbert Spencer** *and* **Walter Bagehot** *(1872), and is known as '*Social Darwinism*'. It blends evolutionary and nationalist elements, and argues that the majority of* groups *which win and conquer are better than the majority of those which fail and perish.*

To a generation that had recently experienced several major wars and that was actively engaged in numerous colonial expeditions against native peoples overseas, Social Darwinism with its glorification of war came as welcome rationalization[419] [e.g. President Theodore Roosevelt held that war alone enabled man to "acquire those virile qualities necessary to win in the stern strife of actual life".]

While Darwin's ideas (1844) were accepted almost everywhere around the world, they were somehow slow to reach the "Bible belt" in the deep south of the U.S. It thus came to pass that 81 years later, in the town of Dayton, Tennessee, a public school science teacher by the name of John T. Scopes was arrested for violating the Anti-Evolution Law *that prohibited teaching Darwin's theory of evolution in public schools of that state.*

*The ensuing trial (known as the "*Monkey Trial*") took place in July 1925. Assisting the state prosecution was* **William Jennings Bryan** *(1860–1925), a famous orator and statesman who strongly advocated literal interpretation of the Bible and who believed in religious fundamentalism.*

Opposite him stood **Clarence Seward Darrow** *(1857–1938), the renowned criminal lawyer who defended Scopes and the right to teach evolution.*

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[419] The maxim that 'might makes right' had little to do with Darwin's original theory. In fact, the emphasis on struggle as a *necessary condition* for progress was a narrow and one-sided interpretation of Darwinism, not shared by its author. In his *Descent of Man*, Darwin had emphasized that a feeling of sympathy and coherence, social and moral activities, were needed for the advancement of society.

When the legal aspects of the case had been fought to conclusion[420]*, when both sides had belabored the right of sovereign people to pass any legislation they saw fit, and when the question of whether the Anti-Evolution Law violated the Constitution of the United States had been obscured, it was the fifty questions that Darrow had put to Bryan, which suddenly flashed the trial into focus an discredited the Anti-Evolution Bill.*

However, the anti-evolution laws remained on the books in half a dozen states for another forty years.

Social Darwinism also led to the clothing of racism *in the disguise of a scientific doctrine. The first 'theorist' of the superiority of the Germanic 'Aryans' over the inferior Slavs and Jews was the French* ***Joseph Arthur de Gobineau*** *in his* Inequality of the Human Races *(1852). The idea of white, specifically Anglo-Saxon, superiority found its main echo in Germany but was also popular in England and America. Ever since, it remained one of the underlying ideological sources and justifications of modern antisemitism.*

Recently, a number of authors[421] *have linked Darwin's theory of evolution to the major mass murders and genocides in the 20th century. It seems that Darwinism has infected the whole culture. Indeed, the world would witness Nazi Germany, Stalinist gulags and the slaughter of 70 million Chinese at the hands of their exalted chairman.*

Scientists such as ***Francis Galton*** *and* ***Ernst Haeckel*** *extended Darwinism to advance their ideas for selective breeding of humans and forced sterilization of "unfit", calling politics "applied biology" — a phrase later appropriated by the Nazis.*

It is impossible to understand Hitler's monstrous views apart from his belief in natural selection applied to races. He believed Darwin's theory of natural selection showed that "science" justified the extermination of the Jews.

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420 The real story of the Scopes trial is told in the book *Summer of the Gods* (by E.J. Larson, Basic Books: New York, 1997): the trial was nothing but a publicity stunt. The idea for a trial on evolution was hatched by the ACLU in New York and seized upon by civic leaders in Dayton, Tennessee as a way to drum up publicity for their town.

421 In his book "*From Darwin to Hitler*" (Palgrave MacMillan, 2004), **Richard Weikart** documents the proliferation of eugenics organization in Germany around 1900. Darwin's theory was quickly and widely accepted among German biologists and Darwinism provided the lingo for "scientific" racism. Not only were all eugenicists Darwinists, but nearly all Darwinists were scientific racist. See also the last chapter of the Ann Coulter's book *Godless* (Crown Forum Publ.: New York, 2006).

Hitler embraced an evolutionary ethic that made Darwinian struggle for existence between races, become the sole arbiter for morality.

Indeed, within one century of the appearance of this book, the 'theory' has been applied by the Germans in a most efficient way, for the "final solution" of the "Jewish problem" in Europe.

1870–1871 CE The *Franco-Prussian War* resulted in the foundation of the German Empire. To it, France ceded Alsace-Lorraine and paid one billion dollars in reparations. The war led France to withdraw the French troops that were protecting Rome for the Pope. The Italian army moved into Rome, and Italy at last included the entire peninsula. The Pope's territory was reduced to Vatican City, and Rome became the capital of Italy in 1871.

1870–1893 CE **Marius Sophus Lie** (1842–1899, Norway). A path-breaking mathematician whose work has found wide applications in 20^{th} century analysis and physics.

Developed his notions of continuous transformation groups and their role in the theory of differential equations. Today the theory of continuous groups is a fundamental tool in such diverse areas as analysis, differential geometry, number theory, differential equations, mechanics, atomic structure and high energy physics. *Lie groups* and *Lie algebras* are named after him.

Lie groups are smooth Riemannian manifolds in which each point is an element in a continuous group of matrices. A tensor, which in ordinary manifolds enters via the study of tangent curves, enters in Lie groups in a two-fold manner: (1) In the ordinary way on manifolds. (2) Each point on the manifold is itself a matrix. Lie invented the so-called 'Lie-derivative'[422].

[422] *The Lie derivative* is a covariant process of *directional* differentiation which is distinct from "covariant differentiation" (absolute differentiation, based on the affine connection). The Lie derivative depends only on the *tensor field* it acts on and on the *vector field* defining the local direction, and in this sense is more natural.

Both derivatives obey the basic laws of differentiation: they are linear, obey Leibniz' rule, and reduce to ordinary directional derivative when acting on a scalar field. Thus, for a scalar field $\phi(x)$ and a contravariant vector field $v^i(x)$, the Lie derivative of ϕ along v is by definition $\mathcal{L}_{\boldsymbol{V}}\phi \equiv v^i \frac{\partial\phi}{\partial x^i} = \boldsymbol{V}\cdot\boldsymbol{\nabla}\phi$. When acting on another contravariant vector field A^i, or on a covariant vector field

Lie was born at Nordfjordeif, near Bergen, and was educated at the University of Christiania, where he took his doctorate degree in 1868 and became an associate professor of mathematics four years later[423]. In 1886 he was cho-

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B_i, the Lie derivative is defined as follows:

$$\mathcal{L}_{\boldsymbol{V}} A^i = v^k \frac{\partial A^i}{\partial x^k} - A^k \frac{\partial v^i}{\partial x^k}; \qquad \mathcal{L}_{\boldsymbol{V}} B_i = v^k \frac{\partial B_i}{\partial x^k} + B_k \frac{\partial v^k}{\partial x^i}.$$

In index-free notation, the above formulae are written as

$$\mathcal{L}_{\boldsymbol{V}} \boldsymbol{A} = \boldsymbol{V} \cdot \boldsymbol{\nabla} \boldsymbol{A} - \boldsymbol{A} \cdot \boldsymbol{\nabla} \boldsymbol{V}; \qquad \mathcal{L}_{\boldsymbol{V}} \boldsymbol{B} = \boldsymbol{V} \cdot \boldsymbol{\nabla} \boldsymbol{B} + \boldsymbol{\nabla} \boldsymbol{V} \cdot \boldsymbol{B}.$$

It is straightforward to verify that $\mathcal{L}_{\boldsymbol{V}} \boldsymbol{A}$, $\mathcal{L}_{\boldsymbol{V}} \boldsymbol{B}$ again transform as contravariant and covariant vector fields respectively, and that $\mathcal{L}_{\boldsymbol{V}}$ acts on the scalar field $A^i B_i$ in accordance with the Leibniz rule. By using this rule on arbitrary *dyadic products* of $\boldsymbol{A}$ and $\boldsymbol{B}$, we easily discover the law of action of $\mathcal{L}_{\boldsymbol{V}}$ on any mixed tensor field; thus

$$\mathcal{L}_{\boldsymbol{V}} T^i_{\cdot j} = v^k \frac{\partial T^i \cdot j}{\partial x^k} - T^k_{\cdot j} \frac{\partial v^i}{\partial x^k} + T^i_{\cdot k} \frac{\partial v^k}{\partial x^j},$$

which in index-free notation reads

$$\mathcal{L}_{\boldsymbol{V}} \mathfrak{T} = \boldsymbol{V} \cdot \boldsymbol{\nabla} \mathfrak{T} - \widetilde{\mathfrak{T}} \cdot \boldsymbol{\nabla} \boldsymbol{V} + \mathfrak{T} \cdot (\widetilde{\boldsymbol{\nabla} \boldsymbol{V}}),$$

where twiddle ($\sim$) denotes dyadic transposition. Note that the price we pay for the freedom of the Lie derivative from the concept of connection, is that $\mathcal{L}_{\boldsymbol{V}}$ depends both on the field $\boldsymbol{V}$ *and* its gradient.

It is possible to introduce the Lie derivative in an alternative way; we associate with the field $\boldsymbol{V}$ the infinitesimal coordinate transformation $\overline{x}^i = x^i + \lambda v^i(x)$. It can then be shown that $\mathcal{L}_{\boldsymbol{V}} T = \lim_{\lambda \to 0} \frac{T(x) - \overline{T}(x)}{\lambda}$.

When $\boldsymbol{V}$ is a *constant vector*, $\mathcal{L}_{\boldsymbol{V}}$ reduces to the ordinary directional derivative

$$\mathcal{L}_{\boldsymbol{V}}(\mathfrak{T}) = \frac{d\mathfrak{T}}{d\boldsymbol{V}} = \boldsymbol{V} \cdot \boldsymbol{\nabla} \mathfrak{T}.$$

If on the other hand the transformation $x \to \overline{x}$ is an infinitesimal rotation $\boldsymbol{V} = \boldsymbol{\omega} \times \boldsymbol{r}$, with $\boldsymbol{\omega}$ as a constant vector, we find for a scalar field

$$\mathcal{L}_{\boldsymbol{V}} \phi = (\boldsymbol{\omega} \times \boldsymbol{r}) \cdot \nabla \phi = \boldsymbol{\omega} \cdot (\boldsymbol{r} \times \nabla \phi).$$

For a contravariant vector field, the result is

$$\mathcal{L}_{\boldsymbol{V}} \boldsymbol{A} = \boldsymbol{\omega} \cdot \{\boldsymbol{r} \times \boldsymbol{\nabla} \boldsymbol{A} - \mathfrak{I} \times \boldsymbol{A}\}.$$

423 Because of the French-German war in 1870, Lie left France and decided to go to

sen to succeed Felix Klein to the chair of geometry at Leipzig. As his fame grew, a special post was arranged for him in Christiania. But his health had deteriorated by a life of assiduous study, and he died in Christiania six months after his return.

Italy. On the way he was arrested as a German spy and his mathematics notes were assumed to be coded messages. Only after the intervention of Darboux was Lie released.

Lie Algebras and Lie Groups[424]

Consider the set of all three-dimensional vectors $\boldsymbol{A}$ *as oriented line segments. On defining the sum* $\boldsymbol{A}+\boldsymbol{B}$*, the negative* $-\boldsymbol{A}$ *and the scalar multiple* $\lambda\boldsymbol{A}$ *such that*

$$\lambda(\mu\boldsymbol{A}) = (\lambda\mu)\boldsymbol{A}\ , \quad \lambda(\boldsymbol{A}+\boldsymbol{B}) = \lambda\boldsymbol{A}+\lambda\boldsymbol{B}$$

and

$$(\lambda+\mu)\boldsymbol{A} = \lambda\boldsymbol{A}+\mu\boldsymbol{A},$$

we create a linear vector space over the field of real numbers. If to this structure we now add the product *of two vectors defined via the* vector product $\boldsymbol{A}\times\boldsymbol{B}$*, then every three vectors satisfy the relations*

$$\boldsymbol{A}\times\boldsymbol{B} = -\boldsymbol{B}\times\boldsymbol{A} \quad \text{(anticommutativity)} \tag{1}$$

$$\boldsymbol{A}\times(\boldsymbol{B}\times\boldsymbol{C})+\boldsymbol{B}\times(\boldsymbol{C}\times\boldsymbol{A})+\boldsymbol{C}\times(\boldsymbol{A}\times\boldsymbol{B}) = 0 \tag{2}$$

We also have

$$(\boldsymbol{A}+\boldsymbol{B})\times\boldsymbol{C} = (\boldsymbol{A}\times\boldsymbol{C})+(\boldsymbol{B}\times\boldsymbol{C}) \quad \text{(distributivity)} \tag{3}$$

$$(\lambda\boldsymbol{A})\times\boldsymbol{B} = \lambda(\boldsymbol{A}\times\boldsymbol{B}) \quad \text{(associativity)} \tag{4}$$

Consider next all square matrices A *of order* n *under the usual vector-space laws of addition and multiplication by a scalar. Define a new 'product'*

$$A\odot B = AB - BA \equiv [A,B]$$

where AB *is the ordinary matrix product. The symbol* $[A,B]$ *is called the* commutator *of* A *and* B*. It satisfies the two conditions*

$$[A,B] = -[B,A]$$

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424 To dig deeper, see:

- Sattinger, D.H. and O.L. Weaver, *Lie Groups and Algebras with Applications to Physics, Geometry and Mechanics*, Springer-Verlag: New York, 1986, 215 pp.
- Altmann, S.L., *Rotations, Quaternions, and Double Groups*, Dover, 1986, 317 pp.
- Srinivasa Rao, K.N., *The Rotation and Lorentz Groups*, Wiley, 1988, 351 pp.

$$[A,[B,C]]+[B,[C,A]]+[C,[A,B]]=0$$

As a third case we specialize to skew-symmetric matrices of order n. They have the general form

$$S=(I\times\boldsymbol{V})=\begin{bmatrix}0 & -c & b\\ c & 0 & -a\\ -b & a & 0\end{bmatrix},$$

where I is the unit matrix and $\boldsymbol{V}(a,b,c)$ is a vector in $\mathbb{R}^3$.

The commutator of any two such matrices

$$[S_1,S_2]=(I\times\boldsymbol{V}_1)\cdot(I\times\boldsymbol{V}_2)-(I\times\boldsymbol{V}_2)\cdot(I\times\boldsymbol{V}_1)\equiv I\times(\boldsymbol{V}_1\times\boldsymbol{V}_2)=-[S_2,S_1]$$

It is not difficult to verify that in this case too

$$[S_1,[S_2,S_3]]+[S_2,[S_3,S_1]]+[S_3,[S_1,S_2]]=0.$$

Note that

$$(I\times\boldsymbol{V}_1)\cdot(I\times\boldsymbol{V}_2)=\boldsymbol{V}_2\boldsymbol{V}_1-I(\boldsymbol{V}_1\cdot\boldsymbol{V}_2)$$

and

$$I\times(\boldsymbol{V}_1\times\boldsymbol{V}_2)=\boldsymbol{V}_2\boldsymbol{V}_1-\boldsymbol{V}_1\boldsymbol{V}_2=\boldsymbol{V}_2\wedge\boldsymbol{V}_1 \quad \text{('wedge product').}$$

A fourth case concerns the linear differential operator

$$X=\boldsymbol{a}\cdot\boldsymbol{\nabla}=a_1\frac{\partial}{\partial x_1}+a_2\frac{\partial}{\partial x_2}+a_3\frac{\partial}{\partial x_3}$$

where $a_i(x_1,x_2,x_3)$ are three differentiable functions. When this operator is applied to a smooth function $f(x_1,x_2,x_3)$, it assigns to it a real number known as the directional derivative *along the vector field $\boldsymbol{a}$:*

$$X(f)=a_1\frac{\partial f}{\partial x_1}+a_2\frac{\partial f}{\partial x_2}+a_3\frac{\partial f}{\partial x_3},$$

where $(\frac{\partial f}{\partial x_1},\frac{\partial f}{\partial x_2},\frac{\partial f}{\partial x_3})$ are the components of a vector normal to the surface $f=\text{const}$, at a given point. To each pair of operators $X=\boldsymbol{a}\cdot\boldsymbol{\nabla}$, $Y=\boldsymbol{b}\cdot\boldsymbol{\nabla}$, we can associate a Lie bracket

$$[X,Y] = XY - YX = (\boldsymbol{a}\cdot\boldsymbol{\nabla})(\boldsymbol{b}\cdot\boldsymbol{\nabla}) - (\boldsymbol{b}\cdot\boldsymbol{\nabla})(\boldsymbol{a}\cdot\boldsymbol{\nabla})$$
$$= (\boldsymbol{a}\cdot\nabla\boldsymbol{b} - \boldsymbol{b}\cdot\nabla\boldsymbol{a})\cdot\boldsymbol{\nabla} = L_1\frac{\partial}{\partial x_1} + L_2\frac{\partial}{\partial x_2} + L_3\frac{\partial}{\partial x_3}$$

where

$$L_i = (\boldsymbol{a}\cdot\nabla\boldsymbol{b}_i - \boldsymbol{b}\cdot\nabla\boldsymbol{a}_i) \qquad i = 1,2,3.$$

Here also

$$[X_1 + X_2, Y] = [X_1, Y] + [X_2, Y],$$
$$[\lambda X, Y] = \lambda[X, Y] \qquad (\lambda \text{ constant})$$

and

$$[Y, X] = -[X, Y],$$
$$[[X,Y],Z] + [[Y,Z],X] + [[Z,X],Y] = 0.$$

Since systems such as the four described above hold importance in mathematics and theoretical physics, it is advantageous to bring them under the umbrella of a common new algebra, the Lie algebra, abstractly defined as follows:

A Lie algebra is a vector space L over some field F (typically the real *or* complex *numbers) together with a* binary operation *$[X,Y] \in L$ called the Lie* bracket, *which satisfies the conditions:*

- *It is* bilinear, *i.e.* $[aX + bY, Z] = a[X,Z] + b[Y,Z]$; *also* $[Z, aX + bY] = a[Z,X] + b[Z,Y]$ *for all* a, b *in* F *and* X, Y, Z *in* L.
- *It is* antisymmetric, *i.e.* $[X,Y] = -[Y,X]$ *for all* X, Y *in* L.
- *It satisfies the Jacobi identity,*

$$J_{XYZ} \equiv [[X,Y],Z] + [[Y,Z],X] + [[Z,X],Y] = 0$$

for all X, Y, Z in L.

This mathematical definition requires some clarifications:

(1) Whenever the binary operation on the vector space is defined as the commutator of ordinary matrix multiplication, then the above three conditions are met automatically. However, one must still impose the closure condition, i.e. that $xy - yx$, for every x and y in L, belong to the same vector space.

Since every vector space has a basis $x_1, \ldots, x_n$, then it suffices to demand that for any i, j $(i, j = 1, \ldots, n)$ the entity $x_i x_j - x_j x_i$ is a linear combination of x_k, where the coefficients belong to the field. We shall soon see how this condition works, for example, in the case of the generators of the rotation group and the Lorentz transformation group.

(2) *Some Lie algebras of importance in physics and applied mathematics are of infinite dimensionality. An example is the original Lie group considered by Lie himself, which is the group of coordinate transformations on a surface. In the case of infinite dimensional Lie groups, the Jacobi identity is non-trivial and there are examples where it exhibits "anomalies".*

(3) *Lie algebras can be represented or realized: representation is usually restricted to a set of matrices or operators. Realization is a generalization of the concept of representation to also include sets of functions or other entities with appropriate group or algebraic properties.*

Note that the Lie bracket is not a multiplication in the usual sense because it is not associative. If an associative algebra with multiplication $(*)$ is given, it can be turned into a Lie algebra by defining the commutator $[X, Y] = X^*Y - Y^*X$. Conversely, every Lie algebra is embedded into one that arises from an associative algebra in this fashion. An example is linear associative algebra w.r.t. ordinary matrix product.

The proof of the Jacobi identity is elementary:

Deleting the star $(*)$ and assuming the associative law under multiplication, we have

$$J_{XYZ} = (XY - YX)Z - Z(XY - YX) + (YZ - ZY)X - X(YZ - ZY) + (ZX - XZ)Y - Y(ZX - XZ) \equiv 0$$

While the entity $[X, Y]$ is known as the commutators of X and Y, the entity J_{XYZ} is known as the associator.

The algebra of commutators leads to expressions for commutators of functions of the elements. Thus if

$$[A, B] = AB - BA = \gamma$$

then

$$[A^2, B] = A^2B - BA^2 = AAB - BAA = \\ A(BA + \gamma) - BAA = ABA + A\gamma - BAA = A\gamma + \gamma A;$$

$$[A^3, B] = A(A^2B) - (BA^2)A = A(A^2B) - (A^2B - A\gamma - \gamma A)A \\ = A\gamma A + \gamma A^2 + A^2(AB - BA) = A\gamma A + \gamma A^2 + A^2\gamma$$

Since $e^A = \sum_{m=0}^{\infty} \frac{A^m}{m!}$, *it is not difficult to show that*

$$e^{-A}Be^A = B + \frac{1}{1!}[B, A] + \frac{1}{2!}[[B, A], A] + \dots, \quad \text{and}$$

$$[e^A, B] = \sum_{m=0}^{\infty} \frac{1}{m!}[A^m, B]$$

If $A\gamma = \gamma A$ *(see an example in the footnote*[425]*) we have* $[A^m, B] = m\gamma A^{m-1}$ *and therefore*

$$[e^A, B] = \gamma e^A, \qquad [e^A, e^B] = (e^\gamma - 1)e^B e^A$$

The ***Campbell-Baker-Hausdorff*** *theorem states that if* A *and* B *are matrices which do not necessarily commute, then*

$$e^A e^B = e^C$$

where

$$C = A + B + \frac{1}{2}[A, B] + \frac{1}{12}([A, [A, B]] - [B, [B, A]]) + \dots$$

and the coefficients are the Bernoulli numbers.

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[425] valid, for example for $A = x$, $B = \frac{d}{dx}$. In this case $[A, B] = -1$, so

$$[A, [A, B]] = 0.$$

Thus

$$AAB + BAA = 2ABA,$$

which is indeed satisfied for any function f since

$$x^2\frac{df}{dx} + \frac{d}{dx}(x^2 f) = 2x\frac{d}{dx}(xf).$$

To prove this theorem we consider $e^{C(t)} = e^{tA}e^{B}$, where t is a parameter. Evaluating via a Taylor's expansion about $t = 0$, i.e.

$$C(t) = C(0) + \frac{1}{1!}C'(0) + \frac{1}{2!}C''(0) + \ldots, \text{ and setting } t = 1,$$

the desired result is obtained by successively evaluating $C^{(n)}(0),\ n = 1, 2, \ldots,$ using $C(0) = B$ and the differential equation

$$e^{-B}Ae^{B} = e^{-C(t)}\frac{d}{dt}\left(e^{C(t)}\right)$$

for $C(t)$.

This theorem is applicable to Wigner's rotation in the Lorentz group (special relativity), the Gibbs formula for addition of finite rotations (geometry and rotational dynamics), and various problems in quantum mechanics.

We have shown that all 3-dimensional skew-symmetric matrices form a Lie algebra. We know that proper active rotations in 3-dimensional space are represented by 3×3 orthogonal matrices R with determinant $+1$. The *Lie group* formed under these operations is denoted by the symbol SO(3). *We also know that*

$$R = e^{(I \times \boldsymbol{V})}$$

where

$$(I \times \boldsymbol{V}) = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix} = v_1 I_1 + v_2 I_2 + v_3 I_3,$$

$$I_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad I_2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \quad I_3 = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Consequently, there is a close connection between the geometry *of the rotations R and their composition law on one hand, and the* algebra *of skew-symmetric matrices on the other. Given R, the vector $\boldsymbol{V}$ can be explicitly constructed from the matrix elements of R. It specifies both the direction of the axis of rotation in space and the finite angle of rotation about that axis, which are $\frac{\boldsymbol{v}}{|\boldsymbol{v}|}$ and $|\boldsymbol{v}|$, respectively. The three matrices (I_1, I_2, I_3) are said to* generate *the Lie group SO(3), as well as furnishing a basis for the corresponding Lie Algebra. The parameters (v_1, v_2, v_3) are the "coordinates" or "components" of $(I \times \boldsymbol{V})$ relative to this basis.*

Since the elements of $(I \times \boldsymbol{V})$ *and* R *determine each other uniquely in small enough neighborhood of* I *through the relation* $R = e^{(I \times \boldsymbol{V})}$, v_i *can be considered as local coordinates of* R *in the group manifold* SO(3).

The matrices I_i *obey the commutation relations*

$$I_1 I_2 - I_2 I_1 = I_3, \qquad I_2 I_3 - I_3 I_2 = I_1, \qquad I_3 I_1 - I_1 I_3 = I_2;$$

one also has $I_1^2 + I_2^2 + I_3^2 = -2I$.

We can write $\boldsymbol{V} = \boldsymbol{e}\theta$, *where* $\boldsymbol{e}$ *is a unit vector along the axis of rotation and* θ *the angle of rotation. Denoting by* (α, β, γ) *the three Euler-like angles representing* R, *we have* $R = M_\alpha M_\beta M_\gamma$ *where*

$$M_\alpha = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix},$$

$$M_\beta = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}, \qquad M_\gamma = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

For an infinitesimal rotation, we expand $R(\alpha, \beta, \gamma)$ *in Taylor series about* $\alpha = 0$, $\beta = 0$, $\gamma = 0$. *We find*

$$R = I + \frac{\partial M_\alpha}{\partial \alpha}\Big|_{\alpha=0} \delta\alpha + \frac{\partial M_\beta}{\partial \beta}\Big|_{\beta=0} \delta\beta + \frac{\partial M_\gamma}{\partial \gamma}\Big|_{\gamma=0} \delta\gamma,$$

where

$$\frac{\partial M_\alpha}{\partial \alpha}\Big|_{\alpha=0} = I_1, \qquad \frac{\partial M_\beta}{\partial \beta}\Big|_{\beta=0} = I_2, \qquad \frac{\partial M_\gamma}{\partial \gamma}\Big|_{\gamma=0} = I_3, \quad \text{and}$$

$$\delta\alpha = e_1 \delta\theta, \quad \delta\beta = e_2 \delta\theta, \quad \delta\gamma = e_3 \delta\theta.$$

Thus, under an infinitesimal rotation $\delta\theta$ *about the* z*-axis* $(e_1 = e_2 = 0, e_3 = 1)$ *we have*

$$R_z = e^{I_3 \delta\theta}.$$

The matrices I_1, I_2, I_3 *are known as generators of the Lie group.*

These generators are associated with the local structure of the Lie group in the neighborhood of the identity element.

In general, in n-dimensional Euclidean space the number of generators (i.e. the number of Euler angles) is $N = \frac{n(n-1)}{2}$. We can then write symbolically for a general rotation:

$$g(x) = \exp[\sum_{i=1}^{N} x_i T_i],$$

where x_i are local coordinates in the N-dimensional group manifold (an abstract set of points, each representing one element of the Lie group). The operators T_i are the generators of the Lie group in these coordinates.

The point $x_i = 0$ $(i = 1, \ldots, n)$ in the manifold is the identity element $g = I$. Each T_i is an operator with a matrix representation. This formalism can be extended to non-vector representations of the rotation groups, and indeed to any Lie group and its corresponding algebra. Examples are:

- *For the vectorial rotation group* SO(3), *we saw that the generators are the 3×3 matrices*

$$(T_j)_{ab} \equiv (I_j)_{ab} = \epsilon_{jba}, \quad 1 \leq j, a, b \leq 3,$$

where ϵ_{jab} is the Levi-Civita symbol.

- *For the $\mathbb{R}^3$ rotation group of 3D spinors,* SU(2), *the 3 generators in the spinor representation are*

$$(T_i)_{ab} = \frac{1}{2} i(\sigma_j)_{ab}, \quad i = \sqrt{-1}, \quad 1 \leq j \leq 3, \quad 1 \leq a, b \leq 2,$$

where σ_j are the Pauli matrices. The SU(2) *and* SO(3) *Lie group manifolds differ in their* global *topology (e.g. the* SU(2) *manifold is topologically equivalent to the 3-sphere S^3), but are* locally *equivalent since they share the same Lie algebra; there is also a* two-to-one *mapping from* SU(2) *to* SO(3) *which preserves the rotation-composition law*[426].

How do we express the "addition law" of the group [namely the law by which $g(x)g(y)$ gives a new $g(z)$] in terms of the $\{T_j\}$?

For that we have the above-mentioned Campbell-Baker-Hausdorff theorem

$$e^{A_1} e^{A_2} = e^{A_1 + A_2 + \phi(A_1, A_2)},$$

[426] To wit, both SU(2) elements $\pm e^{\frac{i}{2}\boldsymbol{v}\cdot\boldsymbol{\sigma}}$ correspond under this map to the same SO(3) element $e^{\boldsymbol{I}\times\boldsymbol{v}}$.

where

$$\phi(A_1, A_2) = \sum_{n=1}^{\infty} \sum_{\{j\}} C_{\{j\}}[\dots[A_{j_1}, A_{j_2}], \dots A_{j_{n+1}}],$$

$C_{\{j\}}$ *are known constants and the second sum ranges over* $\{j_p \mid 1 \leq j_p \leq 2, p = 1, \dots, n+1\}$.

Since ϕ *is built by repeatedly commuting* A_1, A_2 *with each other, it follows that if* $g(x) \cdot g(y)$ *is again to be of the form*

$$g(z) = exp\{\sum_j z_j T_j\},$$

we must have

$$[T_i, T_j] = \sum_k \gamma_{ijk} T_k,$$

where the numbers γ_{ijk} *are called the structure constants of the Lie algebra and fully characterize it.*

Note that all the local structure of a Lie group is contained in its Lie algebra, since for any two elements A, B *of the latter,*

$$[A, B] = AB - BA$$

is also an element of the algebra (i.e. the algebra is closed under commutation); in sufficiently small neighborhood of the identity $g = I$, *we have a one-to-one corresponding between the group and the algebra, namely*

$$g(x) \leftrightarrow X = \sum_{j=1}^{N} x_j T_j, \quad g(x) = e^X$$

(the so-called exponential map).

Since the SU(2) *and* SO(3) *generators have the same commutation relations*

$$[T_i, T_j] = \sum_{k=1} \epsilon_{ijk} T_k, \quad 1 \leq i, j \leq 3,$$

they must have the same Lie algebra and the same local structure as discussed above.

In the foregoing discussion the matrix R *effected the rotation of the coordinate axes or the space coordinates. We may, however, consider the idea of rotation of a function relative to fixed axes (coordinates) under the definition*

$Rf(\boldsymbol{r}) = f(R \cdot \boldsymbol{r})$. Let us, for example, apply it to a rotation of a function about the z-axis with

$$R_z = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

This means that

$$f(x, y, z) \to f(x\cos\varphi + y\sin\varphi, -x\sin\varphi + y\cos\varphi, z).$$

For $\varphi = \frac{\pi}{2}$, $\qquad R(\boldsymbol{e}_z, \frac{\pi}{2})f(x, y, z) = f(y, -x, z)$.

Under an infinitesimal rotation $R(\boldsymbol{e}_z, \delta\varphi)$, a Taylor expansion yields

$$\begin{aligned} R(\boldsymbol{e}_z, \delta\varphi)f(x, y, z) &= f(x + y\delta\varphi, y - x\delta\varphi, z) \\ &= f(x, y, z) - \delta\varphi[x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}]f + O(\delta\varphi)^2 \\ &= \{e^{-i(\delta\varphi)L_z}\}f \end{aligned}$$

where

$$L_z = -i(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}).$$

Similar results are obtained for rotations about the other axes, with

$$L_x = -i(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}), \quad L_y = -i(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}).$$

We note that L_x, L_y, and L_z satisfy exactly the same commutation relation as iT_a: $[L_i, L_j] = i\epsilon_{ijk}L_k$. It can be shown that the Casimir operator $L^2 = L_x^2 + L_y^2 + L_z^2$ has vanishing commutators:

$$[L^2, L_x] = [L^2, L_y] = [L^2, L_z] = 0.$$

In quantum mechanics, L_a is the a-th component of the orbital angular momentum operator, in units where $\hbar = 1$. The quantum angular-momentum Casimir operator is related to the Laplacian operator as follows:

$$\boldsymbol{\nabla}^2 = \frac{1}{r^2}\boldsymbol{L}^2 + \frac{\partial^2}{\partial r^2},$$

where in the partial differentiation $\frac{\boldsymbol{r}}{r}$ is held fixed and $r = \sqrt{x^2 + y^2 + z^2}$.

In general

$$f(\boldsymbol{r}+\boldsymbol{a}) = \sum_{n=0}^{\infty} \frac{1}{n!}(\boldsymbol{a}\cdot\boldsymbol{\nabla})^n f(\boldsymbol{r}) = e^{\boldsymbol{a}\cdot\boldsymbol{\nabla}} f(\boldsymbol{r}).$$

But quantum-mechanically

$$\boldsymbol{p} \text{ (momentum operator)} = -i\hbar\nabla, \qquad \text{and so} \qquad f(\boldsymbol{r}+\boldsymbol{a}) = e^{i\boldsymbol{a}\cdot\boldsymbol{p}/\hbar} f(\boldsymbol{r}).$$

The special case $(r,\theta,\varphi) \to (r,\theta,\varphi+\alpha)$ *designates a rotation about the* z *axis by an angle* α*; therefore* $e^{\alpha\frac{\partial}{\partial\varphi}} = e^{i\alpha L_z/\hbar}$, *where* $L_z = -i\hbar\frac{\partial}{\partial\varphi}$.

It must be emphasized that the basic idea of the Lie group is that from a generator with elements infinitesimally close to I, *such as* L_z, *which shifts the point* $s=(x,y,z)$ *(or in general, the Lie-group representation or realization) to the point* $s+ds$, *one may generate an operator which shifts the point* s *into a point* s' *at a finite distance along the 'path curve' of a one-parameter group generated by the infinitesimal operator.*

This idea can be applied to obtain addition theorems for special functions of mathematical physics from their respective recursion relations. The infinitesimal differential operators which appear in the recursion-relations of the various special functions, can be used to generate from them finite operators. For example, the Bessel functions obey the recursion equations

$$\mathcal{L}_+\{e^{in\phi}J_n(r)\} = e^{i(n+1)\phi}J_{n+1}(r); \mathcal{L}_-\{e^{in\phi}J_n(r)\} = e^{i(n-1)\phi}J_{n-1}(r)$$

where

$$\mathcal{L}_+ = -\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}, \qquad \mathcal{L}_- = \frac{\partial}{\partial x} - i\frac{\partial}{\partial y}; \qquad r = \sqrt{x^2+y^2+z^2}.$$

Since $[\mathcal{L}_+,\mathcal{L}_-]=0$, *the composition law within the Lie group is additive:*

$$e^{\alpha\mathcal{L}_+}\cdot e^{\beta\mathcal{L}_-} = e^{\alpha\mathcal{L}_+ + \beta\mathcal{L}_-}.$$

It is then a trivial matter to derive the addition theorem

$$\frac{J_n[\sqrt{r^2+h^2}]}{(r^2+h^2)^{n/2}} = \sum_{m=0}^{\infty}\frac{1}{m!}\left(\frac{-h}{2}\right)^m r^{-n-m}J_{n+m}(r).$$

Lie algebra and classical mechanics

Hamilton *(1835) converted the* **Lagrange** *equations (1788) into a set of coupled, 1^{st}-order ODE's representing the solution of a conservative mechanical system in phase space:*

$$\dot{q}_i = \frac{\partial H}{\partial p_i}; \qquad \dot{p}_i = -\frac{\partial H}{\partial q_i}; \qquad (p_i = \frac{\partial L}{\partial \dot{q}_i})$$

where $H(q,p,t) = \sum_{i=1}^{n} p_i \dot{q}_i - L$ is the Hamilton function (Hamiltonian) and $L(q,\dot{q},t)$ is the Lagrangian of the system. Let $f(p,q,t)$ be some function of the coordinates, momenta and time. Its total time-derivative is

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \sum_k (\frac{\partial f}{\partial q_k}\dot{q}_k + \frac{\partial f}{\partial p_k}\dot{p}_k) = \frac{\partial f}{\partial t} + [H,f],$$

where

$$[H,f] = \sum_k \left(\frac{\partial f}{\partial q_k}\frac{\partial H}{\partial p_k} - \frac{\partial f}{\partial p_k}\frac{\partial H}{\partial q_k} \right)$$

is known as the Poisson bracket of H and f.

For any two given functions of the dynamical variables, $u(p,q,t)$ and $v(p,q,t)$, the Poisson bracket is defined analogously as

$$[u,v] = \sum_k \left(\frac{\partial u}{\partial p_k}\frac{\partial v}{\partial q_k} - \frac{\partial u}{\partial q_k}\frac{\partial v}{\partial p_k} \right).$$

It has the following basic properties:

$$[u,v] = -[v,u]; \quad [u+v,w] = [u,w] + [v,w];$$
$$[uv,w] = u[v,w] + v[u,w];$$
$$\frac{\partial}{\partial t}[u,v] = [\frac{\partial u}{\partial t}, v] + [u, \frac{\partial v}{\partial t}];$$
$$\frac{d}{dt}[u,v] = [\frac{du}{dt}, v] + [u, \frac{dv}{dt}];$$
$$[u,q_k] = \frac{\partial u}{\partial p_k}; \quad [u,p_k] = -\frac{\partial u}{\partial q_k};$$
$$[q_i,q_k] = 0; \quad [p_i,p_k] = 0; \quad [p_i,q_k] = \delta_{ik};$$

$$[u,[v,w]] + [v,[w,u]] + [w,[u,v]] = 0 \quad \text{(Jacobi's identity)}$$

If u and v do not depend on time (integrals of the motion) then

$$\frac{d}{dt}[u,v] = 0$$

This follows immediately from the Jacobi identity with $w = H$. Note that if $\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{p}$ (regular momentum), then

$$[M_x, M_y] = -M_z, \qquad [M_y, M_z] = -M_x, \qquad [M_z, M_x] = -M_y$$

Lie algebra of the Lorentz group

We know that every Lorentz transformation L between two frames of reference with arbitrary orientation with respect to each other, is represented by a (4×4) orthogonal matrix acting in Euclideanized Minkowski space (space-time) having metric δ_{ij}:

$$L = e^S,$$

where S is the (4×4) skew-symmetric matrix

$$S = \begin{bmatrix} 0 & -h_3 & h_2 & -ie_1 \\ h_3 & 0 & -h_1 & -ie_2 \\ -h_2 & h_1 & 0 & -ie_3 \\ ie_1 & ie_2 & ie_3 & 0 \end{bmatrix}.$$

In S, (e_1, e_2, e_3) are the direction cosines of a vector *and (h_1, h_2, h_3) are the three components of a* pseudo-vector*. We can write*

$$S = \boldsymbol{h} \cdot \boldsymbol{u} + \boldsymbol{e} \cdot \boldsymbol{w} = h_1 u_1 + h_2 u_2 + h_3 u_3 + e_1 w_1 + e_2 w_2 + e_3 w_3$$

where

$$u_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, u_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, u_3 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

and

$$w_1 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, w_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}, w_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix},$$

The six skew-symmetric matrices u_i, w_i form a basis in the vector space of our Lie algebra.

Note that e_i, h_i are functions of the components of the velocity vector $\boldsymbol{v}$ and the Euler angles which determine the relative orientation and motion of the two reference frames. Two limiting cases are simple:

- $h_i = 0$ $\quad$ $\boldsymbol{e} \parallel \boldsymbol{v}$, $\quad |\boldsymbol{e}| = \mathrm{th}^{-1} \frac{|\boldsymbol{v}|}{c}$ $\quad$ *(pure boost)*
- $e_i = 0$, $v_i = 0$; $\quad$ h_i *are functions of the Euler angles (pure rotation)*

The Lie group of Euclideanized matrices L is SO(4)*; in the non-Euclidean (real) Minkowski space of STR, the Lorentz group is called* SO(3, 1).

Lie algebra and the symplectic group (Weyl 1938)

Symplectic transformations preserve skew-symmetric products, which are abundant in physical applications. We know, for example, that the "dot" product of two plane vectors $\boldsymbol{A} = (A_x, A_y)$, $\boldsymbol{B} = (B_x, B_y)$, namely $\boldsymbol{A} \cdot \boldsymbol{B} = A_x B_x + A_y B_y$ is invariant under rotation in the two-dimensional space. On the other hand, the "cross" product $\boldsymbol{A} \times \boldsymbol{B} = \boldsymbol{e}_z (A_x B_y - A_y B_x)$, where $(A_x B_y - A_y B_x)$ is the area of the parallelogram formed by the two vectors, is invariant under a group of transformations which preserves this skew-symmetric product and is known as the symplectic group in 2-dimensional space, or $S_p(2)$. To meet this group, we write

$$\text{area} = A_x B_y - A_y B_x = [A_x, A_y] \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} B_x \\ B_y \end{bmatrix} = AJB \tag{1}$$

where $J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. *Define a* 2×2 *symplectic matrix* M *such that*

$$A \to AM^T, \quad B \to MB. \tag{2}$$

Then, the preservation of the area implies $A'JB' = AJB$, *or*

$$M^T JM = J \tag{3}$$

Even without going into the detailed form of M, *we can deduce that the unit* 2×2 *matrix* $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ *is symplectic, that* M *is nonsingular, that if two matrices* M *and* N *are symplectic, so are* MN *and* NM; *and that the inverse of a symplectic matrix is also symplectic. From these observations we can conclude that the* 2×2 *symplectic matrices form a group. It is called* $Sp(2)$, *and includes the planar-rotations group,* SO(2), *as a subgroup.*

Geometrical illustration of the $Sp(2)$ group

Consider a unit circle *around the origin of a Cartesian system* (x, y), *namely* $x^2 + y^2 = 1$. *Next, consider an* ellipse, *whose equation in another Cartesian system* (X, Y) *is*

$$e^{-\eta}U^2 + e^{\eta}V^2 = 1$$

with

$$U = X \cos\frac{\theta}{2} + Y \sin\frac{\theta}{2}, \qquad V = X \sin\frac{\theta}{2} - Y \cos\frac{\theta}{2}. \tag{4}$$

If $\eta > 0$, *the major and minor axes of this ellipse are* $e^{\eta/2}$ *and* $e^{-\eta/2}$ *respectively. The major axis is along the* $\frac{\theta}{2}$ *direction. The area of the ellipse is* π *(same as that of the unit circle) and remains* invariant *as we change the values of* θ *and* η.

The question now arises as to what transformations carry the (x, y) *into the* (X, Y) *coordinates such that the area is preserved. One class of such transformations is*

$$\begin{bmatrix} X \\ Y \end{bmatrix} = [S(\theta, \eta)] \begin{bmatrix} x \\ y \end{bmatrix}, \tag{5}$$

where

$$S(\theta, \eta) = R(\theta)S(\eta)R(-\theta)$$

$$= \begin{bmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix} \begin{bmatrix} e^{\eta/2} & 0 \\ 0 & e^{-\eta/2} \end{bmatrix} \begin{bmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2} \\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \mathrm{ch}\,\frac{\eta}{2} + \mathrm{sh}\,\frac{\eta}{2}\cos\theta & \mathrm{sh}\,\frac{\eta}{2}\sin\theta \\ \mathrm{sh}\,\frac{\eta}{2}\sin\theta & \mathrm{ch}\,\frac{\eta}{2} - \mathrm{sh}\,\frac{\eta}{2}\cos\theta \end{bmatrix}. \tag{6}$$

The matrix $S(\theta,\eta)$ is symmetric, satisfies the symplectic condition $S^T JS = J$, and its determinant is unity. Geometrically, $S(\theta,\eta)$ elongates / contracts along the Cartesian axes tilted by angle $\frac{\theta}{2}$. It can be shown that the two-parameter scale-transformation matrices $S(\theta,\eta)$ alone cannot *form a group unless they are supplemented by a rotation matrix $R(\alpha)$, where α is determined from θ and η. This rotation does not affect the area-preservation property.*

If we introduce the matrices

$$F_1 = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad F_2 = \frac{1}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad F_3 = \frac{1}{2}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \tag{7}$$

then we can write

$$S(0,\lambda) = e^{\lambda F_1}, \qquad S(\frac{\pi}{2},\eta) = e^{\eta F_2}, \qquad R(\theta) = e^{\theta F_3} \tag{8}$$

where F_1,F_2,F_3 are the generators *of the $Sp(2)$ group. They satisfy the commutation relations*

$$[F_1, F_2] = -F_3, \qquad [F_2, F_3] = F_1, \qquad [F_3, F_1] = F_2. \tag{9}$$

These generators form a system of closed *commutation relations, and are a basis for the* Lie algebra *for the $Sp(2)$ group.*

Assume

$$S(\theta,\eta) = e^{aF_1 + bF_2}. \tag{10}$$

Since F_1 and F_2 anticommute with each other, and since

$$(2F_1)^2 = (2F_2)^2 = -(2F_3)^2 = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$(aF_1 + bF_2)^2 = \frac{a^2+b^2}{4} I,$$

the expansion of the exponential (10) in a power series yields

$$S(\theta,\eta) = I\,\mathrm{ch}\,\frac{\eta}{2} + (2F_1\cos\theta + 2F_2\sin\theta)\,\mathrm{sh}\,\frac{\eta}{2} \tag{11}$$

where

$$\eta^2 = a^2 + b^2, \quad \tan\theta = \frac{b}{a}.$$

Clearly, $S(\theta, \eta)$ in (11) is identical to its form in (6).

Note that

$$F_1 = \frac{1}{2}\sigma_z, \qquad F_2 = \frac{1}{2}\sigma_x, \qquad F_3 = -\frac{i}{2}\sigma_y, \tag{12}$$

where $(\sigma_x, \sigma_y, \sigma_z)$ are the Pauli spin matrices.

It can be shown that the $Sp(2)$ group is locally isomorphic to the group of Lorentz transformations in two-dimensional space (i.e. three-dimensional pseudo-Euclidean Minkowski spacetime). This group is known as SO(2, 1), *and is a subgroup of the group consisting of Lorentz transformations in 4-dimensional spacetime, consisting of three space- and one time-dimension. If we use (x, y, z) to specify the coordinates in this $(2 + 1)$ space, then* SO(2, 1) *consists of Lorentz transformations along the x and y directions and of a rotation on the xy plane around the z axis. The generators of this group are*

$$T_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \qquad T_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \qquad T_3 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{13}$$

with the top-to-bottom rows and left-to-right columns corresponding to the x, y, t directions of Minkowski spacetime, respectively.

They satisfy the commutation relations

$$[T_1, T_2] = -T_3, \qquad [T_2, T_3] = T_1, \qquad [T_3, T_1] = T_2, \tag{14}$$

which are exactly the same like those for the generators of the group $Sp(2)$.

The element $L(\theta, \eta)$ of the Lie-group SO(2, 1) *which corresponds to the element $S(\theta, \eta)$ of $Sp(2)$ is:*

$$L(\theta, \eta) = K(\theta)L(0, \eta)K(-\theta)$$

with

$$K(\theta) = e^{\theta T_3} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$L(0, \eta) = e^{\eta T_1} = \begin{bmatrix} \operatorname{ch}\eta & 0 & \operatorname{sh}\eta \\ 0 & 1 & 0 \\ \operatorname{sh}\eta & 0 & \operatorname{ch}\eta \end{bmatrix} = \text{boost along the } x \text{ direction},$$

$$L(\frac{\pi}{2},\eta) = e^{\eta T_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \operatorname{ch}\eta & \operatorname{sh}\eta \\ 0 & \operatorname{sh}\eta & \operatorname{ch}\eta \end{bmatrix} = \text{ boost along the } y \text{ direction.}$$

1870–1903 CE **Jakob Rosanes (Rosales)** (1842–1922, Germany). Mathematician. Contributed significantly to algebraic geometry and theory of invariants. Some of his important results were later proved independently by **Max Noether** and elaborated on by **Castelnuovo**.

Rosanes was born in Brody, Austria-Hungary (now the Ukraine) to a distinguished Jewish family that originated from *Castallvi de Rosanes* near Barcelona. With the expulsion of the Jews from Spain in 1492, the family went to Portugal and from there dispersed into Europe, North-Africa and the Near-East. In Portugal, they turned into Marranos, and one of their members **Immanuel Rosales** (1593–1668) became a mathematician and a famous physician.

Rosanes studied at the Universities of Berlin and Breslau, obtaining his doctorate from Breslau in 1865. In 1876 he became a professor of mathematics at the latter, where he remained for the rest of his career.

1870–1906 CE **Georg Ferdinand Frobenius** (1849–1917, Germany). Mathematician. Contributed to the theory of algebraic equations, number theory, character theory of finite groups[427] (1896) and ordinary differential equations (*Frobenius method*).

Frobenius was born in Berlin. He was educated at the University of Göttingen (1867–1870) and was a professor of mathematics at the Zürich Polytechnicum (1875–1892) and afterwards at the University of Berlin.

[427] Frobenius' character theory of finite groups was used with great effect by **William Burnside** (1852–1927, England), published by him during 1897–1911. Together they co-founded the theory of finite group representation. Burnside's ('$p^a \times q^b$ theorem') states that every finite group whose order (number of elements) is divisible by fewer than 3 distinct primes is *solvable*. In 1897 Burnside's classic work: "theory of Groups of Finite Order" was published. The second edition (1911) already included a *character theory*. While first developed for finite groups, characters were later extended to (infinite) Lie groups. The so-called *Burnside lemma*, stated in his book was actually discovered earlier by Frobenius. Burnside's conjecture that every finite group of odd order is solvable was proved only in 1962 by **W. Feit** and **J.C. Thompson**. Burnside was a student of Stokes, Maxwell and Cayley.

The 'Cult of Science' (1870–1914)

With the rapid advance in all fields of scientific research, the belief in unlimited progress that had prevailed since the enlightenment seemed happily confirmed. This created an intellectual climate and a cultural trend whose outstanding characteristic may be described as an overriding interest and a deep belief in science. Man had been interested in science before, but it was only since the second half of the 19^{th} *century that a veritable '*Cult of Science*' developed. Science offered a positive alternative to the seemingly futile idealism and Romanticism of the early* 19^{th} *century.*

Scientific research, in the past the domain of a few scientists and gentleman scholars, now became the concern of large numbers of people, especially as the application of science to industry gave an incentive for new inventions. Pure *science continued to be of fundamental importance, but* applied *science — the marriage of science and technology so characteristic of the Industrial Revolution — now took precedence in the minds of most people. A virtually endless series of scientific inventions seemed to provide tangible evidence of man's ability to unveil the secrets of nature.*

The growing concern of modern man with the material aspects of his civilization was also reflected in late 19^{th} *century thought. A few basic scientific discoveries served as a foundation for an essentially materialistic philosophy that appealed to the educated middle class. Chemists and physicists earlier in the century had declared matter and energy to be constant and indestructible.*

These scientific findings were translated by certain popularizers of science into a philosophy of materialism*. An early exponent of this philosophy was* ***Ludwig Feuerbach*** *(1804–1872, Germany). In his* scientific socialism*,* ***Karl Marx*** *(1818–1895, Germany) blended the materialistic doctrine of Feuerbach, the positivism of* ***Comte****, the Evolutionary Naturalism of* ***Spencer*** *and* ***Darwin****, and finally, the dialectic*[428] *of* ***Hegel****.*

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[428] A dynamic logic which finds truth through a series of trials: thesis, antithesis, and synthesis, i.e., every fact will be understood only when related to its opposites, to those things which the thesis is not. Only by pointing out the many relationships of any one object to other objects can we establish the truth about that object. If we unite the idea to its opposite, we discover a different truth about them which transcends their previous separate meaning. It is like two conflicting forces that merge to produce a synthesis in the form of a new and greater reality. Marx applied this logical principle to socio-economic history; the two socio-economical classes which are antithetical to each other are the

According to him, history has been determined primarily by economic factors and punctuated by a series of class-struggles. The existing struggle of capital and labor was rationalized in terms of the theory of surplus value: *profit seeking capitalists pay labor subsistence wages, and take for themselves the surplus value which the workers have added to the product through their labor.*

In the second half of the 19^{th} *century, less than 9 generations after the trial of Galileo Galilei, science began to gain the upper hand in its long war against Christian dogma.*

As the state took over the functions of the churches in social welfare and education, and as some of the material benefits of industrialization spread among the lower classes, the need for the aid and comfort that religion had given in the past was no longer so acute. Furthermore, the tendency of the churches, to favor the political status quo *agonized many liberals, and political anticlericalism became an important issue in most countries. Finally, there was the appeal that nationalism, socialism, and materialism came to have for many people (both socialism and materialism were avowed enemies of religion).*

However, the most important reason for the decline in religious interests was the effect of modern science on Christianity; many scientific discoveries, especially in geology and biology, contradicted *Christian beliefs, and the methods of scientific inquiry, when applied to Christianity itself, produced some disturbing results [e.g.* **Ernest Renan** *in his book* Life of Jesus *(1863) denied that he performed miracles or had arisen from the dead].*

Far more drastic in their effect on the faithful than the attempts to humanize Christ were the findings of **Darwin** *and* **Lyell**. *These scientists challenged the biblical view of creation, by making man a non-unique part of the general process of creation. Why, one might now ask, should man alone of all creatures posses an immortal soul, and at what stage of his evolution was he endowed with it?*

In 1864, *Pope Pius IX issued "A Syllabus of the Principle Errors of Our Times" which condemned most of the new scientific tendencies. In* 1870, *a general church council, in an effort to strengthen the pope's position, proclaimed* the dogma of papal infallibility, *which made the pope infallible in all statements he made officially.*

The impact of scientific discoveries on the Protestant church were felt more deeply, since its doctrine and ritual were almost entirely based on the Bible.

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property-owning class (capitalists) and the proletariat (workers who sell their labor in order to survive). The conflict between them will generate a society of people who work *and* own the means of production.

The further fact, that Protestantism was split into almost 300 *sects, made any uniform stand in the warfare between science and theology very difficult. At the same time, however, Protestant emphasis on the freedom of the individual to work out his own relations with God made it possible for many Protestants to reach their own compromise between faith and reason.*

A minority of Protestant 'fundamentalists', more influential in the United States than in Europe, continued to cling to a literal interpretation of the Bible and insisted on the validity of the account of creation as given in the Book of Genesis.

Interestingly enough, the conflict between science and theology did not seriously interfere with the progress of science. The world in 1914 *was still viewed as the intricate mechanism that Newton had supposedly shown it to be, a mechanism whose secrets would gradually yield to scientific inquiry. Only a handful of scientists realized that new developments — the discovery of X rays (1895), the isolation of radium (1898), and, most important, the formulation of the theory of special relativity and the Planck-Einstein discovery of the quantum (1905) — had opened up an infinite number of new mysteries and had brought the world to the threshold of another scientific revolution.*

The cult of science that dominated the intellectual climate at the end of the 19^{th} *century also had its impact on art and literature. In the early* 19^{th} *century, the Romantic artist escaped from the ugliness of early industrialism into an ideal world of his imagination set by his concept of natural beauty of a more glamorous past. Even before the middle of the* 19^{th} *century, some artists had begun to be interested in the world as it was, not as they felt it ought to be. This shift from Romanticism to Realism was more evident in literature, though less pronounced in painting; and there were hardly any signs of it in music.*

While the Romantic writers had been primarily interested in the unusual individual, the realistic novel was concerned with typical everyday society. Most of the great novels of the 19^{th} *century — by* ***Charles Dickens*** *(1812–1870,* England*),* ***Honore de Balzac*** *(1799–1850,* France*),* ***Gustave Flaubert*** *(1821–1880,* France*)* and ***Lev N. Tolstoy*** *(1828–1910,* Russia*) — fall into the category of social novels. Not only did authors describe the society in which they lived; they dwellt on the problems of that society. Literature increasingly became a form of social criticism.*

The trend toward Realism reached its climax in the 19^{th} *century in a literary movement called Naturalism. It represented a conscious effort on the artist's part to apply scientific principles to art.*

The naturalistic writers — men like ***Emile Zola*** *(1840–1902,* France*),* ***Henrik Ibsen*** *(1828–1906,* Norway*)* and ***Gerhart Hauptman*** *(1862–1946,* *Germany) — were not interested in the creation of* beauty *but, like the scientist, they were interested in* truth. *To get at truth they discarded subjectivity and intuition and strove to describe objectively what they had learned from study and observation. The Naturalist was much impressed with the finding of modern science, especially in biology, and such new fields as sociology and psychology, and he made use of the new knowledge in his writing.*

It was the application of scientific principles to painting that characterized the school of Impressionism. *Influenced by the scientific discoveries about the composition of light, painters like* ***Camille Pissaro*** *(1830–1903,* France), ***Claude Monet*** *(1840–1926,* France), ***Auguste Renoir*** *(1841–1919,* France) *and others, used small dabs of color to depict nature in its ever-changing moods, not as it appeared to the logical mind but as it "impressed" the eye in viewing a whole scene rather than a series of specific objects. An Impressionist painting, examined at close range, thus appeared as a maze of colored dots which, if seen from a distance, merge into recognizable objects with the vibrant quality imparted by light.*

The prevailing school of Naturalism had little use for beauty. To the Naturalist, art had to serve a purpose and preach a message. In opposition to this view of art, a group of French poets at the end of the century claimed that art was sufficient unto itself — "art for art's sake"*.*

*To these Symbolists [****Stephane Mallarme*** *(1842–1898,* France), ***Paul Verlaine*** *(1844–1896,* France), ***Rainer Maria Rilke*** *(1875–1926,* Germany) *and others] — art was not for the masses but only for the few to whom it spoke in '*symbols*', using words not merely for their meaning but for the images and analogies they conveyed, often by sound alone. Symbolism is significant as a sign that there were people before* 1914 *who did not live in harmony with a society that glorified materialistic achievements and accepted the struggle for wealth as a sign of progress.*

The most outspoken critic of the generation before 1914 *was the philosopher* ***Friedrich Nietzsche*** *(1844–1900, Germany) who attacked almost everything his age held sacred — democracy, socialism, materialism, intellectualism, and Christianity. His wholesale condemnation of society was felt far beyond the first World War.*

Rise of Science in Germany (1870–1930)

In the 17th and 18th century, science moved relatively slowly. The number of brilliant scientists was limited. But during the 19th century scientific progress began to be made at a rapidly increasing rate. In the last three decades of the 19th century, and the first three decades of the 20th century, the center of science moved to Germany. There, due to a close working between fundamental science and its technological applications, science grew to become an integral part of the shaping of modern society in the 20th century. This monumental rise of German science and technology transformed Germany from a relative destitute and backward country into one of the great powers on earth. To understand this rapid and unparalleled growth of German science and industry, one must look at the factors that led to these developments.

When the ravages of the Thirty Years' War had been ended by the Westphalian peace treaty (1648), Germany was a devastated country. It took more than a century for it to recover intellectually[429]*, politically, and economically. In the 18th century, Germany was still quite backward compared to France and England. During the period of the Enlightenment (second half of the 18th century), the country started to recover intellectually as illustrated by the names of* ***Schiller****,* ***Goethe****,* ***Kant*** *and* ***Beethoven****. But otherwise the country was still in a depressed condition; the middle class was poor and had virtually no influence, compared to that of its counterparts in France and England. Poverty, starvation, and disease were widespread.*

An unexpected factor in the rebirth of German intellectual forces was the defeat, occupation and humiliation of Germany by Napoleon. Since the rebuilding of a strong army was, under French occupation, out of question, the leadership turned to the creation of a strong intellectual elite through the expansion of the universities. Thus, ***Wilhelm von Humboldt*** *was the driving force in the establishment of the University of Berlin, in the midst of the Napoleonic wars (1810). New universities were soon established in Breslau (1811) and Bonn (1818). In 1820, science began to develop at these universities, but reactionary forces were simultaneously at work: the revolutionary movement among students in favor of progressive ideas was suppressed by a massacre carried by troops under the command of Prince Wilhelm (1848), later to become Kaiser Wilhelm I.*

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[429] With a few exceptions such as **Johann Sebastian Bach** (1685–1750) and his family and **Gottfried Wilhelm von Leibniz** (1646–1716).

The expansion and unification of Germany under **Bismarck** *(1862–1890), created favorable conditions for the rapid growth of industry and agriculture. Unlike France and England, Germany was poor in natural resources and had no empire to exploit. Bismarck and other farsighted leaders recognized the vital importance of developing science and technology to increase the national economic wealth.*

To this end, the universities were granted strong government financial support, on a scale unprecedented in the history of any nation. During 1825–1900, a dozen of fist rate institutes of technology were established throughout the country. Many of the graduates of these excellent schools became dynamic leaders in industry. *They were fully aware that* basic science *was the main source of new inventions and improvements in* technology. *They established large research laboratories attached to their manufacturing enterprises.*

By the turn of the century, Germany had become the leading industrial country. It had a gigantic pharmaceutical and chemical industry; it had an electronic and optical industry of unmatched quality. Close collaboration between industry and universities was frequent and of mutual benefit[430].

Thus, while in 1840, with a population of about 35 million, it was plagued by poverty, misery, starvation and disease – in 1910, with a population of 70 million, it was a rich country with a highly developed middle class and a working class with better living conditions and more advanced social institutions than their counterparts in France and England, although both classes were virtually without political power.

1871 CE Parliament votes to abolish the "*religious tests*" at Cambridge University; from this year on degrees and positions could be earned without the need to adhere to the principles of faith of the Church of England. In the past, candidates for the M.A. degree and persons elected to fellowships were required to make subscriptions and declaration that, for non Christians, were equivalent to conversion[431].

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430 The United States was the first country to follow Germany's lead in joining research and industry as evidences by DuPont, General Electric, among others. France and England soon fell behind.

431 E.g., **James Joseph Sylvester** entered St. John's College, Cambridge in 1831. Being a Jew, and unwilling to sign the *Thirty-nine Articles*, he could not compete for one of the Smith's prizes and was ineligible for a fellowship, nor could

1871–1890 CE Years of social, economical and political instability in Western Europe: *The Paris Commune* (1871) — workers and soldiers took over the government of Paris for 3 months. The Commune was suppressed with the help of the Prussian Army. About 30,000 communards were executed by French authorities.

In 1873, there occurred the great world-wide financial crash. The next 17 years held hardship for ordinary people, great profits and consolidation for a few. Small businessmen (such as Einstein's father) were badly hit. This was a time of labor struggles, immigration, the rise of militant socialism, and above all the beginning of the age of imperialism and monopoly capitalism.

In 1878, **Otto von Bismarck** (1815–1898, Chancellor of Germany, 1871–1890) passed anti-socialist laws to suppress working-class political agitation, and said "*The great questions of the day will not be settled by revolutions and majority votes but by blood and iron*".

In 1879 **Wilhelm Marr** coined the word *anti-semitism* and found the *League of Anti-Semites*. The league blamed the Jews for the financial crisis.

It was a period of tremendous overall industrial expansion. People throughout Europe were forced off the land and into the cities. Central to Germany's industrialization was the growth of the chemical and electrical industries and the formation of the cartels of I.G. Farben, Krupp, etc. (by 1913, half of the world's trade in electro-chemical products was in German hands).

In 1887, the German government opened the *Physikalische-Technische Reichsanstalt* for research in the exact sciences and precision technology. **Werner von Siemens** (1816–1892) donated 500,000 marks to the project. His old friend, **Hermann von Helmholtz** (1821–1894) of the University of Berlin circle, was appointed head.

Arms expenditures in Germany nearly tripled between 1870 and 1890; the officer corps increased from 3000 to 22,500. Three-year military service became compulsory. Socialist literature was forbidden. Youth were subjected to intimidation and humiliation. Veteran organizations were state supported: membership increased from 27,000 in 1873 to 1,000,000 in 1900. Heads of state all appeared in military uniforms; even taxi drivers wore uniforms. The head of this military state was **Wilhelm I, Emperor of Germany** (1871–1888).

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he even take a degree; this last, however, he obtained at Trinity College, Dublin, where religious restrictions were no longer in force. Only in 1883 could he be appointed (as a Jew) a professor at Oxford.

1871–1908 CE *Discovery of the Aegean Bronze-Age Civilization.* In matters legendary and pre-Classical the antiquarian scholars confined their knowledge to the literary evidence of ancient authors: **Homer**, **Plutarch**, **Ovid**, **Pliny** and **Virgil**.

In the 18th and early 19th centuries travelers to Greece were naturally impressed by the great walls of Mycenae and Tiryns, still standing and book were written by scholars who described these impressive remains. But non of these scholars and explorers was concerned with pre-Classical history, with proving the relation of the legends of the heroic age to the visible monuments, by the clear evidence of excavation. For this an entirely new mental attitude was required, not that of antiquarianism, but of archaeology.

It came with **Heinrich Schliemann** (1822–1900, Germany), an archaeologist and linguist who realized his childhood dreams of rediscovering the Homeric world of Troy and Mycenae (1871–1884). His work was followed by **Arthur John Evans** (1851–1941, England) who conducted excavations in Crete (1894–1908), discovering pre-Phoenician script and the prehistoric Palace of Knossos, seat of an early culture he named Minoan. Evans' opening up the whole world of Mycenaean palace and state organization, has been one of the greatest contributions toward our understanding of the Bronze Age.

Scholars all over the world had struggled in vain far 50 years to decipher the Cretan so-called "Linear B". This honor fell to a young English architect[432].

1871–1909 CE **Edward Burnett Tylor** (1832–1917, England and USA). Founder of cultural anthropology. Advanced the new idea that human history is dominated by the concept of *cultures* rather than that of *Nations* that had spread across Europe after the 15^{th} century. This innovation of Modern Social Science would be the key to new ways of thinking about the meaning of history and the future. He saw all cultures as parts of a single history of human thought, and all evolution that Darwin had described in biology, Tylor too now saw in society.

Tylor was born in London, the son of a prosperous English Quaker. As Quaker he could not enter a university and so began life in the family business. Seeking a climate to cure his tuberculosis, he went to Mexico (1856) where he

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[432] The Minoan Linear B script was deciphered (1952) by the architect and cryptographer **Michael George Francis Ventris** (1922–1956, England) who demonstrated that it is Greek in oldest known form, predating Homer by some 500 years. He had heard Evans' lecture in 1936 when he, Ventris, was 14. After the War, in which he served as a pilot in the Royal Air Force, he continued his efforts to decipher the linear scripts. He was killed at the age of 34 in a motor accident.

accidentally joined a study of Toltec remains. So began Tylor's lifelong study of strange and ancient societies and their relation to modern life.

Although he never studied formally at a university, he became a professor of anthropology at Oxford (1896–1909).

He wrote: *Primitive Cultures* (1871); *Early History of Mankind and the Development of Civilization* (1865).

Tylor wrote:

"The past is continually needed to explain the present, and the whole to explain the part. There seems to be no human thought so primitive as to have lost its bearing on our own thought, nor so ancient as to have lost its bearing on our own thought".

His work was continued by **F. Boas** (1911), **O. Spengler** (1918) and **A.J. Toynbee** (1934).

1872 CE **Peter Ludwig Mejdell Sylow** (1832–1918, Norway). Mathematician. Proved a key theorem in the theory of *finite groups*[433]. Sylow studied at the University of Christiania and became a high school teacher (1858–1898). Lie had a special chair created for Sylow at Christiania from 1898.

1872 CE **Ernst Mach** (1838–1916, Moravia). Physicist, philosopher and psychologist. Rejected the Newtonian concepts of absolute space and inherent inertia of material bodies. His qualitative ideas (no quantitative theory!) stemmed from the realization that Newton's observations on inertia were entirely *local*, and no references to the rest of the universe were made.

To measure the rotation of the earth, Newton used a terrestrial experiment, whereas the same rotation can be determined by global or *astronomical* measurement via the apparent motion of the stars. This coincidence, Mach argued, must stem from a causal relationship between the motion of the distant stars and the local inertial frame of reference, and it must imply that the inertia of any body is determined by the distribution of distant matter in the universe.

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[433] *Sylow's Theorem*: If p^n is the largest power of the prime p to divide the order of a group G then

- G has subgroups of order p^n (called Sylow p-subgroups)
- any two such subgroups are conjugate
- G has $n_p = 1 (\mathrm{mod}\ p)$ such subgroups, and n_p is the order of the quotient group of G by the normalizer of any given Sylow p-subgroup

Almost all work on finite groups uses Sylow's theorem.

According to Mach there is no absolute space, and particles' inertia is due to unspecified interactions with the rest of the universe; all that matters in mechanics is the *relative* motions of *all* the masses, near and distant (i.e. an isolated mass has no inertia, centrifugal forces are physical, etc.). The totality of these ideas can be encapsulated into one statement, known as "*Mach's principle*":

A body's inertia and the local structure of space-time are determined by the mass distribution in the rest of the universe.[434]

Thus, according to Mach, stars en masse *cause* inertia[435].

Although Einstein's GTR is local and does not incorporate the 'Mach principle' in any direct way, the accepted cosmological models *based* on GTR, do

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[434] For further reading, see:
Mach, Ernst, *The Science of Mechanics*, The Open Court Publishing Co., 1989, 634 pp.

[435] On an earth covered permanently by clouds, an observer of the peculiar motions of the free gyrocompass with respect to ground would inevitably (after having searched in vain for any visible agency to which the spin axis of the gyro may appear attached) be drawn toward the Newtonian idea of some mysterious "absolute space" with respect to which the earth happens to be in rotation. However, if in this state of affairs the cloud cover were suddenly removed, revealing the stars whose average motion with respect to ground happens to correlate with the precession averaged gyro's axis, the observer would be equally inevitably drawn toward the thought that the stars are the cause of the gyro's previously inexplicable behavior. The formulation of this idea, which goes back to George Berkeley, is known under the name of *Mach's principle*: the local inertial behavior of any object is somehow determined by the entire actual distribution of masses and their motions in the universe.
Mach himself did not specify what kind of interaction ought to be held responsible for inertia. From time to time attempts have been made to put Mach's principle, which is really not more than a program, on a quantitative basis by constructing cosmological models in which a particular kind of interaction between the distant stars and local objects is invoked to explain inertial behavior. Most of these attempts invoke gravitation as the inertia-producing interaction, a rather obvious choice in view of the universal proportionality between inertial and gravitational mass (*equivalence principle* of GTR). There were also attempts to account for inertia through invention of a special kind of "field", other than gravitation, just for this purpose.
None of these attempts has been wholly satisfactory. In particular, no one has yet been able to construct a cosmological model incorporating Mach's principle and also incorporating consistently the velocity of light as the limiting speed with which physical influences may be propagated.

satisfy a limited form of Mach's principle — in the sense that at any location there is a preferred *local frame* of reference in which the distant galaxies of the universe are, on the average, *receding isotropically*. This frame is best defined by the *cosmic microwave* background radiation. Thus, in relativistic big-bang cosmology the local inertial frames of Newtonian mechanics and STR are indeed determined to some extent by distant matter — although a body's inertia is *not*.

The most "Machian" effect in GTR is *frame-dragging* — the ability of a rotating mass to (partially and locally) drag with it the inertial frame; the dragging is not rigid, and decreases with distance from the rotating mass. This is one of the effects to be tested in the "gravity probe B", a Stanford University Collaborative project with industry[436], in which an extremely round niobium coated gyroscope is set spinning in a box within a satellite, and its general-relativistic precession measured via induced quantum eddy-currents in superconducting devices. In this experiment, the frame-dragging, rotating mass is the nearby earth.

An extreme, though impractical, manifestation of this effect occurs in the **Lense-Thirring-Brill-Cohen** thought experiment: an infinitely massive, spherical shell, when set rotating, drags the inertial frames in its interior *rigidly*, at the same angular velocity as that of the rotating shell. Again, the *magnitudes* of objects' inertia are not themselves affected — only the identity of inertial *frames*[437].

There is evidently a consistent line of thought which goes through the philosophical doctrines of Spinoza-Leibniz-Berkeley-Mach, that also has a bearing on the *ideas of quantum mechanics*: namely, the non-separability of interacting systems which forces us to treat the observer as part of the physical system being observed — a *Machian notion*. Yet at the same time, in the quantum mechanical description, one is forced to consider higher-dimensional Hilbert spaces, and when proceeding to gauge theories, one must also reckon with the geometry of fiber bundles — all increasingly remote from direct observation. This latter trend in modern physics is in *contrast* to Mach's positivism: he was steadfast in his rejection of non-empirical concepts.

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[436] This experiment was launched into orbit in April of 2004, and as of this writing is still taking data.

[437] However, according to the standard models of particle physics and cosmology, the rest masses of electrons and quarks – and thus of all ordinary matter existing today – were endowed, a fraction of a second after the Big Bang, by the condensation of a global, vacuum-permeating quantum scalar field; in this sense and to this extent, modern physics does incorporate the Machian notion of cosmologically determined inertia, at least as far as *rest* mass is concerned.

Mach stands out as a unique figure among physicists. He mounted a crusade against a universally accepted approach to classical mechanics and went to extremes in his zeal to purge physics of its scholastic relics[438]. Thus, he rejected the existence of atoms, and died unconvinced of the kinetic theory of gases, Brownian motion and the special theory of relativity. Einstein, however, considered him as the forerunner of the general theory of relativity.

Mach was born in Turas, Moravia and studied in Vienna. He was a professor of mathematics at Graz (1864–1867), of physics at Prague (1867–1895) and of physics at Vienna (1895–1901).

1872–1876 CE *The Challenger expedition.* The first systematic attempt to explore the depth and breadth of the world's ocean from the chemical, physical and biological points of view. Spurred by this international competition (1871), the Royal Society appointed a committee which recommended that funds be requested immediately from Her Majesty's Government for an expedition with the following objectives:

1. To investigate the physical conditions of the deep sea in the great ocean basins.
2. To determine the chemical composition of seawater at all depths in the ocean.
3. To ascertain the physical and chemical characters of the deposits at the sea floor and their origins.
4. To examine the distribution of organic life at all depths in the sea as well as on the sea floor.

To carry out these objectives, it was recommended that a sizable ship, a staff of scientists qualified to carry out the desired investigations, and an ample supply of equipment, instruments, and special apparatus be made available. As a result of these recommendations, the first great oceanographic expedition, a

[438] Mach entered into a debate with Planck about the nature of science, at the beginning of the 20th century. Mach championed an instrumentalist philosophy of science. He lodged science in everyday *psychological* experience and exalted technology. Being a liberal democrat, Mach was intent on empowering "citizen scientists".

Plank, on the other hand, was a realist. He reduced everyday experience to the ultimate constituents of physics and promoted abstract problem solving. He was a state corporatist, who thought ordinary folks had no claim on "real science".

This debate is emblematic of all such debates since.

model for all subsequent efforts, was organized. The expedition was a bold attack upon the unknown in the tradition of the great see explorations of the 15^{th} and 16^{th} centuries.

H.M.S. Challenger was an 18-gun corvette of the British navy that had been stripped of battle gear and fitted for oceanographic research. It had a displacement of 2306 tons and was equipped with both sail and steam power. The British Admiralty provided a crew under the command of Captain George Nares;

Charles Wyville Thomson (1830–1882), professor of natural philosophy at the University of Edinburgh, headed the research staff. The 240-man expedition left Portsmouth, England, in December, 1872. The *Challenger* criss-crossed the North Atlantic, swerved down through the South Atlantic, and eastward into the Antarctic Ocean. Leaving the Antarctic, it continued its way to Australia and the Western Pacific islands, eastward to the Hawaiian Islands, on through the Straits of Magellan, and finally back to England where it landed on May 24, 1876.

In three years the *Challenger* has sailed 110,500 km and made 362 "oceanographic stations", gathering data on weather conditions, surface currents, water temperature, water composition at various depths, marine organisms, and bottom sediments. Expedition scientists charted oceanic topography on the ship's track, netted and classified 4,717 new species of marine life[439], and took a depth measurement of 8183 m in what came to be known as the *Challenger Deep*[440].

The official report of the *Challenger* expedition, filled 29,500 pages in 50 volumes and took 23 years to complete. A total of 76 authors contributed to the report, and numerous other specialists were consulted. No other expedition has made so many important contributions to oceanography.

When she left England, the ocean depths were an almost unfathomed mystery. When she returned, she had sounded the depth of every ocean except the Arctic and laid the foundation for the modern science of oceanography.

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[439] One of the most interesting of those organisms is the *radiolaria*, of which the expedition collected 3508 new species to add to the 600 then known. The Challenger discoveries demonstrated that the oceans were teeming with unknown life waiting to be classified. It proved that life existed at great depth in the sea.

[440] It was measured on March 23, 1875, off the Mariana Islands. The deepest known spot in all the oceans is at 11,033 m below the surface in the *Mariana Trench*.

Oceanography[441] *— The Conquest of Inner Space (1000 BCE–1927 CE)*

I Historical Background

Ancient and medieval navigation had been largely coastal; mariners did not sail many days out of sight of land. They knew the sea but not the Ocean. Indeed, even in the middle of the 15th century, at the time when the Renaissance is supposed to begin, man's knowledge at the face of the earth was still restricted to a very small portion of it, and even in that portion was very superficial. One of the great tasks to be accomplished was the discovery of planet earth.

Schematically, the earth can be regarded as being composed of three materials: rock, water and air, arranged in three layers — the lithosphere, hydrosphere, and atmosphere. The earth is also an astronomical body: the effects of nonuniform distribution of sunlight over the earth and the equally energetic but more uniform radiation of earth heat into space, acting with rotational and gravitational forces, produce a complex of interdependent fluid phenomena which characterize the world as we know it.

Oceanography may be defined as the study of the oceans. It is principally concerned with the various aspects of sea water[442]*: its motions and chemical constituents, its physical properties and behavior, its relationships to the solid earth, the atmosphere, and to living organisms of all kinds, its economic and technical potentialities, its role as part of the earth's outer covering.*

Thus, oceanographers are drawn from four large areas of science: geology, chemistry, physics, and biology. Geologic study of oceanic sediments, rock

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441 For further reading, see:

- Von Arx, W.S., *An Introduction to Physical Oceanography*, Addison-Wesley Publishing Company: Reading, MA, 1962, 422 pp.
- Weyl, P.K., *Oceanography*, John Wiley & Sons: New York, 1970, 535 pp.
- Yasso, W.E., *Oceanography*, Holt, Rinehard and Winston: New York, 1965, 176 pp.

442 Gross properties of sea water: *Characteristic density* = 1.025 gm/cm^3; *Velocity of sound at surface* = 1448.6 m/sec; *Specific heat* = 0.932 cal/gm/°C at salinity of 35%; *Adiabatic lapse rate* $\sim$ 0.1 °C/km; *Maximum surface temperature* = 32 °C; *Minimum surface temperature* = −2 °C.

structure, and topography is referred to as submarine geology *or* geological oceanography. *The study of sea-water chemistry is called* marine chemistry *or* chemical oceanography. *The biological aspects of the ocean environment, such as fish populations and plant life, are studied by the* marine biologist, *or* biological oceanographer. *Atmospheric processes, circulation of the oceans, and the physical properties of ocean water fall within the scope of* marine physics, *or* physical oceanography.

Oceans cover nearly $\frac{3}{4}$ *of the earth's surface; the topography of the ocean floors is more rugged and more mountainous than the topography of the continents; the greatest oceanic depths are greater than the height of Mt. Everest. The distance from the top of this mountain to the bottom of the Mariana Trench is about* 20 *km. Even so, the solid surface of the earth would still be much smoother than the surface of an orange if scaled to that size. But the oceans are even smoother than that.*

The volume of the oceans is roughly 1365 *million cubic kilometers, with a mass of* 1560 *million billion tons. Although these are enormous figures, they represent only* 1/790 *of the volume* and 1/4200 *of the mass* of the earth. *Covering a surface area of* 362 *million square kilometers, the average depth of this water mass is slightly less than* 3.8 *kilometers, or about* 1/1680 *of the earth's radius. Truly, the oceans are hardly more than a film of salt water on the surface of our planet.*

Yet, it was in some shallow, near-shore area of this film that life on earth began two or three billion years ago. Today it is the tremendous abundance of life in the sea that appears to present the surest solution of the world's food problems. Well over half — perhaps as much as 85 *percent — of the food product of all plants is produced by marine plants.*

In time the oceans will certainly become a major source of valuable minerals and chemicals and an important source of fresh water when these are no longer available in sufficient quantities on the continents. Also, in recent years, it has become clear that the oceans play a very important but little-understood role in determining the earth's weather and climate patterns. And finally, it may be that the sediment layers at the bottom of the sea contain a record going back several billion years — a record of the earth's history. For these and many other reasons, the importance of the earth's oceans can hardly be overestimated.

The heat content in the oceans does not vary appreciably from year to year, but maintains a gently shifting balance consistent with external sources and demands. Several processes serve to heat the oceans:

- *Radiation absorbed from the sun and sky.*

- *Condensation of water vapor as dew on the sea surface.*
- *Conduction from the atmosphere.*
- *Conduction from the sea floor.*
- *Conversion of mechanical energy into heat.*

Other processes serve to cool it:

- *Radiation from the sea surface to space.*
- *Evaporation to the atmosphere.*
- *Conduction to the atmosphere.*
- *Conduction to the sea floor.*

Of these processes, the conductive and radiative exchanges with the atmosphere are of greatest importance. The heat of conversion of the mechanical energy of winds and ocean currents is generally less than the heat of incoming radiation from the sun and the sky by a factor of 10^{-4}*, but* locally *may be somewhat greater where tides are accompanied by strong frictional retardation, as in some shallow seas.*

Conduction to the atmosphere is significant where the ocean temperature is higher than that of the atmosphere but not in the reverse case. This difference in conductive efficiency is due to the strong convection that can develop in cool air over a warm ocean, and the marked stability that develops when warm air is chilled by a cold ocean. Conduction may amount to some 10% of the evaporative heat loss of the oceans. Conductive exchanges with the sea floor are thought to be mainly upward, owing to the evolution of radiogenic heat in the earth's crust. The important influences are seen to be associated with radiative exchanges, evaporation processes, and the upward flux of heat in conduction to the atmosphere.

The flux of sensible heat is directed mostly from the oceans to the atmosphere, but not exclusively so. In middle latitudes of the Northern Hemisphere, particularly over the eastern parts of the North Atlantic Ocean, the heat flux in summer is directed from air to sea. This effect places the region of maximum atmospheric warming by conduction in the western and northern portions of the Northern Hemisphere oceans, mainly in association with the poleward transport of tropical waters in the Kuroshio of the North Pacific and the Gulf Stream of the North Atlantic Ocean. Over the middle-latitude portions of these currents the annual average upward flux of sensible heat may exceed 90 $\text{cal/cm}^2\text{/day}$ *over the Kuroshio, and 120* $\text{cal/cm}^2\text{/day}$ *over the Gulf*

Stream. The total energy loss of sensible plus latent heat is about four times as great.

Precipitation, in general, returns sensible but not latent heat to the oceans. Oceanic warming by latent heat can occur only when there is condensation directly on the sea surface as dew, or possibly to some extent in very shallow fogs. In other cases latent heat is released at the level of condensation, and has only the indirect effect of abating radiation losses from the sea surface.

The sea radiates to the atmosphere and space very nearly as a black body and therefore contributes outgoing energy in amounts proportional to the fourth power of its absolute surface temperature. The wavelength of maximum emission for the sea surface is nearly centered on the 10-μ "window" between the absorption bands for atmospheric water vapor and carbon dioxide. Water vapor is, however, such a strong absorber of infrared radiations even in this window that the amount of outgoing radiation from the ocean surface is more closely related to the absolute humidity of the lower air than to the ocean temperature. As the air temperature falls and the absolute humidity of the lower atmosphere is correspondingly decreased, the radiation losses from the ocean tend to increase until a skin of ice is formed. Bubble-free ice is nearly as good a black-body radiator as a free-water surface is, but as soon as bubbles are trapped in sea ice they reflect radiation back into the ocean and reduce the intensity of long wavelength emission to the atmosphere. The latter effect tends to confine heat within the water phase of ice-covered oceans, and indeed explains in part why the ice in the Arctic Ocean is relatively so thin despite the very low air temperatures that sometimes prevail.

The average energy of incoming short-wave radiation from the sun and sky usually exceeds the heat loss through radiative processes from the sea surface. The excess heat in the ocean is first communicated to the atmosphere by the process of evaporation and conduction, whereupon the atmosphere radiates this energy to space.

The movement and circulation of the oceans is tied very closely to the circulation of the atmosphere: Both are ultimately driven by the distribution of available solar energy, and their motions are linked by friction at the sea surface. There exists an imbalance in the latitudinal distribution of energy that produces an equator-to-pole temperature gradient at the surface — the driving force for the pattern of earth's surface wind. These wind patterns are responsible for the circulation of the ocean surface and the formation of the world's major ocean currents. As with the atmosphere, once the ocean starts to move, it comes under the influence of the Coriolis effect, which plays a significant role in the resulting circulation patterns.

The oceans are vertically stratified, with more dense water at the bottoms of the major ocean basins and less-dense water near the surface. The density *is controlled by the temperature and by the salt content (salinity) of the water. The deep-ocean water is separated from the surface layer of the ocean by a transition zone with sharply defined density, temperature, and salinity gradients. This deep-ocean water moves as a response to small changes in density that occur over wide areas, and the movement is largely independent of the surface-ocean circulation. Together, however, both types of ocean circulation contribute to the redistribution of available energy in the earth system, albeit over very different time scales. And both play a major role in the distribution of nutrient supplies in the oceans.*

Like the circulation of the atmosphere, the circulation of the ocean is ultimately driven by solar energy. Figure 12 is a systems diagram of ocean circulation. The distribution of solar energy over space and time results in the formation of the global wind belts. These roughly latitudinal wind patterns in turn produce the ocean currents that determine the circulation patterns of the upper ocean. The distribution of surface-ocean temperatures, which is partly a result of these circulation patterns, strongly influences the density structure of the ocean. It is this density structure that drives the circulation within the deep ocean. As with the atmosphere, the feedback loops in Figure 12 are negative — surface-temperature gradients drive the circulations, but the net effect is to move warmer water poleward and cooler water toward the tropics. Both the surface-ocean and deep-ocean circulations also play a vital role in climate. The surface circulation is one of the primary factors controlling climate variability on the order of years to decades, but even the deep circulation, which generally operates on the time scale of hundreds of years, is being implicated in short-term climate change. The two different circulation systems act together in controlling the distribution of the nutrients that are essential to marine life.

The circulation in the troposphere is caused by atmospheric pressure gradients that result from vertical or horizontal temperature differences. From a global perspective, these temperature variations are caused by latitudinal differences in solar heating. But ocean surfaces are also heated by incoming solar radiation. Do the oceans, therefore, circulate for the same reason as the atmosphere? The answer is no, because the solar heating of the ocean takes place at the upper *surface of the fluid, whereas the solar heating of the atmosphere occurs largely at the* lower *surface of the fluid-near earth's surface, where clouds and the green-house effect warm the atmosphere. Solar heating results in warmer water at the surface of most of the world's oceans. But the sun's rays warm only the top few hundred meters of the ocean; 90% of the radiation that penetrates the surface is absorbed in the first 100 m. The warmer water is less dense than the cooler water below, which is not affected by the surface heating. This situation is inherently stable, so there is very little vertical movement.*

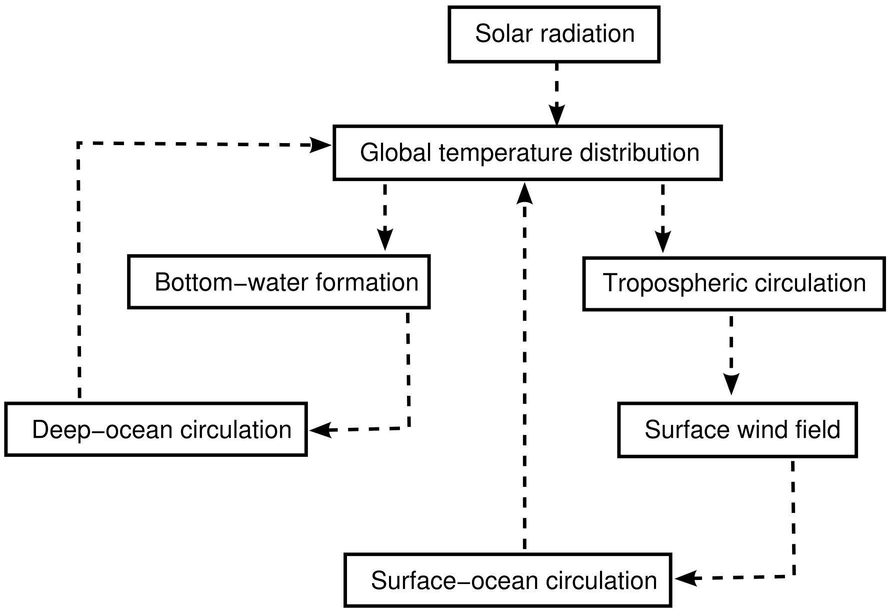

Fig. 12: Ocean circulation diagram

It is similar to the situation in the stratosphere. The atmosphere at this level is stable because the maximum solar heating occurs high in the stratosphere, the site of peak absorption of ultraviolet radiation by ozone. Where temperature increases with height, there is no density imbalance, and convection cannot take place. The fluid – water or air – remains well stratified. The true situation in the ocean is actually more complicated than this, because the density of seawater is also affected by its salt content. It remains true, however, that the ocean overturns very slowly.

At the same time, temperature changes in the ocean occur slowly. The oceans have a high heat capacity — it takes a considerable amount of heat to produce just small changes in temperature. Slight differences in incoming solar radiation from place to place thus have little impact on the surface temperature of the ocean, so lateral temperature and density differences are slight over large areas. Unlike the troposphere, therefore, the surface ocean does not circulate as a direct *response to the surface heating. Instead, surface temperature plays a more indirect role: The surface temperature influences the atmospheric circulation, and the resulting pattern of global winds determines the circulation of the upper ocean.*

The content of oceanography, like that of any other science, depends to a large extent on its history. Man's exploration of the world's oceans was at first largely limited to the surface of the sea and to the mapping of the *distribution of land and water. Later man turned to the exploration of the depths of the ocean.*

Our current knowledge of the oceans developed in stages: At first the map of the world was completed. The geographic exploration of the distribution of land and sea led to an interest in the sea itself. Oceanographic expeditions were conceived and carried out, and then published scientific reports added to our store of knowledge. Finally, puzzling observations led to the development of a dynamic theory which transformed oceanography from a purely descriptive to an analytical science.

Although primitive man had explored all the oceans and most of the land areas of the world, his geographic knowledge was localized. The inhabitant of a Pacific atoll knew his immediate environment and had stick charts indicating the locations of neighboring islands. His oral traditions told of the faraway places from which his ancestors had come to settle the island. While there was extensive knowledge of the local geography, there was no geography of the earth as a whole. The gradual evolution of the picture of the world, as seen from Europe, began with the earliest roots of civilization.

*The study of the sea seems always to have been promoted by a practical rather than an abstract curiosity about the natural world. The maritime commerce of the Phoenicians, Hebrews and Greeks made the Mediterranean and adjacent seas reasonably well known even before the time of the Judean kings [**Solomon** (ca 940 BCE) II Chron* **8***, 17;* ***Jehoshaphat*** *(ca 870 BCE) II Kings* **22***, 49* and *II Chron* **20***, 36]. The waters of this sea enclosed by three continents provided a ready means of transport of goods and soldiers. The conception of the world, about* 850 *BCE, included only the land immediately adjacent to the Mediterranean, surrounded by Oceanus, the indefinite land-encircling ocean which lay everywhere beyond the frontiers of knowledge.*

The master merchant mariners, the Phoenicians, passed through the Pillars of Hercules, the Straits of Gibraltar, into the Atlantic Ocean. They circumnavigated Africa and penetrated north to Great Britain: Indeed, about 600 BCE (according to ***Herodotos****), King Necho of Egypt sent an expedition manned with Phoenician sailors down to the Red Sea and along the east coast of Africa. The ship is said to have returned to the Mediterranean,* 3 *years later by way of Gibraltar. The expedition should have established that Africa is a separate continent, but the chronicle was rejected by scholars of that time, and Greek philosophers continued to support the Homeric map.*

In the 4^{th} *century BCE*[443]*, man's knowledge was somewhat extended due to the Indus River expedition of* ***Alexander the Great*** *(329–325 BCE) and the voyage of* ***Pytheas of Massilia****. The former revealed the relationships*

[443] The book of **Jonah** was composed at about that time.

of several bodies of water to one another, namely, the Caspian Sea, Persian Gulf, and Arabian Sea, and to the then known world around the Red Sea and the Mediterranean. On a map drawn about 300 BCE by ***Dinaearchos of Messina*** *(a pupil of Aristotle), parallels of latitude are used for the first time.*

The geographic knowledge of the Romans was summarized by ***Ptolemy*** *(150). He introduced the concepts of latitude and longitude, and presented a projection of the globe on a map which showed the Indian Ocean surrounded by land in part unknown.*

Although it was relatively easy to measure the latitude by noting the angle the pole star makes with the vertical, the ancients had no way to measure longitude directly. Ptolemy therefore estimated distances in the east-west direction from the time required for voyages.

The Indian Ocean was the first to be used for trade but, strangely, was one of the last to be thoroughly explored. The ancients carried on a brisk trade between the Mediterranean and the East by way of the Red Sea and Indian Ocean. This sea traffic was much influenced by the monsoon. The wet monsoon of the Northern Hemisphere summer permitted the ships of the Greek and Arab traders to penetrate the Arabian Sea and Bay of Bengal, and then during the period of the dry monsoon of the Northern Hemisphere winter to find favorable winds for the homeward journey.

It was the reversing currents associated with the monsoon winds of the Northern Indian Ocean that favored this traffic, made fruitful by the ready markets for oriental products in the Roman world. With the fall of Rome the trade dwindled, but unwittingly the knowledge that the Greeks and Romans possessed had been deposited with Arabian scholars for safekeeping during the Dark Ages[444]*. By this accident the Ptolematic view of the world was kept intact until the* 11^{th} *century CE when, as a by-product of the Crusades, Western civilization was re-educated concerning its own past.*

While southern Europe was preoccupied with matters of theology, the Norsemen were engaged in journeys of discovery, aided by improved climatic conditions, which reduced the amount of ice in Northern waters. Iceland was visited by the Picts and the Celts in 650, and settled by the Celts in 770.

In 835, a papal bull referred to Christian settlements in both Iceland and Greenland. The Vikings began to take over these Northern lands in about

[444] The deterioration of geographic knowledge during the Middle Ages is indicated by the world map of **Cosmas Indicopleustes** (fl. 548 BCE), an Alexandrian navigator of the Indian Ocean. He insisted that the earth was a quadrilateral measuring 20,000 km × 10,000 km.

870. *In* 982 ***Eric the Red*** *crossed the Davis Strait from Greenland to Baffin Island, in Canada. Three years later he established a colony in Greenland. His son* ***Leif Ericsson*** *sailed west from Greenland in* 995 *and spent the winter in Newfoundland.*

Because of a deterioration of the climate beginning in about 1200*, the Viking colonies in Greenland became isolated; the Vikings were therefore never able to exploit their discovery of America. Were it not for the readvance of ice in the North Atlantic, the history of America might have been very different. The Vikings' conception of the Northern ocean are known to us through the map of* ***Sigurd Stefansson*** *(1570).*

While the Vikings were exploring the Northern seas, Arab traders were exploring the Indian Ocean and sailed as far as China. A map by ***Abu ar-Rayhan al-Biruni*** *(1030) reflects the geographic knowledge of his times. The Arabs brought the lodestone from China and thus introduced the magnetic compass to the West. At first it was viewed with suspicion as being under the influence of some infernal spirit. However, the mariner's need for an instrument with which he could steer a fixed course, regardless of visibility, led to the rapid adoption of the magnetic compass.*

The period from 1492 *to* 1522 *is known as the Age of Discovery because geographic knowledge expanded at a very rapid rate during these* 30 *years. The continents of North and South America were added to the globe, and the earth was circumnavigated. These daring voyages of discovery were brought about by a political event. In* 1453 *the Sultan Muhammad II captured the capital of Eastern Christendom, Constantinople. As a result, the Mediterranean ports were cut off from the riches of the East.*

At the same time, learned Greeks expelled from Constantinople brought the geographical knowledge of the ancients to Italy, and the introduction of paper permitted the wide distribution of these works. Thus the Turks indirectly revived old knowledge, which made it possible to find new sea routes while creating an economic motivation for exploration.

Meanwhile, the Portuguese and others had been making preparations for their great voyages of discovery. In 1420 *Prince* ***Henry the Navigator*** *established a maritime observatory and assembled the best Italian map-makers and Jewish astronomers to teach navigation to the Portuguese. Until that time the Portuguese had been afraid to sail out of sight of land, and all expeditions to round Africa had turned back at Cape Bojador (27°N). The Cape of Good Hope was finally rounded by* ***Bartholomeu Dias*** *in* 1488*. In* 1498*,* ***Vasco da Gama*** *extended the trip around Africa to India*[445]*.*

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[445] When **da Gama** rounded the Cape of Good Hope in April, 1498, he acquired the services of an Arab navigator, **Ahmad Ibn Majid**, to guide him across

In 1474, *the Florentine astronomer* **Toscanelli** *wrote to the King of Portugal suggesting an expedition to explore a route to the Spice Islands of the East across the Atlantic Ocean. He appended a map which greatly underestimated the distance to the east coast of Asia, placing it at a longitude just off the west coast of America. Later, on inquiry, he sent a copy of this letter to* **Christopher Columbus**.

In 1492, *Columbus set sail westward to reach the Indies. His underestimation of the distance to China caused him to believe that he had reached the Indies when he had in fact discovered what we know as the West Indies; actually he was farther from his goal than when he had left Spain. The Spaniards and the Portuguese set out to explore the eastern shores of the Americas and the Indian Ocean. The greatest of the oceans, the Pacific, was not discovered until* 1513, *when* **Vasco Núñez de Balboa** *sighted it from a mountain in Panama.*

These early voyages of discovery culminated in the circumnavigation of the globe by **Ferdinand Magellan**. *He left Spain in September* 1519 *with* 5 *ships, on a mission to find a passage between the Atlantic and Pacific Oceans. On the* 21^{th} *of October* 1520 *he found a passage to the great Western ocean at* $52°S$, *now known as the Straits of Magellan. In March* 1521 *he discovered the Philippines, and in April of that year met his death, at the hands of the aborigines of Cebu. On his trip (1521) Magellan made the first recorded attempt to measure the depth of the open ocean by lowering a weighted line to a depth of some* 370 *m, but failed to reach bottom, which is now known to be* 3700 *m*[446].

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the Indian Ocean to the Malabar Coast of India. Although the route was not known to European navigators, the Arabs had an intimate knowledge of sea routes between the Indian Ocean, Arabian Sea, and Mediterranean Sea.

446 Another measuring problem faced by the mariner was how to determine the speed at which his ship was moving through the water. Since there are no fixed reference points at sea, the captain would throw a floating object overboard and time how long it took the object to drift by a measured interval marked off on the deck of the ship. An improved method of measuring speed was introduced by the Dutch near the end of the 16^{th} century. This was the so-called Dutchman's log, which has left its traces in nautical jargon.

The Dutchman's log consists of a piece of wood (the log) attached to a reel of string, with knots tied in at equal, fixed intervals. When the log is thrown overboard, an hourglass is inverted. As the sand in the hourglass runs out, the knots that pass overboard are counted. Thus one obtains the speed of the ship in *knots*. The speed is then entered in the *logbook* with information about the state of the weather and the sea.

By the 16^{th} century, the fact of the earth's rotundity was proven beyond question by Magellan's expedition.

By the year 1600 the surface of the known earth was doubled. It was not only a matter of quantity, but one of quality as well. New climates, new aspects of nature were revealed, new plants, new animals, new men and women.

The psychological reverberation of such new vistas was immense. A man of today can recall the deep emotions he felt when he found himself for the first time in the middle of the ocean, or in the heart of a tropical jungle, or when he tried to cross a desert or a glacier. These discoveries, which are fundamental for each of us individually, were made for the whole of mankind in the fifteenth and sixteenth centuries.

In the meantime, the intense rivalries of colonizing nations encouraged the progress of navigation and of the physical sciences which would increase the accuracy of sailings and minimize their dangers. The main requirements were geodetic, astronomic (better methods of taking the ship's bearings), cartographic; one needed faster ships and better instruments to navigate them. Geodetic improvements were due to ***Jean Fernel*** *(1528) and* ***Gemma Frisius*** *(1533); better maps were due to the* ***Pedro Nuñez*** *(1530), and* ***Gerhard Mercator*** *(1568), and* ***Abraham Ortelius*** *(1568). The splendid geographic atlases which were produced in 1570, provide a large mass of information of vital importance to navigators.*

One of the first fruits of oceanic navigation was a better knowledge of magnetic declination, for the compass was one of the sailor's best instruments, but its readings could not be trusted without taking occasional deviations into account. The magnetic observations and other knowledge useful for navigation were put together by Englishmen like ***Robert Norman*** *(1581) and* ***William Barlow*** *(1597) and by* ***Simon Stevin*** *(1599). At the very end of the Renaissance,* ***William Gilbert*** *published the first great treatise on magnetism (1600).*

Nearly six decades after Magellan, ***Francis Drake*** *found the gap between Tierra del Fuego and the mainland of Antarctica, the* Drake Passage *(1578). This provided the closing link in the discovery of the Southern (Antarctic) Ocean. It remained to be known, however, whether or not land lay to the south. It was* ***James Cook*** *who suggested that an Antarctic continent existed (1772–1775). He was one of the first to lead a journey intended to produce scientific discoveries. [The early discoverers set out, not to discover the secrets of nature, but rather to find the riches of the world and claim them for their royal sponsors. Their successes were measured in treasure-laden galleons rather than in scientific information recorded in expedition reports. While the gold of the Aztecs and the Incas has been largely dissipated, the* *scientific treasures of the newer explorers represent a permanent addition to our store of knowledge.]*

Between 1769 *and* 1779 *Cook commanded British vessels on three major voyages of discovery. On the last Cook set off in search of a northwest passage between the Atlantic and Pacific Oceans and successfully penetrated into the Arctic Ocean by way of the Bering Straits. A short distance beyond the Bering Strait, he was stopped by the Arctic ice pack.*

Cook was the first explorer provided with the proper instruments to determine latitude and longitude accurately. On his second voyage he had four accurate clocks to help in navigation. They had been developed as a result of a naval disaster in 1707*, when* 2000 *men were lost because of faulty navigation. To help avoid such occurrences in the future, Parliament established a prize for a method of determining longitude.*

In 1000 *days at sea, Cook lost only one sailor out of a crew of* 118*. He was the first to conquer the sailor's disease, scurvy, which results from a lack of vitamin C. By watching the diet of his sailors and giving them citrus juice, Cook showed that long sea voyages were possible without detriment to health. Because they were required to drink lime juice to avoid scurvy, British sailors were thenceforth called Limeys.*

Cook's third voyage essentially completed the geographical exploration of the oceans of the world. Only the continent of Antarctica, hidden by a shield of ice, remained to be discovered. (This was accomplished in 1820 *by* ***Nathaniel Palmer****.) Cook's skill as a navigator set new standards and accurately fixed the location of many islands that had previously been known only vaguely. He showed that long voyages of exploration were possible without endangering the health of the crew. Finally, he demonstrated convincingly that the land was not distributed symmetrically about the equator: the great southern continent did not exist unless it was south of* 70°*, protected by ice.*

The depth of the sea, however, remained to be explored. The first expedition to measure the vertical extent of the ocean and one of the last to map its southern boundaries was led by ***James Clark Ross*** *(1800–1862,* England*) during the years* 1839 *to* 1843*.*

A few years before the Ross expedition, the Beagle *(1831 to 1836), with* Charles Darwin *aboard, made its famous voyage in which so much new knowledge of the "natural history" of the ocean islands was obtained. Darwin also looked into the geologic structure and possible origins of ocean islands. This voyage ushered in the succession of cruises devoted to scientific study of the natural history of the seas which culminated in the efforts of* ***Charles Wyville Thomson*** *in* Lightning *(1868),* Porcupine *(1869 to 1870), and finally in the* Challenger *expedition of* 1872 *to* 1876*.*

The American interest in the practical aspects of oceanography were advanced by the efforts of **Matthew Fontaine Maury** *(1806–1873). Having been injured early in his naval career, Maury was placed in charge of the depot of naval charts and instruments. There he found a collection of logbooks of ships' officers containing a wealth of information about currents and weather at sea. Maury proceeded to analyze these data systematically and from them prepared charts of winds and currents which proved to be extremely useful.*

In order to obtain even better data, Maury was instrumental in arranging for the first international oceanographic conference. At the Brussels Maritime Conference of 1853, *uniform methods of making nautical and meteorological observations at sea were agreed upon. These increased the available data and made them more reliable. In* 1855 *Maury published* The Physical Geography of the Sea, *a summary of his findings.*

The chemistry of seawater was investigated by **Johan Georg Forchhammer** *(1794–1865) of Copenhagen, a professor of geology. Over a period of* 20 *years, Forchhammer analyzed surface samples of seawater brought to him by sailors from all over the globe. When he published his findings in* 1865, *he demonstrated that while the total salt content of seawater differs from place to place, the relative amounts of the various major salts remain constant. These findings together with those of the* Challenger *expedition (1872–1876), provided the nucleus of present understanding of the architecture of the oceans. Study of the physical mechanisms that bring these features into existence has been the main concern of physical oceanography ever since*[447].

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447 Since 1953, oceanographic research has been revolutionized due to a number of technological advances:

- "*Real-time*" *recording*: Prior to 1953, most instruments were mechanical devices. A program of sampling was laid out before the ship left port, technicians collected the data, and scientists ashore analyzed it months afterwards. Today, when bathimetry can be surveyed instantly in real time, the course of the ship can be modified instantly to accommodate, say, a newly discovered undersea mountain. Moreover, oceanographers had measured the temperature and salinity of sea water at widely spaced stations by lowering instruments on vertical wires suspended from ships. Now, they trail electronic temperature sensors behind the ship, so they could detect the boundaries between ocean currents as they crossed them.

- *Expansion of instrument capabilities*: With solid-state electronics instruments which are not so sensitive to the environment, oceanographers could conduct many kinds of research that were previously impossible.

- *On-board computers* are used to process many kinds of marine data that formerly could be analyzed only after return to port. Minute by minute and

The last volume of the Challenger Report appeared in 1895 — three years after ***Robert E. Peary*** *discovered that Greenland was an Island, not part of a polar continent; one year before a Norwegian explorer,* ***Fridtjof Nansen****, proved that no such continent existed; 14 years before Peary became the first man to reach the North Pole; 17 years before* ***Roald Amundsen****, another Norwegian explorer, reached the South Pole.*

Although the Challenger expedition is one of the great achievements in the history of scientific exploration, it was nonetheless a candle in a vast darkness. A quarter of a century later, oceanographers had gained a general picture of the earth's oceans; and another quarter of a century after that, oceanography entered upon a new era with the Meteor expedition of 1925–1927.

During the half-century from the Challenger to the Meteor, oceanographic research had consisted mostly of isolated and widely scattered observations. The emphasis was on the amount of territory covered rather than on systematic research. The most notable exceptions to this rule were the Norwegian studies of the North Atlantic and the Norwegian Sea that begun shortly after the turn of the century.

In the field of physical oceanography proper, major contributions came from the school of thought stimulated by ***V.F.K. Bjerkens****. In 1898 Bjerkens published a paper which provided a basis for determining the field of motion in the sea from measurements of the vertical and horizontal distributions of pressure. Currents and volumes of water moving in the oceans are difficult to measure directly because there are no convenient reference marks at sea that are assuredly at rest. The practical methods for applying dynamical principles*

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hour by hour the ships collect data and feed them into the computer. It can print out the ships' position, its speed and much other information, as one desires.

- *SCUBA gear and research submarines*. Marine geologists and biologists can make dives to personally examine at close range the phenomena about which they had previously speculated. Deep water submersibles with remote handling (for manipulating the environment outside the submersibles) are opening new lines of research, such as detailed mapping of the Mid-Atlantic rift.

- *Special types of research vessels* can drill in the deep-sea floor, or be partially flooded so it stands on end for the measurement of the motion of the sea.

- *Artificial satellites*, coupled to the ship's on-board computer, provide a navigational system. The sensors on these satellites are capable of measuring the temperature of the ocean surface, thereby mapping ocean currents almost instantly over enormous areas. Other uses of space technology will surely further revolutionize some oceanography and the utilization of the seas.

at sea were developed during the first quarter of the 20th century by ***Björn Helland-Hansen***, ***J.W. Sandström***, *and several other Norwegian, Swedish, and German oceanographers.*

II Physical Foundations of Dynamic Oceanography (1813–1969)

The important forces which drive the large-scale oceanic motions are the force of gravity ($\boldsymbol{g}$), the Coriolis force, pressure gradient force, and frictional forces ($\boldsymbol{F}$). (The centrifugal force of the earth's rotation is usually included in gravity.) Thus, the Eulerian vector equation for the acceleration of a fluid element relative to a co-rotating frame, is:

$$\frac{d\boldsymbol{V}}{dt} = \frac{\partial \boldsymbol{V}}{\partial t} + (\boldsymbol{V}\cdot\nabla\boldsymbol{V}) = (-2\Omega\boldsymbol{n}\times\boldsymbol{V}) + (-\alpha\nabla p) + \boldsymbol{g} + \boldsymbol{F}, \tag{1}$$

where $\boldsymbol{V} = (u, v, w)$ *is the velocity vector with components along* x *(east),* y *(north) and* z *(upward). Here* $\Omega = 0.729\times 10^{-4}$ *sec*$^{-1}$ *is the angular velocity of the earth's daily rotation,* $\boldsymbol{n}$ *is a unit vector pointing from south to north pole,* p *is the pressure* (dyn/cm^2), $\boldsymbol{F}$ *is force per unit mass, and* $\alpha = \frac{1}{\rho}$ = *specific volume in units* cm^3/gm. *Under conditions of* steady state $\left(\frac{\partial \boldsymbol{V}}{\partial t} = 0\right)$, *small advective terms* $(\boldsymbol{V}\cdot\nabla\boldsymbol{V} \doteq 0)$, *and small frictional force* $(\boldsymbol{F} = 0)$, *the motions are said to be* geostrophic *(earth-turned). In this case the Coriolis force and the pressure gradient force just balance each other. Consequently,*

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = (2\Omega\sin\phi)v, \qquad \frac{1}{\rho}\frac{\partial \rho}{\partial y} = (-2\Omega\sin\phi)u, \quad \frac{1}{\rho}\frac{\partial p}{\partial z} = g. \tag{2}$$

The above equations govern the motion of major ocean currents. These are of two general types: vertical and horizontal (both surface and deep). Lesser and variable currents may be caused by tides and storm conditions. Some currents are of short duration and cover only a small area; others, such as the great oceanic circulation systems, *are permanent.*

A few remarks are needed with regard to Eqs. (1) *and* (2)*:*

- *In the gravitational term,* $\boldsymbol{g} = (0, 0, -g)$ *represents the* apparent *gravitational acceleration, or the true (central) gravitational acceleration modified by the small 'centrifugal' contribution normal to the axis of the earth's rotation. The direction of* $\boldsymbol{g}$ *defines the local vertical; its magnitude varies throughout the ocean from its mean value of approximately 981* $\frac{cm}{sec^2}$ *by less than 0.3%, and for dynamical purposes it can be considered constant.*

- *The term* $\boldsymbol{F}$ *represents the resultant of all other forces acting on a unit volume of the fluid. The most important of these arises from molecular viscosity. In almost all oceanic circumstances where viscous effects are important, the water can be regarded as an isotropic, incompressible Newtonian fluid for which the stress tensor is given by*

$$\boldsymbol{T} = -pI + \mu(\nabla \boldsymbol{V} + \boldsymbol{V}\nabla), \tag{3}$$

and where μ *is the fluid's viscosity. The frictional force per unit volume in therefore (with* $\operatorname{div} \boldsymbol{V} = 0$*)*

$$\boldsymbol{F} = \mu \nabla^2 \boldsymbol{V} \tag{4}$$

If L *is the differential length scale of a given motion in which the velocity varies in magnitude by* U*, the ratio* $R = \frac{pUL}{\mu}$ *(known as the Reynolds number) represents the relative magnitudes of the inertial and viscous terms in the momentum equation. In many oceanic motions, the Reynolds number is very large, and the viscous form is often quite negligible over most of the field of motion.*

- *Using the vector identity* $\boldsymbol{V} \cdot \nabla \boldsymbol{V} = \nabla(\frac{1}{2}\boldsymbol{V}^2) + (\boldsymbol{\omega} \times \boldsymbol{V})$*, where* $\boldsymbol{\omega} = \operatorname{curl} \boldsymbol{V}$*, Eq. (1) becomes*

$$\frac{\partial \boldsymbol{V}}{\partial t} + 2(\boldsymbol{\Omega} + \boldsymbol{\omega}) \times \boldsymbol{V} + \frac{1}{\rho}\nabla p + \nabla\left(\frac{1}{2}\boldsymbol{V}^2\right) - \boldsymbol{g} = \boldsymbol{F}. \tag{5}$$

When ρ *is in effect constant, (5) goes into*

$$\rho\frac{\partial \boldsymbol{V}}{\partial t} = \rho \boldsymbol{g} + \mu\nabla^2 \boldsymbol{V} - \rho(\boldsymbol{\Omega} + \boldsymbol{\omega}) \times \boldsymbol{V} - \nabla(p + \frac{1}{2}\rho \boldsymbol{V}^2). \tag{6}$$

The Eulerian mass-acceleration is thus given as a balance between four forces. The term $\rho(\boldsymbol{\Omega} + \boldsymbol{\omega}) \times \boldsymbol{V}$ *can be called total vortex force.*

- *Sea water is a chemical solution; its density* $\rho = \rho(p, T, S)$ *where* p = *pressure,* T = *temperature and* S = *salinity (the mass of dissolved solids per unit mass of sea water). This dependence has no analytical form, but various empirical approximations. However, for an ordinary range of temperature and salinity encountered in the ocean, the equation of state is approximately given by*

$$\rho = 1 - aT - bS,$$

where a, b *are numerical constants.*

- *If in (2) we retain the Coriolis term we shall have*

$$\frac{\partial p}{\partial z} = g\rho(1 - \frac{2u\Omega}{g}\cos\theta)$$

With fast currents near the equator (e.g. zonal undercurrents of the Pacific and Atlantic Oceans flowing from west to east), $\theta \approx 0°$, $u \sim 150$ *cm/sec, and therefore* $\frac{2u\Omega}{g}\cos\theta \approx 2.3 \times 10^{-5}$, *which may be significant.*

- *In moving fluids where the velocity varies in space, frictional stresses are present as a result of momentum transfer between layers of different velocities. In the case of laminar flow the exchange of momentum is the result of molecular motion. However, if the fluid is stirred by some internal or external cause and individual layers are 'entangled' by macroscopic irregular displacements of water parcels, the rate of momentum exchange (as well as heat exchange, diffusion of dissolved solids, etc.) increase considerably. This is called turbulent flow. Turbulence, stirring, mixing and diffusion play a very important role in both oceanography and meteorology.*

The mechanical energy equation

Forming the scalar product of $\boldsymbol{V}$ *with the respective terms of (5) yields*

$$\rho\frac{\partial}{\partial t}\left(\frac{1}{2}\boldsymbol{V}^2\right) + \boldsymbol{V}\cdot\nabla p - \rho\boldsymbol{V}\cdot\boldsymbol{g} = \boldsymbol{V}\cdot\boldsymbol{F}. \tag{7}$$

Now, if ξ *measures the vertical displacement of a fluid element (measured upwards), then*

$$-\rho\boldsymbol{V}\cdot\boldsymbol{g} = \rho g w = \rho g\frac{d\xi}{dt} \tag{8}$$

and with use of the continuity equation $\frac{d\rho}{dt} + \rho\,\mathrm{div}\,\boldsymbol{V} = 0$, *we can express (8) as*

$$\frac{\partial}{\partial t}\left[\frac{1}{2}\rho\boldsymbol{V}^2 + \rho g\xi\right] + \mathrm{div}\left[\boldsymbol{V}\left(p + \frac{1}{2}\rho\boldsymbol{V}^2 + \rho g\xi\right)\right] = p\,\mathrm{div}\,\boldsymbol{V} + \boldsymbol{V}\cdot\boldsymbol{F}. \tag{9}$$

Since

$$\boldsymbol{\Sigma} = \boldsymbol{V}\left(p + \frac{1}{2}\rho\boldsymbol{V}^2 + \rho g\xi\right) \tag{10}$$

is the energy flux density vector, (9) *is just a statement that the rate of change of the Hamiltonian*

$$H = \frac{1}{2}\rho\boldsymbol{V}^2 + \rho g\xi \tag{11}$$

and the divergence of the energy flux density equal the rate of working in compressing the fluid and against frictional forces. If the fluid is incompressible ($\operatorname{div}\boldsymbol{V}=0$) *and inviscid* ($\boldsymbol{F}=0$)*, the energy balance is* $\frac{\partial H}{\partial t}+\operatorname{div}\boldsymbol{\Sigma}=0$. *Coriolis forces do no work, since their direction is always normal to the velocity* $\boldsymbol{V}$. *They can, however, influence the energy flux* indirectly *by contributing to the pressure variation in the fluid.*

In an incompressible Newtonian fluid $\boldsymbol{F}=\mu\nabla^2\boldsymbol{V}$*, so that the rate of working against viscous forces is*

$$\boldsymbol{V}\cdot\boldsymbol{F}=2\mu\boldsymbol{V}\cdot\operatorname{div}\boldsymbol{E}=2\mu\operatorname{div}(\boldsymbol{V}\cdot\boldsymbol{E})-\epsilon, \tag{12}$$

where $\epsilon=2\mu(\epsilon_{ii})^2$. *While* $2\mu\operatorname{div}(\boldsymbol{V}\cdot\boldsymbol{E})$ *can be interpreted as a* viscous energy flux*, the quantity* ϵ *(essentially* positive*) represents the rate of energy dissipation per unit volume by molecular viscosity.*

Summary

A complete set of basic equations of dynamic oceanography is the following ($\nu=\frac{\mu}{\rho}$)

$$\frac{D\boldsymbol{V}}{Dt}+2(\boldsymbol{\Omega}\times\boldsymbol{V})=\frac{1}{\rho}\nabla p-\nabla\Psi-\beta\nabla\boldsymbol{g}+\nu\nabla^2\boldsymbol{V} \quad \textit{conservation of momentum} \tag{13}$$

$$\frac{D\rho}{Dt}=\frac{\partial\rho}{\partial t}+\operatorname{div}(\rho\boldsymbol{V})=0 \quad \textit{conservation of mass} \tag{14}$$

$$\rho=f(p,T,S) \quad \textit{equation of state} \tag{15}$$

$$\frac{\partial(\rho T)}{\partial t}+\operatorname{div}(\rho T\boldsymbol{V})=-\operatorname{div}\boldsymbol{\Sigma}_T+q_T \quad \textit{conservation of heat} \tag{16}$$

$$\frac{\partial(\rho S)}{\partial t}+\operatorname{div}(\rho S\boldsymbol{V})=-\operatorname{div}\boldsymbol{\Sigma}_S+q_S \quad \textit{conservation of salt} \tag{17}$$

These are 7 scalar equations in the 7 unknown scalar functions ($\boldsymbol{V},p,\rho,T,S$)*, where* Ψ *is the gravitational potential,* T *is the temperature,* S *is the salinity,* $\boldsymbol{\Sigma}_T$ *is the flux of* T *due to heat conduction and diffusion, and* $\boldsymbol{\Sigma}_S$ *is the corresponding salinity flux. Note that since* $\operatorname{div}(\rho T\boldsymbol{V})=\rho T\operatorname{div}\boldsymbol{V}+\boldsymbol{V}\cdot\nabla(\rho T)$*, the advective term replaces the divergence term for incompressible ocean. If Fick's law holds,* $-\operatorname{div}\boldsymbol{\Sigma}_T=\kappa_T\nabla^2T$, $-\operatorname{div}\boldsymbol{\Sigma}_S=\kappa_S\nabla^2S$*, and* free convection *ensues.*

Note that although the law of conservation of the total energy *is not included explicitly, it is* implicit *in the flux vectors* $\boldsymbol{\Sigma}_T$ *and* $\boldsymbol{\Sigma}_S$. *Also* $\beta=-\frac{1}{p}\frac{\partial\rho}{\partial T}$

is the thermal expansion coefficient of the fluid. If $\rho = \rho(p)$ only, the first 5 equations constitute a complete set for the functions $(\boldsymbol{V}, \rho, p)$. In the first equation, incompressibility was assumed for the frictional forces. In Eqs. (16)-(17), q is the total internal source of heat and salinity respectively.

It is clear from (13)-(17) that the equations of motion are quadratically nonlinear, *i.e., contain products of dynamic variables. This implies that in principle it is not possible to simply superpose solutions of the equations. In physical terms, motions on one spatial scale interact with motions on other scales. There is therefore an* a priori *possibility that small-scale motions may influence the large-scale motions. There is in fact evidence that the small-scale motions, which appear sporadic on larger time scales, act to smooth and mix properties on the larger scales by processes analogous to molecular, diffusive transports. This will become apparent in the next section when the Reynolds stress is introduced.*

Nondimensional parameters

Let us return to Eq. (9) and associate with each term a characteristic dimension by means of the correspondence

Fluid velocity	$\boldsymbol{V}$	$\to$	U
Scale-time	t	$\to$	T
Coriolis parameter	$\boldsymbol{\Omega}$	$\to$	$f = 2\Omega \sin\theta$
Frictional force	$\nu\nabla^2\boldsymbol{V}$	$\to$	$\frac{\nu}{L^2}U \quad (\boldsymbol{\nabla} \to \frac{1}{L})$
Gravitational potential	Ψ	$\to$	$gH,$

where L is some lateral characteristic length, H is a vertical characteristic length, T is a characteristic time and U is a characteristic speed. Since each term in equations (13)-(17) is of the same dimension, the relative influence *of terms will be determined by dimensionless numbers. There are seven such numbers, each named after the person who first stressed its importance in some fluid system:*

$$
\begin{aligned}
F_r &= \textbf{Froude number} &&= \frac{\text{Inertial force}}{\text{Gravitational force}} &&= \frac{U^2}{gH} \\
R_0 &= \textbf{Rossby number} &&= \frac{\text{Inertial force}}{\text{Coriolis force}} &&= \frac{U}{fL} \\
R_e &= \textbf{Reynolds number} &&= \frac{\text{Inertial force}}{\text{Frictional force}} &&= \frac{\rho U L}{\mu} \\
E_k &= \textbf{Ekman number} &&= \frac{\text{Coriolis force}}{\text{Frictional force}} &&= L\sqrt{\frac{fp}{\mu_\nu}}
\end{aligned}
\qquad (18)
$$

$$E_u = \textbf{\textit{Euler}} \text{ number} \quad = \frac{\textit{Inertial force}}{\textit{Pressure gradient force}} \quad = \frac{\rho U^2}{|\nabla p|}$$

$$P_r = \textbf{\textit{Prandtl}} \text{ number} \quad = \frac{\mu}{\rho \kappa_T}$$

$$G_R = \textbf{\textit{Grashof}} \text{ number} \quad = \rho^2 \beta g L^3 \frac{T_1 - T_0}{\mu^2}$$

The relative magnitudes of the various forces and accelerations can be compared among each other by dimensional analysis. Ocean currents are slow enough that pressure can be approximated with high accuracy by the hydrostatic equation. Consequently, pressure gradients are produced primarily by slopes of the sea surface and variations of density within the interior of the ocean. Furthermore, the forces due to horizontal gradients of pressure are balanced primarily by the Coriolis forces. The remaining forces due to accelerations and friction are generally smaller than the pressure gradient and Coriolis forces, but can become important in some regions of the ocean.

By evaluating the coefficients R_0, R_e, F_r *for a given flow, we obtain an appreciation of the magnitudes of the forces involved and can decide which force needs to be taken into account for a satisfactory interpretation of the flow in terms of the equations. Conversely, we can examine the coefficients to determine the horizontal and vertical scales for which any given term of the equations becomes comparable to unity. For example, the ratio of the nonlinear accelerations to the Coriolis forces is given by the Rossby number,* R_0*. If we use the observation that steady velocities in the ocean do not exceed 2 or 3 m/sec, we can make the Rossby number comparable to unity only by assuming the current to be sharply confined horizontally (L small), or by assuming a current near the equator (f small). At mid-latitudes, the Rossby number approaches unity for horizontal scales of 20 to 30 km for the maximum velocities given above. Such conditions are approached only in concentrated current systems such as the Gulf Stream and Kuroshio. Another way to interpret the Rossby number is to note that the ratio* U/L *is the characteristic magnitude of the vertical component of relative vorticity,* $\partial U_u/\partial x_1 - \partial U_1/\partial x_2$*. Thus, the Rossby number can be considered as the ratio of the relative vorticity to the Coriolis parameter. Hence, if the relative vorticity approaches the Coriolis parameter, the Rossby number will be near unity and nonlinear accelerations will be of the same magnitude as the Coriolis forces.*

Near the turn of the century two other major steps were taken in the formulation of the modern point of view. Maury's observation of the close relationship between surface winds and ocean surface currents was given a physical explanation by ***Vagn Walfrid Ekman*** *(1874–1954, Sweden). During* 1905–1923 *he determined the ocean's theoretical response both to a steady wind and to an impulsive horizontally uniform wind, examining particularly* *the influence of the Coriolis force on the dynamical behavior of the ice and the upper layers of the ocean.*

Ekman showed not only that there should be a spiral effect (waters moving slower and further to the right with depth), but also that the wind-driven currents should extend only to a depth of about 200 *meters.*

Near the sea-bed, as the flow becomes influenced by friction *with the ocean bottom, the spiraling is such that the direction of flow moves leftward with depth (1905). He later (1923) extended his results to non-uniform winds, variable ocean-bottom topography, and variable latitude.*

It was found that major ocean currents *are basically produced by two factors: distribution of density and effect of wind stress on the sea surface. Variations in density are widely distributed due to* differential heating *and* evaporation*. They cause the waters to move both horizontally and vertically.*

Major ocean currents are set in motion by these variations and by the drag of the wind on the surface layers. Once the water mass begins to move, it is deflected due to the Coriolis force*, and the major surface current circulation is established. Upwelling and sinking of water masses are similarly caused by density differences and the blowing away or piling up of waters under the stress of* air currents*. Density gradients so light that they are difficult to measure, may nevertheless be sufficient to produce or to maintain ocean currents.*

The Coriolis effect, due to the earth's rotation, causes surface currents in the Northern Hemisphere to deflect 45° *to the right of the wind direction, and in the Southern Hemisphere,* 45° *to the left. At the depth at which the current speed is* $\frac{1}{23}$ *of the surface speed, the current moves exactly opposite to the direction at the surface, a phenomenon called the Ekman spiral.*

The surface circulation of the ocean is a direct result of the circulation of the atmosphere, where the pattern of gyres results from the winds and the geography of the continents. While the oceans are separated in the North, the free passage around Antarctica permits a great current to flow from west to east around the globe.

The rate of transport of water by the major ocean currents are (in 10^6 $\text{m}^3\text{sec}^{-1}$*): Antarctic current (200); Gulf Stream — Florida Straits (25); Gulf Stream — Cape Hatteras (100); Kuroshio (50); North Pacific to Arctic Ocean (0.7); All the world's rivers (1); Flux of atmospheric water vapor across a parallel of latitude (0.7).*

Thus, the ocean currents transport a tremendous amount of water and keep the surface water of the sea relatively well mixed. In closed seas, such as the Mediterranean Sea and the Red Sea, an arid atmosphere causes great evaporation: surface water flows in to replace the loss. In humid climate conditions, the system is reversed due to heavy rainfall.

In the Atlantic, deep water circulation takes place: below Greenland, the warm, highly saline waters of the Gulf Stream are cooled, the density increases, and the water sinks to intermediate level where it is displaced to the south. In the tropical regions of the North Equatorial Current, the surface waters are in turn heated, evaporated, and displaced by wind stress. Here the deep waters rise to the surface, completing the cycle[448].

1872–1873 CE **Elias Ney** (1844–1897, England). Explorer of Asia. Led eight major expeditions to Central Asia, all of them hazardous. His most famous journey begun in September 1872, when with a Chinese servant, a camel driver, an interpreter, six camels and two ponies he set out to cross the Gobi Desert from a location NW of Peking (114°E; 42°N). Traveling NW across Mongolia, he reached Uliastay, and then crossed the frozen Lake Haar Us Nuur to reach Hovd. From there he traversed the Altai range to Biysk in Siberia, on the upper waters of the Ob. Finally a horse-drawn sleigh ride in the depth of the Siberian winter took him to Nijni-Novgorod. He traveled altogether some 8000 km.

Ney was born in 1844, the son of Jewish parents. In his time he ranked with Stanley (both received the Royal Geographical Society's Founders Medal for outstanding work in 1873), but is now almost forgotten.

1872–1884 CE **Georg** (**Ferdinand Ludwig Philipp**) **Cantor**[449] (1845–1918, Germany). A great and revolutionary mathematician, whose work on set theory and the theory of the infinite created a whole new field of mathematical research and exerted profound influence on most branches of contemporary mathematics — especially the foundation of mathematics and mathematical logic. In his papers he created an arithmetic of transfinite numbers,

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[448] Movement of the deeper water masses was unknown until deep-current meters, buoys, and radioactive trace-element detectors came into use. In 1952, the *Cromwell Current* in the equatorial Pacific was discovered by scientists from the Scripps Institute of Oceanography. This great current, about 400 kilometers wide, sweeps thousands of kilometers in an eastward direction at a speed of 6.5 km/hr.

[449] For further reading, see:

- Dauben, J.W., *George Cantor*: *His Mathematics and Philosophy of the Infinite*, Harvard University Press: Cambridge, 1979, 404 pp.

analogous to the arithmetic of finite numbers, eliminating all metaphysical elements from the foundations of the exact sciences. Historians of mathematics have ranked his work as "one of the most original contributions to mathematics in the past 2500 years".

Cantor's early interests were in number theory, indeterminate equations and trigonometric series. The subtle theory of trigonometric series seems to have inspired him to look into the foundations of analysis.

Ever since the days of Zeno, men had been talking about infinity, in theology as well as mathematics, but no one before 1872 had been able to tell precisely what he was talking about. **Cauchy** and **Weierstrass** saw only paradoxes in their attempts to identify infinity in mathematics. In fact, there was a considerable '*horror infinite*' and mathematicians were reluctant to accept '*completed infinity*'. Cantor created a theory of the actual infinite which by its apparent consistency demolished the Aristotelian and scholastic "proofs" that no such theory could be found.

Some of Cantor's ideas, simply stated, are as follows:

(1) A transfinite number, unlike a finite number, can *always* be put into 1–1 (one to one) correspondence with some *part* of itself [e.g.: set of all integers with the set of all even integers].

(2) Although the set of rational numbers is *dense* (i.e. between any two rational numbers one can 'pack in' an infinity of other rational numbers), the set of all rationals can be rearranged in such a way that they can be put into 1–1 correspondence with the set of all integers, which is a *discrete set*. [A denumerable infinity is designated by the Hebrew letter $\aleph_0$. This is the "power" (cardinality) of the set of positive integers and also the "power" of the positive rationals.]

(3) Any linear continuum, no matter of what length, can be put in 1–1 correspondence with the line-segment from 0 to 1, i.e. the set of real numbers between 0 and 1 is equivalent to the set of all real numbers[450].

(4) The set of real numbers between 0 and 1 (known as a *continuum*) is not countable, in the sense that it cannot be put in a 1–1 correspondence with the aggregate of natural numbers. Its power equals to the power of the set of *all* real numbers, and is denoted by C.

(5) A 1–1 correspondence can be set up between points on a one-dimensional continuum and any finite-dimensional continuum, i.e. there are no more points in a square or cube than in a line segment $[C^n = C]$.

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[450] The mapping $z' = [1 + e^{-z}]^{-1}$ maps $(-\infty, \infty)$ onto $(0, 1)$; the inverse transformation is $z = \log_e \frac{z'}{1-z'}$.

(6) Since any finite set of m elements has 2^m subsets, one denotes, for any set A of power p, the power of the *power set* of A (set of all subsets) by 2^p. It can be shown that $2^{\aleph_0} = C$, and in fact $N^{\aleph_0} = C$, when $N \geq 2$ is any finite integer.

(7) As in ordinary arithmetic, numbers are of two kinds: *cardinal* and *ordinal* [e.g. cardinal numbers are $1, 2, 3, 4, \ldots$; ordinal numbers are $1^{st}, 2^{nd}, 3^{rd}, 4^{th}, \ldots$]. So, in the arithmetic of transfinites as well, $\aleph_0$ and C are *cardinal transfinite numbers.* Cantor conjectured that there is no cardinal number greater than $\aleph_0$ and smaller than C. This is known as the *continuum hypothesis.*

Cantor's ideas threw new light on the concept of *dimension.* This concept presents no great difficulty as long as one deals only with simple geometrical figures such as points, areas, lines, triangles, and polyhedra. A single point or any *finite* set of points has dimension zero, a line segment is one-dimensional, and the surface of a triangle or of a sphere is two-dimensional.

The set of points in a solid cube is three-dimensional. But when one attempts to extend this concept to more general point sets, the need for a precise definition arises. What dimension should be assigned to the point set R consisting of all points on the x-axis whose coordinates are *rational* numbers? The set of rational points is dense on the line and might therefore be considered to be one-dimensional, like the line itself. On the other hand, there are irrational gaps between any pair of rational points, as between any two points of a finite point set, so that the dimension of the set R might also be considered to be zero.

The problem becomes even more complex as one tries to assign a dimension to the following curious point-set, first considered by Cantor and known as the *Cantor set* C.

From the unit segment remove the middle third, consisting of all points x such that $1/3 < x < 2/3$. Call the remaining set of points C_1. Now from C_1 remove the middle third of each of its two segments, leaving a set which we call C_2. Repeat this process by removing the middle third of each of the four intervals of C_2, leaving a set C_3, and proceed in this manner to form sets C_4, C_5, $C_6, \ldots$. Denote by C the set of points on the unit segment that are left after all these intervals have been removed, i.e. C is the set of points common to the infinite sequence of sets C_1, $C_2, \ldots$. Since one interval, of length $1/3$, was removed at the first step; two intervals, each of length $1/3^2$, at the second step; etc.; the total length of the segments removed is

$$1 \cdot \frac{1}{3} + 2 \cdot \frac{1}{3^2} + 2^2 \cdot \frac{1}{3^3} + \cdots = \frac{1}{3}\Big[1 + \big(\frac{2}{3}\big) + \big(\frac{2}{3}\big)^2 + \ldots\Big].$$

The infinite series in parentheses is a geometrical series whose sum is $1/(1-2/3) = 3$; hence the total length of the segments removed is 1. Still there remain points in the set C. Such, for example, are the points $1/3$, $2/3$, $1/9$, $2/9$, $7/9$, $8/9, \ldots$, by which the successive segments are trisected. As a matter of fact it is easy to show that C will consist precisely of all those points x whose base 3 decimal expansions can be written in the form

$$x = \frac{a_1}{3} + \frac{a_2}{3^2} + \frac{a_3}{3^3} + \cdots + \frac{a_n}{3^n} + \ldots,$$

where each a_i is either 0 or 2, while the triadic expansion of every point removed will have at least one of the numbers a_i equal to 1.

What shall be the dimension of the set C? The diagonal process used to prove the non-denumerability of the set of all real numbers can be so modified as to yield the same result for the set C. It would seem, therefore, that the set C should be one-dimensional. Yet C contains no complete interval, no matter how small, so that C might also be thought of as zero-dimensional, like a finite set of points. In the same spirit, we might ask whether the set of points of the *plane*, obtained by erecting at each rational point or at each point of the Cantor set C a segment of unit length, should be considered to be one-dimensional or two-dimensional[451].

After Cantor, mathematicians based set theory on abstract postulate systems. One such axiomatization, for example, is due to **Ernst Zermelo** (1871–1956, Germany, 1922) and **Abraham Halevi Fraenkel** (1891–1965, Israel, 1927). Then, in 1938, **Kurt Gödel** demonstrated that one can safely assume Cantor's 'continuum hypothesis' as an *additional postulate* in set theory, i.e. he proved that the continuum hypothesis is consistent with the Zermelo-Fraenkel

[451] An inductive definition of dimensionality is also contained implicitly in **Euclid**'s *Elements*, where a one-dimensional figure is something whose boundary consists of points, a two-dimensional figure one whose boundary consists of curves, and a three-dimensional figure one whose boundary consists of surfaces.

Poincaré (1912) first called attention to the need for a deeper analysis and a precise definition of the concept of dimensionality. Poincaré observed that the line is one-dimensional because we may separate any two points on it by cutting it at a single point (which is of dimension 0), while the plane is two-dimensional because in order to separate a pair of points in the plane we must cut out a whole closed curve (of dimension 1). This suggests the inductive nature of dimensionality: a space is n-dimensional if any two points may be separated by removing an $(n-1)$-dimensional subset, and if a lower-dimensional subset will not always suffice. The introduction of *fractal geometry* by **Mandelbrot** (1977) finally made the theory applicable to many physical problems.

axioms. The last word however, had not been said, since Gödel had neither proven the continuum hypothesis[452] nor shown that it is indemonstrable.

In 1963 **Paul Joseph Cohen** (b. 1934, U.S.A.) has shown that Cantor's hypothesis is independent of the other axioms of set theory. Hence, the continuum hypothesis can be assumed or denied depending on the applications one has in mind, i.e. there are at least two types of mathematics possible — one that holds that the continuum hypothesis is true, and another in which it is false.

Cantor was born in St. Petersburg, Russia, of pure Jewish descent on both sides [though his father converted to Protestantism and his mother had been born a Catholic]. In 1856 he moved with his parents to Frankfurt, Germany. He rejected his father's suggestion of preparing for a career in engineering in favor of concentrating on philosophy, physics and mathematics. He studied at Zürich, Göttingen and Berlin, where he came under the influence of **Kummer** and **Weierstrass** and took his Ph.D. degree in 1867.

In 1874 he published his path-breaking paper on the theory of infinite sets. In the same year Cantor married Vally Guttman; six children were born of the marriage. The following 10 years were the period of his most original productivity. All his active professional career was spent at the University of Halle, a distinctly third-rate institution, where he was appointed full professor in 1879. He never achieved his ambition of professorship in Berlin, which at that time was the highest German distinction. It was **Kronecker** who blocked his appointment in Berlin[453] and was instrumental in rejecting the publication of Cantor's papers in Crelle's Journal. In fact, Kronecker regarded Cantor's ideas as a dangerous type of mathematical insanity, and attacked the hypersensitive author vigorously and viciously with every weapon that came to his hand. The tragic outcome was that Kronecker's attack broke the creator of the theory, who died in a mental hospital in Halle.

David Hilbert (1862–1943), himself one of the greatest mathematicians of recent times, considered Cantor's achievement to be: "*the most wonderful flowering of the spirit of mathematics and indeed one of the greatest achievements of human reason*" (1926).

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[452] The *continuum hypothesis* problem was the first of **Hilbert**'s famous 23 problems delivered to the Second International Congress of Mathematicians in Paris (1900).

[453] Kronecker's vicious animosity toward Cantor was basically very personal. It was motivated by a combination of jealousy and fear, disguised under the hypocritical cover of an academic controversy and enhanced by the fact that both belonged to an intellectual elité of an assimilated convert minority.

Infinity, Transfinity and Set Theory[454]

The idea of infinity has been the subject of deep thought from the time of the Greeks. **Zeno of Elea** *(ca 450 BCE), with his famous 'paradoxes', made an early major contribution. Other thinkers who had adduced ideas on the concept of infinity include* **Aristotle**, **Descartes**, **Berkeley** and **Leibniz**. **Albert of Saxony** *in his Questiones subtilissime in libros de celo et mundi (1365) proves that a beam of infinite length has the same volume as 3-space. He proves it by sawing the beam into imaginary pieces which he then assembles into successive concentric shells which fill space. Thus, by the Middle Ages, discussion of the infinite had led to comparisons among infinite sets of objects.* **Nicolas of Cusa** *(1440) studied the infinitely large and the infinitely small and had an intuitive feel for the procedure of the limit.*

By the beginning of the 19th century a clear distinction had been established between analysis (the study of infinite processes) and algebra, which deals with operations on discrete entities such as the natural numbers and polynomials. A major objective of much of the 19th century mathematical effort was to unify — or, at any rate, to build bridges between — these two branches of mathematics. This endeavor was termed 'the arithmetization of analysis'. It was realized that the prime task was to construct a sound logical foundation of the real number system. Although the basic concepts of analysis — function, continuity, limit, convergence, infinity and so on — were progressively clarified and refined during the first half of the 19th century, mathematicians failed to consider the precise structure and properties of the real numbers. Even **Cauchy** *lacked the understanding of the structure of the number system. Indeed — the theory of the arithmetic continuum was needed.*

A step in the direction of an improved understanding of irrational numbers was the mid-19th century work on algebraic and transcendental numbers[455].

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454 For further reading, see:

- Aczel, A.D., *The Mystery of the Aleph*, Washington Square Press, 2001, 258 pp.
- Kaplan, R. and E. Kaplan, *The Art of the Infinite*, Oxford University Press, 2003, 324 pp.
- Lieber, L.R., *Infinity*, Reinhart and Company, 1953, 359 pp.

455 Real numbers can be divided into *algebraic* and *transcendental* numbers. An algebraic number is defined as a number which is a root of a polynomial equation

The interest in this distinction was heightened by the 19th-century work on the solution of equations, because this work revealed that not all irrationals could be obtained by finite sequences of algebraic operators on rational numbers[456]*. This grew out of the question of whether there are indeed any transcendental numbers at all. In 1844,* ***Liouville*** *answered the question in the affirmative by actually constructing such numbers. He proved, e.g. that all numbers of the form*

$$\frac{a_1}{10^{1!}} + \frac{a_2}{10^{2!}} + \frac{a_3}{10^{3!}} + \cdots = 0.a_1a_2000a_3\ldots$$

where a_i are arbitrary integers in the range 0–9, are nonalgebraic, and therefore transcendental.

B. Bolzano *considered sets with the following definition:*

"An embodiment of the idea or concept which we can conceive when we regard the arrangement of its parts as a matter of indifference".

Bolzano defended the concept of an infinite set. At this time many believed that infinite sets could not exist. He gave examples to show that, unlike for finite sets, the elements of an infinite set could be put in one-to-one correspondence with elements of one of its proper sets. This idea eventually came to be used in the definition of a finite set. It was with ***Cantor's*** *work, however, that set theory came to be put on a proper mathematical basis.*

In 1873 ***Hermite*** *proved that e is transcendental; and in 1882* ***Lindemann*** *did the same for π. Finally in 1934,* ***A. Gelfond*** *discovered that if a is an algebraic number (not equal to 0 or 1) and b is an algebraic irrational number, then a^b is transcendental (e.g. $3^{\sqrt{2}}$). However, the Mascheroni-Euler constant*

$$\gamma = \lim_{n\to\infty}(1 + \frac{1}{2} + \cdots + \frac{1}{n} - \log n) = 0.577216\ldots$$

is not known to be rational, irrational, algebraic or transcendental.

Dedekind *(1858) realized that the system of real numbers lacked a firm logical foundation: the Greeks, with their geometrical predilections, identified real numbers (rational and irrational) with line segments.*

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of the form: $a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n = 0$, where n and the a_i are integers. A transcendental number is, therefore, any irrational number that is not algebraic (a rational number is also algebraic, of course). It can be proved that even if a_i are any algebraic numbers (as opposed to integers or rationals), all roots of $\sum_{i=0}^{n} a_ix^{n-i}$ are, nevertheless, algebraic.

[456] Nor could all algebraic numbers be expressed as finite successions of arithmetical operations upon integers, *even* when these operations include the radical operations $\sqrt[n]{\quad}$, n integer.

They tacitly assumed that any such number could be represented by a unique point on an infinitely extended straight line with a specific origin. The Greeks, however, while accepting irrational geometric entities (such as the diagonal of the unit square), could not accept the concept of irrational numbers, because it was less intuitive and required confrontation with the concept of infinity, whether by sequences, decimals, or continued fractions.

Western mathematics seemed to follow this trend and until the 19th century this seemed to be a good reason for considering geometry to be a better foundation continuum for mathematics than arithmetic. Then the problems of geometry came to a head with the advent of non-Euclidean geometry, and mathematicians began to fear geometric intuition as much as they had previously feared infinity.

***Dedekind** then set out to construct irrational numbers from scratch using sets of rationals. The number $\sqrt{2}$ is determined by the two sets of positive rationals:*

$$L_{\sqrt{2}} = \text{ all rational numbers } r \text{ whose square } r^2 < 2$$

$$U_{\sqrt{2}} = \text{ all rational numbers } r \text{ whose square } r^2 > 2$$

He decided to identify $\sqrt{2}$ with this pair of sets. In general, any partition of the positive rationals into sets L, U such that any member of L is less than any member of U and elements of L can get arbitrary close to elements of U, defines a positive real number. This idea (known as a Dedekind cut) gives a complete and uniform construction of all real numbers, or points on a line, using just rationals. It is an explanation of the continuous in terms of the discrete, finally resolving the fundamental conflict in Greek mathematics.

The assumption that the points on a line can be put in one-to-one correspondence with the real numbers is now known as the Dedekind-Cantor axiom. It must be realized that the logical definition of an irrational number is rather sophisticated — being not just a single symbol or a pair of symbols but an infinite collection.

Finally, Dedekind's theory shows that the 'arithmetic continuum' of real numbers is closed under infinite processes (Dedekind theorem): with real numbers we reach, as it were, the end of the road.[457]

Although Dedekind retained the Greek geometrical model of the number system as an aid to thought and exposition, the aim of most 19th century

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[457] However, it turned out there are *other* roads via which infinite sets of rationals may be used to define a continuum; these result in the so-called *p-adic number systems*, studied by **Hensel** and others.

mathematicians was to exclude geometrical considerations altogether, to base virtually the whole of mathematics on the concept of number. Broadly speaking, this objective had been achieved by the mid 20th century, although some foundational difficulties (e.g. with the logical basis of set theory) still remain. The next step in the erection of foundations for the number system was the definition and deduction of the properties of the rational numbers. ***Peano*** *(1889) began this process with five axioms for the* natural numbers:

- *1 is a natural number*
- *1 is not the successor of any other natural number*
- *Each natural number a has a successor*
- *If the successors of a and b are equal, then so are a and b*
- *If a set S of natural numbers contain 1, and if when S contains any number a it also contains the successor of a, then S contains all the natural numbers (axiom of* mathematical induction*)*

On these axioms he built all the familiar properties of natural numbers, and then established the properties of the negative whole numbers and the rational numbers as ordered pairs *of integers. Again, suitable definitions of the operations of addition and multiplication of pairs lead to the usual properties of the rational numbers.*

Thus, once the logical approach to the natural numbers was attained, the problem of building up the foundations of the real number system was completed.

Sets

Dedekind seemed to have settled the ancient problem of explaining the continuous *in terms of the* discrete*, but in penetrating as far as he did, he also uncovered deeper problems. The central problem is the relationship between two concepts:* completeness *and* countability*. To this end,* ***Cantor*** *(1874) introduced the notion of a* set*, which is one of the basic primitive mathematical concepts which does not lend itself to an accurate definition.*

Set is the name for an aggregate, ensemble, or collection of objects that are combined under a certain criterion or rule, e.g. the set of planets of our solar system, the set of all roots of a given equation, the set of all natural numbers, *the set of all points of a line, etc. The mathematical discipline that studies general properties of sets, i.e. properties that do not depend on the nature of the constituent objects, is called the theory of sets. The ideas and concepts of this theory penetrated into all branches of mathematics and changed its face entirely. It is of particularly great significance for the theory of functions of real variable.*

A countable *set is one that can be put in one-to-one correspondence with the set of all natural numbers $N = \{1, 2, 3, 4, \dots\}$. If both sets are infinite and such a correspondence can be set up, then we say that they have the same cardinality. If not, then we say that one of them contains more elements than the other, or that one has a* greater *cardinality than the other.*

Cantor discovered that the set of rationals *and the set of* algebraic numbers *are countable.*[458]*He proved, however, that the set of all real numbers (equivalent to the set of all points of the segment $0 < x < 1$) is not countable.*[459] *Thus the non-countability of the continuum was established. Since the real numbers are uncountable and the algebraic numbers are countable, there must be transcendental irrationals – and in fact all reals but a subset of vanishing small relative size must be transcendental! This is Cantor's nonconstructive existence proof.*

Having demonstrated the existence of infinite sets with the same size (cardinality) and different sizes, Cantor introduced the theory of cardinal and ordinal numbers (1879–1884). He expressed the size (or 'power' as he called it) of an infinite set by means of a transfinite number[460]. *He started with the infinite set of the natural numbers (i.e. the positive integers), and denoted its 'size' by the transfinite number $\aleph_0$, which is the cardinal number of this set. Since the real numbers cannot be put into one-to-one correspondence with*

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[458] To the layman the former may seem rather counterintuitive for the following reason: although the integers consist an infinite set, one cannot "pack in" any integers *between* two successive integers. Yet one *can* "pack in" an *infinity* of other rational numbers *between any two rationals* by simply taking the arithmetic mean between them and repeating the process indefinitely [e.g. take $\frac{1}{2}$ and $\frac{1}{3}$; the first average is $\frac{5}{12}$; the second mean is $\frac{1}{2}(\frac{1}{2} + \frac{5}{12}) = \frac{11}{24}$ and so on.] The rationals are thus called a *dense* set. Intuitively, there are infinitely 'more' rationals than there are integers, yet a 1:1 correspondence is possible between the two sets — e.g. by a lexicographical ordering of positive rationals $\frac{m}{n}$ (m, n positive) by $m + n$ and m, followed by "pruning" from the ordered list all rationals in which m and n possess a common factor.

[459] E.g. by means of the homeomorphism $x \to \tan \pi(x - \frac{1}{2})$.

[460] This theory is *distinct* from the concept of *infinity* wherein one speaks of a *variable becoming* infinitely large in a limit.

the natural numbers, the set of real numbers must have a greater cardinal number which is denoted by C (first letter of the word continuum). Thus $C > \aleph_0$. In Cantor's theory of sets, there is a whole hierarchy of transfinite numbers. Thus:

- *The power of the set of positive rational numbers is also $\aleph_0$.*

- *Since $k + \aleph_0$ is also denumerable*[461]*, we can write $k + \aleph_0 = \aleph_0$ ($k > 0$ integer).*

- *Since $k\aleph_0$ is also denumerable, we can write $k\aleph_0 = \aleph_0$, where k is any positive integer (e.g. the union of the sets of even and odd integers, each countable, is just the set of integers, also countable).*

- *We construct the table*

$\rightarrow i$

$j \downarrow$	1	2	3	4	5	6	...
1	1	2	4	7	11		
2	3	5	8	12			
3	6	9	13				
4	10	14					
5	15						
⋮							

$$i = 1, 2, \ldots, \infty$$
$$j = 1, 2, \ldots, \infty$$

where the entries in the table are assigned natural numbers along the consecutive diagonals, each starting from the upper right and sweeping toward the lower left. Thus, any pair of natural numbers is uniquely associated with a definite box (ij). Clearly, the totality of such pairs is represented by the numbered boxes, and this set of boxes is countable. Hence we see that

$$\aleph_0 \cdot \aleph_0 = \aleph_0$$

In general $\aleph_0^n = \aleph_0$ (n = finite positive integer).

- *Any real number (say between 0 and 1) can be written as the base-2 decimal fraction $0.a_1a_2a_3a_4\ldots$. Each a_i can be filled in by any of the two binary digits 0 or 1. Hence $2^{\aleph_0} = C$.*

Another way of demonstrating this is as follows: Let S be a finite set, and let its number of elements be n. The number of distinct subsets is 2^n.

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[461] e.g. $\begin{array}{ccccccccc} \{a\} & \bigcup & \{ & 1, & 2, & 3, & 4, & \ldots & \} \\ \downarrow & & & \downarrow & \downarrow & \downarrow & \downarrow & & \\ 1 & & & 2, & 3, & 4, & 5, & \ldots & \end{array}$

Extending this notion to infinite sets leads to the result (or actually, definition) that the set of all subsets (known as the power set) of N (the set of natural numbers) has cardinality $2^{\aleph_0}$. But this also equals the cardinality of the reals in the interval $(0, 1)$, since each such real number is uniquely represented by the set of its binary digits that equal 1 and these sets are precisely the distinct subsets of the set of decimal positions to the right of the decimal point – i.e., the subsets of $N = \{1, 2, 3, \dots\}$. Thus we again obtain $C = 2^{\aleph_0}$.

Furthermore, $C \cdot C = 2^{\aleph_0} \cdot 2^{\aleph_0} = 2^{2\aleph_0} = 2^{\aleph_0} = C$, and in general

$$C^n = C \qquad (n \text{ any positive integer})$$

The geometrical interpretation of this statement is that the 'number' of real points in a unit square (or unit hypercube of any dimension) is the same as the 'number' of points in one of its sides. Indeed, it is easy to demonstrate that there exists a 1-1 correspondence between the points on a line segment and the points in a square. Likewise, a 1-1 correspondence can be set between points on a line segment and points in a cube (C^3), and in fact in an n-dimensional continuum C^n for any $n > 1$.

- *A 1-1 correspondence can be set up between all points of a line segment and any part of itself. This equivalence is demonstrated analytically via the transformation $z' = a + (b - a)z$, where $0 \le z \le 1$ and $0 \le a \le 1$, $0 \le b \le 1$, which maps $[0, 1]$ onto $[a, b]$. Likewise, the transformation $z' = \frac{e^z}{1+e^z}$ transforms z in the range $-\infty < z < +\infty$ to $0 < z' < 1$. This means that the set of real numbers between 0 and 1 is equivalent (in its cardinality) to the set of all real numbers.*

- $$C^{\aleph_0} = (2^{\aleph_0})^{\aleph_0} = 2^{\aleph_0 \cdot \aleph_0} = 2^{\aleph_0} = C$$

means that a continuum of a denumerable (countable) infinity of dimensions (such as e.g. the set of all Fourier Series on a real interval) still has the same 'power' as a one dimensional continuum.

- $${\aleph_0}^{\aleph_0} = 2^{\aleph_0} = C$$

Ordinal numbers

The concept of infinity is perplexing to mathematicians and nonmathematicians alike. The ancient Greek philosophers and mathematicians were *quite wary of infinities except potential infinities. Their lines were never completed, always what we call line segments. Aristotle distinguished two kinds of infinity. One can be described by the unlimited extent to which a line segment can be extended. This is a potential infinity, an unboundedness. Aristotle's other infinity can be found within a line segment since that line segment can be divided without bound. Again a potential infinity.*

Through the centuries actual infinities remained suspect. Calculus (infinitesimal analysis) would probably have developed sooner had actual infinities been accepted. Actual infinities gained slow acceptance. For instance, completed lines became standard objects in geometry. Infinity in calculus remained a problem, though, until Cauchy found a way to define limits as a foundation for calculus.

***Cantor** discovered that there were different kinds of infinities in set theory: one was that of the* cardinal numbers *(sizes of sets). The other is that of* ordinal numbers *(counting numbers: first, second, third, etc.). For finite sets, they are the same. They differ, however, for infinite sets.*

A set is simply ordered *if an ordering relation ("$<$") can be defined on it such that any two elements have a definite order — i.e., given m_1 and $m_2 \neq m_1$, either m_1 precedes m_2 or m_2 precedes m_1; the notation is $m_1 < m_2$ or $m_2 < m_1$. Further, if $m_1 < m_2$ and $m_2 < m_3$, then simple order also implies $m_1 < m_3$; that is, the order relationship is* transitive. *An ordinal number of a simply ordered set M is the* order type *of the order in the set.*

Two ordered sets are similar if there is a 1-1 correspondence between them and if when m_1 corresponds to n_1 and m_2 corresponds to n_2 and $m_1 < m_2$, then $n_1 < n_2$ (and vice versa). Two similar sets have the same type of ordinal number. The algebra of ordinal numbers is best expounded in the following words of Morris Kline.[462]

> *"The ordinal number of the set of positive integers* in their natural order *is denoted by ω. On the other hand, the set of positive integers in decreasing order, that is $\ldots, 4, 3, 2, 1$ is denoted by ${}^*\omega$. The set of integers with zero in the usual order has the ordinal number ${}^*\omega + \omega$.*
>
> *Next Cantor defined the addition and multiplication of ordinal numbers. The sum of two ordinal numbers is the ordinal number of the first ordered set plus the ordinal number of the second ordered*

[462] *Mathematical Thought from Ancient to Modern Time*, pp. 1001–1002 (Oxford University Press, 1972).

set taken in that specific order. Thus, the set of positive integers followed by the first five integers, that is,

$$1, 2, 3, \ldots, 1, 2, 3, 4, 5,$$

has the ordinal number $\omega + 5$. Also the equality and inequality of ordinal numbers is defined in a rather obvious way.

He next introduced the full set of transfinite ordinals, partly for their own value and partly to precisely define higher transfinite cardinal numbers. To introduce these new ordinals he restricted the simply ordered sets to well-ordered sets. *A set is* well-ordered *if it has a first element in the ordering and if every subset has a first element*[463]. *There is a hierarchy of* ordinal numbers *and* cardinal numbers. *In the first class, denoted by Z_1 , are the finite ordinals*

$$1, 2, 3, \ldots.$$

In the second class, denoted by Z_2 are the ordinals

$$\omega, \omega+1, \omega+2, \ldots, 2\omega, 2\omega+1, \ldots, 3\omega, 3\omega+1, \ldots \omega^2, \omega^3, \ldots, \omega^\omega, \ldots$$

Each of these ordinals is the ordinal of a set whose cardinal number is $\aleph_0$.

The set of ordinals in Z_2 has a cardinal number. The set is not denumerable and so Cantor introduces a new cardinal number *$\aleph_1$ as the cardinality of the set Z_2. $\aleph_1$ is then shown to be the next cardinal after $\aleph_0$. The ordinals of the third class, denoted by Z_3 are*

$$\Omega, \Omega + 1, \Omega + 2, \ldots, 2\Omega, \ldots.$$

These are the ordinal numbers of the well-ordered sets, having $\aleph_1$ elements. However, the set of ordinals Z_3 has more than $\aleph_1$ elements, and Cantor denoted the cardinal number of the set Z_3 by $\aleph_2$. This hierarchy of ordinals and cardinals can be continued indefinitely.

Now, Cantor had also shown that given any set, it is always possible to create a new set, the set of subsets (power set) of the given set, *whose cardinal number is larger than that of the given set. If $\aleph_0$ is the given set, then the cardinal number of its power set is $2^{\aleph_0}$. As noted above, Cantor proved that $2^{\aleph_0} = C$, where C is the cardinal number of the continuum. On the other hand he*

[463] *Zermelo* proved that in the version of Zermelo-Fraenkel set theory where the *axiom of choice* is included, every set if well-ordered.

introduced $\aleph_1$ through the ordinal numbers and proved that $\aleph_1$ is the next cardinal after $\aleph_0$.

Hence $\aleph_1 \leq C$, but the question naturally arises as to whether $\aleph_1 = C$. The conjecture that this holds, known as the continuum hypothesis*, Cantor, despite arduous efforts, could neither prove nor disprove.*

For general sets M and N it is possible that M cannot be put into one-to-one correspondence with any subset of N and N cannot be put into one-to-one correspondence with a subset of M. In this case, though M and N have cardinal numbers α and β, say, it is not possible to say that $\beta = \alpha$, $\alpha < \beta$, or $\alpha > \beta$. That is, the two cardinal numbers are not comparable.

For well-ordered sets, Cantor was able to prove that this situation cannot arise. But it seemed paradoxical that there should be non-well-ordered sets whose cardinal numbers cannot be compared. But this problem, too, Cantor could not solve.

***Ernst Zermelo** (1871–1953) took up the problem of what to do about the comparison of the cardinal numbers of sets that are not well-ordered. In 1904 he proved, and in 1908 gave a second proof, that* every set can be well-ordered *(in some rearrangement). To construct the proof he had to use what is now known as the* axiom of choice *(Zermelo's axiom), which states that given any collection of nonempty, disjoint sets, it is possible to choose just one member from each set and so make up a new set.*

The axiom of choice, the well-ordering theorem, and the fact that any two sets may be compared as to size (that is, if their cardinal numbers are α and β, either $\alpha = \beta$, $\alpha < \beta$, or $\alpha > \beta$) are all equivalent principles."

*The issue of whether there are any transfinite numbers between $\aleph_0$ and C was finally resolved by **Paul Cohen** (1963) when he proved that the question is* undecidable *in the sense that consistent theories of infinite sets can be constructed which* either accept or deny *the assumption of the continuum hypothesis. Its status is, therefore, that of an independent axiom of set theory, just as Euclid's parallels axiom was shown by Gauss and others to be an independent axiom of classical geometry. Cohen* also *proved that the axiom of choice (and thus also the cardinality-comparability of any two sets) is itself undecidable.*

*In 1930, **Kurt Gödel** proved that no formal axiomatic system adequate to embrace arithmetic (and thus number theory) can be both consistent and complete. If such a system is consistent, then there must be some true statements which can be neither proved nor disproved within this formal system;*

they are undecidable within the system, and so some problems are logically unsolvable within a give axiomatic framework. (Cohen's discoveries, mentioned in the last section, established both the continuum hypothesis and the axiom of choice as such undecidable propositions.) In 1933 Gödel proved a second negative theorem: that there is no constructive procedure whereby an axiomatic system can establish its own consistency, i.e. freedom from internal contradictions.

The study of the properties of infinite sets, with their paradoxes and apparent contradictions, has been a major mathematical activity of the twentieth century. We cannot go into the details of this highly technical field, but to give a glimpse of the kinds of problems that arise, we end this section with a well-known story.

A village has only one barber, who claims that he shaves every man in the village who does not shave himself. The question is: who shaves the barber? Either answer leads to a contradiction. This paradox is a simple example of a situation where the use of such words as 'all' or 'every' can set a trap for the unwary. Indeed, the argument as to how such difficulties are to be resolved has divided mathematical logicians into contending camps for most of the 20th century.

1872–1897 CE **Julius Wilhelm Richard Dedekind** (1831–1916, Germany). One of the most original mathematicians of the 19^{th} century. He was educated at Göttingen and became one of **Gauss**' last students. His most influential teachers were **Riemann** and **Dirichlet**. In 1858 Dedekind was appointed professor of mathematics in Zürich, and from 1862 he taught at the polytechnical school in his native city, Brunswick.

Dedekind's work is associated with 4 main topics: Theory of positive integers (1887), theory of irrational numbers (1872)[464], the theory of algebraic numbers (1871) and the idea of the modular grid (1897). His revolutionary contribution to the first topic is the establishment of the integer concept on exclusive theoretical-logical basis, using set-theoretic concepts and the introduction of the 'definition via induction'.

[464] **W.R. Hamilton** made in 1833 one of the first attempts to define irrationals as partitions of rationals, thus presaging the *Dedekind cut.*

In 1872 Dedekind presented a theory of *real numbers* based on the concept of *Dedekind cut*[465], in which he *proved* that every cut in the domain of rational numbers defines a real number. The principal contribution of Dedekind to mathematical science is his creation of the theory of *algebraic numbers* and the introduction of the concept of the 'ideal', which is fundamental to ring theory.

Dedekind's brilliance was expressed not only in the theorems and concepts that he studied: Because of his ability to formulate and express his ideas so clearly, he introduced a whole new style of mathematics that has been a major influence on mathematicians ever since.

Richard Dedekind was born in Brunswick (the natal town of Gauss), the youngest of the four children of Julius Levin Dedekind, a professor of law of Jewish origin. In 1848 he entered, in the footsteps of Gauss, the Caroline College and from there he went in 1850 to Göttingen, to become one of Gauss' last pupils. He stayed there for 7 years and in 1857 was appointed an ordinary professor at the Zürich polytechnic, returning in 1862 to Brunswick as professor at the technical high school. He stayed there for the next 50 years. Nobody has as yet been able to explain why Dedekind occupied a relatively obscure position for half a century, while men who were not fit to lace his shoes filled important and influential university chairs. Dedekind never married.

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[465] **Dedekind**'s idea is this: all *rationals* can be divided into 2 classes, such that all the terms in one class are less than all the terms in the other. There are 3 possibilities:

(1) There may be a maximum to the lower section (L) *and* a minimum to the upper section (U).

(2) There may be a maximum to L and no minimum to U or vice versa.

(3) There may be *neither* a maximum to L *nor* a minimum to U. Example: The series of decimal approximants to $\sqrt{2}$ where L is the class of all rational numbers whose square is less than 2 and U is the class of all rational numbers whose square is greater than 2. In this case, since L has no maximum rational and U has no minimum rational, there is a *hole* (cut) in the rational series, which must be filled, if one desires continuity of the number line. This is done by *postulating* that every such 'cut' *defines* an irrational number.

Algebraic Numbers and Dedekind's Ideals

The failure of 18^{th} and 19^{th} century mathematics to resolve Fermat's Last "Theorem" led to the development of the new arithmetic of algebraic numbers.

An algebraic number *is a complex number that is a root of an algebraic equation $p(x) = 0$, where $p(x) = a_0x^m + \cdots + a_m$ is a polynomial with rational coefficients with $a_0 \neq 0$ and $m > 1$. An algebraic number is a root of infinitely many equations of various degrees: e.g. $\alpha = \sqrt{3}$ satisfies the equations $x^2 - 3 = 0$; $x^3 - x^2 - 3x + 3 = 0$; $x^4 - 9 = 0$ etc. But the polynomials for the last two equations are* reducible *over the field of rationals, i.e. they can each be factored into lower-degree polynomials with rational coefficients. If it is impossible to factor a polynomial $p(x)$ over the rationals into non-constant factors of lower degree, again with rational coefficients, then p is called* irreducible *over the rationals. Thus $p(x) = x^2 - 3$ is irreducible.*

For any algebraic number α there is exactly one irreducible polynomial *$\phi(x)$ over the rationals with leading coefficient 1 such that $\phi(\alpha) = 0$. The degree of the algebraic number α is defined as the degree of $\phi(x)$. For example, every rational number r is algebraic of the 1^{st} degree, being the root of $x - r = 0$; $\frac{1}{2}(1 + i\sqrt{3})$ is of degree 2, being the root of $x^2 - x + 1 = 0$; and $\sqrt[n]{2}$ is of degree n as a root of $x^n - 2 = 0$. If all the coefficients in $\phi(x) = 0$ are integers, then α is called an* algebraic integer. *It can be shown that the roots of a polynomial with algebraic coefficients, are also algebraic.*

It is customary to denote the ring of integers by the symbol $\mathbb{Z}$; a ring *is a set of numbers in which operations of addition, subtraction and multiplication are performed without restriction. Addition in a ring is always commutative, but multiplication need not be. If it is, the ring is said to be* commutative. *A commutative ring in which every nonzero element has a multiplicative inverse, is called a* field. *The symbol $\mathbb{Q}$ stands for the field of rational numbers. The symbol $\mathbb{Q}(\sqrt{d})$ denotes the field of numbers of the form $a + b\sqrt{d}$, where (a, b) are arbitrary rational numbers: such a field is known as a* quadratic field. *If $d > 0$, we call it a real quadratic field. If $d < 0$, we call it complex quadratic field. If d itself is a square of a rational number, $\mathbb{Q}(\sqrt{d})$ is just $\mathbb{Q}$. Just as $\mathbb{Q}$ is the set of all ratios of elements of $\mathbb{Z}$, so $\mathbb{Q}(\sqrt{d})$ is the set of ratios $\frac{a}{b}$ with $a \in \mathbb{Z}(\sqrt{d})$, $b \in \mathbb{Z}(\sqrt{d})$, $b \neq 0$, where $\mathbb{Z}(\sqrt{d})$ is the ring-subset of $\mathbb{Q}(\sqrt{d})$ consisting of $a + b\sqrt{d}$ with a, b integers. All the rings to be considered here are commutative, have an* identity *(an element $1 \neq 0$ such that $a \cdot 1 = a$ for all a in the ring) and have* no zero divisors *(i.e. $a \cdot b = 0$ implies $a = 0$ or $b = 0$). Such a ring is called an* integral domain.

A number α of $\mathbb{Q}(\sqrt{d})$ is called a quadratic integer (or just integer for short) if either α is in $\mathbb{Z}$ or α is irrational and the coefficient of x^2 in the defining integer-coefficient polynomial equation for α is 1. The numbers in $\mathbb{Z}$ will be called rational integers. Thus $\frac{1}{2}(-1+\sqrt{-3})$ is an integer because its defining equation is $x^2+x+1=0$, while $\frac{1}{2}(-3+6\sqrt{-3})$ is not an integer because its defining equation is $4x^2+12x+117=0$. Note that if d belongs to the ring $\mathbb{Z}(\sqrt{d})$ in $\mathbb{Q}(\sqrt{d})$, it is a quadratic integer (but the converse need not hold).

Consider the Diophantine equation $y^2+2=x^3$ that has exactly two solutions in positive integers: $x=3$, $y=\pm 5$. We wish to show that there are no more solutions in ordinary integers. Although the l.h.s. has no real polynomial factors, we can still factor it into algebraic integers of the form $a+b\sqrt{-2}$, namely $(y+\sqrt{-2})(y-\sqrt{-2})=y^2+2$.

Let us assume for the moment that this factorization has broken y^2+2 into two mutually prime factors in $\mathbb{Z}(\sqrt{-2})$. If that is so, then by the unique factorization theorem[466], each of $(y+\sqrt{-2})$ and $(y-\sqrt{-2})$ must be a cube if their product is to be x^3. That is,

$$y+\sqrt{-2}=(u+v\sqrt{-2})^3=(u^3-6uv^2)+(3u^2v-2v^3)\sqrt{-2},\ \text{with } u,v \text{ integers.}$$

Equating the imaginary parts of both sides we find $1=3u^2v-2v^3=v(3u^2-2v^2)$. Therefore v can only be 1 and $u=\pm 1$. Matching the real parts then yields $y=\pm 1\mp 6=\mp 5$, and thus $x=\sqrt[3]{y^2+2}=3$.

This proof has two major gaps: The first is the assumption that $(y\pm\sqrt{-2})$ are prime in the ring $\mathbb{Z}(\sqrt{-2})$, which can be proved. The second missing step is more serious: how do we know that prime factorization is unique in $\mathbb{Z}(\sqrt{-2})$? The answer to this question is long and interesting, and involves a complicated story. It began with 19^{th} century attempts to prove Fermat's Last Theorem.

Among those who thought for a time that they had proved it was ***E. E. Kummer*** *(1810–1893). He assumed as a matter of course that factorization into primes was always unique, even when the integers were of the form $a+b\sqrt{-5}$ [a,b being regular (rational) integers, i.e. belonging to $\mathbb{Z}$]. But this happens to a ring $\mathbb{Z}(\sqrt{-5})$ in which the Fundamental Theorem of Arithmetic fails, e.g.*

$$6=2\cdot 3=(1+\sqrt{-5})(1-\sqrt{-5}).$$

Each of 2, 3, $(1+\sqrt{-5})$ and $(1-\sqrt{-5})$ can be shown to be a prime number in this ring. So we have two different prime factorizations of the number 6, and it appears that no simple arithmetic theory of the algebraic integers could be possible.

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[466] This theorem turns out to hold for $\mathbb{Z}(\sqrt{-2})$.

In order to resolve this difficulty, Kummer (1846) created a new kind of entity that he called an "ideal number". Although he failed to prove Fermat's Last Theorem, he laid the foundation to the theory of algebraic numbers.

Dedekind *(1871) reformulated the concept of "ideal number" and generalized it to other rings of algebraic numbers. He replaced algebraic integers by a new concept which he coined: that of an ideal, one example of which is the ring R of quadratic integers in a particular quadratic number field. We now consider ideals and their properties in some detail.*

By the use of Euclid's algorithm it is shown that the factorization of a rational integer into prime factors is unique, apart from the order of the factors and their sign. The same is not true of every integral domain. The first difficulty that arises is due to the fact that in some integral domains there exist numbers besides 1 and -1 which have reciprocals. Thus in $\mathbb{Z}(\sqrt{2})$ [i.e., numbers of the form $a + b\sqrt{2}$ with a, b integers], $(\sqrt{2}+1)(\sqrt{2}-1) = 1$. If such factorizations were taken into account, then no number could be prime, e.g. $3 = (3\sqrt{2}+3)(\sqrt{2}-1)$. Moreover, we have $6 = 3 \cdot 2 = (3\sqrt{2}+3)(2\sqrt{2}-2)$ which renders a non-unique factorization. The difficulty is surmounted by regarding $(\sqrt{2}+1)$ and $(\sqrt{2}-1)$ as units. Since $3\sqrt{2}+3 = 3(\sqrt{2}+1)$ and $2\sqrt{2}-2 = 2(\sqrt{2}-1)$, then $3 \cdot 2$ is considered to be equivalent to $(3\sqrt{2}+3)(2\sqrt{2}-2) = 3 \cdot 2(\sqrt{2}+1)(\sqrt{2}-1)$ and unique factorization is restored[467].

In $\mathbb{Z}(\sqrt{5})$ [i.e., numbers of the form $a + b\sqrt{5}$ with a, b in $\mathbb{Z}$] it is found that $(\sqrt{5}-1)(\sqrt{5}+1) = 4 = 2 \cdot 2$. Though it might appear at first sight that unique factorization has failed here (and with it, the theory of congruences and residues!), such is not the case. For $\frac{1}{2}(1 \pm \sqrt{5})$ are not only algebraic integers [satisfying $x^2 - x - 1 = 0$] but also units, on account of $\frac{\sqrt{5}+1}{2}\frac{\sqrt{5}-1}{2} = 1$. Therefore both $\sqrt{5}+1$ and $\sqrt{5}-1$ are equivalent to 2. A complete failure of unique factorization (and of Euclid's algorithm), however, occurs, as we saw, in $\mathbb{Z}[\sqrt{-5}]$. Thus, the two factorizations of 6, namely, $6 = 3 \cdot 2 = (1+\sqrt{-5})(1-\sqrt{-5})$ cannot be reconciled by any convention.

Dedekind evaded the difficulty by the introduction of a new entity, which restored uniqueness of prime factorization to domains for which it fails. Let R be a ring whose elements r may be real or complex. Consider a subset I of R with the following properties:

(i) If a and b are numbers in I, so is $a - b$.

(ii) For every number r in R and every number a in I, the product ra also lies in I.

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467 Note that since $(\sqrt{2}+1)^n(\sqrt{2}-1)^n = 1$, there exist here an *infinite* number of units!

The subsets I of R are called ideals in R. For example, if m is a natural number, the totality of numbers $\{0, \pm m, \pm 2m, \pm 3m, \dots\}$ is an ideal in $\mathbb{Z}$ and will be denoted by $[m]$. Clearly, the difference of two integer multiples of m is again a multiple of m, and every multiple of a multiple of m is itself a multiple of m. Thus, $[6] = \{\dots, -18, -12, -6, 0, 6, 12, 18, \dots\}$.

Ideals consisting of all the multiples of a single element of a ring R (in the present case m) and no other elements, are called principal ideals. It is easy to verify that in the ring $\mathbb{Z}$, every ideal is principal, so that all ideals $[m]$ in $\mathbb{Z}$ are obtained by setting in turn $m = 0, 1, 2, \dots$ (in other rings, however, there exist also ideals which are not principal). The elements of a principal ideal are all multiples of a single element a which is said to generate the ideal (m in the above example). Thus, the principal ideal $[a]$ is the set of all elements xa, when a is a fixed number (element) of the ring and x is any other element.

The product of two ideals is the set of numbers which have the form of a product of two numbers, one from each ideal, together with all numbers that can be formed from these by addition and subtraction. The product is easily shown to itself be an ideal. Thus the product of the principal ideals $[2] \times [3]$ contains the numbers $2 \times 3 = 6$, $4 \times 3 = 12$, $2 \times 9 = 18$, etc., and forms the ideal $[6]$, in full analogy to arithmetic multiplication. In particular, $[1]$ is the unit ideal since it clearly satisfies $[1][m] = [m]$; $[1]$ is in fact the entire ring ($\mathbb{Z}$ in this case). If an ideal cannot be expressed as a product of two ideals in which neither is the unit ideal, we shall say that it is a prime ideal, again in analogy to prime factorization of integers in ordinary arithmetic[468].

[468] Consider the two principal ideals over $\mathbb{Z}$:

$$[3] = \{\dots, -3, \ 0, 3, 6, 9, 12, \dots\}$$
$$[10] = \{\dots, -10, 0, 10, \dots, \}$$

When *any* number of [3] is factorized into two factors a, b of $\mathbb{Z}$ it is found that either a or b belong to [3]. The ideal [10] does not share this property since, for example, $20 = 4 \cdot 5$, where neither 4 nor 5 is in [10]. Dedekind therefore said that [3] is a *prime ideal*. [In general, an ideal P in a ring R is called a *prime ideal* if $ab \in P$ implies either $a \in P$ or $b \in P$, for all elements of P.]
Thus, in the ring $\mathbb{Z}(\sqrt{-5})$, where the elements are of the form $(m + n\sqrt{-5})$, the ideal $[2, 1 + \sqrt{-5}]$ (the sub-ring of elements $2m + (1 + \sqrt{5})n$ with m, n in $\mathbb{Z}$) is not expressible as a principal ideal as there is no element which divides both $(1 + \sqrt{-5})$ and 2. In fact

$$[2] = [2, 2 + 2\sqrt{-5}] = [2, 1 + \sqrt{-5}][2, 1 + \sqrt{-5}]$$

by the law of ideal multiplication, where $[2, 1 + \sqrt{-5}]$ is a *prime ideal* in $R(\sqrt{-5})$, but [2] is not!

The concept of divisibility can also be defined for ideals. One says that an ideal A is divisible by an ideal B if every element a of A is also element b of B. When applied to $\mathbb{Z}$, it is seen that $[a]$ is divisible by $[b]$ iff a is divisible by b.

We next consider a type of non-principal ideal generated by two elements of the integral domain (ring) and denoted as $[a, b]$. It consists of all elements $xa + yb$, where a and b are fixed members of the domain, and x, y are any elements whatsoever therein. In general, given an integral domain D, the set of elements of D which are linear combinations of α_1,$\alpha_2, \ldots, \alpha_i$ with any coefficients in the domain, constitutes an ideal which is denoted $[\alpha_1, \alpha_2, \ldots, \alpha_i]$.

Multiplication of such ideals is defined by the relation

$$[\alpha_1, \alpha_2, \ldots, \alpha_i][[\beta_1, \beta_2, \ldots, \beta_j] = [\alpha_1\beta_1, \alpha_1\beta_2, \ldots, \alpha_1\beta_j, \alpha_2\beta_1, \ldots, \alpha_i\beta_j].$$

As stated earlier, a principal ideal is an ideal with but one symbol in the bracket, such as $[\alpha_1]$, and consists of the products of α_1 with all elements of D. Ideals in $\mathbb{Z}$ are all principal ideals, for any pair of rational integers m, n have a g.c.d. (greatest common divisor) h, such that $Mm + Nn = h$ for some M, N in $\mathbb{Z}$, and m and n are multiples of h. Thus $[m, n] = [h]$. Factorization of ideals in the ring of rational integers is therefore the same as factorization of integers.

With $s = \sqrt{-5}$ we define

$$p = [2, 1 + s]; \qquad q = [3, 1 + s]; \qquad r = [3, 1 - s]$$

and find that

$$pq = [1 + s]; \qquad pr = [1 - s]; \qquad pp = [2, 2 + 2s] = [2, 2s]; \qquad qr = [3, 3s].$$

Hence the principal ideal [6] can be expressed as either the product of the two non-prime ideals [2] and [3] or uniquely as the product of the prime ideals

$$[6] = p^2qr.$$

In general, Dedekind proved his main theorem:

"Every ideal in a ring R, other than R itself and $[0]$, can be represented as a product of prime ideals, uniquely apart from the order of the factors."

For the ring $\mathbb{Z}$ of rational integers, this means that every principal ideal $[m]$ is uniquely (apart from the order of the factors) a product of prime ideals $[p_1][p_2][p_3]\ldots[p_m]$, which is another way of stating the fundamental theorem of elementary number theory: $m = \pm p_1p_2\ldots p_n$ for any $m \in \mathbb{Z}$, where p_j are some (possibly repeating) prime numbers.

1872–1912 CE **Felix Klein** (1849–1925, Germany). Mathematician. A central figure in world mathematics during his lifetime. Known for his novel approach to geometry and his decisive influence upon the development of mathematics in the 19^{th} century.

At the age of 23 he was a full professor at the University of Erlangen. His inaugural lecture there had made mathematical history as the *Erlangen Program* — a bold proposal to use the group concept to classify and unify the many diverse and seemingly unrelated geometries which had developed since the beginning of the 19^{th} century.

Early in his career he had shown an unusual combination of creative and organizational abilities and a strong drive to break down barriers between pure and applied science. His mathematical interests were all-inclusive: geometry, number theory, group theory, invariant theory, algebra — all were combined for the development and completion of the Riemannian ideas on geometric function theory. He made significant contributions to the theory of automorphic functions, topology ("*Klein's bottle*"), group theory (*Klein's group*, *Cayley-Klein parameters*) and gyroscopic theory. Klein solved (1877) the *icosahedral equation* in terms of hypergeometric functions. This allowed him to give a closed-form solution of the quintic.

Klein was born in Düsseldorf, Prussia. At 17 he was chosen by **Julius Plücker** (1801–1868) as his assistant in his physics laboratory at Bonn [the same laboratory where Plücker had invented what today we call the *Geissler tube*]. Plücker had reverted in his later years to his early interest in geometry. When he died in 1868, he left an unfinished manuscript, entitled "*New geometry of space, founded on the straight line as element*".

The task of completing the work and issuing the second half of the book was entrusted to Plücker's young assistant, Felix Klein. During 1869/70 he was at the University of Berlin, where he hoped to profit from personal contact with Weierstrass, but the latter was not receptive to Klein's ideas. Klein participated in the Franco-German war, serving in the ambulance corps.

Klein married the beautiful Anna Hegel, granddaughter of the philosopher **Hegel**. In 1886 he migrated to the University of Göttingen, where he stayed for the rest of his life.

"*All things near and far*
Hiddenly linked are.
Thou canst not stir a flower
Without the troubling of a star".

William Blake (1757–1827)

1872 CE **Francois Felix Tisserand** (1845–1896, France). Astronomer. Suggested that the gravitational force exerted by a moving body might obey the same laws as the electric and magnetic forces exerted by a moving charge. His results predicted that the planets would deviate slightly from their Newtonian orbits around the sun. Such a deviation had been detected in the motion of Mercury by **LeVerrier** in 1845, but Tisserand was unable to match his theory with the observations. The motion of Mercury remained a mystery until 1915.

This was the first effort at a gravitodynamical extension of Newtonian theory.

1872–1921 CE **Gabriel Jonas Lippmann** (1845–1921, France). Physicist and inventor. A multi–talented researcher best known for his contributions to optics, electricity and thermodynamics:

- Invented (1872), at Heidelberg, the *capillary electrometer*, which measures small differences in voltage and was used by **Weller** and **Einthoven** in their early *electrocardiographs*; it is an instrument in which small electric currents are detected by movement of a mercury meniscus in a capillary tube. The instrument consists of a thin glass tube with a column of mercury beneath sulphuric acid. The mercury meniscus moves with varying electrical potential and is observed through a microscope.

- Invented the *coelostat*, a new astronomical tool that compensated for the earth's rotation and allowed a region of the sky to be photographed without apparent movement. Essentially, it allows long-exposure photographs of the sky by compensating for the earth's motion during the exposure: The device consists of a flat mirror that is turned slowly by a motor to reflect the sun's continuously into a fixed telescope. The mirror is mounted to rotate about an axis through its front surface that points to a celestial pole and is driven at a rate of one revolution in 48 hours. The telescope image is then stationary and non-rotating. The coelostat is particularly useful for eclipse expeditions when elaborate equatorial mounting of telescopes is impossible.

- Studied induction in *superconductor* circuits (precursor of **Kamerlingh–Onees'** validations)
- Did early important work in *piezoelectricity* (precursor of **Pierre Curie**'s work). His research furthered developments in this field.
- The beginning of photography came before 1849 due to the efforts of such as Niepce, Daguerre, and Talbot. Even though these men made the foundation for photography, they did not know how to obtain proper color from the early examples. Edmond Becquerel came close to discovering this, but like many others, failed. After the theories and experiments of other men such as Wilhelm Zenker and Otto Weiner, color could finally be duplicated, more or less.

  Lippmann had evolved the general theory of his process for the photographic reproduction of color in 1886, but the practical execution presented great difficulties. However, after years of patient and skillful experiment, he was able to communicate the process to the Academy of Sciences in 1891, although the photographs were somewhat defective due to the varying sensitivity of the photographic film. In 1893, he was able to present to the Academy photographs taken by A. and L. Lumiere in which the colors were produced with perfect orthochromatism. He published the complete theory in 1894.

  Lippmann's color photographic technique was based on interference, the combining of different light waves arriving simultaneously at the same point – the same phenomenon that causes color to appear in colorless substances such as soap bubbles. To receive the image, Lippmann used a glass plate coated on one side with light-sensitive emulsion, a mixture of gelatin, grains of silver nitrate, and potassium bromide. In the camera, the emulsion side of the plate faced a plate holder coated with mercury, which acted as a mirror.

  When the camera lens was opened, light was reflected from the objects in the lens's field of view through the lens to the emulsion-coated plate and through the plate to the mirror; the various wavelengths of this light corresponded to the various colors of the objects in the field of view. The incoming light was then reflected back into the emulsion by the mirror. When the incoming light waves and the light waves reflected by the mirror met on the surface of the emulsion, they created interference patterns in the silver grains of the emulsion. These patterns were then fixed on the plate by chemical baths. When the plate dried, the interference patterns reflected light in various wavelengths corresponding to the original colors of the photographic objects. Lippmann's process was an important experimental milestone although it proved impractical in photography: because exposure times were too lengthy, the image had to be viewed at a precise angle to a light source, and it could not be reproduced.

Gabriel Lippmann was born of Jewish parents at Hollerich, Luxembourg.The family moved to Paris and he received his early education at home. He entered the Lycée Napoleon (1858) and in 1868 was admitted to the Ecole Normale. In 1873, he was appointed to a Government scientific mission visiting Germany to study methods for teaching science: he worked with **Kirchhoff** in Heidelberg and with **Helmholtz** in Berlin.

He joined the Faculty of Science in Paris (1878), was appointed Professor of Mathematical Physics at the Sorbonne (1883) and became a Professor of Experimental Physics[469] there (1886). He then became Director of the Research Laboratory and retained this position until his death.

Lippmann became a member of the Academy of Sciences (1883) and served as its President (1912). He was awarded the 1908 Nobel Prize in Physics for his method of reproducing colors photographically based on the phenomenon of interference, known as the *Lippmann plate*.

He died at sea on July 13, 1921, during his return from a journey to North America.

1873 CE **Louis Joseph May** (England) and **Willoughby Smith** (1828–1891, England) discovered *electrical photoconductivity*, thus enabling to transform images into electrical signals. They found that the electrical conductivity of the element *selenium*[470] changes when light falls on it, i.e. when a selenium bar is exposed to light it becomes a strong conductor of electricity and the ensuing current is proportional to the amount of light hitting the bar. May then used selenium to send a signal through the Atlantic cable (laid in 1865).

Both inventors were at the time telegraph operators in Valentin, Ireland.

In the same year Maxwell published his book *Treatise on Electricity and Magnetism*, expounding the theory of electromagnetic radiation.

Due to the *photoconductive effect*, selenium would become the basis for the manufacture of *photoelectric cells*[471].

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[469] He was **Marie Curie**'s thesis advisor at the Sarbonne. Lippmann let her use his laboratory for her thesis work and helped her find other sources of support. At that time Lippmann did early studies in a field of electrical effects in crystals. It was he that introduced Marie to one of his best students, **Pierre Curie**.

[470] Discovered (1818) by **J.J. Berzelius**.

[471] The first photocell was built by **Charles Sumner Tainter** (1880). The first practical *photoelectric cell* was devised through 1900–1904 by **Julius Elster**

Willoughby Smith was born in Great Yarmouth. During 1850 he superintended the manufacture and laying of a telegraph cable from Dover to Calais, and later assisted Charles Wheatstone with his experiments. In 1865 he was on board the *Great Eastern* and assisted in laying the transatlantic cable from Ireland to Newfoundland.

Photoconductivity and photoelectric cells (1873–1973)

Photoconductivity is an internal photoeffect. The absorption of a photon by an intrinsic photoconductor results in the generation of a free electron excited from the valence band to the conduction band, and a corresponding free hole in the valence band. The application of an electric field in the material results in the transport of both electrons and holes through the material and the consequent production of an electric current in the electrical circuit of the detector.

Photoelectric cell is a device that converts light into electricity. Two main types of photoelectric cell are in use today: the phototube and the solid-state photodetector. The phototube is an electron tube in which electrons initiating an electric current originate through photoelectric emission. In its simplest form the phototube is composed of a cathode coated with a photosensitive material, and an anode. Light falling upon the cathode causes the liberation of electrons, which are then attracted to the positively charged anode, resulting in a flow of electrons (i.e., current) proportional to the intensity of the light.

Phototubes may be highly evacuated, or filled with an inert gas at low pressure to achieve greater sensitivity. In a modification called the multiplier phototube, or photomultiplier, a series of metal plates are shaped and arranged so that the photoelectric emission is amplified by secondary electron emission. The multiplier phototube is capable of detecting radiation of extremely low intensity; it is an essential tool for nuclear research, astronomy, and space guidance systems.

(1854–1920) and **Hans Geitel** (1855–1923). They studied together in Heidelberg and Berlin, and worked as high school teachers of mathematics and physics in Wolfenbüttel.

The second type of photoelectric cell, the solid-state photodetector*, has replaces the phototube for many applications because it is small, inexpensive, and uses little power.*

The simplest type of solid-state photodetector is the photoconductor — a semiconductor *whose resistance changes when it is exposed to light — that is, to a flow of photons. Semiconductors are characterized by an energy gap that separates the electron valence band from the conduction band. When an electron in the valence band absorbs a photon of energy greater than the energy gap, it can move from the valence band into the conduction band and increase the conductivity of the semiconductor. Moving the electron into the conduction band leaves an excess positive charge, or hole, in the valence band, which can also contribute to conductivity.*

The conductivity of a photoconductor increases (while its resistance decreases) as the number of photons increases. When the photoconductor is connected in an electric circuit, the current through it therefore increases in proportion to the intensity of the light striking it.

The photoconductor, popularly known as the electric eye, is employed in operating burglar alarms, traffic-light controls, and door openers. A light source (which may be infrared and invisible to the human eye) at one end of the circuit falls on the photocell located some distance away. Interrupting the beam of light breaks the circuit. This in turn causes a relay to close, which energizes the burglar alarm or other circuit. Other common uses for photoconductors include light switching and dimming, and light meters for cameras.

A more sophisticated photodetector, the CCD (charge-coupled device) is a small capacitor*, composed of metal, oxide, and semiconductor layers, capable of both photodetection and memory storage. When a positive voltage is applied to the metal layer (called the gate), electron-hole pairs created in the semiconductor by the absorption of a photon are separated by an electric field, and the electrons become trapped in the region under the gate. This trapped charge represents a small piece of an image known as a pixel. The complete image can be recreated by reading out a sequence of pixels from an array of CCDs. These arrays are used to capture images in video and digital cameras.*

More stable and precise than a simple photoconductor, the photodiode is a p-n junction formed by placing a p-type semiconductor against the surface of an n-type semiconductor The region around the interface between the two types of semiconductors, called the depletion region, contains an electric field. If a photon of sufficient energy is absorbed in this region, it creates an electron-hole pair; the electric field sweeps the electron toward the n *region and the hole toward the* p *region. If the* p *and* n *terminals are connected, or a reverse voltage is applied, an external current is created.*

The photovoltaic cell, or solar cell, is a well-known application of the photodiode. "Avalanche" diodes are used to amplify the signal from a light source. In these devices, a large reverse voltage is applied so that a photon-created electron in the conduction band gains enough energy to bounce against atoms in the semiconductor and thus liberate additional electrons. A large current is therefore produced when light strikes the diode.

Phototransistors are also used to amplify light signals. Their construction is similar to conventional transistors except that one of the transistor's junctions is exposed to radiation. In bipolar phototransistors, it is the base-emitter junction that is exposed to radiation; in field-effect phototransistors it is the gate junction.

1873 CE **Johannes Diderik van der Waals** (1837–1923, Holland). Theoretical physicist. Established an improved ideal gas law which accounts for the finite size of gas molecules and for the intermolecular attraction forces.

In 1910, he won the Nobel prize in physics for developing the equation of state which bears his name.

Van der Waals was born at Leyden, The Netherlands. He served as professor of physics at Leyden from 1877 until his retirement in 1907.

1873–1876 CE **Carl Paul Gottfried von Linde** (1842–1934, Germany). Engineer. Introduced the first practical cooling compression system (refrigerator), utilizing the Joule-Thomson effect with liquid ammonia as coolant. Earlier experimental cooling systems were produced by **Jacob Perkins** (U.S.A., 1834) who developed the first compression machine, and **Ferdinand Carré**, a French engineer who built the first absorption system using ammonia (1854).

1873–1878 CE **William Kingdon Clifford** (1845–1879, England). Mathematician and philosopher. Generalized the quaternions of Hamilton to biquaternions. These are used for the study of both Euclidean and non-Euclidean spaces. Determined the topological equivalence of many-sheeted surfaces.[472]

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[472] He showed, for example, that the Riemann surface of an n-valued function with w branch-points can be transformed into a topological equivalent of a sphere with p holes where $p = \frac{w}{2} - n + 1$ (i.e. a sphere with p handles).

In his endeavor to graft Hamilton's quaternions on to Grassmann's extensive algebra, he discovered '*Clifford algebra*'. In 1878 he coined the words '*divergence*' and '*curl*'[473]. The term Nabla was contributed by **Robertson Smith**. Clifford confessed his belief (1870) that "*matter is only a manifestation of curvature in a space time manifold*" (!!) He made advances in non-Euclidean geometry. *Clifford parallels* and *Clifford surfaces* are named after him.

Clifford derived his theory of *biquaternions* (through a generalization of quaternions), associating them specifically with linear algebra. In this way he represented motions in three-dimensional non-Euclidean space. He then suggested (1876) that motion of matter may be due to changes in the geometry of space.

Clifford was born at Exeter. He was educated at Kings' College, London and at Trinity College, Cambridge. In 1871 he was appointed professor of mathematics at University College, London and in 1874 he became Fellow of the Royal Society. He died of pulmonary consumption at Madeira.

Clifford Algebras

William Clifford invented his algebras (1876–1879) as an attempt to generalize the quaternions to higher dimensions. To begin with, he started from quaternions over the complex number field with 1 and 0 as its unity and zero elements, i.e. $q = a + bj + ck + dl$, *where* (a, b, c, d) *are allowed to be complex numbers. Here, one must distinguish between the* quaternion conjugate $q^* = a - bj - ck - dl$ *and the ordinary* complex conjugate $\bar{q} = \bar{a} + \bar{b}j + \bar{c}k + \bar{d}l$. *If* $q^* = \bar{q}$, *the quaternion is* Hermitian. *A quaternion for which* $\|q\| \equiv a^2 + b^2 + c^2 + d^2 = 1$ *is called a* unit quaternion. *If* $\|q\| = 0$ *the quaternion is* singular. Hermitian biquaternions *(with a real scalar part) are used to represent space-time in the theory of relativity.*

[473] The corresponding *concepts* are, however, due to **Maxwell**.

On setting

$$a = a_0, \qquad b = ia_1, \qquad c = ia_2, \qquad d = -a_3, \qquad i^2 = -1,$$

$$j = -ie_1, \qquad k = -ie_2, \qquad l = -e_1e_2,$$

$$e_1^2 = e_2^2 = 1; \qquad e_1e_2 = -e_2e_1; \qquad (e_1e_2)^2 = -1$$

and remembering that the quaternion units obey the relations

$$j^2 = -1, \qquad k^2 = -1, \qquad l^2 = -1;$$

$$jk = l, \quad kl = j, \quad lj = k, \quad kj = -l, \quad lk = -j, \quad jl = -k,$$

the quaternion q *is transformed into the new form* $q \to q'$ *where*

$$q' = a_0 + a_1e_1 + a_2e_2 + a_3e_1e_2.$$

The entity q' *has a base* $\{1, e_1, e_2, e_1e_2\}$ *with a 'multiplication table':*

	1	e_1	e_2	e_1e_2
1	1	e_1	e_2	e_1e_2
e_1	e_1	1	e_1e_2	e_2
e_2	e_2	$-e_1e_2$	1	$-e_1$
e_1e_2	e_1e_2	$-e_2$	e_1	-1

This new algebra, known as C_2, *is isomorphic to quaternion algebra such that*

$$\left.\begin{matrix} q \to q' \\ p \to p' \end{matrix}\right\} \text{ implies } \qquad \begin{matrix} q + p \Rightarrow q' + p', \qquad qp \Rightarrow q'p' \\ \lambda q \Rightarrow \lambda q' \quad (\lambda = \text{ complex scalar}) \end{matrix}$$

The algebra can be represented by square matrices A, B *such that*

$$\left.\begin{matrix} q \to A \\ p \to B \end{matrix}\right\} \text{ implies } \qquad \begin{matrix} \lambda q' + \mu p' \Rightarrow \lambda A + \mu B \\ p'q' \Rightarrow AB \end{matrix}$$

$$(\lambda, \mu = \text{ elements of the complex field})$$

To this end we infer from the above 'multiplication table' that the unit elements $\{1, e_1, e_2, e_1e_2\}$ *can be represented by the matrices*

$$1 \Rightarrow E_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad e_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$e_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, \quad e_1e_2 = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

The algebra C_2 *may be generalized to an algebra* C_n *generated by* n *symbols* e_r $(r = 1, 2, \ldots, n)$ *satisfying the relations*

$$e_re_s + e_se_r = 2\delta_{rs}$$

These algebras are known as the Clifford-Dirac *algebras, with applications in quantum mechanics.*

To see the connection we now 'translate' our former 4×4 *matrix representation of the quaternions (*C_2 *algebra) into the language of the Pauli (spin) matrices* σ_k*:*

$$j = -ie_1 \Rightarrow -i\sigma_1 \qquad e_1 \Rightarrow \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$k = -ie_2 \Rightarrow -i\sigma_2 \qquad e_2 \Rightarrow \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$l = -e_1e_2 \Rightarrow -i\sigma_3 \qquad e_1e_2 \Rightarrow i\sigma_3 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

$$\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

We then have the isomorphism:

$$\begin{aligned} q = q_0 + q_1j + q_2k + q_3l &\Rightarrow q_0E - iq_1\sigma_1 - iq_2\sigma_2 - iq_3\sigma_3 \\ &\Rightarrow \begin{bmatrix} q_0 - iq_3 & -q_2 - iq_1 \\ q_2 - iq_1 & q_0 + iq_3 \end{bmatrix} = M, \text{ say} \end{aligned}$$

with $\det M = q_0^2 + q_1^2 + q_2^2 + q_3^2$. *For unit quaternions* $\det M = 1$. *For real unit quaternions* $M = U =$ *unitary matrix:* $U\bar{U} = E$.

If we denote $a = q_0 - iq_3$, $b = -q_2 - iq_1$, *then for a real quaternion*

$$q_0 = \frac{1}{2}(a + \bar{a}); \quad q_1 = \frac{i}{2}(b - \bar{b}); \quad q_2 = -\frac{1}{2}(b + \bar{b}); \quad q_3 = \frac{i}{2}(a - \bar{a})$$

If on the other hand q *is a complex unit quaternion , the elements of* M *are not complex conjugate of each other and we can only write*

$$q \to M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} q_0 - iq_3 & -q_2 - iq_1 \\ q_2 - iq_1 & q_0 + iq_3 \end{pmatrix},$$

where (a, b, c, d) *only satisfy the condition* $ad - bc = +1$.

Clifford C_3 algebra (biquaternions)

An associative noncommutative 8-dimensional algebra can be generated by 3 symbols e_1, e_2, e_3, *satisfying the relations*

$$e_1^2 = e_2^2 = e_3^2 = 1; \qquad e_r e_s + e_s e_r = 0, \quad s \neq r \quad (r, s = 1, 2, 3)$$

The 8 basis elements of the algebra are

$$1, \quad e_1, \quad e_2, \quad e_3, \quad e_1 e_2, \quad e_1 e_3, \quad e_2 e_3, \quad e_1 e_2 e_3,$$

and their linear combinations are known as biquaternions. The algebra can be represented by the 2×2 *Pauli matrices in the following way:*

$$e_1 \to \sigma_1; \qquad e_2 \to \sigma_2; \qquad e_3 \to \sigma_3$$

$$e_1 e_2 \to \sigma_1 \sigma_2 = i\sigma_3; \qquad e_2 e_3 \to \sigma_2 \sigma_3 = i\sigma_1; \qquad e_3 e_1 \to \sigma_3 \sigma_1 = i\sigma_2$$

$$\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = E$$

$$-ie_1 e_2 e_3 \to -i\sigma_1 \sigma_2 \sigma_3 = -i(i\sigma_3)\sigma_3 = \sigma_3^2 = E.$$

Clifford C_4 algebra ('Clifford numbers')

An associative noncommutative 16*-dimensional algebra is generated by the 4 symbols* e_1, e_2, e_3, e_4 *satisfying the relations*

$$e_r^2 = 1; \qquad e_r e_s + e_s e_r = 2\delta_{rs} \qquad (r, s = 1, 2, 3, 4)$$

$$\begin{cases} e_k \to \alpha_k = \sigma_1 \times \sigma_k \qquad k = 1, 2, 3 \\ \qquad e_4 \to \alpha_4 = (\sigma_3 \times E) \end{cases}$$

$$\alpha_k^2 = (\sigma_1 \times \sigma_k)(\sigma_1 \times \sigma_k) = (E \times E) = E_4, \quad \alpha_4^2 = E_4,$$

where $\times$ *is the Kronecker direct product. Hence*

$$\alpha_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \qquad \alpha_2 = \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{bmatrix},$$

$$\alpha_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, \qquad \alpha_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

These 4×4 *matrices are sometimes recast in the shorthand*

$$k = 1,2,3 \qquad \alpha_k = \begin{bmatrix} (0) & \sigma_k \\ \sigma_k & (0) \end{bmatrix}; \qquad \alpha_4 = \begin{bmatrix} E & 0 \\ 0 & -E \end{bmatrix}$$

where $(0) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ *is the zero* 2×2 *matrix. The 16 basis elements of the algebra can be grouped into five sets as follows:*

$$\begin{array}{llllll} 1, & & & & & \\ e_1, & e_2, & e_3, & e_4, & & \\ e_1e_2, & e_2e_3, & e_3e_1, & e_1e_4, & e_2e_4, & e_3e_4, \\ e_1e_2e_3, & e_2e_3e_4, & e_3e_4e_1, & e_4e_1e_2, & & \\ e_1e_2e_3e_4. & & & & & \end{array}$$

Note that the anti-commutation relations $\alpha_r\alpha_s + \alpha_s\alpha_r = 2\delta_{rs}$ *are satisfied by an infinite number of other matrix representations, but they must be at least* 4×4*. It is remarkable that* **Clifford** *(1876) forged the mathematical tools used by* **Dirac** *(1928) in his formulation of the special relativistic free electron quantum mechanical wave equation:*

$$[\alpha_1 \frac{\partial}{\partial x_1} + \alpha_2 \frac{\partial}{\partial x_2} + \alpha_3 \frac{\partial}{\partial x_3} + \frac{i\alpha_4 m_0 c}{\hbar} + \frac{1}{c} E_4 \frac{\partial}{\partial t}]\Psi = 0$$

with x_j, $j = 1,2,3$ *the spatial coordinates,* t *time,* m_0 *the electron rest mass,* c *the speed of light in vacuum, and* Ψ *the electron 4-component complex spinor wavefunction.*

Clifford algebras in linear vector spaces

The quaternions, biquaternions and Clifford numbers are examples of hypercomplex numbers. But just as we have previously found it convenient to order sets of scalar components together to form vectors, so it becomes convenient to define hypercomplex vectors *as linear arrays of components which* *are hypercomplex numbers. We shall next show that the Pauli matrices can be considered as a vector base in a linear vector space L which obeys Clifford algebra in 2^n dimensions.*

Let $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \dots)$ be vectors in L with base $(e_1, e_2, \dots, e_n)$. Then (employing the summations convention)

$$\boldsymbol{\alpha} = \alpha_i e_i, \qquad \boldsymbol{\beta} = \beta_j e_j$$

If the symbols e_i are ordinary vectors, say in real 3D Euclidean space, we define the wedge product as the antisymmetric dyadic

$$\begin{aligned}\boldsymbol{\alpha} \wedge \boldsymbol{\beta} &= \frac{1}{2}(\boldsymbol{\alpha\beta} - \boldsymbol{\beta\alpha}) \\ &= \frac{1}{2}(\alpha_1\beta_2 - \beta_1\alpha_2)(e_1e_2 - e_2e_1) + \frac{1}{2}(\alpha_2\beta_3 - \beta_2\alpha_3)(e_2e_3 - e_3e_2) \\ &\qquad + \frac{1}{2}(\alpha_3\beta_1 - \beta_3\alpha_1)(e_3e_1 - e_1e_3) = -\frac{1}{2}\left[I \times (\boldsymbol{\alpha} \times \boldsymbol{\beta})\right]\end{aligned}$$

also known as a bivector.

If however the symbols e_i are the Pauli spin matrices

$$e_1 \Rightarrow \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad e_1e_2 - e_2e_1 = 2ie_3$$

$$e_2 \Rightarrow \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad e_2e_3 - e_3e_2 = 2ie_1$$

$$e_3 \Rightarrow \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad e_3e_1 - e_1e_3 = 2ie_2,$$

then the wedge product assumes the form

$$\boldsymbol{\alpha} \wedge \boldsymbol{\beta} \Rightarrow i\boldsymbol{\sigma} \cdot (\boldsymbol{\alpha} \times \boldsymbol{\beta})$$

where $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ is a hypercomplex vector. Now, the formal scalar product of $\boldsymbol{\alpha}$ and $\boldsymbol{\sigma}$ is the hypercomplex vector

$$(\boldsymbol{\alpha} \cdot \boldsymbol{\sigma}) = \begin{bmatrix} i\alpha_3 & \alpha_1 - i\alpha_2 \\ \alpha_1 + i\alpha_2 & -i\alpha_3 \end{bmatrix} = a \quad \text{(defined)}$$

It then follows that

$$ab = (\boldsymbol{\alpha} \cdot \boldsymbol{\sigma})(\boldsymbol{\beta} \cdot \boldsymbol{\sigma}) = (\boldsymbol{\alpha} \cdot \boldsymbol{\beta})E + i\boldsymbol{\sigma} \cdot (\boldsymbol{\alpha} \times \boldsymbol{\beta}).$$

Introducing the notation

$$(a \cdot b) \equiv (\boldsymbol{\alpha} \cdot \boldsymbol{\beta})E, \qquad a \wedge b = i\boldsymbol{\sigma} \cdot (\boldsymbol{\alpha} \times \boldsymbol{\beta}),$$

we find that the 'product' rule of two hypercomplex vectors in their vector space is

$$ab = (a \cdot b) + a \wedge b.$$

The wedge product has the usual distributive and associative *properties (λ a scalar)*

$$a \wedge (b + c) = a \wedge b + a \wedge c; \qquad (a + b) \wedge c = a \wedge \ + b \wedge c$$

$$\lambda(a \wedge b) = a \wedge (\lambda b) = (\lambda a) \wedge b; \qquad (a \wedge b) \wedge c = a \wedge (b \wedge c).$$

It is however anti-commutative: $a \wedge b = -b \wedge a$.

The wedge product of two vectors can be written symbolically in the determinant form

$$\boldsymbol{\alpha} \wedge \boldsymbol{\beta} = \frac{1}{2}(\boldsymbol{\alpha}\boldsymbol{\beta} - \boldsymbol{\beta}\boldsymbol{\alpha}) = \frac{1}{2}\sum_{i,j} t_{i,j}(\boldsymbol{e}_i \wedge \boldsymbol{e}_j)$$

$$t_{ij} = \alpha_i\beta_j - \beta_i\alpha_j = \begin{vmatrix} \alpha_i & \beta_i \\ \alpha_j & \beta_j \end{vmatrix}$$

One may then extend the concept of the wedge product to antisymmetric tensors of higher order. For example, the totally antisymmetric 3$^{\text{rd}}$ *rank tensor has the determinant form in* m *dimensions ($i, j, k = 1, \dots, m$):*

$$t_{ijk} = \frac{1}{6}\begin{vmatrix} \alpha_i & \beta_i & \gamma_i \\ \alpha_j & \beta_j & \gamma_j \\ \alpha_k & \beta_k & \gamma_k \end{vmatrix}$$

$$= \frac{1}{6}(\alpha_i\beta_j\gamma_k + \alpha_j\beta_k\gamma_i + \alpha_k\beta_i\gamma_j - \gamma_i\beta_j\alpha_k - \gamma_j\beta_k\alpha_i - \gamma_k\beta_i\alpha_j)$$

$$\text{or} \qquad \mathfrak{T} = \frac{1}{6}(\boldsymbol{\alpha}\boldsymbol{\beta}\boldsymbol{\gamma} + \boldsymbol{\gamma}\boldsymbol{\alpha}\boldsymbol{\beta} + \boldsymbol{\beta}\boldsymbol{\gamma}\boldsymbol{\alpha} - \boldsymbol{\gamma}\boldsymbol{\beta}\boldsymbol{\alpha} - \boldsymbol{\alpha}\boldsymbol{\gamma}\boldsymbol{\beta} - \boldsymbol{\beta}\boldsymbol{\alpha}\boldsymbol{\gamma})$$

$$= \frac{1}{6}[(\boldsymbol{\alpha}\boldsymbol{\beta} - \boldsymbol{\beta}\boldsymbol{\alpha})\boldsymbol{\gamma} + (\boldsymbol{\gamma}\boldsymbol{\alpha} - \boldsymbol{\alpha}\boldsymbol{\gamma})\boldsymbol{\beta} + (\boldsymbol{\beta}\boldsymbol{\gamma} - \boldsymbol{\gamma}\boldsymbol{\beta})\boldsymbol{\alpha}],$$

where t_{ijk} reverses sign under the interchange of any two indices (i.e., interchange of any two rows in the determinant). This tensor is known as a trivector. *It can be represented by the* triple wedge product

$$\mathfrak{T} = \boldsymbol{\alpha} \wedge \boldsymbol{\beta} \wedge \boldsymbol{\gamma} = \frac{1}{6}\sum_{i,j,k} t_{ijk}(\boldsymbol{e}_i \wedge \boldsymbol{e}_j \wedge e_k).$$

This can be naturally expressed in terms of a suitable Clifford algebra. Thus, for $m = 4$ *(e.g. tensors in spacetime) we have in* C_4*:*

$$(\boldsymbol{\alpha}\cdot\boldsymbol{\Gamma})(\boldsymbol{\beta}\cdot\boldsymbol{\Gamma})(\boldsymbol{\gamma}\cdot\boldsymbol{\Gamma}) = (\boldsymbol{\alpha}\cdot\boldsymbol{\beta})\,\boldsymbol{\gamma}\cdot\boldsymbol{\Gamma} + (\boldsymbol{\beta}\cdot\boldsymbol{\gamma})\,\boldsymbol{\alpha}\cdot\boldsymbol{\Gamma} - (\boldsymbol{\gamma}\cdot\boldsymbol{\alpha})\,\boldsymbol{\beta}\cdot\boldsymbol{\Gamma} + \boldsymbol{\alpha}\cdot\boldsymbol{\Gamma}\wedge\boldsymbol{\beta}\cdot\boldsymbol{\Gamma}\wedge\boldsymbol{\gamma}\cdot\boldsymbol{\Gamma},$$

where

$$\Gamma_i\Gamma_k + \Gamma_k\Gamma_j = 2\delta_{jk}, \qquad \Gamma_i \wedge \Gamma_j \wedge \Gamma_k \equiv \epsilon_{ijkl}\Gamma_5\Gamma_l.$$

Here ϵ_{ijkl} *is the totally antisymmetric Levi-Civita tensor* ($\epsilon_{ijkl} \equiv 1$)*, and* $\Gamma_5 = \Gamma_1\Gamma_2\Gamma_3\Gamma_4$.

C_4 *relations such as this are quite useful in particle-physics quantum field theory calculations (such as in QED or the Standard Model).*

Note that if $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$ *are orthogonal*

$$\mathfrak{T} = -\frac{1}{6}[I \times (\boldsymbol{\alpha\alpha} + \boldsymbol{\beta\beta} + \boldsymbol{\gamma\gamma})]$$

and if they are orthonormal

$$\mathfrak{T} = -\frac{1}{6}(I \times I) = \frac{1}{6}\epsilon_{ijk}.$$

The above results can be generalized to entities of the form

$$\begin{aligned} a &= \lambda I + (\boldsymbol{\alpha}\cdot\boldsymbol{\sigma}), \\ b &= \mu I + (\boldsymbol{\beta}\cdot\boldsymbol{\sigma}), \qquad \textit{for which} \\ ab &= \{\lambda\mu + (\boldsymbol{\alpha}\cdot\boldsymbol{\beta})\}I + [\lambda\boldsymbol{\beta} + \mu\boldsymbol{\alpha} + i(\boldsymbol{\alpha}\times\boldsymbol{\beta})]\cdot\boldsymbol{\sigma}. \end{aligned}$$

In this case

$$\begin{aligned} a\cdot b &= \{\lambda\mu + (\boldsymbol{\alpha}\cdot\boldsymbol{\beta})\}I, \\ a\wedge b &= [\lambda\boldsymbol{\beta} + \mu\boldsymbol{\alpha} + i(\boldsymbol{\alpha}\times\boldsymbol{\beta})]\cdot\boldsymbol{\sigma}. \end{aligned}$$

Note that

$$\boldsymbol{\nabla}\wedge\boldsymbol{a} = \frac{1}{2}(\boldsymbol{\nabla a} - \boldsymbol{a\nabla}) = -\frac{1}{2}I \times \mathrm{curl}\ \boldsymbol{a}.$$

The development of a Clifford calculus is completed by defining the differential operator $\boldsymbol{\nabla} \equiv \sigma^k\partial_k$, $\partial_k = \frac{\partial}{\partial x^k}$*, where* σ^k *are base vectors.*

We call $\boldsymbol{\nabla}$ *the gradient operator, and decompose it into a symmetrical and antisymmetrical parts:*

$$\boldsymbol{\nabla} a = \boldsymbol{\nabla}\cdot a + \boldsymbol{\nabla}\wedge a = \boldsymbol{\nabla}\cdot a + i\boldsymbol{\nabla}\times a$$

[in line with the vector analog

$$\frac{1}{2}(\boldsymbol{\nabla}\boldsymbol{a} - \boldsymbol{a}\boldsymbol{\nabla}) = -\frac{1}{2}\mathfrak{I} \times \text{curl } \boldsymbol{a}].$$

In conclusion: Clifford algebra over the field of real numbers is a linear vector space, closed w.r.t. the multiplication operation ab, defined for two hypercomplex vectors a and b.

Localization of Cerebral function (1810–1876)

As the 19th century progressed, the problem of the relationship of mind *and* brain *became especially acute as physiologists and psychologists began to focus on the nature of cerebral function. In a diffuse and general way, the idea of functional localization had been available since antiquity. A notion of* 'soul'*, globally related to the brain, can be found in the works of* ***Pythagoras***, ***Hippocrates***, ***Plato***, ***Aristotle***, ***Herophilos*** *and* ***Galen***. *The pneumatic physiologists of the Middle Ages thought that mental capacities were located in the fluid of the ventricles.*

As belief in animal spirits died, so too did the ventricular hypothesis. By 1784, when ***Jiri Prochaska*** *published his* De functionibus systematis nervosi, *interest had shifted to the brain stem and cerebellum. Despite these early views, the notion that specific mental processes are correlated with discrete regions in the brain and the attempts to establish localization by means of empirical observation – were essentially 19th century achievements.*

The first critical steps toward those ends can be traced to the work of ***Franz Josef Gall*** *(1758–1828,* Germany*). His work was followed by* ***Marie-Jean-Pierre Flourens*** *(1794–1867,* France*) who provided the first experimental demonstration of localization of function in the brain (1824). However, Flourens concluded, erroneously, that while* sensorimotor *functions are differentiated and localized sub-cortically, higher mental functions operate together, spread throughout the entire cerebellum.*

For more than 30 years this was the established view. Then in 1861 the first studies appeared that would lead to the rejection of this idea and to the establishment of patterns of functional localization in the cortex.

In the period between 1861 and 1876, ***Paul Broca*** *(1824–1880),* ***Gustav Theodor Fritsch*** *(1838–1927),* ***Eduard Hitzig*** *(1838–1907),* ***David Ferrier*** *(1843–1928) and* ***John Hughlings Jackson*** *(1835–1911) provided conclusive evidence that circumscribed areas of the cortex are involved in movement of the contralateral limbs. Their findings established* electrophysiology *as a preferred method for the participation of the hemispheres in motor function. Ferrier also examined the functions of the spinal cord, the medulla[474], the corpora quadrigemina, and the cerebellum (1876).*

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474 For further reading, see:

- Bruun, R.D. and B. Bruun, *The Human Body Random House*, New York, 1982, 96 pp.
- Netter, F.H., *Atlas of Human Anatomy*, CIBA-GEIGY Corporation, 1993, 514 pp.

1873–1893 CE **Camillo Golgi** (1843–1926, Italy). Physician and neuropathologist. Discovered dendritic nerve cells called 'Golgi cells' and Golgi tendon spindle (1880). First to use silver nitrate to stain nerve tissue for study (1873). Shared with **Santiago Ramon y Cajal** the 1906 Nobel Prize for physiology or medicine.

Golgi was born at Corteno near Brescia. He studied medicine at the University of Pavia. He was a professor of General Pathology at Pavia (1881–1918) and senator (1900). In 1886 he demonstrated the life cycle and structure of malarial parasites.

1873–1893 CE **Ernst Abbe** (1840–1905, Germany). Physicist. Established the theoretical basis for the design of microscopes and laid the foundation of *Fourier optics* already in 1873, ahead of Rayleigh (1896).

Although the inventions of the telescope and microscope date back to the 16^{th} and 17^{th} centuries — with such eminent scientists as **Galileo**, **Huygens**, and **Newton** contributing to their development — their design was not placed on a strictly scientific basis until the beginning of the 19^{th} century, with the work of **J. Fraunhofer**.

In 1869, Abbe started to develop his theory of *image formation* which gave new insight into the laws underlying the formation of an image in the microscope in terms of light wave amplitudes. Abbe's study revealed that sharp imaging of surface elements perpendicular to the optical axis is achieved only under specific conditions (even though these elements were located close to the axis). Consequently he discovered an unambiguous relationship between the angles of rays of an arbitrarily wide bundle on the object side and those of corresponding rays on the image side (*Abbe sine condition*). The fulfillment of this condition can provide an optical system (capable of imaging an *axial* point without *spherical aberration*) with the additional ability of imaging the points of a small surface element lying perpendicular to the axis without the asymmetrical aberration called *coma*.

He also formulated the theoretical limits of *resolving power* (minimum distance between two points in an object that can be resolved in the image) and found that the resolving power is limited by the wavelength of the light used for producing the image, by the angular aperture 2α of the objective

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- Hole, J.W. Jr., *Human Anatomy and Physiology*, Wm.C. Brown Publishers: Dubuque Iowa, 1987, 966 pp.

- Sherwood, L., *Human Physiology – From Cell to Systems*, Wadsworth Publishing Co., 1997, 947 pp.

and by the refractive index of the medium filling the space between specimen and objective. As a measure for this, he established the *numerical aperture* $N = n\sin\alpha$.

Later, he searched for the basic laws governing the efficiency of optical instruments in general. Going beyond the findings of **Gauss**, **Listing** (1808–1882), **Helmholtz**, and **Neumann** (1832–1925), Abbe established the essential characteristics of any optical image formation in a form of a collinear relationship between image space and object space, in the geometric-optical sense. No one before Abbe had dealt with the basic theory of light intensity in optical instruments.

An illuminated object in front of a thin lens, becomes a source of secondary Huygens wavelets, and is diffracted by the finite lens.

Under the Fraunhofer approximation (far field), the amplitude distribution in the focal plane of a lens is the *spatial Fourier transform* of the amplitude distribution for the light field on the surface of the object, with an error not exceeding a certain insignificant phase multiplier, and a scale factor. The total diffracted field in the *focal plane* is given explicitly by

$$\Phi(x,y;k,L,f) = Be^{i\frac{k}{2f}(1-\frac{L}{f})(x^2+y^2)}\iint_{-\infty}^{\infty}\psi(x_0,y_0)e^{-\frac{ik}{f}(xx_0+yy_0)}dx_0dy_0,$$

where B is an arbitrary amplitude. Here $k=\frac{\omega}{c}$ is the wavenumber of the monochromatic plane wave that falls on the object, f is the focal length of the lens, L is the distance of the object plane in front of the lens, and ψ is the amplitude *aperture function* in the *object plane*.

If $L=f>0$ there is no phase distortion in the focal plane of the lens. If, on the other hand, the amplitude distribution to be imaged is not in the lens' focal plane, but at arbitrary distance ℓ, the condition $\frac{1}{L}+\frac{1}{\ell}=\frac{1}{f}$ will secure (under the *geometrical optics approximation*, and more specifically the *stationary phase* method in the xy integral) that the diffraction pattern intensity distribution $|\Phi(x,y)|^2$ will be proportional to $|\psi(x_0,y_0)|^2$ with $(x,y)=-M(x_0,y_0)$ and M the magnification factor predicted by the ray theory. We then say that an *image* of the object has been formed on a plane normal to the optical axis at ℓ. The relation between them is

$$|\Phi(x,y)|^2 = |B|^2\left|\psi\left(\frac{-x}{M},\frac{-y}{M}\right)\right|^2,$$

where $M=\frac{\ell}{L}$.

Thus, the diffraction formation of an image can be split into two stages:

(1) The formation of a *diffraction pattern* of the object in the focal plane of a lens, and

(2) the transformation of the diffraction pattern in the focal plane of the lens into an *image* of the object in the image plane.

The entire information contained in the image of the object is also contained in the diffraction pattern of the object in the focal plane of the lens. If the diffraction pattern in the focal plane is altered [for example, if some maxima are eliminated or attenuated], the image of the object will change accordingly. The variation in the image of an object through a modification of its diffraction pattern from which the image is subsequently formed is called the *spatial filtering of the image* (Abbe, 1893).

Another model of optical imaging (influenced by earlier work of **Airy** and **Helmholtz**) was proposed by **Rayleigh** in 1896. The model visualizes an image as the superposition of *Airy patterns* (or more complicated patterns if aberration is present). The Airy pattern is the image (response) of the entire optical system to a point light source in the object. The wavefronts from it are limited in their entry into the imaging system by the finite aperture of the imaging lens, and the diffraction pattern of that aperture is formed in the image plane. Each point of the object is therefore imaged *not as a point* but as the Airy pattern of the aperture of the imaging lens.

The advantage of the Rayleigh method is in its validity even for *incoherent* illumination. For then, the Airy intensity patterns due to all the object points are simply additive. If it is coherent, there is interference, and mathematically one deals with the combination of the complex-amplitude Airy patterns.

Ernst Abbe was born in Eisenach. In 1861 he received his doctorate at the University of Jena, where he became a lecturer. In 1866 he met **Carl Zeiss** (1816–1888) and started a relationship which shaped his entire life. Zeiss was at that time running a small microscope factory in Jena. He realized the shortcomings of the trial-and-error development techniques of that era, and therefore employed Abbe to help him to improve the construction of microscopes. They later became partners. After Zeiss' death in 1888, Abbe established the Carl Zeiss foundation and transferred to it his entire fortune, worth millions. In his statute governing the foundation, Abbe introduced the 8-hour work day (before 1900!) and social benefits to his workers, far ahead of his time. He pondered the problems of human co-existence as deeply and as methodically as he approached the natural sciences[475].

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[475] An account of Abbe's life and work is given by H. Volkmann, *Applied Optics* **5**, 1720–1731, 1966.

Whatever Abbe initiated, be it as an economist, as a social scientist, or as a man of pure science, all his innovations led to products of unsurpassed quality.

1873–1895 CE **Friedrich Wilhelm Nietzsche**[476] (1844–1900, Germany). Philosopher and classical scholar. Professor of classical philology, Basel (1869–1879). Worked chiefly on philology, music, Greek antiquity and especially philosophy: *The Birth of Tragedy* (1872); *Essays* (1873–1878); *The Genealogy of Morals* (1887); *The Antichrist* (1888); *Thus Spoke Zarathustra* (1883–1892); *Twilight of Idols* (1889); *Ecce Homo* (1888); *Beyond Good and Evil* (1886); *Will to Power* (1888).

Nietzsche did not produce his own systematic doctrine. However, by virtue of his insight into the existential condition of modern man, his perception of the culture flattening of the industrial era, and his idea of breeding a new aristocracy — he has brought about a considerable impact on 20th century thought; many philosophers, writers and psychologists have been deeply influenced by him.

Nietzsche criticized religion. In his proclamation '*God is dying*' he meant that religion, in his time, had lost its meaningfulness and power over people. Thus, he argued, religion could no longer serve as the foundation for moral values.

Nietzsche sought a re-evaluation of all values; he said that the warriors who originally dominated society had defined their own strength and nobility as "good", and the weakness of the common people as "bad". Later, when the priests and the common people came to dominate society, they redefined their own weakness and humility as "good" and the strength and cruelty of the warriors whom they feared as "evil". Nietzsche criticized this second set of values because it was based on fear and resentment, and he associated these values with the Judeo-Christian tradition, repeatedly criticizing Christianity and Judaism.

Nietzsche's major psychological theory is that all human behavior is basically motivated by the will of people to overpower each other and gain control over their unruly passions. He thought that the self-control exhibited by ascetics and artists was a higher form of power than the physical bullying of the weak by the strong. Nietzsche's ideal, the *overman*, is the passionate man who learns to control his passions and use them in a creative manner.

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[476] For further reading, see:

- *A Nietzsche Reader*, Penguin Books, 1977, 286 pp.
- Strathern, P., *Nietzsche in 90 minutes*, Ivan R. Dee: Chicago, 1996, 83 pp.

Nietzsche unjustly suffered notoriety as a racist and forerunner of Nazism. This is largely due to the editing and misinterpretation of his ideas by Nazi propagandists.[477]

Nietzsche was born in Saxony, the son and grandson of Protestant ministers. He studied at the Universities of Bonn and Leipzig. He became a professor of classics at the age of 24 and retired from academic life (1879) because of poor health. Collapsing under the weight of the questions that he posed for himself, he suffered a mental breakdown (1889) from which he never recovered. Spent his last years in care of his mother at Naumburg and his racist sister Elizabeth at Weimar. A strong opponent of Wagner in art and Schopenhauer in philosophy.

Worldview XXIV: Nietzsche

* * *

"We live in the age of atoms, in an atomistic chaos. In the Middle Ages the opposite forces were held together by the Church, to some extent assimilated

[477] Nietzsche not only admired Jews for their spiritual mastery and grandeur, but vehemently dissociated himself from the '*damnable German antisemitism*', detesting '*the stupidity, crudity and pettiness of German nationalism*'. At the same time he criticized the Jews, whose historic legacy he denounced as being responsible for 'the slave-revolt in morals'. This aspect of his approach to Judaism was posthumously distorted in an effort to turn him into a spiritual godfather of German Nazism. This was mainly due to the making of his sister Elizabeth, who 'edited' (from various notes and rough drafts) and issued, posthumously, a book which Nietzsche had abandoned, *The Will to Power*; she thus recruited her brother to the Nazi antisemitic and racist propaganda machinery, promoting him falsely into a Nazi thinker. Nietzsche, however, detested all kinds of nationalism. Nevertheless, in his book *The Antichrist* (1888) he launched a vicious crusade against Judaism, which had undoubtedly great impact on German Nazis' ideology.

into each other under the strong pressure it exerted. Since this pressure has diminished, the opposing forces have rebelled against each other.”

* * *

“If you want to achieve piece of mind and happiness — then believe, but if you want to be a disciple of truth — search.”

* * *

“There are no facts, only interpretations.”

* * *

“Nobody dies nowadays of fatal truths: there are too many antidotes to them.”

* * *

“Everything that lives — suffers. There you have the essence of existence: To live is to want, to want is to suffer. We are fugitive, doomed to sickness, nostalgia and death.”

* * *

“It is all over with priests and gods when man becomes scientific. Science is the first sin, seed of all sin, the original sin. This alone is morality: ‘Thou shalt not know’ — the rest follows.”

* * *

“Do you believe that the sciences would ever have arisen and become great if there had not beforehand been magicians, alchemists, astrologers and wizards, who thirsted and hungered after recondite and forbidden powers?”

* * *

"What is it: is man only a blunder of God, or God only a blunder of men?"

* *
*

"Morality is the best of all devices for leading mankind by the nose."

* *
*

"In former times, fanaticism of the lust for power was inflamed by the belief that one was in possession of the truth. This lust bore such beautiful names that one could thenceforward venture to be inhuman with a good conscience *(to burn Jews, heretics and good books and exterminate entire higher cultures such as those of Peru and Mexico). The means employed by the lust for power have changed, but the same volcano continues to glow, the impatience and the immoderate love demand their sacrifice: and what one formerly did 'for the sake of God', one now does for the sake of money, that is to say, for the sake of that which now gives the highest feeling of power and good conscience."*

* *
*

"The Jews are the most remarkable nation of world history because, faced with the question of being and not being, they preferred being at any price. Considered psychologically, the Jewish nation is a nation of the toughest vital energy, placed in impossible circumstances, voluntarily, from the profoundest shrewdness of its self-preservation, took the side of all decadence instincts — not *as being dominated by them, but because it divined in them a power by means of which one can prevail* against *'the world'."*

* *
*

"The Jews are indeed the strongest, toughest, and purest race now living in Europe, who could gain mastery over it if so they wished. Yet, they desire nothing but accommodation and absorption, to put an end to their centuries of wandering — to which purpose it might be useful and fair to expel the antisemitic screamers from the country."

* * *

"There was only one Christian and he died on the cross."

* * *

"The deepest sense of existence is to be found in suffering and only art enables us to face this suffering and not run away from it."

* * *

"Insanity in individuals is rare — but in groups, parties, nations, and epochs, it is the rule."

* * *

"A politician divides mankind into two classes: tools and enemies."

* * *

"The Christian conception of God is one of the most corrupt conceptions of God arrived at on earth; perhaps it even represents the low-water mark in the descending development of the God type. God degenerated into the contradiction of life, *instead of being its transfiguration and eternal* Yes!, *into a declaration of hostility toward life, nature and the will to life, into a formula for every calumny of 'this world', for every lie about 'the next world', — into a God of nothingness denied, into a will to nothingness sanctified..."*

* * *

"When on a Sunday morning we hear the bells ringing, we ask ourselves: is it possible that this is going on because of a Jew crucified 2,000 years ago who said he was the son of God? A God who begets children on a mortal woman,

a sage who calls upon us no longer to work, no longer to sit in judgment, but to heed the signs of the imminent end of the world; a justice which accepts an innocent man as a substitute sacrifice; someone who bids his disciples to drink his blood; prayers for miraculous interventions; sins perpetrated against a god atoned for by a god; fear of a Beyond to which death is a gateway; the figure of the Cross as a symbol in an age which no longer knows the meaning and shame of the Cross — how gruesomely all this is wafted to us, as if out of the grave of a primeval past."

Can one believe that things of this sort are still believed in?"

* * *

"One day there will be associated with my name the recollection of something frightful — of a crisis like no other before on earth, of the profoundest collision of conscience, of a decision evoked against everything that until then had been believed in, demanded, sanctified."

* * *

"There will be wars such as there have never yet been on earth."

* * *

"The philosopher is not interested in truth, but only in 'my truth'."

1873–1903 CE *The years of the chromosome*[478]: elucidation of the essential facts of *mitosis* (the division of the cell nucleus into two daughter nuclei, during cell multiplication) *meiosis* (the formation of sex cells, i.e. eggs and sperms) and their relation to *heredity*. About ten main actors took part in this drama, most of them Germans, born in the interval 1831–1862. These were:

- **Anton Schneider** (1831–1890, Germany). Cytologist. Discovered visible changes in the nucleus during cell division. His account was the first accurate description of mitosis in animal cells (1873).
- **Eduard Strassburger** (1844–1912, Germany). Botanist. Observed in plant cells (1875) all the phenomena seen by others in more transparent animal cells. He noted the difference between *mitosis* and *meiosis* and its meaning for heredity. He coined *haploid* (a cell with half the usual complement of chromosomes and *diploid* (with the normal number). Discovered maturation (reduction division) of plant cells (1888).
- **Walther Flemming** (1843–1905, Germany). Coined the name *mitosis* (1879) and made first accurate accounts of chromosome numbers and accurately figured the *longitudinal splitting* of chromosomes (1882). Determined chromosome number as 24 in man (1898).
- **Edouard van Beneden** (1846–1910, Belgium). Zoologist. First studies *meiosis* (1883) and stressed the importance of the qualitative and quantitative equality of chromosome distribution to daughter cells (1887).
- **Wilhelm von Waldeyer-Hartz** (1836–1921, Germany). Anatomist. Coined the name *chromosome* (1888). Proposed the *neuron* theory of the nervous system (1891).
- **August Weismann** (1834–1914, Germany). Biologist. Proposed a theory of heredity (1892) whereby the germ-plasm, located in the sex cells, is the carrier of the heredity endowment, half the germ-plasm for an offspring coming from the mother and half from the father. The germ-plasm is transmitted unmodified to offspring such that acquired characteristics are not inherited. Described the process of meiosis, whereby the number of chromosomes is halved.
- **Oscar Hertwig** (1849–1922, Germany), zoologist, and **Theodore Boveri** (1862–1915, Germany), biologist. Showed independently that

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[478] *Chromosomes*: Dark staining strands in the cell nucleus comprising the material of *heredity* and containing two forms of *nucleic acid*, mostly *DNA* and some *RNA* combined with protein. Lengths of chromosomes constitute the *genes* and carry the *genetic code*. Chromosomes occur in pairs.

the nature of cell-division was one of maturation, i.e. pairs of chromosomes split, replicating each member before dispersing into four separate nuclei.

- **Walter Stanborough Sutton** (1877–1916, U.S.A.), biologist, and **Theodore Boveri** (1862–1915, Germany). Pointed out (1902) the parallelism between chromosome behavior and *Mendelism*, closing the gap between cytology and heredity. Sutton coined the name *gene* (1902), and proposed that chromosomes carry genes (factors which **Mendel** said could be passed from generation to generation).

1874 CE The first practical mechanical typewriter, by present-day standards, was commercially produced[479].

In 1868, **Christopher Latham Sholes** (1819–1890, USA), a Milwaukee senator and former postmaster, with his friend **Carlos Glidden**, an attorney, presented an improved version of their earlier (1867) invention. In 1873, E. Remington and Sons, a gun manufacturer, became interested in Sholes' typewriter and the company put the machine on the market in 1874. The first key-shift model was produced in 1878. The first successful *portable* typewriter appeared in the early 1900's. The *electric* typewriter came into use during the 1920's. Sholes devised the QWERTY keyboard which has been used from 1874 to the present day. It is the dominant survivor of dozen of keyboard designs that competed during the early years of the typewriter. The name derives from the arrangement of the letters in three rows:

Q W E R T Y U I O P
A S D F G H J K L
Z X C V B N M

[479] The first recorded typewriter patent was filed in 1714 by the British engineer **Henry Mill**, but there is no evidence that Mill actually built his proposed machine. The Italian **Pellegrino Turri** constructed a typewriter (1808), which allowed blind people to write more easily. Over the next six decades, several dozen inventors filed patents or built prototypes, but none of the machines entered mass production or attained commercial success.

1874 CE **George Johnstone Stoney** (1826–1911, Ireland). Physicist. Eccentric and original thinker. The first person to show how to deduce whether or not other planets in the solar system possessed a gaseous atmosphere, like the earth, by calculating whether their surface gravity was strong enough to hold on to one.

Postulated that an electric oscillator exists within the atom which generates its characteristic spectra. He called this oscillator "*electron*" and asserted that the magnitude of its charge is the same as that on a hydrogen atom during electrolysis.

In 1899, following the discovery of **J.J. Thomson** in 1897, **Lorentz** suggested that the name electron be given to the newly discovered particle.

In 1874, Stoney first discussed the possibility that there exist particular systems of units picked out by nature itself, what we might term 'natural units'[480]. To this end, he advocated the selection of natural constants that prevail throughout the universe such as the velocity of light, c (because it connects electrostatic and electromagnetic units); Newton's gravitation constant, G, and lastly, e, the unit of electric charge deduced from Faraday's Law. From these entities, a length, a mass, and a time can be constructed.

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[480] Compare with Planck's units (1906):

$$l_P = (\frac{G\hbar}{c^3})^{1/2} \sim 10^{-33} \text{ cm}; \qquad t_P = (\frac{G\hbar}{c^5})^{1/2} \sim 5 \times 10^{-44} \text{ s};$$

$$m_P = (\frac{c\hbar}{G})^{1/2} \sim 10^{-5} \text{ gm}.$$

Since the ratio $\frac{e^2}{\hbar c}$ is dimensionless and approximately equals $\frac{1}{137}$, we see that each Stoney's unit just differs from the corresponding Planck quantities by a numerical factor $\sim \frac{1}{\sqrt{137}}$.
For further reading, see:

- Davies, Paul, *The Accidental Universe*, Cambridge University Press, 1993, 139 pp.
- Cohen-Tannoudji, G., *Universal Constants in Physics*, McGraw-Hill, 1993, 116 pp.
- Rees, Martin, *Just Six Numbers*, Basic Books, 2000, 195 pp.
- Barrow, J.D., *The Constants of Nature*, Vintage, 2003, 352 pp.
- Seife, C., *Alpha and Omega*, Penguin Books, 2003, 294 pp.

The values of the units Stoney evaluated from these three standards were:

$$L_s = (\frac{Ge^2}{4\pi\epsilon_0 c^4})^{1/2} \sim 10^{-34} \text{ cm}; \qquad T_s = (\frac{Ge^2}{4\pi\epsilon_0 c^6})^{1/2} \sim 3 \times 10^{-45} \text{ s};$$

$$M_s = (\frac{e^2}{4\pi\epsilon_0 G})^{1/2} \sim 10^{-6} \text{ gm}.$$

These new natural units attracted little attention. There was no practical use for them and their significance was hidden to everyone, even Stoney himself. Natural units needed to be discovered all over again in the 20^{th} century.

Stoney was a professor at Queen's College, Galway (1852–1857) and Queen's University (1857–1882). His work included also investigations related to physical optics, molecular physics, kinetic theory of gases and planetary atmospheres.

Stoney was the uncle of **Geoge FitzGerald** and also an older distant cousin of **Alan Turing**.

1874–1888 CE **Henry Morton Stanley** (1841–1904, England). Explorer of Africa, discoverer of the course of the Congo River. With **David Livingstone** (1813–1873, England) made the African continent known to the world. Accomplished more geographical discoveries in Africa than any other explorer.

Stanley was born at Denbigh, Wales, and was baptized John Rowlands. His father died when the boy was two, and he spent most of his youth in an orphanage. At 18, he sailed as a cabin boy to New Orleans, La, where he was adopted by a merchant, Henry Morton Stanley, who gave him his name. When the Civil War began (1861), Stanley joined the Confederate Army but was soon captured. He later joined the Union Army.

After the war, Stanley became a newspaper reporter, and the New York Herald sent him to find Livingstone (1869). After many hardships he met Livingstone on Lake Tanganyika (1871), greeting him with the famous words: "Dr. Livingstone, I presume?" They stayed together until March 1872. In 1874 Stanley heard of Livingstone's death and returned to Africa to carry on his work. In November 1874 he left Zanzibar on a dangerous trip down the Congo River from its source to its mouth. The Congo region was rich in rubber and ivory, and Stanley tried to interest the British in the area. But he did not succeed, and, instead, the Belgian colonized the region as the Congo Free State. Stanley then led another expedition for the Belgians (1879–1883). In 1887 he made his last trip to Africa to rescue **Emin Phasha** during the African uprising. Stanley returned to Britain and served in Parliament until 1900.

The main achievements of Stanley are:

- Traced the Nile's source.
- Circumnavigated Lakes Victoria and Tanganyika, showing that the latter was an isolated lake.
- Traced the whole source of the Congo River.
- First to cross Africa from ocean to ocean.
- Traced Lake Albert's source to Lake Edward and identified the Ruwenzori Range as the fabled *Mountains of the Moon*.

1874–1895 CE **Jacobus Hendricus van't Hoff** (1852–1911, Holland). Chemist and physicist whose chief aim was the application of mathematics to chemistry. A founder of *stereochemistry* (1874). Discovered the laws of *chemical kinetics of weak solutions* and *osmotic pressure*[481] (1885–1886), for which he received the Nobel prize for chemistry in 1901.

Van't Hoff was born in Rotterdam. During 1869–1871 he studied at the polytechnic at Delft, in 1871 at the University of Leyden, in 1872 under F. Kekulé at Bonn, in 1873 at Paris and in 1874 at Utrecht. In 1878 he was appointed professor of chemistry in Amsterdam University, and in 1896 he went to Berlin, as professor at the Prussian Academy of Sciences.

In 1848 **Louis Pasteur** (1822–1895, France) discovered molecular asymmetry and demonstrated the existence of *optical isomers*: he had shown that a compound called sodium ammonium tartrate existed in two different crystalline forms. The two crystal types were identical to each other, except that they were mirror images, like right and left hands. They had identical properties, but solutions of one crystal would rotate polarized light in one direction and the other type in the opposite direction. This was among the earliest works dealing with 3-dimensional structure of molecules.

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[481] He stated that the *osmotic pressure* is given by $\delta p = (c_2 - c_1)\frac{kT}{v}$, where $\{c_2, c_1\}$ are the concentrations of solutions on both sides of the neutral semi-permeable membrane, k is the Boltzmann constant, T is the absolute temperature, and v is molecular volume of the pure *solvent*. In particular, if there is pure solvent on one side of the membrane $(c_1 = 0,\ c_2 = c)$, one arrives at van't Hoff's formula: $\delta p = \frac{nkT}{V}$, where n is the number of molecules of the *solute* in a volume V of solvent. (It is similar to the *Clapeyron formula*, if we replace gas pressure by osmotic pressure, volume of gas by volume of solution, and number of particles of gas by number of molecules of the solute.)

Osmosis plays an extremely important role in the world of animals and plants. Most of the partitions in living organisms and plants are semi-permeable. For example: the osmotic pressure in plant cells reaches several atmospheres, owing to which ground water can rise along the trunk of a tree to a large height. In the human body, osmosis plays an important part in the function of the kidneys. It also results in the transfer of water and various nutrients between the blood and the fluid of cells. Chemists use *reverse osmosis* to purify water.

The practical applications of osmosis were apparently known to **Moses** as early as ca 1230 BCE, for it is written in *Exodus* **15**, 23–25: "*And when they came to Marah they could not drink of the waters for they were bitter. And the people murmured against Moses, saying. What shall we drink? And he cried unto the Lord; and the Lord showed him a tree, which when he had cast into the waters, the waters were made sweet*".

Stimulated by Pasteur's work, van't Hoff, and independently **Joseph Achille Le Bell** (1847–1930, France), proposed an explanation for this phenomenon in 1874: the bonds formed by carbon could be considered as pointing to the corners of a regular tetrahedron, where it is attached to 4 different substituents. The molecule thus formed is *nonsuperposable* on its mirror image.

The discovery meant that in order to explain the constitution of certain organic compounds, the tridimensional arrangement of atoms in space must be taken into account.

When more than one C atom is considered, the possibilities become more complex, but they still remain mirror images of each other, identical in all properties except that they rotate polarized light in opposite directions. Such optically active compounds are of vital importance to the chemistry of life. This problem, however, could not be successfully attacked from the theoretical side until knowledge of the structure of atoms had been gained.

From 1874 to 1884 van't Hoff's attention was mainly given to the law of mass action; he classified and defined *orders of reactions*[482] in terms of number of molecules actively involved in the reaction. In 1884 he defined an index of *chemical affinity* as the maximum external work generated from a reversible isothermal reaction.

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[482] Reaction-rate (velocity) is the change in concentration of a reactant or product per unit time. A formula in brackets, for example $[A]$, represents the concentration of the indicated species in moles per liter. The reaction velocity is designated by $\frac{d}{dt}[A]$, and measured in mole/liter/min. Knowledge of factors which influence reaction velocity has practical consequences; such information includes its velocity and the temperature and concentration dependence of the velocity. An equation relating reaction rate $d[p]/dt$ and concentrations is called a *rate law*. The exponent of a concentration factor in the rate is the *reaction order* for that species. For example, the rate of oxidation of bromide ion by bromate ion in acidic aqueous solution

$$5Br^- + BrO_3^- + 6H^+ \rightleftharpoons 3Br_2 + 3H_2O,$$

is given by the law

$$\frac{d}{dt}[Br_2] = k[Br^-][BrO_3^-][H^+]^2,$$

where the rate constant k is fixed for a given temperature. The above reaction is of second order in hydrogen ions. The sum of the exponents is the total reaction order, namely 4. A rate law is a *differential equation*, that can be integrated. The order, and powers of individual species' concentrations in the rate, can be modified from their naive-counting values, e.g. by pre-equilibrium of fast steps in a reaction chain; Thus, in this example, we would have expected the powers of $[Br^-]$ and $[H^+]$ to be 5 and 6, respectively, had the reaction proceed in a *single* step. Empirical orders can be fractional and even *negative*.

From 1885 to 1895 he was engaged in the theory of solutions, and developed an analogy between dilute solutions and gases. He showed (1886) that the *osmotic pressure* of a solution is equal to the gas pressure which the solute would exert if all the solvent were removed, and the dissolved substance were left in the space in the condition of an ideal gas. The consequences of this theory had a remarkable influence on the progress of the science of biology.

1874–1896 CE **Marie-Esprit Léon Walras** (1834–1910, France). Mathematical economist. Made outstanding original contributions to modern economic theory; first to apply comprehensive mathematical analysis to general *economic equilibrium* in 'Elements d'economie politique pure' (1874–1877). Professor at Lausanne (1871–1892).

He showed how to formulate an independent system of equations relating to prices and quantities in all markets for the economy as a whole. These reflected conditions under which the market mechanism maximizes benefits and minimizes costs for the economy generally. Walras' model only recently became a source of inspiration for the new fast-developing specialization of mathematical economists.

1874–1909 CE **Karl Ferdinand Braun** (1850–1918, Germany). Physicist and inventor. Discovered the *crystal rectifier* (1874), and used it for the detection of radio waves (1901). Invented the *cathode*[483]*-ray tube* (1897). Discovered method in *wireless telegraphy* of boosting the outgoing signal at the sending station.

Another example is the reaction:

$$2HI \rightleftharpoons H_2 + 2I \rightleftharpoons H_2 + I_2$$

with its rate law

$$\frac{d[I_2]}{dt} = k_f[HI]^2 - k_r[H_2][I_2].$$

At equilibrium

$$k_f[HI]^2 = k_r[H_2][I_2],$$

yielding the equilibrium constant

$$K = \frac{k_f}{k_r} = \frac{[H_2][I_2]}{[HI]^2}.$$

In this example, the empirical reaction orders of the three relevant species are the same as their naive-counting values.

483 He painted the inside end of a glass tube with fluorescent paint; a cathode inside the tube emitted electrons which made the paint glow.

Braun was a professor of physics at the University of Tübingen (1885–1895), and director of the Physical Institute at Strasbourg (from 1895). Shared the 1909 Nobel prize for physics with G. Marconi.

One Earth — One Language (1670–1983)

1875 CE, May 20 *The Treaty of the Meter. An international conference signed a treaty to adopt* new *measurement standards for the kilogram and meter. Seventeen nations, including the United States, took part in the conference. The treaty set up a permanent organization, the International Bureau of Weights and Measures in Sèvere, France (IBWM).*

In the original metric system (1790), the unit of length equaled 10^{-7} *of the distance from the North Pole to the equator along the line of longitude going through Dunkirk, France and Barcelona, Spain. It was named* metre, *from the Greek word metron, meaning a measure.*

The unit of mass, the gram, was defined (1790) as the mass of a cubic centimeter of water at the temperature where it weights the most, namely at $4°C$ *(39°F).*

Before the development of the metric system, every nation used measurement units that had grown from local customs. However, the rapid development of science and technology made scientists realize that the 'tower of science' could not be built unless "the whole earth was of one language and of one speech" (Genesis **XI**, *1–6).*

Indeed, already in 1670, **Gabriel Mouton**, *the vicar of St. Paul church in Lyons, France proposed a decimal measurement system, namely the length of a minute (1/21,600) of the earth's circumference. In 1671* **Jean Picard** *(1620–1682), a French astronomer, proposed the length of a pendulum that swung once per second as a standard of length. Through the years, other people suggested various systems and standards of measurement.*

In 1790, the National Assembly of France requested the French Academy of Sciences to develop a standard system of weights and measures. A commission appointed by the Academy (including **Laplace**, **Lagrange**, **Lavoisier** *and* **Monge**) *proposed a system that was both simple and scientific. This became* *known as the* metric system, *and France officially adopted it in 1795 [but the government did not require the French people to use the new units until 1840].*

Thomas Jefferson (1743–1826), then the U.S. Secretary of State, recommended that the United States use a decimal system of measurement, but Congress rejected the idea.

In 1792, **Jean Baptiste Joseph Delambre** *(1749–1822, France) and* **Pierre Mechain** *(1744–1804, France), began their measurement of the arc of the meridian from Dunkirk to Barcelona.*

In 1821, **John Quincy Adams** *(1767–1848), the U.S. Secretary of State, proposed conversion to the metric system. Congress again rejected the proposal. In 1866 Congress made the metric system* legal *in the United States.*

The 1875 conference decided that units based on the size of the earth and mass of water are inaccurate for scientific purposes, and replaced it with a standard length demarced on a platinum-iridium bar and a standard mass of platinum-iridium.

In 1899, the new meter and kilogram standards, based on those adopted by the 1875 conference, were made and sent to all countries who signed the treaty. The kilogram standard was established in the form of cylinder made of a platinum-iridium alloy.

In 1960, the meter was redefined on the basis of the frequency of light emitted by a particular isotope of Krypton.

In 1975, the United States Congress passed the 'Metric Conversion Act' which called for voluntary *change over to the metric system. At that time, almost every country in the world had either converted to the system or planned to do so.*

In 1983, IBWM *settled on a new basis for the standard of length by making the meter exactly the distance traveled by light in vacuum in 1/299,792,458 seconds. From then on, every measurement of light's speed in vacuum, is by definition really a measurement of the* length *of one's laboratory distance-yardsticks in terms of the new* meter*!*

In 1992 it was discovered that the standard kilogram changed its mass by the amount 23 microgram. Consequently, a new mass standard will be established in the beginning of the 21st century.

1875–1876 CE **John Kerr** (1824–1907, Scotland). Physicist. Discovered the *electro-optical* Kerr effect (1875) and the *magneto-optical* Kerr effect (1876), among the first *non-linear* optical phenomena.

Kerr was born in Ardrossan, Ayrshire, the son of a fish merchant. He was educated at Glasgow University in theology and became a lecturer in mathematics at the Free Church Training College for Teachers, Glasgow (1857–1901), and set up a modest laboratory there. He was one of the first research students of **Kelvin**.

Amorphous substances (e.g. glass and other insulators, liquids with inversion symmetry such as carbon disulphite, paraffin oil, nitrobenzene etc.) which are isotropic under ordinary conditions, become *doubly refracting* (birefringent) when subjected to intense electric fields[484]. They then resemble uniaxial crystals with their optic axes parallel to the applied field.

Kerr showed that the effect was strongest when the plane of polarization was 45° to the field and zero when perpendicular or parallel. He found that the extent of the effect is proportional to the *square* of the applied field strength.

In the magneto-optical effect, a beam of plane polarized light was reflected from the polished pole of an electromagnet. The beam became elliptically polarized (with the major axis *rotated* from the original plane), when the magnet was switched on. The effect depended on the position of the reflecting surface w.r.t. the direction of magnetization and to the plane of incidence of the light.

In 1893 **F. Pockels** discovered a similar but much weaker effect in several crystals. Isotropic (cubic) crystals became uniaxial, and uniaxial crystals became biaxial, in a steady electric field of sufficient intensity, but the effect is *linearly*[485] proportional to the applied electric field.

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[484] Because the molecules tend to align under the influence of the electric field. The existence of the Kerr-effect makes it possible to construct an electrically controlled "light valve": a cell with transparent walls contains a liquid between a pair of parallel plates. The cell is inserted between crossed *Nicols*; light is transmitted when an electric field is set up between the plates and is cut off when the field is removed. Thus one may *modulate* the intensity of a light beam.

[485] The change in the refractive index n due to an applied electric field $\boldsymbol{E}$ is expressed as $\delta(\frac{1}{n^2}) = r|\boldsymbol{E}| + g|\boldsymbol{E}|^2$. The first term is linearly proportional to the applied electric field and is known as the *Pockels effect*. The second term, which has quadratic dependence on the applied electric field, is known as the *Kerr electrooptic effect*. While the Pockels effect depends upon the *polarity* of the applied electric field, the Kerr effect does not.

The most popular crystals which display the electrooptic effect and are used for electrooptic devices are:

1875–1885 CE **Eduard Suess** (1831–1914, Austria). Geologist. Argued (1875) that mountains and continents were formed not by vertical uplift but by thrusting movements that crumpled and broke outer portions of the earth's crust. Postulated (1883) the existence of *Gondwana*, a great southern continent that broke up to form Africa, Antarctica, Australia, India, and South America. Coined the name *biosphere* as that part of the earth in which life exists.

In his five-volume work *Das Antlitz der Erde* he attempted to explain many geological features in terms of the earth's contraction as it cooled.

Suess was born in London. Professor at Vienna (1857–1901). Liberal[486] member of the *Landtag* of Lower Austria (1869–1896) and of the *Reichsrat* (1872–1896).

Suess' tectonic synthesis is one of the most remarkable achievements of the beginning of the 20th century. For the first time, science acquired an elaborate survey of the *whole earth*, a description of all the irregularities of its crust, the mountains, the seas and lakes, the valleys, the river beds and deltas — an attempt to explain the deformations and foldings which led to the earth's present appearance.

1875–1920 CE **Luther Burbank** (1849–1926, U.S.A.). Naturalist, plant breeder and horticulturist. Developed many new trees, fruits, flowers, vegetables, grains and grasses. He also improved many plants and trees already known. Many common foods we eat every day come from his experiments. Among the plants he developed are the *Burbank potato*, the *Shasta daisy*, the *spineless cactus*, and the *blackberry*.

Burbank was born in Lancaster, Mass. He became a gardener to support his widowed mother. In 1875 he moved to California, and settled in Santa

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- *Lithium niobate* ($LiNbO_3$)
- *Lithium tantalate* ($LiTaO_3$)
- *Potassium dihydrogen phosphate* (KH_2PO_4), known as KDP.
- *Ammonium dihydrogen phosphate* ($NH_4H_2PO_4$), known as ADP.
- *Gallium arsenide* (GaAs).

486 Amidst the surging wave of antisemitism in Austria, Suess had the courage and the integrity to speak firmly in the Reichstrat (April 1894):
"What has been spoken, written, and done against the Jewish people during the last few years, has been a flagrant violation not only of our Constitution, but of the principles of human justice and Christianity."

Rosa. Stimulated by the works of Charles Darwin, he improved his crops by *crossing* and *selection*. He usually grew thousands of plants in the effort to produce one improved species.

The vast range of his experiments (he handled more than a million plants each year, and sometimes would perform more than a thousand simultaneous experiments), and his ability to detect and exploit even the smallest variation in a plant for a fast development of new properties — made him the greatest plant-creator ever. It is amazing that he could do so much without grasping the true *genetic significance* of his achievements, and despite his erroneous belief in the inheritance of acquired characteristics.

1876–1880 CE **Wilhelm Lexis** (1837–1914, Germany). Statistician-economist. A pioneer in the application of statistics to the social sciences.

Graduated from the University of Bonn (1859) in mathematics. Went to Paris (1861) to study social science and subsequently held positions at Strasbourg (1872), Dorpat (1874), Freiburg (1876), Breslau (1884) and finally Göttingen (1887).

1876–1886 CE **Eugen Goldstein** (1850–1931, Germany). Physicist. Performed many valuable experiments upon discharges through gases. Coined the names '*cathode rays*' (1876) and '*canal rays*' (1886). These were eventually shown to be *electrons* and *ions* respectively by Thompson (1897).

Goldstein was born at Gleiwitz. He was a pupil of Helmholtz and worked at the Potsdam Observatory from 1888. He was first to suggest that 'cathode-rays' emanating from the sun produce the *northern lights* and affect the magnetic field of the earth.

1876–1891 CE **Francois Edouard-Anatole Lucas** (1842–1891, France). Mathematician. Best known for his results in number theory (e.g. the converse of Fermat's little theorem[487]). In particular, he studied (1878) the *Fibonacci*

[487] *Fermat's little theorem* (1640), known as FLT, states that for every number a not divisible by the prime p, the congruence $a^{p-1} = 1(\text{mod } p)$ is satisfied. The examples

$$2^{340} \equiv 1(\text{mod } 341), \qquad 3^{90} \equiv 1(\text{mod } 91)$$

where $341 = 11 \cdot 31$ and $91 = 7 \cdot 13$ are sufficient to show that the converse of FLT is not generally valid. However, it was shown by Lucas (1876) that by *imposing additional restrictions* on the number a in the above congruence, it is possible to express a converse form of FLT.

The theorem of Lucas states: When for some number a the congruence $a^{n-1} \equiv 1(\text{mod } n)$ holds, while no similar congruence with a lower exponent $a^t = 1(\text{mod } n)$, $0 < t < n-1$ is fulfilled, the module n is prime.

sequence and the associated *Lucas sequence*[488] named after him. Devised methods of testing primality, and used them (1876) to prove that the Mersenne number $2^{127} - 1$ is prime (*Lucas test*[489]).

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488 The *Lucas sequence*: $L_n = a_{n-1} + a_{n+1}$, where a_n is the general term of the Fibonacci sequence. It follows that $L_n = L_{n-1} + L_{n-2}$, $L_0 = 2$, and consequently

$$L_n = \alpha^n + \beta^n, \quad \alpha = \frac{1}{2}(1+\sqrt{5}), \quad \beta = \frac{1}{2}(1-\sqrt{5}).$$

One can also show that if

$$F(x) = \sum_{n=1}^{\infty} a_n x^{n-1} \equiv (1 - x - x^2)^{-1}$$

then

$$L(x) = \sum_{n=1}^{\infty} L_n x^{n-1}.$$

489 If p is an odd prime, and $N = M_p = 2^p - 1$ is the corresponding Mersenne number, let $\{r_i\}$ be the sequence defined by $r_1 = 4$, $r_i = r_{i-1}^2 - 2$. Then N is prime if $r_{p-1} \equiv 0 (\text{mod } N)$ and otherwise composite.

Example: Suppose that we want to know if the 5^{th} Mersenne number $2^5 - 1 = 31$ is prime. We start with the number 4 and we repeatedly square it, and subtract 2. At each step, however, we reduce the result by taking only the remainder when it is divided by 31. So our test goes like this

$$\begin{aligned} r_1 &= 4 \\ r_2 &= 4^2 - 2 = 14 \\ r_3 &= 14^2 - 2 = 194 = 6 \cdot 31 + 8 \\ r_4 &= 8^2 - 2 = 62 = 2 \cdot 31 + 0. \end{aligned}$$

There is no remainder at the *fourth step*, and so the *fifth* Mersenne number is indeed a prime.

Lucas also sought algorithms for testing the primality of arbitrary numbers, not necessarily of the Mersenne type. He noticed that the sequence A_n defined by the recursion relation $A_{n+1} = A_{n-1} + A_{n-2}$ (known as the *Padovan sequence*) with initial values $A_0 = 3$, $A_1 = 0$, $A_2 = 0$ has the following bizarre property: Whenever n is a prime number, it divides A_n exactly! For example, $A_{19} = 209$ and $\frac{209}{19} = 11$. Until 1982 it was an open question whether or not the converse statement was true, since nobody had found such numbers (known as *Perrin pseudoprimes*).

Unfortunately Perrin pseudoprimes do turn out to exist! Adams and Shanks (1982) discovered the smallest one, $521^2 = 271441$ and J. Grentham proved

He used the algebraic identity

$$4x^4 + 1 = (2x^2 - 2x + 1)(2x^2 + 2x + 1)$$

to effect the factorization

$$2^{4n+2} + 1 = (2^{2n+1} - 2^{n+1} + 1)(2^{2n+1} + 2^{n+1} + 1)$$

with $x = 2^n$.

Lucas is also well known for his invention of the *Tower of Hanoi*[490] (1883) and other mathematical recreations.

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that there are many such pseudoprimes. The conjecture that no Perrin pseudoprimes exist was important, because the remainder on dividing A_n by n can be calculated very rapidly. If the conjecture were true this would have provided a speedy primality test and useful application to secret codes, which nowadays often hinge on properties of large primes.

490 Three pegs are fastened to a stand. There are n (usually 8) wooden discs, each with a hole in the center. The discs are of different radii, and at the start of the game all are placed on one peg in order of size, the biggest at the bottom. The problem is to shift the pile from one peg to another by a succession of steps, at each moving just one disc, and seeing to it that *at no stage is any disc underneath a larger one*. All three pegs may be used.

Let T_n be the minimum number of moves that will transfer the n discs from one peg to another under Lucas' rules. Then obviously $T_0 = 0$ (*no* moves are needed to transfer *no* disc), $T_1 = 1$ and $T_2 = 3$. Experiments with three discs show that the winning idea is to transfer the top two discs to the middle peg, then move the third, then bring the other two onto it. This gives us a clue for transferring n discs in general: We first transfer the $n-1$ smallest to a different peg (requiring T_{n-1} moves), then move the largest (requiring one move), and finally transfer the $n-1$ smallest back onto the largest (requiring another T_{n-1} moves). Thus we can transfer n discs (for $n > 0$) in *at most* $2T_{n-1} + 1$ moves:

$$T_n \le 2T_{n-1} + 1, \qquad \text{for } n > 0.$$

But there is no better way! At some point we must move the largest disc. When we do, the $n-1$ smallest must be on a single peg, and it has taken at least T_{n-1} moves to put them there. We might move the largest disc more than once, if we are not too alert. But after moving the largest disc for the last time, we must transfer the $n-1$ smallest discs (which must again be on a single peg) back onto the largest; this too requires T_{n-1} moves. Hence

$$T_n \ge 2T_{n-1} + 1, \qquad \text{for } n > 0.$$

These two inequalities, together with the trivial solution for $n = 0$, yield

$$T_0 = 0,$$
$$T_n = 2T_{n-1} + 1, \qquad \text{for } n > 0.$$

He was wounded as a result of a freak accident at a banquet when a plate was dropped and a piece flew up and cut his cheek, and he died of Erysipelas a few days later.

Lucas Sequences and Primes

*The number-sequences of **Fibonacci** (1202 CE), **Fermat** (1637 CE), and **Pell** (1668), among many others, occupy a central role in modern number theory. Many particular facts were known about these sequences; however, the general theory was first developed by **Lucas** in a seminal paper which appeared in Volume I of the American Journal of Mathematics (1878). It is a long memoir with a rich content, relating Lucas sequences to many interesting topics, like trigonometric functions, continued fractions, the greatest common divisor and primality tests. **R. D. Carmichael** (1913) corrected errors and generalized results.*

Consider the polynomial $f(t;P,Q) = t^2 - Pt + Q$ *where* (P,Q) *are nonzero integers. From its roots*

$$\alpha = \frac{1}{2}(P + \sqrt{P^2 - 4Q}) \qquad \text{and}$$
$$\beta = \frac{1}{2}(P - \sqrt{P^2 - 4Q})$$

Lucas constructed the sequence of numbers

$$U_n(P,Q) = \frac{\alpha^n - \beta^n}{\alpha - \beta}, \qquad V_n(P,Q) = \alpha^n + \beta^n \qquad n \geq 0$$

which are called the Lucas sequences associated with the pair (P,Q)*. They*

Substituting $U_n = T_n + 1$ we get $U_0 = 1$, $U_n = 2U_{n-1}$ for $n > 0$. A solution to this recurrence relation is $U_n = 2^n$, leading to $T_n = 2^n - 1$ for $n \geq 0$, which can easily verified by mathematical induction. Thus, the problem can always be solved in $2^n - 1$ steps. Assuming that the player can make one transfer every second, with never a mistake, he must work more than 500,000 *million years* for $n = 64$.

obey the recurrence relations

$$U_n = PU_{n-1} - QU_{n-2}; \qquad V_n = PV_{n-1} - QV_{n-2}$$

$$U_0 = 0, \qquad U_1 = 1; \qquad V_0 = 2, \qquad V_1 = P.$$

Special cases:

(1) $P = 1,\ Q = -1;\quad U_n = U_{n-1} + U_{n-2}$

$$U_n = 0, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, \ldots$$

$$\alpha = \frac{1}{2}[1 + \sqrt{5}], \quad \beta = \frac{1}{2}[1 - \sqrt{5}]$$

The $\{U_n\}$ *are immediately recognized as the Fibonacci numbers. They have the properties:*

- $F(x) = \sum_{n=1}^{\infty} U_n x^{n-1} \equiv (1 - x - x^2)^{-1}$
- $G(x) = \sum_{n=1}^{\infty} V_n x^{n-1} \equiv -\log(1 - x - x^2)$
- $(U_m, U_n) = U_{(m,n)}$, *where* $(,)$ *indicate the operation of taking the greatest common divisor:*

$$e.g. \quad (U_{45}, U_{30}) = (1,134,903,170; \quad 832,040) = 610 = U_{15}.$$

The associated series $V_n(1, -1)$ *yields the Lucas numbers*

$$2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, \ldots$$

(2) $P = 3,\ Q = 2;\qquad U_n = 3U_{n-1} - 2U_{n-2}$

$$U_n = 2^n - 1 \quad (\textit{Mersenne numbers}); \quad V_n = 2^n + 1.$$

(3) $P = 2,\ Q = -1;\qquad U_n = 2U_{n-1} + U_{n-2}$

$$\alpha = 1 + \sqrt{2}, \qquad \beta = 1 - \sqrt{2} \quad (\textit{Pell numbers})$$

The Pell numbers $(n = 0, 1, 2, 3, \ldots)$

$$U_n = 0, 1, 2, 5, 12, 29, 70, 169, 408, 985, 2378, 5741, 13860, \ldots$$

and their companions $(n = 0, 1, 2, 3, \ldots)$

$$V_n = 2, 2, 6, 14, 34, 82, 198, 478, 1154, 2786, 6726, 16238, 39202, \ldots$$

are associated with the solutions of the Pell equation

$$z_n^2 - 2x_n^2 = (-1)^n,$$

where $z_n = \frac{1}{2}V_n, \qquad x_n = U_n$ *and*

$$z_n = \frac{1}{2}\{[1+\sqrt{2}]^n + [1-\sqrt{2}]^n\},$$

$$x_n = \frac{1}{2\sqrt{2}}\{[1+\sqrt{2}]^n - [1-\sqrt{2}]^n\}.$$

(4) $P = 4,\ Q = 3$;

$$\alpha = 3, \qquad \beta = 1; \qquad\qquad U_n = \frac{1}{2}(3^n - 1), \qquad V_n = 3^n + 1.$$

(5) $P = 11,\ Q = 10$;

$$\alpha = 10, \qquad \beta = 1; \qquad\qquad U_n = \frac{1}{9}(10^n - 1), \qquad V_n = 10^n + 1.$$

The numbers $10^n + 1$ *are known as* repunits *(repeated units).*

Having defined his sequences, Lucas asked: for what values of p *and* a *do the sequences* $\frac{a^p-1}{a-1}$ *and* $\frac{a^p+1}{a+1}$ *yield prime numbers. We know, for example, that* $10^n + 1$ *yields prime numbers for* $n = 1, 2$*:*

$$10^1 + 1 = 1; \qquad\qquad 10^2 + 1 = 101.$$

Clearly, if n *contains an odd factor,* $10^n + 1$ *cannot be prime because* $10^{(2k+1)d} + 1$ *is divisible by* $10^d + 1$*. Factorizations for* $n > 2$ *then yield*

$$\begin{aligned}
10^3 + 1 &= 11 \times 91,\\
10^5 + 1 &= 11 \times 9091,\\
10^7 + 1 &= 11 \times 909091,\\
10^6 + 1 &= 101 \times 9901,\\
10^{10} + 1 &= 101 \times 99009901,
\end{aligned}$$

and

$$\begin{aligned}
10^4 + 1 &= 73 \times 137, \\
10^8 + 1 &= 17 \times 5882353, \\
10^{16} + 1 &= 353 \times 449 \times 641 \times 1409 \times 69857, \\
10^{32} + 1 &= 19841 \times 976193 \times 6187457 \times 834427406578561, \\
10^{64} + 1 &= 1265011073 \times 15343168188889137818369 \\
&\quad \times 515217525265213267447869906815873, \\
10^{128} + 1 &= 257 \times 15361 \times 453377 \times \text{ a prime of 116 digits.}
\end{aligned}$$

In general, machine calculations have yielded the following results for the Lucas sequences:

- $\frac{a^p-1}{a-1}$ *is prime for:*

$a = 2$; $p = 3$; 5; 7; 13; 17; 19; 31; 61; 89; 107; 127; 521; 607; 1279; 2203; 2281; 3217; 4253; 4423; 9689; 9941; 11,213; 19,937; 21,701; 23,209; 44,497; 86,243; 110,503; 132,049; 216,091; 756,839; 859,433; 1,257,787; 1,398,269; 2,976,221; 3,021,377; 6,972,593

$a = 3$; $p = 3$; 7; 13; 71; 103

$a = 5$; $p = 3$; 7; 11; 13; 47; 149; 181

$a = 6$; $p = 3$; 71; 127

$a = 7$; $p = 5$; 13; 131; 149

$a = 10$; $p = 2$; 19; 23; 317; 1031

$a = 11$; $p = 17$; 19; 73

$a = 12$; $p = 3$; 5; 97; 109

- $\frac{a^p+1}{a+1}$ *is prime for:*

$a = 2$; $p = 3$; 5; 7; 11; 13; 17; 19; 23; 31; 43; 61; 101; 127; 167; 191; 199; 313; 347; 701

$a = 3$; $p = 3$; 5; 7; 13; 23; 43; 281

$a = 5$; $p = 5$; 67; 101; 103

$a = 6; \quad p = 3; \; 11; \; 31; \; 43; \; 47; \; 59; \; 107$

$a = 7; \quad p = 3; \; 17; \; 23; \; 29; \; 47; \; 61$

$a = 10; \quad p = 5; \; 7; \; 19; \; 31; \; 53; \; 67$

$a = 11; \quad p = 5; \; 7$

$a = 12; \quad p = 5; \; 11$

1876–1916 CE **John William Strutt** (**Lord Rayleigh**, 1842–1919, England). Distinguished physicist. In 1904 he was awarded the Nobel prize in physics for his discovery (with **William Ramsay**) of Argon (1894).

Rayleigh's life-work included numerous contributions on a wide range of subjects in chemical physics, capillarity and viscosity, theory of gases, optics, photography, color vision, acoustics, electromagnetism, elasticity, hydrodynamics and mathematical physics. His treatise on sound includes much original work on diffraction and scattering (1877–1878).

In 1885 he predicted the existence of elastic surface-waves produced by natural and artificial sources in the earth that now bear his name: *Rayleigh waves.*

In 1900 he derived the blackbody radiation formula for long waves, known as *Rayleigh's radiation formula*, which later became the starting point for **Planck**'s quantum theory[491]. In 1892 he generalized the principles of *dimen-*

[491] *Rayleigh scattering* applies where the size of the scatterer is much smaller than the wavelength of the radiation, e.g.: scattering of light by molecules. For gases with a moderate number density N and refractive index n, the total scattering intensity per unit volume of material is given by $I = \frac{32\pi^3(n-1)^2}{3N\lambda^4}\langle S_0\rangle$, where $\langle S_0\rangle$, λ are respectively the mean energy flux density and the wavelength of the incident wave. It means that air molecules are more effective scatterers of the shorter wavelength (blue and violet) portion of the 'white' sunlight then the longer wavelength (red and orange) portion.
Thus, when we look in a region of the sky away from direct solar rays, we see predominantly *blue light* which was more readily scattered. On the other hand, the sun appears to have yellowish to reddish tint when viewed near the horizon; the solar beam must travel through a great deal of atmosphere before it reaches the observer. Hence most of the blue and violet will be scattered out, leaving a beam of light composed mostly of red and yellow (crimson). This latter phenomenon is particularly pronounced on a day when fine dust or smoke particles are present, or at sunset.

sional analysis as a logical procedure [*Phil. Mag.* **34**, 59–70].

In scientific stature, he is ranked alongside **Stokes** and **Kelvin**. The special feature of his work is its extreme accuracy and definiteness, combining highest mathematical acumen with refinement of experimental skill.

Possessing an immense range of knowledge, he has filled up lacunae in nearly every part of classical physics, and although he made no discovery which captured the popular imagination, he added analytic refinement to many branches of physics. His papers are often difficult to read but never diffuse or tedious, and his mathematical treatment is never needlessly abstruse, for when his analysis is complicated it is only because the subject-matter is so.

Rayleigh was born in Essex, the son of the second baron of a barony created in 1821 at George IV's coronation. He went to Trinity College, graduated as senior wrangler in 1865, and obtained the first Smith's prize of the year. He married in 1871 and from 1879 to 1884 was a Cavendish professor of experimental physics at the University of Cambridge, in succession to Clerk Maxwell. In 1887 he became a professor of natural philosophy at the Royal Institution of Great Britain. In 1908 he became the chancellor of Cambridge University.

Classical Thermal Physics

Thermal physics[492] *unites the disciplines of heat thermodynamics and statistical mechanics. Heat is a form of energy, and the science of heat deals with the changes in the properties of matter accompanying the transfer of energy through the mechanisms of work and the heat flow. It is an experimental science and the data obtained is represented by empirical laws, many of which can be justified a posteriori by theory.*

The name thermodynamics (from the Greek $\theta\varepsilon\rho\mu o s$ *= hot,* $\delta\upsilon\nu\alpha\mu i s$ *= force) is given to that branch of physics which deals with the relations between*

[492] For further reading, see:

- Kittel, C., *Thermal Physics*, John Wiley & Sons: New York, 1969, 418 pp.

thermal and mechanical energy — the transformations of heat into work and vice versa.

Thermodynamics is an axiomatic science, and a purely mathematical discipline. The laws governing the transformation of energy through work and heat are derived from a few basic postulates, and important relations are obtained between the properties of systems in thermal equilibrium.

Thermodynamics – both its equilibrium and near-equilibrium branches – theory contributes to the understanding of matter and the physical world: it provides quantitative values for various properties of matter, *it gives information about the* possibility and impossibility of processes *and shows the direction of* evolution *of a macroscopic system. It also provides methods for testing the* stability *of a given state of the system.*

Thermodynamics is with us on a daily basis. With its help, we can understand how our car functions, predict why water boils, and understand the formation of clouds, rain, or snow.

The power of thermodynamics lies in the fact that it describes and correlates directly observable properties of diverse substances. This is done without using any detailed *knowledge of the internal structure of the bulk matter. With relatively few laws and variables, an impressive number of remarkable conclusions can be drawn for complex systems containing a great number of individual molecules. The specific nature of substances enters into the theory via a few parameters, such as heat capacities or molar volumes.*

The science of thermodynamics introduces the new concept of temperature; *it is absent from classical mechanics, as well as from the theory of electricity and magnetism and from atomic physics*[493]*. This concept is introduced through the Zeroth Law of thermodynamics:* There exists a property — temperature — such that the equality of temperature is a condition for thermal equilibrium between two systems or between two parts of the same system.

Most empirical laws, however fall outside the scope of thermodynamics, and the irreversibility of thermal processes seems to violate the more basic laws of mechanics. The fact is that thermodynamics would be an empty discipline, with no application in nature, were not matter composed of a myriad of molecules, or at least a large number of degrees of freedom.

The basis for both the empirical laws of heat and the postulates of thermodynamics is found in statistical mechanics, *and the latter gives a concrete picture of the abstractions of thermodynamics, such as entropy, temperature, internal energy and other system variables.*

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[493] Historically, the subject of thermodynamics first arose *before* the atomic nature of matter was understood.

Mechanics is founded on certain general principles, such as the conservation of energy and momentum, that are applicable to the motion of interacting particles. When Newton's second law is translated into mathematical language, the solution of mechanical problem requires in turn the solutions of systems of 2^d *order ordinary differential equations, in which difficulties are encountered already with 3 interacting masses. Properties of matter in bulk (called* macroscopic *properties), as we ordinarily observe them, are the result of a collective actions of a large number of atoms and molecules*[494].

It is not only practically impossible, but also unnecessary to take into account the motions of each of these molecules in detail in order to determine the bulk properties of the matter, such as its pressure and temperature[495]. *Thus, to describe processes involving a very large number of particles, special methods must be devised. These methods are, by necessity, of a statistical nature. An important concept is that of the* probability of distribution *of the particles among the different dynamical states in which they may be found.*

A short historical survey is adequate at this point; Fire has fascinated and terrorized the human race throughout its history, but by the time of the great Ice Ages, humans had learned to tame fire into a constructive source of useful heat.

Until the end of the 18^{th} *century, fire was mainly used for heating, cooking, melting, and as a source of light. In some civilizations of antiquity, fire was a subject of worship and an agent of purification. In ancient Persia fire symbolized Ahura Mazda, the god of Zoroastrianism. In Greek mythology, Prometheus saved the human race by bringing the celestial gift of fire from the sun. The Aztec, Norse, and Hindu pantheons also had their gods of fire.*

That fire generates power can be seen by anyone watching a covered boiling kettle of water. ***Heron of Alexandria*** *made use of hot vapor in the construction of the first aeolipile (early gas turbine), which was used as a miracle agent in temples.*

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494 In one cubic cm of gas at STP there are about 3×10^{19} molecules. The collective behavior of such a gigantic number of particles is basically the result of their quantum-mechanical *electromagnetic interaction*, since gravitational interaction plays only a minor role and the strong and weak interactions affect mainly nuclear processes. Familiar processes, such as melting and vaporization, diffusion, viscosity, thermal and electrical conductivities, thermionic emission, heat capacity, latent heat, etc. fall in this category of collective properties.

495 *The temperature of a system in thermal equilibrium is a quantity related to the average kinetic energy per particle of the system*, the relation depending on the structure of the system.

The new industrial society at the turn of the 18^{th} *century needed coal at an ever increasing rate. Rising water in coal mines had to be eliminated, and muscle power was too slow and inefficient to do this.* ***Denis Papin*** *(1690) conceived the first vacuum-producing steam pump. A few years later such pumps were operational in English mines thanks to the ingenuity of* ***Thomas Savery*** *(1698) and* ***Newcomen*** *(1705). In* 1765, ***James Watt*** *modified the extremely inefficient Newcomen pump into a more efficient device by using a thermodynamic property, the* adiabatic expansion, *and also by introducing an automatic control inside the engine. The pump was transformed by* ***Fulton*** *(1807) into the first* steam engine. *This revolutionary discovery opened up the era of heat engines or* machines. *Engines drastically changed the nature of human societies, turning them into industrial societies. The development of heat engines was followed by a theoretical approach to the interrelationship between heat and mechanical motion.*

The idea that heat is a form of energy was first suggested by the works of ***Count Rumford*** *(1798) and* ***Davy*** *(1799). It was then stated explicitly by* ***R.J. Mayer*** *(1842), but gained acceptance only after the careful experimental work of* ***Joule*** *(1843 to 1849). The first theoretical analysis of heat engines was given by* ***Sadi Carnot*** *(1824), who thus became the founder of the new branch of macroscopic science — thermodynamics.*

Thermodynamic theory was formulated in consistent form by ***R.J.E. Clausius*** *and* ***Lord Kelvin*** *around* 1850, *and was greatly improved by* ***J.W. Gibbs*** *in several fundamental papers (1876–1878).*

The atomic approach to macroscopic problems began with the study of the kinetic theory of dilute gases. This subject was developed through the pioneering work of ***Clausius***, ***J.C. Maxwell*** *and* ***L.E. Boltzmann***. *Maxwell discovered the distribution law of molecular velocities in* 1859, *while Boltzmann formulated his fundamental integro-differential* transport equation *in* 1872. *The kinetic theory of gases assumed its modern form when* ***S. Chapman*** *and* ***D. Enskog*** *(1916–1917) approached the subject by developing systematic methods for solving the Boltzmann equation.*

The more general discipline of statistical mechanics also grew out of the work of ***Boltzmann***, *who (1872) further succeeded in giving a fundamental microscopic analysis of irreversibility and the approach to equilibrium. He was first to give the probabilistic interpretation of entropy.*

The theory of statistical mechanics was then developed further by the contributions of ***J.W. Gibbs*** *(1902). Although the advent of quantum mechanics has brought many changes, the basic framework of the modern theory is still the one which Gibbs formulated.*

Beginning in the 1970*'s, physicists recognized the close mathematical affinity between the statistical mechanics of condensed matter and fluctuations of* *fields in the second-quantized vacuum, and began applying this connection to advance both disciplines.*

1876–1902 CE **Josiah Willard Gibbs** (1839–1903, U.S.A.). Theoretical physicist and chemist. Among the most prominent scientists produced by the United States.

In a path-breaking paper: "On the Equilibrium of Heterogeneous Substances" (1876–1878) Gibbs applied the principles of thermodynamics to the determination of chemical equilibrium (of chemical reactions rates). In this he helped lay the foundations of chemical thermodynamics and modern physical chemistry. He actually converted large parts of the physical chemistry of his day from an empirical to a deductive science. The importance of this work was soon recognized by Maxwell. The new concepts of '*free energy*' and '*chemical potential*' were introduced by Gibbs. Although Gibbs performed few experiments, his theory led to such practical results as the production of ammonia, dyes, drugs and plastics.

In his work he preferred a laconic, mathematician's style, making sure to say what was necessary for the logical structure of his argument — and little more. His spare and abstract style, and unwillingness to include a variety of examples and applications to particular experimental situations, made his work very difficult for potential readers. As a consequence, the literature of the 19^{th} century contains many rediscoveries of results already published by Gibbs. Such major figures as **Helmholtz** and **Planck** independently developed their own thermodynamic methods for treating chemical problems, quite unaware of the treasures concealed in his 1876–1878 paper[496].

While for Clausius and his contemporaries, thermodynamics was the study of heat and work, Gibbs eliminated these concepts from the foundations of the subject in favor of *state functions* — energy and entropy — and thermodynamics became the theory of properties of matter at equilibrium. Among other innovations, he gave an explicit derivation to Liouville's equation.

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[496] In 1892 Rayleigh wrote to Gibbs urging him to expand on his ideas, saying that the original memoir was "*too condensed and too difficult for most, I might say all, readers*". Gibbs answered that he thought that his paper did seem "*too long*"....

In 1884 Gibbs coined the name '*statistical mechanics*', but he had not built molecular concepts into his papers on thermodynamics because he "*had no need for that hypothesis*", to paraphrase Laplace.

Gibbs is the father of our present-day '*vector analysis*'[497] (1881). He abstracted the vector and scalar concepts from the framework of Hamilton and Grassmann (thus disentangling them from the quaternion idea), and put them within a structure convenient to geometry and physics[498]. Similarly, he constructed the algebra and calculus of second-rank tensors (known as dyadics) on the basis of Grassmann's 'gap' products. From the point of view of physics, the Gibbs' vector calculus was a major simplification and improvement of Hamilton's quaternions. When attacked by the quaternionophil P.G. Tait, he replied: "*The world is too large, and the current of modern thought is too broad, to be confined to the* '*ipse dixit*' *even of a Hamilton*".

In 1886 he emphasized the superior generality of Grassmann's indeterminate product in dyadic and matrix algebra, over the unique product insisted upon by Hamilton. The vectors of Gibbs gradually displaced quaternions as a practical applied algebra. In the end, however, quaternions returned to physics under the guise of *Pauli matrices*, representing the action of the angular-momentum operators on *quantum-mechanical spinors*.

Gibbs was born in New Haven, Connecticut. His father was a professor of sacred literature in Yale Divinity School. He entered Yale College in 1854, graduated in 1858, and received his doctorate of engineering in 1863. He taught Latin and natural philosophy until 1866, when he went to Europe, studying in Paris (1866–1867), Berlin (1867), and Heidelberg (1868). Returning to New Haven in 1869, he was appointed professor of mathematical physics at Yale College in 1871[499], a position he held until his death. His interest in thermodynamics arose while endeavoring to improve the governor of James Watt's steam engine. In analyzing its equilibrium, he began to develop methods by which equilibriums of chemical processes could be calculated.

Gibbs remained a bachelor, living in the household of his surviving sister. He was a man of few words. Once, at a gathering of scientists, he was asked to give a talk on the subject "The role of mathematics in the physical sciences". He rose and issued just four words: "*Mathematics is a language*".

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[497] For further reading, see:

- Gibbs, J.W., *Vector Analysis*, Dover, 1960, 436 pp.

[498] Gibbs introduced the *notation* $\nabla \cdot \boldsymbol{a}$, $\nabla \times \boldsymbol{a}$ for the divergence and curl of a vector field $\boldsymbol{a}$, respectively.

[499] Yale University refused for seven years to pay a salary to Willard Gibbs, already famous in Europe, on the ground that his studies were "non relevant".

Origins of Classical Statistical Physics[500] *(1850–1902)*

In 1865, ***Joseph Loschmidt***[501] *(1821–1895) gave the first estimate of Avogadro's number N (number of molecules in 22.4 liters of gas at standard temperature and pressure). Calculating from the newly developed kinetic theory of gases, he obtained the approximate value of* $N = 6 \times 10^{23}$*. Thus, under conditions prevailing (say) in a living room, the number of molecules in 1 cm*^{3} *of nitrogen gas is of the order of* 10^{20}*. This astronomical size of N is the major reason why attempts to arrive at thermophysical results from a purely mechanical point of view are foredoomed to fail without the use of statistics.*

Although a number of statistical problems (e.g., the explanation of some properties of gases on the basis of the notion of molecular motions), were considered by ***Newton****,* ***D. Bernoulli****, and a number of other scientists back in the* 18^{th} *century, the appearance of statistical physics as an independent branch of physics dates to the second half of the* 19^{th} *century.*

In 1857, ***Clausius*** *clearly indicated that heat energy is the kinetic energy of random motions of molecules.*

In 1859 he introduced the useful concept of the mean free path and gave a correct molecular-kinetic explanation of the phenomena of thermal conductivity and viscosity. Also in 1859, ***Maxwell*** *fused together statistical ideas with those of mechanics in a kinetic theory that resulted in the law of distribution of the velocities of gas molecules, that now bears his name. In his kinetic theory he analyzed events involving single molecules, making special assumptions about the nature of inter-particle forces, while assigning to probability notions a mere subsidiary role.*

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500 To dig deeper, see:

- Feynman, R.P., *Statistical Mechanics*, Perseus Books, 1998, 354 pp.
- Ruhla, C., *The Physics of Chance*, Oxford University Press, 1992, 222 pp.
- Harris, S., *An Introduction to the Theory of the Boltzmann Equation*, Dover, 2004, 221 pp.
- Brown, A.F., *Statistical Physics*, Edinburgh University Press: Edinburgh, 1968, 307 pp.
- Zeldovich, Ya.B. et al., *The Almighty Chance*, World Scientific, 1990, 316 pp.

501 In the same year Loschmidt also obtained estimates of *molecular diameters* from measurements of liquid density and gaseous viscosity.

A further fundamental development was due to ***Boltzmann*** *(1877). Instead of scrutinizing separate microscopic events, he supplemented the mechanical laws of general validity by far-reaching* probability hypotheses *of comparable importance, thus subjecting the atomic particles themselves to statistical analysis. This choice was made by* ***Maxwell*** *as well in his kinetic theory, but his efforts were confined to an examination of the distribution of particles w.r.t. their velocity components only.*

Boltzmann generalized this procedure to encompass the distribution of particles in relation to position coordinates as well. ***Gibbs*** *(1884–1902) crowned the achievements of Clausius, Maxwell, and Boltzmann with decisive researches of his own. In his works, statistical physics obtained a fundamental substantiation suitable for arbitrary systems, and not only gaseous ones.*

The Gibbs ensemble *is treated at present as a fundamental principle whose role in statistical physics can be compared with that played by Newton's equations in classical mechanics or by Maxwell's equations in electrodynamics. Gibbs' book* Elementary Principles in Statistical Mechanics *(1902) played the same role in statistical physics as Maxwell's* Treatise *did in electrodynamics; the molecular-kinetic substantiation of the phenomenological science of classical thermodynamics, commenced by Boltzmann, was completed by Gibbs.*

Thus, a new discipline was established that succeeded in deriving the facts of phenomenological thermodynamics from postulates more fundamental *than the laws of thermodynamics. Moreover, it also provided numerical values for individual macroscopic properties, and finally alerted us to the possibility of rarely occurring fluctuation effects — a direct result of the conceptual researches initiated mainly by* ***Maxwell***, ***Boltzmann***, ***Gibbs*** *and* ***Einstein***. *This branch of physics, called* statistical thermodynamics, *seeks to deduce the thermodynamic properties of matter and energy from the laws of governing the behavior of its microscopic, or atomic, constituents.*

Let us highlight two fundamental ideas of this approach, beginning with Gibbs' derivation of Liouville's theorem, *using concepts of* Hamiltonian dynamics.

Consider a closed system of gas (molecules, electrons, stars) described by N generalized coordinate vectors $\boldsymbol{q}_1, \ldots, \boldsymbol{q}_N$, and generalized momenta $\boldsymbol{p}_1, \ldots, \boldsymbol{p}_N$. The $6N$-dimensional space spanned by the vectors $(\boldsymbol{p}_1 \cdots \boldsymbol{p}_N;\ \boldsymbol{q}_1 \cdots \boldsymbol{q}_N)$ shall be called the Γ-space or the phase space of the system. A point *in Γ-space, having $6N$ scalar coordinates, represents a state of the entire gas at a specific moment, and is known as a* representative point *or the* phase point *of the system. In terms of this phase point in Γ-space, the temporal development of the mechanical system, whose state is represented by it, can be surveyed geometrically.*

The position of a phase point at a given time t_0, corresponding to some initial state of the system, might be a matter for arbitrary decision. Once this choice has been made, Hamilton's equations of motion, viz., $\dot{q}_i = \frac{\partial H}{\partial p_i}$, $\dot{p}_i = -\frac{\partial H}{\partial q_i}$ $(i = 1, 2, \ldots, 3N)$, *uniquely determines the position of the phase point at any other (earlier or later) time t. The phase point therefore describes in the course of time a curve in Γ-space, known as a* phase orbit *or* trajectory*; due to the uniqueness of the solutions of the above equations, each point in Γ-space is traversed by only one trajectory — that is, trajectories cannot intersect themselves or one another. Each position of the phase point at any given time describes a* microstate *of the system. Since the gas molecules are continually in motion, every conceivable microstate (compatible with constraints and conservation laws) will be approached arbitrarily closely sooner or later if the allowed Γ-space region is finite in volume.*

*A complete specification of the system's state (phase point) requires knowledge of the motions and positions of all microscopic particles at some time. In practice, however, we do not have (and are not interested in) the detailed information that is required to specify a particular microstate. We are usually observing the "*average*" behavior of a system with a given set of* macroscopic *properties (such as density distribution, velocity profile, pressure, temperature, etc.) which constitute a* macrostate *of the system.*

It is obvious that a very large number of microstates all correspond to a given macroscopic condition of the gas. Through macroscopic measurements we would not be able to distinguish between two different microstates that satisfy the same macroscopic condition. Thus when we speak of gas under certain macroscopic condition, we are in fact referring to a practically infinite number of microstates. In other words, we refer to a collection of systems, identical in composition and macroscopic conditions but existing in different microstates. Gibbs called such a collection of systems an ensemble.

Each member of the ensemble is a virtual *copy of the* real *system. The virtual systems are, of course, not identical in all respects. Indeed, the similarity extends only as far as the Hamiltonian functions of the systems, and the virtual systems may differ vastly among themselves and from the real system with respect to the configuration of the* velocities and positions of their particles*. Thus each member of the ensemble is an independent system* with its own phase point*. So we turn our attention from the individual phase point representing the real system to the assembly of phase points in Γ-space representing the totality of every system of the ensemble.*

Because these systems differ in their microscopic states, their corresponding phase points will correspondingly occupy different positions, so that the assembly of points will spread out in a "cloud" over a finite region in Γ-space.

This diversity reflects our ignorance about the microstate of the real system that serves as a prototype for the construction of the virtual systems.

One of the main goals of statistical thermodynamics is to find the correct statistical distribution of phase points under given macroscopical physical conditions (mechanical, chemical and thermal forces, initial and boundary conditions, etc.). To achieve this goal, one must first inquire how an assembly of phase points, initially arranged in phase space in an arbitrary manner, will evolve with the passage of time.

The situation may be conveniently described by a density function $\rho(p, q, t)$, *where* (p, q) *is an abbreviation for* $(\boldsymbol{p}_1 \cdots \boldsymbol{p}_N;\ \boldsymbol{q}_1 \cdots \boldsymbol{q}_N)$, *so defined that* $\rho(p, q, t)d^{3N}pd^{3N}q$ *is the expected fraction of representative points which at time* t *are contained in an infinitesimal volume element* $d^{3N}p\, d^{3N}q = dq_1 dq_2 \cdots dq_{3N} dp_1 dp_2 \cdots dp_{3N}$ *of* Γ*-space centered about the point* (p, q).

An ensemble is completely specified by $\rho(p, q, t)$: *Given* $\rho(p, q, t)$ *at any given time, the evolution of the random phase point with time is governed by the Hamiltonian* $H(p_1 \cdots p_{3N};\ q_1 \cdots q_{3N})$ *through to the equations of motion*

$$\dot{p}_i = -\frac{\partial H}{\partial q_i}, \quad \dot{q}_i = \frac{\partial H}{\partial p_i} \quad (i = 1, \ldots, 3N).$$

Since H *does not depend on time derivatives of* p *and* q, *these Hamilton equations guarantee that the locus of a phase point is either a simple closed curve or a curve that never intersects itself. Moreover, since the total number of microstates in an ensemble is conserved, the number of phase points leaving an arbitrary volume* ω *in* Γ*-space with surface* S *must be equal to the rate of decrease of the number of phase points in the same volume. Hence*

$$\frac{d}{dt}\int_\omega \rho d\omega = \int_S dS(\boldsymbol{n} \cdot \boldsymbol{V}\rho),$$

where $\boldsymbol{V}$ *is a* $6N$*-dimensional vector with components*

$$\boldsymbol{V} = (\dot{p}_1, \dot{p}_2, \ldots, \dot{p}_{3N};\ \ \dot{q}_1, \dot{q}_2, \ldots, \dot{q}_{3N}),$$

and $\boldsymbol{n}$ *is the vector locally normal to the surface* S. *Using the divergence theorem in* $6N$*-dimensional space, we obtain the continuity equation for the phase-space density function:*

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{V}) = 0,$$

where ∇ *is the* $6N$*-dimensional gradient operator*

$$\nabla \equiv \left(\frac{\partial}{\partial p_1}, \frac{\partial}{\partial p_2}, \ldots, \frac{\partial}{\partial p_{3N}};\ \ \frac{\partial}{\partial q_1}, \frac{\partial}{\partial q_2}, \ldots, \frac{\partial}{\partial q_{3N}}\right).$$

Performing the indicated differentiations and using the equations of motion in the form

$$\frac{\partial \dot{p}_i}{\partial p_i} + \frac{\partial \dot{q}_i}{\partial q_i} = 0,$$

there emerges the mathematical formulation of Liouville's theorem:

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + \sum_{i=1}^{3N} \left(\frac{\partial \rho}{\partial p_i}\dot{p}_i + \frac{\partial \rho}{\partial q_i}\dot{q}_i \right) = 0.$$

Geometrical interpretation: if we 'ride' on the trajectory of a phase-point in Γ*-space, we will at all times measure the same density of representative points in its neighborhood. Hence the distribution of phase-points moves in* Γ*-space like an* incompressible fluid. *Also, the co-moving (Euclidean) spatial volume element* $d^{3N}pd^{3N}q$ *changes its* shape, *but retains its* volume *throughout.*

If the virtual systems are required to be closed *and* conservative[502] *[so that* H *does not depend explicitly on time, and can be put equal to a constant* E*, the energy of the system], and if* $\rho = \rho(H)$ *it then follows that* $\frac{\partial \rho}{\partial t} \equiv 0$ *or* $\rho = \rho(p, q)$*. This means that the density in phase space does not vary with time and depends only on energy.*[503]*The ensembles defined in this way is called a* stationary ensemble, *and is said to be in* statistical equilibrium. *The equilibrium situation is thus guaranteed at all times if the phase points at an arbitrary instant* t_0 *are distributed in* Γ*-space with a density* $\rho(p, q, t_0) = \rho(H)$.

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502 In Gibbs' large number of similar simultaneous systems (ensemble), all the gases are composed of the same number of molecules as the gas in the real system, and they are placed in vessels having the same shape. The energies of the different systems, however, are allowed to extend from E to $E + dE$, where dE is very small. This spread of energy is essential to the proof of Liouville's theorem, but once the incompressibility has been established, we may restrict our attention to the systems of energy E.

503 This means that a sufficiently long time has elapsed so that macroscopic equilibrium is achieved, and it assumes there are no other conservation laws but energy, or that if there are, the system constraints prevent the system from exchanging the other conserved quantities with its environment. It is also assumed that *the phase point spends on average equal amounts of time* in all microstates compatible with the conservation laws (*ergodic hypothesis*). If the system is allowed to exchange with its environment other conserved quantities (molecules of various species, volume, electric charge etc.), then ρ will in general depend on *all* such conserved quantities, not just H.

Note that $\rho(p,q)$ *is actually the probability per unit volume that a phase point be found in an infinitesimal volume of the* Γ*-space. Being an* N*-particle distribution function we change its notation to* f*, where*

$$\int f(\boldsymbol{W}_1, \boldsymbol{W}_2, \ldots, \boldsymbol{W}_{_N}, t) d^6\boldsymbol{W}_1 \cdots d^6\boldsymbol{W}_{_N} = 1$$

and $\boldsymbol{W}_i \equiv (\boldsymbol{q}_i, \boldsymbol{p}_i)$ *denote the location of an individual particle in its 6-dimensional phase subspace. If the forces are conservative*

$$d\boldsymbol{p}_i/dt = -\partial\Phi_i/\partial\boldsymbol{q}_i,$$

where Φ_i *is the potential at particle* i *due to the other particles. Liouville's theorem then assumes the form*

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{N}\left[\boldsymbol{p}_i \cdot \frac{\partial f}{\partial \boldsymbol{q}_i} - \frac{\partial \Phi_i}{\partial \boldsymbol{q}_i} \cdot \frac{\partial f}{\partial \boldsymbol{p}_i}\right] = 0.$$

A special case of Liouville's theorem is known as the collisionless Boltzmann equation. Boltzmann, unlike Gibbs, used a 6-dimensional (not $6N$*-dimensional) phase space of a single representative molecule, which* ***Paul*** *and* ***Tatyana Ehrenfest*** *(1912) later termed the* μ*-space. A point in this space is* $\boldsymbol{W} = (\boldsymbol{q}, \boldsymbol{p})$*, and the velocity of its flow is given by the 6-vector* $\dot{\boldsymbol{W}} = (\boldsymbol{p}, -\nabla\Phi)$*.*

With this notation, the above Liouville's theorem is simplified to

$$\frac{Df}{Dt} \equiv \frac{\partial f}{\partial t} + \boldsymbol{p} \cdot \nabla f - \nabla\Phi \cdot \frac{\partial f}{\partial \boldsymbol{p}} = 0.$$

Given Φ*, it is a differential equation for the unknown distribution function* $f(\boldsymbol{r}, \boldsymbol{p}, t)$*. Here,* $\frac{Df}{Dt}$ *represents the rate of change of the density of phase points as seen by an observer who moves through phase space with a molecule at velocity* $\dot{\boldsymbol{W}}$*, and* $\frac{Df}{Dt} = 0$ *implies that the phase-space density* f *around the moving phase point of a given molecule remains the same.*

Jeans *(1919) first applied the Boltzmann collisionless equation to stellar dynamics, where the role of gas molecules is played by non-colliding stars. Integrating the equation over all possible velocities, he assumed that:*

(1) the range of velocities over which we are integrating does not depend on time,

(2) $\boldsymbol{\Phi}$ *does not depend on* $\boldsymbol{p}$*,*

(3) $f(\boldsymbol{r}, \boldsymbol{p}, t) = 0$ *for sufficiently large* $|\boldsymbol{p}|$ *(there are no stars that move infinitely fast).*

This led him directly to the continuity equation (obtained by integrating the Boltzmann equation over all $\boldsymbol{p}$*)*

$$\frac{\partial \nu}{\partial t} + \operatorname{div}(\nu \boldsymbol{u}) = 0$$

where

$$\nu = \int f d^3\boldsymbol{p}, \qquad \boldsymbol{u} = \frac{1}{\nu}\int f\boldsymbol{p}d^3\boldsymbol{p}.$$

Multiplying the Boltzmann equation by $\boldsymbol{p}$ *and integrating again over all momenta (subject to the former assumptions), one arrives at the analog of Euler's equation of fluid flow:*

$$\nu\frac{\partial \boldsymbol{u}}{\partial t} + \nu\boldsymbol{u}\cdot\nabla\boldsymbol{u} = -\nu\nabla\Phi - \operatorname{div}\left[\nu(\mathfrak{Q} - \boldsymbol{u}\boldsymbol{u})\right],$$

where $\mathfrak{Q} = \frac{1}{\nu}\int \boldsymbol{u}\boldsymbol{u}fd^3\boldsymbol{p}$ *is a symmetric stress tensor.*[504]

When encounters of molecules (stars) are taken into account, the phase-space density of individual molecules changes with time and we may write $\frac{Df}{Dt} = M(f)$*, where the collision term* M *denotes the co-moving rate of change of* f *due to encounters (collisions). Boltzmann derived the explicit form of* M *under the assumptions:*

(1) only binary elastic collisions are taken into account (dilute gas);

(2) the walls of container are ignored;

(3) the effect of the external forces on the collision cross-section is ignored;

(4) the velocity of a molecule is uncorrelated with its position (molecular chaos, valid for sufficiently low gas densities).

When these assumptions are translated into mathematics, and the physics of elastic binary collision is applied, the end result is the Boltzmann transport equation:

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\boldsymbol{\nabla}_{\boldsymbol{r}} + \frac{\boldsymbol{F}}{m}\cdot\boldsymbol{\nabla}_{\boldsymbol{v}}\right)f = \frac{s^2}{2}\int |\boldsymbol{U}\cdot\boldsymbol{e}|(f'f_1' - ff_1)d\omega d^3\boldsymbol{v}_1,$$

where $\boldsymbol{U} = \boldsymbol{v}_1 - \boldsymbol{v}$, $\quad -\nabla\Phi = \frac{\boldsymbol{F}}{m}$, $\quad d\omega =$ *solid angle element about the vector* $\boldsymbol{v}_1$*. Here* s *is the cross-section radius of a molecule,* $\boldsymbol{e}$ *is an arbitrary*

[504] Under some further, often reasonable approximations, macroscopic continuum-mechanics PDE's such as the Navier-Stokes (NS) equation of fluid dynamics, can be derived via momentum-averaging techniques such as described above. The extra term in the NS equation (absent in Euler's equation) manifests viscosity, which is a molecular-collision effect.

unit-vector [that may be taken as $\boldsymbol{e} = (0,0,1)$ *without loss of generality], and*

$$f = f(\boldsymbol{r}, \boldsymbol{v}, t); \qquad f_1 = f(\boldsymbol{r}, \boldsymbol{v}_1, t);$$
$$f' = f\,[\boldsymbol{r}, \boldsymbol{v} + (\boldsymbol{U}\cdot\boldsymbol{e})\boldsymbol{e}, t];$$
$$f_1' = f\,[\boldsymbol{r}, \boldsymbol{v}_1 - (\boldsymbol{U}\cdot\boldsymbol{e})\boldsymbol{e}, t].$$

The mathematical problem of the kinetic theory of gases, now consists of the solution of the preceding non-linear integro-differential equation.

Returning to Gibbs' $6N$*-dimensional* Γ*-space, and having discussed the law of time-evolution of phase points in an ensemble, the next logical step is the establishment of a correspondence between a given* real system *and a suitable virtual ensemble. In other words, we must specify mathematically what is meant by the* average *behavior of a macroscopic system.*

There are two types of averages that are of interest. The first of these is the ordinary average at a given time *over all systems of the ensemble, known as the* ensemble average.

The second average of interest is the average of an observable entity for a given system of the ensemble *over some very large time interval.*

The ergodic[505] *hypothesis, first advanced by* **Boltzmann** *(1887), states that for* stationary *time processes (in which there is no preferred origin in time for the statistical description of observable entities, i.e., the ensemble is invariant under a time shift) the two averages are the same.*

The ergodic hypothesis *expresses the central assumption of classical as well as quantum statistical thermodynamics.*[506] *Attempts to prove it have given rise to the renowned* ergodic problem, *which has bedeviled eminent physicists and mathematicians for the past century.*

So far this hypothesis has been proved for a time average over an infinitely long time (both in classical and quantum mechanics) under certain assumptions that are too abstract to be easily stated and that remain to be justified. In physical experiments, however, we do not average over an infinite time, but over a finite time that is very short by macroscopic standards. It is plausible that this time interval can be effectively considered infinite because it is to be compared with characteristic molecular times, e.g., molecular collision mean-free time.

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505 The term *ergodic* is a combination of the Greek words for *energy* and *path*.

506 In quantum statistical thermodynamics – of which the classical version is a mere approximation – the *phase-space-density* function f is replaced with a *Hilbert space* density operator $\hat{\rho}$, and the Boltzmann transport equation is replaced by the *Fokker-Planck quantum Master-equation*.

***Von Neumann** (1931–1932) was able to formulate a single necessary and sufficient condition for the validity of the ergodic hypothesis. At present, therefore, the ergodic problem appears to have been reduced to the problem of demonstrating that the von-Neumann condition is fulfilled.*

The Rise of the 'New World', III
The Coming of American Technology (1876–1966)

*One hundred years after its inception, the United States of America began the marathon race for world supremacy in technology with the inventions of the telephone (**Bell**, 1876), the phonograph (**Edison**, 1877), the light bulb (**Edison**, 1884), the movie camera (**Edison**, 1889), the vacuum tube (**de Forest**, 1907) and finally the advent of the mass-produced automobile (**Ford**, 1908).*

*The Michelson-Morley experiment (1887), the establishment of the astronomical observatories at Lick (1888) and Yerkes (1900) and the experiments of **Millikan** (1910) have put the United States in the first league of the world's efforts in physics and astronomy. During the second half of the* 19^{th} *century, the U.S.A. continued to share in the leading trends of Europe. American scholars were trained at foreign universities. However, toward the end of the century, cultural exchange between Europe and the United States became less one-sided.*

By the early 1960's America had reluctantly come to realize that it possessed, as a nation, the most potent scientific complex in the history of the world. Eighty per cent of all scientific discoveries in the preceding three decades had been made by Americans. The United States had 75 per cent of the world's computers, and 90 per cent of the world's lasers. The United States had three and a half times as many scientists as the Soviet Union and spent three and a half times as much money on research; the U.S. had four times as many scientists as the European Economic Community and spent seven times as much on research. Most of this money came, directly or indirectly, from Congress, and Congress felt a great need for men to advise them on how to spend it.

1876 CE **Alexander Graham Bell** (1847–1922, U.S.A.). Scientist, educator and inventor. Bell was born in Edinburgh, Scotland and educated at the University of Edinburgh and the University of London. He moved with his father to Canada in 1870. In 1872 he became a professor of vocal physiology in Boston University. In 1876 he exhibited an apparatus embodying the results of his studies in the transmission of sound by electricity[507], and this invention, with improvements and modifications, constitutes the modern commercial telephone.

The telegraph had been invented before Bell's time. Noises, music and signals had been sent over electrified wires. But human *speech* had never been effectively sent by wire[508]. Many inventors were working to accomplish this, but Bell was the first to succeed. His great invention was the result of many years of scientific training. Bell exhibited his telephone at the Centennial Exposition in Philadelphia in June 1876.

The first telephone company, The Bell Telephone Company, came into existence in July 1877. Bell was frequently called upon to testify in lawsuits brought by men claiming they had invented the telephone earlier. Several of these suits reached the Supreme Court of the United States. Bell spent most of his later life at his estate on Cape Breton Island, Nova Scotia. He disliked the telephone because it interrupted his experiments in his laboratory. He died at his Nova Scotia home.

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[507] Bell's primitive telephone transmitter (1876) consisted of a membrane capable of moving in the field of a permanent magnet and an electromagnet that was fed by the dc current of a battery. The undulation of pressure on the membrane, caused by sound, generated an induced current signal which activated another membrane at the receiving end. The apparatus at each end acted both as a receiver and a transmitter. Bell uttered the first telephone message on March 10, 1876. He had spilled some acid on his cloths and was calling to his assistant, Thomas A. Watson, for help: "*Mr. Watson, come here. I want you*". In October 1876 Bell and Watson held the first two-way long distance telephone conversation between Boston and Cambridge, Mass., a distance of 3 kilometers. An American inventor, **Elisha Gray** (1835–1901), disputed Bell's claims as the inventor of the telephone. He filed a claim with the United States Patent Office only two hours after Bell filed a claim for a workable telephone (1876). Western Union supported Gray's claim in a bitter suit, but the claims were disallowed (Gray, however, made a fortune on other devices, such as simultaneous transmission of messages).

[508] With perhaps one exception: In 1860, **Jacob Reis** exhibited the first device which could transmit speech over a 100 meter wire. Bell acknowledged that he drew upon Reis' ideas in the construction of his telephone.

Efforts to improve the telephone transmitter led to the development of the *microphone* — a general device for changing sound waves into electrical signals (a telephone is essentially a simple type of a microphone). Other independent microphone inventors and improvers were: **Emile Berliner** (1877), **David Edward Hughes** (1878; also invented the *teleprinter*), **Thomas Edison**, **Francis Blake**, **Henry Humings**, **Charles Cuttris**, **Jerome Redding** and **E.W. Siemens**.

Note that Bell tried to develop a telegraph capable of sending multiple messages and accidentally discovered the principle of the telephone.

1876 CE *Flooding* in the Bay of Bengal region. Sea-waves 15 meter high, poured into the Ganges River delta and flooded an area of 380 km^2. Loss of life estimated at 215,000. It may have been a tsunami of seismic origin in the Andaman Islands. Again, on Nov. 12, 1970, cyclonic and tidal waves devastate the same region (now *Bangladesh*); 300,000 to 500,000 perished.

1876–1877 CE Great crop failure in India led to outbreak of *cholera* through which ca 3 million perished.

1876–1880 CE **Samuel (Siegfried Karl von) Basch** (1837–1919, Germany). Physician. Laid the foundation for the diagnosis of high blood pressure (hypertension). Invented the first simple and reliable apparatus for taking a person's blood pressure.

Basch was born in the ghetto of the Old City of Prague and obtained his medical degree in Vienna (1862). Became the personal physician of the Emperor Maximilian of Mexico until the latter's tragic end (1867).

In appreciation for Basch's loyal services, Kaiser Franz Joseph knighted him, an honor very rarely bestowed upon a Jew at that time. Two years later Basch was appointed the first lecturer in Experimental Pathology at the University of Vienna, but it took many years before he was given a modest laboratory of his own. Appointed (1877) professor at Vienna. From 1876 he published numerous papers on blood pressure and its measurement. All non-electronic instruments, so-called *sphygmomanometers*, are based on the original one invented by Basch[509].

1876–1883 CE **Paul Emile Appell** (1855–1930, France). Mathematician. Contributed significantly to the fields of analytical mechanics (non-holonomic systems), differential equations and special functions (*Appell's polynomials*). Although his work lacks central themes, seminal ideas and dramatic results,

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[509] Like many other new inventions, the instrument of Basch was at first subjected to scorn and ridicule.

he was a technician who used the classical methods of this time to answer open questions, work out details and make natural extensions in the mainstream of the late 19^{th} century.

Appell was born in Strasbourg. He was educated at the École Normale and became a professor at the University of Paris (1903–1925). He married a niece of Hermite (1881).

1876–1892 CE **Isaac** (**Eduard**) **Schnitzer** (**Emin Pasha**, 1840–1892, Austria). Explorer of central Africa, ornithologist, meteorologist and physician. The southern inlet of Lake Victoria bears his name.

Isaac was born in Oppeln, Upper Schlesia to Jewish parents, and baptized after their death (1846). He later (1870) converted to Islam and took the name Emin Pasha. He was appointed governor of the Equatorial Province of Egypt by General C.G. Gordon (1878) and was murdered (1892) by slave traders whose activities he opposed.

1876–1897 CE **Robert** (**Heinrich Hermann**) **Koch** (1843–1910, Germany). Physician. The father of modern *bacteriology*, which he established as a separate science. He developed new techniques for straining, incubating, and growing bacteria, which remain the basis of the bacteriological study of infections.

Koch discovered and isolated the bacilli of *anthrax* (1876), *tuberculosis* (1882) (the first definite discovery of a specific microbe causing specific human disease), *cholera* (1883), and *bubonic plague* (1897). He won the 1905 Nobel prize for physiology or medicine for his work on tuberculosis.

Koch was born at Klausthal, Hanover and studied medicine at Göttingen. In 1885 he was appointed a professor at the University of Berlin.

The origins of Microbiology (1676–1900)

Stimulated by development of light microscopy, scientists began the study of microscopic organism (microorganisms). These organisms (most of which cannot be seen without a microscope) include: algae, bacteria molds, protozoans, fungi and viruses. They are sometimes called microbes. *Biologists* *specialize in the study of various kinds of microorganisms. For example, bacteriologists work with bacteria, mycologists are concerned with fungi, and virologists with viruses.*

Nearly all microorganisms measure less than 0.1 mm (100 micron) across, and many must be studied with microscopes that magnify objects at least 1,000 times. Most viruses are so tiny that they can be seen only with electron microscopes that magnify many thousands of times.

Viruses are called acellular microorganisms because they do not have true cell structures. All other microorganisms are cellular. They have cell membranes, cytoplasm, and a nuclear body. Bacteria are the smallest single-celled organisms. The smallest bacteria may measure only ca $\frac{4}{10}$ *of a micron.*

About 10,000 small viruses could be packed into a cell the size of one of these bacteria. Over a billion such cells could be packed into one of the largest microbial cells — the cells of a certain algae.

Microbiology as a discipline is defined by the techniques employed:

- *Microscopy and Strains*
- *Sterilization*
- *Getting a pure culture*
- *Composition of culture media*
- *Anaerobe, aerobe, microaerophile*

Bacteria are one-celled organisms, first seen by **Leeuwenhoek** *(1676). Originally confused with protozoa, bacteria were variously called animalcules or microbes.*

During the 18th century bacteria contributed to the 'spontaneous generation controversy' as **Spallanzani** *(1767–1768) refuted Needham's assertion that microbes appeared in sealed flasks of boiled broth. Bacterial studies outside medicine remained superficial until 1872 when* **F. J. Cohn** *(1828–1898) defined and named bacteria, distinguishing four groups on the basis of external form and specific fermentative activity. He recognized bacteria that take nitrogen from simple ammonia compounds, elucidated their life-cycles, identified spores and suggested that bacteria were motile cells devoid of walls. Determining bacterial temperature limits,* **Cohn**, **Pasteur** *and* **Tyndall** *effectively ended the spontaneous generation controversy with their studies on sterilization.*

The link between Leeuwenhoek's microorganisms (1676) and the induction of infectious diseases by bacteria was not recognized for another 200 years![510]*Indeed, proof that microbes cause diseases in humans was first given by* ***Bassi*** *(1835), anticipating Pasteur and Koch.* ***Semmelweiss*** *(1847) proved that puerperal fever is contagious.*

Casimir Davain *(1812–1882, France), physician, was first to produce experimental infection in animals with blood containing the anthrax bacillus and first to suggest that the bacillus caused the disease (1850–1863).* ***Joseph Lister****, influenced by the discoveries of* ***Pasteur****, introduced carbolic acid as an antiseptic in surgery (1867).* ***Robert Koch*** *(1876) confirmed Davain's suggestions. Koch also developed techniques for handling bacteria, improving upon* ***Carl Weigert****'s (1845–1904) original use of methyl violet to stain them, introducing solid nutrient media (agar-agar) to grow pure cultures, and devising methods for fixing bacteria.*

Viruses are sub-microscopic, obligate intracellular parasites: they are produced from the assembly of pre-formed components, whereas other agents 'grow' from an increase in the integrated sum of their components & reproduce by division. Virus particles (virions) themselves do not 'grow' or undergo division. Viruses lack the genetic information which encodes apparatus necessary for the generation of metabolic energy or for protein synthesis (ribosomes). No known virus has the biochemical or genetic potential to generate the energy necessary for driving all biological processes, e.g. macromolecular synthesis. They are therefore absolutely dependent on the host cell for this function.

- *Viroids are small (200–400 nm), circular RNA molecules with a rod-like secondary structure which possess no capsid or envelope.*
- *Virusoids are satellite, viroid-like molecules, somewhat larger than viroids (e.g. approximately 1000 nm).*
- *Prions are infectious agents believed to consist of a single type of abnormally-folded protein molecule with no nucleic acid component. They are believed to infect other, normal proteins by somehow inducing in them their own folding abnormality.*

Viruses infect all types of living cells — animals, plants & bacteria.

[510] **Girolamo Fracastoro** made the first scientific statement (1546) on the true nature of contagion and transmission of diseases by germs, but he had no physical idea about these agents.

Virology began with ***Edward Jenner's*** *vaccination against smallpox (1796).* ***Pasteur*** *(1881) made the first artificially produced virus vaccine (rabies).* ***Dimitri Iwanowski*** *(1864–1920, Russia) explained (1892) the infectiousness of tobacco mosaic disease by showing that it can be transmitted via cell-free filtration from leaves of diseased plants to leaves of healthy plants. During the 1890s increased knowledge of soil and water bacteria was responsible for completing the explication of the nitrogen, sulphur and carbon cycles.*

Nodule-forming bacteria living in the roots of leguminous plants were found to fix atmospheric nitrogen. As a result of ***Winogradski's*** *(1856–1953) and* ***M. Beijerinck's*** *(1851–1931) work on anaerobic bacteria, knowledge of a whole world of organisms able to live on elementary nitrogen, iron or sulphur has emerged.*

In 1898, ***Friedrich Loeffler*** *(1852–1915, Germany) and* ***Paul Frosch*** *(1860–1928, Germany) discovered that a virus is responsible for foot-and-mouth disease. In 1900,* ***Walter Reed*** *(1851–1902) proved that yellow fever was caused by a virus spread by mosquitoes.* ***K. Landsteiner*** *(1900) demonstrated that poliomyelitis is caused by a virus.*

1876–1909 CE **Otto Wallach** (1847–1931, Germany). Organic chemist. Pioneered in the field of alicyclic compounds which formed the basis for the *perfume industry*. Awarded the 1910 Nobel prize in chemistry.

Wallach was born to Jewish parents. Studied under **Kekule**. Professor at Bonn (1876) and Göttingen (1889–1915).

1877–1881 CE **Wilhelm Friedrich Philipp Pfeffer** (1845–1920, Germany). Physiological botanist. First to measure *osmotic pressure*[511] and determine through it molecular weights of proteins. Made pioneering studies of *respiration*, *transpiration*, *photosynthesis*, *metabolism*, *transport in plants*

[511] He used a membrane of $Cu_2[Fe(CN)_6]$ (discovered in 1864 by **Moritz Traube** (1826–1894)), to make accurate measurements of osmotic pressures. The semipermeable membrane container was filled with a sugar solution and immersed in a vessel of water. He then connected a mercury-filled manometer to the top of the semipermeable container, and was able to show that the pressure was directly proportional to *concentration* (and hence inversely to *volume*), and also directly to the absolute *temperature*, i.e. $PV = kT$. Van't Hoff used Pfeffer's measurements (1886) to derive the law of osmotic pressure.

and *mycorhiza* (a fungus entering into symbiotic partnership with roots of trees). His work on *osmosis* was of fundamental importance in the study of cells, because *semipermeable membranes* surround all cells and play a large part in controlling their internal environment.

Pfeffer was born in Grebenstein, near Kassel, and was trained as a pharmacist at Göttingen (Doctorate, 1865), Marburg and Bonn. He became a professor at Bonn (1873) and at Leipzig (1887–1920).

1877–1887 CE **Emile Berliner** (1851–1926, U.S.A.). Electrical engineer and inventor. Invented the variable-resistance *microphone* (1877, ahead of Edison), and the *gramophone record* (1887).

Berliner was born in Hanover, Germany to a Jewish family, and was educated in his native place and Wolfenbüttel, where he graduated in 1865. He emigrated to the United States in 1870 and settled in Washington D.C. There he worked as a clerk, salesman, and assistant in a chemical laboratory. He studied electrical engineering, and in 1876 began experimenting with Bell's newly invented telephone. In 1877 Berliner succeeded in refining it with his invention of the loose-contact telephone transmitter[512]. The Bell Telephone

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[512] In contradistinction to Bell's telephone transmitter, the electrical resistance at the *contact of two conductors* is made to vary with the sound pressure on the membrane. This variable resistance generates, in turn, a variable electric signal that flows to the receiver and creates there the inverse effect. Berliner's patent was issued on April, 4, 1877. Almost simultaneously with Berliner, on July 21, 1877, Edison improved on this idea by replacing the two conductors in contact by a small cell of *carbon powder*.

The overall operation of the modern telephone transmitter (mouth piece) is as follows: behind the mouth piece of the phone lies a thin metal disc called a *diaphragm*. When a person talks into the telephone, the diaphragm vibrates in accordance with the sound pressure waves. Behind the diaphragm lies a small cup filled with tiny grains of carbon. A low-voltage electric current, activated by batteries (at the telephone company), travels through the grains. The electric resistance of the powder depends on its packing (loose of tight), which in turn is determined by the sound pressure on the diaphragm. Thus, the electric current through the powder grains replicates the pattern of the sound waves.

The *receiver* (ear piece) consists of a permanent magnet and an electromagnet located at the edge of a diaphragm and causing it to vibrate. The permanent magnet holds the diaphragm close to it permanently. The electromagnet controls the vibrations of the diaphragm. The electric current from the transmitter flows through the coils of the electromagnet, which pulls the diaphragm away from the permanent magnet. This causes the diaphragm to vibrate and set up sound waves.

Company immediately purchased the rights to his invention, which *for the first time made the telephone practical for long-distant use*. In 1887 he improved Edison's phonograph by introducing the *gramophone record* — a laterally cut shallow grooved disc. He also invented a way to press duplicate records from one master disc (by making a wax disc from which a 'negative' metal matrix was made for producing endless 'positives'). The patent was acquired by the Victor Talking Machine Company and served as a basis for the modern gramophone.

In his later years Berliner engaged in aviation experiments: between 1919 and 1926 he built three helicopters, which he tested in flight himself.

1877–1889 CE **Thomas Alva Edison** (1847–1931, U.S.A.). Distinguished inventor. Invented the *phonograph* (1877), the carbon-powder *microphone* transmitter (1877), the *incandescent electric light* (1879); contributed to the development of motion pictures (1889) and the memeograph machine (1887) and patented 1093 inventions in his lifetime. He had only 3 months of formal schooling in his whole life(!).

Edison was born at Milan, Erie county, Ohio, of mixed Dutch and Scottish descent. His parents moved to Port Huron, Michigan, when he was 7 years old. At the age of 12 he became a train news-boy on the railway to Detroit. At 15 he became a telegraph operator, and was employed in many cities in the United States and Canada, but frequently neglected his duties in order to carry on studies and experiments in electrical science. In 1869 Edison came to New York City, and soon afterwards became connected with the Gold & Stock Company. He invented an improved printing telegraph for stock quotations, for which he received $40,000. He then established a laboratory and factory for further experiments and for the manufacture of his inventions. On Oct. 19, 1879, after many failures, Edison finally succeeded in placing a filament of carbonized thread in a bulb[513]. On Dec. 21, 1879 the news of Edison's

[513] Edison can hardly claim to be the bulb's sole inventor. He was neither the first to come up with the incandescent light bulb idea. Contrary to popular opinion, the key ingredient he used for his light bulb — carbon — was certainly not unique. Carbon had been an ingredient of experimental light bulbs 50 years before Edison. At least 3 or 4 serious inventors, in England, France and the United States, were working on the incandescent lamp in the 1870s. They had the right ingredients and had functioning light bulbs. **Joseph Wilson Swan** (1828–1914, England) had lit residences in 1879 with his British bulb. **Hiram Stevens Maxim** (1840–1916, England and USA) had filed for incandescent light patents in 1878 and 1879 and had carbon incandescent lamps burning for twenty four hours at a time. **Hippolyte Fontaine** (1833–1917, France) displayed his version of a carbon-vacuum bulb in 1876. **William Eve Sawyer**

invention of the electric incandescent light-bulb astounded the world. Nations and individuals honored the modern Prometheus as probably no other person has been honored while alive. He died in West Orange, N.J.

Edison was a poor businessman and his inventions had never made him the money he thought he was entitled to[514].

(England) issued a patent (1878) on a carbon-nitrogen light bulb. It is still debatable whether he or Edison perfected the first light bulb. [He died in prison (1883) while serving a sentence for murder.] However, these pioneers succeeded in producing workable bulbs only on a *small scale*. Edison, from the start, designed his lamp to be part of a total electrical system the size of a city, complete with electric dynamos to produce the electricity and wires and fuses to distribute and control it. Only Edison discerned that the lamp and the system had to work as a unit and had to match. In addition, it was Edison's enormous wealth, influence, and power that allowed him to create the entire system from scratch in his New Jersey laboratories, set up a power station to light New York City with his new bulbs, and influence an eager press and public into believing his bulb to be the superior to all others.

514 He blamed it all on the Jews. As a close associate of the motor-car tycoon Henry Ford, he also propagated anti-Semitic views, habitually grousing about "*Jewish conspiracies*".

History of Sound Reproduction — from antiquity to Berliner

The ancient Greeks knew that sound as heard by the ear consisted of vibrations of air which, at certain frequencies, could even cause objects to vibrate. Records indicate that resonating panels were commonly used to improve acoustics of Greek theater. Thus, the origins of recorded sounds can be traced as far back as the ancient Greeks.

The colossal "vocal" statue of Memnon at Thebes was built about 1500 BCE with the ability to make the sound of the harpstring every day to greet Memnon's mother, the Goddess of the Dawn. The secret of this sound was lost when the original statue was destroyed in 27 CE by an earthquake. Back in the year 18 BCE, the Romans installed metal vases in their amphitheaters, specially tuned to certain frequencies.

The wheel was the first mechanism used to record sound, with pegs positioned to strike chimes as the wheel was rotated by hand. In the Middle Ages, music was reproduced by cylinders with attached pins that would strike certain keys or bells when rotated. Automatic carillons were built in the 14th century and the oldest surviving barrel organ dates from 1502. Renaissance Europe was fascinated with automata, or automatic music boxes that used elaborate clockwork gears to produce motions and sounds.

The most famous automata was a mechanical duck by ***Jacques de Vaucanson*** *in 1745 that flapped its wings, raised up on its legs, stretched its neck and moved its intestines that were visible from the outside. Influenced by Vaucanson and by the flood of new inventions from the Industrial Revolution, the French silk-weaver* ***Joseph Marie Jacquard*** *was awarded a medal at the Paris Exhibition of 1801 for an automatic loom that used punched cards to "record" a complex pattern for textiles woven by a loom with controls connected to spring-loaded keys. The long punched-card strips held down the keys until a hole allowed a key to open and start a loom operation.*

The weavers of Lyons burned Jacquard's loom in 1808 (where his statue is now located in Lyons) but Napoleon recognized its significance and awarded Jacquard a pension and royalties. The punched card was adapted by music instrument designers such as ***Charles Dawson*** *who exhibited a "jacquard organ" at the London Exhibition of 1851 that used cardboard strips to control the air bellows. From this would come the player piano of the 1880s and* ***Herman Hollerith****'s punch card tabulator for the 1890 census, a precursor of the modern computer.*

In 1855, the first successful sound recording device was developed by **Edouard-Leon Scott de Martinville** *(1817–1879, France). He called his invention the 'phonautograph'. It used a mouthpiece horn and membrane fixed to a stylus that recorded sound waves on a rotating cylinder wrapped with smoke-blackened paper. This device did not record the sound itself, only a graphical image of the sound. There was no way at the time to play the sound back*[515]*.*

In 1877 **Thomas Edison** *(1847–1931, USA) designed his "tinfoil phonograph"*[516]*. The device consisted of a cylindrical drum wrapped in tinfoil and*

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[515] In March 2008, a group of American audio historians unearthed in an archive in Paris a recording of the human voice made on April 9, 1860 by **Scott**. It is a 10-seconds recordings of a singer crooning the folk song "Au Clair de la Lune". This phonautogram was made playable (converted from squiggles on paper to sound) by scientists at the Lawrence Berkeley National Laboratory in Berkeley, California. The Berkeley scientists used optical imaging and a "virtual stylus" on high-resolution scans of the phonautogram, deploying modern technology to extract sound from patterns inscribed on the soot-blackened paper almost a century and a half ago he recording was played in public on March 28, 2008 at Stanford University.

Scott's 1860 phonautogram was made 17 years before Edison received a patent for the phonograph and 28 years before an Edison associate captured a snippet of a Handel Oratorio on a wax cylinder.

Scott is in many ways an unlikely hero of recorded sound. He was a man of letters, not a scientist, who worked in the printing trade and as a librarian. He published a book on the history of shorthand, and evidently viewed sound recording as an extension of stenography. In a self-published memoir in 1878, he railed against Edison for "appropriating" his methods and misconstruing the purpose of recording technology. In his memoir, Scott scorned his American rival and made brazen appeals to French nationalism: "What are the rights of the discoverer versus the improver? Come, Parisians, don't let them take our prize." Thus, Scott went to his grave convinced that credit for his breakthrough had been improperly bestowed on Edison.

[516] The invention of this first "*talking machine*" is most commonly attributed to **Edison**, in part because of the publicity that attended his celebrity and the theatrical power of his demonstrations, and in part because previous invention had earned him the means to have the device built. However, the first to *build* a phonograph was his top laboratory mechanic **John Kruesi**. The first to conceive of a workable design was the Frenchman **Charles Cros** who delivered viable plans for a machine that would use *disks* to the French Academy of Sciences in April 1877, several months before Edison happened on his idea.

Note that Edison was seeking to improve the *telephone* in 1877, when he discovered the recording device known as the phonograph.

mounted on a threaded axle. A mouthpiece attached to a diaphragm was connected to a stylus that etched vibrational patterns from a sound source on the rotating foil. For playback, the mouthpiece was replaced by a "reproducer". When the stylus was made to travel over the grooves, it made the membrane vibrate in response to the depressions in the grooves. Hence the motion of the stylus could reproduce the original sound.

So far, electricity was not directly involved in this mechanical sound-recording devices. But after the wheel, electricity would soon become another method of recording sound. To see the evolution of this process, one must go back in time to 1832, the year in which ***Samuel Morse*** *began the design of the telegraph. The electric telegraph was the stimulus for inventors to search for better methods of sending* and recording *all kinds of messages, including voice and music.*

David Edward Hughes*, a Professor of Music at St. Joseph's College in Kentucky, invented in 1855 a keyboard telegraph with rotating type-wheel printer that grew into the modern telex industry. In Germany, telegraph printers were patented as early as 1848 and* ***Philip Reis*** *invented an acoustic transmitter in 1861 that used a diaphragm to open and close an electrical circuit. He called it a "telephone" hoping to use it to reproduce speech and music but was unsuccessful.* ***Elisha Gray*** *and his Western Electric Company in Chicago had also invented an improved telegraph receiver, calling it a "telephone" after 1874 because it produced a wide range of sounds, but failed to make a similar transmitter.*

Herman Helmholtz *made an electric tuning-fork "sounder" device that used an electromagnetic coil, tuning fork and cardboard tube "resonator" to amplify the sound. His purpose was the scientific study of sound that was published in the influential 1862 book "Sensation of Tone".*

Berliner's gramophone (1887) was based on Scott's phonautograph and Cros's disc. But in spite of its superiority over Edison's cylinder machines, the Berliner gramophone was slow to attract attention. However, a new wax engraving process improved recording quality dramatically, and by 1901 the Gramophone company recorded four stars of the Russian Imperial Opera. A few years later, it recorded the voice of ***Enrico Caruso****, and a worldwide recording odyssey began.*

The invention of the phonograph and other sound reproduction machines began a new way of producing historical archives. Expressions of the human voice were no longer limited to their abstraction as words on the page, and the artistry and passion of a musical performance could be presented outside human memory. People could bring the sounds of the world into their homes, *and a global culture began to arise out of the mixture of influences that a broad diversity of recordings could provide. Before radio and sound motion pictures, the phonograph and other "talking machines" reigned for several decades as the great modern innovation in audio culture and entertainment.*

Evolution of Celestial Mechanics[517]

Astronomy is the oldest science, and in a certain sense the parent of all sciences. The relatively simple and regularly recurring celestial phenomena first taught men, in the days of ancient Greece, that Nature is systematic and orderly.

For a long time progress was painfully slow. Centuries of observations and attempts at theories for explaining them were necessary before it was finally possible for **Kepler** *(1571–1630) to derive his laws, which are first approximations to the description of the way in which planets move. The wonder is that, in spite of the distractions of the constant struggles incident to an unstable social order, there should have been so many men who found their greatest pleasure in patiently making the laborious observations which were necessary to establish the laws of celestial motions.*

The work of Kepler closed the preliminary epoch of 2000 *years or more, and the discoveries of* **Newton** *(1642–1727) opened another. The invention of the calculus furnished for the first time a mathematical machinery, which was suitable for grappling with such difficult problems as the disturbing effect of the sun on the motion of the moon, or the mutual perturbations of the planets. It was fortunate that the telescope was invented at about the same time; for without its use, it would not have been possible to acquire the*

[517] For further reading, see:

- Moulton, F.R., *An Introduction to Celestial Mechanics*, Dover: New York, 1970, 436 pp.
- Sterne, T.E., *An Introduction to Celestial Mechanics*, Interscience Publishers, 1960, 206 pp.

accurate observations which furnished the grist of data for the mill of the new mathematical theory.

The history of celestial mechanics during the 18^{th} *century is one of continual triumphs. The analytical foundations laid by* ***Clairaut*** *(1713–1765),* ***d'Alembert*** *(1717–1747) and* ***Euler*** *(1707–1783) formed the basis for the achievements of* ***Lagrange*** *(1736–1813) and* ***Laplace*** *(1749–1827).*

Their successors in the 19^{th} *century further developed, largely by the same methods, the theory of motions of moon and planets. They advanced the theoretical calculations to higher levels of precision, and compared them with more and better observations. In this connection, the names of* ***LeVerrier*** *(1811–1877),* ***Delaunay*** *(1816–1872),* ***Hansen*** *(1795–1874) and* ***Newcomb*** *(1835–1909) are especially noteworthy.*

Near the close of the 19^{th} *century, a third epoch was entered. It is distinguished by new points of view and new methods which, in power and mathematical rigor, surpassed all that went before. It was inaugurated by* ***Hill*** *(1838–1914) in his 'Research on the Lunar Theory', but owes most to the contributions of* ***Poincaré*** *(1854–1912) to the problem of three bodies.*

Celestial mechanics should be regarded as one of the splendid achievements of the human mind. No other science is based on so many observations extending over so long a time. No other scientific theory except Quantum Electrodynamics, has been empirically vindicated to such an outstanding accuracy.

1877 CE **Asaph Hall** (1829–1907, U.S.A.). Astronomer. Discovered two satellites of Mars with diameters 11 km and 6 km respectively, which he named *Phobos* (fear) and *Deimos* (terror). Curiously enough, these satellites were predicted by **Jonathan Swift** (1667–1745) already in 1726, in his "*Gulliver's Travels*"!

1877 CE **George William Hill** (1838–1914, U.S.A.). Mathematical astronomer. Contributed significantly to the theory of lunar motion. Also developed a theory of the motions of Jupiter and Saturn. Hill was first to use infinite determinants to analyze the motion of the moon's perigee. [This work was published in 1877 under the arcane title: "*On the part of the motion of the lunar perigee which is a function of the mean motions of the sun and the moon*".] In his work he came across a class of homogeneous, linear, 2^{nd} order differential equation with real, periodic coefficients. Such an equation is now known as "*Hill's equation*"[518]. It has numerous applications to problems in engineering, physics and astronomy. It includes as special cases the equations of **Mathieu**, **Lamé**, **Whittaker-Hill**, **Hermite** and **Picard**.

Hill was born in New York and educated at Rutgers College. In 1861 he joined the staff of scientists working in Cambridge, Massachusetts, on the *American Ephemeris and Nautical Almanac*. There he was assigned the task of calculating the American ephemeris, work he was later authorized to continue at his rural home in West Nyack, NY.

During 1881–1892 Hill resided in Washington D.C., working for the Navy Department in the Nautical Almanac Office. In 1898 he accepted the chair of astronomy at Columbia University. Since few students were qualified to comprehend graduate-level work in celestial mechanics, Hill objected to receiving pay, and finally resigned in 1901. He remained a recluse in West Nyack, devoted to his researches and to his large scientific library, which he bequeathed to Columbia University. Illness during his last years reduced his physical activity, and a failing heart brought his career to a close.

[518] Its standard form is $y'' + [\lambda + Q(x)]\,y = 0$, where λ is a parameter and $Q(x)$ is a real periodic function of x with period π. The fundamental importance of Hill's equation for stability problems was established by **Lyapunov** in 1907.

Lunar Theory — Part II (1687–1878)

The mathematical theory of the orbital motion of the moon is known as lunar theory. It is one of the most complex and difficult problems of dynamical astronomy. Its solution required the combined efforts of the greatest mathematicians since Newton, and its history extends back twenty centuries or more.

Before Newton, the problem was that of devising empirical curves for anomalies in the motion of the moon around the earth. After the establishment of universal gravitation as the primary law of celestial motions, the problem was reduced to that of integrating the differential equations of the moon's motion, and testing the results by comparison with observations.

Modern research developed naturally from the results of the ancients. In the hands of ***Hipparchos*** *(ca 135 BCE), observations were brought to a degree of precision which is truly marvelous in comparison with the level of other branches of physical science in that age.*

Hipparchos *discovered the 'annual equation' and* ***Ptolemy*** *(150 CE) discovered 'evection'. The 'variation' was discovered by* ***Tycho Brahe*** *in about* 1600. *The inclination of the moon's orbit and the regression of the nodes were discovered in* 1670 *by* ***John Flamsteed*** *(1646–1719,* England*).*

The modern lunar theory began with **Newton**, *and consisted in determining the motion of the moon from the universal theory of gravitation. Newton tried to explain the rotation of the line of apsides in his 'Principia', but his predictions accounted for only about* half *the observed apsidal rotation. In* 1749, ***Clairaut*** *found that Newton had neglected some small terms in the equations, and he brought theory and fact into agreement by taking such terms into account. However, more than a century later, in* 1872, *the correct calculations were also discovered among Newton's unpublished papers: he had detected his own error but had never bothered to correct it in print!*

Since the days of Newton, the methods of analysis have succeeded those of geometry. In the 18^{th} *century the development of lunar theory was almost entirely the work of five men:* **Euler** *(1707–1783),* **Clairaut** *(1713–1765),* **d'Alembert** *(1717–1783),* **Lagrange** *(1736–1813)* and **Laplace** *(1749–1827).*

The first complete explanation of the irregularities in the motion of the moon was given by **Newton** *both in his published and unpublished manuscripts. Newton regarded lunar theory as being very difficult and he confided* *to Halley in despair that it "made his head ache and kept him awake so often that he would think of it no more".*

In the 18th *century lunar theory was developed analytically by* ***Euler***, ***Clairaut***, ***d'Alembert***, ***Lagrange*** *and* ***Laplace***. *This intensive work was motivated by the general demand, in the* 18th *century, for accurate lunar tables for the use of navigators in determining their position at sea. This, together with the fact that the motions of the moon presented the best test of the Newtonian Theory, induced the English Government and a number of scientific societies to offer very substantial prizes for lunar tables agreeing with observations within certain narrow limits.*

Euler *published some imperfect lunar tables in* 1746. *In* 1747, ***Clairaut*** *and* ***d'Alembert*** *presented to the Paris Academy, on the same day, memoirs on the lunar theory. Each had trouble in explaining the motion of the perigee. In* 1749, *Clairaut found the source of the difficulty (also discovered by Euler and d'Alembert a little later). In* 1787, ***Laplace*** *explained the cause of the secular acceleration of the moon's mean motion.*

The immediate successors of Laplace, ***M.C. Damoiseau*** *(1768–1846) and* ***Plana*** *(1781–1864), carried out his method to a high degree of approximation. They integrated the equations of motions by expressing the time in terms of the moon's true longitude. Then, by inverting the series, the longitude was expressed in terms of the time.*

A second method was followed by **Hansen** *(1795–1874),* **Lubbock** *(1803–1865),* **de Pontécoulant** *(1795–1874) and* **Delaunay** *(1816–1872) during the years* 1832–1867. *According to this school, the moon's coordinates are obtained in terms of the time by a direct integration of the differential equations of motion. The expressions for the longitude, latitude and parallax appear as infinite trigonometric series, in which the coefficients of the sines and cosines are themselves infinite power series in small entities [eccentricities of moon and earth orbits, sine of half the moon's inclination etc.]. However, by this method, the series converge slowly and the final expressions of the moon's longitude are overlong and complicated.*

An entirely different approach, based on a method suggested by ***Euler***, *was taken up by* ***George William Hill*** *(1838–1914) and continued by* ***John Couch Adams*** *(1819–1892) and* ***Ernest William Brown*** *(1866–1938).*

Euler conceived the idea of an iterative scheme, starting with a zeroth order solution of the problem in which the orbit of the moon is supposed to lie in the ecliptic and have no eccentricity, while that of the earth is taken to be circular. The additional terms were then found, which were multiplied by the first powers of the eccentricities and of the inclination. Then the terms of the second order were found, and so on to any desired order. This method is *superior by far to the method of Laplace, since the convergence is faster and high precision is achieved even after a small number of iterations.*

Hill improved on Euler's method, and worked it out with greater rigor and detail.

Differential Equations and Special Functions[519] ***(1694–1879)***

Most of the special functions of mathematical physics and their corresponding differential equations were discovered during the 18^{th} *and* 19^{th} *centuries. The differential equations were usually encountered in the solutions of*

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519 To dig deeper, see:

- Andrews, G.E., R. Askey and R. Roy, *Special Functions*, Cambridge University Press, 2000, 661 pp.
- Vilenkin, N.J., *Special Functions and the Theory of Group Representations*, American Mathematical Society, 1968, 613 pp.
- Bell, W.W., *Special Functions for Scientists and Engineers*, Van Nostrand, 1968, 247 pp.
- Erdélyi, A. (Editor), *Higher Transcendental Functions*, 3 Volumes, McGraw-Hill Book Company: New York, 1953–1955.
- Magnus, W., F. Oberhettinger and R.P. Soni, *Formulas and Theorems for the Special Functions of Mathematical Physics*, Springer-Verlag: Berlin, 1966, 508 pp.
- Kamke, E., *Gewöhnliche Differentialgleichungen*, Chelsea Publishing Company: New York, 1959, 666 pp.
- Zwillinger, D., *Handbook of Differential Equations*, Academic Press: Boston, 1989, 673 pp.
- Inch, E.L., *Ordinary Differential Equations*, Dover Publications: New York, 1956, 558 pp.
- Havil, J., *Gamma*, Princeton University Press, 2003, 266 pp.

geometrical, astronomical or physical problems. The pioneers in this field were the illustrious ***Bernoullis*** *and* ***Euler****. They were later followed by* ***Legendre****,* ***Gauss****,* ***Jacobi*** *and others. In principle, the common special functions of mathematical physics evolved in a systematic way from the solutions of the wave-equation, heat-equation and the Laplace equation in the various orthogonal curvilinear coordinate systems. Historically, however, many of the well-known special functions were discovered independently, through solutions of problems in the various branches of science and engineering.*

The historical pattern is as follows:

A. The Gamma function

The first non-elementary function to be discovered after the scientific revolution. Its notation $\Gamma(z)$ *was introduced by Legendre in* 1814*. Euler's formula for this function was given in* 1729 *and most of its properties*[520] *were discovered by* ***Gauss****,* ***Legendre****,* ***Neumann*** *(1848),* ***Weierstrass*** *(1856) [who also showed that the Gamma-function does not satisfy any differential equation with rational coefficients] and* ***Hankel*** *[expression of* $\Gamma(z)$ *as a contour integral, 1864]. The associated Beta-function was introduced by Euler (1772).*

- Watson, G.N., *A Treatise on the Theory of Bessel Functions*, Cambridge University Press, 1966, 804 pp.
- Robin, Louis, *Functions Spheriques De Legendre et Functions Spheroidales*, Gauthier Villars: Paris, 1957–1959, Vols I-III (201 pp., 384 pp., 289 pp.)
- Goursat, E., *Differential Equations*, Ginn and Company, 1945, 300 pp.
- Titchmarsh, E.C., *The Theory of the Riemann Zeta-Function*, Oxford University Press: Oxford, 1951, 346 pp.
- Edwards, H.M., *Riemann's Zeta Function*, Academic Press: New York, 1974, 315 pp.
- Vallée, O. and M. Soares, *Airy Functions and Applications to Physics*, Imperial College Press, 2004, 194 pp.

520 When $\Re\{z\} > 0$, $\Gamma(z) = \int_0^\infty e^{-t}t^{z-1}dt$ is known as the Eulerian integral of the second kind. It leads to the basic recursion relation satisfied by the Gamma-function $\Gamma(z+1) = z\Gamma(z)$. Euler's formula is $z\Gamma(z) = \prod_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^z\left(1+\frac{z}{n}\right)^{-1}$.

B. Bessel functions (1694–1824)

In 1694 ***Johann Bernoulli*** *discovered the so-called Riccati's equation,* $y' = y^2 + x^2$, *in a paper on curves, but did not solve it. In* 1703, ***James Bernoulli*** *communicated to Leibniz a solution of the above equation in the form of infinite power series. In* 1738 ***Daniel Bernoulli*** *was engaged in the problem of the lateral oscillation of a heavy uniform chain. His solution was given in terms of a series, now described as a Bessel function of order zero.*

Euler, *returned to this problem in* 1781, *deriving the equation of motion for the chain's horizontal displacement. By an extremely ingenious analysis he found the three gravest eigenperiods of the chain. Earlier, in* 1764, *Euler had investigated the vibrations of a stretched membrane. He wrote the partial differential equation of its transverse displacement in polar coordinates and then proceeded to obtain the ordinary differential equation of the motion's amplitude, known today as the 'Bessel equation'. He also gave its explicit solution in terms of an infinite series. This investigation of Euler contains the earliest appearance of a Bessel function of general integral order.*

In 1770, *Lagrange encountered the Bessel-function in the astronomical problem of the Kepler equation*[521], *and gave an expansion of the radius vector*

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[521] The planetary elliptic orbit is given parametrically by means of the *eccentric anomaly* angle E, via the relations

$$r = a(1 - e\cos E), \qquad \cos\theta = \frac{\cos E - e}{1 - e\cos E},$$

where e is the eccentricity, θ is the *true anomaly* and r is the radius-vector. The auxiliary angle E is a solution of *Kepler's transcendental equation*

$$E - e\sin E = \frac{2\pi}{p}(t - T) = M,$$

where p is the orbital period and $t - T$ is the time, measured from the perihelion passage T. The quantity M, known as the *mean anomaly*, is the angle which the radius-vector would have described if it had been moving uniformly with average rate $\frac{2\pi}{p}$. Once the *Kepler equation* $E - e\sin E = M$ is solved for given M, the orbit $\{r, \theta\}$ is known for all times.

It turns out that the Fourier-series expansions of the radius-vector and of E are:

$$r = a\left[1 + \frac{1}{2}e^2 + \sum_{n=1}^{\infty} B_n \cos(nM)\right],$$

$$E = M + \sum_{n=1}^{\infty} A_n \sin(nM),$$

of the orbit in terms of coefficients that are infinite-series representations of Bessel functions. ***Fourier*** *(1822) and* ***Poisson*** *(1823) obtained similar series in problems of heat diffusion in solid circular cylinders and spheres. Finally,* ***Bessel*** *(1824) made a systematic study of the functions that now bear his name, in connection with the Kepler's problem.*

C. Legendre functions (1784–1884)

In 1784, ***Legendre*** *studied the gravitational attraction of spheroids. In the course of his work he expanded* $(1-2hz+h^2)^{-1/2}$ *in a power series of* $z=\cos\theta$. *The coefficients of* h^n *in this expansion were named later after him. These coefficients,* $P_n(z)$, *are of frequent use not only in potential theory, but in other branches of analysis as well and are called today Legendre polynomials.*

The algebraic and analytic properties of these polynomials were investigated by Legendre himself and by his followers during the next 100 *years [**Rodrigues** (1814), **Murphy** (1833), **Dirichlet** (1837), **Neumann** (1848–1862), **Heine** (1851–1878), **Christoffel** (1858), **Frobenius** (1871), **Mehler** (1872), **Schläfli** (1881), **Ferrer** (1877), **Hobson** (1891) and many others].*

A more extended class of Legendre functions is the Legendre associated polynomials *and functions. A function connected with the associated Legendre function* $P_n^m(z)$ *is the function* $C_n^\nu(z)$, *which for integral values of* n *is defined to be the coefficient of* h^n *in the expansion of* $(1-2hz+h^2)^{-\nu}$ *in ascending powers of* h. *It has been studied by* ***Gegenbauer*** *(1874–1893).*

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with

$$A_n = \frac{2}{n} J_n(ne),$$

$$B_n = -\frac{e}{\pi n}\int_0^{2\pi} \sin u \sin(nu - ne\sin u)du = -2\left(\frac{e}{n}\right) J_n'(ne),$$

where J_n is the Bessel function of order n and J_n' is its derivative w.r.t. the argument.

D. The hypergeometric functions (1755–1879)

This general function includes as special cases most of the familiar functions of elementary analysis [e.g. Chebyshev polynomials]. The function was known to **Euler** *(1755), who discovered a number of its properties, but it was studied systematically by* **Gauss** *(1811–1812), who gave the earliest satisfactory treatment of the convergence of an infinite series. Gauss' work initiated far-reaching development in many branches of analysis, in infinite series, general theories of linear differential equations and functions of complex variables.*

Another classical differential equation of considerable importance is the confluent hypergeometric equation[522]*, otherwise known as the Kummer equation (***Kummer***, 1836). Special cases and associated functions of this class are: Parabolic cylinder functions (***Weber***, 1869), Hermite polynomials (***Hermite***, 1864), Laguerre polynomials*[523] *and functions (***Laguerre***, 1879), Error functions*[524]*, Gamma functions*[525] *(***Euler***, 1729;* **Legendre***, 1876;* **Schlömilch***,*

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[522] Gauss' hypergeometric equation is

$$x(1-x)y'' + [c-(a+b+1)x]\,y' - aby = 0.$$

A solution which is regular at $x = 0$ is given by the hypergeometric series

$$y = {}_2F_1(a,b;c;x) = 1 + \frac{ab}{c}\frac{x}{1!} + \frac{a(a+1)b(b+1)}{c(c+1)}\frac{x^2}{2!} + \ldots.$$

If the independent variable is changed from x to $u = bx$, the former equation becomes

$$u\left(1-\frac{u}{b}\right)y'' + \left[(c-u) - \frac{(a+1)}{b}u\right]y' - ay = 0.$$

If we let $b \to \infty$ it becomes

$$uy'' + (c-u)y' - ay = 0,$$

which is the *confluent equation*. A solution of this equation is given by the series

$$u = {}_1F_1(a;c;x) = 1 + \frac{a}{c}x + \frac{a(a+1)}{2!c(c+1)}x^2 + \frac{a(a+1)(a+2)}{3!c(c+1)(c+2)}x^3 + \ldots.$$

[523] Laguerre and Hermite polynomials have important applications in quantum mechanics of the hydrogen atom and the linear harmonic oscillator, respectively.

[524] $Erfc(x) = \int_x^\infty e^{-t^2}dt$ and $Erf(x) = \int_0^x e^{-t^2}dt$ occur in connection with the theories of probability, observation errors and heat conduction.

[525] $\gamma(n,x) = \int_0^x t^{n-1}e^{-t}dt$.

1871) and the Logarithmic integral[526] *(**Euler**, 1755). **Thomas Clausen** (1801–1885) introduced the generalized hypergeometric function (1828)*

$$ {}_pF_q(a_1,\dots,a_p;b_1,\dots,b_q;z) = \sum_{k=0}^{\infty} \frac{\Pi_{i=1}^{p}(a_i)_k z^k}{\Pi_{i=1}^{q}(b_i)_k k!}, $$

where the notation $(a)_k = a(a+1)\cdots(a+k-1)$ *is known as the Pochhammer symbol (**Leo Pochhammer**, 1841–1920, Germany).*

Various mathematical constants, all elementary functions, and many special functions can be expressed in the hypergeometric notation; for example:

$$ \cos(z) = {}_0F_1\left(\frac{1}{2};\frac{-z^2}{4}\right) $$

$$ \log(z+1) = z\ {}_2F_1(1,1;2;-z) $$

$$ \mathrm{erf}(z) = \frac{2z}{\sqrt{\pi}}\ {}_1F_1\left(\frac{1}{2};\frac{3}{2};-z^2\right)^2 $$

$$ \pi = 4 - \frac{8}{9}\ {}_3F_2\left(\frac{1}{2},\frac{1}{2},1;\frac{5}{2},\frac{5}{2};-1\right) $$

$$ \pi = \frac{426880\sqrt{10005}}{13591409} \Big/ \left({}_3F_2\left(\frac{1}{6},\frac{1}{2},\frac{5}{6};1,1;\frac{-1}{151931373056000}\right) - \frac{30285563}{1651969144908540723200}\ {}_3F_2\left(\frac{7}{6},\frac{3}{2},\frac{11}{6};2,2;\frac{-1}{151931373056000}\right)\right) $$

E. Elliptic functions and integrals[527] (1655–1920)

The study of the theory of elliptic and associated functions is of great importance for its close relation to the development of the general theory of functions of complex variable and for its important applications in various branches of mathematics, physics and engineering. Extensive and detailed elaboration of the subject has provided a testing ground for discovery and improvement of the general theorems of complex variable: the theorems of

[526] $\ell_i(x) = \int_0^x \frac{dt}{\log t}$.

[527] The terminology for elliptic integrals and functions has changed during their investigation. What were originally called *elliptic functions* are now called *elliptic integrals* and the term elliptic functions is reserved for a different idea. We will use modern terminology throughout this section to avoid confusion.

Liouville[528] *(1847) and* ***Picard*** *(1879), the theorems of multiple periodicity and many other important results of the theory of functions of complex variable.*

*The theory of elliptic and associated functions has a very wide field of applications in the analytical theory of numbers [**Gauss**, **Jacobi**, **Hermite**, **Hardy**, **Ramanujan**] and in the theory of equations [**Hermite**, **Kronecker**, 1858], where the general solution of a quintic equation was obtained in terms of elliptic functions.*

*In the field of geometry, elliptic functions are of great use in studying the properties of certain classes of curves. In mechanics, the earliest applications were to the problems of the simple pendulum with a finite amplitude (**Euler**), the spherical pendulum (**Lagrange**, **Richelot**, 1852) and the motion of a rigid body about a point (**Legendre**). Further applications appear in the theories of potential, elasticity, electrostatics and heat conduction.*

*The first encounter with elliptic integrals resulted from attempts to harness the calculus for the rectification of the ellipse (**Wallis**, 1655). What is now known as an* elliptic integral, *occurs in the researches of **Jakob Bernoulli** on the Elastica (1694). He was first to notice that these integrals cannot be expressed in terms of elementary functions*[529].

Giulio Carlo Fagnano dei Toschi *(1682–1766, Italy, 1715) made extensive research of elliptic integrals and proved that the difference of any two elliptic arcs is algebraic.* ***Euler*** *was acquainted with the results of Fagnano in* 1751 *and obtained from it suggestions for his proof of the addition theorem of elliptic integrals (1761). He systematically studied the geometri-*

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[528] If $f(z)$ is analytic for all values of z and if $|f(z)| < K$ for all z, where K is a constant [so that $|f(z)|$ is bounded as $z \to \infty$], then $f(z)$ is constant. This theorem furnishes short and convenient proofs of some of the most important results in analysis, e.g. that an elliptic function $f(z)$ with no poles in a cell is merely a constant.

[529] An ellipse has the parametric equations $x = a\sin\theta$, $y = a\cos\theta$, where $a > b$ and the eccentric angle θ measured from the minor axis. If s is the arc length parameter measured clockwise around the curve from the end B of the minor axis, then

$$ds^2 = \sqrt{(dx^2 + dy^2)} = \sqrt{(a^2\cos^2\theta + b^2\sin^2\theta)}\,d\theta = a\sqrt{(1 - e^2\sin^2\theta)}\,d\theta$$

where $e = \sqrt{1 - \frac{a^2}{b^2}}$ is the eccentricity. Thus, the length arc from B to any point P were $\theta = \varphi$ is given by $s = a\int_0^{\varphi}\sqrt{1 - e^2\sin^2\theta}\,d\theta = aE(u, e)$, where E is the elliptic integral of the second kind.

cal applications of elliptic integrals, and proposed that they be recognized as primitive new transcendentals to be investigated on their own merits. ***Legendre*** *worked more than* 40 *years on the systematic development of this vast field.*

Abel *(1827),* ***Jacobi*** *(1827) and* ***Gauss*** *(1797, unpublished) revolutionized this subject, and opened the floodgates to* 19^{th} *century analysis, with the simple idea of inverting the elliptic integral*[530]*. Abel's first important discovery in this connection was the double periodicity of this inverse function, known as an elliptic function, which thus started the study of elliptic functions proper. Abel also generalized the elliptic integrals, and considered the integrals (and their inverse functions) later named as Abelian integrals and Abelian functions.*

Jacobi *(1827–1829) was the principal and most accomplished investigator of elliptic and theta functions*[531]*. He obtained their properties by purely algebraic methods. His analysis is so complete that practically most of the results known today are to be found in his works.*

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[530] Instead of directly investigating the elliptic integral

$$y = \int^{x} \left[(1-c^2x^2)(1+e^2x^2)\right]^{-1/2} dx,$$

they proposed to consider the inverse function $x = F(y)$. This procedure is similar to that of defining the inverse circular function and the logarithmic functions by integrals $\sin^{-1} x = \int^{x} \frac{dx}{\sqrt{1-x^2}}$, $\log x = \int^{x} \frac{dx}{x}$ respectively, and then establishing the properties of the circle and exponential functions from the corresponding inverse functions. The advantage of this artifice is that instead of having to consider, say, the ∞-ly *multiple-valued* function $y = \sin^{-1} x$, restricted to the range $-1 < x < 1$ (for real values of x), we may, by writing the same equation as $x = \sin y$, treat x as *single valued* function of y, which is much easier to deal with.

[531] The four functions

$$\theta_1(z,q) = 2\sum_{n=0}^{\infty}(-)^n q^{(n+\frac{1}{2})^2}\sin(2n+1)z,$$

$$\theta_2(z,q) = 2\sum_{n=0}^{\infty} q^{(n+\frac{1}{2})^2}\cos(2n+1)z,$$

$$\theta_3(z,q) = 1 + 2\sum_{n=1}^{\infty} q^{n^2}\cos 2nz,$$

$$\theta_4(z,q) = 1 + 2\sum_{n=1}^{\infty}(-)^n q^{n^2}\cos 2nz,$$

The next epoch in the development of the theory of elliptic functions started with the works of **Liouville** *(1844–1851) and* **Weierstrass** *(1854). In their approach, the elliptic functions are not introduced as inverse functions of primitives of elliptic integrals — instead they are suitably defined by some important properties, such as double periodicity*[532]*, meromorphism etc.*

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are known as the *theta functions*. They solve the *heat-conduction equation*

$$k\frac{\partial^2\theta}{\partial z^2} = \frac{\partial\theta}{\partial t},$$

where k is the *diffusivity* and $q = e^{-4kt}$. For most applications it is convenient to pass to complex time τ such that $q = e^{\pi i\tau}$, $\operatorname{Im}\tau > 0$, $|q| < 1$, to secure convergence. Clearly θ_1, θ_2 are periodic with period 2π and θ_3, θ_4 are periodic with period π. Interestingly enough, the series are *also pseudoperiodic* in z with period $\pi\tau$, namely $\theta_1(z+\pi\tau, q) = -N\theta_1(z,q)$ where $N = q^{-1}e^{-2iz}$, etc. The theta function can also be represented as a product of *partition functions*, e.g.

$$\theta_4(z,q) = \prod_{n=1}^{\infty}(1-q^{2n})(1-q^{2n-1}e^{2iz})(1-q^{2n-1}e^{-2iz}).$$

[Euler was first to study, in 1748, the partition function $\prod_{m=1}^{\infty}(1-q^m x)$.]
The solution of

$$\left(\frac{dy}{du}\right)^2 = (1-y^2)/(1-k^2y^2)$$

is $y = sn(u,k)$, known as the *Jacobian elliptic function of* u. It can be shown that

$$y = \frac{\theta_3(0,q)}{\theta_2(0,q)}\frac{\theta_1(x,q)}{\theta_4(x,q)},$$

where $x = \frac{u}{[\theta_3(0,q)]^2}$ with $k^2 = \left[\frac{\theta_2(0,q)}{\theta_3(0,q)}\right]^4$.

532 In analogy to the relation $\frac{1}{(\sin z)^2} = \sum_{m=-\infty}^{\infty}\frac{1}{(z-m\pi)^2}$, Weierstrass defined a new function

$$\wp(z) = z^{-2} + \sum_{m,n}{}' \left\{\frac{1}{(z-\Omega_{m,n})^2} - \frac{1}{\Omega_{m,n}^2}\right\}$$

where $\Omega_{m,n} = 2m\omega_1 + 2n\omega_2$, and $\sum' = \sum_{m,n}$, $(m,n) \neq (0,0)$.
It turns out that $\wp(z)$ is doubly periodic, with no singularities but poles. Hence it is an elliptic function, and satisfies the differential equation:

$$\left[\wp'(z)\right]^2 = 4\wp^2(z) - g_2\wp(z) - g_3$$

with

$$g_2 = 60\sum_{m,n}{}'\Omega_{m,n}^{-4}, \quad g_4 = 140\sum_{m,n}{}'\Omega_{m,n}^{-6}$$

The final stage in the development of elliptic functions, involving their mysterious relations to number theory, is marked by the contributions of ***Hermite****,* ***Klein****,* ***Kronecker****,* ***Dedekind****,* ***Poincaré*** and ***Ramanujan****. Their approach was based on two novel ideas: one is the concept of* automorphic *function introduced by Poincaré, and the other is the concept of* modular *function, which arises from the behavior of the* modulus *k^2 as a function $f(\tau)$ in the complex τ-plane, where $f(\tau) = \frac{\theta_2^4(0,\tau)}{\theta_3^4(0,\tau)}$.*

F. Differential equations with periodic coefficients (1868–1940)

In 1868, ***Émile Léonard Mathieu*** *(1835–1890,* France*) determined the vibrational modes of a stretched membrane having an elliptical boundary. Solving the two-dimensional wave equation in elliptical coordinates by the method of separation of variables, a second order equation, known today as the Mathieu equation*

$$[y'' + (a - 2q\cos 2z)y = 0]$$

is obtained. Its solutions are the Mathieu functions. Other problems of mathematical physics which lead to the Mathieu equation are:

(1) *Tidal waves in a cylindrical vessel with an elliptical boundary.*
(2) *Certain forms of steady vortex motions in an elliptical cylinder.*
(3) *Diffraction of sound and electromagnetic waves by a right elliptical cylinder.*
(4) *Propagation of waves in elliptical wave guides.*
(5) *Heat conduction in an elliptical cylinder.*
(6) *Amplitude distortion in a moving-coil loudspeaker.*

H.G. Hill *(1887) investigated the mean motion of the lunar perigee by means of a generalized form of Mathieu equation*

$$[y'' + \{a - 2q\psi(2z)\}y = 0\,,$$

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and

$$z = \int_{\wp}^{\infty} [4t^3 - g_2t - g_3]^{-1/2} dt.$$

where

$$\psi(2z) = -\sum_{r=1}^{\infty} \theta_{2r} \cos 2rz \Bigg].$$

In 1883, ***G. Floquet*** *proved an important theorem concerning the general nature of the solutions of Hill's equation. Since then, an extensive literature has accumulated on the exact, asymptotic and numerical solutions of Mathieu functions [**Lindemann** (1883), **Stieltjes** (1884), **Maclaurin** (1899), **Hilbert** (1904), **Sieger** (1908), **Whittaker** (1914), **Ince** (1924–1939), **Erdélyi** (1934–1936), **Bickley** (1940)].*

1877–1902 CE **Benjamin Baker** (1840–1907, England). Engineer and bridge constructor. Involved in the construction of Metropolitan railways and in designing the cylindrical vessel in which *Cleopatra's needle* was brought over from Egypt to England (1877–1878). Designed and erected the *Forth Bridge* in Scotland (with John Fowler) in 1890. Directed the construction of the Assam dam (1902). Pioneered in the construction of intra-urban railways in deep tubular tunnels built up of cast iron segments. Author of many papers on engineering subjects.

Baker was born near Bath and received his early training in a South Wales ironworks. He afterwards became associated with John Fowler in London.

1878–1886 CE **Francois Marie Raoult** (1830–1901, France). Physicist and chemist. Discovered (1878) that the depression of the *freezing points*[533] of liquids due to the presence of a substances dissolved in them is proportional to the solute's *molarity (moles per unit solvent weight), and number of dissociated ions per molecule of solute with a coefficient depending only upon the solvent (cryoscopic coefficient).* Introduced (1886) the law named after him, stating that in a dilute solution, the lowering of the *vapor pressure* of the solvent is proportional to the *molecular weight* of the substance dissolved, unless the solvent is an electrolyte. Both phenomena afforded new methods of determining the molecular weight of substances.

[533] The bare fact that the presence of dissolved substances in water lowers its freezing-point was already known to the English physician **Charles Blagden** (1748–1820). ["Experiments on the cooling of water below its freezing point", *Phil. Trans.* **78**, 120–130, 1788; and "Experiments on the effect of various substances in lowering the point of congelation in water", *Phil. Trans.* **78**, 277–312, 1788.]

W. Ostwald and **J.H. van't Hoff** used Raoult's laws to support the hypothesis of electrolytic dissociation.

Raoult was born at Fournes en-Weppes Nord and taught at Grenoble (1867–1901).

1878–1913 CE **Ferdinand de Saussure** (1857–1913, Switzerland). Linguist. The father of modern linguistics. His book *Memoir on the primitive system of vowels in Indo-European languages* (1878) was a major breakthrough in comparative philology. His most famous work *Course in General Linguistic* was published posthumously by his students (1915).

de Saussure led the *structural movement* against the *comparative*[534] method and formulated many basic principles of structural linguistics (1906) which apply to *all* languages. He urged the development of a general science of signs aided by mathematics — suggestions which were intensively followed by modern philologists.

de Saussure was born in Geneva. He studied Indo-European languages at the Leipzig and Berlin Universities. After several years of teaching in Paris, he returned to his native Geneva (1891) and remained there until his death.

1879 CE **Charles Émile Picard** (1856–1941, France). Mathematician. Advanced research in the fields of analysis, algebraic geometry and mechanics.

Developed an existence theorem for differential equations based on the method of *successive approximations.*

Picard worked on quadratic forms, Abelian functions and the allied theories of discontinuous and continuous groups of transformations. His work led to a study of the algebraic manifold, now known as the *Picard variety*, which play a fundamental role in algebraic geometry.

In 1879 Picard proved the theorem named after him. This theorem became the starting point for many important studies in the theory of complex functions[535].

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[534] The early *structuralists* believed that the comparativists overemphasized languages as written in the past, and ignored languages as spoken today. The structuralists also disagreed with the traditional method of describing languages by *paradigms* (patterns) of conjugations and declensions. They studied many non-Indo-European languages and found that some do not have conjugations and declensions. Thus, these languages could not be described by the traditional method.

[535] In the neighborhood of an isolated essential singularity, a one-valued function takes every value, with one possible exception, an infinite number of times.

During 1878–1899, Picard held various posts with the universities of Paris, Toulouse and the École Normale Supériere. In 1898 he was appointed professor at the University of Paris.

1879 CE **Edwin Herbert Hall** (1855–1938, U.S.A.). Physicist. Discovered the "Hall effect" in which a voltage is produced across a current-carrying conductor in a magnetic field. The Hall voltage is perpendicular to both the direction of the current and the direction of the magnetic field, and proportional to the current and the magnetic field. Since different material produce different Hall voltages, scientists can use the Hall effect as a probe of the electronic structure of various materials[536].

Hall was born at Great Falls, ME. He was a professor at Harvard University (1888–1921).

1879 CE **Joseph Stefan** (1835–1893, Austria). Physicist. Empirically discovered the law that the total energy radiated by a blackbody per unit area-time (u) is proportional to the 4^{th} power of the temperature T: $u = \sigma T^4$. In 1884, **L. Boltzmann** derived the law theoretically[537], and since then it is known as the "*Stefan-Boltzmann law*" and σ as the "*Stefan-Boltzmann constant*". At any rate, Stefan's contribution was the first important step toward the understanding of black-body radiation, from which sprang the idea of the quantum of radiation.

Stefan was born at St. Peter, Austria and did his major work at the University of Vienna [lecturer, 1858; full professor, 1863; director of the Physical Institute, 1866].

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536 **Hall** designed a series of thought experiments which demonstrated that the magnetic force should deflect the charge carriers and cause them to collect on one side of the conductor. Charge carries in most metals bear a negative charge, (*electrons* were discovered by **J.J. Thomson** in 1897).

The *Hall effect* is easily understood in terms of the simple free-electron model of **Drude**, but for metals with valence > 1 (e.g. Be, Mg, In and Al), the explanation of experimental results require a quantum treatment of the effect.

In 1980, **Klaus von Klitzing** discovered the *quantized Hall effect* (Nobel prize, 1985), in which changes in resistance in a plate kept in a magnetic field at temperatures near absolute zero occur in discrete steps instead of continuously; it is an example of quantum behavior that is directly observable macroscopically.

537 The laws of Stefan-Boltzmann and Wien follow purely as a result of the general laws of thermodynamics and the electromagnetic nature of radiation. Indeed, Boltzmann was able to provide their theoretical basis *without* the use of Planck's radiation formula.

1879–1906 CE **Henri Jules Poincaré** (1854–1912, France). Outstanding and versatile mathematician, theoretical astronomer, and philosopher of science. He is known as 'the last of the universalists' since he was the last man to have had a creative command of the whole of mathematics as existed in his day, including: algebra, geometry, arithmetic and analysis — as well as the entire gamut of mathematical physics (celestial mechanics, general analytical mechanics, optics, elasticity, thermodynamics, potential theory, electromagnetism). He will probably be the last man who will ever be in this position.

Poincaré did not dwell on any particular field long enough to round his work. A contemporary said of him: "*He was a conqueror, not a colonist*". He had an unusually retentive memory for everything he read, and could also visualize what he heard. Throughout his life he was able to perform complex mathematical calculations in his head, and could quickly write a paper without extensive revisions. He produced more than 30 books and 500 technical papers. His major achievements are:

- Virtually founded the theory of automorphic functions[538]. He found that these functions are associated with transformations arising in non-Euclidean geometry (1879–1887). Such functions are generalizations of trigonometric functions $(a = d = 1,\ \ c = 0,\ \ b = 2k\pi)$ and elliptic functions. **Hermite** had studied such transformations for the restricted case in which the coefficients a, b, c, d are integers, and satisfy $ad - bc = 1$, and had discovered a class of elliptic modular functions invariant under the restricted transformations. But Poincaré's generalization uncovered a broader category of functions, known as *zeta-Fuchsian functions*, which could be used to solve 2^{nd} order linear differential equations with algebraic coefficients.

  Poincaré found that two automorphic functions, invariant under the same group, are connected by an algebraic equation. Conversely, the coordinates of a point on any algebraic curve can be expressed in terms of automorphic functions, and hence by uniform functions of a single parameter [e.g. $x^2 + y^2 = a^2$ is parametrically represented by $x = a\cos t$, $y = a\sin t$].

- Developed the theory of asymptotic series representation of functions (1886).
- Father of *algebraic topology* (Poincaré Conjecture, 1904).

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538 An automorphic function $f(z)$ of the complex variable z is one which is analytic, except for poles, in a domain D and which is invariant under a denumerably infinite group of linear fractional transformations $z' = \frac{az+b}{cz+d}$, i.e. $f\left(\frac{az+b}{cz+d}\right) = f(z)$.

- Established combinatorial topology, i.e. the study of intrinsic qualitative aspects of spatial configurations that remain invariant under continuous 1–1 transformations[539].

  Among other things, he generalized *Euler's formula* $V - E + F = 2$ to n dimensional space. Instead of vertices (V), edges (E) and faces (F), we then have $0-, 1-, 2-, \dots, (n-1)$-dimensional entities. If the numbers of these entities is $N_0, N_1, \dots, N_{n-1}$ respectively, then the equation $N_0 - N_1 + N_2 - \cdots = 1 - (-1)^n$ applies to the manifolds corresponding to the simple polyhedra. For $n = 3$ this reduces to *Euler's formula*.

Poincaré's generalization furnished a determination of the *regular* polytopes in higher dimensional spaces [in 3 dimensions there are only 5 regular polyhedra: *Tetrahedron* ($V = 4$, $E = 6$, $F = 4$); *Cube* ($V = 8$, $E = 12$, $F = 6$); *Octahedron* ($V = 6$, $E = 12$, $F = 8$); *Dodecahedron* ($V = 20$, $E = 30$, $F = 12$); *Icosahedron* ($V = 12$, $E = 30$, $F = 20$)].

- Advanced the qualitative study of *nonlinear differential equations* by the introduction of *topological* arguments. To describe the nature of a singular point he introduced the notion of an *index*[540].

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539 Transformations which are continuous and which have a continuous inverse transformation. Such transformations are known as *homeomorphisms*. Two homeomorphic manifolds are said to be *topologically equivalent*.

540 Consider a singular point P_0 and a simple closed curve C surrounding it. At each intersection of C with the solutions of

$$\frac{dy}{dx} = \frac{P(x,y)}{Q(x,y)},$$

there is a direction angle of the trajectory, which we shall denote by ϕ and which can have any value from 0 to 2π radians. If a point now moves in a counterclockwise direction around C, the angle ϕ will vary; and after completion of the circuit around C, ϕ will have the value $2\pi I$ where I is an integer or zero (since the direction angle of the trajectories has returned to its original value). The quantity I is the index of the curve. It can be proved that the index of a closed curve that contains several singularities is the algebraic sum of their indices. The index of a closed trajectory (and of no other simple closed curve) is $+1$.
The nature of the trajectories can be determined by the characteristic equation, and so the index I of a curve should be determinable by knowing just the differential equation. One can prove that

$$I = \frac{1}{2\pi}\int_C d\left(\arctan\frac{P}{Q}\right) = \frac{1}{2\pi}\int_C \frac{Q\,dP - P\,dQ}{P^2 + Q^2},$$

where the path of integration is the closed curve C.

- Proved the *recurrence theorem*, stating that a non-dissipative dynamical system having a finite energy and confined to a finite volume, will, after sufficiently long time, return to an arbitrary small neighborhood of almost any initial state.
- Contributed[541] substantially to the classical 'n-body problem' in celestial mechanics: given the present masses, velocities, and mutual positions of n bodies, how long will they remain stable in their present orbits? In his solution, Poincaré initiated the qualitative theory of non-linear differential equations. He developed new mathematical techniques and made fundamental discoveries on the behavior of the integral curves of differential equations near singularities (1889–1895).

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[541] One of Isaac Newton's most important discoveries was that two bodies moving under the influence of each other's gravitational fields, both follow ellipses (or hyperbolas or parabolas). The question was: how do three bodies move under Newtonian gravitational forces? The 'two-body problems' is "integrable" — the laws of conservation of energy and momentum restrict solutions to such an extent that they are forced to take simple mathematical form. The suspicion that three bodies can move *chaotically* (chaos — apparently random motion with purely deterministic causes — too complicated to occur in an integrable system) predates the recognition of chaos in mathematics; indeed, it was one of the key historical steps in its discovery.

In 1887, King Oscar II of Sweden was worried about the stability of the solar system. Will it persist forever, behaving such as it does today, or will a planet escape or crash into the sun?

A prize of 2500 crowns, offered by the king to anyone who could solve the problem, was won by the leading mathematician of the day, Henri Poincaré — even though he did not answer the question. However, what Poincaré achieved was more important — the introduction of qualitative geometrical methods into dynamics. It led him to discover some curious behavior that we now recognize as *chaos*.

He found it in the '*restricted three-body problem*' (an idealization in which one body is assumed to have such a small ("test") mass that the other two are not affected by it). This test body does, of course, respond to the gravitational fields of the two more massive bodies. The question is: does the chaos persist if we make the model more realistic by including the very small but non-zero gravitational effects of the almost massless test body in our calculations?

In 1994, **Zhihong Xia** of the Georgia Institute of Technology, U.S.A., proved that a system of three bodies is not integrable (i.e., it has no conserved quantity other than energy and linear and angular momentum) and that the full three-body problem is chaotic.

- Contributed to the theory of numbers by demonstrating how the concept of binary quadratic forms (developed by Gauss) could be cast in a geometric form (1904).
- Made important contributions to the theory of equilibrium of gravitating rotating fluid masses. In particular he described the conditions of stability of the pear-shaped figures that played a prominent part in evolution models of celestial bodies.

Many of the problems he tackled were seeds of new ways of thinking,which have since grown and flourished in 20^{th} century mathematics.

Poincaré was born in Nancy into a distinguished family [his first cousin, Raymond Poincaré (1860–1934) was president of the French republic during WWI]. He studied at the École Polytechnique (1872–1875), devoting himself to scientific mining, and took his doctorate in 1879. He was lecturer at Caen and then moved to the University of Paris in 1881, where he held several professorships in mathematics and science.

Poincaré died of embolism, a week after a successful prostate operation. He was in his 59^{th} year and at the height of his powers — '*the living brain of the rational sciences*', in the words of Painlevé.

The Poincaré Conjecture[542] *(1904–2003)*

"One of the early successes of topology was to show that just two topological invariants, the Euler characteristic *and* orientability*, are all you need to be able to distinguish any two closed surfaces. That is to say, if two surfaces have the same Euler characteristic and are either both orientable or both non-orientable, then they are in fact the same — even if one is unable to see how to continuously deform one into the other. This result is called the classification theorem for surfaces, since it says that one can classify all surfaces (topologically) by means of just these two attributes.*

542 Quotations are from Keith Devlin's book *The Millennium Problems*, Basic Books: New York, 2002.

Loosely speaking, the classification theorem for surfaces is proved by taking a sphere as the basic surface and measuring the degree to which any given surface differs from a sphere — what would one have to do to a sphere to turn it into that surface. This corresponds to our ordinary intuition that a sphere is the simplest, most basic, and, some might say, the most aesthetically perfect closed surface.

It should be pointed out that in this case, the operations to be performed on a sphere to turn it into some other surface go beyond the normal topological operations of continuous deformations. Indeed, if one changes a sphere by means of twisting, bending, stretching, or shrinking, the resulting object, topologically, will still be a sphere. To classify surfaces by seeing how they can be constructed from a sphere, one has to allow cutting and stitching together in addition to the usual twisting, stretching, etc. Topologists refer to this process as "surgery." The term is apt, since a typical surgical operation involves cutting one or more pieces from the sphere, twisting, turning, stretching, or shrinking each of those pieces, and then sewing those pieces back into the sphere again.

The classification theorem tells us that any orientable surface is topologically equivalent to a sphere with a certain number of "handles" sewn onto it. You get a handle by cutting two holes into the sphere and joining them together by means of a tube. Any non-orientable surface is equivalent to a sphere with a certain number of "crosscaps" sewn in. You get a crosscap by cutting a hole in the sphere and sewing a Möbius band to the boundary of the hole. As with the Klein bottle, in ordinary three-dimensional space one cannot do this without the Möbius band passing through itself; one needs four dimensions to do it properly.

In the early years of the twentieth century, Poincaré and other mathematicians set out to classify higher-dimensional analogues of surfaces — which they called "manifolds." Not surprisingly, they tried an approach similar to the one that had worked for two-dimensional surfaces. They sought to classify all three-dimensional manifolds (called "3-manifolds" for short) by taking a three-dimensional analogue of a sphere (called a "3-sphere") as basic and measuring the degree to which any 3-manifold differs from that 3-sphere.

One has to be careful here. A regular surface such as a sphere or a torus is a two-dimensional object. The figure the surface encloses is three-dimensional, of course, but the surface itself is two-dimensional. Apart from a plane, any surface can be constructed only in a space of three or more dimensions. Thus, any closed surface requires three or more dimensions. For instance, it takes three dimensions to construct a sphere or a torus, four dimensions to construct a Klein bottle. Yet a sphere, a torus, or a Klein bottle is a two-dimensional *object — a surface that has no thickness and can, in principle, be constructed from a flat, perfectly elastic sheet.*

But just as a sphere can be regarded as a two-dimensional analogue (in three-dimensional space) of a circle (which is a one-dimensional object – a curved line – in two-dimensional space), so too we can imagine a three-dimensional analogue (in four-dimensional space) of a sphere. Well, actually, we can't imagine it. But we can write down equations that determine such an object, and study "it" mathematically. Indeed, physicists routinely study such imaginary objects, and use the results to help understand the universe we live in. The 3-manifolds, i.e., the three-dimensional analogues of surfaces (which exist in spaces of four or more dimensions), are sometimes called hypersurfaces, with the three-dimensional analogue of a sphere being called a hypersphere.

There is no mathematical reason to stop at three dimensions. One can write down equations that determine manifolds of 3, 4, 5, 6, or any number of dimensions. Once again, these considerations turn out to be more than idle speculation. The mathematical theories of matter that physicists are currently working on view the universe we live in as having 11 dimensions. According to these theories, we are directly aware of three of those dimensions, and the others manifest themselves as various physical features such as electromagnetic radiation and the forces that hold atoms together.

Poincaré attempted to classify manifolds of three and more dimensions by taking a "sphere" of the respective dimension as a base figure and then applying surgery, A natural first step in this endeavor was to look for a simple topological property that tells you when a given (two-dimensional) surface is topologically equivalent to a sphere. (Even in the simple case of regular two-dimensional surfaces, a surface might appear extremely complicated and yet turn out to be continuously deformable to a sphere.)

In the case of two-dimensional surfaces, there is such a property. Suppose you were to take a pencil and draw a simple closed loop on the surface of a sphere. Now imagine the loop shrinking in size, sliding over the surface as it does so. Is there a limit to how small the loop can shrink? Obviously not. One can shrink the loop until it becomes indistinguishable from a point. Mathematically, one cane shrink it until it actually becomes a point.

The same thing is not necessarily true if one starts with a loop drawn on a torus. One can draw loops on a torus that cannot be shrunk down to a point. No loop that goes right around the ring of the torus can be shrunk down indefinitely, nor can any loop that encircles the torus like a belt.

The shrinkability to a point of any loop drawn in a surface is a topological property of the surface that is unique to spheres. That is to say, if one has *a surface on which every loop (the "every" is important here) can be shrunk down to a point without leaving the surface, then that surface is topologically equivalent to a sphere.*

Is the same true for a three-dimensional hypersphere? This is the question Poincaré asked in the early 1900s, hoping that a speedy positive answer would be the first step on the road to a classification theorem for three-dimensional hypersurfaces. He developed a systematic method — called homotopy theory *— for studying (using methods of algebra) what happens to loops when they are moved around a manifold and deformed.*

Actually, that's not quite what happened. At first, Poincaré tacitly assumed *that the loop-shrinking property for 3-manifolds did characterize the 3-sphere. After a while, however, he realized that his assumption might not be valid, and in 1904 he put his doubts into print, writing (in French): "Consider a compact three-dimensional manifold V without boundary. Is it possible that the fundamental group of V could be trivial, even though V is not homeomorphic to the three-dimensional sphere?" Stripping away the technical terms, what Poincaré asked was, "Is it possible that a 3-manifold can have the loop-shrinking property and not be equivalent to a 3-sphere?" That was the birth of the Poincaré conjecture.*

As it turned out, his question did not get a speedy answer. Nor, indeed, a slow answer, despite the best efforts of a number of leading topologists. As a result, finding a proof (or a disproof) of the Poincaré conjecture rose to become one of the most sought-after prizes in mathematics."

Thus, in 1904, Poincaré conjectured that every simply connected, closed, orientable 3-dimensional manifold[543] *is homeomorphic to the sphere of that dimension*[544] *(the surface of a 4-dimensional solid sphere). This conjecture has been generalized to read:*

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[543] A 3-dimensional manifold is a space such that every point has an open neighborhood homeomorphic to a 3-dimensional Euclidean space. From the point of view of STR, we are living on a 3-dimensional sphere.

[544] Earlier, Poincaré asserted that any two closed manifolds that have the same *Betty numbers* and torsion coefficients are homeomorphic. But he soon gave an example of a 3-dimensional manifold that has the Betti numbers and torsion coefficients of the 3-dimensional sphere but is not connected. Hence he added simple connectedness as a condition.

He then showed that there are 3-dimensional manifolds with the same Betti numbers and torsion coefficients but which have different fundamental groups and so are no homeomorphic. However, **James W. Alexander** showed (1919) that two 3-dimensional manifolds may have the same Betti numbers, torsion coefficients, and fundamental group and yet not be homeomorphic.

> *"Every simply connected, closed, n-dimensional manifold that has Betti numbers and torsion coefficients of the n-dimensional sphere, is homeomorphic to it."*

This generalized conjecture has been proved by ***Stephen Smale*** *(1960) for $n \geq 5$ and for $n = 4$ by* ***Michael Freedman*** *(1982). The case $n = 2$ is classical (and was known even to 19th century mathematicians), and the case $n = 1$ is trivial.*

Thus, by 1982, the only unsettled case of the Poincaré conjecture was the one originally posed by Poincaré, in three dimensions. This happened because 2-dimensional space is too small to have room for any serious complexity, and 4- or higher dimensional space is so big that the complexities can be arranged nicely. In 3 dimensions there is a creative tension: big enough to be complicated; too cramped to be easily simplified. What was needed was a line of attack that exploited the special properties of 3-dimensional manifolds. This feat was finally achieved in 2003 by ***Grigori Perelman****, of the Steklov Institute of Mathematics, of the Russian Academy of Sciences in St. Petersburg. It carried with it a prize of one million dollars, given by the Clay Mathematical Institute. (For an account of recent development in this field see "The Poincare Conjecture" by D. Oshea, Walker and Co. New York 2007, 293 pp.)*

Worldview XXV: Poincare

* * *

"Mathematics is the art of giving the same name to different things. [As opposed to the quotation: Poetry is the art of giving different names to the same thing]."

* * *

"Later generations will regard Mengenlehre (set theory) as a disease from which one has recovered."

* * *

"What is it indeed that gives us the feeling of elegance in a solution, in a demonstration? It is the harmony of the diverse parts, their symmetry, their happy balance; in a word it is all that introduces order, all that gives unity, that permits us to see clearly at once both the ensemble and the details."

* * *

"Mathematicians do not study objects, but relations between objects. Thus, they are free to replace some objects by others so long as the relations remain unchanged. Content to them is irrelevant: they are interested in form only."

* * *

"The mind uses its faculty for creativity only when experience forces it to do so."

* * *

"Mathematical discoveries, small or great, are never born of spontaneous generation. They always presuppose a soil seeded with preliminary knowledge and well prepared by labor, both conscious and subconscious."

* * *

"Absolute space, that is to say, the mark to which it would be necessary to refer the earth to know whether it really moves, has no objective existence... The two propositions: "The earth turns round" and "it is more convenient to suppose the earth turns round" have the same meaning; there is nothing more in the one than in the other."

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"...by natural selection our mind has adapted itself to the conditions of the external world. It has adopted the geometry most advantageous to the species or, in other words, the most convenient. Geometry is not true, it is advantageous."

* * *

"The problem is not what is the ANSWER, the problem is in what is the QUESTION."

* * *

"THOUGHT is the lightening between two infinities of blackness. But it is the lightening which matters."

1879–1918 CE **John Moses Browning** (1855–1926, USA). Inventor and designer of firearms. Designed a series of pistols, rifles and shotguns. The United States Army adopted his machine-gun (1890) and his automatic rifle (1918). He became internationally famous for designing and inventing automatic arms, including the Browning automatic rifle. He was born in Ogden, Utah.

1880 CE **Charles Louis Alphonse Laveran** (1845–1922, France). Physician and parasitologist. Discovered the blood parasite causing *malaria*[545] (Nobel prize for physiology or medicine, 1907).

[545] The British physician-bacteriologist **Ronald Ross** (1857–1932) discovered (1898) in India that mosquitoes can transmit malaria to birds. For this he was awarded (1902) the Nobel prize. The Italian zoologist **Giovanni Battista Grassi** (1854–1925), building on the work of Ross, determined that malaria is spread to humans by the *Anopheles* mosquito (1899). In that year, the complete life-cycle of the parasite became known.

Laveran worked as a military physician (1870–1896). While in Algeria to study malarial fever (1878–1883) he found microscopic parasites in red blood cells of human victims of the disease. He continued his research with the Pasteur Institute in Paris (1896–1922). Author of *Traité des fievres palustres* (1884) and *Traité d'hygienne militaire* (1896).

1880 CE **Aurel Edmund Voss** (1845–1931, Germany). Mathematician. Was first to derive the *contracted* Bianchi identity between the covariant derivatives of the components of the Riemann tensor.

It was discovered independently by **Ricci** in 1889, and then again in 1902 by **Bianchi**. Voss also derived a generalization of Gauss' formula. He was a professor at the University of Münich during 1902–1923.

1880 CE *Piezoelectricity* discovered by **Pierre and Jacques Curie** (certain crystals develop an electric charge on the surface when stretched or compressed along an axis).

1880–1885 CE **Charles Sumner Tainter** (1854–1940, England and USA). Engineer and inventor. Constructed the first system that utilized a photocell to convert sound into light (1880). With **A.G. Bell** developed a working prototype of the '*gramophone*' which used a *wax cylinder* rather than **Edison**'s tinfoil cylinder (1880–1881). Also, with Bell, he invented the '*photophone*' (1881) — an apparatus that transformed sound into light signals which in turn activated a photocell[546].

The name malaria was coined in the 17^{th} century by Dr. **Francisco Torti** by combining the Italian names for "bad" and "air", and it has been called the shakes, the fevers, the ague, and many other things, none affectionate. **Hippocrates** reported several clinical types of malaria. Untreated malaria may kill about one percent of those infected. The survivors, prone to relapse, may suffer from anemia, weakness, sexual impotence, chronic abortion, or secondary infections — all of which lower the value of the individual to self, family, and community.

Throughout men's history, few diseases have played so tragic a role as malaria. It has killed or incapacitated more people than all plagues, wars, and automobiles. Endemic malaria contributed to the downfall of Greece (after 400 BCE). **Alexander the Great** died of it in June 323 BCE, and it was again the malaria that taxed heavily the vitality of Rome in its declining years. **Oliver Cromwell** died of malaria in 1658. Troops in many wars during history were inactivated by malaria, e.g. over 10 percent of the U.S. overseas armies in 1943 had malaria.

[546] According to his futuristic idea, the recording and reproduction of sound utilized photocells and a magnetic induction sensing device. However, in lack of elec-

Tainter developed the *dictaphone* (1885), a machine that could record dictations.

1880–1885 CE **James Alfred Ewing** (1855–1935, Japan and Scotland). Physicist and engineer. Helped **John Milne** to construct in Japan (1880) the first useful *seismograph system* for recording local earthquakes. Investigated magnetic properties of iron, steel etc. Observed and named the phenomenon of *hysteresis*[547] (1885).

Ewing was professor at Tokyo (1878–1883), Dundee (1883–1890), Cambridge (1890–1903) and the University of Edinburgh (1916–1929).

1880–1890 CE **Wilhelm Killing** (1847–1923, Germany). Mathematician. An original and profound mathematical thinker. His work was neglected, but his ideas, results, and methods served as a basis to the later works of **Élie Cartan**, **Hermann Weyl**, **Noether**, **Wedderburn**, **Coxeter** and many others. In his epoch making paper: "*Die Zusammenensetzung der stetigen, endlichen Transformationsgruppen*" [Mathematische Ann. 1888–1890; four parts] he originated such key notions as the rank of an algebra, semi-simple algebra, Cartan algebra, root systems and Cartan integers. Weyl's theory of the representation of semi-simple Lie groups would have been impossible without

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tronic amplification (in 1886), the typical output of a single cell was not enough to be 'audible'. [In fact, it was not enough even to light a small flashlight bulb.]

547 Greek origin meaning: lagging behind. A remarkable aspect of *ferromagnetism*: the tendency of ferromagnetic materials to retain initial magnetization, explained by the fact that the magnetic domains offer *resistance to orientation*. The magnetization of weakly magnetized substances varies linearly with the field strength. However, the magnetization of ferromagnetics (substances capable of having magnetization in the absence of an external magnetic field) depends on $\boldsymbol{H}$ in an intricate way: μ depends on $\boldsymbol{H}$, and consequently $\boldsymbol{B} = \mu\boldsymbol{H}$ depends *nonlinearly* on $\boldsymbol{H}$. If a sample is initially magnetized, we obtain the first portion of the curve in which $\boldsymbol{B}$ increases with $\boldsymbol{H}$ until it begins to flatten off due to *saturation*. On decreasing the external field, the curve does not follow the same path and shows a positive value of $\boldsymbol{B}$ when $\boldsymbol{H} = 0$. This is known as *residual magnetization* in the sample. When $\boldsymbol{H}$ is reversed, it is found that $\boldsymbol{B}$ finally becomes zero at some negative value of $\boldsymbol{H}$, known as the *coercive force*. The other half of the *hysteresis loop* is then obtained by making $\boldsymbol{H}$ still more negative until reverse saturation is reached, and then returning $\boldsymbol{H}$ to the original positive saturation value. From an engineering point of view, these substances are of immense importance and have very important technological consequences.

the results of the above paper. Roughly one third of the extraordinary work of Cartan was based on Killing's paper[548].

Killing's theorem enumerating all possible structures for finite dimensional Lie algebra (which he invented in 1880 independently of **Sophus Lie**) over the complex numbers, was used by Cartan and **Molien** as a paradigm for the development of the structure theory of finite dimensional linear associative algebras.

It was Killing who discovered the exceptional Lie algebra E_8, which today figures prominently in superstring theory. Killing introduced the notion that a vector field on a manifold represents a flow, which induces a continuous change of coordinates on the manifold. A *Killing vector* of a metric is a flow that leaves the metric tensor invariant. It manifests a symmetry (isometry) in the metric. Otherwise stated, the Lie-derivative of the metric tensor along the *Killing vector* vanishes on the manifold.

Under a given coordinate transformation $x \to x'$, the metric tensor $g_{\mu\nu}(x)$ is transformed according to the relation

$$g'_{\mu\nu}(x') = \frac{\partial x^\rho}{\partial x'^\mu}\frac{\partial x^\sigma}{\partial x'^\nu} g_{\rho\sigma}(x),$$

or equivalently

$$g_{\mu\nu}(x) = \frac{\partial x'^\rho}{\partial x^\mu}\frac{\partial x'^\sigma}{\partial x^\nu} g'_{\rho\sigma}(x').$$

If we require that the transformed metric be the *same* function of its argument x'^μ as the original metric $g_{\mu\nu}(x)$ was of its argument x^μ (*form-invariance*), we can write the last equation as

$$g_{\mu\nu}(x) = \frac{\partial x'^\rho}{\partial x^\mu}\frac{\partial x'^\sigma}{\partial x^\nu} g_{\rho\nu}(x').$$

In general, this equation is a very complicated restriction on the function $x'^\mu(x)$. It can be greatly simplified by descending to the special case of an *infinitesimal coordinate transformation* $x'^\mu = x^\mu + \epsilon\xi^\mu(x)$ with $|\epsilon| \ll 1$. Then, to first order in ϵ,

$$0 = \frac{\partial \xi^\mu(x)}{\partial x^\rho} g_{\mu\sigma}(x) + \frac{\partial \xi^\nu(x)}{\partial x^\sigma} g_{\rho\nu}(x) + \xi^\mu(x)\frac{\partial g_{\rho\nu}(x)}{\partial x^\mu}.$$

Note that the r.h.s. of this equation is exactly the *Lie derivative* of the metric tensor w.r.t. the vector field $\xi^\mu(x)$.

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[548] John A. Coleman, *The greatest mathematical paper of all time*. The Mathematical Intelligencer **11**, 29–38, 1989. Cartan was meticulous in noting his indebtedness in 63 references to Killing in his 1894 thesis.

This can be written in terms of derivatives of the covariant components $\xi_\sigma = g_{\mu\sigma}\xi^\mu$:

$$\begin{aligned} 0 &= \frac{\partial \xi_\sigma}{\partial x^\rho} + \frac{\partial \xi_\rho}{\partial x^\sigma} + \xi^\mu \left[\frac{\partial g_{\rho\sigma}}{\partial x^\mu} - \frac{\partial g_{\mu\sigma}}{\partial x^\rho} - \frac{\partial g_{\rho\mu}}{\partial x^\sigma}\right] \\ &= \left[\frac{\partial \xi_\sigma}{\partial x^\rho} - \Gamma^\mu_{\sigma\rho}\xi_\mu\right] + \left[\frac{\partial \xi_\rho}{\partial x^\sigma} - \Gamma^\mu_{\rho\sigma}\xi_\mu\right] = \xi_{\sigma;\rho} + \xi_{\rho;\sigma}, \end{aligned}$$

where $\xi_{\sigma;\rho}$ is the *covariant derivative* of $\xi_\sigma(x)$.

Any four-vector field $\xi_\sigma(x)$ that satisfy the last equation will be said to form a *Killing vector of the metric* $g_{\mu\nu}(x)$.

It can be shown that Killing vectors have two other useful properties:

- If A_i is a Killing vector, then $A_i \frac{dx^i}{ds}$ is constant along any *geodetic.*
- A necessary condition for the metric $g_{\mu\nu}$ to have a *hidden continuous symmetry (isometry)* is that it admits one Killing vector field per parameter of the symmetric (Lie) group.

Killing was born in Burbach, Westphalia. He began university studies in Münster (1865) but quickly moved to Berlin and came under the influence of **Kummer** and **Weierstrass**. In 1872 he completed his dissertation under Weierstrass. From 1868 to 1882 he was teaching at the gymnasium level in Berlin. On the recommendation of Weierstrass, Killing was appointed professor of mathematics at the Lyzeum Hosianum in Braunsberg, East Prussia (now Braniewo in the Ulsztyn region of Poland). The main object of the college was the training of Roman Catholic clergy, so Killing had to teach a wide range of topics — including the reconciliation of faith and science.

Although he was isolated mathematically during his ten years in Braunsberg, this was the most creative period of his mathematical life. He produced his brilliant work despite worries about the health of his wife and seven children, demanding administrative duties as rector of the college and as a member and chairman of the city council, and his active role in the church of St. Catherine.

In 1892 he was called back to his native Westphalia as professor of mathematics at the University of Münster, and he stayed there for the rest of his life.

1880–1894 CE **John Milne** (1850–1913, England and Japan). One of the founders of the science of seismology. Constructed the first seismograph suitable for world-wide use, and set up seismological observatories to measure ground movements on a global basis (1892).

Milne was born in Liverpool. After working in Labrador and Newfoundland as a mining engineer (1873) and serving as a geologist in an expedition to the Sinai desert (1874), he accepted (1875) a position of professor of geology and mining at the Imperial College of Engineering, Tokyo.

An earthquake in 1880 near Yokohama prompted him to create the Seismological Society of Japan, the first of its kind. In 1881 he married Tone Horikawa and stayed in Japan until 1895. During his stay there he traveled all over the islands and set up a network of seismological stations equipped with his seismographs. Upon his return to England in 1894, he settled on the Isle of Wight and established a private seismological station there. His activity in his later years centered around the establishment of a global network of seismic stations. He died in Shide, Isle of Wight.

In 1974, the University of Tokyo donated a number of cherry tree saplings to be planted at Shide and at the Isle of Wight College of Arts and Technology as 'a living memorial' to the pioneer seismologist.

1880–1908 CE **Moritz Benedict Cantor** (1829–1920, Germany). Distinguished historian of mathematics. A professor of mathematics, who devoted only his final academic years exclusively to the history of his field. His monumental work *Vorlesungen über Geshichte der Mathematic* (1880–1908), which carried the study down to 1799, dwarfed all previous endeavors and is still unsurpassed.

Moritz Cantor, a relative of Georg Cantor, was born in Mannheim to a Jewish family; he was appointed a private docent in Heidelberg (1853) and the rest of his active life was spent in the service of the university (1863–1913).

His treatise on the history of mathematics remains the most elaborate ever produced, without any equal of all histories of science.

Science Progress Report No. 11
The Long Arm of Orthodoxy, or did Moses write the Torah?

The Pentateuch, the first five books of the Bible, has always held a special place in both Christianity and Judaism. Islam also owes much to these books. As the Torah (the "five books of Moses"), the books are especially venerated by Jews; they were the first part of the Bible to be admitted to the Hebrew canon.

Christianity accepts the entire Hebrew Old Testament as found in the Masoretic text, including the Pentateuch, as canonical, although there is some variation in organization. Much of early Christianity art depicts events from the Pentateuch. Almost everyone in the Western world is early exposed to stories of Noah and the ark, the passage across the Red Sea, and the story of Abraham and Isaac.

A passage in the book of Deuteronomy (31, 9), relating that Moses wrote the Torah, gave rise to the doctrine of the Mosaic authorship of the whole Pentateuch.

Serious doubts about the traditional Mosaic authorship began to be heard already in the Middle Ages; ***Isaac Ibn Yashush*** *(**Ibn Kastar**; 982–1068, Spain), grammarian, biblical commentator and personal physician to Muwaffak Mudshaid al Amiri in Toledo, Spain, was first to demonstrate that Moses could not have been the author of the Pentateuch in his book "Sefer hatserufim". Consequently, he was ferociously excoriated. His opinion were later shared by the Jewish scholar* ***Joseph Bonfils*** *(fl. 1370, Damascus) and by the Bishop of Avilla,* ***Alonso Tostado*** *(fl. 1450, Spain), at the cost of their being persecuted by their respective religious establishments.*

Even in the 17th century, it would have taken a brave person to deny that Moses personally wrote "his" five books. The philosophers ***Thomas Hobbes*** *(Leviathan, 1651),* ***Baruch Spinoza***[549] *(Tractacus Theologico-*

[549] Spinoza argued that those who believe Moses to be the sole literal author of the entire text must reconcile the following questions and facts:

- Could: "Now the man Moses *was* very meek, above all the men which were upon the face of the earth" [Numbers **12**, 3] been written by the meek man himself?

- The first of Edomite kings [*Genesis*, **36**] includes those who lived many years after Moses. Especially, how could Moses know [**36**, 31] about future Israeli kings? [See also: 2 *Samuel* **8**, 14].

Politicus, 1670) and the theologian ***Richard Simon*** *(1638–1712, France; 1678) pioneered the modern historical method of Biblical study and critique, citing many textural examples.*

- How could Moses write Deuteronomy **34**, 5–12? He certainly could not had known that "... there arose not a prophet like Moses...".
- In Deuteronomy **1**, 1 we read: "These are the words which Moses spoke unto all Israel beyond the Jordan..." This could have only been written by someone on the West bank of the river, but it could not have been Moses, since Moses never set foot on the Western side. [From **1**, 5 we know that "beyond" means the side of *Moab*, namely the Eastern side].
- In *Genesis*, **12**, 6 we read: "And Abraham passed through the land unto the place of Shechem... *and Cannanite was then in the land*". Clearly, this was written in a post-Mosian era when the Cannanite were not there anymore. The discrepancy was already noticed by **Avraham Ibn Ezra** (c. 1140 CE).
- Deuteronomy **3**, 11–15 says: "For only Og king of Bashan remained of the remnant of the Rephaim; behold, his bedstead was a bedstead of iron; is it not in Rabbah... to this day". The style and content of this passage could have been written only after Rabbah was taken by David (a few hundred years later).
- In Genesis **22**, 14 we are told that the mountain is called by Abraham *the mountain of God*, whereas the writer of the story called it mount *Moriah* [**22**, 2]. This last name, however, was chosen in the days of David for the site of the Solomon Temple, a fact unknown to Moses [2 *Chron* **3**, 1].
- In Genesis **14**, 14 we read: "... and pursued as far as Dan". However, the name of this city was given a long time after the death of Joshua [*Judges* **18**, 29].
- The Pentateuch contains many *terms* which Moses could not have unknown, description of *places* which Moses never visited and *linguistic forms* foreign to his time.
- Most of the laws in the Pentateuch and most of its narrative were *not* an integral part of the day to day living habits of the Hebrews during the time of Moses.
- Moses was so busy leading his people around the wilderness that he scarcely had time for extensive writing [*Exodus*, **18**, 14–18].

According to Jewish tradition, the original scrolls of the Pentateuch and additional biblical manuscripts were lost in the fire that destroyed the Solomon Temple (587 BCE). Ezra apparently reconstructed it from duplicate fragmentary documents. [*Hazon Ezra* **12**, 20–22, ca 100 CE].

For that, Spinoza was excommunicated from Judaism and exiled from Amsterdam. Simon's book Histoire critique du Vieux Testament was placed on the Catholic Index and he himself was expelled from his order just because he dared deny that Moses personally wrote "his" five books and because his superiors thought that his attack on Spinoza was insufficiently fierce. Both Catholics and Protestants joined forces to persecute him; from the 1300 copies of his book only six were saved from the stake.

*The politician **John Hampden the younger** (1656–1696, England), who translated Simon's book into English was imprisoned (1688), and later released from the Tower only after he publicly renounced his 'crime'. Depressed and humiliated, he eventually took his own life.*

During the 18th century there were more imprisonments and even assassination attempts against those who dare suggest that Moses was not the literal author. But as Victor Hugo once said, no army in the world can suppress an idea whose time has come. Biblical scholars n Europe, Christian and non Christian alike, began a modern scholarly textual investigation of sources of the Pentateuch.

*This new trend began already in 1315 CE with **Joseph Even Caspi**, [known also by his French name **Don Bonafous de L'argentera**] (1280–1340, France and Spain). In his book* Tirat ha-kesef *(1315) he discovered that in the book of* Genesis *there are instances where the* same *story is being told* twice *in entirely different terms, sometimes with* contradictions *between the two narratives. Moreover, in some versions God is called* Elohim *[e.g. Gen 1, 1 – 3, 23] while in the following chapters it is referred to as YHVH.*

*Caspi, however, did not draw from it any conclusions concerning the layout and uniformity of the biblical narrative. Only 300 years later was this line continued by the German priest **H.B. Witter**. He claimed (1711) that the book of Genesis was composed of two sources,[550] each describing the creation in a different way, a different style and using a different name of God.*

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550 *Examples*:

- *Gen 20*, Vs. **26**, 6–11: If these stories are not independent, they cannot be reconciled: for why should Abraham repeat his excuse ("She is my sister") with Abimelech, when it had already failed with Pharaoh (*Gen* **20**, 10–20). On the other hand why should Abimelech not learn from his bitter experience with Abraham and disbelieve Isaac that Rebekah was indeed his sister. The only logical way out of this dilemma is to assume that the Sarah-Pharaoh and Rebekah-Abimelech incidents belong to one source while the Sarah-Abimelech story belongs to another document.

- *Gen* **15** Vs Gen *21*, 9–21; two different stories on the escape of Hagar

- *Gen* **21**, 22–34 Vs **26**, 26–38; Abimelech makes covenants with both Abraham and Isaac?

The physician ***Jean Astruc***[551] *(1684–1766, France) began the modern scholarly textual investigation of sources of the Pentateuch. In his book: Conjectures sur les mémoires originaux dont il parait que Moyse s'est servi pour composer de livre de la Genèse (1753), he developed a method of separation of the sources in* Genesis *and* Exodus *into two parallel stories.*

Independently, the orientalist and historian ***Johann Gottfried Eich-***

- Gen **30**, 28–43 Vs **31**, 11–; two versions on how Jacob tricked Laban.
- Gen **31**, 44–54; two versions on the Jacob-Laban covenant.
- Gen **37**, 22–30; was Joseph saved by Reuben or Judah?
- Gen **42**, 37 Vs **43**, 9; who was surety for Benjamin, Reuben or Judah?
- Gen **42**, 27 Vs **42**, 35; where did the brothers discovered their money?
- The story of the *deluge* is a remarkable 'scissors and paste' work of two *inconsistent* sources [*Gen* **7**: 1–5, 7, 10, 12, 16–20 Vs *Gen* **7**: 8–9, 11, 13–16, where the editor put a line of his own in the intervals!]
- The *creation* account appears in two *inconsistent* sources: *Gen* **2**, 4–25 Vs **3**, 1–24, where the editor appears in **2**, 4. The reason both versions were included is that both were highly venerated and the redactor unwilling to discard either one.
- *Exodus* **20**, 1–17 Vs *Deuteronomy* **5** are two versions of the Decalogue. the Covenant Code and case law in *Exodus* **21** is at par with the primitive "law" of *Exodus* **34**, which does not agree with either of the Decalogue versions. The case law in *Numbers 5* and the religious law in *Numbers* **28–29** do not agree with case law of Exodus **21**.
- *Joshua* **1–12** and *Judges* **1**, tell two different stories on the conquest of Israel. The first tells us that the land fell in three swift and decisive military campaigns. The second describes it as a series of independent tribal actions that did not result in a complete occupation of the land – a slow and complex process, which is more in line with archaeological findings; *Judges* **2–16** tell of many setbacks and reverses in the process of consolidating control over Israel.
- Could Moses have written both *Gen:* **19**, 31–38 and *Lev:* **18**, 17; **20**, 14? Or else, could he have written both *Gen:* **38**, 18 and *Lev:* **18**, 15?

[551] M.D. Montpellier (1703); Prof. of anatomy at Toulouse (1710–1717); Prof. of Medicine: Montpellier (1717–1730), Paris (1731).

horn[552] *(1752–1827, Germany) founded the modern Old Testament criticism and pioneered scientific study of biblical literature in his book Einleitung in das Alte Testament (1780–1783). His conclusions:*

- *most of the writings of the Hebrews have passed through several hands.*
- *the so-called supernatural facts related to both Testaments were explicable on natural principles. It should be judged from the standpoint of the ancient world and accounted for by the superstitious beliefs which were then generally in vogue.*

The Protestant theologian ***Wilhelm Martin Leberecht de Wette***[553] *(1780–1849, Germany) first applied historical criticism to the Pentateuch in his book Beiträge zur Einleitung in das Alte Testament (1806–7). Showed that the book of Deuteronomy could not have been written earlier than 622 BCE, when it was found by a priest in the Solomon Temple. It may have been first written early in the reign of Josiah (ca 640–609 BCE) and edited later [the general idea was expounded already by* ***Eusebius Hieronymus*** *(c. 380 CE)].*

Following de Wette, about a century of biblical research has been devoted to the analysis of the process by which the books of the Bible emerged from a welter of traditions, oral and written, and the determination of the main stages of transmission until the present received text. The principal result has been the promulgation of the so-called "Documentary hypothesis", which is associated with the name of the German scholar ***Julius Wellhausen***[554] *(1844–1918) who gave it its classical formulation in his book Komposition des Hexateuch (1889).*

In his analysis Wellenhausen demonstrates that the Pentateuch is a composite work of many hands and periods, including priestly editors and professional scribes. It is the result of a long period of growth, compilation and transmission. Much of its narrative content derives from oral traditions which were subsequently reworked and expanded by revisers of various schools. This accounts for the fact that the five books of the Pentateuch contain so many

[552] Educated at Göttingen (1770–1774), Prof. Oriental Languages, Jena (1775), Prof. Exegesies of Testament and Political history, Göttingen (1778–1827).

[553] Educated in Jena. Prof. of Theology, Heidelberg (1807–1810), Univ. of Berlin (1810–1819), Univ. of Basel (1822–1849). In 1819 de Wette was dismissed from Berlin University and banished from the Prussian Kingdom. His subsequent appointment at Basel was strongly opposed by the orthodox party for obvious reasons.

[554] Professor at Halle (1882), Marburg (1845), Göttingen (1892).

duplications, inconsistencies, and even contradictions, not to speak of major stylistic differences, the result of the blending of diverse traditions and disparate points of view.

Wellhausen distinguished four sources in the Pentateuch, each of which is regarded as an independent 'document' which has been composed or compiled by a single author or editor. Various editors then put these sources together with necessary modifications, bridges, and adjustments to produce the connected whole. The dating of these documents is:

- *J, E: before the major prophets of the 9th–8th centuries. Covers the ancient period when festivities, cults, rituals, and ceremonies were connected to agriculture and nature (Exodus* ***23****, 24) and fertility stage of religion. The letter J stands for* Jehova *and the letter E for the* Elohim*, both the divine characteristics of God in the respective passages of the book of* Genesis, Judges *(in part),* Samuel *and* Kings.

- *D (Deuteronomist) in the century before the discovery of the document in the Temple (622 BCE). This was the final period of the Kingdom of Judah, where there was a tendency to make the religion feasts into historical national symbols (Deut* **16***) and turn the rituals into symbols that unite the nation under a single kingdom. It is spiritual and ethical stage of religion.*

- *P(priestly) during and after the exile (6th century BCE[555]). Feasts and rituals are independent of agriculture and secular policy. The corresponding biblical texts are the books of* Leviticus, Numbers *and parts from* Exodus *[Especially: Lev* **23***, Num* **28–29***]. It represents the* legal stage of religion*, principally concerned with rites, ceremonies, priestly duties, genealogy, and measurements.*

During the 19th century the results of the biblical sources research were met with vehement opposition by the religious establishment; the entire weight of Catholicism, Protestantism, and Judaism was arrayed against the scholars: not only were there four authors (name of which was Moses), but these men were actually suggesting that the accounts were written at very different times, centuries removed from each other!

[555] Professor **Yehezkel Kaufmann** (1889–1963, Israel) defended effectively (1937) an earlier date for P (making it roughly contemporary with D) and argued for the effective coalescence of J and E. Archaeological discoveries during the 20th century support his views that although the *books* of the Pentateuch obtained their final form in the days of **Ezra** (ca 445 BCE), the *sources* of the Torah are ancient, some dating as far back as the days of Moses.

One victim of this resistance was **William Robertson Smith** *(1846–1894), Scottish philologist, physicist, archaeologist, Biblical critic, and chief editor of the Britannica (from 1881). He was educated in the Universities of Aberdeen, Edinburgh (1866), Bonn and Göttingen and became Professor of Oriental Languages and Old Testament exegeses at Free Church College, Aberdeen. His articles in the 9th edition of the Britannica on the Documentary hypothesis aroused the anger of his dons and after a Church trial he was removed from his Chair (1881). He was later appointed Prof. of Arabic at Cambridge (1883). By the 20th century, the fury abated. Most mainline Protestants began to see that it really didn't matter who wrote the books; the content was the important part. The oppression of the Catholic Church melted when Pope Pius XII published "Divino Afflante Spiritu" (1943):*

"Let the interpreter then, with all care ... endeavor to determine ... the sources, written or oral, to which he had recourse and the forms of expression he employed".

Mark Twain (1835–1910), the most irreverent of writers, was actually a very religious man, but he did not subscribe to any orthodox set of beliefs, and he did not believe that the Bible was literally the word of God. He once said: "It ain't those parts of the Bible that I can't understand that bother me, it is the parts that I do understand".

1880–1899 CE **Adolf Johann Friedrich Wilhelm von Baeyer** (1835–1917, Germany). Among the major contributors to organic chemistry throughout the 19th century. Did pioneering work on organic dyes and the hydrochromatic compounds. Known especially for synthesis of *indigo* (1880) and formulation of its structure (1883), synthesis of Phthalein dyes, discovery of uric acid derivatives and investigation of polyacetone. He put forward a strain theory of carbon rings (1885) which explained the stability of ring compounds in terms of the distortion of their valence bonds from the normal angles of the tetrahedral carbon atom.

Baeyer's father was a Prussian general and his mother – an apostate Jewess. He studied under **Bunsen** and **Kekule** and held professional positions at Strasbourg, Berlin and Munich (1875–1915). Awarded the 1905 Nobel Prize for chemistry.

1881 CE **John Venn** (1834–1923, England). Logician. His book '*Symbolic Logic*' contained the '*Venn diagrams*', a system of overlapping circles (or ellipses, or other figures) for representing logical propositions.

Venn was born in Drypool, Hull, a descendant from a Devonshire family long distinguished for its erudition. Representing the 8^{th} generation of his family to study at Cambridge, he entered the university in 1853 and studied mathematics. Ordained a deacon in 1858 and priest in 1859, he served for a short period in parochial work. He resigned his orders in 1883 to devote himself entirely to the study and teaching of logic.

1881 CE **Charles Roy**. Made pioneering experiments demonstrating the nonlinear elastic behavior of the arteries.

Without the aid of any electronic devices, he constructed a gravity-driven apparatus that inflated isolated segments of blood vessel from human beings and other mammals, measured instantaneous pressure and volume, and plotted the results on a rotating drum called a *kymograph*.[556] He also tested strips of artery wall with an apparatus that plotted the fork-length curves for the tissue as it was stretched. With these data, Roy determined an artery wall's *nonlinear elasticity* and found that the distensibility of the human aorta decreases as a function of age. He also showed that the arteries distend considerably at resting blood pressure, which means that the *tensile stress is never zero*. Finally, he showed that the aorta is most compliant in the normal range of blood pressure.

In other experiments, he showed that heating makes the artery wall stiffer, so that an applied stress produces less strain. Thus Roy recognized that an artery wall had an elastic mechanism that was thermodynamically like that of natural latex *rubber* (caoutchouc) although the physics of this type of elastic material was not understood in Roy's time.

Modern research on synthetic rubber-like polymers, as well as on animal rubbers like *elastin*, has revealed that the elasticity of such polymer networks arises from changes in the entropy of the molecular chains; an imposed strain *increases order in the molecular network*, thereby decreasing its entropy. The *elastic force* arises from the tendency of the network to return to conformational states of higher entropy (or disorder), according to the laws of thermodynamics.

1881 CE **Clément Ader** (1841–1925, France). Engineer and inventor. Built and flew for 50 meters the *Eole*, a bat-winged, steam-powered airplane, 13 years ahead of the Wright brothers.

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[556] An instrument used to record temporal variations of any physiological or muscular process; it consists essentially of a revolving drum bearing a record sheet (usually of smoked paper) on which a stylus or pen-point travels to and fro at right angles to the motion of the cylinder. The drum is rotated by a mechanism at a uniform rate, or the rate is indicated by a time marker which registers on the sheet.

Discovered the *stereo effect* of sound by recording a live performance with the use of more than one microphone and subsequently delivered it via a number of phase-different signals to the listener's ears.

"The musical telephone" was a major attraction at the International Electrical Exhibition in Paris in 1881, where Ader demonstrated stereophonic transmission by telephone direct from the staged of the Paris Opera House and the Comédie Francaise. He used 12 stage microphones and laid the lines through the Paris sewers to the Exhibition Hall at the Palais de l'Industrie. Up to 48 listeners could hear the opera, using two receivers each, one for each ear. This first public broadcast entertainment was known as the *Theatrophone*.

In 1890, a commercial company, Compagnie du Theatrophone, was established in Paris, distributing music by telephone from various theaters to special coin-operated telephones installed in hotels, cafés etc., and to domestic subscribers.

The service continued until 1932. Elsewhere, trials of concerts by telephone were held, not only on a local basis but also with distribution over longer distances, e.g. from Paris to Brussels in 1887, and from Paris to London in 1891. A mixed service of news, telephone concerts and lectures was opened in Budapest in 1893.

1881–1886 CE **Lucien Gaulard** (1850–1888, France). Engineer and inventor.

Invented the first *power transformer* with annular core. With **John Dixon Gibbs** (England) succeeded in transmitting, for the first time (1883), a voltage of 2000 Volts over a distance of 40 km. This was the first effective device for long-distance transmission of AC power. In 1885, **Westinghouse** imported a number of Gaulard-Gibbs transformers and a Siemens AC generator to begin experimenting with AC networks in Pittsburgh, USA.

During his life, his work was not recognized in France. He fell into sever depression, was sheltered in a clinic and died there in 1888. A street in Paris now bears his name.

1881–1906 CE The great exodus of Jews from Russia. Organized government-incited massacres of the Jews in Russia [1871, 1881, 1903, 1905, 1906] was aimed at diverting public attention from serious interior problems and prevent the collapse of the crumbling Tzar's regime. The Nazis were to use exactly the same technique of violence-led legislation 52 years later.

In 1882 500,000 Jews living in the rural areas of the *Pale of Settlement*, were forced to leave their homes and live in towns and townlets (*shtetls*) in the Pale; 250,000 Jews living along the Western frontiers of Russia were also moved into the Pale; 700,000 Jews living east of the Pale were driven into the Pale by 1881 [e.g. 2000 Jews of Moscow were expelled (1891) and 2000 Jews were deported from St. Petersburg (1891)]. By 1897 there were 5 million Jews living in the Pale and 320,000 outside it in Siberia, Baltic provinces, Caucasus, Russian Central Asia, Astrakhan and Terek regions.

Thus, from 1881 this vicious, mounting and cumulatively overwhelming pressure on Russian Jewry produced a panic flight from Russia westwards. This year was the most important year in Jewish history since the expulsion of the Jews from Spain (1492). It had wide and fundamental consequences in world history too. During 1881–1914, more than 2 million Jews from Russia, Poland and Romania moved to the United States. This was a completely new phenomenon, which in time changed the whole balance of Jewish influence in the world and had great future impact on American and world science.

1881–1912 CE **Paul Ehrlich** (1854–1915, Germany). Bacteriologist, physician and a distinguished 'microbe-hunter'. Revolutionized the whole aspect of the preventive and curative treatment of infections. Founded *chemotherapy* (1910), modern *hematology* (1885), and modern *immunology* (1897). Became known for discovering Salvarsan (arsphenamine)[557], the 'magic bullet' remedy for syphilis (1909). Salvarsan is also called "606" because it was the 606^{th} compound tested. Ehrlich shared the 1908 Nobel prize for physiology or medicine with Elie Metchnikov for their work on immunity. He dominated the first phase of the chemotherapeutic revolution which ended with his death and was not resumed until 20 years later with the discovery by **Domagk** of the antibacterial action of the dye *prontosil rubrum.*

Ehrlich was born in Strehlen, upper Silesia, near Wroclaw to Jewish parents. After graduating in medicine from the University of Leipzig (1878), he spent a period as medical assistant at the Charité Hospital in Berlin. He married the 19 year old Hedwig Pinkus (1884). In 1888 tubercle bacilli were found in his sputum and he was forced to spend the next two years in Egypt, until he was cured. In 1890 Ehrlich joined Koch in the latter's new Institute of Infectious Diseases. In 1896 an institute for serum research and control was created near Berlin under Ehrlich's direction. In 1898 he moved to Frankfurt am Main to head the State Institute for Experimental Therapy. Later the George Speyer House was built nearby, and from 1906 until his death Ehrlich directed both institutions.

Ehrlich was a forceful personality, often engaged in controversy yet inspiring great loyalty, smoking more than 25 cigars a day ["burning his life's candle at both ends" — as he used to joke], but drinking only mineral water. He used to mail himself postcards a few days before every family anniversary lest

[557] *Dioxy-diamino-arseno-benzene-dihydrochloride.*

he forget them, and believed in the four big *G*'s, the formula for successful work: *Geld* (money), *Geduld* (patience), *Geschick* (ability) and *Glück* (luck).

Ehrlich began his work by studying the capacity of aniline dyes for selective staining; and one can trace almost all his later work back to the specific interaction between chemical substances and particular biological structures that dyes reveal, whose full significance he was the first to see. An early discovery (1881) was the so-called 'mast-cell', a large cell with distinctive granules taking up basic dyes, now known to be rich is histamine and to mediate many allergic reactions. A year later (1882) he defined the 'eosinophil' cells that occur in blood; these cells are now known to be particularly involved in resistance to parasitic infections. Extending his work to bacteria, he was the first to stain tubercle bacilli. The same insight led him to recognize the 'blood-brain barrier' (1885), through which only drugs with sufficiently fat solubility can penetrate, but which otherwise prevents many substances dissolved in blood from reaching the brain.

Further discoveries in this period were that the dye *methylene blue* selectively stained nervous tissue *in vivo*; and that other dyes, which bleach on removal of oxygen, allowed the study of the varying oxygen demanded of different tissues. In 1890 he demonstrated that antibodies were transmitted in maternal milk to provide '*passive immunity*' to a newborn animal. He produced (1890) a diphteria antitoxin sufficiently concentrated for its first clinical use. Its success led to the necessity of a standardization of toxin and antitoxin, which Ehrlich solved by preparing a dried, evacuated reference sample of antitoxin as an international standard, with which the toxin was titrated. With this, Ehrlich established the field of immunology.

His '*side-chain theory*' (1897) envisaged that a toxin which could combine with (but not kill) a cell, would give rise to a proliferation of the cellular binding sites involved[558]. As this method of '*immunotherapy*' failed against diseases such a malaria and syphilis, Ehrlich turned again to *chemotherapy*, the use of chemicals – especially constructed 'magic bullets' – to find, bind to, and act on the parasite. A range of phenols proved to be inhibited by serum and too toxic. The dye '*trypan red*' was found effective but '*drug resistance*' developed — the first description of this phenomenon. Compound

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558 **Ehrlich** postulated that the body's cells possess a great many "receptors" by which they combine with the food substances in the body fluids. He theorized that the metabolic products of certain bacteria combine with the receptors of some cells, thus injuring the cells. Ehrlich visualized receptors as unsatisfied chemical side chains. This is not far from the modern idea of receptors as domains in enzymes or other proteins, with which drugs of appropriate structure can combine. In fact, knowledge of cell receptors is now on the cutting edge of pharmacology and drug discovery.

No. 606 proved effective in human syphilis and, as *Salvarsan*, revolutionized its treatment. Adverse reactions, however, led to much controversy, until the safer No. 914, '*Neosalvarsan*' (1912) appeared.

The Ehrlichs has two daughters, the youngest of which, Marianne, was married to the mathematician **Edmund Landau**.

During the first year of World War I — the last of his life — Ehrlich's health deteriorated. Long years of heavy smoking of strong cigars (stimulants *needed* to withstand the enormous strain on his physical and intellectual constitution, and to stave off the effects of exhaustion, irregular meals, and improper food) took their toll and produced a disastrous effects on his system. He died of a stroke in Bad Homburg, Germany, and was buried in the Jewish Cemetery of Frankfurt.

Atop the two high columns at the entrance to the box-edged tomb were, visible from afar, the Star of David and the Snake of Aesculapius. At the head of the tomb, on a high stone carved from a natural block of marble, was a large vase of porphyritic rock containing trailing rose bushes with an abundance of blossoms. So his resting place was covered with the falling petals of these glowing flowers.

Ehrlich's whole life was one long fight for the promotion of medical science in the service of mankind. He had a deep-rooted and unwavering optimism, aiming always at perfection and ever more difficult targets, supported always by an unshakable faith in progress. He could have done much more for humanity had his life not been cut short by his premature death at the age of sixty-one. Striving for the health and happiness of the world he had overtaxed his own physical strength, and burnt the candle at both ends.

Ehrlich was a solitary thinker, inspired by humanitarian unselfish motives rather than by the struggle for power; single-minded, and able to elicit devotion from his followers. He was also a fearless rule-breaker, challenged on all sides by narrow-minded bureaucrats, and ill-treated by skeptical colleagues, stubbornly pursuing his research into a disease regarded with shocked abhorrence in his day[559].

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[559] Ehrlich's secretary, **Martha Marquardt**, worked for him during 1902–1915. Her first book 'Paul Ehrlich als Mench und Arbeiter', was published in memory of his 70^{th} birthday anniversary (1924). Most of the copies of this book were burnt by the Nazis on May 10, 1933. An extension of the previous work was therefore written and finished by her in 1940 in Paris, but remained unpublished owing to the ongoing war. In December 1946 Marquardt was able to revisit Frankfurt for a few weeks, and found a number of Ehrlich's old staff still at their posts in the Institutes in the Paul Ehrlich Strasse.

Edward G. Robinson (1893–1973; born Emanuel Goldenberg in Bucharest,

1881–1916 CE **Edward Emerson Barnard** (1857–1923, U.S.A.). Astronomer. Discovered the 5^{th} satellite of Jupiter[560], *Amalthea* (1892) and *Barnard's star*[561] (1916). Barnard was a pioneer in celestial photography: he made the first photographic discovery of a comet (1881). Barnard was also the first person to report seeing craters on Mars (however, he did not publish these observations for fear of ridicule). Around 1900, Barnard discovered the first of the *dark nebulae*, known today as *Barnard objects*[562].

Romania) played the character of Ehrlich in the movie *Dr. Ehrlich's Magic Bullet* (1940).

560 The first four had been detected by **Galileo Galilei** (1610). *Amalthea* is the last planetary satellite to be discovered without the aid of photography or spaceprobes.

561 The star with the largest known *proper motion* (10.3 arcseconds per year). It is due to the star's motion perpendicular to the line of sight. The largest proper motion of any naked-eye star is that of 61 cygni ($5''$ per year).
Barnard's star is at *distance* of 6.0 light-years from earth, its *absolute magnitude* is 13.22, its *apparent magnitude* is 9.54, *color* red, *surface temperature* 3250 K, *radius* about 0.2 solar radii. The question of whether it has planets, has not been answered.

562 For many years, astronomers have suspected that stars are born in cold, dark clouds of interstellar gas. If an interstellar cloud is warm, its atoms are moving about so rapidly that there is no chance for a protostar to condense from the agitated gases. If the temperature is low, however, then the atoms are moving slowly enough to allow denser portions of the cloud to contract gravitationally into clumps that collapse to form new stars. Many of these cold clouds are scattered across the Milky Way. In some cases they appear as dark regions silhouetted against glowing background nebulosity, such as the famous Horsehead Nebula. In other cases they appear as dark blobs that obscure background stars. These are *Barnard's objects*.

The Ether Hypothesis[563]

Aristotle *(ca 350 BCE) supported the theory that all of space is filled with the four elements: fire, water, earth, air and a fifth element — the ether. The ether was supposed to serve as a medium through which the stars moved in their daily courses around the earth. During the following* 2200 *years, the term 'ether' and 'vacuum' became largely synonymous. With the takeover of the Copernican world-view in the* 16^{th} *century the ether concept, in its Greek sense, disappeared from science. It was given, however, a new 'task': to serve as a medium for the transmission of light and gravitation. This view, known as the 'ether hypothesis', was mainly due to* **Descartes** *(1637) who specified the physical properties of the ether as an elastic, weightless material.*

Huygens *(1678), in his wave theory of light, needed the ether not only as a medium for the wave motion but also to explain its finite velocity and its refraction. The later discoveries of stellar aberration (1728) and the Doppler effect (1842), in which the velocities of the light and of its sources or detectors were combined, added support to the existence of the ether. The years* 1821–1838 *witnessed the development of the elastic ether theory. In* 1821, *the engineer* ***C.L.M.H. Navier*** *established the theory of elasticity of solid bodies, discerning that matter consists of countless particles (mass points, atoms) exerting on each other forces along the lines joining them. [****A.L. Cauchy*** *(1828) derived the equations of elasticity by means of the continuum concept.]*

Further development of this theory was due to **S.D. Poisson** *(1828),* ***G. Green*** *(1838),* **J. Maccullagh** *(1837) and* **Franz Ernst Neumann** *(1798–1895,* Germany, *1835). At this point the ether was assumed to be a luminiferous (light-carrying) elastic solid, capable of accommodating transverse wave-motion. However, all efforts to reach a consensus regarding a* mechanical model *of the ether, met with total failure.*

*Following the advent of the electromagnetic field concept [*introduced by ***Faraday*** *(1846) and* **Maxwell** *(1865)], the discovery of the electromagnetic nature of light and the recognition that the internal forces in material media*

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563 For further reading, see:

- Whittaker, E.T., *A History of the Theories of Aether and Electricity*: *From the Age of Descartes to the Close of the Nineteenth Century*, Longmans, Green and Company: London, 1910, 475 pp.
- Whittaker, E.T., *A History of the Theories of Aether and Electricity*: *The Modern Theories 1900–1926*, Philosophical Library: New York, 1954, 319 pp.

are of electrical and magnetic origin — the ether concept underwent further modification: It was stripped of all its material attributes and left with only the properties of imbuing all space outside and inside *matter, and of carrying electric and magnetic fields.*

The state of the ether prior to 1881 *was this: The space of* mechanics *was regarded as empty wherever material bodies were not present. The space of* optics *was filled with ether, which had a certain mass, density and elasticity. The universe no longer consisted of isolated masses separated by empty space, but was completely filled with a thin rigid, elastic medium of ether in which the masses were floating. Ether and matter acted upon each other with mechanical forces and moved according to Newtonian laws.*

This doctrine was assisted by an additional working hypothesis, stated as follows: The ether in astronomical space, far removed from material bodies, is at rest in an inertial frame. [If this were not the case, parts of the ether would be accelerated which in turn would bring about changes in density and elasticity, detectable through analysis of star light reaching us from those regions.] Hence, absolute space is at rest relative to the ether.

If the ether indeed defined a system of reference which was absolutely at rest, then the motion of the earth (for example) relative to it, should in principle be detectable.

Now, the detection of this motion by means of light waves *depended on whether or not the Galilean principle of relativity remains valid for optical phenomena. The ether theory gives the following answer to this question: the optical Doppler effect is indeed determined by the relative motion of the source of light and of the observer in accordance with Galilean relativity — but only if quantities of second-order are neglected*[564]*. Hence, the classical principle of relativity holds only approximately for optical wave phenomena.*

This furnishes us with a means of establishing motions relative to the ether. Indeed, ***Maxwell*** *(1879) called attention to the fact that by observing the eclipses of Jovian moons, it should be possible to ascertain a motion of the whole solar system relative to the ether, by measuring the variation in the* ***Roemer*** *apparent* 6*-month delay of these eclipses, over a* 6*-year interval (half*

[564] Let a light source, at rest relative to the ether, be monitored by two observers at rest in respective frame S (ether frame) and S' that move with relative uniform velocity $\boldsymbol{v}$. The origins of the frames, O and O', coincide at $t' = t = 0$. Let the train of waves be associated with an electric field $\boldsymbol{E} = \boldsymbol{E}_0 \sin(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t)$, $k^2 = \frac{\omega^2}{c^2}$. Since the phase is invariant (representing as it does the number of wave-crests) we have: $\boldsymbol{k}'\cdot\boldsymbol{x}' - \omega' t' = \boldsymbol{k}\cdot\boldsymbol{x} - \omega t$, as seen by the two observers [$\boldsymbol{k}$ = wave-number vector, $\boldsymbol{x}$ = spatial coordinates, ω = angular frequency, t = time]. Assuming the *Galilean transformation* $\boldsymbol{x}' = \boldsymbol{x} - \boldsymbol{v}t$, $t = t'$, one obtains

the Jovian year). However, the experimental errors in such a measurement are too large to obtain a meaningful result.

The experiments of ***Michelson*** *(1881) and Michelson and Morley (1887) have shown that the velocity of light in terrestrial measurements is not influenced by the motion of the earth* even to the extent involving quantities of the second order.

The immediate result of this experiment was that the ether lost its last material properties, namely — its ability to move and its specific location: For if we must assume that light propagates with a fixed velocity relative to all observers, independent of the velocity of its source and irrespective of the velocities of its observers [which may move with different velocities relative to each other], then this ether is totally superfluous[565].

$$\boldsymbol{k}' = \boldsymbol{k}, \quad \omega' = \omega\left(1 - \frac{\boldsymbol{v}\cdot\widehat{\boldsymbol{k}}}{c}\right), \quad c' = c - \boldsymbol{v}\cdot\widehat{\boldsymbol{k}}, \quad \omega = 2\pi\nu, \quad \omega' = 2\pi\nu',$$

where $\widehat{\boldsymbol{k}}$ is a unit vector normal to the wave-front, c' is the *phase velocity* in frame S', and ν, ν' are the frequencies (in cycles per second) in the respective frames. In the simple case where $\boldsymbol{v}$ is parallel to $\widehat{k}$ (along the x-axis, say),

$$c' = c - v, \quad \nu' = \nu\left(1 - \frac{v}{c}\right).$$

Here c is the velocity of light measured by an observer at rest relative to the ether.

If, on the other hand, an observer at rest in the ether measures the frequency of a *moving source* with intrinsic frequency ν_0 and velocity v_0, the observed frequency is $\nu = \frac{\nu_0}{1-\frac{v_0}{c}}$. For cases where $\frac{v_0}{c} \ll 1$, we find

$$\nu \sim \nu_0\left(1 + \frac{v_0}{c}\right).$$

If we assume a simultaneous motion of the source of light (v_0) and the observer (v), the observed frequency ν' is

$$\nu' = \nu\left(1 - \frac{v}{c}\right) \sim \nu_0\left(1 + \frac{v_0 - v}{c}\right).$$

So, *to first order*, the Galilean relativity principle holds for the optical Doppler effect.

565 In a sense, though, the $3\,^\circ$K cosmic microwave background radiation (CMBR), permeating the universe, has replaced the ether in modern cosmology; at least, the velocity of the solar system w.r.t. it can, and has, been measured through the terrestrially-observed 'Doppler shift' in that cosmic temperature — once

Sic transit gloria ether. Indeed, after **Einstein** *had expounded the special theory of relativity (1905), the ether was banished from classical physics. After* 1915, *when the general theory of relativity interpreted gravitation as the intrinsic geometry of space-time, besides the CMBR, one might say that another, modern version of the ether entered physics – namely, dynamical spacetime itself!*

This "ether", though, is locally invariant under non-accelerating changes of reference frame. Space (the vacuum) is dynamical not just by virtue of GTR (where undulations in it are called gravitational waves), but also in Quantum Field Theory.

1881–1933 CE **Albert Abraham Michelson** (1852–1933, U.S.A.). Experimental physicist. Established that the velocity of light is not influenced by the motion of the earth thereby showing that the ether hypothesis must be abandoned.[566]

Michelson spent 50 years in improving his measurements of the velocity of light, ending in 1933 with a value of 299,744 km/sec [2 km/sec higher than the value accepted in the 1970's]. Michelson was the first American scientist to win the Nobel prize in physics (1907).

Michelson was born to Jewish parents in Strelno, Prussia. He came to the United States with his parents in 1854. At 17 he entered the U.S. Naval Academy, from which he graduated in 1873, serving as a science instructor there until 1879. In 1880 he traveled to Europe and during 1880–1881 built an interferometer by means of which he was able to demonstrate in Berlin (1881) that there was no motion of the earth relative to the ether. He repeated this experiment with **Edward William Morley** (1838–1923, U.S.A.) in 1887,

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the shift due to the earth's motion relative to the sun is subtracted. (The result is a few hundred km/sec.)

566 The experiment's "null" result quite obviously dominated the work of **Lorentz** and many others. But it was *not* the road along which special relativity evolved. **Einstein** said (1945) that at the time he wrote his basic paper on relativity (1905) he had never heard of the experiment. Einstein elaboration on STR began with his rejection of the "luminiferous ether", and in that sense Michelson's experiment was not decisive. Einstein's reasoning is sufficiently simple and logical, and there is every reason to use it in expounding the special theory of relativity.

with an improved interferometer which he built with a grant of 200 dollars obtained from Alexander Graham Bell. In 1883 he accepted a position of professor of physics at the Case School of Applied Science in Cleveland. At 1892 he was appointed professor and head of the department of physics at the University of Chicago, a position held until his retirement in 1929. Michelson's other achievements in physics are:

(1) Measured the earth's mean rigidity[567].

(2) Measured the diameter of the star Betelguese[568] (Alpha Orionis) by means of the partial coherence of light arriving from its opposite edges (1920).

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567 He used an interferometer to measure the bodily tide in the solid earth by the disturbance of the water level in two vertical tubes with a long horizontal connection underground [*Astrophys. J.* **39**, 105–138, 1914; *Astrophys. J.* **50**, 330–345, 1919].

568 *Michelson's stellar interferometer* (1890) measures the small angular dimensions of remote astronomical objects; a star is presumed to be a circular distribution of partially coherent point sources such that it has a uniform brilliance. If one were to perform an interference experiment with this source, in which a double-slit aperture was used, as in Young's experiment, then the distance between the slits would have to be less than the lateral coherence width in order to obtain distinct interference fringes. In practice, a telescope objective, diaphragmed by two equal small apertures, is used to view the starlight, of effective wavelength $\overline{\lambda}$ (mean wavelength of a narrow spectral band) and angular source diameter θ. The star's *visibility* is shown to be

$$\varphi = |\gamma_{12}(0)| = 2\left|\frac{J_1(\pi h\theta/\overline{\lambda})}{\pi h\theta/\overline{\lambda}}\right|,$$

where h is the smallest value of the slit separation for which the visibility of the fringes is minimum. The first zero of φ occur when

$$\pi h\theta/\overline{\lambda} = 3.83, \quad \text{or} \quad h = 1.22\frac{\overline{\lambda}}{\theta}.$$

Once h and $\overline{\lambda}$ are known, θ is calculable. Michelson employed *mirrors* to increase the effective distance between the slits. This enabled him to measure very small angular diameters (even for nearby stars, angular diameters are of the order of hundredths of a second of arc, with corresponding lateral coherence width of the order of several meters). Pointing the 100 inch reflector telescope of the Mount Wilson Observatory toward *Betelguese* (α Orionis), the fringes formed by the interferometer were made to vanish at $h = 121$ inches, and with $\overline{\lambda} = 5800$ Å, $\theta = 0.047$ seconds of arc. Using its known distance (determined from parallax measurements), the star's diameter turned out to be about 280 times that of the sun! (it is a red giant). In the 1990's, Betelguese's disc was optically resolved

(3) Suggested the use of the wave-length of the red line of Cadmium as the basis for a new standard of length (1893). This suggestion was accepted in 1960.

Science and Economy

In 1881, Albert Michelson, using a $ 200 grant from Alexander Graham Bell, had built an instrument called the interferometer, with which he disproved the existence of the mysterious ether that was supposed to fill all space. Today's atom smashers, in dramatic contrast, cost hundreds of millions of dollars to build and operate – and there is no guarantee that anything as momentous as Michelson's discovery will result.

By the mid 1970's, the honeymoon between science and the US Government had ended. Pressures on the Federal Government multiplied, and Congress was becoming tougher about spending. Scientists were told that "... the American people cannot afford to finance science as a hobby horse — science for the fun of it. They envision practical science as a workhorse for the people — research that produces a better quality of life... Congress wants scientific research that gets results".

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and imaged (in the usual, ray-optics manner) by the orbiting Hubble space telescope.

The Operational Calculus[569]

***Leibniz**' differential notation (1672) made it possible to consider the differential operator as an algebraic quantity independent of the function operated upon. Several mathematicians, among them **Lagrange**, **Laplace** (1812) and **Cauchy** (1827) employed this idea, so fundamental for the operational calculus. An explanation of the success of the algebraic treatment of the differential operators was sought in other fields of mathematics. Laplace, for example, explained the operational methods by means of the* Laplace transform, *whereas Cauchy used the* Fourier theorem.

***Servois** (1814) thought that the reason why algebraic treatment was applicable to differential operators was that the latter obeyed the commutative and distributive laws. **Boole** (1859) created his own version of an abstract algebraic approach to differential operators.*

*However, all the above contributions consisted only in the introduction of operational methods into analysis. It was **Oliver Heaviside** (1893), whose work stimulated* a systematic use of operational methods in physical and technical problems. *It was he who presented an abundance of mathematical and physical methods and results. In fact, most of his methods stemmed from his need to solve practical problems associated with work as an operator of the great Northern Telegraph Company. However, Heaviside developed a* formal calculus, *suited for his own purpose.*

The pure mathematicians of his time would not deal with this nonrigorous theory, but in the 20^{th} *century several attempts were made to rigorize Heaviside's operational calculus. These attempts can be grouped into two classes: the one leading to a representation of the operational calculus in*

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[569] For further reading, see:

- Van Der Pol, B., *Operational Calculus*, Cambridge University Press, 1959, 415 pp.
- Scott, E.J., *Transform Calculus*, Harper and Brothers: New York, 1955, 330 pp.
- McLachlan, N.W., *Modern Operational Calculus*, Dover: New York, 1962, 218 pp.
- Carslaw, H.S. and J.C. Jaeger, *Operational Methods in Applied Mathematics*, Oxford University Press, 1953, 359 pp.

*terms of integral transforms [***Bromwich**[570] *(1916),* **Carson** *(1917),* **van der Pol** *(1929)] and the other leading to an abstract algebraic formulation [***P. Levy** *(1926),* **Mikusinski** *(1949)]. Also,* **Schwarz***'s creation of the theory of distributions (1945) was very much inspired by problems in the operational calculus of Heaviside.*

It is remarkable that the theory of linear operators of Hilbert spaces and Banach spaces and the theory of von Neumann algebras, which was developed in the period 1900–1940, *did not interact at first with the development of the operational calculus. There are three reasons for this:*

(1) The operational calculus had its source in practical applications of differential equations whereas operators in Banach spaces developed from theoretical interest in integral equations.

(2) The operational calculus was successful in practice but lacked in rigorous interpretation, whereas the theory of integral equations had clear concepts but no effective methods of solution.

(3) Operational calculus developed at the fringe of the mainstream of mathematics and was scarcely used by practitioners, while the theory of operators in Banach space occupied a central position in the mathematics of the 20^{th} *century and was created by pure mathematicians.*

The development of the operational calculus provides an illustrative example of how a practical problem — long distance telegraphy — can influence mathematical theory. Thus, computation techniques that arose in engineering inspired an essential field of mathematics, namely the theory of distributions.

The Maxwellians (1879–1894)

After Maxwell's death (1879), a tightly knit group of British physicists, the self-styled 'Maxwellians' transformed the rich but confusing raw material of James Clerks Maxwell's Treatise *into a solid, concise and well-confirmed theory. Only with that transformation in the two decades after Maxwell's death*

570 The Bromwich inversion integral $h(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{pt}\frac{f(p)}{p}dp$ of the Laplace transform $f(p) = p\int_0^\infty e^{-pt}h(t)dt$, was known to **Riemann** as early as 1859.

did 'Maxwell's equations' emerge in the form they have since retained, and the accompanying technological developments begin. Thus, the field-theoretical ideas of Faraday and Maxwell were clarified, consolidated, completed and reformulated into the core of thinking and research in physics.

In 1873, Maxwell published a two-volume Treatise on Electricity and Magnetism *that was destined to change the orthodox picture of physical reality. This treatise did for electromagnetism what Newton's* Principia *had done from classical mechanics. It not only provided the mathematical tools for the investigation and representation of the whole electromagnetic theory, but it altered the very framework of both theoretical and experimental physics. It was this work that finally displayed action-at-a-distance physics and substituted the physics of the field.*

Like Newton's Principia*, Maxwell's* Treatise *did not immediately convince the scientific community. The concepts in it were strange and the mathematics was clumsy and involved. Most of the experimental basis was drawn from the researches of Michael Faraday and one of Maxwell's purposes in writing his treatise was to put Faraday's ideas into the language of mathematical physics — precisely so that orthodox physicists would be persuaded in their importance.*

Maxwell died in 1879, midway through preparing a second edition of the treatise. At that time, he had convinced only a very few of his fellow countrymen and none of his continental colleagues. That task fell to his disciples.

During the twenty years that followed Maxwell's ideas were picked up in Great Britain, modified, organized and reworked mathematically so that the Treatise *as a whole and Maxwell's concepts were clarified and made palatable and irresistible to the physicists of the late 19th century.*

James Clerks Maxwell's theory of the electromagnetic field is generally acknowledged as one of the outstanding intellectual achievements of the 19th century. By the mid-1890s the 4 "Maxwell's equations" were recognized as the foundation of one of the most successful theories in all of physics, taking their place as companions to Newton's laws of mechanics. The equations were by then also being put to practical use, most dramatically in the emerging new technology of radio communication, telegraph, telephone, and electric power industries.

Surprisingly enough, Maxwell's Treatise *(1873) doe not contain the 4 famous* Maxwell's equations*, nor does it even hint at how electromagnetic waves might be produced or detected. These and many other aspects of the theory were thoroughly hidden in the version of it given by Maxwell himself.*

The task of digging out the "latent" aspects of his theory and of exploring its wider implications was thus left to a group of younger physicists: Between

1879 and 1894 these "Maxwellians"[571], led by **George Francis FitzGerald** *(1851–1901),* **Oliver Lodge** *(1851–1940) and* **Oliver Heaviside** *(1850–1925), with a key contribution from* **Heinrich Hertz** *(1857–1894), transformed the rich but confusing raw material of the* Treatise *into a solid, concise, and well-confirmed theory (at least for free space) — the "Maxwell's theory" we know today. It was they who first explored the possibility of generating electromagnetic waves and then actually demonstrated their existence; it was they, along with* **J.H. Poynting** *(1852–1914), who first delineated the paths of energy flow in the electromagnetic field and then followed out the far-reaching implications of this discovery; it was they who recast the long list of equations Maxwell had given in his* Treatise *into the compact set now universally known as "Maxwell's"; and it was they who began to apply this revised theory to problems of electrical communications, with results that have transformed modern life.*

Scientific theories rarely sprig fully formed from the mind of one person; a theory is likely to be so refined and reinterpreted by later thinkers that by the time it is codified and passes into general circulation, it often bears little resemblance to the form in which it was first propounded.

1881–1912 CE **Oliver Heaviside**[572] (1850–1925, England). Eminent mathematical physicist. Originated modern operational calculus[573] (1893), laid the foundation of modern electric-circuit design and pioneered the application of vectors to physics. He developed and reformulated the electromagnetic theory of Maxwell, discovered the circuit principle that made the long-distance telephone possible, and foresaw television and over-the-horizon

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571 To dig deeper into the contributions of these "Maxwellians" to Maxwell's heritage, one is advised to read the comprehensive study "The Maxwellians" by B.J. Hunt, Cornell University Press 1994, 266 pp.

572 For further reading, see:

- Nahin, P.J., *Oliver Heaviside*: *Sage in Solitude*, IEEE Press: New York.

573 Heaviside was first to apply the *unit-step function* (sometimes named after him). The *delta-function*, $\delta(x)$, was introduced by Dirac in quantum mechanics (1930), but Heaviside had already used it extensively before him (1893). **Cauchy** (1815) knew the unit-step function in the definition $U(t) = \frac{1}{2}(1 + \frac{t}{\sqrt{t^2}})$ which was called by him '*restricteur*'.

radio. He virtually 'invented' the ionosphere to explain Marconi's transmission of radio signals over the Atlantic in 1901.

Stimulated by Maxwell's "*Treatise on Electricity and Magnetism*" (1873) and working independently of Gibbs until 1888, he developed vector analysis from the quaternion system. Consequently, he vastly simplified Maxwell's 20 equations in 20 variables by squeezing their essence into 4 equations in vector form. He eliminated the potentials and emphasized the primacy of the physical field-vectors[574,575] $\boldsymbol{E}$ and $\boldsymbol{B}$ (he also suggested boldface type to distinguish vectors from scalars).

Heaviside was born in a London slum, "among these dark Satanic mills", at the beginning of the mid-Victorian age. His family was at a low social and economic level. The world of his youth was very grim and Heaviside might have been a character straight out of Dickens: the youngest of four sons of a sickly wood engraver who could barely support his family. An early bout with scarlet fever left his hearing permanently impaired, cutting him off from the society of other children. [That handicap molded a confrontational personality and sarcastic style that would sometimes carry him too far in his published attacks on those with whom he disagreed. Years latter he recalled his youth with great bitterness, declaring that it had permanently deformed the course of his life.]

He left school at 16 and thereafter had no formal education, let alone any university training. After teaching himself the Morse Code and the elements of electricity, he went at 18 to Denmark to work for the Northern Telegraph company. He got this job through his uncle, **Charles Wheatstone**, husband of his mother's sister. In Denmark, Heaviside gained practical experience as a telegraph operator and technical trouble-shooter, and was steadily promoted.

He returned to England in 1871 and embarked on an ambitious program of self-education in science and mathematics. A paper he published in 1873 merited mention in the 2^{nd} edition of Maxwell's treatise on '*Electricity and Magnetism*'. His encounter with this book led him to quit his position in 1874 and devote himself entirely to private study. [Heaviside would never again hold any other job in his life; he spent the next 35 years in scholarly research,

574 To Maxwell, the magnetic vector potential, not the fields, played the central role in electrodynamics [an idea enjoying a comeback in modern Quantum Electrodynamics, and the other gauge theories inspired by it]. Also, Maxwell's original equation display no obvious symmetry in their form.

575 This was done independently by **Hertz** in Germany (1884). However, the two men arrived at the same endpoint by two entirely different paths. Heaviside and Hertz became good friends through an extensive correspondence, but never met.

the publication of technical papers and the carrying on a most interesting correspondence.] This was a momentous decision for a man of 24 without independent means. Since then he was living as a recluse among and off his relatives, devoting himself to the extension of Maxwell's theory.

In 1884, independently of **Poynting**, he described the flow of electromagnetic energy in space [Poynting got into print first, which justifies the modern name of 'Poynting vector']. During 1888–1903, Heaviside pushed Maxwell's theory beyond the limits set by the master himself: he was speculating on 'faster-than-light' charged particles producing a *conical wave* [electromagnetic shock-wave, known today as *Cherenkov radiation* of light in matter].

In 1891 Heaviside was elected a fellow of the Royal Society. Thus, in 17 years he had risen from the obscurity of an unemployed telegraphist to world fame. In 1896 a state pension was awarded him, at the instigation of **FitzGerald** and other distinguished scientists.

Nevertheless, the fact that he was not a university man raised a barrier, a certain antagonism, between him and his contemporaries. The latter reproached him for his notable lack of mathematical rigor. Yet Heaviside did develop an abundance of mathematical and physical methods and results which afterwards, on critical elaboration by various scientists, proved to be substantially true[576]. By 1908 Heaviside moved to Torquay, on the southern

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[576] Heaviside used the abbreviations $p = \frac{d}{dt}$, $p^{-1} = \int_0^t \cdot dt$. One of the problems considered by him was a semi-infinite cable in series with impedance r and a voltage source $V_0 H(t)$. Neglecting the self-induction in the cable and denoting the potential and current by $E(x,t)$ and $I(x,t)$, respectively, the circuit equations are

$$-\frac{\partial I}{\partial x} = C\frac{\partial E}{\partial t}; \qquad -\frac{\partial E}{\partial x} = RI,$$

where C, R are the capacitance and resistance per unit length, respectively. Eliminating I and putting $\frac{\partial E}{\partial t} = pE$, Heaviside obtained the ODE:

$$\frac{d^2E}{dx^2} = (RCp)E$$

with the solution

$$E(x,p) = A(p)e^{\lambda x} + B(p)e^{-\lambda x}, \qquad \lambda = \sqrt{RCp}.$$

After the determination of A and B from the boundary conditions at $x = 0$ and $x = \infty$, he obtained the current I_0 and voltage E_0 at the end of the line in the explicit form

$$I_0 = V_0\left[r + \sqrt{R/Cp}\right]^{-1}; \quad E_0 = V_0\left[1 + r\sqrt{\frac{Cp}{R}}\right]^{-1}.$$

coast of England. There, his F.R.S. and other honors meant nothing to his neighbors, who treated him as a joke. He died there and lies buried in his parents' grave, his name visible on the tombstone only when the grass is closely cut.

Homage to Oliver

"Like all creative scientists, he did it because he could not help it. There were ideas pent up in him which demanded expression at any cost. He developed scientific ideas as naturally as a poet writes or a bird sings."

Norbert Wiener, 1936 (1894–1964)

Expanding in ascending powers of p (valid for $t \to \infty$) and proving heuristically that $p^{1/2}H(t) = (\pi t)^{-1/2}$ [a formula known to **Sylvestre Lacroix** (1819)], he obtained

$$E_0 = V_0 - V_0 r\sqrt{\frac{C}{\pi Rt}}\left\{1 - \frac{r^2C}{2Rt} + 3\left(\frac{r^2C}{2Rt}\right)^2 + \cdots\right\}.$$

For small values of t, Heaviside expanded the expression for E_0 in descending powers of p, arriving at

$$E_0 = 2V_0\sqrt{\frac{Rt}{\pi r^2C}}\left\{1 + \frac{2Rt}{3r^2C} + \frac{1}{15}\left(\frac{2Rt}{3r^2C}\right)^2 + \cdots\right\} - V_0\left(e^{\frac{Rt}{r^2C}} - 1\right).$$

Thus, Heaviside solved the problem of the electrical transmission-line *avoiding the use of the Laplace transform* and without using the term 'asymptotic series'. He never employed any of the theories of divergent series which were introduced at the end of the 19^{th} century, but proceeded quite formally. This caused him a great deal of trouble with the "Cambridge mathematicians", who were so indignant at Heaviside's unrigorous use of divergent series, that they stopped the publication of a sequence of his papers. Nevertheless, Heaviside continued to use his '*experimental mathematics*'.

"We should now place the operational calculus with Poincaré discovery of automorphic functions and Ricci's discovery of the tensor calculus as the three most important mathematical advances of the last quarter of the 19th century. Applications, extension, and justifications of it constitute a considerable part of the mathematical activity of today."

Edmund Taylor Whittaker (1928)

"The next time you make a long-distance call and the voice on the other end comes through loud and clear, reflect for a moment on the man who made it possible."

Paul J. Nahin[577] (1988)

1881–1922 CE **Eliezer Ben-Yehuda** (1858–1922, Israel). Philologist and lexicographer. Father of modern Hebrew. Revived the ancient language into a vernacular that served as a basis for current spoken Hebrew. Compiled the great modern Hebrew Thesaurus (17 volumes; 1910–1922[578]), based on biblical, Talmudic and post-Talmudic sources. It was his fanaticism, aided by a combination of fortunate circumstances, which finally made a reality of his dream. If modern Hebrew became a living tongue with time, it was partly because of his innovations and efforts.

Ben-Yehuda was born as Eliezer Isaac Perelman in Lithuania. Studied at the Sorbonne, Paris (1878–1881). Settled in Jerusalem (1881), established the first Hebrew school (1881) and the Hebrew Language Academy (1890).

1881–1925 CE **Francis Ysidro Edgeworth** (1845–1926, Ireland and England). Economist. Made important contributions to mathematical economics and statistics, notably on general equilibrium theory.

Edgeworth was born in Longford, Ireland (now Irish Republic) of mixed Irish, Spanish and Huguenot descent. Became professor of political economy at King's College, London (1888–1891) and Oxford (1891–1922).

His main achievement (not adequately appreciated until the development of *game theory* and related topics after 1944) was to pioneer an approach to general equilibrium based not (like Walras's scheme) on an explicit economy-wide price mechanism, but on direct *cooperation between individual agents* in the absence of prices. This approach has subsequently been shown to yield an optimum effectively identical with the competitive optimum of **Walras** and **Pareto**.

[577] P.J. Nahin: "*Oliver Heaviside: Sage in Soltitude*" IEEE Press, 1988.

[578] Completed 1959, by his widow and son.

1882 CE Electric light in New York.

1882 CE The *Albatross expedition*, under direction of U.S. Fish Commission, further extended knowledge of the extent and variety of *marine life*.

1882 CE **Moritz Pasch** (1843–1930, Germany). Mathematician. Pioneer of the pure axiomatic approach to geometry. In his book (1882) *Vorlesungen über neuerer Geometrie* (Lectures on modern geometry), he developed a new method of representation of rigorous deductive structures of projective geometry via axioms. The *Pasch Axiom*[579] is named after him. Hilbert was influenced by these ideas of Pasch.

Pasch was born to a Jewish family. He was a professor at Giessen (1873–1911).

1882–1892 CE **Carl Louis Ferdinand von Lindemann** (1852–1939, Germany). Mathematician. Proved that π is transcendental. This proof ended the long odyssey in quest of the squared circle, started by Anaxagoras[580], ca 434 BCE. Showed (1884, 1892) how to express the roots of an arbitrary polynomial in terms of theta functions.

1882–1897 CE **Friedrich Ratzel** (1844–1904, Germany). Geographer and ethnographer. Had principal influence in the modern development of both disciplines.

Ratzel was born in Karlsruhe, Germany and studied zoology at the University of Heidelberg, graduating in 1868. During 1869–1875 he traveled extensively in the Americas, studying urban and cultural life, which later helped him to lay the foundation of *cultural geography* and *political geography*. His most important works are: *Anthropogeographie* (1882, 1891) and *Politische Geography* (1897).

According to Ratzel, cities are the best place to study people because life is "blended, compressed, and accelerated" in cities, and they bring out the "greatest, best, most typical aspects of people." He believed that once these facts about urban life are examined, they can serve as a great aid in the study of cultural history. His interest in cultural geography would soon inspire him to explore the field of *human geography*.

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[579] An axiom of *order*. If a straight line intersects one side of a triangle and does not pass through a vertex, it must intersect another side of the triangle. In the axiomatic system of Pasch, the concepts of point, line and plane are undefined.

[580] The side of a square of equal area to a unit circle is $x = \sqrt{\pi}$. Since π is transcendental, the equation $x^2 - \pi = 0$ is not solvable by an algebraic number and hence π has no construction with compass and straightedge.

In his *Anthropogeographie*, he examined the causes of human population distribution, or the dynamic aspects of geography. He also related geography to history. Physical features, such as mountains or bodies of water, are discussed w.r.t. *human migrations*. According to Ratzel, religious, linguistic, and ethnic groupings also determine population distribution.

As an outgrowth of these studies, he began his study of *political geography*. In this book, Ratzel develops the concept that views the state as "a particular spatial grouping on the earth's surface." The state, as defined by Ratzel, consists of "a human group with definite organization and distribution." From these ideas, Ratzel developed the concept of *Lebensraum* or living space, Ratzel hypothesized that the state naturally seeks to increase its size. If the state's neighbors are weak, the state will grow larger and spread into other states. As evidenced, Ratzel believed that space was a great political force.[581]

1882–1911 CE **Adolph Hurwitz** (1859–1919, Germany). Mathematician. Contributed to the theories of special functions, ordinary differential equations, modular functions, number theory, Riemann surfaces, set theory and Fourier series[582]. In 1882 he defined the generalized Zeta function

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[581] *Geopolitics* is an approach to understanding international politics that seeks to explain the political behavior of states in terms of geographical variables such as size or location (a kind of 'Social Darwinism').

The ideas of Ratzel in this field were extended by **Rudolf Kjellen** (1864–1922) and **Karl Haushofer** (1869–1946). The latter came to be seen in the 1930s and during WWII, as providing the geopolitical ideas for the *Nazis*. However, Nazi geopoliticians rejected Haushofer's geopolitics because it failed to incorporate the '*race principle*' adequately. Like Ratzel, Haushofer had some of his ideas hijacked by the Nazis. Nevertheless, Haushofer is still accused of providing the academic and scientific support for the expansion of the Third Reich. Incidentally, Haushofer's son, Albrecht, was indicted in the July 20, 1944 attempt to assassinate Hitler and was executed in 1945 by the SS.

Following the war, Haushofer was interrogated by the allies and put to trial before the Nuremberg War Crimes Tribunal, but acquitted. Together with his wife (half-Jewish) Haushofer committed suicide on March 13, 1946, in Pähl, W. Germany.

[582] In 1902, Hurwitz gave an elegant solution to the ancient *isoperimetric problem* of finding a simple closed plane curve of given perimeter with maximum area. Confining himself to continuous piecewise smooth close curves, he put $x = x(s)$, $y = y(s)$, $0 \le s < L$ as the parametric representation of such a curve of perimeter L and area F, s being the arc-length.

$[\zeta(s,a) = \sum_{n=0}^{\infty}(a+n)^{-s}]$ and derived an important formula for it[583].

In 1889 he wrote a fundamental paper on the zeros of Bessel functions, Lommel polynomials and analytic functions in general (*Hurwitz theorem*).

In 1896 he showed that any rotation in 4-dimensional space E_4, could be expressed in the form $q \to \ell q r^{-1}$, where q is a quaternion and ℓ, r are unit quaternions.

Assume the Fourier-series expansions

$$x(t) = \frac{1}{2}a_0 + \sum_{\nu=1}^{\infty}[a_\nu \cos(\nu t) + b_\nu \sin(\nu t)],$$

$$y(t) = \frac{1}{2}c_0 + \sum_{\nu=1}^{\infty}[c_\nu \cos(\nu t) + d_\nu \sin(\nu t)]$$

where $t = 2\pi\frac{s}{L}$. The relations

$$(dx)^2 + (dy)^2 = (ds)^2 = \left(\frac{L}{2\pi}\right)^2 dt^2, \qquad F = \int_0^{2\pi} x\dot{y}dt$$

then lead to

$$L^2 = 2\pi^2\sum_{\nu=1}^{\infty}\nu^2(a_\nu^2 + b_\nu^2 + c_\nu^2 + d_\nu^2),$$
$$F = \pi\sum_{\nu=1}^{\infty}\nu(a_\nu d_\nu - b_\nu c_\nu).$$

It follows that

$$L^2 - 4\pi F = 2\pi^2\sum_{\nu=1}^{\infty}\left[(\nu a_\nu - d_\nu)^2 + (\nu b_\nu + c_\nu)^2 + (\nu^2-1)(c_\nu^2 + d_\nu^2)\right] \geq 0.$$

For a given L, the *equality* will hold for a maximal F. In that case one must have $a_\nu = b_\nu = c_\nu = d_\nu \equiv 0$ for $\nu = 2, 3, \dots$ and $b_1 + c_1 = 0$, $a_1 - d_1 = 0$. Hence

$$x = \frac{1}{2}a_0 + a_1\cos t + b_1\sin t, \qquad y = \frac{1}{2}c_0 - b_1\cos t + a_1\sin t,$$

which are the parametric equations of a *circle*. All other curves must satisfy $L^2 > 4\pi F$.

583 $\zeta(s,a) = \frac{2\Gamma(1-s)}{(2\pi)^{1-s}}\left\{\sin\left(\frac{\pi s}{2}\right)\sum_{n=1}^{\infty}\frac{\cos(2\pi a n)}{n^{1-s}} + \cos\left(\frac{\pi s}{2}\right)\sum_{n=1}^{\infty}\frac{\sin(2\pi a n)}{n^{1-s}}\right\}$ for $\Re(s) < 0$.

In 1903 he investigated the properties of Fourier series when the sum does not necessarily converge, and discovered the *Hurwitz-Lyapunov theorem*[584].

He also derived a criterion for the stability of the solutions of ordinary linear differential equations. This criterion (necessary, but not sufficient) required the positivity of certain determinants formed by the coefficients [*Hurwitz criterion*, *Hurwitz polynomials*].

Hurwitz was born at Hildesheim, Germany, into a Jewish family. He studied at Münich, Berlin and Leipzig under **Weierstrass**, **Kronecker** and **Klein**. In 1882 he became privatdocent at Göttingen and in 1892 he was appointed to the vacant chair of **Frobenius** at the polytechnicum of Zürich.

1883–1894 CE **Osborne Reynolds** (1842–1912, Ireland). British engineer, physicist and educator. Known for his work in fluid mechanics. In 1883 he demonstrated that the transition from laminar to turbulent flow depends on a dimensionless characteristic number, known as "Reynolds number"[585]. In 1894 Reynolds introduced *turbulent shearing stresses* or *Reynolds' stresses*, into hydrodynamics in his paper: "*On the Dynamical Theory of Incompressible Viscous Fluids and the Determination of the Criterion*" [*Phil. Trans. A* **186**, 123–164]. These concepts are of great importance in fluid flow modeling experiments and many geophysical phenomena. Reynolds made significant contributions to the theories of heat transfer, turbine pumps, turbulence, tidal motions in rivers and the concept of *group velocity.*

Reynolds was born in Belfast into a family of Anglican clerics. He graduated at Queen's College, Cambridge, in mathematics (1867). In 1868 he became the first professor of engineering at Owen's College, Manchester, a position he held until his retirement in 1905.

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[584] Let $f(x)$ be bounded in the interval $(-\pi, \pi)$ and let $\int_{-\pi}^{\pi} f(x)dx$ exist, so that the Fourier constants a_n and b_n of $f(x)$ exist. Then the series $\frac{1}{2}a_0^2 + \sum_{n=1}^{\infty}(a_n^2 + b_n^2)$ is convergent and its sum is $\frac{1}{\pi}\int_{-\pi}^{\pi}[f(x)]^2\,dx$.

[585] In fluid flow through a pipe, for example, Reynolds number R is given by $\{\overline{v}d\rho/\eta\}$, where $\overline{v}$ is the average flow velocity, d the pipe's diameter, ρ the fluid density and η is the fluid viscosity. The transition from laminar to turbulent flow occurs when $1000 < R < 5000$.

Modern Heat Engines; The Turbine

The chief reason for the low efficiency of steam engines is that they burn their fuel outside the cylinder. Much of the heat is absorbed by the bulky equipment that produces the steam. But if the heat is created inside the cylinder, this major source of energy loss is removed and we have an internal-combustion engine (ICE).

Early ICE used gas instead of gasoline as fuel. In a gas engine, the working fluid is a mixture of atmospheric air and an inflammable gas. Early experiments were described already in 1820 *in a paper entitled:*

"On the Application of Hydrogen Gas to produce a Moving Power in Machinery, with a description of an Engine which is Moved by the Pressure of the Atmosphere Upon a Vacuum Caused by the Explosions of Hydrogen Gas and Atmospheric Air".

It was read by ***W. Cecil*** *before the Cambridge Philosophical Society in England. Cecil mentioned earlier experiments at Cambridge by* ***Farish****, who was said to have operated an engine by gun powder.*

Another English inventor, ***William Barnett****, patented (1838) a gas engine which compressed the fuel mixture. Barnett's engine had a single up-and-down cylinder with explosions occurring first at the top, then at the bottom of the piston.*

In France, ***Jean Joseph Étienne Lenoir*** *built the first practical gas engine in* 1860. *It used street-lighting gas for fuel. This single-cylinder engine had a storage battery ignition system. The piston, moving forward for a portion of its stroke by the energy stored in the fly-wheel, drew into the cylinder a charge of gas and air at ordinary atmospheric pressure. At about half-stroke the valves closed, and an explosion, caused by an electric spark, propelled the piston to the end of its stroke. On the return stroke, the burnt gases were discharged, just as a steam engine exhausts. These operations were repeated on both sides of the piston, and the engine was thus double-acting. These engines were quiet and smooth in running. The gas consumption was, however, excessive. By* 1865 *four-hundred of these engines were in use in Paris for such jobs as powering printing presses, lathes, and water pumps.*

To a Frenchman, ***Alphonse Beau de Rochas****, belongs the credit of proposing the idea of a 4-cycle engine. In a pamphlet published in Paris in* 1862 *he contemplated such an engine which was to be built* 14 *years later. Rochas himself did not, however, put his engine into practice, and probably* *had no idea of the practical difficulties to be overcome.* ***Siegfried Marcus***[586] *built the first successful petrol-driven 4-stroke cycle engine and carriage in 1875, superseding an earlier model (1864). In the same year,* ***Carl Benz*** *also developed a gasoline engine. These engines were basically the same as gasoline engines built today.*

In a gasoline engine the explosion of the fuel produces hot, expanding gases which force the piston to move. Four strokes of the piston are required to complete one cycle of operation — intake, compression, power and *exhaust. Gasoline engines generally use many small cylinders rather than a large one, because vibration is reduced and the engine can run more slowly without stalling.*

A 4-stroke cycle engine in an automobile or airplane can make 30 *revolutions per second. One should imagine the pistons racing up and down, the valves opening and closing, the sparks igniting the gasoline — all at the right time for each cylinder! [It seems a wonder that any mechanism so complex should work at all. Yet such is the skill of modern science and engineering, that we give the matter hardly a thought when we get into a car or airplane: we are confident of arriving at our destination in luxurious comfort and without a hitch.]*

In spite of the advantages of internal combustion, the average automobile engine is only about 15–20 *percent efficient*[587]*. Most of the heat of the explosion is lost in the hot exhaust gases, in the flow of hot air through the radiator, and in the friction of moving parts inside the engine.*

In 1896, ***Rudolf Diesel*** *(1858–1913,* Germany) *invented and built the diesel engine, which increased the efficiency by a simple method: instead of compressing an explosive mixture of air and gasoline, only the nonexplosive air is compressed. Since compression of air alone cannot result in burning, the air may be compressed to as high as ratio as* 18: 1. *The high compression*

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[586] **Nikolaus August Otto** (1832–1881, Germany) built a small experimental gas engine (1861). He devised the 4-stroke cycle (which bears his name) in 1876 and derived a patent for it in 1877. **Gottlieb Daimler** used petrol (1885) to drive the engine.

[587] The efficiency of a steam engine is $\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$ where Q_1 is the total heat supplied (for heating the water and vaporizing it) and Q_2 is the heat given up to the condenser. It is less than $\frac{T_1 - T_2}{T_1}$, which is the ideal efficiency of a reversible Carnot cycle which would operate between two reservoirs of temperatures T_1 and T_2.

In an ICE the efficiency is $\eta = 1 - \left(\frac{v_1}{v_2}\right)^{\gamma - 1}$, where $\frac{v_2}{v_1}$ is the *compression-ratio* and $\gamma = \frac{C_p}{C_v} \simeq 1.4$. For $\frac{v_2}{v_1} = 7$ the limiting theoretical efficiency is 55 percent; the actual achievable efficiency is much lower.

heats the air to about $550\,^{\circ}C$*. Then, at the top of the compression stroke, a powerful pump (fuel injector), squirts fuel into the cylinder and the fuel ignites as it comes into contact with the very hot compressed air. Cheap oil can be used in the diesel engine instead of gasoline and an efficiency of up to* 40 *percent is achieved. However, because of the higher compression and more powerful explosion, the engine requires stronger and heavier cylinders. The extra weight makes the diesel engine less practical for light passenger cars.*

The steam engine, gasoline engine and diesel engine — all require a piston which moves back and forth in a cylinder. These engines are known as reciprocating engines*. A part called a* crankshaft *transforms this reciprocating motion into rotary motion to turn wheels.*

Felix Wankel *(1902–1988,* Germany*) patented in* 1929 *and built during the early* 1950*'s a gasoline engine that uses rotors instead of pistons (known as a* rotary engine *or a* Wankel engine*). The rotors produce rotary motion directly. A Wankel engine operates more quietly and smoothly than a piston engine, needs fewer moving parts, weights less, is smaller than a piston engine of the same power, uses lower octane gasoline but burns more fuel per kilometer. Large-scale production came in* 1968 *with the Japanese 'Mazda* 110 *S'.*

A turbine is a wheel turned by the momentum of a moving fluid such as wind, water, steam or gas. It converts the fluid motion directly into rotary motion [from the Latin turbo*, meaning that which* spins *or* whirls *around]. This rotary motion is used to turn generators that produce electricity and drive huge ocean liners, and serves as an essential component of a jet-propelled aircraft.*

Thus, turbines work on the same principle as the windmill and the water wheel. Unlike the other important types of engines described earlier, they have no pistons or cranks. The motion of a current fluid turns a shaft by means of an arrangement of projecting blades. Steam and internal combustion engines have to change reciprocatory motion into rotatory motion; turbines produce rotatory motion directly.

There are two main kinds of turbines: in some the whole available energy of the fluid is converted into kinetic energy before *the fluid acts on the moving part of the turbine. (In the case of steam, it supplies power* after *it has completely expanded, solely at the expense of its kinetic energy.) Such turbines are termed* Impulse *or* Action Turbines*, and they are distinguished by the fact that the wheel passages are never entirely filled by the fluid.*

In the early de Laval turbines*, the shaft turned at* 30,000 *revolutions per minute — too rapidly for driving most kind of machinery — so it had to be reduced with gear wheels, thus lowering the efficiency. High speeds, of course,* *limit the size of these turbines. The centrifugal force in a large wheel would become so great that the wheel would break apart.*

Turbines in which only part of the available energy is converted into kinetic energy before the fluid enters the wheel are termed pressure *or* Reaction *Turbines.*

In practice, the fluid (steam, say) passes through a ring of fixed blades *fastened to the turbine casing. These blades are so shaped that the space between one blade and the next* acts like a nozzle; *i.e., the blade spacing at the leading edge is greater than the spacing at the trailing edge. This means that as the steam leaves the blades, the pressure falls and the* steam expands. *The steam now hits a ring of moving blades. These blades are again so designed that the spaces between them function as nozzles.*

The moving blades receive a continuous backward thrust *(reaction) from the steam issuing from them. This makes them rotate in the opposite direction. As it leaves the moving blades, the steam again expands. It goes through another ring of fixed blades, onto another set of moving blades and so on. The function of the fixed blades (fastened to the turbine casing) is to aim the steam so that it strikes the wheels at the correct angle.*

Reaction turbines rotate about 3000 *revolutions per minute. A turbine works more efficiently than a piston engine, because the fluid pushes continuously against the turbine wheel. In a 4-cycle piston engine, the exploding fuel pushes against the piston on only one of the pistons four strokes. The turbine is also more efficient than piston engines because of its faster running speed, which makes it possible to deliver more power for its weight and volume. In addition, they do not have moving valves, spark plugs, carburetors and other parts which, in other engines, frequently require repair.*

The turbine finds its most important use today as a generator of electricity in power stations. These powerhouse turbines use coal[588] *— a cheap source of energy. The burning coal heats water and produces high pressure steam to drive the wheels.*

Water turbines *are used to generate electricity at dams and waterfalls. The power of a water turbine depends on the volume of flowing water and the distance (head) that water falls before it strikes the turbine wheel.*

Reaction turbine wheels are mounted on vertical shafts and are completely under water. *They have either spirally curved vanes or blades with variable slant that adjust the wheel to differing amounts of water flow. The wheels of a reaction turbine work best when a large volume of water falls a short*

[588] Nowadays, *nuclear fuel* is also used.

distance, while impulse wheel work best where a small volume of water falls a great distance.

Steam turbines *rank among the most powerful machines in the world: one steam turbine turning a generator can supply all the electricity used by about* 3 *million people. Steam enters many turbines at temperatures up to* $566^{\circ}C$ *and may have pressures of up to* 140 kg/cm^2. *The steam rushes into the turbine at a speed of* 1600 *km/h. It strikes the first wheel, giving it a push, goes on to the next wheel, and so on. A modern steam engine has as many as* 24 *wheels mounted on a horizontal shaft. Steam expands to as much as* 1000 *times its original volume as it passes through the turbine. Therefore, each succeeding pair of nozzles and wheels must be larger than the last one to make use of all the expanding steam. This gives the steam turbine its typical trumpet-like shape. Most modern turbines use both impact-type and reaction-type wheels at different stages along the shaft.*

Gas turbines *use hot gases (oil, kerosene, natural gas) instead of steam, without first using them to heat water into steam. It has three main parts: first a* *compressor*, *a special type of fan that sucks in air and compresses it. This compressed air mixes with fuel and burns in a* *combustion chamber*. *The burning gases expand enormously and rush through a* *turbine*, *spinning the turbine wheels. Part of the rotary power from the turbine wheels drives the air compressor, that is mounted on the same shaft as the wheels. The rest of the rotary power can turn electric generators, run pumps or drive ships and locomotives. In a jet engine most of the power must rush* out *the turbine's tailpiece to give the plane a forward thrust.*

The temperatures generated in gas turbines range from 700°–$800^{\circ}C$. *Thus engineers must make gas turbines from metals or ceramic materials that keep their strength and shape at such temperatures, which would weaken steel. The hotter a gas turbine runs, the more efficiently it operates. This can be a disadvantage when the turbine is used to propel ships or locomotives, which must often move slowly.*

Windmills came into use in the Middle East in the 900*'s and in Europe in the* 1100*'s. In the* 1600*'s people built the first crude gas turbines by mounting fans over a cooking fire to turn roasting meat on a spit. The hot gases from the fire spun the fan, and gears connected the fan to the spit. In* 1629 *an Italian engineer,* ***Giovanni Branca***, *built a crude steam turbine which drove a machine. In* 1791, *the Englishman* ***John Barber*** *patented a gas turbine that was an ancestor of the turbojet. These first forms of turbines worked inefficiently. In 1832,* ***Benoit Fourneyron*** *(1802–1867, France) developed the first fully successful enclosed water turbine. It developed* 37 *KW and drove hammers used to forge metal.*

C.G. de Laval** built a successful impulse steam turbine in* 1883*. In 1884* ***Charles Algernon Parsons[589] *(1854–1931) developed a reaction steam turbine in England. The American inventor* ***Charles G. Curtis*** *(1860–1953) developed (1900) the first steam turbine using many sets of wheels. The first big Curtis turbine was installed in an electric power plant in Chicago in* 1903*. It ran a generator that produced* 5000 *kilowatts of electricity and started a revolution in power production.*

1883 CE, Aug. 26, 1:00 pm and ***Aug. 27***, 10:02 am *The volcanic explosion of Krakatoa* (Sunda Strait, 6.10 °S, 105.42 °E). A volcano island, about halfway between Sumatra and Java, was blown to bits in one of the most stupendous natural explosions ever recorded. About 20 cubic kilometers of material were emitted during the paroxysmal eruption.

An explosion of 150 megaton of TNT is required to produce the equivalent of the ensuing air pressure disturbance, and the total energy released through the Aug. 27 explosion is estimated at 10^{25} ergs. Actual sound from the Aug. 27 explosion was *heard* 5000 km away. Atmospheric ultrasonic *acoustic-gravity waves* circled the earth several times before they attenuated below the recording level. Long-period air-coupled *sea-waves* traveled as far as 18,000 km (through these waves energy was coupled from the atmosphere to the ocean via resonant coupling).

Some 36,000 humans perished in the disaster, mostly by a huge *tidal wave*, 40 meters high, that washed over the shores of nearby islands. A column of stones, dust, and ashes projected from the volcano shot up into the air to a height of 80 km — higher than the ozone layer. The finer particles coming into the higher layers of the atmosphere covered a large part of the surface of the earth, and gave rise to beautifully brilliant sunset glows and multicolored twilight effects, that were observed for 3–4 years after the eruption.

1883–1889 CE **Carl Gustaf Patrik de Laval** (1845–1913, Sweden). Engineer and inventor. Built the first practical *impulse steam turbine* (to power a cream separator of his invention).

Attempts to design a steam turbine had been made by numerous inventors, but all fell short of practical success — mainly because of the difficulty

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[589] Son of the Irish astronomer **William Parsons** (1800–1867), 3^d Earl of Rosse, the first to observe a spiral nebulae (1845).

of arranging for a sufficiently high velocity in the working parts to utilize a reasonably large fraction of the steam kinetic energy [the principle involved requires that, for optimal efficiency, the velocity of the blades should approximate to half the velocity of the jets which strike them]. There was a further difficulty of getting the energy of the steam collimated in a single direction without undue dispersion, when it is allowed to expand through an orifice from a chamber at high pressure.

Laval overcame these difficulties, partly by the special shape of the nozzle used to produce the steam jet speed and partly by features of design which allowed an exceptionally high speed to be reached in the wheel carrying the vanes against which the steam impinged. To increase the velocity of gas in the nozzle beyond the speed of sound, he designed it such that after converging to a minimum cross-sectional area, the nozzle was expanded to a larger area. Laval's principle of nozzle design is widely employed in contemporary turbines and jet engines.

1883–1892 CE **George Francis FitzGerald** (1851–1901, Ireland). Physicist. Concluded, on the basis of Maxwell's equations that an oscillating electric current would produce electromagnetic waves (1883). This finding was later verified experimentally by **Heinrich R. Hertz** (1886) and used in the development of wireless telegraphy.

Independently of **H.A. Lorentz**, FitzGerald studied the results of the Michelson-Morley experiment[590](1887) and suggested that when in motion, a body is shorter (along its line of motion) than when at rest and that such a shortening, or contraction, affects also the instruments used in the experiment. Lorentz arrived at this idea independently (1895) and developed it considerably. The theory is known as the *Lorentz-FitzGerald contraction*, which **Albert Einstein** used in his special Theory of Relativity (1905).

FitzGerald was born and died in Dublin.

1883–1906 CE **Francis Edgar Stanley** (1849–1918, USA) and his identical twin brother **Freelan O. Stanley** (1849–1940, USA). Inventors and manufacturers. Invented (1883) a photographic *dry plate process* and operated a firm to manufacture the plates. Built (1896) the first *steam-engine powered automobile*; founded and directed (1902–1917) Stanley Motor Co. to produce *Stanley Streamers*; broke world's record for fastest mile (28.2 s) in a steam car (1906). Francis was killed in an automobile accident.

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[590] The experiment was an attempt to measure the earth's motion relative to the pervasive luminiferous ether postulated as the medium in which light waves were propagated. The attempt failed to detect any such motion.

1883–1892 CE **Ludwig Gumplowicz** (1838–1909, Poland and Austria). Influential economist and sociologist. Maintained that human history is a result of a continued conflict, first among different ethnic groups, then between states (that were formed as a result of the conquest of habitable lands by the strong groups who subdued the weaker groups), and finally between classes inside the states.

Gumplowicz was born in Krakow, Poland, to a family of Jewish Rabbis, but later converted to Christianity. Professor (at Graz) from 1883.

1883–1908 CE **Svante August Arrhenius** (1859–1927, Sweden). Physicist and chemist. One of the founders of physical chemistry. Won the 1903 Nobel prize for his pioneering contributions to the *electrolytic theory of dissociation*[591] (1883–1887) and *chemical kinetics* (1889–1899).

In 1886 he established the importance of carbon dioxide to the earth's heat balance (maintaining that the doubling of the concentration of CO_2 in the atmosphere would result in an average global temperature increase of about $6\,^\circ C$). He went on to describe the "*greenhouse effect*" brought about by CO_2 in the atmosphere.

According to the *kinetic theory of gases*, the pressure exerted by the gas on the walls of the container is determined by the bulk of molecules, whose energy is of order the *mean value*. This means that molecules with very large energies have practically no perceptible effect on the pressure and also on the total reserve of energy of the gas. In the case of chemical reactions the converse is often true. It turns out that precisely the rare molecules with high energy often determine the course of chemical reactions.

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[591] His theory states that salts, upon dissolution in water, separate into mobile oppositely charged ions, even in the absence of an applied electric field. He used this to rationalize many seemingly puzzling properties pertaining to the behavior of solutions, e.g. the hydrolysis of salts, acids and bases; the solubility of salts and its variation with temperature; the constancy of the observed heat of neutralization of strong acids and strong bases. His theory's greatest triumph was that it could render a physical meaning to the parameter n in the van't Hoff equation for *osmotic pressure* $PV = nkT$, n being the *number of ions in the solution*.

Arrhenius did not know about electrons at the time (1883). Disbelieved by most of the senior chemists in Sweden, his thesis received the lowest grade that the University of Uppsala could bestow. But Arrhenius seems to have been a stubborn fellow, not easily put down by rebuffs. He circulated copies of his work to leading scientists through Europe, and as the years passed the theory gradually became more and more accepted. Eventually it was judged respectable enough for Arrhenius to be elected a member of the Swedish Academy of Sciences.

The mystery of chemical reaction timescales stems from the fact that molecules entering into a reaction collide every 10^{-10} sec whereas a reaction frequently requires several minutes (sometimes hours). It means that at any given time only an extremely small portion of all collisions result in a chemical reaction.

Arrhenius theorized that reactions are initiated only by collisions of molecules whose energy exceeds a definite critical value, the so-called *activation energy* E_A. Taking into account a thermodynamic equation of van't Hoff, he arrived at an equation which expresses the *reaction time*, t is proportional to $\tau e^{\frac{E_A}{kT}}$, where τ is the time between collisions, k is the Boltzmann constant and T is the absolute temperature.

For instance, when molecules of hydrogen and iodine collide, they form two molecules of hydrogen iodide HI. Taking $\tau = 10^{-10}$ sec, $T = 0\,^{\circ}\mathrm{C} = 273$ K, $E_A = 3 \times 10^{-12}$ erg, $kT = 3.8 \times 10^{-14}$ erg, we get $t = 3 \times 10^{17}$ years! This result accords with the fact that at $0\,^{\circ}\mathrm{C}$, the reaction $H_2 + I_2 \rightarrow 2HI$ is practically unobservable. A characteristic feature of Arrhenius' formula is the extremely sharp decrease in reaction-time and increase in reaction rate for slight lowering of the temperature.

The arrhenius formula can also be recast in the form:

$$\textit{probability of reaction} \approx \textit{const.} \times \exp\left[-\frac{E_A}{kT}\right].$$

For example, the reaction $2H_2 + O_2 \rightarrow 2H_2O + 117$ kcal/mole although exotermic does not take place even at several hundreds degrees. A spark, or other such disturbance is required to ignite them: As a result of ignition, a violent explosion takes place, the hydrogen and oxygen being practically instantaneously converted into water vapor (the explosion is an adiabatic process: the heat cannot escape, but will go into raising the temperature, and consequently the pressure of the resulting water vapor will rise enormously).

The spark increases T in a localized region, and by Arrhenius' law, this increases the probability of the reaction. Once started, the heat liberated by the reaction near the spark raises the gas in the neighborhood to such a high temperature, that it in turn can react. This liberates more heat, and allows gas still further away to react, and so on.

The magnitude of E_A for each reaction can be determined *theoretically*, by solving the associated *Schrödinger equation*.

It frequently happens that chemical reactions are much more involved because they may proceed via diverse intermediate stages (reaction pathways). By way of illustration, the reaction $H_2 + Cl_2 \rightarrow 2HCl$ does *not* proceed via collisions of a molecule of hydrogen and a molecule of chlorine, but by the scheme

$$H_2 + Cl_2 \rightleftharpoons H_2 + Cl + Cl; \quad Cl + H_2 \rightleftharpoons HCl + H; \quad H + Cl_2 \rightleftharpoons HCl + Cl.$$

As a result, the actually observed reaction rate involves complicated relationships. However, for each separate reaction, say, for $Cl + H_2 \rightleftharpoons HCl + H$, the Arrhenius law holds true, and the reaction rate is proportional to $e^{-\frac{E_A}{kT}}$, the activation energy E_A having different values for each reaction[592].

The energy of activation E_A, characterizes the *energy barrier* that must be overcome by the system for the reaction to occur. The dependence of the reaction rate on $\frac{E_A}{kT}$ follows from the Boltzmann distribution among energy levels: the exponential indicates the fraction of molecules that possess sufficient energy E_A for the reaction to take place. A chemical reaction proceeds spontaneously (i.e. is exergenic) only if it is accompanied by a *decrease of the free energy* $\Delta G < 0$ (a necessary but not sufficient kinetic condition).

Ahrenius' theory explains how a persence of a *catalyst* can accelerate a chemical reaction by many orders of magnitude – by lowering the activation energy.

Arrhenius was born at Vik, near Uppsala, the son of a surveyor and land agent. He studied at Uppsala (1876–1881), Stockholm (1881–1884), and the Universities of Graz, Amsterdam and Leipzig (1886–1888).

In 1895 he became professor of physics at Stockholm. In 1896 Arrhenius discovered that the amount of CO_2 in the atmosphere determines the global temperature and theorized that the ice ages occurred because some process had reduced the level of CO_2. [In 1938, **G.S. Callendar** determined that human activities were causing *increase* in the amount of CO_2 in the earth's atmosphere.] Similar views of the role of CO_2 in the earth's atmosphere were expounded in 1899 by **Thomas Chamberlin** (1843–1928, U.S.A.).

In 1907 he put forward the so-called '*panspermia theory*' in his book *Worlds in the Making.* In it he lifted the rather undeveloped ideas of **R.E. Richter** (Germany) and **Lord Kelvin** (1881), to a level of a serious scientific theory. Accordingly Arrhenius suggested that life did not originate on earth. He imagined that simple living forms may have drifted from world to world, propelled by *radiation pressure* through interstellar space.

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592 In chemical and biochemical reactions, the fraction $\frac{E_A}{kT}$ in the exponent is usually written as $\frac{E}{RT}$, where the universal gas constant $R = kN_A$ and $E = E_A N_A$ relate to a *mole* of substance rather than to a single molecule $[N_4 = 6.02205 \times 10^{23}/\text{mole}]$.

Panspermia and the Quest for Life's Origin[593]

In the physical sciences, the question of the origin of life is the study of the nature in which life evolved from non-life sometime between 3.9 and 3.5 billion years ago. This topic also includes theories and ideas regarding possible extra-planetary or extra-terrestrial origin of life hypotheses, thought to have possibly occurred over the last 13.7 billion years in the evolution of the known universe since the big bang.

Origin of life studies has a profound impact on biology and human understanding of the natural world. Progress in this field is generally slow and sporadic, though it still draws the attention of many due to the eminence of the question being investigated. A few facts give insight into the conditions in which life may have emerged, but the mechanisms by which non-life became life are still elusive.

Astronomers now believe that the universe began at least 15 billion years ago, when the first clouds of the elements hydrogen and helium were formed. Gravitational forces collapsed these clouds to form stars. These stars converted hydrogen and helium into heavier elements, including those such as carbon, nitrogen, and oxygen, which are necessary for life. These elements were returned to interstellar space by explosions of some of these stars to form clouds in which simple molecules such as water, carbon monoxide, and hydrocarbons were formed. These clouds then collapsed to form a new generation of stars and solar systems. In at least one solar system, our own, a variety of objects were formed, including comets (believed to be the most primitive objects in our solar system), meteorites, asteroids, and planets. One of the planets, the earth, formed at a distance from the sun where conditions were

[593] For further reading one is referred to the following books:

- Hoyle, Fred and C. Wickramasinghe, *Evolution from Space*, Simon and Schuster, 1981.
- Dyson, Freeman, *Origins of Life*, Cambridge University Press, 1989.
- Goldsmith, D. and Tobias Owen, *The Search for Life in the Universe*, Addison-Wesley, 1992.
- De Duve, C., *Vital Dust: The Origin and Evolution of Life on Earth*, Basic Books, 1996.
- Hazen, R.M., *Genesis: The Scientific Quest for Life's Origins*, Joseph Henry Press, 2005.

favorable and the necessary chemical ingredients were available for the origin of life.

The final, most important events leading to the origin of life are perhaps the least understood of the story. Life began during the first billion years of an earth history which is 4.5 billion years old. In the early earth, volcanoes, a gray, lifeless ocean, and a turbulent atmosphere dominated the landscape. Vigorous chemical activity generated heavy clouds, which were fed by volcanoes and penetrated both by lightning discharges and solar radiation. The ocean received organic matter from the land and the atmosphere, as well as from infalling meteorites and comets. Here, substances such as water, carbon dioxide, methane, and hydrogen cyanide formed key molecules such as sugars, amino acids, and nucleotides. Such molecules are the building blocks of proteins and nucleic acids, compounds ubiquitous to all living organisms.

A critical early triumph was the development of RNA and DNA molecules, which directed biological processes and preserved life's "operation instructions" for generations. DNA and RNA first appeared as fragments, then a fully assembled helices. These helices formed some of the living threads. However, other threads derived from planetary processes such as ocean chemistry and volcanic activity. This evolving bundle of threads thus arose from a variety of sources, illustrating that the origin of life was triggered not only by special molecules such as RNA or DNA, but also by the chemical and physical properties of the earth's primitive environments.

The evolution of the plants and animals most familiar to us occurred only in the last 550 million years. The illustration depicts the appearance of marine invertebrates (such as shell-making ammonites), then fish, amphibians, reptiles, mammals, and humanity. The life thread which continues on in the oceans reminds us that the evolution of aquatic life continues even today. The development of land plant communities was manifested in the relatively ancient clubmosses, horsetails, and ferns, and the more recent gymnosperms (for example, conifers) and angiosperms (flowering plants).

Perhaps the most recent significant evolutionary innovation has been humanity's ability to record and build upon its experience, thus triggering the rise of civilization and technology. These developments bring us to the present, and, as the thread of life reaches the summit of a tree-covered hill, we ponder our future.

Most of life's history involved the biochemical evolution of single-celled microorganisms. We find individual fossilized microbes in rocks 3.5 billion years old, yet we can conclusively identify multicelled fossils only in rocks younger than 1 billion years. The oldest microbial communities often constructed layered mound-shaped deposits called stromatolites, whose structures suggest that those organisms sought light and were therefore photosynthetic. These *early stromatolites grew along ancient seacoasts and endured harsh sunlight as well as episodic wetting and drying by tides. Thus it appears that, even as early as 3.5 billion years ago, microorganisms had become remarkably durable and sophisticated!*

Many important events mark the interval between 1 and 3 billion years ago. Smaller volcanic terrains were joined by larger, more stable granitic continents. Life learned how to release oxygen from water, and it populated the newly expanded continental shelf regions. Finally, between 1 and 2 billion years ago, the eukaryotic cells with their complex system of organells and membranes developed and began to experiment with multicelled body structures.

Given the huge number of stars known to exist in the universe, life has very likely also developed elsewhere. If this "other" life can control and transmit energy such as light and radio waves, we just might be able to detect it.

As NASA develops its mission to build a space station and to visit other solar system bodies such as comets, planets, and moons, it responds to humanity's need to return to the cosmos, both to understand life's origins as well as to expand its horizons.

Panspermia

Since the dawn of history, man has speculated about the possibility that intelligent life may exist on other worlds beyond the earth. Perhaps the earliest record of this tradition is found in Genesis **6**, *1–4.*

As astronomy developed, concepts akin to panspermia *(ubiquitous life) were propounded by various philosophers and scientists:* ***Anaximander*** *(ca 560 BCE) asserted that worlds are created and destroyed.* ***Anaxagoras*** *(ca 460 BCE), one of the first proponents of the heliocentric theory, believed that invisible seeds of life were dispersed throughout the universe.* ***Epicuros*** *(ca 300 BCE) and his school taught that many habitable worlds, similar to our world, existed in the vast reaches of space. The Roman philosopher* ***Lucretius*** *(ca 65 BCE) was a firm believer in the existence of 'other earths' inhabited by 'other people'.*

The concept of life on other worlds was incompatible with the Ptolemaic-Christian cosmology, and until the 15^{th} *century CE the geocentric doctrine excluded all 'heretical' notions of panspermia. The first telescopic observations by* ***Galileo*** *(1609) opened a new era in astronomy and dealt a mighty blow to the ideas of many of his contemporaries. It became evident that the planets were similar to the earth in many respects, and this similarity evoked the questions of the existence of cities inhabited by intelligent beings there.*

These bold ideas were advanced by ***Giordano Bruno*** *(1584) who wrote: "Innumerable suns exist; innumerable earths revolve around these suns. Living begins inhabit these worlds".*

During 1650–1800, *such writers, philosophers and scientists including* ***Cyrano de Bergerac*** *(1619–1655,* France*),* ***Christiaan Huygens*** *(1629–1695,* The Netherlands*),* ***Bernard de Fontenelle*** *(1657–1757,* France*), and* ***Voltaire*** *(1694–1778,* France*) published works dealing with life on other planets. Scientists and philosophers such as* ***Immanuel Kant*** *(1724–1804, Germany),* ***Pierre Simon de Laplace*** *(1749–1827,* France*), and* ***William Herschel*** *(1738–1822, England) advocated the hypothesis of the plurality of habitable worlds. By the end of the* 18^{th} *century, this hypothesis had gained almost universal acceptance by scientists and intellectuals.*

In 1854, ***William Whewell*** *(1794–1866, England) , in his book* Of the Plurality of Worlds*, argued against the probability of planetary life. During the late* 19^{th} *and early* 20^{th} *centuries various modifications of the panspermia hypothesis received wide circulation. According to this hypothesis, life in the universe exists eternally; living organisms never arise from nonliving matter, but are transmitted from one planet to another.*

At the turn of the 20th *century, the Swedish chemist,* ***Svante Arrhenius*** *(1907), conjectured that microorganisms — spores or bacteria, probably adhering to small specks of dust — are propelled by the* starlight radiation pressure *from one planet to another. If, by chance, they should land on some planet where conditions for life are favorable, these spores were thought to germinate and initiate the local evolution of life.*

Although such transmission of life from one planet to another within a single planetary system cannot be completely discounted, the feasibility of propagation of panspermia from one planetary system to another has divided the scientific community: most biologists and astronomers consider the hypothesis to be highly unlikely because ultraviolet light and cosmic X-rays would prove lethal to Arrhenius' spores. A minority of diehards, however, still believes that the complexity of terrestrial life cannot have been caused by a sequence of random events, but must have come from elsewhere.

Chief among these was the astronomer ***Fred Hoyle*** *(1915–2001), who resumed the debate in* 1981 *with a new theory of cosmic creationism, overriding most of the objections of the negativists. Others have pointed out that radiation pressure is not the only conceivable mechanism for interstellar transport of living things. One may assume that the Galaxy is populated here and there by advanced technical civilizations that have discovered and exploited space travel. A survey party from such a civilization, landing on a clement planet, may 'contaminate' it with diverse microorganisms — intentionally or otherwise. Alternatively, some have speculated that colonizing starships might* *themselves be nanotechnologically designed vehicles, carrying equipment for automated re-creation of favorable planets. Indeed, it has been suggested that humankind itself engage in such nano-robotic assisted colonization projects!*

Our assessment of the probabilities for the existence of extraterrestrial life are based on knowledge gained from terrestrial biology and the laws of physics and chemistry, which make some scenarios more probable than others. These laws (the summary of our experience in studying the universe around us) appear to be valid as far as we can test them, including the analysis of light from stars and from the most distant galaxies.

Can we make any definite statement about the possible chemistry of alien life — the molecules that form living organisms in faraway places? Life that is based on chemical reactions (i.e. on the interaction of atoms to form complex molecules) appears to require carbon as its key structural element. Only carbon can form chemical bonds with hydrogen, oxygen, and nitrogen (as well as other less abundant elements) in a way that readily promotes the development of a wide variety of information-bearing, structure forming, energy converting polymers.[594]If carbon is the crucial element in all chemical life, we are still not much restricted, since carbon has a high abundance everywhere in the universe — and indeed, some types of stars would not shine without the catalytic effect of carbon nucleus.

Life also seems to require a solvent, a fluid medium in which atoms, ions and molecules can encounter one another and undergo chemical reactions. The great ability of water to dissolve other polar substances makes it one of the most favored solvents. In addition, water's heat capacity, its heat of vaporization, its ability to remain liquid in a temperature range appropriate for many chemical reactions, its cosmic abundance, and its chemical stability — all single it out as exceptionally well suited for use by living organisms[595].

For all types of life, it seems important to have a variety of individuals, and thus some sort of chemical mutation, starting from the simplest progenitor molecules. Otherwise, the processes of natural selection will not have enough material on which to operate, as it discriminates among various living creatures on the basis of reproductive success.

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594 *Silicon* can also form polymers, but these are too stable under ordinary conditions to serve as a basis for life. The chemical affinity of silicon for oxygen implies that at temperatures low enough for complex molecular structures to exist, silicon will bound up as silicates.

595 *Ammonia* or *methyl alcohol* might serve as a solvent instead of water under certain highly specialized conditions, but that would restrict the temperature range that life can tolerate.

The Greenhouse Effect

The atmosphere is largely transparent to visible light (3000–7000 Å) which occupies the peak of the solar spectrum (blackbody radiation at 5800° K). The earth, warmed by solar light, emits a 'blackbody' spectrum (at 300° K) peaking in the infrared, at a wavelength of about $10\,\mu m$. This radiation cannot escape immediately because it is absorbed by the atmosphere, particularly by water vapor[596] *and to a lesser extent by CO_2. The atmosphere therefore acts in the same way as the glass in a greenhouse*[597]*, letting through light but not allowing infrared radiation to escape. It is this infrared radiation that is mostly responsible for heating the atmosphere whereas the visible radiation is an inefficient heater.*

When gas molecules absorbs light waves, this energy is transformed into internal molecular motion, which is detectable as a rise in temperature. The warmed gas eventually radiates this energy away. Some of this reradiated energy travels upwards, where it may be reabsorbed by other gas molecules, a possibility less likely with increasing height, because the concentration of water vapor decreases with altitude. The remainder travels downward, and is again absorbed by the earth. Thus, the earth's surface is supplied continually with heat from the atmosphere, as well as from the sun. Without these absorptive gases in our atmosphere, the earth would not be a suitable habitat for humans and numerous other life-forms.

Of all the gases which comprise our atmosphere, N_2 is a poor absorber of all types of incoming radiation. O_2 is an efficient absorber of shorter ultraviolet

[596] The first scientist to describe the *greenhouse effect* (for *water vapor*), was the physicist **John Tyndall** (1820–1893, Ireland) in a paper "*On Radiation Through the Earth's Atmosphere*" (1863).

[597] *Greenhouse effect*: short-wave solar radiation passes through the glass and is absorbed by the objects in the greenhouse. The long wavelengths radiated by these objects (e.g. infrared) cannot penetrate the glass and are trapped, thus warming the greenhouse. An important factor in keeping the greenhouse warm is the fact that it prevents mixing of the air inside with cooler air outside. In other words, greenhouses are warm mainly because air is not allowed to escape and *convect* heat away; a crop of standing corn, a walled garden, a tree-bordered enclosure, act in the same way and become warmer than their surrounding by reducing free circulation of air.

waves in the high atmosphere. Ozone (O_3) absorbs longer ultraviolet radiation in the stratosphere (10–50 km), which accounts for the high temperature there. Altogether H_2O, O_2 and O_3 absorb 20% of the total solar radiation.

Any reduction in the water vapor or CO_2 content of the atmosphere would weaken the greenhouse effect and allow the earth to cool. However, the concentration of CO_2 in the atmosphere is now increasing, slowly but steadily.

When humans started to burn fossil fuel on a large scale with the onset of the Industrial Revolution, they caused a great deal of "locked up" carbon to be released, and this trend is now being aggravated by burning tropical forests.

Whereas the level of CO_2 in 1850 *amounted to* 265 *ppm, it has now grown to* 340 *ppm, and unchecked it could well reach* 600 *ppm by* 2050. *The result is a steady warming of our planet, projected to rise by $3\,^\circ C$ above normal within* 50 *years. While there will be little change at the equator, the poles may well become $7\,^\circ C$ warmer.*

Thus, carbon dioxide, that gas that puts fizz into soft drinks, is one of the most important components of the atmosphere, and plays a key role in determining the earth's climate, even though it amounts to a mere 0.03 *percent*[598].

[598] Models and analyses of global warming generally agree that human economic activity makes the earth warmer than it would otherwise be. Yet discrepancies between theory and observation persist (1994); the predicted warming based on recent increases in concentrations of greenhouse gases is slightly more than the observed warming of the atmosphere. In addition, the warming trend in North America does not appear to follow the global pattern.

The answer is ironic. In all probability, aerosols primarily composed of sulfates, themselves the result of commercial activity, enhance the ability of the atmosphere to reflect sunlight back into space before it can reach the planet's surface and participate in the warming process. The sulfate particles, about 0.1–1.0 micron in diameter, are particularly concentrated over the industrial areas of the Northern Hemisphere. Their capacity to *cool by scattering sunlight* has become a recognized force in climatic change only recently. Clearly, both the cooling effects of aerosols and the warming caused by greenhouse gases must be taken into account if we are to attain accurate climate models.
In contrast to industrial effects, *agriculture* is directly or, at least in some cases, indirectly responsible for releasing a substantial proportion of greenhouse gases (CO_2, methane, nitrogen oxide and chlorofluorocarbons).
Now, global warming could either *enhance or impede* agriculture: warmer air holds more water vapor, and so global warming will bring about more evaporation and precipitation. Areas where crop production is limited by acid condi-

In addition to absorption, the overall radiation balance depends also on the part of the solar energy that is reflected back into cosmic space. This reflection takes place from the outer layers of the atmosphere, clouds, dust, and the earth's surface. The coefficient of reflectivity (i.e. the percentage of solar energy flux reflected back into space) is called the albedo, and for the earth it averages approximately 0.310. *The most important component affecting the earth's albedo are the clouds*[599].

When both reflection and absorption are taken into account an overview of the radiation balance in the earth-atmosphere system can be set up. Certain complication arise due to the following factors:

- *There are subsystems which can store thermal energy in the hydrosphere and atmosphere, and these can mutually exchange some of the absorbed solar energy.*
- *A significant part of the solar energy is converted to other forms (e.g. mechanical energy) in the moving atmosphere (winds), energy in the movement of the hydrosphere (waves), the chemical energy of photosynthesis, and heat which evaporates water from the biosphere.*
- *There are some small (but non-negligible) sources of energy which are completely independent of solar energy (geothermal, gravitational, radioactive).*

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tions would benefit from a wetter climate. Moreover, given sufficient water and light, increased ambient CO_2 concentrations absorbed during photosynthesis could act as a fertilizer and facilitate growth in certain plants.

If, however, increased evaporation from soil and plants does not coincide with more rainfall in a region, more frequent dry spells and droughts would occur, and a further rise in temperature will reduce crop yields in tropical and subtropical areas.

Finally, global warming will precipitate a thermal swelling of the oceans and melt polar ice. Higher sea levels may claim low-lying farmland and cause higher salt concentrations in the coastal groundwater.

The most recent analysis of the impact of climatic change on the world food supply (1992), concluded that average global food production will decline 5 percent by 2060.

599 Land has an albedo of about 20%, calm sea 8%, stormy sea 40%, fresh snow 80%, clouds 70%, ice 70%, and forest 12%. *Man* has changed 17% of the continent's surface, that is some 5% of the earth's surface. This probably changed the global albedo from 0.305 to the present value of 0.310, corresponding to a decrease in solar flux of approximately 8.6×10^{11} KW with a resulting *global cooling* of $\sim 1\,^{\circ}$K.

All these factors result in a rather complex (and still imperfectly understood) system of energy flows which has existed in stable form for million of years. However, since the Industrial Revolution, these have become somewhat influenced by man's own technology.

A blackbody's radiation is governed by the Stefan-Boltzmann radiation law.

The average total flux (insolation) from the sun, known as the solar constant, is about 1.36 kW/m^2 *[Watt* $=$ *Joule/sec* $= 0.239$ *cal/sec, of which 40% is in the visible light range (0.4–0.7 micron)]. On a sunny day 75% of insolation may reach the earth's surface; on an overcast day only 15%. On average 51% of insolation is absorbed by the surface as thermal energy – 29% as direct radiation, and 22% as diffused radiation. The latter comprises light scattered by atmospheric dust, water vapour, and air molecules. About 4% of the radiation reaching the surface is directly reflected at the same wavelengths,from the surface back into space. Surface reflectance values (albedo) depend on materials (e.g. 5–10% soils, 15–25% grass, 40–90% snow, etc.) Of the mean* $(100 - 51 - 4 = 45\%)$ *of insolation not reaching the surface at all, the breakdown is as follows: 6% (in insolation units) scatters from the atmosphere back into space; 20% reflected from cloud tops; 3% absorbed by clouds, and 16% absorbed by the atmosphere. Eventually, all of the visible (and other) optical radiation absorbed by the atmosphere and earth's surface is re-radiated back into space as infrared rays (3–30 microns) peaking at 10 microns. The average result of radiation absorption, scattering, reflection and re-radiation is that the mean atmospheric surface temperature is maintained at about* $15\,^\circ C$*.*

Ignoring the atmosphere, and assuming a steady state blackbody radiation, where the influx of energy from the sun equals the outflux of energy radiated by the earth, the terrestrial temperature T_e *assumes a value given by the equation*

$$e_a \sigma T_e^4 4\pi R_e^2 = f_e e_s \sigma T_s^4 \left(\frac{R_s^2}{d^2}\right) \pi R_e^2,$$

e_a *= emissivity of earth,* e_s *= emissivity of sun,* σ *= Stefan's constant,* $T_s = 5800\,^\circ K$ *= sun's absolute temperature,* $R_s = 6.96 \times 10^8$ *m = radius of sun,* d *= sun-earth distance* $= 1.5 \times 10^{11}$ *m,* f_e *= fraction absorbed by earth. Therefore,*

$$T_e = T_s \left[f_e \frac{e_s}{e_a} \frac{R_s^2}{4d^2}\right]^{1/4}.$$

To qualify as a perfect black body, the surface of the planet must be non-reflecting; that is, all the sunlight reaching it must be absorbed and later *reradiated as planet light* $(f_e = 1)$. *Also, there can be no gases in its atmosphere that absorbed outgoing planet light. Leaving aside such subtleties the above equation renders a mean surface temperature of about* $278\,^\circ K = 5\,^\circ C$.

If we take into account the sunlight reflected back by clouds, ice caps, and deserts, the proper value for f_e *is* 0.65. *If this be the entire story, the earth's surface temperature would instead be* $-20\,^\circ C$ *(all water is frozen).*

However, the greenhouse blanketing of the absorbing atmospheric gases keeps the earth's surface significantly warmer than $250\,^\circ K$. *To estimate how much warmer, we note that the application of the Stefan-Boltzmann law to the earth-atmosphere system yields* $T^4 = 2(250)^4$, *where the factor* 2 *arises from the fact that the atmosphere radiates outward into space* and *inwards to the earth's surface, whereas the earth's surface radiates only upwards. Therefore* $T = 297\,^\circ K = 24\,^\circ C$, *which is about right. The difference between* $-20\,^\circ C$ *and* $24\,^\circ C$ *is a measure of the greenhouse effect; the empirical result, as noted above, is actually* $15\,^\circ C$.

The energy exchange between the atmosphere and the surface is not entirely radiative: air which is warmed by contact with the surface rises, and transports heat upward by convection. *Also, evaporation of water from the ocean cools the surface, and when water droplets condense, heat (latent heat of condensation) is passed into the atmosphere. Thermal* conduction *also transports heat from the surface into the air, though to a smaller extent than convection and evaporation.*

For every 100 *units of solar radiation incident upon the upper atmosphere, only* 22 *reach the surface directly and are absorbed. Of the rest,* 35 *are reflected back into space (mostly within the troposphere),* 21 *are absorbed by the atmosphere (mostly above the troposphere), and the remainder of* 22 *reaches the surface after scattering and diffusing through clouds.*

The earth radiates some 118 *units: 11 escape directly, and* 107 *are absorbed by the atmosphere.*

The atmosphere radiates 158 *units: 54 travel into space and* 104 *return to earth.*

The outgoing thermal radiation $(11 + 54 = 65)$ *therefore just balance the* $100 - 35 = 65$ *units of solar energy that entered altogether into the system.*

Radiative processes alone, however, leave the atmosphere with a debit *of* 30 *units* $(21 + 107 - 158 = -30)$ *and the surface with a* credit *of the same amount* $(44 + 104 - 118 = 30)$. *A net balance is achieved via the processes of convection, evaporation and conduction.*

1883–1909 CE **Wilhelm Maybach** (1846–1929, Germany). Automobile builder. Produced with **Daimler** (1883) one of the first gasoline motors. Constructed the first Mercedes automobile (1900–1901); credited with the invention of spray-nozzle *carburetor*, *honeycomb radiator*, and change-speed gear. With his son Carl established at Friedrichshafen a company to build aircraft engines (1909). Maybach automobiles were produced from 1922 to 1939.

1883–1923 CE **Alfred Pringsheim** (1850–1941, Germany). Mathematician. made important contributions to the theory of convergence and divergence of infinite series and infinite products. Studied the position of singularities of power series, derived a new test of convergence of complex series and established theorems on multiplication of infinite series and the convergence and summability of Fourier series.

Pringsheim was born of Jewish parents and baptized in order to obtain a university position. All his property was confiscated by the Nazis (1939) and he was forced to flee to Switzerland.

1883–1932 CE **Konstantin Eduardovitch Tsiolkovsky**[600] (1857–1935, Russia). A pioneer of astronautical and space travel theory. In an article written in 1898 [appeared in 1903 in *Nautschnoje obozrenije* (Science Survey)] he described a streamlined, rocket-driven vehicle for space travel which used liquid oxygen and hydrogen as propellants. He was perhaps the first man to base this project on sound principles[601]. His proposal included such practical innovations as gyroscopic control, a jet deflector for navigation in space, proper rocket shapes, nozzles to ensure supersonic exhaust velocities, a multistage operation to escape the earth's gravitational field, the protection of the passenger chamber from atmospheric frictional heating and a rotating

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[600] For further reading, see:

- Kosmodemianskii, A.A., *Konstantin Tsiolkovsky: His Life and Work*, Translated from the Russian by X. Danko, Foreign Language Pub. House: Moscow, 1956, 101 pp.
- Von Braun, W. and F.I. Ordway, *History of Rocketry and Space Travel*, 1967.
- Celnikier, L.M., *Basics of Space Flight*, Editions Frontieres: Gif-sur-Yvette Cedex: France, 1993, 356 pp.

[601] Before him (1881), the German engineer **Hermann Ganswindt** had recognized the fundamental importance of the *escape velocity* $\left(11.3\ \frac{\text{km}}{\text{sec}}\right)$ and had conceived (with the physicist **R.B. Gostkowsky**) vehicles that would propel themselves with a series of chemical explosions and thus escape terrestrial attraction. The name *astronautical* was coined (1912) by the Frenchman **Robert Esnault-Pelterie**.

space station. Most of his suggestions are realized today in the design of space vehicles.

In 1895, Tsiolkovsky introduced the new concept of *space elevator*, He imagined placing a "celestial castle" at the end of a spindle-shaped cable, with the "castle" orbiting the earth in a *geosynchronous orbit*. The tower would be built from the ground up to an altitude of 35,790 kilometers above mean sea level (geostationary orbit).

Tsiolkovsky was born in Izhevsk, southern Siberia, the son of a forester. At the age of nine he became almost completely deaf following a serious illness. He was self-educated and made his living as a science teacher during 1880–1918, first in a sequestered country school in the Borovsk district and then at Kaluga. Throughout this period he devoted most of his free time to scientific investigations.

His advanced ideas were slow to gain acceptance; he was met with indifference and disbelief. However, in 1918 he became a member of the Academy, and in 1921 he was allotted a personal pension. In the mid-1920's, Tsiolkovsky's works on rocket engineering and space flight began to win international recognition. **Hermann Oberth** wrote to him in 1929: "*You have ignited the flame, and we shall not permit it to be extinguished*".

One of the largest craters, on the dark side of the moon, discovered in 1959 by the Soviet spaceship *Lunik 3*, was named after Tsiolkovsky.

Space Elevator — or, Climbing to the Stars[602] *(1895–2005)*

The Biblical Jacob [Genesis ***28****, 12] saw in his dream "a ladder set up on the earth, and the top of it reached to heaven, and ... the angels of God ascending and descending on it." Since his time, ladders have been replaced*

602 For further reading, see:

- Celnikier, L.M., *Basics of Space Flight*, Editions Frontieres: Gif-sur-Yvette Cedex: France, 1993, 356 pp.
- Pearson, J., *The Orbital Tower: a spacecraft launcher using the earth's rotational energy*, Acta Astronautica **2**, 1975, pp. 785–799.

by elevators, and so a modern Jacob might well dream of building an elevator along which astronauts could ride to space.

Indeed, we know that a satellite at altitude of 35,790 km, whose orbit is in the plane of the equator, will appear stationary from the earth's surface: an electromagnetic signal can be sent from one to the other without adjusting the direction of the antenna. This is called a geostationary orbit[603]*, much used by the telecommunication industry.*

In principle, one may draw a material cable from a geostationary satellite to the point immediately below it on the terrestrial equator, effectively creating a physical track along which vehicles could move, driven by an earth-bound electrical generator, thereby lowering rocket-launch costs and risks to a minimum.

Next, we expound the basic physics of such an endeavor: Let m *be a mass point in a circular orbit at distance* r *from the earth's center, and orbital angular velocity* $\omega = \frac{v}{r}$*. No net force acts on the satellite, being in dynamic equilibrium under the opposing centrifugal force* $m\omega^2 r$ *and the gravitational force* $G\frac{M_E m}{r^2}$*.*

Consider now a second mass $\Delta m \ll m$ *attached to* m *by a weightless wire or cable (tether) of length* l*, the link being aligned along the radius vector to the earth's center, with* Δm *closer to earth. Since* $\omega = \frac{v}{r}$ *is common to both* m *and* Δm *(a rigid link constraint), the forces acting on* Δm *are:*

- *a* centrifugal force $(\Delta m)\omega^2(r - l)$ *away from the earth's center*
- *a* gravitational force $GM_E(\Delta m)/(r - l)^2$ *towards the earth's center*

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[603] *A Geosynchronous Satellite*: consider a satellite of mass m in a circular orbit around the earth at a constant speed v at an altitude h above the earth's surface. Since its centripetal acceleration is furnished by the gravitational force, Newton's second law yields $G\frac{M_E m}{r^2} = m\frac{v^2}{r}$, where G is the gravitational constant, M_E is the earth's mass and $r = R_E + h$ is the distance of the satellite from the earth's center.

In order to appear to remain over a fixed position on the earth, the period of the satellite must be $T = 24$ hours and the satellite must be in orbit directly over the equator. Combining the above equation with $v = \frac{2\pi r}{T}$ we obtain $r = \left(\frac{GM_E T^2}{4\pi^2}\right)^{1/3}$. Substituting numerical values $G = 6.67 \times 10^{-11}\,\frac{\mathrm{Nm}^2}{\mathrm{kg}^2}$, $T = 86,400\,\mathrm{sec}$, $M_E = 5.98 \times 10^{24}\,\mathrm{kg}$, we obtain $r = 42,250\,\mathrm{km}$, or $h = 35,790\,\mathrm{km}$.

Thus, there is a net force tending to pull Δm towards the earth:

$$\begin{aligned}\Delta F &= \frac{GM_E(\Delta m)}{r^2}\left[\left(1-\frac{l}{r}\right)^{-2} - \left(1-\frac{l}{r}\right)\right] \\ &\approx \frac{GM_E(\Delta m)}{r^2}\left[3\frac{l}{r}\right] \qquad \text{for } l \ll r,\end{aligned} \tag{1}$$

where the relation $m\omega^2 r = \frac{GM_E(\Delta m)}{r^2}$ was used. The mass Δm is prevented from falling towards the earth by a tension ΔF in the connecting link.

Instead of a small mass joined to a larger one by a mass-less wire, consider next a long, massy cylindrical tether sticking out of m towards the earth, the entire system moving as a rigid body at an angular velocity ω around the earth. Across an elementary segment of tether, mass dm and distance r from the earth, there will be a difference in tension dF given by

$$dF = \frac{GM_E dm}{r^2} - \omega^2 r dm = \frac{GM_E \rho A dr}{r^2} - \omega^2 r \rho A dr, \tag{2}$$

where $dm = \rho A dr$, ρ being the density, A the cross-section, and dr the length of the segment. Suppose that the cable has a uniform cross-section and density, and stretches from a geosynchronous satellite to a point whose distance is a from the earth's center. The tension F_r at a point on the cable at distance r from the terrestrial center is obtained by integrating the last equation:

$$F_r = GM_E A\rho\left(\frac{1}{a} - \frac{1}{r}\right) - \frac{1}{2}\omega^2 A\rho(r^2 - a^2). \tag{3}$$

Since $GM_E \approx 10^{14}$, $\omega^2 \approx 5 \times 10^{-9}$, the second term in the above equation is negligible compared to the first for cables whose length is a finite fraction of the geostationary distance. Thus

$$F_r \approx GM_E A\rho\left(\frac{1}{a} - \frac{1}{r}\right) \approx \frac{GM_E A\rho}{a} \qquad \text{for } r \gg a, \tag{4}$$

directed towards the earth's center. If the material of which the cylinder is made must not rupture under its own weight, we have

$$\sigma A > GM_E A\rho\left(\frac{1}{a} - \frac{1}{r}\right), \tag{5}$$

where σ is the strength. This imposes a limit to the length of the cable which can be suspended in this way. It can be shown that for a uniform tether to hang all the way from a geostationary satellite to the surface of the earth, the *strength of the material from which the cable is made must be in the region of the theoretical upper limit, namely* $10^{11}\,N/m^2$*, or* $100\,GPa$[604].

Since the tension in the tether is greatest at the height of the geosynchronous satellite and tapers down towards earth, it is advantageous to taper the structure from the point of greatest force to that of least force, in such a way that the stress per unit cross-sectional area is constant. Thus, replacing dF *by* σdA *we recast Eq.* (2) *in the form*

$$dF = \sigma dA = \frac{GM_E}{r^2}\rho A dr - \omega^2 r \rho A dr. \tag{6}$$

Dividing by A *and integrating from* ξ *to* r *we obtain*

$$A(r) = A(\xi)\, e^{\frac{\rho}{\sigma}\left[\frac{1}{2}\omega^2(\xi^2 - r^2) + \frac{GM_E}{\xi}(1 - \frac{\xi}{r})\right]} \tag{7}$$

But $\frac{GM_E}{\xi^2} = g(\xi) =$ *the acceleration of gravity at level* ξ*. Also the first term in the exponent (centrifugal acceleration) is much smaller than the gravitational term. Choosing* $\xi = r_0 =$ *earth's equatorial radius* $= 6378\,km$*,* $g_0 = g(r_0) =$ *acceleration due to gravity at the cable's base (earth's surface)* $= 9.780\,m{\cdot}s^{-2}$*;* $A(r_0) =$ *cross-sectional area of the cable on the earth's surface,* (7) *yields*

$$A(r) = A(r_0)\, e^{\frac{\rho}{\sigma} g_0 r_0 (1 - \frac{r_0}{r})}. \tag{8}$$

This equation gives a shape where the cable thickness initially increases rapidly in an exponential fashion, but slows at an altitude a few times the earth's radius, and then gradually plateaus when it finally reaches maximum thickness at geostationary orbit. The cable thickness then decreases again out from geosynchronous orbit.

[604] By comparison, most steel has a tensile strength of under 2 GPa, and the strongest steels no more than 5.5 GPa, but steel is dense, The much lighter material *Kevlar* has a tensile strength of 2.6-4.1 GPa, while *quartz* fiber can reach upwards of 20 GPa; the tensile strength of *diamond* filaments would theoretically be minimally higher.

Carbon nanotubes appear to have a theoretical tensile strength and density that are well above the desired minimum for space elevator structures. The technology to manufacture bulk quantities of this material and fabricate them into a cable is in early stages of development. While theoretically carbon nanotubes can have tensile strengths beyond 120 GPa, in practice the highest tensile strength ever observed in a single-walled tube is 52 GPa, and such tubes tensile strength ranges between 30 and 50 GPa. Even the strongest fiber made of nanotubes is likely to have notably less strength that its components. Improving tensile strength depends on further research on purity and different types of nanotubes.

Thus the taper of the cable from base to the satellite at $\xi = r_G = 42,250\,km$ *is:*

$$\frac{A(r_G)}{A_0} = \exp\left[\frac{\rho}{s} \times 4.832 \times 10^6\, m^2 \cdot s^{-2}\right].$$

Using the density and tensile strength of steel, and assuming a diameter of 1 cm at ground level, yields a diameter of several hundred kilometers (!) at geostationary orbit height, showing that steel, and indeed most materials used in present day engineering, are unsuitable for building a space elevator.

The equation shows us that there are four ways of achieving a more reasonable thickness at geostationary orbit:

- *Using a lower density material. Not much scope for improvement as the range of densities of most solids that come into question is rather narrow, somewhere between 1000 kg* m^{-3} *and 5000 kg* m^{-3}*.*
- *Using a higher strength material. This is the area where most of the research is focused. Carbon nanotubes are tens of times stronger than the strongest types of steel, hugely reducing the cable's cross-sectional area at geostationary orbit.*
- *Increasing the height of a tip of the base station, where the base of cable is attached. The exponential relationship means a small increase in base height results in a large decrease in thickness at geostationary level. Towers of up to 100 km high have been proposed. Not only would a tower of such height reduce the cable mass, it would also avoid exposure of the cable to atmospheric processes.*
- *Making the cable as thin as possible at its base. It still has to be thick enough to carry a payload however, so the minimum thickness at base level also depends on tensile strength. A cable made of carbon nanotubes (a type of fullerene), would typically be just a millimeter wide at the base.*

A space elevator cannot be an elevator in the typical sense (with moving cables) due to the need for the cable to be significantly wider at the center than the tips. While designs employing smaller, segmented moving cables along the length of the main cable have been proposed, most cable designs call for the "elevator" to climb up a stationary cable.

Climbers cover a wide range of designs. On elevator designs whose cables are planar ribbons, some have proposed to use pairs of rollers to hold the cable with friction. Other climber designs involve moving arms containing pads of *hooks, rollers with retracting hooks, magnetic levitation (unlikely due to the bulky track required on the cable), and numerous other possibilities.*

Power is a significant obstacle for climbers. Available energy storage densities, barring significant technological advances, are unlikely to be able to store the energy for an entire climb in a single climber without making it weigh too much. Some potential solutions have involved laser *or* microwave power beaming*, and solar power. Other possible designs involve the use of:*

- *energy from regenerative braking of down-climbers passing energy to up-climbers as they pass,*
- *magnetospheric braking of the cable to dampen oscillations,*
- tropospheric *heat differentials in the cable,*
- ionospheric *discharge through the cable.*

The primary power methods (laser and microwave power beaming) have significant problems with both efficiency and heat dissipation on both sides, although with optimistic numbers for future technologies, they are feasible. Electrical power transmitted from earth or from the geostationary station through the tether cable might require the use of yet to be developed superconducting materials which could complicate the cable design and add potential corrosion and microscopic cracking issues. Carbon nanotubes, while not superconducting, can be extremely conductive and may represent a solution to this problem.

Planetary engineering on a scale needed to realize a working space elevator may seem today hardly more than a science fiction writers' domain. Yet the point is that it is not excluded by any physical law!

Space Elevator Timeline

1895 ***Konstantin Tsiolkovsky*** *imagined the idea of a* space elevator*. Comments from* ***Nikola Tesla*** *at about the same time suggests that he may have also conceived this idea.*

1957 ***Yuri N. Artsutanov*** *suggested to extend a* counterweight *from the geosynchronous satellite in a direction away from earth, keeping the center of gravity of the cable motionless relative to earth. He also proposed* tapering the cable thickness *so that the tension in the cable be* *kept constant. This renders a thin cable at ground level, thickening up towards the geosynchronous satellite.*

There have been two dominant methods proposed for dealing with the counterweight need: a heavy object, such as a captured asteroid or a space station, positioned past geosynchronous orbit, or extending the cable itself well past geosynchronous orbit. The latter idea has gained more support in recent years due to relative simplicity of the task and the fact that a payload that went to the end of the counterweight-cable would acquire considerable velocity relative to the earth, allowing it to be launched into interplanetary space.

1966 *American engineers found that the strength required for a cylindrical tether would be twice that of any existing material including graphite, quartz, and diamond.*

1975 *An American scientist,* ***Jerome Pearson****, designed a tapered cross section that would be better suited to building the elevator. The completed cable would be thickest at the geosynchronous orbit, where the tension was greatest, and would be narrowest at the tips to reduce the amount of weight per unit area of cress section that any point on the cable would have to bear. He suggested using a counterweight that would be slowly extended out to 114,000 kilometers (almost half the distance to the moon) as the lower section of the elevator was built. Without a large counterweight, the upper portion of the cable would have to be longer than the lower due to the way gravitational and centrifugal forces change with distance from earth. His analysis included disturbances such as the gravitation of the moon, wind and moving payloads up and down the cable. The weight of the material needed to build the elevator would have required thousands of Space Shuttle trips, although part of the material could be transported up the elevator when a minimum strength strand reached the ground or be manufactured in space from asteroidal or lunar ore.*

1977 ***Hans Moravec*** *published an article called "A Non-Synchronous Orbital Skyhook," in which he proposed a modification of the space elevator idea into a more feasible tether propulsion system (Journal of the Astronautical Sciences, Vol. 25, Oct.–Dec. 1977).*

1978 ***Arthur C. Clarke*** *introduced the concept of a space elevator to a broader audience in his novel, The Fountains of Paradise, in which engineers construct a space elevator on top of a mountain peak in the fictional island country of Taprobane (which is actually an early name for Sri Lanka).*

1982 *In* ***Robert A. Heinlein****'s novel Friday the principal character makes use of the "Nairobi Beanstalk" in the course of her travels.*

1999 ***Larry Niven*** *authored the book Rainbow Mars which contained a "Hanging Tree" — an organic 'Skyhook' which was capable of interstellar travel. The book skillfully discussed several merits/demerits of such an approach to the Beanstalk — the primary demerit being that the water necessary to sustain such an enormous 'tree' would require the drying up of all of its host planet's water bodies — which is used as a plot device to explain the drying up of Mars.*

2000 ***Min-Feng Yu*** et al., *publish: "Tensile Loading of Ropes of Single Wall Carbon Nanotubes and their Mechanical Properties," Phys. Rev. Lett. 84, pp. 5552–5555.*

T. Yildirim et al., *publish: "Pressure-induced interlinking of carbon nanotubes," Phys. Rev. B 62, pp. 12648–12651.*

David Smitherman *of NASA / Marshall's Advanced Projects Office has compiled plans for such an elevator that could turn science fiction into reality. His publication, "Space Elevators: An Advanced Earth-Space Infrastructure for the New Millennium" is based on findings from a space infrastructure conference held at the Marshall Space Flight Center in 1999.*

2002 *American scientist,* ***Bradley C. Edwards****, suggests creating a 100,000 km long paper-thin ribbon, which would stand a greater chance of surviving impacts by meteors. The work of Edwards has expanded to cover: the deployment scenario, climber design, power delivery system,* orbital debris *avoidance, anchor system, surviving atomic oxygen, avoiding lightning and hurricanes by locating the anchor in the western equatorial pacific, construction costs, construction schedule, and environmental hazards. Plans are currently being made to complete engineering developments, material development and begin construction of the first elevator. Funding to date has been through a grant from NASA Institute for Advanced Concepts. Future funding is sought through NASA, the* United States Department of Defense*, private, and public sources. The largest holdup to Edwards' proposed design is the technological limits of the tether material. His calculations call for a fiber composed of epoxy-bonded* carbon nanotubes *with a minimal tensile strength of 130 GPa (including a safety factor of 2); however, tests in 2000 of individual single-walled carbon nanotubes (SWCNTs), which should be notably stronger than an epoxy-bonded rope, indicated the strongest measured as 52 GPa. Multi-walled carbon nanotubes have been measured with tensile strengths up to 63 GPa.*

1884–1892 CE **Heinrich Rudolf Hertz** (1857–1894, Germany). The last of the great physicists of the 19^{th} century. Discovered electromagnetic waves and opened the way for the development of radio, radar and television. **James Clerk Maxwell** had predicted such waves in 1864.

Hertz used a rapidly oscillating electric spark to produce waves of ultra-high frequency, and showed that these waves induced similar oscillations in a distance wire loop. He measured the velocity of electromagnetic waves and demonstrated that their speed, the transverse nature of their vibrations and their susceptibility to reflection, refraction and polarization are all in complete correspondence with the properties of light waves and infrared radiation. He thus established beyond doubt the electromagnetic nature of light.

Hertz discovered the effect of ultraviolet radiation upon electric discharge and thus laid the foundation to the discovery of the photoelectric effect. In 1892 he experimented with the passage of cathode-rays (electrons) through thin layers of metals. These experiments were crucial for the eventual identification of these rays.

Hertz is remembered today for his experimental discovery of electromagnetic radiation predicted by Maxwell, but he was a gifted theoretician too. Independent of **Heaviside**, he reformulated the 20 original Maxwell equation (in 20 variable), in a compact set, removed the potentials and emphasized the fields $\boldsymbol{E}$ and $\boldsymbol{B}$.

Hertz was born at Hamburg to a Jewish family that had converted to Christianity[605] in 1838. He began to study engineering at Münich in 1877, but soon abandoned it in favor of physical science at Berlin's University. During 1877–1878 Hertz prepared himself for his future work by reading the original works of Laplace and Lagrange and attending the lectures of **G.R. Kirchhoff** and **H. von Helmholtz**. In 1880 he submitted his dissertation and became assistant to Helmholtz in the physical laboratory of the Berlin Institute. In 1883 he became acquainted with Maxwell's theory. He made his discoveries in the Karlsruhe Polytechnic, where he was professor of physics.

In 1889 Hertz was appointed to succeed **R.J. Clausius** as professor of physics at the University of Bonn. He died prematurely, a few weeks short of

[605] Nevertheless, **Gustav Hertz** (1887–1975), a nephew of Heinrich and a physics Nobel Laurete in 1925, was still considered a Jew by the Nazis in 1933. Consequently he was expelled from the Berlin Polytechnical University.

his 37^{th} birthday, from jaw cancer — complicated by blood poisoning caused by surgery.

Standard Time

1884 CE *Worldwide Time Zones were established. The meridian of longitude passing through the Greenwich Observatory in England was chosen as the fiducial point for the world's time zones. [The mean solar time at Greenwich is called the Greenwich Mean Time (GMT) or Greenwich Civil Time (GCT). Astronomers call it Universal Time (UT).]*

The international conference in Washington D.C. (1884) set up 12 *time zones west of Greenwich and* 12 *to its east. These zones divide the world into* 23 *full zones and two half-zones. The* 12^{th} *zone east and the* 12^{th} *zone west are separated by an imaginary line called the International Date Line (IDL). This IDL is halfway around the world from Greenwich. A traveler crossing this line while headed west, toward China, loses a day. If he crosses it traveling eastward, he gains a day. [In his book 'Around the World in Eighty Days",* **Jules Verne** *(1828–1905, France) made use of this fact to dramatize the conclusion of his story.] A few places, such as the polar regions, use GMT but not standard time zones.*

Before the adoption of standard time, each city in the U.S.A. kept the local time of its own meridian. With the growth of railroads, these differences caused difficulties: Railroads that met in the same city *sometimes ran on different times. In* 1883 *the railroads of the United States and Canada adopted a system for standard time.*

In 1918, *the U.S. Congress authorized the establishments of time zones in the United States. Today, nearly all nations keep standard time.*

1884 CE **Paul Gottlieb Nipkow** (1860–1940, Germany). Inventor. Discovered the basic scanning principle of television, in which the light intensities of small portion of an image are successively analyzed and transmitted. This he accomplished through the invention of a rotating disc with one or more spirals of apertures that passed successively across the picture.

Nipkow was born in Lauenburg, Pomerania, and died in Berlin during the second World War.

1884–1885 CE **William Le Baron Jenney** (1832–1907, USA). Architect. Designed Home Insurance Co. Building, Chicago, with type of steel skeleton construction, making it the forerunner of modern *skyscraper*.

1884–1885 CE **John Henry Poynting** (1852–1914, England). Physicist. Defined a vector that quantified the direction and magnitude of energy flow of electromagnetic waves (*Poynting vector*).

Poynting also discovered a theorem that states the conservation of energy for the electromagnetic field[606]. Poynting was a professor of physics at Mason Science College, Birmingham, from 1880 until his death.

1884–1886 CE **Ottmar Mergenthaler** (1854–1899, Germany and USA). Clockmaker and inventor. Invented the Linotype typesetting machine, regarded as the greatest advance in printing since the development of movable type 400 years earlier.

It went through many stages of experimental development and was first successfully used commercially in New York City by *The Tribune* (1886). It gave a great impact to the development of printing.

Mergenthaler was born in Hachtel, Württemberg, Germany and was trained as a watch and clockmaker. He arrived in Baltimore USA (1872) and took a job in a machine shop, eventually working his way up into a partnership.

His device consisted of a keyboard that composed *matrices* (molds) for letters, and then cast an entire line of type at once. He demonstrated the device and patented the Linotype in 1884. Many improvements have been

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[606] The time rate of change of electromagnetic energy within a certain volume, plus the energy flowing out through the boundary surfaces of the volume per unit time, is equal to the negative of the work done by the field on the sources within the volume (provided there are no dissipative effects).

made in the design of the machine since then,[607]and more than 1500 separate patents have been taken out in connection with it. The present Blue Streak Linotypes can set type in all sizes from the very smallest to the larger display sizes and in thousands of designs. More than 850 languages and dialects are set on Linotype machines in all parts of the world.

1884–1888 CE **Max Eastman** (1854–1932, U.S.A.). Inventor. Made it possible for millions of people to become amateur photographers. Perfected flexible roll-films and roll holders for winding them. Produced the first light-weight camera.

1884–1903 CE **Gottlob** (**Friedrich Ludwig**) **Frege** (1848–1925, Germany). Mathematician, logician and philosopher.

Played a crucial role in the emergence of modern logic and analytical philosophy. His writings on the philosophy of logic, mathematics and language are of major importance. His logical works mark a break between contemporary approaches and the older Aristotelian tradition. Created the first fully axiomatic system of propositional and first-order logic and also represented the first treatment of higher-order logic. His theory of *meaning*, especially his distinction between the *Sense* (*Sinn*) and *Reference* (*Bedeutung*) of linguistic expressions, was important in semantics and the philosophy of language.

His major works are: *Begriffsschrift* (Concept-Script, 1879); *Funktion und Begriff* ('Function and Concept', 1891); *Über Sinn und Bedeutung* ('on Sense and Reference, 1892); *Grundgesetze der Arithmetic* ('Basic Laws of Arithmetics, 1893–1902).

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[607] The operator sits before the keyboard which resembles that of a typewriter but has 90 keys. He touches a letter key that releases a *matrix* (brass mold) from the magazine (metal case) at the top of the machine. A moving belt carries the matrix to its proper place in the line. Spaces between words are formed by wedge-shaped spacebands, which are automatically inserted when the operator presses a key. When the operator has composed the line of matrices they are transferred to the casting mechanism. Here they are automatically *Justified* (spaced) and molten metal forced into the faces of the matrices. The metal hardens into a *slug* with raised letters into a shallow, sideless tray that is called *galley*. The slugs are used to print books, newspapers, magazines and other kinds of printed materials. The machine immediately and automatically return the matrices to their places in the magazine, where it will be used over and over again.

Frege's lifelong project, of showing that mathematics was reducible to logic, was not successful[608].

Frege's ideas influenced **Dedekind**, **Zermelo**, **Husserl**, **Russell**, **Carnap** and **Wittgenstein**. He in turn was influenced by **Leibniz**, **Boole** (1847), **de Morgan** (1847), **Cantor** (1872), and **C.S. Peirce** (1878). It seems that Frege and **Peano**, working in parallel along the same time-window 1884–1904, had influenced each other in the field of axiomatic arithmetic and symbolic logic. Frege rederived the *Peano Axioms* (governing the natural numbers) from *Hume's Principle.*

Frege was born in the coastal city of Wismar in Northern Germany, and lived there until 1869. He studied at the University of Jena (1869), receiving his Ph.D. there in 1873. He then spent all his working life at that university, rising to a final level of associate professor in 1894. He married (1880) Margaret Liesenburg (1856–1905). They had two children who died young. His work was unfavorably reviewed by his contemporaries and then completely ignored for 20 years. In his own country he long remained an obscure professor of mathematics[609]. He was known to be rather anti-semitic and wanted to see all Jews expelled from Germany. Had he lasted for another decade he would surely become a Nazi sympathizer. This feature in his personality has gravely disappointed some of Frege's intellectual progeny.

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[608] In 1903, while the second volume of his *Grundgesetze* was still in Press, Frege received a letter from **Bertrand Russell** which left him 'thunderstruck': in it Russell informed him of a contradiction in his logical system (the '*Russell Paradox*'), Frege never did manage to amend his axioms to his satisfaction. After Frege's death, **Kurt Gödel** showed (1930) in his *incompleteness theorems* that Frege's logistic program was impossible.

[609] In her book '*Frege*', Joan Weiner (Oxford University Press 1999, p.3) quotes the diary of Frege, written in 1924:

"One can acknowledge that the Jews are of the highest respectability, and yet regard it as a misfortune that there are so many Jews in Germany and that they have a complete equality of political rights with citizens of Arian descent; but how little is achieved by the wish that the Jews in Germany should lose their political rights, or better yet, vanish from Germany.

If one wanted laws past to remedy this evil, the first question to be answered would be: how can one distinguish Jew from non-Jew for certain? That may have been relatively easy 60 years ago. Now it appears to me to be quite difficult. Perhaps one must be satisfied with fighting the ways of thinking which show up in the activities of the Jews and are so harmful, and to punish exactly their activities with the loss of civil rights, and to make the achievement of civil rights more difficult."

1884–1918 CE **Alexandr Mikhailovich Lyapunov** (1857–1918, Russia). Mathematician and mechanical engineer. Initiated the modern theory of stability of autonomous systems of nonlinear differential equations (both ordinary and partial). By his method, one gains information about the location of the solution in phase-space even without solving the equation. The procedure consists of finding a nonnegative functional of the configuration ("*Lyapunov function*") which has a non-positive time derivative[610]. Then the solution of the differential equation will remain in a region described by the functional and the initial conditions. To date, certain constructive methods are known for obtaining analytic expressions for Lyapunov functionals.

In today's terminology, *Lyapunov's theorem* (1892) asserts that the equilibrium state will be an *attractor* if the Lyapunov function is zero, iff the configurations is at equilibrium and if the time-derivative of this function has a fixed sign opposite to that of the function itself. This function plays an important role in thermodynamic stability theory, since it is identified with the *entropy production function* and is a useful concept even for a system away from thermodynamic equilibrium. Lyapunov's theory is also important in nonlinear dynamics and control theory.

Lyapunov was born in Yaroslavl, a son of the astronomer Mikhail Vasilievich Lyapunov, who worked at Kazan University. Lyapunov's brother

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[610] The concept of the stability of an equilibrium is familiar from elementary mechanics. It is known, for example, that in a system whose mechanical energy is conserved ('conservative system') — an equilibrium position corresponding to a local minimum of the potential energy is a stable equilibrium position (Lagrange, 1788).

This idea was generalized by Lyapunov into a simple but powerful method for studying stability problems in a broader context.

A simple example is provided by the autonomous system

$$\frac{dx}{dt} = f(x,y), \qquad \frac{dy}{dt} = g(x,y)$$

which is assumed to have an isolated critical point at (x^*, y^*), i.e. $f(x^*, y^*) = 0$; $g(x^*, y^*) = 0$.

Let $u = x - x^*$, $v = y - y^*$, (u, v) small. Expanding in Taylor series about (x^*, y^*) we have

$$\dot{u} = \dot{x} = f(x^* + u, y^* + v) = u\frac{\partial f}{\partial x}\Big|_{x^*} + v\frac{\partial f}{\partial y}\Big|_{y^*} + O(u^2, v^2, uv)$$

$$\dot{v} = \dot{y} = g(x^* + u, y^* + v) = u\frac{\partial g}{\partial x}\Big|_{x^*} + v\frac{\partial g}{\partial y}\Big|_{y^*} + O(u^2, v^2, uv),$$

where $\frac{\partial f}{\partial x}\big|_{x^*}$ etc are numbers, *not* functions. Hence the *linearized* system can be written as

$$\begin{bmatrix} \dot{u} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} f_x & f_y \\ g_x & g_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix},$$

or simply $\dot{\boldsymbol{u}} = A\boldsymbol{u}$ where $\boldsymbol{u} = (u, v)$ and

$$A = \begin{bmatrix} f_x & f_y \\ g_x & g_y \end{bmatrix}_{(x^*, y^*)} = \textit{Jacobian matrix}$$

at the fixed point (also known as the *stability matrix*).

The general solution of this linearized ODE system is written in terms of the *eigenvalues* (λ_1, λ_2) of A and the corresponding eigenvectors $(\boldsymbol{v}_1, \boldsymbol{v}_2)$ which are solutions of $A\boldsymbol{x} = \lambda\boldsymbol{x}$, namely:

$$\boldsymbol{u}(t) = c_1 e^{\lambda_1 t}\boldsymbol{v}_1 + c_2 e^{\lambda_2 t}\boldsymbol{v}_2$$

under the initial condition $\boldsymbol{x}_0 = c_1\boldsymbol{v}_1 + c_2\boldsymbol{v}_2$. [Note that not every matrix A *has* two independent eigenvectors – e.g. $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ has $\lambda_1 = \lambda_2 = 1$ but only a *single* eigenvector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, up to a multiplicative constant. In such a case, the analysis presented here must be slightly revised.]
If $\tau =$ trace of $A = \lambda_1 + \lambda_2$ and $\Delta = \det A = \lambda_1\lambda_2$, one finds

$$\lambda_{1,2} = \frac{1}{2}[\tau \pm \sqrt{\tau^2 - 4\Delta}],$$

where λ_1, λ_2 may or may not be real. Note that the transformation $u = x - x^*$, $v = y - y^*$ has virtually moved the isolated critical point to the origin.
If $\lim_{t\to\infty} \boldsymbol{x}(t) = \boldsymbol{x}^*$, $\boldsymbol{x}^*$ is called an *attractor*; this is guaranteed within some neighborhood of $\boldsymbol{x}^*$ (for $\boldsymbol{x}_0$) provided $\operatorname{Re}\lambda_j < 0$, $j = 1, 2$. Assume that we have shifted $\boldsymbol{x}^*$ to $(0,0)$.
Let $x = x(t)$, $y = y(t)$ be a general solution of the above DE system and let $V[x(t), y(t)] \equiv V(t)$ be a function with continuous first partial derivatives in the neighborhood of the origin, such that $V(0,0) = 0$. Then V is said to be *positive definite* if $V(x,y) > 0$ for $(x,y) \neq (0,0)$ in the neighborhood and *negative definite* if $V(x,y) < 0$ for $(x,y) \neq (0,0)$.
Similarly V is called *positive semidefinite* if $V(x,y) \geq 0$ for $(x,y) \neq (0,0)$ and *negative semidefinite* if $V(x,y) \leq 0$ for $(x,y) \neq (0,0)$. Clearly, along trajectory $(x(t), y(t))$ that solves the DE system,

$$\frac{dV}{dt} = \frac{\partial V}{\partial x}f + \frac{\partial V}{\partial y}g.$$

A positive definite function $V(x,y)$ with the property that $\frac{dV}{dt}$ is negative semidefinite is called a *Lyapunov function* of the above DE system.

The *Lyapunov stability theorem* states: If there exist a Lyapunov function $V(x,y)$ for the system $\{\dot{x} = f, \dot{y} = g\}$ in some neighborhood of $(0,0)$, then the critical point $(0,0)$ is stable. If V has the additional property that

$$\frac{dV}{dt} = \frac{\partial V}{\partial x}f + \frac{\partial V}{\partial y}g$$

is negative, then the critical point $(0,0)$ is *asymptotically stable.*
Loosely speaking, a critical point is *stable* if all paths that get sufficiently close to the point stay close to the point at all times. Our critical point is said to be asymptotically stable if it is stable *and* there exist a circle $x^2+y^2=r_0^2$ such that every path inside it for some $t=t_0$, approaches the origin as $t\to\infty$.
In the case of a system of n first order ODE

$$\dot{x}_i = f_i(x_1\cdots x_n;t), \qquad i=1,2,\ldots,n,$$

we write it in a vector form

$$\dot{\boldsymbol{x}} = \boldsymbol{f}[\boldsymbol{x}(t);t].$$

The system is *autonomous* if explicit time dependence of $\boldsymbol{f}$ is absent. *Any* solution of this system is denoted by $\boldsymbol{x}(t)=\boldsymbol{x}(\boldsymbol{x}_0,t_0;t)$ with $\boldsymbol{x}_0=\boldsymbol{x}(\boldsymbol{x}_0,t_0;t_0)$. A *specific* solution $\boldsymbol{x}^*(t)=\boldsymbol{x}(\boldsymbol{a},t_0;t)$ is said to be *Lyapunov-stable* for $t\geq t_0$ if, for any $\epsilon>0$ there exists $\delta(\epsilon,t_0)>0$ such that for any general solution, $|\boldsymbol{x}(t_0)-\boldsymbol{x}^*(t_0)|=|\boldsymbol{x}_0-\boldsymbol{a}|<\delta$ implies

$$|\boldsymbol{x}(t)-\boldsymbol{x}^*(t)|<\epsilon \qquad \text{for all times } t\geq t_0.$$

In the case of instability, there always exists *some* $\epsilon>0$ and some $\boldsymbol{x}_0$ in an arbitrary small neighborhood of $\boldsymbol{a}$ such that $\boldsymbol{x}(\boldsymbol{x}_0,t_0;t)$ will leave the 'ϵ-tube' for some $t>t_0$ (thus, stability is nothing more than a uniformly continuous dependence on the initial conditions). One and the same DE may have both stable and unstable solutions (linear DE are an exception).
If a solution is stable for $t\geq t_0$ and δ is independent of t_0, the solution is *uniformly stable* for $t\geq t_0$. A solution $\boldsymbol{x}^*(t)$ is *attractive* if there exists $\eta>0$ such that $|\boldsymbol{x}(t_0)-\boldsymbol{x}^*(t_0)|<\eta$ implies $\lim_{t\to\infty}|\boldsymbol{x}(t)-\boldsymbol{x}^*(t)|=0$. If $\boldsymbol{x}^*(t)$ is also stable it is said to be *asymptotically stable.*
The transformation $\boldsymbol{y}(t)=\boldsymbol{x}(t)-\boldsymbol{x}^*(t)$ yields

$$\dot{\boldsymbol{y}} = f[\boldsymbol{y}+\boldsymbol{x}^*(t)] - f[\boldsymbol{x}^*(t)] = g(\boldsymbol{y},t).$$

The solution $\boldsymbol{x}^*(t)$ now corresponds to the trivial solution $\boldsymbol{y}=0$ and the stability of this solution corresponds exactly to that of $\boldsymbol{x}^*(t)$. Thus, it is possible to reduce the concept of stability of a motion to that of a treatment of the special case of the stability of an *equilibrium position.* One may therefore restrict oneself to the treatment of the trivial solutions of $\dot{\boldsymbol{x}}=\boldsymbol{f}(\boldsymbol{x};t)$ i.e. assume that $\boldsymbol{f}[0;t]=0$.
The *Lyapunov stability theorem* for this system then states the following: If there is a positive definite function $v[\boldsymbol{x}(t),t]$ for the system $\dot{\boldsymbol{x}}=f[\boldsymbol{x}(t),t]$ such that for $\boldsymbol{x}(t)=(x_1,\ldots,x_n)$

$$\frac{dv}{dt} \equiv \dot{v}[\boldsymbol{x}(t),t] = \frac{\partial v}{\partial t} + \nabla v\cdot\boldsymbol{f}[x(t),t] \leq 0,$$

then the trivial solution is stable.

Sergei was a composer; another brother, Boris, was a specialist in Slavic philology and a member of the Soviet Academy of Sciences.

After graduating from the Gymnasium in Nizhny Novgorod in 1876, Lyapunov enrolled in the Physics and Mathematics Faculty of St. Petersburg University, where **P.L. Chebyshev** greatly influenced him. His master's thesis, suggested to him by Chebyshev, led him to become interested in the stability of ellipsoidal forms of equilibrium of rotating fluids (1884–1885). This, in turn, led to his classical paper on the general problem of the stability of systems having a finite number of degrees of freedom (doctoral thesis, 1892)[611].

In 1885 Lyapunov moved to the University of Kharkov, where he became a professor in 1893. In 1901 he occupied the vacant chair of Chebyshev at the St. Petersburg Academy of Sciences. In a series of papers written between 1903 and 1918, Lyapunov returned to the problem of the figure of equilibrium of rotating nonhomogeneous fluids. He came to the conclusion that 'pear-shaped' figures, that branch off from the *Jacobi ellipsoids*, are unstable. (This instability was confirmed in 1917 by **J. Jeans**, who used Lyapunov's results in his astrophysical models.)

In the summer of 1917 Lyapunov went to Odessa with his wife, who suffered from a serious form of tuberculosis. On the day of her death on 31 October 1918 he shot himself, and died three days later.

Lyapunov and **A.A. Markov**, who had been schoolmates at St. Petersburg University and, later, colleagues at the Academy of Sciences, were Chebyshev's most prominent students, and representatives of the St. Petersburg mathematics school. Both were outstanding mathematicians and both exerted a powerful influence on the subsequent development of science.

1885 CE, Aug. 20 **Ernst Hartwig** (Germany). An astronomer. Observed a new 'star-light' from a supernova explosion near the center of the Andromeda Galaxy that happened ca 2 million years ago.

1885 CE **Gottlieb Wilhelm Daimler** (1834–1900, Germany) and **Carl Benz** (1844–1929, Germany). Engineers. Experimenting separately, they developed successful gasoline engines.

Daimler powered a two-wheeled motorcycle with his engine. Benz installed his engine in a three-wheeled carriage. His vehicle had electric ignition, a water-cooled engine and a differential gear, all of which are still common in

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[611] **Poincaré** tackled similar problems by making wide use of geometrical and topological concepts, while Lyapunov used purely analytical methods. Both works are fundamental to the qualitative theory of ODE.

automobiles today. He also designed a float-type carburetor and a transmission system.

1885 CE **Charles Sanders Peirce** (1839–1914, U.S.A.). Mathematical logician. Introduced the concept of *truth values* of a proposition, the forerunner of later truth tables. Son of Benjamin Pierce. Studied at Harvard. Worked as a physicist and mathematician in the United States Coast and Geodetic Survey (1861–1891). He retired from the USCGS without a pension, to devote himself to writing, and consequently suffered financial hardships during his retirement. After his death, several hundreds of unpublished manuscripts were found.

Pierce worked on the *4-color problem* and problems of knots and linkages. He then extended his father's work on associative algebras and set theory. Invented a map projection using elliptic functions.

1885–1893 CE **William Seward Burroughs** (1855–1898, USA). Inventor. Developed mechanical calculating machine (1885). It could do addition and listing.

He was born in Rochester, N.Y. and began his career as a bank clerk. This made him aware of the need for labor-saving device in accounting. His poor health necessitated a move to a warmer climate and he relocated to St. Louis (1882) where he devoted the next few years to devising an efficient calculating machine. He improved the machine in 1893, including an oil-filled hydraulic governor.

By 1898, the year Burroughs died, more than 1,000 machines had been sold, and by 1926 his company had produced a million machines.

1885–1896 CE **Karl Martin Leonhard Albrecht Kossel** (1853–1927, Germany). Biochemist. Discovered the nucleic acids *adenine* (1885), *thymine* (1894) and the amino acid *hystidine* (1896). Investigated the chemistry of proteins, the cell, and the cell nucleus. One of the first to apply methods of analytical chemistry to examine chemical processes in living tissues. Awarded the Nobel Prize for physiology or medicine (1910). He was a professor at Heidelberg (1901–1924).

His son **Walther Kossel** (1888–1956, Germany) is known for his theory of the physical nature of chemical valence (1916).

1885–1904 CE **Carl Freiherr Auer von Welsbach** (1858–1929, Austria). Chemist and inventor. Discovered (1885) the metallic *rare earth* elements[612] *neodymium* (Greek: "new twin") and *praseodymium* (Greek: "green twin"). Invented (1898) first metallic filament for incandescent gas lamps, which, for a while, competed successfully with Edison's electric light (1879).

Auer separated the so-called element didymium into neodymium (Nd; $A = 60$) and praseodymium (Pr; $A = 59$). The *ceramic industry* uses salts of neodymium to color glass and in glazes. The metal is present in *mich metal* (1904), an alloy with many uses.

1885–1909 CE **Edward Herbert Thompson**(1860–1935, USA). Explorer and archaeologist. Discovered Yucatan Maya remains at *Chichen-Itza*, including Sacred Well, Great pyramid and astronomical observatory. Unearthed many objects of archaeological significance.

Thompson was born in Worcester, Mass. Was US Consul in Merida, Mexico (1885–1909). On March 04, 1904, Thompson began dredging the Cenote of Sacrifice at the ancient Maya city Chichen-Itza and eventually substantiated legends[613] describing this natural, water-filled, limestone well as a repository for the precious objects and human victims offered to the gods by the ancient Maya (ca. 600 AD).

1886 CE The Woods Hole *Biological Station* was established (on Cape Cod, Mass., U.S.A.). Out of it emerged (1930) the oceanographic institution for research and study of marine science.

1886 CE **Albert Ladenburg** (1842–1911, Germany). Chemist. Made the first laboratory synthesis of a natural alkaloid, *coniine*[614]. Coniine is the toxic component of *hemlock*, the poison that ended the life of **Socrates** in 399 BCE.

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612 A group of 14 elements with atomic numbers $A = 58$–71. The name *rare earth* is a misnomer, since they are neither rare nor earths. Rare earths have 3 electrons in the outer shells of their atoms that take part in valence bonding. Because of this property, all rare earths have similar properties in water solutions, and all can exist in the 3-valent state. In nature they are always found in the form of phosphates, carbonates, fluorides and silicates.
The rare earths have many scientific and industrial uses. Tiny amounts of separated rare earths are used in *lasers*.

613 According to the book *Relacion de les Cosas de Yucatan* (1566) by Fray **Diego de Landa** (the bishop of Yucatan) and another book by Don **Diego Sarmiento de Figueroa** (1579): in times of drought and disaster, beautiful maidens were cast into the deep and muddy well as offerings to appease the god of rain. The well was some 6 m across and 30 m deep.

614 $C_8H_{17}N$: one of the simplest alkaloids; found in the plant *Conium maculatum* which was known to the ancient Hebrews as *Rosh* [*Deut* **29**, 17; **32**, 33; *Jer* **8**,

1886 CE An economical way was discovered to make *aluminum* from abundant *alumina* and electric power.

14; **9**, 14; **23**, 15; *Psalms* **69**, 22; *Lament* **3**, 19; **3**, 4; *Hoshea* **10**, 4; *Amos* **6**, 12]. Described also by **Theophrastos** (320 BCE), **Dioscorides** (70 CE) and **Pliny** (75 CE).

Aluminum

Aluminum is the most abundant metal in the earth's crust (about 7.3 percent), and is the third most abundant element. It occurs as silicates in almost all crystalline rocks[615]*, clay and slate*[616]*.*

Alum,[617]*from which the element takes its name, was known to the Greeks and Romans.*

The German chemist **Andreas Marggraf** *(1709–1782) was first able (1754) to isolate alumina (Al_2O_3) from clay.* **Humphry Davy** *(1809), isolated the impure metal, which he called* aluminum*. A purer metal was obtained by* **H.C. Oersted** *(1824) by heating Aluminum chloride with potassium.*

Although aluminum currently costs less than $2.00 per kg, it was considered the most valuable metal in 1827 (16 million dollars for one kilogram!). Indeed, it was so cherished by royalty in the early to mid 1800s that they alone ate with aluminum spoons and forks while their lower class guests dined with cheaper gold and silver service. Why was it originally so expensive?

Aluminum was first prepared by **Friedrich Wöhler** *(1827), using the following reaction*

$$AlCl_3 + 3K \rightarrow Al + 3KCl,$$

where potassium was obtained by passing an electric current (from a voltaic cell) through molten KCl. But copper and zinc used in voltaic cells were expensive in the early 1800's and in addition the great cost of energy required

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615 *Feldspar* [$KAlSi_3O_8$, or K_2O, Al_2O_3, $6SiO_2$] is a constituent of primary rocks such as granite, and by the disintegration of these rocks, either by simple hydrolysis or by combined action of moisture and atmospheric CO_2, soluble alkali salts and insoluble aluminum silicate (clay) pass into the soil; *Cryolite* (Na_3AlF_6); *Hornblende*; *Tourmaline*; *Augite* and Micas.

616 Common clay is a mixture of *kaolin* [Al_2O_3, $2SiO_2$, $2H_2O$] with limestone, quartz, and oxide of iron. The oxide *alumina*, Al_2O_3 is found either anhydrous as *corundum*, or hydrated as *diaspore* [Al_2O_3, H_2O], *gibbsite* [Al_2O_3, $3H_2O$] and *bauxite* [an ore which is a mixture of the minerals $AlHO_2$, $Al(OH)_3$. It contains also some iron oxide, titanium oxide and other impurities]. *Slate* is clay hardened and laminated by pressure.

617 $KAl(SO_4)_2 \cdot 12H_2O$; a mixture of clay and limestone constitutes *marl*, whilst a mixture of clay and sand is called *loam*. *Bauxite* was discovered (1821) by the mineralogist **P. Berthier** near Les Baux in Provence, France.

to melt large quantities of KCl was prohibitive. Thus, it was impractical to produce aluminum by passing an electric current through molten Al_2O_3 *because it has a high melting point of ca* 2000° *C. This high temperature is difficult to achieve and maintain, and even so, the components of most voltaic cells melt below this temperature (zinc at* 420° *C and copper at* 1083° *C).*

The cost of aluminum began to drop in the late 1800s as a result of two major advances:

- *The invention of the commercial direct current* electric generator *(1869) by the Belgian inventor* ***Zénube Théophile Gramme****, which could produce electricity by steam or water. Although this mode of production of electricity was much less costly than electricity generated by voltaic cells, aluminum still cost more than $200,000 a kg.*
- *Chemists discovered*[618] *(1886) the electrolytic method of producing aluminum: they could lower the melting point of aluminum oxides by mixing it with salts such as cryolyte.*

Since 1886, the price of aluminum has decreased markedly because of lower electrical costs, improved production techniques, and recycling of discarded aluminum products.

Aluminum is not only the most abundant metal but also one of the most useful because of its unique combination of properties: low density (2.699$\frac{\mathrm{g}}{\mathrm{cm}^3}$ *at* 20° *C), high resistance to corrosion (by forming a very thin protective layer of* Al_2O_3*), good thermal and electrical conductivity, attractive luster, and lack of toxicity. Its uses range from electric transmission lines to kitchen foil and cooking utensils. Alloys with magnesium, manganese, and copper have high mechanical strength and are easily machined, so that they have come into widespread use in the construction of buildings, automobiles, airplanes, and ships.*

Nowadays, aluminum is of inestimable value in energy conservation: *Around homes one finds storm doors and windows, insulation backed with aluminum foil, and aluminum siding, Because vehicle weight significantly affects gas mileage, substituting aluminum for heavier metals in cars, trucks, trains, and aircraft helps preserve petroleum supplies. Aluminum thus helps*

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[618] **Charles Martin Hall** (1863–1914, USA) and independently, in the same year, by **Paul Louis Toussaint Heroult** (1863–1914, France): A carbon-lined iron box, which serves as a cathode, contains the electrolyte, which is the molten mineral *cryolite* (Na_3AlF_6) in which aluminum oxide Al_2O_3 is dissolved. The aluminum oxide is obtained from the ore *bauxite*. To make a ton of aluminum in this way requires about 20,000 KW-hours of electricity.

conserve energy and improve standard of living at the same time. The largest producers are currently the United States (ca 2 million metric tons in 1990), Russia and Canada.

Alkaloids — Elixirs of Life and Death

Primitive man had no knowledge of the cause of his physical ailments, nor did he have effective means of alleviating his suffering. His life span was relatively short. The slightest injury or the smallest infection frequently brought him great suffering and death. Little did he know that healing and pain-killing medicinal substances lay within arm's length.

Primitive man attributed his ills so "evil spirits" that had invaded and taken control of his body. He sought the services of the tribal medicine man, who he believed was endowed with supernatural powers. The medicine man had many magical ways of "healing" the members of his tribe. Often he danced and chanted around his patient's prostrate body, shaking rattles filled with animal teeth and small bones. Sometimes the patient was instructed to kill a small animal and wear a string of its teeth around his neck as a "charm". Or the "witch doctor" might prepare a foul-tasting concoction of toad's eyes, the dried blood of a bird, and a pulverized bone of a deceased enemy soaked in the urine of a newborn baby. Then he would administer the concoction in various ways; he might have the patient drink it, or he might rub it on the patient's body.

However, as man roamed the land in search of anything edible, he tasted roots, leaves, stalks, and tree barks — almost anything he thought might provide nourishment. Through trial and error, he undoubtedly encountered many plants with both beneficial and malevolent properties. Many of these plants contained alkaloids *that are still valuable in modern medicine. Other plants had a negative effect on him. For example, the heady fragrance and delicacy of the lily of the valley with its white bell-shaped flowers belies its* *toxic properties. The plant can cause severe skin lesions in a susceptible individual. However, modern physicians have found beneficial uses for this plant as well. It contains a substance that sometimes is incorporated into* diuretics *(fluid reducers) and* cardiac *(heart) tonics.*

Many decorative garden plants that we take for granted, such as the rhododendron, also have toxic properties. Children who have sucked the nectar from the colorful flowers have suffered severe shock-like symptoms. At one time rhododendrons were used as an insecticide because of their toxicity. Daffodils, eucalyptus, oleander, azaleas, hyacinth, poinsettia, and bleeding heart are other common plants that can be deadly if eaten. Undoubtedly early man tasted them all in his quest for food.

Thus, the curative, narcotic, hallucinogenic and poisonous effects of certain plant extracts were known already in prehistorical times: The use of psychoactive drugs is very ancient. The peoples of India (ca 1000 BCE) were using a potent psychoactive drug called soma *(possibly derived from mushrooms), and Herodotos records the Scythians inhaling the smoke from burning hemp seeds.*

The linkage between chemistry and the art of healing also goes back to ancient times. Recipe books or antidotaries *of various mixtures believed to have curative powers were known in medieval Europe. The Arabs preserved an essentially unbroken contact with ancient medical science; portions of the works of* ***Hippocrates*** *and other early medical practitioners such as* ***Dioscorides*** *and* ***Galen*** *had been translated into Arabic by the 9^{th} century. The first Arabic pharmacopoeia was brought to Europe in the 11^{th} century.*

Paracelsus *(1493–1541), who was part charlatan and part scientist, played an important role in furthering the medical applications of chemistry and in urging a search for new drugs.*

Poisons play an important part in history. Every historical figure in power lived with the constant fear that his life could be abruptly ended with his next cup of wine or morsel of food. The Duchess of Ferrara, the infamous ***Lucretia Borgia*** *of Italy (1480–1519), was notorious for her use of herb poisons, which she carried concealed in hinged finger rings. The Duchess disposed of an untold number of persons who threatened the power of her family, or persons whom she considered menacing.*

Alchemists were often members of the royal courts. One of their functions was to prepare special poisonous potions, as well as to prepare antidotes for them.

In addition to making protein, carbohydrates, fats, and other compounds familiar to most people, plants synthesize a huge array of substances that *are usually found in relatively low quantities. The general name of these substances is* secondary plant products*, which gives no idea of the chemical range of the materials or any idea of their importance. The medicine products, the natural dyestuffs, products for the chemical industry (gums, resins, etc.), and a wide variety of common, everyday substances used as flavorings and essential oils (perfumes, peppermint) are secondary plant products.*

Among the secondary plant products that have long played important roles in human life are the alkaloids*. They can be extracted from the roots, leaves and seeds of certain plants (Table 4.8) and possess marked physiological activity and are also valuable curative agents. The word alkaloid is a purely chemical word defined as an organic chemical molecule — one containing carbon atoms — which also contains at least one atom of nitrogen. All can be crystallized and, when dissolved in water or in alcohol, they give an alkaline reaction to the solution. The nitrogen in an alkaloid is usually found in combination with a ring of carbon atoms, the so-called* heterocyclic ring*.*

Other common replacement for carbon in these structures are oxygen and sulfur. These compounds are contrasted to carbocyclic *compounds, which contain only carbon in the ring. When an alkaloid is mixed with an acid, such as hydrochloric, the very water-soluble hydrochloride is formed; this is the usual way of getting alkaloids into simple solutions. In solutions or as the crystal, they are colorless, are most soluble in alkaline solutions such as weak sodium hydroxide, and almost invariably have a bitter taste. The flavor of tonic water is due to quinine, one of the alkaloids.*

There are today about 300 *known natural alkaloids and their number increases with the growth of biochemical research of plants. Among the types in common use are the sedatives, and analgesics, the narcotics, the stimulants, the antidepressants, the tranquilizers, and the hallucinogens. Some of these alkaloids are listed in Table 4.12.*

The exploration of the chemical nature of alkaloids has been one of the boldest and most difficult challenges of analytic organic chemistry, occupying the minds of the greatest chemists over the past two centuries. The 19^{th} *century in particular, saw an accelerated growth in the discovery and understanding of drugs.*

A good deal of study of the specific effects of various drugs and of the relationship between chemistry and physiology took place. An interest in the synthesis *of drugs arose, as opposed to simply isolating them from naturally occurring materials.*

Table 4.12: Common Alkaloids and their properties

Alkaloid	Source	Activity or Application
Aconitine	Aconite root (Wolfsbane) (*Aconitum napellus*)	Poison, sedative, useful in all febrile and inflammatory diseases
Atropine	*Atropa belladonna*	Mydriatic, antispasmodic
Caffeine	Coffee (*Caffea arabica*)	Stimulant, diuretic
Cinchonine	Bark of the Quina tree (*Cinchona officinalis*)	Antipyretic
Cocaine	Coca leaf (*Erythroxylom coca*)	Local anesthetic
Codeine	Opium poppy (*Papaver somniferum*)	Cough control, analgesic
Colchicine	(*Colchicum autumnale*)	Anti-cancer teraphy; treatment of gouty-arthritis
Coniine	Hemlock (plant) (*Conium maculatum*)	Poison
Digoxigenin	Purple Foxglove (plant) (*Digitalis purpurea*)	*Digitalis* for treatment of cardiac insufficiency
Ephedrine	Ma huang (*Ephedra vulgaris*)	Decongestant (respiratory ailments); Mydriatic
Epinephrine	Body adrenal medulla	Hormone controls metabolism, and adjustment to stress
Ergonovine	Sclerotia of rye grain fungus (*Claviceps purpurea*)	Ergotism
Ergotamine	Sclerotia of rye grain fungus (*Claviceps purpurea*)	Ergotism
Etoposide	Mandrake (plant) (*Mandragora officinarum*)	Analgesic, soporific, chemotherapy
Himbacine	(*Galbulimima belgraviana*)	Hallucinogenic
Hyoscyamine	Henbane (*Hyoscyamus niger*)	Hallucinogenic, narcotic, mydriatic

Table 4.12: (Cont.)

Alkaloid	Source	Activity or Application
Lobeline	Indian tobacco (*Lobelia inflate*)	
LSD	Sclerotia of rye grain fungus (*Claviceps purpurea*)	Hallucinogenic
Mescaline	Peyote cactus (*Lophophora williamsii*)	Hallucinogenic
Morphine	Opium poppy (*Papaver somniferum*)	Cough control, analgesic
Nicotine	Tobacco (*Nicotiana tabacum*)	Insecticide
Papaverine	Opium poppy (*Papaver somniferum*)	Cough control, analgesic
Piperine	Pepper (*Piper nigrum*)	Spice, seasoning
Psilocybin	Mushroom fungus (Soma) (*Stropharia cubensis* and *Psilocybe mexicana*, *Amanita muscaria*)	Hallucinogenic
Quinine	Bark of the Quinea tree	Antimalarial
Reserpine	Indian snake root (*Rauwolfia serpentina*)	Tranquilizer, sedative
Scopolamine	Jimson weed (*Datura stramonium*)	Sedative, hypnotic, mydriatic soporific, depressant
Serotonin	Body blood platelets, mid brain and enterochromaffine cells	Hormone; prevent bleeding
Strychnine	*Nux Vomica* (plant)	Poison, tonic
Taxol	Bark of yew tree	Ovary and lung cancer
THC	Hemp (Hashish, Marijuana) (*Cannabis Sativa*)	Hallucinogenic, fiber
Thebaine	Opium poppy (*Papaver somniferum*)	Cough control, analgesic

Table 4.12: (Cont.)

Alkaloid	Source	Activity or Application
Tryptamine	Yakee, Yato (*Virala calophylla*)	Hallucinogenic
Tubocurarine	Curare (*Chondodendron*)	Poison, treatment of tetanus and hydrophobia
Valium (diazepam)	Synthetic	Minor tranquilizer
Vinblastine	Periwinkle plant	Chemotherapy
Vincristine	Periwinkle plant	Chemotherapy
Vindesine	Periwinkle plant	Chemotherapy

Medical vocabulary:

Analgesic: pain-reducing without impairing consciousness.

Anesthetic: capable of producing loss of bodily sensations with or without loss of consciousness; used in surgery. Whereas *general* anesthetics produced a state of coma, *local* anesthetics work by depressing sensory endings or blocking the conduction of impulses through the nerves.

Antipyretic: reducing fever.

Antispasmodic: preventing or curing spasms.

Antitoxin: a serum serving to neutralize a toxin.

Aphrodisiac: exciting the sexual organs.

Barbiturate: derivative of barbituric acid, used especially as sedative or hypnotic. Affects all levels of the central nervous system. Can be addictive.

Carminative: easing gripping pains and expelling flatulence.

Decongestant: a drug that reduces excessive circulation in an organ by constricting blood vessels; usually taken to drain nasal passages and alleviate cold symptoms.

Depressant: an agent that reduces activity of bodily function.

Diuretic: fluid-reducing.

Emetic: cause vomiting.

Emollient: having softening and soothing effect.

Hallucinogenic: having the capability of the perception of objects or the experiencing of feelings that have no cause outside one's mind; caused especially as a result of mental disease or effects of a drug. It has been suggested that hallucinogens permit people to enter the "real" world, closed off from childhood by the many layers of culture that surround us from birth.

Medical vocabulary (cont.)

Haemostatic: drugs used to control bleeding.

Hypnotic: (= *Soporific*) sleep-inducing agent.

Mydriatic: a drug that produces dilation of the pupils.

Narcotic: an addictive drug that in moderate doses blunts the senses, relieves pain and induces sleep. In excessive doses causes stupor, coma or convulsions.

Sedative: tending to calm or relieve tension and irritability. Both sedative and hypnotic drugs depress the higher brain centers, decreasing excitement and activity.

Stimulant: an agent that temporarily increases the functional activity or efficiency of a tissue or an organ. Energy producing.

Sudorific: producing copious perspiration.

Therapy: mode of medical healing. Perhaps from the Hebrew *trufa* = medicine; may also be linked to *teraph* = ancient Hebrew household god.

Tonic: substance producing a feeling of well-being.

Tranquilizer: drug used to reduce mental disturbance such as anxiety or tension.

The highly competitive nature of our culture often necessitates the quick lunch, the fast freeway, the "urgent" telephone call — in general, a "burning the candle at both ends" way of life. This fast pace tends to play havoc on many people's "nerves". Consequently some people use tranquilizers to relieve anxiety and tension.

Physicians' offices are lined daily with victims of modern life who find that relaxation is difficult to obtain. Patients complain of many symptoms, from queasy stomach to "sledgehammer" headache. Some fear that they are developing major diseases because their tensions "translate" into symptoms of actual diseases.

Vulnerary: used in healing wounds.

Table 4.13: Milestones of Progress of Alkaloid Research

1776	***William Withering*** *recognized the importance of digitalis in the treatment of heart and kidney diseases.*
1805	***Friedrich Sertürner*** *extracted morphine from opium and used it to relieve pain.* ***Carl Gauss*** *used morphine to relieve pain of mothers in difficult child-birth.*
1816	***Pierre Pelletier*** *and* ***Joseph Caventou*** *isolated strychnine and quinine.*
1817	*The name alkaloid coined by the pharmacist* ***W. Meissner****.*
1818–1840	*Discovery of caffeine, atropine, codeine, curarine and other important alkaloids.* ***Gerhardt****,* ***Regnault****,* ***Laurent****,* ***Andrews*** *and* ***Berzelius*** *developed new methods for the investigation of alkaloid structure.* ***Liebig****,* ***Würtz*** *and* ***Hoffmann*** *(1848) considered alkaloids as acids of amonia in which atoms of hydrogen were replaced by organic radicals.*
1886	***Albert Ladenburg*** *synthesized coniine — the first alkaloid to be synthesized in the laboratory.*
1905	***Robert Willstätter*** *discovered the chemical structure of many alkaloids and synthesized some of them (atrophine, cocaine).*
1925	***Robert Robinson*** *discovered the chemical structure of morphine and other alkaloids. Explained the formation of alkaloids from condensation of amonia, formaldehyd and amino-acids. Suggested that plant-alkaloids are end waste-products of their metabolic chain.*
1938	***Hoffmann*** *and* ***Stoll*** *discovered LSD.* ***Dustin*** *discovered that colchicine was cytotoxic (blocking cell division).*

Table 4.13: (Cont.)

1943	**Hoffmann** *discovered that LSD is hallucinogenic.*
1944–1956	**Robert Woodward** *synthesized quinine (1944), strychnine (1947), lysergic acid (1954) and reserpine (1956).*
1965	*First synthesis of the active hallucinogen THC.*

Additional historical, folkloristic and scientific data concerning alkaloids is given below.

I. The four genera: *Atropa, Datura, Hyoscyamus* and *Mandragora* belong to the *tomato* family (Solanaceae) and each contains one or more of the tropane alkaloids: Atropine, Scopolamine and Hyoscyamine.

Atropine is a stimulant of the central nervous system and depressant of the parasympathetic nervous system.

In minute quantities, atropine is used as an antidote to other poisons; in moderate doses it causes loss of motor coordination; in higher concentrations it leads to hallucinations, delirium, stupor; in large quantities it is a deadly poison.

Greece and Rome knew it as a sedative and an hallucinogen. Bacchanalian orgies utilized new wine spiked with small amounts of the sap. With the spread of Christianity, bacchanalian orgies became a lamented aspect of the golden age of Rome, but belladonna's star rose again as witchcraft and demonology captured people's imagination.

The name belladona *dates from the late Middle Ages: Italian ladies put drops of diluted nightshade sap in their eyes to induce* mydriasis *— the deep, dark mysterious look caused by dilated pupils. Indeed, since atropine blocks the normal transmission of signals across synaptic junctions between nerves, opthalmologists used it to prevent the autonomous closing of the pupil in bright light.*

At one time the jimsonweed, which contains the alkaloid scopolamine, was used in childbirth to alleviate pain. However, it is extremely toxic, and it *often resulted in coma and death for those women who ingested it. Today, scopolamine is incorporated into some motion-sickness remedies. It is also used in the treatment of some symptoms of Parkinson's disease.*

White henbane, another herb with poisonous properties, contains the alkaloids hyoscyamine, hyoscine, and atropine. Ancient men found this herb to be useful in warfare. For example, two enemy camps might hold a "truce" celebration. The conquered army would serve wine laced with this poison to the invading army as a "goodwill" gesture, and the two armies would exchange toasts. The conquered became the conquerors after the first cup of wine. In modern times this drug has been used for treating mercury poisoning and morphine addiciton. In small amounts it produces sleep; in larger amounts it produces death.

Scopolamine as a truth serum *may or may not still be used, depending on whose national intelligence agency is being asked.*

The third tropane alkaloid, hyoscyamine, *is similar in action to atropine, but the clinical responses are sufficiently different to suggest that the receptor sites are not the same. The amounts used medically are very small; practitioners know that even slightly higher dosages, particularly when administered internally, have grave consequences and may lead to death.*

Mandrake, *known in biblical times [Gen* **30**, *14–17; Cant* **7**, *14] also contains large quantities of the alkaloids hyoscyamine and scopolamine, which are capable of causing prolonged stupor and alleviating severe pain. Relatives of victims being crucified brought sponges soaked with a solution of mandrake and other herbs to help numb the victim's pain. This was a merciful release from the tortures of the slow death of crucifixion. It is believed that Christ was administered this drug by his disciples to help relieve his agony in his last hours.*

Long before the Hebrews, the mandrake had been associated with sexuality and sins of the flesh. The Ebers medical papyrus of 1500 *BCE listed it as* dudajm, *the fruit that excites love; Pharaoh Tutankhamen was buried with* 11 *mandrake roots in the sixth row of his floral collarette to ensure his potency in the next world. The Greeks named it* circeium *after Circe who, Homer reported, lured men to her and changed them into swine, that is, into sexual pigs. They referred to Aphrodite as* Dios Mandragoritis. *Mandrake roots, carved into big-hipped and -breasted fertility figurines, have been unearthed at Antioch and Damascus and from tombs in Constantinople and Mersina.*

Further evidence for the medicinal use of mandrake is found in the writings of ***Hippocrates***, ***Plato***, ***Pliny***, ***Theophrastos***, ***Galen*** *and Dioscorides. Even as late as the* 13^{th} *century, a mixture of opium, and mandrake juice compounded in vinegar was taken up in sponges and inserted into the nostrils of patients undergoing surgery, at the Bologna medical school.*

Datura (Jimsonweeds) contains the three tropane alkaloids mentioned above. It has been used ritually in India as far back as records have been kept. It is also known to be used in puberty rites of passage in South and Central America and from there it diffused into the British colonies in North America.

Datura was the original knockout drops, and thieves in India and in Europe used it for centuries. Up until at least the beginning of this century, juice of datura leaves was added to milk given young Indian girls who were to be initiated into prostitution. The drink was narcotic and, it was asserted, aphrodisiac, so that the victim was believed to have actively contributed to her own downfall. The Chinese, believing that datura was a sexual stimulant, administered it to brides on their wedding night to calm their nerves and to make them more sexually receptive.

The European attitude towards datura parallels that in the Far East. Apollo's priests drank datura to achieve sedated, prophetic, and oracular states. The sacerdotal plant of Delphi was undoubtedly datura; the mumbling speech, trance states, and known fears of over-dosage are consistent with datura intoxication. Greek physicians knew it as nuxmetal *or* neura*, a reference to its sedative action, and when extended unconsciousness was desirable, as during surgery, datura was mixed with opium.*

Rome followed Athens' lead in ritual and in medicine and added the drug-induced orgy in which datura mixed with wine was used to induce hallucinogenic states and to heighten sexual activity. Avicenna, a tenth-century Arabian physician, recommended datura *not only for surgery, but as an excellent treatment of anxiety.*

From Arabia, the medical and aphrodisiacal use of datura spread to Spain and to Western Europe; Northern Europe was too cold to support the growth of datura with high tropane content.

Datura was an important ingredient in the poisonings that pervaded Southern Europe from 1400 to 1700. Nobles and merchants sent members of their family to schools teaching the art of poisoning for much the same reasons as we send our children to graduate schools of business administration. No love potion worth paying good money for was without datura, usually supplemented with extracts of other solanaceous plants and, for good measure, attar of roses, marjoram, other herbs, and a newt's tongue. Witches' Sabbats, the infamous black masses that so intrigued prurient Victorians, utilized ointments and unguents containing datura. Whole leaves were inserted into the rectum or vagina where tropanes are quickly absorbed; the broomstick, developed as a symbol of this part of the ritual probably because of the "flying" hallucination experienced by the users of the drug.

Cocaine, like atropine, belongs to the pyrrolidine *family of alkaloids. It is found in the leaves of the South American coca bush, known to the natives before the discovery of America.*

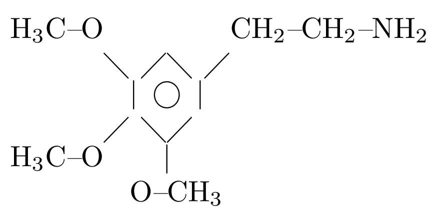
Cocaine acts as a stimulant to the central nervous system. It produces euphoria and insensitivity to pain and is a potent antifatigue agent. Although cocaine has been used as a local anesthetic, its toxic properties have caused a decline in its medicinal use. Many of its derivatives such as Novocaine and Lidocaine are widely used in dentistry and plastic surgery and in nerve blocks to reduced severe pain.

II. THE ALKALOID FAMILY OF *isoquinoline* INCLUDES *quinine, mescaline, chinchonine* AND THE OPIATES — *morphine, codein, thebaine* AND OTHERS.

Quinine is obtained from the bark of the chinchona tree, found primarily in the rain-forests on the eastern slopes of the Andes of Peru, Bolivia and Columbia, at heights of 1000–8000 *m. The name of the genera was coined by Linnaeus (1628) in honor of Count Chinchona, viceroy of Peru. Natives of Peru used it to cure fevers. After Lima was founded (1520), this became known to the Spaniards who carried the curative bark to Europe, whereby* 1640*, it was widely used as an antimalarial.*

During WWII, when Japanese overrun the plantations of south-east Asia, Quinine shortage urged American and British chemists to develop synthetic antimalarial substitutes.

Mescaline is a hallucinogenic alkaloid. It causes hallucinations, sense distortions, elevated blood pressure, and profuse sweating, although it is about 7500 *times less potent than LSD. Mescaline was obtained from the small cactus plant, the peyote, by the Indians in Mexico as early as the* 16^{th} *century for use in their religious ceremonies.*



Mescaline

Peyote grows in the Rio Grande River area. It has buttons or tufts, which are from a small cactus plant. They are dried and sometimes crushed and brewed into a beverage. The Aztec Indians used the brew during their religious festivals, and the early American Indians used it as a hallucinogen to "communicate" with their "divine spirits". The drug reached the North-American Apache Indians (1870), who adopted it as a cult object.

The botanical family Papaveraceae contains 25 *genera and* 120 *species of flowering plants. Most are herbaceous annuals, although a few die back to the ground each year and form new shoots from a perennial rootstock. The family originated in Asia Minor. The genus Papaver contains ten species, several of which, the Iceland poppy (P. nudicaule) and the oriental poppy (P. orientalis), are common garden plants. The California poppy (Eschscholzia), well known to most gardeners, is a member of another genus in the family. None of these plants produce alkaloids of medical interest. The one that does is the opium poppy (P. somniferum) whose specific name was chosen by Linneaus because of the sleep-inducing properties of the gum produced in the young seed capsule of the plant.*

The opium poppy can be grown in many parts of the world where the growing season is sufficiently long with warm, sunny weather. Because of the need for cheap labor, it is presently grown in relatively few countries of the world. India and Pakistan each produce about 100 *metric tons of opium each year, most of it consumed locally, Afghanistan produces the same amount, but the crop is smuggled into other countries of the Middle East. Turkey has traditionally been the major supplier of legal (medicinal) opium for the West, with between* 60 *and* 100 *metric tons produced each year. Mexico has only recently become an opium-producing country, although its production is still under ten metric tons. Less is known about production in the area of southeast Asia called the Golden Triangle, an area embracing parts of Burma, Laos, and Cambodia, but it may well be the greatest producer. Report on production vary from* 250 *to over* 500 *metric tons per year. The world's production of raw opium is close to* 1000 *metric tons per year, of which less than* 250 *tons enters legal medical channels; the United States processes about* 150 *metric tons to isolate morphine and codeine.*

Of the alkaloids found in raw opium, three are of medical importance. Close to 11 *percent of opium is the single alkaloid morphine, while codeine constitutes about* 2 *percent and thebaine a bit less than* 1 *percent of the weight of the opium gum.*

Morphine ($C_{17}H_{19}O_3N$) *is the principal component of opium [from the Greek word opion, for "poppy juice"; mentioned in the Jerusalem Talmud: Avoda Zara, page* 40*, side* 4*], which is obtained as the milky juice that exudes from unripe poppy seed capsules.*

Morphine acts on the central nervous system and can induce addiction. The specific action of morphine appears to be related to the ability of the molecule to fit into and block specific receptor site on a nerve cell; the benzene group of the morphine molecule fits snugly against the flat part of the protein that acts as a receptor site, and the neighboring group of carbon atoms is at the correct distance and orientation to fit into the groove. Beyond the groove is a group with a negative charge, which can attract the positive charge of the nitrogen atom. By fitting the shape of the receptor and binding to it, the incoming morphine molecule eliminates its action. In this respect the molecule mimics the body's natural pain killers, the enkephalins.

Raw opium has used medically for centuries. Summerian tablets of 2500 *BCE noted that when small balls of opium were eaten or taken after mixing with wine, the drug induced sleep and relieved pain. Homer spoke of nepenthe, a substance which will "lull pain and bring forgetfulness of sorrow". Hippocrates, Theophrastos, Pliny, Dioscorides, and other ancient medical writers recommended opium, and it was the most used analgesic up to the* 20^{th} *century.*

In the later half of the 19^{th} *century, pharmaceutical chemists started altering the morphine molecule in order to make a compound which would be more effective than the natural alkaloid and less addictive. They came up with heroin (more "heroic" than morphine), in which morphine's hydrogen atoms of two –OH groups have been replaced by acetyl groups* ($-CO-CH_3$)*. This replacement made heroin more soluble among the hydrocarbon chains of fats and less soluble in water. When injected directly into the blood, it passes more rapidly through the blood-brain barrier, the barrier that prevents water-soluble and large molecules from passing between the two. As a result, it is more potent than morphine, but its effect does not last as long. Once heroin is absorbed into the body, the acetyl groups are removed, forming morphine, which provides its analgesic and euphoric action.*

III. THE *piperidine* ALKALOID GROUP; (includes *coniine* and *nicotine*).

Nicotine, along with about ten other alkaloids, is found in tobacco. When it is taken into the body through smoking, nicotine increases the blood pressure and pulse rate and constricts the blood vessels. Nicotine contains both a piperidine ring and a pyrrolidine ring. In concentrated form, it is extremely toxic and is therefore often used as an insecticide.

IV. THE *ergot alkaloid* GROUP; includes *lysergic acid* (*LSD*), *strychnine* AND *THC* (DELTA-TETRA-HYDRO-CANNABINOL) WHICH IS THE ACTIVE ELEMENT IN THE RESINOUS EXUDATE OF THE HEMP PLANT (CANNABIS).

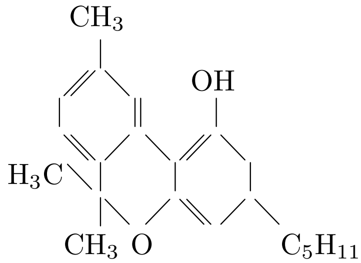
Nux Vomica, a strychnine-type drug, was once widely used to kill rats whose parasitic fleas were responsible for spreading plague diseases, such as the bubonic plague that killed millions of people in Europe for three centuries. These fleas also spread typhus, another deadly disease. Other strychnine-type drugs were used to eradicate unwanted spouses, competing heirs, meddling relatives, and so on. These unfortunate people were often served their last meal with a "seasoning" of this drug.

Although the use of cannabis *as a fiber and food crop dominated its production in the Orient for a long time, its medical uses were known and exploited. In a medical book ascribed to the legendary Emperor Shen-Nung in 2737 BCE (but probably written in the Han dynasty about 100 BCE) , ground, dried leaves of cannabis, known as* ma-yo, *were recommended for malaria, beriberi, constipation, as an anesthetic in surgery, and for that disease of old age — absent mindedness. Shen-Nung noted that its primary medical value was in calming hysterical women. In India and throughout southeast Asia, cannabis was also used medically, and its capacity to ease anxiety was noted in both Hindu and, later, in Buddhist writings, where it was referred to as the "soother of grief".*

It seems that cannabis entered India about 1500 *BCE, where it was first formally exploited as an hallucinogen. Knowledge of its hallucinogenic properties spread from India throughout Asia and Asia Minor about* 500–1000 *BCE. Herodotos described its uses by the Scythians in* 500 *BCE. Thebans, Greeks, Arabs and natives of Africa succumbed in turn to the drug.*

Marijuana entered the New World via Mexico when her French rulers introduced its cultivation for hallucinogenic purposes in the nearly ideal hot, dry lands around Mexico City. Its use spread very slowly to the native peoples because the peon preferred native hallucinogenic plants that were sanctified by usage dating back to Aztec times — marijuana was good for a mild smoke after a long hard day in the fields or mines. The product entered the United States through the Southwest and by 1860 *was introduced to the Eastern Seaboard via the immigrants from the Caribbean Islands. In New York, it was used by the black poor and by the socially chic.*

Marijuana is composed of the dried leaves and flowering tops, stems, and leaves of the female Indian hemp plant. The active component of marijuana is tetra-hydro-cannabinol (THC). Some slang names for this drug are grass, pot, and weed.



Marijuana

Hashish has the same active principle (THC) as marijuana, but it is considered to be about ten times more potent than the marijuana grown in America. Hashish comes from pure resinous exudate of the female hemp plant. The Tunisian cannabis is considered three times more potent than the American varieties. Like marijuana, it is not considered addictive, but can produce psychic dependence, hallucination, and distortion of time and space.

The word "hashish" is derived from the name of a Persian prince, Hasan-ibn-Sabbah, whose pirate army, the Hashashins, were paid off partly with resin [our word "assassins" is derived from these mercenaries].

The introduction of cannabis to "polite" society came when Napoleon's army brought hashish back from conquered Egypt. In 1844 *a private club, the Club de Hachichins opened in Paris, dispensing hashish in candy or mixed with wine. Its founders included* ***P. Gautier****,* ***C. Baudelaire****,* ***Dumas****, and others of the socially elite literary set. Other clubs sprang up and introduced smoking. At the same time, French medical authorities began recommending it as a calming agent for the hysterical, justifying the recommendation of Emperor Shen-Nung.*

Ergot alkaloids

For uncounted centuries, ergot has been one of the cursed scourges of mankind. It has plagued the body and mind ever since we began to use grasses for their edible seeds. In Europe it was called the holy fire (ignis sacer), St. Anthony's Fire, the ignius beatae, Virginis invisibilis, or the infernalis.

Three major cereal grains have been used to make bread. The common bread was an unleavened product made from barley, but only wheat and rye flour make a raised or leavened bread. It is likely that Thrace and Macedonia, but not Greece, grew some rye, but the plant was not introduced into much of Europe until the Christian era. France began to grow rye about 300 *CE,* *and Britain obtained her starting seed when the Teutons invaded the island. Rye was not grown as a major cereal grain in Europe and European Russia until the fifth century, and attempts to pinpoint just when historical records of ergotism began is difficult.*

Thus, an epidemic suspiciously like ergotism broke out among the Spartans in 430 *BCE, and a plague of* 857 *CE in the Rhineland also matches the clinical symptoms. A disease "like fire" was reported in Paris in* 943*, from Aquitaine-Limousa in* 994 *with* 4000 *deaths, and from Rheims in* 1041 *with* 2000 *deaths. From that time on, instances of ergotism have been recorded in sufficient detail so that we can be sure of its cause.*

Ergotism results from the ingestion of sclerotia of ergot ground up in rye flour. Two major types of ergotism are known, gangrenous and convulsive. In the former, severe constriction of the blood vessels results in swelling as blood accumulates in the hands or feet, with burning sensations alternating with intense cold. Numbness follows within a few days and this, in turn, is followed by blackening of the limb, horrible odors, and eventually merciful, but unbearably painful death.

Convulsive ergotism accurately describes the symptoms. Twitching of head, arms, and hands is followed by contractions of muscles throughout the whole body. The afflicted typically roll themselves into a ball and then stretch themselves out at full length, the actions accompanied by terrible pains. Vomiting, deafness, blindness, and hallucinations usually follow. Feats of superhuman strength, and the conviction that flying is possible have been noted. If the victim recovers, and about 30–40 *percent do, hallucinations can continue aperiodically for up to a year. Domestic animals who eat ergot-contaminated grain or table scraps exhibit identical responses. It is said that dogs will tear bark from trees until their teeth fall out and that ducks will strut like roosters, attacking people and other animals.*

Depending upon the weather, the genetic constitution of the host and the fungus, the care taken to eliminate sclerotia before milling grain into flour, and the amount of bread eaten, devastating outbreaks of ergotism could occur. And they did occur in Europe on an average of once every five to ten years.

Innumerable people had ergotism before its etiology was recognized in 1673 *by a Parisian lawyer-physician,* **Denis Dodart***. Up to the middle of the* 11^{th} *century, over* 20 *massive epidemics were reported in France alone, and by the middle of the* 14^{th} *century, over* 50 *epidemics had been reported from central Europe. Lacking any knowledge of its cause, it was reasonable to call upon the saints to intercede with heaven for succor. But which one? St. Anthony the Great was born in Egypt in the first century CE and established the idea of monastic life. Long the patron saint for erysipelas, a bacterial disease of the* *skin which causes swelling and burning, it seemed logical for him to become the intercessor for this disease as well.*

In 1039, a French nobleman, ***Gaston de la Vollaire****, built a hospital in the Rhone Valley, obtained relics of St. Anthony, and asked monks to serve in the hospital. These men formed the Order of St. Anthony and dedicated themselves to nursing the survivors of ergotism. In the* Book of Hours *by the master of Mary of Burgundy*[619]*(1480), St. Anthony is asked for protection against the disease. The Holy Fire disease was soon called St. Anthony's fire. The fantastic paintings of the Dutch painter* ***Hieronymus Bosch*** *(1450–1516) depict victims of ergotism: crawling cripples and "flyers" out of high windows.*

Although Dodart's identification of the cause of ergotism was known among the few educated physicians of the 17^{th} *century, the direct connection between* ergot *and St. Anthony's fire did not become general knowledge until the* 18^{th} *century. The dark, heavy, sour, but very nourishing bread of central and eastern Europe contained so much ground-up weed seed that the dark sclerotia went unnoticed. Since bread was truly the staff of life, the persistence of the peasants in eating ergot-contaminated bread is not surprising. When a high probability of starvation had to be weighed against possible ergotism, people made the only logical choice.*

Between 1580 *and* 1900 *there were* 65 *major ergot epidemics in Europe and the United States. In* 1722*, Peter the Great mounted an invasion of Turkey to obtain for Russia the still-coveted ice-free port to the seas. His cavalry ate ergotized black bread and* 20,000 *men and horses were stricken; the invasion was called off. Between* 1770 *and* 1780*, epidemics raged through Germany and France with over* 8000 *documented deaths. In the winter of* 1812–1813*, Napoleon's troops and horses ate bread baked from rye commandeered from the Ukraine, and the resulting epidemic of ergotism contributed to his Russian defeat and turned the retreat from Moscow into a horror. In* 1812*, Austria passed a law stating that inadequately cleaned rye would be confiscated, and other European countries quickly passed similar legislation.*

There was a severe outbreak on the Soviet Union in 1926 *and a smaller one in England in* 1928–1929 *when the Jewish community imported rye from central Europe. During the well-studied Soviet Union epidemic in* 1926*, flour*

[619] **Mary of Burgundy** (1457–1482) was the daughter of **Charles the Bold** (1433–1477) by **Isabella of Bourbon**. The marriage of Mary to the irascible Maximilian of Austria was a major event in European history. Her accidental death at the age of 25 took place while out hunting with a falcon. The *Book of Hours* was made for her by an anonymous painter.

containing 2 *percent ergot was found to be enough to cause convulsive ergotism and some samples of rye flour contained up to 7 percent sclerotia. It has been claimed that convulsive and hallucinogenic ergotism struck a town in Provence in* 1951, *but the French government denied this, stating that there was an inadvertent contamination of the flour by an insecticide; this bureaucratic explanation does not conform to the symptoms noted.*

Lysergic acid diethylamide (LSD) *is a product of ergot, a parasitic black fungus that grows on rye. Because it acts on the central nervous system and has unpredictable effects,* LSD *is sometimes referred to as a "mind-bending" drug. Many users have experienced visual and perceptual distortions, strange sensations, and difficulty in distinguishing between illusion and reality. Occasionally some users have experienced acute terror and other unpleasant psychological effects.*

LSD *is one of the most potent drugs known to man. Twenty micrograms (1 microgram is one millionth of a gram) will cause physiological changes. Some alleged* LSD *tablets have been known to contain up to one thousand micrograms of the drug. In addition other substances have been mixed with* LSD, *such as arsenic, strychnine, and atropine. These substances often add to the many bad* LSD *"trips".*

LSD *is believed to be structurally similar to serotonin, which is a compound found in the brain tissue that may play an important role in thinking processes. There is some evidence that* LSD *either replaces or blocks serotonin activity in the brain, which may help explain the variety of eccentric and bizarre symptoms some users experience. A compound called* DMPEA, *which is similar to mescaline, has been found in the urine of* 65 *percent of schizophrenic mental patients and* LSD *users, which suggests an explanation for the bizarre behavior of some* LSD *users.*

Researchers are now experimenting with one possible medical use for LSD. *They are trying to determine whether it can be used to make the last weeks, or even months, more tolerable for terminal cancer patients. They believe that under controlled conditions,* LSD *could lessen the harsh reality of impending death and make possible reduced dosages of analgesic drugs.*

V. *Reserpine* has been used in India for a long time under the name CHANDRA *(moon)* TO TREAT "LUNATIC" PEOPLE.

It was also known as an effective reducer of fever, sedative and a curer of dysentery. Since 1940 *it is used in Western medicine to reduce blood pressure. Today it is used successfully to treat and control nervous disorders such as* *schizophrenia and for calming psychotic patients so that they can undertake psychotherapy.*

VI. *Psilocybin (sometimes called the magic mushroom)* IS A WILD FUNGUS DERIVATIVE, WHICH HAS ALSO BEEN USED IN RELIGIOUS RITES.

Aztec, Inca and Mayan priests, since 1000 *BCE, used amanita under the name teonanacatl ("flesh of the gods"). The same drug, under the name soma was introduced by the Aryan people that entered India from the north in ca* 1500 *BCE. The Spaniards brought the drug to North America and it then became known to the American pioneers.*

Early Europeans recognized that fly agaric, a fungus parasite of the amanita mushroom, could also act as a potent insecticide. They boiled these mushrooms in milk and placed saucers of the mixture on their windowsills and in the doorways of their homes and marketplaces. The flies and other insects that spread disease from home to home and from village to village ingested this lethal mixture and died. Unfortunately children, dogs, and cats were also attracted to it and poisoned.

Those addicted to the mushroom cult experience visions, muscular relaxation, hilarity, alteration in perceptions of time, feeling of total isolation from one's environment. Under its influence, priests would chant the "truth" about health, disease, success or failure, and how to remedy the affairs of everyday life. For the Indians of Central America, Psilocybe experience awakened the forces of creation.

Psychoactive components of hallucinogenic mushrooms are related to those found at the junction of nerve cells in the body and the brain. If too much is taken, death by respiratory failure may occur.

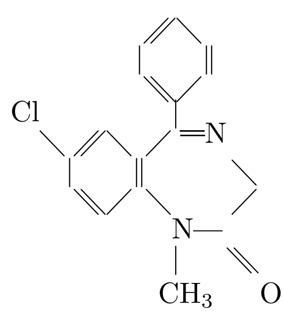
VII. *Digitalis is a mixture of several naturally occurring cardiac glycosides synthesized by* Digitalis purpurea *and related species in the figwort (Scrophulariaceae) family. Native to Europe, Western Asia, and Central Asia, it is grown all over the world.*

The Latin generic name, Digitalis, *means "little finger" from the Latin* digitis *and is directly derived from the German name for the plant, fingerhut. The Latin term was applied by* ***Hieronymus Tragus*** *in* 1539 *and was repeated in* ***Leonhard Fuchs****'s* De Historia Stirpium *in* 1542. *Several species* *are native to Europe, and all bear the common name foxglove or variants on the same theme.*

Digitalis was a medicinal herb for centuries; ***Dioscorides*** *praised it as a plant whose leaves, applied to the skin, could cure many diseases. Juice pressed from the leaves became an ingredient of salves applied to cuts, bruises, and the leg ulcers common in an era of inadequate diets and lack of soap. Rural people made hot water infusions of leaves and drank foxglove tea to experience an inexpensive but dangerous intoxication.*

Among the ills to which flesh is heir is cardiac insufficiency in which a weakened heart fails to pump enough blood through the body. Heartbeat is irregular and fluids collect in the arms, legs, and abdomen because the kidneys cannot perform their normal function. The swelling is known as dropsy or, more formally, as edema. This disease syndrome is not new. Ancient physicians knew of it, but lacking knowledge of the circulation of the blood discovered by ***William Harvey*** *in* 1628 *and information on the function of the kidneys, treatment was limited to usually unsuccessful attempts to reduce edema with medicines which increased urine production (diuretic agents). Today, millions of people pop a small pill which regulates and strengthens the heartbeat and allows the kidneys to expel excess fluid quickly; cardiac insufficiency kills few people since the discovery of digitalis.*

VIII. MINOR TRANQUILIZERS SUCH AS DIAZEPAM *(Valium) are used to alleviate anxiety tensions; they work as skeletal-muscle relaxants and control muscle spasms. Other tranquilizers, such as meprobamate (Miltown), have an antiemetic action that is useful in the treatment of nausea and vomiting, or "morning sickness", of early pregnancy.*



Diazepam (Valium)

IX. *Curare*, A SUBSTANCE OBTAINED FROM A NATIVE SHRUB IN THE SOUTH AMERICAN AMAZON REGION, *was used for centuries by Indians as a weapon in their hunt for small game, such as monkeys. Blowgun darts,* *containing curare-tipped arrows, fatally paralyzed small animals and caused their respiratory muscles to stop functioning. In modern surgery, curare is often used when the complete relaxation of the abdominal muscles is required.*

X. *Colchicin* and *acunitin are* AMONG THE MOST POTENT AND POISONOUS ALKALOIDS*: few milligrams of pure substance can cause a human's death.*

Alkaloids are at the junction of four major sciences — chemistry, botany, physiology and medicine.

In spite of a great deal intensive interdisciplinary research, the physiological mode of action of alkaloids in the animal body is poorly understood. Some, like the caffeine alkaloids in tea and coffee, are stimulants. Others, like the alkaloids in ergot, cause constriction of smooth muscle, and still others, like those in the opium poppy, are powerful pair-killers.

It is likely that all operate on or in some part of the central nervous system and that the responses reflect alterations in control over cellular function by the brain and peripheral nerve network.

The physiology, pharmacology, and psychology of addiction to alkaloids like morphine and heroin is even less well understood. Certainly, not all alkaloids are addictive or even habit-forming. One can get along without a morning cup of coffee without experiencing withdrawal symptoms. Even for those which are addictive, the nature of the addiction and its consequences are poorly understood, and the same can be said for the response to withdrawal from the substance. There are certainly psychological factors as well as biochemical and physiological factors which must be evaluated.

1886–1897 CE **Gustave Victor Robin** (1855–1897, France). Mathematician. Made significant contributions to potential theory (1886) and thermodynamics. Named after him are:

- A third boundary condition of partial differential equations. (*Robin's boundary condition*; the corresponding Green's function is known as *Robin's kernel*.

- The logarithmic *capacity* of a compact set E (*Robin's Constant*). This is related to his original solution to a problem in electrostatics (*Robin's Problem*), where he established a remarkable connection between potential theory and the capacity concept in point-set topology. From it developed later (**N. Wiener**, 1924) the concept of *capacity* in point-set topology.

- A method for evaluating a single-layered charge distribution over a closed bounded surface of a conductor. This leads to *Robin's integral equation* for the charge density which is solved by successive approximation (*Robin's potentials*; *Robin's Principle*, *Robin's function*).

Robin was a professor of mathematical physics at the Sorbonne in Paris. His idiosyncrasies and early death left him almost unremembered and he died in obscurity. His collected works were published posthumously (1899–1903) by his friend and colleague **Louis Raffy**.

1886–1904 CE **Giuseppe Peano** (1858–1932, Italy). Mathematician, linguist and logician. One of the founders of symbolic logic. Endeavored to develop a formalized language which could be used in mathematical logic and mathematics in its entirety. His major work in this field is '*Formulaire de mathémathiques*' (1894–1908) which he wrote with his students and colleagues at the University of Turin. This work was intended to flow from its fundamental postulates using his logic notation. Parts of his method and notation were accepted in the mathematical world and profoundly changed the outlook of mathematicians.

Following the work of **Dedekind**, he established in 1899 a system of axioms for natural numbers that bears his name. In topology, he discovered a continuous function whose points completely fill the unit square (*Peano's curve*).

He made important contributions to the theory of ordinary differential equations. Peano presented an abstract form of the theory of vectors based on Grassmann's calculus of extensions, and introduced the concept of 'Riemann content'.

He created an artificial international language, later called '*Interlingua*'. It is based upon a synthesis of vocabulary from Latin, French, German and English, with a greatly simplified grammar.

Peano was born in Sardinia. He became a professor at the University of Turin in 1890 and was also a professor at the Military Academy in Turin.

1886–1906 CE **Ferdinand-Frederic-Henri Moissan** (1852–1907, France). Inorganic chemist. First to isolate the element *fluorine* (F) in 1886. Introduced an improved arc furnace for metallurgy (1892). Discovered silicon carbide (1893). Was awarded the Nobel Prize for chemistry (1906).

Fluorine, the lightest of the halogens, is the most reactive of all the elements, and it forms compounds with all the elements except the lighter inert gases. Because its *electronegativity*[620](4) is greater than any other element, it cannot be prepared by reaction of any other element with a fluoride. It was by the *electrolysis* of a solution of KF in liquid HF that fluorine was first obtained by Moissan.[621]

Moissan was born in Paris to Jewish parents and studied at the laboratory of the Natural History Museum. Professor, Ecole de Pharmacie, Paris (1886–1900), Sorbonne (1900). Fluorine's poisonous nature is believed to contribute to his early death at the age of 54.

1886–1908 CE **Elie Metchnikov** (1845–1916, Russia). Immunologist. Pioneer 'microbe hunter'. Hypothesized the role of *phagocytes* in vertebrate blood to fight invasion of bacteria. Was awarded the Nobel Prize for physiology or Medicine (1908) jointly with **Ehrlich**.

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[620] A measure of the relative tendency of an atom to attract electrons to form *anions*. Elements with low electronegativities (metals) often loose electrons to form cations. Oxygen is the second most electronegative element (3.5). Then come chlorine (3.0), Nitrogen (3.0), Bromine (2.8), Iodine (2.5), Sulfur (2.5), Carbon (2.5).

Fluorine occurs in large quantities in the minerals *fluorspar* (CaF_2); *cryolite* (Na_2AlF_6); and *fluoroapatite* [$Ca_5(PO_4)_3$]. It also occurs in small amounts in sea water, teeth, bones, and blood. Fluorinated organic compounds, called *fluorocarbons* are stable and nonflammable. They are used as refrigerants, lubricants, plastics (such as Teflon), insecticides, and aerosol propellants. Stannous fluoride, SnF_2, is used as toothpaste.

[621] Only in 1986 was Moissan's method of fluorine production superseded by the discovery of **Carl O. Christe** that F_2 can be obtained in better than 40% yield by the reaction

$$2K_2MnF_6 \text{ (solid) } + 4SbF_5 \text{ (liquid)} \\ \rightarrow 4KSbF_6 \text{ (solid) } + 2MnF_3 \text{ (solid) } + F_2 \text{ (gas)}.$$

Born in Ivanovka, the Ukraine to Jewish parents. Graduated from the University of Kharkov (1864) and received a doctorate from the university of St. Petersburg (1867). Left Russia (1887) to work with **Pasteur**, who offered him the directorship of a laboratory at the Pasteur Institute in Paris.

1887 CE, September to October Yellow River (Huang-ho) in Honan province, China, overflowed, submerging 130,000 km^2 of land and killing about a million people. Flooding was caused by rain.

1887–1890 CE **Augustin** (**Louis Aimé August**) **Le Prince** (1841–1890, France, England and U.S.A.). Engineer, artist and inventor. Constructed the first *moving-picture machine* (camera and projector), predating Edison's claim, and made short moving pictures in Leeds in 1888. He was not the first to have the idea but the first to succeed.

Le Prince was born in Metz, France. His father was a major in the service of Louis-Philippe. He studied chemistry and optics at Leipzig and was trained as a painter. In 1866 he came to Leeds, England to work for a firm that manufactured components for the local locomotive industry.

He first became interested in moving pictures in 1869 under the inspiration of **Muybridge** photography and **Houdin**'s 'magic lanterns' in Paris, but started to realize his ideas upon his immigration to New York in 1882. His serious experiments began in 1885. In 1887 he returned to Leeds to avoid New York's industrial spies and to take advantage of his father in law's offer of support. At first he developed a camera with 16 lenses. The 16 shutters were operated by electromagnets and armatures controlled by a circuit closer. The lenses were made to converge on a single point.

He later developed a single-lens camera but did not have the advantage of celluloid; the paper film from the camera had to be developed into a negative. The negative frames were then stripped from their paper backings and positive transparencies were made of them. These were very flimsy and needed a stronger transparent backing to survive the heat and jerking of the projector, through which they were transported at a rate of 16 frames per second. He used gelatin or glass, but non of these could roll without cracking and only glass was transparent enough. Le Prince had to mount each individual frame onto a specially designed picture-belt which made it all very heavy.

In 1889 he finally got hold of synthetic celluloid which came in coated sheets at a foot square but not in long rolls. He had to make his own. When operating his single-lens camera, the sensitive paper film was intermittently activated at the rear of the lens by providing it with a properly timed intermittently operated shutter.

Le Prince died under mysterious circumstances: he disappeared on Sept. 16, 1890 during a train trip from Dijon to Paris. He never arrived to Paris and was never seen again. No trace of him was ever found. Documents, discovered recently in the city archives of Leeds, point to the possibility that he engineered his own disappearance. Financial and technical difficulties were probably the cause. His failure to patent his single-lens camera in sufficient detail was a fatal oversight and caused his family to loose their legal claim against Edison in 1901.

In 1930 a plaque commemorating Le Prince's pioneering invention was unveiled in the city of Leeds. A second plaque in that city was unveiled in 1988, commemorating the centennial of his great achievement — the first moving picture ever, taken on Leeds bridge on Oct. 14, 1888.

1887–1893 CE **Paul Tannery** (1843–1904, France). The first modern historian of science. He wrote: *Pour l'histoire de la science hellene* (1887), *La géometrie grecque* (1887) and *Recherches sur l'histoire de l'astronomie ancienne* (1893).

Tannery was born at Mantes-la-Jolié and died at Pantin (both localities near Paris). He entered the École Polytechnique (1860), and graduated among the ranking members of his class. For the next 40 years he was in the service of the state monopoly of tobacco, but his evenings and holidays were devoted to the study of the history of science.

It was only in relatively recent times that the importance and centrality of the history of science was realized. There were a few pioneers beginning with the end of the 17th century.

Such men were: **Albrecht von Haller** (1708–1777); **Joseph Priestley** (1733–1804); **Adam Smith** (1723–1790); **Jean Etienne Montucla** (1725–1799) and **Jean Sylvain Bailly** (1735–1793).

But the first man to introduce this theme in a broader context and to increase its circulation was the French philosopher, **Auguste Comte**, who developed it in his *Cours de philosophie positive* (1830–1842). His views were discussed by another French philosopher, **Antoine Augustin Cournot**, in 1861, but the real inheritor of Comte's thought and the first great teacher of the history of science was **Paul Tannery**.

Tannery's philosophy is very different from Comte's, but the greatest difference between them is that Comte's knowledge of the history of science was very superficial, whereas Paul Tannery, being extremely learned and having at his disposal a mass of historical research work which did not exist in the thirties, knew more of the history of science than anybody else in the world. Certainly no man ever was better prepared to write a complete history of science, at least of European science, than Paul Tannery. It was his dream to carry out this great work, but unfortunately he died, before realizing his ambition. During the 20th century his example has been followed by many scholars, notably **George Sarton**.

1887–1898 CE **Woldemar Voigt** (1850–1919, Germany). Mathematician. First to write down, in 1887, a mathematical transformation which leaves the scalar wave-equation, and consequently Maxwell's equations, invariant (later known as the *Lorentz transformation*). The next pre-relativistic mention of this transformations was given by **Hendrik Antoon Lorentz** (1853–1928, Holland) in 1895, and then in 1898 by **Joseph Larmor** (1857–1942, England).

Voigt established the stress-strain relation in a *viscoelastic solid* in which the stress is related to a linear combination of the strain and the rate of strain, known as a Kelvin-Voigt substance[622] (1892). In 1898 he reinstated Hamilton's term '*tensor*' as the entity representing the local state of stress in an elastic continuum.

Voigt was a tall, thin man with a red beard. He was a truly quiet scholar. His lectures were like his book on crystals — very hard to understand, but deep and knowledgeable. He drew beautiful sketches on the blackboard, polishing and correcting them for five or six minutes. He talked in short, concise sentences, never looking at his audience. He had reputation for calculating incredibly and magnificently.

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[622] It is a generalized *Hooke's law* is 3 dimensions:

$$\mathfrak{T} = \overset{4}{C} : \mathfrak{E} + \overset{4}{D} : \frac{\partial \mathfrak{E}}{\partial t},$$

where $\mathfrak{E}$ is the strain tensor and $\{\overset{4}{C}, \overset{4}{D}\}$ are two fourth order tensors. In isotropic homogeneous materials, the above stress-tensor takes the simplified form:

$$\mathfrak{T}(\boldsymbol{r}, t) = \left(\lambda + \lambda' \frac{\partial}{\partial t}\right) \mathfrak{I} \operatorname{div} \boldsymbol{u} + 2\left(\mu + \mu' \frac{\partial}{\partial t}\right) \mathfrak{E}.$$

For $\lambda' = \mu' = 0$ we fall back on the *elastic solid*, while in the limit

$$\lambda \operatorname{div} \boldsymbol{u} = -p, \quad \mu = 0, \quad \mu' = \eta, \quad \lambda' = \overline{\lambda} - \frac{2}{3}\eta$$

($\overline{\lambda}$, η — viscosity coefficients), we fall back on the *Newtonian fluid*.

Likewise, the limiting case $\lambda' = \mu' = 0, \quad \lambda = \mu \to \infty$ leads us back to a *rigid body*, and the limit $\lambda \operatorname{div} \boldsymbol{u} = -p, \quad \mu = \mu' = \lambda' = 0$ renders the *ideal fluid*.

1887–1889 CE **Lois-Gustave Binger** (1856–1936, France). West-African explorer. The first European to cross the watershed of the *Volta River*. Traveled widely in the Ivory Coast, Senegal, and (todays) Upper Volta, Ghana, Guinea and Gambia.

Binger was born to a Jewish family in Strasbourg and became governor of Ivory Coast (1893).

1887–1901 CE **Ernesto Cesàro** (1859–1906, Italy). Mathematician. Contributed mainly to differential geometry, summability of divergent series (1890) and the theory of numbers. Formulated '*Intrinsic geometry*' [*Lezioni di geometria intrinseca*, 1896], in which he derived coordinate-free ("natural") equation of curves by means of the arc length and curvature variables. This goes back to **Euler** (1736) who used it for special curves.

Cesàro was born in Naples and continued his studies at Liege (Belgium) and Paris under **Hermite** and **Darboux**. He received his doctorate from the University of Rome (1887). He held the chair of mathematics at Palermo until 1891, moving then to Rome, where he held the chair until his death.

1887–1907 CE **Vito Volterra** (1860–1940, Italy). One of the top mathematical physicists of his time. Made major contributions in the general theory of functionals[623] (1887–1889), partial differential equations, integral equations, integro-differential equations, theory of dislocations (1907), mathematical biology (1920) and other topics in mathematical physics. His work had strong influence on the general development of modern calculus.

Volterra contributed to the solution of linear equations in multi- or infinite-dimensional linear spaces by means of his multiplicative integral. Developing the general theory of the functional calculus, Volterra invented a way to reduce calculations with functionals to calculations with usual functions with many variables.[624]

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[623] For further reading, see:

- Volterra, V., *The Theory of Functionals*, Blackie and Sons, 1930, 225 pp.

[624] In this procedure one has to divide the interval from the initial time t_{in} to the final time t_{fin} into a large, but finite number N of time instants t_i, and then to approximate the functional $F[x(t)]$ with the functions $F(\dots, x_{i+1}, x_i, \dots)$, where x_i gives the value of $x(t_i)$. Then, one has to work with this function, instead of the functional $F[x(t)]$.

This process is called *finite-dimensional approximation* or discretization, of the functional $F[x(t)]$, which itself may be considered as a function of infinitely many variables $x(t)$, with a continuous label t. After performing operations on the function $F(\dots, x_{i+1}, x_i, \dots)$ in the final result one has to take the limit

Volterra was born in Ancona, Italy, to a poor Jewish family. His interest in mathematics started at the age of eleven. At the age of 13 he began to study

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$N \to \infty$, keeping t_{in} and t_{fin} fixed.

R.P. Feynman (1942) utilized Volterra's theory of functionals in his new calculations of averages of *quantum mechanical quantities.* His formulas give us the expectations of certain functionals on the paths $x(t)$ in the configuration space of the classical mechanical system, where the time t runs from some initial time t_{in} to some final time t_{fin}. As a weight in the averaging procedure, one uses the complex phase with argument equalling classical action divided by $\hbar$.

Volterra introduced, for the first time, the beautiful mathematical idea of a *functional derivative* (1887), which was developed further by **Gateaux** (1919) and **Fréchet** (1925) in the framework of functional analysis.

The functional $F[x(t)]$ gives a *number* for each *function* $x(t)$ that we may choose.Volterra asked: How much does this number change if we make a very small change in the argument function $x(t)$? Thus, for a small $\eta(t)$, how much is $\delta F \equiv F[x(t)+\eta(t)] - F[x(t)]$?

To evaluate this, suppose time is divided into very many steps of small intervals ϵ, the values of the time being t_i where $t_{i+1} = t_i + \epsilon$. The function $x(t)$ can then be approximately specified by giving the values x_i that it takes on at each of the times t_i, namely $x_i = x(t_i)$. The functional $F[x(t)]$ is now a number depending on *all* the x_i, that is, it becomes an ordinary function of the variables x_i,

$$F[x(t)] \qquad \to \qquad F(\dots, x_i, x_{i+1}, \dots).$$

If we alter the path from $x(t)$ to $x(t)+\eta(t)$, we change each x_i to $x_i + \eta_i$, where $\eta_i = \eta(t_i)$. Then, the first-order change in our *multivariable function* is

$$dF \equiv F(\dots, x_i+\eta_i, x_{i+1}+\eta_{i+1}, \dots) - F(\dots, x_i, x_{i+1}, \dots) = \sum \frac{\partial F}{\partial x_i}\eta_i,$$

according to the ordinary rules of partial differentiation.

In the limit $\epsilon \to 0$ (assuming it exists, etc.)

$$dF \to \delta F; \qquad \sum \frac{\partial F}{\partial x_i}\eta_i \to \int \frac{\delta F}{\delta x(s)}\eta(s)\,ds,$$

where $\delta x(s)$ is the differential change in path at $x(s)$, and the functional derivative is taken at the point $t_i = s$. Thus, one can show, for example, that if $S = \int_{t_1}^{t_2} L(\dot{x}, x, s)\,ds$, then for any s inside the range t_1 to t_2

$$\frac{\delta S}{\delta x(s)} = -\frac{d}{ds}\left(\frac{\partial L}{\partial \dot{x}}\right) + \frac{\partial L}{\partial x},$$

where the partial derivatives are evaluated at $t = s$.

the Three Body Problem and made some progress. He attended the University of Pisa (1878–1882) and was appointed professor of rational mechanics there (1883). During that time he began to develop the theory of functionals, which he applied to the solutions of integral and integro-differential equations. The important idea of *harmonic integrals* derives essentially from his functional analysis.

In 1892 Volterra became professor of mechanics at the University of Turin, and from 1900 onward he occupied the chair of mathematical physics at the University of Rome.

In 1905 he became a senator of the Kingdom of Italy. In WWI he joined the Italian Air Force and was first to propose the use of Helium in airships. In 1922 Fascism seized Italy and Volterra fought against it in the Italian Parliament. However by 1930 the Parliament was abolished and Volterra refused to take an oath of allegiance to the Fascist Government. As a Jew in Fascist Italy, he was forced to leave the University of Rome (1931) and resign from all Italian scientific academies. He died in Rome during WWII.

History of Integral Equations[625]

An integral equation is an equation in which an unknown function appears under an integral sign and the problem of solving the equation is to

[625] For further reading, see:

- Polyanin, A. D. and A. V. Manzhirov, *Handbook of Integral Equations*, CRC Press: New York, 1998, 787 pp.
- Hamel, G., *Integralgleichungen*, Springer-Verlag: Berlin, 1949, 166 pp.
- Kanwal, R. P., *Linear Integral Equations*, Academic Press, 1971, 296 pp.
- Moiseiwitsch, B. L., *Integral Equations*, Longman: London, 1977, 161 pp.
- Chambers, Ll. G., *Integral Equations*, 1976, 198 pp.
- Tricomi , F. G., *Integral Equations*, Dover: New York, 1985, 238 pp.
- Kondo, J., *Integral Equations*, Oxford University Press, 1991, 440 pp.

*determine that function. The term 'integral equation' was coined by **Du Bois-Reymond** (1888).*

*At first, solving integral equations was described as inverting integrals. Long before the subject acquired a distinct status and methodology, **Laplace** (1782) considered the integral equation for $g(t)$ given by $f(x) = \int_{-\infty}^{\infty} e^{-xt} g(t)\,dt$, now called the Laplace transform of $g(t)$.*

***Poisson** (1811) discovered its solution, namely,*

$$g(t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{xt} f(x)\,dx$$

for large enough a.

*Another result stems from **Fourier's** (1811) paper on the theory of heat conduction*

$$f(x) = \int_0^{\infty} u(t)\cos(xt)\,dt$$

and the inversion formula

$$u(t) = \frac{2}{\pi}\int_0^{\infty} f(x)\cos(xt)\,dx$$

*The first conscious direct use and solution of an integral equation go back to **Abel** (1823). He considered a mechanical problem which led him to the equation*

$$f(x) = \int_a^x \frac{u(\xi)d\xi}{(x-\xi)^{\lambda}} \qquad 0 < \lambda < 1$$

He then found the solution

$$u(z) = \frac{\sin(\lambda\pi)}{\pi}\frac{d}{dz}\int_a^z \frac{f(x)\,dx}{(z-x)^{1-\lambda}}$$

***Liouville** (1832) showed that the solution of the* differential equation

$$y'' + [p^2 - \sigma(x)]y = 0 \quad \text{or} \quad y'' + p^2 y = \sigma(x)y$$

$$a \le x \le b, \qquad y(a) = 1, \qquad y'(a) = 0 \qquad p = \text{ parameter}$$

is also the solution of the integral equation

$$y(x) = \cos p(x-a) + \frac{1}{p}\int_a^x \sigma(\xi)y(\xi)\sin p(x-\xi)\,d\xi.$$

The conversion of differential equations to integral equations became a major technique for solving initial and boundary-value problems of ODE and PDE, and this was the strongest impetus for the study of integral equations.

Volterra *(1896) is the first founder of a general theory of integral equations. He set out to solve*

$$f(s) = \Phi(s) + \int_a^b K(s,t)\Phi(t)\,dt$$

for $\Phi(s)$, *where* $f(s)$ *is known and* $K(s,t) = 0 \quad$ *for* $t > s$. *His solution can be written in the form*

$$\Phi(s) = f(s) + \int_a^b \overline{K}(s,t) f(t)\,dt$$

where

$$\overline{K}(s,t) = -K(s,t) + \int_a^b K(s,\tau)K(\tau,t)\,d\tau \\ - \int_a^b\int_a^b K(s,\tau)K(\tau,\omega)K(\omega,t)\,d\tau d\omega + \dots.$$

Volterra also observed that the integral equation

$$f(s) = \int_a^b K(x,s)\Phi(x)\,dx$$

is a limiting form of a system of n *linear equations in* n *unknowns as* n *becomes infinite.* **Fredholm** *(1900–3) used this idea to solve*

$$u(x) = f(x) + \lambda\int_a^b K(x,\xi)u(\xi)\,d\xi.$$

Dividing the x*-interval* $[a,b]$ *into* n *equal parts* $(x_1, x_2, \dots, x_n)$, *he presented his solution in the form*

$$u(x,\lambda) = f(x) + \int_a^b \frac{D(x,y,\lambda)}{D(\lambda)} f(y)\,dy, \qquad D(\lambda) \neq 0, \qquad \text{where}$$

$$D(\lambda) = 1 - \lambda\int_a^b K(\xi_1,\xi_1)\,d\xi_1 + \frac{\lambda^2}{2!}\int_a^b\int_a^b \begin{vmatrix} K(\xi_1,\xi_1) & K(\xi_1,\xi_2) \\ K(\xi_2,\xi_1) & K(\xi_2,\xi_2) \end{vmatrix} d\xi_1 d\xi_2 + \dots$$

$$D(x,y,\lambda) = \lambda K(x,y) - \lambda^2\int_a^b \begin{vmatrix} K(x,y) & K(x,\xi_1) \\ K(\xi_1,y) & K(\xi_1,\xi_1) \end{vmatrix} d\xi_1 \\ + \frac{\lambda^3}{2}\int_a^b\int_a^b \begin{vmatrix} K(x,y) & K(x,\xi_1) & K(x,\xi_2) \\ K(\xi_1,y) & K(\xi_1,\xi_1) & K(\xi_1,\xi_2) \\ K(\xi_2,y) & K(\xi_2,\xi_1) & K(\xi_2,\xi_2) \end{vmatrix} d\xi_1 d\xi_2 - \dots$$

***Hilbert** (1904–1912) completed Fredholm's solution by carrying out the limiting process for the infinite number of algebraic equations, dispensing with Fredholm's infinite determinants. He then applied his research to a variety of problems in geometry and physics. Hilbert's work was simplified by **Schmidt** (1907), completed by **Fischer** (1907) and **Riesz** (1907), and extended to nonlinear integral equations. Moreover, the theory was extended to non-continuous functions $f(x)$ and $K(x, \xi)$ and to infinite limits of integration (singular integral equations) by **Weyl** (1908).*

1887–1907 CE **Emil Hermann Fischer** (1852–1919, Germany). Distinguished organic chemist. Analyzed the structure of sugars (1887). First to promote the idea of an encoding of genetic specificity in a spatial arrangements of subunits (1907): proposed the theory of 'lock and key' to explain stereo-specific interaction of enzyme with substrate. Synthesized *polypeptide* (1907), a small protein consisting of 18 amino acids, and showed that it could be broken by digestive juices just as natural proteins are.

His studies on the structures of *purines* (1882–1901) and *polypeptides* (1900–1906), opened the way for an understanding of *nitrogen metabolism*, which was essential before the biochemistry of these substances could be developed.

Fischer was born in Euskirchen, Rhenish Prussia. Studied at Bonn and Strasbourg. Professor at Wirzburg (1885) and Berlin (1892). Awarded the Nobel prize for chemistry (1902).

By the turn of the century, with a dozen amino acids isolated from proteins[626], the time was ripe to try to reverse the process and to form a protein out of amino acids: Fischer, using the technique of organic chemistry as developed over the previous half-century, painstakingly treated amino acid mixtures under such conditions as would encourage combination. By 1907 he had managed to build up a molecule made up of 18 amino acid units, consisting of 15 *glycines* and 3 *leucines*[627].

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[626] The major units of the protein molecule were discovered in the following order: *Glycine* (1820), *Leucine* (1820), *Tyrosine* (1849), *Serine* (1865), *Glutamic Acid* (1866), *Asparic Acid* (1868), *Phenylalanine* (1881), *Alanine* (1888), *Lysine* (1889), *Arginine* (1895), *Histidine* (1896), *Cystine* (1899).

[627] Such relatively small strings of amino acids are called *peptides* (Greek for "digestion") because they are produced in the process of digestion.

In doing this, Fischer demonstrated unequivocally that amino acids are indeed the building blocks of proteins, for his 18-unit product had all the properties that might be expected of a very simple protein molecule.

He was further able to show that, in forming a chain, the carboxyl group of one amino acid combines with the amino group of another via a *peptide bond*[628].

1887–1910 CE **Gaston Jean Darboux** (1842–1917, France). Notable mathematician. Made important contributions to analysis and differential geometry: Founded the general theory of algebraic manifolds and coined the phrase '*repermobile*' (today's '*soldering form*', known in GTR as '*tetrad*' or '*vierbein*'). Applied infinitesimal calculus to geometry in the study of the differential geometry of curves and surfaces.

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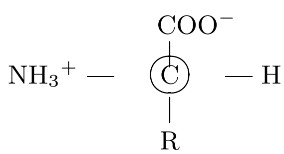
[628] The general structure of amino acids is



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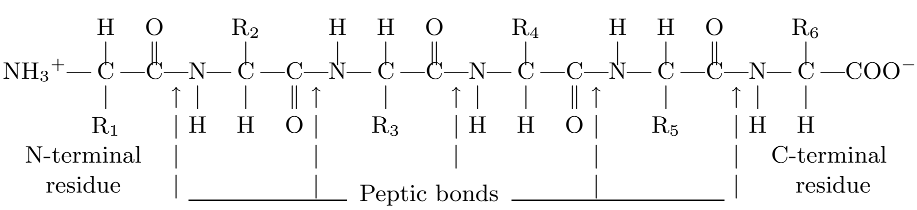
It involves an *amino group* (H_3N^+) and a *carboxyl group* (COO^-), both of which are bonded to the α-carbon Ⓒ, which is also bonded to a hydrogen and to the *side chain group* (R).

The R group determines the identity of the particular amino acid. The two-dimensional formula shown here can only partially convey the common structure of amino acids, because one of the most important properties of these compounds is their 3-dimensional shape (*Stereochemistry*).

Individual amino acids can be linked together by formation of *covalent bonds*. The bond is between the α-carboxyl group of one amino acid and the α-amino group of the next one. Water is eliminated in the process, and the linked amino acid *residues* remain after the elimination of water: e.g.



In a protein, many amino acids (usually more than a hundred) are linked by peptide bonds to form *polypeptide chain*. The peptide bond can be written as a *resonance hybrid* of a *single* bond between carbon and nitrogen and another with a *double* bond between the carbon and nitrogen. The peptide bond is stronger than an ordinary single bond because of this resonance stabilization.

He solved Euler's equations for the rotation of a rigid body about a point, relative to a fixed inertial frame, when the angular velocity vector is known in the rotating frame.

Darboux left his mark on several fields of pure and applied mathematics: We have *Darboux surfaces*, *Darboux vector*, *Darboux theorem*[629] and *Darboux integral* in the infinitesimal calculus, the *Darboux transformation* in the theory of linear differential equations and the *Darboux equation* in modern gas dynamics[630].

In his many papers and books he combined geometrical intuition with mastery of algebra and analysis. His treatises "*Lessons on the General Theory of Surfaces and the Geometrical Applications of Infinitesimal Calculus*" and "*Lessons on Orthogonal Systems and Curvilinear Coordinates*" (originally in French) are a vast source of information, and among the best written mathematical books of the 19^{th} century.

Darboux was born in Nimes, France. He was a professor of mathematics at the Sorbonne during 1873–1890.

1887–1896 CE **Gregorio Ricci-Curbastro** (1853–1925, Italy). Outstanding mathematician. Distilled and perfected the tensor calculus as an independent discipline. He was instrumental in bringing to fruition the ideas of **Christoffel**, **Beltrami** and **Lipschitz**. In his studies of surfaces, Ricci encountered several interesting metric attributes of hyperspaces. One of them was the *Ricci tensor.*

This new invariant symbolism, originally constructed to deal with the transformation theory of partial differential equations and quadratic differential forms, turned into what he now called the *theory of tensors.* He elaborated on the theory and worked out an elegant and comprehensive notation. With the aid of his pupil **Tullio Levi-Civita** (1873–1941) he showed that tensors could provide a unification of many invariant symbolisms, and deal with a wide variety of problems in analysis, geometry and the physical disciplines of elasticity, hydrodynamics, electromagnetism and relativity.

Thus, the mathematical machinery demanded by the theory of general relativity was available a year after the Michelson-Morley experiment, which was partly responsible for the special theory of relativity in 1905. Without the tensor calculus, the general relativity theory of 1915–1916 would have been

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629 If $f(x)$ is differentiable for $a \le x \le b$, $f'(a) = \alpha$, $f'(b) = \beta$, and γ lies between α and β, then there is a ξ between a and b for which $f'(\xi) = \gamma$.

630 $(x+y)\frac{\partial^2\Phi}{\partial x\partial y} + k\left(\frac{\partial\Phi}{\partial x} + \frac{\partial\Phi}{\partial y}\right) = f(x,y), \quad (k > 1).$

impossible. With later modifications and generalizations, tensor methods quickly induced a vast development of modern differential geometry.

Ricci was a professor at the University of Padua during 1880–1925.

From Vectors to Tensors[631]*; the Principle of Covariance*

Vector analysis was born in the middle of the 19^{th} *century in the minds of* ***W.R. Hamilton*** *(1844) and* ***H. Grassmann*** *(1844). The 'pregnancy' of this idea lasted for about 22 centuries, since its 'conception' during the era of Greek science.* ***Aristotle*** *(ca 350 BCE) was aware of the parallelogram of composition of forces.* ***Stevin*** *(ca 1583) employed the same principle in his studies of static mechanics and* ***Galileo*** *(ca 1583) recognized the concepts of*

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631 For further reading, see:

- Sokolnikoff, I. S., *Tensor Analysis* (Theory and Applications to Geometry and Mechanics of Continua), John Wiley & Sons: New York, 1964, 361 pp.
- Lass, H., *Vector and Tensor Analysis*, McGraw-Hill Book Company: New York, 1950, 347 pp.
- Crowe, M. J., *A History of Vector Analysis*: *Evolution of the Idea of a Vectorial System*, University of Notre Dame Press: South Bend, 1967, 270 pp.
- Schwartz, M., S. Green, and A. W. Ruthledge, *Vector Analysis*, Harper and Brothers: New York, 1960.
- Brand, L., *Vector and Tensor Analysis*, John Wiley & Sons: New York, 1948, 439 pp.
- Marsden, J. E., and A. J. Tromba, *Vector Calculus*, W. H. Freeman and Company: New York, 1988, 655 pp.
- Danielson, D. A., *Vectors and Tensors in Engineering and Physics*, Addison-Wesley: Redwood City CA, 1992, 280 pp.
- Lovelock, D., and H. Rund, *Tensors, Differential Forms and Variational Principles*, John Wiley & Sons: New York, 1975, 364 pp.

parallelogram of forces and velocities. However, no scientist until 1844 *ever comprehended the full scope of the vector concept and its latent potentialities.*

From the mathematical point of view, vector and tensor analysis is a study of geometric entities and algebraic forms independent of the coordinate system. This creates a link between the nascence of vectors and the algebraization of geometry through the invention of analytic geometry by ***Fermat*** *and* ***Descartes*** *during* 1629–1637. *These mathematicians combined the notation and problem-solving ability of the algebraist (which originated with the Babylonians) with the geometry of the plane and space developed by the Greeks. [****Apollonios of Perga*** *(ca 230 BCE) produced a characterization of conic sections in terms of what we now call coordinates.]*

The systematic transition from one to another is achieved by means of a system of coordinates.

With the idea of coordinate system established in the first half of the 17th century, there came the first strides taken in the geometric representation of complex numbers[632] *[****Wallis*** *(1673),* ***Wessel*** *(ca 1785),* ***Argand*** *(1806)]. This was the common geometric and algebraic background against which* ***Hamilton*** *and* ***Grassmann*** *operated in* 1884. *Yet their concepts were introduced from quite divergent modes of thought, and in significantly different frameworks. Hamilton seems to have been inspired mainly by a necessity for appropriate mathematical tools with which he could apply Newtonian mechanics to various aspects of astronomy and physics.*

From the strictly mathematical standpoint he was perhaps stimulated by the desire to introduce a binary operation that could be interpreted physically by means of rotation in space. On the other hand, Grassmann's motivations were of a more philosophical nature. His chief desire seems to have been that of developing a theoretical algebraic structure on which a geometry of any number of dimensions could be based. It was Grassmann who introduced for the first time the concept of indeterminate product, *a special case of which was the* 2^{nd} *rank tensor, the* dyadic[633].

In addition to the geometrical and algebraic ingredients of the vector concept, the advent of the infinitesimal calculus added the analytical dimension to its development. The idea of the arithmetical and geometrical limit *appeared*

632 Complex numbers are important in the historical background of vectors because of the analogy between these entities in two dimensions. The term "*complex number*" was introduced by **Gauss**.

633 Grassmann was apparently *not* aware of the fact that his 2^{nd} rank indeterminate product could serve as a mathematical representation of the *inertia tensor*, that had appeared already in 1785 in **Euler**'s equations of rotation of a rigid body about a point.

already in embryonic form in both Babylonian and Greek mathematics. It was explicitly introduced by **Newton** *and* **Leibniz** *during 1665–1679 via the concepts of the derivative and integral. This marked the advent of geometrical analysis and in particular differential geometry, the basic ideas of which were introduced by* **Gauss** *(1827).*

Thus, the triple merger of Newtonian analysis, Euclidean geometry and Cartesian coordinate systems produced the ultimate mathematical vehicle for the development of tensor analysis. Nevertheless, in 1844 the time was not yet ripe for the exposition of the full theory. In spite of the great merit of Grassmann's work, it made little impression on the scientific world and because of a lack of pressing need, the tensor theory was slow in coming into formal being.

However, the first realization of the need for tensors arose with the doctoral thesis of **Riemann** *in 1854, in which he based the metric properties of n-dimensional space on a fundamental quadratic form: $ds^2 = \sum_{\alpha,\beta=1}^{n} g_{\alpha\beta}dx^{\alpha}dx^{\beta}$. He generalized the concept of curvature on a surface to n dimensional space, in terms of the metric coefficients $g_{\alpha\beta}$. His work was followed by* **E. Beltrami** *(1864),* **E.B. Christoffel** *(1869) and* **R. Lipschitz** *(1869), who introduced further concepts into the algebra and calculus of n-dimensional manifolds, including the concept of covariant differentiation. In other veins,* **Cayley** *(1857) created the theory of matrices,* **Aronhold** *and* **Clebsch** *(1858–1861) and* **Gordan** *(1868–1870) developed the theory of algebraic invariants and covariants, and* **Clifford** *(1873–1878) invented his algebra.*

At the close of the 19^{th} century all these ideas were compiled and integrated by **Gregorio Ricci-Curbastro** *(1887) into what is known today as the algebra and calculus of tensors. His pupil* **Tullio Levi-Civita** *(1901), generalized the concept of parallelism to Riemannian spaces.*

In spite of these developments and the many applications of tensor analysis to both mathematics and physics, the subject was, at the beginning of the 20^{th} century, little more than a plaything of a small group of mathematicians. Only since 1916, with the advent of Einstein's theory of general relativity, did tensors come of age. Wide areas of applications in theoretical physics, applied mathematics and differential geometry have been found. Due to the remarkable effectiveness of the tensor apparatus in the study of nature, it is serving as the universal language which **Hamilton** *and* **Grassmann** *envisioned in their original theories.*

In 1915, **Einstein** *said:*

"The magic of this theory will hardly fail to impose itself on anybody who has truly understood it; it represents a genuine triumph of the method

*of absolute differential calculus, founded by **Gauss**, **Riemann**, **Christoffel**, **Ricci** and **Levi-Civita**".*

Non-relativistic physical laws are written in terms of scalars, vectors and tensors. Scalars are entities such as time, volume and mass that are specified by a single number, the magnitude.

Vectors are entities having a direction as well as a magnitude (examples are position, velocity and force). They require more than one number for their specification.

A tensor[634] *is a more complex entity, specified at each point by an array of numbers. A rank-2 tensor is a matrix — such as the state of stress or strain at a given point in an isotropic elastic solid, or the moment of inertia of a rotating rigid body. The tensor that relates stress and strain in a general elastic medium, is a tensor of rank 4.*

Physical laws are usually mathematical statements that establish algebraic, differential and integral relations between tensors. As such they must assume the same mathematical form, irrespective of the position, orientation and state of uniform motion of the observer. This is known as the principle of covariance of physical laws. Had it been otherwise, these laws would be only of limited local value and loose their universality.

To see how the principle of covariance manifest itself in the mathematical properties of tensors, we imagine two observers that view a given physical relation from two different coordinate systems, one being rotated with respect to the other about the common origin O. This rotation is specified mathematically by a 3×3 orthogonal matrix $\mathfrak{R}$. The components (V_x, V_y, V_z) of any vector $\boldsymbol{V}$ are transformed by the rotation into $\boldsymbol{V}' = (V'_x, V'_y, V'_z)$ such that $\boldsymbol{V} = \mathfrak{R} \cdot \boldsymbol{V}'$. The inverse relation[635] $\boldsymbol{V}' = \mathfrak{R}^T \cdot \boldsymbol{V}$ *is the law of vector covariance. A triplet of numbers that transforms in this way under rotation of the axes is defined to be a physical vector.*

634 Of rank 2 or higher; technically, scalars and vectors are tensors of ranks 0 and 1, respectively.

635 The axes $O(x, y, z)$ rotate into their new positions $O'(x', y', z')$. Through this transformation the vector $\boldsymbol{V}$ remains intact, but its *components* relative to the new axes are different. In the inverse relation $\boldsymbol{V}' = \mathfrak{R}^T \cdot \boldsymbol{V}$, $\mathfrak{R}^T$ is the transpose (= inverse) of $\mathfrak{R}$. For a rotation by an angle θ about the z-axis,

$$\mathfrak{R} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Now, consider the law in O, $W = \boldsymbol{F} \cdot \boldsymbol{s}$, *which measures the work W (scalar) done by a force* $\boldsymbol{F}$ *that displaces a body over a finite straight-line path* $\boldsymbol{s}$. *How will the same law be observed in* O'? *To see this we simply substitute therein the relations* $\boldsymbol{F} = \mathfrak{R} \cdot \boldsymbol{F}'$, $\boldsymbol{s} = \mathfrak{R} \cdot \boldsymbol{s}'$ *and* $W = W'$ *(a scalar is* invariant *under rotation of the axes). The result is*

$$W' = (\mathfrak{R} \cdot \boldsymbol{F'}) \cdot (\mathfrak{R} \cdot \boldsymbol{s}') = \boldsymbol{F}' \cdot \{\mathfrak{R}^T \cdot \mathfrak{R}\} \cdot \boldsymbol{s}' = \boldsymbol{F}' \cdot \boldsymbol{s}'.$$

The law thus has the same form *in* O' *as in* O.

The mathematical form of the law is the same for any pair of observers, and therefore to all *inertial-frame observers in space. The law of vector* covariance *therefore secures the* invariance *of the physical law under rotation of the coordinates.*

A tensor of rank 2 in Cartesian 3-dimensional space is an array of 3×3 *components (with respect to a given coordinate system) that obey a* law of tensor covariance. *The tensor is represented by the symbol* σ_{ij}, $i, j = 1, 2, 3$ *(or* σ*), and the law of covariance can be shown to have the mathematical form*

$$\sigma' = \mathfrak{R}^T \cdot \sigma \cdot \mathfrak{R}$$

(or $\sigma = \mathfrak{R} \cdot \sigma' \cdot \mathfrak{R}^T$ *in the reciprocal form).*

Let σ *have the physical meaning of the state of* stress *at a point O in an elastic solid and let the physical law associated with it be given in O as*

$$\boldsymbol{F} = \sigma \cdot \boldsymbol{n},$$

where $\boldsymbol{n}$ *is the normal vector to a plane passing through O and* $\boldsymbol{F}(\boldsymbol{n})$ *the force vector across this plane at O.*

The observer at O' *will read the same law as*

$$\mathfrak{R} \cdot \boldsymbol{F}' = (\mathfrak{R} \cdot \sigma' \cdot \mathfrak{R}^T) \cdot (\mathfrak{R} \cdot \boldsymbol{n}') = \mathfrak{R} \cdot \{\sigma' \cdot \boldsymbol{n}'\},$$

or

$$\boldsymbol{F}' = \sigma' \cdot \boldsymbol{n}'.$$

Again, the physical law is of the same form *in both coordinate systems.*

Tensor analysis deals with abstract objects (entities) that are independent of the choice of the reference frames used to describe them. A tensor is represented in a particular reference frame by a set of functions called components. *As we learned in the above discussion, a given set of functions representing a tensor depends on the* law of transformation *of these functions from one coordinate system to another. But the independence of the form of the laws obeyed by the tensor upon the choice of the reference frame, provides an ideal tool for the study of* natural laws.

Indeed, whether a logical deduction based upon a conglomerate of observational facts deserves the name of natural law is often determined by the generality of such a deduction including its validity in a sufficiently wide class of reference systems. This is intimately bound up with the possibility of formulating the deduction in the form of tensor equations. The concept of covariance of mathematical objects under coordinate transformations, is thus of prime importance in tensor analysis.

*A fundamental concept that permeates the entire calculus of tensors is that of covariant differentiation (***Ricci***, 1884). It constitutes a generalization of partial differentiation that is covariant under general coordinate transformations.*

The basic idea behind the covariant derivative is as follows: consider a vector field in a 3D Cartesian coordinate system. The physical vectors may vary from point to point, but the Cartesian unit vectors are the same at each point. Hence, when we come to compare two field-vectors at two points $P(\boldsymbol{r})$ *and* $Q(\boldsymbol{r}+d\boldsymbol{r})$ *that are infinitesimally close to each other (differentiation is basically an operation of comparison!), the variation of the vector between these points is naturally measured in the same coordinate system, and therefore reflects the observed physical change of the field, as given by the ordinary partial derivative of its Cartesian components w.r.t. to the coordinates.*

Now, suppose that the same physical vector field *is quantified in a curvilinear orthogonal system in which, say, spherical coordinate are used. Since the orientation of the coordinate axes at* any *two neighboring points is now* different *(rotated), the change in the field vector between these points reflects both a true* physical *change and a superposed, artificial, change which arises from the fact that the two measurements are performed in two differently oriented local Cartesian systems.*

This last superfluous effect can be eliminated, simply by displacing the vector at P *parallel to itself to the point* Q *and making the comparison* there! *The result of this process is the so-called* covariant derivative *which expresses the rate of change of physical quantities (vectors and higher tensors) in a way that is independent of the coordinate system used.*

Let us put the above idea into quantitative form and apply it to a covariant[636] *vector field, say, in an affine space, with components* $A_i(x^j)$ *at a point* $P(x^j)$. *At a neighboring point* $Q(x^j+dx^j)$, *the value of the field is* $A_i(x^j+dx^j)=A_i(x^j)+dA_i$. *To make the comparison, we transplant* A_i

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[636] A vector is said to be *covariant* if its component transform from A_i to B_i with $A_i=\frac{\partial y^k}{\partial x^i}B_k$ upon the transformation $y^k(x^i)$. It is said to be *contravariant* if its components transform instead as $A^i=\frac{\partial x^i}{\partial y_k}B^k$.

to Q parallel to itself. However, at Q the local coordinate axes are different then those at P and therefore the new components, $\widehat{A}_i$, are not the same as they were at P. Rather, $\widehat{A}_i(x^j + dx^j) = A_i(x^j) + \delta A_i$. The difference $dA_i - \delta A_i$ will yield the true physical change of A_i. Therefore we write $dA_i - \delta A_i = A_{i\|j} dx$, where $A_{i\|j}$ are the components of a rank-2 covariant tensor known as the covariant derivative of A_i.

Explicitly

$$\begin{aligned} dA_i - \delta A_i &= A_i(x^j + dx^j) - \widehat{A}_i(x^j + dx^j) \\ &= \left[A_i(x^j + dx^j) - A_i(x^j)\right] - \left[\widehat{A}_i(x^j + dx^j) - A_i(x^j)\right]. \end{aligned}$$

The first term on the r.h.s. is simply $\frac{\partial A_i}{\partial x^j} dx^j$. The second term involves the a priori information of the extent to which the Cartesian axes have rotated from P to Q. But this is calculable through the covariance law $A_i = \frac{\partial y^j}{\partial x^i} B_j$, where $y^j(x^i)$ are the transformation functions. If $\{y\}$ is a Cartesian coordinate system, then $\delta A_i = \delta \left[\frac{\partial y^j}{\partial x^i} B_j\right] = \delta \left[\frac{\partial y^j}{\partial x^i}\right] B_j$ since $\delta B_j = 0$ in parallel displacement of Cartesian axes.

However,

$$B_i = \frac{\partial x^j}{\partial y^i} A_j, \qquad \delta\left[\frac{\partial y^j}{\partial x^i}\right] = \frac{\partial^2 y^j}{\partial x^i \partial x^k} dx^k.$$

Therefore

$$\delta A_i = \Gamma^m_{ik} A_m dx^k,$$

where the entity

$$\Gamma^m_{ik} = \frac{\partial^2 y^j}{\partial x^i \partial x^k} \frac{\partial x^m}{\partial y^j}$$

is known as the affine connection[637] *between the points of the space [a space which is affinely connected or an affine space possesses sufficient structure*

[637] The coefficients Γ^m_{ik} of the affine connection are *not* components of a tensor. A given affine connection can always be decomposed into its symmetric and skew-symmetric parts according to the usual rule

$$\Gamma^m_{ik} = \frac{1}{2}(\Gamma^m_{ik} + \Gamma^m_{ki}) + \frac{1}{2}(\Gamma^m_{ik} - \Gamma^m_{ki}).$$

The connection is said to be *symmetric* if $\Gamma^m_{ik} = \Gamma^m_{ki}$. However, $T^m_{ik} = \Gamma^m_{ik} - \Gamma^m_{ki}$ *is* a tensor, and is often referred to as the *torsion tensor* of the connection. Clearly, if the torsion tensor vanishes in some coordinate system, it will vanish in any other system, and accordingly, the symmetry condition is independent of the choice of the coordinate system.

In GTR, spacetime is both a *Riemannian* space (manifold) *and* an affine one, *to permit the operations of tensor calculus. An affine space is more general than a Riemannian space*[638] *since it does not necessarily have a metric]. An*

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with the Christoffel connection determined by the metric (and vanishing torsion):

$$\Gamma^m_{ik} = \frac{1}{2} g^{m\sigma} \left(\frac{\partial g_{\sigma i}}{\partial x^k} + \frac{\partial g_{\sigma k}}{\partial x^i} - \frac{\partial g_{ik}}{\partial x^\sigma} \right).$$

In his efforts to unify the gravitational and electromagnetic field theories, **Einstein** (1928) hoped to be able to identify the *contracted* torsion tensor T^m_{im} with the electromagnetic potential. Previously, **Cartan** (1922), motivated by work of **Cassirer** on non-symmetric stress tensors in magnetic materials, developed an alternative to GTR in which the affine connection is not symmetric.

638 In a flat Riemannian space a Euclidean (Cartesian) vector $\boldsymbol{A}$ can be written as $\boldsymbol{A} = A^i \boldsymbol{g}_i$, where A^i are its *contravariant components in the given curvilinear coordinate system* and $\boldsymbol{g}_i = \frac{\partial \boldsymbol{r}}{\partial x^i}$ are the *base vectors*. Or it can be written as $\boldsymbol{A} = A_i \boldsymbol{g}^i$, where A_i are the *covariant components* and $\boldsymbol{g}^i$ is the *reciprocal base*. Now,

$$\frac{\partial \boldsymbol{A}}{\partial x^j} = \frac{\partial}{\partial x^j}(A^k \boldsymbol{g}_k) = \frac{\partial A^k}{\partial x^j}\boldsymbol{g}_k + A^k \frac{\partial \boldsymbol{g}_k}{\partial x^j}$$
$$= \frac{\partial A^k}{\partial x^j}\boldsymbol{g}_k + A^k \Gamma^m_{ki} \boldsymbol{g}_m = \left[\frac{\partial A^m}{\partial x^j} + \Gamma^m_{ki} A^k\right] \boldsymbol{g}_m \equiv A^m_{\cdot j} \boldsymbol{g}_m$$

Similarly,

$$\frac{\partial \boldsymbol{A}}{\partial x^j} = \frac{\partial}{\partial x^j}(A_k \boldsymbol{g}^k) = \left[\frac{\partial A_m}{\partial x^j} - \Gamma^k_{mj} A_k\right] \boldsymbol{g}^m = A_{m,j} \boldsymbol{g}^m.$$

The relation

$$\nabla \boldsymbol{A} = \left(\boldsymbol{g}^\alpha \frac{\partial}{\partial x^\alpha}\right) \boldsymbol{A} = A^\beta_\alpha \boldsymbol{g}^\alpha \boldsymbol{g}_\beta$$

shows that the curvilinear components of the gradient tensor are the covariant derivatives of the vector components.
Note that $\nabla \boldsymbol{g}^\alpha = \nabla \boldsymbol{g}_\alpha \equiv 0$. Moreover, the fundamental tensors g_{jk} and g^{jk} behave like constants w.r.t. covariant differentiation (*Ricci's Theorem*).

Let $\boldsymbol{A}(\boldsymbol{r}) = \boldsymbol{A}^{(0)}$ be a flat-space Cartesian vector field which does not vary from point to point, i.e. its magnitude and direction are constant. Since in this case $\frac{\partial \boldsymbol{A}}{\partial x^k} = A^m_{\cdot k} \boldsymbol{g}_m \equiv 0$, it follows that $A^m_{\cdot k} = 0$. Thus, the covariant derivative of a uniform vector field vanishes.
But a uniform field can be regarded as the result of displacing the vector $\boldsymbol{A}$ parallel to itself from some fiducial (origin) point to every point of the field. With this interpretation the equation

$$\frac{\partial A^m}{\partial x^j} + \Gamma^m_{kj} A^k = 0$$

affine space in which there exists, at each point, a coordinate system in which $\Gamma = 0$*, is said to be flat. Here the term "space" is synonymous with manifold.*

Altogether,

$$A_{i,k} = \frac{\partial A_i}{\partial x^k} - \Gamma^m_{ik} A_m$$

is the covariant derivative of a covariant vector. Likewise, the covariant derivative of a contravariant vector is

$$A^k_{\cdot j} = \frac{\partial A^k}{\partial x^j} + \Gamma^k_{mj} A^m.$$

The power of these definitions is that they, and the tensor calculus based upon them, hold for any affine space — even if it is not flat, i.e. cannot be locally reduced to a Euclidean space via coordinate transformations. In a Riemannian space, which is endowed with a (covariant, rank 2) metric tensor $g_{\mu\nu}$*, the Christoffel symbol* Γ *is that connection for which* $g_{\mu\nu,\alpha} = 0$*. Thus, with this connection, index contractions (via* $g_{\mu\nu}$*) commute with covariant differentiation.*

1888 CE Telescopic photographs reveal the spiral shape of the Andromeda Nebula. [In 1923, **Hubble** established its galactic nature.]

1888 CE **Henry Louis Le Châtelier** (1850–1936, France). Physicist. Established a principle, named after him, for the behavior of a thermodynamic system at equilibrium [its macroscopic parameters such as temperature, pressure, composition, entropy — do not depend on either time or space, i.e. the system is uniform and either isolated (closed) or in contact with uniform environment]. This principle states: "*An external influence disturbing the equilibrium of the system, induces in it processes tending to weaken the effects of this influence*".

In other words: any change in the equilibrium conditions, results in a shift of that equilibrium in the direction that will partially nullify the perturbation, and thus tend to restore the unperturbed state.

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becomes the *condition* for a vector to be its *own parallel displacement* (in any coordinate system). Such covariantly-constant vector fields can *only* exist (if A is not to be 0 everywhere) in *flat* (curvature-free) spaces.

This law, valid for *quasistatical thermodynamics*, is actually valid for a wider class of phenomena. Example: the law of electromagnetic induction introduced by **Heinrich Friedrich Emil Lenz** (1804–1865, Russia, 1834).

In 1947, **Ilya Prigogine** (1917–2003, Belgium), extended the thermodynamic principle to a wider class of stationary states, namely *open non-equilibrium* stationary states[639], where entropy-producing processes are sustained by a continual flux of energy (or matter and energy) between the system and its surrounding. In that case the stationary state is the configuration of minimum entropy. This generalization enables us to include processes with *thermal diffusion* under the umbrella of the Le Châtelier principle, where the entropy production is a *Lyapunov function* (1892).

The theory of open systems has been applied with success to many specific problems of biology, thus demonstrating that thermodynamic principles related to open systems lie at the core of central biological problems. Prigogine's theory accounts for many features of life, which can thus be treated as physical phenomena.

1888 CE **Wilhelm Hallwachs** (1859–1922, Germany). Physicist. Stimulated by Hertz's work, he showed that irradiation with ultraviolet light causes uncharged metallic bodies to acquire a positive charge (emit electrons). The earliest speculations on the nature of the effect predate the discovery of the electron in 1897.

Hallwachs demonstrated the possibility of using photoelectric cells in cameras. This property, called *photoemission*, was applied in the 20th century in the creation of the electronic television camera.

1888 CE **Frank Julian Sprague** (1857–1934, U.S.A.). Electrical engineer and inventor. Built the first large electric passenger railway system, in Richmond, VA (20 km long).

1888 CE The American Mathematical Society established. Lick Astronomical Observatory established on Mount Hamilton, California, equipped with a 36-inch refractor telescope.

1888–1891 CE **Pierre Paul Émile Roux** (1853–1933, France). Physician, bacteriologist and immunologist. Discoverer of the anti-diphtheria

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[639] Definitions:

Isolated system: Completely disconnected from its surroundings. No exchange of *energy or matter* possible.

Closed system: May exchange *energy* with its surroundings, but not matter.

Open system: May exchange *energy and matter* with its surroundings.

serum, the first effective therapy for this disease. One of the close collaborators of **Louis Pasteur**.

Roux joined Pasteur's laboratory as a research assistant (1878–1883) at the École Normale Supérieure in Paris. He worked with Pasteur in *Avian cholera* (1879–1880), *anthrax* (1879–1890) and *rabies* (1881–1883).

In 1888 he published with **Alexandre Yersin** (1863–1943) the first of his works on the causation of *diphtheria* by the *Klebs-Loeffler bacillus*. He then began (1891) to develop an effective serum to treat the disease, following the demonstration by **Emil von Behring** (1854–1917) and **Shibasaburo Kitasato**(1852–1931) that *antibodies* against the diphtheric toxin could be produced in animals. He demonstrated successfully this antitoxin in the Hospital des Enfants-Malades (1891).

In the following years, Roux dedicated himself to the immunology of *tetanus*, *tuberculosis*, *syphilis* and pneumonia. He became the director of the Pasteur Institute in 1916.

1888–1903 CE **Nikola Tesla**[640] (1856–1943, U.S.A.). An American inventor of Croatian origin. A key figure in the history of electrical technology. Invented the alternating current induction motor (*electric alternator*, known also as the electromagnetic motor) and polyphase power transmission. He also invented the *Tesla coil transformer* (produces high voltage at high frequencies), *arc lightning*, a system of *wireless transmission* (in 1893, two years ahead of Marconi (1874–1937)), a telephone repeater, rotating magnetic field principle, fluorescent light and more than 700 other patents.[641]

Tesla was born of Serbian parents in Smiljan Lika, Croatia and was raised and educated in the Austro-Hungarian kingdom. In 1882 he conceived the ideas that would form the foundation of his only truly successful inventions: the induction motor and polyphase power transmission.

In 1884, while working in Paris for the Continental Edison Company, he obtained a letter of introduction to Edison and immigrated to New York. He worked for Edison for about a year before having some kind of falling out[642].

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640 For further reading, see:

- Cheney, M., *Tesla*: *Man Out of Time*, A Laurel Book, Dell Publishing: New York, 1981, 320 pp.

641 Tesla patented (1903) the electrical logical circuits that become crucial to addition, subtraction, and multiplication in later computer machines.

642 The standard story is that Edison told Tesla it would be worth $50,000 to him if he could improve upon Edison's electric generators significantly. Tesla did this and then asked Edison for his money. "*Tesla*", Edison replied, "*you don't*

Tesla then caught the attention of George Westinghouse (1846–1914), inventor of the air-brake (1868), who was looking to break into electrical technology and thought Tesla's ideas on electric power distribution had merit. Using the ideas of Tesla, Tesla and Westinghouse made commercial use of AC motors, generators and transmission lines (1891) and the polyphase AC power transmission (1893). At this time Tesla further developed his induction motor and his high voltage generator known as the *Tesla coil*[643]. In the next few years Tesla would install the world's first true commercial electric power station at Niagara Falls. He would continue to produce remarkable ideas for decades but would never again be able to finish what he started.

Tesla developed all the components needed to construct a practical radio system, but then seems to have lost interest — he never took his ideas beyond some very short-range demonstrations. This left the field to Marconi, who proved the feasibility of long-range wireless communication just a few years later. Although he anticipated Marconi and others in many ways, histories of early radio make only incidental mention of Tesla.

A U.S. supreme court decision (June 21, 1943) found that Tesla anticipated the four-circuit tuned combination of Marconi, and ruled that Tesla had anticipated all other contenders with his fundamental radio patents. [Yet the Nobel prize in physics for 1909 had gone to Marconi and K.F. Braun.]

Tesla was obsessed with the idea of wireless transmission of electric *power* (in contradistinction to wireless transmission of *information* via low-energy electromagnetic waves). He also talked of making the upper atmosphere fluoresce — abolishing the dark night forever. None of these ideas was ever realized.

Although a millionaire in the 1890's, Tesla had so indulged his appetite for expensive experiments that from the early 1920's until his death in 1943 he was nearly destitute.

In his honor, the physical mks (SI) unit of magnetic flux density, is named the 'tesla'.

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understand our American humor".

643 *Tesla coil*: a specialized electrical transformer and spark gaps, used in circuits that produce high-voltage at high frequencies. Large Tesla coils can produce millions of volts and are used to make spectacular electrical displays, but have no important scientific or industrial applications. Today's scientists and engineers have far superior methods of producing high voltage — methods that do not derive from Tesla's work .

Science Progress Report No. 12

Tesla vs. Edison, or — the 'War of the Currents'

Alternating currents technology is rooted in the discovery of ***Joseph Henry*** *(1830, USA) and* ***Michael Faraday*** *(1831, England) that a changing magnetic field near an electric circuit, or a static one through which the circuit moves, is capable of inducing an electric current in the circuit. Earlier studies had been confined to* static *magnetic fields.*[644] *Faraday is usually given credit for the discovery since he published his results first.*

The principle of the voltage transformer was applied in 1851 by ***Heinrich Daniel Ruhmkorff*** *(1803–1877, Germany and France) in his* induction-coil *(also known as the Ruhmkorff-coil). Through this device he generated a train of unidirectional* high-voltage pulses *in an open secondary coil circuit induced by rapid mechanical make-and-break switching in a primary direct-current low resistance coil circuit. The alternations of the magnetic flux induce an emf between the ends of the secondary coil, and a high voltage is produced that tends to cause a spark or an arc to pass. If, e.g. an X-ray tube is connected between the secondary terminals, the magnetic-field energy is transformed partly into X-ray energy and partly into heat.*

Exploitation of the discoveries of Henry and Faraday began in 1887, with the construction of the first commercial alternating current (AC) power transformer *by the engineers* ***Lucien Gaulard*** *(1850–1888, France) and* ***John Gibbs*** *(England). Improvements were introduced in Budapest by* ***Otto Blathy*** *(1860–1939, Hungary),* ***Max Deri*** *(1854–1938, Hungary) and* ***Karl Zipernowsky*** *(1853–1942, Hungary) during 1881–1885.*

The American electrical engineer and inventor ***William Stanley*** *(1858–1916), using the patents of Gaulard and Gibbs built a transformer system to form an integral part of the first multiple-voltage AC power system in Great Barrington (Massachusetts, U.S.A., 1886). The network was driven by a hydropower generator producing 500 Volts AC. It was* stepped up *to 3 kV for transmission, then* stepped down *to 100 V to power electric lights. Stanley also invented two-phase motors and patented a carbonized filament incandescent lamp.*

George Westinghouse *(1846–1914, USA), Stanley's employer (an adventurous Pittsburgh industrialist and the inventor of railroad air breaks), was an early advocate of AC with great plans for the electrification of America. He bought the American rights to the Gaulard and Gibbs' patents (1885).*

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[644] Whether generated *by* naturally-occurring magnets or electric currents, and whether acting *upon* magnetic/magnetizable materials or electric currents.

Three-phase currents were introduced into electrical engineering by ***Nikola Tesla*** *(1887) and the Italian engineer* ***Gallileo Ferraris*** *(1847–1897), (1888). A decisive factor in bringing about the subsequent almost universal adoption of three-phase currents for the* transmission of power over large distances *was the successful transmission of electric power between Lauffen-on-the-Neckar and Frankfurt-on-the-Main on the occasion of the important exhibition at Frankfurt (1891), a distance of 175 km. It was accomplished by the Berlin engineer* ***M. Von Dolivo-Dobrowolski****.*

Elihu Thomson *(1853–1937, USA), electrical engineer and inventor, invented the standard three-phase alternating current generator, the high-frequency transformer, the high-frequency generator(1890), the centrifugal cream-separator, the common Watt-meter, the street arc lamp (fed by alternating currents, 1878–9) and 700 other patented inventions. He became one of the great pioneers of the electrical manufacturing industry in the USA. Thomson and* ***Edwin James Houston*** *founded the Thomson-Houston Electric Company (1883), which merged with* ***Edison's*** *firm (1892) to form the General Electric company.*

Electricity was first introduced to New York in the late 1870s. Edison's incandescent lamp had created an astonishing demand for electric power, and his DC power station on Pearl Street in lower Manhattan was quickly becoming a monopoly. Edison knew little of alternating current and did not care to learn more about it. In short, AC power sounded like competition to Edison.

In November and December of 1887, Tesla filed for seven U.S.patents in the field of polyphase AC motors and power transmission. These comprise a complete system of generators, transformers, transmissions, motors and lighting. ***George Westinghouse*** *heard about Tesla's invention and thought it could be the missing link in long-distance power transmission.*[645]

[645] The main advantage of AC over DC (direct current) is the ease and efficiency with which AC voltages can be raised or lowered. When electric power is transmitted over long distances it is economical to use high voltage and low current to minimize the I^2R heating losses (R = resistance, I = current) in the transmission lines for the same amount of power transmitted. The voltages are stepped up or down by passive devices called *transformers* which usually operate with an efficiency of 99 percent.

In practice, the generator's voltage is stepped up to around 230,000 V at the generating station, then stepped down to around 20,000 V at a distributing station, and finally stepped down to 110–120 V at the customers utility poles. DC power is useful only in the vicinity of the generator. Its use over larger distances would require very thick wires to decrease resistance to energy flow.

He came to Tesla's laboratory and purchased his patents for $60,000.With the breakthrough provided by Tesla's patents, a full scale industrial war erupted. At stake, in effect, was the path of industrial development in the United States, and whether the Tesla-Westinghouse alternating current or Edison's direct current would be the chosen technology.

It was at this time that Edison launched a propaganda war against alternating current.[646] *He even hired a professor who went around talking to audiences and electrocuting dogs and old horses right on stage, to show how dangerous alternating current was.*

In spite of bad press, good things were happening for Westinghouse and Tesla. The Westinghouse Corporation won the bid for illuminating The Chicago World's Fair, the first all-electric fair in history. The fair was also called the Columbian Exposition — in celebration of the 400th Anniversary of Columbus discovering America. Up against the newly formed General Electric Company (the company that had taken over the Edison Company), Westinghouse undercut GE's million-dollar bid by half. Much of GE's proposed expenses were tied to the amount of copper wire necessary to utilize DC power. Westinghouse's winning bid proposed a more efficient, cost-effective AC system.

The Columbian Exposition opened on May 1, 1893. That evening, President Grover Cleveland pushed a button and a hundred thousand incandescent lamps illuminated the fairground's neoclassical buildings. This "City of Light" was the work of Tesla, Westinghouse and twelve new thousand-horsepower AC generation units located in the Hall of Machinery.

In the Great Hall of Electricity, the Tesla polyphase system of alternating current power generation and transmission was proudly displayed. For the twenty-seven million people who attended that fair, it was dramatically clear that the power of the future was AC. From that point forward more than 80 percent of all the electrical devices ordered in the United States were for alternating current.

[646] This must be attributed to Edison's lack of education and ignorance of some basic principles of physics. Tesla had a formal European education and knew better.

1888–1906 CE **Friedrich Wilhelm Ostwald** (1853–1932, Germany). Chemist. With **Arrhenius** and **van't Hoff** he established physical chemistry as a separate discipline of science. Developed new methods for measuring the rate of chemical reactions. Rediscovered catalysis, pointing out that its essence lay in its accelerating the rate of the reaction, but not creating it. Was awarded the Nobel prize for chemistry (1909). Ostwald was born in Riga, and was a professor at Leipzig University (1888–1906).

1888–1906 CE **Fridtjof Nansen** (1861–1930, Norway). Arctic explorer, marine zoologist, pioneer oceanographer, and statesman. Began the first scientific study of the Arctic Ocean (1893–1896), obtaining information about the ocean's bed, current, ice, weather, and wildlife.

In the summer of 1888, he and five other men crossed Greenland by land from east to west, a feat that experts had declared impossible. This expedition confirmed that Greenland is nearly completely covered with ice. Detailed meteorological conditions compiled during the winter of 1889 led to a better understanding of weather conditions in Northern Europe.

In 1893 he led the *Fram* expedition to the North pole. To this end he had a ship specially built to withstand the grinding ice floes[647]. The *Fram* sailed from Christiania (Oslo) in June 1893, provisioned for 5 years with a crew of 13, sailing along the coast of Siberia. On Sept. 27, upon encountering an impassable ice barrier, its engine was dismantled, a windmill set up to work the dynamo, and the *Fram* froze in and began to drift through the ice, while the crew carried on their various scientific tasks. They took meteorological, astronomical, electrical, magnetic, and hydrodynamical observations, and collected wildlife and underwater specimens.

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[647] In 1881, the steam yacht *Jeannette*, of the De-Long expedition, was crushed by the ice of the Arctic Ocean, and sank 240 km off the New Siberian Islands. Nansen had planned a ship that, skillfully reinforced, would ride up under the pressure of ice and rest on its surface until a thaw released it to float again — "*that the whole craft should be able to slip like an eel out of the embraces of the ice*".

Nansen noted that 1100 days after the sinking of the *Jeannette*, some of its objects were found by Eskimoes in drift-ice near Julianehab, on the southwest coast of Greenland, some 4600 km from where it sunk. He saw this as an evidence of the existence of a slow steady current across the polar basin. It convinced him that it was possible to drift across it in a vessel, traveling with the ice instead of fighting against it, and possibly, at the same time, reach the pole, providing that the right sort of vessel could be constructed. Nansen's plan was greeted with skepticism, if not derision, by most Arctic experts. However, the *Fram* (forward) *did drift* for 35 months, carrying Nansen and his crew to within about 640 km south of the Pole.

In March 1895, at 84 °N, Nansen and Hjalmar Johansen left the *Fram*, taking with them 2 kayaks, three sledges and 28 dogs. On April 8 Nansen hoisted the Norwegian flag in 86.13 °N, 95 °E, 438 km of the North Pole, nearer than anyone before him. They could go no further. On June 17, 1896 they reached Cape Flora and met some of their friends.

Nansen used his fame to facilitate his entry into Norwegian and international politics, as organizer of the Leage of Nations Refugee Work, and inventor of the 'Nansen Passport for Stateless Persons' (resulting from the collapse of the European empires and the revolution of 1917–1922). For that he was awarded the Nobel Peace prize in 1922.

1888–1906 CE **Georges Fernand Isidore Widal** (1862–1929, France). Distinguished physician. Laid the foundations to *citodiagnosis* and contributed to *pathological physiology.* Known for his pioneering work on bacteria agglutination and its applications ('*Widal reaction*') to the serological diagnosis of typhoid fever (1896). Recognized (1906) the value of salt-deprivation in nephritis and cardiac edema.

Widal was born to Jewish parents in Alger, studied medicine in Paris and served as a professor at the University of Paris (1911–1929).

1888–1910 CE **Salvatore Pincherle** (1853–1936, Italy). Mathematician. Founded (together with Volterra) *functional analysis.* Contributed to functional equations, the theory of functions, the expansion of functions in infinite series,[648] and to abstract linear spaces.

Pincherle was born in Trieste of a Jewish family. A student of **Betti**. Professor at the University of Bologna (1881–1928). Pincherle worked on a formal theory of linear operators on an infinite dimensional vector spaces, basing his work on the abstract operator theory of **Leibniz** and **d'Alembert**, but not on that of Peano. His work had little immediate impact. Axiomatic infinite dimensional vector spaces were not studied again until **Banach** and his associates took up the subject in the 1920's.

1889 CE **Sophia (Sonya) Vasilyevna Kovalevsky**[649] (1850–1891, Russia). Outstanding woman mathematician of the 19^{th} century. A favorite pupil

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648 *Pincherle's expansion* (1896): for every $\Phi(z)$, analytic near $z = 0$, the series

$$f(z) = \sum_{n=0}^{\infty} [1 + \lambda_n e^z] \frac{d^n \Phi(z)}{dz^n},$$

where $\lambda_n(z) = -1 + z - \frac{z^2}{2!} + \frac{z^3}{3!} - \cdots + (-)^{n+1} \frac{z^n}{n!}$, is convergent near $z = 0$, and $f'(z) = f(z) - \Phi(z)$.

649 For further reading, see:

of **Weierstrass**. She is remembered today mainly because of her solution of Euler's equations for the motion of a spinning symmetrical top under gravity. She was able to find a third integral for the special case $A = B = 2C$ where the center of gravity lies in the equatorial plane of the body. The solution can be made to depend on integrals of the form $\int \frac{dx}{f(x)}$, where $f(x)$ is an rational function of the fifth degree. She also contributed to the theory of partial differential equations, where the '*Cauchy-Kovalevsky theorem*' bears her name.

Recently her name was assigned to a crater on the moon. She is thus one of less than a dozen women from all of history to be so honored.

1889 CE **Alexandre Gustave Eiffel** (1832–1923, France). Structural and aeronautical engineer. Designed the *Eiffel Tower* in Paris for the World's Fair of 1889. The tower rises 300 meters from a base 101 m^2. Elevators and stairways lead to the top. It contains about 6400 tons of iron and steel and cost over one million dollars[650].

1889 CE **Otto Ludwig Hölder** (1859–1937, Germany). Mathematician. Discovered one of the most useful *inequalities* of analysis, the *Hölder Inequality.* This states that if x and y are positive, if $x + y = 1$ and if the numbers $a_1, \dots, a_n$ and $b_1, \dots, b_n$ are nonnegative, then

$$\sum_1^n a_i^x b_i^y \leq (\sum_1^n a_i)^x \cdot (\sum_1^n b_i)^y$$

- Kennedy, D.H., *Little Sparrow: A Portrait of Sophia Kovalevsky*, Ohio University Press: Athens, OH, 1983, 341 pp.
- Cooke, R., *The Mathematics of Sonya Kovalevskaya*, Springer-Verlag: New York, 1984, 234 pp.

650 In order to minimize construction materials cost (steel, iron), the compressive strength of the structure was fully exploited. To this end, it must be required that the gravitational compressive stress at any horizontal cross-section is made independent of the height of this section above the ground. Mathematically: $\rho g \int_x^\infty A(x)dx/A(x) = K$, where $A(x)$ is the area of the cross-section at level x, ρ is the density of steel, and g is the acceleration of gravity.
Differentiation yield a differential equation for $A(x)$, the solution of which is $A(x) = A_0 e^{-\lambda x}$, where A_0 is the base area, and $\lambda = \rho g/K$.
The shape of the tower's profile is therefore:

$$y(x) = y_0 \exp\left[-\frac{\rho g x}{2K}\right]$$

.

or equivalently

$$\sum_1^n a_i b_i \leq (\sum_1^n a_i^{\frac{1}{x}})^x (\sum_1^n b_i^{\frac{1}{y}})^y.$$

Equality holds iff $a_i^{\frac{1}{x}} = K b_i^{\frac{1}{y}}$, K constant. The special case $x = y = \frac{1}{2}$ is known as Cauchy–Schwarz Inequality (1821) and has a simple geometrical interpretation (i.e. the cosine of the angle between two vectors may not exceed 1).

Hölder's Inequality holds for complex numbers: If $p > 1$ and $\frac{1}{p} + \frac{1}{q} = 1$ then $|\sum_1^n a_i b_i| \leq (\sum_1^n |a_i|^p)^{\frac{1}{p}} (\sum_1^n |b_i|^q)^{\frac{1}{q}}$. Moreover, it holds also for *integrals*, where integration takes the role of summation: if f and g are continuous real-valued functions defined on $[a, b]$, if $p > 1$, and if $\frac{1}{p} + \frac{1}{q} = 1$, then

$$|\int_a^b f(t)g(t)\,dt| \leq \int_a^b |f(t)g(t)|\,dt \leq (\int_a^b |f(s)|^p\,ds)^{\frac{1}{p}} (\int_a^b |g(t)|^q\,dt)^{\frac{1}{q}}.$$

Hölder was born in Stuttgart. He studied at Berlin under Weierstrass, Kronecker and Kummer (1877–1882), and became a professor at Tübingen (from 1889).

1889 CE **Oskar Minkowski** (1858–1931, Germany). Distinguished physician and endocrinologist. With **Joseph von Mering** discovered the direct connection between the pancreas and diabetes which led to the discovery of *insulin* (they found that the pancreas supplies a hormone essential to glucose metabolism).

Oskar was born in Lithuania, to Jewish parents. He was professor at Strasbourg (1891–1904), Cologne (1904), Greifswald (1905–1909), Breslau (1909–1926). He was the brother of the physicist **Hermann Minkowski**. Both converted to Christianity to be able to pursue their academic careers.

1889–1890 CE Great influenza epidemic afflicted 40 percent of the world population. Millions died.

1889 CE **Otto Lilienthal** (1848–1896, Germany). Inventor and aeronaut. Designed (with the assistance of his brother Gustav) and flew the first gliders that can soar above the height of takeoff. Their observation of the takeoff of storks *against the wind*, brought to aviation one of its first breakthroughs at the end of the 19th century.

Born in Anklam, Pomerania, Lilienthal and his brother studied the flight of birds and while still at school succeeded in constructing s glider. Lilienthal's theory was that artificial flight must follow the principles of bird-flight. His experiments extended over a period of 20 years — building many gliders and executing over 2000 flights. He demonstrated (1891) the superiority of curved wings over flat-surfaced type. Wrote pioneering book on aeronautics (*Des Vogelflug als Grundlage der Fliegekunst*, 1889; *Die Flugapparate*, 1894). While on flight on Aug 9, 1896, near Rhinow, Germany, his machine was upset by a sudden gust of wind and he was killed. His work was continued by the Wright brothers (1903) who inherited his tenacity and perseverance.

1889–1899 CE **Rudolf Christian Karl Diesel** (1858–1913, Germany). Mechanical engineer, inventor, industrialist. Invented the *'compression-ignition'* engine. In a paper *'The theory and construction of an economical heat engine'* (1889) he proposed a more efficient engine than the petrol engine in which no carburetor or ignition system would be required since spontaneous ignition would occur as the fresh-air mixture was compressed at constant pressure.[651]

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[651] The ordinary petrol engine draws its heat supply from the combustion of petrol vapor in the pressure of air: vapor and air are mixed in the carburetor. A suitable mechanism causes the inlet and exhaust valves to open and close at the appropriate times, and a spark to pass through the compressed charge at the right moment.

In the Diesel cycle, the working substance (air) is raised to a very high temperature by adiabatic compression. The fuel is injected in a liquid form into the cylinder during the first part of the outward motion of the piston. The rate of injection is carefully controlled so that the pressure on the piston during the supply of the fuel is maintained constant. Thus the air is heated at *constant pressure*, instead of at constant volume as in the petrol engine.

The Diesel cycle, while less efficient than the *Carnot cycle*, is more efficient than the *Otto cycle* working between the same temperatures.

The *thermal efficiency* (ability to convert stored chemical energy in the fuel into mechanical energy) of the *Otto cycle*, assuming the air-fuel mixture to be an ideal gas, is $e = 1 - \left(\frac{v_2}{v_1}\right)^{\gamma-1}$. For a typical compression ratio of $\frac{v_2}{v_1} = \frac{1}{8}$ and $\gamma = 1.4$ a theoretical efficiency of 56% is predicted for an engine operating in the idealized Otto cycle. This is much higher than what is achieved in real engines (15% or 20%) because of such effects as friction, heat loss to the cylinder walls and incomplete combustion of the air-fuel mixture.

The efficiency of an idealized Diesel cycle is given by $e \approx 1 - \frac{1}{\gamma}\left(\frac{v_2}{v_1}\right)^{\gamma-1}$. With $\frac{v_2}{v_1} = \frac{1}{16}$, the theoretical limit is 76.4%. The difference is due to both a higher compression ratio and a higher combustion temperature. The realizable efficiency is about 30 or 35 percent.

A two-stroke gas engine was patented (1881) by the Scottish engineer and

Applying the idea of **Sadi Carnot** (1824), according to which the output from a reversible movement motor depends on the temperature at which it operates, Diesel sought to modify the *piston cycle*, for that was what caused the heat loss. In a first patent (1892), revised in 1893, Diesel proposed to obtain the necessary heat for the air-fuel mixture in the classical combustion engine to burn, not by using a spark but by a very *high compression of air alone* which would be enough to bring the air to the required temperature. The injected fuel then burns to vapor in the cylinder and the pressure of the hot gases pushes the piston.

Up to this point the principle of the diesel engine is not much different from the 4-stroke engine, except that the spark is removed. But in other respects Diesel modified the design of his engine in such a way that the compression of the air took place *outside* the cylinder; thus it was the compressed air which injected directly into the cylinder. Moreover, the injection of compressed air pushes the gases from the burnt fuel through openings at the base of the cylinders; the track of the piston is reduced for it no longer has the space to descend to the base of the cylinder. Thus heat loss was reduced by further reduction of the piston track. In this respect, Diesel was following an idea of **James Joule** (1885) who was first to try to design Carnot's ideal engine, which he did by using a *porous piston* through which the exhaust escaped.

Diesel also introduced a significant factor of *economy*: since the high volatility of high grade petrol (such as gasoline) is not required, heavier and less refined fuel oil are sufficient. Indeed, in his first engine, Diesel had used *coal dust* as a fuel, but he later discarded this along with several other types in a favor of a form of refined mineral oil.

The Diesel motor rapidly proved its *reliability* and the superiority of its output: it is easy to manufacture, strong and hardly ever breaks down and it costs less to operate since unrefined oil is decidedly cheaper. On the other hand, the Diesel engines are heavier than petrol engines of comparable horsepower, for the cylinders must withstand the high pressure and the engine must also accommodate a separate fuel pump to inject the oil into the cylinder at high pressure. This engine thwarted the intellectual habits of the engineers of the time and it took 20 years to become widely used. It was criticized firstly for its weight and for the noise it made when working and the particularly unpleasant smell of its exhaust.

Diesel himself was one of the main reasons why industry took so long to adopt his engine: until his death the engineer actually demanded that the engines built under license fitted his rigorous specifications and, in particular,

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inventor **Dugald Clerk** (1854–1932), known as the *Clerk Cycle engine*. It was used for large gas and small petrol engines.

that they were designed to function at a constant temperature, for he wanted to keep strictly faithful to Carnot's theory. The problem was that the engine functioned much too slowly when kept at constant temperature; in order to attain the desired output it had to be much more powerful (higher running speeds).

The diesel engine fulfilled its true potential when it was improved after its inventor's death. The *automotive* diesel was first built in the US (1923), and became popular among farmers during the Depression.

The diesel engines has greatly increased the efficiency of industry and transportation. They are used chiefly for heavy-duty work: they drive high freight trucks, large buses, tractors, and heavy road-building equipment. They are also used to power submarines and ships, and the generating of electric-power stations in small cities.

Diesel was born in Paris, France of German parents. They moved to Germany after the outbreak of the Franco-Prussian War (1870) and Rudolf studied at Munich Polytechnic. He was trained as a refrigeration engineer and the idea of the compression-ignition first occurred to him at the age of 19 (1878). When he first built his engine (1893), it exploded and almost killed him, but it proved that fuel could be ignited without a spark. At the same year he had taken his first patent. **Friedrich Krupp** backed the project and the engine bearing Diesel's name was created (1897). In 1899 he founded his own manufacturing company in Augsburg.

License fees on Diesel engine soon made him a millionaire. Diesel was a proverbial success for 15 years. He combined his inventive talents with the social skill of a modern executive, being competent, widely traveled, and fluent in various languages. He apparently committed suicide when he vanished without trace from a cross-Channel steamer (29 April, 1913).

1889–1907 CE **Henri Louis Bergson** (1859–1941, France). A philosopher, who at the end of the 19^{th} century undertook a search for an acceptable alternative to the science of his time. His philosophical system represents the revolt against the 19^{th} century materialism and the reduction of psychology to physics.

The primacy of mathematics and mechanics in the development of modern science, and the reciprocal stimulation of industry and physics under the common pressure of expanding needs, lent to speculation a materialistic flavor; and the most successful of the sciences became the models of philosophy. Despite **Descartes**' insistence that philosophy should begin with the self and travel outward, the industrialization of Western Europe drove thought in the direction of material things. It was **Schopenhauer** who first emphasized in modern thought the possibility of making the concept of life more fundamental and inclusive than that of force; it is Bergson who has taken up this idea, and has almost converted a skeptical world to it by the impact of his sincerity and eloquence.

Bergson was born in Paris of Jewish parentage[652]. He specialized first in mathematics and physics, but in 1881 turned spontaneously to philosophy. He was a professor of philosophy at the École Normale Superieure (1897–1900) and the College de France (1900–1921). Awarded the Nobel prize for literature in 1927. His influence extended far beyond the realm of philosophy into such areas as literature, the social sciences and religion (e.g., the various attempts of writers such as Virginia Woolf, Luigi Pirandello, Marcel Proust to penetrate beneath the static images and facsimiles of the self and to render the flux of consciousness, owe much to him). Moreover, Bergson's philosophy originated a new philosophical attitude, revolutionary in its impact on thought. It was a great liberating force from over-intellectualized modes of thought.

Bergson recognized the three weak cleavage planes of modern knowledge: between *matter and life*, between *body and mind*, and between *determinism and choice*. On the first issue, after a hundred years of theory (since **Pasteur**), and many vain experiments, the materialists were no nearer than before to solving the problem of the origin of life. On the second issue, the mode of connection of thought and brain was as mysterious as it had ever been; consciousness could not be yet explained in terms of an electromechanical neural model. Finally, he rejected any materialistic mechanism that would claim that a sonnet of Shakespeare 'evolved' from the primeval nebula of the solar system.

In his three major works: *Time and the Free Will* (1889), *Matter and Memory* (1896), and *Creative Evolution* (1907), Bergson advanced his basic psycho-physical credo, which he believed capable of tackling the above three tasks:

- *Time*: One must take a sharp distinction between "mathematical time" (objective Newtonian time) and lived time (duration). The former is just a succession of instantaneous states linked by a deterministic law, a quantity without quality, a form of space.

  Duration, on the other hand, is the essence of life, and perhaps all of reality. It exhibits itself in *memory*. Lived time (duration) means that

[652] The name Bergson stems from Berkson (the son of Behr), an illustrious Jewish family of Warsaw, Poland, that descended from Samuel Zbitkower, the financial advisor of the last Polish King Stanislas Poniatowski (king: 1764–1795). In 1891, Bergson married a cousin of the novelist **Marcel Proust** (1871–1922), whose own writings were influenced by the philosophy of Bergson.

the past endures and nothing of it is quite lost. Life is a matter of time rather than of space. It is not *being*, it is *becoming* and change. It is not redistribution of matter and motion, it is fluid and persistent creation, a constant flow from the past into the future.

- *Intellect versus intuition* (instinct): Pure *perception*, which is the lowest degree of the mind (mind without memory) is really part of matter.

  The brain is a system of images and reaction-patterns. The part of our minds which we call the *intellect* was developed, in the process of evolution, to understand and deal with material, spatial objects; from this field it derives all its concepts and its "laws", and its notion of a fatalistic and predictive regularity everywhere. Our intellect is intended to secure the perfect fitting of our body to its environment, to represent the relations of external things among themselves. It is at home with solid, inert things; it sees all becoming as being, as series of states; *it misses* the connective tissue of things.

  In other words, the intellect, for practical purposes, introduces measurements and substitutes for qualitative processes as abstract, spatialized representations of reality; whereas the intellect is connected with space, *intuition* is associated with *time*. It is a way of thinking in duration. Intuition apprehends the true nature of things. It is essentially the most trustworthy guide to understanding. It does not falsify things by analyzing them. *Consciousness* is the recall of images and the choice of reactions.

- Strict *determinism* is unacceptable. To break the chain of deterministic evolution one must relate time to life, to mind, to choice and free-will.

- The concepts of *physics* are inappropriate in the world of the mind. The essence of life is mind, not matter; time, not space; action, not passivity; choice, not mechanism. Intuition, as a form of speculative knowledge, is the only means through which we can restore primary flexibility into scientific methods. Geometrical predictability, which is the ultimate goal of a mechanical science, is only an intellectual delusion[653].

At the end of his life, Bergson leaned toward Catholicism, but the persecution of the Jews by the Nazis caused him to identify with the Jewish

[653] Bergson had obviously misunderstood Einstein's theory of relativity. An historic scene took place on April 6, 1922, when Henri Bergson attempted to defend the cause of multiplicity of coexisting "lived" times against Einstein. Einstein's reply was absolute: he categorically rejected "philosopher's time", stating that *lived experience cannot save what has been denied by science*.

cause. After the collapse of France (1940), the Vichy government offered him exemption from the Jewish laws, patterned after the Nuremberg Laws. Bergson declined the offer and resigned his professorship from the College de France. Sick and enfeebled he stood for hours in que lines for food and daily commodities, with his coreligionists, loyal to the end to his brethren.

Over a century has passed since Bergson published his first book. With hindsight perspective we can say that

"*his grand attempt to limit the scope of modern science, as well as to open new avenues alien to those of science — has failed*[654]. *He has failed insofar as the methaphysics based on intuition he wished to create has not materialized, although the problems which he identified are still our problems. The limitation of the science of his day* (*which he erroneously attributed to science in general*) *are beginning to be overcome, not by abandoning the scientific approach or abstract thinking but by perceiving the limitations of the concepts of classical dynamics and by discovering new formulations valid in more general situations.*

Bergson's case convinces us that only an opening, a widening of science can end the dichotomy between science and philosophy. This widening of science is possible only if we revise our conception of time. To deny time — that is, to reduce it to a mere deployment of a reversible law — is to abandon the possibility of defining a conception of nature coherent with the hypothesis that nature produced living beings, particularly man. It dooms us to choosing between an antiscientific philosophy and an alienating science".

1889–1928 CE **Santiago Ramon y Cajal** (1852–1934, Spain). Histologist. A pioneer of modern neurophysiology. First to formulate the *neuronal theory* (based on the individuality of the nerve cell) which replaced the older view of a reticular system of nerve channels through which impulses were distributed. In his research he was able for the first time, to display the structure of individual cells and the contact of dendrites with adjacent cells by modifying a hitherto unreliable method of *staining*. This new staining technique

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654 Quoted from *Order Out of Chaos* by Ilya Prigogine and Isabelle Stengers, Bantam Books, New York, 1984.

Bergson failed in this respect because he was too deeply versed into the physical doctrine of his time: The equilibrium thermodynamics of the 19^{th} century was based on the second law, which predicted a gradual disorganization of the system. It could not account for the daily observations which showed the reverse phenomena. Consequently, *vitalistic theories* were invoked whereby it was suggested that biological organisms obey laws that are not part of ordinary physics and chemistry.

also provided for long-distance *tracing of axons* to other parts of the brain or junction with other nerve bundles.

Cajal was born in Petilla de Aragon, Navarra. After taking his degree in medicine at Zaragoza University in 1873, he joined the Spanish army as a medical officer, serving in Cuba during the Spanish-American war. From 1892 he held the chair of histology at Madrid University, and in 1906 he received the Nobel prize for medicine, shared with **Camillo Golgi**.

1889–1930 CE **Herbert Henry Dow** (1866–1930, USA). Chemist and manufacturer. Discovered electrolytic method for extracting bromine from brine (1889); organized chlorine-extracting firm (1895); founded Dow Chemical Co. (1897). Developed and patented over 100 chemical processes.

Dow was born in Belleville in Ontario, Canada. He graduated from Case School of Applied Science (1888) with a B.S. degree.

During Dow's lifetime, the company obtained its bromine, chlorine, sodium, calcium, and magnesium from the brine (sea water) of ancient seas under Midland, Ohio. But Dow, like **Fritz Haber**, in Germany, developed experimental processes to mine modern seas.

Three years after his death, his company opened its first seawater plant in North Carolina. By WWII, Dow plants on the Gulf Coast were in position to supply magnesium for firebombs and to make lightweight parts for airplanes.

1890 CE **Alfred Marshall** (1842–1924, England). Economist. A founder of the school of *neoclassical economics.* Professor at Cambridge University (1845–1908).

Previously, the mechanism of supply and demand was considered only in a single market that is assumed to be an infinitesimally small but representative fraction of the whole economic system (*microeconomics*). Marshall's analysis covered markets for factors of production (labor, land, etc) as well as commodities; and it made pioneering contributions to the study of adjustment processes and stability, notably in applying the concept of *elasticity*[655] and in distinguishing among time periods required for different types of adjustment (capital costs and the like being fixed in the short run but variable in the long run).

[655] A quantitative method to measure the public's responsiveness to a price change. Such a measure is given by the ratio of the percentage of change in *demand* to the percentage of change in price. The ratio when x units are sold is known as the *price elasticity*:
If the price p is regarded as a function of the demand x, and a change in demand Δx corresponds to a change of price Δp, then the elasticity $E(x)$ of the price

1890 CE **Herman Hollerith** (1860–1929, U.S.A.). Statistician. Invented the electromechanical *punched-card* calculating machine. It was the first major advance of automatic computing since **Babbage**.

In 1886, the returns of the 1880 U.S. census were still being counted and sorted and it was clear that, with the methods then existing, the job would still be unfinished in 1890, when the next census was due. Hollerith, on the staff of the U.S. Bureau of the Census, saw that the solution lay in some measure of mechanization, and set about the task of devising suitable equipment. He was familiar with the punched-card system of control used on the *Jacquard* looms (1805), and realized that the answer to many census questions, which are of the 'yes' or 'no' type, could be represented by the presence or absence of a hole in a particular position on a Jacquard type card. The answers to more complex questions could be represented in coded form by the presence or absence of holes in a *group of positions*. He also realized that the positions of holes in a card could be detected by electrical means: the presence of a hole would allow a current to flow through; the absence of a hole would stop it.

Hollerith experimented with devices based on this principle for sorting and counting — the main census operations — and some of his machines were used for analyzing the U.S. Census in 1890. Thereafter progress was rapid: the range of 'Hollerith' machines was extended to deal with most of the operations of office arithmetic.

During the first half of the 20^{th} century, punched-card equipment has been extensively applied to the ever increasing mass of clerical work in commerce, industry, and administration — and to a lesser extent, to scientific and technical calculations.

1890–1901 CE **Emil Adolf von Behring** (1854–1917, Germany). Microbe-hunter, bacteriologist, physiologist. Pioneer in immunology. Discovered *antibodies*. Explained that both tetanus and diphtheria immunity depend

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with respect to the demand is defined by

$$E(x) = -\frac{p(x)}{xp'(x)}$$

where $p(x)$ is the price per unit of an item when x units are demanded, or sold. The price function is said to be *elastic* when $E(x) > 1$ and *inelastic* for $E(x) < 1$. The second case indicates that a decrease in price is accompanied by a decrease in total revenue, while in the first case a decrease in price will increase the total revenue. The concept of 'elasticity of demand' was previously introduced by **Cournot** (1838).

on the capacity of the cell-free blood serum to neutralize the toxic substance produced by the tetanus/diphtheria bacilli. Developed vaccine against tetanus and introduced the concepts of passive immunization and antitoxins[656] (1890).

Behring was born at Deutsch-Eylau. Worked at the Koch Institute of Hygiene, Berlin (1889–1994); professor at Hale University (1894–1851), Marburg (1895 ff). Won the Nobel prize for physiology or medicine (1901).

Seismology[657] — *Birth of a New Science (1889–1936)*

Early historical records contain references to earthquakes as far back as 2000 BCE. ***Aristotle*** *(ca 340 BCE) gave a classification of earthquakes into six types, according to the nature of the earth movement observed; for example, those which caused an upward earth movement, those which shook the ground from side to side, etc.*

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656 *Antitoxin*: A substance with the ability to counteract the effect of toxin or poison; the specific antibody capable of neutralizing the pathogenic toxin.

Toxin: The Greek word for bow is *toxon*. The Greeks used *toxikon* for the poison in which the arrow was dipped; hence the English *toxin*, *toxic*, *antitoxin*.

Poison: Was originally a harmless draught which the Old French borrowed from the Latin *potionem* from *potare*, *potum* = to drink; but with the medieval practice of lethal beverages it took on its fatal sense.

This etymology has yet another twist: the word tocsin is composed of two parts *toc* (knock on a door) + the Latin *signum* which together implies: alarm, bell!

657 For further reading, see:

- Ben-Menahem, A. and S.J. Singh, *Seismic Waves and Sources*, Dover: New York, 2000, 1102 pp.

The earliest instrument made to respond to earthquake ground motion, known to us, is the seismoscope, invented in 132 *CE by the Chinese scholar* ***Chang Heng****. It consisted of a column so suspended that it could move in one of* 8 *directions; a ball was held lightly along each of these lines and, when thrown down by the rod, was caught in a cup below and so revealed the direction of motion. [Later seismoscopes were designed to give the time of occurrence of a shock: They were equipped with horizontal rod lightly pivoted at one end and provided with teeth below so that, when the rod fell, the teeth caught a pin projecting from the pendulum of a clock.] This instrument is reputed to have detected some earthquakes not felt locally.*

The ancients attributed earthquakes to supernatural powers; indeed, a writer in the Philosophic Transactions of the Royal Society of London, as late as 1750 *CE, deemed it expedient to apologize to 'those who are apt to be offended at any attempts to give a natural account of earthquakes'. Notwithstanding, stubborn facts of earthquake effects continued to accumulate, especially in the wake of the disastrous Lisbon earthquake of* 1755.

Finally it was firmly established in 1760 *by* ***John Michell*** *(England) that earthquakes originate within the earth. He declared that "earthquakes were waves set up by the shifting masses of rock miles below the surface... the motion of the earth in earthquakes is partly tremulous and partly propagated by waves which succeed each another", and he estimated that the earthquake waves after the Lisbon earthquake had traveled outward at* 530 *m/sec.*

Most of the work on earthquakes during 1760–1840 *was concerned with appraisals of geological effects of earthquakes, and of effects on buildings. Early in the* 19^{th} *century, earthquakes lists were being regularly published, and in* 1840 *there appeared the first earthquake catalogue for the whole world.*

Meanwhile a great deal of progress had been taking place on the theoretical front, namely the theory of elasticity. In 1638, ***Galileo*** *investigated the behavior of a loaded beam attached at one end to a wall. He found that with increasing load the beam bends around an axis perpendicular to its length and situated in the plane of the wall. Even though he did not give any mathematical relations between load and deformation, his works were pioneering in elasticity theory.*

In 1660 ***Robert Hooke*** *established the linear relationship between stress and strain in one dimension, which forms the basis for the mathematical theory of elasticity, and still serves as a good first approximation to the elastic conditions in the earth.*

During 1821–1830, *the French mathematicians* ***Navier****,* ***Cauchy*** *and* ***Poisson*** *laid the foundation to the mathematical theory of dynamic elasticity relevant to seismology. In particular, Poisson (1828) predicted the existence* *of longitudinal and transverse waves, moving with different speeds in the interior of perfectly elastic substances (known in seismology as P and S waves, respectively). In* 1845 ***Stokes*** *defined the moduli of compressibility and rigidity for isotropic elastic bodies, and in* 1849 *he conceived the first mathematical model of an earthquake point-source.*

In 1857, *the first true seismologist (as we would now recognize the term in hindsight), appeared on the scene: He was* ***Robert Mallet***[658] *(1810–1881, Ireland), the engineer who laid the foundation of instrumental seismology.*

The first seismometer[659]*, worthy of the name, was designed in* 1841 *by the physicist* ***James David Forbes*** *(1809–1868, Scotland). It consisted of an inverted pendulum, hinged below by a cylindrical steel wire. A pencil attached to the top of the pendulum rod, recorded the motion on paper.*

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658 He was born in Dublin, and after taking his degree at Trinity College in that city, he went into his father's small engineering factory. After building a lighthouse and a number of bridges, he became interested in global seismicity and earthquake engineering problems. His detailed study of the damage caused by the Napolitan earthquake of 1857 led him to suggest the setting up of a network of observatories over the earth's surface. He published the first world seismicity map (including material from many books) and made the first systematic attempt to apply physical principles to earthquake effects (1860–1862). He made estimates of the epicentral depth and also carried out a number of experiments to determine the velocity of earth waves, by setting off charges of explosives in different soils and by measuring the effects on *bowls of mercury* set at varying distances up to 800 meters away.

659 The name was coined by **David Milne Home** in 1841. A few years later, the name *seismograph* was given to an instrument built by **Luigi Palmieri** (1855) in the observatory on Vesuvius.

The word derives from the Greek $\sigma\epsilon\iota\sigma\mu\acute{o}\delta$ = earthquake. A *seismometer* is an instrument that amplifies and records small movements of the ground. Most sensitive seismographs magnify ground motion by as much as ten million times. It consists of a weight suspended from a frame by a spring. The frame moves with the ground, but the mass, due to its inertia, tends to remain stationary (evidently, any instrument containing a pendulum can be considered as a kind of seismograph). The *relative* motion between the mass and the frame is magnified by using an electromagnetic transducer and an electronic amplifier. The amplified signal controls a recording device that displays the ground motion in analog or digital form. Seismographs can detect ground movements of the order of an Angström (10^{-8} cm). Most seismographs are designed to measure ground *velocity*. Others are capable of monitoring ground *displacements*, *accelerations*, and *strains* (extensions, tilts, rotations).

The first useful seismograph system was constructed in Japan in 1880 *by* ***John Milne*** *and his assistants* ***James Alfred Ewing*** *and* ***Thomas Gray****. But this instrument had insufficient magnification and could record only Japanese earthquakes. However, on April 1889,* ***Ernst von Rebeur-Paschwitz*** *(1861–1895, Germany) was experimenting in Potsdam with a modified form of* ***Zöllner****'s horizontal pendulum* $(V_0 = 50,\ T_0 = 18\ sec,\ no\ damping)$ *when an earthquake from Japan was recorded. This event marks the birth of instrumental seismology in its world-wide sense.*

Stimulated by these observations, Milne was able by 1893 *to design, construct and test the now famous seismograph which bears his name. It was capable of detecting earthquake waves which had traveled many thousands of kilometers from their origin. Moreover, it was sufficiently compact and simple in operation to enable it to be installed and used in many parts of the world. It could record all three components of the ground motion (up-down, east-west, north-south). From this time onwards, precise instrumental data on earthquakes began to accumulate, and seismology has developed from the qualitative towards the quantitative side.*

The seismograph is to the earth scientist what the telescope is to the astronomer — a tool for peering into inaccessible regions. For that reasons one may consider the year of the deployment of the Milne seismographs as an important milestone in the history of seismology. Indeed, since 1893*, the number of instrumentally recorded earthquakes steady increased; the earliest known list of earthquakes with computed origin-times and epicenters is that for the period* 1899–1903*. Further improvement in the design of seismographs was due to* ***Emil Wiechert*** *(1861–1928, Germany) who gave a detailed account of his mechanical seismograph*[660] *(1900) and* ***Boris Borisovich Golitzin*** *(1862–1916, Russia) who designed the first electromagnetic seismograph with*

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660 **Wiechert** designed a seismograph in which the pendulum is vertical and inverted, being maintained by small springs pressing against supports rigidly attached to the ground. The mass of the pendulum is large (up to several tons), and the seismograph records both horizontal components at once. A cardinal development took place when **Golitzin** introduced the idea of recording ground motion by means of a ray of light reflected from the moving mirror of a galvanometer: the motion of the mirror is excited by an electric current generated by electromagnetic induction when the pendulum of the seismometer moves. The strain seismometer measures the variation in the distance between two points, some 30 meters apart, caused by the passage of seismic waves. **Benioff**'s recording was electromagnetic, the original galvanometer period being 40 sec, subsequently increased to 480 sec. His strain seismograph was the first to record earth motions with periods up to the order of one hour, such as the gravest mode of the *free oscillations of the earth* (1952).

photographic recording (1906). The next development came in 1935, *when* ***Hugo Benioff*** *(1899–1968, U.S.A.) designed and constructed an instrument to measure a component of ground strain, instead of the usual ground displacement.*

The science of seismology aims simultaneously to obtain the infrastructure of the earth's interior with the aid of seismic wave phenomena, and to study the nature of earthquake sources with the ultimate goal of mitigating and eventually controlling the phenomenon. This double feature is apparent from the early days of the science.

The achievements toward the first goal began in 1799, *when* ***Cavendish*** *employed Newton's law of universal gravitation to estimate the earth's mean density* $\left[\langle\rho\rangle = \frac{3}{4\pi G}\frac{g(R)}{R} \simeq 5.5\ \frac{g}{\text{cm}^3}\right]$. *As this density exceeded the density of surface rocks, the conclusion was that the density must increase with depth in the earth. By means of observations of the tidal effect in the solid earth,* ***Lord Kelvin*** *claimed in* 1863 *that the earth as a whole is more rigid than glass. [This opinion has been confirmed later, when it was found that steel offers a better comparison, where the gravest mode of the earth's free oscillation is concerned.]*

In 1897, ***Wiechert*** *conjectured from theoretical calculations that the earth's interior consists of a mantle of silicates, surrounded a core of iron. The existence of the earth's core was established by* ***Richard Dixon Oldham*** *(1858–1936,* India and England*) in* 1906, *from observations of earthquake waves.*

In 1909, **Andrija Mohorovičic** *(1857–1936,* Croatia*) discovered*[661] *a sharp material discontinuity at some level below the earth's surface (known*

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[661] This was known to **John Milne** already in or prior to 1906! In his *Bakerian Lecture* delivered March 22, 1906, and published in the *Proceedings of the Royal Society of London A* **77**, 365–376, he reported an outcome of recent seismological research in the following words: "*Preceding the large waves of a teleseismic disturbance we find preliminary tremors... for (ray paths) which lie within a depth of* 30 *miles, the recorded speeds do not exceed those which we would expect for waves of compression in rocky material. This, therefore, is the maximum depth at which we should look for materials having similar physical properties to those we see on the earth's surface. Beneath this limit, the materials of the outer part of this planet appear rapidly to merge into a fairly homogeneous nucleus with high rigidity*".

In the same lecture Milne was also the first to observe (1906) that breaks in the trajectory of the secular motion of the earth's North Pole (relative to its mean position) could be correlated with the occurrence of major earthquakes during 1892–1904. A quantitative theory of this effect was only given in 1970.

today as the Moho), which could explain the travel-times of seismic rays from a local earthquake. It was subsequently found to demarcate the base of the earth's crust. This discovery demonstrated that the structure of the earth's outer layers could be deduced from travel-times of reflected and refracted seismic signals.

In 1914, ***Beno Gutenberg*** *(1889–1960,* Germany and U.S.A.*), published his accurate determination of the depth of the boundary of the earth's core at* 2900 *km below the surface. He speculated that this discontinuity divides a liquid core of radius* 3500 *km from a solid mantle*[662]*. [In* 1955 *he discovered a global low velocity zone at depth* 70–250 *km in the earth's mantle*[663]*]. In 1936,* ***Inge Lehmann*** *(1888–1993,* Denmark*) produced the first evidence of the existence of the earth's solid inner core with a radius of ca* 1400 *km.*

The advent of elastodynamics began with the discovery of longitudinal and transverse waves by ***Poisson*** *in* 1828*, and their physical interpretation by* ***Stokes*** *in* 1845*. In* 1885*,* ***Lord Rayleigh*** *discovered, ahead of observations, another type of elastic waves (to be known later as the Rayleigh wave) that is associated with material discontinuities such as a free surface of a body.*

In 1897, ***Oldham***[664] *identified on earthquake recordings (seismograms) the three main types of waves predicted by Poisson and Rayleigh, thus confirming that, at least for short period wave-motion (dominating periods: 0.1–1 sec), the earth indeed behaves like an elastic body for which Hooke's law may*

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[662] In his treatise *Principles of philosophy* (1644), **Descartes** made one of the first attempts to speculate about the earth's interior. He wrote that the earth had a central nucleus made of primordial, sun-like fluid surrounded by a solid, opaque layer. Succeeding concentric layers of rock, metal, water and air made up the rest of the planet. In the current view, the earth possesses a solid inner core and a molten outer core. Both consist of iron-rich alloys. The earth's composition changes abruptly about 2900 km below the surface, where the core gives way to a mantle made of solid magnesium-iron silicate minerals. Another significant discontinuity, located 670 km below the surface marks the boundary between the upper and lower mantle (the lattice structure of the mantle minerals changes across that boundary because of high pressure).

[663] Known as the *Astenosphere*. Now believed to be due to partial melting (1–10%) of basaltic magma. The major mineral in the earth's mantle is *Olivine* (Mg_2SiO_4 with Fe_2SiO_4). In the Astenosphere, shear-wave velocities take low value and seismic waves are more strongly attenuated.

[664] Joined the Geological Survey of India in 1879. Retired in 1903.

apply. In 1899, ***Cargill Gilston Knott*** *(1856–1922, Scotland) derived the general equations for reflection and refraction of plane elastic waves at plane boundaries. This was needed to relate the amplitudes of the waves activating the seismometer to the corresponding seismogram traces, modified by the presence of the free surface of the earth.*

In 1904, ***Horace Lamb*** *(1849–1934, England) came forth with the first mathematical theory of a point-source earthquake in a half-space earth model. He thus laid the theoretical foundation for the propagation of seismic waves in layered media.*

The first inverse problem in geophysics was formulated and solved in 1907 *by* ***Gustav (Ferdinand Joseph) Herglotz*** *(1881–1953, Germany), enabling the intrinsic compressional and shear velocities to be determined from travel-time data*[665]*. By* 1909 ***E. Wiechert***, ***K. Zoeppritz***, *and* ***L. Geiger*** *ex-*

[665] In seismology, observations are mostly made at seismograph stations on the earth's surface. Rays emitted from an earthquake source (*focus*), eventually reach the stations located at various distances from the point of the earth's surface above the source (*epicenter*). The distance from the epicenter to the observing point is the *epicentral distance*. For the case when both the source and the receiver are on the earth's surface, we have the relation:

$$\Delta(p) = 2p \int_{r_m}^{a} \frac{d(\ln r)}{\sqrt{r^2/V^2 - p^2}},$$

where Δ is the angle subtended by the seismic *ray* at the earth's center (equal in this case to the angular source-receiver distance), and $p = \frac{dT}{d\Delta}$ is the ray-parameter. [This relation was discovered by **Hans Benndorf** (1870–1953, Germany) in 1905.] T is the travel-time along the curved ray, r_m is the distance from the earth's center to the lowest point of the ray, and $V(r)$ is the intrinsic wave velocity at radial coordinate r and a is the earth's radius.
Knowing $p(\Delta)$ (travel-time data) for a sufficiently dense grid of points in some interval $0 \le \Delta \le \Delta_1$, the above equation turns into an integral equation for $V(r)$. It leads to the *Abel integral equation*

$$f(x) = \int_x^b \frac{u(y)dy}{(y-x)^k} \qquad (0 < k < 1)$$

for the unknown $u(y) = \frac{d}{dy}\ln r$ with $y = \left(\frac{r}{aV}\right)^2$, $x = \left(\frac{r_m}{aV_m}\right)^2$, and $f(x) = \frac{1}{2\sqrt{x}}\Delta(x)$.

Its explicit solution:

$$u(y) = -\frac{\sin \pi k}{\pi}\frac{d}{dy}\int_y^b \frac{f(x)dx}{(x-y)^{1-k}},$$

ploited this method to obtain for the first time a profile of compressional wave velocity in the earth's mantle.

A significant contribution to theoretical seismology was made in 1911 *by* ***A.E.H. Love***[666] *(1863–1940, England) with his discovery of a horizontally-polarized surface-wave (now known as the Love-wave), from the analysis of which seismologists could derive estimates of the thickness of the earth's crust and its rigidity.*

Further advance during 1915–1936 *was made by* **Harold Jeffreys** *(1891–1989, England), who brought to bear mathematical and statistical methods and a great knowledge of wider geodynamical problems. His attention to scientific method and statistical detail has been one of the main forces through which pre-WWII seismology has attained its level of precision.*

Significant progress in seismology has been made through the first four decades of the 20^{th} *century: In* 1901*, the first Geophysical Institute was founded in Göttingen (Germany), and the number of seismic observatories capable of teleseismic recording did not exceed 25 (compared to* 8 *in 1894). By* 1940*, there were about* 10 *major seismic research centers and* 250 *seismic stations around the globe.*

An international Association of Seismology was founded in 1905 *at a meeting of representative of* 23 *countries in Berlin, and met in Rome in* 1906 *where it was decided to establish an international center at Strasbourg. The year* 1919 *saw the appearance of a bulletin for global recordings of earthquakes, published at Oxford, under the name International Seismological Summary (I.S.S.).*

Following the catastrophic San-Francisco earthquake of April 18, 1906, ***Harry Fielding Reid*** *(1859–1944, U.S.A.), advanced his elastic rebound theory (1910). Earthquakes are associated with large fractures, or faults, in*

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can be recast in the form:

$$V(r_1) = \frac{a}{p(\Delta_1)} \exp\left[-\frac{1}{\pi}\int_0^{\Delta_1} \mathrm{ch}^{-1}\left\{\frac{p(\Delta)}{p(\Delta_1)}\right\} d\Delta\right],$$

where a is the radius of the earth and Δ_1 is the epicentral distance for a ray that bottoms at $r = r_1$. The integration extends over *a family of rays* for *each* specific depth.

666 **Augustus Edward Hough Love** was a Sedleian professor of natural philosophy at Oxford University during 1899–1940. He discovered a horizontally-polarized guided shear wave that propagates in the earth's crust (1911). It was subsequently named after him ('*Love wave*'). His name is also associated with a dimensionless number in the theory of earth tides ('*Love number*').

the earth's crust and upper mantle. As the rock is strained, elastic energy is stored in the same way that it is stored in a wound-up watch spring. The strain builds up until the frictional bond that locks the fault can no longer hold at some point on the fault, and it breaks. Consequently, the blocks suddenly slip at this point, which is the focus of the earthquake.

Once the rupture is initiated it will travel at a speed of about 3.5 *km/sec*[667], *continuing as much as* 1000 *kilometers. In great earthquakes, the slip, or offset, of the two blocks can be as large as* 15 *meters. Once the frictional bond is broken, the elastic strain energy, which had been slowly stored over tens or hundreds of years, is suddenly released in the form of intense seismic vibrations — which constitute the earthquake*[668]*. The process through which the frictional bond is 'lubricated' to enable the commencement of the slip is yet not understood.*

The time between great earthquakes is about 50–100 *years in California and somewhat less in more active seismic regions, such as Japan or the Aleutians. Thus the time required to build up the elastic strain energy in the rocks adjacent to a fault is enormous compared with the time that elapses during the release of stored energy.*

The present state of knowledge of earthquake phenomena precludes the reliable prediction of the time of occurrence of the next major earthquake in any given location. Perhaps the most adequate answer to such questions was given long ago by Mark Twain: "I was gratified to be able to answer promptly, and I did. I said I did not know".

Since 1556, *an estimated* 3.5 *million persons were killed by earthquakes.*

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[667] This was first discovered, both experimentally and theoretically, by **Ari Ben-Menahem** (Ph.D thesis, CALTECH, 1960). It led to establishing of a novel intrinsic magnitude scale of earthquakes based on the physical concept of '*earthquake moment*' (**A. Ben-Menahem** and **D.G. Harkrider**, *Journal of Geophysical Research* **69** 2605–2620, 1964).

[668] About 10^9 erg of strain-energy is released from each cubic meter of the earthquake source volume. The greatest earthquakes release such energy from a strained volume of $1000 \text{ km} \times 100 \text{ km} \times 100 \text{ km} = 10^{16} \text{ m}^3$, which gives a total of 10^{25} erg. This is about the equivalent of 1000 nuclear explosions, each with strength of 1 megaton (1 million tons) of TNT.
It is of interest to note that the few large earthquakes each year release more energy than hundred of thousands of small shocks combined. About 10^{26} erg of seismic energy are released each year. This is about 1 percent of yearly amount of the heat energy reaching the earth's surface from the interior.

The Primeval 'Seismologist'

Homo sapiens invented the seismograph and discovered Rayleigh waves some hundred years ago. Nature, however, produced 60 *million years ago an arthropod, devoid of visual, auditory or olfactory senses, but equipped (in modern terminology) with a mobile array of* 8 *seismometers, amplifiers, and a minicomputer that enables it to locate its subsurface prey from amplitudes and travel-times of P waves and Rayleigh waves in the sand.*[669]

The sand scorpion Paruroctonus mesaentis, a nocturnal hunter of the Mojave Desert, has receptors on its legs that are extraordinary sensitive to subtle disturbances of the sand. With this unusual prey-detection mechanism it derives information needed to locate its prey. It essentially locates the source of a signal by detecting and interpreting minute differences in the time and amplitude of mechanical waves through the sand by means of its spatially separated sensors.

The sand scorpion is one of the largest dune arthropods, growing to a length of 8 *centimeters and a weight of* 4 *grams over the course of its* 5- *to* 6-*year lifetime. It can detect disturbances as far away as* 30 *centimeters. At a distance of* 10 *centimeters or less their estimates of target angle and distance are virtually perfect. It determined the turning angle towards its prey by integrating the input from all its legs.*

As a granular disaggregated medium, sand acts as a reasonably good conductor of mechanical waves up to a distance of several decimeters in the 1 *to* 5 *kilohertz bandwidth; lower frequencies are damped and higher frequencies are scattered. Of the four types of elastic waves that can propagate in solids, sand conducts only P waves and Rayleigh waves. Typical group velocities of P waves are* 150 *m/sec at* 5 *kilohertz and those of Rayleigh waves are* 50 *m/sec at the same frequency. The wavelengths corresponding to these frequencies commensurate with the size of the scorpion.*

Two types of mechanoreceptors on the tarsal (terminal) leg segment of the scorpion are sensitive to subtle vibrations of the substrate: Hairs protruding

[669] Philip H. Brownell, *Compressional and surface waves in sand: used by desert scorpions to located prey*, *Science* **197**, 479–482, 1977. (Also in *Scientific American*, December 1984).

from the sides and bottom of the tarsus rest on and between sand grains. The basitarsal slit sensillum consists of regions where the cuticle folds in on itself. The slit sensillum is particularly sensitive to vibrations that compress the slits in a direction perpendicular to their long axis; it is capable of detecting movements in the substrate that have amplitudes of about one Angström unit (10^{-8} *cm). Experiments have shown the hairs detect the compressional waves, and the slit sensila register the arrival of Rayleigh waves. The adult scorpion's eight legs form a roughly circular sensor "array" about* 4 *to* 6 *centimeters across.*

When the sand is disturbed, the first signals to arrive at the tarsus are compressional waves, which stimulate the tarsal hairs, causing large amplitude action potentials to ascend the leg nerve. A few milliseconds later, the vertical ground particle motion associated with the slower-traveling Rayleigh wave compresses the slit sensillum, triggering smaller-amplitude signals.

Rayleigh-wave stimulation of the slit sensillum appears to be the basis of the scorpion's perception of target direction. It may also exploit the time-delays of both P and Rayleigh waves across his "array"; assuming a sensory field of 5 *centimeters in diameter, this time delay would be about one millisecond for a Rayleigh wave, and* 0.3 *millisecond for a compressional wave. It might then determine the direction of the source from the time delay between stimulation of sensors close to the source and those further away; that is, the scorpion might simply turn in the direction of the sensors that are stimulated first (many animals use smaller time delays to locate the source of compressional waves propagated in the air; humans, for example, can easily judge the direction of a sound source on the basis of a time delay between the two ears of less than* 10 *microseconds).*

Alternatively, a scorpion might gauge the direction of a wave source from differences in the intensity with which the wave stimulates different sensors; as a wave propagates, its amplitude decreases, partly because the wave front expands geometrically, spreading out the energy of the wave, and partly because the signal is absorbed by the medium. Sensors nearest to the source should thus be stimulated most intensely.

Experiments have shown that the scorpion can detect time delays as small as 0.2 *milliseconds, but they respond most consistently to delays of one to two milliseconds — roughly the time it takes for a Rayleigh wave to traverse the span of their legs. There remains only the question of how the scorpion perceives the distance to its prey, i.e., how does it translate time delays into distance.*

Field observations showed that it rarely missed at 10 *centimeters or less. One possibility is that the animal times the delay between the arrival of the fast-moving compressional wave and the slow-moving Rayleigh waves.*

The delay would be proportional to the distance of the source. The second possibility would involve sensing the gradient of the amplitude of the wave across the "array", which increases with the decrease of its distance from the prey.

In any case, the evolutionary process endowed this creature with a suitable "computer" to achieve this task since its mere existence must rely so exclusively on information transmitted through the ground.

1890–1897 CE **David Schwarz** (1845–1897, Germany). Invented, designed and built the first *airship* (*metal* dirigible balloon). It was made of aluminum, filled with gas and driven by a Daimler benzine motor [length = 48 m, diameter = 14 m, volume = 3700 m^3, weight = 3100 kg, speed = 27 km/h].

It was tested in Berlin in 1897: after flying for 4 hours, a driving belt slipped, and in descent the balloon was damaged beyond repair. Among the spectators was **Ferdinand von Zeppelin** (1838–1917), of the German army, who foresaw the potentialities of the airship for the military. He bought all plans and models of Schwarz's airship from his widow, and developed it further.

Schwarz was born in Hungary to Jewish parents and started as a successful lumber merchant in Zagreb. He then studied mechanical engineering on his own. Impressed by the special properties of the aluminum metal (its large-scale industrial production began during 1886–7 in America, England and France), he set forth to harness it to the construction of light airships. In 1890 he flew his first model in Austria and Russia, but failed to interest the respective governments. Finally, when in 1897 the Germans were ready to support his invention, the excitement caused his untimely death.

1890–1903 CE **Samuel Pierpont Langley** (1834–1906, USA). Astronomer, physicist, pioneer in aerodynamics and inventor. His steam-driven aeroplane flew for 90 seconds (1896) — the first flight by an heavier-then-air, engine-equipped, aircraft (uncrewed).

Langley was born in Roxbury, MA and attended Boston Latin School. He spent several years studying architecture and engineering before turning to astronomy. His several inventions included an instrument called *bolometer*, which measures the sun's radiation. He was a professor of physics and astronomy at the Western University of Pennsylvania (1866–1887), studying the infrared portions of the sun's spectrum. In 1890 he turned to pioneering work in aerodynamics, contributing greatly to the design of early aircraft wing shape.

The United States government gave Langley 50,000 dollars to build a man-carrying "aerodrome". After two failed attempts (1903) to get his flying machine off the ground, Samuel Pierpont Langley was criticized by the *new York Times* for wasting government funds on and idle dream.

A third attempt using a smaller model succeeded. The subsequent catapult-launched flights of the Wright brothers at Kitty Hawk owed much to Langley's principles as well as to the more powerful engines available by the early 1900's. The Langley design was tested in later years by using a model with a modern engine; it flew successfully with a pilot aboard.

1890–1908 CE **Edouard-Eugene Branly** (1844–1940, France). Physicist, physician and inventor. Invented the *coherer* (1890), a primitive form of radio detector that made wireless telegraphy possible. He thus established the principles later developed by Marconi. He also evolved the forerunner of the receiving antennae.

Branly was born in Amiens. He obtained a doctorate from the Sorbonne and a medical degree from the University of Paris. By 1908[670], he developed the *remote-controlled* torpedo, fired from a torpedo-boat and operated by electromagnetic waves via a relay system.

1890–1911 CE **Sebastian Ziani de Ferranti** (1864–1930, England). Engineer and inventor. Innovator in the development of electrical engineering who led the application of power generation and distribution.

Ferranti was born in Liverpool, where his father had a photographic art studio. At the age of 22 he became Chief Engineer of the London Electric Supply Corporation, and was deeply involved in the planning, generation and distribution of electricity. He was one of the first people to advocate large power generating stations sited outside of population centers and established the principle of the national grid, using alternating current transmission.

1891 CE *California Institute of Technology* (*Caltech*) founded.

1891 CE **Seth Carlo Chandler** (1846–1913, U.S.A.). Astronomer. Discovered a periodicity of 428 mean solar days in the spectrum of the latitude

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[670] By 1868, **Robert Whitehead** (1823–1905, England), engineer, had developed the first real torpedo. Powered by compressed air, it was completely self-propelled.

variation. This value exceeds Euler's (1765) theoretical value for the free precession of a rigid ellipsoid of revolution, by about 4 months. In 1892 **Simon Newcomb** (1835–1909, U.S.A.) explained this period lengthening as being due to the elastic yield of the earth.

The Chandler Wobble (1765–1909)

The equations of rigid gyroscopic motion were given by ***Euler*** *in* 1758. *On the basis of this theory, he suggested in* 1765 *that the earth might undergo a free precession with period* $A/(C-A)$ *sidereal days. Assuming this to be true, a spectator, partaking in the earth's motion, should observe periodic changes in latitude relative to the fixed stars. Indeed,* ***Lord Kelvin*** *urged astronomers in* 1876 *to look for a period of 10 months, as predicted by Euler. However, no such period could be found.*

Instead, ***S.C. Chandler*** *established in* 1891 *the existence of a* 428*-days period in the spectrum of the latitude variation.*

The lengthening of the period was explained by ***S. Newcomb*** *(1892) to be the result of the earth's elasticity. A theoretical verification was given by* ***A.E.H. Love*** *(1863–1940, England) and* ***J. Larmor*** *(1857–1942, England), based on first order theory of the figure of the earth.*

This 14*-month precessional motion of the instantaneous axis of rotation about the earth's axis of figure is known today as the Chandler Wobble. The source of excitation of this motion has not been fully accounted for.*

The Chandler Wobble is accompanied by two additional observed phenomena:

(1) Secular (transient polar shifts resulting from impulses and jumps in the source of excitation);

(2) Changes in the length of day due to such excitations.

The French novelist ***Jules Verne*** *(1828–1905) cleverly used realistic, believable explanations to support incredible tales of adventure. In his book "Sens dessous dessous" (1890) he concocted a plot in which a colossal missile* *of mass* $180,000$ *tons is launched at latitude* $45\,^\circ$, *in order to displace the pole by* $23.5\,^\circ$ *and so remove the obliquity of the ecliptic. Working out the physics of this problem for an earth model with no equatorial bulge* $(A = B = C)$, *one discovers a little fact which Verne did not bother to tell his readers — the earth will require 'only'* 10^8 *years to creep to the desired state!*

1891–1892 CE **Arthur Moritz Schönflies** (1853–1928, Germany). Mathematician and crystallographer. Classified the complete list of 230 *space groups*. Introduced the known *Schönflies notation* for *point groups*.[671]

Schönflies was born to Jewish parents in Landberg an der Warthe. Was a student at Berlin and did his doctorate (1877) under **Kummer** and **Weierstrass**. He taught in Berlin, Colmar, Göttingen, Königsberg and Frankfurt a. M. He worked mainly on set theory and crystallography.

1891–1892 CE **Almon Brown Strowger** (1839–1902, USA). Undertaker and inventor. Invented, patented and installed the first *automatic telephone exchange system*, known at that time as 'Strowger's switch'. It replaced the switchboard operator for placing local cells.

The first automatic exchange began operating in La Porte, Indiana (1892); the central office switch worked in concert with a similar switch at the subscribers home, operated by push buttons. The contact electromechanical switch, which operated the telephone, could select a line of a wanted subscriber. Later (1894) **A.E. Keith**, **J. Erickson** and **C.J. Erickson** invented the rotating finger-wheel needed for a dial which first began operating in Milwaukee's City Hall (1896).

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[671] The most important type of group in crystallography is the one which consists of the symmetry operations pertaining to molecular structure. For such a group the combining rule is one operation followed by another. Since the application of any symmetry operation leaves a molecule physically unchanged and with the same orientation in space, its center of mass must also remain fixed in space under all symmetry operations. From this it follows that all the axes and planes of symmetry of a molecule must intersect in at least one common point. Such groups are called *point groups*. For a crystal of infinite size we can have symmetry operations, e.g. translation, that leaves *no* point fixed in space; these give rise to *space groups*.

The automatic dial system changed telephony forever — it became "girl-less, cuss-less, out-of-order-less, and wait-less", and expedited the extension of the telephone network.

Strowger was born in Penfield, New York, a suburb of Rochester. He went to a New York state university, served in the Civil War (1861–1865), ending as a lieutenant. He then taught school in Kansas and Ohio and wound up first in Topeka and then in Kansas City as an undertaker (1886), an unlikely profession for an inspired inventor.[672]

1891–1896 CE **Edward Goodrich Acheson** (1856–1931, USA). Engineer and inventor. Produced silicon carbide, or *carborundum*[673] (1891). Invented a process for manufacturing *graphite* by heating a mixture of coke and clay (1896).

The discovery of carborundum, which is the hardest surface made by man and second only to diamond in hardness, ended the search for a highly effective and durable abrasive needed by industry to manufacture precision-ground interchangeable metal parts. One of the byproducts of the carborundum manufacturing process was graphite, which proved useful as a lubricant.

Acheson was born in Washington, Pennsylvania. In 1880 he had secured a position with Thomas Edison in his Menlo Park, N.J. laboratories and was involved in the development and installation of electrical lighting, including working on the lamp exhibit at the Paris Exhibition (1881).

1891–1913 CE **Alfred Werner** (1866–1919, Switzerland). Distinguished chemist. Father of *coordination chemistry.* First to put forward ideas on bonding which were eventually to revolutionize inorganic chemistry. His theory led to the discovery of many cases of *isomerism.* The importance of his ideas was amplified in modern times since it was discovered that the mode of action of many *enzymes catalysts*, that are essential for life processes, depends on the formation of *metal ion coordination complexes.*

The idea of a *privileged* central metal atom or ion surrounded by a group of tightly bound molecules or ions (e.g., as Mg in chlorophyll or Fe in hemoglobin) puts Werner in a class with **Kekulé** and **van't Hoff** before him, and **Pauling** after him as far as our present day understanding of molecular architecture is concerned.

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672 The story surrounding his motivation to invent the automatic switch is odder still: the wife of his competitor, working as a switchboard operator, gave busy signals to customers calling Strowger, thus stealing his business.

673 By mixing clay with carbon and fusing it electrically

Werner argued that the factor determining the structure of coordination compounds was not the primary valency of the central atom but the number of ions, atoms, radicals or molecules directly bonded to the metal[674], now known collectively as *ligands*. The ligands were postulated to be arranged in simple, *spatially geometric structures*, with the *octahedron* as the commonest arrangement. A corollary of this theory was that some coordination complexes should exist as *optically active isomers*.

Werner was awarded the Nobel Prize in chemistry (1913).

1891–1914 CE **Paul Painlevé** (1863–1933, France). Mathematician and statesman. Developed the theory of functions defined by non-linear differ-

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674 Werner was faced with a need to explain a perplexing experimental fact; cobalt chloride can bind itself to ammonia in 4 different ways: $CoCl_3{\cdot}6NH_3$, $CoCl_3{\cdot}5NH_3$, $CoCl_3{\cdot}4NH_3$ and $CoCl_3{\cdot}3NH_3$. There were two questions involved here: first, why was there such arbitrariness about the number of ammonia molecules. Second, when these cobalt complexes were dissolved in water and $AgNO_3$ added, one obtained strikingly different quantities of *insoluble* silver chloride precipitating from one mole of the complex:

$CoCl_3{\cdot}3NH_3 \longrightarrow$ no precipitate at all;
$CoCl_3{\cdot}4NH_3 \longrightarrow$ 1 mole AgCl precipitate;
$CoCl_3{\cdot}5NH_3 \longrightarrow$ 2 mole AgCl precipitate;
$CoCl_3{\cdot}6NH_3 \longrightarrow$ 3 mole AgCl precipitate.

Why not 3 moles of AgCl in each case? After all, aren't there 3 moles of chlorine available?

Werner suggested correctly that the *cobalt ions* form *octahedral* complexes with 6 surrounding groups (octahedron = a regular polyhedron with 6 vertices and 8 equilateral triangles faces). For $CoCl_3{\cdot}6NH_3$, all three chlorines are *loosely held* in an ionic bond $[Co(NH_3)_6]^{+++}\ 3Cl^-$ (like NaCl); for $CoCl_3{\cdot}3NH_3$, all three chlorine atoms are *tightly held* as $[Co(NH_3)_3Cl_3]$ such that Ag could not pull the chlorine atoms from this complex. The other two cases fall in between.

Another important aspect of coordination theory concerns the possible alternate spatial arrangements of the six different ligand groups coordinated about the metal atom. For example, in the case of $[Co(NH_3)_4Cl_2]^+$, the two chlorines can be on the same or opposite sides of the octahedron. That makes for electronic differences, and results in slightly different properties. The two *isomers*, as such compound are called, differ only in the spatial geometry or arrangements of atoms. Most evident immediately are their different colored solutions: the *trans* form, in which the chlorine are opposite, is green while the *cis* form, in which the coordination complex has the chlorines on the same side, is violet. Such spatial isomers are common in nature and of great importance in *living systems*.

ential equations of the first and second order. (*Painlevé property*, *Painlevé transcendents*[675], *Painlevé function*).

Painlevé was born in Paris, the son of a lithographic draughtsman, and was educated at the École Normale Superiere. He became a professor at the Sorbonne and the École Polytechnique in 1898.

His interest in dynamics led Painlevé to take part in the early development of aeronautics, on the practical as well as on the theoretical side: he was, in fact, one of the first passengers of **Wilbur Wright**.

From his 40^{th} year on, public affairs occupied a greater and greater portion of his time. In 1917 he became Minister of War in the government of M. Ribot; on Sept. 1917 he became Prime Minister. In 1925 he became Prime Minister for the second time. In 1930–1931 and 1932–1933 he was Air Minister. He died suddenly of heart failure, and was accorded public funeral in the Panthéon.

1891–1916 CE **Charles Proteus (Karl August Rudolf) Steinmetz** (1865–1923, US). Electrical engineer and inventor. Developed the theory of alternating-current (AC) phenomena (using complex numbers a la Heaviside). This enabled the design of AC machines to be made more efficient and consolidated the victory of AC over DC gained by Tesla in fierce competition with Edison.

Worked on the design of AC transmission, developing lightning arresters for high-power transmission lines; patented over 200 inventions, including improvements on generators and motors. Formulated the *Steinmetz hysteresis law* (1891), which describes the dissipation of energy that occurs when a system is subject to an alternating magnetic field. This made it possible to reduce loss of efficiency in electromagnetic systems.

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675 In the course of classifying nonlinear differential equations, he considered all equations of the form $w'' = R(z,w)(w')^2 + S(z,w)w' + T(z,w)$ where R, S, T are rational functions of w (but have arbitrary dependence on z). The solutions may have various kinds of *fixed singularities* (poles, branch points, essential singularities), but may not have *movable singularities* (its location depending on the initial conditions) except for poles. There are 50 distinct types of equations having these properties. Of those, 44 types are soluble in terms of elementary transcendents (sines, cosines, exponentials), functions defined by linear second-order equations (Bessel functions, Legendre functions, and so on) or elliptic functions. The remaining 6 equations define the 6 Painlevé transcendents, one of which is $y'' = 6y^2 + x$. Painlevé transcendents have recently found an application in the theory of random surfaces and two-dimensional quantum gravity

Steinmetz was born in Breslau (now Wroclaw, Poland) to Jewish parents. Educated there and at the Technical High School, Berlin. Forced to flee Germany (1888) because of his socialist activities just before receiving his Ph.D. from Breslau University, he completed his studies in Zürich. Migrating to the US (1889), he worked for an electrical firm in Yonkers until his monographs attracted wide attention. In 1893 he became chief consulting engineer in General Electric's Schenectady plant, where he spent the rest of his life experimenting with electrical appliances and machinery. He was a Professor of Electrophysics at Union College, Schenectady (from 1902) and authored several books on electrical theory.

Throughout his life, Steinmetz retained his belief in socialism and in later years favored Zionism.

1891–1917 CE **Roland von Eötvös** (1848–1919, Hungary). Experimental physicist. Established through his torsion-balance experiments that inertial and gravitational mass are equivalent to accuracy of 1 part in 10^9.

Eötvös was born in Budapest. In 1872 he was appointed professor of physics at the University of Budapest. During 1894–1895 he was Minister of Education.

1891–1921 CE **Eugene Dubois** (1858–1940, Holland). Physician, anatomist and paleontologist. The man who found the Missing Link in the Darwinian evolutionary trail from ape to human.

While serving as military surgeon in the Dutch East Indies (1887–1895), he discovered in Java the bones of a *hominid*, apparently intermediate between man and simian ancestors, which he named (1891) *Pithecanthropus erectus* (now *Hominid erectus*).

Dubois gave up a promising post at the University of Amsterdam to go to Java with the aim of finding a fossil of a prehuman that would be demonstrably the "Missing Link". After finding what he believed to be such a fossil he had to spend some thirty years defending his claim. He has been an underestimated scientist.

1891–1923 CE **George Ellery Hale** (1868–1938, U.S.A.). Astronomer. Advanced solar and stellar spectroscopy, discovered the existence of magnetic fields in sunspots[676] and founded three large observatories in the United States: Yerkes (1895), Mount Wilson (1904) and Palomar (1948).

[676] *Sunspots* are one of many phenomena associated with the 22-year solar cycle; they are irregularly-shaped dark regions in the photosphere of the sun. Although they vary greatly in size, typical sunspots measure a few tens of thousands of kilometers across. On very rare occasions, a sunspot is so large that it can be seen with the naked eye (using special dark filters!). Ancient Chinese

The last two were formally known as the *Hale observatory*, The Palomar Observatory's Hale telescope has a diameter of 508 cm (200 inch[677]).

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astronomers recorded such sightings 2000 years ago. **Galileo** (1612) was the first person to examine sunspots in detail and **Schwabe** (1843) discovered that the number of sunspots varies in a periodic fashion (*sunspot cycle* of about 11 years). **Maunder** (1904) discovered also a spatial periodicity, i.e. that the location of sunspots varies in a regular fashion over the sunspot-cycle: the first sunspots of a cycle appear at large distance from the solar equator, whereas the last spots of a cycle are formed very near the equator. At *sunspot maximum* in the middle of the cycle, most sunspots occur at latitude of 10° to 15° north and south of the equator.

In 1908, **Hale** observed the splitting of the Fe I spectral line into three lines corresponding to a very intense magnetic field of 4130 Gauss (compared to the terrestrial dipolar field of 0.7 Gauss). Hale also discovered that sunspot groups are *bipolar* and that the polarity pattern reverses itself every 11 years, making a complete cycle of 22 years through which the *solar surface features* vary (the *average number* of sunspots still increases and decreases in a regular 11-year-cycle). Hale's discovery demonstrates that sunspots are places where a powerful, concentrated magnetic field protrudes through the hot gases of the photosphere. Because of the temperature, many of the atoms in the photosphere are ionized, so that the photosphere is a mixture of electric charges. This *plasma* is an extremely good conductor of electricity, and it interacts vigorously with magnetic fields, which in turn, *restricts and contains* the motions of a plasma and *inhibits* the natural convective motions. Since energy cannot flow freely upward from the sun's convective zone, the plasma within sunspot *cools off*. This is why temperatures in a sunspot are typically 4000–4500 K, i.e. more than 1000 K cooler than the surrounding undisturbed photosphere. Because of this lowered temperature, sunspots look *dark* in contrast to their brighter surroundings.

A host of exotic phenomena occur around and above sunspots as a direct result of their intense magnetic fields. One of them – the *solar flare* – is a brief eruption of very hot ionized gases from a sunspot group; vast quantities of particles and radiation are blasted into space. When the resulting UV and X-rays, and solar wind surges, arrive at the earth a day or so later, they produce aurorae and interact with the gases of the upper atmosphere.

In 1960, the astronomer **Horace Babcock** put forward a *magnetic-dynamo* model which makes use of the sun's *differential rotation* and its *convective envelope* to explain the sunspot cycle as the result of the wrapping of a magnetic field around the sun: sunspots appear where the concentrated magnetic field has broken through the solar surface.

677 The Mount Wilson 150 cm reflecting telescope began observations in 1908, and the second, Hooker telescope (250 cm; 1917) was used to revolutionize astronomy, astrophysics, and cosmology in the 1920's and beyond. The 508 cm

Hale was born in Chicago, studied at M.I.T. and was professor at the University of Chicago (1897–1904). He invented the *spectroheliograph* [1891, with **Henri Deslandres** (1853–1948, France)], an instrument used to photograph the sun at a single wavelength. Founded the *Astrophysical Journal* (1895).

In 1908, Hale examined solar magnetic storms and determined that the *Zeeman effect* (1896) is apparent in the spectra of *sunspots*, namely, the splitting of spectral lines due to the strong magnetic fields associated with these sunspots. This led to his discovery (1919) of the periodic *reversal* in the polarization of their magnetic fields.

1891–1933 CE **Sven Andreas Hedin** (1865–1952, Sweden). Central Asia explorer. Drew the first maps and gathered information about areas in Persia, Turkestan, Tibet, China, and Mongolia. During his early travels, Hedin unearthed ancient cities in Turkestan deserts, and the Lop Nor basin of Western China. In 1893, he began a three-year trip over the Pamir, a mountainous plateau in Central Asia and the plateaus of Tibet. In the early 1900's Hedin explored the high sources of the Brahmaputra River, locating mountains and waters never before known. In 1933 he mapped the ancient silk trade route that extended 16,000 km from Asia to Europe.

Hedin was born in Stockholm to Jewish ancestry. By the time he was 22 he had already crossed the Elburz Mountains, traveled through Persia on horseback, crossed the Kara Kum, visited Bokhara and Samarkand, and crossed Tien Shan from Andizhan, in Ferghana, to Kashzar. Even an outline map of the routes he followed looks as if it was the work of a centipede whose feet had been dipped in ink. He described his travels in many books. During WWI, Hedin was a Nazi sympathizer.

1892 CE **Dmitri Iosifovich Ivanowski** (1864–1920, Russia). Microbiologist. Discovered a disease-causing agent smaller than bacteria — the *virus*.

Explained the infectiousness of tobacco mosaic disease (1892) by showing it can be transmitted via cell-free filtrates[678] of diseased plants to leaves of healthy plants.

1892–1894 CE **Richard Friedrich Johannes Pfeiffer** (1858–1945, Germany). Bacteriologist. First to observe a complex *immune reaction* (1894) of the body to an invading microbe. He injected live cholera vibrios (bacteria)

Palomar (Hale) telescope was completed in 1908; It was the world's largest until 1976, and retired in 1987 because of air and light pollution.

[678] An agent in the sap of leaves is not filtered out of the sap even with the so-called chamberlands' bacteriological filter. The term *filterable virus* was thus coined. Later, 'filterable' was dropped and *virus* took its modern meaning.

into guinea pigs which had already been immunized, then extracted some of the germs. Examining the extract under a microscope, he observed the germs becoming motionless, then swelling and finally disintegrating (a process he named *bacteriolysis*).

He showed that the same process occurred in vitro, and that the reaction would cease when heated over 60°C [This let **J. Bordet** to study the immune system and discover the *complement* (1898).] During the influenza epidemic of 1889–1892 he discovered the bacillus *Haemophilus influenzae* (1892), later found to be responsible for many of the complications of the influenza viral infection.

Pfeiffer was born near Posen and educated in Berlin as a military surgeon. He worked under **Koch** at the institute for Hygiene (1894) and became a professor of hygiene at Koenigsberg (1899) and Breslau (1909).

1892–1899 CE **Henri Eugene Padé** (1863–1953, France). Mathematician. Developed an important analytical method through which a function with singularities can be approximated as a ratio of two polynomials.

Padé was educated at the Ecole Normale Superieure in Paris and at Leipzig and Göttingen under **Klein** and **Schwarz** (1889–1890). He returned to France and obtained his doctorate under **Hermite's** supervision. He held positions at Besancon, Dijon and Aix-Marseilles.

Historically, Padé was motivated by the work of **Stieltjes** on the analytic theory of continued fractions (1889), which he came to know on his visit to Göttingen in 1890. His starting point, however, was the work of **Frobenius** (1881) who made a systematic study of those rational fractions.

1892–1905 CE **James Dewar** (1842–1923, Scotland). Chemist and physicist. First to produce liquid hydrogen (1898), later (1899) obtaining it as a solid. Studied the properties of matter at low temperatures. Invented the *Dewar vessel* (1892), forerunner of the vacuum bottle.

Dewar demonstrated (1898) that hydrogen, a gas which at normal temperatures tends to *warm upon expansion*, exhibits the normal Joule-Thomson cooling (1852) at temperatures below −80 °C. Hence, below −80 °C the Joule-Thomson effect allows a mechanism for further cooling of hydrogen to below its critical temperature for liquefaction.

Dewar was born at Kincardine-on-Forth, Scotland. He was educated at the universities of Edinburgh (under **Playfair**) and Ghent (under **Kekulé**). In 1877 he became Fullerian professor of chemistry in the Royal Institution, London. In 1904, he was the first British subject to receive the Lavoisier medal of the French Academy of Sciences. He was also a professor of natural experimental philosophy at Cambridge (1875–1923).

1892–1923 CE **Michael (Mihailo) Idvorsky Pupin** (1858–1935, U.S.A.). Physicist and inventor. His inventions led to great advances in long-distance telephone systems, telegraphy and radio transmission networks.

His main contributions:

- Multiplex telegraphy accomplished by electrical tunning (1892–1894)
- Extending the range of long-distance telephony by amplifying the signal at intervals along the line without distortion (1894)
- A rapid method for X-ray photography, shortening the time of exposure from about an hour to a few seconds (1896)
- Discovered the Secondary X-ray Radiation (1896)

Pupin was born in Idvor, Austria-Hungary (now Yugoslavia), a son of illiterate parents who encouraged his education. He arrived in America, a penniless immigrant, in 1874, and set out to understand the Maxwell theory like a knight in quest of the Holy Grail. First he went to Columbia University, but found nobody there who could explain Maxwell. Then he went to Cambridge, England, where Maxwell had worked; but Maxwell was dead, and Pupin's tutors were mainly interested in getting him good marks in the mathematical tripos.

Finally he went to Berlin, and there he found **Ludwig Boltzmann**, who taught Pupin what he knew about Maxwell's equations. Pupin was amazed to find out how few were the physicists who had caught the meaning of the theory, even 20 years after it was stated by Maxwell in 1865. He obtained his PhD degree at the University of Berlin (1888) and returned to the US in 1889.

After various adventures he became a professor of electromechanics at Columbia University in 1892. In 1923 he published his autobiography *From Immigrant to Inventor*, which won the 1924 Pulitzer prize.[679]

1892–1924 CE **Maxim Gorky** (**Aleksei Maksimovich Peshkov** 1868–1936, Russia). Novelist, humanist, social reformer and pioneer social-democratic thinker. The Socrates of modern times.

[679] It was estimated that Pupin's invention of the 'Pupin's coils' (loading a telephone wire with inductance coils) had saved over 100 million dollars in the first 22 years. Pupin asked: "Where are those one hundred million dollars which the invention has saved? I know that not even a microscopic part of them is in the pockets of the inventor".

Gorky was born in Nizhny Novgorod. He became orphan at age nine and was raised by his grandparents. At age 19 he traveled on foot across the Russian Empire, changing jobs and accumulating impressions used later in his novels, stories and plays.

In 1887 Gorky witnessed a *Pogrom* in Nizhny Novgorod. Deeply shocked by what he saw, Gorky became a life-long opponent of racism. Gorky worked with the *Liberation of Labor* group and in October, 1889 was arrested and accused of spreading revolutionary propaganda. He was later released because they did not have enough evidence to gain a conviction. However, the *Okhrana* decided to keep him under police surveillance.

In 1891 Gorky moved to Tiflis where he found employment as a painter in a railway yard. The following year his first short-story, *Makar Chudra*, appeared in the Tiflis newspaper, *Kavkaz*. The story appeared under the name Maxim Gorky (Maxim the Bitter). The story was popular with the readers and soon others began appearing in other journals such as the successful *Russian Wealth*.

Gorky also began writing articles on politics and literature for newspapers. In 1895 he began writing a daily column under the heading, *By the Way*. In this articles he campaigned against the eviction of *peasants* from their land and the persecution of *trade unionists* in Russia. He also criticized the country's poor educational standards, the government's treatment of the *Jewish* community and the growth in foreign investment in Russia.

His short stories such as *Twenty-six Men and a Girl*, often showed Gorky's interest in social reform. In a letter to a friend, Gorky argued that "the aim of literature is to help man to understand himself, to strengthen the trust in himself, and to develop in him the striving toward truth; it is to fight meanness in people, to learn how to find the good in them, to awake in their souls shame, anger, courage; to do all in order that man become nobly strong."

In 1898 Gorky published his first collection of short-stories. The book was a great success and he was now one of the country's most read and discussed writers. His choice of heroes and themes helped him emerge as the champion of the poor and the oppressed. The *Okhrana* became greatly concerned with Gorky's outspoken views, especially his articles and stories about the police, but his increasing popularity with the public made it difficult for them to take action against him.

Gorky secretly began helping illegal organizations such as the *Socialist Revolutionaries* and the *Social Democratic Labor Party*. He donated money to party funds and helped with the distribution of radical newspapers such as *Iskra*.

On the 4th March, 1901, Gorky witnessed a police attack on a student demonstration in Kazan. After publishing a statement attacking the way the police treated the demonstrators, Gorky was arrested and imprisoned. Gorky's health deteriorated and afraid he would die, the authorities released him after a month. He was put under house arrest, his correspondence was monitored and restrictions were placed on his movement around the country. When he was allowed to travel to the Crimea, he was greeted on the route by large crowds bearing banners with the words: "Long live Gorky, the bard of Freedom exiled without investigation or trial."

After *Blood Sunday* Gorky was arrested and charged with inciting the people to revolt. Following a world-wide protest at Gorky's imprisonment in the Peter and Paul Fortress, *Nicholas II* agreed for him to be deported from Russia.

In 1906 Gorky toured Europe and the United States. He arrived in *New York* on 28th March, 1906 and the *New York Times* reported that "the reception given to Gorky rivaled that of Kossuth and Garibaldi." His campaign tour was organized by a group of writers that included **Ernest Poole**, **William Dean Howells**, **Jack London**, **Mark Twain**, **Charles Beard** and **Upton Sinclair**.

In 1907 Gorky attended the Fifth Congress of the *Social Democratic Labor Party*. While there he met **Vladimir Lenin**, **Julius Martov**, **George Plekhanov**, **Leon Trotsky** and other leaders of the party. Gorky preferred Martov and the *Mensheviks* and was highly critical of Lenin's attempts to create a small party of professional revolutionaries.

Gorky continued to write and his most successful novels include *Three of Them* (1900), *Mother* (1906), *A Confession* (1908), *Okurov City* (1909) and the *Life of Matvey Kozhemyakin* (1910).

Gorky was strongly opposed the *First World War* and he was attacked in the Russian press as being unpatriotic. In 1915 he established the political-literary journal, *Letopis* (Chronicle) and helped establish the Russian Society of the Life of the Jews, an organization that protested against the persecution of the *Jewish* community in Russia.

Gorky started a newspaper, *New Life*, in 1917, and used it to attack the idea that the *Bolsheviks* were planning to overthrow the government of Alexander Kerensky. On 16th October, 1917, he called on Vladimir Lenin to deny these rumors and show he was "capable of leading the masses, and not a weapon in the hands of shameless adventurers of fanatics gone mad."

In January, 1918, Gorky led the attack on Lenin's decision to close down the *Constituent Assembly*. Gorky wrote in the *New Life* that the *Bolsheviks* had betrayed the ideals of generations of reformers: "For a hundred years the best people of Russia lived with the hope of a Constituent Assembly. In this struggle for this idea thousands of the intelligentsia perished along with tens of thousands of workers and peasants."

The Bolshevik government controlled the distribution of newsprint and in July, 1918, it cut off supplies to *New Life* and Gorky was forced to close his newspaper. The government also took action making it impossible for Gorky to get his work published in Russia.

In 1921 Gorky once again clashed with the Soviet government over the suppression of the *Kronstadt Uprising*. Gorky blamed **Gregory Zinoviev** for the way the sailors were treated after the rebellion. Gorky failed to save the life of the writer, **Nikolai Gumilev**, who was arrested and executed for his support for the Kronstadt sailors. He was also unsuccessful in obtaining an exit visa for the poet, **Alexander Blok**, who was dangerously ill. By the time Zinoviev gave permission for Blok to leave the country, he was dead.

During the terrible famine of 1921, Gorky used his world fame to appeal for funds to provide food for the people starving in Russia. One of those who responded was Herbert Hoover, head of the American Relief Administration (ARA).

Gorky continued to criticize the Soviet government and after coming under considerable pressure from Vladimir Lenin, he agreed to leave the country. In October, 1921, Gorky went to live in *Germany* where he joined a community of around 600,000 Russian émigrés. He continued to criticize Lenin and in one article wrote: "Russia is not of any concern to Lenin but as a charred log to set the bourgeois world on fire." In July, 1922, Gorky campaigned against the decision to sentence to death twelve leading members of the *Socialist Revolutionary Party*.

Gorky stayed in Germany for two and half years before moving to Sorrento in Italy.

Joseph Stalin attempted to bring an end to Gorky's exile by inviting him back to his homeland to celebrate the author's sixtieth birthday. Gorky accepted the invitation and returned on 20th May, 1928. Stalin wanted Gorky to write a biography of him. He refused but did take the opportunity to seek help for those writers being persecuted in the Soviet Union.

It is unlikely that Gorky ever discovered the full picture of what Joseph Stalin was doing in the Soviet Union. He was kept under close surveillance by the *NKVD* and his private correspondence reveals that he believed Stalin that Leon Trotsky and his followers were behind the assassination of Sergey Kirov.

Maxim Gorky died of a heart attack on 18th June, 1936. Rumors began circulating that Stalin had arranged for him to be murdered. This story was given some support when Yagoda, the head of the *NKVD* at the time of his death, was convicted of Gorky's murder in 1938.

Asteroid *2768 Gorky*, was named after him.

* * *

Worldview XXVI: Maxim Gorky

* * *

Happiness always looks small while you hold it in your hands, but let it go, and you learn at once how big and precious it is.

* * *

In the carriages of the past you can't go anywhere.

* * *

Be good, be kind, be humane, and charitable; love your fellows; console the afflicted; pardon those who have done you wrong.

* * *

Only mothers can think of the future — because they give birth to it in their children.

* * *

There is no one on earth more disgusting and repulsive than he who gives alms. Even as there is no one so miserable as he who accepts them.

* * *

When everything is easy one quickly gets stupid.

* * *

When one loves somebody everything is clear – where to go, what to do – it all takes care of itself and one doesn't have to ask anybody about anything.

* * *

You can't do without philosophy, since everything has its hidden meaning which we must know.

* * *

You must write for children in the same way as you do for adults, only better.

* * *

When work is a pleasure, life is a joy! When work is a duty, life is slavery.

* * *

A good man can be stupid and still be good. But a bad man must have brains.

* * *

Everybody, my friend, everybody lives for something better to come. That's why we want to be considerate of every man — Who knows what's in him, why he was born and what he can do?

(1902)

* * *

The aim of literature is to help man to understand himself, to strengthen the trust in himself, and to develop in him the striving toward truth; it is to fight meanness in people, to learn how to find the good in them, to awake in their souls shame, anger, courage; to do all in order that man become nobly strong.

(1901)

* * *

Lenin and Trotsky and all who follow them are dishonoring the Revolution, and the working-class. Imagining themselves Napoleons of socialism. The *proletariat is for Lenin the same as iron ore is for a metallurgist. Is it possible, taking into consideration the present conditions, to cast out of this ore a socialist state? Obviously this is impossible. Conscious workers who follow Lenin must understand that a pitiless experiment is being carried out with the Russian people which is going to destroy the best forces of the workers, and which will stop the normal growth of the Russian Revolution for a long time.*

(1917)

* * *

Lenin and Trotsky don't have any idea about freedom or human rights. They are already corrupted by dirty poison of the power, this is visible by their shameful disrespect of freedom of speech and all other civil liberties for which the democracy was fighting.

(1917)

* * *

If the trial of the Socialist Revolutionaries will end with a death sentence, then this will be a premeditated murder, a foul murder. I beg of you to inform Leon Trotsky and the others that this is my contention. I hope this will not surprise you since I had told the Soviet authorities a thousand times that it is a senseless and criminal to decimate the ranks of our intelligentsia in our illiterate and lacking of culture country.

(1922)

* * *

1892–1934 CE **Hayyim Nachman Bialik** (1873–1934, Ukraine and Israel). Poet, essayist, scholar, linguist, translator and rejuvenator of the Hebrew language. In his scientific work, Bialik turned to the lore of Jewish antiquity (especially medieval Hebrew poetry and the Talmudic legendary literature) to bring about the renaissance of the classical Jewish heritage, giving life and verve to modern Hebrew. He restored to the almost defunct Hebrew language its elasticity and originality, showing that it is capable of expressing all the effects of light, sound and color.

Bialik was born in the small hamlet of Radi, in the Volhynia district of the Ukraine and educated by his paternal grandfather in Zhitomir. He studied in the famous Lithuanian Talmudic Academy ("Yeshiva") of Volozhin. He then worked as a timber trader, teacher and publisher. Settled in Tel-Aviv[680] (1923) and became the symbol and leader of Hebrew national and cultural revival in the old-new homeland of Israel. There are few examples in history of real poetry influencing a generation so deeply and so directly. His poem "In the City of Slaughter" prophetically depicts the 1903 Russian pogrom in Kishinev as a prelude to world tragedy. This poem caused thousands of youths in Russia to joint the underground to fight the Czar and tyranny.

With an exceptional mastery of every layer of the Hebrew language, Bialik confronted head-on the struggle of Judaism with other civilizations. Breaking away from the traditional upbringing of the Talmudic scholar, he came under the influence of the enlightenment. He elevated the *mathmid*, that perpetual student of Talmudic scholasticism, to the height of an extraordinary man to whom pure intellect, ascetism, self-sacrifice for 'learning for the sake of learning', had become unity in the highest degree[681]. In this poem Bialik portrayed the rapidly vanishing life of the traditional Jewish past.

To save this heritage and incorporate it into the values of the new age he foreshadowed a new beginning of a more complete life in which not all that is old will be cast away for the sake of the new, but only that part which has become obsolete, and in place of it a new Jewish life would absorb all that is best in the new age. Among his many translations into Hebrew are works of Shakespeare, Schiller and Cervantes.

1893 CE **Albert Londe** (1858–1917, France). Photographer. Published the first book on medical photography.

1893 CE **Wilhelm** (**Carl Werner Otto Fritz Franz**) **Wien** (1864–1928, Germany). Physicist discovered his displacement-law concerning the radiation

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680 **Maxim Gorky** helped him obtain a permission to leave the Soviet Union.

681 From these seeds sprang the great Jewish mathematicians, physicists and biochemists of the 19th and 20th centuries.

emitted by a perfectly efficient blackbody. This law states that the spectral peak wavelength is inversely proportional to the absolute temperature of the body. This law led Planck to discover his quantum theory of radiation.

Wien was born in Gaffken, East Prussia. He served as a professor of physics at the Universities of Giessen (1899) and Münich (1920). He received the Nobel Prize for physics in 1911.

1893–1894 CE Renewed worldwide outbreak of *cholera*. Millions perished.

1893–1896 CE **Mordecai Wolfe** (**Waldemar**) **Haffkine** (1857–1930, Russia, France and England). Bacteriologist, immunologist and microbe-hunter. Discovered and used an improved successful method of inoculation against cholera, plague, and typhoid, which reduced significantly the mortality rate of these diseases.

Haffkine was born to Jewish parents at Odessa, Russia, and graduated from the University of Odessa (D.Sc., 1884). Since he could not obtain a suitable position without conversion, he went to Paris to work under **Pasteur** (1888). Here he developed an attenuated strain of the cholera which he tested on himself (1892). The next year he used it on 45,000 people in India where it reduced the death rate by 70% among those inoculated. In 1896 he was deputed by the Indian government to inquire into the bacteriology of the plague. He discovered an effective method of inoculation, and succeeded in reducing the mortality by 80 percent. The same method was used successfully in Egypt (1947).

1893–1912 CE **Karl Pearson** (1857–1936, England). Mathematician. One of the founders of modern statistics. His work established statistics as a subject in its own right.

Pearson defined the *standard deviation* of a set of measurements and the *Chi-square* test of goodness of fit[682] (1900). Stimulated by the evolutionary

682 The χ^2 (*Chi-squared*) test determines the goodness of fit of a given model to noisy data. Let a sample consist of N trials and let $F(n)$ be the frequency of event n (i.e. the number of occurrences of the value n). If the *parent distribution* (model) which we are testing is $f(n)$, then the frequency predicted by the parent distribution is just $Nf(n)$. The difference $Nf(n) - F(n)$ for each n characterizes the difference in the two frequencies. A measure of *goodness of fit* is

$$\chi^2 = \sum_n \frac{[Nf(n) - F(n)]^2}{Nf(n)},$$

yielding the a weighted mean of the square of the fractional difference of the expected and observed frequency distributions. A mathematical table then writings of **Francis Galton** (1822–1911, England), he became immersed in the application of statistics to biological problems of heredity and evolution[683]. Pearson's other discoveries included the *Pearson coefficient of correlation* (1892), the theory of *multiple and partial correlation* (1896), the *coefficient of variation* (1898), work on *errors of judgment* (1902), and the *theory of random walk* (1905).

Pearson was born in London and educated at University College, London and King's College, Cambridge. He began his career as a lawyer (1881–1884) and also published literary works (1880–1882). During 1884–1933 he taught at University College, under the varied titles of professor of mathematics and mechanics (1884), geometry (1891) and eugenics (1911). He wrote books on the philosophy of science, and on statistics.

1893–1920 CE **Max Weber** (1846–1920, Germany) and **Emile Durkheim** (1858–1917, France). Founding fathers of modern *sociology*.[684]

Durkheim produced the first major sociological work (1853) employing a rigorous scientific methodology and single-handedly established sociology as an independent academic discipline. His thinking derived from French rationalism through which he sought to develop elementary forms as building-blocks of a theory of society; e.g. endeavored to explain *religion* in terms of totemism as an elementary form.

The heart of his sociology is a rejection of the individual basis of society[685]and the notion that society was prior to the individual. Therefore, methodologically, the social could not be reduced to the psychological.

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converts the values of (χ^2) and the number of degrees of freedom (DoF) to the probability that the model is correct. The integer DoF is N – the number of parameters in $f(n)$.

683 He showed that a wide variety of frequency distribution functions (including the 'Gaussian') can evolve from a single differential equation

$$\left[\frac{1}{y}\frac{dy}{dx} = (x+a)/(b_0 + b_1 x + b_2 x^2)\right]$$

by suitable choices of its coefficients.

684 The science of sociology was invented at least twice; once during 1830–1842 by **Auguste Comte** (1898–1857, France), who gave it its name by combining the Latin term *societas* with the Greek *logos*.

685 The *utilitarian* idea of **Hobbes** and **Bentham** that the *individual* and his self-interest comprise the unit of society and that the community is a superstructure that to succeed must bribe individuals into cooperation based on their material self-interest (social contract, minimalist government).

Thus Durkheim attacked the notion that society was simply a 'contract' between individuals, for the norms which govern contracts are embedded in a broader context of *moral* understanding or social solidarity. To him, the *religious* bond was simply the symbolic representation of the social bond, expressed through ritual.

Durkheim was born in Espinal, in the Lorraine, France to Jewish parents. Taught philosophy at Bordeaux (1887–1902) and was professor of sociology and education at the Sorbonne (1902–1917). He lost a son in WWI, where half of the 1913 class of the Ecole Normale (the school of France intellectual elite) was killed.

His books: *Suicide* (1897); *The Elementary Forms of Religious Life* (1912).

Max Weber, economist and sociologist, was born at Erfurt, Germany. Professor at Berlin (1893), Freiburg (1894), Heidelberg (1897–1903) and Munich (1919). His theory derived from German historical thinking. Taking his cue from Nietzsche, he claimed that successful capitalism in the Protestant countries derived from the value-positing of their charismatic founders (Calvin, Luther). What mattered to him was not the truth of religious experiences but the values it instilles in people.

The Modern Bicycle (1893)

Bicycle, a light two-wheeled steerable vehicle, propelled by human muscular power, evolved in the 19^{th} *century into an important popular means of transportation and recreation all over the world.*

Suggestions of vehicles having two or more wheels and propelled by muscular effort of the rider (or riders) are to be found in very early times, even on the bas-reliefs of Egypt and Babylon and the frescoes of Pompeii. There is some evidence for the presence of such vehicles in medieval England.

A primitive version of the bicycle appeared in France in 1779*; it was known as a velocipede, and invented by* **Blanchard** *and* **Magurier**. *It differed little from a later version known as célérifère, proposed by* **Mede de Sivrac** *(1790). His model consisted of a wooden bar rigidly connecting two wheels placed one in front of the other, and was propelled by the rider, seated astride the bar, pushing against the ground with his feet.*

The next advance was made in 1816 *by the Baron* ***Karl Drais von Sauerbronn*** *(1785–1851, Germany). In his contraption, the front wheel was pivoted on the frame so that it could be turned sideways by a handle, thus serving to steer the machine. It was known as the* draisine. *A similar machine, the* celeripede, *also with a movable front wheel, is said to have been ridden by* ***Joseph Nicèphore Niépce*** *(1765–1833,* France) *in Paris in* 1816. *The Scot blacksmith* ***Kirkpatrick Macmillan*** *added in* 1839 *to the* draisine *connecting rods working on the rear axle. Thus fitted, the* draisine *had wooden wheels with iron tires, the leading one about* 75 cm *in diameter and the rear driving one about* 100 cm. *It formed a prototype, though not the ancestor, of the modern rear-driven safety bicycle.*

About 1865, ***Pierre Lallement*** *in Paris constructed a bicycle in which the* front *wheel was driven by pedals and cranks attached directly to its axle, but it is unclear whether the origin of this idea should be attributed to him or to* ***Ernest Michaux***, *the son of his employer, who was a carriage repairer. (Lallement took his machine to the United States, and in* 1866 *was granted a patent which had an important influence on the subsequent course of the cycle industry in that country.) This machine, consisting of a wooden frame supported on two wooden wheels, soon became popular in England as well as in France and America, and came to be called* bicycle (or bysicle) *by those who took it seriously and* bone-shaker *by those who did not.*

Improvements quickly followed, chiefly in England, for the popularity of the machine in America was short-lived, and in France the industry was checked by the Franco-German war. Rubber tires, in place of iron ones, appeared in 1868 *the chain-drive was invented by* ***J.F. Tretz*** *in Germany in* 1869, *and applied to bicycles by* ***Guilmet*** *(France) in* 1870. *Suspension wheels with wire spokes in tension were seen in London.*

During the 1870*'s, a new type of a bicycle appeared with a large driving wheel in front and a small trailing behind. The same type retained its supremacy until* 1885. *The same year saw the first commercially successful* safety bicycle *produced by* ***John Kemp Starley*** *(England); it had equal-sized wire-spoked wheels, "diamond shape" steel-tubing frames, cone (and then ball) bearings at points of friction, crank and pedals in the center with a chain and sprocket drive to the back wheel. The rider sat so far back that he could not be thrown forward over the handles. Finally, the machine was made more* stable *with a* curved front wheel fork[686]

[686] A stable bicycle is one whose *forkpoint* (the point of intersection of a projection along the front steering axis and a horizontal line through the wheel center) falls as the wheel turns into a lean when the bike is tilted. *Gyroscopic effects* have little to do with riding stability, although if the bike is pushed off riderless, then the gyroscopic effect from the wheels will help stabilize the bike for a while.

With the invention of air-filled rubber tires by ***John Boyd Dunlop*** *(1840–1921, Scotland) in* 1888*, and the addition of coaster brakes and adjustable handle bars — the modern version of the bicycle was ready by* 1893*. Early forms of the gear shift came into use soon after* 1900*.*

By 1897 *about* 4 *million Americans were riding bicycles regularly, more than at any previous time. During the early* 1900*'s, the rapid development of the automobile caused many people to lose interest in bicycles, but in the early* 1970*'s bicycle riding in the United States became more popular than ever before, and* 75 *million bike riders were on the roads. In* 1990 *this number climbed to* 100 *million in the United States alone. The Annual Tour de France (began 1903), the most famous bicycle road-race, covers* 4800 *km and takes* 21 *days. The cyclist with the shortest total riding time is the winner.*

About 100 million bicycles were produced worldwide in 2000: China (60 m); India (11 m), Taiwan (7.5 m); Japan (4.7 m); Italy (3.2 m); UK (1.2 m); USA (1.1 m).

The first motorcycle[687] *was invented by* ***Gottlieb Daimler*** *in* 1885*, who attached a 4-stroke piston engine to a wooden bicycle frame. During the* 1900*'s, with continual improvements, motorcycles developed into useful, dependable vehicle.*

[D.E.H. Jones, "*The Stability of the Bicycle*", *Phys. Today*, April 1970; S.S. Wilson, "*Bicycle Technology*", *Sci. Amer.*, March 1973; A.T. Jones, "*Physics and Bicycles*, *Am. J. Phys.* **10**, 332, 1942.]

687 Two-wheeled vehicles powered by internal combustion engines comprise *motorcycles*, *motor scooters* and *mopeds* [abbr. for 'motor-assisted pedal cycle']. These all are similar in principle. The motorcycle comprises four main sections: the frame, the engine with gearbox and drive components (chain or drive shaft), the road wheels, and the petrol tank. Two-stroke and four-stroke engines are used as power units for motorcycles. The power is transmitted to the rear wheel through a gearbox and thence through sprockets and chains or through a drive shaft. Motorcycles and mopeds have wire-spoked wheels, whereas scooters generally have solid wheels like those of a car.

The Last Great Naturalists[688]

"When white man first come to Canada, he shoot all big animals, haul off meat. Next trip he trap all small animals, haul off fur. Third time he cut down all big trees, haul off lumber. Fourth time, cut down all small trees, make paper. Now he haul off all rocks".

Indian Chief's lament

The accelerated advance of science and technology that followed in the wake of the Industrial Revolution was mostly accomplished in the laboratories and the institutions of European universities and industrial research centers. Yet, the 19^{th} *century was still abundant with a different breed of natural scientists who sought to study nature in its own milieu and for its own sake. These were explorers, geographers, ornithologists, entomologists and other naturalists who went out of the cities and away from the centers of higher learning to rediscover nature and our place in it.*

John James Audubon *(1785–1851, U.S.A.). Naturalist and artist. Captured for posterity the images of contemporary birds and animals of North America.*

Henry Baker Tristram *(1822–1906, England). Naturalist, ornithologist, and the first scientific explorer of the Sahara (1855–1857) and the Lands of the Bible (1863–64, 1872, 1880–81, 1894–95). Among the first ardent Darwinists.*

Jean Henri (Casimir) Fabre *(1823–1915, France). One of the greatest entomologists ever. His keen observations, patience, extraordinary intuitive power and unsurpassed ability to transmit the mysteries of the insect world to his fellow men, made him a unique figure in the history of science.*

John Muir *(1838–1914, U.S.A.). Explorer, naturalist and writer. The first man to explain the glacial origin of the Yosemite Valley. Explored Alaska, the Arctic, Africa, Asia and the United States.*

Ernest Thompson-Seton *(1860–1946, Scotland and Canada). Naturalist, artist, animal observer and writer.*

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688 For further reading, see:

- Adams, A.B., *Eternal Quest*: *The Story of the Great Naturalists*, Isaac Putnam's Sons: New York, 1969, 509 pp.

***Charles William Beebe** (1877–1962, U.S.A.). Naturalist, explorer and writer. Explored the tropical jungles of Borneo, Guyana and Trinidad (1916–1925). He was first to dive into the depths of the ocean in a diving chamber (bathysphere), reaching a depth of 800 m (1930).*

Beebe was born in Brooklyn, NY. He became curator of ornithology (bird studies) at the New York Zoological Society in 1899. He helped found the Society's Tropical Research Department in 1916, and wrote numerous books about his adventures.

1894 CE Bacteriologists **Shibasaburo Kitasato** (1852–1931, Japan) and **Alexandre Yersin** (1863–1943, Switzerland) discovered simultaneously and independently the causative organism, *Pasteurella pestis*, of bubonic plague, during an outbreak at Hong Kong.

Prevention was found to be possible by inoculation with a killed vaccine or by injection of a live avirulent organism i.e. a relatively harmless strain of the bacteria. Antibiotic drugs, give good results when administered to infected patients.

1894, Feb 15 A group of anarchists attempted to blow up the *Greenwich Observatory.*[689]

1894 CE **Thomas Jan Stieltjes** (1856–1894, Netherlands and France). Dutch-born French mathematician. Made notable contributions to the analytic theory of continued fractions and integration theory.

Stieltjes was born in Zwolle, Netherlands, and educated at the universities of Delft, Leyden and Groningen. He moved to France in 1885 and became a professor of mathematics at the University of Toulouse, where he remained for the rest of his life. Stieltjes contributed to the fields of divergent and conditionally convergent series, number theory and spherical harmonics. He proposed the *Riemann-Stieltjes integrals*[690] and the *Lebesgue-Stieltjes inte-*

[689] The event prompted **Joseph Conrad** (1857–1924) to write his masterpiece *The Secret Agent* (1907). In this political-detective novel Conrad expressed society's disillusionment from science, the late 19th century 'god-substitute' that failed.

[690] $\int_a^b f(x)dg(x) = \lim_{\max|x_i - x_{i-1}|\to 0} \sum_{i=1}^m f(\xi_i)\left[g(x_i) - g(x_{i-1})\right]$ for arbitrary sequence of partitions

$$a = x_0 < \xi_1 < x_1 < \xi_2 < x_2 < \cdots < \xi_m < x_m = b.$$

grals which have wide applications in probability, distribution and Laplace-transform theories.

1894–1897 CE **George Oliver** (1841–1915, England), physician and physiologist and **Edward Albert Sharpey-Schäfer** (1850–1935, England) first demonstrated the action of a *specific hormone*: the effect of an extract of the adrenal gland (*adrenaline* or *epinephrine*) on blood vessels and muscle contraction. Upon injection into normal animals it produced a striking elevation in blood pressure.

John Jacob Abel (1857–1938, U.S.A.), pharmacologist and physiological chemist first isolated *epinephrine* (1897). He also developed artificial kidney (1914) and crystallized insulin (1926).

1894–1914 CE **Jean Léon Jaures** (1859–1914, France). Social philosopher, father of social democracy and socialist leader. With his political instincts inspired by the French Revolution, Jaures opposed imperialism in all its forms, yet he believed in the rights of the individual over the state.

Jaures was born in Castres and attended the Ecole Normale Superieure in Paris. After graduating he lectured on philosophy at the University of Toulouse (1883–1885) and earned his doctorate in philosophy there (1891). During 1885–1889 and 1893–1914 he was a member of the *Chamber of Deputies* as an independent socialist.

Involved in the Dreyfus affair in 1894 as a supporter of Dreyfus, Jaures argued that Alfred Dreyfus' treason conviction was based upon forged evidence.

A co-founder in 1904 of the socialist newspaper *L'Humanite* (along with Rene Viviani and Aristide Briand, both future French Prime Ministers), Jaures was a man of numerous talents. A prolific writer, he proved himself as capable at giving a speech as penning it.

A firm advocate of the Second International socialist movement, he accepted their argument preventing its members from participating in so-called 'bourgeois' governments. As such he never accepted a position within the French cabinet; which meant, given his leadership of the party (since 1905), that the Socialist Party was also denied a role in government.

As the storm clouds of war approached, Jaures' popularity waned somewhat, as he continued to advocate closer relations with Germany. Indeed, at the height of the July Crisis of 1914 he traveled to Brussels to try to persuade German socialists to strike against potential war in Europe.

The limit exists whenever $g(x)$ is of bounded variation and $f(x)$ is continuous in $[a, b]$.

Shortly after his return from Brussels to Paris, on 31 July 1914, Jaures was murdered by a 29 year old nationalist fanatic, Raoul Villain; three days later Germany declared war with France.

1894–1925 CE **Schlomo Sigmund Freud** (1856–1939, Austria). Neurologist and founder of psychoanalysis. One of the most influential thinkers in modern times. Revolutionized our view of human nature and affected almost every department of our culture. His method of treatment led to the use of psychotherapy, and greatly extended our sensitivity about human relations in general, and between doctor and patient in particular. Freud's work on the origin and treatment of mental illness helped form the basis of modern *psychiatry*. He especially influenced the field of abnormal *psychology* and the study of personality.

Freud's theories on sexual development led to open discussion and treatment of sexual matters and problems. His stress on the importance of childhood helped teach the value of giving children an emotionally nourishing environment. His insight also influenced the fields of *anthropology* and *sociology*. In art and literature, Freud's theories encouraged understanding of *surrealism*, which like psychoanalysis explores the inner depths of the unconscious mind. Freudian concepts have provided subject matter for many authors and artists.

Some of the Freud's theories are controversial. Future science will have to settle these problems, and it will be probably a long time before the value of his achievement is ultimately determined. But at face value, Freud was the discoverer of a new humanistic discipline whose significance went beyond the boundaries of psychiatry. He brought into the world a new definition of human fate, because he placed in the hands of man the means with which to alter impediments which were previously considered irremediable.

Freud at times stated that the psychic apparatus was free from any *anatomical* implications, but it is certain that he hoped for an eventual integration of his theory with neurology and that he always considered the *biological* facts to be quite relevant to his decisions about his own model. The following five biological facts become familiar to us only since his death and they decisively refute the model of *passive* reflex mechanism:

1. The nervous system is perpetually *active*. Electroencephalographic (EEG) data have shown that even in the deepest sleep and in coma the brain does not cease its activity; at these times of minimal input and behavioral output, hypersynchrony seems to produce the most massive discharges, the resting nerve cell periodically fires (produces a spike potential), and its nontransmitted activity waxes and wanes, all without any outside stimulation.

2. Thus, the effect of stimulation is primarily to *modulate* the activity of the nervous system. It may step up the frequency of discharge but mainly imposes an order and pattering on it; that is to say, encodes it.

3. The nervous system does not *conduct* energy; the nervous impulse is rather propagated. An appropriate physical analogy is not current flowing along a wired circuit, but a signal traveling along the axon to the synapses which in turn pass signals on to the dendrites of other cells.

4. The energies of the nervous system, whether or not triggered by the sensory organs, are *different in kind* from the impinging external stimuli. The sensory surface acts as a *transducer*.

5. The tiny energies of the nerves bear encoded information and are quantitatively negligible; their amount bear no relation to the motivational state of the person. The electrical phenomena associated with the neuron are accessible to quantitative study today, but this work offers no basis for the economic point of new — the assumption that mental events might be meaningfully examined from the standpoint of the 'volumes of excitation' involved. Rather than this kind of 'power engineering', '*information engineering*' seems to be relevant discipline.

It stands to reason that most of Freuds provisional ideas in psychology will presumably some day be based upon an organic substructure.

Freud was born to Jewish parents (of Chassidic rabbinic stock on both sides) in the town of Freiberg, Moravia, which is today part of the Czech Republic, but was then part of the Austrio-Hungarian Empire. His father's family was settled for a long period at Cologne, but fled eastward as a result of the persecution of the Jews during the 14th century. In the course of the 19th century they migrated from Lithuania through Galicia (*Buczacz*) into the Habsburg Empire.

In 1859 his father Jacob Freud (1815–1896), moved to Vienna. Earlier (1855) he married, the second time, to Amalie Nathanson (1835–1930), the descendant of a famous Talmudic scholar, Nathan Halevi Charmaz of Brody, Poland. Sigmund was the eldest and favorite of her 8 children. From an early age, Freud dedicated himself to learning which remained a unique trait identified with the Jewish ethics. Yet due to the liberal spirit prevailing then among the Viennese Jews, he was subjected to non-Jewish upbringing.

A youthful interest in science and human personality[691] led him to enter the University of Vienna medical school (1873). He took his degree in medicine (1881) and married (1882) Martha Bernays (1861–1951), a granddaughter of the chief Rabi of Hamburg.

After serving as intern and resident physician in a hospital, he decided to specialize in *neurology* (the treatment of disorders of the nervous system) and went to Paris (1885) to study under **Jean-Martin Charcot**, a leading authority of hysteria.

He returned to Vienna[692] (1886) and began medical practice, specializing in nervous diseases. The case histories of his patients convinced him that *sexual causes* played a major role in many forms of *neurosis*. He gradually formed ideas about the origin of mental illness, using the term *psychoanalysis* for both his theory and his method of treatment.

When he first presented his ideas in the 1890's, other physicians rejected with hostility, but Freud eventually attracted a group of followers (1902), and by 1910, gained international recognition and acclaim. During the following

[691] It is important to note that there is a link between psychoanalysis and *Jewish mysticism*: Freud himself mentioned the 16^{th} century Jewish physician **Solomon Almoli** (1490–1542, Turkey) whose book *The Solution of Dreams* (1516) gives a description of sexual symbolism, wish fulfillment and word-play as elements found in dreams. Many counterparts of Freudian theory were found in the *Zohar* (the mystical writings of **Moshe de Leon**, 1286 CE), such as the portrayal of primordial man where the divine act of creation was given an erotic character and where sex relations were treated as avenues of salvation.

Another great sage, who anticipated so many of Freud's views was **Baruch Spinoza** (1632–1677). Indeed the essence of his philosophy which was expressed in his dictum: *Humanus actiones non ridere, nec lugere, nec detestera, sed intelligere* (Human actions should not be mocked, should not be lamented, nor execrated, but should be understood), could be taken as the source and origin of Freud's whole system.

A more recent connection of psychoanalytic thought to *chassidism* was suggested by Freud, whose father Jacob, came from chassidic stock.

[692] Vienna in the 1890s was famous for its Blue Danube, its wit, sensuality, waltzes and cafés. But it had a darker side: the Empire was in deep economic trouble. The jobless were crowded in slums and flophouses. Karl Lueger, Mayor of Vienna, made anti-Semitism politically fashionable. Austria's first anti-Semitic party was formed in 1880 and during the next 60 years or so anti-Semitism was made the central issue of both municipal and state elections. Between 1880 and 1914, almost two million Jews came to the United States from Eastern Europe. Throughout his entire adult life Freud's Vienna continued to remain a virulently anti-Semitic city. In fact, anti-Semitism pursued him *all his life*.

decade, Freud's reputation continued to grow, but two of his early disciples, **Alfred Adler** and **Carl Jung**[693] split with him. By 1914–1915 Freud had developed his earlier theory of infantile sexuality to cover and explain the distinction between conscious and unconscious functioning by means of the concept of *repression*.

In 1919 Freud was finally made full professor at the University of Vienna. But his appointment did not allow him the privilege of a seat on the board of the faculty.[694]

By the beginning of 1920, the work of Freud had contributed gradually to establish the category of the 'neuroses' in contemporary psychiatry. He went on (1923–1927) to develop his earlier views by stressing the role of the *ego*[695]

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[693] Freud's former disciple (1906–1913). Many years later (ca 1935), Jung contrasted Freud's inferior "Jewish" psychology with Hitler's perfect scientific doctrines of Aryan superiority. After Freud's books were burned in public by the German Nazis (May 1933), Jung published books and articles asserting the negative foundations of Freud's psychology. His primary function was to show that as Jews, Freud and his followers were unable to understand the "*superior German psyche embodied in the powerful National Socialism, at which the whole world looks on in astonishment*" (1935). Freud not wishing to degrade himself in nonsential arguments, remarked slyly: "*What Jung contributed to psychoanalysis, we can dispense with...*".

[694] It took Freud 38 years to climb the academic ladder from MD (1881), through the ranks of *privatdocent* (1885–1902) and associate professor (1902–1919). The fact that he was Jewish was one reason for the delay. The other was that he established himself as a pioneer in a new field of research which was looked upon by the leading men in psychology and psychiatry as fantastic and even indecent! When, at 70 (1926) congratulation arrived from leading scientists all over the world, the University of Vienna did not send him even a letter of felicitation.

[695] The mind consists of three parts:

- *id*: mental representation of the biological instincts, such as the drive to satisfy hunger and the drive to satisfy sexual needs. It does not distinguish between the internal mind (e.g. mental image of food) and the outside environment (the food itself).
- *ego*: controls the behavior that bridges the gap between mental images and the outside world. It distinguishes between the internal mind and the external reality, e.g. the ego directs a hungry person to look for and eat real food.
- *superego*: governs moral behavior. It is the mental representation of society's moral code. It seeks to limit behavior based on the drives of the id.

In mentally healthy individuals, the three parts of the mind work in harmony.

and the *super-ego* and to apply his ideas (1927–1930) to account for religious belief, social discontent, and produce a range of new concepts to describe and explain human reactivity.

In 1923 Freud was operated for cancer of the jaw and palate, the first of 33 operations. For the last 16 years, Freud often suffered agonizing pain; his speech and hearing were affected and eating was difficult.

When the Nazis invaded Austria (1938) they burned his books and banned his theories. Friends got him out of Austria to England.[696] He left his home in Vienna in which he lived continuously for 42 years: in the same house, in the same street, in the same Jewish section. The British Medical Journal said (1938): "The medical profession of Great Britain will feel proud that their

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But in others the parts may conflict, resulting in psychological disturbances.

Freud observed that many patients behaved according to drives and experiences of which they were not consciously aware. He thus concluded that the unconscious plays a major role in shaping behavior. He also concluded that the unconscious is full of memories of events from early childhood — sometimes as far back as infancy; if these memories were especially painful, people kept them out of conscious awareness (defense mechanisms). Freud believed that patients used vast amounts of energy in forming defense mechanisms. This tied energy could affect a person's ability to lead a productive life, causing an illness that Freud called *neurosis*.

Freud also concluded that many childhood memories dealt with sex. He theorized that sexual functioning begins at birth, and that a person passes through several psychological stages of development from infant sexuality to adult sexuality. If for some reason, the normal pattern of sexual development is interrupted in some individuals, mental illness in adulthood could result.

[696] Shortly after the *Anschluss* (Mar. 11, 1939), Freud's home in Berggasse was invaded by a gang of German Storm Troopers who helped themselves to whatever money was in the house, including 6000 Austrian schillings (then about 840 dollars) which belonged to the Psychoanalytic Association. Freud reacted to this "house-call", saying: "I've been a doctor for fifty years, but I never got 6000 schillings for a visit to an old, sick man".

In June 1938, thanks to the intervention of **Marie Bonaparte** (who paid the Nazis a ransom of 35,000 dollars), the American ambassador to France, and the British Home Secretary, Freud and the members of his immediate family received permission to leave Vienna for London. Before his departure, the Gestapo forced him to sign a certificate declaring that he had been well treated by the authorities. Freud complied, but added a sentence of his own in an advertising copywriter style: "*Ich kann die Gestapo jedermann auf das beste empfehlen*" ("I can heartly recommend the Gestapo to anyone").

country has offered an asylum to professor Freud and that he chose it as his new home". He died of the jaw and palate cancer in London on Sept 23, 1939.

His most important writings include: *The Interpretation of Dreams* (1900); *Three Essays on the Theory of Sexuality* (1905); *Totem and Taboo* (1913); *General Introduction to Psychoanalysis* (1920); *The Ego and the Id* (1923); *Civilization and Its Discontents* (1930).

Worldview XXVII: Freud

* * *

"I have often felt as if I had inherited all the passions of our ancestors when they defended their Temple, as if I could joyfully cast away my life in a great cause."

(1886)

* * *

"If you do not let your son grow up as a Jew, you will deprive him of those sources of energy which cannot be replaced by anything else. He will have to struggle as a Jew and you ought to develop in him all the energy he will need for the struggle. Do not deprive him of that advantage."

(to Max Graf, 1895)

* * *

"I found the essential characteristic and most significant part of my dream theory — the reduction of dream-distortion to an inner conflict — later in a writer who was familiar with philosophy though not with medicine, the engineer ***Josef Popper-Lynkeus****. A special feeling of sympathy drew me to him, since he too had clearly painful experience of the bitterness of the life of a Jew and the hollowness of the ideals of present-day civilization."*

(1899)

* * *

"I am not really a man of science, not an observer, nor an experimenter, and not a philosopher. I am by temperament nothing but a conquistador…

with the curiosity, the boldness and the tenacity that belong to that type of person."

* * *

"Poets are masters of us ordinary men, in knowledge of the mind, because they drink at streams which we have not yet made accessible to science."

* * *

"My life and work has been aimed at one goal only: to infer or guess how the mental apparatus is constructed and what forces interplay and counteract in it."

* * *

"I have no concern with any economic criticism of the communistic system: I cannot inquire into whether the abolition of private property is advantageous and expedient. But I am able to recognize that psychologically it is founded on an untenable illusion. By abolishing private property one deprives the human love of aggression of one of its instruments... This instinct did not arise as a result of property; it reigned almost supreme in primitive times when possessions were still extremely scanty..."

* * *

"Hatred of Judaism is at bottom hatred of Christianity."

* * *

"Toward the person who has died we adopt a special attitude: something like admiration for someone who has accomplished a very difficult task."

* * *

"From error to error one discovers the entire truth."

* * *

"Analogies make one venture to regard obsessional neurosis as a private religious system and religion as a universal obsessional neurosis."

(1907)

* * *

"There are no such things as Aryan or Jewish science. Results in science must be identical, though the presentation of them may vary. If these differences mirror themselves in the apprehension of objective relationships in science, there must be something wrong."

(1913)

* * *

"God is nothing other than an exalted father."

(1913)

* * *

"Totemism, with its worship of a father substitute, may be regarded as the earliest appearance of religion in the history of mankind, and it illustrates the close connection existing from the very beginning of time between social institutions and moral obligations."

(1913)

* * *

"You may be sure that if my name were Oberhuber, my new ideas would, despite all the other factors, have met with far less resistance."

* * *

"Only to my Jewish nature did I owe the two qualities which had been indispensable to me on my hard road: because I was a Jew I found myself free from many prejudices which limited others in the use of their intellect, and, being a Jew, I was prepared to enter opposition and to renounce agreement with the 'compact majority'."

(1926)

* * *

"They will always throw stones at me. You see, I have troubled humanity's sleep."

* * *

"Religion is an attempt to get control over the sensory world, in which we are placed, by means of the wish-world, which we have developed within us as a result of biological and psychological necessities. But it cannot achieve its end. Its doctrines carry with them the stamp of the times in which they originated, the ignorant childhood days of the human race... The ethical commands, to which religion seek to lend its weight, require some other foundation instead, since human society cannot do without them, and it is dangerous to link up obedience to them with religion itself."

* * *

"You need not be the victim of your own past, or your own environment."

* * *

"Sometimes a cigar is just a cigar."

History of Biology and Medicine, IV – The 19th century

During this epoch, biology progressed along four major avenues: cytology (cell theory), genetics, bacteriology and physiological chemistry.

Into the 19th century, explorer-naturalists such as ***Alexander von Humboldt*** *tried to elucidate the interactions between organisms and their environment, and the ways these relationships depend on geography-creating the foundations for biogeography, ecology and ethology. Many naturalists began to reject essentialism and seriously consider the possibilities of extinction and the mutability of species. These developments, as well as the results of new fields such as embryology and paleontology, were synthesized in Darwin's theory of evolution by natural selection. The end of the 19th century saw debates over spontaneous generation and the rise of the germ theory of disease and the fields of cytology, bacteriology and physiological chemistry, though the problem of inheritance was still a mystery.*

Wöhler *showed In 1828 that organic molecules, such as urea, can be created by synthetic means that do not involve life, and thus provided a powerful argument against vitalism. The first enzyme, diastase, was described in 1833, and the science of biochemistry may be said to have begun.*

By the mid 1850's the miasma theory of disease was largely superseded by the germ theory of disease, and antisepsis became a medically important invention. Surgery and medicine was advanced in 1858 when Gray's Anatomy was first published.

In about the 1880's the science of bacteriology began to be formed, especially through the work of ***Robert Koch****, who introduced methods for growing pure cultures on agar gels containing specific nutrients in Petri dishes. He also introduced the "Koch's postulates" for the reliable determination of when a proposed microorganism caused a specific disease. The long-held idea that living organisms could easily originate from nonliving matter (spontaneous generation) was finally discredited in a series of experiments carried out by* ***Louis Pasteur****.*

Schleiden *and* ***Schwann*** *proposed the cell theory in 1839: the basic unit of organisms is the cell and all cells come from preexisting cells.*

The British naturalist ***Charles Darwin****'s seminal work On the Origin of Species (1859) described natural selection, the primary mechanism for evolution.*

In 1866 genetics had its beginnings in the work of the Austrian monk ***Gregor Mendel*** *who formulated his laws of inheritance. However, his work* *was not recognized until 35 years afterward. Three years after his publication, in 1869* ***Friedrich Miescher*** *discovered what he called nuclein, which was later realized to be a crude preparation of DNA.*

The cytologist ***Walther Flemming*** *in 1882 was the first to demonstrate that the discrete stages of mitosis were not an artifact of staining, but occurred in living cells, and moreover, that chromosomes doubled in number just before the cell divided and a daughter cell was produced. In 1887* ***August Weismann*** *proposed that the chromosome number must then be halved in the case of the sexual cells, the gametes. This was shortly proved to be the case and the process of meiosis began to be understood.*

In medicine, the rapid advancement of physical and chemical theories together with their corresponding industrial technologies induced important medical inventions, Thus we witness the stethoscope (1816, ***T. Laennec****); dental plate (1817,* ***A. Plantson****); endoscope (1827,* ***P. Segalas****); anesthetics (1846,* ***W. Morton****); ophthalmoscope (1851,* ***H. von Helmholtz****); hypodermic syringe (1893,* A. Wood*); barbiturate (1863,* ***A. Von Bayer****); antiseptic (1865,* ***J. Lister****); rabies vaccination (1885,* ***L. Pasteur****); contact lens (1887,* ***A. Frick****).*

Cytology

It is the study of the internal structure and organization of cells. Microscopic studies of the structure of the cell provided an explanation of cell division *and served as a foundation for* genetics*. These studies also showed that each structure has some function, and that each cell activity is related to changes in chemicals that make up the cell. Structures now recognizable as* cell nuclei *were described by many early microscopists. The term, however, was coined by* ***Robert Brown*** *(1833).*

*The term '*cell theory*' was introduced by* ***T. Schwann*** *(1839) to include the principle of construction of all organic products. The studies of* ***Hugo von Mohl*** *and* ***Max Schultze*** *clarified that plant cells alone possessed walls; what they shared with animals was the material within their walls, the primordial* protoplasm*. Evidence also accumulated that cells were formed by division of existing cells. This new model of cell formation, developed for animals by* ***Robert Remak****, was generalized and popularized by the anatomist* ***Rudolf Virchow*** *(1858).*

Embryonic development appeared as successive cell divisions from an egg cell *formed in the mother. By 1900 it was generally accepted that plants and animals were made up of discrete masses of nucleated protoplasm propagated by division.*

Bacteriology

Bacteria, one-celled organisms, were first seen by ***Leeuvenhoek****. Originally confused with* protozoa*, bacteria were variously called* animalcules*,* microbes *or* vibrionia*.*

During the 18^{th} century bacteria contributed to the spontaneous generation controversy, as ***Spallanzani*** *(1729–99) refuted the assertion that microbes appeared in sealed flasks of boiled broth. Spallanzani demonstrated microbes appeared only after inadequate heating or the admission of air into the vessel. Bacterial studies outside medicine remained superficial until 1872 when* ***F.J. Cohn*** *defined and named bacteria, distinguishing four groups on the basis of external form and specific fermentative activity. He recognized bacteria take nitrogen from simple ammonia compounds, elucidated their life-cycles, identified spores and suggested bacteria were motile cells devoid of walls. Determining bacterial temperature limits,* ***Cohn****,* ***Pasteur*** *and* ***Tyndall*** *effectively ended the spontaneous generation controversy with their studies on sterilization.*

Some bacteria were suggested to be pathogenic by ***Casimir Davaine****'s experiments (1850) indicating anthrax was caused by rod-shaped organisms, 'bacteridia', found in the blood of diseased animals.* ***Robert Koch****'s classic experiments confirmed these suggestions in 1876. Koch also developed techniques for handling bacteria, introducing solid nutrient media (agar-agar) to grow pure cultures, and devising methods for fixing bacteria.*

Dimitri Ivanovski *using* ***Chamberland****'s bacteriological filter (1884) explained the infectiousness of tobacco mosaic disease (1892) by showing it can be transmitted via cell-free filtrates from leaves of diseased plants to leaves of healthy plants. Thus the term 'filterable virus' was coined; later filterable was dropped and 'virus' took on its modern meaning.* ***F. Loeffler*** *and* ***P.Frosch****'s work on foot-and-mouth disease first demonstrated (1898) an animal disease in which a virus was the causative agent. Yellow fever was the first human disease proved (1901) to be caused by a filterable virus by* ***W. Reed****.*

During the 1890's increased knowledge of soil and water bacteria was responsible for completion of the nitrogen, sulfur and carbon cycles. Nodule-forming bacteria living in the roots of leguminous plants were found to fix atmospheric nitrogen. As a result of ***Winogradsky****'s and* ***Beijerinck****'s work on anaerobic bacteria, knowledge of a whole world of organisms able to live on elementary nitrogen, iron or sulfur has emerged.*

Evolution Theory, Darwinism and Mendelian Genetics

The first broad theory of the transmutation of organic forms was by ***Jean-Baptiste Lamarck*** *(1800–1809). He advanced the idea that the simplest forms of life had been* spontaneously *generated and that from there all other forms of life had been successively produced.*

Lamarck explained organic change as the result of two factors: the 'power of life', which was responsible for the general scale of increasing complexity formed by the different animal classes; and the influence of particular environments, accounting for the fact that species and genera could not be aligned in a single series [natural order]. Explaining how animals change in response to different environments Lamarck affiliated himself with the idea of the inheritance of acquired characters. *In his view, animals responded to environmental changes by developing new habits, leading to changes in the animals' structures which were then passed on to offspring. It took many generations for the effects of this to become appreciable.*

Lamarck had relatively little evidence for his theory beyond the structural similarities among living things. He believed the earth's age to be immeasurably greater than his predecessors had supposed, a prerequisite for any theory of the gradual change of living things over time. In Lamarck's day, however, the study of fossils, a primary impetus for enlarged views of the earth's antiquity, could not confirm the reality of evolution. Opponents such as ***Georges Cuvier*** *argued the fossil record did not reveal the translation between forms that theories such as Lamarck's demanded. Also Cuvier's system of classification, identifying four fundamentally different types of animal organization, denied a chain of being and hence the idea of linear progression central to Lamarck's thinking.*

In 1813, **William Charles Wells** *produced essays assuming that there had been evolution of humans, and recognized the principle of* natural selection. ***Charles Darwin*** *and* ***Alfred Russel Wallace*** *were unaware of this work when they jointly published the theory in 1858, but Darwin later acknowledged that Wells had recognized the principle before them.* ***Augustin de Candolle****'s natural system of classification laid emphasis on the "war" between competing species.*

By 1833 the geologist **Charles Lyell** *in the second volume of his* Principles of Geology *had set out a gradualist variation of creation beliefs in which each species had its "center of creation" and was designed for the habitat, but would go extinct when the habitat changed.*

*Lamarckism became discredited as experiments simply did not support the concept that purely "*acquired traits*" were inherited. The mechanisms of* inheritance *were not elucidated until later in the* 19^{th} *century, after Lamarck's* death. Lamarckism in toto has largely been discredited as a mechanism in evolution.

Although paleontologists and embryologists' evidence appeared to confirm the reality of evolution, its mechanism remained unresolved at the end of the century, being vigorously debated by proponents of neo-Darwinism, neo-Lamarckism, orthogenesis and other views. Darwin had not explained the causes of variation or the means by which characters are passed on from one generation to the next. Without an adequate theory of heredity it was unclear how important natural selection was in the evolutionary process. For example, inheritance of acquired characters might account for the creative side of evolution, leaving natural selection with merely the negative function of weeding out the unfit.

Though in retrospect it appears that what Darwin lacked was the theory of particulate inheritance proposed in the 1860's by **Gregor Mendel**, when Mendel's work first came to be appreciated in 1900 people saw it as an alternative rather than complementing Darwin's theory. The three Mendelians most interested in evolution, **Hugo de Vries**, **William Bateson** and **Wilhelm Johannsen**, were all highly critical of the theory of evolution by natural selection.

Thus, while the scientific community generally accepted that evolution had occurred, many disagreed that it had happened under the conditions or mechanisms provided by Darwin. In the years immediately following Darwin's death, evolutionary thought fractured into a number of interpretations, include neo-Darwinism, neo-Lamarckism, orthogenesis, Mendelism, the biometric approach, and mutation theory. Eventually this boiled down to a debate between two camps. The Mendelians, advocating discrete variation, were led by **William Bateson** (who coined the word genetics) and **Hugo de Vries** (who coined the word mutation). Their opponents were the biometricians, advocating continuous variation; their leaders **Karl Pearson** and **Walter Frank Raphael Weldon**, following in the tradition of **Francis Galton**.

An important issue in the debate between the Mendelians and the biometricians was the nature of variation in species. Darwin and Wallace believed that small variations were more important than large ones, since small variations hewed closely to an already-proven model. The biometricians agreed with this position, while the Mendelians insisted that discontinuous species were unlikely to arise from a continuous process of change. While the immediate issue of speciation was resolved in large part by the clear definition of a species as a reproductively isolated population, the rate of evolution would arise again as a point of contention in the late 20^{th} century with the proposal *of punctuated equilibrium. Most other questions resolving variation were resolved with the recognition that the size of a* genotypic *change did not always correspond with the size of the resulting* phenotypic *change.*

Another source of clashes between Mendelians and biometricians was the debate over the origins of variation. Mendelians argued for intrinsic *variations originating from* genetic *transmission; biometricians, observing primarily the* phenotype *of the organism, were not yet prepared to abandon Lamarckian views on the heritability of acquired characteristics.* ***August Weismann*** *was among those who demonstrated that acquired characteristics were not always inherited, pointing out the existence of worker ants and worker bees, and the importance of '*germ plasm*' or* gametes *in the biology of reproduction. The recognition of means of postnatal* adaptation *as inherited traits did much to explain acquired characteristics.*

Heredity

It was known from the 1840's that the organic cell reproduced asexually by fission, the nucleus dividing first. ***Mendel****'s hybridization experiments (1865) showed that independently transmitted characters separated and recombined in hybrid progeny.*

From the 1870's a number of technical advances were made in the field of experimental biology which allowed the processes occurring in the asexual reproduction of cells and in the union of sexual cells to be observed more closely: The achromatic microscope *was further improved by the introduction of the high-power immersion lens and substage illumination, while the newly discovered* aniline dyes*, together with natural dyes and some inorganic salts, were found to stain selectively certain parts of the organic cell, particularly the nucleus.*

For the next four decades, biologists succeeded to close the gap between cytology and heredity: In the 1870's it was shown by ***Hertwig****, at Berlin, and* ***Fol****, at Geneva, working on animals, and* ***Strassburger****, at Bonn, working on plants, that sexual reproduction involved the union of the nuclei of the male and female cells, from which they suggested (1884) that the nucleus of the cell was the physical basis for heredity.* ***Walter Flemming*** *(1879) coined the name* mitosis *and made first accurate accounts of chromosome numbers and figured their longitudinal splitting (1882). He then determined chromosome number (24) in man (1898).*

Edoúard van Beneden*, zoologist, first studied* meiosis *(1883) and* ***Wilhelm von Valdeyer-Hartz*** *coined the name* chromosome *(1888).* ***August Weismann****, biologist, proposed a germ-plasm theory of heredity and described the process of meiosis, whereby the number of chromosomes is halved.*

__Oscar Hertwig__ and __Theodore Bovery__ (1889–1892) showed independently that pairs of chromosome split, replicating each member before dispersing into four separate nuclei.

Finally, __Walter Sutton__ and Theodore Bovery pointed out (1902) the parallelism between chromosome behavior and Mendelism. Sutton coined the name gene *(1902) and proposed that chromosomes carry genes.*

Anatomy and physiology

Form (anatomy) and function (physiology) were traditionally conceived as a single integrated subject, but experimental techniques, particularly in the 19th century, gradually divorced the two: __Francois Magandie__ (1783–1855), __Claude Bernard__ (1813–1878), __Johannes Müller__ (1801–1858), __Carl Ludwig__ (1816–1895), __Emil Du Bois-Reymond__ (1818–1896), __William Sharpey__ (1802–1880), __Michael Foster__ (1836–1907) and __H. Bowditch__ (1840–1911) — helped create an autonomous discipline of physiology, with its research schools, professional societies and specialized journals.

Organic chemistry

Using inorganic chemistry *as its paradigm, 19th century chemists created* organic chemistry, *whence emerged the powerful ideas of* valence *and* structure. *The advent of the* periodic law *in the 1870s finally provided chemists with a comprehensive classificatory system of elements.*

By the 1880s physics and chemistry were drawing closer together in the sub-discipline of physical chemistry. Finally, the discovery of the electron *enabled the chemists to solve the fundamental problem of* chemical affinity.

The dawn of biochemistry

Chemists of the 19th century were so busy with their own science that for a long time they did not attempt to systemize the chemistry of biological processes. Most of their biochemical discoveries were incidental to their major chemical work. The most important result of the development of organic chemistry *at first, from the viewpoint of biochemistry, was the demonstration that natural organic compounds were responsive to the same laws as inorganic substances.*

The urea synthesis of __Wöhler__ (1828) and the subsequent advances in organic synthesis struck telling blows at the vitalistic hypothesis that a special force controlled living matter. Toward the middle of the century a few *chemists (chief among them was **Liebig**) really did begin to integrate their work with that of biological investigators.*

Meanwhile physiology was developing as a science in its own right, much as was chemistry. Physiologists were chiefly concerned with the mechanics of bodily organs and with studies of the nervous system. Nevertheless, it was from the physiologists that most of the advances in biochemistry came until the end of the century. The approach of these men was usually related to their studies of special systems and organs, and so an overall view of the biochemical functioning of the body was not obtained.

Many important discoveries were made in this century, but they were like isolated pieces of a jigsaw puzzle. The science was properly called physiological chemistry *at this period, since it was used mostly to help understand specific physiological problems.*

It was only at the end of the 19th century and in the 20th that the pieces began to fit together so that a unified picture of the chemical changes in the cells and their significance for the body as a whole could be obtained.

The borderline between chemistry and physiology then became a science in its own right, and to this the name biochemistry*, the chemistry of life, can more properly be applied.*

By about 1920 biochemistry possessed the basic principles upon which it is still developing. The chemical nature of the body constituents was fairly well understood, the nutritional requirements could be seen, and the enzymatic *and* hormonal *mechanisms by which metabolic processes occurred were at least known to exist.*[697]

Modern medicine

Medicine was revolutionized in the 19th century by advances in chemistry and laboratory techniques and equipment, old ideas of infectious disease epidemiology were replaced with bacteriology.

[697] *Chlorophyll* was isolated by **Pelletier** and **Caventou** in 1817, though at first its importance was not appreciated because the full significance of the photosynthetic process could not be realized until the concept of *energy* was better understood. Indeed, **J.R. Mayer**, who propounded the law of conservation of energy pointed out in 1845 that plants supplied sunlight energy as a source on which humans depended. In the meantime, the mechanism by which animals released the stored energy of plants, was clarified. By the middle of the 19th century many of the important principles of nutrition has been established, but the nature of "ferments", as *enzymes* were called during the first three quarters of the century, was the subject of much discussion.

Ignaz Semmelweis *(1818–1865) in 1847 dramatically reduced the death rate of new mothers from childbed fever by the simple expedient of requiring physicians to clean their hands before attending to women in childbirth. His discovery predated the* germ theory of disease. *However, his discoveries were not appreciated by his contemporaries and came into general use only with discoveries of British surgeon* ***Joseph Lister****, who in 1865 proved the principles of* antisepsis*; However, medical conservatism on new breakthroughs in pre-existing science prevented them from being generally well received during the* 19^{th} *century.*

After ***Charles Darwin****'s 1859 publication of The Origin of Species,* ***Gregor Mendel*** *(1822–1884) published in 1865 his books on pea plants, which would be later known as* Mendel's laws. *Re-discovered at the turn of the century, they would form the basis of classical genetics. The 1953 discovery of the structure of DNA by* ***Watson*** *and* ***Crick*** *would open the door to* ***molecular biology*** *and modern genetics. During the late* 19^{th} *century and the first part of the* 20^{th} *century, several physicians, such as Nobel prize winner* ***Alexis Carrel****, supported* eugenics*, a theory first formulated in 1865 by* ***Francis Galton****. Eugenics was discredited as a science after the Nazis' experiments in World War II became known; however, compulsory sterilization programs continued to be used in modern countries (including the US, Sweden or Peru) until much later.*

Semmelweis' work was supported by the discoveries made by ***Louis Pasteur****, who produced in 1880 the* vaccine *against* rabies. *Linking microorganisms with disease, Pasteur brought about a revolution in medicine. He also invented with* ***Claude Bernard*** *(1813–1878) the process of* pasteurization *still in use today. His experiments confirmed the germ theory. Claude Bernard aimed at establishing scientific method in medicine; he published An Introduction to the Study of Experimental Medicine in 1865. Beside this, Pasteur, along with* ***Robert Koch*** *(who was awarded the Nobel Prize in 1905), founded* bacteriology. *Koch was also famous for the discovery of the* tubercle bacillus *(1882) and the* cholera bacillus *(1883).*

For the first time actual cures were developed for certain endemic infectious diseases. However the decline in many of the most lethal diseases was more due to improvements in public health and nutrition than to medicine.

Table 4.14: NOTABLE BIOLOGISTS AND MEN OF MEDICINE (1800–1900)

Key:

A = Anatomy	E = Ecology	EN = Entomology
BI = Biochemistry	EB = Evolutionary Biology	MI = Microbiology
BO = Botany	B = Biology	AN = Anthropology
H = Heredity	M = Marine Biology	OL = Origin of Life
PL = Paleontology	P = Physiology	T = Taxonomy
ZO = Zoology	CL = Chemistry of Life	EM = Embryology
MY = Mycology	CY = Cytology	PA = Pathology
S = Surgery	BG = Biogeography	BA = Bacteriology
IM = Immunology	NA = Naturalist	

Name	*fl.*	*Specialization*
Jean-Baptiste Lamarck	*1800–1829*	*EB*
G.R. Treviranus	*1802–1837*	*P*
K.F. Burdach	*1802*	*A, P, M*
P-J. Pelletier	*1820*	*CL*
J-B. Caventou	*1820*	*CL*
Christian Eherenberg	*1820–1875*	*B*
Jan Purkyne	*1823–1839*	*P*
Christian Pander	*1825–1865*	*EM*
Karl von Baer	*1826–1876*	*EM*
Henri Dutrochet	*1826–1839*	*BO, CY*
Robert Brown	*1827–1839*	*BO, CY*
John James Audubon	*1827–1839*	
Friedrich Wöhler	*1828–1832*	*CL*
Jean-Pierre Flourens	*1830–1865*	*B, P*
Marshall Hall	*1830–1833*	*M, P*
Augustino Bassi	*1835*	*MI*
Edward Blyth	*1835–1837*	*EB*
Theodor Schwann	*1835–1839*	*M, P, CY*
Jean Marie Poiseuffe	*1835–1846*	*M, P*
Robert Remak	*1836–1858*	*M, CY, B, EM*
Matthias Schleiden	*1838–1839*	*BO, CY*
Gerhardus Müller	*1838*	*CL*

Table 4.14: (Cont.)

Name	*fl.*	*Specialization*
Friedrich Henle	*1840*	*A, PA*
Karl Schimper	*1840–1865*	*B, BO*
Wilhelm Schimper	*1840–1890*	*BO*
Edward Forbes	*1841–1847*	
David Gruby	*1841–1852*	*MI, PA*
Julius Robert Mayer	*1842*	*M*
Crawford Long	*1842*	*S, M*
Gabril Gustav Valentin	*1844*	*M, P, CL*
Carl Friedrich Ludwig	*1844–1859*	*P*
Alexander von Humboldt	*1845–1859*	*BG*
Hugo von Mohl	*1846*	*BO, P, CY*
Ernest Heinrich Weber	*1846*	*A, P, CL*
Louis Pasteur	*1846–1885*	*MI, B, CL*
Ignaz Semmelweis	*1847*	*BA, M*
Herman von Helmholtz	*1847–1894*	*M, P*
Emil Du Bois-Reymond	*1849–1877*	*P*
Casimir Davaine	*1850–1882*	*M*
Julius Ferdinand Cohn	*1850–1881*	*BO, MI*
Rudolf Ludwig Virchow	*1856–1858*	*PA*
Nathanael Pringsheim	*1855–1868*	*BO*
Adolf Eugen Fick	*1856*	*P*
Claude Bernard	*1857*	*CY*
Max Schultze	*1858–1866*	*A, ZO, CY*
Charles Robert Darwin	*1858–1871*	*EB*
Alfred Russel Wallace	*1858*	*EB*
Jackson St. George Mivrat	*1860–1900*	*B*
Pierre-Paul Broca	*1861*	*M, A, AN*
Carl von Voit	*1861–1884*	*P*
Ernst Hoppe-Seyler	*1862–1871*	*P, CL*
Julius Sachs	*1862–1887*	*BO, P*
Ernst Haeckel	*1862–1899*	*B, M, EB*
Henri Baker Tristram	*1863–1895*	*ZO, OR*

Table 4.14: (Cont.)

Name	*fl.*	*Specialization*
Julius Friedrich Cohenheim	*1864–1884*	*A, PA*
Gregor Johann Mendel	*1865*	*BO, G, EB*
Heinrich Anton de Bary	*1865–1877*	*PA, MY*
Friedrich August Kekulé	*1865*	*P, CL*
John Hughlings	*1865–1911*	
Joseph Lister	*1867*	*M, S*
Johann Friedrich Miescher	*1868–1874*	*P, CY*
John Muir	*1868–1916*	
Jean Henri Fabre	*1870–1913*	*EN*
Carl Weigert	*1870–1904*	
Gustav Theodor Fritsch	*1870–1927*	*P, CL*
Eduard Hitzig	*1870–1907*	*P*
Charles Wyville Thomson	*1872–1876*	*B, A*
Camilo Golgi	*1873–1893*	*P, M, A*
Anton Schneider	*1873*	*CY, G*
Jacobus Van't Hoff	*1874*	*CL*
Joseph-Achille Le Bel	*1874*	*CL*
Santigo Ramon y Cajal	*1875–1928*	*A, P, M*
Luther Burbank	*1875–1920*	*BO*
David Ferrier	*1875–1925*	*P*
Robert Koch	*1876–1897*	*M, BA, P*
Oscar Hertwig	*1876*	*B, CY*
Herman Fol	*1876*	*B, CY*
Wilhelm Friedrich Pffefer	*1877–1881*	*BO*
Emile Roux	*1879–1891*	*BA, IM*
Walter Flemming	*1879–1898*	*CY, G*
Charles Louis Laveran	*1880*	*M, P*
Hugo de Vries	*1880–1935*	*H*
Walter Reed	*1881–1902*	*S, BA*
Charles Roy	*1881*	*P*
Paul Ehrlich	*1881–1912*	*M, MI, BA*
Edouard van Beneden	*1883–1887*	*CY, G, ZO*

Table 4.14: (Cont.)

Name	*fl.*	*Specialization*
Emil Fischer	*1884*	*CL*
Karl Martin Kossel	*1885–1896*	*BI*
Elie Metchnikov	*1886–1908*	*IM*
Ernest Thompson-Seton	*1886–1940*	*NA*
Eduard Strassburger	*1888*	*CY, G, BO*
Wilhelm von Waldeyer-Hartz	*1888–1891*	*CY, G, A*
Georges Fernand Widal	*1888–1906*	*M, P, PA*
Theobald Smith	*1889–1895*	*BA*
Oscar Hertwig	*1889*	*CY, G, ZO*
Oskar Minkowski	*1889*	*M, P*
Joseph von Mering	*1889*	*M, P*
Emil von Behring	*1890–1901*	*IM, P, MI*
Eugene Dubois	*1891–1921*	*M, A, PA*
August Weismann	*1892*	*CY, G, B*
Theodore Boveri	*1892–1903*	*CY, G, B*
Dimitri Ivanovski	*1892*	*MI, BO*
Richard Friedrich Pffifer	*1892–1894*	*MI, IM, BA*
Morde Wolfe Haffkine	*1893–1896*	*BA, IM, MI*
Zigmund Freud	*1894–1925*	*M*
Georg Oliver	*1894–1897*	*M, P*
Edward Sharpey-Schäfer	*1894–1897*	*M, P, BI*
Carl Correns	*1895–1933*	*BO, H*
David Bruce	*1895–1915*	*MI, PA*
Giovanni Batista Grassi	*1895–*	*BA*
John Jacob Abel	*1897–1926*	*P, BI*
Ronald Ross	*1897–1916*	*M, P*
Paul Frosch	*1898*	*MI, BA*
Friedrich Loeffler	*1898*	*MI, BA*
Sergei Winogradsky	*1888–1905*	*MI, E, BA*
Martinus Beijerinck	*1898*	*MI, BO*

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5. *Demise of the Dogmatic Universe*

1895 CE–1950 CE

MATURATION OF ABSTRACT ALGEBRA AND THE GRAND FUSION OF GEOMETRY, ALGEBRA, ARITHMETIC AND TOPOLOGY

LOGIC, SET THEORY, FOUNDATION OF MATHEMATICS AND THE GENESIS OF COMPUTER SCIENCE

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Environmental Events that Impacted Civilization

1898–1923 *Bubonic plague* pandemic kills 20 million people in China, India, North Africa and South America

1900 The Galveston (TX, USA) *hurricane* kills 8000 persons

1902 The eruption of the Mt. Pelée *volcano* (Martinique) kills 30,000 people

1906 The San-Francisco *earthquake*

1908 The Messina *earthquake* kills 160,000 people

1908 The Tunguska *bolide explosion*

1912 The 'Titanic' disaster

1917–1920 Worldwide Influenza pandemic kills 80 million people, mostly in Europe and Asia

1921–1930 *Cholera*, *smallpox* and *typhus* pandemic in India kills ca 2 million people

1923 The Tokyo *earthquake*

1931–1950 *Floods* of the Yellow and Yangtze Rivers in China (1928, 1929, 1931, 1936, 1938, 1950) kill 22 million people

1942 *Hurricane* at the Bay of Bengal kills 40,000 people

1970 *Cyclone storm* and *tsunami* kill 500,000 people at the Bay of Bengal

1976 *Earthquake* in Tangshan (China) kills 650,000 people

1983–1985 Famine in Ethiopia kills ca 1 million people

Political and Religious Events that Impacted World Order

1904–1905	Russia and Japan at war
1917	The Bolshevik Revolution
1914–1918	World War I
1936–1939	The Spanish Civil War
1939–1945	World War II and the Holocaust
1947–1949	The rebirth of Israel
1949	Independence of India
1949	Foundation of the Republic of China
1949–1989	The 'Cold War'
1950–1953	The Korean War
1965–1975	The Vietnam War
1991	The Soviet Union officially ceased to exist
2001	The 'Nine-Eleven' event – Muslim terror hits the USA

1895 CE **Wilhelm Conrad Röntgen** (1845–1923, Germany). Physicist. Discovered X-rays while experimenting with electric current flow in partially evacuated glass tube (cathode-ray tube). In 1912, **Max von Laue** (1879–1960, Germany) determined its wave-lengths by means of diffraction through regularly-spaced atoms in crystals.

Although Röntgen was unaware of the true nature of these 'rays', he found that they affected photographic plates, and took the first anatomical X-ray photograph [the bones of his wife's hand]. His discovery heralded the age of modern physics and revolutionized diagnostic medicine. He was the recipient of the first Noble prize for physics, in 1901.

Röntgen was educated at Zürich and was then professor of physics at the universities of Strasbourg (1876–1879), Giessen (1879–1888), Würzburg (1888–1900) and finally Münich (1900–1920).

1895 CE **Auguste** (**Marie Louis**) **Lumière** (1862–1954, France) and his brother **Louis Jean** (1864–1948) developed a satisfactory camera and projector and made the first motion-picture film-show to the general public.

Inspired by Edison's *kinematoscope* they invented the *cinematograph*: a claw mechanism to pull the film a fixed distance past the projection (and camera) lens while the light was cut off by a shutter. Their choice of 16 frames per second remained the standard filming and projection rate through all the years of silent films.

1895–1896 CE **Hendrik Antoon Lorentz** (1853–1928, Holland). A leading physicist. Established the notion that electromagnetic radiation originates due to harmonic oscillations of charged particles *inside* atoms or molecules[1]. Lorentz suggested that a strong magnetic field ought to affect these oscillations and change the wavelength of the emitted radiation. This prediction was verified in 1896 by **Pieter Zeeman** (1865–1943, Holland), a pupil of Lorentz, and in 1902 they were awarded the Nobel prize in physics for their discovery.

Since the electron theory of Lorentz could not explain the results of the Michelson-Morley experiment, he was forced to concoct the 'Lorentz transformation' equations as an ad hoc device to overcome the difficulty.

Lorentz was born in Arnhem. During 1878–1923 he was a professor of theoretical physics at Leyden University.

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[1] According to Maxwell's theory, electromagnetic radiation is produced by oscillations of electric charges, but charges that produce *visible light* were not known at that time.

1895–1909 CE **Georg (Yuri Viktorovich) Wulff** (1863–1925, Russia). Crystallographer. Discovered that the shape of small crystals can be explained, to some extent, by a variational principle similar to that of the isoperimetric problem[2], and that their remarkable difference in structure results from the difference in the corresponding potential energies; physically, a crystal with small surface irregularities will tend to *lower its free surface energy* and this becomes the dominating factor in its shape formation.

Wulff was born in Nezhin, the Ukraine. He studied at the Universities of Warsaw (1880–1892) and Odessa (1892–1899). He then held professorial positions at Kazan and Moscow (1918–1925). Wulff showed that for every given volume, there is a unique convex body whose boundary surface has less energy than does the boundary surface of any piecewise smooth body of the same volume. In his 1895 thesis, Wulff showed that for a constant volume, the surface energy per unit area at any point on the surface depends only on the direction of the tangent plane to the surface at that point.

Moreover, the total surface energy would be minimized when the specific surface energies for each face (K_i) were proportional to the perpendicular distances (n_i or *Wulff vectors*) from a central point to each face such that $K_1 : K_2 : K_3 : \cdots = n_1 : n_2 : n_3 : \ldots$. In modern studies of crystal growth the geometric algorithm for determining the equilibrium form derived from the theorem is known as *Wulff's construction*.

1895–1901 CE **Guglielmo Marconi** (1874–1937, Italy). Inventor and electrical engineer. Became the first person to send radio communication signals through the air[3]. In 1895 he sent a wireless telegraph code signal to a distance of 2 km and in 1901 he send a code signal across the Atlantic Ocean from England to Newfoundland.

Marconi was the last in the long chain of contributors during 1884–1897: He combined **Ruhmkorff's** induction-coil, the stable *spark oscillator* of **Augusto Righi** (1850–1920, Italy), the *coherer* of **Eugene Branly** (1844–1940,

[2] In three dimensions, the perfectly smooth symmetrical sphere has the smallest free surface energy (area) when compared to all other smooth shape-sake bodies of the same volume. If however a region in space is bounded by a *finite collection of pieces of smooth surfaces* (piecewise smooth) there is an infinite number of possible surface energies; nevertheless, for each such admissible energy, the unique minimum is a convex region bounded by *planes*. The solution to this minimal problem, the optimal crystalline region, can be determined by the *Wulff construction*.

[3] The Russian claim to fame in this field is **Alexander Stepanovich Popov** (1859–1906), a physicist who devised the first *aerial* (1897), although he did not use it for radio communication. He also invented a detector for radio waves (1895).

France), **T.C. Onesti**[4] and **Oliver Lodge**, and the *antenna* of **Alexander Popov** into a workable system.

1895–1904 CE **Horace Lamb** (1849–1934, England). Applied mathematician. A student of **Stokes** and **Maxwell**; Professor at Adelaide, Australia (1875–1885) and University of Manchester (1885–1920). Author of *Hydrodynamics* (1895). Laid the foundation to modern theoretical seismology and contributed to the theory of the tides.

1895–1906 CE **Pierre Curie** (1859–1906, France). Physical chemist. Among the founders of modern physics. Discovered radium and polonium with his wife **Marie Curie** (1898), and the law that relates some magnetic properties to changes in temperature (*Curie's law*; *Curie point*). Established an analogy between paramagnetic materials and perfect gases and between ferromagnetic materials and condensed fluids.

Curie was a son of a Paris physician. Studied and taught physics at the Sorbonne, where he was appointed professor in 1904. He was run over by a dray in the rue Dauphine in Paris in 1906 and died instantly.

1895–1909 CE **Thorvald Nicolai Thiele** (1838–1910, Denmark). Mathematician with interest in astronomy. Derived a continued-fraction expansion of a given function, the convergents of which serve as *rational approximations* of the function (1909). Thiele taught in Copenhagen as well as being chief actuary of an insurance company.

1895–1915 CE **Wallace Clement Sabine** (1868–1919, U.S.A.). Physicist. Founded the science of *architectural acoustics*. Until 1895, criteria for what constituted a good acoustic hall were lacking. Sabine, a young physics instructor at Harvard University, was called on to attempt a remedy of the intolerable acoustics of the auditorium of the recently completed Fogg Art Museum[5]. He defined the parameters for good acoustical qualities before trying to translate them into practical considerations — dimensions, shapes, and building materials. He became later Hollis Professor of Mathematics and Natural Philosophy at Harvard.

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[4] **Temistocle Calzecchi Onesti** (1853–1922).

[5] In the Fogg Art Museum it was almost impossible to understand speakers in the lecture room.

Architectural Acoustics

Before Sabine, good acoustical design consisted chiefly of imitating halls in which music sounded good. Poor acoustic design consisted of superstitious practices, such as stringing useless wires across the upper spaces of a church or auditorium. Sabine identified the persistence of sound *(i.e., the excessive reverberation) as the factor that rendered speech unintelligible. He reduced the reverberation by placing felt on particular walls.*

Sabine was the first to define reverberation time*, one important parameter of lecture halls and auditoriums. His definition was the time that it takes, after a sound is turned off, for the reverberant sound level to become barely audible. (When accurate electronic measurement of sound level became possible many years later, this turned out to be a fall in sound level of 60 db.)*

From a series of ingenious experiments Sabine deduced a mathematical model that has a relation to the full-wave model (wave equation plus boundary conditions) of classical acoustics similar to that of radiative heat transfer to electromagnetic theory or of kinetic theory to classical mechanics. His idealization that sound fills a reverberant room in such a way that the average energy per unit volume in any region is nearly the same as in any other region, applies best to large *rooms whose characteristic dimensions are substantially larger than a typical wavelength.*

It also applies to live *rooms, for which the time determined by the ratio of the total propagating energy within the room to the time rate at which energy is being lost from the room is considerably larger than the time required for a sound wave to travel across a representative dimension of the room.*

When a sound source is in a room, sound waves emanating from the source will propagate until they strike the walls. Some energy will be absorbed by it and a weaker wave will be reflected back. The reflected wave will propagate until it reaches another wall where it is again reflected with partial absorption. This process continues until all the sound energy is eventually absorbed. The overall array of randomly criss-crossing rays is called the reverberant *sound field.*

The basic assumption of geometrical acoustics *is that the room walls are irregular enough so that the acoustic energy density W is distributed uniformly through the room. For this to be true, a large number of standing waves must be involved. Since each standing waves can be considered to be made up of a number of plane traveling waves, reflecting from the walls at appropriate angles, the sound in the room at the point characterized by the position vector*

$\boldsymbol{r}$ can be represented by an assemblage of harmonic plane waves, each going in a direction (φ, θ), each with pressure amplitude $A(\boldsymbol{r}, \varphi, \theta; \omega)$, intensity $I = \frac{|A|^2}{\rho c}$ (ρ = density; c = sound velocity), and energy density $W = \frac{|A|^2}{\rho c^2}$. These entities are given by the corresponding integrals

$$p(\boldsymbol{r}, \omega) = \int_0^{2\pi} d\varphi \int_0^{\pi} A(\boldsymbol{r}, \theta, \varphi) e^{i\boldsymbol{k}\cdot\boldsymbol{r} - i\omega t} \sin\theta d\theta$$

($\boldsymbol{k}$ = wave number vector; k, θ, φ its spherical components),

$$I(\boldsymbol{r}, \omega) = \frac{1}{\rho c} \int_0^{2\pi} d\varphi \int_0^{\pi/2} |A(\boldsymbol{r}, \theta, \varphi; \omega)|^2 \cos\theta \sin\theta d\theta$$

(power per unit area normal to $\boldsymbol{k}$),

$$W(\boldsymbol{r}, \omega) = \frac{1}{\rho c^2} \int_0^{2\pi} d\varphi \int_0^{\pi} |A(\boldsymbol{r}, \theta, \varphi; \omega)|^2 \sin\theta d\theta.$$

Geometric acoustics makes here two basic assumptions:

(1) The energy flow I is homogeneous and isotropic, *i.e., the average value of $|A|^2$ over a small region in space is independent of $\boldsymbol{r}$, θ and φ.*

(2) The absorptive properties of the wall surface are represented by a single parameter, averaged over all directions of incidence and integrated over the total wall area of the room. This quantity a, called the absorption *of the room, has dimensions of area.*

The first assumption leads to the simple relation $I = \frac{1}{4}cW$ for the diffused sound field in a room, when each direction of propagation is equally likely (compared with $I = cW$ for unidirectional *plane wave). The second assumption, when coupled to the energy balance equation*

$$V\frac{dW}{dt} + Ia = 0$$

(V = volume of the room), leads to the solution

$$I = I_0 e^{-\left(\frac{ac}{4V}\right)t},$$

where I_0 is the initial value of the intensity.

The reverberation time τ is defined to be the time at which $I/I_0 = 10^{-6}$, or $\tau = \frac{V}{ac}\,55.3$ (Sabine, 1895). The reverberation time can be calculated from the room's volume and the total absorption. If τ is too long, it can easily be decreased by hanging additional curtains or sound absorbent panels.

When it is too short, electronic feedback system is used; a microphone on the ceiling picks up the incoming signal which is delayed and then re-emitted by loudspeakers.

Although τ is an important parameter related to a room's acoustic behavior, it is by no means the only one. Typical values of τ are about 0.3 sec for living rooms or up to 10 sec for large churches. Most large rooms have reverberation times between 0.7 and 2 sec. If the reverberation time is too short, sounds appear 'dead' as the lack of echo produces a very clipped sound. If τ is too long, speech becomes incoherent, and echoes drown the speaker.

Sabine appreciated both the physical *aspect of measuring and predicting the transmission and decay of sound in concert halls, and the* psychological *aspect: what makes a good hall good? He experimented to find the preferred reverberation time for musical performance.*

There are two general problems in architectural acoustics, and each has many aspects. One problem is, what do we want? *(e.g., what enables performers to play well? When they do play well, what is it that make them sound good?). The other problem is,* how can we attain what is good for the performers and what is good for the audience?

Thus, concert-hall design requires great attention both to excluding external noise and to not producing noise. Satisfactory ensemble playing depends on early reflections of sound from behind and above the performers; each player must hear all the rest by means of reflected sound that is not too much delayed.

In spite of the progress made since Sabine's time, major errors have been committed in the design of music halls. Philharmonic Hall in Lincoln Center, Manhattan, opened on September 12, 1962. There were echoes at some seat locations. The members of the orchestra couldn't hear themselves and others play. There was a lack of subjectively felt reverberation. There was inadequate diffusion of sound through the hall. Worst of all, there was an apparent absence of low frequencies: it was difficult to hear the celli and double basses. In short — it was a disaster.

It turned out that the overhead acoustic panels did not reflect low-frequency components with sufficient strength into the main audience area. This was partly the result of poor scaling *(to properly reflect musical notes of different wavelengths from an acoustic panel, the panel's geometric dimensions must be at least comparable in size with the longest wavelength present in the sound. In actual fact they were much too small).*

During 1967–1974, **Manfred Schroeder** *(Germany) and his collaborators, undertook to compare more than 20 European concert halls. They played back a certain piece of music (a multichannel tape recording of Mozart's*

Jupiter symphony played by the BBC orchestra in an anechoic room) over several loudspeakers on the stages of various concert halls. In each hall they made two channel tapes of what a dummy head heard when seated in several locations. They then put listeners in an anechoic room at Göttingen and played back to them what the dummy head heard in various concert halls. By analyzing the judgments of these listeners, Schroeder and his colleagues learned that listeners liked:

- Long reverberation times (below 2.2 seconds).
- Sound to differ at their two ears.
- Narrow halls better than wide halls. (In a wide hall the first reflected sound rays reach the listener from the ceiling. In narrow halls the first reflections reach the listener from the left and right walls, and these two reflections are different.) The less preferred halls revealed a consistent absence of strong laterally traveling sound waves.

Thus, good acoustics — given proper reverberation time, frequency balance, and absence of disturbing echoes — is mediated by the presence of strong lateral sound waves that give rise to preferred stereophonic sound. In old-style high and narrow halls, such lateral sound is naturally provided by the architecture.

By contrast, in many modern fan-shaped halls with low ceiling, a monophonic sound, arriving in the symmetry plane through the listener's head, predominates, giving rise to an undesirable sensation of detachment from the music. To increase the amount of laterally traveling sound in a modern hall, highly efficient sound scattering surfaces have been recently invented (1990).

These reflection phase gratings are based on number-theoretic principles and have the remarkable property of scattering nearly equal acoustic intensities into all directions. Such broadly scattering surfaces are now being introduced into recording studios, churches, and even individual living rooms. The sound, dispersed from the ceiling, is scattered into a broad lateral radiation pattern (horizontal plane). The far-field (Fraunhofer diffraction) of such grating is approximated by the spatial Fourier transform of the acoustic signal.

The Korteweg–de Vries Equation (1895)[6]

In 1834, *the British engineer* **John Scott Russell** *(1808–1882) was consulted as to the possibility of utilizing steam navigation on the Edinburgh-Glasgow canal. He then undertook a series of experiments, in which he observed and reported (1844) the existence of solitary gravity waves in the canal. He deduced the empirical equation* $U^2 = g(h_0 + \eta_0)$, *where* U *is the wave's speed,* h_0 *the undisturbed depth of water,* g *the acceleration of gravity and* η_0 *the amplitude of the wave.*

A theoretical study of wave motion in inviscid incompressible fluid by **J. Boussinesq**[7] *(1871) and* **Lord Rayleigh** *(1876) verified Russell's equation and showed that the wave profile* $z = \eta(x,t)$ *is given by*

$$\eta(x,t) = \frac{\eta_0}{\cos h^2\{\beta(x - Ut)\}}$$

where

$$\beta = \frac{1}{2h_0}\sqrt{3\frac{\eta_0}{h_0}},$$

$$\frac{\eta_0}{h_0} \ll 1,$$

$$U \approx \sqrt{gh_0}\left(1 + \frac{1}{2}\frac{\eta_0}{h_0}\right).$$

These authors did not, however, write down the equation for which $\eta(x,t)$ *was a solution. This final step was completed in* 1895 *by* **D.J. Korteweg**[8]

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[6] To dig deeper, see:

- Drazin, P.G. and R.S. Johnson, *Solitons*: *An Introduction*, Cambridge University Press: Cambridge, 1990, 226 pp.
- Tabor, M., *Chaos and Integrability in Nonlinear Dynamics*, Wiley, 1989, 364 pp.
- Shen, S.S., *A Course on Nonlinear Waves*, Kluwer, 1994, 327 pp.
- Lamb, G.L. Jr., *Elements of Soliton Theory*, Wiley, 1980, 289 pp.

[7] **Boussinesq** (1842–1929, France).

[8] **Diederik Johannes Korteweg** (1848–1941, Holland). Applied mathematician. Educated at Delft as an engineer but later turned to mathematics. Was a professor of mathematics at the University of Amsterdam (1881–1918). Collaborated with **J.D. van der Waals** on various research topics in statistical mechanics and thermodynamics.

and **G. de Vries***. Their equation, for waves on the surface of shallow water, was*

$$\frac{\partial \eta}{\partial t} + c_0 \left(1 + \frac{3}{2h_0}\eta\right) \frac{\partial \eta}{\partial x} + \frac{1}{6} c_0 h_0^2 \frac{\partial^3 \eta}{\partial x^3} = 0$$

where $c_0 = \sqrt{gh_0}$.

This is essentially a one-dimensional wave equation in which non-linearity and dispersion occur together. It is known today as the 'KdV equation' and it has solutions known as solitons. It is characteristic of non-linear wave propagation in weakly dispersive media governed by a dispersion relation $\omega = c_0 k - \beta k^3$, *in which the relation between frequency* ω *and wave number* k, *involves the amplitude.*

Note that the soliton solution is exact *and describes a* traveling permanent profile, *which does not change its shape and propagates with constant speed. This is due to a balance between the two competing effects of non-linearity and dispersion. It is this property which gives the KdV equation its universal nature.*

Indeed, it was discovered throughout the 20^{th} *century that the equation and its modifications have many diverse applications such as: waves in a rotating atmosphere (Rossby waves), ion-acoustic waves in plasma, pressure waves in a liquid-gas bubble mixture, the non-linear Schrödinger equation and anharmonic lattice vibrations.*

1895–1915 CE **Herbert George Wells** (1866–1946, England). Novelist, sociologist, and historian. Wrote fantastic scientific romances in which he combined scientific speculations with a strain of sociological idealism: *The Time Machine* (1895), *The Island of Doctor Moreau* (1896), *The Wheels of Chance* (1896), *The Invisible Man* (1897), *The War in the Air* (1908), *The War of the Worlds* (1898), *First Men in the Moon* (1901), *The Food of the Gods* (1904), *A Modern Utopia* (1905). His novel *The World Set Free* (1914), predicted atomic bombs, atomic war and world government.

1895–1949 CE **Élie Joseph Cartan** (1869–1951, France). One of the foremost mathematician of the 20^{th} century, and one of the architects of modern mathematics. A principal founder of the modern theory of Lie groups and Lie algebras, a contributor to the theory of subalgebras and discoverer of the general mathematical form of *spinors*[9] (1913), 14 years ahead of physics. His work achieved a synthesis between continuous group, Lie algebras, differential equations and geometry.

Cartan's thesis (1894) was on the structure of continuous groups of transformations, and most of the ideas which directed all his subsequent work are to be found in it. The principal part of the thesis was devoted to the classification of simple Lie algebras over the complex field, and it completed the work of **Lie** and **Killing** on this subject.

About 1897 Cartan turned his attention to *linear associative algebras* over real and complex fields. In 1899 he began his work on Pfaffian forms, including such topics as contact transformations, invariant integrals and Hamiltonian dynamics, and his great contributions to differential geometry. Within this framework he invented the calculus of *differential forms* (1897) and introduced the concept of *wedge product*. His *exterior calculus* is anchored in the pioneering studies of **Poincaré** and **Édouard Jean Baptist Goursat** (1858–1936).

During 1904–1909 Cartan made substantial contributions to the theory of infinite *continuous groups*. In 1913 he developed systematically the theory of *spinors*, by giving a purely geometrical definition of these mathematical entities. This geometrical origin made it easy to introduce spinors into Riemannian geometry, and particularly to apply to them the idea of parallel transport.

Cartan further contributed to geometry with his theory of symmetric spaces which have their origin in papers he wrote in 1926. It developed ideas first studied by Clifford and Cayley and used topological methods developed by Weyl. This work was completed by 1932.

The discovery of the general theory of relativity in 1916 turned the attention of many mathematicians, including Cartan, to the general concept of geometry, and nearly all of Cartan's work from this time onwards is devoted to the development of a general theory of *differential geometry* (1917–1949). It forms a most vital contribution to modern mathematics.

Thus, in 1922 he proposed and developed a gravitational theory with *nonsymmetric connection* (geometry with *torsion*). The result was the *Einstein-Cartan equations* which include, in addition to Einstein's ten equations for the metric $g_{\mu\nu}$, a system of equations for the torsion tensor. The source in the torsion equation is represented by a tensor that is defined by the spin properties of matter.

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9 For further reading, see: Cartan, E., *The Theory of Spinors*, Dover Publications: New York, 1981, 157 pp.

It is indeed remarkable that this work was begun when he was nearly 50, and carried on until he was 80 — a most striking exception to **Hardy**'s dictum that mathematics is a young man's game.

Cartan was born in Dolomien, a village in the south of France. His father was a blacksmith. Cartan's elementary education was made possible by a state stipend for gifted children. In 1888 he entered the École Normale Superieur, where he learned higher mathematics from **Picard**, **Darboux** and **Hermite**. His research work started with his famous thesis on continuous groups, a subject suggested to him by a fellow student, recently returned from studying with **Sophus Lie** in Leipzig.

He was made a professor at the Sorbonne in 1912. The report on his work, which was the basis for this promotion, was written by **Poincaré**. He remained in the Sorbonne until his retirement in 1940.

In 1903 Cartan married Mlle Marie-Louis Bianconi. Besides a daughter, there were three sons of the marriage — **Henri Paul Cartan** (b. 1904), a distinguished mathematician in his own right who made significant advances in the theory of analytic functions, theory of sheaves, homological theory, algebraic topology and potential theory.

His other son, Jean, oriented himself toward music, and had already emerged as one of the most gifted composers of his generation, when he was cruelly taken by death. His third son, Louis Cartan, a professor of physics, was arrested by the Germans at the beginning of the Resistance and murdered by them in 1943.

Besides several books, Cartan published about 200 mathematical papers. His mathematical works can be roughly classified under three headings: group theory, systems of differential equations and geometry. These themes are constantly interwoven with each other in his work. Almost everything Cartan did is more or less connected with the theory of Lie groups.

The Calculus of Differential Forms[10]

The calculus of alternating differential forms (also known as the exterior calculus or Cartan calculus) enables one to make a systematic generalization to n-dimensional spaces of vector analysis in the plane and in three dimensional space. Thus, the theory provides a convenient and elegant way of phrasing Green's, Stokes', and Gauss' theorems. In fact, the use of differential forms shows that these theorems are all manifestations of a single underlying mathematical theory and provides the necessary language to generalize them to n dimensions.

This calculus has applications, among other things, to differential geometry and theoretical physics (e.g., relativity theory, electrodynamics, thermodynamics, analytical mechanics, particle physics). There is a very close connection between alternating differential forms and skew-symmetric tensors. The calculus of differential forms carries to manifolds (especially those that do not include a metric or a covariant derivative) such basic notions as gradient, curl and integral. Further, it enables an index-free treatment of differential geometry.

In the following, the algebraic structure of differential forms (DF) will be outlined from an axiomatic viewpoint. Then, the deep-seated reasons for the apparently arbitrary definitions will be anchored in vector analysis, and finally motivated by physical applications.

- *A real-valued twice-differentiable function* $f(x, y, z)$ *is an 0-form. It can be considered as a rule that assigns to each point in* R^3 *a real number. This generalizes to any n-dimensional manifold.*
- *Formal expressions such as*

$$\omega = A(x, y)dx + B(x, y)dy \qquad \text{and}$$
$$\omega = A(x, y, z)dx + B(x, y, z)dy + C(x, y, z)\,dz$$

are 1-forms. The first is a 1-form *in the* XY *plane (or any* 2-dimensional *manifold), while the second is a* 1-form *in* 3-dimensional *space. [A, B, C will be assumed to be real-valued infinitely differentiable functions, although this is not necessary for the derivation of some of the results.]*

[10] To dig deeper, see:
- Flanders, H., *Differential Forms with Applications to the Physical Sciences*, Dover, 1989, 205 pp.

- *The formal expression*

$$\eta = A(x, y, z)dxdy + B(x, y, z)dydz + C(x, y, z)dzdx$$

is a 2-form in a 3-space (i.e. a 3-dimensional manifold or R^3). The 2-form $\eta = A(x, y)dxdy$ *is a special case for* $B = 0,\ C = 0,\ \frac{\partial A}{\partial Z} = 0$, *and also the general case in two dimensions. The order of the differentials is essential in this product-notation;* $dxdy = -dydx$, *etc. In general* $\omega_1\omega_2 = -\omega_2\omega_1$ *for any pair of 1-forms.*

- *The formal expression* $\mu = A(x, y, z)dxdydz$ *is a 3-form. The order of the differentials is again essential, modulo cyclic permutation:* $dxdydz = dzdxdy = -dzdydx$, *etc.*

In 3-space there exist only 0-forms, 1-forms, 2-forms, and 3-forms, while in 2-space, there are only 0-forms, 1-forms, and 2-forms. In the manifold case, the above expressions hold in any given local coordinate system.

Algebraic structure: *The system of DF in 3-space, for instance, is a linear associative algebra (Grassman algebra) with a basis of 8 elements:*

$$1,\ dx,\ dy,\ dz,\ dxdy,\ dydz,\ dzdx,\ dxdydz$$

whose coefficients belong to the field of continuous functions, and whose multiplication table is specified by:

$$dxdy = -dydx, \quad dydz = -dzdy, \quad dzdx = -dxdz,$$

$$dxdx = 0, \quad dydy = 0, \quad dzdz = 0.$$

The product of a k-form and an m-form is a $(k + m)$ *form, where the integer prefix is the form's degree (dx is a 1-form, dxdy a 2-form, etc.).*

If $m + k > n$, *the number of variables, then there will be repetitions, and such a product will be zero. Since a 0-form is merely a function, multiplication by a 0-form does not affect the degree of the form. [Example:* $(xdx - zdy + y^2dz)(x^2dydz + 2dzdx - ydxdy) = (x^3 - 2z - y^3)dxdydz$.*]*

Geometrical structure: *One can naturally define the line integral of a 1-form* $\omega = Adx + Bdy + Cdz$ *along a curve* γ. *Let* γ *be a smooth simple curve, with parametric equations* $x = x(t),\ \ y = y(t),\ \ z = z(t),\ \ a \le t \le b$, *and oriented in such a way that the positive direction of* γ *is associated with* *the direction which* $x(t), y(t), z(t)$ *traverse as* t *increases from* a *to* b*; the same curve with the opposite orientation is denoted* $-\gamma$*. Then*

$$\int_\gamma \omega \equiv \int_a^b \left[A\{x(t), y(t), z(t)\} \frac{dx}{dt} + B\{x(t), y(t), z(t)\} \frac{dy}{dt} + C\{x(t), y(t), z(t)\} \frac{dz}{dt} \right] dt.$$

In this way a 1-form can be thought of as a rule that assigns a real number to each oriented curve[11]*. Note that* $\int_{-\gamma} \omega = -\int_\gamma \omega$ *since reversal of orientation of a curve changes the sign of the integral.*

A 2-form may be similarly interpreted as a surface functional, namely a function that associates with each oriented 2D *surface a real number. Again, using the local parametrization* $x = x(u, v)$, $y = y(u, v)$, $z = z(u, v)$ *with* (u, v) *belonging to a domain* D *in* R^2*, we have for a surface* S *and 2-form* η*:*

$$\int_S \eta = \int_S (Adxdy + Bdydz + Cdzdx)$$
$$= \int_D \left[A\{x(u,v), y(u,v), z(u,v)\} \frac{\partial(x,y)}{\partial(u,v)} + B\{x(u,v), y(u,v), z(u,v)\} \frac{\partial(y,z)}{\partial(u,v)} + C\{x(u,v), y(u,v), z(u,v)\} \frac{\partial(z,x)}{\partial(u,v)} \right] dudv$$

where

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}; \qquad \frac{\partial(y,z)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} \end{vmatrix}; \qquad \frac{\partial(z,x)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} \\ \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \end{vmatrix}.$$

Note that, if η *would have a* $dxdx$ *term (which formally vanishes), the determinant* $\frac{\partial(x,x)}{\partial(u,v)}$ *has equal rows, and hence vanishes. Also, if we interchange*

[11] In general, given a *1-form* and a smooth curve γ, the value of the line integral will depend on γ. However, when two curves are *parametrically equivalent*, then a *1-form* will assign the same value to both. The same is true for smoothly equivalent surfaces and *2-forms*. All these integrals can be extended to curves and surfaces embedded in any n-space ($n \geq 2$). If the space is a manifold, the integrals need to be broken up into single–coordinate–system pieces.

Parametrical equivalence: if (in any coordinate–system neighborhood through which γ passes) $x = \phi(t)$ on γ_1 with $a \leq t \leq b$ and γ_2 is given by $x = \phi[f(t)]$ over $\alpha \leq t \leq \beta$, and the function $f(t)$ maps the interval $[\alpha, \beta]$ onto $[a, b]$ in a one-to-one, differentiable manner with $f'(t) > 0$, then (and only then) are the curves γ_1 and γ_2 said to be parametrically equivalent.

x and y, the corresponding determinant changes sign; etc. This renders a geometrical motivation for the rules $dxdx = 0$, $dydx = -dxdy$, with similar results for the other Jacobian determinants.

Finally, a 3-form ν in 3-space assigns a real number to each 3D submanifold Ω of the 3-space in question. The number is $\int_\Omega \nu = \int_\Omega f(x,y,z)dxdydz$ which is just the ordinary triple integral of f over Ω.

All these integrals are true geometric entities – because they do not depend on the choice of local manifold coordinate systems, nor upon the parameterizations (coordinate systems) of the submanifolds integrated over (curves, surfaces and higher submanifolds). This concludes the description of the geometrical hierarchy.

Differentiation of forms in 3-space: *In general, if ω is a k-form, its differential $d\omega$ will be a $k+1$ form. Thus if A is a 0-form (function) then dA is (locally) the 1-form $dA = \frac{\partial A}{\partial x}dx + \frac{\partial A}{\partial y}dy + \frac{\partial A}{\partial z}dz$. If ω is a 1-form $Adx + Bdy + Cdz$ whose local coefficients are functions, then $d\omega$ is the 2-form $d\omega = (dA)dx + (dB)dy + (dC)dz$.*

If ω is a 2-form in 3-space $Adydz + Bdzdx + Cdxdy$, then $d\omega$ is the 3-form $d\omega = (dA)dydz + (dB)dzdx + (dC)dxdy$, where $d\omega$ is to be locally computed by evaluating dA, dB, and dC, and then computing the indicated products.

Since dA is a 1-form, $(dA)dx$ is indeed a 2-form and $(dA)dydz$ is a 3-form. [For example: $\omega = x^2ydydz - xzdxdy$, $d\omega = (2xy - x)dxdydz$.] The operator d is known as the exterior derivative[12]*. The derivative of a 3-form may be computed in the same fashion, but since it will be a 4-form in three variables, it will automatically be zero. Differentiation of forms in two variables is done in the same fashion*[13]*. The foregoing rules for multiplication and differentiation have in store for us a number of surprises:*

[12] There are three kinds of derivatives in differential geometry and tensor analysis:
- *The covariant derivative* in the direction of the contravariant vector $\boldsymbol{A}$ (∇_A); acts on *any* tensor; it depends only on $\boldsymbol{A}$ but not its derivatives.
- *The Lie derivative* (L_A); depends on $\boldsymbol{A}$ *and* its derivatives.
- *The exterior derivative*; acts on any totally antisymmetric, covariant tensor (= differential form) to yield a form with rank higher by one; it is also covariant.

The last two derivatives exist even in spaces without an affine connection, Γ^m_{ik}, but ∇_A exist only in an affine space (a space endowed with a connection).

[13] The rules may seem arbitrary, but in fact they "happen" to fit with the rule of the transformation of the area element through a coordinate transformation.

- *The product of two 1-forms in $\mathbb{R}^3$ yields the general formula*

$$\omega_1\omega_2 = (Adx + Bdy + Cdz)(adx + bdy + cdz)$$
$$= \begin{vmatrix} B & C \\ b & c \end{vmatrix} dydz + \begin{vmatrix} C & A \\ c & a \end{vmatrix} dzdx + \begin{vmatrix} A & B \\ a & b \end{vmatrix} dxdy \Rightarrow (\boldsymbol{\omega}_1 \times \boldsymbol{\omega}_2).$$

  In the above equation we treat (A, B, C) and (a, b, c) as the respective components of the vectors $\boldsymbol{\omega}_1 = A\boldsymbol{e}_x + B\boldsymbol{e}_y + C\boldsymbol{e}_z$, $\boldsymbol{\omega}_2 = a\boldsymbol{e}_x + b\boldsymbol{e}_y + c\boldsymbol{e}_z$, and $(\times)$ is the usual vector product operation.

- *The product of a 1-form $\omega = Adx + Bdy + Cdz$ by the 2-form $\nu = adydz + bdzdx + cdxdy$ in a 3-space yields the 3-form $(aA + bB + cC)dxdydz$. Its scalar function coefficient can be written as the scalar product of the local vectors $\boldsymbol{\omega} = A\boldsymbol{e}_x + B\boldsymbol{e}_y + C\boldsymbol{e}_z$ and $\boldsymbol{\nu} = a\boldsymbol{e}_x + b\boldsymbol{e}_y + c\boldsymbol{e}_z$, namely: $(\boldsymbol{\omega} \cdot \boldsymbol{\nu})$.*

- *If $\omega = A(x, y, z)dx + B(x, y, z)dy + C(x, y, z)dz$, then*

$$d\omega = \left(\frac{\partial C}{\partial y} - \frac{\partial B}{\partial z}\right) dydz + \left(\frac{\partial A}{\partial z} - \frac{\partial C}{\partial x}\right) dzdx + \left(\frac{\partial B}{\partial x} - \frac{\partial A}{\partial y}\right) dxdy \Rightarrow \mathrm{curl}\,\boldsymbol{\omega},$$

  where $\boldsymbol{\omega} = A\boldsymbol{e}_x + B\boldsymbol{e}_y + C\boldsymbol{e}_z$.

- *If $\nu = a(x, y, z)dydz + b(x, y, z)dzdx + c(x, y, z)dxdy$, then*

$$d\nu = \left(\frac{\partial a}{\partial x} + \frac{\partial b}{\partial y} + \frac{\partial c}{\partial z}\right) dxdydz \Rightarrow \mathrm{div}\,\boldsymbol{\nu},$$

  where $\boldsymbol{\nu} = a\boldsymbol{e}_x + b\boldsymbol{e}_y + c\boldsymbol{e}_z$.

The above examples hint to a strong link between vector analysis and differential forms. But prior to the establishment of the nature of these connections one must summarize the historical background:

Let $u = f(x, y)$, $v = g(x, y)$, then $du = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$, $dv = \frac{\partial g}{\partial x}dx + \frac{\partial g}{\partial y}dy$ and

$$dudv = \left(\frac{\partial f}{\partial x}\,dx + \frac{\partial f}{\partial y}\,dy\right)\left(\frac{\partial g}{\partial x}\,dx + \frac{\partial g}{\partial y}\,dy\right)$$
$$= \left(\frac{\partial f}{\partial x}\frac{\partial g}{\partial y} - \frac{\partial f}{\partial y}\frac{\partial g}{\partial x}\right) dxdy = \frac{\partial(u, v)}{\partial(x, y)}\,dxdy.$$

This ensures that a surface integral $\int_S \eta$, for any *2-form* η and 2-submanifold S in any 2-space, does not depend on local coordinate–system choices. Similar reasoning applies to forms of any degree in any dimensionality.

Early efforts to establish an algebra for points of the plane *(which has the same rules as the algebra of numbers) met with success because a suitable definition for multiplication could be found, namely, the (commutative) product of ordered–real–pair points*

$$(x_1, x_2)(y_1, y_2) = (x_1y_1 - x_2y_2, \quad x_1y_2 + x_2y_1).$$

The motivation for this rule may be seen by making the correspondence $(a, b) \leftrightarrow a + bi$ *between the plane and the field of* complex numbers. *If* $p = (x_1, x_2)$ *corresponds to* $z = x_1 + ix_2$ *and* $q = (y_1, y_2)$ *corresponds to* $w = y_1 + iy_2$, *then we see that*

$$zw = (x_1 + ix_2)(y_1 + iy_2) = (x_1y_1 - x_2y_2) + i(x_1y_2 + x_2y_1)$$

which corresponds to the point which is given as the product of p *and* q.

With this example in mind, one may attempt to find a similar definition for multiplication of points in 3-space. By algebraic methods, it can be shown that no such formula exists (if we require that the ordinary algebraic rules remain valid).

However, going to the next higher dimension, **Hamilton** *(1843) discovered that a definition for multiplication of points in* E^4 *could be given which yields a system obeying all the algebraic rules which apply to real numbers (i.e., the field axioms) except one; multiplication is no longer commutative, so that* (pq) *and* (qp) *may be different points. This system is called the algebra of* quaternions.

It was soon seen that it could be used to great advantage in analytical mechanics. By restricting points to a particular 3-space embedded in E^4, ***Gibbs*** *and others developed a modification of the algebra of quaternions which was called* vector analysis, *and which gained widespread acceptance and importance, particularly in physics.*

Let us now compare the system of vector analysis with the system of differential forms in three variables based on the four examples given above. We first notice that there is a certain formal similarity between the multiplication table for the unit vectors $\boldsymbol{e}_i \times \boldsymbol{e}_j = \sum_{k=1}^{3} \epsilon_{ijk}\boldsymbol{e}_k$ *and the corresponding table for the basic differential forms* dx, dy, *and* dz. *In the latter, however, we do not have the identification* $dxdy = dz$ *which corresponds to the relation* $\boldsymbol{e}_x \times \boldsymbol{e}_y = \boldsymbol{e}_z$. *This suggests that we link elements in pairs:*

$$\begin{matrix} dx \\ dydz \end{matrix} \leftrightarrow \boldsymbol{e}_x \quad \begin{matrix} dy \\ dzdx \end{matrix} \leftrightarrow \boldsymbol{e}_y \quad \begin{matrix} dz \\ dxdy \end{matrix} \leftrightarrow \boldsymbol{e}_z.$$

To complete these, and take into account 0-forms and 3-forms, we adjoin one more correspondence:

$$\frac{1}{dxdydz} \leftrightarrow 1.$$

We are now ready to set up a two-to-one correspondence between differential forms on one hand, and vector- and scalar-valued functions on the other. To any 1-form or 2-form will correspond a vector function, and to any 0-form or 3-form will correspond a scalar function. The rule of correspondence is indicated below:

$$\left.\begin{array}{r} A\,dx + B\,dy + C\,dz \\ A\,dy\,dz + B\,dz\,dx + C\,dx\,dy \end{array}\right\} \leftrightarrow A\boldsymbol{e}_x + B\boldsymbol{e}_y + C\boldsymbol{e}_z,$$

$$\left.\begin{array}{r} f(x,y,z) \\ f(x,y,z)\,dx\,dy\,dz \end{array}\right\} \leftrightarrow f(x,y,z).$$

In the opposite direction, we see that a vector-valued function corresponds to both a 1-form and to a 2-form, and a scalar function to a 0-form and to a 3-form. It then transpires that the single notion of multiplication among differential forms corresponds to both the scalar and vector products among vectors.

What vector operations correspond to differentiation of forms? Let us compare the differential of an 0-form $df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$ *with the vector gradient of a scalar function* $\operatorname{grad} f = \frac{\partial f}{\partial x}\boldsymbol{e}_x + \frac{\partial f}{\partial y}\boldsymbol{e}_y + \frac{\partial f}{\partial z}\boldsymbol{e}_z$. *These are tied through the relation* $df = d\boldsymbol{r}\cdot\nabla f$.

As we proceed to 1-forms, it was shown earlier that its differential corresponds to the curl of the corresponding vector function. Finally, it was shown that the differential of a 2-form corresponds to the divergence of the corresponding vector function. Briefly, then, the single operation of differentiation in the system of differential forms corresponds in turn to the operations of taking the gradient of a scalar field and taking the curl and the divergence of a vector field. This is indicated schematically as follows:

$$\begin{array}{ccccccc} \begin{array}{c} f \\ \textit{scalar} \\ \textit{function} \end{array} & \longrightarrow & \begin{array}{c} f \\ \textit{0-form} \end{array} & \longrightarrow & \begin{array}{c} df \\ \textit{1-form} \end{array} & \longrightarrow & \begin{array}{c} \mathrm{grad}(f) \\ \textit{vector} \\ \textit{function} \end{array} \end{array}$$

$$\begin{array}{ccccccc} & & \begin{array}{c} \omega \\ \textit{1-form} \end{array} & \longrightarrow & \begin{array}{c} d\omega \\ \textit{2-form} \end{array} & \longrightarrow & \begin{array}{c} \mathrm{curl}(\boldsymbol{V}) \\ \textit{vector function} \end{array} \\ \begin{array}{c} \boldsymbol{V} \\ \textit{vector} \\ \textit{function} \end{array} & \begin{array}{c} \nearrow \\ \searrow \end{array} & & & & & \\ & & \begin{array}{c} \omega^* \\ \textit{2-form} \end{array} & \longrightarrow & \begin{array}{c} d\omega^* \\ \textit{3-form} \end{array} & \longrightarrow & \begin{array}{c} \mathrm{div}(\boldsymbol{V}) \\ \textit{scalar function} \end{array} \end{array}$$

Next, let $\omega = A(x,y,z)dx$. *Then*

$$d\omega = d(A)\,dx = \frac{\partial A}{\partial x}\,dxdx + \frac{\partial A}{\partial y}\,dydx + \frac{\partial A}{\partial z}\,dzdx = \frac{\partial A}{\partial y}\,dydx + \frac{\partial A}{\partial z}\,dzdx$$

and

$$dd\omega = d\left(\frac{\partial A}{\partial y}\right)\,dydx + d\left(\frac{\partial A}{\partial z}\right)\,dzdx = \frac{\partial^2 A}{\partial z \partial y}\,dzdydx + \frac{\partial^2 A}{\partial y \partial z}\,dydzdx = 0,$$

where we have used the symmetry of the mixed derivatives and the fact that $dydzdx = -dzdydx$. *A similar argument holds for* Bdy *and* Cdz.

Using the above results we see that the statement $ddf = 0$, *holding for a 0-form* f, *corresponds to the vector identity* $\mathrm{curl}(\mathrm{grad}\, f) = 0$ *and the statement* $dd\omega = 0$, *holding for a 1-form, corresponds to the vector identity* $\mathrm{div}(\mathrm{curl}\,\boldsymbol{V}) = 0$.

Our final connections between vector analysis and differential forms will be made by relating the integral of a form to integrals of certain scalar functions which are obtained by vector operations.

To see this we let $\boldsymbol{F} = A\boldsymbol{e}_x + B\boldsymbol{e}_y + C\boldsymbol{e}_z$ *define a continuous vector field in some region in space, and let* $\omega = Adx + Bdy + Cdz$ *be the corresponding 1-form. If* γ *is a smooth curve in the said region, and* $\boldsymbol{t}(s)$ *is a unit tangent vector to* $\gamma(s)$ *(where* s *is the arc length) then*

$$\begin{aligned} \boldsymbol{F}\cdot\boldsymbol{t} &= (A\boldsymbol{e}_x + B\boldsymbol{e}_y + C\boldsymbol{e}_z)\cdot\left(\frac{dx}{ds}\boldsymbol{e}_x + \frac{dy}{ds}\boldsymbol{e}_y + \frac{dz}{ds}\boldsymbol{e}_z\right) \\ &= A\frac{dx}{ds} + B\frac{dy}{ds} + C\frac{dz}{ds} \end{aligned}$$

and

$$\int_\gamma \boldsymbol{F}\cdot\boldsymbol{t}ds = \int_0^\ell \left(A\frac{dx}{ds} + B\frac{dy}{ds} + C\frac{dz}{ds}\right)ds = \int_\gamma \omega,$$

with ℓ *the length of the curve* γ. *The situation for integrals of 2-forms is similar.*

The theorems of Green, Stokes, and Gauss can be translated into the language of differential forms. They are all shown to be special cases of what is called the generalized Stokes' theorem, *connecting an integral of a differential form* ω *with an integral of its derivative* $d\omega$. *Symbolically*

$$\int_{\partial M} \omega = \int_M d\omega,$$

where M *is a* $k+1$ *dimensional manifold with boundary manifold* ∂M *and* ω *is a* k*-form.*

The demonstration[14] *of this statement in the case of Green's theorem is immediate since*

$$\int_\gamma \omega = \int_{\partial D}(Pdx + Qdy) = \int_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dxdy = \int_D d\omega,$$

where D *is a region in the* xy *plane and* ∂D *is its boundary curve* γ. *Similarly, for Stokes' theorem, if* Σ *be and oriented 2D surface embedded in* R^3 *and* $\partial\Sigma$ *its closed bounding curve (suitably oriented), we find:*

$$\begin{aligned}\int_{\partial\Sigma}\omega &= \int_{\partial\Sigma}\boldsymbol{F}\cdot d\boldsymbol{s} = \int_{\partial\Sigma}(Adx + Bdy + Cdz)\\ &= \int_\Sigma \left[\left(\frac{\partial C}{\partial y} - \frac{\partial B}{\partial z}\right)dydz + \left(\frac{\partial A}{\partial z} - \frac{\partial C}{\partial x}\right)dzdx + \left(\frac{\partial B}{\partial x} - \frac{\partial A}{\partial y}\right)dxdy\right]\\ &= \int_\Sigma d\omega.\end{aligned}$$

Finally, for a region R *in* R^3 *and its closed-surface boundary* ∂R, *with suitable orientations:*

$$\begin{aligned}\int_{\partial R}\omega &= \int_{\partial R}[Adydz + Bdzdx + Cdxdy]\\ &= \int_R \left[\frac{\partial A}{\partial x} + \frac{\partial B}{\partial y} + \frac{\partial C}{\partial z}\right]dxdydz = \int_R d\omega.\end{aligned}$$

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[14] These are *not* in lieu of rigorous mathematical proofs.

This statement is the supercompact form of Gauss' divergence theorem, stating that the flux of a vector field out of an oriented closed surface equals the integral of the divergence of that vector field over the volume *enclosed by the surface. [In all these cases the orientations of volumes, curves and surfaces are related via the right hand rule]*

EXACT DIFFERENTIAL FORMS *constitute a special class of DF for which* $\omega = d\eta$ *[e.g.,* $\omega = 2xydx + x^2dy + 2zdz = d(x^2y + z)$*]. Clearly,* $\int_\gamma \omega = f(p_1) - f(p_0)$, *where integration extends from* p_0 *on* γ *to* p_1 *on* γ*.*

If γ *is closed and* f *single-valued, then* $p_0 = p_1$ and $\int_\gamma \omega = 0$. *In this case the line integral is* path-independent. *Furthermore, it follows directly from* $\omega = df$ *that* $d\omega = ddf = 0$ *throughout the region. This can be summarized by the flow-diagram*

$$\begin{array}{ccc} \begin{array}{c}\text{1-form } \omega \\ \text{exact in } \Omega\end{array} & \longrightarrow & \displaystyle\int_\gamma \omega \begin{array}{c}\text{independent} \\ \text{of path in } \Omega\end{array} \\ \searrow & & \nearrow \\ & \begin{array}{c} d\omega = 0 \\ \text{in } \Omega \end{array} & \end{array} ,$$

where Ω *is any sub-region in* R^n*.*

The notions of exactness and of path independence may also be given in vector form: If a vector field $\boldsymbol{F}$ *can be written as* $\boldsymbol{F} = \text{grad}\, f$, *where* f *is the* potential *of the field, then* $\int_\gamma \boldsymbol{F} \cdot d\boldsymbol{s} = f(p_1) - f(p_0)$. *For instance, if we let* $U = -f$ *be the* potential energy, *then* $\boldsymbol{F}$ *represents its force field.*

THE WEDGE PRODUCT *(WP): It is convenient to denote the multiplication of differential forms with a special symbol, the wedge* $\wedge$, *instead of just a juxtaposition that we used hitherto.*

Thus, for example, the product of the 1-forms $\omega = a_\mu dx^\mu$ and $\omega' = b_\nu dx^\nu$ *(both covariant vectors!) will be written as*

$$\omega \wedge \omega' = \frac{1}{2}(a_\mu b_\nu - a_\nu b_\mu) dx^\mu \wedge dx^\nu$$

and called the wedge product.

With $x^1 = x$, $x^2 = y$, $x^3 = z$, *we find as before*

$$\begin{aligned} \omega \wedge \omega' = (a_1b_2 - a_2b_1)dx \wedge dy + (a_2b_3 - a_3b_2)dy \wedge dz \\ + (a_3b_1 - a_1b_3)dz \wedge dx. \end{aligned}$$

With this notation, the product of differential 1-forms appear in a new light — as a totally antisymmetric form which takes in the vectors $\boldsymbol{a}(a_1, a_2, a_3)$, $\boldsymbol{b}(b_1, b_2, b_3)$ *and yields a number.*

Now, since in dyadic notation

$$\boldsymbol{ab} - \boldsymbol{ba} = (a_1b_2 - a_2b_1)(\boldsymbol{e}_x\boldsymbol{e}_y - \boldsymbol{e}_y\boldsymbol{e}_x) + (a_2b_3 - a_3b_2)(\boldsymbol{e}_y\boldsymbol{e}_z - \boldsymbol{e}_z\boldsymbol{e}_y) + (a_3b_1 - a_1b_3)(\boldsymbol{e}_z\boldsymbol{e}_x - \boldsymbol{e}_x\boldsymbol{e}_z),$$

we can set a one-to-one correspondence between the components of this totally antisymmetric tensor of the second rank and the terms of the wedge product $\omega \times \omega'$, *provided we also set the correspondence* $dx \leftrightarrow \boldsymbol{e}_x$, $dy \leftrightarrow \boldsymbol{e}_y$, $dz \leftrightarrow \boldsymbol{e}_z$, *and also* $dx \wedge dy \leftrightarrow \boldsymbol{e}_x\boldsymbol{e}_y - \boldsymbol{e}_y\boldsymbol{e}_x$, *etc. In this sense*

$$\omega \wedge \omega' \Leftrightarrow \boldsymbol{ab} - \boldsymbol{ba} \equiv \mathfrak{I} \times (\boldsymbol{b} \times \boldsymbol{a}).$$

Because of this correspondence, we may speak of the wedge product of 1-forms in the sense of DF, *and at the same time write*

$$\boldsymbol{a} \wedge \boldsymbol{b} = \boldsymbol{ab} - \boldsymbol{ba}$$

for the associated vectors. Note that $a_1b_2 - a_2b_1$ *is the signed area of the projection of the* $\boldsymbol{ab}$ parallelogram *on the* xy *plane, etc.*

The wedge product has the usual distributive and associative properties of abstract algebra over a field:

$$\begin{aligned} \boldsymbol{a} \wedge (\boldsymbol{b} + \boldsymbol{c}) &= \boldsymbol{a} \wedge \boldsymbol{b} + \boldsymbol{a} \wedge \boldsymbol{c}, \\ (\boldsymbol{a} + \boldsymbol{b}) \wedge \boldsymbol{c} &= \boldsymbol{a} \wedge \boldsymbol{b} + \boldsymbol{b} \wedge \boldsymbol{c}, \\ \alpha(\boldsymbol{a} \wedge \boldsymbol{b}) &= \boldsymbol{a} \wedge \alpha\boldsymbol{b} = \alpha\boldsymbol{a} \wedge \boldsymbol{b}, \end{aligned}$$

with $\boldsymbol{a}$, $\boldsymbol{b}$ *any forms and* α *is a scalar (0-form).*

But it is anticommutative (if $\boldsymbol{a}, \boldsymbol{b}$ *are 1-forms, or in general, if the degrees of* $\boldsymbol{a}$, $\boldsymbol{b}$ *are both odd):*

$$\boldsymbol{a} \wedge \boldsymbol{b} = -\boldsymbol{b} \wedge \boldsymbol{a}.$$

Specializing again to the case where $\boldsymbol{a}$, $\boldsymbol{b}$ *are 1-forms, the antisymmetric tensor* $\boldsymbol{a} \wedge \boldsymbol{b}$ *can be written symbolically in determinant form*

$$\begin{vmatrix} a_i & b_i \\ a_j & b_j \end{vmatrix}.$$

One can then extend $\wedge$ to triple and higher products. For example, the totally antisymmetric tensor of the third rank $\boldsymbol{a}\wedge\boldsymbol{b}\wedge\boldsymbol{c}$ *has the determinant form*

$$T_{ijk} = \begin{bmatrix} a_i & b_i & c_i \\ a_j & b_j & c_j \\ a_k & b_k & c_k \end{bmatrix}.$$

Explicitly, in triadic form,

$$T = \boldsymbol{abc} + \boldsymbol{cab} + \boldsymbol{bca} - \boldsymbol{cba} - \boldsymbol{acb} - \boldsymbol{bac}$$

is the proper definition of the triple wedge product $\boldsymbol{a}\wedge\boldsymbol{b}\wedge\boldsymbol{c}$. *This rank-3 tensor, or triadic, also known as a trivector, is antisymmetric in all its indices.*

The associated 3-form is obtained by the triple wedge product of three 1-forms corresponding to $\boldsymbol{a}$, $\boldsymbol{b}$ *and* $\boldsymbol{c}$*:*

$$\omega\wedge\omega'\wedge\omega'' = (a_1dx+a_2dy+a_3dz)\wedge(b_1dx+b_2dy+b_3dz)\wedge(c_1dx+c_2dy+c_3dz)$$
$$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} dx\wedge dy\wedge dz,$$

where the determinant is the 3-dimensional oriented volume of the parallelepiped spanned by the vectors $\boldsymbol{a}$, $\boldsymbol{b}$ *and* $\boldsymbol{c}$.

Science and Non-Science

The perspectives of science are, like those of philosophy and religion, thought-structures for viewing the world. Science *aims to attain objective truth, and scientific ideas ought to be ones in which we can trust as a matter of fact rather than opinion, and which are not subject to doubt and fads as are the beliefs of the religions, or even philosophies. Thus, the overall purpose of the scientific method is to make valid distinctions between the false and the true in nature, so as to render a true picture of realities and their underlying mechanisms and principles.*

Nevertheless, the average scientist *(by the very nature of his textbook-oriented education and the fact that his progress and success depend in many cases on the acceptance of given ideas) is as narrow, rigid and dogmatic as the orthodox theologician.*

In contradistinction to science and the scientific method we recognize certain related disciplines: pseudoscience *is false science, a body of 'knowledge' which is a mere belief, like astrology, alchemy, phrenology, Mesmerism, pangenesis, creationism, etc. Many of the theories of science, like the phlogiston and caloric theories, were not so much pseudoscience as* protoscience*: early first guesses that served well in their time. Indeed, the Ptolemaic geocentric astronomy, with its complex system of epicycles, would have quite adequately predicted planetary motion if only those astronomers who credited it had access to Fourier Analysis.*

Theories for which there is no concrete evidence like panspermia belong to the realm of quasi-science.

At the frontiers of science, the scientific method leads us sometimes to a multiplicity of doubts, where questions rather than answers prevail. There — we have no immutable answers, but rather hypotheses embracing the observable evidence[15].

This condition gave rise to a popular kind of imaginative literature known as science fiction (SF) *which deals principally with the impact of actual and imagined science on society and individuals.*

Unlike fantasy*, which deals with the impossible, SF describes events that* could *actually occur, according to accepted* possible *theories. While ordinary*

[15] Thomas S. Kuhn in his book "*The Structure of Scientific Revolutions*", Chicago Univ. Press, 1970) went as far as saying that scientists are 'puzzle solvers', not problem solvers.

'fiction' concerns itself more centrally with faith, psychology or history, SF is motivated by scientific knowledge.

Hence, the essential difference between SF and other forms of literature is, of course, that we are dealing with science *fiction. In some respects the very term seems to suggest a contradiction: how can the known and the make believe be part and parcel of the same creation. How can we reconcile the world of reason, manifest in technology, and the mysticism of spiritual experience?*

The basic themes of SF include space travel, time travel, and marvelous discoveries or inventions. Most modern SF stories are set in the future, but some take place in the past or even in the present day. Some are set in another universe. Some SF stories give detailed scientific explanations. Other stories simply thrust the reader into a strange time or place.

Like all fiction, SF make frequent use of myths, *those archetypal stories which provide the symbols that help us shape our world. The roots of SF, like the roots science itself are in magic and mythology.*

Science fiction is not like other writing about science; it looks forward *where other kinds usually look back, speculate — where other consolidate.*

The good SF writer is essentially a creative artist first, who knows or understands and sympathizes with one or more scientific thought. In a world where even group of scientists (e.g., physicists and geneticists) can scarcely understand each other, the SF writer sets himself as a kind of translator between different ways of seeing the world, not just todays, but tomorrow's world.

SF recognizes the germ of future development and enlarges upon it from different angles and in fanciful ways. It presupposes in its readers a willingness to consider possibility rather than fact. Prophetic accuracy is neither essential nor important ingredient of SF.

History[16]

The history of SF is also the history of humanity's changing attitude toward space and time. It is the history of our growing understanding of the universe and the position of our species in that universe. Like the history of science itself, the history of this literary form is thin and episodic until about four centuries ago, when the scientific method began to replace more authoritarian and dogmatic modes of thought, and people at last could see that the earth is not the center of the universe.

In the following survey of the history of SF we see it as a movement away from mythology toward realism; from a mythic way of seeing the world to a rational or empirical way of seeing it. As human science developed, human fiction changed with it. This movement involves a change in the world from one which lacks a clear distinction between natural and supernatural to a world in which the distinction is very clear and from which supernatural events are excluded[17].

The history of SF until 1950 *is divided into five stages:*

I. Collective prehistoric myths

Human beings felt the world to be alive with spiritual presence: divinities inhabited every bush and waterfall. People learned to fear and worship especially those gods they sensed behind the most awesome of natural phenomena — tempests, earthquakes, and the fertility of plants and animals. Our primitive ancestors knew a world that was timeless in one sense and tightly bound up by time in another.

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[16] For further reading, see:

- Wuckel, D. and B. Cassiday, *The Illustrated History of Science Fiction*, Ungar: New York, 1989, 251 pp.
- Bleiler, E.F., *Science Fiction – The Early years*, The Kent State University Press, 1990.
- Scholes, R. and E.S. Rabkin, *Science Fiction: History. Science. Vision*, OUP, 1977, 258 pp.
- Bleiler, E.F., Ed. *Science-Fiction Writers*, Charles Scribner's Sons: New York, 1982.

[17] Fiction which *is* aware of this difference but deliberately presents supernatural events, is called *fantasy*.

It was a world without history, with no sense of historical change that might lead to situations different from those which people already knew; it was a world bound to the seasonal flow of time, planting and harvesting, sweating and shivering, thanking the gods for blessings and begging them to end punishments. The seasons required religious rituals, which were held to contribute to the great temporal cycle, without which humanity would surely perish. The rituals enacted episodes from the lives of the gods, explaining the creation of the world, and preserving in the memory of humanity the values of which the gods were believed to approve. These memories and values, when separated from their ritual enactment, we call myths. *Myths are the ancestors of all other fictions. They have immense inertia, persisting in time as a conservative force, teaching the old values, the old ways — resisting the new.*

Prehistoric myths abound with tales of fantastic voyages and adventures. The richest source of myths is the Bible *(the creation story etc.). Greek mythology gave us the story of the pioneer aviators Daedalos and Icaros.*

II. Ancient social utopias and fantasies *(ca 800 BCE–200 CE)*

Utopias *were the creation of an age of arbitrary authority and frequent (albeit creative) disorder, in which the security and prosperity of the majority could be imperiled at any moment by the willful behavior of a determined and powerful individual or minority. These were the wishful systems devised, by men of good will, for the constraint of the turbulent individual by means of institutions and laws. Their objective was order, their by-products were general prosperity and peace, and their foundation were a strict hierarchy in which each person not only knew and kept his proper station but enjoyed it. During this period scientific fantasy showed itself as a mingling of literature, science, and social theory.*

Hesiod *(ca 800 BCE) in his* Dreams of the Golden Age, *the Biblical visions of the* Hebrew prophets *(ca 800–300 BCE) and* ***Aeschylos*** *(ca 525–456 BCE) contain such elements.* ***Aristophanes*** *(ca 450–388 BCE) also investigated fundamental problems by distancing them through fantasy, as in* The Birds, The Frogs, *and the* Ecclesiazusae.

Social utopias rose to special importance in the prehellenist period; in the 5th century BCE up to the beginning of the 4th century BCE ***Hippodamos of Miletos*** *and* ***Phaleas of Chalcedon*** *sketched hierarchical social models, which anticipated some of the thoughts of* ***Plato*** *(427–347 BCE). In* The Republic, *Plato advocated the abolition of private wealth and the introduction of a kind of consumer communism.*

Other utopias were sketched by ***Euhemeros*** *(ca 340–260 BCE) and* ***Jambulos*** *(ca 200 BCE).*

The first true SF on record is that of the Greek writer ***Lucian of Samosata*** *(125–190 BCE). In about* 160 *CE he wrote Vera Historica (The True History) in which he described trips to the moon.*

III. The late Renaissance, reformation and the scientific revolution (1492–1752)

Science fiction's roots lie deep in the Renaissance, an epoch characterized above all by violent socio-economical changes caused by a transition from feudalism to capitalism. The explosive technological development seen during the Renaissance was intricately bound up with the progress of science in opening up the world. The new methods of research that were thereby perfected were founded upon experiment, observation, and experience.

Copernicus*' heliocentric cosmos,* ***Gutenberg****'s invention of the printing press with movable type, the voyages of discovery led by* ***Columbus****,* ***da Gama*** *and* ***Magellan*** *— each was an immense achievement in its own right; together they were the basis for a flowering of the sciences, arts, and literature on an unprecedented scale.*

The alterations in the means of production led to a transition from a theocentric to an anthropocentric world views. The emphasis was now on people and the power of the personality, the might of the individual. The revolutions of the epoch became the objective sources for the later humanist movement and for the development of a deeply humanist view of the world and mankind.

Philosophy strove for the liberation of humanity from the fetters of theological dogma. Renaissance art did away with medieval conventions and turned towards the realities of life, to the activities of humans in their changing world. From these contradictory processes new genres of literature peculiar to the new age sprang up. The modern novel began to crystallize; the epic was already loosing its importance. The Renaissance gave rise to the gradual merging of traditional imaginative fantasy with scientific ideas. Thus the Renaissance saw the birth of scientific fantasy, or, what is now called — science fiction.

The main contributors to SF during this period were:

***Thomas More** [Morus, 1478–1535 (executed)]. En English statesman and humanist*[18]*. In his Utopia (1516) he set his ideal society on an island at the very edge of the world. (The word* utopia *literally means: nowhereland).*

***Ludovico Ariosto** (Italy, 1474–1533). In his Orlando Furioso (1532) he describes a fictional voyage to the moon.*

The idea of the first robot, the legendary Golem, is attributed to the chief Rabbi of Prague, the philosopher, savant and Kabbalist, ***Yehuda Liwa*** *(1588). He was an historical figure, a friend of* ***Tycho Brahe*** *and* ***Johannes Kepler****. In real life he was a sober theologian, not a man to meddle with magic, but the legend about him was rather different.*

To protect his people against the pogroms, the tale goes, Liwa and two assistants went in the dead of the night to the River Moldau, and from the clay of the riverbank they fashioned a human figure. When Liwa inscribed the Holy Name upon its forehead, the golem opened its eyes and came to life. It was incapable of speech, but had superhuman strength. It became Liwa's servant and worked as a sanitary within the temple. Only Liwa could control it, but eventually the golem could not be controlled at all. It ran amok, attacking its creator. Its career of destruction ended only when Liwa plucked the sacred name from its forehead. Magically, the golem was once again reduced to clay.

***Thommaso Campanella** (Italy, 1568–1639) was a Dominican friar. His utopia (1602) Civitas Solis (The Sun State) describes an ideal communal society which, like More's, is located at the furthermost reaches of the known world.*

***Christopher Marlowe** (1564–1593, England) wrote (1604) The Tragical History of Dr. Faustus.*

[18] He was born in London. During his college years in Oxford, he became familiar with representatives of the "new learning" (which meant Greek learning), and he would fain have followed in their footsteps, but his father, Justice Sir Thomas, wanted him to make law his career. Toward the end of the century he became acquainted with Erasmus, who influenced him deeply in many ways. His *Epistola ad Martinum Dorpium* was a defense of Erasmus' *Moriae encomium* and of the new learning; his masterpiece, *Utopia*, revealed not only his piety and love of education and learning, but also his consciousness of social wrongs. It is a satire on English (or European) conditions, for life in Utopia is the reverse in almost every respect of English life. More gives an elaborate description of the good society, which brotherhood, universal education, and religion combined with toleration would make possible. Not only was he one of the first defenders of the education of women, but he suggested that women be admitted to the priesthood.

Francis Bacon *(1561–1626, England) wrote New Atlantis (1627). This work uses the theme of a marvelous voyage to describe a society based on experimental science and the practical wonders that science could create.*

Johannes Kepler *(1571–1630, Germany) described a trip to the moon in his Somnium (1634). This book was the first SF that tried to tell a story with scientific accuracy.*

Cyrano de Bergerac *(1619–1655, France), in his L'autre mondes (1642, 1650), combined the philosophical systems of* **Descartes** *and* **Gassendi** *in two SF stories: in the first he describes for the first time a motorized lift-off into space in a rocket propelled spaceship through which he reaches the moon. In his second he describes a flying machine which takes him into the realm of the sun.*

In 1719, after economic and social ups and downs and tireless work in a multitude of fields, **Daniel Defoe** *(1660–1731, England) at 59 published his book The Life and Strange and Surprising Adventures of Robinson Crusoe of York. This book owes its origin to the Renaissance sailor-discoveries and the picaresque fictions of many literary predecessors, and to various portrayals of island life. It follows the philosophy of John Locke, that nature and common sense are motivating forces at the source of all individual and social evolution; knowledge won from experience triumphs, and achieves success for the individual.*

The astronomical discoveries of the 17^{th} *century and* **Torricelli***'s discovery of outer space (1643) have shown how precarious was man's grip of the universe, and enhanced man's primordial fears — fears that science itself helped to create.*

Science fiction tried to deal with these fears in two ways: first, by alleviating it through rationalization (inventing myths to limit and control these fears); second, use of the scientific method to modify his environment and therefore, ultimately, his density. These two themes occur and reoccur, through the history of SF, since the scientific revolution to the present day.

As SF developed during the 1700's, it produced its first literary masterpiece; Gulliver's Travels (1726) by **Jonathan Swift** *(1667–1745, England). In his book Swift subjected to rational analysis, the economic and social aspects of the postrevolutionary age in England*[19].

[19] We find in *Gulliver's Travels* (1726) a literary reference to the two moons of Mars. But these moons were first observed by Asaph Hall in 1877. How could have Swift known? The answer is quite simple: In 1610 **Galileo** used one of the earliest astronomical telescopes to discover the 4 moons of Jupiter. When **Kepler** heard of this, he immediately assumed that Mars must have two moons.

The first story of visitors from other planets was Micromégas (1752) by ***Voltaire*** *(1694–1778, France).*

Many of the basic ingredients for science fiction had appeared in embryo form by the early 18^{th} *century. Even if SF is taken as no more than a kind of fictional humanism, it was clearly not sufficient for its growth merely to have a widespread inculcation of scientific 'facts', acceptance of the experimental method or the stimulus of apocalyptic forebodings.*

These, however tenuously, in the shape of biblical fundamentalist dogma, the beginning of experimental science in ***Roger Bacon****, the experimental laboratories of the Renaissance, and the conviction of an imminent call to final judgment – are all influences on medieval literature which yet produced no science fiction.*

It was additionally necessary for the belief to be established, amongst at least a substantial minority, that Man could, through the use of the scientific method, modify his environment and therefore, ultimately, his destiny.

IV. The industrial revolution, the Victorian period and the turn of the 20^{th} ***century*** *[1776 (first steam engine) –1913]*

The second great epoch for SF literature is closely related to the scientific and technical, political and social, military and intellectual developments in Western civilization during the 19^{th} *century.*

Indeed, the palpable progressiveness of science and technology, and the similar concreteness of political change in revolutionary Europe and America at the end of the 18^{th} *century, forced people to begin perceiving the world in new ways. Above all, humanity was finally faced with a future at once real and unknown, stimulating and terrifying.*

The industrial revolution, via the steam engine, had a profound effect on the very structure of society; on one hand, it increased mass misery, poverty and hardship. These elements influenced the Gothic novel, which featured horror, violence and the supernatural.

After all, the planets are organized according to geometrical law: Venus has no moons, the earth has one, and so Mars — between earth and Jupiter — must have two to form a geometrical progression! This conclusion has always been accepted as true, and well known to Swift.

[Jupiter, incidentally, has 14 moons (1994)]. Kepler was lucky enough to have his belief *seem* right, and hence scientific. But the scientific knowledge was still based, in part, on belief.

On the other hand, most authors were deeply impressed by the fact that the new machines were able to multiply a hundredfold the muscle power of the worker, that new secrets were being wrung from nature every day, that products were being moved to and fro on the world market quicker than ever before, and that radio created the means of immediate communication worldwide.

The belief soon surfaced that science alone would be able to bring into being a superior mode of life. Thus this second phase of SF is characterized primarily by its sense of euphoria. The leading authors of this era are:

***Ernst Theodor Amadeus Hoffmann** (1776–1822, Germany) was intensely preoccupied with the Mesmerian theory of animal magnetism (1813) and intelligent machines (1814). Other fantasy tales by Hoffmann contain ideas that play important roles throughout SF: vampirism, strange beings in animal form, non-decaying dead bodies and much more.*

***Mary Wollstonecraft Shelley** (1797–1851, England) created (1818) the monstrous figure of Frankenstein — the archetype of the restless scientist not to be deflected from his own research and experimentation. Like his famous predecessor, the single-minded quester, **Marlowe**'s Faust, Frankenstein is ready to break down the boundaries of knowledge, giving not a moment's thought to considerations of the rightness or morality of his activities. Not satisfied with half solutions or compromise, he must aim directly at the summit, become a godlike figure, a second creator. This over-reaching, of course, means his eventual fall is so much the greater.*

Mary Shelley blended the old theme of the artificial creation of life with the new contemporary genre of the Gothic novel. She thus introduced the hideous, the heinous, the cryptic, and the criminal into literature and combined them with "scientific" elements.

***Edgar Allan Poe** (1809–1849, U.S.A.) developed the SF short story. In 'The Unparalleled Adventures of One Hans Pfaall' (1835), he describes a journey to the moon. Perhaps under the influence of Cook's voyages (1773–1774) he wrote 'The Narrative of Arthur Gordon Pym at Nantucket'.*

*Under the combined influence of Mary Shelley and Poe, the Scottish poet **Robert Louis Stevenson** (1850–1894) wrote 'Dr. Jekyll and Mr. Hide (1886), where a scientist, obsessed with his pursuit of knowledge and enlightenment, is temporarily changed into a monstrous alter ego. He thus layed the foundations for an investigation into the duality of human nature by splitting it clearly into good and bad.*

***Henry Rider Haggard** (1856–1925, England) wrote 34 novels of history and adventure. His best novels are based on his experience in Africa; King Solomon's Mines (1885) became a young people's classic. It is the story of* *search for the legendary lost treasure of King Solomon. She (1887) is the story of Ayesha ('She who must be obeyed'), a white goddess of Africa who is 2000 years old but still appears young and beautiful.*

Haggard was a firm believer in the evolutionary theories of Darwin. The memorable fantasy element in the book is Ayesh's surprising death as She baths herself in a "life-giving" flame (that seems strangely prophetic of nuclear power!) and slowly reverts — in a reverse of Darwinian "ontogeny recapitulates phylogeny" — from a smashingly beautiful woman to a 2000-year-old ugly ape.

Ayesha's memorable transformation in death recurs in the mainstream utopian novel Lost Horizon (1933) by **James Hilton**[20] *(1900–1954, England) — when a beautiful "immortal" woman living well beyond her years in the salubrious atmosphere of Shangri-La (a secret Tibetian monastery) leaves her home with her new English lover only to turn wrenchingly into an ugly, aged crone during her passage out. The utopianism of Shangri-La has its ingenious combination of the careless rapture of an unpolluted "magic" atmosphere and the practice of passive Eastern mysticism. The message is clear — an utopian place of ideal perfection is an impractical scheme for social improvement.*

When Poe needed a fiction catalyzer to set off his moon voyage, he invented an atomic component of hydrogen discovered by a chemist at Nantes, France. Poe had no way of knowing that there had just been born at Nantes someone who would become the most catalytic figure in the history of SF.

Jules Verne *(1828–1905, France) was the first classic writer of SF literature, who* specialized *in science fiction. His subject is nature. The* voyages extraordinaires *explore worlds known and unknown: the interior of Africa, the interior of the earth, the deeps of the sea, the deeps of space. Characteristically, Verne's voyagers travel in vehicles that are themselves closed worlds, snug interiors from which the immensity of nature can be appreciated in upholstered comfort (e.g., the* Nautilus*). The basic activity in Verne is the construction of closed and safe spaces, the enslavement and appropriation of nature to make place for man to live in comfort. Verne's novel is built upon an unresolvable incompatibility between a fundamental materialistic ideology and a literary form that projects the world as ultimately magical in nature. He thus produced narratives that mediate between spiritualistic and materialistic world views.*

Verne draws on new discoveries, experiments in physics and chemistry, and and technological discoveries of his immediate present. He led his characters to parts of the world that at the time were largely unknown and unexplored.

[20] He went on to write his masterwork *Good Bye, Mr. Chips* (1934). In 1935 he came to live in Hollywood, where he wrote film screenplays.

These backgrounds enhanced the exotic nature of the adventurers described. For instance, Verne describes Africa (Five Weeks in a Balloon, 1863); the delta swamps of Florida (North Against South, 1873); Siberia and China (Michael Strogoff, 1871); the Arctic and the North Pole (The Adventures of Captain Hatteras, 1866); India (Around the World in 80 Days, 1873); the depths of the ocean (20,000 Leagues Under the Sea, 1869); the interior of the earth (Journey to the Center of the Earth, 1864); the deeps of space (From the Earth to the Moon, 1865)[21].

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[21] An unknown manuscript, written by Verne in 1863, was discovered in 1989 by his great grandson Jean Verne in Toulouse. The novel *Paris in the* 20^{th} *Century* centers on the year 1963 and describes a society run by high finance and technology. In the story Verne anticipates the Daimler automobile (invented 1885), the electric chair (invented 1888) and the modern telefax machine.

The *principle* of the 4-stroke internal combustion engine was proposed in 1862 by **Alphonse Beau de Rochas**, of which Verne may have read!

Hard as it is to believe, the fax machine is older than the telephone and patents for the first prototype date back to 1843. The first commercial fax system called the *pantelegraph* was invented by the Italian priest **Giovanni Caselli** (1855). This was a relatively complicated system: an iron point crossed by a current was used to write onto a paper impregnated with a solution of potassium cyanate which is decomposed, leaving a blue mark on the paper. Despite the difficulties in synchronizing the transmitting needle and the receiving needle, the Caselli system was installed between Paris Amiens and Marseille in 1856. Verne must have known about this enterprise prior to 1863.

In 1980, modern fax machines came into being with a fax standard that allows *digital signal* to be sent over regular telephone lines in one minute or less. The pictures or text are converted to binary form and sent via standard telephone lines. On the other end, the fax machine decodes the bits and reconstructs the image.

However, Verne sometimes abused science for primarily fictional purposes: In *From the Earth to the Moon* (1865), for example, Verne has his ballistic spaceship fired from a mine 35 meters deep hole in Florida. He knew perfectly well that a hole of that depth anywhere in Florida would be under water; his straight-faced show of scientific accuracy ironically masked a satire on American ingenuity.

In *Purchase of the North Pole* (1889), some amateur scientists conspire to change the earth's axis by explosives, thus melting the polar ice cap and making accessible vast mineral wealth. Verne chose to ignore what he knew perfectly well — that the experiment would be likely to devastate all coastal cities as it would free the ice-bound land masses — not for the purpose of satire but for the simpler joy of working out the problem of axis-shifting, and the consequences be damned!

The great claim made for 20,000 *Leagues Under the Sea* is that Nemo's *Nautilus*

The era in which Verne was writing was an era of unbounded belief in science. Mankind rules the natural world and the limitless power of technology was the tool through which he ruled. This was the credo of the 19*th* *century bourgeoisie, and was the formula according to which Verne created the characters in his fictions.*

Enlightenment had paved the way to compulsory education which — according to the demands of the new forms of production — was accomplished in more and more countries and led to the growth of the reading public. The need for information in all classes of society, but especially the new middle classes, continued to increase with the growth of international trade — and as traveling became easier.

*Apart from books, there were newspapers and magazines of all kinds, and in particular, family journals, which bridged the gap between knowledge and entertainment. The now very high turnover in books helped finance research in high grade paper production and in printing itself. This led to the invention of the cylindrical paper-making machine (***John Dickinson***, 1809, England), the new steam press (***Friedrich König***, 1810–1811, Germany) and machine-aided book-sewing and book-binding methods.*

Through such developments, book production was simplified and the product made cheaper to buy. Print run multiplied and the structure of literary genres was irrevocably changed, with more and more authors working for the press.

New forms of publicity, pamphlets and early eye-witness reports had an effect on the purer forms of storytelling: the serialization of novels, stories, and travel books in magazine was tried, first of all in France, and found to be highly popular. Writers adapted their techniques to these new conditions by developing literary methods of creating and maintaining suspense. Other fundamental aspects of sale were the increasing members of new lending libraries spreading like wild fire.

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accurately predicts the development of the submarine. In fact, Verne was creating the fictional context, fully *against* the facts of contemporary science, that would give the submarine the thrill of the fantastic — and then he used much of the book to make this fantasy plausible.

Without detracting from such inventive detail as electric lighting, chemical oxygen production, seaweed cigars, and so on, one should note that **David Bushnell**, who coined the name *submarine*, first successfully tested his *Turtle* in 1775; **Robert Fulton** demonstrated a functional steam submarine in the Seine in 1807 (this ship, incidentally, was named *Nautilus*); and the Confederate States of America, in 1864, successfully used the 9-man submarine *Huntley* — to sink the United States frigate *Housatonic*. Verne doubtless knew all this.

Only when set against this background can the far-reaching effectiveness of Jules Verne work be fully appreciated.

Since Verne was so extraordinarily successful with his basic structure, it is really not to be wondered at that many writers sought to borrow his formula. Elements of his concepts were common in adventure fiction up through the middle years of our century. Moreover, Verne's best works still rank at the forefront of SF. Versions are produced on stage, on film, or on television; famous actors do not turn down the chance to play the parts of Verne's characters. The roots of this success may be traced to Verne's skill in combining strenuous and exciting action with accurate observation of human capacity and value under most adverse conditions, in addition to immense optimism conveyed in his books.

In the 130 *years that passed since Verne embarked on his career, science and technology have made advances of which it was impossible for the author of* From the Earth to the Moon *even to dream.*

***Edward Everett Hale** (1822–1909, U.S.A.), an American Unitarian clergyman (and chaplain to the U.S. Senate), editor and humanitarian. In his SF book* The Brick Moon and Other Stories *(1869) he rendered the first serious, extended consideration of an artificial satellite launched into space. This story, however, did not have much historical influence.*

***Edward George Earle Bulwer-Lytton** (1803–1873, England). First Baron. Historical novelist and playwright in Victorian England and politician. Best known for his novel* The Last Days of Pompeii *(1834) and the SF* The Coming Race *(1871). In the last novel he describes a world inside the earth inhabited by a strange underground race with superman technology (robots, death-rays, non-conventional power sources etc.).*

***Samuel Butler** (1835–1901, England) is best known for his satirical SF novel* Erewhon *(1872) that ridicules English institutions and customs through the eyes of a traveler in a strange new world. [Erewhon — a backward rendition (almost) of the word "nowhere".] The Butler's new society is developed not from the scientific penetration of political and social problems but rather from the individual extrapolation of Darwinian ideas, to which Butler vehemently objected.*

The similarities between Butler and Bulwer-Lytton lie in the fact that both envisage a future in which the social structure they know stays largely unchanged and both have little belief in the possibility of a social flowering of mankind. They unite against the loss of individual identity, and place their hope for change principally in the use of technological and scientific discoveries.

Sir Arthur Conan Doyle *(1859–1930, England) is most known for his detective fiction hero Sherlock Holmes (1887–1915), but he wrote also a SF series based on the figure of Professor Challenger. One of these novels is The Lost World (1912), his most important contribution to the literature of SF. It is also most central in illustrating his artistic and scientific vision. The novel tells about an expedition to the upper Amazon where the group discovers an ape-man society thought to be the missing link in human evolution. The power of The Lost World lies in a consummate balance of adventure, skilled characterization, novelty of story line, and adept use of scientific themes. Doyle's use of concepts taken from paleontology and evolutionary theory gives the action verisimilitude.*

In The Poison Belt (1913) Doyle speculates that humanity may not be the culmination of evolution, but only a temporary development to be surpassed and supplanted by other higher organisms unlike it in form. The power of the story lies in Doyle's moral thesis that humanity is given a second chance to fulfill its moral destiny on earth. It must recognize its feebleness before the infinite latent power of the universe. Thus, Doyle's vision of the future of mankind is shaped, not in scientific progress, but in progressing to a higher level of moral awareness through a recognition of a world beyond this life; materialism and conventional religion only further distort our view.

Herbert George Wells *(1866–1946, England). Novelist, historian, science writer and one of the most important pioneers of modern SF.*

He was born some three years after Verne's first major success with Five Weeks in a Balloon. His fiction embodies stimulating ideas of unrivaled originality. Wells the man is as entertaining as his fiction, for he retained until the end a diabolical mixture of a sentimentalist, a moralist, a patriot, a racist, a member of a secret society and a dreamer. Out of the 120 *books that bear his name — a small but significant proportion of them are SF. Among his best known novels in this field are: The Time Machine (1895), The Invisible Man (1897), The War of the Worlds (1898) and The First Men in the Moon (1901).*

Wells made significant strides forward from the Vernian model of SF. He did not confine himself to the fictional conquest of geographical areas of the natural world known to exist though as yet still not fully explored. Wells toyed with ideas wholly new — time travel, contact with other beings, aliens, wars between worlds etc. He thus first introduced into literature those ideas and themes that for nearly a century have formed the basis of SF throughout the world and that still inspire authors now to try new variations.

The Time Machine (1895) includes references to contemporary prerelativistic interpretation of time as a 4^{th} *dimension. Wells was by no means the first writer to confront the present with either the past or the future.*

In **Mark Twain**'s humorous novel *A Connecticut Yankee in King Arthur's Court* (1899) and **Edward Bellamy**'s *Looking Backward* (1888), the heroes are sent into the past and the future respectively.

The Time Machine was however the first SF story wherein a machine is envisaged that enables travel through time at will, and in this respect it can claim to be the first example of a new branch of SF.

The War of the Worlds (1898) was certainly motivated by the "discovery" of **Schiaparelli** (1877) of canals on Mars. Wells took **Lowell**'s premise of intelligent Martians (1896), added to it the aggressive nature attributed since ancient times to the blood red planet named for the god of war, and used Lowell's scientific descriptions of Mars to extrapolate the nature and aims of the race which invades earth. This line was later extended by Edgar Rice Burroughs in his novel *A Princess of Mars* (1912).

Nowadays, people believe anything, and they exist in a world-situation of insecurity. The Victorians of the 1890's were reasonably secure, reasonably arrogant. Wells took advantage of that situation: instead of our being the imperialists, the conquerors — supposing something arrived that fully intended to conquer us? Wells' nastiness really wounds because there is the poison of moral purpose at its tip. The Martians are what *we* may become! The conquering Martians are at once the products and victims of evolution. For all their pride, they fall prey to bacteria.

The most important novel exploring technology and its future in the epoch under consideration is *The Tunnel* (1913) by **Bernhard Kellerman** (1879–1951, Germany). Kellerman's fantasy tells of a tunnel dug out under the Atlantic Ocean connecting Europe and America. It followed the spectacular sinking of the *Titanic* in the Atlantic by only two years. The impression of this catastrophe was still vivid in the minds of readers, rendering them particularly receptive to the notions of the author with his suboceanic borrowings.

Apart from the concessions to the age in which he lived (for example, anti-Semitic elements, which are incidentally also to be found in the work of Jules Verne), this novel is a model of exploiting futuristic technology. Kellerman presents on the one hand the possibilities inherent in modern industrial society, and on the other hand, explores the dangers that threaten humanity through its existing contradictions.

The name of **Edgar Rice Burroughs** (1875–1950, U.S.A.) burst onto the world scene in 1912 with two novels published one after the other in the pulp magazine *All-Story*. The first was a serial titled *Under the Moons of Mars* (republished in book form in 1917 as *A Princess of Mars*); the second was *Tarzan of the Apes*. His biggest success in terms of popularity and monetary reward came from the Tarzan stories and films. However, Burroughs was known from the start, far and wide, for his non-Tarzan science fiction as well.

Although science per se plays little part in his work (and it is safe to say that he knew and cared little about it), owing to their huge commercial success, they found countless imitators and did have a profound effect on the development of SF form in America after WWI. Tarzan became one of the most famous characters in fiction and outlived Burroughs in novels and film.

By 1975*, more than* 36 *million copies of Tarzan books, in* 56 *languages, had been sold, making Tarzan an international superman folk-hero.*

Prior to becoming a best-selling novelist, Burroughs had behind him a career as a soldier, policeman, Sears Roebuck manager, gold-miner, cowboy and storekeeper. He had two towns named after him, but never visited Africa. His stories, according to many critics, belong to the lowest stratum of literature: narrow mental world, weak plotting, paper-thin characters, cumbersome and amateurish style. Yet the readers gobbled up Burroughs, always seemingly hungry for more.

The glorification of strength and the outdoor life and simplistic solutions to the problems of a rapidly changing world were popular ideas during the time of Theodore Roosevelt. Tarzan was also a unique superman, since he reconciled elitism and democracy.

V. Dystopia *(1921–1950)*

Social and political arguments, which appeared in much early science fiction, were emphasized even more in the 1900*'s. One of the main literary currents that dominated SF during the above epoch was the newly formed* dystopia *or anti-utopia; while utopian fiction portrays ideal worlds, anti-utopian fiction sees these ideal worlds as nightmares.*

Since the 18^{th} *century, some prophetic anticipations of scientific achievements were made by physicists and SF writers alike:* ***Newton*** *(1728) in* The System of the World *(a popular version of the third book of the* Principia*) envisaged man-made satellites.* ***Jules Verne*** *(1863) anticipated rocket-launching, non-classical power source through which his* Nautilus *was propelled, incandescent lighting (1870, nine years before Edison's patent was granted) and ocean-landing of spacecraft*[22]*.* ***Mark Twain*** *(1898) forecasted television, which he named* telectroscope*.* ***Cleve Cartmill*** *(1944) correctly hypothesized, in a story called* Deadline*, how one may construct an atom bomb — as the Manhattan project was then doing in extreme secrecy. (He*

[22] **F.R. Molton** stated unequivocally in an astronomy textbook (1930) that SF stories about interplanetary travel were totally impossible and that anyone knowing the physical forces involved would know them to be so!

was interrogated by military intelligence). ***Frank Quattrochi*** *(1955) predicted the heart-lung machine.*

In the United States, magazines called pulps *have played the major role in development of SF.* ***Hugo Gernsback*** *founded the first pulp, Amazing Stories (1926). In* 1930 *he became the first person to use the term* science fiction*. The early pulp magazines concentrated on scientific marvels, but turned increasingly to major social concerns after* ***John W. Campbell*** *Jr. became editor of Astounding Science Fiction (1937). Campbell developed a group of writers who dominated the field in the mid-*1900*'s, including* ***Isaac Asimov*** *and* ***L. Sprague de Camp****.*

Science fiction gained a wider audience after WWII ended in 1945*. Its popularity grew as developments in atomic energy and space exploration showed that much SF was more realistic than many people believed.*

Karel Čapek *(1890–1938, Czechoslovakia), Czech humanist, prolific man of letters and a working journalist throughout his career. A critical observer of certain manifestations of the time. Čapek did most of his writing during the unsettled period between the first and second World Wars. In his travels in numerous countries throughout the world, and in his own country, he recognized an increase in violence, and saw the dangers inherent in manipulating people and subjecting them to faceless power. In these circumstances, it is perfectly understandable that Čapek should choose to estrange himself from individual and social trends by means of SF at the beginning of the* 1920*'s.*

In rapid succession he published a series of works that are of equal interest in terms of the history of mainstream literature and in terms of the history of SF literature. Chief among those are the play R.U.R. (Rossum's Universal Robots[23], 1920) and Krakatit[24] *(1924). In the years succeeding this burst of literary fantasy, Čapek turned his attention to the small everyday things in life. Only in* 1936 *did he revert to the metaphorical mode of SF in The War with the Newts. In all these works, Čapek shows his concern of man's destruction of himself by science; the danger to mankind arising from contradictions between the advances of technology and the stagnation in human ethical maturation.*

In R.U.R. the robots represent a complex of symbolic meanings; the threatening aspects of the industrial dehumanization of the work force as well as the pathos that surrounds the victims of the assembly line. Through this

[23] Taken from the Czech *robota*, meaning 'forced labor'. This word was invented by Čapek's brother Josef.

[24] A name of a *castle* planted in the same unstable soil as **Franz Kafka**'s (1883–1924). Both men inevitably responded to the same cultural traumas as the Austro-Hungarian Empire entered its death throes.

ambivalence, the image of the robot represents the logical outcome for the helpless masses. (In the play, the robots are not mechanical metallic creatures but androids-living organic simulacra–indistinguishable at first glance from humans.)

The title of Krakatit *gives an immediate hint of what is to come by recalling the devastating aftermath of the famous* 1883 *eruption of the volcano Krakatoa. In the novel,* Krakatit *is a superexplosive atomic substance, something like an atomic bomb, through which a dictator wants to conquer the world. Instead,* Krakatit *destroys those who try to misuse it.*

The War with the Newts *is one of the masterworks of SF. It is basically an anti-Nazi satire and a grim sense of what the future might hold in store for mankind. It is not surprising that the Gestapo tried to arrest the dead Čapek. But Čapek died before the Germans could kill him and before WWII could provide him with material that might simultaneously inspire the intensity of* Krakatit *and the complex thrust of* The War with the Newts*. Čapek's influence has been for the most part indirect, although his humane breeziness arguably infuses the work of some SF writers today.*

Aldous Huxley *(1894–1963,* England) *wrote the classic dystopia* Brave New World *(1932), a title taken from Shakespeare's* Tempest*. It describes a male-dominated world in which the population is perfectly controlled and people are genetically engineered in carefully regulated mental and physical sizes and types. The author employs characters as mouthpieces for the dialectic of his tale. They engage in Socratic dialogues. In* Brave New World*, the products of science have overwhelmed the poise of human reason.*

Huxley's great fear was not that what science could do should not be done, but that science would become the only *thing that man did — and, after all, this turns to be too little.*

The brave new world is boring. In the use of science to find safety, discipline and courage have become obsolete. A civilization ordered solely by science, sex, and drugs kills the spirit. People become mere cattle. Savagery may be preferable. The savage knows little joy and ecstasy, but by the almost limitless capacity for pain that he had learned, he can imagine, dream of, and therefore in a way attain a transcendence of the richest possible pleasure of his body. Other persons in the novel might die, but only the death of the savage can be profoundly tragic.

Virtually all that Huxley had to say in his SF centered on earth and on mankind. He tried to anatomize the confusion of human science, art, and spirit.

George Orwell *(1903–1950,* England*), novelist and political writer; can be considered as a SF writer if we define utopia as a socio-political subgenre* *of science fiction. He wrote Animal Farm (1945), a political allegory — a satire on the Russian revolution and its monstrous perversion of the vision of democratic socialism. His last novel 1984 (1949), a nightmare, is a culmination of Orwell's intellectual and artistic development: a dystopian nightmare that fuses all the themes derived from his reading, his personal history, and his involvement with some of he more significant socio-political issues of his time.*

1984 most fully dramatized Orwell's fear that a totalitarian state could legitimize its power by "altering" the past, present and future, that it could control its subjects' perception of reality by consciously manipulating language. The interpretation of language and experience, which constitutes one of the major themes of the novel, has interested thinkers from Aristotle to Karl Marx;

The specific relationship between language and political power had been discussed before, but after the Holocaust, born of the masterful evil rhetoric of the Nazi Führer, Orwell's brilliant dramatization of these themes takes on additional significance and power. (The Nazis used symbols in such a way that people did not only think about hate, but expressed hate. The Führer did not use language to teach the Germans to think; he provided them with forms through which they could act.) Orwell's socio-political legacy is therefore two-fold:

- *"Reality" and "meaning" are not identical with "fact". Meaning arises in social relationship that exist in and through the communication of significant symbols, most notably in language.*
- *History was not something to be created but rather discovered*[25]*, and intellectual freedom lay in being able to report this history. If people cannot know, cannot be certain about events, they all fall victims to the most irresponsible propaganda.*

Orwell meant, 1984 to be a warning, not a prophecy. The book's gloom is often referred to his illness and his growing conviction of the manipulability

[25] At his point Orwell's position is at par with arguments from such historians as **Benedetto Croce** (1866–1952, Italy), who insisted that all history is "present" history in the sense that it is impossible to create an objective historical narrative. History is not a mere description of the past, but an evaluation of it, with EACH GENERATION RENDERING ITS OWN VALUE JUDGMENT OF IT. The interpretation of history creates history, by constructing the mind's self-creative value judgment of events. Since the philosophical interpretation of the present generation will one day be history, philosophy and history must be considered identical.

of the human mind. The book's despair comes not just from the fact that tyranny is universal and that the individual is doomed, but from the bottomless selfishness of the human being.

Born Eric Arthur Blair in Bengal, India, son of an official in the Indian civil service. He returned with his parents to England, and after education at Eton, joined the Burmese police. Returning to Europe (1927) he chose to live among the deprived, and completed his rebirth by adopting a new name. Until 1935 *he lived with the poor, tramped over the English countryside. He participated one year (1936–1937) as a common soldier in the Spanish Civil War on the side of the Republicans, where he became disillusioned with Communism. After* Animal Farm *appeared, Orwell took a house on Jura in the Hebrides. He died of tuberculosis soon after writing his last novel, 1984.*

As scientific ways of understanding the world developed in the 17^{th} *and* 18^{th} *centuries, fiction became more and more realistic, and the realistic novel came more and more to dominate the world of fiction. Fantasy was considered a minor form, suitable for children or as light reading for adults, but not really "literature", not really serious.*

In the 19^{th} *century, realism developed new techniques for representing a whole social scene accurately and finally new ways of making individual psychology available to readers. The realistic novel presented* this *world in* this *time, competing with history and journalism as a way of recording the truth of contemporary experience. So powerful was this fictional form, that many writers and critics believed it to be the end of a long process of evolution. At last we had learned how to tell the truth in fiction! But truth is elusive and has a way of turning to dust and ashes whenever we try to stop it from growing and changing.*

All during the time of the rise of realism, a number of things had been going on which tended to counteract the realistic movement and prepared the way for a great shift in human awareness. The physical scientists, as they perfected their instruments of vision and measurement, began to explore worlds which in relation to ordinary human experience seem fantastic. Cosmic space and atomic space began to reveal their secrets, and in doing so posed problems which only "fantastic" speculation seemed able to solve.

Arthur Charles Clarke *(b. 1917, England). Science fiction novelist, scientist and prophet of space flight. Anticipated (1945) artificial communication satellites in synchronous orbits. Established in his novels futuristic technologies and scientific developments.*

Clarke joined the fledgling British Interplanetary Society at 17*, becoming its chairman while completing his B.Sc. at Kings College, London, in the late* 1940*'s. In* Interplanetary Flight (1950) *and* The Exploration of Space (1951) *he expounded his technical knowledge and enthusiasm for the 'space age' to a wide public and made him a foremost popularizer of space travel.*

At the same time he made his debut in the front rank of postwar SF writers. With Stanley Kubrick he wrote the script for 2001: a Space Odyssey (1968), perhaps the most imaginative of all SF films. In his novels he portrays man's encounter with alien intelligence as the chief turning point in a future which is cosmic and evolutionary rather than mundane and catastrophic. He conceives of man as on a continuous odyssey, facing that giant staircase as a challenge his heritage demands that he accept. Clarke has been a most influential voice in shaping SF in the epoch of the recent and continuing scientific revolution.

***Isaac Asimov** (1920–1992, U.S.A.). Influential writer of SF during the second half of the* 20^{th} *century. He wrote nearly* 500 *books on a wide gamut of scientific and non-scientific subjects. He was a pioneer in elevating the SF genre from pulp-magazine adventure to a more intellectual level that dealt with sociology, history and science.*

The special character in Asimov's work derives from the last fact that the robots stand side by side with the humans. His robot stories are consistent with his 'three rules of robotics' (1942):

(1) A robot may not injure a human being, or through inaction allow a human being come to harm.

(2) A robot must obey the orders given it by human beings except when such orders would conflict with the First Law.

(3) A robot must protect its own existence as long as such protection does not conflict with the First or Second Laws.

Asimov was born to Jewish parents in a suburb of Smolensk called Petrovich. His parents Judah and Rachel Berman Asimov moved to New York in 1923 *and settled in Brooklyn. He graduated from Columbia University (1948) with a Ph.D. in chemistry and gained the rank of associate professor in biochemistry at the Boston University (1955). In* 1958 *he reached an agreement with that University whereby he would perform no substantial duties and receive no salary, but would retain his faculty rank and status.*

Early 20^{th} *century science fiction was inspired largely by* astronomy *and to a lesser extent by physics and mathematics. These sciences, together with the preoccupation with gadgets and with a little chemistry thrown in, dominated the scene until about* 1948.

After WWII the biological sciences emerge as major elements in the genre. Having escaped at last from the unpromising side track into which this fiction *had long been diverted by the man-made bug-eyed monsters sired, or rather dammed, biology began to emerge as an SF inspiration with early* 20^{th} *century writers but it was slow to gain prominence until the WWII, when plastic surgery in particular made a considerable impact.*

By the early 1960s psychology and sociology had become a major source of science fiction stimulus and the mathematical sciences, including economics and cybernetics, were also in great evidence. The 1970s saw an even greater change, the almost total rejection of scientific reason.

Until 1945 science could be seen as the friend of mankind; modern medicine has brought about a great increase not only in longevity but in the capacity to enjoy life physically, an increase which has affected our aspirations.

However, after the explosion of the first atomic bomb its evident capacity to destroy humanity turned science into a potential enemy; the prospect of a sudden cataclysmic end to all human life has destroyed the hope slowly engendered through the 18^{th} *and* 19^{th} *centuries that science and reason would bring about an inevitable millennium. Thus we resurrected the monster of Dr. Frankenstein and the devil of Faust — forbidden, uncontrollable and therefore dangerous knowledge.*

Modern man has now come full cycle again to the more vigorous fears and uncertainties of earlier time. This is amplified by the fact that today, the sheer mass of scientific knowledge is beyond individual comprehension, despite the far higher level of general education. We have to some extent returned to the situation of the primitive man who required his myths and mysteries as protection against forces which he could neither fully understand nor control.

Because of all this, SF was firmly established as a particularly sensitive form of literature for reflecting the moods and psychoses of its host society.

1895–1932 CE **Charles Scott Sherrington** (1857–1952, England). Neurophysiologist. Formed the scientific basis of modern neurology; coined the terms *neuron* and *synapse*; demonstrated that reflexes in higher animals are integrated activities of the total organism; made lifelong study of the mammalian nervous function. Shared (with **Edgar Douglas Adrian**) the Nobel Prize for physiology or medicine (1932).

Postulated (1906) that neural reflexes must use more than one neuron; so he proposed a synapse and a neurotransmitter substance to connect the two nerve cells. Developed (1913–1930) modern techniques for recording nerve activity and outlining the nature of communication between nerves and between nerves and muscles.

Sherrington was born in London. He took his medical degree at Cambridge (1885). Taught at London University, where he became professor of pathology (1891–1895). He was then professor of physiology at Liverpool (1895–1913) and Oxford (1913–1931).

1895–1945 CE Leading Western poets and novelists from the turn of the 20^{th} century to WWII:

• Oscar Wilde	1854–1900
• Joseph Conrad	1857–1924
• Axel Munthe	1857–1949
• Arthur Conann Doyle	1859–1930
• Rudiard Kipling	1865–1941
• Ladislas Stanislas Reymont	1867–1925
• John Galsworthy	1867–1933
• Maxim Gorky	1868–1936
• Felix Salten	1869–1945
• Ivan Bunin	1870–1953
• Marcel Proust	1871–1922
• Hayim Nahman Bialik	1873–1934
• Robert Frost	1874–1963
• R.M. Rilke	1875–1926
• Thomas Mann	1875–1955
• Jack London	1876–1916
• Carl Sandburg	1878–1967
• Upton Sinclair	1878–1968
• James Joyce	1882–1941
• Jaroslav Hasek	1883–1923
• Franz Kafka	1883–1924
• Sinclair Lewis	1885–1951
• S.Y. Agnon	1887–1970
• Fernando Pessoa	1888–1935
• T.S. Eliot	1888–1965
• Karl Capek	1890–1938
• Franz Werfel	1890–1945
• Lajos Zilahy	1891–1974
• Edna St. Vincent Millay	1892–1950
• E.E. Cummings	1894–1962
• Erich Maria Remarque	1898–1970
• Bertolt Brecht	1898–1956

- Ernest Hemingway 1899–1961
- John Steinbeck 1902–1968
- William Saroyan 1908–1978
- Dylan Thomas 1914–1953

1896 CE First modern *Olympic Games* in Athens, Greece; the first known Olympic contest took place in the Stadium of *Olympia* in 776 BCE.

In 394 CE, Emperor Theodosius ordered the games ended, but they continued until 426 CE (304th Olympiad), when an earthquake destroyed the Stadium of Olympia. A second earthquake (521 CE) buried the ruins of the structure.

The Olympics were held every four years, and were used in Greece as a system of dating for literary purposes (but never adopted in every-day life); all events were dated from 776 BCE. The beginning of the year of the Olympiad was determined by the first full moon after the summer solstice, the longest day of the year. The full moon fell about the first of July. Each interval of four years was known as an *Olympiad*.

1896 CE **Arthur Schuster** (1851–1934, England). Applied Fourier analysis to determine periodicities of geophysical and astronomical time-series.

1896 CE **Max von Gruber** (1853–1927, Austria). Bacteriologist and physician. Discovered the specific *agglutination* of bacteria by the serum of an organism immune to a certain disease, such as typhoid fever and cholera. This reaction, which bears his name, is used to identify unknown bacteria and was first utilized by **Fernand Widal** in his test for diagnosis of typhoid fever. This discovery paved the road for clinical diagnosis of many contagious diseases.

Gruber studied in Vienna and Münich. He was professor at Graz (1883–1887), Vienna (1887–1902) and Münich (1902–1923).

1896 CE **Jacques Solomon Hadamard** (1865–1963, France). Outstanding mathematician. Proved the '*Prime-number theorem*'[26], which states that

[26] After proving the 'Prime-number theorem', Hadamard was fascinated by what went on in the mind of a creative mathematician. He set down his ideas in a book entitled *The Psychology of Invention in the Mathematical Field* (1945), in which he made a powerful case for the role of the subconscious. In his book he divided the act of mathematical discovery into four stages: *preparation*, *incubation*, *illumination* and *verification*.

$\pi(n)$, the number of primes less or equal to n, approaches $\left\{\frac{n}{\ln n}\right\}$ for large value of n. This conjecture was made by Gauss (1792) and Legendre (1778).

In 1852 and 1859 **Chebyshev** and **Riemann**, respectively, provided incomplete proofs to this theorem. Riemann was able to link the zeros of the Zeta function to the properties of $\pi(n)$, but he did not supply any proof for this connection. For about thirty years, other mathematicians tried to prove the main result enunciated in Riemann's paper — but to no avail. Only in 1896 was it proven independently and simultaneously by Hadamard and **Charles de la Vallée-Poussin** (1866–1962, Belgium), both using analytic methods[27]. Interestingly enough, the complex-variable methods used by Hadamard in his proof found applications in the theory of radio waves.

In 1932 **Edmund Landau** (1877–1938, Germany) and **Norbert Wiener**, using 'Tauberian theorems', simplified Hadamard's proof. Hadamard introduced the word *functional* (1903) when he studied

$$F(f) = \lim_{n\to\infty} \int_a^b f(x) g_n(x)\, dx.$$

Fréchet (1904) defined the derivative of a functional.

Hadamard also obtained important results in the theory of functions of complex variable [*multiplication theorem*, *3-circles theorem*, *gap theorem* and the *factorization theorem*], partial differential equations, theory of variations, functional analysis, geometry, hydrodynamics, theory of determinants and integral equations.

Hadamard's contributions are partly reflected in such terms as *Hadamard's inequality*, *Hadamard variational formula*, *Hadamard matrices*[28] and *Hadamard transform optics.*

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[27] In 1892 Hadamard proved that $\xi(t) = \Gamma\left(\frac{s}{2}+1\right)(s-1)\pi^{-s/2}\zeta(s)$, is of the form $\xi(t) = C\prod_1^\infty \left(1 - \frac{t^2}{\lambda_n^2}\right)$, $\quad \sum_1^\infty \frac{1}{|\lambda_n^2|} < \infty$ where $s = \frac{1}{2} + it$.

[28] A Hadamard matrix H_n of order n is an $n \times n$ array, the elements of which are either $+1$ or -1, such that the scalar product of any two distinct rows or columns is zero. Thus H_n must satisfy $H_n H_n^T = H_n^T H_n = nI_n$. Examples are $H_1 = [1]$, $H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$. Its determinant is $\pm n^{n/2}$ (n even), its maximum eigenvalue is $\sqrt{n}$ and its inverse is $H_n^{-1} = \frac{1}{n} H_n^T$.
Hadamard matrices are realized in optical spectroscopy and image processing, with applications in chemistry, medical diagnosis, infrared astronomy, high energy physics and radar. The basic method which underlines these various applications is *multiplexing*. In conventional spectroscopy, for example, electromagnetic radiation is sorted into distinct bundles of rays corresponding to different colors. Thus each bundle is labeled by the appropriate frequency, wavelength

Hadamard was born in Versailles. He originated from a Jewish family of Lorraine. There are traces of Hadamards, printers in Metz, in the 18^{th} century, and also a remarkable great grandmother who lived during the French Revolution. Before Jacques was born the family settled in the Paris area. His father taught humanities in high school; his mother was a good pianist. He achieved the highest score ever obtained in the entrance examinations to the École Polytechnique, France's greatest school of science and, in Hadamard's youth, the foremost world institution of its type. He chose, however, École Normale Superier (1884), where he studied under **Jules Tannery**[29] and **Émile Picard**.

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or wavenumber. The spectrum of the radiation is found by measuring the intensity of each bundle. Alternatively, the bundles can be multiplexed: instead of measuring the intensity of each bundle separately, we can measure the total intensity of *various combinations* of *bundles*. After measuring n suitably chosen combinations, the individual intensities of n different bundles can be calculated, and the spectrum obtained. Finally, by combining multiplexed radiation from different parts of the picture and from different frequency bands, it is possible to reconstruct a color picture of a scene.

The primary purpose of multiplexing is to maximize the radiant flux incident on the detector, in order to improve the signal-to-noise ratio of the final intensity display.

So where does Hadamard enter in the scheme of things? — in the design of the *mask* which splits the source beam into bundles! This mask is essentially a two-dimensional grid made of two basic elements: open and closed slots. Each element of the beam is either transmitted or absorbed. [To overcome the difficulty that there is no way of registering a negative signal with an ordinary light detector, a Hadamard matrix can always by written as difference of special matrices whose elements are 1 or 0.] Knowing the special algebra of Hadamard matrices, one can design the multiplexing spectrometer accordingly.

[29] **Jules Tannery** (1848–1910, France). Known primarily for his treatise on elliptic functions and his contributions to the history and philosophy of mathematics. Discovered the summation formula

$$\sum_{n=1}^{\infty} \frac{x^{2^{n-1}}}{x^{2^n}-1} = \begin{cases} \frac{1}{x-1}, & \text{if } |x| > 1, \\ \frac{x}{x-1}, & \text{if } |x| < 1. \end{cases}$$

The special case $x = 2$ yields the interesting result

$$\sum_{n=1}^{\infty} \frac{2^{2^{n-1}}}{2^{2^n}-1} = 1.$$

He was a professor of mathematics at Bordeaux (1893–1896), Sorbonne (1896–1909), College de France (1897–1935) and École Polytechnique (1912–1935).

As a brother-in-law of Alfred Dreyfus, Hadamard took an active interest in the *Dreyfus case*. The dangers of Hitlerism were recognized by Hadamard at an early stage and he worked to alleviate the plight of German Jewry. He escaped from France in 1941 to the United States, and moved to England to engage in *operational research* with the Royal Air Force. He had three sons and two daughters. The two elder sons were killed in action in WWI within an interval of less than two months. The third son was killed in North Africa in WWII.

Hadamard loved music and used to have a small orchestra for amateurs in his house: **Einstein** played in it whenever he was in Paris. Duhamel, the writer, was the flautist, Hadamard played the violin and Mme Hadamard played the piano, supplementing by playing the parts of the brass instruments when required.

1896 CE **Henry Ford** (1863–1947) and **Charles Brady King** drove their first gasoline cars in Detroit, MI. That same year, **Ransom Eli Olds** (1864–1950), drove his first gasoline car in Lansing, MI. Also in 1896, **Alexander Winton** successfully tested his own automobile in Cleveland. In 1903, **David Dunbar Buick** (1854–1929) built his first car in Detroit. Most of these pioneer American automakers later began the mass production of cars in the United States.

1896–1900 CE **Antoine Henri Becquerel** (1852–1908, France). Physicist. Discovered radioactivity in uranium ores and identified beta particles with Thomson's electrons.

He embarked on the subject through his interest in the relation between absorption of light and the stimulated emission of phosphorescence in some uranium compounds. Influenced by the discovery of X-rays by Röntgen, he decided to test an hypothesis that uranium salts emit X-rays when irradiated by sunlight. He found in 1896 that the uranium salts would eject penetrating radiation (as revealed by their effect on a photographic plate) even when they were not excited by the ultraviolet in sunlight. He then postulated the existence of invisible phosphorescence.

Becquerel was a member of a scientific family extending through several generations[30]. He received his formal scientific education at the École Polytechnique (1872–1874) and engineering training at the school of Bridges and

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[30] This family of physicists includes:

- **Antoine-César** (1788–1878). Professor, Musée d'Histoire Naturelle (1837–1878); one of the creators of the science of electrochemistry.

Highways (1874–1877). In addition to his teaching and research posts he was for many years an engineer in the department of Bridges and Highways. He became a professor of physics in 1895. For his discovery of radioactivity he shared the 1903 Nobel prize for physics with the Curies.

1896–1902 CE **Walter Reed** (1851–1902, US). Surgeon and pioneer medical researcher who led to the eventual eradication of Yellow fever and typhoid fever.

Born in Belroi, Virginia and received his MD degree from the University of Virginia. He joined the Army Medical Corps (1875), served as an army surgeon in Arizona (1876–1889) and Baltimore (1890–1893), and was professor of bacteriology at the Army Medical College (1893–1902).

Much of his work was centered on epidemic diseases: malaria, diphtheria, hog cholera, typhoid fever and Yellow fever, showing them to be caused by bacilli or viruses. Discovered (1898) that the Yellow fever virus was transmitted by the mosquito *Aedes aegypti*. Walter Reed Hospital in Washington D.C. was named after him.

1896–1902 CE **Robert Hjalmar Mellin** (1854–1933, Finland). Mathematician. Introduced the *Mellin transform*[31] into linear applied mathematics

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- **Alexandre-Edmond** (1820–1891). Son of the above. Succeeded to his professorship (1878–1891). Contributed to photochemistry. Discovered (1839) that when two pieces of metal were immersed in an electrolyte, an electric charge developed when one of the pieces was illuminated. But although he discovered the electrochemical effects of light, he did not offer any practical suggestion for its use.

- **Antoine-Henry** (1852–1908). Son of the above. Succeeded to his professorship (1892).

[31] Let $f(r)$ be a real function defined in the interval $(0,\infty)$ such that $f(r)$ is piecewise continuous and of bounded variation in every finite subinterval $[a,b]$, where $0 < a < b < \infty$. If in addition both integrals

$$\int_0^1 r^{\sigma_1-1}|f(r)|\,dr, \quad \int_0^1 r^{\sigma_2-1}|f(r)|\,dr$$

are finite for suitably chosen real numbers σ_1 and σ_2, then the Mellin transform of $f(r)$ is defined by the formula

$$F(s) = \int_0^\infty f(r)r^{s-1}\,dr,$$

where $s = \sigma + i\tau$ is any complex number in the strip $\sigma_1 < \text{Re } s < \sigma_2$. The as a powerful tool of solving problems in elasticity theory and potential theory. The Mellin transform arises from the multiplicative structure of the real line in the same way as the Fourier transform arises from its additive structure.

Mellin was pupil of **Mittag-Leffler** and then studied in Berlin (1881–1882). He was later a professor of mathematics at the University of Helsinki.

1896–1916 CE **Arnold (Johannes Wilhelm) Sommerfeld** (1868–1951, Germany). Outstanding physicist, and a prodigious producer of future Nobel prize winners. Introduced the quantization of the action integral $\int pdq$, which paved the way for modern quantum theory (1911). Defined the fine-structure constant of electromagnetic interaction, $\alpha = 2\pi e^2/hc \simeq 1/137$. In classical physics, he is known for his contributions to the theories of the gyroscope, diffraction of light (1896), and propagation of radio waves.

Sommerfeld's investigations of atomic spectra led him to suggest that, in the Bohr model of the atom, the electrons move in *elliptical orbits* as well as circular ones. From this idea he postulated the azimuthal quantum number. He later introduced the magnetic quantum number as well. Sommerfeld also did detailed work on wave mechanics, and his theory of electrons in metals proved valuable in the study of thermoelectricity and metallic conduction.

Sommerfeld was born in Königsberg, Prussia. He was educated in his native city and then became an assistant at the University of Göttingen. He served as a professor of physics at Munich (1906–1931), where he did most of his important work. Sommerfeld was a gifted teacher and educator. His 5-volume treatise '*Lectures on Theoretical Physics*' still serves today as a graduate textbook. Among his students were **W. Pauli**, **W. Heisenberg** and **H. Bethe**.

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inversion of $F(s)$ is given by the formula

$$f(r) = \frac{1}{2\pi i}\int_\Gamma F(s)r^{-s}\,ds,$$

where Γ is a straight line parallel to the imaginary axis lying inside the strip.
The *Mellin transform* is related to the Laplace and Fourier transforms and is the appropriate tool to use for solving problems in two-dimensional elasticity theory and potential theory involving angular regions.
The *Mellin transform pair* appeared in Riemann's famous memoir on prime numbers and it was later formulated more explicitly by **E. Cahen** (1894). But the first one to put it on a rigorous basis and point some of its applications was Mellin, and that is why the transforms bears his name.

1896–1920 CE **Vilfredo Pareto** (1848–1923, Italy and Switzerland). Economist, sociologist and engineer. Developed methods of mathematical analysis in the study of economic and sociological problems.

Pareto extended Walras' theory of general economic equilibrium, and sought to extend it to the entire range of social phenomena. In his sociological theories, Pareto argued for the superiority of the *elite*, claiming that society was always composed of elites and masses.

While his sociological theories are controversial, Pareto's contributions to economics have come to be recognized as immensely important during the second half of the 20th century.

In his economic theory, Pareto rejected the treatment of utility as a *cardinally* measurable quantity whose maximization involved the comparisons of one person's happiness with another's; instead, he treated it as *ordinal* concept (i.e. one implying only a *ranking* by each individual of alternatives available to him), and defined a corresponding optimum as a condition of society from which it is impossible to make any one individual subjectively better off without simultaneously making at least one other individual worse off. This idea of '*Pareto optimum*' — according to which the economic system can, in principle, generate an optimal distribution function of welfare among its individual members — is the fundamental concept of modern welfare economics.

Pareto's work led modern economists to the finding that the conditions for such an optimum will be satisfied by a Walrasian general equilibrium, where *each consumer is maximizing utility and each producer maximizing profits*, all under conditions of perfect competition (i.e. with no single consumer or producer able to influence, on his own, any market price).

Pareto also made significant contributions to the empirical study of *income distribution*, enunciating what came to be known as *Pareto's Law*, or the *Pareto distribution of incomes.* This purported to describe the pattern of inequality of incomes which any society will tend to generate, regardless of its economic system.

Pareto was born in Paris, France, and embarked on his research after a successful career in industry. He then became a professor at Lausanne (from 1893). In his book *Mind and Society* (1916), he stressed the irrational elements in social life and emphasized the role of leading groups (elites) in society. He criticized democracy and saw history as a succession of aristocracies. Because of his antidemocratic attitudes, he is considered an intellectual forerunner of fascism. Indeed, the ideology of Italian fascism was largely based on his theory.

1896–1927 CE **Felix Édouard Justin Émil Borel** (1871–1956, France). Mathematician. Created the first effective and general theory of the measures of sets points in topological spaces, and contributed [with **H. Lebesgue**] to the development of the modern theory of functions of real variable.

Émil Borel was born in Sain-Affrique, France. His father was a Protestant village pastor and his mother came from a family of merchants. In 1889 he enrolled in the École Normale. After graduation he taught at the University of Lille, and was appointed to the faculty of the École Normale Supériere, Paris. One of the famous results of his University thesis is the so called *Heine-Borel theorem*. In 1896, Borel discovered an elementary proof of Picard's theorem. He then formulated the theory of integral functions and was first to develop (1899) a systematic theory of *divergent series*. During 1921–1927, Borel became the first to define games of strategy in *game theory*. Borel was the immediate predecessor of Lebesgue in the development of measure theory.

Borel married Marguerite Appel in 1901, but they had no children. From 1924 to 1940 he was heavily involved in politics; he served as a minister of the navy (1925–1940). In 1940 he was arrested and imprisoned by the Vichy regime. He later returned to his native village and participated in the Résistance.

In 1955, while Borel was returning from a scientific meeting in Brazil, he was injured in a fall aboard his ship and died the following year.

The 'sum' of a divergent series

In 1850, ***Stokes*** *introduced the integral formula*

$$w = \int_0^\infty \cos\frac{\pi}{2}(\omega^3 - m\omega)\, d\omega$$

representing the strength of diffracted light near a caustic (known as an Airy integral). Stokes proved that for large positive $n = (\frac{\pi}{2})^{2/3}m$*, the integral can* *be represented by a divergent series, but one still useful for calculations, if a finite number of terms are retained.*

It thus became clear to him that divergent series could be used to solve differential equations. The full recognition of the nature of those divergent series that are useful in the representation and calculation of functions, and a formal definition of these series, were achieved by ***Poincaré*** *(1886) and* ***Stieltjes*** *(1886). They defined*

$$f(x) \sim a_0 + \frac{a_1}{x} + \frac{a_2}{x^2} + \dots$$

whenever

$$\lim_{x\to\infty} x^n[f(x) - \left(a_0 + \frac{a_1}{x} + \frac{a_2}{x^2} + \dots + \frac{a_n}{x^n}\right)] = 0.$$

Such series are asymptotic expansions[32] *of functions in the neighborhood of $x = \infty$. Likewise one speaks of the series $f(x) \sim a_0 + a_1x + a_2x^2 + \dots$ as asymptotic to $f(x)$ at $x = 0$, if*

$$\lim_{x\to 0} \frac{1}{x^n}\left[f(x) - \sum_0^{n-1} a_i x^i\right] = a_n.$$

Though in the case of some asymptotic series one knows or can estimate the error incurred by stopping at a definite term, no such information about the numerical error is known for general asymptotic series. However, asymptotic series can be used to give rather accurate numerical results for large x by employing only those terms for which the magnitude of successive terms is still decreasing as one includes further terms. The order of magnitude of the error at any stage is equal to that of the first term omitted. The theory of asymptotic series, whether used for the evaluation of integrals or the approximate solution of differential equations, has been vastly extended in recent decades.

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[32] For further reading, see:

- Sirovich, L., *Techniques of Asymptotic Analysis*, Springer-Verlag, 1971, 306 pp.
- Jeffrey, A. and T. Kawahara, *Asymptotic Methods in Nonlinear Wave Theory*, Pitman, 1982, 256 pp.
- Jeffreys, H., *Asymptotic Approximations*, Oxford University Press, 1962, 144 pp.
- Copson, E.T., *Asymptotic Expansions*, Cambridge University Press, 1965, 120 pp.

The work on divergent series described so far has dealt with finding asymptotic series to represent functions either known explicitly or existing implicitly as solutions of ODE's or PDE's.

Another problem that mathematicians tackled from about 1880 on is essentially the converse of finding asymptotic series: Given a series divergent in Cauchy's sense, can a "sum" be assigned to the series? As in so many other cases, the traces lead us back to ***Euler****, who first considered (1755) the 'sum' of a non-convergent series as the finite numerical value of the algebraic expression, from the formal expansion of which the series was derived. For instance:*

$$1-1+1-1+\cdots = \lim_{x\to 1-0}(1-x+x^2-x^3+\dots) = \lim_{x\to 1-0}\frac{1}{1+x} = \frac{1}{2}$$

$$1-2+3-4+5-\cdots = \lim_{x\to 1-0}(1-2x+3x^2-4x^3+\dots) = \lim_{x\to 1-0}\frac{1}{(1+x)^2} = \frac{1}{4}$$

In general, Euler's idea implies that[33]

$$\sum_{n=0}^{\infty} a_n = \lim_{x\to 1-0}\left(\sum_{n=0}^{\infty} a_n x^n\right),$$

for any power series with radius of convergence 1 *and for which the limit exists.*

It is clear that this convention has no general or compelling basis, since there is no reason why the same series should not result from quite different

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[33] For example

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{3n+1} = \lim_{x\to 1-}\sum_{n=0}^{\infty}\frac{(-1)^n}{3n+1}x^{3n+1} = \lim_{x\to 1-}\int_0^x \sum_{n=0}^{\infty}(-1)^n x^{3n}\,dx$$
$$= \lim_{x\to 1-}\int_0^x \frac{dx}{1+x^3} = \lim_{x\to 1-}\left\{\frac{1}{6}\ln\frac{(x+1)^2}{x^2-x+1} + \frac{1}{\sqrt{3}}\arctan\frac{2x-1}{\sqrt{3}} + \frac{\pi}{6\sqrt{3}}\right\}$$
$$= \frac{1}{3}\ln 2 + \frac{\pi}{3\sqrt{3}}.$$

In general

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{\alpha n+1} = \lim_{x\to 1-}\int_0^x \frac{dx}{1+x^\alpha} = \frac{1}{\alpha}\beta\left(\frac{1}{\alpha}\right).$$

analytical expressions which leads to a different value. Take for example

$$\frac{1+x}{1+x+x^2} = 1 - x^2 + x^3 - x^5 + x^6 - x^8 + \ldots.$$

Setting $x = 1$ *renders* $1 - 1 + 1 - 1 + \cdots = \frac{2}{3}$, *different from the result given above. To eliminate such ambiguities,* ***Frobenius*** *(1880) defined summability of a series as:*

$$\lim_{x\to 1-0}\left(\sum_{n=0}^{\infty} a_n x^n\right) = \lim_{n\to\infty}\frac{S_0 + S_1 + \cdots + S_n}{n+1},$$

whenever the r.h.s. limit (of the average of the partial sums) exists. Indeed, in Euler's first example $S_n = 1, 0, 1, 0, 1, 0, \cdots = \frac{1}{2}[1 + (-1)^n]$ *so that*

$$\frac{S_0 + S_1 + \cdots + S_n}{n+1} = \frac{(n+1) + [1 + (-1)^n]}{2(n+1)} \to \frac{1}{2}.$$

Similarly, the above series $1 - x^2 + x^3 - \ldots$ *gives the partial sums of* $1, 2, 3, 4, 5, 6, \ldots$ *terms as* $1, 1, 0, 1, 1, 0, \ldots$ *so that the average sum is*

$$\frac{S_0 + S_1 + \cdots + S_n}{n+1} = \frac{(n+1) - \{\text{integer part of } \frac{1}{3}(n+1)\}}{n+1} \to 1 - \frac{1}{3} = \frac{2}{3}.$$

Considering the fact that Euler and other mathematicians made numerous valid mathematical discoveries by using series which do not converge, we conclude that these "pre–rigor" mathematicians had sufficiently good "experimental evidence" that the use of such series as if they were convergent led to correct results in the majority of cases when they presented themselves naturally.

Consider ***Fourier's*** *own example in his Theorie Analytique de la Chaleur, where he obtained the sine series for the function* $f(x) = \frac{\pi \operatorname{sh} x}{2 \operatorname{sh} \pi}$ *in the interval* $[0, \pi]$, *and found that the coefficient of* $\sin nx$ *is*

$$(-1)^{n-1}\left[\frac{1}{n} - \frac{1}{n^3} + \frac{1}{n^5} - \ldots\right] = (-1)^{n-1}\frac{n}{1+n^2}.$$

Thus, the coefficient of $\sin x$ *appears as* $1 - 1 + 1 - 1 + \ldots$, *and may therefore be expected to be* $\frac{1}{2}$, *if we adopt Euler's summation principle. And indeed, this is correct, since* $\int \operatorname{sh} x \sin x \, dx = \frac{1}{2}(\operatorname{ch} x \sin x - \operatorname{sh} x \cos x)$, *so that* $\frac{2}{\pi}\int_0^{\pi} f(x) \sin x \, dx = \frac{1}{2}$.

We see from these examples that Euler had views which do not differ greatly, at bottom, from those held by modern workers on this subject. It was his unusual instinct for what is mathematically correct which in general saved him from false conclusions.

Clearly, if $\sum a_n$ *converges, then Frobenius' procedure gives the usual sum. The Frobenius summability was generalized by* **Hölder** *(1882) and* **Cesaro** *(1890). Further progress in finding 'sums' for divergent series received its motivation from a totally different direction which, again, started with Euler.* **Euler** *(1754), in seeking a sum for the divergent series*

$$1 - 2! + 3! - 4! + 5! - \dots,$$

proved that $y(x) = x - (1!)x^2 + (2!)x^3 - (3!)x^4 + \dots$ *formally satisfies the differential equation* $x^2y' + y = x$*, for which he obtained the integral*[34]

$$y(x) = \int_0^\infty \frac{xe^{-t}}{1+xt}\,dt.$$

On the other hand, by using the rules which he had obtained for the transformation of convergent series into continued fractions, Euler found

$$y(x) = \cfrac{x}{1+\cfrac{x}{1+\cfrac{x}{1+\cfrac{2x}{1+\cfrac{2x}{1+\cfrac{3x}{1+\cfrac{3x}{1+\dots}}}}}}}$$

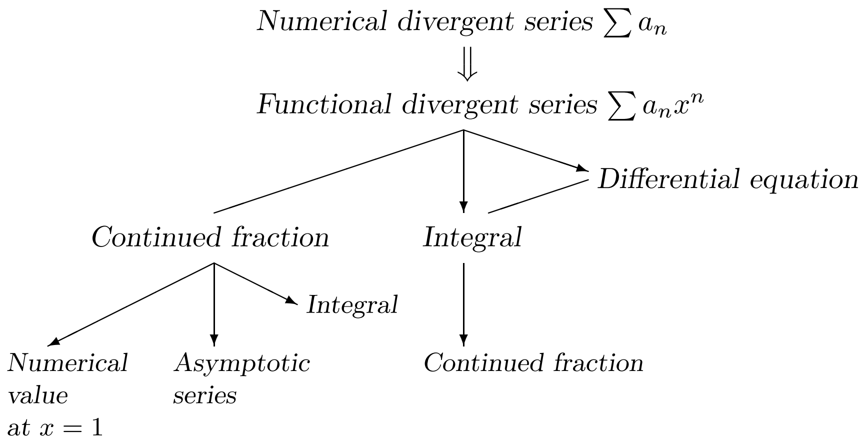
From this he subsequently calculated

$$1 - y(1) = 1 - 2! + 3! - 4! + \dots = 0.596\ 347\ 4\dots$$

This work contains two features. First, Euler obtained an integral that can be taken to be the 'sum' of the divergent series (the latter is in fact asymptotic to the integral). Then, he showed how to convert divergent series into continued fractions.

Euler's pioneering work was continued by **Laguerre** *(1879),* **Stieltjes** *(1886–95),* **Borel** *(1895–99),* **Tauber** *(1897) and* **Fejer** *(1904) along two main routes that can be summed up in the following scheme:*

[34] The substitution $1 + xt = xu$ leads to $y(x) = e^{\frac{1}{x}} \int_{\frac{1}{x}}^\infty \frac{e^{-u}}{u}\,du \equiv e^{\frac{1}{x}} E_1\left(\frac{1}{x}\right)$.



Borel *(1901) discovered a new method of summation of divergent series. First he proved the theorem that if* $f(z) = \sum_{n=0}^{\infty} a_n z^n$ *is analytic when* $|z| < r$, *then* $f(z) = \int_0^{\infty} e^{-t}\Phi(zt)\,dt$ *where* $\Phi(z) = \sum_{n=0}^{\infty} \frac{a_n}{n!} z^n$.

If the integral exists at points z *outside the circle of convergence, we define the 'Borel sum' of* $\sum_{n=0}^{\infty} a_n z^n$ *to mean the integral.*

The function $\Phi(z)$ *is called the* Borel function *associated with* $\sum_{n=0}^{\infty} a_n z^n$. *If*

$$S = \sum_{n=0}^{\infty} a_n$$

is divergent and

$$\Phi(z) = \sum_{n=0}^{\infty} \frac{a_n}{n!} z^n$$

exists and if we can establish the relation

$$S = \int_0^{\infty} e^{-t}\Phi(t)\,dt,$$

with the r.h.s integral convergent, then the series S *is said to be 'Borel summable' or 'summable (B)'.*

The above result implies that a Taylor series representing an analytic function is summable (B).

For example, the 'Borel sum' of the series $\sum_{n=0}^{\infty} z^n$ *is*

$$\int_0^{\infty} e^{-t} e^{tz}\,dt = \frac{1}{1-z}.$$

These studies in the theory of divergent series have clarified a number of hidden connections:

(*i*) *Functional divergent series have functional equivalents in the form of an integral and a continued fraction. The divergent series belongs to one or more functions, which can each be taken to be the sum of the series in a new sense of sum (summability).*

(*ii*) *A divergent series, if summable, can be manipulated precisely as a convergent series.*

(*iii*) *If a given infinite series were to arise in a physical situation, the appropriateness of any definition of its sum would depend entirely on whether the sum is physically significant.*

Cauchy's (classical) definition of a sum is the one that usually fits, because it says basically that the sum is what one gets by continually adding more and more terms in the ordinary sense. But there is no logical reason to prefer this concept to the others that have been introduced.

A general lesson here is that when a concept or technique proves to be useful even though the logic of it is confused or even nonexistent, persistent research will likely uncover a logical justification.

Thus, although concepts like summability may seem artificial, man-made, and contrived, their justification as naturally arising in mathematical solution of physical problems is now sufficient grounds for admitting them into the domain of legitimate mathematics. (This applies also to concepts such as generalized functions, non-standard analysis, and even the Newton-Leibniz non-rigorous, pre-Cauchy calculus.)

***Ramanujan**, devoid of any physical principles and completely innocent of rigor, drove Euler's concepts of divergent series 'ad absurdum'. In 1913 he presented some 'impossible' results of his own:*

$$1 + 2 + 3 + 4 + 5 + \cdots = -\frac{1}{12}$$

$$1^3 + 2^3 + 3^3 + \cdots = \frac{1}{120}$$

He obtained these results regarding the values of the Riemann zeta-function at $s = -1$ *and* $s = -3$ *respectively; they then follow from the general formula*

$$\zeta(1 - 2m) = (-1)^m \frac{B_m}{2m},$$

where B_m are the Bernoulli numbers. Ramanujan also stated (without proof) that

$$1 - 1^1 + 2^2 - 3^3 + \cdots = \int_1^\infty \frac{dx}{x^x} = 0.704\ 169\ 96\ldots,$$

where the integral can be computed by the Euler-Maclaurin formula to yield the numerical value stated on the r.h.s. We have no idea how Ramanujan reached this result, but ***G.N. Watson*** *(1929) offered the following proof: since $\int_0^\infty t^n e^{-t}\,dt = n!$, we write formally*

$$\begin{aligned} 1 - 1^1 + 2^2 - 3^3 + \ldots &= 1 + \sum_{n=1}^\infty \frac{(-1)^n n^n}{n!} \int_0^\infty t^n e^{-t}\,dt \\ &\equiv \int_0^\infty \{1 + \sum_{n=1}^\infty \frac{(-1)^n}{n!} n^n t^n\} e^{-t}\,dt. \end{aligned}$$

When u is sufficiently small, a Lagrange expansion of e^u in powers of ue^u yields

$$e^u = 1 + ue^u + \sum_{n=2}^\infty \frac{(-1)^{n-1}(n-1)^{n-1} u^n e^{nu}}{n!}$$

and hence, by differentiation,

$$e^u = (1+u)e^u[1 + \sum_{n=2}^\infty \frac{(-1)^{n-1}(n-1)^{n-1}(ue^u)^{n-1}}{(n-1)!}].$$

Thus $\frac{1}{1+u} = 1 + \sum_{n=1}^\infty \frac{(-1)^n n^n (ue^u)^n}{n!}$. Putting $ue^u = t$ in the last integral,

$$1 - 1^1 + 2^2 - 3^3 + \cdots = \int_0^\infty \frac{1}{1+u} e^{-(ue^u)} \frac{d(ue^u)}{du}\,du = \int_0^\infty e^{u - ue^u}\,du = \int_1^\infty \frac{dx}{x^x}.$$

Independently, ***John Bernoulli*** *(1696) proved that*

$$s = \int_0^1 x^x\,dx = 1 - 2^{-2} + 3^{-3} - 4^{-4} + \ldots$$

in the following way: $s = \int_0^1 e^{x \ln x}\,dx = \sum_{n=0}^\infty \frac{1}{n!} \int_0^1 x^n (\ln x)^n\,dx$. The integral is then evaluated by means of the substitution $y = \ln \frac{1}{x}$:

$$\int_0^1 x^n (\ln x)^n\,dx = (-1)^n \int_0^\infty dy\, y^n e^{-(n+1)y} = (-1)^n \frac{n!}{(n+1)^{n+1}}.$$

The Automobile (1770–1950)

The first road vehicles that could travel autonomously were powered by steam engines.

In 1770, ***N.J. Cugnot*** *of France successfully operated a three-wheeled steam-powered transportation vehicle. In* 1801, ***Richard Trevithick****, an English inventor, built a 4-wheeled steam wagon. By the mid-*1830*'s, English steam carriages were providing regular passenger service. Some of these carriages carried as many as* 20 *persons. These early cars were noisy, polluted the air with smoke and scattered hot coals. Sometimes the coals set fire to crops or wooden bridges.*

Operators of railroads and stagecoach lines opposed steam carriages because they were becoming successful rivals. The opposition resulted in an English law that put severe limits on the operation of steam carriages. This law, the locomotive act of 1865*, limited the speed of the steam vehicles to* 6 *km/h on country roads and* 3 *km/h in towns. It required that a signalman walk ahead of each steam carriage to warn of its approach. The signalman carried a red flag during the day, and a red lantern at night (Red Flag Law). This law, repealed in* 1896*, discouraged automobile development in England for* 30 *years.*

Steam cars proved impractical because they were hard to start and to operate. Their boilers generated steam too slowly for long-distance travel. Also, many people were afraid to drive a vehicle that depended for power on an open fire and hot steam.

In the late 1890*'s and early* 1900*'s, the electric car became popular in America. It was easy to operate, ran quietly, and did not give off smelly fumes. But few of the cars traveled faster than* 30 *km/h and their batteries had to be recharged about every* 80 *km. The gasoline car gradually replaced the steam and electric cars.*

In 1863, **Jean Joseph Étienne Lenoir** *(1822–1900), a French inventor, installed his one-cylinder internal-combustion engine in a clumsy vehicle that traveled* 10 *kilometers in* 2 *hours.*

In 1864, the electromechanical engineer and inventor ***Siegfried Marcus*** *(1831–1898, Germany) built in his Vienna workshops the first gasoline-driven* *carriage. His second model (1875) was already equipped with most modern technical features. It was patented in Germany (1882). He thus deserves the credit for building the first practical gasoline motorcar applicable for urban transportation. He did not, however, pursue its mass-production, and it had remained just a museum item.*

Marcus was born in Mecklenburg and worked for ***Werner von Siemens*** *during 1848–1860. In* 1885, ***Gottlieb Wilhelm Daimler*** *with* ***Wilhelm Maybach*** *and* ***Carl Benz****, in Germany, separately installed their respective engines in carriages.*

The general design of present day automobiles was developed in France, during 1890–1898 *mainly by* ***René Panhard*** *and* ***Emile Levassor*** *(1891). The cars used chains to carry the engine's power to the rear wheels. In* 1898, ***Louis Renault*** *replaced the chain with a drive shaft.*

Many American inventors experimented with gasoline-powered vehicles in the early 1890*'s. Of these, the brothers* ***Charles E. Duryea*** *(1861–1938) and* ***Frank J. Duryea*** *(1869–1967) built the first successful gasoline-powered automobile in America in* 1893–1894.

Two developments in 1901 *accelerated the growth of the automobile industry: the sharp drop in the price of gasoline and the introduction of mass production methods into the manufacture of cars. Gasoline prices were reduced after the discovery of rich oil fields in eastern Texas. Automobiles could now be operated relatively inexpensively, and they became a popular means of transportation.*

With the assembly-line method, the Olds company built 425 *cars in* 1901 *and* 5000 *in 1903.*

Henry Ford[35] *improved the assembly-line methods to cut production costs. His goal was a low-priced car that many people in all walks of life could*

[35] Ford was the foremost advocate of *anti-Semitism* in America. His weekly *The Dearborn Independence* systematically defamed American Jews for seven years (1920–1927).

afford[36]. Ford achieved this goal with his Model T.

In 1910 ***Hermann Föttinger*** *(Germany) invented the automatic gear changer. It was made using a special drive belt system mounted between the drive shaft and the propeller shaft. It enabled the suppression of the clutch.*

In 1912 General Motors introduced the electric starter. Balloon tires were introduced in 1922, and the automatic transmission in 1939. Finally, power steering was installed in cars in 1950.

During the 1890's automobiles were so new and so strange that they were shown in circuses. In 1990, about 350 million passenger cars traveled on the highways of the world.

1897 CE **Joseph John Thomson** (1856–1940, England). Physicist. Discovered the electron[37], the first known particle smaller than the atom [he called

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Ford had a 4-volume reissue of the anti-Semitic articles of his weekly separately reprinted under the title *The International Jew*; this work was highly praised by Adolf Hitler who said to an American reporter: "*I regard Heinrich [sic] Ford as my inspiration*".

Although Ford retracted publicly [July 7, 1927] all he said about the Jews, he clung to his prejudice for the rest of his life. In any case, a considerable damage had already been done:

Anti-Semitism has come to be a particularly ugly and obscene climax in the 20^{th} century, and if any one American were to be singled out for his contribution to the evils of Nazism, it would have been Henry Ford. His republished articles and the currency which he gave to the *Protocols of the Learned Elders of Zion* had considerable impact on Germany in the early 1920's — a vulnerable and, as it proved, crucial formative time. Hitler, still an obscure figure in those days, read Ford's books, hung Henry's picture on his wall, and cited him frequently as an inspiration. He even based several sections of *Mein Kampf* upon Ford's words and accorded Henry the unhappy distinction of being the only American to be mentioned in that work.

36 Ford produced cars "in any color that the customer wanted — as long as it is black."

37 This statement is an oversimplification. The idea of an atomic unit of charge was probably formulated for the first time by **Faraday** in connection with his experiments on electrolysis. But this idea did not easily fit in with the notion of an electromagnetic field. **Hendrik Antoon Lorentz** modified and completed

them '*corpuscles*']. He came by his discovery in his attempts to explain the nature of cathode rays. By applying improved vacuum techniques, Thomson was able to demonstrate that these rays were composed of identical particles carrying negative charge. He concluded that the particles were present in all kinds of matter and could also be produced from hot metals. In 1906 he received the Nobel prize in physics for his researches into the electrical conductivity of gases.

Thomson was born in a suburb of Manchester, the son of a bookseller. In 1876 he obtained a scholarship at Trinity College, Cambridge, where he remained for the rest of his life. In 1884 he was appointed to the chair of physics at the Cavendish Laboratory.

Thomson entered physics at a critical point in its history. Following the great discoveries of the 19^{th} century in electricity, magnetism, and thermodynamics, many physicists in the 1880's were saying that their science was

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Maxwell's theory of electromagnetism. In his theory, the electric and magnetic properties of matter are interpreted in terms of the motion of charged atomic particles. A magnetic field exerts a force on these particles, now called the "*Lorentz force*". In 1896 **Pieter Zeeman**, then an experimental physicist in Leiden, made a surprising discovery: the splitting of spectral lines by a magnetic field. Lorentz was able to explain the new phenomenon with his electron theory. He concluded that the radiation of atoms consisted of negatively charged particles with a very small mass. Also, German physicists were breathing hard on Thomson's neck throughout 1897. On Jan. 7, 1897, **Emil Wiechert** (1861–1928) issued the first statement that there may exist particles about 2000 to 4000 times lighter than the hydrogen atom. Independent of Thomson, Wiechert determined the ratio of mass to charge of the particles by deflecting them with electric and magnetic fields. Simultaneously, **Walter Kaufmann** determined the ratio of the mass to charge for cathode rays.

The Greek name '*electron*' was used for *amber* for the first time in the third century BCE by **Theophrastos**, a pupil of Aristotle. The name suggests 'lustrous metal'. Perhaps the clear lustrous yellow color of amber, so enhanced with cutting and polishing, led Theophrastos to choose such a name. The ancient Romans knew that when amber was rubbed with a piece of cloth, it picked up small particles. This was actually discovered in the 6th century BCE by the Greek philosopher **Thales of Miletos**. Later, when it was proved that this was the action of *electricity*, the energy was derived from the Greek word for amber — 'electron'. The modern name 'electron' was coined by **Stoney** (1911) for the unit of charge carried by the ion of a monovalent element in electrolysis. **Abraham Pais** coined the collective names *Lepton* (1946) and *Baryon* (1954) to classes of elementary particles, while **L.B. Okun** gave the name *Hadron* (1962) to another class.

coming to an end, much as an exhausted mine. By 1900, however, only elderly conservatives held this view, and by 1914 a new physics was in existence, which raised, indeed, more questions than it could answer. The new physics was wildly exciting to those who saw its boundless possibilities. Probably not more than a half dozen great physicists were associated with bringing about this change.

Thomson was an outstanding teacher; his importance in physics depended almost as much on the work he inspired in others as on that which he did himself. The group of men that gathered around him between 1895 and 1914 came from all over the world, and after working under him, many accepted professorships abroad. Seven Nobel prizes were awarded to those who worked under him. Thomson took his teaching duties very seriously: he lectured regularly to elementary classes in the morning and to postgraduates in the afternoon. He considered teaching to be helpful for a researcher, since it required him to reconsider basic ideas that otherwise might be taken for granted. He never advised a man entering a new research field to begin by reading the work already done. Rather, Thomson thought it wise that he first clarify his own ideas. Then he could safely read the reports of others, without having his own views influenced by assumptions that he might find difficult to throw off.

1897 CE **Alfred Tauber** (1866–1942, Slovakia & Austria). Mathematician. Discovered an important theorem through which ordinary convergence of power series is deduced from some type of summability; known as the *Tauberian theorem*[38].

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[38] "*Abel's theorem*" states that if $\sum_{n=0}^{\infty} a_n$ is convergent with a sum S, then the series $f(z) = \sum_{n=0}^{\infty} a_n z^n$ is uniformly convergent for $|z| < 1$ and $\lim_{z\to 1} \sum_0^{\infty} a_n z^n = S$.

The direct converse of Abel's theorem is false, as is shown by the simple counter-example $f(z) = \sum_{n=0}^{\infty}(-1)^n z^n = \frac{1}{1+z}$ with $f(1) = 2$. Since $\sum_0^{\infty} a_n$ is not convergent. If, however, we impose a *restriction* on a_n as to their order of magnitude, it is possible to prove the converse theorem.

Tauber's theorem: If $a_n = o\left(\frac{1}{n}\right)$ and $f(z) \to S$ as $z \to 1$, then $\sum_0^{\infty} a_n$ converges to the sum S.

In short

$$\sum_{n=0}^{\infty} a_n = \lim_{z\to 1} \sum_{n=0}^{\infty} a_n z^n \qquad \text{(Abel)},$$

$$\lim_{z\to 1} \sum_{n=0}^{\infty} a_n z^n \;\Rightarrow\; \sum_{n=0}^{\infty} a_n, \qquad \text{if } \lim_{n\to\infty} (na_n) = 0 \qquad \text{(Tauber)}.$$

Tauber's theorem became essential to the *operational calculus* (especially for Laplace and Fourier integral transform theory) since it enables one to use the known behavior of a linear system at large times in order to deduce its behavior

Tauber was born on Pressburg (now Bratislava), Slovakia to Jewish parents. He was sent by the Nazis to Theresienstadt concentration camp on June 28, 1942 and was hauled from there to the *Auschwitz Gas Chambers* with another 53,000 Jewish inhabitants of the camp.

1897 CE, August 29 *The First Zionist Congress* opened in Basel, Switzerland, under the leadership of **Theodor Herzl** (1860–1904), the founder of the Zionist movement. It approved the programme for re-establishing a Jewish homeland in the Land of Israel.

Herzl was a product of late 19th century positivism, of the age of springtime of the nations. It was also the time of the pogroms in Russia and the Dreyfuss Affair. Herzl saw the suffering of his people as the outcome of the curse of Exile and held that political sovereignty for the Jews was the solution to the problem of *antisemitism*. After the terrible lesson of the Holocaust (1939–1945), it became the credo of the State of Israel.

Not only did antisemitism not "fall silent immediately", but the Jewish state itself became the focus of world Judeophobia. This is what actually occurred even if the hatred was transformed among the nations into "anti-Zionism". In its short history, the Jewish state has faced those who wanted to destroy her in five large-scale wars (1948, 1956, 1967, 1973, 1982) and through unrelenting terrorism within and on her frontiers (1990–2008).

Many are the praises and merits of Zionism: the renewal of a dead language, the ingathering of the exiles, and the re-establishment of a state for a persecuted, ancient people that co-founded Western Civilization and gave the world so much. The movement of revival of the Jewish nation expressed a combination of will power and the victory of man's spirit, worthy of the grave duty of the descendants of the ancient Israelites, who may take pride in about one fourth of all Nobel prizes for science since 1901[39].

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at zero frequency (and vice versa). For example, in the case of the *Laplace transform* $f(p) = p\int_0^\infty e^{-pt}h(t)\,dt$ we have:

$$\lim_{t\to\infty} h(t) = \lim_{p\to +0} f(p) \qquad \text{(Abel)},$$

$$\lim_{p\to +0} f(p) = \lim_{t\to\infty} h(t), \qquad \text{if } \lim_{t\to\infty} t[h(t) - h(t-1)] = 0 \qquad \text{(Tauber)}.$$

[39] Jews and persons of half-Jewish ancestry have been awarded at least 161 Nobel Prizes, accounting for 22% of all such prizes awarded to individuals worldwide between 1901 and 2003, and constituting 36% of all US Nobel Prize winners during the same period. In the *scientific research* fields of Chemistry, Economics, Medicine, and Physics, the corresponding world and US percentages are 26% and 39%, respectively. (Jews currently make up approximately 0.25% of the world's

1897–1908 CE **Frederick William Lanchester** (1868–1946, England). Aeronautics pioneer who expounded, ahead of his time, the principles of heavier-than-air flight. He was a practical engineer, amateur mathematician and by trade, an automobile builder who began the construction of the first Lanchester motorcar in 1894. In 1897 he developed *the circulation theory of flight*[40]. Two books by him, containing his well-developed ideas, appeared in 1907 and 1908. He contributed to many branches of applied mathematics (e.g. operations research), and continued to produce technical inventions throughout his life.

Lanchester was born in London. After attending college he went in 1891 to work for a gas-engine works in Birmingham. In 1896 he left to set up his own automobile manufacturing firm, producing his first car the same year. He later founded the Lanchester engine company which produced cars which were relatively vibration-free and had a graceful appearance.

1897–1910 CE **Adolf Loos** (1870–1933, Austria). Modernist Viennese architect. Developed a simplified style of architecture marked by uncluttered lines and flat surfaces, setting great value on precision and economy in design. Claimed that buildings must be appropriate to their use and nothing more. His architecture was designed to show in the modern its relationship to the classic. His use of *reinforced concrete* (a new material at the time), utter lack

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population and less than 2% of the US population.)

- *Chemistry* (25 prize winners, 18% of world total, 26% of US total)
- *Economics* (21 prize winners, 40% of world total, 54% of US total)
- *Literature* (11 prize winners, 11% of world total, 27% of US total)
- *Physiology or Medicine* (51 prize winners, 28% of world total, 42% of US total)
- *Peace* (9 prize winners, 10% of world total, 11% of US total)
- *Physics* (44 prize winners, 26% of world total, 37% of US total)

This enumeration constitutes an update and an expansion of the information on Jewish Nobel Prize winners contained in the 1997 CD ROM edition of the *Encyclopedia Judaica*.

40 He discovered that the wing can be effectively replaced by a *vortex system* consisting of a *bound vortex* which travels with the wing and *free vortices* springing from the wing tips. He was also the first to recognize the importance of *aspect-ratio* of the wing in connection with the work required for sustentation. [The slope of the *line of lift* versus the *angle of attack* depends on the aspect ratio and decreases with its decrease.]

of ornament and severe cubic form and treatment of interior space, his work become a landmark in architectural history, chiefly influencing architects in the United States.

Loos condemned *art nouveau*. In his articles in Viennese journals he structured his arguments within broad socio-political and cultural frameworks. He argued that while ornament had a place in the past, it was degenerate in modern culture. He also criticized the modern practice of architecture for lack of craftsmanship and for working in a cultural vacuum. His buildings relentlessly expressed his theories.

Loos was born in Brno (now in the Czech Republic). During his stay in the USA (1893–1896) he was strongly influenced by the Chicago School. He lived in Vienna from 1897 until his death, except for six years (1922–1928) spent in Paris.

1897–1910 CE **Ernest William Barnes** (1874–1953, England). Mathematician and bishop. Discovered a contour integral representation for the hypergeometric function[41] (1908). Made significant contributions to the theory of transcendental functions (such as the G-function[42]) and proved the so-called *Barnes lemma*.

Barnes was born in the small Oxfordshire town of Charlbury, and educated at Trinity College, Cambridge. He graduated Sc.D. in 1907. He was ordained in the Anglican Church (1902). In 1924 he was appointed by Ramsay MacDonald to the Bishopric of Birmingham.

1897–1921 CE **Vilhelm Frimann Koren Bjerkens** (1862–1951, Norway). Meteorologist and physicist. One of the founders of the science of weather forecasting.

Discovered the circulation theorems that led him to the synthesis of hydrodynamics and thermodynamics applicable to large-scale motions in the atmosphere and the ocean. This work ultimately resulted in the theory of motion of air-masses, which is essential to modern weather forecasting (1919–1921)[43].

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41 ${}_2F_1(a,b,c;z) = \frac{\Gamma(c)}{2\pi i\Gamma(a)\Gamma(b)}\int_{-i\infty}^{i\infty}\frac{\Gamma(a+t)\Gamma(b+t)\Gamma(-t)}{\Gamma(c+t)}(-z)^t dt.$

42 $G\left(\frac{z+1}{\tau}\right) = \Gamma\left(\frac{z}{\tau}\right)G\left(\frac{z}{\tau}\right),$ where τ is a constant.

43 This work was done in collaboration with his son **Jacob Aall Bonnevie Bjerkens** (1897–1975, Norway and U.S.A.). In 1919 they coined the term *front* for an advancing discontinuity surface separating air masses of different densities, and formed when warm and cold air masses meet. They suggested that *cyclones* originate as waves along the front.

Bjerkens was born in Christiania, Norway. In 1890 he went to Germany and collaborated with **Heinrich Hertz**. In 1895 he became professor of applied mechanics and mathematical physics at the University of Stockholm. He served later as professor in Christiania (1907), Leipzig (1912), Bergen (1917) and Oslo (1926–1932).

1898 CE **Ivan Stanislavovic Bloch** (1836–1901, Poland). Banker and railway financier who devoted his private life to the study of modern industrial warfare. In his six volume master work, *La Guerre Future* (Paris, 1898) he presented a detailed analysis of modern warfare, its tactical, strategic and political implications. His main argument was that:

- New arms technology has rendered maneuvers over open ground (such as bayonet and cavalry charges) obsolete. War between Great Powers would be a war of entrenchment and that rapid attacks and decisive victories were likewise a thing of the past.

- Industrial societies would have to settle the resultant stalemate by committing armies numbering in the millions, as opposed to the tens of thousands of preceding wars. An enormous battlefront would develop. A war of this type could not be resolved quickly.

- The war would become a duel of industrial fight, a matter of total economic attrition. Severe economic and social dislocations would result in the imminent risk of famine, disease, the break-up of the whole social organization and revolutions from below.

Europe's patriots were unmoved. French cavalry and British infantry commanders only learned Bloch's lessons the hard way during WWI. The Russian and German monarchies proved equally incapable of assimilating Bloch's cautionary words concerning revolutions, paying the price with summary execution and exile, respectively.

Bloch was born into a Jewish family at Radom, Poland and died at Warsaw. In his late twenties he adopted Christianity in the form of Calvinism to ease his way up in the Russo-Polish banking system. With the banker Kronenberg of Warsaw, whose sister he married, Bloch participated in the construction of railroads in Russia.

1898 CE **John Philip Holland** (1840–1914, U.S.A.). An Irish-American inventor who built the first practical submarine. It became a model on which later submarines were developed[44].

In 1898, after 28 years of development, he privately built and launched the *Holland* (so named in 1900 by the U.S. Navy). It was a 16 m submarine, powered by a gasoline engine and electric batteries. It could reach a speed of some 15 km/h, submerged.

Holland was born in County Clare, Ireland. He began work on the idea of a submarine while a school teacher in Ireland during 1858–1872. By 1870 he had completed the first plans for his invention. In 1873, Holland came to the United States and earned his living as a school teacher in Paterson, NJ. He submitted his submarine plans to the U.S. Navy in 1875, but they were rejected. However, the *Fenian Society*, a group of Irish patriots in the United States, who hoped to destroy England's naval power, became interested. They supported Holland's experiments and gave him money to build two submarines. In 1900, the Navy bought the *Holland* and asked the inventor to build several more ships like it.

Holland died in poverty, because the company which he formed ran into difficulties.

1898 CE **Marie Sklodowska Curie** (1867–1934, France). Polish-born French physicist. Discovered the atomic origin of radioactivity and the elements radium and polonium [with **Pierre Curie** (1867–1906, France, 1898)]. She coined the word '*radioactivity*'. In 1911 she was awarded the Nobel prize for chemistry, for her isolation of pure radium from pitchblende.

Marie went to Paris in 1891. She married Pierre Curie in 1895 and gave birth to two daughters, Irène (1897) and Éve (1904). Following the sudden death of Pierre in 1906, she devoted all her energy to completing alone the

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[44] Other pioneers also contributed to the design and development of modern naval submarines:

Gustave Zedé (1825–1891, France). A naval engineer. Built in 1888 an electricity-driven submarine (30 tons, 17 m long) that reached a range of 59 km at a speed of 15 km/h. In 1896 he invented the *periscope*.

Thorsten Nordenfeldt (1842–1920, Sweden) was first to equip submarines with torpedoes (1888).

Simon Lake (1886–1945, U.S.A.) built submarines driven with Diesel engines. His submarine *Argonaut I* was first to make long journeys in the open ocean (1898). In 1902 he developed a periscope with magnifying lenses to sight distant targets. He also built submarines with wheels, so that they could roll along the sea bottom.

scientific work that they had undertaken. In 1908 she was appointed to the professorship that had been left vacant on her husband's death. She died of leukemia caused by the exposure to radiation, a few months after the discoveries of the neutron and artificial radioactivity.

1898 CE **Martinus Willem Beijerinck** (1851–1931, Holland). Botanist. Introduced the name *Virus*[45] for the infective microorganism found in the tobacco mosaic disease. It was coined earlier (1801) by **Dominique Jean Larrey** (head surgeon of Napoleon's Egyptian campaign) as the specific external agent causing diseases.

1898–1900 CE **Joseph Larmor** (1857–1942, England). Irish mathematical physicist. Derived the *Lorentz transformation equations* before Lorentz in his prize-winning essay 'Aether and Matter' [completed 1898, published 1900]. It contains not only the exact transformations, but also the proof that one arrives at the FitzGerald-Lorentz contraction with the help of these transformations. In this work Larmor already suggested that moving clocks must run slow, and by how much.

Larmor was first to calculate the rate at which energy is radiated by an accelerated charge. He also discovered the *classical* effect of an applied magnetic field $\boldsymbol{B}$ on an electron revolving in a circular orbit [it results in the '*Larmor precession*' of the electron's orbital angular momentum about the magnetic field direction. The angular velocity of precession is equal to $\pm\frac{e|\boldsymbol{B}|}{2m}$, where e/m is the ratio of the electron's charge to its mass.] Through this model he explained the splitting of spectral lines by a magnetic field.

Larmor was born at Magheragall in Ireland and was educated at Queen's University, Belfast and St. John's College, Cambridge. He taught at Queen's College, Galway (1880–1885) and at Cambridge (1885–1932), becoming a Lucasian professor there. During 1911–1922 he was member of Parliament for Cambridge.

1898–1923 CE Repeated outbreak of *plague* in India. Death toll as high as 12 million.

1898–1907 CE **Valdemar Poulsen** (1869–1942, Denmark). Electrical engineer and inventor. Became known as the Danish Edison. Developed the

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[45] *Virus* is the Latin word for 'poison'. The average virus is about 0.2 micron in diameter, as compared to 2 micron for the diameter of an average bacterium. It was found in 1936 that the tobacco mosaic virus was 94 percent protein and 6 percent nucleic acid. Today, we may say that just as a bacterium could be viewed as a kind of a isolated cell nucleus, a virus could be viewed as an isolated cell *chromosome*.

first *magnetic sound recorder* (1899). He later invented the arc-transmitter (1902), important for wireless telegraphy.

Poulsen's *Telegraphone* (1898) was the first practical apparatus for magnetic sound recording and reproduction. It recorded, in a magnetized steel piano wire[46], the varying magnetic fields produced by a sound. The magnetized wire could then be used to play back the sound.

The Telegraphone received considerable attention when it was exhibited at the Exposition Universelle in Paris (1900). The few words that the Austrian emperor Francis Joseph spoke into it at that exhibition are believed to be the earliest surviving magnetic recording.

Poulsen's arc-transmitter[47] was the first device for generating continuous radio waves, thus aiding the development of radio broadcasting. In 1904 he was transmitting voice over appreciable distances.

Poulsen was born in Copenhagen. He made his invention while working at the Copenhagen Telephone Company as an assistant in the technical section.

The Telegraphone was offered commercially in the US in the 1920's. Sometimes in the 1950's a workable Telegraphone was found during cleaning in the cellar at the Interior Department. It could play back a part of a radio broadcast from 1921.

In 1927, the American inventor **J.A. O'Neill** replaced the wire with a magnetically coated ribbon and since then magnetic tape recorders have dominated the recording industry.

1898–1919 CE **Jules-Jean-Baptiste-Vincent Bordet** (1870–1961, Belgium). Bacteriologist and immunologist. Discovered the role of blood serum in the human immune system response (1898) [the *complement* is the component of blood needed for antibodies[48] to react with invading bacteria]. His work made possible new techniques for the diagnosis and control of infectious

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46 It recorded continuously for 30 minutes at a speed of 2/3 cm per second. He stretched his wire across his laboratory and put the recording apparatus on a trolley that traveled along that wire. He would run along with the moving trolley, talking into its microphone to record sound on the wire. To play back this sounds, he would roll a second trolley containing the playback equipment along the wire, and it would reproduce the sound.

47 The arc was formed between a copper cathode (positive terminal) and a carbon anode (negative terminal) in an atmosphere of a hydrocarbon gas and a transverse magnetic field. Subsequent efforts with this device by Poulsen and others made long-wave radio broadcasting possible by 1920.

48 This discovery plus the discovery of other serum factors gave rise to *humoral theory* of immunity, and the term *antigen* and *antibody* came into use. This humoral

diseases. In 1901, Bordet further showed that *complement* is used up when an antibody reacts with an antigen. He was first to isolate (1906) the whooping-cough bacillus and isolated (1909) the germs for bovine peripneumonia and Avium diphtheria.

Bordet was born in Soignies and received his MD from the University of Brussels (1892). Went to Paris (1894) to work in Metchnikoff's laboratory at the Pasteur Institute. Became Director of the Pasteur Institute in Brussels (1901), where he remained until he was succeeded by his son Paul (1940). He was awarded the Nobel Prize for Physiology or Medicine (1919) for his studies in immunology.

1899 CE The drug *aspirin* (acetyl-salicylic acid; $C_9H_8O_4$) became available as an effective remedy in relieving minor pain and in reducing fever. It was introduced by the German chemical firm of Bayer.

The Alsatian chemist **Charles-Frederic** (**Karl**) **Gerhardt** (1816–1856) discovered aspirin (1853) as a *natural* by-product of coal tar. It was first synthesized (1859) by the German organic chemist **Adolf Wilhelm Hermann Kolbe** (1818–1884) in a reaction named after him, through which salicylic acid and acetic acid are combined. The result is a colorless odorless powder with bitter taste; its structural formula consists of a benzene-ring in which one hydrogen atom is replaced by the radical $-\underset{\underset{\displaystyle \mathrm{O}}{\|}}{\mathrm{C}}-\mathrm{OH}$, and a neighboring hydrogen atom by the radical $-\mathrm{O}-\underset{\underset{\displaystyle \mathrm{O}}{\|}}{\mathrm{C}}-\mathrm{CH_3}$.

The discovery of aspirin was motivated by the shortage of the drug *quinine*, which led chemists to search for a substitute pain reliever. The medical value of aspirin was recognized only in 1899 when **Heinrich Dreser**, a German scientist, wrote about its effectiveness as a mild sedative. Consequently, its large-scale manufacturing started at the same year by the German Firm Bayer, where **Felix Hoffman** used Kolbe's synthesizing reaction. Today, aspirin is used as a antipyretic analgesic agent in relieving the symptoms of headache, neuralgia, arthritis, rheumatism, common cold and influenza.

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view was vigorously contested by **Elie Metchnikoff** (1845–1916) who described *phagocytosis* (the ingestion of bacteria by white blood cells) and proposed a cellular theory of immunity (inflammation). Later, **Almroth Wright** (1861–1947) combined both views by showing serum factors necessary for phagocytosis.

Aspirin

Aspirin is found naturally in certain plants and trees: e.g. the barks of the willow tree *and the* silver birch tree *as well as the leaves of* meadowsweet *and* wintergreen *plants.*

In the summer of 1758, the Rev. ***Edward Stone****, of Chipping Norton, in Oxfordshire, England, was suffering another of his bouts of fever and rheumatic twinges. By accident, he chewed on a twig of the white willow tree (*Salix alba*), and despite its "extraordinary bitterness", he was astounded to find that it relieved his "ague". He devised a method of drying and pulverizing the bark, and then experimented to discover the best dosage. Over the next five years, he gave his remedy to 50 others, and it "never failed in the cure". Enthusiastic at his discovery, on 25 April 1763 he wrote to the Earl of Macclesfield, President of the Royal Society, but was ignored.*

In the 1820s, the Swiss Pharmacist ***Johann Pagenstecher*** *began extracting a substance from the leaves of the plant* Spirea ulmaria*, commonly called* meadowsweet *and well known as a pain reliever in folk medicine. His report in a scientific journal was read in 1835 by the German chemist* ***Karl Jacob Löwig****, who, using the extract, obtained an acid, later to be known as* salicylic acid.

Earlier (1829), the French pharmacist ***H. Leroux*** *identified the active element in willow bark to be* salicin*. In 1838, the Italian* ***R. Piria*** *extracted pure salicylic acid from methyl salicylate taken from birch tree bark. The Alsatian chemist* ***Karl Friedrich Gerhardt*** *(1816–1856), a chemistry professor at France's Montpellier University, discovered (1853) the molecular structure of salicylic acid. He also tried to modify its rather severe side effect — the painful irritation of the stomach lining — but he found the procedure so time-consuming that he abandoned the drug as "of no further significance".*

But in 1860, the German organic chemist ***Hermann Kolbe*** *(1818–1884) first synthesized salicylic acid — a colorless odorless powder with bitter taste. In the following years the shortage of the drug* quinine *motivated chemists to search for a substitute fever reducing and pain reliever medicine. Salicylic acid, however, continued to be used by people whose pain was worse than that caused by the drug itself. One of these was a Herr Hoffman, who lived in the German town of Elberfeld and was crippled by arthritis. His son,* ***Felix Hoffman****, worked as a chemist at the huge Bayer drug plant nearby, and in 1895 he decided to try to change salicylic acid to end his father's suffering. He simplified Kolbe's method and came up with* acetylsalicylic acid.

Hoffman's colleague, ***Heinrich Dreser****, investigated the overall medical effects of the new drug and recognized it to be effective not only as a mild sedative, but also as a reducer of fever and inflammations. In 1899, Hoffman and Dreser coined a new name for that new drug:* aspirin *— '*a*' for 'acetyl', '*spir*' for the Spirea plant family and '*in*' to round it off.*

The following year, the Bayer drug company took out patents on aspirin, on the intermediate compounds in its manufacture, and on the design of the manufacturing equipment, and began to make huge amounts of what was to become their bestselling product all over the world.

In 1914, in anticipation to the outbreak of war and a halt in supplies from Germany, the British and Australian government offered a prize of £25,000 to anyone in Britain or the British Commonwealth who could come up with a new formulation for aspirin that could circumvent Bayer's patents. The chemist ***George Nicholas*** *took up the challenge and devised a process that yielded exceptionally pure aspirin — and won the prize. Following Germany's defeat, the victors confiscated the name 'aspirin', and the Bayer company lost its exclusive rights to both the name and the manufacture of the drug.*

Since its discovery, aspirin is used as an antipyretic analgesic agent in relieving the symptoms of headache, neuralgia, arthritis, rheumatism, common cold, influenza, inflammation, swelling and even as a blood thinner for preventing ailments — but with no known clue for its effectiveness. It was only in 1971 that researchers in Britain came up with at least one reason why aspirin works. Prostaglandins*, a group of hormone-like substances found in virtually all the tissues of the body, seem to increase the sensitivity of nerve endings at sites of inflammation — and aspirin appears to interfere with the effective action of these substances.*

Aspirin eventually became part of a group known as non-steroidal anti-inflammatory drugs (NSAIDs) which now include the more recent drug ibuprofen*. In the 1980s, aspirin was superseded by* paracetamol *(first used in 1891, but first marketed in 1953) as a popular painkiller for all ages.*

1899–1905 CE **René-Louis Baire** (1874–1932, France). Mathematician. Made significant contributions to the analysis of real functions and *functionals* and the concept of limit. Introduced the new notions of *semicontinuity*[49] of

[49] The condition of *continuity of a function* $f(x)$ at a point x states that, given ϵ, an open interval $(x-h, x+h)$ exists that for any point x' in it, $|f(x')-f(x)|<\epsilon$.

functionals and classified them into the '*Baire-functions*'. His books on analysis include: *Theorie des nombres irrationels, des limites et de la continuite* (1905) and *Lecons sur les theories generales de l'analyse* (1907–8). Baire was a student of **Volterra** and **Darboux**. Became a professor of mathematics at the Universities of Montpellier and Dijon.

The limit function of a pointwise converging sequence of continuous functions defined on a metric space is *not* necessarily continuous; a simple but profound example suffices to exhibit this property: consider an isosceles triangle with sides $AC = BC = 10$ units and base $AB = 5$ units. The sum of lengths of its sides is therefore 20 units. Mark the midpoints of AC, BC, AB at E, D, F respectively and draw the 4-segment zigzag line $AEFDB$ whose length is again 20 units. Repeat this process to each of the triangles AEF and FDB, thus producing a new 8-segment zigzag line of total length 20 units. As the process continues, the zigzag line (whose total length is always 20) will get closer and closer to the 5 unit base AB, and its vertical height will get smaller and smaller.

Clearly, the *length* of the zigzag line is a *functional* of its shape. This functional is *not continuous* at the base of the triangle, since the length of the zigzag line does *not* necessarily approximate closely the length of the base as the position of the lines gets close to the position of the base. However, the functional has the property that Baire called *lower semicontinuity*, since all the different zigzag lines we can possibly draw, as close to the base as we please, have a *lower bound*, namely 5 units, but no *upper bound*.

This can be split into two separate conditions:

(i) $f(x') < f(x) + \epsilon$,

(ii): $f(x') > f(x) - \epsilon$. It is possible that at a point x one of these conditions may be satisfied and not the other. If for every point x' in the above open interval the first condition is satisfied then the point x is said to be a point of *upper semi-continuity* of the function $f(x)$. If an open neighborhood of the point x can be determined for each ϵ, such that $f(x') > f(x) - \epsilon$, then the point x is said to be a point of *lower semi-continuity.* A function $f(x)$ is said to be *upper semi-continuous* in the interval (a, b), if every point in it is a point of upper semi-continuity.

Baire (1899) introduced the following classification: continuous functions are *functions of class* 0; a function that is a pointwise limit of a sequence of continuous functions is a *function of at most class* 1. A function is said to be of class 1 if it is of at most class 1 and not of class 0. He similarly defined the notion of *class* n for arbitrary natural number n. For example the *Dirichlet function* $f(x) = \lim_{\nu\to\infty}[\lim_{k\to\infty}(\cos\nu!\pi x)^{2k}]$ is of class 2; it takes the value 1 at rational points and 0 at irrational points.

The Magnetic Recording Story (1888–1964)

Early attempts to record sound centered about experiments to trace the vibrations of bodies emitting sound, such as tuning-forks and membranes. In 1807 ***Thomas Young*** *described a method of recording the vibration of a tuning-fork on the surface of a drum; his method was fully carried out by* ***Wilhelm Wertheim*** *in* 1842.

Recording the vibrations of a membrane was first accomplished by the French typographer and painter ***Léon Scott de Martinville*** *in* 1857 *by the invention of the phonautograph, which may be regarded as the precursor of the phonograph.*

This ingenious device consisted of a horn with a thin membrane stretched over the end, to which was attached a stiff bristle. Rotating beneath the bristle and advanced by means of a lead screw was a smoked drum. Sounds directed into the horn caused the bristle to move back and forth across the smoked paper and trace out a wave-form. For the first time in history it was possible to see sound.

The dazzling possibility that has escaped both ***Scott*** *and the acoustic experimentalist* ***Karl Rudolph König*** *(who actually constructed the first phonautograph) was the simple idea of reversing the process and retrieving the original sound which had been recorded.*

Edison's phonograph (1877) operated by indenting, or embossing, a tinfoil-covered cylinder to a varying depth corresponding to the sound pressure. The sound track was, therefore, a spiral of varying depth around a cylinder.

In 1888 ***Oberlin Smith*** *wrote an article in the magazine Electrical World in which he suggested, probably for the first time, the use of permanent magnetic impressions for sound recording. Smith visualized a cotton or silk thread in which steel dust or short clippings of fine wire were suspended, these particles to be magnetized in accordance with the undulatory current delivered from a microphone. He discussed the possibility of employing a hard steel wire, but did not believe that a wire would divide itself up properly into a number of short magnets to establish a magnetic pattern that is a replica of the microphone current. Smith never built an instrument to implement his idea.*

In 1887, ***Emile Berliner*** *(1851–1929, Germany and U.S.A.), invented the gramophone. It had a flat disc instead of a cylinder. At the turn of the century, Berliner made further commercial development of the phonograph by working out methods of mass-producing shellac records.*

It is at this very moment in the history of recorded sound that ***Poulsen*** *appeared on the scene; he came from a good family — his father was supreme court judge in Copenhagen and wanted his son to be a doctor. Valdemar accordingly enrolled in medical school, but in* 1893 *he left that school to work in the technical section of the Copenhagen Telephone Company.*

It is not known how he got the idea of recording sound by varying the magnetization of a steel wire. It is believed that he never read Smith's article. Moreover, there was no theoretical basis for believing that one could magnetize just one spot on a bar and leave the rest unaffected, and granted that, for knowing how permanent the record would actually be.

It was an amazing and counter-intuitive idea, and it was Poulsen's alone! In contradistinction to the telephone and countless other major inventions throughout the history of science, nobody else, so far as is known, even claimed to have invented the telegraphone, *as Poulsen named his invention.*

In Poulsen's machine, a large brass cylinder had spiral grooves running the length of the cylinder. Lying in the groove was a steel wire, against which rested two poles of an electromagnet, which carried the current generated by the microphone. The electromagnet was rotating, and thus magnetizing portion after portion of the spiral wire. When the recording was completed, the microphone was switched out of the circuit and a telephone receiver connected in its place. Then, by placing the recording head (which now served as a reproducing head) at the beginning of the wire spiral, the original message was heard, as the varying magnetization of the wire generated current in the windings of the electromagnet.

The low playback level (which required the use of earphone) was the besetting weakness of early magnetic recording. When it became obvious with the passage of a number of years that this obstacle was basic, magnetic recording dropped out of sight, and interest was not revived until electronic amplifiers became available.

In the late 1920*'s, engineers found a more efficient and convenient substitute for the steel wire used in Poulsen's recorder. They worked out a method for using* plastic tape *coated with magnetic material. Thus* ***J.A. O'Neil*** *(1927) replaced wire with a diamagnetic ribbon (coated with a metallic substance such as ferric oxides, chrome dioxide on a mylar tape) while* ***F. Pfleumer*** *introduced a magnetic tape made of paper coated with magnetic particles. By 1930, tape recorders based on this principle were developed in Germany. In 1935, AEG (Germany) introduced the first model of commercial tape recorder using plastic tape coated with magnetic particles; it was called the Magnetophone.*

Further improvements were made by ***Joseph Begun*** *(1905–1995, Germany and USA) who built the first tape recorder for broadcasting, later used* *in the 1936 Berlin Olympics. During WWII (1939–1945), the Germans further developed the process of recording on magnetic tape. After the war, engineers in the United States continued the German experiments.*

Begun, who was born in Danzig (now Gdansk, Poland) and graduated (1929) with a PhD from the Berlin Institute of Technology, moved to the United States in 1935 and continued to develop tape recording.

By 1950, *tape had largely replaced phonograph records for radio recording. Stereo tape recorders were introduced in the United States in* 1955. *About the same time, television stations began recording programs on videotape. In the early* 1960*'s, engineers in the Netherlands developed the cassette audio tape recorder, which was introduced in the United States in 1964.*

1899–1913 CE **Jacques Loeb** (1859–1924, Germany and USA). Biologist and pioneer of the mechanistic view of life (*The Mechanistic Conception of Life*, 1912). Found a way to remove the eggs from a female of sea-urchin and make them start their embryonic development, just as if they had been fertilized by a sperm — but without sperm[50]. He found that a dose of certain lifeless chemicals would launch the development of an organism from a single cell. This he used as a confirmation of the mechanistic view of life, i.e. that life is essentially a mechanical process which can be *entirely* explained by the laws of physics and chemistry.

Loeb was confident that the mechanics of life would prove simple enough to create life in the laboratory! He in fact prophesied *self-assembly*, i.e. that certain molecules assemble themselves into specific, predictable structures, because their shapes, their own physics and chemistry, will allow them to assemble *only* in those patterns. In the next 50 years biochemists would apply a barely reductionist approach to the study of life, and vindicate the mechanistic view with the discovery of the double helix of DNA and the cracking of the genetic code.

Loeb was born to a Jewish family in Mayen, Germany. Educated in philosophy at Berlin and in medicine at Strasbourg (MD, 1884). Assistant professor at Würzburg (1886). An interest in the philosophy of the will led to research which attempted to show, in the animal world, phenomena analogous to plant tropism. Emigrated to the USA (1891), and held various university

[50] Nowadays the procedure is quite routine and is performed even in high school biology classes to demonstrate the early stages of embryonic development.

appointments before becoming head of the general physiology division of the Rockefeller Institute for Medical Research (1910–1024). **Sinclair Lewis** made Loeb the model for Max Gottlieb, in his book "Arrowsmith". Loeb will be remembered as a champion of materialism in philosophy and the mechanical view of life in biology.

1899–1916 CE **Karl Schwarzschild** (1873–1916, Germany). A gifted astronomer whose contributions, both practical and theoretical, were of primary importance in the development of 20^{th} century astronomy and cosmology. His name is associated with a number of discoveries: Schwarzschild *radiation equilibrium*, '*distribution function*', '*velocity ellipsoid*', '*metric*', '*horizon*', '*radius*' and '*exponent*'.

Schwarzschild was first to suggest that heat and light in stellar atmospheres are transported mainly by radiation. He enunciated the principle of radiative equilibrium[51], and was first to reorganize clearly the role of radiative processes in energy transport in the stellar atmospheres.

He found, already in 1899, that the changes in luminosity of cepheids are accompanied by changes in effective temperature, suggesting that the variability is a temperature effect. In 1900, when GTR was still unknown, Schwarzschild apprehended the importance of *non-Euclidean geometry*.

He concluded that "*it is possible, without contradicting the evidence, to think of the universe as contained within a hyperbolic space with a radius of curvature larger than* 4,000,000 *radii of the earth's orbit, or within a finite, elliptical space with a radius of curvature larger than* 1,000,000 *radii of the earth's orbit, while assuming in the latter case an absorption of light equal to* 40 *magnitudes in a journey around this space*".

Although it is now known that the possible radius of curvature of the universe must be much larger than the minimum values quoted by him, his paper represents a pioneering effort in the field now referred to as *cosmology*.

In 1906 he showed that in the sun's photosphere (the layers that send out most of the radiation) the transport of energy outwards from the interior is performed by radiation. He calculated the increase of temperature with optical depth on the assumption of *radiative equilibrium* and showed that one thereby derives the correct center-to-limb darkening of the solar disc.

Schwarzschild's work on the solar eclipse of Aug. 30, 1905 is a masterpiece of insight into observation and theory.

In 1914 he investigated theoretically, and by spectrophotometric measurements, the radiative exchange and the broad *H*-and *K* lines (3933, 3968 Å)

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[51] The idea was later followed up by **A.S. Eddington** (1916).

of the solar spectrum. He clearly saw that the further prosecution of his undertakings required an *atomistic theory of absorption* coefficients, i.e. of the *interaction of radiation and matter*. He thus turned with enthusiasm to the quantum theory of atomic structure found by **N. Bohr** in 1913. This effort resulted in his famous works on the *quantum theory of the Stark effect* and on band spectra. Independently of **A. Sommerfeld**, he developed the general rules of quantization and initiated the quantum theory of molecular spectra.

In December 1915, one month after the publication of Einstein's series of four papers outlining GTR, Schwarzschild derived (1916) the first rigorous solution of Einstein's full gravitational field equations for the static isotropic field surrounding a spherical mass. In a second paper of that year, he gave the solution for the motion of a mass point in the gravitational field of an incompressible fluid sphere. It is there that the '*Schwarzschild radius*' is introduced for the first time.

Schwarzschild sent his paper to **Einstein** to transmit to the Berlin Academy. In his reply Einstein wrote: "*I had not expected that the exact solution to the problem could be formulated. Your analytical treatment of the problem appears to be splendid*".

Neither of them, nor anybody else at that time knew that Schwarzschild's solution contained a complete description of the external field of a spherically symmetric, electrically neutral nonrotating black hole. Today we refer to it as "*Schwarzschild's black hole*".

Schwarzschild was born in Frankfurt am Main and attended a Jewish school until the age of 11. His exceptional ability in science became evident at the age of 16, when his paper on the theory of celestial orbits was published. He was then educated at the University of München. In 1901 he became professor and director of the observatory at the University of Göttingen, and in 1909 was appointed director of the Astrophysical Observatory at Potsdam. In 1909 he married a non-Jewish woman, against the objections of both families.

He participated actively in WWI as a German soldier and died on May 11, 1916, following a short illness contracted at the Russian front. His son **Martin Schwarzschild** (1912–1997) fled Nazi Germany in 1935 and emigrated to the U.S. in 1937. He became a professor of astrophysics at Princeton in 1947 and made significant contributions to our knowledge of variable stars and stellar evolution.

1899–1917 CE **Georg Alexander Pick**[52] (1859–1942, Austria and Czechoslovakia). Mathematician. Contributed to linear algebra, invariant

[52] To dig deeper, see:

- Varberg, D.E., *Pick's Theorem Revisited*, Am. Math. Monthly, 1985, *92*, 584–587 pp.

theory, potential theory, functional analysis, differential geometry and discrete (reticular) geometry. Terms like 'Pick matrices', 'Pick–Nevanlinna interpolation', and the 'Schwarz–Pick lemma' are sometimes used today. But from a lifelong work covering a wide range of topics he is best remembered, however, for 'Pick's theorem' [53] (1899).

Georg Pick was born in Vienna into a Jewish family. He entered the University of Vienna (1875) and was awarded his doctorate there in mathematics (1880). He was then appointed as an assistant to **Ernest Mach**, at the Charles–Ferdinand University of Prague and later promoted to full professorship at the German University in Prague (1892)[54]. After his retirement (1927) he returned to Vienna, only to return to Prague after the Anschluss (1938).

- Funkenbusch W.W., *From Euler's Formula to Pick's Formula etc.*, Am. Math. Monthly, 1974, *81*, 647–648 pp.

- Bruckheimer, M. and A., Arcavi, *Farey Series and Pick's Area Theorem*, The Mathematical Intelligencer, 1995, *17*, 64–67 pp.

- Coxeter, H.S.M, *Introduction to Geometry*, John Wiley & Sons: New York, 1969, 469 pp.; 208–210 pp.

- Grünbaum, B. and G.C., Shepard, *Pick's Theorem*, Am. Math. Monthly, 1993, *100*, 150–161 pp.

[53] *Pick's theorem*: The area of any simple polygon whose vertices are lattice points, is given by the formula $\frac{1}{2}b + c - 1$, where b is the number of lattice points on the boundary while c is the number of lattice points inside (by 'simple' polygon we mean one whose sides do not cross each other). There is *no* analog of Pick's theorem in 3 dimensions that expresses the volume of a polytope by counting its interior and boundary points.
This theorem was brought to wide attention in 1969 through the popular *Mathematical Snapshots* by **H. Steinhaus**. It then attracted much attention due to its simplicity and elegance. Pick's theorem is linked to several other beautiful results like the celebrated *Euler's formula* and the basic property of the *Farey Series*.

[54] There is another aspect of Pick's life which merits attention: Pick was the driving force behind the appointment of **Albert Einstein** to the chair of mathematical physics at the German University of Prague (1911–1913). In 1911 Pick introduced Einstein to the tensor calculus of **Gregorio Ricci–Curbastro** and **Tullio Levi–Civita**, which later (1915) helped Einstein formulate General Relativity.
During the years the two were close friends who also shared passionate interest in music (in fact Pick's quartet consisted of four professors from the University).

In July 1942 the Nazis moved him to Theresienstadt and he died there two weeks later aged 82.

1899–1918 CE **Kurt Hensel** (1861–1941, Germany). Mathematician. First to introduce *p-adic numbers*[55] (1899) and p-adic analysis. In recent years, p-adic arithmetic and analysis has been recognized as an important, widely generalizable field of mathematics and mathematical physics.

Hensel was born in Königsberg, a descendant of a Jewish illustrious family[56]. He studied mathematics at Bonn and Berlin and came under the influence of **R. Lipschitz**, **K. Weierstrass**, **G. Kirchhoff**, **von Helmholtz** and especially **L. Kronecker**, under whose guidance he took his Ph.D. in 1884. In 1901, Hensel became full professor at the University of Marburg, where he remained for the next 40 years.

From 1901 he was editor of the influential *Crelle's Journal*. Hensel's scientific work is based upon Kronecker's arithmetical theory of algebraic number fields and the Weierstrass method of power series development for algebraic functions. This led Hensel in 1899 to the conception of an analogue in the theory of algebraic numbers: p-adic numbers[57] which he developed into a *sys-*

[55] Though they are fore-shadowed in the work of his predecessor E. Kummer (ca 1829).

[56] The four mathematicians: **P.G. Lejeune-Dirichlet**, **E.E. Kummer**, **H.A. Schwarz** and **K. Hensel** were all members of the Mendelssohn family in the following way: Dirichlet married Rebecca (1811–1858), a sister of the composer Felix Mendelssohn. Felix's other sister, Fanny (1805–1847) was the paternal grandmother of Kurt Hensel. The second wife of Kummer, Ottilie Mendelssohn (1819–1848), was a daughter of a cousin of Fanny. Their daughter, Marie Elisabeth Kummer (1842–1921), was married to H.A. Schwarz.

[57] *The Archimedean property of the real-number system*: If $x > 0$ and if y is an arbitrary real number, there exists a positive integer n such that $nx > y$.
To prove the theorem we first show that the set of positive integers in unbounded above, from which it follows that for every real x there exists a positive integer n such that $n > x$. In this theorem we replace x by $\frac{y}{x}$, which proves the Archimedean property (AP) (also known as *The theorem of Eudoxos*).
Geometrically, AP means that any line segment, no matter how long, may be covered by a *finite number* of line segments of a given positive length, no matter how small. In other words, a small ruler used often enough can measure arbitrarily large distances. Archimedes realized that this was a fundamental property of the straight line and stated it explicitly as one of the *axioms of geometry*.
In the 19^{th} and 20^{th} centuries, *non-Archimedean* geometries have been constructed in which this axiom is rejected.

tematic theory. His pupil, **Helmut Hasse** further developed p-adic analysis (1923), and applied it to the theory of algebras over number fields.

Non-Archimedean mathematics — the strange world of p-adic numbers

The sequence of rational numbers

$$\begin{array}{l} 1.414\ 213\ 5 \\ 1.414\ 213\ 56 \\ 1.414\ 213\ 562 \\ 1.414\ 213\ 562\ 3 \\ 1.414\ 213\ 562\ 37 \\ 1.414\ 213\ 562\ 373 \\ \ldots \end{array}$$

get ever closer to $\sqrt{2}$. *The central idea here is that we have a sequence of numbers whose members get closer and closer to each other – they constitute what is called a* Cauchy sequence *[i.e. upon picking a positive distance no matter how small, there always exists some member beyond which any two members will be within that distance of each other] and therefore* ought to *converge to something.*

Normally (i.e., with distance defined in the standard way on the real axis), if we want to know whether two rational numbers are close to each other, and we have their decimal expansions, we can start from the left*, comparing digits, and the further right you can get before you run into a discrepancy, the closer together the numbers are.*

Now suppose we choose to translate decimal numbers into the binary system, e.g.

$$\begin{aligned} 519 = \quad & 1\cdot 2^9 + 0\cdot 2^8 + 0\cdot 2^7 + 0\cdot 2^6 + 0\cdot 2^5 + 0\cdot 2^4 + 0\cdot 2^3 \\ & \qquad + 1\cdot 2^2 + 1\cdot 2^1 + 1\cdot 2^0 \\ 7 = \quad & \qquad 1\cdot 2^2 + 1\cdot 2^1 + 1\cdot 2^0 \end{aligned}$$

and write these in the 2-adic form

$$\begin{array}{lr} 100\ 000\ 011\ 1. & (519) \\ 000\ 000\ 011\ 1. & (7) \end{array}$$

If we agree to read these numbers from right to left, we may think of them as being relatively close to each other, differing only in a single digit 1 *in the tenth place. On the other hand, when applying the same thinking to*

$$\begin{array}{lr} 111. & (7) \\ 101. & (5), \end{array}$$

we may think of them as being rather far apart, since they differ already in the second digit from the right. We may therefore establish the 'rule' that two 2-adic numbers are 'close' if their difference is a multiple of some large power of 2*. This rule can be made more precise, as we will see, and thus be elevated into a new (so-called 2-adic) norm for rationals – an alternative to the standard definition of norms as absolute value.*

Carrying this logic one step further we can consider sequences of 2-adically expressed integers that are Cauchy sequences in the 2-adic (though not standard) sense, and thus – one feels – "ought" to converge in the 2-adic sense, for example

$$\begin{array}{rr} 111. & (7) \\ 1\ 111. & (15) \\ 11\ 111. & (31) \\ 111\ 111. & (63) \\ 1\ 111\ 111. & (127) \\ 11\ 111\ 111. & (255) \\ \cdots & \end{array}$$

This looks as if we claim that the number $\ldots 11\ 111\ 111.$ *tends to some limit. Of course, this looks a bit silly; obviously the sequence* $\{7, 15, 31, 63, 127, 255, \ldots, 2^k - 1, \ldots\}$ *marches straight off to infinity as measured by the standard norm.* ***Hensel*** *asserted that this sequence converges (2-adically) to the rational number* -1 *with the understanding that two numbers are close if their difference is a multiple of some large power of* 2*, and they get closer and closer as the power of* 2 *increases. Indeed,*

$$\begin{array}{rl} 63 = & (-1) + 2^6 \\ 255 = & (-1) + 2^8, \qquad \textit{etc.,} \end{array}$$

so these numbers are getting closer, converging, to -1 *such that we may write*

$$\ldots 111\ 111\ 111\ 111. = -1$$

We might then ask about $\ldots 101010101,$ *with its '10' pattern repeating off to the left. Multiplying this number by 3 (multiplying by 2 and adding the original) we get*

$$\begin{array}{rl} \ldots 1\ 010\ 101\ 010. & \\ \ldots\ \ 101\ 010\ 101. & \\ \hline \ldots 1\ 111\ 111\ 111. & = -1 \end{array}$$

leading to the results:

$$\ldots 101\ 010\ 101. \quad = -\tfrac{1}{3}$$

$$\ldots 010\ 101\ 010. \quad = -\tfrac{2}{3}$$

$$\ldots 010\ 101\ 011. \quad = 1 + 2 \cdot (-\tfrac{1}{3}) = \tfrac{1}{3}$$

Now, ordinary decimal (or digital in any other base) expansions of fractions repeating off to the right converge in the standard sense to rationals with denominators that are not powers of 10 *–* e.g.[58]*:*

$$0.\overline{142\ 857} \quad = \tfrac{1}{7}$$

$$0.\overline{09} \quad = \tfrac{1}{11}$$

In similar fashion, the binary expansions repeating off to the left converge 2-adically to rationals with denominators that are not powers of 2:

$$\overline{0\ 011}01. \quad = \tfrac{1}{5}$$

$$\overline{011}1. \quad = \tfrac{1}{7}$$

$$\overline{0\ 111\ 010\ 001}1. \quad = \tfrac{1}{11}$$

$$\overline{011\ 101\ 100\ 010}1. \quad = \tfrac{1}{13}$$

It can be shown that non-periodic binary expansions going off to the left do not 2-adically converge to rational numbers. Yet such expansion are Cauchy sequences under the 2-adic norm. So, the next logical development was to define *non-periodic 2-adic expansions to be* new *2-adic numbers*[59]*, just as irrational reals are augmented to the rationals using standard norm. These new*

[58] A bar over a contiguous of digits, means the group is to repeated *ad infinitum.*

[59] Formally, this procedure is referred to as the topological completion of the set $\mathbb{Q}$ of rationals under the new, 2-adic norm.

numbers, together with the rationals $\mathbb{Q}_1$ comprise the topologically complete algebraic field (under ordinary arithmetical operations) $\mathbb{Q}_2$ of 2-adic numbers. Proceeding similarly with any other prime base p creates the $\mathbb{Q}_p$ field of p-adic numbers. $\mathbb{Q}_p$ really is a field, since within it one may add, subtract, multiply and divide (except by 0).

Following this heuristic introduction we turn to a summary of the mathematical foundations of p-adic analysis. Since ***Euclid****'s time the three-dimensional Euclidean space (and its geometry) has been described by means of real numbers and treated as the physical space. Since the time of* ***Newton*** *and* ***Leibniz****, differential equations, integrals, and other limits involving the real (and complex) algebraic fields have been used in mathematical physics.*

Important extensions of this point of view have been introduced by ***Riemann****, and later by* ***Einstein*** *using the pseudo-Riemannian geometry, but locally $\mathbb{R}^3$ is still our mathematical model for space and $\mathbb{R}^4$ for space-time. It is not customary to discuss why exactly real numbers should be used, and why this happened. The point is that physical processes take place in space and time, and space-time coordinates are usually considered as real numbers – obtained from rationals via topological completion under the standard norm.*

In computations of everyday life, in scientific experiments and in computer based representation of numbers, one is dealing with integers and fractions, that is with rational numbers; irrational numbers — infinite nonperiodic decimal expressions — are not manipulated directly (only via formal procedures or rational approximations). Results of any practical action we can express only in terms of rational numbers. There exists, however, a generally accepted expectation that if we carry out measurements and calculations more and more precisely, then in principle we can measure and compute physical quantities out to any large number of decimal digits and interpret the result as a real number. This of course is an idealization[60].

Thus, let us take as our starting point the field $\mathbb{Q}$ of rational numbers. A geometric notion of distance *corresponds to topological notion of* norm *(and thus also a* metric *– i.e. a distance – defined as the norm of a difference) on $\mathbb{Q}$. A* norm *on a field is a real-valued function $|x|$ with the following properties:*

1) $|x| \geq 0, \qquad |x| = 0 \quad \textit{iff } x = 0$

2) $|xy| = |x||y|$

3) $|x + y| \leq |x| + |y|$

[60] In the case of purely mathematical manipulations – solving equations and the like – these expectations are rooted in rigorously-proven theorems.

The last property is the well-known triangle inequality, which – for the usual absolute-value norm on the real or rational fields – is a direct consequence of the definition of the absolute value[61]*.*

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[61] *Basic concepts:* consider a set X of elements. For each two elements (x, y) we form a real function $d(x, y)$ with the following properties:

(i) $d(x, y) \geq 0, \qquad d(x, y) = 0 \quad \text{iff } x = y;$

(ii) $d(x, y) = d(y, x);$

(iii) $d(x, y) \leq d(x, z) + d(z, y)$ for all z of X.

The function d is called a *distance* or *metric* on X. The set X, when endowed with a metric $d(\cdot, \cdot)$, is a *metric space* (X, d). The same set X can give rise to many different metric spaces (X, d). The set X will mostly be a *field* in this discussion of p-adics. [A field F is a set together with two operations $+$ and $\cdot$ such that F is a commutative group under $+$, $F - \{0\}$ is a commutative group under $\cdot$, and the distributive law holds.] The most common examples, apart from $\mathbb{Q}$, are the field of real numbers $\mathbb{R}$ and the field of complex numbers, $\mathbb{C}$. In the first case the standard distance between two points on the number line is the norm $d(x, y) = |x - y|$. In two (real) dimensions, the norm of the complex number $|a + ib| = \sqrt{a^2 + b^2}$ leads to the distance between two complex numbers $d(a + ib, c + id) = \sqrt{(a - c)^2 + (b - d)^2}$. Property (iii) is then the *actual triangular inequality* $d(x, y) \leq d(x, z) + d(z, y)$ with the geometrical interpretation that the sum of two sides of any triangle is never smaller than the third side.

The proof of 3) and thus iii) for the real line with the standard absolute-value norm is simple: adding the inequalities $-|x| \leq x \leq |x|$ and $-|y| \leq y \leq |y|$ we obtain $-[|x| + |y|] \leq x + y \leq |x| + |y|$.

But if $x + y > 0$, we have $|x + y| = x + y \leq |x| + |y|$;
whereas if $x + y \leq 0$, we have $|x + y| = -(x + y) \leq |x| + |y|$.

Hence we conclude that $|x + y| \leq |x| + |y|$ always.

Since a field possesses a unit element "1" (its identity when considered as a multiplicative group), 2) implies $|1| = |-1| = 1$ and thus also $|-x| \equiv |x|$; the latter implies (ii) for the metric $d(x, y) = |x - y|$.

Standard notation

$\mathbb{P}$ Set of prime numbers

$\mathbb{N}$ Set of natural numbers

$\mathbb{Z}$ Ring of integers

$\mathbb{Q}$ Field of rational numbers

$\mathbb{R}$ Field of real numbers

Are there any other norms on $\mathbb{Q}$ besides the standard absolute-value one? There is a remarkable theorem by **Ostrowski** *(1921) to the effect that apart from the standard norm, there is but one other nontrivial way to construct a metric (norm) on the field of rational numbers — or, more precisely, one such way per prime number. This norm is called non-Archimedean, and is based on a non-Archimedean absolute value with the properties* 1)–3) *and with the additional requirement that*

4) $|x+y| \leq \max\{|x|,|y|\}$ *for all* x, y.

This condition implies 3), *since* $\max\{|x|,|y|\}$ *is certainly smaller than the sum* $|x|+|y|$.

To understand the new, non-standard, p-adic $\mathbb{Q}$*-norms, we begin with a basic property of all rational numbers: Let* p *be any prime number* $(p = 2, 3, 5, 7, 11, \ldots)$. *Then any rational number* x *can be uniquely represented in the form*

$$x = p^{\nu} \frac{a}{b},$$

where ν *is an integer and* a, b *are integers not divisible by* p, *i.e.* ν *is the highest positive, or lowest negative power of* p *which divides* x. *We denote this number as*

$$\nu = \mathrm{ord}_p x \qquad \text{or} \qquad \nu_p(x) = p\text{-adic } \textit{valuation} \text{ on } \mathbb{Q},$$

and by definition,

$$x = p^{\nu_p(x)} \frac{a}{b}, \qquad p \not| a, b.$$

For example:

$$\begin{array}{rll}
35 = & 5 \cdot 7 \cdot 1 & \nu_5(35) = 1 \\
250 = & 5^3 \cdot 2 & \nu_5(250) = 3 \\
96 = & 2^5 \cdot 3 & \nu_2(96) = 5 \\
97 = & 2^0 \cdot 97 & \nu_2(97) = 0 \\
\frac{20}{3} = & 2^2 \cdot \frac{5}{3} & \nu_2(\frac{20}{3}) = 2 \\
1 = & 2^0 \cdot 1 & \nu_2(1) = 0 \\
0 = & p^{\infty} \cdot 0 \ (p\text{-adically}) & \nu_p(0) = +\infty \ (\text{formally!})
\end{array}$$

$\mathbb{C}$ Field of complex numbers

$\mathbb{R}^n$ n-dimensional Euclidean space

$\mathbb{Q}_p$ Field of p-adic numbers

$\mathbb{Z}_p$ Ring of p-adic integers

The basic properties of the p-adic valuation are readily proven to be:

(1) $\nu_p(xy) = \nu_p(x) + \nu_p(y)$;

(2) $\nu_p(x + y) \geq \min\{\nu_p(x), \nu_p(y)\}$

with the convention $\nu_p(0) = +\infty$. *We note that* ν_p *behaves somewhat like a logarithm would. If we compare the above two properties with the conditions 2) and 4) in the definition of the non-Archimedean norm, we see that they are very similar, except that the product in 2) has been turned into a sum (as when taking a logarithm), and that the inequality in 4) has been reversed. We can "un–reverse" the inequality by changing the sign, and then turn the sum into a product by putting it into an exponent. This suggests the following realization of the non-Archimedean norm (otherwise known as the p-adic norm):*

$$|x|_p = \begin{cases} \frac{1}{p^{\nu_p(x)}}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

Examples:

$6 = 3^1 \cdot 2$;	$\nu_3(6) = 1$;	$\lvert 6\rvert_3 = \frac{1}{3^1} = \frac{1}{3}$
$15 = 3^1 \cdot 5$;	$\nu_3(15) = 1$;	$\lvert 15\rvert_3 = \frac{1}{3}$
$137 = 2^0 \cdot 137$;	$\nu_2(137) = 0$;	$\lvert 137\rvert_2 = 1$

Since for $x = \frac{a}{b}$, $\nu_p(x) = \nu_p(a) - \nu_p(b)$, *we have for example:*

$$\frac{1}{4} = \frac{2^0}{2^2}; \qquad \nu_2(\frac{1}{4}) = \nu_2(1) - \nu_2(4) = 0 - 2 = -2$$

$$|\frac{1}{4}|_2 = \frac{1}{2^{-2}} = 4; \qquad |\frac{1}{2^n}|_2 = 2^n.$$

Thus, for a given prime p, two rational numbers are considered to be p-adically close if their difference is divisible by a large power of p.

The p-adic norm possesses the characteristic properties 1) through 4) above. Geometrically, the p-adic norm provides us with a notion of "size": we can use it to measure distances between numbers, i.e. to put a metric on our field. Having the metric, we can define open and closed sets and in general investigate the topology of the field $\mathbb{Q}_p$. *On other hand,* $|\cdot|_p$ *measures the degree of p-divisibility of any rational number; hence the importance of the p-adic norm concept in both number theory and topology. The p-adic metric is non-Archimedean, – meaning an interval cannot be segmented into shorter ones; and this metric also leads to rather strange topological properties of plane figures and 3-dimensional objects, in* $\mathbb{Q}_p$, *such as:*

- *all triangles are isosceles;*
- *any point in a disc is its center;*
- *any two balls are either disjoint or contained one in another (like two drops of mercury!)*

Thus the geometry of the topological space $\mathbb{Q}_p$ (the field obtained by topological completion of $\mathbb{Q}$ under the norm $|\cdot|_p$) is surprisingly unlike the geometry of the field $\mathbb{R}$ (reals) obtained by topological completion of $\mathbb{Q}$ under the standard *norm. That is a consequence of the stricter version* 4) *of the third property of the norm.*

p-ADIC NUMBERS AND ARITHMETIC

Any standard-norm real number x can be expanded in an infinite decimal representation

$$\begin{aligned} x &= \pm 10^{\nu}[x_0 + x_1\tfrac{1}{10} + x_2\tfrac{1}{10^2} + \dots] \\ &= \pm 10^{\nu}\sum_0^{\infty} x_n 10^{-n} \qquad , \qquad x_0 \neq 0, \quad x_j = 0, 1, \dots, 9 \end{aligned}$$

where ν is the highest power of 10 such that $10^{\nu} \leq |x| < 10^{\nu+1}$. In general, an infinite sequence a_n of digits $[0, \dots, p-1]$ (p prime) can represent an element of two distinct additive groups:

- *a real fractional number in the interval $[0, 1]$ expressed to base p*
- *a formal power series* $a_0 + a_1x + a_2x^2 + \dots + a_nx^n + \dots$

The latter polynomial form a ring, in which the two group operations are formal power-series addition and multiplication modulo *p.*

***Hensel** (1908) introduced a* third option *based upon the observations:*

(1) *Any ordinary integer D can be uniquely expressed as a finite sum of powers of a prime p. That is*

$$D = d_0 + d_1p + \dots + d_kp^k$$

in which d_i *is some integer from* 0 *to* $p-1$ *[e.g.* $14 = 2 \cdot 3^0 + 1 \cdot 3^1 + 1 \cdot 3^2$*;* $216 = 2 \cdot 3^3 + 2 \cdot 3^4$*].*

(2) *Any nonzero* rational number x *(not* 0*) can be written in the form*

$$x = p^{\nu} \frac{a}{b}, \qquad p \nmid a, b; \qquad\qquad \nu = \text{ integer}$$

where the integer ν *is the above defined* p*-adic valuation.*

Hensel then generalized from these two observations and introduced the Hensel series, known as p*-adic numbers*

$$x = \sum_{i=-\rho}^{\infty} c_i p^i$$

where ρ *is an integer,* $c_{-\rho} \neq 0$*,* p *is a prime and the coefficients, the* c_i*, are ordinary rational numbers reduced to their lowest form whose denominator and numerator are not divisible by* p*. Such expressions need not in general have values as ordinary rational numbers– nor are they in general real numbers (for the series is as likely as not to* diverge *in the standard norm). However, Hensel's* p*-adic numbers do converge under the* p*-adic norm. We can regard the Hensel series as analogous to the decimal (or other basis) representations of real numbers, and rewrite it as*

$$x = p^{\nu} \sum_{0 \leq n < \infty} x_n p^n$$

where x_n *are integers* $0 \leq x_n \leq p-1$*. If* $x_0 \neq 0$*, then the representation is unique and since the* p*-adic norm of each partial sum is* $p^{-\nu}$ *(again assuming* $x_0 \neq 0$*), we may extend the* p*-adic valuation (2) to all* p*-adic numbers* $x \in \mathbb{Q}_p$ *whether rational or not:*

$$\left| p^{\nu} \sum_{n=0}^{\infty} x_n p^n \right|_p = p^{-\nu}.$$

Hensel's series converges *with respect to the norm* $|x|_p$ *because one has*

$$|p^{\nu} x_n p^n|_p = p^{-\nu-n}, \quad n = 0, 1, \ldots$$

The above p*-adic number representation means that any* p*-adic number – i.e. any* x *in* $\mathbb{Q}_p$ *– is a limit (w.r.t. the* p*-adic norm) of a sequence* $\{x^{(n)}, n \to \infty\}$ *of rational numbers*

$$x^{(n)} = p^{\nu}(x_0 + x_1 p + \cdots + x_n p^n).$$

According to this scheme, any rational number, including negative numbers, can be represented in the p-adic form. For example: (subscript indicate the prime p used)

$$
\begin{aligned}
-1 \quad &= (p-1)+(p-1)p+(p-1)p^2+\dots \\
\left(\tfrac{1}{2}\right)_7 \quad &= \tfrac{8-7}{2} = 4-\tfrac{1}{2}\cdot 7 = 4+\tfrac{7\cdot 3}{1-7} \\
&= 4+3(7+7^2+\dots) = 4+3\cdot 7+3\cdot 7^2+\dots \\
\left(\tfrac{1}{24}\right)_3 \quad &= \tfrac{2}{3}+\tfrac{2+3}{1-9} = 2\cdot\tfrac{1}{3}+(2+3)[1+3^2+3^4+\dots] \\
&= 2\cdot\tfrac{1}{3}+2+3+2\cdot 3^2+3^3+\dots \\
(-5)_3 \quad &= 4-9 = 4+\tfrac{18}{1-3} = 4+2\cdot 3^2+2\cdot 3^3+\dots \\
\left(\tfrac{24}{17}\right)_3 \quad &= \tfrac{2\cdot 3+2\cdot 3^2}{2+2\cdot 3+3^2} = 3+3^3+2\cdot 3^5+3^7+3^8+2\cdot 3^9 \\
\left(-\tfrac{1}{2}\right)_3 \quad &= \tfrac{1}{1-3} = 1+3+3^2+3^3+\dots
\end{aligned}
$$

This last example can also be derived thus[62]*:*

[62] Another example serves to show that sometimes p-adics allow a more conceptual proof of a fact that seems obscure and hard to prove otherwise. Consider the usual Taylor series $\ln(1+x) = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\dots$. Working in the field $\mathbb{Q}_2$ of 2-adic numbers, we know that powers of 2 are "small". We then put $x=-2$ to compute the natural logarithm of -1: $\ln(-1) = \ln(1-2) = -(2+\frac{2^2}{2}+\frac{2^3}{3}+\frac{2^4}{4}+\dots)$. This is of course divergent in $\mathbb{R}$, but it turns out to be convergent in $\mathbb{Q}_2$. Now, if the series converges, it must converge to zero, by the property of the logarithm:

$$2\ln(-1) = \ln(-1)^2 = \ln(1) = 0$$

This last does not hold in $\mathbb{R}$, because $ln(-1)$ (like $\sqrt{-1}$) is not in $\mathbb{R}$ but rather in $\mathbb{C}$, and is multi-valued. But under $|\cdot|_2$, $ln(-1)$ *is* in $\mathbb{Q}_p$ and is unique – and in fact it vanishes ($\sqrt{-1}$ does not exist in $\mathbb{Q}_p$, though).

We conclude that the partial sums

$$2+\frac{2^2}{2}+\frac{2^3}{3}+\frac{2^4}{4}+\dots+\frac{2^n}{n}$$

must tend (2-adically) to zero as n grows. What this means is that the terms in the 2-adic expansion "disappear to the right", – that is to say: the partial

Consider the equation $X = 1 + 3X$. This is of course easy to solve, but let us look at it as a fixed-point problem, i.e., as the problem of finding a solution for $f(x) = x$ for some function $f(x)$. Such problems are often solved by iteration, plugging in an arbitrary initial value, then computing $f(x)$ repeatedly in the hope of converging to a fixed point. To try that in our case, we take $x_0 = 1$ and iterate, so that $x_{n+1} = 1 + 3x_n$. We get:

$$\begin{aligned} x_0 &= 1 \\ x_1 &= 1 + 3x_0 = 1 + 3 \\ x_2 &= 1 + 3x_1 = 1 + 3 + 3^2 \\ &\dots \\ x_n &= 1 + 3 + 3^2 + ... + 3^n. \end{aligned}$$

The sum of the p-adic numbers

$$x = p^{\nu(x)}(x_0 + x_1 p + x_2 p^2 + \dots), \qquad 0 \le x_j \le p - 1, \qquad x_0 > 0$$

and

$$y = p^{\nu(y)}(y_0 + y_1 p + y_2 p^2 + \dots), \qquad 0 \le y_j \le p - 1, \qquad y_0 > 0$$

is again represented in the canonical form

$$x+y = p^{\nu(x+y)}(c_0+c_1 p+c_2 p^2+\dots), \qquad 0 \le c_j \le p-1, \qquad c_0 > 0,$$

where the numbers $\nu(x + y)$ and c_j are uniquely determined from the equation

$$p^{\nu(x)}[x_0 + x_1 p + \dots] + p^{\nu(y)}[y_0 + y_1 p + \dots] = p^{\nu(x+y)}[c_0 + c_1 p + \dots]$$

by the method of indefinite coefficients modulo p.

Thus, addition, subtraction, multiplication and division of p-adic numbers is carried out as for power series. In practice, the mechanics of these operations

---

sums, written in base 2, begin with longer and longer stretches of zeros. The translation of this into $\mathbb{R}$ results in the theorem:

For each integer $M > 0$ there exists an n such that the partial sum

$$2 + \frac{2^2}{2} + \frac{2^3}{3} + \frac{2^4}{4} + \dots + \frac{2^n}{n}$$

is divisible by 2^M.

What this example points to is that using p-adic methods, and in particular the methods of the calculus in the p-adic context, we can often prove facts about divisibility by powers of p which are otherwise quite hard to understand.

is therefore very much like the corresponding operations on decimals – with the exception that "carrying", "borrowing", "long multiplication", etc. go from left to right rather than from right to left. For example:

$$
\begin{array}{rrrl}
 & 3+ & 6\cdot 7+ & 2\cdot 7^2+\dots \\
\times & 4+ & 5\cdot 7+ & 1\cdot 7^2+\dots \\
\hline
 & 5+ & 4\cdot 7+ & 4\cdot 7^2+\dots \\
 & & 1\cdot 7+ & 4\cdot 7^2+\dots \\
 & & & 3\cdot 7^2+\dots \\
\hline
 & 5+ & 5\cdot 7+ & 4\cdot 7^2+\dots
\end{array}
$$

$$\frac{1+2\cdot 7+4\cdot 7^2+\dots}{3+5\cdot 7+1\cdot 7^2+\dots} = 5+1\cdot 7+6\cdot 7^2+\dots$$

Thus, in contradistinction to operations with power series *(no carrying) and* real numbers *(carrying proceeds leftward), carrying is done to the* right. *This fact explains why small perturbations can change every digit in the real case but not in power series and not in Hensel series – where one cannot disturb digits lying before those that are changed.*

Extracting roots *in p-adic arithmetic is interesting: Let us try to extract* $\sqrt{6}$ *in* $\mathbb{Q}_5$*, i.e. we want to find* $\{a_0, a_1, a_2, \dots\}, \quad 0 \le a_i \le 4$ *such that*

$$a^2 = [a_0 + a_1\cdot 5 + a_2\cdot 5^2 + \dots]^2 = 1 + 1\cdot 5.$$

Comparing coefficients of $1 = 5^0$ *on both sides gives* $a_0^2 \equiv 1 \pmod 5$*, which admits the two solutions* $a_0 = 1, 4$*. Let's select* $a_0 = 1$*. Then, comparing coefficients of 5 on both sides gives* $2a_1 \times 5 \equiv 1 \times 5 \pmod{5^2}$*, so that* $2a_1 \equiv 1 \pmod 5$*, and hence* $a_1 = 3$*. At the next step we have:*

$$1 + 1\cdot 5 \equiv (1 + 3\cdot 5 + a_2\cdot 5^2)^2 = 1 + 1\cdot 5 + 2a_2\cdot 5^2 \pmod{5^3}.$$

Hence $2a_2 \equiv 0 \pmod 5$*, and* $a_2 = 0$*. Proceeding in this way, we obtain a series*

$$a = 1 + 3\cdot 5 + 0\cdot 5^2 + 4\cdot 5^3 + a_4\cdot 5^4 + a_5\cdot 5^5 + \dots$$

where each a_i *after* a_0 *is uniquely determined. If we had chosen 4 instead of 1 for* a_0 *we would have obtained*

$$a = 4 + 1\cdot 5 + 4\cdot 5^2 + 0\cdot 5^3 + (4 - a_4)5^4 + (4 - a_5)5^5 + \dots$$

The fact that we had two choices for a_0*, and then, once we chose* a_0*, only a single possibility for* a_1*,* a_2*,* $a_3, \dots$*, merely reflects the fact that a nonzero element in any field – such as* $\mathbb{Q}$ *or* $\mathbb{R}$ *or* $\mathbb{Q}_p$ *– always has exactly two square roots in the field if it has any.*

Do all numbers in $\mathbb{Q}_5$ have square roots? We saw that 6 does; what about 7? If we had

$$(a_0 + a_1 \times 5 + \dots)^2 = 7 = 2 + 1 \cdot 5,$$

it would follow that $a_0^2 \equiv 2 \pmod 5$. But this is impossible, as we see by checking the possible values $a_0 = 0, 1, 2, 3, 4$. Thus $(\sqrt{7})_5$ does not exist (as $\sqrt{-1}$ does not exist in $\mathbb{R}$!).

The method, displayed above, of solving the equation $x^2 - 6 = 0$ in $\mathbb{Q}_5$ — by solving the congruence $a_0^2 - 6 \equiv 0 \pmod 5$ and then solving for the remaining a_i in a step-by-step algorithm — is actually quite general[63].

The next table lists some 2-adic square and cube roots that exist in $\mathbb{Q}_p$ out to 32 bits, expressed as decimals (base 10) integers.

Note that the $\mathbb{Q}_2$ square root of (-7), for instance, has nothing to do with the square root of (-7) that lives in $\mathbb{C}$, namely $0 + i2.645\ 751\ 311\ 06\dots$. Thus $\mathbb{R}$, $\mathbb{C}$ and $\mathbb{Q}_p$ are distinct topological spaces that all happen to be fields and all happen to contain $\mathbb{Q}$ (the field of rationals) as a subset. But the question remains — can one adjoin to $\mathbb{Q}_p$ roots that do not exist within it, in the same way that one adjoins $\sqrt{-1}$ to the real numbers?

In the standard norm one can adjoin a single root *of $x^2 + 1 = 0$ to the field $\mathbb{R}$ and get an entity that is both* algebraically closed *(i.e. has all roots of all polynomials with coefficients in that field) and* topologically complete *(i.e. all sequences that ought to converge – i.e. are* Cauchy sequences *– do indeed converge). For the p-adics, on the other hand, one has to adjoin an infinite number of roots and then take* another topological completion *to fill the gaps. But the resulting space is, again algebraically incomplete! The procedure can be iterated, but never actually terminates.*

---

63 *Hensel's lemma* (also known as the p-adic Newton's method of finding a real root of a polynomial):
Let $F(x) = c_0 + c_1x + \dots + c_nx^n$ be a polynomial whose coefficients are p-adic integers (that is, elements $x \in \mathbb{Q}_p$ such that $|x|_p \leq 1$, so that x is the p-adic-norm of a sequence of integers $x_n = \sum_{k=0}^{n} a_k p^k$, with a_k integers and $0 \leq a_k \leq p-1$. Any ordinary integer is also a p-adic integer). Let $F'(x) = c_1 + 2c_2x + 3c_3x^2 + \dots + nc_nx^{n-1}$ be the derivative of $F(x)$. Let a_0 be an integer $0 \leq a_0 \leq p-1$ such that $F(a_0) \equiv 0 \pmod p$ and $F'(a_0) \not\equiv 0 \pmod p$. Then there exists a unique p-adic integer a such that

$$F(a) = 0 \quad \text{and} \quad a \equiv a_0 \pmod p.$$

Conversely, if a_0 does *not* exist neither does a.
In the special case treated above, we had
$p = 5, \quad F(x) = x^2 - 6 \quad F'(x) = 2x, \quad a_0 = 1.$
The ring (not field!) of p-adic integers, is denoted $\mathbb{Z}_p$.

Table 5.1: 2-ADIC SQUARE AND CUBE ROOTS

x	$\sqrt{x} \pmod{2^{32}}$
−103	900474053
−95	280134481
−87	667878003
−79	188523961
−71	539081963
−63	708165089
−55	162937693
−47	593439177
−39	884702309
−31	597776241
−23	203854803
−15	34716455
−7	479772853
1	1
9	3
17	869476073
25	5
33	633169809
41	13395661
49	7
57	176171605
65	766918177
73	208539805
81	9
89	662884443
97	1052232783
105	638489235
113	75016423

x	$\sqrt[3]{x} \pmod{2^{32}}$
−41	2012075911
−39	840847113
−37	−1681700029
−35	−664015323
−33	−1677128033
−31	903300769
−29	−1627424485
−27	−3
−25	−502320585
−23	1013543353
−21	1982165363
−19	−1098587307
−17	236600911
−15	−1369322415
−13	1526247499
−11	207361069
−9	−259969305
−7	−1885318551
−5	1690153379
−3	819859077
−1	−1
1	1
3	−819859077
5	−1690153379
7	1885318551
9	259969305
11	−207361069
13	−1526247499
15	1369322415
17	−236600911
19	1098587307
21	−1982165363
23	−1013543353
25	502320585
27	3
29	1627424485
31	−903300769
33	1677128033
35	664015323
37	1681700029
39	−840847113
41	−2012075911

In general, the p-adic expansion of a in $\mathbb{Q}_p$ terminates (i.e. $a_i = 0$ for all i greater than some N) iff a is a positive rational number whose reduced–form denominator is a power p. Also, the p-adic expansion of a in $\mathbb{Q}_p$ has repeating digits from some point on iff a is rational.

Hensel defined the four basic arithmetical operations with the p-adic numbers in $\mathbb{Q}_p$ and showed that it is a field. A subset of the p-adic numbers can be put into one-to-one correspondence with the ordinary rational numbers, and in fact this subset is isomorphic to the rational numbers in the full sense of an isomorphism between two fields. In the field of p-adic numbers, Hensel defined units, integral p-adic numbers, and other notions analogous to those of the ordinary rational numbers.

By introducing polynomials whose coefficients are p-adic numbers, Hensel was able to speak of p-adic roots of polynomial equations and extend to these roots all of the concepts of algebraic number fields. Thus there are p-adic integral algebraic numbers and more general p-adic algebraic numbers, and one can form fields of p-adic algebraic numbers that are extensions of the "rational" p-adic numbers. In fact, all of the ordinary theory of algebraic numbers is carried over to p-adic numbers. Surprisingly perhaps, the theory of p-adic algebraic numbers leads to results on ordinary algebraic numbers. It has also been useful in treating quadratic forms and has led to the notion of valuation fields.

The p-adic numbers can be regarded as a completion of the rational numbers in a different way than the usual completion which leads to the real numbers. Over the last century the p-adic universe became the meeting point of algebra and analysis; p-adic numbers and p-adic analysis have come to play a central role in modern number theory. This importance results from the fact that they afford a natural and powerful language for talking about congruences between integers.

More recently, p-adic numbers have turned up in other areas of modern mathematics (algebraic geometry, representation theory) and even in physics. There is a natural scale–hierarchical and fractal-like structure in the field of p-adic numbers; and the complications of the interplay between algebraic and topological completions, and between discreteness and continuum, for those found in standard real and complex analysis.

Therefore p-adic analysis and non-Archimedean geometry might find a use the description of quantum–gravity spacetime geometry at small distances (of order of the Planck scale); indeed, there have been attempts to apply p-adics to string theories. They have also been applied to study the behavior of complicated systems such as spin-glasses, as well as chaos, in the framework of traditional theoretical and mathematical physics.

1899–1929 CE **Edmund (Georg Hermann) Landau** (1877–1938, Germany). A leading pure[64] mathematician of his time. Was educated in Berlin and was appointed professor of mathematics at Göttingen (1909–1933), following the death of Minkowski.

Apart from important work on Lambert and Dirichlet series[65] (1899–1906) and the theory of functions, his main interest was in analytic number theory, especially the distribution of prime numbers and prime ideals. In his books: "*Handbuch der Lehre von der Verteilung der Primzahlen*"[66] (1909) and "*Vorlesungen über die Zahlentheorie*" (1927), he presented for the first time a

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64 Landau had an absolute contempt for applied mathematics.

65 Landau showed that the function

$$\xi(s) = \left(\frac{4}{\pi}\right)^{\frac{1+s}{2}} \Gamma\left(\frac{s+1}{2}\right) L(s)$$

where

$$L(s) = \frac{1}{1^s} - \frac{1}{3^s} + \frac{1}{5^s} - \cdots = \sum_{n=0}^{\infty} (-)^n \frac{1}{(2n+1)^s}$$

obeys the functional equation $\xi(s) = \xi(1-s)$. The proof hinges on the relations

$$L(1-s) = \left(\frac{2}{\pi}\right)^{s} \Gamma(s) \sin\left(\frac{\pi s}{2}\right) L(s)$$

$$\Gamma\left(\frac{s}{2}\right) \Gamma\left(1-\frac{s}{2}\right) \sin\left(\frac{\pi s}{2}\right) = \pi$$

$$\sqrt{\pi}\Gamma(s) = 2^{s-1}\Gamma\left(\frac{s}{2}\right) \Gamma\left(\frac{1+s}{2}\right)$$

66 In 1909 Landau introduced the $\{o, O\}$ notation: the notation $f(x) = O\{g(x)\}$ as $x \to \infty$ means that there is a constant K and a value x_0 of x such that $|f(x)| < Kg(x)$ whenever $x \geq x_0$, i.e. the modulus of f *grows no faster* than a constant times g as $x \to \infty$.
The notation $f(x) = o\{g(x)\}$ as $x \to \infty$, means that for every $\epsilon > 0$ there is a value x_0 of x such that $|f(x)| < \epsilon g(x)$ whenever $x \geq x_0$. In words: the modulus of f *grows more slowly* than g as $x \to \infty$.
There are various obvious extensions of this notation, such as $f(x) = O\{g(x)\}$ as $x \to 0$, which means that there exist K, x_0 such that $|f(x)| < Kg(x)$ whenever $|x| < x_0$, or $f(x) = F(x) + O\{g(x)\}$ as $x \to \infty$, which means $f(x) - F(x) = O\{g(x)\}$ as $x \to \infty$, etc. Thus the function $2x + 3x^2$ is $O(x)$ when $x \to 0$, but $O(x^2)$ when $x \to \infty$. If x is an infinitesimal quantity, then $x^2 = o(x)$, $1 - \cos x = o(x)$. If x is an infinitely large quantity, then $x = o(x^2)$.

systematic account of the analytic number theory. These books stimulated further research in the field.

During the 1920's he was the center of a fantastic flowering in the analytic theory of numbers. In lectures, as in books, his ideal was absolute rigor and completeness. The assistant was instructed to interrupt if the professor omitted anything at all. Standing before the big blackboards in the lecture halls of the new building, Landau wrote rapidly — theorem, proof, theorem, proof — while a menial with a sponge hastened after him to erase what he had written so that there would be room for him to write more. He never gave any explanation of where he was going, but he had organized his material so well that there was a quality of incredible clarity about his lectures.

Landau was a full member in most European academies. From 1927 to 1928 he was a visiting professor at the Hebrew University in Jerusalem. It was hoped that he would head the Institute of Mathematics there, but he declined and returned to Göttingen, only to be forced to resign his chair by the Nazi regime in 1933.

Landau continued to lecture, but when he announced a course in calculus, an unruly mob prevented his entering the lecture hall. Hardy arranged for him to deliver a series of lectures in England, but he returned to Germany, being tied to his native land by the fact of his wealth and possessions. He died heartbroken in 1938.

Landau was married to Maria Ehrlich, daughter of **Paul Ehrlich** (1854–1915) [discovered the "magic bullet" treatment for syphilis]. His father Leopold (1848–1920) was a known professor of gynecology at the University of Berlin, and a descendant of the famous Talmudic scholar **Ezekiel Landau** (1713–1793).

Does a Cicada 'Know' that 17 is a Fermat-Prime?

History shows us that mathematics has, to a great extent, been developed by pure mathematicians who were under the spell of the beauty of it; who were entranced by its mysterious generality and who spent their lives discovering new facts and relations in this very wide and wondrous domain. Whether *their results could be applied to astronomy, physics, chemistry, biology or technology was not, as a rule, their primary concern.*

However, the state of the art of the Natural sciences has today reached a stage where ideas, methods and results, belonging even to such an arcane branch of mathematics as advanced Number Theory can be applied with great success. The following examples are instructive:

- *In X-ray crystallography of cubic crystals it had been noted that certain reflections are absent. These correspond to the integers* $n = 7, 15, 23, 28, \ldots$, *all of the form*

$$n = 4^k(8m+7) \qquad\qquad k, m = \text{ positive integers.}$$

Number theory tells us that these are numbers which cannot be represented as the sum of three or less squares. Clearly, this fact is closely related to the 3 dimensions of our Euclidean space.

- *The theory of potentials inside a crystal lattice (e.g., the well-known Madelung constant) can best be derived with the aid of the Jacobi elliptic theta-functions which at the same time form the basis of the analytical derivation in Number Theory of the representation of an integer as a sum of squares.*

- *Several aspects of Planck's radiation formula and of the Bose-Einstein and Fermi statistics are closely related to Riemann's zeta-function* $\zeta(s)$ *defined by*

$$\zeta(s) = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots = \frac{1}{\Gamma(s)}\int_0^\infty \frac{x^{s-1}}{e^x - 1}\,dx \qquad (\text{Re } s > 1)$$

and which is at the base of practically all arithmetical investigations on the distribution of the prime numbers[67].

- *The partition function* $p(n)$ *enumerates the number of ways in which the integer* n *can be written as a sum of any positive integers, repetition being allowed. It is defined by the generating function*

$$\prod_{k=1}^{\infty} \frac{1}{1-x^k} = 1 + \sum p(n)x^n \qquad (|x| > 1)$$

e.g.

$$\begin{aligned} 6 \quad &= 5+1 = 4+2 = 4+1+1 = 3+3 = 3+2+1 \\ &= 3+1+1+1 = 2+2+2 = 2+2+1+1 \\ &= 2+1+1+1+1 = 1+1+1+1+1+1, \end{aligned}$$

and therefore $p(6) = 11$.

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67 During WWII, even purely mathematical researches on the properties of the ζ-function were classified

Application of this partition function and its asymptotic behavior are now being made in different parts of physics, such as the theory of crystal growth.

- *Kelvin's expression for the capacitance* $C_{a,b}$ *of two mutually external spheres of radii* a *and* b *respectively, with their centers a distance* c *apart, can be written in the form*

$$C_{a,b} = \frac{EI}{c}\sum_{n=1}^{\infty}\left\{d(n) - d\left(\frac{n}{2}\right)\right\}\alpha^n, \qquad \alpha = \frac{E-I}{E+I}$$

where E *and* I *are the length of the external and internal tangents, respectively, to the circles obtained by cutting the spheres with a plane through their centers. The symbol* $d(n)$ *stands for the number of divisors of* n *[e.g.* $d(6) = 4$ *since the four divisors are* 1, 2, 3, 6*]. Here* $d\left(\frac{n}{2}\right) = 0$ *for any odd* n*.*

- *The "sawtooth" function* $\mathrm{Sa}(x) = [x] - x + \frac{1}{2}$ *where* $[x]$ *is the "staircase" function, equal to the integer part of* x *(i.e.* $[x] = n$ *for* $n \le x < n+1$*). This function of time is used in any television transmitter and receiver in order to scan periodically the lines and frames of the picture. As it turns out,* $\mathrm{Sa}(x)$*, used so extensively in Radio technology, is a most fundamental function in Number Theory. Some of the properties of* $\mathrm{Sa}(x)$ *are:*

(*i*) $$\frac{1}{s}\zeta(s) = \int_0^{\infty}\frac{\mathrm{Sa}(u)}{u^{s+1}}\,du \qquad -1 < \mathrm{Re}\ s < 0$$

(*ii*) $$\mathrm{Sa}(x) = \sum_{k=1}^{\infty}\frac{\sin 2\pi kx}{\pi k}$$

(*iii*) $$\sum_{k=1}^{n}\mathrm{Sa}(x - \frac{k}{n}) = \mathrm{Sa}(nx)$$

This means that a superposition of n *sawtooth functions, each one shifted over a phase difference of* $\frac{1}{n}$ *w.r.t. the preceding one, yields a further sawtooth function, but of* n *times the frequency.*

(*iv*) $$\mathrm{Sa}(x) - \mathrm{Sa}(x - \frac{1}{2}) = \frac{1}{2}\,\mathsf{S}in(2\pi x),$$

where S*in represents the 'square-sine' function which jumps from* -1 *to* $+1$ *at* $x = \ldots, -1, 0, 1, 2, 3, \ldots$ *and from* $+1$ *to* -1 *at* $x = \ldots, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$ *whereas* *it stays constant at 1 or −1 in the intervals between the jumps. It can be also written in the forms*

$$\mathfrak{S}in(2\pi x) = \frac{\sin 2\pi x}{|\sin 2\pi x|} = (-1)^{[2x]}$$
$$= 4\mathrm{Sa}(x) - 2\mathrm{Sa}(2x)$$

It can be shown that Sa(x) *can be expanded in a series of square-sine functions of frequency* 2^n

$$(v) \qquad \mathrm{Sa}(x) = \sum_{n=1}^{\infty} \frac{1}{2^{n+1}} \mathfrak{S}in(2^{n-1} \cdot 2\pi x) = \sum_{n=1}^{\infty} \frac{1}{2^{n+1}} (-1)^{[2^n x]}$$

In contradistinction to the Fourier synthesis, the square-sine synthesis has the clear advantage that the so-called Gibbs' phenomenon is absent here! (no 'overshoot' at the discontinuities).

$$(vi) \qquad \begin{aligned} \mathrm{Sa}(\mathrm{Sa}(x)) &= -\mathrm{Sa}(x + \tfrac{1}{2}) \\ \mathrm{Sa}(\mathrm{Sa}(\mathrm{Sa}(x))) &= \mathrm{Sa}(x) \end{aligned}$$

Thus, any odd number of iteration always reproduces the original function. ***Edmund Landau*** *(1927) discovered the relation*

$$(vii) \qquad \int_0^1 \mathrm{Sa}(mx) \cdot \mathrm{Sa}(nx)\, dx = \frac{1}{12} \frac{(m,n)}{\{m,n\}}$$

In this beautiful 'orthogonality relation',

$$\begin{aligned} m, n &= \text{ positive integers} \\ (m,n) &= \text{ highest common divisor of } m, n \\ \{m,n\} &= \text{ least common multiple of } m, n \end{aligned}$$

The Sa(x) *function is connected to the first Bernoullian polynomial* $B_1(x) = x - \frac{1}{2}$ *when the latter is extended periodically outside the range* $0 < x \leq 1$*, namely*

$$(viii) \qquad \mathrm{Sa}(x) = -B_1(x - [x]).$$

The Landau 'orthogonality relation' can then be extended in the form

$$(ix) \qquad \begin{aligned} &\textstyle\int_0^1 B_k(mx - [mx]) \cdot B_k(nx - [nx])\, dx \\ &\qquad = (-)^{k+1} \tfrac{(k!)^2}{(2k)!} B_{2k}(0) \left(\tfrac{(m,n)}{\{m,n\}}\right)^k . \end{aligned}$$

These examples suffice to show that the theory of integers, properly combined with modern analysis, can link the beautiful and the applicable, and thus provide unexpected answers to real-world problems.

Number theory has been considered since time immemorial to be the very paradigm of pure (and useless) mathematics. This attitude is reflected in the name integers *— meaning the 'untouched ones'. Yet the role of integer ratios in musical scales had been widely appreciated ever since Pythagoras first pointed out their importance.*

*The occurrence of integers in biology — from plant morphology (e.g. Fibonacci Numbers) to the genetic code — is pervasive. It has been hypothesized (**Robert M. May**, 1979) that the North American* 17*-year cicada selected its life-cycle because* 17 *is a prime number, prime cycles offering better protection from predators than nonprime cycles. The suggestion that the* 17*-year cicada 'knows' that* 17 *is a **Fermat** prime has yet to be touted though.*

1900 CE, Sept. 08 A *hurricane* killed about 8000 persons in the Galveston, Texas, area. It was the worst natural disaster in United States history. The strength of the storm, coupled with the lack of adequate warning, caught the population by surprise[68].

1900 CE **Ivar Erik Fredholm** (1866–1927, Sweden). Mathematician. Among the founders of modern integral-equation theory.

He was educated at the universities of Uppsala (1886) and Stockholm (1888–1893) and became interested mainly in mathematical physics. In 1898 he received his Ph.D. from Uppsala, and turned to integral equations. He worked as an actuary until 1906, when he was appointed professor of theoretical physics at the University of Stockholm. In 1900 he developed the essential part of what is now known as the theory of Fredholm integral equations. This theory inspired the later investigations of **Hilbert**.

Previously, integral equations had received the attention of **N.H. Abel**, **J. Liouville** and **Eugène Rouché** (1832–1910) of Paris, but were quite neglected. In 1823, Abel had proposed a generalization of the *tautochrone*

[68] Since the successful launching of TIROS (Television and Infra-Red Observation Satellite) in April 1960, which inaugurated the era of weather observation by satellites, meteorologists have been able to identify and track tropical storms even before they become hurricanes.

problem, the solution of which involved an integral equation that has since been designated as being of the first kind. Liouville (1837), showed that a particular solution of a linear differential equation of the second order could be found by solving an integral equation, now designated as of the second kind. A method of solving integral equations of the second kind was given by **C.G. Neumann** (1877). Fredholm studied integral equations from the point of view of an immediate generalization of a system of linear algebraic equations.

1900 CE Yerkes Astronomical Observatory (established 1895) in Williams Bay, Wisconsin, U.S.A., was equipped with a 40-inch refractor telescope.

1900 CE **David Hilbert**[69] (1862–1943, Germany). The man who set the course for 20^{th} century mathematics. A professor of mathematics at Göttingen during 1895–1930.

His mathematical interests were wide ranging: algebraic invariant theory (until 1892); algebraic number theory (1892–1899); foundations of geometry (1898–1903); calculus of variations and the Dirichlet principle (1899–1905); integral equations (1901–1912); mathematical foundations of physics (1912–1917); logic (1917–1943); foundations of mathematics (1934–1935, with Paul Bernays[70]). Hilbert developed the theory of integral equations into a tool which enabled scientists to make breakthroughs in regions once muddy with confusion. At the International Congress of Mathematics in 1900 Hilbert presented his famous list of 23 difficult but inspiring problems, to which he believed mathematicians should address themselves. Several of these, including the Riemann conjecture, remain unsolved to this day.

Hilbert was born in Whelau, near Königsberg, East Prussia, into a Protestant family. Their Biblical names seem to indicate that they were Pietists, members of a fundamentalist sect which emphasized faith and an attitude of the heart. His father was a country judge. His mother Therese was an unusual woman — interested in philosophy and astronomy, and fascinated by prime numbers.

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[69] For further reading, see:

- Reid, C., *Hilbert*, Springer-Verlag: New York, 1970, 290 pp.
- Gustafson, K.E., *Introduction to PDE and Hilbert Space Methods*, Dover, 1999, 448 pp.

[70] **Paul Isaac Bernays** (1888–1977). Mathematician and philosopher. Of illustrious rabbinic ancestry. Collaborated with **Hilbert** in his research on the foundations of mathematics. Contributed to Axiomatic Set Theory and symbolic logic. Professor of mathematics at Göttingen and Zürich.

Hilbert grew up in Königsberg, the city of Immanuel Kant and of the famous seven great bridges through which this city entered the history of mathematics[71] [they had provided the problem, solved a century before by Euler, which lies at the foundation of the field now known as '*topology*'].

In 1880 Hilbert entered the University of Königsberg where he studied under **Heinrich Weber** (1804–1891) and **Adolf Hurwitz** (1859–1919) and met fellow-student **Hermann Minkowski** (1864–1909). In 1885 he received his Ph.D. and went to Paris, where he met the French mathematicians **Poincaré**, **Darboux**, **Picard** and **Gordan**. He then returned to his native town and stayed at the university, where he was appointed ordinary professor of mathematics in 1895. In the same year he accepted a professorship in mathematics at the University of Göttingen, where he remained for the rest of his life.

The University of Göttingen had a flourishing tradition in mathematics and physics which persisted over 150 years of uninterrupted greatness. It is associated with the names of: **Gauss**, **Weber**, **Dirichlet**, **Riemann**, **F. Klein**, **Hurwitz**, **Hilbert**, **Minkowski**, **Nernst**, **Landau**, **Noether**, **Born**, **Debye**, **Weyl**, **Courant**[72], **Heisenberg**, **Franck**, **Runge**, **Carathéodory**, **Voigt**, **Prandtl**, **Wiechert**, **Schwarzschild**, **Zermelo**, **Landé**, **von Neumann**, **Wiener**, **Jordan**, **Pauli**, **Wigner**, **Birkhoff**, **von Laue**, **von Kármán**, **Siegel**, **Veblen** and many others.

The last decade of Hilbert's life was darkened by the tragedy brought upon himself, Göttingen and many of his students and colleagues by the Nazi regime. In 1935, Hilbert's antecedents were examined. There was a joke that there was only one Aryan mathematician in Göttingen and in his veins Jewish blood was flowing. The joke was based upon the fact that during Hilbert's illness, he had received a blood transfusion from Courant. Now the question was seriously raised if it was not suspicious for an Aryan mathematician to have the name David. It finally became necessary for Hilbert to produce the autobiography of his great grandfather Christian David Hilbert to show that David was a family name.

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[71] There was at that time a rare concentration of youthful scientific talent in Königsberg: at one point, **Wilhelm Wien**, **Arnold Sommerfeld** and **Hermann Minkowski** were all simultaneously in attendance at the Altstadt Gymnasium (1882–1884). David Hilbert attended another school and did not have the opportunity to become acquainted with any of these boys during his school days.

[72] For further reading, see:
Courant, R. and D. Hilbert, *Methods of Mathematical Physics*, Interscience Publishers, 1953, Vols I-II.

Hilbert died of complications arising from the physical inactivity that resulted from an accident. Not more than a dozen people attended the morning funeral service in the living room of the house on Wilhelm Weber Strasse in Göttingen. Arnold Sommerfeld spoke of Hilbert's work.

At the time of his death, there was scarcely a mathematician in the world whose work did not derive in some way from that of Hilbert. There are *Hilbert spaces*, *Hilbert inequality*, *Hilbert transform*[73], *Hilbert invariant integral*, *Hilbert irreducibility theorem*, *Hilbert base theorem*, *Hilbert axiom*, *Hilbert subgroups*, *Hilbert class-field*.

A *Hilbert space* (H) is a linear vector space of infinite dimensionality. By a vector $\boldsymbol{x}$ in such an infinite-dimensional space, we mean an infinite sequence of complex numbers $\boldsymbol{x} = (x_1, x_2, \ldots)$, where we always assume that these numbers obey the condition that the series $\sum_1^\infty |x_k|^2$ converges.

The basic operations on vectors are defined for the vectors of H in just the same way as for ordinary vectors, since the convergence of the above sum guarantees the meaningfulness of the sum of vectors and their product by a finite scalar, as well as of the *scalar product*

$$(\boldsymbol{x}, \boldsymbol{y}) = \sum_{k=1}^{\infty} x_k \bar{y}_k$$

of two vectors in H (the bar denoting complex conjugate).

[73] The *infinite Hilbert transform* of a function $f(x)$ is defined as

$$g(u) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{f(x)dx}{x-u}.$$

When viewed as an integral equation for the unknown $f(x)$ in terms of the known $g(x)$, it has the explicit solution: $f(x) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{g(u)du}{x-u}$. The transform is thus its own inverse.

The *finite Hilbert transform* is defined as $g(u) = \frac{1}{\pi} P \int_{-1}^{+1} \frac{f(x)dx}{x-u}$, $-1 \leq u \leq 1$. Its solution (inverse) takes the form $f(x) = \frac{1}{\sqrt{1-x^2}} \left\{ C + \frac{1}{\pi} P \int_{-1}^{+1} \sqrt{1-u^2} \frac{g(u)du}{u-x} \right\}$, where $-1 < x < 1$, and C is an arbitrary constant. Note that $\frac{C}{\sqrt{1-x^2}}$ is a solution of the homogeneous equation $\frac{1}{\pi} P \int_{-1}^{1} \frac{f(x)dx}{x-u} = 0$.

Finite Hilbert transforms are encountered in the *theory of dislocations* and in the aerodynamic theory of a thin airfoil. In this context it is known as the *airfoil equation* (**Prandtl**, 1918).

The sum

$$(\boldsymbol{x},\boldsymbol{x}) = \sum_{k=1}^{\infty} |x_k|^2 \equiv \|\boldsymbol{x}\|^2 \geq 0$$

defines the square of the *norm* (*length*) of the vector $\boldsymbol{x}$. The scalar product obeys the same basic rules as in finite dimensional spaces. In particular, the inequality

$$|(\boldsymbol{x},\boldsymbol{y})| \leq \|\boldsymbol{x}\|\|\boldsymbol{y}\|$$

holds, and so does the triangle inequality

$$\|\boldsymbol{x}+\boldsymbol{y}\| \leq \|\boldsymbol{x}\| + \|\boldsymbol{y}\|.$$

Also, the square of the norm of a sum of orthogonal vectors equals the sum of the squares of the norms of the summands (*Pythagorean theorem*).

The fundamental *basis vectors* in the Hilbert space H described above are the unit orthogonal (orthonormal) vectors $\boldsymbol{a}^{(1)} = (1,0,0,\ldots)$, $\boldsymbol{a}^2 = (0,1,0,\ldots)$, etc. The *components* x_k of a vector $\boldsymbol{x}$ can be expressed as the scalar products $x_k = (\boldsymbol{x},\boldsymbol{a}^{(k)})$. Another Hilbert space, denoted by F, is the *function space*, where continuously varying functions of one or several variables play the role of vectors.

Let $f(x)$ be a function defined on the interval $a \leq x \leq b$. We can regard $f(x)$ as a vector, where the value of $f(x_0)$ is associated with the point x_0 in the interval and gives the "component of $f(x)$ with index x_0".

Thus in this case the independent variable x, which varies continuously through all values in the interval, plays the role of an index for the components, i.e., $f(x)$ has a continuous set of components. In general H is complex: $f(x) = f_1(x) + if_2(x)$, with $f_1(x)$, $f_2(x)$ real functions defined on the finite interval $a \leq x \leq b$ of the real axis.

To define the norm and the scalar product in F, we need only replace summation by integration everywhere in the previous formulas. We define the *scalar product* by

$$\{\varphi(x),\psi(x)\} = \int_a^b \varphi(x)\overline{\psi(x)}dx$$

and the square of the *norm* by

$$\|f(x)\|^2 = \{f(x),f(x)\} = \int_a^b |f(x)|^2 dx.$$

Let $\varphi_k(x)$ (k – a discrete index) be an orthonormal system, i.e.,

$$\int_a^b \varphi_p(x)\varphi_q(x)dx = \delta_{pq},$$

and let $f(x)$ be any function (vector) in F. We introduce the (generalized) *Fourier coefficients*

$$a_k = \{f(x), \varphi_k(x)\} = \int_a^b f(x)\overline{\varphi_k(x)}dx$$

of the function $f(x)$, which are the magnitudes of the projections of $f(x)$ onto the vectors $\varphi_k(x)$ in F. It can be shown that if

$$\sum_{k=1}^{\infty} |a_k|^2 = \int_a^b |f(x)|^2 dx,$$

then the infinite sum $\sum_{k=1}^{\infty} a_k \varphi_k(x)$ is the generalized Fourier series of the function $f(x)$.

In direct analogy with the definition of the distance between points in an n-dimensional space as the length of the vector $\boldsymbol{x} - \boldsymbol{y}$ [namely, $\sqrt{\sum_1^n |x_k - y_k|^2}$], one defines the "distance" between two functions $f(x)$ and $g(x)$ in a functional space as $\sqrt{\int_a^b |f(x) - g(x)|^2 dx}$. The expression $\int_a^b |f(x) - g(x)|^2 dx$ is (up to a normalization constant) the *mean-square deviation* between the functions $f(x)$ and $g(x)$.

Now, in an n-dimensional space, the angle between the vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ is defined as

$$\cos\phi = \frac{\sum_1^n x_k y_k}{\sqrt{\sum_1^n x_k^2 \sum_1^n y_k^2}}.$$

The function-space analog is (for real-valued functions)

$$\cos\phi = \frac{\int_a^b f(x)g(x)dx}{\sqrt{\int_a^b f^2(x)dx \int_a^b g^2(x)dx}}, \quad |\cos\phi| \le 1 \,,$$

the latter following from the *Cauchy inequality*.

Much of modern quantum theory employs infinite-dimensional vector spaces (Hilbert spaces) in which the unit vectors are quantum *state vectors*[74].

Hilbert was a small, quiet, unpretentious man, but with such an alert mind and air of deep concentration that in a little while one was under his spell. He

[74] These are often describable as complex *wave functions* of particle positions or velocities, or (in quantum field theories) as complex *functionals* of a field distribution (e.g. electric or magnetic) in $\mathbb{R}^3$ space at a fixed time-slice of an $\mathbb{R}^4$ spacetime.

quickly saw relationships which illuminated dark areas and led to solutions of problems on which other mathematicians had worked without success for years.

Hilbert's interests were purely scientific. He cared only for basic theory. He was not too interested in undergraduate students, but could be easily upset if a promising graduate student decided to leave mathematics for marriage or such similar frivolity. He cared little for broad educational questions and less for practical applications of science, but he was a great teacher, one who puts his finger on the heart of a problem and lights up the mind with a flash of understanding.

Students flocked to him as to no one else on the continent. Many of the 20^{th} century's most influential mathematicians and physicists, including **von Kármán**, **Toeplitz**, **Zermelo**, **Courant**[75], **Born**, **Heisenberg** and **Oppenheimer**, were his devoted disciples. Hilbert held the view that nature is inherently mathematical, and therefore urged his students to search for mathematical solutions in areas where practical men saw only insurmountable chaos.

Hilbert's greatest heritage is the notion that there is no gap between pure and applied mathematics and that between mathematics and science as a whole, a fruitful community can be established. This optimism echoes in an epitaph of his own words (1930) that has been placed over his grave in Göttingen:

"Wir müssen wissen. Wir werden wissen".

[75] For further reading, see:

- Courant, R. and D., Hilbert, *Methods of Mathematical Physics*, Interscience Publishers, 1953, vols I–II.

Worldview XXVIII: David Hilbert

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"Before beginning I should put in three years of intensive study, and I haven't that much time to squander on a probable failure."
[On why he didn't try to solve Fermat's last theorem]

* * *

"Galileo was no idiot. Only an idiot could believe that science requires martyrdom – that may be necessary in religion, but in time a scientific result will establish itself."

* * *

"Mathematics is a game played according to certain simple rules with meaningless marks on paper."

* * *

"Physics is much too hard for physicists."

* * *

"How thoroughly it is ingrained in mathematical science that every real advance goes hand in hand with the invention of sharper tools and simpler methods which, at the same time, assist in understanding earlier theories and in casting aside some more complicated developments.
The art of doing mathematics consists in finding that special case which contains all the germs of generality."

* * *

"The further a mathematical theory is developed, the more harmoniously and uniformly does its construction proceed, and unsuspected relations are disclosed between hitherto separated branches of the science."

* * *

"One can measure the importance of a scientific work by the number of earlier publications rendered superfluous by it."

* * *

"Mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country."

* * *

"The infinite! No other question has ever moved so profoundly the spirit of man."

1900–1906 CE **Reginald Aubrey Fessenden** (1866–1932, U.S.A.). Pioneer radio engineer and inventor. Made first radio broadcast of human voice and music over the air (Dec. 24, 1906). The first to experiment with modulation of radio waves (1902).

Fessenden was born in East Bolton, Quebec, Canada, of American parentage. He was chief chemist of Edison Laboratory (1887–1890); professor at the University of Pittsburgh (1893–1900); headed National Signaling Co. (1902–1910). Invented the electrolytic detector (1900), and the heterodyne receiver.

Patented some 300 inventions, including a radio compass, sonic depth finder, submarine signaling devices and turbo-electric drive for battleships.

In his 1906 radio broadcast, he spoke from Brant Rock, Mass. to ships offshore in the Atlantic Ocean on a carrier frequency of 50 kHz with a transmitter of power 1 kilowatt. He put two musical tunes, a poem and a talk on the air, and they were heard by radio operators several hundred kilometers away.

1900–1907 CE **Hugo Marie de Vries** (1848–1935, Holland). Botanist and geneticist. Rediscovered and verified Mendel's principle[76], marking the beginning of modern genetics (1900–1903). His studies of osmosis in plant cells brought to light the crucial role of osmosis in animal and plant physiology.

After a lengthy series of experiments in plant breeding (1890–1900), de Vries deduced the laws of heredity. He then began to reexamine Darwin's concept of evolution in the context of Mendel's results, and asserted that *mutations*[77] account for the key evolutionary steps in the creation of new species.

de Vries was highly critical of the theory of evolution by natural selection and stressed the importance of mutations in plant evolution, believing that the *genes* provide a mechanism through which natural selection can operate.

He was born in Haarlem, Holland and was a professor at Amsterdam (1878–1918).

1900–1908 CE **Ernst Friedrich Ferdinand Zermelo** (1871–1953, Germany). Mathematician. Made important contributions to the development of set theory, particularly in developing the *axiomatic set theory* that now bears his name. Following the pioneering work of Georg Cantor he provided (1900)

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[76] *Mendelism*: The study of heredity stemming from the ideas of **Gregor Mendel** whose key assertion was that the reproductive cells of living organism contain 'factors' transmitting discrete characters. A hybrid pea-plant, for example, formed by crossing tall and dwarf varieties, would receive a factor for tallness from one parent, and one for dwarfness from the other. Half of its reproductive cells would contain one factor, and half the other, with no blending. Such suppositions explained observable distributions of types found in the offspring of hybrids. Mendel's work was neglected until rediscovered by **de Vries** (1900). Modern Mendelism, however, stems from **T.H. Morgan** and his colleagues who successfully identified 'factors' as *genes* (parts of chromosomes) present in the nucleus of every cell. At first, genes were thought to control *visible* characters of the organism on a one-for-one basis. Later it was realized that the genotype-phenotype relation is more complex.

[77] *Mutation*: Sudden discontinuous changes in an organism which are transmitted to offsprings, as opposed to the very slight variations described by Darwin (1859).

an ingenious proof to the '*well-ordering theorem*'[78] which led him to identify (1904) a crucial, but hitherto unrecognized, set-theoretic axiom: the *axiom of choice.*

Before Zermelo's work, this axiom was implicitly assumed in mathematical logical reasoning, and it is, in fact, a necessary hypothesis in his well-ordering theorem, as well. It states that if A is a set of non-null disjoint sets a, there exists a set C containing precisely one element of each $a \in A$.

In 1908 he gave the first axiomatic description of set theory including seven axioms: Axiom of *extensionability*, Axiom of *elementary sets*, Axiom of *separation*, *Power set* axiom, *Union* axiom, Axiom of *choice* and Axiom of *infinity.* Though later modified to avoid the paradoxes discovered by Bertrand Russell and others, it remains one of the standard methods of axiomatizing set theory. In particular, the axiom of choice remained a key result in many mathematical applications of set theory. Zermelo also contributed to the calculus of variations and to statistical mechanics.

Zermelo was born n Berlin and studied at Halle and Freiburg. Became a professor at Göttingen (1905), eventually moving to Freiburg, where he resigned his post (1935) in protest against the Nazi regime, but was reinstated in 1946.

1900–1912 CE **Max (Karl Ernst Ludwig) Planck** (1858–1947, Germany). Distinguished theoretical physicist. Originated quantum theory. Asserted that radiated energy is emitted in discrete, finite, irreducible units, or *quanta*.The amount of energy E contained in each quantum is proportional to the radiation's frequency ν, the proportionality factor being a universal constant, h. It is termed "the elementary quantum of action", and became known later as *Planck's constant*[79] [$h = 6.626\ 075\ 5 \times 10^{-27}$ erg-second]. In

[78] The '*well-ordering theorem*': Every set can in principle be arranged in a series (endowed with a total ordering) for which each subset has a least term. A *total ordering* is a relation $a \leq b$ (a comes before b) so that for any two statements a and b, either $a \leq b$ or $b \leq a$, and $a = b$ iff $a \leq b$ *and* $b \leq a$. If there are three elements a, b and c such that $a \leq b$ and $b \leq c$, then $a \leq c$ (*transitivity* property of the relation).

[79] Max Planck first pointed out that a new fundamental length scale could be constructed from the constants $\hbar$, c, and G ($\hbar = h/(2\pi)$).
According to the Heisenberg uncertainty principle, the measured energy of a system is subject to a minimal uncertainty ΔE if measured over a time period Δt, where ΔE and Δt are related by $\Delta E \Delta t \sim \hbar$. If this principle is applied to the gravitational field, one can expect sudden and unpredictable changes in the measured energy, momentum and stress distributions, which in turn cause fluctuations in spacetime curvature by GTR. To estimate the scale of these fluc-

1900 Planck formulated the correct mathematical description of thermal electromagnetic radiation from a perfect absorber (black body) in which energy is emitted and absorbed in *discrete energy packets*, or '*quanta*', each of energy $E = h\nu$.

Einstein later showed (1905) that the radiation is also *propagated* in such discrete quanta (named '*photons*') and that these photons carry momentum (and angular momentum) as well as energy.

A hypothetical body (assumed for simplicity to be surrounded by vacuum) that completely absorbs all radiant electromagnetic energy falling on it, reaches some steady-state temperature, and then re-emits that energy as quickly as it absorbs it. Since this scenario, although realistic enough in many actual cases, is thermodynamically out-of-equilibrium, it is usually recast as the following thought experiment: one considers a box (shape unimportant), with thick walls composed of the perfect absorber. The *equilibrium* radiation spectrum inside this box ('blackbody radiation') is then measured outside by drilling a small hole in the blackbody wall.

Planck assumed that the sources of radiation are oscillating atoms, capable of jumping from one discrete energy level E_2 to another discrete lower level E_1, such that the *frequency* of the ensuing radiation is $\nu = \frac{E_2 - E_1}{h}$. The *amount* of radiant energy in the box per unit volume and per unit wavelength is given by a distribution law that depends both on the wavelength $\left(\frac{c}{\nu}\right)$ *and*

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tuations we may assume that the ripples of geometry produced by the quantum effects propagate like gravitational waves at the speed of light.

If the uncertainty in spatial position is represented by a sphere of radius r, the corresponding uncertainty in time is $\Delta t = \frac{r}{c}$. The corresponding energy fluctuations are then of order $\Delta E \sim \frac{\hbar c}{r}$. The *gravitational self-energy* of these fluctuations, i.e., the negative of the energy required to pull the energy ΔE (mass $\frac{\Delta E}{c^2}$) apart against its own gravity is of order $-\frac{GM^2}{r} = -\frac{G}{r}\left(\frac{\Delta E}{c^2}\right)^2 \sim -\frac{G\hbar^2}{r^3 c^2}$. If we choose the length scale r small enough, a point is reached where the quantum energy will occasionally fluctuate into existence (zero-point motion) without any external work. This happens when the gravitational binding self-energy reaches a value comparable to the energy ΔE itself, so that the net "energy penalty" incurred by the quantum fluctuation may vanish (which is also the regime where black holes, baby-universes and spacetime wormholes may arise). This disruptive regime is approached when $\frac{G(\Delta E)^2}{rc^4} \sim \Delta E \sim \frac{\hbar c}{r}$, or when $\Delta E \sim \sqrt{\frac{\hbar c^5}{G}} \sim 2 \times 10^9$ Joule $\sim 10^{17}$ GeV. This corresponds to a mass of $\sqrt{\frac{\hbar c}{G}} \sim 2 \times 10^{-8}$ kg, a characteristic fluctuation time $\sqrt{\frac{\hbar G}{c^5}} \sim 10^{-44}$ sec, and a length scale $\frac{\hbar c}{\Delta E} \sim \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-35}$ m, known as the *Planck length*. At Planck scales, it is expected that quantum gravity effects would be of great importance.

the equilibrium absolute temperature T, according to the universal law of '*blackbody radiation*':

$$E_\lambda(\lambda;T) = \frac{8\pi hc}{\lambda^5}\left[e^{\frac{hc}{k\lambda T}} - 1\right]^{-1},$$

where c, h, k are three universal constants: the speed of light in vacuum, Planck's constant and Boltzmann's constant, respectively.

For blackbody temperatures up to several hundred degrees Kelvin, most of the radiation is in the infrared region of the electromagnetic spectrum. At increasingly higher temperatures, the total radiated energy increases and its peak shifts to shorter wavelengths, so that a significant portion of the blackbody spectrum is radiated as visible light (beginning with *red* and progressing to cover more of the visible colors of the rainbow as T increases).

The total energy density (per unit volume) is $E = \int_0^\infty E_\lambda d\lambda$. The introduction of a new variable $\xi = \frac{1}{\lambda}\left(\frac{hc}{kT}\right)$ yields the "*Stefan-Boltzmann law*" (1884):

$$E = 8\pi\frac{(kT)^4}{h^3c^3}\int_0^\infty \frac{\xi^3 d\xi}{e^\xi - 1} = \sigma T^4,$$

where

$$\sigma = \frac{8\pi^4 k^4}{15c^3h^3} = 7.56\times 10^{-15}\text{erg}\cdot\text{cm}^{-3}\cdot\text{deg}^{-4}$$

is the "Stefan-Boltzmann constant".

The peak of the curve $E_\lambda(\lambda,T)$ is at $\lambda=\lambda_m$, where $\left(\frac{\partial E}{\partial\lambda}\right)_{\max} = 0$. It yields "*Wien's displacement law*" (1896):

$$\lambda_m T = \frac{hc}{(4.965)k} = (0.2898)\ cm\cdot{}^\circ K.$$

This law relates a star's color to its surface temperature: the intensity of light from a cool star peaks at long wavelengths, and thus the star appears *red*.

A hot star's intensity curve is skewed toward short wavelengths, and thus the star appears *blue*. The maximum intensity of a star of intermediate temperature (such as the sun) occurs near the middle of the visible spectrum, giving the star a yellow-white color.

Planck was born in Kiel. He studied at the universities of Münich[80] and Berlin and was appointed a professor in Kiel (1885) and in Berlin (1889),

[80] Planck's teacher, **Philipp Johann Gustav von Jolly** (1809–1884, Germany), advised him (1876) against the study of physics, since after all it was essentially *finished* (!), so that for anyone who wanted to do active scientific research, it would scarcely be worthwhile to go into this field.

where he resided for the rest of his life. In 1918 he was awarded the Nobel prize in physics. **Albert Einstein** and **Niels Bohr** applied Planck's quantum theory to problems of photoelectric emission and atomic structure, respectively. The new theory succeeded in explaining the structure of the outer part of the atom.

1900–1918 CE **Georg Simmel** (1858–1918, Germany). Philosopher and sociologist. Insisted that the *natural sciences* as well as history offer only an image of reality that is transformed by the theoretical or historical *a priori*; Philosophy and sociology offer two different aspects of the situation of man in the world; They are two autonomous interpretations of mental life. He admitted the influence of economic facts on intellectual attitudes but insisted that the effects of intellectual patterns on economics act likewise.

He maintained that the decisive factor of *human attitude* is antecedent to changes of social and economic institutions. *Sociology* is conceived by Simmel as the doctrine of the forms of the relations between individuals, independent of spiritual contents which are subject to historical change. It is the "geometry of social life".

Religion and the *arts* represent to Simmel autonomous worlds which are independent of *science* but accessible to the philosopher, provided he does not disregard their autonomous foundations; the poet and the artist while forming their own image of life, (although determined by the historical situation of his lifetime), transcends historical conditions and testifies that life always hints beyond itself. The principal problem of *influence* is formulated by Simmel as the difficulty to seize life without violating it.

Simmel was born in Berlin to Jewish parents who converted to Christianity. Taught at the University of Berlin (1885–1914) and the University of Strasbourg. He did not form a school; many of his former pupils died on the battlefields of WWI. His works have been influential in the United States.

1900–1921 CE **Louis Bachelier** (1870–1946, France). Mathematician. In his dissertation (1900) he described the mathematical theory of *Brownian motion*, namely — the one-dimensional diffusion of probability. (5 years before **Einstein**!). Prior to **Wiener** he obtained (1913) some important properties of the so-called Wiener-Lévy process (1923). Bachelier applied the theory to economy and claimed that successive price-changes are statistically independent and follow a one-dimensional Brownian motion.

Bachelier was a professor at the University of Besancon.

1900–1922 CE **Max Wilhelm Dehn** (1878–1952, Germany and USA). Mathematician. Contributed to the foundation of geometry (1900–1906), theory of groups, and topology (1906–1922). Solved the third of Hilbert's 23 Paris problems on the *congruence of polyhedra*.

He showed, as Hilbert conjectured, that a regular tetrahedron cannot be cut up and reassembled into a cube of equal volume (1900–1901). Dehn wrote one of the first systematic expositions on *topology* (1907) and later formulated important problems on *group presentations*, namely the *word problem*[81] and the *isomorphism problem*.

Dehn was born in Hamburg to Jewish parents. He studied at Göttingen under Hilbert's supervision, obtaining his doctorate in 1900. Converted to Christianity (1900) under Hilbert's influence. Held academic positions at the Universities of Münster, Kiel and Breslau (1900–1915). Served in the German army during WWI (1915–1918). Held the chair of Mathematics at the University of Frankfurt on M. (1921–1935). Dismissed (1935) by the Nazis and lived as a refugee in various European countries (1935–1939). Immigrated to the US (1940) in the wake of the brain-drain[82] and held minor academic positions at the Universities of Idaho, Chicago and Maryland. Finally he became a professor of Mathematics in Black Mountain College, North Carolina (1945–1952), where he died at the age of 74.

Two areas, or two volumes, P and Q, are said to be *congruent by addition* if they can be dissected into corresponding pairs of congruent pieces (e.g. familiar proofs of the Pythagorean theorem). Any two equal polygon areas are congruent by addition, and the dissection can always be carried out with a straightedge and compasses. Likewise, *congruence* by *subtraction* refers to a case where corresponding pairs of congruent pieces can be added to P and Q to give two new figures which are congruent by addition. Max Dehn showed that two equal *polyhedral volumes* are not necessarily congruent by either addition or subtraction.

1900–1936 CE **Edmund Husserl** (1859–1938, Germany). Philosopher and mathematician. The father of modern *existentialism*. Developed the

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[81] The *word problem* asks the fundamental question of whether there is an algorithm to determine if a word in a group given by a presentation is trivial. It has since been shown that no such algorithm exists in general.

[82] The United States found itself immeasurably enriched, for almost all the members of the Hilbert school and many other European scientists emigrated to that country. This exodus included: **Artin**, **Courant**, **Debye**, **Einstein**, **Ewald**, **Feller**, **Franck**, **Friedrichs**, **Gödel**, **Hellinger**, **von Karman**, **Landé**, **Lewy**, **Neugebauer**, **von Neumann**, **Emmy Noether**, **Nordheim**, **Ore**, **Polya**, **Szegö**, **Szilard**, **Tarski**, **Olga Taussky**, **Teller**, **Weyl**, **Wigner**.

philosophy of *phenomenology* (the study of relationship of the conscious mind to objects, stressing the existence of an inner reality in the mind). Claimed that the correct procedure for any philosophy must be the understanding of things as they are presented in the form of empirical evidence to experience, and that such understanding must be *intuitive*.

Concluded that the proper objects of all exact disciplines, philosophy as well as *logic and mathematics*, are *essences* (abstract entities present to the mind but not themselves states of mind). Where Husserl mainly concerned himself with the mind's intellectual activities and their objects, Heidegger applied the phenomenological method to the emotions.

Husserl was born to Jewish parents in Prossnitz, Moravia and was baptized as a Christian (1887). Studied in Leipzig and Berlin (where he worked with **Karl Weierstrass**) before coming under the influence of the philosopher **Franz Brentano** in Vienna (1883). He held posts at Halle (1887–1901), Göttingen (1901–1916), where he was a colleague of **David Hilbert**. He finally settled as professor at Freiburg (1916–1928).

Husserl fell victim to the Nazi academic purge, while his successor and former pupil, Martin Heidegger, became a Nazi sympathizer. His most influential books: *Logische Untersuchungen* (Investigation in Logic, 1900), *Ideen zu einer reinen phänomenologie* (Ideas concerning pure phenomenology, 1913), *Die Krisis der europäischen Wissenschaften* (The crisis of European Science, 1936).

1900–1938 CE **Franz Boas** (1858–1942, Germany and USA). Founder of the science of cultural anthropology. Contributed to ethnology, linguistics and physical anthropology, especially to the understanding of the mentality, art, religion, and folklore of primitive societies.

Boas was born to Jewish parents in Minden, Rhine-Westphalia, Germany and received his doctorate at Kiel (1881). Trained initially as a geographer, he shifted his interests to ethnology as a result of contacts with Eskimos in the course of an Arctic expedition. Work in British Columbia, among the Kwakiutl, led him to devote himself entirely to anthropology and also to emigrate to the USA (1886).

Between 1900–1905 he provided the first specific proof of cultural relationship between Siberians, Eskimos and American Indians. He demonstrated that races of man are mixed to some extent and objected to *evolutionary assumptions*, *geographical determinism* and *racial determinism*.

Boas maintained that culture was a more or less autonomous realm which could not be explained in terms of other factors and that the complexity of cultural phenomena was inevitably overwhelmed by detail.[83] His writings include *The mind of Primitive Man* (1911) and *Anthropology and Modern Life* (1928).

1900–1942 CE **Marc Aurel Stein** (1862–1943, Hungary and England). Archaeological surveyor, scholar and explorer. Provided the basis of modern knowledge of the geography, history and cultures of Central Asia.

As a result of more than 15 journeys on foot and horseback (two of which alone totaled 40,000 km) through some of the highest and most isolated mountains and deserts in the world, he established for the first time the extent of the intercourse between Indian, Iranian, Hellenistic and Chinese civilizations, and the overland lines of communication between China and the West during the many centuries before these were finally replaced by the sea route.

In his first three journeys (1900–1916) he crossed the Hindukush and Pamirs, and explored Chinese Turkestan. His most famous discovery was the *Cave of the Thousand Buddhas* at Ch'ien-fo-tung, from which he brought back a great cache of documents in several languages and works of art, dating from the 5^{th}–10^{th} centuries and which had been walled up in the 11^{th} century. It was the greatest archaeological discovery ever made in Asia.

Stein undertook a series of jurneys to trace the vestiges of the prehistoric Indus civilization westward to the Tigris. Other expeditions established the line of Alexander's route and of Marco Polo's journeys. At the age of 80 he was exploring the gorges of the Indus climbing a succession of passes 4600 meters high.

Stein was born to Jewish parents in Budapest, and educated in his native city, and at Vienna, Tübingen, Oxford and London, where he took up oriental studies. In 1888 he was appointed registrar of the Punjab University at Lahore, India, and principal of the Oriental College in the same university. In 1900–1901 Stein directed archaeological expeditions in Chinese Turkestan for the Indian government. He lived most of his life in India and died at Kabul, Afghanistan.

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[83] Boas' critique of evolutionary and racial assumptions sprang from his Germanic heritage and a disillusionment with Victorian civilization. This led him to insist that 'laws' of cultural development must be derived from the comparison of reconstructed histories of particular cultures rather than of characteristics of various racial groups assumed to be at different stages of development (cultural relativism).

After WWII, there was a reaction against cultural relativism and a reassertion of evolutionary themes. After more than a century of serious study, the concept of *culture* still evidences the unresolved tension between an evolutionary perspective and a humanistic orientation toward distinctive species characteristic of man.

1901 CE On the invitation of **Felix Klein** (1849–1925, Germany), **Ricci-Curbastro** and **Levi-Civita** published a joint expository memoir illustrating the application of the tensor methods to geometry and physics, and offering abundant evidence of the utility of tensor analysis in applied mathematics.

They showed that tensor calculus comes near to being a universal language in mathematical physics. It allows for compact expression of equations and provides a guide to the selection of physical laws. In particular, if a system of equations is expressible as the vanishing of a tensor, then the system will be invariant under a wide class of transformations of all variables in the system. But this is precisely the *covariance* condition imposed by one of the postulates of *General Relativity* on a system of equations, if the system is to be an admissible mathematical formulation of an observable sequence of events in physics.

Indeed, the theory of tensors was presented essentially in the form used by Einstein fifteen years later. **Marcel Grossmann** (1878–1936, Zürich), a Swiss geometer, mastered the new calculus and taught it to **Einstein**. This was timely, since tensor calculus was the particular kind of generalized vector analysis appropriate for expressing the differential equations of the gravitational field, and the effects of gravity on the flat four-dimensional space of STR.

1901–1902 CE **Martin Wilhelm Kutta** (1867–1944, Germany). Applied mathematician. Known for the *Runge-Kutta* method (1901) for numerically solving ordinary differential equations and for his important pioneering work on the aerodynamic theory of aerofoils (1902). His particular aim was to understand the effect of curvature — why a horizontally placed curved surface produced a positive lift.

Kutta studied at Breslau (1885–1890), Munich (1891–1894) and Cambridge (1898–1899). He became a professor at Stuttgart (1914) and remained there until his retirement (1939).

On the Wing[84]

The circulation theory of lift for two-dimensional wings of infinite span (1902–1909) is anchored in the studies of ***Lord Rayleigh*** *(1878) on the flow around a circular cylinder and the discovery by* ***Helmholtz*** *(1858) concerning the creation of vorticity by sharp edges.*

In general, a fluid element in a flow can experience translation, rotation and distortion. If rotation is absent, the result is a potential flow *or a* vortex-free flow; *whereas, if the element also rotates, we have a* rotation flow *or a* vortex flow. *Parallel flow with uniform velocity is the simplest example of vortex-free flow, because the fluid elements just travel parallel with no rotation nor distortion. In a* parallel shear-flow *in two dimensions, the velocity is uniform in the stream direction but non-uniform in a perpendicular direction; consequently the element is both rotated and distorted. This is the simplest example of vortex-flow (every element rotates, but the fluid* as a whole *does not necessarily rotate).*

In a circulatory flow *(circular streamline) with constant angular velocity, there is a vortex flow with no distortion. In this case the tangential velocity u of each element, though constant in magnitude along a given streamline, increases linearly from the center outwards.*

It is easy to show that if one chooses a velocity distribution along a radial ray of radius r such that the product $ru =$constant, the flow is vortex-free. The expression $2\pi ru$ is known as circulation; *so constant circulation guarantees a potential flow.*

To avoid a singularity at $r = 0$, let us assume a circulatory flow around a ball or cylinder. Then we already know through the Magnus effect *that when the body is given an additional translatory motion, the combined flows produce (by Bernoulli's theorem) a pressure difference, and hence a 'lift' force, perpendicular to the trajectory of the body.*

The cylinder, however, has an unsuitable shape *for a wing, because the air's viscosity creates around it a boundary layer that greatly increases the body's resistance to motion. To minimize this effect, the cylinder must be*

[84] For further reading, see:

- von Karman, T., *Aerodynamics*, McGraw-Hill, 1963, 203 pp.
- Ashley, H. and M. Landahl, *Aerodynamics of Wings and Bodies*, Dover, 1965, 279 pp.

'streamlined' in order to make it a good aerofoil. Observations of anatomy of birds and fishes, together with numerous experiments, have taught man that the boundary of the leading edge *of the moving object should curve as gradually as possible (large radius of curvature) while the* trailing edge *be made as sharp as possible.*

All this was known before 1902. But then ***Kutta*** *(1902) and independently* ***Joukowski*** *(1906) discovered that when a cylindrical body of* arbitrary cross-section *moves with velocity U in a fluid*[85] *of density ρ and there is a circulation of magnitude Γ around it, a force is produced equal to the product $\rho U\Gamma$ per unit length of the cylinder. The direction of the force is normal to both the velocity U and the axis of the cylinder.*

For a cylindrical body with arbitrary cross-section the circulation is understood as the product of the mean value of the component of the velocity along an arbitrary closed curve encircling the body, multiplied by the length of arc of that curve.

If the flow is vortex-free, this product has the same value independent of the choice of curve. The calculation of the lift thus reduces to the mathematical problem of the determination of the magnitude of the circulation as a function of the velocity and as a function of the shape of the wing section.

Now, apart from being able to calculate the circulation, there is the physical problem of how to generate *circulation in the first place. The answer to this last question was, however, known already to* ***Helmholtz****; he showed that if there is no initial vorticity in the fluid (e.g., if the fluid is originally at rest), vorticity can only be created by* friction *or by a presence of sharp edges on a body.*

This means that if the process of putting a wing section in motion creates a vortex (i.e. a rotation of a part of the fluid), a rotation in the opposite sense is created in the rest of the fluid, simply by Newton's principle of action and reaction. This rotary motion of the fluid appears as the circulation around the wing section.

So Kutta and Joukowski set forth (1902–1909) to calculate the circulation generated by a 2-D wing-section with infinite span and having a sharp trailing edge. In earlier times, the instinctive impression was that air hits the inclined wing surface and that the airplane is therefore carried by the air below. It is now clear that the airplane wing is at least partially hung up or sucked up *by the air passing along the upper surface. In fact, the contribution to the total lift from the negative pressure or suction developed at the upper surface is larger than the contribution from the positive pressure at the lower surface.*

[85] U is also equivalently equal to the wind (fluid) speed past a *stationary* object.

What happens to the vortex created near the trailing edge? Kutta and Joukowski showed that this starting vortex *is swept away and that conditions of smooth flow prevail at the trailing edge as the motion develops ('Kutta-Joukowski condition'). This allows one to calculate the circulation as if the vortex did not exist, turning the whole problem of lift into a purely mathematical exercise*[86]*. The assumption was indeed borne out by visual observations, and the fit of circulation theory with experimental results.*

A comparison between Newton's theory of lift and circulation theory shows that Newton's result is in error by an order of magnitude.

If the span of the wing is infinite, no work is required to obtain lift. Moreover, if we neglect friction and assume that the fluid closes around the wing, the motion can be described by the mathematical solution for nonviscous *fluids ('d'Alembert's Paradox'). In real fluids, however, because of frictional effects, the streamlines do not follow the surface of the body back to the rear end, but separate from the surface somewhere, thus leaving downstream an eddying part called the* wake*.*

Consequently, the pressure over the rear part of the body cannot reach such high values as are calculated for the nonviscous fluid. Because the pressures at the front and the rear are no longer balanced, a pressure drag *occurs, known as the* wake drag*. In addition, work must be expanded to obtain lift, known as* induced drag*.*

To sum up: When the streamlines passing around an obstacle in their path are not symmetric, the velocity of the flow on one side of an object may be different from the velocity on the other side. If the flow satisfies Bernoulli's law, then there will be lower pressure on the side with higher velocity. The resulting force on the object will have a major component perpendicular to the flow. There is a way to create asymmetry of streamlines with a symmetrical obstacle by imparting it with a rotational motion relative to the surrounding fluid.

When the body, however, has an intrinsic geometrical asymmetry, like an aerofoil for example, it is the fluid that is 'rotated' around the body, thus creating the lift. The 'lift' on an aircraft's wing or aerofoil enables it to rise from the ground. The shape of the wing is designed to produce the circulation.

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[86] In order to avoid the mathematical difficulty of calculating the circulation integral for the aerofoil, Kutta and Joukowski found an ingenious method that automatically reduces the problem to that of a flow past a circular cylinder; because of the two-dimensional nature of the problem, the *conformal transformation* $z = \zeta + \frac{\ell}{\zeta^2}$ (with ℓ constant) maps the aerofoil's ζ-plane onto the circular cylinder z-plane, for which an exact solution was known for an incompressible inviscid fluid (Rayleigh 1878).

The circulation adjusts itself so that the velocity at the trailing edge (where a singularity can occur) is finite. Thus a definite value of the lift is obtained for any given wing speed when steady motion is reached.

1901–1906 CE **Henri Léon Lebesgue** (1875–1941, France). A leading mathematician of his day. Revolutionized the field of integration with his generalization of the Riemann integral. Introduced the 'Lebesgue measure', which deals with the determination of the content of geometrical configurations, or more generally, of point sets.

He began his career at the universities of Rennes and Poitiers. In 1912 he became a professor at Collège de France. Also contributed to the fields of topology ('pavement theorem'), Fourier series and potential theory.

Towards the end of the 19^{th} century, mathematical analysis was effectively limited to continuous functions, and artificial restrictions were necessary to cope with discontinuities that cropped up with greater frequency as more exotic functions were encountered. The Riemann method of integration was applicable only to continuous (and a few discontinuous) functions.

Influenced by the work of **E. Borel**, **C. Jordan** and other theories of measure and content, Lebesgue formulated his theory of measure in 1901. In 1902 he framed a new definition of the definite integral. With the Lebesgue integration, any bounded summable function is the derivative of its indefinite integral, except possibly at an ensemble of points with zero measure.

Lebesgue integration was also instrumental in greatly expanding the scope of Fourier analysis.

Measure and modern integration (1894–1906)

(A) The Riemann-Stieltjes Integral

Fourier (1822) discovered that the Fourier series expansion depends on integrals. Assuming that

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n \cos n\pi x + b_n \sin n\pi x),$$

Fourier derived the formulas

$$a_n = \int_{-1}^{1} f(x)\cos n\pi x\,dx, \qquad b_n = \int_{-1}^{1} f(x)\sin n\pi x\,dx$$

Thus the existence of the series depends on the existence of the integrals for a_n and b_n, and this in turn depends on how discontinuous f is. It was known (though not rigorously proved) that every continuous function had an integral, so the next question was how the integral should, or could, be defined for discontinuous functions. The first precise answer was the Riemann [1854] integral concept, based on approximating the integral by step functions.

We recall the definition of the Riemann integral of a bounded function in the closed interval $[a, b]$. Suppose that $f(x)$ is bounded over (a, b); we subdivide this interval by means of the points $x_0, x_1, \dots, x_n$ so that $a = x_0 < x_1 < x_2 \cdots < x_{n-1} < x_n = b$. Let m_ν, M_ν be the lower and upper bounds of $f(x)$ in the interval $x_{\nu-1} < x \le x_\nu$, and let

$$s = \sum_{\nu=1}^{n} m_\nu(x_\nu - x_{\nu-1}), \qquad S = \sum_{\nu=1}^{n} M_\nu(x_\nu - x_{\nu-1}).$$

When the number of division points is increased indefinitely so that the greatest interval $x_\nu - x_{\nu-1}$ tends to zero, each of the sums s and S tends to a limit. If the limits are the same *for* every *sequence of partitions*

$$a = x_0 < \xi_1 < x_1 < \xi_2 < x_2 \cdots < \xi_n < x_n = b$$

as $\max|x_\nu - x_{\nu-1}| \to 0$, then their common value is the Riemann Integral

$$I = \lim_{\substack{\max|x_\nu - x_{\nu-1}|\to 0\\ n\to\infty}} \sum_{\nu=1}^{n} f(\xi_\nu)(x_\nu - x_{\nu-1}) \equiv \int_a^b f(x)\,dx.$$

Stieltjes *(1894) showed that it is also possible to integrate $f(x)$ w.r.t. a function $g(x)$ over the bounded interval $[a, b]$. He defined*

$$\lim_{\substack{\max|x_\nu - x_{\nu-1}|\to 0 \\ n\to\infty}} \sum_{\nu=1}^{n} f(\xi_\nu)(g(x_\nu) - g(x_{\nu-1})) = \int_{x=a}^{b} f(x)\,dg(x)$$

for an arbitrary sequence of partitions

$$a = x_0 < \xi_1 < x_1 < \xi_2 < x_2 \cdots < \xi_n < x_n = b$$

This generalization is known as the Riemann-Stieltjes integral. One must not write here the limits of integration as $g(a)$ to $g(b)$ because $g(x)$ may not be monotonic. It is x, not $g(x)$, that is required to increase steadily throughout the integration domain.

Stieltjes integrals often have an "intuitive" meaning (line integrals, surface integrals, volume integrals; integrals over distributions of mass, charge, and probability). Note that Stieltjes integrals include ordinary integrals and sums as special cases:

$$\int_a^b f(x)\,dg(x) = \int_a^b f(x)g'(x)\,dx$$

whenever $g(x)$ is continuously differentiable on (a, b), and

$$\sum_k f(k) = \int_{-\infty}^{\infty} f(x)\,d\sum_k U_-(x-k) \qquad \left[U_-(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}\right]$$

The Riemann-Stieltjes integral $\int f\,dg$ exists when f is continuous and g is of bounded variation. It is then shown that

$$\int_{x=a}^{b} g\,df = [fg]_a^b - \int_{x=a}^{b} f\,dg,$$

which is of much wider validity than the Riemann integration by parts result

$$\int_a^b f(x)g'(x)\,dx = [f(x)g(x)]_a^b - \int_a^b f'(x)g(x)\,dx,$$

valid when both f and g are differentiable for all $a \leq x \leq b$.

(B) Countable sets; measure

The Riemann integral is valid for a wide class of functions. Any function with a finite number of discontinuities belongs to this class, and indeed so do all functions bounded over a finite interval and having a countable *infinity of discontinuities*[87]*. Yet, the Riemann integral suffers from several difficulties; Consider, for example, the classic function of Dirichlet (1829):*

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

Now, since there are "very few" rationals, only a countable number in fact, we strongly suspect that the integral of this function is zero. However, if we form the upper and lower Riemann integrals by partitioning $[0, 1]$ *into small segments* Δx_i *and write*

$$\overline{\int} f(x)\,dx = \sum_i \Delta x_i \max[f(x)], \qquad x_i \le x \le x_i + \Delta x_i$$

$$\underline{\int} f(x)\,dx = \sum_i \Delta x_i \min[f(x)], \qquad x_i \le x \le x_i + \Delta x_i$$

in the usual way, we see that no matter how small the subinterval, Δx_i*, the maximum of* $f(x)$ *on this interval is always* 1 *and the minimum is always zero. Thus*

$$\overline{\int} f(x)\,dx = 1 \quad \text{and} \quad \underline{\int} f(x)\,dx = 0$$

so the Riemann integral does not exist.

To cope with such functions, a more general integral, the Lebesgue integral*, was introduced. To this end, mathematicians have invented a new concept — measure.*

Measure *generalizes the concept of length (on a line), area (in a plane), and so on, to quite general set of points. It has its roots in metric geometry, where a number is assigned to a length, an area or a volume.*

In antiquity, measurement was at first considered just a case of comparison with a standard unit. Then the problem of incommensurables revealed that

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[87] A necessary and sufficient condition that a bounded function $f(x)$ on (a, b) be Riemann-integrable is that the *points of discontinuity* of $f(x)$ form a set of *measure zero.*

the question was not as simple as intuition suggests, and that it requires a consideration of infinite processes. When calculus was fully developed, there came the more sophisticated point of view that, for most figures, measures do not *exist a priori but are contingent on the existence of associated limits. The evaluation of such limits became the task of integral calculus, a tool that also gives measures for many physical entities such as mass, work, force, charge, etc.*

As scientific history advanced, both physical and abstract geometrical concepts became more complicated, and there arose an ever greater need for precise mathematical formulation.

In Cantor's theory of the infinite, one-to-one correspondence is the criterion for determining whether two sets have the same cardinal number or whether one aggregate "has more elements" than the other. But this does not give the "length" (or area or volume) of a point set. In fact, the interval $[0,3]$ *has the same cardinal number of points,* C*, as* $[0,1]$*, although Euclidean geometry says the former is three times the latter in length. (We have used the bracket* [ ] *symbolism to indicate that the intervals in question include the end-points:* 0 *and* 3 *in the first case,* 0 *and* 1 *in the second.)*

What sort of rule can we apply to infinite sets to obtain a suitable measure for such abstractions as 'length', 'area' or 'volume', so that there may be applications to their counterparts in the physical world? In dealing with the set of all real numbers, the Cartesian picture is the number axis. Here one uses a standard length for the unit interval $[0,1]$ *and marks it off repeatedly to obtain the intervals* $[1,2]$, $[2,3]$, *etc.*

If measure *is to be just a generalization of length, it would seem a good idea to say that the* measure *of the set of real numbers in each of these intervals is also one unit. Thus one can readily measure point sets that are either simple intervals or finite unions of nonoverlapping intervals.*

But certain other questions naturally present themselves: If from the set of all real numbers between 0 *and* 1 *we remove the end-points* 0 *and* 1*, what is the length or measure of the remaining set, that is, the open interval* $(0,1)$*? Or suppose that we remove all the rational fractions that have* 1 *as numerator:* $1/1$, $1/2$, $1/3$, $1/4$, $1/5, \ldots$*; what is the measure of the residue? Or suppose that we go further and remove all the rational points in* $[0,1]$*; what length remains?*

In the last years of the nineteenth century, ***Émil Borel*** *gave much creative thought to these and many more difficult questions of the same kind. Then, in 1902, with Borel's ideas as background,* ***Henri Lebesgue*** *(1875–1941) established a* general *theory of measure. He abstracted its structure from all the particular "measure theories" of the past — empirical, abstract geometrical, Borelian, etc.*

In geometrical measurements, perimeters of polygons are obtained by totaling the lengths of individual sides, and area is sometimes found by subdividing a polygon into triangles, measuring these, then adding up. Such procedures assume: The whole is equal to the sum of its parts. In Lebesgue's generalization this becomes: The measure of the logical sum, or union, of a finite or countably infinite number of nonoverlapping sets is equal to the sum of their measures. (Where the number of sets is countably infinite, the existence of a measure will involve the question of convergence of a series.)

On the other hand, generalizations of measures do not illustrate the classic postulate which asserts that the whole is greater than one of its parts. That axiom applies to finite sets only. The essence of infinity is that the whole of an infinite collection does equal a part. Then, in general measure theory, a statement which combines finite and infinite attributes is: The measure of a set is either equal to or greater than the measure of a proper subset.

Again, abstraction from particular measures leads to the assumption: The measure of a set is zero or some positive real number. Also, it is postulated that the measure of the empty set $\emptyset$ *is zero.*

That "nothing" and zero are not identical is indicated by the fact that the converse of the axiom just stated is false. If the Lebesgue measure of a set is zero, this aggregate is not necessarily empty. In general measure theory, a set consisting of a single point like the origin, or the point $x = 1$ *on the* X*-axis, or the point* $x = 3$*, etc. measures zero. (The reason for this is that the interval is the basis of linear measure, and a single point can be covered by an arbitrarily small interval.)*

If we accept a postulate stating that for finite and countably infinite sets the whole is equal to the sum of its parts, then the measure of a set containing two isolated points is zero, as is the measure of any finite or countably infinite collection of isolated points.

Now, referring to a question raised earlier, the aggregate of all fractions with numerator 1 can be shown to measure zero, and the same is true of the class of all rational numbers between 0 and 1. For this reason, when such sets are removed from the interval $[0, 1]$*, the residual set still measures 1 unit. Thus the aggregate of irrational numbers in* $[0, 1]$ *measures 1 unit. More formally, if we symbolize the class removed by* E *and the residue by* E'*, the logical sum or union of* E *and* E' *is the unit interval. Therefore*

$$\begin{aligned}\text{(measure of } E\text{)} + \quad \text{(measure of } E'\text{)} &= \text{measure of } [0,1] \\ 0 + \quad \text{measure of } E' &= 1 \\ \text{measure of } E' &= 1\end{aligned}$$

Here the measure of the whole interval is equal to the measure of the part E', but is greater than the measure of the part E.

Let us agree that the measure of the interval $[0,1]$ is its length, 1. The measure of an arbitrary interval $[a,b]$ is obviously its length, $b-a$. Similarly, if we have two disjoint intervals $[a_1,b_1]$ and $[a_2,b_2]$, it is natural to interpret the length of the set E consisting of these two intervals as the number $(b_1-a_1)+(b_2-a_2)$.

However, the concept of length (measure) of a set in the line requires a rigorous mathematical definition. The problem of defining the length of sets or, as we may now say, of measuring sets is very important, because it is of vital significance in generalizing the concept of an integral[88]*. The concept of measure of a set also has applications to other problems in the theory of functions, in probability, topology, functional analysis, etc.*

Harnack *(1885) discovered that any countable subset $\{x_0, x_1, x_2, \dots\}$ of the line continuum $\mathbb{R}$ could be covered by a collection of intervals of arbitrarily small total length: For one can cover x_0 by an interval of length $\frac{\epsilon}{2}$, x_1 by an interval of length $\frac{\epsilon}{2^2}$, x_2 by an interval of length $\frac{\epsilon}{2^3}, \dots$, so that the total length of the intervals used is $\leq \epsilon$.*

This seemed to show that countable sets were of measure zero. Yet, mathematicians were reluctant to say this of dense countable sets, like the rationals. **Jordan** *(1892) demonstrated that dense countable sets like the rationals also have measure zero.*

It can be shown that a set may have the cardinality of the continuum and yet have measure zero! Indeed, construct the set P (known as Cantor's perfect set) as follows: delete from the closed interval $[0,1]$ the open interval $\left(\frac{1}{3},\frac{2}{3}\right)$, which forms the middle third. From each of the remaining intervals $\left[0,\frac{1}{3}\right]$ and $\left[\frac{2}{3},1\right]$ we delete its middle third. This process of deleting the middle third of the remaining intervals can be continued infinitely and generally, at the nth step we have thrown out 2^{n-1} adjacent intervals of length $\frac{1}{3^n}$. The sum of all removed intervals is then equal to

$$S = \frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \dots \frac{2^{n-1}}{3^n} + \dots = \frac{1/3}{1-2/3} = 1$$

Thus, the sum of the lengths removed to form the Cantor set is 1; Therefore, the set itself has the measure zero[89].

[88] Since an integral can be viewed as the *area* under a graph, its dependence on the concept of measure is clear.

[89] *Alternative proof*: Consider all numbers of the form $0.a_1a_2a_3\dots$ (in base 3) with $a_i = 0$ or 2. Evidently, this set covers *part* of the real numbers in $[0,1]$. Now, the

Armed with this preliminary background, we may next proceed to a more rigorous treatment of the Lebesgue integral.

(C) The measure of a set of points

We shall now define a new generalization of 'length', starting from an open set, which may contain an infinity of intervals.

The measure of an open subset of the interval (a, b) is defined to be the sum of the lengths of its constituent intervals. It is always convergent, since the sum of any finite number of terms is the sum of the lengths of a finite number of non-overlapping intervals, all contained in an interval (a, b), and so is not greater than $b - a$. Hence, the measure of any open set contained in (a, b) does not exceed $b - a$.

The exterior measure $m_e(S)$ of any bounded linear point-set S is the highest lower bound of the combined length of any set of intervals covering S. Clearly,

$$0 \leq m_e(S) \leq b - a$$

The interior measure $m_i(S)$ of S is defined as the difference between the length $b - a$ of any bounded interval (a, b) containing S and the exterior measure of the complement of S w.r.t. (a, b):

$$m_i(S) = b - a - m_e\{(a, b) - S\}$$

The set S is a measurable set with the Lebesgue measure $m(S)$ if and only if

$$m_e(S) = m_i(S) = m(S)$$

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set is *non-denumerable* since each a_i can take two values such that the cardinality is $2^{\aleph_0} = C$. Yet it can be shown that the *measure* of this set is zero.

(D) The measure of a function

A bounded function $f(s)$ defined on (a,b) is measurable on (a,b) if and only if the set of points x in (a,b), such that $f(x) \le c$, is measurable for every real value of c.

A continuous function is measurable. All the ordinary functions of analysis may be obtained by limiting processes from continuous functions, and so are measurable. An extreme example is

$$f(x) = \lim_{m\to\infty} \left[\lim_{n\to\infty} \{\cos m!\pi x\}^{2n} \right]$$

If x is rational, $m!x$ is an integer if m is large enough. Hence

$$f(x) = \begin{cases} 1, & x \text{ rational} \\ 0, & \text{otherwise} \end{cases}$$

This function presented by Dirichlet — which, as we saw earlier, has no Riemann integral — is therefore measurable.

(E) The Lebesgue integral

Let there be given a real function $y = f(x)$, measurable and bounded on the bounded interval (a,b). Let c and d be the highest lower bound and the lowest upper bound of $f(x)$, respectively. As in the Riemann integration, the integral is obtained by dividing the interval of variation of $f(x)$

$$c = y_0 < \eta_1 < y_1 < \eta_2 < \cdots < \eta_n < y_n = d.$$

Let e_ν be the set of points x in (a,b) such that $y_{\nu-1} < f(x) \le y_\nu$ $(\nu = 1,2,\ldots,n)$. Since $f(x)$ is measurable, the sum $\sum_{\nu=1}^{n} \eta_\nu m(e_\nu)$ can be shown to tend to a unique finite limit I for every sequence of partitions as $\max|y_\nu - y_{\nu-1}| \to 0$. *The quantity*

$$I = \lim_{\max|y_\nu - y_{\nu-1}|\to 0} \sum_{\nu=1}^{n} \eta_\nu m(e_\nu) \equiv \int_a^b f(x)\,dx$$

is the definite integral of $f(x)$ over (a,b) in the sense of Lebesgue. The Lebesgue integral can also be defined for an unbounded function, as well *as for an unbounded interval. If a function has a Riemann integral, then it also has a Lebesgue integral, and the two are equal.*

We have demonstrated above that the Dirichlet function does not have a Riemann integral. Since it is measurable it has a Lebesgue integral, the value of which is readily seen to equal zero. Thus, a function can be discontinuous almost everywhere (except on a set of measure zero) and yet have a Lebesgue integral.

During the first quarter of the 20th century, the notion of the integral was further extended. **Radon** *(1913) invented an integral transform that bears his name. His transform embraces both the Stieltjes and Lebesgue's integral and is in fact known as the Lebesgue-Stieltjes integral. The generalizations cover not only broader or different notions of integrals on point sets of n-dimensional Euclidean spaces but even domains in more general spaces such as spaces of functions. The applications of these more general concepts are now found in the theory of probability and stochastic processes, statistical mechanics, quantum mechanics and quantum field theory, ergodic theory, spectral theory, and harmonic analysis (generalized Fourier analysis).*

Genetics and Evolution (1901–1909)

The theory of evolution did not arise fully formed in 1859. *Of course neither Wallace nor Darwin knew exactly how isolation and heredity could create new species, because they did not know genetics*[90]. *Modern evolutionists con-*

[90] Only in the 1940s, as the nature of the gene was becoming clear could Harvard biologist **Ernst Mayr** offer an explanation: a new species is a result of some change in the environment that splits the population. The sudden rise of a mountain, a prolonged climatic change, the shift of a river — any of these will suffice. If the two groups are kept apart for a sufficiently long time, unable to breed with each other, natural selection and random mutations will force their gene pools to diverge. If the old barrier to the two populations then drops and the organisms mingle again, they might no longer be able to interbreed because their genes are no longer compatible (or their interbreeding might result in sterile individuals). They have become two species.

tinue to improve on **Darwin** *and* **Wallace***:* **Hugo Marie de Vries** *(1848–1935, Holland), a Dutch botanist, discovered mutations and pointed out the importance of mutations in the process of evolution (1900). Shortly thereafter (1903) it was recognized that chromosomes, minute threadlike structures in the cell nucleus, are the carriers of hereditary characteristics. This discovery, which also showed a linkage of characteristics that* **Mendel** *had not suspected, was made independently by* **Walter Stanborough Sutton** *(1877–1916, U.S.A.),* **Theodor Boveri** *(1862–1905, Germany), and* **Thomas Hunt Morgan** *(1866–1945, U.S.A.).*

Finally, in 1909, **Wilhelm Ludvig Johannsen** *(1857–1927, Holland), a plant physiologist, discovered the unit of heredity — the gene — and coined the terms genotype and phenotype. The particular combination of genes that an organism has – is called its genotype (genos = race, kind). An organism's genotype represents the actual genes that are present in the cells. The effect caused in the organism by these genes is called its phenotype (pheno = to show).*

The word phenotype refers to what you can see [e.g. the genotype of a pure tall plant is TT; its phenotype is tall. The genotype of a pure short plant is tt; its phenotype is short. The genotype of an F_1 pea plant is Tt; its phenotype is tall. In such a plant the dominant trait appears, or is expressed. The recessive trait is hidden. Thus the phenotype tall can result from two different genotypes, TT or Tt. Plants pure for tallness have two T genes. When both genes of the pair are the same, the organism is said to be homozygous for that trait. An organism may be homozygous dominant or homozygous recessive.

If the paired genes are not the same, the organism is heterozygous for that trait. Heterozygous organisms are also called hybrid. Plants that have the mixed genotype Tt are heterozygous (or hybrid) tall.

Genes that have contrasting effects on a characteristic are called alleles. Tallness (T) and shortness (t) are alleles. Genes that have more than two alleles are said to have multiple alleles (e.g. genes for blood type in humans have 3 alleles). However, an individual can have only 2 alleles for a particular characteristic, even when the gene has more than 2 alleles. An individual inherits just 2 alleles, one from each parent, for each paired gene location on its chromosomes.

Over the generations, the gene pool of a population (i.e. the total genetic material) changes. A gene pool changes when the frequency of the alleles in the gene pool change. Allele frequency is the term used to describe how often a particular allele occur in a population.

Evolution by natural selection depends on the genetic makeup of an organism. If a trait is determined by genes, then when a trait is selected, the genes *for it are also selected. The frequency of traits in a population is determined simply by observing the members of the population.*

In 1908, **G.H. Hardy** *(1877–1947) and* **W. Weinberg** *(1862–1937) discovered independently the mathematical result that in a sexually-reproducing population the frequency of the allele does not change if:*

- *mating is random*
- *mutations do not occur*
- *there is no immigration or emigration*
- *the population is large and the allele frequencies are high (to prevent genetic drift, i.e. random changes in gene pool).*

Under these ideal conditions, the allele frequencies are related to trait frequencies through an algebraic equation. The conclusion is known as the Hardy-Weinberg principle and implies that under the above conditions evolution does not occur.

Random mating means that there is no preference shown for any particular phenotype when mating [e.g. an individual that is Aa is not preferred over an aa individual].

Mutations are changes in the genetic material. When mutations occur in reproductive cells, they can be inherited. Gene mutations introduce new alleles into a population and are the main source of genetic variation. The rate is generally very low in any population, but its effects may be amplified by natural selection.

Migration, the movement of organisms from one location to another, allows for evolutionary change in two ways. First, it causes gene flow in or out of a population and consequently the appearance of new traits. Since some traits may be beneficial for survival in the new environment, gene combinations that cause these traits would persist or even be preferentially enhanced in succeeding generations, thus changing their frequency in the gene pool. In small populations, random events may drastically affect gene frequencies even in the absence of migration or mutations and cause genetic drift.

The Hardy-Weinberg principle is important in understanding evolution because it implies that reproduction by itself does not lead to evolutionary change in large populations. However, changes in the conditions listed above will result in changes in the gene pool and thus cause evolution. Since the rigid conditions required for the operation of the Hardy-Weinberg principle are probably never met in the real world, one mathematically expects evolution to occur.

Natural selection is probably the most important factor in changing gene pools. When environmental conditions favor a particular trait, the frequency of the alleles that produce that trait will tend to increase in the next generation. If the environment does not favor a particular trait, the frequency of its alleles will tend to decrease. Thus, natural selection can lead to the formation of new species and the extinction of old ones.

1901–1912 CE **Wilhelm Weinberg** (1862–1937, Germany). Physician and founder of *population genetics*. Discovered in 1908 (independently of G.H. Hardy and slightly ahead of him) the equilibrium law of monohybrid populations[91] and the varied processes of attainment of equilibria in polyhybrid population. In his studies of population genetics, Weinberg's derivations of the correlation between relatives expected under Mendelian heredity took into account both genetic and environmental factors: he was the first to partition the total variance of phenotypes into genetic and environmental components. He was also first to construct morbidity tables modeled after the long-known mortality tables.

Weinberg was born to a Jewish father and a Protestant mother. He studied medicine at the Universities of Tübingen and Münich and obtained his Ph.D. in 1886. After clinical experience in Berlin, Vienna and Frankfurt he established himself in Stuttgart as a general practitioner and obstetrician (1889).

[91] *Hardy-Weinberg principle*. In the absence of mutation and selection, the frequencies of genés allele in any large, randomly mating population will reach an equilibrium distribution in one generation and remain in equilibrium thereafter regardless of whether the alleles are dominant or recessive. Let A and a be two different alleles that are passed down in the population from one generation to the next in a given gene (in standard Mendelian notation, 'A' is dominant and 'a' is recessive). A given individual could then have one of the three combinations AA, Aa or aa. Let mating be random and let all three combinations have an equal likelihood of surviving to produce offsprings. If p and $q = 1 - p$ be the relative frequencies of A and a respectively in the population, the three genotypes AA, Aa and aa, occur with relative frequencies p^2, $2pq$, q^2, respectively. The proportions $p^2 : 2pq : q^2$ are reached already in a single generation of random mating, and are kept stable for all successive generations as long as the basic assumptions underlying the process do not change.

When there is no alternative way to ascertain whether mating is random, the Hardy-Weinberg ratio becomes a useful criterion for random mating.

For 42 years he had a large private practice and also acted in public capacities as physician to the poor. He attended more than 3500 births, including more than 120 twin births. He thus came to be interested in the incidence of monozygotic and dizygotic twin births in his statistical data (1901). He proceeded to discover differences between mono- and dizygotic twins in a variety of traits, including the inheritance of a twinning tendency for dizygotic but not for monozygotic twins.

When Weinberg became aware of Mendelism he asked himself "how would different laws of inheritance influence the composition of the relatives of given individuals". This eventually led him to the Hardy-Weinberg law. Weinberg had no personal collaborators or students. Only in his later years did a new generation begin to explore his field of research.

1901–1928 CE **Roald Amundsen** (1872–1928, Norway). Polar explorer. First to navigate and chart the *Northwest passage* and to fix the position of the *North Magnetic Pole* (1903–1905); First to reach the *South Pole* (Dec 14, 1911). Flew across the North Pole with **Lincoln Ellsworth** and **Umberto Nobile** (1926); Disappeared on flight to rescue Nobile who was lost returning from the North Pole.

Author of *North West Passage* (1908); *The South Pole* (1912); *The North East Passage* (1918–1920); *Our Polar Flight* (1925); *First Crossing of the Polar Sea* (1927); *My Life as an Explorer* (1927).

Amundsen was born at Borge, a village near Oslo to a family of Viking ancestry. From boyhood days his life was singularly purposeful — he wished to be a polar explorer. Like Fridjof Nansen before him he devoted a great deal of time to training and strengthening his physique to be ready for the hazardous adventures he was determined to undertake. He bowed however to his mother's wish that he study medicine. But at the age of 21, when both his parents died, Roald Amundsen sold his medical textbooks, packed away the cranium he had studied and announced his intention of becoming a polar explorer.

From his painstaking study of polar expedition literature, Amundsen had learned their inability to captain a vessel. Consequently he went to sea (1894) aboard a sealing vessel, gaining his master's ticket commanding the "*Belgica*" expedition to Antarctica (1897–1899). His next project was to head an arctic expedition, in search of the sea route north of the North American Continent (so-called *Northwest Passage*). His scientific goal was to reach the *magnetic north pole*. He therefore left for Hamburg, where he studied earth magnetism, and at the same time laid meticulous plans for his expedition.

He set out from Christiana (now Oslo) in June 1903 on board the *Gjoa*. For two years the expedition stayed on King William Island, studying the polar magnetic field and learning the Eskimos' way of life (clothing, food, customs, dog-team transportation, etc.). In October 1905 he traveled 800 km with dog-teams across the ice to Eagle City in Alaska to tell the world of the expedition's achievement. While making plans to drift with Nansen's *Fram* with the ice from Siberia toward the North Pole, he changed his plans (1910) and won the race to the South Pole, beating **Robert Falcon Scott** to it.

Amundsen's victory in the race for the South Pole by no means satisfied his desire to reach new goals. On his return from Antarctica, he immediately set preparations in motion for a new expedition. The Arctic was still Amundsen's first love, and he aimed to explore its remaining unknown areas and to repeat Nansen's attempt to drift over the Pole. WWI delayed the execution of the plan, but in June 1918 the expedition left Norway. The "Fram" was no longer seaworthy, so Amundsen designed his own ship, the "Maud", christening it — characteristically enough — not with champagne, but with a block of ice.

The "Maud" expedition, loaded with apparata for oceanographic, meteorological and earth-magnetism measurements, was the biggest and best equipped geophysical expedition ever to have embarked on a polar exploration. But the project was to bring one disappointment after another.

Sailing into the Arctic, the ship froze into the coastal ice and lay helpless for the two first winters. It soon needed extensive repairs. These were carried out in Seattle where the "Maud" was equipped for more years in the ice. In June of 1922 the ship again moved north, only to freeze fast by Wrangel Island, on the far northeast of the USSR.

The ship moved with the ice onto the continental shelf off northeastern Siberia, where it remained for three years.

The ambitious expedition had failed to attain its geographical goals, but the geophysical data which was compiled, largely by meteorologist/oceanographer **Harald Ulrik Sverdrup**, earned the "Maud" expedition the reputation of being one of the most important research projects ever carried out in the Arctic.

In May 1926 Amundsen left Spitsbergen aboard the airship *Norge*. With him were Lincoln Ellsworth, the Italian Umberto Nobile — who had constructed the vessel and flew it — and the pilot Hjalmar Riiser-Larsen, who served as navigator. In addition there was a crew of 12.

After a flight of only 16 hours, the men were able to drop the Norwegian, American and Italian flags over the North Pole. On 14 May the "Norge" landed at Teller in Alaska. The crew had covered 5,456 kilometers in 72 hours, and were the first men to have flown from Europe to America. The route of the "Norge" had been plotted right across unknown polar territory, and Amundsen was able to state that there were no land areas there. The last remaining blank on the world map had been filled in.

The acclaim of the world reached new heights. In the USA and Japan in particular, his name was especially revered. But the period was saddened by an unfortunate enmity that had arisen between Amundsen and Umberto Nobile, who tried to detract from Amundsen's part in the "Norge" flight, while Amundsen criticized the airship.

Nevertheless, he showed his magnanimity to the full when the news came in May, 1928 that Nobile's new airship, the "Italia" had crashed in the Arctic.

Without hesitation Amundsen volunteered to take part in a rescue attempt, and in June he was one of the six men who took off from the town of Tromsø in a French aircraft, the Latham. Nobile and his crew were rescued on 22 June. But three hours after Amundsen's plane took off it transmitted what were to be its final signals. The aircraft never returned.

In a letter, describing his reactions at the time he reached the South Pole on Dec 14, 1911, Amundsen openly confessed that "*no man has ever stood at the spot so diametrically opposed to the object of his real desires*". For Amundsen, the man who went where none have gone before, a new goal always beckoned. He himself described his life as a "*contest journey towards the final destination.*"

1901–1935 CE **Issai Schur** (1875–1941, Germany and Israel). Mathematician. Contributed to number theory and algebra. Laid the foundations to the theory of linear representations of groups. Extended finite group theory to compact groups. Named after him are: *Schur's canonical form* (triangular matrices), *Schur index*, '*S-functions*', *Schur's inequality*, *Schur's lemma* (irreducible matrices) and *Schur-Toeplitz theorem*[92].

Schur was born in Mohilev, the Ukraine, and was educated at the University of Berlin (1894–1901). He was a pupil of **Frobenius**. In 1920 he was appointed a professor of mathematics in Berlin, but was forced to relinquish his position in 1935 because of the Nazi race laws. Schur came to Israel in 1939, and died in Tel-Aviv.

1901–1940 CE **Karl Landsteiner** (1868–1943, Austria-Hungary and U.S.A.). Immunologist and pathologist. Elucidated the fundamental basis of immunological processes. One of the founders of *immunochemistry* and the discoverer of the basic blood types. Defined the relationships between *antibody* and *antigen* with chemical precision. Awarded (1930) the Nobel prize for physiology or medicine for the discovery of the blood groups.

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[92] A matrix is unitarily similar to a diagonal matrix iff it is normal.

Discovered (1901) the A, B, O human blood types, the ABO classification, and showed that there are also many subsidiary groupings (1927), one of which, Rh (Rhesus) is important in the aetiology of hemolytic disease of the newborn (1940). This discovery made possible the development of safe blood transfusions. It also provided useful techniques for establishing paternity and for genetic studies on the origins of human populations. Discovered (1908) that a virus causes poliomyelitis.

Landsteiner explored and defined the various types of immune response and was the first to realize that allergies have an immunological basis. In 1927 he reported (with **Philip Levine**) the discovery of the M and N *agglutinogens*.

Landsteiner was born in Vienna to Jewish parents[93]. After graduating in medicine (1891) he studied chemistry under **Emil Hermann Fischer**. In 1898 he returned to Vienna to begin the lifelong studies which were to transform the science of immunology and greatly increase our understanding of the body's mechanism for protection against disease. From 1909 he was a professor of pathological anatomy, but in 1922 he fled the chaos of postwar Vienna and eventually reached the U.S.A. to join the Rockefeller Institute in New York (1922–1929).

1902 CE, May 08, 07:52 LT Eruption of *Mont Pelée* on the Island of Martinique destroyed the Caribbean coastal town of St. Pierre, and all but two of its 28,000 inhabitants perished within minutes by a *nuée ardente*[94]. The hot-gas hurricane avalanche plunged down the slopes at a speed of about 100 km/h and engulfed the town with a searing 800 °C emulsion of gas, glass and dust. The gas was mostly CO_2. The city was burned to ashes by a *fire storm*, similar to that which occurred in several German cities during WWII and the atomic destruction of Hiroshima (and perhaps the *fire and brimstone* of Sodom and Gomorrah!).

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[93] **Landsteiner** filed an injunction to prevent his inclusion in *Who's Who in American Jewry*. He explained:
"It will be detrimental to me to emphasize publicly the religion of my ancestors, first as a matter of convenience, and, secondly, I want nothing in the slightest degree to cause any mental anguish, pain, or suffering to any members of my family. My son is now nineteen years old and he has no suspicion that any of his ancestors were Jewish."

[94] Hot ash and dust fragments and gases are ejected in a glowing cloud that runs downhill with amazing speed. The solid particles are actually buoyed up by the hot gases, so that there is little frictional resistance to this incandescent, fluidized avalanche. It is a silent and swift killer. **Frank Alvord Perret** (1867–1943, U.S.A.) made extensive observations of hundreds of *nuées ardentes* at close quartes since 1902, and provided the basis for much of the later work in this field.

It is sobering to scientists who render advice to others to recall the statement of Professor Landes of St. Pierre College, issued a day before the cataclysm: "*The Montagne Pelée presents no more danger to the inhabitants of Saint Pierre than does Vesuvius to those of Naples*". Professor Landes perished with the others.

1902–1903 CE **Walter Stanborough Sutton** (1877–1916, USA). Physician and geneticist. Provided first conclusive evidence that chromosomes carry units of inheritance and occur in distinct pairs. His work formed the basis for the chromosomal theory of *heredity*.

He suggested that Mendel's "factors" are located on chromosomes. After observing chromosomal movements during meiosis, Sutton developed the chromosomal theory of heredity. He noticed that chromosomes occur as pairs, and that gametes (egg and sperm cells) receive only one chromosomes from each pair when they form during meiosis. This corroborated Mendel's theory that the genetic "factors" were segregated. Sutton gave Mendel's "factors" the name we use today: "genes".

In 1903 Sutton [and independently **Theodor Boveri** (1862–1915, Germany) proposed that each egg or sperm cell contains only one of each chromosomes pair. This connected two phenomena: the patterns by which pairs of Mendel's factors assort themselves and the precisely similar sorting and recombination of the chromosomes in the formation of the germ cells and the fertilization of the egg.

Sutton was born in Utica, NY. Had private surgical practice in Kansas city.

1902 CE **Luigi Bianchi** (1856–1928, Italy). Mathematician. Obtained cyclic relations between the covariant derivatives of Riemann's symbols. These identities have proven to be of the greatest importance for subsequent researches, both in differential geometry and relativity. In any space with an affine connection, not necessarily Riemannian, one can define the Riemann curvature tensor. The Bianchi identity is then a differential relation satisfied by the Riemann tensor components, their first derivatives and the connection.

These identities can be extended to the curvature on an arbitrary *fiber bundle*, where they take on the simple form[95] $D\Omega = 0$, in the powerful

[95] Starting from the Riemann-Christoffel curvature tensor

$$R^i_{jk\ell} = \Gamma^i_{rk}\Gamma^r_{j\ell} - \Gamma^i_{r\ell}\Gamma^r_{jk} + \frac{\partial \Gamma^i_{j\ell}}{\partial x^k} - \frac{\partial \Gamma^i_{jk}}{\partial x^\ell},$$

we assume that the affine connection Γ^i_{jk} is symmetric in its lower indices and employ geodetic coordinates (in which locally $\Gamma = 0$) at the point being consid-

Cartan notation. The deep significance of the Bianchi identity is its pivotal role in proving the relation between analytical integrals over a manifold and topological indices.

Bianchi was born in Parma and died at Pisa. He studied under **Betti** and **Dini** at the University of Pisa and then continued his studies at Münich and Göttingen under **Klein**. From 1881 onwards he was a professor at Pisa.

In the context of general relativity, the *contracted form* of these identities ensure automatic local energy-momentum conservation in the **Einstein** field equations.

1902–1908 CE **William Maddock Bayliss** (1860–1924, England). Physiologist. With **Ernest Henry Starling** (1866–1927, England), physiologist, found (1902) the first evidence of the existence of *hormones* through their discovery of the digestion hormone *secretin* (secreted by the intestine and activating the pancreas to liberate digestive fluids). Starling used the word *hormone*[96] (1905) to describe secretin and other substances produced in one

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ered. The covariant derivative of the above expression is then simply

$$R^i_{jk\ell;m} = \frac{\partial R^i_{jk\ell}}{\partial x^m} = \frac{\partial^2 \Gamma^i_{j\ell}}{\partial x^m \partial x^k} - \frac{\partial^2 \Gamma^i_{jk}}{\partial x^m \partial x^\ell},$$

since the Γ^i_{jk} (but not their derivatives necessarily) all vanish at this point. Cyclically permuting the indices k, ℓ, m in the above equation and adding the three resulting equations yields the identity

$$R^i_{jk\ell;m} + R^i_{j\ell m;k} + R^i_{jmk;\ell} = 0.$$

But this is a tensor equation and, having been proved true in the geodetic frame, must be true in all frames. Also, since the chosen point can be any point, it is valid at all points in space (i.e. on the manifold). It is the *Bianchi identity*, and applies not only to affine manifolds but to *fiber bundles*, constructed on a spacetime "base manifold" with a particular symmetry Lie group as the "fiber". In modern *Gauge Theories*, thought to describe all known particle and field interactions, the *vector potentials* constitute the bundle connection, the *field strengths* make up the curvature tensor, and the *Bianchi identity* $D\Omega = 0$ is simply a generalization of the two source-free Maxwell equations in electromagnetism. For an n-dimensional base manifold, the fiber-bundle version of Bianchi's identity reduces to the Riemann-curvature version if the *fiber* is chosen to be the Lie group $GL(n, R)$ of local tangent-space base changes; the resulting fiber bundle is known as the *frame bundle* of the given base manifold.

96 From the Greek *Horman* = to set in motion, to arouse. Name chosen because of the marked stimulating effect of the endocrine secretions on various organs of the body.

part of the body but active elsewhere. Their joint work helped to establish *endocrinology* as a special area of scientific knowledge. Bayliss was the first to describe (1908) the action of hormones and put forward the theory of hormonal control.

Starling was a professor at University College, London (1899–1923). Bayliss taught physiology at the same university (1888–1924).

Hormones

Endocrinology is concerned with the study of the biosynthesis, storage, chemistry, and physiological function of hormones and with the cells of the endocrine glands and tissues that secrete them. In medicine, endocrinology deals with disorders of the endocrine system and its specific secretions, hormones. Most hormones reach their targets via the blood. Although every organ system secretes and responds to hormones (including the brain, lungs, heart, intestine, skin, and the kidney), the clinical specialty of endocrinology focuses primarily on the endocrine organs, meaning the organs whose primary function is hormone secretion. These organs include the pituitary, thyroid, adrenals, ovaries and testes, and pancreas.

All multicellular organisms need coordinating systems to regulate and integrate the function of differentiating cells. Two mechanisms perform this function in higher animals: the nervous system and the endocrine system. The endocrine system acts through the release (generally into the blood) of chemical agents and is vital to the proper development and function of organisms. The integration of developmental events such as proliferation, growth, and differentiation (including histogenesis and organogenesis) and the coordination of metabolism, respiration, excretion, movement, reproduction, and sensory perception depend on chemical cues, substances synthesized and secreted by the specialized cells within the animal.

Hormones serve as a means of communication among various parts of an organism. They act as 'chemical messengers' that help these parts function together in a coordinated way. In plants, hormones regulate many aspects of growth. In man and other animals, hormones control and regulate such important body activities as growth, development, metabolism and reproduction.

Since 1902, more than 30 hormones produced by the human body have been identified, e.g.: insulin, thyroxine, cortisone, testosterone ($C_{19}H_{28}O_2$), estrogen, adrenalin ($C_9H_{13}O_3N$), relaxin. Ways were found of removing hormones from living tissues. The first hormone to ever be synthesized was oxytocin (1954).

Most human hormones are either steroids (sex hormones and the hormones of the adrenal cortex) or contain some form of amino acids. The chemical structure of hormones enable it to combine with a receptor in the cells of its target. The union of the hormone with the receptor triggers a change in the chemical processes of the cell. This change, in turn, modifies many of the hundreds of chemical activities of the cell and cause the target to behave in a certain way.

Hormones are biologically effective in small amounts and are produced by endocrine glands within the body that empty their secretions directly into the lymph and the bloodstream. They can be divided into three classes of compounds: proteins, amino-acid derivatives, and steroids.

Protein type of hormones are produced by the pituitary glands, parathyroid glands, islets of Langerhans and cells of the gastrointensinal tract.

Amino-acid based types are produced by the thyroid glands and the adrenal glands. These hormones unite with receptors on the outer membrane of the target cell and this union may change the structure of the cell membrane, allowing certain substances to enter or leave the cell. These substances alter the chemical activities of the cell. In other cases, the union seems to influence the activities of enzymes located on the membrane.

Steroid types of hormones are produced by the testes, ovaries, placenta, corpus luteum, and adrenal cortex. These hormones become attached to receptors and then enter the nucleus of the target cell. In the nucleus, the hormone affects the activity of the genes.[97]

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[97] An estimated 1 million people in the United States, half of them adolescents, abuse anabolic steroids, a group of synthetic hormones. Anabolic steroids are synthetic androgens (male reproductive hormones) that were developed in the 1930s to prevent muscle atrophy in patients with diseases that prevented them from moving about. In the 1950s, anabolic steroids became popular with professional athletes, who used them to increase muscle mass, physical strength, endurance, and aggressiveness. In truth, their athletic performance was probably enhanced, at least in part, by drug-induced euphoria and increased enthusiasm for training.

As with other hormones, the concentration of steroids circulating in the body is precisely regulated, so use of anabolic steroids interferes with normal physiological processes. Even during short-term use and at relatively low doses, anabolic

If an organism fails to produce the proper kind or amount of hormones, serious disturbances may result[98].

steroids have a significant effect on mood and behavior. At higher doses, users experience disturbed thought processes, forgetfulness, and confusion, and often find themselves easily distracted. The term “steroid rage” refers to the mood swings, unpredictable anger, increased aggressiveness, and irrational behavior exhibited by many users.

Anabolic steroids elevate blood pressure, damage the liver, and increase low-density lipoprotein (LDL) concentration, raising the risk of cardiovascular disease. In adolescents, these steroids cause severe acne and stunt growth by premature closing the growth plates in bones. Abuse of these hormones reduces sexual function and can shrink the testes, leading to sterility. These drugs remain in the body for a long time. Their metabolites (breakdown products) can be detected in the urine for up to six months.

When their serious side effects became known in the 1960s, anabolic steroid use became controversial, and in 1973 the Olympic Committee banned their use. They are now prohibited worldwide by amateur and professional sports organizations. However, according to the U.S. Drug Enforcement Administration, a multimillion-dollar black market exists for these synthetic hormones. They are both injected and taken in pill form. Steroid abusers who share needles or use nonsterile techniques when they inject steroids are at risk for contracting hepatitis, HIV, and other serious infections.

The typical anabolic steroid user is a male (95%) athlete (65%), most often a football player, weight lifter, or wrestler. Surprisingly, though, about 10% of male high school students have used anabolic steroids and about one third of these students are not even on a high school team. These adolescents use the hormone only to change their physical appearance — to pump up their muscles (“bulk up”) — and increase endurance. Many anabolic steroid users have difficulty realistically perceiving their body images. They remain unhappy even after dramatic increases in muscle mass.

People also abuse other hormones, including human growth hormone (GH) and erythropoietin. Human growth hormone, like anabolic steroids, helps build muscle mass but also causes acromegaly. Erythropoietin increases the concentration of red blood cells. Although increased oxygen transport enhances the performance of an endurance athlete up to 10%, abnormally high concentrations of red blood cells can cause serious cardiovascular problems. Erythropoietin abuse has caused the deaths of several athletes.

[98] In primitive men, dysfunction of their hormones resulted in disturbed behavior that was sometimes interpreted by their tribesmen as a manifestation of supernatural powers; perhaps such primitive man merely had a severe hyperthyroid condition, causing his eyeballs to bulge so strangely. His sometimes rapid, excitable speech may have given the impression that he was ‘communicating’ with

The hormone concept consolidated earlier anatomical, physiological and clinical observations: 17^{th} century anatomists [e.g., **Marcello Malpighi** *(1628–1694, Italy);* **Thomas Wharton** *(1614–1673, England)] had been fascinated by the 'glands' (pancreas, thyroid, parotid, ovaries, etc.), postulating various physiological roles.*

The study of endocrinology began when ***A.A. Berthold*** *(1812–1886) noted (1849) that castrated cockerels did not develop combs and wattles or exhibit overtly male behavior. He found that replacement of testes back into the abdominal cavity of the same bird or another castrated bird resulted in normal behavioral and morphological development, and he concluded (erroneously) that the testes secreted a substance that "conditioned" the blood that, in turn, acted on the body of the cockerel. In fact, one of two other things could have been true: that the testes modified or activated a constituent of the blood or that the testes removed an inhibitory factor from the blood. It was not proven that the testes released a substance that engenders male characteristics until it was shown that the extract of testes could replace their function in castrated animals.*

The concept of internal secretion developed in the 19^{th} century; ***Claude Bernard*** *described it in 1855, but did not specifically address the possibility of secretions of one organ acting as messengers to others. Still, various endocrine conditions were recognized and even treated adequately (e.g., hypothyroidism with extract of thyroid glands).*

Hermann Boerhaave *(1668–1739, Holland) elaborated mechanical models of secretion. Glands' secretions were firmly implicated in disease by 19^{th} century clinicians.*

In 1894, **Edward Albert Sharpey-Schäfer** *(1850–1935, England) and* **George Oliver** *(1841–1915, England) identified a physiologically active substance called* adrenaline*, secreted by the adrenal medulla.*

Many of the hormones' metabolic effects have been elucidated. Some have been synthesized and replacement therapy is frequently possible in cases of insufficiency.

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unseen spirits, and he would consequently be allowed to take over the role of the witch doctor. Sometimes a disturbance of the pituitary glands would result in either giantism or dwarfism, through which an individual could convince others of his special 'powers'.

The Insulin Story – or, the Nobel Committee goofs again

In 1869 ***Paul Langerhans****, a medical student in Berlin, was studying the structure of the pancreas under a microscope when he noticed some previously unidentified cells scattered in the exocrine tissue. The function of the "little heaps of cells," later known as the* Islets of Langerhans*, was unknown, but* ***Edouard Laguesse*** *later suggested that they may produce a secretion that plays a regulatory role in digestion.*

In 1889, the physician ***Oscar Minkowski*** *in collaboration with Joseph von Mehring removed the pancreas from a healthy dog to demonstrate this assumed role in digestion. Several days after the dog's pancreas was removed, Minkowski's animal keeper noticed a swarm of flies feeding on the dog's urine. On testing the urine they found that there was sugar in the dog's urine, demonstrating for the first time a relationship between the pancreas and diabetes. In 1901, another major step was taken by* ***Eugene Opie****, when he clearly established the link between the Islets of Langerhans and diabetes: Diabetes mellitus is caused by destruction of the islets of Langerhans and occurs only when these bodies are in part or wholly destroyed. Before this demonstration, the link between the pancreas and diabetes was clear, but not the specific role of the Islets.*

Over the next two decades, several attempts were made to isolate the secretion of the Islets as a potential treatment. In 1906 ***George Ludwig Zuelzer*** *was partially successful treating dogs with pancreatic extract, but was unable to continue his work. Between 1911 and 1912,* ***E.L. Scott*** *at the University of Chicago used aqueous pancreatic extracts and noted a slight diminution of glycosuria, but was unable to convince his director and the research was shut down.* ***Israel Kleiner*** *demonstrated similar effects at Rockefeller University in 1919, but his work was interrupted by World War I and he was unable to return to it.* ***Nicolae Paulescu****, a professor of physiology at the Romanian School of Medicine, published similar work in 1921 that was carried out in France and patented in Romania, and it has been argued ever since that he is the rightful discoverer.*

However, the Nobel Prize committee in 1923 credited the practical extraction of insulin to a team at the University of Toronto. In October 1920, ***Frederick Banting*** *was reading one of Minkowski's papers and concluded that it is the very digestive secretions that Minkowski had originally studied that were breaking down the Islet secretion(s), thereby making it impossible to extract successfully. He jotted a note to himself: Ligate pancreatic ducts* *of the dog. Keep dogs alive till acini degenerate leaving Islets. Try to isolate internal secretion of these and relieve glycosurea.*

*He traveled to Toronto to meet with **J.J.R. Macleod**, who was not entirely impressed with his idea. Nevertheless, he supplied Banting with a lab at the University, an assistant (medical student **Charles Best**), and ten dogs, while he left on vacation during the summer of 1921. Their method was tying a ligature (string) around the pancreatic duct, and, when examined several weeks later, the pancreatic digestive cells had died and been absorbed by the immune system, leaving thousands of Islets. They then isolated the protein from these Islets to produce what they called* isletin. *Banting and Best were then able to keep a pancreatectomized dog alive all summer.*

*Macleod saw the value of the research on his return from Europe, but demanded a re-run to prove the method actually worked. Several weeks later it was clear the second run was also a success, and he helped publish their results privately in Toronto that November. However, they needed six weeks to extract the isletin, dramatically slowing testing. Banting suggested that they try to use fetal calf pancreas, which had not yet developed digestive glands; he was relieved to find that this method worked well. With the supply problem solved, the next major effort was to purify the extract. In December 1921, Macleod invited the biochemist **James Collip** to help with this task, and, within a month, the team felt ready for a clinical test.*

On January 11, 1922, Leonard Thompson, a fourteen-year-old diabetic, was given the first injection of insulin. However, the extract was so impure that he suffered a severe allergic reaction, and further injections were canceled. Over the next 12 days, Collip worked day and night to improve the extract, and a second dose injected on the 23rd. This was completely successful, not only in not having obvious side-effects, but in completely eliminating the symptoms of diabetes. However, Banting and Best never worked well with Collip, regarding him as something of an interloper, and Collip left the project soon after.

*Over the spring of 1922, Best managed to improve his techniques to the point where large quantities of insulin could be extracted on demand, but the extract remained impure. However, they had been approached by **Eli Lilly** with an offer of help shortly after their first publications in 1921, and they took Lilly up on the offer in April. In November, Lilly made a major breakthrough, and they were able to produce large quantities of purer insulin. Insulin was offered for sale shortly thereafter.*

Macleod and Banting were awarded the Nobel Prize in Physiology or Medicine in 1923 for the discovery of insulin. Banting, insulted that Best was not mentioned, shared his prize with Best, and Macleod immediately shared his with Collip. The patent for insulin was sold to the University of Toronto *for one dollar. The exact sequence of amino acids comprising the insulin molecule, the so-called primary structure, was determined by British molecular biologist* ***Frederick Sanger****. It was the first protein to have its structure be completely determined. He was awarded the Nobel Prize in Chemistry in 1958. In 1967, after decades of work,* ***Dorothy Crowfoot Hodgkin*** *determined the spatial configuration of the molecule, by means of X-ray diffraction studies. She had been awarded a Nobel Prize in Chemistry in 1964 for the development of crystallography.* ***Rosalyn Sussman Yalow*** *received the 1977 Nobel Prize in Medicine for the development of the* radioimmunoassay *for insulin.*

1902–1909 CE **Archibald Edward Garrod** (1857–1936, England). Physician and biochemical geneticist. Pioneered the study of inherited human metabolic diseases and first to link it to genetic causes (1908). Recognized that gene products are proteins and that a gene is a recipe for a single chemical. Proposed the mode of action of the hereditary substance known as '*one gene-one enzyme*'[99] (1902). By this it is meant that every gene acts by initiating the synthesis of an enzyme which catalyzes a specific reaction.

Professor at Oxford (1920–1928).

1902–1911 CE **Willis Haviland Carrier** (1876–1950, USA). Engineer and inventor. Designed the first scientific system to clean, circulate, and control the temperature and humidity of air (1902). He developed the first safe, low pressure centrifugal refrigeration machine using nontoxic, nonflammable refrigerant.

Born near Angola in western New York, Carrier attended Cornell University, graduating in 1901. He formed (1915) the Carrier Engineering Corporation. By controlling humidity as well as temperature, he invented *air conditioning*[100] as we know it today.

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99 The idea was not developed, however, until the introduction of the genetic analysis of the lower organisms and of techniques of mutagenesis. When in the 1960's the relation between *gene and enzyme* was analyzed at the molecular level, it became clear that the *one gene-one enzyme* hypothesis needed to be replaced by *one gene-one polypeptide chain*. At any rate, Garrod was far ahead of his time.

100 The term "*air conditioning*" was used for the first time by **Stuart W. Cramer**, a textile engineer from Charlotte, N.C. It became a recognized branch of engineering in 1911.

1903 CE **Hantaro Nagaoka** (1865–1950, Japan). Theoretical physicist. Postulated an atomic model in which electrons revolve around a positive charged nucleus, ahead of Rutherford[101] (1871–1937).

Nagaoka adopted his idea from Maxwell's paper (1859) on the stability of motion of *Saturnian rings*. His atomic model consists of a central attracting mass surrounded by rings of revolving electrons. The model was soon rejected because of its mechanical instability[102]. By 1911, however, quantum effects had brought physicists to believe that ordinary mechanics and electrodynamics could not describe the behavior of atoms. In 1913, **Niels Bohr** (1885–1962) quantized the Nagaoka-Rutherford atom, restricting the electrons to paths in which the ratio of their kinetic energy to their orbital frequency equals a multiple of Planck's constant.

1903–1913 CE **Willem Einthoven** (1860–1927, Holland). Dutch physiologist. Discoverer of the electrical properties of the heart and the founder of electrocardiography[103]. In 1903 he invented his famous string galvanometer, with which he was able to measure changes of electrical potential caused by contractions of the heart's muscle. On this basis he developed, during 1908–1913, the graphical diagnostics of cardiopathology. He received the 1924 Nobel prize for his investigations of the electrical currents of the heart.

Einthoven was born in Semarang, Java. His family moved to the Netherlands in 1870. He studied at the University of Utrecht and became a professor of physiology at the University of Leyden, in 1885.

1903–1964 CE **Waclaw Sierpinski** (1882–1969, Poland). Mathematician. Made important contributions to set theory and number theory. His

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[101] Rutherford's biographers agree that Rutherford was unaware of Nagaoka's paper until March 11, 1911. But Nagaoka *did* visit him at Manchester sometime prior to July 1910.

[102] The rings of Saturn are stable because the force operating between the particles of debris that make them up is *attractive*. In Nagaoka's model, the force operating between the electrons is *repulsive*.

[103] The first *electrocardiogram*, however, was obtained earlier by the physiologist **Augustus D. Waller** (1856–1922) in 1887. He reported: "I dipped my right hand and my left foot into a couple of basins, ... which were connected with the two poles of the electrometer, and at once had the pleasure of seeing the mercury column pulsate." This pulsating mercury recorded the heart's electrical tracings via a light beam that then created an image on photographic plates being carried by a toy train and thus provided the first noninvasive procedure to examine the heart.

contributions to number theory (e.g. in the theory of equipartitions) were continued and developed by **G.H. Hardy**, **E. Landau** and **H. Weyl**. His many papers (ca 700) contain new and important theorems (some of which bear his name), geometrical constructions (*Sierpinski curves*[104]), and original and improved proofs of earlier theorems. His findings stimulated further research by mathematicians throughout the world.

Sierpinski was educated at the University of Warsaw during 1900–1904 under the guidance of **G. Voronoi**. He lectured at the University of Lvov until 1914 and then, after WWI, at the University of Warsaw, where he established with others the Polish School of Mathematics.

Birth of Cinematography (1883–1903)

The idea of portraying things in motion has interested man since earliest times. In painting in Altamira Cave in Spain, prehistoric artists tried to show animals running by painting them with many legs. Ancient Egyptian and Greek bas-reliefs portray figures in the act of moving.

*About 65 BCE, the Roman poet **Lucretius** discovered the principle of persistence of vision*[105]*. About 200 years later, the astronomer **Ptolemy of Alexandria** experimentally proved the principle.*

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104 In todays terminology these are the *triadic Sierpinski carpet* (with fractal dimension $D = 1.8928$), *Sierpinski gasket* and *Sierpinski arrowhead*.

105 When light falls on the retina, a chemical change takes place in its tissues, which is manifested by a change in color in the retinal pigment or *visual purple*. How this chemical change gives rise to the sensation of light is not known. Immediately after being affected by light, the retina loses its power of vision at the region excited, owing to the chemical change in the visual purple. The brain does not see a light until about $\frac{1}{10}$ sec after the light is turned *on*. The image *persists* (lasts) about $\frac{1}{10}$ sec after the light is turned *off*. This persistence of the visual image explains why a glowing matchstick seems to leave a trail behind it when thrown away in the dark. A motion-picture projector throws about 24 still pictures on the screen in a second, but we see a continuous movement. Each picture on the screen is presented to the eye before the previous image in the brain fades out.

In 1798, the Belgian physicist **Etienne Gaspard Roberts** *developed a sophisticated version of the* Magic Lantern *which he called* phantascope, *presenting shows to the public.*

During the 1800's many men experimented with devices that would make pictures appear to move. However, a scientific study of the optical appearances of moving objects had began in 1824 with the rediscovery of the persistence of vision by the physician **Peter Mark Roget**[106] *(1779–1869, England). Roget's paper, presented before the Royal Society, inspired several scientists to engage in experimental research, among them the Belgian physicist* **J.A.F. Plateau**.

In 1832, Plateau discovered a method for viewing a series of pictures, representing phases of motion. The pictures were mounted in chronological sequence on the rim of a disc and were observed through slots in a similar disc mounted on the same rotating shaft. The pictures, antedating photography, were necessarily drawings of assumed phases of motion. **Franz von Uchatius** *(1811–1881), an Austrian artillery officer, in 1853, combined the disc device with the magic lantern and projected the pictures upon a screen.*

In 1860 **Coleman Sellers**, *a mechanical engineer, in Philadelphia, made the first known endeavor to harness photography to the recording of motion. He posed his sons in a series of photographs showing them, in successive phases of a cycle of action, driving a nail into a box. The photographs were mounted on the blades of a paddle wheel, which when revolved from a given point of view produced the illusion of motion. The Machine was patented as the* kinematoscope *in 1861. However, photography at that time required exposures so long that a true record of motion was not possible.*

The first successful photography of motion *was made in 1872 by the San Francisco photographer* **Eadweard Muybridge** *(1830–1904), who recorded the progressive motion of race-horses through the use of a series of coordinated cameras [he set 24 cameras in a row, with strings stretched across the race track to the shutter of each camera. When the horse ran by, it broke each string in succession, tripping the shutters]. His experiments were carried out at the ranch of* **Leland Stanford** *— the future site of the campus of Stanford University.*

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106 **Roget** was a physician and philologist, remembered for his *Thesaurus of English Words and Phrases* (1852) — a comprehensive classification of synonyms or verbal equivalents that is still popular in modern editions. He studied medicine at the University of Edinburgh and later helped found the medical school at Manchester. From 1808 to 1840 he practiced in London.

During the late 1800*'s, inventors in France, Great Britain, and the United States tried to find ways to make and project motion pictures. These experimenters included* ***Thomas Armat*** *(1866–1948),* ***Charles F. Jenkins****, and* ***Woodville Latham*** *of the United States;* ***William Friese-Greene*** *and* ***Robert W. Paul*** *(1869–1943) of Great Britain,* ***Charles-Émile Reynaud***[107] *(1844–1918), the brothers* ***Lumière***[108] *of France and* ***Max Skladnowsky*** *(1863–1939) of Germany. After many failures, success came to several pioneers at about the same time.*

The first motion-picture camera was invented in 1887 *by* ***Augustin Le Prince*** *in Leeds, England, after ten years of laborious efforts. However, in lack of a suitable material for projection (a coated celluloid film of sufficient strength), he could not perfect his machine for commercial uses.*

At about the same time, the French physiologist ***Étienne Jules Marey*** *(1830–1904), who was in communication with Muybridge, demonstrated (1888) to the French Academy of Sciences his* chambre chronophotographic*, embodying the essential principles of the cine-camera. His camera, however, was inferior to that of Le Prince and non of his films were projected until ten years later. [Marey is known for his work on the physiology of the heart and circulation, human and animal locomotion and the flight of birds and insects.]*

In 1887 *Thomas A. Edison began work on a device to make pictures appear to move. Influenced by the technological achievements of* ***Le Prince*** *and* ***Marey****, he made some progress in* 1889*, after* ***Hannibal W. Goodwin*** *had developed the transparent nitro-celluloid film base that was tough but flexible such that it could hold a coating, or film, of chemicals sensitive to light (celluloid film first introduced in* 1883 *by* ***John Carbutt*** *of Philadelphia);*

Thus, a series of pictures could be photographed on the film and moved rapidly through a camera. Previously, most photographs were taken on glass

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[107] This inventor presented (1892) in Paris moving cartoons of animated drawings. One cartoon involved more than 600 painstakingly drawn images on a 45 meter ribbon projected on a permanent background scene. To this end he used two projectors: the animated drawing passed in front of a lens and were then projected off a mirror into the screen. The Lumières saw it and promptly made use of his ideas.

[108] The pioneer motion-picture tycoon **Charles Pathé** (1863–1957, France) acquired the Lumière patents (1902) and by 1912 had established one of the largest film production organizations in the world. He introduced the *newsreel* in France (1909) and shortly after in the USA and Britain. He dominated the world film market during the first years of the 20th century. Before 1918, 60 percent of all films were shot with Pathé.

plates that had to be changed after each exposure. ***George Eastman****, a pioneer in making photographic equipment, manufactured the film.*

Using the Eastman film, Edison's assistant ***William Kennedy Laurie Dickson*** *invented the kinetoscope, a cabinet in which 15 meters of film revolved on spools. A peephole in the cabinet enabled a person to watch the pictures move.*

In 1892, ***Ottomar Anschutz*** *(1846–1907, Prussia) was first to show in public (Europe and USA) moving photographs. In 1894 he projected moving sequences of animals and human figures on large screens. His viewing machines were developed from 1886. The first had a wooden disc with 24 glass positives fixed onto it; a Gessler tube fashioned into a spiral form and powered by a Ruhmkorff induction coil fed from batteries, was the light source. This flashed briefly as each picture passed the viewing aperture.*

Birt Acres *(1854–1918, USA and England) was an inventor who developed (1895) a modified version of Edison's kinetoscope. This was the first amateur Cine Camera. On March 30, 1895 Acres filmed the Oxford-Cambridge boat race and on May 29, 1895 he filmed the Derby.*

Projected on a screen picture at a rate of 40 frames per second, he showed short films of men boxing, a review of the German Emperor, Epsom Downs Derby race, serpentine dancing and the sea breaking against the embankment.

In 1894, the Kinetoscope Parlor was opened in New York City and later that year in London and Paris, including coin-operated kinetoscopes. In spite of its success, ***Edison*** *believed that moving pictures were only of passing interest. However, other inventors disagreed. On Dec. 28, 1895, motion pictures were projected for the first time on a screen in a Paris café by* ***the Lumière brothers****. Some simple scenes, including that of a train arriving at a station, were shown.*

Within few months, 'movies' were being shown in all the major cities of Europe. Edison was quick to adapt to the new situation and using a projector invented by ***Armat****, he presented the first public exhibition of motion pictures projected on a screen in New York City on April 23, 1896. The program included a few scenes from a prize fight, a performance by a dancer, and scenes of waves rolling onto a beach. By 1900, motion pictures had become a popular attraction in music halls, fairs, museums and theaters in many countries.*

Soon, however, spectators became bored, attendance declined, and the motion picture faced extinction. One development saved the movies — they began to tell stories. As early as 1899, the movies received their next impetus *from* ***George Méliès*** *(1861–1938), French magician and theater owner. He saw the movie as a mechanical extension of magical illusion, with which he could achieve effects never before conceived. Within a few years Méliès invented or stumbled upon double exposure, stop motion, fast and slow motion, animation, fades, dissolves, almost the entire repertory of the optical tricks used in film making from that time until the nascence of computer-generated special effects.*

In 1900 *he filmed the old fairy tale Cinderella. Though the film, less than* 350 *meters long, was little more than picture-book illustration, it had a beginning, middle, and end — it told a story. Méliès' imaginative films astonished and delighted movie-goers the world over. He flung himself into the work of writing scenarios, designing and painting scenery, and appearing in his own films. But this versatile and ingenious pioneer was no businessman. In* 1914 *the war ended his career as a producer; in* 1925 *he lost his theater, destroyed all the films in his possession and vanished.*

Edwin S. Porter *— filming for the Edison company in America — was influenced by Méliès and, in* 1903, *made the first motion picture using modern film techniques to tell a story; "The Great Train Robbery", an* 11*-minute Western describing a train robbery and the pursuit and capture of the robbers.*

Porter was perhaps the first director to recognize that a motion picture need not be filmed in the strict sequence of the action. The story switched back and forth between a number of settings, and so Porter realized that it was impractical to shoot the story in sequence. His pioneer work in filming and editing set the standard for directors throughout the world for several years.

In 1904, *Edison made the first attempt to produce a sound moving picture by synchronizing the projector with the phonograph. In 1906,* ***Eugen Augustin Lauste*** *(France) invented the production of sound from photographed vibrations on a film, projected upon a selenium cell. This was a crucial step in the development of the sound motion picture.*

In the mid 1920*'s, Bell Telephone Laboratories finally developed a system that successfully coordinated sound on records with the projector: The era of the silent movie ended in* 1928 *when sound was directly photographed on the film.*

1903 CE, Dec. 17 The brothers **Wilbur** (1867–1912) and **Orville** (1871–1948) **Wright** (U.S.A.). Made the world's first successful flight in a power-driven, heavier-then-air machine which they invented and built.[109]

The flight was made at Kitty Hawk, NC. The plane flew about 40 meters and remained airborne for 12 seconds.

The brothers became interested in aviation after reading about the death of pioneer glider **Otto Lilienthal** in 1896. They experimented with model wings in a small wind tunnel, which they built in their shop, in order to obtain reliable data of air pressure on curved surfaces. Then, in the summer of 1902, they built a glider based on their new figures. This glider had aerodynamic qualities far in advance of any tried before. With it they solved most of the problems of balance in flight. They then made some 1000 glides in this model, before equipping it with an engine. They were so sure of the accuracy of their calculations that they were not surprised when the machine flew.

1903–1911 CE **Carl Neuberg** (1877–1956, Germany, Israel and U.S.A.). Physician, engineer, jurist, chemist, biologist and biochemist. First used the term *Biochemistry* (1903) and founded the first biochemistry journal: *Biochemische Zeitschrift* (1906). Discovered (1911) *carboxylase* in yeast which was the first indication that the energy on the cellular level is essentially supplied by burning *hydrogen* rather then carbon. Made other contributions to the understanding of fermentation.

Neuberg was born in Hanover to a Jewish family. Studied in Berlin and Breslau. Professor at Berlin (1919–1938), Jerusalem (1939–1941), New York (1941–1950), Brooklyn Polytechnic (1951 ff.).

[109] In 1906, *Scientific American* magazine sneered at the "alleged" flights of the Wright brothers. One hundred years later (in connection with another matter) the magazine apologized, saying scientists had "dazzled us with their fancy fossils, their radiocarbon dating, and their tens of thousands of peer-reviewed journal articles. As editors, we had no business being persuaded by mountains of evidence."

Development of Aeronautical Science

Part I: Aviation, from Aristotle to the Wright Brothers (1903)

"There are three things which are too wonderful for me, yea, four which I know not: The way of an eagle in the air..."

Proverbs **30**, 18

Aeronautics is the art of 'navigating the air'. It is divisible into the main branches — aerostation, dealing properly with machines which, like balloons, are lighter than air, and aviation, dealing with the problem of artificial dynamic flight by means of flying machines (aircraft) which, like birds, are heavier than air.

*Historically, aviation is the older of the two. In the legends and myths of men or animals which are supposed to have traveled through the air, such as: Pegasus, Medea's dragons, Daedalus and Ezekiel's flying creatures [**Ezekiel** I, 5–26] as well as in Egyptian bas-reliefs — wings appear as the means by which aerial locomotion is effected.*

The above quotation serves to show that as early as the 4^{th} *century BCE, man admitted his ignorance of aerodynamics.* ***Aristotle*** *(384–322 BCE) is no exception: although he mentioned the problem of solid bodies moving in the air, he believed there is always a force necessary to sustain a uniform or even decelerated motion. He thus looked for a force which pushes forward a flying ball, instead of looking for a force which resists the motion. With such a "negative" approach to dynamical processes, no understanding of flight was possible.*

Many great men with artistic imagination studied the fundamentals of bird flight and speculated on the possibilities of human flight. The drawings and notes of ***Leonardo da Vinci*** *(1452–1519) represent an excellent example of such studies. He considered two methods of flight. One method is an imitation of bird flight (a man equipped with a pair of wings, beating them like a bird. An aircraft of this type is called today an ornithopter).*

The other method was based on a screw, called now the screw of ***Archimedes*** *— which would penetrate the air. This is the predecessor of the* *present day helicopter. The characteristic feature underlying both proposed systems, was the general belief that lift and propulsion should be accomplished by the same mechanism. This is true for the bird, whose propulsion and sustentation are produced by the motion of the same wings. It is also correct in the case of the helicopter.*

The idea of imitating bird flight was predominant for centuries in the minds of inventors. Some, however, recognized the limitations of mere imitation of nature, as one pioneer in aeronautics once remarked: "The successful locomotive was not based upon an imitation of an elephant". It is thus clear that the concept of sustentation by flapping wings or by a screw preceded that of a rigid *(fixed-wing)* airplane.

A step forward in the latter direction was taken by the father of modern dynamics: ***Galileo Galilei*** *(1564–1642) recognized the law of inertia and had a correct notion of* air resistance. *He observed that the movement of a pendulum was slowly amortized by air resistance, and actually tried to determine the dependence of air resistance on velocity. This task was however left to* ***Isaac Newton*** *(1642–1727). From the fundamental laws of mechanics he derived the formula, known generally as* Newton's sine-square law of air resistance, *for the force acting on an inclined flat plate exposed to a uniform air-stream*[110].

Newton stated clearly that the forces acting between the solid and the fluid depend on their relative *motion. [This was already understood by* ***da Vinci****, who said: "The resistance of an object against air at rest is equal to the resistance of the air moving against the object at rest".]*

In the 216 *years that elapsed between the publication of Newton's* Principia *(1687) and the date of the first mechanical flight (1903), a great number of observations were made to determine the resistance experienced by a body, in water as well as in air.*

In the long list of experimenters, engineers, and physicists we find the names of many generally known scientists: ***Edme Mariotte*** *(1620–1684),*

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110 $F = \rho S V^2 \sin^2\alpha$, where F is the change of momentum of the fluid mass hitting the plane in unit time at inclination α. Here V is the *relative* velocity of fluid and plate, S is the plate's area and ρ is the fluid's density. The force F is directed normal to the plate. The quantity $(\rho S V \sin\alpha)$ is evidently the mass flow in unit time through a cross-section $(S\sin\alpha)$, equal to the projection of the plate area perpendicular to the original flow direction; and this must be multiplied by the velocity component $(V\sin\alpha)$ created by the impact. The dependence of the force on $\sin^2\alpha$ is *not* found in Newton's work and was deduced by other investigators, based on a method of calculation which Newton used for comparison of the air-resistance of bodies of different geometrical shapes.

***Benjamin Robins** (1707–1751), **Jean le Rond d'Alembert** (1717–1783), **Charles Bossut** (1730–1814), **Jean Charles de Borda** (1733–1799), and **Antoine Condorcet** (1743–1794).*

Remarkable experiments were carried out at the end of the 19^{th} *century and the beginning of the* 20^{th} *century by **Alexandre Gustave Eiffel** (1832–1923, France) and his collaborators, who used the tower named after Eiffel in Paris. The best method for measuring air resistance is to put a model in an artificial stream of air, i.e. the method of the* wind tunnel. *The first man to make such an installation was **Francis Herbert Wenham** (1824–1908, England), who in* 1871 *designed it for the Aeronautical Society of Great Britain.*

*In 1891 **Nikolai E. Joukowski** (1847–1921), at the University of Moscow, built a tunnel two feet in diameter. No wind tunnel built before* 1910 *had more than* 100 *horsepower [today, wind tunnels may employ* $250,000$ *horsepower for driving the air-stream].*

The experimental evidence has shown that the dependence of the resistance on the sine of the inclination angle is nearly linear, and not square as Newton had stated. It is believed that Newton's law contributed to pessimistic forecasts on the possibilities of powered flight and thus delayed its development[111].

*The idea that lift can be accomplished by moving inclined surfaces in the flight direction, provided we have mechanical power to compensate for the air resistance that hinders this motion, was clearly enunciated for the first time by **George Cayley** (1773–1857, England) in* 1809–1810. *He was first to announce the* principle of the airplane *as we know it today. It can be said*

[111] According to Newton, the *lift* (vertical component of F) is proportional to $\sin^2\alpha\cos\alpha$ while the *drag* is proportional to $\sin^3\alpha$. Being very small for small values of α, the airplane designer needs tremendous wing area to obtain a sufficient amount of lift. On the other hand the ratio between lift and drag (horizontal component of force) is equal to $\cot\alpha$, which is large only if α is small. If Newton's law is correct, the poor designer has only the choice of either making a huge contraption having a very large wing area, and therefore a heavy structural weight, or building a machine with reasonable wing area but low lift-drag ratio, which means a heavy engine for propulsion.

that Cayley was the man who founded the science of aerodynamics[112]*, and was probably the first man to describe fixed-wing, powered airplane moved by propellers.*

In 1842, ***William Samuel Henson*** *(1805–1888), a British inventor, patented plans for the first plane with a steam engine, two propellers, fixed wings and a passenger cabin. But Henson's "airliner" was never built. In* 1848 *a friend of Henson,* ***John Stringfellow****, built a small model plane using Henson's design. The model was successfully launched but was able to stay in the air for only a short time.*

Throughout the 19^{th} *century, two practically unrelated developments took place side by side. On one hand, flight enthusiasts developed their own rather primitive theories of bird flight, and tried to apply their results to the requirements of human flight. On the other hand, a mathematical theory of fluid dynamics was developed by scientists, who did not provide much useful advice to those who wanted to fly.*

In the experimental vein, interdisciplinary efforts by physicists, aeronauticists, and physiologists, during 1868–1910*, were directed towards the determination of the power required for flight. The fact that birds actually fly through the air furnished a certain solid support for the speculations.*

It was shown by ***Herman von Helmholtz*** *(1868–1873),* ***Charles Renard*** *(1847–1905, France, 1889–1903),* ***Étienne Jules Marey*** *(1830–1904, France, 1873–1890) and* ***R. Henry*** *(1891) that the ratio* W/S *[*$W =$ weight, $S =$ wing area, $\rho =$ *density of the air], known as wing loading, is related to the ratio* W/P *[*$P =$ expended power*], known as power loading, through the equation*

$$P/W = constant \times \sqrt{\frac{W}{\rho S}}.$$

For soaring birds it was found that wing-loading increases like the cube of the weight, which means that flying becomes more of a problem for a large bird than for a small one and that there is a certain size beyond which a living creature is unable to fly.

A second problem that bothered the experimentalists was to find the most efficient shapes for wings. ***Helmholtz*** *(1858) showed that if there is no initial*

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[112] For further reading, see:

- Von Karman, T., *Aerodynamics*, McGraw-Hill Book Company: New York, 1954, 203 pp.
- Milne-Thomson, L.M., *Theoretical Aerodynamics*, Dover Publications: New York, 1958, 430 pp.

vorticity in the fluid, e.g., if the fluid is originally at rest, vorticity can only be created by friction or by the presence of sharp edges on the body. Following this lead, experiments began either in wind tunnels or by means of actual flying in gliders.

*It was found by **H. Phillips** (1885), **Octave Chanute** (1832–1910, U.S.A., 1894–1897), **Alphonse Pénaud** (1850–1880, France), **Samuel P. Langley** (1834–1906, U.S.A.), **Otto Lilienthal** (1848–1896, Germany, 1889) and **Charles M. Manly** (1876–1927, U.S.A.) that curved wing surfaces are superior to flat surfaces in their lift-drag ratio, and also because they show positive lift in the case of zero angle of attack.*

***Wilbur** (1867–1912) and **Orville** (1871–1948) **Wright** were familiar with all practical aerodynamical ideas developed before them by various researchers. In addition to a remarkable talent for construction, they had the ability to utilize model experiments for their full-scale design*[113].

However, at the time of the first human flight (1903), no theory existed that would explain the support obtained by means of a curved surface. It seems that the mathematical theory of fluid motion was unable to explain the fundamental facts revealed by experimental aerodynamics.

On the theoretical front, it became clear to the scientists of the 18^{th} *and* 19^{th} *centuries that the problem is not as simple as Newton thought, and that one cannot replace the flow by parallel motion. In 1878, **Lord Rayleigh** (1842–1919) found that the superposition of a circulatory flow around a circular cylinder, upon a uniform flow perpendicular to it, produces a force perpendicular to the direction of the original flow (or to the direction of motion of the cylinder through stationary air). This result was used to explain the so-called Magnus effect, which had been known to artillerists since the beginning of the* 19^{th} *century. Rayleigh himself undertook his study to elucidate the swerving flight of a "cut" tennis ball*[114].

***Anton Flettner** (1885–1961, Germany), an engineer from the University of Göttingen, harnessed the Magnus effect to drive a boat by wind power (1924). In lieu of the usual sail (which is nothing but an airfoil), a circular cylinder was erected vertically on the boat and made to spin around its axis some 2 revolutions per second. This spin created the circulation, that when added to the laminar wind flow invoked the 'Magnus-force', that drove the*

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113 Their airplane had the following parameters: $W = 340$ kg, $S = 46.45$ m^2, $W/S = 7.32$ kg/m^2 (a little larger than that of a vulture), $P = 595$kg·m/sec, $P/W = 1.75$ m/sec. According to Renard's formula, the value of the power required per unit weight would be 1.35 m/sec.

114 Lord Rayleigh, "*On the Irregular Flight of a Tennis-Ball*", Messenger of Mathematics **7**, 14–16 (1878).

ship in a direction that is normal both to the cylinder-axis and the wind flow. By rotating two tandem cylinders in opposite directions, the boat could be made to turn around. In 1925 *a Flettner 'rotorship', 680 tons, crossed the Atlantic Ocean. The ultimate failure of the invention, however, was due to economic reasons*[115]*.*

The connection between the lift of airplane wings and the circulatory motion of the air around them was recognized and developed by 3 *persons of very different mentality and training:* ***Frederick W. Lanchester*** *(1878–1946),* ***Wilhelm Martin Kutta*** *(1867–1944)* and ***Nikolai Egorovich Joukowski*** *(1847–1921, Russia).*

Kutta, a pure mathematician, became interested in Otto Lilienthal's gliding experiments and therefore in aerodynamic theory. His particular aim was to understand the effect of curvature — why a horizontally placed curved surface produces a positive lift (1902). Joukowski had extensive training in mathematics and physics, obtained originally in Russia and Paris. In 1872 *he became professor of mechanics at the Polytechnical Institute, and in* 1886 *at the University of Moscow. During* 1902–1909*, independently of Kutta and Lanchester, he developed the mathematical foundations of the theory of lift, at least for two-dimensional flow, i.e. for wings of infinite span and constant cross section.*

Although ***Cayley*** *described the propeller*[116] *already in* 1809 *and* ***Henson***

[115] In 1926 Flettner established in Berlin an aircraft company that produced helicopters much used in WWII. After the war he emigrated to the U.S.A., and became president of the Flettner Aircraft Co., Queens, NY.

[116] A propeller is a device which converts the engine torque into a thrust, which must be enough to overcome the drag of the entire aircraft and also provide additional power to enable the aircraft to climb. It does this by making use of the resistance of the air, whole *reaction* on the blades produces two forces: one of which is the desired *thrust* along the axis of rotation, while the other acts as a brake on the engine shaft. In steady flight, the torque developed by the engine exactly balances the breaking action due to air resistance.

The thrust of the propeller is obtained by giving a backward momentum to the air with which it comes in contact, and to do this effectively, the propeller blades are given first an *aerofoil* shape and then a twist. The actual thrust is caused by the difference in pressure in front and behind the propeller disc. From the viewpoint of general principles of mechanics, the propeller, like the rocket, is a device for propulsion by reaction.

There is an economic upper limit to the speed at which the blades can rotate and still work efficiently. This limit is reached when the tip speed approaches that of sound. A forward speed of 950 km/hour may represent the limit which a propeller-driven aircraft can attain in still air at altitude. Apart from these *designed it as an integral part of powered flight in* 1842*, the physics of the propeller began to be understood during* 1865–1909 *due to the* momentum theory *of* ***W.J.M. Rankine*** *(1865) and the* blade-element theory *of* ***William Froude*** *(1878, England) and* ***Stefan Drzewiecki*** *(1844–1938, Poland and France, 1892–1909).*

Part II: Balloons and Airships (1783–1937)

The dove of ***Archytas of Tarentum*** *(c. 428–347 BCE) is the earliest suggestion of true aerostation. It may conceivably represent an anticipation of the hot-air balloon.*

In the middle ages, vague ideas appear of some ethereal substance so light that vessels containing it would remain suspended in the air. ***Roger Bacon*** *(1214–1294) wrote that man might fly if he were attached to a large hollow globe made of very thin metal and filled with ethereal air or liquid fire, which would float on the atmosphere like a ship on water. But Bacon never tried to put this idea into practice.*

This state of affairs persisted for the next 500 *years. It was generally believed that if a substance lighter than air were found, human flying would be possible. Thus, when* ***Henry Cavendish*** *discovered hydrogen in* 1766 *and* ***Joseph Priestley*** *elaborated in* 1774 *on the lighter density of hot air, these discoveries were immediately put to use;*

In 1766*, it occurred to* ***Joseph Black*** *of Edinburgh, that a thin bag filled with hydrogen gas would rise to the ceiling of a room. But for some reason the experiment failed and Black did not repeat it, thus allowing a great discovery, almost within his reach, to escape him.*

In 1782, ***Tiberius Cavallo*** *(1749–1809), an Italian experimental physicist living in England, was first to show that* soap bubbles *filled with hydrogen*

considerations, however, the propeller shows serious faults of an aerodynamic nature even at moderate speeds: the vortex system developed by the blades represent so much wasted power, for the rotational motion adds nothing to the thrust, and the disturbances thus created can interfere seriously with the otherwise smooth flow over the wings.

would float upward when they were released. A year later, the French brothers ***Joseph Michel Montgolfier*** *(1740–1810) and* ***Jacque Étienne Montgolfier*** *(1745–1799) made a hot-air balloon*[117] *that successfully carried a man [Jean Francois Pilâtre de Rozier, historian to King Louis XVI of France; Oct.* 15, 1783*. He was killed in* 1785 *in the explosion of a combination hydrogen and hot-air balloon].*

The Montgolfier brothers arrived at their idea of hot air in their balloon after a careful study of ***Priestley****'s book on gases. In this balloon, no source of heat was taken up, so that the air inside rapidly cooled and the balloon soon descended.*

In the same year (1783), another group launched a hydrogen-filled balloon. The group consisted of two brothers of the Robert family under the supervision of the French physicist ***J.A.C. Charles*** *(who discovered one of the gas laws, which bears his name). The balloon,* 4 *m in diameter, was made of thin silk varnished with a solution of elastic gum. The hydrogen gas was obtained by the action of dilute sulphuric acid upon iron filings, and was introduced through leaden pipes.*

Since 1784 *the balloon has been applied to the study of the atmosphere, via barometric, thermometric, hygrometric, gravimetric and magnetic observations. Thus, for example,* ***J.L. Gay-Lussac*** *and* ***J.B. Biot*** *ascended up to the height of* 7 *km in* 1804 *to measure the variation of the earth's magnetic and gravity fields with elevation.*

No sooner was the balloon discovered than it received a military status. It was used in the French Revolutionary was (1794), the French campaign in Italy (1859), the American Civil War (1861) and the Franco-Prussian War (1870–1871) for reconnaissance, transportation (e.g. carrying pigeons), communication and even bomb-throwing. The first air-raid in history took place in 1849*, when the Austrians sent balloons over the city of Venice during the Italian uprising. Each balloon carried some* 14 *kg bomb with a time fuse [the first bombs dropped from airplanes in warfare were released over Lybia in* 1911 *by Italian aviators during a war between Italy and Turkey].*

In 1914*, the balloonist* ***Hans Berliner*** *traveled some* 3053 *km from Germany to the U.S.S.R. In* 1961 ***Victor G. Prather*** *and* ***Malcolm Ross*** *of the United States ascended to the height of* 34.6 *km.*

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[117] An airtight bag that is able to rise in the air because it is filled with light gases. Early balloons were usually made of silk or cotton cloth that was coated with rubber to make the balloon airtight. Today, balloons are made of plastic. The earliest balloons were filled only with heated air, since it is about half as dense as cold air.

From the very first invention of balloons, the problem has been how to navigate them by propulsion. Experiments with airships[118] *started with the French engineer* ***Henri Giffard*** *(1825–1882), who in* 1852 *ascended in a cigar-shaped balloon (length = 44 m; diameter = 12 m; total weight = 1475 kg) driven by a* 3 *horsepower (2.2 kilowatt) steam engine linked to a propeller. It drove the craft about* 8 *km/hour for* 27 *km, from Paris to the city's outskirts. Giffard's engine lacked the power to turn the balloon completely around and return to the starting point.*

Many men continued to work on airships through the 19^{th} *century. The first rigid ship was built by the Austrian engineer* ***David Schwarz*** *in Berlin. It crashed on its first flight on Nov.* 3, 1897. *The work of Schwarz influenced* ***Ferdinand von Zeppelin*** *(1838–1917), a retired German army officer, to begin work on airships. Throughout* 1900–1937, *rigid airships built by Zeppelin and other Germans regularly plied the world skies.*

During 1910–1914, *the German Airship Transportation Company carried about* 35,000 *passengers without a single death, though there were several accidents. The fastest airship built by Zeppelin could fly about* 80 *km/hour. During WWI, the German used airships for scouting, observation and supply work. They made over* 50 *bombing raids on England.*

In 1928, *the Germans completed the* Graf Zeppelin, *a giant airship [length = 240 m, diameter = 30 m] that carried* 50 *passengers and their baggage at a speed of* 110 *km/hour. It was used for regular commercial service between Germany and South America. The largest airships ever built was the* Hindenburg *[length = 247 m, diameter = 41 m, 1936]. It burst into flames on May* 6, 1937, *while approaching Lakehurst, NJ. This marked the end of the use of airships for regular passenger service.*

In World war II, small airships called blimps *played a vital part in protecting ships against submarine attacks: Blimps located submarines and attacked them with depth charges and other weapons. Today, blimps are used mainly for advertisement over the skies of major cities.*

[118] Lighter-than-air aircraft that have their own motive power and can be steered in any direction by their crews. Airships were once called *dirigibles*, which comes from the Latin word *dirigere*, meaning to *steer*.

Part III: Rocket and Jet Propulsion (1232–1945)

When air is allowed to escape from an inflated balloon, the balloon moves in a direction opposite to the escaping air. This principle, in its varied forms, is so obvious that it could not have escaped the observant eyes and the inquisitive minds of erudite men of antiquity. Granted the invention of the wheel, it must have occurred to some of them to watch the recoil of a cart, say, when an object was being thrown from it in the opposite direction.

Indeed, ca 100 *BCE,* ***Heron of Alexandria*** *built a steam jet engine, called an aeolipile: a spherical cauldron was supported by a vertical axis. Escaping from the cauldron through elbow-shaped pipes, the steam pushed these pipes in opposite directions, and the sphere rotated.*

Nature, of course, preceded man in the field of rocket propulsion: it is used for locomotion by jellyfish and mollusks like the octopus and squid. The large brown jellyfish swims by pulsations of its bell, which expands and contracts like an umbrella being opened and closed. Cephalopods, on the other hand, contract their mantle, forcing a narrow stream of high pressure water out through a flexible siphon.

The rocket probably evolved in a simple way from the incendiary arrow. It is known that the Greeks used flying incendiary objects to burn enemy cities in their wars. Thus it is even possible that they already knew how to prepare explosive mixtures. As early as 1044*, the Chinese learned that salpeter added to charcoal and sulfur made it fizz alarmingly.*

Somebody in China, between 1044 *and* 1232*, discovered that if charcoal, sulfur and salpeter are grounded very finely, mixed thoroughly in the proportion of* 1: 4: 4 *and the mixture packed into a close container it will, when ignited, explode with a delightful bang. The mixture was applied both to fireworks and to primitive military devices in the war of* 1232 *against the Mongols. There we encounter the earliest version of a 'rocket-arrow': The powder was packed in a long thin tube to keep it from going off all at once. The tube was open at the rear end such that the reactive thrust made the use of the bow redundant.*

The new weapon spread quickly, and by 1300 *it was well established in Europe, the realms of Islam and the Far East. As with printing and the clock escapement, we do not know just how this knowledge traveled from China westward: whether over the northern route through Russia (then under the Mongol yoke), or south through Muslim lands.*

The next step was taken by ***William Congreve*** *(1805), the father of military rocketry. Although Newton's laws had by then been known for more than* *a century, it was the practical needs of warfare which brought about the development of the rocket and* not *the indulgence of savants in the consequences and uses of Newtonian theory.*

The Congreve rockets were about 100 *cm in length,* 9 *cm in diameter, weighted as much as* 30 *kg and were equipped with a stabilizing wooden tail rod, some* 5 *m in length. It could travel* 2.4 *km with rather poor accuracy, and chiefly effective for its noise, glare and incandescent power. It was thus used against masses of troops within easy range, or to set fire to buildings. The rocket consisted of two parts: a head projectile made of hollow metal that could be filled either with explosives (acting as a bursting anti-personnel shell) or incandescent material. Screwed to the head was an iron casing in the rear, containing the propellant gun-powder.*

An English inventor, ***William Hale****, improved the accuracy of military rockets by substituting three fins for the long wooden tail that had been used to guide the rocket.*

The modern era of rocketry begins with the pioneering work of ***Konstantin E. Tsiolkovsky****, who first stated the correct theory of rocket propulsion in* 1903. *But the indifference of the Russian government to his new ideas, and the political events that overwhelmed Russia during* 1914–1932*, prevented the penetration of his revolutionary ideas into Europe and the United States.*

Consequently, his notions were rediscovered by ***Robert H. Goddard*** *(1918) in the United States and* ***Hermann Oberth*** *in Germany (1923). Of these three pioneer thinkers, Goddard was the first to undertake specific translation of his theories into shootable rockets and patentable devices. But even so, Goddard's development of a successful solid-fuel ballistic rocket by* 1918*, lay unused for* 25 *years.*

Oberth began his study of the space flight problem about the time of WWI, and presented his first treatment in his book Die Rakete zu den Planetenraum *(The rocket into interplanetary space), published in* 1923*. It is the first book to contain the notion of* escape velocity *(although the concept was recognized by Goddard already in 1912). This book stimulated the foundation in Germany of* The Society for Space Travel *(1927).*

In 1926*, Goddard conducted the first successful launch of a liquid propellant rocket. It climbed* 56 *meters into the air at a speed of about* 100 *km/hour.*

The three pioneers had one thing in common — their funds were extremely limited. None had a financially strong sponsor (eccentric inventors with new ideas, do not usually get such sponsors). They all had excellent ideas imagination, and even skill, but they failed to perceive the development costs and the amount of hard work required before attaining convincing results.

The history of technology proves that when the time is ripe, people are to be found thinking about or working on the same problems in almost all civilized countries. Indeed, since the early 1920's, with the development of inexpensive, mass-produced light metals, highly efficient oxidizers which could be handled, and reliable, accurate electronic equipment — three basic elements became available for the revival of the ancient art of rocketry.

Thus we see, since 1928, private groups, inventors and engineers in many countries working on rocket propulsion. Particularly in Germany, serious study had begun on rocket-propulsion as a means of aircraft locomotion. In 1928, ***Max Valier*** *became one of the first individuals to experiment with a liquid-propellant rocket motor, which he used to drive a small racing car. In 1928, rocket powered winged flight was demonstrated in its first primitive form with glider flights in Germany. In 1929, a seaplane made a solid-propellant rocket-assisted takeoff at Dessau, Germany. Later,* **Fritz von Opel** *(1899–1971) flew a glider almost 3 km from Frankfurt-am-Main, Germany, with 16 rockets of 25 kg thrust each.*

Above all, however, it was the military potential*, an art lost to artillery in the* 19^{th} *century, which created missilery as a strategic weapons system, and brought forth the technology that made possible the birth of practical astronautics.*

The treaty of Versailles prohibited military aviation, and thus prompted the German Army in 1931 to initiate a serious, albeit modest and secret, investment in the possible military potential of the rocket as a carrier of explosives. Now, for the first time, rocketry had a sponsor: private industry or government would not have spent hundreds of millions of dollars for a new technical idea which, in the foreseeable future, would not produce any profit. The Germans were looking for a new superior weapon system which has not prohibited to them by the Treaty of Versailles. This called for strict secrecy, and hence no involvement of private industry.

In 1935, **Werner von Braun** *worked on liquid propellant rocket engines for aircraft application, and in 1936 he mounted a 300 kg thrust engine onto small aircraft in the first airplane ground test, using a propulsion system of this type.*

Hardly anyone in the world knew before 1943 that such a development was underway. Yet by 1945 the Peenemuende team (1936–1945) had a rocket lead of approximately ten years. The product of this effort, namely, the V-2 guided ballistic missile, was used by von Braun and his colleagues to bombard London. Some 1100 such flying bombs fell in England between Sept. 1944 and March 1945. American forces captures many of these V-2 missiles and sent them (together with some 200 German rocketeers) to the United States for use and research. The history of rocketry is summarized in table 5.2.

Table 5.2: Rockets (1232–1981 CE)

- *c. 25,000 BCE* *The* bow *and* arrow *are invented; The idea of the* rocket *probably followed the sight of* flaming arrows *flying through the air — a military technique of ancient times (c. 3500 BCE).*

- *c. 360 BCE* **Archytas of Tarentum** *built the first known device that operated on* reaction principle: *thrusts from jets of stream moved pigeon-shaped device.*

- *c. 210 BCE* *Earliest recorded mention of gunpowder from* China. *Bamboo tubes filled with salpeter, sulphur and charcoal used as primitive bombs tossed into ceremonial fires.*

- *1045 CE* *Use of gunpowder and rockets formed an integral aspect of Chinese military tactics.*

- *1232 CE* *Rocket fire-arrows used by the Chinese Sung Dynasty to repel Mongol invaders at the battle of Kai-fung-fu (capital of the Chinese kingdom of Honan). The huge rockets could be heard for about 24 km away and at the point of impact, devastated an area with radius of 700 m. Apparently these large military rockets carried incendiary material and iron shrapnel.*

- *1248 CE* *Mongols brought the Chinese rockets to Europe, and used them against Magyar forces at the battle of Sejo which preceded their capture of Buda (Dec 25), and at the battle of Legnica in Silesia. [Accounts also describe Mongols use of a noxious smoke screen — possibly the first instance of chemical warfare.]*

- *1258 CE* *Mongols use rockets to capture the city of Baghdad (Feb 15).*

- *1268–1288 CE* *Arabs adopted the rocket into their own arm inventory and used them against the French army of King Louis IX during the 7th Crusade (1268). The Arabs used them again during their attack on Valencia, Spain.*

- *1300–1750 CE* *Rockets found their way into European arsenals, and thenceforth many armies adopted and improved the technology of these 'flying bombs'. The French used rockets at the Siege of Orleans during the Hundred Years War against the English (1429). Rockets reached Italy (1500), England (1647), The Netherlands (1650) and Germany (1668). By 1730 the Germans were manufacturing rockets weighing 24–54 kilograms.*

It is known that the construction of these bombs was rudimentary, and their use undoubtedly hazardous, for they were made of paper *which was lacquered or pasted with starch, thus forming a tube which was narrow at one end, the mouth, and was sealed at the front. The tube, loaded with charcoal, salpetre and sulphur, was the ancestor of solid fuel rockets, in which the combustion forced the ejection of a mass of warm air which propelled the whole rocket.*

- *1379 CE* The word rocket was first used (in Italian, rocketta).
- *1650 CE* Writer **Cyrano de Bergerac** suggested rockets as means of traveling from earth to moon.

  Rockets reached a remarkable degree of perfection:

  - Rockets have fins.
  - Multi-stage accelerating rockets were invented: this consisted of a main rocket to which back-up rockets were connected.
- *1687 CE* **Isaac Newton** published his laws of motion. His third law — for every action there is an equal and opposite reaction — explains the basic motive force of rocket engines.
- *1715 CE* The first military rocket factory in the world was built near St. Petersburg by Peter the Great.
- *1792–1798 CE* In the Battle of Seringapatam, India between the English and the army of Tippu Sultan, the English ranks are overwhelmed by a barrage of thousands of rockets. These rockets were metal-made with a range of about 1 km; improvements were made in their manufacture and firing method.
- *1804–1865 CE* **William Congreve** (England), the true pioneer of modern rockets, improved their ballistic stability, range and destructive power, turning rockets into a powerful military weapon. He was first to think of igniting the explosive charge separately from the propellant charge, which were placed in different compartments. His 27 kg models were launched from inclined ramps for aerial bombing and horizontal ramps for field bombing. His rockets played a major role in the bombing of Boulogne (1806), Copenhagen (1807), Leipzig (1813), Baltimore (1814), Mexico (1847), and in the North American Civil War (1861–1865).
- *1850 CE* Further improvement of rockets by **William Hale** (England).

- *1883 CE* ***Konstantin Tsiolkovsky*** *(Russia) pioneered in application of rocketry to space travel. He laid the theoretical groundwork for space flight such as calculating the* escape velocity *and use of* liquid fuel rocket.

- *1909–1941 CE* ***Robert H. Goddard*** *(USA) built and tested for the first time* liquid-propellant rockets*, thus making many basic advances in rocketry. Speculated (1919) on sending rockets to the moon. First launched rocket carrying instruments (camera, barometer, thermometer, 1926).*

- *1923 CE* ***Hermann Oberth*** *(Germany) promoted development of rocketry and space travel.*

- *1933 CE Soviet liquid-propellant sounding rocket reached 5 km altitude.*

- *1942 CE First successful test of German V-2 rocket at Peenemünde. Powered by* liquid oxygen and alcohol. *It traveled 200 km, reached an altitude of over 80 km and landed* $3\frac{1}{2}$ *km from target.*

- *1957 CE US* Jupiter C, *three-stage ICBM (Inter Continental Ballistic Missile) was launched. Recoverable nose cone traveled 460 km up into space.*

- *1958 CE US* Atlas *ICBM missile fired; had maximum range up to 14,400 km.*

- *1959 CE US two-stage ICBM,* Titan 1, *was tested successfully. It had a maximum range of 9600 km.*

- *1969 CE* Saturn 5 *rocket launched 43.2 ton Apollo 11 command module and lunar lander on successful manned voyage to the moon. The 110 m tall, three-stage liquid-fuel rocket developed about 3.5 million kg of lift-off thrust from five first-stage engines. Four smaller second-stage engines produced about 0.45 million kg of thrust, while a single third-stage engine produced enough thrust to orbit the earth and later to break away for the moon.*

- *1981 CE First flight of the first* reusable *rocket, the US space shuttle* Columbia. *Shuttle was powered by three variable-thrust liquid-fuel engines (burning liquid oxygen and liquid hydrogen). Shuttle's two solid-fuel rocket boosters (for lift-off) developed 0.8 million kg of thrust. The* Columbia *exploded during its return voyage on Feb 01, '03, killing all seven astronauts on board at height of 65 km over Texas.*

Aircraft Jet Propulsion (1909–1960)

The theory of the propeller came to completion during the period 1918–1929 *in Germany, England and Italy. It thus took over half a century for a progressive clarification of ideas on the functioning of the propeller: from the analogy with a screw jack to a complete theory based on the principles of scientific fluid mechanics, and using all the mathematical methods of this science.*

From a practical point of view, great progress had also been made in the construction of propellers. However, the difficulties that arose in relation to the supersonic propeller *caused both theoreticians and engineers to look elsewhere for different solutions. In fact, the quest for jet propulsion goes back to* 1909, *the year in which the French engineer* ***René Lorin*** *first proposed the idea of the* ramjet *engine*[119], *the simplest of all jet engines.*

If we imagine that an airplane is flying very fast, *say over* 650 *km/h (e.g. if the plane is first being set into motion by an auxiliary rocket), then the air that enters the engine through a front inlet is being compressed (rammed) without any auxiliary device, since the air cannot move through the engine fast enough to make room for new air coming in. The compressed air becomes very hot. The hot air then flows past the fuel inlets and ignites the fuel. The mixture of air and fuel burns* continuously *in a combustion chamber. The pressure generated by the burning fuel and air sends a flaming exhaust out the jet nozzle and drives the engine forward.*

Thus, using the ram effect, *one can simplify the turbojet engine by virtually discarding the compressor and the turbine. The resulting device is called a* ramjet. *It possesses extreme mechanical simplicity but is penalized in comparison with other jet engines by higher fuel consumption — at least up to the flight-speed range of high supersonic Mach-numbers; and by the fact that without a specific starting drive it functions only above a certain threshold flight velocity.*

The ramjet was displayed for the first time in Paris by the French engineer ***René Leduc*** *in* 1938. *It has since been used for the propulsion of guided missiles such as the U.S. Bomarc.*

An ingenious device that functions right from zero flight speed *is the* pulse-jet. *Like the ramjet, it works without compression, and therefore does not need a turbine for compressor drive. It differs from the ramjet in that the*

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119 Lorin, R., "La propulsion à grande vitesse des véhicules aériens: Étude d'un propulseur à réaction directe", *L'Aérophile* **17** (1909) 463–465. "Propulsion par réaction directe et son application à l'aviation", *ibid* **18** (1910) 322–325.

process is not continuous, but periodic. This device has intake valves which open and close somewhat as in a reciprocating engine, but the valves are controlled automatically, principally by resonance with the periodic process of successive compression, combustion, and outflow.

*The first practical application of this idea was made by the Germans in the design of their V-*1 *flying bomb (1944). These were automatically controlled unmanned aeroplanes, airborne by wings.The V-*1 *carried an explosive charge of* 300 *kg to a distance of the order of* 240 *km, at a speed of* 650 *km/h. The propulsive unit of this device, which was carried above the fuselage and aft of the short and extremely simple wings, consisted mainly of a slightly tapered sheet steel tube, the forward portion of which formed the combustion chamber.*

As the aircraft moved forward (usually launched by a catapult), the dynamic pressure of the air on the forward end of the steel tube forced open an assembly of spring-leaf flap valves; a quantity of fuel was then injected into the chamber and the mixture fired by an electric spark. The resultant gas pressure was large enough to close the spring-leaf valves on the forward end, so that a high-speed jet of gas rushed out at the rear end — giving the machine a forward impulse. As the gas emerged, the pressure in the combustion chamber fell, the valves opened and the whole process started again, giving the characteristic 'motor-cycle' sound (frequency of about 40 *per second) which Londoners soon learned to recognize. This primitive engine developed* 600 *h.p.*

It is hard to conceive of anything simpler and more suitable for rapid production in time of war[120]*, but as a weapon it failed completely once the initial surprise was over. An aircraft which flies straight and level and at constant speed is a choice target for anti-aircraft gunners, who with the aid of radar, reaped a rich harvest.*

Certainly, the ramjet and the pulsejet were not the answer for all the needs of commercial and military aviation. Already in the early 1920*'s engineers began to realize that jet propulsion, and propellers driven by gas turbines, showed promise.*

Indeed, aircraft designers had long lamented the fact that the task of propelling a plane by pushing air from fore to aft (propeller) was achieved inefficiently; obviously dissipating energy by additional friction, and requiring an extra amount of engine weight. The blast of the exploding air-petrol mixture is first used to push a piston, whose reciprocating movement is transformed by means of a connecting rod and crank into rotation of the main shaft, which finally produces by means of a propeller, the necessary thrust to

[120] With the exception of the Japanese "suicide planes".

push the plane forward. Quite a number of links, gliding parts, and bearings are required for each of the many cylinders of the 2, 4 or 6 engines of a plane, each item incurring frictional losses.

Was there no shortcut for transforming the blast of the burnt gases into the final forward thrust on the plane?

***A.A. Griffith**, one of the pioneers in aircraft gas-turbine research expounded his early theories in England (1926). In 1930 **Frank Whittle** (England) patented a design for a jet-aircraft engine. In the early 1930's, engineers in the United States and several European countries worked to develop a practical jet engine. Whittle had tested some of his designs by 1938. Finally, in 1939, the Heinkel Company in Germany built and flew the first jet-engine airplane designed by **Hans Joachim Pabst von Ohain**. Its compressor was of the centrifugal type and the turbine had radial inflow.*

The first successful turbojet airplane (with an engine designed by Whittle) was flown in Great Britain in 1941. The first successful jet-propelled combat airplane was flown in Germany in 1942. In 1944, jet airplanes began to be used by the German fighter squadrons, eliciting the astonishment of allied aviators.

In the turbojet engine, air is compressed mechanically instead of ramming it by the aircrafts motion through the air. A compressor takes air from the outside and brings it to a certain pressure in order to make the combustion and the transformation of heat into mechanical energy more economical. The air stream mixed with the injected fuel is burned in a combustion chamber. The exhaust of this chamber drives the turbine and then escapes from the rear. The shaft-output of the turbine drives the compressor. Ordinarily the gas leaves the turbine at high velocity, and forms the jet that furnishes the thrust. Thus, the turbine-compressor combination ultimately serves as a gas generator for producing the jet.

Turbojets produce thrusts ranging from 10,000 to 150,000 Newton. A turbojet of moderate size uses about 250,000 kg of air per hour. It also requires about 4500 liters of fuel per hour. If a 4-engine turbojet airplane flies at 1000 km/h, it travels only about 50 meters per liter. The turbine wheel must be made of materials which can withstand temperatures as high as 870°C, caused by the hot gases (such materials were not available in the pre WWII years).

Two kinds of compressors may be used in turbojets:

(1) A centrifugal-flow compressor squeezes the air by bringing it into the center of a rapidly spinning wheel, which throws the air towards the rim. There it enters a nearly circular expanding passage, where its speed decreases and its pressure increases.

(2) An axial-flow compressor which raises the pressure of the air up to 12 times that at the inlet.

Turbojet engines power most military aircraft. An afterburner gives the turbojet extra power by spraying additional fuel into the burning mixtures of air and fuel.

A turboprop engine is similar to the turbojet, except that most of its thrust comes from a propeller driven by the gas turbine. The jet exhaust adds only a slight extra thrust. It usually contains two turbines; one drives the propeller and the other drives the compressor. The turboprop is most efficient for short flights at relatively low speeds, and is well suited for business aircraft.

A turbofan is another modification of the turbojet, and is employed by many commercial airliners, including the Boeing 727. Turbofan engines have fans at their front end that act like ordinary propellers. These fans take in large amounts of air through the engine, making it more powerful at low speeds than a turbojet. As a result, the airplane can take off in a shorter distance. Also, the fanjet burns less fuel for the thrust it produces, increasing the distance the airplane can fly without having to refuel.

The fans (propeller-like wheels) at the front end push air back into the engine. This air divides into two streams: one stream goes through the compressor, combustion chambers and turbines of an enclosed turbojet engine. The other stream flows around the engine. The two air streams finally combine at the jet nozzle to produce the thrust. A low speeds, turbofan engines are more powerful than turbojets because the huge fans suck greater amounts of air through the engine.

The first scheduled airline flights by jet transports were launched in 1952 by Great Britain. In 1958, commercial jet passenger service began between New York and London. The turbofan engine was introduced into commercial use in 1960.

A rocket engine ejects a stream of rapidly moving gas particles through nozzles located at its rear. There hot gases are the result of burning fuel in a combustion chamber of the rocket as the fuel and oxidizer are consumed and ejected.

Let $m(t)$ *be the mass of the burnt fuel and oxidizer that is ejected up until time* t *through the rocket nozzle, and* $M(t)$ *the mass of the rocket (including the remaining fuel and oxidizer).*

Let $\boldsymbol{u}$ *and* $\boldsymbol{v}$ *be the respective center of mass velocities of* m *and* M *at time* t*, relative to some suitable inertial frame (the same for both) and let* $\boldsymbol{F}^e$ *represent the total external force acting on the craft in the same frame.*

During a time interval Δt, a mass Δm is ejected and the rocket's velocity changes by $\Delta \boldsymbol{v}$. Clearly $\Delta M = -\Delta m$. Consider the system of all particles of which the rocket is composed at time t. Its momentum is $M\boldsymbol{v}$. At time $t + \Delta t$ the momentum is $(M - \Delta m)(\boldsymbol{v} + \Delta \boldsymbol{v}) + (\Delta m)\boldsymbol{u}$. The total rate of change of momentum (of ejecta plus craft) is then $\left\{M \frac{d\boldsymbol{v}}{dt} + \frac{dm}{dt}(\boldsymbol{u} - \boldsymbol{v})\right\}$, and this must be equal to $\boldsymbol{F}^e$, according to Newton's second law.

Denoting by $\boldsymbol{w} = \boldsymbol{u} - \boldsymbol{v}$ the relative instantaneous velocity of the center of mass of the ejected mass w.r.t. the center of mass of the rocket, one arrives at the rocket equation

$$M \frac{d\boldsymbol{v}}{dt} = \boldsymbol{F}^e + \boldsymbol{w}\frac{dM}{dt} = \boldsymbol{F}^e - \boldsymbol{w}\frac{dm}{dt}.$$

The term $\left\{\boldsymbol{w}\frac{dM}{dt}\right\}$ is referred to as the thrust (or reactive force) of the rocket engine.

The rocket equation can be written in the alternative form

$$\frac{d(M\boldsymbol{v})}{dt} = \boldsymbol{F}^e + \boldsymbol{F}^R,$$

where $\boldsymbol{F}^R = \boldsymbol{u}\frac{dM}{dt} = -\boldsymbol{u}\frac{dm}{dt}$ is the reactive force relative to the inertial frame. In this form, Newton's second law is applied to the rocket proper, and the thrust appears as an additional external force on the system of the instantaneous mass $M(t)$.

Since $\frac{dM}{dt} < 0$ and the direction of $\boldsymbol{w}$ is opposite the rocket's motion, the thrust gives a contribution to $M\frac{d\boldsymbol{v}}{dt}$ that produces a positive acceleration. To achieve large accelerations design engineers must deal with ejection speeds $\boldsymbol{w}$ that are very high, with combustion chambers and nozzles that are capable of large through-put.

For one-dimensional motion with $\boldsymbol{F}^e = 0$, fixed $|\boldsymbol{w}| = w_0$, initial velocity v_0 and initial rocket mass M_0, the rocket equation reduces to $-\frac{dM}{M} = \frac{dv}{w_0}$, yielding the Tsiolkovsky formula (1914) applicable to spaceflight,

$$v = v_0 + w_0 \ln \frac{M_0}{M}.$$

In order for v to exceed the rocket-frame exhaust speed w_0 by a factor N, we must have $\{M_{\text{initial}}/M_{\text{final}}\} = e^N$ (assuming $v_0 = 0$); thus, to exceed the exhaust velocity by any substantial factor, the payload M_{final} must be a small fraction of the entire rocket mass.

Consequently, a rocket is a jet-propelled missile which carries along with it the source of its propulsion energy, and its basic functioning is independent of the presence of the atmosphere.

Rocket propulsion is the only autonomous active propulsion system that can function in vacuum. It is in this respect that a rocket differs from an aircraft jet engine*. The latter uses air to burn its fuel, and can therefore operate only in the atmosphere.*

The equation of the jet engine *is therefore somewhat more involved: air enters at the* front *intake of the engine and, after compression, supports the combustion of the fuel; the exhaust gases are ejected with high velocities at the* rear *of the engine. The above rocket equation is then modified to read (ignoring air drag)*

$$M\frac{d\boldsymbol{v}}{dt} = \left\{-\boldsymbol{w}\frac{dm}{dt} - \boldsymbol{u}\frac{d\mu}{dt}\right\} \equiv thrust,$$

where $\mu(t)$ *is the mass of air processed by the engine up to time* t*,* $m(t)$ *is the mass of fuel ejected up to time* t*,* $\boldsymbol{w}$ *is the relative velocity of the exhaust gases w.r.t. the* engine *and* $\boldsymbol{u}$ *is the ejection velocity relative to the inertial frame.*

Since both $\frac{dm}{dt}$ *and* $\frac{d\mu}{dt}$ *are positive and* $\boldsymbol{u}$ *is in the same direction as* $\boldsymbol{w}$*, both terms of the thrust represent positive acceleration. In fact, even though* $|\boldsymbol{w}| > |\boldsymbol{u}|$*, the term* $\boldsymbol{u}\frac{d\mu}{dt}$ *contributes the major part of the thrust because* $\frac{d\mu}{dt}$ *is usually* 20 *or so times greater than* $\frac{dm}{dt}$ *(that is, the air-to-fuel mass ratio is* 20 *or more). The relative exhaust speed,* w*, is usually at least several times greater than the forward speed of the engine w.r.t. the still air.*

The propulsive efficiency *of engines that furnish a reactive thrust is estimated as follows: let the craft (ship or aircraft) propel itself by taking in a mass* M *of fluid per second, of velocity* u_1 *relative to itself, and eject it behind it at higher velocity* $u_2 > u_1$*. The thrust of the craft is* $M(u_2 - u_1)$*, and the power employed in the propulsion is* $Mu_1(u_2 - u_1)$*, since the craft is moving* forward *with velocity* u_1*.*

Now, the engine must supply the power $\left\{\frac{1}{2}Mu_2^2 - \frac{1}{2}Mu_1^2\right\}$ *per second in order to sustain the motion. This is equal to* $\frac{1}{2}M(u_2 + u_1)(u_2 - u_1)$*. The power efficiency is then:*

$$\eta = \frac{\text{power useful in propulsion}}{\text{power expended}} = \frac{Mu_1(u_2 - u_1)}{\frac{1}{2}M(u_2 + u_1)(u_2 - u_1)} = \frac{2u_1}{u_2 + u_1} < 1.$$

This ratio may be made close to unity by making u_2 *tend to* u_1 *and by making* M *greater in order to keep the thrust* $M(u_2 - u_1)$ *invariant. If* $u_2 = 2u_1$*, i.e., if the acceleration is* 100 *percent, the efficiency is only* 67 *percent.*

The above calculation ignores the fuel mass relative to that of the air; it also does not include all *the losses — such as, for example, the loss due to* *the rotational motion imparted to the fluid or the friction on the propeller blades.*

The principle that an efficient propulsion requires as small a value as possible for the velocity increment of fluid passing through it, applies to other propulsive devices based on the reaction principle. We often have to tolerate jet velocities that are high compared to the flight velocity, although we know that the propulsive efficiency will be poor.

For example, with rockets the outflow velocity of gas may be equal to 2 *km/sec whereas the flight velocity may be only around* 300 *m/sec. One can easily calculate how poor a rocket airplane would be for commercial purposes.*

1904 CE Mount Wilson Astronomical Observatory founded by **George Ellery Hale** (1868–1938, U.S.A.) on Mount Wilson, California (1740 m above sea level). Equipped with a 100-inch reflector telescope in 1917.

1904 CE **John Ambrose Fleming** (1849–1945, England). Engineer. Inventor of the diode vacuum tube that acts as a rectifier for the detection of "wireless" radio signals. The Fleming *valve* was the first practical radio tube, and the first practical application of the *Edison effect*. Fleming made numerous contributions to electronics, photometry, electric measurements and wireless telegraphy.

Fleming was born in Lancaster, Lancashire. He was educated at University College, London and Cambridge University under **James Clerk Maxwell**. He was first to hold the title of professor of electrical engineering at University College.

1904–1907 CE **Bertram Borden Boltwood** (1870–1927, U.S.A.). Scientist. First proposed to use radioactivity data to date minerals. Determined *radium* to be a decay product of uranium (1904) and developed a radioactive dating method for uranium bearing rocks (1907). Discoverer of *ionium*, isotope of thorium (atomic mass 230), which naturally transforms into radium (1907).

Boldwood was born at Amherst, MA. Professor at Yale (1897–1927).

1904–1930 CE **Ludwig Prandtl** (1875–1953, Germany). Pioneer in modern fluid mechanics. Made fundamental contributions in the theory of *boundary layers* (1904) and the field of *wing theory* (1918). An engineer by training, he was endowed with an unusual ability of putting physical phenomena into relatively simple mathematical form and establishing systems of simplified equation which expressed the essential physical relations.

Through his far-reaching concept of the *boundary layer*, he showed the way to treat satisfactorily the flow past a streamlined body at high Reynolds numbers[121]. This discovery led to an understanding of skin friction drag, and the way in which streamlining reduces the drag of airplane wings and other moving bodies. In 1918, Prandtl formulated the flow past a finite wing for straight wings with large aspect ratio. This was done independent of a similar work published in 1897 by **F.W. Lanchester**[122], and is known today as the *Lanchester-Prandtl wing theory.*

Prandtl made important advances in the theories of heat transfer in incompressible fluid (Prandtl number[123]), subsonic and supersonic flows and turbulence (*mixing length*). He also made notable innovations in the design of wind tunnels and other aerodynamic equipment. His advocacy of monoplanes greatly advanced heavier-than-air aviation.

Prandtl was born in Freising. In 1901 he became professor of mechanics at the University of Hanover, where he continued his earlier efforts to provide a sound theoretical basis for fluid mechanics. He served as professor of applied mechanics at the University of Göttingen from 1904 to 1953, and established there a school of aerodynamics and hydrodynamics that achieved world renown. In 1925 he became director of the Kaiser Wilhelm (later the Max Planck) Institute for Fluid Mechanics.

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121 When a fluid flows past a fixed body, because of the effect of viscosity, no matter how small, the layer of the fluid immediately adjacent to the body's surface is at rest. The viscosity is then virtually confined to a very thin layer close to the body and a thin wake extending from the body. In this region, the spatial velocity gradient normal to the body is large, and consequently the viscous forces would not be negligible even if the viscosity were small. In the boundary layer the flow may be laminar, turbulent or a combination of both. Under certain circumstances a *reverse flow* can develop close to the wall.

122 Both established a conceptual model for lift and drag based on *vorticity.* It is hard for an active and creative mind to remember from which readings or conversations the first inspiration arose. Some feel however, that Prandtl did not fully acknowledge his debt to Lanchester in his publications.

123 *Prandtl's number* $\sigma = \frac{\mu/\rho}{\kappa/\rho c_p} = \frac{\mu c_p}{\kappa} = \frac{\text{kinematic viscosity}}{\text{thermal diffusivity}}$, is an index of a fluid's capacity to diffuse momentum as compared with its capacity to diffuse heat. It is a material property that depends on its pressure and temperature [air = 0.733; water = 6.75; mercury = 0.044; glycerin = 7250, at standard temperature and pressure].

1904–1930 CE **Frigyes (Friedrich) Riesz** (1880–1956, Hungary). Mathematician. Foremost among the founders of functional analysis, which has found important applications to mathematical physics [*Riesz-Fischer theorem*, *Riesz representation theorem*]. Discovered general L^p spaces for $p > 1$, $p \neq 2$ and their adjoints. Was first to introduce the concepts of the abstract operator, the *adjoint operator* and the sufficient conditions for the existence of the *inverse operator*. During 1924–1930 Riesz created the theory of subharmonic functions.

Riesz was born to Jewish parents in Györ. He studied at the polytechnic in Zürich and then at Budapest and Göttingen before taking his doctorate at Budapest. He then taught mathematics at the University of Cluj (1911–1922) and was appointed professor of mathematics at the University of Budapest in 1946.

1905 CE The first *geothermal power* station was built at Larderello, Italy.

Energy: Availability, Consumption, and Conversion

Prior to the Industrial Revolution, the sun *was the only source of energy widely available to humankind.* Wood-burning *has been used, where available, since prehistory.* Sails *to harness sun-created wind power were first hoisted* 5000 *years ago,* windmills *were erected* 2000 *years later, and* water-wheels, *which use water raised by the sun, were used* 2000 *years after that.* Coal *came into general use just* 300 *years ago, and* oil *and* gas *only in the last* 100 *years. Not until the* 20^{th} *century did non-solar derived energy arrive, in the forms of* geothermal *and* nuclear power.

The natural flows of energy that have been used for millennia are known as renewable *sources. The amount of energy fossil fuels can supply is ultimately limited by geology. These are known as* non-renewable *sources. As global energy demand grows and non-renewable sources are depleted, attention is turning back to the renewables [*biomass *(plant or animal matter that can be* *converted into fuel), hydropower, solar, power from the sea, geothermal, wind, and nuclear power]*[124].

The standard of living of a population seems to be, in first approximation, a function of the amount of power available per capita. Judging by the present energy consumption in countries such as the United States or Canada, a supply of about 10 *KW per capita would appear to satisfy the energy requirement in an industrial society. This number includes all forms of energy put to work in the service of man, such as the fuel powering our cars, the food we ingest, and the power needed to operate machine tools and household appliances, heat and light habitats, etc.*

Supposing one can stabilize the world population at about 9×10^9 *people, one arrives at a figure of* 9×10^{10} *KW as the power requirement of humanity. The power accessible at present is still far short of this goal. Large areas of the world are still underdeveloped, as indicated by the estimated total present power of* 3×10^9 *KW, corresponding to a world average of only about* 1 *KW per capita.There is a need, then, to increase the world power supply about thirtyfold*[125].

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124 Nearly half of the world's population rely on *biomass*, mostly in the form of wood. *Falling water* generates 25% of the world's electricity, yet this technology is still underexploited. The *sun* already contributes significantly to the energy needs of buildings through walls and windows, and there is a massive increase in investment in technologies to make efficient use of the sun's energy.

Ocean power comes in four main forms: wave power, tidal power, current power and ocean thermal energy conversion — which exploits temperature differences between the surface and depths. The ultimate energy potential is massive but only a small fraction is likely to be harnessed.

The earth's temperature rises 1 °C every 30 m down: more in geologically active areas. *Geothermal power* makes use of this heat, either directly as hot water or to produce electricity.

Winds are caused by uneven heating of the earth's surface. The power of winds is proportional to the cube of wind speed. Windmills can be used either to generate electricity or to do mechanical work.

Oil is the world's largest energy source, but the time when this will no longer be so is already in sight. *Coal* is the most plentiful fossil fuel. Its use is growing at a rate (3% per year) that will intensify problems of acid rain and carbon dioxide. *Natural gas* accounts for 18% of the world's current annual energy budget.

125 If one wants to make 9×10^{10} KW available entirely through conversion of nuclear energy into useful work, one gram of rest mass must be converted into other forms of energy each second!

A possible source of nuclear energy is the heavy hydrogen or "deuterium", D,

***Volta**'s battery (1795) that converts chemical energy (without moving mechanical parts) into electrical energy, gave **Ampère**, **Oersted**, and **Faraday** their first experimental supplies of electricity. The lessons they learned about electrical energy and its intimate relation with magnetism spawned the mighty turboelectric energy converters — steam and hydroelectric turbines — which power modern civilization.*

Forms of energy are interchangeable. When gasoline is burned in an automobile engine, potential chemical energy is first turned into heat. A portion of this heat, say 25%, is then converted into mechanical motion. The remainder of the heat is wasted and must be removed from the engine. A multitude of processes and devices have been found which facilitate these transformations from one form of energy to another. To date we recognize seven forms of macroscopic interconvertible energy: Electromagnetic, chemical, nuclear, thermal, kinetic, electrical and gravitational.

Some examples are: Gravitational to kinetic to electric via watermills and hydroelectric plants; chemical to thermal to kinetic to electric via turbines and electric power stations; electromagnetic to chemical via photosynthesis (in plants) and photochemistry (in photographic films); Electromagnetic to electrical via photoelectricity, radio antenna and solar cells; chemical to electromagnetic via chemiluminescence (fireflies); chemical to kinetic in muscles, rockets and firearms; electrical to electromagnetic via electromagnetic radiation (radio, TV etc.) and electroluminescence; thermal to chemical via boiling and dissociation; electrical to chemical through electrolysis and battery charging; chemical to electrical through batteries and fuel cells; thermal to electromagnetic via thermal radiation; nuclear to thermal via radioactivity fission and fusion. (From the 42 combinatorial conversion possibilities, 13 are as yet unknown.)

The general laws governing energy conversion are the laws of thermodynamics, which can be paraphrased thus:

- *You can't win (conservation of mass-energy and other quantities).*
- *You can't break even (some energy will unavoidably be lost in all heat engines — the 'Second Law of Thermodynamics'). This law is less restrictive for some processes, such as those where kinetic or chemical energy are converted into or produced from electricity without turning into heat first. We can then escape the Carnot efficiency straitjacket. Chemical*

present in seawater. In certain fusion reactions about 1/200 of the rest mass of D is converted into useful energy. The total amount of D in the sea is about 6×10^{16} kg. If this supply is used at a rate of 9×10^{10} KW, it will last for 10^{10} years.

> *batteries perform this trick. So do fuel cells, solar cells, and many other direct conversion devices (energy transformation without moving parts, such as shafts and pistons).*

Direct conversion is desirable in places where energy conversion equipment must run for years without maintenance or breakdown. Also, direct conversion is required where the ultimate in reliability is required, such as on scientific satellites and manned space flights. Under these circumstances, direct conversion will be more reliable and trustworthy than dynamic conversion.

Some methods of direct energy conversion include nuclear-heated thermoelectric generators, thermionic converters and magnetohydrodynamic converters, hydrogen-oxygen fuel cells and nuclear batteries[126].

An example of man's effort to harness natural energy sources is the tapping of geothermal energy.

For years man has viewed with awe the spectacular bursts of natural steam from volcanoes, geysers, and boiling springs. Although the use of hot springs for baths dates to ancient times, the use of natural steam for the manufacture of electric power did not begin until the turn of the 20^{th} *century. For the next several decades, there was no other major development in the field. During* 1950–1972, *production of power from geothermal sources began in the United States, Japan, New Zealand, Iceland, and the Soviet Union.*

Most of the promising areas for geothermal power development are within belts of volcanic activity. A major belt, called "the ring of fire", surrounds the Pacific Ocean. The "hot spots" favorable for geothermal energy are related to volcanic activity in the present and the past 10 *million years.*

Volcanoes produce the most dramatic displays of natural steam. Water that comes into contact with molten lava (temperatures of $2000\,^\circ C$ *and higher) near the earth's surface can exist only as steam. Rapid expansion of steam and other gases below the surface cause some of nature's most violent and explosive eruptions. That of Mt. Vesuvius in* 79 *CE, for example, destroyed the city of Pompeii.*

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[126] In a *nuclear battery*, a central rod is coated with an electron-emitting *radioisotope* (a beta-emitter; say strontium-90). The high-velocity electrons emitted by the radioisotope cross the gap between the cylinders and are collected by a simple metallic sleeve and sent to the load. Space charge effects do *not* prevent the electrons from crossing the gap as they do in the thermionic converter because the nuclear electrons have a million times more kinetic energy than those boiled off the thermionic emitter surface: Consequently, they are too powerful to be stopped by any space charge in the narrow gap.

Almost all active volcanoes have fumaroles, or vents, that discharge steam and other hot gases. But, despite the large quantities of steam discharged during active volcanism, the energy cannot be harnessed as a dependable source of power. In some areas the emission of steam cannot be controlled, and in other areas the costs of controlling the steam would exceed the value of the power obtained.

More promising sources of commercial steam *are certain in other subsurface hot spots or geothermal reservoirs that are generally found in areas of volcanism. These reservoirs contain larger and more dependable volumes of steam or hot water. Wells are drilled into the reservoirs to tap the naturally hot fluids that may drive power generators.*

Most known geothermal reservoirs contain hot water rather than steam. Water at depth and under high pressure remains liquid at temperatures far above $100\,^{\circ}C$ *(the boiling point of water at sea level). When this water is tapped by drilled wells and rises to the surface, the pressure falls. As the pressure decreases, the water boils, perhaps violently, and the resulting steam is separated from the remaining liquid water. Because the well itself acts as a continuously erupting geyser, the expanding steam propels the liquid water to the surface and pumping costs are obviated.*

Generally speaking, geothermal fields are either hot-spring systems or deep insulated reservoirs that have little leakage of heated fluids to the surface. Yellowstone National Park and Weirakei, New Zealand are examples of large hot spring systems. Larderello in Italy and the Salton Sea area of California are examples of insulated reservoirs.

Hot springs *have a plumbing system of interconnected channels within rocks. Water from rain or snow seeps underground. If the water reaches a local region of greater heat it expands and rises, pushed onward by the pressure from new cold and heavy water that is just entering the system. The hot water is discharged as hot springs and geysers.*

1905 CE **Emanuel Lasker** (1868–1941, Germany). World chess champion[127] (1894–1921) and mathematician. He proved (1905) that every polyno-

[127] Unlike many chess geniuses, Lasker's interests were far from narrow, and his concern with philosophical matters led to a deep consideration of what he called the "philosophy of struggle". For Lasker, the chessboard was a stage reflecting the struggle of life in its purest form, a view encapsulated in his well-known

mial ideal is a finite intersection of primary ideals. In 1921, **Emmy Noether** generalized this result in the framework of her foundation of abstract algebra.

Lasker, a mathematician by training, gave up mathematics for chess because, as a Jew, his chances of obtaining a professorship in a German university were practically nill. Later, fleeing the Nazi Holocaust, he had to leave all his possessions in Germany. Neither experience made him bitter. As he explained to a friend: "*In mathematics, if I find a new approach to a problem, another mathematician might claim that he has a better, more elegant solution. In chess, if anybody claims he is better than I, I can checkmate him*".

Lasker was born in Poland. He studied mathematics and philosophy at several German universities and was a student of **Hilbert**, receiving his Ph.D. in 1902. When the Nazis came to power (1933), he was forced to leave Germany and had to come out of retirement in chess to earn his living. He first stayed in Russia during 1934–1938 as a professor of mathematics. Emigrated in 1938 to the United States, where he taught mathematics in several universities. Died in New-York.

1905–1917 CE Years of revolution and terror in Russia which had great impact on World history throughout the 20^{th} century. The traits of the revolutionary situation was engineered by industrial revolution and unwittingly sponsored by a doomed regime. One result of this transformation was the attempt made by the new technicians of espionage, provocation and counter-revolution to dominate the new proletarian masses by creating government rather than police-sponsored worker's unions which were managed and directed by police agents. All this paved the way for chaos, intrigue, hypocrisy and betrayal and heralded upcoming social upheavals.

The 1905 revolution was the melting pot from which emerged one of the greatest revolutionary transformations in modern Jewish history. It proved to be a renaissance for political, spiritual, literary and ideological factors.

The trends which were born from this great upheaval encompassed a new Jewish revival from which a new Zionist revolution sprang. Side by side, an enormous mass emigration to the New World laid the foundation for the flourishing Jewish centers in America. The sons, grandchildren and great

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remark, "On the chessboard lies and hypocrisy do not long survive." He went on to note that "there are 64 squares on the chessboard, if you control 33 of them you must have an advantage." While this is a vast oversimplification of the situation it points out the importance of positional play in the thinking of chess masters.

grandsons of these immigrant would later contribute most significantly to the cultural and scientific milieu of the United States of America.

1905–1925 CE **Richard Martin Willstätter** (1872–1942, Germany). One of the leading organic chemists and biochemists of the early 20^{th} century. Discovered the structure of many natural substances of biological significance. Laid the foundations for the later elucidation of the structure of *alkaloids*, *chlorophyll* and *photosynthesis*. His chemical researches on *enzymes*, ferments and *cholesterol* (in relation to longevity) led to further important developments in this field. The preeminence of the German dye industry was to a great extent due to his discoveries. His overall research on complex organic substances, especially on the coloring matter (anthocyanins and chlorophyll) in plants and on enzymes earned him the Nobel prize for chemistry (1915).

Willstätter was born in Karlsruhe to Jewish parents of rabbinic ancestry. In 1890 he entered the University of Münich, and later served as a professor of organic chemistry in Zürich (1905–1912). He then became the director of the Kaiser Wilhelm Institute in Berlin (1912–1916) and eventually a professor at Münich University. Willstätter retired prematurely (1924) in protest of the increasing anti-Semitism among his colleagues. His property was confiscated and he fled to Switzerland (1939) to escape the Gestapo[128].

[128] Of all the prominent Jewish scientists in Germany, Willstätter presents the most complex, most puzzling, and probably the most tragic picture in his imperviousness toward the Jewish problem in Germany. In 1933 Willstätter visited England and the United States. He was received enthusiastically everywhere, honored in many ways and offered several excellent positions. He turned them down and stayed in Germany, even when the situation became threatening and humiliating for the Jews. It seems difficult to understand the psychology of a man, conscious of his illustrious Jewish ancestry, who had suffered all his life from anti-Semitism and witnessed the horrible persecution of the Jews prevailing in Germany. When **Chaim Weizmann** asked him, “Professor Willstätter, why do you not leave Germany?”, he replied that “one does not leave his mother even when she behaves badly.” He thus continued to stay in Germany until the end of 1938 and decided to leave only when conditions became unbearable and his life was threatened.

The Chemistry of Plant Metabolism

The overall process by which plants absorb, use, and store radiant energy is called photosynthesis. Through this process the chlorophyll of green plants catalyzes the formation of carbohydrates from atmospheric carbon dioxide and water through the action of sunlight. Schematically

$$sunlight \; + \; CO_2 \; + \; H_2O \xrightarrow{\text{chlorophyll}} O_2 \; + \; carbohydrates.$$

In the reverse process (arrow points in the opposite direction) known as animal respiration, cellular oxidation of food (carbohydrates proteins, fats) releases carbon dioxide, water and liberate thermal energy at a rate of 683 kcal per mole.

The concepts involved in these metabolic processes developed slowly alongside the fundamental mainstream ideas of physics and chemistry in the past three centuries:

***Van Helmont** (1620) had been the first to discover the existence of different gases; in particular, he described the properties of carbon dioxide. After conducting many experiments, he concluded that plants were nourished not from the soil but exclusively from water. Through the next century, research on various gases continued at an intensifying pace and in 1723 **Stephen Hales** (1677–1761, England) found that plant growth depended on the very carbon dioxide[129] that Van Helmont had discovered.*

***Joseph Priestley** concluded (1772) that plants not only consumed CO_2 but gave off oxygen, while animals consumed oxygen and gave off CO_2.*

***Jan Ingenhousz** (1779) showed that plants absorbed CO_2 from the atmosphere and water in the presence of sunlight, while their green portion gave off oxygen. In the absence of light the roots, flowers and fruits gave off CO_2. He was the first to recognize (1796) clearly two distinct respiratory cycles in*

129 Although CO_2 makes up only 0.03 percent of the volume of the atmosphere, the total amount of *carbon* in the CO_2 of the atmosphere is 4×10^{14} kilograms. Fifty times as much as this is dissolved in the ocean, either as CO_2 or CO_3^-, so that the total mass of carbon available for life, in the air and sea together is 2×10^{16} kilograms. This amounts to 80 times the carbon existing in all living things. If the carbon of CO_2 is considered the general food supply of all life, there is enough of it to spare to support all the individual organisms our planet now carries.

plants, and that sunlight was essential for the production of oxygen by the leaves.

At about the same time (1782), ***Jean Senebier*** *(1742–1809, Switzerland), botanist, demonstrated that green plants convert CO_2 to oxygen under the influence of light and suggested that CO_2 nourishes the plant.*

With the growing appreciation of catalysis that came about in the early 19^{th} century, it was suspected that some catalyst is associated with the "greenness" of plants.

Consequently, the chemists ***Pierre Joseph Pelletier*** *(1778–1842, France) and* ***Joseph Bienaimé Caventou*** *(1795–1877, France), extracted (1817) the green substance of plants and named it chlorophyll (from the Greek green leaf). Caventou also discovered strychnine (1818), quinine (1820) and caffeine (1821).*

Nicholas de Saussure *(1767–1845, Switzerland) showed (1804) that green plants absorb water and require CO_2 from the air and nitrogen from the soil. It was asserted in 1840 by the agricultural chemist* ***Jean Baptiste Joseph Dieudonné Boussingault*** *(1802–1887, France) that higher plants cannot utilize atmospheric nitrogen but only nitrogen from nitrates in the soil.*

It was later found that other elements, too, are engaged in cyclic processes: sulfur, phosphorus, iron, chlorine, magnesium, potassium, sodium and calcium are absorbed from the soil by the plants and incorporated into their tissue. All these elements must be in a water-soluble salt or ionic form in the soil solution, being taken up with the water that all living cells require[130].

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[130] These minerals originate in the soil as it is formed from rock, and they are normally replaced in the soil as plants and animals die and decay. Small amounts arrive as rain washes down particulate matter suspended in the air. Thus, there are *mineral cycles* in which a molecule of, say, potassium phosphate moves from the soil to plant to animal to bacterium or fungus and then back to soil. If the cycle is broken when a plant is harvested, a need to replenish the soil is established. Of the required minerals, *nitrogen* is frequently in limited supply. Although 78 percent of air is nitrogen gas (N_2), elemental nitrogen must be transformed into an *ionic form* [ammonium ion $-NH_4^+$ or nitrate ion $-NO_3^-$] before plants can absorb and use it. Since most chemical constituents of life are either nitrogen-containing (e.g., 16 percent of protein is nitrogen) or have their synthesis and degradation controlled by proteinaceous enzymes, cellular growth and development is directly limited by the supply of nitrogen.

Phosphorous, as the phosphate ion ($-PO_4^{+3}$) is built into nucleic acid and the phosphate-phosphate bond in ATP, bears the chemical-physical energy needed for every energy-requiring process in cells.

Potassium plays many different roles: It is the primary regulator of the move-

Henri Dutrochet *(1776–1847, France) showed (1837) that CO_2 is absorbed only by those plant cells that contain chlorophyll and only in the presence of light.*

Julius von Sachs *(1832–1897, Germany) discovered (1865) that chlorophyll in plants is found only in small bodies (later termed chloroplasts), and that chlorophyll is the key compound that turns CO_2 + water into starch while releasing oxygen.*

The concept of photosynthesis was introduced in 1893 *to signify the new biochemical approach. It stood for the chemical activity of plants and their ability to synthesize carbohydrates. It was coined by the German-born* ***George Engelmann*** *(1809–1884, U.S.A.). He was a meteorologist, physician and botanist, who practiced medicine in St. Louis (from 1835) and made meteorological and botanical observations during the rest of his life.*

A century after the isolation of chlorophyll, its structure was finally worked out (1910) by ***Richard Willstätter*** *(1872–1942, Germany) and*

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ment of water into and out of plant cells; it is part of many enzymes, and it forms complexes with organic acids within the cells. *Sodium* acts much like potassium.

Calcium and magnesium are enzyme activators, serve as carriers for other ions through *plant cell membranes*, and reduce the toxicity of other ions which may be in excess.

Magnesium, as part of the chlorophyll molecule, has a vital role in *photosynthesis.*

Sulfur's main roles are as constituents of certain amino acids, cysteine and methionine, that determine the 3-dimensional shape of proteins and as parts of compounds that regulate the oxidation-reduction state of cells.

Chlorine is needed in the form of a chloride ion as part of the enzyme complex that breaks water in photosynthesis, releasing molecular oxygen into the atmosphere.

Iron ions are involved in cellular energy transmission in both respiration and photosynthesis. It is also required for the synthesis of chlorophyll.

In addition to these minerals, at least five other are required in very small or trace amounts:

Zinc and *copper* are required as parts of certain enzymes.

Manganese, is part of the enzyme complex that releases oxygen in photosynthesis.

Plant cells will not divide without *boron*, and growing tissues of the root and shoot meristem will die without continuous supplies of borate ions.

All plants require *molybdenum* in order to carry on normal nitrogen metabolism, and many plants also require *silicon*, *germanium*, *vanadium*, or *gallium* for reasons which are completely obscure.

Hans Fischer[131] *(1881–1945, Germany). They took the molecule of chlorophyll apart and deduced its composition from the nature of the fragments ($C_{55}H_{72}MgN_4O_5$).*

As it turned out, chlorophyll closely resembles, in its basic structural pattern, the molecule of heme, which is found in hemoglobin, catalase, *and the* cytochromes. *Its chief points of difference are, first, that it contains an atom of* magnesium *in the center of the molecule, where heme contains an atom of iron. Secondly, attached to it is a long hydrocarbon molecule of the type known as* carotenoid.

The chlorophyll molecule absorbs both violet and red light, hence vegetation is green. It thus acts as a selective antenna that plants use to harvest the sun and open the way to the processes of life.

Carotene ($C_{40}H_{56}$) accompanies chlorophyll in photosynthetic organisms. Its role is partly to harvest some sunlight that is not absorbed by chlorophyll, as well as to react with energetic oxygen molecules so as to protect the cell from degradation. The yellow-orange of carotene remains masked by the chlorophyll until the fall, when the chlorophyll molecule decays and is not replaced; that leaves sturdier carotene molecule to exhibit its powers of light absorption, and the leaves turn yellow.

The light-activated chlorophyll is raised to a high-energy state. It can then expend its energy and return to "ground state" by bringing about some energy-consuming reaction *that is central to the process of photosynthesis. The discovery of this key chemical reaction had to await the days of isotope* tagging.

1905 CE **Arthur Harden** (1865–1940, England). First to detect and identify inorganic phosphate in *metabolic intermediates*[132]. It marked the birth of the systematic study of intermediary metabolism. Discovered (1904) the

[131] Won the Nobel prize (1930) for chemistry for studying the coloring matter of *blood* and *leaves* and synthesizing *hemin*.

[132] Compounds required for the reaction to take place but are not involved as the original compound or as the final ones.

first coenzyme[133] *zymase* (with **W.J. Young**) which they have obtained from yeast.

Harden was professor of biochemistry (from 1912) in London. Awarded the Nobel prize for chemistry (1929).

He found (1905) that when the rate at which yeast cells produce CO_2 begins to fall off, it can be restored by the addition of an inorganic phosphate (e.g. KH_2PO_4); as the evolution of CO_2 proceeded, the quantity of the phosphate remaining in solution decreased. The organic compound found to combine with the phosphate group was later identified as *fructose diphosphate*, the first known metabolic intermediate.

1905 CE **Albert Einstein**[134] (1879–1955, Switzerland, U.S.A.). One of the greatest scientists of all times. Revolutionized scientific thought with new conceptions of time, space, mass, energy and gravitation. He treated matter and energy as interchangeable, rather than distinct categories, and showed that energy and momentum on the one hand, and time and space on the other, are "rotated" into each other when the observer changes his state of motion. He also deduced, on purely theoretical grounds, the relativity of measured time durations and of simultaneity, as well as the dynamical nature of space-time and the effects of gravitation upon geometry and the flow of time. His deductions concerning the inter-convertibility of mass and energy revealed the theoretical possibility of releasing vast amounts of energy bound up in atomic nuclei. Thus, Einstein was one of the harbingers of the atomic age.

In his two theories of relativity, as in his bold contributions to statistical physics and to early quantum theory, he established new paradigms for how hypotheses and theories of modern physics should be enunciated, thus playing a central role in the *demise of the dogmatic universe*. His most astonishing

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133 A compound that is not a protein but which is required for the protein to act as an enzyme. Beginning with the 1920's and continuing to the present day, the chemical nature of various coenzymes was worked out. Most coenzymes proved to contain phosphorus atoms as part of their molecules.

134 For further reading, see:

- Fölsing, A., *Albert Einstein*, Penguin Books: New York, 1997, 882 pp.
- Pais, A., '*Subtle is the Lord...*', *The Science and the Life of Albert Einstein*, Oxford University Press: Oxford, 1983, 552 pp.
- Calaprice, A. (ed), *The Quotable Einstein*, Princeton University Press, 1996, 269 pp.

achievement, the General Theory of Relativity (GTR), finally made *cosmology* a full-fledged branch of physics. GTR has also initiated a far-reaching geometrization and algebraization of our conception of physical reality, that is still in full swing at the dawn of the 21^{th} century.

He was the first to grasp that Planck's discovery had far-reaching implications for the nature of light, and he introduced the revolutionary concept of the wave-particle duality of light, in the process explaining the photo-electric effect and developing the theoretical underpinnings of *laser* theory and *photonics*.

In 1905 Einstein contributed 4 papers to **Annalen der Physik**, each of which became the basis of a new branch of physics[135]:

(1) The paper: "*Die von der molekularkinetischen Theorie der Wärme gefordete Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen*" (On the movement of small particles suspended in a stationary liquid governed by the molecular kinetic theory of heat)[136]. Between 1905 and 1908, Einstein published 5 papers on this subject. The 1905 paper developed the mathematical theory of the irregular motion of microscopic particles suspended in a liquid or gas[137]. Einstein used this phenomenon to support both the atomic hypothesis and the molecular kinetic theory of heat. This work, along with later experimental confirmation by Jean

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[135] The year 1905 is known in science as the "*miraculous year*" with its Latin counterpart "*annus mirabilis*", long used to describe the year 1686, during which Isaac Newton laid the foundations for much of physics and mathematics and revolutionized 17th-century science. It seems entirely fitting to apply the same phrase to the year 1905, during which Albert Einstein not only brought to fruition parts of that Newtonian legacy, but laid the foundation for the break with it that has revolutionized 20th-century science.
Incidentally, the phrase was coined without reference to Newton. In a long poem entitled *Annus Mirabilis*: The Year of Wonders, 1666, **John Dryden**, the famed Restoration poet, celebrated the victory of the English fleet over the Dutch as well as the city of London's survival of the Great Fire.

[136] For further reading, see:
Einstein, Albert, *Investigations on the Theory of the Brownian Movement*, Dover, 1956, 119 pp.

[137] *Einstein's equation* for the mean square displacement $\overline{r^2}$ of spherical particles of radius r_0, moving in a gas of viscosity η at an absolute temperature T after an observation time t, is $\overline{r^2} = \left[\frac{kT}{3\pi\eta r_0}\right] t$.

Perrin (1909), provided the most convincing evidence available up to that time for the existence of molecules and atoms.

In this paper Einstein did *not* set out to explain old 19^{th} century observations, as he stated very clearly at the beginning of his paper: "*In this paper, it will be shown that according to the molecular-kinetic theory of heat, bodies of microscopically visible size suspended in a liquid, will perform movements of such magnitude that they can be easily observed in a microscope, on account of the molecular motions of heat. It is possible that the movements to be discussed here are identical with the so-called* "*Brownian molecular motion*"; *however, the information available to me regarding the latter is so lacking in precision, that I can form no judgment in the matter*".

It took a long time for *Brownian motion* to work itself into the main stream of physics. But when it was finally recognized as a phenomenon worthy of serious study, the consequences were striking[138]. Eventually, the theories of Brownian motion made contact with the quantum theories of the 20^{th} century.

(2) The paper: "*Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt*" (On the heuristic viewpoint concerning the production and transformation of light). Scientists before Einstein had discovered that certain metals eject electrons when struck by light[139], but scientists could not explain this phenomenon on the basis of the classical electromagnetic theory. Einstein boldly adopted Planck's hypothesis of 1900 (that radiant energy is emitted and absorbed not in continuously divisible amounts, but rather in discrete units (*quanta*) depending on the frequency of light. Einstein, however, took it more seriously than Planck ever did — by positing that light also *exists* and *propagates* in these discrete quanta (later called *photons*), and that these light-particles carry momentum as well as energy — both determined

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[138] The physical phenomenon described by **Robert Brown** (1827) was the complex and erratic motion of grains of pollen suspended in a liquid. In the many years which have passed since this description, *Brownian motion* has become an object of study in pure as well as applied mathematics. Even now many of its important properties are being discovered.

[139] *The photoelectric effect* (1902). The number of electrons emitted per second is proportional to the *intensity* of the incident light. The maximum energy of the electrons, however, does not depend on the intensity but on the *frequency* of the incident radiation. No photoelectric emission occurs at all below a certain threshold frequency, whatever the light intensity. Note that Planck's discovery remained isolated until Einstein presented the first general interpretation of h.

from its frequency via Planck's constant, h. In so doing, Einstein introduced into physics the revolutionary concept of *particle-wave duality*. Later (1920's) **de Broglie** and **Schrödinger** extended these concepts to *matter* and enshrined it in Quantum Mechanics. Einstein supposed that in the photoelectric effect, each ejected electron received its energy and momentum from one quantum of the incident light.

Einstein's photoelectric equation $\left[\frac{1}{2}mv^2 = h(\nu - \nu_0)\right]$ was tested experimentally by **Hughes** and **Millikan**, who established its accuracy over a wide range of frequencies, and measured the value of h. It was thus made clear that the new quantum theory of radiation gave a correct account of the photoelectric effect in metals, and that the classical wave theory was inadequate to deal with the interactions between atoms and radiation.

(3) In a paper: "*Zur Elektrodynamik bewegter Körpen*" (On the electrodynamics of moving bodies)[140], Albert Einstein introduced his special theory of relativity (STR). It replaced Newtonian concepts of space and of separate absolute time, with a single geometrical framework of space-time. It is the greatest contribution, since the 17^{th} century, to our understanding of time and of classical (non-quantum) dynamics.

In formulating the STR, Einstein was motivated by two basic facts:

140 For further reading, see:

- Landau, L.D. and G.B. Rumer, *What is Relativity?*, Basic Books: New York, 1962, 72 pp.
- Pauli, W., *Theory of Relativity*, Dover Publications: New York, 1958, 241 pp.
- Aharoni, J., *The Special Theory of Relativity*, Dover Publications: New York, 1965, 331 pp.
- Smith, J.H., *Introduction to Special Relativity*, Dover: New York, 1965, 218 pp.
- Ugarov, V.A., *Special Theory of Relativity*, Mir: Moscow, 1979, 406 pp.
- Sartori, Leo, *Understanding Relativity*, University of California Press, 1996, 367 pp.
- Jammer, M., *Concepts of Space*, Dover: New York, 1993 (Forward by Albert Einstein).
- Jammer, M., *Concepts of Force*, Dover: New York, 1999.
- Jammer, M., *Concepts of Mass*, Dover: New York, 1997.

- The equations of the electromagnetic field are *not* covariant w.r.t. the Galilean transformation.
- Experimentally, the velocity of light is the same for all unaccelerated observers, independent of the apparatus employed for its measurement.

His idea was then to look for *another* linear transformation between inertial frames [it must be linear, because only for a linear transformation is the unaccelerated motion of a particle in one frame, seen as unaccelerated motion in the other]. Maxwell's equations then are covariant with respect to that new transformation. Simple algebra leads one to the so-called *Lorentz transformation*[141].

Thus, STR is based on two fundamental principles:

(1) A principle of covariance under change of inertial frame (generalization of the Newton-Galilei *relativity principle* to encompass *all* laws of physics).

(2) The constancy of the velocity of light[142].

When these principles are translated into mathematics, the results yield the Lorentz transformation law between the coordinates (x, y, z, t) and (x', y', z', t') of a physical event as seen in any two inertial systems moving relative to each other with uniform velocity[143].

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141 $x' = \frac{x-vt}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$; $y' = y$; $z' = z$; $t' = \frac{t-\frac{v}{c^2}x}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$. These formulae connect the coordinates (x, y, z, t) and (x', y', z', t') of two reference systems K' and K, such that K' moves relative to K with uniform velocity v in the positive x-direction.

142 This second principle is really an amalgam of two sub-hypotheses: the independence of the speed of light upon (a) source motion, and (b) observer (inertial) motion. If we assume, as Einstein did, that light propagation obeys Maxwell's equation in any inertial frame, this makes (a) obvious; (b) then follows as special case of the *first* principle.
Furthermore, since Maxwell's equations are assumed to hold in all inertial frames, the raison d'etre for the ether hypothesis 'softly and suddenly vanishes away', as Lewis Carroll might have put it!

143 The famous "*Twin paradox*"; one of a pair of twins stays at home, remaining at rest in an inertial frame of reference at all times, while the other goes on a space flight, traveling with, say, speed $v = \frac{3}{5}c$ for two years of his own time, then switching to *another inertial frame* for the return journey at the same speed $\frac{3}{5}c$. The total time ticked off by the traveler's clock will be four years, while an identical clock carried by the homebody twin will have ticked off five years

If one accepts that the equations of the electromagnetic field are true for all observers in inertial frames (they are *Lorentz covariant*), then one must discard the principle of *Galilean covariance*, and the equations of particle mechanics must be rendered *Lorentz covariant* instead.

Although the mathematical results are expressed in simple algebra that can be understood by high-school students, its physical implications are revolutionary and profound. For example: the rate of flow of time depend on the state of motion of the observer. Also, two events at two different locations, can be seen to be simultaneous by one observer, and yet will *not* appear so to a second observer moving w.r.t. the first.

Since the Lorentz transformation depends only on the relative velocity and *not* on acceleration, STR holds for an accelerated observer, provided the rules for synchronizing clocks at different locations are suitably modified to include acceleration effects — similar to the way in which Newton's second law must be modified by non-inertial forces in an accelerated frame. However, it is much easier to work out the physics of an accelerated laboratory in a continuous succession of instantaneous comoving inertial frames, performing a succession of infinitesimal Lorentz transformations between such frames.

A physicist in an accelerated laboratory knows he is being accelerated, because his accelerometers do not read zero and he cannot play three dimensional billiards. But his instantaneous inertial frame is *continuously changing* according to a specific, known rule, and the Lorentz transformation can be adjusted for him each instant anew.

The Lorentz-covariantization of the laws of dynamics, necessitated by STR, caused a fundamental change in our notions of the properties of space, time, velocity, acceleration, force, mass, momentum, energy, power and field.

(4) In a paper: "*Ist Die Trägheit eines körpers von seinem Energiegehalt abhänging*?" (Does the inertia of a body depend on its energy content?) Einstein expounded the revolutionary idea that the conservation of total energy is equivalent to the conservation of mass. That is, the invariance of total energy implies the invariance of total relativistic mass. The ensuing

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between departure and arrival of the traveling twin. There is no contradiction with the principle of equivalence of inertial observers because the traveling twin switches Newtonian frames at one point in the journey. The twins are therefore *not* equivalent observers, because one can tell from inertial effects which of the twins changed rest frames and which did not.

relation $E = mc^2$ expresses the fact that mass-energy can be expressed in energy units as E, or equivalently in mass units, as[144] $m = E/c^2$.

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[144] Around the year 1850 physicists began to realize that one can extend the law of conservation of energy from the realm of mechanics into other branches of physics, by conceiving, for example, of heat and electricity as other forms of energy. The heat content of an object was soon recognized as the *disordered* kinetic energy of its atoms or molecules, moving randomly in all directions so that the net momentum vanishes. (In contrast, the ordinary kinetic energy $\frac{1}{2}mv^2$ is always accompanied by a net momentum mv of the object.) Other forms of energy were noticed soon afterward that truly deserve the name of "internal" energies because they are not associated with any (classical) motion of any objects. Such a form of internal energy is represented by the various chemical bonds of molecules, which are of essentially electromagnetic and quantum-mechanical nature (although they *do* involve motions of electrons and nuclei). By convention, one talks about a binding energy of a molecule if one has to do work to break the bond between its constituents. Accordingly, during the formation of such a bond, positive energy must be given up to the environment, for example, as heat in most oxidation processes.

Nevertheless, matter and energy were considered two separate and distinct entities existing in an all-pervading ether.

This was justified because the speed of light c is so large, that for any *non-nuclear process* observed before the advent of STR, the mass changes $\frac{\Delta E}{c^2}$ are negligible. Thus, for all energy-conversion processes then known, rest-mass and energy may be assumed to be separately conserved to a high degree of accuracy.

The relation $E = mc^2$ can be derived theoretically in STR, in many different ways. For example, if a point-mass' momentum is to be $\boldsymbol{p} = m\boldsymbol{v}$ as in Newtonian mechanics, and if momentum is to be conserved in billiard-ball collisions, the conservation law $\Delta\left(\sum \boldsymbol{p}_i\right) = 0$ must be Lorentz covariant, and thus is mathematically possible if the inertia of a mass m depends upon its velocity in any given frame in accordance with the expression $m = m(v) = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$; m_0 is called the *rest mass*.

Applying Newton's second law in one dimension,

$$\boldsymbol{F} = \frac{d}{dt}m(\boldsymbol{v})\boldsymbol{v} = \frac{d}{dt}\left[\frac{m_0\boldsymbol{v}}{\sqrt{1-\frac{v^2}{c^2}}}\right],$$

the work done by accelerating the body from $v = 0$ to $|\boldsymbol{v}| = V$ is

$$W = \int_{v=0}^{V}(\boldsymbol{F}\cdot\boldsymbol{v})\,dt == m_0\int_0^V \boldsymbol{v}\,d\left[\frac{\boldsymbol{v}}{\sqrt{1-\frac{v^2}{c^2}}}\right] = [m(\boldsymbol{v}) - m_0]c^2 = E - E_0.$$

Classical electromagnetism is consistent, ab initio, with STR: Maxwell's

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Hence $E = mc^2 = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}$, with the accompanying momentum

$$\boldsymbol{p} = m(\boldsymbol{v})\boldsymbol{v} = \frac{m_0\boldsymbol{v}}{\sqrt{1-\frac{v^2}{c^2}}}.$$

Furthermore, E and $\boldsymbol{p}$ are tied up by the algebraic relation $E^2 - p^2c^2 = m_0^2c^4$. For particle of zero rest mass (e.g., a photon), $m_0 = 0$ and therefore $E = |\boldsymbol{p}|c$ or $p = \frac{E}{c}$. Clearly, $\frac{E^2}{c^2} - p^2 = m_0^2c^2$ is invariant under the Lorentz transformation.

At rest, or low (non-relativistic) speeds $|\boldsymbol{v}| \ll c$, the relation $E = mc^2$ yields the "rest energy" of an object of mass m — some of which is transferred into other forms of energy whenever the object takes part in some exothermic transmutation, such as in a nuclear reaction, in which this mass is not conserved. (This is true for chemical reactions too, but the conversion of energy into rest-mass is negligible in this case.)

The great technological impact made by the discovery of methods for converting rest energy into useful work stems from the fact that light's speed c happens to be so large in everyday units. The work that can in principle be extracted from the conversion of 1 kg of mass is, in mks units ($c = 3 \times 10^8$ m/sec; $1\ J = \frac{5}{18} \times 10^{-6}$ kilowatt-hour): $W = 1 \times (3 \times 10^8)^2 = 9 \times 10^{16}\ J = 2.5 \times 10^{10}$ KWh. Otherwise stated: one gram of matter is equivalent to 3000 tons of coal!

The relation $E = mc^2$ also states the mass equivalence of the binding gravitational energy between masses. For example, when two spherical masses m_1 and m_2 form a two-body system, part of their combined mass is eaten up by the gravitational binding energy: $G\frac{m_1m_2}{2Rc^2}$. Here R is the separation between the mass-centers of the co-orbiting masses and the factor $\frac{1}{2}$ indicates that the *gravitational mass-defect* arising from the mutual potential energy is partially offset by the masses orbital kinetic energy. The mass-equivalent of the binding energy of the earth-moon system is about 4×10^{14} grams. It is the mass of a small mountain, ca 400 meters high, which constitutes about $5 \cdot 10^{-12}$ of the moon's mass.

Similarly, the mass defect of a self-gravitating non-rotating earth of mass M and radius a is $\Delta M = \frac{2}{5}\frac{GM^2}{ac^2}$. If a deranged accountant could tally the total mass defect ΔM from the knowledge of the exact distribution and composition of its constituent materials, he would discover a shortfall of 1.6×10^{18} grams, which is of the order of $3 \cdot 10^{-10}$ of the earth's mass. GTR introduces some extremely small correction to the STR expression for mass defects we have used here.

equations are invariant under the Lorentz transformation and do not need to be modified. However, STR gives us a new point of view, enhances our understanding of electromagnetism and introduces relativistic corrections to the interactions of charged masses with electromagnetic fields.

Other realms of physics are also affected, including the microscopic world of elementary particles that abide by the rules of quantum mechanics, special relativity and quantum electrodynamics. All laws are written so as to be covariant under the Lorentz transformation. This encompasses also the theories of weak interactions, responsible for β-decay, and the strong nuclear interactions which hold the nucleus together. Nowhere yet is there evidence for the breakdown of STR over the many scale decades in energy, time, distance and mass accessible to modern particle accelerators.

The metric of the new, "**Minkowski**" 4-dimensional space-time introduced into physics with STR is pseudo-Euclidean[145]. Laws of Euclidean geometry are still valid for the spatial part of space-time, but only in inertial frames; they are *violated* in a non-inertial frame. [To see this, consider a rigidly rotating disc in a Minkowski space-time. An inertial observer, who is *not* participating in the rotation, attaches a non-rotating circle under the rotating disc, measures the circumference L and diameter D of the disc, and finds $L/D = \pi$. But in a frame set up by an observer on the rotating disc, one finds that $L'/D' > \pi$, as a careful use of the Lorentz transformation shows.]

The year 1905 was ripe for the discovery of relativity. In 1900, **Larmor** (1857–1942) had already suggested that moving clocks must run slow, and by how much. Lorentz had published the final version of his transformation in 1903. In 1904 Poincaré attached the name of Lorentz to the transformation, and stated that there must be a new dynamics in which no velocity can exceed the velocity of light.

The sun loses a mass-equivalent of radiation at the rate of 4.5 million tons per second. At this rate it loses a moon-mass once every 500,000 years, and an earth-mass every 40 million years. The sun's radiation incident each second on the earth's surface has a mass equivalent of order 2 kilograms.

145 The distance (interval, ds) between two closely neighboring points is given by $ds^2 = -c^2dt^2 + dx_1^2 + dx_2^2 + dx_3^2$. Writing $-c^2dt^2 = -dx_0^2$, we have $ds^2 = -dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2$, which is the analog of the Pythagorean theorem in *Minkowski geometry*. For two events having a time-like separation — i.e. there exists an inertial frame in which they occur at the same spatial point — $ds^2 < 0$, so technically ds is imaginary in this case.

Lorentz himself, in spite of his early involvement, was not happy with relativity and felt, to his last day, that time and space are completely distinct and a universal *true* time must exist.

Einstein was born in Ulm, Württemberg, Germany, the son of Hermann and Paulina (neé Koch) Einstein. After public school in Münich and in Aarau, Switzerland, Einstein studied mathematics and physics at the Swiss Polytechnic Institute in Zürich. He graduated in 1900.

From 1902 to 1909 he worked as an examiner at the Swiss Patent Office, in Bern. This job allowed him much free time, which he spent in scientific investigations. In 1903 Einstein married a fellow member of his class at Zürich — Mileva Maric (1875–1948) from Serbia. They had two sons: Hans Albert (1904–1973) and Edward (1910–1964). He became a Swiss citizen in 1905. The year 1905 was Einstein's '*annus mirabilis*' — a year to rank alongside 1543 (when Copernicus published '*De Revolutionibus Orbium Coalestium*') and 1686 (when Newton completed his '*Principia*').

One of the great ironies in the history of physics, is that Einstein submitted his 1905 paper on special relativity to the University of Bern in support of his candidacy for an affiliation giving him the right to practice as a *Privatdocent* (entitled to offer instruction under the auspices of the university) — and it was rejected! However, his work attracted the interest of greater men, in particular **Max Planck** in Germany and **H.A. Lorentz** in Holland. It is amazing how a junior civil servant with only the formal training of a schoolteacher could rearrange the structure of physics created by such giants as **Maxwell**, **Boltzmann**, **Lorentz** and **Planck**.

Einstein once remarked that he had never met a real physicist until he was 30. The only person he was able to discuss his ideas with was an engineer, **Michelangelo Besso** (1873–1955), then also an employee at the patent office, whom Einstein had known since his student days in Zürich, and whom he has immortalized in the last sentence of his 1905 paper: "*In conclusion I wish to say that in working at the problem here dealt with, I have had the loyal assistance of my friend and colleague M. Besso, and that I am indebted to him for several valuable suggestions*".

Immediately after the publication of that paper, Einstein was offered an associate professorship at the University of Zürich, which he held for three semesters[146]. In 1911 he joined the German University in Prague, where he held the position of professor ordinarius in physics, the highest academic rank.

[146] His boss at the Patent Office had no idea what a great scientist he was harboring within his walls, and when Einstein announced his resignation from his post he was greatly surprised and asked him for his reasons. Einstein told him that he had been offered the post of Professor at the University of Zürich. 'Now

Despite his absorption in his scholarly pursuits, he could not fail to notice the political strife and quarrels between the rival nationalists, and felt great sympathy for the Czechs and their aspirations. In 1912 Einstein returned to Switzerland, where he taught at the Polytechnic, the same place to which he had come as a poor student in 1896. His friend and colleague, Max Planck, succeeded in obtaining for him a professorship at the Prussian Academy of Science in Berlin, a research institute where Einstein could devote all his time to research.

Einstein took up his new position in April 1914. Very soon after making the move to Berlin with her husband, Mileva Einstein returned to Zürich with their two boys. As the war raged on unabated, Einstein remained separated from his family in the German capital, where he was putting the finishing touches to the general theory of relativity.

The stress of the war[147], many years of overwork and a catalogue of personal catastrophes were to take their toll on Einstein with unexpected ferocity. In the early part of 1917, he fell into a period of a severe sickness from which he did not fully recover until 1920 and which severely hampered his work. With the help of his cousin, Elsa Löwenthal, Einstein dragged himself out of his depression which had accompanied the physical symptoms. He divorced Mileva in 1919, giving her and the children the entire proceeds from the Nobel prize which he was certain to receive within a few years (1922). He later married Elsa who became his lifelong companion.

International fame came to Einstein in 1919, with the announcements that a prediction of his general theory of relativity was verified. In 1921 he was awarded the Nobel prize in Physics for his work on the photoelectric effect. In 1932 Einstein accepted an invitation to spend the winter term at the California Institute of Technology. By January 1933, Einstein resigned from his position at the Royal Prussian Academy of Sciences, and never returned to Germany. Many positions were offered him, but he finally accepted a professorship at the new Institute for Advanced Study in Princeton[148], New Jersey, and later became an American citizen.

Einstein', he answered, 'don't make any foolish jokes. Nobody would believe such an absurdity.'

147 One of his friends, a physician, described Einstein's lifestyle during the first half of the war thus: "... *he sleeps until he is wakened; he stays awake until he is told to go to bed; he will go hungry until he is given something to eat; and then he eats until he is stopped*". During the first three months of his illness, diagnosed as a stomach ulcer, Einstein lost some 25 kg.

148 His first impressions of Princeton were: "*A quaint ceremonious village of puny demigods on stilts*".

During WWII secret news reached U.S. physicists that the German uranium project was progressing. Einstein, when approached by his friend **Leo Szilard** (1898–1964), signed a letter to President Roosevelt pointing out the feasibility of atomic energy. It was that letter which sparked the Manhattan Project and future developments of atomic energy. In 1945 Einstein retired from his position at the institute, but continued to work there. Despite his advancing age he continued to work on the "Unified Field Theory", which attempted as a first step to unify gravitation and electromagnetism into one theory.

Einstein was not only one of the greatest scientists of all time, but also a generous person, who took out time and effort to help others and spoke out openly for his beliefs and principles. He never forgot that he had been a refugee himself, and lent a helping hand to the many who asked for his intervention. The man who refused to write popular articles for his own profit, devoted hours to raising money for refugees and other worthwhile causes.

Einstein was a Jew, not only by birth but also by belief and action. He was in full harmony with the two fountain heads of Judaism: Justice and Charity. The democratic and humanitarian character of the Mosaic Law was deeply embedded in his conscience, and the magnificent poetry of the Old Testament prophets filled him with awe. He took an active part in Jewish affairs, wrote extensively, and attended many functions in order to raise money for Jewish causes. He was first introduced to Zionism during his stay in Prague, where Jewish intellectuals gathered in each other's homes talking about their dream of a Jewish homeland.

In 1921, **Chaim Weizmann** (1874–1952), then president of the World Zionist movement, asked Einstein to join him on a fund-raising tour of America to seek aid for the nascent Hebrew University. Einstein readily agreed, since his interest in the university had been growing. The tour was highly successful. He visited the land of Israel and was greatly impressed by what he saw. Einstein appeared before the Anglo-American Committee of Inquiry in 1946, and entered a strong plea for a Jewish homeland.

When the State of Israel was established, he hailed the event as the fulfillment of an ancient dream, providing conditions in which the spiritual and cultural life of a Hebrew society could find free expression. After Weizmann's death he was asked by **David Ben-Gurion** (1886–1973), then the Prime Minister of Israel, to stand as a candidate for the presidency of the young state, which he declined "being deeply touched by the offer but not suited for the position". When he was hospitalized for the illness which proved to be his last, he took with him the notes he had made for the television address he was to give on Israel's seventh Independence Day.

Some of the theoretical physicists who collaborated with Einstein in Princeton were: **Boris Podolsky** (1896–1966; coll.: 1931–1935), **Leopold Infeld** (1898–1968; coll.: 1936–1939), **Nathan N. Rosen** (1909–1992; coll.: 1932–1937). Rosen worked with Einstein on the foundations of Quantum Mechanics, gravitational lenses, and on two-sheeted spaces ('*Einstein-Rosen bridge*'). Infeld also collaborated with Max Born on non-linear corrections to Maxwell's equations under conditions of strong fields.

As Einstein's own mathematical skills were not exceptional, he had employed the assistance of an outstanding mathematician in his work on GTR. He was **Jakob Grommer** (ca 1883–1933), with whom he coauthored a number of papers and was acknowledged in others, during 1917–1929. It means that Grommer collaborated with Einstein for twelve years — longer than anyone else. Grommer was born in Brest-Litovsk. He was a studious disciple in the local Yeshiva, intending to become a Rabbi. A burning interest in mathematics brought him to Göttingen. In an incredibly short time, he not only acquired a deep knowledge of mathematics but produced an essay which so impressed **David Hilbert** that the faculty decided to grant him a doctorate in spite of the fact that he had never graduated from high school. If one considers that he was disfigured as the result of a malignant disease, and that he was, moreover, physically weak, then one can appreciate how uncommon the talents were which this man brought into the world. In 1929 Grommer returned to his homeland. He became a professor in Minsk and a member of the Academy of the Belorussian Soviet Republic.

Worldview XXIX: Einstein

God, Nature and Quantum Mechanics

* * *

"The eternal mystery of the world is its comprehensibility."

* * *

"How insidious Nature is when one is trying to get at it experimentally."

(1915)

* * *

"Coincidence is God's way of remaining anonymous."

* * *

"Nature conceals her secret by exaltedness, but not by cunning."

(1930)

* * *

"Nature is showing us only the tail of the lion. But I have no doubt that the lion belongs to it, even though, because of its Colossal dimensions, it cannot directly reveal itself to the beholder."

(1914)

* * *

"I want to know how God created this world. I am not interested in this or that phenomenon, in the spectrum of this or that element. I want to know His thoughts, the rest are details."

* * *

"Raffiniert ist der Herr Gott, aber boshaft ist er icht. ("God is subtle, but he is not malicious.")

* * *

"What really interests me is whether God could have made the world differently; in other words, whether the demand for logical simplicity leaves any freedom at all."

* * *

"When I am judging a theory, I ask myself whether, if I were God, I would have arranged the world in such a way."

* * *

"The more success the quantum theory has, the sillier it looks."

* * *

"...undoubtedly a piece of definite truth – but not the whole truth, let alone the definite truth."

(1931, on quantum mechanics)

* * *

"It is hard to sneak a look at God's cards. But that he would choose to play dice with the world is something that I cannot believe for a single moment."

(1942)

* * *

"I can, if the worse comes to the worst, still realize that God may have created a world in which there are no natural laws. In short, a chaos. But that there should be statistical laws with definite solutions, i.e., laws that compel God to throw dice in each individual case, I find highly disagreeable."

* * *

"Quantum mechanics is very worthy of regard. But an inner voice tells me that this is not the true Jacob. The theory yields much, but it hardly brings us close to the secrets of the Ancient One. In any case, I am convinced that He does not play dice."

(1926)

* * *

"I admire to the highest degree the achievement of the younger generation of physicists which goes by the name of quantum mechanics and believe in the deep level of truth of that theory; but I believe that the restriction to statistical laws will be a passing one."

(1929)

* * *

"You believe in a God who plays dice, and I in complete law and order in a world which objectively exists, and which I, in a wildly speculative way, am trying to capture... Even the great initial success of the quantum theory does not make me believe in the fundamental dice game."

Albert Einstein, in a letter to Max Born, December 4, 1926

Mathematics

* * *

"As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality."

* * *

"How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the objects of reality?"

* * *

"Since the mathematicians have invaded the theory of relativity, I do not understand it myself anymore."

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“Do not worry about your difficulties in mathematics, I assure you that mine are greater.” (replying to a letter from a little school-girl)

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“I don’t believe in mathematics.”

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“God does not care about our mathematical difficulties. He integrates empirically.

Music

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“Music does not influence *research work, but both are nourished by the same sort of longing, and they complement each other in the release they offer.”*

(1928)

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“First I improvise and if that doesn’t help, then I seek solace in ***Mozart****; but when I’m improvising and it appears that something may come of it, I require the clear constructions of* ***Bach*** *in order to follow through.”*

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*“***Bach*** *and* ***Mozart*** *are my favorites; also* ***Schubert****, because of the composer’s ability to express emotions. I am considerably less fond of* ***Beethoven****, since his music is too dramatic and personal.*
Handel *is technically good but displays shallowness.* ***Schumann****’s shorter works are attractive because they are original and rich in feelings.*

Mendelssohn *had considerable talent but lacked depth. I like some lieder and chamber music by* ***Brahms****.*
I find ***Wagner****'s musical personality indescribably offensive so that for the most part I can listen to him with disgust. I consider* ***Richard Strauss*** *gifted but without inner truth and concerned too much with outside effects."*

(1939)

Science and mankind

* * *

"Why does this magnificent applied science which saves work and makes life easier bring us so little happiness? The simple answer: because we have not yet learned to make sensible use of it. Concern for man himself and his fate must always be the chief interest of all technical endeavors... in order that the creations of our mind shall be a blessing and not a curse to mankind. Never forget this in the midst of your diagrams and equations."

From an address at the California Institute of Technology,

Pasadena, February 1931

* * *

"One thing I have learned in a long life: that all our science, measured against reality is primitive and childlike – and yet it is the most precious thing we have."

* * *

"Science without religion is lame; religion without science is blind."

* * *

" Science will stagnate if it is made to serve practical goals."

* * *

" Physical concepts are free creation of the human mind and are not, however it may seem, uniquely determined by the external world."

(1937)

* * *

"The most beautiful thing we can experience is the mysterious. It is the source of all true art and science."

* * *

"The whole of science is nothing more than the refined thinking... Most of the fundamental ideas of science are essentially simple, and as a rule, can be expressed in a language comprehensible to everyone."

(1936)

* * *

"A theory, in order to deserve confidence, has to be based on generalizable facts... Never has a truly useful and profound theory been discovered by pure speculation."

(1918)

* * *

"Only the theory decides what can and what cannot be observed."

(1926)

* * *

"No amount of experimentation can ever prove me right; a single experiment can prove me wrong."

* * *

"Development of Western Science is based on two great achievements: the invention of the formal logical system (in Euclidean geometry) by the Greek Philosophers, and the discovery of the possibility to find out causal relationships by systematic experiment (Renaissance)."

(1953)

Judaism

* * *

"The Jewish religion is... a way of sublimating everyday existence... It demands no act of faith – in the popular sense of the term – on the part of its members. And for that reason there has never been a conflict between our religious outlook and the world outlook of science."

(1930)

* * *

"The support for cultural life is of primary concern to the Jewish people. We would not be in existence today as people without this activity in learning."

(1950)

* * *

"The pursuit of knowledge for its own sake, and almost fanatical love of justice and the desire for personal independence – these are the features of the Jewish tradition which makes me thank my stars that I belong to it."

(1934)

* * *

"Were I wrong, one professor would have been quite enough."

In response to a manifesto in which 100 Nazi professors charged him with scientific error

On the Human Condition

* * *

"Comfort and happiness have never appeared to me as a goal... The trite objects of human efforts – possessions, outward success, luxury – have always seemed to me contemptible, since early youth.

I live in that solitude which is painful in youth, but delicious in the years of maturity."

* * *

"Of what is significant in one's own existence, one is hardly aware, and it certainly should not bother the other fellow. What does a fish know about the water in which he swims all his life?

The bitter and the sweet come from the outside, the hard from within, from one's own efforts. For the most part I do the thing which my own nature drives me to do. It is embarrassing to earn so much respect and love for it. Arrows of hate have been shot at me too, but they never hit me, because somehow they belonged to another world, with which I have no connection whatsoever."

* * *

"The years of anxious searching in the dark, with their intense longing, their alternations of confidence and exhaustion and final emergence into light – only those who have experienced it can understand it."

* * *

"Personalities are not formed by what is heard or said, but by labor and activity. The most important method of education accordingly always has consisted of that in which the pupil was urged to actual performance."

* * *

"I believe with Schopenhauer that one of the strongest motives that leads men to art and science is escape from everyday life with its painful crudity and hopeless dreariness from the fetters of one's own ever-shifting desires. A finely tempered nature longs to escape from personal life into the world of objective perception and thought."

* * *

"I cannot imagine a God who rewards and punishes the objects of his creation, whose purposes are modeled after our own – a God, in short, who is but a reflection of human frailty... It is enough for me to contemplate the mystery of conscious life perpetuating itself through all eternity, to reflect upon the marvelous structure of the universe which we can dimly perceive and try humbly to comprehend even an infinitesimal part of the intelligence manifested in Nature.

(1932)

* * *

"The most precious things in life are not those one gets for money."

(1946)

* * *

"Morality is of the highest importance – but for us, not for God."

(1927)

* * *

"Try not to become a man of success, but rather try to become a man of value."

(1955)

* * *

"I soon learned to ferret out that which might lead to the bottom of things, to disregard everything else, to disregard the multitude of things that the mind but detract from the essential."

(1948)

* * *

"To us believing physicists the distinction between past, present, and future has only the significance of a stubborn illusion."

(1955)

* * *

"I believe that the terrible decline in man's ethical behavior is due primarily to the mechanization and depersonalization of our lives – a disastrous by-product of the development of the technological-scientific intellect."

(1946)

* * *

"Politics is for the present, while an equation is for eternity."

* * *

"Death is an old debt that one eventually pays. Yet instinctively one does everything possible to postpone this final settlement. Such is the game that nature plays with us."

(1955)

* * *

"If all efforts are in vain and mankind ends in self-destruction, the universe will not shed a single tear over it."

(1946)

* * *

"I do not know how the Third World War will be fought, but I do know how the Fourth will: with sticks and stones."

* * *

"If you want to live a happy life, tie it to a goal, not to people or things."

* * *

"A life directed chiefly toward the fulfillment of personal desires sooner or later always leads to bitter disappointment."

(1954)

* * *

"What is the meaning of human life, or for that matter, of the life of any creature? To know an answer to this question means to be religious."

* * *

"Mankind remains as idiotic as it always was, and it's no great pity; but that no one would then play Bach or Mozart any more – that is a pity."

(1950)

* * *

"The rarest and most valuable of all intellectual traits is the capacity to doubt the obvious."

* * *

"Everything is determined, the beginning as well as the end, by forces over which we have no control. It is determined for the insect as well as for the star. Human beings, vegetables, or cosmic dust, we all dance to a mysterious tune, intoned in the distance by an invisible player."

* * *

"Academia places a young person under a kind of compulsion to produce impressive quantities of scientific publications – a temptation to superficiality, which only strong characters can resist."

* * *

"Great spirits have always encountered violent opposition from mediocre minds."

* * *

"Imagination is more important than knowledge."

* * *

"Everything should be made as simple as possible, but not simpler."

* * *

"The search for truth is more precious than its possession.

* * *

"I never worry about the future. It comes soon enough."

(1945)

Others on Einstein

* * *

"Einstein is a typical Old Testament figure, with the Jehovah-type attitude that there is a law and one must find it."

Abraham Pais

* * *

"Einstein's struggle is our struggle today; It is the search for a final theory."

Steven Weinberg, 1993

* * *

"Few scientists have ever poised their ambitions as poetically or as nakedly as Einstein who spoke sometimes as if God were someone he met for coffee every day... To Einstein, God was a code word for the mystery and grandeur of the universe, the wellspring of awe, a reminder that there was something at the core of existence that all his equations could only graze, as he said once, "something we cannot penetrate"."

(Dennis Overbye, 1998)

* * *

"When I read a book on Einstein's physics of which I understood nothing, it doesn't matter: that will make me understand something else.*"*

(Pablo Picasso, 1956)

* * *

"I was able to appreciate the clarity of his mind, the breadth of his documentation, and the profundity of his knowledge... One has every right to build the greatest hopes on him and to see in him one of the leading theoreticians of the future."

Marie Curie, 1911

* * *

"Einstein would be one of the greatest theoretical physicists of all time even if he had not written a single line on Relativity."

Max Born

* * *

"What we must particularly admire in him is the facility with which he adapts himself to new concepts and knows how to draw from them every conclusion."

Henri Poincaré, 1911

* * *

"Even though without writing each other, we are in mental communication for we respond to our dreadful times in the same way and tremble together for the future of mankind... I like it that we have the same given name."

Albert Schweitzer, 1955

* * *

"The revolutionary proletariat salutes the great revolutionary of the natural sciences as a fellow fighter against the dark forces of ignorance, barbarism and reaction."

(Soviet 'Pravda' on Einstein 50^{th} birthday, March 14, 1929)

* * *

"An Englishman would scarcely have produced this theory; perhaps it reflects the abstract-conceptual character of the Semite..."

(Sommerfeld to Wien (1907) on the Theory of Relativity)

* * *

"Unconfined by conventional restrictions, he confronted the world spirit as a laughing philosopher, and his witty sarcasm mercilessly castigated all vanity and artificiality".

Hans Byland (Einstein's classmate), 1928

* * *

"As befits a genius, Einstein ideas on gravitation had been irrational – a mixture of philosophical requirements with a powerful physical insight and a progressive penetration into the preparatory studies of the mathematicians."

(Felix Klein, 1921)

* * *

"Ptolemy made a universe, which lasted 1400 years. Newton, also, made a universe, which lasted 300 years. Einstein has made a universe, and I can't tell you how long that will last."

George Bernard Shaw

* * *

"One of the greatest – perhaps the greatest – of achievements in the history of human thought."

Joseph John Thomson, discoverer of the electron,

referring to Einstein's work on General Relativity, 1919

* * *

"Einstein was a physicist and not a philosopher. But the naive directness of his questions was philosophical."

C.F. von Weiszaecker

* * *

"Einstein explained his theory to me every day, and soon I was fully convinced that he understood it."

Chaim Weizmann, 1929

* * *

"Einstein was indisputably one of the greatest men of our time. He had, in a high degree, the simplicity characteristic of the best men of science – a simplicity which comes of a single-minded desire to know and understand things that are completely impersonal."

* * *

"He removed the mystery from gravitation, which everybody since Newton had accepted with a reluctant feeling that it was unintelligible."

* * *

"Of all the public figures that I have known, Einstein was the one who commanded my most wholehearted admiration... Einstein was not only a great scientist, he was a great man. He stood for peace in a world drifting towards war. He remained sane in a mad world, and liberal in a world of fanatics."

Bertrand Russell, 1955

* * *

"A generation or more ahead of his time, he gave physicists the theoretical tools to describe the big bang, quasars, pulsars and black holes. We live in Einstein's universe, and the only way to end our account of his scientific achievements is by looking at his description of the universe in which we live."

* * *

"At the end of the 1980s, a satellite known as COBE (from Cosmic Background Explorer) was launched by NASA to study the background radiation with more precision than ever before. In 1992, the NASA team announced that they had discovered exactly the kind of ripples in time that the theory

had predicted. It was headline news around the world – the combination of Einstein's general theory, the big-bang model, and the added ingredient of dark matter, had been vindicated. This was, and is, the most compelling evidence ever that the universe we live in is described by the equations of the General Theory of Relativity – that it is, indeed, Einstein's universe."

M. White and J. Gribbin, 1993

1905–1911 CE **Ejnar Hertzsprung** (1873–1967, Denmark). Astronomer. Discovered the most important graph ever plotted by astrophysicists. It relates the total energy output of each group of stars to the corresponding stellar temperature (known as "*Hertzsprung-Russell diagram*" (or 'R-H diagram'). His discovery, although just a method of two-dimensional spectral classification[149] of stars, led astronomers to extremely important discoveries concerning the relations between the luminosities and surface temperatures of stars which eventually resulted in the understanding of processes of stellar evolution.

Hertzsprung specialized in the study of binary stars and star clusters, and in 1905–1907 announced the discovery of *giant stars* (more voluminous than the mean; brightness and spectral class increase together). He was the first to recognize the distinction between giant and *dwarf* stars.

In 1908 Hertzsprung plotted a scatter diagram of stellar absolute magnitude against spectral class and in 1911 he compared colors and luminosities of stars within several clusters by plotting their magnitudes against their colors. In 1913 he calculated the distance of the *Small Magellanic Cloud* from the solar system.

Hertzsprung was born in Fredericksberg, Denmark. In 1898 he graduated from the Polytechnical Institute in Copenhagen in chemical engineering and then spent the next several years as a chemist in St. Petersburg, Russia. In

[149] "*Classification is one method, probably the simplest method, of discovering order in the world. By noting similarities between numerous distinct individuals as forming one class or kind, the many are in a sense reduced to one, and to that extent simplicity and order are introduced into the bewildering multiplicity of nature*". (A. Wolf, "Classification", *Encyclopedia Britannica* **5** (1954), 778.)

1901 he went to Leipzig to study photochemistry and returned to Denmark in 1902 to begin his study of astronomy. In 1909 he was invited by **Karl Schwarzschild** to Göttingen and later followed him to the Potsdam Observatory. From 1919 to 1944 he was a professor at the University of Leyden, Holland.

1905–1933 CE **Otto Toeplitz** (1881–1940, Germany and Israel). Mathematician. Toeplitz was born in Breslau (now Wroclaw, Poland). His father, Emil, and his grandfather, Julius, were both Gymnasium teachers of mathematics; and they themselves published several mathematical papers. In Breslau, Toeplitz completed his classical Gymnasium and then studied at the university, where he specialized in algebraic geometry and received his Ph.D. in 1905.

He began his academic career in Göttingen as a disciple of **David Hilbert**. Toeplitz was later a professor at Kiel (1913–1927) and Bonn (1927–1933). His scientific work centered around the theory of integral equations, theory of matrices[150], theory of quadratic forms and the theory of functions of infinitely many variables, fields to which he has made lasting contributions. The Nazis expelled him from Germany in 1938, whereupon he became a professor of mathematics at the Hebrew University in Jerusalem.

The Harvard Women (1882–1924)

There is some irony to the fact that Harvard University, long a stronghold of the all-male tradition in American colleges, was the institution that nurtured some of America's first leading women astronomers.

Edward Charles Pickering *(1846–1919, U.S.A.) was a pioneer astronomer in the study of stellar spectroscopy, and especially in the study of the spectra of binary stars. In* 1877 *he became director of the Harvard College Observatory. The work of* ***Pietro Angelo Secchi*** *(1818–1878, Italy), based*

[150] Introduced the $n \times n$ *Toeplitz matrices* having equal elements along diagonals parallel to the *principal* diagonal; *circulant matrices* are special cases of Toeplitz matrices.

on visual inspection of stellar spectra through a spectroscope, was the only attempt to classify stars according to the appearance of their spectra.

***Henry Draper** (1837–1882, U.S.A.), an amateur astronomer, had in* 1872 *became the first to photograph the spectrum of a star. Upon Draper's death, his widow endowed a new department of stellar spectroscopy at Harvard. As director, Pickering hired among his assistants a number of women, several of whom went to work on the problem of classifying spectra of stars:*

***Williamina P. Fleming** (1857–1911) published in* 1890 *the first Draper Catalog of Stellar Spectra. This catalog assigned some* 10,350 *stars in the Northern Hemisphere to spectral classes A through N, in a simple elaboration of a rudimentary classification scheme adopted earlier by Secchi.*

***Antonia C. Maury** (1866–1952), a niece of Henry Draper, classified during* 1889–1905 *the brighter stars from the north pole to declination* 30° *south of the equator. One of her classes, denoted by the letter C, was later recognized as* giant stars. *Maury's discovery helped* **Hertzsprung** *confirm the distinction he had found between giant and main-sequence stars, and he thought her work to be of fundamental importance. Another of her discoveries was a periodic doubling of lines in the case of Mizar, one of the stars in the handle of the Big Dipper. This was the first star to be recognized as a "spectroscopic binary".*

The most important of the Harvard workers in spectral classification arrived on the scene in 1896. *Her name was* ***Annie Jump Cannon** (1863–1941), and she gradually modified the classification system to the present sequence, finding that the arrangement O, B, A, F, G, K, M*[151] *was a logical ordering, with smooth transitions from one type to the next*[152]. *Cannon was able to distinguish the gradation so finely that she established the ten subclasses for each major division that are in use today. During her career at Harvard she personally classified* 500,000 *stars whose spectra appeared on survey plates covering both hemispheres. Cannon layed the groundwork for modern stellar spectroscopy, so fundamental to our understanding of the stars.*

***Henrietta Swan Leavitt** (1868–1921) joined the group as a volunteer in* 1894 *and subsequently played a leading role in the study of variable stars.*

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[151] **H.N. Russell** proposed a scheme by which every student can remember the order of classes in the spectral sequence: The class letters are the first letters in the words, "*Oh, Be A Fine Girl, Kiss Me*!"

[152] At this time, there was still no inkling of the fact that this was a temperature sequence.

The Harvard women, including those mentioned here and several whose names are now obscure, were a remarkable group. They were responsible for a number of major advances in the science of astronomy at a time when its basis in physics was just becoming clear.

Conquest of the Polar Regions (1906–1912)

The explorers who followed Columbus soon found that North America was not a part of Asia, as they believed at first. At this time, British, French, and Dutch adventurers were more interested in finding the easy route to Asia than they were in exploring and settling North America. So they began to look for the Northwest passage, or waterway, that would take them through the continent.

Man had long known that the North Pole lay in the midst of the Arctic Ocean, 640 *kilometers from Cape Columbia, Canada. The Arctic is a frozen sea; tides and currents keep the ice pack in motion, making surface travel hazardous. During the brief Arctic summer, when the sun never sets, the surface of the ice pack melts. Great stretches of water open between the ice floes. Many men braved these waters in search of the Northwest passage and their quest is a tale of adventure and heroism. No country tried harder than England to find the passage.* ***Martin Forbisher*** *(1535–1594) began a series of English expeditions in* 1576*. Other Englishmen continued these explorations for* 300 *years.*

After the defeat of Napoleon at Waterloo in 1815*, Britain pledged herself to maintain the freedom of the seas, and the new Hydrographic Department of the Royal Navy, which has been established in* 1811*, began to carry intensive scientific surveys to make navigation safer for merchant shipping. Thus the Navy returned to the quest for the Northwest passage. It was interested in the passage for scientific reasons, but there was also alarm at the rapid extension* *of Russian influence on the shores of the Arctic Ocean, the Bering Sea and the Pacific seaboard.*

It seemed possible that the British and Americans might have to contend with a third power on the American continent and it was only a matter of time before the Russians began to search for the passage between the Pacific and the Atlantic. In 1847, **John Franklin** *(1786–1847, England) perished with his* 129 *men, and were only recently found. Finally, in 1906,* ***Roald Amundsen***[153] *(1872–1928, Norway) completed the first trip through the Northwest passage with his ship Gjöa, traveling from east to west, bringing to an end a* 400 *year long saga.*

At the turn of the 20th century it was realized that the best time for polar exploration was not during the summer, but during the long, dark winter. Temperatures plunge and savage winds rage across the Arctic in winter, yet the sea is frozen solid, providing a route to the pole for men with dog sleds.

In 1891*, the Philadelphia Academy of Natural Sciences put* ***Robert Edwin Peary*** *(1856–1920, U.S.A.) in charge of an expedition to northern Greenland. The knowledge gained from his experiments on this trip was proof that Greenland is an island. Other expeditions between* 1893 *and* 1897 *resulted in important scientific discoveries about the nature of the polar regions. Peary tried to reach the pole in* 1898 *and* 1905 *but failed. The final assault began in July* 1908*; with four Eskimos and his chief assistant,* ***Matthew Henson****, he reached the North Pole on April* 6*,* 1909[154]*.*

The South Pole lies in a high, mountain-rimmed plateau in the midst of the frozen Antarctic continent. It is the coldest and most desolate region

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[153] During the International Geophysical Year (1957–1958), the U.S. Navy discovered a deep-channel route close to Amundsen's path. Through this channel, the U.S. atomic submarine *Seadragon* made in 1960 the first *underwater* crossing of the Northwest passage. It traveled 1368 km from Lancaster Sound, through the Canadian Arctic Islands, and into the McClure Strait. In 1969, the U.S. icebreaker-tanker *Manhattan* became the first commercial ship to complete the passage.

[154] Nearly 50 years after Peary's expedition reached the North Pole, the U.S. Navy atomic-powered submarine *Nautilus* slipped under the polar icecap off Point Barrow, Alaska, on Aug. 1, 1958. Guided by inertial navigation devices, Commander **William R. Anderson** pointed the submarine toward the North Pole. It was cruising as fast as 40 km/hr, with closed-circuit television plotting the outline of the icecap overhead. At 11:45 PM on Aug. 3, the *Nautilus* reached the North Pole, some 120 meters beneath the ice. Two days later the submarine surfaced in the Greenland Sea. The 5069-kilometer voyage extended through the back door of the *Northwest passage*, from the Pacific to the Atlantic.

on earth, larger in area than Europe and the United States together. In complete contrast to the Arctic — an ocean hemmed in by land — Antarctica is a landmass, hemmed in by sea and, for most of the year, by a rampart of pack-ice. Two great indentations, the Ross and Weddell seas, on the coastline of the continent closest to the South Pole itself, offered explorers the most promising entrances to the heart of the continent.

Roald Amundsen left Norway in 1910 *aboard his ship, the Fram. He hoped at first to reach the North Pole by drifting with the ice floes in the Arctic seas. But just as he was about to sail, he learned that Admiral R.E. Peary had arrived at the North Pole. Amundsen changed his plans and headed south to the Antarctic. In January* 1911 *he pitched camp on the Ross Ice Shelf and in October, when spring came to the Antarctic, Amundsen and four companions started inland across the ice. They had four sledges drawn by dogs. They traveled much of the time across the lofty plateau (3350 meters high and crisscrossed by dangerous cracks in the ice). On Dec.* 14, 1911, *the little group reached the South Pole.*

A month later, on Jan. 18, 1912, **Robert Falcon Scott** *(1868–1912, England), leading a rival expedition, found Amundsen's tent and flag still standing. Broken in spirit and body they began the long march to their base. It ended in a tragedy: Scott and his two remaining companions died on March* 29, 1912, *of hunger an cold, only* 18 *km from a supply depot. The three bodies, as well as Scott's own* Journal *were found by a search party on November* 12, 1912. *The epitaph on Scott's tomb reads: "To strive, to find, and not to yield".*

This race for the South Pole was one of the most dramatic events of the 20^{th} *century; the protagonists — Scott and Amundsen — were both highly capable, dedicated professionals. Their backgrounds, however, were very different, as were their characters. Scott was, by nature and training, a man in whom the desire for discovery for its own sake overlaid by the realization of the importance of scientific discovery, which had perhaps to some extent been forced upon him by the nature of the sponsored, semi-official expeditions that he had commanded.*

Amundsen was a free lance who had been inspired by his spirit of adventure to make the almost impossible Northwest passage in a puny vessel. In the Arctic he had learned that survival could best be ensured by copying the way of life of its inhabitants, the Eskimos, and this he did with conspicuous success. Like Peary, he employed dog teams to draw his sledges, sacrificing them in what to dog lovers seems a ruthless way, and he used Eskimo clothing.

Scott seems to have had no intention of taking dogs. He was a believer in woolen clothing, worn with waterproof gabardine smocks, although these tended to freeze solid.

In May 1926 Amundsen and Lincoln Ellsworth, an American explorer, crossed the North Pole in a dirigible, The Norge. The pilot of the Norge was Umberto Nobile, an Italian explorer. He organized a North Pole expedition in 1928, but it met with disaster. For six weeks, Nobile was believed dead. Amundsen led a party to find him. It was Amundsen's last expedition. He was lost in the Arctic while flying in an airplane with five other men. No one knows how the men died. Soon after their disappearance, Nobile was rescued.

1905–1907 CE **Joseph Henry Maclagen Wedderburn** (1882–1948, USA). Mathematician. Made advances in algebra, especially in the theory of rings, algebras and matrix theory. Discovered two fundamental theorems known by his name, one on the classification of semi-simple algebras, and the other on finite division rings: He showed (1907) that every semisimple algebra is a direct sum of simple algebras and that a simple algebra was a matrix algebra over a division ring. He showed (1905) that non-commutative finite field could not exist. This had as a corollary the complete structure of all finite projective geometries, showing that in all these geometries *Pascal's theorem* is a consequence of *Desargues' theorem*.

Wedderburn was born in Forlar, Angus, Scotland. He entered Edinburgh University (1898), obtaining a degree in mathematics (1903). He then pursued postgraduate studies in Leipzig, Berlin and Chicago, and returned to Edinburgh as lecturer (1905–1909). In 1909 he moved to Princeton, New Jersey, but returned to fight in the British army during WWI. After the war he settled at Princeton University until his retirement in 1945.

1905–1936 CE **Alexis Carrel** (1873–1944, France and U.S.A.). Surgeon and biologist. Developed techniques for rejoining severed blood vessels (1905) and transplantation of organs. Performed the first successful heart surgery on a dog (1914). Developed a form of artificial heart that was used during cardiac surgery (1936). Successful in cultivating chicken heart tissue outside the body for many years. Constructed a perfusion pump, for keeping organs alive outside the body.

Carrel was born in Lion, France and educated there. Professor at the University of Lion (1900–1902); to U.S.A. (1904). On the staff of the Rockefeller Institute for Medical Research (1906–1938). Awarded the Nobel prize for physiology or medicine (1912) for vascular grafting of blood vessels.

1905–1945 CE **Henry Joseph Round** (1881–1966, England). Electronic engineer. Discoverer of the phenomenon of electroluminescence[155] (the emission of light from a semiconductor diode). After the application of a potential of ten volts between two points on a crystal of carborundum, the crystal gave out a yellowish bright light (1907).[156,157]

Round is also known for numerous inventions that contributed to the development of radio communications. He worked with Marconi's Wireless Telegraph Company, both in the United States and England (1902–1914), where he build early radio direction finders and radio telephones. During WWI he installed networks of radio direction finders for military intelligence purposes.[158]

Rejoining the Marconi company after the war, Round designed and installed several important transmitters: from one, the first radio telephone messages were sent from Europe across the Atlantic. As a consultant to the Admiralty during WWII, Round worked on submarine-detection through echo sounding.

1906 CE **Lord Kelvin** disputed the developing theory of radioactive disintegration of atoms, suggesting that radium (discovered 1898) is not an element but a molecular compound of lead and helium. In 1911, **Marie Curie** proved him wrong.

1906 CE, April 18, 05:12 PST A major earthquake hit the city of San Francisco, causing 700 casualties, massive destruction, and marking the end of the Golden West[159].

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[155] See: Loebner, E.E., *Subhistories of the light-emitting diode*, IEEE Transactions on Electron Devices, pp. 675–699 (1976).

[156] Round, H.J., *A Note on Carborundum*, Electrical World, 49, p. 308 (1907).

[157] In 1962, four research groups in the U.S. simultaneously reported a functioning LED semiconductor laser based on gallium arsenide crystals, thus opening the field of solid-state optoelectronics.

[158] One of these alerted the British Admiralty to the departure of the German fleet from Wilhelmshaven on May 30, 1916; the fleet's interception on the following day by the British occasioned the Battle of Jutland.

[159] Without California's mountain-building and faulting it would be hard to imagine the Central Valley, San-Francisco Bay, and the Peninsula. The same features that make California earthquake-prone make it resource-rich as well as visually beautiful!

1906–1913 CE **Eugen Augustin Lauste** (1857–1935, France). Inventor. The Father of Sound-on-Film. Invented the production of sound from photographed vibrations *on a film*, projected upon a selenium cell. This was a crucial step in the development of the sound motion picture.

Although he achieved synchronization by photographing the sound *and* picture on the same strip of film, his invention was hindered by problems of amplification.

In 1913, Lauste took his equipment to the USA in search of support and funds, but he was met with lack of enthusiasm from the film industry.

In the mid 1920's, Bell Telephone Laboratories finally developed a system that successfully coordinated sound on records with the projector: The era of the silent movie ended in 1928 when sound was directly photographed on the film.

1906 CE **August Paul von Wassermann** (1866–1925, Germany). Physicist, bacteriologist and immunologist. One of the founders of immunology. Discovered and gave his name to a blood-serum test for syphilis (1906): an infected person will produce syphilis antibodies in the blood and in the *Wassermann test* these will react with known antigens to form a chemical complex.

Wassermann was born in Bamberg, Germany to Jewish parents and remained in close contact with Judaism throughout his life. Worked in Koch Institute for Infectious Diseases, Berlin (1890–1913) and in the Kaiser Wilhelm Institute in Berlin-Dahlem (1913–1925). Professor at the University of Berlin (1906–1911); Appointed full professor there in 1911. Worked under **Robert Koch** (1901).

1906 CE **Hermann Walther Nernst** (1864–1941, Germany). Physicist and chemist. Made extensive contributions to thermodynamics and electrochemistry. A founder of modern physical chemistry.

Formulated the third law of thermodynamics. It states that the entropy of any body vanishes at absolute zero. This theorem is deduced from quantum statistics and depends on the concept of discrete quantum states. It cannot be derived from purely classical statistics, in which entropy is only defined to within an arbitrary additive constant.

He also contributed to the theory of galvanic cells, thermodynamics of chemical equilibrium, theories of ions[160], properties of vapors at high temperatures and of solids at low temperature, and the mechanism of photochemistry.

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[160] Nernst's application of the *Boltzmann factor* (by which the number of microstates in a reservoir decrease when the reservoir gives up energy) is widely used in physiology. Suppose that certain ions can pass easily through a semipermeable membrane of a neuron cell. An electrical potential is then established

His work had important applications in industry and science. He received the Nobel prize for chemistry on his researches in thermochemistry (1920).

Nernst was born in Briesen, Prussia and was educated at the universities of Zürich, Graz and Würzburg. In 1887 he became assistant to **Wilhelm Ostwald**, who with **Jacobus van't Hoff** and **Svante Arrhenius** established the independence of physical chemistry. He was a professor of physical chemistry at Göttingen (1890–1905) and Berlin (1905–1933).

Nernst was an exception to all the formality of Göttingen. He frightened everyone with his automobile. It was the first car in Göttingen, and Nernst was proud of it. One day he tried to show his students that it was safer than a fiacre (a coach and two horses). While several of them watched, he waved, started the motor, over-optimistically turned a corner with a roar, and promptly crashed into a shop window. There were no injuries, except for Nernst's dignity, but the incident helped to humanize life in Göttingen.

Nernst had two sons and three daughters. The two sons were killed in WWI. Two of his daughters were married to Jews and forced to leave Germany under the Nazis. Nernst retired in 1933. Thanks to the generosity of Lindemann he was invited to Oxford (1937). As for countless other German scientists, the Nazi period was for him a personal disaster in many respects.

Nernst was one of the first scientists to recognize Einstein, following the latter's publication in the Miraculous Year (1905). By 1909, Nernst had verified experimentally Einstein's conclusions concerning the law of Dulong and Petit that followed from the quantum-theoretical interpretation of the specific heat of solids.

Thus, Nernst came to believe that Einstein quantum theory of solids best fitted both the measured values and his theorem, and was ready to follow, pragmatically, Einstein's quantum ideas in his own specialized field.

In 1922 Nernst and Einstein issued together a new refrigerating patent.

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across the cell. If the concentrations of the ions are C_1 and C_2, inside and outside respectively, then the resulting potential is $V = \frac{kT}{q} \log_e \left(\frac{C_1}{C_2}\right)$, where q is the charge on each ion, T is the absolute temperature, and k is Boltzmann's constant (*Nernst equation*).

The ions most important to the cell membrane potential are sodium (Na^+) and potassium (K^+). Each ionic species has associated with it a membrane potential that is maintained by a 'pump' in the membrane that forces Na^+ out and K^+ into the cell. At equilibrium, the Nernst equation shows that $V_{\text{Na}} = \frac{kT}{q} \log_e(C_0^{\text{Na}}/C_i^{\text{Na}}) = 55$ millivolt, $V_{\text{K}} = \frac{kT}{q} \log_e(C_0^{\text{K}}/C_i^{\text{K}}) = -70$ millivolt. These are referred to as the sodium and potassium resting potentials.

In an obituary in 1942, Einstein paid tribute to Walther Nernst, who died in 1941; Just as if there were no war, he gave homage to the liberal-mindedness of his sometime patron and colleague.

1906 CE **Greenleaf Whittier Pickard** (1877–1956, USA). Electrical engineer and inventor. Discovered the *crystal detector* — the point-contact rectifier that was the forerunner of the *transistor* (1948).

He found that the contact between a fine metallic wire ("cat whisker") and the surface of a certain crystalline materials (notably silicon) rectifies and demodulates high-frequency alternating currents, such as those produced in a receiving antenna of radio waves. This device, called a crystal detector (patented by Pickard in 1906) was an essential component of the crystal set, a form of radio receiver that was popular until it was replaced by the triode vacuum tube.

Pickard was one of the first to demonstrate the wireless electromagnetic transmission of speech. In 1899, at the Blue Hills Observatory at Milton MA, he transmitted spoken messages by radio over a distance of 15 km, using a carbon-steel detector to recover the audible signal that had been impressed on the *radio-frequency carrier waves.*

Pickard (a grandnephew of the poet John Greenleaf Whittier, 1807–1892) was educated at Harvard University and M.I.T., Cambridge. He worked as an engineer at the American Telephone and Telegraph Company (1902–1906) and after 1945 headed the electronics engineering firm of Pickard and Burns.

Crystal radio receivers were inconvenient, however, because the signal received was very weak and in order to hear it, it had to be connected to a loudspeaker, which itself was connected to a headset. The completion of the first *diaphragm and trumpet loudspeakers* encouraged **David Sarnoff** to undertake the industrial manufacture of diode and triode radio sets, which were enclosed in wooden boxes so that it was possible to listen in as a group without any special equipment. This was an enormous success and it prepared the arrival of radiotransmission to a large audience. Most of these receivers appeared in 1921.

1906 CE **Henry Harrison Chase Dunwoody** (1842–1906, USA). Army General and inventor. Patented *silicon carbide* (carborundum) crystal radio detector. It allows current to flow in one direction, so that only the upper half of the modulated wave is allowed to pass. It lead to the advent of 'crystal set' (1910), making it possible for amateurs to build their own wireless receivers and listen to early radio transmissions.

1906 CE **Bernard Brunhes** (1867–1910, France). Physicist. Discovered the phenomenon of *geomagnetic reversal* in ancient lava flows from the Massif Central mountain range in France.

Many rock samples (containing iron-rich minerals particles) of different ages and from different localities have shown a magnetic polarization opposite to that of the present field. Brunhes found evidence of the latest of such reversals, occurring some 700,000 years ago, in iron-rich particles in an ancient lava flow in France.

Since then, examples have been found in almost every part of the world. It is believed that the earth's field reversed its polarity several times during the past 20 million years — the duration of the reversal process being of the order of 10,000 years.

1906–1907 CE **Andrei Andreyevich Markov** (1856–1922, Russia). Mathematician. Developed the theory of linked probabilities where the occurrence of one event determines the probability distribution of the following event, so that the future distribution of a variable depends only upon the present value and not on the sequence of past events. These sequences are named for him since he was first to study them systematically. Usually the term *Markov process* is restricted to sequences of events in which the random variables can assume continuous values. The analogous sequences of discrete-valued variables are called *Markov chains*.[161]

Markov was born in Ryazan, Russia. From 1886 he taught at the University of St. Petersburg. His early work was devoted to number theory, analysis and continued fractions, but after 1900 he was chiefly occupied with probability theory. He derived his results *without* using the notion of measurable functions and modern theories of integration. His theory is widely applied in many natural phenomena, especially in the biological and social sciences.

1906–1908 CE **William Sealy Gossett**[162] (1876–1937, England). Mathematician. Made notable contributions to statistical theory, especially to *small-*

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[161] For further reading, see:

- Wentzel, E. and L. Ovcharov, *Applied Problems in Probability Theory*, Mir Publishers, Moscow, 1986, 413 pp.
- Parzen, E., *Modern Probability Theory and its Applications*, Wiley, 1960, 464 pp.

[162] For further reading, see:

- Larsen, R. and M. Marx, *An Introduction to Mathematical Statistics and its Applications*, 1981, 536 pp.
- Bailey, N.T., *The Elements of Stochastic Processes*, Wiley, 1964, 249 pp.

sample theory. Discovered the "*Student's*" *t-distribution*[163] (1907). His work with the t-ratio was destined to become a cornerstone of modern statistical methodology due to its fundamentality for inference. In particular it paved the way for the analysis of variance, which was to occupy an important place in the next generation of statistical research.

Gossett was born in Canterbury. After earning an Oxford degree in mathematics and chemistry under **Airy**, he began working in 1899 for Messrs. Guinness, a Dublin brewery.

Fluctuations in materials and temperature and the necessity for more accurate statistical analysis of a variety of processes, from barley production to yeast fermentation, urged him to seek mathematical advice. In 1906 he was therefore sent to work under **Karl Pearson** at University College, London. In the next few years Gossett made his most important contributions to statistics. He worked for the Guinness Company throughout his entire life.

1906–1914 CE **Boris Borisovich Golitzin** (1862–1916, Russia). Physicist and seismologist. Invented and built the *electromagnetic seismograph*[164]

[163] Because of his company's policy, which forbade publication by employees, it was necessary for Gossett to publish his scientific papers under a pen name. The pseudonym he chose was "*Student*". The *Student*'s distribution with ν degrees of freedom is specified by the probability density function $f(x) = \frac{1}{\sqrt{\pi\nu}} \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{x^2}{\nu}\right)^{-(\nu+1)/2}$. For $\nu = 1$ it coincides with the Cauchy probability law. The n^{th} moment $E[x^n]$ exists only for $n < \nu$. If $n = \nu$ and n is odd then $E[x^n] = 0$, but if $n < \nu$ and n is even, then $E[x^n] = \nu^{n/2} \frac{\Gamma(\frac{\nu+1}{2})\Gamma(\frac{\nu-n}{2})}{\Gamma(\frac{1}{2})\Gamma(\frac{\nu}{2})}$. If $X_0, X_1, \ldots, X_n$ are $(n+1)$ independent random variables, each normally distributed with parameters $\mu = 0$ and $\sigma > 0$, then the random variable $\frac{X_0}{\sqrt{\frac{1}{n}\sum_{k=1}^{n} X_k^2}}$ has as its probability law the Student's distribution with parameter n, which is *independent* of σ.
Gosset discovered the t-distribution by a combination of mathematical and empirical work with *random numbers*, an early application of the *Monte-Carlo method*.

[164] In older seismographs (of the *mechanical* type), the motion is transferred from the pendulum to the recording pens in a purely mechanical way, and simultaneously magnified. In addition, the recording is mechanical (stylus on smoked paper). These requirements imply that the seismograph becomes rather bulky, and that friction is introduced both in the mechanical transmission and recording. To overcome friction it is necessary to use large pendulum masses, and indeed, pendulum masses of as much as 20 tons were used. In the Golitzin

with photographic recording (1906). This invention was a cardinal development in the science of seismology, since it enabled seismologists to obtain higher-quality data, with greater magnification over a wider spectral window.

Golitzin was born and educated in St. Petersburg. He studied first at the Naval Academy and then at the University. He completed his studies at the University of Strasbourg (1887–1891), and became in 1893 a professor of physics at Dorpat. In 1912 he received a D.Sc. degree from the University of Manchester. He died of pneumonia, contracted while traveling across Russia to establish a network of seismic stations equipped with his seismographs.

1906–1915 CE **Paul Langevin** (1872–1946, France). Physicist and inventor. Known for his fundamental work on the theory of *paramagnetic* substances (1906), the *Kerr effect* (1907), the *Bohr magneton* (1911) and the invention of *sonar* (1918). In 1908, Langevin derived a stochastic ordinary differential equation modeling *Brownian motion*.

Langevin's approach to the Brownian motion was different than those of **Einstein** and **Smoluchovski**: A Brownian particle of radius a, moving under the action of a random force, will obey the equation

$$m\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} = F(t),$$

where $x(t)$ is the position of the particle, m its mass, $\gamma \approx 6\pi\eta a$ is the frictional resistance coefficient and $F(t)$ represents the random force which fluctuates both in direction and magnitude, transferring momentum via collisions to m.

The force $F(t)$ is microscopically complicated, since each Brownian particle is typically subjected to some 10^{16} consecutive random impacts per second. However, on a macroscopic scale it can be considered as a stochastic process with independent, stationary distribution at each instance. One notes that the averages

$$\bar{x}(t) = \frac{1}{\Delta t}\int_t^{t+\Delta t} x(\tau)d\tau;$$

electromagnetic seismograph one or several coils are attached to the pendulum such that they are moving in the field of a permanent magnet. The magnet is attached to the frame and the coils are connected to a galvanometer. On the arrival of seismic waves, the coil is set in motion relative to the magnet, and induced current is used to deflect a galvanometer mirror. In this construction, friction is avoided.

$$\bar{F}(t) = \frac{1}{\Delta t}\int_t^{t+\Delta t} F(\tau)d\tau$$

satisfy the above equation of motion, when $x(t)$ and $F(t)$ are the *actual* position and force.

Our eye, or whatever measuring instrument is used, observes not $x(t)$ but rather the average $\bar{x}$. A solution of the Langevin's equation leads directly to Einstein's equation

$$\langle x(t)^2\rangle = kTt/3\pi\eta a,$$

where the absolute temperature of the ambient fluid enters via equipartition.

Einstein's own treatment of Brownian motion is formulated as follows. Denote the average density of particles at point x and time t as $n(x,t)$, and the fraction of particles which shift from point x to within $(x+\Delta, x+\Delta+d\Delta)$ in a *short time* τ as $\varphi(\Delta)$.

It is assumed that $\int_{-\infty}^{\infty}\varphi d\Delta = 1$ (particle conservation), $\int_{-\infty}^{\infty}\varphi\Delta d\Delta = 0$ [symmetry: $\varphi(-\Delta) = \varphi(\Delta)$], and that the probability of large shifts is small, i.e. $\varphi(\Delta)$ is concentrated at small values of Δ.

Clearly,

$$n(x, t+\tau) = \int_{-\infty}^{\infty} n(x-\Delta, t)\varphi(\Delta)d\Delta.$$

Expanding the density in Taylor series

$$n(x-\Delta, t) = n(x,t) - \frac{\partial n}{\partial x}\Delta + \frac{\partial^2 n}{\partial x^2}\frac{\Delta^2}{2} - \cdots;$$

substituting this expansion into the integral and performing the integration, one obtains, in the limit $\tau \to 0$, for $n > 1$:

$$\frac{\partial n(x,t)}{\partial t} = D\frac{\partial^2 n}{\partial x^2},$$

$$D = \frac{1}{2\tau}\int_{-\infty}^{\infty}\varphi\Delta^2 d\Delta \qquad (diffusivity).$$

It follows from this equation, that if initially the particles were concentrated at the point x_0, then at moment t later we have

$$n(x,t) = \frac{1}{\sqrt{2\pi Dt}}e^{-\frac{(x-x_0)^2}{2Dt}}.$$

This implies that the root mean square of the deviation of the Brownian path is $\langle (x-x_0)^2\rangle^{1/2} = \sqrt{2Dt}$.

To derive an explicit expression for D, Einstein assumed that the particles suspended in the fluid fall under gravity and rise again by diffusion; the two competing tendencies balance on average to yield a Boltzmann distribution in the heights of the suspended particles, on account of the concentration gradient that gravity tends to set up.

The *steady-state* velocity of a particle subject to force F is given by Stokes' law $v = \frac{F}{6\pi\eta a}$, where a is the particle's radius and η the viscosity of the medium. Now, the statistical equilibrium condition is $nv = (-)D\frac{\partial n}{\partial x}$ where the $+x$ direction is *up*, since the r.h.s. is the average number of particles crossing a unit surface area downwards per unit time (due to gravity) and the l.h.s. is the number of particles which cross by diffusion in the *opposite* direction (by definition of D). Also

$$n = n_0 \exp\left((-)\frac{\chi}{kT}\right),$$

where χ is the potential of the force F, i.e. $F = \frac{\partial \chi}{\partial x}$. Hence:

$$(-)D\frac{\partial n}{\partial x} = Dn\frac{1}{kT}\frac{\partial \chi}{\partial x} = \frac{D}{kT}(nF) = nv = \frac{Fn}{6\pi\eta a},$$

namely

$$D = \frac{kT}{6\pi\eta a}$$

(*Einstein-Smoluchovski* relation). This relation, when combined with the above root mean square derivation, yields the Langevin expression for $\langle x(t)^2\rangle$.

1906–1916 CE **Marian Ritter von Smolan-Smoluchovski** (1872–1917, Poland). Outstanding theoretical physicist and a fine experimentalist as well. Developed the theory of the Brownian motion on the basis of the molecular kinetic theory, independently of Einstein, using a somewhat different method[165].

Smoluchovski, born to a Polish family, spent his early years in Vienna, where he also received his university education. After finishing his studies

[165] Smoluchowski showed that in the presence of a force-field, Einstein's equation $\frac{\partial P(t,x)}{\partial t} = D\frac{\partial^2}{\partial x^2}P(x,t)$ for the probability density $P(x,t)$ that a particle is at x at time t, should be replaced by $\frac{\partial P}{\partial t} = -\frac{1}{f}\frac{\partial}{\partial x}(PF) + D\frac{\partial^2 P}{\partial x^2}$. Here f is a frictional coefficient which can be expressed in terms of the viscosity of the ambient fluid and the size of the particles. The symbol F stands for either a constant force $(F = -a)$ or an elastic restoring force $(F = -bx)$.

in 1894, he worked in several laboratories abroad, then returned to Vienna, where he became Privatdocent. In 1900 he became a professor of theoretical physics in Lemberg (now Lviv), where he stayed until 1913. In that period he did his major work. In 1913 he took over the directorship of the Institute for Experimental Physics at the Jagiellonian University in Cracow. There he died, the victim of a dysentery epidemic. Einstein called him an ingenious man of research and a noble and subtle human being.

Functional Analysis (1900–1928)

Functional analysis (FA) seeks to discover deep-seated abstract relations that are common to certain branches of analysis, such as integral equations, the calculus of variations, differential equations, operator calculus, the theory of functions of a real variable, the theory of approximation of functions, linear algebra, and topology.

The rise and spread of functional analysis in the 20^{th} *century had two main causes. On the one hand it became desirable to interpret from a uniform point of view the copious factual material accumulated in the course of the* 19^{th} *century in various, often hardly connected, branches of mathematics.*

The beginnings of FA lies in the recognition that widely different kinds of mathematical operations*, from the basic operations of arithmetic to differentiation and integration, have strikingly many features in common; and that the* mathematical objects *subjected to these operations exhibit the same or similar properties in relation to the operations, although they originate from quite different fields of mathematics.*

Moreover, FA permitted an understanding of many results in the above domains from a single point of view and often promoted the derivation of new ones. In recent decades the preparatory concepts and apparatus were then used in a new branch of mathematical physics — quantum mechanics.

On the other hand, the investigation of mathematical problems connected with quantum mechanics became a crucial feature in the further development of FA itself; It created, and still creates fundamental branches of this development: numerous new concepts, which have become the basis of FA, are frequently used in modern mathematics.

Thus, FA has not yet reached its completion by far. On the contrary, undoubtedly in its further development the questions and requirements of contemporary physics will have the same significance for it as classical mechanics had for the rise and development of the infinitesimal calculus in the 18^{th} *century. Some of the main topics of FA are: Abstract spaces (metric spaces, normed spaces); functionals, approximation theory, operators (quantum mechanics makes extensive use of the mathematical apparatus of the theory of self-adjoint-operators).*

A brief history

FA was born in the early years of the 20th century as a part of a larger trend toward abstraction. This same trend contributed to the foundations of abstract linear algebra, modern geometry *and* topology. *Historically, the roots of FA must be sought in the period 1822–1837 in connection with early attempts to solve integral equations:* ***Fourier*** *(1822) offered the solution* $g(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-itx} f(t)\,dt$ *to the equation*

$$f(x) = \int_{-\infty}^{\infty} e^{itx} g(t)\,dt.$$

Abel *(1823) followed with the solution* $g(x) = \frac{1}{\pi}\int_a^x \frac{f'(y)}{\sqrt{x-y}}\,dy$ *to the equation*

$$f(x) = \int_a^x \frac{g(y)}{\sqrt{x-y}}\,dy$$

where $a \le x \le b,\ \ f(a) = 0.$

Liouville *(1837) discovered that the ODE* $f''(x) + f'(x) = g(x)$ *with the initial conditions* $f(a) = 1,\ f'(a) = 0$ *can be converted into the integral equation*

$$f(x) = \cos(x-a) + \int_a^x \sin(x-y) g(y)\,dy.$$

By the middle of the 19th century, interest in integral equations centered around the solution of certain boundary-value problems involving the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,$ *which were known to be equivalent to integral equations. It was then discovered that the study of integral equations is closely related to the study of systems of linear equations of infinitely many unknowns.*

The first rigorous treatment of the general theory of integral equations was given by the Swedish astronomer and mathematician ***Ivar Fredholm*** *in a series of papers from 1900 to 1903. His work was quite influential and attracted a great deal of attention although, curiously, his techniques were largely ignored for years.*

David Hilbert *continued Fredholm's work in a series of seminal papers (1904–1910). The importance of Hilbert's contribution is that he completely abandoned the integral equations, showing it to be a special case of the theory of systems of linear equations, thus beginning the* algebraization of analysis.

In another vein, ***Fréchet****, building on* ***Voltera's*** *concept of* functionals, *advanced a new '*Functional Calculus*' which was based on two very general principles:*

- *Basic notions from set theory,*
- *A notion of* limit, *which was assumed to be available in the particular class of spaces he considered.*

*In modern terms, Fréchet's 'Functional Analysis' was an early example of what we would now call '*point set topology*'. In particular, it was Fréchet who formalized the notion of a* metric space. *The main results in his 1906 thesis were generalizations of the work of*

▷ ***Cantor*** *(by generalizing the notions of the* interior of a set, a derived set, compactness, perfect set*)*

▷ ***Baire*** *(by generalizing the notion of* semi-continuous functions*)*

▷ ***Cesare Arzelà*** *(by extending the notion of* compactness *to set of functions)*

All these developments did not *stem from Hilbert's work. Fréchet's approach was so revolutionary, that he felt the need to justify it even as late as 1950.*

Hilbert's successors were ***Erhard Schmidt*** *(1907),* ***Friedrich Riesz*** *(1907) and* ***Ernst Fischer*** *(1908) who considered the notion of* convergence in the mean *for square-summable functions. Other important players in the story of FA were* ***Eduard Helly*** *(1922) and* ***Hans Hahn*** *(1922), but it was* ***Stefan Banach*** *(1922) who gave the first complete treatment of* abstract normed vector spaces.

As an example we consider the generalization of the classical concept of differentiation such that it can meet the needs of wider applications:

Traditionally, differentiation enables us to find the tangent to a curve at a given point. Very often, in a diagram showing a curve with a tangent, there is a considerable stretch where the eye cannot distinguish the curve from the tangent. In other words, over a single interval the tangent gives an excellent approximation to the curve. If (x, y) *is a point on the tangent to the graph* $y = f(x)$ *at the point* (x_0, y_0), *then the equation of the tangent is* $y - y_0 = f'(x_0)(x - x_0)$, *which is a* linear relation. *So, the problem of differentiation is equivalent to the problem of finding a linear function, relating the change in input to the approximate change in output, the approximation being good for small changes.*

In order to clarify what we mean by 'a good approximation' we turn to the traditional calculus. For instance, when $f(x) = x^3$ *we find that* $f(x_0 + h) - f(x_0) = 3hx_0^2 + (3h^2x_0 + h^3)$. *If* x_0 *is of order 1 and* h *is of the order, say, of* 10^{-6}, *then* h^2 *and* h^3 *are of the orders* 10^{-12} *and* 10^{-18} *respectively, and we may well write* $f(x_0 + h) - f(x_0) = 3hx_0^2 + \cdots$, *where the dots represent nonlinear terms in* h *that are small compared with* h. *If we use* $e(h)$ *to represent the terms omitted — "e" for error — a formal statement is that if* $f(x_0 + h) - f(x_0) = mh + e(h)$, *where* $e(h)/h \to 0$ *as* $h \to 0$, *then* $f'(x_0)$ *exists and has the value* m.

We now take a very small step in the direction of generalization, and consider a function f *that maps the vector* $u = (x, y)$ *to* $w = (q, p)$, *where* $p = xy$, $q = x^2 + y^2$. *Let* $h = (a, b)$. *When* (x, y) *change to* $(x + a, y + b)$, *the corresponding changes in the mapping are* $\Delta p = ya + xb + \cdots$ *and* $\Delta q = 2xa + 2yb + \cdots$, *where the omitted terms involve* a^2, ab *and* b^2. *Thus we may write* $\begin{pmatrix} \Delta q \\ \Delta p \end{pmatrix} = \begin{pmatrix} 2x & 2y \\ y & x \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$. *The vector* (a, b) *in this equation represents* h *(the change in input) and the approximate change in output is given by* Mh, *where* M *is the* 2×2 *matrix in the above equation. The error in our approximation is given by* $e(h) = (ab, a^2 + b^2)$.

We cannot keep our earlier condition, $e(h)/h \to 0$, *since in general it is not possible to divide one vector by another. However, we are not really interested in the* direction *of the vector* $e(h)$, *since we propose to neglect this vector. All we want to be sure of, is that its size is negligible compared with that of* h. *Sizes are measured by norms*[166]. *So the condition,* $\|e(h)\|/\|h\| \to 0$, *as* $\|h\| \to 0$, *is meaningful and is all we need demand. We are thus led to reformulate our definition of differentiation in a way that suits the above example and works equally well for a function* f *in any* Banach space:

If

$$f(u_0 + h) - f(u_0) = Mh + e(h),$$

[166] $\|v\|$, the norm, is the *length* of the vector v in a Euclidean space.

*where M is a bounded linear operator and $\|e(h)\|/\|h\| \to 0$ as $\|h\| \to 0$, the function f is called Fréchet-differentiable at the point u_0, and we define $f'(u_0) = M$ as the Fréchet derivative at a point u_0. (**Maurice Fréchet**, 1925).*

Requiring M to be bounded is much like requiring the number $m = f'(x_0)$ in traditional calculus to be finite. Note that the Fréchet derivative in our example is a matrix, and not a single number, since it transforms a vector into another nonparallel vector (in general). Multiplication by a number cannot do this, for it leaves the direction of the vector invariant. Multiplication by a matrix can change both length and direction, and $f'(u)$ turns out to be such a transformation.

The availability of a generalized derivative does not mean that all theorems of the traditional calculus have counterpart in functional analysis. Roll's theorem, for one, does not have such counterpart[167]. This means that the mean value theorem can only be generalized as an inequality.

1906 CE **Albert Einstein** completed a paper on the *specific heat of solids*, the first paper ever written on the quantum theory of the solid state (published, 1907) and the first attempt to deal with the problem of specific heats in the context of the quantum theory. Until 1906, Planck's quantum had played a role only in the rather isolated problem of blackbody radiation and in Einstein's explanation for the photoelectric effect in terms of the light quantum (photon). Einstein's work on specific heats made it clear for the first time that quantum concepts have a far more general applicability.

In the classical model of a monatomic solid, each atom vibrates about an equilibrium position; these vibrations constitute the thermal agitation of the crystal. The specific heat at constant volume is then defined as $C_v = \left(\frac{\partial E}{\partial T}\right)_v$,

[167] *A counter-example*: let ϕ map $t \to (x, y)$ with $x = t - t^2$, $y = t - t^5$. Then $\phi'(t) = (1 - 2t, 1 - 5t^4)$. Now $\phi(0) = (0, 0)$ and $\phi(1) = (0, 0)$, but there need be no T with $0 < T < 1$ such that $\phi'(T) = 0$. When we are dealing with a single number that starts at 0 and returns to 0, there will be a moment when its stops growing and begins to decrease, or vice versa. But when the output involves two numbers, x and y, there need be no such special moment (the number x ceases growing when $t = 0.5$, but y continues to grow until $t = 0.67$ approximately).

where E is the energy of vibration of the atoms and T is the absolute temperature. To within an additive constant, this energy E is the internal energy U of the solid.

Invoking the principle of equipartition of energy, adumbrated by **Clausius** and stated clearly by **Maxwell** (1860), each atom has $\frac{1}{2}kT$ of energy per degree of freedom (on average), with 6 degrees of freedom. Therefore N atoms have a total of $E = 6N\left(\frac{1}{2}kT\right) = 3NkT$. The heat capacity associated with these atoms is $C_v = \frac{dE}{dT} = 3Nk$ in this classical model.

The value $3Nk$ agrees with experimental results fairly well for most elemental solids at *high temperatures*; at low temperatures C_v is much lower than the constant $3Nk$ value. Experimentally, C_v for non-metals is found to be approximately proportional to T^3 at low temperatures, approaching zero as $T \to 0$.

Einstein set forth to explain the discrepancy and his work, like his innovative articles of 1905, proceeded directly and succinctly to the heart of the matter: He realized that the gradual disappearance of the atomic heat in approaching absolute zero is a *typical quantum effect*.

It is due to the fact that, in contrast to the fundamental assumptions of the classical law of *equipartition*, the oscillating lattice constituents cannot take up arbitrarily small amounts of thermal energy but (according to quantum mechanics) only integral energy quanta[168] $h\nu$ true for every oscillator of eigenfrequency ν. With decreasing temperature, the mean thermal energy kT finally becomes smaller than one vibrational quantum $h\nu$ for any given oscillation mode of frequency ν. Below that temperature, less and less energy quanta are available to the crystal — more and more modes of vibration are "frozen out".

Consequently, with decreasing number of the degrees of freedom which are able to absorb thermal energy, the specific heat decreases toward zero. The lighter the atoms of a specific crystal are, the higher is the temperature at which this "starving" of degrees of freedom occurs. This is due to the fact that the vibrational energy quanta are the larger the smaller the oscillating masses are.

The statistical computation is very similar to that of the cavity oscillations which lead to Planck's radiation formula and the result reflects this similarity. If we assume *a single eigenfrequency* ν_0 of the crystal atoms, Einstein's

[168] These mechanical vibrational quanta are known today as *phonons*, rhyming with *photons*. A phonon is the quantum of *sound* or *elastic* waves in a crystal lattice.

theory[169] leads to the atomic heat

$$C_v = 3Nk\xi^2 \frac{e^\xi}{(e^\xi - 1)^2},$$

where $\xi = \frac{h\nu_0}{kT}$, instead of the classical result $C_v = 3Nk$. At sufficiently high temperature $(\xi \to 0)$, this expression approaches $3Nk$ in agreement with the law of **Dulong** and **Petit**. The characteristic eigenfrequency ν_0 was determined by Einstein from a comparison of his formula with the empirical temperature dependence of the specific heats. (For alkali halide crystals, he obtained values of ν_0 that agreed within 20 percent with the frequencies of the optical fundamental vibrations.)

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169 Einstein wrote the energy per each of the $3N$ one-dimensional oscillators as

$$U(\nu, T) = \frac{\int \epsilon e^{-\epsilon/kT}\omega(\nu,\epsilon)d\epsilon}{\int e^{-\epsilon/kT}\omega(\nu,\epsilon)d\epsilon}.$$

The exponential factor denotes the statistical Boltzmann factor for the energy ϵ. The weight factor ω contains the dynamic information about the density of states between ϵ and $\epsilon + d\epsilon$. For the case at hand (linear oscillators), the classical theory requires $\omega \equiv 1$, which leads to the *equipartition* result $U = kT$. Einstein's assumption, written in modern notation, is $\omega = \sum_n \delta\left[\epsilon - \left(n + \frac{1}{2}\right)h\nu\right]$.

A straightforward integration then yields $E = 3NU = \frac{3N}{2}h\nu + 3N\frac{h\nu}{e^{h\nu/kT}-1}$. Taking the derivative w.r.t. T, Einstein's final expression for C_v is obtained.
Debye's analysis (1912) begins with the results for the characteristic vibrations of an enclosure of volume V in which two kinds of elastic waves can propagate with the respective velocities v_ℓ (longitudinal) and v_t (transverse). The number of characteristic frequencies and of corresponding lattice models (coupled-atom modes) between ν and $\nu + d\nu$, counting all types of waves, is $dn = 4\pi V\left(\frac{1}{v_\ell^3} + \frac{1}{v_t^3}\right)\nu^2 d\nu$. Altogether, $3N$ of them are found, going up to ν_0 such that $3N = \frac{4\pi V}{3}\left(\frac{1}{v_\ell^3} + \frac{2}{v_t^3}\right)\nu_0^3$. Hence $dn \equiv g(\nu)d\nu = \frac{9N}{\nu_0^3}\nu^2 d\nu$. To each degree of freedom (mode) having frequency ν, Debye assigned the average energy found in Einstein's single-mode analysis. Hence $dn' = \left[\frac{9N}{\nu_0^3}\nu^2 d\nu\right]\frac{1}{e^{\frac{h\nu}{kT}}-1}$. The total vibrational energy of the solid is $E - E_0 = \int_0^{\nu_0} h\nu dn' = \frac{9Nh}{\nu_0^3}\int_0^{\nu_0}\frac{\nu^3 d\nu}{e^{h\nu/kT}-1}$ where E_0 is the energy at absolute zero, corresponding to atoms executing zero-point motion. The expression for $C_v = \left(\frac{\partial E}{\partial T}\right)_v$ then leads to the Debye integral.

In spite of this success, it was clear that the basic assumption of Einstein's theory, the existence of only one eigenfrequency of the crystal, is too much of a simplification. Thus it was not surprising that this model failed to yield the observed relation $C_v \propto T^3$ at low temperatures, since the higher frequency did not "freeze" at the same temperature values for lower frequencies.

Debye (1912) first carried out the quantization of the eigenfrequencies of the entire lattice (coupled-atoms eigenmodes of vibration). The crystal in his theory is treated as *one* atomic system the states of which are characterized by different eigenfrequencies, with a limiting maximal cut-off vibration frequency.

His final formula is

$$C_v = 9Nk\left(\frac{T}{\Theta_{_D}}\right)^3 \int_0^{\Theta_D/T} \frac{x^4 e^x}{(e^x-1)^2}dx,$$

where $\Theta_{_D} = \frac{h\nu_c}{k}$ is the *Debye temperature*, and the cut-off frequency ν_c is determined from $3N = \frac{4\pi}{3}V\left(\frac{1}{v_\ell^3} + \frac{2}{v_t^3}\right)\nu_c^3$. Here V is the volume of the sample, and $\{v_\ell, v_t\}$ are the respective longitudinal and transverse propagation velocities of elastic waves in the crystal.

Certain deviations from the Debye theory are still to be observed at low temperatures as well as very high temperatures, due to the still insufficient treatment of *details of the vibrational spectrum*. These include the anharmonicity of the vibrations, which causes a decrease of the higher vibrational quanta and may be though of as due to collisions among phonon pairs. For metals, a small contribution of free electrons to the observed specific heat has also to be taken into account at low temperatures.

1906–1916 CE **Pierre Maurice Duhem** (1861–1916, France). Historian of science. Uncovered unsuspected aspects of medieval science and contradicted the myth of medieval scientific backwardness. He brought to light the Parisian school, in particular the work of **Jean Buridan** (1295–1358), **Nicole Oresme** (1323–1382) and **Albert of Saxony**[170] (1316–1390, Germany) and showed how their work on mechanics was known to **Leonardo da Vinci** and subsequently led in the 17^{th} century to the development of such basic notions of Galilean and Newtonian physics as *impetus* (momentum) and *inertia*.

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[170] Also known as Albert von Helmstädt. German scholastic philosopher. Rector of University of Paris (1353–1362) and of University of Vienna (1365–1366). Bishop of Halberstadt (1366). Wrote on mathematics, physics, logic. Helped spread the logic of William of Ockham.

Duhem taught at Lille (1887–1893), Rennes (1893–1894), and Bordeaux (1894–1916). In his early work (1903) he made pioneering attempts to establish a theory of irreversible thermodynamic processes, but these ideas were in advance of their time and did not receive recognition in his lifetime. This was partly because he disproved a favorite theorem of **Berthelot**, an influential chemist and minister of education who saw to it that Duhem was never elected to a chair at Paris.

1906–1925 CE **Maurice Rene Fréchet** (1878–1973, France). Distinguished mathematician. Created a geometry of abstract metric spaces. Developed further the functional calculus of **Volterra** and introduced the concept of the '*Fréchet derivative*'.

Fréchet was born in Maligny, France. He was a professor of mechanics at the University of Poitiers (1910–1919), a professor of higher calculus at the University of Strasbourg (1920–1927) and a professor of mathematics at the University of Paris (1928–1948).

The *Fréchet derivative* of a nonlinear operator is a generalization of the derivative of a real-valued function based on the idea of *local linearization*; it enables us to find a linear approximation to a nonlinear operator in a neighborhood of some given point. Regarding this as a point in the appropriate function space, we can find a local linearization of the original nonlinear operator equation in a neighborhood of this point. We can iterate this process, and it is a generalization of *Newton's method* for solving an equation in a single real variable.

Let $P\colon S \to B_2,\ S \subseteq B_1$ be an operator mapping of a subset S of Banach space B_1 into a Banach space B_2. Let x_0 be an element of B_1 such that S contains a neighborhood of x_0.

Then P is said to be *Fréchet differentiable* at x_0 if there is a continuous *linear* operator $L\colon B_1 \to B_2$ such that $P(x) = P(x_0) + L(x - x_0) + G(x, x_0)$, with $G\colon B_1 \to B_2$ defined by $G(x, x_0) = P(x) - P(x_0) - L(x, x_0)$, satisfying

$$\lim_{\|x-x_0\|\to 0} \left\{ \frac{\|G(x,x_0)\|_{B_2}}{\|x - x_0\|_{B_1}} \right\} = 0.$$

If such a continuous (bound) *linear operator* L exists for a particular x_0 in B_1 we denote it $P'(x_0)$, the Fréchet derivative P at x_0, and write

$$P(x) = P(x_0) + P'(x_0)(x - x_0) + G(x, x_0)$$

where $G(x, x_0)$ satisfies the condition given above.

Consider the following examples:

- Denote the real line by $\mathbb{R}$ and let $[a,b]$ be an interval in $\mathbb{R}$. Now $\mathbb{R}$ is a *Banach space* with norm $\|x\| = |x|$ for x in $\mathbb{R}$. Let $f\colon [a,b] \to \mathbb{R}$ be a real valued function, and let x_0 be in $[a,b]$; then the Fréchet derivative of $f(x)$ at x_0 is the ordinary derivative $f'(x_0)$. Since $f'(x_0) = \lim_{|x-x_0|\to 0} \frac{f(x)-f(x_0)}{x-x_0}$, it follows that $f(x) = f(x_0) + f'(x_0)(x-x_0) + G(x,x_0)$ with $\lim \frac{|G(x,x_0)|}{|x-x_0|} = 0$.
- The n-dimensional Euclidean space, E^n, with norm $\|x\| = \left(\sum_{i=1}^n x_i^2\right)^{1/2}$ is also a Banach space.

Let $P\colon S \to E^n$, $S \subseteq E^n$ be a mapping defined on a subset S of E^n which contains a neighborhood of a point x_0. It is not hard to show that $P'(x_0)$, if it exists, is the *Jacobian* matrix with elements $\left.\frac{\partial P_i}{\partial x_j}\right|_{x_0}$, $i,j = 1,2,\ldots,n$.

A sufficient condition for the existence of $P'(x_0)$ is the continuity, at x_0, of the partial derivatives. In this case, we have $P_i(x) = P_i(x_0) + \sum_{j=1}^n \left.\frac{\partial P_i}{\partial x_j}\right|_{x_0} (x-x_0)_j + G_i(x,x_0)$, $\quad i = 1,2,\ldots,n$. The limit $DP(x_0)(y) = \lim_{\eta\to 0} \frac{P(x_0+\eta y)-P(x_0)}{\eta}$, if it exists, is called the *Gâteau derivative* (1919) of P at x_0 in the direction y, where $y \in E^n$.

As a special application of the foregoing theory consider the mapping of the complex number $x+iy$ in E^2 to $f(x,y) = u(x,y) + iv(x,y)$. This function corresponds to $f(x,y) = \begin{pmatrix} u \\ v \end{pmatrix}$ in column vector notation for the mapping f.

The Fréchet derivative $f'(z_0)$ exists at $z_0 = x_0 + iy_0$ and is expressible as the Jacobian matrix $f'(z_0) = J(z_0)$ given by $J(z_0) = \begin{pmatrix} u_x & u_y \\ v_x & v_y \end{pmatrix}$ where $u_x = \left.\frac{\partial u}{\partial x}\right|_{z_0}$, etc., provided that for every w in E^2 the limit $Df(z_0)(w) = \lim_{a\to 0} \frac{f(z_0+aw)-f(z_0)}{a}$ exists and $Df(z_0)w = J_0(z)w$.

Now suppose that the limit does exist for *every* w in E^2 and that $Df(z_0)w = J_0(z)w$. Then, regarding w and $f'(z_0)w$ as complex numbers, we have (with $w = w_1 + iw_2$),

$$f'(z_0)w = J_0(z)w = \begin{pmatrix} u_x & u_y \\ v_x & v_y \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} u_x w_1 + u_y w_2 \\ v_x w_1 + v_y w_2 \end{pmatrix}.$$

Applying the rules of complex multiplication, we find

$$f'(z_0) = \frac{u_x w_1^2 + v_y w_2^2 + (u_y + v_x)w_1 w_2 + i\{v_x w_1^2 - u_y w_2^2 + (v_y - u_x)w_1 w_2\}}{w_1^2 + w_2^2}.$$

Since $f'(z_0)$ is independent of w we must have $u_x = v_y$ and $u_y = -v_x$ (the *Cauchy-Riemann relations*!), leading to $f'(z_0) = u_x + iv_x = v_y - iu_y$. Thus

$$f'(z_0) = \begin{pmatrix} u_x & -v_x \\ v_x & u_x \end{pmatrix} = u_x I + v_x J$$

where

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

This matrix representation can be seen to be algebraically equivalent to the complex number $f'(z_0) = u_x + iv_y$ by means of an isomorphism between the complex number field and 2×2 matrices of the form $aI + bJ$. (In fact, we have $I^2 = 1$, $J^2 = -1$ so that, putting $I \sim 1$ and $J \sim i$ we have $aI + bJ \sim a + bi$ for any real numbers a and b.)

1906–1931 CE **Paul Ehrenfest** (1880–1933, Austria). Theoretical physicist. Clarified many of the basis assumptions of statistical mechanics[171], and contributed a variety of particular results[172]. He tackled the basic problem of reconciling the reversibility of classical mechanics with the irreversibility of the events of ordinary experience, showing that this is due to the extreme improbability of processes in which order spontaneously increases.

Ehrenfest was born in Vienna to Jewish parents who moved to the capital in 1860 from their home in Loschwitz, a little village in Moravia. He was educated in the University of Vienna and was a student of **Ludwig Boltzmann**. In 1912 he succeeded **H.A. Lorentz** to the chair of theoretical physics in Leyden.

During his entire academic life he was fully committed to maintaining clarity and intelligibility in the flood of new development, and in doing so extended himself up to and beyond the limits of his resources. Despite his

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171 Collaborated in part of his work with his wife **Tatyana Alexeyevna Ehrenfest-Affanasyeva** (1876–1964).

172 For example, in 1913 Ehrenfest was first to suggest the correct quantization of the energy levels of the rigid molecular rotor with moment of inertia I: starting from the classical expression $E = \frac{1}{8\pi^2 I}$ (action)^2, he derived $E_n = \frac{n^2 h^2}{8\pi^2 I}$, $n = 1, 2, \ldots$.

achievements in science he was dissatisfied with his own development. Thus, the problems of his own life, exacerbated by the persecution of the Jews by the Nazis, led him to take his own life.

Albert Einstein, a dear friend of Ehrenfest, sought the origin of the tragedy in that "*he suffered incessantly from the fact that his critical faculties transcended his constructive capacities*".

1906–1932 CE **Oswald Veblen** (1880–1960, U.S.A.). Distinguished mathematician. Made major contributions to projective geometry, and differential geometry; laid the foundation to modern topology.

Was among the few American scholars who came forth publicly against the Nazi persecution of Jewish mathematicians.

He was the nephew of **Thorstein Bunde Veblen** (1857–1929, U.S.A.), the known economist and social critic. Weblen taught mathematics at Princeton University (1905–1932). In 1932 he helped organize the Institute of Advanced Study in Princeton and became a professor there.

1906–1936 CE **Harry Bateman** (1882–1946, U.S.A.). Applied mathematician. Made important contributions to the theory of partial differential equations of mathematical physics, especially in hydrodynamics and electrodynamics. He also wrote much on integral equations and special functions. One of the first to apply Laplace transforms to integral equations (1906).

Bateman was born in Manchester, England to Jewish parents and studied at Cambridge (1900–1904), Göttingen (1905) and Paris (1906). In 1910 he removed to the United States, and in 1917 became a professor of mathematics, theoretical physics and aeronautics at the California Institute of Technology in Pasadena, CA. He accumulated a vast store of information on all the familiar special functions of mathematical physics and on his death the publication of his manuscript was undertaken by **Arthur Erdélyi** and his associates in the form of the series *Higher Transcendental Functions* and *Tables of Integral Transforms.*

1906–1955 CE **Ernst Frederik Werner Alexanderson** (1878–1975, U.S.A.). Electrical engineer and inventor. Pioneer in electrical power engineering and broadcasting.

Born in Uppsala, Sweden and studied at Lund University (1896), Royal Institute of Technology, Stockholm (1897–1900), and received a Ph.D. from the Technical University of Berlin (1901). He came to the US (1901) and designed for Fessenden the high-frequency generator (1906). Associated with General Electric Co. (1902–1948), and with RCA from 1952.

Invented a high-frequency (2 kW, 100 kHz) *alternator*[173] that greatly improved transoceanic radio (1906), multiple-tuned antenna, and selective radio tunning circuit (1916). Improved transoceanic communication and firmly established the wireless as an important tool in shipping and warfare. Made the first transmission (1924) of a fax message across the Atlantic. Among his other inventions: vacuum-tube telephone transmitter; electric ship propulsion, railroad electrification, and power transmission. Also made significant contributions to television (1927) and color television (1955). Issued 322 patents in radio, television and computer technologies.

1907 CE **Pierre Weiss** (1865–1949, France). Physicist. Contributed to modern theory of *magnetism*, especially *ferromagnetism*.

Suggested the theoretical existence of small *magnetic domains*[174] and developed a domain theory of ferromagnetic materials: when the dipoles of the domain are aligned, a strong and stable magnetic field results. Weiss treated phenomenologically spontaneous magnetization of metals like iron by his theory[175] — a prototype of many attempts to describe cooperative condensed–matter effects like melting of solids, order-disorder transition in alloys etc.

The Curie-Weiss Law of ferromagnetism describes the behavior of the magnetic *susceptibility* of many solids. Weiss also determined a unit of magnetic moment known as *Weiss magneton*. He was first to introduce the useful concept of *mean-field approximation*, called *Weiss field*.

Weiss was born in Mulhouse, France, and was a professor at Zürich (1902) and Strasbourg (1919).

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173 *Alternator*: a device that converts direct current into alternating current capable of producing continuous radio-frequency waves and thereby revolutionizing radio communication.

174 He suggested that each piece of a paramagnetic material consists of regions (domains) already magnetized and that the direction of their magnetization differs from one region to another so as to cancel out in the whole piece. When the piece is subjected to a uniform magnetic field from outside, the direction of magnetization within each domain turns toward alignment with that field, and a net magnetization appears in the material. It turns out that a domain cannot choose the direction of its magnetization at random; it must choose that direction from among a very few easy directions of magnetization in the crystalline structure of the material.

175 According to this scheme, each magnetic atom experiences a field proportional to the magnetization $\boldsymbol{B}_{\text{effective}} = \lambda \boldsymbol{M}$ where λ is a constant independent of temperature. This means that each spin sees the average magnetization of all other spins (in truth, it can only "see" its nearby neighbors).

History of Magnetism III[176] *(1895–1928)*

*Matter is essentially electrical in nature, and magnetic effects arise from the distribution of currents of the electrical charges in matter and from the quantum-mechanical spin of charged electrons and nuclei. According to the view now accepted by physicists, the magnetic properties of matter are explained in terms of the quantum states and orbitals of electrons within atoms and molecules and (macroscopically) in multi-atom condensed matter. As early as 1820, **Ampere** suggested that magnetism was due to electric current circulating within matter. However, the identification of these "Amperian currents" with orbital motion of electrons is a later achievement due principally to **J.J. Thomson** (1897), **Lord Rutherford** (1911), and **Niels Bohr** (1913). In fact, **Maxwell** himself endeavored (1861) to detect such gyroscopic effects, but without success. The Stern-Gerlach experiment (1920-1) showed that the electron's intrinsic quantum angular momentum (spin) makes it a permanent magnet, and in fact it is spin rather than orbital electron motion that came to be understood as the main source of macroscopic magnetic effects. Spin is not directly associated with actual spatial motion, but is interconvertible with orbital angular momentum and, like it, exhibits gyroscopic inertia.*

*In 1915, **Albert Einstein** and **Johannes de Haas** (1878–1960) conducted an experiment that gave the first proof of the existence of mechanical rotation induced by magnetization: An iron cylinder hung vertically by means of a wire. A fixed solenoid was placed coaxially around the cylinder. The iron was magnetized by an alternating current run through the solenoid. If the magnetic moment* $\boldsymbol{M}$ *of the magnetized body (at rest) is due to circulating hidden (molecular, atomic and/or macroscopic) electric currents, the current due to the flow of electrons in their closed orbits with orbital angular momentum* $\boldsymbol{J}$ *will induce the magnetization* $\boldsymbol{M} = -\frac{e}{2m} g\boldsymbol{J}$ *per electron, where* (e, m)

[176] For further reading, see:

- Livingston, J.D., *Driving Force – the Natural Magic of Magnets*, Harvard University Press, 1996, 311 pp.
- Chaikin, P.M. and T.C. Lubenski, *Principles of Condensed Matter Physics*, Cambridge University Press, 1997, 699 pp.
- Kittel, C., *Introduction to Solid State Physics*, Wiley, 1986, 646 pp.
- Epifanov, G.I., *Solid State Physics*, Mir publishers: Moscow, 1979, 333 pp.

are the charge and mass of the electron respectively and the dimensionless factor $g > 0$ is now called the Lande factor or gyromagnetic ratio.

Einstein argued that $g = 1$ per classical theory. To see this consider one electron moving with uniform velocity $v = 2\pi r\nu$ in a circular orbit with radius r and frequency ν. The angular momentum has the magnitude $mvr = 2\pi r^2 m\nu$. An amount of electric charge $(-e\nu)$ passes per second through a point of the orbit. The magnetic moment is therefore equal to $(-e\nu)(\pi r^2)$. Hence $g = 1$.

Einstein and de Haas obtained the mean experimental value $g = 1.02$, apparently confirming the classical value. By the 1920's however, their measurements were considered to have been in error, because the new experimental value of g was found to lie close to 2. Ampere's molecular currents were then abandoned in favor of the spin theory of electron magnetism. However, in the pre-spin days of 1915, any dynamical theory of ferromagnetism had necessarily to be incorrect. Einstein could not know that his theoretical prediction for iron was wrong by a factor of 2.

Electrons may give rise to a magnetic field in two ways:

(1) An electron revolving in an orbit about the nucleus of an atom *(or the nuclei of a molecule) is equivalent to a tiny current which gives rise to a magnetic field; this classical phenomenon extends to quantum–mechanical orbitals, as well as to conduction electrons in meso- or macroscopic matter (e.g. in metallic conduction – band orbitals).*

(2) It has also been found necessary to make the hypothesis that electrons always spin *about any given axis in a manner loosely analogous to the spin of the earth about its axis. This quantum-mechanical spin also gives rise to a magnetic field quite independently of any orbital motion the electron may possess. The observed Cartesian components of both spin an orbital angular momenta take discrete values: $\{-\frac{\hbar}{2}, \frac{\hbar}{2}\}$ for the former, $\{-l\hbar,\ -(l-1)\hbar,\ \dots\ l\hbar\}$ for the latter, $\hbar = h/2\pi$ being the reduced Planck's constant. The pure-spin Lande factor* twice *the classical value, while the purely orbital value equals the classical result.*

In an atom containing many electrons, it is possible that the orbits of the electrons and their spins may be so oriented that the atom as a whole possesses a resultant magnetic moment, i.e., acts like a tiny magnet. We say that the atom then possesses a permanent magnetic moment. *(The same holds for an ion or molecule.) If such an atom is placed in an external magnetic field, two things occur. First, the external field tends to turn the tiny magnet parallel to the field, thus adding the field due to the Amperian current to the external field. This phenomenon is called* paramagnetism.

Second, the atom, because of its intrinsic electrical character, will have induced currents set up due to asymmetric response of oppositely-orbiting electrons to the applied field. The net magnetic field due to these currents is opposite *to the external field, resulting in the phenomenon of* diamagnetism. *In general, the diamagnetism effects are masked by the larger paramagnetic effects if the latter are present, though their relative strengths depend on the temperature.*

If an atom or molecule does not possess a permanent magnetic moment — that is to say, its orbits and spins are so oriented that their individual fields cancel in the absence of an external field[177] *— such an atom or molecule exhibits diamagnetism only.*

In the case of ferromagnetic *materials such as iron, not only is the underlying atom or molecule paramagnetic, but a new and unusual phenomenon is present that accounts for the very conspicuous magnetic properties of such (usually solid) materials. It was found from the experimental work of* ***Weiss*** *(1907) that iron in the bulk, even when not magnetized as a whole, consists of a large number of grains or* domains *that act as permanent magnets. These domains are small compared to ordinary macroscopic dimensions but large compared with atomic sizes.*

There are typically some 10^{17} *to* 10^{21} *atoms in each domain. At sufficiently low temperatures the magnetic moments of the atoms, ions or molecules within a domain are forced by quantum nearest-neighbor couplings (such as covalent bonds and Pauli exchange forces) to be parallel, rise to a large magnetic moment per domain. The process of magnetization of macroscopic samples of iron consists in bringing the magnetic moments of the domains into parallelism with the external field.* Magnetic saturation *is present when*

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[177] One reason for that cancellation is that electrons usually occupy their permitted state in *pairs*; in each pair one electron's spin is oriented 'up' and the other's 'down' (along any given axis of measurement). In some molecules or atoms, however, the elementary magnets are odd in number and/or they do not cancel one another's effect which thus *add up* to yield a net atomic or molecular magnetic–dipole moment. The material may then be *paramagnetic* or *ferromagnetic*. The former involves only independent responses of each molecule to an external field, and so can occur in any phase (gaseous, liquid, solid). *Ferromagnetism* involves collective behavior of nearby molecular magnets, so does not arise in gases and disappears above the material's *Curie temperature*.

Iron, Cobalt, Nickel and the loadstone of the ancients are among the ferromagnetic materials that had long been known, and more are constantly being discovered. *Antiferromagnetism* and *ferrimagnetism* are allied, similar phenomena.

all the domains have their moments parallel. is explained by the fact that the domains offer resistance to orientation.

Experimental evidence for this grain-like structure is found in the Barkhausen effect (1919), observed when a sample of iron is being magnetized. If a B-H curve is determined very accurately, it is found that the increase in magnetization of iron occurs in jumps rather than continuously. These jumps correspond to reorientation of the magnetic moments. By suitable amplification of the induced currents it is possible to actually observe (or rather, listen to) the orientation of a simple domain.

A domain cannot select the direction of its magnetization at random; it must choose that direction from among a very few easy directions of magnetization in the crystalline structure of the material. More recently, it has become clear that the magnetization within a domain does not "flop" in so abrupt a fashion. Instead, the magnetizing force makes the favorably disposed domains grow at the expense of those less favorably disposed. The wall between two adjacent domains moves more or less smoothly, providing a gradually increasing region of favorable magnetization.

Even on the atomic scale the wall itself is not abrupt; the elementary magnets in the wall also change their directions smoothly. The wall moves by a smooth and orderly change in the directions of the elementary magnets, subject to the "fuzziness" of quantum mechanics (there is an uncertainty relation *between different Cartesian components of spin magnetization). The approximate, semiclassical picture is as follows: induced by a magnetizing force applied from outside, the directions of magnetization of the elementary magnets* rotates *about a line perpendicular to the wall, and thus the wall moves, overcoming various obstacles: pinning, inertia, eddy-current losses, etc.*

The Critical Temperature

With the aid of this picture of moving domain walls, the process of magnetizing a ferromagnet can be traced:

As the external magnetizing field Hext increases, the magnetization increases because more domain walls are enabled to move. But the magnetization cannot increase indefinitely; when there are no walls left to be moved, the torque due to the field (Hext plus that due to the already–aligned domains) can do little to increase the magnetization further. That final saturation magnetization *represents the magnetization that each domain had already, which* *did not appear macroscopically because the many domains were magnetized in random directions.*

When the magnetizing external field is removed, the piece still exhibit a net magnetization (Fig. 5.1.1b) because the domains cannot easily return to randomized orientations. This remanence *magnetization can be made to disappear only by exerting a magnetizing field in the opposite direction (Fig. 5.1.1c). Increasing that reverse field still further (Fig. 5.1.1d) can magnetize the piece in the other direction until it is again saturated. If this process is continued by cycling the direction of the magnetizing field, the plot of magnetization against field (Fig. 5.1.1e) traces a closed curve called a* hysteresis loop.

In the technological uses of magnets, the form of that loop is extremely important. The magnetization that remains when the magnetizing field is removed, for example, is a measure of how strong a permanent magnet can be made of the material. The area of the loop is a measure of how much power will be lost to heat, sound etc. if the material is used to make cores in transformers.

Of more fundamental concern, however, is the saturation magnetization of the material. Since that represents the spontaneous magnetization within each domain, it is a property of the substance that is independent of how it is fabricated into pieces. Again, as with electrical conduction, the clearest insights come from examining how the property varies with temperature.

The form of that variation in nickel is shown in Fig. 5.1.2. As the metal is heated, the saturation magnetization declines, and at a critical temperature *(631 degrees absolute, or 358 degrees centigrade), it falls precipitously to zero. Above that temperature, the substance is no longer a ferromagnet; it behaves like many other metals and non-metals, and is* paramagnetic.

When it is cooled again, the magnetization within the domains reappears and traverses the same curve in the opposite direction. In other words, the spontaneous magnetization of the material varies simply and reversibly with the temperature and vanishes above a critical temperature. Iron behaves similarly but with a different critical temperature (1043 degrees absolute, or 770 degrees centigrade).

The Internal Field

This phase–transition behavior is somewhat reminiscent of melting. When a solid material melts, the orderliness characteristic of crystallinity abruptly *disappears and the crystals fall apart into a disorderly liquid. The vigor of the atomic vibrations becomes sufficient to overcome the forces tending to hold the atoms in their orderly arrays.*

The analogy between the critical temperature at which a crystal melts and the critical temperature at which ferromagnetism disappears suggests examining the heat capacity of a ferromagnet. Heating an ordinary solid will raise its temperature progressively up to its melting point. Then, as the solid begins to melt, enough heat must be supplied to melt it completely before the temperature of the liquid will rise further. In other words, at that melting temperature the heat capacity of the material is infinitely large; supplying heat makes no change in its temperature.

The heat capacity of a ferromagnet behaves somewhat similarly (Fig. 5.1.3); it shows a sharp spike at the critical temperature. To be sure, the heat capacity is not infinite; the temperature of the material will not stay constant while heat is supplied. Nevertheless, the behavior of the heat capacity is anomalous — a conspicuous departure from the smooth behavior observed in other metals.

It was Weiss again who suggested in 1907 a way to account for the variation of the spontaneous magnetization in the domains with varying temperature. He imagined a force originating in the material itself and tending to align all the elementary magnets in a domain in the same direction — a force directly proportional to the magnetization already present in the domain.

Notice how neatly that idea can explain the behavior diagrammed in Figs. 5.1.2 and 5.1.3. The heat vibrations tend to disturb the alignment of the elementary magnets, increasingly so as the temperature rises. The more that alignment is disturbed, the lower is the spontaneous magnetization. But, according to Weiss, the force (actually torque) tending to align the elementary magnets is itself proportional to the extent of alignment that it achieves. Hence, as the temperature increases, the magnetization decreases calamitously; as the temperature decreases, the magnetization again lifts itself "by its own bootstraps."

At the critical temperature, the heat capacity displays a peak because a small increase of temperature induces a large increase of disorder in the arrangement of the elementary magnets. For disordering that arrangement, heat is required, just as heat is required to disorder the atomic arrangement in a melting solid.

In order to explain the origin of the internal forces that Weiss imagined, it seemed natural at first to turn to the magnetic forces between the elementary magnets; the van der Waals forces between molecules suggested ways in which those magnets might tend to align one another. If the magnets were arranged *in strings, for example, they would adopt a head-to-tail arrangement (Fig. 5.1.4a) of the sort required rather than an arrangement (Fig. 5.1.4b) that has no net magnetization.*

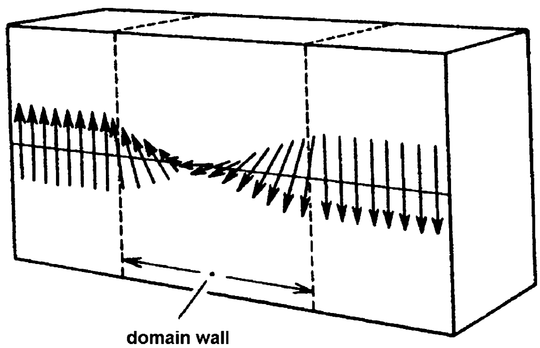
Unfortunately, when numbers were put into Weiss' theory, it turned out that magnetic forces between the elementary magnets are 10^3 *times too small to account for the measured behavior of ferromagnets. The Weiss internal field, so necessary to the picture of ferromagnetic behavior, had to be accepted for many years without explanation of its origin.*

It was clear that the needed factor of 10^3 *could not be found in the magnetic forces between the elementary magnets. And it was also clear that electrostatic forces operating on the electronic constituents of atoms might have the needed magnitude. Is there a way for electrostatic forces to align the spinning electrons?* ***Werner Heisenberg*** *was the first to recognize that quantum mechanics offers a positive answer to this question.*

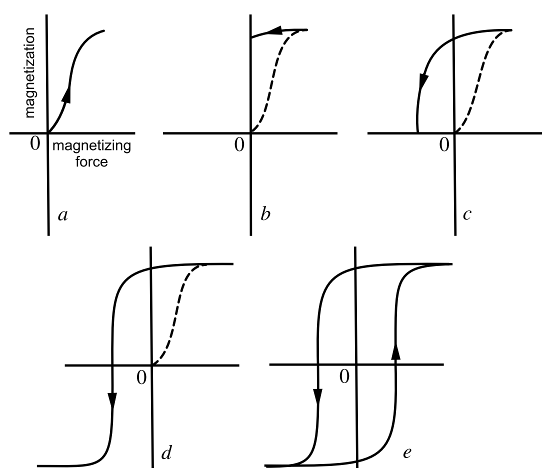
Until now we have spoken of quantum states, orbitals and spins for each electron separately, as if we could lay hands on it and label it. But when many electrons are crowded into a solid, there is no way to distinguish one from another. The Pauli exclusion principle (Fermi-Dirac statistics) comes into play in atoms, molecules and conduction bands, dictating that no two electrons with their spins aligned may occupy the same orbital. This result in "exclusion" or "exchange" forces among electrons, a key effect in chemical bonds and all aspects of condensed matter and atomic physics. In particular, a conjunction of exclusion and electrostatic forces between electrons in molecular orbitals often causes their spins to align.

The principles of quantum mechanics have been brought to bear in order to treat the entire assembly of electrons (and nuclei) as a single physical system. But as so often happens in physical science, the application of those principles to special cases is difficult – and the quantum many-body models we have for describing ferromagnetism, ferrimagnetism and antiferromagnetism, while promising, are greatly oversimplified. Much of the present research in the physical theory of solids and other phases of condensed matter is devoted to perfecting methods for applying quantum-mechanical principles to elucidate the collective behavior of electrons, atoms, ions, molecules, and then mesoscopic and macroscopic structures.

1907 CE **Lee de Forest** (1873–1961, U.S.A.). Inventor. Pioneered in wireless telegraphy and radio broadcasting. Invented the *triode vacuum tube*



Smooth change in direction of magnetization within a domain wall.



The magnetization of a ferromagnet with changing magnetizing force.

Fig. 5.1.1

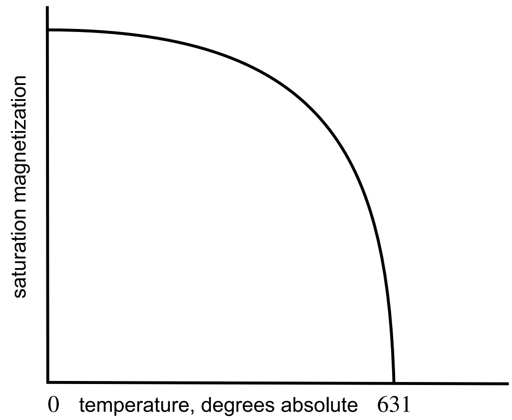


Fig. 5.1.2: Changing of saturation magnetization with temperature in nickel

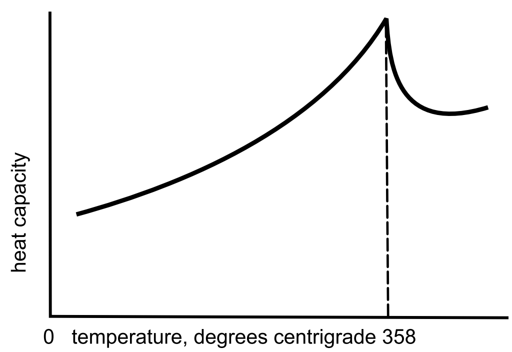


Fig. 5.1.3: Change of heat capacity with temperature in nickel

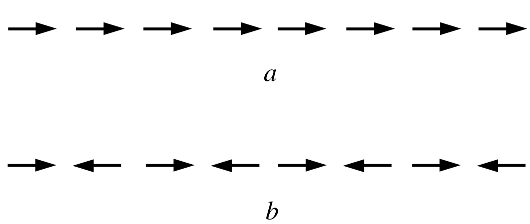


Fig. 5.1.4: The arrangement of magnets *a* has a lower energy than the arrangement *b*

and with it the radio amplifier, as a direct application of the *Edison effect* (1883). He obtained patents on more than 300 inventions. Forest staged the first musical radio broadcast in history, from the Metropolitan Opera House in New York City (1910).

Forest was born in Council Bluffs, Iowa. He graduated from Yale University in 1896 and moved to the Pacific coast in 1911. He worked on methods for photographing sound waves on motion picture films.

How the Triode was Invented[178] (1907–1914)

The triode vacuum tube was the first electronic device capable of amplification. Unfortunately, de Forest did not understand how his invention actually worked, having stumbled upon it by way of circuitous (and occasionally unethical) routes:

The vacuum tube actually traces its ancestry to the lowly incandescent light bulb of **Thomas Edison**. *Edison's bulbs had a problem with progressive darkening caused by the accumulation of soot (given off by the carbon filaments) on the inner surface of the bulb. In an attempt to cure the problem, he inserted a metal electrode, hoping somehow to attract the soot to this plate rather than to the glass. Ever the experimentalist, he applied both positive and negative voltages (relative to one of the filament connections) to this plate, and noted in 1883 that a current mysteriously flowed when the plate was positive, but none flowed when the plate was negative. Furthermore, the current that flowed depended on how hot he made the filament. He had no theory to explain these observations (remember, the word electron wasn't even coined until 1891, and the particle itself wasn't unambiguously identified until J.J. Thomson's experiments of 1897), but Edison went ahead and patented in 1884 the first electronic (as opposed to electrical) device, one that exploited the dependence of plate current on filament temperature to measure line voltage indirectly. This instrument never made it into production since it was inferior to a standard voltmeter; Edison just wanted another patent, that's all (that's one way he ended up with over 1000 of them).*

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178 Adapted from: Lee, Thomas H., *The Design of CMOS Radio-Frequency Integrated Circuits*, Cambridge University Press, 2003.

The funny thing about this episode is that Edison arguably had never invented anything in the fundamental sense of the term, and here he had stumbled across an electronic rectifier but nevertheless failed to recognize the implications of what he had found. Part of this blindness was no doubt related to his emotional (and financial) fixation on the DC transmission of power, where a rectifier had no role.

At about this time a consultant to the British Edison Company named ***John Ambrose Fleming*** *happened to attend a conference in Canada. He dropped down to the U.S. to visit his brother in New Jersey and also stopped by Edison's lab. He was greatly intrigued by the "Edison effect" (much more so than Edison, who found it difficult to understand Fleming's excitement over something that had no obvious promise of practical application), and eventually published papers on the Edison effect from 1890 to 1896. Although his experiments created an initial stir, Röntgen's announcement in January 1896 of the discovery of X-rays as well as the discovery of natural radioactivity later that same year soon dominated the interest of the physics community, and the Edison effect quickly lapsed into obscurity.*

Several years later, though, Fleming became a consultant to British Marconi and joined in the search for improved detectors. Recalling the Edison effect, he tested some bulbs, found out that they worked all right as RF rectifiers, and patented the Fleming valve (vacuum tubes are thus still known as valves in the U.K.) in 1905. The nearly-deaf Fleming used a mirror galvanometer to provide a visual indication of the received signal, and included this feature as part of his patent.

While not particularly sensitive, the Fleming valve was at least continually responsive, and required no mechanical adjustments. Various Marconi installations used them (largely out of contractual obligations), but the Fleming valve never was popular (contrary to the assertions of some poorly researched histories) — it needed too much power, filament life was poor, the thing was expensive, and it was remarkably insensitive detector compared with, say, Fessenden's barretter, and well-made crystal detectors.

De Forest*, meanwhile, was busy in America setting up shady wireless companies whose sole purpose was to earn money via the sale of stock. "Soon, we believe, the suckers will begin to bite," he wrote in his journal in early 1902. As soon as the stock in one wireless installation was sold, he and his cronies picked up stakes (whether or not the station was actually completed), and moved on to the next town. In another demonstration of his sterling character, he just outright stole Fessenden's barretter (simply reforming the Wollaston wire into the shape of a spade) after visiting Fessenden's laboratory, and even had the audacity to claim a prize for his invention. In this case, however, justice did prevail and Fessenden won an infringement suit against de Forest.*

Fortunately for de Forest, **Dunwoody** *invented the carborundum detector just in time to save him from bankruptcy. Not content to develop this legitimate invention, though, de Forest proceeded to steal Fleming's vacuum tube diode, and actually received a patent for it in 1905. He simply replaced the mirror galvanometer with a headphone, and added a huge forward bias (thus reducing sensitivity of an already insensitive detector). De Forest repeatedly and unconvincingly denied throughout his life that he was aware of Fleming's prior work (even though Fleming published in professional journals that de Forest habitually and assiduously scanned) and to bolster his claims, de Forest pointed to his use of bias, where Fleming used none. Conclusive evidence that de Forest had lied outright finally came to light when historian Gerald Tyne obtained the business records of* ***W. McCandless****, the man who made all of de Forest's vacuum tubes (de Forest called them audions). The records clearly show that de Forest had asked McCandless to duplicate some Fleming valves months before he filed his patent. There is thus no room for a charitable interpretation that de Forest independently invented the vacuum tube diode.*

His crowning achievement came soon after, however. He added a zigzag wire electrode, which he called the grid, between the filament and wing electrode (later known as the plate), and thus the triode was born. This three-element audion was capable of amplification, but de Forest did not realize this fact until years later. In fact, his patent application only mentioned the triode audion as a detector, not as an amplifier. Motivation for the addition of the grid is thus still curiously unclear. He certainly did not add the grid as the consequence of careful reasoning, as some histories claim. The fact is that he added electrodes all over the place. He even tried "control electrodes" outside of the plate! We must therefore regard his addition of the grid as merely the result of haphazard but persistent tinkering in his search for a detector to call his own. It would not be inaccurate to say that he stumbled onto the triode, and it is certainly true that others had to explain its operation to him.

From the available evidence, neither de Forest nor anyone else thought much of the audion for a number of years (1906–1909 saw essentially no activity on the audion). In fact, when de Forest barely escaped conviction and a jail sentence for stock fraud after the collapse of one of his companies, he had to relinquish interest in all of his inventions as a condition of the subsequent reorganization of his companies, with just one exception: the lawyers let him keep the patent for the audion, thinking it worthless.

He intermittently puttered around with the audion and eventually discovered its amplifying potential, as did others almost simultaneously (including rocket pioneer **Robert Goddard***). He managed to sell the device to AT&T in 1912 as a telephone repeater amplifier, but initially had a tough time because of the erratic behavior of the audion. Reproducibility of device characteristics* *was rather poor and the tube had a limited dynamic range. It functioned well for small signals, but behaved badly upon overload (the residual gas in the tube would ionize, resulting in a blue glow and a frying noise in the output signal). To top things off, the audion filaments (made of tantalum) had a life of only about 100–200 hours. It would be a while before the vacuum tube could take over the world.*

Fortunately, some gifted people finally became interested in the audion. ***Irving Langmuir*** *at GE Lab in Schenectady worked to achieve high vacua, thus eliminating the erratic behavior caused by the presence of (easily ionized) residual gases. De Forest never thought to do this (in fact, warned against it, believing that it would reduce the sensitivity) because he never really believed in thermionic emission of electrons (indeed, it isn't clear he even believed in electrons at the time), asserting instead that the audion depended fundamentally on ionized gas for its operation.*

After Langmuir's achievement, the way was paved for a bright engineer to devise useful circuits to exploit the audion's potential. That bright engineer was ***Edwin Howard Armstrong*** *who invented the regenerative amplifier/detector in 1912 at the tender age of 21. This circuit employed positive feedback (via a "tickler coil" that coupled some of the output energy back to the input with the right phase) to boost the gain and Q of the system simultaneously. Thus high gain (for good sensitivity) and narrow bandwidth (for good selectivity) could be obtained rather simply from one tube. Additionally, the nonlinearity of the tube demodulated the signal. Furthermore, over-coupling the output to the input turned the thing into a wonderfully compact RF oscillator.*

In a 1914 paper titled "Operating Features of the Audion," Armstrong published the first correct explanation for how the triode worked, and provided experimental evidence to support his claims. He followed this paper with another ("Some Recent Developments in the Audion Receiver") in which he additionally explained the operation of the regenerative amplifier/detector, and showed how to make an oscillator out of it. The paper is a model of clarity and quite readable even to modern audiences. De Forest, however, was quite upset at Armstrong's presumptuousness. In a published discussion section following the paper, de Forest repeatedly attacked Armstrong. It is clear from the published exchange that, in sharp contrast with Armstrong, de Forest had difficulty with certain basic concepts (e.g., that the average value of a sine-wave is zero), and didn't even understand how the triode, his own invention (more of a discovery, really) actually worked.

The bitter lifelong enmity that arose between these two men never waned.

1907–1908 CE **Hermann Minkowski** (1864–1909, Germany). Outstanding mathematician and theoretical physicist. Developed the geometrical theory of numbers and used geometrical methods to solve difficult problems in number theory, mathematical physics and the Theory of Relativity. His concept of 'space-time' has proved to be one of the most valuable contributions ever made to theoretical physics by a mathematician, and made it possible for **Einstein** to formulate the theory of General Relativity.

Minkowski's early work was in number theory and n-dimensional geometry. In 1883, while only 18 years old, he shared with **Henry Smith** the Grand Prix des Sciences Mathematiques of the Paris Academy, for his work on the problem of representation of a number as a sum of 5 squares. In his book *Geometrie der Zahlen* (1896), the connection between geometry and number theory was forged into a strong link. In this book he proved many beautiful relationships at the interface of geometry and number theory.

His most famous result, known as *Minkowski's theorem*, states that any convex planar region symmetrical about $(0, 0)$ and having an area greater than 4 contains integer lattice points apart from $(0, 0)$. This theorem, and its generalization to higher dimensional spaces, is particularly useful in proofs concerning the representation of numbers by quadratic forms, such as the decomposition of certain primes into sums of squares.

In an address before the Göttingen Mathematical Society in 1905, Minkowski hypothesized that someday soon, number theory would triumph in physics and that, for example, the decomposition of primes into the sum of 2 squares would be seen to be related to important properties of matter.

Another intriguing geometrical concept due to Minkowski is that of a '*ray body*' (strahlkörper), defined as a region in n-dimensional Euclidean space containing the origin and whose surface, as seen from the origin, exhibits only one point in any direction. In other words, if the inner region were made of transparent glass and only the surface were opaque, then the origin would be visible from each surface point of the 'ray body' with no intervening surface points (any convex region is a ray body, but the converse does not hold). Minkowski proved the bizarre theorem that if the volume of such a body does not exceed $\zeta(n)$ [Riemann zeta function], a volume-preserving linear transformation exists such that the transformed body has no points in common with the integer lattice, other than the origin.

Around 1902, Minkowski became increasingly fascinated by the riddles of electromagnetism, as had recently been formulated in the works of **H.A. Lorentz**. His earlier friendship with **H. Hertz** may have also contributed to these interests. In 1907 he was able to show that the electromagnetic scalar and vector potentials as well as the charge-current densities are 4-vectors w.r.t. the Lorentz symmetry group of Special Relativity, while the electromagnetic field-strengths form a second rank skew-symmetric 4-tensor. On Nov. 5, 1907 he gave in Göttingen a colloquium about relativity, in which he identified the Lorentz transformation with pseudorotations[179].

Then, in 1908, he presented the Maxwell-Lorentz equations in four-dimension tensor form[180].

At a scientific gathering in Cologne in 1908, Minkowski introduced the concept of *space-time*. A point in space at an instant in time (an *event*) he called a 'world point', and the totality of all conceivable world-points is a

[179] In 1905 **Hilbert** and **Minkowski** conducted a joint seminar on the 'electrodynamics of moving bodies', with **Max Born** as one of the students. The FitzGerald contraction, Lorentz time dilation and the Michelson-Morley experiments were discussed. However, the name of Einstein was never mentioned. When the work of the Bern patent clerk finally reached Göttingen, Minkowski recalled his former student in Zürich and remarked: "*Oh, that Einstein, always missing lectures — I really would not have believed him capable of it*!" (translated).

[180] The Lorentz covariant tensor formulation of the Maxwell theory was a natural consequence of Einstein's postulate of equivalence of inertial systems, since it implies that all the equations of physics must be form-invariant under Lorentz transformations. Consequently, it should be possible to express them as absolute or covariant relations between four-tensors, and the transformation laws of the tensors become the transformation laws of the fields, current-charge distribution, positions, times, etc.

Consider first the inhomogeneous pair of Maxwell equations in empty space with sources: $\operatorname{curl}\boldsymbol{B} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} = \frac{1}{c^2}\boldsymbol{J}$, $\operatorname{div}\boldsymbol{E} = \rho$ and introduce $x_4 = ict$, $J_4 = ic\rho$, where $i = \sqrt{-1}$ and boldface quantities are spatial vectors (3-vectors). These equations become,

$$\begin{array}{lllll}
0 & +\dfrac{\partial(cB_3)}{\partial x_2} & -\dfrac{\partial(cB_2)}{\partial x_3} & -\dfrac{\partial(iE_1)}{\partial x_4} & = \dfrac{1}{c}J_1, \\
-\dfrac{\partial(cB_3)}{\partial x_1} & +0 & +\dfrac{\partial(cB_1)}{\partial x_3} & -\dfrac{\partial(iE_2)}{\partial x_4} & = \dfrac{1}{c}J_2, \\
\dfrac{\partial(cB_2)}{\partial x_1} & -\dfrac{\partial(cB_1)}{\partial x_2} & +0 & -\dfrac{\partial(iE_3)}{\partial x_4} & = \dfrac{1}{c}J_3, \\
\dfrac{\partial(iE_1)}{\partial x_1} & +\dfrac{\partial(iE_2)}{\partial x_2} & +\dfrac{\partial(iE_3)}{\partial x_3} & +0 & = \dfrac{1}{c}J_4.
\end{array}$$

Here, the left-hand sides of the four equations can be recast in terms of a 4×4 antisymmetric matrix, which hints at the possibility of defining a four-tensor of second rank, known as *the field-strength tensor*, according to the scheme

$$F = \begin{bmatrix} 0 & B_3 & -B_2 & -iE_1/c \\ -B_3 & 0 & B_1 & -iE_2/c \\ B_2 & -B_1 & 0 & -iE_3/c \\ iE_1/c & iE_2/c & iE_3/c & 0 \end{bmatrix}.$$

We can then write the above pair of Maxwell equations in the form

$$\sum_{\nu=1}^{4} \frac{\partial F_{\mu\nu}}{\partial x_\nu} = \frac{1}{c^2} J_\mu \qquad (\mu = 1, 2, 3, 4),$$

or simply

$$\operatorname{div} F = -\frac{1}{c^2}\mathrm{J}.$$

The remaining (homogeneous) Maxwell equations $\operatorname{curl} \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = 0$, $\operatorname{div} \boldsymbol{B} = 0$, can be similarly expressed in tensor form if we introduce a second antisymmetric tensor $G_{\mu\nu}$ defined as follows:

$$G = \begin{bmatrix} 0 & -E_3/c & E_2/c & -iB_1 \\ E_3/c & 0 & -E_1/c & -iB_2 \\ -E_2/c & E_1/c & 0 & -iB_3 \\ iB_1 & iB_2 & iB_3 & 0 \end{bmatrix}.$$

We then write the homogeneous Maxwell equation pair as

$$\sum_{\nu=1}^{4} \frac{\partial G_{\mu\nu}}{\partial x_\nu} = 0 \qquad (\mu = 1, 2, 3, 4),$$

or $\operatorname{div} G = 0$.

The tensor $G_{\mu\nu}$ is known as the *dual* to $F_{\mu\nu}$ and is related to it via the equation $G_{\alpha\lambda} = -iF_{\mu\nu}$, where α, λ, μ, ν is an even permutation of 1, 2, 3, 4. It can be shown that G is a 4-tensor (transforms as a tensor under Lorentz-transformations) F is. It is readily seen that $\operatorname{div} G = 0$ is equivalent to

$$\frac{\partial F_{\alpha\lambda}}{\partial x_\mu} + \frac{\partial F_{\lambda\mu}}{\partial x_\alpha} + \frac{\partial F_{\mu\alpha}}{\partial x_\lambda} = 0.$$

In the absence of sources, $\operatorname{div} F = 0$, $\operatorname{div} G = 0$ express the full set of Maxwell's equations in covariant form in terms of a single tensor and its dual. The covariance of Maxwell's equations w.r.t. Lorentz transformations is thus made manifest.

'world'. A particle of matter or light enduring for a finite time interval will correspond to a curve which he called 'world line', the points of which can be labeled by successive readings of the time that would be exhibited by a clock carried by the particle.

In the language of Cartan's exterior-form calculus,

$$F = \sum_{\mu,\nu} F_{\mu\nu} dx_\mu \wedge dx_\nu$$

and

$$^*F = G = \sum_{\mu,\nu} G_{\mu\nu} dx_\mu \wedge dx_\nu$$

are a pair of dual *2-forms* which are both closed in the no-source case: $dF = dG = 0$. With a source J_μ present, $dF = 0$ still, but the inhomogeneous Maxwell equations become $d^*F = \frac{1}{c}{}^*J$, the 3-form $^*J$ being the dual of the 4-current 1-form, $J = \sum_\mu J_\mu dx_\mu$. Viewing the *2-form* F as the curvature of a *fiber bundle*, with Minkowski spacetime as its base manifold and the abelian $U(1)$ group of complex phases as its fiber, the homogeneous half of Maxwell's equations, $dF = 0$, is seen to be the Bianchi identity for the connection A_μ (= 4-vector potential $(\boldsymbol{A}, \frac{i}{c}\Phi)$). This is valid because $F = dA$ is an *exact*, and therefore also *closed*, 2-form in Minkowski 4-space, where the 1-form A is defined as $A = \sum_\mu A_\mu dx_\mu$. For non-abelian fiber groups, this Bianchi identity and the corresponding inhomogeneous field equations generalize to

$$dF_a + \sum_{b,c} g f_{abc} A_b \wedge F_c = 0$$

and

$$d^*F_a + \sum_{b,c} g f_{abc} A_b \wedge {}^*F_c = \frac{1}{c^2}\,{}^*J_a\ ,$$

where: a, b, c are indices in the adjoint representation of the (Lie) group, $A_a = \sum_{\mu=1}^{4} A_{\mu a} dx_\mu$ is the connection 1-form, f_{abc} are group structure constants, and g is a coupling constant (generalization of electric charge). These are the field equations of the Yang-Milles (YM) gauge theory for the given (Lie) group. In the 1970s and 1980s it was discovered that all known subnuclear forces are described by Yang-Milles gauge field theories. Unlike Maxwell's equations the YM eqs. are nonlinear and the abelian relation between fields (curvature) and potentials (connection) is likewise modified:

$$F_\alpha = dA_\alpha + \sum_{b,c} g f_{abc} A_b \wedge A_c$$

The Newtonian absolute time and space, discarded by Einstein, are supplanted by the absolute 'world' of fused *space-time*. The distance between points in this space-time is a *space-time interval* that is invariant, as measured by all observers (via their respective reference frames) in inertial motion. This invariant space-time interval can be real, zero or imaginary, and is defined by the rule that its square is equal to the difference between the squares of the temporal interval and spatial distance between the two world points concerned, where c, the speed of light in *vacuum*, is used to inter-convert units of space and time. One of the peculiarities of the geometry of Minkowski space (space-time) is that on the world-lines of light, the space-time intervals are of zero length.

Following Minkowski, Einstein came to the conclusion that the objective world of physics is essentially a 4-dimensional structure. Its resolution into three-dimensional space and one-dimensional time is not the same for all observers.

Minkowski was born in Alexotas, near Kovno, to a family that left their native Russia [because of the persecution to which Jews were subjected by the Czar's government] and moved to Königsberg, Prussia (1872). He studied in the local high school, and completed his higher education at the universities of Königsberg and Berlin.

Among his teachers were **Kronecker**, **Weierstrass** and **Kirchhoff**. He received his Ph.D. at the University of Königsberg and served as a professor at Zürich (1896–1902) and Göttingen (1902–1909).

On Jan. 10, 1909 he was suddenly stricken with acute appendicitis. He died two days later, not having yet attained his 45^{th} year, at the height of his scientific creativity. On Wednesday, Jan. 11, 1909, the announcement was made to the students. One of the students recalled: "*Because of the great position of a professor in those days and the distance between him and the students, it was almost more of a shock for us to see Hilbert weep than to hear that Minkowski was dead*".

The Special Theory of Relativity[181] *(1905–1908)*

(I) Historical background and fundamental principles

At the end of the 19th century an optimistic view was taken of the achievements of theoretical physics. Few problems, so it seemed, remained to be solved. There were, however, some details which marred this general picture. For example, the rate of advance of the perihelion of Mercury's orbit exceeded the predicted amount by nearly 10 percent. Attempts to describe the interaction of radiation and matter led to a formula which disagreed with experiment. Certain problems in the optics of moving media were still unresolved, and atomic spectra as well as the dynamical stability of atoms were not yet understood. The billions–of–years age of the solar system required that the sun shine steadily for a duration exceeding by far that accounted for by its gravitational collapse upon formation from cold gases. But few physicists would have thought at the time that these particular features of nature would require for their explanation a complete revolution in physical ideas. To see why this was unavoidable, a reexamination of the fundamentals of Newtonian mechanics and the wave theory of light is called for.

In Newton's hand mechanics was formulated based on the notions of absolute space and time. *The time variable occupied a unique position as the independent variable in terms of which the position of a particle is described. Its measure and flow is unrelated to the choice of the particular* reference system *used for the description of mechanical problems.*

By a system (or frame) of reference we mean a system of coordinates used to measure and indicate the position of a particle in space, as well as mutually synchronized clocks fixed in this system, serving to indicate time. Reference systems which move with uniform velocity relative to the idealized "fixed

[181] To dig deeper, see:

- Landau, L.D. and E.M. Lifshitz, *The Classical Theory of Fields*, Pergamon Press, 1962, 404 pp.
- Low, F.E., *Classical Field Theory*, Wiley, 1997, 427 pp.
- Stephenson, G. and C.W., Kilmister, *Special Relativity for Physicists*, Wiley, 1962, 108 pp.
- Jammer, M., *Concepts of Simultaneity: From Antiquity to Einstein and Beyond*, John Hopkins University Press, 2006.

stars" (in which Newton's first law is valid) are known as inertial frames. Consider a reference frame S and another reference frame $\bar{S}$ moving uniformly relative to S with finite velocity $\boldsymbol{v}$.

A physical event occurs at a point in space and at an instance of time and is specified by the entities $(\boldsymbol{r}, t)$ in S and $(\boldsymbol{r}', t')$ in $\bar{S}$. According to classical mechanics these entities are related by the Galilean transformation equations

$$t' = t, \qquad \boldsymbol{r}' = \boldsymbol{r} - \boldsymbol{v}t \tag{1}$$

Clearly, if the first law is valid in S, then it is also valid in $\bar{S}$, and in all other inertial frames as well. In fact, all three Newton's laws are the same in all inertial frames, and no frame has any preference over another.

The laws of mechanics are thus independent of the uniform rectilinear velocity of the system, and one cannot determine it by means of any internal mechanical experiment. This last statement is known as Newton's principle of relativity.

It can thus be said that the idea of an absolute coordinate-system can be replaced by that of the whole set of inertial frames, no one of which is preeminent. The situation in Newtonian dynamics at the end of the 19th century was therefore that the need for an absolute space had been replaced by that of a set of inertial frames. The problem of specifying the frames was still unresolved. Likewise, the concept of absolute time has been left untouched.

How fared (then relatively new) the Maxwell equations w.r.t. the Galilean transformation and the ensuing Newtonian principle of relativity?

In empty space the fields $\boldsymbol{H}$ (and $\boldsymbol{E}$) satisfy the wave equation $\nabla^2 \boldsymbol{H} = \frac{1}{c^2}\frac{\partial^2 \boldsymbol{H}}{\partial t^2}$, etc. where c, with dimensions of velocity is the ratio of electromagnetic and electrostatic units of charge. Already in 1856 ***Weber*** *and* ***Kohlrauch*** *compared the capacity of a Leyden jar as measured by an electrostatic method, with that calculated from the effects of current produced by discharging the jar. These experiments gave a value of c equal to $3.1 \times 10^{10} \frac{\text{cm}}{\text{sec}}$, close to the speed of light in vacuo. This suggested the identification of light and electromagnetic radiation, and such an identification gave a very satisfactory explanation to optical phenomena.*

The wave equation, however, contains no reference to the velocity of the source of the light and this naturally suggests that the velocity of light must be independent of the velocity of its source. This is in agreement with observations. For example, there exist certain 'double stars' consisting of two stars moving in orbits about their common center of gravity. At one point in the orbit one star will be traveling towards the earth, and the other away from it.

If their center of gravity is at a distance h from the earth, the light will reach the earth at a time of order h/c after it has been emitted, where c is the speed of light. For any small change Δc in c, we have a change $\delta t = -(h/c^2)\Delta c$ in the time of arrival. This change would produce apparent irregularities in the motion of such stars. No such irregularities have ever been observed and we are forced to conclude that the velocity of light is independent of the velocity of the source.

However, this independence of the velocity of light on the velocity of its source poses the problem of the coordinate-system with respect to which c is to be measured. In the theory of sound a similar problem arises, but there it is easily resolved since the speed is to be measured relative to the still air.

In the nineteenth century, it seemed reasonable to give a similar answer in the case of light. This required the postulation of an unobserved, all-pervasive medium — the luminiferous aether or ether *— in which the wave motion took place (first introduced by Hooke in 1667). This ether could have the great advantage of linking the hitherto separated theories of mechanics and electromagnetism.*

But alas, Maxwell's equations are not invariant in form w.r.t. the Galilean transformation! In other words: the speed of light is not invariant under this transformation. Indeed, since

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t'} - \boldsymbol{v}\cdot\nabla'; \qquad \nabla = \nabla',$$

the wave equation for $\boldsymbol{H}$, say, in $\bar{S}$, becomes

$$\nabla'^2\boldsymbol{H} - \frac{1}{c^2}\frac{\partial^2\boldsymbol{H}}{\partial t'^2} = \left(\frac{\boldsymbol{v}}{c}\cdot\nabla'\right)\left(\frac{\boldsymbol{v}}{c}\cdot\nabla'\boldsymbol{H} - \frac{2}{c}\frac{\partial\boldsymbol{H}}{\partial t'}\right).$$

There are therefore three logical possibilities:

(a) *In electrodynamics there* is *a preferred inertial frame, the so-called ether rest-frame S, in which Maxwell's equations are valid and in which light is propagated at exactly the speed c. In a frame $\bar{S}$ moving at a constant velocity v with respect to this ether frame an observer would measure a different light velocity, ranging in magnitude from $c-v$ to $c+v$. By an optical experiment in $\bar{S}$ one could determine the value of v. Galilean transformations apply to all physical laws.*

(b) *A relativity principle exists for both mechanics and electrodynamics but the laws of electrodynamics are not correct. Experiment should be able to detect deviations from Maxwell's laws, which should then be reformulated. The Galilean transformation would apply to the corrected laws.*

(c) *A relativity principle exists for both mechanics and electrodynamics but Newton's laws are not correct. Experiments should show deviations from Newtonian mechanics and those laws must be reformulated. The Galilean transformation does not apply and a new transformation must be found which leaves Maxwell's equations and the new mechanical laws invariant.*

The critical experiment of Michelson and Morley (1887) failed to detect a motion of the earth relative to a preferred reference frame. This led Einstein to assume that (1) must be replaced by new equations such that consistency with experiment is attained without having to discard either Maxwell's equations or Newton's principle of relativity. Einstein's theory of relativity accepts (c) and is based on two fundamental postulates:

(A) *The principle of relativity*[182]

[182] Note that *accelerated* motion of a reference frame relative to an inertial frame of reference can, of course be detected. A *mechanical* experiment of this kind was designed (1851) by **Foucault**, *optical* variants of the experiment were carried out by **Francis Harress** (1911) and the French physicist **George Sagnac** (1869–1926) in 1913 (Published in Comptes Rendus de l'Academie des Sciences, **157**, 708–710, 1410–1413, 1915).
According to the *Sagnac-Harress experiment*, two pulses of light are sent (by means of reflections) from a split source in opposite directions around a stationary circular loop of radius R. Clearly, since they will travel the same distance at the same speed, they will each travel full-circle and arrive at the light detector, near the light source, simultaneously.
If the loop itself is rotating during this procedure, however, the pulse traveling in the same direction as the rotation of the loop must traverse a slightly greater distance than the pulse traveling in the opposite direction. As a result, the counter-rotating pulse arrives at the end point slightly earlier than the co-rotating pulse. Quantitatively, if we let ω denote the angular speed of the loop, then the circumferential tangent speed of the end point is $v = \omega R$, and the relative speed of the wave front and the receiver at the end point is $c - v$ in the co-rotating direction and $c + v$ in the counter- rotating direction. Both pulses begin with an initial separation of $2\pi R$ from the end point, so the difference between the travel times is

$$\Delta t = 2\pi R \left(\frac{1}{c - v} - \frac{1}{c + v} \right) = \frac{4\pi R v}{c^2 - v^2} = \frac{4A\omega}{c^2 - v^2} \approx \frac{4A\omega}{c^2}$$

where $A = \pi R^2$ is the area enclosed by the loop. This analysis is perfectly valid in both the classical and the relativistic contexts. The *interference* of the two coherent beams will result in a *fringe shift* $\Delta\varphi = \Delta t \frac{c}{\lambda} = \frac{4A\omega}{c\lambda}$. Indeed Sagnac, using an area of $863 cm^2$, a mercury lamp with wavelength $\lambda = 0.436$ micron as light source and an angular velocity of about 14 rad/sec, predicted a fringe shift of $\Delta\varphi = 0.037$ – which agreed nicely with the observations.

One cannot determine the uniform rectilinear motion of a reference system by means of any *internal experiment (non existence of absolute motion).*

(B) *The principle of the constancy of the velocity of light*[183]
The velocity of light is one and the same in all inertial systems, independent of the observer's velocity with respect to the frame in which the light source is at rest.

(A) has an equivalent form known as the principle of covariance*. It states that if a physical event or process is observed from any inertial frame, then the physical entities such as time, spatial coordinates, velocities etc., will change in such a way that the laws of nature remain invariant with respect to transformation of these entities from one inertial frame to another. When these laws are written in terms of the coordinates, times and other physical attributes in different inertial frames they will have one and the same form. If it were otherwise, the experimental laws deduced by means of internal measurements would depend on the frame's uniform rectilinear motion.*

(A) has two important consequences. The first states that there is no preferred coordinate system *in which the laws of nature are simpler than in*

The shift can be used to determine the angular velocity ω.

If the Earth is used as the turntable its angular velocity can likewise be determined. This experiment was carried out in 1925 by **Michelson** and **Gale**. The angular velocity corresponded to the component of the angular velocity of rotation of the Earth along a plumb line at the point of observation. For the experiment two kilometers of pipes were laid and a second circuit was built to determine the zero point of displacement of the fringes.

Michelson and Gale used a circuit of area $0.2km^2$. The formula becomes $\Delta\varphi = \frac{4A\omega}{c\lambda} sin\phi$, where ϕ is the latitude. Michelson was at latitude 41.8° and looked for a fringe shift of $\Delta\varphi = 0.236 \pm 0.002$. The observed fringe shift was 0.230 ± 0.005. Excellent agreement! Thus, unlike uniform translational motion of the earth, its rotation can be determined by various physical experiments.

The fringe shift is known as the *Sagnac effect* and the modern fiber-optic experimental set-up is known as *Sagnac interferometer*.

Sagnac belonged to a group of friends that notably included **Pierre** and **Marie Curie**, **Paul Langevin**, **Jean Perrin** and the mathematician **Emil Borel**. The Sagnac effect is at the basis of interferometers and *laser gyroscopes* developed since the 1970s.

183 In recent years [see e.g Amelino–Camelia, G., *Doubly – Special Relativity*, Int. J. Mod. Phys., 2002, D11, 1643 pp] there arose a modified theory of Special Relativity in which there is not only an observer-independent maximum velocity (the speed of light), but an observer-independent minimum length (the Planck length). Doubly–Special Relativity is also called *Deformed Special Relativity*.

other inertial frames, since the existence of such a system is ruled out by the principle of covariance. The second consequence of (A) is that a light ray emitted from a source at rest in S will appear to an observer at rest with respect to S, to move in one and the same velocity c irrespective of the constant rectilinear motion of S relative to the fixed stars. This is in fact an application of the principle of relativity to electromagnetic phenomena.

This consequence is not *trivial. It does not hold in acoustics. For if a sound source is placed on a uniformly moving platform the velocity of sound with respect to the stationary air is always c, say, irrespective of the source motion. But it is $c - v$ relative to the source observed in the line of motion ahead of the source and $c + v$ in the other direction.*

Note that (B) follows from the principle of covariance provided that the validity of Maxwell's electrodynamics is assumed, for both Maxwell's equations and the velocity c are laws of nature, and as such must be the same for all inertial frames.

***Einstein** (1905) then set forth to deduce a new transformation law in accord with his two postulates. Based on a few simple optical thought-experiments, it was found to be*

$$\boldsymbol{r}' = \gamma(\boldsymbol{r}_1 - \boldsymbol{v}t), \qquad t' = \gamma(t - \frac{\boldsymbol{v}\cdot\boldsymbol{r}}{c^2}) \tag{2}$$

where

$$\boldsymbol{r}_1 = \frac{1}{\gamma}\boldsymbol{r} + \frac{\gamma - 1}{\gamma v^2}(\boldsymbol{v}\cdot\boldsymbol{r})\boldsymbol{v}, \qquad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

This is known as the Lorentz transformation.

Defining the dyadic

$$\Phi = I + \frac{\gamma - 1}{v^2}\boldsymbol{v}\boldsymbol{v}, \qquad \Phi^{-1} = I - \frac{\gamma - 1}{v^2}\boldsymbol{v}\boldsymbol{v}, \tag{3}$$

the transformation law and its inverse are

$$\begin{aligned} \boldsymbol{r}' &= \Phi\cdot\boldsymbol{r} - \gamma t\boldsymbol{v}; & \boldsymbol{r} &= \Phi\cdot\boldsymbol{r}' + \gamma t'\boldsymbol{v}; \\ t' &= \gamma\left(t - \frac{\boldsymbol{r}\cdot\boldsymbol{v}}{c^2}\right); & t &= \gamma\left(t' + \frac{\boldsymbol{r}'\cdot\boldsymbol{v}}{c^2}\right) \end{aligned} \tag{4}$$

Also

$$\begin{aligned} \frac{\partial}{\partial t} &= \frac{\partial t'}{\partial t}\frac{\partial}{\partial t'} + \frac{\partial \boldsymbol{r}'}{\partial t}\cdot\frac{\partial}{\partial \boldsymbol{r}'} = \gamma\left(\frac{\partial}{\partial t'} - \boldsymbol{v}\cdot\nabla'\right) \\ \nabla = \frac{\partial}{\partial \boldsymbol{r}} &= \frac{\partial \boldsymbol{r}'}{\partial \boldsymbol{r}}\cdot\frac{\partial}{\partial \boldsymbol{r}'} + \frac{\partial t'}{\partial \boldsymbol{r}}\frac{\partial}{\partial t'} = \nabla' + \boldsymbol{v}\left[\frac{\gamma - 1}{v^2}\boldsymbol{v}\cdot\nabla' - \frac{\gamma}{c^2}\frac{\partial}{\partial t'}\right]. \end{aligned} \tag{5}$$

Some algebra is needed to show from this that indeed

$$\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} = \nabla'^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t'^2} \tag{6}$$

This result was expected since it just reflects the postulate of the constancy of the velocity of light, upon which the Lorentz transformation is based.

(II) The 4-dimensional 'space-time' world (Minkowski 1908) and relativistic electrodynamics

It follows directly from (2, *I*) *that*

$$r'^2 - c^2 t'^2 = r^2 - c^2 t^2. \tag{1}$$

This can be rewritten as

$$x_1'^2 + x_2'^2 + x_3'^2 + x_4'^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2, \tag{2}$$

provided we introduce the coordinates $(x_1, x_2, x_3, x_4 = ict)$, *or simply* $(\boldsymbol{r}, ict)$. *In other words: the* pseudonorm *of the spacetime positional 4-vector, is invariant under both ordinary 3-rotation and Lorentz transformations. (It is called a* pseudonorm *because it lacks the positive–definiteness of an actual norm.) Eq.* (6, *I*) *can be recast in terms of a 4-dimensional* gradient operator,

$$\nabla_\mu = (\nabla, \frac{1}{ic}\frac{\partial}{\partial t}) \qquad (\mu = 1, 2, 3, 4) \tag{3}$$

to read

$$\sum_{\mu=1}^{4} \nabla_\mu'^2 = \sum_{\mu=1}^{4} \nabla_\mu^2. \tag{4}$$

These considerations prompted ***Minkowski*** *to define a general 4-vector (real 3-vector and a imaginary scalar) as an ordered pair* $(\boldsymbol{A}, i\theta)$ *which transforms like* $(\boldsymbol{r}, ict)$ *under a general Lorentz transformation; namely*

$$\left.\begin{aligned} \boldsymbol{A} \to \boldsymbol{A}' &= \boldsymbol{A} + \frac{\gamma-1}{v^2}(\boldsymbol{v}\cdot\boldsymbol{A})\boldsymbol{v} - \frac{1}{c}\gamma\boldsymbol{v}\theta \\ \theta \to \theta' &= \gamma\big(\theta - \frac{\boldsymbol{v}\cdot\boldsymbol{A}}{c}\big) \end{aligned}\right\} \tag{5}$$

For example $(\boldsymbol{J}, ic\rho)$ *is a 4-vector. To see this we recall the* continuity equation $\nabla\cdot\boldsymbol{J} + \frac{\partial\rho}{\partial t} = 0$ *(expressing local charge conservation) Upon substitution of the operators* ∇ *and* $\frac{\partial}{\partial t}$ *from* (4, *I*), *we obtain*

$$\frac{\partial}{\partial t'}\big(\gamma\rho - \frac{\gamma}{c^2}\boldsymbol{v}\cdot\boldsymbol{J}\big) + \nabla'\cdot\big[\boldsymbol{J} + \frac{\gamma-1}{v^2}\boldsymbol{v}(\boldsymbol{v}\cdot\boldsymbol{J}) - \gamma\rho\boldsymbol{v}\big] = 0.$$

This will yield $\nabla' \cdot \boldsymbol{J}' + \frac{\partial \rho'}{\partial t'} = 0$ *guaranteeing the covariance of charge conservation – if and only if:*

$$\rho' = \gamma\big(\rho - \frac{1}{c^2}\boldsymbol{v}\cdot\boldsymbol{J}\big); \qquad \boldsymbol{J}' = \boldsymbol{J} + \frac{\gamma-1}{v^2}\boldsymbol{v}(\boldsymbol{v}\cdot\boldsymbol{J}) - \gamma\rho\boldsymbol{v} \tag{6}$$

which complies with the definition $(5, II)$*. In a similar way, it is shown that* $(\boldsymbol{A}, i\frac{\phi}{c})$ *is a four-vector on the strength of the invariance of the Lorentz-gauge condition* $\boldsymbol{\nabla}\cdot\boldsymbol{A} + \frac{1}{c}\frac{\partial\phi}{\partial t} = 0$.

The covariance of 4-vectors suggests that for any two 4-vectors, their scalar product is invariant under the Lorentz transformation. Indeed, using $(5, II)$*, it is easily shown that*

$$(\boldsymbol{A}, i\theta)\cdot(\boldsymbol{B}, i\phi) = (\boldsymbol{A}\cdot\boldsymbol{B}) - \theta\phi \tag{7}$$

is invariant. In particular, the square of the pseudonorm of a 4-vector is a 4-scalar or Lorentz invariant:

$$(\boldsymbol{A}, i\theta)\cdot(\boldsymbol{A}, i\theta) = |\boldsymbol{A}|^2 - \theta^2 = |\boldsymbol{A}'|^2 - \theta'^2 \tag{8}$$

4-vectors thereby divide into 3 categories: space-like $|\boldsymbol{A}|^2 - \theta^2 > 0$, *time-like* $|\boldsymbol{A}|^2 - \theta^2 < 0$ *and light-like* $|\boldsymbol{A}|^2 = \theta^2$.

We saw earlier (cf. Minkowski) that the electromagnetic fields, $\boldsymbol{E}$ *and* $\boldsymbol{B}$ *can be arranged into* $F_{\mu\nu}$*, the skew-symmetric rank 2 field-strength 4-tensor. This transforms as* $a_\mu b_\nu - a_\nu b_\mu$ *where* a, b *are any two 4-vectors. From this and Eq.* $(5, II)$*, the Lorentz transformation law for* $\boldsymbol{E}$ *and* $\boldsymbol{B}$ *is readily found:*

$$[c\boldsymbol{B}, -i\boldsymbol{E}] \Rightarrow \begin{cases} \boldsymbol{E}' = \gamma\boldsymbol{E} - \dfrac{\gamma-1}{v^2}(\boldsymbol{v}\cdot\boldsymbol{E})\boldsymbol{v} + \gamma(\boldsymbol{v}\times\boldsymbol{B}) \\ \boldsymbol{B}' = \gamma\boldsymbol{B} - \dfrac{\gamma-1}{v^2}(\boldsymbol{v}\cdot\boldsymbol{B})\boldsymbol{v} - \dfrac{\gamma}{c^2}(\boldsymbol{v}\times\boldsymbol{E}) \end{cases} \tag{9}$$

These equations were first obtained by ***Minkowski*** *in 1908.*

As we saw, the field–strength (or electromagnetic field) 4-tensor is

$$F_{\alpha\beta} = [\boldsymbol{B}, -i\boldsymbol{E}/c] = \left[\begin{array}{ccc|c} & & & -iE_x/c \\ & -(I\times\boldsymbol{B})_{ij} & & -iE_y/c \\ & & & -iE_z/c \\ \hline iE_x/c & iE_y/c & iE_z/c & 0 \end{array}\right] \tag{10}$$

The tensor $F_{\alpha\beta}$ is called the electromagnetic field tensor and $F_{\alpha\beta} = \nabla_\alpha A_\beta - \nabla_\beta A_\alpha$ in terms of the 4-vector potential. The sourceless Maxwell equations, $dF = 0$, is then satisfied automatically; while the source Maxwell equations,

$$\sum_{\alpha=1}^{4} \nabla_\alpha F_{\alpha\beta} = -\frac{1}{c^2} J_\beta$$

become (in the Lorentz gauge the 4-vector wave equation)

$$\Box A_\mu = -\frac{1}{c^2} J_\mu \tag{11}$$

where $\Box \equiv \boldsymbol{\nabla}^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$ is the D'Alembertian operator.

- *Since $\nabla_\mu J_\mu = 0$ is a 4-scalar while ∇_μ is a vector, J_μ must transform like a 4-vector (Eq. (6, II)). The charge in a volume element is $q = \rho\, dx_1\, dx_2\, dx_3$. It can be shown that q integrated over a volume τ outside of which $\rho = 0$, is a Lorentz invariant. This is the law of invariance of electric charge. Moreover*

$$\begin{aligned} \frac{d}{dt}\int_\tau \rho\, dx_1\, dx_2\, dx_3 &= \int_\tau \frac{d\rho}{dt}\, dx_1\, dx_2\, dx_3 \\ &= \int_\tau (\frac{\partial\rho}{\partial t} + \text{ div } \boldsymbol{J})\, dx_1\, dx_2\, dx_3 \equiv 0, \end{aligned} \tag{12}$$

meaning that the total charge present in all space is time-independent.

(III) Matrix representation

The Lorentz transformation in (2, I) can be put in a convenient matrix form. Introducing a unit vector $\boldsymbol{n}$ in the direction of the velocity $\boldsymbol{v}$, $\boldsymbol{n} = \frac{1}{|\boldsymbol{v}|}\boldsymbol{v} = n_1\boldsymbol{e}_1 + n_2\boldsymbol{e}_2 + n_3\boldsymbol{e}_3$, the transformation equations are recast as

$$\begin{aligned} \boldsymbol{r}' &= \Phi\cdot\boldsymbol{r} - \gamma\beta\boldsymbol{n}(ct), & \Phi &= I + (\gamma-1)\boldsymbol{n}\boldsymbol{n} \\ ct' &= -\gamma\beta(\boldsymbol{n}\cdot\boldsymbol{r}) + \gamma(ct), & \beta &= \frac{v}{c}\ ,\ \gamma = \frac{1}{\sqrt{1-\beta^2}} \end{aligned} \tag{1}$$

or

$$\begin{bmatrix} y' \\ z' \\ x' \\ ct' \end{bmatrix} = \begin{bmatrix} 1+(\gamma-1)n_2^2 & (\gamma-1)n_2n_3 & (\gamma-1)n_1n_2 & -\gamma\beta n_2 \\ (\gamma-1)n_2n_3 & 1+(\gamma-1)n_3^2 & (\gamma-1)n_1n_3 & -\gamma\beta n_3 \\ (\gamma-1)n_1n_2 & (\gamma-1)n_1n_3 & 1+(\gamma-1)n_1^2 & -\gamma\beta n_1 \\ -\gamma\beta n_2 & -\gamma\beta n_3 & -\gamma\beta n_1 & \gamma \end{bmatrix} \begin{bmatrix} y \\ z \\ x \\ ct \end{bmatrix} \tag{2}$$

The case in which the direction of $\boldsymbol{v}$ *is chosen in the* $\boldsymbol{e}_1$ *direction* $(n_2 = n_3 = 0)$ *degenerates into*

$$\begin{bmatrix} y' \\ z' \\ x' \\ ct' \end{bmatrix} = \begin{bmatrix} 1 & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \gamma & -\beta\gamma \\ \cdot & \cdot & -\beta\gamma & \gamma \end{bmatrix} \begin{bmatrix} y \\ z \\ x \\ ct \end{bmatrix} \tag{3}$$

Eq. $(1,\ III)$ *can be written in the compact form*

$$x^{\nu\prime} = \Lambda^{\nu}_{\mu} x^{\mu} \qquad (\nu, \mu = 1, 2, 3, 4) \tag{4}$$

or in index-free form

$$\boldsymbol{x}' = L(\boldsymbol{n},\ \beta) \cdot \boldsymbol{x}, \qquad \boldsymbol{x} = L^{-1} \cdot \boldsymbol{x}' = L(-\boldsymbol{n},\ \beta) \cdot \boldsymbol{x}' \tag{5}$$

We recall next the Lorentz invariant

$$ds^2 = c^2\, d\tau^2 = c^2\, dt^2 - dx^2 - dy^2 - dz^2 = -\sum_{\mu,\nu} g_{\mu\nu}\, dx_\mu\, dx_\nu \qquad \mu, \nu = 1, 2, 3, 4$$

The tensor

$$G = g_{\mu\nu} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{bmatrix} \tag{6}$$

is called the metric tensor *corresponding to the line element. It is a metric of a pseudo-Euclidean 4-dimensional space-time world known as* Minkowski space*. It is immediately noticed that*

$$LG\widetilde{L} = G, \qquad (\sim \text{ is transpose}), \tag{7}$$

There is an alternative way to represent the Lorentz transformation as an imaginary rotation *in 4-dimensional Euclidean space. To see this we choose*

$$x_4 = ict, \quad \gamma = \cos\eta, \quad i\beta\gamma = \sin\eta \qquad n_2 = n_3 = 0 \tag{8}$$

The Lorentz transformation equations in the new variable, can be exhibited as

$$\begin{bmatrix} x_2' \\ x_3' \\ x_1' \\ x_4' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\eta & \sin\eta \\ 0 & 0 & -\sin\eta & \cos\eta \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \\ x_1 \\ x_4 \end{bmatrix} \tag{9}$$

The orthogonal matrix in (9, III) represents a rotation in the x_1x_4 plane by an imaginary angle $\eta = i\mathrm{th}^{-1}\left(\frac{v}{c}\right)$. The corresponding metric is $g_{\mu\nu} = \delta_{\mu\nu}$. The transformation in (9, III) is then written as

$$x_\mu' = q_{\mu\nu} x_\nu \tag{10}$$

where

$$q_{\mu\nu} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & \gamma & i\beta\gamma \\ & & -i\beta\gamma & \gamma \end{bmatrix} \tag{11}$$

(IV) The Lorentz Group; Wigner's rotation

In (9, III) we put $\eta = i\theta$. Then $\gamma = \mathrm{ch}\ \theta$, $\gamma\beta = \mathrm{sh}\ \theta$ and the transformed coordinates (x_1, x_4) undergo the 2×2 orthogonal matrix rotation

$$\begin{bmatrix} x_1' \\ x_4' \end{bmatrix} = \begin{bmatrix} \mathrm{ch}\ \theta & i\mathrm{sh}\ \theta \\ -i\mathrm{sh}\ \theta & \mathrm{ch}\ \theta \end{bmatrix} \begin{bmatrix} x_1 \\ x_4 \end{bmatrix} \quad \text{or} \quad [\boldsymbol{x}'] = A(v)[\boldsymbol{x}] \tag{1}$$

The set of all such matrices $A(v)$ forms a Lie group. $A(v)$ depends continuously on the velocity parameter v such that

$$A(v_1)A(v_2) = A(v_3), \qquad v_3 = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}} \tag{2}$$

(law of relativistic collinear velocity addition) and the matrices $A(v)$ have a determinant of $+1$ for $\gamma > 0$.

Since

$$A(v) = \text{ch}\ \theta \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \text{sh}\ \theta \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix} = \exp\left\{\theta \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}\right\}, \tag{3}$$

we say that the matrix $\begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$ *is the generator of this representation.*

Returning to the general case, when the relative velocity $\boldsymbol{v}$ *between frames is not parallel to any space axis, it can be shown that the generator is the* 4×4 *matrix*

$$\sigma = \begin{bmatrix} 0 & 0 & 0 & i\lambda \\ 0 & 0 & 0 & i\mu \\ 0 & 0 & 0 & i\nu \\ -i\lambda & -i\mu & -i\nu & 0 \end{bmatrix} \tag{14}$$

where (λ, μ, ν) *are the direction cosines of* $\boldsymbol{v}$. *The transformation itself is given by the* 4×4 *matrix*

$$L(\boldsymbol{v}) = e^{\theta\sigma} = I + \sigma \text{sh}\ \theta + \sigma^2(\text{ch}\ \theta - 1) \tag{5}$$

In this general case, however, the product of two Lorentz transformations matrices $L(\boldsymbol{v}_1)$ *and* $L(\boldsymbol{v}_2)$, *yield a third Lorentz transformation* $L(\boldsymbol{v}_3)$ *only if the two velocities* $\boldsymbol{v}_1$ *and* $\boldsymbol{v}_2$ *are parallel. If not, we find that*

$$L(\boldsymbol{v}_3) = RL(\boldsymbol{v}_2)L(\boldsymbol{v}_1), \tag{6}$$

where R *is a* 3×3 *real space rotation matrix, representing the so-called Wigner's rotation (***Wigner** *1939). The parameters of the rotation are*

$$R = R(\boldsymbol{e}, \psi); \quad \boldsymbol{e} = \frac{\boldsymbol{v}_2 \times \boldsymbol{v}_1}{|\boldsymbol{v}_2 \times \boldsymbol{v}_1|}; \quad \psi = 2\tan^{-1}\left\{\frac{\sin\varphi}{\tau + \cos\varphi}\right\}, \tag{7}$$

where $\boldsymbol{e}$ *is a unit vector in the direction of the axis of rotation,* φ *is the angle between the vectors* $\boldsymbol{v}_1$ *and* $\boldsymbol{v}_2$, $\tau = \sqrt{\frac{(\gamma_1+1)(\gamma_2+1)}{(\gamma_1-1)(\gamma_2-1)}}$ *and* ψ *is the Wigner rotation angle. Wigner's rotation is a purely kinematic relativistic phenomenon, with the following dynamical consequence: a rigidly accelerated body (for which the instantaneous acceleration is not parallel to the instantaneous velocity exhibits an instantaneous rigid rotation relative to an external inertial frame, even in the absence of torques as measured in an instantaneous inertial rest frame. A comoving observer will observe no centrifugal or Coriolis effects, yet an external observer would find that the body at rest in it, is rotating.*

*Because of the presence of the Wigner rotation, the $L(\boldsymbol{v})$ matrices themselves do not form a group. This rotation is also the origin of the Thomas precession (**Thomas** 1926). The latter is the phenomenon according to which a spinning mass like an electron in an atom, say, or the earth orbiting the sun, exhibits a precession of its axis of spin about the orbital axis as it follows its orbit (or orbital) in a central potential field, such that the ratio of the precessional period to the orbital period is* $2\frac{c^2}{v^2}$.

The Thomas precession is manifested as a factor 2 reduction in the torque related spin-orbit coupling effect in atomic physics. It is encountered in the fine structure of atomic levels, where it causes the magnetic field, as seen by the electron, to be half as effective as one would naively expect. This field itself results by Lorentz–transforming the nuclear electric *field to the electron's comoving frame; all possible Lorentz* and *3-rotation transformations do, however, form one – the Lorentz Lie group $SO(3,\ 1)$ – of which 3-rotations comprise the $SO(3)$ subgroup.*

*When this result became known, it surprised many, including **Pauli** and even **Einstein** himself. The gravitational (classical) version of the Thomas precession, called* geodetic precession *in GTR, is one of several effects being tested by the relativistic spinning–gyro experiment which was put into orbit in 2004.*

As stated previously, the matrix L has the property

$$LG\widetilde{L} = G, \qquad (\sim \text{ is transpose})$$

where G is the metric matrix 4-tensor of Minkowski space.

The real contravariant space time position 4-vector $x^\mu,\ 0 \le \mu \le 3\ ;\ x^0 = ct_1$ $\boldsymbol{x} = \boldsymbol{r} = (x^1, x^2, x^3)$ *can be represented by the hermitian* matrix

$$X = \begin{bmatrix} ct+z & x-iy \\ x+iy & ct-z \end{bmatrix}, \tag{8}$$

whose determinant $|X| = c^2t^2 - x^2 - y^2 - z^2$ is the Lorentz invariant pseudonorm. We note that

$$\begin{aligned} X &= ct\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + x\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + y\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} + z\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \\ &= ctI + x\sigma_x + y\sigma_y + z\sigma_z = x^\mu\sigma_\mu, \end{aligned} \tag{9}$$

where $\sigma_\mu = (I, \boldsymbol{\sigma})$, I being the 2×2 unit matrix and σ_j $(j = 1, 2, 3)$ are the Pauli spin matrices and the Einstein summation convention (for repeating indices) is used. Note that in this representation of 4-vectors, imaginary 4^{th} coordinates are replaced with real 0^{th} coordinates, and a covariant 4-vector is related to its contravariant counterpart via $a_\mu = \eta_{\mu\nu} a^\nu$, where

$$\eta = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

is the pseudo–Euclidean Minkowski metric tensor. It can easily be shown that if we represent the Lorentz transformation by the 2×2 matrix

$$L = \begin{bmatrix} \text{ch}\,\frac{\theta}{2} - n_3 \text{sh}\,\frac{\theta}{2} & -(n_1 - in_2)\text{sh}\,\frac{\theta}{2} \\ -(n_1 + in_2)\text{sh}\,\frac{\theta}{2} & \text{ch}\,\frac{\theta}{2} + n_3 \text{sh}\,\frac{\theta}{2} \end{bmatrix} \equiv \exp\left[-\frac{1}{2}\theta(\boldsymbol{n} \cdot \boldsymbol{\sigma})\right] \tag{10}$$

then

$$\begin{bmatrix} ct' + z' & x' - iy' \\ x' + iy' & ct' - z' \end{bmatrix} = L \begin{bmatrix} ct + z & x - iy \\ x + iy & ct - z \end{bmatrix} L^* \tag{11}$$

Note that L in this representation is not unitary, but its determinant is unity. All 4-vectors $\boldsymbol{A}$ will transform like X,

$$A' = LAL^*; \quad \textit{with } A = a^\mu \sigma_\mu,\ A' = a'^\mu \sigma_\mu. \tag{12}$$

(V) Relativistic mechanics

Consider a point moving with velocity $\boldsymbol{u}(t)$ relative to S. In $\bar{S}$, connected to S by the Lorentz transformation $(2,\ I)$, the transformed velocity is $\boldsymbol{u}'(t')$, where

$$\boldsymbol{u} = \frac{d\boldsymbol{r}}{dt}, \qquad \boldsymbol{u}' = \frac{d\boldsymbol{r}'}{dt'}$$

But since $d\boldsymbol{r} = \Phi \cdot d\boldsymbol{r}' + \gamma \boldsymbol{v}\, dt'$ *and* $dt = \gamma\,(dt' + \frac{1}{c^2}\boldsymbol{v} \cdot d\boldsymbol{r}')$

$$\boldsymbol{u} = \frac{\boldsymbol{v} + \frac{1}{\gamma}\Phi \cdot \boldsymbol{u}'}{1 + \frac{1}{c^2}(\boldsymbol{v} \cdot \boldsymbol{u}')} \tag{1}$$

We observe that $\boldsymbol{u}$ *in* $(1,\ V)$ *does not transform like a part of a* 4*-vector. Instead, it has much more complicated transformation properties. The reason for this complexity is that although* $\boldsymbol{r}$ *is a part of a* 4*-vector* $(\boldsymbol{r}, ict)$, dt *is not invariant as it is in the Newtonian theory, and consequently* $\frac{d\boldsymbol{r}}{dt}$ *is not a part of a* 4*-vector. In place of* $\frac{d\boldsymbol{r}}{dt}$ *we thus consider* $\frac{d\boldsymbol{r}}{d\tau}$, *where*

$$d\tau^2 = dt^2 - \frac{(d\boldsymbol{r})^2}{c^2} \tag{2}$$

is a Lorentz invariant. In fact, $d\tau = dt$ *when* $d\boldsymbol{r} \equiv 0$, *i.e. in an inertial frame where the particle is momentarily at rest; in other words, in a frame co-moving with the particle – one of its instantaneous rest frames.* $d\tau$ *is an infinitesimal increment of the particle's proper time. The entity* $\frac{d\boldsymbol{r}}{dt}$ *is the spatial part of a 4-vector and thus transforms simply. We have*

$$d\tau = dt\sqrt{1 - \frac{1}{c^2}(\frac{d\boldsymbol{r}}{dt})^2} = dt\sqrt{1 - \frac{\boldsymbol{u}^2}{c^2}} \tag{3}$$

Since $(\boldsymbol{r}, ict)$ *is a* 4*-vector, we are led to consider the* 4*-vector*

$$u_\mu = \left(\frac{d\boldsymbol{r}}{d\tau}, ic\frac{dt}{d\tau}\right) = (\gamma\boldsymbol{u}, i\gamma c), \qquad \sum_{\mu=1}^{4} u_\mu^2 = -c^2,$$

where $\gamma = \left(1 - \frac{\boldsymbol{u}^2}{c^2}\right)^{-1/2}$, *and* $\boldsymbol{u} = \frac{d\boldsymbol{r}}{dt}$ *is the Newtonian velocity.*

Next, we define a 4-scalar m_0 *to be the mass of the particle at rest (i.e. in its instantaneous rest frame* $\bar{S}$*). Then* p_μ *is also a* 4*-vector – known as the* 4*-momentum:* $p_\mu = (\gamma m_0 \boldsymbol{u},\quad i\gamma m_0 c)$*. Furthermore, the entity* $\frac{dp_\mu}{d\tau}$ *is also a* 4*-vector, known as the Minkowski force 4-vector or 4-force*

$$K_\nu = \frac{dp_\mu}{d\tau} = m_0\frac{du_\nu}{d\tau} = m_0\frac{d^2x_\nu(s)}{d\tau^2} = m_0\gamma\frac{d}{dt}[\gamma\boldsymbol{u},\quad i\gamma c] = [\gamma\boldsymbol{F}, K_4], \tag{4}$$

where $\boldsymbol{F}$ *is the* 3*-dimensional vector* $\frac{d}{dt}(m_0\gamma\boldsymbol{u})$ *equal to the ordinary time derivative of the momentum (because the instantaneous inertia of the particle in a general inertial frame is* $m(\boldsymbol{u}) = m_0\gamma(\boldsymbol{u}) \geq m_0$*). Differentiating the identity* $\sum u_\mu^2 = -c^2$ *w.r.t. particle proper time* τ *we obtain the identity* $\sum K_\mu u_\mu = 0$, *from which we derive*

$$K_4 = \frac{i}{c}\gamma(\boldsymbol{F}\cdot\boldsymbol{u}). \tag{5}$$

Eq. $(4,\ V)$ *is the covariant equation of motion for a massive particle. It relates the Minkowski force, which is determined by the environment (fields* *and other particles) to the rate of change of the 4-velocity w.r.t. the proper time τ of the particle. The spatial components of $(4,\ V)$ generalize Newton's 2^{nd} law of motion, whilst its 4^{th} (time) component is a statement of relativistic energy conservation.*

We next apply the above to a "particle" defined as a spatially–infinitesimal volume element of a charged medium subject to electromagnetic fields. The force $\boldsymbol{F}$ per unit volume (in any frame) is then the Lorentz force per unit volume $\boldsymbol{f} = \rho\,[\boldsymbol{E} + (\boldsymbol{u} \times \boldsymbol{B})]$ and

$$\frac{i}{c}\boldsymbol{E} = (F_{41}, F_{42}, F_{43}); \quad \boldsymbol{B} = (F_{23}, F_{31}, F_{12}); \quad \left(\frac{\rho\boldsymbol{u}}{c}, i\rho\right) = \frac{1}{c}(J_1, J_2, J_3, J_4)$$

and $\boldsymbol{f} = (f_1, f_2, f_3, f_4)$, we thus find the covariant form: (summation convention employed)

$$f_\nu = F_{\nu\mu}J_\mu = F_{\nu\mu}c^2\frac{\partial F_{\mu\lambda}}{\partial x_\lambda} = \boldsymbol{F} \cdot \mathrm{div}\ \boldsymbol{F}$$

The 4th component of f_ν in this equation is simply

$$f_4 = \frac{i}{c}(\boldsymbol{E} \cdot \boldsymbol{J}) = i\frac{\rho}{c}(\boldsymbol{E} \cdot \boldsymbol{u}) = \frac{i}{c}(\boldsymbol{f} \cdot \boldsymbol{u}).$$

Apart from the Minkowski–geometric conversion factor $\frac{i}{c}$, this has the meaning of the rate at which the field does work on the sources per unit volume. The spatial part of f_μ is the rate of change of mechanical momentum (i.e. 3-force) per unit volume. Altogether, the force density is the 4-vector density (i.e., its volume integral transforms as a 4-vector)

$$f_\mu = \rho\left[(\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}), \quad \frac{i}{c}\boldsymbol{E} \cdot \boldsymbol{u}\right]. \tag{6}$$

From $(4,\ V)$ and $(5,\ V)$ we have

$$\frac{d}{dt}\left[\frac{m_0c^2}{\sqrt{1 - \frac{\boldsymbol{u}^2}{c^2}}}\right] = \boldsymbol{F} \cdot \boldsymbol{u} \tag{7}$$

Since $\boldsymbol{F} \cdot \boldsymbol{u}$ is the total rate of work done by the force or the particle; the expression in square brackets on the left is the total motion energy (rest plus kinetic) of the particle. If we write

$$\frac{m_0c^2}{\sqrt{1 - \frac{\boldsymbol{u}^2}{c^2}}} = m_0c^2 + m_0c^2\left[\frac{1}{\sqrt{1 - \frac{\boldsymbol{u}^2}{c^2}}} - 1\right], \tag{8}$$

the part m_0c^2 *is the rest energy, or intrinsic energy, whereas the remaining part is the relativistic kinetic energy which is approximated by* $\frac{1}{2}mu^2$ *for* $\frac{|\boldsymbol{u}|}{c} \ll 1$.

It is easily seen that the spatial part of $(4,\ V)$, *namely*

$$\frac{d}{dt}\left[\frac{m_0\boldsymbol{u}}{\sqrt{1-\frac{\boldsymbol{u}^2}{c^2}}}\right] = \boldsymbol{F}, \tag{9}$$

approximates Newton's second law in an inertial coordinate system when $\frac{|\boldsymbol{u}|}{c} \ll 1$, *in which regime* $m(\boldsymbol{u}) \approx m_0$ *is approximately motion-independent.*

If we define

$$E = \frac{m_0c^2}{\sqrt{1-\frac{\boldsymbol{u}^2}{c^2}}},$$

Then, since the spatial part of p_μ *is* $\boldsymbol{p} = \gamma m_0\boldsymbol{u}$, *it follows that*

$$E^2 = c^2\boldsymbol{p}^2 + m_0^2c^4, \tag{10}$$

a generalization of the classical nonrelativistic relation $E = \frac{\boldsymbol{P}^2}{2m}$.

Note that

$$p_\mu = (m\boldsymbol{u}, \quad i\frac{E}{c}) = (\boldsymbol{p}, i\frac{E}{c}), \quad m = m(\boldsymbol{u}) = \frac{m_0}{\sqrt{1-\frac{\boldsymbol{u}^2}{c^2}}} \quad (motion - mass) \tag{11}$$

where

$$p_\mu^2 = -m_0^2c^2.$$

(VI) The 'Twin Paradox' — a one-way time machine

Einstein's STR brought with it some remarkable implications. There were consequences that seemed opposed to our intuition and common sense, in a way that classical theories were not; e.g., the increase of inertia with speed and the so-called 'twin paradox'. It was aspects such as these which conferred on it a glamor and popular interest probably never equaled in the whole history of physics.

Of all the supposed paradoxes engendered by relativity theory, the 'twin paradox' (or 'clock paradox') is most famous, and has been the most controversial (although it is extremely well supported by experiments). It asserts that if one clock remains at rest in an inertial frame, and another, initially synchronized with it, is taken off on any sort of path and finally brought back to its starting point, the second clock will have lost time as compared with the first. It is not that the traveling clock is somehow damaged by the excursion; indeed a co-traveling astronaut will end up by becoming younger than his twin brother who stayed with the stationary clock.

This result, which was stated by Einstein in his very first relativity paper (1905), became the subject of raging controversy in the physical literature during the years 1939–1959. The matter was finally settled by experimental tests in nuclear physics during 1960–1963, and later tests – involving the hauling of atomic clocks aboard aircraft – established the effect for macroscopic objects[184].

*The experiments thus far accessible to us are not those of spaceships traveling at near-light speeds; rather, they involve high-precision nuclear and atomic processes, and atomic clocks moving at aircraft and satellite speeds. Thus, in the early 60's, several measurements utilizing the Mössbauer effect — recoilless, extremely narrow-linewidth gamma ray emission from radioisotopes embedded in crystal lattices — were carried out to verify the special-relativistic time dilatations. In one such experiment (**Hay** et al. 1960), a rotating cylinder carried a gamma-ray source* (^{57}Co) *wrapped in a band around it. A larger, concentric band, containing the isotope* ^{57}Fe, *was supported via a pair of perpendicular discs.*

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[184] In both types of experiments, a *gravitation* part of the twin–paradox effect, predicted by Einstein's GTR, must sometimes be added to the *motional* (STR) effect, to obtain agreement with experiment. Both effects must be (and are) accounted for in GPS technology, since it depends on satellite–borne atomic clocks.

When the apparatus was at rest, the ^{57}Fe *nuclei could optimally absorb photons emitted by the inner* ^{57}Co *band. But when the apparatus was spun about the cylinder axis (at 500 revolutions per second) the different linear velocities of emitter and absorber resulted in a degraded absorption efficiency — by an amount found to agree with the prediction of STR. In effect, the inner-radius emitter atoms constitute the stay-at-home twin, while the absorber atoms represent the traveling twin. The central frequencies of the emission and absorption bands represent the rate of "ticking" of the respective twins' gamma-ray "clocks".*

In this language, the outer clock ticks slower *than the inner one — the "traveling twin" ages less, as predicted by STR. Actually,* both *clocks in the experiment execute (literally) round trips; but the emitter is accelerated to lower speeds in the (approximately inertial) laboratory frame. For that reason, its "clock ticks" will be more closely spaced in time — as reckoned by a non-rotating observer – than the "ticks" of the absorber "clock".*

The Mössbauer effect also figured in the celebrated Pound-Rebka experiment (1959)*, which verified that clocks tick faster the higher their altitudes in the Earth's gravitational field — as predicted by GTR. Later, in the 70's,* both *types of time dilatation were also verified using pairs of atomic (MASER) clocks – one of which went on a round-trip aboard a plane.*

Consider a flat pseudo-Euclidean 4-space (a.k.a. Minkowski space); a point is labeled $(t, \boldsymbol{x})$*, where* $\boldsymbol{x}$ *belongs to ordinary Euclidean 3-space. (Such a point is called an "event".) The difference between two events,* $(t_2, \boldsymbol{x}_2) - (t_1, \boldsymbol{x}_1) = (t_2 - t_1, \boldsymbol{x}_2 - \boldsymbol{x}_1)$*, is a 4-vector, and is labeled "timelike", "lightlike" or "spacelike" according as the value of* $c^2(t_2 - t_1)^2 - (\boldsymbol{x}_2 - \boldsymbol{x}_1)^2$ *is positive, zero or negative. Each point* $(t, \boldsymbol{x})$ *is the apex of a local* future light-cone*, consisting of all future points* $(t', \boldsymbol{x}')$ *so that the interval* $(t', \boldsymbol{x}') - (t, \boldsymbol{x})$ *is lightlike; the* interior *of this cone consists of all future points for which the interval is* timelike*.*

For any vector $a = (\alpha, \boldsymbol{\beta})$*, we define its "square" or "pseudo-norm" to be:* $a^2 = \alpha^2 - \boldsymbol{\beta}^2$ *where* $\boldsymbol{\beta}^2$ *is the usual norm in 3-dimensional Euclidean space. Accordingly, a* scalar product *in Minkowski space is defined: if*

$$a^{(i)} = (a_0^{(i)}, \boldsymbol{a}^{(i)}), \quad i = 1, 2,$$

then

$$a^{(1)} \cdot a^{(2)} \equiv \frac{1}{2}\{(a^{(1)} + a^{(2)})^2 - (a^{(1)})^2 - (a^{(2)})^2\}.$$

The above-defined square is clearly not a norm*, since* $a^2 = 0$ *for all lightlike vectors; nor is it even a* seminorm*, as* $a^2 < 0$ *for all spacelike vectors. That is why we refer to it as a "pseudo-norm".*

We next investigate how the triangle inequality *of Euclidean norms fares for this pseudo-norm of Minkowski space, & how this relates to the STR Twin Paradox.*

Consider three points (events) A, B, C in Minkowski space; let C and B be inside the future light cone of A. We also assume C to be in the future light cone of B (Fig. 5.2). We denote the timelike vectors AB, BC, AC by c, a, b, respectively; in some inertial frame, let

$$\begin{aligned} c &= (c_0, \boldsymbol{c}) \\ b &= (b_0, \boldsymbol{b}) \\ a &= (a_0, \boldsymbol{a}), \end{aligned}$$

where the 0-components are the (real) time coordinates of the three events in this frame, and we select units in which the speed of light equals 1.

Since $b = a + c$ *we have*

$$b^2 = (a_0 + c_0)^2 - (\boldsymbol{a} + \boldsymbol{c})^2 = a^2 + c^2 + 2a \cdot c = a^2 + c^2 + 2(a_0 c_0 - \boldsymbol{a} \cdot \boldsymbol{c}). \quad (1)$$

Now, the "length" of these three intervals by their pseudo-norms, are:

$$\tau_1 = \sqrt{c^2} = \sqrt{(c_0)^2 - \boldsymbol{c}^2}; \quad \tau_2 = \sqrt{a^2}; \quad \tau_3 = \sqrt{b^2}.$$

Physically, these are the proper times of the respective intervals; e.g. τ_1 *is the time interval between events A and B as reckoned aboard a spaceship of uniform velocity whose worldline intersects events A and B.*

We rewrite the above equation $(1,\ VI)$ *as*

$$(\tau_3)^2 = (\tau_2)^2 + (\tau_1)^2 + 2(\sqrt{(\tau_2)^2 + \boldsymbol{a}^2}\sqrt{(\tau_1)^2 + \boldsymbol{c}^2} - \boldsymbol{a} \cdot \boldsymbol{c}) \quad (2)$$

But on the strength of the identity

$$(\alpha^2 + \beta^2)(\gamma^2 + \delta^2) \equiv (\alpha\gamma + \beta\delta)^2 + (\alpha\delta - \beta\gamma)^2,$$

we have:

$$\begin{aligned} [(\tau_2)^2 + \boldsymbol{a}^2][(\tau_1)^2 + \boldsymbol{c}^2] &\equiv [|\boldsymbol{a}||\boldsymbol{c}| + \tau_1\tau_2]^2 + [\tau_2|\boldsymbol{c}| - \tau_1|\boldsymbol{a}|]^2 \\ &\geq (|\boldsymbol{a}||\boldsymbol{c}| + \tau_1\tau_2)^2 \geq (\boldsymbol{a} \cdot \boldsymbol{c} + \tau_1\tau_2)^2, \end{aligned}$$

so

$$\sqrt{(\tau_2)^2 + \boldsymbol{a}^2}\sqrt{(\tau_1)^2 + \boldsymbol{c}^2} \geq \boldsymbol{a} \cdot \boldsymbol{c} + \tau_1\tau_2,$$

hence (2) implies

$$(\tau_3)^2 \geq (\tau_1 + \tau_2)^2 \Rightarrow \tau_3 \geq \tau_1 + \tau_2.$$

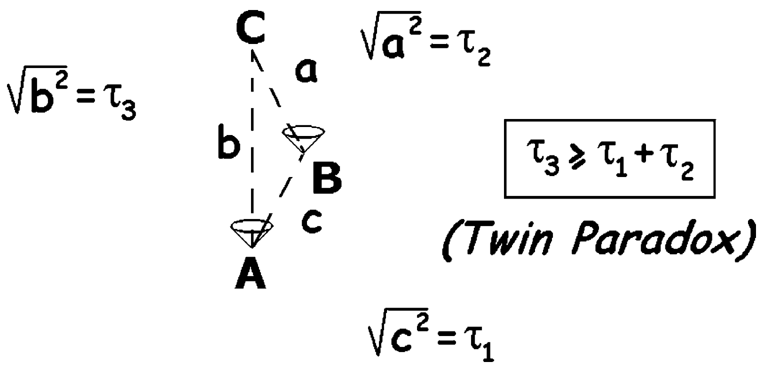

Fig. 5.2: A triangle of timelike vectors (=intervals) in Minkowski space. Local future light-cones are shown; the vertical direction is time (in some reference frame), while the horizontal directions are spatial in the same frame

By the above interpretation of τ_i as proper times, it follows that the journey $A \to B \to C$ (which is accelerated in the immediate vicinity of B) takes less proper time than the unaccelerated worldline $A \to C$. Hence the Twin Paradox!

In this way a variant of the Euclidean triangle inequality, which holds for Minkowski space is used to furnish a 4-geometrical proof of the twin paradox of STR (Note that in a Euclidean space one would instead have $\tau_3 \leq \tau_1 + \tau_2$).

(VII) Relativistic wave phenomena

Acoustic Doppler shifts are derived on the assumption that the radiation propagation speed is constant relative to the medium, whereas the wave's frequency is affected by the motion of both source and observer. Electromagnetic theory, however, requires a modification of these assumptions for two reasons:

- *Non-existence of 'medium' for vacuum propagation;*
- *The replacement of the Galilean transformation by the Lorentz transformation*

Consider a source of electromagnetic waves moving with uniform velocity $\boldsymbol{v}$ *relative to an observer in* S. *If the source is at rest in* S', *then an observer at* $0'$ *(the origin of* S'*) describes a monochromatic plane-wave emitted from the source as being proportional to* $e^{i(\boldsymbol{k}'\cdot\boldsymbol{r}'-\omega' t')}$. *Similarly, an observer at* 0 *in* S *describes the same wave as being proportional to* $e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}$.

Due to the form-invariance of the basic wave equation

$$\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = \nabla'^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t'^2},$$

the phase of the plane-wave solution of this equation is itself a 4-scalar

$$\boldsymbol{k}'\boldsymbol{r}' - \omega' t' = \boldsymbol{k}\boldsymbol{r} - \omega t. \tag{1}$$

Working in the imaginary–4^{th}*–component representation of Minkowski space, this scalar can be written as a dot product of the position 4-vector* $(\boldsymbol{r},\ ict)$ *and the 4-vector*

$$k_\mu = (k_1, k_2, k_3, \frac{i\omega}{c}) = (\boldsymbol{k},\quad i\frac{\omega}{c}), \tag{2}$$

namely

$$(\boldsymbol{r},\quad ict)\cdot(\boldsymbol{k},\quad \frac{i\omega}{c}) = k_1x_1 + k_2x_2 + k_3x_3 + k_4x_4 = \boldsymbol{k}\cdot\boldsymbol{r} - \omega t \tag{3}$$

The entity k_μ *is therefore a covariant 4-vector of zero magnitude*

$$k_\mu k_\mu = k_1^2 + k_2^2 + k_3^2 + k_4^2 = |\boldsymbol{k}|^2 - \frac{\omega^2}{c^2} = 0 \tag{4}$$

It transforms according to the equations

$$\boldsymbol{k}' = \boldsymbol{k} + \left[\frac{\gamma-1}{v^2}(\boldsymbol{v}\cdot\boldsymbol{k}) - \gamma\frac{\omega}{c^2}\right]\boldsymbol{v}; \qquad \omega' = \gamma(\omega - \boldsymbol{v}\cdot\boldsymbol{k}) \tag{5}$$

Putting $\boldsymbol{v}\cdot\boldsymbol{k} = \omega\frac{v}{c}\cos\theta$, *we obtain*

$$\omega' = \omega\frac{1-\frac{v}{c}\cos\theta}{\sqrt{1-\frac{v^2}{c^2}}} \tag{6}$$

and by symmetry

$$\omega = \omega'\frac{1+\frac{v}{c}\cos\theta'}{\sqrt{1-\frac{v^2}{c^2}}} \tag{7}$$

where θ' *is the angle between* $\boldsymbol{v}$ *and* $\boldsymbol{k}'$.

Eliminating both ω *and* ω' *between these equations we derive the relation:*

$$\cos\theta' = \frac{\cos\theta - \frac{v}{c}}{1 - \frac{v}{c}\cos\theta}; \qquad \cos\theta = \frac{\cos\theta' + \frac{v}{c}}{1 + \frac{v}{c}\cos\theta'} \tag{8}$$

Finally, from (5, VII), $\boldsymbol{k}\times\boldsymbol{v} = \boldsymbol{k}'\times\boldsymbol{v}$, *which implies*

$$\left.\begin{aligned} \sin\theta' = \frac{k}{k'}\sin\theta = \frac{\omega}{\omega'}\sin\theta = \sin\theta\frac{\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v}{c}\cos\theta} \\ \sin\theta = \sin\theta'\frac{\sqrt{1-\frac{v^2}{c^2}}}{1+\frac{v}{c}\cos\theta'} \end{aligned}\right\} \tag{9}$$

Combining (8, VII) *and* (9, VII), *we have*

$$\tan\theta = \sin\theta'\frac{\sqrt{1-\frac{v^2}{c^2}}}{\cos\theta' + \frac{v}{c}}; \qquad \tan\frac{\theta'}{2} = \tan\frac{\theta}{2}\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} \tag{10}$$

Equations (6, VII)–(10, VII) *give the apparent frequency and direction of an electromagnetic radiation source which is moving uniformly relative to the observer* S. *For* $\frac{v}{c} \ll 1$, *the relativistic formulae reduce to the well-known classical ones:*

$$\omega \approx \omega'(1 + \frac{v\cos\theta'}{c}); \qquad \omega' \approx \omega(1 - \frac{v\cos\theta}{c}); \tag{11}$$

For light in vacuo, as contrasted with sound, it has been proved impossible to identify a medium of transmission relative to which the source and observer are moving. Hence, the statements 'source receding from observer' and 'observer receding from source' are physically identical situations and must exhibit the same Doppler shift. The familiar observed effects of 'red-shift', 'blue-shift', and the aberration of star light, are all direct consequences of (6, VII)–(10, VII). *Note however that at* $\theta = \frac{\pi}{2}$, $\omega = \omega'\sqrt{1-\frac{v^2}{c^2}} = \omega'(1 - \frac{1}{2}\frac{v^2}{c^2} + \dots)$ *predicts the purely relativistic transverse Doppler shift.*

(VIII) Lagrangian and Hamiltonian of Electrodynamics

*Consider first the noncovariant Lagrangian formulation of electrodynamics (**Schwarzschild** 1903). In the presence of charged bodies or a charged medium, and in the Lorentz gauge, Maxwell's equations mean that the potentials comply with the equations*

$$\left.\begin{aligned} \nabla^2\boldsymbol{A} - \frac{1}{c^2}\frac{\partial^2\boldsymbol{A}}{\partial t^2} = -\frac{1}{c^2}\boldsymbol{J}, \qquad \nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = -\rho \\ \text{div }\boldsymbol{A} + \frac{1}{c^2}\frac{\partial\phi}{\partial t} = 0 \end{aligned}\right\} \tag{1}$$

For a charge distribution with local density ρ *and velocity* $\boldsymbol{v}$, $\boldsymbol{J} = \rho\boldsymbol{v}$. *The Lagrangian density of sourceless electromagnetic fields is* $\mathcal{L} = \frac{1}{2}(\boldsymbol{E}^2 - c^2\boldsymbol{B}^2)$ *where* $\boldsymbol{E} = -\nabla\phi - \frac{\partial\boldsymbol{A}}{\partial t}$, $\boldsymbol{B} = \text{curl }\boldsymbol{A}$. *In the presence of sources, we must add to it the term* $(\boldsymbol{A}\cdot\boldsymbol{J} - \rho\phi)$, *resulting in*

$$\mathcal{L} = \frac{1}{2}\left[\left(-\nabla\phi - \frac{\partial\boldsymbol{A}}{\partial t}\right)^2 - c^2(\text{curl }\boldsymbol{A})^2\right] + (\boldsymbol{A}\cdot\boldsymbol{J} - \rho\phi) \tag{2}$$

Then the Euler-Lagrange equations – resulting from the condition that the action $\int d^4x\mathcal{L}$ *be extremal at the actual field configuration in Minkowski space – are:*

$$\frac{\partial\mathcal{L}}{\partial\phi} - \text{div }\left[\frac{\partial\mathcal{L}}{\partial(\nabla\phi)}\right] - \frac{\partial}{\partial t}\left(\frac{\partial\mathcal{L}}{\partial\dot{\phi}}\right) = 0 \tag{3}$$

$$\frac{\partial\mathcal{L}}{\partial\boldsymbol{A}} - \text{div }\left[\frac{\partial\mathcal{L}}{\partial(\nabla\boldsymbol{A})}\right] - \frac{\partial}{\partial t}\left(\frac{\partial\mathcal{L}}{\partial\dot{\boldsymbol{A}}}\right) = 0 \tag{4}$$

To establish that Maxwell's equations (1, $VIII$) *indeed follow from* (2, $VIII$)–(4, $VIII$) *we verify that*

$$\frac{\partial\mathcal{L}}{\partial\phi} = -\rho; \qquad \frac{\partial\mathcal{L}}{\partial\dot{\phi}} = 0; \qquad \frac{\partial\mathcal{L}}{\partial(\nabla\phi)} = \left(\nabla\phi + \frac{\partial\boldsymbol{A}}{\partial t}\right);$$

$$\text{div }\frac{\partial\mathcal{L}}{\partial(\nabla\phi)} = \left(\nabla^2\phi + \frac{\partial\text{div }\boldsymbol{A}}{\partial t}\right) = \left(\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}\right);$$

$$\frac{\partial \mathcal{L}}{\partial \boldsymbol{A}} = \boldsymbol{J}; \qquad \frac{\partial \mathcal{L}}{\partial \dot{\boldsymbol{A}}} = \left(\nabla\phi + \frac{\partial \boldsymbol{A}}{\partial t}\right); \qquad \frac{\partial}{\partial t}\frac{\partial \mathcal{L}}{\partial \dot{\boldsymbol{A}}} = \left[\frac{\partial^2 \boldsymbol{A}}{\partial t^2} - c^2 \text{ grad div } \boldsymbol{A}\right] \tag{5}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial(\nabla \boldsymbol{A})} &= -\frac{c^2}{2}\frac{\partial}{\partial \nabla \boldsymbol{A}}(\text{curl } \boldsymbol{A})^2 \\ &= \frac{c^2}{4}\frac{\partial}{\partial \nabla \boldsymbol{A}}[(\nabla \boldsymbol{A} - \boldsymbol{A}\nabla) : (\nabla \boldsymbol{A} - \boldsymbol{A}\nabla)] = (I \times \text{curl } \boldsymbol{A});\end{aligned}$$

$$\text{div } \frac{\partial \mathcal{L}}{\partial(\nabla \boldsymbol{A})} = c^2 \text{curl curl } \boldsymbol{A}$$

*In order to build the Lorentz-invariant Lagrangian density (**Born** 1909), we start from the expression (equation(2, $VIII$))*

$$\boxed{\mathcal{L} = \frac{1}{2}(\boldsymbol{E}^2 - c^2\boldsymbol{B}^2) + (\boldsymbol{A}\cdot\boldsymbol{J} - \rho\phi)} \tag{6}$$

where $\{\boldsymbol{E}, \boldsymbol{B}, \boldsymbol{A}, \boldsymbol{J}\}$ are ordinary 3-vectors, and express it in terms of 4-vectors and 4-tensors. We have already shown that (summation convention)

$$(\boldsymbol{A}, \frac{i}{c}\phi)\cdot(\frac{1}{c}\boldsymbol{J}, i\rho) = \frac{1}{c}(\boldsymbol{A}\cdot\boldsymbol{J} - \rho\phi) = \frac{1}{c}J_\alpha A_\alpha \qquad \alpha = 1,2,3,4 \tag{7}$$

We also know that

$$\boxed{F_{\mu\nu} = \frac{\partial A_\mu}{\partial x_\nu} - \frac{\partial A_\nu}{\partial x_\mu}} \tag{8}$$

It then follows that

$$-\frac{c^2}{2}(F_{\mu\nu}F_{\mu\nu}) = \boldsymbol{E}^2 - c^2\boldsymbol{B}^2. \tag{9}$$

Hence manifestly covariant (relativistic) form of the Lagrangian density is:

$$\boxed{\mathcal{L} = -\frac{c^2}{4}(F_{\alpha\beta}F_{\alpha\beta}) + J_\alpha A_\alpha} \tag{10}$$

From the principle of least (or more precisely extremal) action one derives the covariant form of the Euler-Lagrange equation

$$\frac{\partial \mathcal{L}}{\partial A_\alpha} - \frac{\partial}{\partial x_\beta}\left(\frac{\partial \mathcal{L}}{\partial(\partial A_\alpha/\partial x_\beta)}\right) = 0 \tag{11}$$

where

$$\frac{\partial \mathcal{L}}{\partial A_\alpha} = J_\alpha, \qquad \frac{\partial \mathcal{L}}{\partial(\partial A_\alpha/\partial x_\beta)} = c^2 F_{\alpha\beta}, \qquad \frac{\partial}{\partial x_\beta} F_{\alpha\beta} = \frac{1}{c^2}\, J_\alpha \tag{12}$$

We must still add to the Lagrangian density the matter kinetic energy. Thus, in the case of a charged point-particle of mass m, charge e and trajectory $\boldsymbol{r}(t)$ and velocity $\boldsymbol{v}(t) = \frac{d\boldsymbol{r}}{dt}$, *the particle–dependent term in the Lagrangian* $L = \int d^3\boldsymbol{x}\, \mathcal{L}$ *is:*

$$L_p\{\boldsymbol{r}(t), \boldsymbol{v}(t); t\} = \frac{1}{2} m\boldsymbol{v}^2(t) + \frac{1}{c} e\boldsymbol{v}(t)\cdot \boldsymbol{A}\{\boldsymbol{r}(t), t\} - e\phi\{\boldsymbol{r}(t), t\} \tag{13}$$

(the particle is assumed to move much slower then light, $|\boldsymbol{v}| \ll c$, so the Newtonian kinetic energy can be used).

In this representation, the principle of least action renders the particle equation of motion

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \boldsymbol{v}}\right) - \frac{\partial L}{\partial \boldsymbol{r}} = 0 \tag{14}$$

Carrying out the differentiation with

$$\frac{\partial L}{\partial \boldsymbol{v}} = m\boldsymbol{v} + \boldsymbol{A}, \qquad m\boldsymbol{v} = \boldsymbol{p} = \text{ mechanical momentum}$$

$$\frac{\partial L}{\partial \boldsymbol{r}} = e \text{ grad } (\boldsymbol{A}\cdot\boldsymbol{v}) - e\nabla\phi = e\ (\boldsymbol{v}\cdot\nabla\boldsymbol{A}) + e\ \boldsymbol{v}\times \text{curl } \boldsymbol{A} - e\nabla\phi,$$

the particle equation of motion becomes

$$\frac{d}{dt}(\boldsymbol{p} + e\boldsymbol{A}) = e\ (\boldsymbol{v}\cdot\nabla\boldsymbol{A}) + e\boldsymbol{v}\times \text{curl } \boldsymbol{A} - e\nabla\phi, \tag{15}$$

But the total time-derivative $\frac{d\boldsymbol{A}}{dt}$ consists of two parts: the change $\frac{\partial \boldsymbol{A}}{\partial t}$ of the vector potential with time at a fixed point in space, and the change due to motion of the charged particle:

$$\frac{d\boldsymbol{A}}{dt} = \frac{\partial \boldsymbol{A}}{\partial t} + \boldsymbol{v}\cdot\nabla A \tag{16}$$

Therefore

$$m\frac{d^2\boldsymbol{r}}{dt^2} = m\frac{d\boldsymbol{v}}{dt} = \frac{d\boldsymbol{p}}{dt} \quad = -e\left[\frac{\partial \boldsymbol{A}}{\partial t} + \nabla\phi\right] + \boldsymbol{v}\times \text{curl } \boldsymbol{A} \\ = e\boldsymbol{E} + e\left(\boldsymbol{v}\times\boldsymbol{B}\right) = \text{ the Lorentz force} \tag{17}$$

($\boldsymbol{A}$, ϕ, $\boldsymbol{E}$ and $\boldsymbol{B}$ are evaluated at the particle's trajectory).

The foregoing results can be made Lorentz invariant upon the introduction of the relativistic particle Lagrangian

$$L_p = -m_0c^2\sqrt{1-\frac{v^2}{c^2}} + e\,\boldsymbol{A}\cdot\boldsymbol{v} - e\phi \tag{18}$$

The corresponding particle Hamiltonian is

$$\mathcal{H}_p = \boldsymbol{v}\cdot\frac{\partial L}{\partial \boldsymbol{v}} - L = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} + e\phi \tag{19}$$

with the 3-vector *mechanical* momentum

$$\boldsymbol{p} = \frac{m_0\boldsymbol{v}}{\sqrt{1-\frac{v^2}{c^2}}} \quad , \tag{20}$$

and the corresponding canonical 3-vector *momentum*

$$\boldsymbol{\pi} = \frac{\partial L}{\partial \boldsymbol{v}} = \boldsymbol{p} + e\,\boldsymbol{A} \tag{21}$$

Note that the potential $\boldsymbol{A}$ does not appear explicitly in $\mathcal{H}$ because the magnetic field is perpendicular to the Lorentz force and thus does not exert work upon the charged particle. The first term on the r.h.s. of (18, $VIII$) *arises from the Lorentz-invariant of the action integral of a particle*

$$-m_0c^2\int_{t_1}^{t_2} d\tau = -\int_{t_1}^{t_2} m_0c^2\sqrt{1-\frac{v^2}{c^2}}\,dt$$

where τ is the proper time (time elapsed in the particle's rest-frame).

We express the Hamiltonian in terms of the canonical momentum defined above:

$$(\mathcal{H}_p - e\phi)^2 = c^2(\boldsymbol{\pi} - e\boldsymbol{A})^2 + m_0^2c^4. \tag{22}$$

The canonical (Hamilton) equations are then equivalent to the relativistic equation of motion of the charged particle acted upon by the Lorentz force law.

(IX) The equations of electrodynamics in the presence of a gravitational field

The Lorentz-covariant electromagnetic field equations can easily be generalized so that they are applicable in an arbitrary 4-dimensional curved pseudo-Riemannian (i.e. locally Minkowskian) manifold, i.e., in the presence of an external gravitational field.

As seen above the Lorentz–covariant equations of the electromagnetic field in STR (no gravity) are

$$\text{div } F = -\frac{1}{c^2}\, J, \qquad \text{div } J = 0, \qquad J_\mu = (\boldsymbol{J}, ic\rho)$$

$$F = F_{\mu\nu} = \frac{\partial A_\nu}{\partial x_\mu} - \frac{\partial A_\mu}{\partial x_\nu} = (\boldsymbol{B}, -\frac{i\boldsymbol{E}}{c})\ ,$$

where, as usual, Greek letters are used for 4-space indices. Switching from flat Minkowski space to locally Minkowski, 4-dimensional manifold of metric $g_{\mu\nu}$ and curvilinear coordinates x^μ –also switching from imaginary–4^{th}–coordinate notation to the more modern real–0^{th}–coordinate representation for the local Minkowski space – these equations take on the form:

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\nu}(\sqrt{-g}F^{\mu\nu}) = \frac{1}{c^2}\, J^\mu \qquad \text{(Maxwell's source equations)}$$

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}(\sqrt{-g}J^\mu) = 0 \qquad \text{(Continuity equation)}$$

where $g = -det|g_{\mu\nu}|$ and

$$\mathbf{g}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

In a locally free-falling coordinate system (frame).

The equations of motion of a charged particle in the presence of both gravitational and electromagnetic fields is then a generalization of $m_0 \frac{du^\mu}{d\tau} = e\, u_\nu\, F^{\nu\mu}$ ($u^\mu =$ components of the 4-velocity vector), namely

$$\frac{e}{c}\, u_\nu F^{\nu\mu} = m_0 c\left(\frac{du^\mu}{d\tau} + \Gamma^\mu_{\nu\lambda} u^\nu u^\lambda\right),$$

where Γ is the manifold's affine connection (expressible in terms of $g_{\alpha\beta}$ and their derivatives) and

$$g^{\mu\lambda} g_{\lambda\alpha} = \delta^\mu_\alpha (Kronecker\ delta), \qquad F^{\alpha\beta} = g^{\alpha\mu} F_{\mu\nu} g^{\nu\beta}$$

(X) Stress-energy-momentum tensor

***Minkowski** (1908) recast the electromagnetic field equations via a continuum–mechanics covariant representation by means of a single entity — the symmetric zero-traced 2-indexed 4-dimensional, covariant electromagnetic stress-energy-momentum tensor in free space*

$$S_{\mu\nu} = \left[\begin{array}{ccc|c} -T_{11} & -T_{12} & -T_{13} & -icQ_1 \\ -T_{21} & -T_{22} & -T_{23} & -icQ_2 \\ -T_{31} & -T_{32} & -T_{33} & -icQ_3 \\ \hline -icQ_1 & -icQ_2 & -icQ_3 & W \end{array}\right]; \quad S_{\mu\nu} = S_{\nu\mu}; \quad S_{\mu\mu} = 0 \quad (1)$$

with

$$\begin{aligned} T_{ij} &= \boldsymbol{EE} + \boldsymbol{c}^2\, B\boldsymbol{B} - \frac{1}{2} I(E^2 + c^2\, B^2), \qquad i,j = 1,2,3 \\ \boldsymbol{Q} &= [\boldsymbol{E} \times \boldsymbol{B}] \\ W &= \frac{1}{2}(E^2 + c^2\, B^2) \end{aligned} \quad (2)$$

Where $\boldsymbol{Q}$ is the Poynting vector, W is the electromagnetic field energy density and τ_{ij} is the electromagnetic field stress 3-tensor, and where the imaginary–4^{th}–component representation of Minkowski 4-space is used. In terms of the skew – symmetric electromagnetic field 4-tensor, this symmetric stress–energy–momentum tensor can be elegantly represented as follows:

$$S_{\mu\nu} = c^2(F_{\mu\alpha}F_{\alpha\nu} + \frac{1}{4}F_{\alpha\beta}F_{\alpha\beta}\delta_{\mu\nu}) = c^2(F{\cdot}F + \frac{1}{4}(F : F)I)$$

where $\delta_{\mu\nu}$ is the Kronecker delta and $I = I_4$ is the 4×4 unit matrix. Taking the 4-divergence and using the covariant form of Maxwell's equations, we readily obtain

$$\frac{\partial S_{\mu\nu}}{\partial x_\mu} = -J_\alpha F_{\alpha\nu} = f_\nu, \quad (3)$$

where J_μ is the 4-current density and f_μ is the Lorentz 4-force density; that is $J_\mu = (\boldsymbol{J}, ics)$ and $f_\mu = (\boldsymbol{f},\ f_4 = \frac{i}{c}P)$ with $\boldsymbol{f} = \rho\boldsymbol{E} + \boldsymbol{J} \times \boldsymbol{B}$ the Lorentz 3-force per unit volume exerted by the fields on the sources, and $P = \boldsymbol{J}{\cdot}\boldsymbol{E}$ the power (work per unit time) exerted by the fields upon a unit volume of sources.

In terms of the Newtonian (3D space, 1D time) block–decomposition of the 4-matrix S (given by Eqs. (1, X) *and* (2, X) *above)* (1, X) *becomes:*

$$\begin{cases} \boldsymbol{\nabla}(c^2\boldsymbol{Q}) + \frac{\partial W}{\partial t} = -P & \nu = 4 : \text{ local energy conservation} \\ \boldsymbol{\nabla}\cdot\boldsymbol{T} + \frac{\partial \boldsymbol{Q}}{\partial t} = -\boldsymbol{f} & \nu = 1,2,3 : \text{ local momentum conservation} \end{cases} \tag{4}$$

From Eqs. (4, X) we see that the Poynting vector has a dual interpretation: it is both the local 3-momentum density and the local energy – current density of the electromagnetic fields in empty space (i.e. without a medium).

(XI) Quantum correction to the Maxwell equations

An electromagnetic field (classical or quantum) may produce charged electron-positron pairs (as well as heavier pairs such as proton-antiproton) as a consequence of quantum effects and these charged particles – whether real or virtual *– have a real effect upon the fields themselves (back–reaction). This means that the dynamics of a classical field contain nonlinear quantum corrections to the Maxwell equations, even in empty space.*

A way to think of these corrections is to realize that the vacuum is not empty, but rather it is a complicated nonlinear *medium — especially when probed with high fields and/or at high frequencies. It is, however, an* isotropic, homogeneous *and* Lorentz invariant *medium — not an "ether" in the 19th century sense.*

One may define an effective Lagrangian density $\mathcal{L}_{\text{eff}} = \mathcal{L}_0 + \delta\mathcal{L}$, *where* $\mathcal{L}_0$ *is the classical term while* $\delta\mathcal{L}$ *includes the quantum corrections and can be expanded perturbatively as a joint Taylor expansion in* $\alpha = \frac{e^2}{4\pi\hbar c}$ *(fine-structure constant),* eE *and* eB.

W. Heisenberg *and* ***H. Euler*** *have derived (1936) an analytical approximation to the nonlinear correction term, valid in the* low frequency limit *(*$f \ll \frac{m_e c^2}{h} \sim 10^{20}$ *Hz,* $m_e =$ *the electron mass) and to all orders in* eE, eB *but zeroth order in* α *(at fixed* eE, eB*). To lowest (4th order) in the fields, their transcendental–function expression for* $\delta\mathcal{L}$ *reduces to the quartic form*

$$\delta\mathcal{L}^{(4)} = \left(\frac{\alpha}{4\pi}\right)^2 \frac{2\hbar^3}{45 m_e^4 c^5} \left[(\boldsymbol{E}^2 - c^2\boldsymbol{B}^2)^2 + 7(\boldsymbol{E}\cdot\boldsymbol{B})^2 c^2\right].$$

This expression is Lorentz covariant, *since*

$$\boldsymbol{E}^2 - c^2\boldsymbol{B}^2 = -\frac{c^2}{2} F : F \quad \text{and} \quad \boldsymbol{E}\cdot\boldsymbol{B} = -\frac{c}{4} F : G\,.$$

in terms of the (imaginary–4^{th}–component representation) Minkowski skew–symmetric electromagnetic field 4-tensor, $F_\mu\nu$, and its dual, $G_{\mu\nu} = {}^*F_{\mu\nu}$.

Their approximation applies at wavelengths which are much longer than the electron's Compton wavelength. The correction becomes non-negligible only at field strengths of the order $\frac{m_e^2 c^3}{e\hbar} \sim 10^{16}\ \frac{\text{Volt}}{\text{cm}}$. The latter quantity has been known as the Schwinger critical field *ever since* ***Julian Schwinger*** *(1951) reanalyzed the problem using his elegant functional methods. The non-quadratic correction $\delta\mathcal{L}$ to the classical Lagrangian, results from the virtual production of an* electron-positron pair *in the external EM field. Two examples of observable physical effects which result from the Euler-Heisenberg correction are: 'light by light' scattering (the collision of two γ rays), and the Delbrück effect (1933; scattering of a photon by the Coulomb field of the nucleus).*

The fundamental reason that quantum theory engenders nonlinear corrections to the Maxwell empty-space equations is this: the coupled Maxwell-Dirac operator field equations are themselves nonlinear[185]. Even in the absence of physical ('on shell', i.e. non-virtual) electrons or positrons, the vacuum itself has a finite quantum amplitude to occasionally-produced virtual $\{e^+, e^-\}$ pairs for short periods of time (of order 10^{-21} sec). Upon integrating out these electronic vacuum fluctuations, the nonlinearity of the original Maxwell-Dirac equations is manifested as nonlinear corrections to the purely EM sector of the theory. At the classical level Maxwell's equations in vacuum receive nonlinear corrections. At the level of quantized EM fields, the nonlinearities result in effects such as photon splitting *in a strong magnetic field, as well as the above-mentioned light-by-light and light-by-field scattering processes.*

We note in passing that the nonlinear modifications to classical EM theory in vacuo are not all quantum in origin; the minimal framework needed to encompass both Maxwell's theory and Einstein's GTR is the Einstein–Maxwell coupled field equations, which are nonlinear and yet completely classical (i.e. non-quantum) in origin.

Further work on nonlinear corrections to Maxwell's equations under conditions of strong fields was done by ***Infeld*** *and* ***Born****.*

[185] As are even the coupled *classical* Maxwell-Lorentz equations of motion for a charged electron interacting with dynamical electromagnetic fields.

1907–1909 CE **Leo Hendrik Baekeland** (1863–1944, U.S.A.). Chemist and inventor. Invented the first synthetic plastic material, *Bakelite*[186], that had no counterpart in nature. Bakelite is the prototype of a phenomenally comprehensive family of synthetic polymers called plastics and resins.

Baekeland was born in Ghent, Belgium. He received his Ph.D. from the University of Ghent (1884) and emigrated to the U.S. (1889). Before he was 30 he sold a new type of photographic paper of his own invention to Eastman Kodak for a million dollars (which in those days was really worth a million – no income tax either). Baekeland, an extraordinary gifted young chemist and entrepreneur, began his search for a synthetic substitute for *shellac*[187].

Earlier work by **Adolf von Baeyer** (1835–1917, Germany), who had mixed phenol with formaldehyde, pointed Baekeland down the right road to his discovery. What was a nuisance material for Baeyer, became a cornerstone for Baekeland. Bakelite was unusually strong in spite of its relatively light weight and it was inert to acids and bases, unaffected by heat and effective as an electrical insulator. It could be colored or dyed to suit customer taste.

1907–1923 CE **Luitzen Egbertus Jan Brouwer** (1881–1966, Holland). Dutch mathematician. The founder of topology[188] and mathematical neo-intuitionism. Some of his well known results are the *plane translation theorem* (1907) and the *fixed-point theorems*[189] (1907) that are of special importance in game theory and differential geometry.

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[186] Made from phenol and formaldehyde. The result of the polymerizing reaction is *polyphenolformaldehyde*. Upon heating, cross-links are formed between the chains of the polymer and a thermosetic material is obtained.

[187] *Shellac* is a natural product made from the resinous secretions of the tiny *lac* bugs common to India and Southeast Asia. It is found in lacquers, varnishes, waxes, and other protective agents for coating a variety of surface materials.
Lac bugs pierce the bark of a tree and feed on the sap, discharging a quantity of *lac* as a protective agent. This sticky resin accumulates about the bug until it is all but completely encapsulated, leaving only an opening or two for breathing and for the worm-like larvae of the next generation to exit. The female dies after feeding, leaving colonies of bodies to be collected and processed to make shellac. A few hundred thousands of these little bugs are needed to each pound of shellac produced. At the turn of the 20^{th} century, more than 3000 ton of shellac were imported to the United States each year.

[188] The study of the most basic properties of geometrical surfaces and configurations, that remain unchanged upon continuous transformations.

[189] A *Fixed-Point property* is illustrated in the following simple example: consider the square $0 \leq x \leq 1$, $0 \leq y \leq 1$ in the first quadrant of a Cartesian coordinate system. We place one dot anywhere on each of the two vertical sides of the square (the lines $x = 0$ and $x = 1$). We then draw a continuous curve connecting

In 1911 he discovered his *theorems of topological invariance*. In addition he merged the ideas of **Cantor** with the early topological concepts that existed before his time. In his doctoral thesis "*On the Foundations of Mathematics*" he shaped the beginnings of the neo-intuitionist school. In 1908 he rejected as invalid the use in mathematical proofs of the *principle of the excluded middle*, according to which every mathematical statement is either true or false, no other possibility allowed. During 1918–1923 he reformulated set theory, theory of measure and a theory of functions, without using the above principle.

In 1907 Brouwer also took up Kronecker's program of the "*arithmetization*" of mathematics through the elimination from it of all "*non-constructive concepts*"[190], but his "*intuitionism*" made little headway among practicing math-

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the two dots. This curve must necessarily cut the diagonal ($x = y$) in at least one point. Here is why: Any continuous curve $C(x)$ can be regarded as a *mapping* of the unit interval $0 \leq x \leq 1$ *into itself*, since it associates every point x on this interval with another such point $C(x)$ that also lies in the range between 0 and 1. The *diagonal line*, however, corresponds to points that map to themselves under this transformation; they *remain fixed*. Consequently, any point x that maps to a point on the diagonal is a fixed point of the transformation. In other words, it is a point for which $C(x) = x$. It is then argued that every such curve mapping the unit interval to itself must possess at least one such point. Since the fixed points are given by $C(x) = x$, it follows that determining fixed points is the same as *solving equations*.

Brower's "fixed-point" theorem': If X is a closed disc, then every *continuous* map into itself $X \to X$ has a fixed point. In general, we say that a subset X of 3-dimensional space has a *fixed point property* iff every continuous transformation of X into itself has a fixed point. It is easy to see that a closed disc with the center point removed does not have this property (e.g. rotation).

Every vector field on an even-dimensional sphere must have at least one singular point. In applications of topology to other branches of mathematics, "fixed point" theorems play an important role. In particular, these theorems provide a powerful method for the proof of many mathematical 'existence theorems' which at first sight may not seem to be of geometrical character (e.g. the fundamental theorem of Algebra). A famous example is a fixed point theorem conjectured by **Poincare** (1912) and proved by **G.D. Birkhoff** (1916). This theorem has as an immediate consequence the existence of an infinite number of periodic orbits in the restricted problem of 3 bodies. Since then topological methods have been applied with great success to the study of the qualitative behavior of dynamical systems.

[190] In Kronecker's view transcendental numbers do not exist, since they require an infinite number of fractions or operations for their representation. According

ematicians, who feared Brouwer's views would unnaturally and unnecessarily limit the development of mathematics.

Hermann Weyl (1951) summarized this state of affairs: "*At the end of the* 19^{th} *century it became clear that the unrestricted formation of sets, subsets, sets of sets, etc., together with an unimpeded application as to the original elements of the logical quantifiers* '*there exists*' *and* '*all*" ... *inexorably leads to antinomies.*

The 3 *most characteristic contributions of the* 20^{th} *century to the solution of this Gordian knot are associated with the names of L.E.J. Brouwer, David Hilbert and Kurt Gödel. Brouwer's critique of* '*mathematical existentialism*' *not only dissolved the antimonies completely but also destroyed a good part of classical mathematics that had heretofore been universally accepted...*

Brouwer served as a professor of mathematics at the University of Amsterdam during 1909–1951, with the exclusion of 1945–1946 since he was suspected of collaborating with the Nazis in WWII.

to him nothing could be said to have a mathematical existence unless it could actually be constructed in terms of *finite* number of positive integers. Kronecker is thus a mathematician in whom a computer can believe.

"Have You Seen a Molecule?" (1803–1908)

"This conclusion, that heat consists in mechanical processes, in motion, has spread over the whole cultivated world like wildfire. There is a huge mass of literature on this subject, and now people are everywhere eagerly bent on explaining heat by means of motions. They determine the velocities, the average distances, and the paths of the molecules, and there is hardly a single problem which could not, people say, be completely solved in this way by means of sufficiently long calculations. If, then, we are astonished at the discovery that heat is motion, we are astonished at something which has never been discovered. It is quite irrelevant for scientific purposes whether we think of heat as a substance or not".

Ernst Mach, 1909 (1838–1916)

"Atoms are only hypothetical things".

Friedrich Wilhelm Ostwald, 1906 (1853–1932)

"... merely statistical validity of the Second Law is not good enough; irreversibility is a fundamental property of natural processes, and any molecular hypothesis — or perhaps all conceivable molecular hypotheses based on Newtonian mechanics — that permits any exception, must be wrong".

Ernst Zermelo, 1906 (1871–1956)

The success of Newton's planetary laws of motion led to attempts at more theories similarly based on the laws of motion. Could not some theory of gases be constructed, to account for Boyle's law (1660) by "predicting it", and to make other predictions and increase our general understanding? Gases move easily, diffuse among each other and seep through porous walls. Could these properties be "explained" in terms of some mechanical picture? Newton's contemporaries revived the Greek philosopher's idea of matter being made of "fiery atoms" in constant motion.

Indeed, in 1738, ***Daniel Bernoulli*** *published a 'bombardment' theory in which he pointed out that* moving *particles would produce pressure by bombarding the container, and he suggested that heating it, must make its* *particles move faster. However, his outline was incomplete. This idea stayed dormant for more than a century, when* ***Joule*** *(1847),* ***Clausius*** *(1850),* ***Maxwell*** *(1860) and* ***Boltzmann*** *(1860) set forth the kinetic theory of gases based on the assumption that a gas consists of small elastic particles in rapid motion, and the pressure on the walls is simply the accumulated effect of bombardment.*

*In the field of chemistry, molecules seemed useful: a helpful concept that made the regularities of chemical combinations easy to understand and provided a good start for a simple theory of gases (**Dalton**, 1803; **Avogadro**, 1811). But did they really exist? There was only circumstantial evidence that made the idea plausible. Many scientists were skeptical, and at least one great chemist maintained his right to disbelieve in molecules and atoms even until the beginning of the* 20^{th} *century[191]. Yet one piece of experimental evidence appeared as early as 1827: the Brownian motion.*

*The Scottish botanist **Robert Brown** (1773–1858) made an amazing discovery — he practically saw molecular motion: Looking through his microscope at small specks of solid suspended in water, he saw them dancing with an incessant jiggling motion. The microscopic dance made the specks look alive. Brown was in fact watching the effects of water molecules jostling the solid specks. If the molecules were infinitely small and infinitely numerous, they would bombard a big speck symmetrically from all sides and there would be no Brownian motion to see. At the other extreme, if there were only a few, very big molecules of surrounding water, the speck would execute infrequent, violent jumps when it did get hit. From what we see, we infer something between these extremes: there must be many molecules in the container, hitting the speck from all sides, many times a second.*

In a short time, many hundreds of molecules hit the speck from every direction, and occasionally a few hundreds more hit one side than the other and drive it noticeably in one direction. A big jump is rare, but several tiny random motions in the same general direction may pile up into a visible shift. Detailed observations and calculations from later knowledge tell us that what we see under the microscope are those gross resultant shifts; but, although the individual movements are too small to see, we can still estimate their speed by cataloging the gross staggers and analyzing them statistically.

The kinetic theory was successful not only in providing a theoretical interpretation to the ideal gas equation and the Avogadro hypothesis, but also in yielding a gamut of molecular properties such as speed, number, mass and

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[191] It is a matter of record that as late as 1908, distinguished scientists such as **Ernst Zermelo**, **Wilhelm Ostwald** and **Ernst Mach**, doubted the atomic theory of matter. Mach died unconvinced.

size. Already in 1865, ***Loschmidt*** *obtained estimates of molecular diameters from measurements of liquid density and gaseous viscosity, a method which is now of historical interest only.*

A number of independent methods were devised to measure ***Avogadro's number*** N_0*: Since* $N_0 = R/k$ *[where* k *is the Boltzmann constant and* R *the universal gas constant],* k *was determined from physical laws in which it appeared [e.g. Stefan-Boltzmann law, Planck's law, Einstein's mean-square displacement law of Brownian motion, and Boltzmann's energy partition formula as applied by* ***Perrin****]. In addition,* N_0 *could be directly obtained from Faraday's law of electrolysis and the calculation of a voluminal element of a crystal lattice* $[N_0 = (6.0225 \pm 0.0003) \times 10^{23}$ *molecules/mole].*

So, by 1908, molecules have graduated from speculative tiny ad-hoc entities into full-fledged 'citizens' of the real physical world, except for one attribute — visibility. Could we actually see a molecule? That would indeed be convincing. Scientists of the 19^{th} *century agreed that seeing is hopeless — not just unlikely but impossible, for a sound physical reason.*

Seeing uses light, which consists of waves, ranging in wave-length from 7000 Å for red to 4000 Å for violet: with the naked eye we can see the shape of a pin's head, a millimeter across (10^7 Å $= 10^3$ *microns). With a magnifying glass we examine a fine hair, 0.1 millimeter thick* (10^6 Å $= 10^2$ *microns). A smoke speck is 0.01 millimeter in size* (10^5 Å $= 10$ *microns). With a high-power microscope we see the smallest bacteria* (10^3 Å $= 0.1$ *microns) or a typical particle in Brownian motion. But beyond that scale, vision using ordinary light stops, as the light's wave-length becomes comparable to the object's size. So, viruses (ca 100 Å) and molecules (ca 1–10 Å) cannot be seen with light waves.*

However, early in this century, X-rays offered indirect information: X-ray diffraction patterns revealed both the arrangements of atoms in crystals and the spacing of their layers. Such measurements confirmed estimates on molecule's size and even the general shape of some big molecules.

With the advent of the electron microscope, virus particles and big molecules are "seen", and recently the tunneling microscope has allowed us to see, under favorable conditions, individual atoms. These glimpses of molecular structure agree well with the speculative pictures drawn by chemists, arguing from chemical behavior.

The vicissitudes of scientific atomism is clearly reflected in the quotations at the head of this article. It took almost the whole of the nineteenth century to establish atomism on a sound experimental basis and this required an immense amount of chemical investigation. When success was finally in sight, a number of men of science and philosophers rejected atomism as a kind *of illusion. Antiatomic views were published by such men as* ***Ernst Mach*** *(1838–1916),* ***Pierre Duhem***[192] *(1861–1916), even by a practical chemist like* ***Wilhelm Ostwald*** *(1853–1932); these men were fighting a rearguard action at the very time when atomism had ceased to be a hypothesis, when atoms could be counted and weighed, yet ceased to be atoms in the literal sense, for they were reduced to other elements incredibly smaller than themselves.*

Our Material Culture — Metals, Ceramics and Plastics[193]

Metals have been known since ancient times [Au, Ag, Cu, Fe, Pb]. The basic characteristic of metals is that they loose their electrons rather easily. A sample of a pure metal may be regarded as a framework or lattice of positive metal ions suspended in a sea of electrons. The "free electrons" composing the sea have been liberated by the metallic atoms. Unlike covalent compounds and many nonmetals, the electrons are not localized between pairs of atoms but move freely about through the whole system. These freely moving electrons account for the high electrical conductivity of metals. Electrons carry kinetic energy as well as negative charge; hence metals tend to be good conductors of

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[192] Duhem also erroneously pictured the mechanism of an explosive reaction as the breakdown of a "false-equilibrium", failing to see that it is a question of *activation energy*, or energy necessary to break some bonds in the molecule of the explosive substance [see "Thermodynamique" by **Y. Rocard**, Masson et Cie 1952].

[193] For further reading, see:

- Dobbs, F.W., *The Age of the Molecule*, Harper and Row, 1976, 336 pp.
- Atkins, P.W., *Molecules*, W.H. Freeman, 1987, 197 pp.

heat as well as electricity. Because metals have no specifically directed bonds, layers of metal ions can glide over each other with relative ease. For this reason metals can be deformed readily without breaking, and they possess such properties as high malleability and ductility.

Metallurgy is concerned with obtaining pure metals from naturally occurring materials. First it is necessary to separate the ore (chemical compound containing the metal) from the various other substances with which it is found. Then, it is necessary to carry out a chemical reaction or reactions to liberate the metal from its compound. Metals almost always occur naturally in ionic compounds in which they have lost their electrons and become positive ions.

Obtaining the pure metal means, in chemical terms, restoring the electrons to the metal ion so that it becomes a neutral atom. This process of gaining electrons is called reduction. The most easily reduced metals, those lowest in the activity series, were discovered first. The most active metals, such as sodium (1807) and aluminum (1825), were discovered more recently. The most active metals are reduced by electrolysis; less active ones are produced by reduction with carbon. Aluminum is an example of the former, iron of the latter.

The earth's crust is made of about 8 *percent (by weight) of aluminum,* 5 *percent of iron and* 4 *percent of calcium. Potassium, sodium, and magnesium (extracted from sea water and dolomite) also occur in large amounts. The core of the earth is believed to be made up mainly of nickel and iron.*

Gold was used for ornaments, plates, and utensils as early as 3500 *BCE. Silver was used as early as* 2400 *BCE, and many ancients considered it to be more valuable than gold, because it was rarer in the native state. Native copper also was used at an early date in tools and utensils, because it was found near the surface of the ground, and could easily worked and shaped. Since about* 1000 *BCE, iron and steel have been the chief materials for construction.*

There are 81 *metals in the periodic table of the elements.*

Ceramics (from the Greek Keramos = potters, clay) are generally complex silicates that have been heated so that they are hard and fire resistant. The chief feature of ceramics is their great resistance to chemicals and high temperatures. China, pottery, porcelain, brick, tile, sewer pipe and refractories are all typical ceramics. The three principal raw materials used in making ceramics are clay, feldspar, and sand.

Pottery, the oldest form of ceramic products, dates back to prehistoric times. Examples of pottery more than 6000 *years old have been found in several parts of the world. Industrial uses of ceramics began during the* 1900*'s. Military requirements of WWII (1939–1945) created a need for high performance materials and helped speed the development of ceramic science and* *engineering. During the* 1960*'s and early* 1970*'s, advances in atomic energy, communications, and space travel required new kinds of ceramics.*

Within the last 200 *years, man's knowledge of the nature and structure of matter has had an impact on his ability to create or alter materials for particular purposes. According to a recent estimate there are some* 6 *million known chemical compounds. Man's ability to synthesize many of them and construct such a variety of new materials from them has had an enormous impact on his material culture. A great many of the substances that are directly involved in our daily lives have been developed in their present-form through a knowledge of chemistry.*

The manufacture of chemical compounds and materials is so complex and varied that it defies any effective categorization. Some of the different types of materials in whose manufacture or processing chemistry plays some role are: drugs, agrichemicals *(fertilizers, pesticides),* coloring materials *(paints, dyes, inks),* explosives, photographic materials, industrial chemicals *(H_2SO_4, HCl, HNO_3, petrochemicals),* modified naturally occurring substances *(sugar, starch, gelatin, leather, soap, paper),* materials used for building, constructing and manufacturing *(ceramics, metals, polymeric materials), and a* miscellany of other materials *(adhesives, oils, waxes, detergents, flavors, fragrances, cosmetics).*

A polymer[194] *(from the Greek* polis *= many, and* meros *= part, together "something of many parts") is a giant molecule composed of recurring identical units called* monomers*. The number of monomers in a polymer may reach hundreds of thousands and its molecular weight varies between* 5000 *and* 20 *million. [Water has a molecular weight of* 18*, polyisoprene (natural rubber) has a molecular weight of about one million; that means that a mole of water weights about an ounce, while that of rubber weights a ton!]*

Polymers are created by a process of polymerization*, in which many identical molecules are combined together, sometimes to form a long chain, sometimes to form a more complex network with cross-linkages.*

Recognition of the special properties of the natural polymers *began already in the middle of the* 19^{th} *century, long before their structure became known.* ***Berthelot*** *discussed (1863) the polymerization of olefins and effected the polymerization of ethylene in 1869.* ***Eugen Bamberger*** *(1857–1932, Switzerland) synthesized polyethylene in* 1900*.*

The most important kind of non-living materials that are polymeric in their structure are: plastics, synthetic fibers and synthetic rubber.

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[194] The name was coined by **Berzelius** (1833) for a member of a subclass of *isomers*.

Plastics (from the Greek plastikos = able to be molded) are synthetic polymers. Polymers used in plastics may be classified into two types based on the type of linkage of the monomers — addition polymers, and condensation polymers.

Another classification divides polymers into two groups based on their behavior when subjected to heat. A polymer that is readily softened by heat is referred to as being thermoplastic. Each polymer can be repeatedly heated and softened (e.g., polyethylene, rubber, synthetic fibers, polypropylene, teflon, polyvinyl chloride (PVC), acrylic, cellulose, acetate).

In contrast, some polymers are composed of chain of polymers which are cross-linked. These polymers do not soften upon heating and remain rigid. They are called thermoset plastics (e.g., a Melamine dinner ware, Epoxy, Polyester, Silicone).

The product formed by the polymerization of different monomers is called a copolymer. An example of this type of reaction is the production of S rubber. The addition of one part styrene to three parts butadiene polymerizes in the presence of the catalyst sodium and results in an excellent rubber substitute.

The properties of plastics can also be affected by the use of various additives. Some types used include foamers, fillers, solvents, plasticizers, dyes, lubricants, and stabilizers.

Manufacturers make plastics from raw materials such as coal, limestone, petroleum, salt, and water. Solid plastics can be made to look and act like glass, wood, metal, and other materials. But they can usually be manufactured more cheaply than these materials. Liquid plastics may be used as adhesives and paints. Scientists and engineers have developed hundreds of plastics. These made materials have a wide variety of characteristics such as hardness, softness and transparency.

Fabrics woven from plastic fibers feel soft, but the fibers are made from hard plastics. Nylon is hard enough to be used to make gears for machinery, yet when drawn into fine threads, nylon can be used to weave delicate stockings and lingerie.

Some plastics are foamed to make soft sponge-like materials used in cushions for furniture and automobiles.

Transparent plastics such as vinyls, polystyrenes and polyethylenes are used as envelops to package food, medicines, toys, and many other products. Contact lenses made of acrylic are more transparent and less fragile than lenses made of glass. Decorative plastics often look like gold, silver, marble, wood or leather.

Plastics are used in industry, home building, medicine and science. In medicine, certain plastics have important uses because they do not harm the body and are not affected by its chemicals. Doctors use plastics rivets, screws and plates to join broken bones. They sew up wounds and surgical incisions with plastic threads. In science polyethylene plastics made good shields for nuclear reactors; they absorb neutrons better than an equally thick shield of concrete or water, yet are much lighter. Other plastics, called scintillation plastics, can detect radioactivity or passage of charged particles.

Polymer science and the plastic industry, both of which continue to exert an incredibly large impact on Western culture, trace their beginning to natural rubber.

On his second voyage (1494), Columbus found the natives of Haiti playing with rubber balls. Earlier, in the 13^{th} *century, articles of rubber (including balls for games), were in common use among the Mayas and Aztecs. The explorers learned that the Indians made "water proof" rubber shoes by dipping their feet in latex, the milky white ooze of the rubber tree. The Indians also made waterproof bottles by smoothing latex on a bottle-shaped clay mold. They dried the latex over a fire, and then washed out the clay. The South American Indians called the rubber tree cahuchu, which means weeping wood.*

Since 1615, *the Spaniards conquerers were themselves using rubber to weatherproof soldiers' cloaks, but no one in Europe was aware of the existence of rubber. It was left for the French, more than a century later, to carry the study and the use of rubber to Europe;* ***Charles de la Condamine*** *(1701–1774), who had been sent by the French Academy of Sciences on an expedition to Peru, brought back (1736) caoutchouc, as the substance was then called. But only in* 1751, *fifteen years after the expedition, he gave an account of it to the Academy. Rubber received its name when (1770)* ***Joseph Priestley*** *found it would rub off pencil marks.*

By the late 1700*'s, chemists had found that hardened latex dissolved in turpentine made a waterproofing liquid for cloth. In* 1823, *the Scottish chemist* ***Charles Macintosh*** *(1766–1843) began manufacturing the "mackintosh" raincoats that became world-famous. Early rubber products became sticky in hot weather, and stiff and brittle in cold weather.*

In 1826, ***Michael Faraday***, *discovered that rubber is a hydrocarbon.*

In 1839, ***Charles Goodyear*** *(1800–1860, U.S.A.) made a serendipitous discovery of the process of vulcanization through which rubber could be made stronger and resistant to heat and cold, by heating it together with sulfur [Vulcan, the Roman god of fire]. With vulcanization, the rubber industry grew rapidly: vulcanized rubber was elastic, airtight and watertight.*

In 1860, ***Greville Williams*** *(1829–1910, England) discovered that natural rubber yields, upon heating, a colorless liquid that he called isoprene. His was, in fact, a process of depolymerization.*

At first, manufacturers used only wild rubber which came mostly from the Amazon Valley of Brazil. At the request of the British government, an amateur botanist ***Henry A. Wickham*** *(1846–1928) took about 70,000 seeds of the Hevea brasiliensis tree from Brazil to England (1876). Seedling from the sprouted seeds in a Kew Gardens greenhouse were then taken to Ceylon and Malaya for replanting on plantations.*

The invention of the automobile in the late 1800's created a tremendous demand for rubber. By 1914, the yearly production of plantation rubber had exceeded that of wild rubber.

The importance of rubber in wartime became obvious during WWI. Armies needed rubber-tired vehicles to carry troops and supplies. The German were cut off from their natural-rubber supplies by the Allied blockade, and began to make synthetic rubber. But it did not work well. Experiments in producing synthetic rubber continued in the 1920's chiefly by scientists in Germany and the United States.

In 1942, the Japanese invaded Southeast Asia and cut off over $\frac{3}{4}$ of the West's supply of natural rubber. As early as 1939, perceptive governments and rubber corporations realized the dangers inherent in fighting wars that would involve reduction of rubber supplies. Germany, the Soviet Union, Britain, and the United States began stepping up their efforts to produce synthetic rubbers.

The first synthetic was based on the discovery of the priest ***Julius Arthur Nieuwland*** *(1878–1936, Belgium and U.S.A.) at Notre Dame University. He worked on (1904–1924) the synthesis of rubber from acetylene and sold his patents to the DuPont Company in 1925. At DuPont,* ***Wallace Carothers*** *(1896–1936, U.S.A.) further developed Nieuwland's ideas into neoprene (1932), and made important contributions to the understanding of polymers in general. Isoprene, the monomer of natural rubber, is difficult to synthesize, and the attention of the German I.G. Farben group and the Standard Oil was directed toward rubber synthesis from butadiene.*

Alongside with the development of synthetic rubber, chemists had the good fortune to discover other plastic materials, based also on natural polymers.

Natural cellulose $(C_6H_{10}O_5)_n$, like starch and glycogen, is a polymer of glucose ($C_6H_{12}O_6$), but neighboring glucose units are linked differently. This one difference — a simple twist of a link — makes cellulose indigestible by *humans (termites and herbivorous animals* can *digest cellulose if certain enzymes are present). The major part of the rigid portion of many plants is cellulose. Cotton and linen are about* 98 *percent cellulose. The explosive cellulose nitrate [$C_6H_7O_2(O{\cdot}NO_2)_3$] was discovered (1846) by* ***Schönbein*** *by treating cotton fibers with certain acids.*

Alexander Parkes *(1813–1890, England) tried to create a moldable material from it by treating it with a variety of solvents. The plastic material that he created, called Parksine (1862), was used to produce a variety of objects, such as combs. His associate* ***Daniel Spill*** *(1832–1887, England), invented Xylonite (1867). However, Parksine had an unfortunate habit of shriveling and wrinkling.* ***John Wesley Hyatt*** *(1837–1920, U.S.A.), a printer in Albany, N.Y., who sought to win a $10,000 prize for developing the best substitute for ivory in billiard balls remedied the deficiencies of Parksine. Although he did not win the prize, the material, Celluloid (1869), which he prepared by mixing camphor with cellulose nitrate and a solvent, was a superior product. Celluloid could be sawed, carved and made into sheets. As a result, new plastic products appeared on the market. Common Celluloid articles included combs, collars, dentures, carriage curtains, clock cases and the first photographic roll film. But Celluloid was hard to mold and it caught fire easily.*

Soon after the invention of Celluloid, chemists developed other products made from plant fibers. In 1884 *the French chemist* ***Hilaire Chardonnet*** *(1839–1924) invented viscose rayon, the first man-made fiber. The chemist* ***Jacques Edwin Brandenberger*** *(1872–1954, Switzerland) invented cellophane (1908) [when one presses on a cellophane sheet or tries to tear it, one is experiencing the strength of the hydrogen bonds!]*

During the 1800*'s English and German chemists experimented with combinations of carbolic acid (phenol, C_6H_5OH) and formaldehyde (CH_2O). This combination produced a resin, but the chemists could not control the violent reaction.* ***Leo Hendrik Baekeland*** *(1863–1944, U.S.A.) succeeded (1909) in controlling the reaction and invented the first completely synthetic resin, Bakelite.*

Baekeland produced his resin while trying to make a better kind of varnish. At first, he did not recognize the value of Bakelite as a plastic material. ***Richard W. Seabury*** *(1883–1970, U.S.A.), a rubber manufacturer, showed that the new resin could be molded. He mixed Bakelite with asbestos fibers, and molded a part for an electrical instrument.*

Bakelite became widely used to make telephones and handles for pots and irons. The electrical and automative industries used Bakelite for many products. Unlike celluloid, Bakelite was not flammable.

Synthetic fibers, like plastics, had their origin in experiments with cellulose. At the end of the 19^{th} *century, two different processes for preparing a cellulose* *thread called rayon were developed. In 1931,* **Wallace Carothers** *(1896–1937, U.S.A.) synthesized a material stronger than silk, which is now called nylon.*

Other plastic materials were synthesized all through the 20^{th} *century, each with its own specific use: Cellulose Acetate (1927); Urea-formaldehyde (1928); Perspex (1936); Polyurethane (1939); Polyester resins (1942); Silicone (1943); Teflon (1943); Epoxy resins (1947); Polypropylene (1957); Polycarbonate (1958).*

Polymers[195]

Unsaturated monomers under high pressure and temperature and in the presence of a catalyst, link to each other to form long chains. For example, with the aid of a catalyst, one molecule of ethene combines with a second molecule of ethene to form a dimer:

$$
\begin{array}{ccccc}
 & H & & H \\
 & | & & | \\
H - & C & = & C \\
 & & & | \\
 & & & H
\end{array}
\quad + \quad
\begin{array}{ccccc}
 & H & & H \\
 & | & & | \\
H - & C & = & C \\
 & & & | \\
 & & & H
\end{array}
\longrightarrow
\begin{array}{cccccccc}
 & H & & H & & H & & H \\
 & | & & | & & | & & | \\
H - & C & - & C & - & C & = & C \\
 & | & & | & & & & | \\
 & H & & H & & & & H
\end{array}
$$

This molecule in turn links to a third molecule, and a fourth, and continuous on to form a giant molecule consisting of up to several thousands units of ethene, called polyethylene

$$
\begin{array}{cccccccccccccccccc}
 & H & & H & & H & & H & & H & & H & & H & & H & & H \\
 & | & & | & & | & & | & & | & & | & & | & & | & & | \\
H - & C & - & C & - & C & - & C & - & C & - & C & - & C & \ldots \text{etc.}, \ldots - & C & = & C \\
 & | & & | & & | & & | & & | & & | & & | & & & & | \\
 & H & & H & & H & & H & & H & & H & & H & & & & H
\end{array}
$$

[195] To dig deeper, see:

- Perepechko I.I., *An Introduction to Polymer Physics*, Mir Publishers: Moscow, 1981, 266 pp.

We thus write that the monomer $CH_2 = CH_2$ *was transformed into the polymer* $(\text{—}CH_2\text{—}CH_2\text{—})_n$. *The brand name of this plastic is Handi-Wrap and it is used for food wrappings, tubing, molding objects, and electrical insulation.*

If the hydrogens of the monomer ethylene are replaced by fluorines, the monomer tetrafluoroethylene $(CF_2 = CF_2)$ *becomes upon polymerization polytetrafluoroethylene (teflon) used for lining in cooking utensils.*

A number of additional polymers are made from hydrocarbon compounds with a carbon-carbon double bond (alkenes); propylene is similar to ethylene, but one of the ethene hydrogens is replaced by a methyl group

$$\begin{array}{ccccc} & & & & H \\ & & & & | \\ CH_3 & \text{—} & C & = & C. \\ & & | & & | \\ & & H & & H \end{array}$$

When polymerization occurs, polypropylene is obtained $[CH(CH_3)CH_2]_n$.

Special catalysts are used in synthesis to ensure that there is little chain branching and that $\{\text{—}CH_3\}$ *groups all point in the same direction. This yields an extremely orderly solid (crystalline-like) with many useful properties, namely — stiffness, resistance to abrasion and high enough melting point for objects made from it to be sterilized. However, because the* CH_3 *groups are liable to oxidation, polypropylene articles usually have antioxidants incorporated into them to divert attack by oxygen. Polypropylene is used for ropes, fishing-nets, toys, housewares, baby bottles, wire insulation, pipe and fittings.*

Vinyl chloride (C_2H_3Cl), *is derived from an ethylene molecule by replacing a hydrogen atom with a chlorine atom. It can be polymerized to form polyvinyl chloride* $(CHClCH_2)_n$ *or PVC, one of the most versatile and adaptable of all plastics. The addition to it of certain synthetic resins (plasticizers), makes PVC softer and more flexible. Moreover, PVC can be mixed with a very wide range of additives chosen to tailor its properties to many different applications. When properly protected by additives, it is chemically resistant to attack and degradation.*

PVC is used for imitation leather, phonograph records, packaging, pipes, credit cards, baby pants, rain coats. About 3 million tons are used annually in the United States alone, of which 10,000 tons are used for making credit cards.

Vinylidene chloride $(C_2H_2Cl_2)$ *may be polymerized on its own or copolymerized with vinyl chloride to give the polymers known collectively as saran* $(CCl_2CH_2)_n$, *used for upholstery covers and as a protective film (saran wrap).*

If the monomer is styrene (a benzene ring in which one hydrogen atom was replaced by $CH = CH_2$*; discovered in 1839 by a Berlin pharmacist by the name of* **Simon***), the polymer is polystyrene used for styrofoam, insulation, combs, toys, models, and household articles. Likewise, the monomer acrylonitrile*

$$\begin{array}{c} H - CH{=}CH \\ \qquad\quad | \\ \qquad\quad CN \end{array}$$

yields upon polymerization the polymer polyacrylonitrile

$$\begin{array}{c} (-CH_2 - CH-)_n \\ \quad | \\ \quad CN \end{array}$$

known commercially as Acrilan (or Orlon). It is used for fibers in the production of carpeting, clothing, etc.

Rubber is a polymer of a five-carbon compound called isoprene

$$\begin{array}{c} CH_2{=}CH - C = CH_2 \\ \quad | \\ \quad CH_3 \end{array}$$

The polymer may be linear, cross-linked, or it can exist in more complex configurations. Polymer molecules of unstretched rubber fold back on themselves somewhat like irregular coils. Stretching the rubber straightens the chain of folded molecules. Releasing the rubber lets the chain return to its coiled position. The sulfur that combines with the rubber during vulcanization sets up "cross links" between the rubber chains so that the chains are bound together and cannot slip past one another. This gives elasticity and strength to the vulcanized product.

The number of cross links increases with the amount of sulfur, such that the rubber becomes stiffer, tougher, and less stretchable (chemists use the word elastomer for any substance, including rubber, that stretches easily to several times its length, and returns to its original shape).

The molecule that has proved most useful in synthetic rubber is butadiene: $CH_2{=}CH{-}CH{=}CH_2$*, easily obtained from coal tar, coal gas or from ethyl alcohol (obtained from grain fermentation). It was later found that addition of styrene stabilizes synthetic rubber. These styrene butadiene rubbers (SBR) are the most common general purpose rubbers. Another synthetic product, polymerized chloroprene* ($CH_2{=}CCl{-}CH{=}CH_2$)*, known as neoprene is particularly heat and weather resistant.*

Methyl methacrylate ($C_5H_8O_2$) *is the monomer from which the polymer known as lucite, plexiglass, or perspex is derived. Bulky, irregular side groups* *attached to the basic ethylene fragment cause the polymer chains to lie together in a very irregular way, so that the solid is internally very chaotic. Since the solid is amorphous on a molecular scale, it does not scatter light that passes through. Consequently, blocks of the polymer are brilliantly transparent, like clean water.*

If one of the side groups ($—CH_3$) *of the monomer, that is responsible for its rigidity, is removed, the resulting substance serves as a base for* acrylic paints.

The polymerization of adipic acid *($C_6H_{10}O_4$) with* hexamethylene diamine *($C_6H_{16}N_2$) by condensation, yields the polymer* polyhexamethylene adipamide *$[CO(CH_2)_4CONH(CH_2)_6NH]_n$, known as* nylon, *the first completely synthetic fiber. It is strong, springy, resists abrasion, and has good electrical qualities. Used for fabrics, gears, bearings, hardware, brush bristles, electrical appliances.*

Artificial (as well as natural) fibers should consist of long molecules that can be made to lie parallel *to each other as they are drawn out into a thread. This can be achieved by linking together an acid and an alcohol molecule as in the process of ester formation, but in such a way that the ester molecule can go on growing at each end. This results in indefinitely long molecules of repeating units called* polyesters. *Thus, if* terephthalic acid *($C_8H_6O_4$) is allowed to react with* ethylene glycol *($C_2H_6O_2$), the result is the polymer* polyethylene terephthalate *$[O_2CC_6H_4CO_2C_2H_4]_n$. It can be made into either a fiber (Dacron) or a film (Mylar).*

The first synthetic heart *was made of Dacron. Mylar is used in* cassette tapes.

1907–1920 CE **Ernest Rutherford** (1871–1937, England). Physicist. Proposed the concept of a nuclear atom in which a smaller center is surrounded by electrons (1911). Discovered the *proton* (1919) and suggested the existence of the *neutron*. Rutherford inspired two generations of physicists, and his discoveries had a major influence on the scientific thought of his era[196].

Rutherford was born in Spring Grove, New Zealand. He was educated at Canterbury College, Christchurch, and went to the Cavendish Laboratory at Cambridge to work under **J.J. Thomson**. In 1902 he discovered Thorium X with **Frederick Soddy** (1877–1956) and during 1906–1909 he showed alpha-particles to be nuclei of helium atoms. For these discoveries he won the Nobel prize for chemistry in 1908. In 1919 he succeeded J.J. Thomson to the Cavendish chair at Cambridge.

1907–1945 CE **Ellsworth Huntington** (1876–1947, U.S.A.). Geographer and explorer. Was concerned with the complex origins and causes of the world's major civilizations and why they had emerged in particular locations. He laid particular emphasis on *climatic* factors and contended that certain types of climate were favorable to a high level of civilization. This climate is characterized by a moderate temperature, and by the passage of frequent barometric depressions which give a sufficient rainfall and changeable stimulating weather. He argued that the location of the hearths of particular civilizations could be matched with critical environmental limits, and saw in *climatic changes* the clues to unexplained shifts in these centers[197].

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[196] Yet, four years before his death (1933), Rutherford said, firmly and explicitly, that he did not believe the energy of the nucleus would ever be released. Nine years later (1942), in Chicago, the first fission pile began to run.

[197] Huntington gives three interesting examples:

- The level of civilization in ancient Greece was critically affected by the average *rainfall*: Up to about 400 BCE Greece had been well watered and forested, with perennial streams unsuited to the development of mosquitoes, but after that date the rainfall diminishes greatly. The streams were reduced in summer to stagnant pools and swamps, with the result that malaria became endemic and undermined the vitality of the population.

- The level of civilization in *Rome* fluctuated with the average *rainfall*: The rigorous Roman life of the early Republic (450–420 BCE) was based on intensive agriculture. This was a period of very high civilization *and* very heavy rainfall. Towards 250 BCE the spirit of discipline and rural simplicity began to decay. The period from 225 to 200 BCE was one of economic stress — marked by decreasing rainfall. The second century BCE with its low fain-fall witnessed great decline in agriculture. During this period malaria became endemic because of formation of stagnant pools and marshes. From 100–50 BCE heavy rains again brought an increase in general luxury and comfort. From 80 CE onwards, as rain fall decreases there was a gradual decline and 180–190 CE were years of famine and pestilence. 193–210 CE saw a slight increase in prosperity, but then began in full force the long "decline and fall of the Roman Empire".

Huntington was born in New England into a congregational family, studied at Harvard and later held a post at Yale. Took part in expeditions to Mesopotamia, Turkestan and later to all the continents. Authored many books and papers, among them: *The Pulse of Asia* (1907), *Civilization and Climate* (1915), *Mainsprings of Civilization* (1945).

Huntington advanced (1923) an hypothesis that solar activity has an ongoing effects on terrestrial climates. He went further to suggest a correlation of *sunspot activity* with terrestrial weathers and especially linked high sunspot with intensified terrestrial atmospheric circulation. He pointed out that in the northern hemisphere, *cyclonic storm*-belt centers more nearly on the magnetic pole rather than on the geographic pole, just as do auroral displays which are more obviously related to sunspots.

Moreover, he found correlation between sunspot extremes and severe droughts and indicated that hurricane activity increased at times of high sunspots. On the other hand he pointed out that during the Little Ice Age in Europe (1430–1850 *CE*) the sun seems to have gone through a calm period, in which sunspots were practically absent[198].

Due to the great complexity of the problem, work on climatic changes must be restricted more to *correlation* than to *mechanism*. Yet, Huntington suggested a crude model for the interaction of the sun's photosphere with the earth's atmosphere: Increasing sunspot indicate increased emission of charged particles. Many of these become trapped in the earth's magnetosphere from which they filter down to the lower atmosphere, mainly near the magnetic poles. From there they exert pressure over air-mass formations, as in northern Canada. These rapid pressure changes may occur simultaneously in several widely separated areas and intensify cyclonic activity in the mid latitudes; under favorable conditions, comparatively small causes may have disproportionately large effects.

- There were four great waves of emigration from *Arabia*; the first during the 4^{th} millennium BCE, the second (Amorite) about 2000 BCE, the third (Aramean) mainly around 1350 BCE, and the fourth (Arabian) culminated in the Islamic expansion of the 7^{th} century CE. All of them are attributed to dry periods preceding the actual outbursts.

[198] Sunspot-free periods, known as *Maunder-minimum* occurred at irregular intervals in earlier times.

A recent study of tree-rings has revealed an apparent correspondence between periods of drought in the high plains of the Western U.S. and the minimum of the 22-year sunspot cycle.

1908–1911 CE **Jean Baptiste Perrin** (1870–1942, France). Physicist. Designed experiments that established the reality of atoms and gave a macroscopic confirmation to the **Einstein-Smoluchovski** theory of Brownian motion. Determined the '*Avogadro number*' and molecular size through an ingenious experiment in which he used colloidal suspensions of gum resins of uniform size in water, and registered via microscope[199] the dependence of the concentration of these grains on height.

Since the suspended particles obtained kinetic energy from their collision with the water molecules (Brownian motion), they attain a stationary suspension at constant temperature in a uniform gravitational field.

The application of Boltzmann's *energy partition function* for the number of molecules at energy level ϵ $[N = N_0 e^{-\epsilon/kT}]$ where ϵ is the gravitational potential energy mgH, [m, mass of a colloidal particle and H, height above a reference level], yields the density of gum particles (concentration) at any level, through the Laplace 'isothermal atmosphere' formula $\rho = \rho_0 e^{-mgH/kT}$. When m is corrected for buoyancy of the liquid and ρ/ρ_0 is measured, k can be calculated, and through it the Avogadro number.

In 1895 Perrin found evidence that cathode rays are negatively charged particles. In 1901 he presciently proposed a "nucleo-planetary" model of atomic structure, with a positively-charged "sun" surrounded by many smaller negatively charged "planets". The periods of rotation of these planets might correspond to different wavelengths in the emission spectrum.

Perrin emphasized an analogy of Brownian movement paths with non-differentiable functions (previously studied by **Riemann** and **Weierstrass**[200]). This analogy seems to have stimulated some of the later research on functional integrals, notably by **Wiener**.

1908–1928 CE **Wilhelm Geiger** (1882–1945, Germany). Physicist. Developed the *Geiger counter* [used for the detection of radioactivity]. His work

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[199] The introduction of the *ultramicroscope* in 1903 rendered visible small colloidal particles whose greater activity could be measured more easily.

[200] **Weierstrass** (1872) constructed a function, defined by the Fourier-series: $f(x) = \sum_{n=0}^{\infty} a^n \cos(\pi b^n x)$ where a, b are real, $b > 1$, $0 < a < 1$, $ab \geq 1$. The series is uniformly convergent in any interval, so that $f(x)$ is everywhere continuous. However, the series obtained via term by term differentiation is divergent. This property is maintained if we choose $a = b^{D-2}$, $1 < D < 2$, where D has the modern meaning of *fractal dimension*. One can consider the above function to be the real part of the *complex Weierstrass function* $W(t) = \sum_{n=0}^{\infty} a^n e^{\pi i b^n t}$. It yields a 3-dimensional *Brownian surface* that renders a visual realization of the stochastic Brownian process (**Mandelbrot**, 1982).

on the deflection of α-particles by thin metal foils led **E. Rutherford** to discover his model of atomic structure in 1911. Geiger and **Walther Müller** improved the 1908 counter and made it more sensitive and durable (1928).

The counter is usually in the form of a thin metal cylinder enclosed in a glass tube. The metal will serve as one electrode. A straight wire projected into the cylinder is the other electrode. The electrodes are maintained at a voltage which is just short of breakdown potential of the air or other gas in the cylinder. If ionizing radiation enters the cylinder, the gas will ionize, setting up a weak current, which is revealed by light signals, by clicks picked up by earthphones or by readings on a meter.

Geiger was born in Neustadt, Germany. He was Rutherford's assistant from 1906 to 1912. He later became a professor of experimental physics at Tübingen University. When **Hans Bethe** was dismissed (1933) from his position as theoretical physicist at that university, Geiger turned his back on him. With **Heisenberg**, **Hahn** and other notable physicists, Geiger joined the Nazi nuclear war-effort.

1908 CE, June 30, 00:14:28 GMT A mysterious object exploded in the sky at altitude ca 8 km over the basin of the River Podkamennaya Tunguska in central Siberia ($60°55'$N, $101°57'$E).

The explosion itself took the form of a vertical column of fire, and spewed up incandescent matter to a height of 20 km. *Heat* radiation was felt at 70 km from the epicenter. The event was *seen* in the sunlight at distance of 500 km and *heard* over a distance of 1270 km.

Near the epicenter, trees of a coniferous forest were uprooted within a radius of 25 km and burnt inside a radius of 15 km, their tops charcoaled. The total energy of the object was calculated to be equivalent to 12 megaton of TNT[201]. There was lack of evidence for either an impact crater or sizable debris. Had the object arrived 4^h27^m later, it would have landed on the city of Petrograd, then capital of the Russian empire. No other event of its kind has ever been recorded.

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- Ben-Menahem, Ari, *The Siberian Explosion of June 30, 1908*, Phys. Earth Planet. Interiors, 1975, **11**, 1–35 pp.
- Foschini, L., *A solution for the Tunguska Event*, Astron. Astrophys,, 1999, **342**, L1–L4 pp.
- Farinella, P. *et al.*, *Probable Asteroidal Origin of the Tunguska Cosmic Body*, Astron. Astrophys., 2001, **377**, 1081–1091 pp.

Punctuated Evolution — the Evils and Blessings of Planetesimal Bombardment

The surfaces of Mercury, Venus, Mars and moon are dotted with many thousands of craters, most of which were made within a few hundred million years after the solar system formed. During this epoch, meteoroids by the millions must have rained down on all the planets of the inner solar system as the last chunks of matter to reach the planets encountered the new planetary surfaces. This earlier bombardment included comets *as well as* asteroids. *The earlier bombardment essentially ended* 3.5×10^9 *years ago. Our own planet shows only a few large meteorite craters, representing relatively recent impact.*

Although the earth surely did not escape this rain of terror, the first few hundred million years of our geological record have vanished because of erosion and the movement of crustal plates. Comets (which spend most of their lives in the frozen depths of space at the boundaries of the solar system) represent the best preserved primitive material, that is relatively accessible to us, because they orbit the sun in highly elongated trajectories that carry them much further from the sun than any of the planets.

At this huge distance from their central star, no heating has disturbed the original state of the condensed material. Individual comet nuclei are generally only a few kilometers in diameter and consist of frozen "snowballs" of water, carbon dioxide and other gases, silicate dust plus more complex molecules including organic compounds.

Upon approaching the sun, some of the cometary ice vaporize, producing a gauzy cone around the nucleus and a long, rarefied tail that streams away from the sun for millions of kilometers. Another source of somewhat primitive matter resides in the lumps of debris called asteroids and meteoroids, which, like short period comets, orbit the sun in rather elongated trajectories, but which always remain fairly close to the sun.

Although the delivery of life seeds to earth via comets *seems unlikely, comets may have provided earth with the basic materials necessary for life to form. Scientists have suggested that collisions of comets and the accretion of cometary matter to the earth provided a plausible mechanism for supplying part of the* earth's early atmosphere. *The same collisions might form local concentrations of organic molecules and liquid water on the earth's surface, where, in the presence of solar ultraviolet radiation, amino acids and other biochemical compounds could form spontaneously.*

Comets may have provided much of the earth's water, volatiles, *and* organic molecules, *thus depositing some of the biogenic material from which primitive* *life ultimately formed. Subsequent impacts, however, could easily have erased many of the earth's life forms, leaving only the most adaptable to develop further.*

This duality of comets, as both life giving and life threatening, is a theme that began with ***Isaac Newton*** *and* ***Edmund Halley*** *(1688), the latter suggesting (1694) that cometary collisions might well cause a major extinction of species, but the fine debris would then settle onto the earth's surface and render the soil more suitable for vegetable production and animal life. In 1750* ***De Maupertuis*** *wrote that comets could crash into the planets, and the resultant heat and contamination of atmosphere and water would lead to mass extinction.*

It has been concluded recently from the fossil record (1992) that throughout the history of multicellular life — from 600 *million years ago — some* 60 *percent of all species extinctions may have been caused by impacts of asteroids, comets or other extraterrestrial bodies. Some of these impacts would have eliminated* $\frac{2}{3}$ *or more of species living at the time. The mass extinction at the end of the Cretaceous period,* 65 *million years ago, that accounted for the demise of the dinosaurs, is widely thought to be an example of such a calamity*[202].

Other collisions would have destroyed between 5 *and 60% of the standing population of species. Extinction levels up to 5% fall into the range of "background" extinction, the result of biotic effects including the overwhelming success of one species over another, or local disasters such as a hurricane that destroys animals and plants unique to one island.*

Throughout history, background extinction has cumulatively accounted for the loss of 40% of species, a much lower figure than most biologists have guessed. The above conclusions were reached by dating asteroid collisions from their impact craters and checking them against known mass extinctions. If it is true that throughout earth history, 60% of all species have gone extinct through asteroid impact, then the odds are better than even that Homo sapiens *will end its tenure in the same way.*

It might then turn out that those planets where mass extinction periodically "clear the undergrowth" for new species to dominate, will allow evolution to proceed more rapidly than it can on planets where no mass extinction occur. In other words, a precondition for a planet to evolve intelligent life

[202] It has been estimated that the said extinction could have been precipitated by an asteroid measuring about 10 kilometers across that slammed into the Yucatan with a yield equivalent of 100 million megatons of TNT. This event is just one of a dozen or more such mass extinctions found at intervals of about 26 million years through the fossil record of life.

sooner rather then later could be that at long intervals, large impacts did occur, eliminating most species of life, which were then replaced with new species.

In 1978, *an explosion equivalent to* 100 *kilotons of TNT was detected in the South Pacific. Once suspected of being a clandestine nuclear test, the event is now considered to have been an asteroid strike. In Jan.* 1991 *an asteroid* 10 *meters in diameter passed between the earth and the moon, scoring a near miss.*

Countless millions of asteroids hug an orbit between Mars and Jupiter, forming a belt that normally poses no threat to life on earth. Occasionally, however, some are jostled out of the belt to assume earth-crossing orbits. Some 150 *such asteroids with a diameter of at least* 1 *km have been detected, with* 2 *or* 3 *new ones discovered every month.*

It is estimated that between 1000 *and* 4000 *earth-crossing asteroids exist with diameters equal to or larger than* 1 *km. An equal number of comets may pose a similar threat. Most asteroids with diameters ranging from* 10 *to* 100 *meters explode on hitting the earth's atmosphere, usually with no harmful effects. The larger and denser ones in this range penetrate further.*

The Siberian bolide, is thought to have been 90–190 *meters across*[203]*. It is estimated that impacts of this magnitude occur once every* 2000 *years. The impact of a* 1*-km or larger object occur once every* 300, 000 *years, and carries the potential to severely disrupt or even terminate our civilization; huge quantities of ash, dust and vapor would penetrate into the atmosphere, shrouding the sun and triggering the equivalent of a "nuclear winter". Objects up to* 5 *kilometers in diameter arrive once every million years and cause mass extinction; agriculture and civilization would certainly collapse and the future existence of the human species would be in doubt.*

1908 CE, Dec. 28 A major earthquake destroyed the cities of Messina and Reggio, killing some 80,000 people in Calabria and Sicily[204].

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[203] Z. Sekanina, The Tunguska event: No cometary signature in evidence, *Astrophys. Jour.* **88**, 1382–1414, 1983.

[204] Global statistics confirm that earthquakes in the 20th century still take an average toll of 10,000 lives and cost $400 million a year worldwide. Earthquakes are still difficult to forecast: seismology, soil mechanics, and engineering cannot prevent vast damage. But most engineers believe that improved education,

1908 CE **Hermann Anschütz-Kaempfe** (1872–1931, Germany). Engineer. Invented the gyrocompass.

1908–1909 CE **Fritz Haber** (1868–1934, Germany). Chemist. One of the first chemists to bridge the gap between pure and theoretical chemistry on one hand and its industrial applications on the other. His most important work was on high-pressure synthesis of *ammonia* from its elements nitrogen and hydrogen[205]. This process known as the *Haber process of ammonia synthesis* (1909) (for which he was awarded the Nobel prize in 1919), *laid the foundation of the fertilizer and explosive industries*.

Haber was born in Breslau, Silesia, Germany (now Wročlav, Poland), the son of a Jewish dye-stuffs merchant. His early training was in organic chemistry with a view to his entering the family firm. At Karlsruhe, however, he took up the study of physical chemistry, and started to apply it to chemical problems of practical importance. He became a professor at Karlsruhe (1898), and director of the Kaiser Wilhelm Institute for Physical Chemistry, Berlin (1911–1933). During WWI he put all his energies into the German war effort, working on explosives, petrol, and the poison-gas chlorine[206].

Haber believed that his development of the use of *chlorine gas* for the German general staff would help bring a swift victory and thus limit overall suffering. On the eve of the first use of the gas against Allied troops in 1915, Haber's wife committed suicide, tormented by her husband's horrific contribution to the war.

Haber[207] then came up with a far more potent poison, the *mustard gas* ($C_4H_8SCl_2$) which was used with devastating results by Germans at Ypres in 1917. It is a volatile liquid, odorless and therefore is not immediately detectable by smell. Where it touches the skin and is inhaled, it forms blisters.

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warning systems, maps of fault lines and expected hazards, better building design, careful zoning, and stricter building codes can reduce risk to life.

205 The process operates at a high temperature (ca 450°C) and pressure (up to 1000 atmospheres) with *iron* as a catalyst: $N_2+3H_2 \leftrightharpoons 2NH_3$. The gases are cooled and the ammonia is separated. Only about 15% of the mixture is converted to ammonia. The unreacted substances are recycled.

206 Consequently, the award to him of the Nobel prize for chemistry led to some criticism from scientists in the allied countries. His work is said to have prolonged the first World War by two years, as it enabled Germany to manufacture explosives long after natural supplies of ammonia-yielding compounds had been exhausted.

207 His colleague **Gerhard Schroeder** produced the *nerve gas* ($C_6H_{14}O_3PF$) but it was never used by the Germans in WWII.

Those who do not die at once, or from infections that follow the blistering, suffer a generalized poisoning that renders them ill for the rest of their lives.

About 100,000 allied soldiers in WWI died due to gas poisoning. After the Armistice, the Allied considered Haber a war criminal. Haber was demoralized, but he continued to conduct research and provided the Nazis with *Cyclone B*, used in their *Gas Chambers* to murder six million of his own people during WWII. In 1920 he became involved in the futile effort to pay off the German war debts by means of gold extracted from seawater(!)

Upon the rise of the Nazis to power in Germany (1933), Haber was forced to resign his directorship of the Kaiser Wilhelm Institute[208]. He removed to Switzerland and died there, heartbroken, a year later.

1908–1922 CE **Carl Vilhelm Ludvig Charlier** (1862–1934, Sweden). Astronomer. Developed a cosmological theory of an "hierarchical" fractal universe along the lines of **Lambert** (1761), in which galaxies form clusters, clusters form superclusters and so on ad infinitum. By arranging the dimensions suitably, it is possible in this way to construct a universe with *zero average density*. [Example: A cluster of order n contains p^n stars of mass m in a volume kq^n; its average density $\rho = \left(\frac{m}{k}\right)\left(\frac{p}{q}\right)^n \to 0$ as $n \to \infty$ if $p < q$.] Thus, one avoids the infinities inherent in the Newtonian treatment of an homogeneous universe with a finite average density.

Charlier universes are inhomogeneous, and no volume V is large enough to be typical. This model eliminates *Olbers' paradox* if the average density of matter in a cluster is less for higher order clusters than for lower order clusters,

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[208] Haber's conversion to Christianity and all the services rendered by him to Germany's war-effort, industry and science did not make him less of a Jew in the eyes of the Nazis.

In May 1933, **Max Planck** went to congratulate the newly elected Führer. He seized upon the opportunity to plead on behalf of his friend Haber, who was already blacklisted. Planck spoke of Haber's major contributions to Germany's war-effort and postwar chemical industry. "*There are*", he said, "*nevertheless, Jews of various kinds, and distinction should be made in favor of those among them who are of great value to humanity and of distinguished German families of high culture...*". The Führer stopped him there and remarked: "*It is not true, a Jew is a Jew, ... I have to act against all of them in the same way*".

Planck then went on to say that Germany needs its Jewish scientists and their dismissal will cause many of them to leave Germany, which will have a grave consequences for German science. But the Führer went into a frenzy and Planck had to leave.

and is probably the only explanation which does not invoke a systematic motion of all stars[209].

Charlier was educated at the University of Uppsala and the Stockholm Observatory. He then became the observatory director at the University of Lund (1897–1927). After working in celestial mechanics, the calibration of photographic photometry, and the theory of lenses, he turned to statistics where he made extensive statistical studies of the distribution and motion of stars in the solar neighborhood. Named after him are: *Gram-Charlier series*, *Lunar crater Charlier*, *Martian crater Charlier* and Minor Planet #8677 Charlier. He was a *1933 Bruce Medalist*.

1908–1928 CE **Godfrey Harold Hardy**[210] (1877–1947, England). One of the greatest pure mathematicians of the 20^{th} century. Helped foster, more

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[209] Carl Charlier's first publication on hierarchy "*Wie eine unendliche Welt aufebauen sein kann*" (1908), contains mathematical errors. The correct result was first derived in 1909 by **Hugo von Seeliger** in a letter to Charlier, who acknowledged and used this result in a second article (1922).
John Herschel (1848), in his review of the first volume of Alexander von Humboldt's *Kosmos*, hinted at the possibility of hierarchical structure as a solution to Olbers' paradox. **Richard A. Proctor** (1837–1888, England), an astronomer and popularizer of science, presented in *Other Worlds than Ours* (1871) a semiquantitative treatment of a hierarchical solution. But neither Herschel nor Proctor derived the conditions with any generality and precision.
Edward Fournier d'Albe (England), in his book *Two New Worlds* (1907), dismissed absorption as a possible rescue mechanism for the Olbers' paradox and revived a long-forgotten idea of **Lord Kelvin** (1901): if the universe began in the finite past, then only a finite part is visible. The rest — the part beyond a certain distance — cannot be seen because the light from this part has not had time to reach us. Moreover, Fournier d'Albe championed the notion of hierarchy and elaborated on the idea that our visible universe is just one in a multiuniverse. All universes have similar structure, he suggested, and differ only in scale.
Fournier d'Albe's fractal theory of the universe, greatly influenced Charlier and stimulated him to derive the mathematical conditions for a hierarchical solution to the riddle of darkness.

[210] For further reading, see:

- Hardy, G.H., *A Mathematician's Apology*, Cambridge University Press, 1976, 153 pp.
- Hardy, G.H. and E.M. Wright, *An Introduction to the Theory of Numbers*, Oxford University Press, 1989, 426 pp.
- Hardy, G.H., *Ramanujan*, Chelsea: New York, 1940, 236 pp.

than anyone else, analytic methods for the resolution of discrete problems of higher arithmetic.

Hardy contributed fundamentally to many realms in mathematics, including analytic number theory, Diophantine analysis, Fourier series[211], divergent series, integral equations, inequalities, distribution of primes and the Riemann zeta function[212]. His books and articles had a decisive influence on the development of the above topics.

Hardy collaborated with **J.E. Littlewood** (1885–1977), **S. Ramanujan** (1887–1920) and **E. Landau** (1877–1938).

Although Hardy boasted that he had never done anything useful in the sense of practical applications, he discovered [concurrently with the German physician **Wilhelm Weinberg**] an important and useful law in population genetics. It became centrally important in the study of many genetic problems, including Rh blood group distribution and hemolytic diseases.

He showed that in large enough populations obeying Mendelian laws of heredity, and in the absence of outside influences and of mutations, random mating will produce within *one generation*, a *stationary* genotype distribution with unchanged gene frequency.

Hardy was born in Cranleigh, Surrey. Both his parents were extremely able people and mathematically minded, but want of funds had prevented them from acquiring a university training.

- Hardy, G.H., *A Course of Pure Mathematics*, Cambridge University Press, 1967, 509 pp.

[211] **Riemann** conjectured that the function $\sum_{n=1}^{\infty} \frac{\sin(\pi n^2 t)}{n^2}$ is continuous but nowhere differentiable. However, neither he nor **Weierstrass** succeeded in proving this. **Hardy** (1916) showed that the function is not differentiable at any irrational point and at some specific rationals. Finally (1971), mathematicians were able to show that it is differentiable at any rational point $\frac{P}{Q}$ with $P \equiv Q \equiv 1 \pmod 2$. Weierstrass (1872) introduced his famous function $\sigma(t) = \sum_{n=1}^{\infty} \alpha^n \cos(\beta^n t), \quad 0 < \alpha < 1$. He showed that this function is continuous but nowhere differentiable whenever $\alpha\beta$ exceeds a certain value. Hardy (1916) showed that $\alpha\beta > 1$ is all that is needed.

[212] He proved that there are an infinity of roots of $\zeta(x + iy) = 0$ on the line $\frac{1}{2} + iy$ $(y \neq 0)$. The *Riemann hypothesis* claims that *all* zeros of $\zeta(x + iy)$ are of that form.

He was a child prodigy[213]. He went to Trinity College, Cambridge, in 1896, was first wrangler in 1900 and was awarded (with **Jeans**) the Smith's prize in 1901. He taught mathematics at Cambridge during 1900–1919. In 1919 he was appointed to the Savilian chair of geometry at Oxford University. He returned to Cambridge in 1931 as Sadleirian professor of pure mathematics, and remained there until his death.

Hardy is credited with the following toast in a meeting of the London Mathematical Society: "*Here is to pure mathematics, let it be of no use to anybody*". Hardy described himself as a *problem-solver*, and did not claim to have introduced any new system of ideas. Yet he had a profound influence on the mathematics of his time.

[213] May this explain his interest in Ramanujan and also his preoccupation with population genetics?

Worldview XXX: Hardy

* * *

"The function of a mathematician is to do something, to prove new theorems, to add to mathematics, and not to talk about what he or other mathematicians have done. Painters despise art-critics, and physicists, or mathematicians have usually similar feelings. Exposition, criticism, appreciation, is the work for second rate minds."

* * *

"Most people can do nothing at all well; perhaps five or even ten percent of men can do something rather well.It is a tiny minority who can do anything really *well, and the number of people who can do two things well is negligible."*

* * *

"Mathematics, more than any other art or science, is a young man's game. Newton, who was one of the world's three greatest mathematicians, gave up mathematics at fifty ... His greatest ideas of all, fluxions and the law of gravitation, came to him when he was twenty-four(1666). He made big discoveries until he was nearly forty, but after that he did little but polish and perfect. Galois died at twenty-one, Abel at twenty seven, Ramanujan at thirty-three, Riemann at forty. There have been men who have done great work a good deal later; Gauss' great memoir on differential geometry was published when he was fifty (thought he had the fundamental ideas ten years before), I do not know an instance of a major mathematical advance initiated by a man past fifty."

* * *

"If intellectual curiosity, professional pride, and ambition are the dominant incentives to research, then assuredly no one has a finer chance of gratifying them than a mathematician. His subject is the most curious of all – there is none in which truth plays such odd pranks. It has the most elaborate and the

most fascinating technique, and gives unrivaled openings for the display of sheer professional skills. Finally, as history proves abundantly, mathematical achievement, whatever its intrinsic worth, is the most enduring of all. We can see this even in semi-historic civilization. The Babylonian and Assyrian civilizations have perished; Hammurabi, Sargon, and Nebuchadnezzar are empty names; yet Babylonian mathematics is still interesting, and the Babylonian scale of 60 is still used in astronomy. But of course the crucial case is that of the Greeks.

The Greeks were the first mathematicians who are still 'real' to us today. Oriental mathematics may be an interesting curiosity, but Greek mathematics is the real thing. The Greeks first spoke a language which modern mathematicians can understand; as Littlewood said to me once, they are not clever schoolboys or 'scholarship candidates', 'but fellows of another college'. Immortality may be a silly word, but probably a mathematician has the best chance of whatever it may mean... Mathematical game, if you have the cash to pay for it, is one of the soundest and steadiest of investments."

* * *

"Immortality is often ridiculous or cruel: few of us would have chosen to be Og or Ananias or Gallio. Even in mathematics, history sometimes plays strange tricks; Rolle figures in the text-books of elementary mathematics like Newton; Farey is immortal because he failed to understand a theorem which Haros have proved perfectly fourteen years before; the names of five worthy Norwegians still stand in Abel's Life*, just for one act of conscientious imbecility, dutifully performed at the expense of their country's greatest man. But on the whole the history of science is fair, and this is particularly true in mathematics. No other subject has such clearcut of unanimously accepted standards, and the men who are remembered are almost always the men who merit it."*

* * *

"A mathematician like a painter or a poet, is a maker of patterns... The mathematicians' patterns, like painters' or the poet's, must be beautiful*; the ideas, like the colors or the words, must fit together in a harmonious way. It may be very hard to* define *mathematical beauty, but that is just as true of beauty of any kind – we may not know quite what we mean by a beautiful poem, but that does not prevent us from recognizing one when we read it. Every chess-player can recognize and appreciate a 'beautiful' game or a problem. Yet a chess problem is* simply *an exercise in pure mathematics."*

* * *

A chess problem is genuine mathematics, but it is in some way 'trivial' mathematics. However ingenious and intricate, however original and surprising the moves, there is something essential lacking. Chess problems are unimportant. *The best of mathematics is* serious *as well as beautiful... The 'seriousness' of a mathematical theorem lie, not in its practical consequences, which are usually negligible, but in the* significance *of the mathematical ideas which it connects. We may say, roughly, that a mathematical idea is 'significant' if it can be connected, in natural and illuminating way, with a large complex of other mathematical ideas. Thus a serious mathematical theorem, a theorem which connects significant ideas, is likely to lead to important advances in mathematics itself and even in other sciences. No chess problem has ever affected the general development of scientific thought. The chess problem is the product of an ingenious but very limited complex of ideas, which do not differ from one another very fundamentally and had no external repercussions. We should think in the same way if chess had never been invented, whereas the theorems of Euclid and Pythagoras have influenced thought profoundly even outside mathematics."*

* * *

Euclid's theorem *(existence of the infinity of prime numbers) is vital for the whole structure of arithmetics. The primes are the raw material out of which arithmetics is built, and Euclid's theorem assures us that we have plenty of material for the task. But the* theorem of Pythagoras *(irrationality of* $\sqrt{2}$*) and its extensions (to very wide class of 'irrationals') has wider applications: Euclid's theorem tells us that we have a good supply of material for the construction of a coherent arithmetics of the integers. Pythagoras' theorem and it's extensions tell us that, when we have constructed this arithmetic, it will not prove sufficient for our needs, since there will be many magnitudes which obtrude themselves upon our attention and which it will be unable to measure. Pythagoras discovery led to the construction of the much more profound theory of Eudoxos , the finest achievements of Greek mathematics. This theory is astonishingly modern in spirit, and may be regarded as the beginning of the modern theory of irrational numbers, which has revolutionized mathematical analysis and had much influence on recent philosophy... It is obvious that irrationals are uninteresting to an engineer, since he is concerned only with approximations, and all approximations are rational."*

* * *

“A serious theorem is a theorem which contains ‘significant’ ideas. There are two things which seem essential, a certain generality *and a certain* depth*:*

The idea should be one which is a consistent in many mathematical constructions, even if stated originally in a fully special form, is capable of considerable extension and is typical of a whole class of theorems of its kind. The relation revealed by the proof should be such as connect many different mathematical ideas.

Depth is difficult to define. It has something to do with difficulty*. It seems that mathematical ideas are arranged somehow in strata, the ideas in each stratum being linked by a complex of relations both among themselves and above and below. The lower the stratum, the deeper (and is general the more difficult) the idea. Thus the idea of an ‘irrational’ is deeper than that of an integer, but there are many theorems about integers which we cannot appreciate properly, and still less prove, without digging deeper considering what happens below”.*

* * *

“A physicist is trying to correlate the incoherent body of crude fact confronting him with some definite orderly scheme of abstract relations, the kind of schema which he can borrow only from mathematics.

A mathematician, on the other hand, is working with his own reality. Of this reality, I take a ‘realistic’ and not an idealistic view.

This realistic view is much more plausible of mathematical than of physical reality, because mathematical objects are so much more what they seem. A chair or a star is not in the least like what it seems to be; the more we think of it, the fuzzier its outline become in the haze of sensation which surrounds it; but 2 or 317 has nothing to do with sensations, and its properties stand out the more clearly the more we scrutinize it. Pure mathematics seems to me a rock on which all idealism founders: 317 is a prime, not because we think so, or because our minds are shaped in one way rather than another, but because it is so*, because mathematical reality is built that way.”*

* * *

“A science or an art may be said to be ‘useful’ if its development increases, even indirectly, the material well-being and comfort of men, if it promotes happiness. It is undeniable that a good deal of elementary mathematics (including calculus) has considerable practical utility; the engineers could not

do their job without a fair working of mathematics, and mathematics is beginning to find applications in medicine and biology and the social sciences. These parts of mathematics are, on the whole, rather dull; they are just the parts which have least aesthetic value. The 'real' mathematics of the 'real' mathematicians, the mathematics of Fermat and Euler and Gauss and Abel and Jacobi and Riemann and Ramanujan, is almost wholly 'useless'. It is the dull and elementary parts of applied mathematics, as the dull and elementary parts of pure mathematics, that work for good and ill[214]. It is the commonplace and dull that counts for practical life."

* * *

"In great mathematics there is a very high degree of unexpectedness, combined with inevitability and economy."

* * *

"Young Men should prove theorems, old-men should write books"

* * *

"The mathematician's patterns, like the painter's or the poet's must be beautiful; the ideas, like the colors or the words must fit together in a harmonious way. Beauty is the first test: there is no permanent place in this world for ugly mathematics."

* * *

"I believe that mathematical reality lies outside us, that our function is to discover or observe it and that the theorems which we prove, and which we describe grandiloquently as our "creations," are simply the notes of our observations."

* * *

214 During WWI Hardy took this to extreme, saying that: "a science is said to be useful if its development tends to accentuate the existing inequalities in the distribution of wealth, or more directly promotes the destruction of human life."

“Archimedes will be remembered when Aeschylos is forgotten, because languages die and mathematical ideas do not. “Immortality” may be a silly word, but probably a mathematician has the best chance of whatever it may mean.”

* * *

“The fact is that there are few more “popular” subjects than mathematics. Most people have some appreciation of mathematics, just as most people can enjoy a pleasant tune; and there are probably more people really interested in mathematics than in music. Appearances may suggest the contrary, but there are easy explanations. Music can be used to stimulate mass emotion, while mathematics cannot; and musical incapacity is recognized (no doubt rightly) as mildly discreditable, whereas most people are so frightened of the name of mathematics that they are ready, quite unaffectedly, to exaggerate their own mathematical stupidity.”

* * *

“I had better say at once that by ‘mathematics I mean real mathematics, the mathematics of ***Fermat*** *and* ***Euler*** *and* ***Gauss*** *and* ***Abel****; and not the stuff that passes for mathematics in an engineering laboratory. I am not thinking of “‘pure mathematics” (though naturally that is my first concern), I count* ***Maxwell*** *and* ***Einstein*** *and* ***Eddington*** *and* ***Dirac*** *among real mathematicians.”*

* * *

“It seems to me that no philosophy can possibly be sympathetic to a mathematician which does not admit, in one manner of other, the immutability and unconditional validity of mathematical truth. Mathematical theorems are true or false; their truth or falsity is absolutely independent of our knowledge of them. In some sense, mathematical truth is part of objective reality.”

Hardy-Littlewood conjecture — the clustering of Prime Numbers (1919–1922)

The prime number theorem tells us that the average density of primes around x is approximately[215] $\frac{x/\ln x}{x} = \frac{1}{\ln x}$*. This means that if we consider an interval of length Δx about x and choose any integer t in this interval, then the probability of t being a prime will approach* $\frac{1}{\ln x}\Delta x$ *as* $x \to \infty$, *if* Δx *is small compared to* x.

This implies that the primes tend to thin out as x grows larger. The law that governs the distribution of primes as we go higher up in the number series has boggled the minds of number-theorists since the time of Euler. We know, for example that the so-called "twin primes" (i.e., pairs of primes of the form x, $x + 2$), occur very high up in the number series. The largest pair discovered up to 1995 is

$$242,206,083 \cdot 2^{38,880} \pm 1,$$

and has 11,713 *decimal digits. Statistics indicate that twins tend to thin out as we move higher up in the number series since, if p is a prime, it becomes less and less likely that $p + 2$ is also a prime on account of the thinning out of the primes themselves. Indeed,* **Viggo Brun** *(1885–1978) proved that the sum of* $\frac{1}{p}$ *taken over all twin primes converges:*

$$B = \left(\frac{1}{3} + \frac{1}{5}\right) + \left(\frac{1}{5} + \frac{1}{7}\right) + \left(\frac{1}{11} + \frac{1}{13}\right) + \left(\frac{1}{17} + \frac{1}{19}\right) + \dots$$
$$= 1.902\ 160\ 54\dots$$

where B is known as the Brun constant. This theorem (1919) tells us that there are not very many twin primes compared with the total number of primes, since $\sum \frac{1}{p}$ taken over only the twin converges, while $\sum \frac{1}{p}$ extended over all primes diverges.

A study of prime tables has shown that constellations of primes other than twins reoccur in the number series. Of special interest are prime triplets $(p, p+2, p+6)$ *and* $(p, p+4, p+6)$, *and prime quadruplets* $(p, p+2, p+6, p+8)$. *These constellations are exemplified in* $(41, 43, 47)$, $(37, 41, 43)$ *and* $(11, 13, 17, 19)$, $(101, 103, 107, 109)$ *respectively.*

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[215] The nth prime has therefore the approximate value $p_n \sim n \ln n$.

***Hardy** and **Littlewood** (1922) made the following conjecture: P_x, the number of prime constellations $p \le x$, as $x \to \infty$ is asymptotic to*

$$\pi_2(x) = P_x(p, p+2) \sim 2C_2 \int_2^x \frac{du}{(\ln u)^2} \le 2C_2 \frac{x}{(\ln x)^2}$$

$$C_2 = \prod_{p \ge 3} \left\{1 - \frac{1}{(p-1)^2}\right\} = \prod_{p \ge 3} \frac{p(p-2)}{(p-1)^2} = 0.660\ 16\ldots$$

The 'twin-prime constant' C_2 was calculated in 1961. Numerical evidence shows (1976) that $\pi_2(10^{11}) = 224,376,048$. For other prime constellations, the Hardy-Littlewood formulae are:

$$\pi_3(x) = P_x(p, p+2, p+6) \sim P_x(p, p+4, p+6) \sim \frac{9}{2}C_3 \int_2^x \frac{du}{(\ln u)^3}$$

$$\frac{9}{2}C_3 = \frac{9}{2} \prod_{p \ge 5} \frac{p^2(p-3)}{(p-1)^3} = 2.858\ 248\ 596\ldots$$

$$\pi_4(x) = P_x(p, p{+}2, p{+}6, p{+}8) \sim \frac{1}{2}P_x(p, p{+}4, p{+}6, p{+}10) \sim \frac{27}{2}C_4 \int_2^x \frac{du}{(\ln u)^4}$$

$$\frac{27}{2}C_4 = \frac{27}{2} \prod_{p \ge 5} \frac{p^3(p-4)}{(p-1)^4} = 4.151\ 180\ 863\ 237\ 4\ldots$$

Some of the largest known prime quadruplets $(p, p+2, p+6, p+8)$ are:

$$p = 10^{99} + 349,781,731 \qquad (1995)$$

$$p = 10^{499} + 883,750,143,961 \qquad (1996)$$

$$p = 10^{599} + 1,394,283,756,151 \qquad (1997)$$

The table below compares the computer-count against calculation made by means of the Hardy-Littlewood formulas for $x \le 10^8$:

Constellation	*Count*	*Approximation*
$(p, p+2)$	$440,312$	$440,368$
$(p, p+2, p+6)$	$55,600$	$55,490$
$(p, p+4, p+6)$	$55,556$	$55,490$
$(p, p+2, p+6, p+8)$	$4,768$	$4,734$

The Hardy-Littlewood conjecture can be extended to longer finite constellations, for example it can be shown that

$$P_x(p+11, p+13, p+17, \ldots, p+59, p+61, p+67) \sim 187{,}823 \cdot 7 \int_2^x \frac{du}{(\ln u)^{15}}$$

where the numerical constant is equal to

$$\frac{2^{14}}{1^{15}} \cdot \frac{3^{14}}{2^{15}} \cdot \frac{5^{14}}{4^{15}} \cdot \frac{7^{14}}{6^{15}} \cdot \frac{11^{14}}{10^{15}} \cdot \frac{13^{14}}{12^{15}} \cdot \frac{4 \cdot 17^{14}}{16^{15}} \cdot \frac{6 \cdot 19^{14}}{18^{15}} \cdot \frac{9 \cdot 23^{14}}{22^{15}} \cdot \prod_{p \geq 29} \frac{p^{14}(p-15)}{(p-1)^{15}}.$$

Despite the fact that the primes are not all individually known, these constants can be computed to any desired accuracy*! This astonishing result is due to the fact that the constants are expressible in terms of the* prime zeta-function.[216]

An Unbelievable Identity (1917)

The classical theory of numbers was based, up to the 18th century, on the unique representation of an integer as a product of prime numbers*; these* multiplicative *building blocks, and the concept of* divisibility *— are central to the theory.* ***Euler*** *(1748) laid the foundations to a new branch of number theory centered on the representation of integers as* sums of other integers *–*

216 *e.g.:*

$$\begin{aligned}
\ln C_2 &= \sum_{p\geq 3} \ln \frac{p(p-2)}{(p-1)^2} = \sum_{p \geq 3} \left\{ \ln\left(1 - \frac{2}{p}\right) - 2\ln\left(1 - \frac{1}{p}\right) \right\} \\
&= \sum_{p\geq 3} \left\{ -\frac{2}{p} - \frac{1}{2} \cdot \frac{4}{p^2} - \frac{1}{3} \cdot \frac{8}{p^3} - \cdots + \frac{2}{p} + \frac{1}{2} \cdot \frac{2}{p^2} + \frac{1}{3} \cdot \frac{2}{p^3} + \ldots \right\} \\
&= \sum_{p\geq 3} \sum_{j=2}^{\infty} \frac{2 - 2^j}{j} \cdot \frac{1}{p^j} = -\sum_{j=2}^{\infty} \frac{2^j - 2}{j} \sum_{p \geq 3} \frac{1}{p^j}
\end{aligned}$$

an additive theory, based on the concept of partition[217]*. The word "partition" has numerous meanings in mathematics. Any time a division of some object into subobjects is undertaken, the word partition is likely to pop up. In the present context, the partitions of a number are the ways of writing that number as a sum of positive integers*[218]*. For example, the five partitions of* 4 *are*

$$4, \quad 3+1, \quad 2+2, \quad 2+1+1, \quad 1+1+1+1$$

and we write $p(4) = 5$*. Surprisingly, such a simple matter requires some deep mathematics for its study.*

The theory of partitions has an interesting history. Certain special problems in partition date back to the Middle Ages; however, the first discoveries of any depth were made when **Euler** *proved many beautiful and significant partition theorems. Other great mathematicians —* **Lagrange** *(ca 1775),* **Legendre** *(1801),* **Gauss** *(1808),* **Jacobi** *(1829),* **Sylvester** *(1855),* **Cayley** *(1876),* **Ramanujan** *(1917) and* **Hardy** *(1917) have contributed to the development of the theory. The various theoretical aspects of this subject have recently found applications to statistical mechanics, combinatorics, analysis and number theory. In these diverse applications, one is struck by the interplay of combinatorial and asymptotic methods.*

Returning to the concept of partition of a positive integer n*, we define it as a finite nonincreasing sequence of positive integers* $\lambda_1, \lambda_2, \ldots, \lambda_r$*, such that* $\sum_{i=1}^{r} \lambda_i = n$*. The partition function* $p(n)$ *is the number of partitions of* n*.*

For example

$p(0) = 1$ *[an empty sequence forms the only partition of zero]*

$p(1) = 1$

$p(2) = 2$;2, $1+1$

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[217] To dig deeper, see:

- Andrews, G.E., *The Theory of Partitions*, Cambridge University Press, 1998, 255 pp.

[218] In the *general problem of additive arithmetic* we are given a system A of integers $a_1, a_2, a_3, \ldots$, where A might contain all the positive integers, or the squares, or the primes. We then consider all possible representations of an arbitrary positive integer in the form $n = a_{i_1} + a_{i_2} + \cdots + a_{i_s}$, where s may be fixed or unrestricted, the a's may or may not be necessarily different, and order may or may not be relevant, according to the particular problem considered. We denote by $p(n)$ the number of such representations. We then ask, what can be said about $p(n)$? e.g., is there always one representation for every n?

$p(3) = 3$;3, 2 + 1, 1 + 1 + 1

$p(4) = 5$;4, 3 + 1, 2 + 1 + 1, 1 + 1 + 1 + 1

$p(5) = 7$;5, 4 + 1, 3 + 2, 3 + 1 + 1, 2 + 2 + 1, 2 + 1 + 1 + 1, 1 + 1 + 1 + 1 + 1

$p(6) = 11$

$p(7) = 15$

$p(8) = 22$

$p(9) = 30$

$p(10) = 42$

$p(20) = 627$

$p(50) = 204,226$

$p(100) = 190,569,292$

$p(200) = 3,972,999,029,388$

$p(721) = 161,061,755,750,279,477,635,534,762$

To evaluate $p(n)$, ***Euler*** *invented a powerful weapon provided by the theory of generating functions; namely, he was seeking a function* $F(x)$, *whose power-series expansion has* $p(n)$ *for its general coefficient. Indeed, he discovered that*

$$F(x) = \prod_{n=1}^{\infty}(1-x^n)^{-1} = \frac{1}{(1-x)(1-x^2)(1-x^3)\dots} = 1 + \sum_{n=1}^{\infty} p(n)x^n = \sum_{n=0}^{\infty} p(n)x^n. \tag{1}$$

To see this, one writes the infinite product as

$$(1 + x + x^2 + \dots)(1 + x^2 + x^4 + \dots)(1 + x^3 + x^6 + \dots)\dots$$

and multiplies the series together: Every partition of n *contributes just* 1 *to the coefficient of* x^n. *It is obvious that the finite product* $\left[(1-x)(1-x^2)\dots(1-x^m)\right]^{-1}$ *enumerates the partitions of* n *into parts which do not exceed* m *or (what is the same thing) into at most* m *distinct parts.*

Hardy *and* ***Ramanujan*** *(1917) set forth to invert (1), namely solve for* $p(n)$ *in terms of* $F(x)$. *Clearly, Cauchy's integral theorem implies that*

$$p(n) = \frac{1}{2\pi i}\int_C \frac{F(x)}{x^{n+1}}\,dx, \tag{2}$$

where, say, C is a circle centered in the origin and inside *the unit circle $|x| = 1$. The evaluation of this integral leads eventually to the* exact *result (as completed by* **Rademacher** *in 1937):*

$$p(n) = \frac{1}{\pi\sqrt{2}} \sum_{k=1}^{\infty} A_k(n)\sqrt{k}\Big[\frac{d}{dx}\frac{\operatorname{sh}\left\{\frac{\pi}{k}\sqrt{\frac{2}{3}(x-\frac{1}{24})}\right\}}{\sqrt{x-\frac{1}{24}}}\Big]_{x=n}, \tag{3}$$

where

$$A_k(n) = \sum_h \omega_{h,k} e^{-\frac{2\pi i n h}{k}},$$

the sum being over h values prime to k and less then it, $\omega_{h,k}$ being a certain 24^{qth} *root of unity*[219].

As one may discern, $p(n)$ grows astronomically. Actual enumeration of the $3,972,999,029,388$ *partitions of* 200 *by hand would certainly take more than a lifetime. However, the first* 5 *terms of the remarkable Hardy-Ramanujan formula give the correct value of* $p(200)$.

This unbelievable identity wherein the left-hand side is the humble arithmetic function $p(n)$ and the right-hand side is an infinite series involving π, square roots, complex roots of unity, and derivatives of hyperbolic functions provided not only a theoretical formula for $p(n)$ but also a formula which admits relatively rapid computation.

For example, comparing the first 8 *terms of the series for $n = 200$, we find that the result is*

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219 Hardy and Ramanujan actually proved that $p(n)$ is the *integer nearest* to $\sum_{k=1}^{\nu}$, where ν is of the order $\sqrt{n}$. One thus obtains the *exact* value of $p(n)$ with a *finite* number of terms. **Ramanujan** (1913), in his famous letter to Hardy (problem 14), actually suggested that the first term of the series may suffice. "Ramanujan's false statement" Hardy later said, "was one of the most fruitful he ever made."

1st	*term*	$+3,972,988,993,185.896$
2nd	*term*	$+36,282.978$
3rd	*term*	-87.555
4th	*term*	$+5.147$
5th	*term*	$+1.424$
6th	*term*	$+0.071$
7th	*term*	$+0.000$
8th	*term*	$+0.043$
		$3,972,999,029,388.004$

which is the correct value of $p(n)$ *within* 0.004.

The Hardy-Ramanujan formula is connected also to the theory of modular functions. To see this we put $x = q^2 = e^{2\pi i\tau}$ $(\mathrm{Im}\ \tau > 0)$ *into Euler's formula. It then follows that* $q^{\frac{1}{12}} \prod_1^\infty (1 - q^{2n}) = x^{\frac{1}{24}}/F(x)$.

Prior to 1917, $p(n)$ *were calculated either directly from Euler's formula* (1) *or from the known recursion formula*[220]

$$p(n) = \frac{1}{n} \sum_{k=1}^{n} \sigma(k) p(n-k), \tag{4}$$

where $\sigma(k)$ *is the sum of divisors of* k. *Thus* $p(n)$ *was calculated for* $n \leq 200$ *by* ***Percy Alexander MacMahon***[221] *(1916). Ramanujan used his data to discover, with the aid of the theory of modular functions, that:*

$$p(5m + 4) \equiv 0 \pmod{5}$$
$$p(7m + 5) \equiv 0 \pmod{7}$$
$$p(11m + 6) \equiv 0 \pmod{11}$$
$$p(25m + 24) \equiv 0 \pmod{5^2}$$
$$p(49m + 47) \equiv 0 \pmod{7^2}$$

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220 Proved with the aid of Euler's formula

$$(1 - x)(1 - x^2)(1 - x^3) \cdots = \sum_{n=-\infty}^{\infty} (-1)^n x^{\frac{1}{2}n(3n+1)}.$$

221 Soldier and mathematician (1854–1928, England). Served as a major in the Artillery before seriously taking up mathematics. He then became a professor of mathematics at the Woolrich army school. Since 1904 had been associated with St. John's college in Cambridge. Contributed to combinatorics.

For large values of n, one obtains from (3) the asymptotic law (with $k = 1$, $A_k(n) = 1$):

$$p(n) \approx \frac{1}{4n\sqrt{3}} e^{\pi\sqrt{\frac{2n}{3}}} \qquad (5)$$

For example, with $n = 243$, *(5) yields* $p(n) \approx 1.38 \times 10^{14}$ *while the exact result is*

$$p(243) = 133,978,259,344,888.$$

In deriving the analytic expression for $p(n)$, Ramanujan supplied the central idea in the form of a conjecture and Hardy furnished the technical skill. ***Hans Adolph Rademacher*** *(1892–1969, Germany) made the formula exact in 1937.*

Hardy and Ramanujan were a formidable pair; as a mathematical team, they would remind us of the story of the two men, one blind and the other lame, who together could do what no normal man could. Whatever the proper assignment of credit, "we owe the theorem" Littlewood would write, "to a singularly happy collaboration of two men, of quite unlike gifts, in which each contributed the best, most characteristic, and most fortunate work that was in him. Ramanujan's genius did have this one opportunity worthy of it."

1909 CE **William Bateson** (1861–1926, England). Geneticist. Coined the term *genetics*. One of the founders of the science of genetics. Experimentally confirmed Mendel's reports and demonstrated *genetic linkage*.

1909 CE **Axel Thue** (1863–1922, Norway). Mathematician. Proved[222] that for all integers $m \neq 0$ and $n \geq 3$, the Diophantine equation with integer coefficients

$$g(x, y) = a_0x^n + a_1x^{n-1}y + \cdots + a_{n-1}xy^{n-1} + a_ny^n = m$$

has at most a finite number of integer solutions x, y.

Thue was born in Tönsberg. He enrolled at Oslo University in 1883. He was a professor of applied mathematics at Oslo from 1903 to 1922. During

[222] Thue's theorem was characterized by **Edmund Landau** (1922) as "*the most important discovery in elementary number theory that I know*". In 1967, **Alan Baker** (England) proved that for any solution, both $|x|$ and $|y|$ cannot exceed a certain constant C whose value can be calculated explicitly in terms of m, n and the coefficients $a_0, a_1, \ldots, a_n$.

1890–1891 he studied at Leipzig under **Sophus Lie**, but his works do not reveal Lie's influence, probably because of Thue's unwillingness to follow anyone else's line of thought.

1909 CE **A. Wieferich** (Germany). Mathematician. Proved that if p is a prime and if xyz is not a multiple of p, the equation $x^p + y^p = z^p$ cannot be solved in integers unless p^2 divides $(2^{p-1} - 1)$. This criterion is sufficient to preclude solutions for all $p < 100,000$, except 1093 and 3511. (No one has proven that there are infinitely many p values which satisfy $p^2 \nmid 2^{p-1} - 1$). In 1910, **D. Mirimanoff** showed that $p^2 \nmid 3^{p-1} - 1$ is an equally valid criterion.

In 1914, **H.S. Vandiver** proved that $p^2 \nmid (5^{p-1} - 1)$ is also a necessary condition for $x^p + y^p = z^p$ to be soluble. Then **Frobenius** arrived with 11 and 17, and it has also been shown that if p is a prime of the form $6x - 1$, then $7^{p-1} - 1$, $13^{p-1} - 1$ and $19^{p-1} - 1$ must each be divisible by p^2 if Fermat's conjecture is to be violated for power p. Under such restrictions, the minimal prime exponent p required was continually raised, and in 1941 it was shown that if $x^p + y^p + z^p = 0$ and no member of the triple x, y, and z is a multiple of p, then p must be not less than $253,747,889$.

Prime Numbers[223] *— The Inexplicable Secret*

* * *

"And Joseph gathered corn as the sand of the sea, very much, until he stopped numbering; for it was without number."

(*Genesis 41*, 49; ca 1750 BCE)

"The sequence of Prime numbers is a mystery that the human mind will never penetrate."

(**Leonhard Euler**)

"There are two facts about the distribution of prime numbers which I hope to convince you so overwhelmingly that they will be permanently engraved in your hearts. The first is that despite their simple definition and role as the building blocks of the natural numbers, the prime numbers belong to the most arbitrary and ornery objects studied by mathematicians: they grow like weeds among the natural numbers, seeming to obey no other law than that of chance, and nobody can predict where the next one will sprout.

The second fact is even more astonishing, for it states just the opposite: that the prime numbers exhibit stunning regularity, that there are laws governing their behavior, and that they obey these laws with almost military precision."

(**Don Zagier**, Inaugural lecture, Bonn University; May 05, 1975 CE)

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[223] To dig deeper, see:

- Du Sautoy, M., *The Music of the Primes*, Perennial, 2004, 335 pp.
- Derbyshire, J., *Prime Obsession*, Joseph Henry Press: Washington, D.C., 2003, 422 pp.
- Conway, J.H. and R.K. Guy, *The Book of Numbers*, Copernicus, 1995, 310 pp.
- Landau, E., *Hendbuch Der Lehre Von Der Veteilung Der Primzahlen*, Chelsea, 1953, vol I–II.

An integer greater than one is called a prime number if its only positive divisors (factors) are one and itself. For example, the prime divisors of 10 *are* 2 *and* 5*; and the first six primes are* 2*,* 3*,* 5*,* 7*,* 11 *and* 13*. The Fundamental Theorem of Arithmetic states that the primes are the building blocks of the positive integers: every positive integer is a product of prime numbers in one and only one way, except for the order of the factors. (This is the key to their importance: the prime factors of an integer determine its properties.)*

The ancient Greeks proved (ca 300 BC) that there were infinitely many primes and that they were irregularly spaced (there can be arbitrarily large gaps between successive primes). On the other hand, in the nineteenth century it was shown that the number of primes less than or equal to n *approaches a constant times* $n/(\ln n)$ *(as* n *becomes very large); so a rough estimate for the* n*th prime is* $n \ln n$*.*

The Sieve of Eratosthenes is still the most efficient way of finding all very small primes (e.g., those less than $1,000,000$*). However, most of the largest primes are found using special cases of Lagrange's Theorem from group theory.*

In 1984 Samuel Yates defined a titanic prime to be any prime with at least $1,000$ *decimal digits. When he introduced this term there were only* 110 *such primes known; now there are over* 1000 *times that many! And as computers and cryptology continually provide new impetus to search for ever larger primes, this number will continue to grow. Before long we expect to see the first ten-million digit prime.*

The factorization of a number into two large prime numbers, found in 1997 by Samuel Wagstaff and colleagues at the University of Indiana.

$$(3^{349} - 1)/2$$
$$=$$
$$163,790,195,580,536,623,921,741,301,546,724,495,839,239,656,848,$$
$$327,040,249,837,817,092,396,946,863,513,212,041,565,096,492,200,$$
$$805,419,718,247,075,557,971,445,689,690,738,777,729,730,388,837,$$
$$174,490,306,288,873,892,840,41$$
$$=$$
$$940,428,508,899,845,109,982,891,523,204,385,417,985,$$
$$320,180,216,539,562,837,411,932,116,540,252,801,854,59$$
$$\times$$
$$174,165,497,408,752,564,647,463,889,994,805,339,909,$$
$$443,342,668,496,870,546,115,249,228,788,407,082,066,088,604,99$$

The ten largest known primes as of this writing are:

Prime	Decimal Digits	Discovered
$2^{13466917}-1$	4053946	*2001*
$2^{6972593}-1$	2098960	*1999*
$2^{3021377}-1$	909526	*1998*
$2^{2976221}-1$	895932	*1997*
$2^{1398269}-1$	420921	*1996*
$1483076^{65536}+1$	404434	*2003*
$1478036^{65536}+1$	404337	*2002*
$54767 \cdot 2^{1337287}+1$	402569	*2002*
$1361846^{65536}+1$	402007	*2002*
$1266062^{65536}+1$	399931	*2002*

Up to 1800, the theory of numbers consisted of a collection of isolated results. ***Gauss****' work (1801) heralded the modern theory of numbers and determined the directions of work in the subject up to the present time. Subsequent innovations were made by* ***Dirichlet*** *(1837–41) and* ***Riemann*** *(1859) in analytic number theory, which uses analysis in addition to algebra to treat problems involving the integers.*

Positive integers can be partitioned into three classes: the unit, 1*; the* primes, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, . . .; *and the* composite numbers, 4, 6, 8, 9, 10, ... *A number greater than* 1 *is* prime *if its only positive divisors are* 1 *and itself; otherwise it is* composite.

Primes have interested mathematicians at least since Euclid, who proved they were infinitely many. The general problem of determining whether a large number is prime or composite, and in the latter case determining its factors, has fascinated number theorists down the ages. With the advent of high speed computers in the second half of the 20th century, considerable advances have been made, although most fundamental problems in number theory still remain unsolved[224]*, awaiting the mathematicians of genius in fu-*

224 For example:

- To find a prime number greater than a given prime.
- To find the prime number which follows a given prime.

ture centuries[225].

Prime numbers do not distinguish themselves from composite numbers in any obvious way. If we look, for example, at the list of all prime and some odd composite numbers up to 100*,*

prime:	2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
composite:	9, 15, 21, 25, 27, 33, 35, 39, 45, 49, 51, 55, 57, 63, 65, 69, 75, 77, 81, 85, 87, 91, 93, 95, 99,

and list the primes among the 100 *numbers immediately preceding and the same number immediately following* 10 *million:*

9,999,901	10,000,019
9,999,907	10,000,079,
9,999,929	
9,999,931	
9,999,937	
9,999,943	
9,999,971	
9,999,973	
9,999,991,	

we see no apparent reason why one number is prime and another is composite. To the contrary, upon looking at these numbers one has the feeling of being in the presence of one of the inexplicable secrets of creation. That mathematicians have not penetrated this secret is perhaps most convincingly shown by

- To compute directly the nth prime number, when n is given.
- To find the number of primes not greater than a given prime (exact formula).

[225] The twelve greatest number theorists during the past 25 centuries are: **Pythagoras** (ca 580–500 BCE, Greece); **Euclid** (330–260 BCE, Greece); **Diophantos** (206–290 CE, Greece); **Fermat** (1601–1665, France); **Euler** (1707–1783, Switzerland); **Lagrange** (1736–1813, France); **Legendre** (1752–1833, France); **Gauss** (1777–1855, Germany); **Jacobi** (1804–1851, Germany); **Riemann** (1826–1866, Germany); **Hardy** (1877–1947, England); **Ramanujan** (1887–1920, India).

the ardor with which they search for bigger and bigger primes. With numbers which grow regularly, like squares or powers of two, nobody would ever bother writing down examples larger than the previously known ones, but for prime numbers, people have gone to a great deal of trouble to do just that.

Moreover, it was found that the inquiry into composite numbers is not easier than investigations about prime numbers! Thus, for example, we are not yet able to answer the question whether or not, among the Fermat numbers $F_n = 2^{2^n} + 1 \quad (n = 1, 2, 3, \dots)$, *there are infinitely many composite numbers. So far we only know about* 100 *such composites, of which the greatest is* F_{23471} *(year 1999).*

A composite number C *can always be written as a product in at least two ways (since* $1 \cdot C$ *and* $C \cdot 1$ *are always possible). Call these two products*

$$C = ab = cd,$$

then it is obviously the case that c *divides* ab. *Set*

$$c = mn,$$

where m *is the part which divides* a, *and* n *the part which divides* b. *Then there are integers* p *and* q *such that*

$$a = mp$$
$$b = nq.$$

Solving $ab = cd$ *for* d *gives*

$$d = \frac{ab}{c} = \frac{(mp)(nq)}{mn} = pq.$$

It then follows that

$$\begin{aligned} S &\equiv a^2 + b^2 + c^2 + d^2 \\ &= m^2p^2 + n^2q^2 + m^2n^2 + p^2q^2 \\ &= (m^2 + q^2)(n^2 + p^2). \end{aligned}$$

Consequently, $a^2 + b^2 + c^2 + d^2$ *is never prime! In fact, the more general result that*

$$S \equiv a^k + b^k + c^k + d^k$$

is never prime for k *an integer* ≥ 0, *also holds.*

Divisors, sums of divisors and aliquot parts

Let n be any natural number greater than 1 and let the distinct primes in its factorization be $p_1, p_2, p_3, \ldots, p_k$. Suppose that the prime p_1 occurs α_1 times in the factorization of n, the prime p_2 occurs α_2 times, and so on. Then, the fundamental theorem of arithmetic[226] *states that the unique prime factorization of n is:*

$$n = p_1^{\alpha_1} p_2^{\alpha_2} \ldots p_k^{\alpha_k}. \tag{1}$$

The divisors of n are all possible products of prime powers where the power of p_j can assume the values $0, 1, 2, \ldots, \alpha_j$. We can then count $d(n)$, the number of different divisors of a number n (including 1 and n)

$$d(n) = (\alpha_1 + 1)(\alpha_2 + 1) \ldots (\alpha_k + 1). \tag{2}$$

One may also consider the sum of all divisors (again, including 1 and n)

$$\begin{aligned} \sigma(n) &= (1 + p_1 + p_1^2 + \cdots + p_1^{\alpha_1})(1 + p_2 + p_2^2 + \cdots + p_2^{\alpha_2}) \\ &\quad \times \ldots (1 + p_k + p_k^2 + \cdots + p_k^{\alpha_k}) \\ &= \frac{p_1^{\alpha_1+1} - 1}{p_1 - 1} \cdot \frac{p_2^{\alpha_2+1} - 1}{p_2 - 1} \cdot \ldots \cdot \frac{p_k^{\alpha_k+1} - 1}{p_k - 1}. \end{aligned} \tag{3}$$

Thus e.g.

$$d(p^\alpha) = \alpha + 1; \qquad \sigma(p^\alpha) = \frac{p^{\alpha+1} - 1}{p - 1}.$$

The function $\sigma(n) - n$ yields the sum of divisors of n other than n itself. This is known as aliquot parts of a number.

Examples of theorems:

- *Prove that $\log n \geq k \cdot \log 2$, where n is any natural number and k is the number of distinct primes that divide n.*

  This obviously holds for $n = 1$ $(k = 0)$ Let $n = p_1^{\alpha_1} p_2^{\alpha_2} \ldots p_k^{\alpha_k}$, where n is a natural number greater than 1 and $p_1, p_2, \ldots, p_k$ its prime divisors. Since none of the p_i is less than 2 and each $\alpha_i \geq 1$, then $n \geq 2^{\alpha_1 + \alpha_2 + \cdots + \alpha_k} \geq 2^k$. Taking the natural logarithm of this yields the desired result: $\log n \geq k \cdot \log 2$.

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[226] This theorem does not seem to have been stated explicitly before **Gauss**. It was, of course, familiar to earlier mathematicians, but Gauss was the first to develop arithmetic as a systematic science.

- *Every composite number n has at least one prime divisor less than or equal to $\sqrt{n}$. Indeed, if n is a composite number, then $n = ab$, where a and b are natural numbers $< n$. Without loss of generality, we may assume that $a \leq b$. Hence $n = ab \geq a^2$ so that $a \leq \sqrt{n}$. But the number a is > 1, because if $a = 1$, then we should have $n = b$ while at the same time $b < n$. However, the number a has a prime divisor p which is obviously $\leq a$ and so $\leq \sqrt{n}$. But p, being a divisor of a divisor a of the number n, is also a divisor of n. The number n has therefore a prime divisor $p \leq \sqrt{n}$.*

- *For any number, the least divisor greater than 1 must be a prime number.*

  If the number is a prime, its least divisor greater than 1 is the number itself, which is a prime. If it is not a prime, and it has a composite least divisor, this number has lesser divisors (> 1) of its own so it cannot be the least.

Degree of compositeness

There are two ways in which it is natural to "measure", or quantify, the degree of compositeness of a number[227] n:

- *By its number of divisors, $d(n)$.*
- *By its number of prime factors: Let $f(n)$ denote the number of different prime factors and $F(n)$ denote the number of total prime factors. For example,*

  $$f(2^3 \cdot 3^2 \cdot 5) = 3; \qquad F(2^3 \cdot 3^2 \cdot 5) = 6; \qquad F(2^k) = k.$$

  [For a prime number n: $f(n) = F(n) = 1$.]

Dirichlet *(1838) showed that*

$$\lim_{n\to\infty} \left\{ \frac{1}{n}[d(1) + d(2) + \cdots + d(n)] \right\} \Rightarrow \ln n.$$

[227] In this section, "number" means "natural number" unless stated otherwise.

Ramanujan *(1917) proved that*

$$\lim_{n\to\infty} \frac{1}{n}[f(1) + f(2) + \cdots + f(n)] \Rightarrow \log\log n,$$

$$\lim_{n\to\infty} \frac{1}{n}[F(1) + F(2) + \cdots + F(n)] \Rightarrow \log\log n.$$

Twin primes

Clearly, there are no successive primes except 2 *and* 3*, since all primes* > 2 *are odd. However, there are many pairs of successive odd primes, known as twin primes, such as* $(3,5)$, $(5,7)$, $(11,13)$, $(17,19)$, $(29,31)$, $(41,43)$*. There are* $152,892$ *such pairs of numbers less than* 30 *million. We do not know whether the number of such primes is infinite. In other words, we do not know whether the number* 2 *can be written as a difference of two primes in an infinity of ways.*

The probability that two random numbers near x *are both prime is of order* $\left(\frac{1}{\log x}\right)^2$*; i.e. in an interval from* x *to* $x+a$ *one may expect of order* $\frac{a}{(\log x)^2}$ *such numbers. Actually, we should expect a bit more, since the fact that* n *is already a prime slightly changes the chance that* $n+2$ *is prime (e.g.* $n+2$ *is certainly odd). An easy heuristic argument gives* $c\frac{a}{(\log x)^2}$ *where* $c = 1.320\ 323\ 631\ 6\ldots$*. Thus, for example, in the interval between* $10^{15} \to 10^{15} + 150,000$*, the expected number of twin primes is* 166 *against* 161 *actually found. This extremely good agreement is especially surprising, since it has not yet even been proved that there are infinitely many such pairs, let alone that they are distributed according to the conjectured law.*

According to ***Hardy*** *and* ***Littlewood*** *(1922), the number of prime quadruplets (4-tuples of successive odd primes) with* $p < x$ *is asymptotic to*[228]

$$Q(x) = c\int_2^x \frac{dt}{(\log t)^4}, \qquad c = 4.151\ 180\ 863\ 237\ 4\ldots,$$

[228] In this chapter and elsewhere in this book we sometimes use the British notation $\log x$ for $\ln x$.

where c is known as the Hardy-Littlewood constant. In the vicinity of a large x*, the average density of prime quadruplets* $\sim \frac{dQ}{dx} = \frac{c}{(\log x)^4}$.

It is easy to prove that there exist arbitrarily long sequences of natural numbers which contain no prime numbers. A sequence of m *such successive numbers is, for example, the sequence* $(m+1)!+2,\ (m+1)!+3,\ (m+1)!+4, \dots, (m+1)!+(m+1),$ *because the first number of this sequence is divisible by* 2*, the second by* 3*, etc., and the last by* $m+1$*; thus they are all composite. For* $m = 100$ *the numbers would be gigantic, but between the prime numbers* 370,261 *and* 370,373 *there lie* 111 *successive composite numbers. Among the hundred and one successive numbers from* 1,671,800 *to* 1,671,900 *there is no prime number.*

It is difficult to prove that there exist prime numbers on both sides of which there are arbitrarily many composite numbers, i.e. that for every natural m *there exist prime numbers* p *such that each of the numbers* $p-k$ *and* $p+k$*, where* $k = 1, 2, \dots, m$*, is composite.*

In 1975, the largest known twin primes were

$$76 \cdot 3^{139} \pm 1 = \begin{array}{l} 158,733,282,881,841,916,274,491,012,923,328,901,749, \\ 236,259,319,203,520,296,443,150,620,292 \pm 1. \end{array}$$

By 1995, the record grew to numbers with 11,713 *digits!*

Fermat's Last Theorem[229] *(250–1980 CE)*

The history of FLT starts in ca 250 CE with **Diophantos**, *whose Arithmetica considered many problems in elementary number theory. A typical problem, taken from Book II, Problem 8, would be to divide a given square into the sum of two squares. His solution, in modern notation, is as follows: let a^2 be the given square for which one wants to find x and y such that $a^2 = x^2 + y^2$. As usual, Diophantos asks for rational solutions. For a suitably chosen number m, one can write $y = mx - a$. When this is substituted into the former equation, one finds $x = \frac{2am}{m^2+1}$. Here m may be any rational number. Diophantos must have proceeded only by illustrating the method on a sample. He chooses $a = 4$ and takes a solution that corresponds to $m = 2$ in our formula, giving $x = \frac{16}{5}$, $y = \frac{12}{5}$. One verifies that indeed $\left(\frac{16}{5}\right)^2 + \left(\frac{12}{5}\right)^2 = 4^2$.*

To us this problem is quite straightforward, but it was not always so. In the oldest preserved Diophantos manuscript, copied in the 13^{th} century, we find at this point the following heartfelt remark by the scribe: "Thy soul, Diophantos, to Satanos, for the difficulty of thy problems and this one in particular".

The Arithmetica was one of the last Greek mathematical works translated into Latin (1575). **Fermat** *(1601–1665) had a copy of Bachet's translation of 1621 and made a series of intriguing annotations in its margins. Sometime in the late 1630's, while reading the section which solves the problem given above, he added in the margin: "On the other hand, it is impossible to separate a cube into two cubes, or a biquadrate into two biquadrates, or generally any power except a square into two powers with the same exponent. I have discovered a truly marvelous proof of this, which however the margin is not large enough to contain".*

Thus, the basic claim of Fermat's Last Theorem is that the equation $x^n + y^n = z^n$ has no solutions when x, y, z are nonzero integers and $n > 2$.

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229 To dig deeper, see:

- Stewart, I. and D. Tall, *Algebraic Number Theory and Fermat's Last Theorem*, A K Peters, 2002, 218 pp.
- Van der Poorten, A., *Notes on Fermat's Last Theorem*, Wiley, 1996, 222 pp.
- Ribenboim, P., *13 Lectures on Fermat's Last Theorem*, Springer-Verlag, 1979, 302 pp.

Generations of mathematical historians have debated whether Fermat really did have a proof, through many experts doubt that he did. For one thing, the equation $x^n + y^n = z^n$ *was atypical for Fermat — the vast majority of the other equations he studied dealt with exponents* ≤ 4. *Also, in his correspondence, he only stated FLT for the exponent* $n = 3$. *As for Fermat's "marvelous proof", it probably used his technique of infinite descent.*

His 'descent proof' for $n = 4$ *is actually known: it follows from a theorem of Fermat's on the area of a right triangle with integral sides not being able to be a square. This proof is given in one of his marginal notes — this time the margin was big enough. It seems likely that Fermat thought that his proofs for* $n = 3$ *and 4 can be generalized, but that they actually did not*[230].

After Fermat's death, in 1670, *his marginal notes were published by his son.*

In 1729, ***Goldbach*** *wrote Euler and mentioned the conjectures of Fermat presented in the published notes. This got* ***Euler***, *only* 22 *at the time, thinking about number theory. Three years later, Euler wrote his first paper on number theory, disproving a conjecture of Fermat on primes of the form* $2^{2^n} + 1$. *For the next forty years, Euler proved many of Fermat's conjectures and in so doing, transformed number theory from a collection of miscellaneous facts and results into an organized body of knowledge at the very center of mathematics.*

Consider an example of what Euler did: In Book VI of the Arithmetica, *Fermat had claimed in a marginal note that the only integer solutions to* $x^3 = y^2 + 2$ *are given by* $(x, y) = (3, \pm 5)$. *To prove this, Euler uses numbers of the form* $a + b\sqrt{-2}$, *with* a, b *integers.*

Here is his proof: $x^3 = y^2 + 2 = (y + \sqrt{-2})(y - \sqrt{-2})$. *One can show that* $y + \sqrt{-2}$ *and* $y - \sqrt{-2}$ *are relatively prime*[231], *and since their product*

230 Fermat claimed to have found a proof of the theorem at an early stage in his career (1637). Much later (1659) he spent his time and effort proving the cases $n = 4$ and $n = 5$. Had he had a proof to his theorem earlier, there would have been no need for him to study specific cases.

Furthermore, if he really thought he had a proof, he would have announced the result publicly, or challenge some English mathematician to prove it. It is likely that he found a flaw in his own proof, and never bothered to erase the marginal comment because it never occurred to him that anyone would see it there. All the facts indicate that Fermat quickly became aware of the incompleteness of the (allegedly general) "proof" of 1637. Of course, there was no reason for a public retraction of his privately made conjecture.

231 No common factor $p + q\sqrt{-2}$ for any integer p, q. These complex numbers $p + i\sqrt{2}q$, form a *ring*.

is a cube of $(x+0\sqrt{-2},)$ *each of them must also be a cube of a number of the form: integer + integer*$\sqrt{-2}$*, so*

$$y+\sqrt{-2}=(p+q\sqrt{-2}\,)^3=p^3-6pq^2+(3p^2q-2q^3)\sqrt{-2}$$
$$\Rightarrow 1=3p^2q-2q^3=q(3p^2-2q^2).$$

The last equation implies $p=\pm1$ *and* $q=1$*. Substituting this in, we get* $y=p^3-6pq^2=\pm5$ *and* $x=3$*, as claimed.*

This proof, while elegant, is incomplete, for we have not shown that numbers of the form $a+b\sqrt{-2}$ *have unique factorizations into primes, or even for that matter, that there are primes in this ring, or that any ring element factorizes into them (although it is relatively easy to prove that the numbers* $a+b\sqrt{-2}$ *do have these properties). There are three reasons why the above example is important:*

- ▷ *It reminds us that there are many Diophantine equations besides just FLT, and that what we really want is a method for dealing with as many of them as possible.*
- ▷ *It generalizes the integers to a set of numbers which has much of the same arithmetic structure (addition, multiplication, etc.). This sort of generalization occurs frequently in mathematics.*
- ▷ *Finally, the equation* $y^2=x^3-2$ *is an example of an elliptic curve. Elliptic curves play a crucial role in the eventual 1990s proof by A. Wiles of FLT, which uses analytical number theory.*

Clearly it is sufficient to prove FLT for $n=4$ *(done by Fermat) and for* n *an odd prime, since we can factor the exponent. It can also be assumed that* x*,* y*,* z *are nonzero relatively prime integers, because we can cancel common factors. By the early 1800's, all of Fermat's problems were solved except for FLT (thus engendering the name, Fermat's Last Theorem). That being said, the highlights of the* 19^{th} *century work on the FLT is given by the following chronological list:*

- *1816 — The French Academy announces a prize for resolutions of FLT (either proof of the conjecture or of its falsehood).*
- *In the 1820's,* ***Sophie Germain*** *showed that if* p *and* $2p+1$ *are prime, then* $x^p+y^p=z^p$ *has no solution with* $p\nmid xyz$*. This latter condition defines the so-called Case I of FLT. [Case II is where* $p\mid xyz$ *and is usually regarded as being much harder.]*
- *1825 —* ***Dirichlet*** *and Legendre prove FLT for* $n=5$*.*

- *1832* — ***Dirichlet***, *after trying to prove it for* $n = 7$, *proved it for* $n = 14$.
- *1839* — ***Lamé*** *proved FLT for* $n = 7$.
- *1847* — *Lamé and Cauchy presented false proofs of FLT.*
- *1844–1857* — ***Kummer****'s work on FLT.*

Kummer (and Cauchy and Lamé) started, á la Euler, by factoring the right hand side of the FLT equation as

$$x^p = z^p - y^p = (z - y)(z - \zeta y)(z - \zeta^2 y \cdots (z - \zeta^{p-1} y)$$

where $\zeta = e^{2\pi i/p} = \cos(2\pi/p) + i\sin(2\pi/p)$ *is a* p^{th} *root of unity* *and satisfies* $\zeta^p = 1$. *In general, working with roots of unity will require us to use number rings of the form* $a_0 + a_1\zeta + \cdots + a_{p-1}\zeta^{p-1}$, $a_1 \cdots a_{p-1} \in \mathbb{Z}$ *which are called cyclotomic integers* *(*$\mathbb{Z}$ *is the ring of integers:* $0,\ \pm 1,\ \pm 2,\ \ldots$*). But a problem arises when unique factorization, one of our main tools, fails for the cyclotomic integers. As Kummer discovered in* 1844, *this first occurs for* $p = 23$ *(and now we know that unique factorization fails for all larger primes as well).*

Kummer's solution to this was twofold. First, he introduced a generalization of cyclotomic integers, called ideal numbers, *which make up for the lack of unique factorization. Second, he defined the* class number h, *which measures how badly unique factorization is violated.*

Here is a summary of Kummer's results:

- *1847* — *Theorem: FLT holds for* p *if* $p \nmid h$ *(such* p *are called* regular primes*).*
- *1847* — *Theorem:* p *is regular iff* p *does not divide the numerator of the Bernoulli numbers* $B_2, B_4, \ldots, B_{p-3}$.

  We can define the Bernoulli numbers by the Taylor expansion $\frac{x}{e^x - 1} = \sum_{n=1}^{\infty} \frac{B_n}{n!} x^n$. *A corollary of this result is that for* $p < 100$, *only* 37, 59, *and* 67 *are irregular.*
- *1850* — *The French Academy offers a second prize for a solution to FLT, withdraws it, and then awards a medal to Kummer.*
- *1857* — *Kummer develops complicated criteria for proving FLT for certain irregular primes. There are some gaps in his proofs which are later filled by* ***H.S. Vandiver*** *in the* 1920*'s. These results establish FLT for* $p < 100$.

Further developments occurred as follows:

- *1908 — The Wolfskehl prize for a solution to FLT is announced. Later inflation of the Deutschmark reduced the value of this prize considerably, but did not reduce the flow of crank solutions submitted.*
- *1909 —* ***Wieferich*** *proved that if* $x^p + y^p = z^p$ and $p \nmid xyz$ *(Case I of FLT), then* $2^{p-1} \equiv 1 \bmod p^2$*. This is a strong congruence which is particularly easy to check on a computer.*
- *1953 —* ***Kustaa Adolf Inkeri*** *(1908–1997, Finland) proved that if* $x^p + y^p = z^p$ and $x < y < z$, *then* $x > p^{3p-4}$.
- *1971 —* ***Brillhart***, ***Tonascia*** and ***Weinberger*** *showed that Case I of FLT is true for all primes less than* $3 \cdot 10^9$.
- *1976 —* ***Wagstaff*** *showed that FLT is true for all primes less than* 125, 000.

The conclusion of all this work is that any counter-example to FLT must involve $p > 125,003$ *and* $z > y > x > (125,003)^{375,005} \approx 4.5 \cdot 10^{1,911,370}$.

Missing from the present account of FLT is the work of the many mathematicians who created the theories of reciprocity theorems, class field theory, elliptic curves, modular forms and Galois representations, and searched out the amazing connections between them[232]; and also the work in the 1980s and 1990s, including the first valid proof of FLT by A. Wiles in the 1990s.

1909–1913 CE **William David Coolidge** (1873–1975, USA). Physicist and inventor. Developed a method of producing fine tungsten wires for use as filaments of light bulbs (1909). Invented the '*Coolidge tube*', a device for emitting X-rays (1913). This invention completely revolutionized the generation of X-rays and remains to this day the model upon which all X-ray tubes for medical applications are patterned.

232 For example, it was discovered only in 1984 by **R.C. Mason** that the proof of Fermat's theorem for *polynomials* is rather trivial!
Let $x(t)$, $y(t)$, $z(t)$ be relatively prime polynomials in one variable over the field of complex numbers. Then $[x(t)]^n + [y(t)]^n = [z(t)]^n$ is valid only for $n \le 2$. The proof follows as a corollary of *Mason's theorem* (1984): If $a(t)$, $b(t)$, $c(t) = a(t) + b(t)$ be relatively prime polynomials, then $\max\ \deg\{a, b, c\} = n_0(abc) - 1$, where $n_0(f) =$ number of *distinct* roots of f.

Coolidge was born in Hudson, MA and graduated from M.I.T. (1896), majoring in electrical engineering. He received his Ph.D. from the University of Leipzig. During WWII he contributed to the development of radar.

1909–1917 CE **Sören Peter Lauritz Sörensen** (1868–1939, Denmark). Chemist. Introduced the symbol pH to denote the negative logarithm of the concentration of the hydrogen-ion in a *Sörensen scale* that serves as a measure of acidity or alkalinity of a solution. Pointed out the effect of pH on enzyme activity.

Sörensen was director of Chemical Department, Carlsberg Laboratory, Copenhagen (1901–1939).

Acidic solutions contain hydrogen-ions, H^+ (actually *hydroniom-ions*, H_3O^+). A basic solution contains hydroxide-ions, OH^-. *Pure water* at 25°C contain hydrogen-ions in concentration 1×10^{-7} mole/liter, and hydroxide-ions in the same concentration. These ions are formed by the dissociation of water[233]: $H_2O \rightleftharpoons H^+ + OH^-$. *Acidic solutions* contain hydrogen-ion in large concentration and hydroxide-ion in a very small concentration. The range of acidities and basicities for aqueous solutions fall between 1 mole/liter solutions of strong acids and 1 mole/liter of strong bases. It includes practically all the solutions associated with the *metabolism of living organisms* as well as most of the aqueous solutions taking part in *geological processes*. The corresponding hydrogen-ion concentrations lie between 1 and 10^{-14} mole/liter.

Sörensen expressed the acidities of aqueous solutions on a logarithmic scale, defined as: $\mathrm{pH} = -\log_{10}[H_3O^+]$. The symbol pH was intended originally as an abbreviation for the phrase "potential of hydrogen-ion". Electric potential difference between the electrodes for certain types of galvanic cells vary linearly with this particular property of the electrolyte. So called "pH meters" operate on this principle.

Sörensen reported in 1917 that *egg albumin* has a molecular weight of 34,000 (determined through osmotic pressure measurements).

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233 The reaction $H^+ + OH^- \rightleftharpoons H_2O$ has an *ion-product*

$$K_W = [H] \times [OH^-] = 1.0 \times 10^{-14} \mathrm{mole}^2/\mathrm{l}^2$$

at room temperature (25°C). The quantity pH is defined as the *negative logarithm of the hydrogen ion-concentration*. At the *neutral point* where $[H^+] = [OH^-]$, the pH is 7. If $pH < 7$, the solution is *acid*; if $pH > 7$, the solution is *alkaline*.

1909–1925 CE **Karl Bosch** (1874–1940, Germany). Chemist. Developed the nitrogen fixation process for the synthesis of ammonia (invented by **Fritz Haber**) to an industrial scale (1914), enabling the Germans to manufacture explosives without relying on foreign imports of nitrogen bearing materials, capable of being blocked by the allies in WWI. In peacetime, the process allowed the cheap manufacture of fertilizers, equally vital to the survival of a country. Invented a process for manufacture of hydrogen (1925).

Bosch was born in Cologne and studied chemistry at the University of Leipzig, taking his doctorate in 1898. He became chairman of the vast industrial conglomerate IG Farbenindustrie AG (1925). In 1931 he received the Nobel Prize for Chemistry (jointly with **Friedrich Bergius**) for their work on high-pressure synthesis reactions.

1909–1926 CE **Peter** (**Joseph William**) **Debye** (1884–1966, Netherlands and U.S.A.). Physicist. Contributed widely to physical chemistry and particularly to our understanding of solids. His theory on the thermal vibration of a solid (1911) is among his best known works. Studied molecular structure through investigations on *dipole moments* and on diffraction of X-rays and electrons in crystals and gases (1916). Proposed (with **Giauque**, 1926) that paramagnetic salts could be used for achieving very low temperatures.

Revised **Pierre Dulong** and **Alexis Petit**'s value of 6 cal/mole/°C for the molar heat capacity of most substances (1819) by correcting at low temperatures for quantum freeze-out of vibrational modes of the atoms (1912). The resulting theoretical curve, which descends to 0 cal/mole/°C at 0°K, is found to agree with experiment and was seen as an early success for quantum physics.

Discovered (1909) a useful asymptotic expansion of Bessel functions for large real argument x and index ν (ν/x fixed). Awarded the 1936 Nobel prize in chemistry.

Debye was born in Maastricht, Holland. Director of Kaiser Wilhelm Institute for Physics, Berlin (1935–1940). Professor at Cornell University (1940–1950). Debye was a Nazi sympathizer and intensely supported the German Nazi Party and Hitler from May 1933 together with his friends **Werner Heisenberg** and **George Braque**.

However, being a Dutch citizen, he was forced to leave Germany after the German occupation of the Netherlands.

1909–1929 CE **Phoebus Aaron Theodor Levene** (**Fishel Aaronovich Levin**, 1869–1940, U.S.A.). Biochemist. Discoverer of RNA and DNA.

First to identify *sugar ribose* (1909) and *deoxyribose* (1929) in nucleic acids[234]. Levene maintained that, in the nucleic acid molecule, each ribose (or deoxyribose) portion has a phosphorous group attached to one side and a *purine* or *pyrimidine* to the other. This combination of groups is called a *nucleotide*. In his pioneering work Levene determined the formation of nucleotides and how they combine in *chains*.

Levene was born in Sagor, Russia to Jewish parents and immigrated to the U.S. in 1892. He was a member of the Rockefeller Institute (now Rockefeller University) during 1905–1939.

1909–1930 CE **Otto Selz** (1881–1943; Germany). Father of modern cognitive psychology. Had decisive influence on the key ideas of Popper's philosophy of science with its emphasis on the method of trial and error. His theory of problem solving and scientific discovery laid the foundation for much of contemporary cognitive psychology. Expounded his views in his books: "*Über die Gesetze des geordneten Denkverlaufs*" (1913) and "*Zur Psychologie des produktiven Denkens und des Irrtums*" (1922).

Otto Selz was born in Munich to Jewish parents. Studied law and philosophy in Munich and Berlin (Ph.D. 1909; Habilitation at the University of Bonn 1912).

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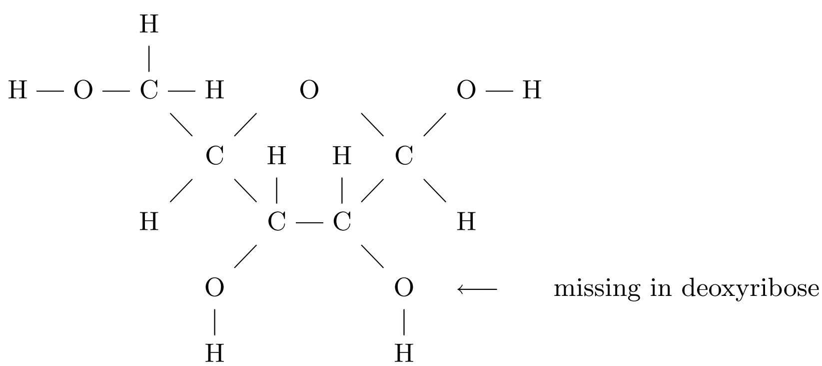
[234] *Ribose* differs from glucose, fructose, and galactose in that it has 5 carbons instead of 6. This five-carbon chain tends to form a ring with an oxygen atom connecting the first and 4-th carbon atoms



Later, Levene discovered that not all nucleic acid molecules contain ribose; some contain a closely related sugar, which differs only in the absence of one of the oxygen atoms of ribose. Its name is, therefore, *deoxyribose*.
It is on the basis of these two sugars that nucleic acid came to be divided into two groups: *ribonucleic acid* (RNA), which contained ribose; and *deoxyribonucleic acid* (DNA), which contained deoxyribose.

After serving on the Western front in WWI (1915–1918) he occupied positions in psychology and philosophy at the Universities of Bonn and Mannheim (1920–1932).

In 1933 he was dismissed from his academic positions because of his Jewish descent and he went into exile in the Netherlands. In 1942 the Nazis confined him to the Amsterdam Ghetto. He was arrested (1943) and soon thereafter perished with his entire family in the gas chambers of Auschwitz.

1909–1933 **Alfred Haar** (1885–1933, Hungary). Mathematician. Worked in analysis, studying orthogonal systems of functions, partial differential equations, Chebyshev approximations, linear inequalities and compact continuous groups.

In his doctoral thesis (under Hilbert, 1909) he discovered the first *wavelet*[235] *basis* (system of compactly supported orthonormal functions generated from a single function by translation and dilation) known as *Haar-basis*. Introduced (1932) a measure on groups, now called the *Haar measure*, which allows an analogue of Lebesgue integrals to be defined on locally compact topological groups [it was used by **von Neumann**, by **Pontryagin** (1934) and **Weil** (1940) to set up an abstract theory of commutative harmonic analysis].

Haar was born in Budapest and died in Szeged. He traveled to Germany (1904) to study at Göttingen. He taught there until 1912, when he returned to Hungary and held chairs at Budapest and Szeged universities.

1909–1937 CE **Constantin Carathéodory** (1873–1950, Greece and Germany). Pure and applied mathematician. Made significant contributions to the calculus of variations, theory of real and complex functions, conformal mapping and the theory of point-set measure. An *outer measure* and a *pseudomeasure* in complex analytic space are named after him. His interests also extended beyond pure mathematics into applications to mechanics, thermodynamics, relativity theory and geometrical optics.

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[235] *Wavelet theory* is based upon a multiresolution spectral decomposition that allows information in both time and frequency domains (or space and wavenumber domains) to be retained. Earlier pioneers (except **Alfred Haar**) include **Paul Levy**; **Eugene Wigner** (*Wigner distribution*, 1932); **Dennis Gabor** (*Gabor expansion*, 1946). **Jean Morlet** and **Alex Grossman** (1975–1981) initiated modern wavelet theory while developing the previous idea of *Windowed Fourier Transform* (D. Gabor). It has since found extensive applications in digital signal processing, image processing and music. Important later contributions were made by **Y. Mayer** and **Stephane Mallot** (1986), **I. Daubechies** (1988) and **L. Cohen** (*Cohen classes*, 1989).

Carathéodory was born in Berlin and attended Belgium's École Militaire (1891–1895), then worked as engineer on the Asyut Dam in Egypt. In 1900 he begun to study mathematics at the University of Berlin. Transferred (1902) to Göttingen and received his Ph.D. (1904) there under **Hermann Minkowski**. He held professorships at various universities in Germany and, 1920–1924, in Greece. From 1924 he was at th University of Munich; where he stayed throughout the entire Nazi regime and WWII.

1909–1944 CE **Richard Martin von Mises** (1883–1953, Austria and U.S.A.). Applied mathematician. Contributed significantly to the fields of statistics, probability, mechanics, aerodynamics and philosophy of science.

His primary work in statistics concerned the theory of measure, and introduced the use of *statistical functions*. Suggested axiomatic systems for the logical foundation of the theory of probability [in his book: "Probability, Statistics and Truth"]. In aerodynamics, he made fundamental advances in boundary-layer-flow theory and air-foil design. In elasticity theory, he originated criteria for fracture (*Von Mises yield-criteria*).

Von Mises was born to Jewish parents in Lvov, Poland and was educated in Vienna. He was a professor of applied mathematics at the University of Strasbourg (1909–1916). He served as a pilot in the German army during WWI. After the war he moved to the University of Dresden and then became professor of applied mathematics and director of the Institute for Applied Mathematics at the University of Berlin (1920–1933). With the rise of the Nazis to power in 1933 he became a professor of mathematics at the University of Istanbul (1933–1939). Finally, in 1939, he joined the staff of Harvard University, where in 1944 he became professor of aerodynamics. His philosophical views are summarized in his book: "*Positivism*: *A Study in Human Understanding*" (1951).

1909–1915 CE **Howard Robard Hughes, Sr.** (1869–1924, USA). Inventor. Revolutionized the process of drilling for oil with his invention of the first *rotary rock bit*.

Hughes was born in Lancaster, Missouri and spent his boyhood in Keokuk, Iowa, where his father maintained a law practice. He studied law at Harvard University and the University of Iowa, but in 1901 he joined the 'Oil Rush' in Texas.

Around 1906, Hughes became interested in finding a solution to the problem of drilling through hard rock formations and began experimenting. While working in 1908 in Oil City, Louisiana with his business associate, **Walter Sharp**, Hughes produced a small wooden model of a roller-type bit with cone-shaped tooth cutters. The following year, he successfully tested the first rotary bit in an oil well at the Goose Creek field, just east of Houston. Since it was the first cone-type rock bit featuring rolling cutters, capable of drilling faster and more efficiently, he formed, with Walter Sharp, the Sharp-Hughes Tool Company to manufacture and market his new bit and tool joints.

Hughes is credited with the invention of numerous other time and money saving drilling services. With his operation headquartered in Houston, his early advances in rotary drilling technology focused attention on the city as the world's leading manufacturer of drilling equipment and tool joints. After Sharp's death in 1912, Hughes bought Sharp's share of the business, and in 1915 renamed the firm the Hughes Tool Company.

Hughes died at the age of 54, leaving the bulk of his million dollar oil well drilling equipment firm to his son **Howard Robard Hughes, Jr.**[236] (1905–1976), an 18-year old student at Rice Institute.

1909–1949 CE **Maria Montessori** (1870–1952, Italy). Physician and educator. Developed a special method of teaching young children through

[236] Aviation-pioneer film producer and industrialist. At one time he owned the *Hughes Tool Company*, the *Hughes Aircraft Company*, *RKO Pictures Corporation*, and a controlling interest in *Trans World Airlines*. At the time of his death, his financial empire was worth about two billion dollars, becoming one of the richest men in the world.

He designed and raced airplanes and set several speed records including (1938) around-the-world mark of 3 days, 19 hours, 14 minutes. In the 1940's he designed the largest plane ever built up to that time. This 8-engine wooden flying boat (nicknamed the *Spruce Goose*), had room for 700 passengers and weighed about 200 tons.

His aircraft company pioneered many innovations in aerospace technology. In the 1950's and beyond, Hughes manufactured spy satellites.

In the mid-1950's, Hughes deliberately dropped out of sight. He became a mysterious figure who never appeared in public and even refused to be photographed.

Hughes died en route by private jet to a hospital in Houston. His drastically changed appearance and the fact that he had been seen by so few people for so long forced the Treasury Department to use fingerprints to identify his body.

Four hundred prospective heirs tried to inherit his $2 billion estate, but it eventually went to 22 cousins on both sides of the family. Texas, Nevada and California claimed inheritance-tax in disputes reviewed by the Supreme Court three times. *Hughes Aircraft* ended in the hands of *Hughes Medical Institute*, which sold it to *General Motors* (1985) for $5 billion.

Hughes never graduated from high-school. Nonetheless, his father arranged for him to audit mathematics and engineering classes at CalTech in Pasadena, California, by donating money to the school.

Houston's airport was renamed in his honor.

work rather than play, and aimed at creating a scientific pedagogy. According to the *Montessori method*, children should be helped to learn by themselves and gain confidence in themselves while making use of their abilities.

Maria Montessori was born in Ancona and was the first woman in Italy to earn a medical degree (1894). She became a professor of anthropology (1904–1908) and opened the first Montessori school for children in Rome (1907). Later she traveled throughout the world, writing and lecturing about her teaching method. She inspired many devoted supporters in the USA, Britain and on the Continent who sought to implement her ideas in schools, societies and associations.

1910 CE, May 20 *Comet Halley*[237] came within 23 million km (0.153 AU) of earth, traveling with relative velocity of 83 $\frac{\text{km}}{\text{sec}}$ (52 $\frac{\text{km}}{\text{sec}}$ w.r.t. sun). On May 21, earth passed through the tail of the comet with no ill effects; many people feared that the inhabitants of earth would be wiped out by cyanogen gas detected in the nucleus of the comet.

1910 CE *The International Geological Congress at Stockholm* first made the majority of geologists familiar with the existence of a warm period interlaced between the ice-age and the present. Up to the turn of the 20^{th} century the post-glacial period was supposed to show merely a more or less rapid warming up to the present level, followed by a long period in which the climate of the different parts of the world were exactly as we now find them.

About the same time (1908), a number of investigations in different countries combined to prove that the ice-age itself was not so remote as it had seemed to be, and that in fact the post-glacial "geology" of Europe was partly contemporaneous with the "history" of Egypt. The beginning of the "period of unchanging climate" has advanced later and later until it stopped only a few centuries BCE.

Moreover, it became clear that the present does not differ from the past in the sense that variations of climate are still in progress, which are similar in kind, though not in extent, to the climatic vicissitudes of the ice-age. There is, however, one point in which the "historical" period may be said to differ from the "geological" periods — any climatic changes in the "historical" period

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[237] Mark Twain told his biographer, Albert Bigelow Paine (1909):
"I came in with the Halley Comet in 1835. it is coming again next year, and I expect to go out with it. The Almighty had said, no doubt: 'Now here are these two unaccountable freaks; they came in together, they must go out together.' Oh! I am looking forward to that."
As fate would have it, it happened just that way.

must be attributed to *non-geographical* factors, and most probably due to variations in solar radiation.

1910 CE **Georges Claude** (1871–1966, France). Chemist, physicist and inventor. Invented *neon light* (first used commercially in 1923). Showed that acetylene dissolved in acetone can be safely transported (1897). Produced liquid air by expansion method (1902) and separated from it the various gases of the air; invented new method for synthesis of ammonia (1917). Imprisoned (1945–1949) as Nazi collaborator in WWII.

1910 CE **L. Southerns** showed that the ratios between mass and weight for uranium oxide and lead oxide, respectively, differed at most by 5 parts per million. Since the binding energy of uranium oxide is particularly high, this experiment proves that the proportionality of inertial and gravitational mass ("weak equivalence principle") applies also to atomic binding energy (via $E = mc^2$).

1910–1913 CE Bubonic plague in China. Death toll reached millions.

1910–1927 CE **Thomas Hunt Morgan** (1866–1945, U.S.A.). Geneticist. Proposed a theory of sex-linked inheritance including the *principle of linkage*. Established the importance of the gene and the chromosome in transmitting inherited characteristics. From his examination of hundreds of mutant characters in *Drosophila* he established that inherited characters are connected to paired Mendelian factors or *genes* and that genes are linearly joined into chromosomes. Although the inheritance of characteristics from genes positioned along different chromosomes occurs independently, as in **Gregor Mendel**'s second law, this does not apply for characteristics from genes along the same chromosome. These later characteristics tend to be inherited as a group.

Morgan was born in Lexington, KY. He was professor at Columbia University (1904–1928) and from 1928, at CALTECH. Awarded the Nobel prize for physiology or medicine (1933).

1910–1928 CE **Ernst Steinitz** (1871–1928, Germany). Mathematician. Made substantial contributions to the algebraic theory of fields (Steinitz's '*replacement theorem*'), algebraic geometry and the theory of polyhedra.

Steinitz was born to Jewish parents in Silesia and was educated at the University of Breslau (Ph.D., 1894). He began his career at the Technical College in Berlin-Charlottenburg and ended it at the University of Kiel.

1910–1939 CE **Jan Lukasiewicz** (1878–1956, Poland). Influential mathematical logician. One of the founders of the school of logicians in Poland after WWI. Made important contributions to *modal logic* (in which 'is possible' is allowed as a fundamental notion) and *many-valued logics* (in which truth-values additional to 'true' and 'false' are admitted). In particular he developed a *three-valued propositional calculus* (1917) and devised a notation system for syllogistic propositions. Made detailed study of Aristotle's syllogistic (1910) and reevaluated ancient and medieval logic by employing modern formal techniques of logic. He also introduced into logic the '*Polish notation*' which has found favor in computing for its avoidance of brackets.

Lukasiewicz was born in Lvov (Lemberg). Was Polish Minister of Education (1919) and professor at Warsaw University (1920–1939). When the Russians occupied Poland (1945), Lukasiewicz was politically persecuted and had to seek asylum in Dublin (Irish Republic), where he was appointed to the Royal Irish Academy. He died in Dublin.

1910–1943 CE **Janusz Korczak** (1878–1943, Poland). Physician, child psychologist, writer and educator. Founder of an original system of education and one of the finest examples of selflessness and caring for others in human history.

Born in Warsaw as **Henryk Goldszmit**, he was one of the few Jews to be accepted for medical studies of the University of Warsaw, graduating in 1905. He continued his training in pediatrics both in Berlin and Paris under Virchow, Marfan and Charcot. During the Russo-Japanese War he served in field hospitals and wards in the Ukraine and in Manchuria.

Early in his medical career he began to write poetry and fiction under the pseudonym Janusz Korczak. In 1910 he decided to give up both his successful pediatric practice and his literary career to become the director of orphanages. In these "children's republics" he experimented with his progressive ideas by allowing children to govern themselves with as much independence as possible. Disinterested in personal affairs, he lived a monastic existence.

With the German take-over of Poland, the orphanages were closed on Aug 06, 1942 and Korczak with his two hundred children were deported to Treblinka where they perished in the Gas-Chambers on August 5, 1943. (Because of his fame,, he was offered safe passage out of the Warsaw Getto, but he refused a chance to save himself and was voluntarily deported with the children).

1910–1940 CE Pre WWII Western music. Its leading composers are:

- Ermanno Wolf-Ferrari 1876–1948
- Earnst Bloch 1880–1959
- Bela Bartok 1881–1945
- Zultan Kodály 1882–1967

• Igor Stravinsky	1882–1971
• Joseph Achron	1886–1943
• Arthur Honegger	1892–1955
• George Gershwin	1898–1937
• Francis Poulenc	1899–1963
• Aaron Copland	1900–1990
• Aram Khachaturian	1903–1978
• Richard Addinsel	1904–1977
• Samuel Barber	1910–1981

1911 CE **Robert Andrews Millikan** (1868–1953, U.S.A.). Distinguished experimental physicist. Determined the charge of the electron and established it as a fundamental unit of electricity. Verified the Einstein photoelectric equation, and obtained a precise value for the Planck constant.

Prior to 1911, the ratio e/m of the electron's charge to its mass had been determined experimentally by **J.J. Thomson** and others. It remained to find the magnitude of the charge (e) so that the mass (m) of a single particle might be calculated.

In his famous 'oil-drop experiment'[238] Millikan obtained the first *direct* and compelling measurement of the electric charge of a *single* electron, with an accuracy of one percent [owing to errors in the viscosity coefficient as applied to very small droplets].

Millikan found that the charge of an oil drop is an integral multiple, ne, of some basic unit, which is identified as the charge on one electron. The most important feature of the 'oil-drop experiment' is its clear demonstration that electric charge is *quantized*. With the aid of e and $\frac{e}{m}$, the electronic mass could finally be fixed, and was found to be $\frac{1}{1837}$ of the mass of a

[238] A vertical electric field E (which could be switched on and off) was set up between two parallel plates. The upper plate had at its center a few small perforations through which oil drops, produced by an atomizer, could pass. Most of the oil drops were charged by friction with the nozzle of the atomizer. With the electric field switched off, a single droplet of mass $m = \frac{4\pi}{3}r^3\rho$, density ρ, and radius r is acted upon by its own weight mg and the opposing frictional Stokes-force $6\pi\eta rv$. In equilibrium (no acceleration), the drop's downwards terminal velocity is $v_1 = \frac{2r^2g}{9\eta}(\rho - \rho_a)$, where ρ_a is the air density accounting for buoyancy. With the (upward) field switched on, the new terminal velocity (upward) is $v_2 = \frac{neE}{6\pi\eta r} - v_1$, where ne is the charge (taken positive). Expressing r in terms of v_1, η, g and $(\rho - \rho_a)$, one finds $ne = \frac{6\pi\eta(v_1+v_2)}{E}\sqrt{\frac{9\eta v_1}{2g(\rho-\rho_a)}}$.

neutral hydrogen atom. The electron is by far the most important particle in theoretical chemistry, as well as in atomic physics.

Millikan was born in Morrison, Illinois. After studying at the Universities of Berlin and Göttingen, he joined the faculty of the University of Chicago. In 1921 he became director of the Norman Bridge Laboratory of Physics at the California Institute of Technology, Pasadena. He won the Nobel prize for physics in 1923.

1911 CE **Heike Kamerlingh Onnes** (1853–1926, Holland). Dutch physicist. Discovered '*superconductivity*'[239] of metals with the aid of liquefied helium. Won the Nobel prize for physics in 1913 for his work on low-temperature physics and his production of liquid helium (1908).

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[239] The complete and sudden vanishing of electrical resistance in certain pure metals, when cooled to a temperature near absolute zero. For materials that are good conductors, such as copper and silver, the resistivity decreases essentially linearly with temperature down to 10 K. As the temperature decreases further, the resistivity assumes a constant value that is maintained up to the lowest temperatures investigated (ca 0.05 K). In certain materials, called *superconductors*, there occurs at low temperatures a remarkable interaction among the electrons and the ionic lattice. In the superconducting state, *pairs* of electrons with spin and momenta *antialigned*, develop an extremely weak electrical attraction for each other by interaction with the lattice [the attraction of the two electrons may be thought of as a two-step process in which the passage of one electron creates a distortion in the lattice to which the second electron is attracted]. The bound state of the two electrons is called a *Cooper pair*. The size of the Cooper pair is far larger than the size of an atom, and they all have the same center-of-mass drift speed. A Cooper pair is bound by an energy of the order 10^{-3} eV, and the energy required to break this bond is not available at low temperatures. Thus, at least in principle, the mean free path of the pair becomes infinite and the resistivity becomes zero (not just very small, but *zero*!). The temperature that marks the transition between the normal phase and the superconducting phase of a material is called the *critical temperature* T_c (e.g. 4.15 K for mercury, observed by Onnes). A current, once initiated in a superconducting material, will continue to flow (essentially) forever, even in the absence of any driving electric field.

The superconductors discovered by Onnes are not merely *perfect conductors*, they have the additional property that there is a zero magnetic field inside the superconductor (except for a thin surface layer). This class of superconductors is called *type-1*. Thirty pure elements (Be, Al, Ti, V, Zn, Ga, Zr, Nb, Mo, Tc, Ru, Rh, Cd, In, Sn, Lu, Hf, Ta, W, Re, Os, Ir, Hg, Tl, Pb, La, Gd, Th, Pa, U) are type-1 superconductors at low temperatures.

During 1871–1873 Kamerlingh Onnes studied and worked at Heidelberg University, notably with **Kirchhoff**. From 1882 to 1923 he served as professor of experimental physics at the University of Leyden.

1911 CE, July 24 **Hiram Bingham** (1875–1956, USA). Explorer, historian, archaeologist and politician. Found the "lost city" of *Machu Picchu* near Cuzco in the Peruvian Andes, the fortified city that was the sanctuary of the Inca king Pachacuti (d. 1470). It lies in the saddle between two peaks, 660 m above the rapids of the Urubamba river. Machu Picchu was one of a series of stone-laid cities which constituted a veritable chain of fortress-sanctuaries, built to defend the empire from raids of the wilder jungle tribes. All the cities, approximately 15 km apart, are bound together by a stone-laid road.

Machu Picchu was never mentioned by the Inca to the Spaniards, and dates from the first Inca, about 1000 CE.

Bingham was born in Honolulu. He graduated from Yale University and later taught history there. From 1926 to 1933, he served as a Republican U.S. Senator from Connecticut.

1911 CE **Victor Hess** (1883–1964, Austria). Physicist. Discovered *cosmic radiation* (cosmic rays) by sending electroscopes aloft in balloons. He was able to show that the ionization effect was more pronounced at high altitudes rather than less so. To explain this phenomenon, Hess advanced the hypothesis that the ionizing radiation responsible for the observed effects is incident upon the earth's atmosphere from *outside*.

1911–1914 CE **Elmer Ambrose Sperry** (1860–1930, U.S.A.). Inventor and industrialist. Developed the gyrocompass and the gyroscopic stabilizer for ships and aircraft. From his gyrocompass, Sperry developed the gyropilot, which steers a ship automatically. Later he installed giant gyroscopes which could steady the rolling motions of ships. He also produced an aerial torpedo controlled by a gyroscope. Today's naval gunnery methods would be impossible without inventions which grew out of Sperry's original gyroscope. His inventions were equally important for use in navigating aircraft.

Sperry was born in Cortland, New York. He set up his Sperry Gyroscope Company in Brooklyn in 1910. His gyrocompass was first installed on the U.S. battleship **Delaware** in 1911. In his lifetime, Sperry founded 8 manufacturing companies and took out more than 400 patents.

1911–1919 CE **John Edensor Littlewood**[240] (1885–1977, England). Distinguish mathematician. Made significant contributions to the theory of

[240] For further reading, see:
- Du Sautoy, M., *The Music of the Primes*, Perennial, 2004, 335 pp.

functions, theory of numbers, summability of series, inequalities and Fourier series theory. Along with **G.H. Hardy**, with whom he collaborated for 35 years, he was instrumental in establishing the school of mathematics in 20^{th} century Great Britain[241]. The Hardy-Littlewood partnership was extremely successful and the individual contributions are mostly inseparable, especially as far as proofs of the theorems are concerned[242].

In 1912, Littlewood showed that $\int_2^n \frac{dx}{\log x}$ switches from being an overestimate (to the number of primes less than or equal to n) to an underestimate, and back again, an infinite number of times as n keeps increasing (Littlewood's theorem)[243].

Littlewood was born in Rochester, England. He entered Trinity College, Cambridge in 1903, and started research in 1906 under the tutorship of **E.W.**

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241 Compared with the mathematics of continental Europe, 19^{th} century British mathematics was rather barren of significant figures, and was most emphatically subordinate to the natural sciences.

242 **Norbert Wiener** (1894–1964) spend the early years of his career working in England. The story goes that when he met Littlewood he said: "*Oh, so you really exist. I thought that 'Littlewood' was just a pseudonym that Hardy put on his weaker papers*". It is also told that **Edmund Landau** (1877–1938) so doubted the existence of Littlewood that he made a special trip to Great Britain to see the man with his own eyes.

243 **Gauss** conjectured that the logarithmic integral $Li(N)$ would *always overestimate the number of primes* – it would never predict that there were fewer primes than there really were in the range from 1 to N.
Littlewood proved Gauss to be wrong (although Gauss' conjecture held true (1912) for all numbers up to 10 million!). He proved that as you counted higher, you would eventually come to regions of numbers where Gauss' guess would switch from overestimating to underestimating the number of primes.

S. Skewes proved in 1955 that the first switch occurs *before* n reaches $10^{10^{10^{34}}}$, though he had to assume the truth of the Riemann conjecture. This is known as the *Skewes number* [by way of comparison — there are "*only*" about 10^{80} protons in the observable universe]. **G.H. Hardy** thought it '*the largest number which has ever served any definite purpose in mathematics*', and suggested that if a game of chess was played with all the particles in the universe as pieces, one move being the interchange of a pair of particles and the game terminating when the same position recurred for the third time, the number of possible games would be about Skewes' number.
In 1986, **H.J.J. te Riele** demonstrated that the "*Littlewood constant*" can be much better bounded, to some n below 6.69×10^{370}, still an enormously large number, well beyond human grasp. [A computer search, made as far as 10^9, failed to reach the first switch.]

Barnes (1874–1953), who assigned to him the problem of proving the Riemann hypothesis(!) [It is an amazing illustration of the insularity of British mathematics at that time that Barnes should have thought it suitable for a research student to tackle this problem.]

Characteristically, Littlewood was able to report: "*As a matter of fact this heroic suggestion was not without result... there was a consolation prize*". During 1915–1919, Littlewood collaborated with Hardy in Cambridge. He authored a steady flow of papers well into the 9^{th} decade of his life. Many of the later papers are on differential equations, and many show his interest in astronomy, physics and probability, as well as in problems of pure analysis which filled most of his life. His power at the age of 85 was shown when he solved a problem which "*raised difficulties which defeated me for some time. I have now overcome them*".

Some of his keen observations, based on long experience, are: (quoted from his publication "*The Mathematician's Art at Work*", 1967)

- "*Try a hard problem. You may not solve it, but you will prove something else*".
- "*There are four phases in mathematical creativity*: *preparation, incubation, illumination and verification. Preparation needs roots in an intense curiosity; the essential problem, stripped of its accidentals, must be brought and kept before the mind. The resulting drive is communicated to the subconscious*, '*which does all the real work and would seem to be always on duty*'. *Incubation is the work of the subconscious during the waiting time, which may be several years. Illumination, which can happen in a fraction of a second, is the emergence of the creative idea into the conscious*".
- "*Mathematics is very hard work, and dons tend to be above the average in health and vigor. Below a certain threshold a man cracks up, but above it — hard mental work makes for health and vigor* (*also — on much historical evidence throughout the ages — for longevity*)".
- "*A Ph.D. was a degree which you had to take if you failed to get a Research Fellowship, just to show that you had been at Cambridge*".

1911–1935 CE **Julius Edgar Lilienfeld** (1881–1963, Germany and USA). Physicist and inventor. Pioneer of semiconductor devices. Proposed the basic principle behind the MOSFET[244] (Metal – Oxide – Semiconductor Field Effect Transistor) in 1925, some twenty years ahead of **Shockley** and **Bardeen**. Patented a high-vacuum X-ray tube with hot tungsten filament in 1911, five years ahead of **W.D. Coolidge** in the USA.

Lilienfeld was born in Lemberg, Galicia (now Lvov, Ukraine) to Jewish parents. He entered the Polytechnic University of Berlin (1899) and earned a Ph.D. degree in experimental physics (1905). He then joined the Physics Institute at the University of Leipzig and patented a high-vacuum X-ray tube (1911). At the same time he operated the first large-scale hydrogen liquification facility used to fill the Zeppelines and for cryogenic research.

He continued to work on electrical discharges, extending his investigations to field-emission of electrons, and applied this to the development of cold-cathode high-vacuum X-ray tubes. He discovered (1919) a polarized light coming from the vicinity of the target of an X-ray tube due to time variation of the virtual *dipole* between the electrons and their image charges formed near the surface of the tube's target.

In 1926, Lilienfeld resigned his professorship position at Leipzig and emigrated to the USA. He then began experimenting with solid-state electronic devices. During 1925–1928 he developed and patented (1930) several devices which could now be referred to as *field-effect* (or, *point junction*) *transistors*[245].

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[244] A *unipolar* device where current flows through a narrow *channel* between two electrodes (the *gate*) from one region called the *source*, to another called the *drain*. A modulating signal is applied to the *gate* (Figure 5.3).

In practice, a wafer of a semiconductor material has two highly doped regions of opposite polarity diffused into it, to form a source and drain regions.

An insulating layer of silicon oxide is formed on the surface between these regions and a metal conductor is evaporated on to the top of this layer to form the *gate*.

When a positive voltage is applied to the *gate*, electrons move along the surface of the p-type substrate below the gate, producing a thin surface of n-type material, which forms the *channel* between the *source* and the *drain*. This surface layer is called an *inversion layer*, as it has opposite carriers to that of the substrate. The number of induced electrons is directly proportional to the gate voltage, thus the conductivity of the channel *increases* with gate voltage.

[245] The word 'transistor' had its beginning in 1946 at the Bell Telephone Laboratories that used high-purity germanium to create solid-state amplifying devices. While Lilienfeld's devices did not perform to todays standards, signal amplification was detected. In 1935, **Oscar Heil** described a structure similar to the junction field-effect transistor. However, practical implementations were impossible until 1960 due to material-related problems. The technology to produce such devices on a commercial basis did not yet exist.

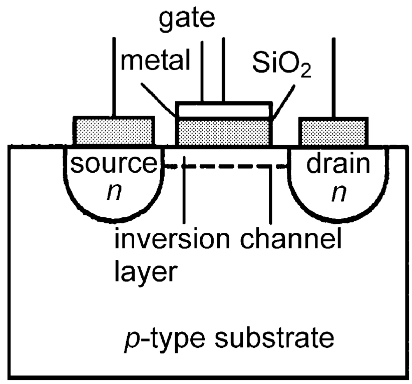

Fig. 5.3: MOSFET

In 1928, Lilienfeld began work on the *electrochemistry* of anodic aluminum-oxide films and their application in the manufacture of *electrolytic capacitors*, essential compounds in much electronic components. In 1935, he moved with his wife to the US Virgin Islands, where he died in 1963.

Although Lilienfeld's work on high-vacuum X-ray tubes and on field-effect transistors came at the wrong time to bring him fame, he deserves recognition as a pioneer scientist and prolific inventor.

In 1988, the American Physical Society established the Julius Edgar Lilienfeld Prize, in recognition of a most outstanding contribution to physics.

1911–1938 CE **Alfred North Whitehead** (1861–1947, England) and **Bertrand Arthur William Russell** (1872–1970, England), both mathematicians, logicians, metaphysicians and philosophers, published their joint work: "*Principia Mathematica*" — an attempt to anchor arithmetic to a system of formal procedures, based on Peano's axioms and symbolic logic. The program, intended to prove that all pure mathematics can be derived from a small number of fundamental logical principles, is in line with Russell's view that mathematics is indistinguishable from logic.

This system left unanswered Hilbert's query whether the numerical continuum can be considered a well ordered set, to which Gödel in 1931 gave a definite answer.

Whitehead was born at Ramsgate, Isle of Thanet, Kent, England. His father, a vicar, taught him at home until he was 14. He was then sent to Sherborne School, Dorset, where he received classical education. In 1880 he entered Trinity College, Cambridge. He met **Russell** in 1890 when the latter was a freshman studying mathematics at Trinity. Gradually, the two men became close friends.

In 1910 Whitehead left Cambridge: his future there was uncertain, since he did not produce any discoveries that could earn him a Cambridge professorship in mathematics [his interest was always philosophical, grasping the nature of mathematics in its widest aspects and organizing its ideas rather then discovering new theorems]. He moved to London and became a teacher of applied mathematics at the Imperial College of Science and Technology (1914). He became interested in education, university administration, and philosophy of science. His shrewdness, common sense and goodwill put him in great demand as a committee man.

Whitehead was a pacific man but not a pacifist; he felt that WWI was hideous but that England's part in it was necessary. His elder son, North, fought throughout the war, and his daughter, Jessie, worked in the Foreign Office. In 1918 his younger son, Eric, was killed in action, and after that it was only by immense effort that Whitehead could go on working. To Whitehead, Russell's pacifism was simplistic; yet he visited him in prison, remained his friend, and, as Russell later said, showed him greater tolerance than he could return.

Whitehead emigrated to the U.S. in 1924 and became a professor of philosophy at Harvard University (1924–1938). His metaphysical system was influenced by **Plato**, **Aristotle**, **Leibniz**, **William James**, **Bergson**, **Minkowski** and **Einstein**. He set out in his philosophy to rectify what he considered to be the great error in the philosophical tradition — the doctrine of the duality of reality (which held that reality is a compound of mind and matter). This "bifurcation of matter", as he called it, was initially posited by **Descartes**, and, according to Whitehead, had poisoned philosophical thinking ever since.

Whitehead claimed that there is only *one reality*: reality consists only in what appears, in what is perceived, in whatever is in the experience of a subject (subject meaning any actual entity). There are neither concepts nor substances in the world: only a network of events. All such events are actual extensions of the unified whole of the space-time relation. Hence, reality is based on the patterned process of events.

His writings did much to narrow the gap between philosophy and science. He insisted that scientific knowledge, though precise, is incomplete. It must be supplemented, he said, by philosophical principles and the insight of poets(!)

Whitehead's nephew, **J.H.C. Whitehead**[246] (1904–1960, England), was a mathematician who specialized in topology and applied the subject to the study of generalized spaces. He was born in Madras, India and taught at Oxford (1932–1960).

Bertrand Russell was born near Trelleck, Wales. His father was a viscount and his grandfather, John Russell, was twice prime minister and became the 1^{st} Earl Russell. His parents died when he was 3 years old, and he and his brother were brought up by their grandparents. He entered Trinity College in 1890, where he graduated in 1894. His first wife was a Quaker from Philadelphia (1894). He married again in 1921.

Russell outspokenness and liberal views involved him in many controversies. During WWI he was dismissed from Cambridge University and imprisoned because of his pacifist views. In 1940, protests against his radical views on religion and morals caused the college of the city of New York to cancel his appointment as a professor, and he was almost dismissed from Harvard University. In 1961 he was imprisoned for his political activities.

Russell maintained that logic is not a function of philosophy but a general theory of science. He further insisted that the proper function of philosophy is to deal with the problems raised by the sciences, not with theological or ethical problems. Hence, philosophy should devote itself to analyzing the empirical data of science, because the primary problems of human life consist in the relation between individual experience and general scientific knowledge. Knowledge itself is a relatively unimportant feature of the universe; It can also be a corruptive influence in the search for truth, since it is so subject to the interpretations of human experience. To counter this, philosophy must limit itself to simple, objective descriptions of the phenomena of the world, keeping such descriptions free of the self-same corrupt influences of experience.

Russell defined himself thus: "*Liberal, anarchistic, left-wing skeptical atheist*".

[246] J.H.C. Whitehead was often asked for his views on the work of his uncle. Eventually he developed a standard answer. When asked "*What do you think of your uncle's philosophy*?", he would reply "*I really haven't thought much about it — but what do you think of your uncle's philosophy*?".

Worldview XXXI: Russell

* * *

"Aristotle maintained that women have fewer teeth than men; although he was twice married, it never occurred to him to verify this statement by examining his wives' mouths."

* * *

"The desire to understand the world and the desire to reform it are the two great engines of progress."

* * *

"It can be shown that a mathematical web of some kind can be woven about any universe containing several objects. The fact that our universe lends itself to mathematical treatment is not a fact of any great philosophical significance."

* * *

"The fact that all Mathematics is Symbolic Logic is one of the greatest discoveries of our age; and when this fact has been established, the remainder of the principles of mathematics consists in the analysis of Symbolic Logic itself."

* * *

"With equal passion I have sought knowledge. I have wished to understand the hearts of men. I have wished to know why the stars shine. And I have tried to apprehend the Pythagorean power by which number holds sway about the flux. A little of this, but not much, I have achieved."

* * *

"A good notation has a subtlety and suggestiveness which at times make it almost seem like a live teacher."

* * *

The conventional wisdom is that systems of reasoning must be consistent. That is, no statement can be both true and false. If so, then the system collapses because there remain no restrictions on what is true or false: every statement can be proved true (and false as well!).

When Bertrand Russell once made this claim during a public lecture he was challenged by a skeptical heckler to prove that the questioner was the Pope if twice 2 were 5. Russell replied, 'if twice 2 is 5, then 4 is 5, subtract 3; then 1 = 2. But you and the Pope are 2; therefore you and the Pope are one.'!

* * *

"How dare we speak of the laws of chance? Is not chance the antithesis of all law?"

* * *

"Mathematics takes us into the region of absolute necessity, to which not only the actual word, but every possible word, must conform."

* * *

"Perhaps the oddest thing about Modern Science is its return to Pythagoreanism."

* * *

"At the age of eleven, I began Euclid, with my brother as my tutor. This was one of the great events of my life, as dazzling as first love. I had not imagined there was anything so delicious in the world. From that moment until I was thirty-eight, mathematics was my chief interest and my chief source of happiness."

Worldview XXXII: Whitehead

* * *

"The aim of science is to seek the simplest explanation of complex facts. We are apt to fall into the error of thinking that the facts are simple because simplicity is the goal of our quest. The guiding motto in the life of every natural philosopher should be 'seek simplicity and distrust it'."

* * *

"The aims of scientific thought are to see the general in the particular and the eternal in the transitory."

* * *

"Now it cannot be too clearly understood that, in science, technical terms are names arbitrarily assigned, like Christian names to children."

* * *

"Not ignorance, but ignorance of ignorance, is the death of knowledge."

* * *

"We think in generalities, we live in detail."

* * *

"Everything of importance has been said before by somebody who did not discover it."

1911–1942 CE **Sergei Natanovich Bernstein** (1880–1968, Russia). Mathematician. Contributed to the theory of probability and the theory of approximation of functions. There are polynomials[247], an inequality and a constant named after him.

Bernstein was born in Odessa to Jewish parents. He went to Paris (1898) and studied at the Sorbonne. In his doctoral dissertation (1904) he solved Hilbert's 19th problem (to determine whether the solutions of "regular" problems in the calculus of variations are necessarily analytic). On his return to Russia (1905) he had to do a second doctorate (1913) in order to qualify for a university post. Through this work he solved Hilbert's 20th problem (concerning the analytic solution of Dirichlet's problem for a wide class of non-linear elliptic equations). Bernstein then taught at Kharkov University (1907–1932), Leningrad University, the Mathematical Institute of the USSR Academy of Sciences (1933–1942), and Moscow University (1943–1950).

1911–1947 CE **Charles Franklin Kettering** (1876–1958, USA). Engineer and inventor. Developed such varied industrial products as a self-starter for automobiles (1911), engine ignition system, ethyl gasoline, lightweight two-cycle diesel engine for trams, a high-compression automobile engine, four-wheel brakes, safety glass, the refrigerant *Freon* and many other items.

Kettering was born near Loudonville, Ohio. Bad eyesight slowed his education, but he graduated from Ohio State University in 1904 as an engineer and then joined the National Cash Register Company. In 1909 he left NCR and set up the Dayton Engineering Laboratories Company or Delco, where he invented his most significant engine devices. Kettering's engine-driven generator, named the 'Delco', provided electricity on millions of farms.

1911–1950 CE **Theodore von Kármán** (1881–1963, Hungary, Germany and U.S.A.). Founder of aeronautical and astronautical sciences in the 20^{th} century. Father of the supersonic age. Made important contributions to fluid mechanics, turbulence theory, supersonic flight, mathematics in engineering,

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[247] Let $f(x)$ be defined on $[], 0, 1\,]$. The n-th ($n \geq 1$) *Bernstein polynomial* for $f(x)$ is given by

$$B_n(f;x) = \sum_{k=0}^{n} f\left(\tfrac{k}{n}\right)\binom{n}{k} x^k (1-x)^{n-k}.$$

Notice that $B_n(f;0) = f(0), \qquad B_n(f;1) = f(1).$

$B_n(1;x) = 1; \qquad B_n(x;x) = x; \qquad B_n(x^2;x) = \frac{1}{n^2}\binom{n}{1}x + \frac{2}{n^2}\binom{n}{2}x^2;$

$B_n(e^{\alpha x};x) = [xe^{\alpha/n} + (1-x)]^n; \qquad \lim_{n\to\infty} B_n(f;x) = f(x).$

The Bernstein expansion often provides useful *approximations* to various functions.

aircraft structures and wind erosion of soil. In 1940 he showed together with **Frank J. Malina**, for the first time since the invention of the black-powder rocket in China in 969 *CE*, that it was possible to design a stable, long-duration, solid-propellant rocket engine.

Von Kármán participated in a number of major contributions to rocket technology, America's first assisted takeoff of aircraft with solid- and liquid-propellant rockets, flight of an aircraft with rocket propulsion alone, and development of spontaneously igniting liquid propellants of the kind that were to be used in the Apollo Command and Lunar Excursion modules in the late 60's.

Von Kármán was born in Budapest to Jewish parents, and was educated in Budapest and Göttingen. He became an assistant to **Ludwig Prandtl** on dirigible research, and came under the influence of **David Hilbert** and **Felix Klein**. The latter stressed the fullest use of mathematics and the basic sciences in engineering to increase technological efficiency.

In 1911 he discovered that at large Reynolds-numbers, the eddies in the wake behind an elliptic cylinder do not remain stationary with the cylinder but shed alternatively to form the '*Von Kármán vortex street*' (consisting of two staggered rows of counter-rotating vortices). In 1921 he obtained an approximate solution for two-dimensional laminar boundary layer flow, which he converted into a momentum integral equation (*Kármán-Polhausen equation*).

A law describing the shear forces which *turbulent flows* exert at boundaries was proposed by von Kárman (1930) and **Prandtl** (1933). It states that a fluid's average velocity is proportional to the logarithm of the distance from the boundary and known as Universal *Law of the Wall*. The logarithmic velocity distribution was first found for flow between two walls, but it could be applied to the calculation of the skin friction of a flat plate which is covered by a *turbulent boundary layer*[248].

In 1912 he became director of the Aeronautical Institute at Aachen, Germany, remaining there until 1930. He then moved to the United States, assuming the direction of the Guggenheim Aeronautical Laboratory at the California Institute of Technology, which in 1944 became the NASA Jet Propulsion Laboratory in Pasadena. Shortly after his arrival at CALTECH, his laboratory became a Mecca of the world of aeronautical sciences.

In 1932 he extended the *Joukowski transformation* so as to obtain an airfoil with a finite trailing edge [Kármán-Trefftz airfoil]. In 1938 he put forward a statistical theory of isotropic turbulence [*Kármán-Howarth theory*],

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[248] Some scientists claimed that the law may be in disagreement with experiment by up to 65 per cent, but this conclusion remains controversial.

and in 1941 he advanced the *Kármán-Tsien approximation* for steady two-dimensional subsonic flow. Also in 1941, von Kármán participated in the founding of the Aerojet General Corporation, the first American manufacturer of liquid- and solid-propellant rocket engines. In 1960, his efforts brought into being the International Academy of Astronautics. A crater on the moon has carried his name since 1970.

Von Kármán was a unique man in many ways: as a classroom teacher his clarity and imagination were superbly effective. At the blackboard he would run into, and extricate himself from, mathematical traps and blind alleys of his own making with such agility as to make the class hold its breath and then applaud. Mathematical equations were almost alive, performing like puppets before the class.

As head of the Aeronautical Institute of Aachen, and late of CALTECH's Guggenheim Aeronautical Laboratory at Pasadena, von Kármán guided two generations of scientists and engineers into pioneering areas that led to the establishment of aviation and astronautics on firm scientific foundations. His vivid and dynamic personality held people spellbound with his Old World wit, showmanship, and crusty insights into natural phenomena and human nature. The following story serves to illuminate both his wisdom and the 'name of the game' in applied mathematics:

Once, during a laboratory session, a student reported that he had made a peculiar observation which he couldn't explain. Von Kármán adjusted his hearing aid and listened intently. Then he gave a plausible explanation for the student's puzzling observation. The fellow thanked him profusely. As he turned to leave, von Kármán called him back. "*Wait*", he said. "*Just in case you made a mistake and the effect is negative* [*just the opposite*], *then the explanation is as follows*", and he proceeded to give an equally clear and plausible explanation for the opposite effect.

Von Kármán expected great respect from the students, and he had clever ways of chiding those who were deceived by his gentle ways and tried to take advantage of him. Once a student came to class, unfolded a newspaper and proceeded to read while the professor was lecturing. He did this for several days, and each day von Kármán's annoyance grew. Finally, unable to stand the inattention any more, he hired a boy to bring up a cup of coffee and set it in front of the offensive student. The class roared with laughter. The poor fellow never opened his paper in class again.

When President Kennedy presented him with a special medal in the White House (February 1963), von Kármán responded:

"I hope that I have shown that the college professor is of use."

1912 CE, April 14–15 The *Titanic* disaster; the largest ship in the world at the time (269 meters long; 46,328 tons), struck an iceberg on its maiden voyage from England to New York City, and sank to a depth of 4000 m. The *Titanic* sighted the iceberg just before the crash (11:40 PM ships-time, April 14; 2570 km northeast of New York), but too late to avoid it. The lifeboats held less than half of the approximately 2200 passengers. The ship sank at 41°44′N, 49°57′E in 2^h40^m hours (02:20 AM). The liner *Carpathia* picked up 705 survivors. Among the victims were 670 crew members, including the captain E.J. Smith. It was a moonless night and the iceberg was *not seen*.

Not until 1985, searches led by oceanographer **Robert Ballard** revealed the *Titanic* resting in two pieces on the ocean floor.

In 1914, International Ice Patrol was established as a result of the sinking of the *Titanic*. Thirteen maritime nations, using shipping lanes in the ice regions, met at the International Conference on the Safety of Life at Sea, and on January 20 signed an agreement to establish the patrol and pay its expenses.

1912 CE **Max Theodor Felix von Laue** (1879–1960, Germany). Physicist. Discovered the diffraction of X-rays in crystals and as a result was able to measure the wavelengths of X-rays and to study the structure of crystals. Contributed to the developments of the theories of relativity, electromagnetism, and diffraction of light. Awarded (1914) the Nobel prize for physics.

Prior to 1912, X-rays had resisted being diffracted even by the most precise gratings. While studying the theory of 3-D gratings, von Laue recognized that since atoms had dimensions and separations that were so much smaller than the wavelength of visible light, a crystal might very well serve as a *natural diffraction grating* for X-rays. Accordingly, his associates directed a beam of X-rays at a crystal of copper sulfate (they also later tested zinc sulfate, diamond, rock salt, and copper) and exposed a photographic plate for several hours to the *transmitted* radiation. The outcome was a remarkable concentric pattern of spots that showed that the X-rays were indeed subject to diffraction.

By working backwards from the observed diffraction angles, it became possible to deduce the geometry of the "grating". These experiments confirmed the electromagnetic wave nature of X-rays, allowed measurements of their wavelengths, *and* opened the way to the determination of interatomic distances in crystals. It was one of the most important scientific discoveries of the 20^{th} century, and illustrates once again the role of *creative imagination* in scientific discovery.

Far from simply "looking at the facts", a popular misconception of the scientific method, Max von Laue was inspired to a new world of facts to examine.

His discovery immediately led to Moseley's investigation of X-ray spectra and his discovery of atomic number. Many scientists were soon working in the new field of crystal structure determination, and X-ray crystallography rapidly expanded into a major branch of chemistry[249] and physics.

Von Laue's method had some series disadvantages and has been superseded by other methods in which X-rays of a single wavelength are *reflected* from crystal faces. Because X-rays penetrate deeply into the crystal, the geometric condition for diffraction differ from that of optical diffraction: the grating or *lattice* has 3 effective dimensions instead of only two, and the diffraction angles θ are determined by the ratio of wavelength λ to the spacing d between parallel planes through sites occupied by atoms in the crystal according to *Bragg's equation* (1912) for *constructive interference*: $2d\sin\theta = n\lambda \ (n = 1, 2, 3, \ldots)$. If monochromatic X-rays were used, then, even if their wavelength is not known, the *relative* interplanar spacing for different set of planes within the crystal can be determined[250].

[249] Some of the most exciting applications of X-ray diffraction have been in *molecular biology*: **J.C. Kendrew** and **M.F. Perutz** were awarded the Nobel prize in chemistry (1962) for their studies of myoglobin and hemoglobin. The award for 1964 went to **Dorothy Crowfoot Hodgkin** for her solution of the structure of vitamin B_{12}, work that required the application of an electronic computer to analyze the complex crystallographic data.

[250] Absolute determination of d requires, however, a preknowledge of λ. Early attempts to measure the wavelength of X-rays with ruled gratings failed because of the large disparity between the grating spacing (several thousands Ångstroms) and the wavelengths (a few Ångstroms). Finally, **Compton** (1923) was successful in obtaining diffraction patterns when X-rays fell on ruled gratings at *grazing incidence*: by passing the radiation in a fine beam very close to the surface of the grating it is possible to obtain *total reflection* of the beam. In addition to the reflected rays, *diffracted beams* of different orders are found and these are used to standardize the X-ray wavelengths in terms of metric units. In this way lattice constants can be measured very accurately from X-ray crystal diffraction patterns and the method further leads to a determination of the electronic charge via the ionic theory of solids.

Crystals and Quasicrystals

For the majority of chemical compounds, a most striking aspect of the solid state is crystallinity: the solid consists of crystals having a recognizable natural shape, bounded by plane surfaces which meet at characteristic angles. Because of the forces acting between the atoms or ions of the solid (crystal), a stable equilibrium is possible only if constituents are arranged in a regular repetitive geometric pattern called a crystal lattice. Group-theoretical methods show that there is a finite but large number of crystal systems which are possible arrangements of the lattice constituents in a crystal. In which one of these systems a particular material crystallizes, depends upon the size of the atomic or ionic constituents and upon the magnitude and orientation of the forces acting between them. The same material may crystallize in different configurations depending upon the type of binding forces (carbon, for example, can crystallize in the graphite or diamond lattice).

A fundamental property of crystals is their symmetry. It is possible to pick the smallest part of the structure that has all the symmetry elements of the entire 3*-dimensional pattern. This is called the unit cell, or elementary cell, and the entire crystal can then be built by translational repetition along the crystal axes. In the cubic NaCl crystal, for instance, the elementary cell is a small cube. For a cubic lattice, the smallest distance of two equivalent constituents is designated as the lattice constant a; the distance of two neighboring lattice planes as the lattice distance, d. If a coordinate system is introduced with its axes parallel to the edges of the crystal, each crystal plane (and thus the whole crystal as determined by the crystal planes) may be characterized by specifying the lengths from the origin to the intersections of the crystal plane with the axes. These lengths are measured in units of the corresponding edges of the elementary cell.*

The principal classification of crystals is on the basis of their symmetry. An object has symmetry if some operation can be carried out on it that converts it into itself. For example, a three-bladed propeller can be rotated about its axis by 120° *(one-third of a revolution), and it is then indistinguishable from its original condition, provided that the three blades are exactly identical to one another. Similarly it can be rotated by* 240° *(two-thirds of a revolution), again becoming indistinguishable from its original condition. These operations of rotation by $\frac{1}{3}$ of a revolution and rotation by $\frac{2}{3}$ of a revolution, together with the original operation involving no change — constitute the symmetry group corresponding to a* 3*-fold axis of symmetry.*

Only a few symmetry elements are manifested by crystals. These include: center of symmetry, 2*-fold axis,* 3*-fold axis,* 4*-fold axis,* 6*-fold inversion axis,* *3-fold inversion axis, and symmetry plane (a 5-fold axis does not occur in crystals because the angle of a pentagon, 108°, is not a factor of 360°).*

There are 32 combinations of these symmetry elements that are represented by crystals. These combinations are called the 32 crystal classes. They can be divided in 6 crystal systems, as follows:

(a) *Cubic crystals, with both 3-fold and 4-fold symmetry axes (the 4-fold axis can be of the rotation-inversion type); three equal mutually orthogonal axes of symmetry. The body-centered cubic lattice has one lattice point at each corner plus one lattice point in the center of the cubic volume. The face-centered cubic lattice has one lattice point at each corner plus one lattice point at the center of each of the 6 faces of the cube [octahedral, tetrahedral and dodecahedral crystals belong to the cubic system].*

(b) *Tetragonal crystals, with one 4-fold axis; two equal symmetry axes with length a, and a third axis with length c, all at right angles.*

(c) *Hexagonal (or trigonal) crystals (including rhombohedral crystals), with one 6-fold axis or 3-fold axis; two equal axes of symmetry with length a and oriented at 120° to each other, and a third axis with length c, at right angles to the other two [bipyramidal, rhombohedral and trigonal trapezohedral crystals belong to the hexagonal system].*

(d) *Orthorhombic crystals, with two or three planes of symmetry or twofold axes at right angles to each other; three axes, with lengths a, b, c at right angles to one another.*

(e) *Monoclinic crystals, with one plane of symmetry, or one twofold axis, or both; two axes of symmetry, a and c, at the angle β with one another, and the third axis b, at right angles to a and c.*

(f) *Triclinic crystals, with either a center of symmetry or none; three axes of symmetry a, b, c, with angles α, β and γ between them.*

The faces of the crystals must be related to the axes in a rational way; the intercepts of a face with the three axes are related to the lengths of the axes a, b, c, in the ratio of integers.

The use of symmetry arguments can tell us whether a molecule has a dipole moment and along which line it lies. Since a symmetry operation leaves a molecule in a configuration physically indistinguishable from the one before the operation, the direction of the dipole moment vector must also remain unchanged after the symmetry operation. Therefore, if a molecule has a n-fold axis of rotation, the dipole moment must lie along this axis. But if we have two or more non-coincident symmetry axes, the molecule cannot have a dipole moment because it cannot lie on two axes at the same time.

Methane (CH_4) has 4 non-coincident axes and therefore has no dipole moment. If there are several symmetry planes, the dipole moment must lie along their intersection. In ammonia (NH_4) the dipole moment lies along the 3-fold symmetry axis which is also the intersection of 3 symmetry planes. A molecule having a center of symmetry cannot have a dipole moment, since inversion reverses the direction of any vector.

X-ray diffraction studies have shown that nearly all metallic *elements crystallize in either a* cubic face-centered *lattice,* hexagonal close-packed *lattice, or a* body-centered cubic *lattice.*

The noble gases neon, argon, krypton and xenon at sufficiently low temperatures crystallize in a face-centered cubic lattice.

Quasicrystals

Until the early 1980's, scientists believed that all *crystal solids were arranged* periodically *at a* microscopic level. *It was assumed that the only way to achieve order in the bulk is by having some basic structural unit which repeats itself infinitely in all directions, filling up all space. This is much like the way in which a floor can be covered in patterns of identical squares or hexagons, without leaving gaps. However, covering the floor with pentagon or decagons results in a patchwork of empty space because under this arrangement the tiles cannot stretch to infinity in a regularly repeated pattern, i.e. the pattern lacks* periodicity. *(In periodic patterns, if one tile is surrounded, say, by six neighbors, then* every *tile is surrounded by six neighbors.)*

In nature we observe this regularity in the way in which bees arrange their honeycombs in periodic hexagonal arrays. (The artist ***M.C. Escher*** *has exhibited many periodic 2D and 3 D structures in his drawings.)*

A perfect crystal consists of a space-filling array of periodically repeated identical copies of a single structural unit containing some distribution of matter and charge. In the simplest case, the structural unit contains a single atom. More generally, it may contain many different atoms or a continuous variation in the mass density about some mean. The repeated structural unit is called the unit-cell.

Equivalent points in unit cells in a D-dimensional perfect crystal lie on a periodic lattice*, called a* Bravais lattice*, consisting of a* mathematical *array of points.*

Any lattice point can be specified by an integral linear combination of independent translation vectors $\boldsymbol{a}_1, ..., \ \boldsymbol{a}_D$

$$\boldsymbol{R}_l = l_1\boldsymbol{a}_1 + l_2\boldsymbol{a}_2 + \cdots + l_D\boldsymbol{a}_D$$

where $\boldsymbol{l} = (l_1...l_D)$ *is a D-dimensional vector with components* l_i *(*$\boldsymbol{l}$ *indexes a particular unit cell,* $\boldsymbol{R}_l$ *specifies its position in real space). The set of vectors* $\boldsymbol{a}_1...\boldsymbol{a}_D$ *completely define the mathematical lattice. A translation vector (or lattice vector) connects equivalent points in the lattice:*

$$\boldsymbol{T} = \boldsymbol{R}_l - \boldsymbol{R}_{l'}.$$

If an infinite crystal is translated through a lattice vector $\boldsymbol{T}$*, it will be absolutely indistinguishable from the untranslated crystal to a fixed observer in some laboratory frame of reference.*

Translations by lattice vectors are symmetry operations that leave the physical properties of the crystal invariant. The set of all lattice translations form a (commutative) group*, and the crystal is said to be invariant under operations of this group.*

Crystals are also invariant under point-group *operations consisting of* rotations*,* reflections*, and* inversions *about special* symmetry *points.*

Molecules and other finite size objects can have symmetry axes of arbitrary order. But the requirement that a crystal be invariant under translations through any vector on its lattice (the set of which can be shown to contain no vector shorter than some minimum length vector) places severe restrictions on possible rotational symmetries. Thus, e.g., it is impossible for a periodic crystal to have a 5-fold symmetry*, i.e. to be invariant w.r.t. rotations through* $2\pi/5$.

To see this assume that a crystal does have 5-fold symmetry and let $\boldsymbol{a}_0 = (1, 0)$ *be the shortest vector in the lattice. Since the crystal is assumed to have a 5-fold symmetry, the vectors*

$$\boldsymbol{a}_n = \left(\cos\frac{2\pi n}{5}, \ \sin\frac{2\pi n}{5}\right),$$

with n *an integer, must also be in its lattice. But by the closure property of any lattice, the vector*

$$\boldsymbol{T} = \boldsymbol{a}_4 + \boldsymbol{a}_1 = \left(\cos\frac{8\pi}{5} + \cos\frac{2\pi}{5}, \ \sin\frac{8\pi}{5} + \sin\frac{2\pi}{5}\right) = \tau^{-1}(1, 0) = \tau^{-1}\boldsymbol{a}_0$$

where

$$\tau = 2\cos\frac{\pi}{5} = \frac{1}{2}(1+\sqrt{5}),$$

must also be in the lattice, [$\tau^2 = \tau + 1$, τ = the golden mean].

However, $\boldsymbol{T}$ is shorter than $\boldsymbol{a}_0$, contradicting the assumption that $\boldsymbol{a}_0$ was the shortest vector in the lattice. Thus, it is impossible for a periodic lattice in two dimensions to have a 5-fold symmetry. Similar arguments rule out all periodic lattices in two dimensions with other than 2-, 3-, 4-, or 6-fold symmetry. It can be shown that these restrictions lead to only five distinct two-dimensional Bravais lattices: square, oblique, rectangular, centered rectangular and hexagonal.

In three dimensions, the existence of a periodic crystal with the point-group symmetry of an icosahedron[251] *(six 5-fold, ten 3-fold and fifteen 2-fold axes) is similarly ruled out.*

As a result, a fundamental tenet of classical crystallography[252] *was that* MATERIALS EXHIBITING ICOSAHEDRAL SYMMETRY COULD NOT EXIST.

*In 1984, **Dan Schechtman** and his colleagues shook the foundation of crystallography*[253] *when they reported an electron diffraction pattern for an alloy of aluminum and manganese formed by rapid cooling of a mixture of two molten metals; it showed a clear point-group symmetry of an icosahedron, namely: 5-, 3-, and 2-fold axes characteristic of icosahedral symmetry. The density of Bragg peaks in each plane was* higher *than one would expect from*

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[251] One can stack atoms together in 3D lattice arrays that have 3-fold, 4-fold and 6-fold symmetries, but 5-fold symmetric periodic stacking is not possible: A cubic lattice – a stacking of perfect cubes – exhibits not only the easily recognized 4-fold symmetry, but also 3-fold, when viewed along cube *diagonals*. But the two regular polyhedra with 5-fold symmetry – the *dodecahedron* and *icosahedron* – cannot be stacked together in a way that fills 3-dimensional space without gaps.

[252] The idea that crystals are periodically ordered first emerged in the works of **Kepler** (1619) and **Hooke** (1660) as a direct result of the advent of microscopy. These ideas were then formalized into the theory of *crystallography* by **René-Just Haüy** (1822). With the discovery of X-ray diffraction in crystals (**von Laue**, 1912) and the subsequent development of X-ray crystallography (**W.H. and W.L. Bragg** 1915), the periodicity of crystals received an unequivocal stamp of approval because light diffracted from crystals produced sharp, crisp patterns indicative of periodicity.

[253] **Linus Pauling**, however, was not convinced and in the ensuing years slowed the bandwagon. Finally his alternative theory lost to the prevailing views of quasicrystalists.

a periodic crystal[254]*. There were 10 bright spots in the ring around the bright central blob, implying that the alloy was a crystal with a "forbidden" 10-fold symmetry. Rotating the crystal lattice by 72° would leave it unchanged.*

The aluminum/manganese alloy, as well as many other metal alloys that have since been found to produce diffraction patterns with forbidden 5-, 8-, 10-, and 12-fold symmetries, cannot be true crystals in the sense of possessing a genuine unit cell which repeats throughout the material. In other words, the position of their constituent atoms cannot be orderly over large distances. The geometric ban on crystals with these symmetries is mathematically absolute.

These alloys, which appear to be neither crystalline nor wholly noncrystalline, have been given the name "quasicrystals"[255] *(short for "quasiperiodic crystals").*

We have seen that the closure property of any lattice implies that a lattice with 5-fold symmetry has collinear vectors with irrational magnitude ratios, and has vectors with arbitrary small separations. However, atoms in real space cannot be arbitrary close together. In a periodic solid, the existence of a shortest length in a unit cell ensures that distances between atoms are greater than some minimum distance. How can there be a minimum distance in a quasicrystal?

The answer is provided by tiling of a 2-dimensional plane with 5-fold symmetry invented by ***Roger Penrose***[256] *(1974) and by their generalization to*

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[254] Disordered, amorphous materials, in contrast, produce smeared-out scattering patterns, from which rather little can be deduced about the structure. Symmetry properties of the diffraction pattern give an indication of that of the crystal.

[255] In 1991 the International Union of Crystallography decided to *redefine* the term "crystal" to mean any solid having an essentially discrete diffraction diagram, thereby shifting the essential attribute of crystallinity from position space to Fourier space. This broader definition reflects our current understanding that *microscopic periodicity* is a sufficient but not a necessary condition for crystallinity.

One can also use the diffraction diagram to distinguish between periodic crystals and quasicrystals as follows: Each Bragg peak in the discrete diffraction diagram defines a *wave vector* which points from the center of the diagram to the peak, and at which the density has a nonvanishing coefficient in its Fourier expansion.

[256] **Penrose**, a mathematical physicist, had become captivated by the post-WWII work of artist **Maurits C. Escher**. His tilings represent an exploration of ways in which a plane can be filled completely by identical tiles *without* producing long-range order (that is, in an aperiodic manner).

*icosahedral symmetry in 3 dimensions. The basic idea here is to fill the plane with two types unit cells (tiles) rather than a single unit cell required to tile a plane periodically. Adjacent tiles must be joined so that they obey certain matching rules. There is a shortest distance between tile vertices and it is possible to "decorate" the tiles with atoms in such a way that the atoms have a minimum separation. The diffraction pattern (Fourier transform), first calculated by **D. Levine and P.J. Steinhardt** (1984), of the icosahedral generalization of Penrose tiles, agrees well with the experimental observed pattern.*

Why do atoms form a complex, quasiperiodic pattern rather than a regularly-repeating, crystal arrangement? Scientists suggested that the conditions necessary to form quasicrystals are significantly more complex than conditions for forming crystals. Yet, the energetics of the process are as yet not well understood.

There is no doubt that quasicrystals represent a new kind of a solid. Much of the research now focuses on how their unusual structure affects properties such as electrical conductivity or magnetism. A stimulating aspect of the discovery of quasicrystals is that it has led to a new appreciation of the importance of 5-fold symmetric objects in many spheres of science, from the shape of viruses[257] *and flowers to the patterns of fluid flow close to the onset of turbulence.*

[257] Simian virus 40 (SV40) is an example of a DNA virus. It appears to be spherical, but it is actually an icosahedron, a geometric figure with 20 faces that are equilateral triangles. The genome of this virus is a closed circle of double-stranded DNA, with genes that encode the amino acid sequences of five proteins. Three of the five proteins are coat proteins. Of the remaining two proteins, one, the large-T protein, is involved in the development of the virus when it infects a cell. The function of the fifth protein, the small-T protein, is not known. In the complete virus particle (called a virion), the coat proteins are packed around the DNA to give the observed icosahedral shape.

1912 CE **Casimir Funk** (1884–1967, Poland, U.S.A.), and **Frederick G. Hopkins** (1861–1947, England, 1906) independently discovered the vitamin concept. Funk isolated the crystalline B-complex and named it *vitamine* (= vit amine = amine essential to life). Later, the name was changed to *vitamin* when it was determined that the labeled substances were not all amines.

1912 CE **Chaim Weizmann** (1874–1952, England and Israel). Chemist and statesman. Found (1912) a bacterium *Clostridium acetobutylum* which would convert carbohydrate into acetone. This process proved of great importance in WWI as acetone is used in large quantities to plasticize the propellant cordite.

Weizmann was born in Motol, Russia. He obtained his Doctorate from the University of Fribourg, Switzerland (1899) and then moved to the University of Geneva, where he produced a number of patents on dyestuffs (1901). By this time he was already an important figure in the Zionist movement. In 1904 he moved to Manchester, England, to work with **William Henry Perkin, Jr**.

Weizmann became the president of the World Zionist Movement (1921). He founded (1934) the *Weizmann Institute of Science* in Rehovot and became the first President of Israel (1948).

1912–1924 CE **Franz Kafka** (1883–1924, Czechoslovakia). Novelist and one of the most influential thinkers of the 20-th century. Virtually unknown during his lifetime, his surreal works have become synonymous with the grotesque alienation of modern man in an unintelligible, hostile, or at least indifferent world.

In his novels '*The Trial*' (1914–1916, published posthumously 1925) and '*The Castle*' (1922–1924, published posthumously 1926), one finds authority a hierarchical, abstract, and impersonal "apparatus" whose operation is controlled by procedures which remain shadowy even to those carrying out its orders and *a fortiori* to those being manipulated. A modern citizen realizes that his fate is being determined by brutal, petty, and sordid characters (the bureaucrats), who themselves are cogs in this machine.

Kafka's critique of the state touches anonymous impersonal character insofar as this alienated, hypostatized and autonomous bureaucratic system is becoming transformed into an end-in-itself.

Kafka had a profound insight into the way the bureaucratic machine operates like a blind network of gears. When he speaks to us of the *state*, it is in the form of "administration" or "justice" as an impersonal system of domination which crushes, suffocates, or kills individuals. This is an agonizing, opaque, and unintelligible world where unfreedom prevails.

"*The Trial*" is often presented as a prophetic work. With his visionary imagination, the author had foreseen the 'justice' of the totalitarian state and the Nazi or Stalinist show trials.

It is no accident that the word *"kafkaesque"* has entered our current vocabulary. The term denotes an aspect of social reality that sociology and political science tend to overlook. With his libertarian sensibility, Kafka has succeeded marvelously in capturing the *oppressive* and *absurd* nature of the bureaucratic nightmare, the opacity, the impenetrable and incomprehensible character of the rules of the state hierarchy as they are seen *from below and the outside*. This runs contrary to social science which generally confines itself to examining the bureaucratic machine from the "inside" and taking the point of view of those "at the top."

Social science has not yet formulated a concept for the "oppressive effect" of a reified bureaucratic apparatus which undoubtedly constitutes one of the most characteristic phenomena of modern societies which millions of men and women run across daily. Meanwhile, this essential dimension of social reality will continue to be conjured up by reference to Kafka's work.

In his story "*Penal Colony*" (1915), written three months after the outbreak of WWI, the main character is the machine itself — a sinister apparatus which does not exist to execute a man, but rather the victim exists for the sake of the apparatus. Thus saw Kafka the Great War as an "apparatus of Authority," sacrificing human lives.

The work of Franz Kafka cannot be reduced to a political doctrine of any kind. The symbolic world of literature cannot be reduced to the discursive world of ideologies. It is not an abstract conceptual system similar to philosophical or political doctrines, but rather the creation of a concrete imaginary universe of individuals and things.

Kafka's works have been interpreted through certain schools as literary criticism such as *existentialism*, *Marxism*, *anarchism*, *Freudianism* and *Judaism*. It seems, however, that a combination of major biographical factors permeated and dominated the Kafkaesque world. These are

- Ill-health and consequent perpetual fear of physical and mental collapse. He was a bit of hypochondriac and anorexic. His prodigious consumption of unpasteurized milk was the source of his tuberculosis, which finally killed him at age 41.

- The complex relations with his father. Herman Kafka was a domestic tyrant, who directed his anger against his son. Kafka himself attributed much of his outlook on life to the affects of the relationship with his father. In *Letter to His Father* (1919) he stated: "My writing was all about you; all I did there, after all, was to bemoan what I could not bemoan upon your breast. It was an intentionally long-drawn-out leave-taking from you." His work was often fraught with cold, authoritarian figures that persecuted and threatened for reasons barely understood and often unexplained — a situation which perfectly summed up Kafka's childhood sense of his own father.

- The institutionalized system of control of the post Austro-Hungarian Empire, composed of: "a stand of soldiers, a sitting army of officials, a kneeling priests, and creeping army of informers."

- The Virgin/Whore complex[258] that made him virtually incapable of a "normal" sexual affair with a "nice girl" owing to his neurotic attitude toward sex.

- The historical Hassidic-Kabbalistic Jewish mystique.[259]

- The pervasive Czech-German anti-semitism in Prague [Kafka grew up in an atmosphere of familial tensions and social rejection that he experienced as a member of Prague's Jewish minority].

Franz Kafka was born in Prague to the Jewish family of Hermann (1852–1931) and Julie Loewy (1856–1934), who operated a small business for linen thread and cotton. In 1901 he graduated from a German High School and went on to the Karl Ferdinand University in Prague, where he graduated (1906) with a doctorate in law. During 1908–1922 he was employed by the "Institute for Worker Accident Insurance" in Prague. There he made reports on industrial accidents and health hazards. His profession marked the formal, legalistic language of his stories which avoided all sentimentality and moral interpretations — all conclusions are left to the reader.

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[258] In this syndrome, every woman is either a "nice girl" or slut, with no room in between. So a normal adult affair with a woman he liked and respected would prove all but impossible, as **Felice Bauer** soon found out.

[259] Kafka was not a religious writer, and at face value, his writings have nothing to do with Judaism. However, his works incorporate and reinterpret many Jewish religious themes. Kafka had a positive relationship vis-à-vis his Judaism – it was not one of rejection or apathy. He had a strong Jewish identity albeit a highly troubled one.

He attended synagogue but four times a year, spoke mainly German and had little cultural involvement with the Jewish community, with the exception of Jewish theater groups. In 1911 he made a trip to Paris, Italy and Switzerland.

Kafka had three sisters: Gabriele (Eli): 1889–1942, Valerie (Vali): 1892–1943 and Ottilie (Ottla): 1890–1943. All of them, with their families, perished in the Nazi Holocaust: Eli and Vali at the Lodz Ghetto and Ottla gassed in Auschwitz on Oct 5, 1943.

His attitudes toward sexuality was marked by extreme neurosis. Unable to reconcile his physical urges (which were visited upon prostitutes and loose women) with his romantic longings, he had a series of prolonged, probably chaste, engagements that invariably ended in his breaking off the relationship. Despite this fact, and despite his being overwhelmed with feelings of inadequacy and self-loathing, he was much liked by friends (including women friends[260]) for his gentle, cool demeanor, his wry wit, and his obvious intelligence. He was further viewed not as weak and repulsive, his own fear, but as boyishly good looking, as well as neat and austere.

During 1907–1923, Kafka had intimate relations with at least eight women, some of whom impacted his creative work in one way or another. In order of occurrence they are:

- Hedwig Weiler (1887–1953), 1907–1908
- Flice Bauer (1887–1960), 1912–1917; engaged to
- Greta Bloch (1892–1944), 1913–1915
- Gerti Wasner (1895–?), 1913
- Julie Wohryzek (1891–1944), 1919–1920; engaged to
- Minze Eisner (?–?), 1920
- Milena Jesenská (1896–1944), 1920–1922

260 He once met a little girl in the streets of Berlin — five years old, or something like that — who was crying, and he asked her why she was crying, and she said it was because she'd lost her doll. And he said, 'You haven't lost your doll; she's gone on holiday.' And for the next weeks, Kafka wrote letters as from the doll to this little girl. Well, a man who can do that has a heart as big as all outdoors, as they say. So it was no wonder that his friends loved him, there's no wonder that his girlfriends loved him, despite all this maddening neuroticism and indecision.

- Dora Diamant (1898–1952), 1923–1924; lived with

Of these, Gerta Bloch, Julie Wohryzek and Milena Jesenska perished in the Holocaust.

Franz Kafka died in 1924 in the Kierling Sanatorium from complications related to tuberculosis. It has been noted that due to the great pain in his throat, from this condition, he was unable to eat and barely to drink in his last days. He often complained of thirst in his letters home to his father, and at the end may have simply starved to death. His body is buried in Prague, in the Jewish section of the Strasnice cemetery.

He remained self-deprecating about his writing, as he was about himself in general, and on his death bed asked his close lifelong friend **Max Brod** to burn all his unpublished works, including journals and correspondence. It was a promise Brod did not keep. Instead Brod, who had always been impressed with Kafka's literary abilities, edited a portion of what remained and had it published. Interest in Kafka's work soared and, over the years, has continued to garner a following. While Kafka holds a considerable following among existentialists, who find in his gloom and doom anxiety about life an expression of deepest truth (that we are meaningless and alone, yet can't help but yearn for it all to matter), he also finds sympathetic response among magical realists and others who favor an absurdest, surreal representation of life.

Kafka has become an icon of sorts, emblematic of modern times. His popularity increased exponentially after publication of his stories in the 20s and 30s of the 20^{th} century. He is now an institution, his own adjective. In 1995, somebody bought the manuscript of *The Trial* for to two million dollars, not bad for an uncompleted manuscript meant for the flames. Few writers had such an effect on their times as he had.

The asteroid 3412 Kafka, a small main belt bolide (discovered 1988) was named after him.

Worldview XXXIII: Franz Kafka

* * *

Logic is doubtless unshakable, but it cannot withstand a man who wants to go on living.

(The Trial)

* * *

The metaphysical urge is only the urge toward death.

(1912)

* * *

Beyond a certain point there is no return. This point has to be reached.

* * *

In the fight between you and the world, back the world.

* * *

Every revolution evaporates and leaves only the slime of a new bureaucracy.

* * *

The chains of the tortured humanity are made of the official papers of ministers.

* * *

If it had been possible to build
the Tower of
Babel without ascending it, the
work would
have been permitted.

* * *

Anyone who believes cannot experience miracles. By day one cannot see any stars.

* * *

We were expelled from Paradise, but Paradise was not destroyed. In a sense our expulsion from Paradise was a stroke of luck, for had we not been expelled, Paradise would have had to be destroyed.

* * *

You are free and that is why you are lost.

* * *

Anything that has real and lasting value is always a gift from within.

* * *

Either the world is so tiny or we are enormous; in either case we fill it completely.

* * *

All knowledge, the totality of all questions and all answers is contained in the dog.

* * *

There is a goal, but no way; but what we call a way is hesitation.

* * *

In science one tries to tell people, in such a way as to be understood by everyone, something that no one ever knew before. But in poetry, it's the exact opposite.

* * *

Altogether, I think we ought to read only books that bite and sting us. If the book does not shake us awake like a blow to the skull, why bother reading it in the first place? So that it can make us happy, as you put it? Good God, we'd be just as happy if we had no books at all; books that make us happy we could, in a pinch, also write ourselves. What we need are books that hit us like a most painful misfortune, like the death of someone we loved more than ourselves, that make us feel as though we had been banished to the woods, far from any human presence, like a suicide. A book must be the ax for the frozen sea within us. That is what I believe.

* * *

To die would mean nothing else than to surrender a nothing to the nothing, but that would be impossible to conceive, for how could a person, even only as a nothing, consciously surrender himself to the nothing, and not merely to an empty nothing but rather to a roaring nothing whose nothingness consists only in its incomprehensibility.

(1913)

* * *

It is entirely conceivable that life's splendor forever lies in wait about each of us in all its fullness, but veiled from view, deep down, invisible, far off.

(1921)

* * *

Youth is happy because it has the ability to see beauty. Anyone who keeps the ability to see beauty never grows old.

* * *

Discoveries have forced themselves on people.

(1913)

* * *

The meaning of life is that it stops.

* * *

The life of society moves in a circle. Only those burdened with a common affliction understand each other.

(1914)

* * *

We are sinful not merely because we have eaten of the tree of knowledge but also because we have not eaten of the tree of life.

* * *

Many a book is like a key to unknown chambers within the castle of one's own self.

(1903)

* * *

Writing sustains me. But wouldn't it be more accurate to say that it sustains this kind of life? Which does not, of course, mean that my life is any better when I don't write. On the contrary, at such times it is far worse, wholly unbearable, and inevitably ends in madness. This is, of course, only on the assumption that I am a writer even when I don't write — which is indeed the case; and a non-writing writer is, in fact, a monster courting insanity.

(1922)

* * *

When it became clear in my organism that writing was the most productive direction for my being to take, everything rushed in that direction and left empty all those abilities which were directed toward the joys of sex, eating, drinking, philosophical reflection and above all music.

(1912)

* * *

God doesn't want me to write, but I — I must.

* * *

These revolting doctors! Businesslike, determined and so ignorant of healing that, if this businesslike determination were to leave them, they would stand at sickbeds like schoolboys. I wished I had the strength to found a nature-cure society.

(March 5, 1912)

* * *

The priest: "We are not obliged to accept everything he says as true. It suffices that it is accepted as necessary."

Josef K.: “A mournful opinion; It elevates the lie to the stature of a world principle.”

* * *

Coitus is the punishment for the happiness of being together.

On Kafka

* * *

The look in Kafka's eyes was always a little puzzled, full of the wisdom of children and of melancholy slightly counterpointed by an enigmatic smile. He always seemed to be somewhat embarrassed.

John Urzidil

* * *

The fact is we all seem capable of living, because at some time or other we have taken refuge in a lie, in blindness, in enthusiasm, in optimism, in some conviction, in pessimism or something of the sort. He has never taken refuge in anything. He is absolutely incapable of lying, just as he is incapable of getting drunk.

Milena Jesenská

* * *

He marvels at everything, including typewriters and women. He will never understand.

Milena Jesenská

* * *

My Franz was a saint.

Felice Bauer

* * *

I couldn't read it for its perversity. The human mind isn't complicated enough.

Albert Einstein, after returning a Kafka novel loaned to him by Thomas Mann.

* * *

And his is — on another plane — the curse and grace of an almost saint-like abstemiousness . . . a readiness to let go, not to cling, which may be offensively modest and gentle in its inevitable gesture of refusal — a saintly, mild, almost Christlike gesture which is nonetheless ambiguous since it arouses, at the same time, the suspicion of a diabolical arrogance to the point where one would want to shout at him: "Don't pretend to be that small, you are not that great!"

Peter Heller

* * *

To do justice to the figure of Kafka in its purity and its peculiar beauty, one must never lose sight of one thing: it is the purity and beauty of a failure.

Walter Benjamin

* * *

In Kafka we have the modern mind, seemingly self-sufficient, intelligent, skeptical, ironical, splendidly trained for the great game of pretending that the world it comprehends in sterilized sobriety is the only and ultimate real one — yet a mind living in sin with the soul of Abraham. Thus he knows two things at once, and both with equal assurance: that there is no God, and that there must be God.

Erich Heller

* * *

A final paradox decrees that Kafka's books are among literature's least difficult. There is nothing subjective, arbitrary, or doubtful in them: Amerika *is not the truth according to Karl, or* The Trial *the truth according to Joseph K. Dickens is hard to understand, not Kafka. We have only to keep in mind all the events and characters of* Amerika *or* The Trial *or* The Castle *— a truth that stands far above or lies much deeper than Karl, Joseph K., K. and Kafka — will burst forth by itself, dazzlingly.*

Pietro Citati

* * *

1912 CE **Charles** (**Thomson Rees**) **Wilson** (1869–1959, England). Physicist. Invented the *Wilson Cloud Chamber*. This device made the tracks of high-speed atomic and nuclear particles *visible*, by means of trails of water droplets condensed on the ions produced along particle paths. For this achievement he won the physics Nobel prize in 1927. Wilson was born in Glencorse, Scotland.

1912–1925 CE **Henry Norris Russell** (1877–1957, U.S.A.). One of the leading astrophysicist of the era when physical understanding of stars was first emerging. Discovered, independently of **Hertzsprung**, the main sequence of stars by plotting the absolute magnitudes of stars against their spectral classes[261].

In 1912 he presented the earliest systematic analysis of the variation of the light received from eclipsing binary stars; he later pointed out the importance of the motion of the periastron of the orbit which provides information about the internal structure of the component stars. On the basis of his parallax studies, Russell developed the theory of stellar evolution that at the time was in good agreement with the data. This work stimulated **Eddington** and other astrophysicists. In 1925 Russell presented a reliable determination of the abundance of various chemical elements in the solar atmosphere. This work provided clear evidence of the predominance of hydrogen in the sun and, by inference, in most stars.

Russell was born in Oyster Bay, New York. He graduated from Princeton University in 1897 and remained there to obtain his doctorate in 1900. During 1902–1905 he stayed at Cambridge University, and since 1911 served as a professor of astronomy at Princeton. In 1921 Russell began his association with the Mt. Wilson Observatory, where he was a research associate until his retirement.

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[261] The spectral types O, B, A, F, G, K, M and N, are essentially measures of the surface temperatures of the stars, running from about 20,000° at spectral class B to about 2500° at class N.

Fundamental Stellar Properties — H–R Diagrams

A number of fundamental quantities characterize a star; these include luminosity, temperature, radius and above all, mass. It is also important to know the distance to stars — in order to determine luminosity from apparent magnitude.

The luminosity *of a star is the* total *rate of energy it emits from its surface, and is usually measured in units of ergs per second. To measure it, it is necessary to take into account the distance to a star and the radiation received from it at* all *wavelengths, not just those to which the human eye responds. While stellar magnitudes (both apparent and absolute) refer to visible light, the bolometric magnitude is based on the star's radiation emitted over a much broader wavelength band than the eye can see. On the other hand, the wavelength at which the star emits most strongly depends on its* temperature. *Thus, by measuring a star's brightness at two different wavelengths, it is possible to learn something about its temperature.*

One can thus speak about a B (blue) magnitude and a V (visual, or yellow) magnitude. Since a hot star emits more light in blue than in yellow, its B magnitude is smaller *than its V magnitude. For a cool star, the situation is reversed. The difference between the B and V magnitudes is called the* Color index. *Stellar surface temperatures range from about 2000 K for the coolest M stars to $50,000$ K or more for the hottest O stars.*

In the first decade of the 20^{th} century ***E. Hertzsprung*** *and* ***H.N. Russell*** *began to consider how luminosity and spectral class might be related to each other. Each gathered data on stars whose luminosities (or absolute magnitudes) were known, and found a close link between spectral lines (temperatures) and absolute magnitudes (luminosities). This relationship is now called the Hertzsprung-Russell or H-R diagram. In this plot of absolute magnitude (on the vertical scale) versus spectral class (on the horizontal scale), stars fall into narrowly defined regions, rather than being randomly distributed.*

A star of a given spectral class cannot have just any absolute magnitude, and vice versa. In fact, the great majority of stars fall into a diagonal strip running from the upper left (high temperature, high luminosity) to the lower right (low temperature, low luminosity) of the H–R diagram. This strip has been given the name main sequence.

One group of a few stars fails to fall on the main sequence but appears in the upper right (low temperature, high luminosity) of the diagram. The *only way one star can be much more luminous than another of the same temperature is if it has a lot more surface area. Hertzsprung and Russell realized that these extra-luminous stars sitting above the main sequence must be much larger than those on the main sequence, and they called them* giants *and* supergiants*. Another group of stars appear off the main sequence in the lower left (high temperature, low luminosity) of the diagram. Since these stars are hot but not very luminous, they must be very small, and they have been given the name* white dwarfs*.*

The H–R diagram can be used to find the distance to a star, provided we can place it on the H–R diagram by determining its spectral class (i.e. as long as we know whether it is on the main sequence or is a giant, supergiant or dwarf). Once it has been placed on the diagram, its absolute magnitude is read off the vertical axis. A comparison of its absolute magnitude with its observed apparent magnitude then yields the star's distance from earth.

In a telescope all stars appear simply as points of light, so it is usually impossible to measure the size of a star directly. The radii of a few stars have been measured with an instrument called a stellar interferometer. Others can be deduced for stars that are members of binary systems. For most stars, however, the H–R diagram provides means to estimate their size in the following way: since luminosity is related to total surface area, a star's position in the H–R diagram depends partly on its size. We know that the Stefan-Boltzmann law specifically relates the three quantities luminosity, temperature and radius; use of this law allows the radius to be determined if the other two quantities are known[262]*.*

It has already been noted that the temperature attained by a star depends almost entirely on the initial mass *of the star. Indeed, studies of* binary stars *have provided knowledge of the masses of over a hundred individual stars. When the masses and luminosities of those stars for which both of these quantities are determined are compared, it is found that, in general, the more*

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[262] The Stefan-Boltzmann law states that the emitted flux (measured in units of erg·cm^{-2}·sec^{-1}) is related to the temperature (in kelvins) by $E = \sigma T^4$ where $\sigma = 5.67 \times 10^{-5}$ erg·cm^{-2}sec^{-1}($^\circ$K)$^{-4}$ is the Stefan-Boltzmann constant. The corresponding luminosity is $L = 4\pi R^2 \sigma T^4$, where R is the star's radius. Solving for R we find $R = \frac{1}{T^2}\sqrt{\frac{L}{4\pi\sigma}}$. Scaling the quantities L, R, and T to those of the sun, we obtain $\frac{L}{L_\odot} = \left(\frac{R}{R_\odot}\right)^2 \left(\frac{T}{T_\odot}\right)^4$. These relations make it possible to transform the original H–R diagram into another in which the horizontal scale is not the temperature but the radius. In this representation (radius-luminosity relation), the main characteristics of the H–R diagram are retained, but the sequences are tilted in slightly different manner.

massive stars are also more luminous. This observation is known as the mass-luminosity relation.

The law seems to be $L \propto M^{3.7}$. *It provides a useful means of estimating the masses of stars of known luminosity that do not happen to be members of visual or eclipsing binary systems. The range of stellar masses seems to be between 0.1 and 50 solar masses. According to the mass-luminosity relation, the corresponding luminosities range from* 0.01 *to* 10^6 *solar luminosities — a much greater range indeed.*

During 1913–1962, *a large amount of information has been accumulated concerning the parameters of stars. It was then deduced that the relations are* statistical *in nature; actually there are very few, if any, parts of the radii-masses-luminosities space which are "prohibited", i.e. representative points may be found in many parts of the R-M-L octant which where previously believed to be completely empty. This result should not be confused with the well-established* statistical tendency *of the stars* in our galaxy *to occur in preferential bands within the model.*

It has been estimated that approximately 10^{11} *stars of the Milky Way system belong to the main sequence and obey the mass-luminosity relation. The total number of white dwarfs may be one hundred times less. The giants are probably* $10,000$ *times less frequent than the main sequence stars.*

1912 CE **Henrietta Swan Leavitt** (1868–1921, U.S.A.). Astronomer. Elucidated the properties of a class of pulsating stars (Cepheid variables[263]) via predictable relation between period and intrinsic brightness and used them for calculating distances to remote and even extra-galactic stars. This enabled **Hubble** in 1929 to formulate his law of expansion.

Leavitt graduated from the women's college that was affiliated with Harvard College, in 1892. In 1902, after working as a volunteer research assistant, Leavitt became a permanent staff member of the Harvard College Observatory. She conducted studies of stellar magnitudes (star brightness) and became head of the department of photographic stellar photometry.

[263] Pulsating stars that vary regularly in brightness, with periods ranging from a few days to several months. First identified in the 18^{th} century. We now know that the variations are caused by periodic expansions and contractions of the stellar surface.

In 1912, after about five years of single-minded study, she found that the periods of the Cepheids were proportional to their luminosity. Such a relationship can be calibrated for nearby Cepheids, where the distance is known. Then it can be used to gauge the distances to remote Cepheid stars. If it is necessary to know the distance to a globular cluster or nebula or galaxy, the astronomer simply has to identify a Cepheid star inside it. For this reason, Leavitt's relationship became important to the measurement of interstellar and intergalactic distances. Leavitt also discovered four novae (stars that suddenly become brighter and within a few months gradually fade away) and more than 2,400 variable stars.

1912–1913 CE **Gunnar Nordström** (1881–1923, Finland). Physicist. Presented a pre-general-relativity theory of gravitation, which was the first logically consistent relativistic field theory of gravitation ever formulated. The theory is Lorentz invariant, and satisfies the conservation laws, but the equivalence principle appears in it as a statistical law. Later (1914) Nordström proposed to use a five-dimensional space for the unification of electromagnetism with a *scalar* gravitational field[264].

1912–1915 CE Einstein's work stimulated an outpouring of short-lived abortive gravitational and unified-field theories, by physicists and mathematicians alike. Noted among them were **Max Abraham** (1875–1923), **Gustave Mie** (1868–1957) and **David Hilbert** (1862–1943).

1912 CE Opening of the *Kaiser-Wilhelm Institutes* in Berlin-Dahlem, Germany, marks the pinnacle of German science. A year earlier (1911) the Kaiser Wilhelm Society was established. The first president of the Society and the Institute was the historian **Adolf von Harnack**[265] (1851–1930), to be followed by Max Planck.

The first institute was that of *chemistry*, including **Ernst Beckman**, **Richard Willstätter**, **Otto Hahn**, and **Lise Meitner**. The second institute was that of *physical chemistry* of which **Fritz Haber** became the director. The third institute was that for *experimental therapy* under the directorship of **August von Wasserman**. The building of the fourth institute, for *biology*, was finished in 1915. Einstein was supposed to be the director of the institute for *physics*, but the plans were suspended because of the War and taken up only in the 1930s, when it was built with the financial support of the Rockefeller Foundation. When it was finished, Einstein had already left.

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264 This is one of the abortive 'STR gravitation' theories.

265 Harnack belonged to a large family, prominent in universities, industry, and government. Because of its well-known liberal and progressive attitude many members of the family were executed by the Nazis.

The Kaiser Institutes were destined to become one of the greatest and most brilliant centers of that era.

After the turmoil of the first postwar years, an extraordinary renaissance also took place in literature, art, music, theaters, operas and other areas. It is thus not surprising that Berlin, in the 1920s, impressed visitors from abroad as the scientific and intellectual capital of the world, in spite of the lost war. Indeed, in the mid 1920s the Kaiser Wilhelm Institutes became one of the world's greatest centers, particularly in chemistry, physical chemistry, and the biological sciences.

In physics there were centers of great attraction, especially in atomic physics:

at the University of Berlin, with **Planck**, **Einstein**, **Schrödinger**, **von Laue**, and **Nernst**;

at the University of Göttingen, with **Franck** and **Born** and their intimate collaboration with the famous mathematicians there;

at the University of Hamburg, with **Otto Stern**;

at the University of Munich, with **Sommerfeld**;

and at the University of Leipzig, with **Werner Heisenberg**.

1912–1922 CE **Alfred Lothar Wegener** (1880–1930, Germany). Meteorologist, geophysicist and Arctic explorer who mixed research with adventure. First to propound in exhaustive detail his then-revolutionary hypothesis of *continental drift* based on fossil and glacial evidence.

He contended that evidence of past climates, supported by evidence from widely scattered disciplines, could not be reconciled with the fixed position of the continents and of the north and south magnetic poles.

Wegener was then led to assume that the now-separated continents must at one time have formed a single large mass, which he named *Pangaea* (pan=all, gaia=earth; meaning "all land"). He maintained that some 200 million years ago, Pangaea began breaking into smaller continents that "drifted" to their present position. Ancient climatic similarities, fossil evidence, and rock structures — all seemed to bridge together these now separate landmasses.

Other geologists refined Wegener's theory, arguing that Pangaea first split into two smaller supercontinents they called *Laurasia* and *Gondwanaland*, separated by what was called the *Tethys.* Gondwanaland later split into Africa and South America, while Laurasia divided to become North America and Eurasia. The Mediterranean Sea, according to this theory, is a surviving remnant of the ancient Tethys Sea.

Wegener was born in Berlin and took his doctorate in astronomy. He was a Renaissance man, also studying meteorology, biology, paleontology and much besides. He was selected as meteorologist to a Danish expedition to northeast Greenland. A second expedition to Greenland in 1912, included the longest crossing of the ice cap ever undertaken.

Wegener first presented his continental-drift hypothesis in 1912 in a lecture before the German Geological Association in Frankfurt am Main. His fuller development of the theory appeared in his 1915 book *Die Entstehung der Kontinente und Ozeane* (The Origin of Continents and Oceans).

Wegener's ideas were initially met with ridicule and scorn and he suffered criticism from the pillars of orthodoxy in geophysics and geology. His theory was too radical for the time, and his book brought him more notoriety than praise. Although it was generally accepted that the continents do float in a denser, somewhat plastic mantle beneath them, few could accept the idea that entire continents could move around the earth at speeds that must be as great as several centimeters per year. He was also ridiculed for failing to explain the origin of a force that would permit "continents of granite to flow through oceans of rock".

Wegener lost his life on the Greenland ice sheet during a mission to establish a mid-ice observatory. The proofs he was seeking were found years later, quite by chance and on another frontier — at the bottom of the sea.

Continents Adrift[266] *(1912–1968)*

The idea of continental drift is very old[267]*; the notion that continents, particularly South America and Africa, fit together like pieces of a puzzle has*

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266 For further reading, see:

- Tarling, D. and M. Tarling, *Continental Drift*, Anchor Books: New York, 1975, 142 pp.

267 Genesis **1**, 9–10 "And God said, Let the waters under the heaven be gathered together unto one place, and let the dry *land* appear: and it was. And God called the dry *land* Earth; and the gathering together of the waters called he Seas: and God saw that it was good".

Genesis **10**, 25: "And unto Eber were born two sons: the name of one was Peleg; for in his days was the earth divided". [Also, *Chronicles* **1**, 19.]

been around as long as world maps. ***Francis Bacon*** *(Novum Organum, 1620) had already wondered about the coincidence of coastlines. In* 1668 *a French monk,* ***Francois Placet****, suggested that the continents had been broken apart by the Deluge. Another theological scholar,* ***Theodore Lilienthal*** *(1756), also found biblical evidence for the separation of a single land mass into the present continents.*

In 1858, ***Antonio Snider-Pelligrini*** *made the first geological (as opposed to geometrical) survey of similarities of the continents on either side of the Atlantic. Charles Darwin found convincing signs of* vertical *movements of land masses, but did not see evidence for large-scale horizontal movements. His son,* ***George Howard Darwin*** *(1845–1912), suggested in* 1878 *that the moon originated by being thrown off from the Pacific area of the spinning earth or being drawn from it by gravitational attraction of a passing star. This idea was extended by* ***Osmond Fisher*** *in* 1882, *when he suggested that the continents broke up at the same time of the moon's separation, and subsequently readjusted their positions to the new shape of the earth.*

In 1908 ***Frank B. Taylor*** *(U.S.A.) suggested that the tidal pull of the moon might have tugged the continents about on the surface of the planet. In 1911,* ***Howard B. Baker*** *(U.S.A.) revived Darwin's idea that the continent might have been set in motion by the planet Venus, who had swooped close enough to tear out a chunk of the Pacific floor, which became the moon.*

Wegener's intriguing idea did not die with him. After decades of skepticism, evidence has come to light to confirm the essence of his theory. Indeed, during the years that followed his proposal, great strides in technology permitted mapping of the ocean floor, and extensive data on seismic activity and the earth's magnetic field became available. By 1968 *these developments led to the unfolding of a far more encompassing theory, known as* plate tectonics. *It became one of the most important geological discoveries of the* 20^{th} *century; in essence it is the realization that our planet has an active, constantly changing crust. This crust is divided into huge* plates *that jostle each other, producing earthquakes, volcanoes and oceanic trenches. Plate tectonics is responsible for many details of the earth's surface, including mountain ranges.*

A careful examination of the currently active features of the land and the ocean floor has suggested that the earth now consists of 6 *large blocks which are in motion relative to each other. These are:*

- The Pacific block *consists of most of the Pacific Ocean and includes small fragments of the west coast of North America.*

- *The American block* consists of North and South America as well as the western half of the Atlantic Ocean. The Caribbean region forms a small subblock.
- *The Eurasian block* is almost entirely continental, consisting of all of Europe and most of Asia, including the East Indies and the Philippines.
- *The African block* consists of the continent of Africa, Madagascar, the eastern half of the South Atlantic Ocean, and the western half of the Indian Ocean.
- *The Indian block* stretches from the Arabian peninsula to New Zealand. The block contains the portion of Asia south of the great mountain range from Turkey on the west to the Himalayas, and the west coast of Burma on the east. The eastern half of the Indian Ocean, New Guinea, and Australia are included in this block.
- *The Antarctica block* consists of the continent of Antarctica and the Southern Ocean. It includes a strip of the Pacific between the East Pacific Rise and the west coast of South America.

The differential motions of these 6 blocks lead to the formation of new ocean floor where the blocks are moving apart, and to a loss of surface where the blocks are forced against each other. The areas where new surface is created correspond to the *mid-ocean ridges*. Where all blocks are driven into each other, the surface expression is more complex: Some of these areas correspond to ocean trenches. Along other segments where the blocks are pressing against each other, we find active mountain belts, such as the Himalayas. While these appear to be compressional features, trenches seem to be the result of tension.

It appears that the rigid surface of the earth, the *lithosphere*, which is tens of kilometers thick, rests on a weaker layer, the *asthenosphere*. The lithosphere is divided in large blocks which move relative to each other. The boundaries between the blocks are zones of weakness, where earthquake activity reveals relative motion. The fact that the present blocks have existed for *tens of million of years* indicates that the breaks in the lithosphere tend to be preserved. However, over long periods, these breaks heal and new breaks develop; thus the pattern of blocks may change.

The existence of a more plastic asthenosphere under the relatively rigid lithosphere (though elastic as far as seismic waves are concerned) facilitates the motion. Nevertheless, frictional forces at the bottom of the blocks and at the points where the blocks are converging would tend to stop the drift in the absence of a driving force.

The main force within the earth is thermal energy, which results from the radioactive decay of uranium, thorium, and an isotope of potassium. This *outward flux of heat amounts to about* $1.5 \times 10^{-6}\ \frac{\text{cal}}{\text{sec}\cdot\text{cm}^2}$. *This is much less heat than is received from the sun, so that the heat flux from the interior does not directly affects the climate on earth. The total* heat flux, *however, is* $60\ \frac{\text{ergs}}{\text{sec}\cdot\text{cm}^2}$, *which is* 10^9 *times the kinetic energy of the lithosphere in motion.*

The heat from the interior can be transferred by conduction, radiation, and convection. Of these, only convection is associated with motion, and so could be the driving force *for continental drift.*

Convection currents result if a fluid is heated from below. The density difference between the heated and cool fluids results in vertical motion, since the heated fluid rises and the cooled fluid sinks. The vertical motions become spatially organized and result in a pattern of vertical convection cells. *The horizontal motion at the top of the asthenosphere exert forces on the blocks of the lithosphere. The direction of the net force will depend on the disposition of the blocks relative to the convection cells*[268].

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[268] Although the theory of *plate tectonics* is well established, the engine that drives the motion of the lithospheric plates continues to defy easy analysis because it is so utterly hidden from view.

Among the regions offering the best access to the earth's insides are the mid-ocean ridges. These ridges dissect all major oceans. They actually make up a system that winds around the globe, stretching a total of more than 60,000 km. The *Mid-Atlantic Ridge* (MAR) is a part of that global ridge system; a huge north-south scar in the ocean floor, it forms as the eastern and western parts of the Atlantic move apart at a speed of roughly $1\ \frac{\text{cm}}{\text{year}}$. In addition to the frequent earthquakes that take place there, the summit of the MAR spews out hot magma during frequent volcanic eruptions. The magma cools and solidifies, thus forming new oceanic crust. The magma that rises in the MAR originates in the upper mantle and forms a common kind of rock known as *basalt*. It is known, however, that the upper mantle is made of *peridotite* which consists mostly of three silicon-based minerals: *olivine* (a dense silicate containing magnesium and iron); *orthopyroxene* (a similar and less dense mineral); *clinopyroxene* (incorporates some aluminum and calcium) and *spinel* (an oxide of chromium, aluminum, magnesium and iron).

Hot peridotite rises under the mid-ocean ridges from depths exceeding 100 km below the seafloor. As it moves upward, it decompresses and partially melts. The melted part takes on the composition of basaltic magma and separates from the peridotite that did not melt.

1912–1925 CE **Vesto Melvin Slipher** (1875–1965, U.S.A.). Astronomer. Made systematic observations of radial velocities of spiral galaxies[269] (then known as Nebulae). These observations provided the first evidence supporting the expanding-universe theory.

Slipher was born at Mulberry, Indiana. In 1901 he joined the staff of the Lowell Observatory at Flagstaff, Arizona, and in 1916 became its director. There he organized and guided the search that resulted in the discovery of Pluto, the ninth known planet of the solar system. Slipher's extensive investigations led to the determination of the rotational periods of several planets. His discovery of dark absorption bands in the spectra of Jupiter, Saturn, and Neptune led to the identification of some of the chemical constituents of their atmospheres. He demonstrated that many diffuse nebulae (clouds of dust and gas) shine by the reflected light of nearby stars.

1912–1930 CE **Eduard Helly** (1884–1943, Austria and USA). Mathematician. Worked on functional analysis and proved the *Hahn-Banach theorem* (1912), 15 years before Hahn published essentially the same proof, and 20 years before **Banach** gave his new setting. He is remembered for the *Helly Theorem*[270] (published by him in 1923).

Helly came from a Jewish family in Vienna. He studied at the University of Vienna and was awarded his doctorate in 1907 after writing his thesis under the direction of **Mertens**. He worked in the field of mathematics until he enlisted in the army at the beginning of World War I. He was wounded in September 1915 and became a Russian prisoner of war in Siberia. After returning to Vienna in 1920, Helly was unable to secure permanent employment as a mathematician and worked as a bank clerk and insurance actuary until 1938 when Austria was taken into the Third Reich and Helly and his family escaped to the U.S. He was able to work at various mathematically-related positions until his death of a heart attack in 1943.

Had Helly succeeded in staying in the mainstream of mathematics, as an academician who published and participated in seminars, he would have undoubtedly have capitalized on his earlier contributions. He not only might

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[269] He found that the Andromeda galaxy approaches us at a speed of 200 km/sec ('*violet shift*').

[270] It states that if there are given n convex subsets of a d-dimensional euclidean space with $n \geq d+1$ and if each collection of $d+1$ of the subsets has a point in common, then there is a common point of the n subsets. His 1912 paper also includes the *Helly Selection principle* which says that given a sequence of functions of bounded variation which are of uniform bounded variation and uniformly bounded at a point, then there exists a subsequence which converges to a function of bounded variation.

have seen to it that proper credit should be ascribed, but it is likely that he would have extended his results further. In most careers there are some disappointments and failures, but Helly's career derailed early, and life never gave him a chance to get back on the right track.

1912–1933 CE **Edwin Howard Armstrong** (1890–1954, U.S.A.). Inventor and electrical engineer who made important contributions to radio communications. He developed the superheterodyne circuit which became widely used in radio receivers (1918). In 1933 he invented the *frequency modulation* (FM) system for short-wave broadcasting. Armstrong was born in New York city.

His invention of the regenerative (feedback) circuit[271] in 1912, while he was still in college, was challenged by **Lee de Forest** in a series of lengthy patent suits. Although Armstrong lost the case, the scientific community continued to support his claims. His invention of the heterodyne principle (1918) was also challenged in a patent suit. In poor health, with most of his money gone, he committed suicide[272].

Armstrong has posthumously received increasing recognition for his many important inventions. He was the father of FM radio, the grandfather of radar and a great grandfather of space communication.

1912–1935 CE **Otto Heinrich Warburg** (1883–1970, Germany). Distinguished biochemist and cell-physiologist. His researches had a seminal effects

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[271] When the feedback was increased beyond a critical level, the triode tube turned into an *oscillator* (instead of just being a receiver) which not only amplified radio signals but generated them as well, and this advance made all the difference. As a radiowave generator (transmitter), this circuit is still at the heart of all radio-television broadcasting. By the end of WWII, Armstrong developed his continuous wave FM radar to a point where he was able to bounce a radio signal 400,000 km to the moon and back again. He had proven that FM waves, unlike AM waves, could penetrate the ionosphere. That paved the way to radio communication in space.

[272] By 1934, Armstrong had helped create an industry which was worth almost 2 billion dollars. Thus, RCA, Zenith, Philco, Magnavox and everybody else were all turning fantastic profits, using Armstrong inventions without paying him royalties. In 1948 he took RCA to court but legal fees exhausted his assets. After his suicidal death, RCA offered his widow a million dollars (much less than one percent of their profits from Armstrong inventions), the same amount David Sarnoff (RCA president) offered Armstrong in 1940. In effect Sarnoff had finally got an answer to the question: "Do you want me to pay you, or your widow?"

on the development of biochemistry throughout the 20^{th} century. Elucidated the main biochemical processes of *respiration*, *fermentation* and *photosynthesis*, together with the enzymatic mechanisms that activate them. Discovered the respiratory enzymes (1912–1923) and was first to note the action of *coenzyme* (1935).

Warburg's major achievements:

- Devised techniques, particularly of manometry[273] and spectrophotometry, which have been the mainstays of many biochemical laboratories over a span of 60 years and used them to investigate oxidative processes occurring in living cells. In particular, developed a method of studying respiration in thin slices of tissue.
- Clarified the biocatalytic function of *iron* in the enzymes of biological oxidation. Discovered the 'respiration ferment'[274] in which he saw the activator of the molecular oxygen for the absorption of hydrogen from the substrate (dehydrogenization).
- Isolated and characterized many of the enzymes and their coenzymes, thus laying much of the foundation on which rests the present understanding of the *cell's energy metabolism*. The methods he devised for the isolation of enzymes have found widespread industrial applications and the techniques he developed for their assay are still widely used, especially in clinical diagnosis.
- Developed a theory for the biochemical basis of *cancer* formation. This theory was not fully verified[275] but fertilized the biochemical research of tumors.

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[273] Metabolic processes are accompanied by production or absorption of gases. The manometric method accurately measures changes in the pressure of these gases. His manometric apparatus became a standard tool for measuring metabolism in living cells.

[274] Oxidation-reduction reactions are concerned with the production of heat and energy in the body. The enzymes involved include the *oxidases*, which activate molecular oxygen; the *dehydrogenases*, which catalyze the removal of hydrogen of a substrate to an easily reducible substance; and the *peroxidases*, which catalyze the decomposition of organic peroxides and hydrogen peroxide. Warburg's 'respiration ferment' was identified with the *cytochrome oxidase*, at the end of the oxidation chain.

[275] Warburg's intolerance of criticism and obsession with some of his own theories estranged him from the mainstream of thought in these areas. Toward the end of his life he was clinging to views almost universally rejected by other scientists.

Warburg was a descendant of an illustrious Jewish family of scientists, bankers and philanthropists, whose ancestors came to the town of Warburg in Westphalia from Bologna, Italy in the 17^{th} century. His father **Emil** (1846–1931), a known physicist in Strasbourg and Berlin, converted to Christianity. His uncle **Otto** (1859–1938), professor of botany, became the president of the World Zionist Organization (1912–1920).

Warburg was born in Freiburg-im-Breisgau. Studied chemistry under **Emil Fischer** (Ph.D.: Berlin, 1906) and medicine at Heidelberg (M.D.: 1911); professor in Berlin (1915–1952), Göttingen (1953 ff.). Awarded two Nobel prizes in physiology or medicine (1931, 1944[276]).

Warburg continued to work during the whole duration of the Nazi regime and was never disturbed or harmed by the Nazis. According to Nazi ideology, he would be considered as a *half-Jew* (his father was baptized and his mother came from an old distinguished non-Jewish German family). However, influential friends succeeded in convincing Hitler that Warburg was the only scientist who offered serious hope of producing a cure for cancer some day. Hitler apparently suffered from a strong phobia of cancer, and this factor induced Göring to arrange a recalculation of Warburg's ancestry — with the result that he was considered to be a *quarter-Jew*.

In 1943, air attack made his life in Berlin dangerous and Warburg moved his laboratory to an estate 50 km north of Berlin. Here he worked undisturbed until 1945, when the Russians occupied the area and removed all the equipment from the laboratory. The Russian commander-in-chief, Marshall Zhukov, told Warburg in the name of the Russian government that the dismantling of the laboratory was an error. Although the Marshall ordered the return of the equipment and the books, they could not be traced.

Enzymes

None of the chemical reactions taking place in the human body such as digestion, respiration and metabolism, would be possible without enzymes, which are the most important tools of the living cell. It has been estimated

[276] By Hitler's decree he was unable to receive the second.

that there are as many as 1000 separate enzymes in a single cell, each responsible for a specific chemical reaction. The absence or inaction of a single enzyme can disrupt a key process and result in dysfunction or death of the organism.

Enzymes are organic catalysts formed by living cells, but their actions are independent of the presence of living cells. All enzymes have been found to be proteins in nature. While some enzymes consist entirely of proteins, other require non-protein components in order to be active. The component firmly attached to the enzyme that makes it active is said to occupy the active site. If the activating component is easily separated, it is known as a coenzyme. Several vitamins of the B complex group have been found to be constituents of certain coenzymes. The most important coenzymes are:

Nicotinamide adenine dinucleotide (NAD),

Nicotinamide adenine dinucleotide phosphate (NADP),

Flavin mononucleotide (FMN),

Flavin adenine dinucleotide (FAD),

Coenzyme A (CoA).

The first four coenzymes are involved in oxidation-reduction reactions. NAD and NADP accept hydrogen atoms from a substrate and transfer them to FMN and FAD, and FMN and FAD transfer the hydrogen atoms to the cytochromes. The cytochromes in turn transfer the hydrogen atoms to an enzyme called cytochrome oxidase, which activates oxygen so that it may combine with the hydrogen to form water.

Coenzyme A is involved in the metabolism of carbohydrates (citric acid cycle) and the biological synthesis and degradations of fatty acids.

Some enzymes are first produced in an inactive form. Such a precursor of an active enzyme is called a proenzyme (or zymogen). The proenzyme must be activated by some other substance. For example: the inactive proenzyme pepsinogen is converted into active pepsin by the HCl in the gastric juice.

The generally accepted theory for enzyme action is that the enzyme (E) first combines with the substrate (S) to form an enzyme-substrate complex (ES). The enzyme substrate complex undergoes a chemical change and then dissociates, yielding the reaction products and liberating the original enzyme: $E + S \to ES \to E + \text{products}$.

Enzyme specificity has been explained by the lock and key theory, which postulates that each enzyme has an active catalytic center of precise chemical structure or surface shape to which the substrate fits perfectly.

Enzymes catalyze a reaction by lowering the activation energy required to initiate it. Without enzymes, chemical reactions in the body would proceed too slowly to maintain life.

Unlike inorganic catalysts, such as platinum, which catalyze many reactions, enzymes are highly specific in their action. Thus lipase will catalyze the hydrolysis of lipids, but not of carbohydrates or proteins. Sucrase will hydrolyze sucrose, but not lactose or maltose.

Enzymes are also more efficient than inorganic catalysts. Sucrase, for example, is 10^6 times more powerful than the hydrogen ion in the hydrolysis of sucrose. Another difference between enzymes and inorganic catalysts is that enzymes are easily destroyed by heat, while inorganic catalysts are not affected by high temperatures. Among the factors influencing the rate of enzyme action are: the concentrations of the substrate and of the enzyme, the temperature, the pH, and the accumulation of end products which slow down the reaction according to the law of mass action.

Some enzymes require the presence of a metallic ion for their activity (e.g. Fe^{+2}, Fe^{+3}, Co^{+2}, Zn^{+2}, Mn^{+2}, Mg^{+2}, Mo^{+2}).

Certain chemicals have a toxic or inhibitory effect upon enzyme activity. Among these are formaldehyde, chloroform, carbon tetrachloride, arsenic compounds, cyanides, and heavy metals like mercury and silver.

Other inhibitors include antibiotics (streptomycin, auremycin), antienzymes (e.g. antitrypsin from soybeans), and antimetabolites (such as sulfanilamide).

Example of a specific enzyme: oxygen enters the red blood cells where it combined with hemoglobin to form oxyhemoglobin. This oxyhemoglobin reacts with bicarbonates to form carbonic acid (H_2CO_3) and a basic form of oxyhemoglobin. The carbonic acid, in the presence of the enzyme carbonic anhydrase, decomposes into carbon dioxide and water:

$$H_2CO_3 \xrightarrow[\text{unhydrase}]{\text{carbonic}} CO_2 + H_2O$$

The basic oxyhemoglobin is carried to the tissues by the blood. Because of the low partial pressure of oxygen in the tissues (40 mm Hg as against 100 mm Hg in the lungs), the basic oxyhemoglobin decomposes to oxygen and basic hemoglobin. The oxygen, thus released, diffuses into the tissue cells where it enters into metabolic reactions. The reduced hemoglobin returns to the lungs in the venous blood.

The CO_2 formed by metabolic reactions in the tissues diffuses into the plasma and enters the red cells where the enzyme carbonic anhydrase, now working in reverse, catalyzes its combination with water to form H_2CO_3. The *carbonic acid reacts with the basic hemoglobin to form bicarbonates and acid-reacting hemoglobin. As the bicarbonate ion concentration increases, these ions diffuse out of the red cells and into the plasma. To balance the loss of the negative ions from the red cells, an equal number of chloride ions enter the red cells from the plasma. CO_2, mainly as bicarbonates, is returned to the lungs.*

1912–1942 CE **Milutin Milankovitch**[277] (1879–1958, Yugoslavia). Applied mathematician, engineer and astronomer. Proposed (1920) an astronomical theory of Ice Ages. He concluded that variations in *summertime* radiation at high latitudes in both hemispheres result primarily from variations in *axial tilt* (41,000 year cycle), but include also the effect of equinoxial precession (22,000 year cycle). Taking into account changes in the reflective power of the earth, he calculated how the geographic positions of ice-sheet margins varied over the past million years. The essential feature of the Milankovitch theory is a curve that shows how the intensity of *summer sunlight*[278] varied over the past 600,000 years at any given latitude. He identified certain low points on the 65° curve with 4 European Ice Ages.

[277] For further reading, see:

- Milankovitch, M., *Canon of Insolation and the Ice-Age Problem*, Translated from German by the Israel Program for Scientific Translations, Jerusalem, 1969, 484 pp.
- Imbrie, J. and K. Palmer Imbrie, *Ice Ages: Solving the Mystery*, Enslow Publications: New Jersey, 1979, 224 pp.

[278] He reasoned that changes in winter radiation could hardly have much effect on the annual snow budget, because temperatures in Arctic regions are cold enough for snow to accumulate even in modern times. During the summer, however, modern glaciers melt. Therefore any decrease in the intensity of summer sunlight would inhibit melting, render the annual snow budget positive, and lead to glacial expansion.

The effect of tilt on the distribution of sunlight was as follows: when the tilt is decreased from its present value of $23\frac{1}{2}^\circ$, the polar regions receive less sunlight than they do today. When the tilt is increased, polar regions receive more sunlight. The possible limits of these effects (never actually achieved) would be a tilt of 0°, when the poles would receive no sunlight and a tilt of 54°, when all points on earth would receive the same amount of sunlight annually.

With the publications of these radiation curves, geologists understood for the first time how two of the astronomical cycles influenced the pattern of incoming solar radiation: It was now clear that *the strength of these effects varied systematically with latitude*.

The influence of the tilt cycle (41,000 years) is large at the poles and decreases towards the equator. In contrast, the influence of the precession cycle (22,000 years) is small at the poles and becomes large near the equator. This meant that the curves calculated for high latitudes are dominated by the 41,000-year cycle, while those for low latitudes are dominated by the 22,000-year precession cycle[279].

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[279] The radiant energy available at the top of the atmosphere at latitude φ, is a single-valued function of the *solar constant* S_0, the *semi-major axis* a of the earth's elliptic orbit in the ecliptic, its *eccentricity* $e = \sqrt{a^2 - b^2}$, the earth's axial tilt (*obliquity*) relative to the normal to the ecliptic, ϵ, and the *longitude of the perihelion* $\varpi = \Pi + \Psi$, measured from the *moving* vernal equinox. [Ψ is the precession in longitude which describes the absolute motion of the vernal equinox along the earth's orbit relative to the fixed stars. Π is the longitude of the perihelion, measured from the reference vernal equinox of 1950 CE, and describes the absolute motion of the perihelion relative to the fixed stars.]
The time variation of insolation thus requires the long-term variations of the orbital elements of the earth. These are expanded in trigonometric series

$$e = e_0 + \sum E_i \cos(\lambda_i t + \phi_i),$$
$$e \sin\varpi = \sum P_i \sin(\alpha_i t + \beta_i),$$
$$\epsilon = \overline{\epsilon} + \sum A_i \cos(\gamma_i t + \delta_i),$$

with $e_0 = 0.028\,706$, $\overline{\epsilon} = 23.320\,556\,^\circ$ and time $t = 0$ refers to 1950 CE. In these series, the values of the $\{E_i, \lambda_i, \phi_i, P_i, \alpha_i, \beta_i, A_i, \gamma_i, \delta_i\}$ have been calculated for many terms. The series provide us directly with a power spectral analysis of the time variation of the orbital elements. It turns out that the four largest terms of the *precessional component* $e \sin\bar{\omega}$ have period of 23,716; 22,428; 18,976; 19,155 years. The first two and the last two clearly correspond, respectively, to the 24,000- and 19,500-years periods found in 1976 in *deep sea cores*.
In the obliquity term, not only does the dominant component has a period of 41,000 years, but also 5 of the next 10 largest components have periods very close to 41,000 years.
For the eccentricity the situation is more complicated: the *mean period* over the past 5 million years is 95,800 years, while the leading first term has a periodicity of 412,085 years — too long to stand out clearly in the analysis of deep-sea cores.

Milankovitch had acquired his Ph.D. in 1904 at the Vienna Institute of Technology. After graduating he worked for five years as a practical engineer. In 1909 he joined the University of Belgrade as a professor of applied mathematics. He started to work on Ice Age problems in 1911 and it took him 25 years to complete his calculations. Basing his work on earlier results of **Pilgrim**[280], he still had to calculate how much solar radiation strikes the surface of the planet during each season and at each latitude, taking into account its spinning, revolving, wobbling and tilting motions. Later he wrote: "*I set out on this hunt in my best years. Had I been somewhat younger I would not have the necessary knowledge and experience. Had I been older I would not have had enough of that self-confidence that only youth can offer in the form of rashness*".

The most valuable feature of the Milankovitch theory was that it made testable predictions about the geological record of climate. It predicted how many Ice-Age deposits geologists would find, and it pinpointed when these deposits had been formed during the past 650,000 years.

During the 1930's and 1940's, most European geologists were won over to the Milankovitch theory since both the radiation diagrams and the climatic diagrams matched at four Pleistocene Ice Ages. But the early 1950's saw a dramatic about-face. By 1955 the astronomical theory was rejected by most geologists: the downfall of the theory was the development of the radiocarbon dating method[281] developed by **Willard F. Libby** during 1946–1949.

Geologists, by using the radiocarbon dates beyond the reliable range of the method (40,000 years), and basing their case solely on evidence collected from the surface of the land, have found that major glacial advances occurred 60,000, 40,000, and 18,000 years ago. Only the youngest of these advances had been predicted by Milankovitch. There were more glacial advances during an interval of time *they believed to be* the last 80,000 years than could be explained by the Milankovitch theory. By 1965, the astronomical theory of the Ice Ages had lost most of its supporters.

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280 The German **Ludwig Pilgrim** calculated in 1904 how the eccentricity, tilt and precession of the earth's orbit varied over the past million years.

281 A radioactive isotope of carbon, ^{14}C, is produced in small quantities in the upper atmosphere by cosmic rays. Eventually, the radiocarbon atoms in the atmosphere are absorbed into the bodies of all living plants and animals. But organisms continue to acquire ^{14}C only as long as they live. *After death*, the radiocarbon atoms in the organic tissues disintegrate, changing into stable-nucleus atoms of nitrogen at a measurable rate. Libby reasoned that it should be possible to use this rate to calculate the *time of death* for any fossil: all that was necessary was to measure what proportion of carbon atoms in the fossil were still radioactive.

However, during 1965–1976 a global record of Pleistocene climate was extracted from *deep-sea cores*. This investigation established that major changes in climate have followed variations in earth's tilt and precession over the past 500,000 years — as predicted by the astronomical theory of the Ice Ages[282]. In 1950, **Dirk Brouwer** and **A.J.J. van Woerkom** recalculated Milankovitch solar radiation curves on the basis of more recent solutions for the planetary masses and orbital coefficients, and showed, among other things, how sensitive such computations can be to small errors when extended over periods of geologic time.

1912–1943 CE **Max Wertheimer** (1880–1943, Germany and USA). Psychologist and philosopher. Founded Gestalt[283] Psychology when he published *Experimental Studies of the Perception of Movement* (1912). Gestalt psychology was concerned with the organization of mental processes. It believes that humans (and animals) tend to perceive organized patterns, not individual parts that are merely added together. Accordingly, relationships between different parts of a stimulus, which we perceive as a pattern, gives it perceived meaning. This applies to seeing, hearing and feeling. In contradistinction, the *structuralists*' view is that experience can be broken down into its component parts.

Gestalt theory applies to all aspects of human *learning*, although it applies most directly to perception and *problem-solving*. The classical example is the illusion that there is apparent movement when a series of separate still images are seen rapidly (the basis for 'movies' at a rate of 28 frames per second).

Wertheimer was born in Prague to Jewish parents. After studying philosophy and psychology at the universities of Prague, Berlin and Würzburg (1901–1904), he received his doctorate in 1905. He then became a professor at Berlin (1918) and Frankfurt (1929). The rise of the Nazi regime in Germany forced him to emigrate to the United States (1933), where he settled in the New School for Social Research in New York City (1933–1943).

Gestalt psychology came under attack from bio-psychologists and neuropsychologists. Authorities such as **John Searle** and **Patricia Smith Churchland** claim that there is no need to suppose that there are any rules on top of the *neurophysiological structures*. They claim that mental states may be functional states, but this does not imply that the specification of their

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[282] Spectral analysis of the isotopic record of two Indian Ocean cores showed a dominant climatic peak at a 100,000-year cycle and three other peaks at 43,000 years, 24,000 years, and 19,000 years long — confirming predictions of the Milankovitch theory. [J.D. Hays, J. Imbrie, and N.J. Shackelton, Variations in the earth's orbit: pacemaker of the ice ages, *Science* **194**, 1121–1132, 1976.]

[283] *Gestalt* is a German word which loosely means "form", "shape", "pattern".

functional profile based on *folk psychology* is correct, either in general or in detail. Nor does it imply that psychology cannot be reduced to *neuroscience*.

1912–1962 CE **Ludwig von Mises** (1881–1973, Austria and USA). Economist and social philosopher. One of the most influential economists of the 20^{th} century. Best known for his part in a debate that raged during the early part of the 20^{th} century about the possibility of successful economic coordination under *socialism*: Misses claimed (1920) that socialism must fail economically, i.e. a socialist government (state ownership of the means of production) could never make the best assignment of capital goods, due to the absence of a market price system to calculate profits and losses. He was vindicated by history.

In the course of a long and highly productive life, he developed an integrated, deductive science of economics based on the fundamental axiom that individual human beings act purposely to achieve desired goals. Mises concluded that the only viable economic policy for the human race was a policy of unrestricted *laissez-faire*, of free markets and the unhampered exercise of the right of private property, with government strictly limited to the defense of person and property within its territorial domain.

Mises was able to demonstrate that:

- Free markets, the division of labor, and private capital investment is the only possible path to the prosperity and flourishing of the human race.
- Socialism would be disastrous for modern economy because the absence of private ownership of land and capital goods prevents any sort of rational pricing or estimate of costs.
- Government intervention, in addition to hampering and crippling the market, would prove counter-productive and cumulative, leading inevitably to socialism unless the entire tissue of interventions was repealed (*Libertarian*[284] economic view).

[284] *Libertarians* favor separation of government and economy and oppose all collusion between government and corporations that would override the free market and would seek to forcibly redistribute resources in an egalitarian manner. They believe that welfare programs serve as a perverse incentive to keep individuals from working to earn a living and that they tend to perpetuate unemployment and poverty. Government interventions such as taxation and regulation are at best necessary evils.

Holding these views, and hewing to truth indomitably in the face of a century increasingly devoted to statism and collectivism, Mises became famous for his intransigence in insisting on a non-inflationary gold standard and on laissez-faire.

The major works of Ludwig von Mises are:
The Theory of Money and Credit (1912); *Nation, State and Economy* (1919); *Socialism* (1922); *Liberalism* (1927); *Critique of Interventionism* (1929); *Epistemological Problems of Economics* (1933); *Bureaucracy* (1944); *Omnipotent Government* (1944); *Human Action* (1949); *Planning for Freedom* (1952); *Theory and History* (1957); *The Ultimate Foundation of Economic Science* (1962).

Mises was born to Arthur Mises and Adele neé Landau, devout practitioners of their Jewish faith. The year he was born (1881), his grandfather was ennobled with the title *Edler* (The Noble), a distinction for Jews in the Austro-Hungarian Empire. His birthplace, *Lemberg*, became "Lwow" (a part of Poland) after WWI, and after WWII, "Lvov" (a part of Ukraine in the USSR); then in December 1991, "Lviv" in the newly independent republic of the Ukraine.

Ludwig's father[285], educated at Zürich Polytechnic, was a construction engineer, employed in the Austrian Railroad Ministry. Ludwig was the oldest of three boys, one died as a child; **Richard von Mises** became well known as an applied mathematician.

He moved to Vienna with his family and attended there a gymnasium (1892–1900) and the University of Vienna[286], where he was awarded his Dr.Jur. (1906). He was a front-line soldier in WWI (1914–1918), an unsalaried lecturer at the University of Vienna (1913, 1918–1934), a member of the Austrian Chamber of Commerce (1918–1938) and a professor of Economics in the Graduate Institute of International Studies (Geneva, Switzerland, 1934–1940).

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[285] The recorded history of the Mises family as real estate owners and businessmen in Lemberg goes back to the 17^{th} century. Before WWII, Jews made up 50 percent of the business community in Central Europe and 90 percent of the business community in Eastern Europe. In Eastern Europe, modern civilization was predominantly an achievement of Jews.

[286] In the 1890s, during Mises' time at the school, 44 percent of the student body was Jewish. In the first decade of the 20^{th} century, almost 21 percent of the student body of the Vienna University was Jewish. Professors of Jewish descent constituted 37 percent of the law faculty, 51 percent of the medical faculty, and 21 percent of the philosophy faculty.

He emigrated to the United States (1940) and was a visiting professor at the New York University Graduate School of Business Administration (1945–1969). He married Margit Sereny (1938) in Geneva.

Mises was effectively barred from any payed university post both in Austria and the U.S., i.e. he never became a full professor. In Austria it was pure antisemitism. In the U.S., he seemed an eccentric 'Germanic' thinker with far too systematic, rigid and uncompromising way of reasoning which would not "assimilate" to his surroundings.[287] During those years his salary was paid by a private foundation. His work was continued by others, notably by **F.A. von Hayek**[288].

Worldview XXXIV: Ludwig von Mises

* * *

The essential characteristic of Western civilization that distinguishes it from the arrested and petrified civilizations of the East was and is its concern for freedom from the state. The history of the West, from the age of the Greek polis down to the present-day resistance to socialism, is essentially the history of the fight for liberty against the encroachments of the officeholders.

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[287] Mises' ideas on economic reasoning and on economic theory were out of fashion during the *Keynesian revolution* that took over American economic thinking during ca 1935–1965. A resurgent Austrian school in the United States owes itself in no small part to Mises' persistence.
Many brilliant minds were not connected with the university, e.g.: **Schopenhauer**, **Freud**, **Spengler**. Mises quickly discovered that in the U.S., academic America was rearing to the left.

[288] **Friedrich August von Hayek** (1899–1992), Austrian-born British economist found solutions to problems proposed by Keynesian economics. Held that inflation, unemployment and recession result from government interference. Awarded the Nobel Prize in Economics for *Misesian Cycle theory* in 1974 (one year after Mises' death!).

* * *

Human civilization is not something achieved against nature; it is rather the outcome of the working of the innate qualities of man.

* * *

Science does not give us absolute and final certainty. It only gives us assurance within the limits of our mental abilities and the prevailing state of scientific thought.

* * *

Each epoch has found in the Gospels what it sought to find there, and has overlooked what it wished to overlook.

* * *

The criterion of truth is that it works even if nobody is prepared to acknowledge it.

* * *

The methods of the natural sciences cannot be applied to human behavior because this behavior... lacks the peculiarity that characterizes events in the field of the natural sciences, viz., regularity.

* * *

Against nature and within nature there is no freedom.

* * *

Education rears disciples, imitators, and routinists, not pioneers of new ideas and creative geniuses. The schools are not nurseries of progress and improvement, but conservatories of tradition and unvarying modes of thought.

* * *

... economic history is a long record of government policies that failed because they were designed with a bold disregard for the laws of economics.

* * *

Economics is not about things and tangible material objects; it is about men, their meanings and actions.

* * *

Economics is a theoretical science and as such abstains from any judgment of value. It is not its task to tell people what ends they should aim at. It is a science of the means to be applied for attainment of ends chosen, not, to be sure, a science of the choosing of ends. Ultimate decisions, the valuations and the choosing of ends, are beyond the scope of any science. Science never tells a man how he should act; it merely shows how a man must act if he wants to attain definite ends.

* * *

The rich adopt novelties and become accustomed to their use. This sets a fashion which others imitate. Once the richer classes have adopted a certain way of living, producers have an incentive to improve the methods of manufacture so that soon it is possible for the poorer classes to follow suit. Thus luxury furthers progress. Innovation "is the whim of an elite before it becomes a need of the public. The luxury today is the necessity of tomorrow." Luxury is the roadmaker of progress: it develops latent needs and makes people discontented. In so far as they think consistently, moralists who condemn luxury must recommend the comparatively desireless existence of the wild life roaming in the woods as the ultimate ideal of civilized life.

* * *

Profits are the driving force of the market economy. The greater the profits, the better the needs of the consumers are supplied.

* * *

. . . it is solely bigness in business which makes it possible to supply the masses with all those products the present-day American common man does not want to do without. Luxury goods for the few can be produced in small shops. Luxury goods for the many require big business.

* * *

If history could teach us anything, it would be that private property is inextricably linked with civilization.

* * *

There is in the universe something for the description and analysis of which the natural sciences cannot contribute anything. There are events beyond the range of those events that the procedures of the natural sciences are fit to observe and describe. There is human action.

* * *

People do not cooperate under the division of labor because they love or should love one another. They cooperate because this best serves their own interests. Neither love nor charity nor any other sympathetic sentiments but rightly understood selfishness is what originally impelled man to adjust himself to the requirements of society, to respect the rights and freedoms of his fellow men and to substitute peaceful collaboration for enmity and conflict.

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Facts per se can neither prove nor refute anything. Everything is decided by the interpretation and explanation of the facts, by the ideas and the theories.

* * *

Reason's biological function is to preserve and promote life and to postpone its extinction as long as possible. Thinking and acting are not contrary to *nature; they are, rather, the foremost features of man's nature. The most appropriate description of man as differentiated from nonhuman beings is: a being* purposively *struggling against the forces adverse to his life.*

* * *

Society cannot contribute anything to the breeding and growing of ingenious men. A creative genius cannot be trained. There are no schools for creativeness. A genius is precisely a man who defies all schools and rules, who deviates from the traditional roads of routine and opens up new paths through land inaccessible before. A genius is always a teacher, never a pupil; he is always self-made.

* * *

Society is joint action and cooperation in which each participant sees the other partner's success as a means for the attainment of his own.

Science Progress Report No. 13
The Piltdown Forgery, or — the 'War of the Skulls', or — the greatest scientific fraud of all time (1912–1953)

Piltdown was an archaeological site in England where in 1908 and 1912 fossil remains of human, ape and other mammals were found. In 1913 at a nearby site was found an ape's jaw with a canine tooth worn down like a human's. To make a long story short, papers were published and the general community of paleoanthropologists came to accept the idea that the fossil remains belonged to a single creature who had a human cranium and an ape's jaw. The lower jaw was too big for a human skull but, significantly, the upper jaw was entirely missing, and with it part of the lower jaw — and the important lower canine teeth. Also missing were the mating parts for the jaw hinge. That which was missing was exactly that which would have shown

(1) whether or not the lower jaw, which was ape-like, was from a human or a ape and

(2) whether the lower jaw fitted with the upper skull bones, which were obviously human.

The skull itself had only several pieces. This meant that the size of the braincase could not be determined. The pieces might fit a larger braincase or a small one; there was no way of knowing.

With this miserly collection of a few bone fragments, the scientists "reconstructed" the entire head of what they proudly proclaimed to be "Piltdown Man." Here at last, they triumphantly declared, was the "long-awaited missing link." Since Latin names are always supposed to prove something, they named it Eoanthropus Dawsoni, *which stands for "Dawson's Dawn Man." That name made everything sound scientific.*

On December 16, 1912, the discovery was officially announced at the Geological Society. The press went wild. Here was a sensation that would sell newspapers. Many people accepted it; many others did not.

By August 1913, when the British Association for the Advancement of Science discussed the Piltdown bones, another molar tooth and two nasal bones "had been found" in that same gravel pit. It was marvelous how many pieces of bone kept appearing in that gravel pit!

Here we have bones well-preserved after 300,000 years in that damp gravel, whereas all the other millions upon millions of bones of animals and men who had lived and died in that area during that supposed time span were not to be found. Just that one set of skull pieces, jawbone, and teeth, and that was it. So close to the surface.

In their final reconstruction of the bones, the men put their solitary canine tooth on the right side of the lower jaw at an angle suggestive of an ape. That helped the cause! It does not take much to fool people, and the reconstructionists worked with care and forethought. With a human skull and an ape skull jaw before them as they worked, they shaped the plaster to produce an "ape-man."

At first, fraud wasn't even suspected. The fossils were, after all, cleverly done, and no money was involved. There were other European finds — Neanderthal, Cro-Magnon, and Heidelberg — so another European "missing link" wasn't too surprising. But not all were satisfied. Some scientists argued that the jaw and skull did not belong to the same individual. It was also observed that the few skull pieces could be arranged in a number of shapes and sizes to match any desired braincase and head shape that might be desired.

In reality, that is exactly what had been done. The parts had been carefully selected with consummate skill to provide only certain evidence while omitting certain other facts. The objective was to afterward reconstruct the head along ape lines, for the nearer the "reconstruction" could be pushed toward the brute beast, the more convincing it would appear as "scientific evidence" of evolution.

The objections offered were tossed aside and given little attention in scientific societies and even less in the public press. A whole generation grew up with "Piltdown Man" as their supposed ancestor. Textbooks, exhibits, displays, encyclopedias — all spread the good news that we came from apes after all.

Oil paintings of the discoverers were executed. The bones were named after Dawson, and the other men (Keith, Woodward and Grafton) were knighted by British royalty for their part in the great discovery.

As for the bones of Piltdown Man? Too many people were finding fault with them, so they were carefully placed under lock and key in the British Museum. Even such authorities as Louis Leakey were permitted to examine nothing better than plaster casts of the bones. Only the originals could reveal the fraud, not casts of them. Plaster casts of the half-man / half-ape "reconstruction" were sent to museums all over the world.

As recently as 1946, the Encyclopedia Britannica (Vol. 14, p. 763) stated authoritatively:

"Amongst British authorities there is agreement that the skull and jaw are parts of the same individual."

More than 500 articles, memoirs (including Doctoral Dissertations!) have been written about the Piltdown man.

Adding to the embarrassment of a government and nation, three years before the expose the National Nature Conservancy had spent a sizable amount of taxpayers' money in transforming the area in and around that pit into the Piltdown Gravel Pit National Monument.

Turn-of-the-century evolutionary theory predicted and believed in the imminent discovery of the "missing link", an ape-man earlier than Neanderthal, who would truly connect modern man with beast. What was needed was a half-million-year-old half-ape / half-man. And indeed, this 'magnificent discovery' came at just the right time: Suddenly, in 1912, the fossil-prophet appeared, not in Africa or Asia, but conveniently in Sussex, England. And what better a place to find such old bones than in perpetually damp England, where even bones half a century old normally have already turned back to dust.

The Piltdown forgery is perhaps the most engaging display of ratiocination since The Gold Bug. It beats Sherlock Holmes at its best. Indeed, a Darwinian might have said that if Piltdown man did not exist, we'd have to invent him. The hoax was so successful that for 40 years, anatomists, paleontologists, theologicians and brilliant academics everywhere attempted to define when Eoanthropus Dawsoni (man-like cranium, ape-like jaw) *fit in the evolutionary scheme. Yet the 'evidence' that early Man developed intelligence before developing in certain other way — sunk in.*

In 1953, chemical tests proved that the fossils were frauds. Someone had taken a slightly odd "modern" human skull, and the jaw of an orangutan. They had been stained, filed, smashed, and so on, in a fairly clever way. The fluorine test is a method of determining whether several bones were buried at the same time or at different times. This is done by measuring the amount of fluorine they have absorbed from ground water. It cannot give ages in years, but is a high-tech method of establishing ages of bones relative to each other.

Microscopic analysis of the wear found on the molars determined it to be unnatural. In fact it looked very much like the type of wear one would expect not from the normal chewing process but from a metal file. Other tests revealed that the specimens had been covered with an ochre colored paint to make them look more authentic. Obviously Piltdown was a well planned out forgery and hoax.

Additional examination revealed that the bones of Piltdown Man had been carefully stained with bichromate in order to make them appear aged. Drillings into the bones produced shavings, but should have produced powder if the bones had been ancient; powder, however, was not produced. Then that canine tooth was brought out — and found to have been filed, stained brown with potassium biochromate, and then packed with grains of sand. No *wonder it took so long before the discovery could be announced; a lot of work had to first be done on those bones and teeth.*

The House of Commons was so disturbed by the announcements of the fraud, that it came close to passing a measure declaring "that the House has no confidence in the Trustees of the British Museum ... because of the tardiness of their discovery that the skull of the Piltdown Man is a partial fake". A member of the British Parliament proposed a vote of 'no confidence' in the scientific leadership of the British Museum. The motion failed to carry when another member of Parliament reminded his colleagues that politicians had 'enough skeletons in their own closets'.

We will never be completely sure why anyone would try to fool the world into believing that Piltdown was the long sought – after Missing Link. We do know that whoever it was that planned such a scheme had to have a pretty extensive understanding of chemistry, geology and human anatomy in order to pull it off at all. They also had to have contacts that would provide them with access to bones outside Great Britain, for many of the animal bones found at the site came from places such as Malta and North Africa.

So, who dunnit? The list of suspects includes the following characters:

- ***Charles Dawson**, a Sussex lawyer and amateur archaeologist, who brought the first cranial fragments from Piltdown; a person with a long record of fraud.*

- ***Sir Arthur Smith Woodward** (1864–1944), head of the Department of Geology at the British Museum of Natural History (BMNH). Woodward was motivated by his desire to obtain the directorship of the BMNH. What better mean to obtain public acclaim than discovering the "missing link" on English soil, thus creating the English cradle of humankind.*

Given the many possible risks to his reputation and career, Arthur Smith Woodward has been considered an implausible confederate to Charles Dawson in the Piltdown affair. Woodward's seeming lack of motive has distracted many. Yet it is clear that ASW did serve to benefit from the acclaim of the "discovery" and had undertaken many other questionable practices in order to advance his desire to be appointed to the Directorship of the Natural History Museum.

Woodward maintained a thirty year association with Charles Dawson, which suggests a close and complex relationship beyond that of any other "suspect". Without such ties the trust essential for the conspiracy to occur would have been inexplicable. Dawson had access to the Sussex localities but lacked appropriate specimens and expertise to succeed alone. Woodward's close association with sites that serve as plausible sources for materials used *in the fraud provide important physical evidence pointing toward his involvement.*

Woodward's participation in the fraud also explains many of the puzzling episodes and "oversights" that surround the discoveries. A Dawson-Woodward nexus appears to draw together all the necessary elements to provide a satisfying resolution of the Piltdown fraud.

Woodward was with Dawson on the day he found the all-important jawbone at the gravel pit. As Woodward looked on — Dawson dug down and there it was!

- ***Sir Arthur Keith**, MD (1866–1955). Anatomist. One of the most highly respected scientist in England. Author of several classic works. Fellow of the Royal College of Surgeons, President of the Royal Anthropological Institute and member of the British Association for the Advancement of Science.*

- ***Grafton Eliot Smith**, a renown brain specialist.*

- ***Pierre-Tielhard de Chardin** (1881–1955). Philosopher and paleontologist. Accompanied Dawson on Aug 29, 1913 to the Piltdown pit where they 'discovered' the missing link canine teeth. It was duely reported in the 1913 meeting of the British Geological Society.*

- ***W.J. Solass**. Professor of Geology at Oxford.*

- ***Martin A.C. Hinton**, a curator of zoology. A trunk with Hinton's initials was recently found in an attic in London's Natural History Museum. The trunk contained bones stained and carved in the same way as the Piltdown fossils.*

- ***Arthur Conan Doyle** (1859–1930). Physician and writer. Creator of Sherlock Holmes. It is believed that he was along with Dowson in initially developing the idea for the fraudulent placement and later "discovery" of the bones.*

1913 CE **Archibald Vivian Hill** (1886–1977, England). Physiologist. Discovered that muscle cells respire (use oxygen) *after* contraction is finished. Awarded the Nobel prize for physiology or medicine (1922).

The muscle, after having become charged with lactic acid during anaerobic glycolysis, consumes oxygen while resting, at a rate greater than normal. The oxygen that would ordinarily have produced energy by combining directly with glucose, and which was short-circuited during the glycolysis, is now making up for what it had missed ("oxygen debt"). Oxygen is supplied as rapidly as possible to discharge the debt[289].

Hill was a professor at Manchester (1920–1923) and University College, London (1923–1925) and secretary of the Royal Society (1935–1945).

1913 CE **Leonor Michaelis** (1875–1949, Germany and U.S.A.). Physiologist and chemist. Discovered the first important mechanism of *enzyme kinetics*. Worked out the *Michaelis-Menton*[290] *equation* that describes the dependence of the rate at which *enzymes* catalyze reactions on the concentration of the substrate. In developing the theory, Michaelis adopted the assumption[291] that the enzyme and the substrate form an intermediate temporary complex. [It was only in 1965 that this mechanism was verified at the atomic level by **David Phillips** and his coworkers in London.]

Michaelis was born in Berlin, Germany to Jewish parents. He was assistant to **Paul Ehrlich** (1898–1899). During 1902–1926 he worked in various research centers, and later at Johns Hopkins University and the Rockefeller Institute (1929–1940).

1913 CE **William Henry Bragg** (1862–1942, England) and his son **William Lawrence Bragg** (1890–1971, England). Physicists. Shared the Nobel prize in 1915 for research on the structure of crystals by means of X-rays. They developed the *X-ray spectrometer*, and discovered much about the structure of the atom and the atomic arrangement in crystals[292].

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289 The debt is discharged by the breakdown of *lactic acid* to water and carbon dioxide, releasing 650 kilocalories per mole. This yields 18 times more energy than that which arises from the conversion of glucose to lactic acid.

290 **Maud Menton** (1879–1960), Michaelis' assistant.

291 First suggested (1902) by the French biochemist **V. Henri**. The *enzyme-substrate* free radical is a far more stable system than the substrate free radical. The net effect is that the enzyme successfully catalyzes the reaction.

292 X-rays of wavelength λ are incident on a crystal face, at angle θ to the normal. The face plane contains rows of surface atoms spaced at nearest-neighbor distances ℓ. The waves are scattered, and by Huygens' principle reinforce each other in the direction making the angle θ with the normal, such that the scattered beams obey the law of reflection. If successive atom layers lie beneath each other at spacing d, the path difference between waves scattered by a surface atom and its interior neighbor at angle θ to the normal is simply $2d\cos\theta$.

Bragg the elder was born at Wigton, Cumberland, and studied at Cambridge University. He served as a professor of physics at the universities of Adelaide, Leeds and London and became professor of chemistry at the Royal Institution, director of the Davy-Faraday Research Laboratory, and director of the Royal Institution.

1913 CE **Frederick Soddy** (1877–1956, England). Chemist. With **Rutherford** he developed the theory of radioactive elements. Investigated the origin and nature of *isotopes*, the name of which he coined himself (*Isotope* = 'same place' in Greek). Their existence was discovered after nearly a decade of experimenting with naturally radioactive materials[293].

Isotopes of a given element are atoms with the same number of protons and electrons but different numbers of neutrons. Having the same number of electrons, they are almost identical in chemical behavior, and having different number of neutrons, they differ in weight. Certain isotopes are unstable and undergo a process of decay during which they emit ionizing radiation. Such isotopes are called *radioisotopes*.

Soddy began his career in McGill University, Montreal, where he did research in radioactivity with Rutherford (1900–1902). He worked at the Universities of Glasgow (1904–1914) and Aberdeen (1914–1919) before moving to Oxford[294] (1919–1936). He was awarded the Nobel prize for chemistry in the same year that Albert Einstein got it for physics (1921).

If this path difference is an integral multiple of the wave length, strong *Bragg reflection* peak will occur from this crystal face; the condition is that $2d\cos\theta = n\lambda$ $(n = 1, 2, 3, \ldots)$. Hence d can be measured by observing the angles where reflection peaks, provided λ is known.

293 **William Crookes** (1832–1919, England) speculated (1886) that not all atoms of a given element have the same atomic weight. **J.J. Thomson** (1910, 1911) showed (by measuring deflections of positive rays in a cathode ray tube) Neon, with average atomic weight 20.183, to consist of appropriate proportions of isotopes of masses 20 and 22.

294 In one of his extracurricular activities, Soddy rediscovered a geometrical theorem, which he published in the form of a poem in *Nature* (**137**, 1021; June 20, 1936). The theorem states that if 4 circles touch each other externally, and if ϵ_1, ϵ_2, ϵ_3, ϵ_4 denote their curvatures (that is, the reciprocals of their radii), then the following relation holds between them: $2(\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2 + \epsilon_4^2) = (\epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4)^2$. The theorem is an interesting one because it can be viewed from a number of different mathematical angles. Furthermore, it can be generalized to n hyperspheres in $(n-2)$-dimensional Euclidean space $(n \geq 3)$, such that each is tangent to all the others at distinct points. The theorem then states: "Let $C_1, \ldots, C_n$ be the centers of these spheres; $r_1, \ldots, r_n$ their radii, and $\epsilon_1, \ldots, \epsilon_n$ their curvatures. Then either all of the spheres touch each other externally, or one of them contains all the oth-

1913–1914 CE **Henry Gwyn-Jeffreys Moseley** (1887–1915, England). Atomic physicist. First to discover a definite way of determining the atomic number of elements. It enabled scientists to determine the atomic number of unknown elements and to correct Mendeleev's periodic table. Discovered experimentally a simple relation between the atomic number of an element and its corresponding X-ray radiation frequency, thus establishing conclusively that the atomic number does truly represent the number of electrons of each neutral atom.

Moseley belonged to a scientific family; his father and his two grandfathers were fellows of the Royal Society. He entered Eaton at the age of 13 and Trinity College, Oxford at 18. He graduated in 1910 and soon after was appointed lecturer in the Physics Department of the University of Manchester, working under the guidance of **Rutherford**. His life work was done in four years.

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ers, and the curvatures satisfy the formula $\left(\sum_{i=1}^{n}\epsilon_i\right)^2 = (n-2)\sum_{i=1}^{n}\epsilon_i^2$ [for $n = 4$ we have 4 circles in 2D, for $n = 5$ we have 5 spheres in 3D, etc.]. The radius and curvature of a containing sphere (if any), are taken to be negative. Soddy published the theorem for $n = 4$ with no proof. It was later found that the theorem for this case was known to **Descartes** and **Steiner**. A simple proof, using just vector algebra was given by S. Brown [*Am. Math. Monthly* **76**, 662, 1969] and runs as follows: Let $\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_{n-1}$ be vectors from C_n to $C_1, \ldots, C_{n-1}$ respectively. The *tangency conditions* are

$$\begin{aligned} x_i^2 &= (r_i + r_n)^2 \qquad (i = 1, \ldots, n-1) \\ (\boldsymbol{x}_i - \boldsymbol{x}_j)^2 &= (r_i + r_j)^2 \qquad (i \neq j). \end{aligned}$$

On the other hand, since the vectors lie in a space of $n-2$ dimensions, they are linearly independent, and therefore $\det[\boldsymbol{x}_i \cdot \boldsymbol{x}_j] = 0 \quad (i, j = 1, \ldots, n-1)$. From the above tangency conditions, we find

$$\begin{aligned} x_i^2 &= \frac{(\epsilon_i + \epsilon_n)^2}{\epsilon_i^2 \epsilon_n^2} \qquad (i = 1, \ldots, n-1) \\ (\boldsymbol{x}_i \cdot \boldsymbol{x}_j) &= \frac{(\epsilon_i + \epsilon_n)(\epsilon_j + \epsilon_n) - 2\epsilon_n^2}{\epsilon_i \epsilon_j \epsilon_n^2} \qquad (i \neq j). \end{aligned}$$

With these relations, the determinant can be reduced to

$$\det[\boldsymbol{x}_i \cdot \boldsymbol{x}_j] = \frac{2^{n-2}}{\epsilon_1^2 \cdots \epsilon_n^2}\left[\left(\sum_{i=1}^{n}\epsilon_i\right)^2 - (n-2)\sum_{i=1}^{n}\epsilon_i^2\right]$$

which proves the theorem. The converse of the theorem is also true.

When WWI broke out, Moseley was offered work suited to his scientific capacity at home, but he preferred to share with others the dangers of active service. He took part in the severe fighting of the *battle of Sari Bair* against the Turks in the Dardanelles, and was killed on Aug. 10, 1915. Because of this loss, Britain restricted its scientists to noncombat duties during WWII.

In a paper *The high-frequency spectra of the elements* [*Phil. Mag.* **26** (*1913*) *1024–1034*. Part II: *Ibid.* **27** (*1914*) *703–713*] Moseley investigated the X-ray spectra of over 50 elements and found that the wavelength of the shortwave principal line K_α became shorter in a regular manner in going from the light to the heavy elements. He determined the frequency ν of the K_α lines of all atoms to be $\nu = \frac{3}{4}R(N-1)^2$, where R is the so-called Rydberg constant and N increases by one in going from one element to the next in the Periodic Table, and was identified with the atomic number N of the element. It was shown a little later that Moseley atomic number N was identical with the charge of the Nucleus Z, a conclusion confirmed (1920) by the scattering experiment of **Chadwick**.

Upon the reordering of the periodic table according to the atomic numbers N rather than atomic weights, hydrogen remains the first element, with atomic number 1, but a few elements do change position (e.g., argon now precedes potassium), improving the periodicity.

Moseley predicted the discovery of 3 missing elements, those with atomic numbers 43, 61 and 75.

While the Braggs were using X-rays to explore the intricacies of crystalline structure, Moseley used them to detect the fundamental modes of vibration of the atoms. His discovery completed the new atomic theories.

1913–1917 CE **Johann Radon** (1887–1956, Germany and Austria). Mathematician. Contributed to measure theory (1913), but is mainly known today for his discovery of the *Radon transform*[295] (1917), which furnishes

[295] The 3D *surface integral*

$$\widehat{f}(p,\boldsymbol{\xi}) = \int f(\boldsymbol{x})\delta(p-\boldsymbol{\xi}\cdot\boldsymbol{x})d^3\boldsymbol{x}$$

is known as the *Radon transform* of a function $f(x_1,x_2,x_3)$ over the plane

$$p = \boldsymbol{\xi}\cdot\boldsymbol{x} = \xi_1x_1+\xi_2x_2+\xi_3x_3.$$

Geometrically, p is the perpendicular distance from the origin to the plane, and $\boldsymbol{\xi}$ is a unit vector along p, that defines the orientation of the plane. The symbol δ denotes the Dirac δ-function.

This transform has a unique inverse and is linked to $F(\boldsymbol{k})|_{k\boldsymbol{\xi}}$, the 3D Fourier the basic mathematical framework common to a large class of reconstruction (imaging) methods, such as the modern computer assisted tomography (CAT).

Born in Tetschen (Bohemia), Radon was educated at the University of Vienna. Taught mathematics at the Universities of Brno and the École Polytechnique and later became a professor at the Universities of Hamburg (1919), Greisswald (1922), Erlangen (1925) and Breslau (1928). He returned to Vienna in 1947.

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transform of $f(\boldsymbol{x})$, via the one-dimensional Fourier integral

$$F(k\boldsymbol{\xi}) = \int_{-\infty}^{\infty} \widehat{f}(p, \boldsymbol{\xi}) e^{ikp} dp.$$

When the Radon transform is defined in 2 dimensions, it reads

$$\widehat{f}(p, \boldsymbol{\xi}) = \int_D f(x, y)\delta(p - \xi_1 x - \xi_2 y) dx dy.$$

Here $p = \xi_1 x + \xi_2 y$ is an equation of a *line* L whose normal is $\boldsymbol{\xi}$. The Radon transform for this case reduces to a *line integral* of $f(x, y)$ along L.
Introduce the normal form of the line L, $p = x\cos\phi + y\sin\phi$, where $\boldsymbol{\xi} = (\cos\phi, \sin\phi)$ and (p, s) are Cartesian axes, rotated by an angle ϕ relative to (x, y).
Suppose that we can evaluate the line integral of the unknown function $f(x, y)$ along the given direction ϕ in the xy plane, and for a fixed value of p, i.e.

$$\widehat{f}(p, \phi) = \int_L f(x, y) ds = \int_{-\infty}^{\infty} f(p\cos\phi - s\sin\phi, p\sin\phi + s\cos\phi) ds.$$

As we move the line L parallel to itself, the corresponding line integrals of $f(x, y)$ will define $\widehat{f}(p, \phi)$ for all p-values but fixed ϕ. The fundamental *Projection-Slice theorem* (**R.N. Bracewell**, 1956), then states: the one-dimensional Fourier transform of $\widehat{f}(p, \phi)$ w.r.t. p is equal to the two-dimensional Fourier-transform of $f(x, y)$ in the (K_x, K_y) plane, along the slice $\phi = \mathrm{tg}^{-1}(K_y/K_x)$. When this process is carried out for all angles ϕ, we may calculate $F(K_x, K_y)$ in the whole (K_x, K_y) plane. The Fourier inversion of F then recovers $f(x, y)$.
In medical applications, for example, $f(x, y)$ may be identified with some physical property of tissues on a plane section of the organ under examination.
To dig deeper, see:

- Deans, S.R., *The Radon Transform and Some of Its Applications*, John Wiley & Sons: New York, 1983, 289 pp.

Radon's original paper went almost unnoticed for half a century. The major developments in areas of application of the transform, did *not* come as a result of Radon's work and were already highly developed before connection with Radon's transform were recognized. It was **Bracewell** (1956) who truly illuminated the connection between the Radon and the Fourier transforms. During 1956–1971, there were many *rediscoveries* of Radon's results throughout the applied literature. These rediscoveries ended in 1972 when it was pointed out that Radon's work was fundamental to the problem of reconstruction from projections (**Allan M. Cormack**, Nobel prize address, 1979).

Indeed, it was found since then that reconstruction problems in optics, astronomy, molecular biology, geophysics, magnetic resonance imaging, material science and medicine, may be united within the framework of the theory of Radon's transform in Euclidean space. The common denominator to the situations that arise in these diverse fields is the need to determine (reconstruct) certain structural properties of an object, utilizing data obtained by methods that leave the entity under consideration in an undamaged and undisturbed state.

Computerized Tomographic Imaging[296] *(1895–1985)*

Since 1956 there has been renewed interest in what is known as the reconstruction problem. *This is the problem of determining the internal structure (or, more precisely, some property of the internal structure) of an object without having to cut, crack, or otherwise macroscopically damage the object.*

Various probes, including: X-rays, gamma rays, visible light, microwaves, sound waves, electrons, protons, neutrons, heavy ions, *and* nuclear magnetic resonance signals *have been used to study a large variety of objects whose size vary over an enormous range, from* complex molecules *studied by the electron microscopist to distant* radio sources *studied by the radio astronomer.*

[296] To dig deeper, see:

- Kak, A.C. and M. Slaney, *Principles of Computerized Tomographic Imaging*, IEEE Press, 1988, 327 pp.

These nondestructive reconstruction methods have been applied to a wide range of fields such as: medicine, astronomy, astrophysics, geophysics, molecular biology, electron microscopy, optics, aerodynamics, archaeology, material science, engineering (e.g. stress analysis, air-pollution monitoring), plasma physics, and various military applications.

Perhaps in no other field of modern science were the results so overwhelming as in medicine. *Here the advent of the digital electronic computer and new imaging techniques such as* ultrasound, magnetic resonance imaging *and* computerized tomographic imaging, *have combined to create an explosion of diagnostic techniques in the past 25 years.*

Tomography *refers to the cross-sectional imaging of an object from either transmission or reflection data collected by illuminating the object from many different directions. The impact of this technique in diagnostic medicine has been revolutionary, since it has enabled doctors to view internal organs with unprecedented precision and safety to the patient. The first application utilized X-rays for forming images of tissues based on the spatial distribution of their X-ray attenuation coefficient*[297]. *More recently, however, medical imaging has also been successfully accomplished with* radioisotopes, ultrasound, *and* magnetic resonance; *the imaged parameter being different in each case.*

There are numerous nonmedical imaging applications which lend themselves to the use of methods of computerized tomography. Researchers have already applied this methodology to the mapping of underground resources via cross-borehole imaging, some specialized cases of cross-sectional imaging for nondestructive testing, *the determination of brightness distribution over a celestial sphere, and* 3-dimensional imaging *with electron microscopy.*

Fundamentally, tomographic imaging deals with reconstructing an image from its projections. *In the strict sense of the word, a projection at a given angle consists of the integrals of the image along many parallel lines at the common direction specified by that angle (in the 3D case two angles are required, of course). However, in a loose sense, projection means the information derived from the transmitted energies, when an object is illuminated from a particular angle.*

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[297] X-ray photons interact with matter in several ways. At photon energies in the neighborhood of 70 keV, which are typical of CT scanners, the combined effects of *scattering* (Compton effect and Rayleigh scattering) and *absorption* (photoelectronic effect) result in exponential attenuation of the beam as it passes through homogeneous matter. The great drawback to *X-ray photographs* of the body is that they provide only an integrated value of the attenuation coefficient along lines of sight, which makes 3D reconstruction non-unique.

In recent years, the X-ray image formation technique has been extended to nuclear medicine and magnetic resonance on the one hand, and ultrasound and microwaves on the other.

In nuclear medicine, our interest is in reconstructing a 3D radioactive isotope distribution *within the human body. In imaging with magnetic resonance we wish to reconstruct* the local magnetic properties of the object. *In both these cases, the problems can be set up as reconstructing an image from its projections, as in X-ray tomography.*

This is not the case when ultrasound *and* microwaves *are used as radiant energy sources, though the aim is the same as with X-rays (viz. to reconstruct the cross-sectional images of, say, the attenuation coefficient). In body tissues, X-rays are* nondiffracting, *i.e. they travel in straight lines, whereas microwaves and ultrasound are diffracting. When an object is illuminated with a diffracting source, the wave-field that impinges on the object is* scattered *in practically all directions. When, however, the inhomogeneities are much larger than the wavelength, one might be able to get away with the assumption of straight-line propagation.*

For situations when one must *take diffraction effects into account, tomographic imaging can in principle be accomplished for weakly diffracting objects if reconstruction theory is based on the* wave equation *rather than on ray theory (geometric optics).*

Timeline

1895 ***Wilhelm Röntgen*** *(Germany) discovered X-rays; produced the first radiation image of the body (his wife's hand).*

1897 ***Pierre and Marie Curie*** *(France) isolated* radium.

1900 *Chest X-ray used to diagnose tuberculosis; Radiology began as a medical sub-specialty.*

1903 ***Willem Einthoven*** *(Holland) created the prototype of the modern* electrocardiograph *(EKG) to monitor and record the electric signature of the heart.*

1906–1912 *Contrast media agents were administered (orally or via vascular injection) to help visualize organs and blood vessels with better image contrast.*

1917 ***Johann Radon*** *(Germany) discovered the Radon transform. He thus laid the foundation for the basic mathematical framework common to a large class of image reconstruction problems. His paper went unnoticed for 40 years.*

1924–1958 *Development of radiology: primary examination creates an image by focusing X-rays through the body part of interest and directly onto a single piece of film inside a special cassette. In the earliest days, a head X-ray could require up to 11 minutes of exposure time (now, modern X-ray images are made in milliseconds and the X-ray dose currently used is as little as 2% of what was used then). Radiographic imaging of the gallbladder, bile duct and blood vessels began in 1924 and coronary artery imaging was first done in 1945.*

The next development involved the use of fluorescent scans and special glasses so that the doctor could see X-ray images in real time. In 1946, the film cassette changer allowed a series of cassettes to be exposed at a movie frame rate of 1.5 cassettes per second. By 1953, this technique had been improved to allow frame rates up to 6 frames per second.

In 1955, an X-ray image intensifier was developed which allowed the pick-up and display of the X-ray movie using TV camera and monitor. Together with the cut-film changer, the image intensifier opened the way for a new radiological sub-specialty known as angiography, allowing the routine imaging of blood vessels and the heart (1958).

1932 *Transmission electron microscope constructed by* ***M. Knoll*** *and* ***E. Ruska*** *(Germany).*

1956–1971 *Rediscoveries of* ***Radon's*** *results throughout the applied literature.*

1956–1979 ***Ronald N. Bracewell*** *and* ***Allan Cormack*** *(1956) show how* ***Radon's*** *theory could be used for image reconstruction. Bracewell (1956) illuminated the connection between the Radon and Fourier transforms ('Projection-slice theorem'). He then applied the theory to obtain a solution to a practical problem in radio-astronomy, namely, to map the regions of emitted microwave radiation from the sun's disc (1956–1979)*[298].

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298 From solar eclipse observations, it was clear by the early 1950's that microwaves were emitted from rather compact regions in the chromosphere and lower corona. But due to the inadequate resolving power of microwave antennas, it was not possible to study these emissions in much detail during the period

1972 **Godfrey Hounsfield** *(England) invented the computed tomography scanner or CT scanner — the first practical tomographic machine for clinical use. He showed that it is possible to compute high quality cross-sectional images with an accuracy of 1 part in 1000. Hounsfield used gamma rays (and later X-rays) and a detector mounted on a special rotating frame together with a digital computer to create detailed cross-sectional images of objects. His original CT scan took hours to acquire a single slice of image data and more than 24 hours to reconstruct data into a single image (today's state of the art CT systems can acquire a single image in less than a second).*

1973–1984 **Paul Lauterbur** *(England) developed magnetic resonance imaging or MRI (1973). It is a tomographic techniques in which for example the density distribution of hydrogen nuclei is reconstructed. The proton density in turn is diagnostic of tissue structure*[299]*. MRI of the brain was first done on a clinical patient in 1980. It was cleared for commercial clinical use by the FDA in 1984 and its use throughout the world has spread rapidly since.*

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between two eclipses.

As seen from earth, the angular diameter of the sun is about 30 minutes of arc. To map the emitted microwave radiation over the solar disc, an antenna with an angular resolution small compared to 30 minutes of arc is needed. Adequate spot resolution using a single antenna would require an antenna of enormous dimensions, impractical to construct. The way around this is to use several antennas along a line, thus achieving good resolution in one direction and poor resolution in a perpendicular direction. It thus proved practical to construct *arrays* of antennas with a beamwidth, 3 minutes of arc. Such an array would be sensitive to microwave radiation from sources inside a narrow strip in the sky, 3 minutes of arc wide and several degrees long. In radio astronomy, a reception pattern of this type is known as a *fan beam*. When the fan beam system is aimed toward the sun, the received signal approximates a line integral of microwave intensity. Since the antenna array is attached to the earth, the sensitive strip region sweeps across the sun as the earth rotates, and the received power as a function of angle represents a two-dimensional map of microwave intensity. To obtain scans for different angles, it is only necessary to allow some time and do the scan again. As time passes, the position angle ϕ varies automatically and may be computed from $\cot\phi = (\sin\delta)(\tan h)$, where δ is the declination and h the hour angle of the source.

[299] "Chemical shifts" in the resonance radio frequency yields further data about the molecular environment of the H atoms.

1974–1980 *Ultrasound computer tomography* was developed by **J.E. Greenleaf** and **S.A. Johnson** *(1974) to look at the abdomen and kidneys, fetal tissues, carotid blood vessels and the heart. The process involves placing a small device (transducer) against the skin of the patient near the region of interest. The transducer produces inaudible high frequency sound waves which penetrate into the body and bounce off the organs inside. The transducer detects the echo from the internal structures and contours of the organs.*

When diffraction effects are ignored, ultrasonic CT is similar to X-ray tomography: in both cases a transmitter illuminates the object and a line integral of the attenuation can be estimated by measuring the energy on the far side of the object.

Ultrasound, however, differs from X-rays because the propagation speed is much lower and much more dependent upon medium composition, and thus it is possible to measure and extract information from not only the attenuation *of the pressure field but also the time-delay of the signal induced by the object. This yields* both *the* attenuation coefficient *and the* refractive index *of the object.*

Most medical ultrasonic images are done using reflected signals *and the discipline belongs to the field of* reflection tomography. *As in radar, the* echoes *are sent along a* narrow beam *and the image is formed by displaying the reflected signal as a function of time and direction of the beam. It is not necessary to encircle the object with transmitters and receivers for gathering the 'projection' data; transmission and reception are now done from the same side.*

1978–1985 *Emission computed tomography (ECT): Isotopes are administered to the patient in the form of radio-pharmaceuticals either by injection or by inhalation. The decay of these radio isotopes is associated with the emission of quanta, which in turn yields information on their distribution (location and concentration) as a function of time. The radioactive isotopes used are characterized by the emission of* gamma-ray photons *or* positrons. *The concentration of such an isotope in any cross section changes with time due to radioactive decay, flow, and biochemical kinetics within the body.*

This implies that all the data for generating one cross-sectional image must be collected in a time interval that is short compared to the time constant associated with the changing concentration. By analyzing the images taken at different times for the same cross-section we can determine the functional state of various organs in a patient's body. There are two types of emission CT:

(a) *Single Photon Emission Computed Tomography (using e.g. Iodine-131, technetium-99m) or SPECT (1979)*

(b) *Positron Emission Tomography (e.g. Carbon-11; Oxygen-15) or PET (1985). Two gamma-ray photons, traveling in opposite directions, are created by the annihilation*[300] *of an electron and a positron.*

1913–1922 CE **Elmer Verner McCollum** (1879–1967, USA). Biochemist and nutritionist. Pioneered in the study of *vitamins* and originated the letter system of naming them. Identified (with **Margaret Davis**) vitamin A (1913). Collaborated in the discovery of vitamin D in cod liver oil (1922), and used it for treating rickets.

McCollum was born in Fort Scott, Kansas and studied at Yale University. From 1917 he was Professor of Biochemistry at John Hopkins University.

1913–1928 CE **Irving Langmuir** (1881–1957, U.S.A.). Versatile scientist, engineer and inventor. Invented the high-vacuum electron tube and the gas-filled incandescent electric lamp[301] at atmospheric pressure. Used the term *plasma* for the first time to describe a collection of charged particles in his

[300] *Pair annihilation* is the opposite phenomenon of *pair production*. A pair of oppositely charged particles – an electron and its anti-particle, a *positron* – briefly attract one another to form a kind of "atom" called *positronium*. Within 10^{-10} seconds, the two particles spiral into one another and annihilate. In their place two (or occasionally three) γ-ray photons are produced. In the positronium rest frame, the two γ-rays move away from each other in opposite directions with equal energies, frequencies, and wavelengths. Therefore, the vector sum of their linear momenta is zero (just one γ-ray could not have zero momentum, so two or three must be created). Since the orbital e^- and e^+ speeds prior to annihilation are negligible, the conservation of mass-energy yields $m_0^2c^2 + m_0^2c^2 = 2h\nu_{\min}$ or $m_0c^2 = h\nu_{\min}$. The minimum γ-ray energy is thus 0.511 MeV, equal to an electron's rest-mass energy; for proton-antiproton pair annihilation it is 938.3 MeV.

[301] He found that the higher pressure did reduce evaporation of the tungsten, but so much heat was conducted away by the gas that the lamp efficiency was reduced. He discovered that coiling the filament decreased the effective area exposed to the gas and thus minimized the loss of heat. Coiled filament gas-filled lamps in 500, 750 and 1000 watt sizes were introduced in 1913. They gave a much better light at higher efficiency with the same life as former lamps. Nitrogen

study of oscillations in electric discharges. He realized that the matter inside the glow discharge was different from ordinary gases.

Langmuir had an unusually wide range of interests: While he is known to physicists for his work with plasmas, he won in 1932 a Nobel prize in chemistry for studies on molecular surface effects that have had important applications in medical research. Toward the end of his life he pioneered experiments on weather modification, and was the guiding force behind the first successful cloud seeding in which man-made snow was produced.

Langmuir was born in Brooklyn, NY and studied at Columbia University (1903) and in Göttingen, Germany under **Nernst** (1906). He conducted research in General Electric Laboratory in Schenectady, NY during 1909–1950.

Langmuir was a man of brilliant ideas, for which he always sought practical applications. With the rapid advance of technology, his scientific achievements were used for the developments of many instruments and production processes.

1913–1931 CE **George David Birkhoff** (1884–1944, U.S.A.). The only American mathematician of his generation with major international reputation. Birkhoff proved in 1913 a conjecture made by Poincaré in 1912 (known as Poincaré last "theorem")[302]. In 1923 he proved a theorem which states that any spherically symmetric vacuum solutions of Einstein equations must be static and must agree with the Schwarzschild solution, apart from a coordinate transformation. This means that when the spacetime surrounding any object has spherical symmetry and is free of matter and of all fields other than gravity, then one can introduce coordinates in which the metric is that of Schwarzschild.

Thus, the exterior field of *any* electrically-neutral, spherical *star* (regardless of whether it is static, collapsing, expanding or pulsating), satisfies the conditions of Birkhoff's theorem.

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gas was used in the first lamps but argon was substituted in 1914. Argon has lower heat conductivity than nitrogen. These lamps could be made smaller than carbon lamps and produced three times the light per watt.

302 The conjecture, which Poincaré did not live to prove, belongs to the field of *topological dynamics*: if a one-to-one continuous transformation carries the ring bounded by two concentric circles into itself in such a way as to preserve areas and to move the points of the inner circle clockwise and those of the outer circle anticlockwise, then at least two points must remain fixed. This theorem has important applications to the classical problem of 3-bodies. Birkhoff astonished the French, who had not believed that Americans were mathematically capable. Birkhoff later told a student that he had lost thirty pounds working out that proof.

The theorem generalizes to the case where the exterior space is spherically symmetric, free of matter *but* allowed to have an electromagnetic field. In this case the exterior metric must be (again, in appropriate coordinates) the *Reissner-Nordström* metric (characterizing a *charged* black hole).

In 1931, Birkhoff proved the *general ergodic theorem*, building on the *mean ergodic theorem* proven earlier by **John von Neumann**. Birkhoff systematically kept Jews out of his department and acted generally to hinder their entry into Harvard. He thus opposed the appointment of **Oscar Zariski** (1926). He vehemently objected to the appointment of **Solomon Lefschetz** for president of AMS (1934) on the grounds that the Jews "... will use the Annals as a good deal of racial prerequisite. The racial interests will get deeper as Einstein's and all of them will do".

Albert Einstein said: "G.D. Birkhoff is one of the world's great anti-semites".

1913–1935 CE **Georg (György) Karl von Hevesy** (1885–1966; Hungary, Switzerland and Denmark). Chemist. The first[303] (1913) to use a radioactive isotope to follow the steps of chemical and biological processes, for which he won the Nobel Prize for Chemistry (1943). Discovered (1923, with **Dirk Coster**) the element *Hafnium*[304].

Hevesy was born to Jewish parents in Budapest and educated in Germany, Switzerland and England, studying in Manchester under **E. Rutherford** (1911). He moved to Vienna (1912), and after service in the Austro-Hungarian army during WWI, he worked in Copenhagen (1920–1926) under **N. Bohr**. He then became a professor at Freiburg (1923–1933), Copenhagen (1933–1942), and Stockholm (1942–1945) to where he fled during WWII. He died in Freiburg.

Hevesy's work on isotopes has been very influential in physics, chemistry and medicine: he worked to find ways of separating isotopes by physical means. Thus in 1934 he used radioactive *phosphorus* isotope to study phosphorus metabolism in plants and in the human body and used heavy water to study the mechanism of water exchange between goldfish and their surroundings, and also within the human body. He then showed that chemical changes are continually taking place in all living tissues. In 1935 he began to calculate the relative abundance of elements in the universe.

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303 With **K.A. Paneth** he used *radioactive lead* to trace the solubility of lead salts, demonstrating that even a small amount of the chemical can be used as a radioactive tracer.

304 The Latin name for Copenhagen, where the discovery was made.

1913–1928 CE **William Mulholland** (1855–1935, USA). Chief architect of the *Owens Valley Aqueduct* that brought water to semi-arid Los Angeles from the lush Owens Valley (1913), thus making this modern metropolis possible and forever changing the course of Southern California's history. His career was ended on March 12, 1928 with the collapse of the St. Francis Dam (over 400 people killed and damage estimated at $20 million).

William Mulholland was born in Belfast, Ireland. In 1877 he arrived in Los Angeles, having worked his way from Ireland as a sailor, lumberjack, Apache fighter and mine prospector. He would, in time, engineer a historic feat and have a great impact upon the future of Los Angeles.

Mulholland saw that a burgeoning and thirsty Los Angeles would soon need much more water than it had available and foresaw the need of opening a new water source by tapping into the Eastern Sierra water from the Owens Valley, thus becoming obsessed with an engineering challenge of epic proportions.

Although he possessed no formal training in engineering, Mulholland pursued intense personal interest in geology, hydraulics and engineering by educating himself at the public library. Living in a shack, he worked as a ditch tender. By 1886, he had worked his way up to become the City Water Company's superintendent.

After the City of Los Angeles bought out the Los Angeles City Water Company (1902), Mulholland oversaw the formation of the new Los Angeles Bureau of Water Works and Supply (which would eventually become Los Angeles Department of Water and Power in the 1920s). He was the department's first superintendent and chief engineer. He also became the first American engineer to build a dam utilizing *hydraulic sluicing* (Silver Lake Reservoir, 1906).

Calling forth the deepest resources of his character: organization, vision and dogged determination, he would, over the next years, be entrusted with building a 375 km *aqueduct* (the world's longest at the time), to bring water from the Owens River north of Los Angeles to the San Fernando Valley, where developers awaited conversion of dry land into farms and housing tracts.[305]

[305] While Mulholland began to look longingly at the *Owens River*, more than 320 km away, the residents of *Owen Valley* had plans for the water as well. Most of them raised crops and ranched, and they were anticipating an economic bonanza once the newly-founded *Reclamation Service* completed its Owens Valley irrigation project. Mulholland realized that to acquire the Owens River for Los Angeles, they would have to put an end to this irrigation project. Thus started "the Owens Valley War" which reached its climax in 1924. It was a struggle for economic and political power between the landowners of the San Fernando

The project required massing of 3900 workers and digging and blasting of 164 tunnels, almost 86 km in all, over desert and mountain terrain. It called for carving out sluiceways, clearing roads, laying railroad tracks and running power lines. When machines broke down, Mulholland used mules, when men perished, he hired more. He was creating one of the engineering marvels of the age, and nothing would get in his way. No wonder, this project was dubbed the "Panama Canal of the West". It was the largest and most difficult municipal engineering project in U.S. history at the time.

The first Owens River water flowed into a San Fernando Valley reservoir on Nov 5, 1913. At the ceremony marking the occasion, the laconic Mulholland uttered to exuberant crowds what may be the five most famous words in the city's history "*There it is. Take it.*". This was the first step toward making Los Angeles into an international metropolis: the achievement gave the city the ability to grow beyond a population of 500,000 and leverage water to expand city territory into San Fernando Valley and other surrounding communities. The massive project was completed ahead of schedule under budget of $24.5 million in municipal bonds approved by voters.

In 1923, the City of Los Angeles honored Mulholland by means of a new scenic highway that ran along the spine of the Santa Monica Mountains. It was named Mulholland Drive.

Five years later, on March 12, 1928, Mulholland's career took a tragic turn when the St. Francis Dam, one of several dams built to increase storage of Owens River water, collapsed, sending about 50 million m^3 of water into the Santa Clara Valley, north of Los Angeles — one of the greatest civil disasters in American history.[306]

Valley growers (who benefited from Mulholland's plan) and the Owen Valley farmers and ranchers.

At the end the residents of the Owens valley were out-maneuvered by Mulholland and his aids, who eventually took ownership of 95 percent of farmland and towns of Owen's Valley.

The film "*Chinatown*" (starring Jack Nicholson) portrays in fictional form some of the events surrounding Mulholland's quest for water. Mulholland's persona in the film is loosely split between two characters in the film: water department chief Hollis Mulwray and water tycoon Noah Cross (played by John Huston).

306 The waters swept through the Santa Clara Valley toward the Pacific Ocean, about 87 km away. 104 km of Valley was devastated before the water finally made its way into the Ocean between Oxnard and Ventura. At its peak the wall of water was said to be 23 m high. By the time it hit Santa Paula, 67 km south of the dam, the water was estimated to be 8 m deep. Almost everything in its path was destroyed: livestock, structures, railways, bridges and orchards. By the time it was over, parts of Ventura County lay under 20 m of mud and debris.

In the end, the jury found that the disaster was caused by the failure of the rock formations on which the dam was built, but responsibility was placed on the governmental organizations behind the construction of the dam, and on its chief designer, William Mulholland. No criminal charges were brought against him. A 1992 examination of the disaster concluded that, given the geological knowledge of the time, Mulholland was in fact innocent of criminal negligence — that the break was caused by the anchoring of the dam's eastern edge to an ancient landslide impossible to detect in the 1920's.

Mulholland was forced to resign in disgrace. He took full responsibility, saying: "If there is an error of human judgment, I am human". His final years were lived in the shadow of the disaster. He died in 1935.

1913–1940 CE **Igor Ivanovich Sikorsky** (1889–1972, Russia and U.S.A.). An aircraft designer and manufacturer. Pioneered in multi-engine airplanes, helicopters and transoceanic flying boats. Designed the world's first 4-engine aircraft in 1913. Built the first practical single-rotor helicopter in 1939 and flew it in 1940.

Sikorsky was born in Kiev, the Ukraine. He was educated at the Petrograd Naval College and at engineering schools in Paris and Kiev. He rose to prominent position in Russian aviation, designing one of the most successful bombers of WWI.

Sikorsky came to the United States in 1919. In 1923 he founded a company which produced flying boats. He then designed and built helicopters.

1913–1942 CE **Bela Schick** (1877–1967, Austria and U.S.A.). Distinguished physician. Known for a skin test for diphteria immunity which carries his name (*Schick Test*, 1913)[307]. Did important research on such childhood diseases as scarlet fever, tuberculosis, serum sickness.

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[307] In the *Schick Test* a very small amount of diphteria toxin is injected under the skin. There is no effect if the person is immune to the disease, but the area around the injection becomes inflamed if he is not.

Clemens von Pirquet (Austria), pediatrician, discovered that the adverse reaction of the diphteria antitoxin was caused by the *horse serum* in which the antitoxin was carried.

Von Pirquet devised the term "*allergy*" by combining two Greek words: *allos* (= different or changed) and *ergos* (= work or action). An allergy denoted that, in this adverse reaction, the action of a substance in the body was somehow changed.

Schick worked in Vienna with **Theodor Escherich** and **Clemens von Pirquet**. With the latter he laid the foundation of our knowledge of serum sickness and allergy.

Schick was born in Bogllár, Hungary to Jewish parents. He was a clinical professor of childhood diseases at the University of Vienna (1902–1923). Came to the United States (1923) and became chief pediatrician at Mt. Sinai Hospital, N.Y.C. (1923–1943) and professor of medicine at Columbia University (1923–1942).

Prequantum Atomic and Subatomic Physics

During the 20^{th} century the detailed study of atomic and nuclear structure has developed into a subject of immense scope and forbidding complexity. But the atomic theory of matter is over two thousand years old, having originated in the philosophy of the ancient Greeks. For centuries the idea that matter consists of indivisible atoms was a commonplace of speculative thought, but the concept became valuable only when it could be applied quantitatively — to an explanation of the gas laws (commencing in the 18^{th} century) and the laws of chemical combination (in the early 19^{th} century).

Even in the 1900's scientists as eminent as ***Mach*** *and* ***Ostwald*** *could still regard the existence of atoms as a hypothesis, with little experimental support. In the same period, however, a series of brilliant discoveries opened the way to novel investigations that established the atom as the smallest unit of a chemical element, although it could no longer be thought of as indivisible.*

The atomic theory of matter originated in the speculations of ***Leucippos*** *and* ***Democritos****, who lived in Greece during the 5^{th} century BCE. Nowadays we may wonder why these philosophers should have adopted so bold a theory without possessing any experimental evidence on the subject. The answer may lie in the difficult logical questions raised by the Eleatics, an earlier school of philosophers headed by* ***Parmenides****. The Eleatic thinkers became famous for their use of uncompromising logic in analyzing the nature of matter and motion, and they certainly reached some astonishing conclusions.*

As an example of Parmenides' arguments we may consider his contention that all space is filled with matter, that is, a vacuum cannot occur. The *statement was based on the proposition “That which does not exist does not exist”. If we accept this and the additional hypothesis that no two bodies can occupy the same place, it follows that motion is impossible, or at best illusory. This conclusion was supported by* ***Zeno****, a disciple of Parmenides, in four famous “paradoxes” concerning the concepts of time and space.*

Possibly the most significant paradox concerns a flying arrow. The arrow is imagined to be at rest at any given instant of time; but, if it is stationary at all possible instants, how can it ever move from place to place? Of course the validity of such a result depends on the assumption granted — in this case, the concept of an “instant” is essential to the analysis. In their historical context, Zeno’s paradoxes were powerful arguments for treating space and time as intrinsically continuous, whatever the nature of matter may be.

The work of the Eleatics caused a crisis in Greek thought and out of this crisis new theories of matter arose. Leucippos and Democritos boldly postulated the existence of a vacuum separating the indivisible atoms of matter. In this way they were able to account for the compression and rarefaction of gases, and also to explain the properties of different substances as being due to different atomic species. Democritos clearly enunciated the principle of conservation of matter, based on the indestructibility of atoms.

Later, the ideas of the atomists were revived by ***Epicuros****, and they found remarkable expression in* ***Lucretius****’ poem* De Rerum Natura*, which includes a sustained panegyric on the beauties of atomic theory.*

However, several philosophers, including ***Aristotle****, argued in favor of a continuous theory of matter, partly on the grounds that atoms separated by a vacuum cannot conceivably interact except at contact. In the third century BCE the* Stoic *philosophers observed waves spreading on the surface of water and supposed that sound travels in a similar way through air. By considering the propagation of waves in a medium, they were led to postulate the existence of a rarefied continuum, later called “the ether”, pervading all space and linking material bodies together in some way. Ideas based on the properties of a continuum were not easily reconciled with the tenets of atomic theory.*

It is important to recognize that the existence of particles in the physical world raises complex problems when we try to describe their behavior mathematically. For example, relativity theory is framed in terms of point-particles and continuous electromagnetic fields, but a charged point-particle should have infinite self-energy if the self-reaction of its electromagnetic field is taken into account. Moreover, the entire question of interaction between different particles requires elaborate treatment in the context of any field theory.

These difficulties persists in modern physics, were we have, on the one hand, field theories (such as Maxwell's electromagnetic theory and its quantum extension, QED*) and, on the other, particles interacting with and emitting, absorbing or annihilating each other, and also produced in pairs from the vacuum. Despite the many successes of quantum mechanics and of quantum field theories, the interactions of particles and fields are still not described completely satisfactorily in mathematical terms.*

The philosophical problems raised by Zeno still require consideration, although the original mathematical difficulties he raised in his paradoxes have been resolved. It can be argued that, in our range of experience, all actual events are atomic or discrete in nature, but potential occurrences must be described in terms of continuity. Thus a particle is regarded in quantum mechanics as potentially located anywhere in the universe, yet its existence is recognized experimentally only by discrete events localized in space-time. Some kind of reconciliation between these two paradigms is necessary for any understanding of present-day physics; needless to say, the last paper on the subject has not yet been written.

The study of nuclear physics commenced in 1896 *with the discovery of* ***Becquerel*** *that salts of uranium emit rays which penetrate matter to some extent and affect photographic plates. The study of* radioactivity *was pursued energetically by* ***Pierre*** *and* ***Marie Curie****, who isolated the new radioactive elements Polonium and Radium, and by* ***E. Rutherford****, who with various collaborators investigated radioactive series and the properties of alpha rays. Remarkable features of the new phenomenon were the apparently spontaneous release of considerable amounts of energy, and the random sequence exhibited by individual radioactive decay processes. It was found that the activity displayed by a given quantity of a radioactive element is quite unaffected by chemical combination and by normal changes in physical conditions. Moreover, radioactive characteristics differ markedly from one isotope to another of the same element; it follows that radioactivity is a property of the massive, positively-charged part of an atom.*

Although immense progress was made between 1900 *and* 1910 *in the quantum theory of radiation, no successful model of atomic structure was evolved, possibly because too much importance was attached to the old concept of the atom as an impenetrable entity. The essential new idea was provided in* 1911 *by Rutherford, who analyzed the results of alpha-particle scattering experiments and showed that the positive charge in an atom is concentrated in a very small* nucleus *at the center. It then became possible to regard the electrons as loosely bound planetary particles, organized into shells by virtue of their orbital motion.*

In 1913 ***Niels Bohr*** *succeeded in giving a description for electron orbits in terms of quantized angular momentum and action, and he used this model to explain the spectrum of atomic hydrogen. Despite the inconsistencies of the Bohr theory, it remains a major landmark in scientific thought and it is still valuable as a model for calculating the orders of magnitude of classically conceived quantities, such as electron speeds and orbital radii, in simple atoms (the so-called "semiclassical" analysis of a quantum system).*

1913–1915 CE **Niels** (**Henrik David**) **Bohr** (1885–1962, Denmark). One of the leading physicists of the 20^{th} century. Linked the new quantum physics to the structure of atoms (and later of molecules). Major contributor to the development of quantum physics for almost 50 years. Suggested a model for the hydrogen atom with discrete energy levels for the orbiting electron, which enabled him to explain the '*Balmer lines*' (discovered in 1885). He is also responsible for the liquid-drop model of the atomic nucleus.

The great achievement of the Rutherford group in establishing the existence of a small positively charged nucleus in each atom, led physicists to the next problem — to discover the correct description of the possible electron orbits about the nucleus. In 1912 **J.W. Nicholson** showed that the angular momentum of a "planetary" electron should change by a definite amount whenever it emitted or absorbed radiant energy. The quantum law of spectroscopy $\hbar\omega_{mn} = E_m - E_n$ [ω_{mn} — angular frequency of spectral line emitted or absorbed when the atom makes a transition from state m of energy E_m to state n with energy E_n] was known to be accurate in several instances, including the infrared spectra of molecules.

The combination of these ideas enabled **Bohr** to expound the first partially successful theory of atomic structure. In a series of papers between 1913 and 1915, Bohr outlined an elementary *semiclassical* theory of orbital electrons, which accounted for the spectrum of the hydrogen atom and certain other phenomena in atomic physics. His theory was based on two postulates:

(1) Each electron in an atom revolves about the nucleus in a fixed orbit satisfying the condition that the angular momentum is an integral multiple of the quantum unit $\hbar = \hbar/2\pi$. The rational for this quantization law comes from the Hamilton-Jacobi formulation of mechanics. Imposing it guarantees that the photon emitted when the orbiting electron transitions from orbit $(n + 1)$ to orbit n, has a frequency approximating that of the two orbits for $n \gg 1$ (classical limit).

(2) An electron does *not* radiate while occupying one of the quantized orbits, but light is emitted or absorbed when an electron changes from one orbit to another, the angular frequency of the radiation ω_{mn} being given by the known equation $\hbar\omega = E_m - E_n$.

Bohr made no attempt to describe the process whereby electrons are assumed to change from one orbit to another. Moreover, the basic postulate of fixed orbit for electrons is *contrary to the classical electromagnetic theory*, since any electron which is accelerated should radiate energy away, and the continual loss of energy would cause the electron to eventually spiral into the nucleus. Nonetheless, the theory remained classical in the sense that details of the electron motions are worked out mechanically, in close analogy with the system of planets revolving the sun[308].

Bohr's theory failed to explain the *fine structure* of the spectral lines. **Sommerfeld** was able to explain it by treating the electrons as *relativistic particles*. Later, the *hyperfine* structure of the spectral lines was explained in terms of the very small *magnetic* interaction between the spins of the electron and the nucleus. This type of interaction is often important in determining the structure of spectral lines emitted by *heavy* atoms.

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[308] In the single-electron hydrogen atom, the attractive Coulomb force e^2/r^2 supplies the centripetal acceleration, i.e. $e^2/r^2 = mr\omega^2$ where (e, m) are the electron's charge and mass respectively, r is the radius of its (assumed circular) orbit and ω is the orbit's angular frequency. The quantization of its angular momentum reads $mvr = n\hbar$ where $v = \omega r$ is its orbital speed. The two equations yield $r_n = \frac{h^2 n^2}{4\pi^2 me^2}$; $\omega_n = \frac{8\pi^3 me^4}{h^3 n^3}$; $v_n = \frac{2\pi e^2}{hn}$. In the hydrogen atom, the first orbit $(n = 1)$ has a radius $r_1 = 0.5291\text{Å}$ in which the electron moves with the velocity $v_1 = 2.2 \times 10^8$ cm/sec. The energy of the atom, E_n, is the sum of the kinetic and potential energies, the sum being negative for any bound electron [on this energy scale, the zero corresponds to the case of an electron at rest at an infinite distance from the nucleus, and the negative E values are referred to as *binding energies* of the electron]. Thus $E_n = \frac{1}{2}mv_n^2 - \frac{e^2}{r_n} = -\frac{2\pi^2 me^4}{h^2 n^2}$, with $E_1 = -13.6$ electron-Volts. The expression for the wavenumbers of the spectral lines of the H atom (reciprocal wavelengths) is $\frac{1}{\lambda_{\ell n}} = \frac{1}{hc}(E_\ell - E_n) = \frac{2\pi^2 me^4}{h^3 c}\left(\frac{1}{n^2} - \frac{1}{\ell^2}\right)$, $\ell > n$. When this relation was compared with the empirical formula for the Balmer series for visible hydrogen spectral lines, $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$, $n = 3, 4, 5, \ldots$, the constant R [known as the *Rydberg constant*] was found to be equal to $\frac{2\pi^2 me^4}{ch^3(1+m/M)}$ [corrected for the motion of *both* electron and proton about their common mass-center]. The calculated value of $109,677.576 \pm 0.012$ cm^{-1} for R was found to be in excellent agreement with the empirical spectroscopic value. This was an outstanding triumph for the Bohr theory.

Bohr was the son of the physiologist Christian Bohr and a Jewish mother. He studied physics at Copenhagen, earning his doctorate in 1911. He then went to England, and worked at Cambridge under **J.J. Thomson** and in Manchester under **E. Rutherford**, where he stayed until 1916, returning to Copenhagen in 1916 as professor, and in 1922 was awarded the Nobel prize for physics. Bohr took active part in the anti-Nazi resistance movement (1940–1943). In 1943, under threat of immediate arrest, he escaped to Sweden on his family fishing boat.

Bohr's brother, **Harald August Bohr** (1887–1951, Denmark), was an outstanding mathematician, known for his studies of the Riemann zeta function and *almost-periodic functions*[309]. Bohr's son **Aage Niels Bohr** (b. 1922) is a physicist who shared the Nobel prize for physics in 1975 for his work in determining the asymmetrical shapes of certain atomic nuclei.

Bohr contemplated the philosophical significance of quantum physics, especially the cognitive contradictions inherent in the wave-particle duality. He thus came to suggest his *principle of complementarity* stating that the model we use is determined by the nature of the measurement: if a measurement proves the wave character of radiation or matter, then it is impossible to observe the particle character in the same measurement, and conversely. Hence, radiation and matter are not simply waves nor simply particles. A more general and complicated model is needed to describe their behavior, even though in extreme situations a simple wave model *or* a simple particle model may apply.

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309 *Almost periodic function*: a function which satisfies $f(x+\tau) = f(x)$ with an error that can be made arbitrary small, when τ denotes any number of an infinite set of values spread over the whole range $(-\infty, \infty)$ in such a way as not to leave empty intervals of arbitrary great length.

Example: $f(x) = \sin px + \cos qx$, where p and q are noncommensurable numbers. Indeed, let integers (m, n) be found such that $p\tau = 2\pi n + \epsilon$, $q\tau = 2\pi m + \epsilon$ where ϵ is arbitrarily small positive number. Then

$$f(x+\tau) = \sin(px+\epsilon) + \cos(qx+\epsilon) = \sin px + \cos qx + 2\theta\epsilon, \quad |\theta| < 1.$$

An almost periodic function can be expanded in a series $f(x) = \sum_{n=0}^{\infty} A_n e^{i\lambda_n x}$ provided $\sum |A_n|^2$ converges. This is a special case of *Dirichlet series*, which in turn includes *Fourier series* as a special case. The *Bohr-Landau theorem* (1914) describes the conditions under which the *Riemann zeta function* is equal to zero. Bohr founded the theory of *almost periodic functions* in 1924 through his studies of representation of functions by *Dirichlet series*. He was a professor at the Copenhagen College of Technology (1915–1930) and then a professor of mathematics at the University of Copenhagen.

Bohr was also preoccupied with the philosophy of biology, regarding life, free will and consciousness as manifestations of a principle that deviates from the domain of the physical sciences.

Spectroscopy (1666–1913)

The history of science is intimately connected with the history of tools and instruments. With a telescope *he had made,* ***Galileo*** *observed the four small bodies which revolve around the planet Jupiter — a miniature solar system which helped substantiate the Copernican theory. Indeed, without optical and radio telescopes, modern astronomy would be virtually nonexistent. Bacteriology is equally dependent on the* microscope. *Cloud,* bubble *and* drift chambers *trace the paths of invisible subatomic particles. The* cyclotron *and its descendants infuse them with energies which result in wholly new phenomena. The atomic* clock *and the* interferometer *provide measurements of extraordinary accuracy. The* camera *fixes visual evidence for permanent study and reference. The* computer *analyzes data and models with a thoroughness and speed otherwise impossible.*

One of the most important of scientific instruments is the spectroscope. *Spectroscopy is the scientific development of the simple fact of observation that light has different colors. In essence, the spectroscope is a sophisticated development of the triangular glass prism which* ***Isaac Newton*** *used in* 1666 *to discover that "white" light is a combination of all other colors.*

Thomas Melville *(1752), observed that a flame in which a salt or metal was placed, gave out a spectrum of bright lines whose pattern varied with the material used; however, it was not until* 1823 *that* ***John Herschel*** *made the seemingly obvious suggestion that the technique might be applied to chemical analysis. Meanwhile,* ***Whollaston*** *(1802) had observed that the solar spectrum contained a number of dark lines.* ***Fraunhofer*** *(1814) perfected the spectroscope by using several prisms to increase the dispersion of light, substituting a slit for a hole, and using a telescope for observation. He also made the first diffraction gratings by ruling glass plates with diamond points. As a result of these improvements he discovered many hundreds of dark lines in the solar spectrum — still known as* Fraunhofer lines *— mapped them carefully, and noticed that certain lines varied from star to star.*

In 1859, *the two types of observation, terrestrial and astronomical, made contact.* ***Bunsen*** and ***Kirchhoff*** *showed that a double line — the D line — in the solar spectrum was due to the element sodium and concluded that it must therefore be present in the sun; and by similar methods that lithium could be present, if at all, only in quantities too small for them to observe. Now it became possible to determine the constituents of objects hundreds of millions of miles away in space! In the course of their investigations, the collaborators discovered two new elements,* caesium *and* rubidium.

In 1868, ***Lockyer*** *and* ***Frankland*** *went further still. Observing a hitherto unknown line in the sun's spectrum, they attributed it to an undiscovered element which they named helium. In* 1895, ***Ramsey*** *discovered the new element as a constituent of the mineral* cleveite. *A thorough history of spectroscopy would include numerous other scientists, such as* ***Anders Jonas Angström*** *(1814–1874, Sweden) who worked out accurate measurements of wavelengths;* ***Henry Augustus Rowland*** *(1848–1901, U.S.A.) who invented the concave diffraction grating; and* ***George Ellery Hale*** *(1868–1938, U.S.A.) whose spectroheliograph offered a new technique for observation of the sun. The discoveries made possible by these developments were startling. Among other achievements, they contributed to the founding of the modern science of astrophysics.*

Prior to ***Bohr*** *(1913), physicists failed to interpret the physical mechanisms underlying discrete spectra. In the first place, they did not conceive the possibility of discrete states for an atom. They simply assigned dipole radiation due to orbital revolutions of the electron, and identified the frequency of this revolution with the frequency of the emitted light. Consequently, they attributed the whole spectrum simultaneously to each atom.*

1914 CE **Adrian Daniel Fokker** (1887–1972, Holland). Dutch physicist. Derived the Fokker-Planck equation for Gaussian-Markov processes[310]. Collaborated with Einstein on his first treatment of general relativity.

[310] Consider unrestricted random walk on a discretized x-axis, with spacing Δx and discrete time-step Δt, starting at $x = 0$, at $t = 0$ with probabilities p, $q = 1 - p$ of a transition at any stage of one unit to the right or left. Let $u(x, t)$ be a *probability* that the particle be at coordinate x at time t. In this Markov process $u(x; t + \Delta t) = pu(x - \Delta x; t) + qu(x + \Delta x, t)$, since the coordinate x can be reached either from the left *or* the right. The boundary conditions are $u(0, 0) = 1$, $u(x, 0) = 0$, $x \neq 0$. The particle makes $[t/\Delta t]$

Fokker was born in Buitenzorg, Dutch Indies. He received his Ph.D. in 1913 under **Lorentz**. His thesis dealt with Brownian motions of electrons in a radiation field and contained an equation which later became known as the *Fokker-Planck equation.* After this work was completed, Lorentz sent Fokker to Zürich to work with **Einstein**. Their collaboration led to a brief paper, which contains Einstein's first treatment of a gravitation theory in which general covariance is strictly obeyed. In later years, Fokker wrote several papers on relativity. He held a professorship at Leyden.

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independent steps during time t, each taking the value Δx or $-\Delta x$ with probabilities p and q respectively. Each step thus has a mean $(p-q)\Delta x$ and variance $4pq(\Delta x)^2$. The overall mean and variance of the displacement in time t are therefore approximately $t(p-q)\frac{\Delta x}{\Delta t}$ and $4pqt\frac{(\Delta x)^2}{\Delta t}$, respectively. If the mean and variance are to remain finite in the continuum limit $\Delta x \to 0$, $\Delta t \to 0$, we must have $(\Delta x)^2/\Delta t = O(1)$, $p-q = O(\Delta x)$. These conditions will be satisfied if $\frac{(\Delta x)^2}{\Delta t} = 2D$, $p = \frac{1}{2} + \frac{c}{2D}\Delta x$, $q = \frac{1}{2} - \frac{c}{2D}\Delta x$ where $2c$ and D are constants with the interpretations of *drift velocity* and *diffusion coefficient*, respectively. The mean and the variance are now given in the limit by $2ct$ and $2Dt$ respectively. Expanding the Markov equation by means of Taylor's theorem yields $\frac{\partial u}{\partial t}\Delta t = \frac{\partial u}{\partial x}(q-p)\Delta x + \frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!}$, neglecting terms on the left of order $(\Delta t)^2$ and on the right of order $(\Delta x)^3$. The limiting form then yields the *Fokker-Planck equation*

$$\frac{\partial u}{\partial t} = -2c\frac{\partial u}{\partial x} + D\frac{\partial^2 u}{\partial x^2}$$

for diffusion with drift. In this continuum limit, $u(x,t)$ becomes a probability *density.* It has the particular solution

$$u(x,t) = \frac{1}{\sqrt{4\pi Dt}}e^{\frac{-(x-2ct)^2}{4Dt}}.$$

For $c = 0$ one finds

$$\int_{-\infty}^{\infty} u(x,t)dx = 1, \quad \int_{-\infty}^{\infty} xu(x,t)dx = 0 \ \ \textit{and} \ \ \int_{-\infty}^{\infty} x^2 u(x,t) = 2Dt = \langle x^2 \rangle.$$

If one interprets the single particle probability density $u(x,t)$ as the number of particles per unit volume around x at time t, the Fokker-Planck equation then becomes *Einstein's diffusion equation* for the *Brownian motion* with $\langle x^2 \rangle = 2Dt$, where $2D = \frac{kT}{3\pi\eta a}$. Today we could say that in 1905, Einstein treated diffusion as a Markovian process, thereby establishing a link between the random walk of a single particle and the diffusion of an ensemble of Brownian particles.

1914 CE **Edgar Buckingham** (1867–1940, U.S.A.). Physicist. Developed the present basic philosophy of the nature of dimensions, and the algebraic theorems[311] involved in the applications of dimensional analysis[312]. These ideas were presented in his paper: *On Physically Similar Systems* (*Phys. Rev.* **4**, 345, October 1914), which he authored while working at the National Bureau of Standards.

1914 CE **James Franck** (1882–1964, Germany and USA) and **Gustav Ludwig Hertz** (1887–1979, Germany). Physicists. Confirmed experimentally Bohr's atomic theory, for which they received the Nobel Prize for physics (1925); in their simple experiment (1914) they rendered one of the most striking proofs of the existence of stationary atomic states by means of *inelastic collisions* between electrons and atoms. In this process, part of the kinetic energy of the electron is transferred as internal energy of the target atom.

Their experimental arrangement was thus: a heated filament F emits electrons which are accelerated toward the grid G by a variable potential V. The

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311 *Buckingham's Pi Theorem*: If a physical phenomenon involves N_1 variables, and if these variables can be expressed in terms of N_2 fundamental dimensions, the physical law describing the phenomenon can be expressed as a function of $(N_1 - N_2)$ dimensionless products of the variables, called "Pi terms" (π).
Stated as an equation, $F(\pi_1, \pi_2, \pi_3, \ldots, \pi_n) = 0$, where $n = N_1 - N_2$.

312 The concept of *dimensional analysis* can be traced back at least as far as 1822, when **Joseph Fourier** used it in his *Theorie Analytique de la Chaleur*. Various scientists, notably **Osborne Reynolds** and **Lord Rayleigh** made use of some form of dimensional technique in the latter half of the 19^{th} century. All physical quantities can be reduced to combinations of basic properties which are referred to broadly as dimensions. The dimensions M (mass), L (length), and T (time) are sufficient for all branches of mechanics. In electrical and electromagnetic measurements, it is necessary only to add one dimension — that of electric charge Q — to the M, L, and T dimensions of mechanics in order to create a system which will provide dimensional combinations for all electrical and magnetic entities. In thermal problems, we need to add only the dimension of temperature — symbolized usually by θ — to M, L, and T.
Specifically, *dimensional analysis* involves the deduction of information about a physical phenomenon from the premise that it can be described by an equation defined only dimensionally and relating the pertinent dimensional variables in a *homogeneous*, linear algebraic form (although dependencies on dimensionless quantities can be much more complicated). The analysis entails the formation of complete sets of *dimensionless* monomials of the variables. These products are often called *numbers* and, because of their frequent use in engineering analysis and physical theory, have come to be recognized by the names of their inventors — **Reynolds**, **Cauchy**, **Prandtl** etc.

space between F and G is filled with mercury vapor. Between the grid G and the collecting plate P a small retarding potential V', of approximately 0.5 Volt, is applied so that those electrons which are left with very little kinetic energy after one or more inelastic collisions, cannot reach the plate and are not registered by the galvanometer. As V increases, the plate current I fluctuates, the peaks occurring at a spacing of about 4.9 Volts. The first dip corresponds to electrons that lose all their kinetic energy after one inelastic collision with the mercury atom, which is left in an excited state. The second dip corresponds to those electrons that suffered two inelastic collisions with two mercury atoms (one with each) losing all their kinetic energy, and so on.

The excited mercury atoms return to their ground state by emission of a photon according to $\mathrm{Hg}^* \rightarrow \mathrm{Hg} + h\nu$ with $h\nu = E_2 - E_1$. From spectroscopic evidence we know that mercury vapor, when excited, emits radiation whose wavelength is 2536 A, corresponding to a photon of energy $h\nu$ equal to 4.86 eV. Radiation of this wavelength is observed coming from the mercury during the passage of the electron through the vapor.

Thus, the quantized energy transfer from kinetic energy to electromagnetic light energy was demonstrated. It proved the reality of the energy quantum, postulated by **Planck** (1900), adopted by **Einstein** (1905) and modeled by **Bohr** (1913).

James Franck was born in Hamburg to a Jewish rabbinical family. He received his Ph.D. at the University of Berlin (1906) where he met his younger colleague, Gustav Hertz (1911). He was a professor at Göttingen (1920–1933).

His position as the leading experimental physicist in Germany, a holder of a Nobel Prize and his active military duty as an officer in the German army in WWI meant nothing to the Nazis. What counted was his "non Aryan" status. He left Germany (1933) and emigrated to the United States (1935), settling eventually in the University of Chicago (1938–1949). He died in Göttingen.

Gustav Hertz was also born in Hamburg to a half-Jewish family; his uncle was **Heinrich Hertz**, the physicist who demonstrated the existence of electromagnetic waves. He studied mathematics and mathematical physics under **Hilbert** at Göttingen and under **Sommerfeld** in Munich and received his Ph.D. in experimental physics in Berlin (1911).

During WWI, Hertz was gravely wounded and returned to academic life only in 1925 at Halle (1925–1928). He then moved to Berlin (1928–1945) and when the Nazis came to power he refused to take the loyalty oath and was removed (1934) from his position — to them he was a Jew. He became the chief physicist of the Siemens concern. After WWII he went to the Soviet Union (1945–1954) to help build the atom bomb for Stalin. He returned to East Germany (1954–1961) to the University of Leipzig.

1914–1917 CE **Ernest Dunlop Swinton** (1868–1951, England). Military engineer, historian and inventor. Invented and designed the first *tank* — a motor driven combat vehicle enclosed in an armor plate, mounted on heavy caterpillar tracks and carrying heavy and light weapons, such as cannon, machine guns, and flame throwers.

Swinton took part in the South African war as commander of the 1st railway pioneer regiment (1904). After the outbreak of WWI he went to France as the official military correspondent. His proposals (1914) became the first link in the evolution of the tank. He became the secretary of the war cabinet (1915) and had much to do with the preparation of the first tank. Appointed (1925) professor of military history at Oxford University.

The prototype '*Little Willie*' was built with the collaboration of Lieutenant **Walter Gordon Wilson** and **William Tritton** in Lincoln, UK. For secrecy's sake, they were called '*Water Carriers*', hence the word '*tank*'. The British first used the tanks in the *Battle of the Somme* (1916). These tanks were slow and clumsy (Mark I), but they terrified the Germans. By Nov. 1917 some 450 improved British Mark IV tanks attacked the Hindenburg Line at Cambrai. Finally, in the decisive *battle of Amiens* (22 August 1918), 580 tanks overrun the German lines and inflicted the coup de grace.

Tanks can cover any kind of ground. They can climb and descend slopes as steep as 35 degrees. They can travel 48 km/hour on level ground, and turn in their own length. They can travel in water and in land. Others can be carried to the front by air. Modern military tanks combine fast-moving attack with the fire-power of light artillery. Tanks are used against infantry, to destroy armored vehicles, and to demolish positions.

Leonardo da Vinci (1482) designed a tank-like vehicle.

1914–1919 CE **Felix Hausdorff** (1868–1942, Germany). Distinguished mathematician. Extended the work of Fréchet and introduced the concept of topological and metric spaces. In 1919 he discovered the '*Hausdorff dimension*', the forerunner of todays concept of '*fractal dimension*'[313]. He also worked in set theory and introduced the concept of *partially ordered set* (1906–1909).

Hausdorff was born in Breslau to a Jewish family. He studied mathematics and astronomy at Leipzig, Freiburg and Berlin, receiving his Ph.D. from Leipzig in 1891. Until the age of 35, he devoted most of his time to philosophy, poetry, writing, directing plays and similar endeavors. In 1902 Hausdorff

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[313] A *fractal* is by definition a set for which the Hausdorff-Besicovitch dimension strictly exceeds the topological dimension.

became associate professor at Leipzig. In 1910 he went to Bonn, where he wrote the monograph *Grundzüge der Mengenlehre*, which appeared in 1914.

Felix Hausdorff perished with his family in the Nazi Holocaust.[314]

1914–1920 CE **Srinivasa Ramanujan Aiyangar**[315] (1887–1920, India). An extraordinary mathematical genius who left a remarkable arithmetic legacy to posterity. His contribution to mathematics could have been much greater had he received a proper university education, and had his health not failed him at the age of 32. Few individuals in the annals of human intellectual endeavor have excited more admiration for their sheer genius and their achievements under adverse conditions.

An obscure and poor young Hindu, from a sequestered town in India, wrote a letter to **Godfrey Harold Hardy** (1877–1947), the leading English mathematician of the day. Hardy, accustomed to receiving crank mail, was inclined to disregard Ramanujan's letter at first glance the day it arrived, January 16, 1913. But after dinner that night, Hardy and a close colleague,

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[314] *Sins of omission*: Modern textbooks often hide from their readers certain gruesome biographical facts. Thus, in "*Elementary Linear Algebra*" (Wiley, 2000), the authors Howard Anton and Chris Rorrers write (p. 663):

"In 1919 the German mathematician Felix Hausdorff (1868–1942) defined the '*Hausdorff dimension*'..."

The fact that the Nazis exterminated Hausdorff in 1942 just for being a Jew AND IN SPITE OF HIM BEING a 'German mathematician' does not merit mention. Instead, the innocent reader is led to believe that Hausdorff died naturally in 1942. In this connection, it is of interest to mention that the Nazis denounced Hausdorff's dimension as being a '*fraudulent Jewish mathematics*' as opposed to their '*Pure Arian mathematics*'.

[315] For further reading, see:

- Ramanujan, S., *Collected Papers*, Cambridge University Press, 1927, 355 pp.
- Berndt, B.C., *Ramanujan's Notebooks*, Springer-Verlag: New York, 1985–1991, *Part I*, 357 pp., *Part II*, 510 pp., *Part III*, 510 pp., *Part IV*, 451 pp., Part V, 624 pp.
- Hardy, G.H., *Ramanujan*, Chelsea Publishing Company:New York, 1940, 236 pp.
- Kanigel, R., *The Man Who Knew Infinity (A life of the genius Ramanujan)*, Charles Scribner's Sons: New York, 1991, 438 pp.

John E. Littlewood, sat down to puzzle through a list of 120 formulas and theorems Ramanujan had appended to his letter: The letter betrayed signs of inadequate training, it was intuitive and disorganized, but Hardy recognized in it brilliant pearls of mathematics. Consequently he invited Ramanujan to London on a special fellowship in 1914. Through these efforts Ramanujan became known to the mathematical world.

One of the "pearls" which Ramanujan challenged Hardy to prove was the continued fraction [g is the *golden ratio*, $\frac{1}{2}(5^{1/2}+1) = 1.618\cdots$]

$$1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-4\pi}}{1+\cfrac{e^{-6\pi}}{1+\cdots}}} = e^{-\frac{2\pi}{5}}(5^{1/4}g^{1/2}-g)^{-1}.$$

Another magic result is the following: given the infinite continued fractions,

$$u = \cfrac{x}{1+\cfrac{x^5}{1+\cfrac{x^{10}}{1+\cfrac{x^{15}}{1+\cfrac{x^{20}}{1+\dots}}}}}, \quad v = \cfrac{\sqrt[5]{x}}{1+\cfrac{x}{1+\cfrac{x^2}{1+\cfrac{x^3}{1+\cfrac{x^4}{1+\dots}}}}}, \tag{1}$$

x can be eliminated (!) to tie u and v in the simple algebraic relation

$$v^5 = u\frac{1-2u+4u^2-3u^3+u^4}{1+3u+4u^2+2u^3+u^4}. \tag{2}$$

Nobody knows how Ramanujan arrived at this result. He was always fearful that English mathematicians would steal his secrets while he was in England. It seems that not only did English mathematicians not steal his secrets, but generations of mathematicians since have not discovered his secrets either.

Nine years after Ramanujan's death, G.N. Watson (1929), going through the late mathematician's notebooks, constructed a proof that Ramanujan *could have* concocted. Using certain known identities of Euler and Jacobi, plus new identities of his own, Watson reduced the problem to the relations

$$u^{-1}-1-u = x^{-1}\prod_{m=1}^{\infty}\left(\frac{1-x^m}{1-x^{25m}}\right); \quad u^{-5}-11-u^5 = x^{-5}\prod_{m=1}^{\infty}\left(\frac{1-x^{5m}}{1-x^{25m}}\right)^6;$$

$$v^{-5}-11-v^5 = x^{-1}\prod_{m=1}^{\infty}\left(\frac{1-x^m}{1-x^{5m}}\right)^6.$$

Of (1) and (2) Hardy commented: "*It defeated me completely; I had never seen anything in the least like it before. A single look at it is enough to show that it must be true because, if it were not true, no one would have had the imagination to invent it*".

Another result of Ramanujan,

$$\int_0^\infty e^{-3\pi x^2}\frac{\sinh(\pi x)}{\sinh(8\pi x)}dx =$$
$$= \frac{1}{e^{2\pi/3}\sqrt{3}}\sum_{n=0}^{\infty}\frac{e^{-2n(n+1)\pi}}{[1+e^{-\pi}]^2[1+e^{-2\pi}]^2\cdots[1+e^{-(2n+1)\pi}]^2},$$

prompted G.N. Watson to say: "*It gives me a thrill which is indistinguishable from the thrill which I feel when I enter the Sagrestia Nuova of the Capella Medici and see before me the austere beauty of the four statues representing* '*Day*', '*Night*', '*Evening*', *and* '*Dawn*', *which Michelangelo has set over the tomb of Giuliano dé Medici and Lorenzo dé Medici*".

Ramanujan, like his great predecessors Euler and Jacobi, was a grand master of summation and integration. Many of his unique results are still buried in his *Notebooks*[316]. These are personal records in which he jotted down

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316 Ramanujan arrived, for example, at the following bizarre identity: $\tan^{-1}\left\{e^{-\frac{\pi n}{2}}\right\} = \frac{\pi}{4} - \left\{\tan^{-1}\frac{n}{1} - \tan^{-1}\frac{n}{3} + \tan^{-1}\frac{n}{5} - \cdots\right\}$. He apparently worked his way *backwards* from a known infinite sum through integration term by term: $\frac{-\frac{\pi}{2}e^{-\frac{\pi n}{2}}}{1+e^{-\pi n}} = -\left[\frac{1}{1+(\frac{n}{1})^2} - \frac{1/3}{1+(\frac{n}{3})^2} + \frac{1/5}{1+(\frac{n}{5})^2} - \cdots\right]$. This can be recast as the well-known result: $\frac{1}{\text{ch}\, x} = 4\pi\sum_{\ell=0}^{\infty}\frac{(-)^\ell(2\ell+1)}{4x^2+(2\ell+1)^2\pi^2}; \qquad x = \frac{\pi n}{2}$.

Another example is: given $f(\lambda) = \sqrt{\lambda}\int_0^\infty \frac{e^{-x^2}}{\text{ch}\ \lambda x}\,dx$, to show that $f(\lambda) = f(\frac{\pi}{\lambda})$. This he proved as follows: since

$$\frac{1}{\text{ch}\,\lambda x} = 2\int_0^\infty \frac{\cos 2\lambda xz}{\text{ch}\,\pi z}\,dz,$$

then

$$\sqrt{\lambda}\int_0^\infty \frac{e^{-x^2}}{\text{ch}\,\lambda x}\,dx = 2\sqrt{\lambda}\int_0^\infty\int_0^\infty e^{-x^2}\frac{\cos 2\lambda xz}{\text{ch}\,\pi z}\,dx\,dz$$
$$= \sqrt{\lambda\pi}\int_0^\infty \frac{e^{-\lambda^2 z^2}}{\text{ch}\,\pi z}\,dz = \sqrt{\frac{\pi}{\lambda}}\int_0^\infty \frac{e^{-z^2}}{\text{ch}\left(\frac{\pi z}{\lambda}\right)}\,dz$$
$$= \sqrt{\frac{\pi}{\lambda}}\int_0^\infty \frac{e^{-x^2}}{\text{ch}\,\frac{\pi}{\lambda}x}\,dx.$$

his formulas, which embody subtle relations among numbers and functions. Others have had to compile, edit and prove them (Ramanujan did not bother to include formal proofs). One such result from his *Notebooks* is

$$\zeta(3) = 1 + \cfrac{1}{2.2 + \cfrac{1^3}{1 + \cfrac{1^3}{6.2 + \cfrac{2^3}{1 + \cfrac{2^3}{10.2 + \cdots}}}}}.$$

Ramanujan's unique capacity for working intuitively with complicated formulas enabled him to plant seeds that are only now coming into bloom: some of his results are being applied today in other fields and even in theoretical physics. Some of his conjectures have just recently been verified[317].

Hardy on Ramanujan

"He had been carrying an impossible handicap, a poor and solitary Hindu, pitting his brains against the accumulating wisdom of Europe".

"... a man whose career seems full of paradoxes and contradictions, who defies almost all the canons by which we are accustomed to judge one another, and

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317 *The tau conjecture*: In 1916, Ramanujan was studying the arithmetical function $\sigma_s(n)$, which denotes the sum of the s^{th} powers of the divisors of n (1 and n included). Through his efforts to calculate $\sigma_s(n)$, he was led to define the *tau function* $\tau(n)$ via the relation

$$\sum_1^\infty \tau(n)x^{n-1} = \left[(1-x)(1-x^2)(1-x^3)\cdots\right]^{24} = [1 - 3x + 5x^3 - 7x^6 + \cdots]^8.$$

He calculated the first 30 values of $\tau(n)$ [1; −24; 252; −1472; 4830; −6048; −16, 744; 84, 400; −113, 643; −115, 920; etc.] and conjectured that

$$\tau(n) = O(n^{\frac{11}{5}+\epsilon}).$$

This conjecture was proved only in 1974 by Pierre Deligne (b. 1944, Belgium), using tools supplied by algebraic geometry.

about whom all of us will probably agree in one judgment only, that he was in some sense a very great mathematician".

"He would probably have been a greater mathematician if he could have been caught and tamed a little in his youth; he would have discovered more that was new, and that, no doubt, of greater importance. On the other hand he would have been less of a Ramanujan, and more of a European professor, and the loss might have been greater than the gain".

"It was Littlewood who said that every positive integer was one of Ramanujan's personal friends[318]*. I remember going to see him once when he was lying ill in Putney. I had ridden in taxi-cab No.* 1729*, and remarked that the number seemed to me rather a dull one and that I hoped that it was not an unfavorable omen. "No", he replied, "it is a very interesting number; it is the smallest number expressible as a sum of two cubes in two different ways"* $(1729 = 12^3 + 1^3 = 10^3 + 9^3)$.

"It was his insight into algebraical formulae, transformation of infinite series, and so forth, that he was most amazing. On this side most certainly I have never met his equal, and I can compare him only with Euler and Jacobi... with his memory, his patience, and his power of calculation he combined a power of generalization, a feeling for form, and a capacity for rapid modification of his hypotheses, that were often really startling, and made him, in his own peculiar field, without a rival in his day".

[318] On this, Littlewood remarked:
I read in the proof sheets of Hardy on Ramanujan: "As someone said, each of the positive integers was one of his personal friends." My reaction was, "I wonder who said that; I wish I had." In the next proof-sheets I read (that now stands), "It was Littlewood who said...".

Ramanujan's Approximation to π

The theory of the modular functions had its origin in the pioneering works of ***John Landen***[319] *in* 1771–1775 *and* ***A.M. Legendre***, ***C.G.J. Jacobi*** *and* ***N.H. Abel*** *during* 1825–1829. *Later developments were due to* ***L. Kronecker*** *(1857–1863),* ***K.T.W. Weierstrass*** *(1860),* ***H. Weber*** *(1842–1913, Germany, 1881) and* ***F. Klein*** *(1890).*

Just how much of this 'accumulated wisdom of Europe' was known to Ramanujan in 1913, *is a great mystery. On this issue,* ***L.J. Mordell*** *surmised that the English textbooks by Greenhill and A. Cayley on the subjects of elliptic functions were easily accessible to him. But even so, his own innovations and intriguing results in the field of modular functions surpassed those of his predecessors.*

In 1914 *he developed new ways of calculating π with extraordinary efficiency. His starting point was the interesting result that for any given rational number n, the infinite product*

$$G_n = 2^{-\frac{1}{4}} e^{\frac{\pi\sqrt{n}}{24}}(1+e^{-\pi\sqrt{n}})(1+e^{-3\pi\sqrt{n}})(1+e^{-5\pi\sqrt{n}})\cdots$$

can always be expressed as the root of an algebraical equation. (Explicit values of G_n are known for many n's, e.g. $G_1 = 1$; $G_3 = \sqrt[12]{2}$; $G_5 = \left(\frac{1+\sqrt{5}}{2}\right)^{1/4}$; $G_7 = \sqrt[4]{2}$; $G_9 = \left[\sqrt[4]{2}(1+\sqrt{3})\right]^{1/3}$; $G_{13} = (3+\sqrt{13})^{1/4}$.)

The G_n's have the remarkable property that $G_n = G_{1/n}$, namely,

$$q^{-1/24}(1+q)(1+q^3)(1+q^5)\cdots = p^{-1/24}(1+p)(1+p^3)(1+p^5)\cdots$$

where $p = q^{1/n} = e^{-\pi/\sqrt{n}}$. For large values of n,

$$G_n = 2^{-\frac{1}{4}} e^{\frac{\pi\sqrt{n}}{24}}\left[1 + O(e^{-\pi\sqrt{n}})\right],$$

and hence $\pi \approx \frac{24}{\sqrt{n}}\log_e(2^{1/4}G_n)$, with an error of nearly $\frac{24}{\sqrt{n}}e^{-\pi\sqrt{n}}$. For[320] $n = 1225$, *this value of π is correct to 47 decimals. (39 places of π suffice*

[319] English surveyor (1719–1790) whose interest in mathematics was a leisure activity. Elected fellow of the Royal Society (1766).

[320] $G_{1225} = \frac{1+\sqrt{5}}{2}(6+\sqrt{35})^{1/4}\left\{\frac{7^{1/4}+\sqrt{4+\sqrt{7}}}{2}\right\}^{3/2}\left[\sqrt{\frac{A+43}{8}}+\sqrt{\frac{A+35}{8}}\right]$, where $A = 15\sqrt{7} + (8+3\sqrt{7})\sqrt{10\sqrt{7}}$.

for computing the circumference of a circle girdling the known universe with an error no greater than the radius of a hydrogen atom.)

When Ramanujan was dealing with these large values of n (for which no modular equation was even available), he must have been guided by his superb intuition. For otherwise, how could one produce an exact solution to an algebraical equation of degree 1225*, without even having the equation on hand? G.N. Watson believed that Ramanujan was in possession of general formulae by means of which he constructed his solutions.*

Although this evaluation is not without a certain charm, it is not in itself a milestone in the history of π for two reasons: Firstly, the method of approximating π by means of equations $e^{\pi\sqrt{n}} = m$ (where m is nearly an integer), was already known to ***C. Hermite*** *(1859); and secondly, the value of π was known correctly to* 527 *decimals, as early as* 1853*.*

But Ramanujan's paper[321]*, barely exposed the tip of the iceberg. In his Notebook*[322]*, he combined it with an iterative algorithm, through which π was calculated in* 1987 *on superfast computers, to many millions of digits*[323]

This algorithm led Ramanujan from the innocent-looking
$\pi \approx \frac{24}{\sqrt{n}} \log_e(2^{1/4}G_n)$ *to the formidable expression for π that appears in his Notebook, namely,*

$$\frac{1}{\pi} = \frac{\sqrt{8}}{9801} \sum_{n=0}^{\infty} \frac{(4n)![1103 + 26390n]}{(n!)^4(396)^{4n}}.$$

It was found that each successive term in this series adds roughly 8 *more correct digits [a pocket calculator yields for the first term $\pi = 3.141\ 592\ldots$].*

321 *Quart. J. Math.* **45** (1914) 350–372.

322 1957, Tata Institute, Bombay, India.

323 J.M. Borwein and P.B. Borwein, *Scientific American*.

Science Progress Report No. 14
World-War I (July 28, 1914 – Nov. 11, 1918) — The Inauguration of Chemical Warfare

The era of the splendor and glory of Europe came abruptly to an end: two pistol shots signaled the start of a war that ended four years later. The 'central powers' of Austria-Hungary, Bulgaria, Germany and the Ottoman Empire fought and lost against the 'allied forces' of Belgium, Brazil, the British Empire, China, Costa Rica, Cuba, France, Greece, Guatemala, Haiti, Honduras, Italy, Japan, Liberia, Montenegro, Nicaragua, Panama, Portugal, Romania, Russia, San-Marino, Serbia, Siam and the United States.

65 *million servicemen on both sides were engaged in the war. By the end of hostilities,* 9 *millions had died,* 20.5 *millions were wounded and* 7.2 *millions were prisoners or missing in action; among the dead — two million Frenchmen (of a population of* 38 *million) and over two million Germans (of a population of* 70 *million). This large-scale slaughter*[324] *wiped out a whole generation, among them the elite and the many geniuses in all fields. In addition,* 5 *millions civilians*[325] *died in areas of actual combat and about* 30 *million people*

[324] The appalling bloodbaths of WWI were augmented by the introduction of the first truly *automatic machine-gun*, invented in 1884 by **Hiram Stevens Maxim** (1840–1916; USA and England). It spewed more than 600 rounds a minute. The British learned its devastating power the hard way, loosing 60,000 men on the first day of the *Battle of the Somme* (July 01, 1916).

The 19^{th} century officers and commanders were accustomed to thinking in terms of human intrepidity and courage as the most important attributes to carry the day in the battlefields. Machine-guns were the first specific application of the technique and logic of the industrial revolution in military combat. Firing an inordinate stream of bullets, machine-guns came to be the definitive symbols of the machine age in military history, regardless of marksmanship or easy targets. Nevertheless, ingrained beliefs die hard. The militaries in all major powers continued to cling to the idea of the irreplaceability of the infantry and cavalry charges, with bayonets, swords and lances, as the final judge of victory or defeat in military matters.

The first *mechanical-gun* (hand-turned) was invented in 1861 by **Richard Jordan Gatling** (1818–1903, USA), firing 616 shots in two minutes. This gun was used late in the *American Civil War* and by the British army in the *Matabele War* (1893–1894), where 50 soldiers fought off 5,000 Zulu warriors with just four Maxim guns.

[325] The historian Eric Hobsbawm ("Age of Extremes", 1994) wrote:

"The first World War led to the killing of an uncounted number of Armenians *died in India, Europe, USA and South Africa of the Spanish influenza epidemic that broke out in 1918 as a direct result of undernourishment and lack of coal caused by the war*[326]. *Costs of the war (both direct and indirect) amounted to 337 billion dollars.*

In 1919, Austria-Hungary was no more; Vienna, once the imperial center of 50 million subjects of mixed race, was now a capital of a small, impoverished and insignificant Alpine republic of little more than six million, mostly German, inhabitants[327].

*WWI interfered with the full implementation of the momentous discoveries of the turn of the century. Also, among the war casualties were scientists, artists and poets of the first rank (**Franz Marc**, **Egon Schiele**, **Rupert Brooke**, **Henri Gaudier**, **Karl Schwarzschild**[328], **Henry Gwyn-Jeffreys Moseley**).*

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by Turkey – the most usual figure is 1.5 millions – which can count as the first modern attempt to eliminate an entire population."

326 This pandemic was one of the worst disasters in history! Influenza deaths tolled as many as 650,000 in the United States alone. World War I killed 15 million in four years, flu killed twice as much in six months. Even bubonic plague did not kill so many people so fast. The word "influenza" entered English from Italian in 1743. It means influence of the stars. Individual pandemics have often been named for their supposed origins, sometimes accurately (Asian flu) and sometimes not (Spanish flu). The infection's ultimate origin is farm animals (pigs, ducks and horses). Human flu probably dates no further back than their domestication in the period 2000–5000 BCE. The virus owes much of its biological success to surface mutation that occur as it shuttles between humans and various domesticated animals.

The first pandemic probably occurred in the 16th century; there were 5–10 more in the 18th and 19th centuries. Most of these began in Russia and Central Asia and traveled by land and by ship, even to remote Pacific Islands. The pandemic of 1833 was especially virulent in European cities. The one of 1889 moved with the speed of trains and steamships. The mortality rate is usually 0.01 percent, mostly from ensuing pneumonia.

327 In 1919 the majority voted for *Anschluss* with Germany. The option was denied them by the Allies, who also, through the war reparations demanded by the Treaties of Versailles and St. Germain, ensured that the German people of both German states would remain poor, resentful and revengeful throughout the inter-war period.

328 100,000 Jews fought on the German side, 80,000 of them in the trenches. Of these, 12,000 died for the '*Vaterland*' and 35,000 merited decorations [as compared with 7000 Jewish patriots who fought in the 19th century against France and Austria]. Among the slain were many university teachers and students. The greatest War Ace of WWI, **Baron Manfred von Richthofen** (1892–

WWI was a scientific disaster for France: more than half the students of École Polytechnique and École Normale Supérieure were killed (among them the two sons of Jacques Hadamard and the son of Emile Durkheim).

The major reason why there was little movement on many fronts during the first world war was that defensive strategy and weapons were better developed than those of an offensive nature.

The opposing armies found it better to stay put than to attempt to advance. When an army did attempt to advance, the defensive capabilities of the opposing army meant that massive casualties would occur for the sake of a few hundred meters gain of ground[329].

This situation beget chemical gas warfare, intended to force soldiers out of their trenches: on April 23, 1915, the Germans first used poison gas *(chlorine) in the Second Battle of Ypres. They then used the more potent* mustard gas *(1917) in the Third Battle of Ypres. About 100,000 perished by gas during the war*[330].

The alliance between the natural sciences and the military has a tradition of at least 2200 years, if one can believe the stories of the terrifying stone

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1918), who shot down 80 enemy warplanes, was of Jewish descent. This did not prevent the German generals who lost the war to blame the defeat on a "stab in the back" by the Jews. But the fact was that the German armed forces had mutinied, unwilling to die for either Kaiser or *Vaterland*! The *Wehrmacht* mutiny did not begin with exhausted soldiers on the front, but with sailors resting in Kiel, where the German Navy had been tucked away ever since its inglorious defeat by the British in 1916. There were no Jews on the decks of those German warships, only Arian Germans hoisting the Red flag of rebellion that spread through German military columns. Even the Kaiser's most trusted regiments deserted. Not even the Kaiser was willing to die for the *Vaterland*; In the dark of the night he slipped across the frontier of Holland, begging asylum, and Germany sued for peace.

[329] By the time World War Two had arrived, the offensive capability of the military had overtaken the defensive capabilities of the opposing army, allowing new tactics such as the Blitzkrieg, (lightning war).

[330] The following gases were also used: Acrolein, Benzyl Bromide, Blue-Cross Gas, Bromacetone, Bromo-Benzyl-Cyanide, Chloropicrin, Cyanogen Bromide, Cyanogen Chlorine, Dimethyl Sulphate, Diphenylchlorarsine, Ethyldichlorarsine, Green Cross Gas, Phenyldichlorarsine, Phosgene, Xylyl Bromide. *For example*, Dimethyl Sulphate, $(CH_3O)_2SO_2$, known also as $DMSO_4$, is a colorless liquid having a vapor with faint onion-like odor. It is extremely toxic — a few whiffs could be fatal. Corrosive to tissues and carcinogenic. Lethal concentrations as low as 97 ppm/10 minutes have been reported in humans.

slings and burning mirrors which ***Archimedes*** *is said to have built for the defense of Syracuse against the besieging Romans (215 BCE). Clearly, war was a stimulus in the development of modern physics: the study of parabolic trajectories by* ***Niccolo Tartajlia*** *and* ***Galileo Galilei*** *stemmed from the need to calculate the trajectories of cannonballs.*

Fritz Haber *had inaugurated chemical warfare, to be used subsequently by Italy in Ethiopia (1935–1936), by the Nazis in the gas chambers of Poland (1942–1945), by Egypt in Yemen (1962–1965) and by Iraq in Iran (1980–1987).*

WWI showed that science could play an important role in the outcome of a war. In Germany, more than 100 laboratories were involved in scientific research for the military. During the years after the war, governments east and west started actively funding science, thus speeding up the growth of applied science during the 20^{th} *century, especially in the United States.*

The rapid growth of German chemical industry occurred during the 19th century. It led her to become the world's greatest chemical power. The German solution to industrial scale up involved teaming research chemists and mechanical engineers to take a reaction from the laboratory bench to the factory floor. They believed that this allowed the research chemist to remain creative by not being tied down with the drudgery of engineering practice. Because of this method the chemical engineer *was entirely unneeded, being instead replaced by a chemist and a mechanical engineer.*

Thus, prior to WWI, Germany had reigned supreme in organic chemistry and chemical technology. It was said that in 1905 America lagged 50 years behind the Germans in organic chemical processing. Even America's chemistry and chemical engineering professors had been primarily trained in German universities, and a working knowledge of the German language was essential to keep up with the latest chemical advances. All in all, America's chemical industry was very narrow, concentrating in only a few high volume chemical products, such as sulfuric acid[331]*.*

[331] American Chemical Society (ACS) was organized in 1876. However, the *American chemical industry* was *fundamentally different* from the German industry in several ways. Instead of specializing in fine chemicals or complicated dyestuffs (often made in batch reactors, something all chemists are familiar with), the American industries produced *only a few simple but widely used chemicals* such as sulfuric acid and alkali (both made in continuous reactors, something chemists have little experience with). These bulk chemicals were produced using *straightforward chemistry*, but required *complex engineering* set on vast scales. American chemical reactors were no longer just big pots, instead they involved complex plumbing systems where *chemistry and engineering were*

When the war started in Europe in 1914, only 528 workers were employed in U.S. plants producing such coal-tar chemicals as dyes and drugs. America was importing more than 90 percent of its dyes, mainly from Germany. There was not one U.S. plant for extracting nitrogen from the air and transforming it into the chemicals so vital for the armed forces, agriculture and industry in general. The U.S. depended on Chile for natural nitrates used in fertilizers and explosives. Starting almost from scratch to gain independence from foreign producers during the war, the U.S. chemical industry moved to a position of world leadership in the years thereafter.

After the war, many large corporations established laboratories in which scientists were encouraged to seek knowledge for its own sake without worrying about whether their work would be of practical use. Most advances in more abstract fields, such as physics, however, continued to be made in university environments.

1914–1921 CE About 3 million people in Europe die of *typhus*[332]. At the start of WWI (Nov 1914 – April 1915) it wiped out 150,000 soldiers in Serbia,

inseparably rolled together. Because of this, the chemistry and engineering aspects of production could not be as easily divided as they were in Germany. The *chemical engineer therefore found a role to play in America* despite their absence in the German system.

The *American chemical industry* (initially following the German example) *employed chemists and mechanical engineers* to perform the functions that would later be the chemical engineer's specialty. However these chemists were of an entirely different nature. The *prominent research chemists* employed in Germany were almost non-existent in *America* until after World War I. *Instead* the American chemical industry employed *analytical chemists* (involved in materials testing and quality control) and a few *production chemists*.

With the goal of claiming their industrial territory, the American Institute of Chemical Engineers (AICHE) was formed in 1908 to bridge the gap between laboratory processes and full-scale industrial production.

332 In the years 1490–1920 typhus had killed more people than armies had. Bacteriologist **Hans Zinsser** (1878–1940, USA) wrote:

"Soldiers have rarely won wars. They more often mop up after the barrage of epidemics. And typhus, with its brothers and sisters — plague, cholera, typhoid, dysentery, — has decided more campaigns than Caesar, Hannibal, Napoleon and the inspectors general of history. The epidemics get the blame for defeat, the generals get the credit for victory. It ought to be the other way around."

virtually removing that region from the war. Typhus later spread through eastern Europe, accelerating with the collapse of civil order in Russia. From 1917 through 1921, it infected 20 million Russians and killed 2.5–3 million. At the height of the epidemic, Lenin declared, “Either socialism will defeat the louse or the louse will defeat socialism”. The louse lost, but it had been a close call.

Public health, hygiene and social developments prevented further outbreaks in the Soviet Union during WWII. However, Italian soldiers returning from North Africa and Sicily brought the disease to Naples (1943). The disease was also common in many of the Nazi concentration camps.

H. da Roche Lima isolated the causative bacteria (1916) and named it after the American **Howard Taylor Ricketts** who died while investigating the disease. The epidemic spread by the human body louse. Crowding, uncleanliness, and human misery (malnutrition and sordid living conditions) during wartime favor the transfer of the infection from one person to another.

1914–1955 CE **Beno Gutenberg** (1889–1960, Germany and USA). Geophysicist. Made the first correct determination of the size of the earth’s core (1914), which he concluded to be liquid[333].

He deduced the existence of a global low-velocity zone in the earth’s upper mantle (1955) from the analysis of travel-times and amplitudes of seismic waves.

Gutenberg was born in Darmstadt, Germany to Jewish parents and gained his doctorate from Göttingen University (1911). Professor, Frankfurt (1926–1930); to U.S. (1930); professor, California Institute of Technology (1930–1957).

1915–1917 CE **Frederick William Twort** (1877–1950, England) and independently **Felix d’Herelle** (1873–1949, Canada) discovered *bacteriophage*[334] (viruses that attack bacteria).

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[333] The existence of some sort of core had already been deduced by **Richard Dixon Oldham** (1906). In 1936 **Inge Lehman** produced the first evidence of the existence of an *inner solid core* with a radius of ca 1400 km.

[334] A fictional but relatively accurate account of the discovery of bacteriophages and of efforts to use them to treat plague epidemic is included in *Arrowsmith*, a novel by **Sinclair Lewis** (1885–1951, USA). As in Lewis’ story, phage therapy did not prove effective.

Twort, a bacteriologist, superintendent of Brown Animal Sanatory Institute, London (1907–1944), was first to describe a bacteriophage (1915).

d'Herelle, a microbiologist, conducted field studies in Guatemala and Mexico (1901–1909) and later worked as assistant at Pasteur Institute, Paris (1909–1921), professor at Leiden (1921–1923), director of Bacteriological Service, Egypt (1923–1927), and professor at Yale (1928–1934). He discovered and *named* the bacteriophage (1916–1917); Author of *Le Bacteriophage* (1921).

1915–1919 CE **James Hopwood Jeans** (1877–1946, England). Mathematician, theoretical physicist and astronomer. Produced a theory for the formation of galaxies and clusters of galaxies by *gravitational instability* ('Jeans instability'): small, large-scale density fluctuations are enhanced and finally result in the formation of a star, a galaxy or even a cluster of galaxies. The minimum mass required for the onset of instability is the 'Jeans mass' with a 'Jeans radius' (1915).

In 1919 he applied the collisionless Boltzmann transport equation to galactic dynamics, in particular for models of evolution of spherical stellar systems.

Jeans was first to propose that matter is continuously created throughout the universe. His work included investigations of spiral nebulae, the source of stellar energy, binary and multiple star systems, and giant and dwarf stars. He also analyzed the breakup of rapidly spinning bodies under the stress of centrifugal force and concluded that the nebular hypothesis of Laplace, which stated that the planets and sun condensed from a single gaseous cloud, was invalid. He proposed the catastrophic or tidal theory, according to which a star narrowly missed colliding with the sun and, in passing, drew away from the sun stellar debris that condensed to form the planets. (However, Laplace's hypothesis now appears to have been correct.)

Jeans was born in London. He taught at Cambridge (1904–1905, 1910–1912) and Princeton (1905–1909). During 1923–1944 he stayed at Mt. Wilson Observatory, Pasadena, California.

History of Creation Theories — I (2500 BCE–1916 CE)

In cosmology an attempt is made to answer several questions. How is the universe built up as a whole? Do the laws of nature, which we derive from experience gained in our 'neighborhood' — be it the earth, solar system or the galaxy — remain applicable if we imagine this 'neighborhood' extended until it comprises the whole universe? is an infinite world *compatible with the laws of nature, or must we restrict the universe to a finite size if we wish to avoid insurmountable difficulties of principle?*

Cosmology has to face the problem of infinity not as a mathematical abstraction but as physical reality. This has always conferred on all its problems a particularly speculative character and, at the same time, a particular attraction. Through the history of science men were anxious to obtain the answer to these questions.

Prologue: Epoch of Early Myths *(2500–600 BCE)*

The earliest creation myths that we know of were written down in Mesopotamia and Egypt about 2500 BCE. These civilizations were based on extensive irrigation agriculture, organized by a centralized priesthood headed by an all-powerful and divine king. The creation stories tell of these societies' origin, how their people had organized the lands (between the Tigris and the Euphrates rivers and in the Nile valley) by literally separating the earth from the waters by channeling swamps into canals. It was this channeling which superseded the chaos of agriculture dependent on fickle and sparse rains. The priests gave the credit for this vast social enterprise to the gods.

According to these myths the task was accomplished not with reason and planning, but by fertility-based magic; *creation is a magical-biological reproduction: gods emerge from a primeval ocean and mate with one another to produce additional deities — the earth, the sky, the heavens, and the oceans.*

Thus, although the earliest civilizations developed essential inventions such as metallurgy, writing, arithmetic, geometry, and astronomy — their societies had little use for reason. And so, once these agricultural improvements were instituted by neolithic farmers or by the first priesthoods that organized the irrigation works, these societies persisted without further technical advance for over 1500 years. The social organization set up to create the irrigation *works — a king and priesthood directing the works of thousands of peasants — itself prevented further progress.*

The peasants who grew food and the artisans who worked in royal workshops were totally isolated from the literate priesthood, who held absolute power. The material traditions of peasants and artisans, and the scientific knowledge of the priesthood, separated from each other, were passed unchanged from generation to generation mystified by ritual and magic. So the myths of the priests gave divine sanction to the workings of society. The kings and priests inherited magical powers from the gods and this justified and enforced their absolute power over society. Magic and ritual ruled here on earth, and so it must have been in the heavens, in the beginning[335].

Act I: The Greek Secular Infinite Universe — Observation vs. Pure Reason *(600–150 BCE)*

With a fixed technical basis, the priestly authorities of the Bronze Age civilization could support its increasing population only by expanding geographically, and when the natural limits of cultivation within the alluvial valleys were reached — it began to collapse. The efforts of kings and pharaohs to squeeze more wealth out of a stagnant production systems led to rapid depletion of the population, decay of irrigation works, and finally the disintegration of society.

Egypt and the Near East, however, gave rise to a new society which sprang into existence out of the ruined shell of the old. The new society brought with it new technology related to new perceptions of the cosmos. It required new ideas, because it was based on trade *and, in part, on* free labor*. While reliance*

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[335] The *Hebrew* account of creation as reflected in *Genesis*, recalls Babylonian, Ugaritic and Canaanite cosmogonies. [From *Isaiah II* (**40**, 26–28; **42**, 5; **45**, 7, 12, 18; **54**, 9) we gather that the Biblical creation narrative must have been *canonized* by 550 BCE at the latest. In fact, some modern Biblical scholars believe that these texts had previously been *written* by the scribes of the early Judean kings.] But while most non-Biblical myths deal with gods and goddesses who take sides in human affairs, each favoring rival heroes — the Bible acknowledges only a single universal God who created the heavens and the earth by *organizing a preexisting chaos*. (The Bible nevertheless still harbors vestigial accounts of ancient gods and goddesses — disguised as men, women, angles, or demons.) In contradistinction to the *Platonic* creation stories (ca 400 BCE), Genesis demotes the sun and the moon to mere functional objects, created in the same manner as the earth.

on authority may suit a priesthood, it is a poor guide for an enterprising trader or craftsman. Instead, the merchant had to learn by observing the world around him — the winds and tides. And the free craftsmen learned by changing nature, by experimenting with new materials and methods.

The strict division between those who learned and those who worked began to break down; learning was democratized to serve the needs of independent merchants and artisans. The economical Phoenician alphabet superseded the elaborate hieroglyphics and cuneiforms of the ancient priests. Nowhere were the changes so thorough as in the trading colonies established by Greeks in Ionia. As is generally the case with colonies, the inherited social patterns were left behind in favor of a more adventurous setup.

By 700 BCE, *the Ionian trading cities, increasingly dependent on trade in specialized agriculture and craft products such as textiles, had thrown off the earlier subordination to the great landowners of mainland Greece. They established new societies of traders, craftsmen, and free-holding peasants — the first limited attempts at democracies and republics. They needed new ideas to run such new societies.*

This change is evident in the Ionian conception of the universe and its origin. Around 580 BCE ***Thales****, a native of the trading and textile center of Miletos, first asserted that the world was formed by natural processes which could be* observed *in the world. He secularized the old creation myths — natural processes without divine intervention.*

While Bronze Age priests had seen a static society ruled by the unchanging cycles of the seasons, the Ionian saw a society in the midst of convulsive changes as aristocratic landholders, merchants, artisans, and peasants battled for power. Having experienced tumultuous overthrows of government, ***Heraclitos*** *(ca 500 BCE) concluded that the universe was in constant flux, like a fire, ever changing. After Ionia was conquered by the Persians, the new ideas spread to Athens in mainland Greece. Here some of the most striking theories of early cosmology were born.*

Anaxagoras *(ca 460 BCE) derived his theory of origins from close observations of nature: Seeing how whirlpools in nature order the chaotic flow of water and separate materials of different densities — mud and wood are drawn to the center, while stones and pebbles are flung outward — he reasoned that such vortices, driven by primeval power, could separate by air from the earth. The sun and stars would have been torn loose from earth, flung outward, and heated by friction to their present fiery state. Stars, he correctly guessed, are suns too far away from us to feel their heat*[336]*. Then from observations of*

336 In his own words (ca 450 BCE): "*The formation of the world began with a vortex, formed out of chaos by energy. This vortex started at the center and*

whirlpools, the glowing hot metal of the blacksmith's forge, the distant light of merchant's signal fires — he hypothesized a naturalistic theory of cosmic origins which was essentially correct in its broad outline.

In Anaxagoras' view the universe is infinite, populated by a host of different worlds — many of them inhabited. His cosmos, imagined by extrapolation of his earthbound observations to remote parts of the universe, was unlimited in space and time. Thus, the Ionian school of Thales, Democritos and Anaxagoras assumed a world knowable by observation, where thought and work joined together. It was the world view of the free craftsman and peasant. Knowledge was available to all.

On the other hand, Plato's doctrine described a cosmos knowable only by pure reason of the few — who thus had the right to rule over the many, even as the heaven rules earth, the soul rules the body, or as the master rules the slave. According to the cosmology of ***Plato*** *(ca 385 BCE) the creator molded preexisting, chaotic matter into approximations of ideal geometrical forms, creating a universe ruled by eternal mathematical laws.*

The battle between these two views of the cosmos have been linked to the most crucial questions of society and history: Is progress, the continual betterment of human life, possible? Must there always be rulers and ruled, or should those who work decide what work is to be done?

In hindsight view, neither authoritarian Sparta nor free Ionia became the model for the social evolution of the Mediterranean world. Instead, slavery, free labor, and expanded trade all coexisted in the centuries that followed the fall of Athens. Similarly, there resulted a synthesis of the two rival cosmologies, incorporating mathematical myth with the observational method into one system.

How did all this come about? Where Athenian imperialism failed, Macedonian imperialism succeeded spectacularly. Beginning in 330 BCE, Alexander the Great conquered the area now occupied by Turkey, Syria, Israel, Jordan, Iraq, Iran, and Egypt, and established colonies of Greek freeholders,

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gradually spread. It separated matter into two regions, the rare, hot, dry and light material, the ether, in the outer regions, and the heavier, cooler, moist material, the air, in the inner regions. The air condensed in the center of the vortex, and out of the air, the clouds, water and earth separated. But after the formation of earth, because of the growing violence of the rotary motion, the surrounding fiery ether tore stones away from the earth and kindled them to stars, just as stones in a whirlpool rush outward more than water. The sun, moon and all the stars are stones on fire, which are moved round by the revolution of the ether".

artisans, and merchants. The free population increased, Mediterranean-wide trade flourished, and living standards rose.

Unlike the nobles of Sparta, the merchants of the Hellenistic world needed observations of nature to speed their ships across the Mediterranean and to ports in India. Even before Alexander's conquests, Plato's students had begun systematic astronomical observations in order to convert his ideas about perfect circular motions into an explanation of the observed motions of the planets. One such disciple, ***Eudoxos*** *(ca 360 BCE), created a system of moving spheres, with the earth at their center, which carried the planets, sun, and moon on their complex travels. This was the cosmological system of perfect motion that* ***Aristotle*** *then popularized (ca 340 BCE).*

Following Alexander's death, the ruling Ptolemies in Egypt established the Museum at Alexandria as a liberally endowed research library to generate and centralize systematic observations. Alexandrian astronomy, for example, used observation to solve practical problems of navigation.

From these observations, startling theoretical results followed. Using ***Euclid****'s discoveries in geometry,* ***Aristarchos*** *(ca 250 BCE) estimated from astronomical observations that the sun was 8 million km away and 6 times as large as the earth (the correct figures being each about 18 times larger). To him, the idea that a much larger sun should circle at a great distance around a small earth did not seem sensible. More important, the increasing accuracy of observations led him to conclude that the idea of heavenly bodies moving in perfect circles around the earth must be wrong. Instead, observation could be much better accounted for if it was assumed that the earth and planets orbit the sun, the moon orbits the earth, and the earth spins on its axis.*

Aristarchos' correct views were rejected by other ancient astronomers including ***Hipparchos*** *(ca 150 BCE), who himself calculated that the sun is far larger than even Aristarchos thought. The time was simply not ripe yet for the acceptance of the heliocentric system. Since the astronomers could neither abandon the Platonic hierarchy of the heavens and the earth, nor wholly accept Plato's disdain for observations, they compromised — and in the process forged a scientific method which contained within it the tensions of ancient society: On one hand, the basic assumptions about the universe must come from pure reason, which can accommodate the perfect mathematical laws of the heavens. On the other hand, observation serves to correct these basic mathematical laws (uniform motion in a circle etc.) in practice, modifying them as needed to "save the phenomenon" or to fit observations.*

Interlude: Geocentric, Static and Finite Universe *(100 BCE–200 CE)*

The immense accumulation of wealth in the Mediterranean after 100 *BCE were based on conquest and imperialism. In the battle over Alexander's empire Rome emerged to swallow up the whole of the Mediterranean. The Roman legions enforced ruinous taxation, looting existing wealth but creating none. Slavery was massively extended and living standards dropped precipitously throughout the empire. Hellenistic society's dependence on slavery prevented any further advance in the technologies of production.*

By the end of the first century CE Roman defeats at the hands of the Germanic tribes terminated the northern expansion of the empire. With the supply of slaves cut off, Rome's internal depredations increased: taxes soared and the population began to decline. Toward the end of the second century CE, the empire entered a period of crisis and revolt, and persecution spread everywhere.

Against this stormy background, the advance of the cosmological ideas of the Greeks which culminated in the brilliant discoveries of Aristarchos, was arrested. Rather than junking geocentricism and the Platonic philosophy that went with it, Hipparchos and his successors — notably ***Ptolemy*** *(ca 150 CE) — added new assumptions consistent with their mathematical ideas of how best to bridge the gap between theory and observations.*

Act II: Creation 'ex nihilo' *(200–400 CE)*

The early Christians began attacking slavery together with Platonic dualism, thus eroding the main ideological, social, and economic obstacles that had impeded scientific advance in the preceding centuries. In the long epoch of increasing misery and oppression that extended through the first two centuries CE, Christianity became the only empire-wide opposition to Rome's slave system.

The early Christian message of universal brotherhood of all humanity, the antithesis of the legion's robber-rule, appealed to the enslaved and the poor. As wider sections of the population defected from allegiance to Rome, educated Christians formulated a potent antidualistic rationalistic argument against the ideology that justified the empire.

Clement of Alexandria *(150–200 CE) attacked the Platonic division between heaven and earth, freeman and slave. While he admired Plato's glorification of reason, he denied the Platonic view of matter as the origin of evil. It was in this social context that the Biblical idea of creation from* *nothing (Genesis,* **1***) was revived by the early Christians who, being clearly related to second Temple Judaism and of partially Jewish stock, were deeply versed in the Hebraic story of creation.*

According to this doctrine the universe had begun at a moment in time, out of nothing, and it would end at a certain moment, returning to nothing. Although such statements were later quoted in the Talmud, they were first clearly formulated by ***Tertullian***[337] *(ca 200 CE). Creation ex nihilo was for him what separated the finite and decaying earth from the infinite and divine heaven*[338].

Augustine *(ca 400 CE) adopted this doctrine as a cosmological justification for his political philosophy of the reconciliation of Christianity with the worldview of pagans who ran the empire, the new alliance of church and state. The foundation of this doctrine was a new cosmological myth, and creation ex nihilo was central to that myth. Thoroughly impregnated with the Judaic notion of creation out of nothing, Augustine set himself the task of countering the Greek pantheistic view, for which God is the world, and adopt the creator of the Old Testament, a God outside the world, a timeless spirit, not himself subject to causality or historical development; when he created the world, he created time and eschatology along with it.*

Interlude: Dominance of the Hierarchical Christian Cosmos *(400–1400 CE)*

The medieval universe in men's minds, finite in time and space, graded into celestial spheres, and knowable through reason and authority was hostile to

337 **Quintus Septimius Florens Tertullianus** (155–222 CE). Latin ecclesiastical writer. Born at Carthage. Worked as jurist in Rome; returned to Carthage and converted to Christianity (190 CE). Withdrew (210 CE) from the orthodox church and formed his own sect.
Tertullian embraced Platonic dualism but rejected its rationalism. To him, as to the pagan neoplatonists, the material world is evil. Such a world could have been created by an omnipotent and beneficent God only for a limited period of time; finiteness implies imperfection, source of evil, and eventual decay. By contrast, only God, who is eternal and infinite, can be wholly good and divine.

338 Modern cosmology, thermodynamics and quantum physics – while neutral in matters of good and evil – support the inherent connection between an initial preparation of a physical system and its tendencies to decay – although they *also* allow for the possibility of local order and life arising *spontaneously* (if only for a limited cosmological epoch).

the least vestige of science. Yet, during the long dark ages that preceded the Renaissance and Reformation, there were three abortive attempts to develop a new scientific view of the universe:

- *Revival of the ancient Ionian idea of a nature knowable by* observation and experiments *by* **Pelagius** *(409 CE)* and **John (Joannes) Philoponus** *(500 CE).* **Severus of Antioch** *(518 CE) revived the idea of* causality, *countering the anti-historical Platonic-Augustinian worldview. His philosophical notions implied a cosmos that unified heaven and earth, a universe not created by fiat, but developing by a historical process.*

- The Muslim renaissance *(1000–1100 CE), centered around the scientific achievements of* **Alhazen** *and* **Avicenna**. *But despite the great strides made by Islamic science, their renaissance lacked a staying power; the* 11^{th} *century thinkers had attacked an important part of the existing worldview, but had not formulated a comprehensive alternative. Their scientific method did not probe too deeply into matters that, in the Islamic east as well as in the Christian west, were so closely linked to religious orthodoxy. For the Muslim empires were just as closely linked to religious hierarchies as were the European feudal states. And while the Muslims encouraged trade, and, to a limited extent, manufacturing, political power rested with a land-lording class, whose power was centralized in the powerful caliphs.*

The conflicts between the wealthy landlords, who exploited enserfed peasants and slaves, and tradesmen and manufacturers, who relied on free labor, broke out again in violent struggle. In the end, the power of the caliphs was gathered into the hands of the invading Turks, who crushed the budding merchant economies and dispersed the scientific institutions they had supported. Fundamentalists attacked philosophers like Ibn Sina as impious and heretical. The first serious effort to establish self-sustaining scientific enterprises had failed. The crucial breakthrough — a new scientific cosmology — would be achieved by the West.

- **Robert Grosseteste** *(1220 CE)* and **Roger Bacon** *(1267 CE), under the influence of the scientific method of Islam, advocated observations and experimentation, and asserted that the highest purpose of scientific work is its eventual practical application. This new trend in Western thinking reflects the rise of more developed technology, economic growth, free labor, and expansion of trade. Yet, the scope of technological and economic expansion was limited by the old order and the scientific theory of Bacon was more utopian than practical. On the social front — feudalism needed new lands to cultivate.*

As arable land became scarce around 1300, *the nobility borrowed on a grand scale to finance their luxuries and wars, taxing their subjects to pay* *the debts. Peasant grain reserves were squandered, famine repeatedly swept over Europe, hunger pervaded the filthy towns, and in 1348 feudal society collapsed in the grip of the Black Death. This catastrophic event cleared the way for the development of science and for modern society. Over the next 300 years, the old cosmology crumbled and a new worldview took over.*

Act III: Revival of the Ionian Non-Geocentric Infinite Universe *(1440–1660 CE)*

The catastrophe of the Black Death undermined the ideological authority of both church and state. The tremendous shortage of labor created by the plague made serfdom unworkable in much of Europe. The doctrines of Augustine and Aquinas, in which the people owed obedience to secular and ecclesiastical authority, no longer held sway. Revolts in England and continental Europe shook the trading towns. The free towns of artisans, merchants, and manufacturers, allied with the free peasants, clashed with the great lords and bishops, kings, and popes.

The first serious attempt to undermine the basic notions of the hierarchical geocentric medieval cosmos was made by a German-born bishop, ***Nicolas of Cusa****. In his major work entitled* On Learned Ignorance *(1440 CE) he returned to the central idea of* ***Anaxagoras*** *— an infinite, centerless universe, unlimited in space and time. The earth, he reasoned, is no different from the stars, moving like everything else in the universe. Although his work (radical as it was in its implications) remained abstract philosophy — his influence spread in a number of parallel channels. It finally led through* ***Leonardo da Vinci*** *(1482 to 1519) to the modern experimental method, and to the cosmology of* ***Copernicus*** *(1543).*

While Copernicus worked, the voyages of discovery provided a sharp incentive for a new astronomy — moreover, a practical *astronomy. If the motions of the moon and planets could be accurately known, they could act as a celestial clock, enabling sailors to gauge their course precisely in crossing the Atlantic. For this task the Ptolemaic system with its epicycles and deferents was far too cumbersome and inaccurate.*

During the centuries-long effort to conform the geocentric worldview to the observations of planetary motions, complexity after complexity had been added. It was well known that the geocentric view accounted approximately for the positions of the planets and moon. Yet the obvious changes in their brightness *(a direct consequence of their changing distance from earth) was inexplicable. For the moon, whose distance is actually nearly constant, the epicycles introduced a variation in the distance — thus in its apparent size —*

that was not observed. It was so absurd that King Alfonso of Spain remarked, "If I had been present at the creation, I could have rendered profound advice".

Nicolaus Copernicus *studied in Italy from* 1501 *until* 1506*. There he absorbed the writings of Aristarchos, Nicolas of Cusa's idea that the earth moves, and Leonardo's conception that the sun is immobile. It is likely that he connected these new-old ideas with the well-known inadequacy of the Ptolemy system; indeed, before Copernicus left Italy, he had developed the basis of the heliocentric system.*

So radical were the implications of Copernicus' view that the leaders of the Reformation rejected it in horror, even as their followers in the universities turned to it with interest. But in England, where the power of the church had been uprooted by Henry VIII's decrees, the new ideas found fertile soil.

Yet despite its widespread acceptance in England, there was still relatively little observational evidence for the Copernical model. ***Tycho Brahe****, the most accurate observer of his day, formulated a compromise alternative in which the planets revolve around the sun, which in turn revolves around an immobile earth.*

After Brahe's death ***Johannes Kepler*** *used his observations, which were* 100*-fold more precise than Ptolemy's, to find an accurate description of the solar system. Starting with the traditional conception of perfect circles, Kepler labored for years. After enormous struggle he broke with this last remnant of the ancient cosmology.*

By trial and error, he discovered in 1609 *that the planets moved in ellipses, not circles, about the sun and not at constant speeds, but at such a rate that the areas swept by the sun-planet line in a given time remained constant throughout the orbit of a given planet. Furthermore, a simple algebraic formula governs the relation of the planetary orbital periods to their sizes. According to the later mechanistic understanding of these three laws of Kepler by Newton: As a planet approaches the sun, which occupies one focus of its elliptical orbit, the gravitational attraction increases, and it speeds up; when it has passed the perihelion (closest approach to the sun), its trajectory carries it further away from the sun, and the (weakened) force of gravity then slows it down. The immensely complex system of epicycles, deferents, and eccentric spheres was replaced by simple ellipses.*

Kepler's system was far more accurate than any other. It could not be translated to Tycho Brahe's, since then the paths of the planets would not be simple ellipses but complex compound motions.

That same year, ***Hans Lippershey*** *introduced the telescope in Holland. Within a year,* ***Galileo*** *in Italy and other astronomers had trained the new instrument upon the heavens. Galileo discovered the existence of the moons* *of Jupiter, the phases of Venus, and the mountains of the moon. The changeless, perfect heaven so crucial to Aristotelian cosmology was shattered by observation.*

Armed with his new observations, Galileo immediately became a propagandist for the Copernical worldview, actively trying to win over the Catholic hierarchy. Cardinal Bellarmine, warned by the case of Bruno, moved to quash Galileo's effort. No conflict with the literal interpretation of scripture is possible, he informed Galileo: the sun is described in the Bible as moving, rising, and setting — anything else is heretical. In 1616 Copernicus' work was added to the index of prohibited works: the new doctrine was officially condemned.

Galileo, however, continued his efforts, which culminated with the publication in 1632 *of his great defense of Copernicus, the* Dialogue on Two World Systems*. The response came swiftly: he was forced, with the example of Bruno before him, to recant and was placed under house arrest. The new science remained forbidden in Catholic countries for over a century.*

It was only in those countries where the new society was victorious that the new science became self-sustaining — above all, in England.

Act IV: Infinite, Gravitating, Evolutionary, Quasi-Static, Euclidean Universe *(1687–1860 CE)*

The English revolution of 1642 *led to a decisive (though to some extent temporary) defeat of the aristocratic landowning classes and their absorption into the new mercantile and manufacturing regime. During the period of the Commonwealth the revolutionaries, though led by Bible–inspired Puritans, also proudly identified their movement with scientific rationalism and the rejection of myths and superstitions. English scientists rapidly synthesized Kepler's laws of planetary motion with Galileo's investigations in mechanics, published in* 1638.

Taken together, these two scientific developments led ***Robert Hooke*** *and others in England to ask whether planetary motion could be explained by attraction spreading from the sun with an inverse-square dependence on the distance (1679).* ***Isaac Newton*** *(1680 to 1687) elevated the notion from the merely speculative to the quantitatively predictive, and extended the realm of its application to universal gravitation.*

Thus, the scientific revolution of the 16^{th} *and* 17^{th} *centuries had, at least in England, displaced the hierarchical, finite universe with an infinite one, and the appeal to authority and pure, scholastic reason with the observational method. But, unlike the Ionians,* 17^{th}*-century scientists had not developed a* *naturalistic theory of the origins of the world, an alternative to the creation from nothing of the medieval cosmology. Philosophers such as Nicolas of Cusa and Giordano Bruno had advocated the idea of a universe unlimited in time and space, eternal and without beginning. But no scientist could corroborate these notions with hard data.*

For many scientists, it was in this realm of origins that religion still intersected with science. Isaac Newton, for example, argued that God is needed to form the solar system and to maintain it.

In the period after the English revolution, the Restoration, and the ensuing Glorious Revolution (ushering in constitutional monarchy), English society settled into a conservative phase. The idea of change, implicit in any concept of evolution in nature, lost its popularity. The universe was a finished product brought into being by events that could not recur.

It was not until the middle of the 18^{th} *century, when the winds of change started to blow in Europe and America, that the problem of origins was again tackled. In* 1755 *the philosopher* ***Immanuel Kant*** *formulated a naturalistic explanation for the origin of the earth and solar system in many ways strikingly similar to that of Anaxagoras.*

Kant, who was familiar with the latest astronomical research, argued that observation showed that stars are not randomly scattered throughout the universe, but appear to be grouped into a huge disc, the Milky Way. He speculated, correctly, that the distant fuzzy nebulae astronomers were then studying are similar vast agglomerations of stars, what we now term galaxies. By analogy he reasoned that these, too, probably formed still larger systems of clusters — once again a guess later confirmed by observation.

Starting with this concept of an infinite universe, arranged into a hierarchy of larger and larger spinning agglomerations of matter, Kant proposed the idea that in the remote past the universe was a nearly homogeneous, infinite gas. Certain regions, which by accident happened to be denser than others, started to attract matter by gravitation. The random motions of the gas gave to each agglomeration a slight spin, creating huge vortices, within which galaxies, then stars, then planets coalesced[339]*. Since Kant assumed that this process started in one place in the universe, and spread outward, he believed that creation was and remains a continuous process, which spreads through the infinite universe.*

339 Newtonian mechanics, as well as further astronomical research, confirmed these mechanisms, basing them upon *conservation of angular momentum* and *growth of gravitational instabilities*.

In the years following Kant's "Theory of the Heavens", Europe and America were convulsed by sweeping revolutions that sought to complete the overthrow of the old hierarchical societies and to replace them with democracies. By the century's end, the spectacular changes of government and society brought about by these revolutions led their supporters to conceive of a general and continual process of human social change — the idea of *progress*. To both the Founding Fathers in the United States, and the French revolutionaries, their respective revolutions were part of the inevitable advance of society, perfecting its institutions and improving without limit the material well-being of mankind.

The revolutionary political concept that society is not a fixed entity, that it continuously evolves through effort and struggle, through science and technology, toward higher forms of organization and material well-being, was swiftly taken up in the field of science. In late-18^{th}-century England geological knowledge advanced rapidly as coal became central to the steam-powered industry of the industrial revolution. Geological observation led **James Hutton**, an amateur scientist, to develop a theory of the continuous evolution of the earth itself.

By observing such processes as the compaction of clay into sedimentary rock, Hutton concluded in his 1795 work, *Theory of the Earth*, that mountains, rivers, oceans, and the sedimentary and igneous rocks of the world today were formed over many millions of years, not by miraculous floods or one-time cataclysms. He emphasizes that a scientific history of the world can be obtained only by examining current processes and working backward in time, not by speculating about origins and working forward. The idea of a world finite in time, with a supernatural origin, is rejected: "The result, therefore, of this physical inquiry is that we find no vestige of a beginning, no prospect of an end".

Within a decade, the French mathematician **Pierre Simon de Laplace** had taken Hutton's approach a step further into the past and given a firm scientific basis to Kant's vortex theory of origin. Using Newtonian mechanics, Laplace demonstrated in 1796 that, if the sun had condensed from a spinning sphere of gas, it would have thrown off material as it contracted, since as it contracted it would have spun faster (by angular–momentum conservation). The material thrown off would form into rings, which would, in turn, condense gravitationally into planets. The nearly circular orbits of the planets would therefore be neatly accounted for.

Hutton and his supporters rapidly accepted Laplace's nebular theory, producing an integrated approach to the history of the world since its origins. Others quickly applied the historical approach to the development of life itself. **Erasmus Darwin** (Charles' grandfather), in the same year as Laplace's *theory, proposed that the fossils found in geological strata represent the evolution of various species of animal from one another, leading to a greater and greater perfection of life over vast stretches of time.*

By the 1840s the new geology and cosmology enjoyed widespread acceptance among scientists and the public, and socialist concepts of societal evolution spread throughout Europe.

In 1859 ***Charles Darwin*** *systematized and popularized the theory of biological evolution, ironically seizing on Malthus' theory of limited resources to formulate a vision of continual evolution and change. By the 1860s, despite continued religious opposition, the evolutionary and historical approaches in the sciences had become dominant, as had the related idea of human progress.*

Neither religion nor philosophy could place limits on the natural universe in time or space, yet it was clearly evolving, not static. The triumph of the scientific revolution was the triumph of the infinite universe — a universe of unlimited progress from an infinite past to an infinite future.

Interlude: Vestige of a Beginning and Prospect of an End *(1823–1916 CE)*

The concept of an infinite universe was questioned even after the scientific revolution. Newton was undecided on whether his laws of gravitation preclude an infinite collection of matter. He thought that only a divinely precise positioning of all the stars could prevent such an infinite collection of matter from collapsing into a series of heaps.

The efforts to solve the cosmological problem within the framework of Newton's Law of Gravitation and Newtonian mechanics met a serious obstacle; if space is infinite and the density of matter is everywhere finite, an infinite world filled with an infinite amount of gravitating matter would result. Is such a universe conceivable? Are we permitted to perform this transition to infinity without encountering difficulties of principle[340]*?*

If the world contained infinitely many bright stars, then, if one's line of sight were extended far enough in any direction from earth, it would intersect a star. This implied that the sky should be uniformly bright — as bright as the surface of the sun, which it obviously is not.

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[340] In a world in which the mean density ρ of matter is finite, even if its value is as small as we please, the gravitational potential $\phi = \int_V \rho \frac{dV}{r}$ has no definite value when V, the volume, tends to infinity. Also, the expression for the gravitational stress becomes indefinite.

This objection (Olbers paradox) was first raised by ***J.P.L. de Cheseaux*** *(1744) and again by* ***H.W.M. Olbers*** *(1826). It can be removed, within the realm of classical physics, by applying the idea of a hierarchic structure of the universe which prescribes a special type of distribution of the infinite amount of matter over the infinite Euclidean space, such as to avoid the singularities mentioned above. The first idea in this direction came from* ***J.H. Lambert*** *(1761).*

By a "hierarchic structure" the following is meant: matter is distributed in space so that the stars combine to form greater systems, star-systems or galaxies; galaxies again combine to form still greater systems, supergalaxies and so on. From each rank on the hierarchic ladder we can step to a rank of higher order consisting of elements of the preceding rank, and so forth to infinity. If we impose certain additional restrictions on such a hierarchy[341]*, the singularities mentioned above need not appear. In the resulting universe the mean density of matter becomes zero.*

In this hierarchic regime, the gravitational potential at each point converges to a finite value in spite of the infinity in the universe's size, mass, and number of stars. Likewise, the brightness of the night sky could be kept sufficiently low, providing a solution to Olbers' Paradox. In such a universe the mean density of matter, taken over a volume increasing toward the infinite Euclidean space, converges to zero, although the total amount of matter would increase to infinity. [The hierarchic structure was hypothesis revived in the 20^{th} *century by* ***Hugo von Seeliger*** *(1909) and* ***C.V.I. Charlier*** *(1922).]*

Another difficulty which was encountered in the first attempts to solve the cosmological problem, was the expectation of unduly high velocities in an infinite universe containing an infinite amount of matter. However, at the time when the idea of a hierarchic world order was developed, the high and systematic velocities of distant spiral systems were still unknown — and the necessity had not yet been realized of formulating the cosmological problem as a dynamical[342]*, and not a static problem.*

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[341] Let us attribute to all systems spherical shapes of radii $R_0, R_1, R_2, \ldots, R_i, \ldots$, each system having the total mass $M_0, M_1, M_2, \ldots, M_i, \ldots$ and consisting respectively of $N_0, N_1, N_2, \ldots, N_i, \ldots$ objects such that $M_1 = N_1 M_0$, $M_2 = N_2 M_1 = N_1 N_2 M_0$ etc. Then the condition is $\frac{R_i}{R_{i-1}} \geq \frac{1}{\gamma} N_i$, where $\gamma < 1$. The convergence of the gravitational potential can be shown to be independent of the assumption of the spherical shape of each system.

[342] The universe could be called '*static*' if — despite local motions of celestial bodies (for instance, the orbital motions of the planets around the sun) — no large-

*The idea that the universe had a finite lifetime also existed in the mid-*19*th* *century, although only on the popular fringes of science. The first suggestion that the universe originated in a creative explosion — the first Big Bang — actually came from the pen of* ***Edgar Allan Poe*** *in* 1849*. Poe was not only a well-known poet and writer, he was also a scientific popularizer who kept himself up-to-date on the latest in astronomical research. In the book-length essay Eureka Poe rejected the idea of an infinite universe, citing Olbers' objections. He reasoned that a universe governed by gravitation would collapse in a heap if not kept apart by some form of repulsion. He postulated that God had, in an enormous explosion at the creation, thrust all the stars apart. Like a rocket racing into the sky, the universe of stars and galaxies would first expand, and then contract into a final catastrophe, the end of the world.*

*However, questions about the infinity of the cosmos remained marginal to the mainstream of science through the mid-*19*th* *century. The swift advance of technological progress and the equally swift transformation of society convinced most scientists that the basic methods of science correctly yield results provable in practice, and that the thesis of an unlimited, evolutionary universe is valid. It was not until social and economic progress slowed that the corresponding scientific assumptions came under serious attack.*

In the latter third of the 19*th* *century, from around* 1870 *on, the nature of the rapid social and economic evolution of Western society began to change. By this time, the last institutional vestiges of compulsory labor had been wiped out by social revolutions in Europe, the Civil War in the United States, and the liberation of the serfs in Russia. After the defeat of the Paris Commune — the* 1871 *attempt to establish a workers' rule — Europe entered a period of relative political stability.*

The sixty years from 1820 *to* 1880 *witnessed the fastest economic growth in history. But by* 1880*, the limits of capitalist markets were being reached: European and American goods were penetrating virtually every corner of the*

scale changes in the distribution of matter in the universe were occurring. In such a "*static*" world, the mean density in the distribution of matter would remain constant when referred to sufficiently large volumes of space and sufficiently long intervals of time.

When our knowledge extended far beyond the limits of our galaxy and the high velocities of distant spiral stellar systems were discovered, not small even when compared with the velocity of light — these velocities were of a character very different from the high velocities expected on account of increasingly large potential differences. This discovery, together with the new methods of determining stellar distances, extended our "neighborhood" far beyond the limits reached until then and changed the whole outlook on the cosmological problem to such an extent that all previous results had to be completely revised.

globe, as Britain, France, and Germany rushed to carve up the only remaining land — Africa. While the actual need for goods remained immense, the market for goods that could be sold at a profit was nearing the end of its growth. For centuries, millions of new farmers and peasants had been drawn into the developing capitalist market system as feudal regimes fell apart and as new colonies were conquered and absorbed. When this expansion lost its frontiers with the formation of the first global market at the end of the 19^{th} *century, the industrial economies could no longer continue their vigorous expansion.*

After 1880, *the production of iron and steel and the laying of new rail lines practically ceased their growth. Real wages continued to increase, but more slowly, peaking in Europe by* 1900. *Manufacturers turned to the European states for new markets, leading to the growth of a gigantic arms industry. Manufacturers found that two battleships are always better than one, unlike two railroads from Liverpool to London. These arms, in turn, were used to maneuver for a greater share of the precious world markets and of the resources of the colonies.*

It was in this era of slowing growth that the first real scientific challenge to the unlimited universe appeared. Steam power had developed throughout the 19^{th} *century, as did the study of heat and its transformations — thermodynamics. In the early part of the century, scientists had discovered that energy can be transformed in various ways, but never created or destroyed, a fundamental principle that came to be known as the* first law of thermodynamics.

In 1850, ***Rudolf Clausius*** *discovered another fundamental principle, the* second law of thermodynamics. *A quantity (macroscopic state–variable) which Clausius dubbed "entropy", always exhibits an overall increase in any transformation of energy — for example, in a steam engine. This principle encodes the* irreversible *aspect of thermal processes. The entropy of a physical subsystem was defined by its infinitesimal change — the quantity of heat transferred into the subsystem from the outside, divided by its absolute temperature.*

In 1877, ***Ludwig Boltzmann*** *attempted to derive the macroscopic second law from the newly emerging atomic theory of matter. He redefined entropy as a function of the probability distribution of* microscopic *states compatible with a given* macroscopic *state of matter and energy: if the microstates are more numerous for one macrostate then for another, the former macrostate has a higher entropy. Thus, if a million atoms of oxygen mixed with a million atoms of nitrogen, it would be far more probable to find them evenly mixed than segregated. The well-mixed state has a higher entropy; and left to itself, a container with oxygen on one side and nitrogen on the other will rapidly go to the higher entropy state of an even mixture.*

Boltzmann, using his new definition of entropy, went on to demonstrate, so he claimed, that all closed systems tend toward a state of thermodynamic equilibrium — defined as the state in which there is no net flow of energy. Thus a hot object and a cold object placed in contact are not in equilibrium, since heat will flow from one to another until they are at the same temperature, which is a state of equilibrium — and also a state of maximal entropy if the two-object system is isolated from its environment.

From this proof, Boltzmann propounded a new concept with profound cosmological implications. The universe as a whole must, like any closed system, tend toward an equilibrium state of maximal entropy: it will be completely homogeneous, the same temperature everywhere, the stars will cool, their life-giving energy flow will cease and their atoms and molecules will evaporate to fill space. The universe will suffer a "heat death". Any closed system must thus go from an ordered to a less ordered state — the opposite of progress.

Boltzmann was aware that his ideas contradicted the notion, then widely accepted, of a universe without beginning or end. The present-day universe is far from a state of equilibrium, comprising as it does hot stars and cold space. If all natural systems "run down" to disorder, the present state of order must have been created by some process that violates the second law at a finite time in the past. Conversely, at a finite time in the future, the universe will become a lifeless homogeneous mass: human progress is but an ephemeral and inconsequential episode in a universal decay.

Boltzmann found his results disturbing. Since he rejected a supernatural origin of the universe, he tried to argue that, in an infinite amount of time, extremely improbable events do occur, such as the spontaneous organization of a universe, or a large section of it, from a prior state of equilibrium. The second law is, after all, a statistical one stating what is likely to happen, not what must *happen. Just as there is an incredibly small chance that all the air molecules in a room will rush to one side, there is a smaller chance that all the atoms in a homogeneous part of an infinite universe will suddenly rush together into one spot of low entropy. Boltzmann's argument did not much impress his fellow scientists, since by his own theories the probability of these occurrences was, in fact, so tiny that it was equivalent to an impossibility.*

But scientists had other reasons for not accepting the second law's implication that the universe necessarily had a beginning from which it was now running down. The predictions of thermodynamics appeared to contradict what was known of geological and biological evolution. In the 1890s *a debate broke out between thermodynamicists and geologists over the age of the earth. The physicist* ***Lord Kelvin*** *argued that, from the cooling rate of the earth as estimated from measurement of heat in mines, the earth must have been nearly molten as recently as twenty million years ago. Geologists countered* *that the formation of certain rock deposits must have taken at least twenty times as long, four hundred million years. Backed up not by theory but by a vast accumulation of observation, geologists doubted the physicists' theories.*

In addition, some thermodynamicists pointed out that Boltzmann had proved far less than he claimed. He assumed that gas began in a high degree of disorder, close to equilibrium, and never got far from it. Moreover, he only allowed for atomic collisions, but took no long-range forces, such as electromagnetism or gravity, into account. In most real physical situations, though, these restrictions are not valid, so Boltzmann's proof is not applicable. A century later scientists were to demonstrate that Boltzmann's law of increasing disorder does not apply to systems far from any state of equilibrium.

Beyond these scientific objections, though, were cultural ones. At any moment, scientists must decide which problems or apparent paradoxes are worthwhile and which should simply be dismissed — it is here that the ideology of the age — of society as a whole — sometimes affects what scientists feel "makes sense". And Boltzmann's concept of a world running down simply did not make sense to most 19^{th}*-century scientists.*

In the late 19^{th} *century, while material advance had slowed and the ominous trends leading toward the crises of the* 20^{th} *century were beginning to emerge, progress remained the overwhelmingly dominant idea of the epoch. Standards of living continued to rise, albeit more slowly, until* 1900. *Technological progress was more rapid than at any other time in human history: someone born in* 1870 *would have grown up in a world of gaslight and horse-drawn carriages, but by age forty he or she would live in a world of electricity, telephones, phonographs, movies, radio-telegraphs, automobiles, and airplanes.*

Science, too, advanced dramatically in the same period. Biology and medicine were transformed in the 1880*s by the germ theory of disease, leading to the widespread use of antiseptics in surgery, and the general use of vaccination. Physics saw the blossoming of the study of electromagnetism, put on a firm foundation in* 1865 *by* ***James Clerk Maxwell****, and later radioactivity, X-rays,* ***Einstein****'s special theory of relativity, and the beginning of quantum theory.*

The reality of progress in science and society was so apparent to the average scientist, that Boltzmann's vision of a universe in continual decay seemed too bizarre. In practice, Boltzmann's laws were very useful in dealing with steam engines and simple gaseous systems, and were widely applied. But his broad generalizations about cosmology, which implied that the universe must have had a beginning, must have been "wound up"and will inevitably wind down and decay — had no significant impact for more than a generation.

Nevertheless, the finite universe did *return: The European and American confidence in progress was shattered in August of* 1914*. In the following four years the vast economic power and technological achievements of the prior century were thrown into the barbaric enterprise of slaughtering twenty million human beings. In the wake of war came revolution and counter-revolution: working-class living standards had plummeted during WWI, and workers' movements had seized power in Russia and tried to do so in Germany. Throughout Europe and America, employers and governments battled strikers.*

At this very moment in history, came the first public announcement of the observational verification of Einstein's Theory of Relativity. On Nov. 9, 1919*, the* New York Times *announced that the solar eclipse of May* 29, 1919 *had confirmed Einstein's prediction of the bending of light from distant stars by the sun's gravity.*

1915–1925 CE **Bedrich Hrozny** (1879–1952; Czechoslovakia). Archaeologist, orientalist and philologist. Deciphered the Hittite language and laid the foundation to a new scientific branch — Hittitology.

Born in Lysa na Labem (near Prague), and attended grammar school in Prague and in Kolin. He then studied in Vienna Semitic and Oriental Philology and worked there as librarian and lecturer (1905–1918). During 1919–1952 he was a professor at Charles University in Prague.

Working with inscriptions from the *Hittite royal archives* discovered (1906) at the ancient capital site of Hattusas (near the Turkish village of Bogazköy, east of Ankara), Hrozny concluded (1915) that Hittite was an Indo-European language because of the similarity of its endings for nouns and verbs to those of other early Indo-European languages. Hittite has provided significant information about the early Indo-European sound system. Some years earlier the existence of an Indo-European idiom in some cuneiform[343] letters found

[343] *Cuneiform*: system of writing used in the ancient Middle East. The name, a coinage from Latin and Middle French roots meaning "wedge-shaped", has been the modern designation from the early 18th century onward. Cuneiform was the most widespread and historically significant writing system in the ancient *Middle East*. Its active history comprised the last three millennia BCE; its long development and geographic expansion involved numerous successive cultures and languages. Its overall significance as an international graphic medium of civilization is second only to that of the Phoenician-Greek-Latin alphabet.

in the Egyptian diplomatic archives of the 18th dynasty at Tell el-Amarna, had been suspected by **Johan Knudtzon** (1902).

The Hittite language is the most important of the extinct Indo-European languages of Anatolia; it was closely related to Luwian, Lydian, Lycian, and Palaic. Hittite is known primarily from the approximately 25,000 cuneiform tablets or fragments of tablets preserved in the archives of Bogazköy, the majority of which are from the period of the Hittite empire (c. 1400–c. 1190 BCE) and are concerned with religious and other subjects. Old Hittite texts, from about 1650 to 1595 BC, are preserved in copies from the empire period and are the earliest Indo-European texts that have thus far been found.

In his *Sprache der Hethiter* (1915) and *Hethitische Keilschrifttexte aus Boghazköi* (1919), Hrozny substantiated his claim by translating a number of documents, including a Hittite legal code. In 1925 he led a Czechoslovak expedition to Kültepe, Turkey, recovered some 1,000 Old Assyrian tablets nearby, and excavated the ancient city of Kanesh, revealing much about its everyday life. During the remainder of his career, he addressed himself to problems of deciphering.

1915–1938 CE **Adolf Otto Reinhold Windaus** (1876–1959, Germany). Chemist. Discovered the structure of *cholesterol*[344] (1932), its relation to *vitamin D* and linked the roles of sunlight and vitamin D in the prevention of *Rickets*. Isolated vitamin D_1 from yeast, and discovered *histamine*.

Received the Nobel Prize for Chemistry (1928). Windaus' influence on junior colleagues, led to studies of steroidal sex hormones by **Adolf Butenandt** (1903–1995, Germany) and the study of the stereochemistry of steroids by **Erich Hückel** (1896–1980, Germany).

Windaus first studied medicine at the University of Berlin (1895) but abandoned medicine for chemistry (1899). He was professor of medical chemistry at Freiburg (1905–1913), Innsbruck (1913–1915) and Göttingen (1915–1944).

1915–1975 CE **Harold Jeffreys** (1891–1989, England). Mathematician, geophysicist and astronomer. Discovered the discontinuity between the

[344] *Cholesterol* ($C_{27}H_{46}O$) is synthesized in the liver and is present in edible food. It is found particularly in the brain, nerve tissues, and gallstones of animals. It aids in absorption of fatty acids from the small intestine. Since it does not produce ions, and therefore does not carry a current, cholesterol is a good electrical insulator in the brain and the nervous system. It is also essential for the production of sex hormones, vitamin D, and the adrenal cortex hormones. It is suspected that excessive ingestion of carbohydrates may result in cholesterol deposits on the wall of the blood vessels, leading to heart disease.

Earth's upper and lower mantle, found evidence for the fluid nature of the core and did much pioneering work on the shape and strength of the earth. His analysis of seismic travel times was published as the *Jeffreys-Bullen Tables* (1940), which remains a standard reference. This work led him to Bayesian theory of probability applicable to a wide range of sciences, including econometrics and statistics. Among his works are *The Earth: Its Origin, History and Physical Constitution* (1924); *Methods of Mathematical Physics* (1946; with B.S. Jeffreys).

Jeffreys was born in Fatfield (near Durham), England. He was educated in Armstrong College, Newcastle upon Tyne, and St. John College, Cambridge, where he graduated in 1913. Jeffreys worked in the Cavendish Laboratories on war related work (1915–1917) and then joined the Meteorological Office (1917–1922), working on hydrodynamical problems. Returning to Cambridge (1922), he held various teaching appointments and later became Plumian Professor of Astronomy (1946–1958). He was knighted in 1953.

1915–1939 CE **David Sarnoff** (1891–1971, USA). A Pioneer in radio and television broadcasting (1915); formed *National Broadcasting Corporation* (1926) as subsidiary of RCA; established experimental television station (1928); gave demonstration of television at New York World's Fair (1938).

Sarnoff was born in Minsk, Russia to Jewish parents. Emigrated to USA (1900); a myth has it that while a telegraph operator (from 1906), was first to pick up distress signal from the *Titanic* (1912). Rose from commercial manager (1919) to president of Radio Corporation of America (RCA, 1947–1970).

Sarnoff, largely self-taught, was perhaps the key figure in the shaping of the modern broadcast and communication system in USA. He was involved with the whole history of 20th century communication system from marine telegraphy, through radio to television. He was not an engineer and never held a single patent, but he had a crucial flair for foreseeing and shaping the applications of the new electronic technology.

1916, February 21–December 19 CE *The Battle of Verdun*: Perhaps the bloodiest and most demanding battle in history, between Germany and France. In the end, the front lines were nearly the same as when the battle started while over 260,000 French and Germans were killed and over 750,000 were wounded.

During the *First World War*, Verdun was a fortified French garrison town on the River Meuse 200 km east of Paris. The town, surrounded by a ring of forts, was an important stronghold that projected into the German lines and guarded the direct route to Paris. Rather than a traditional military victory, Verdun was planned as a vehicle for destroying the French army on account of its holding a great symbolic value in the minds of the French people. The German plan was to subject Verdun to intense bombardment (1,000,000 artillery shells were fired by 1200 guns on a front of 40 km on the day of 21, February 1916, alone!), thus drawing in and diverting French troops from all over the Western Front to the 12 km wide front around Verdun. On the first day of the German attack, a million troops faced only about 200,000 French defenders.

Vowing that "they shall not pass", the French were eventually able to stop the German advance, and after ten months of bloody combat the German gains had been virtually wiped out.

Verdun was a "victory" that would haunt French military and political leaders for a generation.

1916–1918 CE Physicists **H. Reissner** (Germany, 1916) and **Gunnar Nordström** (1881–1923, Finland, 1918) presented a study of the gravitational field of an electrically-charged, non-spinning point mass (i.e., a point singularity of the Einstein field-equation) with an energy-momentum tensor due to its electromagnetic field. This is known as the *Reissner-Nordström solution.*

1916–1923 CE **Gilbert Newton Lewis** (1875–1946, USA). Theoretical physical chemist. Advanced the theory of the *chemical bond.* Did pioneering work on the electronic theory of *valency*, in which he developed the concept of *electron-pair bond.* Broadened the definitions of *acids* and *bases.* In the field of *chemical thermodynamics* he listed the free energies of 143 substances. His later work was on deuterium (heavy hydrogen), photochemistry, and the excited electron states of organic molecules. Through these studies he contributed to the understanding of the *color* of organic substances, and the complex phenomena of *phosphorescence* and *fluorescence.*

Lewis was born in Weymouth, MA, USA and obtained his Ph.D. from Harvard University (1899). He later worked at Leipzig (under **Ostwald**), and Göttingen (under **Nernst**), and MIT (1905–1912); he subsequently moved to the University of California, Berkeley, where he remained until his death.

Lewis showed (1916) the significance of completed shells of 2 and 8 electrons. He defined a *base* as a substance that supplies a pair of electrons for a chemical bond, and an *acid* as a substance that accepts such a pair. He postulated that the atoms of elements whose atomic mass is higher than Helium's have *inner shells* of electrons with the structure of the preceding rare gas. The valency electrons lie outside these shells and form ionic bonds, preferably *covalent bonds.* The covalent bond was identified with a pair of electrons shared by two atoms and occupying jointly an outer-shell orbital belonging to each atom (1919, with I. Langmuir).

Lewis coined the name 'photon'.

"I hope I shall not shock the experimental physicists too much if I add that it is also a good rule not to put over-much confidence in the observational results that are put forward until they been confirmed by theory."

Arthur Eddington

1916–1926 CE **Arthur Stanley Eddington** (1882–1944, England). A leading astronomer of the first half of the 20^{th} century and the most distinguished astrophysicist of his time.

His work on the internal constitution of stars formed the prelude to the whole development of modern astrophysics. Following the older work of **J.H. Lane** (1819–1880, U.S.A.; 1870), **A. Ritter** (1878–1889), **R. Emden** (1862–1940, Switzerland; 1907) and **K. Schwarzschild** (1906–1914), Eddington succeeded in combining their classical results with the theory of *radiative equilibrium* and Bohr's theory of atomic structure (which had meanwhile been developed) into a set of equations governing stellar interiors and luminosities[345].

[345] Normal stars are assumed to be in hydrostatic equilibrium and thermodynamic steady state. Hydrostatic equilibrium arises when there is a balance between the attractive force of gravity and the outward force caused by the pressure gradient $\frac{dP}{dr}$. If spherical symmetry is assumed, only spherical mass shells need be considered. Assuming the temperature gradients are sufficient to trigger convection, and ignoring non-radiative diffusive heat conduction, the standard equations of the theory of the internal constitution of stars are as follows (see legend below):

(1) Hydrostatic equilibrium $\frac{dP}{dr} = -\rho\frac{GM(r)}{r^2}$; $\quad \frac{dM(r)}{dr} = 4\pi\rho(r)r^2$;

(2) Net steady-state energy-flux balance equation $\frac{dL}{dr} = 4\pi r^2\rho\epsilon$;

(3) Radiative energy transfer $\frac{dT}{dr} = -\frac{3}{16\sigma}\frac{\chi\rho}{T^3}\frac{L(r)}{4\pi r^2}$;

(4) Convective energy transfer $\frac{dT}{dr} = \left(1 - \frac{1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr}$, $\quad \gamma = \frac{c_p}{c_v}$;

Eddington empirically verified Einstein's prediction of the deflection of starlight in the sun's gravitational field (1919). In 1920 he suggested that the sun might derive its energy from the fusion of hydrogen into helium. In 1923 he tried to unify electromagnetism and gravitation via a geometrization of the electromagnetic field.

In 1924 he formulated the mass-luminosity law for stars and defined the *Eddington limit* (critical luminosity above which radiation pressure exceeds gravity, preventing further accretion by small-particle infall). He then estimated the size of a *white dwarf star* — one such being the companion of Sirius. (White dwarves were identified ca. 1913 by **Hertzsprung**.)

Also in that year, he put the Schwarzschild solution (to the field equations of GTR) in a new form, from which the later **Kerr** solution for the spacetime metric outside a spinning mass could be derived (1963).

Yet, since 1935, **Eddington** rejected *Chandrasekhar's limit* and never accepted it. He saw that it would lead to the '*black hole*' concept, which he thought to be a physical absurdity. Eddington's shortsightedness regarding black holes delayed the development of this field some 30 years, due to his supreme authority at that time.

Eddington was born in Kendal, England. He was educated at Owen's College, Manchester and Trinity College, Cambridge, where he was senior Wrangler in 1904 and Smith's prizeman in 1907. From 1906 to 1913 he held the post of chief assistant at the Royal Observatory at Greenwich. In 1913 he became Plumian professor of astronomy at Cambridge and in the same year was elected fellow of the Royal Society.

His work on stellar evolution, relativity, gravitation and cosmology led him to search for a relationship between all the fundamental constants of nature[346].

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(5) Gas equation of state $P_g = \rho RT/\mu$;

(6) Nuclear rate equation $f(\epsilon, \rho; T) = 0$.

The functions and parameters appearing in (1)–(6) are: T (absolute temperature), P_g (gas partial pressure), P (combined gas *and* radiation pressure), ρ (mass density), M (mass), r (radial coordinate), μ (mean molecular/ionic weight), L (luminosity, in ergs per second), σ the Stefan-Boltzmann constant, ϵ (energy generated per second per gram of stellar matter by nuclear reactions at T, ρ), χ (mass absorption coefficient of radiation); c_p and c_v denote the specific heats at constant pressure and volume, respectively.

[346] Eddington's quest was a remarkable combination of the profound, the mystic and the fantastic, setting in motion a never-ending progression of attempts to explain constants of nature by feats of pure Pythagorean numerology. He

Eddington wrote several books that explained the nature of the universe in popular terms. In one of them, "The Nature of the Physical World" (1928) he said: "*I know passages written in mathematical symbols which in their sublimity might vie with a sonnet of Shakespeare*".

1916–1928 CE **Thomas John I'Anson Bromwich** (1875–1929, England). Mathematical physicist. Contributed significantly to the theory and applications of the operational calculus (*The Bromwich Integral*).

Bromwich spent his youth in Natal and was educated in Durban. He studied at Cambridge and became a university lecturer from 1909 to 1926. He died by suicide.

1916–1932 CE **Willem de Sitter** (1872–1934, Holland). A noted Dutch astronomer and cosmologist. Solved Einstein's field equations for a static universe and found that his solution possessed the peculiar property that light from great distances is redshifted. In 1928 **Howard Percy Robertson** (1903–1961, U.S.A.) showed the de Sitter's universe could be mathematically transformed into an expanding universe.

In 1916, de Sitter applied GTR to the earth-moon-sun system to find the modification of the moon's orbit required by the new law of gravitation. He found that Einstein's theory predicted new radial and transverse perturbing forces *in addition* to Newtonian dynamics. Accordingly, the leading correction to the lunar theory obtained from Einstein's equations is a *precessional motion* in which the moon's node and perigee advance about 0.02 arcseconds per year.

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thus 'proved' (1930) that the reciprocal of the *fine structure constant*, α (modern experimental value: $\alpha^{-1} = 137.035989561\ldots$) is a whole number equal to $\frac{16^2-16}{2} + 16 + 1 = 137$. He also concocted a "theory" (1935) for the mass ratio of the proton and electron (experimental value = 1847.6...), according to which this number is equal to the ratio of the solutions of the quadratic equation $10m^2 - 136m + 1 = 0$, the form of which he believed to be dictated by the number of directions that characterize our 4 dimensions of space and time.

Few if any of Eddington's peers and colleagues accepted his views. The great theoretical physicists of his day, such as **Einstein**, **Dirac**, **Bohr** and **Born**, found his approach useless and politely confessed that they couldn't understand it. Yet, his attempts to 'explain' the constants of nature by algebraic and numerical gymnastics had enduring effects on readers of his popular books; for he conveyed to them the overwhelming impression that it might be possible to unlock some of the most deeply hidden secrets of the universe by a little bit of inspired 'Kabbalah-style' guesswork and numerology. Indeed, the physicist V. Weisskopf jokingly pointed out that the numerical value of the Hebrew word "Kabbalah" itself equals 137!

According to GTR this phenomenon is a property of the space surrounding the earth — a *precession of the inertial frame* in this region relative to the asymptotic inertial frame of the sidereal system. This was first pointed our in 1918, by **Jan A. Schouten** (1883–1971, Holland), and later by **Eddington**.

Sitter was born at Sneek, Holland and was educated at the University of Gröningen. He went (1897) to the Cape of Good Hope to work at the Cape University for 2 years. He became professor of astronomy at the University of Leyden (1908) and the director of its observatory (1919–1934). His papers, published in London during WWI, aroused scientific interest in Einstein's GTR among British scientists.

During Einstein's second visit to California (1932) he published a joint note with de Sitter in which a spatially flat universe (Euclidean space, curved spacetime) was proposed without a cosmological term and with zero pressure.

1917 CE, Nov. 07 The so-called *October*[347] *Revolution* began in Russia. A forerunner (the so-called *February Revolution*) began on March 08 with strikes, riots, and mutinies by the troops in the capital Petrograd (later called Leningrad, now called St. Petersburg). The unrest was sparked by Russian defeats in WWI and bad governance which had led to food shortages.

The revolution strongly affected the history of Europe and the rest of the world during the next 70 years. Under Communism, free thought and expression were suppressed in Russia (and after WWII — in Eastern Europe and East Germany too) for three generations. During this time, acceptable science was exclusively that endorsed by the Communist party of the USSR, whose intellectual despotism rivaled that of the Catholic Church in the Middle Ages. Consequently, the development of certain sciences was arrested and even perverted. For example, the whole of biology (especially genetics) was ravaged by ideological conflict; serious scientists were pilloried, and laboratories were closed or transformed into havens of 'Communist-style' biology led by the agronomist Trofim Denisovich Lysenko[348] (1898–1976).

The Bolshevik Revolution was the *bloodiest ever in human history*. About 30 million people were executed in the Soviet Union during 1917–1987, es-

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[347] Oct. 25 in Russia was the same as Nov. 7 elsewhere. By 1917, the Russian calendar was 13 days behind the rest of the world; the new regime brought it into line, but the old name for the uprising, persisted.

[348] With Joseph Stalin as his chief supporter, Lysenko gained control of Soviet biological research (1928–1965), and imposed his view regarding heritability of acquired characteristics.

pecially under the dictatorships of Lenin[349] (1870–1924) and Stalin[350] (1879–1953).

1917 CE, Dec. 06 *The Halifax Explosion*, Nova Scotia, Canada. Largest, most devastating man-made explosion in the pre-nuclear age.

During WWI, A Belgian ship, *Imo*, collided with a French ammunition ship, *Mont Blanc*, causing an explosion of about 200 ton of TNT in the city's harbor. The unfortunate disaster killed about 2000 persons, injure 9000 more, and wrecked much of Halifax (about 1630 houses were obliterated and 12,000 more damaged).

1917 CE **Albert Einstein** (1879–1955, U.S.A.) established the theory of spontaneous and induced (stimulated) radiative transitions resulting from the interaction of radiation with atoms. During 1953–1960, this theory was applied to develop *masers* and *lasers*.

Consider two atomic or molecular states of energies E_1 and $E_2 > E_1$, occupied by N_1 and N_2 atoms (or molecules), respectively. The photons corresponding to transitions between these two levels must have an energy close to $h\nu = E_2 - E_1$, ν being the photons' frequency. Atoms at level E_2 may jump spontaneously into level E_1, emitting a photon. Let us call A_{21} the corresponding spontaneous emission transition probability per unit time per atom. If ambient radiation of frequency ν is present and its energy density is $\epsilon(\nu)$, absorption transitions from E_1 into E_2 are also produced.

It is natural to assume that the number of such transitions per unit time is proportional to the energy density $\epsilon(\nu)$; that is, the induced absorption transition probability per unit time is $B_{12}\epsilon(\nu)$, where B_{12} is the transition

[349] Born in Simbirsk, Russia, the son of Ilya Nikolaevich Ulyanov (1831–1886, a Russian civil service official) and Maria Alexandrovna Blank (1835–1916). Her father was **Srul Moishevich Blank**, born to a Jewish family in Zhitomir and baptized as Alexander Dimitrievich Blank. Vladimir Ulyanov, renamed himself *Lenin* after the River Lena. He was 1/4 Jewish, 1/4 German, 1/4 Mongol and 1/4 Russian.

[350] Jews played a major part in the Bolshevik Revolution. Among them: **Leon Trotsky** (Bronstein) 1879–1940; **Lazar M. Kaganovich**, 1893–1991; **Grigory Zinoviev** (Apfelbaum), 1883–1936; **Maxim Litvinov** (Wallach), 1876–1951; **Lev Kamenev** (Rosenfeld), 1883–1936; **Nikolai Yezhov**, 1895–1939; **Alexei Rykov**, 1881–1938; **Genrikh Yagoda**, 1891–1938; **Yakov Sverdlov** (Solomon), 1885–1919; **Mikhail I. Kalinin**, 1875–1946; **Yuri Andropov**. Also, **Stalin**, **Molotov**, **Kruschev**, **Voroshilov** and **Brezhnev** — were married to Jewish women.

probability per unit time, unit intensity of the radiation, and per single lower-level atom.

But the radiation, because of its interaction with excited atoms at level E_2, also produces emission transitions from E_2 to E_1 with an induced emission transition probability per unit time and per atom $B_{21}\epsilon(\nu)$. Therefore the total emission probability per atom per unit time from level E_2 to level E_1 is $A_{21} + B_{21}\epsilon(\nu)$. If there are N_2 atoms in level E_2, the mean number of atoms that jump per unit time from E_2 to E_1 is $[A_{21} + B_{21}\epsilon(\nu)]\, N_2$. At the same time, the mean number of atoms that jump from E_1 to E_2 per unit time is $B_{12}\epsilon(\nu)N_1$. If N_1, N_2 are large enough, the statistical uncertainties in these rates are small as fractions of the rates themselves, and the *actual* jump (transition) rates may be taken to equal the respective *mean* rates.

Therefore the net change per unit time of the atom population in level E_2 is equal to the rate of gain by absorption minus the rate of loss by emission, or

$$\frac{dN_2}{dt} = \underbrace{B_{12}\epsilon(\nu)N_1}_{\text{Absorption}} - \underbrace{[A_{21} + B_{21}\epsilon(\nu)]N_2}_{\text{Emission}}$$

with an equal (but opposite) gain for the lower level.

When equilibrium is established between the atoms and the radiation (which will eventually occur at fixed thermodynamic conditions), we must have $-dN/dt = dN_2/dt = 0$, or $B_{12}\epsilon(\nu)N_1 = [A_{21} + B_{21}\epsilon(\nu)]\, N_2$, so that the numbers of absorption and emission transitions per unit time (that is, the transition rates, in the $1 \to 2$ and in the $2 \to 1$ directions) are the same.

If the atoms are in thermal equilibrium and follow Maxwell-Boltzmann statistics (which is a reasonable approximation in most cases), then

$$N_1/N_2 = e^{(E_2-E_1)kT} = e^{h\nu/kT},$$

so that

$$B_{12}\epsilon(\nu)e^{h\nu/kT} = A_{21} + B_{21}\epsilon(\nu)$$

or

$$\epsilon(\nu) = \frac{A_{21}/B_{12}}{e^{h\nu/kT} - B_{21}/B_{12}}.$$

When we compare this spectrum with Planck's formula

$$\epsilon(\nu) = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/kT} - 1},$$

which gives the energy density for electromagnetic radiation in equilibrium with matter, we find that

$$\frac{A_{21}}{B_{12}} = \frac{8\pi h\nu^3}{c^3} \qquad \textit{and} \qquad \frac{B_{21}}{B_{12}} = 1,$$

a result first obtained by Einstein in 1917. The second relation shows that the induced emission and absorption probabilities per unit time are equal and actually follows merely from time-reversal symmetry plus the wave aspect of light. The above calculation does not allow us to obtain the absolute values of A_{21}, B_{21}, and B_{12}; they must be derived using quantum-mechanical considerations. With $B_{12} = B_{21}$, we obtain the ratio between the spontaneous emission probability rate A_{21} and the induced emission probability rate $B_{21}\epsilon(\nu)$ when matter is in equilibrium with radiation as

$$\frac{\text{Spontaneous emission probability}}{\text{Induced emission probability}} = \frac{A_{21}}{B_{21}\epsilon(\nu)} = e^{h\nu/kT} - 1.$$

Therefore, if $h\nu \gg kT$, spontaneous emission is much more probable than induced emission, which can then be completely neglected. This usually holds true in the case of electronic transition in atoms and molecules and in the case of radiative transitions in nuclei unless a far-from-equilibrium regime is set up (e.g. by *inverting* the normal level populations, as done by *pumping* in lasers and masers). But if $h\nu \ll kT$, as in the microwave region of the spectrum at room temperature, induced or stimulated emission becomes important.

In the same year[351], Einstein found a coordinate-invariant generalization of the *Bohr-Sommerfeld semiclassical quantization conditions* for a system with l degrees of freedom.

Bohr obtained the semiclassically–stable orbits of an electron revolving around the nucleus, for the case of circular orbits, by assuming that the total *orbital angular momentum* of these orbits must be quantized, i.e. be an integer times $\hbar = \frac{h}{2\pi}$. But it was soon realized that more generally, the quantity which should be quantized is the (classical) *action* — i.e. $\oint p\,dq = nh$ where p is the relevant canonical momentum and q the conjugate coordinate, integrated over a cycle (period) of the motion. This was an acceptable answer for *circular* orbits, which could be characterized by a single parameter, namely the radius of the orbit. But we can equally well have elliptical orbits, which had to be characterized by two parameters (apart from the space orientation of the orbit).

Generally, the question arose as to how to quantize systems of arbitrary numbers of degrees of freedom. Sommerfeld gave a partial answer for the case

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351 'Zum Quantensatz von Sommerfeld und Epstein'. *Verh. Deutsch. Phys. Ges. 19*, 82, 1917. A little known remark of Einstein, made in a lecture at a meeting of the German Physical Society and only published in its proceedings. He never returned to this subject nor, for a long time, did others show much interest in it. However, since 1980, the importance and pioneering character of this work has been recognized by mathematicians, quantum physicists and quantum chemists.

of '*separable systems*'. This is the case in which the classical Hamiltonian function could be written as a sum of functions, each one involving only a pair of conjugate variables:

$$H = H_1(p_1, q_1) + H_2(p_2, q_2) + \cdots + H_k(p_k, q_k).$$

In this case we could write down the necessary and sufficient quantization conditions as $\oint p_i \, dq_i = n_i h, \quad i = 1, 2, \ldots k$. Thus, the conditions, if at all realizable, depend on the choice of a particular coordinate system. Moreover, such a procedure was not *unique* since if a system was separable in different coordinate system (e.g., an isotropic 3-D harmonic oscillator is separable in both Cartesian *and* polar coordinates), inequivalent quantization rules might be obtained.

Einstein could not believe that the particular coordinate system, in which the Hamiltonian function was accidentally separable, should have a decisive significance for the physical phenomenon. He found a *coordinate-invariant generalization* of these quantization conditions which, moreover, did not require the motion to be separable, but only *multiply periodic*; he formed the *sum* of all the equations $\oint p_i \, dq_i = n_i h$, obtaining $\oint \sum_{i=1}^{k} p_i \, dq_i = nh$, where n is a new integer. Since $p_i = \frac{\partial S}{\partial q_i}$ with S the Hamilton-Jacobi *action*, we obtain

$$\oint \sum_{i=1}^{k} p_i \, dq_i = \oint \sum_{i=1}^{k} \frac{\partial S}{\partial q_i} \, dq_i = \oint dS = \Delta S = nh,$$

where the integral is taken over *any* possible closed path in configuration space. The '*action*' function S has absolute meaning. If S were a single-valued function, then, traversing a periodic orbit in configuration space and returning to the initial point, ΔS would be zero. But S is, in fact, a *multivalued function.*

While classically it is in principle possible to proceed along *any* closed circuit, in quantum theory only those circuits in configuration space have physical significance for which the change of S in this process is equal to a multiple of h. If we now choose a path for which all coordinates remain constants except p_1 and q_1, then we obtain the first quantum condition, and so on. Thus, Einstein's single condition is equivalent, in (any) separable classical phase–space coordinates, to the entire set of the Bohr-Sommerfeld quantum conditions[352].

352 Einstein, of course, was not aware at the time of wave mechanics and the Schrödinger Equation (1924), and therefore missed the connection between the Bohr-Sommerfeld semiclassical theory and the exact (fully quantum-

The General Theory of Relativity[353]

During 1915–1917 **Albert Einstein** *presented the general theory of relativity (*GTR*), on which he had been working since 1907. It is an extension of his special theory to include gravitational effects*[354].

In addition to electromagnetic fields, there exist in nature long-range fields of another type — so-called gravitational fields, or fields of gravity. These fields have the basic property that all (small and light enough) test bodies move in them in the same manner, independent of mass, composition or charge, provided the initial conditions are the same. For example, the laws

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mechanical) picture, yielding (in the WKBJ or semiclassical approximation) $\oint dS = (n + k/2)h$. Interestingly enough, *Einstein's remark* became relevant in recent years to *classical chaos theory* in the following way: In a holonomic *integrable* mechanical system the number of conservation laws is equal to the number of degrees of freedom of the system. Here, the trajectory can be written analytically and no chaos is possible. A necessary condition for the existence of chaos is that the number of conservation laws is less than the number of degrees of freedom. But each quantum number corresponds to a certain conservation law. Hence the connection between integrability, conservation, chaos and quantization conditions.

[353] For further reading, see:

- Weinberg, S., *Gravitation and Cosmology*, Wiley, 1972, 657 pp.
- Ohanian, H.C., *Gravitation and Spacetime*, W.W. Norton, 1976, 461 pp.
- D'inverno, R., *Introducing Einstein's Relativity* Clarendon Press: Oxford University Press, 1992, 383 pp.
- Eddington, A.S., *The Mathematical Theory of Relativity*, Cambridge University Press: Cambridge, 1963, 270 pp.
- Martin, J.T., *General Relativity*, Ellis Horwood, 1988, 176 pp.
- Kenyon, I.R., *General Relativity*, Oxford University Press, 1990, 234 pp.

[354] If we regard his STR paper (1905) as the unification of the concepts of *space* and *time*, and his $E = mc^2$ paper (1905) as the unification of the concepts of *energy* and *matter*, then GTR (1915) is the unification of the above four concepts with gravitation and with the Newtonian concept of *inertial frames.* GTR is the greatest discovery in connection with gravitation that has been made since Newton first enunciated his universal law of gravitation.

of free fall in the gravity field of the earth (in vacuo) are the same for all bodies; whatever their mass, all acquire one and the same acceleration under identical initial conditions.

The theory of gravitational fields, constructed on the basis of this principle and the local applicability of the theory of Special Relativity, is called the general theory of special relativity *(GTR). It was established by Albert Einstein and finally formulated by him in 1916. It represents the most pristinely beautiful of all existing theories in the natural sciences. It was developed by him in a purely deductive manner from a small set of axioms and assumptions, and only later was substantiated by astronomical observations.*

GTR is the culmination of theories of time and space in modern times; like STR before it, it is also a framework *into which non-gravitational forces and fields may be accommodated. Three roads led to this summit:*

- *the* physics *trail with the beacons of* ***Galilei*** *(1609),* ***Newton*** *(1687) and* ***Maxwell*** *(1873).*
- *The* mathematics *trail of* ***Gauss*** *(1827),* ***Riemann*** *(1854) and* ***Ricci*** *(1887).*
- *The* Philosophy *trail of* ***Spinoza*** *(1676) and* ***Mach*** *(1872).*

Einstein's theories of relativity (special and general) represented – and precipitated further – major revolutions in physics and astronomy during the 20th century. They introduced to science the modified concept of 'relativity', thus superseding the 200 year old Newtonian mechanics. Einstein showed that we reside not in a flat Euclidean space and uniform absolute time of everyday experience, but in another environment: curved, locally Minkowski space-time. The theory of STR played a role in advances in physics that led to the nuclear era, *with its potential for benefit as well as for destruction. Its unification with* quantum *mechanics led to QED and other* quantum field theories, *predicted* antimatter, *and made possible an understanding of the microworld of* elementary particles and fields *and their interactions. The differential geometry of GTR inspired quantum field theories based upon the* Gauge Principle, *and GTR itself has also revolutionized our view of* cosmology *with its predictions (alone and in conjunction with nuclear and particle physics) of apparently bizarre astronomical phenomena such as the* big bang, neutron stars, *cosmological constant (dark energy), (primordial and eternal) inflation,* black holes[355], *gravitational lensing and gravitational waves.*

[355] Albert Einstein's general theory of relativity and the discoveries by himself and others in quantum theory are the foundation for all speculations about the

Einstein was not satisfied with STR *because it arbitrarily selected a certain class of reference frames, namely, inertial frames, among which the laws of physics are invariant. He believed that physical laws should not depend at all upon the choice of reference frame. He was also strongly influenced by Mach's criticism of the foundations of mechanics, and the resulting Mach Principle according to which inertial forces have their origin in the total distribution of mass in the universe. For example, Newton had argued that the centripetal force of a rotating mass was a demonstration of its absolute motion. Mach, however, ascribed that (and other inertial) force to motion w.r.t. other masses in the universe, especially those of the fixed stars.*

Einstein knew that it is possible to make gravity disappear in certain localized *regions in space and time by a nonlinear transformation of the reference frame. For example, freely falling sky divers (with acceleration = g) do not initially observe the force of gravity in their* moving *reference frame. Similarly, observers in an orbiting space shuttle are weightless, as the shuttle is free-falling and has a constant acceleration directed towards the earth and shared by its passengers. These results are consequences of the equivalence of gravitational mass and inertial mass, but there is no single accelerated frame of reference that can cause gravity to disappear* at all points of space-time.

Thus Einstein asked himself, "Is it possible to find a geometry of spacetime that accounts for all the gravitational effects of the masses in the universe, and yet in a small enough region of spacetime reduces to an ordinary accelerated, instantaneously inertial, STR frame of reference?"

Part of the mathematical answer to this question was already at hand, in the generalized geometry of curved spaces devised by Riemann in 1854. For example, the surface of a sphere is a 2-dimensional curved space embedded in an Euclidean space of 3 dimensions. A very small area of the surface has essentially the properties of a flat Euclidean space. Einstein required a 4-dimensional curved space-time. Such a space-time can be embedded in a 10-dimensional Euclidean space, but it is much easier to deal with the intrinsic curvature of 4 dimensions and to discard Euclidean (or even Minkowskian) geometry altogether.

Einstein used tensor calculus in the mathematical *formulation of* GTR, *since both are concerned with the behavior of various entities under the transition from a given coordinate system to another. In his own words (1915):*

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physics of black holes. Yet Einstein rejected the idea of such bizarre singularities and repeatedly argued against their existence. In the late 20^{th} century numerous galactic – center and stellar black hole candidates have been identified by astronomers, but the close observation (let alone manipulation or creation) of black holes with empirical verification of their more exotic properties – remain a distant dream.

"Sie (the gravitational equations) bedeutet einem wahren triumph der durch Gauss, Riemann, Christoffel, Ricci, ..., begründeten methoden des allgemeinen differentialkalkulus".

Through this theory, tensor analysis came into vogue. The combined efforts of ***Grassmann****,* ***Riemann****,* ***Christoffel****,* ***Ricci****,* ***Minkowski*** *and others were blended here in harmony to form the most beautiful physical theory yet concocted by the human mind. It gave a great impetus to tensor theory and opened wide new areas in theoretical physics and applied mathematics.*

After 1916, with the advent of general relativity, the several special brands of vector analysis were supplanted by tensor algebra and analysis. Riemann and ***Clifford****, in the intermediate stage leading from Grassmann to tensors, had presciently predicted the 20th century* geometrization of some parts of mathematical physics.

This remarkable prophecy was indeed realized in Einstein's theory of gravitation and its developments since 1916, as well as in the 'non-Abelian' generalizations of electrodynamics (Yang-Mills field theories) partially inspired by it. Furthermore, in the 1920's, the general theory of relativity stimulated development of differential geometry.

Underlying principles

Newton's law of gravitation, which requires instantaneous *action at a distance is* not compatible *with STR (= Special Theory of Relativity) since the latter requires that the velocity of signal propagation be* finite *and that the gravitational laws be* Lorentz-covariant. *During 1912–1918 various physicists*[356] *attempted to formulate gravitation within the framework of STR. But all these theories employ fields that inhabit the same Minkowski space-time as does the Maxwell field, and as such suffer from two fatal flaws — no* gravitational redshift *and no* light deflection. *They are thus untenable by our current experimental knowledge.*

Einstein based his theory upon four basic principles:

- *The principle of* equivalence

356 **Max Abrahem** (1912); **G. Nordström** (1918) and others. In these attempts gravitation was formulated as carried by a scalar, vector or even a tensor field, inhabiting the same Minkowski space-time as does the Maxwell field.

- *The principle of general covariance (alternate form of Equivalence Principle)*
- *The principle of correspondence*
- *The principle of minimal gravitational coupling*

In Newtonian theory, a system in a homogeneous gravitational field is completely equivalent to a uniformly accelerated reference frame from a mechanical point of view [famous 'elevator in free fall thought – experiment']. Einstein then postulated that this equivalence be extended to all physical processes embedded in a locally homogeneous gravitational fields. This is the restricted form of the principle of equivalence.

Combining this postulate with STR, Einstein derived the result that the rate of clocks at points of lower gravitational potential is slower than that for higher gravitational potential, and already then he pointed out that this entails a shift toward the red spectra of light emitted by the sun, compared with that emitted by terrestrial sources. A further result was that the velocity of light is not constant[357] *in a gravitational field, so that light rays curve (Fermat's principle). He also argued that on the strength of the equivalence principle and STR, energy E must be ascribed not only an inertial mass but also a gravitational mass* $m = E/c^2$.

The next step was to generalize the principle of equivalence to apply to inhomogeneous gravitational fields. This, he claimed, can be done in an infinitesimally small region of space-time. Here gravitation can be transformed

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[357] In GTR the principles of STR hold in any instantaneous, local, inertial Lorentz frame. In particular, this implies the *local* constancy of the speed of light. However, when one describes the geodetic world line of light ray or massive particle, *any* curvilinear coordinate system used, cannot be a Lorentz frame, because it extends over a *finite* volume of spacetime.

Therefore, any experiment in GTR that involves finite lengths and time–intervals will, in general, violate the postulates of STR. This was demonstrated quite clearly in the sun – grazing radar – ranging experiment by **I. Shapiro** (1970s), in which radar beams were found to slow down in the vicinity of the sun relative to a reference frame defined by earth and Mars, as predicted by GTR.

The converse effect may also occur: in a de Sitter universe, any two particles of matter recede from each other exponentially fast, and thus transluminally, as reckoned by an extended reference system. The now-accepted inflationary version of Big–Bang cosmology states that such a de Sitter phase indeed occurred in the very early universe and satellite observations in the 1970s showed that we may be entering a new de Sitter phase.

away in a local coordinate system realized by a freely falling, sufficiently small cabin, which is not subjected to any external forces apart from gravity. The formulation of the equivalence principle in this case is:

"For every infinitely small world region, in which spatial and temporal variations of gravity can be neglected, there always exists a coordinate system in which gravitation has no influence either on the motion of particles or any other physical processes".

*This 'transforming away' is only possible because the gravitational field has the fundamental property that it imparts the same acceleration to all bodies, or stated differently, because the gravitational mass is always equal to the inertial mass (**Newton**, **Eötvös**). This is sometimes called the 'weak equivalence principle' when the falling mass (or cabin) has a low enough mass for self-gravity to be negligible.*

These two different statements are equivalent: When gravitation acts on a body, it acts on the "gravitational mass", but the result of the Newtonian gravity force is an acceleration determined by the body's inertial mass. Both statements imply at once that the path followed by a test particle in space and time under purely gravitational forces is independent of the mass and composition of the test-particles. Moreover, the curvature of space-time is a direct consequence of the fact, evident in Newtonian gravity, that the equivalence principle breaks down over finite world-volumes for a non-uniform gravitational field ("tidal effects").

When Einstein proposed his equivalence principle in 1907, he was not aware of all the experiments that predated him, and arrived at it by noticing that inertial forces (centrifugal, Coriolis) share the same property as gravitational forces, in that they all are proportional to the masses they act upon, unlike electric or magnetic forces. This similarity makes it impossible to tell them apart in the small cabin postulated above.

The equality of inertial and gravitational mass for different materials is quite mysterious in Newton's work.

At this point, Mach's principle comes to mind as a feasible explanation of the identity of gravitational and inertial mass, via Mach's idea that the origin of inertia is gravitational. Indeed, in 1912 Einstein calculated the effect of a heavy spinning spherical shell on an enclosed particle. The calculations showed that the effect of the rotating massive shell is to change the inertial frame in the spatial vicinity of the particle, which now tends to rotate in the same sense as the shell. This calculation foreshadowed the 'Lense-Thirring effect' (1918).

With the aid of the equivalence principle, STR was reinstated as a tool for handling physical problems in the presence of gravity on a local basis. *An alternative statement for the principle of equivalence is:*

"The laws of special relativity hold locally in a space-time, in a sufficiently small, freely falling frame of reference".

Clearly, the Principle of Equivalence is strictly valid only in the presence of a static homogeneous gravitational field. Had the gravitational field depended on $\boldsymbol{r}$ *or* t*, we would not have been able to eliminate it from the equation of motion by the acceleration. For example, the earth is in free fall about the sun, and for the most part we on earth do not feel the sun's gravitational field, but the* slight inhomogeneity *in this field (about 1 part in 6000 from noon to midnight) is enough to raise impressive tides in our ocean. Even the observers in Einstein's freely falling cabin would, in principle, be able to detect the earth's field because objects in the cabin would be falling radially toward the center of the earth, and hence would approach each other laterally as the elevator descended.*

Nevertheless, it is sufficient for the construction of GTR *(General theory of Relativity) to accept the* weak Principle of Equivalence *stating the* observed *equality of gravitational and inertial mass for systems of small enough mass.*

Equivalently, the Principle of Equivalence says that at any point in space-time we may erect a locally inertial coordinate system in which matter and energy satisfy the laws of special relativity.

In this form, there exists a certain resemblance to the Gauss assumption that at any point on a curved surface we may erect a locally Cartesian coordinate system in which distances obey the laws of Euclid Pythagoras. Because of this analogy, we should expect the laws of gravitation to bear strong resemblance to the formulas of Riemannian geometry.

In particular, Gauss' assumption implies that all intrinsic geometric properties of a curved surface *can be described in terms of the quantities* g_{ij} *(metric tensor) in some "atlas" of overlapping, mutually-transformable, local coordinate systems.*

In light of this analogy, the principle of equivalence can be reformulated as the principle that gravitation is just an effect of the curvature of Minkowskian spacetime *— a principle from which Einstein's theory of gravitation follows almost uniquely.*

An alternative version of the Principle of Equivalence is known as the Principle of General Covariance. *It states that a physical equation holds in a general gravitational field if two conditions are met:*

(1) The equation holds in the absence of gravitation, i.e. it agrees with the laws of special relativity when the metric tensor $g_{\alpha\beta}$ equals the Minkowski tensor $\eta_{\alpha\beta}$ (i.e. $diag\{-1,1,1,1\}$ in the real–0^{th}–component representation) and when the affine connection $\Gamma^{\alpha}_{\beta\gamma}$ vanishes.

(2) The equation is generally covariant; i.e., it preserves its form under a general local coordinate transformation.

It should be stressed that part (2) of general covariance by itself is devoid of physical content. Any law of nature can be made *generally covariant by writing it in any one coordinate system, and then working out what it looks like in other arbitrary coordinate systems. Indeed, physicists have long been familiar with the appearance of physical equations in non-Cartesian systems, such as polar coordinates, and in noninertial systems, such as rotating coordinates. The significance of the Principle of General Covariance lies in its statement about the effects of gravitation: that a physical equation, by virtue of its general covariance, will hold in a gravitational field if its flat–Minkowski–space limit represent the correct dynamics in the absence of gravitation and if, in the latter case, it is Lorentz* covariant *and obeys the principles of Special Relativity.*

The meaning of general covariance can be elucidated by comparing it with Lorentz invariance/covariance. Just as any equation can be made generally covariant, so any equation can be made Lorentz-covariant, by writing it in one inertial coordinate system and then working out what it looks like after a Lorentz transformation.

However, if we do this with a nonrelativistic equation like Newton's second law, we find after making it Lorentz-covariant that a new quantity has entered the equation, which of course is the velocity of the coordinate frame with respect to the original, privileged reference frame. The requirement that this velocity not appear in the transformed equation is what we call the Principle of Special Relativity, or "Lorentz invariance" for short, and this requirement places very powerful restrictions on the original equation. Thus part (1) *of the General Covariance Principle is what bestows content upon it.*

Similarly, when we make an equation generally covariant, new entities will enter — that is, the metric tensor $g_{\mu\nu}$, the affine connection $\Gamma^{\lambda}_{\mu\nu}$, the Riemann curvature tensor, and its covariant derivatives. The difference is that we do not require that these quantities drop out at the end, and hence we do not obtain any new restrictions on the equation we start with; rather, we exploit the presence of $g_{\mu\nu}$ and $\Gamma^{\lambda}_{\mu\nu}$ to represent gravitational fields.

To put this succinctly: The Principle of General Covariance is not *an invariance principle, like the Principle of Galilean or Einsteinian (Special) Relativity. Taken together with part (1), however, it becomes a statement* *about the effects of gravitation and its local (approximate) equivalent to acceleration relative to STR inertial frames. In particular, part (2) of general covariance does not imply Lorentz invariance — there are generally covariant theories of gravitation that allow the construction of inertial frames at any point in a gravitational field, but that satisfy Galilean relativity rather than special relativity in these frames.*

Any physical principle, such as general covariance, which takes the form of a local covariance principle but whose content actually involves the introduction of a new class of interactions into otherwise known dynamics, is called a dynamical local symmetry*. There are other dynamic symmetries of importance in physics, such as the GTR-inspired* local gauge invariance*, which governs the interactions of the electromagnetic field and all nuclear and subnuclear forces save gravitation.*

The Principle of General Covariance can only be applied on a scale that is small compared with the space-time distances typical of the gravitational field, for it is only on this small scale that we are assured by the Principle of Equivalence of being able to construct a coordinate system in which the effects of gravitation are absent. However trough the magic of differential geometry, this allows one to deduce gravitational effects at all *scales, provided a few other simple assumptions are made.*

There are in general many generally covariant systems of equations, that reduce to a given special-relativistic theory (e.g. the Maxwell–Lorentz equations) in the absence of gravitation. However, once two additional assumptions are made – namely, that Newton's theory of gravity is recovered for speeds $v \ll c$ *and weak fields, and that only* $g_{\mu\nu}$ *and its first and second order partial derivatives enter our generally covariant equations – we shall see that the Principle of General Covariance makes an almost unambiguous statement about the effects of gravitational fields on any physical system (planets, galaxies, fluids, or even the universe as a whole).*

Observers are intimately tied up with their reference systems or coordinate systems. One ramification of GTR is that any observer can discover the laws of physics, employing any coordinate system. The situation is somewhat different in special relativity, where, because the metric is flat and the connection integrable, there exists a set of canonical *or preferred coordinate systems; namely, Minkowski coordinates. In a curved space-time, that is, a manifold with a non-flat metric, there is in general no canonical coordinate system. This is just another statement of the non-existence of a global inertial frames.*

However, this statement needs to be treated with caution, because in many applications, there will be preferred coordinate systems. For example, many problems possess symmetries and the simplest thing to do is to adapt the coordinate system to the underlying symmetry.

Any new theory must be consistent with any acceptable earlier theories within their range of validity. GTR must therefore agree on the one hand with STR in the absence of gravitation and on the other hand with Newtonian gravitational theory in the limit of weak gravitational fields and low velocities (compared with the speed of light). This gives rise to a correspondence principle *for general relativity.*

The principles discussed so far almost suffice to obtain equations governing systems in general relativity when the corresponding equations are known in special relativity. The principle of minimal gravitational coupling *– which is somewhat stronger than the above restriction on the number of times the metric is differentiated – is a* simplicity principle *or Occam's razor that essentially states that we should not add unnecessary terms in making the transition from the special to the general theory.*

Consider, for example, the law of local conservation of energy-momentum in STR

$$\frac{\partial T^{\mu\nu}}{\partial x^{\mu}} = 0$$

The simplest generalization of this to GTR is the tensor equation

$$\nabla_{\mu} T^{\mu\nu} = 0$$

where ∇_{μ}, denotes covariant differentiation. However, we could equally well posit

$$\nabla_{\mu} T^{\mu\nu} + g^{\nu\lambda} R^{\mu}_{\alpha\mu\beta} \nabla_{\lambda} T^{\alpha\beta} = 0$$

since the Riemann curvature tensor $R^{\mu}_{\alpha\mu\beta}$ vanishes in STR.

Einstein, although not stating it explicitly, used this minimal-coupling principle in establishing the gravitational field equations.

The principle of minimal gravitational coupling can thus be stated: "no terms explicitly containing the curvature tensor should be un-necessarily introduced in making transition from STR to GTR, and no covariant derivatives of the curvature tensor shall be introduced at all".

This 'principle' is equivalent to the a priori assumption *that the equations of general relativity must be* second-order partial differential equations *in the metric tensor and in all non-gravitational dynamical fields and variables and that they are linear in second-order partial derivatives of $g_{\mu\nu}$. In 1915 – and indeed to this day – the most important equations of non-gravitational fundamental theoretical physics were second-order PDE's obeying this restriction, including of course, the Maxwell equations themselves.*

Any terms in the curved–spacetime equation involving non-minimal gravitational couplings would not make much of a difference in any astronomical *observations. Einstein could have made his equations fourth-order differential equations, but he did not. In this sense Einstein's theory, as we view it today, is an approximation valid at long distances, and as such, cannot be expected to deal successfully with infinities and singularities — any more then the Navier–Strokes PDEs of fluid dynamics can be expected to account for the molecular motions at the center of a vortex flow.*

We note that arbitrary coordinate transformations obviously include a subset of transformations that take us locally from any non-inertial frame to an inertial frame. In the latter frames — locally freely–falling frames — gravitational forces on small enough objects are negligible; this is nothing but the equivalence principle. The abstract form of this statement is "general covariance" — if a generally covariant equation is known to be true in a locally inertial frame (in absence of gravitation, curvature or connection), it is true in every frame, i.e. the equation is true in the presence of gravitation.

This is not strictly true for finite laboratories. The precise statement is that gravitational forces are not detectable in the limit where:

(I) the freely–falling laboratory, containing the measuring apparata, is infinitesimally small;

(II) the total mass of the said laboratory is also infinitesimal.

Gravitational effects are *observed in a* non*-ideal 'locally inertial' frame, such as Einstein's famous freely-falling elevator. Such an elevator is not infinitesimal — it encompasses a sufficient volume of space-time to reveal tidal effects in any realistic (non-uniform) gravitational field.*

We must dwell on this important issue in some detail: Consider a non-spinning space capsule, whose center of mass moves in a free-falling orbit of instantaneous distance r from the earth of mass M (or other attracting center of mass). A test particle at the center of the capsule, will experience a zero-g environment: it is 'weightless', acted upon by no gravitational force as reckoned in the capsule frame.Thus, the astronaut does not know whether he is falling in a gravitational field or at rest in some region far away from any attracting masses. (Clearly, this elimination of the earth's gravitational field is only possible under the assumption of equivalence of inertial and gravitational mass for all bodies.

Now, to another observer, at rest relative to the (idealized) "fixed stars", the capsule is in accelerated motion, and such an observer will refute the astronaut's claim that he is in an inertial frame by saying that both the space capsule and the test particle in it are falling at the same rate and that the astronaut is being fooled by appearances.

Suppose, however, that the astronaut places a drop of liquid of radius R at the center of the capsule. He will find that this drop is not exactly spherical *but has two bulges; one bulge points toward the earth, one away. Since, in the absence of external forces, surface tension would make the drop spherical, the deviation from a sphere indicates the existence of a gravitational field. The bulges result from the* inhomogeneity *of the gravitational field; the capsule as a whole is accelerated toward the mass M by an amount $a_G = \frac{GM}{r^2}$, the end of the drop nearer to the earth is subjected to a slightly larger acceleration $a_{\text{near}} = \frac{GM}{(r-R)^2}$, while the far end experiences the slightly smaller acceleration $a_{\text{far}} = \frac{GM}{(r+R)^2}$.*

Accordingly, the relative acceleration of the two ends will cause them to drift away from each other and the center. *Assuming $R \ll r$, each will drift with a* tidal acceleration $a_{\text{tidal}} = \frac{1}{2}(a_{\text{near}} - a_{\text{far}}) \approx 2\frac{GMR}{r^3} = a_G \frac{2R}{r}$. *Similarly the tidal effect on two diametrically opposite points on the drop's surface which lie on the plane* perpendicular *to the first pair, is shown to be $a_G \frac{R}{r}$, and these points will tend to drift* toward each other *at one-half the former ('longitudinal') rate.*

Thus, the drop experiences a deformation due to a quadrupole *force-field such that it is stretched at its 'poles' and compressed at its 'equator'. Next, imagine the drop is a planetesimal, devoid of surface tension but held together by its self-gravity. Denote by g the acceleration of gravity that the drop produces at its own surface, and by Δh the differential tidal deformation (difference between 'high-tide' and 'low-tide'). The equilibrium equation $g\Delta h = \frac{GMR^2}{r^3}[2 - (-1)] = \frac{3GMR^2}{r^3}$ in which $g = \frac{4\pi}{3}\rho GR$, then yields $\frac{\Delta h}{R} = \frac{9M}{4\pi\rho r^3}$. This shows that the* shape *of the tidal ellipsoid is independent of its size. Even in the limit $R \to 0$, the tidal deformation persists. We can therefore regard the prolateness of the tidal ellipsoid as a* local *measure of the gravitational tidal force.*

This principle has been embedded in a sensitive instrument known as the gravity gradiometer *(***R.L. Forward***, 1971). The device consists of a Greek cross with four masses at the ends of its arms. The arms are held together at the center by a torsional spring; when the arms are pressed together and then released they oscillate with their own natural frequency. If the cross is placed in a tidal field, it will be deformed.*

The tidal force is then used to drive the cross at resonance*, where large amplitudes can be built up. Such an instrument, if made small enough, can be used to make* local *measurements of the tidal field, in an arbitrarily small neighborhood of a given point. The limitations on the minimum size of the neighborhood that is needed to perform measurements of a given precision do* *not arise from any intrinsic properties of the gravitational field; rather, these limitations arise from the* quantum nature *of matter.*

To summarize, local *experiments can distinguish between a reference frame in free fall in a gravitational field and a truly inertial reference frame placed far away from all gravitational fields. Gravitational effects are* not truly equivalent *to the effects arising from the observer's acceleration. Thus, the equivalence of gravitation and acceleration is only true in a limited sense: they are equivalent only as far as the translational motion of infinitesimal-mass point particles is concerned.*

If the rotational degrees of freedom of the motion of masses are taken into account, then the equivalence fails in yet other ways. However, the unambiguous equality $m_I = m_G$ *is necessary and, to a large extent, sufficient for the construction of* GTR. *And this becomes a* physical principle *when one posits that all physical laws (including gravitational dynamics itself) are expressible via generally – covariant equations, and that* STR *holds in local free–falling frames for any non–gravitational experiment. This reformulation is very important because it is strictly valid in general relativity; whereas the equivalence principle itself, which inspired it, is not even strictly true in Newtonian mechanics — as the above example demonstrates.*

On the other hand, one way ask whether residual gravitational effects within a small enough frame – such as the tidal effects in the liquid drop discussed above, or a Cavendish–type experiment to measure G – can reveal the difference between a free–falling frame and an inertial frame far from any external masses. If so, we say that the strong *equivalence principle (*SEP*) hold; and it has indeed been shown to hold in* GTR.

The principle of general covariance gives us a precise[358] *prescription through which we can introduce gravitational forces into a physical situation, if we know the equations describing the system in the absence of gravitation. For the latter equations, one takes the laws of special-relativistic physics. To modify these non–gravitational dynamical equations in the presence of gravitation, one takes the corresponding Lorentz–covariant equations and renders them generally covariant. As for gravitation itself, one chooses the minimally–complicated generally covariant extension of Newtonian gravitation, in which Newton's gravitational potential becomes an element in the metric tensor.*

[358] Though not unique: one may add to the covariantized physical equations any number of arbitrary–coefficient, covariant terms involving the Riemann curvature tensor or its covariant derivatives, since such terms vanish in the absence of gravitation. Such terms can, however, be estimated and bounded, and their effects are negligible in all post-Newtonian observational and experimental tests of GTR carried out to date.

The mathematical tool through which the principle of general covariance can be easily implemented and verified is that of general tensor calculus.

In Newtonian mechanics, the trajectory of a free particle is a straight spatial (Euclidean) line, traversed with constant velocity. In STR *it is a straight line in the Minkowski spacetime. In* GTR*, the world line of an infinitesimal inertial (freely falling) observer is a time-like curve with extremal proper time — the* geodesic *curve, or locally* straightest world line*. Along this line the* proper time *of the particle (the register of a clock attached to it) is extremal.*

The geometrical interpretation of the gravitational field is that a gravitating body actually distorts the very fabric of space-time around it. A small test-body entering the vicinity of another, finite mass merely responds to the distortion of space-time that it encounters. This distortion is the geometrical curvature of space-time.

In GTR *the earth, for example, does not rotate around the sun because of the sun's gravity. Instead, the earth is moving in a locally-straight line, but in a space-time curved by the mass of the sun.*

The Field Equations

The other crucial ingredient of Einstein's GTR*, apart from general covariance, is the prescription that determined exactly* how *a given matter distribution distorts space-time. The prescription consists of the* Einstein gravitational field equations[359] *— consisting, in any local spacetime coordinate system, of 10 nonlinear, second order, partial differential equations, for the ten components of the metric tensor.*

The field equations determine the Ricci tensor *of space-time (contracted Riemann curvature tensor) in terms of the energy-momentum tensor, and require local energy-momentum conservation (in local freely falling frames) for consistency. If one extends the equivalence principle to also mean that* gravitational *experiments in the small, freely-falling cabin yield the same results as Newtonian theory (for slowly-moving, sufficiently small masses), it is in fact possible to derive the* minimal GTR *field equations from this emended equivalence principle.*

[359] According to Einstein: "*Fields* are not states of a medium (the ether) and are not bound down to any bearer, but they are independent realities which are not reducible to anything else".

Let us take a brief look at the mathematics underlying space-time-gravitation. In a non-Euclidean but locally Euclidean (Riemannian) space in two dimensions, in any local coordinate system (x_1, x_2), *an infinitesimal vector is drawn from a point* P *to a point* Q. *The square of the distance between* P *and* Q *is a quadratic form in* dx_1 *and* dx_2, $ds^2 = g_{11}dx_1^2 + 2g_{12}dx_1dx_2 + g_{22}dx_2^2$. *For a space of any dimension, the corresponding expression is* $ds^2 = g_{ij}dx_idx_j$, *where* g_{ij} *is the metric tensor and the Einstein summation convention is understood. The* g_{ij} *are functions of* x_k *— they may vary from point to point in spacetime. The metric tensor provides a distance definition between neighboring points in space. Thus Riemannian geometry is a metric geometry.*

In GTR, *the curvature of space-time is quantified by the curvature tensor*[360] *(sometimes known as the Riemann-Christoffel tensor), which is de-*

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[360] The anti-symmetric part of the *affine connection* $\Gamma^{\nu}_{\beta\mu}$, namely $T^{\nu}_{\beta\mu} = \Gamma^{\nu}_{\beta\mu} - \Gamma^{\nu}_{\mu\beta}$ is the *torsion tensor*. It is – aside from the curvature tensor and their contractions, covariant derivatives and combinations thereof – the tensor that can be formed from the affine connection. $\Gamma^{\alpha}_{\beta\gamma}$ is *not* a tensor. If the torsion tensor vanishes, then the connection is *symmetric*. Einstein formulated his theory of gravitation in terms of the symmetric (Schwartz–Christoffel) connection derived from the spacetime metric tensor, partly because the spin of the electron was as yet unknown in 1915.

In the simplest extension of GTR to include torsion, it couples to intrinsic spin, but has no direct effect on cosmology/astrophysics because torsion vanishes outside stars and planets. That leaves open the possibility of some torsion effects *inside* stars or in the *early universe*. Although torsion appears to play no significant role in the *macroscopic* realm, it features prominently in *supergravity* models of quantum fields. In these models, when one looks at the *Planck scale* of distances, times and energies, conventional GTR is modified. One feature of *supergravity field theories*, is that any spinning particle creates a torsion in spacetime in addition to curvature. So far (2008), supergravity theories were not shown to be valid.

The classical spin of a *Kerr black hole*, as well of a spinning planet or galaxy etc. does not count as far as torsion goes, because torsion is only excited by genuinely *intrinsic* spin, whereas the former examples are all manifestations of *orbital* spin.

As far as observational signatures in the real world go, torsion turns out to be a disappointment — the deviations it causes from GTR are miniscule, and are only significant in extreme conditions, such as the *Planck or GUT eras* right after the Big Bang.

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$$R^{\alpha}_{\beta\mu\nu} = \Gamma^{\alpha}_{\beta\nu,\mu} - \Gamma^{\alpha}_{\beta\mu,\nu} + \Gamma^{\alpha}_{\sigma\mu}\Gamma^{\sigma}_{\beta\nu} - \Gamma^{\alpha}_{\sigma\nu}\Gamma^{\sigma}_{\beta\mu},$$

where the spacetime manifold is now locally Minkowskian, and Greek indices (covariant and contravariant) range from 0 through 3. Covariant and contravariant Greek "world" indices are converted into one another via contractions with the metric tensor, and $g^{\alpha\beta}$ *is the inverse* 4×4 *matrix to* $g_{\alpha\beta}$*. The* Γ*'s, called Christoffel symbols, are related to partial derivatives of the components of the metric tensor:*

$$\Gamma^{\nu}_{\beta\mu} = \frac{1}{2} g^{\alpha\nu} (g_{\alpha\beta,\mu} + g_{\alpha\mu,\beta} - g_{\beta\mu,\alpha}),$$

where $g_{\alpha\beta,\mu} \equiv \frac{\partial g_{\alpha\beta}}{\partial x^{\mu}}$.

The distribution and flux of mass-energy-momentum-stress *in the world is represented by the* energy-momentum tensor*,* $T_{\alpha\beta}$*, defined as the flux of* α *component of 4-momentum across a surface of constant* x_{β} *(since* x^0 *is the time coordinate, and* $T_{\alpha 0}$ *is a 4–momentum density. Yet by symmetry* $T_{\alpha 0} = T_{0\alpha}$ *so it is also a 4–current–density of energy. Thus, for free Maxwell fields in a locally free-falling frame* $T_{\mu\nu}$ *is the electromagnetic energy–momentum–stress tensor (cf. Minkowski and STR entries).*

Einstein was striving to relate the curvature of space-time to the distribution of $T_{\alpha\beta}$*. He found that the only two tensors related to space-time geometric structure that follow the (covariant) divergence-theorem (local conservation) of the energy-momentum tensor (and involve at most* 2^{nd}*-order spacetime derivatives of the metric — and is linear in those) are* $g_{\mu\nu}$ *and*

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}.$$

Here $G_{\mu\nu}$ *is called the Einstein tensor,* $R_{\mu\nu}$ *is the Ricci tensor (a contraction of the curvature tensor), and* $R = g^{\mu\nu} R_{\mu\nu}$ *is the curvature scalar (covariant trace of* $R_{\mu\nu}$*). Thus, he set* $G_{\mu\nu}$ *proportional to* $T_{\mu\nu}$*;*

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \equiv G_{\mu\nu} = \kappa T_{\mu\nu}.$$

Due to a now–irrelevant cosmological prejudice against an expanding-universe solution, Einstein later added a term $\lambda g_{\mu\nu}$ *where* λ *was called* *the cosmological constant*[361]*, thus obtaining the final form of his field equations as*

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$

Requiring that this reduce to Newtonian gravity for slowly ($v \ll c$) *moving masses, sub-cosmological distances and low spacetime curvatures, it is found that* $\kappa = \frac{8\pi G}{c^4}$, *where* G *is Newton's gravitational constant.*

If the tensor $T_{\mu\nu}$ *is zero everywhere, that is, if there is no matter-energy in the universe, one solution of the field equations is the Minkowski "flat" space-time. Another relatively simple exact solution of the field equations concerns a spherical body in an empty space.*

If we consider the sun as a spherically symmetric, finite-radius mass–energy–momentum–stress distribution, and assume that space is empty around it, the field equations render the curvature of space-time in that space. The metric tensor around the sun is called the Schwarzschild metric (1916)

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361 The cosmological constant of GTR physically represents the possibility that there is an energy density and pressure associated with "empty" space. The relative sign and magnitude of mass–energy density and pressure are dictated by the STR relativity principle, as applied to the vacuum in locally free-falling frames. The inclusion of this term (although first allowed by Einstein as a mathematical fix) can greatly affect cosmological theories. In its simplest form, GTR predicts that the universe must either expand or contract. Einstein thought the universe was *static*, so he added this new term to balance the self-gravitational collapse of a uniform dust (pressure-free) universe. **Friedmann** realized that this was an unstable solution and proposed an expanding universe model, now part of the *Big Bang* theory. When Hubble's survey of galaxies showed that the universe was expanding, Einstein regretted having modified his elegant theory and viewed the cosmological constant term as his "greatest mistake". The net effect of a *positive* cosmological constant is to create a *repulsive* gravitational force that acts to expand the universe (eventually at an exponential rate). Also, λ need not be an actual constant; it is now thought to have been evolving, as a function of local proper time. Modern field theory associates this term with the *energy density of the vacuum*. Modern observational cosmology has revealed that the cosmological constant today ("dark energy") comprises about 70% of the energy density of the universe, implying that the extrapolated *age* of the universe is larger then for $\lambda \equiv 0$. Adding an early-universe cosmological constant term during a very brief cosmological era (when the proper–time age of the universe was much less than a picosecond) is a hallmark of *inflationary models* (a class of extensions of the Big Bang theory); such models appear to be consistent with the observed large-scale distribution of galaxies, and with the observed microwave background fluctuations.

and corresponds to a line element which, in a particular *system of polar coordinates (one of many!) is*

$$ds^2 = c^2\left(1 - \frac{r_s}{r}\right)dt^2 - \frac{dr^2}{\left(1 - \frac{r_s}{r}\right)} - r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

where $r_s = \frac{2GM}{c^2}$ *is the Schwarzschild radius.*

We observe that if the total mass M *of the body which gives rise to the gravitational field is very small, relative to* $\frac{rc^2}{G}$ *at a given distance* r *from the body's center of symmetry, space-time geometry is approximated there by the Minkowski space-time. As a next step we omit terms of relative order* $\frac{1}{c^2}$ *assuming that* $\frac{2GM}{c^2} \ll r$, *and obtain*

$$ds^2 \approx \left(c^2 - \frac{2GM}{r}\right)dt^2 - dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2).$$

When working with this line-element to lowest order in $\frac{GM}{c^2 r}$ *this is the* Newtonian approximation, *because it reproduces the results of Newton's theory for the motion of the planets. That is, in this approximation the orbits of the planets are the ellipses given by Newton's law of gravitational attraction. However, gravitational red-shifts already appear, even at this level of approximation (although no gravitational bending of light).*

Einstein did not 'derive' his field equations from pre-existing principles, nor was he guided by a need to explain particular sets of experimental data or observations. He was guided only by his quest for two abstract ideals: beauty and simplicity. The equivalence of gravitational and inertial mass and the need to make all the equations independent of any particular choice of coordinates were basic requirements as well as the conditions that Newton's theory be recovered in the small-G, large-c regime; and that STR be locally recovered in the fixed $-c, \quad G \to 0$ *limit.*

The second and fourth of these four conditions are met automatically through the general covariance of GTR, and the fact that it predicts a vanishing Riemannian curvature at spacetime regions far from matter and energy. The first condition was met by requiring that the dynamic paths of bodies be geodesics *in a pseudo-Riemannian space-time whose curvature is determined by the distribution of mass-energy. And we have just seen how the* 3^{rd} *condition is verified (although this needs to be, and has been, generalized to any non-symmetric mass distribution).*

As an added Occamian bonus, Einstein and **Infeld** *have shown that the geodetic motion of test particles is a* consequence *of the field equations!*

As mentioned above, one may always add to the gravitational field equations (as to the covariant matter equations of motion) an infinity of non-minimal terms which cannot be determined by the principles discussed here.

The Einstein field equations in the presence of matter-energy are the simplest possible under the above postulates of GTR, *but they lack the stark beauty of the empty-space field equations, since the source term (proportional to the energy-momentum tensor) is non-geometrical. It is this esthetic and conceptual blemish that unified theories, from Kaluza-Klein to superstring models, have sought to redress.*

In 1919 Einstein suggested that perhaps electrically-charged particles are held together by gravitational forces, such that electromagnetism constrains gravitation. This idea may be considered Einstein's first attempt at a unified field theory[362]*. Since this idea implies that the energy momentum tensor* $T_{\mu\nu}$

[362] Einstein did not contribute much to the progress of physics after 1926: the last thirty years of his life were largely devoted to a search for a so-called *Unified Field Theory* that would unify Maxwell's theory of electromagnetism with GTR. Einstein's attempt was not successful, and with hindsight we can now see why this was so: *electromagnetism* and *gravitation* were the only forces that were known when Einstein was young, but we know today that there are other kinds of forces in nature, including the weak and strong *nuclear forces*. Indeed, the progress that has been made toward unification has been in unifying Maxwell's theory of the electromagnetic force with the theory of the weak nuclear force, not with the theory of gravitation, which is a much harder problem to solve.

A solution of the coupled Einstein–Maxwell (EM) equations representing a source possessing mass, electric charge, and an angular momentum was given by **E.T. Newman** (1965). The solution has a *ring singularity* on a circle lying in a plane normal to the direction of the angular momentum. **F.J. Ernst** (1968, 1974) showed that the coupled EM equations can be reduced to the solution of two coupled equations for a pair of complex potentials $\mathcal{E}$ and Φ

$$(\text{Re}\ \mathcal{E} + \Phi\Phi^*)\nabla^2\mathcal{E} = (\nabla\mathcal{E} + 2\Phi^*\nabla\Phi)\cdot\nabla\mathcal{E}$$

$$(\text{Re}\ \mathcal{E} + \Phi\Phi^*)\nabla^2\Phi = (\nabla\mathcal{E} + 2\Phi^*\nabla\Phi)\cdot\nabla\Phi$$

In 1925, Einstein missed the opportunity of predicting the positron (antielectron) before **P.A.M. Dirac**. In his paper entitled *Electron and General Relativity* he proved that for every elementary particle of mass m and charge e there must exist an "antiparticle" of the same mass m but with charge $-e$. If only Einstein had interpreted his theorem to mean that alongside the negative electron there must also exist an as yet undiscovered positively charged "antiparticle" of identical mass, he would have predicted antimatter! This was left to P.A.M. Dirac in 1928.

is due purely to electromagnetism, Maxwell–Minkowski energy-momentum-stress tensor is substituted for $T_{\mu\nu}$ in Einstein's field equations. The result is known as the Einstein-Maxwell equations. In 1927, Einstein wrote a further short note on the mathematical properties of this model, but never returned to it again. Others, however, have continued since then to seek solution to this hybrid theory.

Light rays follow curvilinear paths in space (and spacetime) when bent in the presence of gravitational fields, and their velocity may vary accordingly (although not as reckoned by a locally inertial frame!). Thus, GTR *extends the Faraday-Maxwell field concept as it does the Galilei-Newton principle of equivalence.*

The GTR *has much to say about the possible structure and history of the universe as a whole. In 1917, Einstein began to apply his theory to the large-scale features of the entire universe. At that time, this was a bold step, because it was not yet known whether there was anything but void outside our own Milky Way galaxy. Thus, the Andromeda galaxy (then called a nebula) was still believed to lie inside the Milky Way. Yet Einstein assumed that the universe could be idealized as a homogeneous distribution of matter. Eventually, this assumption was seen to be valid, at least to first approximation. For a variety of philosophical reasons, he chose a model for the universe having a 3D geometry that is spatially closed (in the sense of a spherical surface: finite but unbounded), and for which both 3-geometry and mean–energy distribution are static (unchanging with time).*

Since his theory did not admit static solutions, but only expanding or contracting universes, he felt compelled to modify the original field equations by adding an ad hoc term called the cosmological constant term[363]*. The modified*

Other ramifications of GTR are:

- Feynman derived the Einstein Field Equations without the use of curved massless space. He only assumed the equivalence principle and the existence of a spin-2 (dyadic) graviton field, minimally coupled.

- Using an idea of Einstein-Infeld (1932–1934), **A. Papapetru** generalized the relativistic point-mass into a small extended particle, obtaining a *relativistic version of McCullagh moments–expansion*. He achieved this via a multipolar expansion for the motion of a small mass in an external field. The first term is a geodesic and the *second term* in the *momentum expansion* is the first interesting one.

[363] Ad hoc yet *unique*: it is the only term that can be added to Einstein field equations without violating any of his assumptions and principles, although it does have an undetermined constant coefficient – the cosmological "constant", which is now knows to actually exist and to *not* be constant (it varies with the

equations did accommodate a static universe. The discovery of cosmological expansion by Hubble and his colleagues in the 20's seemed to obviate the need for this term, but its small observed value (discerned only in the 1990's) remains an enduring mystery for cosmologists and particle theorists to resolve.[364]

The non-linear field equations of GTR *can accommodate a variety of mathematical solutions, among which physicists must choose those that can be verified by observations. Cosmological solutions applying to the universe as a whole and based on the assumption that the large-scale structure of the universe is uniform (homogeneous and isotropic) are said to obey the 'cosmological principle'.*

Before relativity, Lorentz had conjectured (1900) that gravitation "can be attributed to action which does not propagate with a velocity larger than that of light". The term gravitational wave (onde gravifique) appeared for the first time in 1905, when Poincaré discussed the extension of Lorentz invariance to gravitation. In June 1916, Einstein became the first to cast these qualitative ideas into explicit forms, showing that gravitational waves are a direct consequence of the GTR *field equation. In standard Newtonian theory, gravitational interaction between two bodies in instantaneous, but according to* STR *this should be impossible, because no process can propagate with a velocity higher than c. Since* GTR *was designed to be compatible with* STR*, it must incorporate such a limiting velocity for gravitational interaction.*

It was not until the 1960's, with the revival of astronomical interest in GTR*, that physicists began to look for ways and means to detect gravitational waves.*

Einstein was greatly influenced by Spinoza, to whom he is bound ideologically and philosophically. In fact, Einstein completed the Copernican revolution and the work which Spinoza had started, namely: destroying the mechanistic, anthropocentric concepts of a dogmatic universe and adopting a holistic-deterministic view into which space (geometry), time and matter are integrated. Yet he said: "The most incomprehensible thing about the world is that it is comprehensible".

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local, proper–time age of the universe). It is believed that when we know the full, correct quantum field theory (including gravitation) governing the evolution of our observable universe, it will be possible to predict the cosmological evolution of this term in Einsteins GTR field equations.

[364] Generic quantum field theories predict a value of λ so large as to explode any macroscopic sample of matter, and space itself, to emptiness within a small fraction of a second. The observed value of λ is extremely small in comparison.

He sided with Newton in the matter of causality: "It is only in the quantum theory that Newton's differential method becomes inadequate, and indeed strict causality fails us. But the last word has not yet been said".

Einstein was motivated by the quest for simplicity (Occam's Razor), beauty and unity and he was unique among scientists of the 20^{th} *century in the degree to which his work bears the mark of his individual identity.*

Asked how he would have felt had there been no experimental confirmation of his general theory of relativity, Albert Einstein remarked, "Then I would have felt sorry for the dear Lord — the theory is correct"[365,366].

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365 Planck embraced STR and was the first to apply relativity to quantum theory. But when Einstein told him in 1913 what he was trying to accomplish in his GTR, he reacted: "*As an older friend I must advise you against it for in the first place you will not succeed, and even if you succeed, no one will believe you*".

366 Most physicists today agree that GTR must be modified in order to accommodate the consequences of Big-Bang theory, singularities due to gravitational collapse, (if true) quantum field theory and string theory:

- Big-Bang theory, combined with the singularity theorems of **Hawking** and **Penrose** (1973–1980), lead to the conclusion that unless GTR is modified, there existed (early enough in the history of the Universe) places and times at which physical quantities (density, pressure, temperature etc) *diverge* — an unacceptable attribute of any physical theory (such divergences are called "singularities"). This problem also arises in the GTR description of the formation of *Black Foles* in the gravitational collapse of heavy stars and galactic centers.

- Since, by the equivalence principle, all forms of matter couple to the metric tensor, the existence of any *quantum field* leads automatically to modifications of Einstein field equations caused by *quantum vacuum fluctuations*. It is well known that finite "*radiative corrections*" to GTR can eliminate singularities. However, no one has been able to tame the short–distance divergences in any quantum version of GTR, so we do not yet know how to compute such radiative corrections.

- The currently popular *String Theory* of *quantum gravity* modifies the *field* concept, by replacing point-particles with extended *strings*. This tends to smear any singularity over a finite region of spacetime, and there is mathematical evidence that this might render all physical quantities finite. However three decades of intense work by thousands of people has not yet unified these scenarios into a single, well–defined and predictive theory, and there is recent evidence that string theorists are despairing of ever achieving this goal, or of making contact with experiment.

Experimental Tests of General Relativity (1919–1975)

Because the effects predicted by GTR are very small in the solar vicinity, they are difficult to establish experimentally. However, the so-called 'crucial tests' (predicted by Einstein himself) have been confirmed with reasonable accuracy. They are:

(1) The perihelion advance of Mercury.

(2) The relativistic red-shift.

(3) The gravitational deflection of starlight rays grazing the sun.

These three tests of GTR are not equally stringent: (2) only tests the weak equivalence principle, while (1) and (3) also test Einstein's field equations[367].

We next review the status of these tests, and others devised later.

The earliest attempts to provide experimental proofs of GTR were made by Einstein's disciple, assistant and colleague, the astronomer ***Erwin Finlay Freundlich***[368] *(1885–1964, Germany and Scotland), during* 1911–1918. *Years before the theory was perfected and published in* 1916, *Einstein was advising and encouraging Freundlich, an astronomer at the Royal Observatory in Berlin, to conduct experiments to measure effects (1) and (3) that could prove or disprove his revolutionary concept of space, time and gravitation. Freundlich was perhaps the first scientist to become thoroughly acquainted with the fundamental principles of Einstein's theory. As such he was a moderately important figure in Einstein's career during its early years.*

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367 The *qualitative* fact that light is deflected in a gravitational field, though, *does* follow from the equivalence principle alone. This can easily be seen by considering a small, freely falling frame in which we shine a beam of light. The equivalence principle then demands that the beam describes a straight line in this frame, which implies that the beam is bent in a non-inertial frame.

368 Freundlich was born in Biebrich, Germany to a German father and Scottish mother. After earning a doctorate at Göttingen University, he went to work at the Berlin Observatory, where he was introduced to Einstein, who at the time was having trouble finding astronomers to search for experimental proofs to his still unpublished general theory. In 1918, Freundlich became Einstein's full-time assistant. After the war time he raised money to establish the Einstein Observatory in Potsdam, becoming its first director and working alongside Einstein. Freundlich left Germany after the Nazis came to power and wound up his career as an astronomer at St. Andrew University in Scotland.

Freundlich's telescopic studies between 1911 *and* 1913 *confirmed that Mercury's position conformed to Einstein predictions based on the equations of GTR as they pertained to the sun's gravitational effect in warping space-time. These Mercury results left Einstein speechless for several days with excitement, but remained controversial until they were duplicated by* ***Eddington*** *(1916).*

In 1914*, Freundlich was in Russia on his way to the Crimea to observe a total eclipse of the sun, to test (3), when the outbreak of WWI canceled the whole project. Not until* 1919 *was the crucial eclipse test conducted by Eddington, applying the very technique elaborated by Freundlich in 1914. (Eddington later said: "It seems unfair that Dr. Freundlich, who was first in the field, should not have had the satisfaction of accomplishing the experimental test of GTR".)*

Freundlich drew attention to test (2), though he lacked the facilities for making the necessary observations.

A. Perihelion/Periastron advance of planets or stellar companions

Astronomical observation show that the elliptical orbit of the planet Mercury rotates in its plane of motion about $5,599.7$ *seconds of arc per century. Using Newtonian mechanics, one can ascribe this phenomenon to the perturbing effects of other planets, mostly Venus, and obtain a theoretical value of* $5,557.2$ *seconds of arc per century. The difference of about* $43''$*/century, has puzzled astronomers since* 1860. *An oblateness of the sun amounting to* .028 *percent could account for this "anomalous" precession, but would lead to additional phenomena which have not been detected.*

Einstein has shown that GTR predicts an added $\Delta\Phi = 43''$*/century [*$\Delta\Phi = 6\pi GM/ac^2(1-e^2)$ *per revolution, with* M *the solar mass,* $GM/c^2 = 1.4$ *km,* $a = 58 \times 10^6$ *km, (semi-major axis of Mercury's orbit), and* $e = 0.2$ *(that orbit's eccentricity)]. Other planets, such as Venus or earth, have smaller GTR-caused precession rates; Mercury is most suitable because of its proximity to the sun.*

In 1972 ***Irvin I. Shapiro*** *(b.* 1929*, U.S.A.) et al., succeeded in a determination of the anomalous perihelion advance of Mercury, after five years of observation with Haystack (Massachusetts) and Arecibo (Puerto Rico) radio telescopes. They found that the observed and GTR-calculated values agreed to within the uncertainty of the observation (*$\sim$ *2%).*

In 1975 ***Hulse*** *and* ***Taylor*** *measured the periastron advance of the binary pulsar, PSR* $1913+16$*, and found it to agree with the GTR value within the observational error. Further observations of this system over the following* *two decades, have improved the agreement.What is more,* ***Hulse*** *and* ***Tylor*** *found an additional anomalous effect in this stellar system – a slow secular orbital decay unaccounted for by any known process – which agrees very well with kinetics energy loss due to gravitational-wave radiation as predicted by GTR.*

Previously, ***Clemence*** *(1947) showed that the Mercury precession produced by the solar oblateness is negligible because of the non-uniform density of the sun, but the final verdict is not yet in on this issue.*

B. Gravitational time dilation (red shift)

According to GTR, the gravitational field effects the rate of clocks ['clock' in this context stands for any localized time measurement procedure], such that $\Delta\tau_2 = \Delta\tau_1\left[1 + \frac{\Delta\Phi}{c^2}\right]$ *where* $\Delta\Phi$ *is the change in gravitational potential*[369] *difference between clock II (time interval* $\Delta\tau_2$*) and clock I (time interval* $\Delta\tau_1$*).*

Hence, if clock II is at the higher potential $(\Delta\Phi > 0)$, *then* $\Delta\tau_2 > \Delta\tau_1$*. The signals sent out from clock I at (say) one-second intervals, are measured by clock II to arrive at intervals larger than one second. Clock I, which is closer to the source of the field, is thus observed to run slow. If the two clocks are near the earth's surface,* $\Delta\Phi \approx g(\Delta r)$ *where* Δr *is the vertical distance (altitude difference) between the clocks. If we denote* $\Delta\tau_1 = T$, $\Delta\tau_2 - \Delta\tau_1 = \Delta T$, *the above relation will yield* $\Delta T/T \approx g\frac{\Delta r}{c^2}$, *which can be verified experimentally with modern atomic clocks. In terms of clock-rate, or frequency,* $\frac{\Delta\nu}{\nu} \approx -\frac{\nabla T}{T} \approx -g\frac{\Delta r}{c^2}$.

369 Gravitational potential is a Newtonian concept. What is meant here is that, when the two clocks are infinitesimally close and at rest in a *static* space time which is nearly Newtonian (such as that outside the sun), then their rates are related in the above manner – which follows simply from the weak equivalence principle (no need to invoke the field equations).
By considering a string of neighboring clocks, we can define $\Delta\tau_2/\Delta\tau_1$ for any two stationary clocks, even far apart. The relation $\Delta\tau_2 = \Delta\tau_1\left[1 + \frac{\Delta\Phi}{c^2}\right]$ is then modified to $\Delta\tau_2 = \Delta\tau_1\sqrt{\frac{g_{00}(2)}{g_{00}(1)}}$. In the 'post Newtonian' (weak gravity) approximation, which holds in the solar system, $g_{00}(\boldsymbol{r}) \approx 1 + \frac{2\Phi(\boldsymbol{r})}{c^2}$, and the previous expression is recovered.

It then follows that a periodic signal emitted at r_1 in step with the ticking of clock I, will appear to have reduced its frequency when it arrives and is measured at r_2 by clock II. In particular, the spectral lines in the light emitted by an atom placed deep in a gravitational potential, should be found to be red-shifted when compared with the spectral lines emitted by an identical atom placed at higher in the potential[370].

The experimental tests of this effect serve as direct evidence that space-time is not flat but curved. Two kinds of earth-bound experiments were performed. In the first, by ***Pound*** *and* ***Rebka*** *(1959), an emitter of γ-rays (the isotope Fe^{57}) was placed at ground level, and these γ-rays detected via absorption by another Fe^{57} sample, placed at the top of a tower $\Delta r = 22.6$ m above the emitter $\left[\frac{\nabla\nu}{\nu} = -g\frac{\Delta r}{c^2} = -2.46 \times 10^{-15}\right]$. The measurement of such a small frequency shift necessitates a high-Q resonant absorption of Fe^{57}; the experiment thus relies on the Mössbauer effect to prevent shifts of the γ's energy through recoil of the nuclei during emission or absorption. The most recent version of the experiment (****Pound*** *and* ***Snider****, 1964) gave the result $(\Delta\nu)_{\text{exp}}/(\Delta\nu)_{\text{theo}} = 1.00 \pm 0.01$.*

In the second kind of experiment, ***J.C. Hafele*** *and* ***R.E. Keating*** *(1972) took Cesium-beam clocks on flights around the world in commercial jets. The clocks, carried to high altitude by the aircraft, were found to have* gained *time when brought back to a laboratory clock that stayed on the ground [purely gravitational 'gain' was obtained from the 'apparent gain' after subtracting special relativistic time-dilation ('kinematic correction')]. Here $(\Delta\nu)_{\text{exp}}/(\Delta\nu)_{\text{theo}} = 0.9 \pm 0.2$.*

Gravitational red-shifts have also been measured for light emitted by atoms on the surface of the sun $\left[\frac{\Delta\nu}{\nu} = -\frac{GM_\odot}{R_\odot c^2} = -2.12 \times 10^{-6}\right]$ by ***J.W. Brault*** *(1962) and* ***J.L. Snider*** *(1971), and found to be in good agreement with theory.* ***Greenstein*** et al. *(1971) measured red-shifts of H lines on Sirius B, with a figure of merit 1.07 ± 0.2.*

Such measurements are sometimes difficult, since the gravitational red-shift is masked by large and uncertain kinematic Doppler shifts.

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[370] *In other words*: the wavelength of light should be increased while climbing a strong gravitational well such as the sun's, owing to a loss of some energy by photons (recall $E = h\nu$) as the light or any other radiation escapes the gravitational field.

C. Deflection of light by the gravitational field of a star

The GTR value for the angular deflection of light that passes at distance $b \geq R$ *from the center of a spherical mass* M *of radius* R*, is* $\alpha = 4GM/c^2b$ *radians. For a ray grazing the limb of the sun, this amounts to 1.75 seconds of arc. A simple calculation of this effect, based on Newtonian gravitation theory*[371] *and the corpuscular theory of light (**Soldner**, 1801), yielded half the above value. [He evaluated the transverse momentum acquired by a photon in passing the sun, by integrating the component of the gravitational force normal to the approximated linear trajectory.]*

*The first relevant astronomical measurement was done by **A. Eddington** and **Frank Dyson** during the solar eclipse of May 29, 1919 on the islands of Sobral (off Brazil) and Principe (Gulf of Guinea). They photographed the star field around the sun during total eclipse, and measured the displacement of star image relative to those on plates taken when the same stars are supposed to appear in the same place in the night sky (about six months later or earlier). The observational problems were:*

(1) observations are limited to $b > 2R$ *because of glare from the Corona;*

(2) 'paths of totality' for solar eclipses usually do not pass through observatories with big telescopes;

(3) it is necessary to compare separate plates acquired and developed independently at an interval of some months.

During that time the magnification of the system must be kept constant to within one part in 10^4*! Because of the extreme difficulty of the experiment, the systematic errors are much larger than the errors indicated by the consistency of the data.*

The experiment has been performed 12 times in the interval between 1919 and 1973, and the results have the spread $1.3''$–$2.7''$*. It may be taken as consistent with* $\alpha(R) = 1.75''$ *to within an error ∼25%.*

More precise results have been obtained during 1970–1975 with the use of radio-waves rather then optical light. In this case it is not necessary to wait for an eclipse; rather one must wait for the sun's limb to approach a radio source. Results were consistent with $\alpha(R) = 1.75''$ *with errors ∼10%, and*

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[371] Newton, in the appendix to his book *Opticks* (1704) asked:

"Do not bodies act upon light at a distance, and by their action bend its rays, and is not this action, strongest at the least distance?"

In view of Newton's concept of light as a stream of minute particles, this seemed a reasonable question.

provided evidence against the Brans–Dicke scalar-tensor theory of gravitation (an alternative to GTR).

The technology of these radio-wave measurements utilized long baseline interferometry to determine the change in the apparent position in the sky of the quasar[372] $3C279$ *during its annual occultation by the sun (October 8). The relative phase of the signals received by two radio telescopes is monitored during occultation and compared with the relative phase of the signals received from the quasar* $3C273$ *when it is not occulted, and is* $9.5°$ *away from* $3C279$*. The wavelengths used are in the range* 3–15 *cm, and most baselines are in the range* 1–845 *km.*

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372 *Quasars*: With the advent of new radio telescopes in the late 1950s and the improvement of the techniques for using them, radio astronomers were able to map more accurately the coordinates of radio sources in the sky.

In 1960, **Thomas Matthews** (CalTech, U.S.A.) discovered the first QUASAR denoted 3C48 (*Qua*si-*S*tell*ar* *S*ource; a double misnomer since they are quasi-stellar only in the sense that they were first mistaken for stars, and most are not radio sources. By the time that quasars were better understood, it was too late to change the term).

Allan Sandage (U.S.A.), **Jesse L. Greenstein** (U.S.A.) and **Maarten Schmidt** (U.S.A.) investigated the line spectra of observed quasars and found that the lines could be identified, provided huge Doppler *red-shifts* were assumed. These in turn indicated that quasars could be among the most distant objects known — billions of light years away. An absorption line in the 21 centimeter region showed that quasars were indeed beyond the hydrogen cloud constellation Virgo (40 million light years away). It was deduced that quasars must be of small volume but extraordinary luminosity (30 to 100 times that of an entire typical galaxy) to be able to emit such huge quantities of microwaves and visible light.

The possibility that quasar red-shifts were not caused by Hubble–expansion velocity at all but were gravitational in origin (GTR effect) was dismissed since no characteristic side effects supporting it were observed. The year 1960 also saw the establishment of the *Kitt Peak National Observatory* (Arizona, U.S.A.; 2096 m). It houses the largest solar telescope in the world (146 m diameter): It uses a system of three mirrors to produce a real image of the sun that is 86 cm in diameter. A flat mirror, called a '*heliostat*', 208 cm in diameter, follows the path of the sun. It reflects the sun light to a curved mirror 152 cm in diameter at the bottom of the 146 m telescope shaft. Another mirror reflects the sun's image into an observing room. Astronomers use the telescope to study sunspots and solar flares, both of which affect radio communication on earth.

In 1973, an optical telescope with a main mirror of 401 cm in diameter started to operate for the study of faint galaxies, quasars and other objects important to man's understanding of the universe.

D. Retardation of light

GTR predicts that gravitation reduces the speed of propagation of light signals grazing the sun (this velocity change is frequency independent — no dispersion]. This is an "effective" retardation, measured with respect to asymptotically-Minkowskian standards of time and length, far from the source of the gravitational field. An observer measuring the speed of light in situ, near the sun, with local clocks and length standards, would find the usual speed of 3×10^{10} *cm/sec. The theoretical extra time delay for the round trip of a sun-grazing radar signal bounced off one planet from another is* $\left\{2GM \log \frac{4z_1 z_2}{b^2}\right\}$, *where* b *is the grazing distance (closest radar-beam approach to sun's center) and* z_1, z_2 *are the distances of the two planets from the sun (they are assumed to be on diametrically opposed sides of the sun).*

For the earth-sun-Mercury system the maximum possible extra time delay is about 0.11 *milliseconds. To compare this value with experiment, one must make a correction to convert the above expression to proper time on earth by multiplying it by* $\left\{1 - \frac{GM}{rc^2}\right\}$, *where* r *is the radius of the earth's orbit. In addition, since the earth is moving, the time-dilation of special relativity must also be taken into account. Further, non-gravitational corrections must be made for the change of velocity of propagation of the radio signal caused by the solar corona and interplanetary plasma.*

Shapiro et al. *(1968, 1971) measured delays of radar echoes from Mercury, Venus and Mars, using the Haystack and Arecibo radio telescopes. The theoretical and experimental values agreed to within two percent.*

E. Gyroscopic precession

This test of GTR was proposed by ***Georg E. Pugh*** *(U.S.A.) and* ***Leonard I. Schiff*** *(1905–1971, U.S.A.) in* 1959.

The experiment (launched in spring 2004 and still taking data) consists of two gyroscopes in a 800*-km polar orbit around the earth (period of revolution* $\simeq 1.5$ *hours). The first gyroscope, with its spin axis in the orbital plane, is subjected to the so-called 'geodetic precession', namely a slow rotation of its inertial spin axis relative to the inertial axes at infinity (or the background microwave radiation frame, or some fixed stars*[373]*) about the normal to the polar orbital plane. The direction of this 'precession' is in the same sense as that of the orbit, i.e.: after one orbit, the direction of the gyroscope axis has rotated relative to its initial direction in the same sense. The cumulative net*

[373] In the actual experiment, an on-board telescope is used to measure the precession w.r.t. a star.

effect over one year (5000 orbits) is ~ 6.9 *arcseconds*[374]. *It amounts to a full rotation of* 360° *in* $200,000$ *years. This effect depends only on the earth's mass but not on its spin.*

The gyroscope is in a freely falling frame, but the principle of equivalence [which translates the problem into an equivalent STR one in a locally flat inertial frame, in which the spinning mass is instantaneously at rest] must be applied with caution, since the 'Einstein elevator' has a window through which the telescope watches a fixed star! In fact the non-flat *metric causes the gyroscope to precess at a rate that is* three fold *faster than the ordinary, (STR) Thomas precession.*

The second gyroscope has its spin axis parallel *to the equatorial plane of the earth and normal to the plane of its orbit around the earth. A classical GTR spin-orbit interaction cause the gyroscope's spin-axis to rotate slowly with the angular velocity vector* $\boldsymbol{\Omega} = \frac{G}{c^2 r^3}[3\boldsymbol{e}_r(\boldsymbol{e}_r \cdot \boldsymbol{J}) - \boldsymbol{J}]$, *where* $\boldsymbol{e}_r$ *is a unit radius vector from earth's center to the gyro and* $\boldsymbol{J}$ *is the angular momentum vector of the earth. When the gyro is at the pole,* $\boldsymbol{\Omega} = \frac{2G}{c^2 r^3}\boldsymbol{J}$, *and when it is at the equator* $\boldsymbol{\Omega} = -\frac{G}{c^2 r^3}\boldsymbol{J}$, *with the opposite sign. The net precession is* $0.055'' \left(\frac{R}{r}\right)^3$ *sec/year, where* R *is the earth's radius. This is precisely the* Lense-Thirring *effect of* frame-dragging *— the partial dragging of the local, free–falling inertial frames at the gyroscope's instantaneous location to the earth's spin — that can be thought of in terms of the so-called Mach principle (it also plays the same role for a gravitational orbit as does the* hyperfine interaction *in an atomic orbital caused by electromagnetism).*

According to this way of thinking, the gyroscope's inertial frame is acted upon by the earth: the inertial frame fixed to the gyroscope tries to follow the rotation (spin) of the earth, but lags behind due to the persistent interference of the cumulative effect of the entire universe.

The goal of the combined experiment is to measure both the geodetic precession and the fame-dragging effect to an accuracy better than a milliarcsecond per year, which is of order 2 percent of the smaller frame-dragging effect. The task of building an orbiting gyroscope laboratory that can measure such tiny effects has put scientists at the very frontiers of experimental physics and precision-fabrication technology.

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374 The theoretical value of this precession is $\frac{3}{2}\frac{GM}{c^2r^2}\sqrt{\frac{GM}{r}} \approx 6.9''/year$, where M, r are the earth's mass and the gyro's orbital radius, respectively. The above expression can also be written as $\left(\frac{R}{r}\right)^{5/2} \times 8.4''/year$, R being the earth's radius.

The gyroscopes are spheres of fused quartz, about 4 cm in diameter. Their uniformity in shape and density must be better than one part in 10^7 in order to prevent interaction of stray gravitational forces with material irregularities, which can cause enormous precessions[375] *relative to the effects sought. The gyros are coated with a thin uniform layer of niobium, a material that becomes a superconductor when cooled to temperatures approaching absolute zero. Eddy currents trapped in this layer create a magnetic field that is aligned with the spin axis of the ball.*

Very precise (so called SQUID) magnetometers, also utilizing superconductors and likewise operating near absolute zero, can determine the orientation of the magnetic field, and thereby the spin axis. The gyros are suspended in free fall to keep them away from the walls of the satellite. A liquid helium Dewar surrounding the gyros keeps the balls at the required low temperature. Each ball is sprayed tangentially with tiny jets of helium, using friction to set them spinning. The air around the gyros was pumped, so that they spin in an almost perfect vacuum; their rotation should slow down by less than one part in 10^3 per year.

Since the gyros precess relative to distant stars, a very accurate telescope aboard the spacecraft is constantly kept trained on the bright star Rigel in the constellation Orion. The instantaneous orientation of the gyros is continually determined relative to this fixed direction to the desired milliarcsecond-level (per year) precision.

This experiment is believed to be the most important ever to be performed in space, and a crucial test for GTR. In recent years there was a growing feeling among physicists that GTR has never been completely *verified. The difficulties encountered in unifying it with quantum mechanics and its mathematical complexity added to the feeling that the theory is incomplete*[376]*. It is hoped that the result of this experiment may help to decide some of these crucial issues, although no mere post-Newtonian measurement is likely to shed much light on the puzzles of quantum gravity.*

[375] If any of these spheres could be inflated to the size of the earth, its highest mountain would not surpass 64 centimeters!

[376] Although some physicists believe that it is *quantum mechanics* which is to blame for the difficulties in unification, and that gravitational physics must underline quantum mechanics itself!

F. Black holes[377]

A black hole is a self-gravitating object whose gravitational field is so strong that even light cannot escape. The event horizon is the surface where light loses the ability to escape from the black hole. Nothing that crosses to the interior of the event horizon can ever get back out again — not even light.

Black holes can be created by the gravitational collapse of large stars that are at least twice as massive as our sun. Normally, stars balance the gravitational force with the pressure from the nuclear fusion reactions inside. When a star gets old and burns up all of its hydrogen into helium, and then fuses the helium into heavier elements (carbon, oxygen, and eventually iron and nickel), it can have three fates. The first two fates occur for stars less than about twice the mass of our sun (and one of them will be our sun's eventual fate). These two fates both depend on the fermionic (Pauli-exclusion) repulsion pressure prescribed by quantum mechanics –two same – species fermions cannot be in the same quantum state at the same time. This means that the two possible stable destinies for the collapsing star will be:

> *(1) A white dwarf supported by the fermionic repulsion pressure of the electrons in the heavy atoms in the core.*
>
> *(2) A neutron star supported by the fermionic repulsion pressure of the neutrons in the nuclei of the heavy atoms in the core (all the protons having been 'squeezed' into reverse-beta-decaying to neutrons, positrons and neutrinos by the collapse pressure).*

If the mass of the collapsing star is too large (bigger than twice the mass of our sun), the fermionic repulsion pressure of the electrons and the neutrons are not enough to prevent the ultimate gravitational collapse into a black hole.

The estimated age of the universe is several times the lifespan of an average star. This means there must have been many stars heavier than twice the mass of our sun that have burned their nuclear fuel and collapsed since the universe began. Our universe ought to contain many black holes, if the model that astrophysicists use to describe their formation is correct. Black holes created by the collapse of individual stars should only be about 2 to 100 times as massive as our sun.

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[377] For further reading, see:

- Shapiro, S.L. and S.A. Teukolsky, *Black Holes, White Dwarves and Neutron Stars*, Wiley, 1983, 645 pp.

Another way that black holes can be created is the gravitational collapse of the center of a large cluster of stars. These types of black holes can be very much more massive than our sun. There may be one of them in the center of every galaxy, including our galaxy, the Milky Way. There is one in the middle of the galaxy called NGC 7052, surrounded by a bright cloud of dust 3,700 light-years in diameter. The mass of this black hole is about 300 million times the mass of our sun.

The Schwarzschild solution (1916)

Certain problems in GTR can not be treated by a linear approximation of the field equations and require an exact solution. The first and most important exact solution was found by ***Schwarzschild*** *(1916). It is a metric for the spacetime around a spherically symmetric mass distribution of total mass m.*

Near a body of mass m (e.g., the sun), spacetime is curved. The world lines of particles and lightrays in the gravitational field of m are approximately geodesics.

To find these, it is necessary to know the metric tensor $g_{\mu\nu}$ in some useful coordinate system. We choose x^0 to be the time and (x^1, x^2, x^3) to be some type of spherical polar coordinates (r, θ, ϕ) with the mass m at the origin 0.

Thus, we have a series of concentric spheres which, in a flat 3D space, would have radii r and surface areas $4\pi r^2$. If m were zero spacetime would be flat and Minkowskian and thus, from STR, the pseudo–norm of the separation of two infinitesimally close events would be

$$\eta_{\mu\nu}\,\Delta x^\mu \Delta x^\nu = (\Delta t)^2 - \frac{1}{c^2}\left[(\Delta x)^2 + (\Delta y)^2 + (\Delta z)\right] = (\Delta t)^2 - \frac{(d\sigma)^2}{c^2} \quad (1)$$

with $g_{\mu\nu} = \eta_{\mu\nu}$, the Minkowski metric tensor. For $m = 0$ the 3-space is Euclidean, and it has the metric

$$(d\sigma)^2 = (dr)^2 + r^2\left[d\theta^2 + sin^2\theta d\phi^2\right]. \quad (2)$$

If we can now 'switch on' the mass m, two things happen: the position (3D) space become curved, so that the r-spheres no longer have radii related to the respective areas via $A = 4\pi r^2$; and clocks on each r-sphere are no *longer observed from other r-spheres to run at the same rate. To allow for these effects we write the separation pseudo–norm as*

$$g_{\mu\nu}\,\Delta x^\mu \Delta x^\nu = e(r)(\Delta\tau)^2 - \frac{1}{c^2}\left[f(r)(\Delta r)^2 + r^2(\Delta\theta)^2 + r^2 sin^2\theta(d\phi)^2\right], \quad (3)$$

where $e(r)$ and $f(r)$ are positive functions to be determined, subject to the boundary conditions that far from m spacetime is asymptotically flat, so that $e(\infty) = f(\infty) = 1$. We have chosen a coordinate system where a 3-sphere of radius $\sqrt{f(r)}$ has area $4\pi r^2$.

Schwarzschild, by rigorously solving the Einstein field equations, obtained

$$e(r) = 1 - \frac{2Gm}{c^2 r}; \qquad f(r) = \frac{1}{1 - \frac{2Gm}{c^2 r}} \quad (4)$$

The corresponding metric tensor, known as the 'Schwarzschild metric', is

$$g_{\mu\nu} = \begin{pmatrix} 1 - \frac{2Gm}{c^2 r} & 0 & 0 & 0 \\ 0 & -\frac{1/c^2}{1-\frac{2Gm}{c^2 r}} & 0 & 0 \\ 0 & 0 & -\frac{r^2}{c^2} & 0 \\ 0 & 0 & 0 & -\frac{r^2 \sin^2\theta}{c^2} \end{pmatrix} \quad (5)$$

and

$$g_{\mu\nu}\,\delta x^\mu \delta x^\nu = d\tau^2 = \left(1 - \frac{2Gm}{c^2 r}\right) dt^2 - \frac{1}{c^2}\left(\frac{dr^2}{1 - \frac{2Gm}{c^2 r}} + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2\right) \quad (6)$$

In (6), dt is the interval of time between two events at the same distance from the mass centroid (e.g. events aboard a spaceship held at fixed r by firing its rockets towards the $r = 0$ mass centroid), as measured by an observer using a clock which is in a region remote enough that space-time is effectively flat. However, $d\tau$ (when a real number) is the proper time interval measured on a clock carried by someone moving in such a way as to be present at the two events labeled as (t, r, θ, ϕ) and $(t + dt, r + dr, \theta + d\theta, \phi + d\phi)$. (This is only possible if the two events have a timelike separation: $d\tau^2 > 0$.)

Note that the metric components do not depend on time, and so the Schwarzschild metric is static. It can, however, be shown that even when the mass distribution is undergoing spherically symmetric radial motions or flows, the external structure of space-time is described by the Schwarzschild metric *and is hence static. This result applies in the case of a stellar core collapsed to a neutron star or to a black hole while maintaining perfect spherical symmetry.*

Furthermore, it is believed that the gravitational collapse *of any nonrotating, electrically neutral star will necessarily lead to the Schwarzschild geometry; any deviation from the symmetric field will ultimately be radiated away in the form of gravitational waves.*

The solution in the interior *of a mass distribution can be carried out analytically only in some exceptional cases, such as the case of a fluid without pressure (cloud of dust). The solution for the case of matter with a realistic equation of state can usually only be carried out numerically.*

The dimensionless quantity $\frac{Gm}{rc^2}$ *appearing in* (5) *and* (6) *may be regarded as a measure of the local strength of the gravitational field. This quantity enters into the formulae for* light deflection, light retardation, redshift, perihelion precession, *etc.*

Note that the Schwarzschild solution develops a singularity *at* $r \to \frac{2Gm}{c^2}$ $(g_{00} \to 0, \quad g_{11} \to -\infty)$. *The quantity*

$$r_s = \frac{2Gm}{c^2} \tag{7}$$

is called the Schwarzschild radius *of the mass. For the mass of the sun* ($m = 2 \times 10^{33} g$) *we calculate* $r_s = 3.0$ *km. The corresponding value for earth is 8.86 mm. In most cases* r_s *is far smaller than the actual radius of the body concerned. However, the Schwarzschild metric applies only in the space* outside *the gravitating body; inside, a different formula holds, and there* g_{11} *is not singular at* r_s. *Furthermore, even if the sphere* $r = r_s$ *is exterior to the mass distribution, it can be shown that the* $r = r_s$ *singularity is only* apparent *– an artifact of the coordinate choice. The '*Schwarzschild singularity*' is thus unphysical (a small enough object can fall through the event horizon with no ill effects, though it can never exit again, nor communicate to the outside universe). However, but a real physical singularity occurs at* $r \to 0$ *when the mass distribution is pointlike. That in itself is unremarkable, as it is the case in Newtonian gravitation too.*

However, astrophysicists have calculated that the ultimate stage in the evolution of stars several times more massive than the sun would be 'gravitational collapse'. In this process, the star has exhausted its nuclear fuel and no longer radiates, and the pressure of its matter cannot resist the gravitational self-attraction of the star, which therefore shrinks down within its Schwarzschild sphere at $r = r_S$. *Such collapsed stars, and other objects smaller than their Schwarzschild radii, are called* black holes. *No known mechanism can halt such a collapse, and GTR must be somehow modified to describe the fate of the collapsed energy–matter near* $r = 0$.

Black holes are very strange objects. Their most astonishing property is that anything inside the Schwarzschild sphere must fall into the center $r = 0$. This applies to the matter constituting the star whose collapse formed the black hole, and implies that the collapse continues until the star is a point singularity at $r = 0$. (The implication holds if general relativity remains valid in the unimaginably dense and hot conditions near the end point of the collapse. If quantum or other effects intervene, the singularity might be avoided. However, no consistent theory of 'quantum gravity' has yet been devised, and this is still very much an open question.) Thus, the $r = 0$ singularity is worse in GTR than in Newton's physics, as nothing can prevent it.

The 'collapse' property also applies to any light emitted by the star when its radius is less than r_S, and implies that no light can escape, so that these objects cannot be seen from outside – hence the name 'black holes'. (This is not to say that black holes exert no influence on their surroundings; their gravitational effects on external bodies are perfectly normal, because these effects arise from the Schwarzschild metric for $r > r_S$.)

The simplest (Schwarzschild) black holes have no angular momentum or charge. The most general black hole known mathematically has mass, angular momentum (spin) and electric charge, and is a solution of the Einstein–Maxwell field equations. Anyone outside a black hole cannot follow what happens to material or energy once they cross the horizon. This raises the question of whether it is possible to make any distinction between black holes, beyond differences in mass.

***Hawking** (1972) and others have proven rigorously that a full description of any black hole requires just three parameters; these are the total mass M, the total charge Q, and the total angular momentum or spin J of the black hole. In a neat phrase, black holes are said to 'have no hair', which means that they have no externally observable detailed features*

As far as we know, matter on the large scale, and in particular stellar matter, appears to be electrically neutral – though it is almost always spinning (and thus, so are astrophysical black holes). The space-time around a neutral, but rotating collapsed star is described by the Kerr metric (1963).

Table 5.3: TIMELINE OF BLACK-HOLE PHYSICS

1784	***John Michell*** *discussed classical bodies which have escape velocities greater than the speed of light*
1795	***Pierre Laplace*** *discussed classical bodies which have escape velocities greater than the speed of light*
1916	***Karl Schwarzschild*** *solved the* ***Einstein*** *vacuum field equations for uncharged spherically symmetric systems*
1918	***H. Reissner*** *and* ***G. Nordstrøm*** *solved the Einstein-Maxwell field equations for charged spherically symmetric systems*
1923	***George Birkhoff*** *proved that the Schwarzschild spacetime geometry is the unique spherically symmetric solution of the Einstein vacuum field equations*
1939	***Robert Oppenheimer*** and ***Hartland Snyder*** *calculated the collapse of a pressure-free homogeneous fluid sphere and found that it cuts itself off from communication with the rest of the universe*
1963	***Roy Kerr*** *solved the Einstein vacuum field equations for uncharged rotating systems*
1964	***Roger Penrose*** *proved that an imploding star will necessarily produce a singularity once it has formed an event horizon*
1965	***Ezra Newman****, et al. solved the Einstein-Maxwell field equations for charged rotating systems*
1968	***Brandon Carter*** *used Hamilton-Jacobi theory to derive first-order equations of motion for a charged particle moving in the external fields of a Kerr-Newman black hole*
1969	***Roger Penrose*** *discussed the Penrose process for the extraction of the spin energy from a* ***Kerr*** *black hole*
1969	***Roger Penrose*** *proposed the cosmic censorship hypothesis (no "naked singularities", i.e. no singularities un-enveloped by event horizons).*

Table 5.3: (Cont.)

1971	*Identification of Cygnus X-1/HDE 226868 as a binary black hole candidate system*
1972	***Stephen Hawking** proved that the area of a classical black hole's event horizon cannot decrease*
1972	***James Bardeen**, **Brandon Carter**, and Stephen Hawking proposed four laws of black-hole mechanics in analogy with the laws of thermodynamics*
1972	***Jacob Bekenstein** suggested that black holes have an entropies proportional to their surface area due to information loss effects*
1974	***Stephen Hawking** applied quantum field theory to black hole spacetimes and showed that black holes will radiate particles with a blackbody spectrum, which can cause black hole evaporation*
1989	*Identification of GS2023+338/V404 Cygni as a binary black hole candidate system*

1917 CE **Gerhard Hassenberg** (1874–1925, Germany). Mathematician. Stressed the advantages of representing tensors as homogeneous invariant multilinear forms in primary base-vectors (polyadic form), and consequently as hyper-numbers in the Grassmannian sense.

1917 CE **D'Arcy Wentworth Thompson** (1860–1948, England). Classical naturalist and scientist, whose impact thrives in evolutionary biology, biomechanics and architecture. Was mainly concerned with the explanation of biological growth and form in physico-mathematical terms and disregarded the mainstream reductionist approach based on *biochemistry* and *genetics*. He believed that forms are shaped by *physical causes*, such as gravity, surface tension etc., and that growth is governed by *physical laws*.

In his masterpiece *On Growth and Form*, he pursued the undeniable unity of living organisms, and tried to reduce all patterns to a single system of generating forces. He thought of life as always in motion, always responding to rhythms — the "deep-seated rhythms of growth" which he believed created universal forms. He seemed to sense what this unity, if proven, might mean for the science of organic form. Moreover, he argued that *mathematical patterns* exist in living organisms, and that these patterns must therefore have *mathematical causes*!

In this sense, modern biology bypassed him; at the turn of the century, biology was already turning toward methods that reduced organisms to their constituent functioning parts: *Reductionism* prevailed in molecular biology, everywhere from evolution to medicine. Processes in cells were being interpreted in terms of membranes, nuclei, proteins, enzymes and chromosoms.

Against this tide, Thompson valiantly put his credo: "*It may be that all the laws of energy, and all the properties of matter, and all the chemistry of all the colloids are as powerless to explain the body as they are impotent to comprehend the soul. For my part, I think not*".

On Growth and Form has become a classic in natural philosophy. Biologists have conceded that it has had a great, though "*intangible and indirect*", influence. Mathematicians, on the other hand, were and still are fascinated by Thompson's considerations.

Worldview XXXV: D'Arcy Thompson

* * *

"The reflection of mathematical beauty is such that whatsoever is most beautiful and regular is also found to be most useful and excellent."

* * *

"Forms are not simply the result of heredity and behavior, but are shaped by physical forces."

* * *

"We known immeasurably more about the universe than our ancestors did, and yet it increasingly seems they knew something more essential about it than we do."

* * *

"Ancients knew the stars far better than we do; our city streets shut out the sky, and new lamps blind us to the old. Our calendar is ready-made for us, and ask not how."

* * *

"It behooves us always to remember that in physics it has taken great men to discover simple things. They are very great names indeed which we couple with the explanation of the path of a stone, the droop of a chain, the tints of a bubble, the shadows of a cup."

On D'Arcy Thompson

* * *

"An aristocrat of great learning whose intellectual endowments are not likely ever again to be combined in one man."

Peter Medawar (1958)

* * *

"Hovered, as it were, on the fringes of both the scientific and the classical worlds, making, apparently, no deep impression on either."

"He ignored chemistry, misunderstood the cell, and could not have predicted the explosive development of genetics... No modern biologist has to read him. Yet somehow the greatest biologist find themselves drawn to this book. This classicist, polyglot, mathematician, zoologist tried to see life whole, just as biology was turning so productively towards methods that reduced organisms to their constituent functioning parts. Reductionism triumphed."

Ruth Thompson (1958)

* * *

"Thompson is no Euclid of the plant world, and there is no Thompson's equation for an animal. Nonetheless, circumstantial evidence in every corner of the living world convinced Thompson that there is real mathematical patterns in living organisms and that the organic world is just as mathematical as the inorganic world."

Ian Stewart (1998)

The mathematics of life[378] — *structures, patterns and processes*

The universe has its endless gamut of great and small, of near and far of brief and enduring, of sudden and slow; of many and few. Yet, the physical laws that govern it, depend to a large extent on the relative scales of the physical entities that partake in these laws.

It was known already to ***Archimedes*** *(ca 250 BCE), that in similar figures the surface increases as the square, and the volume as a cube, of the linear dimensions. Thus, a fish, in doubling in length and breadth, multiplies its weight no less than eight times. Some physical forces are proportional to the surface area of the body on which they act (such as the wind force on a ship's sail or the lift of a bird's wing) or else, like gravity, exert a force proportional to the volume (actually mass) of the body.*

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[378] For further reading, see:

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- Berg, H.C., *Random Walks in Biology*, Princeton University Press: Princeton, N.J., 1993, 152 pp.
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- Nicolis, G. and I. Prigogine, *Self-Organization in Nonequilibrium Systems*, John Wiley & Sons: New York, 1977, 491 pp.
- Vogel, S., *Life's Devices*, Princeton University Press, 1988, 367 pp.
- Smith, J. Maynard, *Mathematical Ideas in Biology*, Cambridge University Press, 1980, 152 pp.
- Lotka, A.J., *Elements of Mathematical Biology*, Dover, 1956, 465 pp.

*It was **Galileo** (1638) who first laid down the general principle of similitude; and he did so with the utmost possible clarity, and with a great wealth of illustrations drawn from structures in both the animate and inanimate worlds. He said that if we tried building ships, palaces or temples of enormous sizes, yards, beams and bolts would cease to hold together; or can nature grow a tree nor construct an animal beyond a certain size*[379]*, while retaining the proportions and employing the materials and design principles which suffice in the case of smaller structures. The thing will fall to pieces of its own weight unless we either change its relative proportions (which will at length cause it to become clumsy, monstrous and inefficient) or else find new materials, harder and stronger than used before*[380] *(e.g., the discoveries of cement and steel).*

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379 A trivial, but amusing, example concerns the size limits on prehistoric creatures. We know that the largest land animal of all time was the giant sauropod dinosaur named *diplodocus*. It has been compared to a walking suspension bridge with an overall length of 40 meters, height of 18 meters (as high as a 6-story building), and weighing about 100 tons. It lived in the Jurassic (208–114 million years ago) and became extinct some 65 million years ago. Managing its vast bulk was partly an engineering problem and partly mechanical, a matter of force and leverage. It needed powerful muscles, solidly anchored, for walking, running and eating.

It is clear that the interplay of gravity and the strength of biological materials imposes limits on the size to which animals can grow. To calculate this we construct the following toy model: A land animal is, basically, a mass (the trunk) $M \approx \rho L^3$ standing in a column (the legs combined) of height h, having a square section of side $d \leq L$, where ρ is the density.

To support the animal trunk above ground level requires the expenditure of energy $\rho h d^2 Q$, where Q is the amount of energy per unit mass required by the support. If the structure collapses under its own weight, then gravity will supply an energy Mgh, where g is the acceleration of gravity at earth's surface. If the structure is to survive under its own weight $\rho d^2 h Q > Mgh$, whence: $\frac{d}{L} > \sqrt{L}\sqrt{\frac{g}{Q}}$. The smaller the animal, the thinner its legs can be; inversely, the larger it is, the fatter the legs. An ultimate limit is reached when the legs are as thick as the trunk — but then the animal becomes immobile.

At this limit $\left(\frac{d}{L} \approx 1\right)$ we have $L(m) \approx \frac{Q(\mathrm{m}^2 \cdot \mathrm{sec}^{-2})}{g(\mathrm{m} \cdot \mathrm{sec}^{-2})}$. For typical biological structures $\frac{Q}{g} \approx 44$ m, which is close to the right answer. This shows that the size of a land animal is strictly limited; it also shows that structures can be studies without knowing the details of their constitution and construction, just some general principles and constraints.

380 Galileo was, of course, careful to explain that besides the questions of pure stress and strain there is the important question of *bending moments* which affect the whole form of the skeleton, and set limits to the height of a tall tree.

Another phenomenon, and one which is observed throughout the whole field of morphology, is the tendency (due to some definite cause) for body surface to keep pace with volume, through some alteration of its form. The lobulation of the kidney in large animals, and the vast increase of respiratory surface in the air-sacs and alveoli of the lung, are a few of the many cases in which more or less constant ratios tends to be maintained between mass and surface. A leafy wood, a piece of sponge, a reef of coral, are all instances of a like phenomenon, namely, devices for an increased surface in order to stimulate diffusion of molecules or absorption of energy.

Consider, for example, an organ of azimuthal symmetry, whose cross-section in the plane $\phi = const.$ *is given by the polar equation* $r(\theta, t) = a(t)f(\theta)$, *with the origin at* $r = 0$. *Assume that the material, needed for growth, is drained from this center toward the periphery at a rate independent of the polar angle* θ *in such a way that the quantity deposited per unit time per unit diameter length is fixed*[381]. *What is the expected shape of* $f(\theta)$? *Clearly,* $f(\theta) = const.$ *would correspond to a sphere. But the above condition can accommodate a wider class of plane curves, since it is required only that at any given moment* t *we shall have*

$$r(\theta) + r(\theta + \pi) = D = const.$$

One can show that the general solution has the form

$$r(\theta) = b_0 + \sum_{1,3,5,\ldots}^{\infty} (a_n \sin n\theta + b_n \cos n\theta).$$

Apart from a circle $(a_n = b_n = 0,\ n = 1, 3, 5)$, *the simplest such curve is the leaf-shaped* $r(\theta) = a[1 + \sin\theta]$. *This figure has all its diameters the same — yet it is obviously not round! The brain, heart and kidney possess shapes approximated by this equation.*

Another example is the Fibonacci sequence 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, … *manifested in the number of petals, sepals, stamens, and other components of flowers. For instance, lilies have* 3 *petals, buttercups have* 5, *delphiniums have* 8, *marigolds* 13, *asters* 21, *and most daisies have* 34, 55, *or*

[381] The majority of *marine shells* are based on a spiral form. e.g. the *Nautilus* has a shape known to mathematicians as the *logarithmic spiral*, first studied by **Jacob Bernoulli**. It has this shape because the animal pattern of growth increases by a *fixed proportion* as the growing shell turns through a *fixed angle*. On the other hand, the shell of the fossil *Ammonite* (an extinct relative of *Nautilus*) is closer to an Archimedean spiral, in which the growth increases by *fixed amounts* for a *fixed angle*.

89. *Fibonacci numbers are also hidden in sunflower-seed patterns. Although the biological origin of these numbers is not understood, their pattern might provide the plant with some evolutionary advantage, or it may have evolved through certain physical constraint (i.e. constraints imposed by the laws of physics) on what biology can do, in the same sense that the form of a bird's tail depends heavily on aerodynamics.*

The human body contains about 10^{14} cells, of more than a hundred types — nerve cells, blood cells, liver cells, bone cells, muscle cells and so on — but the body is not merely a vast cellular conglomerate. In order to function collectively as a human being, those cells must be put together in a specific, complex manner. The human brain, for instance, has abilities that the largest supercomputer cannot match — such as being able to recognize a face, or look at a landscape and instantly pick out a sheepdog in a distant field. No human–devised technology can as yet assemble a single human cell.

D'Arcy Thompson was concerned with some mathematical aspects of cell-division. He assembled some of the microscopists' pictures and discerned, amidst the daunting complexity, clear patterns and regularities.

He noticed arithmetical patterns in the arrangement of those organelles known as chromosomes, *which contain (most of) the cell's genetic material. He pointed out analogies between cell division and the theories of electricity and gravitation; he also found analogies with chemical diffusion, in a homely experiment using ink in salt water. The shape of a cell — just before, during, and just after division — is mathematical. In cross section, the shape is a simple curve: a circle that develops a waist, which narrows, pinches into a figure-eight shape, and breaks apart to create two circles. This simple shape indicated to Thompson that there must be a connection between the division of cells and the physical principles that govern the forms of soap bubbles and foams.*

One of the key principles of physics is the idea that the inorganic world generally behaves in whatever manner that requires the least expenditure of energy. The energy of a soap bubble comes from the tension that holds its molecules together. (In blowing a balloon one has to provide energy to set up elastic tension in the balloon's rubber surface. The formation of soap bubbles requires much the same type of expenditure of energy). Just as it takes more effort to inflate a bigger balloon than a smaller one, it takes relatively more energy to produce a soap bubble or film with a larger surface area than a smaller one — so soap films with the least energy also have the least surface area.

The French mathematical physicist ***Joseph Plateau*** *discovered that the form of soap bubbles and films can be deduced completely from the principle that they adopt the shape that has the smallest surface area subject to* *boundary – curve or volume constraints. For example, an isolated bubble is spherical because the surface of a sphere is the smallest surface area that contains a specified volume of trapped air. Dividing cells start as a sphere and end as two, passing from one minimal-area surface to another. The figure-eight shape in between is also a minimal-area surface, but a more esoteric one.*

Thompson was fascinated by such surfaces, and he saw them — or at least thought he saw them — everywhere in living creatures. He glimpsed them in cell membranes, in the shapes of jellyfish, in algae, in fungi — even in the skeletons of microscopic creatures. When four soap bubbles meet, they do so along six common surfaces, meeting each other in pairs at an angle of $109°$*. If a fifth, smaller bubble is trapped at the common intersection, it distorts into a rounded pyramid. Exactly the same shape, with the correct angles, can be seen in the silica skeleton of the diminutive organism* Callimitra agnesae.

Thompson saw other mathematical patterns, too. There were obvious ones in radiolarians, which are also microscopic marine creatures with a hard silica skeleton. The bodily scaffolding of these tiny animals displays innumerable and beautiful mathematical patterns, some of which bear a striking resemblance to Euclid's regular solids — the octahedron, dodecahedron, and icosahedron.

Clearly, his discussion made no contact with what is now the dominant theme in modern biology: genetics. What Thompson could not possibly have known is what Crick and Watson recognized in 1953. Their epic discovery was that every cell of every living organism contains a symbolic recipe for that kind of organism, written in a molecular code. This recipe is the organism's genome*, its total genetic makeup, written in the language of DNA.*

DNA plays two linked but distinct roles that make living organisms different from the rest of the physical universe. First, DNA prescribes the growth and the form — and indeed even the behavior — of the organism, using a molecular code. Second, that code recombines and occasionally mutates — changes because of random chemical mistakes — allowing the organism to evolve. So instead of the first–principles mathematics of growth with which D'Arcy Thompson was familiar, life is perpetually and blindly tinkering with its own mathematical basis — nudging its development in preferred directions and away from undesirable ones.

How much of the mathematics of growth survives this nudging? The DNA sequence for an organism can, in principle, create any physically realizable form or pattern — and, in principle, any form whatsoever can also evolve, provided the evolutionary change offers an advantage.

Yet Thompson's key argument is still valid — mathematical patterns exist in living organisms, and these patterns must therefore have mathematical causes.

There are known today (2008 CE) good reasons why the underlying mathematics should have survived such tinkering: living creatures make use of the structures and processes that unfettered physics provides, but those processes have to be modified and controlled before a true living organism can result. Chemical reactions, for instance, tend to slowly run out of important ingredients and grind to a halt.

Organisms solve this problem by replenishing their supply of key chemicals, a trick known as "food." No simple physical or chemical system goes looking for food, but some *of the necessary ingredients of such a system can be found in the inorganic world. Chemicals can diffuse, spread out from their source; the farther away you are from that source, the weaker the concentration of the chemical is. Reversing that process, anything looking for that chemical can climb the gradient, moving in whichever direction increases the concentration. So the physical universe supplies a trick that* could *be used by an organism to search for food.*

Genes come into the picture by making sure that this trick is *used. Genes add a lot of flexibility to growth and form because they control and select the physical patterns that the organism needs. With the help of evolution, any genetic tinkering that works — does something new and useful — may gradually becomes more and more sophisticated. Nevertheless, most of the time, nature employs relatively uncontrived mathematical patterns: spots, stripes, patches, blobs, and other patterns of a type that are explainable by mathematical models. The surface markings of tropical fish, despite their intricacy, are similar to those produced by entirely straightforward mathematical processes, and the same goes for markings on seashells, insects, and mammals.*

It is true that in birds, the forms and patterns do indeed become considerably more exotic. Birds of paradise, for instance, are famous for their fringes, curlicues, spikes, and crests. Also, the basic unit of bird pattern is the feather, which is not a shape traditionally employed by mathematicians. Bird pattern can even change, often in dramatic ways, when the bird changes its position and alters the register *of the feathers — the way they are configured next to each other.*

Nevertheless, the patterns found on birds are made from mathematical ingredients. *The pattern-making mechanism fits those ingredients together in rather arbitrary ways — but that's because evolutionary pressures on markings seem to have been very strong for birds. An exotically marked bird is* *rather like what would be produced by a mad mathematician with a big pattern book and a pair of scissors — a collage of different mathematical forms, rather than a unified whole. Nonetheless, the mathematics is still present.*

Although genetics and evolution are very flexible, they cannot actually do anything. *They can find clever ways to harness physical laws to counterintuitive ends, such as reproduction, but they can't* break *those laws.*

Consider also the hemoglobin molecule, which picks up oxygen from our lungs, carries it in our bloodstream, and releases it where it is needed. Hemoglobin is a highly complex molecule, a protein made by sticking together a large number of units known as "amino acids." A complete hemoglobin molecule acts rather like a pair of pincers: It snaps shut around an oxygen molecule, holds it in its jaws, and opens to release it again sometime later. These abilities depend very sensitively on the precise shape of the hemoglobin molecule, and on its ability to click into slightly different positions — open and shut, so to speak.

Genes prescribe the amino acids that must be put together to make a hemoglobin molecule, but they do not prescribe its actual shape. Indeed the protein folding problem — to predict the three-dimensional geometry of a protein molecule from its sequence of amino acids — lies at the very frontiers of today's science, and we do not understand it at all well. We do understand, however, that the shape of a protein molecule is controlled by more than its genetic code. The shape is a consequence of deep laws of physics and chemistry, which are expressible in mathematical form. So, ultimately, the shape of hemoglobin depends on mathematics.

Genes are not the laws of life; they are what the laws use to operate. As a loose analogy, the current state of the solar system is determined by two things: mathematical laws of motion and gravitation, plus *a list of initial conditions that tell us where all the planets were at some chosen instant. Plug the initial conditions into the laws, and all of the intricacies of the solar system follow.*

There is a strong tendency to think of genes as constituting the laws of biological development, but that's not so; their role is much closer to that of initial conditions. In other words, genes are not the *key to life. They are* a *key, and an enormously important one, but behind them lies something much deeper. There must be more fundamental theories, the true laws of biology, the mathematical rules into which the genetic code is plugged.*

Physics provides a range of patterns and structures that are available. Evolution and genetics can modify those structures and patterns, fine-tune them, and put them together in ways that would not be natural for raw physics; nevertheless, the mathematical patterns provide building blocks and a point *from which genetics and evolution can operate. Moreover, if it so happens that these freely available forms do their job effectively without modification, then evolution will select them, and genetics will respect them. In this manner, these structures and patterns have been built into the very fabric of life.*

Quantification of Nonlinear Biological Phenomena (1917–1985)

Biology deals with phenomena that are intrinsically more complex and more difficult to investigate than those normally studied in the 'hard' sciences such as physics, and not everything which seems obvious is true. Consequently, it is more susceptible to the introduction of hypotheses whose correctness cannot be adequately tested.

Furthermore, mathematical models of biological systems, with their inherent propensity for simplification and idealization, are sometimes so speculative that they have only the merest pretense of accounting for to biological observations.

One must therefore take the mathematical models of these phenomena only as metaphors, and not as "laws" as in astronomy or physics. Nevertheless, there is a body of mathematical ideas and models that is so closely intertwined with biological observations, that it has established a place in the literature of modern biology. The day, furthermore, may come when biological forms and processes can be described with the same mathematical precision as those in physics.

We shall treat here eight topics:

- *Dynamics of non-interacting populations.*
- *Dynamics of interacting species and stability theory.*

- *The Fisher-Kolmogorov equation.*
- *Enzyme kinetics.*
- *Heart-beat equation.*
- *General models of epidemics*
- *Cellular slime molds*
- *Chemotaxis*

Dynamics of non-interacting populations

The use of mathematics in attempts at formulating biological principles has a long history. As far back as 1202, ***Leonardo of Pisa*** *(**Fibonacci**)had clearly thought about population growth in connection with rabbit populations.* ***Giovanni Alfonso Borelli*** *(1608–1679, Italy) presented a quantitative geometrico-mechanical approach to animals' motion in his book De motu animalium (posthumous,* 1680*; several years before Newton's Principia).*

During the 19^{th} *century there was an upsurge of interest in interdisciplinary research among several of the great mathematicians and scientists of the age.* ***D'Arcy Thompson****, influenced by the* 19^{th} *century attempts at more rigorous biological formalisms, published (1917) his work On Growth and Form, a book which marks the advent of modern theoretical biology.*

The works of ***Hardy*** *(1908),* ***Lotka*** *(1920),* ***J.B.S. Haldane*** *(1892–1964, England, 1924),* ***Volterra*** *(1926, 1931),* ***R.A. Fisher*** *(1930)* and ***N. Rashevsky***[382] *(1933 to 1946) advanced the diffusion of mathematical tools and techniques into biological research. In return, biology has confronted mathematicians with a variety of challenging problems which have stimulated developments in the theory of nonlinear (ordinary and partial) differential equations. A few typical examples, chosen from the vast literature of classical mathematical biology, illuminate some of the scenes of nature's drama of life.*

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382 **Nicolas Rashevsky** (1899–1972, U.S.A.). Pioneering mathematical biologist. Among the first to model the detailed structure of *individual organisms* and the relations of the fundamental parts of each organism to the physical inorganic world. Contributed to mathematical biophysics of *nerve conduction, excitation and inhibition*.

Let N *represent the number of individuals* $N(t)$ *in a given spatial region at any time* t. *The growth law* $\frac{dN}{dt} = g(N) = N\lambda(N)$, *represents the excess of birth rate over death rate, and* $\lambda(N) = \frac{1}{N}\frac{dN}{dt}$ *is the growth rate. If* $\lambda = \text{const} > 0$, *the ensuing law* $N(t) = N_0e^{\lambda t}$ *predicts an exponential growth without limit (**Euler** 1760; **Malthus** 1798), where* $N(0) = N_0$ *is the initial size of the population.*

Although this law may accurately reflect experiments in the early stages of population increase, it obviously cannot hold over an infinitely long period[383]*. For once the population grows sufficiently large, it will begin to interact with the environment, with itself, and with other species due to limited sources of nutrients and also due to competition. All this will cause the birth rate to decrease, the death rate to increase, or both. Thus, crowding may have the same effect as limiting the food supply; space can be considered necessary to sustain life for many species. The simplest model of this kind (**Verhulst**[384], 1838) assumes that normal growth is inhibited by a term proportional to* N^2.

At the beginning of the 20^{th} *century, mathematical models were widely used for describing the growth of bacterial populations and the progress of various diseases, such as malaria. These problems were discussed in terms of ordinary differential equations in which time is the dependent variable and the unknown functions are the average values of the populations densities.*

An example is the growth of a microbial colony; when cells divide, they grow in number but as we know, a colony of bacteria or other microorganisms embedded in a nutrient medium will not grow indefinitely. There are many possible reasons for this: first, multiplication in numbers introduces crowding effects. Biochemically, this may be due to lack of nutrients, shortage of oxygen, the appearance of toxic substances, or changes in ion concentration in the medium, especially pH. Whatever the cause, the natural growth is inhibited due to inter-cell interaction.

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383 The bacterium **E. Coli** would, under ideal circumstances, divide (on average) every 20 minutes. In this way, one cell (mass $\approx 3 \times 10^{-10}$ gm) could produce $2^{144} \approx 2 \times 10^{43}$ cells within 48 hours — a supercolony whose size and weight are comparable to those of the sun! This never actually happens, for the simple reason that growth cannot continue indefinitely under "ideal circumstances". Even in the absence of competition and predators, nutrients runs out and oxygen runs out: local conditions within the colony change and check the growth of the organisms.

384 **P.F. Verhulst** (1804–1849, Belgium). Initiated his investigations under stimulation from **L.A.J. Quetelet** (1796–1874, Belgium), a statistician and astronomer. The term '*logistic*' was first used by **Edward Wright** (1599) to describe an S-shaped curve.

Thus, the population $N(t)$ *at any time* t *is represented by the ordinary differential equation* $\frac{dN}{dt} = \lambda N - \beta N^2, \ \lambda > 0, \ \beta > 0$. *Here again, the term* λN *represents the excess of the birth rate over the death rate in the absence of interactions. Since the number of intercellular interactions of* N *cells is of the order* N^2, *the plausibility of the inhibition term is explained (e.g.: a given cell detects the cumulative toxic effect of all* N *cells, if the toxic material diffuses freely throughout the intercellular medium. Thus, the toxic effect on a given cell is proportional to* N. *The toxic effect on all* N *cells is* N *times the effect on one cell and hence proportional to* N^2*).*[385]

The stationary value $\left(\frac{dN}{dt} = 0\right)$ *occur at both* $N = 0$ *and* $N_e = \frac{\lambda}{\beta}$, *of which the first is an unstable equilibrium point while the second is stable*[386]. *The exact solution is known as the logistic* law *of growth*

$$N(t) = \frac{N_e}{1 + (\frac{N_e}{N_0} - 1)e^{-\lambda t}}.$$

As $t \to \infty$, *a stable population* $N = N_e$ *is asymptotically attained, with zero growth. The logistic law found applications to various problems in chemical kinetics, biology and economy. In ecology, the growth of populations is inhibited by intra-species deleterious paired encounters (due to competition for food, habitat and other limited resources) proportional to* N^2.

The alternative functional dependence $\lambda(N) = a_1 + a_2 N - a_3 N^2$ $(a_1, a_2, a_3 > 0)$ *found a biological application in populations having a maximum growth rate at intermediate density, an effect that may stem from a difficulty of finding mates at very low density of population.*

A law similar to the Verhulst model applies to the law of mass action *in the chemical kinetics of reactions of the second order:* $\frac{dN}{dt} = \lambda(A - N)(B - N)$, *where* $\lambda, A, B > 0$, λ *now being the* specific reaction rate *at a constantly held temperature. A solution satisfying the initial condition* $N(0) = 0$ *is*

$$N(t) = A\left[1 + \frac{B - A}{A - Be^{\lambda(B-A)t}}\right].$$

When $t \to \infty$, N *tends either to* A, *if* $A < B$, *or to* B, *if* $A > B$.

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385 Thus, population-dynamics differential equations — whether it be of genome allele frequencies, cells or larger organisms — bear a strong resemblance to *chemical kinetics* equations, governing reaction rates of species of simple *molecules*.

386 In general, the equilibrium points of a system governed by the differential equation $\frac{dN}{dt} = g(N)$ are determined from $g(N_e) = 0$. Near each of these points $g(N) \simeq g'(N_e)(N - N_e)$, from which the stability can be determined by the sign of $g'(N_e)$.

The autocatalytic reaction $\frac{dN}{dt} = \lambda(N + N_0)(B - N)$, *with* $N(0) = 0$, *yields the logistic curve* $N_0 + N = \frac{B+N_0}{1+\frac{B}{N_0}e^{-\lambda(B+N_0)t}}$. *Here, an initial concentration* $N(0) = 0$ *increases initially, as the molecular species in question is produced and eliminated by several pathways: production from steadily–supplied reagents (with or without its own participation), and its elimination (via reactions involving one or two molecules of the species).*

The law $\lambda(N) = -\lambda_0 \ln N$ *(**Gompertz**[387], 1825) is popular in clinical oncology. It is also used by actuaries to estimate the risk of death in life insurance.*

Dynamics of two interacting species

In most ecosystems the conflict between different species must be taken into account, since the growth and decline of populations in nature is strongly affected by the struggle of species to predominate over or prey upon one another. In addition, environmental effects, chance random effects, and spatial heterogeneity, cannot be ignored.

Deterministic mathematical models for the behavior of two interacting species (e.g., trees in a forest, fishes in the oceans, animals on land, etc.) are governed by a system of differential equations

$$\frac{dN_1}{dt} = g_1(N_1, N_2),$$

$$\frac{dN_2}{dt} = g_2(N_1, N_2).$$

387 **Benjamin Gompertz** (1779–1865, England). Mathematician and actuary. Collected data on mortality by natural causes (number of surviving individuals as a function of time).
Gompertz was self-educated, reading Newton and Maclaurin, since he was denied admission to universities on account of being Jewish. He showed (1825) that mortality rate increases in a geometric progression. He became a Fellow of the Royal Society (1819). **Gompertz** — an illustrious Jewish family (since 1600) in Germany, Holland, Britain, United States and Austria-Hungary. The name derived from **Gumpel** — a medieval nickname for **Mordechai**.

If we ignore the effects of each population on itself, there are three distinct types of interaction between two species: both populations enhance each other (symbiosis), both populations conflict with each other (competition), and the hybrid predator-prey interaction such as the plant-herbivore system, the parasite-host system and the fish-shark system.

In 1925, the Italian biologist ***Umberto d'Ancona*** *observed a puzzling biological trend in the fish population of the upper Adriatic: During WWI (1914–1918), commercial fishing in the Adriatic Sea fell to rather low levels. It was anticipated that this would cause a rise in the availability of fish for harvest. Instead, the population of commercially available fish declined on average while the number of sharks and other voracious species, which are their predators, increased! The two populations were also perceived to oscillate. D'Ancona then interested his mathematical colleague,* ***Vito Volterra***[388] *in this problem, and the latter suggested (1926) a somewhat naive model to describe the predator-prey interaction:*

Let $N_1(t)$ denote the fish population (prey) and $N_2(t)$ the predator (shark) population at any given time t. The fish population finds ample food (plankton) all the time, and if not for the sharks would grow in proportion to their number $N_1(t)$. However, their number is constantly reduced due to encounters with sharks, and this decrease rate is proportional both to their density and the density of their predators. Hence

$$\frac{dN_1}{dt} = aN_1 - bN_1N_2 \qquad a, b > 0.$$

On the other hand, in absence of prey, shark will naturally be demised at a rate proportional to their own number. Their rate of increase will depend solely on their food intake, which is again proportional to the rate of encounters with their prey, namely in proportion to N_1N_2.

All told

$$\frac{dN_2}{dt} = -cN_2 + dN_1N_2 \qquad c, d > 0.$$

These two simultaneous differential equations in $\{N_1, N_2\}$, known as the Lotka-Volterra predator-prey equations, have no exact solutions in terms of elementary functions. (The most that can be shown through a straightforward

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388 Volterra, V., Variazioni e fluttuazioni del numero d'individui in specie animal conviventi, *Mem. Acad. Lincei* **2**, 31–113, 1926.

A similar set of equations was independently derived by **Alfred James Lotka** (1880–1949, U.S.A.) to model observed oscillations in concentrations observed in second order chemical reactions. [Undamped oscillations derived from the law of mass action, *J. Amer. Chem. Soc.* **42**, 1595–1599, 1920.]

analysis is that $t = \int \frac{dv}{vu(v)}$, *where* $u(v)$ *is a solution of the implicit transcendental relation* $(a-u)^a e^u = Ke^{cv}v^{-c}$, $K =$ *integration constant, and* $N_1 = \frac{c}{d}v$. *Similar results are obtained for* N_2, *and* $u = a - bN_2$.*)*

Yet, Volterra was able to extract the salient features of the system's behavior without the explicit knowledge of $N_1(t)$ *and* $N_2(t)$. *Denoting* $x(t) = N_1$, $y(t) = N_2$ *and excluding the trivial solution* $x = 0$, $y = 0$, *there is one nontrivial, static solution with equilibrium populations* $x_e = \frac{c}{d}$, $y_e = \frac{a}{b}$. *Effecting a small perturbation* $x = x_e + X$, $y = y_e + Y$ *about equilibrium, the linearized equations yield the small-amplitude solution*

$$X = \sqrt{x_e Q_x}\cos[\omega(t-t_0)],$$

$$Y = \sqrt{y_e Q_y}\cos\left[\omega(t-t_0) - \frac{\pi}{2}\right],$$

with

$$Q_x = \frac{(x_0-x_e)^2}{x_e} + \frac{(y_0-y_e)^2}{y_e}\frac{b}{d},$$

$$\omega = \sqrt{ac} = \frac{2\pi}{T}, \quad Q_y = \frac{d}{b}Q_x.$$

This result means that, starting from given initial populations $\{x_0, y_0\}$ *at time* t_0, *the trajectories of* $x(t)$ *and* $y(t)$ *are closed curves enclosing* (x_e, y_e), *whose scales shapes asymptotically become ellipses as they shrink to size zero; moreover, the motion* $[x(t), y(t)]$ *becomes simple harmonic motion in that regime, and the family of ellipses are centered at* (x_e, y_e). *The initial state places the system at a point on an approximate ellipse and this point moves, with increasing time, on the ellipse, completing one revolution with a period* $T = \frac{2\pi}{\sqrt{ac}}$.

Volterra then proceeded to show that the qualitative behavior of the system at points which are not necessarily near equilibrium retains the same character: the paths in the xy plane are still closed curves enclosing the equilibrium point, meaning that both $N_1(t)$ *and* $N_2(t)$ *are periodic functions of time, with* N_1 *lagging a quarter of a period behind* N_2. *Thus*

$$N_1(t+T) = N_1(t) \quad , \quad N_2(t+T) = N_2(t).$$

Integrating $\frac{\dot{N}_1}{N_1} = a - bN_2$ *from* 0 *to* T, *we find*

$$\frac{a}{b} = \frac{1}{T}\int_0^T N_2(t)dt = \textit{average value of } N_2(t)$$

over one cycle. Likewise,

$$\frac{c}{d} = \textit{average value of } N_1(t) \textit{ over one cycle.}$$

Thus, both populations vary periodically such that their time-averages coincide with their respective equilibrium values.

This mathematical picture is reasonable: assuming the population of the fish to increase, the sharks would have enough food to sustain a larger population and thus pose a severe threat to the fish. Eventually the population of the plankton-eating fish would diminish. After a while food becomes sparse for the sharks, causing their own population to decrease. A shortage of predators then leads to a resurgence of the fish population. This process may continue indefinitely, in which case, the ecosystem would consist of periodic population variations — an example of biological rhythms.

Fishing *decreases the populations of both species. To account for this we modify the equations of the system into*

$$\dot{N}_1 = aN_1 - bN_1N_2 - \epsilon N_1,$$

$$\dot{N}_2 = -cN_2 + dN_1N_2 - \epsilon N_2,$$

where ϵ is a parameter that reflects the density of fishing (number of boats, nets, etc.). For $a - \epsilon > 0$, the new system is of the same type as before with new average values $\overline{N}_1 = \frac{c+\epsilon}{d}$, $\overline{N}_2 = \frac{a-\epsilon}{b}$ and a new period. So, on average, a moderate amount of fishing increases *the number of fish*[389] *and* decreases *the number of sharks. If, however, fishing is reduced from the new fiducial level*

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389 The consequence that the prey always recovers more rapidly from a catastrophic event that decimates both species in proportion to their population sizes (e.g., overhunting, fishing, forest fire, etc.), is known as the *Volterra principle*.

A remarkable confirmation comes from the cottony cushion *scale insect* (Icerya purchasi), which, when accidentally introduced from Australia in 1868, threatened to destroy the American citrus industry. Thereupon, its natural Australian predator, a *ladybird beetle* (Novius Cardinalis) was introduced, and the beetles reduced the scale insects to a low level. When DDT was discovered to kill scale insects, it was applied by the orchardists in the hope of further reducing the scale insects. However, in agreement with Volterra's principle, the effect was an increase of the scale insect. Sometimes the predator is so efficient that no cycle emerges at all, though other unexpected side effects may evolve:

An island in the Pacific was a pleasant place and fertile but it had one drawback; it was infested by snakes. In desperation, the natives imported a cage full of mongooses, brought by some sailors especially for them. In no time, the mongooses bred and lived on the snakes to the complete satisfaction of the is-

$\{c+\epsilon, a-\epsilon\}$, *then w.r.t. this new level, the number of sharks will* increase, *and the number of food fish will decrease. This explains the observations of d'Ancona.*

These are somewhat counter-intuitive results. For assume a situation where farmers are dissatisfied with the large number of rabbits in a fox-rabbit ecosystem. Their instinctive impulse would be to introduce more foxes. But the above theory shows that this will only increase the magnitude of the oscillation without changing the mean-values of both species. Only a gradual elimination of both *species by (selective!) animal traps will do. Thus, man's intervention in the role of a superpredator, preying with equal or unequal intensity on* both *species, can changes nature's delicate balance.*

Volterra's theory has spectacular applications to insecticide treatments which destroy both insect predators and their insect prey. It implies that the application of insecticides will actually increase the population of those insects which are kept in control by other predatory insects. Oddly enough, many ecologists and biologists refused to accept Volterra's model as accurate.

One objection is based on the fact that the model predicts an oscillation whose amplitude *is sensitive to the* initial conditions. *Although this seems reasonable for the sharks and fish of the Adriatic, it is entirely unbelievable that lynx and hare populations should oscillate in a manner determined by events a hundred (or more) years ago. Thus a predator-prey model is sought which perhaps indicates oscillation of a type inherent to the* system *rather than determined by initial conditions. (Predator-prey models can be constructed in which the amplitude of oscillation decay as time goes on, as though nature seeks to restore predators and prey to an ecological balance.)*

A more general model of predator-prey interaction is the system

$$\dot{x} = ax - bxy - ex^2,$$

$$\dot{y} = -cy + dxy - fy^2,$$

which include competition among prey for their limited external resources, and also competition among the predators for the limited number of prey. The solutions of this system are not in general periodic. Indeed, if $a, b, c, d, e, f > 0$

landers. But as the snakes were disappearing, the mongooses were multiplying. There were none of their natural enemies on the island, so there was no check to their growth. With diminishing snakes and increasing mongooses, food for the later grew scarcer and they started to feed on the islander's chickens. So the last state of these islanders was worse than the first. They ended up trying to import some animal that kills mongooses...

and $x(0) > 0$, $y(0) > 0$, *every solution approaches the equilibrium solution* $x = \frac{a}{e}$, $y = 0$ *as* $t \to \infty$ *if* $\frac{c}{d} > \frac{a}{e}$.

Finally, special atypical types of predators have their own equations; and several predator-prey interactions in nature cannot be modeled by any system of ODE. *These situations occur when the prey are provided with refuge that is inaccessible to the predators. In these situations it is impossible to make any definitive statements about the future numbers of predators and preys, since we cannot predict how many prey will be stupid enough to leave their refuge. In other words, this process is* random *rather than* deterministic, *and therefore cannot be modeled by a system of* ODE.

Not all species form predator-prey relationship. Two species in an ecosystem may compete for the same *limited source of nutrients. It is assumed that the effect of competition is to reduce each species' growth-rate by an amount proportional to the other species' population. Consequently,*

$$\dot{N}_1 = aN_1 - bN_1^2 - kN_1N_2,$$

$$\dot{N}_2 = cN_2 - dN_2^2 - \sigma N_1N_2.$$

Volterra showed that if $\frac{k}{d} < \frac{a}{c} = \frac{b}{\sigma}$, *species* N_2 *will die out and* N_1 *will approach the value* $\frac{a}{b}$. *This is known as the* Volterra principle of competitive exclusion.

The stability theory *of the* Volterra-Lotka *equations is analyzed with the aid of the* Lyapunov function. *Consider the nonlinear system of ODE governing the interaction of* n *species*

$$\dot{N}_i = k_iN_i + \frac{1}{\beta_i}\sum_{j=1}^{n} a_{ij}N_iN_j = N_i\left[k_i + \frac{1}{\beta_i}\sum_{j=1}^{n} a_{ij}N_j\right]$$

where $a_{ii} \le 0$, $\beta_i > 0$, $i = 1, 2, \ldots, n$, $\det|a_{ij}| > 0$ *and* $a_{ij} = -a_{ji}$ *(a loss for one species in an interaction produces a gain for the other).*

The different efficiencies of the species are accounted for by the factors β_i. *If we set* $V_i = \ln\left(\frac{N_i}{q_i}\right)$, *where* $q_i > 0$ *are arbitrary, then*

$$\dot{V}_i = k_i + \frac{1}{\beta_i}\sum_{j=1}^{n} a_{ij}q_je^{V_j}.$$

Assume (without loss of generality) that $k_i\beta_i = -\sum_{j=1}^{n} a_{ij}q_j$ *is satisfied. Then*

$$\beta_i\dot{V}_i = \sum_{j=1}^{n} a_{ij}q_j(e^{V_j} - 1),$$

and $V_j = 0$ *is the only fixed point. Following* **Volterra**, *we multiply the last equation by* $(e^{V_i} - 1)q_i$ *and sum on* i *to obtain (using* $a_{ij} = -a_{ji}$ *and* $a_{ii} \le 0$*),*

$$\frac{d}{dt}\sum_{i=1}^{n} \beta_i q_i \left[e^{V_i} - V_i\right] \le 0$$

But since $(e^{V_i} - V_i) \ge 1$ *for all values of* V_i*, the definition of the new function*

$$L(V) = \sum_{i=1}^{n} \beta_i q_i \left[e^{V_i} - V_i - 1\right]$$

ensures $L(0) = 0$, $L(V) > 0$ *and* $\frac{dL}{dt} \le 0$ *for all* $|\boldsymbol{V}| > 0$.

Hence $L(V)$ *is a global Lyapunov function. Since no* V_i *can become arbitrarily large, the system is of bounded stability under the said conditions.*

Moreover, if $a_{ii} < 0$ *(does not vanish),* $V_i = 0$ *is asymptotically stable.*

Stability of food chains: an ecosystem is composed of n *members (chains with up to 6 members are found in nature). The first population is the prey for the second, which is a prey for the third etc. ... up to the* n*-th, which is at the top of the food chain. Taking competition within each species into account, and assuming interaction terms with constant coefficients, one obtains*

$$\dot{N}_1 = N_1(k_1 - a_{11}N_1 - a_{12}N_2),$$
$$\dot{N}_j = N_j(-k_j + a_{j,j-1}N_{j-1} - a_{jj}N_j - a_{j,j+1}N_{j+1}), \qquad j = 2, \ldots, n-1,$$
$$\dot{N}_n = N_n(-k_n + a_{n,n-1}N_{n-1} - a_{nn}N_n).$$

If all $k_j, a_{ij} > 0$*, it can be shown that the above system of ODE admits a stable equilibrium point, just as in the case of the prey-preditor case* $n = 2$.

Cyclic competition: If three or more species compete, a rather curious thing can occur. It may look for some time as if species 1 were bound to be the unique survivor; then, suddenly, its density drops, species 2 takes its place and seems to dominate the ecosystem; after some time, it in turn collapses, and leaves the field to species 3, which appears to be the ultimate winner; but then, species 1 suddenly rallies and outcompetes its rivals, and so another "round" starts. The species supersede each other in cyclic fashion: the time *spans during which one species predominates grows larger and larger. Such a behavior is represented by the ODE system:*

$$\dot{N}_1 = N_1(1 - N_1 - \alpha N_2 - \beta N_3),$$
$$\dot{N}_2 = N_2(1 - \beta N_1 - N_2 - \alpha N_3),$$
$$\dot{N}_3 = N_3(1 - \alpha N_1 - \beta N_2 - N_3)$$

The special symmetry assumption behind the model is that of a cyclic interaction between the species: if we replace 1 *by* 2, 2 *by* 3 *and* 3 *by* 1, *the equation will remain unchanged. The system admits a unique* interior rest point $\boldsymbol{m}$ *at* $m_1 = m_2 = m_3 = \frac{1}{1+\alpha+\beta}$.

The Jacobian at the point $\boldsymbol{m}$ *is the* circulant matrix[390]

$$\frac{1}{1+\alpha+\beta}\begin{bmatrix} -1 & -\alpha & -\beta \\ -\beta & -1 & -\alpha \\ -\alpha & -\beta & -1 \end{bmatrix}.$$

Its eigenvalues are $\gamma_0 = -1$ *[with eigenvector* $(1,1,1)$*] and*

$$\gamma_1 = \bar{\gamma}_2 = \frac{1}{1+\alpha+\beta}(-1 - \alpha e^{\frac{2\pi i}{3}} - \beta e^{\frac{4\pi i}{3}}).$$

The real part of γ_1 *and* γ_2 *is thus* $\frac{1}{1+\alpha+\beta}(-1 + \frac{\alpha+\beta}{2})$, *assumed positive. Hence* $\boldsymbol{m}$ *is a* saddle point. *There are four other rest points: the origin* 0 *and the saddles* $\boldsymbol{e}_1$, $\boldsymbol{e}_2$, $\boldsymbol{e}_3$ *(the coordinate axes unit vectors).*

A study of stability in the $N_1N_2N_3$ *space then shows that the state remains for some time close to the rest-point* $\boldsymbol{e}_1$, *then travels to the vicinity of the rest-point* $\boldsymbol{e}_2$, *lingers there for a still longer time, then transitions to the vicinity of*

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[390] An $n \times n$ matrix is said to be a *circulant* if it is of the form

$$\begin{bmatrix} c_0 & c_1 & c_2 & \dots & c_{n-1} \\ c_{n-1} & c_0 & c_1 & \dots & c_{n-2} \\ \vdots & \vdots & \vdots & & \vdots \\ c_1 & c_2 & c_3 & \dots & c_0 \end{bmatrix},$$

where a cyclic permutation sends the elements of each row into those of the next one. Its *eigenvalues* are $\gamma_k = \sum_{j=0}^{n-1} c_j \lambda^{jk}$, $k = 0, \dots, n-1$ and *eigenvectors* $y_k = \left(1, \lambda^k, \lambda^{2k}, \dots \lambda^{(n-1)k}\right)$ where λ is the nth root of unity $\lambda = e^{2\pi i/n}$.

the rest-point $\boldsymbol{e}_3$, *and so on in cyclic fits and starts. This model thus suggests a surprising mechanism for sudden upheavals in ecological communities.*

The Fisher-Kolmogorov equation

A single species grows to saturation and diffuses according to the partial differential equation

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} + au - bu^2, \qquad D > 0, \quad a > 0, \tag{1}$$

where D *is the diffusion coefficient (diffusivity) and* (a, b) *are the parameters of the logistic growth.*

This equation describes nonlinear evolution of a population u *in a one-dimensional habitat. It was introduced by* ***R.A. Fisher*** *(1936) to describe the propagation of a virile mutant in an infinitely long habitat. It also represents, with a minor variation, a model equation for the evolution of a neutron population in a nuclear reactor (a finite domain). A complex version (Nonlinear Schrödinger Equation) models laser dynamics in a medium of two-state atoms or molecules.*

Equation (1) describes a balance between linear diffusion and nonlinear local multiplication and admits shock-like solutions.

Special case:

$b = 0$ *(simple growth with diffusion)* $0 \le x \le L$ *(finite region).*

Diffusion is proportional to surface area while growth (reproduction) is proportional to volume. For a sphere of radius R, *the ratio of surface area to volume is proportional to* $\frac{1}{R} \approx \frac{\text{diffusion}}{\text{growth}}$. *As* R *decreases, diffusion plays an increasingly important role and eventually a limit is reached beyond which growth can no longer compensate for loss due to diffusion. This is the critical size* $L_c = f(D, a)$. *Dimensional analysis yields*

$$[D] = L^2T^{-1}, \quad [a] = T^{-1}, \qquad \Rightarrow \qquad L_c = \gamma\sqrt{\frac{D}{a}} \quad \text{or} \quad (a - \gamma^2\frac{D}{L_c^2}) = 0,$$

where γ *is a non-dimensional constant of order unity. Assuming boundary conditions* $u(0, t) = u(L, t) = 0$ *and initial condition* $u(x, 0) = f(x)$, *a Fourier-series solution for the one-dimensional case is*

$$\left.\begin{aligned} u(x,t) &= \textstyle\sum_{n=1}^{\infty} A_n \sin(\frac{n\pi x}{L}) e^{(a-\frac{D\pi^2 n^2}{L^2})t} \\ A_n &= \textstyle\frac{2}{L}\int_0^L f(x)\sin(\frac{\pi n x}{L})\,dx \end{aligned}\right\}$$

If $a < \frac{D\pi^2}{L^2}$*, the population will be unable to maintain itself against diffusion and disappear. If* $a > \frac{D\pi^2}{L^2}$*, at least the first term will increase indefinitely with time. Therefore*

$$L_c = \pi\sqrt{\frac{D}{a}} \qquad \text{in one dimension}$$

and it can be shown that:

$$L_c = 4.81\sqrt{\frac{D}{a}} \qquad \text{in two dimensions} \tag{2}$$

Now, the exact solution of $\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} + au$, $a > 0$ *with* $u(x,0) = f(x) = u_0\delta(x)$ *(Dirac delta–function) and as the boundaries are removed to infinity, is*

$$u(x,t) = \frac{u_0}{2\sqrt{\pi D t}} \exp\left(at - \frac{x^2}{4Dt}\right).$$

The exponent is zero at $\frac{x}{t} = 2\sqrt{aD} = V$ *(defined). We can say that the surface* $u = \text{const}$ *propagates with approximate velocity* v*. A similar result holds in higher dimensions.*

Traveling wavefront solutions (Kolmogorov, 1936)

The substitution

$$u = \frac{a}{b}\bar{u}, \qquad t = \frac{1}{a}\bar{t}, \qquad x = \sqrt{\frac{D}{a}}\bar{x}$$

transforms the Fisher equation into the nondimensional *form*

$$\frac{\partial \bar{u}}{\partial \bar{t}} = \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \bar{u}(1-\bar{u}) \tag{3}$$

We seek solutions of (3) *which have a 'plane wave' form of speed c:*

$$\bar{u}(\bar{x},\bar{t}) = U(\bar{x}-\bar{t}) = U(s), \qquad c = \sqrt{aD}\,\bar{c}, \quad \bar{c} > 0 \tag{4}$$

Substitution into (3)*, shows* U *to satisfy equation of the* ***Liénard*** *type*

$$U'' + \bar{c}U' + U(1-U) = 0, \tag{5}$$

where $\bar{c}$ *is chosen such that*

$$\begin{aligned} &U(\infty) = 0 \text{ (before wavefront's arrival)}, \\ &U(-\infty) = 1 \text{ (after front's departure)}. \end{aligned} \tag{6}$$

Eq. (5) can be replaced by a system of two coupled first-order ODEs

$$\left.\begin{aligned} \frac{dU}{ds} &= V \\ \frac{dV}{ds} &= -\bar{c}V - U(1-U) \end{aligned}\right\} \tag{7}$$

This flow has two singular points in the phase plane (U, V), *namely* $(0,0)$ *and* $(1,0)$, *which are obviously steady states:* $U' = 0$, $V' = 0$. *A linear stability analysis shows that the eigenvalues* λ *for the singular points are*

$$\begin{aligned} &(0,0) \qquad \lambda_{\pm} = \tfrac{1}{2}[-\bar{c} \pm \sqrt{\bar{c}^2 - 4}] \Rightarrow \quad \text{stable node} \quad \text{if} \quad \bar{c}^2 \geq 4 \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{stable spiral} \quad \text{if} \quad \bar{c}^2 < 4 \\ &(1,0) \qquad \lambda_{\pm} = \tfrac{1}{2}[-\bar{c} \pm \sqrt{\bar{c}^2 + 4}] \Rightarrow \quad \text{saddle point} \end{aligned} \tag{8}$$

From this, one is able to show that for all wave speeds

$$c \geq c_{\min} = 2\sqrt{aD} \tag{9}$$

there exists a progressive wave that satisfies the Fisher equation such that $0 \leq U \leq 1$, $U(\infty) = 0$, $U(-\infty) = 1$. *The remarkable result here is that there is a continuum of waves, all having different speeds (as with solitary waves in hydrodynamics).*

A key question still to be answered is: what kind of initial conditions $u(x, 0)$ *for the original Fisher equation will evolve into a traveling wave solution, and if such a solution exists, what is its wave speed c.* ***Kolmogorov*** *(1937) gave the following answer:*

If $\bar{u}(\bar{x}, 0) = \begin{cases} 1 & \bar{x} \leq \bar{x}_1 \\ 0 & \bar{x} \geq \bar{x}_2 \end{cases}$ *where* $\bar{x}_1 < \bar{x}_2$ *and* $\bar{u}(\bar{x}, 0)$ *is arbitrary but continuous in* $\bar{x}_1 < \bar{x} < \bar{x}_2$, *then the solution* $\bar{u}(\bar{x}, t)$ *of (3) evolves to a traveling wavefront with* $\bar{z} = \bar{x} - 2t$ *(minimal speed). It can also be shown that in general, the wavefront solution of*

$$\frac{\partial \bar{u}}{\partial \bar{t}} = f(\bar{u}) + \frac{\partial^2 \bar{u}}{\partial x^2}, \tag{10}$$

[where $f(\bar{u})$ has only two zeros (say u_1 and $u_2 > u_1$), $f'(u_1) > 0$ and $f'(u_2) < 0$] evolve with $\bar{u}$ going monotonically from u_1 to u_2 with wave speeds

$$\bar{c} \geq \bar{c}_{\min} = 2\sqrt{f'(u_1)}. \tag{11}$$

Enzyme kinetics

Biochemical reactions are continually taking place in all living organisms and most of them involve large proteins molecules called enzymes, *which act as remarkably efficient* catalysts. *A catalyst accelerates the rate of a reaction but is not consumed in the overall process. Enzymes react selectively on definite compounds undergoing chemical reaction, called* substrates. *For example, hemoglobin in red blood cells is an enzyme and oxygen, with which it combines, is a substrate. Enzymes are important in regulating biological processes, for example as* activators *or* inhibitors *in a reaction.*

Biological catalysts differ from all other catalysts known to chemistry in two essential ways. First, they are exceptionally efficient *under the mild conditions holding in the normal physiological state: aqueous medium, standard pressure, and physiological pH and temperature ranges. Second, a single enzyme molecule can transform 10^3 to 10^6 molecules of substrate per minute. That is why their catalytic function in a cell can be performed* rapidly *and why* extremely small quantities *of enzyme suffice to enable cellular processes.*

One of the most basic enzymatic reaction mechanisms, first proposed (1913) for the yeast-catalyzed conversion of sucrose to glucose and fructose by **Leonor Michaelis** *(1875–1949,* Germany, U.S.A.*) and* **M.I. Menten** *[and later represented mathematically by* **Haldane** *(1925)], has been successful in describing most enzymatically controlled reactions.*

A substrate molecule S reacts with an enzyme molecule E to form a complex molecule C which in turn is converted into a product P and the enzyme. The reactions in question can be symbolized by

$$E + S \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} C \overset{k_2}{\rightarrow} E + P.$$

Let $e(t)$, $s(t)$, $c(t)$ and $p(t)$ denote the molar concentrations of the respective molecular species. Applying the law of mass action, *one finds that the* *following are the differential equations and initial conditions that correspond to the above reaction:*

$$\frac{ds}{dt} = -k_1 es + k_{-1}c,$$

$$\frac{de}{dt} = -k_1 es + (k_{-1} + k_2)c,$$

$$\frac{dp}{dt} = k_2 c,$$

$$\frac{dc}{dt} = k_1 es - (k_{-1} + k_2)c,$$

$$s(0) = s_0, \quad e(0) = e_0, \quad c(0) = 0, \quad p(0) = 0.$$

Since $e(t) + c(t) = e_0$, *and* $p(t) = k_2 \int_0^t c(t')dt'$, *the above system, after scaling out dimensionful quantities, reduces to the system*

$$\frac{du}{d\tau} = -u + (u + K - \lambda)v,$$

$$\epsilon\frac{dv}{d\tau} = u - (u + K)v,$$

$$u(0) = 1, \qquad v(0) = 0, \qquad \tau = (k_1 e_0)t,$$

$$u(\tau) = \frac{s(t)}{s_0}, \qquad v(\tau) = \frac{c(t)}{e_0}, \qquad \lambda = \frac{k_2}{k_1 s_0},$$

$$K = \frac{k_{-1} + k_2}{k_1 s_0}, \qquad \epsilon = \frac{e_0}{s_0}.$$

This system consists of two nonlinear differential equations in the unknown concentration of the substrate S and the complex C.

Biochemists commonly wish to determine the velocity of reaction, *which is usually defined as either the rate of appearance of the product P, or the rate of disappearance of the substrate S. Since an exact analytical solution is not available, and even the numerical solution of the equations is hampered*[391] *by the smallness of ϵ, an approximation is sought which renders a theoretical value for the measured initial velocity of reaction.*

An assumption is made that a quasi-steady state *("pre-equilibrium") is established very rapidly, so that the concentration of C is changing very*

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391 ϵ is typically in the range 10^{-2} to 10^{-7}. Since ϵ multiplies the highest derivative in the second equation, this is a *singular perturbation problem* with an ensuing *boundary layer* near the time origin $\tau = 0$.

slowly with time. Thus, we assume that $\frac{dv}{d\tau} = 0$*. With this assumption the second equation becomes algebraic, rendering* $v = \frac{u}{u+K}$.

Substituting this value into the first equation we obtain $\frac{ds}{dt}\,|_{t=0} = -\frac{k_2 e_0 s_0}{s_0 + k_m}$, *where* $k_m = \frac{k_{-1}+k_2}{k_1}$ *is the Michaelis constant. This rate, based on the quasi-steady state hypothesis, is what is usually needed from a biological point of view.*

The above set of nonlinear equations serve to model a variety of reaction pathways in molecular biology; for example — the way in which bacteria consume organic substance such as glucose[392].

Thermodynamic principles can be invoked to show that catalysts lower the free energy of activation, thus exponentially increasing the effective reaction rate constant which, in turn, speeds up the process. Still, the lowering of the free energy barrier (activation energy) must be explained in terms of a plausible molecular physical mechanism*. Obviously, the energy that can be expended by the enzyme in order to speed up the reaction can come only from one source — it is a part of the free energy liberated upon binding of the substrate to the enzyme*[393]*. That energy is recovered when the enzyme separates from the reaction product(s), and the enzyme molecule is then ready to catalyze the next reaction.*

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[392] Most water-soluble molecules are unable to pass through the hydrophobic environment of the cell membrane directly and must be carried across by special means. Typically, molecular receptors embedded in the bacterial cell membrane are involved in "capturing" these polar molecules in a *loose complex*, conveying them across the membrane barrier, and releasing them to the interior of the cell. The mechanism for nutrient uptake can be described by *Michaelis-Menten kinetics.*

[393] No universal *physical* model valid for all known enzyme reactions has been found. Thus, *electrostatic* dipole interactions were found to be important in *lysozyme* catalysis. In other cases, a hypothesis has been advanced, according to which the liberated free energy is transformed to the energy of *elastic vibrations* of the enzyme globule, which behaves like a liquid drop. The frequencies of such vibrations lie in the hypersonic region — up to 10^{13} sec^{-1}. The standing waves in the liquid drop can then activate the substrate molecule.

Heart-beat equations

The heart is a large hollow muscle which acts as a machine that pumps blood to all parts of the body. Tubes called veins *bring blood to the heart. Other tubes called* arteries *carry blood away from the heart. Four regulators called* valves *control the flow of blood through the heart itself. The heart is divided, lengthwise, by a muscular wall (*septum*). Two chambers, one above the other, are on each side of the septum. The upper chamber on each side is called an* atrium*. The atria collect the blood flowing into the heart from the veins. Below each atrium is another chamber called a* ventricle*. The two ventricles pump the blood into the arteries.*

The tricuspid valve *is between the right atrium and the right ventricle. The* mitral valve *is between the left atrium and the left ventricle. The* semilunar valve *controls the flow of blood from the ventricles to the arteries.*

The right hand side of the heart is a low pressure pump *that takes blood* from the body *and pumps it to the lungs (where carbon dioxide is removed and oxygen is added). It is a low-pressure circuit to avoid damaging the delicate membranes in the lungs.*

The left-hand side is a high-pressure pump *that collects blood from the lungs and delivers it* to the body*. The high pressure is needed in order for the blood to get down to the feet and up again.*

*The two sides of the heart simultaneously undergo two basic states: relax and fill (*diastole *state), contract and empty (*systole *state). The atria contract only a split second before the ventricles do. The action felt as a heartbeat is the systole. The sequence of events is divided into four phases:*

(1) *Blood flows into the heart from the veins, filling both atria. The tricuspid and mitral valves are closed.* The heart relaxes *in the diastolic phase.*

(2) The atria contract*. The mitral and tricuspid valves open, and through them the blood flows into the ventricles (one way only!). Ventricles are still relaxed.*

(3) Ventricles contract*, mitral and tricuspid valves shut, semilunar valves open. Blood enters the body via the aorta on the left side and to the lungs via the pulmonary artery on the right side.*

(4) Semilunar valves close *and the atria expand and fill with blood. A new diastole begins.*

The blood in the circulatory system is always under pressure, as is water in the pipes of a water system. Blood pressure depends upon the amount of *blood* in the system, the *strength and rate* of the heart's contraction, and the *elasticity* of the arteries. The *systolic pressure* is the blood pressure when the heart is contracted (high). The *diastolic pressure* is the pressure when the heart relaxes between beats.

What makes the heart beat is the presence of a *pacemaker* which is located on the top of the atria. It triggers an electrochemical impulse which spreads *slowly* over the atria, causing the muscle fibers to contract and push blood into the ventricles, and then spread *rapidly* over the ventricles causing the whole ventricle to contract into systole and deliver a big pump of blood down the arteries. The muscle fiber then rapidly relax and return the heart to diastole.

In order to develop a mathematical model which reflects the behavior of the heartbeat action described above, the following features are singled out:

I. The *rate of change of the muscle fiber* contraction depends, at any particular instant, on the tension of the fiber and on the chemical control.

II. *The chemical control changes* at a rate directly proportional to the muscle fiber extension.

III. The model must exhibit an equilibrium state corresponding to *diastole*.

IV. The model should contain a *threshold*, i.e., a definite point where the ventricles begin their rapid contraction. While this contraction is going on, the chemical control variable will be rising to a new value corresponding to a *systole*.

V. The model should reflect the rapid return to the equilibrium state.

Let $x(t)$ denote the length of a typical muscle fiber, and $y(t)$ the chemical control that governs the electrochemical pulse. A coupling that incorporates all the above features renders the *heart-beat equations*

$$\epsilon \frac{dx}{dt} = -(x^3 - ax + y),$$

$$\frac{dy}{dt} = x - x_0 + (x_0 - x_1)u = x - x_m.$$

Here, ϵ is a small parameter, which guarantees the rapid decrease of $x(t)$ (and at the same time the fast increase of $\frac{dx}{dt}$) through the transition from diastole (x_0, y_0) to systole (x_1, y_1). u is a control parameter associated with the pacemaker, and is defined as follows:

$$u = 1 \quad \text{for} \quad \begin{Bmatrix} y_0 \le y \le y_1 & \text{and } x^3 - ax + y > 0 \\ \text{or: } y > y_1 & \text{and all } x \end{Bmatrix}$$

$$u = 0 \quad \text{otherwise.}$$

The elimination of y from the above equations yields the Van der Pol equation[394] $\epsilon\ddot{x} - \dot{x}(a - 3x^2) + x - x_m = 0$.

[394] In 1927, **Van der Pol** was first to suggest a *relaxation oscillation* model of heartbeats. His electrical model of the heart consisted of three connected relaxation oscillators (one for the pacemaker, one for the atria, and one for the ventricles), each governed by the equation $\ddot{x} - \epsilon(1 - x^2)\dot{x} + \omega^2 x = 0$. The model given in the text is due to **E.C. Zeeman** (1972).

The Van der Pol equation also appears in certain ecological problems, where a population of herbivores interacts with vegetation in such a way that a *stable limit cycle* is established. The equations are $\frac{du}{dt} = \epsilon\left(u - \frac{u^3}{3}\right) + v$, $\frac{dv}{dt} = -u$, where ϵ is a fixed small, positive number, and $\{u, v\}$ are interpreted as (scaled) deviations of herbivore and vegetative biomasses from reference equilibrium values.

General models of epidemics

The Great Plague of London in 1665 *started in June; its peak came in September and its decline in October. The secondary rise occurred in November and cases of the disease were reported as late as March of the following year. The people of the city followed with anxiety the rise and fall in the number of deaths from the plague, hoping always to see the sharp decline which they knew from past experience indicated that the epidemic was nearing its end.*

When the decline came, the refugees, mostly from the nobility and wealthy merchants, returned to the city, and then for a time the mortality rose again as the disease attacked these new arrivals. The cause of these periodic outbreaks was, however, attributed to evil spirits. Even as late as 1865 *it was still believed in scientific circles that the periodicity of outbreaks of pestilence corresponds with the period of revolution of the lunar node.*

The first mathematical model, involving a nonlinear ordinary differential equation, was produced by ***Daniel Bernoulli*** *(1760), who considered the effect of cow-pox inoculation on the spread of smallpox.*

Some of the earliest classic works on the theory of epidemics is due to ***Kermack*** *and* ***McKendrick***[395] *(1927), who for the first time modeled some general aspects of disease transmission and the temporal development of epidemics. In their model, the total population is taken to be constant (the disease is of short duration relative to natural birth and death processes).*

Consider a host population, subdivided at time t *into three distinct classes according to the health state of its members:* $S(t)$ susceptible *individuals (not yet sick),* $I(t)$ infected *who may transmit the disease, and the* removed *class,* $R(t)$*, who can no longer contract the disease because they have recovered with immunity, have been placed in isolation, or have died. The progress of individuals is schematically described by* $S \to I \to R$*, and therefore the model is called the* SIR *model. The assumptions made about the transmission of the infection and incubation are these:*

(1) The gain of the infected class is at a rate proportional to the numbers of infected and susceptible.

(2) The rate of removal of the infected to the removed class is proportional to the number of infected.

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395 **Kermack, W.O.** and **A.G. McKendrick**, 'Contribution to the mathematical theory of epidemics', *Jour. Roy. Statis. Soc.* **115**, 700–721, 1927.

(3) *The incubation period is short enough to be negligible; that is — a susceptible who contracts the disease is infected right away (not a valid assumption for many diseases!).*

(4) *Every pair of individuals has equal probability of coming into contact with each other.*

The rate equations then become

$$\frac{dS}{dt} = -\beta SI, \qquad \frac{dI}{dt} = \beta SI - \alpha I,$$

$$\frac{dR}{dt} = \alpha I, \text{ with } S(t) + I(t) + R(t) = N = \text{total size of population.}$$

$$\alpha, \beta > 0, \qquad S(0) = S_0, \qquad I(0) = I_0 > 0, \qquad R(0) = 0.$$

Here β is the rate of transmission while α is the rate of removal. A key question in any epidemic situation is: given $\{\beta, \alpha, S_0, I_0\}$, will the infection spread or not, and if it does — when will it start to decline.

As in many other examples of nonlinear differential equations, the exact analytical solution of the Kermack-McKendrick system cannot be presented in a form which renders answers to the above questions. On the other hand, the questions can be answered by squeezing out information from the equations themselves without actually solving them. The following logical steps are taken:

(A) *Since $S(t)$ is nonnegative, the equation $\frac{dS}{dt} = -\beta SI$ implies that for $t > t_1 > 0$, $S(t) < S(t_1) < S(0)$. Furthermore, the limit of $S(t)$ exists as $t \to \infty$. Only solutions for which $S(t) > 0$, $I(t) > 0$ are of concern.*

(B) *From $\frac{dI}{dt} = (\beta S - \alpha)I$ and (A), it follows that if $\beta S_0 < \alpha$, $\frac{dI}{dt} < 0$ for all t, $I_0 > I(t) \to 0$ as $t \to \infty$ and the infection is eventually wiped out, and no epidemic can occur.*

For a disease to spread (i.e., to have an epidemic), S_0 must exceed the critical threshold value $S_c = \frac{\alpha}{\beta}$. The term epidemic *means that $I(t) > I_0$ for some $t > 0$.*

In this model, as time proceeds, S decreases and I increases until $S = S_c$ and $I = I_{\max}$. The function $I(t)$ then starts to decrease *until $\lim_{t\to\infty} I(t) = 0$, so the epidemic asymptotically stops.*

(C) $\frac{dR}{dt} = \alpha I$ *tells us that* $R(t)$ *increases monotonically while* $R \leq N$. *Hence* $R(\infty)$ *exists, and so does* $I(\infty) = 0$. *The quantity* $\frac{1}{N}[I(\infty) + R(\infty)]$ *is a measure of the extent to which the infection swept through the population.*

(D) *From the first and the third equations we deduce* $\frac{dS}{dR} = -\frac{\beta}{\alpha}S$ *so that* $S = S_0 e^{-\frac{\beta}{\alpha}R}$. *Since* $R \leq N$, *it is true that* $S \geq S_0 e^{-\frac{N\beta}{\alpha}}$, *which in turn implies that* $S(\infty) > 0$, *i.e., the disease dies out from lack of potential infecteds and not from a lack of susceptibles.*

(E) *The first two equations yield, upon their division,*

$$\frac{dI}{dS} = -1 + \frac{\alpha}{\beta S}$$

whence

$$I = N - S + \frac{\alpha}{\beta}\log_e\left(\frac{S}{S_0}\right).$$

Clearly

$$I_{\max} = N - S_c + \frac{\alpha}{\beta}\log_e\left(\frac{S_c}{S_0}\right).$$

As time goes on $I(t)$ *declines and eventually* $I(\infty) = 0$. *The value of* $S(\infty)$ *is determined from the transcendental equation*

$$S(\infty) = S(0)e^{-\frac{\beta}{\alpha}[N - S(\infty)]}.$$

This equation is satisfied by only one positive value of $S(\infty) < \frac{\alpha}{\beta} = S_c$. *Once* $S(\infty)$ *is known,* $R(\infty)$ *can be calculated from* $R(\infty) = N - S(\infty)$.

(F) *The* exact *equation for* $R(t)$ *is*

$$\frac{dR}{dt} = \alpha\left[N - R - S_0 e^{-\frac{\beta}{\alpha}R}\right], \quad R(0) = 0,$$

which can be integrated numerically provided α, β, S_0 *and* N *are known.*

(G) *When the infection spreads to unacceptable levels, immunity can be introduced by means of* vaccination. *It is then assumed that vaccination removes the individual instantaneously from* S *to* R *without joining group*

I, and a screening is available which prevents the infected from being vaccinated. Consequently, the first equation is modified into

$$\frac{dS}{dt} = -\beta SI - \gamma(t),$$

while a fourth equation is added $\frac{dV}{dt} = \gamma(t)$, where $V(t)$ is the group of vaccinated individuals; the conservation constraint then becomes: $S + I + R = N - V$.

So far it has been assumed that the population is thoroughly mixed, so that there is no distinction between individuals in one place and those in another. When this is not so, the disease may spread faster in some regions than in others, and it is necessary to allow for the dependent variables to depend on space *as well as on* time*. The geographic spread of infections has indeed been well observed in the form of* spatial epidemic waves*, in addition to the temporal variation in specific regions that was discussed above.*

The best known historical example is the medieval Black Death of 1347–1350*. A less known example is the plague bacillus that was brought by ship to the northwest of America around* 1900*; It has been carried ever since, with an average speed of about* 55 *km/year, via a large number of wild native animals (rats, squirrels, coyotes, mice, bats and domestic pets). Epidemics such as Asian flu appear to travel like a wave across the earth.*

If the spatial dispersal of I and S is modeled by simple diffusion *with the same diffusion coefficient, and the population is assumed to consist only of susceptibles $S(x,t)$ and infecteds $I(x,t)$, the Kermack-McKendrick equations are generalized into*

$$\frac{\partial S}{\partial t} = -\beta SI + D\frac{\partial^2 S}{\partial x^2},$$

$$\frac{\partial I}{\partial t} = \beta SI - \alpha I + D\frac{\partial^2 I}{\partial x^2},$$

where $\alpha, \beta, D > 0$. The non-dimensionalization

$$\overline{I} = \frac{I}{S_0}, \qquad \overline{S} = \frac{S}{S_0},$$

$$\overline{x} = \left(\sqrt{\frac{\beta S_0}{D}}\right)x, \qquad \tau = (\beta S_0)t, \qquad \lambda = \frac{\alpha}{\beta S_0}$$

yields

$$\frac{\partial \overline{S}}{\partial \tau} = -\overline{I}\,\overline{S} + \frac{\partial^2 \overline{S}}{\partial \overline{x}^2}, \qquad \frac{\partial \overline{I}}{\partial \tau} = \overline{I}\,\overline{S} - \lambda\overline{I} + \frac{\partial^2 \overline{I}}{\partial \overline{x}^2}.$$

We assume the existence of a traveling wave solution of constant shape, through which the infection spreads spatially into a uniform population of susceptibles, i.e.,

$$I(x,t) = I(\overline{x} - \bar{c}\tau), \quad S(x,t) = S(\overline{x} - \bar{c}\tau),$$

where c *is the dimensionless wave speed, as yet unspecified. Substituting these into the above partial differential equations yields a pair of coupled nonlinear ordinary differential equations:*

$$\overline{I}'' + \bar{c}\overline{I}' + \overline{I}(\overline{S} - \lambda) = 0,$$

$$\overline{S}'' + \bar{c}\overline{S}' - \overline{I}\,\overline{S} = 0,$$

$$\overline{I}(\infty) = \overline{I}(-\infty) = 0,$$

$$0 \le \overline{S}(-\infty) < \overline{S}(\infty) = 1, \qquad I \ge 0, \qquad S \ge 0.$$

Here, a prime denotes differentiation w.r.t. $\xi = \overline{x} - \bar{c}t$.

It can be shown that the minimum (dimensionful) wave speed is given explicitly by

$$V = 2\sqrt{D(\beta S_0 - \alpha)} \quad \text{if} \quad S_0 > \frac{\alpha}{\beta} = S_c.$$

Also:

(1) There is a minimum critical population density $S_c = \frac{\alpha}{\beta}$ *necessary for an epidemic wave to occur (e.g., a sudden influx of population can raise* S_0 *above* S_c*, and hence initiate an epidemic).*

(2) For a given population S_0 *and mortality rate* α*, there is a critical transmission coefficient* $\beta_c = \frac{\alpha}{S_0}$ *such that for* $\beta < \beta_c$ *the infection cannot spread.*

(3) With a given transmission rate β *and an initial susceptible population* S_0 *there is a threshold mortality rate* $\alpha_c = \beta S_0$*, which if exceeded, prevents an epidemic. So, the more rapidly fatal the disease is, the less chance there is of an epidemic wave propagating through a population.*

During 1975–1985, advances were made by ***Robert May*** *and his coworkers in our understanding of the role of infectious diseases in the regulation of natural populations of plants and animals and the interactions between population of viruses and immune system cells.*

Slime molds

Occasionally during summer, gardeners notice a jelly-like mass situated on the lawn. The same type of matter is often seen in the woods on decaying logs.

Known as 'slime molds' these organisms live in cool, shady, moist places on decaying wood, leaves or other organic matter, retaining abundant moisture. Over 700 species are reported to exist.

The colonies of slime mold living on logs and bark mulch can be strikingly colorful in yellow, orange or red. Some produce cream-colored masses of cells along grass blades. Slime molds often appear in the same area of the lawn year after year, in 10 to 15 cm patches in various shades of purple, grey, white or cream.

Slime molds feed on decaying organic matter, bacteria, protozoa, and other minute organisms which it engulfs and digests.

*Historically, slime molds were first regarded as members of the Fungi Kingdom but later classified with the Protista (*Protoctista*) Kingdom. In a recent system of classification based on analysis of nucleic acid (genetic material) sequences, slime molds have been classified in a major group called the Eukarya Domain (*Superkingdom*) including the four kingdoms of* Protista, Fungi, Plantae *and Animalia.*

Mycologists now consider slime molds to belong to a class called Myxomycetes[396] *[*myxa = slime; myketes = fungi*].*

Slime molds have left almost no fossil record. However, fossils of members of the Kingdom Protoctista, known as Ediacaran biota, produced deposits that date back 600 million years ago. Some of these ancient protoctists may have been ancestral to certain animal and plant phyla.

There are two main groups of slime molds:

- *Plasmodial slime molds: basically enormous single cells with thousands of nuclei and no definite cell wall. They are formed when individual flagellated cells swarm together and fuse. The result is one large bag of cytoplasm with many diploid nuclei.*

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[396] **Heinrich Anton de Bary** (1831–1888, Germany). Botanist and founder of the science of *mycology* and plant pathology. He called slime molds *Mycetozoa* (1887), form the Greek *myketes* (fungi), and zoon (animals). This name was in use until the 1970's.

- *Cellular slime molds spend most of their lives as separate single-celled amoeboid protists, but upon the release of a chemical signal, the individual cells aggregate into a great swarm. Cellular slime molds are thus of great interest to cell and developmental biologists, because they provide a comparatively simple and easily manipulated system for understanding how cells interact to generate a multicellular organism.*

What these two groups have in common is a life cycle. Before entering the reproductive stage, a plasmodium moves to a drier, better-lit place, such as the top of a log. In the amoeba-like or cellular slime molds, up to 125,000 individual cells aggregate and flow together, forming a multicellular mass called a pseudoplasmodium that resembles a slug and crawls about before settling in a location with acceptable warmth and brightness. When conditions become unfavorable, these slime molds form sporangia clusters of spores, often on the tips of stalks. Spores from the sporangia are dispersed to new habitats, "germinate" into small amoebae, and the life cycle begins again.

Certain aspects of the 'life' of a slime molds lend themselves to mathematical formulation.

***Marston Morse** (1949) mentioned slime molds as an example of "equilibria in nature". **Evelyn Keller** and **Lee A. Segel** (1970), using classical PDE ideas in a biological context, first formulated and analyzed equations to show how aggregation might be regarded as an instability of secreting chemotactic cells.*

***Athanasius F.M. Marée** and **Paulien Hogeweg** (2001) have provided a computer simulation of the transformation of slug into fruiting body by a process of self-organization.*

We now present a simplified model governing the slime mold amoebae aggregation as an instability. Let us first recapitulate the essential behavioral and anatomical data:

As mentioned above, slime molds have a complex life cycle that may be divided into an animal-like motile phase (in which growth and feeding occur) and a plantlike, immotile, reproduction phase. The motile phase is commonly found under rotting logs and damp leaves where cellulose is abundant. It consists in the cellular slime molds of solitary, amoeba-like cells, and in Myxomycota of a multinucleate mass of protoplasm called plasmodium, which creeps about by ameboid movement.

Plasmodia often grow to a diameter of several inches. Both types ingest solid food particles using a process called phagocytosis. They feed on living microorganisms, such as bacteria and yeasts, as well as decaying vegetation.

In the reproductive stage, the plasmodium or pseudo-plasmodium is transformed into one or more reproductive structures called fruiting bodies, each consisting of a stalk topped by a spore-producing capsule that resembles the reproductive structure of many fungi.

Eventually, the cellulose-walled spores are released and dispersed; They germinate in wet places, releasing naked cells. In a typical plasmodial slime mold, the germinated spores go through an ameboid or flagellated swimming stage, followed by sexual fusions and cell-divisions. The diploid ameboid cell grows and its nucleus divides repeatedly, resulting in the formation of a new plasmodium. Under adverse conditions, a plasmodium may be transformed into a hard, dry, inactive mass which becomes a plasmodium again when favorable conditions return.

In the case of the cellular slime molds, each spore released becomes a single amoeba, which feeds individually until starving cells release a chemical signals that causes them to aggregate into new pseudo-plasmodium, and the progress is repeated.

In sexual reproduction two haploid amoeba fuse, then engulf surrounding amoebas, forming a single organism called a macrocyst. It then undergoes meiosis and released haploid individuals.

In the morphogenetic development of many species of cellular slime mold (Acrasiales) some interesting effects of long-range intercellular interaction can be observed. The interaction may be of a repulsive or attractive nature, depending on the stage of the cells' life cycle. Immediately following germination, the cells disperse as if acting under a mutual repulsion. When a source of food (bacteria) is present, the cells move toward it with a high positive chemotactic index. After exhausting their food supply, the amoebae first tend to distribute themselves uniformly over the space available to them, but later they begin to aggregate in a number of "collecting points" or centers. At each center a slug forms, migrates and eventually forms a multicellular fruiting body.

The basic biological facts which serve as a basis for the mathematical model are:

- *Amoebae spread out uniformly when sufficient food (bacteria) is available.*
- *Shortage of food causes the amoebae to start secreting a certain chemical (an attractant) which attracts other amoebae.*
- *Further depletion of the food supply leads to an aggregation followed by formation of spores.*

- *Spores develop into new amoebae when conditions are favorable.*

It is therefore necessary to quantify the following statements describing the aggregation process:

- *Amoebae diffuse due to concentration gradients*
- *Amoebae move towards higher concentration of attractant*
- *Amoebae secrete more attractant when there is shortage of food*
- *The attractant breaks down in time due to a chemical reaction*
- *The attractant itself diffuses in the solution due to diffusion gradients*

We thus define:

$a(x, y, t) =$ *concentration (mass per unit area) of amoebae. The function is continuous (and sufficiently differentiable) in the two-dimensional spatial coordinates (x,y) and the time t.*

$\rho(x, y, t) =$ *concentration of attractant.*

We now invoke:

(i) The conservation (continuity) equations for the concentration of amoebae $a(x, y, t)$ *and the chemical attractant* $\rho(x, y, t)$*;*

(ii) The linear dependence of the transport flux of amoebae upon both diffusive amoeba concentration gradient and the concentration gradient of the attractant.

These statements translates into a pair of coupled nonlinear partial differential equations (two space, one time dimension):

$$\frac{\partial a}{\partial t} = div[D_2 \text{grad } a - D_1 \text{grad } \rho], \tag{1}$$

$$\frac{\partial \rho}{\partial t} = -k(\rho)\rho + af(\rho) + D_\rho \nabla^2 \rho. \tag{2}$$

The term $D_2 \nabla a$ *represents a diffusion-like contribution to the amoebae flux due to random motion, where* $D_2(\rho, a)$ *is a measure of the vigor of the random motion of the individual amoebae; while* $D_1(\rho, a)$ *is a measure of the strength of the influence of the attractant gradient on the flow of the amoebae. The signs of the terms in (1) are chosen so that* D_1 *and* D_2 *will be positive, since amoebae flow toward a relatively high value of attractant concentration* *and a relatively low value of amoebae concentration. In (2),* $af(\rho)$ *is the formation rate of attractant by the amoebae, and* $-\rho k(\rho)$ *is the decay rate, while* D_ρ *is the constant attractant diffusivity.*

Eqs. (1)–(2) possess uniform equilibrium solutions

$$a = a_0, \qquad \rho = \rho_0, \tag{3}$$

where

$$a_0 f(\rho_0) = k(\rho_0)\rho_0. \tag{4}$$

We now look for solutions of (1) and (2) near equilibrium. Of particular importance is the time dependence of these solutions. If the deviations $\rho-\rho_0,\ a-a_0$ *decrease with time, then any random perturbation which might occur in the equilibrium state would decay. If, on the other hand, we found a solution which these deviations increased with time, the appearance of any spontaneous perturbation would mark the onset of instability. The perturbation would grow until the system could no longer be described by equilibrium or near-equilibrium equations.*

Near equilibrium, we assume that the right-hand sides of equations (1) and (2) can be replaced by Taylor expansions in ρ *and* a *around the equilibrium point* (ρ_0, a_0). *That is, we consider small perturbations of* a_0 *and* ρ_0

$$\begin{aligned} a &= a_0 + \overline{a}(x,y,t) \qquad & |\overline{a}| << a_0, \\ \rho &= \rho_0 + \overline{\rho}(x,y,t) \qquad & |\overline{\rho}| << \rho_0 \end{aligned} \tag{5}$$

and expand any term in which a *and* ρ *appear as follows*

$$F(\rho,a) = F(\rho_0,a_0) + \left(\frac{\partial F}{\partial \rho}\right)_{\rho_0,a_0} \overline{\rho}(x,y,t) + \left(\frac{\partial F}{\partial a}\right)_{\rho_0,a_0} \overline{a}(x,y,t) + \cdots$$

As long as $\overline{a}$ *and* $\overline{\rho}$ *are small we can ignore higher order terms containing factors such as* $\overline{a}^2,\ \overline{a\rho},\ \overline{\rho}^2,\ \overline{a}^3$ *etc. The linearized (small–signal) equations which result are*

$$\frac{\partial \overline{\rho}}{\partial t} = -\overline{k}\overline{\rho} + a_0 f'(\rho_0)\overline{\rho} + f(\rho_0)\overline{a} + D_\rho \nabla^2 \overline{\rho}, \tag{6}$$

$$\frac{\partial \overline{a}}{\partial t} = -D_1(a_0,\rho_0)\nabla^2\overline{\rho} + D_2(a_0,\rho_0)\nabla^2\overline{a}, \tag{7}$$

where prime denotes differentiation and $\overline{k} = k(\rho_0) + \rho_0 k'(\rho_0)$. *We look for solutions to (6) and (7) of the form*

$$\overline{a} = \hat{a}\cos(q_1 x + q_2 y)e^{\sigma t}; \qquad \overline{\rho} = \hat{\rho}\cos(q_1 x + q_2 y)e^{\sigma t}, \tag{8}$$

where σ, $\hat{a}$, $\overset{\wedge}{\rho}$, q_1, q_2 *are constants. Substitution of (8) into (6) and (7) leads to a quadratic equation for* σ

$$\sigma^2 + b\sigma + c = 0$$

with

$$\begin{aligned} b &= (q_1^2 + q_2^2)D_2(a_0, \rho_0) + [\overline{k} + (q_1^2 + q_2^2)D_\rho - f'(\rho_0)a_0] \\ c &= -(q_1^2 + q_2^2)D_2[f'(\rho_0)a_0 - \overline{k} - (q_1^2 + q_2^2)D_\rho] - (q_1^2 + q_2^2)f(\rho_0)D_1 \end{aligned}$$

The roots of the quadratic equation are real (since $b^2 > 4c$*) and will both be negative iff* $b > 0$ *and* $c > 0$*. It thus follows that instabilities (*$\sigma > 0$*) can arise when:*

- *The diffusive mobility of the amoebae is low.*
- *The decay of attractant is slow.*
- *The production of attractant is high.*
- *The mobility of the amoebae is high.*

It is conceivable that this mechanism is qualitatively related to what we see in reality. However, the form of the aggregation patterns may be quite complicated. Yet, the central contribution of the theory is the idea that the onset of slime mold aggregation can be regarded as an instability of the uniform distribution, triggered when the amoebae cease feeding. The merit of this point of view is independent of the particular simplifying assumptions which the model employs.

Chemotaxis

A taxis is an innate response by an organism (or cell) to a stimulus from a particular direction, whereby the motile organism moves either toward or away from the stimulus. For example, flagellate protozoans of the Genus Euglena move towards a light source – a phenomenon known as phototaxis (a response to light stimulus). Many other types of taxis that have been identified are: Anemotaxis (stimulated by wind); Barotaxis (stimulated by pressure); Chemotaxis (stimulated by chemicals); Galvanotaxis (stimulated by electrical current); Geotaxis (stimulated by gravity); Hydrotaxis (stimulated by moisture); Rheotaxis (stimulated by current flow); Thermotaxis (stimulated by temperature change); Thigmotaxis (stimulated by contact).

Taxis is classified as klinotaxis when an organism continuously samples the environment to determine the direction of the stimulus.

In an assembly of particles (cells, bacteria, chemicals, animals) each particle moves around in a random way. When this microscopic irregular movement results in some macroscopic gross regular motion of the group, it is manifested as a diffusion process governed by the diffusion equation in the continuum limit.

A large number of insects and animals rely on an acute sense of smell for conveying information between members of the species. Chemicals which are involved in this process are called pheromones. For example, the female silk moth Bombyx mori exudes a pheromone, called bombykol, as a sex attractant for the male, which has a remarkably efficient antenna filter to measure the bombykol concentration, and it moves in the direction of increasing concentration. This chemically directed movement, that is chemotaxis, directs (unlike diffusion) the motion up a concentration gradient.

It is not only in animal and insect ecology that chemotaxis is important. It can be equally crucial in many biological processes, e.g.:

(a) A bacterium find food (e.g. glucose) by swimming towards the highest concentration of food molecules, or employs the opposite behavior to flee from poisons (e.g. phenol).

(b) Leukocyte cells in the blood move towards a region of bacterial inflammation, to counter it, by following a chemical gradient caused by the infection.

(c) Single-cell amoebae in the slime mold Dictyostelium discoideum move toward regions of relatively high concentration of a chemical called cyclic-AMP, which is produced by the amoebae themselves.

The first erudite description of chemotaxis and phototaxis was made by the German botanists ***T.W. Engelmann*** *(1881) and* ***W.T.P. Pfeffer*** *(1845–1920, 1884) in bacteria and the American biologist* ***H.S. Jennings*** *(1868–1947, 1906) in ciliates.* ***Eli Metchnikov*** *(1845–1916) also contributed to the study of the field with investigations of chemotaxis as an initial step of phagocytosis.*

The kinetics involved in (c) can be derived via a continuum model equation for the global behavior in terms of a particle concentration:

Assume that the presence of a gradient in an attractant concentration, $a(\boldsymbol{x}, t)$*, gives rise to a movement, of cells say, up that gradient. We then write the chemotaxis–caused flux vector* $\boldsymbol{j}$ *of cells as*

$$\boldsymbol{j_{chemotaxis}} = n\chi(a)\,\mathrm{grad}\,a, \tag{9}$$

where $\chi(a)$ is a function of the attractant concentration. The conservation of matter equation for $n(\boldsymbol{x}, t)$, then becomes

$$\frac{\partial n}{\partial t} + div\, \boldsymbol{j} = f(n), \tag{10}$$

where $f(n)$ represents the growth term for the cells, and

$$\boldsymbol{j} = \boldsymbol{j_{diffusion}} + \boldsymbol{j_{chemotaxis}} = -D\,\mathrm{grad}\, n + n\chi(a)\,\mathrm{grad}\, a. \tag{11}$$

Here D is the diffusion coefficient of the cells. Combining (10) and (11) one obtains a PDE for chemotaxis

$$\frac{\partial n}{\partial t} = f(n) - div[n\chi(a)\,\mathrm{grad}\, a] + div[D\,\mathrm{grad}\, n] \tag{12}$$

Since the attractant $a(\boldsymbol{x}, t)$ is a chemical, it also diffuses and is produced (by the amoebae), so we need a further equation for $a(\boldsymbol{x}, t)$. Typically it takes the form

$$\frac{\partial a}{\partial t} = g(a, n) + div[D_a\,\mathrm{grad}\, a], \tag{13}$$

where D_a is the diffusion coefficient of a and $g(a, n)$ is the source term. Normally we would expect $D_a > D$. In many cases: $f(n) = 0$, $g(a, n) = hn - ka$, $\chi = \chi_0 = const.$ and D, D_a, h, k are also constants. Then, the model in one space dimension is governed by the system

$$\frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial x^2} - \chi_0 \frac{\partial}{\partial x}\left(n\frac{\partial a}{\partial x}\right); \quad \frac{\partial a}{\partial t} = hn - ka + D_a\frac{\partial^2 a}{\partial x^2} \tag{14}$$

Note the sign difference in (12) and (14), between the diffusion and chemotaxis terms. Each has a Laplacian contribution. Whereas diffusion *in generally a stabilizing force,* chemotaxis *is generally destabilizing – a negative diffusion, as it were. It is therefore reasonable to suppose that the balance between stabilizing and destabilizing forces in the model system (14) could result in some steady-state spatial patterns in n and a, or in the appearance of nonuniform spatial patterns in the cell density.*

1917–1919 CE Worldwide *Influenza* epidemic; estimates of deaths range as high as 50 million. *Typhus* claims as high as 3 million lives in Russia.

1917–1921 CE **Albert Wallace Hull** (1880–1966, U.S.A.). Physicist and inventor. Discovered (1917), independently of **Peter Debye** and **P.H. Scherrer** [1916; Swiss physicist, 1890–1969], the method of diffraction analysis in

X-ray crystallography. Invented the *thyratron*[397] and *magnetron* (1921) electron tubes.

The Hull *magnetron* consisted of an electron-emitting cathode surrounded concentrically by a cylindrical anode. Electrons move from the cathode to the anode under the action of an applied voltage V and form a current I in the external circuit. The path of the electron is radial. If a magnetic field B is applied in the direction of the tube axis, the electron will travel in curved path, and if the field strength is increased, a condition will be reached in which no electrons reach the anode and the current is cut off. Since the velocity of thermal agitation is small in comparison, all electrons will have approximately the same velocity. Thus there will exist a critical field B_c, above which the anode current is wholly suppressed. The Hull magnetron could therefore be used as a relay, but was not meant to act as an oscillator. Sometimes, however, by setting B to barely exceed B_c, feeble oscillations were found to be generated at very high frequencies.

The physical basis of the Hull *magnetron* hinges on the theory of motion of charged particles in crossed electric and magnetic fields: Let a particle be emitted from a surface in the plane $x = 0$ and accelerated by a uniform electrostatic field E toward a collector plate in the plane $x = \ell$. A uniform magnetic field B is applied parallel to the OZ direction with the result that the particle is deflected in the negative OY direction. The motion of a particle of mass m and charge e in the XY plane is then described by the coupled ODE

$$m\ddot{x} = eE - Be\dot{y}; \qquad m\ddot{y} = Be\dot{x}.$$

Applying the initial conditions $\dot{x}(0) = \dot{y}(0) = 0$, the solution of the above ODE system yields the cycloid

$$x(t) = x_0(1 - \cos\omega t), \qquad y(t) = x_0(\omega t - \sin\omega t)$$

$$x_0 = \frac{E}{\omega B}, \qquad \omega = \frac{eB}{m}.$$

The cut-off condition $x_{\max} = \ell = 2x_0 = \frac{2mE}{B_c^2 e}$ yields $B_c^2 = \frac{2mV}{e\ell^2}$, where $V = E\ell$ is the interplate voltage-drop.

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[397] A gas-filled triode with a thermionically heated cathode. It operates under arc discharge conditions: The gas ionizes when sufficient current flows, reducing the internal resistance. When the grid is sufficiently negative no electron current flows. Less negative potential values cause electrons to flow, ionizing the gas; the valve conducts heavily until the anode potential is reduced. The valve can thus be used as a relay, or as a rectifier of variable output.

A similar procedure may be followed in analyzing a cylindrical magnetron, with the approximate result $B_c^2 = \frac{8mV}{er_a^2}$, where r_a is the anode radius.

Further study of the oscillations generated by the Hull magnetron led a group of Japanese physicists (1927) to devise the *split-anode magnetron*. The *multi-cavity magnetron* was developed in England (1941) by several research groups for use in wartime RADAR systems.

Hull was born in Southington, CT. He was on the research staff of General Electric Co. (1914–1950).

1917–1930 CE **Harlow Shapley** (1885–1972, U.S.A.). Astronomer. Revolutionized galactic astronomy by obtaining the first realistic estimate for the actual size of the Milky Way galaxy and "removing" the solar system from its center. This was achieved through Shapley's investigation of the distribution of the *globular clusters*[398].

In 1917–1918, Shapley determined the location of the solar system in the Milky Way galaxy. In 1930 he calculated the diameter of the galaxy to be about 100,000 light years. Shapley developed a method to measure the distance to globular clusters using variable stars as yardsticks.

It is possible to determine the distance to an object if both its absolute and apparent magnitudes are known. It was discovered in 1912, through observations of *variable stars* in the Magellanic clouds, that there is a definite relationship between the pulsation period of these stars and their luminosities. For *Cepheids*, the period increases with increasing luminosity (or decreasing absolute magnitude) while with another type of pulsating variable, the *RR Lyrae stars*, all have the same luminosity. This had a profound implication:

When a star is recognized as a variable belonging to one of these classes, its absolute magnitude can be determined simply by measuring its period and using the established period-luminosity relations. Once the absolute magnitude is known, the distance can be found by comparing the absolute and apparent magnitudes. Because Cepheids are giant stars, they are very luminous, and

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[398] Before Shapley, the galaxy was generally believed to be centered approximately at the sun and to extend only a few thousand light years from it. Shapley placed the sun near the galactic central plane, at the outer edge, some 30,000 light years away from the center. Globular clusters are immense, densely packed groups of stars (some containing as many as a million members). Because of their brilliance and the fact that they are not confined to the central galactic plane (where they would be largely obscured by interstellar dust), they can be observed out to very large distances.

can be observed out to great distances. This makes them very powerful tools for measuring distances beyond those reached by other techniques.

From their directions and derived distances, Shapley, was able (1917) to map out the 3-dimensional spatial distribution of the 93 globular clusters then known. He found that the clusters formed a roughly spheroidal system, with the highest concentration of clusters at the center. *That center was not near the sun*, but a point in the middle of the Milky Way in the direction of *Sagittarius*.

Shapley was born in Nashville, MO. He joined the staff of the Mt. Wilson Observatory, California, in 1914. He became professor of astronomy at Harvard University and director of its observatory (1921–1952).

1917–1933 CE **Tullio Levi-Civita**[399] (1873–1941, Italy). Outstanding mathematician. One of the founders of the tensor calculus. Introduced the concept of *parallel displacement* in Riemannian spaces; this was instrumental in the development of the modern differential geometry of generalized spaces.

Parallel displacement (or *parallel transport*): Let C be a curve in three-dimensional space $\mathbb{R}^3$ and let the coordinates $x^i(t),\ t \geq 0 \qquad i = 1, 2, 3$ of a point P on C be functions of a parameter t along it (e.g., arc-length, time, etc.), in any particular (possibly curvilinear) coordinate system.

Define[400] a contravariant vector field $\boldsymbol{A}(P) = A^k(x)\boldsymbol{g}_k(x)$ at a general point P in $\mathbb{R}^3$, with $\boldsymbol{g}_k$ the natural basis of contravariant vectors (frame) corresponding to the given[401] coordinate system. A^k are the components of $\boldsymbol{A}$ in this system.

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[399] For further reading, see:

- Levi-Civita, T., *The Absolute Differential Calculus*, Dover, 1977, 452 pp.
- McConnell, A.J., *Applications of Tensor Calculus*, Dover, 1957, 318 pp.

[400] We employ the *summation convention*: pairs of identical-symbol indices are summed over.

[401] The definition of parallel transport readily generalizes to any n-dimensional manifold, not necessarily equivalent topologically to $\mathbb{R}^n$; thus the coordinates and frames might need to be differently defined on different "patches" of the manifold, with suitably smooth *transition functions* furnished between any pair of coordinate systems on the overlap of their respective patches. Furthermore, such a multiplicity of coordinate–systems patches may be required even in an $\mathbb{R}^n$ manifold, in the case of curvilinear coordinates (e.g spherical coordinates in $\mathbb{R}^3$).

The contravariant – vector basis $\{\boldsymbol{g}_k\,(x)\}$ is in general not orthogonal, but we can construct a *dual base* $\{\boldsymbol{g}^k(x)\}$, such that $\boldsymbol{g}^j \cdot \boldsymbol{g}_k = \delta^j_k$ (Kronecker delta) at each point in $\mathbb{R}^3$; this dual basis is a local basis (frame) for *covariant* vectors, in the same coordinate system.

Next, we construct at every point $P\big(x(t)\big)$ of C a vector equal in magnitude to $\boldsymbol{A}(Q)$ (where $Q \equiv P\big(x(0)\big)$ and parallel to it in direction (in the Euclidean $\mathbb{R}^3$ geometry). We thus obtain a contravariant vector field $\boldsymbol{B}(t|C)$ on the curve C, where t is a parameter along C.

Clearly, while this curve–restricted vector field is a constant in absolute terms [$\boldsymbol{B}(t|C) \equiv \boldsymbol{A}(Q)$ is $t-$independent], its *local components* $B^k = \boldsymbol{B} \cdot \boldsymbol{g}^k\big(x(t)\big)$ will vary along C, since they are measured at each point w.r.t a local base that changes along the curve. There changes are determined by the equation

$$\frac{d\boldsymbol{B}}{dt} = \frac{d}{dt}(B^k \boldsymbol{g}_k) = \left[\frac{\partial B^k}{\partial t} + \frac{dx^i}{dt} B^j \Gamma^k_{ij}\right] \boldsymbol{g}_k\big(x(t)\big) \equiv 0,$$

where we have used the chain rule of differentiation along the curve C,

$$\frac{d\boldsymbol{g}_k\big(x(t)\big)}{dt} = \frac{dx^i}{dt}\,\frac{\partial \boldsymbol{g}_k}{\partial x^i} \quad ,$$

and defined anywhere in $\mathbb{R}^3$:

$$\Gamma^k_{ij}(x) \equiv \boldsymbol{g}^k(x) \cdot \frac{\partial \boldsymbol{g}_j(x)}{\partial x^i} \; .$$

It can be shown that $\Gamma^k_{ij} \equiv \Gamma^k_{ji}$.
The condition of parallel transport of a vector along C – which defined $\boldsymbol{B}(t|C)$ – thus results in the system of coupled linear ODE's:

$$\frac{dB^m}{dt} + \frac{dx^\ell}{dt} B^k \Gamma^m_{k\ell} = 0.$$

for the components $\{B^k(t|C)\}$, in any given coordinate system.

The solution of this system of differential equations yields the parallel vector field along C for given fiducial point Q and $\boldsymbol{A}(Q)$.

Expressed in other terms, if we are given a *closed curve* and, starting with a given vector at a point of this curve, we parallel-transport the vector along the curve, then there is no *a priori* reason why we should arrive at the same initial vector when we have completed the circuit! In fact, the parallel transport of a vector round a closed circuit in a curved Riemannian space generally results in a *new vector* when we have returned to at the initial point.[402]

Consider, for example, the curved-triangle boundary of one octant of a sphere: as we move a vector parallel to itself along the equatorial side of the curved triangle (one quarter of the equatorial circumference) such that it is always normal to the equator (say, pointing north), our intuitive concept of parallelism is not violated. If, however, we further parallel–transport the vector along the *two meridional portions of the closed curve*, then when we

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[402] The concept of parallel vector fields along a curve embedded in Euclidean $\mathbb{R}^3$-space (E_3) was generalized by Levi-Civita to curves embedded in a (possibly curved) Riemannian space (manifold) of dimension n: The Riemannian space has an *affine connection* $\Gamma^i_{jk}(x)$, which defines the *covariant derivatives* $A^m_{;l} = \frac{\partial A^m}{\partial x^l} + \Gamma^m_{lk} A^k$, transforming as a tensor under a general coordinate transformation (while Γ is *not* a tensor, nor is $\frac{\partial A^m}{\partial x^l}$).
For an infinitesimal displacement dx^i of P along C, $DA^m = A^m_{;l}(P)\,dx^l$ can be interpreted geometrically as the difference between $A^m(x+dx)$ and the *parallel transport* of $A^m(P)$ to the point P' with coordinates $x+dx$. The coordinates x^i, dx^i depend on an arbitrary choice of local coordinate system and the local patches in which they hold; yet the *points* P, P', and the *tensor* of which A^m and $A^m_{;l}$ are local components, are genuine and invariant *geometrical entities* on the manifold. In a *curved* region of a Riemann space, a vector field A^m not identically zero cannot satisfy $A^m_{;l} \equiv 0_j$. One *can* however, create a new field $\tilde{A}^m(t)$, defined only along C, by parallel-transporting A^m from $t = t_1$ to any point along C; and this construction is *covariant* under change of coordinates[403]. $\tilde{A}^m$ satisfies the 1st-order system of nonlinear ODE's

$$\frac{D\tilde{A}^m}{Dt} \equiv \frac{d\tilde{A}^m}{dt} + \Gamma^m_{ij}\,\frac{dx^i(t)}{dt}\,\tilde{A}^j(t) = 0,$$

with initial conditions $\tilde{A}^m(t_1) = A^m\big(x(t_1)\big)$.
When $n = 3$ (x^1, x^2, x^3) is Euclidean space, $A^m(x)$ is the velocity field and t is Newtonian time, one example of parallel transport (in a flat space) is free-streaming in fluid dynamics, where $\frac{D}{Dt}$ is the *material* (*comoving*) derivative. For $n = 3N$, an example is the *Boltzmann transport equation* (for N particles) without collision and external-field terms.
In GTR, $n = 4$, t becomes proper time, and the parallel-transport condition is an approximation to the equations of motion (for 4-velocity, intrinsic spin and other energy-momentum moments) in the case of a small object free-falling in an external gravitational field.

[403] Indeed, the variation of tensor fields caused by parallel transport around infinitesimal closed loops at different points P, can be used to define and fully characterize the rank-4 *Riemann curvature tensor* on the manifold.

return to the equatorial starting point, it is easily seen that the vector, originally pointing north, now points parallel to the equator. This is, obviously, due to the uniform scalar curvature of the sphere.

Levi-Civita's researches covered a vast field in the no-man's land between pure and applied mathematics. Their subjects include the absolute differential calculus, integral equations, differential geometry, general relativity (1929), hydrodynamics, partial differential equations, N-body problems and problems of engineering.

Levi-Civita was born in Padua, Italy to a Jewish family, a son of Giaccomo Levi-Civita and his wife Bice Lattis. The family was a wealthy one, well known for its strong liberal tradition. His father was a barrister, jurist and politician and was for many years Mayor of Padua and a Senator of the Kingdom of Italy. As a young man he had fought with Garibaldi in the campaign of 1866.

At the age of 17 Tullio entered the University of Padua. One of his teachers was Ricci-Curbastro, with whom he later collaborated. In 1902 he became a professor of rational mechanics, and in 1918 he was called to the chair of mechanics at the University of Rome, a post which he held for another 20 years.

Levi-Civita was a man of small stature, handicapped throughout his life by defective eyesight. Nevertheless, he was very robust and in his younger days was an avid mountaineer and cyclist. In 1914 he married Libera Trevesani, a former pupil. He always remained true to the liberal tradition of his family. The bond between him and his father was very close, and the father's portrait always hung, beside that of Garibaldi, in his study. He viewed with strong displeasure the advent of fascism in Italy.

His scientific renown protected him from persecution until the introduction of the anti-Jewish laws in Italy in 1938, when he was removed from his chair in Rome. This was a heavy blow to him, from which he never recovered. He received offers of asylum from many parts of the world, but severe heart trouble prevented his traveling to accept any of these. He died in Rome as a result of a stroke.

1917–1953 CE **Hermann Staudinger** (1881–1965, Germany). Organic and macromolecular chemist. Deciphered the polymer structure. His pioneer work on macromolecular chemistry constitutes a major foundation of modern molecular biology.

His early (1917) synthesis of the hydrocarbon isoprene (the constituent monomer of rubber) led him to the long-chain-molecule theory of the structure of polymers (1920). As early as 1926, Staudinger appreciated the importance of macromolecules to biology and visualized the molecular biology of the future (1947). Recognition of his work as a whole came belatedly with the award of a Nobel prize in 1953.

Before Staudinger it was accepted that polymers consist of an aggregate of small molecules connected through mysterious secondary forces that nobody could define. Staudinger was first to put forward a model through which the molecular chain is held together by *covalent* bonds. Subsequently he produced in his laboratory synthetic polymers used as models for natural polymers. Thus was born the concept of polymer as a *macromolecule*. This breakthrough led to a wide theoretical, experimental and industrial development of macromolecular chemistry.

1917–1971 CE **Robert Robinson** (1886–1975, England). Organic chemist. One of the founders of modern organic chemistry. Made fundamental contributions in four major areas: the structure of natural substances (many of major biological importance); synthesis; biosynthesis; and mechanistic organic chemistry. For his work on natural products of biological importance he was awarded the Nobel prize in chemistry (1947). It covered alkaloids [*morphine* (1925), *strychnine* and *tropinone*], the plant pigments of the *anthocyanin* and *anthoxantin* group, *steroids* and *penicillin*.

His work led to the elucidation of *patterns of biosynthesis;* he suggested the importance of the aldol condensation of carbinolamines in alkaloid biosynthesis, and emphasized that the biosynthesis of organic compounds follows recognizable chemical reactions and mechanisms. He also worked out an *electrochemical* (electronic) *theory of organic reactions*.

Robinson was born in Chesterfield, Derbyshire, England. From 1912 he held a series of appointments at Sydney, Liverpool (1915–1919), St. Andrews (1920), Manchester (1922), University College, London (1928), and culminating in his appointment at Oxford (1930–1955).

The Third 'Copernican Revolution' — The Extragalactic Universe (1918–1924)

In 1543, ***Copernicus*** *shifted the center of the universe from the earth to the sun. In* 1610, ***Galileo*** *described his telescopic observations of the Milky Way, which showed it to be composed of a multitude of individual stars. In* 1750 ***Thomas Wright*** *published a speculative explanation which turned out to be substantially correct — that the sun is located within a disc-shaped system of stars, and that the Milky Way is the light from the surrounding stars that lie more or less in the plane of the disc. The disc shape of the stellar system to which the sun belongs (the galaxy*[404]*) was demonstrated quantitatively in* 1785 *by* ***Herschel's*** *"star gauging".*

At the opening of the 20th century, very little was known about our galaxy's size and shape and where we are located in it, let alone that there were galaxies beyond our own. ***Harlow Shapley*** *established in* 1918 *the existence of clusters of stars out at the edge of the Milky Way and "pushed" the sun away from the center of the galaxy towards its rim. He insisted, however, that the visible nebulae did not extend beyond the Milky Way.*

Heber Doust Curtis *(1872–1942, U.S.A.) discovered in* 1918 *that the great spiral nebula in Andromeda was far beyond our galaxy*[405]*. On April* 27, 1920 *these two astronomers met in a formal debate before the NAS in Washington D.C., to decide whether the newly observed so-called nebulae, were inside (Shapley) or outside (Curtis) the 'Milky Way'. Shapley used correct arguments but came to the wrong conclusion. Curtis, whose intuition was better in this case, gave a rather weak and sometimes incorrect argument but reached the correct conclusion. The nebulae were indeed distant galaxies.*

Curtis' findings substantiated the predictions of ***Kant*** *(1755) and* ***Eddington*** *(1914), and were finally confirmed at a meeting of the American Astronomical Society by* ***E.P. Hubble*** *(1924). Before long, astronomers had determined that many of the faint nebulae were, in fact, entire galaxies. With*

[404] From the Greek *galaxias* = gala-aktos = milk. The word *Galaxy* pertains to the *Milky Way*, which contains about 10^{11} visible stars, as well as 10^{10} solar masses ($M_\oplus$) of gas, distributed in tens of thousands of gas clouds with a wide range of masses and sizes ($M_\oplus = 1.99 \times 10^{33}$ gram).

[405] The diameter of the '*Milky Way*' is about 10^5 LY; the center of *Andromeda* is at a distance of 2×10^6 LY from earth; the farthest star in the *Local Group* is about 3×10^6 LY from earth.

this knowledge came a new and larger view of the cosmos. Attention turned to galaxies.

On average, galaxies are separated by about 10^7 *light-years, or about* 100 *times the diameter of one galaxy. Thus, space would appear to be mostly empty, with islands of stars, scattered here and there.*

From another perspective, however, galaxies are far closer together than stars. Individual stars in a galaxy, are separated by an average toll of 10 *light-years, or* 10^8 *times the diameter of one star*.

1918 CE **Hans Thirring** (1888–1976, Austria) and **J. Lense** (Germany). Physicists. Tested whether the theory of general relativity incorporates the 'Mach principle'. They solved the GTR linear 'weak field approximation' inside a large spherical shell rotating relative to an asymptotic inertial frame[406] in an otherwise empty universe.

A small dragging effect from the shell, partially tugging the interior local-inertial frames, was found — in accord with Mach's principle; i.e. a test mass inside the shell behaves as if it was acted upon by centrifugal[407] and other inertial forces usually attributed to absolute space, but caused by the rotating shell. The predicted effect is known as the *Lense-Thirring effect* or *frame dragging*.

1918 CE **Paul Finsler** (1894–1970, Germany). Mathematician. Provided a generalization of Riemannian geometry, in which the metric form is replaced by a more general function of the coordinates and the differentials. The restrictions on this function $F(x, dx)$ are mostly those that insure the regularity of the problem of minimizing the integral $\int F\left(x, \frac{dx}{dt}\right) dt$. His work is related to that of **Emmy Noether** (1882–1935) on differential invariants.

Many attempts have been made since 1918 to reformulate the theory of the gravitational field on the basis of Finsler's metric, but there is yet no

406 Far outside the matter distribution, space-time is Minkowskian and one can choose a fiducial STR-type inertial frame covering this exterior region. The Minkowskian approximation becomes asymptotically exact in this region.

407 A sphere of mass M and radius R rotating with angular velocity ω w.r.t. the distant stars produces near its center a centrifugal acceleration of amount $\omega^2 r\left(\frac{GM}{c^2 R}\right)$.

experimental evidence to suggest that Riemannian geometry should be supplanted by the Finslerian generalization to describe the properties of physical space-time.

1918 CE **Otto Fritz Meyerhof** (1884–1951, Germany and U.S.A.). Biochemist. Broke new grounds in the field of conversion and transport of energy in biological systems. Showed that muscular activity involves the anaerobic conversion of glucose and glycogen (a process known as *glycolysis*) into *lactic acid*[408]. During muscle rest lactic acid combines with oxygen to restore glycogen level.

Furthermore, Meyerhof showed that *yeast* and muscle possessed the same *coenzymes*. This made it appear possible that both types of cells broke down glucose by similar series of reactions. It was the beginning of the demonstration that the metabolic pathways of all organisms were essentially similar. All researches since Meyerhof's time have strengthened this view.

Meyerhof was born in Hanover to Jewish parents. He first studied psychology and philosophy but later switched to medicine and biochemistry. Professor of physiological chemistry at the University of Kiel (1918–1924); member of Kaiser Wilhelm Institute for Biology, Berlin (1924–1928); director of department of physiology there (1929–1938).

Fled Germany (due to Nazi persecutions) to Paris (1938–1940). Came to the U.S. in 1940 and became professor at the University of Pennsylvania (1940–1951). Awarded the Nobel prize for physiology or medicine (1922).

1918–1920 CE **Gaston Maurice Julia** (1893–1979, France). Mathematician. Forefather of modern *dynamical system theory* and is best remembered for his *Julia set*. Opened the floodgates of what is now known as '*fractal geometry*'.

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[408]
$$\begin{array}{ccccccc} & & \mathrm{H} & & \mathrm{OH} & & \\ & & | & & | & & \\ \mathrm{H} & — & \mathrm{C} & — & \mathrm{C} & — & \mathrm{COOH} \\ & & | & & | & & \\ & & \mathrm{H} & & \mathrm{H} & & \end{array}, \qquad \text{or simply } C_3H_6O_3 .$$

It is formed in *sour milk* (its taste!) when lactose (milk sugar) is fermented by bacteria. Lactic acid is used for leavening dough (a process where dough is made to rise) because the acid reacts with the sodium bicarbonate to produce carbon dioxide, which lightens the dough by raising it (increase of volume). In the fermentation action of *yeast*, lactic acid is subjected to a further split $C_3H_6O_3 \rightarrow C_2H_6O$ (ethyl alcohol) $+ CO_2$.

With **Pierre Fatou**[409] (1878–1929) he was first to study the iteration in the complex plane of the mapping $z \to z^2 + c$, where $z = x + iy$ and c is a complex parameter. However, making his debut some 63 years before the advent of computer graphics, he could not visualize the startling results and the striking beauty of his maps.

Julia was born in Sidi-Bel-Abbès, Algeria. As a soldier in the first World War, Julia was severely wounded in a German attack on the French front designed to celebrate the Kaiser's birthday. Consequently he lost his nose and had to wear a leather strap across his face for the rest of his life.

Between several painful operations he carried on his mathematical researches in the hospital. In 1918 Julia published *Memoir sur l'iteration des functions rationalles* [*Journal de Math. Pure et Appl. 8*, 47–245], for which he received the Grand Prix de l'Academie des Sciences.

In this work he gave a precise analytical description of the set of complex points in $\mathbb{C}$ for which the nth iterate of the function $f(z)$ stays bounded as $n \to \infty$. Seminars were organized in Berlin (1925) to study his work, which included the first visualization of a Julia set. Although Julia became famous in the 1920's, his work was essentially forgotten until **Benoit Mandelbrot** brought it back to prominence in the 1970's through his computer experiments.

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[409] **Pierre Fatou** (1906) 'Sur les solutions uniformes des certain equations functionales'. *Comptes rendus* (Paris) *143*, 546–548.

Deterministic Chaos and Fractal Geometry[410] — *the Map is the Treasure*

Most natural phenomena are nonlinear*; yet even today, theoretical analysis of physical systems is usually based on linear mathematical models, or ones allowing only small deviations from linearity. Linear models are still routinely used because they are much easier to solve than the correct nonlinear ones. Within the last several decades, however, both theoretical and experimental investigations of nonlinear phenomena have shown that often, behavior which* appears *to be random or chaotic is actually deterministic in its origin. Nonlinear deterministic systems under these conditions are predictable only for short times. This paradoxical situation exists because the deterministic solutions depend very sensitively on initial conditions. Such systems are said to exhibit deterministic chaos.*

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[410] For further reading, see:

- Mandelbrot, B., *The Fractal Geometry of Nature*, W.H. Freeman and Company: New York, 1983, 468 pp.
- Lauwerier, H., *Fractals*, Penguin books, 1991, 203 pp.
- Takayasu, H., *Fractals in the Physical Sciences*, Manchester University Press, 1989, 170 pp.
- Schroeder, M.R., *Fractals, Chaos, Power Laws*,W.H. Freeman and Company: New York, 1990, 429 pp.
- Peitgen, H.O., Jürgens, H. and D. Saupe, *Chaos and Fractals*, Springer-Verlag, 2004, 864 pp.
- Glass, L. and M.C. Mackey, *From Clocks to Chaos, The Rhythms of Life*, Princeton University Press: Princeton, NJ, 1988, 248 pp.
- Pickover, C.A., *Computers, Pattern, Chaos and Beauty* (Graphics From an Unseen World), St. Martin's Press: New York, 1990, 391 pp.
- Hall, N. Ed., *Exploring Chaos*, W.W. Norton & Company: New York, 1993, 223 pp.
- Holden, A.V. (Editor), *Chaos*, Princeton University Press: Princeton, NJ, 1986, 324 pp.
- Peterson, I., *Newton's Clock*, W.H. Freeman, 1993, 317 pp.

It has also been found that several classes of systems show universal behavior at the onset of chaos[411]*. Thus, systems as diverse as a dripping faucet and a heart in ventricular fibrillation show many common features in their dynamics.*

Fractal structure

A set of points which fully (or "almost" fully, in a measure-theoretical sense) covers a line segment is said to be of topological dimension *unity; a set of points which fills a plane area is of* topological dimension *2, etc. One may, however, construct sets of points which intuitively may be assigned a fractional 'dimension'.*

Consider the following example: start with a line segment of unit length, which we shall call the zeroth generation *curve $(n = 0)$ or the* initiator*. From this initiator we construct the* generator *curve by removing the inner third of the initiator and replacing it by the two other sides of an equilateral triangle, built with the removed segment serving as the base; each new side also has length $\frac{1}{3}$. This is the* first generation *curve $(n = 1)$: it has $N_1 = 4$ segments, each of length $\delta_1 = \frac{1}{3}$ and total length $L_1 = \frac{4}{3}$ [the initiator had $\delta_0 = 1$, $N_0 = 1$, $L_0 = 1$].*

We now take each *segment of the generator and build upon it the* same *pattern reduced by a factor of 3; we then obtain a second generation curve with $N_2 = 4^2$ segments, $\delta_1 = \frac{1}{3^2}$ and $L_2 = \left(\frac{4}{3}\right)^2$.*

By replacing with a suitably reduced generator each segment of a given generation of the curve, a new generation is obtained. The n^{th} generation of this construction is a normal polygonal curve with a finite length, having the parameters $N_n = 4^n$, $\delta_n = 3^{-n}$ and $L_n = \left(\frac{4}{3}\right)^n$.

The set of points defined in the limit of infinite number of iterations $(n \to \infty;\ \delta_n \to 0)$, known as the Koch curve[412]*, is a curve for which* length *is not a useful parameter $(L_n \to \infty)$. It is continuous but does not possess a tangent at any point!*

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[411] The word *chaos* was first used in a technical context by **T.Y. Li** and **J. Yorke** in Amer. Math. Monthly **82**, 985–992, 1975.

[412] **Helge von Koch** (1870–1924, Sweden). Mathematician. Principally known for his work on the theory of infinite systems of linear equations. **E.I. Fredholm** used his results in his method of solution of linear integral equations.

Instead of length, we define a new characteristic parameter in the following way: we force N_n *into the algebraic form* $N_n = (\delta_n)^{-d}$, *where* d *is found from* $\log N_n = -d \log \delta_n$. *This yields the Hausdorff-Besicovitch dimension*

$$d = \frac{\log N_n}{\log\left(\frac{1}{\delta_n}\right)}.$$

In our example $d = \frac{\log 4}{\log 3} \simeq 1.262\,8\ldots$. *The definition leads to* $L_n = \delta_n N_n = (\delta_n)^{1-d}$ *(the normalization is such that for* $d = 1$, $L_n \to$ *constant, as the case should be for a curve whose length is independent of* n, *i.e. independent of the resolution at which it is studied, in the continuum limit* $n \to \infty$*). A fractal*[413] *is a shape made of parts similar to the whole in some way. The Koch curve is a fractal set with a fractal dimension* $d = \frac{\log 4}{\log 3}$. *Note that if the initiator is an equilateral triangle of unit side with the same generator as above, the resulting shape is the Koch 'snowflake' curve, having the same fractal dimension. In this variant, while again* $L_n \to \infty$, *the enclosed area tends to a limit* $S_n \to \frac{8}{5} S_0$.

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413 **Georg Cantor** (1833) thought up the oldest *fractal*. This is the so-called *Cantor point-set*. To construct an example, we start with a line-segment (including its end points). Of this we leave out the middle third, but not its end points. We are then left with two line-segments with a total of 4 endpoints. We treat each of these two line-segments like the original one: the middle thirds are removed, so that a total of 4 line-segments with 8 end-points remain. We continue this procedure. Eventually we will be left with discrete points only: the Cantor point-set (or *fractal*) built up from discrete points. It is now called a Cantor set or *Cantor dust*. If we take the length of the original line-segment as 1, then after n steps we have generated 2^n line-segments, each of length 3^{-n} with total length of $\left(\frac{2}{3}\right)^n \to 0$ as $n \to \infty$. Clearly, the Cantor set consists of the real numbers $x = \sum_{k=1}^{\infty}(a_k/3^k)$, where each base-3 digit a_k equals either zero or two (but not one). In base three fraction notation this becomes $x = .a_1a_2\ldots$. Note, for example, that the point $x = \frac{1}{3}$, which is in the Cantor set, is given by $x = 0.022\ldots$, because the above sum becomes $\frac{(2/3^2)}{1-\frac{1}{3}} = \frac{1}{3}$.

Kolmogorov (1958) defined the *capacity* of a point-set on a straight line [which Mandelbrot later (1977) called the *fractal dimension*] as $d = \lim_{h\to 0} \frac{\log N(h)}{\log\left(\frac{1}{h}\right)}$ where $N(h)$ is the smallest number of neighborhoods of size h needed to cover the set. In our example $h = 3^{-n}$, $N(h) = 2^n$, leading to $d = \frac{\log 2}{\log 3} = 0.6309\ldots$.

While sets of fractional Hausdorff dimension have been studied for many years in harmonic analysis, geometric measure theory and in the theory of singularities, the new term 'fractal' was coined and popularized by ***Benoit Mandelbrot*** *(1975).*

However, fractal geometry is today at the stage perhaps analogous to that at which the Newtonian calculus was in the days of ***George Berkeley*** *(1734): it is still a discipline with few rigorous definitions or theorems, and – unlike the early calculus – not a single problem has been solved by it. In fractal geometry one uses some mathematics to generate a picture — which generates more pictures. Then one asks more questions about the new picture, and so on. It thus consists mainly of algorithmic techniques for generating graphic images on a computer systems.*

The Logistic Map

The simplest model for continuous growth of a species with population $N(t)$, *limited by self-interaction effects (such as competition for food supply, or toxicity effects), is* $\frac{dN}{dt} = cN(1 - \frac{N}{k})$, $N \geq 0$, *where* c *and* k *are positive constants and the initial condition satisfies* $0 \leq N_0 = N(0) \leq k$. *This so-called logistic model (**Verhulst**, 1845), an example of the Riccati equation, is a relatively unexciting equation, having the deterministic solution*

$$N(t) = \frac{N_0 k e^{ct}}{k + N_0(e^{ct} - 1)} \to k \qquad \text{as} \quad t \to \infty.$$

The form of an ordinary differential equation presupposes that population variations at any given time t *only depend upon their instantaneous values at that same time. (It also assumes that the population itself is sufficiently numerous to allow a continuous, non-stochastic approximation.) However, in most natural populations the delayed effects of a number of regulating factors prove to be fairly important. Thus, presentation of a population in the form of discrete variable* N_t, *taking on different values at fixed times (e.g. at yearly intervals), is more realistic.*

This scheme exactly reflects the census-taking process for natural populations (in laboratory or field studies), which is usually realized at discrete points in time. The appropriate mathematical tool is thus that of difference *equations, where the population at time t depends in some definite way upon its values in a discrete set of past times.*

Consider the case where the population of every successive generation N_{t+1} depends only on the population of the previous generation N_t, namely $N_{t+1} = f(N_t)$ $(t \geq 0$ integer). Of course, this is valid only under the assumption of non-overlapping generations, i.e. when a new generation reaches maturity, the previous one already declines to extinction. It is also assumed that the main environmental properties are unchanging (thus no explicit t dependence), and that no part of the population is laid aside in diapause for a length of time that exceeds the span of one generation.

Some restrictions on f immediately follow from obvious considerations:

Since for biological reasons $N_t \geq 0$, clearly $f(N) \geq 0$ for all possible $N > 0$. Naturally $f(0) = 0$, and at small N the population increases (natural competition, negligible competition).

On the other hand, resource limitations require that $f(N) \to 0$, as $N \to \infty$.

Substituting $\frac{\Delta N}{\Delta t}$ for $\frac{dN}{dt}$, where $\Delta N = N_{t+1} - N_t$ and $\Delta t = 1$, the above logistic equation becomes

$$N_{t+1} = N_t \left[1 + c(1 - \frac{N_t}{k}) \right].$$

If at some time $N_t > k(1 + \frac{1}{c})$, this equation yields a negative value of N_{t+1}, which is not the case in the continuous model. From this point of view the discrete analogue of the continuous logistic growth model is biologically incorrect. But even if we disregard this, it turns out that their solutions are very different. The reason for this is that since t increases in discrete steps, there is an inherent delay in the development of changes, and the difference equation is in fact equivalent to a delay functional equation which is known to have oscillatory solutions [e.g. $N(t) = \frac{1}{a}N(t - T)$, $t \geq T$, $a \geq 1$ is solved by $N(t) = e^{-\frac{\ln a}{T}t} f(t)$, with $f(t)$ periodic with period T such that $f(t) = a^{t/T} N(t)$ in the initial interval $0 \leq t < T$].

Our logistic difference equation can be simplified into

$$x_{n+1} = r x_n (1 - x_n),$$

where $r = 1 + c$, $t = n$ units of time, and $N_t = k(1 + \frac{1}{c}) x_t$.

In this form it is known as the logistic map. Here $x_n \geq 0$ is a dimensionless measure of the population of the n-th generation and $r > 0$ is the *intrinsic growth rate, appearing as an adjustable parameter; a realistic solution must have* $x_n \leq 1$ *as well, to ensure* $x_{n+1} \geq 0$*. Though* $r = 1 + c > 1$ *in the above derivation, the logistic map is usually studied for all* $r > o$*.*

Given and initial population value $x = x_0$*, and a particular value of* r*, the equation is solved numerically by iteration to any desirable* n *value. The graph of* $rx(1 - x)$ *is a parabola with a peak value* $\frac{r}{4}$ *at* $x = \frac{1}{2}$*, restricting the control parameter* r *to the range* $0 \leq r \leq 4$ *(to ensure* $x_n \leq 1$*). It will map the interval* $0 \leq x \leq 1$ *into itself, so the iteration will not take us out of the interval* $[0, 1]$*.*

Suppose that we fix r*, choose some initial population* x_0*, and then use* $x_{n+1} = rx_n(1 - x_n)$ *to generate the sequence* $x_1, x_2, x_3, \ldots x_n, \ldots$

What happens?

Numerical experiments exhibit the following results:

- *The initial value* $x_0 = 0$ *invariably renders* $x_n \equiv 0$ *at all later times (*n *values* ≥ 1*) and for all* r *values. The initial value* $x_0 = 1$ *likewise yield the same result.*

- *For small growth rate* $r < 1$*, the population always goes extinct:* $x_n \to 0$*. We say that 0 is an attractor (limit point). Thus, extinction is nature's 'punishment' for insufficient fertility.*

- *For* $1 < r < 3$ *the population grows and eventually reaches a nonzero steady state, as can be seen when we plot the resulting time-series* x_n *vs.* n*. Let us take a closer look at this interval:*

Define an equilibrium point $\bar{x}$*, of the system as a fixed point of the map, i.e. one for which* $x_n = \bar{x}$ *guarantees* $x_{n+1} = x_n = \bar{x}$*. Mathematically it means that* $\bar{x} = r\bar{x}(1 - \bar{x})$*. This equation has two roots, one at* $\bar{x} = 0$ *(which we already encountered before) and the other at* $\bar{x} = 1 - \frac{1}{r}$*. For* $r < 1$*, the second fixed point renders negative values for* x_n*, which we dismiss as being unrealistic.*

For $r > 1$*, however, there will be two fixed points, one at* $\bar{x} = 0$*, and the other at* $\bar{x} = 1 - \frac{1}{r}$*. Now, an equilibrium (fixed) point may be either stable or unstable, according as small perturbations about the point result in dynamics that drive the numbers back to it or away from it, respectively.*

It can be shown that in the general case $\left|\frac{df(x)}{dx}\right|_{\bar{x}} < 1$ *implies stability (attractor), while* $\left|\frac{df(x)}{dx}\right|_{\bar{x}} > 1$ *implies instability (repeller). In our case, since* $f'(x) = r(1 - 2x)$*, we have for the first fixed point* $f'(0) = r$ *and for the second* $f'(1 - \frac{1}{r}) = 2 - r$*.*

Consequently, when $r < 1$, $\bar{x} = 0$ *is the only fixed point, and it is stable;*

But for $1 < r < 3$, *the origin becomes an unstable fixed point (repeller), while the second fixed point is stable (attractor). The map at* $r = 1$ *represents a bifurcation in the systems behavior: as* r *passes through this value, the single, stable fixed-point* $\bar{x} = 0$ *splits continuously into two fixed points – an unstable one remains at* $\bar{x} = 0$*, while the new fixed point (at* $\bar{x} = 1 - \frac{1}{r}$*) is stable.*

Similarly, there is a second bifurcation at $r = 3$ *corresponding to the onset of instability of the non-zero equilibrium point at* $\bar{x} = 1 - \frac{1}{r}$*.*

Thus, for $1 < r < 3$ *there are two equilibrium points of which the origin is unstable, and the other is stable. For* $2 < r < 3$ *numerical experiments show that the sequence* $x_1, x_2, x_3, \ldots$ *approaches the limit value* $1 - \frac{1}{r}$ *through oscillations on both sides of this limit. Our biological model is stable — there is balance in nature.*

However, as soon as r *becomes even a little greater than 3, a new phenomenon appears: the population builds up again and oscillates about the fixed point, alternating between a large population in one generation and a smaller population in the next. This type of oscillation, in which* x_n *repeats every two iterations, is called a period 2-cycle.*

Thus at $r = 3.1$, *after 200 iterations,* x_n *repeatedly hops back and forth between the two values* $0.76456\ldots$ *and* $0.55801\ldots$*, while the (now unstable) fixed point* $\bar{x} = 1 - \frac{1}{3.1} = 0.6774\ldots$ *remains unapproached. The 2-cycle orbit is stable*[414] *up to* $r = 1 + \sqrt{6} = 3.449\ldots$

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[414] Asking for a fixed point of the *second-iterate map*, $f[f(\bar{x})] = f^2(\bar{x}) = \bar{x}$, renders a quartic polynomial equation for $\bar{x}$. However since $f(\bar{x}) = \bar{x}$ implies $f^2(\bar{x}) = \bar{x}$, the two solutions of the first-iterate map [namely, $\bar{x} = 0$, $\bar{x} = 1 - \frac{1}{r}$] are automatically included. After factoring out x and $x - (1 - \frac{1}{r})$ by long polynomial division, the equation $f^2(x) - x = 0$ simplifies to a quadratic equation with the solutions

$$p, q = \frac{1}{2r}[(r+1) \pm \sqrt{(r-3)(r+1)}],$$

which are real for $r > 3$. Thus a 2-cycle exists for all $r > 3$ as claimed,and $f(p) = q$, $f(q) = p$. At $r = 3$, the roots coincide and equal $\bar{x} = 1 - \frac{1}{r} = \frac{2}{3}$, which shows that the 2-cycle bifurcates *continuously* from $\bar{x}$. For $r < 3$ the roots are complex, which means that a 2-cycle does not exist.
To determine the r region of *stability* for the 2-cycle, we impose that p and q are stable fixed points of the second-iterate $f^2(x)$. Analytically,

$$\frac{d}{dx}\left(f^2(x)\right)_{x=p} = f'[f(p)]f'(p) = f'(q)f'(p) = 4 + 2r - r^2.$$

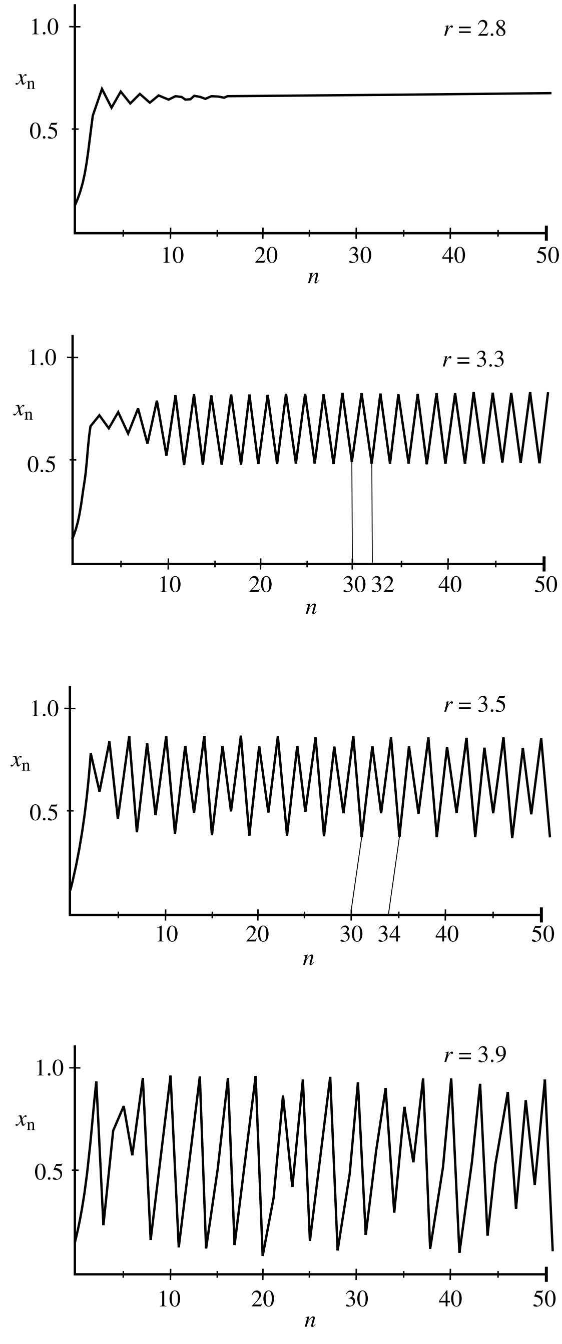

Fig. 5.4: The solution sequence of the Logistic – map recursion $x_{n+1} = rx_n(1 - x_n)$ for some values of the parameter r

At still larger r, the population approaches a stable cycle that now repeats every four generations: the previous cycle has doubled its period to period-4 [e.g. at $r = 3.45$ we find the values $0.87499\ldots$; $0.38281\ldots$; $0.82694\ldots$; $0.50088\ldots$, while the fixed point is at $0.71014\ldots$]. Further bifurcations results in progressive period-doubling to stable cycles of period 8, 16, 32... as r increases, each cycle becomes unstable at the $r-$value at which the next, longer-period cycle continuously "peels off" it – replacing it (until the next bifurcation) as the stable limit cycle.

Specifically, let r_n denote the value of r where a 2^n-cycle first appears. Computer experiments reveal that these threshold parameter values are:

$r_1 = 3$	(period 2 is born)
$r_2 = 3.449\cdots = 1+\sqrt{6}$	4
$r_3 = 3.54409\ldots$	8
$r_4 = 3.5644\ldots$	16
$r_5 = 3.568759\ldots$	32
$\vdots$	$\vdots$
$r_\infty = 3.569946\ldots$	∞

Note that the successive bifurcations occur at progressively smaller r intervals as we sweep through increasing values of r. Ultimately the sequence r_n converges to a limiting value r_∞. The convergence is asymptotically geometric: in the limit of large n, the distance between successive transitions shrinks by a constant factor

$$\delta = \lim_{n\to\infty} \frac{r_n - r_{n-1}}{r_{n+1} - r_n} = 4.669\ldots$$

We list the attractors corresponding to the various r-values:

Therefore the 2-cycle is linearly stable for $|4 + 2r - r^2| < 1$, i.e. for $3 < r < 1 + \sqrt{6} = 3.449\ldots$.

Interval	*Attractor at fixed point X*
$0 \leq r \leq 1$	$X = 0$
$1 < r \leq 3$	$X = 1 - \frac{1}{r}$
$3 < r \leq r_2$	2^1*-cycle*
$r_2 < r \leq r_3$	2^2*-cycle*
$r_3 < r \leq r_4$	2^3*-cycle*
...	...
$r_k < r \leq r_{k+1}$	2^k*-cycle*
...	...
$r_\infty = 3.56994\ldots$	

The limit ratio δ is called the Feigenbaum number[415]*, after the physicist who discovered the properties of this map (M. Feigenbaum, 1978). Beyond*

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[415] The emergence of *chaos* as a research subject unto itself was a story not only of new theories and new discoveries, but also of the belated understanding of old ideas. Many pieces of the puzzle had been understood long ago by **Poincaré**, **Maxwell**, even by **Einstein** — and then forgotten. But when **Feigenbaum** first discovered the universality of chaos, the discovery was greeted with surprise, disbelief, and excitement. It soon became evident that there were structures in nonlinear systems that are fundamentally the same if one looked at them the right way. However, reserved attitudes still prevailed among mathematicians, who were unhappy with numerics and demanded a proof. In 1979, Feigenbaum supplied a 'reasonable proof' that finally convinced the diehards[416].

[416] The *universality* of certain scaling laws – manifested in period–doubling bifurcation sequences when general, iterated nonlinear maps are varied – is closely related to the universality of scaling laws of physical observables as a *phase transition* is approached in quasi–equilibrium, many-body interacting systems, such as condensed matter systems. Many such phenomena are known empirically; the longest-known example is the $\sqrt{T - T_c}$ behavior of bulk magnetization as a ferromagnetic sample's temperature T approaches the critical Curie temperature, T_c, from below.

Through the pioneering work of **L. Kadanoff**, **K. Wilson**, **Migdal** and others in the 1970s (involving theory and simulations), it was found that such *critical behavior* involving *power laws* is usually universal, in the sense that many dis-

r_∞ chaotic evolution can occur ("strange attractor"); that is, the long-term behavior of the iterated sequence $\{x_n\}$ does not settle down to any simple periodic motion.

The interval $(r_\infty, 4)$ contains an infinite number of narrow windows of r-values for which there exist stable m-cycles. The first such cycles to appear beyond r_∞ are of even period. Next odd cycles appear in descending order. The period 3-cycle[417] *first appears for $r = 3.828427\dots$ and stays stable up to $r = 3.841499\dots$. At the end of the 3-window begins a stable 6-cycle*

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parate condensed–matter physical systems – each with a different microscopic Hamiltonian (classical or quantum) – all undergo the same dynamics near criticality. And through the deep analogy between condensed–matter systems and relativistic Quantum Field Theories – especially the mathematical correspondence between *partition sums* and *path integrals* – such critical behavior analyses were also fruitfully applied to the physics of elementary particles and their fields, the quantum vacuum, astrophysics and the early universe.

Apart from advance Monte-Carlo computer simulations, the main mathematical tool used in such studies is the *renormalization* group: a method of iteratively mapping the relevant effective Hamiltonian or action across many scales of space and or time. The universality of critical behavior was thus related to fixed points and asymptotic behaviors of nonlinear maps in various dimensions. The dimensions referred to here are those of thermodynamic phase diagrams; but renormalization–group methods were also applied to condensed–matter and quantum-field systems in various numbers of spacetime dimensions. These methods rely on the fact that thermodynamical phase transitions involve cooperative phenomena at many distance scales (fluctuations, excitations, correlations etc.)

The idea of universal bifurcation, fixed points, limit cycles, stability analysis, etc., was also extended to continuous – time maps (e.g. systems of ODEs describing mechanical, optical, chemical or biological systems), and to the dynamics of systems continuous in both space and time. For example, **I. Prigogine** and coworkers applied these methods to the study of the spontaneous emergence of spatial and temporal order and structure in far–from–equilibrium open thermodynamic systems, described by nonlinear systems of PDEs (Reaction – Diffusion – Advection equations).

In general, bifurcations – whether in the discrete Logistic map or more complex systems – are intrinsically *topological* in nature.

417 The period-3 window that occurs in $3.8284\dots \le r \le 3.8415\dots$ is the most conspicuous. Suddenly, against a backdrop of chaos, a stable 3-cycle appears out of the blue. This is explained as follows: starting from an n-iterate $x_{n+1} = f(x_n)$, we generate the next iterate $x_{n+2} = f(x_{n+1}) = f^2(x_n)$. Similarly $x_{n+3} = f^3(x_n)$. Now, any point p in a period-3 cycle repeats every three iterates, by definition. So such points satisfy $p = f^3(p)$ and are therefore

followed by further period doubling. The same is true for all other stable cycles of odd order. Figure 5.4 exhibits the behavior of the solution $x_n(n,r)$ *for some* r *values in the parameter range* $2.8 \le r \le 3.9$.

Outside the windows there are no stable periodic orbits, although there are an infinite number of unstable cycles. The dynamic behavior of the map is then called chaotic. The most chaotic case, $r = 4$*, deserves special attention since in that case the iterative map can be represented explicitly by an elementary function. Indeed,* $x_{n+1} = 4x_n(1 - x_n)$ *is solved exactly by* $x_n = \sin^2(2^n\theta_0\pi)$*, where* $0 \le \theta_0 < 1$*,* $\theta_0 = \frac{1}{\pi}\sin^{-1}\sqrt{x_0}$*. It is helpful to write* θ_0 *in base-2 form as*

$$\theta_0 = 0.b_1b_2b_3\cdots = \frac{b_1}{2^1} + \frac{b_2}{2^2} + \ldots$$

where

$$x_0 = \sin^2(\pi\theta_0), \qquad x_1 = \sin^2(2\pi\theta_0)\ \ldots etc.$$

Then at each iteration step the foremost binary digit is lost. If x_0 *is an arbitrary starting point, then as a rule* θ_0 *is an irrational number with an infinite string of zeros and ones, as in tossing a coin. This means that as a rule the orbit is aperiodic. Periodic orbits, always unstable, are produced by rational values of* θ_0*. Indeed, defining* $\theta_n = 2^n\theta_0 \pmod 1$ *we have* $x_n = \sin^2(\pi\theta_n)$.

For example, if $\theta_0 = \frac{1}{3}$ *(or* $x_0 = \frac{3}{4}$*), then* $\{\theta_n\} = \{\frac{1}{3}, \frac{2}{3}, \frac{1}{3}, \frac{2}{3}, \ldots\}$ *and* $x_n = \frac{3}{4}$ *is a fixed point. If* $\theta_0 = \frac{1}{5}$*, then* $\{\theta_n\} = \{\frac{1}{5}, \frac{2}{5}, \frac{4}{5}, \frac{3}{5}, \frac{1}{5}, \frac{2}{5}, \ldots\}$ *and* x_n *settles on the 2-cycle* $\frac{5\pm\sqrt{5}}{8}$*. For* $\theta_0 = \frac{1}{7}$ *we have the 3-cycle* 0.188*;* 0.611*;* 0.950*, and for* $\theta_0 = \frac{1}{9}$ *the 3-cycle* 0.117*;* 0.413*;* 0.970.

fixed points of the third-iterate map. Since $f^3(x)$ is an 8-th degree polynomial, we cannot solve it explicitly for its fixed points. We may however plot $y = f^3(x)$ vs x in the interval $0 \le x \le 1$ and intersect it with the diagonal $y = x$. There are in general 8 intersections, of which two are imposters [period-1 points for which $f(\bar{x}) = \bar{x}$]. Of the remaining six, only 3 are consistent with the stability of the cycle, while the other 3 are unstable. The graphical solution shows that as r decreases through $1 + \sqrt{8} = 3.8284\ldots$, the graph of $f^3(x)$ becomes tangent to the diagonal. At this critical value of r, the stable and unstable pair of period-3 limit cycles (which have been continuously approaching each other as r decreases) *coalesce and annihilate in a* so called *tangent bifurcation*! This transition marks the beginning of the periodic window. As as r increases it lasts until r exceeds $3.84149\ldots$

All the periodic sequences are unstable, because each rational value of θ_0 *is arbitrary close to an irrational value of* θ_0 *which generates an aperiodic sequence* $\{\theta_n\}$*. This means that the sequence* $\{x_n\}$*, for large values of* n*, depends sensitively on the initial value* x_0*, with exponentially growing separation of neighboring orbits.*

This sensitivity was demonstrated above for the special value $r = 4$ *because there it can be made explicit, but a similar situation occurs for other values of* r *for which there is an attractor which is infinite. The attractor is the same set for all initial values within the domain of attraction, but the* position *of* x_n *within the attractor for a given large value of* n *depends so sensitively upon* x_0 *as to be, in effect, random. Many other one-dimensional and higher-dimensional nonlinear ordinary differential equations and other nonlinear systems also display sensitive dependence on initial conditions.*

An issue associated with the logistic map is of fundamental importance that goes beyond this particular example:

A given recursion equation determines x_n *precisely in terms of* x_0*, yet in some respects a sequence* $\{x_n\}$ *which wanders nearby an infinite set of unstable limit–cycles may be regarded as a sequence of random numbers. Thus, although a particular value* x_n *may depend sensitively on* x_0*, the* statistical *properties of the sequence* $\{x_n\}$ *are the same for all values of* x_0 *which correspond to irrational values of* θ_0*. And yet, for a given value of* x_0*, the sequence* $\{x_n\}$ *is* in principle *determined completely and precisely by an algorithm, namely iteration of the map* f*.*

But in practice the sequence may be indistinguishable from a sequence of random numbers. In laboratory experiments, however careful, observations are not absolutely precise, and there is no possibility of distinguishing an irrational from rational value of a datum. Also, numerical errors (such as round-off error) are inevitable in the practice of processing the data to find x_{n+1} *from* x_n*.*

So we could not predict the precise long-range future of a chaotic system, even if we had an exact model on which to base our predictions. The errors in prediction grow exponentially in time (or in n*, in the discrete-time case at hand) because of our lack of exact knowledge of the present, so that doubling the accuracy of our measurements and data processing will avail only a little: we may predict with confidence only the near future of chaotic solutions and their statistical properties over a long time. It is because of this feature that such chaos is called* deterministic chaos*.*

It is therefore possible to make such statements as: "In the next 100 iterations, the following approximate state values are more likely to occur than others. . . ."

Though they are less exact than one might expect of deterministic time series, such predictions are potentially valuable nonetheless. Even though it may never be possible to precisely predict phenomena like the weather, earthquakes, population dynamics, the stock market or the sudden collapse of sand piles, one might foresee the global patters of their behavior — the "order within the chaos".

The logistic map is the simplest example that exhibits chaotic dynamics, and indeed serves as a paradigm for chaos.

It has been shown that other maps $x_{n+1} = f(x_n)$, *behave in a similar manner with the same scaling law. Thus, the phenomenon of period-doubling coupled with bifurcation-parameter scaling is an example of a* universal *property for a certain class of one-dimensional difference equations modeling dynamic processes.*

Moreover, period doubling and Feigenbaum scaling have been observed in many physical experiments.

The forgoing analysis shows that simple and fully deterministic models in which all the biological parameters are exactly known, *can nonetheless (if the nonlinearities are sufficiently severe) lead to population dynamics which are in effect indistinguishable from sample drawn from a* random process. *Apparently chaotic population fluctuations need not necessarily be due to random environmental or genetic fluctuations, or sampling errors, but may reflect the workings of some* deterministic, *but strongly density dependent, population model.*

Taking a practical point of view we can say that randomness *occurs to the extent that something cannot be predicted.* Unpredictability *can arise for many reasons; prime among these is shear ignorance: If we do not know the forces that cause something to change, then we cannot predict its behavior. For example, a common way to choose something randomly is to* flip a coin: *In the absence of any other information "heads" is just a likely as "tails", and the outcome is unpredictable. However, if we made precise measurements of the motion of the coin as it left our hand, we could predict the final outcome. People who are skilled in flipping a coin properly can do this.*

Another classical example of randomness is the game of roulette *— the final resting position of the ball is uncertain, but in fact the motion of roulette balls* can *be predicted using simple physical laws, to a sufficient degree of accuracy to give a significant advantage over the house. Thus we see that when one knows the dynamics and has enough information about the state of the system, some classical examples of randomness cease to be random.*

Aside from ignorance, three basic causes of randomness are known (at this point of time):

- *Quantum mechanics: there exists an intrinsic limit to the accuracy of measurements and to our knowledge of, or even the in-principle* knowability *of, intermediate physical variables between measurements. These are limits that no observer, sentient or automated, can go beyond, no matter how much we know or how careful we are.*

- *Complexity: a dynamic system that involves many irreducible degrees of freedom as measured in terms of the* dimension *of the motional phase or* state space*, e.g. the motion of gas particles or atoms in condensed–matter systems. Even setting Quantum Mechanics aside, we can never hope to keep track of all degrees of freedom in such a system, even if we know all relevant microscopic interactions. Yet, the simplest collective modes of behavior of the system – such as spatially smooth wave excitations, low-order statistical moments of various distributions, and the like – may be described by simple physical laws and their behavior predicted through tools such as equilibrium or near–equilibrium statistical mechanics, Landau–Ginsburg effective field theory, etc.*

Consequently, while the behavior of the individual particles is random, many bulk properties of the system are not.

Sometimes, however, microscopic fluctuations interact with collective dynamics at many scales of space and times, to produce complex behavior that is difficult to analyze; an example of this is turbulence *in fluid dynamics.*

- *Chaos: a sensitive dependence on initial conditions. For a* deterministic *dynamical system, chaos occurs when on the average, nearby trajectories separate at an exponential rate. This is measured by the* Lyapunov exponents: *if any of them are positive, then there is exponential separation and the system is chaotic. Chaos can cause random behavior even in systems with only a few degrees of freedom (such as the logistic map, the Lorentz, attractor – a dissipative nonlinear system of 3 coupled ODE's – or the* kicked rotor*). Here randomness emerges through the agency of deterministic evolution, without the need for a large number of system components, ignorance–caused uncertainties or quantum effects.*

Chaos and complexity by no means preclude each other. Chaos also occurs in systems with many degrees of freedom. On the other hand, complex dynamical systems can often be approximately truncated into simplified systems with only a few degrees of freedom, thereby losing much of their complexity, while maintaining chaos and randomness. Thus, some of the randomness of turbulent motion derives from chaos.

If in the recursive logistic map $x_{n+1} = rx_n(1 - x_n)$ *we change variable to* $x_n = \frac{1}{2} - \frac{1}{r}z_n$, *we obtain the new form* $z_{n+1} = z_n^2 + c$, *where* $c = \frac{r}{2} - \frac{r^2}{4}$. *In the logistic map,* x_n *and* r *are, of course,* real *numbers since they typically represent populations and reproduction rates.*

Fatou *(1906) and* ***Julia*** *(1918) formally extended such maps into the complex plane by letting* $z = x + iy$, $c = c_r + ic_i$, *in the hope of discovering new dynamic features and gaining better understanding of the classical logistic case.*

Indeed, the compact form $z_{n+1} = z_n^2 + c$ *then splits into a system of two coupled equations in real variables:*

$$\begin{aligned} x_{n+1} &= x_n^2 - y_n^2 + c_r \\ y_{n+1} &= 2x_n y_n + c_i \end{aligned}$$

This map produces dynamics in the so-called phase plane (x, y) *which depends on the value of the parameter* c *in the* control plane (c_r, c_i). *For fixed* c, *the set of points in the phase plane which remain bounded for all* n *is called the filled-in Julia set. Points which start outside this set rapidly iterate away to infinity. The* boundary *of the filled-in Julia set is often itself called the Julia set. For some points of the control plane set may not have any interior points (as occurs for a* cloud *of points or a curve).*

If $c = 0$, *all point in the* z*-plane either go to* $z = 0$ *[if* $|z_0| < 1$*], to* $|z| = \infty$ *[if* $|z_0| > 1$*] or to* $|z| = 1$ *[if* $|z_0| = 1$*]. Thus, the* unit circle *is the boundary between the two* basins of attraction *at* $z = 0$ *and* $z = \infty$.

However, if $c \neq 0$, *regions of the filled-in Julia set can take on an incredible variety of forms, highly self–similar, that may resemble clouds, branches of a bramble bush, sea horses, etc.*

In 1982, mathematicians extended the logistic map even further, into the field of quaternions *— iterating* $Q_{n+1} = Q_n^2 + q$, *where for each* n $Q_n = a_0 + a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k}$ *is a 4-dimensional variable quaternions and* q *is a constant quaternion; a 2-dimensional slice in the* (a_0, a_2) *plane at fixed* (a_1, a_3), *then exhibits beautiful Julia sets.*

*The beauty of these sets has stimulated the esthetic senses of mathematicians, scientists, and artists alike (**Gaston Julia** himself, never* saw *a Julia set!) Using computer graphics it was found, for example, that if* $c = -0.124 + 0.565i$, *the Julia set has a* fractal structure *which is a self-similar connected set, in the sense that any arbitrary piece of the set can be used to* *construct the entire set by finite number of iterations. For $c = 0.12 + 0.74i$, the Julia set becomes more involved but is still connected.*

***Benoit Mandelbrot** (1980) was first to consider the changes in the Julia set as a function of the complex control parameter. Just as we can distinguish two sets of points in the phase-plane (x, y), two sets can also be distinguished in the control-plane. In the c-plane the Mandelbrot set is the set of points (c_r, c_i) such that $z_0 = 0$ yields a bounded sequence of planar points z_n.*

Like the Julia set, the Mandelbrot set is a connected set, but it has an amazingly complex structure in the c-plane. A new geometry, known as fractal geometry, thus emerged. More general studies involve the complex maps $z_{n+1} = z_n - \frac{f(z_n)}{f'(z_n)}$, known as Newton maps[418], since Newton's method of obtaining the zeros of $f(z)$ is based on the convergence of the iterates of the above recursion to a complex root of the function $f(z)$.

1918 CE, Dec.–1921, Apr. Sixty thousand Jews were killed in the Ukraine in antisemitic riots.

In March and April 1918 the Ukraine declared its independence and the "independent" citizens killed the Jews. Subsequent fightings between the Bolsheviks and the Ukrainian separatists led to the worst riots that had taken place since the 17^{th} century.

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[418] Consider the first-order differential equation $\frac{dx}{dt} = f(x)$. Choosing h as a suitable small positive number, we define $x_n = x(nh)$ for $n = 0, 1, \dots$ and approximate the derivative $\frac{dx}{dt}$ at $t = nh$ by the Euler forward difference $\frac{1}{h}(x_{n+1} - x_n)$. It follows that $\frac{x_{n+1} - x_n}{h} = f(x_n)$ approximately, or $x_{n+1} = x_n + hf(x_n)$. Numerical analysts have studied the efficiency of this method and its convergence as the 'time' step $h \to 0$. Yet if we seek only equilibrium solutions, then we need only seek the zeros of $f(x)$. A well-known equation follows by the *Newton-Raphson method*. For this we assume that $f(x)$ is continuously differentiable, take x_n to be estimates of zeros of $f(x)$ and construct an improved estimate x_{n+1} by approximating the curve $y = f(x)$ by its tangent $y = f(x_n) + (x - x_n)f'(x_n)$ at the point $(x_n, f(x_n))$. This gives the next approximation $x_{n+1} = x_n - f(x_n)/f'(x_n)$, if $f'(x_n) \neq 0$. In practice, we start by *guessing* x_0 from what knowledge of $f(x)$ we have at the start, and compute successive approximations $x_1, x_2, \dots$, stopping when we have reason to believe that x_n approximates the exact zero with desired accuracy.

1918–1922 CE **Oswald Spengler**[419] (1880–1936, Germany). Philosopher of history and a mathematical scholar. His reputation rests on his book *The Decline of the West* (*Der Untergang des Abendlandes*), conceived before WWI, and finished in 1917.

Spengler sought to initiate a 'Copernican revolution' in historiography: history is devoid of any fixed point of reference or unifying meaning. The unit of history is not a nation but a *civilization*, and instead of the traditional linear succession of ancient, medieval and modern times he regarded history as a study of comparative *cultures* or *civilizations*[420].

Each culture was, in his view, an organism which like any other living thing went through a regular and *predictable* life cycle[421] of birth, growth, maturity and decay. *History has not happened to others*; Western civilization and its scientific revolutions are no less vulnerable than the now extinct civilizations of the Aztecs and the Incas, the Sumerians and the Hittites, the early civilization

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[419] For further reading, see:

- Spengler, O., *The Decline of the West* (Abridged Edition by A. Knopf), New York, 1962, 415 pp.

[420] Spengler makes a distinction between *culture* and *civilization*; the former is a period of creative activity of a society where the '*soul*' of the country-side predominates. It comprises the spring, summer, and autumn of a society. Civilization is the era of theoretical elaboration and material comfort, dominated by the '*intellect*' of the city. It comprises the winter of the society. Most civilizations continue hundreds, even thousands of years after their creativity is spent; so long as the culture phase lasts, the leading figures in a society manifest a sure sense of artistic 'style' and a personal 'form'. The breakdown of style and form most clearly marks the transition from culture to civilization. In this sense Rome is the civilization stage of the Greek culture. Together they form the classical world.

[421] As a cyclic historian, Spengler's three precursors were: The monk **Joachim of Floris** (1145–1202, Italy). He taught that each stage of history rose to its own climax, and each successive stage represents a higher level of spiritual development. Yet within each stage, the course of its unfolding bore a detailed resemblance to the course of its predecessor. The second precursor was **Giambattista Vico** (1668–1744, Italy), who advanced (1725) the doctrine that history is a *spiral of progress*: every turn is higher than the one before; history never repeats itself because it is viewed from a loftier position than before, enabling us to see wider horizons; history is the process of total liberation of human spirit, with each spiral bringing it closer to freedom.

The third precursor, **Nikolai Danilevsky** (1822–1885, Russia), most directly anticipated Spengler's major theories.

of China, and the so-called Greco-Roman civilization. The West is doomed to extinction, and another Chinese civilization, having entered its proper phase, will in time replace it.

Each culture is deeply rooted in its natural environment. It is a spiritual phenomenon, manifested in a world-view common to a specific society. If engulfs the entire field of its activity and is characterized by a *specific perception of depth and space*. Of these cultures, the two about which he knew most were the classical Greco-Roman and his own Western society. What was traditionally accepted as ancient history Spengler redefined as the history of the classical culture, plus a brief sketch of two preceding cultures, the Egyptian and the Babylonian, and the garbled account of a successor culture, which he called Magian (Iranian, Hebrew, Arabic).

Similarly, the conventional medieval and modern periods, together formed the history of the West, with side-glances at such non-European societies as China, India, and Aztec-Mexico. Every culture formed a distinct bloc of spiritual and physical reality, clearly delimited from its predecessors, contemporaries and successors. Yet each one went through the same *morphological stages*.

Within each culture, certain basic attitudes permeated all of life and thought. Properly defined and understood, these attitudes would give the key to the history of the whole culture. While they could most readily be identified in the realm of aesthetics (in the plastic arts and music, and above all in architecture) they exercised an equally pervasive influence over the forms of economics, war, and politics, and even over so unlikely a field as *mathematics*). Taken together, these basic attitudes formed a master pattern — a characteristic cast of human spirit working itself out in the history of every culture of which any record remains.

Spengler calls the first stage in the life cycle of a culture, a *spring*: Agricultural-based economy, life centered around the village in a feudal system. Its people exist in a 'precultural stage', characterized by mystical symbolism and primitive expressions. There is yet no philosophy, ideology, or technology.

Next comes *summer*: an effervescent aristocracy living in provincial towns brings about the decline of religion and the rise of science (the Greek Polis, the Renaissance). This period culminates in the first philosophical frameworks and new intellectual clubs.

It is followed by *autumn*, the zenith of intellectual creativeness, the era of big systems. Power is shifted from elite to business, from property to money. It is the era of *maturity*, the final attempt to refine all forms of intellectual development. Culture showing signs of fatigue is centered in big cities — a centralized monarchistic regime (Athens at its peak; 18^{th} century in Western Europe.)

The only way to go from here is down to the final *winter* stage: Culture declines into a civilization that exploits materialistic and organizational accomplishments. Uprooted proletariat inhabit world-cities subjected to expansionist war-mongering despots (e.g. Caesar, Napoleon). It is a dying culture characterized by a cult of science, degradation of artistic creativity and abstract thinking, dissolution of old norms, meaningless luxuries, outlets in sports, and rapidly changing fashions.

Each new cycle, however, was not simply a repetition of its predecessor. In the meantime, fresh cultural elements had appeared, which gave new spiritual content to the cycle. Thus the *Apollonian* man (classical culture) invented geometry, *Magian* man algebra, and *Faustian* man (Western) the calculus. For different culture, numbers mean totally different things. The space perception of the Apollonian man was local and limited and its artistic expression was the free-standing nude statue. It was manifested also in the political structure of the city-state.

The Faustian man, on the other hand, experienced the endless vistas of limitless space. He has lived in eternal restlessness, and was longing for the unattainable. It began with skyward striving of the medieval cathedrals, found a new outlet in the perspective and color of Renaissance and 17^{th} century painting, and ended in music, which alone spoke a language sufficiently abstract to convey a sense of spiritual infinity. In the will to conquer distance, Faustian man has created his most eloquent symbols: *the Copernical view of the universe, the faith of the explorer, and the machines that decade by decade have produced more and traveled faster than ever their inventors had considered possible*. The entire activity of the Faustian man aims to fill his unlimited, endless space.

For Spengler, Western history does not start, as in schoolbooks, with the fall of Rome in 476 *CE*. The first 5 centuries of medieval history he regards as a kind of twilight era, in which the memories of Greco-Roman civilization, the omnipresence of the Judeo-Islamic forms, and the stirrings of a new indigenous spirit, struggled for possession of the Western European soul.

It was not until the 10^{th} century that the Faustian culture was born. With the reform of the Papacy, the reestablishment of an imperial authority, the articulation of feudal society, and the emergence of Romanesque architecture, the new culture manifests itself in clear and vigorous form. Its focal point is Christianity — an aspiring faith which gives to the Middle Ages, the springtime of Western culture, a quality of high tension overflowing with the excitement of passionate deeds and spiritual discovery.

After unfolding its full possibilities in the triumph of Ghotic architecture, and the theological constructions of scholastics and mystics, the summer phase of Faustian culture breaks down in internal contradictions. One of these is the so-called ‘Renaissance’ (essentially an affair of the *nobility*). Its spiritual counterpart, the Reformation, represented a change of orientation for the *clergy*. It formed part of the shift in the cultural center of gravity from the countryside to the city, and provided a religious foundation for a third social class, the new *bourgeoisie*.

In the new society of the cities, Faustian culture reached its maturity. The artistic embodiment of ripe culture is the architecture of the Baroque and the art of the great painters of the 17^{th} century. Ultimately the Faustian spirit found its manifestation in the realm of music which, since ca 1670, dominates the cultural life of the West. Intellectually, the era is one of free inquiry and scientific speculation. Its characteristic thinker is Descartes, the philosophical equivalent of the pre-Socratic.

The 18^{th} century is the autumn of the Faustian soul. It offers the last, most exquisite creations of fully-realized style and form: the art of the Rococo; the music of Mozart, the philosophical writings of Kant and Goethe, who like Plato and Aristotle in the Apollonian world, give a *conclusive formulation* to the deepest speculation of their culture.

As against this positive, creative aspect, the contrasting tendency of criticism and destruction comes more and more to predominate as the century advances. Where earlier a host of cities, strongly differentiated and with intense local consciousness, produced the most varied artistic and intellectual life, now a few great cities like Paris and London draw all aspiring talents into an ever-tightening circle. The century ends in the great 1789 revolution in which the middle class assumes authority. In the struggle between the monarchy and its enemies, the victory goes to Napoleon — the “romantic” tyrant and “contemporary” of Alexander the Great.

With the 19^{th} century begins the winter of the West, the ‘civilization’ phase of the Western spirit. Its 900 years of ‘culture’ have passed, and there is no creativity left in it: The popular preachers of materialism and skepticism are to the 19^{th} century what the Cynics and Epicurians were to antiquity. Socialism — a philosophy of resignation — performs the same function of ethical transvaluation as Stoicism in the Apollonian world and Buddhism in China.

Equally meaningless are the forms of political life. The most characteristic of them, parliamentarism, is nothing more than a transition device, serving to obscure with hollow rhetoric the basic political reality — the triumph of money. Before the power of financial speculation, everything else must give way: Politicians have no choice but to become the paid agents of the financiers.

With the 20^{th} century an age of wars is opening. This phase, which regularly occupies at least the first two centuries of each ‘civilization’, has actually been in progress ever since the time of Napoleon. As opposed to the old wars between national armies, they will be the battles grouped around born leaders of rare political and military talent — the new Caesars, struggling for the mastery of the world. Eventually one of the Ceasars will win out over all his rivals and establish a universal imperium.

Long before this time, life will have descended to a level of general uniformity, in which local and national differences will have virtually ceased to exist. The only places that will matter will be a handful of world-cities — the 'megalopolis', like New York or London, as opposed to the 18^{th}-century city of culture, which still retained some connection with the living tradition. These 'barrack cities' will be what Hellenistic Alexandria and imperial Rome were to the ancient world — vast assemblages of people living all on top of each other, a shiftless mob, willing to obey any leader who will keep them amused.

Their life will be a meaningless repetition of purely mechanical tasks and vulgar brute diversions. Even intellectual activity will have become mechanized, practical, cold, and merely 'clever'. The educated will have lost their feeling for language, and the same 'basic' speech will be on the lips of intellectuals and common laborers alike. Eventually, when every trace of form and style will have disappeared, a new primitivism will begin to pervade all human activity. Even the feeling for scientific norms — which will have survived the dissolution of culture — will grow vague and uncertain. Men will be ready to believe anything; they will regain their appetite for the mysterious and the supernatural. In vulgar credulity, they will find an escape from the universal drabness and mechanization. Out of the desolation of the cities there will arise a 'second religiosity', a fusion of popular cults and the memory of nearly forgotten piety.

We cannot choose our destiny, and we have no alternative but to make the best of the historical situation in which we have been placed. But within the established master-plan, there still is room for individual initiative[422]: the 'themes' are foreseen, but their 'modulations' — the precise fashion is which predetermined development will play themselves out in the actual performance of history — depend on the character and capacities of the individual players.

Hopefully, the Vico-Spengler cyclic 'law' can be modified and generalized such that we can still *buy time* to develop the proper methodology and employ it to improve our long-term chances to stem the tide of decay.

Oswald Spengler was born at Blankenburg, in the Harz, Germany. On his father's side he came from a line of mining technicians, to which he owed his

[422] In line with the famous maxim of **Rabbi Akiva** (ca 100 CE) "All is foreseen and free will is given".

mathematical and scientific talents. His mother's family provided the artistic bent. After his graduation from a classical high school in Halle, Spengler followed the customary German practice of attending two or three universities in turn: Münich, Berlin and finally Halle, majoring in mathematics and the natural sciences. Meanwhile he had upheld another tradition of making several trips to Italy, which the august example of Goethe had established as the goal of the young German's cultural pilgrimage.

This rare blend of education in mathematics, physics, natural history, history and art is the foundation of the peculiar character of Spengler's work. In it, unexpected parallels between scientific truths of physics and mathematics and the artistic and other cultural achievements of an epoch of history, are drawn.

Spengler received his doctor's degree from the University of Halle in 1904. In 1908 he received a teaching appointment in a Hamburg high school, teaching mathematics, science and history. In 1911 he gave up his teaching career and moved to Münich, making his living as a private scholar. With the outbreak of the First World War, Spengler fell into serious financial difficulties. Since he suffered from both a heart condition and acute nearsightedness, he was never called up for military service.

During the war years, Spengler lodged in a dreary slum, took his meals in cheap working-class restaurants, and wrote much of the *Decline* by candlelight. As a bachelor and city-dweller, he found it difficult to obtain even the bare necessities of food, heat and clothing. He was sustained by the conviction that a great, inchoate idea was germinating inside him and that he must fight his way through the laborious process of bringing it to expression.

In the summer of 1918, only a few months before the final defeat of the German Empire in WWI, the *Decline* began to appear in the bookstores of Germany and Austria. After a few weeks of public hesitation, the book started to sell, and it has continued to sell ever since.

The year 1919 was the "Spengler year". Everyone seems to be reading him; everyone was wondering just who he was. Within 8 years after the original publication, total sales had reached 100,000. Spengler, like Schopenhauer and Nietzsche before him, had become the philosopher of the hour. Quite understandably, the *Decline* appealed to lay people who where frantically seeking rationalizations for the despair they already felt. The scholars and specialists, however, recoiled from the notion of predetermined future, and reproached him for shallowness, incompetence and charlatanism.

As early as 1920, the general public had begun to lose interest, and by 1924 even the scholarly furor had largely subsided. The German public — driven to distraction by a galloping inflation and endemic civil war — had turned to more immediate and practical concerns. Spengler never won an academic appointment and his attempts to establish himself as a political commentator were unsuccessful. His political ideas had some affinity with those of the National Socialists (which fact they indeed exploited), but his hopes of influencing them came to nothing: after their rise to power he was denied, and later died in isolation.

In retrospect, one may conclude that Spengler's failure to establish a number of vital links in the sequence of future events (relative to 1923), reflects the inadequacy of his personal preconceptions. His faulty economics, his 'metaphysical' and unrealistic definition of social classes, drastically limit his comprehension of 20^{th} century political movements.

Yet, the basic idea is there, even when the formulation is faulty: More poignantly than any of his predecessors, Spengler has sensed the unprecedented character of our time — the resurgence of those primitive values that so sharply divide the 20^{th} century from the centuries that went before it.

Under the crude phraseology of a "colored peril", for example, Spengler expresses something of a tragic cultural misunderstanding between Asia and the West — an incompatibility far transcending the clash of political institutions and economic interests. And beyond this inter-continental struggle, he sees the terrible outlines of a whole world delivered over to conquest and virtually perpetual war. He grasps the dilemma of creative endeavor in an era of mass culture — its fatal division between a merely repetitive popular art and the esoteric experiments of the 'progressive' schools. And he understands the implications of mass culture itself.

He sees the whole 'phoneyness' of contemporary life — the depressing uniformity of great city society and its deadening effect on democratic procedures. Finally, he comprehends the emptiness and despair that are leading so many of our contemporaries — the untutored and the highly sophisticated alike — to seek solace in a return to dogmatic religion.

It is somewhere between literature and prophecy that the *Decline* has made its most telling contribution. It is a symptom, a synthesis, a symbol of a whole age that Spengler's book remains one of the major works of the 20^{th} century. Indeed, it has gained in stature as the passage of time has enabled us to place it in the context of the events of the past nine decades and the further catastrophes that many anticipate. For when everything else has been said, *The Decline of the West*, and the state of mind it expresses, set before us the spectre of a new barbarism. It formulates, more comprehensively than any other single book, the now familiar pessimism of the 21^{th} century West with regard to its own historical future.

Worldview XXXVI: Spengler

* * *

"Egyptian pyramids, Doric temples, and Gothic cathedrals are mathematics in stone".

* * *

"The sense of form of the sculptor, the painter, the composer, is essentially mathematical in its nature; The same inspired ordering of an infinite world which manifested itself in the projective geometry of the 17^{th} *century, could vivify, energize and suffuse contemporary music with the* harmony *that it developed out of sound physics, and contemporary painting with the principle of* perspective *(the felt geometry of space that only the West knows). For it was the wish, intensified to the point of longing, to fill a spatial infinity with sound which produced (in contrast to the classical lyre and reed) the two great families of keyboard instruments (organ, pianoforte etc.) and bow instruments, and that as early as the Gothic time. It was then that the organ was developed into the* space-commanding *giant that we know, an instrument the like of which does not exist in all musical history. The free organ-playing of Bach and his time was nothing if it was not analysis — analysis of a strange and vast tone-world".*

* * *

"When, about 540 BCE*, the circle of the Pythagoreans arrived at the idea that* number is the essence of all things*, a wholly new mathematics was born. It came forth from the depths of the Classical soul as a formulated theory, a mathematics born in one act at one great historical moment.*

To Euclid, the triangle is the bounding surface of a figure, never a system of 3 intersecting lines or a group of three points in 3-dimensional space. Euclidean geometry is only in agreement with the phenomenal world within the limits of the drawing board. For Euclidean parallels meet already at a line of the horizon (a simple fact upon which all our art perspective is grounded), let alone the triangle formed by an observer and two parallel lines to distant stars.

In the famous treatise on the grains of sand, Archimedes proves that the filling of Aristarchos' Cosmos with atoms of sand leads to very high, but not *infinite number. The exhaustion method of Archimedes through which he achieved the quadrature of the parabola section by means of inscribed triangles, are in sharp contrast to the idea of the Riemann integral. Nowhere else did the two mathematical ideas approach each other more closely then in this instance, and nowhere is it more evident that the gulf between the two is impassable. It was the instinct that guided Nicolaus Cusanus (ca 1450), from the idea of the* unendingness of God in nature *to the elements of the* infinitesimal calculus*".*

* * *

"The Cartesian geometry expresses the emancipation of geometry from servitude to optically realizable constructions and measurable lines. With that the analysis of the infinite became a fact. The clearest example of this is the conversion of angular functions into periodic functions*, and their passage into the realm of infinite numbers".*

* * *

"The time of the great *mathematicians is past. Our tasks today are those of preserving, rounding off, refining, selection — in place of big dynamic creation, the same clever detail which characterized the Alexandrian mathematics of the late Hellenism".*

* * *

"Only in the 19^{th} *century was the infinitesimal calculus made logically secure by Cauchy's definitive elucidation of the limit idea; the limit is no longer that which is approximated to but the approximation,* the process*, the operation itself. It is not 'an infinitely small quantity' but the 'lower limit of* every possible *finite magnitude — a relation".*

* * *

"The liberation of geometry from the visual, and of algebra from the notion of magnitude, and the union of both in the great structure of the theory of functions — this was the grand course of Western number — thought. The

constant number of the classical mathematics was dissolved into the variable. Geometry became analytical, replacing the mathematical figures by abstract spatial relations".

1918–1926 CE **Arthur Scherbius** (1878–1929, Germany). Electrical engineer and inventor of the rotary electro-mechanical enciphering machine, the *Enigma*. It played a major role in the Intelligence-war of WWII.

Scherbius was born in Frankfurt-am-Main and was the son of a small businessman. He studied electricity at the Technical College in Munich, and then went on to study at the Technical College in Hanover, finishing in March 1903. The next year, he completed a dissertation, "Proposal for the Construction of an Indirect Water Turbine Governor", and was awarded a doctorate in engineering.

Scherbius subsequently worked for several electrical firms in Germany and Switzerland. In 1918, he founded the firm of Scherbius & Ritter. He made a number of inventions, e.g. asynchronous motors, electric pillows and ceramic heating parts; his research contributions led to his name being associated with the Scherbius principle for asynchronous motors.

The Enigma machine was primarily intended to allow business to communicate confidential documents efficiently without the need for slow codebooks. There were several commercial models (1918–1923), and one of them (in somewhat modified version) was adopted by the German Navy (1926). Another modified version was adopted a few years later by the German Army. Scherbius was killed (1929) in a horse carriage accident.

The *Enigma* was essentially an electrical version of *Alberti's* cipher disc. The basic form of Scherbius' invention consists of 3 elements connected by wires: a keyboard for inputting each *plaintext* letter, a scrambling unit that encrypts each plaintext letter into a corresponding *ciphertext* letter, and a display board consisting of various lamps for indicating the ciphertext letter.

In order to encrypt a plaintext letter, the operator presses the appropriate plaintext letter on the keyboard, which sends an electric pulse through the central scrambling unit and out the other side, where it illuminates the corresponding ciphertext letter on the lampboard. Now, the scrambler disc automatically *rotates* by $\frac{1}{26}$ of a revolution each time a letter is encrypted. Thus, the cipher alphabet changes after each encryption. Consequently, the *encryption of the same letter is constantly changing.* With this rotating setup, the scrambler essentially defines 26 cipher alphabets, *implementing a polyalphabetic cipher that beats frequency analysis.*

However, as it stands, the machine suffers from one obvious weakness: typing a given letter 26 times will return the scrambler to its original position, and typing that letter again will repeat the pattern of encryption. In general, cryptographers are keen to avoid repetition because it leads to regularity and structure in the ciphertext, symptoms of a weak cipher.

Scherbius alleviated this weakness by introducing a second scrambler disc. Through this device, the pattern of encryption is not repeated until the second scrambler is back where it started, which requires 26 complete revolutions of the first scrambler, or the encryption of 26×26 letters in total. In other words, there are 676 distinct scrambler settings, which is equivalent to switching between 676 cipher alphabets.

For extra complexity, Scherbius added a *third disc*, providing $17,576$ distinct scrambler arrangements. Now, the *initial settings* of these 3 discs provide the *key*, and are dictated by a codebook, which lists the key for each day.

Once the scramblers have been set according to the codebook's daily requirement, the sender can begin encrypting. He types in the first letter of the message, sees which letter is illuminated on the lampboard, and notes it down as the first letter of the ciphertext. Then, the first scrambler having automatically stepped on by one place, the sender inputs the second letter of the message, and so on. Once he has generated the complete ciphertext, he hands it to a radio operator who transmits it to the intended receiver.

In order to decipher the message, the receiver needs to have another Enigma machine and a copy of the codebook that contains the initial scrambler settings for that day. He sets up the machine according to the book, types in the ciphertext letter by letter, and the lampboard indicates the plaintext. In other words, the sender typed in the plaintext to generate the ciphertext, and now the receiver types in the ciphertext to generate the plaintext — encipherment and decipherment are mirror processes.

It is clear that the key, and the codebook that contains it, must never be allowed to fall into enemy hands. It is quite possible that the enemy might capture an Enigma machine, but without knowing the initial settings used for encryption, they cannot easily decrypt an intercepted message. Without the codebook, the enemy cryptanalyst must resort to checking all the possible keys, which means trying all the 17,576 possible initial scrambler settings. The desperate cryptanalyst would set up the captured Enigma machine with a particular scrambler arrangement, input a short piece of ciphertext, and see if the output makes any sense. If not, he would change to a different scrambler arrangement and try again. If he can check one scrambler arrangement each minute and works night and day, it would take almost two weeks to check all the settings. This is a moderate level of security, but if the enemy set a dozen people on the task, then all the settings could be checked within a day. Scherbius therefore decided to improve the security of his invention by increasing the number of initial settings and thus the number of possible keys.

He could have increased security by adding more scramblers (each new scrambler increases the number of keys by a factor of 26), but this would have increased the size of the Enigma machine. Instead, he added two other features. First, he simply made the scramblers removable and interchangeable. So, for example, the first scrambler disc could be moved to the third position, and the third scrambler disc to the first position. The arrangement of the scramblers affects the encryption, so the exact arrangement is crucial to encipherment and decipherment. There are six different ways to arrange the three scramblers, so this feature increases the number of keys, or the number of possible initial settings, by a factor of six.

The second new feature was the insertion of a *plugboard* between the keyboard and the first scrambler. The plugboard allows the sender to insert cables which have the effect of swapping some of the letters before they enter the scrambler. For example, a cable could be used to connect the `a` and `b` sockets of the plugboard, so that when the cryptographer wants to encrypt the letter `b`, the electrical signal actually follows the path through the scramblers that previously would have been the path for the letter `a`, and vice versa. The Enigma operator had six cables, which meant that six pairs of letters could be swapped, leaving fourteen letters unplugged and unswapped. The letters swapped by the plugboard are part of the machine's setting, and so must be specified in the codebook.

Each month, Enigma operators would receive a new codebook which specified which key should be used for each day. For example, on the first day of the month, the codebook might specify the following *day key*:

(1) *Plugboard settings*: `A/L-P/R-T/D-B/W-K/F-O/Y.`
(2) *Scrambler arrangement*: 2-3-1.
(3) *Scrambler orientations*: `Q-C-W.`

Together, the scrambler arrangement and orientations are known as the scrambler settings. To implement this particular day key, the Enigma operator would set up his Enigma machine as follows:

(1) *Plugboard settings*: Swap the letter A and L by connecting them via a lead on the plugboard, and similarly swap P and R, then T and D, then B and W, then K and F, and then O and Y.

(2) *Scrambler arrangement*: Place the 2^{nd} scrambler in the 1^{st} slot of the machine, the 3^{rd} scrambler in the 2^{nd} slot, and the 1^{st} scrambler in the 3^{rd} slot.

(3) *Scrambler orientations*: Each scrambler has an alphabet engraved on its outer rim, which allows the operator to set it in a particular orientation. In this case, the operator would rotate the scrambler in slot 1 so that Q is facing upward, rotate the scrambler in slot 2 so that C is facing upward, and rotate the scrambler in slot 3 so that W is facing upward.

The following list shows each variable of the machine and the corresponding number of possibilities for each one:

Scrambler orientations. Each of the 3 scramblers can be set in one of 26 orientations. There are therefore $26 \times 26 \times 26$ settings:	$17,576$
Scrambler arrangements. The three scramblers (1, 2 and 3)can be positioned in any of the following six orders: 123, 132, 213, 231, 312, 321.	6
Plugboard. The number of ways of connecting, thereby swapping, six pairs of letters out of 26 is enormous:	$100,391,791,500$
Total. The total number of keys is the multiple of these three numbers: $17,576 \times 6 \times 100,391,791,500$	$\approx 10,000,000,000,000,000$

Scherbius believed that Enigma was impregnable. The Poles (1933) and the British (1940) proved him wrong.

1918–1929 CE **Alexander Marcus Ostrowski** (1893–1986, Germany and Switzerland). Mathematician. A leading contributor to the theory of p-adic numbers[423]. Established important theorems in the fields if algebraic

[423] *Ostrowki's Theorem*: The norms $|x|$ and $|x|_p$ $p = 2, 3, \ldots$ exhaust all nonequivalent norms on the field of rational numbers.

In 1918, Ostrowski created the novel concept of the *p-adic valuation*, defined as follows: For each integer $n \neq 0$ and fixed prime number p, the valuation $\nu_p(n)$ is the unique positive integer satisfying:

$$n = p^{\nu_p(n)} n' \qquad \text{with } p \text{ not dividing } n'.$$

geometry, and the theory of functions (quasi-analytic functions, meromorphic functions and convex functions).

Ostrowski was born in Kiev (Ukraine) to Jewish parents. Studied at the University of Marburg and Göttingen (Ph.D. 1921). Assistant to **Felix Klein** at Göttingen (1921–1923). Oxford and Cambridge Universities (1924–1926). Professor at Basel University since 1927. On the staff of the National Bureau of Standards, Washington D.C. USA, 1927–1958.

1918–1933 CE **Emmy** (**Amalie**) **Noether**[424] (1882–1935, Germany, U.S.A.). Distinguished mathematician. A founder of modern algebra. Her innovations gained her recognition as the most creative abstract algebraist of modern times.

During 1920–1927 her investigations centered on the general theory of *ideals* (special subsets of rings) for which her *residual theorem* is an important part. She put forward an axiomatic basis for a completely general theory of ideals. From 1927 on, Noether concentrated on non-commutative algebras, their linear transformations and their application to commutative number fields by means of the concept of cross-product. She also studied hypercomplex number systems and their representation. Much of her work appeared in the publications of students and colleagues. [**Emil Artin** (1898–1962); **Helmut Hasse** (1898–1979); **Richard Brauer** (1901–1977); **Werner Schmeidler** (1890–1969); **Van der Waerden** (1903–1996); **Max Deuring** (1907–1984); **Issai Schur** (1875–1941); **P.S. Alexandrov** (1896–1982); **L.S. Pontryagin** (1908–1988).]

Some of her pupils became famous algebraists in their own right and spread her ideas throughout the world. Oftentimes, a suggestion or even a casual

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The p-adic *absolute value* of a rational number x is then defined via $|x|_p = p^{-\nu_p(x)}$ with $x = p^{\nu_p(x)}\frac{a}{b}$, p not dividing a, b.

424 For further reading, see:

- Van der Waerden, B.L., *A History of Algebra From Al-Khwarizmi to Emmy Noether*, Springer-Verlag: Berlin, 1980.
- Dick, A. (Editor), *Emmy Noether*, Birkhäuser Verlag: Basel, 1970, 72 pp.
- Doughty, N.A., *Lagrangian Interaction*, Addison-Wesley, 1990, 569 pp
- Low, E.F., *Classical Field Theory*, Wiley, 1997, 427 pp.
- Itzykson C. and J-B. Zuber, *Quantum Field Theory*, McGraw-Hill, 1980, 705 pp.
- Zee, A., *Quantum Field Theory*, Princeton University Press, 2003, 518 pp.

remark revealed her great insight, and stimulated another to complete and perfect the idea.

In 1918, Noether discovered a theorem (*Noether's theorem*) in which she stated that conservation laws of certain physical systems are consequences of corresponding symmetry transformations (such as translations, rotations etc.). The theorem provides a method for constructing (under certain conditions) a complete set of integrals of motion for any system of fields for which the action integral is invariant with respect to a certain group of infinitesimal continuous transformations (*Lie group*).[425]

Emmy Noether was born in Erlangen, Germany. She was the daughter of the mathematician **Max Noether** (1844–1921). [Professor in Erlangen since 1875. One of the leaders of 19th century algebraic geometry.] She received her Ph.D. in 1907, with a dissertation on algebraic invariants. At Erlangen she published half a dozen papers, and had lectured to her father's classes from time to time when he was ill. She came to Göttingen in 1916. Being a woman, she could not obtain any academic position — in spite of the efforts made on her behalf by the greatest mathematicians of the day[426]. Finally, in 1919, she won formal admission as an academic lecturer. In 1922 she became an unofficial associate professor (with no salary and no obligations).

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425 A *conservation law* is any rule, derived from the basic laws of physics, that says that the total amount of some quantity is constant and does not change with time. A notable example is energy which can be neither created nor destroyed but only transformed from one form to another. What Emmy Noether showed was that for every symmetry of the laws of physics there is a corresponding conservation law (with some exceptions). We *now* know that the converse is also true: Every conservation law must be associated with a corresponding symmetry.

Thus Noether's theorem can be promoted to *Noether's principle*: The laws of physics must be such that every symmetry of nature corresponds to a conservation law, and vice versa.

During the last 50 years it was discovered that many, and maybe all, of the laws of physics themselves can be generated from symmetry principles. It now seems that all the interactions of physics are caused by a special kind of field called a *gauge field*, whose structure and behavior are completely dictated by a new symmetry requirement of *local symmetry*.

426 Even Hilbert failed to avert the decision of the Göttingen senate, to which he reacted: "*Meine Herren, I do not see that the sex of the candidate is an argument against her admission as a Privatdocent. After all, the Senate is not a bathhouse*". He solved the problem of keeping her at Göttingen in his own way: Lectures would be announced under his name but delivered by Fräulein Noether.

When the Nazis came to power in 1933, Noether, along with many other Jewish professors at Göttingen, was dismissed. She then left for the United States, to become visiting professor of mathematics at Bryn Mawr College and to lecture and conduct research at the Institute for Advanced Study, Princeton, NJ.

She died at Bryn Mawr, PA following an operation. In his office at the Institute for Advanced Study, Einstein wrote a letter to the editor of The New York Times, in which he reported her death only briefly:

"*In the judgment of most competent living mathematicians, Fräulein Noether was the most significant creative female mathematician*[427] *thus far produced*".

Her brother, **Fritz Noether** (1884–1941), was a professor of mathematics at the Breslau Technical Highschool and left Germany in 1933 for a research institute in Tomsk, Siberia[428]. Thus, the Noether family is a striking example of the hereditary nature of mathematical talent.

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427 **Edmund Landau**, when asked for a testimony to the effect that Emmy Noether was a great woman mathematician, said:
"I can testify that she is a great mathematician, but that she is a woman, I cannot swear."

428 Arrested (1937) in Tomsk by the NKVD and executed (shot) in the summer of 1941 on the charge of being a German spy(!). This became known only after the dissolution of the Soviet Union (1991).

Noether's Theorem — Symmetry and Conservation Laws

When the equations of motion of a physical theory are derivable from a variational principle (Hamilton's principle), a general and systematic procedure for the establishment of the theory's conservation theorems can be developed from a direct study of the integral over dynamical history ("action functional"), the variation of which is set to zero. Since the general equations of mechanics, electromagnetic theory, quantum field theories, GTR etc. are derivable from such variational principles, this procedure furnishes the most suitable basis for the systematic study of conservation laws in physics. NOETHER'S THEOREM *results from such considerations.*

Her theorem is used, for example, in classical or quantum electrodynamics to show that electric charge conservation is the result of the equations of motion of the field theory, rather then proving this directly. A simpler example is the derivation of the conservation of energy *from the invariance of a mechanical system under a* time-translation transformation $t \to t + \delta\tau$, *with* $\delta\tau$ *an infinitesimal real constant.*

Under this transformation, and assuming a closed system (no explicit *time dependence in the Lagrangian* $\mathcal{L}$*), the action integral is transformed from* $A = \int_{-\infty}^{\infty} \mathcal{L}(q_i, \dot{q}_i)\, dt$ *into*

$$A + \delta_\tau A = \int_{-\infty}^{\infty} \mathcal{L}\Big(q_i\{t + \delta\tau(t)\}, \frac{d}{dt} q_i\{t + \delta\tau(t)\}\Big)\, dt \,.$$

Noether's approach was to make the constant time shift $\delta\tau$ *into a function of* t, $\delta\tau = \delta\tau(t)$, *where* $\delta\tau$ *is still an infinitesimal —* although δA *vanishes identically only for* $\delta\tau = const.$

Redefining the integration variable to $t' = t + \tau(t)$, *and noting that by the chain rule:*

$$\frac{d}{dt} q_i(t + \delta\tau(t)) = \dot{q}_i(t + \delta\tau(t))(1 + \delta\dot{\tau}(t)),$$

we obtain (quadratic and higher order terms in $\delta\tau$ *ignored):*

$$\begin{aligned} \delta_\tau A \quad &= \int_{-\infty}^{\infty} \left(\sum_{i=1}^{n} \frac{\partial \mathcal{L}}{\partial \dot{q}_i} \dot{q}_i - \mathcal{L} \right) \delta\dot{\tau}\, dt \\ &= \int_{-\infty}^{\infty} \left(\sum_{i=1}^{n} p_i \dot{q}_i - \mathcal{L} \right) \delta\dot{\tau}\, dt \,, \end{aligned}$$

where $p_i = \frac{\partial \mathcal{L}}{\partial q_i}$ *are the generalized momenta ant* t' *has been renamed "*t*".*

Although δA *is not identically zero for any non-physical trajectory* $\{q_i(t)\}_{i=1}^n$ *if* $\delta\dot{\tau} \neq 0$, *it does vanish for any actual system trajectory, because the latter satisfying the equations of motion (Euler-Lagrange equations),*

$$\frac{\partial \mathcal{L}}{\partial q_i} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_i} \, .$$

Thus, integrating by parts the above integral expression for δA *and assuming* $\delta\tau(t) \to 0$ *as* $t \to \pm\infty$, *we conclude that* $\frac{d}{dt}\left[\sum_{i=1}^n p_i \dot{q}_i - \mathcal{L}\right] = 0$ *for any actual system trajectory; thus*

$$H = \sum_{i=1}^n p_i \dot{q}_i - \mathcal{L} = const.,$$

which is the equation for the conservation of energy in a closed system (since H *is the Hamiltonian of the system).*

Other simple symmetry transformations and their attendant conservation laws are: rigid spatial translations (conservation of linear momentum); rigid spatial rotations (conservation of angular momentum); Lorentz transformations (rectilinear motion of relativistic center-of-mass). In any of these symmetry transformations, the "rigid" (spacetime independent) transformation parameters — such as displacement vector, rotation angles, frame velocity, etc. — are made time dependent, and the Noether method is then used to prove that the corresponding quantity is conserved for actual trajectories of the physical system, by using $\delta A = 0$.

In this connection, a brief discussion on the significance of the concepts of energy, momentum and angular momentum in modern physics is called for:

In quantum mechanics *in general, and in field theory (both classical and quantum) in particular, apart from the* orbital *part of the density of angular momentum, there is also the* spin density *of angular momentum; neither of these alone need be conserved — in general the conservation applies to their sum only. A scalar field possesses no spin. Furthermore, the conservation of orbital angular momentum in a scalar field theory implies that the canonical energy – momentum stress 4–tensor is of necessity* symmetrical *(and vice versa).*

In field theories involving non-scalar fields, there are sometimes several useful ways to define a conserved energy-momentum tensor — and some of these (e.g., the one resulting from a straightforward application of Noether's theorem) are indeed non-symmetrical.

The question as to which of these definitions is the 'correct' one is moot, since an experiment designed to measure, say, the energy content of an imaginary volume element, will change the field configuration. This is true even in classical electromagnetism, let alone in quantum physics, where the uncertainty principle renders the space-time localization of energy-momentum a notoriously tricky enterprise.

*The above is true for theories where gravitation is negligible. In the presence of gravitation (*GTR*),* global *energy, momentum, and angular momentum are not even well defined in general, let alone conserved, owing to the curvature of spacetime and the local nature of the Equivalence Principle.*

The only generally-valid vestige of these conservation laws is the local *conservation of the non-gravitational energy-momentum tensor in an infinitesimal, locally inertial spacetime frame. Through general covariance, this is translated into the statement that the absolute (covariant) 4-divergence of the energy-momentum–stress tensor vanishes.*

But as for global *conservation laws, there are only four special cases where one* can *extend the conservation of energy-momentum and/or angular momentum to* GTR*. These are:*

- Weak gravity *(post-Newtonian) approximation. Here, global conservation is restored by adding to the energy-momentum tensor a term representing the contribution of the gravitational field. This term, however, is coordinate-system dependent, and hence the resulting conserved global entities are* not *covariant.*
- Asymptotically-flat spacetimes*; this case occurs when the non-gravitational field–matter distribution is localized in space — although special care must be used in treating the contribution of outgoing radiation at infinity (both gravitational and electromagnetic) to the conserved quantities. This case includes the former as a special case.*
- Motion of localized energy-momentum packets *(either classical or quantum) through an external, slowly – varying (in space and time) and classical gravitational field. In this case the motion can be analyzed by the Maccullagh-like moment-expansion of the packets (the first moments are total 4-momentum, total angular momentum, relative to center–of–mass (classical or quantum spin), moments of inertia, etc.). In this formalism, the total momentum and angular momentum of the packet are* not *conserved, since they comprise a non-closed system due to the influence of the external (background) curved spacetime; however, the rates of change of these quantities can be calculated. This case includes the former two as special cases.*

- *Spacetimes with special symmetries; when spacetime has Killing vector fields (isometries), it also possesses true, covariant, global conservation laws — one for each Killing vector field. This correspondence stems from Noether's theorem. Each such symmetry, when it exists, is an extension to* GTR *of the invariance under shifting time or space by a constant, or rigid rotations of space, or other symmetry transformations (such as a combined shift and Lorentz–transformation (boost)* for certain expanding-*universe cosmologies).*

Thus, all homogeneous, isotropic, expanding-universe *spacetimes have versions of global linear and angular momentum conservations, but few of them enjoy a version of global energy conservation – because shifts of time coordinates can never leave the physics of such spacetimes invariant. Furthermore, since cosmological isotropy and homogeneity hold only in an average sense, none of these conservation laws is as useful as it is for sub-cosmological scales of space and time.*

The global conservation of energy, momentum and angular momentum are very useful concepts for describing terrestrial experiments, the dynamics of a star, or even the calorimetry, rotation, evolution and collisions of entire galaxies and galaxy clusters or super clusters — but they are apparently invalid *for the universe as a whole, or else hold only in an average (homogenized sense).*

Finally, it is of interest to note that there is an extension of GTR *in which intrinsic spin angular momentum is regarded as being as fundamental as energy-momentum. This is known as the* Einstein-Cartan-Kibble-Sciama *theory; in it the standard Einstein field equations are* augmented *by new ones, in which the r.h.s. represents* spin density *whereas the l.h.s. involves the* torsion *(i.e., the nonsymmetrical piece of the affine connection).*

In the 2^{nd} half of the 20^{th} century, the deep Noether relation between symmetries and conservation laws became a powerful tool in developing the new field theories needed to describe the interactions and structure of subnuclear particles and fields. At the same time, it was discovered that quantum effects sometimes lead to the breaking of Noether symmetries (and associated conservation laws) that hold in the classical limit. Such anomalies *often involve deep differential–geometric and topological principles, and play important roles in the Standard Model of Particle Physics, as well as in* string theory.

Table 5.4: Timeline of Symmetry in Mathematics, Physics and Chemistry

ca 400 BCE	Description of the *5 Platonic solids*.
ca 300 BCE	The 3D geometry of *polyhedra* described by **Euclid**.
1528 CE	'De symmetria partium' by **Albrecht Dürer**, a study of symmetry in art.
1596 CE	In 'Mysterium Cosmographicum' **Johannes Kepler** suggested that the orbits of the then – known planets are defined by nested inscribed Platonic solids.
1609 CE	**Kepler** published 'Astronomia Nova' where he announces his three famous laws of planetary motion. The second law we can now understand as the conservation of angular momentum, a consequence of the $SO(3)$ symmetry of the gravitational force from the sun.
1611 CE	In 'De nive sexangula' **Kepler** studied the hexagonal symmetry of *snowflake crystals*.
1669 CE	Investigation of crystal angles published in 'De solido intra sodium naturaliter contendo' by **Nicolaus Steno**. Probably the first instance of the 'law of constancy of angles' for quartz crystals.
1687 CE	'Principia' by **Isaac Newton**, where the first law states the conservation of momentum due to the homogeneity of space (translation invariance).
1770 CE	Permutations first studied by **Joseph-Louis Lagrange** in a paper on algebraic equations.
1772 CE	**Jean-Baptiste Rome de Lisle** published 'Essai de Cristallographie'. He confirmed the observations of Steno, and later tried to order crystals into symmetry classes.
1784 CE	**Rene-Just Hauy** published 'Essai d'une theorie sur la structure des cristaux' describing experiments on the cleaving of crystals. Proposed that a crystalline solid consists of replicas of a unit cell, and the 'law of rational indices'.

Table 5.4: (Cont.)

1830 CE	**J.F.C. Hessel** derived the 32 crystal classes, starting from the law of rational indices.
1832 CE	**Evariste Galois** is the first to understand the relation between the algebraic solutions of an equation and the structure of a group of permutations associated with the equation. This work was not published until 1846.
1844 CE	**A.L. Cauchy** studied the group properties of permutations. The permutations of a fixed number of N elements is now called the symmetric group S_N.
1849 CE	**Auguste Bravais** derived the 14 space lattices in 3 dimensions.
1860 CE	**Louis Pasteur** discovered the connection between optical activity and enantiomorphic molecular structures. Chiral molecules which are mirror images of one another rotate light in opposite senses.
1872 CE	**Felix Klein** proposed the Erlangen program where geometry is classified by invariance groups.
1878 CE	**Arthur Cayley** formulated the abstract *group concept*.
1890–1891 CE	Derivation of the 230 space groups in 3 dimensions by **A.M. Schonflies**.
1893 CE	**Sophus Lie** and **Friedrich Engel** published 'Theorie der Transformationsgruppen'.
1886–1904 CE	**FitzGerald** suggested what is later called the FitzGerald-Lorentz contraction.
	Larmor, **Lorentz** and **Poincaré** introduced the transformations which make up what is now called the Lorentz group. It was shown that they leave Maxwell's equations invariant. The Lorentz group (Lorentz transformations and rotations), together with spacetime translations, is a group of spacetime symmetry transformations often called the *Poincare group*.

Table 5.4: (Cont.)

1905 CE	In his most famous paper **Einstein** adduced a set of physical assumptions, from which the Lorentz transformations follow via a few simple thought experiments. He thus created *Special Relativity* as a physical theory and an alternative to the Newtonian theory. The latter uses a different set of symmetry transformations connecting the inertial reference frames — namely the Galilean group.
1895–1910 CE	**F.G. Frobenius** and **I. Schur** created the theory of group representations.
1912 CE	Experimental evidence for the lattice structure of crystals, following the discovery of X-rays diffraction in crystals by **von Laue**.
1918 CE	**Emmy Noether** showed the general connection between symmetries and conserved quantities in Lagrangian dynamics.
1918 CE	**Hermann Weyl** introduced a classical unified field theory for gravitation and electromagnetism. It includes invariance under local scale transformations, called gauge invariance, which implies the conservation of electric charge.
1924 CE	**S.N. Bose** introduced what is now called Bose-Einstein statistics for photons. In 1925, **Einstein** generalized the results to particles or quanta we now call *bosons*. Any two same–species bosons in a many-quanta state *are indistinguishable* by any labeling except momenta, positions, polarizations and the like; i.e. such states are invariant under all permutations among bosons.
1925 CE	**Wolfgang Pauli** proposed the 'exclusion principle' — later called the 'Pauli principle' — for the allowable quantum states of multi–electron systems.
1926 CE	**Max Born**, **Werner Heisenberg** and **Pascual Jordan** introduced the quantum theory of particles with angular momentum and spin 1/2.

Table 5.4: (Cont.)

1926 CE	*Fermi-Dirac* statistics introduced for half–integer–spin particles and systems (e.g. electrons) we now call *fermions*. Their many-particle quantum state vectors (wave functions) change sign under odd permutations. Later, the list of fermions grew to include many other elementary particles (such as protons, neutrons, quarks, neutrinos, antiparticles) besides electrons, as well as some nuclei and atoms – and similarly for bosons.
1927–1928 CE	**Fritz London** and **Weyl** introduced gauge transformations (and the associated local symmetry) into quantum mechanics, with total electric charge as the conserved quantity.
1928 CE	**Dirac** proposed a relativistic wave equation for spin 1/2 particles, i.e. one covariant under the *spinor representation* of the Poincaré group.
1928 CE	**Weyl** published 'Gruppentheorie und Quantenmechanik'.
1929 CE	**Felix Bloch** described the electron wave functions in periodic potentials, allowing a quantum–mechanical understanding of crystalline symmetry groups.
1929 CE	**Hans Bethe** derived the splitting of atomic levels resulting from the crystal field symmetry.
1930 CE	**Eugene Wigner** studied the effects of the symmetry of molecular configurations on their vibrational spectrum.
1931 CE	**Wigner** introduced the discrete *time–reversal* symmetry operation symmetry (T) into quantum theory and publishes 'Gruppentheorie und ihre Anwendung auf der Quantenmechanik der Atomspektren'.
1931 CE	**L.C. Pauling** studied the theory of chemical bonding using the symmetries of orbitals.
1932 CE	**W. Heisenberg** proposed an approximate internal (non-spacetime) a symmetry between protons and neutrons (to be understood as two "orientations", in an internal space, of a single fermion species — a *nucleon*), in nuclear theory, — later called *isospin symmetry*.

Table 5.4: (Cont.)

1932 CE	**Carl Anderson** discovered the *positron* in a cosmic ray experiment, the first of the antiparticles (predicted by Dirac in 1931).
1932 CE	**B.L van der Waerden**: 'Die gruppentheoretische Methode in der Quantenmechanik'.
1935 CE	**V. Fock** derived the spectrum of the hydrogen atom from an accidental SO(4) (4-dimensional rotation) symmetry possessed by its Hamiltonian operator.
1936 CE	**Heisenberg** introduced *charge conjugation* (C) as a discrete symmetry operation connecting particle and antiparticle states.
1936 CE	**Frederick Seitz** worked out the representation theory of space groups, the symmetry groups of crystal lattices.
1937 CE	**H.A. Jahn** and **E. Teller** derived a connection between the symmetries of molecular configurations and the stability of degenerate molecular electron orbitals (*Jahn-Teller effect*): for a non-linear molecule there is always a distortion into a shape of lower symmetry to remove any orbital degeneracy of its electronic state.
1939 CE	**Wigner** studied the unitary representations of the Poincaré group. The results allow the classification of all relativistic wave equations and of the transformation properties of quantum fields.
1940 CE	**Pauli** proved the spin-statistics theorem: particles with half-integer spin have Fermi-Dirac statistics, those with integer spin are Bosons.
1954 CE	**C.N. Yang** and **Roger Mills** introduced local isospin transformations as an internal symmetry, i.e. non–spacetime Lie–group transformations of fields which depend on the spacetime point. They showed how such a *non–Abelian local (gauge) symmetry* prescribes a minimal–Coupling dynamics, in a manner similar to the role of general covariance in GTR.
1954 CE	**G.C. Wick**, **A.S. Wightman** and **E. Wigner** introduced the notion of *superselection rule.*

Table 5.4: (Cont.)

1954–1955 CE	The CPT theorem was proved by **G. Lüders** and **W. Pauli**, involving space inversion of *Parity* (P), charge conjugation (C) and time reversal (T): in a local, relativistic quantum field theory the composition product CPT of these transformations is always an unbroken symmetry.
1956–1957 CE	A parity breaking weak nuclear interaction is proposed by **C.N. Yang** and **T-D. Lee** and verified experimentally by **C.S. Wu**.
1959–1961 CE	**W. Heisenberg**, **J. Goldstone** and **Y. Nambu** suggested that the ground state (vacuum) of a relativistic quantum field theory may lack the full global (rigid) internal symmetry manifested by the Hamiltonian, and that massless excitations (*Goldstone bosons*) must accompany this 'spontaneous symmetry breaking'. In 1964 **P. Higgs** and others find that for spontaneously broken *gauge* (local) symmetries there are no Goldstone bosons but instead massive vector mesons (*Higgs mechanism*).
1961 CE	**Murray Gell-Mann** proposed the Lie group of unit–determinant, unitary, complex 3×3 matrices – as a global (rigid) internal symmetry for the strong interactions (the *Eightfold Way*). This includes the isospin symmetry as part of a larger symmetry Lie group which also acts on the so-called *strangeness* quantum number. In 1964 Gell-Mann and **G. Zweig** proposed a new sub-nucleon level of quanta – which they named *quarks* – to account for the SU(3) symmetry.
1964 CE	The discrete–symmetry composition product CP — and thus, by the CPT theorem, also time -reversal symmetry, T — is found to be violated (broken); this is observed experimentally –via certain anomalous K-meson decays – by **J.W. Cronin** and **W.L. Fitch**.
1965 CE	**R.B. Woodward** and **R. Hoffmann** described how the conservation of orbital symmetry influences the course of molecular reactions, the 'Woodward-Hoffman rules'.
1973–1974 CE	The essential features of the presently accepted *Standard Model* of particle physics are established.

Table 5.4: (Cont.)

1977 CE	**Roger Penrose** mathematically demonstrated an *aperiodic tiling* of the plane using only two different tiles and an approximate 5-fold symmetry.
1984 CE	**Dan Schechtman** found the first *quasicrystal* phase in the laboratory, produced via slow cooling (annealing). Its X-ray crystallography exhibited a 5-fold symmetry – one not expected to exist by conventional wisdom.
1985 CE	**Robert F. Curl**, **Harold W. Kroto** and **Richard E. Smalley** produced the first observed C60 molecules by laser-vaporizing graphite in a jet of helium (Nobel Prize in chemistry, 1996). This useful "Buckyball" cage structure is highly symmetrical – a truncated icosahedron with 12 perfect–pentagon and 20 perfect–hexagon faces.

1918–1935 CE **Hermann Weyl** (1885–1955, Germany and U.S.A.). Distinguished mathematician. Made major attempts to embed the theory of the electromagnetic field into the geometric framework of an extended theory of general relativity. Established the importance of gauge invariance in classical and quantum electrodynamics (1929).

Weyl was first to construct a non-Riemannian geometry in an effort to produce the required unified field theory. Extended the concept of parallel displacement of a vector for manifolds in which the line element is undefined. He rendered a full treatment of spinors[429] with reference to finite rotations.

[429] The collective designation 'spinor' has been given to these quantities because of the role they play in the theory of the spinning electron. The name 'spinor' was coined by **Paul Ehrenfest** (1880–1933, Leyden) in 1929. In the same year, **van der Waerden** (1903–1996) introduced a notation of spinors by means of dotted and undotted indices. This latter formalism is based on the fundamental representation of the pseudorotation group in 4-dimensional Minkowski space (or, more precisely, of $SL\ (2, c)$, the maximal cover of the Lorentz group $SO\ (3, 1)$) in terms of *two* sets of Pauli spin matrices and complex rotation angles.

During 1923–1938 Weyl used Clifford algebras in order to obtain two-valued matrix representation of the group of rotation in n-dimensions, thus developing Cartan's theory of spinors in n dimensions. He evolved a general theory of continuous groups, and found that many of the regularities of quantum phenomena on the atomic level can be most simply understood using group theory. Weyl made original contributions in many areas of mathematics. His findings were fundamental to later progress in the analytic theory of numbers.

Weyl was born in Elmshorn, near Hamburg. As a student at Göttingen, he came under the influence of **Hilbert**[430]. He graduated in 1908 and in 1913 became a professor of mathematics at the Technische Hochschule, Zürich, where he was a colleague of **Albert Einstein**. In 1930 he was appointed a professor of mathematics at the University of Göttingen. The Nazi dismissal of many of his colleagues prompted him to leave Germany in 1933 and accept a position at the Institute for Advanced Study, Princeton. He became a U.S. citizen in 1939. After his retirement he divided his time between Princeton and Zürich.

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[430] **H.A. Lorentz** conjectured in a Göttingen seminar (1910) that the asymptotic behavior of the eigenvalues for differential equations with constant coefficients does not depend on the shape but only on the size of the vibrating domain. His host, **Hilbert**, immediately predicted the problem to be unprovable in his lifetime. Weyl, who listened to the lecture in the audience, did not share Hilbert's pessimism. Indeed, within a short time (1912) Weyl was able to prove that, asymptotically, for large value of the frequency f and for 3D vibrating domains (e.g. a concert hall) with sufficiently smooth boundaries, but otherwise of *arbitrary shape*, the number of eigenvalues (room modes) whose frequencies are less than a given value of f is determined by $N_3(f) = \frac{4\pi}{3}V\left(\frac{f}{c}\right)^3$, where V is the domain volume and c is the wave velocity inside the domain. The corresponding formula for 2-dimensional enclosures is $N_2(f) = \pi A\left(\frac{f}{c}\right)^2$, where A is the area of the resonator. The result is asymptotically correct, to order f^2, again independent of the shape of the boundary (perimeter). These results are important in thermodynamics (for calculating specific heat of solids) and concert hall acoustics. It was later shown (1939) that Weyl's formula can be improved for *rectangular rooms* to yield $N_3(f) = \frac{4\pi}{3}V\left(\frac{f}{c}\right)^3 + \frac{\pi}{4}S\left(\frac{f}{c}\right)^2 + \frac{1}{8}L\frac{f}{c} + \frac{1}{8}$, where S is the total surface area of the room and L is the total length of the edges of the room. In the limit $V \gg \frac{3S}{16(f/c)}$, the first term predominates, independent of the room's shape.

The Gauge Principle

Introduction

Gauge theory developed, during 1917–1954, by a group of leading theoretical physicists and mathematicians, among them: ***A. Einstein*** *(1917);* ***T. Levi-Civita*** *(1917);* ***E. Noether*** *(1918);* ***H. Weyl*** *(1918, 1929);* ***Th. Kaluza*** *(1920);* ***E. Schrödinger*** *(1922),* ***E. Cartan*** *(1923).* ***O. Klein*** *(1926, 1939);* ***F. London*** *(1927);* ***V. Fock*** *(1927);* ***P. Dirac*** *(1928);* ***W. Pauli*** *(1933);* ***C.N. Yang*** *(1954);* ***R. Mills*** *(1954).*

Two interrelated concepts are involved:

- *Gauge principle: The requirement that all physical quantities, including actions and equations of motion, should be invariant or covariant under certain local internal transformations, known as gauge transformations. This property is called gauge invariance. It has evolved into a uniform organizing principle of the standard model of particle physics. The gauge principle can also be viewed as an algorithm used to build a dynamical (classical or quantum field) theory starting from the postulate of gauge-invariance.*

- *Gauge choice: A way to get rid of the ambiguity due to gauge freedom (the freedom of arbitrary local gauge transformations) by fixing the gauge [e.g. the Lorentz gauge or Coulomb gauge in EM theory].*

The gauge principle works differently in different theories. There are three basic kinds of relevant physical theories:

(a) *Macroscopic classical electrodynamics of particles or effective fields, currents and charges: i.e. Maxwell equations in free space and matter and/or plasma. These theories can be nonlinear, as in the case of MHD, nonlinear optics, Landau–Ginsburg effective field theories in superconductors, or models of ferromagnetic media.*

(b) *Classical or quantum electrodynamics of fundamental charged fields (always nonlinear).*

(c) *Non-Abelian Yang-Mills field theories (classical or quantum): there are several 4-potentials (gauge fields), with their associated field–strength* *tensors, interacting with one another and with (optional) matter fields. Always nonlinear.*

In theories of type (a), gauge invariance is just a mathematical artifice that is not essential, since $\boldsymbol{E}$ *and* $\boldsymbol{B}$ *are the observable fields. The vector potential* $\boldsymbol{A}$ *and scalar potential* Φ *need not appear explicitly in the field equations.*

In (c), even on the classical level alone and with no other non-gauge fields, the field equations can be written only by means of the covariant derivatives of (the several copies of) $\boldsymbol{E}$ *and* $\boldsymbol{B}$*, and these involve the (several) four-vector potential explicitly and non-linearly.*[431]

Electromagnetism as gauge theory

The variational formulation of electromagnetism was expounded by ***K. Schwarzschild*** *(1903),* ***A. Lorentz*** *and* ***H. Poincare*** *(1905–6), while the corresponding relativistic theory was developed by* ***H. Minkowski*** *(1908) and* ***M. Born*** *(1909). The theory of the classical electromagnetic field, that began with* ***Maxwell's*** *equations (1865–1873) and their simplification in the hands of* ***O. Heaviside*** *(1873–1888),* ***H.R. Hertz*** *(1884) and* ***J.H. Poynting*** *(1884) – was thus complete.*

We recall that in vacuum, Maxwell's equations are:

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431 Similarly, in theories of type (b), A_μ appears in the field equations for the charged fields. In both cases (b) and (c) (and also some effective field theories in category (a)), A_μ plays the role that the *affine connection* does in GTR, but the corresponding differential-geometric structure is different: a *vector fiber bundle* together with a *Lie-group* principal fiber-bundle (In the special case of GTR, these are the *tangent* and *frame* bundles of the spacetime manifold, respectively).

In a field theory of type (b) or (c), the gauge potentials are *essential* not only to make the underlying differential geometry manifest, but in order for the dynamical wave equations (of the one or several EM-like field types and of the charged fields, if any) to be at all local. And if a theory of type (b) or (c) is *quantized*, the vector potentials become even more essential for the mathematical formulation of the dynamics. Thus in the *Aharonov–Bohm effect*, an electron wave function senses (and observably responds to) the $\boldsymbol{B}-$field in a thin current solenoid, even far from the $\boldsymbol{B} \neq 0$ region; this is most easily described as the electron being acted upon directly by $\boldsymbol{A}$, which is nonzero in that region.

$$curl\ \boldsymbol{B} = \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} + \frac{1}{c^2}\boldsymbol{j} \qquad \textit{Ampere's law} \tag{1}$$

$$div\ \boldsymbol{E} = \rho \qquad \textit{Coulomb's law} \tag{2}$$

$$-curl\ \boldsymbol{E} = \frac{\partial \boldsymbol{B}}{\partial t} \qquad \textit{Faraday's law} \tag{3}$$

$$div\ \boldsymbol{B} = 0 \qquad \textit{Gauss' law} \tag{4}$$

Here ρ is the charge density, $\boldsymbol{j}$ is the current density, and no magnetic monopole charges are present. From (1) and (2) one derives the local charge conservation

$$div\ \boldsymbol{j} + \frac{\partial \rho}{\partial t} = 0. \tag{5}$$

Since the last two of Maxwell's equations make no reference to charge or current (the so-called sourceless pair of field equations), they can be identically satisfied by introducing potentials *through the definitions [**Lord Kelvin** 1851, **Maxwell**[432] 1855]*

$$\boldsymbol{B} = curl\boldsymbol{A}, \qquad \boldsymbol{E} = -\frac{\partial \boldsymbol{A}}{\partial t} - grad\ \Phi \tag{6}$$

The potentials $\boldsymbol{A}$ and Φ are not uniquely determined, since the transformations

$$\boldsymbol{A} \to \boldsymbol{A} + grad\ \lambda, \qquad \Phi \to \Phi - \frac{\partial \lambda}{\partial t}, \tag{7}$$

where λ is an arbitrary function, leave $\boldsymbol{B}$ and $\boldsymbol{E}$ unchanged.

The transformed set of potentials is as acceptable as the original one since the fields $\boldsymbol{B}$ and $\boldsymbol{E}$ are the physically measurable EM fields, and they are unaffected. This arbitrariness in the choice of the potentials is called the gauge freedom *of the theory or* gauge invariance, *while the corresponding transformations are called* gauge transformations.

[432] The vector potential in $\boldsymbol{B} = curl\ \boldsymbol{A}$ was first introduced, as a mathematical artifice, by **W. Thomson** (Lord Kelvin) in 1851. **Maxwell** borrowed the idea from Thomson in 1855 when he wrote $\boldsymbol{E} = -\frac{\partial \boldsymbol{A}}{\partial t} - \nabla\Phi$, upon integrating **Faraday's law**: $curl\ \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$.

We exploit this freedom in the following manner: substitute (6) into (1) and (2) to obtain a pair of coupled partial differential equations for the potentials $\boldsymbol{A}$ *and* Φ*:*

$$-\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\boldsymbol{A} = -\nabla\left(div\boldsymbol{A} + \frac{1}{c^2}\frac{\partial\Phi}{\partial t}\right) + \frac{1}{c^2}\,\boldsymbol{j} \tag{8}$$

$$-\nabla^2\Phi - \frac{\partial}{\partial t}\left(div\boldsymbol{A}\right) = \rho \tag{9}$$

We may now simplify (8)–(9) by utilizing the gauge freedom in defining the potentials. The two most convenient and common choices are:

I. The Coulomb gauge[433]

$$div\boldsymbol{A} = 0 \tag{10}$$

In this gauge (8)–(9) reduce to

$$\Box\boldsymbol{A} = \frac{1}{c^2}\boldsymbol{j} - \frac{1}{c^2}\frac{\partial}{\partial t}\left(grad\Phi\right), \qquad \Box \equiv -\nabla^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \tag{11}$$

$$-\nabla^2\Phi = \rho \tag{12}$$

The equation for Φ *is just the same as that in electrostatics (Poisson equation): Hence the term 'Coulomb gauge'. If we take the divergence of (11), using the Coulomb gauge condition, and also use (12), we regain the law of local charge conservation, equation (5).*

II. The Lorentz gauge

$$div\boldsymbol{A} + \frac{1}{c^2}\frac{\partial\Phi}{\partial t} = 0 \tag{13}$$

In this gauge, the source differential equations for $\boldsymbol{A}$ *and* Φ *have the simpler form*

$$\Box\boldsymbol{A} = \frac{1}{c^2}\boldsymbol{j}, \qquad \Box\Phi = \rho. \tag{14}$$

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433 This can be always chosen, because if one starts with $\boldsymbol{A_0}$ such that $div\boldsymbol{A_0} \neq 0$, the transformation $\boldsymbol{A_0} \to \boldsymbol{A_0} + grad\lambda$ leads to $\nabla^2\lambda = -div\boldsymbol{A_0}$, which can be solved for λ

Again, charge is locally conserved:

$$0 = \Box\left(div\boldsymbol{A} + \frac{1}{c^2}\frac{\partial\Phi}{\partial t}\right) = \frac{1}{c^2}\left[div\boldsymbol{j} + \frac{\partial\rho}{\partial t}\right] \tag{15}$$

Thus far we dealt with Maxwell's theory in its non-Lorentz-covariant form. In its real-0^{th}*-component Minkowski form, we compute the change in the skew-symmetric electromagnetic field-tensor* $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ *following the gauge transformation*

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \partial_\mu\lambda(x),$$

is

$$\begin{aligned} F_{\mu\nu} \to F'_{\mu\nu} &= \partial_\mu(A_\nu + \partial_\nu\lambda) - \partial_\nu(A_\mu + \partial_\mu\lambda) \\ &= \partial_\mu A_\nu - \partial_\nu A_\mu = F_{\mu\nu}; \end{aligned} \tag{16}$$

where $\partial_\mu \equiv \frac{\partial}{\partial x^\mu}$; $x^0 = ct$, $x^i = 3D$ *(spatial)* i^{th} *position component; Greek (Latin) indices range from* 0 *to* 3 *(*1 *to* 3*);*

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

is the covariant–indices Minkowski (pseudo-) metric tensor; Minkowski ("world") indices are raised and lowered via $v^\mu = \eta^{\mu\nu}v_\nu$, $v_\mu = \eta_{\mu\nu}v^\nu$ *with* $\eta_{\mu\nu} = \eta^{\mu\nu}$ *numerically;* $\Box = \partial_\mu\partial^\mu = \eta^{\mu\nu}\partial_\mu\partial_\nu$ *and the summation convention is understood; and* $(c\ \rho, \boldsymbol{j}) = j^\mu$ $(\phi/c,\ \boldsymbol{A}) = A^\mu$ *are the 4-current and 4-potential, respectively*[434].

Note that the Coulomb gauge $div\boldsymbol{A} = 0$ *where* $\boldsymbol{A}$ *is a spatial 3-vector, is* not *invariant under a general gauge transformation; for* $\boldsymbol{A} \to \boldsymbol{A} + \nabla\lambda$ *to preserve this gauge, it is required that* $\nabla^2\lambda = 0$. *This restricts* λ *to be of the form (*λ_i *functions of time alone):*

$$\lambda = \lambda_0 + \lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3$$

if the gauge transformation is to hold throughout space.

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[434] Eq. (16) confirms that the field strengths are gauge invariant. Note that $F_{0i} = \frac{1}{c}F_i$, $F_{ij} = -\epsilon_{ijk}B_k$, where ϵ_{ijk} is the 3D *Levi–Civita tensor*: $\epsilon_{ijk} = \epsilon_{jki} = -\epsilon_{jik}$, $\epsilon_{123} = 1$.

The Lorentz gauge, on the other hand, when applied to the Lorentz covariant 4-vector potential A_μ*, is invariant under the gauge transformation* $A_\mu \to A'_\mu = A_\mu + \partial_\mu \lambda$ *provided* λ *satisfies the wave equation*

$$\Box \lambda = 0, \tag{17}$$

Thus, the Lorentz gauge fixes the potentials modulo a sum of fictitious waves propagating with the speed of light.

The Lagrangian density for the EM fields in the presence of given external sources, is:

$$\mathcal{L} = -\frac{1}{4}(c^2 F_{\mu\nu} F^{\mu\nu} - j_\mu A^\mu), \tag{18}$$

This covariant form is due to Minkowski (1908) and Born (1909)

The action integral is then

$$S = \int d^4 x \mathcal{L} \tag{19}$$

Upon setting the variation of this action w.r.t. A_μ *to zero, the Euler–Lagrange equation again yield the two Maxwell equations with sources*

$$\partial_\mu F^{\mu\nu} = \frac{1}{c^2} j^\nu \tag{20}$$

Eq. (20) implies that the current is conserved $\partial_\mu j^\mu = 0$ *and this, in turn, leads to gauge invariance of the action (19) via integration by parts, provided* $\lambda(x)$ *decreases rapidly enough at spacetime infinity. Indeed, the action for the field* $A'_\mu = A_\mu + \partial_\mu \lambda$ *is equal to*

$$S(A') = S(A) + \int d^4 x j^\mu \partial_\mu \lambda \tag{21}$$

The last integral reduces, when $\partial_\mu j^\mu = 0$ *is taken into account, to an integral over an infinitely distant surface in 4-dimensional space-time*

$$\int d^4 x j_\mu \partial_\mu \lambda = -\int d\Sigma_\mu \lambda(x) j_\mu$$

Maxwell's equations have profoundly influenced most aspects of 20th *century technology. What may not have been sufficiently emphasized is that they have also, through their two invariances (symmetries), — profoundly* *shaped basic theoretical physics in that century. These invariances are the STR Lorentz invariance (invariance under a rigid, finite-dimensional Lie group of symmetry transformations) and gauge invariance (invariance under a local, infinite-dimensional Lie group of symmetry transformations).*

Gauge Theory of Charged Scalar or Spinor Fields

In 1926 **Schrödinger** *was endeavoring to generalize his wave–mechanical equation such that it becomes invariant under the Lorentz transformation, and thus suitable to govern the quantum matter-waves of a free particle that has speeds approaching that of light. Starting with the relativistic kinematical equation*

$$E = \sqrt{c^2\boldsymbol{p}^2 + m_0^2c^4} \qquad (m_0 = \textit{rest mass}) \tag{1}$$

and replacing $E \to \hbar\omega \to i\,\hbar\,\frac{\partial}{\partial t}$ *(Planck) and* $\boldsymbol{p} \to \hbar\,\boldsymbol{k} \to \frac{\hbar}{i}\,\boldsymbol{\nabla}$ *(de Broglie), one obtains a new Schrödinger equation:*

$$i\hbar\frac{\partial\Phi}{\partial t} = H\Phi = c\sqrt{m_0^2c^2 - \hbar^2\boldsymbol{\nabla}^2} \tag{2}$$

which, however, is fundamentally asymmetrical w.r.t space and time derivatives and hence not relativistic.

Schrödinger transformed (2) instead into

$$\boxed{\frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} = \nabla^2\Psi - \frac{m_0^2c^2}{\hbar^2}\Psi} \tag{3}$$

to be known later as the Klein-Gordon wave equation. *For* $m_0 = 0$ *(e.g. a photon) it reduces to Maxwell's electromagnetic wave equation (but for the complex* single-photon probability amplitude *— although the real* $\boldsymbol{E}$, $\boldsymbol{B}$ *fields satisfy the same equation).*

In order to derive a relativistic Schrödinger-like equation for an electron matter-wave interacting with an external electromagnetic field, Eq.(1) is replaced by

$$(E - e\Phi)^2 = c^2\left(\boldsymbol{p} - e\,\boldsymbol{A}\right)^2 + m_0^2c^4, \tag{4}$$

where $\boldsymbol{A}$ *is the vector potential and* Φ *the scalar potential.*[435]

Putting again $\boldsymbol{P} \to \frac{\hbar}{i}\nabla, \quad E \to i\hbar\frac{\partial}{\partial t}$ *we obtain:*

$$\boxed{\left(\partial_\mu + \frac{ie}{\hbar}A_\mu\right)\left(\partial^\mu + \frac{ie}{\hbar}A^\mu\right)\Psi + \frac{m_0^2c^2}{\hbar^2}\Psi = 0} \tag{5}$$

where, as seen above $\partial_\mu\partial^\mu = \Box$

If we specify the Lorentz gauge condition $\partial_\mu\, A^\mu = 0$*, equation (5) is equivalent to*

$$\left(\Box + \frac{m_0\, c}{\hbar}\right)^2 \Psi = \frac{e^2}{\hbar^2\, c^2}\left(-c^2\, \boldsymbol{A}^2 + \Phi^2\right)\Psi - \frac{2ie}{\hbar}\left[\boldsymbol{A}\cdot\nabla\Psi + \frac{1}{c^2}\Phi\frac{\partial\Psi}{\partial t}\right], \tag{6}$$

which was Schrödinger's result for the interacting relativistic spinless Klein-Gordon (1926). However, at that time it was immediately rejected – along with its free ($A_\mu = 0$) *version – for two reasons: first, it possesses negative energy solutions (since it admits negative frequencies). Second, it leads to negative probability densities. Note that (5) is an inhomogeneous version of the free Klein–Gordon equation in which the four-indexed partial derivative operator* ∂_μ *is replaced by a gauge–covariant (and Lorentz covariant*[436] *spacetime derivative operator)*

$$D_\mu = \partial_\mu + \frac{ie}{\hbar}A_\mu \tag{7}$$

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[435] This is justified at the classical (non-quantum) level by the fact that the relativistic Lagrangian of a charged point-particle in an external EM field

$$-m_0c^2\sqrt{1-\frac{\boldsymbol{v}^2}{c^2}} - e\Phi\left(\boldsymbol{r}(t),t\right) + e\boldsymbol{v}(t)\cdot\boldsymbol{A}\left(\boldsymbol{r}(t),t\right)\ ,$$

leads to the Hamiltonian

$$H = e\Phi + c\sqrt{\left(\boldsymbol{p} - e\boldsymbol{A}\right)^2 + m_0^2c^2}\ ,$$

where $\boldsymbol{p} - e\boldsymbol{A} = \frac{m_0\boldsymbol{v}}{\sqrt{1-\frac{v^2}{c^2}}}$ is the particle's *physical* 3-momentum and $\boldsymbol{p}$ is its *canonical* momentum (the one obeying the uncertainty principle).

[436] The correct *quantum* gauge transformation – under which both (5) and the Dirac Eq. (13) (below) are *covariant*, is $A_\mu \to A_\mu + \partial_\mu\lambda(x)$, $\Psi \to \Psi\cdot exp\{-\frac{ie}{\hbar}\lambda(x)\}$, the latter being a local phase change. In retrospect, gauge theories should have actually been named *phase* theories!

The foibles of the Klein–Gordon equation seemed at the time to make it a poor candidate for a matter–wave equation for the electron[437]*, but those imagined weaknesses helped to point Dirac in the right direction and to develop the correct relativistic field equation for the electron.*

The relativistic first-order Dirac equation (1928), which governs the electromagnetic interaction of the electron (or any other spin – 1/2 charged fermion) field, is

$$i\hbar\frac{\partial\Psi}{\partial t} = \left[c\,\boldsymbol{\alpha}\cdot\left(\frac{\hbar}{i}\boldsymbol{\nabla} - e\,\boldsymbol{A}\right) + \beta m_0 c^2 + e\,\Phi I\right]\Psi. \tag{8}$$

Here Ψ *is a 4-wave-component electron field* $\Psi = \begin{bmatrix}\Psi_1\\ \Psi_2\\ \Psi_3\\ \Psi_4\end{bmatrix}$,

(with spin, not spacetime, indices!); $e,\ m_0$ *are respectively the charge and rest-mass of the electron,* c *is the speed of light in vacuum, and* $\{\boldsymbol{\alpha},\beta\}$ *are the* 4×4 *complex hermitian matrices*

$$\alpha_1 = \begin{bmatrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{bmatrix},\ \alpha_2 = \begin{bmatrix}0&0&0&-i\\0&0&i&0\\0&-i&0&0\\i&0&0&0\end{bmatrix},\ \alpha_3 = \begin{bmatrix}0&0&1&0\\0&0&0&-1\\1&0&0&0\\0&-1&0&0\end{bmatrix} \tag{9}$$

$$\beta = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{bmatrix} \qquad I = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} \tag{10}$$

where $i = \sqrt{-1}$. *Introducing the further notation (*$\mu,\ k$ *are Minkowski and 3D Cartesian indices respectively)*

$$\gamma^\mu = (\gamma^0, \boldsymbol{\gamma}) \quad , \quad \gamma^0 = \beta \quad , \tag{11}$$

437 It was later realized that the correct interpretation for Ψ in (5) was the operator-valued quantum field of the (electron/positron or other) charged (or neutral if $e = 0$) particle–antiparticle pair – and, *not* a probability amplitude. Thus, today *both* the Klein-Gordon (KG) *and* the Dirac equations are used as operatorial field equations; both hold (with spin–dependent corrections in the KG) for the *electron*, while only the KG equation holds for *boson* fields.

we denote $m = \frac{m_0 c}{\hbar}$ *(the inverse reduced Compton wavelength of the spinor field), and recast (8) in the compact form*

$$(i\gamma^\mu D_\mu - mI)\,\Psi(x) = 0 \tag{12}$$

with the 4-vector of gauge–covariant spacetime derivatives, D_μ*, again given by (7); the* γ_μ *matrices form a Clifford Algebra*

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu} \tag{13}$$

Equation (12) – the Dirac equation for charged spinor field – is gauge covariant (as in (5)). It also looks *Lorentz covariant – as does (13). And they actually* are*, since a spinor version of the* $SO(3,1)$ *Lorentz–transformation Lie group exists.*

Thus, under a Lorentz transformation $x^\mu \to x'^\mu = \Lambda^\mu_\nu x^\nu$*, the electron (or any spinor) field transforms as* $\Psi(x) \to \Psi'(x') = U\Psi(x)$*, where the* 4×4 *unitary matrix* U *obeys* $U^{-1}\gamma^\mu U = \Lambda^\mu_\nu \gamma^\nu$*.*

Therefore, γ^μ *(the Dirac Matrices) can indeed be considered a Lorentz 4-vector.*

Upon left-multiplying both sides of (13) by the operator $(-i\gamma^\mu D_\mu - mI)$ *and taking into account the anti-commutation relations of the* γ*-matrices, we obtain*

$$\left(D_\mu D^\mu + m^2 I + \frac{e}{\hbar}\sigma_{\mu\nu}F^{\mu\nu}\right)\Psi(x) = 0 \tag{14}$$

where

$$\sigma_{\mu\nu} = \frac{i}{4}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu); \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{15}$$

Eq. (15) *is just the electromagnetic Klein-Gordon equation Eq.(5) obtained by Schrödinger,* except *that it includes the missing* spin magnetic moment *term that is the result of the interaction of the electron (or other charged spinor particle) intrinsic spin with the external magnetic field. (Note that the relativistic Klein-Gordon does include the interaction of the electromagnetic field with the* orbital *angular momentum of the charged particle represented by the quantum field* Ψ*).*

In this connection, it is worth emphasizing that the appearance of the correct gyromagnetic ratio in the spin magnetic moment for the electron or positron in Dirac's theory – namely twice the classical value – is not only *due to the relativistic nature of the theory, but also due to the fact that the* *gauge principle is applied to obtain the minimal coupling*[438] *of a spin one-half charged field to an external EM field.*

Mathematically, the reason a 4D Dirac field has four components is that the all–important Clifford algebra [Eg. (13)] cannot be realized with smaller matrices[439].

Physically, these $4 = 2 \times 2$ *components represent two spin states for the electron, plus two for the opposite–charge antiparticle (positron).*

It can be shown that the correct action for the minimally-coupled electrodynamics of our charged Dirac field is the following: (from here on we use a unit system in which $\hbar = c = 1$, *so* $m = m_0$*).*

$$S = -\frac{1}{4}\int F_{\mu\nu}(x)F^{\mu\nu}(x)d^4x + \int \Psi^*(i\gamma_\mu D^\mu - mI)\Psi d^4x$$

where $d^4x = d^3\boldsymbol{x}\,dt$ *and* $\Psi^*(x)$ *is the hermitian conjugate of the field* $\Psi(x)$ *if* Ψ *is interpreted as an operator in Hilbert space, or the complex conjugate if* Ψ *is interpreted as a wavefunction; either way* Ψ *has a Dirac 4-value index, so e.g.*[440]

$$\Psi^*\gamma_\mu\Psi = \sum_{a=1}^{4}\sum_{b=1}^{4}\Psi_a^*(\gamma_\mu)_{ab}\Psi_b\ .$$

If Ψ *is instead a bosonic spin-0 charged field — e.g. the field of a charged pion, or the wavefunction of a Cooper pair of bound conduction electrons in a superconductor, etc. — the fundamental charged-field equation of motion is the gauge-covariant KG equation (15) (without the spin term); then the minimally coupled "Maxwell-Klein-Gordon" action becomes:*

$$S = -\frac{1}{4}\int F_{\mu\nu}(x)F^{\mu\nu}(x)d^4x + \frac{1}{2}\int \left[(D_\mu\Psi)^*(D^\mu\Psi) - m^2\Psi^*\Psi\right]d^4x \quad (16)$$

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438 In both the Dirac and KG equations, minimal coupling to the EM field consist in simply replacing any spacetime derivative with its gauge covariant version. This minimal coupling to EM in exactly the same sense that one converts any STR–compatible theory to a GTR–compatible theory, by replacing spacetime derivative with generally covariant derivative.

439 The equality of this number to spacetime dimensionality is accidental; in general the Dirac field has $2^{[D/2]}$ components in D-dimensional spacetime, with $\lfloor x \rfloor \equiv$ *maximal integer* $\leq x$. This is relevant in higher–dimensional quantum theories – such as Kaluza–Klein, supergravity or superstring theories.

440 From here on, *Latin* indices will sometimes be used to represent non-spatial degrees of freedom. *Greek* indices will still represent Minkowski spacetime components.

The Euler-Lagrange equations corresponding to S are Maxwell's equations (1)–(4) (but in units $\epsilon_0 = 1$), and the spinless minimally coupled KG equation,

$$(D_\mu D^\mu - m^2)\Psi = 0 \ .$$

In (16):

$$\begin{aligned} D_\mu \Psi &= (\partial_\mu + ieA_\mu)\Psi \\ D_\mu \Psi^* &= (D_\mu \Psi)^* = (\partial_\mu - ieA_\mu)\Psi^* \\ (*) &= \text{ complex or hermitian conjugate} \end{aligned} \tag{17}$$

The accompanying Maxwell's equations are

$$\partial^\nu F_{\nu\mu} = j_\mu(x) \equiv \frac{ie}{2}\left[-\Psi(x)D_\mu\Psi^*(x) + \Psi^*(x)D_\mu\Psi(x)\right] \tag{18}$$

Eq. (17) are known as the gauge covariant derivatives of $\Psi(x)$. Note that in contradistinction to the classical Maxwell's equations, the four-current $j_\mu(x)$ depends explicitly on the gauge-potential A_μ as well as on the field $\Psi(x)$. Because of Noether's theorem, the invariance of the action (18) under the global symmetry operation represented by the transformation

$$\Psi(x) \to e^{i\epsilon}\Psi(x) \ , \tag{19}$$

where ϵ is an infinitesimal real constant parameter, implies the local conservation of the electric 4-current: $\partial_\mu j^\mu = 0$. This conservation (charge continuity equation) is also guaranteed from Eq. (18) by making use of the KG wave equation $(D_\mu D^\mu + m^2)\Psi = 0$. Note that the 4-current $j_\mu(x)$ is real.

Furthermore, the action (for either the Dirac or KG cases) is also invariant under the following simultaneous local transformations

$$\begin{aligned} \Psi(x) &\to e^{-i\Lambda(x)}\Psi(x), \\ A_\mu &\to A_\mu + \tfrac{1}{e}\partial_\mu\Lambda(x), \end{aligned} \tag{20}$$

where $\Lambda(x)$ is an arbitrary real function of space and time. The local gauge symmetry transformation (20) – already introduced as a symmetry under which both (5) and (12) are covariant – is known as an Abelian gauge transformation.

Mathematically, it is the analogue, in a general Fiber Bundle, to general coordinate transformation in a Riemannian manifold and its associated *frame bundle. It is readily verified that the gauge covariant derivative defined above is indeed covariant under the gauge transformation (20). In other words $D_\mu\Psi(x)$ transforms under the gauge transformation exactly as does $\Psi(x)$ itself. The covariant gauge derivatives are clearly analogous to the covariant derivatives of Riemannian geometry (and thus of general relativity), and the four-vector potential field $A_\mu(x)$ (also known as a 'gauge field') is thus analogous to the Christoffel symbols. Also, the field-strength tensor $F_{\mu\nu}$ plays the same role played by the Riemann curvature tensor in the tangent (or frame) bundles of Riemannian geometry.*

So far we have considered Lagrangians densities (in the action integral) which are quadratic in the charged fields Ψ; also, the purely EM part (first term) of the action (17) (or its Dirac counterpart) is quadratic in $\{A_\mu\}$ (although the minimally-coupled KG term in (17) is quartic in the full set of fields $\{A_\mu, \Psi, \Psi^*\}$). The Euler–Lagrange equation stemming from S thus lead to linear equations for the charged field Ψ for fixed external EM fields, for both the Dirac and KG cases.

However, when the Lagrangian contains terms of order higher then quadratic in the Ψ fields there arise field interactions with corresponding nonlinear terms in $\{\Psi, \Psi^*\}$ appearing in the KG or Dirac wave equation.

Such a theory of interacting fields will be invariant under translations (shifts) in space and time and under Lorentz transformation, if the Lagrangian – density term for the interaction is a Lorentz scalar which does not depend explicitly on space-time coordinates. The simplest example arises in the theory of a real scalar field, if for the Lagrangian of the interaction one chooses some non-quadratic function of the field, $V_I(\Psi)$, such that the action is

$$S = \int d^4x \left[\frac{1}{2}(\partial_\mu\Psi)^2 - V(\Psi)\right], \tag{21}$$

where

$$V(\Psi) = \frac{m^2}{2}\Psi^2 + V_I(\Psi).$$

V_I should contain terms of type Ψ^3,Ψ^4, etc.

In quantum field theory, compelling technical considerations (renormalizability) favor choosing $V_I(\Psi)$ to be a polynomial in Ψ of degree at most four in four-dimensional space-time (in two-dimensional space time there are essentially no restrictions on the form of $V_I(\Psi)$). We shall see in the next section that in generalizations of the "scalar electrodynamics" theory (18) obeying a non-Abelian local gauge principle, the action and wave equations of motion become inherently more nonlinear than in the Abelian (EM) case.

To lay the foundation for the non-Abelian case, we return to Eqs. (22) *and recast them in the form*[441]

$$\begin{gathered}\Psi(x) \to g^{-1}(x)\Psi(x) \\ eA_\mu(x) \to g^{-1}(x)eA_\mu(x)g(x) - ig^{-1}(x)\partial_\mu g(x)\end{gathered} \tag{22}$$

where $g(x) = e^{i\Lambda(x)}$.

One advantage of this notation is that $g(x)$ *at any point x can be interpreted as an element of an Abelian group* $U(1)$*, which is the multiplicative group of complex numbers of unit modulus. Then, the non-derivative parts of transformation* (24) *looks like a transformation of the fundamental representation of this group, whereas* $g^{-1}(x)\partial_\mu g(x)$ *at any point is an element of the Lie algebra (tangent space) of this group. Generalizations of the transformation* (23) *to cases of other (non-Abelian) Lie groups leads to non-Abelian gauge fields, and their associates transformation and dynamics, as we explore in the next section.*

We stress again that in the theory of interacting fields, the Lagrangian density contain both terms quadratic in the fields and terms of degree three, four and higher in the fields. The quadratic terms lead to equations linear in the fields, while higher-order terms lead to nonlinear terms. It is usually impossible to find general solutions of nonlinear field equations (integrable models form an exception).

In quantum field theories, excitations corresponding to elementary particles are sometimes small. In that case the nonlinear terms in the field equations are small in comparison with the linear terms and may be treated perturbatively.

Non-Abelian Gauge Theory – Yang-Mills Fields (1954)

Let us first restrict our attention to $SU(2)$, *the Lie group of unitary* (2×2) *complex matrices of unit determinant. By the well known Cayley-Klein*

[441] For the remainder of this and the next section, we restrict our attention (in the inherent of simplicity) to the KG (scalar matter field) case although the minimal coupling of Dirac matter fields to non-Abelian gauge fields is just as straightforward – and is, in fact, of more fundamental importance in particle physics.

parameters representation any such matrix can be thought of a representing a real, 3×3 orthogonal rotation matrix. The general Cayley-Klein $SU(2)$ matrix can be written as

$$G = e^{i \sum_{a=1}^{3} b_a \tau_a / 2} \tag{1}$$

where: b_a $(a = 1, 2, 3)$ are three real numbers related to the Euler angles of the corresponding 3D rotation matrices, and τ_a are the three 2×2 Pauli matrices:

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{2}$$

obeying

$$\tau_a \tau_b = \delta_{ab} I + i\epsilon_{abc}\tau_c \tag{3}$$

where $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, δ_{ab} is the Kroneker delta and ϵ_{abc} is the Levi-Civita symbol, and the summation convention is henceforth understood over both Lorentz and internal ("isospin") indices — denoted by Greek and Latin indices ($\mu, \nu, \gamma, \ldots$ and $a, b, c, \ldots$), respectively.

Defining the hermitian matrix field

$$\Psi(x) = \Psi_a(x) \frac{\tau_a}{2} \tag{4}$$

with[442] $\Psi_a = \Psi_a^*$,*the Non-Abelian Gauge Transformation* law, *becomes*

$$\Psi(x) \to G^{-1}\Psi(x)G \tag{5}$$

[442] Ψ_a can either be *real* classical fields, or *hermitian* quantum field–operators. In the latter case(*) denotes *hermitian* conjugation, rather then *complex* conjugation. We emphasize that neither the 3D rotations nor the spin-like Pauli matrices referred to in this example, have anything to do with spatial dimensions $\mu = 1, 2, 3$ of Minkowski space, nor with angular–momentum spin. Rather, they pertain to continuous rotation-like transformations in an internal, mathematical space which is the so-called *fiber* of the Fiber–Bundle referred to above, at each spacetime point x. Minkowski space itself, M_4 is referred to as the *base manifold* of the Fiber Bundle. The internal fiber-space *could* arise from extra curled-up of a "primordial" Minkowski space M_N with $N > 4$ dimensions – as in Kaluza–Klein theories – but this need not be the case.

$$A_\mu \to G^{-1}A_\mu(x)G - \frac{i}{e}G^{-1}(\partial_\mu G) \tag{6}$$

where $G = G(x)$ *is a general spacetime-dependent element of* $SU(2)$,

$$A_\mu(x) \equiv A_\mu^a(x)\frac{\tau_a}{2} \tag{7}$$

are hermitian matrix-fields, $(A_\mu^a)^* = A_\mu^a$ *encode three copies of EM-like 4-vector potentials, and the non-Abelian gauge covariant spacetime derivative operators are defined as follows:*

$$D_\mu\Psi \equiv \partial_\mu\Psi + ie[A_\mu\Psi - \Psi A_\mu] \tag{8}$$

By (3), (4), (7) *and* (8)

$$D_\mu\Psi_a = \partial_\mu\Psi_a - e\epsilon_{abc}A_\mu^b\Psi_c \tag{9}$$

One readily verifies that the just-defined non-Abelian gauge covariant derivative via (8) – (9), *are indeed gauge–covariant, in the sense that* $D_\mu\Psi(x)$ *transforms just as* $\Psi(x)$ *does under the gauge transformations*[443] (5)–(6).

Recall that, the Abelian gauge-invariant action for the complex charged scalar matter field in an external Abelian ($U(1)$) *gauge field is given by the second term on the r.h.s. of Eq.* (18) *of the previous section, namely the second (Klein–Gordon) term in*

$$\begin{aligned} S_{total} &= S_{gauge\ field} + S_{matter} \qquad (10)\\ &= -\frac{1}{4}\int F_{\mu\nu}(x)F^{\mu\nu}(x)d^4x + \frac{1}{2}\int \left[(D_\mu\Psi)^*(D^\mu\Psi) - m^2\Psi^*\Psi\right]d^4x. \end{aligned}$$

This can be generalized to the non-Abelian case (for instance, to the $SU(2)$ *gauge group of the present example) by replacing the real fields* $Re\,\Phi, Im\,\Phi$ *by the three real fields* Ψ_a $(a = 1, 2, 3)$, *and then updating the covariant derivatives* $D_\mu\Psi_a$ *to their non-Abelian* $SU(2)$ *versions (Eq. (9)):*

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[443] It is also straightforward to verify that equations (4) through (7) are equivalent to the non-Abelian transformations law (23) of the previous section, provided A_μ and Ψ of that latter equation are now interpreted as the matrix $iA_\mu^b\epsilon_{abc}$ and the vector $\boldsymbol{\Psi_a}$, respectively, and we also identify the 3×3 matrix g^{-1} as

$$(g^{-1})ab = \frac{1}{2}tr(G\tau_a G^{-1}\tau_b).$$

$$S_{\text{matter}} = \frac{1}{2}\int d^4x\left[(D_\mu\Psi_a)(D^\mu\Psi_a) - m^2\Psi_a^2\right]. \tag{11}$$

The matter action in (11) is invariant under the general local gauge transformation (5)–(6)*, and is the $SU(2)$ non-Abelian version of the minimally coupled charged-field KG action term in (18) of the previous section. To complete the action formulation – i.e. to also extend the pure-gauge action term in (10) to the non-Abelian case – a non-Abelian version of the electromagnetic field strength tensor $F_{\mu\nu}$ is needed, which will be gauge covariant.*

It can readily be shown that the $SU(2)$ matrix $F_{\mu\nu}$ of spacetime skew-symmetric tensors

$$F_{\mu\nu} \equiv F^a_{\mu\nu}\frac{\tau_a}{2} = \partial_\mu A_\nu - \partial_\nu A_\mu + ie[A_\mu A_\nu - A_\nu A_\mu] \tag{12}$$

transforms in the same way as does $\Psi(x)$ under the local gauge transformations (6).

Note that (12) *consists of 3 real field tensors, $F^a_{\mu\nu}(x)$. Each of those is antisymmetric in its two spacetime indices $\mu\nu$, as in the $U(1)$ or Abelian (electromagnetic) case. However, these non-Abelian field-strength tensors are non-linear in the gauge potential fields.*[444]

We can also write

$$F^a_{\mu\nu} = (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) - e\epsilon_{abc}A^b_\mu A^c_\nu \tag{13}$$

The simplest possible ("minimal") non-Abelian generalization of the EM action is thus the pure Yang-Mills action:

$$S_{YM} = S_{gauge\ fields} = -\frac{1}{4}\int F^a_{\mu\nu}F^{a\mu\nu}d^4x \tag{14}$$

It can be proven that (14) is the unique action for the $SU(2)$ non-Abelian gauge fields which satisfies the following three conditions:

- *Invariant under the local gauge transformation* (6).

[444] From the differential-geometric point of view, the matrix-field skew-symmetric field strength tensor $F_{\mu\nu}(x)$ is interpreted as the Cartan curvature 2-form of the principal fiber bundle, with Minkowski space as its *base manifold*, and the Lie group $SU(2)$ (homeomorphic to the 3-sphere) as the fiber erected at each point x of Minkowski space (=spacetime).

- *Yields field equations which are 2^d order PDE and are linear in the highest-order spacetime derivatives.*
- *Reduces to 3 uncoupled non-interacting copies of Maxwell's EM theory in either the limit of weak gauge fields $(A_\mu^a \to 0)$ or the $e \to 0$ limit.*

We may now combine (14) with (11) *to yield the generalization of the total (gauge-fields plus charged no space matter) action (10) to $SU(2)$ non-Abelian YM gauge fields* minimally *coupled to a scalar charged field in the adjoint (vector) representation of the Lie group $SU(2)$:*

$$S = -\frac{1}{4}\int F^a_{\mu\nu}(x)F^{a\mu\nu}(x)d^4x + \frac{1}{2}\int\left[(D_\mu\Psi_a)(D^\mu\Psi_a) - m^2\Psi_a\Psi_a\right]d^4x \quad (15)$$

Because the field strength tensors $F^a_{\mu\nu}$ are nonlinear in the gauge potentials, even the pure Yang-Mills action term S_{YM} yields nonlinear field equations: The non-Abelian generalizations of Maxwell equations are nonlinear even in the absence of matter (non-gauge-fields) sources.

This establishes that the gauge fields act as their own source[445]*. In fact, the nonlinearity of the pure YM theory (means that the matter–independent currents in the three copies of Maxwell's equations are quadratic in the gauge fields).*

By infinitesimally varying both the gauge fields and the scalar fields in Eq. (15) and requiring that the action S be stationary under any such variation, the field equations for both types of fields are obtained as the Euler-Lagrange equations of the action S:

$$D_\mu F^{a\mu\nu} = e\epsilon_{abc}\Psi_b(x)D^\nu\Psi_c(x) \quad (16a)$$

$$(D_\mu D^\mu + m^2)\Psi_a = 0 \quad (16b)$$

Eq. $(16a)$*, the field equations for the gauge fields, is the $SU(2)$ non-Abelian version of* (20) *(previous section), while* $(16b)$*, the field equation for the scalar*

[445] Even in the Abelian theory, the conserved four-current $j_\mu(x)$ (Eq. (20), previous section) – which *generates* the gauge field strength through Maxwell's equations — itself *depends* on these same gauge fields – through the covariant derivative. However, in the Abelian case, the four-current vanishes when the charged scalar field does; whilst in the non-Abelian case, the gauge field strength can be entirely self-generated.

field, is the non-Abelian version of the minimally-coupled charged field KG wave equation.

Eq. (16b) *is also the free Klein-Gordon field equation in which partial derivatives have been replaced by non-Abelian gauge–covariant derivatives; the scalar charged field is thus said to be minimally coupled to the non-Abelian gauge fields in the action* (15).

It should be noted that although our derivation of the non-Abelian gauge (YM) theory and its underlying non-Abelian gauge principle (local gauge symmetry and minimal coupling) was developed for the particular gauge group $SU(2)$, *the main results – those expressed in matrix form – hold without modification for any gauge group, whether Abelian or not.*

These main results are as follows: the gauge transformation laws for matter and gauge fields, Eqs. (5), (6)*; the covariant derivatives for matter fields,* (8)*; the field-strength (a.k.a. fiber-bundle curvature) tensor in terms of the gauge potential fields,* (12)*; and the field equations* (16a)–(16b), *provided they are written in the following matrix form:*

$$D_\mu F^{a\mu\nu} = -ie[\Psi, D^\nu \Psi], \tag{17a}$$

$$(D_\mu D^\mu + m^2)\Psi = 0. \tag{17b}$$

1918–1936 CE **Leonid Isaakovich Mandelstam** (1879–1944; Russia). Physicist. Established the Soviet school of *nonlinear dynamics*. Did significant research in optics, radiophysics and quantum mechanics.

Discovered *Raman scattering* in crystals independently of Raman and Krishnan's work on liquids. Contributed to electrotechnology, including the theory of oscillators with time-varying parameters.

Mandelstam was born in Mogilev (now Belarus) to Jewish parents. Expelled from Odessa University after student riots in 1899, he continued his undergraduate and postgraduate studies in Strasbourg, returning to Russia in 1914. A variety of scientific and academic posts followed, culminating in the chair of Theoretical Physics at Moscow State University in 1925 and full membership of the Soviet Academy of Sciences in 1929.

Due to his Jewish ethnic origin and his close academic links to Germany, he was politically attacked during the Stalinist purges (1937–1938).

1918–1964 CE **Le Corbusier** [ps. of **Charles-Edouard Jeaneret-Gris** (1887–1965, Switzerland and France)]. Architect and city planner. Often considered the most important architect of the 20th century. Used certain characteristics of reinforced concrete construction to enclose and use space in new ways.

Le Corbusier was born in Switzerland and trained initially as an engraver and goldsmith. He was largely self-taught during travels in Greece, Italy, France, Germany. Settled in Paris (1917) and developed *purism* (1918) based on architectural equilibrium and functional simplicity, in which he expounded his revolutionary concept of mass-produced housing based on a ferroconcrete modular skeleton. In his *Vers une architecture* (1923) he developed a personal version of *International style*, utilizing pillar supports, roof terraces, unornamental facades, strip windows. It contained the idea of high-rise residential complexes surrounded by green spaces. This he implemented in numerous private residences in Europe, North Africa and South America.

In the 1920's Le Corbusier published the first of his town-planning projects for futuristic cities focused upon a central complex of identical skyscrapers. His new approach to a structure's form and use was part of what he called the *New Spirit* that man set free for his own full development (1920–1925).

During the 1930's and 1940's, Le Corbusier built few buildings and centered his interests on city planning. He proposed the demolition of urban areas and their complete rebuilding according to his ideas. His major achievement was his plan and design for the new city of Chandigrah, India in the 1950's.

After WWII, he moved away from functionalism to explore possibilities of an irrational, expressionistic sculptural style. In his final buildings (1952–1964), Le Corbusier continued to demonstrate his understanding for architectural forms interacting with a variety of functional and social conditions.

1919 CE **Karl Ereky** (Hungary). Agricultural engineer. Coined the word *Biotechnology*. He defined it as

- "*All lines of work by which products are produced from raw materials with the aid of living things.*"

Ereky envisioned a *biochemical age* similar to the stone and iron ages.

Newer and more comprehensive definitions are:

- “Any technique that uses living organisms to make or modify products, to improve plants or animals, or to develop microorganisms for specific purposes” (Office of Technology Assessment of the U.S. Congress)
- “The integration of natural sciences and engineering sciences in order to achieve the application of organisms, cells, parts thereof and molecular analogues for products and services” (European Federation of Biotechnologists)
- “A collection of scientific techniques that use living cells and their molecules to make products or solve problem intended to modify human health and the human environment”.
- “Systematic industrial use of biological processes to manufacture medical, agricultural and consumer products”.

Biotechnology Chronicles, I

I: Early Applications and Speculations
6000 BCE–1700 CE

Biotechnology is technology based on biology, especially when used in agriculture, food science and medicine. It includes the directed use of organisms for the manufacture of organic products[446].

Biotechnology is not new — humans have been manipulating living things to solve problems and improve their way of life for millennia.

The origins of biotechnology date back nearly 10,000 years ago to early agrarian societies in which people collected seeds of plants with the most desirable traits for planting the next year. There is evidence that Babylonians, Egyptians and Romans used these same selective breeding practices for improving livestock.

Certain practices that we would now classify as applications of biotechnology have been in use since man's earliest days. Nearly 6000 years ago, our ancestors were producing wine, beer, and bread by using fermentation, a natural process in which the biological activity of one-celled organisms plays a critical role.

In fermentation, microorganisms such as bacteria, yeasts, and molds are mixed with ingredients that provide them with food. As they digest this food, the organisms produce two critical by-products, carbon dioxide gas and alcohol.

In beer making, yeast cells break down starch and sugar (present in cereal grains) to form alcohol; the froth, or head, of the beer results from the carbon dioxide gas that the cells produce. In simple terms, the living cells rearrange chemical elements to form new products that they need to live and reproduce. By happy coincidence, in the process of doing so they help make a popular beverage.

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[446] One aspect of biotechnology is the directed use of *organisms* for the manufacture of organic products. Examples include *beer* and *milk* products. For another example, naturally present *bacteria* are utilized by the mining industry in *bioleaching*. Biotechnology is also used to recycle, treat waste, clean up sites contaminated by industrial activities *(bioremediation)*, and produce *biological weapons*.

There are also applications of biotechnology that do not use living organisms. Example are *DNA microarrays* used in *genetics* and *radioactive* tracers used in medicine.

Bread baking is also dependent on the action of yeast cells. The bread dough contains nutrients that these cells digest for their own sustenance. The digestion process generates alcohol (which contributes to that wonderful aroma of baking bread) and carbon dioxide gas (which makes the dough rise and forms the honeycomb texture of the baked loaf).

Discovery of the fermentation process allowed early peoples to produce foods by allowing live organisms to act on other ingredients. But our ancestors also found that, by manipulating the conditions under which the fermentation took place, they could improve both the quality and the yield of the ingredients themselves.

By 4000 B.CE, the Chinese *were using lactic-acid-producing bacteria making yogurt, molds for making cheese and acetic acid bacteria for making wine vinegar.*

By 500 B.CE the Chinese *were using moldy soybean curds as an antibiotic treatment for pustules and open sores. By 250 B.CE* Greeks *practiced crop rotation to preserve soil fertility. One hundred and fifty years later the first insecticide was used by the* Chinese*, namely — that of powdered Chrysanthemum.*

The first millennia CE witnessed a consolidation of food processing technologies, with the Romans *in particular, diversifying and exploring cheese, and winemaking methods throughout their Empire. Beer making too, became more sophisticated and spread rapidly throughout Europe. The Dark Ages stemmed the flow of biotechnology and its products, with the Church coveting wine and beer making techniques. In 1300 CE the Aztecs in Mexico harvested algae from lakes as a food source. By 1400 CE, distillation of a variety of spirits from fermented grain was widespread. Egypt and Persia, however largely gave up brewing as a result of the influence of Islam. Fermented breads and cereals still maintained their hold in the African diet.*

By 1500 CE, plant-collecting expeditions became quite common across the globe. The collections led to the establishment of the first plant gene banks. Plants with desirable traits, including resistance to disease, were stored for future breeding purposes.

These examples suffice to show that humankind has been using Mother Nature's own remedies for thousands of years to preserve the environment and heal damage done to it.

It was not until 1590 that the first microscope was engineered and 75 years later, that ***Hooke*** *described cells. In 1675* ***Leeuwenhoek*** *discovered bacteria.*

Major events throughout the stage I *are summarized in Table 5.5.*

Table 5.5: Milestones in the progress of biotechnology 4000 BCE–1900 CE

ca 4000 BCE	• *Yeast was used to brew beer by Sumerian and Babylonians.* • *Egyptians discovered how to bake bread using yeast.* • *Other fermentation processes were established in the ancient world, notably in China: The preservation of milk by lactic acid bacteria resulted in yogurt. Molds were used to produce cheese, and vinegar and wine were manufactured by fermentation.*
ca 1000 BCE	*Babylonians celebrated the pollination of palm trees with religious rituals.*
ca 500 BCE	*The Chinese used moldy soybeans curds as an antibiotic to treat boils.*
ca 400 BCE	**Hippocrates d***etermined that the male contribution to the child's heredity is carried by a semen. He guessed there is a similar fluid in women, since children clearly receive traits from each in approximately equal proportion.*
ca 250 BCE	*The Greek practice crop rotation to maximize soil fertility.*
ca 100 CE	*Powdered chrysanthemum is used in China as an insecticide.*
ca 1000 CE	*Hindus observed that certain diseases may "run in the family" and that children inherit all their parent's characteristics.*
ca 1300 CE	*Aztecs in Mexico harvested algae from lakes as a food source.*
1590 CE	*The microscope was invented by* **Janssen** *in the Netherlands.*
1630 CE	**William Harvey** *speculated that plants and animals alike reproduce through the joining of an egg and sperm, 200 years before a mammalian egg was finally observed.*
1663 CE	*Cells were first described by* **Robert Hooke***: he observed the cellular structure of cork. But it was not until almost 200 years later hat scientists, armed with better microscopes, realized that the human body is divided into very small components.*

1660–1675 CE ***Marcello Malpighi*** *used the microscope to study details of blood capillaries, nerve fibers, silkworm anatomy and plant anatomy.*

1668 CE ***Francesco Redi*** *disproved spontaneous generation through pioneering controlled experiments.*

1673 CE ***Anton van Leeuwenhoek*** *used the microscope to discover protoza and bacteria. First to recognize that such microorganisms might play a role in fermentation.*

1701 CE ***Giacomo Pylarini****, the first immunologist, practiced inoculation*[447] *of children with smallpox in Constantinople.*

1724 CE *Cross-fertilization in corn was discovered.*

1750 CE ***Lazzaro Spallanzani*** *suggested preserving food by sealing it in heated containers.*

1797 CE ***Edward Jenner*** *vaccinated*[448] *a child with a viral vaccine to protect him from smallpox.*

1802 CE *The word "biology" first appeared.*

1809 CE ***Nicolas Appert*** *devised a technique using heat to sterilize and can food.*

1820 CE *First amino acid discovered.*

1827 CE *First observation of canine eggs.*

1830 CE *Proteins were discovered.*

1833 CE *The first enzymes were discovered.*

1836 CE ***Heinrich von Valdeyer-Hartz*** *coined the name chromosome.*

1847 CE ***Ignaz Semmelweis*** *used epidemiological observations to propose that childhood fever can be spread to mother by physicians.*

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[447] *inoculation*: intentionally infecting humans with a putatively mild strain of smallpox to induce resistance to severe strain of the disease, thus preventing a serious case later in life.

[448] *vaccination*: intentionally infecting humans cowpox to induce resistance to smallpox.

1851 CE ***Charles Chamberland*** *discovered organisms smaller then bacteria, later known as viruses.*

1855 CE *The Escherichia Coli bacterium is discovered. It later became a major research, development and production tool for biotechnology.*

1855 CE ***Louis Pasteur*** *first to introduce the notion that microbes are the cause, not the result of a disease; asserted that microbes are responsible for fermentation (result of activity of yeasts and bacteria); invented the process of pasteurization (heating wine sufficiently to inactivate microbes, while at the same time not ruining the flavor of the wine); theorized that decayed organisms are found as 'germs' in the air; developed the germ theory of disease (1865); developed a rabies vaccine.*

1856 CE ***Karl Ludwig*** *discovered a technique for keeping animal organs alive outside the body, by pumping blood through them.*

1859 CE ***Charles Darwin*** *theorized his principle of "natural selection": only the creatures best suited to their environment survive to reproduce.*

1865 CE ***Gregor Mendel*** *discovered that traits were transmitted from parents to progeny by discrete, independent units, later called genes.*

Mendel's work remained unnoticed, languishing in the shadow of Darwin's more sensational theory (1859), until 1900, when ***Hugo de Vries***, ***Erich von Tschermak***, *and* ***Carl Correns*** *published research corroborating Mendel's mechanism of heredity.*

1868 CE ***Friedrich Miescher*** *successfully isolated nuclein, a compound that includes nucleic acid, from pus cells. He was not, however, investigating heredity. Instead, he was trying to identify the chemicals in cells. Several generations of scientists would pass before the connection would be made between the DNA found by Miescher and the laws of heredity described by* ***Mendel*** *just three years previously.*

1870 CE ***Walther Flemming*** *discovered the mechanism of cell division (mitosis) and the notion of chromosomes.*

1871 CE ***Ernst Hoppe-Seyler*** *discovered invertase, an enzyme that cuts disaccharide sucrose into glucose and fructose.*

1877–1885 CE ***Robert Koch*** *developed a technique for staining and identifying bacteria, a single most important discovery in the rise of bacteriology (1877).*

He also became the first to uncover the cause of a human microbial disease – tuberculosis (1882).

1878 CE *The term 'microbe' was first used.*

1882 CE ***Elie Metchnikoff*** *developed a cell theory to explain the action of vaccines.*

1883 CE ***August Weismann*** *asserted that chromosomes must be the bearers of heredity.*

1885–1895 CE ***Robert Koch*** *and* ***Paul Ehrlich*** *identified a host of human disease-causing organisms.* ***Emil von Behring*** *developed the first antitoxin for diphtheria.*

1887 CE ***Eduard van Beneden*** *discovered that each species has a fixed number of chromosomes; he also discovered the formation of haploid cells during cell-division of sperm and ova (meiosis).*

1892 CE ***Dmitri Ivanovsky*** *reported that the causal agent of the tobacco mosaic disease is transmitted and is smaller than bacteria. Such agents are later called viruses.*

1898 CE ***Friedrich Löffler*** *and* ***Paul Frosch*** *discovered that foot-and-moth disease is caused by viruses.*

1899 CE ***Walter Reed*** *established that yellow fever is caused by a virus.*

II: The Age of the Industrial Revolution 1700–1900

The 18th and 19th centuries witnessed technological and scientific explosions, the diversity and inventiveness of which will likely never be equaled. The Industrial Revolution was matched by the Scientific Revolution. These phenomena were to ultimately fuse in the later part of the 19th century, especially in the agricultural, medical and environmental settings. Thus, the empirical method and the Industrial Revolution brought monumental changes to farming and industry, while the biological sciences were inspired by the work of ***Darwin*** *and* ***Pasteur****. The microbial nature of many diseases was established.*

*The 18th century beheld the first inoculation techniques (****Pylarini****, 1701), successful vaccinations (****Jenner****, 1797) and an explosion in engineering processes. The 1800s were the dawning century of modern scientific thought. The word biology was coined in 1802,* ***Appert*** *invented the canning process (1809), proteins were discovered (1830), the first enzymes were isolated (1833), and during the 1850s industrially processed animal foods, inorganic fertilizers and a wide range of seed drill harrows, automated mowers, cultivators etc. came into use.*

In 1855 Escherichia coli was isolated, a bacterium which has remained an indispensable tool in a wide range of production and research fields. In 1856 ***Karl Ludwig*** *described a method by which animal organs could be maintained outside the body — necessary prerequisite to organ transplants and* in vitro *methods.* ***Mendel*** *laid the groundwork of the modem field of genetics in 1863, while in 1865,* ***Lister*** *began using disinfectants.*

1869 bore witness to the first isolation of DNA — from brown trout sperm — by ***Friedrich Miescher*** *(1844–1895) and a year afterwards,* ***Flemming*** *recorded mitosis. Between 1870 and 1880,* ***Hoppe-Seyler*** *discovered* invertase*, which is still widely used for making sweeteners;* ***Koch*** *(1893) developed staining methods for bacteria;* ***Lister****'s* antiseptic surgery *was an important leap in the understanding of infectious diseases; the word "microbe" was used for the first time; chromosomes were described by* ***Flemming****;* ***Pasteur*** *published his work on attenuated strains of bacteria, and the first centrifuge was manufactured by* ***C.G.P. de Laval****.*

From 1881–1900, the first rabies vaccines appeared, ***Metchnikoff*** *laid the foundations of the study of immunology;* ***Vavilov*** *mapped the worlds centers of biodiversity;* ***Galton*** *coined the term* eugenics*;* ***C. Gram*** *described his differential staining techniques for bacteria;* ***von Behring*** *developed the first antitoxin;* ***Petri*** *introduced the glass plates still widely used in microbiology; the self-propelled tractor was marketed, and nitrogen fixation described.*

Toward the end of the 19th century, **Buchner** *demonstrated that fermentation can occur with yeast extracts — a defining moment in modern biochemistry and enzymology.*

1919 CE

- First commercial airplane service between London and Paris.
- First transatlantic flight: *Vickers Vimy*, piloted by the British aviators Captain **John Alcock** and Lieutenant **Arthur Whitten-Brown** from Newfoundland to Ireland, taking almost 16 hours.

1919 CE **Heinrich Georg Barkhausen** (1881–1956, Germany). Physicist. Discovered that the magnetization of iron proceeds in small discrete steps accompanied by tiny clicks and devised a loudspeaker system to render this discontinuity audible. This phenomenon is now known as the *Barkhausen effect*[449]. It can be explained by the domain theory of magnetization: as small magnetic domains become aligned, one at a time,the magnetic field changes

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449 Because the ferromagnetic elements Fe, Ni, and Co, have an unusual configuration of electrons orbital and spin states, the angular momenta of these atoms have *abnormally large electron spin components*. These produce a strong quantum mechanical and electrical interaction between neighboring atoms that leads to *maximum stability* when the atomic magnetic moments are aligned. This spontaneous tendency to align produces *mesoscopic regions* (*domains*) in which the moments are all aligned. In these domains, which have volumes of about order 10^{-7} mm^3 and 10^{15} atoms, the local magnetization has its maximum value Nm_0 [m_0 = atomic magnetic moment for a paramagnetic substance, typically about one Bohr magneton, m_B; $N = \text{atoms/m}^3$]. If an increasing external magnetic field is applied to an unmagnetized sample of iron below the critical (Curie) temperature, those domains that have moments aligned with the field, increase in size at the expense of those not favorably aligned. At first, when the applied field is relatively weak, this process is reversible. That is, if the external field is reduced to zero, the domain boundaries return to their original positions and the bulk sample is again unmagnetized. For stronger applied fields, however, the domain boundaries do not recover completely and the sample acquires a *permanent magnetism* that remains after the external field is removed.

A strong field can also produce permanent magnetization by causing unaligned domains to *rotate* and become nearly parallel to the field direction. The *shifts of the domain boundaries and the domain rotations both occur in discrete steps.*

induce electric pulses, and forces between neighboring domains also directly generate sound vibrations – causing the audible clicks. The Barkhausen effect is the main source of noise and hiss in almost all magnetic recording equipment.

Barkhausen was born in Bremen, son of a district judge. In 1911 he was appointed Professor of Low-Current Technology in the Technische Hochschule, Dresden, the first chair anywhere devoted to the relatively new field of electrical communication. He carried out fundamental research on electron tubes and electrical oscillations at ultra-high frequencies phenomena which were later utilized in high–power microwave devices. His Institute of High-Frequency and Electron-Tube Technology was destroyed by Allies bombing in 1945.

1919–1921 CE **Theodor Franz Eduard Kaluza** (1885–1954, Poland and Germany). Mathematician. A pioneer of unified field theories. First to suggest that unification of electromagnetism and gravitation might be achieved by extending space-time to a *five-dimensional cylinder world*. The electromagnetic vector-potential appears in his scheme as the cross components of the metric tensor (between the 5^{th}, compact dimension and the usual four spacetime dimensions). This scheme also gives rise to a scalar ('Brans-Dicke' or *dilaton* field), and the Einstein–Maxwell field equations in the four macroscopic space–time dimensions follow directly from the pure vacuum Einstein field equations of general relativity in 5 dimensions.

In April 1919, Einstein received a letter that left him speechless. It was from an unknown mathematician, Theodor Kaluza, at the University of Königsberg (today Kaliningrad in Russia). In a short article, only a few pages long, this obscure mathematician was proposing a union of Einstein's theory of gravitation with Maxwell's theory of light by introducing a 5th dimension (4 dimensions of space and one dimension of time). Kaluza's idea was to write down the pseudo-Riemann metric in 5 spacetime dimensions. The 5th column and 5th row are identified as the electromagnetic vector and scalar potentials of Maxwell, the g_{55} component is the Lorentz–scalar dilation field, while the remaining 4×4 block is the 4-dimensional metric of Einstein. It seemed incredible to Einstein that such a simple idea could explain the (then known) two fundamental forces of nature: gravity and electromagnetism.

In fact, it is possible to detect the *electric impulses* due to the induction effect as these realignments occur — this is called the *Barkhausen effect*.

The magnetization of an iron sample by an applied external field is a complicated *nonlinear process* that depends on the particular way that the sample was formed (cast, hot-rolled, alloyed etc).

After the initial shock of confronting the 5th dimension, Kaluza's theory was found to raise more questions than it answered. Since all observations and experiments – then as now – show that we live in a universe with $3+1$ dimensions of space-time, the embarrassing question: "Where is the 5th dimension?" still remained. Kaluza responded that the 5th dimension collapsed down to a *circle* so small that even atoms could not fit inside it. It was a *physical* dimension (not a mathematical trick) that provided the glue to unite the two fundamental long–range forces of nature, yet it was simply too small to measure. Since the universe (Kaluza claimed) is topologically identical to the direct product space $M^4 \times S^1$ (a type of "hyper cylinder") with M^4 the ordinary Minkowski 4-space (spacetime) of STR, and the 5th dimension topologically equivalent to a circle, anyone walking in the direction of the 5th dimension would find himself back where he started after traveling a subatomic (indeed subnuclear!) distance.

In 1926, **Oscar Klein** made several improvements upon the theory, stating that perhaps *quantum theory* could explain why the 5th dimension rolled up. On this basis he calculated that the 5th dimension should be of the order of 10^{-33} cm (The *Planck length*).

By the 1930's the Kaluza-Klein theory was "clinically dead" although destined to be resurrected, in the context of the other (weak and strong nuclear forces) 10 or 11 dimensions string–theory scenarios and Grand Unified quantum field theories, in the 1980's. On the one hand, physicists were not convinced that the 5th dimension really existed (Klein's conjecture was and still is untestable), and it has never been clear why the hidden (curled–up) dimensions should not expand to macroscopic – indeed cosmological–scales via a Big–Bang like process. On the other hand, the rising quantum mechanics challenged the smooth, geometric interpretation of forces, replacing it with stochastic quanta and Hilbert-space states and operators[450].

Kaluza was born in Ratibor, Germany (now Raciborz, Poland) to a family which had lived there for around 300 years. He was educated at the University of Köningsberg (1902–1909). After writing his habilitation thesis on Tschirnhausen transformation (1909) he became a Privatdozent and remained in that position for the next 20 years. Only after the intervention of **Einstein** on his behalf was he promoted to the rank of a professor at Kiel university (1929).

[450] The Kaluza-Klein theory furnished one of the several avenues – linking the *gauge principle* to differential geometry – along which Yang-Mills theories were ultimately discovered. It was recently revealed that Oscar Klein had independently invented the Yang-Mills field (1954) already in 1938! A Kaluza-Klein renaissance began in the 1980's as part of *supergravity* and *superstring theories*, but these remain untested (indeed, perhaps untestable), non-unique and mathematically ill-defined conjectures.

In 1935 he was made a full professor at Göttingen, where he remained until his death in 1954.

Merger of Gravitation with Electromagnetism – Kaluza-Klein Theory (1919–1926)

The ancient Greek philosophers began the quest for the source *of all material diversity from which all the different forms and laws of nature emerge. This quest for unification persisted throughout the evolution of physics, and was a major drive that motivated the great achievements of* ***Newton*** *(1687),* ***Maxwell*** *(1865) and* ***Einstein*** *(1917).*

Thus, in the mid 19th century ***James Clerk Maxwell*** *formulated the first genuine field – that of theory of electromagnetism, unifying the then separate classical phenomena of electricity, magnetism and optics. Then, in the early 20th century,* ***Albert Einstein*** *developed general relativity, a field theory of gravitation. Later, Einstein and others attempted to construct a unified field theory in which electromagnetism and gravity would emerge as different aspects of a single fundamental field.*

Back in 1919, the strong and weak nuclear interactions were unknown except in barest outline, and full–fledged quantum mechanics had not yet been discovered. In searching for a unified theory of fundamental forces, it was therefore natural to attempt to merge gravity with Maxwellian electrodynamics and generalize Einstein's GTR while remaining within the framework of pseudo–Riemannian geometry. The idea was that since the increase of dimensions from 3 to 4 had led to Einstein's gravitational theory, a further increase from 4 to 5, using the 5-dimensional version of ***Einstein****'s theory, might describe both gravitation* and *electromagnetism as (unified) geometrical effects.*

The first attempt at such a theory was made by ***Kaluza*** *in his 1921 paper.*

To construct a five-dimensional General Relativity, Kaluza considered a five-dimensional pseudo–Riemannian line element for which

$$ds^2 = \gamma_{jk} dx^j dx^k,$$

where the summation convention is understood; Latin indices will range from 0 *through* 4*, while Greek ones will range from* 0 *through* 3*. (Thus* x^4 *is the "fifth", curled–up dimension in this notation). The* $\binom{5}{2}+5=15$ *quantities* γ_{jk} *are the covariant components of a five-dimensional symmetric tensor. To relate them to the usual quantities* $g_{\mu\nu}$ *and* A_μ *of standard GTR and generally-covariant electrodynamics, one must make special assumptions.*

First, the extra spatial dimension x^4 *must coordinatize a topological circle. Second, the quantities* γ_{ik} *should not depend on this fifth*[451] *coordinate* x^4*.*

From this it follows that the permitted general coordinate transformations are restricted to the following group

$$\begin{aligned} x'^4 &= x^4 + \Psi\left(\{x^\nu\}\right), \\ x'^\mu &= x'^\mu\left(\{x^\nu\}\right) \end{aligned} \tag{1}$$

It can then easily be shown that γ_{44} *is a 4-dimensional scalar field w.r.t. the transformation (1). The assumption* $\gamma_{44} = const. = \alpha$ *is therefore permissible. It can also be shown that for fixed* x^4 $(\Psi = 0)$ *the four* $\gamma_{4\mu}$ *transform as the covariant components of a* 4*-vector. But if* x^4 *is transformed, yet* x^μ *are not, then the 4-gradient of the 4-scalar* $\alpha\Psi$ *is added:*

$$\gamma'_{4\mu}\left(\{x^\nu\}\right) = \gamma_{4\mu}\left(\{x^\nu\}\right) - \alpha\partial_\mu\Psi \ ,$$

So $\gamma_{4\mu}$ *can be interpreted as the EM gauge-potential 4-vector, with the 4-scalar* $\alpha\Psi$ *playing the role of the local gauge-transformation Lorentz scalar function. This means that the quantities*

$$\frac{\partial\gamma_{4\nu}}{\partial x^\mu} - \frac{\partial\gamma_{4\mu}}{\partial x^\nu}$$

transform like the covariant components $F_{\mu\nu}$ *of the electromagnetic field-strength skew-symmetric 4-tensor.*

In fact it is convenient to rescale:

$$\gamma_{4\mu} = \alpha\beta A_\mu, \qquad\qquad \beta = const. \tag{2}$$

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[451] Since the fifth dimension is assumed to be curled-up into a circle of circumference $\sim 10^{-33}$ cm, all fields $\gamma_{ij}(x^\mu, x^4)$ $(\mu = 0, 1, 2, 3, 4)$ must be periodic in x^4. It can be shown that the non-constant (in x^4) Fourier components are Klein–Gordon fields in the ordinary spacetime dimensions x^4, with masses of order of the *Planck mass* $(10^{19} GeV/c^2)$, which can be neglected at normal scales of energy, space and time (and even those encountered in accelerators, cosmic rays and the most violent cosmological and astrophysical cataclysms known).

According to the generally-covariant formulation of electromagnetism, the electromagnetic field-tensor in a source-free medium is given by

$$F_{\mu\nu} = \frac{\partial A_\mu}{\partial x_\nu} - \frac{\partial A_\nu}{\partial x_\mu} \tag{3}$$

We would like to relate the 5D differential ds^2 *with the usual 4D relativistic one. We therefore combine* $ds^2 = \gamma_{jk} dx^j dx^k$ *with*

$$\gamma_{44} = \alpha \ , \quad \gamma_{4\mu} = \gamma_{\mu 4} = \alpha\beta A_\mu \ , \quad \gamma_{\mu\nu} = g_{\mu\nu} + \alpha\beta^2 A_\mu A_\nu \tag{4}$$

The 4D metric $g_{\mu\nu}$ *has been chosen so that in locally Minkowskian coordinates we have*

$$(ds^{(4)})^2 \equiv g_{\mu\nu} dx^\mu dx^\nu = -(dx^2 + dy^2 + dz^2) + c^2 dt^2 \tag{5}$$

There remains the problem of constructing field equations for the γ_{jk} *which will lead to the usual Einstein-Maxwell field equations for* $g_{\mu\nu}$ *and* A_μ*. To this end one constructs the Lagrangian density*

$$\mathcal{L} = \gamma^{jk} \mathcal{R}^l_{jkl}, \tag{6}$$

where γ^{jk} *are the contravariant components of the five-dimensional metric tensor determined by,* A_μ*, matrix-inversion and Eq. (4);* $\mathcal{R}^l_{jkl}$ *is the 5D Ricci tensor, and (6) is the generalization of the Hilbert-Einstein Lagrangian density of GTR to the Kaluza 5-dimensional world.*

As explained above, we assume that all γ_{jk} *(and thus* γ^{jk}*) components, and therefore the 5D Ricci tensor as well, are independent of* x^4 *and that* $\gamma_{44} = \alpha$*. Let us now consider the 5D Hilbert-Einstein action integral*

$$J = \int \sqrt{-\gamma}\, dx^0 dx^1 dx^2 dx^3 dx^4 \mathcal{L} \ , \tag{7}$$

taken over the five-dimensional space, where γ *denotes the determinant of* γ_{jk}*. We express* δJ *in terms of a variation of the quantities* γ_{jk} *and* $\frac{\partial \gamma_{jk}}{\partial x^l}$ *where the variation vanishes sufficiently rapidly at 4D spacetime infinity and* α *is regarded as constant (not varied). The variational principle* $\delta J = 0$ *then leads to the following Euler-Lagrange equations:*

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R + \frac{\alpha\beta^2}{2} T^{\mu\nu} = 0 \ , \tag{8}$$

$$\frac{\partial\left(\sqrt{g} F^{\mu\nu}\right)}{\partial x^\mu} = 0 \ , \tag{9}$$

where R is the Einstein 4D scalar curvature, $R^{\mu\nu}$ are the contravariant components of the Einstein 4D Ricci tensor, $g^{\mu\nu}$ are the contravariant components of the 4D metric tensor, $T^{\mu\nu}$ are the contravariant components of the 4D electromagnetic energy-momentum-stress tensor, g is the determinant of $g_{\mu\nu}$, and finally $F^{\mu\nu}$ are the contravariant components of the electromagnetic field tensor. If we set

$$\frac{\alpha\beta^2}{2} = k \tag{10}$$

where $k = \frac{8\pi G}{c^4}$ is the usual 4D Einstein gravitational constant, we see that equations (8) and (9) are in fact identical with the matter-free field equations for the gravitational field (with only EM fields as sources) and the generally-covariant Maxwell equations, respectively.

While the Kaluza-Klein theory was an impressive accomplishment, it was a failure as a unification attempt, at least in its original form. This is because unlike Maxwell's unification (and the 20th century unifications of quantum mechanics with STR, and of Feynman et al.*'s QED with the weak nuclear force), no new physics is predicted (apart from the possible* scalar-tensor *or* Brans-Dicke *extension of GTR).*

Both the original Kaluza–Klein model and more modern versions (in which several *spatial dimensions are curled–up into compact, sub–nuclear–sized symmetrical spaces[452]) have the serious – and thus for unmet – challenge of explaining why the extra dimension or dimensions are curled up, and why nature would settle on a particular version of this "compactification" among the infinite possibilities.*

The enduring value of the Kaluza-Klein model to theoretical physics, however, lies in the tantalizing hint it provides that not only gravity, *but also* energy *and* matter *could be geometrical in origin. Furthermore, the connection – pointed to by this model – between the EM gauge principle and general covariance, helped point the way to the mathematical existence and physical relevance of* non-Abelian, *fiber-bundle-based gauge field theories.*

It's plausible, though by no means certain, that the varied and complicated chemical species and interactions necessary for life to arise would not be possible if one had fewer than three uncompactified spatial dimensions. similar "anthropic"–type arguments are nowadays adduced by many leading string

[452] In such higher–dimensional extensions of the Kaluza–Klein theory – motivated by *supergravity* and *superstring* candidate theories and studied in detail in the 1980s – GTR is unified, superficially at least, with all sorts of (matter and non-Abelian gauge–) fields. Such extensions are thought necessary in order to unify (at least) the four known forces, and all matter besides, into a single, geometrical framework.

theorists to attempt to pare down and choose among the myriad possible scenarios uncovered in their own higher–dimensional speculations. However, it is not clear that life could not thrive in a universe of more *than three open dimensions (although some philosophers have valiantly tried to adduce such arguments!). Perhaps one clue is that* non-Abelian gauge field theories *cannot be consistently quantized at low energies (of order those governing chemistry and nuclear physics) in any spacetime dimensions higher than 4, without fine-tuning the constants of nature at the Planck scale. (*This *is an example of a "Platonic" (first–principles) argument in physics that may – or may not – have anthropic overtones.)*

So if one were to imagine there were an infinite or very large number of universes (or different spacetime regions of one universe) with different numbers of dimensions or uncompactified dimensions, we would be looking at a biased sample – since our existing here to ask such questions, already implies that certain attributes and laws must locally hold in our observable universe.

In fact, even for fixed dimensionality, different compactifications schemes could result in different manifest laws of (4D) nature at low energy scales.

1919–1925 CE **Avraham (Adolf) Halevi Fraenkel** (1891–1965, Germany and Israel) Mathematician. Put set theory into an axiomatic setting that avoids paradoxes, improving the definitions of **Zermelo** and proving the independence of the *axiom of choice* within the Zermelo axiom system (1919). His system of axioms was modified by **Skolem** (1922) to give what is today known as the ZFS system[453] (Zermelo-Fraenkel-Skolem). The independence of the axiom of choice within this system was proved by **Cohen** (1963).

Fraenkel was born in Munich and studied at the Universities of Munich, Marburg, Berlin and Breslau. He became a professor of Mathematics at Marburg University (1922) and spent one year at Kiel (1928). Being a fervent Zionist he moved to the newly established Hebrew University of Jerusalem

[453] Within this system it is harder to prove the independence of the axiom of choice and this was not achieved until the work of **Paul Cohen** (1963). By virtue of *Gödel's incompleteness theorem*, there is no possibility of *proving* that the axioms of ZFS are consistent, but they certainly appear to avoid paradoxes, such as *Russel's paradox*, and most mathematicians believe that they will not lead to any contradictions at all.

(1929) and spent there the rest of his career. His major work is *Einleitung in die Mengenlehre* (1919).

1919–1927 CE **Francis William Aston** (1877–1945, England). Physicist. Discovered a number of isotopes in several nonradioactive elements by means of a *mass spectrograph*[454] of his own devising (1919). In all, discovered 212 out of 287 natural isotopes.

Aston was assistant to J.J. Thomson (1910–1919) and fellow of Trinity College, Cambridge (from 1919). Awarded the Nobel prize for chemistry (1922).

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[454] J.J. Thomson (1913) demonstrated that charged particles with different masses (nuclei with the same number of protons but with different number of neutrons are *isotopes* of the same chemical element) can be separated by using combined electric and magnetic fields. Thomson's crude device was improved upon by Aston and by A.J. Dempster. By 1919, atomic masses were being measured with a precision of 1 part in 10^3; the best modern instruments are capable of precision of about 1 part in 10^7. The principle of operation of the mass spectrograph is as follows: positively charged ions are produced in a source chamber and are accelerated through a potential difference V. The ions enter a region with uniform magnetic field $\boldsymbol{B}$ such that they are deflected in semicircular arcs in the plane of motion.

The ions are then recorded on a photographic plate so that the distance x from the entrance slit (the diameter of the orbit) can be measured. The entire system is maintained under vacuum to prevent scattering of the ions by gas molecules. Consider singly ionized positive ion $(q = +e)$ with a mass M that enters the magnetic field with a velocity v obtained from $eV = \frac{1}{2}Mv^2$. Within the magnetic field the force on the ion is always perpendicular to its velocity vector, which thus has a constant magnitude (i.e. no work is done upon the charge by the magnetic field). Therefore, the ion follows a circular path with radius $R = \frac{1}{2}x$. The centripetal acceleration is provided by the magnetic field. Thus $a_c = \frac{v^2}{R} = \frac{v^2}{\frac{1}{2}x} = \frac{F}{M} = \frac{evB}{M}$. Expressing v in terms of V and solving for M gives $M = \left(\frac{eB^2}{8V}\right)x^2$. Thus, the mass of the ion is obtained from measurements of B, V, and x. The *relative* masses of two ions accelerated at the same time involve only measurements of x and so can be obtained with high precision.

Clearly, the mass spectrograph places each of the *isotopes* of the same chemical element at different distances x along the plate.

The Berlin Colloquia (1919–1933)

One of the most characteristic and central features of the scientific and intellectual life in Berlin-Dahlen Kaiser Wilhelm Institute became the famous Haber Colloquia, which took place every second Monday afternoon. There were many famous guests speakers in different fields and many of the department heads or senior scientists of different institutes presented papers.

Haber's aim was to break down the barriers between physics, chemistry, physical chemistry, and the biological sciences. The presentation of a paper was followed by a vigorous discussion, deliberately stimulated by Haber, in which scientists of different disciplines participated. Thus the discussions automatically stimulated interests beyond one's own field and frequently suggested ways for trying a new approach. Young people were encouraged to participate. Controversies were considered an essential element for clarifying a subject. The young people taking part in these colloquia became immunized against authoritative and dogmatic thinking. Science in all fields was in rapid expansion. Nobody could claim to have the right answers.

It was an illuminating experience to listen to the opposing views of so many illustrious people — a fascinating demonstration of the limitation of our knowledge, a warning to be flexible and to avoid rigidity of views. Moreover, non of these great scientists hesitated to admit errors. The spirit of searching for the truth, not worrying about reputation or prestige when one was wrong, was one of the most extraordinary features of the Colloquia. They became an intellectual center in the lives of scientists of various fields, outlooks, and interests of exchanging views and learning about developments in other fields.

Walter Nernst*'s department at the University of Berlin had similar Colloquia in which an exciting atmosphere prevailed and in which vigorous discussions and controversies took place*[455]*. Sitting in the front row were* ***Max Planck****,* ***Albert Einstein****,* ***Max Von Laue*** *and* ***Erwin Schrödinger****. Students sat in the back and for them it must have been an experience similar to that felt at the Haber Colloquia.*

[455] In Anglo-Saxon countries, controversies seem to be considered "unpleasant and distasteful". The Haber colloquia were a particularly great stimulus for the remarkable development of biological sciences in the Kaiser Wilhelm Institutes. In them, the revolutionary developments in the fields of physics, chemistry, and physical chemistry were brought to the attention of biologists, thus providing them with new insight and later with important tools for the analysis of the processes taking place in the living cell.

1919–1928 CE **Edgar Douglas Adrian** (1889–1977, England). Physician and psychologist. His studies on the electrophysiology of the brain and the nervous system provided a new quantitative basis of nerve's behavior.

Adrian was born in London and studied at Cambridge University. Was professor of Clinical neurology at Cambridge (1937–1951), master of Trinity College (1951–1965) and chancellor (1968–1975). Shared the Nobel Prize for physiology and medicine (1932) with **Charles Sherrington**.

1919–1938 CE **Léon Theremin** (**Lev Sergeyevich Termen**, 1896–1993, Russia). Inventor, electronic engineer and electronic music pioneer. Invented the *theremin*, one of the earliest fully electronic musical instrument. The theremin is unusual in that it requires no physical contact in order to produce music and was designed to be played without being touched.[456]

Léon Theremin was born in St. Petersburg. His ancestors were Huguenots who escaped France in 1572, following the Massacre of Saint Bartholomew's Day. His invention came at a time when his country was in the midst of the Russian Civil War. After a lengthy tour of Europe, during which time he demonstrated his invention to packed houses, he found his way to the U.S., playing the theremin with the New York Philharmonic (1928). He patented his invention (1929) and subsequently granted production rights to RCA. Although the RCA Theremin, released immediately following the Stock Market

[456] The original design resembled a gramophone cabinet on 4 legs with a protruding metal antenae and a metal loop. The instrument was played by moving the hands around the metal loop for volume and around the antenae for pitch. The output was a monophonic continuous tone modulated by the performer. The timbre of the instrument was fixed and resembled a violin string sound. The sound was produced directly by the heterodyning combination of two radio-frequency oscillators: one performing at a fixed frequency of 170,000 Hz, the other with a variable frequency between 168,000 and 170,000 Hz, the frequency of the second oscillator being determined by the proximity of the musician's hand to the pitch antenna. The difference of the fixed and variable radio frequencies results in an *audible beat frequency* between 0 and 2,000 Hz. The audible sound came from the oscillators, later models adding an amplifier and large triangular loudspeaker. This Theremin model was first shown to the public at the Moscow Industrial Fair in 1920 and was witnessed by **Lenin** who requested lessons on the instrument. Lenin later commissioned 600 models of the Theremin to be built and toured around the Soviet Union.

Crash (1929), was not a commercial success, it fascinated audiences in America and abroad. **Clara Rockmore** (1911–1998; born Reisenberg in Vilnius, Lithuania), widely considered the greatest therminist ever, met Theremin in 1927 and toured to wide acclaim, performing a classic repertoire in concert halls around the United States.

In 1938, Theremin was kidnapped from his New York apartment by Soviet agents, and forced to return to the U.S.S.R.. He was then sent to work in the gold mines in Kolyma and later put to work in a *sharashka*, together with **Tupolev**, **Korolev** and other well-known scientists and engineers on several tasks.[457] He was rehabilitated in 1956.

After his rehabilitation Theremin took up a teaching position at the Moscow conservatory of music. However he was ejected for continuing his researches in the field of electronic music. Post war Soviet ideology decreed that modern music was pernicious. Theremin was reportedly told that electricity should be reserved for the execution of traitors. After this episode Theremin took up a technical position, and worked upon non-music related electronics. Ironically his invention, the Theremin, was becoming vastly influential in America, a development of which he was completely unaware.

Before his death in 1993 Theremin made one final visit to America lecturing, and demonstrating his Theremin. Indeed the instrument is still being used today, and has an avid following of Theremin-o-philes[458].

1919–1953 CE **Lewis Fry Richardson** (1881–1953, England). Distinguished applied mathematician. A most original and versatile mind that

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457 There he invented a sophisticated electronic eavesdropping device. His "bug" was the first to use induced energy from radio waves of one frequency to transmit an audio signal to another. This made the device difficult to detect since it did not radiate any signal unless it was actively being powered and listened to remotely. This feature also endowed it with (potentially) unlimited life.
"The Thing", as it was called, was very simple by today's standards, having only a capacitive membrane (a *condenser microphone*) connected to an antenna. Thus the *impedance* seen by the quarter-wavelength *antenna* was altered by sound, and the reflections of the 330 *MHz* signal impinging on the device were *modulated*, allowing the audio to be detected. A bug of this nature was embedded in a wooden plaque and presented to the American ambassador in Moscow by Russian schoolchildren where it hung in his office until detected by a professional bug sweeper. For this work Theremin was awarded the Stalin Prize.

458 The famous theme music of the original 1960's *Star Trek* television series, which seems to be hummed by a choir of female voices, is actually Theremin music!

concerned himself with the solutions of difficult problems, and made fundamental contributions in physics, mathematics, meteorology and psychology. Published about 90 papers and 4 books. His novel ideas have led to the following results:

- *Fractal geometry*[459]: Rechardson is certainly among the progenitors of the fractal concept, and had much influence on the genesis of fractal geometry.
- *Weather prediction by numerical process* (1922): He was the first man to compute the weather. Developed the application of the method of *finite differences* to the major problem of meteorology viz., the computation of the physical state of the atmosphere (rainfall, transfer of heat and moisture, etc.) for an epoch finitely subsequent to that for which the state is known by observations.
- *Atmospheric turbulence* (1920–1926). We are indebted to him for some of the most profound and most durable ideas regarding the nature of turbulence, notably the notion that over a wide range of scales, turbulence is decomposable into a hierarchy of self-similar eddies. He introduced a fundamental dimensionless parameter that controls eddy-diffusion in the atmosphere, later termed the *Richardson number*.
- *Studies of the causation of wars* (1919–1953). Developed the application of mathematics to the study of relations between nations, especially to elucidate the effects of armaments, trade, communications, rivalry and grievances on the stability of the regime and the feasibility of armed conflicts. He published 3 books on this subject [*Mathematical Psychology of War* (1919), *Arms and Insecurity* (1949), *Statistics of Deadly Quarrels* (1950)]. It was one of the first serious applications of science to human situations.

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459 Richardson had noticed that the results of measuring the length of a coastline from a map depends heavily on the scale of the map used. Thus, he set out to make an experimental measurement of the west coast of Britain as a function of the map's scale. The same phenomenon can also be expressed by considering one map only, on which all detail can be seen, but using a smaller measuring unit each time. Denoting the measured length by L and the length of the measuring unit by ϵ, the plot of L against ϵ yielded the line: $\log_{10} L = -0.22 \log_{10} \epsilon + \log_{10} L_0$, where L_0 is the coastal length when using a measuring unit of 1 km. This can be expressed as $L = L_0 \epsilon^{-0.22}$ or as $L = L_0 \left(\frac{1}{\epsilon}\right)^{0.22}$. If we reduce ϵ by a factor of 32, s will double. Clearly $\lim_{\epsilon \to 0} L = \infty$. This is known as the *Richardson effect*, and denoted as $L(\epsilon) = L_0 \left(\frac{1}{\epsilon}\right)^{D-1}$, where D is now called the *fractal dimension* of that coastline.

Richardson constructed a mathematical model which describes the relation between two nations, each determined to defend itself against a possible attack by the other. Each nation considers the possibility of attack quite real, and reasonably enough, bases its apprehensions on the readiness of the other to wage war. The theory is not an attempt to make scientific statements about foreign politics or to predict the date at which the next war will break out. It only predicts what will occur if *instinct* and *tradition* were allowed to act uncontrolled.

Let $x = x(t)$ denote the war potential or armaments, of the first nation (called X) and let $y(t)$ denote the war potential of the second nation, Y. The rate of change of $x(t)$ depends on the war readiness $y(t)$ of Y and on the grievances that X feels towards Y. We represent these terms by ky and g respectively, where k and g are positive constants. On the other hand, the cost of armament has a restraining effect on $\frac{dx}{dt}$. We represent this term by $-\alpha x$, where α is a positive constant. A similar analysis holds for $\frac{dy}{dt}$. Consequently $\{x(t)\ y(t)\}$ is a solution of the linear system of ordinary differential equations

$$\frac{dx}{dt} = ky - \alpha x + g, \qquad \frac{dy}{dt} = \ell x - \beta y + h.$$

This model is not limited to two nations; it can also represent the relation between two alliances. Note also that these equations accommodate conflicting political theories about the *causes* of war. Those who maintain that armaments cause war (like **Thucydides**) will take $g \ll k,\ h \ll \ell,$ whereas those who believe that the grievance factor dominates will take $k \ll g,\ \ \ell \ll h$). The equations imply:

- If $g = h = 0,$ then $x(t) = 0,\ y(t) = 0$ is an equilibrium solution of the system, i.e., if x, y, g, h are all made zero simultaneously, then $x(t)$ and $y(t)$ will always remain zero. This ideal condition is *permanent peace by disarmament and satisfaction.*

- If at some time $t = t_0,\ x(t_0) = y(t_0) = 0,$ then at this stage $\frac{dx}{dt} = g$ and $\frac{dy}{dt} = h$. Thus, x and y will not remain zero if g and h are positive, and both nation will rearm. Hence: *mutual disarmament without satisfaction is not permanent.*

- If $y = 0$ at some time (unilateral disarmament), then at this time $\frac{dy}{dt} = \ell x + h$. This implies that y will not remain zero if either h or x remain positive. Thus, *unilateral disarmament is never permanent.*

- A race in armament occurs when the "defense" terms predominate. In this case $\frac{dx}{dt} = ky,\ \frac{dy}{dt} = \ell x,$ having the solution $x(t) = Ae^{\lambda t} + Be^{-\lambda t},$

$y(t) = \sqrt{\frac{\ell}{k}}[Ae^{\lambda t} + Be^{-\lambda t}]$, $\lambda = \sqrt{k\ell}$. Therefore, both $x(t)$ and $y(t)$ approach infinity if $A > 0$. This can be interpreted as war.

Richardson was born in Newcastle-on-Tyne, the youngest of seven children of a Quaker family, well known for owning a profitable leather works for about 300 years. He attended Cambridge University (1900), where he earned his B.A. in mathematics and the natural sciences (1903). His formal education then ended, but 20 years later he studied psychology as an external student of University College, London, where he received D.Sc. in physics on his published researches (1926), and a B.Sc. in psychology (1929). During 1903–1940 he held a sequence of different appointments as physicist and meteorologist in colleges and government institutions. In 1940 he retired to do research on wars and eddy-diffusion.

1919–1956 CE **Balthazar Van der Pol** (1889–1959, Holland). Applied mathematician. Pioneer in the field of radio; pursued the mathematical problems encountered in radio applications so far that his work has formed the basis of much of the modern theory of *nonlinear oscillations*[460] (1924). His name was given to a typical equation of that theory.

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[460] To dig deeper, see:

- Jackson, E.A., *Perspectives of Nonlinear Dynamics*, Cambridge University Press, 1993, vol I–II (496 pp + 633 pp).
- Shen, S.S., *A Course on Nonlinear Waves*, Kluwer, 1993, 327 pp.
- Jordan, D.W. and P. Smith, *Nonlinear Ordinary Differential Equations*, Oxford University Press: Oxford, 1977, 360 pp.
- Davis, H.T., *Introduction to Nonlinear Differential and Integral Equations*, Dover Publications: New York, 1962, 566 pp.
- Saaty, T.L., *Modern Nonlinear Equations*, Dover Publications: New York, 1981, 471 pp.
- Drazin, P.G., *Nonlinear Systems*, Cambridge University Press, 1992, 317 pp.
- Huntly, I. and R.M. Johnson, *Linear and Nonlinear Differential Equations*, Ellis Horwood: Chichester, England, 1983, 190 pp.
- Strogaz, S.H., *Nonlinear Dynamics and Chaos*, Addison-Wesley, 1994, 498 pp.
- Verhulst, F., *Nonlinear Differential Equations and Dynamical Systems*, Springer-Verlag, 1985, 277 pp.

The *Van der Pol equation* is the differential equation of the *triode oscillator*

$$\ddot{x} - \epsilon(1 - x^2)\dot{x} + x = 0, \quad \epsilon > 0,$$

or with a forcing term

$$\ddot{x} - \epsilon(1 - x^2)\dot{x} + x = F \cos \omega t.$$

For arbitrary initial conditions, the solution to the unforced equation approaches a *limit cycle*, with radius approximately 2 and period approximately 2π. The limit cycle is generated by the balance between internal energy loss and energy generation, and the forcing term will alter this balance.

If F is small (weak excitation), its effect depends on whether or not ω is close to the natural frequency. If it is, an oscillation is generated which is a perturbation of the limit cycle. If F is not small (hard excitation) or if the natural and imposed frequencies are not close, the '*natural oscillation*' will be extinguished, as occurs with the corresponding linear equation.

Rayleigh (1883) was first to treat equations of this kind. He arrived at them in the following way: The solution of the equation

$$u'' + ku' + \omega^2 u = 0$$

defines a steady vibration if $k = 0$; but if $k > 0$, the vibrations will die down, and if $k < 0$ they will increase without limit. Let us add to the above equation a term proportional to the cube of u', i.e.,

$$u'' + ku' + \lambda {u'}^3 + \omega^2 u = 0.$$

If $k > 0$, $\lambda > 0$, the resulting motion will again die out, and if $k < 0$, $\lambda < 0$, the motion will increase without limit.

But if k and λ have *different* signs, then the two terms which contain them can be written $ku'(1 - a{u'}^2)$, $a > 0$ and the motion is no longer unidirectional. If k is initially negative and the initial value of u' is sufficiently small so that the term in parentheses is positive, the motion will expand until $(1 - a{u'}^2)$ becomes negative. Thereupon the motion will begin to damp, u' will diminish until the term $ku'(1 - a{u'}^2)$ is again negative, and the motion once more increases.

For small value of k and λ Rayleigh gave the following approximate solution

$$u = A \sin \omega t + \frac{\lambda \omega A^3}{32} \cos 3\omega t,$$

where A is defined by $k + \frac{3}{4}\lambda\omega^2 A^2 = 0$.

Writing Rayleigh's equation in the form

$$\frac{d^2u}{dt^2} + \left[-b + c\left(\frac{du}{dt}\right)^2\right]\frac{du}{dt} + \omega^2 u = 0,$$

the variables $pt = x$, $q\frac{du}{dt} = y$, yields $\frac{p}{q}\frac{dy}{dx} + \left(-b + \frac{c}{q^2}y^2\right)\frac{y}{q} + \omega^2 u = 0$. Differentiating this equation, simplifying, and introducing the notation $p = \omega^2$, $q = \sqrt{\frac{3c}{b}}$, $\epsilon = \frac{b}{p}$, we finally arrive at the *van der Pol equation*

$$\frac{d^2y}{dx^2} - \epsilon(1 - y^2)\frac{dy}{dx} + y = 0.$$

Van der Pol was born at Utrecht. He studied at the University of Utrecht during 1911–1916. In 1917 he worked at the Cavendish Laboratory in Cambridge under **J.J. Thomson**, and met with **E.V. Appleton** (1892–1965, England). From 1919 to 1922 he was assistant to **H.A. Lorentz** in Haarlem. He was concurrently head physicist in the Phillips Research Laboratory at Eindhoven and a professor of theoretical electricity at the Technical University, Delft.

1920 CE, July 12 Official opening of the *Panama Canal*, a 81.6 km long waterway that cuts across the Isthmus of Panama and connects the Atlantic and Pacific oceans. It ranks as one of man's greatest engineering achievements and is perhaps the greatest construction project in 4000 years.

The Panama canal was built by the United States at a total cost of 600 million dollars. Work started in 1904 with improving sanitary conditions and required the removal of 161 million cubic meters of earth and rock. The canal has three sets of *locks*, or water-filled chambers, that raise and lower ships from one level to another; each lock has a usable length of 300 m, a width of 34 m and a depth of 21 m. (U.S. Navy supercarriers are too wide to pass through the canal.)

Dreams of a canal cutting through the isthmus of Panama date from the 16th century. Concrete plans began to take shape in the American mind about 1825. The Mexican War and the discovery of gold in California turned the attention of the people of the United States to the importance of improvement of communications between the coasts. The French organized a company in 1876 to build a canal across the isthmus and began working in 1882, spending more than 200 million dollars on it. The Canal was only about 40 per cent complete when the United States took over the project in 1903. The French failed in their efforts to complete the canal because of improper financing, bad sanitary conditions (Yellow fever, malaria) that killed their workmen by the thousands and the lack of system for removal of excavation debris.

President Theodore Roosevelt gave great impetus to the project. Operations began in 1904 with the appointment of **John F. Stevens** (1853–1943), a railroad civil engineer, to direct the excavation and **William C. Gorgas** (1854–1920) to direct the effective eradication of the mosquitoes that transmit Yellow fever and malaria[461].

The physical difficulties at Panama were enormous: The climate is deadly to most races. On the Atlantic side a tempestuous river, the Chagras, had to be controlled. Next came the Continental Divide, a ridge over 100 meters high which must be sliced before the canal could be dropped down to the Pacific.

Stevens was the right man to overcome these difficulties and command the conquest of both nature and geography. Before resuming the excavation he directed a wide sanitation plan to eradicate the mosquitoes, built an adequate railway system for the removal of earth rocks and debris, acquired heavy dredging and excavation machinery and mobilized a new work force of over 100,000 men, He then blocked off the Chargas River by a huge earth dam, forming the inland Gatan Lake, at an elevation of 30 meters above sea level. A devious channel through this lake constitutes the middle half of the entire canal. Beyond the lake the Culbera Cut was sliced through the ridge.

Nothing like this excavation, carried to a total depth of 90 m, had ever been attempted anywhere. The excavation and transportation of the ridge material presented problems of a magnitude never before encountered. Since sea-level passage is not feasible under the said geographical conditions, Stevens devised the *lock gates system*, enabling ship transportation at a level of 30 m above sea-level by means of three steps of twin concrete locks with chambers 35×330 m built on the Atlantic side and three more on the Pacific side giving a total of twelve lock chambers in all, six in each direction. Stevens resigned in 1907, and his plans were completed by the United States Army Core of Engineers.

1920 CE, Dec. 16 A *landslide* in Kansu Province, China, kills ca 200,000 people.

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461 The success of the American effort was due not only to adequate generous financing, but also of two monumental medical discoveries made in 1898 and 1900. The first was that the Anopheles mosquito transmits malaria from one person to another; the second was that the *Stegomyra fasiata* mosquito carries Yellow fever.

1920 CE Licensed radio broadcasts on a regular basis began in the United States.

1920 CE **Andrew Elliot Douglass** (1867–1962, USA). Astronomer, father of *dendrochronology* — a technique for dating objects based on the characteristic growth rings of trees in their given region.

Douglas was born in Windsor, Vermont. After research work at the Lowell Observatory at Flagstaff, Arizona, he became professor of Physics and Astronomy at Arizona University (1906) and later Director of the Stewart Observatory (1918–1938). He investigated the relationship between sunspots and climate by examining and measuring the annual growth-rings of long lived Arizona pines and sequoias; he noted that the variations in their width correspond to the specific climatic cycles, creating patterns which can be discerned in timbers from prehistoric archaeological sites providing a time-sequence for dating purposes. He coined the term '*dendrochronology*' (tree-dating).

1920–1940 CE **Thoralf Albert Skolem** (1887–1963, Norway). Mathematician. Did important work in Diophantine equations and helped to provide the axiomatic foundation for set theory in logic. Before such subjects as model theory, recursive function theory, and axiomatic set theory had become separate branches of mathematics, he introduced a number of the fundamental notions that gave rise to them. Contributed also to Diophantine equations, group theory and lattice theory. Wrote 182 scientific papers, but they remained largely unread.

Skolem was born at Sandsvaer and educated at Oslo, where he became professor (1938). His main work was in the field of mathematical logic. He created what is now known as the *Löwenheim-Skolem theorem*, one consequence of which is *Skolem's Paradox*[462]. In this work, Skolem was ahead of his time. From 1933 he did pioneering work in *metalogic* and contributed a nonstandard model for arithmetic.

1920 CE CE **Quirino Majorana** (1871–1957, Italy). Physicist. Proposed a modification of Newton's inverse-square law, including a factor of shielding. The weak equivalence principle is violated in his model.

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[462] If an axiomatic system (such as Ernst Zermelo's axiomatic set theory, which intends to generate arithmetic, including the natural numbers, as part of set theory) is consistent, then it must be satisfiable within a countable domain; but Georg Cantor had shown the existence of a never-ending sequence of transfinite powers in mathematics (that is, uncountability). Skolem resolved this paradox by saying that there is no complete axiomatization of mathematics: certain concept must be interpreted only relatively, i.e. they can have no 'absolute' meaning.

1920–1927 CE **William Draper Harkins** (1873–1951, U.S.A.). Nuclear chemist. Discovered *nuclear fusion*[463], the fundamental process underlying stellar luminosity and the thermonuclear bomb. Previously, in 1920, he proposed (independent of simultaneous notions by Rutherford) the existence of the *neutron*. He believed that the neutron could be formed from a proton and an electron. Chadwick's discovery of the neutron (1932) confirmed his prediction. He also predicted the existence of deuterium, and introduced the concept of *packing fraction*, a measure of energy involved in the association of protons and neutrons within the nucleus.

Utilizing Einstein's concept of the equivalence of mass and energy, he demonstrated that by combining four hydrogen atoms to produce one helium atom, a small amount of mass would be converted to energy; he correctly theorized that this process was a source of stellar energy. Harkins made one of the first attempts to calculate the abundance of elements in the universe.

Harkins was born in Titusville, PA. He received his Ph.D. from Stanford University (1908) and spent most of his career at the University of Chicago.

1920–1933 CE **Otto Stern** (1888–1969, Germany and USA). Physicist. Demonstrated clearly and directly the fundamental fact of the dual nature of matter, i.e. the wave-like vs. matter property of elementary particles, thus verifying the space-quantization theory of atoms. For that he was awarded the Nobel Prize in Physics (1943).

Stern was born in Sohrau, Germany to Jewish parents. He studied physical chemistry at Breslau University (Ph.D. 1912) and later held posts at the Universities of Prague (1912), Zurich (1913), Frankfurt (1914–1921), Rostock (1921–1923) and Hamburg (1923–1933). He worked in Prague and Zurich with **Albert Einstein**. With the rise of the Nazis, he moved to the USA and appointed Professor at the Carnegie Tech., Pittsburgh (1933–1945). Stern's main experimental results are:

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463 Fusion takes place when light nuclei fuse to form a heavier nucleus, releasing energy in the process. Fusion reactions are often called *thermonuclear* reactions because they are endothermic and are ignited only at temperatures higher than 100 million degrees. The sun's energy comes, essentially, from the fusion of light hydrogen atoms to form a helium atom. The energy of the hydrogen bomb comes from the fusion of deuterium and tritium (two heavy isotopes of hydrogen) into helium. Efforts have been underway for decades to achieve sustained *controlled* fusion in suitable reactors, a process that may become an important new source of energy for commercial use.

• 1920–1: by projecting a beam of neutral silver atoms through a non-uniform magnetic field [Stern-Gerlach[464] apparatus] showed that two distinct beams could be produced[465]. this provided proof of the quantum theory prediction that atoms possessing a magnetic moment could be oriented in two fixed directions to the external magnetic field [atoms and molecules behave like tiny magnets on account of the electric charges of the proton and the electron].

• 1933: Using molecular beams he measured the magnetic moment of the *proton* and *deuteron* and demonstrated that the proton's magnetic moment was 2.5 times greater than predicted by **P.A.M. Dirac**[466].

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[464] **Walther Gerlach** (1899–1979, Germany).

[465] Since Bohr arrived at the appropriate energy levels for the hydrogen atom by postulating that *angular momentum* is quantized, it is reasonable to expect that in quantum mechanics *other quantities than energy might be subjected to quantum conditions*: A beam of neutral atoms is passed through an evacuated region between two specially designed magnetic pole faces which set up a strongly inhomogeneous field *transverse to the beam*. The field exerts a deflecting force on the magnetic-dipole moment $\boldsymbol{\mu}$ of each of the moving atoms of magnitude $F_z = \mu_z \frac{\partial B_z}{\partial z}$, where μ_z is the component of $\boldsymbol{\mu}$ along the *field-gradient* direction. Classically, the random orientations of the dipole moments should be expected to render *spreading* of the beam i.e a continuous range of deflections on either side of the original beam. Stern and Gerlach observed, however, a *splitting* of the beam into *two discrete components*, indicating the existence of *only two possible values for* μ_z. Now, according to quantum theory, spatial quantization requires the vectors to adopt only certain angles of orientation and the components μ_z must all be multiple of the *Bohr magneton*. Accordingly, the beam should split into $(2n+1)$ different beams, each one corresponding to a certain value of the magnetic quantum number m_n, where $\mu_z = m_n \mu_B$ with $\mu_B = \frac{e\hbar}{2m}$. The results of the experiment show that $n = \frac{1}{2}$. Since the experiment was done four years before the *intrinsic spin* angular momentum of the electron was suggested, the Stern-Gerlach experiment indicates that while the principal quantum number n fixes the energy levels in hydrogen-like atoms, it does *not* determine the angular momentum of individual electrons. In due course this result led to the recognition of the *electron spin* as an extra angular momentum variable, with an associated magnetic moment.
In hindsight we say that the since the Ag atom with its ground state ${}^2s_{\frac{1}{2}}$ has a total angular momentum $J = \frac{1}{2}$, the only possible orientations in the magnetic field are $M = \pm\frac{1}{2}$, i.e. $M = +\frac{1}{2}$ for one beam and $M = -\frac{1}{2}$ for the other.

[466] No existing theory of the proton is able to account for this discrepancy. However, physicists believe that it arises because the proton is, unlike the electron, not

The Ice-Age Mystery[467] *(1787–1987)*

"Great clocks of eternity beat ages as ours beat seconds".

Anon

The present is the key to the past, but there have been many strange meteorological situations for which we have no present parallel to guide us. Moreover, the meteorology of the present has yet to solve many problems of its own, and we are even encouraged to hope that the meteorology of the past may at times help in the study of the present. The theory of the circulation of the earth's atmosphere, for instance, is not yet complete; and it may be that data on modifications of the circulation during the varied climatic history of the globe, as deduced from past distributions of rainfall and temperature, will provide just the additional information required for a solution.

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a point particle obeying the Dirac equation, but rather a composite, made up of 3 valence quarks, in addition to a so-called 'sea' of quark-antiquark pairs and gluons, as described by modern QCD (1964). The theory, however, is able to explain the observed *ratio* of the magnetic moments of the proton and the neutron, namely $\mu_P/\mu_N = 2.79/(-1.91) \approx -\frac{3}{2}$.

Victor Weisskopf told the following story: There was a seminar held by the theoretical group at Göttingen in 1920, and Otto Stern gave a talk on the measurements he was about to finish of the magnetic moment of the proton. He explained his apparatus, but did not tell us the result. He took a piece of paper and went to each of us, saying, "What is your prediction of the magnetic moment of the proton?" Every theoretical physicist from Max Born down to Victor Weisskopf said, "Well, of course the great thing about the Dirac equation is that it predicts a magnetic moment of one Bohr magneton for a particle of spin one-half." Then he asked us to write down the predictions; everyone wrote "one magneton." Then, two months later, he came again to give a talk about the finished experiment, which showed that the value was 2.5. He projected the paper with our predictions on the screen. It was a sobering experience.

[467] For further reading, see:

- Imbrie, J. and K.P. Imbrie, *Ice Ages*, Harvard University Press, 2002, 224 pp.
- Graedel, T.E. and P.J. Crutzen, *Atmosphere, climate and Change*, Scientific American Library, 1995, 196 pp.

Swiss naturalists, who lived and worked in the mountains, had long been coming into daily contact with evidence of an extensive past glaciation. As early as 1787, **Bernard Friedrich Kuhn** *(1762–1825, Switzerland), a Swiss minister, concluded that the Grindelwald glacier had been more extensive at some time in the past. In 1794,* **James Hutton** *(1726–1797) visited the Jura and reached the same conclusions, namely: that erratic boulders far downstream from current glaciers suggested the former existence of more extensive glaciation. In 1815,* **Jean Pierre Perraudin** *(1767–1858, Switzerland), a mountaineer and guide in the Swiss Alps, conjectured that Alpine glaciers formerly extended well beyond their present limits. In* 1829 *he announced to the Swiss Society of Natural Sciences his thesis that the glaciers of the Alps once covered the Jura Mountains and extended northward beyond the mountains into the plains. These and other early pioneers developed their ideas completely independently, through personal observation and deduction. On the other hand, until the* 18^{th} *century, it was conventional to assume that the observed blanket of glacial sediments had been transported and deposited by the great flood described in the Bible!*

So deeply entrenched was this accepted explanation, that none of these men was able to make their revolutionary ideas widely known. It would demand the combined efforts of some of the greatest scientific minds of the age, over a period of 25 *years, to overthrow the established "theory". It is not surprising that in such a religious age scientists and laymen alike believed that the local boulders had been transported by unimaginably huge currents of water and mud deriving from the biblical deluge of Noah's time.*

In 1834, **Johann (Jean) von Charpentier** *(1786–1855, Germany), in a talk before the Swiss Society of Natural Sciences, outlined evidence supporting the claims that Alpine glaciers have extended to lower elevations in the past. In* 1836 *he convinced* **Louis Agassiz** *(1807–1873, Switzerland and U.S.A.) that many features in the currently unglaciated landscape were formed in the past by glaciers. Once convinced, Agassiz assimilated the evidence, developed a broader-scale theory, and moved toward publication and vigorous advocation of the theory.*

In 1837*, Agassiz announced his theory of a Great Ice Age*[468] *at a meeting of the aforementioned society in Neuchatel. So it was that glacial theory — born from observations of amateur scientists, developed by* **Ignace Venetz** *(1788–1859, Switzerland, 1829–1836), and systematized by* **Charpentier** *— at last found a forceful spokesman in the person of Louis Agassiz. By* 1840*,*

[468] Defined as a period in the earth's history during which ice sheets covered large regions of land. The name *Eiszeit* was coined by **Karl Schimper** (1803–1867, Germany), in 1836.

Agassiz was able to convert the British geologists **William Buckland** *(1784–1856) and* **Charles Lyell** *(1797–1875) to his ice-age theory, and by* 1860*, most European geologists were on his side.*

Why did this theory, whose validity now seems self-evident, encounter so much resistance? In part, its slow acceptance may be attributed to natural resistance to new ideas — particularly if those ideas run counter to long-held scientific principles or to religious convictions. The Agassiz theory challenged both biblical dogma and scientific orthodoxy.

Once science was persuaded that an age of ice had occurred in the past, there began a period of intensive search for clues that would enable geologists to deduce details of what happened thousands of years earlier. It seemed that Agassiz had chosen an opportune moment in history to propose his theory. For in the prosperous years of Queen Victoria's reign, the wealth generated by the industrial revolution and the resources of a far-flung empire made possible the organization of geological expeditions to the farthest corners of the earth.

Victorian geologists had both theoretical and practical motives for searching so persistently for evidence about the ice-age world. There was, of course, a natural desire to fill in the pieces of the puzzle with which Agassiz had presented them, but economics *provided an additional motive: In every civilized country, geological surveys were organized to assess the potential economic value of little-known regions. (Nowhere is this better illustrated than in the United States where, in the years following the Civil War, the West was explored and mapped by geologists on horseback. To carry on this work, the U.S. Geological Survey was created by act of Congress in 1879.)*

Victorian geologists were surprised to find that the great ice sheets in the Northern Hemisphere had a northern, as well, as southern a boundary. Thus, Agassiz' idea that a single ice sheet had spread out to cover most of the Northern Hemisphere from a center at the North Pole, was found to be incorrect. In fact, individual ice sheets had expanded from different spreading centers.

Calculations showed that continental ice sheets in the Northern Hemisphere were about 1.5 *km thick. Worldwide lowering of sea levels caused shorelines to move downwards by about* 110 *m. By* 1875 *geologists had completed their initial survey of what the world of the last ice age was like. They had mapped its glaciers, measured its sea level, and determined which areas had been cold and wet, which cold and dry. They had also discovered that the ice age was* not *a unique event — that, in fact, there had been a succession of ice ages, each separated by warmer,* interglacial *ages similar to the present one. Armed with these observations geologists were ready to turn their attention from facts to theories.*

Since instrumental records of climatic elements go back only a couple of centuries (at best), how do scientists study climates and climatic changes prior to that time?

They must reconstruct past climates from indirect evidence, i.e., they must examine and analyze accessible records of phenomena that respond to and reflect changing atmospheric conditions.

Today the hippopotamus is confined to the tropics of East Africa, but $100,000$ *years ago herds of these giant mammals lived far to the north, in present-day England. The plant* Armeria sibirica*, found today only in the Arctic tundra of northern Canada, grew in southeastern Massachusetts* $12,000$ *years ago; clearly, England must have been more tropical, and Massachusetts must have resembled Arctic Canada during those respective epochs.*

The fossilized remains *of animal and plant life as those just cited, provide important clues from which inferences are made about the duration and geographic extent of climatic conditions in the geologic past. However, since fossil evidence is meager or nonexistent in the geologic record prior to about* 550×10^6 *years ago (the so-called* pre–Cambrian explosion *of new life–forms), the very history of our planet, from its formation* 4.6×10^9 *years ago until the start of the Paleozoic era, is not sufficiently well known to determine these most ancient climatic changes.*

In addition to fossils, other types of geological *evidence yield indications of past climatic conditions:* coral reef *terraces in New Guinea,* glacial features *in Africa,* solidified sand dune *outcroppings in humid regions, and countless other examples have been made available to climatology from the science of geology. Much of this evidence provides us with only a* broad and generalized picture *of past climatic changes on the scale of thousands to millions of years.*

Among the most interesting and important techniques for analyzing the climatic history on earth on time scales of 100*'s to* 1000*'s of years, are the study of* ocean floor sediments *and* oxygen isotope analysis*.*

Although seafloor sediments are of many types, most contain the remains of organisms that once lived near the sea surface (the ocean-atmosphere interface). When such near-surface organisms die, their shells slowly settle to the floor of the ocean where they become part of the sedimentary record. Working on the basic assumption that changes in climate in the ocean/atmosphere interface (i.e., temperature equilibrium between surface seawater and the air above it) should be reflected in changes in organisms living near the surface of the deep sea leads us to the notion that the temperature sensitivity of the shell organisms found in the sediments is somehow connected to climatic changes on earth.

The second technique, oxygen isotope analysis, is based on a precise measurement of the ratio between two isotopes of oxygen — ^{16}O *(which is the most common), and the heavier* ^{18}O*. Since the lighter isotope* ^{16}O *evaporates more readily from the oceans, precipitation (and hence the glacial ice that it may form) is enriched in* ^{16}O*. Of course, this leaves a greater concentration of the heavier isotope,* ^{18}O*, in the ocean water. Thus, during periods when glaciers are extensive, the concentration of* ^{18}O *in seawater increases. Conversely, during the warmer interglacial periods, when the amount of glacial ice drops dramatically, the ratio* $\{^{18}O/^{16}O\}$ *in ocean water also drops. As certain microorganisms secrete their shells of* $CaCO_3$ *which later settle in seafloor sedimentary layers, the prevailing* $\{^{18}O/^{16}O\}$ *ratio at the epochs in which the organisms lived and metabolized, can be inferred from the isotope ratios of corresponding layers. Consequently, periods of glacial activity can be determined from variations in* $\{^{18}O/^{16}O\}$ *found in the shells of microorganisms buried in deep sea sediments.*

A second use of the $\{^{18}O/^{16}O\}$ *ratio technique is applied to the study of cores taken from ice sheets, such as the one that covers Greenland. In this application another cause for variation of the oxygen isotope ratio is used, namely its temperature dependence; more* ^{18}O *is evaporated from the oceans when the temperatures are high and less is evaporated when temperatures are low. Thus, the heavy isotope is more abundant in the precipitation of warm eras and less abundant during colder periods. Using this principle, scientists studying the layers of ice and snow in Greenland have been able to produce a record of past temperature changes*[469].

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[469] Since the 1980's several studies of ice cores drilled from thick glaciers in Greenland and Antarctica have offered evidence of a correlation between CO_2 and global climate. Those cores showed that CO_2 levels in the atmosphere were much lower during ice ages than during comparatively warm periods such as the present. The finding has amplified concerns regarding the allegedly ominous implications of the large quantities of CO_2 that humans continue to dump into the air. However, in 1993, the ice core data on atmospheric CO_2 have come under assault: an alternative method of extracting air from a 35,000-year-old ice sheet from the Greenland ice sample yielded 250 ppm of CO_2, only slightly below the modern but preindustrial levels of about 270 ppm. This indicates that the relation between CO_2 and ice ages is still far from clear.

Studies of ice cores also revealed that the *temperatures* recorded in the Greenland ice cores *fluctuated rapidly* during the last ice age, warming and cooling over the course of a decade or less. These short-term climate fluctuations may result from changes in atmospheric circulation patterns, which in turn may result from *variations in the brightness of the sun.*

The theories that have been proposed to explain climatic changes are many and varied. Four of the leading theories, unrelated to human activities, include changes brought about by: continental drift, volcanic activity, fluctuations in solar output and changes in the earth's orbital elements. No single theory explains climatic change on all time scales; A theory that explains variations over millions of years, for example, is generally not satisfactory when dealing with fluctuations over a span of mere hundreds of centuries.

It is now believed that during the geologic past, continental drift *accounted for many climatic changes as land masses shifted in relation to one another and moved to different latitudinal positions. Changes in oceanic circulation must also have occurred, altering the transport of heat and moisture, and consequently the climate as well.*

Since the rate of movement of the continent–carrying tectonic plates *is very slow (of the order of a few centimeters per year), appreciable changes in position of the continents occur only over great spans of geological time. Thus, climatic changes brought about by continental drift are extremely gradual and happen on a scale of millions of years. As a result, the theory of plate tectonics is not useful for explaining climatic variations that occur over periods less than, say, a million years.*

Some aspects of climatic variability can be explained by the volcanic dust theory: *Explosive volcanic eruptions emit great quantities of fine-grained debris into the atmosphere. Some of the biggest are sufficiently powerful to inject dust and ash into the stratosphere, where it is spread around the globe and where it may remain for several years. The basic premise of the volcanic dust theory is that this suspended volcanic material will filter out a portion of the incoming solar radiation which, in turn, will lead to lower air temperatures.*

For volcanic activity to have a pronounced impact, many great eruptions, closely spaced in time, would have to occur[470]*. (Episodes like the Krakatoa*

[470] About 50 to 100 major eruptions per century would be needed to maintain solar radiation levels at about 20 percent below normal. It has been calculated that the rate at which eruptions were occurring at the end of the last century was probably comparable with that during the last Ice Age. For example, many of the great volcanoes in the Andes seem to have been constructed wholly within the past few million years, but few of them show much evidence of activity *after* the end of the Ice Age — which lasted in all about a million years. Sediment cores obtained by the *Deep Sea Drilling Project* (1974) revealed evidence for a world-wide wave of volcanic activity during the period of the last Ice Age. However, there is always the possibility that an ice age could *cause* volcanic activity!

The eruption of Mt. Pinatubo in the Philippines on June 15–16, 1991 provided *event of Aug.* 27, 1883 *or the Mount St. Helens eruption of May* 18, 1980 *— do not count.) Since no such period of explosive volcanism is known to have occurred in historic times, the volcanic dust theory is most often mentioned as a possible cause for prehistoric climatic shifts, such as the Ice Ages.*

Although the analysis of deep-sea sediment cores indicated that there was a much higher rate of explosive volcanism during the past 2 *million years than during the previous* 18 *million years, the data is not sufficiently accurate to deal with glacial-interglacial episodes which require alternating periods of explosive volcanism. The* variable sun theory *is based on the idea that the sun is a variable star and that its output of energy varies through time. Unfortunately for the theory, no major variation in the total intensity of solar radiation has yet been measured outside the atmosphere, and each ground-based observation must be corrected for large atmospheric effects.*

Nevertheless some correlation has been established between climatic changes during the past 5000 *years and a certain aspect of solar activity, namely:* sunspot cycles[471]*. In 1890,* **Edward Walter Maunder** *(1851–1928, England), superintendent of the Royal Greenwich Observatory in London, after a search through old books and journals, discovered that during* 1645–1715*, sunspots and other solar activity had all but vanished from the sun (no induced aurora borealis activity and no coronal streamers). This meant that the sun was not the regular and predictable star as had previously been believed.*

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definite evidence for the cooling of the earth's surface. The 20×10^6 tons of sulphur oxides that were ejected into the stratosphere caused dramatic climatic changes in the Middle East and Europe, manifesting itself through stormy winter and cooler summer in 1991–1992. The Pinatubo eruption affected also the *ozone layer*, through the emission of chlorine. Although volcanic eruptions tend to cool the earth, in contrast to greenhouse gases, Pinatubo is nonetheless providing critical data for climate models that seek to predict the effect of human induced changes.

Some mega-explosions of the past, however, dwarf the last century's eruptions. These biggest perturbations of climate really show what a large volcanic eruption can do. Eruptions of massive flood basalts, for example, seem to have been linked to mass extinctions of life. Huge asteroid impacts may also alter climate.

471 *Sunspots* — dark blemishes on the surface of the sun. Although their origin is uncertain, sunspots have been established to be huge magnetic storms that extend from the sun's surface deep into the interior. Further, these spots are associated with the sun's ejection of huge masses of particles which, upon reaching its upper atmosphere, interact with the gases there to produce auroral displays. Data since the early 1700's indicates an almost regular *cycle of 11 years* in annual number of sunspots.

The above sunspot-quiescence period is now known as the 'Maunder minimum'[472]. *This minimum corresponds very closely to the coldest portion of a 400-year interval known in climatic history as the 'Little Ice Age' (1450–1850 CE). It is a well-documented cold period that saw Alpine glaciers in Europe, Alaska and New Zealand advance farther than at any time since the last major glaciation* 15,000 *years ago*[473]. *There is a very powerful, modern technique that sheds further light upon this subject and enables us to extend the discussion to very ancient times.*

Radioactive ^{14}C *is continually formed in the upper atmosphere through the action of cosmic rays. When the sun is very active, its extended magnetic fields shield the earth from cosmic rays (beyond the shielding effect of earth's own field), with the result that very little* ^{14}C *is formed. When the sun is less active, as during the Maunder minimum, the reverse is true. It is fortunate that trees provide a record of the amount of* ^{14}C *in the atmosphere. By analyzing the wood in the annual growth rings of very old trees, the ratio of* ^{14}C *to the common stable isotope of carbon,* ^{12}C*, can be determined.*

From an analysis of ^{14}C *data over the past* 5000 *years it is found that there have been at least* 12 *solar excursions as prominent as the Maunder minimum; A search of old records of Alpine glacial advances and retreats correlates exactly with period of greater or lesser sunspot activity as determined from* ^{14}C *data. Therefore it appears that for the past* 5000 *years, all climatological curves rise and fall in response to the long-term level of solar activity.*

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[472] In 1887, **Gustav Spörer** (Germany) noticed the sunspot-deficient period, and first brought it to the attention of Maunder. In 1977, J.A. Eddy *et al.* (*Scientific American* **236**, 80–92) compared old drawings of sunspots made by **Christoph Scheiner** in 1625–1626 with those made by **Johannes Hevelius** in 1642–1644, and concluded from these patterns that the rotation of the sun's equator speeded up (completing a rotation one full day faster than it had in 1625) just before the onset of the Maunder minimum! It is not known whether this was a cause or an effect of the Maunder minimum.

[473] Snow lay for months on the high mountains of Ethiopia (where it is now unknown). Global climate generally was 1 °C cooler than now. Colonists in New England endured winters far more severe than any today. The 17^{th} century Dutch grandmasters **Rembrandt van Rijn**, **Frans Hals**, and **Jan Vermeer** painted winter landscapes, which feature spectacular snowdrifts, *frozen canals*, and skaters everywhere. (The canals were built in the early 17^{th} century to link the Netherlands' biggest cities. During many winters in those days, the canals really were frozen and impassable, sometimes for as long as three months. Today, the canals of Holland rarely freeze over — yet the great painters did not lie.) The detailed books kept by French winemakers show that the abnormally cold period 1617–1650 were bad years for the wineries of Bordeaux.

In addition to solar variability, terrestrial climate could be affected by changes in earth's circumsolar orbit. ***Newton*** *(1687) worried that the behavior of the solar system (a many-body problem in celestial mechanics) is inherently unstable – and in order for it not to fly apart, he postulated that God might be obliged to step in every now and then to set things right.* ***Laplace****, however, proved (1773) in his treatise Mécanique Céleste that long-term stability of the solar system has "no need for that hypothesis". Laplace used perturbation theory (that he invented). It begins from planetary (planetary–moon) orbits that are precisely Keplerian, and then worked out the (assumed second–order) effects of perturbation – due to the gravitational effects or other planets and moons upon the exactly calculable,* $1^{st}-$*order orbits. In the case of the orbits of moons about their planets, the sun itself is viewed as one of the perturbing bodies.*

As late as 1820, ***William Buckland****, then an Oxford Professor of Mineralogy and Geology, wrote:*

> *[The objective of geology is] "to confirm the evidences of natural religion; and to show that the facts developed by it are consistent with the accounts of the creation and deluge recorded in the Mosaic writings."*

Yet in 1840 it was understood that one cause of climatic changes could be astronomy*. So when the existence of long-term changes of climate were discovered, it was natural to investigate whether they could be attributed to astronomical causes. Indeed, by the 1840s, astronomers had already shown that the orbit of the earth undergoes slow changes.*

It is not the same ellipse this year that it was last year. The orbit would be an unchanging ellipse, if the Sun were the only source of attraction[474]*. The main deviations come about because of the presence of the moon and other planets. Gravitational perturbations due to these objects means that the force on the earth is not a simple inverse square centered on the sun, and the natural result is that the orbit is not a simple repeating ellipse.*

However the effects of the planets are relatively small. Jupiter has 10^{-3} *of the mass of the sun, and it is, on average, about* 5 *times further away. Venus, although having a mass* 390 *times smaller than Jupiter, can come much closer, within* 0.28 *AU (where the Astronomical Unit, or AU, is the average earth-sun distance) vs.* 4.2 *AU for Jupiter. Since gravity varies as l/(distance squared) and tidal gradients vary as* $1/(\text{distance cubed})$*, the effect*

474 We ignore effects due to Einsteinian GTR, which are negligible for earth, and secular variations due to tidal forces which take billions of years to effect significant orbital changes.

of Venus is often more important than that of Jupiter. But all these effects are small enough that they can be treated as perturbations, small changes, to the classical elliptical orbit. A convenient consequence of this is that the earth's orbit is always approximately *an ellipse, and we can treat the perturbations of the planets as extra forces that gradually alter the parameters of that ellipse. For example, the major axis of earth's orbit slowly precesses (rotates) relative to the "fixed" stars. This effect is big enough that it was discovered experimentally in 120 BCE by the astronomer Hipparchos, who found differences between his own measurements and those of earlier Babylonian records.*

By 1749, ***Alexis Claude Clairaut*** *had shown, using Newton's laws, that the north pole of the earth precesses with a period of 25,800 years. So, for example, 13,000 years from now, the North Pole of earth will not be pointing towards the "North Star", but will be pointing in a direction close to the star Vega.*

This happens because the Sun and Moon exert a torque on the (spin–rotation caused) equatorial bulge of the Earth. This causes the axis of rotation of the Earth to wobble, an effect completely analogous to the wobble of a tilted top – spinning while supported on a table or pivot – under the torque exerted by terrestrial gravity.

This spin precession *causes a* precession of the equinoxes *– a gradual calendrical advance of the autumnal and vernal equinoxes, solstices and the seasons in general. This also implies that the signs of the zodiac change, or "advance". When astrology was defined, about two thousand years ago, a person born in January was said to be under the sign of Capricorn, since the sun was in the constellation Capricorn at that time of year. Since then, the precession of the axis of the earth has changed by* $2000/26000 = 1/13$ *of a cycle; this corresponds to a change by about one sign of the zodiac. This means that a person who is born in January any time in the last few hundred years, was born when the sun was actually in Sagittarius — not Capricorn. Nevertheless, following tradition, such a person is still said to be "born under the sign of Capricorn". The more educated astrologers are aware of this change (and, of course, claim that they compensate for it!).*

Another consequence of the Earth's precession, noted above, is that the celestial location of the sun at the spring equinox also changes. It is presently leaving the constellation Pisces and entering Aquarius. Astrologers say that this change could have a profound effect on our lives, and it is why they talk about the future (and sometimes the present; it depends on exactly where you draw the constellation boundaries) as "the Age of Aquarius".

The astronomical theory of climate variation is based on the idea that changes in the amount of solar energy received by the earth are caused by quasi-periodicities in the earth's orbital elements. These are[475]*:*

1. *Variations in the shape (eccentricity) of the earth's elliptic orbit about the sun, through a cycle of ca* 100,000 *years.*
2. *Changes in the season–controlling angle (obliquity) that the earth's elliptic axis of rotation makes with the plane of the ecliptic (plane of earth's orbit) during a cycle that averages about* 41,000 *years.*
3. *The slow axial precession of the earth's axis that takes place during a cycle of ca* 22,000 *years.*

This theory was originated by ***James Croll*** *(1864) and developed by* ***Milankovich*** *(1920). To establish the validity of the theory, the following questions must be addressed:*

- *Are the periodicities of the earth's orbital elements significantly reflected in the geological record?*
- *Is there a significant correlation between insolation curves (sunlight received at given latitude as a function of time) and geological data?*
- *Can these insolation changes be correlated with climatic changes?*

Although variations in the distance between the earth and sun are of minor significance in understanding current seasonal temperature fluctuations, they may play a very important role in producing global climatic changes on a time scale of tens of thousands years. A difference of only 3% exists between aphelion (∼July 4^{th}, in the middle of Northern Hemisphere summer), and perihelion (∼January 3^{rd}, in the middle of Northern Hemisphere winter).

This small difference in distance amounts to about 6% in excess solar energy. However, this is not always the case. The shape of the earth's orbit changes during a cycle that astronomers say takes between 90,000 *and* 100,000 *years: It stretches into a longer ellipse and returns to a more circular shape. When the orbit is very eccentric, the amount of radiation received at closest approach (perihelion) could be on the order of 20–30% greater than at*

[475] Apart from the just–discussed spin–axis precession, which shifts the seasons through the calendar. Both this and the ellipse – axis (perihelion) precession can vary the climate, since they both affect the synchronization of the interplay between earth's distance from the sun and the angle at which its rays fall.

aphelion. This would most certainly result in a substantially different climate from what we now have.

The theory predicts a change in obliquity (tilt of the rotation axis) that varies between $22.1\,^{\circ}$*–*$24.5\,^{\circ}$ *during a cycle of* $41,000$ *years (current value:* $23.5\,^{\circ}$*). The smaller the tilt, the smaller the temperature difference between winter and summer. It is believed that a* reduced seasonal contrast *(warmer winters, cooler summers) could promote the growth of ice sheets: since winters could be warmer, more snow would fall because the capacity of air to hold moisture increases with temperature. Conversely, summer temperatures would be cooler, meaning that less snow would melt. The result could be the growth of ice sheets.*

The combined equinoctial and elliptic–axis precessions eventually resulted (and will again in the future) in the Northern Hemisphere experiencing winter near aphelion and summer near perihelion. Thus, seasonal contrasts will be enhanced *in the future, because winters will be colder and summers will be warmer than at present.*

Using these factors, Milankovich calculated variations in insolation and the corresponding surface temperatures of the earth back into the past, in an attempt to correlate these changes with the climatic fluctuations of the Ice Ages. In explaining climatic changes that result from these orbital variations, it should be pointed out that they cause little or no variation in the total *annual amount of solar energy reaching the ground. Instead, their impact is felt because the same energy is redistributed in time and coordinates on the earth's surface, i.e., they change the degree of contrast between the seasons and the latitudes.*

Among studies that have added credibility and support to the astronomical theory is one in which deep-sea sediments were examined (1976). Through oxygen isotope analysis of certain climatically sensitive microorganisms, a chronology of temperature changes going back $450,000$ *years was established. This time – scale of climatic change was then compared to astronomical calculations of eccentricity, obliquity, and precession in order to determine if a correlation did indeed exist. It was found that major variations in climate over the past half a million years or so, were closely associated with changes in geometry of the earth's orbit.*

Clearly, any predictions based on these results cannot account for climatic changes over scales of tens to hundreds of years, because the cycles in the astronomical theory are too long for this purpose. Also, prediction apply only to the natural component of climatic change, since human influence when and if significant was ignored. In addition, the astronomical theory does not provide the full mechanisms *via which the climate is modified by the orbital* *variables – only the insolation changes, which are assumed to trigger these mechanisms.*

If the astronomical theory does indeed explain alternating glacial-interglacial periods, the question may arise as to why glaciers have been absent throughout most of the earth history. The advent of plate-tectonics can supply the answer: since glaciers can form only on the continents, land masses must exist somewhere in the higher latitudes before an ice age can commence. Long-term temperature fluctuations are not great enough to create widespread glacial conditions in the tropics. Consequently, ice ages have only occurred when the earth's shifting crustal plates have carried the continents from tropical latitudes to more poleward positions.

The results of all studies indicate that the earth may have undergone 6 *to* 20 *glaciations during the past* 2 *million years. A typical glaciation lasted about* 40–60 *thousand years, and a typical interglacial lasted about* 40,000 *years. The last ice retreat began less than* 20,000 *years ago. In North America, the main center was near Hudson Bay. Ice piled up from* 2400 *to* 3000 *meters thick. The pressure of its weight caused the ice to flow westward and southward. It spread over most of North America down to about the present valleys of the Missouri and Ohio rivers. In Europe, the Scandinavian Peninsula was the center of glaciation, and it flowed southeast about* 1300 *km, almost to Moscow. It also covered northern England, Denmark, and Germany. As the glaciers retreated, the low places they had scoured out filled up with water, forming lakes, such as the Great Lakes of North America.*

During the Ice Ages Europe was for long periods extensively connected to Africa, either across the Straits of Gibraltar or by way of the Italian peninsula via Sicily and Malta. Likewise North America was connected to Asia via the Bering Strait; when the icecaps pushed down from the North, they drove the animals southward. Then, during interglacial periods, the animals could follow the melting ice northward. The ancestors of today's horse and camel originated in North America, then crossed the Bering Strait millions of years ago and spread into Asia.

Putting together all the information gathered over the last two centuries on past climates, we have learned that changes of various durations occurred. The planet has gone through alternating periods of cold and warm climates. Some changes extended over millions of years, others over tens of thousands of years; others still over one or several centuries. The causes of these variations are diverse, and the processes that regulate them numerous. Sooner or later, earth will undergo another cold period that may last for thousands of years, but it is difficult to predict when.

At present[476]*, we have no absolute proof that human activities actually influence our climates. The human influence on climatic processes has perhaps been exaggerated, recent political fashions notwithstanding. Some scientists have estimated that termites release during their respiration as much* CO_2 *as all industrial activities combined! The human species may thus play only*

[476] Milankovitch concluded, somewhat prematurely, that the problem was completely solved.

But Milankovitch's theory was abandoned when precise age estimates, made possible by **Willard Libby**'s invention of radiocarbon dating, appeared to show that the timing of the ice ages were in conflict with Milankovitch's detailed calculations. In retrospect, this was not justified. The Milankovitch theory actually explains many of the phenomena that we now see in the data. Do we throw out the astronomical theory of the seasons, simply because the first day of Spring is not always Spring-like? The warm weather of Spring can be delayed by a month, or it can come early by a month; the important fact is that it always comes. With complex phenomena, it is sometimes too much to demand of a theory that it predict all the details in addition to the major behavior.

In fact, it was the observations of the regularity of the ice age cycles that led to the revival of the insolation theory. Scientists developed and promoted the use of isotopes to measure records of past changes in the earth. The technology for obtaining sea floor cores rapidly improved. In 1970, it was shown for the first time that the dominant variation in the ice ages was a repeating cycle of 100,000 years. This was a frequency that appeared in the insolation theory. The use of geomagnetic reversals in sea floor cores allowed a vastly improved time scale. In 1976, it was shown that the presence of both a 41,000 and 23,000 years cycle existed in the data derived from sea floor sediments. The same frequencies were dominant in spectral analysis of the insolation predicted by the theory. Even if the details of the theory were wrong, the presence of the same frequencies as those present in the orbits of the planets was a strong reason to revive the astronomical theory.

But the growing number of problems with the insolation theory is cause for serious concern. It may be the fact that insolation theory predicted the correct values of the frequencies that leads to its tenacity in holding the minds of paleoclimatologists. But there are alternatives now appearing. In 1993, it was discovered that there is another astronomical oscillation (orbital inclination) that could contribute to climate variation; it has a spectrum that is an excellent match to the narrow 100 kyr peak. The theory based on this does not have a causality problem. It does not predict a nonexistent 400 kyr peak, and it accounts in a natural way (no adjustable parameters) for the shift of frequencies that took place about one million years ago, when the dominant frequency of ice age oscillation changed from 41 kyr to the present value of 100 kyr.

a minor role in the history of climate. Furthermore, compared to natural catastrophes and variations, human activities are perhaps minor: our scientific knowledge and technological prowess do not allow us as yet to channel solar winds, to prevent volcanic eruptions, regulate the rotation of the earth, change the size of giant ice-caps, decimate the deserts encroaching upon our meadows, push back the oceans drowning our coastlines, ward off endless droughts or avert hurricanes and torrential rains.

To forecast the climates of coming centuries remains a difficult task because too many poorly known factors have to be dealt with: volcanic eruptions, increase of CO_2 in the atmosphere, jet stream and ocean currents, nebulousity, albedo, changes in polar ice cover, and human activities. Our forecasts therefore remain theoretical, and our working models vague. Much remains to be known before climatic forecasting can become an exact science.

1920–1929 CE **Otto Loewi** (1873–1961, Germany and USA) and **Henry Hallet Dale** (1875–1968, England) independently isolated *acetylcholine*, the substance released by the vagus nerve, thus providing evidence for chemical transmission of nerve impulses across the synapse[477]. Both focused their

[477] *The neuromuscular junction*: Each skeleton muscle fiber is connected to a fiber of a nerve cell. Such a nerve fiber is an extension of a *motor neuron* that passes outward from the brain or spinal cord. Usually a skeleton muscle fiber contracts only when it is stimulated by the action of a motor neuron. The site where the nerve fiber and the muscle meet is called a *neuromuscular junction*. At this junction the muscle fiber membrane is specialized to form a *motor end plate*. The end of the motor nerve fiber is branched, and the ends of these branches project into recesses (synaptic clefts) of the muscle fiber membrane. The cytoplasm at the ends of the nerve fibers is rich in mitochondria and contains many tiny vesicles that store chemicals called *neurotransmitters*. When a nerve impulse traveling from the brain or spinal cord reaches the end of a motor nerve fiber, some of the vesicle release a neurotransmitter into the gap between the nerve and the motor end plate. This action stimulates the muscle fiber to contract.

The *neurotransmitter* is a compound called *acetylcholine*, the general formula of which is

$$CH_3\text{—}\overset{\overset{\displaystyle O}{\|}}{C}\text{—}O\text{—}CH_2CH_2\text{—}N^+(CH_3)_3$$

This substance is synthesized in the cytoplasm at the distal end of the motor work on the physiology and pharmacology of chemicals occurring naturally in animals.

Loewi was born in Frankfurt to a Jewish family and educated at Strasbourg and Munich and appointed professor of pharmacology at Graz (1909–1938). He was forced to leave Nazi Germany (1938) and became research professor at New York University College of Medicine (1940–1961). He demonstrated (1920) the release of stimulating and inhibitory substance (acetylcholine) from terminal branches of nerve fibers. This discovery led to the concept of *nerve impulse transmission* across junctions by means of chemical mediators or neurotransmitters.

Dale was born in London and studied at Trinity College, Cambridge, qualifying in medicine from St. Bartholomew's Hospital (1902). He became director of National Institute for Medical Research, London (1928–1942). Identified *histamine* (1910); Isolated acetylcholine from biological material (1914) and later (1929) showed it to be produced at the ending of parasympathetic nerves. Both shared the Nobel Prize for Physiology or Medicine (1936).

1920–1947 CE **Emil Leon Post** (1897–1954, USA). Mathematical logician. Initiated the modern metamathematical method in logic. Obtained results similar to **Gödel**, **Turing** and **Church** in the 1920's but did not publish them. Modern proof theory, and likewise the modern theory of machine

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neuron and stored in its vesicles. When a nerve impulse reaches the end of the nerve fiber (it travels at a speed of 91 $\frac{\text{m}}{\text{sec}}$ in man's largest nerves and at a speed of 0.5–1.8 $\frac{\text{m}}{\text{sec}}$ in the smaller fibers of the autonomic nervous system), many of these vesicles discharge their acetylcholine into the gap between the nerve fiber and the motor end plate. The acetylcholine diffuses rapidly across the gap ($\sim 10^{-3}$ sec), combines with certain molecules (receptors) in the muscle fiber membrane, and thus stimulates the membrane. As a result of this stimulus, a *muscle impulse* (action potential), very much like a nerve impulse, passes in all directions over the surface of the muscle fiber membrane and deep into the fiber. In response to the muscle impulse, the membrane becomes more permeable to calcium ions (which are present there). This motion of Calcium ions within the muscle membrane cause, in turn, to the muscle contraction. Due to this very contraction, the calcium concentration is lowered, and within a fraction of a second the muscle fiber relaxes. Meanwhile, the acetylcholine that stimulates the muscle fiber in the first place is rapidly decomposed by the action of an enzyme called *cholinesterase*, which is present at the neuromuscular junction within the membrane of the motor end plate. This action prevents a single nerve impulse from causing a continued stimulation of the muscle fiber. The energy used in muscle fiber contraction comes from ATP molecules, which are supplied by numerous mitochondria positioned within the myofibriles.

computation, hinge on the concept of the *recursive function*. This important number-theoretic concept was discovered independently by four mathematicians, and one of these was Post. Subsequent work by Post was instrumental to the further progress of the theory of recursive functions.

Post was born in Augustow, Poland to Jewish parents. He arrived in New York City (1904), and lived there for the remainder of his life. His life was plagued by tragic problems: he lost his left arm while still a child and was troubled as an adult by recurring episodes of disabling mental illness.

While still an undergraduate at City College, he worked out (1917) a generalization of the differential calculus which later turned out to be of practical importance. In his doctoral dissertation at Columbia University (1920) he proved the *consistency of the propositional calculus* described in Whitehead and Russell's 'Principia Mathematica'. His work marks the beginning of *proof theory*. His researches while a postdoctoral fellow at Princeton (1921–1923) anticipated later work by Gödel and Turing, but remained unpublished until much later, partly because of the lack of a receptive atmosphere for such work at the time. In 1924 Post went to Cornell. Because of recurring bouts of his illness he resumed work as a high school teacher in New York (1929), but in 1935 was appointed a professor of mathematics at City College NY and remained there until his death. In 1936 he proposed what is now known as a *Post machine*, a kind of automaton which predates the notion of a program which von Neumann studied in 1946. Post showed (1947) that the word problem for semigroups was recursive insolvable, a problem posed by Thue in 1914.

His work in computability theory includes the independent discovery of Turing's analysis of the computation process, various important unsolvability results, and the first investigations into degree of unsolvability, which provide a classification of unsolvable problems. He died quite unexpectedly while under medical care.

1920–1951 CE **James Waddell Alexander** (1888–1971, USA). Mathematician. Contributed to algebraic topology. Began his research career by putting the ideas of **Poincare** on a more rigorous foundations. In collaboration with **Veblen** he showed (1920) that topology of manifolds could be extended to polyhedra and that the homology of a simplicial complex is a *topological invariant*. Around that time he also made fundamental contributions to the theory of algebraic surfaces and to the study of *Cremona transformations*.

In his work on the *Jordan-Brower separation theorem* he discovered his *Alexander Duality Theorem* and *Alexander's lemma on the* $n-sphere$. In 1924 he introduced the *Alexander horned space*. In 1928 he discovered the

Alexander polynomial which is much used today in *knot theory* and opened the field of *combinatorial theory of complexes*. In 1935 he discovered *Cohomology theory* (independently discovered by Kolmogorov in 1936).

Alexander was born in Sea Bright, New Jersey. Educated at the universities of *Princeton* (B.S. 1910, M.S. 1911, Ph.D. 1915), *Paris* and *Bologna*. He spent most of his academic career as a member of the Institute for Advanced Study in Princeton (1933–1951). He had become a millionaire through inherited wealth and never drew a salary from the Institute.

1920–1965 CE **Solomon Lefschetz** (1884–1972, USA). Mathematician. Made major contributions to the theories of algebraic topology, differential equations and the stability of nonlinear control systems. Lefschetz had widened the use of topological methods in algebraic geometry as well as in differential equations, and consolidated the use of algebraic methods in topology.

Lefschetz was born in Moscow to Jewish parents. He studied engineering in Paris but later turned to mathematics[478]. He took his doctorate and taught at Kansas University, USA (1913–1925), where he soon made a reputation by his work in algebraic topology. In 1925 he moved to Princeton where he remained until his retirement (1953), when he became a visiting professor at Brown University. Lefschetz was the leading topologist of his generation in the USA and an important theorem on the existence of *fixed points* of mapping bears his name.

His work during WWII roused his interest in differential equations, and he continued to work on their *qualitative theory*. In his 80's he worked on the topology of Feynman Integrals.

1921 CE Development of the *teleprinter* greatly speeded the transmission of long-distance information.

1921–1923 CE **Arthur Holly Compton** (1892–1962, U.S.A.). Physicist. One of the pioneers of high-energy physics. Discovered the '*Compton effect*' through which a high energy photon is scattered by an electron. The electron, initially stationary, recoils with some energy, and the scattered photon, initially with energy $h\nu_0$, is left with a smaller energy $h\nu$, which varies with angle θ. The ensuing change in the photon's wavelength is $\Delta\lambda = \frac{\hbar}{m_0 c}(1 - \cos\theta)$,

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478 He had the misfortune to lose both his hands in a laboratory accident (1910). This mishap was fortunate for mathematics for at the age of 36 Lefschetz went to the USA and turned to mathematics. He had two artificial hands over which he always wore shiny black gloves. First thing every morning, a graduate student had to push a piece of chalk into his hand and remove it at the end of the day.

where $\frac{\hbar}{m_0 c} = 0.024\ \mathring{A}$ is the electron's *Compton wavelength*. This effect was detected by Compton experimentally, using X-rays scattered from carbon with a Bragg spectrometer (1923). The agreement between theory and experiment indicates that the STR energy-momentum formulation is correct.

The Compton effect has also been detected with protons as the scattering particles. The *probability* of occurrence of the Compton effect (including the probability distribution of the scattering angle θ) must be calculated with the aid of *quantum mechanics*, which was developed in the years immediately after Compton's discovery. In 1921 Compton *speculated* that "the electron is spinning like a tiny gyroscope".

During 1913–1915 Compton devised (at Princeton) an elegant method providing a nonastronomical measurement of latitude and an additional experimental proof of the earth's rotation[479].

Compton was born at Wooster, Ohio, the son of Elias Compton, Professor of Philosophy and Dean of the College of Wooster. He was educated at the college graduating B.Sc. (1913) and spent three years in postgraduate study at Princeton University; M.A (1914); Ph.D. (1916). Professor of Physics at Washington University, St. Louis (1920); University of Chicago (1923); Washington University (1945–1961).

He won the Nobel prize for physics in 1927.

1921–1924 CE **Pavel Samuilovich Uryson** (1898–1924, Russia). Mathematician. Creator of theories of abstract topology that influenced the subsequent development of this field[480]. His main results are the introduction and investigation of a class of so-called *normal spaces*, and metrization theorems, including a theorem on the existence of a topological mapping of any normed space with a countable base into a Hilbert space. The principal tool used in all the recent investigations of normed spaces is the classical *Uryson's Lemma*[481]. In his theory, Uryson presented an inductive definition of *dimensionality* (1921–1922).

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[479] A.H. Compton, 1913; "A Laboratory Method of Demonstrating the Earth's Rotation", *Science*, 37: 803–806; "A Determination of Latitude, Azimuth, and the Length of the Day Independent of Astronomical Observations", *Phys. Rev.*, 2, 5: 109–117. 1915.

[480] In particular, by **Andrei N. Tikhonov** (1906–1993) and **Juliusz P. Schauder** (1899–1943).

[481] *Uryson's Lemma*: Given two disjoint closed sets in a normal (T_4) topological space, there exists a continuous real-valued function that is 0 on one of the sets and 1 on the other.

Uryson was born in Odessa to Jewish parents. He was educated at the University of Moscow (1915–1919) and in 1921 became an assistant professor there. The reports he delivered at the Mathematical Society of Göttingen in 1923 attracted the attention of Hilbert, and in the summer of 1924, while touring Germany, Holland and France, he met L.E.J. Brouwer and Felix Hausdorff, who praised his works highly. Uryson drowned off the coast of Brittany (*Batz-sur-Mer*, a small town on the Atlantic Ocean) at the age of 26, while on vacation.

Interaction of Radiation with Matter (1860–1921)

One of the greatest achievements of 19^{th} *century physics was the electromagnetic theory of radiation, formulated by* ***Maxwell*** *in the early 1860's. Not only did this theory put physical optics on an entirely new basis, but it suggested a means by which radiation can act and be acted upon by atoms, namely, through the interactions of electric and magnetic fields with charged particles forming part of the atomic structure. The growing body of evidence concerning the existence of such particles led in due course to the classical electron theory of* ***Lorentz****, in which the classical interactions of electrons with the electromagnetic field were fully developed. This theory was able to account for many observed phenomena like dispersion, scattering, and the* ***Zeeman*** *effect.*

However, a detailed theory of atomic interactions must aim to explain the vast amount of spectroscopic data concerning spectral lines emitted by the atoms of different elements. The work of ***Balmer****,* ***Rydberg***[482]*, and others showed that hydrogen and the alkali metals emit discrete spectral lines which form distinct series, but all attempts to account for these series by classical methods failed. It became clear that light possesses certain properties which are not embodied in the electromagnetic theory, despite the successes of that theory in the realms of radio propagation and physical optics.*

[482] **Johannes Robert Rydberg** (1854–1919, Sweden). Physicist. Discovered (1890) the *Rydberg constant*, a spectral parameter that appears in Bohr's atomic theory (1913).

The new properties of radiation were realized in a different context by ***Planck*** *in 1900. His studies of the continuous "blackbody" radiation convinced him that the equipartition theorem of classical statistical mechanics cannot apply to an assembly of electromagnetic waves in an enclosure, and he boldly introduced the quantum hypothesis, according to which radiant energy is emitted and absorbed only in discrete amounts. The provisional quantum theory accounted for the blackbody energy distribution, and was successfully applied by* ***Einstein*** *to the photoelectric effect in metals and the specific heats of solids. Not only did the new theory explain thermal phenomena in which equipartition breaks down, but it also provided the essential clue toward solving the problem of the interaction between atoms and radiation.*

The discovery of X-rays by ***Röntgen*** *in 1895 stimulated research in many fields, but the electromagnetic character of the rays was not established for several years. Indeed, the development of quantum theory raised profound questions concerning the nature of radiation in general.*

As early as 1910 ***W.H. Bragg*** *pointed out that X-rays probably partake in the nature of both electromagnetic waves and corpuscles possessing definite energy and momentum. On the one hand, experiments on polarization and diffraction phenomena demonstrated the wavelike behavior of X-rays; on the other hand, studies of the photoelectric and* ***Compton*** *effects revealed properties similar to those of particles. The introduction of quantum theory therefore meant that models based on Maxwell's theory are incomplete in the realm of optics — a fully developed theory must include within its formalism both the wave and corpuscular aspects of radiation.*

1921–1926 CE **Max Born** (1882–1970, Germany). Distinguished physicist. Gave the probabilistic and statistical interpretation to quantum (*wave*, *matrix*) mechanics. He was first to recognize that the square of the modulus of the complex Schrödinger wave-function can be interpreted as a statistical distribution which describes the behavior of a single subatomic particle in space and time.

The time-independent form of Schrödinger's wave equation in three dimensions is

$$\nabla^2\psi + \frac{8\pi^2 m}{h^2}\left[E - U(\boldsymbol{r})\right]\psi = 0.$$

Here h is Planck's constant, m is the mass of the particle, E is the energy level (discrete or continuous), $U(\boldsymbol{r})$ the potential energy and ψ the wave-function[483]. Although this formalism is successful in determining energy levels in atomic systems, there remains the question of the significance to be attached to the function $\psi(\boldsymbol{r})$. For bound states, or for free (near-continuum) states in a large but finite box, a physically meaningful solution is achieved only when $\psi(\boldsymbol{r})$ is square-integrable over the entire space. Each such solution is a *state function* describing the state of the atomic system. This complex function – also known as a *probability amplitude* – is related to the probability density for a particle being detected at any chosen point, when the system is in the appropriate state.

Born's interpretation of the state function is that the *square* of its modulus at any point,

$$|\psi(\boldsymbol{r})|^2 = \psi^*(\boldsymbol{r})\psi(\boldsymbol{r}) \qquad [* \ = \ \textit{complex conjugate}],$$

is the *probability density function*, that is, the probability per unit volume of the particle being at the point $\boldsymbol{r}$. This principle enables us to impose a normalization condition for the particle, since the total probability is unity if we integrate over all space

$$\int \psi^*(\boldsymbol{r})\psi(\boldsymbol{r})d^3r = 1.$$

This interpretation extends to a time-dependent probability distribution, over space and internal degrees of freedom, for a particle (or system of particles) in a non-stationary state, that is, a superposition of different eigenvectors. The spatial *Fourier transform* of the wave function $\Psi(\boldsymbol{r}, t)$ is yet another kind of probability amplitude; its modulus-squared is the probability density function for values of the particle's *momentum* vector, when measured at time t.

Born's interpretation served to emphasize that a particle cannot in general be precisely located, especially when it is in a well-defined energy state (Heisenberg's *uncertainty principle*). The motion of particles follows probability laws, but probability amplitudes themselves propagate in conformity

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483 A value of E for which mathematically well-behaved solutions ψ exist, is an *eigenvalue* of the Hamiltonian operator $H = -\frac{\hbar^2}{2m}\nabla^2 + U$ ($\hbar = h/2\pi$ being the reduced Planck constant), and ψ is then the corresponding *eigenvector* (in the sense of functional vector-spaces) or eigenfunction for that energy level. ψ may be an undulatory wave spread over all space (for E in the *continuum* — e.g. for an ionized electron), or a localized wave ('bound state' — e.g. an electron occupying an orbital in an atom).

For *bound* states, E takes discrete values and $E_{\min}$ corresponds to the '*ground state*'; for *continuum* states, E ranges freely over some interval $E_c < E < \infty$.

with the laws of causality. When the particle is free (a 'continuum state'), an initially-localized state function spreads with time over the entire accessible volume. The degree of spread depends, in general, on the mass of the particle, and is greatest for small values of the mass, since the particle's (de Broglie) wavelength is inversely proportional to the momentum[484].

One result of this general principle is that electrons cannot be completely localized by any method inside an atom without completely destroying its original energy state (orbital). On the other hand, atoms themselves are much more massive than electrons and can be located with fair precision in most physical situations, for example, by X-rays in crystal analysis, or individually using tunneling microscopes. Macroscopic or even mesoscopic or nanoscopic objects (DNA nucleotides, Buckeyballs, quantum wires, protein molecules, viruses, cells, magnetic domains, colloidal particles, etc.) possess very short de Broglie wavelengths, so a classical description of the motion of such objects is accurate enough for all ordinary purposes.

Born arrived at his interpretation by analogy with *optics*, where the calculated *intensity* of the light at a given place is taken as a measure of the *probability* of finding a *photon* at that place. He suggested a similar probability interpretation of *matter waves*.

He also introduced a useful technique, known as the *Born approximation*[485], for solving scattering problems in quantum mechanics by perturbations.

Throughout his life Born was a quick and prolific writer, publishing more than 300 scientific papers, about 31 books, as well as numerous articles on nonscientific topics. He had an encyclopedic knowledge of physics and whatever problem one brought to him, he was able to offer a useful insight or suggest a pertinent reference.

Born was born in Breslau[486], Germany, where his father Gustav Jacob Born was a professor of anatomy.

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484 More generally, one may *superpose* many states (continuum and/or bound) to form localized *wave packets*; the higher the mass, the slower the spread of the packet, and the more accurate it becomes to treat the wave packet as a classical particle.

485 Although Born wrote the basic paper on the subject, he was rather irritated when the *Born approximation* was mentioned. He once said to his collaborator Emil Wolf: "*I developed in that paper the whole perturbation expansion for the scattered field, valid for all orders, yet I am only given credit for the first term in the series*"!

486 *Wroclaw* (Breslau); a city on the Oder River. Founded around 1000 CE. Early in the 11th century it was made the seat of a bishop and its cathedral dates

He became professor of theoretical physics at Göttingen in 1921. Although he had dissociated himself publicly from the Jewish community (baptized, 1913), he was not spared by the Nazis and was ejected from Göttingen in 1933 because of his Jewish origin. He settled in England, lecturing on applied mathematics at Edinburgh University[487] (1936–1953). On his retirement in 1953, he returned to Göttingen and remained there. He shared the Nobel prize for physics in 1954.

Unlike **Einstein**, Born was not proud of his Jewish heritage and did not draw the necessary conclusions from the Nazi holocaust. The establishment of the state of Israel (1948) meant nothing to him, and he never bothered to visit it. He rushed, however, back to Germany at the first opportunity and died in his 'Fatherland'.

1921–1933 CE **Alfred (Habdank Skarbek) Korzybski** (1879–1950, Poland and USA). Scholar and philosopher of language. Originator of a system of *linguistic philosophy* and expression based on man's "time-binding capacity" to transmit ideas from generation to generation. Highly eclectic and in some respects eccentric thinker who nevertheless had considerable influence. He linked a wide range of social, psychological, intellectual and even

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from the 1100's. After having formed part of Poland, it became the capital of an independent duchy (1163). Destroyed by the Mongols (1241) but soon recovered its former prosperity and received a large influx of Germans. When Henry VI, the last duke of Breslau, died (1335), it was bought by John, king of Bohemia, whose successors retained it until 1460. Austria took over the city (1526) and Prussia seized it in 1741. It was, however recovered by the Austrians (1757) but regained by Frederick the Great in the same year. The French took over (1807) after the battle of Jena, and again (1813) after the battle of Bautzen. It became Prussian through 1814–1870 and then German (1871). Under the new German Empire it turned into one of the most important cultural centers, having some of the best academic institutions in Germany [p. 630,000 (1938)]. The city was almost totally destroyed under the Red Army siege in 1945. It then came under Polish rule. Jews lived in Wroclaw since the end of the 12th century. The entire community was eliminated (1453) by the bloody inquisitor Giovanni di Capistrano, and was Bereft of Jews for the next 200 years. In the 19th century it began to absorb Jews from both Poland and Prussia, but the entire Jewish community was destroyed in the Holocaust [p. 4400 (1817); 20,300 (1900); 20,200 (1933); 0 (1940); 8000 (1962)]. Its community furnished Germany with many distinguished scientists.

[487] One of his best students there was **Klaus Fuchs**. He and his father Emil, a protestant pastor, were German refugees because they were avowed communists. Fuchs later transmitted secrets of the atom bomb to the Soviets.

medical ills to linguistic causes and emphasized the importance of language in the transmission of knowledge.

Author of *Manhood of Humanity* (1921), *Science and Sanity* (1933). Korzybski proposed a system called "*general semantics*"[488], whose basic postulates are:

- words are not to be confused with things,
- words can never say all about anything,
- words about words about words, and so on, can go on indefinitely.

He thought that his philosophy could significantly improve the quality of life of the individual who freed himself from the confusions engendered by language.

Korzybski was born in Warsaw. Educated as an engineer, he served in Russian military intelligence in WWI. Sent to the USA in 1915 on a Russian military mission, he remained there, becoming naturalized American citizen in 1940.

1921–1942 CE **Ronald Aylmer Fisher** (1890–1962, England). Statistician and geneticist. Prominent contributor to the development of both applied and mathematical statistics, especially in the theory of statistical inference, test of significance, estimation and experimental design.

His main contributions:

- Modernized Darwin's theory of evolution through the development of *quantitative* arguments in support of the theory of natural selection[489] (1930–1937). In his book "*The Genetical Theory of Natural Selection*" (1930) he applied his new statistical techniques to genetics in order to show that Mendel's

[488] *Semantics*: the study of the conditions under which signs, symbols and words are meaningful. It is also the study of how human behavior is affected by words (spoken to others or to oneself in thought), In *philology* (the scientific study of languages), semantics is concerned with the historical study of changes in the meaning of words. Semantics deals with the relations between words and the things talked about and with meanings as a factor in human relations. Humans are the only creatures that talk themselves into trouble, and semantics is concerned with how to avoid doing so.

[489] *Fisher's Equation*: Consider a population of individuals carrying an advantageous allele (call it a) of some gene and migrating randomly into a region in which only the allele A is initially present. If p is the frequency of a in the population and $q = 1 - p$ the frequency of A, it can be shown that under the assumption of Hardy-Weinberg genetics, the rate of change of the frequency p

discoveries support Darwin's theory. He thus secured the key biological concept of genetic change by natural selection.

• Introduced the concept of *likelihood* (1921). The likelihood of a parameter involves a function usually having a single maximum value, which he called the *maximum likelihood*.

• Gave a new definition of statistics; It's purpose was the reduction of data and he identified three fundamental problems:

(i) specification of the kind of population that the data came from;

(ii) estimation;

(iii) distribution.

Developed methods suitable for *small samples* (like those of Gosset), discovered the precise distributions of many sample statistics and invented the analysis of variance.

• Evolved statistical rules for *decision making* that are now used universally, and many other methods that have since been extended to virtually every academic field to which statistical analysis can be applied.

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at a given location is governed by the equation

$$\frac{\partial p}{\partial t} = D\frac{\partial^2 p}{\partial x^2} + \alpha p(1-p),$$

where α is a constant coefficient that represents the intensity of selection. Historically, this model elicited considerable interest and was thoroughly investigated. The equation has a variety of solutions depending on other constraints (such as boundary conditions). Among these are *traveling waves* solutions on an infinite domain. Indeed, putting $z = x - Vt$, $p(x,t) = P(z)$, one obtains the second-order ODE

$$V\frac{dP}{dz} = D\frac{d^2 P}{dz^2} + \alpha P(1-P).$$

It can be converted into a non-linear system of first order ODEs:

$$\frac{dP}{dz} = -S; \quad \frac{dS}{dz} = \frac{\alpha}{D}P(1-P) - \frac{V}{D}S.$$

Kolmogorov (1937) has shown that if suitable initial conditions are assumed, the solution of the Fisher Equation would evolve into a traveling wave that propagates at the minimal speed $V_{\min} = 2\sqrt{\alpha D}$.

- Contributed to the contemporary understanding of genetic dominance

Fisher was born in London, educated at Harrow and graduated in mathematics at Cambridge. In 1919 he became a statistician at the Rothamsted Agricultural Research Institute. There he developed his techniques for the design and analysis of experiments which he expanded in *Statistical Methods for Research Workers* (1925). He also studied the genetics of human groups, elucidating the *Rhesus factor*. He became professor of Eugenics at University College London (1933–1943) and Professor of Genetics at Cambridge (1943–1957).

1921–1944 CE **Emil Artin** (1898–1962, Germany and U.S.A.). Mathematician. Played a major role in the development of the class field theory, abstract algebra and hypercomplex numbers (noncommutative rings). Expanded the theory of algebras of associative rings (*Artin rings*), and the theory of *braids*. He also made contributions to the theory of the Zeta function, algebraic number theory (Artin's conjecture[490]) and Dirichlet series.

Artin was born in Vienna, a son of an art dealer and an opera singer. He grew up in Bohemia and held professorial positions at the Universities of Leipzig and Hamburg (1923–1937). In 1937 he emigrated to the United States and became a professor of mathematics at Princeton University (1946–1958). He returned to Hamburg in 1958.

1921–1947 CE **Ludwig Josef Wittgenstein**[491] (1889–1951, Austria and England). A thinker interested in the limits of meaning, language and thought, whose philosophical perspectives differ radically from that of professional philosophers of the 20^{th} century. Had considerable influence on *linguistic philosophy* and the *philosophy of mathematics*, through which he inspired

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490 **Artin** (1927) conjectured that if n is not (-1) and not a square, then the set $S(n)$ of all primes for which n is a *primitive root*, must be *infinite*.

Artin further conjectured that if n is not an r^{th} power for any $r > 1$, then, *independently of the choice of* n

$$\prod_{k=1}^{\infty}\left[1-\frac{1}{p_k(p_k-1)}\right] = 0.373\ 955\ 813\ 6\cdots = \text{ Artin's Constant},$$

where the product is restricted to values of k such that p_k are members of the set $S(n)$.

491 For further reading, see:

- Monk, Ray, *Ludwig Wittgenstein*, Penguin Books, 1990, 654 pp.

other thinkers like **Bertrand Russell**, **Alan Turing**, **Moritz Schlick** and **Alfred Whitehead**.

Wittgenstein's philosophy of mathematics is not a contribution to the debate on the foundations of the subject that was fought during the first half of the 20^{th} century by the opposing camps of *logicists* (led by **Frege** and **Russell**), *formalists* (led by **Hilbert**) and *intuitionists* (led by **Brouwer** and **Weyl**). It is, instead, an attempt to undermine the whole basis of this debate — to undermine the idea that mathematics *needs* foundations.

All the branches of mathematics that were inspired by this search for 'foundations' — *Set Theory, Proof Theory, Mathematical Logic, Recursive Function Theory*, etc. — he regarded as based on philosophical confusion[492].

In his book *Tractatus Logico – Philosophicus* (1921) he offered a general means of removing philosophical difficulties by investigating the logical structure of language. He developed the view that all truths of logic are tautological, claiming that the inability to see through the logic of language is the cause of many apparently insoluble problems. He set up a system of *linguistic analysis* by which any statement must satisfy certain logical conditions before being admitted as proper philosophical statement.

Many Western philosophers considered *Tractatus* their 'bible' and much of American and British philosophy in our times has been affected in one way or another by Wittgenstein philosophy of language. It consists of remarks on the

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492 Hilbert once said: "No one is going to turn us out of the paradise which Cantor has created".

Wittgenstein told his class (1939): "I would try to show you that it is not a paradise — so that you'll leave of your own accord".

Indeed, Wittgenstein's quixotic assault on the status of pure mathematics reached a peak during the academic year 1932–33, at Cambridge. In his lecture "Philosophy for Mathematicians" he would read out extracts from **Hardy**'s *Pure Mathematics* and use them to illustrate the philosophical fog that he believed surrounded the whole discipline of pure mathematics. Since the time of **Plato**, philosophers have traditionally been divided between those who say that mathematical statements are true about the *physical* world and those who say that they are true of the *mathematical* world — Plato's eternal world of ideas or forms. To this division **Kant** added a third view, which is that mathematical statements are true manifestation of the 'form of our intuition' and this was roughly the view of **Brouwer** and the intuitionist school. But for Wittgenstein, the whole idea that mathematics is concerned with the discovery of truths is a mistake that has arisen with the growth of pure mathematics and the separation of mathematics from physical science (in the words of **Willard Gibbs** — "Mathematics is a language").

essence of language, the nature of logic, mathematics, science and philosophy, and ending with comments on ethics, religion and mysticism.

There are many reasons for the wide acceptance of the *Tractatus*. Chief among them are:

- The failure of Einstein's attempts to develop an all-embracing unified field-theory and his futile efforts to crush the subjectivistic 'psychological ghosts' introduced into the scientific interpretation of quantum theory, marked the withdrawal of philosophy from physics and physics from philosophy.
- The collapse of the Austrian Empire and consequently Vienna's loss of political and intellectual leadership, caused members of the 'Vienna Circle' to direct their mind upon themselves rather than face external events.
- The rise of Soviet Communism and dialectical materialism.

The outcome was inevitable: Western thought declined, philosophy was cornered, scientism flourished, and ideology ended up in a dead-end street. The 'inward trend' of the Tractatus seemed the adequate answer.

Wittgenstein was a descendant of a wealthy Jewish family in Vienna. His parental great grandfather, Moses Maier, worked as a land-agent for the princely family of Wittgenstein and after the Napoleonic decree of 1808 (which demanded that Jews adopt a surname), took the name of his employers. Ludwig's grandfather, Hermann, and his family converted to Protestantism (1838), moved to Vienna and were assimilated into the local German elité, shunning any contact with their former fatesakes.

Wittgenstein's father, Karl, became one of the most astute industrialists in the Austro-Hungarian Empire. Because of the Catholic denomination of his half-Jewish spouse, all his offsprings were baptized into the Catholic faith and raised as accepted and proud members of the Austria high-bourgeoisie. The Wittgensteins were thus at the center of Vienneese Germanic cultural life for three generations[493]. [**Johannes Brahms** gave piano lessons to Ludwig's aunts and **Gustav Klimt** was commissioned to paint Ludwig's sister portrait

[493] After the *Anschluss* of Austria by the Nazis (1938), the Wittgensteins discovered, to their horror, that according to the *Nuremburg Laws* they still count as Jews, unless they produced evidence for a second Aryan grandparent! So, some fled to Switzerland, some gave a sworn affidavit that Hermann was known as an antisemite who, in adult life, avoided association with the Jewish community and did not allow his offsprings to marry Jews (the Nazis saw some advantage to themselves in accepting it), while others (like Ludwig's sisters) earned their

(1905); **Mahler** was a frequent visitor to the musical evening at Ludwig's childhood home and young **Casals** played there.]

The *fin de siécle* Vienna of Ludwig's childhood was characterized by a 'nervous splendor': beneath the all-pervading atmosphere of culture and humanity, lay doubt, tension and conflict. Its tensions prefigured those that have dominated the history of Europe during the 20^{th} century. From these tensions sprang many of the intellectual and cultural movements that have shaped that history: It was the birthplace of both Zionism and Nazism, the place where Freud developed psychoanalysis, where Klimt, Schiele and Kokoschka inaugurated the *Jugendstil* movement in art, where Schönberg developed atonal music and Adolf Loos (1870–1933), the pioneer of modern architecture, introduced the starkly functional, unadorned style that characterized the buildings of the modern age.

Wittgenstein came to England in 1911, studying aeronautical engineering until he moved to Cambridge to pursue a philosophical interest in mathematics. Frege, to whom he had written, advised him to seek out Bertrand Russell and the contact was fruitful.

Wittgenstein served in WWI in the Austrian army. He was made an officer, involved in heavy fighting during 1916–1918 and taken prisoner on the Italian front (1918); he thus experienced the dehumanizing results of modern, mechanized combat, the 'grand strategies' of Total War which engineered the slaughter of millions of men in conditions of unimaginable horror.

In 1919 he dedicated himself to a life of simple asceticism (perhaps under the influence of the religious writings of Leo Tolstoy). He gave his money away to his relations and abandoned philosophy. In the following years he worked as an elementary school teacher in the Austrian countryside (until that ended in disaster), an amateur architect and a convent-gardener.

He returned to Cambridge (1929), where he was the center of a small, intensely loyal cult[494]. In 1939 he succeeded G.E. Moore as professor of phi-

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Aryan papers by handing over all their foreign currency to the Third Reich. Sic transit the Wittgenstein's wealth.

494 "*God has arrived. I met him on the* 5:15 *train*". Thus was Wittgenstein's return to Cambridge announced by one of his disciples.

Soon after his arrival, he was hurriedly awarded a Ph.D. for his thesis, the *Tractatus*, a work that had been in print for seven years and was already regarded by many as a philosophical classic. The examiners were Moore and Russell and the exam was a farce: as Russell walked into the examination room with Moore, he smiled and said: "I have never known anything so absurd in my life". The examination began with a chat between old friends. Then Russell, relishing the absurdity of the situation, said to Moore: "Go on, you've got to ask him some losophy and logic, but he was absent on menial war work soon afterwards and resigned altogether in 1947. Four years later he died from cancer of the prostate after a period of severe illness. For a long time he had meditated suicide[495] fearing that he shared the pronounced strain of insanity in his family (all his three brothers killed themselves).

Wittgenstein was an imposing figure with a fearless independent spirit. People tended to be fascinated or repelled by him, as he was very direct in his approach and impatient of any pretentiousness. The magic of his personality and style was infectious and pupils tended to imitate him. He was a deeply serious man and put his soul into everything he did. He was not learned or widely read, but would only read what he could wholeheartedly assimilate. He was not religious in the conventional sense, but had a deep respect to the Bible and some religious authors (e.g. **Kierkegaard**).

He was an engineer by training and his knowledge was intimately connected with doing. Music was central to his life, and he had no interest in modern music: the music of **Bach**, **Beethoven**, **Schubert** and **Schumann** were amongst his favorites. He intensely disliked academic life, avoided any publicity and regarded the Press as one of the disasters of modern life. Modern times to him were a dark age: the idols of progress and the belief that technology will solve all our problems, he felt were profoundly wrong. He believed that only a change in our way of life would heal the sickness of our age — and this is only to happen when disaster confronts us.

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questions — you're the professor". There followed a short discussion in which Russell advanced his view that Wittgenstein was inconsistent in claiming to have expressed unassailable truths by means of meaningless propositions. He was, of course, unable to convince Wittgenstein, who brought the proceeding to an end by clapping each of his examiners on the shoulder and remarking consolingly: "Don't worry, I know you'll never understand it".

[495] Some of his biographers believe that his homosexuality provided a key to his tormented personality, and even his philosophy.

Worldview XXXVII: Wittgenstein

* * *

"We could present spatially an atomic fact which contradicted the laws of physics, but not which contradicted the laws of geometry."

* * *

"Whereof one cannot speak, thereof one must be silent."

* * *

"In order to draw a limit to thinking, we should have to think both sides of the limit."

* * *

"Is logic the foundation of mathematics? In my view mathematical logic is simply part of mathematics. Russell's calculus is not fundamental; it is just another calculus. There is nothing wrong with a science before the foundations are laid."

* * *

"The theory he[496] *has constructed, is not metamathematics, but mathematics. It is another calculus, just like any other one. It offers a series of rules and proofs, when what is needed is a clear view. A proof cannot dispel the fog. If I am unclear about the nature of mathematics, no proof can help me and the question about its consistency cannot arise at all."*

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496 **Hilbert** (1904) endeavored to construct a formalist approach to the foundation of logic and arithmetic. It was a 'meta-theory' of mathematics, seeking to lay a *provably* consistent foundation for arithmetic.

* * *

"The sole remaining task of philosophy is the critique of language."

* * *

"To pray is to think about the meaning of life. To believe in God means to see that life has a meaning; The meaning of life, i.e. the meaning of the world, we can call God."

* * *

"I am my world."

* * *

"Don't try to improve the world. Just improve yourself; that is the only thing you can do to better the world."

* * *

"Philosophy is not a theory but an activity; the purpose of philosophy is the logical clarification of thoughts; a philosophical work consists mainly of elucidations; philosophy should take thoughts that are otherwise turbid and blurred, so to speak, and make them clear and sharp."

* * *

"I never use notes – thoughts become stale that way."

* * *

"I strongly advise you against becoming academic philosophers. The temptation to fake thinking amongst them is very great."

* * *

"Philosophy is a battle against bewitchment of our intelligence by means of language."

* * *

"There is no compulsion making one thing happen because another has happened. The only necessity that exists is logical *necessity. The whole modern conception of the world is founded on the illusion that the so-called laws of nature are the explanations of natural phenomena."*

* * *

"I should not like my writing to spare other people the trouble of thinking. But, if possible, to stimulate someone to thoughts of his own."

* * *

"The riddle does not exist. If a question can be put at all, then it can also be answered."

* * *

"Mathematics is a logical method... Mathematical propositions express no thoughts. In life it is never a mathematical proposition which we need, but we use the mathematical propositions only in order to infer from propositions which do not belong to mathematics to others which equally do not belong to mathematics."

1921–1948 CE **Norbert Wiener**[497] (1894–1964, U.S.A.). Outstanding mathematician and logician. Invented the science of *cybernetics*. He opened up to mathematical analysis many practical areas in physics and engineering which are basically statistical in nature. Wiener's work led to the treatment, solution and interpretation of many physical problems, as well as to the development of computer and communication equipment.

Wiener had a major influence on advances in radar, high-speed electronic computers, control theory, and quantum theory. He contributed in a most significant way to the mathematical analysis of the theory of feedback and automated processes, set theory, group theory, probability and mathematical logic.

Wiener was born in Columbia, MO. His father, Leo Wiener, was a historian of Yiddish culture, who later became a professor of Slavic languages and literature. He made incessant intellectual demands on his son (and did not reveal to him that they were Jews — a fact discovered by Norbert only when he was in his teens).

Wiener was a child prodigy. He could read and write at the age of 3 and read scientific books at 4. He entered Tufts University at 11, and obtained his Ph.D. from Harvard at 18. He then went to Europe and was strongly influenced by **Bertrand Russell**, **G.H. Hardy** and **D. Hilbert**, under whom he studied mathematical logic and pure mathematics.

For five years (1914–1919) he tried various occupations in search of his self realization. Finally, in 1919, he was hired as an instructor by the mathematics department at the Massachusetts Institute of Technology (MIT). Wiener remained at MIT until his retirement in 1960.

His main work started in the early twenties, on the subject of linear spaces and his development of the theory of Banach spaces. In 1930 he published a paper in *Acta Mathematica* on "*Generalized harmonic analysis*"[498], in which he generalized the theory of the Fourier integral to functions whose average

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[497] For further reading, see:

- Wiener, Norbert, *The Fourier Integral*, Dover, 1958, 201 pp.
- Wiener, Norbert, *Time Series*, M.I.T. Press, 1964, 163 pp.

[498] From about 1900 to 1930, Heaviside's methods dominated the whole of communication engineering, and their rigorous mathematical justification was considered a moot question. Toward the end of this period, rigorous mathematical justifications of the formal Heaviside calculus were found, and with this it came to be appreciated that Heaviside's work belonged together with the theory of the Laplace and Fourier integrals.

power is finite. He defined the autocorrelation, cross correlations, and power spectral density functions and established relations between them (*Wiener-Khinchin theorem*).

He demonstrated convincingly for the first time that the Fourier integral could be used as a link between two distinct branches of mathematics — statistics and analysis. Through 1921–1931, Wiener did highly innovative work on stochastic processes, in particular on the theory of the Brownian motion. During WWII Wiener worked on gunfire control (the problem of pointing a gun at a moving target). The ideas that evolved led to the theory of prediction of stationary time-series. There he introduced certain statistical methods into control and communication engineering, which led him eventually to the formulation of his concept of *cybernetics* [a word coined from the Greek work for "*Navigator*", meaning the study of control. The word is etymologically related to the English "govern" and also appears in Hebrew in the connotation of a *ship captain*].

In his book *Cybernetics* (1948), Wiener opened a new branch of science that deals with control mechanisms and the transmission of information. Wiener noted that the means for internal control and communication in an animal, such as its nervous system, were similar to those in a machine. He also realized that biologists who studied animals, and engineers who designed automatic control equipment, did not usually know each other's fields of work. He proposed that control and communication in both fields be studied together as the science of cybernetics. An important part of cybernetics is the study of *feedback*, where information concerning the error in the operation of a system is fed back to the controlling device, which then acts to correct the error.

Wiener lived up to the stereotype of the absentminded professor. He was worried about the quality of his work and his standing as a mathematician. He was a vegetarian and something of a puritan. He was also an extremely generous man, with a strong sense of social and moral responsibility.

He will be remembered for his marvelously versatile contributions, profoundly original, ranging from pure to applied mathematics, and penetrating boldly into engineering and biological sciences.

Worldview XXXVIII: Norbert Wiener

* * *

"A professor is one who can speak on any subject – for precisely fifty minutes."

* * *

"The modern physicist is a quantum theorist on Monday, Wednesday, and Friday and a student of gravitational relativity theory on Tuesday, Thursday, and Saturday. On Sunday he is neither, but is praying to his God that someone, preferably himself, will find the reconciliation between the two views."

* * *

"Progress imposes not only new possibilities for the future but new restrictions."

* * *

"We are raising a generation of young men who will not look at any scientific project which does not have millions of dollars invested in it... We are for the first time finding a scientific career well paid and attractive to a large number of our best young go-getters. The trouble is that scientific work of the first quality is seldom done by go-getters, and that the dilution of the intellectual milieu makes it progressively harder for the individual worker with any ideas to get a hearing... The degradation of a position of the scientist as an independent worker and thinker to that of a morally irresponsible stooge in a science-factory has proceeded even more rapidly and devastatingly than I had expected."

The Wiener Process (1922–1930)

Whereas from a physical viewpoint Einstein's calculations and Perrin's experiments (1909) had explained Brownian motion quite adequately, from a mathematical viewpoint the subject was still tantalizingly confused. The heart of the difficulty was to make precise mathematical sense out of the notion of a particle moving "at random". Everyone knows what it means to pick between heads and tails at random; it means each alternative has a probability of $\frac{1}{2}$ *(fair toss). The Brownian particle follows a path that is in some sense chosen at random from among all possible paths.*

The set of all possible paths, however, is a very large and complicated one, and it was one of the Wiener's major achievements in pure mathematics to show in what sense one can speak about choosing an element from this set at random. He studied the phenomenon of Brownian motion from a new point of view (1922–1930), expanding the concept of probability further than his predecessors, thereby bringing the term 'Brownian motion' into the language of mathematics.

An intuitive notion of what is involved can be obtained by considering the path traced out in a finite time period (say, an hour) by a one-dimensional motion, which changes direction only at the instances $t =$ *one second,* $t =$ *two seconds and so on. In this case there are only a finite number of possible paths (*2^{3600}*, to be exact), and one could say that the Brownian particle chooses one path at random in the sense each path has a probability of* 2^{-3600}.

In the Wiener process, the distances traveled are distributed according to a Gaussian curve, just as they are in Einstein's physical model of Brownian motion. Moreover, Wiener proved that almost certainly (with a probability 1*) the path is continuous but nowhere smooth.*

When a series of discrete events is taking place at random times [e.g. cosmic rays striking a detector or raindrops falling on a demarked area of ground], the probability that the time interval (between two consecutive events or between a fiducial time and a given event) will fall within the range dt *centered around* t *is* $\rho(t)\,dt$, *where* $\rho(t)$ *is the corresponding probability distribution. We write the probability distribution of* x *as* $\rho(x)$ *if* $\rho(x)\,dx$ *is the probability that the variable lies in the range* dx *about* x. *In two variables, the probability distribution of* x *and* y *is* $\rho(x,y)$, *which means that the probability of finding the variables* x *and* y *in the region* A *of the* xy *plane is given by* $\int_A \rho(x,y)\,dx\,dy$.

Thus Wiener asked:

"What is the probability of the Brownian particle following a certain path in space?" Since in one dimension, the time history of the particle is represented by a continuous function $x(t)$, the question is equivalent to: "What is the probability of obtaining a particular time history $x(t)$?" To this end he considered the space of paths, focusing not on the distribution of a single variable but of complete curves. This entails the construction of functionals[499]*, using mathematical tools provided by the Volterra theory of functionals (1887).*

If $x = 0$ at $t = 0$, then at time t the probability that the particle's position is between a and b is $\int_a^b \rho(x,t)\,dx$. Similarly, the probability that a particle starting at $(0,0)$ is between a_1 and b_1 positions at time t_1, between a_2 and b_2 at time t_2, ..., and between a_N and b_N at time t_N $(a_i < b_i,$ $t_i < t_{i+1})$ is

$$\int_{a_1}^{b_1} dx_1 \ldots \int_{a_N}^{b_N} dx_N \rho(x_1,t_1)\rho(x_2-x_1,t_2-t_1)\ldots\rho(x_N-x_{N-1},t_N-t_{N-1}),$$

where the appearance of the arguments $x_i - x_{i-1}$ and $t_i - t_{i-1}$ reflects the homogeneity in space and in time of this diffusion process and $\rho(x,t)$ is the Gaussian distribution defined above. This distribution fits physical phenomena which are the result of the combination of a large number of independent events occurring randomly. This is the conclusion of the central limit theorem of probability theory.

For us, the quantity of interest is the conditional probability that the path of the particle [starting at $(0,0)$] satisfies

$$a_i < x(t_i) < b_i \qquad \text{at times } t_i, \qquad i = 1,2,\ldots,N-1$$

given that $x(t_N) = x_N$. This conditional probability[500] *density is given by*

[499] *Functional*: A mapping from a space of functions to some set of numbers (such as the real or complex fields). Thus $\rho[f(t)]$ assigns to each function a *number*, in the same way that $\rho(t)$ assigns a number $\rho(t)$ to each value of t. In fact, $\rho(t)$ is a special case of $\rho[f(t)]$ for $f(t) = t$.
Phenomena in which the *probability distribution of a function* is involved may include such diverse cases as the voltage in a resistor, the price of a commodity (both as functions of time), or the shape of the surface of the sea as a function of latitude and longitude.

[500] *Conditional probability* of an event A, given the event B, is

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

It tells us the probability of an event within a *restricted* set of possible outcomes.

the above multivariable integral without the integral over x_N and divided by $\rho(x_N, t_N)$. It measures the probability that the path of the particle passes through each of the given "portals" at the associated times, given the path's endpoints at times 0 and t_N.

As we increase the density at which a specification of (a_i, b_i) is made as $|a_i - b_i| \to 0$, the path of the particle is pinned down more and more precisely. Substituting the Gaussian distribution for each factor in the integral, we can write the product of the densities for the individual intervals

$$\rho(x_1, t_1) \ldots \rho(x_N - x_{N-1}, t_N - t_{N-1}) = \prod_{j=1}^{N} \frac{1}{\sqrt{\pi(t_j - t_{j-1})}} \exp\Big\{-\frac{(x_j - x_{j-1})^2}{t_j - t_{j-1}}\Big\}.$$

In the limit $N \to \infty$, as the time intervals go to zero, the integration of the above expression defines a measure *on the set of all paths $x = x(t)$, known as the* Wiener measure *which we denote in the abbreviated notation $\int d_W\,[x(t)]$, where*

$$d_W\,[x(t)] = \frac{\rho\,[x(t)]\; Dx(t)}{\int_A \rho\,[x(t)]\; Dx(t)} = \frac{1}{C} e^{-\int_0^1 \dot{x}(\tau)^2\, d\tau} \prod_0^t dx(t).$$

Here, A is the space of allowed paths, $\prod_0^t dx(t)$ is the limit of $\prod_1^N dx_i$ and the integral in the exponent is just twice the time integral of the kinetic energy of the particle with unit mass. C is a constant which normalizes the infinite-dimensional integral under the condition $\int_A d_W\,[x(t)] = 1$.

The concept of measure *underpins the modern interpretation of probability, defined in terms of a measure on the space Ω of elementary events. A general event is a set of elementary events, i.e. a subset of Ω, and its probability is equal to the measure of the corresponding set of events, divided by the measure of the whole space Ω. As a rule, the measure is conveniently chosen such that the measure of the whole Ω is unity. Over the space of elementary events we can construct certain functions whose properties are practically independent of the structure of this space and the detailed definition of probability.*

As an example, we consider the simplest case where the Brownian-motion path is unrestricted: $(a_i, b_i) \to (-\infty, \infty)$. Then the iterated integral (with

$\Delta t = t_j - t_{j-1},$ *fixed)*

$$\lim_{N\to\infty}\Big[\frac{1}{\sqrt{\pi\Delta t}}\Big]^N \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} \exp\Big\{-\sum_{j=1}^{N}\frac{(x_j - x_{j-1})^2}{\Delta t}\Big\}\, dx_1 \ldots dx_N$$

can be transformed, with $y_j = x_j - x_{j-1},$ *into the product of ordinary integrals, which all equal* $\frac{1}{\sqrt{\pi\Delta t}}\int_{-\infty}^{\infty}\exp\left\{-\frac{y^2}{\Delta t}\right\}dy = 1.$ *The considered Wiener integral is equal to unity, which equals the total volume of the space of elementary events (particle paths in this case). Thus, the measure of the total space of paths is equal to unity.*

The Wiener measure of an event A is given by the Wiener path integral $\int_A \rho\,[x(t)]\; Dx(t),$ *which weighs each path with a Gaussian probability density functional, and then sums the weighted contribution for all possible paths between two given end-points. This process can, of course, be generalized to probability density functions other than the Gaussian. In fact, we could say that, in general, the integrand has the form* $\rho(x_1, x_2 \ldots x_N)\, dx_1\, dx_2\, \ldots\, dx_N.$ *This, in turn, can be viewed as a discretization of the functional* $\rho\,[x(t)],$ *where* $x_i = x(t_i).$ *Once the path integral has been defined, the analogy to the probability density function of a variable can be stretched further to include the concept of the mean value of any functional* $Q\,[x(t)]$ *on the random path* $x(t).$

Thus, the expectation of the function $Q(x)$ *of the random variable* $x,$ *namely* $\int_{-\infty}^{\infty} Q(x)P(x)\,dx = \langle Q(x)\rangle,$ *with* $P(x)$ *the probability distribution function, takes on an analogous form for the functional* $Q[x(t)]$*:*

$$\begin{aligned}\langle Q\,[x(t)]\rangle &= \int Q\,[x(t)]\; d_W\,[x(t)] \\ &= \frac{\int Q\,[x(t)]\,\rho\,[x(t)]\; Dx(t)}{\int \rho\,[x(t)]\; Dx(t)},\end{aligned}$$

where the functional integrals extend over the allowed histories (paths) of the particles.

Likewise, for the mean-square value of the path function at a particular time, $t = a$*:*

$$\left\langle [x(a)]^2 \right\rangle = \frac{\int [x(a)]^2\,\rho\,[x(t)]\; Dx(t)}{\int \rho\,[x(t)]\; Dx(t)}.$$

Also, the concept of the characteristic function (i.e. the Fourier transform of $p(x)$, *known as the moment-generating function)*

$$\phi(k) = \langle e^{ikx} \rangle = \int_{-\infty}^{\infty} e^{ikx} p(x)\, dx$$

has its analog in the characteristic functional

$$\Phi\,[k(t)] = \frac{\int e^{i \int k(t)x(t)\,dt} \rho\,[x(t)]\; Dx(t)}{\int \rho\,[x(t)]\; Dx(t)},$$

where $e^{i \int k(t)x(t)\,dt}$ *is the limit of the product* $e^{ik_1x_1}e^{ik_2x_2}\ldots$ *for an infinite number of time intervals, with* $k_i = k(t_j)\,\Delta t_i$. *The possibility of performing the inverse transform* $\rho(x) = \int_{-\infty}^{\infty} e^{-ikx}\phi(k)\,dk$, *leads, in principle, to the analogue expression*

$$\rho\,[x(t)] = \int e^{-i \int k(t)x(t)\,dt} \Phi\,[k(t)]\; Dk(t),$$

where the path integral is now carried out over the space of possible k-*functions. In this way Wiener showed the close connection of the Brownian motion to the Fourier transform.*

Returning to the Brownian motion, we assume that $Q\,[x(t)]$ *is some functional of the path* $x(t)$. *Let* $Q(x_1, \ldots, x_n)$ *be the value* Q *takes on the broken (discrete) line path from* $(0,0)$ *to* (t_1, x_1) *to* $\ldots$ (x_{N-1}, t_{N-1}) *to* (x,t), *with* $t_j = j\frac{t}{N}$. *Then, the mean value of* Q *is*

$$\langle Q \rangle = \lim_{N\to\infty} \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_2 \ldots \int_{-\infty}^{\infty} dx_{N-1}\, \rho(x_1,t_1)\rho(x_2-x_1,t_2-t_1)\ldots \\ \times \rho(x_N - x_{N-1}, t_N - t_{N-1}) Q(x_1, \ldots, x_{N-1})$$

It can be shown that the special case $W(x,t) = \left\langle e^{\int_0^t U[x(t)]\,dt} \right\rangle$ *obeys the integral equation*

$$W(x,t) = W_0(x,t) + \int_0^t d\tau \int d\xi\, W_0(x-\xi, t-\tau) U(\xi) W(\xi,\tau)$$

with

$$W_0(x,t) = \langle 1 \rangle = (4\pi D t)^{-1/2} e^{-\frac{x^2}{4Dt}} = \rho(x,t).$$

Recalling that $\left(\frac{\partial}{\partial t} - D\frac{\partial^2}{\partial x^2}\right) W_0 = 0$ *and that* $W(x,0) = \delta(x)$, *this integral equation for* W *implies that* W *satisfies the differential equation governing* *diffusion with random coordinate-dependent growth*

$$\frac{\partial W(x,t)}{\partial t} = D\frac{\partial^2 W(x,t)}{\partial x^2} + U(x)W(x,t)$$

For constant $U = U_0 > 0$, *the explicit deterministic solution has two remarkable features:*

(1) *there exists a critical length scale* $L_c = \pi\sqrt{\frac{D}{U_0}}$ *at which gain (amplification) and dissipative loss are in equilibrium.*

(2) *The surface* $\psi(x,t) = \frac{1}{\sqrt{4\pi Dt}}\exp\left[U_0 t - \frac{x^2}{4Dt}\right] = constant$ *propagates with the velocity* $v = 2\sqrt{DU_0}$.

The physical meaning of this result is as follows: in the course of evolution, the field W *is diffusively transported in space and simultaneously changes at a rate* U *which, although constant in this example, generally varies from one position to another; e.g. the evolution of a bacterial population in a medium with randomly distributed food supply and other environmental factors when the population is able to spread via diffusion in space. This model predicts that the bacteria gather in comparatively stable clusters at places where the environment is most favorable for their development. This picture is a surprisingly good description of the distribution of human populations on levels of concentration that range from villages to towns, cities and megalopolises.*

Path integrals are used today in fields ranging from finance to radiophysics and statistical mechanics to quantum field theory.

Although the task of calculating a path integral by superposing an infinite number of paths may seem insurmountable, the Wiener integrals can be calculated numerically, and pioneering results in this direction began to appear in the 1950s as computational mathematics grew in the wake of the advent of electronic computers. The most convenient approach considers the integral as an average over the Wiener measure rather as a limit of an iterated integral.

It is sufficient to consider three dozens typical Wiener paths and calculate the average over them — the central limit theorem guarantees that the result approximates the Wiener integral. There are however certain inherent difficulties here: first, the construction of the Wiener paths requires generators of random numbers which is sometimes accompanied by the onset of chaos.

Another difficulty is that the numerical calculation of Wiener integrals is associated with intermittent[501] *random quantities, and is due to the fact that some of the Wiener integrals are determined by unlikely events (non-typical paths).*

A similar difficulty is also known for ordinary integrals of rapidly oscillating functions. These two kinds of difficulties are actually intrinsically connected: the more exotic the paths that make principal contributions to the Wiener integral, the tougher are the requirements that arise for the reproduction of these paths via random number generators.

Notwithstanding these difficulties, the simplest Wiener integrals are within the reach of modern numerical methods. In this connection we point out a remarkable and surprising peculiarity of computational mathematics: This field of science is generally connected with pure mathematics, the science which exists in the name of rigorous proof.

In modern computational mathematics, however, even if in some cases irreproachable mathematical proofs can be given, they prove to be almost useless after a deeper analysis. As a result, modern computational mathematics is also unexpectedly close to experimental physics in the nature of its heuristic arguments: in both cases, the principal instrument of verification is experimentation, either in a laboratory or numerical.

Let us clarify this by the following example: When computing an ordinary integral, one replaces the integrand by a polygonal line or by using the more precise Simpson's parabolic rule. One should know how many mesh points must be taken in order to reach the desired accuracy. This proof can be made as rigorous as possible for theorems of calculus. However, the result itself is practically useless. At least, nobody thinks of incorporating it into standard software. The reason for this is simple: the desired estimate of the required fineness of the mesh depends on the maximum continuous derivative of the integrand.

For other interpolations, even higher derivatives are involved. The computation of those is much more complicated than the computation of an integral.

Therefore, instead of this rigorously proved estimate, the so-called Runge's empirical estimate is usually applied, which is based on the presumption that the result of the computations is close to the true value of the integral when the doubling of the number of mesh points does not lead to a considerable change in the result. This estimate cannot be proved while the examples that

[501] Similar to a *Gaussian-distributed* random quantity which typically arises as the *sum* of many random numbers, the *intermittent* random quantity is typically a result of the *multiplication* of many random numbers.

directly contradict to it can be given explicitly. Nevertheless, in the hands of a judicious person, this empirical estimate gives excellent results.

1921–1954 CE **Léon Nicolas Brillouin** (1889–1969, France and U.S.A.). Physicist. Made significant contributions to *mathematical physics* (WKBJ, 1926), *electromagnetic wave theory* (signal velocity; diffraction of light by ultrasonic waves, 1921; optics of metals, 1949), and *quantum theory of solid state physics* (Brillouin scattering of light, Brillouin function, Brillouin zones).

He was a professor at College de France (1932–1939), Brown University (1941–1943), Harvard University (1946–1949), and director of research at I.B.M. (1949–1954). His father **Marcel Louis Brillouin** (1854–1948, France) was a physics professor at College de France (1900–1931), and known for his work on structure of crystals, kinetic theory, and the viscosity of liquids and gases.

1921–1994 CE **Leopold Vietoris** (June 04, 1891 – April 09, 2002, Austria). Mathematician. Contributed to general and algebraic topology and other branches of mathematics. Originated the modern *convergence* concepts in topology. The *Mayer-Vietoris sequence* (1927), *Vietoris complex*, the *Vietoris-Begle theorem* and the *Vietoris inequalities*[502] (1958) are named after him.

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502 *Vietoris inequality*: If $C_{2k} = C_{2k+1} = \frac{1}{2^{2k}}\binom{2k}{k}$, $k \geq 0$, then

$$\sum_{k=1}^{n} C_k \sin kx > 0, \qquad 0 < x < \pi$$
$$\sum_{k=1}^{n} C_k \cos kx > 0, \qquad 0 < x < \pi$$

An *equivalent* theorem states: Let $a_0, a_1, \dots, a_n$ and t be real numbers. If $a_0 \geq a_1 \geq a_2 \geq \cdots \geq a_n > 0$, and $2ka_{2k} \leq (2k-1)a_{2k-1}$ where $1 \leq k \leq \frac{n}{2}$, then

$$\sum_{k=1}^{n} a_k \sin kt > 0, \qquad \text{and} \qquad \sum_{k=1}^{n} a_k \cos kt > 0 \qquad (0 < t < \pi)$$

Putting $a_0 = 1$, $a_k = \frac{1}{k}$ $(k = 1, 2, \dots, n)$ gives the *Fejer-Jackson inequality* $\sum_{k=1}^{n} \frac{\sin kt}{k} > 0$ $(0 < t < \pi)$, and the *W.H. Young inequality* $1 + \sum_{k=1}^{n} \frac{\cos kt}{k} > 0$, $(0 < t < \pi)$.

Vietoris was born in Radkersburg, Austria, and was educated at the Technical University of Vienna (Ph.D. 1919). He was a soldier in WWI (1914–1918). He then worked with **Brouwer** in Amsterdam (1925–1926) and became a full professor at the University of Innsbruck (1930), where he remained for the rest of his life.

He was an avid skier (up to the age of 92!), keen alpinist and an accomplished mountain climber. He also became an expert on the formation of glaciers in his later years, and researched into the physics of blocks of glaciers.

He lived to be 110 years and 309 days old; In his time he was believed to be the oldest living mathematician and the oldest living Austrian. His last paper was written at the age of 94. He died a few weeks after the death of his wife (101), with whom he had been married for 66 years.

History of Meteorology and Weather Prediction[503]

I. Historical Background

Weather phenomena have profoundly affected the social customs, economic activities and personal experiences of human beings everywhere and in every age; weather prophets and 'weather makers' have practiced their arts for thousands of years. In ancient times, weather was often associated with religious cults, and so we have the thunder-God Zeus and the Germanic God Thor. In the old Testament Jehovah spoke to Moses from a cloud (Ex **19***), accompanied by flashes of lightning, and to Job out of the whirlwind (Job* **38***, 1;* **40***, 6)*[504].

[503] For further reading, see:

- Hess, S.L., *Introduction to Theoretical Meteorology*, Krieger, 1978, 362 pp.
- Lutgens, F.K. and E.J. Tarbuck, *The Atmosphere*, Prentice-Hall: New Jersey, 1982, 478 pp.
- Humphreys, W.J., *Physics of the Air*, Dover Publications: New York, 1964.
- Brancazio, P.J. and A.G.W. Cameron (Editors), *The Origin and Evolution of Atmospheres and Oceans*, John Wiley & Sons: New York, 1964, 314 pp.
- Thompson, P.D., *Numerical Weather Analysis and Prediction*, Macmillan and Company: New York, 1961, 170 pp.
- Whitaker (Ed), R., *Weather Watching*, Fog City Press, 2003, 480 pp.
- Firor, J., *The Changing Atmosphere*, Yale University Press, 1990, 145 pp.
- Reynolds, Ross, *Guide to Weather*, Firefly Books, 2005, 208 pp.
- Lawrence, E. and B. van Loon, *An Instant Guide to Weather*, Gramercy Books: New York, 2000, 125 pp.

[504] When the results of modern meteorological observations are compared with the picture of weather conditions reflected in the Bible, it seems that the climate and weather phenomena of Israel in the first millennium BCE were essentially the same as today. Incidentally, first references to rainfall records had been made in the Old Testament.

In Britain, it was not too long ago that a 16^{th} *century law requiring 'witches and weather prophets to be condemned to death, was struck from the books'. Even today there are primitive tribes to whom weather is still an act of the gods, and many of them are really quite expert at weather forecasting: The inhabitants of the Marshall Islands in the Pacific, for example, can recognize the characteristic high clouds, composed entirely of ice crystals, which precede a hurricane.*

Aristotle *(ca 335 BCE) was first to attribute the global winds to the heating of the sun in his Meteorologica. It was the first systematic study on meteorology. The ancient Greeks knew their* weather signs, *as we know from* ***Theophrastos***[505].

Not much was added to weather lore during the next 2000 *years, and the progress of meteorology has been coincident with the progress of physics and chemistry in general. Exceptions are the works of* ***Alhazen*** *(1050) on* twilight *and* ***Vitelo*** *(1250) on the rainbow. Minor activity is noted during 1300–1600: the first extant record of daily visual weather observations (no instruments yet available!) was due to* ***William Merle***[506] *(d.* 1347, *England) during* 1331–1338. ***Nicolas of Cusa*** *designed in* 1450 *the first Western Hygrometer.*

With the rise of the new physics in the 17^{th} *century, the discipline of meteorology entered its empirical stage. The main stimuli came from the works of* ***Galileo*** *(1607) on the* thermometer, *on the laws of inertia, and on the weight of air, the work of* ***Torricelli*** *(1642) on the* barometer, *the work of* ***Boyle*** *(1659) on fluid pressure, and the work of* ***Newton*** *(1673) on optics.*

The response was immediate: ***Benedetto Castelli*** *(1578–1643, Italy) used, in* 1639, *a cylindrical glass container to measure* rainfall. *It was the first rain-gauge in Western civilization. In* 1657, *the newly founded society Accademia del Cimento (1657–1667) established a network of meteorological observers in Italian cities (later extended to Paris, Warsaw and Innsbruck) for the recording of pressure, humidity, wind direction and the state of the sky.*

Earlier, in 1653, ***Ferdinand II of Tuscany*** *(1616–1670, Italy) organized a local system of stations for daily recording of weather conditions (1655–1670). He provided the observers of the Accademia with thermometers, barometers, hygrometers and other instruments. This effort was discontinued, however, in* 1670.

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505 **Theophrastos of Eresos** (372–288 BCE, Greece). Philosopher and scientist. The founder of botanical science. Pupil of Plato and Aristotle and successor of the latter as head of the Lyceum (323–288 BCE). Wrote a number of books on many different subjects. Wrote on *winds* and *weather signs*.

506 Rector of Dirby monastery. The records were found in the monastic diary.

In England, **Robert Hooke** *(1663) encouraged the systematic recording of standardized weather observations, including: wind direction and strength, moisture, temperature, pressure, movement of clouds, and precipitation. In response to his call,* **Richard Townely** *(1629–1707, England) recorded detailed rainfall measurements during* 1677–1704.

Edmund Halley *published (1686) the first meteorological chart: a map indicating prevailing winds over the tropical oceans. He outlined a thermal circulation (over a non-rotating earth) with warm air rising in the zone of maximum heating in the low latitudes, producing simple equator-ward flow in the trade winds*[507] *and a poleward flow aloft. This model was improved by* **George Hadley** *(1735). He incorporated the effects of the earth's rotation, deflecting flows to the right in the Northern Hemisphere and to the left in the Southern Hemisphere, with northeasterly and southeasterly trade winds resulting at the surface. Thus, the circulation according to Hadley consisted of convection cells.*

In 1699, **William Dampier** *(1652–1715, England) suggested that major ocean currents are caused by winds.* **James Jurin***, secretary of the Royal Society (England) organized in* 1723 *a system of meteorological observatories in Europe, North America and India. These were equipped with specific instruments for standard daily measurements and recordings. Monthly and yearly averages of these measurements were calculated at each station. In 1728,* **Isaac Greenwood** *(1702–1745, pre-U.S.A.) recommended that the Royal Society extend its meteorology observations to ships on the high seas.*

Jean Jacques d'Ortous de Mairan *(1678–1771, France) suggested in* 1733 *that Northern Lights (Aurora Borealis) are caused by the sun's atmosphere.* **Charles Le Roy** *(1726–1779, France) introduced (1751) the concepts of relative humidity (the actual amount of water in the air relative to*

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[507] We owe the classification of cloud forms to the English apothecary **Luke Howard** (1803). They were accepted partly because of their simplicity, but also because of the support they received from the German poet **Johann Wolfgang von Goethe** (1749–1832), who was an enthusiastic cloud watcher because of his great sensitivity to weather changes. He mentioned these cloud forms in several of his poems, as well as in a guide to weather watching which he wrote. The first network of weather stations promoted and financed by a government was in the small state of Sachsen-Weimar, where Goethe was a minister responsible for this activity. Unfortunately, however, shortly after his withdrawal from government, financial difficulties forced the dissolution of the organization he had created.

the maximum possible at a given temperature) and Dew point[508]*, and conjectured that the atmospheric moisture is the origin of* precipitation *(rain, hail, snowfall). In the same year,* ***Henry Ellis*** *(1721–1806, England) found that ocean temperature at depth (1630 m;* $25°13'N$, $25°12'W$*) was lower* ($12°C$) *than the surface value* ($29°C$)*.*

In Germany, the meteorological society of the Palatinate in Manheim (Societas Meteorologica Palatine) was founded in 1780 *by* ***Karl Theodor of Bavaria****. The society (disbanded in 1795) established* 57 *observatories in the Northern Hemisphere, each equipped with barometer, thermometer, hygrometer, rain gauge, wind vane and electrometer. In 1784,* ***Benjamin Franklin*** *(1706–1790, U.S.A.) attributed the severe winter of* 1783–1784 *to the volcanic eruption of Laki fissure in Iceland (June 8, 1783), where solar insolation was reduced by the spread of volcanic ash through the atmosphere.*

In 1783, ***Horace Benedict de Saussure*** *(1740–1799, Switzerland) developed a sensitive hair-hygrometer for the measurement of* relative humidity*. In England,* ***John Dalton*** *conducted weather observations during* 1787–1844*. The period* 1752–1783 *is marked by a collective effort to study the nature of the atmosphere and finally establish its character as a simple mixture of gases. This was contributed by* ***Black*** *(1752),* ***Cavendish*** *(1760–1777),* ***D. Rutherford*** *(1772),* ***Priestley*** and ***Scheele*** *(1775), and* ***Lavoisier*** *(1783).*

To obtain a true picture of the seemingly random sequence of diverse weather conditions, it is necessary to record on one chart the observations made simultaneously at many points. This was done for the first time by ***Brandes*** *(1817, Germany) at the University of Leipzig. He based his work on the observations made over many decades by the first international network of weather stations (that of the Meteorological Society of the Palatinate in Manheim, 1781–1795). We call the recording of simultaneous meteorological phenomena '*synoptic meteorology*'. [The invention of the telegraph and all the means of communication based on it – such as radio and teleprinters – , has made it possible for synoptic meteorology to cover vast areas, and nowadays the entire globe.]*

Fourier *suggested in* 1827 *that human activities have an effect on earth's climate.* ***Matthew Fontaine Maury*** *(1806–1876, U.S.A.) was one of the first to treat the oceans and the atmosphere as a unified system: in* 1847 *he published wind and current charts for the North Atlantic, and during* 1850–1857 *he extended his charts to include surface winds over world oceans.*

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[508] The *Dew point* is that temperature at which the relative humidity becomes 100% (i.e., the air is saturated) and condensation accordingly takes place, invisible water vapor changing into visible water drops. It is the temperature at which an invisible air-stream will breed clouds, and decreases with humidity.

A major advance in the understanding of the motion of air-masses[509] *in the*

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[509] The Polar Regions can be regarded as the two great '*air-conditioning*' areas for the earth's atmosphere, while the Tropics are the great '*central heating*' areas. Within the Tropics, from Cancer across the equator to Capricorn, day and night are nearly equal throughout the year. The heating by day and the cooling by night are similarly balanced and it is always hot. The daily average temperatures remain steadily high. The path of the rising sun is a steep climb upward in the east. Within an hour it is high enough and strong enough to cancel the heat-loss of the night. Although the downward path of the sun to its setting in the west is as steep as its morning rise, not until several hours after sunset do surface temperatures fall appreciably. (Surprisingly, really intolerable heat is found not on or near the equator but between latitudes 15° and 30° north and south of the equator, and well away from the sea — in the hot desert lands of the world.)

Air masses are huge masses of air with approximately uniform temperature and humidity. They are caused when an area of air, hundreds of kilometers wide, rests on a sea or land mass that has fairly even temperature and humidity. The air takes on the characteristics of the surface below. There are two extreme kinds of air masses — *tropical* ones, which are warm, and *polar* ones, which are cold.

Air moves over the earth's surface, tending to even out the distribution of heat. As air masses leave the place where they formed, they become modified, warming up or cooling down, becoming drier or moister, according to the different surfaces they travel over. Meanwhile, fresh supplies of polar or tropical air are being produced at the Poles and in the Tropics.

Very different air masses do not mix together when they meet. There is a *polar front* in each hemisphere, a boundary between modified polar and tropical air masses. These boundaries are belts of unsettled weather.

On weather maps, air masses are labeled to show where they came from. Air masses from the Poles are labeled P; air masses from the Tropics, T. If they formed over land, they are labeled c; if they formed over sea or ocean, m. So an air mass labeled cP came from a polar land mass, like Alaska, for example, while an air mass labeled mT came from a tropical sea, like the Caribbean.

Air masses play a vital role in the type of weather experienced all over the world. The basic reason for these air masses moving the way they do is *pressure*, the compressive atmospheric force per unit area at any point and upon any surface, and equivalent at *equilibrium* to the weight of a unit-area-base column of air extending upwards from the given point.

This weight varies, depending on *temperature* (warm air lighter than cold air) and *humidity* (wet air lighter than dry air). Barometers react to changes in local pressure by 'rising' when the pressure is high (usually indicating good weather), and falling when low (bad weather).

atmosphere came with the discovery of the Coriolis force[510] *(1835);* ***William Ferrel*** *(1817–1891, U.S.A.) improved on Hadley's model, producing the first reasonably complete model of major wind systems — the first* 3*-cell model. He made a quantitative use of the Coriolis effect to explain the easterly trade winds in low latitudes. Thus, Ferrel included a belt of southwesterly winds in middle latitudes, between the easterlies of high and low latitudes, with subsidence in the subtropical highs and ascent at higher latitudes, at the limit of what is now known as the Ferrel cell.*

He also explained the counterclockwise rotation of cyclons *(low pressure areas) in the Northern Hemisphere and the clockwise rotation of* anticyclones *(high pressure areas) in the Southern Hemisphere. Ferrel also described the effects of the earth's rotation on the distribution of ocean currents caused by the wind, and derived, in* 1874*, the equation relating the barometric gradient of pressure to the velocity of the wind. In the same year* ***M. Leverrier*** *presented a* weather map *of* France*, constructed from observations at* 10 *stations. The modern method of mapping the weather was introduced in* 1863 *by* ***Francis Galton*** *(1822–1911, England). In* 1869*, the meteorologist* ***Cleveland Abbe*** *(1838–1916, U.S.A.) began to send out* weather bulletins *from the Cincinati observatory. In 1885* ***Albert I of Monaco*** *(1848–1922,* Monaco*) provided evidence (through the expedition of his ship Hirondelle) that the Gulf-stream is transatlantic.*

While atmospheric pressure and air density decrease rapidly with altitude, the vertical temperature structure of the atmosphere is not as simple as was once believed. At the end of the 19^{th} *century it seemed reasonable that a decrease in temperature should accompany the lowering of pressure at the outer*

Arctic and sub-tropical areas are normally recognized as being *high pressure zones*, while the temperate latitude area (ca 40 °N–60 °N) and the equatorial belt (ca 15 °N–15 °S) as being *low pressure zones*. High pressure forces air masses towards low pressure: this is a vital clue to all movements of air masses.

510 It is a direct consequence of the conservation of the component of the angular momentum of the air-mass, parallel to the earth's rotation axis: to maintain the constancy of the quantity $\{V_\Theta R \cos\Theta\}$ per unit mass [R = earth's radius, Θ = latitude, V_Θ = linear velocity in west-east direction tangential to latitude circle], V_Θ must increase as $\cos\Theta$ decreases; hence the effect. The magnitude of the *Coriolis force* is shown to be $\{2V\omega\sin\Theta\}$ per unit mass, where $\omega = 7.29 \times 10^{-5}$ rad/sec and V is the wind's speed. Upper-level winds are deflected until the Coriolis force just balances the *pressure gradient force* (normal to isobars). Above 600 meters, where friction is negligible, these winds will flow nearly *parallel to the isobars*, and are called *geostrophic winds*. This was asserted in 1857 by **Christoph Buys Ballot** (1817–1890, Holland).

limits of the atmosphere. The lower temperatures recorded on mountain summits confirmed this assumption. At the turn of the 20^{th} *century* ***Teisserenc de Bort*** *(1855–1913, France), working from an observatory near Versailles, explored the structure of the atmosphere using balloons carrying recording thermometers. His observations during* 1898–1902 *showed that above 11 km, over Europe, temperatures cease to fall and may even increase with height. He had in fact discovered the stratosphere*[511]*, a name which he gave to this region of the atmosphere in* 1908. *In 1918,* ***Vladimir Peter Köppen*** *(1846–1940, Germany) developed an empirical climatic classification based on temperature and precipitation, yielding* 5 *major climate types: tropical, dry, warm temperature, snow, and ice*[512].

The basic features of Atlantic circulation were explained in 1922 *by* ***Alfred Merz*** *(1880–1925, Germany), and* ***Georg Adolf Wüst*** *(1890–1977, Germany), emphasizing cross-equatorial exchange. In the same year,* ***Frederick Alexander Lindemann*** *(1886–1957, England) and* ***G.M.B. Dobson*** *(England) determined atmospheric temperatures at levels above those reached by balloon soundings, by examining meteor trail records, for brightness, height and length of trail. From these they derived air densities, and then calculated temperatures from the density and other variables. They concluded that air is warmer at* 50 *km than at lower levels.*

During 1919–1936, ***Andrew Douglass*** *(1867–1962, U.S.A.) correlated climatic cycles and tree growth, using the tree-rings for dating.*

Alongside with the development of the art of measurement and observation, meteorology benefited from the independent evolution of the sciences of hydrodynamics and thermodynamics developed from the works of ***Newton*** *(1670),* ***Euler*** *(1736),* ***Laplace*** *(1780),* ***Fourier*** *(1785),* ***Poisson*** *(1815), and* ***Stokes*** *(1851).*

[511] Today it is customary to divide the lower part of the atmosphere into two layers: a lower layer called the *troposphere* (from the Greek '*tropos*', meaning 'turn'), which is descriptive of the layer's convective and mixing characteristics. This, the lowest layer of the atmosphere, contains 75% of the total atmospheric mass, and is the most important in terms of weather. It contains virtually all the water vapor and clouds, and the atmospheric pollution. The layer above it is the *stratosphere* (from the Latin '*stratum*' means 'a layer'), which is descriptive of its stratified, non-convective nature. It extends up to about 50 km. It contains the ozone (O_3), which although present in only very small amounts is vital for the existence of life on earth, for it absorbs and filters out the ultraviolet radiation wavelength (0.23–0.32 microns).

[512] **Strabo**, in ca 5 BCE, divided the world into frigid, temperate and tropic zones.

II. The Physical foundations

The theoretical branch of meteorology is based upon the fundamental postulate that the behavior of the atmosphere is capable of being analyzed and understood in terms of the basic laws and concepts of physics. The three fields of physics which are most applicable to the atmosphere are thermodynamics, optics *and* hydrodynamics.

From the field of thermodynamics *we set down three laws:*

- *The equation of state of a perfect gas.*
- *The first law of thermodynamics.*
- *The second law of thermodynamics.*

The relevant laws governing the emission, propagation and absorption of optical radiation *are:*

- *Kirchhoff's law.*
- *Planck's law.*
- *Beer's law.*
- *The equation of radiative transfer.*

In hydrodynamics *we make use of:*

- *Newton's three laws of motion and the law of universal gravitation.*
- *The laws of conservation of mass and energy.*
- *The Navier-Stokes equation incorporating Newtonian viscosity.*

The ultimate aim of theoretical meteorology is to express the above laws and ideas in forms applicable to the atmosphere, and to apply the resultant equations to the modeling and prediction of atmospheric dynamics.

III. The Helmholtz system of equations and its consequences (1858)

For ease of application to meteorological data, it is convenient to choose a Cartesian coordinate system tangent to the earth surface, and co-rotating with the earth: the x-y plane coincides with a hypothetical sea-surface, with the y-axis pointing northward*, the x-axis pointing* eastward *and the z-axis pointing* upward*. If $\boldsymbol{V}$ is the velocity vector relative to this local co-rotating frame, the corresponding acceleration is*

$$\frac{D\boldsymbol{V}}{Dt} = -\frac{1}{\rho}\,\mathrm{grad}\,p - 2[\boldsymbol{\Omega}\times\boldsymbol{V}] - \mathrm{grad}\,\phi + \boldsymbol{F}$$

Here, $\boldsymbol{\Omega}$ is the earth's rotation vector ($|\boldsymbol{\Omega}| = 7.29\times 10^{-5} rad/sec$); ϕ is the geopotential *= sum of the terrestrial Newtonian gravitational potential and the centrifugal potential, and is equal to a constant at standard sea level. Within the first several hundred km above sea level, the surfaces of constant ϕ are sufficiently spherical and $\Delta\phi = \boldsymbol{g} - \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\boldsymbol{r})$ is sufficiently uniform that for purposes of atmospheric dynamics the earth may be considered to be a sphere of constant radius a (6370 km) and ϕ can be set equal to gz, where g is constant and z is height above sea-level. Spherical coordinates λ (eastward longitude), Θ (latitude), and $r = a + z$ can thus be used with little error. $\boldsymbol{F}$ represents friction per unit mass (and any additional forces, if needed) that is caused by* molecular viscosity *of the air and* eddy stresses *due to vortex motions.*
The first term on the r.h.s. of the above equation derives from the pressure gradient, where ρ is the density of the air. The second term is the Coriolis acceleration*, which causes only a* change of direction *of the moving air relative to the ground.*
Denoting the local Cartesian components of the air velocity by $\boldsymbol{V}(u,v,w)$, we have

$$-2[\boldsymbol{\Omega}\times\boldsymbol{V}] = 2(v\Omega_z - w\Omega_y)\boldsymbol{e_x} + 2(w\Omega_x - u\Omega_z)\boldsymbol{e_y} + 2(u\Omega_y - v\Omega_x)\boldsymbol{e_z}$$

with

$$\Omega_z = \Omega\sin\Theta, \quad \Omega_y = \Omega\cos\Theta, \quad \Omega_x = 0$$

Therefore, the above Helmholtz equation decomposes into the three scalar equations:

$$\frac{Du}{Dt} = \frac{\partial u}{\partial t} + \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = 2\Omega(v\sin\Theta - w\cos\Theta) - \frac{1}{\rho}\frac{\partial p}{\partial x} + F_x$$

$$\frac{Dv}{Dt} = \frac{\partial v}{\partial t} + \left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -2\Omega u\sin\Theta - \frac{1}{\rho}\frac{\partial p}{\partial y} + F_y$$

$$\frac{Dw}{Dt} = \frac{\partial w}{\partial t} + \left(u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = 2\Omega u\cos\Theta - \frac{1}{\rho}\frac{\partial p}{\partial z} - g + F_z$$

In 1858 ***Helmholtz*** *began the study of the complete set of hydrodynamical equations as a possible means of dealing with meteorological problems. It comprised a system of 6 partial differential equations involving the six unknown functions: temperature, pressure, density, and 3 components of particle velocity. The form of these equations is such that their solution is determined* for all time *by the values of all 6 variables at a single* instant *and at every point in the atmosphere.*

For large-scale motions *of the atmosphere* relative to the earth, *various terms in Newton's second law can be neglected. In particular, since* vertical accelerations *are very much less than the gravitational acceleration, the gravitational acceleration is almost exactly balanced by the acceleration due to the atmosphere's buoyancy.*

Thus, for meteorological purposes, it is permissible to treat the atmosphere as if it were always in a state of local vertical hydrostatic equilibrium. *Next, owing to the large horizontal scale of weather disturbances, it turns out that the* horizontal *components of the force of internal viscosity (due to the molecular transfer of momentum) are also negligible, when compared with the Coriolis and horizontal pressure-gradient forces.*

Moreover, although the horizontal accelerations of air are not *negligible, there is a tendency for the Coriolis and horizontal pressure-gradient forces to balance each other. Thus, the approximate form of the equations of motion that are generally employed in the analysis of meteorological problems is*

$$\frac{Du}{Dt} - fv + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0,$$

$$\frac{Dv}{Dt} + fu + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0,$$

$$\frac{\partial p}{\partial z} + g\rho = 0$$

where $\boldsymbol{v}(u, v)$ *is the horizontal velocity field,* $f = 2\Omega \sin \Theta$, Θ *is latitude,* p *is the pressure,* ρ *the density and* Ω *is the angular velocity of the earth's rotation*[513]*. To the above equations, we add the equation of continuity (local conservation of mass):*

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho \boldsymbol{V}) = 0.$$

The last of the physical principles essential to the theory of dynamic weather prediction are the ideal gas law (equation of state) and the first law of thermodynamics for adiabatic non-viscous flow, through which the temperature, pressure and specific volume of a moving air-mass may change without addition of heat energy.

These processes are governed by the two equations

$$p\alpha = RT, \qquad \frac{D}{Dt}\left\{ T \left(\frac{p_0}{p} \right)^k \right\} = 0,$$

where α *is air volume per mole,* R *the universal gas constant,* T *is the absolute temperature (°K),* $k = R/C_p$, C_p = *specific heat at constant pressure, and* p_0 *is some fiducial pressure.*

In all, there are 6 equations in 6 variables. Although the system is complete, it remains to be investigated whether or not it is determinate. If heat is being added to and transported in the atmosphere through physical processes of absorption, radiation, condensation, and eddy heat conduction, the last equation is modified into the general expression for non-adiabatic flow

$$\frac{d\theta}{dt} = \frac{\theta}{C_p T} \frac{dq}{dt},$$

where

$$\theta = T \left(\frac{p_0}{p} \right)^k$$

is known as potential temperature, and $\frac{dq}{dt}$ *is the rate at which heat is added to a unit mass of air.*

In local Cartesian coordinates (x, y, z)*, the horizontal gradient of* $p(x, y, z)$ *for any fixed level* z *is denoted by*

$$\nabla_z p = \boldsymbol{e}_x \left(\frac{\partial p}{\partial x} \right)_z + \boldsymbol{e}_y \left(\frac{\partial p}{\partial y} \right)_z.$$

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513 Since $\boldsymbol{V} \approx \boldsymbol{e}_z \times \frac{1}{\rho f} \nabla p$, this '*geostrophic wind*' *relationship shows that winds tend to blow parallel to the isobars.*

However, on a given (fixed) isobar, we can define

$$\nabla_p z = \boldsymbol{e}_x \left(\frac{\partial z}{\partial x}\right)_p + \boldsymbol{e}_y \left(\frac{\partial z}{\partial y}\right)_p,$$

where $z = z(x, y; p)$, *for a fixed pressure* p, *is the equation of the isobaric surface itself. Since* $\frac{\partial p}{\partial z} = -g\rho$, *it follows that* $\left(\frac{\partial z}{\partial p}\right)_{x,y} = -\frac{1}{g\rho}$, *and hence*

$$\frac{1}{\rho}\nabla_z p = g\nabla_p z.$$

Consequently, the geostrophic wind relationship takes the form

$$\boldsymbol{e}_z \times f\boldsymbol{V} + g\nabla_p z \approx 0.$$

Differentiating this equation w.r.t. p *and making use of the vertical hydrostatic equation and the ideal-gas equation of state, one obtains*

$$\boldsymbol{e}_z \times f\frac{\partial \boldsymbol{V}}{\partial p} \approx \nabla_p \left(\frac{1}{\rho}\right) = \beta\frac{R}{p}\nabla_p T,$$

where β *is the number of atmospheric-composition moles per unit atmospheric mass.*

This equation, known as the thermal-wind equation, *tells us that* horizontal temperature gradients *induce pressure gradients that in turn produce* change in the local geostrophic wind (with height and across isobars).

This variation of flow 'blows' parallel to the mean isothermals

$$f\frac{\partial \boldsymbol{V}}{\partial p} \approx \beta\frac{R}{p}\nabla_p T \times \boldsymbol{e}_z$$

with low temperatures to the right of the flow variation $\frac{\partial \boldsymbol{V}}{\partial p}$ *(i.e. to the left of* $\frac{\partial \boldsymbol{V}}{\partial z}$*) in the Northern Hemisphere, and its magnitude proportional to the thermal gradient in the layer.*

Thus, high-level isobars depend not only on the pattern of sea-level isobars, but also on the horizontal temperature distribution. This situation will arise when cold and warm air lie alongside each other [as they do in a frontal system, and in general wherever polar and sub-tropical air meet]. The thermal wind must then be added vectorially to the geostrophic wind.

In principle, the above system of equations (including the inertia term deviations from the approximate geostrophic wind relationship) provides the basis for a mathematical system of weather forecasting; *the problem is essentially one of integrating a system of partial differential equations, starting* *with known initial conditions. At this point in the history of science (1858), meteorology was thus finally put on equal footing with astronomy, where the positions of suns, planets, and satellites are predicted from Newton's equations of motion.*

However, the interest in the equations of dynamic meteorology in 1858 *was somewhat premature; in the first place, the equations are too difficult to solve. In mathematical terms, the difficulty is one of solving a general boundary and initial-value problem for a simultaneous system of* 6 *non-linear partial differential equations in* 3 *dimensions. Even today, there are no known methods by which the solution of such equations can be related explicitly to general boundary and initial conditions.*

With exact analytical methods failing, the most satisfactory course would have been to solve the equations by purely numerical methods. These, however, were not fully developed until early in the 20^{th} *century, and in any case, would have required enormous computational power.*

Caught between two complementary difficulties, several generations of dynamical meteorologists were forced to tailor their problems to fit their limited mathematical means — i.e., to introduce such approximations and simplifications into the equations as were necessary to make them amenable to known methods of exact mathematical analysis. It is not surprising that the solutions of the approximate equations reflected little of the actual meteorological behavior of the atmosphere.

The second, and more important, obstacle to early development of dynamical weather prediction was the lack of adequate data; Before the turn of the 20^{th} *century, the main source of meteorological data was a rather loose network of surface observation stations, with only scattered and spasmodic kite and tethered-balloon soundings through the lower troposphere.*

With the advantage of long hindsight, it is easy to see that the dynamical meteorologist of that day did not really know what kind of phenomena he had to explain, *and could not fully test any theory that he might propose to account for the behavior of large-scale weather disturbances (which as we now know, extend through great depths of the atmosphere). He had no direct way of checking his assumptions, either by preliminary estimate or by verification of their mathematical consequences. To some extent, although upper-air observations are now much more frequent, more dense, and more accurate, and extend to higher altitudes, the meteorologist is still confronted by the problem of inadequate data.*

The first concerted attack on the problem of dynamical weather prediction was begun by the Norwegian (Bergen) school of frontal meteorology, led by **Vilhelm Bjerkens** *(1862–1951) and continued by* **Jacob Bjerkens** *(fils;*

1897–1975). Their outstanding achievement was to build a bridge between classical hydrodynamics and thermodynamics, which brought the atmosphere and the oceans within the scope of those two disciplines and opened up the field of applications for the quantitative treatment of motion on various scales in the atmosphere and the oceans.

Building on Helmholtz's theoretical foundations, the Norwegian school addressed the problem of general circulation of the atmosphere and carried out a systematic study of idealized mathematical models, directed toward the classification of atmospheric motions and their identification with the solutions of linearized forms of the hydrodynamic equations. But although their work contributed to our understanding of the kinds *of phenomena occurring in the atmosphere, they still failed to find a satisfactory formulation to the central problem of weather prediction.*

One of the turning points in the development of dynamic weather prediction was the meteorologists' realization that the general hydrodynamical equations could be solved in principle by purely numerical methods. *The form of these equations is such that the* instantaneous local time derivative *of each variable can be expressed in terms of* space derivatives *of variables in the same set.*

Accordingly, if we observe the initial values of all variables *at a network of discrete points,* filling the entire atmosphere, *we can approximate the relevant space derivatives by the method of finite differences and* compute the initial rates of spatial change of each variables. *Knowing the initial value* and *initial spatial rate of change of each variable, we can then extrapolate its value over a very short interval of time at each point in the network. Finally, we regard this very short-range forecast as a new set of initial data and repeat the process. Thus, it is possible (in principle) to build up a prediction scheme over any desired period of time as a series of successive forecasts over very short intervals of time.*

This possibility occurred first to ***Lewis Fry Richardson*** *(1881–1953), a highly original applied mathematician who also had a lively interest in the new finite-difference methods. Richardson's experiment failed, however, for reasons which became clear only in the late* 40*'s. Richardson himself estimated that it would take about* 64,000 *human computers just to predict weather as fast as it happened, let alone gain on nature. This figure was alarming enough that no one had the courage or enthusiasm to repeat his calculations under more closely controlled conditions, and interest in dynamical weather prediction withered and lay dormant for almost* 20 *years.*

The study of the atmospheric sciences has conventionally been divided into two main subject areas: meteorology and climatology.

Meteorology seeks to analyze, explain and ultimately to predict atmospheric processes and their behavior over *time*. It is the science of the atmosphere, and thus the science of the *weather*.

Climatology endeavors to document, analyze and explain the variations of meteorological processes with geography and on a number of time-scales, as related to the human environment. The *climate* of a particular place is an abstraction; it is a statistical generalization for the place rather than an actual reality, for it is misleading in most cases to assume that the climate of a place is simply its 'average weather'.

For many locations, particularly in temperate latitudes, the average weather is rarely the weather that is experienced at any given time. The weather systems which determine the climate of a locality are governed by many interactions and complex feedback processes involving the underlying surface (whether land, sea, snow, ice, mountains, forest or cities), and the overlying atmosphere, with its variable winds and clouds. All these elements and their interactions vary with time; climatic variability about the average state is therefore only to be expected. This variability is the very essence of the climate. It is thus preferable to consider a climate as being the integration of the spectrum of weather, likely to be experienced over time at a particular place.

Traditionally, meteorology has tended to be taught and studied in a university department of physics and mathematics, where students are familiar with the laws of radiation, thermodynamics and hydrodynamics. Because of the spatial and environmental aspects of climatology, the subjects has tended to be taught in university departments of geography, where it is studied as a component of physical environment.

The division between the two branches of the atmospheric sciences is now fading. A climate can only be understood in terms of meteorological mechanisms, no matter what the spatial and temporal scales. Climatology in its modern form can be looked upon perhaps as *spatial long-term meteorology* on a variety of spatial scales (global, regional and local) and time periods..

Moreover, there is a growing interest in the interaction of man and his climatic environment. Climatic extremes such as drought, floods, storms, frosts, fogs and blizzards have economic repercussions for industry, transport, agriculture, tourism and health. Increases in global concentrations of carbon dioxide, sulfur – and nitrogen oxides and methane, depletion of stratospheric *ozone* levels, and the regional problems of *acid rain* may also have serious and fully understood consequences for man's well-being (contributions to global warming, global dimming, etc.), while the nightmare possibility of the *nuclear winter* is one scientific experiment which must never be tested.

Linked to these two developments is the study of climate change. *This may operate on a variety of temporal scales, and the problems of understanding and modeling the meteorological and complex feedback processes in the earth-atmosphere-ocean system, together with the documentation of past climatic changes, represent some of the most important areas of climatological research today.*

The general circulation of the atmosphere manifests itself in the persistent easterly (from the east) wind belt of the trade winds in the tropics, and the prevailing westerly wind belt of temperate latitudes. Apart from long-term fluctuations, this is a constant arrangement, indicating an underlying order in the general pattern of circulation of the global atmosphere. The general circulation can be considered as the long-period average circulation of the atmosphere, free from all but the largest-scale seasonal trends of airflow. It is this which determines the patterns of world climate *and their main characteristics.*

Nevertheless, major climatic anomalies *perturb the world's weather from time to time. One of these is the* El-Niño *syndrome — the largest disturbance oceanographers and meteorologists have recognized in the planet's weather patterns. El-Niño (Spanish for "The Child", because it usually appears off the coast of Peru around Christmas time) is a heat wave in the Pacific, coupled with heavier than usual rainfall in the western portions of the Americas. Some recent occasions were in 1965, 1972, 1982–3, 1986–8. The opposite configuration is termed La-Niña; both seem to have become more frequent as of recent decades.*

Sometimes before the onset of El-Niño, the trade winds stop blowing, or even reverse themselves and blow from west to east.

Then, the warm water piled up off the coast of Asia, sloshes back to the Americas (trade winds, by blowing from east to west, normally heap up surface water on the eastern shores of the Pacific Ocean. Sea level near New Guinea and the Philippines is several feet higher than near Peru.) It spreads out over the Eastern Pacific, creating a ceiling of hot water some 150 *meters thick. When this happens, the cold, nutrient-rich waters from the bottom cannot penetrate the hot layer and are sealed in. The hot surface waters warm the air above them, preventing the trade winds from reasserting themselves. In a long chain reaction, the disruption of the atmosphere above the Pacific sets off a series of storms and droughts elsewhere.*

Thus, the El-Niño of 1982–3 *signaled vast changes in the atmosphere's global circulation, – alterations that made for strange weather around the world; there were torrential storms in parts of Ecuador, Bolivia, Brazil, and Peru — some desert regions got* 4000 *mm of rain. In the United States, there were huge storms and rains along the West Coast, and even in Florida —*

causing more than \$ 1 billion in property damage, and killing at least 100 *people. The tranquil island of Tahiti, which had not seen a single typhoon earlier in the century, was struck by one after another. Meanwhile, in Southern Africa, perturbations of the global weather, coming after two dry years, contributed to the terrible drought that has devastated Botswana, and also led to droughts in Indonesia, India, Sri Lanka, and Australia.*

The underlying causes of the El-Niño are not completely understood yet. It is known, however, that average global temperatures have risen about $0.5\,^\circ C$ *since* 1880. *The year* 1987 *was the warmest, on average, around the globe in the* 100*-year record of instrumentally recorded temperatures. The recent warming surge since* 1965 *is pushing the earth to temperatures that rival the warmest since the last ice age.*

1922, Nov 04 CE Discovery of the tomb of Egyptian Pharaoh **Tutankhamen** in Valley of the Kings near Thebes by archaeologist **Howard Carter** (1873–1939, England). Inside he found an abundance of ancient Egyptian artifacts that have laid untouched since 1342 BCE.

Tutankhamen died at the age of 19 in 1352 BCE.

1922 CE The mathematicians **Oswald Veblen** (1880–1960, U.S.A.) and **Luther Pfahler Eisenhart** (1876–1965, U.S.A.) generalized Levi-Civita's parallelism into *displacement* and the concept of geodesic into *path* in a non-Riemannian geometry, and established the 'geometry of paths'.

1922 CE **Louis Joel Mordell** (1888–1972, England). Pure mathematician. Proved important theorems in number theory. Conjectured[514] (1922)

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[514] Mordell's conjecture is actually more general: Let K be an algebraic number field and let C be a nonsingular projective curve over K with genus $g \geq 2$. The set of points on C which are K-rational (that is, have coordinates in K) is necessarily finite. If this turns out to be true [taking K = field of rationals and the Fermat curve $x^n + y^n + z^n = 0$, with $n \geq 4$], then there would be only finitely many such solutions in rational numbers, or equivalently, in integers. Since C is projective, (x, y, z) and $(\lambda x, \lambda y, \lambda z)$ are identified $(\lambda \in K)$.
A simple example of a Diophantine equation with finite number of solutions was found by W. Ljunggrens (1942): $x^2 + 1 = 2y^4$ has *exactly* two solutions

$$(x, y) = (1, 1);\ (239, 13).$$

that if, for some $n > 2$, the equation $x^n + y^n = z^n$ has a solution in nonzero integers [with (x, y, z) mutually prime], then it has at *most* finitely many such solutions for that value of n [proved in 1983 by **G. Faltings** (Germany)].

Mordell was born in Philadelphia of Jewish Lithuanian parents who had emigrated to the U.S. in 1881. His father, Phineas Mordell (1861–1934) was a Hebrew grammarian and scholar. In 1906 Louis went to St. John's College, Cambridge, where he graduated in 1912. He was a professor of pure mathematics at the Universities of Manchester (1923–1945) and Cambridge (1945–1958).

1922–1924 CE **Alexandr Alexandrovich Friedmann** (1888–1925, Russia). Applied mathematician and meteorologist. Showed that **Einstein**'s field equations of general relativity admit *non-static* solutions with isotropic and homogeneous matter distributions, corresponding to an expanding universe. His solution was the first to postulate the 'big-bang' cosmology.

Friedmann was born in St. Petersburg to a Jewish family. He entered the university of his native city in 1906 and obtained his master degree in applied mathematics in 1914. He took part in WWI (1914–1918) in an aviation detachment and participated in military flights. During 1918–1922 he did research in the field of theoretical meteorology at the physics laboratory of the U.S.S.R. Academy of Sciences. He died of typhoid fever at the age of 37.

1922–1928 CE **Philo Taylor Farnsworth** (1906–1971, USA). Invented electronic television. Farnsworth was a 16-year-old Mormon boy from Rugby, Idaho, with virtually no knowledge of electronics when he first sketched his idea for electronic video system on a blackboard for his high-school science teacher in 1922. He successfully displayed his first TV picture on Sept 07, 1927, and soon after patented the idea at a tender age of 21. He then successfully fought off the combined might of one of the largest corporations, RCA, and its massive team of lawyers and finally won the battle. The whole of RCA's research effort — at an expense that cost more than $50 million — was intended to circumvent Farnsworth patents[515]. Unfortunately, RCA

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515 In 1923, Zworykin *applied* for a patent. In 1927, Farnsworth produced the first successful transmission of a television image by wholly electronic means — and Zworykin application was still pending. Farnsworth's patent #1,773,980 was issued in August 1930 — and Zworykin's application was *still* pending. In fact, the 1923 Zworykin application would be all but forgotten — except that a patent was finally issued in 1938 — a long fifteen years after the original application, and then only after many modifications has been made to the original application. Furthermore, the eventual patent on the 1923 application — #2,141,059 was issued by a court, not the patent office. In the end, RCA was forced to pay

managed to maneuver its way around the patents and effectively degraded Farnsworth's status to that of 'just another contributor' in the field.

Born in Beaver, Utah, Farnsworth was educated in the Utah and Idaho public school system. He attended high school at Provo (1923) and enrolled in Brigham Young University (1924) but left it at the end of his second year due to the death of his father. In 1926 he joined the Crocker Research Laboratories in San Francisco. Farnsworth's basic television patents covered scanning, focusing, synchronizing, contrast, controls, and power. When he died at age 64, he held more than 300 U.S. and foreign patents[516].

1922–1932 CE **Stefan Banach** (1892–1945, Poland). Mathematician. One of the founders of modern functional analysis, and among the developers of the theory of topological vector spaces. Utilizing concepts of **Riesz**, he introduced the notion of *complete* normed linear spaces, whose elements need not be defined with respect to the complex number field. These are now called '*Banach spaces*'. The spaces named after **Hilbert** are special

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royalties to Farnsworth for the Image Dissector.

Zworykin did not have a clue how to create a high-resolution television signal by wholly electronic means until he visited Farnsworth's lab in 1930. As soon as he saw what Farnsworth had achieved, he got busy, not only duplicating Farnsworth's equipment, but using all the might of RCA to claim Farnsworth's achievement for his own. He failed in that effort and RCA was left with no choice but to accept a patent license from Farnsworth in 1939. At the end, however, Zworykin created the standards on which television was based.

[516] One of his patents, known today as the *Farnsworth-Hirsch Fusor*,is a device for producing nuclear fusion reactions which has since become a practical *neutron-source*, and is produced commercially for this role.

In the late 1950's and early 1960's Farnsworth was adapting an earlier device of his, connected with his television work, thus arriving at his fusion reactor. **Robert Hirsch** worked an Farnsworth's development team in the mid 1960's and made significant contributions to the device, which was patented in June 1966.

A deuterium gas is introduced into a high vacuum chamber. An accelerating voltage of several kV is high enough to cause the deuterium (electron orbiting a nucleus containing a proton and a neutron) nuclei to fuse, thus producing free neutrons. Farnsworth and Hirsch could obtain a neutron flux up to 10^{12} neutron/sec. Although hopes were high that it could be developed into a practical power source, later experiments that intended to turn it into a generator have failed.

The resulting neutrons are cheaper and lighter than standard ones and therefore could find applications in material science, nuclear research and medicine.

cases of normed linear spaces whose norm has the parallelogram property $\|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2$.

Banach contributed to the theory of orthogonal series and made innovations in the theory of measure and integration. His major publication is his book *Théorie des Opérations linéaires* (1932). His textbook *Mechanics*[517] (1938) is among the best treatises ever written on this subject.

Banach was born at Kraków. He started his academic career as a lecturer in mathematics at the Lwów Polytechnic Institute (1919), and became a full professor in 1927.

1922–1941 CE **Oleg Vladimirovich Losev**[518] (1903–1942, Russia). A self-taught radio engineer and inventor. Pioneer of semiconductor electronics: first to discover (1927) that semiconductor junctions emit light[519] (what we now know as LED) and first to foresee its use in telecommunications[520]. His observations languished for half a century before being recognized in the late 20^{th} and early 21^{th} century. In addition, Losev developed novel kinds of

[517] For further reading, see:

- Banach, S., *Mechanics*, Nakladem Polskiego Towarzystwa Matematycznego: Warszawa-Wroclaw, 1951, 546 pp.

[518] See: Loebner, E.E., *Subhistories of the light-emitting diode*, IEEE Transactions on Electron Devices, pp. 675–699 (1976). Also: Zheludev, N., *The life and times of the LED – a 100-year history*, Nature Photonics 11, pp. 189–192 (2007).

[519] See:

- Losev, O.V., *Luminous Carborundum Detector and Detection with Crystals*, Phil. Mag. 6, pp. 1024–1044 (1928) and Telegrafia i Telefoniya bez Provodov 44, pp. 485–494 (1927).
- Losev, O.V., Physik Zeitschr 30, pp. 920–923 (1929); 32, pp. 692–696 (1931); 34, pp. 397–403 (1933).
- Losev, O.V., Soviet Patent #12191 (1929).

[520] In February 1907, **Henry J. Round** (1881–1966), one of Marconi's assistants in England, published a 24-line note in *Electrical World* reporting a "bright glow" from a carborundum diode. There was no follow-up publication, and apparently this small note was not known to Losev. It is not appropriate to credit Round with the invention of LED, but he should be recognized as the discoverer of the phenomenon of electroluminescence. In 1962, four research groups in the U.S. simultaneously reported a functioning LED semiconductor laser based on gallium arsenide crystals, thus opening the field of solid-state optoelectronics.

crystal radio sets (with new crystals he fabricated himself) and was the first to study the effects of bias voltage upon the functioning of crystal diodes in circuits, essentially discovering "*negative resistance*" before the tunneling diode, as well as a pre-ATT version of the transistor and associated amplifiers. He then constructed completely solid-state radios that function up to 5 MHz, a quarter century before the transistor (1956).

Losev was born in Nizhniy to a high-ranking family in imperial Russia and served as a captain in the Czarist military. He received no formal education but during the span of his short research career he published 43 papers in leading Russian, British and German research journals and was granted 16 patents, of which he was the sole author.

In the mid 1920s Losev observed light emission from zinc oxide and silicon carbide crystal rectifier used in radio receivers when a current was passed through them. Losev's first paper on the emission of silicon carbide diodes, entitled "Luminous carborundum [silicon carbide] detector and detection with crystals," was published in 1927 by the journal *Telegrafiya i Telefoniya bez Provodov* (*Wireless Telegraphy and Telephony*) in Nizhniy Novgorod, Russia. Important publications in British and German journals soon followed. In his first paper on the LED, Losev established the current threshold for the onset of light emission from the point contact between a metal wire and a silicon carbide crystal, and recorded the spectrum of this light. In the 16 papers published between 1924 and 1930 he provided a comprehensive study of the LED and outlined its applications. Losev understood the 'cold' (non-thermal) nature of the emission, measured its current threshold, recognized that LED emission is related to diode action and measured the current-voltage characteristics of the device in detail. He also studied the temperature dependence of the emission down to the temperature of liquefied air (a predominantly nitrogen-based mixture of gases used at the time) and modulated the LED emission up to the frequency of 78.5 kHz by applying an a.c. current to the contact.

Most remarkably for a young technician with no academic qualifications, Losev was acutely aware of many contemporary developments in physics. He used Einstein's quantum theory to explain the action of the LED and called the emission process the "inverse photo-electric effect." In addition, he proposed a formula relating the voltage drop on the diode contact, V, the electronic charge, e, and the light emission frequency, ν, through Planck's constant, h, that is $\nu = eV/h$. The formula is still in use to this day. Despite the fact that semiconductor band theory had not yet been fully developed, Losev was able to relate the effect in silicon carbide to the diffraction of the electron de Broglie matter waves. According to the prominent Russian physicist Abram Ioffe, Losev wrote to Einstein asking him for help in further developing the theory, but received no reply.

In 1929, Losev published detailed measurements of LED spectra and clearly observed their dependence on current. It is incredibly interesting to see this data now, with the hindsight that the narrowing of the spectrum is evidence of laser action: the 1962 reports of laser action in gallium-arsenide-based diodes were underpinned by such spectral measurements. Did Losev see, without understanding its importance, coherent laser radiation from an LED in 1929? Perhaps not, but remarkably, the first significant blue LEDs reinvented at the start of the 1990s used silicon carbide.

Losev was the first to understand the potential of the LED for telecommunications. In the introduction to his patent entitled 'Light Relay', which was filed in 1927 and granted on 31 December 1929, he wrote: "The proposed invention uses the known phenomenon of luminescence of a carborundum detector and consists of the use of such a detector in an optical relay for the purpose of fast telegraphic and telephone communication, transmission of images and other applications when a light luminescence contact point is used as the light source connected directly to a circuit of modulated current." Unknown and uncelebrated, this should have been the beginning of the photonic telecommunication revolution.

Losev was a lonely scientist who left no disciples and never had a co-author. Being born into the noble family of a Russian Imperial Army officer was not exactly a good starting point in Bolshevik Russia, where people of such descent were banned from the career ladder. A self-made scientist who attended a few university courses but never formally completed his education, Losev was eventually awarded a PhD without a formal thesis by the Ioffe Institute in 1938. The happiest and most productive years of his research work were spent at the Nizhniy Novgorod Radio Laboratory before he moved to Leningrad where, after years of hardship, he eventually found himself a position as a humble technician at the Leningrad Medical Institute. According to Losev's recently discovered autobiography, he was nevertheless able to continue his research in Leningrad. He discovered that "using semiconductors, a tree-terminal system may be constructed analogous to a [vacuum] triode". However, in November 1941 he was unable to pass the finished paper (presumably on an important silicon device) from the besieged city of Leningrad to the evacuated editorial office of the *Soviet Physics JETP* (now *Journal of Experimental and Theoretical Physics*) in Kazan. Was it a paper on what we now know as a transistor? We shall never know for certain unless his manuscript is found. Sadly, after he passed away in Leningrad during World War II, his name was simply forgotten. In the difficult years that followed nobody took the opportunity to propagate his knowledge or follow up the potential revealed by his discoveries.

1922–1942 CE **Enrico Fermi** (1901–1954, Italy and U.S.A.). Distinguished physicist. One of the chief architects of the nuclear age. Developed the correct quantum-mechanical statistics for a class of subatomic particles, discovered neutron-induced radioactivity and directed the first controlled nuclear chain-reaction in the framework of the *Manhattan Project*[521]. He was awarded the 1938 Nobel prize for physics.

In 1922, Fermi generalized and extended the concepts of parallel vector transport and geodetic coordinates. His constructions are known today as 'Fermi transport' and 'Fermi coordinates', respectively.

In 1926 he was first to develop a quantum-mechanical treatment of a gas obeying the Pauli exclusion principle ('*Fermi-Dirac statistics*). The class of particles obeying these statistics are known as 'fermions', in his honor. In 1933, Fermi suggested for the first time the existence of the *weak interactions* in the theory of beta decay, and coined the name '*neutrino*' ("little neutral one").

Bombarding many elements with neutrons, Fermi showed in 1934 that slow thermal neutrons are very effective in producing radioactive elements. He presumed them (erroneously) to be heavier than uranium, not realizing that he had actually split the atom. This work was taken up by **Otto Hahn** at Berlin, who by 1937 had claimed the preparation of several trans-uranic elements with atomic numbers ranging from 93 to 96. However, **Frèdèric Joliot** (1900–1958, France) and **Irène Joliot-Curie** (1897–1956), working in Paris in 1938, pointed out that the radioactive characteristics of the substances produced by bombarding uranium with neutrons resemble those of much lighter radioactive elements. They further showed in 1939 that the fission of uranium produces

[521] Among the many stories told about Fermi, the following is most instructive: Enlisting in the Manhattan nuclear weapon project, the newly arrived immigrant was brought face-to-face with U.S. flag officers. On being told that so-and-so is a great general, Fermi asked for the definition of a 'great general'. "A general who won many battles", he was told. "How many", Fermi continued to ask. They settled on five. "What fraction of American generals are great", asked Fermi again. The officers agreed on a few percent. Fermi was totally unimpressed and explained why: Assume that there is no such thing as a great general, that all armies are equally matched, and that winning a battle is purely a matter of chance, like flipping an unbiased coin. Then the chance of winning five consecutive battles is 1/32 which is about 3 percent. On the other hand, the chance of winning ten consecutive battles is 1/1024, which is about 0.1 percent. So few percent of American generals are *expected* to win five consecutive battles, purely by chance, and that does not make them *great*! Now, has any of them won *ten* consecutive battles..?

neutrons, which under suitable circumstances, could cause further fission in other atoms in their vicinity.

Fermi designed the first atomic piles and produced the first nuclear chain reaction in 1942. He later worked on the atomic bomb project at Los Alamos, NM. After WWII, he pioneered in research on high energy physics.

Fermi was born in Rome. He studied at the University of Rome and received a doctor's degree from the University of Pisa in 1922. He became a professor of theoretical physics at the University of Rome in 1926.

Fermi left Italy in 1938 to escape the Fascist regime, and settled in the United States. He became a professor of physics at Columbia University in 1939. He then moved to the University of Chicago as a professor of physics in 1942. Element number 100 was named after him (fermium).

1922–1964 CE **Leo Szilard**[522] (1898–1964, Hungary, Germany and USA). Physicist, biophysicist, inventor and "scientist of conscience". A most creative, versatile and practical scientist, and politically far-sighted. A key-figure in the birth of the nuclear age — "The man behind the Bomb".

Pioneered in the development of nuclear energy. Szilard's ideas included *modern information theory*[523], *electromagnetic pump* for refrigeration (with Einstein), the *linear accelerator*, *cyclotron*, *electron microscope*, *nuclear chain-reaction*, *isotope separation*, *chemostat*. He was instrumental in establishing the *Manhattan Project*, but at the same time insisted that scientists accept moral responsibility for the consequence of their work.

Born Leo Spitz[524] in Budapest to an assimilated Jewish family. Young Szilard grew up with no personal religious beliefs or traditions. Yet from

[522] For further reading, see:

- Lanouette, W., *Genius in the Shadows*, University of Chicago Press, 1992, 587 pp.

[523] Some 30 years ahead of **Claude E. Shannon**, Szilard (1922) linked the concepts of *information* and *entropy*. In 1947, **John von Neumann** reconsidered Szilard's ideas and later urged Shannon to use the term"entropy" in his work. By the early 1950's, around Columbia University, Szilard also discussed his information-entropy ideas with physicist **Léon Brillouin**. Szilard did not seek to tie his early insights to later commercial developments. Not until the 1970s and 1980s, however, did Szilard's role in information theory gain the attention of a wider scientific community.

[524] In 1900, his father **Louis Spitz**, a prosperous building contractor, changed his name to **Szilard**, yielding to growing government pressure for the magyarization of foreign-sounding names. In Hungarian, Szilard means "solid".

his mother he did acquire a strongly held ethical views, abstract ideals such as honesty and loyalty, independence of mind and defiance of social conventions. He escaped from Hungary (1919) to study engineering at the Technical Institute in Berlin. He transferred (1920) to the physics department of the University of Berlin, where he studied under physicists **Max Planck**, **Max von Laue**, **James Franck** and **Albert Einstein**. His thesis, under von Laue (1922), became a cornerstone of modern information theory. Filed a patent with Einstein (1927) for an electromagnetic pump, which became the basis of cooling systems in "breeder" nuclear reactors in the 1950's and 1960's.

A refugee in England (1933–1938), he turned to nuclear physics, seeking the element which would yield a nuclear chain-reaction and lead to atomic power and bombs. Developed the idea of a nuclear "chain-reaction" (1933; patented 1934) and the concept of "critical mass" to create it.

With **T.A. Chalmers**, developed (1934) first method of separating isotopes of artificial radioactive elements.

Szilard came to the US in 1938 and collaborated with Fermi to design the first nuclear reactor at Columbia University (1939). Early in 1939 (after the discovery of Uranium fission in Germany) prompted, with **Wigner**, Einstein's letter to President Roosevelt warning him of atomic weapons — an alert that led to the creation of the *Manhattan Project* to develop A-bombs.

With Fermi (1942), he put at Chicago into operation the world's first chain-reaction atomic "pile" (reactor) of their design.

Szilard became a central figure in the Manhattan Project, and after WWII became a strong proponent of the peaceful uses of atomic energy. He then turned to biology, developed the chemostat and theories of aging and of memory and recall. He died at La Jolla, California.

Worldview XXXIX: Szilard

* * *

"An expert is a man who knows what cannot be done."

* * *

"If you want to succeed in the world you don't have to be much cleverer than other people; you just have to be one day earlier..."

* * *

"Here in America you are expected to keep busy all the time - it does not matter so much what you are doing as long as you are doing it fast."

* * *

"In our society, there is a market for skills and knowledge. But I have some doubts if there is much of a market for wisdom."

* * *

"A university runs on the happiness of the faculty: see that they are well paid, that their offices are comfortable, their graduate assistants are bright and eager, and that the faculty club food is appetizing. Then you will have a first-rate university."

* * *

"That is not how my brain works. I have no idea where my thought come from and no control over where they go"

(As he refused to sign an agreement that his inventions were 'General Atomic' property)

* * *

"Logical thinking and an analytical ability are necessary attributes to a scientist, but they are far from sufficient for creative work. Those insights in science which have led to a breakthrough were not logically derived from preexisting knowledge; the creative process on which the progress of science is based operate on the level of the subconscious."

* * *

"There were those who would never forgive me for being right."

On Szilard

* * *

"During a long life among scientists, I have met no one with more imagination and originality, with more independence of thought and opinion".

(Eugene Wigner)

* * *

"If the uranium project could have been run on ideas alone, no one but Leo Szilard would have been needed".

(Eugene Wigner)

* * *

"The only secrets then, worth protecting were not in government files but solely in the minds of "enemy aliens" Fermi and Szilard".

(William Lanouette)

[Refereeing to U.S. military security report (1940) on Szilard, recommending to discontinue his work on the A-bomb for being an enemy agent]

* * *

"What he had to say was always of profound and original kind".

(Erwin Schrödinger)

* * *

"Szilard belongs to the group of people who, through their richness of ideas, create an intellectual environment for others".

Albert Einstein

* * *

" Exotically original, versatile, and innovative intellect. A very rare example of a man, because of his combination of great purely scientific acumen and his ability to immerse himself in and solve technical problems".

(Paul Ehrenfest)

* * *

"Szilard was as generous with his ideas as a Maori chief with his wives."

(Jacques Monod)

1923 CE **Johannes Nicolaus Brönsted** (1879–1947, Denmark). Chemist. Professor at Copenhagen (1908–1947). Developed (with **T.M. Lowry**) a theory of acids and bases[525] (1923); suggested that acids give up a hydrogen ion in a solution and that bases accept a hydrogen ion in a solution, virtually the modern concept of acids and bases. It is known as the *donor-acceptor theory* and is especially useful for reactions in aqueous solutions. It is widely used in medicine and the biological sciences.

According to the theory an *acid* is a proton donor, H^+, and a *base* is defined as a proton acceptor. These definitions are sufficiently broad that any hydrogen-containing molecule or ion capable of releasing one or more protons, H^+, is an acid, whereas any molecule or ion that can accept a proton (or several) is a base. An acid-base reaction is the transfer of a proton from an acid to a base.[526]

Brönsted and Lowry presented logical extensions of the **Arrhenius** theory: H^+ ions in water are not bare ions but exist as $H^+(H_2O)_n$ in which n is a small number (due to *hydrogen bonds*). This is because of the attraction of the H^+ ions (protons) to the oxygen end of the water molecule. Indeed, when two water molecules collide, the collision sometimes results in *auto-protolysis*, creating a *hydronium*

$$H_3O^+ \sim H\text{—}\ddot{\underset{\underset{\displaystyle H}{|}}{O}}\text{—}H^+ \sim H^+(H_2O)$$

and a hydroxide:

$$2H_2O \rightleftharpoons H_3O^+ + OH^-.$$

For example, the complete ionization of hydrogen chloride, HCl (a *strong* acid) in water, is an acid-base reaction in which water acts as a base (a proton acceptor):

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525 The history of this concept goes back to **Robert Boyle** (1680) who noted that acids dissolve many substances, change the colors of some natural dyes and lose their characteristic properties when mixed with alkalis (bases). By 1814, **Gay-Lussac** concluded that acids *neutralize* bases and that the two classes of substances should be defined in terms of their reaction with each other.

In 1844 **Svante Arrhenius** presented his theory of base reactions. In his view *acid* is a substance that contains hydrogen and produces H^+ in aqueous solution. A *base* is a substance that contains the OH group and produces hydroxide ions, OH^-, in aqueous solution. *Neutralization* is defined as a combination of H^+ ions with OH^- ions to form H_2O molecules.

526 However, another definition by **Lewis** holds that a base *donates* electron pairs (*Lewis base*), while an acid accepts electron pairs (*Lewis acid*).

$$\begin{array}{rll} \text{Step 1:} & \text{HCl (aq)} \rightarrow \text{H}^+ \text{ (aq)} + \text{Cl}^- \text{ (aq)} & \text{(Arrhenius description)} \\ \text{Step 2:} & \text{H}^+ + \text{H}_2\text{O} \rightarrow \text{H}_3\text{O}^+ & \\ \hline \text{Overall:} & \text{HCl (aq)} + \text{H}_2\text{O} \rightarrow \text{H}_3\text{O}^+ + \text{Cl}^- \text{ (aq)} & \end{array}$$

Due to this reaction, every liter of pure neutral water at STP has 10^{-7} mol[527] of protons in the form of hydronium, counterbalanced by 10^{-7} mol of OH^-. This is responsible for the *electrical conductivity* of pure water. When the water is not pure it may become more conductive due to the presence of ions.

1923, Sept 01 CE *Earthquake* destroys Yokohama, Japan and much of Tokyo; over 140,000 perish in the quake and its subsequent fire.

The industrial society of the 20th century made earthquakes more hazardous, mainly by gathering more people into cities[528].

1923–1928 CE **Juan de la Cierva Codorniu** (1896–1936, Spain). Aeronautical engineer. Invented the first successful *Autogiro*, forerunner of the helicopter. This aircraft was first flown in 1923. In 1928, Cierva piloted an Autogiro across the English Channel.

Autogiro is a type of heavier-than air craft that is supported in the air by a rotor instead of by fixed wings, as an airplane is. The rotor spins by itself *freely* as it passes through the air. The Autogiro's engine rotates a propeller on the front of the fuselage.

In contradistinction to the *helicopter*, which always has its rotor revolve by an engine, an autogiro must run along the ground before the rotors can revolve fast enough to lift it. While flying, the rotor is disconnected from the engine, but the blades continue to revolve because of the air pressure against them. Thus, while the nose propeller pulls the plane forward, the generated air stream creates a lift force on the rotor blades. The blades are *hinged* at their hubs and made slightly flexible; the advancing blades thus yield to the air pressure and rise, while the retreating blades flap downward against the rise. The tilt of the rotor blades can be controlled, in order to turn the craft in a desired direction.

1923–1929 CE **Vladimir Kosma Zworykin** (1889–1982, Russia and U.S.A.). Physicist, electronic engineer and inventor. Invented the *iconoscope*, the first television transmission tube [an electronic tube that converts

527 We say that the pH of pure water is 7, where $\text{pH} = -\log[\text{H}^+]$.

528 In a sparsely settled rural areas, huge release of energy may harm few. The New Madrid earthquakes of Dec 16, 1811 – Feb 07, 1812 were strong enough to reshape the landscape, alter the course of the Mississippi and cause damage hundreds kilometers away from the epicenter. Yet but a few lives were lost.

light rays into electric signals and acts as a television camera suitable for broadcasting] and the *kinescope* [the picture tube used in television receivers].

He was also largely responsible for developing and perfecting the *electron microscope* (1939).

Zworykin was born in Murom, 320 km east of Moscow. He studied at the St. Petersburg Institute of Technology. There he came under the influence of **Boris Rosing**, a professor who was trying in 1910 to transmit pictures by wire in his own laboratory employing a mechanical disc scanner in the transmitter and the primitive K. F. Braun cathode ray tube in the receiver.

After graduation (1912), Zworykin went to College de France in Paris where he studied X-rays under **Paul Langevin**. With the Russian Revolution, Rosing went into exile and died. Zworykin carried on his work: he left for the United States (1919) and soon joined the staff of the Westinghouse laboratory in Pittsburgh. In 1923 he demonstrated his scanner before officials at Westinghouse, but the company did not find his system worthy of investment.

He continued to perfect it and after using a key idea of **Farnsworth** (1930), demonstrated his *all electronic* television system to the Institute of Radio Engineers. In attendance was **David Sarnoff** who hired Zworykin to develop his commercial television system for RCA. By 1933 a complete electronic system was being employed with a resolution of 240 lines, Zworykin's television system provided the impetus for the development of modern television as an entertainment and educational medium.

He also developed a *color-television* system (1928) and made innovations in the *electron microscope*. His electron image tube, sensitive to infrared light, was the basis for the *sniperscope* and the *snooperscope*, devices first used in WWII for seeing in the dark. His *secondary-emission multiplier* was used in the *scintillation counter*, one of the most sensitive radiation detectors.

In later life Zworykin lamented the way television had been abused to titillate and trivialize subjects rather that being used for the educational and cultural enrichment of audiences.

The Advent of Television

Black-and-white television camera (iconoscope) utilizes the cinematographic projection principle; a picture rate of at least 25 *frames per second* *produces the visual impression of a continuous motion: The image is divided into a large number of "picture elements". This means that the picture is divided into a number of lines (say, 625), and each line must contain several hundred individually identifiable light values. This is known as* scanning. *To obtain a reasonably good picture, the image must be thus analyzed into at least* 200,000 *picture elements.*

In the television camera the image is focused on a plate called the signal plate whose surface is covered with a mosaic of photosensitive elements. Each of these points, corresponding to one picture element, acquires a positive electric charge whose magnitude depends on the strength of the illumination falling on it. An electron beam, forming a scanning spot on the signal plate, zig-zags its way, line by line, across the plate every $\frac{1}{25}$ *second and thus* discharges *each photosensitive point* 25 *times per second. Each point thus gives an electric impulse whose strength corresponds to the strength of illumination at that point at that particular instant. These impulses (forming the picture signal) are amplified and transmitted.*

In the television receiver, *the incoming impulses, after amplification, are fed to the control electrode of the picture tube* (cathode ray tube), *in which an electron beam is zig-zagged across a fluorescent screen synchronously with the beam in the camera tube and with an intensity varying with the strength of the incoming electric impulses. In this way a pattern of luminous points of varying brightness, and formed in rapid succession, is produced on the screen, thus making the picture that the viewer sees.*

The picture signal can be conveyed to the receiver by cable or by wireless broadcasting of high-frequency short waves. These signals are only able to travel in straight paths from the transmitter, so that, because of the earth's curvature the range is limited to the visual horizon. It is for this reason that television transmitters are installed on tall masts or towers, which have to be spaced about 75 km *apart in order to provide good television coverage throughout the region*[529].

1923–1929 CE **Hermann Julius Oberth** (1894–1989, Romania and Germany). Among the three pioneering progenitors of the modern space age.

[529] At the turn of the 21st century, commercial television broadcasts *via satellites* are widely being used.

Published *The Rocket into Interplanetary Space* in 1923. In this book, Oberth discussed many technical problems of space flight. He even described what a spaceship should look like.

Interest in Oberth's book in Germany led to the formation of the *German Society for Space Travel*. The members of this society helped develop the first successful guided missile during WWII. In 1924 Oberth published his second book, *Way to Space Travel* (Wege zur Raumschiffahrt).

Oberth was born in Sibiu, on the northern slopes of the Transylvania Alps (now in Romania). After serving in the German medical corps in WWI, he graduated in physics and astronomy from Heidelberg University. His preoccupation with spaceships and space-travel problems was not appreciated by his peers and he spent most of his scientific career as a university teacher in Romania. During WWII he was distrusted by the Nazis and was not permitted to do serious work in Peenemuenda.

After the war he was arrested by the Americans, together with **Werner von Braun** and **Walter R. Dornberger**. But while the latter were moved to the U.S.A. to help develop American space rockets, Oberth was ignored, despite his potential theoretical capabilities. Yet, when new theoretical problems hindered the development of the big rockets at Huntsville, Alabama, Oberth was rushed there. He returned to West Germany in 1958.

1923–1947 CE **Victor Moritz Goldschmidt** (1888–1947, Norway and England). A pioneer geochemist. Justly called the *father of geochemistry*. Devoted most of his professional life to an attempt to find the laws underlying the frequency and distribution of the various chemical elements in the earth. His discovery of the fundamental relationship between crystal structure and chemical constitution laid the foundation of crystal chemistry. He was first to predict the formation of specific minerals from specific combinations of elements and geological conditions.

Goldschmidt was born in Zürich, a son of a distinguished physical chemist. In 1905 he moved with his family to Oslo. After studying geology, mineralogy and chemistry at Oslo, he was appointed full professor and director of the Minerological Institute (1914). In 1929 he accepted a similar appointment at Göttingen, Germany. As a Jew, he was forced to resign shortly after the Nazis came to power. He was immediately granted a chair at Oslo but became a refugee when the Germans invaded Norway. He escaped to England, where he spent most of the remainder of his life.

1923–1955 CE **Francesco Giacomo Tricomi** (1897–1978, Italy and USA). Applied mathematician. Contributed to the theories of differential and integral equations, *functional transforms*, special functions, probability theory and its applications to number theory. Certain equations[530] and special functions are named after him.

Tricomi studied at the University of Bologna, then at the University of Naples. Prior to WWII he held academic appointments at the universities of Padua, Rome, Florence and Torino.

In 1946 Tricomi joined the *Bateman-project* team (headed by **Arthur Erdelyi** at the California Institute of Technology). He returned to Torino (1950) and continued to work there on various topics of applied mathematics.

1924 CE **Ethel Browne Harvey** (1885–1965, U.S.A.). Biologist and embryologist whose studies of *induction* preceded those of Nobel laureate **Hans Spemann** (1935) by more than 10 years. An investigator at Princeton University for 25 years, she was never made a full professor.

Harvey, working on the mechanics of embryologic development, was first to discover the directive function (embryonic induction) of certain tissues.

1924 CE **Paul Marsh Ramey**[531] (b. 1896, United States). Inventor and electrical engineer. Invented and completely worked out the signal transmission method of *Pulse Code Modulation* (PCM). It is the basics of digital audio and used in voice transmission and reproduction. This ground-breaking work was then apparently forgotten. The idea was reinvented (1939) by **A.H. Reeves** (1902–1971; England), forgotten again and finally resurrected during WWII by the Bell Labs during a research into the method of encoding phone conversation.

Modulation, in communication, is a process in which some characteristic of a wave (the carrier wave) such as: amplitude, frequency or phase, is made to vary in accordance with an information-bearing signal wave (the modulating wave); *demodulation* is the inverse process by which the original signal is recovered from the modulated wave. The original, unmodulated wave may be of any kind, such as sound or, most often, electromagnetic radiation including

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[530] He studied (1923) the equation $y\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ now known as the '*Tricomi equation*'. This equation became important in describing an object moving at supersonic speed, but in 1923 there were no supersonic aircraft yet.

[531] The **Ramey** (Remi, Remy) family tree goes back to a tribe in Gaul (Northern France and Belgium). The family was represented in French history by Bishop Saint Remi of Rheims (437–533). A millennium later, Jacques Remy (b. 1547) became a Hugenot and was killed, and so was his son Pierre. Pierre's son, Jacques fled to England and them emigrated to Virginia (1654), establishing the Remy-Ramey branch in North America.

optical waves. When the carrier is a train of regularly recurrent pulses, PCM ensues.

The modulation, in this case, might vary the amplitude (PAM = Pulse Amplitude Modulation), the duration of the pulse (PDM = Pulse Duration Modulation) or the presence of the pulses (PCM). PCM can be used to send digital data[532]; (audio signals on a *compact disc* use pulse code modulation). PCM is the most important form of pulse modulation because it can be used to transmit information over long distances with very little interference or distortion. For this reason it has become increasingly important in the transmission of data in the space program and between computers. Although PCM transmits digital instead of analog signals, the modulating wave-train is continuous.

1924 CE First circumnavigation of the globe by four United States Army airplanes.

1924 CE **Louis Victor de Broglie** (1892–1987, France). Distinguished physicist. Expressed the idea of general *wave-particle dualism* and attributed, for the first time, wave qualities to material particles. This was the final synthesis of the 300-year old debate that started in the days of **Newton** and **Huygens**.

Broglie was led by two trains of thought to this assumption:

(1) An interpretation of Bohr's stationary state of the atom in terms of *standing waves* of discrete vibrational states, like those of a string or a drum but with *azimuthal* sinusoidal variation (for circular orbits).

(2) The application of the basic Planck energy relation $E = h\nu$ to *material particles* as well as to photons. By using special relativity, this becomes the covariant relation $p^\mu = \hbar k^\mu$, where $\hbar = \frac{h}{2\pi}$, $p^\mu = (E, \boldsymbol{p}c)$ is the particle four-momentum, and $k^\mu = (\omega, \boldsymbol{k}c)$ is the wave 4-vector.

[532] The input is sampled periodically at a rate of 8000 times per second for telephone systems (8 bit) and at a rate of 44,000 times a second for audio CD (16 bit). Shannon's sampling criterion (which state that the sampling frequency must at least be double the highest frequency to be transmitted) has been taken into account. Since digitizing assigns a specific (binary) number for any input amplitude, and only a given number of amplitudes are available (e.g. 256 for 8-bit digitizing of phone conversations), chances are that the assigned number will be a little too big or a little too low compared to the actual input. This creates an *error* called *quantization error*, producing *quantization noise* at the output. To partially remedy this situation, digitization values are assigned closer together for long signal levels.

This idea led him to associate with a particle of mass m a wave $\exp\{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{x})\}$, with the *de Broglie wavelength* $\lambda = \frac{h}{p}$, where $p = |\boldsymbol{p}|$ is the particle's momentum. These matter-waves satisfy the uncertainty principle, and *possess a phase velocity* $u = \frac{\omega}{k} = \frac{E}{p}$, where $E = \sqrt{m^2c^4 + p^2c^2}$. Here u exceeds c, but is dispersive; hence (just as in an electromagnetic waveguide), the *group-velocity* of a wave-packet of average momentum p, is $\boldsymbol{V}_g = \frac{d\omega(\boldsymbol{k})}{d\boldsymbol{k}} = \frac{\boldsymbol{p}c^2}{E}$, the correct relativistic *particle* velocity.

Thus, although the crucial Schrödinger equation and *probabilistic interpretation* are still lacking in de Broglie's matter-wave theory, it already contains much of the machinery of the nascent quantum mechanics. For the non-relativistic regime, one obtains

$$\lambda = \frac{h}{p} = \frac{h}{mv}.$$

The theoretical predictions of de Broglie were verified through the first experiments of electron diffraction by **Clinton Joseph Davisson** (1881–1958, U.S.A.), **Lester Halbert Germer** (1896–1971, U.S.A.) and **G.P. Thomson** in 1927. These diffraction patterns are completely analogous to the X-ray diffraction patterns of **von Laue**.

From electron diffraction measurements with crystal lattices of known lattice constants, the wavelength of the material waves corresponding to a beam of electrons may be determined. For electrons having velocity v after being accelerated through a potential difference V, one uses $eV = \frac{1}{2}mv^2$ and $\lambda = \frac{h}{mv}$ to obtain $\lambda_e = \frac{12.3}{\sqrt{V}}\ \mathring{A}$, where V is measured in volts. For example, an electron which has passed through a potential increase of 10,000 volts has a wavelength of 0.12 $\mathring{A}$, corresponding to hard X-rays.

De Broglie was born in Diepp [his grandfather, Albert, was prime minister of France in 1877]. He was educated in Paris, became a professor of theoretical physics at the Sorbonne in 1928 and won the Nobel prize for physics in 1929.

Broglie is the name of a noble French family which emigrated to France from Italy in 1643. The head of the family, Francois Marie (1611–1656), took the title of comte de Broglie. He distinguished himself as a soldier and died as a general at the siege of Valenza. During the next 200 years, members of this family served as marshals of France, statesmen, diplomats and cabinet members.

The development of quantum theory in France was aborted by de Broglie, despite his personal contribution to this theory in its early stages[533].

[533] **S.P. Novikov**, Amer. Math. Soc. Trans. (2) vol. **212**, 2004.

1924 CE **Satyendra Nath Bose** (1894–1974, India). Physicist. First investigated the statistics of identical and indistinguishable particles that are *not* restricted by the exclusion principle[534]. This yielded the quantum theory of the ideal gas (following the classical statistical thermodynamics of **Boltzmann**, with the sole change being the counting rule for microscopic states).

Bose gave a new derivation of Planck's radiation formula, divesting it of all supererogatory elements of electromagnetic theory and basing it on the bare essentials[535]. Together with Einstein[536] he established the new quantum

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534 A principle unknown to Bose at that time (Pauli, 1925).

535 His derivation rests on replacing the *counting of wave frequencies* by the *counting of cells in one-particle phase space*. Bose's theory assumed a spin value of 1 for the light quanta (photon), at a time when the concept of the photon spin had not yet been established (!). In all, Bose made three bold assumptions (actually, shots in the dark) which later proved to be correct: photon number *nonconservation*, statistical independence of cells (not particles!) and the removal of Boltzmann's axiom of distinguishability.

536 Maxwell-Boltzmann statistics gives the probability distributions for systems of distinguishable particles that are well described by the laws of classical Newtonian physics. However, when the quantum nature of the particles becomes important (such as at high enough densities or low enough temperatures), other types of statistics must be used.

Quantum systems of indistinguishable, identical particles are distributed differently depending on whether only one particle can be put into each state (fermions) or there is no such limitations (bosons). *Bose-Einstein statistics* are derived for indistinguishable, identical particles of *integral spin angular momentum quantum number*, s (particles called *bosons* for which $s = 0, 1, \dots .$) Examples of bosons include photons, ^{4}He atoms and Cooper electron pairs. The *Bose-Einstein distribution function* is $f(E) = \frac{1}{e^{\alpha}e^{E/kT}-1}$, where $\alpha = 0$ if the total number of particles is not conserved within the overall closed system (such as photons in a cavity in blackbody radiation). For systems having a constant total number of particles, α decreases to zero as the temperature is lowered. Near a particular low temperature, this behavior causes the particles to drop rapidly into the ground energy level. This "condensation" of particles is called *Bose-Einstein condensates (BEC)* and is part of the explanation for *superfluidity* (zero viscosity) of liquid helium, for BEC and for *superconductivity*.

Another "particle" that follows the Bose-Einstein statistics is the *phonon*, which is the quantum of energy of the mechanical vibrational mode of a solid-state lattice (actually a *quasiparticle* as it can exist only in solid lattice crystals). Since the total number of phonons is not conserved we have $e^{\alpha} = 1$. It then follows directly from the expression for $f(E)$ that the average number of phonons per lattice vibration mode (energy state), for frequencies that are much less than statistics, known thereafter as the Bose-Einstein statistics. It is one of two known ways in which a collection of indistinguishable particles may occupy a set of available discrete energy states. Aggregation of particles in the same state, characteristic of particles obeying Bose-Einstein statistics, accounts for the coherent streaming of laser light and the frictionless creeping of superfluid helium as well as for the novel optical and acoustical properties of *Bose-Einstein Condensates* (BEC).

Bose was born in Calcutta, India, and was educated there. He sent his 1924 paper to the *Philosophical Magazine*. It was rejected, and Bose sent it to **Einstein**, asking him to consider it. Einstein personally translated it into German and submitted it to the *Zeitschrift für Physik*.

1924–1926 CE **Edward Victor Appleton** (1892–1965, England). Physicist. Explorer of the ionosphere. Discovered that the upper layer of the ionosphere, called the F-region or *Appleton layer*, reflects radio waves. The discovery established the possibility of radio communication over long distances, and was also fundamental to the development of radar. He was awarded the Nobel Prize for physics (1947) for his discovery.

Appleton was born in Branford and matriculated at the University of London at the age of 16. He was trained at St. John's College, Cambridge and studied under J.J. Thomson and Rutherford. Since 1919 he devoted himself to scientific problems in atmospheric physics, using mainly radio waves. In 1924 Appleton began a series of experiments which proved the existence of a reflecting layer in the upper atmosphere. By periodically varying the frequency of the BBC transmitter at Bournemouth and measuring the intensity of the received transmission 100 km away, Appleton found that there was a regular fading in and fading out of the signals at night but that this effect diminished considerable at dawn as the *Kennely – Heaviside layer* broke up.

Radio waves continued to be reflected by the atmosphere during the day but by a higher-level ionized layer. By 1926 this layer (250 km above the earth's surface — the first distance measurement made by means of radio) became generally known as the *Appleton layer*.

Appleton was involved in the initial work on the atom bomb.

$\left(\frac{k}{h}\right)T$, is proportional to the temperature. At the other extreme (low temperatures), the average number of phonons per energy state will be proportional to $e^{-h\nu/kT}$.

1924–1929 CE **Hans Berger** (1873–1941, Germany). Psychiatrist. Recorded the first human electro-encephalogram (EEG). Discoverer of the electric activity of the human brain[537].

Electroencephalography (EEG) uses electrodes (made of lead, zinc, platinum, etc.) attached to the intact skull and connected to an oscillograph. The result is a visual picture of brain wave rhythms. Berger made seventy-three EEG recordings from his fifteen-years-old son, Klaus. The first frequency he encountered was the 10-hertz range, (8 to 12 Hz) which at first was called the Berger rhythm, currently called *Alpha* rhythm brain wave. After five years of investigation and re-examination of his results, he published his findings.

He reported that the brain generates electrical impulses or 'brain waves'. The brain waves changed dramatically if the subject simply shifts from sitting quietly with eyes closed (short or alpha waves) to sitting quietly with eyes opened (long or beta waves). Furthermore, brain waves also changed when the subject sat quietly with eyes closed, "focusing" on solving a math problem (beta waves). That is, the electrical brain wave pattern shifts with attention. The publication of Hans Berger's "On the Electroencephalogram of Man" in 1929 changed neurophysiology forever. Hans Berger thus earned recognition as the "Father of Electroencephalography".

Berger was born in Neuses, Thüringia, Germany. Received his M.D (1897) from the University of Jena and in 1919 was appointed to the chair of psychiatry and neurology. He was rector of the University (1927–1928) and professor (1935–1938). He retired in 1938 and committed suicide in 1941.

After his findings were confirmed, the electroencephalogram was launched into use for the study of normal and abnormal human brain activity. The EEG revolutionized neurological and psychiatric diagnosis and made possible specialized research in the neurological sciences. Today, the EEG is used in

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[537] In developing electroencephalography, Berger was fully aware that **Richard Caton** (1842–1926), a Liverpool surgeon, had succeeded in 1875 in measuring electrical potentials on the exposed cortex of experimental animals (rabbit and monkey), and that he was thus the discoverer of the electrical activity of the brain. Berger also knew about the further successes along this line achieved by the Polish physiologist **Adolf Beck** (1863–1939) in 1891, and of the findings of Russian workers. In 1912 a paper by the Russian physiologist **Pravdich-Neminski** (1879–1952) for the first time illustrated a photographic record of the electrical activity of the brain. He called it an "electrocerebrogram". Pravdich-Neminski's electrocerebrogram was recorded from dogs with intact skulls by means of the string galvanometer (**Willem Einthoven**, 1860–1927). Having suffered many setbacks in his experiments, Berger's reaction to this demonstration was that he should work harder.

the clinical diagnosis of serious head injuries, brain tumors, cerebral infections, epilepsy, and various degenerative diseases of the nervous system.

1924–1929 CE **Edwin Powell Hubble** (1889–1953, U.S.A.). Astronomer. Provided the first conclusive observational evidence for the expansion of the universe, thus confirming the Friedmann-Lemaître solution of the Einstein field equations of general relativity (1929). Founder of extragalactic astronomy (1924).

Hubble was born in Marshfield, MO. At the University of Chicago he earned undergraduate degrees in mathematics and astronomy, and was inspired by the astronomer **George E. Hale** (1910). However, he turned away from astronomy and athletics (he was a fine boxer too), preferring to continue the study law at Oxford University. In 1913 he joined the Kentucky bar, but soon abandoned his law practice and returned to continue the study of astronomy at the University of Chicago and its Yerkes Observatory. He obtained his Ph.D. in astronomy in 1917, and then served in WWI before settling down to work at the Mount Wilson Observatory.

During 1922–1924 he centered his research on extragalactic phenomena, and discovered that not all nebulae in the sky are part of the Milky Way Galaxy, the vast star system to which the sun belongs. He found that certain nebulae contain stars called Cepheid variables, for which a correlation was already known to exist between periodicity and absolute magnitude.

Using the further relationship among distance, apparent magnitude, and absolute magnitude, Hubble determined that these Cepheids are several hundred thousand light-years away, and thus outside the Milky Way system, and that the nebulae in which they are located are actually galaxies distinct from the Milky Way.

This discovery, announced in 1924, forced astronomers to revise their ideas about the cosmos. He thus settled the *Shapley-Curtis* debate once and for all: the universe was recognized to be far larger and populated with far bigger objects than anyone had seriously imagined — the realm of the galaxies.

Soon after discovering the existence of these external galaxies, Hubble undertook the task of classifying them according to their shapes (1926) and exploring their stellar contents and brightness patterns. In studying the galaxies Hubble made his second remarkable discovery — namely, that these galaxies are apparently receding from ours, and that the further away they are, the faster they are receding (1927).

The implications of this discovery were immense: the universe, long considered static, was expanding. Even more remarkably, Hubble discovered in

1929, from observations of *spectral redshift* of starlight[538], that the universe was expanding in such a way that the ratio of the speed of a galaxy to its distance from us is nearly a constant (*Hubble's law*). This constant is now called

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[538] A distant galaxy emits light that astronomers later (in today's epoch) detect on Earth. As the light travels through expanding space toward our Galaxy, its wavelengths are steadily stretched. Finally, the light enters a telescope, and the astronomers compare its spectrum with the spectra of other sources of light in the laboratory and within our Galaxy, and in this way they measure the amount of redshift. Invoking the *cosmological principle*, they assume that the light-emitting atoms in the distant galaxy are identical to the corresponding atoms in our Galaxy. The amount of redshift detected depends on how much the universe has expanded between the time of emission and the time of reception. The redshift applies to the whole spectrum, extending from radio waves to visible light to X-rays, and when one wavelength is doubled, all wavelengths are doubled in the spectrum of the source.

The *cosmic redshift* is best interpreted as a result of the expansion of space in the framework of GTR. Although the frequency shifts are in agreement with a Special-Relativistic Big Bang model for low redshifts $\left(z = \frac{\Delta\nu}{\nu} \ll 1\right)$, the frequency of light is also affected by the gravitational field (and attendant spacetime curvature) of the universe, and it is not strictly correct to interpret the frequency shifts of light from very distant sources in terms of a special-relativistic Doppler effect alone (i.e., the frequency shift is not *wholly* the result of a relative motion in Minkowski spacetime that results in an STR Doppler redshift). Distances between locally-comoving galaxies in an expanding homogeneous universe are proportional to the metric scale factor $R(t)$.

The locally-measured wavelength λ of a ray of light traveling in extragalactic space varies as $R(t)$ (in the adiabatic approximation), t being the epoch — the standard cosmological time (i.e. proper time since the Big Bang, as reckoned by a comoving local frame) at the observing creatures' galaxy. If λ_0 is the present (detected) wavelength and $R(t_0) = R_0$ the value of the scaling factor at the present epoch, then $\frac{\lambda}{\lambda_0} = \frac{R(t)}{R(t_0)}$, where t, λ are, respectively, the *epoch* at which the light was emitted and its *wavelength* as reckoned in the locally-Minkowskian coordinate system comoving with its galaxy of origin. *Redshift* is defined by $z = \frac{\lambda_0 - \lambda}{\lambda}$, and hence the cosmic redshift is $z = \frac{R(t_0)}{R(t)} - 1$.

Assuming that $(t_0 - t)$ is small, we may Taylor-expand $R(t)$ to find $R(t) = R_0 - \dot{R}_0(t_0 - t) + \cdots$, where $\dot{R}_0$ denotes the derivative of R w.r.t. time, calculated at $t = t_0$. It then follows that $z = \frac{\dot{R}_0}{R_0}(t_0 - t) + \cdots$. We also have approximately, for the *comoving coordinate of radial distance* r, $R_0 r = c(t_0 - t) + \cdots$. (In this comoving coordinate system $r =$ const. for an average galaxy, ignoring intra-cluster "peculiar" motions.)

Hubble's constant, H. Derived from theoretical considerations and confirmed by observations, most astronomers believe the velocity-distance law has made secure the concept of an expanding universe. (There are nonlinear GTR corrections to the Hubble law at large redshifts.) It is not really a constant, since it varies with the epoch.

Hubble's original value for H in our present epoch was 150 km per second per 1,000,000 light-years. Modern estimates, using more precise distance measurements, place the value of H between 15 and 30 km per second per 1,000,000 light-years. The reciprocal of Hubble's constant thus lies between 10 billion and 20 billion years, and this cosmic time scale serves as an approximate measure of the age of the universe. More precisely, $H = \dot{R}(t)/R(t)$ and as $R(t) \sim t^{2/3}$ in today's nearly flat, matter-dominated universe (it has been thus since $t \approx t_0/10^3$), $H \approx \frac{2}{3t}$; so, the current age of the universe as tallied by a co-moving clock in our local galaxy cluster is estimated at $t \approx \frac{2}{3H}$.

Measurements in the 1990's, utilizing tools such as the Hubble Space Telescope and certain types of well-understood supernova explosions (replacing the old Cepheid-variable stars as a distance yardstick to remote galaxies), have narrowed the range of values considerably, and the age of the universe is now thought to be about 14 billion years.

Whether in its Newtonian, STR or GTR form, Hubble's law implies that, on average, the universe and its evolution are seen to be independent of the galaxy in which the observer happens to reside.

Since for small values of $\frac{v}{c}$ (c is the velocity of light in empty space, v is the velocity of the receding source of light relative to the observer) we can write $z = \frac{\lambda_0}{\lambda} - 1 \approx \frac{v}{c}$ from the known STR Doppler formula, the relation $v = cz = HR_0r$ embodies Hubble's experimental results. Thus, $z \approx \frac{\dot{R}_0}{R_0}(t_0 - t) \approx \frac{\dot{R}_0}{R_0}\frac{R_0 r}{c}$ implies $cz \approx \frac{\dot{R}_0}{R_0} \cdot R_0 r$, which in turn yields $H = \frac{\dot{R}_0}{R_0}$ ($R_0 r$ is the approximate *Euclidean* distance from observer to the location of the emitting object at the time of emission).

These linear relations, *including* Hubble's law, require nonlinear modifications for z of order 1 and higher. Quasars and many known galaxies have such high z values.

The latest astronomical surveys have pushed the largest observable z values to between 2 and 3 for *discrete* objects. The $2.7\,^\circ$K background radiation has undergone a redshift of order 10^3, since that radiation was emitted in the recombination epoch, when the comoving-frame cosmic average temperature was $T \sim 3000\,^\circ$K. ($T(t)$ scales with $\frac{1}{R(t)}$ and $R(t) \sim t^{2/3}$ in the post-recombination, matter-dominate epoch, so $t_0/t_{\text{recomb}} \sim (10^3)^{3/2}$; with $t_0 \sim 1.4 \times 10^{10}\,y$, $t_{\text{recomb}} \sim 400,000\,y$.)

Thus, the prophetic words of cardinal **Nicolas of Cusa** (1401–1464) and **Blaise Pascal** (1623–1662) were vindicated: "*The fabric of nature has its center everywhere and its circumference nowhere*".

How old is the Universe?

The age of the Universe has been a subject of religious, mythological and scientific importance. On the scientific side, ***Newton****'s guess for the age of the Universe was only a few thousand years.* ***Einstein****, the developer of the General Theory of Relativity, preferred to believe that the Universe was ageless and eternal. However, in 1929, observational evidence proved his fantasy was not to be fulfilled by nature.*

In order to understand this evidence, let's think about how a train sounds to a person standing on the platform. An arriving train emits tones that are higher pitched as the train approaches the listener while a departing train emits tones that get lower pitched as the train recedes from the listener. This change in the observed pitch of the train sounds is called a Doppler shift.

The Doppler shift happens with light as well as with sound. A source of light that is approaching the viewer will seem to have a higher frequency than a source of light that is receding from that viewer. In 1929, observations of distant galaxies showed that the light from distant galaxies behaved as if they were going away from us. If all the distant galaxies are receding from us on the average, that means that the Universe as a whole could be expanding. It could be blowing up like a balloon, retaining the relative *distances of its main "landmarks", on average.*

This is what tells us that the Universe probably does have a finite age; it probably is not eternal and ageless as Einstein wanted to believe.

We know from studies of radioactivity of the earth and nuclear reactions in the sun that our solar system probably formed about 4.5 billions years ago, which means that the universe must be at least twice that old, because before our solar system formed, our Milky Way galaxy had to form, and that probably took several billions years by itself (besides which, our galaxy contains older stars than the sun, in some of which the heavier-than-Lithium atoms comprising the sun were formed).

However, we can't do radioactive dating on distant stars and galaxies. The best we can do is balance a lot of different measurements of the brightness and distance of stars and the red shifting of their light to come up with some ballpark figure. The oldest star clusters whose age we can estimate are about 12 to 15 billions years old.

So it seems safe to estimate that the age of the universe is at least about 15 billion years old, but probably not more than 20 billion years old.

As far as we can tell, the expansion of the universe started from a very hot and dense state. From that state, it mushroomed and evolved into the universe we know today. Cosmologists call that process of expansion the Big Bang *because in some phases, especially in the beginning, the process was rather like an explosion.*

Much of our understanding the Big Bang is gleaned by extrapolating between knowledge of particle physics today, projections from the mathematical models of an expanding universe in general relativity, and astronomical and astrophysical data. The Einstein equations give us a mathematical model for describing how fast the universe would be expanding at what size and time, given the energy densities of matter and radiation at that time. Our estimates of the matter and radiation density of the early universe are based on the ancient light reaching us from the past in our night skies, and what we have learned about elementary particle physics, through theory and experiment.

1924–1930 CE **Frank Plumpton Ramsey** (1903–1930, England). Philosopher, mathematician and economist. Made important contributions to the above-named disciplines. In each case the fields of investigation that he opened came to be extensively developed only some thirty years after his death. In mathematics he established a pair of theorems about infinite sets which gave rise to an extensive new field, subsequently called 'Ramsey theory'. He is also known for translating Wittgenstein's *Tractatus* into English (1922).

1924–1935 CE **Pavel Sergeevich Alexandrov** (1896–1982, USSR). Mathematician. Contributed to the development of *algebraic topology* (as founded by Poincaré) and created a homological theory of dimension.

1924–1936 CE *The Vienna Circle* (*Wiener Kreis*): A philosophical movement active during the period between the two World Wars; brought together a group of *Logical Positivists*, led by **Moritz Schlick**[539] (1882–1936). The majority were scientists and mathematicians uninterested in metaphysical problems as such.

Leading figures in the Vienna Circle were: **Otto Neurath** (1882–1945); **Rudolf Carnap** (1891–1970); **Richard von Mises** (1883–1953); **Hans Reichenbach** (1891–1953); **Kurt Gödel** (1906–1978) and others. With the rise of Nazism, many of the Vienna group fled Austria and joined the faculties of British or American universities.

According to the Vienna Circle, *Logical Positivism* (alias *Logical Empirism*, *Neo-Positivism* or *Analytic Philosophy*) dethrones all philosophies, except the philosophy of language. It differs from **Comte**'s positivism in holding that all rational doctrines are meaningless. It claims that the unanswerable questions about causality and determinism are unanswerable just because they are not genuine questions at al; What is left for philosophy is the critique of language. Its result is to show that all genuine knowledge about nature can be expressed in a single language common to all sciences; Emphasis was given to the study of the limitations of the true sentences about nature. Thus, any 'necessary truth' was conceived as derivable from a rule of language or mathematics.

Although Wittgenstein (1889–1951) was not a member of the Vienna Circle, his *Tractatus Logico-Philosophicus* (1921) became one of the foundation treatises of Logical Positivism.

The Chemical Structure of Sex Hormones (1927–1935)

The sex hormones, cortisone, cholesterol and vitamin D — all have a 'bee's honeycomb' structure containing 4 *saturated hydrocarbon rings, known as the steroid ring structure.*

The male sex hormones (androgens) are produced in the testes. Of these, testosterone ($C_{19}H_{28}O_2$) *stimulate the development of the secondary male sexual characteristics, such as a deep voice and facial hair. It is converted in*

[539] Assassinated by a Nazi student on the steps of the Vienna University library on June 22, 1936.

the body into androsterone ($C_{19}H_{30}O_2$)*, which is excreted in the urine. Its principal function is to stimulate the development of the male reproductive organs.*

The female sex hormones are formed in the ovaries, It consists of estrogens [estrone ($C_{18}H_{22}O_2$)*; estradiol* ($C_{18}H_{24}O_2$)*; estriol] and progesterone* ($C_{21}H_{30}O_2$).

Estrogens help to develop the secondary female sexual characteristics that take place at puberty, control the menstrual cycle and are active during pregnancy, e.g. estrone is responsible for the cycle of ovulation. Both estradiol and estrone are found in the urine during pregnancy. Progesterone inhibits the release of ova (eggs) from the ovaries and aids in the maintenance of pregnancy. Relatively small quantities of sex hormones are required to instigate a particular activity. A great deal of effort, however, is required to exhibit even small quantities of sex hormones from animal tissues. Only 16 *mg of estradiol can be obtained from* 20 *tons of hog ovaries.*

The road to chemical identification of the sex hormones was opened in 1927 when ***Selmar Aschheim*** *(1878–1965) and* ***Bernhard Zondek*** *(1891–1966), then in Berlin, discovered that the urine of pregnant women contained amounts of the hormone, sufficient to produce sexual heat in mice or rats.* ***Adolf Windaus***[540] *(1876–1959) was asked by a German chemical firm to explore this substance. Involved at the time in the vitamin D research, he elected to turn over the new problem to his student* ***Adolf Butenandt*** *(1903–1995). The student plunged into the task of isolating the hormone. In the meantime* ***Edward Adelbert Doisy*** *(1893–1986, USA), of the ST. Louis University School of Medicine, also set to identify the active substance. Working independently, both Butenandt and Doisy succeeded to isolate the first known sex hormone — estrone. Soon after, Butenandt deduced the correct structure of estrone.*

The discovery and analysis of the male sex hormone testosterone followed a similar pattern. In 1931 Butenandt isolated from 15,000 *liters of male urine,* 15 *milligrams of a hormonal substance which he named androsterone. From his tiny pile of crystals, hardly enough to cover the tip of a small spatula, Butenandt derived a great deal of information about the nature of the molecule. He tentatively deduced its structure, and his deduction was independently proved correct by* ***Leopold Ruzicka*** *(1887–1976) of Zurich, who produced androsterone by splitting off the eight-carbon side chain from a derivative of cholesterol through oxidation.*

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540 Received the Nobel Prize for chemistry (1928) for studying *sterols* and their connection with vitamins.

Androsterone was obtainable only in very small amounts, either by synthesis or by extraction from urine, but Butenandt, Ruzicka and others soon succeeded in synthesizing a related substance which could be produced in more plentiful yield. This substance, named hydroepiandrosterone, was made from cholesterol by burning off the side chain, while the essential hydroxyl group attached to the first ring and the double bond at the 5, 6 position in the second ring were protected by stable chemical combinations of those positions. With the more plentiful working material at hand, the investigators were able to synthesize a number of interesting products, some of which proved more potent than androsterone in hormonal activity.

Meanwhile ***Ernst Laqueur*** *extracted pure testosterone from bull's testicles. Shortly thereafter, Butenandt and Ruzicka synthesized testosterone from dehydroepiandrosterone. It became clear that testosterone was the true hormone.*

In 1934, four research groups isolated from the corpus luteum tissue in sow ovaries the pregnancy hormone — progesterone. Butenandt obtained 20 *milligrams of the hormone from the ovaries of* 50,000 *sows. The structure of progesterone, very similar to that of testosterone, was soon inferred from its chemical properties and ultraviolet analysis. Butenandt promptly synthesized progesterone by two methods, one of which was the oxidation of a substance called pregnenolone, a by-product of the production of testosterone from cholesterol.*

At this dramatic point in the development of the chemistry of the sex hormones, and after the establishing of the structure of cholesterol (1932), there came a breakthrough in another field which was to open a more fertile route for production of the hormones — a new surge of intensive research on the extraction of hormones from plants.

1924–1939 CE **Walter Andrew Shewhart** (1891–1967, USA). Statistician. Pioneered modern thinking and developed statistical methods for *quality control* of industrial processes, which were subsequently applied to measurement processes in science. To this end he brought together the disciplines of statistics, engineering and economics. His main idea was to use statistics to distinguish between 'legitimate' statistical fluctuations and sub-performances that are more likely to be due to systematic errors such as a faulty machine, under-qualified workers etc. In the sciences, his methods came handy in distinguishing 'suspect' data sets in empirical work.

Shewhart was born in New Canton, Ill. He studied at the universities of Illinois and California, receiving his Ph.D. in Physics from UC Berkeley (1917). Most of his professional career was spent as an engineer at Western Electric (1918–1924) and at Bell Telephone Laboratories (1925–1956).

Shewhart presented a powerful and convincing case for establishing standards of economic quality using a statistical methodology that recognized the importance of achieving and maintaining stability in the processes of production and supply. And he showed how these standards of quality must be *standards by which the consumer may judge the quality of product, and which in themselves represent the goal of the producer*.

In conclusion, Shewhart observed that the engineer always likes to have a goal to attain, and suggests that he and his superiors must have available quantitative equations based on statistical understanding to define standards of economic quality. Shewhart also noted that in the development of productive science and the meeting of human want there is a balance between economic value to the consumer of a development, and the cost to the producer of such a development.

Quality Control

While studying physics ***Shewhart*** *had become inculcated with the concept of the exactness of physical laws, but when he became a practicing engineer with Western Electric he increasingly realized that another, fresher, concept was needed which matched more closely the real world of production and business.*

In May 1924 he presented his concept of a production control chart *which would "give at a glance the greatest amount of accurate information" about an ongoing process and how to ensure that it is maintained in a state of* economic control.

He was thus the first person to identify the vital importance of removing variation from all processes of production of goods and services, while at the same time recognizing that at the human level *this same understanding would inevitably favor those organization that chose to replace win-lose competition with win-win cooperation.*

In his 1931 book on the subject, Economic Control of Quality of Manufactured Product*, he encouraged his readers to ponder the subjects of psychology, philosophy and logic so as to better understand the mechanism by which the mind, its reasoning, and the practical world all work.*

In May 1932 Shewhart was invited to London to present a series of papers at University College where his work received a warmer welcome than in the United States. The English response to Application of Statistical Methods to Industrial Standardization and Quality Control *(1935) stimulated a reawakening of interest in the subject in the US.*

The win-win cooperation is a better strategy *from the point of view of* game theory*: Indeed, before Shewhart, a great part of the thought and interest of management and workmen in manufacturing establishments has been centered upon the proper division of surplus resulting from their joint efforts. The* management *have been looking for as large a* profit *as possible, and the* workers *have been looking to maximize* wages*: the surplus was the balance of selling* price *and the non-payroll* costs *associated with the manufacture of an article sold. From the surplus would come both the workmen's wages and the management's profit; the division of this surplus has generally been the source of most trouble between management and labor.*

Frequently, when management found the selling price going down they have turned toward a cut in wages — toward reducing the workman's share of the surplus — as their way of preserving their profits intact. Gradually the two sides have come to look upon one another as antagonists, and at times even as enemies — putting their strength against each other bargaining, strikes, layoffs etc.

While profit is clearly a prime measure of success of any organization operating in an open market, it is not the only parameter within the complex equation which defines human happiness and which must guide the thinking of engineers and managers alike. There are other, less tangible parameters by which management's efforts must be gauged, such as quality*, satisfaction etc., but profit is the most broadly accepted and therefore most universally useful.*

Yet profit may be seen in two ways. Firstly, it can be the margin by which the grand total cost of supply differs from the market price. Or it can be seen as the margin between the market price and the lowest possible cost of supply. In the first case profit is the seller's margin; in the second it is the buyer's margin.

It was the early telephone that served to drive managements to discover new levels of reliable performance.

In the early part of the 20th century America was pioneering the widespread use of telephony with transcontinental landlines and improving levels *of reliability. Conventional handsets consisted of a few hundred pieces while the switching arrangements between handsets would account for upward of 100,000 separate, but interdependent, parts — undreamed of complexity! Excessive failures of handsets, switching units and buried amplifiers caused by variations in manufacture could not be tolerated if public demand for reliable and instant communication at a distance was to be met.*

When America entered WWII, the demand for war supplies increased greatly and the need for weapons perfection was of paramount importance.

But with the war ended and a massive redirection of industrial capacity from war to peace, the marketplace of the mid-forties shifted from one of national survival to one of a sellers' paradise. The hard work and discipline of designing quality into controlled production dissipated, and the more laissez faire approach of sorting bad quality out from ad hoc *production, or even shipping products regardless of quality, took over. Quantity was now the issue, moving boxes the preoccupation, easy profits the Pyrrhic prize. In the five postwar years until 1950, when this run-down of quality management took place in the US, precisely the opposite was happening in the Far East.*

There are two ways to accept or reject lots of merchandise in a production line:

- Sampling inspection *is a method for protecting the purchaser against poor quality* after *the product has been manufactured. From a consumer point of view, there is a maximum percentage of defectives that he or she will tolerate. This percentage is expressed by p. Clearly, without a 100 percent inspection, it may be impossible to be certain that the quality is better then the level expresses by p. However, the sampling practice arises from the fact that it is often more economical to tolerate small percentages of defectives than to bear the cost of 100 percent inspection. The sampling inspection procedure ensures the quality of the product* with a certain probability.

- Quality control (QC) *is a method of finding and correcting flaws in the* manufacturing process *to ensure that the product meets the standards of the company. After the bugs are ironed out of a production process, its output is stable and is said to be "in control".* Keeping *it in control is a major statistical and engineering task.*

Statistical theory tells us that the proportion of successes $\frac{x}{n}$ will be approximately normally distributed with mean p and standard deviation $\sigma = \sqrt{\frac{pq}{n}}$ (where $q = 1 - p$), if n is sufficiently large. Now, suppose we want to test whether daily percentages of defectives p may be treated as independent trials of an experiment for which p is constant from trial to trial.

One plots a graph of the observed p *vs time (in units of days) and then draws three parallel lines parallel to the time axis at* $\bar{p}+3\sigma,\ \bar{p},\ \bar{p}-3\sigma$ *where* $\bar{p}$ *is the mean of past daily proportions* $\frac{x}{n}$, *and* $\sigma^2 = \frac{1}{n}\bar{p}(1-\bar{p})$.

Now, if the production process behaves in an idealized manner and if the normal approximation to the binomial distribution may be used, the probability that a daily proportion (when plotted on this chart) will fall outside the control band is approximately equal to the probability that a normally-distributed variable will assume a value more than three standard deviations away from its mean, which is calculated to be 0.03.

Because of this small probability, it is reasonable to assume that the production process is no longer behaving properly when a point falls outside the control band; consequently, the production engineer checks over the various steps in the process when this event occurs. Industrial experience shows that only rarely does a production process behave in this idealized manner when the control-chart technique is first applied. Nevertheless, the technique is highly useful because it enables one to discover causes of a lack of control and thus improve on the production process until gradually statistical control *has been obtained.*

Note that a lot of items is sampled according to a scheme guaranteed to reject a good lot with a certain probability α *(supplier's risk), and to accept a defective lot with a certain probability* β *(consumer's risk). A lot is considered under control if the demerit parameter that characterizes its quality (such as* p*) does not* exceed *a certain limiting value and defective if this parameter has a value not* smaller *than another limiting value.*

1924–1940 CE **Wolfgang Pauli** (1900–1958, Switzerland). One of the most influential physicists of the 20^{th} century. Won the Nobel prize for physics in 1945 for his discoveries in quantum theory. His major achievements are:

(1) In 1924 he was first to propose a fourth *spin* quantum number for the specification of the atomic single–electron energy state[541], which may take

[541] The orbital angular momentum is quantized both in *magnitude* and *direction*. In classical physics, the angular momentum $\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}$ of a system relative to the center of a central or contact force is constant in magnitude and direction. In quantum mechanics the magnitude of the orbital angular momentum

is $|\boldsymbol{L}| = \sqrt{\ell(\ell+1)}\ \hbar$ with the quantum number ℓ a non-negative integer. In a nuclear Coulomb field (or that of a nucleus plus closed inner electron shells), for each value of n specifying a single–electron energy level, there are n distinct values of the orbital angular momentum of each electron, from $\ell = 0$ to $\ell = n-1$. The quantum restriction of $\boldsymbol{L}$ shows that the vector $\boldsymbol{L}$ cannot make arbitrary angles with a given direction (the z-direction, say); allowed values of its observed component in the z direction are quantized, $L_z = m_\ell \hbar$. For each value of ℓ, there are $2\ell+1$ values of m_ℓ $(-\ell, -\ell+1, \dots, \ell)$, or $2\ell+1$ different orientations of $\boldsymbol{L}$. It is *impossible* to know exactly more than one component of the angular momentum; i.e. if we know $|\boldsymbol{L}|$ and L_z, our knowledge of L_x and L_y is at best within the uncertainties ΔL_x and ΔL_y such that $(\Delta L_x)(\Delta L_y) \geq \frac{\hbar}{2}|L_z|$. Thus, for given (ℓ, m_ℓ), $\boldsymbol{L}$ points in an unknown direction along a *cone*. Such uncertainty relations follow from $\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}$ and from the position-momentum uncertainty relation.

The so-called *magnetic quantum number* m_ℓ manifests itself when the electron's motion is disturbed by an applied magnetic field (in the z direction, say). Under a magnetic field, each spectral line in a one-electron atom is observed to split into a number of closely spaced lines [1897; **Pieter Zeeman** (1865–1943, Holland)]. The orbital magnetic dipole moment of the electron is $\boldsymbol{M}_L = -\frac{e}{2m_e}\boldsymbol{L}$, which entails $\{M_L\}_z = -\mu_B m_\ell$ where $\mu_B = \frac{e\hbar}{2m_e}$ is the *Bohr magneton.* Thus, when an atom is placed in a weak magnetic field $\boldsymbol{B}$, the interaction of this field with the magnetic dipole moment of the orbiting electron causes $\boldsymbol{L}$ to precess around $\boldsymbol{B}$, where the tilt of $\boldsymbol{L}$ is quantized. In addition, the electron acquires the energy shift $-(\boldsymbol{M}_L \cdot \boldsymbol{B}) = -\mu_B B m_\ell$ *which is also quantized*, taking $2\ell+1$ distinct values, corresponding to the $2\ell+1$ possible orientations of $\boldsymbol{L}$ relative to $\boldsymbol{B}$ (the application of the external field $\boldsymbol{B}$ automatically selects its direction as the z-axis appropriate for quantization). That is, each energy level $\{n, \ell\}$ splits into $2\ell+1$ levels. This explains the *Zeeman effect*, which thus turns out to provide evidence for (orbital) angular momentum quantization.

A major shortcoming of the Schrödinger theory of hydrogen-like atoms was its failure to account for the *fine structure* of spectral lines, including those of hydrogen. Several lines of evidence converged to indicate that an electron possesses intrinsic angular momentum. In 1925 **Uhlenbeck** and **Goudsmit** suggested that this intrinsic (spin) angular momentum has observable component values $\pm\frac{1}{2}\hbar$. In retrospect, this explained the *Stern-Gerlach* experiment (1921) where the neutral silver atoms could only carry spin angular momentum, due to the single S-orbital $(\ell = 0)$ valence electron in each atom (the other electrons residing in a inner core with zero overall spin and orbital angular momenta). The nonuniform magnetic field in that experiment thus provided a spin-dependent semiclassical force, which split the beam into two components.

The spin angular momentum of the electron $\boldsymbol{S}$ adds another level of complexity to atomic physics. Because the electron is a charged particle, its spin results in an intrinsic magnetic dipole $\boldsymbol{M}_S$, related to $\boldsymbol{S}$ via $\boldsymbol{M}_S = -g_e\frac{e}{2m_e}\boldsymbol{S}$, the numerical values[542] $+\frac{1}{2}$ or $-\frac{1}{2}$ in $\hbar$ units.

(2) In 1925, Pauli introduced his *exclusion principle*, which states that there can be no two electrons in any atom that have the same set of quantum numbers (spin included). With the aid of this principle it was possible to explain the *atomic shell structure and the periodic system of the elements*. In 1940, Pauli generalized the exclusion principle to include all fermions.

(3) In 1927 Pauli proceeded to incorporate the electron spin into the Schrödinger formalism. In his theory the state of the electron was specified by a two–component wave-function, known as the *Pauli spinor*, with a transformation law under rotations given by the general unitary unimodular complex matrix in 2 dimensions. This led to the recognition (by physicists!) that for every unitary unimodular transformation in two complex dimensions there is an orthogonal transformation in three dimensions corresponding to a proper rotation. This fact had been previously known in mathematics, under the guise of the relation between Euler angles and the Cayley-Klein parameters. In this formalism, the three Pauli matrices $(\sigma_1, \sigma_2, \sigma_3)$ act as generators. When multiplied by $(-i)$, these new matrices obey the multiplication laws of *quaternions*.
Van der Waerden later extended Pauli spinors to 4 spacetime dimensions such that the entire Lorentz group $SO(3,1)$ can be represented by them (via the locally-isomorphic $SL(2,C)$ Lie group). However, spin angular momentum does not arise naturally from the solutions of the nonrelativistic Schrödinger wave equation; a better understanding of it thus had to await the advent of the *Dirac equation*, in 1928.
Pauli's theory helped to explain the *Stern-Gerlach experiment* (1921), the

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where g_e is the *(spin) gyromagnetic ratio* of the electron, with an experimental value of 2.0024. The *total* magnetic dipole moment of an orbiting electron is $\boldsymbol{M} = \boldsymbol{M}_L + \boldsymbol{M}_S = -\frac{e}{2m_\ell}(\boldsymbol{L} + g_s\boldsymbol{S})$. Since the Stern-Gerlach experiment showed that the electron spin may have only two orientations relative to the magnetic field, the solution of the equation $2 = 2s + 1$ yields $s = \frac{1}{2}$, where $m_s = \pm\frac{1}{2}$. The only two permitted values of $S_z = \hbar m_s$ with regard to a selected measurement (z) axis correspond to the two possible orientations of $\boldsymbol{S}$. For brevity, they are referred to as spin up (↑) and spin down (↓), although the spin is never actually directed along the z-axis or opposite to it, due to the uncertainty in S_x and S_y.

[542] Quantum physicists often employ units $\hbar = c = 1$ where mass, energy, momentum, length and time units are all inter-convertible. In $\hbar$ units, Bohr's *orbital* angular momentum takes integer values.

atomic Zeeman effect (1896) and the experiment of **Phipps** and **Taylor** (1927).

(4) In 1931 Pauli proposed a new particle, to account for the apparent violation of the law of conservation of energy in nuclear β decays[543]. It was

[543] Around the year 1930 the skill of experimental physicists had revealed an apparent non-conservation of energy (and momentum) in the radioactive decay of the bismuth nucleus (^{210}Bi), which emits an electron (e^-) and thereupon becomes a polonium nucleus (^{210}Po). The combined total energy of the ^{210}Po and the electron e^- turned out to vary in amount in various such events, but was always *less* than the energy of the initial ^{210}Bi. The angular momentum balance also seemed to be upset by this reaction, because both ^{210}Bi and ^{210}Po nuclei are bosons, possessing integer spin in atomic units, (because they each have the even number 210 of fermionic nucleons, which – like the electron – have spin $S = \frac{1}{2}$); whereas the electron is a fermion possessing half integer ($S = \frac{1}{2}$) spin. Since the orbital angular momentum had never been observed to be anything but an integer number of $\frac{\hbar}{2}$ units, the reaction seemed to violate the conservation of angular momentum as well. The faith in conservation laws drove Pauli to predict the neutrino as the carrier of the "missing" energy, momentum and angular momentum of the reaction.

Strong experimental confirmation of Pauli's hypothesis was gained in 1956, when physicists from Los Alamos succeeded in inducing reactions of a beam of neutrinos from a nuclear reactor with nuclei. It is now known that the neutrinos emitted in the radioactive decay of nuclei consists of two types. The quanta appearing in conjunction with the emission of positive electrons (e^+) always have their angular momentum pointing in a direction opposite the velocity, i.e. their spin aligned opposite to their momentum ("left-handed") and are called neutrinos (ν). The quanta of the neutrino field appearing in conjunction with the emission of negative electrons (e^-) always have their spin aligned *parallel* to their momentum ("righthanded") and are called antineutrinos ($\overline{\nu}$). Thus the all but invisible quantum emitted in the decay of ^{210}Bi is actually an antineutrino. In later decades two more species of ν and $\overline{\nu}$ (*muon* neutrinos – ν_μ, $\overline{\nu}_\mu$ – and τau neutrinos ν_τ and $\overline{\nu}_e$), associated with much heavier (and radioactive) versions of the electron (the μ^-, τ^- leptons and their antiparticles). All the neutrinos have small but finite masses, and slowly *interconvert* (mix coherently) when traveling astronomical distances, or through the earth or sun.

The *helicity* of a particle is defined as the component of its intrinsic angular momentum ($\boldsymbol{S}$) in the direction of its velocity vector ($\boldsymbol{v}$) : $H = \frac{1}{S|\boldsymbol{v}|}(\boldsymbol{S} \cdot \boldsymbol{v})$.

All neutrinos observed in nature are found to have *negative helicity*, while all antineutrinos observed in nature are found to have *positive helicity*. Note that since the neutrino is almost massless (several $\frac{eV}{c^2}$), its speed is almost equal to

> later named '*neutrino*' ("little neutral one") by **E. Fermi**. The neutrino was finally observed in 1956; it is a spin-$\frac{1}{2}$ fermion, like the electron but devoid of electron charge, nearly massless, and interacting only weakly and gravitationally.

Pauli was born in Vienna, son of a Jewish physician whose name was Paschkes before he converted to Christianity. At the age of 21 he wrote a 200-page encyclopedia article on the theory of relativity, which today is still one of the best expositions of this subject. He studied at Münich and was a student of **Sommerfeld**. Later he was an assistant to **Max Born** and **Niels Bohr**.

In 1928 Pauli became professor of theoretical physics at the Federal Institute of Technology, Zürich. In 1940 he was appointed to the chair of theoretical physics at the Institute for Advanced Study, Princeton, NJ. Following WWII he returned to Zürich.

1924–1941 CE **Hendrik Anthony Kramers** (1894–1952, Holland). Physicist. Made significant contributions to mathematical and theoretical physics.

In 1924 he predicted the existence of the *Raman effect* (an inelastic scattering of light off molecules). Discovered, independently of **H. Jeffreys** (1923) **G. Wentzel**[544] and **L.N. Brillouin**, the WKBJ approximation method (1926). Together with **R.L. Kronig**, in their study of optical X-ray dispersion, derived basic relations between the real and imaginary parts of the Fourier transform of a causal function (1927). These are known today as the *Kramers-Kronig dispersion relations*[545] and play an important role in many branches of physics and engineering [they are linked to the Cauchy integral formula and the Hilbert transform].

In 1937 Kramers introduced the quantum–mechanical *charge conjugation operation* (replacing particles by their antiparticles) and the law of invariance

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the speed of light; in the limit in which it were truly devoid of (rest) mass, it could not be stopped nor reversed by a Lorentz transformation (i.e. there would be no frame in which the neutrino is moving in the opposite direction with *opposite helicity*).

[544] **Gregor Wentzel** (1898–1978; Germany, Switzerland and USA). A student of Sommerfeld in Munich (Ph.D. 1921). Professor of Mathematical physics at Leipzig (1926), Zurich (1928–1947), Chicago (1948–1970).

[545] Originally, they derived relations between absorption and dispersion of EM radiation. In the context of electrical engineering, these causality constraints are known as the *Bode integral theorem*.

(symmetry) under this operation which turned out, like *E. Wigner's parity invariance* (spatial reflection), to be valid for all known interactions except the weak nuclear force. Kramers contributed to the mathematical structure of quantum mechanics, theory of phase transitions in ferromagnetism and the one-dimensional Ising model (1941) [a chain of N spins, each spin interacting with its two nearest neighbors].

Just before he died, he had surgery postponed because he wanted to finish a paper.

1924–1953 CE **Richard Courant** (1888–1972, Germany and USA). Mathematician and educator. Founded the famous Mathematical Institute at Göttingen University and directed it (1920–1933). Founded (1953) and directed the *Courant Institute of Mathematical Sciences* — an applied mathematics research center in New York, based on the Göttingen model.

Courant was born in Lublinitz, Prussia (now Lubliniez, Poland) to Jewish parents. He obtained his doctorate under Hilbert's supervision. He taught mathematics at Göttingen, where he was **Klein**'s successor, until the start of WWI. A few years after the war he returned to Göttingen, becoming a professor (1920). Contributed to various branches in mathematical physics.

In 1924 he published, jointly with **Hilbert**, an important text *Methoden der mathematischen Physik*. When the Nazis purged Göttingen of its Jewish mathematicians (1933), he left for England, and thence to New York (1934).

Although Courant did not originate new ideas or techniques in mathematics, his contribution lies mainly in transplanting the heritage of the German school of mathematics to the United States at a most opportune time on the eve of WWII, thus securing the uninterrupted continuity and growth of mathematical physics in the free Western culture.

1924–1957 CE **Felix Heinrich Wankel** (1902–1988, Germany). Inventor of the *Wankel rotary engine* – a type of internal combustion engine which uses a rotor instead of pistons.

In the Wankel engine, the 4 strokes of a typical Otto cycle occur in the space between a three-sided rotor [having a shape approximating a *Reuleaux Triangle*[546]] and the inside of an oval-like housing (Fig. 5.6).

[546] The *Reuleaux Triangle* (Fig. 5.5) forms the basis of the Wankel rotary internal combustion engine. It is named after the engineer **Franz Reuleaux** (1829–1905, Germany) who did pioneering work on ways that machines translate one type of motion into another (1861–1875).

Reuleaux believed that *machines could be abstracted into chains of ideal elements* constrained in their motions by adjacent parts in the kinematic chain.

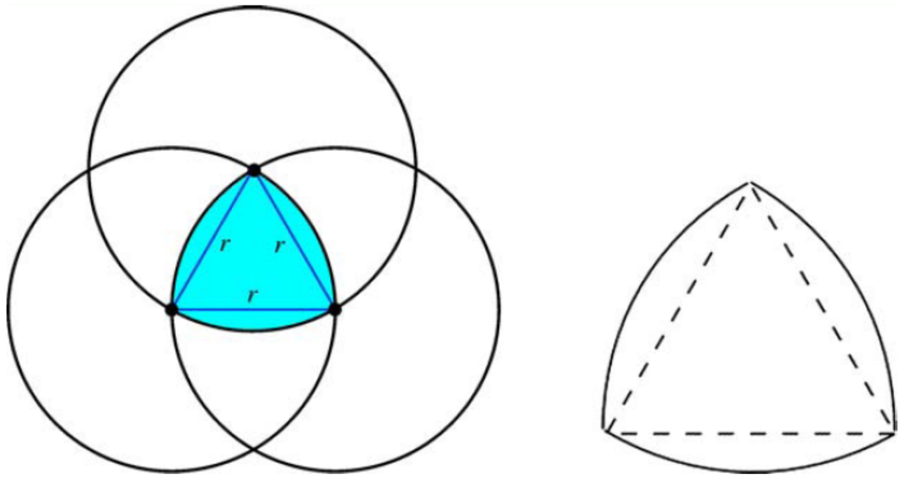

Fig. 5.5: The *Reuleaux triangle* is a constant – width curve based on an equilateral triangle. The distances from any point on a side to the opposite vertex are all equal

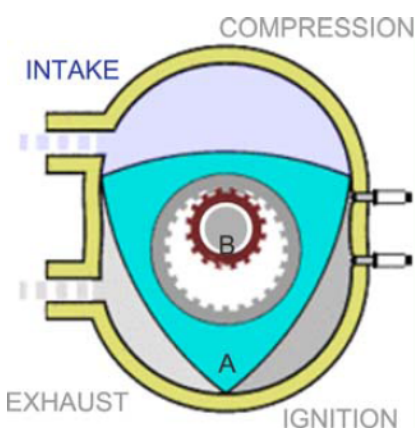

Fig. 5.6: The Wankel cycle. "A" marks one of the three apexes of the rotor. "B" marks the eccentric shaft, and the inner white region – a gear – toothed circular hole bored about the rotor center – is the lobe of the housing– centered eccentric shaft. The shaft turns three times for each clockwise spin of the rotor around the lobe and once for each orbital revolution of the rotor within the housing

The central drive shaft, also called an eccentric shaft or E-shaft, passes through the central borehole (offset lobe) of the rotor; it is half the lobe's radius and engages its boundary via gear teeth, and is supported by bearings. The rotor both rotates about the offset lobe and makes orbital revolutions around the engaged central shaft. Seals at the corners apexes of the rotor seal against the periphery of the housing, dividing it into three moving combustion chambers. Fixed gears mounted on each side of the housing engage with ring gears attached to the rotor to ensure the proper orientation as the rotor moves.

As the rotor rotates and orbitally revolves, each side of the rotor gets periodically closer and farther from the oval walls of the housing, in the process compressing and expanding the combustion chambers similarly to the strokes of a piston in a reciprocating engine. While a four-stroke piston engine makes one combustion stroke per cylinder for every two rotations of the crankshaft (that is, one half power stroke per crankshaft rotation per cylinder), each combustion chamber in the Wankel generates one combustion stroke per each driveshaft rotation, i.e. one power stroke per rotor orbital revolution and three power strokes per rotor rotation. Thus, power output of a Wankel engine is generally higher than that of a four-stroke piston engine of similar engine displacement in a similar state of tune, and also higher than that of a four-stroke piston engine of similar physical dimensions and weight.

He developed a compact symbolic notation to describe the topology of a very wide variety of mechanisms, and showed how it could be used to classify them and even lead to the invention of new useful mechanisms. At the expense of the German government, he directed the design and manufacture of over 300 beautiful models of simple mechanisms, such as the four bar linkage and the crank. These were sold to universities for pedagogical purposes. Today, the most complete set are at Cornell University College of Engineering. Using his notation and methods for systematically varying the elements (e.g. inversions, changing relative sizes of the links, etc.) he showed how the four bar linkage could be mutated into 54 mechanisms, which fall within 12 classes.

Today, he may be best remembered for the *Reuleaux triangle*, a curve of constant width that he helped develop as a useful mechanical form.

To construct the Reuleaux triangle, start with an equilateral triangle. Center a compass at one vertex and sweep out the (minor) arc between the other two vertices. Do the same with the compass centered at each of the other two vertices. Delete the original triangle. The result is a curve of constant width.

The total area of the Reuleaux Triangle is $\frac{1}{2}(\pi - \sqrt{3})r^2$, where r is the circular arc radius. It can be rotated inside a square of side r, covering a fractional area of $2\sqrt{3} + \frac{\pi}{6} - 3 = 0.9877003908\ldots$. By the *Blaschke-Lebesgue theorem*, the Reuleaux Triangle has the least area of any curve of given constant width.

Wankel engines have several major advantages over reciprocating piston designs, in addition to having higher output for similar displacement and physical size. Wankel engines are considerably simpler and contain far fewer moving parts. For instance, because valving is accomplished by simple ports cut into the walls of the rotor housing, they have no valves or complex valve trains; in addition, since the rotor is geared directly to the output shaft, there is no need for connecting rods, a conventional crankshaft, crankshaft balance weights, etc. The elimination of these parts not only makes a Wankel engine much lighter (typically half that of a conventional engine of equivalent power), but it also completely eliminates the reciprocating mass of a piston engine with its internal strain and inherent vibration due to repeated acceleration and deceleration, producing not only a smoother flow of power but also the ability to produce more power by running at higher RPM.

In addition to the enhanced reliability by virtue of the elimination of this reciprocating strain on internal parts, the engine is constructed with an iron rotor within a housing made of aluminum, which has greater thermal expansion coefficient. This ensures that even a severely overheated Wankel engine cannot seize, as would likely occur in an overheated piston engine. This is a substantial safety benefit in aircraft use, since no valves can burn out. A further advantage of the Wankel engine for use in aircraft is the fact that a Wankel engine can have a smaller frontal area than a piston engine of equivalent power. The simplicity of design and smaller size of the Wankel engine also allows for savings in construction costs, compared to piston engines of comparable power output.

As another advantage, the dynamical shape of the Wankel combustion chambers and the turbulence induced by the moving rotor prevent localized hot spots from forming, thereby allowing the use of fuel of very low octane number or very low ignition power requirement without preignition or detonation, a particular advantage for hydrogen cars. Four-stroke reciprocating engines are less suitable for hydrogen. The hydrogen can misfire on hot parts like the exhaust valve and spark plugs. Another problem concerns the hydrogenate attack on the lubricating film in reciprocating engines. In a Wankel engine this problem is circumvented by using a ceramic apex seal against a ceramic surface: no oil film means no hydrogenate attack. Since a piston ring of ceramic material is not possible, a similar fix is unworkable with the reciprocating engine. The piston shell must be lubricated *and* also cooled with oil, which substantially increases the lubricating oil consumption in a four-stroke engine.

Disadvantages

Compared to piston engines, the time available for fuel to be injected into a Wankel engine is significantly shorter, due to the way the three chambers rotate. The fuel-air mixture cannot be pre-stored as there is no intake valve. This means that to get good performance out of a Wankel engine, more complicated fuel injection technologies are required than for regular four-stroke engines. This difference in intake times also causes Wankel engines to be more susceptible to pressure loss at low RPM compared to regular piston engines.

In terms of fuel economy, Wankel engines are generally less efficient than four stroke piston engines. Problems also occur with exhaust gases at the Wankel's peripheral port exhaust, where the concentrations of hydrocarbons can be higher than from the exhausts of regular piston engines.

The reason Wankel-cycle engines have higher fuel consumption and emissions than Otto-cycle engines is that the combustion chambers in a Wankel are quite large at ignition, so the compression ratio is lower. This lowers the thermal efficiency and thus the fuel economy. Additionally, some fuel may get too far from the flame front during combustion to be fully burned. This is why there can be more carbon monoxide and unburnt hydrocarbons in a Wankel's exhausts stream.

Wankel was born in Lahr (upper Rhine Valley), Germany. Although he had no university education, he was able to teach himself technical subjects and conceived the idea of the Wankel engine in 1924. In the 1930s, he was imprisoned by the Nazis for some months.

During World War II, he developed rotary valves for German air force aircraft and navy torpedoes. After the war, he was imprisoned by the Allies for several months; his laboratory was closed, his work confiscated, and he was prohibited from doing further work. In 1951, he began development of this engine, leading to the first running prototype on February 1, 1957. The engine has been successfully used by Mazda in several generations of their RX-series of coupés.

1924–1957 CE **Alexander Ivanovich Oparin** (1894–1980, Russia). Biochemist. A pioneer of modern theories of the origin of life. Put forward the idea that life evolved in random chemical processes in the ocean, which became a '*biochemical soup*' conducive to early life forms. His ideas became the basis of many modern scientific theories of the origin of life.

Oparin's theory of the origin of life rests on the belief that the earth's early atmosphere contained mostly ammonia, hydrogen, methane, and water vapor — not nitrogen and oxygen as it does today. He suggested that the chemicals necessary for life formed spontaneously in such an atmosphere. These molecules combined and formed more complex molecules. These molecules formed still larger compounds and structures, and finally they developed into the first living cells over the course of hundreds of millions of years.

It is, of course, a long leap from an atmosphere containing methane and ammonia to even the simplest living systems, but at least the atmospheric evidence suggests that most of the elements now found in organisms were also abundant on the earth early in its history.

In 1953, **Stanley L. Miller** performed a laboratory experiment on the origin of life. He constructed an apparatus for producing amino acids under primitive earth conditions: steam was passed through a mixtures of gaseous methane (CH_4), ammonia (NH_3), and hydrogen (H_2), and then exposed to a high-energy electric spark before being recondensed as water. After a week of operation of the apparatus, the water was found to contain amino acids, the fundamental building blocks of proteins and organisms.

Miller's simple experiment showed that electrical discharges, such as lightning, in an early reducing atmosphere, could have led to the production of some complex molecules of living systems[547].

Oparin was born in Uglich, near Moscow. He graduated from Moscow State University in 1917. His theory was described in a book *The Origin of Life on Earth* in 1924, long before anything was known about the structure and chemical nature of genes. In the third edition of his book (1957) he proposed that the order of events in the origin of life was: *cells first, enzymes second, genes third*.

The Oparin picture was generally accepted by biologists for half a century, because it seems to be the only alternative to biblical creationism. But since the discovery of the double helix (1953) showed that genes are structurally simpler than enzymes, it became natural to think of the nucleic acids as primary and of the proteins as secondary structures. For that reason, the sequential order of Oparin was changed to: *genes first, enzymes second and cells third* (**Manfred Eigen**, 1981), and Oparin's theory was neglected.

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[547] Yet, for the next 20 years, scientists could not get close to the next step, which is to produce *proteins*. Simple amino acids are not even proteins, much less life, so the bridge between nonlife and life remained elusive.

In the early 1970s, geochemists realized that the earth's early atmosphere was probably nothing like the gases used in the Miller-Urey experiment. This experiment used an "atmosphere" modeled on what we knew of Jupiter, composed of methane, ammonia, hydrogen, and water. In the 1970s, geochemists discredited this theory of the earth's early atmosphere and concluded that it probably contained more *carbon dioxide*, almost no hydrogen, and possibly some oxygen. Creation of even simple amino acids would have been impossible in such an environment. There is still (2006) no plausible account for the origin of life. Yet, the Miller-Urey experiment is still taught in biology textbooks!

Recently, however, **Freeman Dyson** (1985) chose to base his mathematical model of the origin of life on Oparin's theory, because it allows early evolution to proceed in spite of high transcription-error rates.

Origin and Evolution of Terrestrial Atmospheres

As soon as the solid earth formed, the natural decay of radioactive elements below the surface heated up the crustal rocks, causing the chemical decomposition of some of the minerals present there. In the process water (H_2O), carbon dioxide (CO_2), and other gases were liberated from chemical compounds. These substances outgased to the surface, especially through volcanism. Water was the most plentiful compound to be released from the earth's crust; once it reached the cooler surface, it condensed to form the oceans where most of it remains today.

Next most important was CO_2, with about $\frac{1}{10}$ of the abundance of water. Much of it dissolved in the oceans and some recombined (at surface temperatures) with surface rocks to re-form carbonates, which remain on the ground today. Perhaps only $\frac{1}{5}$ of the abundance of the CO_2 represented nitrogen gas, but nitrogen is relatively inactive chemically, and now remains in the atmosphere as its major constituent (about 78 percent). Another, even more inert gas to be released was argon-40, formed by the radioactive decay of potassium-40. Argon today comprises about 1 percent of the atmosphere.

The early atmosphere of the earth may have been made of nitrogen, CO_2, argon, H_2O, methane (CH_4), ammonia (NH_3), and hydrogen, the last three having originated either from outgassing or from chemical activity at the surface. These would quickly escape or decompose, but their temporary presence, bathed in solar radiation, would create a chemical environment favorable to the formation of more complex molecules.

With energy supplied by sunlight, some water vapor in the upper atmosphere may have broken down to hydrogen and oxygen by a process called photolysis *('breaking by light'). Any oxygen so formed would not have remained free (uncombined) because it is highly reactive and would have quickly combined with gases like methane and carbon monoxide to form water and carbon dioxide. It would have also combined with crustal materials (with metals like iron in olivines and pyroxenes) to form iron oxides like hamatite (Fe_2O_3). The production of significant amounts of* free oxygen, *and its persistence in the atmosphere, probably came about only after life evolved at least to the complexity of green algae*[548].

[548] Although the general course of events described here is based on physical plausibility and extrapolation from experiment, the scenario for the details of each stage is but one of several hypotheses currently extant.

Of all the solar-system's planets, the earth alone has appreciable free oxygen in its atmosphere. It came about because of the development of life. Green plant life, mainly in the oceans, flourished during the first few billion years of the earth's existence. This vegetation removed the CO_2 *from the air by the process of* photosynthesis*, building itself with the carbon and releasing the oxygen into the atmosphere. When the vegetable organisms died, they decayed (or oxidized), removing the oxygen from the air again.*

However, part of the dead vegetation escaped the decay process, by being preserved in the ground in the form of fossil fuels*, where much of it remains today. Thus, most of the oxygen is removed from the air by the decay of dead plant matter (and later by decay of animals, by combustion and by respiration), but a little (somewhere between one part in* 10^4 *and one part in* 10^5*) of that produced by photosynthesis remains in the atmosphere, gradually building up the oxygen concentration. It is estimated that oxygen comprised only about* 1 *percent of the atmosphere* 600 *millions years ago, but since then it has gradually accumulated to about* 21 *percent.*

In recent years man has upset this delicate balance. It is estimated that the amount of oxygen used in the technosphere in relation to the net production of oxygen in the biosphere is about 11 *percent. This arises from our consumption of fossil fuels (coal and oil) extracted from the earth. At the present rate it would still take thousands of years to use up our oxygen (the easily available coal and oil will be used up much sooner than this), but the resulting increase in the concentration of* CO_2 *in the atmosphere could have a profound effect on the climate by increasing the world's average temperature enough to melt the polar caps and flood several countries (the greenhouse effect).*

All life today is protected from deadly doses of the sun's ultraviolet radiation by atmospheric oxygen; under the intense radiation present in the upper atmosphere, this oxygen forms a layer of ozone *(*O_3*) which absorbs most of the sun's ultraviolet rays and prevents them from reaching the lower atmosphere and the earth's surface. Without this ozone screen, life would be possible only under rocks, in deep water, or in other places where direct sunlight could not penetrate. Only after free oxygen, and a consequent ozone screen, began accumulating in the atmosphere was life first possible in shallow waters and, ultimately, on the surface of the land.*

Land life did not become abundant before the time when free atmospheric oxygen and ozone first attained approximately their present-day levels (ca 400 *millions years ago).*

1924–1962 CE **Tibor Radó** (1895–1965, Hungary and U.S.A.). Mathematician. Made important contributions to the calculus of variations, algebraic topology, measure and integration theory and "Turing programs". He proved (1924) that every manifold of dimension 2 can be triangulated. Proved (1930) the existence of a surface of minimal area bounded by a given closed contour in space (*The Plateau Problem*). Discovered (1962) a noncomputable function (Turing Machine).

Radó was born in Budapest, Hungary to Jewish parents. He began his university studies in civil engineering at the Technical University in Budapest. In 1915 he enlisted in the army, was trained and then commissioned as second lieutenant in the infantry. He took part in two major battles on the Russian front before being captured on a scouting mission. His four years in prison camps (1916–1920) read like a scenario of an action movie: prior to the revolution he was imprisoned in Tobolsk, Siberia, where the only books he could obtain happened to be on mathematics. After the revolution he was transported thousands of kilometers under harrowing conditions. During the confusion he and three fellow officers traded names with four private soldiers. As far as his family knew, Radó was dead. He spent the next year working as a laborer in railroad yards. Then, he and a group of prisoners escaped by hijacking a train.

Finally, in 1920 he returned to Budapest on an American-financed boat which was assisting the return of war prisoners. Back at the University of Szeged, he re-enrolled, this time as a mathematics major, and in 1922 he received his Ph.D. under **Frigyes Riesz**. In 1929 he emigrated to the United States and in 1930 moved to Ohio State University as full professor of mathematics. At the end of WWII, he went to Europe to recruit German scientists needed by the United States.

1924–1962 CE **Edward Charles Titchmarsh** (1899–1963, England). Pure mathematician. A dominant figure in Oxford mathematics during 1931–1949. Did important work on Fourier integrals, integral equations, Fourier series, integral functions, the Riemann zeta-function and eigenfunctions of second order differential equations.

Titchmarsh was born in Newbury and educated at Oxford. He served in World War I (1917–1919) and succeeded Hardy as Savilian professor at Oxford.

1924–1968 CE **John Desmond Bernal** (1901–1971, England). Crystallographer and biophysicist. Developed modern crystallography and was a founder of molecular biology. First to take X-ray photographs (1934) of biologically important molecules, plants, viruses, amino acids, proteins, sterols and nucleoproteins. First to determine the structure of graphite (1924).

Bernal was born Nenagh, Tipperary County, Ireland to a Sephardic Jewish family on his father's side. He graduated (1922) from Emmanuel Collage, Cambridge. After graduating he started research under **William Bragg**. He was later a professor of physics at the University of London (1937–1968). It was here that he did the major pioneering work in crystallography. Other prominent scientists who worked or studied with him were **Dorothy Hodgkin**, **Rosalind Franklin**, **Aaron Klug** and **Max Perutz**.

Bernal was convinced that from an understanding of the physical molecular structure of biologically important molecules would come a clearer insight into the way the living processes worked. His researches were dominated by the quest for the *origin of life*.

During his wartime service, he contributed substantially to the scientific underpinning of the invasion of the European Continent.

He joined the Communist Party of Great Britain (1923) during his student days and was awarded the Lenin Peace Prize (1953) for being active in the international peace movement during the Cold War.

White Dwarf — Death of a Small Star

White dwarves are faint stars with absolute magnitudes typically more than 10, making them at least 100 times fainter than the sun. They have temperatures around $10,000\,^{\circ}K$ and appear to be white.

A typical such star is about one solar mass with a density of ca 10^6 g/cm^3. It has exhausted all its thermonuclear fuel and contracted to a size roughly that of the earth. It is slowly cooling as it radiates away its residual thermal energy, and supports itself against gravity by the pressure of degenerate electrons. Its radius is inversely proportional to the cube root of its mass, i.e. the more massive a white dwarf is — the smaller it is.

The first white dwarf was discovered as early as 1844: the motion of the bright, nearby star Sirius had been found to be slightly irregular. Its proper motion of roughly 1 second of arc per year was not along a perfectly straight line across the sky, but rather a slightly wavy path. Astronomers concluded that Sirius was a double star, and that its wavy proper motion is due to an orbital motion around a companion.

This hypothesis was confirmed in 1862 when ***Alvan Graham Clark*** *(1832–1897, U.S.A.) saw it through a new 46 cm refracting telescope at the Dearborn Observatory in Illinois. The star's companion was given the name Sirius B. The strange characteristics of this star were soon discovered: Its effective temperature is* $32,000\,^{\circ}K$*, its radius 5400 km and its mean density* 3×10^6 *g/cm*^{3}*. Astronomers of the late* 19^{th} *century were justifiably skeptical. However, new discoveries of white dwarves soon poured in. A renewed interest in the internal structure of these stars arose after the new theories of matter and gravitation were established during the first quarter of the* 20^{th} *century.*

Walter Sydney Adams *(1876–1956, U.S.A.) measured, in 1925, gravitational red-shifts of several spectral lines from Sirius B, and* ***A.S. Eddington*** *(1926) used GTR to obtain the* M/R *ratio. Since the mass* M *was known from the binary orbit, Eddington could estimate its radius* R *and hence its density. In this way both the validity of GTR and the star's geometry were tested simultaneously.*

In 1926 ***R.H. Fowler*** *applied Fermi-Dirac statistics to explain the puzzling nature of the white dwarves. He identified the pressure, preventing gravitational collapse, with electron degeneracy pressure.*

In 1930 ***S. Chandrasekhar*** *calculated that a white dwarf can exists only if its mass is less than 1.4 solar masses [known as the Chandrasekhar limit].*

The demise of a normal star in that mass range, is thought to proceed as follows: As it grows older, it steadily converts protons into helium nuclei at its center. As the supply of protons there runs low, the star loses its ability to maintain a constant rate of energy liberation. For a time, the star can compensate for its dwindling resources of protons by contracting its central region, thereby raising the temperature to fuse the remaining protons at an ever-increasing rate. This contraction actually increases the rate of energy liberation: part of the extra kinetic energy expands the star's outer layers, cooling them slightly and producing a red-giant star. Its core will eventually contract to the point where helium begins to fuse into carbon. This helium ignition temporarily expands the core, and sets the stage for a period of instability during which the star is likely to pulsate in size and brightness. This is the stage of the Cepheid stars.

When the helium nuclei have fused into carbon, most stars will become degenerate in their centers. At this point, the central regions have such high density that the Pauli exclusion principle (1925) prevents the electrons from packing any tighter. The electrons hold the nuclei by electromagnetic forces, so the exclusion principle supports the entire star against its self-gravitation. After the star's outer layers evaporate, the core remains as a white dwarf.

Quantum Mechanics — The Formative Years[549]

Despite the initial success of the ***Bohr*** *theory, and other developments of the original quantum theory of* ***Planck****, it became increasingly evident in the early* 1920*'s that many features of atomic physics cannot be described in semiclassical terms. The discovery and interpretation of the* ***Compton*** *effect in* 1923 *stressed the photon aspect of radiation, but the necessary connection between the wave and corpuscle treatments remained elusive. Shortly after this discovery* ***de Broglie*** *suggested that beams of particles should exhibit wave properties, the effective wavelength being inversely proportional to the momentum per particle. This idea was widely accepted, but it was not until* 1927 *that full experimental confirmation was obtained, by* **Davisson** *and* **Germer** *and by* **G.P. Thomson**.

The initial development of full-fledged quantum mechanics *stemmed from the problem of calculating the probabilities of transitions between various atomic states. The theory was put in the form of* matrix mechanics *by* ***Heisenberg****,* ***Born****, and* **Jordan**, *whose ideas expressed a new philosophy of physical epistemology and ontology. Briefly, quantum mechanics is connected only with the calculation of observable quantities (including statistical probabilities); detailed mechanical models of observable entities are regarded as misleading and unnecessary.*

An alternative approach, explored by ***Schrödinger*** *in* 1926*, led to a system of* wave mechanics *based on de Broglie's hypothesis, and this system admits of a somewhat different interpretation than matrix mechanics. In wave mechanics an atomic state is described by a function* Ψ*, which is related to (but contains more information than) the probability distribution of the positions of (one or several) electrons in the given atomic state.*

An important consequence of any form of quantum mechanics is the "uncertainty" principle, stated by Heisenberg in 1927*, limiting the accuracy with*

[549] For further reading, see:

- Guillemin, V., *The Story of Quantum Mechanics*, Charles Scribner's Sons: New York, 1968, 332 pp.
- Finkelnburg, W., *Structure of Matter*, Springer-Verlag: Berlin, 1964, 511 pp.

which physical measurements can ever ascertain the point occupied by a physical system in classical phase space. The principle stresses the essentially statistical nature of the new quantum theory, which cannot, in general, predict the exact behavior of any atomic system, but is rather confined to the calculation of probabilities, such as the transition probabilities of Heisenberg's matrix mechanics.

While the quantum mechanical scheme was still being worked out, many difficulties in atomic theory were removed by ***Uhlenbeck*** *and* ***Goudsmit****'s hypothesis that the electron possesses intrinsic angular momentum or "spin". The entire periodic classification of the chemical elements could now be understood with the aid of the spin variables and* ***Pauli****'s exclusion principle. The incorporation of electron spin into a fully relativistic theory of particles was achieved in* 1928 *by* ***Dirac****, whose treatment of the electron was particularly significant in that it predicted the existence of an antiparticle, in this case the positron — discovered soon thereafter.*

Among the many applications of quantum mechanics to systems containing several particles, the most general are the laws governing the statistical behavior of large assemblies. A new form of physical statistics was discovered in 1924 *by* ***Bose*** *and* ***Einstein****, who treated all identical particles in an assembly as strictly indistinguishable (such particles are known as* bosons*; examples are the photon, ^4He, H_2, the graviton, π-mesons). The special properties of particles which are indistinguishable and in addition*[550] *obey Pauli's exclusion principle, were expressed in the* **Fermi-Dirac** *statistics of 1926 (these*

550 Let a system (unit volume, say) consist of a total of N noninteracting fermions distributed among a finite number of energy states, such that there are n_i of them at energy level ϵ_i $[\sum n_i = N;\ \sum n_i\epsilon_i = E =$ total energy of system$]$.
Each energy level ϵ_i has g_i different *quantum states* with the same energy; g_i is known as the *degeneracy* of that level. For free particles of spin $\frac{1}{2}$, not subject to magnetic fields, each particle may have spin up or down, so $g_i = 2$ for all levels. We specialize to this case in what follows; m will denote the fermion mass.
The number n_i/g_i is the *occupancy number* of each degenerate state.
According to the *Fermi-Dirac* distribution law for fermions, the statistical expectation of n_i is $n_i = g_i f_i(\epsilon_i)$, where $f_i(\epsilon_i)$ expresses the probability that a given quantum state of energy ϵ_i is occupied by a fermion $[0 \le f_i \le 1]$. Explicitly, for the *Fermi-Dirac* distribution in equilibrium, one has

$$f_i = \frac{1}{1 + \exp\left(\frac{\epsilon_i - \epsilon_F}{kT}\right)},$$

where k is the Boltzmann constant, T the absolute temperature in °K and ϵ_F is the *Fermi energy*; T and ϵ_F are adjusted to satisfy the two constraints *particles are fermions, e.g.: electron, neutrino, muon, proton, neutron). Two or more bosons, in contradistinction, may occupy the same quantum state. Under suitable almost-classical conditions, both these forms of quantum statistics can be replaced approximately by the classical* ***Boltzmann*** *scheme, but the quantum theory is essential in dealing with problems such as the behavior of photons in an enclosure, atoms in a superfluid or a Bose-Einstein Condensate, electrons in a metal or white dwarf star, nucleons in an atomic nucleus or neutron star, or in atomic and molecular electron orbitals.*

Quantum mechanics provides a method for the theoretical treatment of atomic or subatomic systems, for calculating their energy levels and for predicting the probability of transition from one state to another. When we consider the properties of matter in bulk, new problems arise because of the complicated interactions between atoms.

Even in the gaseous phase, intermolecular forces affect the behavior considerably, although it is usually possible to describe deviations from the gas laws in terms of simple two-body interactions. In the solid or liquid state, the close proximity of atoms gives rise to strong forces affecting many particles.

At the same time, the fundamental rules of quantum statistics become important in dealing with the vibrations of the crystal lattice or with conduction electrons in metals. Despite the mathematical difficulties, condensed-matter physics has made great progress in accounting for many collective (many-body) phenomena which admits of no classical explanation. Moreover, its results have great practical significance and wide implications in many fields of physical research.

As an illustration, consider the study of magnetic effects in solids. The classical statistics of Boltzmann can be applied to assemblies of paramagnetic atoms which are weakly interacting. The basic theory of level splitting in magnetic fields is well known, and can be used directly in such cases to calculate the paramagnetic susceptibility.

However, in the solid state, atoms and ions interact strongly with each other and complex collective phenomena are observed, notably the phenomenon of ferromagnetism. The spontaneous magnetization of metals like iron was treated phenomenologically in Weiss' theory, the prototype of many at-

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$\sum n_i = N,\ \sum n_i\epsilon_i = E$. At zero temperature and for free electrons, one finds

$$\epsilon_F = \frac{h^2}{8m}\left(\frac{3N}{\pi}\right)^{2/3}.$$

For $T = 0$, all energy states up to $\epsilon = \epsilon_F$ are fully occupied $(n_i = g_i)$ while

all states with $\epsilon > \epsilon_F$ are empty $(n_i = 0)$. In contradistinction, at $T = 0$, in *Maxwell-Boltzmann* statistics, all particles should be at the ground energy level with $T = 0$. In *Fermi-Dirac* statistics, this accumulation at the ground level is prevented by the exclusion principle, and particles at $T = 0$ occupy the lowest energy levels available up to energy ϵ_F. At higher temperatures, the population of the fermions is spread out among the quantum states: electrons with energies within $O(kT)$ of ϵ_F typically absorb thermal energy of the same order, and move to higher levels, in accordance to the distribution $f_i(\epsilon_i)$.
Thus, for temperatures $kT \ll \epsilon_F$, only states with energies close to ϵ_F are affected since the low-energy states are fully occupied and the exclusion principle prevents the addition of further electrons to those states. Consequently, only those fermions with energy close to ϵ_F can move into higher unoccupied states by absorbing a relatively small amount of energy, of order kT.
At sufficiently *high* temperatures, the *Fermi-Dirac* distribution becomes essentially a *Maxwell-Boltzmann* distribution, i.e.

$$n(\epsilon_1)/n(\epsilon_2) = e^{(\epsilon_2-\epsilon_1)/kT}.$$

This occurs in the regime

$$T \gg T_0 = \frac{h^2}{8mk}\left(\frac{3}{\pi}N\right)^{2/3};$$

for electrons when $T_0 = 300\,^\circ\mathrm{K}$ (room temperature), $N \sim 10^{13}$ per cm^3.
In cases where the energy spectrum of the fermions is practically continuous, such as free electrons in a macroscopic box, there are numerous discrete levels (with $g_i = 2$ each) in any reasonable energy interval $\Delta\epsilon$, so we may replace g_i with the continuous distribution $g(\epsilon)d\epsilon$, and n_i by dN. One finds that

$$g(\epsilon) = \frac{8\pi}{h^3}\sqrt{2m^3\epsilon}$$

(again per unit volume) and thus the number of electrons with a kinetic energy between ϵ and $\epsilon + d\epsilon$ is

$$dN = \frac{g(\epsilon)d\epsilon}{1 + e^{(\epsilon-\epsilon_F)/kT}}.$$

This theory can be applied successfully to the problem of finding the energy distribution of a large number of independent nearly free electrons in an enclosure (applicable to conduction electrons in a metal too). Indeed, at low temperatures for $\epsilon \leq \epsilon_F$,

$$N = \sum_{\epsilon=0}^{\epsilon=\epsilon_F} g_i(\epsilon_i) \Rightarrow \frac{8\pi}{h^3}(2m^3)^{1/2}\int_0^{\epsilon_F}\epsilon^{1/2}d\epsilon = \frac{16\pi}{3h^3}(2m^3)^{1/2}\epsilon_F^{3/2},$$

from which the above expression for ϵ_F was derived.

tempts to describe cooperative effects, which include the melting of solids and order-disorder transitions in alloys.

Some success has been achieved in theoretical descriptions of such effects, but it is not always easy to formulate a fundamental account of the interactions responsible for them.

Our Galaxy (1925–1983)[551]

"Poor mankind. We began with the belief that we were the center of the universe. At each step of our growing knowledge, we become less central. Frankly, whenever I think of our insignificant place in the cosmos, and contemplate the fact that we live on one of its hundred billion stars, I become more firm in my belief that we cannot be the only intelligent creatures in the Galaxy".

Robert DeWitt Chapman (1978)

Our sun is located in a stellar system called the Milky Way or simply the Galaxy (on clear dark nights[552] *the cumulative light from the myriad of its faint stars is visible as a luminous band stretching across the sky).*

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551 For further reading, see:

- Kaufmann, W.J., *III. Universe*, W.H. Freeman, 1985, 594 pp.

552 During WWII cities on the west coast of the United States were routinely blacked out at nights to hamper attacks from the sea. This included Los Angeles, whose city lights normally interfered with observations from Mount Wilson, site of the 2.5 meter *Hooker Telescope*. At the time this was the biggest and best telescope in the world, but the increasingly bright skies due to the development of Los Angeles limited its usefulness. Thus, the war provided excellent opportunities for

The Milky Way, a spiral galaxy, is a dynamic entity with its own structure and evolution governed by complex interactions of stars with each other and with the interstellar gas and dust. It is vastly more complicated than a single star, and its life story is correspondingly more difficult to unravel.

About the turn of the century, following upon the star gauging of ***W. Herschel*** and ***J. Herschel***, *astronomers tried to investigate the structure of the Milky Way system by the methods of stellar statistics. Even if the goal was not attained, the incredible labor of these undertakings has nevertheless proved to be very valuable in other connections.*

H. Shapley*'s method (1918) of photometric distance determinations using cepheids, brought the decisive advance. The period-luminosity relation (i.e. the relation between the period P of the luminosity variation and the absolute magnitude M), made it possible to measure the distance of every cosmic system in which one could detect any sort of cepheids. The distance of the cepheids determined by Shapley implied that these clusters form a slightly flattened system whose center lies at a distance of about* 30,000 *light years in the direction of Sagittarius. The present picture of our Milky Way system developed rapidly from these beginnings.*

Shapley's picture of our Galaxy was not universally accepted among astronomers at first, but when in 1923 ***Hubble****'s observations showed that the Great Andromeda galaxy was similar in size and content to our galaxy, astronomers accepted the hypothesis that the universe is filled with galaxies and turned their attention to studying the detailed structure of our own.*

In 1926, **Bertil Lindblad** *(1895–1965, Sweden) came forward with the hypothesis of* differential rotation *of the Galaxy. The first clues of this rotation came from examining the motions of stars in the sky. Because of differential rotation, the sun is like a car on a circular freeway with the fast lane on one side and the slow lane on the other side; stars on the fast lane are passing the sun and thus appear to be moving in one direction, while stars in the slow*

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astronomers to observe faint objects. Most American astronomers were unable to take advantage of these opportunities, however, being assigned to war-related duties elsewhere. One exception was **Walter Baade** (1893–1960) (exempted from war duties), who took excellent photographs of the Andromeda galaxy, a spiral much like our own. This allowed him to determine the characteristics of individual stars in that galaxy, something never before possible. From the study of these stellar characteristics emerged the recognition of *stellar populations* and their profound implications for galactic evolution.

After the war the conditions at Mount Wilson became worse and worse due to city lights and pollution, and the 100-inch telescope was closed in 1985.

lane are being overtaken by the sun and therefore appear to be moving in the opposite direction.

However, not all of the stars in the sky move in this orderly pattern. Extra-galactic objects and some components of our galaxy do not seem to participate in the general rotation of the Galaxy; instead they display more-or-less random motions.

Lindblad took the average of these random motions as a background, using Doppler shifts of optical spectral lines to measure the sun's speed w.r.t. the background in various directions. From these measurements, he concluded that sun moves along its orbit about the galactic center at a speed of $250\ \frac{\text{km}}{\text{sec}}$*.*

Observational evidence confirming Lindblad's hypothesis was provided a year later (1927) by ***Jan H. Oort****. The work of Lindblad and Oort thus showed that our galaxy rotates around a point which agrees with the center as determined by Shapley. The agreements of these results established the sun's off-center position in the Galaxy once and for all.*

Toward the 1930's, astronomers gathered sufficient data to be able to quantify the salient geometrical and kinematical features of the Galaxy: the main body of stars forms a flat disc of about 30 kpc in diameter ($\sim 10^{23}$ *cm or* $100,000$ *light years) and about* 2000 *light years-thick. The* galactic center *(nucleus) is surrounded by a spherical distribution of stars called the* galactic bulge *which is hidden from us by clouds of dark interstellar matter. We ourselves are situated far out in the disc, some 10 kpc from its center. The disc is surrounded by the much less flattened* halo *to which the* globular clusters *and certain other classes of stars belong.*

The stars of the disc revolve round the galactic center under the gravitational attraction of the masses that are concentrated there. In particular, the sun describes a circular path of radius about 10 *kpc with a speed of about* $V = 250\ \frac{\text{km}}{\text{sec}}$ *in about* 250 *million years*[553]*. The Galaxy contains about* 10^{11}

[553] The Galaxy must rotate — otherwise all stars would have fallen into the galactic center. Suppose that the sun S moves in a Keplerian circular orbit with radius r_s about C, the galactic center. Draw a line-of-sight to a star P (tangent to the circular orbit of this star about C). The polar coordinates of the star relative to C are (r, θ). Erect a Cartesian coordinate system, with origin at the sun such that the star's motion is in the galactic plane with SC as the x-axis, and the sun's velocity vector along the y-axis. Let ℓ be the angle CSP (galactic longitude). It then follows from elementary plane-geometry considerations that the velocity of approach of P along the line-of-sight is: $U(r, \ell) = \left[\dot{\theta}(r) - \dot{\theta}(r_s)\right] r_s \sin \ell$. Since for a *Keplerian motion* $\frac{d}{dr}\dot{\theta} < 0$, it is evident that $U(r, \ell)$ is positive in quadrants I and III of the Cartesian system

visible stars, gas and dust with a total mass of about 1.5×10^{11} *solar masses* ($1M_\odot = 1.99 \times 10^{33}$ *g*). *Since the gas contains only* $\frac{1}{6}$ *of the total mass, it has little direct influence on the dynamics, but the dense gas clouds are the sites of new star formation and thereby play an important role in the long-term evolution of the Galaxy.*

Since the age of the Galaxy is about 10^{10} *years, a typical disc star has completed over* 30 *revolutions, and hence it is reasonable to assume that the Galaxy is approximately in a steady state at the present time. Simple calculations show that collisions between stars are exceedingly rare and have absolutely no importance to the dynamics of the Galaxy. (This rarity of encounters is fortunate, since the passage of a star within even* 10^9 *km of the sun would cause major perturbations to the earth's orbit and hence have disastrous consequences for life on earth.)*

Because interstellar dust effectively obscures our visual views in the galactic plane, a detailed understanding of the structure of the galactic disc had to await the development of radio astronomy. *Because of their long wavelength, radio waves easily penetrate the interstellar medium without being scattered or absorbed.*

Indeed, radio observations (via Doppler shifted 21*-cm interstellar hydrogen emission*[554], *1951) revealed that our Galaxy has* spiral arms *(concentrations of gas and dust) unwinding from the center in a shape reminiscent of*

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so that stars and gas in these direction should appear to *approach*, and their spectra should be *blue-shifted*. In quadrants II and IV stellar spectra should appear *red-shifted*. This, in fact, is what Oort observed in 1927. At any given galactic longitude, the highest velocity should be observed at P, where the line-of-sight is tangential. By noting the maximal velocity at given ℓ one can derive the distance of the sun from the galactic center. Present results yield $r_s \sim 9.5$ kpc $\pm$ 1.5 kpc. Using Lindblad's value of $V_s = 250 \ \frac{\text{km}}{\text{sec}}$, and $r_s = 10$ kpc, $G = 6.668 \times 10^{-8}$ dyn cm^2 g^{-2}, Kepler's law $V_s^2 = \frac{GM}{r_s}$ yields the mass of the matter inside the sun's orbit: $M = 2.9 \times 10^{44}$ g $= 1.5 \times 10^{11}$ solar masses. Also $T = \frac{2\pi r_s}{V_s} = 2.3 \times 10^8$ years is the period of the sun's orbit around the galactic center.

554 When the electron and proton spins of a hydrogen atom are parallel, the atom is in a slightly higher energy level than when they are antiparallel. As a result, an atom with parallel spins will eventually flip the electron spin over to reach a lower energy level. This spin flip is accompanied by the emission of a photon that carries away just the energy difference between the two spin states. An average hydrogen atom will spend 10 million years in the parallel spin state before the electron spin flips over, but our Galaxy contains so many hydrogen atoms that about 10^{54} spin flips, and accompanying photon emissions, occur each sec-

a pinwheel. By measuring Doppler shifts, astronomers can determine speeds parallel to our line-of-sight across the Galaxy. These observations clearly indicate that our Galaxy does not rotate like a rigid body but rather exhibits differential rotation.

Thus, the important result of the work of Lindblad and Oort was that the portion of the galaxy in the vicinity of the sun behaves like a fluid, with each star moving in a Keplerian orbit as an independent particle. Near the center of the galaxy this is not true: here the entire system does rotate like a rigid object. The stars in the inner part *of the Galaxy are subject to the combined gravitational pull of all the stars around them (there, speeds of individual stars* increase *with distance from the center, whereas they* decrease *with distance in the outer portion); There is an intermediate distance, just inside the sun's orbit, where a transition between rigid-body and Keplerian orbits occurs, and it is there that stars have the greatest orbital velocities*[555]*.*

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ond. Collisions among atoms tend to flip some of the antiparallel-spin atoms back into the parallel-spin configuration. At any given time, $\frac{3}{4}$ of the hydrogen atoms have parallel spins, while the remaining ones have antiparallel spins. The parallel-antiparallel spin energy difference amounts to 9.5×10^{-18} erg, and the corresponding frequency is $\frac{\Delta E}{h} = 1420.4058$ megahertz, corresponding to a wavelength $= \frac{hc}{\Delta E} = 21.10611$ centimeters. The astronomer points his radio telescope in a particular direction in the plane of the Galaxy and measures the intensity of the 21-cm hydrogen emission received (proportional to the number of hydrogen atoms in that direction). The receiver is swept in frequency around the 21-cm frequency, and a profile of intensity versus frequency is compiled. The detected emission is not exactly at 21-cm wavelength: The telescope receives 21-cm emission from each segment of spiral arm in that direction, but because of differential rotation, a given arm segment has a distinct velocity from the others that are closer to the center or farther out. Therefore, instead of a single emission peak at 21.1 cm, what we see is a cluster of emission lines near this wavelength but separated from each other by the Doppler effect.

Now, by measuring Doppler shifts away from the standard frequency, astronomers can determine the relative velocity toward or away from us that characterizes a particular group of hydrogen atoms. If we assume that all the atoms move in circular orbits around the galactic center, we can correlate a distance along our line-of-sight to each particular Doppler shift. (The assumption of perfectly circular orbits introduces some imprecision, but the distances derived from this assumption are thought to be rather accurate.)

Using this method, astronomers have mapped the hydrogen distribution in the galactic plane. It showed that hydrogen does concentrate in particular "arms" that exhibit a vaguely spiral pattern.

[555] In the Galaxy, the deviation of a star's motion from a perfect circular orbit is called its *peculiar velocity*. The sun has a velocity of about 20 km/sec w.r.t. a

However, the orbital velocities do not *drop off to lower and lower velocities beyond the sun's orbit, (as it would if all the mass of the galaxy were concentrated at its center). In fact, the orbital velocity of stars and gas is observed to* increase *out to a distance of at least* 60,000 *light years from the galactic center. This indicates that a surprising amount of matter must be scattered around the edges of our Galaxy, raising the total galactic mass to a total of some* $6 \times 10^{11}\ M_{\odot}$. *This outlying matter is dark; it does not show up on photographs. Many astronomers suspect that it is spherically distributed all around the galaxy, along with the globular clusters. Thus our Galaxy halo is more massive than previously expected.*

Almost half the galaxies we know are spiral galaxies like the Milky Way, with a spiral-arm pattern of young, bright stars and interstellar hydrogen gas. When we consider the fact that the inner parts of these spiral galaxies rotate with a higher angular velocity than the outer parts, we would expect that the spiral arms would be wound up after a few revolutions (the farthest regions complete one revolution in 400 *million years, while close to the center it takes only* 100 *million years). Why, then, does the spiral pattern persist? We know that it* does *persist, for we would not otherwise see so many spiral galaxies.*

Bertil Lindblad *was first to argue that the spiral arms of a galaxy are merely a* pattern *(density wave) that propagates among the actual stars. Lindblad struggled with the problem from* 1927 *until his death in* 1965. *He correctly recognized that spiral structure arises through the interaction between the orbits and the gravitational forces of the stars of the disc, and thus should be investigated using stellar dynamics.*

Lindblad recognized that such density waves *would be caused by gravitational perturbations on the circular Keplerian orbit of a star by the motion of other matter in the galaxy. This will cause the star to rotate counterclockwise around an epicycle while the epicycle itself moves clockwise along the undisturbed path. The final path of the star is a* precessing ellipse. *The gravity of this star in turn affects the motions of its neighbors, and thus a wave disturbance, called a* kinematic density wave, *propagates from one stellar orbit to the next.*

The density-wave theory was greatly elaborated and mathematically embellished by the American astronomers ***C.C. Lin*** *and* ***Frank Shu*** *during* 1964–1969. *They made the crucial step that elucidated the theory toward which Lindblad had been groping. They have shown that the spiral structure represents a local increase in density and that this enhanced density spiral*

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circular orbit, in a direction about 45° from the galactic center and slightly out of the plane of the disc.

travels around the galaxy like a wave, at a "pattern" velocity different from that of the speed of the stars involved[556].

Stars and gas overtake the spiral pattern, and as they enter it, the gas density increases to 5 *or* 10 *times its density outside. This increase in gas density has an important effect. It triggers the formation and collapse of large gas clouds, which condense into* protostars *within a million years or so — producing clumps of young, bright stars. Thus, within a few million years after the gas enters a dense part of the pattern, some of it becomes incorporated into one of the bright stars that outline spiral arms for a few million years before fading away into obscurity. After a few million (or tens of millions) of years, these stars, known as Blue giants, burn themselves out, because they burn their nuclear fuel so prodigiously.*

By the time the pattern has moved a significant distance around the galaxy relative to the stars, the stars that once outlined the spiral arm will be fading away, while a new group of stars forms the gas that has entered the dense part of the pattern more recently. Thus the density-wave pattern persists even as the individual bright stars perish. It is not known how such a spiral density-wave pattern gets started; one theory suggests that it originates with a close encounter between galaxies. Another theory suggests that spiral galaxies generate their own density-wave pattern.

In 1973, ***A.J. Kalnajs*** *elaborated on the theory of Lin and Shu and showed that each of their precessing elliptical orbit is tilted w.r.t. its neighbor through a specific angle, resulting in a spiral pattern which arises in those locations where the ellipses are bunched together. Thus, in spite of the natural random scatter of the stars in their orbits, the orbits become correlated such that some of the stars happen to get close together along the high arching spiral arms. This in turn enhances the gravitational attraction upon the lightweight atoms and molecules in the interstellar gas and dust, which are readily sucked into the gravitational well along the spiral — forming the crest of the density wave.*

In Kalnajs' spiral pattern process, the density wave moves through the material of the Galaxy at a speed of about 30 *km/sec, slower than the stars themselves. On its own, however, the interstellar gas can transport a compressional disturbance at a speed of only* 10 *km/sec, which is the speed of sound in the interstellar medium. The density wave is therefore* supersonic

[556] When a slow truck heads uphill on a highway, with cars passing as they can, we have a one-dimensional analogue to a kinematic density wave: The pattern of bunched cars persists, even though a given car passes into and then out of the denser part of the pattern. Furthermore, the pattern moves at a slower speed than that of the average car.

— creating a shock wave *along the leading edge of the density wave, which causes a violent compression of the gas.*

As the spiral density waves sweep through the plane of the Galaxy, they recycle the interstellar medium. Old dust and gas left behind from ancient, dead stars are recompressed into new nebulae in which new stars are formed. Because the material left over from the deaths of ancient stars is enriched in heavy elements, new generations of stars are more metal-rich than were their predecessors.

This theory still leaves many open questions. For example: what keeps the density wave going? Why don't they dissipate? The compression of the interstellar gas and dust requires enormous amounts of energy. What mechanism is constantly replenishing this energy? A possible source is the galactic nucleus.

The center of our Galaxy is a mysterious region forever blocked from our view by the intervening interstellar medium. From Shapley's work on the distribution of the globular clusters, as well as Oort's analysis of stellar motions, it was known in the 1920's that the center of our great stellar pinwheel lies in the direction of the constellation Sagitarius. The distribution of infrared radiation *from the nucleus at wavelengths around 2.2 microns looks just like the distribution of visible light from the nucleus of the Andromeda Galaxy. The center of our Galaxy contains a million stars per cubic persec [1 pc equals $206,000$ AU], compared to one star per cubic parsec near the sun.*

Longer-wavelengths infrared observations (around 20 microns) show several intensely bright sources near the center of the Galaxy. The center is also a copious emitter of electromagnetic radiation of other kinds. It emits synchrotron radiation, thermal radiation, 21-cm radiation, molecular radiation, and X-rays. The motion of gas clouds within a few light years of the galactic center were studied in the 1970's. It was shown that a certain spectral line of singly ionized neon, which normally has a wavelength of about 12.80 microns in the infrared, is extremely broad. This line is broadened by motion of gas clouds in which neon is a minor constituent. The broadening has been interpreted as a result of the speed at which ionized gas orbits the galactic nucleus; radiation from gas coming toward us is blueshifted, while radiation from receding gas is redshifted. The final result is to smear out a spectral line over a range of wavelengths corresponding to a range of line-of-sight velocities. The broadening of the line then reveals a velocity spread of about 400 km/sec. On one side of the galactic nucleus, gas is coming toward us at speeds up to 200 km/sec, while on the other side of the galactic nucleus, it is rushing away from us at speeds up to 200 km/sec.

Something must be holding this high-speed gas in orbit about the galactic center. Using Kepler's third law, it is estimated that $10^6\ M_{\odot}$ *of material* *is needed to prevent this gas from flying off into interstellar space. These observations therefore imply that an object with the mass of million suns is concentrated at Sagittarius A West. This object must be extremely compact — much smaller than a few light years over which the infrared neon source extended.*

1925 CE, March 18 The most devastating *tornado* on record, and the greatest tornado disaster in the United States. Known as the *Tri-State tornado* (Indiana, Illinois and Missouri), it remained on the ground for 350 km. The resulting losses included 695 dead, 2027 injured, and damages of about 43 billion in 1970 dollars.

1925 CE, April 1 *The Hebrew University in Jerusalem* opened with a keynote address by **Albert Einstein**. The first academic institution for secular studies in Israel, ever.

1925 CE, Oct. 13 *Scripps Institution of Oceanography* of the University of California came into existence at a site overlooking the Pacific Ocean, just north of La Jolla, California.

1925 CE **Samuel Abraham Goudsmit** (1902–1978, Holland) and **George Eugene Uhlenbeck** (1900–1988, Holland) suggested the electron spin angular momentum. At the same time **Ralph de Laer Kronig** (1904–1995, Holland) had the same idea. He asked W. Pauli for his opinion before publication, and Pauli convinced him that the idea had no merit.

1925–1927 CE *The Atlantic Meteor expedition*: One of the first systematic studies of a single ocean was carried out by the German research ship *Meteor*[557], which made 13 crossings of the Atlantic between 20 °S and 60 °S. The results of this survey were published in a series of atlases. The *Meteor* was the first expedition to use the *echo sounder* (sonar) to measure the depth of the ocean almost continuously along its track. As a result, a more accurate idea of the shape of the ocean floor was obtained than it was possible to form with isolated sounding.

557 Designed to recover gold from the ocean to repay Germany's WWI debts, although it was an unsuccessful enterprise in this respect. The ship was commissioned by the Hydrographic Department of the German Navy.

In 1925, the expedition discovered the *Mid-Atlantic Ridge*. The data obtained — gathered day and night, in all weathers and all seasons — included some 70,000 soundings of the ocean depths over a period of 25 months. There was nothing to equal it for another 30 years: it was not until the International Geophysical Year of 1957–1958 that oceanographers once again undertook to survey an entire ocean.

1925–1932 CE *Rise and decline of the 'talking machines'*. Research in *wireless telephony* conducted during WWI yielded *viable microphones* and *amplifiers* that made *radio broadcast* possible. By 1925, the studio experience and the quality of the recordings improved dramatically. Individual microphones replaced shared recording horns, and artists could now overdub mistakes. Electric amplification made it possible for studio acoustics to emulate the atmosphere and clarity of live performances. A much-expanded frequency range allowed for the improved definition of sharper treble and the weighty force of deep bass.

These innovations sparked another surge in enthusiasm for recorded music that now appeared to complement the popularity of radio. A number of radio-phonograph combination machines were marketed successfully. The grandest symbol of corporate confidence in the alliance was *RCA Victor*, the result of the Radio Company of America's acquisition in early 1929 of the Victor Talking Machine Company.

Later in the same year, however, the predicted death of the phonograph seemed to suddenly become a reality. The industry ground to a halt almost overnight in October when the stock market crashed. People saw little point in spending bread money on records when the radio continued to provide free entertainment. In November, eighty-two year old Edison and his corporate allies discontinued production of records and phonographs. Cylinder records had already begun a sharp and steady decline since the advent of electronic recording. The Edison announcement finally rendered them extinct. Thomas Edison died in 1931.

In 1927, $987,000$ machines were produced and $104,000,000$ records were sold. In 1932 those numbers dropped to $40,000$ and $6,000,000$ respectively. With the exception of a few die-hard collectors, consumers not only quit buying records, they also began to think of the whole phenomenon of "canned music" as part of an outdated culture. Free live radio and the first sound motion pictures (the first feature-length "talkie", *The Jazz Singer*, was released in 1929) seemed to provide more vibrant, immediate and modern cultural outlets. Millions of machines and records found their way into attics and junkpiles. In decades to come, of course, recording would be revived and go through even more dramatic technological, cultural and corporate transformations. However, the Depression, the death of the phonograph's inventor, the drastic decline in consumer interest, and the competition from new forms of audio technology marked the end of the beginning for talking machines.

1925–1934 CE **Ida Eva Tacke Noddack** (1896–1979, Germany). Chemist. Discovered the element *Rhenium* [Re, atomic weight = 186.2; atomic number = 75] with **Walter Karl Noddack** (1893–1960) and **Otto Berg**. It was discovered through X-ray spectra, in a sample of the mineral Columbite.

In 1934 she proposed, counter to Enrico Fermi, that heavy nuclei bombarded by neutrons break down into isotopes of known elements but not neighbors of transuranium element, as Fermi and all other nuclear physicists then believed. In that *she was ahead of her time*; Bohr's liquid-drop model of the nucleus had not yet been formulated, and so there was at hand no accepted way to calculate whether breaking up into several large fragments was energetically allowed.

Noddack's physics was *avant garde*. By 1938 her article was gathering dust on back shelves. But in 1939, her belief in *nuclear fission* was confirmed by Lise Meitner and Otto Hahn.

1925–1944 CE **Erwin Schrödinger**[558] (1887–1961, Austria). Pathbreaking theoretical physicist. Created quantum wave-mechanics[559] and established the fundamental wave equation governing submicroscopic phenomena. The formulation of the '*Schrödinger equation*' (1925) put quantum theory on a firm mathematical basis, and provided the foundation for its further rapid development. It plays a role in modern physics comparable to that played by the equations established by Newton, Lagrange and Hamilton in classical physics.

The Schrödinger equation describes the evolution of the *probability-amplitude function* that governs the dynamics of any physical system (the simplest case being that of a single particle), and specifies how these waves are altered by external influences.

The probabilistic aspect of quantum theory made Schrödinger and other leading physicists profoundly unhappy, and he devoted much of his later life to formulating philosophical *objections* to the generally accepted interpretation of the theory that he has done so much to create!

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[558] For further reading, see:

- Moore, W., *Schrödinger*, Cambridge University Press, 1990, 513 pp.

[559] Its equivalence to Heisenberg's *matrix-mechanics* was later established by **Dirac**, **Jordan** and **Born**.

The Schrödinger equation had an enormous impact on *chemistry* since the nature of the chemical bond could be formulated in terms of physical concepts.

Schrödinger established the correctness of his equation by applying it to the hydrogen atom, predicting many of its properties with remarkable accuracy. The equation is used extensively in atomic, molecular, nuclear, particle, and condensed-matter physics.

During 1941–1943, while in exile in Dublin, Schrödinger devoted considerable effort to working out a *nonlinear* classical electromagnetic theory previously outlined by Born[560] and Infeld (1934). His conviction that nonlinear theories would be essential for future progress of physics has turned out to be abundantly justified.

From 1943 to 1951, Schrödinger's work was dedicated almost exclusively to a search for a *unified field theory* that would encompass both gravitation and electromagnetism[561]. Like Einstein, he was inspired by a metaphysical belief in the unity of nature which induced in him a feeling of wonder at the simplicity and beauty of the universe as revealed through the window

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560 The **Maxwell** equations are linear, i.e. the fields and their derivatives occur only in terms of the first degree. A fundamental property of such linear equations is that the fields cannot generate themselves — sources (dynamical or external) must be introduced. In this case, for example, if one introduces a point charge such as an electron, the fields diverge at this point — which is physically unreasonable. Moreover, the mass and charge of the electron are not deducible from the linear theory.

The idea of a nonlinear modification of Maxwell's equations had occurred to Born in 1933. He aimed at developing a theory in which an electron of *finite radius* arises *naturally* out of the field equations. **Born** and **Infeld** (1934) pointed out that the relation of matter to the electromagnetic field can be interpreted from two opposite standpoints: (1) *Unitarian* — only the field exists and particles are singularities of the field, their masses are derivable from the field energy. (2) *Dualistic* — particles are sources of the field, acted upon by the field but not part of it; they have the characteristic intrinsic property of inertia, measured by mass.

The development of *quantum electrodynamics* followed a quite different path, starting with **Dirac**'s relativistic quantum theory of the electron and culminating in the work of **Schwinger**, **Feynman** and **Tomonaga** from 1948 to 1953.

561 Schrödinger and Einstein turned to unified field theories because of their disenchantment with the prevailing state of quantum mechanics. Einstein hoped that field theory would eventually include both the macroscopic systems of cosmology and the microscopic systems of elementary particles and quanta.

of mathematical theory[562]. Schrödinger believed that technology had caused deterioration in the quality of man's relation to the deeper sources of his being.

In 1944, he wrote his popular masterpiece '*What is Life*'. It deals with the impact of quantum ideas on biology, and above all on the molecular processes that underline the laws of heredity. While a good deal of what the book had to say is now dated, it has had a great influence on physicists and biologists because it suggested how the two disciplines join together at their base.

Schrödinger was born in Vienna. He entered the University of Vienna in 1906, served in WWI and moved to Zürich, where he stayed until 1926. In 1927 he was invited to succeed **Max Planck** at the University of Berlin, thus becoming a colleague of **Albert Einstein**. In 1933 he was deeply affected by the political climate in Germany, and realized that he could no longer live in a country in which the persecution of the Jews had become national policy. Consequently he began a seven-year odyssey that took him to Austria, Great Britain, Belgium, the Pontifical Academy of Science in Rome, and finally — in 1940 — the Dublin Institute for Advanced Studies (founded under the influence of Premier Eamon de Valera, who had been a mathematician before turning to politics). Schrödinger remained in Ireland for the next 15 years, doing research both in physics and in the philosophy and history of science. In 1956 Schrödinger retired and returned to Vienna as professor emeritus at the University.

Schrödinger was a man with great charm and a fascinating personality and thus attracted many interesting women in his life. He married Annemarie Bertel in 1920. It was a childless marriage based on friendship alone, providing him a secure haven from importune mistresses. Out of his numerous amorous affairs he had three daughters from three different women: Ruth (b. 1934),

562 Einstein was 36 years old when he published his general theory of relativity; Schrödinger was 38 when he discovered wave mechanics. By 1940, when he began to consider generalized field theory, he was 53. Einstein, then 61, had been working on the problem for twenty-five years without apparent success. In the annals of physics it is unusual to find anyone who had made a major theoretical discovery after the age of 40. Thus it might seem that Einstein and Schrödinger were facing an insurmountable psychological barrier. Revolutionaries in physics must be young people whose minds have not had time to become habituated to well-worn pathways of thought. Thus Schrödinger and Einstein may have had little chance of success in their efforts to discover a unified field theory in the 1940's using methods of the 1920's. Richard Feynman summed up this notion, saying (1962): "*None of these unified field theories has been successful... Most of them are mathematical games, invented by mathematically minded people... and most of them are not understandable*".

Nicole (b. 1945), and Linda (b. 1946). He believed that scientific creativity would be promoted and sustained by erotic excitement, and that love is not an impediment to great efforts but its carrier.

Schrödinger was also an amateur poet, and a book of his poems appeared in 1949. Many of these were love poems which he wrote for his women. Appreciation of one of his poems gave him much more pleasure than any amount of praise for his scientific papers.

By the end of his life, he must have mastered as much general culture — scientific and nonscientific — as it is possible for any single person to absorb in this age of technical specialization. He read widely in several languages, and wrote perceptively about the relation between science and the humanities and about Greek science, in which he was particularly interested.

Of all the physicists of his generation, with the possible exception of Einstein, Schrödinger stands out on account of his extraordinary intellectual versatility and his significant contributions to nearly all branches of science and philosophy.

Worldview XL: Erwin Schrödinger on all that and physics too

* * *

"I don't like it, and I'm sorry I ever had anything to do with it."

(on "Schrödinger's Cat")

* * *

"Only metaphysics can inspire the hard work of theoretical physics."

* * *

"Actions are transitory while works remain."

* * *

"What is Life?, I asked in 1943. In 1944, Sheila May told me. Glory be to God!"

* * *

"In Germany, if a thing was not allowed, it was forbidden. In England, if a thing was not forbidden, it was allowed. In Austria and Ireland, whether it was allowed or forbidden, they all did it if they wanted."

* * *

"One cannot derive philosophical conclusion from physics. In contrast, however, philosophy could influence physics."

* * *

"All great things in the world are worked through love. It produces everything. Love is not an impediment to great effort but its carrier."

* * *

"Thus, the task is, not so much to see what no one has yet seen; but to think what nobody has yet thought, about that which everybody sees."

* * *

"When you feel your own equal in the body of a beautiful woman, just as ready to forget the world for you as you for her – oh my good Lord – who can describe what happiness then. You can live it, now and again – you cannot speak of it."

The Schrödinger Equation[563] *(1925)*

For a free, non-relativistic spinless particle of mass m, the associated de Broglie wave can be written $\psi = e^{-i\frac{1}{\hbar}\chi(\boldsymbol{r},t)}$, with χ satisfying $\frac{\partial\chi}{\partial t} = \frac{1}{2m}(\boldsymbol{\nabla}\chi)^2$. If this particle/wave moves in a potential field with potential $V(\boldsymbol{r},t)$ one may draw parallels with the Hamilton-Jacobi equation of analytic mechanics and identify χ with the action*. This analogy leads to an equation*

$$\frac{\partial\chi}{\partial t} = \frac{1}{2m}(\boldsymbol{\nabla}\chi)^2 + V(\boldsymbol{r},t).$$

But on the other hand, one would like the de Broglie wave to satisfy a wave equation.

Schrödinger then (1925) came up with a wave equation that no one had seen before:

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[563] For further reading, see:

- Schiff, L.I., *Quantum Mechanics*, McGraw-Hill Book Company: New York, 1968, 544 pp.
- Park, D., *Introduction to the Quantum Theory*, Dover, 2005, 601 pp.
- Griffiths, D.J., *Introduction to the Quantum Mechanics*, Prentice Hall, 1995, 394 pp.
- Zettili, N., *Quantum Mechanics*, Wiley, 2001, 649 pp.
- Blinder, S.M., *Introduction to the Quantum Mechanics in Chemistry, Materials Science and Biology*, Elsevier, 2004, 319 pp.
- Treiman, S., *The Odd Quantum*, Princeton University Press, 1999, 262 pp.
- Becker, R., *Quantum Theory of Atoms and Radiation*, Blackie & Son: London, 1964, 403 pp.
- Bethe, H. and E.E. Salpeter, *Quantum Mechanics of One and Two-Electron Atoms*, Plenum Publishing Corporation: New York, 1977, 369 pp.
- Feynman, R.P., R.B. Leighton and M. Sands, *The Feynman Lectures on Physics*, 3 Volumes, Addison-Wesley Publishing Company: Reading, MA, 1963–1965.
- Landau, L.D. and E.M. Lifshitz, *Quantum Mechanics*, Pergamon Press.

$$\boxed{i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V(\boldsymbol{r},t)\psi \;=\; H\psi} \tag{1}$$

He arrived at this equation by starting from the classical energy equation for a particle $E = \frac{1}{2m}\boldsymbol{p}^2 + V(\boldsymbol{r},t)$ *and representing the energy and momentum by the respective differential operators*

$$E \to i\hbar\frac{\partial}{\partial t}, \qquad \boldsymbol{p} \to -i\hbar\nabla,$$

acting on the wave function $\psi(\boldsymbol{r},t)$, *where* $V(\boldsymbol{r},t)$ *is a real potential, not depending on* $\boldsymbol{p}$ *or* E*. The Hamiltonian operator now bears the form*

$$H = -\frac{\hbar}{2m}\nabla^2 + V(\boldsymbol{r},t). \tag{2}$$

Note that there is no source term *for* ψ*; rather,* ψ *acts as a source for itself in conjunction with the external field* V*.*

Born (1926) interpreted ψ *as a* probability wave-function*, in the sense that it is large where the particle is 'likely to be' and small elsewhere. In the physical microworld it replaces the deterministic classical trajectory* $\boldsymbol{r}(t)$*. Thus (1) is assumed to provide a quantum-mechanical complete description of the behavior of a particle of mass* m *with potential energy* $V(\boldsymbol{r},t)$*.*

It turned out that in the semiclassical *limit* $\hbar \to 0$, *an approximation (known in optics as the* eikonal approximation*, and in quantum mechanics as the* WKB or WKBJ approximation*) holds, wherein the Schrödinger equation becomes the* Hamilton-Jacobi equation*. To see this we put* $\psi(\boldsymbol{r},t) = Ae^{i\frac{1}{\hbar}W(\boldsymbol{r},t)}$ *into (1) (A constant) and obtain*

$$\frac{\partial W}{\partial t} + \frac{1}{2m}(\nabla W)^2 + V - \frac{i\hbar}{2m}\nabla^2 W = 0 \tag{3}$$

In the limit $\hbar \to 0$*, (3) is the same as the Hamilton-Jacobi PDE for the principal function* W*:*

$$\frac{\partial W}{\partial t} + H(\boldsymbol{r},\boldsymbol{p}) = 0 \qquad \boldsymbol{p} = \nabla W. \tag{4}$$

Since the momentum of the particle is the gradient of W*, the possible trajectories are orthogonal to the surfaces of constant* W *and hence, in the classical* *limit, to the surfaces of constant phase of the wave function ψ. Thus, in this limit, the* rays *associated with ψ (orthogonal trajectories to the surfaces of constant phase) are the possible paths of the classical particle.*

Let us now split the variables in $W(\boldsymbol{r},t)$, putting $W(\boldsymbol{r},t) = S(\boldsymbol{r}) - Et$, namely

$$\psi(\boldsymbol{r},t) = Ae^{i\frac{1}{\hbar}S(\boldsymbol{r})}e^{-i\frac{1}{\hbar}Et} = u(\boldsymbol{r})e^{-i\frac{1}{\hbar}Et}.$$

We shall then have an equation for $S(\boldsymbol{r})$

$$\boxed{\frac{1}{2m}(\nabla S)^2 - [E - V(\boldsymbol{r})] - \frac{i\hbar}{2m}\nabla^2 S = 0} \tag{5}$$

The WKB approximation starts from (5) and expands $S(\boldsymbol{r})$, the logarithm of the wavefunction, in powers of $\hbar$ in the one dimensional case. The leading term, S_0, is real and approximates the phase *of the wave-function (times $\hbar$), while the next term, S_1, both corrects the* phase *(real part) and approximates the* amplitude *(imaginary part).*[564]

Returning to (1) we note that the wave function ψ^* satisfies the complex-conjugate equation

$$-i\hbar\frac{\partial\psi^*}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi^* + V(r)\psi^*. \tag{6}$$

For any fixed volume τ, the combined use of (1), (6) and Green's third identity yield,

$$\begin{aligned}\frac{\partial}{\partial t}\int_\tau \psi\psi^*\,d\tau &= \int_\tau\left[\psi^*\frac{\partial\psi}{\partial t} + \frac{\partial\psi^*}{\partial t}\psi\right]d\tau \\ &= \frac{i\hbar}{2m}\int_\tau\left[\psi^*\nabla^2\psi - \nabla^2\psi^*\psi\right]d\tau \\ &= \frac{i\hbar}{2m}\int_\tau \text{div}\,[\psi^*(\nabla\psi) - (\nabla\psi^*)\psi]\,d\tau \\ &= \frac{i\hbar}{2m}\int_S [\psi^*(\nabla\psi) - (\nabla\psi^*)\psi]\cdot\boldsymbol{n}\,dS.\end{aligned} \tag{7}$$

If we let τ engulf the entire space and recede S to infinity under proper behavior conditions of Ψ at infinity, we obtain

[564] The phase-correction part of S_1 vanishes, but this is not so for higher $\hbar^n$ terms.

$$\frac{\partial}{\partial t}\int\limits_{\substack{\text{all}\\ \text{space}}} \psi^*\psi\, d\tau = 0 \qquad\qquad \Rightarrow \qquad\qquad \int\limits_{\substack{\text{all}\\ \text{space}}} \psi^*\psi\, d\tau = constant.$$

By proper normalization the constant can be chosen as unity. The entity $\rho = \psi^*\psi$, *which obeys*

$$\int\limits_{\substack{\text{all}\\ \text{space}}} \rho(\boldsymbol{r}, t)\, d^3\boldsymbol{r} = 1 \tag{8}$$

is given the interpretation of position probability density (Born 1926).

Returning to (3), one defines the probability current density vector

$$\boldsymbol{J} = \frac{\hbar}{2im}\left[\psi^*(\nabla\psi) - (\nabla\psi^*)\psi\right]. \tag{9}$$

It is linked to the probability density $\rho(\boldsymbol{r}, t)$ *via the differential relation*

$$\frac{\partial\rho(\boldsymbol{r}, \boldsymbol{t})}{\partial t} + \operatorname{div}\boldsymbol{J}(\boldsymbol{r}, t) = 0. \tag{10}$$

This has the familiar form associated with the conservation of flows of fluid of density ρ *and current density* $\boldsymbol{J}$, *in which there are no sources or sinks. The vector* $\boldsymbol{J}$ *is thus interpreted as a probability current density. Also, with* $H = -\frac{\hbar^2}{2m}\nabla^2 + V$ *we have*

$$\psi^* H\psi - \psi H\psi^* = i\hbar\frac{\partial\rho}{\partial t}. \tag{11}$$

Ehrenfest *(1927) gave a semiclassical interpretation of the Schrödinger field by linking the motion of a wave packet centroid and the trajectory and dynamics of the corresponding classical particle, whenever the potential* V *changes by negligible amount over the dimensions of the packet (which is the condition for the WKBJ expansion to be useful). This is equivalent to the statement that Newton's second law of motion holds for average quantities.*

To see this we note that if the quantities ρ *and* $\boldsymbol{J}$ *are both multiplied by* m, *the mass of the particle, we obtain mass density* $\rho_m = m\rho$ *and momentum density* $\boldsymbol{P} = m\boldsymbol{J}$, *and the equation of continuity may be interpreted as the law of mass conservation. In the same way, a multiplication by the particle charge* $e^{\cdot}$, *yields the charge density* $\rho_e = e\rho$ *and the electric current density* $\boldsymbol{J}_e = e\boldsymbol{J}$, *and (10) becomes the law of charge conservation. The conservation laws of both mass and charge are identical because one particle, by its probability flow, caused both.*

Mathematically, we get from (9) the expression of the statistical expectation of the total momentum operator in the quantum state Ψ*:*

$$\begin{aligned}\langle \boldsymbol{p}\rangle &= \int_\tau \Psi^* \boldsymbol{p}\Psi \, d^3x = \tfrac{\hbar}{2i}\int_\tau \left[\psi^*\nabla\psi - \psi\nabla\psi^*\right] \, d^3x \\ &= \tfrac{\hbar}{i}\int_\tau \psi^*\nabla\psi \, d^3x = m\int_\tau \boldsymbol{J}\, d^3x\end{aligned} \tag{12}$$

where the second term in the middle equation is reduced by partial integration. It can also be shown (by use of (1), (6) and partial integration) that the semiclassical force acting on the particle is

$$\boldsymbol{F} = -\langle \boldsymbol{\nabla} V\rangle = \frac{d\langle \boldsymbol{p}\rangle}{dt} = \frac{\hbar}{i}\int_\tau \frac{\partial}{\partial t}\left(\Psi^* \boldsymbol{\nabla}\Psi\right)\, d^3x. \tag{13}$$

By a similar derivation it is found that if we define the expected (mean) particle position as

$$\langle \boldsymbol{r}\rangle = \int_\tau \psi^* \boldsymbol{r}\psi \, d^3x, \tag{14}$$

then

$$\frac{d}{dt}\langle \boldsymbol{r}\rangle = \frac{1}{m}\langle \boldsymbol{p}\rangle, \qquad \frac{d^2}{dt^2}\langle \boldsymbol{r}\rangle = \frac{1}{m}\langle -\nabla V\rangle. \tag{15}$$

These relations are analogous to the classical equations of motion

$$\frac{d\boldsymbol{r}}{dt} = \frac{1}{m}\boldsymbol{p}, \qquad \frac{d\boldsymbol{p}}{dt} = -\nabla V. \tag{16}$$

Likewise, the quantum analogs of the laws of the classical angular motion conservation are derived from $\boldsymbol{T}(\text{torque}) = \boldsymbol{r}\times\boldsymbol{F}$ *and* $\boldsymbol{T} = \frac{d\boldsymbol{L}}{dt}$ *($\boldsymbol{L}$ = particle angular momentum) where*

$$\boldsymbol{L} = \frac{\hbar}{i}\int_\tau \Psi^*(\boldsymbol{r}\times\boldsymbol{\nabla})\Psi\, d^3x = \int_\tau m\boldsymbol{r}\times\boldsymbol{J}\, d^3x\ , \tag{17}$$

$$\boldsymbol{T} = -\hbar i\int_\tau \frac{\partial}{\partial t}\left(\Psi^*(\boldsymbol{r}\times\boldsymbol{\nabla})\Psi\right)\, d^3x = \int_\tau \boldsymbol{r}\times(-\nabla\boldsymbol{V})\Psi^*\Psi\, d^3x\ , \tag{18}$$

where the time derivatives in the 2^{nd} *equation can be calculated from (1), (6) (note that in general* $\boldsymbol{L}\neq\langle\boldsymbol{r}\rangle\times\langle\boldsymbol{p}\rangle$, $\boldsymbol{T}\neq\langle\boldsymbol{r}\rangle\times\boldsymbol{F}$*).*

Finally, we define the total energy

$$E = \int_\tau \psi^* H \psi \, d^3x = \int_\tau \psi^* \left\{ -\frac{\hbar^2}{2m}\nabla^2 + V \right\} \psi \, d^3x. \tag{19}$$

Making use of the identity $\psi^* \nabla^2 \psi = \mathrm{div}(\psi^* \nabla \psi) - (\nabla \psi^* \cdot \nabla \psi)$, *assuming that at large distances* Ψ *tends to zero rapidly enough, and using Gauss' divergence theorem,* $\int \mathrm{div}(\psi^* \nabla \psi)\, d^3x$ *vanishes when taken over an infinitely remote sphere, so that* (19) *can be recast in the form*

$$E = \int d^3x \left[\frac{\hbar^2}{2m} (\nabla \psi^*) \cdot (\nabla \psi) + \psi^* V \psi \right] = \int d^3x \, W, \tag{20}$$

where the energy density is

$$W = \frac{\hbar^2}{2m} (\nabla \psi^*) \cdot (\nabla \psi) + \psi^* V \psi. \tag{21}$$

Using the Schrödinger equation, it is easy to demonstrate that the law of energy conservation should be

$$\frac{\partial W}{\partial t} + \mathrm{div}\, \boldsymbol{\Sigma} = 0, \tag{22}$$

where the expected (mean) energy flux vector is

$$\boldsymbol{\Sigma} = \frac{\hbar^2}{2m} \frac{h}{2mi} \left(\boldsymbol{\nabla} \Psi^* \cdot \boldsymbol{\nabla\nabla} \Psi - \nabla\nabla \Psi^* \cdot \nabla \Psi \right) + V \boldsymbol{J} \, . \tag{23}$$

The Ehrenfest analogy [(12)–(17)] provides an example of the correspondence principle, since it shows that the probability-centroid of a wave packet moves like a classical particle subject to the position and velocity uncertainties inherent in the wave function and the uncertainty principle. The correspondence is mainly useful in the macroscopic limit in which the finite size and the internal structure of the packet can be ignored.

When V *is time-independent (as we have so far been assuming), any solution* $\psi(\boldsymbol{x}, t)$ *of Schrödinger's equation can be decomposed as*

$$\psi(\boldsymbol{x}, t) = \sum_j e^{-i(E_i/\hbar)t} \psi_j(\boldsymbol{x}), \tag{24}$$

where j *ranges over all eigenstates,* $\psi_j(\boldsymbol{x})$, *of the Hamiltonian operator* $H = \frac{1}{2m}\boldsymbol{p}^2 + V(\boldsymbol{x})$, *with corresponding eigenvalues (energy levels)* E_j.

Note that $\sum_j$ *usually includes a discrete summation over the bound states, and an integration over the continuum states. The eigenstate condition on* ψ_j *is known as the time-independent (stationary) Schrödinger equation*

$$-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2\psi_j + U(\boldsymbol{x})\psi_j = E_j\psi_j, \tag{25}$$

and describes e.g. a single, stationary atomic energy state.

A transition between different energy levels (induced e.g. by an external time-varying electromagnetic field) is described by the time-dependent equation, with time-dependent U*. It governs atomic and molecular processes that vary with time, such as absorption and emission of photons and electrons, scattering, and chemical reactions. The Schrödinger equation (SE) in its above form (which is not the most general) is not Lorentz-covariant, and hence requires relativistic corrections.*

For the stationary case, if we multiply both sides of the equation $H\psi = E\psi$ *by* ψ^* *and integrate over the coordinates, we obtain:*

$$E = \frac{\int \psi^* H\psi d\tau}{\int \psi^*\psi d\tau}.$$

This formula enables us to calculate E *when* ψ *is known. If* ψ *is not known exactly, the Rayleigh-Ritz variational principle*[565] *asserts that the above ratio*

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[565] We seek a complex wave-function ψ for which the functional

$$J[\psi] = \iiint \psi^*(H\psi)dx\,dy\,dz$$

is stationary, subject to the probability-normalization constraint $\iiint \psi\psi^* dx\,dy\,dz = 1$, and with

$$H = -\frac{\hbar^2}{2m}\nabla^2 + V(x,y,z).$$

Assuming that ψ satisfies appropriate spatial boundary conditions we find $\int \psi^*\nabla^2\psi d^3\boldsymbol{r} = -\int(\nabla\psi^*)\cdot(\nabla\psi)d^3\boldsymbol{r}$. Utilizing a Lagrange multiplier λ for the constraint, the condition $\delta J = 0$ now reads $\delta\int F d^3\boldsymbol{r} = 0$, where

$$F = \frac{\hbar^2}{2m}(\nabla\psi^*)\cdot(\nabla\psi) + V\psi^*\psi - \lambda\psi^*\psi.$$

The Euler-Lagrange equations are:

$$\frac{\partial F}{\partial\psi} - \frac{\partial}{\partial x}\frac{\partial F}{\partial\psi_x} - \frac{\partial}{\partial y}\frac{\partial F}{\partial\psi_y} - \frac{\partial}{\partial z}\frac{\partial F}{\partial\psi_z} = 0$$

and a similar (equivalent) equation for ψ^*. These equations reduce to $H\psi = \lambda\psi$.

has a (global) minimal value equal to the ground state energy of the system; choosing a reasonable trial form for ψ *with some free parameter, one can find approximations to* E *by varying w.r.t. these parameters.*

A solution of the time-independent SE *for the electron in a hydrogen atom (Coulomb field) is obtained through the method of separation of variables in spherical coordinates* (r, θ, ϕ). *It yields eigenfunctions of the form* $R_{n\ell}(r)P_\ell^m(\cos\theta)e^{im\phi}$, *where* $\{n, \ell, m\}$ *are known as the orbital quantum numbers.*

Of these, n *is associated with the total energy (permissible values* $= 1, 2, 3, \ldots$*);* ℓ *is the magnitude of the orbital angular momentum in units of* $\hbar$ *(permissible values* $= 0, 1, 2, \ldots, n-1$*), and* m_ℓ *is the projection of the orbital angular momentum along a given axis in the same units (axis chosen to be* z*; permissible values* $= -\ell, -\ell+1, \ldots, 0, \ldots, \ell-1, \ell$*). It was established experimentally that a fourth quantum number is needed to describe the quantized spin (internal) angular momentum of the electron (this is the component of spin angular momentum in units* $\hbar$*; permissible values* $m_s = \pm\frac{1}{2}$*).*

The electron orbital angular momentum states of atoms are designated by the letters s, p, d, f *(and alphabetically ordered thereafter:* g, h, $\ldots$*) which were borrowed from early spectroscopic work and describe those states which give rise to* SHARP, PRINCIPAL, DIFFUSE, *and* FUNDAMENTAL *spectral lines. The letter* s *is used for* $\ell = 0$, p *for* $\ell = 1$, d *for* $\ell = 2$, f *for* $\ell = 3$, *etc.*

To identify λ we multiply it by ψ^* and integrate over x, y, z. The result is $\lambda = E$ where

$$E = \iiint \psi^*(H\psi)d^3\boldsymbol{r}.$$

Further inspection shows the energy E to be a minimum rather than a maximum (except when the spectrum of the operator H is bounded and E is the maximal energy level). Note that the expression for F (above) can be modified to yield the *Lagrangian density* for the time-dependent Schrödinger equation

$$L = -\frac{\hbar^2}{2m}(\nabla\psi^*)\cdot(\nabla\psi) - \frac{\hbar}{2i}\left(\psi^*\frac{\partial\psi}{\partial t} - \frac{\partial\psi^*}{\partial t}\psi\right) - \psi^* V\psi.$$

Indeed, applying the *principle of least action* to $\iiiint Ld^3\boldsymbol{r}dt$ yields:

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t}.$$

In his dissertation (1942), **R.P. Feynman** derived the Schrödinger equation in this way, thus founding Quantum Mechanics on the Lagrangian method in contradistinction to the traditional Hamiltonian theory of classical mechanics.

Each different combination of n, ℓ *and* m *corresponds to a unique (except for spin) quantum state called an orbital. For cataloging the elements it suffices to designate only the numbers of electrons having various values of these two quantum numbers in the atom's ground (lowest energy) state. Thus,* $(1s)^1$ *represents the ground state of the hydrogen atom with one electron in the level* $n = 1,\ \ell = 0$. *Likewise* $(1s)^2$ *represents helium with 2 electrons in the orbital* $n = 1,\ \ell = 0$ *(one each for spin* $\pm\frac{1}{2}$*).*

The maximum number of electrons in energy level n *is* $2n^2$ $(2, 8, 18, 32, \dots$ *etc.); the* (l, m) *orbitals of given* n *are together called a shell (because mean orbital radius increases with* n*). The configurations and designations of filled shells for the first four energy levels are:* $(1s)^2$ $(n = 1)$; $(2s)^2(2p)^6$ $(n = 2)$; $(3s)^2(3p)^6(3d)^{10}$ $(n = 3)$; $(4s)^2(4p)^6(4d)^{10}(4f)^{14}$ $(n = 4)$.

Due to inter-electron repulsion, inner electron shell partially shield outer electrons from the nuclear charge, and the amount of shielding increases with the ℓ*-value of the outer electrons. Consequently, the single-electron energy levels do vary somewhat within a given* n *shell. Thus* np *is higher in energy than* ns *and filled later;* nd *is only filled after* $(n+1)s$ *(but before* $(n+1)p$*) for* $n = 3, 4$.

In Oxygen, for example, the second shell is not filled and has only 6 electrons. Two of them are in the states $\left\{n = 2,\ \ell = 0,\ m_\ell = 0,\ m_s = \pm\frac{1}{2}\right\}$, *while the other four may, for example, occupy the states* $\left\{n = 2,\ \ell = 1,\ m_\ell = \pm 1,\ m_s = \pm\frac{1}{2}\right\}$. *This shell is not closed, and its deficiency (2) is the valence of oxygen.*

The directionality, rigidity, length, etc. of an inter-atomic bond depends on the spatial symmetry of these orbitals around the nucleus. The quantum numbers ℓ, m *determine the angular dependence of the non-radial factor of the Schrödinger probability amplitude; only the orbital with* $\ell = 0$ *has a spherically symmetric probability distribution*[566]*. The sum of the probability distributions for the three orbitals* $n = 2,\ \ell = 1,\ m_\ell = -1, 0, +1$ *is spherically symmetric; in general, a full shell of electrons (for any given value of* n*) presents a spherically symmetric charge distribution around the nucleus.*

The three orthogonal basis $2p$ *orbitals* $(n = 2,\ \ell = 1)$ *are sometimes chosen to be* p_x, p_y, *and* p_z, *which correspond to certain linear combi-*

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[566] The fact that $\ell = 0$ does not mean that the electron is at rest but only that its motion is as probable in any one direction as in any other. The spherically symmetric "probability cloud" is the statistical amalgam (a sort of "time exposure" photograph) of all possible classical $\ell = 0$ orbits, which are line segments (degenerate ellipses) with all possible spatial orientations, emanating from the origin (nucleus).

nations of the waves for the m_ℓ *values* $+1,\ -1$ *and* 0[567]. *Their probability amplitude patterns are* $\{\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta\}$, *and contain one angular node each on a plane with null electron density. The 3d orbitals* $(n=3,\ \ \ell=2,\ \ m=\pm 2,\pm 1,0)$ *split into* d_{z^2} $(m=0,\ 3\cos^2\theta-1)$, d_{xz} $(m=\pm 1,\ \sin\theta\cos\theta\cos\phi)$, d_{yz} $(m=\pm 1,\ \sin\theta\cos\theta\sin\phi)$, d_{xy} $(m=\pm 2,\ \sin^2\theta\sin 2\phi)$ *and* $d_{x^2-y^2}$ $(m=\pm 2,\ \sin^2\theta\cos 2\phi)$.

If two electrons in the same atom have the same values of $\{n,\ell,m_\ell\}$, *they share the* same *orbital and are said to be* paired. *This is because they must have opposite spins (by the* EXCLUSION PRINCIPLE*), but are identical in every other respect. Since one electron has the spin quantum number* $m_s=\frac{1}{2}$ *and the other has* $m_s=-\frac{1}{2}$, *the* magnetic moments *associated with these spins cancel.*

For an atom or molecule with an unpaired *electron, the electron spin interacts with an external magnetic field, because the spin magnetic moment of the unpaired electron tends to align itself in the direction of the external field*[568]. *A substance is* paramagnetic, *if its bulk magnetization is entirely due to the average alignment by an external magnetic field of its atomic (or molecular) moments. These moments may be due to orbital angular momentum, spin, or both; the atomic nucleus, too, may possess spin.*

The following examples serve to show possible electron configurations of two key atoms:

divalent Oxygen atom: $(1s)^2(2s)^2(2p_z)^2(2p_x)^1(2p_y)^1$

2 paired electrons in $n=1,\ \ell=0,\ m_\ell=0,\ m_s=\pm\frac{1}{2}$

2 paired electrons in $n=2,\ \ell=0,\ m_\ell=0,\ m_s=\pm\frac{1}{2}$

2 paired electrons in $n=2,\ \ell=1,\ m_\ell=0,\ m_s=\pm\frac{1}{2}$

2 unpaired electrons in $n=2,\ \ell=1$ $\begin{cases} m_\ell=+1 & m_s=\frac{1}{2}\text{ or }-\frac{1}{2}\\ m_\ell=-1 & m_s=\frac{1}{2}\text{ or }-\frac{1}{2}\end{cases}$

divalent Carbon atom: $(1s)^2(2s)^2(2p_x)^1(2p_y)^1$

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567 The linear combinations are chosen such that the corresponding probability distributions for the electron are peaked about the x, y, z axes, respectively. This makes it easier to visualize the orbitals geometrically — especially when it comes to multi-atomic bonds and molecules.

568 This also happens when two or more unpaired electrons, each occupying a distinct atomic (or molecular) orbital, because such electrons tend to have mutually aligned spins (due to a combination of electrostatic and Pauli Exclusion effects) and can thus align *as one magnet* with the external field.

2 paired electrons in $n = 1,\ \ell = 0,\ m_\ell = 0,\ m_s = \pm\frac{1}{2}$

2 paired electrons in $n = 2,\ \ell = 0,\ m_\ell = 0,\ m_s = \pm\frac{1}{2}$

2 unpaired electrons in $n = 2,\ \ell = 1\ \begin{cases} m_\ell = +1 & m_s = \frac{1}{2} \text{ or } -\frac{1}{2} \\ m_\ell = -1 & m_s = \frac{1}{2} \text{ or } -\frac{1}{2} \end{cases}$

tetravalent Carbon atom: $(1s)^2(2s)^1(2p_x)^1(2p_y)^1(2p_z)^1$

2 paired electrons in $n = 1,\ \ell = 0,\ m_\ell = 0,\ m_s = \pm\frac{1}{2}$

2 unpaired electrons in $n = 2\ \begin{cases} \ell = 0 & m_\ell = 0 & m_s = \frac{1}{2} \text{ or } -\frac{1}{2} \\ \ell = 1 & m_\ell = 0 & m_s = \frac{1}{2} \text{ or } -\frac{1}{2} \\ \ell = 1 & m_\ell = +1 & m_s = \frac{1}{2} \text{ or } -\frac{1}{2} \\ \ell = 1 & m_\ell = -1 & m_s = \frac{1}{2} \text{ or } -\frac{1}{2} \end{cases}$

In the last case a $2s$ *electron was excited into the (slightly higher-energy) empty* $2p_z$ *orbital, thus conferring on the carbon the ability to bond to more surrounding atoms (hybridization) on its transition from* $\ell = 0$ *to* $\ell = 1$*. The maximum number of electrons in the* n^{th} *energy shell is* $2n^2$.

The solution of the SE for a one-electron atom is of little practical value. All atoms — except hydrogen and certain ions of the light elements — contain several electrons. In these atoms, the Hamiltonian of the whole atom includes the interaction of each electron with the nucleus, and in addition the interaction of the electrons among themselves. Since the motion and quantum state of each electron depends on those of all the others, any modification in the state of one electron must affect the state of all the other electrons.

Therefore we can talk only of the energy of the whole atom (or ion or, indeed molecule) and for the same reason, of a wave function for the complete atom or molecule. But the inter-dependence is more insidious in quantum-mechanics than in a classical system (say, the planets in the solar system), since the motion of a single planet is a valid concept; here there is no single-electron state (although this is often a useful approximate *concept). This impossibility of describing the motion (or even quantum state) of an individual electron means that many-electron atoms, ions and molecules are difficult to quantify exactly, and certain approximations are required. In general, this leads to complex numerical calculations.*

As a first approximation, one may ignore the electron-electron electrostatic interaction term $\sum_{\text{all pairs}} \frac{e^2}{r_{ij}}$*. This is equivalent to assuming that each electron moves independently of the others. Thus we may call this approximation the 'independent-particle model' in which each electron can be described by a hydrogen-like wave function.*

In cases where the known energy levels E_n of a system with Hamiltonian H_0 are slightly perturbed due to an additional term in the Hamiltonian, the first order perturbation in the energy levels is:

$$(\Delta E)_n \approx \int \psi_n^* H' \psi_n d\tau,$$

where H' is the first order perturbation of H_0 and ψ_n are the eigenfunctions of H_0. The new eigenfunctions are given by $\psi_n' \approx \sum_j a_j^{(n)} \psi_j$ where

$$a_k^{(n)} = \frac{1}{E_n - E_k} \int \psi_k^* H' \psi_n d\tau, \qquad k \neq n \quad (a_n = 1).$$

The foregoing technique may be used to solve approximately the problems of the helium atom and the normal Zeeman effect. In the latter case the classical Hamiltonian for an electron of mass m_e carrying a charge $-e$ and moving in a magnetic field whose vector potential is $\boldsymbol{A}$ is

$$H = \frac{1}{2m_e}\{\boldsymbol{p}^2 + 2e\boldsymbol{A}\cdot\boldsymbol{p} + e^2\boldsymbol{A}^2\} + U(r).$$

For a weak field we may neglect the $\boldsymbol{A}^2$ term, whence H becomes the operator $-\frac{\hbar^2}{2m_e}\nabla^2 - \frac{i\hbar e}{m_e}\boldsymbol{A}\cdot\nabla + V(r)$. In the non-relativistic limit of the Dirac equation, H is found to acquire the additional term $\frac{e\hbar}{2m_e}\boldsymbol{\sigma}\cdot\boldsymbol{B}$, with $\boldsymbol{\sigma}$ the Pauli spin matrices and $\boldsymbol{B} = \boldsymbol{\nabla}\times\boldsymbol{A}$ the external field.

First order perturbation of the energy levels of hydrogen is found to be $\Delta E = \left[\frac{e\hbar}{2m_e}B\right](m_e + 2m_s)$, in agreement with experiment. When an atom containing more than one electron is treated quantum-mechanically, a useful first approximation is that the electrons do not interact with one another. In this approximation, the wave function describing each electron is a 'hydrogen-like' wavefunction, characterized by the quantum numbers $\{n, \ell, m_\ell, m_s\}$. The Pauli exclusion principle states that in a multielectron system no two electrons can have identical sets of quantum numbers. In other words, no two electrons may have the same spatial distribution and spin orientation of their wavefunctions[569].

Thus, for example, for the shell with $n = 1$, we must have $\ell = 0$, $m_\ell = 0$ and $m_s = \pm\frac{1}{2}$. Hence, at most 2 electrons may have $n = 1$ in a single atom. For $n = 2$ there are $2(2\ell + 1) = 6$ possible sets for $\ell = 1$

569 Where spin is included in the wavefunction by definition.

$(m_\ell = -1, 0, +1;\ m_s = \pm\frac{1}{2})$ and 2 possible sets for $\ell = 0$, yielding at most 8 electrons in the $n = 2$ shell.

The Pauli principle correctly predicts the numbers of electrons in the closed atomic shells of the periodic table of the elements.

Further elaboration on this important principle is linked to the concept of exchange symmetry of wave functions.

In classical mechanics, the existence of sharply definable trajectories for individual particles makes it possible in principle to distinguish between particles that are identical except for their paths, since each particle can be followed during the course of an experiment. In quantum mechanics, the uncertainty principle limits our ability to follow the motions of the particles without disturbing the system, so that we can never be certain which one of a number of identical particles we have actually found at a given point — because the particles are indeed physically indistinguishable. This places certain restrictions upon the mathematical form that the wave-function for several identical particles may have.

In the modern view of atoms, the electrons in the space surrounding the dense, point-like nucleus may be thought of as existing in motional orbitals, but these are smeared into continuous, overlapping spatial distributions. At most two electrons, in opposite spin states, can occupy a single orbital — according to Pauli's exclusion principle. The orbital occupied by a pair of electrons of opposite spin is filled: no more electrons may enter it until one of the pair vacates the orbital. Moreover, the exclusion principle extends to a wider exchange-symmetry rule.

An example will serve to explain this principle: A helium-like atom or ion has a nucleus of charge Ze surrounded by two electrons, which we label as 1 and 2. In the 'independent-particle

model' it follows from the Schrödinger equation that the wave function of the atom should be the product of the wave functions for each electron, or a linear combination of such products. If we designate the orbital quantum numbers $\{n, \ell, m_\ell\}$ of electron 1 by a and the corresponding quantum numbers of electron 2 by b, we may then write $\psi_{\text{atom}} = \psi_a(1)\psi_b(2)$, where the arguments 1 and 2 of the wavefunctions are shorthand for $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$, respectively. This result in a joint spatial probability distribution $|\psi_a(1)|^2|\psi_b(2)|^2$.

Since the electrons are identical and indistinguishable, ψ_{atom} must be constructed in such a way that the exchange of electrons 1 and 2 will not affect $|\psi_{\text{atom}}|^2$, i.e. the joint spatial distribution will be symmetric between the two electrons. This will be achieved if we construct either of the two linear combinations $\psi_{\text{atom}} = \psi_a(1)\psi_b(2) \pm \psi_a(2)\psi_b(1)$. (All wave functions thus far are *spatial — no reference to spin, yet). The plus sign yields a symmetric wavefunction whereas the negative sign represents the antisymmetric counterpart, namely*

$$\psi_S(1,2) = \psi_a(1)\psi_b(2) + \psi_a(2)\psi_b(1), \quad \psi_A = \psi_a(1)\psi_b(2) - \psi_a(2)\psi_b(1).$$

It can be shown that the energy of the atom associated with ψ_A is larger[570] *(less strongly bound) than that associated with ψ_S.*

Thus, a two-electron atom has two sets of stationary states and energy levels, one described by symmetric orbital wave functions and the other by antisymmetric orbital wave functions. An exception occurs when $a = b$ (the two sets of orbital quantum numbers of the electrons are identical), for which $\psi_A = 0$.

When the electron's spin is taken into account, the vectorial sum of the spins of the two electrons can either be zero ($S = 0$; singlet state-Parahelium*) or $S = 1$ [triplet, orthohelium; with $M_S = +1$, (both spins with positive components), or $M_S = 0$ (vanishing z component, vectorial sum normal to z axis), or $M_s = -1$ (both spins with negative z component)].*

The total spin wave function of the singlet state ($S = 0$) is antisymmetric *in the two electrons*

$$\chi_A = \frac{1}{\sqrt{2}}\left[\chi_+(1)\chi_-(2) - \chi_+(2)\chi_-(1)\right],$$

where χ are spin wavefunctions; $\chi_\pm(1)$ are the 1 electron wavefunctions for the cases of spin up $\left(m_s = +\frac{1}{2}\right)$, for subscript "+", or spin down for "−". On the other hand, the total spin wave functions of the triplet ($S = 1$) are symmetric *in the two electrons: $\chi_S = \chi_+(1)\chi_+(2)$ for $M_S = 1$, $\chi_S = \chi_-(1)\chi_-(2)$ for $M_S = -1$ and*

$$\chi_S = \frac{1}{\sqrt{2}}\left[\chi_+(1)\chi_-(2) + \chi_+(2)\chi_-(1)\right]$$

for $M_S = 0$.

The total *wave function of the atom for each state is the direct product*

$$\psi_{\text{total}} = [\textit{orbital (space) wave functions}] \times [\textit{spin wave function}].$$

570 Because Ψ_A vanishes at more spatial configurations and thus confines the electrons to a smaller average volume, resulting in higher *zero-point kinetic* energy by virtue of the uncertainty principle.

Now, an empirical examination of the energy levels of the helium atom (e.g. by spectroscopic methods) reveals that the state described by symmetric orbital wave functions ψ_S *are always spin singlets* $(S = 0)$, *while the states described by antisymmetric orbital wave functions* ψ_A *are always triplets* $(S = 1)$. *In either case* ψ_{total} *is antisymmetric. We know from spectroscopic experience that this result is not restricted to the helium atom, and can be elevated to the level of a general principle, stating that the total wave function of a system of electrons must be totally antisymmetric.*

This may be considered as an alternative, and more general, statement of the exclusion principle since the antisymmetry of the wave function implies the validity of the exclusion principle.

This can be seen in a simple way by writing the total wave function of an atom with N *electrons in the independent-particle model. Designating by a single letter (a, say) the orbital-spin state* $\{n, \ell, m_\ell, m_s\}$, *the configuration in which one electron is in state* a, *another in state* b *etc. may be expressed by the Slater determinant* representation of the wave-function:

$$\psi_{abc\cdots} = \frac{1}{\sqrt{N!}} \begin{vmatrix} \psi_a(1) & \psi_a(2) & \psi_a(3) & \cdots \\ \psi_b(1) & \psi_b(2) & \psi_b(3) & \cdots \\ \psi_c(1) & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots \end{vmatrix}.$$

This expression is antisymmetric since the interchange of any two columns (two electrons) changes the sign of the determinant. Moreover, if any two electrons have the same quantum number, the determinant is identically zero; thus the exclusion principle follows from the antisymmetry of the multielectron wave-function when the independent-particle model is used.

For a concrete example we return to the helium atom with $S = M_S = 1$. *Then*

$$\begin{aligned} \psi = \psi_A \chi_S &= \frac{1}{\sqrt{2}} \left[\psi_a(1)\psi_b(2) - \psi_a(2)\psi_b(1)\right] \chi_+(1)\chi_+(2) \\ &= \frac{1}{\sqrt{2}} \begin{vmatrix} \psi_a(1)\chi_+(1) & \psi_a(2)\chi_+(2) \\ \psi_b(1)\chi_+(1) & \psi_b(2)\chi_+(2) \end{vmatrix}. \end{aligned}$$

In this triplet state (spins parallel, antisymmetric spatial wave-function, $S = 1$), *when electrons 1 and 2 are close together, the value of* $|\psi_A|$ *is very small. The result is that the probability density is such that there is little chance of finding them close together. This has nothing to do with a Coulomb repulsion because we assumed ab initio that there is no explicit interaction between the electrons.*

This tendency of the electrons in the triplet state to be relatively far apart, can be interpreted as being due to an exchange *or* Pauli force*, that* repels *the electrons from each other. In the singlet state* ($S = 0$, *spins antiparallel, symmetric spatial wave-function), when electrons have almost the same coordinates, the probability density has the value* $2|\psi_a(1)|^2|\psi_b(2)|^2$, *which is* twice *the value expected without Pauli's principle.*

Thus, in the singlet state, the two electrons of opposite spin act as if they attract *each other, i.e. are under the influence of an* attractive exchange force*; again, this force has nothing to do with the electromagnetic forces between the two like charged, oppositely-magnetized electrons.*

Exchange forces arise not only between two electrons of the same *atom; they are an important factor in inter-atomic and molecular forces. The Pauli force is of a purely quantum origin and has no classical counterpart.*

Exchange symmetry rules can be generalized as follows:

(1) Identical particles having an integral quantum number for their intrinsic spin can be described only by wave functions which are symmetric *with respect to the interchange of the space* and *spin coordinates of any two such identical particles. [Such particles are said to obey the Bose-Einstein statistics; examples are the photon (spin 1), graviton (spin 2),* α*-particle (spin 0),* π*-meson (spin 0) of given charge, etc.]*

(2) Identical particles having a half integral quantum numbers for their intrinsic spin can be described only by wave functions which are antisymmetric *with respect to the interchange of the space* and *spin coordinates of any two such identical particles. [Particles obeying the Fermi-Dirac statistics – such as electrons, protons, neutrons, neutrinos,* μ*-mesons, quarks, etc.]*[571]

The Pauli principle, in both of its alternative formulations, cannot be derived from a more general basic principle. We do not know yet why nature

[571] In case the identical particles are endowed with other intrinsic quantum numbers besides spin – such as *isospin*, *color* (in the sense of quark-gluon forces), *strangeness*, *charm*, etc. – the above rules are generalized to include these further degrees of freedom within the spin coordinates.

seems to prefer antisymmetric state functions for fermions rather than symmetric functions and vice-versa for bosons; although the problem is definitely linked to causality via the *'spin-statistics' theorem*[572].

The above rules have the effect of limiting our choice of wave functions for a system containing several identical particles, to those which exhibit the proper type of exchange symmetry. This introduces fundamental new physical effects which are of importance in many phenomena. Some of them are:

- *As compared with the behavior of hypothetical equal but distinguishable* particles, *Bosons exhibit an additional mutual attraction. Identical fermions of the same spin state, on the other hand, repel one another.*
- *At a given temperature, the energy content and pressure of a system of Bose particles are less than (and those of a system of Fermi particles are greater than) the respective energy content and pressure of a corresponding system of equal but distinguishable* particles.[573]
- *If the individual particles of a system are acted upon by some outside force (but do not interact with each other), the quantum states of the system correspond to the various particles occupying certain of the eigenstates available to a single particle. Bosons tend to occupy the same quantum states, while fermions cannot occupy the same quantum states.*

The above properties are responsible for such diverse phenomena as the saturation of chemical bonding forces, the condensation of liquid helium into a superfluid state, the periodic system of the elements, superconductivity and lasers.

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572 This theorem states that in a relativistic quantum field theory, any attempt to assign the wrong statistics to particles results in the violation of *causality*. It follows that the Dirac field (spin $\frac{1}{2}$ relativistic field) *must* be quantized using anticommutator relations. This was unknown to Pauli when he formulated his exclusion principle. Furthermore, a strong candidate for a spontaneously broken symmetry of unified field theories of particle physics, is supersymmetry — in which each boson is linked to a corresponding fermion via an intrinsic "rotation".

Supersymmetry extends the concepts of differential geometry to non-commuting coordinates. It is related to such abstract algebra structures as *graded Lie algebras* and *affine Lie algebras*, and plays a key role in superstring theories and (probably) in the ultimate theory of *quantum gravity*.

573 This has important consequences for *lasers*, superconductors, metals, semiconductors, superfluids, Bose-Einstein condensates, neutron stars, white-dwarf stars, and early-universe cosmology.

The quantitative *analysis of the helium two-electron system proceeds schematically as follows: take the nucleus (positive charge 2e) at coordinate 0, and two electrons at* $P_1(\boldsymbol{r}_1)$ *and* $P_2(\boldsymbol{r}_2)$*. The potential function will be*

$$V(\boldsymbol{r}) = -\frac{2e^2}{r_1} - \frac{2e^2}{r_2} + \frac{e^2}{r_{12}}, \qquad r_{12} = |\boldsymbol{r}_1 - \boldsymbol{r}_2|.$$

If we write $\nabla_i^2 = \frac{\partial^2}{\partial x_i^2} + \frac{\partial^2}{\partial y_i^2} + \frac{\partial^2}{\partial z_i^2}$ $(i = 1, 2)$, *the time independent Schrödinger equation is*

$$-\frac{\hbar^2}{2m}(\nabla_1^2\psi + \nabla_2^2\psi) - 2e^2(\frac{1}{r_1} + \frac{1}{r_2})\psi + \frac{e^2}{r_{12}}\psi = E\psi,$$

with $\psi = \psi(\boldsymbol{r}_1, \boldsymbol{r}_2)$.

Since the spin does not occur explicitly in this equation[574]*, one need not carry along the spin nomenclature. In the absence of the term* $\{e^2/r_{12}\}$, *solutions are known in the form*

$$\psi = \psi_{n,\ell,m}(1)\psi_{n',\ell',m'}(2), \quad \Psi_{n,\ell,m;i} = R_{n,\ell}(r_i)Y_{m,\ell}(\theta_i, \phi_i), \quad E_n = -\frac{8}{n^2}R_\infty.$$

For $n = 1$ *we have* $E_0 = -8 \times 13.6\ eV = -108.8\ eV$. *Here* R_∞ *is the Rydberg constant.*

Consider now the case for which both *electrons are in the lowest (ground) state, corresponding to* $n = 1$, $\ell = 0$ *(this is allowed since the two spins are opposite); and* $H = e/r_{12}$ *is considered as a perturbation to the Hamiltonian. The first order correction to the energy,* W'*, is given by the* Coulomb integral

$$W' = \int_{\tau_1}\int_{\tau_2} [\psi_{1,0,0}(1)\psi_{1,0,0}(2)]^2 \frac{e^2}{r_{12}} d\tau_1 d\tau_2,$$

$$\psi_{1,0,0}(r_i) = Y_{0,0}R_{1,0}(r_i) = \frac{1}{\sqrt{\pi}}\left(\frac{2}{a_0}\right)^{3/2} e^{-\frac{2}{a_0}r_i},$$

$$a_0 = \frac{\hbar^2}{me^2} = first\ Bohr\ radius.$$

This integral represents the positive *energy contribution due to the mutual electrostatic repulsion of the two electrons.*

The exact integrals in this approximation yield $W' = \frac{5}{4}\frac{e^2}{a_0} = 34\ eV$. *Using the earlier zeroth-approximation, the energy of the ground state of helium is*

[574] We are ignoring all spin effects, for the moment, *except* that the two electrons have opposite spins, so they may be spatially *distinguishable*.

calculated to be $W = E_0 + W' = -108.8 + 34.0\ eV = -74.8\ eV$. *The experimental value is* $-78.62\ eV$.

*More refined calculations (**E.A. Hylleraas**, 1930), using a variational method, gave a theoretical value in excellent agreement with the observed value.*

If the two electrons do not have the same orbital quantum numbers, and are in a singlet or triplet spin state, the total energy perturbation integral is

$$W'_{\pm} = \frac{1}{2}\int_{\tau_1}\int_{\tau_2} [\psi_a(1)\psi_b(2) \pm \psi_b(1)\psi_a(2)]\,\frac{e^2}{r_{12}}\,[\psi_a^*(1)\psi_b^*(2) \pm \psi_b^*(1)\psi_a^*(2)]\,d\tau_1 d\tau_2$$

or $W'_{\pm} = J_{a,b} \pm K_{a,b}$, *where*

$$J_{a,b} = \int_{\tau_1}\int_{\tau_2} \frac{e^2}{r_{12}}|\psi_a(1)|^2|\psi_b(2)|^2 d\tau_1 d\tau_2;$$

$$K_{a,b} = \int_{\tau_1}\int_{\tau_2} \frac{e^2}{r_{12}}\psi_a(1)\psi_b(2)\psi_a^*(2)\psi_b^*(1) d\tau_1 d\tau_2.$$

The quantity $J_{a,b}$ *is the Coulomb integral, calculated earlier for the ground state. It is the interaction energy of the two electrons, assuming that they are distributed with charge densities* $\rho_1 = -e|\psi_a(1)|^2$, $\rho_2 = -e|\psi_b(2)|^2$, *respectively.*

The quantity $K_{a,b}$, *the exchange integral, giving the interaction energy of charges with effective (sometimes complex!) densities* $\rho'_1 = -e\psi_a(1)\psi_b^*(1)$ *and* $\rho_2 = -e\psi_b(2)\psi_a^*(2)$. *It results from quantum interference between the two exchanged configurations.*

*An important solution of the SE with a periodic potential field, was obtained in 1931 by **Kronig** and **Penney**. It could be directly applied to the electron motion in a periodic structure, such as a crystal lattice.*

Consider a periodic square-well potential in one dimension with height V_0, *width* b *and spacing* a. *If this is caused by regularly spaced atomic nuclei, we would expect to find a similar periodicity in the electron charge distribution, and thus in the wave function modulus. The wave equation will be*

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V_0)\psi = 0 \qquad (0 \le x \le b)$$

and

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}E\psi = 0 \qquad (b \le x \le a + b).$$

Let us consider a lattice with volume $L = N(a+b)$. *We look for periodic solutions* $\psi = u(x)e^{i\alpha x}$ *with* u *periodic with period* $(a+b)$ (***F. Bloch***[575], *1928). Imposing also the periodic boundary condition* $\psi(x+L) = \psi(x)$, *we find* $\alpha = \frac{2\pi K}{N(a+b)}$ $(K = 0, \pm 1, \pm 2, \ldots, \pm M)$, *with* $M = \frac{N-1}{2}$ *(N odd),* $M = \frac{N}{2}$ *(N even).*

A substitution of the 'Bloch function' into the above pair of Schrödinger equations, yields a pair of ordinary second-order differential equation for $u(x)$, *whose solutions are*

$$u_1 = e^{-i\alpha x}[A\,\mathrm{ch}\,\gamma x + B\,\mathrm{sh}\,\gamma x] \qquad (0 \le x \le b),$$

$$u_2 = e^{-i\alpha x}[C\cos\beta x + D\sin\beta x] \qquad (b \le x \le a+b),$$

subject to the boundary conditions

$$u_1(b) = u_2(b), \quad u_1'(b) = u_2'(b), \quad u_1(0) = u_2(a+b), \quad u_1'(0) = u_2'(a+b).$$

The parameters β, γ *are defined as follows:*

$$\beta = \sqrt{\frac{2mE}{\hbar^2}}, \qquad \gamma = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}.$$

These conditions impose an algebraic system of 4 equations in A, B, C, D. *For non-trivial solution this requires the determinant to vanish, leading to the characteristic equation*

$$\left(\frac{\gamma^2 - \beta^2}{2\beta\gamma}\right)\mathrm{sh}\,\gamma b \cdot \sin\beta a + \mathrm{ch}\,\gamma b \cdot \cos\beta a = \cos\alpha(a+b).$$

It is instructive to consider the limiting case $V_0 \to \infty$, $b \to 0$ *such that* $\lim \frac{\gamma^2 ab}{2} = P$ *(a finite constant); physically, this means that the lattice atoms (nuclei and their localized valence electrons) interact with a moving (conduction) electron through a periodic delta-function potential.*

The characteristic equation then reduces to

$$P\frac{\sin\beta a}{\beta a} + \cos\beta a = \cos\alpha a.$$

[575] **Bloch Theorem**: The eigenfunctions of the SE for a periodic potential are of the form $\psi_{\boldsymbol{k}}(\boldsymbol{r}) = u_{\boldsymbol{k}}(\boldsymbol{r})e^{i(\boldsymbol{k}\cdot\boldsymbol{r})}$, where $u_{\boldsymbol{k}}(\boldsymbol{r})$ has the periodicity of the crystal lattice, i.e., $\psi_{\boldsymbol{k}}(\boldsymbol{r}+\boldsymbol{\Lambda}) = e^{i(\boldsymbol{k}\cdot\boldsymbol{\Lambda})}e^{i(\boldsymbol{k}\cdot\boldsymbol{r})}u_{\boldsymbol{k}}(\boldsymbol{r}+\boldsymbol{\Lambda}) = e^{i(\boldsymbol{k}\cdot\boldsymbol{\Lambda})}\psi_{\boldsymbol{k}}(\boldsymbol{r})$.

This is an implicit equation for a set of energy levels that depend on the geometrical and physical parameters of the lattice. The possible values of βa (i.e., those solving the equation for some Bloch wavenumber α) fall into separate bands. *Thus the spectrum of β, and therefore of E, consists of a number of* continuous bands *separated by intervals in which there are* no *energy levels. Note that α is continuous, since we take the infinite volume limit $N \to \infty$.*

The number P represents the strength of the binding of the moving (conduction) electrons to the lattice. Thus, $P = 0$ leads to the solutions $\beta a = 2n\pi \pm \alpha a$ (n integer), and since α can vary continuously, this means that all continuous energies are allowed; the conduction electrons are now free.

On the other hand, as $P \to \infty$ we must have $\sin \beta a = 0$, that is, $\beta a = n\pi$ $(n = \pm 1, \pm 2, \ldots)$ and the energy spectrum is now discrete *with values $E_n = \frac{n^2 h^2}{8ma^2}$. These are exactly the levels of a particle confined to move between two impenetrable potential barriers a distance a apart. The electrons are now completely bound.*

For finite P values, only certain bands of energy are allowed to an electron, and these bands are separated by forbidden regions representing energies which the electron cannot have.

The problem of solving the SE with a periodic structure in 3 dimensions is very complicated. Nevertheless, the problem has been solved in many cases of special interest. The general result, however, is qualitatively very similar to that which has been obtained for a one-dimensional crystal.

Consider, for example, bringing together a large number of isolated atoms to form a crystal. The electrons in each isolated atom can occupy a series of discrete energy levels. As the atoms are brought closer and closer together, these discrete energy levels split up into bands of energy, so that the resulting allowed energies for that separation of the atoms which finally correspond to the atomic spacing in the crystal are qualitatively similar to the energy bands which we have seen in the one-dimensional case above.

Modern material science allows scientists to adjust and select the effective lattice spacings almost at will, enabling the creation of so-called engineering bandgaps *— with numerous applications in microelectronics, photonics, telecommunications and computers.*

History of the Theories of Light IV (1900–1925)

D. The Ultimate Synthesis: Wave-Particle Duality[576]

It follows directly from Kirchhoff's law of blackbody radiation (1859) that the spectral EM energy density can be written as $u_\nu(T) = \frac{8\pi}{c}K_\nu(T)$, *where* c *is the velocity of light in vacuum,* T *is the absolute temperature,* ν *the radiation's frequency and* $K_\nu(T)$ *is a universal function independent of the nature of the bodies in equilibrium with the radiation field. The four decades that followed Kirchhoff's discovery of the said law, witnessed many unsuccessful attempts to derive the correct form of* $K_\nu(T)$, *or what amounts to the same thing — the spectrum of blackbody radiation as a function of frequency and temperature.*

The first formula, derived by **W. Wien** *(1893) on the basis of thermodynamics is known as Wien's displacement law. According to this law* $K_\nu(T) = \nu^3 F\left(\frac{\nu}{T}\right)$. *However, thermodynamics did not specify the form of this function. Using a somewhat heuristic argument, Wien proposed (1896) the explicit form* $F = \alpha e^{-\beta\frac{\nu}{T}}$, *leading to the so-called Wien's radiation law. The values of the constants* α *and* β *that appear in this law did not follow from Wien's arguments and had to be determined empirically.*

Subsequently (1900), **Lord Rayleigh** *showed that the application of the equipartition theorem of classical statistical mechanics to electromagnetic vibrations in a cavity leads to a certain radiation law. When later corrected by* **J.H. Jeans** *(1905) for a simple error, it became known as the Rayleigh-Jeans law:* $K_\nu(T) = \left(\frac{\nu}{c}\right)^2 kT$.

We note that both Wien's radiation law and the Rayleigh-Jeans law are of the form required by Wien's displacement law. However, careful measurements carried out around the turn of the century revealed that neither of them completely agrees with the experimentally determined energy distribution in the spectrum of blackbody radiation. It turned out that Wien's radiation law was a good approximation for sufficiently high frequencies and low enough temperatures, whereas the Rayleigh-Jeans law was a good approximation for the opposite regime.

Thus, both thermodynamics and electromagnetic theory failed to predict the energy distribution in the spectrum of blackbody radiation.

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[576] Adapted from: "Einstein's Researches on the Nature of Light" by **Emil Wolf**, *Optics News* **5**, 24–39, 1979.

Just a few months after the 1900 *publication of Rayleigh's paper,* ***Max Planck*** *discovered the correct law that is in complete agreement with experiment:*

$$K_\nu(T) = \frac{h\nu^3}{c^2}(e^{h\nu/kT} - 1).$$

(He outlined the essential features of his new derivation at a meeting of the German Physical Society in Berlin on Dec. 14, 1900.)

Planck's radiation law reduces to the Wien's radiation law when $\nu/T \gg 1$ *with* $\alpha = \frac{h}{c^2}$*;* $\beta = \frac{h}{k}$*. When* $T/\nu \gg 1$ *it yields precisely the Rayleigh-Jeans law. In deriving the law, Planck found it necessary to introduce the notion of the* quantum of energy $E = h\nu$, *which represents the smallest amount of energy that an oscillator can emit or absorb at a given frequency. The need of introducing such a quantum of energy was in flat contradiction to Maxwell's electromagnetic theory and Lorentz' classical electron theory, which places no restriction on the amount of energy that an oscillator can emit or absorb.*

Even though Planck's introduction of the concept of energy quanta led eventually to one of the greatest scientific revolutions of all times, his heuristic derivation did not, at first, attract much attention. One of the first scientists who clearly recognized that Planck's discovery initiated a new era in physics was a young man who was just appointed (1902) to the rank of "Technical Expert, Third Class" at the Swiss patent office.

In 1905, *this man, as yet unknown to the scientific world, published a paper in the* 17^{th} *volume of the journal* Annalen der Physik *entitled (in translation)* "On the heuristic point of view concerning the creation and conversion of light". *In modern textbooks it is usually referred to as "Einstein's paper on the photoelectric effect". Actually it contains appreciably more. In fact, Einstein's whole discussion of the photoelectric effect covers less than 4 pages; but as in most of his writings, Einstein was able to get to the root of the problem in a few lines, with simple language that was remarkably free of complicated mathematics.*

What Einstein essentially did in this paper was to put forward a great deal of evidence that not only do the processes of emission and absorption of radiation take place in discrete amounts of energy (as appears to have been established by Planck) but that radiation itself behaves under certain circumstances as if it consisted of a collection of particles (photons)[577]*. Thus in this*

[577] Einstein's argument runs as follows: If n particles are thrown into a box of volume V, the probability that *all* n particles will end up in a subregion ΔV is $p(n) = \left(\frac{\Delta V}{V}\right)^n$. On the other hand, invoking Wien's radiation law and some general principles of thermodynamics, Einstein showed that the probability that at a given instant *all* the energy E will be concentrated in this subregion is

paper Einstein reintroduced a corpuscular theory of light — first advanced by Newton in the 17^{th} *century. This was 90 years after the corpuscular theory was completely discredited by* ***Fresnel****'s wave theory and 40 years after* ***Maxwell*** *put the wave theory of light on firm foundations.*

Another example that Einstein gave in this paper in support of his view regarding the corpuscular nature of radiation, was the photoelectric effect*. This is the phenomenon of ejection of electrons from a metal when electromagnetic radiation of short enough wavelength impinges on a metal surface. The effect was discovered by* ***Heinrich Hertz*** *(1887).*

Experiments conducted during 1899–1902 by P.E.A. Lenard (1862–1947, Germany) disclosed that the electron energy did not depend on the intensity of the light illuminating the metal surface, but did depend on the frequency of the light. The number *of ejected electrons was found to increase with the light intensity. Einstein's photoelectric equation[578]explained at once Lenard's experiments.*

Einstein's analysis showed the need for more drastic changes than those brought about by Planck's assumption of quantized energy of the emitting and absorbing oscillators. It indicated that not only do emission and absorption of energy take place in discrete energy quanta, but that the radiation field itself *behaves, in certain situations, as if it consisted of such corpuscles of energy.*

In spite of the clarity and simplicity of Einstein's arguments, his views about the particle structure of radiation were strongly opposed at that time — and for a long time afterwards — by many eminent physicists, including Planck. Undeterred by opposition, Einstein continued to explore the consequences of his corpuscular theory, and to probe more deeply into the nature of radiation.

In 1909, Einstein published a paper with the title: "On the present status of the problem of radiation", in which he showed that Planck's radiation law itself implies that the radiation field exhibits not only wave features but also corpuscular features. This result was the first clear indication of the so-called

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p (all $E \in \Delta V) = \left(\frac{\Delta V}{V}\right)^{E/h\nu}$. Comparison of the two results shows that this probability is the same as if the radiation field consisted of n particles, where $n = \frac{E}{h\nu}$, i.e., of n particles each carrying energy $h\nu$.

578 $(E_{\text{kin}})_{\text{max}} = h\nu - W$, where W is the work required to remove the electron from the metal. In 1905, when Einstein put forward his equation, quantitative studies of the photoelectric effect were in their infancy. It took nearly a decade of difficult experimentation before Einstein's equation could be tested. It was largely confirmed by the work of **R.A. Millikan**, who at first completely disbelieved Einstein theory.

wave-particle duality, that many years later became an accepted feature of modern quantum physics.

Bohr's quantum theory of the hydrogen atom (1913) did not give any indication of the laws governing the *amplitudes* of the atomic spectral lines. Moreover, Bohr's frequency condition $\Delta E = h\nu$ was simply *assumed*. Einstein's epochal paper "On the quantum theory of radiation" (1917) provided the first real insight into the laws that govern the transitions from one allowed state of the atom to another. Einstein showed therein that Planck's law follows directly from the interaction between radiation and matter. The probabilities that Einstein assumed for each of the elementary processes taking part in the interaction are examples of what is known today as *transition probabilities* between states.

In the above paper, Einstein also considered the question of *momentum transfer* between gas molecules and the radiation field. He showed that, when a molecule absorbs or emits a quantum of energy $h\nu$ under the influence of external radiation from a *definite direction*, momentum of magnitude $\frac{h\nu}{c}$ is transferred to the molecule; and that the change in the momentum of the molecule is in the direction of the incident radiation if the energy is absorbed, or in the opposite direction if the energy is emitted. More surprisingly, Einstein showed that if an energy quantum is emitted in the absence of any external influence (spontaneous emission), the momentum transfer is also a *directed process*.

In Einstein's words (1917):

"There is no radiation in spherical waves. In a spontaneous emission process the molecule suffers a recoil of magnitude $\frac{h\nu}{c}$ in a direction that in the present state of the theory is determined only by 'chance'... These properties of elementary processes... make it seem practically unavoidable that one must construct an essentially quantum mechanical theory of radiation".

Einstein's conclusion that a quantum of energy $h\nu$ carries a momentum whose magnitude is $\frac{h\nu}{c}$ and has a definite direction, was verified by **H.A. Compton** (1923) in his experiments on scattering of X-rays.

Successful as Einstein's notion of radiation field quanta was in elucidating various phenomena involving the interaction of light and matter, many puzzles surrounded it. All the derivations of Planck's radiation law, including Einstein's 1917 derivation, appealed at some point to classical electromagnetic theory, which is in direct contradiction with the quantum features of the radiation field. Being well aware of these difficulties (quantum mechanics was only formulated about 8 years later), Einstein took a few important steps in this direction that have a bearing on the question of the nature of light, particularly with regard to its statistical properties.

S. Bose *(1924) published a manuscript in which he essentially treated the light quanta as particles of a gas, with the difference that those particles that belong to the same elementary cell of phase space, of volume h^3, are intrinsically indistinguishable. This assumed property of the quanta led to a statistical procedure that differs from that of classical statistic mechanics, and indeed provided a complete derivation of Planck's radiation law —, without any appeal to classical electromagnetic theory.*

Einstein (1925) applied Bose's method not to a gas of light quanta, but to a real gas, consisting of monoatomic molecules, thus establishing the quantum theory of an ideal gas[579]*.*

Bose's and Einstein's papers are the foundation of the Bose-Einstein statistics. It applies to photons and to many other elementary particles and antiparticles which were as yet unknown in 1925.

Between the appearance of Einstein's first and second papers on the quantum theory of an ideal gas (a period spanning less than five months), ***Louis de Broglie*** *put forward his theory of matter waves. In his second paper, Einstein outlined the connection that he believed to exist between de Broglie's hypothesis of matter waves and his own investigations on the quantum theory of the ideal gas. These remarks have stimulated* ***Ervin Schrödinger*** *only a few months later to develop one form of modern quantum mechanics — namely wave mechanics.*

Thus, the structure that Einstein built with his 1905 "photoelectric paper", his 1909 *paper containing the first clear evidence for the wave-particle duality, his* 1917 *paper on the elementary processes of interaction between molecules and radiation, and his* 1924 *and* 1925 *papers on the quantum theory of an ideal gas — not only elucidate the nature of light and radiation in general, but also proved to be of fundamental importance to the gestation and final formulation of wave mechanics.*

[579] Einstein showed that the variance of the energy fluctuations of an ideal gas is expressible as the sum of two terms; the first term can be attributed to classical particles and can be understood on the basis of the *Maxwell-Boltzmann* statistics of noninteracting molecules. The second term, which is analogous to the contribution from wave interference in the radiation problem, cannot be understood from the classical particle theory.

1925–1956 CE **Marietta Blau**[580] (1894–1970, Austria and USA). Experimental nuclear physicist. Pioneer in the field of nuclear emulsions (i.e. tracking nuclear particles in photographic emulsions). First to use nuclear emulsions to detect neutrons by observing recoil protons in the emulsions (1925). First to *identify* (1925) proton tracks resulting from either the elastic scattering of α-particles by protons in the hydrogen in the emulsion or the reaction of α's with the nuclei of the emulsion. Also determined for the first time the spectrum of neutrons resulting from specific nuclear reaction processes.

Her life and career were tragically disrupted by the Nazi Germans and Austrians in 1938. She maintained, however, a life line to the world physics through her simple portable technique she subsequently created. **Einstein** and **Born** praised her work highly, and Erwin **Schrödinger** nominated her for the Nobel Physics Prize, to no avail — others received the prize she so rightly deserved.

Blau was born in Vienna and received her Ph.D. (1919) from the University of that city. Between 1923 and 1938 Blau's investigations were centered at the Institut für Radiumforschung in Vienna and at the Second Physical Institute. But she was never paid and never promoted because, as she was told by a professor: "...you are a woman and a Jew, and the two together are simply too much".

On Friday, 11 March 1938, the Germans entered Vienna. Blau fled first to Oslo, but could not find a permanent employment there. Desperate to rescue her mother from Vienna she began exploring the possibility of getting to Mexico. With the recommendation of **Einstein**, Blau was appointed professor at the Technical University in Mexico City. She took a German ship to America; en route the Gestapo confiscated all her scientific papers, including work on particle tracks in nuclear emulsions.

In May 1944 Blau moved to New York City and then moved (1950) to the Brookhaven National Laboratory. Although this place initially seemed congenial, even that resting spot did not last: personal friction with some staff members, coupled with dire financial difficulties and health problems, led her back to Vienna (1960).

Marietta Blau had moved some ten times, lost all her scientific papers and notebooks and still had no clear path to a permanent position. Several physicists tried to gather funds for her. Erwin Schrödinger put her up for the Schrödinger Prize (which she won) and Otto Frish tried to convince the big film companies to grant her a sinecure in recognition of the industry of nuclear emulsions that she helped create. All they were willing to contribute

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[580] Peter L. Galison: "Marietta Blau: *Between Nazis and Nuclei*" Physics Today, November 1997, pp 42–48

were £100 per annum which she declined despite her poverty (1964). She died five years later poor and virtually unknown.

1925–1958 CE **Raymond Dart** (1893–1988, South Africa). Anatomist and paleoanthropologist. Discovered and pioneered the pre-human evolutionary stage formed by the *australopithecines*. This species, found in Southern Africa and recognized as *Australopithecus africanus*, spans a period from 3 to 2.5 million years ago.

At the start of the 20th century the search for the supposed "missing link" between apes and human focused on the Far East and Europe, where finds such as "*Pithecanthropus erectus*" ("Java man") had been made. This is now identified as *Homo erectus*, some way along the human line.

In 1925, the anatomist Raymond Dart dared to identify a new kind of ape-man on the basis of the isolated skull of a young primate found during mining work at *Taung*, near Kimberley, South Africa, the previous year. He knew he was challenging a powerful, if poorly founded assumption that human first evolved in Europe, and that their big brains developed before other human features.

According to Dart, the shape of the skull and probable shape of the brain, together with the design of the teeth and jaws and the upright angle of the skull where it would have topped the (vanished) spinal column, all announced a recruit that was more human than any known ape. Dart labored for decades before he and another researcher **Robert Broom** (1866–1951, South Africa) eventually began to establish a new genus with the help of more skulls, jaws, teeth, and post-craniel bones from additional cave sites.

Dart was born in Brisbane, Queensland, Australia. He graduated from the universities of Brisbane and Sydney, and after holding a number of posts and fellowships in the UK and the USA, was appointed to the chair of anatomy at Witwatersrand University, Johannesburg (1923).

It is now well established that the australopithecines represent an early stage in the evolutionary differentiation of man and that these fossils demonstrate that one of the first features to be evolved was terrestrial bipedalism. Only later came the substantial brain expansion and jaw reduction which characterize modern man.

The significance of bipedalism was that it completely emancipated the arms and hands from support of and locomotive function. Whether the South African australopithecines actually made tools is uncertain.

After an initial burst of praise, the scientific establishment in Britain rejected the Taung baby[581] as an ape. Virtually the only supporter of Dart was Robert Broom. Dart did travel to London (1930) to try and win support for his Taung baby, but his find was overshadowed by the recently discovered *Peking Man* skull.

Dart gave up fossil hunting for many years, concentrating instead on his work at the Witwatersrand anatomy department. In the late 1930s and early 1940s Broom found many more australopithecines fossils in South Africa, and in the late 1940's Dart's position was vindicated when many scientists finally accepted that australopithecines were hominids. Once Piltdown Man was out of the picture, the center of gravity of physical anthropology moved to Africa, where it belonged.

1925–1965 CE **Jan Hendrik Oort** (1900–1992, Holland). Astronomer. Elucidated the kinematics and dynamics of the Milky Way system through his pioneering work on the *differential galactic rotation* (1925–1927) and the 21 cm radiation associated with the hydrogen flow along the *spiral arms* of the galaxy (1951). Established (1950) the now accepted hypothesis about the *source* (if not the ultimate origin) *of comets*; He concluded that the observed distribution of cometary orbits could be explained by supposing a cloud of 1.9×10^{11} comets surrounds the sun as far as 50,000 to 150,000 AU and affected from time to time by the perturbing effects of passing stars. This reservoir of cometary nuclei is called the *Oort cloud*[582].

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581 At the time, *Piltdown Man* was widely accepted as a human ancestor, and *Taung*, with its apelike skull and human-like teeth, seemed difficult to reconcile with Piltdown's human skull and apelike jaw.

Piltdown Man: name given to fossil remains found (1908) at Piltdown, England, thought to be human and 200,000 to 1 million years old. In 1950, fluorine tests and X-ray analysis proved that Piltdown Man was a forgery. The fraud endured for more than 40 years in part because of the desire of European scientists to find evidence for the European origin of man.

582 Astronomers discover *long-period comets* (orbital periods of 100,000 years to 1 million years) at a rate of about one per month. These comets travel along extremely elongated orbits and consequently spend most of their time at distances of ca 4000 to 20,000 AU from the sun — comprising the *inner* Oort cloud; the tenuous, weakly bound *outer* cloud extends out to ca 50,000 AU from the sun, i.e about $\frac{1}{5}$ of the way to the nearest star. Thus it is reasonable to suppose that there is an enormous population of comets on the outskirts of the solar system. Only with such a large reserve of cometary nuclei can we understand why we see so many long-period comets even though each one takes up to 1 million years to travel once around its orbit.

Oort determined that stellar perturbations of cometary orbits over the lifetime

Oort was born in Franeker, Friesland. He completed his training at Leyden Observatory, where he remained for the rest of his career, becoming director in 1945.

In 1932 he estimated the mass density of the galactic disc in the solar neighborhood, known as the *Oort limit*[583]. By measuring the expansion rate of the dissipating material from the Crab Nebula he confirmed that it was the remnant of the exploding star seen as the 1054 *CE* supernova.

1925–1966 CE **Andrei Nikolaevich Tikhonov** (1906–1993, Russia). Mathematician. Obtained fundamental results in a wide range of modern mathematical fields: topology and functional analysis (1925–1935), ordinary and partial differential equations and their application to problems in mathematical physics (1948–1960), and computational mathematics (1960–1966).

Tikhonov was born in Gzhatska, Smolensk region, Russia. He graduated from Moscow University (1927) and became a professor there (1936).

His initial works (1925–1930) are related to the pioneering results of **Uryson** on the condition for the metrization of a topological space. He

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of the solar system would completely rearrange the orbital orientations into a random distribution, even if their initial orbits began with a preference for the ecliptic plane. Thus the wide variation of orbital inclinations for observed comets with nearly parabolic orbits was explained in a natural way. Oort then asked how many comets would be required in the cloud to explain the observed flux of one dynamically new comet passing each year inside a sun-centered sphere with a radius of 1.5 AU. Assuming only the effects of stellar perturbations, he determined some 1.9×10^{11} comets with a total mass of about 10^{27} grams, of order of the mass of the earth! Much of the work on the source of comets since 1950 has supported Oort's basic ideas concerning the provenance of long-period comets.

583 It is possible to estimate the *mass density* of the galactic disc in the solar neighborhood, using *Jeans' equations* in cylindrical coordinates R, z (which in turn are derived from the collisionless Boltzmann transport equation of statistical mechanics), in conjunction with the *Poisson equation*. Oort's analysis yielded the relation

$$\frac{\partial}{\partial z}\left[\frac{1}{\nu}\frac{\partial}{\partial R}(\nu\overline{V_z^2})\right] = -4\pi G\rho,$$

where $\nu = \nu_0 e^{-2.4R/R_0}$ is the *spatial density* of stars, R_0 the sun's distance from the galaxy's rotational axis, and $\overline{V_z^2}$ is the mean-square vertical velocity. He thus concluded (1932) that

$$\rho_0 = \rho(R_0, z = 0) \simeq 0.15\ M_\odot pc^{-3},$$

which is known as the *Oort limit*.

then defined the product-space of topological spaces and went on to prove that the product-space of any set of compact topological spaces is compact. Tikhonov's '*embedding theorem*' concerns the mapping of a topological space into an infinite-dimensional space[584]. He later (1935) found conditions for a topological space to be metrizable. His work led from topology to functional analysis with his fixed-point theorem for continuous maps (1935). These results are of importance in both topology and functional analysis and were applied by him to solve problems of mathematical physics. In the 1960's he shifted his attention to numerical methods for the solution of nonlinear ill-posed problems.

1925–1969 CE **Salomon Bochner** (1899–1982, Germany and USA). Mathematician. Made important contributions to *harmonic analysis*, *probability theory*, *algebraic geometry* and *topology of harmonic vector fields*. His research profoundly influenced development of a wide area of analysis in the second half of the 20^{th} century such as:

- Classification of compact manifolds.
- Higher-dimensional geometry in String Theory.
- Infinite-dimensional representation of non-compact semi-simple Lee groups.
- Complex analysis in several variables.

He also presaged the Zorn Lemma (1933) already in 1928.

Bochner was born into an orthodox Jewish family in Podgorze (near Krakow), Poland. In 1915, the family moved to Germany, seeking greater security. After graduating from a Berlin Gymnasium, Bochner entered the University of Berlin (1918) and received his doctorate there in mathematics (1921). He then worked with Harald Bohr in Copenhagen, G.H. Hardy in Oxford, and J.E. Littlewood in Cambridge. In 1933 he came to the United States and spent the next 35 years at Princeton University. During 1968–1975 he was the chairman of the Mathematical Department of Rice University, Houston, Texas.

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[584] Infinite dimensional spaces are relevant to mathematical physics, with applications to differential equations and to functional equations via *fixed-point theorems* extending those long known for finite–dimensional spaces. In 1930 **Schauder** proved a fixed-point theorem for continuous mapping of a closed convex subset of a Banach space onto a countably-compact subset of itself. In 1935 Tikhonov proved a similar theorem. In 1922, **Birkhoff** and **Kellos** extended Brouwer's fixed point theorem to functional spaces.

During 1966–1969, Bochner contributed to the *history of Science* through his books: *The Role of Mathematics in the Rise of Science* (1966) and *Eclosion and Synthesis: Perspective in the History of Knowledge (1969).* Therein, he attempted to differentiate periods in science according to their peculiar thought-patterns.

1926 CE **Vladimir Ivanovich Vernadsky** (1863–1945, Russia). Geochemist and mineralogist. Founder of modern *biogeochemistry*. In his book *La Biosphere* he developed the ideas of **Eduard Suess** (1875) and **Jean Baptiste Lamarck** to those we accept today.

Moscow University (1898–1911); State Radium Institute, Leningrad (1926–1938); Founder and director of biogeochemical laboratory of Leningrad Academy of Sciences.

Vernadsky was a Russian liberal who grew up in the 19^{th} century. Accepting the Russian Revolution, he did much of his work after 1917, although his numerous philosophical references were far from Marxist.

1926 CE **Carl Eckart** (1902–1971, U.S.A.). Physicist. Establishes independently that **Schrödinger**'s wave mechanics is mathematically equivalent to the matrix mechanics of **W. Heisenberg**, **M. Born** and **P. Jordan**.

1926–1928 CE **Kalman Tihanyi** (1897–1947, Hungary). Electrical engineer and inventor. Electronic television pioneer. Patented his fully electronic television system in 1926.

The idea to utilize the cathode ray tube as image converter on the side of transmission surfaced in 1908, and was described in detail by **A.A. Campbell Swinton** in a paper he published in 1911. In the 1920's, variations on the Campbell Swinton design were proposed by Zworykin. All were electrical analogues of mechanical scanners, in that electron emission would occur only during the momentary contact by the scanning ray of each elemental area of the photocell. In 1925 a demonstration by Zworykin with his system produced discouraging results.

The decisive solution — television operating on the basis of continuous electron emission with accumulation and storage of released secondary electrons during the entire scansion cycle — was first described by Kalman Tihanyi in 1926, with further refined versions patented by him in 1928.

Tihanyi was born in Uzbeg and studied electrical engineering and physics in Budapest.

1926 CE **Robert Hutchings Goddard** (1882–1945, U.S.A.). Among the pioneers of modern rocketry and space flight. Experimented during 1909–1945 with solid and liquid propellant rockets which led to the development of intercontinental missiles, earth-orbiting satellites and the exploration of space. He was first to develop rockets equipped with propellant pumps, gyro-controls, and other instrumentation. Goddard foresaw many of the space flight ideas that later became reality.

In his treatise "*A Method of Reaching Extreme Altitudes*" (1919) he proposed trying to reach the moon by rocket. In 1926 he successfully launched a liquid fuel rocket. Ridiculed[585] as a "moon man", Goddard lived to see his work win recognition for putting man on the threshold of space.

1926–1927 CE *The WKBJ method*[586] for the approximate treatment of

585 On January 30, 1920, an anonymous editorial-page writer from the *New York Times* mocked **Robert Goddard** for suggesting that a rocket could someday reach the moon: "*That Professor Goddard, with his 'chair' in Clark College, and the countenancing of the Smithsonian Institution, does not know the relation of action to reaction and of the need to have something better than a vacuum against which to react — to say that would be an absurd. Of course, he only seems to lack the knowledge ladled out in high schools*".

The *Times* went on to cite "the same mistake" in Jules Verne's description of firing a rocket to adjust the course of a manned moonship: "*The Frenchman, having got his travelers to the moon in a desperate fix of riding a satellite of a satellite, saved them from circling it forever by means of an explosion, rocket fashion, where it could not have had in the slightest degree the effect of releasing them from their dreadful slavery*".

Such ignorant criticisms of Goddard's work scared off many supporters for ten years, until Charles Lindbergh courageously laid his own prestige on the line to boost Goddard's.

Almost 50 years later, after two manned lunar expeditions had already used a pure Vernesian rocket maneuver to escape from lunar orbit and return to earth, the Apollo 11 moon-landing expedition was launched. In a special section of the newspaper, the *Times* printed a small box titled: "*A Correction*". In it, the newspaper's original Goddard criticism was quoted and retracted: "*Further investigation and experimentation have confirmed Isaac Newton in the* 17^{th} *century and it is now definitely established that a rocket can function in a vacuum as well as in the atmosphere. The Times regrets the error*".

586 The method treats the asymptotic solutions of the equation

$$\frac{d^2W}{dz^2} + \nu^2 Q(z,\nu)W = 0,$$

where the parameter ν is taken to be large and positive and where $Q(z,\nu)$ tends to a limit as $\nu \to \infty$ for fixed z. A change of variables from (z,W) to (u,X) through the relations $z = z(u)$, $W = X\left(\frac{dz}{du}\right)^{1/2}$ trans-the Schrödinger wave equation is discovered independently by **Léon Brillouin** (1889–1969, France), **Hendrik Anthony Kramers** (1894–1952, Holland) and **Gregor Wentzel** (1898–1978, Germany and U.S.A.). This general mathematical technique had been used earlier by **Liouville** (1837), **Rayleigh** (1912) and **Harold Jeffreys** (1891–1989, England) in 1923.

1926–1929 CE **John Logie Baird** (1888–1946, Scotland). Engineer and inventor. Pioneer of television. A self-taught inventor, who matched inventive wits against the accumulated wisdom and vast resources of great laboratory physicists and engineers. Invented the *noctovision* (1926), a television system that uses infrared rays to take pictures in the dark. In the same year he gave the first public demonstration of television broadcasting in England, using a high-speed *electro-mechanical* scanning system.

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forms the original equation into $\frac{d^2X}{du^2} + \left[\nu^2 Q z'^2\right] X = \left[\frac{3}{4}\left(\frac{z''}{z'}\right)^2 - \frac{1}{2}\frac{z'''}{z'}\right] X$, $\left(' = \frac{d}{du}\right)$. One then chooses $z(u)$ such that $\nu^2 Q \left(\frac{dz}{du}\right)^2 = -u$, thereby yielding $u = (\pm i)^{2/3}\nu^{2/3}\Phi(z)$ where $\Phi(z) = \left[\frac{3}{2}\int_{z_0}^{z}\sqrt{Q}dz\right]^{2/3}$. The differential equation thus assumes the form $\frac{d^2X}{du^2} - uX = \nu^{-4/3} r_1(u) X$ with $r_1(u) = -\frac{1}{2}\nu^{4/3}\{z,u\}$ and $\{z,u\} = \frac{z'''}{z'} - \frac{3}{2}\left(\frac{z''}{z'}\right)^2$ is the *Schwarzian derivative* of z with respect to u. Since $\frac{dz}{du} \propto \nu^{-2/3}$, $\frac{d^2z}{du^2} \propto \nu^{-4/3}$, $\frac{d^3z}{du^3} \propto \nu^{-2}$, it follows that $r_1(u)$ tends to a ν-independent limit. Neglecting the r.h.s. of the last differential equation, the solutions of this equation tend to those of the *Airy equation* $\frac{d^2X}{du^2} - uX = 0$. Its general solution is known to be (in the original variables)

$$W \approx \nu^{-1/3}\left[\frac{\Phi}{Q}\right]^{1/4}\left[c_1 Ai\left\{e^{-2\pi i/3}\nu^{2/3}\Phi\right\} + c_2 Ai\left\{-e^{-2\pi i/3}\nu^{2/3}\Phi\right\}\right]$$

where c_1 and c_2 are constants and $Ai(z) = \frac{1}{\pi}\int_0^\infty \cos\left(zs + \frac{1}{3}s^3\right)ds$. Away from *turning points* [roots of $Q = 0$], and in regions where $Q > 0$, the above solution simplifies to the WKBJ modified plane wave

$$W \approx \nu^{-1/2}Q^{-1/4}\left[c_1 e^{i\nu\int_{z_0}^{z}\sqrt{Q}dz} + c_2 e^{-i\nu\int_{z_0}^{z}\sqrt{Q}dz}\right].$$

In optics, this is related to the ray approximation for a high-frequency wave, propagating in an inhomogeneous medium; whereas in quantum mechanics, it describes the semiclassical motion of a particle when $\hbar$ is small relative to the classical scales set by the energy, potential, mass etc.

Baird's electro-mechanical system consisted of a light sensitive camera behind a rotating disc. It delivered a crude picture consisting of 30 lines at 12 frames per second to a television receiver that displayed an uneven and tiny orange and black image. By 1932 Baird had developed the first commercially viable television system and had sold 10,000 sets. He was unable to obtain a patent protection to his device, because it contained the already patented Nipkow disc. He transmitted a television image between London and Glasgow by telephone lines in 1927, and between London and New York by radio in 1928. He also experimented with color and three-dimensional television.

The British Broadcasting Corporation adopted Baird's system for their first television programme in 1929, but in 1937 it was abandoned in favor of an *electronic* scanning system.

Baird was born in Helensburgh, Scotland and studied at Glasgow University. He came to Trinidad (1919), escaping from the harsh Scottish climate that was plaguing him with colds and fevers and bronchial infections. During his nine months in Santa Cruz he made his basic breakthrough in television.

Back in Britain, he made his first public demonstration in 1925. He developed a system for televising onto large cinema screens, and covered the 1931 and 1932 Derby live. Although by the mid 1930's mechanical television was being overshadowed by the all-electronic technology developed in the United States, what really interested governments all over the world was not the entertainment or commercial value of television, but its military applications.

Thus, Baird was secretly working on new uses for television which had profound impact on WWII[587].

Baird probably had a lot to do with the chain of radar stations built along Britain's vulnerable coasts in preparation for war as early as 1935, with radar-controlled anti-aircraft guns that appeared around London in 1940. He also contributed to the system which guided high-flying bombers to their targets while flying above cloud and with high-speed wartime signaling techniques which used television for *facsimile transmission of maps* and written material without interception.

In 1943, when the battle for the Atlantic was at its height, Baird was seen in Port of Spain, Trinidad, for several weeks. Trinidad was an assembly point for trans-Atlantic oil convoys, and the battle against the German submarines which preyed on them was being won, largely through improved methods of detection and radar-surveillance. He was now being deployed in defense of

587 Some even claim the 1936 Crystal Palace fire, where Baird's research and production was based, was the work of Nazi Germany, intended to destroy Baird's work.

Trinidad's lumbering convoys and the Western World's struggle against Nazi Germany.

1926–1929 CE **Oscar Klein** (1894–1977, Sweden and USA). Theoretical physicist. His work had broad impact on 20th century physics and his name is associated with several major accomplishments: *Klein-Gordon Equation* (1926), *Kaluza-Klein Theory* (1926), *Jordan-Klein matrices* (1927), *Klein-Nishina scattering* (1929). Contributed also to statistical mechanics, thermodynamics, superconductivity and cosmology.

Klein was born in Mörby, Sweden, the youngest son of Sweden's chief Rabbi, Gottlieb Klein, who arrived there from Germany. He was educated at the Universities of Stockholm (Ph.D., 1921) and Copenhagen. Collaborated from an early age with **Arrhenius**, **Bohr** and **Kramers**. Held professorial positions at the Universities of Ann Arbor, Michigan (1923–1925), Copenhagen (1927) and Stockholm (1930–1962).

Let us summarize briefly the achievements of Klein:

- In his work with **Y. Nishina** on the Compton scattering of photons by free electrons in the framework of quantum mechanics, he derived the differential and total cross-sections. Through this work he was able to convince physicists of the soundness of Dirac's relativistic wave equation.

- His work with **Pascual Jordan** (1902–1980, Germany) on the *second quantization* problem in quantum mechanics demonstrated the close connection between quantum fields and quantum statistics. It was shown that second quantization guarantees that photons obey Bose-Einstein statistics, and that one can quantize the *wave function* of the non-relativistic Schrödinger equation.

- With **Walter Gordon** (1893–1939) introduced a Lorentz-covariant relativistic wave-equation for the electron, known as the *Klein-Gordon equation*

  $$\frac{\partial^2 u}{\partial t^2} = c^2\nabla^2 u - \frac{m_e^2 c^4}{\hbar^2} u$$

  with m_e the electron rest–mass. This marks the beginning of *relativistic quantum mechanics*. Oddly enough, Schrödinger himself privately developed the equation slightly earlier (1926) – even including minimal coupling to EM fields – but he never published his results. The equation turned out to be *fundamental* only for *spinless* bosonic particles.

- In 1938, suggested that a spin-1 particle mediates beta-decay and plays a role in weak interactions in a similar manner to a photon in electromagnetic theory. This idea was resurrected by J. Schwinger only in 1957, and a decade later it was incorporated into the unified ("electroweak") theory of weak–nuclear and electromagnetic forces.

- Introduced major modifications to the 5-dimensional Kaluza unification scheme of Maxwell's electromagnetic theory with Einstein's GTR. Kaluza's theory suffered from two defects: first, classical behavior of the fields was assumed without considering the effects of quantum mechanics. Second, the theory did not describe the nature of the 5th dimension.

  To rectify this, Klein assumed that the extra fifth dimension was curled up into a circle that was of the order of *Planck's length*, 10^{-33} cm. This extra dimension, albeit curled up, was still *Euclidean* in nature. Klein also assumed it to be periodic with a period $\tau = \frac{\hbar c}{e}\sqrt{2\kappa}$, with e the electron's charge (in absolute value) and κ Einstein's gravitational constant in 4-dimensional spacetime, $\frac{8\pi G}{c^4}$. The extra dimension was not observable but was a physical quantity conjugate to the electron's charge. By this Klein attempted to explain the atomicity of electricity as a quantum law.

The combined theory is known as the *Kaluza-Klein* Theory[588]. Over the 80 year period since its inception, physicists have had difficulty correlating the Kaluza-Klein Theory or the now-popular higher dimensional extensions of it, to physical reality. However, the last decades of the 20th century witnessed a revival of the theory and its amalgamation into various unification programs. In particular, those involving *superstrings* and supergravity could e.g. have 6 or 7 (rather than 1) curled-up dimensions. But it now seems doubtful that any given scheme for curling up the "small" spatial dimensions can be shown to be unique, stable, or dynamically feasible.

A Centennial Nobel Symposium was held in September 19–21, 1994 in honor of Oscar Klein.

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[588] A 5-dimensional unification scheme was apparently discovered independently by two other physicists at about the same time. The first was **Heinrich Mandel** (Germany) and the second was **Vladimir Alexandrovich Fock** (1898–1974, Russia). Fock, a theoretical physicist, was born in St. Petersburg, graduated from its University in 1922, and became a professor there (1932).

Applied Group Theory[589]

In the 19^{th} *century the theory of groups arose primarily as the theory of transformation groups. However, in the course of time it became more and more clear that the most significant of the results obtained depend only on the fact that transformations can be multiplied and that this operation has a number of characteristic properties. On the other hand, objects were found having nothing to do with transformations, but to which the main theorems of the theory of transformation groups could be applied. As a result, the concept of a group was applied not only to systems of transformations, but also to systems of arbitrary elements.*

For the physicist, group theory is an extraordinarily useful tool for formalizing semi-intuitive concepts and for exploiting symmetries. Group theory became a useful tool for the development of crystallography, solid state physics, atomic, nuclear and particle physics and even cosmology.

The extension of group theory to continuous groups (groups with an infinite number of elements, where a general element depends on one or more parameters which vary continuously), has led to applications to special relativity, quantum theory and the particles, fields and high energy physics.

As knowledge of our physical world expanded explosively in the first third of the 20^{th} *century, new mathematical structures were revealed which were already anticipated by mathematicians of the* 19^{th} *century. Chief among them was* ***M.S. Lie****.*

The importance of Lie groups (and their associated algebras) in modern physics *(especially quantum field theory, STR and GTR) stems from the demand for* covariant[590] *physical laws* under space and time translations, ro-

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589 To dig deeper, see:

- Bishop, D.M., *Group Theory and Chemistry*, Dover Publications: New York, 1993, 300 pp.
- Tinkham, M., *Group Theory and Quantum Mechanics*, McGraw-Hill, 1964, 340 pp.
- Lyubarskii, G.Ya. *The Application of Group Theory in Physics*, Pergamon Press, 1960, 381 pp.

590 Covariant equations have the same *form* in different coordinate systems so that there is *no preferred fiducial reference system* w.r.t. the given group of transformations.

tations in real 3-dimensional space, Lorentz transformations and rotations in various abstract spaces.

The demand for covariance under translation is based on the homogeneity *of space and time. Covariance under rotation is an assertion of the* isotropy *of space. The requirement of Lorentz covariance is based on the acceptance of special relativity. Together, the above three transformations form the* inhomogeneous Lorentz group, *also known as the* Poincaré group.

From the point of view of pure mathematics, *a number of results were obtained for Lie algebras at the turn of the* 20^{th} *century that are similar to the fundamental results on associative algebras, although the proofs and statements are here more complicated. This was mainly due to the efforts of* ***Killing*** *and* **Cartan**, *who classified all simple Lie algebras over the field of real and complex numbers.*

In the early 1930*'s the theory of representations of Lie algebras by matrices was constructed, principally by* **Cartan** *and* **Weyl**, *and proved to be a remarkable tool for the solution of many problems.*

A Lie group that plays an important role in relativity theory, electrodynamics and relativistic quantum mechanics is the Lorentz group, *the "one-dimensional boosts" subgroups of which depend on a single real parameter. It is represented by the one-dimensional* (x, t) *Lorentz transformation (LT)*

$$\begin{bmatrix} x' \\ t' \end{bmatrix} = \gamma \begin{bmatrix} 1 & -v \\ \frac{-v}{c^2} & 1 \end{bmatrix} \begin{bmatrix} x \\ t \end{bmatrix}, \qquad \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}.$$

Indeed, the set of all matrices

$$A(v) = \begin{bmatrix} 1 & -v \\ -\frac{v}{c^2} & 1 \end{bmatrix}$$

form a continuous group with $A(v_1)A(v_2) = A(v_3)$ *where* $v_3 = \frac{v_1+v_2}{1+\frac{v_1 v_2}{c^2}}$. *The inverse transformation is obtained by replacing* v *by* $-v$.

Upon the substitution $\gamma = \cosh\theta, \quad \gamma\frac{v}{c} = \sinh\theta, \quad x_4 = ict,$ *the LT can be recast in the form of an orthogonal transformation*

$$\begin{bmatrix} x_1' \\ x_4' \end{bmatrix} = \begin{bmatrix} \cosh\theta & i\sinh\theta \\ -i\sinh\theta & \cosh\theta \end{bmatrix} \begin{bmatrix} x_1 \\ x_4 \end{bmatrix}.$$

A LT for which the determinant of the transformation is equal to $+1$ *is said to be* proper *(choice* $\gamma > 0$*). This condition is equivalent to requiring that*

$t' \to \infty$ as $t \to \infty$ for fixed x_1, which corresponds to the invariance of the direction in which time is measured. Such a LT is said to be orthochronous[591].

Since

$$\begin{bmatrix} \cosh\theta & i\sinh\theta \\ -i\sinh\theta & \cosh\theta \end{bmatrix} \equiv I\cosh\theta - \sigma_2\sinh\theta = e^{-\theta\sigma_2},$$

where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$, *we can say that σ_2 generates representation of this Lorentz transformation.*

Returning to the case of three space dimensions, when the relative velocity $\boldsymbol{v}$ is not parallel to any coordinate axis, it can be shown that the generator is the 4×4 matrix

$$\sigma = \begin{bmatrix} 0 & 0 & 0 & -i\lambda \\ 0 & 0 & 0 & -i\mu \\ 0 & 0 & 0 & -i\nu \\ i\lambda & i\mu & i\nu & 0 \end{bmatrix},$$

where (λ, μ, ν) are the direction cosines of $\boldsymbol{v}$.

The transformation itself is given by the 4×4 matrix

$$L(\boldsymbol{v}) = e^{-\theta\sigma} = I - \sigma\sinh\theta + \sigma^2(\cosh\theta - 1).$$

In this general case, however, the product of two Lorentz transformation matrices $L(\boldsymbol{v}_1)$ and $L(\boldsymbol{v}_2)$ yields a third Lorentz transformation $L(\boldsymbol{v}_3)$ only if the two velocities $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ are parallel. If $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ are not parallel we find that $L(\boldsymbol{v}_3) = RL(\boldsymbol{v}_2)L(\boldsymbol{v}_1)$, where R is a 3×3 space rotation matrix (Wigner's rotation).

The matrix L has the property that $LGL^T = G$, where

$$G = \begin{bmatrix} 1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & -1 \end{bmatrix}$$

is the metric matrix of Minkowski space (in its imaginary – 4^{th} component representation).

591 The basic relation $x^2 - c^2t^2 = x'^2 - c^2t'^2$ can accommodate both $\gamma > 0$ and $\gamma < 0$.

Together with the spatial-rotations Lie group SO(3), the set of orthochronous boosts $L(\boldsymbol{v})$ forms a 6 parameter Lie group of linear transformation in $3+1$ dimensions, $SO(3,1)$ the Lorentz group.

Another important Lie group applicable to relativistic spinors is $Sp(2n)$, the symplectic group of all matrices A of dimension $2n \times 2n$ which satisfy the equation $AGA^T = G$, where G is a given skew-symmetric matrix and is usually taken as

$$G = \begin{bmatrix} 0 & 1 & & & & & \\ -1 & 0 & & & & & \\ & & 0 & 1 & & & \\ & & -1 & 0 & & & \\ & & & & \ddots & & \\ & & & & & 0 & 1 \\ & & & & & -1 & 0 \end{bmatrix}.$$

1926–1932 CE **Werner Karl Heisenberg** (1901–1976, Germany). Distinguished theoretical physicist. One of the major figures in the transition from classical to quantum physics. Together with **M. Born** and **P. Jordan** developed *Quantum Matrix Mechanics* based on non-commutative matrix algebras[592], the germ of which had existed in the quaternions of **Hamilton**, 83 years before.

In 1927 he published his *Uncertainty principle*, for which he is best known; he showed that quantum mechanics embodies a very general principle through which the least possible errors in measurement of position are necessarily correlated with the least possible errors in measurement of momentum. A similar

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592 When Heisenberg discovered *matrix mechanics* (1925), neither Heisenberg nor Born knew what to make of the appearance of matrices in the concept of the atom. David Hilbert told them to go look for a differential equation with the same eigenvalues. They did not follow Hilbert's advice and thereby may have missed discovering the Schrödinger wave equation.

'*uncertainty*' relation exists between energy and time, angle and angular momentum, different components of spin or orbital angular momentum, phase and intensity of a light wave, electric and magnetic fields, as well as other pairs of so-called 'canonically conjugate observables'.

Heisenberg studied theoretical physics at Münich under **A. Sommerfeld**, receiving his doctorate degree in 1923. In the same year he became assistant to **Max Born** at Göttingen. He then worked under **Niels Bohr** at Copenhagen (1924–1927). During 1927–1941 he was professor of theoretical physics at Leipzig. From 1942 on he was associated with the Max Planck Institute for physics at Göttingen. He was awarded the Nobel prize for physics in 1932.

Werner Heisenberg was among the few renowned scientists who stayed in Nazi Germany. In his youth, he was associated with organizations whose philosophy was close to that of the Nationalist socialist party (e.g. *The White Knights*). The organizations were elitist, opposed to the Weimar Republic, willing to submit unquestioningly to a leader, fanatically devoted to a romanticized German culture, and strongly convinced of German superiority. He decided early on that he would not leave Germany. When Jewish professors were dismissed from the universities, he made vigorous efforts to replace them by Dutch professors (non-Jewish, of course); they all turned him down.

Clearly, to Heisenberg at that juncture, maintaining a strong physics establishment took precedence over human rights, justice or dignity. From the beginning, Heisenberg was sympathetic to the Nazi's national aims. He later participated in the German war effort and made propaganda trips to occupied countries, thus becoming a willing, active participant in Germany's politics and war aims. Even after the war Heisenberg never gave up his strong belief in the superiority of German culture and science. There was perennial conceit, continued arrogance and more than a touch of condescension in his attitude toward U.S. science, as well as a strong tendency to trivialize U.S. scientific accomplishments during the war.

He was accused by physicists of collaboration with the Nazis, and after the war was virtually ostracized by the international scientific community.

The Uncertainty Principle and the Copenhagen formulation of Quantum Mechanics (1925–1930)

__Schrödinger__ (1925–1926) transformed __de Broglie's__ (1924) rather vague ideas about electron waves into a precise and coherent mathematical formalism that applied to electrons or other particles. At the heart of Schrödinger's approach was a dynamical equation that dictated the way any given particle or system of particles would evolve with time. Schrödinger's equation is mathematically of a kind similar to those that had been used since the 19th century to study waves of sound, light and radio. Physicists in the 1920's were immediately able to set about calculating the energies and other properties of all sorts of atoms and molecules. Despite this success, nobody at first knew what physical quantity was oscillating in an electron wave, especially since the wave amplitude is complex. __Born__ (1926) suggested that the wave function at any point tells us (through its squared modulus) the probability density that the electron is near that point.

Neither Schrödinger nor de Broglie were comfortable with this interpretation of electron waves. However, __Heisenberg__ (1927) adopted the Born interpretation to arrive at his uncertainty principle, claiming that electrons have neither a definite position nor a definite momentum.

Physicists continued to wrangle over the interpretation of quantum mechanics for years after they had become used to solving the Schrödinger equation; they still do. While __Einstein__ rejected quantum mechanics altogether, most physicists were simply trying to understand it.

Much of this debate went on at the University Institute for Theoretical Physics in Copenhagen, under the guidance of __Niels Bohr__. By 1930 the discussion at Bohr's Institute led to an orthodox 'Copenhagen' formulation of quantum mechanics. The essence of this interpretation is a sharp separation between the system itself and the apparatus used to measure its configuration:

During the time intervals between measurements the wave function of the quantum system evolves in a continuous and deterministic way, dictated by its Schrödinger equation. While this is going on, the system cannot be said to be in any definite configuration of classical phase space. If we measure the configuration of the system (e.g. by measuring all the particles' position or all their momenta, or one's position and another's momentum, or a combination of momentum and position for each particle), the system jumps into a state that is definitely in one configuration or another of the measured variables, *with joint probability density given by the square modulus of the one– or many– particle wave function*[593] *just before the measurement.*

While human beings had no special status in Newtonian physics, in the Copenhagen interpretation of quantum mechanics humans (or at least, macroscopic measuring devices) play an essential role in giving meaning to the wave function by the act of measurement. Where the Newtonian physicist spoke of precise prediction, the quantum mechanician now offers mainly calculation of probabilities.[594]

Although it seems weird at first, quantum mechanics provides a precise *framework for calculating energies, spectra, bulk properties of matter, transition rates and probabilities.*

An idealized plane electromagnetic wave of total energy E transmits a linear momentum $p = \frac{E}{c}$. Since for a single photon (of frequency ν and wavelength λ) $E = h\nu$, the momentum of a single photon is $p = \frac{h\nu}{c} = \frac{h}{\lambda}$, i.e. $\lambda = \frac{h}{p}$. This particle aspect of radiation – first understood by Einstein – is borne out by many experiments, such as the photoelectric and Compton effects.

The complementary wave *aspects of material particles, predicted by de Broglie and Schrödinger, were confirmed via the experiments of* **Davisson** *and* **Germer** *(1927), who showed that a beam of electrons reflected from the surface of a nickel crystal form diffraction patterns, exactly analogous to the Bragg diffraction of X-ray electromagnetic radiation from the same crystal. This is successfully explained by assigning a* de Broglie wavelength $\lambda = \frac{h}{mv}$ *to an electron of mass m and velocity v.*

Thus, for both light waves/photons and electron particles/waves, the dualism of wave and particle is governed by the simple relation $p = \frac{h}{\lambda}$. It has a fundamental consequence regarding the unambiguous and complete description of a subatomic system's behavior in space and *time:*

Consider, for example, the free electron in one dimension: the corresponding time-dependent Schrödinger equation is

$$\frac{\partial^2\psi}{\partial x^2} = \frac{4\pi m}{hi}\frac{\partial\psi}{\partial t}$$

[593] Or some linear transform of it. In case it is the particle momenta that are being measured, it is the $3n$ dimensional Fourier transform, with n the number of particles.

[594] Although many predictions — such as atomic spectra, laser or maser frequencies and some low-temperature many-body observables — are *more* precise in quantum mechanics than is possible in classical physics.

and its general solution is

$$\psi(x,t) = \int_{-\infty}^{\infty} g(p) \exp\left[\frac{2\pi i}{h}(px - Et)\right] dp$$

where

$$E = E(p) = \frac{p^2}{2m}$$

is the classical Newtonian relation of momentum to kinetic energy.

We assume that $g(p)$ is only appreciably different from zero in the neighborhood of a given momentum p_0 and has the form of a Gaussian wave-packet,

$$g(p) = A \exp\left[-\frac{(p-p_0)^2}{4s^2}\right].$$

Clearly the packet group velocity is $\left(\frac{\partial E}{\partial p}\right)_0 = v_0 = \frac{p_0}{m}$. *Then, the stationary-phase approximation to the integral yields ($\hbar = \frac{h}{2\pi}$ is the reduced Planck's constant):*

$$\psi(x,t) = 2sA\left[1 + \frac{2its^2}{m\hbar}\right]^{-1/2} \exp\left\{\frac{i}{\hbar}(p_0x - E_0t)\right\} \cdot \exp\left\{-\frac{s^2}{\hbar^2}\frac{(x-v_0t)^2}{1+\frac{2its^2}{m\hbar}}\right\}.$$

This means that the coordinate probability distribution of the electron is

$$|\psi(x,t)|^2 = 4s^2|A|^2\left[1 + \frac{4t^2s^4}{m^2\hbar^2}\right]^{-1/2} \exp\left\{-\frac{2s^2}{\hbar^2}\frac{(x-v_0t)^2}{1+\frac{4t^2s^4}{m^2\hbar^2}}\right\}$$

while its momentum probability distribution is

$$|g(p)|^2 = |A|^2 \exp\left\{-\frac{(p-p_0)^2}{2s^2}\right\}.$$

Both distributions are represented by Gaussian functions: the root mean square deviations from the most probable values (x_0 and p_0 respectively) are given by the respective widths $\Delta p = s$, $\quad \Delta x = \frac{\hbar}{2s}\left[1 + \frac{4t^2s^4}{m^2\hbar^2}\right]^{1/2}$ *at time t.*

At $t = 0$, *these indeterminacies are reciprocal, such that*

$$\Delta x \cdot \Delta p = \frac{1}{2}\hbar.$$

As t increases, Δp stays unchanged, but Δx increases towards the asymptote $\Delta x \approx \frac{st}{m} = \frac{(\Delta p)t}{m}$, which is merely the classical *position uncertainty due to the initial spread in velocities.*

It can be shown that Gaussian wave packets are actually optimal*, as far as simultaneous measurements of x and p is concerned; thus in general, $(\Delta x)(\Delta p) \geq \frac{1}{2}\hbar$, which is the celebrated* Heisenberg uncertainty relation *(Mathematically, it follows from the* Cauchy–Schwartz inequality*).*

The simultaneous measurement of the position and momentum of an electron necessarily involve minimal uncertainties which are related in accordance with this inequality.

If one attempts to localize a particle of mass m to within a Δx of order or less than its Compton wavelength, $\Delta x \lesssim \hbar/(mc)$, one enters the realm of field quantization*, or in other words,* quantum field theory *— the union of special relativity and quantum mechanics. In that regime ($\Delta x \lesssim 10^{-11}$ cm for electrons), the very number of particles becomes uncertain, for the zero-point energy associated with the act of measurement — i.e. the kinetic energy due to the "fuzziness" in momentum created by measuring x to an accuracy of Δx — culls out of the vacuum* virtual *particles rendering them* real *(actual).*[595]

Field quantization introduces its own set of uncertainty relations; these are simple functional extensions of Heisenberg's original, 'first-quantized' version. Thus, in Quantum Electrodynamics*, the electric and magnetic fields measured in the same small volume element, obey such an uncertainty relation; so do the* intensity *and* phase *of a light wave (the latter fact has found applications in the technology of* quantum optics*).*

Quantum Electrodynamics itself was originated, along with second quantization (known today, more accurately, as "field quantization") by ***Heisenberg*** *and* ***W. Pauli****.*

Descending to even smaller values of Δx (implying more accurate position measurements), it is known today — from considerations involving quantum gravity *— that problems with the very concept of a smooth spacetime continuum, dictate a lower bound to Δx, of order $\ell_P = \sqrt{\frac{\hbar G}{c^3}}$. (Here $\hbar$ is the reduced Planck constant, c the speed of light in vacuo, and G the constant of universal gravitation.) ℓ_P is known as the* Planck length *and it is an extremely small length scale — approximately 10^{-33} cm; twenty orders of magnitude smaller than typical nuclear sizes.*

Qualitatively, the implied inequality is of fundamental importance since it states that measuring distances smaller than the Planck length is meaningless*,*

[595] This is possible since for $\Delta x \lesssim \frac{\hbar}{mc}$, the zero-point energy is of order $\gtrsim mc^2$.

at least within the classical (or even Minkowskian or General-Relativistic) description of spacetime. This circumstance is somewhat reminiscent of the situation in 'p-adic' number systems — in which the Archimedean axiom (according to which any given segment on a straight line can be refined by dividing it into smaller segments) is abandoned.

This has suggested to some physicists that spatial geometry on a small enough scale is not only non-euclidean *(as follows from GTR) and* quantized *(as shown from what little we know of quantum gravity), but is furthermore based on* non-Archimedean arithmetic. *However, these latter speculations have thus far not proven fruitful.*

It is inaccurate to regard the uncertainty relations as a restriction which nature places on our knowledge of the microworld[596]. *The meaning of the relation* $(\Delta x)(\Delta p) \geq \frac{1}{2}\hbar$ *(and the other similar relations in first–quantization and for quantized fields) is rather that instantaneous, simultaneous and sharply defined numerical pairs of values of position and momentum (or angle and angular momentum, electric and magnetic field, etc.) are simply not objective properties of micro-systems — and that this is just the way things are. Micro-objects are neither particles nor classical waves and it is impossible to fully describe them in terms of "common sense" classical notions. The wave-particle duality of micro-objects should be interpreted as their potential ability to behave differently under different conditions, or even in two classically-contradictory ways at the same time. The following consequences of the uncertainty principle will illustrate this:*

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[596] Prior to 1925, physics recognized only one type of fundamental uncertainty: statistical mechanics (thermal) uncertainty. It stems from the physical fact that there are of order 10^{23} particles in one mole of matter, subject to statistical laws summarized by the *second law of t hermodynamics* and its non-equilibrium extensions. While Maxwell posited that super-fast, microscopic and sentient "Demon" might micro–manage molecules and violate the Second Law, **Szilard** and **Brillouin** showed that the Demon's requisite information processing must generate heat – thereby restoring the "thermal uncertainty". While we do not yet possess a theory of quantum gravity, its severe challenges have led to various bold approaches to the problem of spacetime description at or below the Planck (ℓ_p) length scale. One approach posits that *twistors* (spinor variables underlying local light cones) replace *spacetime* as the fundamental variables of quantum gravity.

The *Heisenberg uncertainty relations* express the fact that while we inhabit a deterministic infinite-dimensional Hilbert-space, our macroscopic sense perception force a description of nature in terms of a 4-dimensional space-time, and the former fact severely constrains one's ability to so describe a *microscopic* system.

- *Why does an atom not collapse? This question remained unanswered in Bohr's model; why does an electron, when revolving about the nucleus in the lowest Bohr orbit, not radiate EM wave-energy (which would lead to its eventual fall into the nucleus)?*

An electron in its ground state (lowest–energy quantum orbital) is moving in a localized region of about 10^{-8} *cm in size. If it falls into the nucleus, it will be localized in a much smaller region, of about* 10^{-13} *cm in size. But then, on the strength of the uncertainty principle, the random components added to its velocity by this very precise "measurement" of its position by the nucleus electric field, increases by a factor of order 100,000. The electrostatic attractive force of the atomic nucleus would not be able to hold such a fast-moving electron within the nucleus. The uncertainty principle thus does not allow the electron to fall. Had the velocity uncertainty been a mere statistical matter (i.e., were the electron's velocity is well defined, albeit unknown to us), then occasionally,* some *atom in its ground state would radiate. Yet in quantum mechanics this* never *happens (another example of an instance in which quantum reality actually renders physics* less *uncertain!).*

- *Do electron orbits exist? From classical physics, an electron in an atom has a speed of the order of* 10^8 *cm/sec. Since the dimensions of the atom are* $\sim 10^{-8}$ *cm, the uncertainty relation yields a* velocity uncertainty *of* $\Delta v \sim \frac{\hbar}{m(\Delta x)} \simeq \frac{10^{-27}}{9\times10^{-28}\times10^{-8}} \simeq 10^8$ *cm/sec, that is to say, the error in determining the velocity is comparable with the velocity itself!*

Thus, the notion of electron orbits turns out to be invalid (except for very high quantum numbers n*), and we see again that, to describe micro-objects, new concepts must be introduced. (Atomic physicists and physical chemists speak instead of smeared-out* orbitals.*) Of these, the concept of probability is, perhaps, the most important in quantum mechanics; we may not speak of orbits, it is only meaningful to speak of the* probability density *of finding the electron at a given velocity or given distance from the nucleus. Yet, as we noted above, the stochasticity of quantum mechanics is different in character from classical (or thermal) stochasticity.*

In some sense, the use of probabilities to describe quantum mechanics is due to our own predilection, as macroscopic, almost classical atomic aggregates, for a phase-space description of subatomic motion! The fact that the probabilistic interpretation of quantum mechanics is not required as an independent postulate of quantum mechanics, was first exposed clearly by **Hugh Everett III** *(1930–1982), in his so-called 'Many-World' interpretation of quantum mechanics.*

Systems containing large numbers of particles, whether classical or quantum, require for their description an entirely new approach, that of statistical physics. *In classical physics one solves Newton's equations and, provided the* *initial conditions are specified, the future motion of all particles is fully determined – as so vividly expressed in Laplace's famous dictum. The laws of classical physics are therefore deterministic, and elements of randomness (statistical description) are only relevant when we study systems of large number of particles (e.g. gases) — to compensate for our imperfect knowledge of initial conditions.*

In quantum mechanics of a single or a few microparticles, the situation is quite different. The statistical, or probabilistic, approach is applied even to the motion of one single particle; *whether or not we shall find the electron at a given region near the nucleus — this is a matter of chance. However, unlike statistical mechanics, few-particle quantum mechanics is fully deterministic — as long as we describe its evolution in terms of wavefunctions rather than classical phase space! In* quantum mechanics *of a many-body system, the two kinds of uncertainty are both operative. To illustrate this, we consider the following example:*

Conversion of a disordered state into an ordered state, such as putting a mixed deck of cards back into a properly ordered sequence, may be accomplished by any intelligent person (or computer). Similarly, the task of converting the probable Maxwellian velocity distribution into an improbable distribution might be accomplished if there existed a supremely deft intelligent agent, traditionally known as "Maxwell's demon", keeping watch at a small sliding panel between two halves of a box (each half enclosing molecule samples at nearly equal temperatures), which he opens briefly whenever a fast molecule approaches from the left or a slow molecule from the right. By this procedure the demon should be able to separate slow and fast molecules in a time very much shorter than the time needed for this separation to occur by a statistical fluctuation without any panel.

Clearly, the demon cannot operate successfully unless he can distinguish fast molecules from slow ones. Now, knowledge whether a molecule is fast or slow requires about one bit of information. This has led some authors to consider "information" as a source of "negative entropy", which Maxwell's demon transfers to the gas whenever he uses one bit of information during one operation of the panel. In this fashion the total entropy of the system need not decrease during the demon's operations, provided the demon and his store of negative entropy are counted as part of the system.

This proposal becomes quantitatively feasible if one assigns to one bit of information a negative entropy of amount $S_0 = -k \log 2$ *(k = Boltzmann's constant), with the understanding that entropy is additive — thus making two bits of information equivalent to a negative entropy of amount*

$2S_0 = -k \log 2 - k \log 2 = -k \log 2^2$, *and generally* n *bits of information represent a store of negative entropy of amount* $nS_0 = -k \log 2^n$. *This interconvertibility of information and entropy is further justified on the grounds that the demon's velocity measurement, and subsequent action (opening, or closing the panel), involve* data processing, *which in turn converts some useful energy source into heat.*

The above discussion implies that Maxwell's demon will always be capable of acquiring the information necessary for performance of his task. However, quantum mechanics places a further fundamental limitation on any such demon. If the temperature of a gas consisting of N *molecules of mass* m *in a volume* V *is held below a critical value*

$$T_0 = \frac{\hbar^2}{mk} \left(\frac{N}{V} \right)^{2/3} \quad (\hbar = 1.05 \times 10^{-27} \; erg \; sec)$$

the demon is incapable of distinguishing fast and slow molecules with sufficient accuracy, on account of the uncertainty relation $\Delta p \Delta x = m \Delta v \Delta x \gtrsim \hbar$ — *which states that localization of a molecule to within a length* Δx *engenders an* inevitable *uncertainty in the knowledge of its velocity by the amount* $\Delta v \gtrsim \hbar / m \Delta x$.

To see this we note that at temperature T, *the difference in speed between the slowest and the fastest molecules is of order* $(kT/m)^{1/2}$, *and the uncertainty in the demon's knowledge of the velocity, must clearly be less than that for his project to succeed:* $(kT/m)^{1/2} \gtrsim \Delta v$. *However, if the average speed of the molecules is* v *and the area of the panel* d^2, *about* $vd^2 N/V$ *molecules will hit the panel per second, giving the demon not more time than* $\Delta t \approx V/(vd^2 N)$ *between successive openings and closings of the panel.*

This amounts to a localization of the passing molecule of amount $\Delta x \approx v \Delta t \approx V/(d^2 N)$, *thus engendering an uncertainty in the knowledge of its velocity* $\Delta v \gtrsim \frac{\hbar d^2 N}{mV}$.

The dimension d *of the opening will also require a (transverse) localization of the particle during passage; in order that the corresponding quantum uncertainty in the particle's velocity is not larger than the overall* Δv, *one must give this opening at least the size* $d \gtrsim \frac{\hbar}{m \Delta v}$; *so that one has the combined inequality* $\Delta v \gtrsim \frac{\hbar^3 N}{m^3 V (\Delta v)^2}$, *which when solved for* δv *yields* $\Delta v \gtrsim \frac{\hbar}{m} \left(\frac{N}{V} \right)^{1/3}$. *By substitution into* $(kT/m)^{1/2} \gtrsim \Delta v$, *one thus finds that successful operation of the demon requires a temperature*

$$T \gtrsim \frac{\hbar^2}{mk} \left(\frac{N}{V} \right)^{2/3} \equiv T_0.$$

The value of T_0 for the case of helium molecules $(m = 6.5 \times 10^{-24}$ g), compressed to a density near that of liquid water (about 1 g/cm^3) is $T_0 \approx 3.5\,^\circ K$.

Heisenberg based his new mechanics on the postulate that the atomic theory rests solely on such geometrical or physical entities as are measurable; e.g. the locus of the electron's entire orbit must be excluded, but frequencies and amplitudes of spectral lines that are emitted by the atom are legitimate observables, as are (for instance) any position or momentum Cartesian components (but not both along non-perpendicular directions) of the electron at a given time.

He thus developed a mathematical scheme, based on matrices, that was suitable for the description of the new sub-atomic reality[597]*. This new mathematical framework enabled him to explain various observation that defied Bohr's semiclassical approach.*

Schrödinger's and Heisenberg's work on quantum theory had a profound impact on the development of physics and many other branches of fundamental and applied science – in particular molecular, atomic, nuclear and particle physics; material science; chemistry; biology; optics; electrical engineering, astrophysics, and cosmology.

In 1932 Heisenberg introduced the concept of isospin[598] *and expounded its mathematical theory. [*isospin = *isotopic spin* or *isobaric spin. The former is a*

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[597] As an illustration of how matrices entered his scheme, consider a hydrogen atom emitting a photon in a transition between the Bohr orbits m, n. Classically, the strength of the radiation is proportional to $\left[\ddot{\boldsymbol{X}}(t)\right]^2$, where $\boldsymbol{X}(t)$ is the trajectory of the emitting electron (*Larmor's law*). Heisenberg treated $\boldsymbol{X}(t)$ as an operator (an infinite-rank matrix) and suggested that the strength of the spectral line is related to the square of the *modulus* of the *complex* amplitude $[\boldsymbol{x}(t)]_{mn}$, and that the time dependence of this complex matrix element is proportional to $e^{i\omega_{mn}t}$, where $\omega_{mn} = \frac{1}{\hbar}(E_m - E_n)$. This explains nicely, among other things, the hitherto-unexplainable (empirical) '*Ritz Combination Law*' of atomic spectra.

[598] Since the strong nuclear forces are charge-independent in the sense that the proton-proton and neutron-neutron strong forces are the same (and qualitatively similar to the neutron-proton strong forces), and since these two constituents of the nucleus may be transformed into each other and are close in mass, it is physically meaningful to consider them to be 2 different "*states*" of one heavy nuclear particle — the *nucleon*. In contradistinction to our common concept of different states of one system, these 2 nucleonic states *differ w.r.t. their charge*. It is convenient for the theory of the nucleus to distinguish the 2 states *formally* by the components $[T_z = \frac{1}{2}$ (proton) and $T_z = -\frac{1}{2}$ (neutron)] of a new quantum number called *isospin*. The (weak–nuclear–force mediated)

misnomer since the proton and neutron are not really isotopes, as they differ in charge.] The isospin symmetry, although approximate, predicts various successful relations between nuclear energy levels and probabilities of different nuclear reactions and transitions.

In the standard model *of particle physics, the proton, neutron, pions and other* hadrons *are understood to be made of* quarks; *isospin invariance (symmetry) is understood to be an approximate* accidental *symmetry, but other "internal" (i.e. non-spacetime rotations in internal space)* are *fundamental symmetries in this theory. These are the* non-abelian gauge transformations *under which a quark's "color" or a lepton's* charge *are changed; and under which (for example) an up-quark (charge $+\frac{2}{3}e$) can be converted to a quantum – coherent mixture of down- and strange quark states (both of charge $-\frac{1}{3}e$), with color unchanged.*

β-decay, where a neutron is transformed into a proton, then corresponds to the quantum transition $\Delta T_z = 1$ (isospin flip). Since nucleons are fermions, the nuclear wave function must be totally antisymmetric w.r.t. the combined spatial, spin and isospin degrees of freedom.

The *mathematical* transformation of a proton into a neutron, or vice versa, is then a special case of rotation in abstract isospin space. The strong nuclear interactions are covariant under such iso-rotations, and the corresponding "angular momentum" $\boldsymbol{T}$ (of which T_z is but one component) is thus conserved by these interactions. This limited charge-independence does *not* mean that the *nn* and *pp* forces are exactly the same. The (small) electromagnetic and weak corrections to these forces violate isospin conservation, and are responsible for the observed charge-dependence of nuclear masses, lifetimes and scattering cross sections (electromagnetic effects are by far the dominant isospin-violating nuclear effect).

Causality and Determinism in Quantum Theory[599] — *or, Where is the particle when no one is looking? (1925–1965)*

Science, the systematic effort of man to explore natural phenomena, started with the Greek school of ***Thales*** *and Anaximander in the 6th century BCE. It reached its pinnacle during the period 400–200 BCE, the period of Democritos, Socrates, Plato,* ***Aristotle*** *and* ***Chrysippos****, referred to as the classical period of Greek philosophy. It was followed by the Hellenistic period, which lasted up to the 2nd century CE. Then an era of decline set in, although the investigative efforts continued to some extent until the 6th century CE.*

Except in a very few fields such as astronomy and geometry, Greek philosophy is based on speculations, hypotheses and theories. However, the extraordinary intuition and perspicacity of Greek philosophers produced concepts that are in some respects amazingly close to the ideas of modern science, particularly physics.

Thus, the concept of the atom was first proposed by ***Democritos*** *and* ***Leucippos****.* ***Anaximander*** *was first to envision the possibility of the transformation of one primary substance into another one.* ***Heraclitos****' foremost fundamental principle — that everything is in constant flux (the famous* $\pi\alpha\nu\tau\alpha\ \rho\epsilon\iota$*) — is the forerunner of the notion that* energy *transformations are at the base of all changes in the world. He assumed fire to be the basic element, but if one replaces 'fire' with 'energy', the doctrine of Heraclitos is in some ways quite close to the story told by modern physics.*

Plato*, contrary to Democritos, held that the smallest units of matter are geometrical forms (platonic regular solids) transformable into each other. In the last analysis, the elementary particles are for Plato not substance but* mathematical forms*. In modern physics, too, elementary particles are considered as forms — moving configurations of energy, irreducible representations*

[599] For further reading, see:

- Jammer, M., *The Conceptual Development of Quantum Mechanics*, Amer. Inst. Phys.: New York, 1989.
- Jammer, M., *The Philosophy of Quantum Mechanics*, Wiley–Interscience: New York, 1974.
- Rae, A., *Quantum Physics*: *Illusion or Reality*? Cambridge University Press: Cambridge, 1986, 123 pp.

of symmetry groups, states in Hilbert space, et cetera. In Greek philosophy, however, these forms were considered to be static, whereas modern physics stresses their dynamic nature.

Since the beginning of modern science in the 16th and 17th centuries, there has been a continuous parallel development of new philosophical ideas and systems stimulated by concepts emerging from the new scientific knowledge[600]*.*

Among the outstanding philosophers of the early period were ***Francis Bacon*** *(1561–1626) in England and* ***Rene Descartes*** *(1596–1659) in France. Bacon, primarily a philosopher, understood the importance of mathematics, and stressed empiricism and the skeptical observation of Nature. He wrote the first classical works in which serious attempts were made to build bridges between the new information resulting from scientific observations and knowledge on the one hand; and philosophical thinking — hitherto still strongly influenced by Aristotelian philosophy — on the other.*

Since the days of Descartes and Newton scientists have regarded the physical universe as a system in which all events proceed in an unbroken chain of cause and effect, according to a universal principle of causality. In a universe in which the principle of strict determinism is meaningful and holds, the laws of nature are at least partially deterministic[601]*; experimental verification of determinism is furnished by the predictability of events. The principle of strict determinism asserts that, if all pertinent details concerning the present state of an isolated system are known, it is possible by means of suitable laws of nature (governing causal agencies) to predict that system's state at any later (or, indeed, earlier) time. The conclusion is drawn that everything that ever did or will happen in such a system, is completely determined (at least in principle) by its present state.*

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600 These movements became particularly strong in the 20th century; quantum mechanics and the theory of relativity had a profound impact on philosophical thinking. Indeed, **Einstein**, **Bohr**, **Born**, **Heisenberg** and **Pauli** analyzed in their writings and lectures the philosophical implications of the newly gained knowledge and experience.

601 *Determinism*: the ability to determine the state of a system at a given time on the basis of information about the state at an earlier time.

Newtonian Causality: causes precede effects (and are best described via mechanical contact forces or influence).

Classical Causality: If a system is perturbed at time t_0, as reckoned in a locally–inertial frame, the perturbation at infinitesimal distance R away (as measured in that frame) cannot arrive before time $(t_0 + \frac{R}{c})$, where c is the velocity of light in vacuo.

Within the confines of classical physics, Newtonian (or, later, relativistic local) causality and determinism were thus largely synonymous. One could, perhaps, imagine a universe in which successful quantitative prediction schemes — such as Babylonian astronomy — exist without any underlying mechanism ('laws of nature'). In such a putative universe, causation *would reduce to its narrow empiricist definition as rendered by* ***David Hume****: A* causes *B whenever it is found to always precede it, all other circumstances being allowed to vary.*

One could also imagine a causal universe in which some casual agencies are inherently difficult to measure or predict, thus rendering determinism less than perfect (as, indeed, happens in thermal physics).

However, the universe in which Newton and his successors thought they found themselves, was one in which relatively simple mechanisms could, and did, account for far-flung chains of causation. The concepts of 'causality' and 'determinism' were thus conflated in classical physics — a situation which was to change with the rise of quantum mechanics, as we shall see.

Determinism was automatically built into Newtonian mechanics in the 17th century. Thus, treating the solar system as an isolated system, the planetary positions and velocities at one moment suffice to determine uniquely, via Newton's laws, their positions and velocities at all subsequent times. Moreover, since these laws contain no directionality of time, the present state suffices to uniquely fix all past *states as well: one may predict eclipses into the far* future*, and also retrodict their occurrences in the past.*

In a causal, deterministic world, the past and future are contained in the present in the sense that the data needed to construct the past and future states of the world are implicit in its present state. The entire cosmos is then, in effect, a gigantic machine or clockwork, slavishly following a pathway of change already laid down from the beginning of time; God is thereafter reduced to a mere archivist turning the pages of a cosmic history book already written. In the words of the astronomer-poet Omar Khayyam in his Rubaiyat *(ca 1100 CE):*

'With Earth's first clay they did the last man knead,
And there of the last harvest sow'd the seed:
And the first morning of creation wrote
What the last Dawn of reckoning shall read.'

Immanuel Kant *(1724–1804) concluded (1781), like other philosophers before him, that only part of our knowledge is based upon experience. What we would now call the 'raw data' of experience is acquired by our senses, but it is our mind (reason, thought, intelligence) that brings law, order and* *regularity into the phenomena observed. We understand these phenomena because we approach them with certain notions and concepts which Kant characterized as a priori. Among such necessary notions are space, time and the law of causality.*

Without these three a priori notions we would be unable to perceive a well-ordered universe. These a priori concepts are, of course, based on Newtonian mechanics, which strongly influenced not only Kant's philosophical thinking but that of 19th century thinkers as well. Their limited applicability only became apparent in the 20th century, through the results of modern physics. Moreover, the geometry that formed an essential basis of Newton's physics was that of Euclid.

*Not till the 19th century were new kinds of geometry developed, particularly through the pioneering work of **Gauss** (1777–1855); this greatly influenced later thinking in physics and astronomy. Kant obviously could not have foreseen the startling developments of modern physics — neither the theories of relativity (which forced changes in the 'a priori' concepts of space and time) nor quantum mechanics, which profoundly transformed the law of causality for microscopic systems and divorced it from any direct connection with the determination of results of future measurements.*

Indeed, prediction is quite impossible for the trajectory of a photon or electron in Young's or similar apparata; its motion after passing the two slits is not determined by its condition immediately before it reached the plane of the slits. Two photons, identical in every observable way, could pass from source to plane in the same way and yet reach quite different destinations thereafter. And yet, despite this quantum indeterminacy, any single photon passing through the apparatus obeys the principle of causality! The seeming contradiction is resolved as follows. The only attribute of the photon that evolves causally is its wave function (a 'ray' in an abstract, infinite dimensional vector space known as Hilbert space).

This (complex) wave function is created, along with the photon it describes, at the light source; thereafter it evolves smoothly in time, according to a Schrödinger equation, subject to the boundary conditions imposed by the two slits. However, this mathematical wave is quite different in nature from classical waves (such as acoustic or elastic waves in bulk matter, many-photon electromagnetic waves in vacuo, etc.). The difference is that the amplitude of the quantum wave does not represent any observable physical attribute of the photon.

Its only relation to such observables is indirect and stochastic in nature — when the photon is observed (e.g. by allowing it to impinge upon a photographic screen at some distance downstream of the slits plane), the distribution of its measured attribute (whether position, momentum etc.) can be *predicted from the pre-measurement quantum wave amplitude. Thus causality is relegated to operate in an abstract mathematical space; and should we nonetheless insist (as we must!) on describing a quantum system in terms of observable entities (such as positions, velocities, electric fields, polarizations, etc.) — the price that must be paid is* determinism: *future (or past) states of the system* cannot *be fully predicted, irrespective of how much we know about its present state.*

These remarkable observations hold quite generally for optical, electrical, mechanical or any other phenomena: it is impossible to predict the behavior of individual photons, electrons etc., no matter how carefully conditions are controlled. Yet the quantum wave pattern does *predict, to any desired precision, the statistical distribution of large numbers of the quanta per given arrangements of the measurement apparata. This is true whether the particles are passed through the apparatus one by one or in a volley (for the case of light, if the volley is sufficiently dense, quantum theory predicts that the light is approximately describable as a classical electromagnetic wave — the (real) amplitudes of which are directly observable as electric or magnetic field components.)*

Starting in the 19th century — long before the photon aspect of light was recognized — statistical methods were used to describe the observable properties of gases *in terms of the (Newtonian) mechanical behavior of their component* molecules. *Yet in spite of the superficial similarity, there is a fundamental difference between the two situations. In the case of gases, statistical methods are employed because the large number of molecules involved makes it impractical to ever carry out a detailed analysis. But for the* photons, *electrons etc., in our above discussion – and for molecules too, since they obey quantum mechanics as well – statistical considerations enter in a more fundamental manner. It is entirely possible to preserve all wave-optical effects in an optical experiment while making the light intensity so low that there is only one* photon *present at a time in the entire apparatus. Nonetheless, it is impossible (as we saw above) to determine the behavior of this single photon — for the laws to which its quantum evolution is ascribed, yield no more than statistical prediction concerning the position, energy or polarization of the photon at the downstream screen*[602].

[602] It is worth noting, though, that for phases of bulk matter cold and/or dense enough — such as many liquids and solids, as well as the recently achieved *Bose-Einstein Condensate* phase of monoatomic gases at ultra-low temperatures — the quantum 'fuzziness' of individual atoms competes with (or even take precedence over) the Newtonian (thermal) uncertainties. And conversely, photon gases in which individual photons interact frequently with matter or with each other — as happens inside the sun or an incandescent light bulb —

Modern physics has taught us that this peculiar quantum description holds for matter *particles as well as for light. Thus electrons, neutrons, whole atoms and other particles can be diffracted (one by one or en masse) as waves through crystal lattices; again it is found that causal evolution applies to an abstract 'quantum amplitude' wave, and that the information encoded in this wave concerning future or past experiments is indirect, incomplete and statistical in nature.*

As another example, consider a radioactive radium nucleus, which can (and eventually must) emit an α*-particle. The precise emission time of the* α*-particle cannot be predicted; only the average time of emission is determinable. In observing the emission one does not — indeed cannot — identify any preceding event that determined the emission to occur at the precise instant it did.*

Nevertheless, the causal mechanism for the α*-decay* is *known — but it operates at the rarefied heights of abstract Hilbert space; this precise knowledge, when converted to actual observation of the actual decay, becomes diluted into a mere statistical distribution.*

Thus, practically speaking, the forces in the radium nucleus responsible for the α*-decay cannot be determined accurately, but only as a probability distribution of possible outcomes (decay time and* α*-particle momentum).*

According to ***Max Born****'s statistical interpretation of the complex de Broglie-Schrödinger wave, the squared modulus of the wave gives the probability density for finding the particle at that position. All that quantum theory could do, he realized, was predict the wave-shape, and hence the probability distribution of various attributes of a quantum particle or system. In most cases it could not predict with certainty the outcome of any single measurement of those properties, as did the old classical physics. Thus, the description of the motion of a quantum particle is inherently* statistical*, and the waves are not material, as de Broglie and Schrödinger wrongly supposed. Furthermore, not all observable attributes are even simultaneously measurable; there is no sense in which an electron's position and velocity vectors, for example, both have definite values at the same instant of time —* even in principle.

How are we supposed to think about the atomic world of the quanta? Atoms, photons, and electrons really exist as particles, but their attributes — such as their locations in space, momentum, energy, discrete angular momentum state and sometimes even their numbers and types — exist only on a contingency basis (unless we are willing to view these observables as operators *rather than knowable* numbers*).*

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often behave similarly to Newtonian ideal gases, with the photons playing the role of atoms (a mole of photons is called an *einstein*).

An important feature of quantum probability distributions — one which distinguishes them from the probability distributions of card hands, dice or roulettes — is that quantum probabilities propagate through space as waves rather than diffusively; This is the Schrödinger wave. The predictive power of quantum theory resides in the fact that it determines precisely the shape of the wave and how it moves — how the potential probabilities of all possible measurements change in space and time. Here we see for the first time a new meaning of causality in quantum theory — it is probability of possible outcomes of potential measurements that is causally determined into the future, not individual events[603].

The randomness at the foundation of the material world does not mean that knowledge is impossible or that physics has failed. To the contrary, the discovery of the indeterminate universe is a triumph of modern physics and reveals a wholly novel aspect of nature; one without which matter would not have the stability allowing an emergent classic approximation to arise. The new quantum theory makes many predictions — all in agreement with experiment. But most of these predictions are for distributions of events, not individual events — like predicting how many times a specific hand of cards gets dealt on the average.

In the wake of Born's statistical interpretation, physicists struggled to deepen the understanding of the new quantum theory. What knowledge of nature was possible in the framework of the new theory? For example, the mathematics of quantum theory permitted both particle-like and wave-like representations for the electron. But clearly these two representations were opposed, and this duality thus comes into conflict with any common sense ideas.

Is the electron a wave or a particle? ***Bohr***, ***Heisenberg***, *and* ***Pauli*** *in Copenhagen, and many others debated these questions for over a year. Frustration set in, but Bohr's persistent optimism kept up a spirit of inquiry.*

[603] However, as stressed by **Everett** and others, wavefunctions are more than probability amplitudes. For one thing, a quantum measurement is properly viewed as a continuous (though limited in time) interaction with a macroscopic (many–body system) measuring apparatus. This measurement is itself describable (in principle) by a deterministic Schrödinger equation of the combined system (observed system plus apparatus). Thus, the notorious subjective "collapse" of the wave function upon measurement is merely a useful simplification — whereas with cards, the probabilities really are subjective (dependent on one's state of knowledge). Furthermore, a wavefunction contains more information than its modulus — its *phase* too can, in some cases, be partially measured (e.g. in **Bohm-Aharonov** type experiments.)

Finally in 1927, Bohr and Heisenberg each in his own style had come to new ideas which were conceptually equivalent: Heisenberg had discovered the uncertainty principle*, and Bohr had discovered the* principle of complementarity*. Together these two principles constituted what became known as the* Copenhagen interpretation *of quantum mechanics. This interpretation revealed the internal consistency of the quantum theory, a consistency which was purchased at the price of modifying age-old notions of determinism and objectivity in the natural world.*

Heisenberg's uncertainty relation (first of many such) states that the product of the simultaneous uncertainties (root-mean-square random deviation) in a position component (Δq) and in the corresponding momentum component (Δp) (were one or *the other to be exactly measured at any given, common instant of time) must be greater than a fixed quantity — essentially, Planck's constant. If this constant were equal to zero in the real world rather than a tiny finite number, then we could simultaneously measure both the position and momentum of a particle. But because Planck's constant is not zero, this is impossible[604]. Heisenberg's relation does* not *apply to a single measurement on a single particle: it is a statement about a statistical average over many measurements of position and moment.*

Let us consider Zeno's arrow paradox *in the light of quantum mechanics: Suppose that the arrow were of the dimensions of a single atomic particle. According to classical Newtonian physics the '*arrow-particle*' has determinate and measurable volume, velocity and position at all times. Zeno supposed that because the particle always occupies a determinate position, the possibility of motion is thereby excluded. Contemporary physics tells us that the precise simultaneous determination of the particle's position and velocity is impossi-*

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[604] To get a feel for what the Heisenberg relation implies for various objects, we can compare the product of the size of an object times its typical momentum to Planck's constant, h — a measure of how important quantum effects are. For a flying tennis ball, the uncertainties due to quantum theory are only of order one part in 10^{34}. Hence a tennis ball, to a high degree of accuracy, obeys the deterministic rules of classical physics. Even for a bacterium the effects are only about one part in a billion (10^{-9}), and it really does not experience the quantum world either as a whole body. For atoms in a crystal we are getting down to the quantum world: the uncertainties can be of order one part in a hundred (10^{-2}) although it can be larger (e.g for Helium atoms in the solid state). Finally, for electrons moving in an atom, molecule or metal the quantum uncertainties completely dominate, and we have entered the true quantum world governed by quantum mechanics.

ble[605]*. Zeno's insight seems to have been that motion and precise localization at an instant have an element of contradiction between them. (Indeed, this element appears in our common sense visualization of a moving body, since when we think of it as being at a definite position we cannot simultaneously form an image of what is its rate of motion!) In this sense, then, Zeno is closer to modern physics than to Newtonian physics.*

However, Zeno's paradox is fallacious also in the realm of quantum mechanics, because there is nothing in quantum physics to prevent our measuring the precise positions of a particle at two different times*, and then calculating what the intervening motion was on average.*

But there is one more twist in this comparison of quantum mechanics with Eleatic philosophy. Theoretically an over-zealous observer, applying a sequence of too-frequent measurements to the arrow's wavefunction, may completely alter its Schrödinger equation, and actually prevent the arrow from ever hitting its target! A version of this seemingly Carrollian scenario was actually observed experimentally at the U.S. National Bureau of Standards. The experiment involved, instead of an arrow, a transition of one atomic energy level into another. It was found, in agreement with quantum theory, that looking for the decay too frequently (by spectroscopic means, in this case) tends to inhibit it!

Bohr emphasized that when we are asking a question of nature we must also specify the experimental apparatus that we will use to determine the answer. In classical physics we do not have to perennially take into account the fact that in answering the question (doing an experiment) we alter the state of the object. We can ignore the interaction of the apparatus and the object under investigation, at least in principle. For quantum objects like electrons this is no longer the case; the very act of observation changes the state of the electron in a way that drastically alters its subsequent evolution.

Particle and wave are what Bohr called complementary descriptive concepts, meaning they exclude each other. Bohr's principle of complementarity asserts that there exist complementary pairs of properties of the same object (such as position and momentum), one of which, if known, to some accuracy will exclude precise knowledge of the other. We may therefore describe an object such as an electron in ways which are mutually exclusive, without logical contradiction: for instance, as a wave of definite de Broglie wavelength and momentum, or as a particle of definite position, or sometimes as a wave packet

[605] Indeed, the particle does not even *possess* these attributes simultaneously, and any assumption to the contrary leads to predictions at variance with both quantum mechanics *and* experiment, as we discuss below.

with fuzzy, but approximately defined, values for both momentum and position. The contradiction is eliminated because the experimental arrangements that determine these descriptions are similarly mutually exclusive.

Thus, two crucial points about the quantum world emerge from the Copenhagen interpretation:

(1) Quantum reality — or, more precisely, its classical phase-space manifestations — is statistical, not deterministic. Even after *the experimental arrangement bas been specified for measuring some quantum property with utmost precision, individual measurement outcomes are generally unpredictable. The microworld is empirically knowable only as statistical distributions of measurements, and these distributions can be predicted and verified by physics. The attempt to form a mental picture of the position and momentum of a single electron consistent with a series of measurements results in the* fuzzy *electron. This is a human construct, attempting to fit the quantum world into the limitations of macroscopic observations, sense-perceptions and mental conceptualizations.*

(2) It is meaningless to talk about the physical properties of quantum objects without precisely specifying the experimental arrangement by which one intends to measure them. Quantum reality is in part an observer-created reality — although observing apparata are themselves subject to its laws. The idealized classical physical world unfolds itself according to immutable laws independent of the experimenter, who may watch the physical process much as the audience watches a play in a theater. In quantum theory the observed structure of a microscopic system is inherently dependent upon the nature of its interaction with the 'observing' macro-system — which, as noted above, could be human, artificial or natural[606]*.*

In summary, at the level of observable attributes of a microscopic system, the Copenhagen interpretation of the quantum theory rejected determinism, accepting instead the statistical nature of reality; and it also rejected

[606] In the early decades of quantum mechanics, it was supposed by some practitioners (such as Wigner) that *human consciousness* somehow plays a special role in quantum measurement. However, the following two lines of investigations have gradually made it quite clear that *any* entanglement of the micro-system wavefunction with that of a macroscopic object is a form of quantum measurement. **Von Neumann** pioneered the theoretical modeling of mechanized (automated) measurement, while **Everett** systematically described measurement interactions without wavefunction collapse in his 'Many Worlds' interpretation of quantum mechanics.

> *objectivity — accepting instead that material reality depended in some sense on how we choose to observe it. After hundreds of years of holding sway in the Western mind, the world view of classical physics fell — although causality, determinism and objective reality survive, albeit at a more abstract, holistic level of mathematical description. Thus, from the very substance of the universe — the atom — the physicists learned a new lesson about reality*[607].

***Einstein**, however, was unconvinced. Distressed by the indeterminism of the new quantum theory, he disputed that the theory gave a complete and objective description of nature. But that objection was not the main one that stood in the way of his accepting the theory's picture of reality. A principle of physics that he held even more dear than determinism was the* principle of relativistic local causality *— that distant events cannot instantaneously influence local objects without any mediation.*

*Together with **N. Rosen** and **B. Podolsky** he produced (1935) an argument (known as the* EPR paradox[608]*) which endeavored to show that quantum theory violated local causality. The EPR argument seemed to indicate that quantum theory had either to violate the principle of local causality or be*

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[607] In the *Many-World* interpretation objectivity and determinism are retained, though at the heavy price of renouncing *either* the separate objectivity of any system smaller than the entire universe, *or* the ontological reality of a single universe (as opposed to an infinite ensembles of *possible* universes).

[608] **Nathan N. Rosen** (1909–1992, USA and Israel). Collaborated with Einstein at Princeton during 1935–1937 on the foundations of Quantum Mechanics, gravitational lenses, and the singularity-free solution of the gravitational-electromagnetic field equations.

Boris Podolsky (1896–1966, U.S.A.). Born in Taganrog, Russia; emigrated to the United States in 1913. Collaborated with Einstein during 1931–1935 at Caltech (Pasadena, CA) and at the Institute of Advanced Study, Princeton.

The EPR paper showed how quantum physics requires that a property, such as the polarization of a photon, could be measured at a distance by measuring the polarization of a second photon that had interacted with the first some time in the past. If it is deemed unacceptable that the polarization measurement could instantaneously influence the distant object, it follows (EPR claimed) that the first (now distant) photon must have possessed the measured property *before* the measurement was carried out. As the property measured can be varied by the experimenter adjusting the polarization – measurement apparatus, EPR concluded that all physical properties (in our example — values of polarizations in all possible directions) must be 'real' *before* they are measured, in direct contradiction to the Copenhagen interpretation.

incomplete; the EPR team concluded that quantum theory had to be incomplete.

Bohr himself considered the EPR paper as an 'onslaught that came down upon us like a bolt from the blue'. He made valiant efforts to refute the new challenge, commenting: 'They (EPR) do it smartly, but what counts is to do it right'.

For over thirty years physicists debated the conclusions of the EPR article. Then, in 1965, ***John Bell*** *proposed a (doable) EPR-type thought experiment (involving an entangled pair of particles), the outcome of which — assuming it verified the predictions of quantum mechanics — would require a choice between two physical interpretations: either the world was not describable in terms of a classical-physics set of variables (abandonment of physical realism), or it was nonlocal (i.e. instantaneous observations or other disturbances at distinct spatial locations are not independent). Bell derived a mathematical formula — an inequality — which holds classically, is violated in quantum mechanics, and could be checked experimentally.*

Variants of the experiment were repeatedly performed starting in the 1970s; and Bell's inequality — and thus by implication the central assumption used in its proof (that both locality and physical realism hold in the microworld) — was found to be violated. The world, it seems, is either not locally describable, or not classically objective. Either alternative — a nonobjective or nonlocal reality — is somewhat unpalatable, but perfectly consistent. A deeper analysis of the Bell thought-experiment shows that regardless of which of the two interpretations of reality we choose, relativistic local causality is preserved, i.e. influences cannot be transmitted instantaneously, or indeed faster than the speed of light in vacuo[609]*.*

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609 The local causality principle, which Einstein insisted upon, is thus *weaker* than the *locality* assumption. In fact, if Bell's inequality is violated (as all experiments have hitherto demonstrated), there exist *instantaneous statistical correlations* between the results of various potential polarization measurements on the two entangled EPR photons, however far they have flown apart. However — as is well known even in classical physics — correlation does not imply causation; more specifically, one can prove that the Bell quantum correlations do not allow an observer measuring one of the photons to thereby send an instantaneous message (through the collapsing joint wavefunction of the two photons) to the observer who subsequently measures some polarization component of the other photon. Such a superluminal communication scheme — which is possible in some modified, nonlinear-Schrödinger equation versions of quantum mechanics — are whimsically known as "Bell Telephone".

Contrary to what EPR believed, the Hilbert space description of reality used in quantum mechanics can be complete without violating relativistic local causality; in fact, quantum field theory — the marriage of quantum mechanics and special relativity — is not only locally causal, but fully local when described as interacting operator–valued fields: local wave equations hold for all quantum fields (such as the electromagnetic fields, the electronic Dirac field, etc.), although the value of such a field at any point in spacetime should be understood as a Hilbert–space operator *rather than a real number. Hilbert-space* correlations *of observables, though, are still nonzero even outside the light cone, just as in the non-relativistic quantum description of EPR-type experiments.*

In retrospect, although EPR misconstrued the implications of quantum mechanics for the principle of local causality, their thought experiment led to a refined understanding of these implications. In addition, the EPR/Bell analyses are the lithmus test against which all extensions or generalizations of quantum mechanics are tested, and continue to inspire high-precision experiments to probe the foundations of quantum reality.

1926–1935 CE **William Francis Giauque** (1895–1982, U.S.A.). Chemist. Won the 1949 Nobel prize in chemistry for pioneer work in the field of very low temperatures. He was first to achieve (1933) a temperature close to $-273\,^{\circ}$C, the absolute zero, by applying the new principle of *cooling by adiabatic demagnetization* which he and **P. Debye** discovered independently (1926). From information gained at these low temperatures, he accurately predicted the existence of two *isotopes of oxygen* O^{17} and O^{18}, eventually resulting in abandonment of the oxygen-16 standard for atomic mass (1961) and its replacement by carbon-12 standard. In 1935 he succeeded in the magnetic cooling of helium to a temperature of $0.1\,^{\circ}$K.

Giauque was born in Canada, in Niagara Falls, Ontario. He was professor at the University of California, Berkeley (1934–1981).

Cooling by adiabatic demagnetization: there are elements which, when in a crystal lattice, can form ions whose outer shells are only partially filled. The unpaired electron spins give rise to independent ionic magnetic moments (paramagnetism).

Gadolinium, iron, chromium, and cerium are examples. When these elements are chemically combined with a large number of nonmagnetic molecules or radicals, the individual magnetic ions are so far removed from other magnetic ions that their magnetic moments behave almost independently, like the atoms of an ideal gas, even though they vibrate about equilibrium positions in the lattice. The paramagnetic salt Gadolinium sulfate octahydrate $[Gd_2(SO_4)_3 \cdot 8H_2O]$ is an example in which one magnetic ion is surrounded by many nonmagnetic atoms and is therefore, magnetically, very dilute.

Pierre Curie, and later **Léon Brillouin**, established the dependence of the magnetization M of such materials on the ratio $\frac{H}{T}$ (H — applied external magnetic field; T — absolute ambient temperature) in the form $M = C\frac{H}{T}$ for small values of $\frac{H}{T}$. This is known as *Curie's law* and C is called the *Curie constant*. The larger the Curie constant, the smaller the value of $\frac{H}{T}$ that is needed to produce a given magnetization.

The nature of the method of magnetic cooling is made clear by comparing it with a mechanical analogue:

Suppose that it is desired to cool a gas by means of mechanical work. The gas is in a piston–equipped container, and is initially in thermal contact with a heat bath at temperature T_1. In the first step of the cycle, the gas is *isothermally compressed* from volume V_0 to a volume V_1. In this process work is done on the gas, but since it can give off heat to the bath its temperature is not changed. In the second step, the gas is thermally insulated (e.g., by removing the bath) and is allowed to expand adiabatically back to $V = V_0$. In this process the gas does work at the expense of its internal energy and, as a result, its temperature falls to some value $T_2 < T_1$.

One can then complete the cycle by re-attaching the gas container to the heat bath, allowing it to absorb heat at fixed volume until it returns to temperature $T = T_1$. This cycle can be repeated to continually cool (extract heat from) the gas–container's environment.

The method of *magnetic cooling by adiabatic demagnetization* is very similar: The system of interest is a magnetic sample (e.g., Gadolinium sulfate) initially in thermal contact with a heat bath at temperature T_1. In practice this heat bath is liquid helium near $1\,^\circ$K, and thermal contact of the sample with the bath is established by heat conduction via helium gas at low pressure. A magnetic field is now switched on, until it attains some value H_1. In this process the sample becomes magnetized and energy is released in orienting the magnetic moments of the ions along the direction of the magnetic field (lowering of entropy, increase of order).

During the magnetization, the sample gives off heat to the bath, as the ion's magnetic potential energy lowers. The sample remains at temperature T_1 after equilibrium has been reached. The sample is then thermally insulated (e.g., by pumping off the helium gas which provided the contact with the bath) and the magnetic field is reduced to a lower value H_2. While the first step of *isothermal magnetization* is an "entropy squeezing" operation, like compressing a gas, the second step, namely the *adiabatic demagnetization*, takes place at *constant entropy*.

For the entropy to remain constant, a "disorder-increasing" process like demagnetization, in which orientations of ionic magnets are *randomized*, must be compensated by a "disorder decreasing" process, which can only be a drop in temperature. Thus, the temperature of the sample drops to $T_2 < T_1$. The sample is then put in contact again with the heat bath (still at $H = H_2$) and allowed to absorb heat from it, until the sample has returned to $T = T_0$. If this process is repeated several times, the continual absorption of heat from the sample's environment can result in the attainment of temperatures as low as $0.01\,^\circ$K. Indeed, temperatures close to 10^{-6} °K have been achieved by elaboration of this method. In the 1990's, other cooling methods — using combinations of lasers, RF, magnetic fields and gravity — were developed. The latest low-temperature record (2004) using such techniques is about 0.5 pico-Kelvins.

1926–1938 CE **Llewellyn Hilleth Thomas** (b. 1903, England). Physicist. Discovered a relativistic kinematic phenomenon, according to which a spinning *and* orbiting relativistic mass (electron, say) exhibits a precession of its axis of spin, such that the ratio of the precessional period to the orbital period[610] is $2c^2/v^2$. The effect is known as '*Thomas precession*' and is a special case of the more general '*Wigner rotation*'[611] (Wigner, 1939): an accelerated frame *rotates*, relative to an inertial frame, even in the absence of torques as measured in the instantaneous inertial frame. A comoving observer will

610 This expression is modified when $\frac{v}{c}$ is of order unity; it is valid as it stands for $v \ll c$.

611 *Wigner's rotation*: Consider 3 STR reference frames S, S' and S'', with S and S' related by a Lorentz transformation $L(\boldsymbol{v}_1)$, and S' and S'' related by $L(\boldsymbol{v}_2)$. If the velocity of S'' relative to original system S is $\boldsymbol{v}_3$, S'' is *not* obtained from S by $L(\boldsymbol{v}_3)$ alone. Rather, we find that $L(\boldsymbol{v}_3) = RL(\boldsymbol{v}_2)L(\boldsymbol{v}_1)$, where R is a 3D space rotation. With $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ not parallel, $R \neq I$ and the final system S'' is *rotated* relative to S (*Wigner's rotation*). This rotation is the origin of the *Kinematical Thomas precession* which is manifested as a spin-orbit coupling terms in atomic and nuclear physics. Because of the presence of the *Wigner rotation*, the $L(\boldsymbol{v})$ ("pure boosts") themselves do not form a group – but together with the Lie group of 3D rotations (SO(3)), the set of boosts $L(\boldsymbol{v})$ *do* form a Lie group: the 4D Lorentz group (denoted SO(3, n)), of which SO(3) forms a subgroup.
Explicitly, $L(\boldsymbol{v}) = I - \sigma \operatorname{sh}\theta + \sigma^2(\operatorname{ch}\theta - 1)$, where I is the unit tensor in 4

observe no centrifugal or Coriolis effects, yet an external observer would find that the frame, and any body at rest in it, is rotating.

An application of the Thomas precession is encountered in the fine structure of atomic levels, where it causes the magnetic field, as seen by the electron, in its instantaneous inertial frame, to be half as effective at causing the electron's spin to precess. When this result became known, it surprised many, including **Goudsmit**, **Pauli** and even **Einstein** himself.[612]

Thomas suggested (1938) a modification in the design of cyclotrons such that the magnetic field should be varied in azimuth (i.e. round the perimeter of the orbit), as well as in the radial direction. This AVF (Azimuthally Varying Field) concept has been realized in the '*spiral ridge cyclotron*', where especially strong magnetic field regions are produced by spiral sectors built into the pole faces. This provides extra focusing action to overcome the defocusing effects of the radial field variations.

1926–1939 CE **Alexander Aitken** (1895–1967, New Zealand and Scotland). Mathematician. Influential researcher in statistics, numerical analysis and the theory of determinants and matrices; *Aitken's δ^2 process* and *Aitken's iterative interpolation* in numerical analysis, are named after him. Showed how invariant theory came under the theory of groups.

Aitken was born in Dudedin, New Zealand. His university career was interrupted by WWI; he enlisted (1915) and served in Gallipoli, Egypt and

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dimension, $\operatorname{ch}\ \theta = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \gamma(\boldsymbol{v}), \quad \operatorname{sh}\theta = \frac{v}{c}\gamma, \ \ v = |\boldsymbol{v}|$, and,

$$\sigma = \begin{bmatrix} 0 & 0 & 0 & -\lambda \\ 0 & 0 & 0 & -\mu \\ 0 & 0 & 0 & -\nu \\ \lambda & \mu & \nu & 0 \end{bmatrix},$$

with (λ, μ, ν) the direction cosines of $\boldsymbol{v}$. The parameters of the above–defined Wigner rotation are: $R = R(\boldsymbol{e}, \psi)$, $\boldsymbol{e} = \frac{\boldsymbol{v}_2 \times \boldsymbol{v}_1}{|\boldsymbol{v}_1 \times \boldsymbol{v}_2|}$, $\psi = 2\tan^{-1}\left\{\frac{\sin\phi}{\tau + \cos\phi}\right\}$, where $\boldsymbol{e}$ is a unit vector in the direction of the axis of rotation and ψ is the *Wigner angle*. Here, $\tau = \sqrt{\frac{(\gamma_1+1)(\gamma_2+1)}{(\gamma_1-1)(\gamma_2-1)}}$ $\big(\gamma_1 = \gamma(v_1),\ \gamma_2 = \gamma(v_2)\big)$ and ϕ the angle between $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$. For a more detailed analysis see **A. Ben-Menahem**, "Wigner's rotation revisited", Am J. Phys. **53**, 62–66, 1985.

612 A gravitational variant of Thomas precession is known as "geodetic precession" and is one of the GTR effects being investigated in the relativistic gyro (gravity Probe B) experiment, launched into orbit in 2004.

France, being wounded at the battle of the Somme. His war experience was to haunt him for the rest of his life. Aitken came to Edinburgh, Scotland (1923) and studied for a Ph.D. under **Whittaker**, to whose chair he was appointed in 1946. He spent the rest of his life there.

Aitken had an incredible photographic memory; he knew π to 2000 places[613] and could instantly multiply, divide and take roots of large numbers. But his memory was also a problem for him. For most people memories fade in time which is particularly fortunate for the unpleasant things that happen. However, with Aitken memories did not fade and his horrific memories of the battle of the Somme lived with him as real as the day he lived them. These memories have contributed to the ill health he suffered and eventually led to his death.

1926–1939 CE **Eugene Paul Wigner** (1902–1995, Hungary, Germany and U.S.A.). Influential mathematician and physicist whose contributions had a decisive impact on modern theoretical physics, especially in quantum mechanics and nuclear physics.

In 1926 he was first to apply *group theory* to quantum mechanics. In 1927 he created the concept of *parity*[614] for atomic states and formulated

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613 Before the days of computing machines there was a kind of human competition in seeing how far one could calculate π. In 1873, **Shanks** carried this out to 707 decimals; it was not until 1948 that it was discovered that the last 180 of them were wrong. In 1927, Aitken had memorized those 707 digits for an informal demonstration to a student society and naturally was chagrined in 1948 that he had memorized something erroneous. When π was calculated anew to 1000 decimals Aitken rememorized it, but had to suppress his earliest memory of those 180 erroneous digits which he could not forget! For this reason, Aitken found it much harder to recite the *second* 500 digits.

614 For all isolated atomic or nuclear systems, *parity* is defined as the multiplicative sign acquired by their wavefunction upon reflection of all coordinates w.r.t. the origin: *even* or positive parity for a "+" sign, *odd* or negative parity for a "−" sign. The parity of any state remains unchanged throughout its time evolution, provided that the Hamiltonian of the system is unchanged by the reflection of the coordinates (*conservation of parity*). The concept of parity was later extended to quantum field theories. The parity of a field or a system of particles is not just a matter of whether the relevant wavefunction is an even or odd function of spatial position: in analogy with angular momentum, particles and their fields often also possess *internal* parity. Thus, the *electric* and *magnetic* field transform as *polar* and *axial* vector fields, respectively:

$$\boldsymbol{E}(\boldsymbol{r},t) \longrightarrow -\boldsymbol{E}(-\boldsymbol{r},t)$$

$$\boldsymbol{B}(\boldsymbol{r},t) \longrightarrow \boldsymbol{B}(-\boldsymbol{r},t)$$

the *law of conservation of (or, equivalently, invariance under) parity*. In 1932 he evolved the concept of invariance under time reversal, that marks the behavior of subatomic particles[615]. In 1936 he worked out the theory of neutron absorption which later proved important in nuclear fission and the operation of nuclear reactors.

His book on group theory (1931) served as a basis for research on the molecular structure and band theory in solids.

With his Hungarian compatriots **Leo Szilard** and **Edward Teller**, Wigner helped persuade **Albert Einstein** (1939) to write the historic letter to President F.D. Roosevelt that set in motion the U.S. atomic bomb project. During WWII, he helped **Enrico Fermi** construct the first atomic pile.

Wigner was born in Budapest to a Jewish family. He received his higher education at the Technische Hochschule in Berlin. In 1930 he moved to the United States, and spent most of his academic life at Princeton University as a professor of mathematical physics (1938–1971). In 1963 he won the Nobel prize in physics.

1926–1944 CE **John von Neumann**[616] (1903–1957, Hungary and U.S.A.). A phenomenal mathematician (both pure and applied) who made major contributions in quantum physics, computer science, game theory and meteorology. About 20 of his 150 papers are on physics. The rest are distributed evenly throughout pure mathematics [set theory, logic, topological

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the *pseudoscalar pi*–meson fields have intrinsic parity values of (−1), etc. In the spirit of *Noether's theorem*, one may view the conservation of the parity quantum number either as a conservation law *or* as a symmetry law. (The theorem refers to continuous symmetry groups, and thus does not strictly apply to discrete symmetries such as parity — although it can be modified to accommodate them.) Electromagnetic, gravitational and strong nuclear forces conserve parity; the world of physics was astonished when it was discovered in the 1950's that the *weak nuclear* force *violates* this symmetry.

[615] Again, with the exception of (some forms of) the weak nuclear interactions.

[616] For further reading, see:

- Von Neumann, J., *Theory of Self-Reproducing Automata*, Urbana, Ill, University of Illinois Press, 1966.
- Von Neumann, J., *The Computer and the Brain*, Yale University Press: New Haven, CT, 1958, 82 pp.

groups, measure theory, ergodic theory, operator theory (*von Neumann's algebras*)] and applied mathematics [statistics, numerical analysis, shock waves, flow problems, hydrodynamics, aerodynamics, ballistics, problems of detonation, meteorology, games and computers].

Von Neumann's thought processes were rapid, and his associates often found it difficult to keep up with his rapid flow of ideas. He was also a linguist, and could converse in 7 European languages. He preferred general to special problems, and rarely worried about mathematical elegance[617]. He got to the root of the matter by concentrating on the basic properties (axioms) from which all else follows. His insights were illuminating and his statements precise.

His book "*Theory of Games and Economic Behavior*" (1944, jointly with **Oskar Morgenstern**) had a significant influence upon economics. In the framework of their theory, the authors modeled the behavior of individuals in economic and adversarial situations. An individual's choices are ranked according to some payoff function, which assigns a numerical worth to the consequences of each choice.

Within game theory, individuals behave rationally: they choose the action that yields the highest payoff[618]. (Real people may not be consistently

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617 In connection with a long-winded but straightforward proof, he is quoted as saying that he *did not have the time to make the subject difficult*.

618 Beginning in the second half of the 20^{th} century, powerful computers have been pressed into service to simulate of *social behavior of groups* in an overall effort to understand the *dynamics of social dilemmas*. A social dilemma involves a group of people attempting to provide themselves with a common good in the absence of central authority. The computer experiments gloss over the complexities of human nature, but it is believed that they can help elucidate some of the principles that govern interaction of many participants. The results indicate that overall cooperation cannot generally be sustained in groups that exceed a *critical size*. The size depends on how long individuals expect to remain part of the group as well as on the amount of information available to them. Moreover, both general cooperation and defection can appear *suddenly and unexpectedly* (nonlinear phenomena!). These results can serve as aids for interpreting *historical trends* and as guidelines for constructively reorganizing cooperations, trade unions, governments and other group enterprises.

To study the evolution of social cooperation, methods were borrowed from *statistical thermodynamics*. This branch of physics attempts to derive the macroscopic properties of matter from the interaction of its constituent molecules and other quanta.

Diversity strongly affects the dynamics of social dilemmas. A heterogeneous group can display two different types of diversity: variations around a common rational, but they do behave that way when presented with simple choices and straightforward situations.) In 1929 he gave *Hilbert spaces* their name, their first axiomatization and their present highly abstract form.

Neumann's war effort convinced him of the need for high-speed computers. He was instrumental in the development of *MANIAC* (Mathematical Analyzer Numerical Integrator And Computer). In computer theory, he did much of the pioneering work in logical design, in the problem of obtaining reliable answers from a machine with unreliable components, the function of *memory*, machine imitation of *randomness*, and the problem of constructing automata that can reproduce their own kind. These contributions were important in the development of the hydrogen bomb.

Neumann was born in Budapest to a Jewish family. Many anecdotes, from childhood on, tell of his phenomenal speed in absorbing ideas and solving problems, and of his equally phenomenal memory. He received his high school education at the *Minta*, the famous Budapest school, known for its high standards, liberal teacher-pupil relations and competitive training for its science students[619]. He studied chemistry at the University of Berlin and at

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average or segregating into factions. Relations within subgroups may powerfully influence the evolution of cooperation, especially in large *hierarchical organizations*.

The study of social dilemmas provides insight into a central issue of behavior: how *global cooperation* among individuals confronted with conflicting choices can be secured. Recent studies (1994) have shown that cooperative behavior can rise *spontaneously* in social settings, provided that the groups are small and diverse in composition and that their constituents have *a long outlook*. Even more significantly, when cooperation does appear, it does so *suddenly and unpredictably* after a long period of stasis.

The fall of the Berlin wall and the breakdown of the centralized Soviet Union into many autonomous republics are examples of abrupt global defections from prevailing social compacts. The member countries of the European union currently face their own social dilemma as they try to secure supranational cooperation.

Hopefully, nonlinear dynamics will play an ever growing role in solving problems in the social sciences.

619 Most of the other famous expatriate Hungarian scientists, such as: **Edward Teller** (b. 1908), **Leo Szilard** (b. 1898), **George Polya** (b. 1888), **Theodore von Kármán** (b. 1881), **Eugene Wigner** (b. 1902), **Michael Polanyi** (b. 1891) and **Georg von Hevesy** (b. 1885) also graduated from this school. All eight were of Jewish middle class stock. All left Hungary as young men; all proved unusually versatile and made major contributions to 20^{th} century science and technology. Two among them eventually won Nobel prizes (see Table 5.6).

the Technische Hochschulle in Zürich. In 1926 he received both a diploma in chemical engineering and a Ph.D. in mathematics from the University of Budapest. He accepted a chair at Princeton University in 1931. In 1933 he was appointed the first professor of mathematical physics at the newly formed Institute of Advanced Study at Princeton. In 1954 his health began to deteriorate, and he died after a prolonged illness.

Game theory — Analysis of Conflicts and Cooperation

(A) Overview and history

Game theory, one of the most useful branches of modern mathematics, was anticipated in 1921 by ***Émil Borel****, but it was not until 1926 that* ***John von Neumann*** *gave his proof of the* minimax theorem*, the fundamental theorem of game theory. On this cornerstone he built almost single-handedly the basis structure of the theory. His classic work (1944),* Theory of Games and Economic Behavior*, written with the economist Oscar Morgenstern, created a tremendous stir in economic circles. Since then game theory has developed into an amalgam of algebra, geometry, set theory, and topology, with applications to competitive and cooperative situations in business, warfare, politics, biology and economics. It has been employed by folks as diverse as philosophers, political scientists, arms-control negotiators and evolutionary biologists.*

Game-theoretic analysis has insidiously penetrated the literature of macro- and microeconomics, international trade, labor, public policy, natural resources and development. Recurring themes include: threatening, bluffing, punishing, rewarding, building reputations, signaling one's unobservable "type" and sustaining cooperation in apparently noncooperative environment through repeated interactions. Answers through Game Theory have been sought to such questions as: "What is the nation's optimal strategy in the Cold War Game?"; "Is the Golden Rule the best strategy for maximizing happiness payoffs in the Great Game of Life?"; "How can a scientist best play the Induction Game against his formidable opponent, Nature?"

The theory of games in intended mainly as a theory of rational behavior *in practical situations that involve conflicting interests and in which the outcome* *is determined by the "best" strategies chosen by intelligent opponents. The players may be commercial rivals engaged in a competition, union leaders and management in an industry trying to reach a labor contract, campaign managers of candidates trying to get elected to a political office, military generals in an army trying to win a war, attorneys in a lawsuit trying for the best settlement for their clients, and so on; The theory of games finds applications in all situations in which parties involved have opposing goals, use their best strategy*[620]*, and yet cannot completely dominate the final outcome.*

In the life sciences, game theory is used as a mathematical tool for understanding the behavior of a species and how it evolved and interact with other species.

A game[621] *begins with, and is centered around, a specified set of decision makers who are called "players". Each player has some array of resources at his disposal, some spectrum of alternative courses of action (including attempts to communicate and collaborate), and some inherent system of preferences or utility concerning the possible outcomes. Outside the game model all players are alike — the theory refuses to distinguish among them. This is known as the principle of* external symmetry*. In such game models it will be always assumed that the identified players are rational, conscious decision makers having well-defined goals and exercising freedom of choice within prescribed limits. What they do with that freedom is a question for the solution theories to answer, not the rules of the game. The answers will generally fall far short of a deterministic prescription of behavior. Despite this essential element of free will, the rules of the game may nevertheless severely restrict the player's behavior.*

Virtually all game models involve a special kind of uncertainty that is caused by not knowing what other players are going to do: this is called strategic uncertainty*. Some games also allow random moves, by players or by nature, thus bringing the factor of* risk *(uncertainty with known probabilities) into the player's calculations. Historically, game theory has operated for the*

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620 *Strategy*: a programmed way of behavior in pairwise contests. In the context of *chess*, for example, or war, strategy suggests a nicely calculated sequence of moves. The term *two-person* means that two players with conflicting interests are making use of their ingenuity to outwit each other. *Zero sum* means that any loss of one player is the gain of another.

621 The word "*game*" was an unfortunate choice for a technical term. The usual sense of the word has connotations of fun and amusements, and of removal from the mainstream and the major problems of life. These connotations should not be allowed to obscure the more serious role of game theory in providing a mathematical basis for the study of conflict, competition and cooperation.

most part under the assumption of complete information: all the players know all the rules and can make all necessary calculations[622].

The modern history of Game Theory goes back to the 18th century: ***James Waldegrave*** *(1684–1741, England) provided (1713) the first known minimax mixed strategy solution to a two-person game.* ***Augustin Cournot*** *(1838) discussed the special case of duopoly and utilized a solution concept that is a restricted version of the Nash equation.* ***Francis Edgeworth*** *(1881) proposed the contract curve as a solution to the problem of determining the outcome of trading between individuals. The concept of the core is a generalization of Edgeworth's contract curve.*

Zermelo *(1913) asserted (Zermelo's Theorem) that chess has only one individually rational payoff profile in pure strategies, i.e. chess is strictly determined.* ***Borel*** *(1921–1927) gave the first modern formulation of a mixed strategy along with finding the minimax solution for specific two-person games.* ***John von Neumann*** *(1928) proved that every two-person zero-sum game with finitely many pure strategies for each layer is determined, i.e. when mixed strategies are admitted this variety of game has precisely one individually rational payoff vector.*

Game theory continued its rapid development after WWII; The game now known as the Prisoners Dilemma was introduced at the Rand Corporation in California (1950) and in the same year ***John Nash*** *made important contributions to both non-cooperative game theory and bargaining theory. He proved the existence of a type of strategic equilibrium — the Nash equilibrium for non-cooperative games. The first explicit application to evolutionary biology was made by* ***R.C. Lewontin*** *(1961).*

(b) Theoretical background

We consider non-cooperative zero-sum games:

A zero-sum game is one in which the pay-offs to all the players add up to zero (similarly for a constant-sum game). Such games can be completely specified by a rectangular array of numbers (known as the game or payoff matrix) together with certain conventions on how to read it. The entries a_{ij} of this matrix represent the pay-offs to one of the players, call him R. Since

622 Since 1957, progress has been made in extending the theory to game-like situations in which the rules are incompletely known, such as forecasting of demand (i.e. unknown preferences).

the game is zero-sum, the pay-offs to the other player, C *are* $-a_{ij}$. *The index* i *conventionally ranges over* R*'s strategies (i.e. rows of the matrix), while* j *ranges over* C*'s strategies (i.e. columns of the matrix).*

To be more specific, it is assumed that player R *has a choice of* m *moves, which may be identified as* $R_1, R_2, ..., R_m$. *After he has selected a move, player* C *(column), makes his own move (based on whatever knowledge characterizing the game, and on whatever strategy or algorithm) among several alternatives* $C_1, C_2, ..., C_n$. *The moves of the two players are then compared and the winner declared according to the rules of the game. The payoffs, which are assigned numerical values, are generally presented in the form of an* $m \times n$ *matrix* $A = [a_{ij}]$ *such as the following:*

$$\text{player } R \quad \overset{\text{player } C}{\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ & & \dots & \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}}$$

Each row of the matrix corresponds to one of the m *possible moves of player* R, *while each column corresponds to one of* n *possible choices* C *may move for his own move. The element* a_{ij} *represents the payoff to* R *when he selects move* R_i *and* C *selects move* C_j. *Positive amounts correspond to payments* C *makes to* R *(or benefit occurring to* R *at the expense of* C*), while negative amounts represent amounts or benefits player* R *makes or loses to player* C.

An important question that arises in the theory of games is whether one can determine a best move for players who wish to maximize their gains *or at least* minimize their losses, *i.e. whether one player should prefer one move over another to maximize his resulting advantage.*

To illustrate, let us consider the following example, in which we propose to determine the best move for each player.

R *and* C *agree to play a game: First* R *chooses a number from a set* $\{1,2,3\}$, *then* C *(not knowing the choice of* R*) chooses a number from the set* $\{1,2,3,4\}$. *The numbers are then compared by a referee who determines the winner and announce the reward or penalty according to the following* payoff matrix *(ignore the circle and cross-hairs for now):*

$$\begin{array}{cc} & C \\ & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ R \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 4 & 2 & 3 & 9 \\ 5 & 6 & \textcircled{5} & 7 \\ 6 & 8 & 4 & 3 \end{bmatrix} \end{array}$$

Assuming that both players wish to maximize their profits, the best move for each player is determined by the following rationale. First consider the game from R's point of view:

- *If I choose the first row, I am sure to win at least \$2, no matter what C will do. If I chose the second row, I'm guaranteed at least \$5 and if I choose the third row, I will gain at least \$3. These minima are* $\{2, 5, 3\}$*; so my best move is to choose the maximum of these minima — i.e.* $\underset{i}{max}\ \underset{j}{min}\ [a_{ij}] = 5$. *I therefore select the second row.*

  C, unaware of R's choice, argues as follows:

- *If I choose the first column, I will lose at most \$6; likewise \$8 in the second column, 5 in the third, and \$9 in the fourth column. Since I'm interesting in minimizing my losses, I must choose the minimum of the maxima;* The maxima are $\{6, 8, 5, 9\}$, *i.e.* $\underset{i}{max}\ \underset{j}{min}\ [a_{ij}] = 5$. *Therefore I select the third column.*

Thus R would play row 2 while C would play column 3. *Those choices guarantee R to win* 5 *dollars – the amount that C loses in the game.*

As long as either player uses his optimal strategy he is sure to receive a payoff equal to or better (from his perspective) than 5.

Note that the intersection of the second row and the third column indicated in the above matrix is the element that holds the minimax = maximin. In other words, the element $a_{23} = 5$ *which determines the winning of R (and the loss of C) has the unique property of being the minimum in its row and the maximum in its column. Such an entry is called a saddle point and the numerical value* $V = 5$ *associated with it is the value of the game. In this case the game matrix A is said to be strictly determined.*

If $V = 0$ *the game is called fair (e.g. with* $A = \begin{bmatrix} 0 & 2 & 4 \\ -2 & 0 & 2 \\ -4 & -2 & 0 \end{bmatrix}$, $V = a_{11} = 0$).

The mode of play of R and C in this case is known as pure strategy with R having a dominant strategy. It appears that the solution of a strictly determined game is easy to find, since each player can determine his opponents strategy and make his moves accordingly. However, most payoff matrices do not have saddle-points and the theory must be extended. i.e. one must consider general conditions under which games have values but players do not have good pure strategies. Consider, for example, the following pay-off matrix that has no saddle point,

$$A = \begin{bmatrix} -1 & 7 \\ 6 & -2 \end{bmatrix}$$

If players choose to play this game only once, there exist no professional advice for them at this stage and they will have to use their own rational thinking as to what strategy is best for them.

But suppose that the game is played more than once. A reasonable approach for player R in the above game is to choose row 1, since he has a chance of winning \$7 or may at worst lose \$1 in some cases. If he uses this strategy too often and plays it most of the time, his opponent can foil him by choosing column 1 and thus receive \$1 from him instead of loosing \$7 to him. It is thus reasonable for R to sometimes play row 2 so as to win \$6 at best or loose \$2.

Thus, C would be advised to chose column 1 if his opponent selects row 1 and select column 2 if R chooses row 2 This would work very nicely for C if he knew in advance precisely what R plans to do – which he generally does not.

How then can each player outsmart his opponent? The only way for them is to mix their strategies so as not to establish any pattern at all. The mixed strategies will keep R from winning two much, and protect C from losing too much, in the long run.

Let us consider a version of a game known as two-finger Morra; there are two players (R and C). Each player holds up either one or two fingers. If they hold up the same number of fingers, R gets the sum (in dollars) of the digits, and if they hold up a different number, C gets the sum. R's pay-off matrix is

$$\begin{array}{ccc} & & C \\ & & \begin{array}{cc} \text{1 finger} & \text{2 finger} \end{array} \\ R & \begin{array}{c} \text{1 finger} \\ \text{2 finger} \end{array} & \begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix} \end{array}$$

Neither R nor C has a dominant strategy, and the maximin of R (-3) in not equal to the minimax of $C(2)$. This means that the game is not solvable in terms of pure strategies.

Suppose that R will show 1 finger for a fraction p_1 of the time and 2 fingers for a fraction p_2 of the time, such that his average pay-off will be $(2p_1 - 3p_2)$ if C shows 1 finger and $(-3p_1 + 4p_2)$ if C shows 2 fingers. But this only holds if C does adopt pure strategy. However, if C adopts a mixed strategy of his own, say with probabilities $[q_1, q_2]$, the average pay-off to R will be $(2p_1 - 3p_2)q_1 + (-3p_1 + 4p_2)q_2$ where $q_1 + q_2 = 1, p_1 + p_2 = 1$. The expected value of the game, $E(P, Q)$, can be written conventionally as the matrix product

$$E(P,Q) = [p_1, p_2] \begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = PAQ^T$$

Where P is a row vector of probabilities that the row player selects for his strategies, and Q^T is the column vector of probabilities the column player selects for his own strategies. The object of the row-player is to choose P so as to maximize his expected pay-off (PAQ^T) given that the column player can choose any Q^T; while the column player tries to minimize it, given that the row player van choose any P.

This is just a generalization of the former maximin and minimax of pure strategy cases. For any given vector P, R sees what happens when C picks the $Q = Q^*(P)$ that does R worst damage (=least benefit) and subsequently picks that P such that, when $Q = Q^*(P)$ is selected by C, will do R the least harm (or most good). This strategy for R is achieved by the equality $2p_1^* - 3p_2^* = -3p_1^* + 4p_2^*$ yielding $p^* = \left[\frac{7}{12}, \frac{5}{12}\right]$.

Player R should therefore play one finger with probability $\frac{7}{12}$. If R adopts this strategy his expected pay-off is $2p_1^* - 3p_2^* = -\frac{1}{12}$ whatever C does.

So the best R can guarantee himself via his statistical strategy, is to expect to make on the average a small loss[623] *from every play of the game.*

The strategy for C is given by the solution of the equation $2q_1^* - 3q_2^* = -3q_1^* + 4q_2^*$, namely $Q^* = \left[\frac{7}{12}, \frac{5}{12}\right]$. C's expected loss is then $2q_1^* - 3q_2^* = -\frac{1}{12}$ i.e. a gain of $\frac{1}{12}$.

As long as R plays his best strategy, the 7 : 5 mixture, he holds his average loss per game to at most $\frac{1}{12}$ of a dollar. As long as C plays his best mixture, the 7 : 5, he ensures an average win per game of at least $\frac{1}{12}$ of a dollar. Had

623 Pay-offs are always given as payments from C to R even when the money (or benefit) actually goes that way, in which case the payment to R is indicated by a minus sign.

the pay-off matrix been $\begin{bmatrix} 1 & -2 \\ -7 & 8 \end{bmatrix}$, *the best strategy for* R *would be the mixture 5:1 and that of* C $5:4$, *with* $E(P,Q) = -\frac{1}{3}$.

In the general case of 2×2 *nonstrictly determined game matrix* $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ *one has*[624] $(a,d) > (b,c)$ *or* $(b,c) > (a,d)$, *where the strategies of the players* R *and* C *are* $P = [x, 1-x]$ *and* $Q = [y, 1-y]$ *respectively, the expected value of the game is*

$$E(x,y) = [x, 1-x] \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} y \\ 1-y \end{bmatrix}$$
$$= (a+d-b-c)xy + x(b-d) + y(c-d) + d$$
$$= (a+d-b-c)(x-x_0)(y-y_0) + E(x_0,y_0),$$

where

$$x_0 = \frac{d-c}{a+d-b-c} \text{ is a solution of } \quad ax_0 + c(1-x_0) = bx_0 + d(1-x_0)$$

$$y_0 = \frac{d-b}{a+d-b-c} \text{ is a solution of } \quad ay_0 + b(1-y_0) = cy_0 + d(1-y_0)$$

The explicit value of the game is

$$E(x_0,y_0) = [x_0, 1-x_0] \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} y_0 \\ 1-y_0 \end{bmatrix} =$$
$$= ax_0 + c(1-x_0) = \frac{ad-bc}{a+d-b-c}$$

From the algebraic form of $E(x,y)$ *it is clear that* R*'s choice of the (probabilistic) strategy,* $x = x_0$ *assures him of a maximal expectation of* $E = E(x_0,y_0)$, *no matter what* C*'s strategy is. But whereas in the case of pure strategy he was guaranteed a maximum payoff, here he selects a mixed strategy that will guarantee him the maximum expectation or maximum average winning in the long run. This is* R*'s optimum strategy. On the other hand, if* C *chooses the strategy* $y = y_0$, *no strategy of* R *can increase* C*'s average loss beyond* $E(x_0,y_0)$.

Thus the mixed strategy $y = y_0$ *is optimum for* C, *since it corresponds to* $E(x_0,y_0)$, *the minimum expected payoff to* R.

Note that $E(x,y)$ *has a saddle-point at* $x = x_0$, $y = y_0$, *since* $\Delta = E_{xx}E_{yy} - E_{xy}^2 = -(a+d-bc)^2 < 0$. *In this sense, the case of mixed strategies is a*

[624] In this notation $(x,y) > (z,w)$ means $min(x,y) > max(z,w)$.

generalization of the simple case of pure strategies: the equal minimax and maximin are associated with an expectation function $E(x, y)$ rather than with the payoff itself.

Mixed strategy introduced an all-important aspect of game theory: to be effective, the mixing must be done by a randomizing device. It is easy to see why nonrandom mixing is dangerous; Suppose that in two-finger Morra, R mixes by using the pattern $\{1\ 1\ 2\ 1\ 1\ 1\ 2\ 2\ 1\ 1\ 2\ 2\}$. C catches on and plays to win every time. R can adopt a subtler mixing pattern, but there is always a chance that C will discover it. If he tries to randomize in his head, unconscious biases creep in. The only way to achieve optimal strategy is to use a randomizer (die or machine).

Von Neumann proved that in every matrix game, regardless of size, there exists at least one optimal strategy for each player. This fundamental proposition is called the minimax theorem. It states that for all matrix orders $[\boldsymbol{x} = (x_1, x_2, \dots, x_n); \quad \boldsymbol{y} = (y_1, y_2, \dots, y_n)]$

$$\underset{\text{for all } \boldsymbol{x}}{\textit{maximum}} \left[\underset{\text{for all } \boldsymbol{y}}{\textit{minimum}}\ E(\boldsymbol{x}, \boldsymbol{y}) \right] = \underset{\text{for all } \boldsymbol{y}}{\textit{minimum}} \left[\underset{\text{for all } \boldsymbol{x}}{\textit{maximum}}\ E(\boldsymbol{x}, \boldsymbol{y}) \right]$$

Most two-person board games, such as chess and checkers, are played in a sequence of alternating moves that continues, until either one player wins or the game is drawn. Since the number of possible sequences is vast and so is the number of possible strategies, the matrix is much too enormous to draw. Even as simple a game as ticktacktoe would require a matrix with tens of thousands of cells, each labeled 1, -1 or 0.

If the game is finite (each player has a finite number of moves and a finite number of choices at each move) and has perfect information (both players know the complete state of the game at every stage before the current move), it can be proved (von Neumann was the first to do it) that the game is strictly determined. This means that either there is at least one best pure strategy that always wins for the first or for the second player, or that both of the players have pure strategies that can ensure a draw.

Example 1: Aerial Warfare

White repeatedly sends two-plane missions to attack one of Blue's installations. One plane carries bombs, and the other (identical in appearance) flies *cover for the plane carrying the bombs. Suppose the lead plane can be defended better by the guns of the plane in the second position than vice versa, so that the chance of the lead plane surviving an attack by Blue's fighter is 80 %, while the chance of the plane in the second position surviving such an attack is only 60 %. Suppose further that Blue can attack just one of White's planes and that Blue's sole concern is the protection of his installation, while White's sole concern is the destruction of Blue's installation. Which of White's planes should carry the bombs, and which plane should Blue attack?*

Let White's payoff be the probability of accomplishing the mission. Then

$$A = \begin{bmatrix} 0.8 & 1 \\ 1 & 0.6 \end{bmatrix}, \qquad p_1 = \frac{0.4}{0.6} = \frac{2}{3}; \quad p_2 = \frac{0.2}{0.6} = \frac{1}{3}$$

Thus always putting the bombs in the lead plane is not White's best strategy, although this plane is less likely to be shot down than the other. In fact, if White always puts the bombs in the lead plane, then Blue will always attack this plane and the resulting probability of the mission succeeding will be 0.8. On the other hand, if White adopts the optimal mixed strategy and puts the bombs in the lead plane only two times out of three he will increase his probability of accomplishing the mission by $\frac{1}{15}$*, since the value of the game is:*

$$v = \frac{2}{3} \cdot \frac{8}{10} + \frac{1}{3} \cdot 1 = \frac{13}{15}$$

By the same token, Blue's best strategy is to attack the lead plane only one time out of three and the other plane the rest of the time.

Example 2: The Prisoner's Dilemma

Two prisoners kept in separate cells are asked to confess a joint crime. If both confess, both remain in jail for 9 years. If only one confesses, he will instantly be freed (as witness for the prosecution), while the other gets sentenced to 10 years. If none of them confess, both will be sentenced for one year, pending investigation. Writing the pay-offs as -n (for n years in prison), we get for R

$$\begin{array}{ccc} & & C & \\ & & \textit{confess} & \textit{keep quiet} \\ R & \begin{array}{c}\textit{confess} \\ \textit{keep quiet}\end{array} & \left[\begin{array}{c} -9 \\ -10 \end{array}\right. & \left.\begin{array}{c} 0 \\ -1 \end{array}\right] \end{array}$$

The value of the game is -9 *(saddle point), indicating that the best policy for both* R *and* C *is to confess which is worse than* $(-1, -1)$ *if both keep quiet. Most people would regard the latter as the 'best' solution but the self-interest of the two 'players' leads to an outcome which is disastrous for both.*

This game has exercised an overwhelming fascination upon game theorists and psychologists. It encapsulates some of the major dilemmas in conflict situations and also models problems as diverse as nuclear disarmament*, predator/pray competition, "arms races" in evolutionary biology,* wage negotiation*, and the controversy of* whooping cough vaccinations*. The first dilemma is what should be the player's objective – to do what is best for* him *as an individual, or* him *as part of a group? This conflict is between* individual rationality *which would lead one to confess, and* group rationality *which would suggest keeping quiet. Which tendency predominates depends very much on the individuals involved, and their previous experience with other people, including each other. This obviously explains psychologist's interest in the game.*

The second problem is whether to think of Prisoner's Dilemma as a one-off game or as one that will be played repeatedly*. In a one-off game it seems best to confess, because there is no reason to build up ones opponent's trust in oneself. If, however, the number of games to be played is* not known *by the players the 'keep silent' strategy should be played all the time!*

Example 3: Evolutionary Biological systems – battle of the sexes

Much of game theory is static in the sense that time does not enter explicitly and dynamic considerations, if any, are implicit. In certain situations, however, the underlying dynamics can be modeled by nonlinear differential equations *of the Volter-Lotka type.*

In our section on zero-sum games, It was shown that the expected value of the game for the case of mixed strategies was given by the mathematical expression

$$E(P, Q) = \sum_{ij} p_i a_{ij} q_j$$

with a_{ij} *the pay-off matrix and* $\{p_i\}$, $\{q_j\}$ *the strategies (probability patterns) of the two players. When applying the theory to evolutionary biological systems, one must interpret the relevant entities in accord with the following "dictionary":*

probabilities	$\longrightarrow$	*frequencies of phenotypes in the population, or of behavioral strategies for given phenotype*
pay off matrix	$\longrightarrow$	*Darwinian fitness; eventually manifested in the number of offsprings*
Realization of strategy	$\longrightarrow$	*phenotype (behavioral pattern determined by the genes or genotype)*
Pure strategy	$\longrightarrow$	*(e.g.:'produce only sons', 'produce only daughters' in the sex-ratio game.)*

Assume a population divided into n *phenotypes* $E_1, E_2, ..., E_n$ *with time-dependent frequencies* $x_1(t), x_2(t), \ldots, x_n(t)$. E_i *corresponds to a (pure or mixed) strategy, and its fitness* f_i *will be a function of the state* $\boldsymbol{x}(x_1, x_2, \ldots, x_n)$ *of the population.*

If the population is very large, and if the generations blend continually into each other, we may assume that the state $\boldsymbol{x}(t)$ *is a differentiable function of* t*. The fractional rate of increase* $\frac{\dot{x}_i}{x_i}$ *of the phenotype* E_i *is a measure of its evolutionary success. Following the basic ideas of Darwinism, we may express this success metric as the difference between the fitness* $f_i(\boldsymbol{x})$ *of* E_i *and the average fitness*

$$\overline{f}(\boldsymbol{x}) = \sum x_i f_i(\boldsymbol{x})$$

of the population (i.e., the so-called field of game theory). Thus we obtain

$$\dot{x}_i = x_i \left[f_i(\boldsymbol{x}) - \overline{f}(\boldsymbol{x}) \right], \quad i = 1, 2, ..., n.$$

This model does not explicitly treat sexual reproduction, which – together with competition, cooperation and conflict, is implicitly encoded within the f_i functions.

Consider a linear game with n strategies (i.e phenotypes) and pay-off matrix U. The pay-off for a "strategist" (category of phenotype) with genotype–determined strategy vector $\boldsymbol{p}^i$ in a field $\boldsymbol{p}^j$ opponents is

$$a_{ij} = \boldsymbol{p}^i \cdot U\boldsymbol{p}^j$$

and the fitness $f_i(\boldsymbol{x})$ of the phenotype E_i is the superposition (mean) expressed by

$$f_i(\boldsymbol{x}) = \sum_j a_{ij}x_j = (A\boldsymbol{x})_i$$

where A is the $n \times n$ fitness pay-off matrix. The differential equations for the linear game are then

$$\dot{x}_i = x_i\left[(A\boldsymbol{x})_i - \boldsymbol{x}A\boldsymbol{x}\right], \qquad i = 1, 2, 3, ..., n$$

In the case $n = 2$, by setting $x = x_1$ and $1 - x_1 = x_2$ we obtain

$$\dot{x} = x(1-x)\left[(A\boldsymbol{x})_1 - (A\boldsymbol{x})_2\right].$$

If $a_{ij} = -a_{ji}$ hold for all i and j, the game is a zero-sum game. In that case $\dot{x}_i = x_i(Ax)_i$.

Conflicts among animals (especially with heavily armed species) are often settled by displays rather then all out fighting, with escalated contests rare. The following thought experiment explains the high frequency of conventional contests, and with it evolutionary stability*, in terms of game theory.*

Consider a conflict between males and females concerning their respective share in parental investment*.*

In many species, raising an offspring requires a considerable amount of time and energy. Each parent might attempt to reduce its own share at the expense of the other. The outcome might depend on which sex is in a position to desert first. Whenever fertilization is internal, for example, females risk being deserted even before giving birth to the offspring. The game is still further "rigged" against females by the fact that they produce relatively few, large gametes, and males many small ones. Females are thereby much more committed and can less afford to lose an offspring. Thus males are in many cases in a better position to desert. They can invest the corresponding gain in *time and energy into increasing the number of their offsprings with the help of new mates*[625]*.*

The female counterstrategy is "coyness", i.e. the insistence upon a long engagement period before copulation. Rather than undergoing a second costly engagement (for which it might be too late in the mating season), males would do better to stay faithfully home and help raise their offspring. Roughly speaking, in a population of coy females, males would have to be faithful.

Among faithful males, it would not pay a female to be coy, however: the long engagement period is an unnecessary cost. Thus the proportion of "fast" females would grow. But then "philandering" males will have their chance and spread. Females, therefore, would do well to be coy. The argument thus runs full circle.

In order to model this game theoretically, let us assume that there are two phenotypes in the male population X*, namely* E_1 *("philandering") and* E_2 *("faithful"), with frequencies* x_1 *and* x_2*; and two phenotypes in the female population* Y*, namely* F_1 *("coy") and* F_2 *("fast") with frequencies* y_1 *and* y_2*. Let us suppose that the successful raising of an offspring increases the fitness of both parents by* G*. The parental investment* C *will be entirely borne by the female if the male deserts. Otherwise, it is shared equally by both parents. A long engagement period represents a cost* E *to both partners.*

If a "faithful" male mates with a "coy" female, the pay-off is $(G - \frac{C}{2} - E)$ *for both. A "faithful" male and a "fast" female skip the engagement cost and their pay-off is* $(G - \frac{C}{2})$*.*

But a "philandering" male meeting a "fast" female makes off with G*, while her payoff is* $G - C$*. Finally, if a "philandering" male encounters a "coy" female nothing much happens and their payoff for both is zero. The male (*A*) and female (*B*) payoff matrices therefore are*

$$A = \begin{bmatrix} 0 & G \\ G - \frac{C}{2} - E & G - \frac{C}{2} \end{bmatrix}; \quad B = \begin{bmatrix} 0 & G - \frac{C}{2} - E \\ G - C & G - \frac{C}{2} \end{bmatrix};$$

where: the rows of A *and columns of* B *are, in order,* E_1 *and* E_2*; whilst the columns of* A *and rows of* B *are the two female categories (*F_1 *and* F_2*).*

No pair of male and female phenotypes is evolutionary stable in conjunction. There does, however, exist a unique pair of mixed strategies $\boldsymbol{p}$ *and* $\boldsymbol{q}$

[625] The underlying assumption is that each individual's behavior aims – or is genetically programmed – to maximize his or her progeny. The genotypes *not* resulting in such a drive tend, of course, to have their frequencies severely curtailed over the generations.

(of males and females, respectively) in Nash equilibrium. It is given by the solution of

$$a_{11}q_1 + a_{12}q_2 = a_{21}q_1 + a_{22}q_2 \qquad (q_2 = 1 - q_1)$$

$$b_{11}p_1 + b_{12}p_2 = b_{21}p_1 + b_{22}p_2 \qquad (p_2 = 1 - p_1)$$

i.e. by

$$p_1 = \frac{E}{C - G + E}\,, \qquad q_1 = \frac{C}{2(G - E)}\,.$$

This equilibrium is not stable. If a fluctuation decreases the frequency of philandering males, then the payoff of the males will not change; each phenotype has the same payoff, which depends only on the state of the female population. One cannot expect the frequency of philanderers to return to p_1.

As to the female population, their payoff will even increase but "fast" females gain more than "coy" ones since their risk of being deserted decreases. It is only when the "fast" female population increases that the male payoff changes. Again, they increase: but philanderers gain more than faithful males; hence more philanderers, hence more coy females, hence fewer philanderers, and so on.

This looks like an oscillating system. The appropriate differential equations are

$$\begin{aligned} \dot{x} &= x(1-x)\left[\frac{C}{2} - (G-E)y\right] \\ \dot{y} &= y(1-y)\left[-E + (C+E-G)x\right] \end{aligned}$$

This resembles a Volterra-Lotka prey-predator system

$$[\dot{x} = x(a - by); \qquad \dot{y} = y(-c + dx)]$$

and, like it, is doomed to perpetual oscillations.

Example 4: The Voter's Paradox - or, is Democracy mathematically Sound?

In the following we review the applications of Game Theory to the social sciences from ***Condorcet*** *(1788) to* ***Arrow*** *(1951).*

Much of the economic and social behavior in which we are interested is either ***group behavior*** *or that of an individual acting for a group. Group preferences may be regarded either as derived from individual preferences by some process of aggregation or as a direct attribute of the group itself.*

Game-theoretic methods provide an intriguing alternative to treating a group as though it were a sentient individual: we can cast the members of the group as players in an internal organizational subgame, vying for control of the group's action in the larger game.

As early as 1785, the French mathematician ***Marie-Jean-Antoine-Nicolas Caritat*** *(****Marquis de Condorcet****) discovered that society often has collective preferences that, if held by an individual, would be dismissed as irrational.*

This is the famous Voter's Paradox: three individuals use simple majority rule to decide what to do as a group. Their personal preferences orderings are

First individual: $A > B > C$
Second individual: $C > A > B$
Third individual: $B > C > A$

In this example, if the choice is between policies A *and* B*, then the group will choose* A *by a* $2:1$ *vote. Similarly, they will choose* B *over* C*, and* C *over* A*. hence the preferences of the group are described by the relations*

$$A > B, \qquad B > C, \qquad C > A$$

But this relation is not transitive! Hence a group utility scale cannot be constructed. Given that all possible individual preferences are equally likely, the chance of intransitivity is $\frac{12}{3!^3} \approx 5.6$ *percent. This may not seem like much, but one must keep in mind that this percentage is only for the simplest case of three people and three alternatives. It turns out that the probability increase both as the number of alternatives increases and as the number of voters increase (being more sensitive to the number of alternatives) as the following table show:*

NUMBER OF ALTERNATIVES	NUMBER OF VOTERS					
	3	*5*	*7*	*9*	*11*	∞
3	5.6%	6.9%	7.5%	7.8%	8.0%	8.0%
4	11.1%	13.9%	15.0%	15.6%	16.0%	17.6%
5	16.0%	20.0%	21.5%	23.0%	25.1%	25.1%
6	20.2%	25.5%	25.8 %	28.4%	29.4%	31.5%
7	23.9%	29.9%	30.5%	34.2%	34.3%	36.9%
∞	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%

Clearly, group preferences come solely from the preferences of the individuals, unless one wishes other considerations to enter. In either case there is one simple and compelling condition called the principle of unanimity or the ***Pareto*** *Principle: If every member of the group prefers x to y, then the group itself, in its corporate judgment, also prefers x to y.*

This principle, obvious thought it may appear, has some far-reaching logical consequences. In some settings it may even lead to transitivity for the group preferences relation, contrary to what the Voters' Paradox would lead us to expect.

Building upon Condorcet's Voters' Paradox and the Pareto Principle, ***Kenneth Arrow*** *(1951) astounded mathematicians and economists alike with his "impossibility theorem" - a landmark in the application of rigorous mathematical methods to the social sciences.*

The thrust of the theorem is that no mathematical function exists that satisfies all of a certain set of arguably reasonable or desirable properties (restrictions); These are: monotonicity, independence, unanimity (Pareto Principle) *and nondictatorship. An immediate corollary of this theorem is the statement that any conceivable democratic voting system can yield undemocratic result*[626].

626 Arrow's unsettling game-theoretic demonstration was commented on by **Paul Samuelson** (1952) in these words: "The search of the great minds of recorded history for perfect democracy, is the search for a chimera, for a logical self-contradiction. Now scholars all over the world – in mathematics, politics, philosophy, and economics – are trying to salvage what can be salvaged from Arrow's devastating discovery that is to mathematical politics what **Kurt Gödel**'s (1931) impossibility-of-proving-consistency theorem is to mathematical logic." Arrow's demonstration helped earn him the Nobel Prize in economics (1972). It is one of the earliest astonishing results in game theory.

Conflicts between individual and group behavior, already encountered earlier, has an interesting analogue in sociology *through the bizarre phenomenon of* altruism*, which is a particular challenge to theories of evolution by* natural selection*.*

An animal behaves in a altruistic *way if it promotes the welfare of another at the expense of it's own. This seems difficult to reconcile with the notion of survival value, but it occurs nevertheless — and indeed quite frequently. The alarm call which warns a flock but attracts the attention of the predator upon the caller, is a common example. True, a group may benefit from altruistic traits among its members: it reduces the probability of extinction of the group. But selection at the group level is, as a rule, slow and much less effective than on the level of the individual: if a mutant gene, promoting altruistic behavior reduces the fitness of it's carrier, it will tend to be eliminated in spite of the boost it provides for the group. One must therefore find explanations in terms of* gene selection *and individual survival values. One such explanation is* kin selection: *a gene complex programming altruistic acts which benefit* relatives *may spread because it occurs, with a certain probability, amongst relatives whose overall (familial) reproductive success is thereby increased.*

Apart from this genetic *explanation there is a* strategic *explanation, which claims that in certain situations an altruistic act may increase the reproductive success of the individual performing it;* Game-theoretical *considerations show that conflicting interests can lead to the evolution of stable and apparently cooperative traits of behavior (e.g., wolves refrain from dealing the killing bite if their opponent offers his throat in a gesture of surrender). This strategic explanation relies on the notion of* frequency-dependent fitness: *the fitness of an individual may depend on what the* others *are doing.*

Finally, an economic *explanation relies on the notion of* reciprocal altruism; *e.g. unrelated young male baboons team up – while one of them mounts a female, the other one fights off its consort. The roles are reversed on a later occasion.*

Consider the voting system consisting of a plurality election *followed by a runoff between the top two vote getters out of three candidates. Suppose the votes of 17 voters distribute as follows:*

Class	Number of votes	Preferences (Best to worse)		
I	6	A	B	C
II	5	C	A	B
III	4	B	C	A
IV	2	B	A	C

If all the voters vote sincerely, and do not change their preference rankings, A (with 6 votes) and B (also with 6) will end up in the runoff, which A will win, 11 votes to 6.

Now imagine that the preferences are the same, except that the last class of voters elevate A from second choice to first choice:

Class	Number of votes	Preferences (Best to worse)		
I	*6*	*A*	*B*	*C*
II	*5*	*C*	*A*	*B*
III	*4*	*B*	*C*	*A*
IV	*2*	*A*	*B*	*C*

On the first ballot, A (8 votes) and C (5 votes) make the runoff. But A then loses, 8 votes to 9, because B's 4 supporters switch to C. Thus A's increased support has perversely torpedoed his victory! We witness here a situation where a candidate may be hurt if he receives additional votes: more votes can make a winner a looser! Similar situation arises in a straightforward plurality election without a runoff when public announcement of how candidates fared in a preelection are made.

Other methods of voting, such as the "Hare voting system"[627]*, are also not immune to perverse results. It can be shown that in the Hare system, a candidate who wins in two separate districts can loose in a combined tally of the two districts!*

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[627] Advocated by **Thomas Hare** (1806–1891). English reformer and barrister. Best known for his proposed election system, giving each class of votes in the electorate a representation in proportion to its numerical strength (1858). Defined a *quota* as the greatest integer less then $\left[1+\frac{V}{n+1}\right]$, where V is the number of voters and n the number of open seats. Each voter lists a number of m candidates in his own order of preference. The first -choice votes are tabulated and the candidates who achieve the quota are the winners.

If, however, a candidate does not meet the quota, the least popular candidate on the first-preference list is eliminated and his/her supporters transfer their votes to the next higher choice from bottom. If this transfer causes another candidate to meet the quota, he is elected. If seats remain unfilled, the process continues until all the seats are filled. If at any point there is an open seat but no surplus of votes to transfer, the process continues until the candidates with lowest number of votes is eliminated and his supporters simply transfer their votes to their next-higher choice who still is in the race. The idea is that no vote should be wasted.

1926–1946 CE **Douglas Rayner Hartree**[628] (1897–1958, England). Applied mathematician. Developed ingenious approximation methods for the calculation of atomic wavefunctions of many-electron atoms[629]. He also applied his methods of numerical analysis to problems in ballistics, atmospheric physics, hydrodynamics and control of chemical engineering processes. Much of his work was of importance to Britain's war effort (1939–1945). Hartree was one of the first (1945) to use the electronic computer (ENIAC = Electric Numerical Integrator and Calculator) as a general-purpose computer.

Hartree was born in Cambridge and received his higher education there. During 1929–1937 he held the chair of applied mathematics at the University of Manchester. From 1946 until his death Hartree was Plummer professor of mathematical physics at Cambridge University.

1926–1949 CE **Pascual** (**Ernst**) **Jordan** (1902–1980, Germany). Physicist. Founded, with **Max Born** and **Werner Heisenberg**, quantum matrix mechanics. In 1949, Jordan suggested a cosmological model of '*creation out of nothing*', such that the sum of the mass-energy in the universe is *always* zero.

Jordan was a professor of theoretical physics in Rostock, Germany (1929–1944) and after 1947 in Hamburg.

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628 For further reading, see:

- Hartree, D.R., *The Calculations of Atomic Structures*, John Wiley & Sons: New York, 1957, 181 pp.

629 In the *Hartree approximation* (1928) it is assumed that each electron moves in a central field that can be calculated from the nuclear potential and the wave functions of all the other electrons, by assuming that the charge density associated with an electron is (-e) times its position probability density. The Schrödinger equation is then solved for each electron in its own central field. Clearly, the approximation neglects correlations between the position of the electrons, since the entire wave function for all electrons is assumed to be a simple product of one-electron functions (but antisymmetrized in spin and spatial dependence, to confirm to Pauli's exclusion principle). From the atomic wave-functions it is possible to calculate the average distribution of negative electronic charge as a function of distance from the nucleus, which in turn determine the self-consistent electrostatic potential in which each electron orbital evolves. These charge distributions are of great importance in the theoretical calculation of macroscopic properties of matter.

Table 5.6: The magnificent Hungarians (1832–1994)

When Nobel Laureate Enrico Fermi was asked if he believed in extraterrestrials, he replied: "They are already here... they are called Hungarians!" The following list of 33 scientists, mathematicians and engineers provides the answer: [(∗) = Jewish]

Name	Life-span	Years of peak activity	Nobel prize	Field	Detail
Janos Bolyai	1802–1860	1832		Mathematics	Non-Euclidean hyperbolic geometry
Joseph Petzval	1807–1891	1840		Optics	Precision optical systems
Lorand von Eötvös	1848–1919	1891–1917		Physics	Gravitational vs. Inertial mass
Karl Zipernowsky	1853–1942			Engineering	AC Power transmission
Max Deri	1854–1938			Engineering	AC Power transmission
Otto Blathy	1860–1939			Engineering	AC Power transmission
Richard Zigmondi	1865–1929		1925	Chemistry	Colloidal chemistry
Robert Barany(∗)	1876–1936	1910	1914	Physiology	Inner ear
Lippot Fejer(∗)	1880–1959			Mathematics	Fourier series
Frigyes Riesz(∗)	1880–1956	1924		Mathematics	Functional Analysis
Theodor von Karman(∗)	1881–1967	1911–1950		Aeronautics	Supersonic flight: Jets and rockets
Georg von Hevesy(∗)	1885–1966	1923–1940	1943	Chemistry	Isotope tracers
Alfred Haar(∗)	1885–1933	1932		Mathematics	Measure on groups
Georg Polya(∗)	1887–1985			Mathematics	
Leopold Ruzicka	1887–1976		1939	Chemistry	Polymethylenes; Sex hormones
Albert Szent Gyorgyi	1893–1986	1933	1937	Physiology	Vitamin C and oxidation in tissues

Table 5.6: (Cont.)

Name	Life-span	Years of peak activity	Nobel prize	Field	Detail
Tibor Radó(*)	1895–1965	1924–1930		Mathematics	Algebraic topology; Plateau Problem
Kalman Tihanyi	1897–1947	1926–8		Engineering	Television pioneer
Leo Szilard(*)	1898–1964	1922–1964		Physics	Atomic bomb; Nuclear 'chain-reaction'
George von Bekesy(*)	1899–1972		1961	Physiology	Inner ear
Laszlo Biro	1899–1985	1938		Engineering	Ball pen; Automatic transmission
Dennis Gabor(*)	1900–1979		1971	Physics	Holography
Eugene Wigner(*)	1902–1995		1963	Physics	Quantum Theory; Elementary particles; Nuclear engineering
John von Neumann(*)	1903–1957	1921–1957		Mathematics	Binary code and stored-program computer; Game theory
Peter Carl Goldmark(*)	1906–1977			Engineering	Color TV, LP records, video recorder
Edward Teller(*)	1908–2003			Physics	'Father of H-bomb'
Arthur Erdélyi(*)	1908–1977			Mathematics	Analysis
Karl Ereky		1917–1919		Engineering	Horticultural biotechnology
Paul Erdos(*)	1913–1996			Mathematics	Number theory
John Harsanyi(*)	1920–2000		1994	Economics	Game theory
Imre Lakatos(*)	1922–1974	1948–1964		Mathematics and Philosophy	Methodology of scientific research
George Olah(*)	1927–		1994	Chemistry	Carbocation chemistry
John Polanyi(*)	1929–		1986	Chemistry	Reaction dynamics

Quaternions and Spinors: Hamilton to Pauli[630] *(1843–1927)*

If one tries to define three–dimensional vector division by seeking a vector $\boldsymbol{C}$ *such that* $\boldsymbol{B} \times \boldsymbol{C} = \boldsymbol{A}$ (or $\boldsymbol{C} \times \boldsymbol{B} = \boldsymbol{A}$), *for two given vectors* $\boldsymbol{A}$ and $\boldsymbol{B}$, *one discovers that this operation is:*

(I) well-defined only when $\boldsymbol{A} \cdot \boldsymbol{B} = 0$*;*

(II) non-unique, on account of the identity $\boldsymbol{B} \times \boldsymbol{C} = \boldsymbol{B} \times (\boldsymbol{C} - \alpha \boldsymbol{B})$.

It was this effort, to extend 3*-dimensional vector analysis to include both multiplication and division, which led* **Hamilton** *(1843) to invent a new division algebra for quadruples of numbers.*

Hamilton considered a 4*-dimensional real vector space with abstract unit base elements* $\{e_0, e_1, e_2, e_3\}$. *A general vector in this space, known as a quaternion, is written in the form*

$$q = q_0 e_0 + (q_1 e_1 + q_2 e_2 + q_3 e_3) = q_0 e_0 + \boldsymbol{q}.$$

with $q_0, \boldsymbol{q}$ *real.*

Quaternions obey the rules of ordinary algebra w.r.t. addition and scalar (in this case, real number) multiplication. With the definitions

$$e_1^2 = e_2^2 = e_3^2 = -e_0, \quad e_1 e_2 = -e_2 e_1 = e_3,$$

$$e_0^2 = e_0; \quad e_2 e_3 = -e_3 e_2 = e_1,$$

$$e_3 e_1 = -e_1 e_3 = e_2, \quad e_k e_0 = e_0 e_k = e_k, \quad k = 1, 2, 3$$

the product of two quaternions assumes the form

$$pq = (p_0 q_0 - \boldsymbol{p} \cdot \boldsymbol{q}) e_0 + p_0 \boldsymbol{q} + q_0 \boldsymbol{p} + (\boldsymbol{p} \times \boldsymbol{q}).$$

This reduces to the ordinary vector cross-product for $p_0 = q_0 \equiv 0$, *if we ignore the invariant part (scalar product) of the total product. If on the*

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[630] For further reading, see:

- Cartan, E., *The Theory of Spinors*, Dover, 1966, 157 pp.
- Altmann, S.L., *Rotations, Quaternions and Double Groups*, Dover, 2005, 317 pp.

other hand $p_2 = q_2 = p_3 = q_3 \equiv 0$, $e_0 = 1$, $e_1 = \sqrt{-1}$, *it reduces to the complex multiplication of* $p_0 + ip_1$ *and* $q_0 + iq_1$.

Quaternions can thus be viewed as 4-dimensional numbers forming a vector space generated by unity $(e_0 = 1)$ *and three other base vectors, each of the latter being an independent square root of* -1. *We shall soon see that this number system provides a description of rotation in 3-dimensions, just as ordinary complex numbers do in 2 dimensions.*

This is achieved at the cost of sacrificing the law of commutative multiplication (since $e_1e_2 = -e_2e_1$, *etc.).*

Division of quaternions is realized through the definition of the inverse quaternion

$$q^{-1} = \frac{q_0e_0 - \boldsymbol{q}}{q_0^2 + q_1^2 + q_2^2 + q_3^2},$$

yielding $qq^{-1} = e_0$. *Thus, quaternions form a non-commutative division algebra over the real numbers.*

The further useful definitions

$$\boldsymbol{n} = \frac{q_1e_1 + q_2e_2 + q_3e_3}{\sqrt{q_1^2 + q_2^2 + q_3^2}} \qquad (\boldsymbol{n}^2 = -e_0),$$

$$q_0 = h\cos\frac{\varphi}{2}, \qquad \sqrt{q_1^2 + q_2^2 + q_3^2} = h\sin\frac{\varphi}{2},$$

$$h = \sqrt{q_0^2 + q_1^2 + q_2^2 + q_3^2}$$

reduce every quaternion to the standard form

$$q = h\left(\cos\frac{\varphi}{2}e_0 + \sin\frac{\varphi}{2}\boldsymbol{n}\right).$$

It then follows that

$$q\boldsymbol{r}q^{-1} = \mathfrak{R}\cdot\boldsymbol{r}, \qquad \text{for all} \quad \boldsymbol{r} = x\boldsymbol{e}_1 + y\boldsymbol{e}_2 + z\boldsymbol{e}_3$$

where $\mathfrak{R}$ *is a q-dependent orthogonal, real* 3×3 *matrix that represents a rotation of the axes by an angle* φ *about the axis* $\boldsymbol{n}$ *relative to the fixed axes* $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3\}$.

In fact, it turns out that one can set up a $2{:}\,1$ *correspondence between unit modulus* $(h = 1)$ *quaternions and the continuous Lie–group of finite* *rotations, in the sense that for every pair* $\pm q$ *of unit-modulus quaternions we can generate a distinct rotation about the axis* $\boldsymbol{n}$ *defined above, by an angle*

$$\varphi = 2\tan^{-1}\left\{\frac{1}{q_0}\sqrt{q_1^2+q_2^2+q_3^2}\right\}$$

[e.g. $q = \frac{e_0+e_3}{\sqrt{2}}$ *represents a rotation by* 90° *about the* z*-axis].*

So far we have not specified the nature of the abstract base elements $\{e_0, e_1, e_2, e_3\}$, *except through their 'multiplication table'. We saw, however, that in two limiting cases they can assume either the role of unit vectors in the 3-dimensional Euclidean vector space* $\mathbb{R}^3$ $(e_1 = \boldsymbol{e}_1,\ e_2 = \boldsymbol{e}_2,\ e_3 = \boldsymbol{e}_3)$ *or unit complex numbers in* $(e_0 = 1,\ e_1 = i)$.

Another interesting case arises when we represent the base elements as 2×2 *matrices such that:*

$$e_0 = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \equiv I,$$

$$e_1 = -i\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} \equiv -i\sigma_1,$$

$$e_2 = -i\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix} \equiv -i\sigma_2,$$

$$e_3 = -i\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} \equiv -i\sigma_3,$$

where $\boldsymbol{\sigma} = \{\sigma_1, \sigma_2, \sigma_3\}$ *later became known as the* 3 *Pauli matrices (in the context of quantum mechanics). These* e_i *matrices obey the quaternionic laws of multiplication. The Pauli matrices are Hermitian*[631] $(\sigma_i^\dagger = \sigma_i)$ *and traceless.*

Keeping this in mind, we can write the standard form of a unit-modulus quaternion as

$$q(\boldsymbol{n}, \varphi) = I\cos\frac{\varphi}{2} - i\sin\frac{\varphi}{2}(\boldsymbol{\sigma}\cdot\boldsymbol{n}),$$

---

631 $A^\dagger \equiv$ transpose of the element–wise complex conjugate of A, for any complex matrix A.

where

$$\boldsymbol{\sigma}\cdot\boldsymbol{n} = \begin{bmatrix} n_3 & n_1 - in_2 \\ n_1 + in_2 & -n_3 \end{bmatrix}$$

and $(\boldsymbol{n}, \varphi)$ *are, as before, the (real) parameters of a rotation of the axes about the unit vector* $\boldsymbol{n}$ *with an angle* φ.

Any such rotation can be decomposed into 3 successive rotations by Euler angles (α, β, γ) *about the respective fixed-space axes* $\{\boldsymbol{e}_z, \boldsymbol{e}_y, \boldsymbol{e}_z\}$, *and indeed we find that*

$$q(\boldsymbol{n}, \varphi) = q(\boldsymbol{e}_z, \alpha) q(\boldsymbol{e}_y, \beta) q(\boldsymbol{e}_z, \gamma) =$$

$$\begin{bmatrix} e^{-i\frac{\alpha}{2}} & 0 \\ 0 & e^{+i\frac{\alpha}{2}} \end{bmatrix} \begin{bmatrix} \cos\frac{\beta}{2} & -\sin\frac{\beta}{2} \\ \sin\frac{\beta}{2} & \cos\frac{\beta}{2} \end{bmatrix} \begin{bmatrix} e^{-i\frac{\gamma}{2}} & 0 \\ 0 & e^{+i\frac{\gamma}{2}} \end{bmatrix} =$$

$$\begin{bmatrix} \cos\frac{\beta}{2} e^{-\frac{i}{2}(\gamma+\alpha)} & -\sin\frac{\beta}{2} e^{\frac{i}{2}(\gamma-\alpha)} \\ \sin\frac{\beta}{2} e^{-\frac{i}{2}(\gamma-\alpha)} & \cos\frac{\beta}{2} e^{\frac{i}{2}(\gamma+\alpha)} \end{bmatrix} = \begin{bmatrix} q_0 - iq_3 & -(q_2 + iq_1) \\ q_2 - iq_1 & q_0 + iq_3 \end{bmatrix}$$

which is the most general 2×2 *complex, unimodular (unit–determinant) and unitary matrix. If we shift* φ *to* $\varphi + 2\pi$ *(or by any odd multiple of* 2π*),* $\mathfrak{R}(\boldsymbol{n}, \varphi)$ *remains the same while* $q(\boldsymbol{n}, \varphi)$ *changes its sign. Thus, the quaternions* $\pm q$ *represent the same 3D rotation, and the* $2 \leftrightarrow 1$ *correspondence between unimodular quaternions and 3D rotations is thereby established.*

Spinors in 3 Dimensions (Pauli)

We have seen that the relations $\boldsymbol{r}' = \mathfrak{R}\cdot\boldsymbol{r}$ *and* $\boldsymbol{r}' = q\boldsymbol{r}q^\dagger$ *(since* $q^{-1} = q^\dagger$ *– in other words q is* unitary*) represent the* same *rotation, if one uses the base quaternions (*$e_k = -i\sigma_k; \quad k = 1,2,3$*) as unit basis vectors in 3D space. The quaternion relation can be put in a convenient form that involves multiplication of* 2×2 *matrices:*

$$S' = USU^\dagger, \qquad S = \begin{bmatrix} z & x - iy \\ x + iy & -z \end{bmatrix}$$

Let U^i_j *denote the* (i,j) *element of the unitary matrix* $q(\boldsymbol{n}, \varphi)$. *Denoting the elements of S by* $S^{k\ell}$, *and carrying out the matrix multiplication, we find the explicit transformation equations (a bar represents complex conjugation)*

$$
\begin{aligned}
S^{11\prime} &= U_1^1\overline{U}_1^1S^{11} + U_2^1\overline{U}_1^1S^{21} + U_1^1\overline{U}_2^1S^{12} + U_2^1\overline{U}_2^1S^{22} \\
S^{21\prime} &= U_1^2\overline{U}_1^1S^{11} + U_2^2\overline{U}_1^1S^{21} + U_1^2\overline{U}_2^1S^{12} + U_2^2\overline{U}_2^1S^{22} \\
S^{12\prime} &= U_1^1\overline{U}_1^2S^{11} + U_2^1\overline{U}_1^2S^{21} + U_1^1\overline{U}_2^2S^{12} + U_2^1\overline{U}_2^2S^{22} \\
S^{22\prime} &= U_1^2\overline{U}_1^2S^{11} + U_2^2\overline{U}_1^2S^{21} + U_1^2\overline{U}_2^2S^{12} + U_2^2\overline{U}_2^2S^{22}.
\end{aligned}
$$

Now we recall that under a rotation of the axes $\mathfrak{R}$, *the components of a vector transforms as* $\boldsymbol{r}' = \mathfrak{R}\cdot\boldsymbol{r}$, *while the components of a second rank tensor* $\mathfrak{T}$ *(represented as a matrix) transform according to* $\mathfrak{T}' = \mathfrak{R}\cdot\mathfrak{T}\cdot\mathfrak{R}^T$. *We may then draw an immediate analogy to* $S' = USU^\dagger$, *and posit a two component complex entity* ξ *(analogous to a vector* $\boldsymbol{r}$ *in 3D) that transforms according to the law*

$$\xi' = U\xi,$$

or explicitly

$$\begin{bmatrix}\xi^{1\prime}\\ \xi^{2\prime}\end{bmatrix} = \begin{bmatrix}U_1^1 & U_2^1\\ U_1^2 & U_2^2\end{bmatrix}\begin{bmatrix}\xi^1\\ \xi^2\end{bmatrix}.$$

Let another entity

$$\eta = \begin{bmatrix}\eta^1\\ \eta^2\end{bmatrix}$$

transform according to the different law $\eta' = \overline{U}\eta$. *We can then form Cartesian (tensor) products which transform as:*

$$
\begin{aligned}
\xi^{1\prime}\eta^{1\prime} &= (U_1^1\xi^1 + U_2^1\xi^2)(\overline{U}_1^1\eta^1 + \overline{U}_2^1\eta^2) \\
&= U_1^1\overline{U}_1^1(\xi^1\eta^1) + U_2^1\overline{U}_1^1(\xi^2\eta^1) + U_1^1\overline{U}_2^1(\xi^1\eta^2) + U_2^1\overline{U}_2^1(\xi^2\eta^2),
\end{aligned}
$$

etc.

Comparing this, term by term, with the laws of transformation of S^{AB}, *we find that* $\xi^A\eta^B$ *transform in exactly the same way; here* $A, B = 1, 2$.

Since S *is also a* 3*D vector, we may call* ξ^A *and* η^B semi-vectors, *since the transformation law of* S *was* split *into two separate transformation laws*

$$\xi'^A = U_B^A\xi^B, \quad \eta'^A = \overline{U}_B^A\eta^B,$$

where U is a unitary 2×2 matrix $(U^{-1}=\overline{U}^T)$ and the summation convention is employed for repeating indices. In terms of the Euler angles, the transformation law of ξ (for example) is

$$\begin{bmatrix}\xi^{1'}\\ \xi^{2'}\end{bmatrix}=\begin{bmatrix}\cos\frac{\beta}{2}e^{-\frac{i}{2}(\gamma+\alpha)} & -\sin\frac{\beta}{2}e^{\frac{i}{2}(\gamma-\alpha)}\\ \sin\frac{\beta}{2}e^{-\frac{i}{2}(\gamma-\alpha)} & \cos\frac{\beta}{2}e^{\frac{i}{2}(\gamma+\alpha)}\end{bmatrix}\begin{bmatrix}\xi^1\\ \xi^2\end{bmatrix}.$$

The norm of ξ is defined as

$$\xi^1\overline{\xi}^1+\xi^2\overline{\xi}^2=\xi^{1'}\overline{\xi}^{1'}+\xi^{2'}\overline{\xi}^{2'}$$

and is invariant under rotations.

We can improve the above notation by using dotted index ($\dot{1},\dot{2}$, or $\dot{A},\dot{B}$ etc.) for the $\overline{U}$ – transforming semi–vectors, for then one may distinguish the two transformation laws at a glance. Thus for instance $\xi^{\bar{A}}=\bar{\xi}^{\dot{A}}$, where a dotted index[632] *is taken to mean that the spinor transforms via $\overline{U}$; it ranges over $(\dot{1},\dot{2})$, while A,B etc. take values $(1,2)$.*

---

[632] This notation was introduced in 1929 by **van der Waerden** (1903–1996) to accommodate 4-dimensional relativistic spinors and the action of rotations and Lorentz-transformations on them; in this case U is *not* necessarily unitary but is still unimodular (that is, $\det U=1$). In this application, there are *four* types of spinors and the algebraic notation requires both upper and lower indices, each dotted *or* undotted, with four types of transformation laws. Thus, similar to the practice in ordinary differential – geometric tensor algebra, one introduces covariant and contravariant components (of both dotted and undotted spinors). Raising and lowering of spinor indices is done by the skew-symmetric spinor metric (known as the *symplectic*)

$$\epsilon^{BA}=\epsilon_{AB}=\epsilon^{\dot{B}\dot{A}}=\epsilon_{\dot{A}\dot{B}}=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$$

which satisfies:

$$\epsilon_{AB}\epsilon^{BC}=\delta_A^C,\quad \epsilon_{\dot{A}\dot{B}}\epsilon^{\dot{B}\dot{C}}=\delta_{\dot{A}}^{C}=\delta_{\dot{A}}^{\dot{C}},$$
$$\epsilon_{AB}=-\epsilon_{BA},\quad \epsilon_{\dot{A}\dot{B}}=-\epsilon_{\dot{B}\dot{A}},$$
$$\xi^A=\epsilon^{AB}\xi_B,\quad \xi_B=\xi^A\epsilon_{BA},$$
$$\overline{\xi}^{\dot{A}}=\epsilon^{\dot{A}\dot{B}}\overline{\xi}_{\dot{B}},\quad \overline{\xi}_{\dot{B}}=\overline{\xi}^{\dot{A}}\epsilon_{\dot{B}\dot{A}}.$$

with δ_A^C, etc. representing the Kronecker delta. The scalar product of two

Having introduced spinors as building-block entities from which vectors can be constructed[633]*, one may ask whether one can construct a spinor from a given vector. The answer is that one* cannot *build a unique spinor from*

dotted or two undotted spinors may then be defined as

$$\phi_A\psi^A = -\phi^A\psi_A = \epsilon^{AB}\phi_A\psi_B = -\epsilon_{AB}\phi^A\psi^B,$$

$$\overline{\phi}_{\dot{A}}\overline{\psi}^{\dot{A}} = -\overline{\phi}^{\dot{A}}\overline{\psi}_{\dot{A}} = \epsilon^{\dot{A}\dot{B}}\overline{\phi}_{\dot{A}}\overline{\psi}_{\dot{B}} = -\epsilon_{\dot{A}\dot{B}}\overline{\phi}^{\dot{A}}\overline{\psi}^{\dot{B}}.$$

In particular we have

$$\phi_A\phi^A = [\xi_1 \quad \xi_2]\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}\xi_1\\ \xi_2\end{bmatrix} = -\xi_1\xi_2 + \xi_2\xi_1 = 0.$$

In the 4D (Minkowski/STR) case the matrix $S = \boldsymbol{r}\cdot\boldsymbol{\sigma}$ is replaced with

$$S_{A\dot{B}} = c\,t\cdot I + \boldsymbol{r}\cdot\boldsymbol{\sigma} = \begin{bmatrix}ct+z & x-iy\\ x+iy & ct-z\end{bmatrix},$$

and its transformation law by

$$S'_{A\dot{B}} = U_A^C\, U_{\dot{B}}^{\dot{D}}\, S_{C\dot{D}} \quad ,$$

where U_A^C is the (A, C) component of a quaternion $q_0 - i\boldsymbol{q}\cdot\boldsymbol{\sigma}$ with complex $(q_0, \boldsymbol{q})$ (corresponding to complex Euler angles); the angles are *real* for pure 3D rotations, and *complex* for pure boosts (Lorentz transformation).

Returning to the case where U is both unimodular and unitary (representing 3D rotations), $\overline{\phi}^{\dot{1}}\psi^1 + \overline{\phi}^{\dot{2}}\psi^2$ invariant under the transformation, so we may identify, for any spinor, ϕ, $\phi_A = \xi_{\dot{A}}$, where ξ is a different spinor. Hence, only two types of indices arise for 3D rotations, but four kinds are needed to accommodate Lorentz transformations as well.

633 For any given spinor $\begin{bmatrix}a+ib\\ c+id\end{bmatrix} = \begin{bmatrix}u\\ v\end{bmatrix}$ with unit norm $(a^2+b^2+c^2+d^2=1)$, we may create the real unit vector $\left[2(ac+bd),\ \ 2(ad-bc),\ \ a^2+b^2-c^2-d^2\right]$. For example, $(0, 0, 1)$ is obtained from the spinor $\begin{bmatrix}1\\ 0\end{bmatrix}$ and also from $\begin{bmatrix}(1+i)/\sqrt{2}\\ 0\end{bmatrix}$. In general, from $u = u_0\cos\frac{\beta}{2}e^{-\frac{i}{2}(\gamma+\alpha)}$, $v = u_0\sin\frac{\beta}{2}e^{-\frac{i}{2}(\gamma-\alpha)}$, there emerges the vector $[u_0^2\sin\beta\cos\alpha,\ \ u_0^2\sin\beta\sin\alpha,\ \ u_0^2\cos\beta]$. Denoting $u_0^2 = S$, the length of the 3D position vector, the magnitude of the corresponding spinor must be $\sqrt{\overline{u}u+\overline{v}v} = \sqrt{S}$.

three given components of a single vector since a spinor is specified by 4 *real parameters, whereas a vector requires only* 3.

A simple geometrical description of a spinor can, however, be given, analogous to that of a vector. A vector can be visualized as an arrow of length R *and two polar angles* φ *(azimuth) and* θ *(colatitude) relative to some fixed axes.*

To visualize a spinor we may think of an axe, letting the handle *represent the magnitude* S *and direction* (θ, φ)*, just as the arrow does in the case of a vector. Once we have fixed these specifications, the axe still has one degree of freedom left, since it can still rotate about the axis of the handle.*

To fix this degree of freedom, we imagine a short line segment perpendicular to the handle at its tip and call it the blade*. The angular position is now measured as the angle* ψ *between the southbound local meridian [passing through the point* (θ, ψ) *on the unit sphere], directed from the north pole toward the south pole, and the blade itself.*

1927 CE **Eugène Freyssinet** (1879–1962, France). Civil engineer. Developed *prestressed concrete* and made it into a successful worldwide building material for bridges, shell roofs, airship hangars, concrete sea-going ships and mass-produced pylons.

1927 CE First actual transmission of television signals (New York to Washington) by the American Telephone and Telegraph Company.

1927 CE First transatlantic telephone service opened.

1927 CE **Bernardus (Dominicus Hubertus) Tellegen** (1900–1990, Holland). Engineer and inventor. Invented the *Pentode*: a 5-electrode electric vacuum tube. It was developed from the *tetrode* by inserting the fifth electrode, called the *suppressor grid* (or grid No 3) in order to avoid tetrode secondary emission which reduces plate current. The suppressor grid has negative voltage w.r.t. the other electrodes because it is connected directly to the cathode. When the electrons emitted from the cathode strike the anode plate, *secondary emission* will occur.

In the tetrode this secondary emission is attracted to the screen grid, creating a screen grid current. The negative voltage of the suppressor grid will push the secondary electrons back to the plate, thus eliminating the dip (kink) found in the voltage-current curve of the tetrode.

Consequently the pentode may be considered as a *constant-current device* over a wide range of plate voltages, with non-linear characteristics only below the knee of the curve.

Tellegen worked at the Phillips Research Laboratory and was a professor at the University of Delft.

1927–1928 CE **Warren Alvin Marrison** (1896–1980, U.S.A.) and **Joseph W. Horton** (U.S.A.) built the first clock based on a quartz crystal oscillator.

Marrison was born in Kingston, Ontario. After earning a Master's degree from Harvard (1921), he went to work for Western Electric in New York City, and then for Bell Laboratories in New Jersey (1925). During his career, he invented 65 patents.

Marrison and others demonstrated that the quartz oscillator used in this way was more accurate than the best existing mechanical clocks used in astronomical observatories as time standards. During the 1940s, time standard laboratories throughout the world switched from mechanical clocks to quartz. The fundamental standard of time remained the rotation of the earth relative to the stars, but quartz clocks confirmed that the earth was an unreliable timekeeper.

Today electronic watches, cell phones, computers and many other devices use the same timekeeping standard as Marrison's clock — the regular vibrations of a quartz crystal. The world's fundamental time standard, though, is now based on *atomic* clock.

1927–1929 CE **Martin Heidegger** (1889–1976, Germany). Metaphysical thinker. The founder of modern *existentialism*. Exercised a great influence on the philosophers of continental Europe, South America, and Japan. His work stimulated much that is original and compelling in modern thought.

Heidegger found the obscure and fragmentary writings of the *pre-Socratic* philosophers to be an agreeably, plastic sort of raw material for his speculations, which came to be expressed in an increasingly sibylline form.

His *Sein und Zeit* (1927), a 20^{th} century version of Kierkegaard's[634] anguished acknowledgment of the contingent and the irrational, was the most potent and influential presentation of the existentialism that dominated Europe in WWII and for some years thereafter.

[634] **Sören Kierkegaard** (1813–1855, Denmark). Religious philosopher and social thinker. Held that Christianity stands opposed to the world, to time, and to reason, and that the interminable paradoxes of life are the inevitable result of man's reflections.

Heidegger's work is an attempt to understand the nature of Being (*sein*). To this end he analyzed human existence (*Dasein*), because it is the form of Being we can best know. His extensive discussions of human existence emphasizes anxiety (*angst*), alienation and death. This particular kind of existence peculiar to human beings is what differentiates men from the inert material surroundings within which they find they have been arbitrarily 'thrown'; it is a condition characterized by anxious awareness of the future, and as containing both the necessity of choice and death, the cessation of being.

Most people distract their attention from the fact of death and extinction and trivialize their freedom of choice, satisfied to follow conventional routine. But authentic life is only possible if death is resolutely confronted and freedom exercised with a sense of its essentially creative nature.

The superficial, practical business of man has the effect of hiding Being from him, although it is always present and we can make ourselves 'open' to it. The basic mood of man is *anxiety*, and the fundamental structure of man is *concern*. Anxiety is caused by man's encountering the indeterminate and indefinable *nothingness*.

Man's life is oriented from a standpoint of his consciousness of death, which makes the difference in the choices an individual makes during life. Death is a singular experience in that each person must encounter his own, without any possibility of delegating it to another.

Man, being summoned by his consciousness to the numerous possibilities among which he may choose, experiences the frustrating awareness that whatever choice he makes leaves others behind. The realization of certain choices allows the unfulfilled choices to plague him with *guilt*. Guilt is an indelible quality of *Dasein*; human Beings always feel guilty.

However, despite his anxiety, guilt, finitude and the nothingness of the world, man's present existence can attain values by moving through time with *resoluteness* against his *background of historical fate*, i.e. gaining authentic existence by being prepared for anxiety.

In this state, which Heidegger calls *histericality*, the individual moves from the past into the future, being driven by the past and oriented by the future; the past is futural in the sense that it is not finished, because it holds future possibilities and things which bear repetition.

While Heidegger's philosophy itself is comprehensible and, in its psychological orientation, quite sensible — the method of presenting his philosophy is extremely abstract, complex and obscure: In his attempt to understand Being, he often sought philosophical enlightenment in the etymologies of words and the insights of poets, especially his favorite — **Friedrich Hölderlin** (1770–1843, Germany).

Heidegger was born in Messkirch, Baden-Württemburg, the son of a Catholic sexton. He entered the Jesuit order as a novice and later studied philosophy at the University of Freiburg under **Edmund Husserl**. Then became a professor at Marburg (1923–1927) and Freiburg (1927–1944), where he succeeded his old teacher.

Possessing a seductive classroom presence, he attracted Germany's brightest young intellects during the 1920's.[635] He was thus able to inspire gifted disciples who produced political theories very different from the ideology endorsed by the master. Yet troubling residues remain not far beneath the surface of their influential work[636].

In 1933, Heidegger cast his lot with National Socialism. He squelched the careers of Jewish students and denounced fellow professors whom he considered insufficiently radical. For years, he signed letters and opened lectures

[635] Many were Jews, who would ultimately have to reconcile their philosophical and, often, personal commitments to Heidegger with his nefarious political views. Four of his most influential students came to grip with his Nazi association and it affected their thinking: **Hannah Arendt** (1906–1975), who was Heidegger's lover as well as his student; **Karl Lowith** (1897–1973) returned to Germany in 1953; **Hans Jonas** (1903–1993) grew famous as Germany's premier philosopher of environmentalism; **Herbert Marcuse** (1898–1979) gained celebrity as mentor to the New Left. Why did these Heideggerians fail to see what was in Heidegger's heart and Germany's future? In his book "*Heidegger's Children*", **Richard Wolin** locates this paradox in the wider cruel irony that European Jews experienced their greatest calamity immediately following their fullest assimilation, and he finds in their responses answers to questions about the nature of existential disillusionment and junction between politics and ideas.
Another important student of Heidegger was **Hans-Georg Gadamer** (1900–2002), who elaborated on the subject of human understanding. In his book *Truth and Method* (1960) Gadamer argued that 'truth' and 'method' were at odd with each other. He maintained that people have a 'historically effected consciousness' (*wirkungsgeschichtliches Bewußtsein*) and that they are embedded in the particular history and culture that shaped them.

[636] Much of the damage that this intellectual poison – disseminated by Jewish and non-Jewish thinkers alike, all deeply influenced by Heidegger and ultimately Nietzsche – caused in post–WWII America, can be traced to failure to see the deep connection between German philosophy and the rise of Nazism. On this blinkered view, the only trouble with Weimar was that the wrong side just "happened" to win. According to **Leo Strauss** and his school, this enables a Weimar – like moral catastrophe to recur; the 1960's anarchy was one step down that road, while *post–modernism* and todays rampart moral relativism and appeasement of tyranny are another.

with "Heil Hitler!" He paid dues to the Nazi party until the bitter end. Equally problematic for his former students were his sordid efforts to make existential thought serviceable to Nazi ends, and his failure to ever renounce their actions.

Heidegger hailed Hitler as the great protagonist of a new European culture and leaned toward the socio-political views of the Third Reich. On the strength of these views he was made rector of Freiburg University (1933), but soon quarreled with his new masters, resigned and moved to Switzerland (1934).

Heidegger was an active and fanatic member of the Nazi party until the end of World War II. His involvement with the party and his support of its world-view are undisputed. Even more disturbing than his active participation in Nazism[637], Heidegger never attempted to account for his support of the Nazis outside of calling his involvement with them "a blunder".

Moreover, he never publicly condemned Hitler nor the horrible crimes of Nazi Germany against the Jews. In this light Heidegger stands as a great embarrassment for philosophers. The key focus of recent years, however, has been to decide whether or not his philosophy somehow reflects his political ideology, to see if *Being* and *Nazism* are somehow related. The most likely connection is his account of human beings.

If human beings are *Dasein*, meaning they have no common essence, then there is no reason to expect that a particular group of *Dasein* will respect the rights of another. The only sense of security a *Dasein* can attain, comes from their given society. Consequently, Heidegger's account of *Dasein* can lead to absolute nationalism. At this point we can only wait to see if Heidegger the philosopher can be salvaged from Heidegger the political figure; it might ultimately not be worth the effort.

After the war, Heidegger was prohibited from teaching until the ban was lifted in 1951.

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[637] Heidegger used the phrase '*Verjudung*', coined by Hitler in his 'Mein Kampf'. He informed on **Staudinger** to the Gestapo and insisted that they take action against him. He also had great influence on a whole generation of young German students and mobilized them to the Nazi movement.

Worldview XLI: Heidegger

* * *

"Mathematics itself is only a particular formulation of the mathematical."

* * *

"The mathematical is the fundamental presupposition of the knowledge of things."

* * *

"We are too late for the Gods and too early for Being."

* * *

"You never come to thoughts. They come to you."

* * *

"The oldest of the old follows behind us in our thinking, and yet it comes to meet us."

* * *

"Why is there something rather than nothing at all?"

* * *

"Teaching is more difficult then learning, for only he who can truly learn – and only as long as he can do it – can truly teach. The genuine teacher differs

from the pupil only in that he can learn better and he more genuinely wants to learn. In all teaching the teacher learns the most.”

* * *

“It is not we that speak the language but it is the language that speaks us.”

* * *

“We are constantly projecting ourselves into the future.”

1927–1931 CE **Georges Henri Lemaître** (1894–1966, Belgium). Astronomer and cosmologist. Proposed that the universe was created by an explosion of concentrated energy and may still be expanding. This became known as the '*Big Bang*' hypothesis. He inferred it from the instability of the static general-relativistic models of the universe. Thus (independently of **Friedmann**), he discovered the simplest family of solutions to **Einstein**'s field equations of relativistic gravitation (GTR) that describe the expanding universe.

Lemaître was educated as a civil engineer and served as an artillery officer in WWI. After the war he entered a seminary and in 1923 was ordained a priest. He studied at the University of Cambridge's solar physics laboratory (1923–1924) and then at the Massachusetts Institute of Technology, Cambridge, Massachusetts (1925–1927), where he became acquainted with the findings of the American astronomers **Edwin P. Hubble** and **Harlow Shapley** on the expanding universe.

In 1927, the year he became professor of astrophysics at the University of Louvain, he proposed his Big Bang theory, which explained the recession of the galaxies within the framework of **Einstein**'s theory of general relativity.

Models of the expanding universe had been considered earlier, notably by the Dutch astronomer **Willem de Sitter** (1872–1934) but Lemaître's theory — as modified (on the nuclear/particle physics side) by **George Gamow** (1904–1968) and by *inflationary* scenarios (1980's and 1990's), has explained much empirical data and thus became the leading theory of cosmology.

Cosmological solutions of Einstein equations

One can show that for a *homogeneous* and *isotropic* universe, the *pseudo-norm* line element ds^2 (proper–time interval squared) – encoding the space-time metric tensor – can be written as

$$ds^2 = c^2dt^2 - R^2(t)d\sigma^2, \qquad d\sigma^2 = \frac{dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)}{(1+\frac{1}{4}\epsilon r^2)^2}$$

where ϵ can be always chosen 0 or ± 1. Here $R(t)$ is the *cosmic scale factor* as function of standard cosmic time, t; (r, θ, φ) are curved spherical coordinates on the spatial submanifolds ("slices") at fixed t, with the origin at $r = 0$. The range of r is $[0, 2)$, except when $\epsilon = 0$, for which case r may range from 0 to ∞.

This line element was first derived by **Howard Percy Robertson** (1903–1961, U.S.A.) and **Arthur Geoffrey Walker** (1909–2001, England), and is known as the

Friedmann–Robertson–Walker (FRW) metric (1935–1936). This metric holds for *every* homogeneous and isotropic spacetime, independently of whether Einstein's equations hold or not. In it, the scale factor $R(t)$ describes the universe's expansion. The true distance of a given galaxy from a galaxy fixed at the origin is $rR(t)$, where R increases with time [with $R(0) = 0$ for realistic solutions, where $t = 0$ at the initial Big Bang singularity]; r is a constant dimensionless parameter depending on the galaxy. If $\epsilon = 0$, the space is Euclidean, if $\epsilon = -1$ the space is "*hyperbolic*", while if $\epsilon = 1$, it is "*hyperspherical*". In the first two cases the universe is spatially infinite, and topologically a $t = const.$ "slice" of it is $\mathbb{R}^3$; while for $\epsilon = 1$ it is spatially finite, and constant-t slice is topologically 3-spheres (S^3).

We substitute the Friedmann–Robertson–Walker metric into Einstein's field equations

$$G_{\mu\nu} + \lambda g_{\mu\nu} = -kT_{\mu\nu}.$$

In the above equation $\mu, \nu = 0, 1, 2, 3$, $k = \frac{8\pi G}{c^2}$, G is the constant of universal gravitation, λ is the cosmological constant, $g_{\mu\nu}$ is the metric tensor, $G_{\mu\nu}$ is the Einstein tensor (related to the symmetric, rank-2 contracted (Ricci) curvature tensor), and $T_{\mu\nu}$ is the stress-energy-momentum tensor.

We then find for a pressure-free ("matter-dominated dust") universe that $R(t)$ must satisfy these two conditions:

$$M = \frac{4\pi}{3} R^3 \rho = \text{constant} > 0$$

with ρ the mass density in any of the locally-comoving (galaxy–centered) frames; and

$$\frac{1}{2}\left(\frac{dR}{dt}\right)^2 = \frac{GM}{R} + \frac{\lambda}{6} R^2 c^2 - \frac{1}{2}\epsilon c^2$$

(**Lemaître**, 1927).

The first condition means that the mass-energy of a sphere of radius R does not change as R increases with the expansion of the universe, i.e., no matter or energy is created out of nothing; instead, the density drops because of the expansion.

The second condition leads to a solvable ODE for $R(t)$[638]. Every solution of this equation represents a different model of the universe.

For *a static* universe

$$\frac{dR}{dt} = 0, \quad \epsilon = 1, \quad \lambda = \lambda_c = \frac{64\pi^2}{9k^2M^2}, \quad R_c = \frac{1}{\sqrt{\lambda_c}}$$

(**Einstein**, 1917).

Lemaître showed that this solution is unstable. Indeed, it is easy to show that if R (or λ) is slightly perturbed away from the value R_c (or λ_c), then $\frac{dR(t)}{dt}$, as calculated from the above ODE, will not remain zero but start increasing (or decreasing) with time at an ever faster rate. That is, the universe will either start expanding, or contracting, at a continuously accelerated rate.

Depending on the values of ϵ and $\lambda \neq 0$, there are two fundamental optional scenarios for an expanding universe:

- *Continuously expanding universe*: $\epsilon = 0,\ \lambda > 0$; $\epsilon = -1,\ \lambda > 0$; $\epsilon = 1,\ \lambda > \lambda_c$. The density of the universe tends to zero as $t \to \infty$ and its scale factor expands exponentially fast. This is the process thought to have occurred during the brief *inflationary period* in the particle physics era of the early universe (a small fraction of a second after $t = 0$). In the first case $R = R_0 \exp\left(\sqrt{\frac{\lambda}{3}}c\,t\right)$ in the zero–mass–density ($\rho \to 0$) limit (**de Sitter**, 1917].
- *Pulsating ("anti-de Sitter") universe*: $\epsilon = 0,\ \lambda < 0$; $\epsilon = -1,\ \lambda < 0$; or $\epsilon = 1,\ 0 < \lambda < \lambda_c$. The function $R(t)$ increases initially from zero up to a maximum value $R_{\max}$ and afterwards decreases to zero again.

For $\lambda = 0$ we have the following possibilities:

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[638] Our universe at the present epoch is quite well approximated by a pressure-free self-gravitating dust, with galaxy clusters playing the role of 'dust particles'. It is only during the so-called 'radiation dominated' era (before the recombination of ionized matter into neutral hydrogen at ca 300,000 years after the Big-Bang) that pressure was cosmologically important. Exact Friedmann-Robertson-Walker solutions of Einstein's equations are also obtainable for the radiation-dominated universe. A pulsating universe *may* or *may not* "bounce" back from its final "Big Crunch" $R < 0$ singularity; if it does, the Big–Bang/expansion/contraction/Big Crunch cycle repeats ed infinitum, perhaps with random variations in each new repetition. Our present understanding of quantum gravity is too meager to tell what happens before a Big Bang (or after a Big Crunch) singularity.

1. The universe is pulsating if $\epsilon = 1$ (**Friedmann**, 1922).
2. The universe is continuously expanding if $\epsilon = 0$, or -1.

Thus, in this case, the spatial geometry of the universe is clearly correlated to whether the universe is expanding continuously or pulsating; it may be hyperbolic $(\epsilon = -1)$ or spatially Euclidean $(\epsilon = 0)$ and continuously expanding, or else it is hyperspherical $(\epsilon = 1)$, and pulsating. The universe is spatially *infinite* in the first case, and finite without boundaries in the second. (The case of a Euclidean universe $(\epsilon = 0)$, which is continuously expanding, is a limiting special case of the hyperbolic universe.)

The light which comes to earth from a galaxy follows a *null geodesic* along which $ds = 0$ and $d\theta = d\varphi = 0$. (in the Milky-Way centered frame). The Friedmann–Robertson–Walker line element equation then yields

$$0 = c^2 dt^2 - \frac{R^2(t) dr^2}{\left(1 + \frac{\epsilon}{4} r^2\right)^2}$$

or

$$\frac{dt}{R(t)} = \frac{-dr}{c\left(1 + \frac{\epsilon}{4} r^2\right)},$$

where the minus sign is employed to indicate that the light comes towards us, i.e., propagates to smaller r.

For galaxies whose light has taken a small fraction of the age of the universe to reach earth, the above relation leads directly to *Hubble's law* for the redshift (relative wavelength lengthening) of light arriving from a galaxy at current distance $d = R(t_0) r$ from earth:

$$c\frac{\Delta \ell}{\ell} \approx Hd - \frac{H^2 d^2}{2c}(1 - q), \qquad q = -\frac{R_0 \ddot{R}_0}{\dot{R}_0^2},$$

where ℓ is light wavelength, $R_0 = R(t_0)$, t_0 is the time that a light ray (that left the galaxy at time t) reaches earth, $H = \dot{R}_0 / R_0$ is the Hubble's "constant" at the present cosmic epoch, and q is the present–epoch (dimensionless) *deceleration parameter*.

Using the above cosmological–dynamics ODE of Lemaître together with the assumption $\lambda = 0$, we find:

$$q = \frac{4\pi G\rho}{3H^2}, \qquad \epsilon = \frac{8\pi G\rho R^2}{3qc^2}\left(q - \frac{1}{2}\right).$$

The universe, therefore, is spatially hyperspherical if $q > \frac{1}{2}$ (i.e., $\epsilon > 0$), hyperbolic if $q < \frac{1}{2}$ (i.e., $\epsilon < 0$), and flat if $q = \frac{1}{2}$. Since q is related to the density of the universe, there is a critical value of ρ corresponding to $q = \frac{1}{2}$, and given by $\rho_c = \frac{3H^2}{8\pi G}$.

If $\rho > \rho_c$ the universe is hyperspherical (finite) and pulsating, while for $\rho \leq \rho_c$ it is hyperbolic or flat and expanding forever. If we put $H = 50$ km/sec/megaparsec (approximately corresponding to the latest empirical value for the universe's age t_0) in the last formula, we obtain $\rho_c = 5 \times 10^{-30}$ gm/cm^3.

Refined cosmological observations since the mid-1990's have established that the cosmological "constant" λ is *small* and *positive* at the present epoch ($\lambda \approx 5 \times 10^{-53}$ meter^{-2}). (We know from particle physics and the success of the inflationary scenarios for the very early universe, that λ is actually *dynamical* and epoch–dependent).

Therefore, the above $\lambda = 0$ equations must be modified accordingly. It is also believed, based on these observations, that we live in a $\epsilon = 0$ universe. And yet, $\rho \approx 0.3\rho_c$; the balance of the spatial curvature needed to close the universe, is supplied by the cosmological constant (nowadays referred to as *dark energy*).

1927–1937 CE **Fritz Wolfgang London** (1900–1954, Germany and USA). Physicist. Contributed to the quantum theory of valence, and the theories of superconductivity and superfluidity.

With **Walter Heitler**[639] (1927) he derived the first quantum-mechanical model of the hydrogen molecule (H_2), their wave-equations forming the basis for valence-bond approach to *molecular quantum mechanics*.

With his brother Heinz London (1935), he discovered a phenomenological modification of Maxwell's electrodynamics for a superconductor; it included both infinite conductivity and the *Meissner effect*. The *London equation* states that the time-derivative of the superconducting current density $\boldsymbol{J}$ is proportional to the electric field: $\frac{\partial \boldsymbol{J}}{\partial t} = a\boldsymbol{E}$, which implies that in a superconductor, the current can be nonzero even if the electric field is zero; He was then able

[639] **Walter Heitler** (1904–1981, Germany and Switzerland) one of the pioneers of quantum field theory and quantum chemistry. A student of A. Sommerfeld in Munich (Ph.D. 1926). Escaped the Nazis to Dublin (1941–1948). Professor of theoretical physics in Zurich since 1949.

to show that the magnetic field is zero inside the superconductor, except for a thin skin layer near its surface[640].

London (1936) was first to point out that *superfluidity*[641] may be related to Bose statistics.

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640 In a normal conductor and a Drude-type (classical) conduction model, the electric field $\boldsymbol{E}$ provides the force needed to keep the electrons moving with a drift speed $v_d = \frac{eE\tau}{m}$, where e and m are the charge and mass of an electron, respectively, and τ is a time proportional to the electron mean free time between collisions; Thus $\boldsymbol{J} = \sigma\boldsymbol{E}$, where the conductivity σ is directly proportional to the mean free path. In the superconducting phase, all the Cooper (electron) pairs (call their density n) have the same drift velocity $\boldsymbol{v}_d$, yielding a current $\boldsymbol{J} = 2ne\boldsymbol{v}_d$. Taking the time derivative and writing the electron acceleration as the electric force divided by the electron mass, we have

$$\frac{\partial \boldsymbol{J}}{\partial t} = 2ne\frac{\partial \boldsymbol{v}_d}{\partial t} = 2ne\left(\frac{e\boldsymbol{E}}{m}\right) = 2\frac{ne^2}{m}\boldsymbol{E} = a\boldsymbol{E}.$$

Taking the curl of both sides of the London equation, and using *Faraday's law* $\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$ the *magnetic Gauss'– law* $(\boldsymbol{\Delta} \cdot \boldsymbol{B}) = 0$ and *Ampere's law* (with $\frac{\partial \boldsymbol{E}}{\partial t} = 0$) $\nabla \times \boldsymbol{B} = \mu_0\boldsymbol{J}$, we obtain: $\nabla^2\boldsymbol{B} = a\mu_0\boldsymbol{B}$. The only uniform magnetic field that satisfies this equation is $\boldsymbol{B} = 0$, which is Meissner's result.
If an external magnetic field is applied to the superconducting sample, the London theory predicts the $\boldsymbol{B}$ field will decay exponentially inwards of the sample's surface, with skin-layer thickness $\frac{1}{\sqrt{a\mu_0}}$. Note that the *London theory* is in full accord with quantum physics. Indeed, the expression for the current density in terms of the Cooper-pair macroscopic (complex) wavefunction $\Psi(\boldsymbol{r})$ and the vector potential $\boldsymbol{A}$ is known to be

$$\boldsymbol{J}(\boldsymbol{r}) = \boldsymbol{J}_0(\boldsymbol{r}) - \frac{2e^2n}{m}\Psi^*\Psi\boldsymbol{A}$$

where

$$\boldsymbol{J}_0(\boldsymbol{r}) = \frac{ne\hbar}{2mi}(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*)$$

Since Ψ is a pure phase $(\Psi^*\Psi = 1)$,

$$\frac{\partial \boldsymbol{J}}{\partial t} = -\frac{2e^2n}{m}\frac{\partial \boldsymbol{A}}{\partial t} = \frac{2e^2n}{m}\left(-\frac{\partial \boldsymbol{A}}{\partial t}\right) = \frac{2e^2n}{m}\boldsymbol{E}$$

(the gauge $A_0 = 0$ was assumed) and London's result again follows.

641 According to the quantum-mechanical treatment of the *Pauli Exclusion Principle*, a gas of a normal He^4 atoms, each with its two protons, two neutrons, and two electrons is described by a wave-function which is symmetrical w.r.t. Helium–atom exchanges; it obeys *Bose-Einstein statistics* (since the total num-

He was born to Jewish parents in Breslau (now Wroclaw, Poland) and was educated at the universities of Bonn, Frankfurt, Göttingen and Munich (Ph.D., 1921). He was a Rockefeller research fellow at Zurich and Rome and lecturer at the University of Berlin. During 1933–1936 he was a research fellow at the Universities of Oxford and Paris, but Nazi persecution caused him to leave Europe. In 1939 he emigrated to the United States, becoming a professor of chemical physics at Duke University (1939–1954).

1927–1942 CE **R.V.L. Hartley** (1890–1970, U.S.A.). Electrical engineer, inventor and applied mathematician. Made the first serious attempt to introduce a scientific measure of *information* in the field of electrical communica-

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ber of fermions in each atom is even).

Since the He^4 atoms are *bosons*, they may in principle "*condense*" in the lowest per-atom quantum state of the system as absolute zero temperature is approached. In this state they would not carry out any *thermal* motion, but they would nevertheless not form a crystal lattice since the *zero-point energy* of the light helium atoms, according to the *uncertainty principle*, is larger than their very small *Van-der-Waals bond energy* in a lattice.

The vanishing viscosity of superfluid Helium-4 (i.e., the vanishing dissipative collisions between its helium atoms) would then be due to the impossibility to transfer energy from one atom to another if the thermal energy is smaller than the first energy excitation gap of the system. This is so because the energy of the many-particle system of the superfluid helium atom is quantized.

Thus, treating the system of Helium atoms as a degenerate (ground state) boson gas, the lowering of the temperature below a critical value (T_c) of several degrees–Kelvin condenses a macroscopic fraction of the helium atoms into the ground state, forming what is called a *superfluid*. The process is known as *Bose-Einstein condensation*. An atom of the rare He^3 isotope – containing one less neutron – is a *fermion*; but it was discovered (D. Osheroff *et al.*, 1970's) that at very low temperatures (a few *milli*-Kelvin), He^3 atoms form bosonic (Cooper-like) pairs; these condense, and therefore pure He^3 liquid, too, can become superfluid.

The dynamics of inter–atomic interactions in liquid (and thus also superfluid) phase helium is quite complicated. Beginning with the pioneering work of **E. Cornell** (1995), however, physicists have been able to use advanced cryogenics (in Magneto–Optical–Gravitational traps) to produce mesoscopic samples of Bose–Einstein Condensates (BEC's) comprising gases of alkaline atoms (e.g Rb or Cs). This new state of matter – being gaseous and thus weakly–interacting and easily analyzed via simple quantum mechanics – has remarkable electromagnetic and mechanical properties, with many possible technological applications.

tion. To this end he defined (1927) what we now call the information capacity of a message[642].

Hartley formulated (1942) a real integral transform that is fully equivalent to the Fourier integral transform, but dispenses with complex representation. It is known as the *Hartley transform*[643]. It has certain advantages over the traditional Fourier transform, especially in saving computer time.

1927–1942 CE **Bronislaw Kasper Malinowski** (1884–1942, England). Founder of *Social anthropology.* Known for his intensive studies of the culture of Southwest-Pacific and African natives, and for his contributions to the theories of human culture in general.

Malinowski was born in Poland and studied at the University of London. He taught there for many years, and at Yale University from 1939.

Author of *Argonauts of the Western Pacific* (1922), *Myth in Primitive Psychology* (1926), and *A Scientific Theory of Culture* (1944).

1927–1948 CE **George Alfred Léon Sarton** (1884–1956, Belgium and U.S.A.). One of the outstanding historians of science in modern times.

Embarked on a mission to make the history of science an articulate discipline, believing that this would provide a history of human thought and a better understanding of the nature of man. In this he intended to achieve the 'new humanism', a holistic and all-embracing synthesis based on appreciation of science in history.

He began a monumental *Introduction to the History of Science* (3 volumes, ca 6000 pp., 1927–1948), which only reached the third volume, and the year 1400.

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642 He regarded the sender of a message as equipped with a set of symbols (the letters of the alphabet for instance) from which he selects symbol after symbol, thus generating a *sequence* of symbols. He then defined H, the *information* of the message, as the logarithm of the number of possible sequences of symbols which might have been selected; clearly $H = s\log n$. Here s is the number of symbols selected, and n is the number of different symbols in the set from which symbols are selected.

643 The Hartley transform reads:

$$F(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)[\cos\omega t + \sin\omega t]\,dt,$$

$$f(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(\omega)[\cos\omega t + \sin\omega t]\,d\omega.$$

Sarton's legacy, in addition to the subject's professional identity, lay primarily in his bibliographies, documentation and fact finding (15 books, over 300 articles and notes, and 79 critical bibliographies).

Sarton was born in Ghent, E. Flanders, Belgium and educated in philosophy and the natural sciences. He obtained his doctor's degree in mathematics at Ghent University in 1911, and emigrated to the United States in 1915. His lifework at Harvard was financed by the Carnegie Institution. In 1912 he founded *Isis*, still a major journal in the subject. In the U.S.A. he created a learned society and established the identity of science history and its claim to a place in universities.

Sarton worked on the three volumes of his history for 37 years, finishing the last one, on the 14^{th} century, in 1948. He figured that dealing with the 15^{th} century alone would cost him 15 additional years, of which he said: "*At my age this would be tempting Providence*".

He then added:

"*Looking back to the dreams of my youth, the creation of a chronological survey of scientific efforts down to the* 20^{th} *century, to be followed by two series of complementary surveys.*

I may seem to have failed egregiously, for my Introductions stops five centuries short of the goal. The scholarly reader will agree with me that it is better to do something as well as one can than to do considerably more less well. Other scholars will complete my task and may be able to do it much better".

Worldview XLII: Sarton

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"Early historians of science were tempted, in their ignorance and their bumptiousness, to consider the science of the 16^{th} and 17^{th} centuries as a continuation of ancient science, and so it was, but a continuation which would have been impossible or utterly different without the medieval gropings which intervened. Galilean physics is the climax of centuries of such gropings, and even the Newtonian fluxions and gravitation have medieval roots (whether Newton was explicitly aware of them or not does not matter much). The 17^{th}-century mathematicians were generally well acquainted with Greek mathematics down to Diophantos and Pappos, and they believed in good faith that they were taking up the work where those ancients had left it. They did not realize how much they owed to the slow incubation of ideas which was the medieval indispensable contribution. This was hidden from them because Renaissance scholars had tried to obliterate the Middle Ages; that obliteration, whether conscious or not, continued in the field of art and letters until the Romantic age, and in the field of science until our very own."

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"The history of science describes man's exploration of the universe, his discovery of existing relations in time and space, his defense of whatever truth has been attained, his fight against errors and superstitions. Hence, it is full of lessons which one could not expect from political history, wherein human passions have introduced too much arbitrariness. Moreover, it is an account of definite progress, the only progress clearly and unmistakably discernible in human evolution. Of course, this does not mean that scientific progress is never interrupted; there are moments of stagnation and even regression here or there; but the general sweep across the times and across the countries is progressive and measurable. The history of science includes the most glorious, the purest, and the most encouraging deeds in the whole past."

* * *

"In order to explain 14^{th}-century thought, it is not enough to deal with 14^{th}-century authors; one must deal as well with many authors of earlier times

whose thought was still living in the 14^{th} century. It has already been remarked that though Archimedes was killed in 212 BCE, his spirit is still living today. As a matter of fact that particular spirit was not much in evidence in the 14^{th} century, but older ones, those of Hippocrates of Cos, Plato, Aristotle, Euclid were going strong. Medieval men were not chronologically minded, they spoke of Hippocrates and Aristotle in the same way as of their own contemporaries, and those ancient sages were indeed to that extent their contemporaries."

* * *

"The scientific results of the past may generally be neglected, for they have been incorporated into later ones; the scientific achievements, however, can never be superseded, and they keep their value eternally. Mathematicians have no technical reasons for reading Euclid, but they will never cease to admire him, and their appreciation of his genius is bound to increase together with their own knowledge."

* * *

"The history of science is to a large extent a history of rationalism, rationalism in action. It is a history of the gradual emancipation of men from superstition and ambiguities, a history of the growth of light in dark or darkened corners, a history of our salvation not only from lies but also from other evils, from servitude and intolerance. The medieval part of that history is meager in tangible results, but such results are not always a fair measure of the efforts. The difficulties which the medieval heroes of thought had to overcome were immense. Results are the fruit of all preceding efforts, not only of the latest ones. The triumph of modern science was partly due to medieval efforts.

Great men were not rarer in the 14^{th} century than in the 20^{th}, but the old institutions (the church and the empire) were shaking and the new institutions (universities, parliaments, etc.) not yet sufficiently well established."

* * *

"One universal characteristic of mystical thinking is the distrust of intellectualism. The great weakness of mysticism, always and everywhere, is its individualism. It is necessarily and paradoxically dogmatic, for the mystic, however sincere and convinced he may be, cannot give his reasons. His conviction cannot be communicated to others, except if they are exactly in the same

mood. He cannot completely justify himself; he can only oppose dialectical dogmatism with his own intuitive dogmatism.”

* * *

“The Greeks laid stress upon truth and beauty; the Romans upon strength and usefulness. The ruin of science, begun by Roman utilitarianism, was in danger of being completed by Christian piety which considered scientific research not only useless but pernicious.”

* * *

“From the point of view of the history of science, transmission *is as essential as* discovery*. If the results of Ptolemy’s investigations had been hidden instead of published, or if they had been lost in transit, they would be almost as if they had never been.*

Now, the question “How did Ptolemy’s knowledge came down to us?” opens up the study of medieval science and justifies it. If there were no other reason to study medieval science than to find out how ancient knowledge was handed down to us, that reason would be sufficient”.

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“In the beginning astrology was a sound body of knowledge, based upon a premise, which proved to be erroneous, but which was not unreasonable, namely, that planets and stars can and do influence human events. Thus, a scientific study of planetary motions would enable one to interpret and to foretell these events.

It is easy enough to understand the growth of that fallacy if one realizes the appeal which periodical phenomena have never ceased to make upon people of all kinds, whether educated or not. Symmetry and periodicity form the very substance of science and also of art, and the more intricate, the subtler they are, the more impressive once they have been discovered. We may assume that even at the very dawn of civilization, the more thoughtful men had been awed by the extraordinary periodicities in the motion of the stars, of the moon, and the sun. And, by and by, as they discovered the more complicated periodicities involved in the apparently erratic displacements of the planets, their awe and trust in cosmic harmony increased in proportion. Thus were *the astrological assumptions naturally introduced. They received an extraordinary confirmation from two terrestrial phenomena which exhibited similar periodicities and were immensely impressive because of their universality, of their complexity, and of their mathematical rigor: the tides of the sea and the menstruation of women. Both phenomena were explained by planetary influences; the explanation was essentially right in the first case and wrong in the second. That error was a very pardonable one. And if some of the planets could thus affect the bodies of women nay, their very souls, was it unreasonable to assume that they might influence as well the destinies of men? These theories, being erroneous, were naturally sterile and unprogressive; as they could not progress, they deteriorated, and as they blocked the stream of thought, they gathered around them all the superstitions which it carried."*

* * *

"The historian of science can not devote much attention of the study of superstition and magic, that is, of unreason, because this does not help him very much to understand human progress. Magic *is essentially unprogressive, and conservative;* science *is essentially progressive. We can not possibly deal with both movements at once except to indicate their constant strife, and even that is not very instructive, because that strife had hardly varied throughout the ages. Human folly being once unprogressive, unchangeable, and unlimited, its study is a hopeless undertaking!*

The history of astrology*, however, must be carefully considered since a large part of it is so intimately connected with the history of science that it cannot be dissociated from it. This applies with even greater force to* alchemy*, for in this case there was considerably more scope for the continual integration of new experimental facts, which were valuable in spite of wrong interpretations: the alchemical facts outlasted the alchemical structure, and many of them are now integral parts of our chemical knowledge."*

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"To take a step forward in the right direction is always a great thing, and the first steps are always the most difficult and the most creditable. We forget it but too often, and our histories are full of injustice, because we are almost always too generous toward those who made the last steps and reaped the result to all antecedent efforts, and too little generous to those who made the first and least profitable steps."

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"A large part of our knowledge and of our technique was attained, not at all in a logical way, but by the method of trial and error, which works well, but is exceedingly slow. The whole history of thought points to the conclusion that some errors at least were unavoidable, that is, mankind could learn how to avoid them only by making them. Thus we were spared some errors only because our ancestors had made them before us. Indeed, these medieval scientists are our direct ancestors; if they had been such idiots, how could we be so clever? It is extremely probable that if we had been living under the same circumstances as they, we should not have proceeded much faster. The progress of science is on the whole, an accelerated one; thus, in any retrospective survey, we must expect the progress to become slower and slower as we penetrate more deep by into the past. And above all we must remember that science could not progress along certain lines without traversing vested interests and prejudices and without hurting the feelings of the community. To proceed in the face of such opposition has always required a great deal of intellectual courage. There were many more such sensitive lines in the Middle Ages than now, and thus there was a far greater need for that particular kind of heroism. In the whole sweep of history there is nothing more impressive than the spectacle of noble men who had the spirit to fight unreason and ignorance and who did not hesitate, not only to renounce material advantages, but even to jeopardize life and happiness in order to increase the amount of beauty, justice, *and of* truth *which is the essential part of our patrimony."*

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"And this is the very spirit of science — the continual alteration of experimental research, of mathematical elaboration, of theoretical deduction and discussion suggesting new experiments. Or, in other words, the continual alterations of analysis and synthesis: analytical investigations without synthetic attempts must necessarily degenerate into crude empiricism and into superstition; synthetic constructions without periodic experimental contact must necessarily degenerate into a sterile dogmatism. Science is not a being, but a becoming."

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"There are no discontinuities in the intellectual life of the world if we take into account the achievements of all peoples in every direction. Whenever a

nation dropped out of the race, another was ready to take up the torch and to continue mankind's eternal quest."

* * *

"Science slowly emerges from philosophy and religion, chemistry from alchemy, astronomy from astrology."

1927–1950 CE **Barbara McClintock** (1902–1992, US). Geneticist. Revolutionized the field through her observation in maize genetics that genes are not stable — thus overturning one of the main tenets of heredity laid down by **Gregory Mendel**. This had enormous implications and explained, for example, how resistance to antibiotic drugs can be transmitted between entirely different bacterial types. McClintock novel ideas were not accepted for many years. She won, however, the Nobel Prize in Physiology or Medicine (1983), the 5th woman to be so honored.

McClintock was born in Hartford, CT and received a Ph.D. in botany (1927) from Cornell University – where she stayed (1927–1935). Later held posts at the University of Missouri (1936–1941) and the Carnegie Institute, New York (1941–1992).

She showed (1927–1931, with **Harriet Creighton**) that genes can change their position on a chromosome ('*jumping genes*'[644]) from generation to generation. This would explain how originally identical cells take on specialized

[644] She observed that the patterns on twin sectors of maize seedlings were the inverse of one another, and that pigmentation of certain kernels did not correspond to their genetic makeup. Realizing that a single cell divided into sister cells, one gained what the other had lost, she deduced that *not all genes behave in the same way: some genes can switch others on and off*, moving from one place to another on the chromosome, or even 'jumping' from one chromosome to another. These jumping genes act as regulators and were later discovered in bacteria and fruit flies.

functions as skin, muscle, bone, and nerve cells, and also how evolution could give rise to multiplicity of species.

In the 1940's she showed how genes in maize are activated and deactivated by 'controlling elements' — genes that control other genes and which can be copied from chromosome to chromosome. She presented her work in a symposium in 1951, but its significance was lost on the attendees, who mainly worked with bacteria. It was not until the 1970's, after the work of **Jacob** and **Monod**, that her work began to be appreciated.

1927–1956 CE **Bernhard Zondek** (1891–1966, Germany and Israel). Gynecologist and endocrinologist. Discovered gonadotrophine. Developed the first reliable hormonal pregnancy test [1927, with **Selmar Aschheim** (1878–1965)] and the assistance of his brother **Herman Zondek** (1887–1979).

Zondek was born in Wronke, Germany. He was trained in medicine at the University of Berlin, becoming a professor (1926–8) and Director of its Department of obstetrics and Gynecology (1929–1933). He left Nazi Germany (1933) to become Professor at the Hebrew University, Jerusalem (1934–1966).

During this period he discovered that the anterior pituitary gland produced hormones called gonadotrophins, which in turn stimulated other endocrine glands, such as the ovary, to release their hormones. This work provided important evidence of control mechanisms in reproduction, which has had widespread significance in the development of medical and social attitudes towards questions of fertility, infertility, contraceptions and abortion.

1927–1957 CE **Otto Neugebauer** (1899–1990, USA). A foremost historian of premodern science. Made important contributions to the history of mathematics and astronomy. His coverage of Egyptian and Babylonian science and its transmission to the Hellenistic world released the surprising sophistication of certain areas of early science.

Many of his discoveries have revolutionized earlier understandings. He thus showed that Babylonian strength in algebraic and numerical work reveals a level of mathematical development in many aspects comparable to the mathematics of the early Renaissance in Europe (in contrast to the relatively primitive Egyptian mathematics). In the realm of astronomy, too, Neugebauer discovered an unexpected sophistication which he ascribed to a competent mathematical approach rather than to the result of millennia of observations (as used to be the interpretation).

Neugebauer was born in Austria to Jewish parents. He emigrated to the USA (1940).

1927–1958 CE **Ludwig Mies van der Rohe** (1886–1969, Germany and USA). Architect and teacher. One of the founders of the *modern movement* *in architecture*; won fame for the clean, uncluttered design of his buildings of brick, steel, and glass. The sparse appearance of his buildings illustrated his motto: "*Less is more*".

Mies was born in Aachen (N. Rhine – Westphalia), Germany. He built his first steel-framed building in 1927. In 1929 he designed the German pavilion at the Barcelona exhibition, with its marble walls reaching out beyond the building, its hovering roof slab, and its great expanse of glass. In 1930 he became director of the Bauhaus school in Dessau, but closed the school in 1933 as a protest against the Nazis.

In 1938 he emigrated to the USA and was appointed director of the school of architecture at the Illinois Institute of Technology. He is said to have based his attitudes to architecture and the teaching of it on Thomas Aquinas' proposition: '*Reason is the first principle of all human work*'.

The curriculum at his school was designed to teach general principles with a strong emphasis on construction, encouraging general rather than specialized solutions. This approach contrasted with most other contemporary schools of architecture in which the aim was to engender individuality of expression.

Mies' buildings are models of structured clarity and simple geometry. They had enormous influence on his contemporaries and indeed on many present-day architects, who respect his handling of the high-rise skeleton-frame tower block, exemplified in the twin towers on Lake Shore Drive (Chicago, 1948–51) and the Seagram building (New York, 1954–8).

1927–1960 CE **Abram Shmulovich Besicovitch** (1891–1970).
Karaite[645] mathematician. A geometric analyst of extraordinary power[646].

[645] *Karaism* — a sect that broke away from Judaism in 760 CE. They bear the name *Karaim*, "Scripturalists", believers in the Bible only, excluding later traditions. They claimed that the original intentions of the Holy Book were lost in the countless laws built on top of the biblical words. Their belief was that each Jew had a right to explain the biblical statements in accordance with his own views, without regard to the "official" explanation offered by the scribes and their followers.

[646] The following anecdote may serve as a typical example of his sharp wit: One day, during his lecture at Cambridge, the class chuckled at his fractured English. Besicovitch turned to the audience and said: "*Gentlemen, there are fifty million Englishmen speak English you speak; there are two hundred million Russian speak English I speak*". The chuckle ceased.

One of the founders of the theory of *fractals* (*Hausdorff-Besicovitch dimension*), on which he published a series of path-breaking papers during 1928–1937. In 1927, Besicovitch solved the *Kakeya problem*[647], proposed in 1917 by **Soichi Kakeya** (1886–1947, Japan). [It required to find a figure of least area in which a segment of unit length could be turned through 360° by a continuous movement.] Known for his work on *almost periodic functions*.

Besicovitch was born in Berdyansk, Russia. He was taught by **Markov** at St. Petersburg. He left Russia (1924) and worked with **Harald Bohr**. In 1927 he moved to Cambridge. His wife remained in Russia and the marriage was dissolved (1928). He then married (1930) the 16 year old daughter of a previous girlfriend.

1928 CE **Alexander Fleming** (1881–1955, Scotland). Bacteriologist. Discovered *penicillin* at St. Mary's Hospital, Paddington, London. (It was made stable enough for medical use only in 1943.) Shared the Nobel prize for physiology or medicine (1945) with **Howard Walter Florey** (1898–1968, England) and **Ernst Boris Chain** (1906–1979, England) for isolating and purifying penicillin for general clinical use.

1928 CE **Otto Paul Hermann Diels** (1876–1954, Germany) and **Kurt Alder** (1902–1958, Germany). Chemists; developed a synthesis known as the *Diels-Alder reaction*[648]: a technique for combining atoms into molecules that is useful in forming many compounds, especially synthetic rubber and plastics. For this they were awarded the Nobel Prize for Chemistry (1950).

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[647] For a time it was believed that a hypocycloid of area $\frac{\pi}{8}$ was the desired figure. However, Besicovitch proved that Kakeya's problem has no solution by showing that there are figures of arbitrary small area having the Kakeya property.

[648] *Alkanes* are those hydrocarbones in which adjacent carbon atoms are joined together by a *stable single bond*. They are called *saturated hydrocarbons* because the carbon atoms are covalently bonded to as many hydrogens as the carbon-backbone connectivity allows. They have the general formula C_nH_{2n+2} (e.g. Methane CH_4, Ethane C_2H_6 etc.)

Alkenes contain two carbon atoms joined together by a *double bond*, and therefore two hydrogen atoms less than the corresponding alkanes. They are *unsaturated*, having the general formula C_nH_{2n} (e.g. Ethylene C_2H_4).

Dienes contain two double carbon bonds (e.g. Butadiene C_4H_6). Because of the *unstable* double bond, Alkenes and Dienes are chemically more active and form *addition compounds* with H_2, Cl_2, Br_2, HCl, HBr, H_2SO_4, etc.

In the Diels-Alder reaction, a conjugated diene reacts with an alkene:

Reactions that create new carbon-carbon bonds are very important in organic synthesis because only through such reactions can small organic molecules be converted into larger ones. The Diels-Alder reaction is a particularly important reaction because it creates two new carbon-carbon bonds in a manner that results in a *formation of a cyclic molecule* (*aromatic compound*).

1928 CE **Chandrasekhar Venkata Raman** (1888–1970, India). Physicist. Observed experimentally the existence of displacement (lowering or raising) of frequencies in the electromagnetic radiation scattered by a molecule, known as the *spontaneous Raman effect* or *Raman scattering*. The effect had been predicted (1923) by **Adolf Smekal** (Germany) and involves at least three quantum states, and three transitions among them, in a molecule or a radical. The magnitude of the frequency change, and its dependence upon the properties of the absorbing species, constituted a compelling proof for the veracity of the quantum theory which had, at the time, only recently emerged.

The *Raman effect* is related to molecular vibrations and rotations and to electron orbitals. When a gas sample (for example) is illuminated with monochromatic radiation of frequency ν_0, it is observed that the radiation scattered in a direction at right angles to that of incidence, contains, in addition to the primary frequency component ν_0 (which is the result of coherent *Rayleigh scattering* of the incident radiation), secondary radiation components of frequencies $\nu_0 \pm \nu_s$, where ν_s corresponds to a frequency of the vibrational or rotational spectrum of the molecule. This is called *Raman scattering*[649].

Its quantum-mechanical interpretation is as follows: suppose that a molecule is initially in a (ground or excited) vibrational and/or rotational

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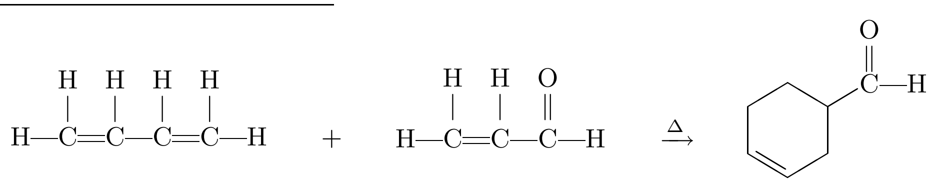

where Δ signifies the addition of heat.

[649] The Raman effect can be explained in the framework of classical physics on the basis of the polarizability of the electron orbitals of molecules: A light wave with frequency ν_0 incident upon an isotropic molecule induces in this molecule an electric *dipole moment* $M_i = \alpha E_0 \sin(2\pi\nu_0 t)$, where α is characteristic of a specific molecule and is called *polarizability* (it is a measure of the deformability in the electron orbitals), and E_0 is the amplitude of the electric field vector of the impinging light. Assume that α is not constant but changes linearly with the intra-molecular atomic positions as the atoms in the molecule *vibrate* against each other with the frequency ν_s, or as optically anisotropic molecules *rotate* around their center of gravity. By setting $\alpha = \alpha_0 + \alpha_1 \sin(2\pi\nu_s t)$ and inserting

state[650], and also in some electronic-orbital configuration (ground state or excited). When an electron in the molecule absorbs a photon of frequency ν_0 it transitions to an excited electronic state (with the atomic-nuclei quantum state unchanged). If the excited state is not a metastable one, there is a virtually immediate re-radiation of a photon.

The molecule may return to the initial state, emitting radiation of the same frequency as the incident light; this is *Rayleigh scattering*. The molecule may, however, return to *another* vibrational or rotational level immediately above or below the initial level, with the emitted radiation having the frequency $\nu_0 - \nu_s$ or $\nu_0 + \nu_s$, respectively; this is the quantum explanation of *Raman Scattering*[651]. The Raman effect has been observed for both vibrational and rotational spectra; for the latter the selection rule for the electronic-orbital sector of the transition is $\Delta\ell = 0, \pm 2$ (compared to $\Delta\ell = \pm 1$ in the vibrational case)[652].

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the expression into the equation for M_i, we obtain

$$M_i = \alpha_0 E_0 \sin(2\pi\nu_0 t) + \frac{1}{2}\alpha_1 E_0 \left[\cos 2\pi(\nu_0 - \nu_s)t - \cos 2\pi(\nu_0 + \nu_s)t\right].$$

While the first term represents an induced dipole moment that vibrates with the exciting frequency ν_0 and thus causes the classical *Rayleigh scattering*, the two terms in brackets are responsible for radiation of two light waves, the frequencies of which are shifted against that of the exciting wave by $\pm\nu_s$. This is the *Raman effect*, which is very important in molecular physics.

650 Vibrational and rotational states are orbitals of the *atoms* within a molecule, i.e. they are solutions of the Schrödinger equation for the nuclear-position wavefunction. Since electron motions are much more rapid than those of atoms (because nuclei are much heavier), the *Born-Oppenheimer* approximation allows one to first solve for the electron's wavefunctions with nuclei held fixed, and then use the dependence of the electronic energy levels upon nuclear positions to derive *effective* inter-atomic potentials — which are in turn used to set up and solve the nuclear-position wave equation.

651 Such a transition involves changes of *both* the electron wavefunction *and* the nuclear-positions wavefunction. (ν_s is a frequency characteristic of the vibrational or rotational transition.)

652 The effect had been predicted earlier by **Kramers**, **Heisenberg**, **Schrödinger** and **Dirac**. In fact it was seen by **Lommel** (1878), but discounted as noise.
At first, the effect was difficult to put to actual use because one needed strong sources (usually Hg discharges were used) and large samples. Often the ultraviolet from the source would further complicate matters by decomposing the specimen. And so it is not surprising that little sustained interest was aroused by the promising practical aspects of the Raman effect.

The $\nu_0 + \nu_s$ spectral component of the Raman effect depends on the initial molecular state being *excited* in its atomic-nuclear-positions sector; such vibrational and rotational excitations occur naturally due to thermal fluctuations.

The Raman effect provides a direct method for the investigation of the structure of molecules and molecular fragments in solid, liquid and gaseous state. It permits measurements of their natural vibration frequencies, and (through the transition selection rules) helps in studying the symmetries of molecules, intramolecular forces, molecular dynamics etc.

The Raman spectra characterize a molecule with such precision that their analysis can be used for determining the composition of mixtures of molecules when the ordinary methods of chemical analysis fail to provide needed results.

Raman was born in Tiruchirappali, Tamil Nadu, India. He received his bachelor degree at Presidency College, Madras (1904). During 1907–1917 he was employed as a civil servant in a finance department in Calcutta. In his spare time he cultivated his interest in acoustical problems and their relevance to the theory of musical instruments, and worked irregular hours in the laboratory of the Indian Association for the Cultivation of Science. In 1917 he became professor of physics at the University of Calcutta.

His most renowned contribution came in 1928 when he telegrammed his letter to *Nature* describing '*A New Type of Secondary Radiation*', for which he was awarded the Nobel prize for physics (1930). From 1933 onwards he lived and worked in Bangalore, where he founded the *Indian Academy of Science* (1934), and later directed the *Raman Research Institute* (1948).

The situation was changed dramatically when *laser* light sources became a reality. *Raman spectroscopy* is now a unique and powerful analytical tool; the laser is an ideal source for spontaneous Raman scattering. It is bright, highly monochromatic, and available in a wide range of frequencies.

In 1962, **Eric J. Woodbury** and **Wan K. Ng** fortuitously discovered a related effect known as *stimulated Raman scattering*, where part of the incident energy (at the wavelength 6943 Å in their experiment) was shifted in wavelength and appeared as a *coherent* scattered beam at 7660 Å. It was subsequently determined that the corresponding frequency shift of about 45×10^{12} Hz was characteristic of one of the vibrational modes of the molecule of nitrobenzene (the scatterer used by them).

By the turn of the 21^{st} century, active and passive photonic-device products became available that utilize intense *pump lasers* to provide the initial-frequency photons; the stimulated longer-wavelength lightwaves, present to begin with but amplified, *near-infrared telecommunication* signals propagating in ordinary optical fibers, with the fiber medium itself providing the scatterer molecules.

He was a prolific writer (there are more than 500 articles bearing his name), and a key figure in the foundation of modern Indian science. Along with **Ramanujan**, **Tagore**, **M. Gandhi** and **S. Radhakrishnan**, he was conspicuous in the flowering of Indian culture in the first part of the 20^{th} century. The astrophysicist S. Chandrasekhar (1910–1995) was his nephew.

1928–1933 CE **Raymond (Edward Alan Christopher) Payley** (1907–1933, England and USA). Mathematician. Made important contributions to the theories of *Fourier series*, *Fourier transforms*, *quasi-analytic functions* and related topics.

Paley was educated at Eton. From there he entered Trinity College, Cambridge where he was taught by **Hardy**. He then collaborated with **Littlewood**, **Zygmund** and **Polya**. In the United States he worked with **Norbert Wiener**[653].

While skiing near Banff he was killed by an avalanche. Thus ended the meteoric career of a brilliant mathematician who had the potential to rise to the level of the great mathematicians with whom he collaborated.

1928–1931 CE **Jesse Douglas** (1897–1965, USA). Mathematician. First to prove the existence of a surface of minimal area bounded by a contour (the *Plateau problem*, 1873; first posed by Lagrange in 1760 and studied by Riemann, Weierstrass and Schwarz).

Before Douglass' solution only special cases had been solved.

Douglass studied at Columbia College (1920–1926). Visited Princeton, Harvard, Chicago, Paris and Göttingen (1926–1930).

1928–1937 CE **Jerzy Neyman** (1894–1981, Poland and U.S.A.). Statistician. Produced the *Neyman-Pearson* system of hypothesis testing[654]: a set of

653 Payley-Wiener condition: A necessary and sufficient condition for a square-integrable function $A(\omega) \geq 0$ to be the Fourier spectrum of a causal function is the convergence of the integral

$$\int_{-\infty}^{\infty} \frac{|\ln\, A(\omega)|}{1+\omega^2}\, d\omega < \infty$$

654 Based on the *Neyman-Pearson lemma* which states (verbally) that an hypothesis is not invalidated because it makes observed events improbable; there must be a realistic alternative hypothesis that does better.

While the structure of this lemma is too restrictive for the result to have much *practical* significance, it does have enormous importance from a conceptual standpoint.

criteria for maximizing efficiency in the design of tests which is the foundation of modern *quality control*.

Faulty items found in an inspected sample may be ineradicable errors or a signal to close down and reset a machine. The theory then looks at the cost and risk of stopping a good machine and compares it to the cost and risk of running a bad one, to obtain the best criterion and sample size.

In 1934 Neyman tackled the problem of using random samples in *human populations*. He proposed a general principle: to take linear combinations of data items and balance them, first to eliminate bias and then to minimize variance. This provides rules for deciding how intensively to sample, and where.

In 1937 he formulated the classical theory of *confidence intervals*.

1928–1937 CE *Antisemitism in Austria* soared to unprecedental levels; violent antisemitic riots. Dr. Seipel, leader of the clericals and Prime minister, preceded to profess his hatred of the Jews. The Archbishop of Vienna endorsed the appeal to boycott Jewish merchants (1928). Governmental discrimination against appointment of Jews to the civil service continued.

By 1933, Jews were virtually eliminated from the university life. An edict was promulgated forbidding Jews to practice medicine in Austrian hospitals. In May 1934, a new Austrian constitution established a fascist-corporatist state on Christian principles. Chancellor **Dollfuss** was shot to death by Nazi storm troopers in Vienna (1934).

By the end of the year only 12 Jewish teachers in the Vienna elementary school system remain of a former total of 5000. Nearly 60,000 out of the 176,000 Vienese Jews were registered (1936) in the welfare department as applicants for relief. As of July 1937, Jewish doctors were not allowed to practice privately. The elimination of "Jewish influence" in education, the theater, the press, the arts, and the sciences was now achieved (1937).

On March 11, 1938 Vienna becomes the second largest city of the Third Reich; the fate of the Jews was finally sealed.

1928–1938 CE **Paul Adrien Maurice Dirac** (1902–1984, England). Distinguished physicist. A major contributor to relativistic quantum mechanics and quantum statistics, and the originator of the concept of antimatter.

Egon Sharpe Pearson (1895–1980, England; son of **Karl Pearson**) contributed to the theory of *statistical inference* and developed the concepts of *likelihood ratio* test of an hypothesis.

In 1926, while still a graduate student, he derived his own version of quantum mechanics, lagging only a few months behind Born and Jordan. In this same year he formulated, independently of **Fermi**, what is known today as Fermi-Dirac statistics. His major work was presented in 1928: he discovered a first-order partial differential wave equation for a free particle, which is invariant under the Lorentz transformation and incorporates spin in a natural way. This *Dirac equation* is consistent with the *Klein-Gordon equation*, and Dirac suggested the former as the fundamental relativistic wave equation for the electron – for which the Klein-Gordon equation is not sufficient[655].

He found that his new linear equation is a matrix equation of rank 4, the wave-function solutions of which are 4 component *spinors*. The associated algebra is the *Clifford algebra* in 4 dimensions, and each spinor component also obeys the Klein-Gordon equation.

Dirac's equation — when interpreted as a first – quantized (Schrödinger) wave equation — showed that there must be states of *negative energy*. The latter conclusion did not seem to correspond to physical reality. In a later paper, in 1929, Dirac suggested that a deficiency of an electron is one of these (otherwise completely *filled*) states, would be equivalent to a positively charged particle.

According to this idea, the vacuum is not empty, but rather contains an infinite sea of completely-filled negative energy states; localized energy, such as a high-energy photon, can occasionally knock one of these electrons into a positive energy state. To an observer, this would appear as the materialization, out of the vacuum, of a pair of particles — an ordinary electron, and a "positive electron" — a hole in the sea of negative, otherwise — filled energy

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[655] One of the motivations at that time to reject the Klein-Gordon equation as a fundamental quantum-mechanical wave equation, was that the Born probabilistic interpretation seemed to lead to probability distributions that can go negative, clearly a nonsensical result. Soon, however, Dirac's own work on antimatter led to *second quantization* (nowadays called *field quantization*), in which there is a distinction between a particle's *field equation* on the one hand, and the *Schrödinger wave equation* of the whole (multiparticle) system on the other. Thus, in quantum electrodynamics — a second-quantized theory — we distinguish between Maxwell's equations (the photon's field equations) and the Schrödinger equation. When all this became clear, the Klein-Gordon equation was restored to grace as a valid equation, on equal footing for some (integer–spin, bosonic) particles with the Dirac equation (which holds for fields of spin-$\frac{1}{2}$ fermions). The Klein-Gordon equation follows from the Dirac equation, since the former is simply the quantum-mechanical version of Einstein's STR kinematical relation $p_\mu p^\mu = m^2c^2$, obeyed by *any* free particle of rest mass m.

levels. Similarly, an electron can *annihilate* its positive counterpart, by dropping from positive to negative energy (emitting radiation in the process) and re-filling a negative level.

This picture — while literally true for quasiparticles in conducting solids, for which the positive quasiparticles is the *hole* familiar from semiconductor physics[656] — requires some modifications in the more elegant field–quantization picture later adopted, in which the *Dirac sea* of negative–energy states is dispensed with. In the modern picture, fermions and anti–fermions are treated on equal footing, with a manifest *charge–conjugation* discrete symmetry between them.

Dirac's theory was confirmed when **C.D. Anderson** obtained a cloud chamber photograph with tracks showing the existence of *positrons*. Thus, an apparent fatal flaw of Dirac's theory turned into the successful prediction of *antimatter*. This development made it clear that the successful marriage of STR and quantum-mechanics requires a new foundation of the latter — in which particle numbers are not conserved, and even the vacuum is a complex many-body system[657]. Dirac's theory emerged triumphant.

Dirac also (1933) raised the question of what corresponds in the quantum theory to the *Lagrangian* method in classical theory. He pointed out that the function $K(\boldsymbol{x}_2, t_2;\ \boldsymbol{x}_1, t_1)$ which carries the wave function $\psi(\boldsymbol{x}_1, t_1)$ at time t_1 to the wave function $\psi(\boldsymbol{x}_2, t_2)$ at time t_2, is analogous to $e^{\frac{i}{\hbar}S}$, where $S(\boldsymbol{x}_2, t_2;\ \boldsymbol{x}_1, t_1)$ is the classical *action*.

This function S was first introduced by **Hamilton**, and was computed from the classical trajectory $\boldsymbol{x}(t)$ linking the coordinate $\boldsymbol{x}_1$ of the classical particle at the instant t_1 to the coordinate $\boldsymbol{x}_2$ at time t_2; the related **Hamiltonian** enables one to describe the classical evolution as a canonical transformation developing in time. Hamilton's action is obtained by integrating the Lagrangian of the system over the yet unknown classical path from the point with the coordinate $\boldsymbol{x}_1$ at instant t_1 to he point with the coordinate $\boldsymbol{x}_2$ at time t_2,

$$S = S(\boldsymbol{x}_2, t_2;\ \boldsymbol{x}_1, t_1) = \int_{t_1}^{t_2} L\left[\dot{\boldsymbol{x}}(t), \boldsymbol{x}(t), t\right]\, dt.$$

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656 However, in the condensed–mater case there is less symmetry: no STR (the lattice frame is preferred!); no actual negative energies expending to $-\infty$ (just a valence band); and electrons have different effective mass than holes.

657 At first, Dirac erroneously identified the 'positive electron' with the proton! It is now accepted, on very strong theoretical and experimental grounds, that the mass and spin of any particle are exactly equal to those of its anti-particle (*CPT theorem*).

The *principle of least action* then states that the classical path $\bar{\boldsymbol{x}}(t)$ chosen by the system out of *all* possible paths is that for which S is extremal. This condition of extremum leads, mathematically, to the path-equation $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\boldsymbol{x}}}\right) - \frac{\partial L}{\partial \boldsymbol{x}} = 0$, where L is a priori *known* [e.g. for a single particle of mass m, moving in a potential $V(\boldsymbol{x}, t)$, we have $L = \frac{m}{2}\dot{\boldsymbol{x}}^2 - V(\boldsymbol{x}, t)$].

During 1937–1938 (and later, 1973–1979), Dirac proposed a new cosmological theory ("large number theory") based on the notion that there are some very large dimensionless numbers in physics with hitherto unknown significance, which are approximately equal to one another. These numbers (all of which are of the order 10^{40}) are:

(1) The ratio of the electromagnetic force between a proton and an electron to the gravitational force between them.

(2) The square root of the number of particles N in the observable universe.

(3) The age of the universe, in units of the time required for light to traverse the classical 'electron radius'.

Dirac suggested that this coincidence expresses some fundamental (although still unexplained) truth, and that therefore all three numbers should be equated.

It then followed that one or more of the 'constants' $\hbar$ (Planck's reduced constant), G (gravitational constant), e (the fundamental charge unit), c (velocity of light in vacuo), m_e and m_p (electron and proton masses) must vary over cosmic time-scales. In order to avoid the reformulation of atomic and nuclear physics, Dirac chose G as that fundamental 'constant' that varies with time. This requires that $G(t)$ *decrease* with time like t^{-1}, where t is time elapsed since the Big Bang in any frame of reference at rest relative to the local Cosmic Microwave Background Radiation. [In GTR, G *must* be kept fixed and unalterable, since there is no mechanism within the theory for G to depend either on the distribution of matter or time. In the *Brans–Dicke* (scalar–tensor) modification of GTR, in which the *strong* equivalence principle is relaxed, the effective G is a function of the scalar field, and so may depend on time (or even space). In modern inflationary and/or higher–dimensional QFT versions of Big–Bang cosmology, G, m_e, m_p and (to some extent) e may vary with cosmological epoch; $\hbar$ and G (in most theories) may not.]

Thus, Dirac's cosmology requires that GTR be replaced with some other field theory of gravitation, which he never supplied. Previously, **Eddington** tried unsuccessfully to explain the fine-structure constant $\left\{\frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137}\right\}$ and other dimensionless parameters.

Dirac's theory is now in conflict with observations on two counts. First, his relation $t = \frac{1}{3}H_0^{-1}$ (H_0 is Hubble's constant at the present epoch) leads to an age of the universe that is too small. Second, recent observations (1981) have shown that $\frac{1}{G}\left|\frac{dG}{dt}\right| \leq 6 \times 10^{-11}$/year, and even as small a value for $\frac{1}{G}\left|\frac{dG}{dt}\right|$ as this bound, if extrapolated backwards over billions of years, is unacceptable: it predicts a surface temperature of the earth 10^9 years ago, that reach that of *boiling water*, which in turn, preclude the evolution of life to its present form.

Carter (1974) gave an entirely different explanation for the coincidence of large numbers: perhaps, at epochs much before these numbers approximately coincided, the physical conditions were such that life, and especially intelligent life like man, could not yet exist! Therefore, the coincidence of Dirac's large numbers is just a *necessary prerequisite* for man to appear in the universe. This is a version of the "*anthropic principle*".

Dirac was born in Bristol. His mathematical ability manifested itself at an early age. At the school he attended in Bristol, he was given rather advanced books on mathematics to study independently. His father, a Swiss by birth who was the French master at the same school, encouraged his son to develop his mathematical ability. He wished him to also become fluent in French, to the extent that, according to the son's report, the elder Dirac refused to speak to him unless he was addressed in the French language. This may have fostered Dirac's pronounced tendency to seldom speak, and to choose his words with utmost care. He avoided company, preferring to work alone. His main leisure pastime was solitary walks.

Toward the practical end of earning a living, Dirac studied engineering at the University of Bristol. The use of approximations that he acquired in these studies had a strong influence on his later work: it strengthened his confidence in the intuitive approach to problem solving. He came to believe that a theory expressing fundamental laws of nature could be constructed solely on the basis of approximations, guided by intuition rather than exact knowledge of the actualities. He declared that the actual phenomena were too complex ever to be pinned down in a precise way; a physicist must be satisfied to work with only approximate knowledge of reality.

Dirac's study of theoretical physics began only after he received a degree in electronic engineering, failed to find work in this field, and, aided by a grant, entered St. John's College, Cambridge. From R.H. Fowler, his faculty supervisor, who had collaborated with Niels Bohr in his pioneering work in atomic physics, Dirac learned the current state of that science. Dirac was awarded the Nobel prize for physics in 1933.

Dirac taught at Cambridge after receiving his doctorate there, and in 1932 was appointed Lucasian professor of mathematics, the chair once held by Isaac Newton. He served in that capacity until 1968, shortly after which he moved to the United States. In 1971 he was made professor emeritus at Florida State University, Tallahassee, FL.

Dirac had no school or following and had produced (like Einstein) very few students. He had essentially no collaborators and once, when asked about this, had remarked that "the really good ideas in physics are had by only one person".

The Dirac Equation

The development of quantum mechanics took place historically through the wave-mechanics of ***Schrödinger****, based on the pioneering work of* ***de Broglie****, and simultaneously through the matrix mechanics of* ***Heisenberg****,* ***Born*** *and* ***Jordan****. The latter scheme was concerned chiefly with the literal observables of an atomic system, such as the spectral lines obtained in transitions between pairs of states or the particle velocities and positions. The two approaches were unified in the work of* ***Dirac****, who based quantum mechanics on the classical mechanics of Poisson, Hamilton and Jacobi, with a new interpretation of Hamilton's 'canonical'* variables.

The successful representation of electron spin by the matrices of ***Pauli****, paved the way for many advances in quantum mechanics. However, the theory was still inadequate for dealing with fast-moving particles, because the Schrödinger equation is not invariant under the Lorentz transformation of STR.*

In order to derive a relativistic Schrödinger-like wave equation[658] *for an electron interacting with an electromagnetic field (atomic and/or external), one must first derive a suitable Hamiltonian function for a particle of rest mass*

[658] At that time, prior to the discovery of second (field) quantization, no distinction was made between probability – amplitude wave-equations and field equations.

m, and charge e, free or in an external electromagnetic field characterized (in a particular gauge) by a vector potential $\boldsymbol{A}$ *and scalar potential* ϕ*. The classical Lorentz-covariant Hamiltonian is*

$$H = \sum_{i=1}^{3} p_i \dot{q}_i - L \quad , \quad p_i = \frac{\partial L}{\partial \dot{x}_i}$$

with $x_i(t)$ *the spatial particle trajectory in a particular inertial frame, where* $\boldsymbol{p}(t)$ *is the canonical (not mechanical) 3-momentum and the Lagrangian is*

$$L = -mc^2 \left(1 - \frac{v^2}{c^2}\right)^{1/2} + e(\boldsymbol{v} \cdot \boldsymbol{A}) - e\phi.$$

This leads to

$$H = e\phi + c\left[(\boldsymbol{p} - e\boldsymbol{A})^2 + m^2c^2\right]^{1/2},$$

or,

$$(H - e\phi)^2 - (\boldsymbol{p} - e\boldsymbol{A})^2 c^2 = m^2 c^4.$$

With the usual quantum mechanical correspondence

$$\boldsymbol{p} \to \frac{\hbar}{i}\boldsymbol{\nabla}, \qquad H \to \hbar i \frac{\partial}{\partial t},$$

the relativistic wave equation for a charged particle devoid of intrinsic angular momentum (spin) becomes:

$$\left[\left(\frac{\hbar}{i}\frac{\partial}{\partial t} + e\phi\right)^2 - \left(\frac{\hbar}{i}\boldsymbol{\nabla} - e\boldsymbol{A}\right)^2 c^2\right]\psi = m^2c^4\psi \ ,$$

with $\psi(\boldsymbol{x}, t)$ *complex wave function. This is the Klein-Gordon equation for a charged scalar particle under the influence of an external EM field. For* $e = m = 0$ *(photon), the Klein-Gordon equation simply reduces to Maxwell's electromagnetic wave equation.*

The Klein-Gordon equation was first discovered by Schrödinger, but was almost immediately rejected by him for two reasons: firstly, it possesses negative-frequency solutions, which are also (by $H \to \hbar i \frac{\partial}{\partial t}$*) negative-energy solutions. This is not really fatal, since one can discard all such solutions, although this is a suspiciously artificial procedure, and leads to problems with causality.*

A more serious objection is that it would lead to negative probabilities in Born's interpretation of the wave function.

Dirac sought to avoid both problems by finding a wave equation with a differential operator linear in $\boldsymbol{\nabla}$. That is, he sought to replace

$$\left[(\boldsymbol{p} - e\boldsymbol{A})^2 + m_e^2 c^2\right]^{1/2}$$

with

$$\boldsymbol{\alpha} \cdot (\boldsymbol{p} - e\boldsymbol{A}) + \beta m_e c,$$

in such a way that the squares of both operators coincide. Clearly, such an equality cannot be valid if the quantities $\boldsymbol{\alpha}$ and β are regarded as an ordinary vector and scalar, since squaring both expressions and equating coefficients of similar terms leads to the contradictory requirements

$$\alpha = (\alpha_1, \alpha_2, \alpha_3), \quad \alpha_1^2 = 1, \quad \alpha_2^2 = 1, \quad \alpha_3^2 = 1, \quad \beta^2 = 1,$$

$$\alpha_1\alpha_2 + \alpha_2\alpha_1 = 0, \quad \alpha_1\beta + \beta\alpha_1 = 0,$$

$$\alpha_2\alpha_3 + \alpha_3\alpha_2 = 0, \quad \alpha_2\beta + \beta\alpha_2 = 0,$$

$$\alpha_3\alpha_1 + \alpha_1\alpha_3 = 0, \quad \alpha_3\beta + \beta\alpha_3 = 0.$$

However, it is possible to satisfy these equations if α_1, α_2, α_3 and β are not numbers, but rather 4×4 anticommuting matrices (called Dirac matrices[659]*):*

[659] This is only one of an infinite number of matrix representations for $\boldsymbol{\alpha}$, β; however, the matrices *must* be at least 4×4 (in 4 dimensions, the *Clifford algebra* can only be implemented using at least 4×4 matrices). The three 2×2 Pauli matrices are the simplest realization of a Clifford algebra in *three* dimensions. In D spacetime dimensions, the Dirac matrices must be at least 2^n by 2^n, where n is the largest integer not greater than $D/2$. In our 4 spacetime dimensions, 2×2 Dirac matrices are allowed *if* the fermion has zero rest mass (*Weyl spinors*). *Neutrinos* were once thought to be massless. Now, these spin $-1/2$ *leptonic* fermions are known to have small but finite masses, but — due to peculiarities of the weak nuclear forces that endowed them with their rest-masses a fraction of a second after the Big Bang – must still be described by (several copies of) a Weyl spinor. But charged leptons (e.g electrons) are described as Dirac spinors, with 4×4 Dirac matrices.

$$\boldsymbol{\alpha} = \left\{ \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \right\}$$

$$\beta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The corresponding Dirac equation for a charged particle in an electromagnetic field then assumes the form

$$H\psi = \left[c\boldsymbol{\alpha} \cdot \left(\frac{\hbar}{i} \boldsymbol{\nabla} - e\boldsymbol{A} \right) + \beta mc^2 + e\phi \right] \psi = \hbar i \frac{\partial \psi}{\partial t},$$

where the quantum Hamiltonian matrix-differential operator on the L.H.S. acts on both the position dependence and the discrete index of the four-component, complex Dirac–spinor wavefunction,

$$\psi = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{bmatrix} \quad ; \quad \psi_a = \psi_a(\boldsymbol{r}, t), \qquad a = 1, 2, 3, 4.$$

The Dirac equation is equivalent to 4 simultaneous, linear, first-order differential equations in the components of ψ. It has been solved exactly for several important problems, including that of the hydrogen (or any hydrogen–like) atom with infinitely–massive nucleus, and an electron in any plane EM wavepacket. In the special case of a free particle

$$\psi = \exp \left[\frac{i}{\hbar} (\boldsymbol{p} \cdot \boldsymbol{r} - Et) \right] \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix},$$

with $E^2 = -\boldsymbol{p}^2 c^2 = m^2 c^4$ and two of the four complex constant c_a components being independent per given value of $\boldsymbol{p}$ and given energy sign. This represents the two possible spin states ("spin up", ↑, and "spin down", ↓) of an electron (or positron) relative to any given rest-frame spatial direction. Thus, the

Dirac equation actually predicts the existence of electron spin[660]. *Better yet: in the non-relativistic* $(|\boldsymbol{p}| \ll mc)$ *regime in a weak external magnetic field, the Dirac equation predicts the electron spin gyromagnetic ratio of* $g_e = 2$ *— very close to the empirical value, which before Dirac was totally inexplicable (the orbital ratio has the classical value of* 1 *in Dirac's theory, in agreement with experiment). For the hydrogen atom, Dirac's equation correctly predicts the fine structure of line spectra (due to the spin-orbit magnetic coupling also predicted by semiclassical relativistic electrodynamics; and also due, in part, to the STR velocity dependence of mass).*

Substituting the above representation for ψ *into the Dirac equation (with* $\boldsymbol{A} = \phi = 0$*) we obtain a system of* 4 *simultaneous homogeneous, linear algebraic equations, whose solubility condition is:* $E = \pm(p^2c^2 + m^2c^4)^{1/2}$. *From this we conclude that the permissible energy values for a free particle in Dirac's theory range from* mc^2 *to* $+\infty$ *and from* $-mc^2$ *to* $-\infty$. *The first of these results is of course just what we expect for a free relativistic particle*

660 A *Weyl* spinor has only *one* spin state when moving at extremely relativistic speeds ($|\boldsymbol{p}| \gg mc$), and correspondingly it needs only *two* Dirac components; such is the case for *neutrinos.* These spin $-1/2$ fermions have very small masses, so they are usually emitted (and absorbed) traveling vary near light–speed, and spinning clockwise relative to their direction of motion. If one could slow them down substantially — or chase them in a frame in which they are at rest — one would, of course, observe them to occur in *both* spin states (if all else fails, one could then use a *magnetic field* to rotate them – like neutron, neutrinos are electrically neutral but have magnetic dipole moments – or, overtake them. So they will now appear to spin *counter–clockwise* relative to their direction of motion!)

This apparent paradox can be resolved by noting that, in any quantum field theory in which a Weyl spinor acquires mass, there is a certain probability (especially at non-relativistic speeds) for a neutrino to turn into an *anti*-neutrino (or vice versa) — and antineutrinos (at near-light speeds) are *always* observed to spin *counter*–clockwise relative to their velocity direction. This postulated weak inter–conversion of matter and antimatter may partly explain cosmological *baryogenesis* — the phase transition (a fraction of a second after the Big Bang) which generated a small (ca 1 part in 10^{10}) *excess* of matter over antimatter.

Subsequently, the *antimatter* annihilated almost entirely with matter, leaving only radiation and the small *matter* excess: the universe we observe today.

In some extensions of the Standard Model of particle physics, the existence of counter–clockwise–spinning ("right handed"), low–mass neutrino states, is due in part to a quantum mixing between *massless*, *left*-handed neutrinos moving at light–speed, and *ultra-heavy*, *right*–handed, Weyl spinor neutrinos.

— that its total energy can have any value greater than or equalling its rest energy. But the second allowed energy band is quite puzzling, since it implies the existence of states of negative energy!

Furthermore, there is nothing to prevent — and indeed the theory requires — the occurrence of transitions between states of positive and negative energy. These rather remarkable features of Dirac's theory led him to predict the positron (e^+), which was indeed discovered in 1932 *by* ***Anderson***, *but not really understood until the advent of quantum electrodynamics (QED) in the* 30*'s and the* 40*'s.*[661]

1928–1941 CE **Frank Whittle** (1907–1996, England). Engineer, inventor and fighter pilot. Father of the jet age. The leading pioneer in the development

[661] QED results when the coupled Maxwell and Dirac equations are second-quantized. According to QED, a positron propagating *forward* in time can be interpreted as a *negative energy electron* propagating *backward* in time (!) — and yet, overall the theory does obey STR's local causality. The small deviations of empirical results from Dirac's predictions – e.g. for g_e – have been explained to very high accuracy by QED effects ("radiative corrections"), such as *vacuum polarization* due to virtual e^+e^- pairs.

It is ironic that the two reasons for the original rejection of the Klein-Gordon (KG) equation, were invalid! for the negative energy solutions occur in Dirac's equation as well, and turn out to be necessary to describe *antimatter*; while the negative-probability problem disappears when one second-quantizes the KG equation.

Today, the KG is used as a field equation for fundamental *scalar* and other integer–spin (bosonic) quantum fields, while the Dirac equation is used for fundamental *spinor* (fermionic) fields (such as the spin $-1/2$ neutrinos, electrons and quarks). Hypothetical fundamental spin $-\frac{3}{2}$ fermions — the so-called *gravitinos* of *supergravity theories* — are described by the Rarita–Schwinger equation, which is essentially the Dirac equation for a field $\psi_{\mu a}(x)$ with both a spacetime *and* a Dirac index, and with a few additional auxiliary conditions. The *spin-statistics theorem* of Quantum Field Theory states that local causality requires that integer-spin fields (such as scalars, photons non-abelian gauge fields and gravitons) describe *bosons* (i.e. obey Bose-Einstein quantum statistics), while half-odd-integer-spin fields (such as electrons, neutrinos, quarks, protons) describe *fermions* (i.e. their associated particles obey Fermi-Dirac statistics and the Pauli Exclusion principle).

of the turbojet engine. This engine powered Britain's first jet plane in 1941 and became the model for the first U.S. turbojet.

Whittle was born in Coventry, England, the son of an inventor. He joined the Royal Air Force when he was 16 and distinguished himself already in 1928 with his first patent, after the Air Ministry rejected his jet engine proposals.

Whittle's basic patents lapsed in 1935 because he did not have enough money to pay patent fees. Later that year, a group of engineers became interested in his work and, with the British government and Whittle, formed *Power Jets Ltd*, to produce engines. Whittle had joined the RAF in 1924 as an apprentice, later training as a fighter pilot.

During 1937–1946 he worked on jet propulsion. In May 1941, the Gloster E 28/39 first flew with the Whittle engine. Both the German and the US jet aircraft were built using his principles. He retired from the RAF (1948) and took up a university appointment in the USA.

While others (e.g. the British Rolls-Royce Co.), made many billions off his invention — which revolutionized transportation in the 20th century — the inventor himself was merely granted the lump sum of $100,000. Even in his own country he never received the recognition he deserved. The jet age started without Whittle.

1928–1952 CE **Felix Bloch** (1905–1983, U.S.A.). Distinguished physicist. Among the first to apply quantum theory to solid state physics, particularly to the magnetic properties of matter. Discovered (with **Edward Mills Purcell**) the phenomenon of nuclear magnetic resonance (1946), and developed it as a means of studying solids and liquids by measuring the magnetic behavior of their atomic nuclei. For this work they were both awarded the Nobel prize for physics (1952).

Bloch was born in Zürich, Switzerland, to Jewish parents. He studied first at the Federal Institute of Technology in Zürich and in 1927 under **Heisenberg**, at Leipzig, where he became a lecturer in physics (1932). He left Germany in 1933 and in 1934 went to Stanford University where, apart from a period during WWII, he remained as a professor of physics (1934–1971). In 1954–5 he took leave to become director of CERN, Geneva. His main contributions to quantum physics were:

- Modern theory of electrical conduction in metals (1928). Developed and applied methods for solving the Schrödinger equation of an electron in periodic structure (metallic lattice). Introduced *Bloch theorem*; *Bloch wavefunctions*; and *Bloch wavenumber*. Showed that the *electrical resistance* of a metal is mainly due to imperfections in the arrangements of its constituent atoms. Investigated the conductivity of metals at low temperatures and determined the dependence of spontaneous magnetization upon temperature (1930).

- Measured the magnetic moment of the *neutron* (with **L.W. Alvarez**, 1939); devised methods of polarizing neutrons, separating them according to the direction of their intrinsic spin (neutron beams).
- Solved several problems concerned with the properties of *ferromagnetic* materials (such as iron). These materials are divided into small mesoscopic regions called *domains*, in each of which the magnetization points in a different direction. Bloch showed which factors influenced the formation of the boundaries (or 'walls') between these domains, and calculated their thickness and dynamics.
- Introduced the model of '*spin waves*' in magnetic materials, which has proved extremely valuable in understanding how magnetic properties deteriorate when the temperature of the material is raised.
- First successful NMR experiment with bulk matter; *Bloch equations* (1946).

1928–1968 CE **George Anthony Gamow** (1904–1968, Russia and U.S.A.). Distinguished astrophysicist. One of a handful of scientists who made a distinctive mark on 20^{th} century science:

- Developed the quantum theory of radioactivity (1928) including the *tunneling effect*[662].

[662] An important quantum-mechanical phenomenon (directly observable), also called *barrier penetration* – which is responsible for such physical phenomena as field emission of electrons from metals, nuclear fusion in the solar core, and α-particle radioactivity. The particle wave-function acquires an exponentially decreasing spatial dependence — the matter-wave analogue of optical *evanescence*. This, in turn, makes it possible for a particle with insufficient *classical* kinetic energy for passing over a potential barrier, to occasionally penetrate it nonetheless, with a computable, finite probability per "attempt". Our sun *could not shine* without quantum tunneling, since its core temperature is too low for thermal energy alone to allow fusing protons to overcome their mutual Coulomb repulsion at sufficient rates.

In 1933, **Clarence Melvin Zenner** (1905–1994, USA) first explained the electrical breakdown of insulators in terms of tunneling effects [known as *Zenner effect* or *Zenner breakdown* and put to use in the *Zenner diode*]. A very similar effect occurs in the *Esaki diode* (1957), where an external electric field is applied across a thin film of insulator separating heavily doped p-type and n-type semiconductors.

- Originated (with **Edward Teller**) the *liquid-drop model* of atomic nuclei (1928–1929).
- Postulated (with **Fritz Houtermans**) that the sun's radiant energy derives from thermonuclear processes (1929), i.e., that *nuclear fusion* is the primary source of energy in stars.
- Formulated the Gamow-Teller theory of *beta decay* (1936) and the theory of internal structure and energy source of red giant stars (1942).
- Sorted out the role played by neutrinos in the supernova explosions of stars.
- His application of nuclear processes to cosmology led him to become a leading advocate of the '*Big-Bang*' theory of the origin of the universe[663]. Already in the late 1930's he applied his wide knowledge of nuclear physics to stellar evolution studies and in work concerning the mass-luminosity relationship for stars.

As early as 1946 Gamow and his collaborators began theoretical research regarding the origin of the elements during the first 4 minutes of the expansion of the universe, hypothesizing that the elements were created by neutron capture[664]. By 1948, Gamow had coined the term 'Big Bang' for the cosmological model that described the Universe as beginning in a highly dense, compact fireball, whose explosive expansion we are still witnessing through the recession of the distant galaxies.

Gamow's work on big-bang cosmology led him to predict the *cosmic abundance of Helium* and the presence of a low-temperature *cosmic microwave*

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663 Einstein himself did *not* believe in the existence of singularities in nature [*Ann. Math.* **40**, 922, 1939]. On the subject of the *big bang*, Einstein's last words were: "One may… not assume the validity of the equations for very high density of field and matter, and one may not conclude that the 'beginning of explosion' must mean a singularity in the mathematical sense" [A. Einstein and N. Rosen, *Phys. Rev.* **48**, 73, 1935]. Modern Quantum Field Theorists concur; for example, in *string theories* Einstein's GTR is but a long-wavelength approximation, to be replaced by an effective, non-local field theory at epochs approaching the Planck scale (times of order 10^{-43} sec after Big Bang). These effective field theories avoid the far-past singularities which **S. Hawking** and others proved must afflict *any* GTR-based cosmology (under reasonable assumptions).

664 Expounded in a paper by **A**lpher, **B**ethe and **G**amow (1948), in which Hans Bethe's name was borrowed for comical effect.

background radiation[665]. [The latter prediction was confirmed observationally by **Arno Penzias** and **Robert Wilson** (1965); light-element abundance predictions based on Big Bang models have also been brilliantly confirmed by astrophysical and cosmological observations.]

- First to suggest (1954) a three–nucleotide–symbol for genetic coding scheme; it was inaccurate, but later acknowledged as a key milestone in the development of molecular biology. The correct DNA genetic code — which indeed utilized codons three nucleotides long — was worked out by **M. Nirenberg** *et al.* in the 1960's.

Gamow was born in Odessa, Russia. He was educated in the USSR, studied nuclear physics at Göttingen, Germany (1928) and spent some time in Cambridge at the Cavendish Laboratory (1929), where **Rutherford** asked him to calculate the energy required to split the atom. He then emigrated to the United States (1933). Became a professor of physics at George Washington University (1934–1956), and later professor of physics at the University of Colorado (1956–1968).

1928–1960 CE **Georg von Békésy** (1899–1972, Hungary and U.S.A.). Physicist. Established a new theory of hearing based on waves formed by fluids in the *cochlea* of the inner ear. This auditory mechanism replaced a theory put forward by **Helmholtz** (1857), and won Von Békésy the Nobel Prize for physiology in 1961.

He showed that the tympanic membrane is almost critically damped, that the middle ear is nonlinear, and that receptions in various regions of the cochlear duct are sensitive to different frequencies of vibrations. He was first to suggest (1960) that the *inner hair cilia* in the cochlea were the transducers

[665] This radiation, which bathes the earth almost exactly isotropically from all directions in the universe, corresponds at the current epoch to the microwave radiation of a black body at a temperature of 2.7 °K. The only plausible explanation for the origin of this radiation is that it is the remnant of an early phase in the expansion of the Universe – the so-called *recombination era* (ca 3×10^5 years after the Big Bang). The near–visible photons of that epoch – when electrons combined with photons to form neutral hydrogen and the universe became transparent to EM radiation and also matter–dominated gravitationally – spread throughout space and lost their energy through the adiabatic cosmic expansion (i.e through red–shifting), so that the corresponding temperature dropped – from about 3000 °K at recombination to its present value of 2.7 °K.

which convert an acoustic signal into a neural signal (traveling along a nerve fiber), and all recent evidence has fully confirmed that hypothesis.

Békésy became interested in hearing as director of the Hungarian Telephone System Research Laboratory (1923–1946), and was later Senior Research Fellow in psychophysics at Harvard.

Mechanics of Hearing (1561–1991)

"Anyone familiar with how extraordinarily various systems have been refined during 200, 000, 000 *years of mammalian evolution by natural selection, has to sit up very respectfully indeed in front of well attested evidence of anything so potentially valuable to an animal's survival as an auditory response curve which, around its peak, becomes more and more sensitive as sound levels get lower and lower. Immediately one is tempted to puzzle over what special sort of mechanism might have evolved to allow such an advantageous increase of sensitivity at low stimulus level".*

James Michael Lighthill, *J. Vib. Acoust.* **113**, 1–13 (1991)

Four hundred years had to pass from the first application of the scientific method to the subject before man could gain some basic knowledge of the structure and auditory mechanism of the human ear. This is not surprising in view of the complexity of the inner ear and the strong dependence of its physiology upon acoustic principles and phenomena that were not fully understood before the end of the 19^{th} *century.*

The first period (1561–1772) is marked by the efforts of Italian physicians and anatomists to 'map' the anatomy of the ear: ***Gabriele Fallopio*** *(1523–1562) gave the first modern description of the organs of the inner ear (1561). His contemporary* ***Bartolomeo Eustachi*** *(1520–1574) published a treatise on the organ of hearing in which he rediscovered the* Eustachian tube *(named after him although it was discovered some* 2000 *years earlier by* ***Alcmaeon of Crotona****), the* tympanic membrane *and the* cochlea *(1564).*

Giulio Casseri *(1522–1616) wrote a book on the anatomy of voice and hearing in which he discussed the physiology of hearing. The Italian anatomist* ***Antonio Maria Valsalva*** *(1666–1723) provided (1704) the first detailed description of the physiology of the ear, and* ***Antonio Scarpa*** *(1747–1832) discovered in* 1772 *the semicircular canals, vestibule and the internal structure of the cochlea.*

During the second half of the 19^{th} *century, physicists and physiologists began to harness acoustical theory to investigate the passage of sound through the ear. Chief among these was* ***Helmholtz*** *(1857–1864), who proposed a* resonance theory of hearing, *based on the argument that the transverse fibers of the* basiliar membrane *in the cochlea of the inner ear act as* tuned resonators *(1857). In* 1864 *he advanced the theory that the pitch (frequency) is detected by a series of resonators of different sizes in the cochlea, and that overtones and beats based on different frequencies, determine the quality of the perceived sound. In* 1870, ***Galtz*** *recognized that the vestibular process is for maintenance of* directional equilibrium *and is not involved in hearing. In 1886,* ***William Rutherford*** *(1839–1899) discovered that tiny hairs in the cochlea are set in motion and thus convey sound.*

The first practical commercial electrical hearing aid was patented in 1902 *by* ***Miller Reese Hutchinson*** *(U.S.A.).*

On other fronts of the science of hearing, the pioneering work of ***W.C. Sabine*** *(1898–1900) laid the foundation of the* acoustics of buildings and rooms; *The first systematic explorations of* binaural hearing *were made by the physicist* ***Irving Langmuir***, *while working on the detection of submarines during WWI. The physicist* ***Harvey Fletcher*** *(1884–1981, U.S.A.) first used precise and effective electronic apparata in studies of sound and hearing (1923–1929). At Bell Laboratories, he led a wide range of experiments on speech, hearing and sound reproduction, using vacuum-tube electronics, microphones, headphones, amplifiers and loudspeakers.*

Like the eyes, the ears perform two functions — they detect sound and gather information about the position and movement of the body. But unlike the eyes, two separate receptors are involved — the so-called cochlea *is sensitive to sound, and the* semicircular canals *and vestibule detect movement.*

As with vision, the body faces the problem of converting an external stimulus into a code understood by the brain. Sound is produced by waves of alternating-sign pressure fluctuations spreading out from a source, rather like ripples on a pond. These waves can pass through solids, liquids and gases. Sound reaches the ears by the vibration of molecules in the air (or in water, if the listener is diving).

Before nerve impulses are produced in response to sound, the vibrations are first converted into the mechanical movement of 3 *tiny bones, the last* *of which sets up a wave in a tube (cochlea) filled with liquid. Rows of tiny nerve cells respond to this wave movement by firing off signals which carry information about the amplitude, phase and frequency content of the incoming sound. This* 3-*stage auditory system gives humans a dynamic range of* 10^{14} *from the softest to the loudest detectable signal. The frequency range of the human ear is between* 20–20,000 *Hz (although it is significantly narrower, in actual fact, for most individuals).*

Many animals can detect sound of far higher frequencies than those detected by humans. (Some bats can produce and detect frequencies up to 120,000 *Hz.) Humans are most sensitive to sounds between* 1000 *and* 6000 *Hz, the range in which speech sounds fall. In this region, the ear can distinguish between two different pitches separated by* 2 *to* 6 *Hz, and between two clicks when the second follows the first after only* 10 *milliseconds.*

The anatomy and physiology of the ear is briefly as follows: The external ear consists of a funnel-like structure (auricle) which helps direct the air pressure changes down into an S-shaped canal that leads inward for about 27 *millimeters. Without the auricle one could not hear very well (and would not have any place to hang one's glasses!). The auditory canal (meatus) leads to the diaphragm, or eardrum (tympanic membrane). Thus, the meatus acts like an organ pipe with one closed end, and sets up a standing wave whose fundamental wavelength is four times its length, or about* 108 *millimeters.*

Since the speed of sound is 354 *m/sec at body temperature, the frequency is* 3280 *Hz. The meatus*[666] *provides an amplification of* 5–10 *dB for frequen-*

[666] During the 1920's, when routine measurements of sound amplitudes first became practical, the wide dynamic range of magnitudes made it customary to plot data on a logarithmic scale. **Harvey Fletcher** introduced (1923) a definition of a *sound-pressure level* (s.p.l) relative to an arbitrary reference standard, $L = 10\log_{10}\left[\frac{(p^2)\text{ave}}{p^2_{\text{ref}}}\right]$, the resulting number having the units of *decibels* (db). This definition implies that $L = 0$ at the level $p^2 = p^2_{\text{ref}}$ and $L = 1$ (dB) for $p^2 = 10^{1/10}p^2_{\text{ref}} = 1.2589P^2_{\text{ref}}$. In general, a second pressure level exceeds the first by 1 dB unit if there is an increment of 0.1 in the logarithm to base 10 of their corresponding *mean square pressures*. The mean square pressure may correspond either to the acoustic pressure, to that of one frequency component, or to a band of frequencies.

The above definition is sometimes written in terms of an *intensity level* in the form $10\log_{10}\frac{I}{I_0}$, where $I = \frac{\langle p^2\rangle_{\text{ave}}}{2\rho_0 c}$ is the *sound intensity* (acoustic energy flow per unit area per unit time), and ρ, c are respectively the mass density and speed of sound in air at the ambient temperature and pressure; the denominator $z = \rho_0 c$ is the acoustic impedance. The reference intensity is usually chosen to be $10^{-12}\ \frac{\text{watt}}{\text{m}^2}$ which is about the threshold of human hearing at *cies between 2000–5000 Hz, and this frequency range is the region in which the ear is most sensitive.*

While the outer ear helps direct the miniscule air-pressure changes (as small as 10^{-9} atmospheres) down into the ear canal with some initial amplification, the problem still confronted by the ear is how to convert (transduce) air-motion into some other physical excitation that will stimulate the nerves to send messages on to the brain. This is done in two steps: in the first step

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1000 Hz. [Note that if the actual intensity doubles $(I = 2I_0)$, the ear hears an increase of $10 \log_{10} 2 = 3$ dB, whereas if the intensity increases tenfold, there is an increase of 10 dB.]

The following auditory conditions are parametrized by the corresponding sound intensity (in $\frac{\text{watt}}{\text{m}^2}$) and dB level as heard by our ears: *human breathing* (10^{-11}, 10); *rustling of leaves*, or *whispers* (10^{-10}, 20); *two person conversation* (10^{-6}, 60); *vacuum cleaner* (10^{-4}, 80); *accelerating motorcycle* (at 5 meters: 10^{-1}, 110); *rock concert* (1, 120); *threshold of pain* (100, 140); *large rocket launch vehicle* (10^7, 190).

Some amusing comparisons can be constructed from these figures: since a human shout generates a power of 10^{-5} watts, the acoustic power generated by all the world's population shouting at once is about 10^5 watts. This power is emitted by a single large jet transport at take-off! Also, the total energy radiated by the combined shouts of the Wembly cup final crowd during an exciting soccer game is about that required to fry one egg! Finally, a 40 W light bulb illuminating an area of 1 cm^2 at distance of 1 cm produces the same power as 1500 bass voices singing fortissimo.

In the human ear, the range of acoustic intensities at the threshold of *audibility* on the one hand and the threshold of *pain* on the other, spans over 14 orders of magnitude; no single mechanical instrument is capable of such a vast dynamic range [*Analogy*: imagine a balance sensitive enough to weigh a single human hair with reasonable precision. If this same balance had the wide range of the human ear, it could also weigh an aircraft-carrier.] Note also that at the threshold of hearing, the vibration *amplitude* is about 10^{-9} cm at 1 KHz, that is, only some 10^{-3} of the mean free molecular path length in the surrounding air, or 0.1 of the Bohr atomic radius (*detail*: assume a displacement of the tympanic membrane $y = A \sin \omega t$ with $u = \dot{y} = A\omega \cos \omega t$; $I_0 = \frac{(p^2)_{\text{ave}}}{2z} = 10^{-12}$ watt/m^2; $(p^2)_{\text{ave}} = z^2 (u^2)_{\text{ave}} = \frac{1}{2} z^2 A^2 \omega^2$. Hence $A = \frac{2}{\omega}\sqrt{\frac{I_0}{z}}$, with $\omega = 2\pi \times 10^3 \,\text{sec}^{-1} = 6280 \,\text{sec}^{-1}$, $z = \rho_0 c = 450 \,\text{kg}\,\text{m}^{-2}\text{s}^{-1}$, yields $A = 1.5 \times 10^{-11}$ meters. Since one atmosphere is equal to $p_0 = 10^5 \frac{\text{Newton}}{\text{m}^2}$, and I_0 corresponds to a peak-to-trough pressure change of $\Delta p \simeq 4\sqrt{I_0 \rho_0 c} = 8 \times 10^{-5} \frac{\text{Newton}}{\text{m}^2}$, we find that $\frac{\Delta p}{p_0} \approx 10^{-9}$ at the threshold of audibility].

the vibrations of the tympanic membrane are converted into vibrations in the liquid, in which receptors are located.

Under normal circumstances the efficiency of vibration transfer from air to liquid is less than 0.1 *percent. The ear gets around this by using three small bones (ossicles)*[667] *of the so-called middle ear, which form a bridge connecting the eardrum to the inner ear and function to transmit vibrations between these parts. Because of the lever action of the bones, the force delivered to the last bone is* 1.4 *times greater than that applied to the tympanic membrane.*

The force delivered by the shapes is further increased because the surface area of the membrane is about 25 *times that of the oval window, which is where the bone applies its movement. Overall, then, the construction of the middle ear serves to amplify the air vibrations by* 35 *— a very clever mechanical structure indeed!*

The inner ear consists of a complex system of intercommunicating chambers and tubes called the labyrinth. The parts of the labyrinth include a cochlea[668] *that functions in hearing, and three semicircular canals that function in providing a sense of equilibrium. A bony chamber, called the vestibule, which is located between the cochlea and the semicircular canals, contains membraneous structures that serve both for hearing and equilibrium.*

If one cuts the cochlea tube lengthwise, it is seen that it is divided into two major compartments by the basiliar membrane; the two compartments interconnect. When the stapes pushes the membrane of the oval window, the fluid in the two compartments is compressed, so much so that the membrane covering the round window[669] *at the other end of the tube bulges out. The motion of the fluid sets the basiliar membrane into motion. A standing wave*

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[667] The auditory ossicles (*malleus*, *incus*, *stapes*) never grow, and keep their size invariant throughout life. The malleus vibrates in unison with the eardrum, and causes the incus to vibrate, which in turn passes the movement onto the stapes. Vibration of the stapes at the *oval window* of the inner ear causes motion in a fluid within the inner ear.

[668] The total length of the uncoiled cochlea is about 35 mm. Its coiled diameter is about 1.5 mm. Its two galleries are filled with *perilymph fluid*. The partition between the galleries consists of 3 membranes of which the basiliar membrane is the lowest one. The partition between the galleries is filled with a highly viscous substance called the *endolymph*.

[669] The role of the *round window* is to relieve any pressure of the perilymph fluid due to *mass movement* of the fluid (caused by low-frequency thumps) from the oval window. High-frequency clicks do not produce a mass movement of perylimph, but cause the whole partition between the galleries to move.

is established in the membrane, the characteristics of which are determined by the frequency of the sound.

The location of nodes and antinodes of the vibrating basiliar *membrane will depend on the frequency of the exciting vibration in the fluid, while the displacement at each point along the membrane will depend on the spectral amplitude of the corresponding frequency.*

Thus, receptors that are linked up along the length of the basiliar membrane could "read" the frequency and amplitude of a given Fourier component of an incoming sound signal from the amplitude pattern formed along the axis of the membrane.

The basiliar membrane *is not as simple as we have implied: At its upper surface is located the* organ of Corti *which stretches from the apex to the base of the* cochlea. *Its receptor cells, which are called* hair cells, *are arranged in rows, of which there is a total of* 23,500 *in each ear. Above these cells is a* tectorial membrane *that is attached to the bony shelf of the cochlea and passes like a roof over the receptor cells, making contact with the tips of their hairs.*

As sound vibrations pass through the inner ear, the hairs shear back and forth against the tectorial membrane, *and the mechanical deformation of the hairs stimulate the receptor cells, thereby initiating a* nerve impulse *from the hair cells, along the sensory fibers to the* cochlear nerve, *which later forms the* auditory nerve. *Various receptor cells, however, have slightly different sensitivities to such deformation of their hairs.*

Thus, a sound that produces a particular frequency of vibration will excite certain receptor cells, while sound involving another frequency will stimulate a different set of cells. The inner hair cells are less sensitive than the outer row and therefore require greater mechanical stimulation before discharging. This arrangement therefore adds a further dimension of intensity discrimination. The auditory nerve carries auditory information to the temporal lobe of the auditory cortex, *passing through the* thalamus *on the way. Both ears send information to the cerebral hemispheres.*

In recent decades (1970–1990) great progress has been made in our knowledge of the macro- and micromechanics of the cochlea. First, experiments have shown that waves propagated along the basilar membrane are indeed dispersive; ***William Rhode*** *(1971, University of Wisconsin, U.S.A.) used surgical procedure allowing two small* Mössbauer sources *(measuring time-variation of Doppler shifted frequencies of source-emitted gamma rays) of diameter* 0.06 *mm, and* 1.5 *mm apart, to be implanted on the basilar membrane of a live squirrel monkey. Another Mössbauer source was placed on the point of the eardrum which is in direct contact with the handle of the malleus bone in the middle ear.*

Thus, when the ear was stimulated acoustically by a pure tone, the sinusoidal vibrations of both *the basilar membrane and the malleus — which drives it — could be determined in both amplitude and phase. From the amplitude ratio and the phase difference, the* group velocity *U could be evaluated at each frequency as a function of a coordinate x_r along the basilar membrane. It was found that, for any given frequency ω, U tended to* zero *at a different location, thus permitting the energy to "pile up" at a characteristic, frequency-dependent position $x_r(\omega)$! In other words, each Fourier component of an acoustic signal will propagate along the cochlea as far as its characteristic place.*

How is the basilar membrane able to achieve this feat of frequency selectivity? Experiments (1978) have shown that the mechanical behavior of the basilar membrane in vivo *is highly anisotropic, to the extent that neighboring short sections of the membrane vibrate almost independently of one another. This results in a continuous variation in* stiffness *along its length by about four orders of magnitude. This massive increase in compliance from base to apex is primarily due to a gradual increase in width (from* 0.1 *mm to* 0.5 *mm) and a corresponding decrease in thickness.*

This property facilitates the dispersive behavior mentioned above, as it permits the traveling wave at a given frequency to undergo a progressive reduction in wavelength, and therefore in group velocity, to zero as the characteristic position is approached. Then its amplitude must build up in a way limited only by viscous dissipation, which allows a very sharp tuning in the basilar membrane vibrations. This is how the different frequency components of an acoustical signal "find their way" to different characteristic positions along the cochlea.

Nevertheless, even the modest level of energy dissipation by viscous action is now believed to prevent this remarkable mechanism of frequency selectivity (based on passive *mechanical properties of the basilar membrane) from entirely accounting for the extreme sharpness of tuning observed in the best recent measurements. Compelling evidence has been amassed to the effect that an* active feedback mechanism, *residing in the* outer hair *cells, produces a further sharpening of the basilar membrane's response at* low amplitudes.

Some 4000 *inner hair cells are stretched along the length of the human cochlea, and each sends a signal to the brain along a group of nerve fibers, which together form part of the complete auditory nerve. Each of these nerve fibers comes from a particular place along the cochlea and predominantly carries information about acoustic signals at frequencies for which that is the characteristic position. Indeed, electrophysiological measurements of the neural activity in one of those fibers show a level of response to acoustic signals which has a* very sharp maximum *at the corresponding frequency (1965–1974).*

In addition, there are 12, 000 *outer hair cells. These cells are endowed with efferent nerve fibers carrying to them (from more central parts of the nervous system) neural signals which can be presumed to control their function.*

Experiments using Mössbauer source implantation on the basilar membranes of guinea pigs (1982) have shown that the threshold sound (sound pressure level in db) required to generate vibrations of the basilar membrane (at a particular place where a Mössbauer source was implanted) decreases with the decrease of frequency.

In addition, the basilar membrane response has a very sharp maximum at the frequency for which that point is the characteristic position. The (also very sharp) tuning of auditory nerve fibers — that is, their sharp maximum of response to acoustic signals at a characteristic frequency for each fiber — is found to be fully matched by the sharp tuning of the basilar-membrane mechanism in the inner hair cells which simply converts vibration velocities into neural activity.

Experiments (1987) have revealed that pure tone signals are able to evoke from the ear an acoustic response, *taking the form of an* emission *of sound from it at the same frequency, known as the* otoacoustic emission. *This means that the low-level incoming acoustic signal stimulates in the cochlea an active process, utilizing metabolic energy, that gives rise to a* backward traveling wave. *This backward wave exerts a pressure on the oval window which, through the same linkage acting in reverse, causes the eardrum to generate a signal that enhances the weak external signal.*

How then is the otoacoustic emission generated? It is believed that the vibrations of the outer hair cells vibrations are the source of the positive mechanical feedback (1987–1989).

So far we mainly discussed that part of the ear which responds to acoustic signals. *The ear, however, is also designed to respond to* accelerations and rotations of the head; *the relevant sensors are used by the brain to control the position, attitude and movements of the body — much as* inertial navigation *systems do in human-engineered air, sea and space craft. The sense of equilibrium actually involves two senses — a sense of* static equilibrium *(stability of the head and body when these parts are motionless), and a sense of* dynamic equilibrium *(maintaining balance upon sudden motion or rotation).*

The organs of static equilibrium are located within the vestibule, *the bony chamber between the* semicircular canals *and the* cochlea. *They are the* utricle *and the* saccule. *The dynamic equilibrium is monitored by the three bony* semicircular canals.

These two groups of receptors all work in basically the same way — that is, there are hairlike nerve cells within the tubes that pick up the swirl of internal *fluid caused by tilting or rotational movement of the head. The* saccule *and* utricle *are simply short tubes in the same plane. The canals, on the other hand, are three tubes arranged at right angles to each other. In this way they can detect movement of the head in any plane*[670]*. Signals from these two groups of receptor organs are largely responsible for inflicting motion sickness on the occasional unfortunate traveler.*

Among other interesting features of the ear which determine its quality as an auditory sensor is the persistence of hearing*. In the optical case, persistence of* vision *is of the order of* 0.02 *sec. In the aural case, regardless of the original intensity, the sound level decays to threshold in about* 0.14 *sec after stimulation has ceased.*

1928–1968 CE **Maurits Cornelius Escher** (1898–1971, Holland). A most original, unique and imaginative artist who created images of the observed world through which he represented concepts of *groups*, *fractals*, *manifolds*, *non-Euclidean geometries* and *topology*.

Most of his life was spent in making various kinds of analytic compositions out of his subjects by means of graphic processes such as wood-engraving or lithography. He was at the same time photographer, architect and visionary, and his images are of equal interest to cognitive psychologists, mathematicians and laymen. Some of his works treat landscapes and natural forms in a fantastic fashion using distorted geometries. Others combine seemingly meticulous realism with paradoxical visual and perspective effects.

Scientists are fascinated by Escher's work because they recognize in it elements of the world with which they are familiar[671]. For them the plurality of Escher's world signifies neither absurdity nor chaos, but a challenge to look for new logical relationships between phenomena: the strangeness or absurdity that seems at first sight to be present in his work can, in the final analysis, be resolved and explained.

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670 It is amazing how nature, via the process of evolution, created a dynamical system that man could not improve on, if he had to construct it himself on the basis of modern science and technology.

671 It is bizarre how very little of 20^{th}-century science has been assimilated into 20^{th}-century art. Besides Escher, the *topological sculptures of* **Henry Spencer Moore** (1898–1986, England) come to mind.

The main source of fascination in Escher's prints is in the obvious message they convey: *reality is wondrous and at the same time comprehensible.*

Escher was born in Leeuwarden, the capital of the province of Friesland in the northern part of the Netherlands. He spent most of his youth in the city of Arnheim, where he attended secondary school. He later went to Haarlem, on his father's advice, to study architecture; there he came under the influence of the Jewish artist **Samuel Jessurun de Mesquita**, (1868–1944) who advised him to drop architecture and pursue his education in the graphic medium under his guidance (1919–1922).

Escher lived in Rome from 1923 to 1935, being greatly fascinated by Italy and its cultural heritage. He traveled extensively throughout the country. The rise of fascism caused him to leave Italy in 1935 and travel in Switzerland and Spain. He returned to Holland in 1941, and stayed there for the rest of his life.

Radio Communication and the Fourth State of Matter

During the early years of the 19^{th} *century, several major discoveries were made concerning the properties of electric current. In 1800* ***W. Nicholson*** *(1753–1815, England) and* ***A. Carlisle*** *(1768–1840, England) discovered the phenomenon of electrolysis, in which the chemical bonds of certain compounds (known as electrolytes) are broken by the passage of a current.*

The laws of electrolysis were stated by ***Faraday*** *in* 1832, *and these could be explained most simply in terms of ionic migration. According to the ionic theory, an electrolyte consists of free positive and negative ions which move to opposite electrodes during electrolysis. This idea was applied successfully to many problems of physical chemistry by* ***Arrhenius*** *and others. As a result, it appeared probable that in certain substances chemical combination depends on the electrostatic attraction between opposite charges attached to different atoms or parts of molecules.*

Some of the most important advances in late 19^{th} *century physics came from the study of electrical conduction in gases. After* 1855, *improved vacuum techniques enabled* ***Plücker***, ***Hittorf*** *(1824–1914, Germany), and* ***Crookes*** *(1832–1919, England) to investigate the properties of cathode rays. In* 1897,

***J.J. Thomson** produced strong evidence in favor of the theory that cathode rays are beams of subatomic particles, now called electrons (and identical to the earlier–discovered negatively charged* beta rays *emitted by radioactive substances). The electron theory was rapidly applied in many branches of physics, and **Millikan**'s experiments (from* 1911 *onward) made it clear that the electronic charge is the fundamental unit of electricity.*

J.J. Thomson went on to analyze the positive rays occurring in discharge tubes and by 1912 *he had evolved a technique for separating ions of different atomic masses and charges (mass spectrometry). In isolating two stable isotopes of neon he provided part of the explanation of fractional atomic weights in chemistry. By this time both the massive positive ions and the much lighter electron had been identified; **E. Rutherford** (1911) adduced strong evidence indicating that electrons orbit oppositely–charged* nuclei *within neutral atoms. Although the details of atomic structure remained obscure until the maturation of the quantum theory in the 1920s. It was evident that the stability of atoms depends on electrostatic forces.*

*With the advent of radio communication, **Arthur Kennely** (1861–1939, U.S.A.) and **Oliver Heaviside** (1850–1925, England) tried to explain the perplexing observation that radio-waves could bend around the earth's curvature and get across the ocean. [Maxwell's theory seemed to predict that radio waves should sail off into space!]*

Both Kennely and Heaviside, neither knowing of the other's work, suggested in 1902 *that there was a layer of electrified gases high up in the earth's atmosphere that* reflected radio waves*. This was quickly found to be true, and that region of the atmosphere is known today as the* Heaviside layer*, or simply* the ionosphere*.*

The ionosphere consists of various layers of partially ionized gases — plasmas *— that reflect a certain range of radio frequencies. The altitudes of the different layers vary considerably from day to day, depending largely on the activity of the sun. Low, medium and high frequency radio waves are reflected at different altitudes, and thus allow long-range radio broadcasts to span oceans and continents. But very high frequencies (VHF) and ultra high frequencies (UHF), which are used for television broadcasts, are not reflected by the ionosphere. Thus, TV signals can only be sent as far as the horizon, unless they are relayed. The* aurorae *are thought to be caused by ions and electrons of the ionosphere that are excited to the point where they glow.*

By the 1920*'s, radio communication was big business. Large industrial research laboratories such as* General Electric *Laboratories were deeply engaged in investigating the basic physical phenomena of radio communication, and seeking ways to make better electronic equipment.*

It was at the GE Labs that **Irving Langmuir** *(1881–1957, U.S.A.) carried out basic studies of electrified gases in vacuum tubes (as did Crookes half a century earlier) and coined the term* plasma.

One of the prime differences between gases and plasmas is that plasmas can conduct electricity, and its constituent particles can exert electromagnetic forces on each other. A plasma as a whole is usually electrically neutral on average (although charged particle beams, generated in particle accelerators or even in simple vacuum tubes, are not neutral). Most plasmas consist of a mixture of free electrons, positive ions and neutral atoms. A plasma may be lightly ionized (neutral atoms outnumbering the electrons and ions), or it may be fully ionized (almost no neutral atoms). Free electrons and ions may carry bulk electrical currents, and both electrons and ions can be energized by electromagnetic fields or by injected particle-beams.

Plasma physics is the study of charged particles collected in sufficient numbers, so that the long-range Coulomb force is a factor in determining their statistical properties, yet low enough in density so that short–range forces due to nearest-neighbor particles are negligible in comparison with the long-range electromagnetic forces exerted by the many distant particles. It is the study of low-density ionized gases.

The most characteristic aspect of the plasma state — to wit, the long rang of the Coulomb (and other electrodynamic) forces and the consequent collective behavior of the charged particles — was known much earlier, and was probably first described by Lord Rayleigh, in 1906, *in his analysis of electron oscillations in the Thomson model of the atom.*

The term "fourth state of matter", often used to describe the plasma state, was coined by **W. Crookes** *in* 1879 *to describe the ionized medium created in a gas discharge. The term follows from the idea that as heat is added to a solid, it eventually undergoes a phase transition to a new state, usually liquid. If heat is added to a liquid, it eventually undergoes a phase transition to the gaseous state. The addition of still more energy to the gas results in the ionization of some of the atoms. At a temperature above* $100,000\,^\circ K$ *most matter exists in an ionized state — the fourth state. A plasma state can exist at temperatures lower than* $100,000\,^\circ K$ *provided there is a mechanism for ionizing the gas, and if the density is low enough so that recombination is not too rapid.*

Early researches into the nature of plasmas, from glow discharge tubes to the Heaviside layer, dealt essentially with plasma at rest. *When WWII ended, many physicists turned their attention to the nature of plasmas flow. This study is called* plasma dynamics.

During the late 30*'s and the War-torn* 40*'s, astronomers and physicists began to realize that the matter in the* universe *(stars, nebulae and galaxies)* *is made up almost entirely of plasma: from the ionosphere, a* 100 *km or so over our heads, out to the deepest reaches of the cosmos (outward in space and backward in time), plasma is by far the most common known form of matter. Planets and interstellar dust, made up of the more familiar forms of matter, constitute a minor portion of the universe's mass*[672]*. Yet this means that plasma must be artificially created in terrestrial laboratories in order to be studied, as we happen to inhabit a planet!*

Plasma physics generally involves the well-known physics of classical mechanics, electromagnetism, and nonrelativistic statistical mechanics.[673] *The challenge of plasma physics comes from the fact that many plasma properties arise from the long-range Coulomb and magnetic interactions, and therefore involve complex collective phenomena with many particles interacting simultaneously via, and with, long-range EM fields.*

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[672] Nevertheless, it now appears quite likely that the universe also contains large quantities of *dark matter*, not yet directly observed or studied directly; and according to the successful Big Bang cosmological theory, dark matter is far more massive overall than visible matter – by about two orders of magnitude. Some of this dark matter is theorized to be novel not just in its *phase* (state) but even in the very elementary particles it is made of — such as neutrinos, 'strange' quarks, axions, heavy "supersymmetric partners" of known particles, black holes, etc. These are not made of electrons, protons and neutrons (and/or their antiparticles), as is all known (so-called *baryonic*) matter that is either observable from afar or can be examined directly.

Apart from these exotica, the observed matter – from the earth's vicinity to the farthest galaxies – seems to be not only baryonic but also overwhelmingly made of *matter* as apposed to *antimatter* (i.e. positrons, antiprotons, etc.). The latter are few and short-lived due to their rapid annihilation (by their normal-matter counterparts) into pure radiant energy (mostly photons and neutrons, ultimately).

However, modern cosmology asserts that, a fraction of a second after the Big Bang, the entire universe was a plasma, but with no nuclei – nor even nucleons; rather it consisted of equal (or almost equal) amounts of particles and antiparticles, as well as quanta that are *their own* antiparticles such as photons and other so-called gauge bosons. The mechanisms that ultimately gave rise to our present matter – dominated observed universe ($\sim 10^{10}$ electrons and nucleons per photon, and almost no antiparticles) are not yet fully understood; they are being studied at several accelerator laboratories around the world.

[673] Yet, STR relativistic effects are important in the sun, in many nuclear reactions and decays, in some aspects of chemistry and atomic physics, and in plasmas naturally produced by cosmic ray cascades; and also in modern klystron tubes and particle accelerating structures.

In its simplest form, a plasma is a collection of protons and electrons at sufficiently low density so that two-body (short-range) interactions are negligible. Many-body theory, or the many-body problem, is the proper framework for the study of the properties of such a medium. When a collection of cations (protons and other atomic ions) and electrons coexist in an equilibrium state, the properties of this state are described by equilibrium statistical mechanics with the appropriate Gibbs ensemble. However, most of the interesting features of plasmas occur for nonequilibrium situations.

Equations of Magnetohydrodynamics[674] (Magnetic Fluid Dynamics)

Consider the behavior of an (approximately) locally electrically neutral, conducting fluid in ambient, dynamical electromagnetic fields. It is described by a matter density $\rho(\boldsymbol{t}, t)$, a velocity profile $\boldsymbol{v}(\boldsymbol{r}, t)$, a pressure field $p(\boldsymbol{r}, t)$, a current density $\boldsymbol{j}(\boldsymbol{r}, t)$, a magnetic permeability μ and a real constant conductivity σ. We assume that the displacement current can be neglected (slow motions), that μ and σ are approximately non-dispersive and homogeneous, and that the fluid is non-relativistic and inviscid. The combined hydrody-

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[674] To dig deeper, see:

- Landau, L.D. and E.M. Lifshitz, *Electrodynamics of Continuous Media*, Addison-Wesley, 1960, 417 pp.
- Thompson, W.B., *An Introduction to Plasma Physics*, Pergamon Press, 1962, 256 pp.
- Jackson, J.D., *Classical Electrodynamics*, Wiley, 1975, 848 pp.
- Alfven, H., *Cosmic Electrodynamics*, Oxford University Press, 1950.
- Cowling, T.G., *Magnetohydrodynamics*, Interscience, 1957.

namical and Maxwell equations, under adiabatic conditions, then read:

$\dfrac{\partial\rho}{\partial t} + div(\rho\boldsymbol{v}) = 0$	*Equation of fluid mass continuity*	(1)
$\rho\frac{D\boldsymbol{v}}{Dt} \equiv \rho\left(\frac{\partial\boldsymbol{v}}{\partial t} + \boldsymbol{v}\cdot\nabla\boldsymbol{v}\right) = -\nabla p + (\boldsymbol{j}\times\boldsymbol{B})$	*Local conservation of linear momentum where* $(\boldsymbol{j}\times\boldsymbol{B})$ *is the magnetic (Lorentz) force per unit volume*	(2)
$\boldsymbol{j} = \sigma[\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B}]$	*Generalized Ohm's law for a conducting fluid (Ohm's law in local co-moving frame)*	(3)
$\dfrac{1}{\mu}\ curl\ \boldsymbol{B} = \boldsymbol{j}$	*Ampere's law*	(4)
$div\ \boldsymbol{B} = 0,\ div\ \boldsymbol{E} = 0$	*Magnetic and Electric Gauss' law*	(5)
$curl\ \boldsymbol{E} = -\dfrac{\partial\boldsymbol{B}}{\partial t}$	*Faraday's law*	(6)
$\dfrac{DS}{Dt} = \dfrac{\partial S}{\partial t} + \boldsymbol{v}\cdot\boldsymbol{\nabla}S = 0$	*Isentropic flow,*	(7)

with $S(\boldsymbol{r},t)$ *the entropy density. This system (together with an equation of state* $S = S(\rho,p)$*), comprising 15 independent scalar equations in 15 unknown scalar functions, can be further reduced*[675]*. Indeed, combining (3) through (6) we obtain*

$$\frac{\partial\boldsymbol{B}}{\partial t} = curl(\boldsymbol{v}\times\boldsymbol{B}) + \frac{1}{\mu\sigma}\nabla^2\boldsymbol{B} \tag{8}$$

Combining (2) and (4) we further have

$$\rho\frac{D\boldsymbol{v}}{Dt} = -\nabla p - \frac{1}{2\mu}\nabla(\boldsymbol{B}^2) + \frac{1}{\mu}(\boldsymbol{B}\cdot\nabla\boldsymbol{B}) \tag{9}$$

Equations (1), (5), (7), (8) and (9) constitute (together with the equation of state) a system of 8 independent scalar partial differential equations in the 8 unknown scalar functions ρ, p, $\boldsymbol{B}$, $\boldsymbol{v}$, *under condition of isentropic*

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675 Note that due to the approximations involved, Eqs. (3)–(4) are not exactly consistent unless *irrotational flow* is assumed. Indeed, (4) implies $div\ \boldsymbol{j} = 0$, consistent with strict local charge-neutrality; yet taking the *div* of (3) and neglecting a term of order $\frac{1}{c^2}$ yields $\mathrm{div}\,\boldsymbol{E} = -(curl\ \boldsymbol{v})\cdot\boldsymbol{B}$, which by the *electric* Gauss' law part of (5) contradicts strict charge neutrality.

flow.[676] *It can be shown that these equations permit the propagation of small perturbations as undamped hydromagnetic waves.*

676 Eq. (8) yields only *two* independent scalar PDE's, since taking the divergence of both sides results in an *identity* due to (5).

History of Creation Theories — II

Act V: *Expanding Pseudo Riemannian Universe and Big Bang Cosmology* *(1917–1983)*

Cosmology is the study of the large-scale structure of the Universe in space and time — what it is now, what it was in the past and what it is likely to be in the future. Since the only long–range forces at work among the galaxies and (as yet unknown) dark–matter particles that make up the material universe are those forces of gravity, the cosmological problem is closely connected with the theory of gravitation — in particular with its modern version as formulated in ***Albert Einstein****'s general theory of relativity. In the framework of this theory the properties of space, time and gravitation are merged into one harmonious and elegant picture.*

Thus, from 1917 *on, cosmology has again been in the foreground of interest in connection with the development of the general theory of relativity. It seemed as if this theory could bring us a decisive step nearer to the solution of the cosmological problem. The actual situation, however, was not so simple. The progress made was predominantly due to the amazing advance of astronomical knowledge beyond the realm of our galaxy into remote depths of the universe. Our 'neighborhood', until then confined to small parts of the galaxy, was extended thereby into the extragalactic space, to distances which possibly may no longer be considered negligibly small as compared with the dimensions of the whole universe. Cosmological theory responded to this enrichment of our observational knowledge:*

The general theory of relativity was published in 1916 *and its first application to our solar system had already been made before* 1920. *But only a very few bold mathematicians and astronomers had the idea of extending its consequences to the whole universe. The first people to construct models of an expanding universe were* **de Sitter** *(1917),* **Friedmann** *(1922)* and **Lemaître** *(1927). However, the majority of astronomers did not take these models seriously. The situation lasted until* **Hubble** *discovered (1929) the expansion of the universe. This indeed is the event which we must mark as heralding beginning of the new science of cosmology.*

The whole way of thinking about the origin, evolution and destiny of the cosmos was altered after that. For the first time the study of the universe as a whole ceased being the realm of subjective speculations and became the subject of scientific research. The great advancement of cosmology that followed *was due to systematic research involving both observations and theory. However, the phenomenon of the expansion of the universe was so enormous, so amazing, that many people disputed it. Many efforts were made to attribute the red shift of light from distant galaxies to causes other than the expansion of the universe. All these efforts failed and today there is no serious dispute of the reality of the expansion.*

The observational basis for our belief in the expansion of the universe and the evaluation of its density distribution is based upon the use of various standard candles (widespread luminous objects such as variable stars, supernovae etc. with known absolute magnitude), in conjunction with galaxy recession – velocity measurements. The latter are done by measuring red shifts.

- *The redshift: Observed systematic increase in wavelength λ of the spectral lines with cosmic source distances, considered to result from a recession of the light source. It is related to the source-observer distance r and the instantaneous speed of recession (at emission epoch t) by the approximate relation $\frac{dr}{dt} = c\frac{\Delta\lambda}{\lambda} = Hr$, where H is constant (Hubble law; requires GTR non-linear corrections for redshifts $z = \frac{\Delta\lambda}{\lambda}$ of order 1). This law for the general expansion of the universe (calibrated for the nearer systems with known distances), furnished a further possibility of estimating distances when only observed red-shifts are available. In this way our knowledge concerning the distribution of matter in space and time was widened step by step through a great variety of methods of measuring or estimating distances (the so-called 'cosmic distance ladder').*

When the problem of cosmology was revived in connection with the theory of General Relativity, the surveyed part of the observable universe extended to distances of nearly 3×10^{21} km, containing in its volume about 10^8 galaxies, each consisting of some 10^{11} stars. This was an empirical background very different from that on which astronomy had to rely when it first attempted to tackle this complex problem.

Experimental evidence, based on this vast reservoir of matter, has shown that beyond a certain distance positive values of $\frac{\Delta\lambda}{\lambda}$ prevail, indicating a general recession of the more distant galaxy clusters which definitely no longer belong to the cluster of which our galaxy is a member. Hence, a general expansion of the universe is indicated. Moreover, observations disclose that to the first approximation, this expansion is isotropic (as viewed from any observatory at rest relative to his local cluster).

In addition, theoretical considerations show that no stable solution of the cosmological problem is expected to yield a static world in which gravitating matter fills the universe with a fixed average density. A universe at rest, filled *with gravitating matter, is not possible. This conclusion is common to both GTR and Newtonian gravity.*

Newtonian Cosmology

Assume the universe to be an infinite Euclidean space filled by galaxies, which will be treated like the molecules of a fluid distributed uniformly (on average) through space. Assume at first that Newtonian physics holds; we also impose for philosophical reasons, the cosmological principle — which states that the universe appears, on average, identical to two observers residing in any two of its galaxies. This principle is an extension of the Copernican notion — that earth is not privileged — to the cosmos at large. It follows that the mass–density ρ and the hydrostatic pressure p are functions of the time t only. Moreover, it also follows from this principle that — in any inertial frame with its spatial origin within any galaxy and co-moving with it — the mean hydrodynamical streaming velocity field $\boldsymbol{v}(\boldsymbol{r})$ of all other galaxies must be a homogeneous linear function of the space coordinate vector $\boldsymbol{r} = (x_1, x_2, x_3)$.

Furthermore, such a cosmos expanding (or contracting) symmetrically in all directions (i.e. isotropically) corresponds to a trajectory $x_i(t) = x_i^0 R(t)$, for any galaxy (or center–of–mass of local galaxy cluster) relative to any given galactic frame as described above; here $R(t)$ is a universal scale factor, while x_i^0 are constants that, however, depend on the galaxy whose trajectory is traced (and also on the galaxy where our observer sets up his comoving metrical frame).

The expansion is described by the equations:

$$v_i = \frac{dx_i}{dt} = x_i^0 \frac{dR}{dt} = x_i \frac{\dot{R}}{R}.$$

We thus obtain a law of expansion corresponding to that derived from the observed red-shift of the galaxies but with the "constant" H actually depending on the emission epoch t.

So far the considerations have been of a purely kinematical nature; they have not taken into account of the fact that all matter in the universe is gravitating. The following equations thus have to be satisfied:

- *Local conservation of mass:* $\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho \boldsymbol{v}) = 0.$

- *Local conservation of linear momentum:* $\frac{\partial}{\partial t}(\rho \boldsymbol{v}) + \mathrm{div}(\rho \boldsymbol{v}\boldsymbol{v}) + \rho\,\mathrm{grad}\,\phi = 0$ *where* ϕ *is the Newtonian gravitational potential; and no term depending on the hydrostatic pressure*[677] *enters, since* $p = p(t)$.
- *Poisson's equation:* $\nabla^2\phi = 4\pi G\rho$.

If we impose the cosmological principle and restrict consideration to the case of isotropic expansion, namely $v_1 = ax_1,\ v_2 = ax_2,\ v_3 = ax_3,$ *it then follows from the above equations that* $a = \frac{\dot{R}}{R} = -\frac{1}{3}\,\frac{\dot{\rho}}{\rho},\ \ \frac{d}{dt}\left(\frac{\dot{\rho}}{\rho}\right) = 4\pi G\rho + 3a^2.$ *In these equations,* $\dot{\rho} = 0$ *entails* $a = 0$ *and* $\rho = 0$. *Hence, it is impossible to construct a Newtonian universe in which gravitating matter remains at rest; it is also impossible to let matter stream in such a way that the density remains constant.*

Now, due to the cosmological principle, the gravitational potential must be such that its derivatives $\frac{\partial\phi}{\partial x_i} = 0$ *for* $x_i = 0$ *(no preferred direction!). Consequently, the representation of* ϕ *by power series in* x_i *must begin with terms of at least the second order. In fact the above system of differential equations, along with* $\boldsymbol{v} = a(t)\boldsymbol{r}$, *imply*

$$\phi(x) = \frac{2\pi G\rho(t)}{3}(x_1^2 + x_2^2 + x_3^2)$$

up to an additive constant.

This quadratic potential with a time–dependent "spring constant" is an exact solution to the above equations of self gravitating flow.

In any of our galactic comoving inertial frames, the gravitational force upon a unit test-mass at position $\boldsymbol{r}$,

$$-\nabla\phi(\boldsymbol{r}, t) = -\frac{4\pi G\rho(t)}{3}\,\boldsymbol{r},$$

is readily seen to be the Newtonian gravity force due to a spherical radius – $|\boldsymbol{r}|$ *ball of galaxies, centered at the frame's galaxy-origin; the test mass lies on this ball's boundary sphere, and galaxies external to it exert no net force upon the test mass.*

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677 In our dilute–gas fluid approximation of the universe, "pressure" stems from the random motions of galaxy clusters relative to their local comoving galactic frames. Cosmologists refer to these as *peculiar motions*. Strictly speaking the R.H.S of the streaming–motion law $\boldsymbol{v}(\boldsymbol{r}) = \frac{\dot{R}}{R}\boldsymbol{r}$ should have a random peculiar–motion term added to it.

This "screening", or cancellation, of the gravitational effects of galaxies outside such a ball is well known – Newton himself derived it; but it is problematical in its cosmological application, for the outside galaxies extend to infinity and the relevant integrals diverge. This problem can only be resolved within the General Relativistic (GTR) formulation of cosmology.

But since the screening assumption is vindicated in GTR, we will adopt it within the Newtonian cosmology, as well.

Introducing the trajectory $\boldsymbol{r} = \boldsymbol{r}_0 R(t)$ for a general galaxy in any one of our frames into the equation of motion[678]

$$\frac{d^2\boldsymbol{r}}{dt^2} = -\frac{GM(r)}{r^2}\,\frac{\boldsymbol{r}}{r}\,,$$

for a galaxy at the surface of a radius - r sphere ($r = |\boldsymbol{r}|$) containing mass $M(r)$, the differential equation for R follows: $\frac{d^2R}{dt^2} = \frac{-GM}{R^2}$, where $M = \frac{4}{3}\pi\rho R^3$ is a constant; it is the mass contained in a volume of one of our expanding spheres of radius $r = r_0 R(t)$ with $r_0 = 1$. The mass remains unchanged within any expanding volume with the galaxies upon its surface fixed relative to that surface.

The differential equation immediately yields the integral $\frac{1}{2}\dot{R}^2 = \frac{GM}{R} + h$, where the constant of integration h may be negative, positive, or zero. Accordingly, we obtain various types of expansions which may be called "elliptic", "hyperbolic", and "parabolic", since the character of the solutions closely resembles solutions obtained in the theory of rectilinear motion of two gravitating bodies.

From the preceding general discussion it follows that the dynamical problem of an expanding or contracting universe always gives rise to a singularity when, for $R = 0$, ρ becomes infinite. The singularity can be removed by amending Poisson's equation for the gravitational potential by an additional term.

If the quadratic gravitational potential $\phi(\boldsymbol{r}, t)$ derived above is supplemented by a term with the new constant λ:

$$\phi = \frac{2\pi G}{3}\,\rho(t)r^2 - \frac{\lambda}{6}\,r^2 = \frac{1}{2}\,\frac{GM(r)}{r} - \frac{\lambda}{6}r^2\,,$$

[678] The equation results *either* from the above "screening" assumption or (equivalently) it can be derived from the equation for $\frac{d}{dt}\left(\frac{\dot{\rho}}{\rho}\right)$, which was itself derived above from local conservation laws, the Poisson equation, and the (isotropic) cosmological condition.

the ordinary differential equation for the scale factor $R(t)$ *(with all other assumptions left unchanged) becomes*

$$\frac{d^2R}{dt^2} = -\frac{GM}{R^2} + \frac{\lambda}{3}R$$

and yields the integral

$$\frac{1}{2}\dot{R}^2 = \frac{GM}{R} + \frac{\lambda}{6}R^2 + h = \left(\frac{4\pi G\rho}{3} + \frac{\lambda}{6}\right)R^2 + h \; .$$

The singularity $\rho \to \infty$, *which formerly occurred for* $R = 0$, *does not appear now, provided* λ *is chosen* > 0, $h < 0$, *and the mean cosmic density is suitably small* $(\rho < \frac{\lambda}{4\pi G}$ *at any given epoch* $t)$.

In this case the equation $\frac{GM}{R} + \frac{\lambda}{6}R^2 + h = 0$, *corresponding to the case* $\dot{R} = 0$, *yields in general two solutions, say,* $R = R_1$ *and* $R = R_2$, *where* $R_1 < R_2$. *For* $R > R_2$, *real solutions exist for which R, coming from infinity — corresponding thus to a contracting universe — approaches the limit* $R = R_2$, *where* $\dot{R}$ *becomes zero. The contraction comes to a standstill at a finite value of* ρ *and changes into an expansion; at this turning point the new repulsive force (characterized by the cosmological constant* λ*) just balances the attraction due to the gravitational attraction.*

According as λ *and* h *are chosen to have various sign combinations and the appropriate density regimes are chosen, a variety of solutions results. But only the case, just mentioned, corresponding to* $\lambda > 0$, $h < 0$, *and* ρ *small enough, yields solutions which are free from singularities and hence do not necessitate the assumption that the universe has to pass through a phase of infinitely large densities, pressures and temperatures. But it must be emphasized that such regular solutions are obtainable only by introducing the ad hoc hypothesis of a new, and otherwise not motivated, universal long–range repulsive force in the universe.*

The constant λ *is the Newtonian manifestation of the GTR cosmological constant. Its introduction, however, is rather ad hoc within the Newtonian framework. In the Einsteinian universe, on the other hand, it is the observed smallness of* λ *which is "unnatural" especially when quantum effects are considered; and this smallness is one of the deep outstanding puzzles of our current models of particles and fields.*

Studies of the CMBR (Cosmic Microwave Background Radiation) and its anisotropies support the assumption of uniform matter density; while galactic surveys indicate (if the evidence in favor of a hierarchic structure is disregarded) a more or less uniform distribution of luminous (non-dark) matter in space in accordance with the equation $\log N_m = 0.6\ m +$ *const., where*

N_m is the number of galaxies brighter than the limiting magnitude m. If the unit volume is taken sufficiently large to smooth out local irregularities in the distribution, the resulting number of star-systems per unit volume is roughly constant at all distances and in all directions. Spreading the non-dark matter, concentrated mainly in stars, evenly over the whole space, a constant value of ρ for the density of matter is obtained[679].

The widely accepted current theory for the origin of the universe is the Big Bang theory. We accept, based on many pieces of evidence, that the universe started with a huge explosion from a superdense and superhot stage. Theoretically, if GTR is accepted without modifications, the universe must have started from a mathematical singularity with infinite density ($R(t) \to 0$ as $t \to 0$). Further, the derivative of R is infinite at this time; that is, the initial explosion happened with infinite velocity. However, it is virtually certain that classical GTR requires major modifications for t less than the Planck time ($\sim 10^{-43}$ sec), so singularities predicted by GTR, are not very worrisome for theorists.

The Big Bang was not an ordinary explosion of the familiar kind, where material is ejected from some central point into a pre-existing space. Here, space itself is created by the explosion, and there is, no central point! The

[679] Strictly speaking, observations do *not* unconditionally agree with the hypothesis of an expanding universe filled with matter of a constant mean density ρ. Nevertheless, the cosmological models of the universe have hitherto been developed on this assumption. Otherwise, the problem would still defy all our efforts to find a solution. In fact, all cosmological problems are approached by making the *postulate* that the observed phenomena (averaged over distance scales typical of galaxy super-clusters) are representative of the universe at large; only with this bold extrapolation can we obtain a truly cosmological theory. Cosmologists' faith in this postulate is greatly buttressed by the remarkable degree of observed isotropy of the Cosmic Microwave Background Radiation (to several parts of a million). This cosmological *principle*, precisely stated, is:

Every observer, wherever placed in the universe, describes the observed (large-scale) phenomena identically, and observes itself to inhabit an isotropically expanding universe in which matter is evenly distributed.

This postulate (which also includes the statement that the *microscopic constants* and laws of nature are the same for observers in any galaxy) brings to conclusion a development in science which had its beginning with Copernicus and Kepler. In much the same way as they removed the earth, and thus man as well, from the central position which he had hitherto claimed for himself in his restricted picture of the world, the cosmological principle deprives the picture of the entire universe of any features that tend to grant to man (or even his local galaxy super-cluster!) any privileged or central position.

material blasted out in the Big Bang is spread out uniformly over the entire *spatial extent of the observable universe, and every point in this universe appears to be the center of the explosion.*

How did this theory evolve? Einstein had first formulated his model of a static, finite universe in 1917, two years after developing the general theory of relativity. But he soon saw the flaws of this model. A static, closed universe could not remain static, because its own gravitation would cause it to collapse. This was a problem not only for his theory, but any theory of gravity, including Newton's. As the poet Edgar Allen Poe had noted seventy years earlier, unless a body of matter rotates, it will collapse under its own gravity — only rotation stabilizes bodies such as the galaxy and the solar system. But Einstein ruled out a rotating universe on philosophical grounds.

First, he believed that rotation itself is relative, like all other motion, and the universe's putative rotation would have to occur relative to a frame external *to it, an impossibility by definition.*

Second, rotation implies a central axis, but such an axis would be a distinct direction in space, different from all others — this contradicted his belief that space is the same everywhere and in every direction.

Clearly, Einstein reasoned, something prevents the collapse of the universe — something like the centrifugal force of rotation, but not rotation itself. This force must somehow increase with distance: it had never been observed on earth or in the solar system, but it must be strong enough at cosmological distances to overcome gravity. He introduced a new term into his equations of gravity, the "cosmological constant", a repulsive force whose strength increases proportionally to the distance between two objects, just as the centrifugal force in a rigidly rotating body increase proportionally to distance from its axis of rotation. But this new force, he reasoned, must act in all directions equally, like gravity, so it does not disturb the rotational symmetry of the universe.

To preserve his conception of a static universe, Einstein set the cosmological constant to a level that would exactly balance gravity, so that its repulsive force neutralized the tendency of the universe to collapse.

In 1924 new observations changed the picture radically. For a decade, astronomers had been measuring the spectra of stars in nearby galaxies. In nearly all cases, the spectra shifted slightly toward the red. Scientists had long known the simplest explanation for these redshifts is that the galaxies are moving away, shifting the frequency of light to the red (the Doppler shift; an analogous phenomenon makes the pitch of a train whistle rise as it approaches and fall as it recedes). It seemed strange that, rather than moving randomly, the galaxies seemed to mostly be moving away from each other and from us.

Carl Wirtz, *a German astronomer, put all the forty-odd observations together in* 1924 *and noted a correlation — the fainter the galaxy the higher its redshift, thus the faster it is receding. Assuming that fainter galaxies are more distant, then velocity increases with distance. The conclusion was tentative, since the distances to the galaxies were uncertain. But the American astronomer* ***Edwin Hubble*** *and his assistant* ***Milton Humason*** *soon began to systematically examine Wirtz's findings. Hubble had developed a new way of measuring the distance to a galaxy, based on the known intrinsic brightness of certain peculiar stars called Cepheid variables. Word soon filtered through the astronomical community that Hubble's data seemed to confirm the relation between redshift and distance.*

The news was of immense interest to a young Belgian, ***Georges Henri Lemaître***. *Lemaître received his doctorate in physics in* 1920, *and shortly thereafter entered a seminary to study for the priesthood. While at the Seminary of Maline, he became fascinated with the new field of general relativity, and after being ordained in* 1923, *went to England to study under Eddington. He then spent the winter of* 1924–1925 *at Harvard Observatory, where he heard Hubble lecture and learned of the growing evidence for the redshift-distance relation.*

Over the next two years Lemaître developed a new cosmological theory. Studying Einstein's equations, he found, as others had before him, that the solution Einstein proposed was unstable; a slight expansion would cause the repulsive force to increase and gravity to weaken, leading to unlimited further expansion; or a slight contraction would, vice versa, lead to collapse. Lemaître, independently reaching conclusions achieved five years earlier by ***Alexandr Friedmann***, *showed that Einstein's universe is only one special solution among an infinity of possible cosmologies — some expanding, some contracting, depending on the value of the cosmological constant and the "initial conditions" of the universe.*

Lemaître synthesized this purely mathematical result with Wirtz's and Hubble's tentative observations, and concluded that the universe as a whole must be expanding, driving the galaxies apart. And if the universe is expanding, then any of the cosmological scenarios that led to expansion could be a valid description of the universe. Cosmic repulsion and gravity are not delicately balanced within a static large–scale cosmos; rather the combined effect of repulsion and relative galactic motions predominate in an expanding universe.

Lemaître put forward his hypothesis of an expanding universe in a little-known publication (1927), and within two years his work and Friedmann's had become widely known and accepted in the tiny cosmology fraternity. By *this time (1929) Hubble had published the first results showing the redshift–distance relation, apparently confirming Lemaître idea of an expanding universe.*

This was not yet the Big Bang, though. The solutions of the equations of general relativity derived by Friedmann, and later by Lemaître, only showed that many solutions led to universal expansion. Some solutions did indeed produce a singularity — a collapse into, or an expansion from, a universe of zero radius. If the universe were dense enough, and repulsion weak, the universe would collapse to a point. But if the repulsive force were strong (or the mean cosmic density small enough), there would be no singular state: the universe could be diverging from a state near Einstein's balance, moving away faster and faster with the passage of time; or it could have contracted from an indefinitely large radius in the infinite past to a minimal radius, and then begin expanding. These nonsingular solutions would imply a universe of infinite age. Indeed not all possible solutions are spatially finite, closed hyperspheres, as Einstein envisioned — some are infinite in spatial extent.

In general when equations describing physical reality produce singularities — solutions involving infinite values for observable quantities — it is a sign that something is wrong, since scientists assume that only measurable, finite quantities should be predicted. So initially the solutions without singularities attracted the most attention.

This, then, was as far as general relativity alone could take the cosmological problem.

In 1928, ***James Jeans****, one of the most prominent astronomers of the time, revived Boltzmann's old arguments about the "heat–death" fate of the universe. The second law of thermodynamics, Jeans reasoned, shows that the universe must have begun from a finite time in the past, and must move from a minimal to a maximal entropy. Incorporating Einstein's equivalence of matter and energy, Jeans argued that entropy increases when matter is converted to energy, because energy is more chaotically dissipated. Thus the end state of the universe must be the complete conversion of matter to energy. "The second law of thermodynamics compels the materials in the universe to move ever in the same direction along the same road, a road which ends only in death and annihilation", he gloomily wrote.*

At the same time ***Eddington*** *was reaching a similar conclusion. Curiously enough, he begins his book* The Nature of the Physical World *with philosophical premises similar to those used by Giordano Bruno's enemies three centuries earlier. Like Bruno's persecutors, Eddington was viscerally repelled by an infinite universe: "The difficulty of an infinite past is appalling", he writes. "It is inconceivable that we are the heirs of an infinite time of preparation". He too concludes that the second law implies a beginning in time. He was not* *pleased by this idea either, but felt that it follows naturally from Boltzmann's laws.*

Lemaître, hearing his former teacher's views in March of 1931, was deeply impressed. He had been viewing his recent mathematical work in a philosophical light; Einstein's ideas of a hyperspherical space showed that a finite universe was again conceivable.

But if the universe is finite in space, then it should be finite in time as well, Lemaître argued (and the GTR equations support this latter inference for weak enough repulsion). Thus the nonsingular solutions that Lemaître had found — in which the universe has no beginning — were unacceptable. The only ones that corresponded to Lemaître's philosophical views were closed in space and *limited in time. Eddington had given him a further rationale for looking at the singular solutions — the second law indicates that the universe must have originated in a state of low entropy.*

From these two philosophical premises, Lemaître developed his concept of the "primeval atom", the first version of the Big Bang. At a 1931 *meeting of the British Association of the Evolution of the universe, he put forth these ideas for the first time. Beginning from the idea that entropy is everywhere increasing, he reasoned, quantum mechanics (developed in the twenties) shows that as entropy increases, the number of quanta — individual particles in the universe — increases.*

Thus, if we trace it back in time, the entire universe must have been a single particle, a vast primeval atom with zero radius. He identified this instant with the singularity of some relativistic solutions. Just as uranium and radium nuclei decay into subatomic particles, so this giant nucleus, as the universe expanded, explosively split up into smaller and smaller units, atoms of the eventual size of galaxies decaying into atoms that later become suns, and so on down to the scale of actual, present-day atoms.

During WWII cosmological research was suspended, along with other peacetime pursuits, as scientists were drawn into the war effort. By the war's end, though, the field was transformed. Prior to the war the creation of elements that compose the universe had been a speculative theoretical subject — too little had been known of nuclear reactions. Now, with the successful production of atomic bombs — the creation of the elements was no longer a hypothesis, but a technological fact. The fuel for one of the bombs unleashed on Japan was itself a created element — plutonium — generated from uranium.

Nuclear piles (reactors) and the A-bombs had transformed common elements into new and exotic elements and isotopes, which scientists found in analyzing the fallout from the bombs — especially that of the Trinity test in *the New Mexico desert. And the vast expansion of nuclear research that grew out of the Manhattan Project continued to yield data about nuclear reactions.*

To one of the Manhattan Project scientists, ***George Gamow****, the detonation of an A-bomb constituted an analogy for the origin of the universe: if an A-bomb can, in a hundred-millionth of a second, create elements still detected in the desert years later, why can't a universal explosion lasting a few seconds have produced the elements we see today, billions of years later, throughout the cosmos?*

In a paper in the fall of 1946, Gamow put forward his idea, a second version of the Big Bang. Unlike Lemaître, he took as observational verification of his hypothesis the abundance of the elements; but like him, Gamow assumed that this abundance could not have been produced by any process continuing in the present-day universe.

Gamow proceeded (1948) to investigate the characteristics of the superdense condition of the first moments of the universe. He concluded that the temperature must have been enormous at this stage. Under these conditions, the protons and neutrons must have formed the various chemical elements. The theory satisfactorily explains the formation of deuterium and helium. Perhaps it might also explain the formation of all the elements up to uranium, by progressive fusion with protons and neutrons to yield more and more complex nuclei.

It soon became clear, however, that since there is no stable nucleus with mass number 5, the formation of the elements just after the Big Bang must have stopped at helium. Thus, cosmic nucleosynthesis was necessarily restricted to light elements, up to helium[680] *^4_2He.*

After this failure, the theory was put aside altogether. The next theory about the formation of the chemical elements was the B^2FH theory, so named after ***G. Burbidge****,* ***M. Burbidge****,* ***W.A. Fowler*** *(1911–1995; Nobel prize, 1983) and* ***Fred Hoyle****. According to this theory, all the elements beyond*

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680 It was later shown that the isotopes of *Lithium*, as well, have present-day cosmic abundances that can be accounted for by the cosmological *nucleosynthesis* reactions which the Big Bang theory predicts to have occurred during the first few minutes of the universe.

The isotopes thought to have been largely created at that epoch are: ^1H (proton); ^2H (deuterium); ^3He; ^4He; ^6Li; and ^7Li. Some of the higher-mass isotopes (created in exploding stars) catalyze the fusion of hydrogen into Helium in the nuclear combustion within later generations of stars, but the accumulated impact of stellar fusion upon the cosmic helium abundance (which is $\sim 25\%$ by mass) is calculated to be small.

^{1}H (proton) have been formed in stellar interiors. In particular, the heavier elements were formed during supernova explosions.

***Edwin E. Salpeter** had already shown (1952) that three nuclei of helium $^{4}_{2}He$ may combine to produce carbon, and other heavier elements may similarly be formed by addition of further helium nuclei, in the interiors of stars rich in helium, where temperatures may reach $10^8 K$. Still heavier elements may be formed during the final stages of stellar evolution, when a supernova explosion may occur.*

*The B^2FH theory was very successful. However, as **Hoyle** and **Taylor** realized (1964), it could not account for the amount of helium observed in stars, which constitutes 25% of their mass. According to the theory of nuclear reactions in stellar interiors, only 1–4% of the amount of matter locked up in stars would be helium if this element were produced entirely inside stars — and this is 6–25 times less than the spectroscopically observed amount. This result made **Hoyle** revisit the formation of elements in the Big Bang theory.*

*Thus, in 1967, **R.V. Wagoner**, **Fowler** and **Hoyle** calculated once again the amount of helium which may be formed in the early universe, and came to the same conclusion as **Gamow** and his collaborators.*

The situation is similar for deuterium, which is observed to have a cosmic abundance of 2×10^{-5}. This amount is much more than the amount expected to form in stars, because deuterium is not expected to survive for very long in stars. The deuterium we observe today, therefore, must have been formed in the early universe.

There are therefore two ways by which the elements of matter were formed. The first way, the cosmological one, produced only the light elements (mainly deuterium and helium) during the first four minutes after the Big Bang. The elements heavier than helium were formed later on, in the interiors of stars. This secondary process for generating elements started as soon as the first stars were formed ($t \sim 10^9$ yr after the Big Bang), and continues today.

Another indication in support of the Big Bang theory is the estimated age of the universe. Independent estimates, based on the expansion of the universe, the age of the oldest stars in the galaxies, or the age of terrestrial rocks based on the lifetimes of radioactive elements, give numbers of the same order of magnitude. All three methods agree that the age of the universe is between 10 and 20 billion years. If the universe did not have a beginning, there would not be an a priori reason for such a good agreement between these three different calculations.

Finally, an important piece of evidence for the Big Bang is the cosmic microwave background radiation. This radiation comes to us from all directions with almost uniform intensity (isotropically), and corresponds to the *radiation from a black body at a temperature of approximately 2.7°K. The radiation does not appear to be clumpy — unlike the distribution of matter; furthermore, the (parts per million) deviations from isotropic flux seem to be well explained by inflationary cosmology*[681]*. The only credible explanation for this radiation is that it consists of the photons which filled the universe during the "radiation dominated era", early in cosmic history, and became largely decoupled from matter during the "recombination era" ($t \approx 3\times10^5$ years after the Big Bang) — when the electron-proton plasma of the universe combined to form transparent, neutral hydrogen. These photons have since (during the $t > 3 \times 10^5$ yr "matter dominated" era we are still in now) undergone a cosmological redshift due to universal expansion, with their wavelengths thus stretched to their current value ($\sim 3\ mm/2.7 \sim 1$ millimeter by Wien's Law). No other plausible explanation has thus far been suggested. The powerful combination of* quantum theory*, GTR, and* nuclear *and* particle physics *allows detailed quantitative calculations — such as the abundances of light elements — that can be compared with empirical observations.*

To recapitulate, we note that the basic evidence in support of the Big Bang theory is:

- *Solutions of Einstein's equations (themselves well-confirmed in independent, solar–system and astrophysical tests).*
- *The observed helium, deuterium and lithium cosmic abundances.*
- *The agreement between the various independent estimates of the age of the universe.*
- *The cosmic microwave background radiation.*

None of the above-listed arguments in support of the Big Bang theory became immediately accepted. In particular, the solutions of Einstein's equations referred to homogeneous and isotropic models of the universe. So the question arises, what happens if the universe is not entirely homogeneous and isotropic after all? is it possible, in that case, to avoid the mathematical singularity and the initial explosion, by accepting (for example) that the universe has

[681] A variant of the Big Bang theory, developed in the 1980's, involving a brief reheating episode – a fraction of a second after the initial $t = 0$ instant, well before nucleosynthesis – in which the universe expanded exponentially. This event was powered by the energy released when a super cooled vacuum converted to its true ground state.

some rotation? (In Newton's theory, a rotating star which collapses does not form a singularity — unlike the collapse of a non-rotating star.)

Much effort has been expended by mathematicians to answer this question in the framework of GTR. The most important advances were made by ***S.W. Hawking*** *and* ***Roger Penrose***[682] *(1969), who showed that* any *reasonable model of the universe which has the observed characteristics of (approximate) homogeneity and isotropy, must start from a singularity. This theorem, which does not require absolute homogeneity and isotropy, is one of the most important achievements in the field of relativity.*

We see, therefore, that the general theory of relativity leads to an initial singularity of the universe. Would this change if we were not using Einstein's theory? Several competing theories have been developed[683]*, including the Brans-Dicke scalar-tensor theory and (more fundamentally) various*

682 The Hawking-Penrose theorem holds under the following assumptions: (1) The general theory of relativity holds. (2) The total energy density is locally positive. (3) There are no closed timelike or lightlike geodesics (i.e. no time paradoxes). (4) Space is not everywhere flat along all timelike or lightlike geodesics (it is unlikely that this is not the case). (5) There is at least one closed spacelike surface.

Since the assumptions on which the theorem is based are not very restrictive, there is little doubt that they apply to the actual universe.

683 Cosmologists have speculated that if we add the non-relativistic *kinetic energy of the expansion* of the observable universe, the *rest mass* energy of all masses in it and its *Newtonian potential energy* (which is negative), the resulting sum is:

$$E = \sum_i \frac{1}{2} m_i v_i^2 + \sum_i m_i c^2 - \sum_i \sum_j \frac{G m_i m_j}{2 r_{ij}} \approx 0,$$

to a high degree of accuracy. [The summations $\sum$ refer to all particles in the universe and r_{ij} is the distance between any two particles.]

This seems to be *roughly* true for the known (luminous) matter – which led **P. Jordan** (1949) to introduce the assumption that the sum of the mass-energy in the universe is *always* zero. Since the universe expands, however, the potential and kinetic energy terms both become smaller. For the total mass-energy of the universe to remain zero, Jordan then assumed that new matter is created, at appropriate distances and with appropriate velocities. This theory is essentially a revival of the old view that the universe has been created *ex nihilo*. The theory cannot explain the microwave background radiation and other observational data and is not relativistic. The *steady–state cosmology* of Hoyle Bondi and Gold is a variant of brans–Dicke theory (thus generally covariant) which allows for such continuous creation, but it fails to account for the CMBR or light–element nucleosynthesis.

string-theory-inspired modifications of GTR. However, so far none of them has managed to replace general relativity. Whenever observational tests were carried out in order to distinguish between relativity and another theory, relativity was vindicated. Consequently, most researchers today work on GTR cosmology, rather than on other competing theories.

It is quite likely that the singular beginning of the Universe is avoided via quantum mechanical phenomena. Such phenomena were very important when the age of the universe was of order 10^{-43} *sec (Planck time), but we cannot yet calculate their effects since we lack a theory of gravity.*

Objections concerning the formation of helium, lithium and deuterium, arise from doubts as to whether the observed abundances are universal or not. Much effort has been devoted to detecting stars with a helium abundance well below the generally accepted value (~25%). It seems, however, that the "exceptions" observed are not due to reduced helium content but rather to peculiarities in the spectra of certain stars. Besides, the most recent satellite observations confirm the idea that the observed deuterium has primordial origin, rather than having been formed in more recent stages of the evolution of the universe.

The objections with respect to the age of the universe are based on the uncertainties involved in the various methods of calculating or bounding it. However, despite some false alarms in the early 1990's, the best cosmological estimates for the present age of the universe–based on data gathered by the Hubble Space Telescope – have converged on a figure of $\sim 1.4 \times 10^{10}$ *yr, exceeding the age of that of the oldest known stars or galaxies. Other theories, like the theory of continuous creation, claim that there is an infinite number of galaxies older than this limit (although they may be too distant for us to observe) – but, again there is no evidence that such is the case.*

Finally, attempts were made to attribute the microwave background radiation to other (non-cosmological) effects. Observations, however, have shown the amazingly high degree of isotropy of this radiation and therefore, any non-cosmological origin of it is highly unlikely. Indeed, if this radiation were due to galaxies or stars, its anisotropy would be much higher. It is particularly significant that it has a black-body spectrum, something which would be very unlikely if it were non-cosmological in origin. And finally, even the minute deviations *of the CMBR from strict isotropicity, seem to be explainable in terms of the Big Bang scenarios known as* inflationary cosmology.

Is the universe spatially finite or not? In principle, we can answer this question by calculating the "deceleration parameter" $q = \frac{4\pi G\rho}{3H^2} = \frac{\rho}{2\rho_c}$ *from* *the redshift of the galaxies of known distances and by using the GTR correction to Hubble's Law (r = galaxy's distance at emission)*

$$c\frac{\Delta\lambda}{\lambda} \approx Hr + \frac{H^2r^2}{2c}(q-1).$$

If $q \le \frac{1}{2}$ the universe is spatially infinite, otherwise it is finite[684].

In practice it is very difficult to measure the distances of the farthest galaxies, so the calculation of q from the above equation is very inaccurate; *the observational data are not yet accurate enough to show any meaningful deviation from $c\frac{\Delta\lambda}{\lambda} = Hr$. We may only say that $1.5 > q > 0$, so this method cannot yet be used in practice to answer the question of whether the universe is finite or not. Furthermore, the above redshift-distance relation assumes that the cosmological constant, λ, vanishes — yet recent (1990's) observations indicate that the cosmic expansion is actually somewhat* accelerating, *meaning that λ must be nonzero and positive. However, the redshift equation can easily be modified to account for finite λ.*

To answer the question of whether the universe is open or closed in view of these difficulties, we must determine the density ρ and examine if it is above or below the critical density ρ_c.

We may then say that if $\rho > \rho_c$, the gravitational attraction of the matter in the universe would be enough to decelerate its expansion (were it not for the cosmological constant), and to eventually stop it and cause a contraction — leading to a pulsating universe or a "Big Crunch".

On the other hand, if $\rho \le \rho_c$ the gravitational attraction is inadequate to stop the expansion which will continue forever, whether $\lambda = 0$ or $\lambda > 0$. [This phenomenon is similar to the ejection of a particle from a celestial body: the escape velocity from a body of mass M is $v^2 = \frac{2GM}{r}$. If we set $v = Hr$ from Hubble's law, and $M = \frac{4\pi}{3}r^3\rho$, we obtain $\rho_c = \frac{3H^2}{8\pi G}$ as above.]

An independent determination of the average galactic mass density leads to a value of $0.12\rho_c$ for the density of luminous matter. To this we must add intergalactic gas and dust, intergalactic stars, possible exotic matter (made of particles other than electrons, protons and neutrons), small black holes, brown dwarves, etc., so-called dark matter *as well as the effect of λ ("dark energy"). The* inflationary *versions of the Big Bang theory suggest that ρ has been naturally tuned to be very close to ρ_c in the first fraction of a second after the initial explosion, which explains why ρ/ρ_c is roughly of order unity.*

[684] This formula should be corrected for the cosmological constant as discussed below.

Also, the recent (1990's) evidence for positive cosmological constant suggests that the universe is spatially open, although just barely.

1929 CE **José Ortega y Gasset** (1883–1955, Spain). Philosopher and essayist. In his works he purported to find patterns of development in European history by which the present could be explained or its ills exposed. This approach to historical situations was also an extension of his philosophical concern with the interrelationship of individuals and their circumstances.

Ortega claimed life to be more important than thought; because life is ever-shifting and mutating, a proper understanding of man demands the abandonment of the immobile concepts postulated by logical theory and the development of mobile thinking processes.

To him, the conceptual reality posited by idealism is not reality at all; reality is to be found in history, especially in personal history (i.e. in individual autobiographies). Thus, history considered through reason is the proper approach to reality; the two are co-existent components of truth.

Hence, reality does not consist in *Being*, but rather in *Becoming*, for what does the rational consideration of history demonstrate except the eternal evolutionary processes in nature? Consciousness is historical, but the importance of history is not exhausted with the past. Historical knowledge is valued as a preparation for the future.

His key works are: *The Theme of Our Time* (1923), *The Revolt of the Masses* (1929) and *Leibniz and Evolution of Deductive Theory* (1959).

Ortega was born in Madrid. After an early Jesuit education, he studied at the Central University of Madrid, graduating in 1902. Following further studies at the Universities of Leipzig, Berlin and Marburg he became professor of metaphysics at the University of Madrid (1910 to 1936). After the establishment of the republic (1931) he became a member of Parliament. Because of his opposition to the Franco regime he embarked on voluntary exile, living for a time in Buenos Aires and later settling in Lisbon.

In his *The Revolt of the Masses* (1929) and *Mission of the University* (1930), Ortega foresaw the ever-growing impact of science on Western culture and civilization. Beginning with the observation that from 1200 to 1800 *CE*, the population of the Western world remained almost constant, while in the last one hundred years the population in Europe and America has tripled in number, Ortega showed how out of this has risen the phenomenon of the *mass-man*. Can Western culture survive the encroachment of the mass-man? Can republican institutions survive this chaotic democracy? These are problems to which he sought a realistic solution.

Ortega addressed the question of the role of genius in science. To what extent are new ideas, and the whole progress of science, determined by the work of scientists of genius? Building on a similar idea of **Francis Bacon** (1620), Ortega asserts that genius is not necessary and that "*experimental science has progressed — thanks in great part to the work of men astoundingly mediocre, and even less than mediocre*".

Science accommodates and even needs the intellectually commonplace. According to this view, science proceeds, in certain areas at least, by addition of small if not tiny steps, and there are no real breakthrough[685].

[685] Some evidence *against* this idea comes from analysis (1990) of the use of the scientific literature. It turns out that 85 percent of papers in scientific journals are quoted in other papers once or not at all each year, while only 1 percent are quoted five or more times. This supports the argument that an extremely small proportion of the literature is dominant. However, it is not clear to what extent this dominant literature relies on the infrastructure created by lesser scientists. The question is less one of breakthroughs than of significant contributions.

Worldview XLIII: Ortega y Gasset

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"Civilization becomes more complex and difficult in proportion as it advances. The problems which it sets before us today are quite intricate. The number of people whose minds are equal to these problems becomes increasingly smaller. This disproportion between the complex subtlety of the problems and the minds that should study them will become still greater if a remedy is not found, and it constitutes the basic tragedy of our civilization. By reasons of the very fertility and certainty of its formulative principles, its production increases in quantity and subtlety, so as to exceed the receptive powers of normal man. This has never happened in the past. All previous civilizations have died through insufficiency of their underlying principles. That of Europe is beginning to succumb for the opposite reason."

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"In Greece and Rome it was not man as such that failed, but principles. The Roman Empire came to an end for lack of technique. When it reached a high level of population, and this vast community demanded the solution of certain material problems which technique alone could furnish, the ancient world started on a process of involution, retrogression and decay.

But today it is man who is the failure, because he is unable to keep pace with the progress of his own civilization."

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"I do not believe in the absolute determinism of history. On the contrary, I believe that all life, and consequently the life of history, is made up of simple moments, each of them relatively determined in respect of the previous one, so that in it reality hesitates, walks up and down, and is uncertain whether to decide for one or other of various possibilities. It is this metaphysical hesitancy which gives to everything living its unmistakable character of tremulous vibration."

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"We live with our technical requirements, but not by them. These give neither nourishment nor breath to themselves, they are not causae sui, but a useful, practical precipitate of superfluous, unpractical activities."

1929 CE **Max Knoll** (1897–1969, Germany) and **Ernst August Friedrich Ruska** (1906–1988, Germany). Physicists and inventors. Invented the *electron microscope*. Three years earlier they had set out to investigate the discovery made by **Hans Busch** (1884–1973) of Jena that when a beam of electrons passes through a wire-coil, which acts as a magnet, the beam can be focused. By 1933, Ruska's 'supermicroscope' achieved magnifications up to 12,000, and *a linear resolution*[686] well beyond that of the optical microscope. Ruska was awarded the Nobel prize for physics in 1986.

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[686] The *linear resolution* of a microscope (**Abbe**, 1868) is the smallest distance of two object points that can still be discerned as being separate. It is given by the number $\frac{\lambda}{2n\sin\alpha}$, where λ is the wavelength of the light used, n is the refractive index of the fluid filling the space between the object and the objective lens, and α is half the angle of the lens subtended at the object. The *numerical aperture* $\{n\sin\alpha\}$ is a measure of the effective angular opening of the lens for gathering light from the object (with oil immersion numerical apertures up to about 1.6 are possible). For visible light with $\lambda = 5500$ Å, one then finds a linear resolution of 1700 Å for the minimum separation between resolved objects. This imposes a limitation on the ability to resolve specific detail in the image. This limitation is due to *diffraction effects*.

As the resolution of the light microscope is limited by the wavelength of light, efforts were made to utilize rays of shorter wavelength which can also be detected and used to form images. The fact that energetic electrons have extremely short de Broglie (quantum matter–wave) wavelengths has been put to practical use in *electron microscopes*. In these devices, electric and magnetic fields are used to focus electrons by means of electromagnetic forces that are exerted on moving charges. The resulting deflections are similar to the refraction effects produced by glass lenses used to focus light in optical microscopes. By using

1929–1935 CE **Bernhard Voldemar Schmidt** (1879–1935, Estonia and Germany). Optical instrument maker and inventor. Invented a telescope which is named after him. It uses a *spherical mirror* (not a parabolical reflector), and employs a spherically shaped correcting plate at the telescope aperture to compensate for spherical aberration; thus it is a combination of reflector-refractor system.

The *Schmidt-Cassegrain telescope* is the most popular among amateur astronomers because of its compact design and large aperture, and because the optics are completely enclosed. The effect of the correcting plate was to eliminate 'coma' (the optical distortion of focus away from the center of the image) and thus to bring the entire image into a single focus.

Schmidt was born on the island of Naissaar near Tallin, Estonia. He lost most of his right arm in a childhood experiment with gunpowder. He studied engineering in Göteburg, Sweden, and at Mittweida in Germany (1901), where he stayed making lenses and mirrors for astronomers. From 1926 he was attached to the Hamburg Observatory. He worked on the mountings and drives of the telescopes, as well as their optics. It was there that he perfected his lens and built it into the observatory telescope, specifically for use in photography (1932).

By replacing the parabolic mirror with a spherical one plus his correcting lens, Schmidt could produce an image that was sharply focused at every point on a curved photographic plate. In later models he used a second lens to compensate for the use of a flat photographic plate.

Schmidt's invention was of great importance to *optical astronomy*, as it provided extremely fine image definition over a field of several degrees. The best known Schmidt telescope is that on Mount Palomar, with an aperture of 120 cm and a focal length of 300 cm, used for photographic survey of the Northern sky (built, 1948).

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electrons (with wavelengths of 0.05 Å or less, depending upon the voltage used to accelerate them) instead of visible light (with wavelengths near 5000 Å), the limitations on resolution imposed by diffraction effects can be largely overcome. Since the de Broglie wavelength for electrons accelerated through a potential difference V is $\frac{h}{\sqrt{2m_e eV}}$, accelerating potentials from 30 kV to several MV give extremely short wavelengths and also give the electrons sufficient energy to penetrate specimens of reasonable thickness. The resolution that is achievable by an electron microscope is limited by *lens aberrations* and by *scattering* in the specimen, so it is somewhat poorer than predicted on the basis of *diffraction* effects. Nevertheless, a linear resolution of 5 to 10 Å is possible with a 50-kV instrument, and 2 Å is possible with special, high-voltage microscopes. Magnifications range from about 10^3 to 10^5.

The *Schmidt-Cassegrain* (or catadioptric) telescopes use a combination of mirrors and lenses to fold the optics and form an image: Incoming light enters through the *aspheric Schmidt correcting lens*, then strikes the *spherical primary mirror* and is reflected back up the tube. The light is then intercepted by a small *secondary mirror* which reflects the light out of an opening in the rear of the instrument, where the image is formed at the eyepiece.

1929–1948 CE **Carl Ludwig Siegel** (1896–1981, Germany and U.S.A.). A distinguished mathematician. Most of his contributions are in number theory and functions of complex variable. Wrote books on Riemann matrices, geometry of numbers, transcendental numbers, symplectic geometry and analytic functions of several complex variables.

In 1929 Siegel extended *Waring's problem* to algebraic numbers. He also proved that if $P(x)$ is a polynomial with integer coefficients, than the Diophantine equation $y^2 = P(x)$ has at most a finite number of integer solutions (x, y), if $P(x)$ has at least 3 different complex roots. In 1932 he deciphered and extended Riemann's unpublished papers on the zeta function. This work resulted in the *Riemann-Siegel formula*, which proved to be useful for the computation of the zeroes of the *Riemann zeta function*.

Siegel was born in Berlin. He came to Göttingen in 1919 and left in 1940 for the United States, where he became a professor of mathematics at the Institute of Advanced Study, Princeton, NJ.

1929–1952 CE **Alexandr Osipovich Gelfond** (1906–1968, Russia). Mathematician. Originated basic techniques in the study of transcendental numbers and advanced the theory of interpolation and approximation of functions of complex variable. In 1929 he *conjectured* that if a_n and b_n $1 \leq n \leq m$ are algebraic numbers where $\{\ln a_n\}$ are linearly independent over $\mathbb{Q}$, then $b_1 \ln a_1 + b_2 \ln a_2 + \cdots + b_m \ln a_m \neq 0$ [proved in 1966 by **A. Baker**]. In 1934 he proved the *Gelfond theorem*, which states that a^b is transcendental if a is an algebraic number (different from 0 and 1) and b is an irrational algebraic number. This statement solves the 7^{th} Hilbert problem[687] (1900).

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687 Thus it was finally shown that numbers such as $2^{\sqrt{2}}$, $(\sqrt{2})^{\sqrt{2}}$, $e^{-\pi} = (-)^i$ are transcendental.

An *algebraic number* α is a complex number that is a root of an algebraic equation $f(x) = 0$, where $f(x)$ is a polynomial over the field $\mathbb{Q}$ of rational numbers. Examples: $\alpha = \frac{1}{2} + \sqrt{11}$ is a root of $4x^2 - 4x - 43 = 0$; $\alpha = \frac{2}{1+\sqrt[3]{5}}$ is a root of $3x^3 - 3x^2 + 6x - 4 = 0$ $\big[$all the roots are $\alpha_k = \frac{2}{1+\rho_k \sqrt[3]{5}}$ where ρ_k, $k = 1, 2, 3$, are the three roots of unity: $\rho_k = \exp\left\{\frac{2\pi i}{3} k\right\}\big]$.

Gelfond was born in St. Petersburg to Jewish parents. He was a professor of mathematics at Moscow State University from 1931.

It can be shown that if x is the root of a polynomial with algebraic coefficients, then x is itself algebraic. A number that is not algebraic is called *transcendental*. It satisfies no algebraic equation with integer coefficients. It is not immediately clear that transcendental numbers exist. **Liouville** was first to construct some explicitly, for example, $\sum_{n=1}^{\infty} \frac{1}{2^{n!}}$. **Hermite** (1873) was first to prove that e is transcendental. **Lindemann** (1882) followed with the proof that π is transcendental (which showed that the quadrature of the circle was impossible).

It follows from Gelfand's theorem and Hermite's result that for $\alpha \neq 0$, α and e^{α} cannot both be algebraic. Consequently the functions e^{x} (for $x \neq 0$) and $\ln x$ $(x \neq 0, 1)$ have transcendental values for algebraic arguments x. This result is proved with the aid of complex analysis, which was the method used by **Gelfond** to show that e^{π} is transcendental. It is still not known whether e^{e} or $(e + \pi)$ or the Euler-Mascheroni constant γ are transcendental.

π, e and other transcendental numbers

Pi is transcendental.
The endless number cannot be expressed by any algebraic equation.
No pattern has been found in its digits,
Yet it cannot be proven in a finite amount of time that no pattern exists in an infinite number of digits.
Pi goes beyond our reality.
The nonexistence of humans would not preclude the existence of pi.
For the circle will always exist
In the shape and orbit of a planet,
In the path of a wave.
And where there is a circle, there is pi
Intrinsically embedded in it.
Pi is mysterious;
It evades all attempts of capture.
It is a line of digits like an endless snake that you can keep pulling at without ever reaching its tail.
Pi is perfect;
Each seemingly random digit is exactly where it belongs.
Whether a circle is as big as the universe,
Or as small as a quark,
Its diameter fits around its boundary exactly pi times.
Because pi is found in waves, every color, every sound is an expression of pi.
Because pi is found in circles, the moon, the sun, every planet, and every star is an expression of pi.
Because pi is found in each atom, pi is present in all physical sensation.
Pi is absolute beauty.

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The above computer-poetry summarizes succinctly the value of π in human culture. Yet one may still wonder why this number, representing the constant ratio of a circumference of every circle to its diameter, rose to such eminence? Clearly, it is the most famous ratio in mathematics – here on earth, and probably for any advanced civilization in the universe. But its real significance goes much deeper — Pi is fundamental to the way in which our universe functions; practically everything is dependent on π at some basic level: light, sound, energy, gravity, electromagnetic fields, matter itself... In fact, π is so central that it can be seen as a symbol of our universe.

Humans like to think that we live in a rational world. We like to take the often chaotic cosmos and discover or create order, so that we can understand it. Notwithstanding, π transcends rationality, and in doing so, it disturbs the order that we like to see. Even today, a small minority of people still try to prove that π is rational, although ***Lambert*** *was able to demonstrate the irrationality of π in 1776.*

In 1882 ***Lindemann*** *proved that π is transcendental*[688]*, finally putting an end to 2500 years of speculation. This means that π cannot satisfy any polynomial equation with rational coefficients. It further means that π cannot be expressed in any finite series of arithmetical or algebraic operations; using a fixed-size font, it cannot be written on a piece of paper as big as the universe. Lindemann's proof also showed the impossibility of squaring the circle.*

Lambert *(1776) also proved the irrationality of e and* ***Hermite*** *(1873) followed with the proof of its transcendentality. But it still remains unknown whether $\pi + e$ is transcendental. It is, however, known that πe is irrational and that e^π is transcendental (Gelfond, 1934).*

Other famous transcendental numbers of classical mathematics are:

- *Euler-Mascheroni constant*

$$\gamma = 0.577\,215\cdots = \lim_{n\to\infty}\left[1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \ln n\right].$$

(It was not proved to be transcendental, but is generally believed to be so by mathematicians.)

688 A real or complex number z is called *algebraic* if it is the root of a polynomial equation $z^n + a_{n-1}z^{n-1} + \cdots + a_1 z + a_0 = 0$, where the coefficients $a_0, a_1, \ldots a_{n-1}$ are all rational; if z cannot be a root of such an equation, it is said to be transcendental. The number $\sqrt{2}$ is algebraic because it is a root of the equation $z^2 - 2 = 0$; similarly, i, a root of $z^2 + 1 = 0$, is also algebraic.

- *Catalan's constant*

$$G = \sum_{k=0}^{\infty} (-1)^k \frac{1}{(1+2k)^2} = 1 - \frac{1}{9} + \frac{1}{25} - \frac{1}{49} + \dots .$$

  (Not proven to be transcendental, but generally believed to be so by mathematicians.)

- *Special values of the Riemann zeta function, such as* $\zeta(3)$.
- $\ln 2$.
- *Hilbert's number* $2^{\sqrt{2}}$; *proven to be transcendental [Gelfond, 1934].*
- e^{π}; *proven to be transcendental [Gelfond, 1934].*
- π^e *(Not proven to be transcendental, but generally believed to be so by mathematicians.)*
- $i^i \equiv e^{-\pi/2}$; *proven to be transcendental [Gelfond, 1934].*

1929–1954 CE **Andrei Nikolaevich Kolmogorov** (1903–1987, Russia). Distinguished mathematician. Made important contributions to the axiomatic foundations of probability theory through the use of the *Lebesgue measure* (1929).

During (1931–1933), Kolmogorov's research centered on analytical methods of probability theory: he formulated two systems of partial differential equations that bear his name. They describe transition probabilities for **Markov** processes in continuous time (forward and backward equations). These *Kolmogorov equations* are used in many applications to obtain the required probability distribution, especially in diffusion processes (e.g. population growth) and Brownian motion. This work marked a new phase in the development of probability theory and its implementation in problems of physics, chemistry, engineering and biology.

Kolmogorov made important investigations in topology (homological rings, nabla operator, nabla groups, continuous maps, duality law), information theory, function theory, functional analysis, turbulent flow of fluids and random stationary processes.

Kolmogorov was born in Tambov, Russia. In 1920 he started his studies at the Moscow state university in the faculty of physics and mathematics. After his graduation (1925) he stayed there as a research associate and was elected professor in 1931. In 1933 he became director of the Institute of Mathematics at the University. In 1939 Kolmogorov was elected an academician of the Academy of Sciences of the U.S.S.R. and later, an academician-secretary of the department of physical and mathematical sciences of the academy.

The KAM theorem

The combined work of ***Kolmogorov*** *(1954), his student* ***Vladimir I. Arnol'd****, and* ***Jürgen Moser*** *(1962–3) provided precisely defined mathematical criteria for determining whether (and which) perturbations can push a dynamical system into instability. The result is known as the KAM theorem.*

From Poincaré's work and that of his successors, mathematicians already had some sense that the phase-spaces of physical systems containing three or more bodies are characterized by an intricate interweaving of regular and chaotic regions. Yet some simple systems, such as the two–body (Kepler) problem is celestial mechanics, are integrable *(admit closed–form solutions), and their solutions are regular. In the solar system, for instance, one may reasonably regard inter–planetary interactions as weak perturbations to the (otherwise independent and Keplerian) motions of each planet about the sun. The question that remained was whether the feeble perturbations of the planets, compared with sun's overwhelming effect, are sufficient to lead to true instability.*

The KAM theorem assures us that motion in any dynamical system remains for the most part *regular, or quasi-periodic, if perturbations stay sufficiently small. Applied to our solar system, it means that any of a large set of initial conditions leads to quasi-periodic rather than chaotic orbits, provided the masses of the planets are sufficiently small compared with the sun's mass.*

An integrable *n-dimensional system is a system whose equations of motion can be reduced to n uncoupled one-dimensional (and in general non-linear) oscillators by means of a* canonical transformation *in $2n$-dimensional phase space is effected.*

The dynamics of a conservative, finite, lumped mechanical system can be described by the canonical system of ODE's (Hamiltonian equations of motion):

$$\dot{p}_k = -\frac{\partial H}{\partial q_k} \quad , \quad \dot{q}_k = \frac{\partial H}{\partial p_k} \qquad (k = 1, \dots, n),$$

where $\{q_j\}$ are generalized coordinates, $\{p_j\}$ are the corresponding canonically conjugate momenta, and H is the Hamiltonian function, given by

$$H(p, q, t) = \sum_{j=1}^{n} p_j \dot{q}_j - L$$

in terms of the Lagrangian $L(q, \dot{q}, t)$; $\{\dot{q}_j\}$ as functions of $\{q_k, p_k\}$ can also be obtained by solving $p_k = \frac{\partial L}{\partial \dot{q}_k}$ for the generalized velocities $\{\dot{q}_k\}$.

Such a dynamical system is in general not integrable*, that is, not explicitly solvable in the form*

$$p_k = f_k(t;\ p^0, q^0); \qquad q_k = g_k(t;\ p^0, q^0),$$

where $p^0 \in \mathbb{R}^n$, $q^0 \in \mathbb{R}^n$ are the initial conditions. There are exceptions to this general rule — among them the so-called integrable Hamiltonian systems*. A simple example of an integrable system occurs when the Hamiltonian is a function only of the p_k: $H(p, q, t) = K(p) \quad (p \in \mathbb{R}^n)$, known as the* normal form*. In this case*

$$\dot{p}_k = -\frac{\partial H}{\partial q_k} = 0; \qquad \dot{q}_k = \frac{\partial K}{\partial p_k};$$

thus all the p_k are constants, and we readily obtain the general solution in the form

$$p_k(t) = p_k^0 \qquad q_k(t) = \Big[\Big(\frac{\partial K}{\partial p_k}\Big)_{p=p^0}\Big] t + q_k^0,$$

so that we essentially have a generalized 'free particle' situation. Another standard example of an integrable system is a collection of n harmonic oscillators *with unit masses:*

$$H(p, q) = \sum_{k=1}^{n} \frac{1}{2}(p_k^2 + \omega_k^2 q_k^2).$$

Then

$$\dot{p}_k = -\omega_k^2 q_k; \qquad \dot{q}_k = p_k$$

where ω_k are the frequencies; this system admits the general solution

$$p_k = A_k \cos(\omega_k t + \phi_k); \qquad q_k = \frac{A_k}{\omega_k} \sin(\omega_k t + \phi_k)$$

$$q_k^0 = p_k^0 \frac{\tan \phi_k}{\omega_k}, \qquad A_k = \left[\left(p_k^0\right)^2 + \omega_k^2 \left(q_k^0\right)^2 \right]^{1/2}$$

Upon changing variables to polar 'action-angle' variables (I_k, θ_k) via

$$p_k = \sqrt{2\omega_k I_k} \cos \theta_k, \qquad q_k = \sqrt{\frac{2I_k}{\omega_k}} \sin \theta_k,$$

the Hamiltonian simplifies to $H(I, \theta) = \sum_{k=1}^{n} \omega_k I_k$, *which is the* normal form. *In these variables, the solution is*

$$I_k(t) = I_k^0; \qquad \theta_k(t) = \omega_k t + \theta_k^0 \qquad (k = 1, \ldots, n).$$

Note that the angle variables θ_k are only defined modulo 2π, i.e. $\theta_k' = \theta_k + 2\pi m_k$ (where for any k, $m_k = \pm 1, \pm 2, \ldots$) are all the same point in phase space. If $n = 2$, the motion is in 4-dimensional phase space, but it is restricted to a surface which satisfies the two conditions $p_k^2 + \omega_k^2 q_k^2 = 2\omega_k I_k$ $(k = 1, 2)$. The parametric equations of this surface are those of a 2-dimensional torus *embedded in $\mathbb{R}^4$:*

$$\begin{array}{ccc} \omega_k q_k = r_k \sin \theta_k; & p_k = r_k \cos \theta_k, & k = 1, 2, \\ r_1 = \sqrt{2\omega_1 I_1}, & r_2 = \sqrt{2\omega_2 I_2}, & r_1 > r_2 \end{array}$$

There are two distinct possible types of trajectories which may occur on this torus:

- (ω_1, ω_2) *are* rationally independent, *that is,* $m_1\omega_1 + m_2\omega_2 = 0$ *(with m_k : integers) has no solutions except $m_1 = m_2 = 0$ (example: $\omega_1 = \sqrt{2}$; $\omega_2 = 4$). The motion is then* quasi-periodic *and the trajectory is* dense everywhere *on the torus. These disjoint 2-tori (one per each (I_1, I_2) ordered pair) are called invariant tori; their union is the entire $\mathbb{R}^4$ space.*
  These frequencies are rationally *dependent, i.e. $m_1\omega_1 + m_2\omega_2 = 0$ has solutions for nonzero integers; e.g. $\omega_1 = \sqrt{2}$, $\omega_2 = \sqrt{18}$. The motion is then* periodic*; The trajectory will not visit some regions of the 2-torus.*

One can generalize this example to the case of the motions in a $2n$-dimensional phase-space which take place on disjoint, invariant n-tori.

A third example of an integrable system is the 'Kepler problem' in $\mathbb{R}^n$*, defined by the Hamiltonian*

$$H = \frac{1}{2}\sum_{k=1}^{n} p_k^2 - \left(\sum_{k=1}^{n} q_k^2\right)^{-1/2}.$$

This system is invariant under n *dimensional rotations and therefore has* $\frac{n(n-1)}{2}$ *angular–momentum type constants of motion:* $\kappa_{ij} = p_i q_j - q_i p_j$ $(1 \le i < j \le n)$.

The Kolmogorov-Arnold-Moser (KAM) theorem treats near-integrable system, with a Hamiltonian

$$H = H_0(I) + H_1(I,\theta) \qquad (I \in \mathbb{R}^N),$$

where H_1 *is taken to be periodic [i.e.* $H_1(I, \theta + 2\pi m) = H_1(I,\theta)$, $m = \{m_k\}$ *a vector of integers] and is required, in some sense, to be "small enough" (i.e.* $\|H_1\| \ll 1$ *for some norm). Hamilton's equations are then*

$$\dot{I}_i = -\frac{\partial H_1}{\partial \theta_i}; \qquad \dot{\theta}_i = \omega_i(I) + \frac{\partial H_1}{\partial I_i},$$

where ω_i *are the unperturbed frequencies, that is* $\omega_i = \frac{\partial H_0}{\partial I_i}$*. For most initial data (i.e. all except a set of small measure) Kolmogorov sketched a proof that the motion remains predominantly periodic or quasiperiodic, that is, confined to invariant tori; in other words the complement of the quasi-periodic motion (i.e. chaotic motion) has as small a Lebesgue measure as desired, provided* H_1 *is small.*

The KAM theorem is formulated by assuming that the Hamiltonian is analytic in a complexified domain of phase space, and that the unperturbed motion is non-degenerate; that is

$$\det\left(\frac{\partial \omega_i}{\partial I_j}\right) = \det\left(\frac{\partial^2 H_0}{\partial I_j \partial I_j}\right) \neq 0.$$

Clearly, an integrable system, by definition, cannot be chaotic. In general, when the perturbation H_1 *is large enough, the invariant tori are destroyed, and most initial–conditions data sets results in trajectories not limited to lower–dimensional submanifolds of* $2n$*–dimensional phase space.*

1929–1956 CE **Adolf Friedrich Johann Butenandt** (1903–1995, Germany). Biochemist. Isolated the first sex hormones (*estrone*, *androsterone* and *progesterone*), and determined their structure. In 1939 he shared the Nobel prize for Chemistry with **Leopold Ruzicka** (1887–1976), although he was forbidden by the Nazi regime to accept it. He discovered the first insect hormone, *ecdysone* (1956) and soon afterwards, *bombykol* — the scent produced by female silkworm to attract the males.

Butenandt was born in Lehe, near Bremerhaven, and studied medicine, biology and chemistry in Marburg and Göttingen. In 1936 he became head of the Kaiser Wilhelm Institute of Biochemistry in Berlin.

The Discovery of Planet Pluto (1906–1930)

The name Pluto comes from Pluton, an alternate Greek name for Hades, the god of the dead. The Romans borrowed and preserved without change almost all the myths about Hades and his underworld kingdom.

Pluto is the most distant known planet from the sun[689]*. Pluto and Neptune are the only planets that cannot be seen without a telescope. Both planets*

[689] Mean distance from the sun: 5914.3 million km; diameter: ca 2300 km; length of year: ca 248 earth years; rotation period: 6.4 earth days; *inclination* of orbit to the ecliptic: $i = 17^\circ$; *orbital eccentricity*: $e = 0.25$; *tilt* of spin axis to orbit: 177° (i.e. its north pole, defined by counterclockwise rotation, lies below the plane of its orbit); *density*: ca $2\ \frac{\text{gm}}{\text{cm}^3}$ (rocky material and ice).

A revolution in the understanding of Pluto began in 1978 when **James W. Christy** discovered that Pluto has a large satellite, which he names *Charon* (after his wife, *not* the Greek eponymous mythological figure that ferried the souls of the dead to the underworld dominion of Pluto). It revolves around the planet over a 6.4-day period, the same as Pluto's known period of rotation. Consequently, Pluto and Charon keep the same hemisphere facing toward each other, just as in the earth-moon system – in both cases as a result of tidal torques. The diameter of Charon was found to be 1186 km, while its orbital radius (about the Charon–Pluto center of mass) is 19640 km. Measurements of Charon's orbit revealed that the total mass of the Pluto-Charon system is about $\frac{1}{400}$ the mass of the earth.

Shortly after Charon was discovered, astronomers realized that twice during *were predicted mathematically prior to their actual discovery, based upon the gravitational perturbation they cause.*

*In 1905, **Percival Lowell** (1855–1916, U.S.A.), an American astronomer, made an elaborate mathematical study of the orbits of Uranus and Neptune. He then attributed the perturbations in their motions to the presence of an unseen planet beyond Neptune, and calculated its position. A systematic search for the planet by the staff of his private observatory (Flagstaff, Arizona — todays 'Lowell Observatory') failed to find it.*

*Fourteen years after his death, **Clyde William Tombaugh** (1906–1997; born in Streator, Illinois, a year after Lowell's own prediction) discovered **Pluto** on photographic plates which he took with a more powerful telescope at Lowell's Observatory (February 18, 1930).*

***Lowell** was born in Boston, a member of the distinguished Lowell family of Massachusetts*[690]*. Until* 1890 *he devoted himself to literature and travel in*

Pluto's 248-year circuit about the sun, the alignment between the earth and Pluto causes Charon's orbit to be seen edge-on. At those times, Charon appears to pass in front of Pluto (*transit*) or behind it (*occultation*) at 3.2 day intervals. These events are commonly known as *eclipses*. Fortunately for astronomers, they occurred close to Charon's discovery, in 1987. Due to the total occultation of Charon behind Pluto it became possible to resolve the individual spectra of the two bodies.

Careful timings of the transits and occultations can be translated into dimensions of the objects, provided one knows the distance between Charon and Pluto. This, combined with the new measurements of the total mass of the system, yielded the above–quoted estimate of Pluto's density.

A star occulted by Pluto in 1988 flickered before it vanished — revealing that the planet has a thin, hazy atmosphere, composed of methane, argon, nitrogen, oxygen and carbon monoxide. Pluto's equatorial surface is at a temperature of about 58° Kelvin; its surface is covered with frozen methane and a wispy atmosphere that may periodically precipitate snow.

Calculations of Pluto's orbital motion covering a period of 845 million years [**G.J. Sussman** and **J. Wisdom**, *Science*, 22 July 1988] indicate that Pluto's orbit is *chaotic* over long periods. It is most likely that Pluto formed in the outer solar system and that chaotic dynamics led to its current eccentric and highly inclined orbit. It may thus be a relic from the formation of the solar system.

Recently, (2000's) there has been a trend among astronomers to "demote" Pluto from the ranks of *bona fide* planets of Solar System.

690 His brother **Abbott Lawrence Lowell** (1856–1943) was president of Harvard University (1909–1933). His sister **Amy Lowell** (1874–1925) was an American poet and critic.

the far east. Afterwards he became best known for his belief in the possibility of life on Mars and the existence of canals there[691]*. In* 1894 *he completed the construction of his private observatory at Flagstaff, Arizona.*

Tombaugh *served at Lowell Observatory, and spent most of his academic career at the New Mexico State University.*

[691] The canal theory was put to rest by data received from U.S. spacecraft *Mariner 4* when it flew past Mars in July 1965. As to Martian life, the chemical and biological analyses performed, via robotics, by the *Viking I* and *II* lander spacecrafts (July 20, 1976 and Sept. 3, 1976), proved inconclusive. In 1996, an analysis of a meteorite discovered (1984) in Antarctica, strongly suggested that it had come from Mars, possibly bearing fossils of ancient organisms. Future missions to Mars may finally settle the question whether life exists, or has ever existed on the red planet.

Exploration and Discovery of the Solar System[692] ***(1610–2008 CE)***

From the dawn of history until the beginning of the 17th century the known universe consisted of only 7 heavenly bodies:[693]

Sun
Mercury
Venus
Moon
Mars
Jupiter
Saturn

plus the "fixed" stars with earth itself considered the universe's center, in accord with the Church–sanctioned Ptolemaic system.

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[692] For further reading, see:

- Caprara, G., *The Solar System*, Firefly Books, 2003, 255 pp.
- Ridpath, I., *Stars and Planets*, DK Publications, 1998, 224 pp.
- Watters, T.R., *Planets – A Smithsonian Guide*, Macmillan, USA, 1995, 256 pp.
- Moore, P., *Philip's Atlas of the Universe*, Phillips, 2005, 288 pp.
- Gallant, R.A., *Universe*, National Geographic Society, 1995, 284 pp.
- Kump, L.R. et al., *The Earth System*, Prentice Hall, 1999, 351 pp.
- Rees, M. ed, *Universe*, DK Publications: New York, 2005, 512 pp.

[693] In *Genesis* **37**, 9 Joseph tells his brothers of a dream about "the sun and the moon and the eleven stars..." If we discount the above list of naked-eye bodies, Joseph must have included another naked-eye "fixed" stars. Candidates may be chosen from the list of brightest stars in the Northern skies, namely: *Sirius*, *Canopus*, α *Centauri*, *Vega* and *Arcturus*.

In 1610, ***Galileo Galilei*** *first pointed a telescope upon the heavens. By the end of the 17th century,* 9 *new bodies had been discovered and the heliocentric theory of* ***Copernicus*** *was widely accepted. The total number of known bodies had more than doubled, (now including planet earth) to* 17. *The new additions were:*

Callisto	*1610*	*Galileo Galilei*
Europa	*1610*	*Galileo Galilei*
Ganymede	*1610*	*Galileo Galilei*
Io	*1610*	*Galileo Galilei*
Titan	*1655*	*Christiaan Huygens*
Iapetus	*1671*	*Giovanni Domenico Cassini*
Rhea	*1672*	*Giovanni Domenico Cassini*
Dione	*1684*	*Giovanni Domenico Cassini*
Tethys	*1684*	*Giovanni Domenico Cassini*

Only 5 *new bodies (not including comets) were discovered in the 18th century (all by* ***William Herschel****), bringing the total to* 22*:*

Uranus	*1781*	*William Herschel*
Oberon	*1787*	*William Herschel*
Titania	*1787*	*William Herschel*
Enceladus	*1789*	*William Herschel*
Mimas	*1789*	*William Herschel*

The number of known bodies in the solar system increased dramatically in the 19th century, with the discovery of the asteroids (464 *of which were known by 1899), but only* 9 *more "major" bodies were discovered. The number of major bodies rose thereby to* 31 *(almost doubling the 17th century total):*

Neptune	*1846*	*Johann Gotfried Galle,* *Urbain Jean Joseph Le Verrier*
Triton	*1846*	*William Lassell*
Hyperion	*1848*	*William Cranch Bond*
Ariel	*1851*	*William Lassell*
Umbriel	*1851*	*William Lassell*
Phobos	*1877*	*Asaph Hall*
Deimos	*1877*	*Asaph Hall*
Amalthea	*1892*	*Edward Emerson Barnard*
Phoebe	*1898*	*William Henry Pickering*

In the 20th century (up to 1990), 40 more major bodies (and thousands of comets and asteroids) have been discovered (27 by the Voyager probes), more than doubling the count again to 71:

Himalia	*1904*	*C. Perrine*
Elara	*1905*	*C. Perrine*
Pasiphae	*1908*	*P. Melotte*
Sinope	*1914*	*S. Nicholson*
Pluto	*1930*	*Clyde W. Tombaugh*
Carme	*1938*	*S. Nicholson*
Lysithea	*1938*	*S. Nicholson*
Miranda	*1948*	*Gerard Kuiper*
Nereid	*1949*	*Gerard Kuiper*
Ananke	*1951*	*S. Nicholson*
Janus	*1966*	*Audouin Dollfus*
Leda	*1974*	*Charles T. Kowal*
Charon	*1978*	*J. Christy*
Adrastea	*1979*	*D. Jewitt & E. Danielson*
Metis	*1979*	*Stephen Synnott*
Thebe	*1979*	*Stephen Synnott*
Epimetheus	*1980*	*R. Walker*
Atlas	*1980*	*R. Terrile*
Calypso	*1980*	*Pascu et al.*
Helene	*1980*	*P. Laques & J. Lecacheus*
Pandora	*1980*	*S. Collins et al.*
Prometheus	*1980*	*S. Collins et al.*
Telesto	*1980*	*Reitsema et al.*
Puck	*1985*	*Voyager 2*
Belinda	*1986*	*Voyager 2*
Bianca	*1986*	*Voyager 2*
Cordelia	*1986*	*Voyager 2*
Cressida	*1986*	*Voyager 2*
Desdemona	*1986*	*Voyager 2*
Juliet	*1986*	*Voyager 2*
Ophelia	*1986*	*Voyager 2*
Portia	*1986*	*Voyager 2*
Rosalind	*1986*	*Voyager 2*
Despina	*1989*	*Voyager 2*
Galatea	*1989*	*Voyager 2*
Larissa	*1989*	*Voyager 2*
Naiad	*1989*	*Voyager 2*
Proteus	*1989*	*Voyager 2*
Thalassa	*1989*	*Voyager 2*
Pan	*1990*	*Mark Showalter*

Timeline

432 BCE — **Meton** introduced his calendar in Athens.

c. 270 BCE — **Aristarchos** of **Samos** estimated the distance and size of the sun and proposed that earth goes around it.

c. 250 BCE — **Erathosthenes** estimated the size of the earth.

c. 135 BCE — **Hipparchos** discovered the precession of the equinoxes and estimated the distance to the moon.

c. 46 BCE — **Julius Caesar** commanded the reform of the Roman Calendar.

c. 140 AD — **Claudius Ptolemaeus** (Ptolemy) wrote "*Megale Syntaxis tes Astronomias*" (known 1000 years later as "*Almagest*"), proposing his world system.

1543 — **Nicolaus Copernicus** published his theory of the solar system.

1582 — **Pope Gregory the 13th** commanded the reform of the calendar.

1609 — **Galileo Galilei** built the first astronomical telescope and observed for the first time craters on the moon, satellites around Jupiter and the moon–like apparent phases of Venus.

1609–1619 — **Johannes Kepler**, using **Tycho's** observations, formulated his first two laws of planetary motion (1609), and the third in 1619.

1686 — **Isaac Newton** established the *law of universal gravitation.*

1781 — **William Herschel** discovered planet *Uranus* with a mirror telescope of his invention.

1801 — **Giuseppe Piazzi** discovered main–belt–asteroid *Ceres.*

1807 — Discovery of main–belt asteroid *Vesta.*

1838 — **Friedrich Bessel** first measured the distance to a star (61 Cygni).

1839 Invention of *photography* by **L.J.M. Daguerre**.

1846 Discovery of a new, theoretically predicted planet — *Neptune*.

1930 Discovery of planet *Pluto*.

1957 Soviet Union launched the first man-made satellite, *Sputnik 1*.

1961 **Yuri Gagarin** became the first human to orbit earth.

1969 United States lands a man on the moon (*Apollo 11* mission)

1976 The U.S. *Viking 1* soft-landed on *Mars*; took pictures and searched for chemical signatures of life.

1977–1989 The U.S. *Voyager 1* and *Voyager 2* embark on a space odyssey which takes them to encounters with Jupiter, Saturn, Uranus, Neptune and beyond.

1989–1999 U.S. space probe *Magellan* sent on a mission to orbit *Venus* and map it, using a radar imaging system.

1989–1991 U.S. craft *Galileo* (with some European sub–systems) sent on a mission to orbit *Jupiter* and study its atmosphere, satellites and surrounding magnetosphere.

1990–1994 U.S. (HST); launched the *Hubble Space Telescope*; it returned high-resolution images of Mars and the other outer planets of the Solar system as well as deep–space imagery; its surveys of supernovae throughout the observable universe played a key role in refining and confirming the Big Bang cosmological theory, establishing the universe's age (14Gy) and discovering its residual *dark energy* (vacuum expansion). In 1994 the HST photographed the spectacular collision of a fragments of comet Schumacher–Levy with Jupiter.

1990–1995 U.S./European craft *Ulysses* sent on a solar fly-by mission to study the poles of the sun and the interstellar space above and below the poles. It used Jupiter for a gravity-assist boost to swing out of the ecliptic plane and onwards to the poles of the sun.

1995 Europe and U.S. Solar Probe *SOHO* (Solar and Heliospheric Observatory) was launched to study the sun's internal structure, by observing velocity oscillations and radiance variations, and to look at the physical processes that form and heat the sun's corona and that give rise to the solar wind — using imaging and spectroscopic diagnosis of the plasma in the sun's outer regions. SOHO was placed in a "halo orbit" around the *L1 Lagrangian point* (a point 1.5 million km away from us at which the gravitational pull of the earth balances that of the sun).

1996 NASA made a startling discovery that points to the *possibility* that a primitive form of microscopic life *may have existed* on Mars more than 3 billion years ago. As evidence, NASA presented a potato-size stone (designated ALH84001), a rare type of meteorite (recovered from Antarctica) that had its genesis on the planet Mars. The stone's Martian provenance was determined by its isotope ratios; it was likely ejected from Mars due to the impact of a asteroid, landing on earth thousands of years ago. Over the course of the preceding $2\frac{1}{2}$ years, a team of NASA researchers and outside collaborators used sophisticated techniques of physical chemistry and optics to minutely examine the meteorite; they uncovered mineralogical, chemical, and structural oddities which they interpreted as evidence for biological activity in Mars' distant past.

The stone's journey began some 16 million years ago when an asteroid slammed into Mars and hurled chunks of the planet into space. About 13,000 years ago at least one of those pieces plummeted onto the frozen wastes of Antarctica, where it was found on Dec 27, 1984.

1996 U.S. Asteroid Orbiter *NEAR* (Near Earth Asteroid Rendezvous) launched to the near–earth asteroid *433 Eros*. The spacecraft studied the asteroid for one year after entering orbit in Feb 1999.

1997 U.S. and Europe Saturn Orbiter and Titan Probe *Cassini – Huygens* was launched to explore the whole Saturnian system — the planet itself, its atmosphere, rings and magnetosphere, and some of its moons. Titan is especially interesting because its atmosphere is supposed to have properties very close to those of the terrestrial atmosphere in its pre-biotic phase.

1998–9 U.S. *Mars Surveyor Project*: spacecraft '*98 Orbiter*' (1998) studied the planet from polar orbit for about 2 years, using a variety of advanced instruments; '*98 Lander*' (1999) studied the environment at the Martian south pole, seeking to understand the planet's climate and soil. It was equipped with meteorological equipment to study the weather, and a robotic arm to dig trenches in the soil.

1999–2006 U.S. Comet Sample Return *Stardust*. Scheduled to rendezvous with Comet P/Wild-2 in 2004, study the object and collect material for analysis on earth. Capsule is scheduled to return in January 2006.

2007 NASA launched the probe *DAWN* via Delta II ion–engine rocket from Cape Canaveral (Sept 27). The DAWN will travel 5.1×10^9 km to asteroids *Vesta* and *Ceres*, reaching *Vesta* sometime in August 2011.

1930 CE, Aug. 13 ca 12^h:04^m GMT (07^h:04^m LT). A mysterious explosion over the Curucá River[694], in the upper reaches of the Amazon, West Brazil (near the border junction of Peru, Columbia and Brazil); ca 5 °S, 71.5 °W.

A glowing bolide arrived from a northerly direction and apparently exploded at low altitude (5–10 km) in the atmosphere over the Brazilian rain forest. The earthquake generated by the impact of the main detonation-wave was recorded at *La Paz* (1322 km away) at 07:05:03 LT, and the explosion was heard in cities 240 km away (Atalaia do Norte and Esperanca). At the source –

"... the sun turned blood red and a darkness spread overhead; a fine red dust (ash) began to fall onto the forest and into the river; several ear-piercing whistles then filled the air, becoming louder and louder; from the sky fell large balls of fire like thunderbolts; there were three distinct explosions, each causing tremors like earthquakes".

Astronomers estimated a yield equivalent of about one megaton TNT (about a tenth of the yield of the Tunguska event of June 30, 1908 in Central Siberia). They believe that the bolide originated in the comet *P/Swift-Tuttle* and that the fireballs were associated with its *Perseid meteroid stream*[695].

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694 A tributary of the Rio Yavari, itself a large tributary of the Amazon River.

695 Huyghe, P., *Incident at Curucá*, The Sciences, March/April 1966, pp. 14–17.

Apart from the major *Tunguska explosion* (June 30, 1908) and the smaller *Brazilian explosion* (Aug. 13, 1930), at least three other similar events, albeit of smaller yield, are known to have happened over land:

British Guiana, Dec. 11, 1935
British Columbia, March 31, 1965
New Guinea, March 04, 1975.

Thus, it is quite feasible that bolide-earth encounters in the yield range 100–1000 KT take place ten times more often than astronomers and geophysicists had thought, with the earth subjected to 3–4 such events per century.

1930 CE

- Superconductivity was discovered in lead-bismuth alloys by **W.J. de Haas** and **J. Voogd**. This class of superconductors is called *type II*, and its members differ from *type I* superconductors in their magnetic properties, which involve *Abrikosov Flux tubes* penetrating the medium bulk.
- The *photographic flashbulb* was patented by a German inventor, **Johannes Ostermeir**. A small filament in the 'flash lamp' heated to ignite foil inside the bulb, providing a bright, smokeless, flash of light. This provided a much safer and more practical means of photographic illumination than did previous methods, which employed flash powder.

1930 CE **Vannevar Bush** (1890–1974, U.S.A.). Electrical engineer. Developed and built the first mechanical analog computer ('*differential analyzer*') designed to solve differential equations.

Bush was born in Everett, MA. He taught at Tufts University (1914–1917). After conducting submarine-detection research for the U.S. Navy, he joined the faculty of the Massachusetts Institute of Technology (MIT) at Cambridge in 1919. From 1930 he worked with a team at MIT to build the differential analyzer. This machine foreshadowed the electronic computers developed after WWII. During 1940–1948 he served as a high ranking government official in charge of scientific research and development. His famous "unfettered research" memorandum set the tone for generous U.S. government support for "Big Science" basic research in the post–war are – a policy that greatly benefited fields such as nuclear and particle physics, in particular.

1930 CE **Ernest Orlando Lawrence** (1901–1958, U.S.A.). Distinguished physicist. Invented the circular particle accelerator (cyclotron). It was the first non-electrostatic accelerator to reach high energies (ca 12 MeV for protons).

The principle of operation of the cyclotron[696]: positive ions from a central source are *repeatedly accelerated by an alternating electric field* applied across the gap between two D-shaped half-circles ("*dees*", diameter ca 90–230 cm). These ions describe spiral paths in a *perpendicular magnetic field* (ca 20,000 gauss) and are finally directed by a deflector plate onto a target.

With the cyclotron, Lawrence produced *technetium*, the first artificially produced element. He later contributed to chemistry, biology and *medicine* by producing new artificial elements (e.g. radioactive phosphorus and iodine) and also generated neutron beams used in cancer treatment. During WWII, he worked in the Manhattan project, in which he was in charge of developing the electromagnetic process for separating uranium-235 for the atomic bomb. Lawrence also invented the color-television picture tubes. The Lawrence Berkeley Laboratory at Berkeley; Lawrence Livermore National Laboratory at Livermore, CA; and element 103, Lawrencium, were named in his honor.

Lawrence was born in Canton, South Dakota. He was an assistant professor at Yale University (1921–1928) and became a full professor at the University of California, Berkeley, in 1930. He built the Radiation Laboratory at Berkeley in 1936 and won the Nobel prize for physics in 1939.

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[696] In the non-relativistic approximation, the revolution frequency of a particle in a uniform magnetic field is independent of its kinetic energy. As the particle is accelerated via synchronized electric fields during short segments of its orbit, it travels faster, experiencing a magnetic deflection force proportional to its velocity. The centripetal acceleration is proportional to the square of its velocity; Newton's second law then becomes $\frac{mv^2}{r} = Bev$ or $m\omega^2 r = Be\omega r$. In these equations m is the mass of the accelerated particle, v its linear velocity, e its charge, r is the radius of revolution, B the magnetic field, and $\omega = \frac{v}{r} = \left(\frac{e}{m}\right) B$ is the angular velocity *which is independent of the speed or radius*, and fixed for a group of particles with fixed $\left(\frac{e}{m}\right)$ ratio. The time of revolution $t = \frac{2\pi}{B}\left(\frac{m}{e}\right)$ is thus also independent of the speed. If the frequency of the alternating potential difference between the "*Dees*" (the D-shaped halves of the cyclotron) is correctly adjusted, the ions emerge from each *dee* after exactly one-half cycle and are attracted into the opposite *dee*. The energy characteristic of the machine is limited by the diameter $2R$ of the *dees* and the magnetic field strength B: $E_{\text{lim}} = \frac{m}{2} v_{\text{lim}}^2 = \frac{e^2 B^2 R^2}{2m}$. Above 10 MeV for protons, the relativistic increase in mass begins to de-synchronize the revolution frequency with the oscillator frequency.

Big Science I — Particle Accelerators[697] *(1930–1971)*

High energy particle physics is the study of fundamental particles and their interactions, transmutations and associated fields. This involves probing short distances, which in turn is accomplished with probes of short de Broglie wavelength, i.e. high momentum and energy $(p = \frac{h}{\lambda})$. *"High energy" varies from a few keV (in the spectrometer of J.J. Thomson, in which he discovered the electron in 1897) to 100 GeV (in the spectrometer where the* Z^0 *particle was discovered in 1983) and beyond — a range of more than 8 orders of magnitude*[698] *which will continue to grow as accelerator technology marches on.*

Particle accelerators are devices used to accelerate charged elementary particles or ions to high energies. Particles are accelerated through their

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[697] For further reading, see:

- Sandin, T.R., *Essentials of Modern Physics*, Addison-Wesley Publishing Company: Reading, MA, 1989, 575 pp.
- Frauenfelder, H. and E.M. Henley, *Subatomic Physics*, Prentice-Hall: Englewood Cliffs, NJ, 1974, 554 pp.
- Thornton, S.T. and A. Rex, *Modern Physics*, Saunders College Publishing, 2000, 556 pp.
- Rohlf, J.W., *Modern Physics from* α *to* Z^0, John Wiley and Sons, Inc: New York, 1994, 646 pp.

[698] *Energy scales in modern physics*: a proton or electron accelerated through 1 Volt acquires a kinetic energy of $1 \text{ eV} = 1.602 \times 10^{-19}$ Joule. Thus: $\text{keV} = 10^3$ eV, $\text{MeV} = 10^6$ eV, $\text{GeV} = 10^9$ eV, $\text{TeV} = 10^{12}$ eV. The electron *rest-mass energy* is $E_0 = m_e c^2 = 0.511$ MeV, while the proton rest-mass energy is 938 MeV. The amount of rest–mass energy stored in 1 kg of matter is 9×10^{16} Joule [at a cost of 10 cents per kilowatt-hour, the energy stored in one kilogram of dirt – if it could be released – would be worth about 2.5 billion dollars!].

Energies of the order of eV are associated with outer electrons in atoms, keV — with inner electrons in heavy atoms, MeV — with neutrons and protons (nucleons) inside nuclei and released in fission, fusion and other nuclear reactions; 10^2 MeV to GeV's — with quarks inside nucleons, light quark-antiquark hadrons, heavy-lepton production, and nucleon-antinucleon production. Energies of order 10^2 GeV resolve the weak nuclear forces' range (via the uncertainty principle) and produce their mediating particles.

electromagnetic interactions. Only electrically charged particles that are relatively stable against spontaneous decay may be readily accelerated. The only particles that fit this description are the electron*, the* proton*, and relatively stable nuclei (heavy ions) plus their antiparticles. However, beams of charged* unstable *weakly–decaying particles (such as the* muon*) can be accelerated, stored and collided with targets and each other, since the highly relativistic energies of such beams result in time–dilation factors that render them stable enough*[699]*.*

Particle accelerators today — along with their attendant peripheral equipment, such as detectors*, and stand–alone detectors designed to study cosmic rays and exotic particles and decays — are some of the largest and most expensive instruments used by scientists. Accelerators all have three basic parts: a source of charged particles, a tube pumped to a high vacuum in which the particles can travel freely until they reach their target, and some means of speeding up the particles. The vacuum is needed so that the particles are not scattered by air molecules.*

Due to many innovative technological advancements in the field of accelerators, the maximum achievable particle energy has grown by many orders of magnitude over the decades since the early accelerators of 1930. Each new accelerator technique has reached a limit in the maximum energy within a few years, only to be overtaken by a new invention. The main stages of development were as follows:

1. Electrostatic machines (1–10 MeV; 1927–1933)

The success of the nuclear model of the atom, proposed in 1911 *by* ***Rutherford*** *and developed by* ***Niels Bohr*** *and others, served to emphasize the great difference in energy scale between atomic processes involving only the extranuclear electrons and nuclear processes proper. During the first* 20 *years of the* 20^{th} *century, the chief source of information about nuclei was natural radioactivity, as exhibited by the chemical elements of highest atomic and mass numbers.*

[699] Also, it is possible to form collimated *secondary* beams – composed of charged or neutral particles, whether stable or not – through in–flight radioactive decays of accelerator–produced collision products. Thus pion, kaon, neutrino, and other secondary particle beams are routinely created in accelerators and fission reactors, and used to induce further reactions.

In 1919 *Rutherford carried out some simple experiments to test whether the positively charged nucleus of a light element could be disrupted by bombardment with the alpha particles emitted from a radioactive preparation. This would require the alpha particles to penetrate the Coulomb field of repulsion exerted by the charge of the light nucleus, and it was known from scattering experiments that such penetration is unlikely. Nevertheless Rutherford was able to show that the element nitrogen, under alpha-particle bombardment, occasionally emitted protons, probably due to the reaction* $^{14}_{7}N + \ ^{4}_{2}He \rightarrow \ ^{1}_{1}H + \ ^{17}_{8}O$.

This hypothesis was verified by ***Patrick (Maynard Stuart) Blackett*** *(1897–1974, England), who in* 1925 *started his experiments with colliding atoms in a Wilson cloud chamber and took first photographs of nuclear reactions. Many basic discoveries were made with radioactive sources, but it was soon realized that particle accelerators would provide a much higher yield of nuclear reactions under more controllable conditions.*

Linear accelerators (linacs) accelerate charges along a straight line in an evacuated tube. In the case of electrostatic linac, the acceleration results from the force applied by a static electric field provided by a large potential difference. A particle of charge e *moving through a potential drop of* V *can gain kinetic energy* $E_k = eV$. *The high voltage (up to* 2 *MV) may be obtained by charging capacitors with a voltage multiplier circuit. In 1930, an international race was underway to produce the first artificial disintegration of the nucleus*[700]*. This goal was achieved at the Cavendish Laboratory, England: a huge voltage divider was built and used to linearly accelerate protons to kinetic energies of some* 800, 000 eV.

In 1932 ***John Douglas Cockroft***[701] *(1897–1967) and* ***Ernest T. S. Wal-***

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[700] Already in the late 1920s, study of the structure, excitation, and disintegration of the nucleus awaited new technology. Calculations by **George Gamow** (1904–1968) based on the new wave mechanics, indicated that protons of relatively low energy could penetrate barriers of light nuclei. This, in turn, encouraged Cockroft and Walton to achieve proton-induced disintegration of lithium nuclei with a conventional voltage multiplier.

[701] **Cockroft** studied at Manchester University under **Horace Lamb**. After serving in WWI he continued his studies at Cambridge and then worked under **Rutherford** in the Cavendish Laboratory, where he collaborated with **P. Kapitsa** on the production of intense magnetic fields at low temperatures. In 1928 he returned to work on the acceleration of protons by high voltages with E.T.S. Walton. He became professor of Natural Philosophy at Cambridge in 1939. In 1940 he was appointed Head of the Air Defense Research and Development Establishment and in 1946 he became Director of the Atomic Energy Research Establishment at Harwell.

ton[702] *(1903–1995) in England were first to accelerate protons by means of an electrostatic generator and demonstrate possibilities for production of many types of reactions in light nuclei, such as*

$$ {}^{1}_{1}H + {}^{7}_{3}Li \rightarrow 2\,{}^{4}_{2}He + Q $$

with Q being the heat of (nuclear) reaction. For this feat of 'atom splitting' they were awarded the Nobel Prize in Physics for 1951. This provided also the first experimental proof (1927) of ***Einstein****'s equation $E = mc^2$.*

The maximum energy achievable in an electrostatic linac, of order 1 MeV, *is limited by electrical discharge. The brute force acceleration by direct current (DC) high voltage was further improved with the development of the* ***Van de Graaff*** *accelerator (1933).*

The rectifier circuit of the Cockroft-Walton system was replaced with an electrostatic charging belt. The effects of sparking were reduced by insulating the accelerating tube with compressed gases. Van de Graaff accelerators can produce kinetic energies of about 10 MeV. *Since kinetic energies of particles from nuclear decays are typically a few* MeV, *the machine can produce kinetic energies that are larger than those available from natural radioactive decays.*

2. RF Linear Accelerators (up to 50 GeV; 1928 to 1990s)

To reach very high energies, particles must be accelerated, stepwise, many times over. Conceptually, the simplest system is a linear accelerator *which uses* alternating electric fields *of high magnitude to push particles along a straight line.*

To achieve this feat, particles pass through a line of hollow metal tubes enclosed in an evacuated cylinder. An alternating electric field of constant frequency [usually in the radio (RF) or microwave range] is timed so that a particle is pushed forward each time it goes through a gap between two of the metal tubes. Since the velocity increases at each gap, the cylinder lengths must increase also. This new concept of RF acceleration was introduced in 1928 by the Norwegian physicist ***Rolf Wideröe*** *(1902–1996, Norway and USA) [but originated with* ***Gustaf Ising*** *(1883–1960) in 1924].*

[702] **Ernest Thomas Sinton Walton** went to the Belfast Trinity College (1922) and later to the Cambridge Cavendish Laboratory (1927). He became professor of Natural and Experimental Philosophy at Trinity College, Belfast (1947).

In his pioneering apparatus Wideröe passed electrons across two gaps. The voltage across both gaps was made to oscillate such that the electrons arrive at each gap when the voltage difference across it is a maximum. The electrons are accelerated in each of the two gaps. Let the gaps be separated by a distance d *under voltage difference* $V_{\max}$. *Then the total kinetic energy of the electron after the first gap is* $E = mc^2 + eV_{\max}$, *its momentum* p *is given by* $pc = \sqrt{E^2 - (mc^2)^2}$ *and its velocity is* $\frac{v}{c} = \frac{pc}{E}$. *To be accelerated in both gaps, the voltage oscillation frequency must be* $f = \frac{v}{d}$. *Substituting* $d = 1$ m, $V_{max} = 100$ kV *one finds* $f = 160$ MHz *— a radio frequency. Since the energy gain from each gap is* $eV_{\max}$, *the electron's overall gain in kinetic energy is* $2eV_{\max}$. *There is no limit in principle to the number of acceleration gaps that can be added.*

The RF acceleration technique is useful in practice largely because of a phenomenon called longitudinal phase stability: *in practice electrons travel in* bunches *of finite length and are collectively accelerated. Consequently, all particles do not arrive at the acceleration gap at the same time. Choosing the accelerating voltage* V_0 *to be* smaller *than the maximum voltage, particles that arrive* early *are accelerated by a* smaller *voltage difference, while particles that arrive* late *are accelerated by a larger voltage. Only particles in the* center *of the bunch are accelerated by* V_0. *But both late and early arrivals are always* synchronized *with the center of the bunch.*

The key ingredient in later improvements of the RF acceleration technique was a novel high-frequency power source: the Klystron *amplifier (1937–1940) can provide several megawatts at a frequency of a few gigahertz. One refinement of the RF technique is to accelerate relativistic particles using electromagnetic waves in a* wave-guide cavity. *Acceleration occurs in such a cavity when the* group velocity *of the wave is equal to the* particle velocity. *The phase and group velocities of the electromagnetic waves are adjusted by appropriate geometric shaping of the wave-guide cavity.*

Theoretically a linac of any energy can be built. However, RF technology required the availability of large RF power sources for the acceleration, and enormous technical problems had to be solved before linear accelerators became useful machines.

The largest linac in the world, at Stanford University (Stanford Linear Accelerator Center – SLAC), is 3.2 km *long. It is capable of accelerating electrons to an energy of* 50GeV *(*50 *billion, or giga, electron volts). Stanford's linac is designed to accelerate two beams of particle bunches (electrons and positrons), which "surf" on regions of a traveling TM* 01 *waveguide mode with oppositely-oriented longitudinal electric fields. The* e^+ *and* e^- *beams are then separated magnetically, made to curve around, and collided.*

Early SLAC experiments (1960's and early 1970's), which established the quark model, involved colliding linac-produced e^- beams with stationary protons and other fixed–target nuclei (e.g. in a liquid hydrogen bubble chamber).

3. Cyclotron and Synchrocyclotron (1929–1952)

Uniform static magnetic bend *charged particles around circular arcs, while uniform electric fields* speed them up. *When a charged particle of mass m and charge e is moving with velocity v along a circular path of radius r perpendicular to the magnetic field $\boldsymbol{B}$, Newton's second law of motion becomes $evB = \frac{mv^2}{r}$, $p \equiv mv = eBr$; hence $v = \frac{eBr}{m}$. In practice, the tracks are not simple circles, even if $\boldsymbol{v}$ is perpendicular to $\boldsymbol{B}$, because the particle loses energy and momentum via its accelerator-caused EM radiation; actual tracks are inward–spiraling helices. The expression $p = eBr$ shows that a decreasing momentum leads to a decreasing radius about the magnetic–field axis.*

A 500-GeV RF linear accelerator would have to be about 75 km long, with enormous construction, maintenance and power challenges and costs. It thus makes more sense to magnetically guide the charged particles into traversals of multiple closed, circular paths, along which their kinetic energies can be gradually increased via the synchronized application of RF fields. This required an altogether new approach, which eventually came in 1929 with the invention of the cyclotron *by* ***E.O. Lawrence***. *It made use of the RF technique pioneered by* ***Rolf Wideröe*** *(1928).*

Lawrence's idea was to accelerate ions or electrons stepwise, using a uniform magnetic field to move the particles in a spiral path which repeatedly crosses an accelerating gap in the plane of the spiral. An applied, radio-frequency electric field, synchronized in phase with the orbital frequency of the ions, is used to accelerate the particles on each trip across the gap.

In 1930, Lawrence's students, ***Niels Edlefsen*** *(1893–1971)* and ***Milton Stanley Livingstone*** *(1905–1986) built the first cyclotron: it was essentially a linac wrapped into a tight spiral.*

But instead of many tubes, the machine has only two flat D-shaped hollow vacuum chambers (called dees), placed between the poles of a magnet (actually a powerful electromagnet). The magnetic field is constant and is perpendicular to the trajectory of the charged particle. This configuration makes the charged particle move in semicircular *paths within each dee. The dees are connected* *to a source of alternating voltage, is synchronized so that whenever a charged particle is moving from dee* 1 *to dee* 2, *the electric field between the oppositely charged dees is maximum in a direction that will accelerate the particle. One-half cycle later, the charged particle is moving in the opposite direction from dee* 2 *to dee* 1 *and the electric field has been reversed to again align itself with the momentary direction of the particle's motion, so as to give it a maximum increase in speed.*

When inside the metal dees, each charged particle is within a conductor where the electric field is zero. As the beam particles gain energy and momentum from its trips between the dees, their orbital radius will increase, according to $p = eBr$. *As the particles gain energy, they will spiral out toward the other peripheries of the accelerator dees, until they gain enough energy to exit the accelerator.*

If the maximum radius is R *and if* $E_k \ll m_0c^2$ *(*$m_0 =$ *charged particle's rest mass), then relativistic effects may be neglected and* $E_k \approx \frac{p_{\text{max}}^2}{2m_0} = \frac{e^2B^2R^2}{2m_0}$. *For example if* $B = 1.3$ Tesla *a singly–charged particle species, and* $R = 11$ cm, *(the 1932 Lawrence-Livingstone machine) we find the cyclotron frequency* $f_{\text{cyc}} = \frac{\omega_{\text{cyc}}}{2\pi} = 20$ MHz *and* $E_k = 1.0$ MeV *for a proton with* $m_0 = 1.67 \times 10^{-27}$ kg. *Its rest energy is* $m_0c^2 = 938$ MeV.

In a cyclotron, the time (T) *between acceleration burst is the time for the particle to make one-half of a revolution. For a particle traveling at a speed* v *in a circle of radius* r, $T = \frac{\pi r}{v}$. *Therefore, the frequency (*ω_{cyc}*) of the oscillating voltage is chosen to be* $\omega_{\text{cyc}} = \frac{\pi}{T} = \frac{v}{r}$, *so that the voltage changes sign each half-revolution. This frequency is called the* cyclotron frequency.

For a non-relativistic particle the orbital frequency *(*ω_{orb}*) is a constant (independent of particle energy),* $\omega_{\text{orb}} = \omega_{\text{cyc}} = \frac{v}{r} = \frac{veB}{p} = \frac{eB}{m_0}$. *Hence, the particles may be accelerated each time they cross the gap by selecting* $\omega_{\text{AC}} = \omega_{\text{cyc}}$.

If, however, the particle's kinetic energy begins to become an appreciable fraction of its rest energy, the mass begins to increase significantly, in accordance with STR. As a result, ω_{orb} *will decrease when the particle is speeded up at a constant* B *field and the particle traversals of the dees will fall out of phase with the accelerating electric field.*

Indeed, **Hans Bethe** *and* **M.E. Rose** *(1911–1967) showed that a fixed-frequency cyclotron of the Lawrence-Livingstone design has a practical upper* *energy limit of about* 25 *MeV* *(for protons), owing to the said relativistic mass increase*[703].

[703] For a *relativistic* particle, $\omega_{\rm orb}$ depends on the particle speed because the momentum has Lorentz factor $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$ via $p = \gamma m_0 v$; $\omega_{\rm orb} = \frac{veB}{p} = \frac{eB}{\gamma m_0}$. Thus, if $\omega_{\rm cyc}$ is kept fixed, particles cannot be accelerated to extreme relativistic energies because the γ factor becomes important.
This effect can be readily calculated; if

$$m_0 c^2(\gamma_1 - 1) = 2eV_{max}$$

is the energy gained per revolution, the increase of γ after the first revolution is $\delta\gamma = \gamma_1 - 1$.
This increase in γ causes orbital oscillation to be out of phase with the cyclotron oscillation by $(2\pi)\delta\gamma$ radians. In the next revolution, γ increases further by $(\delta\gamma)$ and the total phase difference of the orbital and cyclotron oscillations increases to $2\pi(1+2)\delta\gamma$. After N revolutions the phase difference is $2\pi\,\frac{1}{2}\,N(N+1)(\delta\gamma) \approx \pi N^2(\delta\gamma)$ When the phase difference is π, the particles are no longer accelerated because they arrive at the gap when the voltage has the wrong polarity. The maximum number of revolutions that the particles make while still being accelerated is

$$N_c = \sqrt{\frac{1}{\delta\gamma}} = \sqrt{\frac{m_0 c^2}{2eV_{max}}}\ ,$$

while the corresponding kinetic energy after N_c revolutions is
$E_k = N_c \cdot \delta\gamma \cdot m_0 c^2 = \sqrt{m_0 c^2 \cdot 2eV_{max}}$.
For $V_{max} = 150$ kV, the maximum kinetic energy for protons is about 17 MeV. For electrons, the maximum kinetic energy is much smaller, so that the conventional cyclotron is not useful for electron accelerations.
One may estimate the size of magnetic field and cyclotron frequency for any desired energy. Let p be the maximum momentum and E_k the maximum kinetic energy of the proton corresponding to a maximum radius R. Using the relations

$$E = m_0 c^2 + E_k, \qquad pc = \sqrt{E^2 - (m_0 c^2)^2} \approx \sqrt{2E_k m_0 c^2}$$

(for $E_k \ll m_0 c^2$), we find

$$B = \frac{1}{eR}\sqrt{2m_0 E_k}, \qquad v = \sqrt{\frac{2E_k}{m_0}}, \qquad \omega_{\rm cyc} = \frac{v}{R} = \frac{c}{R}\sqrt{\frac{2E_k}{m_0 c^2}}.$$

Thus for $E_k = 1$ MeV and $R = 22$ cm, one obtains $B = 0.65$ Tesla, $f_{\rm cyc} = \frac{\omega_c}{2\pi} \approx 10$ MHz.

A way to circumvent this limit was found (1945) independently by ***Edwin McMillan*** *(1907–1991) and* ***Vladimir Veksler*** *(1907–1966) who proposed to sustain the resonance by frequency-modulating the electric field. Under this arrangement, the RF oscillator that accelerates the particles around the dees is automatically adjusted to stay in step with the accelerated particle; as the particle gains mass, the RF frequency is lowered slightly to keep in phase with the slightly more sluggish orbital traversals*[704].

This means that the frequency of the field is varied synchronously with the inverse of the particle energy $(\gamma m_0 c^2)$*. Such a device is called a synchrocyclotron. Here there is no longer an advantage to having a very large voltage per turn as in a conventional cyclotron; the acceleration per revolution is usually chosen to be about* 10 KeV *per turn. Since there is a limit to how large of a magnetic field can be produced, the maximum energy achievable is limited by how big one can make the device: as the maximum energy of the synchrocyclotron increases, so must its size, for particles must have more space in which to spiral.*

4. Betatron and beta-synchrotron (up to 100 GeV; 1940–1947)

Electrons cannot be easily accelerated in a cyclotron, since the operating principle of this latter device is based in the constancy of the ratio e/m *of the electron. The relativistic variation of the mass of an electron with velocity becomes important at such low energies as to render the cyclotron useless for accelerating these particles (e.g. already at a kinetic energy of* 1 MeV,

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704 Other remedies (suggested in the pre-WWII years) took advantage of the focusing action of *inhomogeneous magnetic fields* at the outer edge of the vacuum chamber, e.g. curvature of magnetic lines of force create a magnetic force on particles away from the central plane directing them back into the central plane. In 1938 **Llewellyn Hilleth Thomas** (1903–1992) suggested that the magnetic field should be varied in *azimuth*, that is, around the perimeter of the orbit, as well as in the *radial* direction. This "AVF" (azimuthally varying field) concept has been realized in the *spiral ridge cyclotron*, where especially strong magnetic-field regions were produced by spiral sectors built into the pole face: the magnetic field strength increases radially everywhere inside the vacuum chamber and is designed to satisfy the fixed-frequency condition into the relativistic region. At the same time the strong azimuthal field variations provide extra focusing action to overcome the defocusing affects of the radial field variation.

the electron has about 3 times as much inertia as an electron at rest). Even conventional synchrocyclotrons cannot be adapted to make allowance for such large increases in mass.

A machine whose operation is independent of relativistic considerations was in order. This necessity was the mother of the invention of the betatron, *a special circular accelerator for electrons, by* **Donald William Kerst** *(1911–1993) in 1940. The operation of the betatron involves simultaneously using a static (DC) magnetic field to hold moving electrons in a stable orbit, while the field also has an AC component that oscillates rapidly. The changing flux results in a tangential electromotive force (EMF) that accelerates the electrons along their orbit.*

The particles are injected with a certain initial velocity distribution from an electron "gun", tangent to a horizontal circular orbit of fixed radius within a ring-shaped vacuum chamber known as the "doughnut". This chamber is placed between the poles of a magnet whose field lines are approximately at right angles to the plane of the orbit. The magnetic field has a static component, known as the 'guide field', and a time-varying AC component (at a fairly low frequency, e.g. 60 *Hz) obtained by applying an alternating current to an electromagnet.*

The guide field keeps the electrons in a circular path, while the time-varying field creates an induced electric field *which accelerates the electrons (over a certain range of each cycle) due to changing magnetic flux through the orbit (Faraday's law). The main condition for successful operation of the betatron is the confinement of the orbital radius within strict limits despite the large difference between the initial and final electron energies. This is achieved by arranging for the magnetic field to vary* radially *in a suitable manner.*[705]

[705] The static magnetic field exerts a centripetal force on the orbiting electrons. From the time of injection the magnetic field varies. According to Faraday's law this results in a tangential electric field E_t, causing the electrons to accelerate. The static 'guide field' must be made radially inhomogeneous in order to force the electron into an orbit of approximately fixed radius R in spite of its increasing momentum. To find the type of function $B = B(R)$ required, we write Faraday's law in the form

$$-\oint d\boldsymbol{\ell} \cdot \boldsymbol{E} = 2\pi R E_t = \frac{d\Phi}{dt},$$

where E_t is the clockwise tangental E-field component and Φ is the magnetic flux through the radius-R orbit, and the EMF line integral is taken in the anticlockwise sense as viewed from above the doughnut. Hence the anticlockwise tangential accelerating force on an electron in a betatron is

$$\frac{dp}{dt} = \frac{d}{dt}(mv) = \frac{e}{2\pi R}\frac{d\Phi}{dt} \ .$$

with p momentum and e the (positive) magnitude of an electron's charge.
The condition that will ensure that the electrons pursue a circular path of fixed radius R while being accelerated is

$$\frac{mv^2}{R} = evB \ ,$$

where $m = m_0/\sqrt{1 - v^2/c^2}$ is the electron's relativistic mass and $B = B(R)$ is the magnitude of the magnetic field at the circular path (of radius R). Hence $mv = eRB$; combining with the EMF acceleration equation yield $eR\frac{\partial}{\partial t}B = \frac{e}{2\pi R}\frac{d\Phi}{dt}$. Integration w.r.t. time (recall R is by design t-independent) yields $B = \frac{1}{2\pi R^2}\Phi = \frac{\Phi}{2A}$, with $A = \pi R^2$ the area enclosed within the orbit.
The condition for a constant orbit of radius R, then, is that the magnetic field at the position of the orbit should equal half the average field (magnetic flux per unit area) through the orbit. This result is achieved by shaping the pole pieces of the magnet so that the flux density decreases with distance from the symmetry axis.
More extensive analysis shows that this orbit is *stable*, i.e. if electrons are displaced away from it, restoring forces will arise to move them back to the orbit (causing so-called *betatron oscillations* and *phase oscillations*). To extract the electrons after their acceleration, the condition $B = \frac{\Phi}{2A}$ may be violated by momentarily adding or subtracting some auxiliary flux that will cause the electrons to *spiral* in or out. While at a fixed radius R, the electron accelerated at a way such that its Kinetic energy rises at a rate

$$\frac{dE_k}{dt} = v\frac{d}{dt}(mv) \approx \frac{2\pi R}{T}\frac{dp}{dt} = \frac{2\pi R}{T}|eE_k|$$

where T is the time the electron takes to complete its current traversal of the circle. Since $v(t)$ increases asymptotically toward c after many traversals, T *decreases* asymptotically toward $2\pi R/c$.
A typical betatron that accelerates electrons to 100 MeV might have an orbit of radius $R = 1$ m, and a magnetic field at that orbit which changes at a rate of $100 \ \frac{\text{Webers}}{\text{m}^2\cdot\text{sec}}$ during the acceleration process. Hence the induced emf is about 628 V. An electron in this betatron acquires 628 eV each time it traverses a complete circle; it must take about $160,000$ revolutions before it has an energy of 100 MeV! The non-relativistic limitation on particle speed that affects the cyclotron, does not affect the betatron, since the electrons in the latter are *accelerated continuously* and need not (indeed do not) make each turn in precisely the same period of time.

In the betatron, there are small displacements of the particles due to collisions with the dilute gas inside the beam pipe. The accelerator must be designed to have electromagnetic restoring forces so that particles slightly out of orbit get pulled back into orbit. To this end a more general treatment of the problem shows that, with radial field variations of the form $B \propto r^{-n}$ $(0 < n < 1)$, *the electron beam executes* oscillations *in both the radial direction ('phase oscillations') and in the direction perpendicular to the orbit ('betatron oscillations'). These oscillations about the mean orbital radius are important in synchrotrons as well as betatrons, because their amplitudes determines the minimum size of the vacuum chamber. The machines have to be designed in such a way that oscillations are of* small *amplitude throughout the acceleration cycle, and this involves a proper choice of the index* n.

The betatron principle applies to particles of all speeds, and it enables electrons to be brought rapidly from the injection energy to energies of several MeV. When the designed maximum energy is reached, the orbit is artificially collapsed, usually by arranging for part of the magnetic pole assembly to reach saturation. The electrons then spiral inwards *to strike a target, which commonly consists of a tungsten rod. At the end of each acceleration cycle the target emits an intense burst of X-rays tangentially to the electron beam, produced by the* bremsstrahlung *process*[706].

Since the electrons in a betatron reach speeds close to c *(the speed of light in vacuum) early in the acceleration cycle, they circulate at nearly constant intervals of time during the major part of the cycle. It is then a comparatively simple matter to supply extra energy to the beam by applying an electric field of fixed frequency, as is done in the* beta-synchrotron. *The beam passes repeatedly through a tuned cavity forming part of the doughnut, and RF power is switched on in this cavity at a suitable moment in the cycle. Many electrons enter the cavity when it is at positive potential relative to the grounded internal surface of the doughnut and these receive extra energy, mainly expressed as a mass increase (their increase in* speed *is negligible). The process is repeated for all electron bunches possessing phase stability, and the final energy greatly exceeds the betatron limit.*

One notable (and unavoidable) feature of electron accelerators of the circular type is the emission of electromagnetic radiation from the beam itself.

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[706] When the electron passes near a target nucleus, and is thus accelerated sideways and radiates X-rays or gamma rays. At the quantum level, *Quantum Electrodynamics* (QED) describes the process via a *Feynman diagram* involving a virtual photon exchange (between electron and nucleus) and the emission of a (real) photon by the electron. If the latter is virtual, it can decay into a real electron–positron pair – resulting in antimatter production, as routinely done at e^+e^- colliders such as SLAC and DESY.

Since the particles are continually accelerated toward the center of the orbit (even when not accelerated tangentially), they radiate according to nearly – classical laws, and an intense synchrotron radiation *is seen. The energy losses from this effect are not serious until the electron energy approaches the GeV scale. Electromagnetic theory predicts that the energy radiated per revolution*[707] *by a particle of rest mass* m_0, *charge* e, *velocity* $v \approx c$ *(ultra–relativistic* regime) *and orbital radius* r *is* $\Delta E \cong \frac{e^2}{3\epsilon_0 r}\left(\frac{E}{m_0c^2}\right)^4$, *with* ϵ_0 *the vacuum permittivity.*

Synchrotron radiation places a practical limit on the maximum energy of a betatron. A high energy electron machine is therefore made large $(\Delta E \propto \frac{1}{r})$ *in order to reduce synchrotron radiation.*

While merely a nuisance from the particle physicist's standpoint,, synchrotron radiation is nowadays being used as a well-controlled collimated X-ray source, with wide applications in medicine, chemistry, biology, material science, and condensed-matter physics. Thus old betatrons and storage rings (see below), no longer useful as nuclear or particle-physics probes, are outfitted with extra groups of magnets called "wigglers" to enhance *synchrotron*

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[707] Classical electromagnetic theory states that the power radiated by an orbiting charge e as observed in the laboratory Lorentz frame is $P = \frac{2}{3c^3}\frac{e^2a^2}{4\pi\epsilon_0}$, (Larmor's Law), where the acceleration is $a = \frac{v^2}{r}$ (valid for a nonrelativistic particle); while for an ultra-relativistic particle, the Lorentz–transformation laws of radiated fields and power mean that the lab–frame energy lost per revolution is $\approx \frac{4\pi\alpha}{3}\frac{\hbar c}{r}\gamma^4$, where $\gamma = (1-\beta^2)^{-1/2}$, $\alpha = \frac{e^2}{4\pi\hbar c}$ (we work in an electromagnetic unit system in which $\epsilon_0 = 1$). On account of their small rest mass, synchrotron radiation is much more important for electrons than for protons, and must be taken into account in the design of the betatron.
Assuming $r = 1$ m, $E_k = 300$ MeV, we have for an electron $(m_e = 0.511\ \frac{MeV}{c^2})$

$$\beta \approx 1; \qquad \gamma = \frac{E}{m_ec^2} = \frac{E_k + m_ec^2}{m_ec^2} \approx 588; \qquad \Delta E = 730 \text{ eV}$$

For the proton $(m = m_p)$ at the same kinetic energy, the exact version of the above synchrotron radiation energy–loss formula (valid for any particle velocity) yields:

$$\gamma = \frac{E}{m_pc^2} = 1.3; \qquad \beta \equiv \frac{v}{c} = \frac{\sqrt{\gamma^2-1}}{\gamma} = 0.64; \qquad \Delta E = 5\times 10^{-9} \text{ eV}$$

The classical approximation we have been using for the synchrotron radiation is valid as long as a typical radiated photon has energy $\hbar\omega_{ph} \ll E_k$ in the laboratory frame. This is usually the case for both electron and proton machines.

radiation by decreasing the radius of curvature of the electron beam. The word's largest electron synchrotron is the Large Electron Project (LEP) at CERN, which has a radius of about 4 km and energy of 100 GeV.

5. High Energy strong-focusing proton synchrotrons (1000 GeV; 1952–1971)

By the last decades of the 20^{th} century, accelerators could produce precisely timed and collimated, nearly 'mono–energetic' beams of charged particles with energies ranging from a few MeV to several hundred GeV. Intensities can be as high as 10^{16} particles/sec (of order 10^9 per bunch), and the beams can be concentrated onto targets of only a few square microns in area. The particles that are most often used as primary projectiles are protons and electrons.

Two tasks can be performed well only by accelerators, namely: the controlled production of new particles and new quantum states, and the investigation of the detailed structure of subatomic systems.

Only a very few stable and free particles exist in nature — the proton, the electron, neutrinos, the photon and the graviton[708]*. Only a limited number of nuclides are available in terrestrial matter, and they are usually in the ground state. To escape the narrow limitations of what is usually available, new, unstable particles and nuclear states must be produced artificially. To create a state of mass m, we need at least the energy $E = mc^2$; Very often, considerably more energy is required, as well as high beam fluxes (luminosities). The expression $p = eBr$, valid for any circular accelerator, shows us that increasing the momentum proportionally increases the trajectory radius, for a given magnetic field. Therefore, high-energy circular accelerators, such as the* 1-TeV *Tevatron accelerator (at the Batavia, Illinois Fermilab facility) require a radius of about a kilometer. To increase the energy to* 20 TeV *will require a radius of* 13.5 *km!*

Obviously, no laboratory can afford a magnet having square kilometers of pole areas. Therefore, huge-radius accelerators keep the curvature of the charged particles path constant by utilizing dipole magnets placed at inter-

[708] Of those, the proton may yet prove, and the three neutrino species oscillate into each other over sufficiently long travel distances through vacuum or matter.

vals[709] *along the path. Since* $B = \frac{mv}{er}$, B *must increase as both* m *and* v *increase, for given* r. *The actual radii of these behemoth accelerators are larger than that calculated from* $p_{\max} = eB_{\max}r$, *because dipole bending magnets alternate with quadrupole and other focusing magnets to keep the particle beam from spreading.*

Beam Optics

Beams of particles typically travel multiple traversals around circular accelerators, with total distances exceeding $10,000$ *km. Thus they must be* collimated *by suitable "lens systems" that operate on charged particle beams in analogy to optical lenses (this is true even in linacs, and "single pass colliders" such as the SLC at SLAC, Stanford). In light optics, the path of a monochromatic light ray through a system of thin lenses and prisms can be found easily by using geometrical optics:*

Consider, for instance, the combination of a positive (converging) and a negative (diverging) thin lens, with equal focal length f *and separated by a distance* $d \ll f$. *This combination is always focusing, with an overall focal length given by* $f_{\text{comb}} \approx \frac{f^2}{d}$.

In principle, one could use electric or magnetic lenses for the guidance of charged particle beams. The electric *field strength required for the effective focusing of high-energy particles is, however, impossibly high, and only* magnetic *elements are used*[710]. *The deflection of a monochromatic (mono-energetic) beam by a desired angle, or the selection of a beam of a desired momentum, is performed with a* dipole magnet *(the optical analog is a prism).*

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709 This method was first realized in the construction of the first proton synchrotron (1 GeV) in Birmingham, England by **Marcus Laurence Edwin Oliphant** (1901–2000; England and Australia) and his collaborators (1947): The orbital radius was kept nearly constant over a large part of the acceleration cycle, and magnetic guidance was used only in the later stages of acceleration. Thus, *ring-shaped magnets* represented the most efficient deployment of a given amount of magnetic material.

710 However, *electric quadrapole* electrostatic lenses are routinely used in *low*–energy (several eV) mass spectrometer, which are used to measure minute amounts of trace chemicals. In this case, the accelerated particle beams are complex mixture of ionized or excited molecules. Due to the low particle velocities involved ($v \ll c$), magnetic fields would not be effective at all, so AC quasi–electrostatic multipole fields are used to guide and separate the different molecular species.

The radius of curvature, ρ, is obtained by equating the centripetal force $\frac{mv^2}{\rho} = \gamma \frac{m_0 v^2}{\rho}$ to the Lorentz force evB, where v is the particle velocity in the plane of motion normal to the field B. This yields $\rho = \frac{p}{|e|B}$. Clearly, an ordinary dipole magnet bends particles in only one plane; and even in that plane, B must be non-uniform for focusing to be achieved (since ρ is otherwise uniform for a mono-energetic beam). Such considerations lead to the conclusion that no magnetic lens with properties analogous to that of an optical focusing lens can be designed.

However, ***E.D. Courant***, ***M.S. Livingstone***, ***H.S. Snyder***, *and* ***N. Christophilos*** *invented during 1950–1952 a system of magnets that can effect a 3-D focusing (known as* strong focusing[711]*) such that focusing occurs simultaneously in two planes perpendicular to each other. Strong focusing is especially important in a large-radius (high energy) machine where the particle path length is very long and where one needs to keep the magnet aperture as small as possible to reduce the costs of magnets.*

In the strong focusing technique, oscillations are greatly reduced by the introduction of magnets that alternately focus and defocus in the horizontal and vertical planes containing the local beam line (i.e one magnet focuses *in one plane and de-focuses in the other, then the next exchanges the roles of the two planes, and so on. The magnets can be combined to yield a net focusing effect.*

There are two ways of achieving this in practice: In the alternating gradient synchrotron *alternating magnet sections have the net effect similar to that of a converging lens system in optics. This feature results in a large saving of magnetic material (4000 tons as against 36,000 tons in the Dubna machine, say) but it also necessitates* extreme accuracy *in the fabrication and alignment of the magnetic sections.*

A second option is the quadrupole doublet system: *the magnet system is composed of two quadrupole magnets, where one is rotated around the central (beam) axis by 90° w.r.t. the other. This arrangement forms an essential element of all modern particle accelerators, and also of* beam lines *that lead from the accelerating machines to the experiment halls (where beams are collided with fixed targets or each other). With the aid of focusing devices, a beam can be transported over distances of many km with small intensity loss.*

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[711] In contradistinction to the *phase stability* arrangement that is named *weak focusing*. In weak focusing, fringe magnetic fields, applying vertical and radial restoring forces on a particle slightly out of orbit, are provided by *shaping* the magnetic fields.

In a proton synchrotron, the energy for fixed radius is limited by the field strength of the bending magnets. Thus, a very high energy machine is made with high-field magnets and a large radius.

6. Storage Rings and Colliding Beams Machines (100 GeV; 1961–1971)

In order to collect a large number of accelerated particles, each output beam pulse from an accelerator can be magnetically guided into a circular ring, where the beam energy can be further increased via RF fields. This ring, called a storage ring, *contains magnetic dipole structures to keep the particles moving in a circle.* Large numbers of particles per bunch, *and short bunches, increase the probability of desired (and sometimes rare) interactions occurring when two counter-rotating beams (of the same or different particle type) are magnetically deflected into* interaction regions, *usually spaced regularly around the storage-ring perimeter.*

The storage ring consists of a vacuum pipe passing through a ring of dipole magnets *that maintain a constant field, so that particles circulate* continuously; *other magnetic devices, also spaced along the ring, are responsible for focusing the beam or beams. The storage ring usually doubles as a synchrotron, so that particles are both accelerated and* stored *in the same machine.*

Two storage rings that intersect at one or more places can be used to study the collision of two stored beams. Particle-antiparticle collisions may be studied with a single storage ring, with particles and antiparticles circulating in opposite directions. (e.g. electrons and positrons – e^+e^- – or protons and antiprotons).

For a colliding beam machine to work, the particles must be accumulated and stored in stable orbits for durations on the scale of hours *— compared to the few* seconds *they typically spend in the synchrotron acceleration process (hence the name 'storage rings'). This requires an* extremely high vacuum *compared to that needed in a synchrotron. In a storage ring, the magnets are continuously operating, whereas in the normal operation of a synchrotron they are pulsed briefly every few seconds. Also, the beams in a storage ring must be* focused *to a small cross-sectional area and contain a large number of particles.*

A major increase in useful energy was achieved with the colliding beam machines: *Two colliding ultra-relativistic protons, for instance (or an electron and positron), each with energy $E^* \gg m_0c^2$, have the same* center-of-mass *energy as a particle of energy* $E = 2\frac{(E^*)^2}{m_0c^2}$ *colliding with a stationary counterpart, where* m_0 *is the proton or electron rest mass. Colliding beams are therefore used to achieve the highest achievable center-of-mass COM collision energies. In general, colliding beams of accelerated particles can yield more COM-frame energy (for instance, to create new particles) than collisions in which one particle is at rest. To understand this, we consider the classical Newtonian (non-relativistic) completely inelastic collision of a particle of mass* m *moving at speed* v *(relative to a fixed laboratory frame of reference) with an identical particle at rest in the same frame. Conservation of linear momentum requires that the two particles move together after the collision with a common velocity* $\frac{1}{2}v$*. The law of conservation of energy then implies that kinetic energy converted during the collision to other forms of energy (e.g. heat, inelastic deformations, elastic vibrations etc.) is*

$$\frac{1}{2}mv^2 - \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 \equiv \frac{1}{2}\left(\frac{1}{2}mv^2\right).$$

Thus, only one-half of the initial translational kinetic energy has been converted to other forms of energy.

On the other hand, if two particles with equal and opposite momenta collide and stick together, they are motionless after the collision and all the initial translational kinetic energy is converted into other kinds of energy. Consequently, if one wants to convert, say, 1 MeV *of kinetic energy, there are two options (classically):*

- *Hit a particle at rest with an equal-mass particle of* 2MeV *kinetic energy*
- *Effect a head-on collision of two equal-mass each with a kinetic energy of* 1/2 MeV *particles*

For the first option one needs a 2-MeV *accelerator, but for the second option one needs only a* $\frac{1}{2}$-MeV *accelerator, i.e. the counter-moving particles in the colliding beams each need only* $\frac{1}{4}$ *of the kinetic energy of the particles bombarding a fixed target in the classical case. In the relativistic case, this ratio decreases rapidly below* $\frac{1}{4}$ *as the required energy increases – in fact it is* $1/\gamma$*, with* γ *being the time–dilation factor for each colliding–beam particle. Therefore, existing accelerators have been altered, and new ones designed and built, to provide head-on collisions.*

7. Use of accelerators in basic and applied science

In about twenty laboratories all around the globe, scientists operate various linear and curved synchrotron–type accelerators, both to induce and study inter–particle collisions and to generate powerful and precise pulses of radiation in the X-ray range, using 3rd generation synchrotron X-ray sources. The primary charged–particle beams and various secondary beams (produced from them or independently) – the latter including, but not limited to, synchrotron X-rays are used at experimental stations to perform novel experiments in five disciplines:

- *Fundamental Physics:*

Accelerators are used to explore atomic nuclei and elementary particles, thereby allowing nuclear and particle physicists to identify new elements, particles and interactions. Machines exceeding 1 GeV *are used to study stable and unstable fundamental particles and resonances. Several hundred of these particles have been identified. High-energy physicists have discovered, and will doubtless continue to discover, rules and principles that govern the classification, inter-conversion, composition and interactions of subnuclear particles. Such schema are as useful to nuclear and particle physics as the periodic table of the chemical elements is to chemistry. They permit scientists to study violent particle collisions that mimic the state of the universe a fraction of a second after the Big Bang; and to elucidate the symmetries and invariances governing the fabric of the universe at the smallest spacetime scales, as well as study astrophysical objects and radiations. The collisions effected in such research involve primary charged–particle beams from accelerators, as well as: (charged and neutral) secondary beams from decays and collisions in accelerators and nuclear reactors; laser beams; specially prepared bulk–matter samples (used at targets); and cosmic rays and other (known and conjectured) particles from space, such as neutrinos, gamma rays, WIMPs etc.*

- *Material Science and Molecular Structure:*

Using synchrotron X-radiation from electron accelerators, studies are made of the atomic structure of natural and artificial polymers using a variety of X-ray techniques, such as spectroscopy, diffraction and imaging.

- *Chemistry:*

Chemistry relates atomic and molecular structure of matter with the transformations occurring when different substances are brought in contact, heated, *pressurized, subjected to electric fields, etc. To go beyond the results of static analysis before and after a chemical reaction, chemists apply synchrotron techniques in absorption spectroscopy and diffraction as well as in surface scattering in order to follow a chemical reaction as it happens. Here, X-rays yield new insight into the kinetics of chemical reactions by obtaining time-resolved structural information directly from the reaction zone (in situ investigation).*

The highly monochromatic, collimated and brief X-ray pulses available from synchrotron sources allow time-resolved studies of extremely small samples even for poorly interacting compounds. This approach has, for instance, improved our understanding of the polymerization process *of the superconducting polymer, polysulphur nitride. In another experiment, the use of hard X-rays penetrating into the bulk of matter revealed details of the* hydration process *of Portland cement that are of some technological importance.*

Modern synchrotron radiation sources contribute to the progress of chemical analysis, directly yielding information about the kinetics of chemical reactions whilst substantially extending the detection limits.

- *Medicine:*

Novel imaging techniques that overcome the limitations of conventional X-ray sources. The small spotsizes of synchrotron X-ray beams allow Computed Micro Tomography (CMT) to be carried out. This non-destructive measurement yields 3-dimensional reconstructions of human tissues with spatial resolution in the micron range (10^{-6} m). *In absorption mode, the technique was used to image the diminution of* bone structure *with aging, while in-line holography setups allowed the monitoring of* coronary artery *plaque and trombosis.*

Also, X-ray based clinical research is performed in the fields of medical imaging and radiation therapy.

- *Molecular Biology:*

The basic principle of molecular biology is that the biological functions of an organism are governed by large and complex molecules, such as proteins, enzymes, lipids, nucleic acids, etc. To understand how these macro-molecules govern the processes of life, the determination of their three-dimensional spatial structures is essential.

The technique used for this purpose is X-ray diffraction. Scientists routinely perform structural analyses of biological molecules (e.g. the nucleosome core particle or *the Blue-tongue virus) and develop new techniques to study structural modifications as a function of time (the biological movie, e.g. of carbon monoxide photo-desorption from myoglobin).*

Synchrotron-radiation studies at SSRL (Stanford Synchrotron Radiation Laboratory, part of SLAC) have yielded important information about the toxicity of various mercury compounds found in fish from polluted habitats.

1930 CE *Woods Hole Oceanographic Institution* chartered as a private, non-profit organization devoted to scientific study of the world's oceans. On Dec. 31, its ship *Atlantis* was launched from the ship-building yards in Copenhagen, and sailed July 02, 1931.

1930–1931 CE **Subrahmanyan Chandrasekhar** (1910–1995, India and U.S.A.). Distinguished astrophysicist. Shared the 1983 Nobel prize for physics for his work on late evolutionary stages of massive stars. He calculated that a *white dwarf* can exist only if its mass is less than 1.4 solar masses (*Chandrasekhar limit*).

Chandrasekhar was born in Lahore, India, a relative of Venkata Raman (who won the Nobel prize for physics in 1930). Educated at the University of Madras and Trinity College, Cambridge University. Joined the staff of the University of Chicago (1938) and became a professor there in 1952. He did important work on radiative energy transfer in stellar atmospheres and convection on the solar surface.

1930–1941 CE **Juliusz Pawel Schauder** (1899–1943, Poland). Mathematician. Made important contributions to topology and the links of topology with the theory of semilinear and quasilinear elliptic partial differential equations (PDE). Named after him are: *Schauder fixed-point theorem*[712]*; Leray-Schauder fixed point theorem; Riesz-Schauder theorem; Schauder basis* in Banach space and *Schauder energy inequality* of PDE of hyperbolic type.

Schauder was born in Lvov to Jewish parents. He was drafted into the Austro-Hungarian army (1917) and fought in Italy, where he was taken prisoner. He then joined the Polish army in France before returning to Lvov to begin his university studies (1919). After obtaining his doctorate (1923) he taught both in a secondary school and at the University of Lvov.

During 1932–1933 he studied at Leipzig and with **Hadamard** in Paris. In 1941 the German army entered Lvov and the systematic murder of its Jews began. Schauder sent pleas for help to **Hopf** and **Heisenberg** saying that he had many important results but no paper to write them on. He was shot by the Gestapo in September 1943.

1930–1949 CE **Kurt Friedrich Gödel** (1906–1978, Austria and U.S.A.). Mathematician and logician.

He discovered a theorem which states that within any rigid logical mathematical system rich enough to contain arithmetic, there either exist propositions that cannot be proved or disproved on the basis of the axioms within that system, or the system's basic axioms give rise to contradictions.[713]

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[712] Proved (1930) a major extension to Brouwer's (1910) fixed-point theorem for the case of infinite-dimensional topological space.

[713] The precise assumptions that underline Gödel's *incompleteness theorem* are these: if a formed system is (1) finitely specified, (2) large enough to include arithmetic, and (3) consistent — then it is incomplete.

Condition (1) means that there is a countable infinity of axioms, with a definite algorithmic procedure for listing them. We could not, for instance, choose our system to consist of all the true statements about arithmetic, because this collection cannot be finitely listed in this sense.

Condition (2) means that the formal system includes all the symbols and axioms used in arithmetic. The symbols are 0, ('zero'), S, ('successor of'), $+$, $\times$, and $=$. Thus, the number two is the successor of the successor of zero, written as the term $SS0$, and 'two plus two equals four' is expressed as $SS0 + SS0 = SSSS0$. It is instructive to see how these requirements might fail to be met. If we picked a theory that consisted of references to (and relations between) only the first ten non-negative numbers (0, 1, 2, 3, 4, 5, 6, 7, 8, 9), then Condition (2) fails and such a mini-arithmetic is complete. Arithmetic makes statements about individual numbers, or terms (like $SS0$, above). If a system does not have individual terms like this but, like Euclidean geometry, makes statements only

This "*Gödel's incompleteness theorem*" put an end to **Hilbert**'s dream of providing, for all of mathematics, a formal axiomatization that is both complete and absolutely consistent.

This discovery was revolutionary in a negative fashion, similar to **Galois**' proof of the impossibility of finite solvability for polynomials of degree higher than 4, and **Hamilton**'s relinquishing of commutative multiplication (for quaternions). But Gödel's theorem is even more 'destructive', for it undermines the very foundation of all pure mathematics[714]. Gödel's so-called

about points, circles, lines etc., then it cannot satisfy Condition (2). Accordingly, as **Alfred Tarski** first showed, Euclidean geometry is complete. There is nothing magical about the flat, Euclidean nature of the geometry either: the non-Euclidean geometries on curved surfaces are also complete. Similarly, if we had a logical theory dealing with numbers that used only the concept of 'greater than' without referring to any specific numbers, then it would be complete: we can determine the truth or falsity of any statement about numbers involving the 'greater than' relationship. The simplest system of formalized mathematical logic – involving relations such as "p and (q or s)" (the symbols referring to *True* or *False* statements) and axioms such as "(p and q) implies p"— has likewise been proven to be complete.

Another example of a system that is smaller than arithmetic is arithmetic without the multiplication, $\times$, operation. This is called *Presburger arithmetic* (the full arithmetic is called *Peano arithmetic*, after the mathematician who first expressed it axiomatically, in 1889). At first this sounds strange. In our everyday encounters with multiplication it is nothing more than a shorthand way of doing addition (for example, $2+2+2+2+2+2 = 2 \times 6$). But in the full logical system of arithmetic, in the presence of logical quantifiers such as 'there exists' or 'for any', multiplication permits constructions which are not merely equivalent to a succession of additions.

Gödel showed, as part of his doctoral thesis work, that *Presburger arithmetic* (**M. Presburger**, 1929) is complete: all statements about the addition of natural numbers can be proved or disproved; all truths can be reached from the axioms. Similarly, if we create another truncated version of arithmetic which does not have addition but retains multiplication, this is also complete. It is only when addition and multiplication are simultaneously present that incompleteness emerges. Extending the system further by adding extra operations (such as exponentiation) to the repertoire of basic operations, makes no difference. Incompleteness remains, but no intrinsically new form of it is found. Arithmetic is the watershed in complexity in mathematics.

714 On the other hand, it seems to safeguard the role of intuition in mathematics, since the theorem shows that doing mathematics cannot be reduced to mechanical symbol-manipulation.

"meta–mathematical" result was made possible by a novel method, discovered by him, of mapping statements *about* the mathematical axiomatic system into arithmetic statements *inside* that system. Gödel's theorem, with the essential incompleteness of knowledge it entails, has been likened to Heisenberg's uncertainty principle in physics[715].

Gödel's monumental demonstration that formal systems of mathematics have limits, gradually infiltrated the way in which philosophers and scientists viewed the world and our quest to understand it. Superficially, it appears that all human investigations of the Universe must be limited. Science is based on mathematics; mathematics cannot discover all truths; therefore science cannot discover all truths. Indeed, some scientists acknowledge that Gödel's incompleteness theorem places limits of our ability to discover the truths of mathematics and science, and therefore acts as a fundamental barrier to human understanding of the universe.

In 1938 Gödel demonstrated that one can safely assume **Cantor**'s continuum hypothesis as an *additional postulate* in set theory, i.e. he proved that the continuum hypothesis is consistent with the **Zermelo-Fraenkel** axioms.

In 1949, Gödel discovered a peculiar exact solution of Einstein's gravitational field equations[716], which leads to a model dust-filled universe that is homogeneous but anisotropic. In Gödel's universe, space-time appears normal locally, but in any sufficiently large region of space-time there exist closed timelike curves, which allow an event to affect its own causal past. Thus, in his model universe, a person could kill one of his ancestors, in time to prevent his own eventual birth! A number of such space-time solutions have since been found, and physicists are still unsure as to their physical significance. This circumstance highlights our as–yet incomplete understanding of gravity, and especially how it should be unified with quantum mechanics.

Gödel collaborated for some time with Albert Einstein in an effort to establish a unified field theory.

Gödel was born in Brünn (now Brno), Austria-Hungary, the second of the two children of Rudolf and Marianne Gödel. His father was a director of a textile factory and his mother descended from a family of weavers. Neither Kurt nor his brother enrolled in optional courses in the Czech language, and gave

[715] One of Hilbert's young students, **Gerhard Gentzen** (1900–1945) showed (1940) that it was possible to circumvent Gödel's conclusion and deduce all the truths of arithmetic – *provided* one allows a more powerful form of mathematical induction (*transfinite induction*) based upon **Cantor**'s ordinals.

[716] Gödel's universe: $ds^2 = -dt^2 + dx^2 - \frac{1}{2}e^{2\sqrt{2}\omega x}dy^2 + dz^2 - 2e^{\sqrt{2}\omega x}dydt$, where $-\Lambda = \omega^2 = 4\pi\rho$; ρ is the proper mass density of the pressureless dust, Λ the cosmological constant, and the natural ($G = C = 1$) system of units is used.

up their Czech citizenship after WWI becoming students at the University of Vienna.

Gödel enrolled there in 1924, intending to major in physics, but switched into mathematics in 1926, receiving his doctorate degree in 1930. He was a member of the faculty of the University of Vienna from 1930. He emigrated to the United States in 1940, and from 1953 served as a professor at the Institute of Advanced Study, Princeton, NJ.

Scientists on the consequences of Gödel's Theorem

"One may speculate that undecidability is common in all but the most trivial physical theories. Even simply formulated problems in theoretical physics may be found to be provably insoluble."

(Stephen Wolfram)

* * *

"Gödel proved that the world of pure mathematics is inexhaustible; no finite set of axioms and rules of inference can ever encompass the whole of mathematics; given any set of axioms, we can find meaningful mathematical questions which the axioms leave unanswered. I hope that an analogous situation exists in the physical world. If my view of the future is correct, it means that the world of physics and astronomy is also inexhaustible; no matter how far we go into the future, there will always be new things happening, new information coming in, new worlds to explore, a constantly expanding domain of life, consciousness, and memory."

(Freeman Dyson)

* * *

"Clearly then no scientific cosmology, which of necessity must be highly mathematical, can have its proof of consistency within itself as far as mathematics goes. In the absence of such consistency, all mathematical models, all theories of elementary particles, including the theory of quarks and gluons... fall inherently short of being that theory which shows in virtue of its a priori *truth that the world can only be what it is and nothing else. This is true even if the theory happened to account with perfect accuracy for all phenomena of the physical world known at a particular time.*

It seems on the strength of Gödel's theorem that the ultimate foundations of the bold symbolic constructions of mathematical physics will remain embedded forever in that deeper level of thinking characterized both by the wisdom and by the haziness of analogies and intuitions. For the speculative physicist this implies that there are limits to the precision of certainty, that even in the pure thinking of theoretical physics there is a boundary. An integral part of this boundary is the scientist himself, as a thinker."

(Stanley Jaki)

* * *

"It is by no means obvious that Gödel places any straightforward limit upon the overall scope of physics to understand the nature of the Universe just because physics makes use of mathematics. The mathematics that Nature makes use of may be smaller and simpler than is needed for incompleteness and undecidability to rear their heads. Yet, within science, it is the smaller individual problems that are at the mercy of computational intractability and undecidability."

(John D. Barrow, "*Impossibility*", 1999)

1930–1956 CE **Witold Hurewicz** (1904–1956, Holland and USA). Mathematician. Contributed to set theory and topology:

His main topics of research were: *topological embedding* of separable metric spaces into compact spaces of the same finite dimension (1930); *dimension theory* (1941) which presents the theory of dimension for separable metric spaces;

Discovered of *higher homotopy groups* (1935–1936) and *exact sequences* (1941). His work led to *homological algebra*.

Hurewicz was born in Lodz, Poland to Jewish parents. He was educated at the Universities of *Warsaw*, and *Vienna* (Ph.D. 1926) and held positions at the Universities of *Amsterdam* (1928–1936), *Princeton*, *North Carolina* (1939–1944), and *M.I.T.* (1945–1956).

Hurewicz died falling off a ziggurat (a Mexican pyramid) at Uxmal, after attending a conference outing at the International Symposium on algebraic topology in Mexico.

1930–1960 CE **Pier Luigi Nervi** (1891–1979, Italy). Engineer and architect. Influenced modern architecture through his imaginative use of *reinforced-concrete* structures that combined economy with great clear span structures. He experimented continually to find ways to push concrete construction techniques to new limits.

Nervi believed that the process of creating form was identical for both the technician and the artist.

During 1947–1961 Nervi was a professor of technology and construction in the faculty of architecture at Rome University.

Condensed-Matter Physics[717] — *from antiquity to quantum electronics*

Baryonic matter in the bulk[718] — *in macroscopic quantities that can be directly sensed by us — is an aggregate of a very large number of atoms. On earth, it appears in three basic states: gas, liquid and solid. In gases, the average distance between molecules is much greater than the molecular sizes, and intermolecular forces are on average much weaker than the forces which hold the atoms together inside molecules. Thus in gases, the molecules largely retain their individuality.*

In a solid, atoms and molecules are tightly packed and held in more or less fixed relative positions by electromagnetic forces. Consequently, the shape and volume of a solid remain essentially constant under a range of physical conditions characteristic of each particular specimen.

A liquid has intermediate properties; in it inter-molecular interactions are important, and position of neighboring molecules are highly correlated, but they can slide past each other, and far-away molecules are not correlated in position or quantum state.

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[717] For further reading, see:

- Chaikin, P.M. and T.C. Lubensky, *Principles of Condensed Matter Physics*, Cambridge University Press, 1997, 699 pp.
- Ziman, J.M., *Principles of the Theory of Solids*, Cambridge University Press, 2^{nd} edition, 1972, 435 pp.
- Holden, A., *The Nature of Solids*, Dover Publications: New York, 1992, 241 pp.
- Epifanov, G.I., *Solid State Physics*, Mir Publications: Moscow, 1979, 333 pp.
- Harnwell, G.P. and W.E. Stephens, *Atomic Physics*, McGraw-Hill, 1955, 401 pp.

[718] "Baryonic matter" is made up of protons, (optionally) neutrons, and electrons. Most baryonic matter in the observable universe is in the form of neutral or ionized atoms and molecules – including all known matter in our solar system. Under extreme conditions (Big Bang, while dwarf stars, neutron stars, accelerator experiments) baryonic matter can *break down* into quarks, leptons and other particles, or aggregate into star–sized atoms or even nuclei. Cosmology indicates that only a small fraction of matter in the universe is baryonic.

Solid state physics is the area of quantum physics which is concerned with the mechanical, thermal, electrical, magnetic and optical properties of solid matter; *condensed matter* physics is more general and covers liquids as well. Sometimes the distinctions are blurred: glass is really a kind of liquid, and the degenerate electron gases in a metal or white dwarf star, or the neutrons in a neutron star are also liquids.

In most solids, each atom (or molecule) is mostly affected only by its close neighbors. Moreover, the structure of solids usually exhibits (at least mesoscopically, and excluding impurities, dislocations, interstitial atoms, etc.) a regularity or periodicity due to a repetitive, three dimensional arrangement of atoms or ions, known as a *crystal lattice*. Therefore, to understand the structure of a solid, it is often necessary to study only the basic unit, or cell, of the lattice.

In a sense, the solid can be regarded as a large molecule, the forces between atoms being due to interaction between atomic electrons, and the structure of the solid being determined by those arrangements of nuclei and electrons which yields a *quantum-mechanically stable system*.[719]

The ways in which atoms are arranged in solid materials are determined primarily by the strength and directionality of the interatomic bonds. Qualitatively, we can understand why an atomic bond is strong or weak, directional or non-directional, from a knowledge of the *energetics* of the bonding electrons and their orbital shapes with respect to the positively charged ion cores. The more negative the bonding energy, the stronger the bond. Thus, a pronounced lowering of the *electron energies* results in a strong, or primary bond; a slight lowering of the energy results in a weaker, or secondary, bond.

Crystalline solids are classified according to five *primary* types of bonding: *molecular*, *ionic*, *covalent*, *metallic* and *pure semiconducting*. One is distinguished from another by the ways in which the bonding electrons are localized in space. All *secondary* bonding may be viewed in terms of weak *dipole* interactions.

Molecular solids (and liquids) consist of stable molecules or atoms that are bound by weak *Van der Waals attraction* such that they retain much of their individuality when brought into close proximity. The physical mechanism involved here is an attraction between *electric dipoles* [because of the fluctuating quantum-mechanical behavior of the electrons in a molecule, all molecules have a fluctuating electric dipole moment, even though for many of them, symmetry considerations require that it fluctuate about an average value of zero; the *interaction energy* results from induced correlations between

[719] Certain solids such as diamond really *are* one giant molecule — hence their strength.

moments of two molecules]. The resulting attraction energy is of the order of 10^{-2} eV, and the forces generally vary with the inverse 7^{th} powers of the intermolecular separation. Many organic compounds, inert gases and ordinary diatomic gases such as I_2, O_2, N_2, H_2 form molecular solids in the solid state. The weak bonding makes molecular solids easy to deform, and the absence of free electrons makes them very poor conductors of heat and electricity.

Ionic solids, such as NaCl (rock salt), consist of close regular 3-dimensional array of alternating positive and negative ions having a lower energy (enthalpy) than the separated ions. The structure is stable because the binding energy of the entire crystal due to the net electrostatic attraction exceeds the sum of the binding energies of the individual, well-separated ionic molecules. Ionic binding in solids is not directional because spherically symmetrical closed-shell ions are involved. Hence the ions are arranged like close-packed spheres with various packing schemes. The actual crystal geometry depends on which arrangement minimizes the energy, and this in turn depends principally on the relative sizes of the ions involved.

In the absence of free electrons to carry energy or charge from one part of the solid to another, such solids are poor conductors of heat and electricity. However, because of the strong electrostatic forces between the ions, ionic solids are usually hard and have high melting points, although they can be solvated – and often in almost ionic form – by polar solvent liquids such as water. Lattice vibrations can be excited by far-infrared energies, so that ionic solids show strong absorption properties in that spectral region, but are mostly transparent to visible radiation.

In covalent solids, atoms are bound by shared valence electrons – as in covalent bonds within molecules. The bonds are directional and determine the geometrical arrangements of atoms in the crystal structure. The rigidity of their electronic structure makes covalent solids hard and difficult to deform, and it accounts for their high melting points. Because there are no free electrons, covalent solids are not good heat or electrical conductors, either. Most covalent solids (diamond excepted) absorb in the visible spectrum and are therefore opaque.

*A metallic solid is a regular lattice of spherically symmetrical positive ions arranged like close-packed spheres, through which the conduction and valence electrons – those contributed from the outer shells of the individual atoms – move (**Bloch**, 1928). This type of solid exhibits a bond that is a limiting case of covalent binding, in which electrons are shared by all the ions in the crystal. As a result, metals are good electric and heat conductors, strong, yet ductile, and strongly reflect in the visible and radio parts of the spectrum.*

A semiconductor is like a metal, except the number of shared valence electrons per atom exactly fills the solid's valence band, so its conduction *band is empty. The energy gap between these bands, plus the Pauli exclusion principle, thus render electronical and thermal conduction very dependent upon impurity content.*

There are solids whose binding is a mixture of the above principal types. Such is the hydrogen bond, where a single hydrogen atom appears to be bonded to two distinct atoms, although (because neutral hydrogen has only one electron) it should form a covalent bond with only one other atom. The hydrogen bond is particularly important because its energy is only of order 0.1 eV ($\sim$ 6 *kcal/mole) and also because the hydroxyl group occurs so frequently in most biological systems. It is especially important in molecular genetics by virtue of controlling in part the possible pairings between the two strands of the DNA molecule (***F.H.C. Crick** *and* **J.D. Watson***, 1954).*

It is believed that the hydrogen bond is largely ionic in character. In the extreme ionic form the hydrogen atom loses its electron to another atom in the molecule and the bare proton forms the hydrogen bond. The small size to the proton permits only 2 *nearest-neighbor atoms, because the atoms adjacent to the proton are so close that more than* 2 *of them would get in each other's way. Thus the hydrogen bond connects only two atoms (belonging to two distinct molecules).*

Together with electrostatic attraction between the electric dipole moments of individual polar covalent–bonded molecules, the hydrogen bond is responsible for the striking physical properties of water and ice.

Departures from ideal crystal structure, so-called lattice imperfections, lead to many properties of solids which have practical consequences.

The history of man's efforts to unveil the 'mystery' of the solid state of matter can be divided into 5 *principal periods:*

(1) DESCRIPTION AND CLASSIFICATION, 315 BCE–ca 1600 CE;

(2) RECOGNITION OF INTERNAL GEOMETRICAL FEATURES, 1665–1800;

(3) OPTICAL, ELECTROMAGNETIC AND CHEMICAL PROPERTIES, 1801–1894;

(4) CLASSICAL ATOMIC MODELS, 1895–1931;

(5) QUANTUM-ELECTRONIC MODELS, 1930–present. We have grouped the first four stages under the single heading of classical studies.

A. Classical Studies (315 *BCE*–1931 CE)

Owing to their numerous applications for useful and decorative purposes, minerals[720] *(especially crystals*[721] *and gems) have attracted the attention of mankind for several thousands of years. Egyptian paintings of* 5000 *years ago show that minerals were used in weapons and jewelry, and in religious ceremonies: The breastplate of the high priests of the ancient Israelites were studded with four rows of gemstones (ca 1200 BCE).*

The oldest existing treatise on minerals is that written about 315 *BCE by* ***Theophrastos****, a Greek philosopher.* ***Pliny the Elder*** *of Rome, in his Historia Naturalis (77 CE), wrote about metals, ores, stones and gems.*

Other early writings about minerals were produced by the German scientists ***Albertus Magnus*** *(De Mineralibus, 1262) and* ***Georgius Agricola*** *(De Re Metallica, 1556).* ***Andreas Libavius*** *(1597), pointed out that salts present in mineral waters crystallize upon evaporation.*

The first important step in the study of crystals was made by ***Robert Hooke*** *in his Micrographia (1665), where he noticed the regularity of the minute quartz crystals found lining the cavities of flints. Independently, the Danish naturalist and physician* ***Nicolaus Steno*** *(Niels Stenson, 1631–1686) described various gems, minerals and fossils enclosed within solid rocks in his book De solido intra solidum naturaliter contento (Florence, 1669). He was first to discover that the angles between the faces of quartz crystals were the same even though the crystals had different shapes. At about the same time,* ***Erasmus Bartholinus*** *(1625–1692, Denmark, 1669) and* ***Christiaan Huygens*** *(1690) studied double refraction of calcite and Iceland-spar crystals.*

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720 Natural, inorganic, solid constituents of the earth's crust. Most minerals also have definite crystalline forms.

Common minerals and organic gems are: *Sapphire*, *Ruby* (Al_2O_3 — hexagonal); *Topaz* ($Al_2F_2SiO_4$ — orthorombic); *Emerald* [$Al_2Be_3(SiO_3)_6$ — hexagonal]; *Opal* ($SiO_2{\cdot}nH_2O$ — amorphous); *Amethyst* (SiO_2 — hexagonal); *Turquoise* [$CuAl_6\{(OH)_2PO_4\}_4{\cdot}4H_2O$ — triclinic]; *Malachite* [$Cu_2CO_3(OH)_2$ — monoclinic]; *Azurite* [$Cu_3(CO_3)_2(OH)_2$ — monoclinic]; *Amber* [$C_{12}H_{20}O$ — amorphous]; *Diamond* (C — cubic); *Lapis Lazuli* [$Na_8(AlSiO_4)_6S_2$ — cubic]; *Alabaster* ($CaSO_4{\cdot}2H_2O$); *Ivory* [$Ca_3(PO_4)_2$]; *Pearl* [84–92% $CaCO_3$ + organic substances + water]; *Coral* (mainly $CaCO_3$ — hexagonal).

721 The word *crystal* is of Greek origin, meaning *clear ice*. The name was also applied to the clear transparent quartz (rock crystal) from the Alps, under the belief that it had been formed from water by intense cold. It was not until the 17^{th} century that the name was extended to other bodies. Quartz crystals from crowns have been preserved since 768 CE in Japan.

In 1695, ***Anton van Leeuwenhoek*** *(1632–1723) observed under the microscope that different forms of crystals grew from solutions of different salts. A French scientist,* ***Jean-Baptist Louis Romé de l'Isle*** *(1736–1790) suggested in* 1772 *that Steno's discovery could be explained only if the crystals were composed of identical units stacked together in a regular way. He described the process of crystallization, and classified crystals into six groups according to their symmetry properties. About* 1780, *chemists began to develop correct ideas about the nature of chemical elements and other substances. These ideas helped scientists understand the chemical makeup of minerals, but did not remove the mystery about crystal shape and internal structure.*

The science of crystallography *was founded in* 1801 *by* ***René Just Haüy*** *(1743–1821, France) in his* 4*-volume treatise* Traité de cristallographie. *There he showed that the arrangement of identical particles in a* 3*-dimensional periodic array could account for the fact that the index numbers of the direction of all faces of a crystal are exact integers* (law of rational indices). *During the* 19^{th} *century, most of the minerals known today were described, optically studied and chemically analyzed.*

The discovery of X*-rays in* 1895 *provided physicists with the tool they needed to study crystal structure, but it was not until June* 8, 1912 *that* ***Max von Laue*** *and his assistants developed an elementary theory of diffraction of* X*-rays by a periodic array of scatterers. They applied it to explain experimental observations of* X*-ray diffraction in crystals, and thus obtained clues to their internal structure. [At that time, scientists did not fully understand* either X-rays or *crystals, and the work of von Laue demonstrated both the wave nature of* X*-rays* and *that crystals are composed of a periodic array of atoms.] The first determinations of crystal structures by* X*-ray diffraction analysis were reported by* ***W.L. Bragg*** *in* 1913.

While all this was going on, the pre-quantum physics of the solid state was advancing along other avenues as well: In 1900, *the physicist* ***Paul*** *(****Karl Ludwig****)* ***Drude*** *(1863–1906, Germany) suggested for the first time that electrical and thermal properties of metals might be correlated, by assuming that metals contain* free electrons *in thermal equilibrium with atoms in the solid. In his model, he introduced the concept of* mean free path *for collision of the free electrons*[722] *and furnished a classical microscopic explanation of*

[722] In a typical metal such as copper or silver, the atoms are arranged in a systematic array to form a crystal. The atoms are in such close proximity that the outer, loosely bound electrons are attracted to numerous neighboring nuclei and, therefore, are not closely associated with any one nucleus. These conduction electrons are visualized as being free to wander through the crystal structure, or lattice. At absolute zero temperature, the conduction electrons encounter no

opposition to motion and the resistance is zero (assuming no crystal defects); this fact is understood in quantum mechanics, but has no *classical* explanation. At nonzero temperatures, the electron-deficient atoms (ions) possess kinetic energy in the form of vibration about equilibrium positions in the lattice; this vibrational energy is measured by temperature. There is a continual interchange of energy between the vibrating ions and the free electrons in the form of elastic and inelastic collisions. The resulting electron motion is random (for both thermal *and* quantum reasons), there is no net motion, and the net current is zero. Quantum – mechanically, the lattice vibrations are quantized into "quasiparticles" called *phonons* (sound quanta), absent at $0\,°$K, and in a perfect crystal electrons can only collide with phonons – they do not 'see' individual lattice ions. If a uniform external electric field $\boldsymbol{E}$ is applied, the electrons are accelerated; superimposed on the rapid random motion, there is a small component of *drift* velocity along $\boldsymbol{E}$. Classically, upon each inelastic collision with an ion, the electron loses most of its kinetic energy: it then accelerates again, gains a velocity component along $\boldsymbol{E}$, and loses its energy at the next inelastic collision. The time between collisions is determined by the random velocity component – typically much larger than the drift term – and the length of the mean free path. On average, the electron gains a directed drift velocity that is directly proportional to $\boldsymbol{E}$.

Thus, the classical theory of metals, as expounded by Drude, assumes a model in which some of the electrons are detached from their parent atoms and become free to move in the material in much the same way as an *electron gas*. When an electric field $\boldsymbol{E}$ is applied, these free electrons drift with an average velocity $\boldsymbol{v}_d$ and give rise to a current density $\boldsymbol{J} = -en\boldsymbol{v}_d$ where n is the number of free electrons per unit volume, each of charge $\{-e\}$. In this classical electron-gas model of conductors, an electron collides with the metal ions, and between collisions moves in accordance with the force produced by $\boldsymbol{E}$, such that the electron's velocity at time t *after the most recent collision* will be $\boldsymbol{v} = \boldsymbol{v}_0 - \left(\frac{e}{m_e}\right)\boldsymbol{E}t$. Here m_e is the electron's mass; $\boldsymbol{v}_d$ is then the *average* drift velocity between collisions. Since $\boldsymbol{v}_0$ is distributed randomly in direction, $\langle \boldsymbol{v}_0 \rangle = 0$ and $\boldsymbol{v}_d = \langle \boldsymbol{v} \rangle = -\frac{e\tau}{2m_e}\boldsymbol{E}$, where angular brackets indicate averaging over time and electrons, τ is the 'mean free time' between collisions, and the factor $\frac{1}{2}$ arises from the 'saw-tooth' shape of $\boldsymbol{v}(t)$. Substituting the expression for $\boldsymbol{v}_d$ into the equation for $\boldsymbol{J}$, one obtains $\boldsymbol{J} = \sigma\boldsymbol{E}$, where σ is known as the *conductivity* and the *resistivity* $\rho = \frac{1}{\sigma} = \frac{2m_e}{e^2 n\tau}$ is inversely proportional to τ, the average time between collisions (in copper $\tau = 5.4 \times 10^{-14}$ sec at room temperature, $T = 273\,°$K).

Many metals obey the above linear relationship between $\boldsymbol{J}$ and $\boldsymbol{E}$. For copper with an atomic weight of 63.6 and a density of $8.9\ \frac{\text{g}}{\text{cm}^3}$, by Avogadro's law, the density of atoms per mole cu is [3] is

$$n_a = \frac{6.022 \times 10^{23}\ \text{atoms/g–mole cu} \times 8.9\ \text{g/cm}^3}{63.6\ \text{g/g–mole cu}} = 8.43 \times 10^{22}\ electrons/cm^3\ .$$

Assuming there is one free electron per atom, and considering a current of $I = 4$

Ohm's law, still taught today (although supplanted by the quantum field theory). The foundations of the classical theory of ionic crystals were laid by **Erwin Madelung** *(1881–1972, Germany) in* 1909 *and by* **Max Born** *in* 1910. *In these crystals (e.g. rock salt: NaCl) the binding depends on the Coulomb electrostatic attraction between the singly charged ions (e.g.* Na^+ *and* Cl^-*), and equilibrium is achieved by a counteracting short-range repulsive force.*

In a classical model of the ideal rock salt crystal, the effective potential of the forces acting between the Na^+ *and* Cl^- *ions is* $U(r) = -0.29\frac{e^2}{r} + \frac{c}{r^9}$. *Here* c *is a constant which is determined from the equilibrium condition* $\left[\left(\frac{\partial U}{\partial r}\right)_{r=r_0} = 0\right]$. *The interaction potential then assumes the form*

$$U(r) = -0.29\frac{e^2}{r}\left[1 - \frac{1}{9}\left(\frac{r_0}{r}\right)^8\right].$$

From this, one evaluates the lattice energy (the total energy per mole of the crystal, equals to minus the energy per mole liberated upon forming the

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Ampere in a copper conductor with a cross-sectional area of $A = 1\ \text{mm}^2$, we find $v_d = \frac{I/A}{n_a e} = 0.03\ \frac{\text{cm}}{\text{sec}}$.

The average drift velocity in a good conductor is thus very low compared to the random thermal electron velocities which are of the order of 10^5 m/sec at room temperatures. As temperature increases, random thermal motion increases, the time between energy-robbing collisions decreases, mobility decreases, and therefore conductivity decreases. It is characteristic of metal conductors that resistance increases with temperature.

The relation $\sigma = \frac{ne^2\tau}{2m_e}$ can be put into a more useful form by defining a measurable quantity, the *mobility* μ, given by the ratio of the drift velocity to the applied field, i.e. $\mu = \frac{v_d}{|\boldsymbol{E}|} = \frac{e\tau}{2m_e}$. Therefore $\sigma = ne\mu$. For conduction by positive carriers as well as negative carriers, the conductivity is given by $\sigma = nq_n\mu_n + pq_p\mu_p$, in which μ_n and μ_p are the mobilities of negative and positive carriers, respectively q_n and q_p are their charges (absolute values), and n and p are the numbers of these carriers per unit volume. The sign of the charge of a species of electric current carrier in a metal can be determined from measurements of the *Hall effect*. Measured values of the resistivity for copper show a *linear* dependence on the temperature.

In general, the classical theory of metals is *unable to predict correctly the linear relationship between the resistivity* ρ *and the temperature* T. However, calculations based upon modern quantum theory *can* account satisfactorily for this linear dependence.

crystal from widely separated Na^+ *and* Cl^- *ions). To this end, one considers that each* Na^+ *ion in the lattice has 6 neighboring* Cl^- *ions, each of which is bound to it by the* same spherically symmetric *interaction* $U(r)$.

One mole of an NaCl crystal has L Na^+ *ions, from each of which binding forces extend to each of its 6 neighbors. Hence the lattice energy, computed assuming an equilibrium distance* r_0 *between adjacent constituents, is* $E = 6LU(r_0) = -1.74\left(\frac{8}{9}\frac{e^2}{r_0}\right)$, *with* L *representing the Avogadro number. This energy is negative because it is* released *if the crystal is formed from its ions (stated otherwise — it is the energy needed to* break *the crystal apart into a state of zero potential energy).*

The lattice energy of any ionic crystal can be expressed by the general formula $E = C\frac{e^2}{a}$ *where* $a = 2r_0$ *is the* lattice constant. *The constant* $\alpha = 1.74$ *that appears in the above expression for* E *is known as the* Madelung constant *(1918) and is of central importance in the theory of ionic crystals; it determines the constant* c*. It is defined via the relation* $\frac{\alpha}{r_0} = \sum_j \frac{(\pm)}{r_j}$, *where* r_j *is the distance of the* j^{th} *ion from the reference ion and* r_0 *is the nearest-neighbor equilibrium distance. For an infinite line of ions of alternating signs, with the negative ion as the reference ion and* r_0 *as the distance between adjacent ions,*

$$\frac{\alpha}{r_0} = 2\left[\frac{1}{r_0} - \frac{1}{2r_0} + \frac{1}{3r_0} - \cdots\right] = \frac{2}{r_0}\log_e 2,$$

yielding

$$\alpha = 2\log_e 2 = 1.386\ldots.$$

In three dimensions, the summation is more involved: a direct summation leads to convergence problems unless one arranges to work with neutral or nearly neutral groups of ions, *by dividing an ion among different groups and using fractional charges. [Physically, it is equivalent to grouping of ions into units of higher electrical multipoles such as dipoles (potential* $\propto r^{-2}$*), quadrupole (potential* $\propto r^{-3}$*), etc.]*

In the sodium chloride structure, we obtain nearly neutral groups by considering the charges on cubes, *counting charges on cubic faces as shared between two cells* $\left(+\frac{1}{2}\right)$, *on edges as shared between four cells* $\left(+\frac{1}{4}\right)$, *and on corners as shared between eight cells* $\left(+\frac{1}{8}\right)$. *The first cube thus contributes*

$$6\frac{\frac{1}{2}}{1} - 12\frac{\frac{1}{4}}{\sqrt{2}} + 8\frac{\frac{1}{8}}{\sqrt{3}} = 1.46.$$

Taking into account the next larger cube enclosing the original cube, the next iteration yields $\alpha = 1.75$, *close to the accurate value of* 1.747 565 *for NaCl.*

When two species of atoms are bonded primarily with either ionic or covalent bonds, it is possible for them to form discrete molecules. When the primary bonds are satisfied completely within a subunit, the subunits must then be held together by a type of bond different from the primary bond.

In such molecular crystals, *subunits are held together with weak, secondary intermolecular forces. The largest class of molecular crystals is that in which covalently bonded molecules have weak intermolecular bonding. When the molecules are approximately spherical (because of molecular rotation), the crystal is usually a close-packed array of these molecules held together by non-directional forces. This occurs, for example, in crystals of* CH_4 *and* NH_3 *at low temperatures. To this class belong also crystals of the inert-gas atoms, whose outer shells are complete (He, Ne, Ar, Kr, Xe).*

All these solids are made of substances whose molecules are not polar. Since all valence electrons in these molecules are paired, covalent bonds between atoms of different molecules (or, for the inert case, between any *two atoms) are essentially impossible, and* the molecules retain their individuality. *They are bonded by the same intermolecular forces that exist between molecules of a gas or liquid, known as* Van der Waals force. *They are weak, and corresponds roughly to a force between two fluctuating, mutually-inducing* electric dipoles[723]. *The interaction energy between the dipoles is always negative (i.e. attractive), varies as the mean square* $\langle p^2 \rangle$ *of the dipole moment of the inducing molecule, and is inversely proportional to the* 6^{th} *power of the distance between the molecules.*

A molecule whose electronic distribution is perfectly symmetrical will yield a finite value for $\langle p^2 \rangle$ *even though* $\langle \boldsymbol{p} \rangle$ *is zero. That is, every possible instantaneous position that the electrons of the molecule can occupy, will*

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723 Suppose that a given molecule possesses, at some instant, an electric-dipole moment $\boldsymbol{p}_1$. This molecule will then be surrounded by an electric-dipole field $\boldsymbol{E} = -\operatorname{grad}\left(\frac{\boldsymbol{p}_1 \cdot \boldsymbol{r}}{r^3}\right) = -\frac{\boldsymbol{p}_1}{r^3} + 3\left(\frac{\boldsymbol{p}_1 \cdot \boldsymbol{r}}{r^5}\right)\boldsymbol{r}$. This field will induce an instantaneous dipole moment $\boldsymbol{p}_2 = k\boldsymbol{E}$ in a second molecule, where k is the *polarizability* (dipole moment per unit electric field) of the second molecule. The mutual energy of interaction of the two dipoles, separated by displacement $\boldsymbol{R}$, is $U(R) = -\boldsymbol{p}_2 \cdot \boldsymbol{E} = \frac{\boldsymbol{p}_1 \cdot \boldsymbol{p}_2}{R^3} - 3\frac{(\boldsymbol{p}_1 \cdot \boldsymbol{R})(\boldsymbol{p}_2 \cdot \boldsymbol{R})}{R^5}$. Using the above expression for $\boldsymbol{p}_2$, we find $U(R) = -k(1 + 3\cos^2\theta)\frac{p_1^2}{R^6}$, where $(\boldsymbol{p_1} \cdot \boldsymbol{R}) = p_1 R \cos\theta$. This indicates the existence of an attractive *force* between the two molecules *varying as* R^{-7}.

lead to a dipole moment of some size and orientation for the molecule as a whole, and although these rapidly fluctuating instantaneous moments average to zero, their mean square *must have a finite, positive value.*

In conclusion, two non-polar molecules should exhibit a characteristic r^{-7} attractive force (the gradient of their $\sim r^{-6}$ interaction potential). The strength of this force in a given case depends both on the mean-square fluctuation of the electric-dipole moment and on the polarizability of the molecules. In general the range of variability of these parameters is narrow, and consequently the Van der Waals' interaction will be rather insensitive to the type of molecule involved, though it tends to increase with molecular or atomic geometric size.

B. The Quantum Physical Basis of Chemistry or "What is a Molecule?" (1927–1937)

"Carbon gives biology, but silicon gives geology".

Charles Kittel

Solid state physics is a natural extension of molecular physics and physical chemistry, which in turn are concerned with the structure, properties and interactions of molecules, so far as they are determined by physical methods. In this sense, solid state physics is a logical continuation of atomic physics proper.

Molecular physics is closely related to chemistry. The chemist attempts to determine the composition of a compound and its structure. He is also interested in certain characteristic quantities, such as the heat of formation of the molecule and the energy freed by its decomposition into its elements, or the rates at which reactions between molecules occur. Chemistry proper is not, however, able to explain the valency, stability and bonds in molecules. Neither can it answer questions such as: Why is NH_3 pyramidally shaped? Why does benzene (C_6H_6) have the form of a hexagon? Why can hydrogen atoms join together to form the molecule H_2, but never form H_3? Why do carbon atoms combine with 4 hydrogen atoms? Why are the spectra of molecules so complex when compared with atomic spectra, ranging from microwaves up to the ultraviolet?

These and many other questions could not be answered satisfactorily before quantum mechanics was developed. It was the development of this theory *since* 1927 *that furnished a basis for answering such questions via the theory of chemical bonding, molecular dynamics and spectroscopy, and other aspects of physical chemistry.*

Molecular physics determines the spatial geometry of the atoms in the molecule, its modes of rotation and vibration, its dissociation energies, the arrangement of the electrons in the orbitals of the molecule, the possibilities of exciting electrons, the molecule's interactions with EM radiation, and the rapid rearrangements of electronic configurations attendant upon inter-molecular collisions and reactions. The experimental methods used by physicists to deal with these problems, such as spectroscopy, various forms of molecular-scale microscopy and X-ray, neutron and electron diffraction are now used also by chemists, with the result that the demarcation lines between molecular physics, modern inorganic and organic chemistry and physical chemistry have almost disappeared.

Moreover, contemporary physics, chemistry and biology meet on the molecular level: molecular physics, chemical physics, theoretical chemistry and molecular biology are but different aspect of one common reality.

During the first stage of this development, scientists were eager to use the new quantum mechanics to establish the theory of the chemical bond. The main contributors were ***W. Heitler*** *and* ***F. London*** *(1927),* ***M. Born*** *and* ***R. Oppenheimer*** *(1927),* ***D.R. Hartree*** *(1928),* ***P.M. Morse*** *(1929),* ***J.E. Lennard-Jones*** *(1929),* ***E.A. Hylleraas*** *(1930, 1931),* ***J.C. Slater*** *(1930),* ***C. Zenner*** *(1930),* ***E. Teller*** *(1930),* ***Kronig*** *and* ***Penney*** *(1931),* ***H.E. White*** *(1931, 1937),* ***J.H. Van Vleck*** *(1933–1936) and* ***L.C. Pauling*** *(1928–1937).*

A molecule is a well defined collection of atoms that are attracted to each other such that the whole collection may be thought of as a single, stable dynamical and structural unit. The attractive interaction between two atoms is called a chemical bond. Clearly, the chemical bond must be something more specific than a simple attraction between atoms, or we would have to refer to all the water in a glass as a single molecule. It is really only a question of degree; but generally, if the attraction between two atoms is such that an energy of at least 10 kcal/mole (≈ 1 eV per pair of atoms) is required to move them an infinite distance apart – a chemical bond is said to exist between these atoms, and they usually may be considered as belonging to the same molecule.

Since electrons obey the laws of quantum mechanics, our understanding of their behavior must be based on a knowledge of these laws (in addiction to electrodynamics). The most important aspects of the quantum mechanical nature of electrons, atoms and molecules, are: the Pauli exclusion principle, *the uncertainty principle, the superposition principle, and the fact that nuclei are much more massive than electrons (and thus move much more sluggishly and can often be treated as semi classical masses holding each other via anharmonic "springs").*

When two or more atoms combine to form a molecule, the more tightly bound, or inner, electrons of each atom (which fill complete shell of their respective atoms) are practically undisturbed, remaining attached to their original nuclei. Only the outermost, or valence, electrons in the unfilled shells are affected, and they move under the resultant forces due to the ions (composed of the nuclei and inner shells), as well as their mutual electrostatic repulsion, the various quantum effects, and other (non-electrostatic and weaker) electromagnetic effects. These valence electrons are responsible for chemical bonding and for most physical properties of the molecule (but not its mass).

In principle, the Schrödinger Equation (SE) and the postulates of quantum mechanics are all that is required to calculate the properties of any molecule. In practice, however, the exact analytical solution of the SE for complicated molecules has not been achieved (and is probably impossible), and experiments are required to determine the structure and behavior of molecules. Various analytical and numerical approximation schemes, though, lead to some important valid conclusions.

Let the center of mass of the molecule be at rest, so its overall the translational kinetic energy is removed from consideration. The time-dependence of any stationary solution for the molecular wave function is taken to be $e^{-i\frac{E}{\hbar}t}$*, where* E *represents one of the permissible total energy values. Then, a system of* p *different nuclei of masses* M_α *and* q *electrons of mass* m *is governed by the following PDE for the complex spatial wavefunction* $\Psi(\{\boldsymbol{R}_\alpha\}, \{\boldsymbol{r}_i\})$*: (*$\boldsymbol{R}_\alpha$ *and* $\boldsymbol{r}_i$ *are nuclear and electronic positions, respectively)*

$$\sum_{\alpha=1}^{p} \frac{1}{2M_\alpha} \nabla_\alpha^2 \Psi + \frac{1}{2m} \sum_{i=1}^{q} \nabla_i^2 \Psi + \frac{1}{\hbar^2}(E - U)\Psi = 0,$$

where the overall potential energy is $U(|\boldsymbol{R}_\alpha - \boldsymbol{r}_i|)$ *if magnetic forces, external EM fields and spontaneous emission of photons are ignored.*

This represents a very complicated physical system, even if the potential energies considered are limited to those of electrostatic origin. If magnetic and electromagnetic forces are ignored, one can write

$$U = U_{ee} + U_{nn} + U_{ne} = \frac{1}{2} \sum_{k,k'} \frac{e^2}{r_{kk'}} + \frac{1}{2} \sum_{\alpha,\alpha'} \frac{Z_\alpha Z_{\alpha'} e^2}{r_{\alpha\alpha'}} - \sum_{\alpha,k} \frac{Z_\alpha e^2}{r_{k\alpha}}.$$

where

$$r_{kk'} = |\boldsymbol{r}_k - \boldsymbol{r}_{k'}| \,,$$

$$r_{k\alpha} = |\boldsymbol{r}_k - \boldsymbol{R}_\alpha|$$

and

$$r_{\alpha\alpha'} = |\boldsymbol{R}_\alpha - \boldsymbol{R}_{\alpha'}| \ .$$

Ordinary perturbation methods cannot be applied directly, because no solutions of an equation simpler than but comparable to the above are known (except in the single-atom case).

The best approximate method of dealing with the above is the Born-Oppenheimer approximation (1927). At the foundation of this treatment of molecular problems is the great disparity between electronic and nuclear masses. This enables one to separate the SE, and with it the problem, in two parts. First, the energy levels and the wave functions corresponding to the outer electrons moving under the influence of stationary ("clamped") protons (and their closely bound electron shell) are found. These electrons move much more rapidly than of the nuclei, and complete many cycles of their motion (classically speaking) during an interval in which the nuclei move only slightly.

The second step entails treating the quantum mechanical dynamics of the protons (and their rigidly attached closely bound inner electronic shells, if any) in an effective potential determined by the electronic wavefunction (treated as charge-density clouds). This potential consists of the electrostatic and magnetic energies of the nuclei and their shells, and the electronic energy obtained in the first (electronic) step of the approximation. This finally yields the rotational and vibrational energy states of the molecule.

The simplest example of a chemical binding that occurs in nature is the hydrogen molecule ion (also known as H_2^+ *molecule or ionized hydrogen molecule) where a single electron is shared between two protons. No exact solution to this '3-body problem' has been found, but most of the salient features of the molecular system can be established in an approximate way.*

In the Born-Oppenheimer approximation we substitute $\Psi = \psi_e \psi_n$ *into the SE, where the electronic wave-function* $\psi_e(\boldsymbol{r}_i, \boldsymbol{R}_\alpha)$ *[*$\boldsymbol{r}_i$ *are the electron position vectors and* $\boldsymbol{R}_\alpha$ *are the position vectors of the nuclei] is the eigenfunction of*

$$H_e = -\frac{\hbar^2}{2m}\sum_i \nabla_i^2 + U_{ee} + U_{en},$$

and $\psi_n(\boldsymbol{R}_\alpha)$ *depends only on* $\boldsymbol{R}_\alpha$*.*

The original SE then approximately decouples into the two equations

$$-\frac{\hbar^2}{2}\sum_{\alpha=1}^{p}\frac{1}{M_\alpha}\nabla_\alpha^2\psi_n + [U_{nn} + E_e(\boldsymbol{R}_\alpha)]\,\psi_n = E\psi_n,$$

and

$$-\frac{\hbar^2}{2m}\sum_{i=1}^{q}\nabla_i^2\psi_e + (U_{ee}+U_{en})\psi_e = E_e(\boldsymbol{R}_\alpha)\psi_e,$$

where E_e is the total electronic energy (the ground-state eigenvalue of the operator H_e). These equations reflect the approximation that the nuclei move in a field of an effective potential which is composed of the electronic energy, which varies adiabatically as a function of the internuclear distances, plus the internuclear potential $U_{nn}(\boldsymbol{R}_\alpha)$.

Consider the special case of H_2^+: The SE for the electron reads

$$H_e\psi_e = -\frac{\hbar^2}{2m}\nabla^2\psi_e - \left(\frac{e^2}{r_a}+\frac{e^2}{r_b}\right)\psi_e = E_e\psi_e$$

where r_a, r_b are the respective distances of the electron from proton a and proton b, and E_e in the energy of the clamped-nuclei electron orbital.

It is convenient to scale all distances to the first Bohr radius ($a_0 = \frac{\hbar^2}{me^2}$ = 'atomic unit') and all energies to the Rydberg. The above equation then becomes

$$H_e\psi_e = -\frac{1}{2}\nabla^2\psi_e - \left(\frac{1}{r_a}+\frac{1}{r_b}\right)\psi_e = E_e\psi_e.$$

Thus E_e, ∇^2, r_a, r_b and R are now dimensionless, and E_e is electronic energy in units of $\left(\frac{e^2}{a_0}\right)$ (in $\epsilon_0 = \frac{1}{4\pi}$ units). For fixed internuclear distance R, the effective potential energy of the molecule is: $U_{\text{mol}} = \frac{1}{R} + E_e$. Using the Rayleigh-Ritz variational method, the 'trial energy' $U_{\text{mol}} = \int \psi_e H_e \psi_e d\tau + \frac{1}{R}$ is minimized subject to the constraint $\int \psi_e^*\psi_e d\tau = 1$ ($d\tau$ is an electron-positron volume element).

As a 'trial wave-function' ψ_e, one uses a linear combination of the radially symmetric trial eigenfunctions of two virtual hydrogen atoms

$$\psi = \alpha\psi_a + \beta\psi_b, \quad \psi_a = \frac{\gamma^{3/2}}{\sqrt{\pi}}e^{-\gamma r_a}, \quad \psi_b = \frac{\gamma^{3/2}}{\sqrt{\pi}}e^{-\gamma r_b}$$

(with γ dimensionless). We then have the normalization relations

$$\int \psi_a^*\psi_a d\tau = \int \psi_b^*\psi_b d\tau = 1$$

and

$$\int \psi_a^*\psi_b d\tau = \int \psi_b^*\psi_a d\tau = S$$

(overlap integral). Straightforward substitution then yields two equations involving α and β:

$$(\alpha^2 + \beta^2) + 2\alpha\beta S = 1,$$

$$E_e(R) = (\alpha^2 + \beta^2) \int \psi_a H_e \psi_a d\tau + 2\alpha\beta \int \psi_b H_e \psi_a d\tau.$$

Since the Hamiltonian operator in the electronic sector is an even function *of the x, y, and z coordinates (components of $\boldsymbol{r} - \frac{1}{2}(\boldsymbol{r}_a + \boldsymbol{r}_b)$, the electron's position relative to the COM of the molecule), the energy eigenfunctions of the system must all be even or odd functions of these coordinates. That is, every nondegenerate energy eigenfunction must have the property that*

$$\psi_n(x, y, z) = \pm\psi_n(-x, -y, -z).$$

On the other hand, for degenerate *energy eigenfunctions, arbitrary* linear *combinations of even and odd functions of the coordinates are acceptable as eigenfunctions. Thus, the hydrogenic wave functions in which the electron is localized near one proton or the other, are good electronic wave-functions for large proton separation, and these wave functions are approximately degenerate.*

But when the nuclei separation decreases to a value where pure hydrogenic wave functions are no longer a good approximation, the degeneracy is lifted, and the correct electronic eigenfunctions must exhibit the required symmetry or antisymmetry with respect to $\boldsymbol{r}_a \leftrightarrow \boldsymbol{r}_b$. One then constructs two solutions: symmetrical

$$\alpha = \beta = \frac{1}{\sqrt{2(1+S)}}; \qquad E_e = \frac{\int \psi_a H_e \psi_a d\tau + \int \psi_b H_e \psi_a d\tau}{1+S};$$

and antisymmetrical

$$\alpha = -\beta = \frac{1}{\sqrt{2(1-S)}}; \qquad E_e = \frac{\int \psi_a H_e \psi_a d\tau - \int \psi_b H_e \psi_a d\tau}{1-S}.$$

Of these, the symmetrical solution corresponds to the lower energy level, *thus leading to the* ground state *of the molecule.*

Performing the necessary integrations, the explicit expression for $E_e(R)$ (the effective proton-proton Born-Oppenheimer potential) for the symmetrical *solution become*

$$E_e(R) = -\frac{1}{2}\gamma^2 + \frac{\gamma(\gamma-1) - J + (\gamma-2)K}{1+S}$$

where

$$S = \frac{\gamma^3}{\pi} \int d\tau e^{-\gamma(r_a+r_b)} = \left(1 + \rho + \frac{1}{3}\rho^2\right) e^{-\rho},$$

$$\rho = \gamma R, \qquad J = \frac{\gamma^3}{\pi} \int d\tau \frac{e^{-2\gamma r_a}}{r_b} = \frac{1}{R}\left[1 - (1+\rho)e^{-2\rho}\right],$$

$$K = \frac{\gamma^3}{\pi} \int d\tau \frac{e^{-\gamma(r_a+r_b)}}{r_b} = \gamma(1+\rho)e^{-\rho}.$$

Here, J is the integral describing the Coulomb attraction of proton b and the electron cloud centered about proton a. K is the exchange integral, having no classical equivalent and being a consequence of the fact that the electron in the molecular ground state is present near both protons (the same applies to S). Note that K is appreciable only if ψ_a and ψ_b overlap substantially.

Since E_e depends on the two variables γ and ρ, a minimization w.r.t. these two variables shows (numerically) that the energy minimum lies at the equilibrium distance $R_0 = 2.08$ atomic units (or 1.10 Å) and $E_0 = -0.5866\ \frac{e^2}{a_0}$. The corresponding dissociation energy of $H_2^+ \to H + H^+$ is calculated to be 2.24 eV, as compared with the experimental value of 2.65 eV.

The Born-Oppenheimer approximation of fixed nuclei can be similarly employed for the evaluation of the binding energy and equilibrium distance of the neutral hydrogen molecule H_2.

The Hamiltonian operator for the system can be divided into 3 parts: one relating to the motion of the center of mass, one to the electronic motion for a given proton separation, and the third to the relative motion of the protons under an effective potential energy consisting of their own mutual Coulomb energy and the electronic energy.

The Hamiltonian operator for the electronic motion is symmetrical in x (the coordinate measured along the direction of the line joining the two protons) so that the electronic eigenfunctions will again have even or odd parity w.r.t. x.

The case of the neutral H_2 molecule differs from the case of the ionized molecule in one important respect: the presence of two identical electrons in the neutral molecule requires the application of the exclusion principle. The electronic eigenfunctions must now not only exhibit either even or odd parity w.r.t. their separate space coordinates, but must also be antisymmetric w.r.t. an interchange of the space and spin coordinates of the two electrons.

Thus, the exclusion principle leads to a physical effect, namely an effective repulsion of the electrons if their spins are parallel (in addition to their electrostatic repulsion).

Let the two nuclei (protons) be denoted by a and b and the two electrons by 1 and 2. We then have the following electronic Hamiltonian in atomic units (after subtracting the motion of the center of mass):

$$H_e = -\frac{1}{2}(\nabla_1^2 + \nabla_2^2) + \frac{1}{r_{12}} - \left(\frac{1}{r_{a1}} + \frac{1}{r_{b1}} + \frac{1}{r_{a2}} + \frac{1}{r_{b2}}\right).$$

At large inter-nuclear distances R, the wave function should go over into the product of the separate atomic eigenfunctions, either becoming of the form $f(\boldsymbol{r}_{a1})f(\boldsymbol{r}_{b2})$ if electron 1 forms an atom with nucleus a, and 2 with b, or of the form $f(\boldsymbol{r}_{b1})f(\boldsymbol{r}_{a2})$, if the two electrons are exchanged.

A reasonable approach at finite *distance R will be to take a linear combination of two such products, and symmetry considerations lead to the choice of the symmetrical solution*

$$\psi(1,2) = \alpha\,[f(\boldsymbol{r}_{a1})f(\boldsymbol{r}_{b2}) + f(\boldsymbol{r}_{b1})f(\boldsymbol{r}_{a2})]$$

for the ground state *(with* antiparallel *electron spins, according to the Pauli principle). We are again neglecting all spin dependent interactions, except those resulting from the Pauli principle. The antisymmetric combination, would lead to a larger energy with no attraction and no formation of a molecule at all.*

We apply the operation $\int d\tau_1 \int d\tau_2 f^*(\boldsymbol{r}_{a1})f^*(\boldsymbol{r}_{b2})\cdots$ to the equation $H_e\psi = E_e\psi$ with the ansatz $f = \frac{\gamma^{3/2}}{\sqrt{\pi}}e^{-\gamma r}$, and use the abbreviations:

$$S = \int d\tau_1 f^*(\boldsymbol{r}_{a1})f(\boldsymbol{r}_{b2}) \qquad \textit{(overlap integral)}$$

$$\left.\begin{aligned} J &= \int d\tau_1 \tfrac{1}{r_{b1}}|f(\boldsymbol{r}_{a1})|^2 \\ J' &= \iint d\tau_1 d\tau_2 \tfrac{1}{r_{12}}|f(\boldsymbol{r}_{a1})|^2|f(\boldsymbol{r}_{b2})|^2 \end{aligned}\right\} \text{classical interaction integrals}$$

$$\left.\begin{aligned} K &= \int d\tau_1 \tfrac{1}{r_{a1}} f^*(\boldsymbol{r}_{a1})f(\boldsymbol{r}_{b1}) \\ K' &= \iint d\tau_1 d\tau_2 \tfrac{1}{r_{12}} f^*(\boldsymbol{r}_{a1})f(\boldsymbol{r}_{b1})f^*(\boldsymbol{r}_{a2})f(\boldsymbol{r}_{b2}) \end{aligned}\right\} \text{exchange integrals}$$

$$A = \int d\tau_1 f^*(\boldsymbol{r}_{a1})\left[-\frac{1}{2}\nabla_1^2 - \frac{1}{r_{a1}}\right] f(\boldsymbol{r}_{a1})$$
$$A' = \int d\tau_1 f^*(\boldsymbol{r}_{a1})\left[-\frac{1}{2}\nabla_1 - \frac{1}{r_{b1}}\right] f(\boldsymbol{r}_{b1}).$$

All the above integrals can be evaluated in terms of elementary functions. The final result has the form

$$E_e = -P(\rho)\gamma + Q(\rho)\gamma^2$$

where $\rho = \gamma R$ *and* P, Q *are known functions of* ρ. *The energy minimum is at* $E_e = -\frac{P^2}{4Q}$. *Numerical computations yield an equilibrium state at* $R = R_0 = 0.77$ Å, *against an experimental value of 0.742 Å. The energy is then* $E = -1.139\frac{e^2}{a_0}$ *as compared with* $2E_0 = -\frac{e^2}{a_0}$ *of two* separate *hydrogen atoms in the ground state.*

In polyatomic molecules, molecular orbitals are obtained by the superposition of the atomic orbitals. The situation becomes interesting when we mix orbitals with different directionalities, such as s and p states. The principle of superposition is based on the recognition that the addition of atomic orbitals will be most effective (producing the greatest bond energy) when the overlap of the electron wave-functions will be maximal. The process of mixing s*- and* p*-orbitals is called* hybridization.

The most striking example of its significance occurs in carbon. This element possesses 4 electrons in the L-shell, and since its valency is 4 we must suppose that these electrons are distributed among the states defined by $\psi(2s)$, $\psi(2p_x)$, $\psi(2p_y)$ *and* $\psi(2p_z)$. *The most stable configuration in this valence state will occur when the respective molecular orbitals exhibit* maximum overlapping *(**Pauling**, 1937).*

It therefore follows that to study the possible hybrid orbitals, we need only discover the conditions for maximum charge-cloud density in an assigned direction. This occurs when wave functions are used which contain 4 equal 'weights' of the constituents. If we write

$$\psi = c_1\psi(2s) + c_2\psi(2p_x) + c_3\psi(2p_y) + c_4\psi(2p_z),$$

then, since the separate wave-functions on the r.h.s. are mutually orthonormal, the contributions to $|\psi|^2$ *are in the ratio of* $c_1^2 : c_2^2 : c_3^2 : c_4^2$. *When these are equal we have* $c_1^2 = c_2^2 = c_3^2 = c_4^2 = \frac{1}{4}$ *which renders (with any arbitrary phase choice for* c_1*)*

$$c_1 = \frac{1}{2}, \quad c_2 = \pm\frac{1}{2},$$

$$c_3 = \pm\frac{1}{2}, \quad c_4 = \pm\frac{1}{2}.$$

Upon choosing $c_1 = \frac{1}{2}$, *there are four independent combinations of the remaining signs. These give the 4 wave functions*

$$\psi_1 = \frac{1}{2}\left[\psi(2s) + \psi(2p_x) + \psi(2p_y) + \psi(2p_z)\right],$$
$$\psi_2 = \frac{1}{2}\left[\psi(2s) + \psi(2p_x) - \psi(2p_y) - \psi(2p_z)\right],$$
$$\psi_3 = \frac{1}{2}\left[\psi(2s) - \psi(2p_x) + \psi(2p_y) - \psi(2p_z)\right],$$
$$\psi_4 = \frac{1}{2}\left[\psi(2s) - \psi(2p_x) - \psi(2p_y) + \psi(2p_z)\right]$$

where (ignoring the common radial factors),

$$\psi(2s) = 1, \qquad \psi(2p_x) = \sqrt{3}\sin\theta\cos\phi,$$
$$\psi(2p_y) = \sqrt{3}\sin\theta\sin\phi, \quad \psi(2p_z) = \sqrt{3}\cos\theta.$$

Now, the maximum of ψ_1 *occurs where*

$$\frac{\partial\psi_1}{\partial\theta} = \frac{1}{2}\sqrt{3}\{\cos\theta\cos\phi + \cos\theta\sin\phi - \sin\theta\} = 0,$$
$$\frac{\partial\psi_1}{\partial\phi} = \frac{1}{2}\sqrt{3}\{-\sin\theta\sin\phi + \sin\theta\cos\phi] = 0.$$

The solution is $\tan\phi = 1 \quad \left(\phi = \frac{\pi}{4}\right)$, $\tan\theta = \sqrt{2} \quad (\theta = 54^\circ 44')$. *Similarly, the maxima of the remaining wave functions occur at* $\phi = -\frac{\pi}{4}$, $\theta = 125^\circ 16'$ *for* ψ_2, $\phi = \frac{3\pi}{4}$, $\theta = 125^\circ 16'$ *for* ψ_3, $\phi = \frac{5\pi}{4}$, $\theta = 54^\circ 44'$ *for* ψ_4. *The corresponding vectors in these directions will be*

$$\boldsymbol{e}_1 = \boldsymbol{e}_x + \boldsymbol{e}_y + \boldsymbol{e}_z, \qquad \boldsymbol{e}_2 = \boldsymbol{e}_x - \boldsymbol{e}_y - \boldsymbol{e}_z,$$
$$\boldsymbol{e}_3 = -\boldsymbol{e}_x + \boldsymbol{e}_y - \boldsymbol{e}_z, \qquad \boldsymbol{e}_4 = -\boldsymbol{e}_x - \boldsymbol{e}_y + \boldsymbol{e}_z.$$

The angle between (say) $\boldsymbol{e}_1$ *and* $\boldsymbol{e}_4$ *is given by* $\cos\theta_{14} = -\frac{1}{3}$, *therefore* $\theta_{14} = 109^\circ 28'$. *It is the angle between the lines joining the centroid (carbon) to any two of the 4 H atoms bonded to the carbon; the 4 lines form the vertices of a regular tetrahedron. The hybridization scheme just described is known as* sp^3, *because all three* $2p$ *orbitals of the central carbon atom participate. The other possible hybridization schemes are:* sp^2 *(trigonal planar* symmetry, *as for the methyl radical* CH_3, *linear shape, as for* CO_2 *or acetylene* C_2H_2*), and schemes involving* s, p *and* d *orbitals (for 3*[rd]*-row central atom, e.g. phosphor and sulfur).*

The simplest stable molecule is that of hydrogen, H_2. *If we start with two ground state hydrogen atoms an infinite distance apart, electron 1 is in a* $1s$ *orbital around proton* a *and electron 2 is in a* $1s$ *orbital around proton* b. *As the distance between the two atoms decreases, we use a linear combination of the* $1s$ *atomic orbitals to construct a molecular* orbital *for which each electron is equally likely to be in the neighborhood of each proton.*

There are only two possible combinations that satisfy the condition that each electron be associated equally with each nucleus, one by taking the sum and the other *by taking the difference of the two* $1s$ *atomic orbitals. Addition leads to constructive interference between the two wave functions in the region between the two nuclei and results in increased electron density in this region. This orbital is called the* BONDING MOLECULAR ORBITAL.

In the opposite situation the two atomic orbitals undergo destructive interference in the region between the two nuclei, and the electron density is very low in this region (it is zero on a nodal plane bisecting the inter-nuclear line segment). This orbital is called the ANTIBONDING MOLECULAR ORBITAL.

The electronic SE can now be used to calculate the energy E_e *of the* hydrogen molecule, *which depends on the internuclear distance. The arbitrary fiducial zero of energy is taken to be the energy of the two hydrogen atoms when infinitely far apart.*

When the molecule is in the bonding state $(1s_a + 1s_b)$ *it will have an energy less than that of the dissociated atoms, i.e. it will be a stable entity. The molecule in this state has a dissociation energy* E_0 *and a bond length* R_0. *The antibonding orbital* $(1s_a - 1s_b)$ *yields a higher energy than that of the dissociated atoms for all* values *of* R.

Therefore, no stable molecule can exist in that state. In the bonding orbital, there is high electron density between the two protons, meaning that the electrons serve as the bonding agent. In the ground state, with both electrons occupying the same molecular orbital, they must have opposite spins.

The electrons in a molecular orbital that has a high density in the region between the two nuclei create a COVALENT BOND, *which results from an overlap of two atomic orbitals. A similar technique is used for diatomic molecules (such as* O_2, N_2*) containing many valence electrons. In this type of bond, known also as* HOMOPOLAR BINDING, *two electrons in the ground state are equally shared between two identical atoms in a manner described by a wave function which is symmetric in the space coordinates of the two electrons, but antisymmetric in the spins. Successive pairs of electrons (pooled from both atoms) similarly occupy successively higher molecular orbitals — both bonding* and *antibonding, in general. This "buildup" (aufbau) scheme holds for molecules involving two different atoms as well, and even for multi-atomic molecules.*

In diatomic molecules, electrons do not move in a central field of force, and therefore the orbital angular momentum operator $\boldsymbol{L}$ of an electron does not remain constant during its motion. However, because of the axial symmetry about the line (say, z axis) passing through the two nuclei, the component of $\boldsymbol{L}$ in the z direction is conserved (except for spin-orbit and inter-electron coupling effects), i.e. $L_z = m_\ell \hbar$; here m_ℓ is, as usual, quantized to $m_\ell = 0, \pm 1, \pm 2, \dots$. The sign of m_ℓ determines the sense of rotation of the electron about the z-axis, but since the energy is independent of this direction one need only give the absolute value, $|m_\ell| = \lambda$.

The different angular momentum states are denoted according to the following scheme: $m_\ell = 0$, $\lambda = 0$ (σ states); $m_\ell = \pm 1$, $\lambda = 1$ (π states); $m_\ell = \pm 2$, $\lambda = 2$ (δ states); $m_\ell = \pm 3$, $\lambda = 3$ (ϕ states). Thus, except for σ-states, all angular momentum states are doubly degenerate because of the possible signs of m_ℓ. In addition, in each of the above states the electron may have its spin up or down relative to the molecular axis. So, σ-states can accommodate two electrons with opposite spins whereas the remaining states, π, δ, ϕ can accommodate up to four electrons each, two with spin up and two with spin down.

For molecular orbital states of the electron, one uses the notation $\lambda n\ell m_\ell$, where $n\ell$ serves to indicate the atomic orbitals from which each of the molecular orbitals has been formed by linear combination. Each of these molecular orbitals corresponds to a different energy.

In the case of molecules composed of two identical nuclei, such as H_2, the aforementioned structural cylindrical symmetry is augmented by a further symmetry (or antisymmetry) of the electron probability amplitude under exchange. The shape of the molecular orbitals resulting from a linear combination of atomic orbitals will depend on the atomic quantum numbers.

When the charges of the two nuclei composing the molecule are different, the Coulomb interaction of each nucleus with its electrons is different and the molecule no longer has a center of symmetry. In general, only the unpaired electrons in the last unfilled shell in each atom will participate strongly in the chemical bond.

In the case of NaCl, for example, these are the $3s$ electron in Na and one of the $3p$ electrons in Cl. Since the Cl nucleus produces a stronger attractive field and its atomic shells are closer to it, the Chlorine's electronegativity is bigger than that of the Na atom, which means that the electronic charge of the bonding electron pair is displaced toward the Cl nucleus. This results in an uneven charge distribution, and hence an electric dipole moment of the NaCl molecule. The NaCl molecule may thus be considered as being composed of two ions held together by their Coulomb attraction. We express this situation by writing Na^+Cl^-.

This type of molecular bonding is called an ionic bond; in it the classical model yields an adequate approximation to the energetic state of the stable bond.

For most heteronuclear diatomic molecules, the situation is intermediate between pure covalent bond and pure ionic bond. The more ionic the bond, the larger the electronic dipole moment of the molecule.

For molecules with more than two atoms, the geometry of the molecule is determined quantum-mechanically. It was found experimentally that a bond between any two of its atoms occurs in the direction in which the respective atomic wave functions have maximum overlap.

Thus, consider the water molecule H_2O*: if for simplicity we ignore hybridization, the active ingredients in the oxygen atom are the two unpaired electrons, each with* $n = 2,\ \ell = 1,\ m_s = \frac{1}{2},\ m_\ell = \pm 1$*, i.e. (say) one O electron in the* p_x *state and the other in the* p_y *state (the remaining 2p-electrons of O are along the z-axis in* p_z *state, with their spins paired). The other active elements, [one (1s) per H atom] are located so that they couple with maximum wave-function overlap to the two respective unpaired electrons in O. The result is a molecule having, to the first approximation, a right-angle shape. The presence of the H atoms polarizes the motion of the p electrons of O such that the lobes nearer to the H atoms are larger than the far lobes. Also, the angle between the O–H bonds increases from* 90° *to* 104.5° *because of the repulsion among the O atom's two lone pairs (non-bonding electron pairs) and the two O-H bonding pairs.*[724]

Detailed calculations show that the hydrogen 1s-electrons are pulled toward the O atom, so that the centroids of the negative and positive charges do not coincide, producing a net electric dipole moment along the line bisecting the bent H-O-H angle.

A similar situation occurs in the ammonia molecule NH_3*, where the interaction is (again, ignoring the* sp^3 *hybridization) between the 3 unpaired* $(2p_x)$*,* $(2p_y)$ *and* $(2p_z)$*-electrons in the N atom and the 3 (1s)-electrons of the H atoms. The result is a trigonal pyramidal structure, with the N atom at the apex and the H atoms forming the base (the angles at the vertex of the pyramid between any two H atoms is* 107.3°*).*

The bonds of methane (CH_4*) can only be understood in terms of hybridization. First a slightly excited state of carbon, consisting of one 2s and three unpaired 2p-electrons, is mandatory to engage the four H atoms. But these four bonds would not have the same energy and the same directionality (2s is spherically symmetric).*

[724] The last two featured can also be understood in terms of an sp^3 (tetrahedral) hybridization of the oxygen orbitals.

To account for the experimental results, which indicate 4 bonds of the same energy and directionality, suitable solutions of the SE were found. By making 4 linearly-independent linear combinations of the four wavefunctions so-called hybridized wavefunction in the form

$$\psi_{1,2,3,4} = \frac{1}{2}\left[\psi(2s) \pm \psi(2p_x) \pm \psi(2p_y) \pm \psi(2p_z)\right]$$

we obtain suitable wave-functions with maxima pointing toward the vertices of a tetrahedron, as explained above.

Clearly, since s and p wave functions correspond to different values of the angular momentum, the hybrid wave-functions do not *describe states of well-defined electronic angular momentum.*

C. Quantum Theory of Solids and The Semiconductor (1930–1936)

The many and varied properties of solids have intrigued us for centuries. Technological developments involving metals *and* alloys *have shaped the course of civilizations, and the symmetry and beauty of naturally occurring, large single* crystals *have consistently captured our imagination. However, the origins of the physical properties of solids were not understood at all until the development of* quantum mechanics*. The application of quantum mechanics to solids has provided the basis for much of the technological progress of modern times.*

Quantum mechanics *provides a "technology" to analyze atomic systems, in order to calculate their energy levels, shapes and interactions and to predict the probability of transition from one state to another. When we consider the properties of matter in bulk, new problems arise because of the complicated interaction between atoms and molecules. Even in the gaseous state, intermolecular forces affect the behavior considerably, although it is possible to describe deviations from the gas laws in terms of simple two-body interactions. In the liquids and solid states, the close proximity of atoms gives rise to strong forces affecting many particles.*

At the same time, the fundamental rules of quantum mechanics *become important in dealing with vibrations of the crystal lattice and with condition of electrons in metals. Despite intense mathematical difficulties, solid-state physics has made great progress in accounting for several phenomena which received no classical explanation. Moreover, its results have great practical* *significance and wide implications in many fields of physical research, including deep and fruitful mathematical similarities to Quantum Field theories (QFT) of particle physics and cosmology.*

The quantum theory of solids includes the following topics:

- *Crystal's lattice vibrations — theory of phonons.*
- *Heat conduction, specific heats and thermal expansion coefficients.*
- *Metallic electric conductivity.*
- *Paramagnetism, diamagnetism, ferrimagnetism, and ferromagnetism.*
- *Band theory of solids and semiconductivity.*
- *Superconductivity.*

The classical Drude model (1900) was based on concepts borrowed from the kinetic theory, suggesting that electrons in a metal behave like molecules of a gas (at ordinary temperatures) and participate in thermal equilibrium according to the Maxwell-Boltzmann velocity distribution law. The greatest success achieved by Drude's theory consisted in the derivation of the Wiedemann-Franz law[725]*, which states that the ratio of thermal conductivity to electrical conductivity is given by the expression* $3\frac{k^2}{e^2}T$.

***Drude** (1863–1906, Germany) was also able to derive an expression for electrical conductivity, which is of some importance even today. However, a theory that was adequate for Newtonian particles was not expected to accommodate electrons, which since the 1920's were recognized as* quantum *entities, obeying* quantum *laws.*

Indeed, Drude's theory soon ran into conflict with experimental results: the hypothesis that every free electron should possess a mean kinetic energy of $\frac{1}{2}kT$ *per degree of freedom in a state of thermal equilibrium leads to a corresponding electronic contribution to the molar heat capacity of* $\frac{3}{2}R = 3$ *cal/mol deg which is in flat contradiction with the Dulong-Petit rule. This and other difficulties disappear only if it is assumed* ad hoc *that the number of free electrons is* considerably smaller *than that of atoms.*

*The first step toward the modification of Drude's theory was taken (1928) by **Arnold Sommerfeld** (1868–1951) with the new assumption that the electron gas in metals possesses the properties of a* highly degenerate Fermi-Dirac

[725] **Gustav Wiedemann** (1826–1899, Germany), **Rudolf Franz** (1827–1902, Germany).

gas. The SE for the free electron, $\nabla^2\psi + \frac{2m}{\hbar^2}E\psi = 0$, admits a de Broglie plane-wave solution for the electron's complex wave function $\psi = ce^{i(\boldsymbol{k}\cdot\boldsymbol{r})}$, where c is a complex normalization constant and $\boldsymbol{k}$ is the vector wavenumber; $\hbar = \frac{h}{2\pi}$ is the reduced Planck constant and m denotes the electron mass. The electron's vectorial momentum operator is defined by $\boldsymbol{p} = -i\hbar\boldsymbol{\nabla}$ and the SE yields $\boldsymbol{p}\psi = -i\hbar\boldsymbol{\nabla}\psi = \hbar\boldsymbol{k}\psi$; in other words, the wavefunction (Hilbert-space state vector) is a simultaneous eigenvector of the operators p_x, p_y, p_z with eigenvalues

$$\boldsymbol{p} = \hbar\boldsymbol{k} = m\boldsymbol{V},$$

where $\boldsymbol{V}$ is the velocity [in one dimension $p = \frac{h}{\lambda} = \hbar k$].

A free electron has only a kinetic energy E, and its *dispersion relation* (dependence of E on $\boldsymbol{k}$) is the parabola

$$E(k) = \frac{p^2}{2m} = \frac{\hbar^2}{2m}k^2$$

(also, the Planck relation $E = h\nu = \hbar\omega$ yields $\omega = \frac{p^2}{2m\hbar} = \frac{\hbar}{2m}k^2$). Since $|\psi|^2 = c^2$ is a constant, the probability density of detecting the single free electron is the same everywhere when it is in the pure $\boldsymbol{k}$ state Ψ. Note that the energy is a *continuous* function of k.

However, the gas of shared electrons is not actually free: ignoring inter-electron interaction (apart from the Pauli exclusion principle), the electrons still interact with the metal's lattice ions.

This causes the individual atomic valence orbitals to become spread over the entire metallic bulk, via superposition, into gigantic, macroscopic, molecular orbitals.

Identical atomic orbitals thus split into a very large ($\sim$ Avogadro's number per mole) shared orbitals, which are for all intents and purposes a continuum — called a *band*.

Since there is more than one atomic orbital involved, more than one electron band results. Bands are separated by finite energy gaps (band gaps); within a band, the dispersion relation is *not* parabolic.

Next, we present a simple-minded model illustrating how such electron bands arise. We begin by restricting the motion of a single electron to a segment $0 \leq x \leq a$. This can be thought of as a model for conduction electron in a one-dimensional metal crystal lattice, where we neglect interactions of these electrons with each other and with the positive ions and assume that the height of the potential barrier is much above the electron's kinetic energy.

The electron can move freely inside each repetitive unit cell of the lattice, assumed of size a, bouncing back and forth between the cell boundaries, but cannot escape to a neighboring cell.

This simplified model is physically relevant to electron condition only when a (possibly small) coupling is allowed between cells; such a coupling occurs because an electron has a finite probability to tunnel across the inter-cell potential barrier, so the conduction electrons — usually one or more per cell — mix and propagate throughout the lattice.

The back and forth motion of an electron inside any given cell — ignoring tunneling and the presence of other electrons — is accommodated by the solution of the SE,

$$\psi = Ae^{ikx} + Be^{-ikx}$$

with the approximate boundary conditions $\psi(a) = \psi(0) = 0$.

The probability amplitude (wavefunction) is thus no longer a propagating wave, but rather a standing wave $\psi = c\sin kx$ *where k, p, E may only range over the sets of discrete values*

$$k_n = n\frac{\pi}{a},$$

$$p_n = \hbar k_n = \frac{\pi n\hbar}{a},$$

$$E_n = \frac{p_n^2}{2m} = \frac{n^2\pi^2\hbar^2}{2ma^2}.$$

Also, we may choose the wavefunction phase convention so that

$$\psi_n = \sqrt{\frac{2}{a}}\sin\left(\frac{\pi n x}{a}\right)$$

on account of the per-cell total probability normalization

$$\int_0^a |\psi_n|^2 dx = 1.$$

The positive wavenumber (e^{ikx}) *term in* Ψ *corresponds to an electron moving in the positive x direction, while the* e^{-ikx} *term corresponds to an electron moving in the opposite direction. In accord with the superposition principle, the electron is doing both at once. Other mobile valence electrons have identical wavefunctions in other cells:* $ma \leq x \leq (m+1)a$, *with* m *ranging over* $\ldots, -2, -1, 0, 1, 2, \ldots$. *It turns out that due to inter-cell tunneling, each* $\Psi_n(x)$ *in each cell* m *acquires a phase factor* e^{iqx}, *where* q *is the Bloch wavenumber for the n-th band;* q *is continuous (unlike* k_n*)*

and ranges over $-\frac{\pi}{a} \leq q \leq \frac{\pi}{a}$, *since* $\Psi_n(x; q + \frac{2\pi}{a})$ *is just* $\Psi(x; q)$ *for a different band.*

The energy E_n *of the electron is not entirely kinetic, as is the case of the free electron, because of the potential energy due to the lattice ions and the resulting inter-cell tunneling effects;* $E_n = E_n(q)$ *depends on the Bloch wavenumber* q *for each band* n. *The expression for the energy in terms of* q *is complicated and depends on the geometry of the lattice. The important general result is that the range of values of the* n*-th band energy* $E_n(q)$ *has discontinuities or* gaps *such that near* $q = 0$ *the shape of* $E_n(q)$ *closely resembles that of a free particle. Therefore, the lattice significantly affects the motion of the non-bound electron only when* q *is close to* $n\frac{\pi}{a}$. *At intermediate values of* q, *the electrons move freely through the lattice.*

The above one-dimensional treatment can easily be generalized to three dimensions. Thus, for crystals having a cubic repetitive unit-cell which is a cube of side a, *the single-cell electron wavefunction (before inter-cell transmission is taken into account) is*

$$\Psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{\pi n_1 x}{a} \sin\frac{\pi n_2 y}{a} \sin\frac{\pi n_3 z}{a}$$

with energy levels $E = \frac{n^2\pi^2\hbar^2}{2ma^2}$, $n^2 = n_1^2 + n_2^2 + n_3^2$, *and* n_1, n_2, n_3 *are positive integers.*

When the cells are allowed to interact, $\Psi_{n_1,n_2,n_3}(x)$ *and* E_{n_1,n_2,n_3} *again become functions of a continuous Bloch wavenumber,* $\boldsymbol{q}$, *which is now a vector belonging to the* reciprocal momentum-space *unit cell:* $-\frac{\pi}{a} \leq q_j \leq \frac{\pi}{a}$, $j = 1, 2, 3$. $E_{n_1,n_2,n_3}(\boldsymbol{q})$ *are periodic band dispersion relations, with approximately parabolic shape near* $\boldsymbol{q} = 0$.

The simplest exactly-soluble quantum-mechanical model for electronic bands in a crystal lattice is the ***Krönig-Penney*** *periodic-potential model.*

In the band, the dispersion relations are not exactly *quadratic, and the electronic energy levels do not fill the whole range* $0 \leq E < \infty$, *but only certain* bands *in the range. However, when only a single, partially-filled band is important, the free box approximation is useful, since the band is always approximately quadratic near its bottom. The 'effective' electron mass is then calculated from the reciprocal of the band's curvature at the bottom,* $\nabla_{\boldsymbol{q}}^2 E$, *and may differ from the electron's physical (inertial and gravitational) mass. We may call this the 'quasi-free' approximation for the conduction electrons.*

For a small molecule, the molecular-orbital energy levels are widely spaced, but for a very large lattice (as is the case for electrons in a metal) successive levels are so close that they practically form continuous spectra (bands). We ask: how many mobile-electron energy levels are there in a small energy range

dE, when the lattice is very large? (For the time being we assume a quasi-free dispersion relation.)

In k-space $\frac{\pi}{a}(\xi, \eta, \zeta)$, each point with $\xi = n_1$, $\eta = n_2$, $\zeta = n_3$ (positive integers) represents an electron state. The total number of points lying inside a surface of a sphere of radius k give the number of different states associated with energies $\leq E$. We evaluate first the total number of states (including a factor of 2 for spin) in a volume of an octant of radius k,

$$N(E) = \frac{2a^3}{\pi^3}\frac{1}{8}\left(\frac{4}{3}\pi k^3\right) = \frac{2a^3}{3\hbar^3\pi^2}(2m^3)^{1/2}E^{3/2}.$$

The number of states with energy between E and $E + dE$ per unit volume of the lattice (a^3) is thus

$$\frac{1}{a^3}dN(E) = g(E)dE,$$

where

$$g(E) = \frac{1}{\pi^2\hbar^3}(2m^3)^{1/2}E^{1/2}$$

is the number of states per unit volume per unit energy interval, at the energy E (since each level can accommodate two electrons with opposite spins, the values of $N(E)$ and $g(E)$ were doubled).

If N, the total number of electrons per unit volume, is less than the total available number of energy levels in the band, the electrons will occupy all band energy levels up to a threshold level, called the Fermi level $\epsilon_F = E(N)$. From

$$N = \frac{2}{3\pi^2\hbar^3}(2m^3)^{1/2}\epsilon_F^{3/2}$$

we derive

$$\epsilon_F = \frac{\hbar^2}{2m}(3\pi^2 N)^{2/3}.$$

[Example: silver, density $= 10.5\frac{\text{g}}{\text{cm}^3}$ and atomic weight 108 g/mole has one free electron per atom. The total number of free electron per cm^3 is thus

$$N = \frac{6.02 \times 10^{23}\ \frac{\text{atom}}{\text{mole}} \times 10.5\frac{\text{g}}{\text{cm}^3}}{108\frac{\text{g}}{\text{mole}}} = 5.9 \times 10^{22}$$

free electrons/cm^3. The Fermi energy is $\epsilon_F = 5.5$ eV, taking the approximation that m is the physical electron mass.]

We can estimate the relative number of conduction electrons in a metal which are thermally excited to higher energy states; most of the excited electrons are in a range ΔE *above the Fermi energy* ϵ_F, *where* $\Delta E \cong kT$. *Assuming that* $kT \ll \epsilon_F$, *the number* ΔN *of excited electrons per unit volume can be calculated from* $\Delta N \simeq g(\epsilon_F)kT$. *The* fraction *of conduction electrons that is thermally excited is small. Simple algebraic manipulations then lead to the result*

$$\Delta N/N \simeq kT/\epsilon_F .$$

At room temperature $kT \simeq 0.025\ eV$ *and typically* $\epsilon_F \simeq 4\ eV$, *so that* $\Delta N/N \simeq 1/160$. *The* absolute *number of excited conduction electrons is, however, very large.*

An estimate can also be made of the relative number of electrons in the conduction band of an insulator or semiconductors at temperature T. *If in the Fermi distribution* $n(E)$ *we have* $E - \epsilon_F \gg kT$, *then*

$$n(E) = \frac{1}{exp\left[(E-\epsilon_F)/kT\right]+1} \simeq exp\left(-\frac{E-\epsilon_F}{kT}\right)$$

so that in such an energy range the Fermi distribution varies with energy like the Boltzmann distribution.

We know that $E - \epsilon_F = \frac{1}{2}E_g$ *at the bottom of the conduction band in an insulator. Thus the condition* $E - \epsilon_F \gg kT$ *is met* ($E_g \gg kT$ *for an insulator), so we can take*

$$n(E) = e^{-\frac{E_g}{2kT}}$$

as the number of electrons per state in the conduction band of an insulator.

Since the Fermi distribution falls in value by an order of magnitude in an energy range of about $\Delta E = 2kT$, *we get a good estimate of* ΔN, *the number of conduction electrons, by evaluating those in the range* $2kT$ *above the bottom of the conduction band. Since*

$$\Delta N = n(E)g(E)\Delta E,$$

we must now evaluate $g(E)$, *the density of states. Because* $g(E)$ *starts at zero at the bottom of the conduction band, a typical value over the range* $\Delta E = 2kT$ *is obtained by evaluating* $g(E)$ *at* $E = kT$. *Hence,*

$$\Delta N \simeq e^{-\frac{E_g}{2kT}} g(kT) 2kT.$$

Using our earlier results for a metal, $N = \frac{2}{3}\epsilon_F g(\epsilon_F)$ *and* $g(kT)/g(\epsilon_F) = \left(\frac{kT}{\epsilon_F}\right)^{1/2}$, *we have*

$$\Delta N/N \simeq \left(\frac{kT}{\epsilon_F}\right)^{3/2} e^{-\frac{E_g}{2kT}}.$$

This is the relative number of conduction electrons for an insulator. This fraction is much smaller than the corresponding result $\frac{kT}{\epsilon_F}$ *for a metal, partly because the density of states* $g(E)$ *is smaller near the bottom of the conduction band in an insulator than at the Fermi energy in a metal — but principally because of the occupation fraction* $e^{-\frac{E_g}{2kT}}$. *Let us take* $E_g = 6$ eV *as the gap in a typical insulator; at room temperature this factor is then* $e^{-\frac{E_g}{2kT}} = e^{-120} \approx 10^{-52}$. *Not only is the fraction* $\Delta N/N$ *insignificant, but the absolute number of conduction electrons is also negligible for an insulator. If, however,* $E_g = 1$ eV, *as for a semiconductor, then although* $e^{-\frac{E_g}{2kT}} \doteq 10^{-9}$ *gives a very small fraction, the number of conduction electrons is no longer insignificant.*

On the basis of the Fermi-Dirac theory, Sommerfeld worked out the thermal *properties of an assembly of electrons, obtaining quite good agreement with observations. In this sense, the Sommerfeld model was an*

important step in the development of the theory of metals. It was, however, less successful in explaining the electrical *properties of metals: it failed to explain why some elements are good electrial conductors and others are not.*

The next important improvement in understanding the physical properties of solids was the development of band theory. *This theory is based on a careful* quantum analysis *of the role of the* atomic lattice *centers in the solid. These centers were found to be far more important than assumed in the previous theories, in which they served only to balance electric charge and to act as scattering sites for the free conduction electrons.*

It was shown earlier that quantum theory provides the means for describing the energy levels of the electrons surrounding an atom. Mathematical complications for systems involving many atoms and many electrons are so great that they prevent a rigorous mathematical treatment. Nevertheless, the use of approximation methods makes it possible to obtain considerable insight into the behavior of complicated many-body systems.

It is of great interest, for example, to consider the behavior of many atoms brought together to form a solid. A useful approximation which bears on the *difference between conductors and insulators is the so-called band approximation, which was discussed earlier in connection with the solutions of the SE.*

*It was shown that the SE exhibits band structure as a direct result of the periodicity of the potential function (**R. de L. Kronig** and **W.G. Penney**, 1931). The qualitative existence of bands was first pointed out by **M.J.O. Strutt** (1927). Band theory was further extended by **F. Bloch** (1928), **L. Brillouin** (1930, 1931), **C. Zenner** (1934), **N.F. Mott** and **H. Jones** (1936).*

The ideas of these scientists can be summarized in the following qualitative manner: An energy band in a one-dimensional crystal is made up of a large number of waves of different energies moving with equal numbers in both directions. Thus each energy level corresponds to two Schrödinger waves, one traveling to the left and the other to the right. Since by the exclusion principle any particular wave function can be shared by two electrons having opposite spins, each energy level can accommodate two electrons (of opposite spins) traveling to the left and two electrons (of opposite spins) traveling to the right, and no more. Each energy level is thus a sort of two-way street by means of which an electron can travel with a particular energy and speed to the right or to the left with its spin either up or down.

Some crystals are insulators (nonconductors) of electricity; Absolutely pure silicon and germanium held at very low temperatures are quite good insulators. How can this be?

In insulators the energy bands lying above a certain energy are completely empty. Obviously, no electric current can flow as the result of an empty energy band. It turns out to be equally true that no current can flow as the result of a completely filled energy band since there is an electron going to the right for every electron going to the left. (If a very strong electric field were applied to the material, an electron might jump from the highest filled band to the lowest empty band and so become free to move.)

In conductors such as pure metals, a particular conduction band is only partially filled with electrons. In lithium, for example, there is only one valence electron per atom, so the lowest band is only half-full; the crystal should therefore be a conductor. [In the case of diamond, which is an insulator, there are 4 valence electrons per atom.]

When an electric field is applied to a conductor, some electrons going one way can transfer to slightly higher empty energy levels and travel in the other direction. Since more electrons will be traveling in one direction than in the other, an electric field can cause a net electric current to flow.

Metals are good conductors. Some substances obdurately remain insulators. A third class of materials, called semiconductors, *can conduct electricity when they contain certain impurities in very minute quantities. Silicon and germanium are semiconductors. The presence of small amounts of elements such as phosphorus, antimony, and arsenic, adds electrons to the empty bands. When there is no electric field applied, these electrons settle down four to a level, two traveling to the right and two to the left*[726]*. But if a small electric field is applied so as to force electrons towards the right (left), some electrons which initially were traveling to the left (right) jump into vacant levels of a shade higher energy and travel to the right (left).*

Another group of substances, including boron and gallium, also make semiconductors. They take away *electrons from a filled energy band, leaving the band in an "almost filled" condition. When an electric field is applied, some electrons traveling against the field will jump to vacant levels and travel in the direction in which the electric field urges them to go. When the mathematics of the "almost filled"energy band is worked out, it shows a behavior exactly like that we would expect from* positive charges *(= absence of negative charge) traveling through the crystal. Experiments confirms this mathematical picture.*

Thus, there is a real reason to think of the conduction in this "acceptor" type of semiconductors as due to holes *in the "almost filled" band — holes that act as free positive charges would act. The effective mass of holes is found to differ from that of electrons.*

Materials such as phosphorus, antimony, or arsenic, are called n-type *impurities or* donors *because of the negative electrons they add. Materials such as boron or gallium, removing electrons from filled bands and create holes that behave like positive charges and are free to move, are called* p-type *impurities, or* acceptors.

Electrons will normally enter the conduction band either by thermal agitation, or excitation via external bombardment by particles or radiation. The energy gap between the valence and conduction bands in semiconductors is no more than about 1 eV [1.14 eV for silicon and 0.67 eV for germanium]. Although the value of the Fermi distribution function (governing the relative population of an energy states in the conduction and valence bands) is small (since $kT \simeq 0.025$ *eV at room temperature), the number of available states in the conduction band is high. Hence the thermal excitation from the valence band into the conduction band occurs for a significant number of electrons,*

726 In the three-dimensional case, there is an almost continuous *shell* of electron momenta vectors sharing a single given energy (per given spin). This shell is spherical for isotropic lattices.

this number being the product of the mean number of electrons per quantum state and the number of quantum states per relevant energy interval.

Furthermore, the conductivity of a semiconductor increases rapidly with temperature; the number of excited electrons in silicon, for example, increases by a factor of about 10^9 *with a doubling of temperature from* $300\,^\circ K$ *to* $600\,^\circ K$. *Thus a semiconductor can be defined as a non-metallic covalent substance which possesses measurable conductivities, dependent upon the temperature and impurity levels.*

Photovoltaic (PV) cells — tapping solar energy (1839–1999 CE)

Introduction

The sun's energy is vital to life on earth, It determines the Earth's surface temperature and supplies virtually all the energy that drives natural global systems and cycles. Although some other stars are enormous sources of energy in the form of X-rays and radio signals, our sun releases 95% of its energy as visible light. Yet, visible light represents only a fraction of the total radiation spectrum; infrared and ultraviolet rays are also significant parts of the solar spectrum.

The sun emits virtually all of its radiation energy in a spectrum of wavelengths that range from about 2×10^{-7} *to* 4×10^{-6} m. *The majority of this energy is in the visible region. Each wavelength corresponds to a frequency and an energy; the shorter the wavelength, the higher the frequency and the greater the energy (expressed in eV, or electron volts).*

Photovoltaic cells convert light energy into electricity at the atomic level, *bypassing thermodynamic cycles and mechanical generators. Although first discovered in 1839, the process of producing electric current in a solid material with the aid of sunlight wasn't truly understood for more than a hundred years. Throughout the second half of the 20th century, the science has been refined and the process has been more fully explained. As a result, the declining cost of these devices has put them into the mainstream of modern energy producers. This was caused in part by advances in the technology, where PV* conversion coefficients *have improved considerably.*

*Most commonly known as "*solar cells*", PV systems are already an important part of our lives. The simplest systems power many of the small calculators and wrist watches we use every day. More complicated systems provide electricity for pumping water, power communication equipment, heat our kitchen and bathroom waters, and even light our homes and run our appliances. In a surprising number of cases, PV power is the cheapest form of electricity for performing these tasks.*

Today, solar cells power virtually all satellites, *including those used for communications, navigation, defense, and scientific research. The* computer *industry, especially transistor semiconductor technology, also contributed to the development of PV cells. Transistor and PV cells are made from* similar materials *and operate on the basis of similar physical mechanisms. As a result, advances in transistor research provide a steady flow of new innovations in PV cell technology.*

Photovoltaic cells are an up and coming alternative energy source. In the 1950s, when oil was thought to be able to last the world forever, scientists at NASA began researching and developing photovoltaic cells for use in space exploration. Power systems applications for earth were not explored until the 1970s, during the oil crisis. At that time scientists also started researching other forms of power systems in a race to find an economical and clean alternative to coal – and oil – burning power plants.

The first person to observe the photovoltaic effect was **Edmund Becquerel***, a French physicist, in 1839. As time progressed, so did interest in this newly discovered phenomenon. In the 1880's photovoltaic cells built from selenium were used to convert light into electricity, with a very poor efficiency of about 1% to 2%.*

The next remarkable development to affect photovoltaic technology was the invention of techniques to produce highly pure crystalline silicon (Bell Telephone Laboratories, 1954). Since then, other scientists have investigated the properties of crystalline silicon and have used it to make photovoltaic cells with acceptable efficiency levels. Early applications involved powering radio systems on orbital satellites.

The benefits of solar energy speak for themselves. It has been estimated that the solar energy available to be generated from the sun is nearly 10,000 times more than the total energy consumption of the world.

Advantages of photovoltaics also include:

- *Low maintenance*
- *No moving parts*
- *High mobility*
- *Effective for a variety of applications*
- *Environmentally friendly*
- *Working life of 20 to 30 years*

The use of silicon crystals in Photovoltaic cells makes them expensive. First of all, silicon crystals are currently assembled manually. Secondly, silicon purification is difficult and a lot of silicon is wasted. In addition, the operation of silicon cells require a cooling system, because performance degrades at high temperatures.

Research is underway for new fabrication techniques, such as those used for microchips. Alternative materials like cadmium sulfide and gallium arsenide are at an experimental stage.

Underlying Physical Mechanisms

Free electrons can be generated near surfaces (of metals, insulators and semiconductors) by four processes:

- *The interaction of photons with matter may cause* photon absorption *or scattering by either of three mechanisms: internal photoelectric effect, Compton effect and electron-positron pair production.*

In all three cases, an electron *is moving after the effect, so the moving electron can be used to detect the photon. At low energies the* photoelectric effect *is very strong, but its probability drops off rapidly at high energies. As photons increase in energy and momentum, the* Compton effect *becomes more important in scattering them out of the beam. Pair production does not begin until* 1.022 MeV, *but it becomes more and more effective after that. In toto, the transmitted intensity* I *is related to the incident intensity* I_0 *along an axis* x, *via the experimental absorption law* $I = I_0 e^{-\mu x}$, *where*

$$\mu = \mu_{\text{photoelectric}} + \mu_{\text{Compton}} + \mu_{\text{pair production}}.$$

The light absorbed in a semiconductor increases the total free carrier concentration and causes the transport of electrons from the valence band into the conduction band. The electron-hole pairs thereby generated are free quasi-particles and can take part in the semiconductor's conductivity [phototubes, photodiodes, photocathodes].

- *Secondary electron emission: If electrons impinge upon the surface of a solid, a certain fraction is reflected, whereas the rest penetrate into the solid and may cause emission of secondary electrons. This phenomenon is common to metals, semiconductors and insulators.*

- *Field-aided emission* by applying an electric field to the surface.

- *Thermionic emission:* heating the surface until some electrons have enough energy to escape the potential barrier [thermionic diodes].

Photoconductivity is the increase of conductivity due to a photon absorption, or otherwise stated — the decrease in electrical resistance when exposed to light. The effect is particularly strong in selenium, in metal sulfides, oxides and halides, as well as in germanium and silicon. The release of valence electrons by photoabsorption is called *internal photoelectric effect*, in contrast to the *external photoelectric effect*, which is the release of electrons from *metal surfaces* (Einstein 1905). The production of conducting electrons and holes by photoabsorption is possible in all insulator or semiconductor crystals. The majority of the good and practically important photoconductors, however, are semiconductors. Electronic semiconductivity, photoconductivity, and phosphorescence are three closely related crystal phenomena.

The application of an external electric field to the material results in the transport of both electrons and holes through the material and the consequent production of an electric current in the electrical circuit of the detector.

A *photoelectric cell* is a device that converts light into electricity. Two main types of photoelectric cell are in use today: the phototube and the solid-state photodetector.

The *phototube* is an electron tube in which electrons, initiating an electric current, originate through photoelectric emission. In its simplest form the phototube is composed of a cathode coated with a photosensitive material (known as a *photocathode*), and an anode. Light falling upon the cathode causes the liberation of electrons, which are then attracted to the positively charged anode, resulting in a flow of electrons (i.e., current) proportional to the intensity of the light. Phototubes may be highly evacuated, or filled with an inert gas at low pressure to achieve greater sensitivity.

In a modification called the multiplier phototube, or *photomultiplier*, a series of metal plates are shaped and arranged so that the photoelectric emission is amplified by secondary electron emission. The multiplier phototube is capable of detecting ionizing radiation of extremely low intensity; it is an essential tool for nuclear and particle research, astronomy, and space guidance systems.

Another important application is the *image converter:* The image of an object which emits only infrared or X-ray radiation is projected onto an infrared- or X-ray sensitive layer (photo-cathode). According to the varying intensity of the incident radiation, a varying number of electrons are emitted from the *photo-cathode. These electrons are electrically accelerated and concentrated on a fluorescent screen by means of electron optics. There, the electronic image excites fluorescence and thus creates a visible image of the infrared or X-ray object.*

The second type of photoelectric cell, the solid-state photodetector, has replaced the phototube for many applications because it is small, inexpensive, and uses little power. The simplest type of solid-state photodetector is the photoconductor — a semiconductor whose resistance changes when it is exposed to light — that is, to a flow of photons.

More stable and precise than a simple photoconductor is the photodiode: it is made by joining together an n-type and p-type semiconductors. In practice, the two types are often single silicon crystals doped with donor impurities on one side and acceptor impurities on the other. The region in which the semiconductor changes from p-type to an n-type is called a junction. The initial concentrations of electrons and holes on opposite sides of the junction results in the diffusion of electrons across the junction from the n-side to the p-side and of holes in the opposite direction, until equilibrium is established. The result of this diffusion is a net transport of positive charge from the p-side to the n-side.

Unlike the case when two different metals are in contact, the electrons cannot travel very far from the junction region because the semiconductor is not a particularly good conductor. The diffusion of electrons and holes therefore creates a double layer of charge at the junction similar to that of a parallel-plate capacitor. There is thus a potential difference across the junction which tends to inhibit further diffusion. In equilibrium, the n-side with its net positive charge will be at a higher potential than the p-side with its net negative charge. In the junction region, there will be very few charge carriers of either type, so the junction region has a high resistance. The junction region is also called the depletion region because it has been depleted of charge carriers.

A semiconductor with a p-n junction can be used as a simple diode rectifier. When an external potential difference is applied across the junction (by connecting a battery and a resistor to the semiconductor) such that the positive terminal of the battery is connected to the p-side of the junction, the diode is said to be forward biased. Forward biasing lowers the potential across the junction, thus enhancing the diffusion of electrons and holes as they attempt to reestablish equilibrium, resulting in current in the circuit.

If the positive terminal of the battery is connected to the n-side of the junction the diode is said to be reverse biased. This tend to increase the potential difference across the junction, thereby further inhibiting diffusion. Essentially, the junction conducts only in one direction, similar to a vacuum-tube diode.

In a photodiode, in the absence of illumination, a negligible dark current flows through the junction. When the p-n junction is illuminated, excess carriers are generated and the current rises in proportion to the light intensity, causing a voltage drop across the load resistor. Photodiodes are used as an electric eye in operating burglar alarms, traffic-light controls, and door openers. A light source (which may be infrared and invisible to the human eye) at one end of the circuit falls on the photocell located some distant away. Interrupting the beam of light breaks the circuit. This in turn actuates a relay, which energizes the burglar alarm or other circuit. Other common uses for photoconductors include light switching and dimming, and light meters for cameras.

It is observed that as the reverse bias is increased and reaches an extreme value, the current suddenly increases. In such large electric fields, electrons are stripped from their atomic bonds and accelerated across the junction. These electrons, in turn, cause others to break loose. This effect is called avalanche breakdown. Although such a breakdown can be disastrous in a circuit where it is not intended, the fact that it occurs at a sharp voltage value makes it of use as a voltage regulator known as a Zenner diode.

"Avalanche" diodes are used to amplify the signal from a light source. In these devices, a large reverse voltage is applied so that a photon-created electron in the conduction band gains enough energy to bounce against atoms in the semiconductor and thus liberate additional electrons. A large current is therefore produced when light strikes the diode. Phototransistors are also used to amplify light signals. Their construction is similar to conventional transistors except that one of the transistor's junctions is exposed to radiation. In bipolar phototransistors, it is the base-emitter junction that is exposed to radiation; in field-effect phototransistors it is the gate junction.

A solar cell (solar battery) *is a photodiode working under conditions of forward bias and converts radiant energy directly into electrical energy. It consists of a thin layer of p-type silicon on an n-type silicon base. The n- and p-regions are heavily doped so that the resistance of the cell is small. There is an optimum thickness of the p-region* (≈ 0.025 mm) *so that as much as possible light is absorbed near the junction. When a photon is absorbed in the p- or n-region, it can create a hole-electron pair. Usually the hole and electron quickly recombine, but in a solar cell the internal electric field in the depletion layer at the p-n junction (directed from the n-side to the p-side) can separate the hole and electron before they have a chance to recombine.*

The separation of the carriers produces a forward voltage across the barrier (forward, because the electric field of the photoexcited carriers is opposite the built-in field of the junction). The appearance of a forward voltage across the illuminated junction is called the photovoltaic effect.

Thus, the two sandwiched semiconductors function as a battery, creating an electric voltage at the surface where they meet (junction). It is this field that causes the carriers to move from the semiconductor bulk toward the surfaces and makes them available for the external electrical circuit. The application of an external voltage and a local resistance in series with the junction makes the charge carriers do work which has come directly from the energy of the incident radiation.

The light-emitting diode *(LED; or luminescent diode) is the reverse of a solar cell: the passage of a* forward *current through the p-n junction involves minority carrier injection of electrons into the p-region and holes into the n-region. The injected carriers recombine with the majority carriers of the respective region, their intensity decreasing with the distance from the p-n junction. In many semiconductors the recombination is nonradiative, i.e. the energy liberated in the recombination process is absorbed by the crystal lattice, that is, turns eventually into* heat. *However, in such semiconductors as* SiC, GaAs, InAs, GaP *and* InSb *the recombination is* radiative: *the energy of recombination is liberated in the form of photons. Because of that, a forward current flowing through the p-n junction made of such materials is accompanied by the emission of light from the junction region. LED are used in displays, and in flash lights.*

A special type of photodetector is the Charge-coupled device *(CCD). It contains an array of light detectors, each registering variations of light intensity as small changes of voltage. The unit has the form of a small capacitor, composed of metal oxide and semiconductor layers, capable of both photodetection and* memory storage. *When the subtle change of voltage (created by the photoconductive electrons) is applied to the metal layer (called the 'gate'), electron-hole pairs created in the semiconductor are separated and the electrons become trapped in the region under the gate. This trapped charge represents a small piece of a digital image known as a* pixel *(picture element). The complete image can be recreated by reading out a sequence of pixels from an array of CCD's.*

Modern computer systems incorporate auxiliary optoelectronic devices in their scanning *and* printing *peripherals. Consider, for example, the principles of operation of a scanner: As one presses the "scan" button on a typical hand-held scanner, a* light-emitting diode *(LED) illuminates the image beneath the scanner. An inverted, angled* mirror *directly above the scanner's window reflects the image onto a* lens *at the back of the scanner.*

The lens focuses a single line of the image into a charge-coupled device *(CCD), which is a component designed to detect minute changes of voltage. The CCD contains a row of light detectors. As the light shine onto these detectors, each registers the local amount of light as a voltage level that* *corresponds to a grey-scale value (black/white or color components). The voltages generated by the CCD are then sent to a specialized chip for gamma correction, a process that enhances the black tones in an image so that the eye, which is more sensitive to dark tones than to light ones, will have an easier time recognizing the image (with some scanners, gamma correction is performed as a software process).*

The single line of the image now passes to an analog-to-digital converter (ADC). In a grayscale scanner, the converter assigns 8 *bits to each pixel, which translates into* $2^8 = 256$ *levels of gray in the final digitized image. The A-D converter on a monochrome scanner registers only* 1 *bit per pixel (either on or off), representing, respectively, black or white.*

As the operator's hand moves the scanner, a hard rubber roller (the main purpose of which is to keep the scanner's path straight) also turns a series of gears that rotate the slotted disc. As the disc turns, a light shines through the slits and is detected by a photomicrosensor on the other side of the disc. Light striking the sensor throws a switch that sends a signal to the ADC converter. The signal tells the converter to send the line of bits generated by the converter to the PC. The converter clears itself of the data, and is ready to receive a new stream of voltages from the next line of the image.

Timeline

1839 **Edmund Becquerel** (France) discovered the *photovoltaic effect*: he observed that shining light on an electrode in an electrolytic cell increased the generation of electric current between the metal electrodes. His discovery, however, remained a curiosity of science for the next 65 years.

1873 **Louis May** and **Willoughby Smith** (England) discovered the *photoconductivity of selenium*.

1876 **W.G. Adams** and **R.E. Day** observed the *photovoltaic effect* in solid selenium.

1883–6 **Charles Fritts** (USA) described and developed the first *solar cells* made from selenium wafers. Fritts envisioned that solar cells may one day compete with the large electrical generating plants (which were just then being established in the United States), since solar cells were compact, self-sustaining and their fuel, solar energy, is both without limit, and without cost.

The fledgling solar cell industry of the 19th century, however, never developed as Fritts predicted. Most engineers at the time felt that Fritts' experiments violated the principle of conservation of energy. This bias within inside the engineering community inhibited any serious large scale research and development.

Fritts and other solar cell experimenters of the period *could not formulate a theoretical defense* against their colleagues' objections; they simply knew that these cells worked. Why they worked (i.e., the photovoltaic effect), however, could not be adequately explained by 19th century classical physics.

Researchers in photovoltaics were to remain the black sheep of the scientific community until *quantum mechanics* (which explained how solar cells worked) won general acceptance among scientists and engineers.

1887 **Heinrich Hertz** (Germany) discovered that ultraviolet light altered the lowest voltage capable of causing a spark to jump between two metal electrodes.

Selenium PV cells were adopted in the emerging field of *photography* for use in *light* measuring devices.

1890–1900 *The photoelectric effect*: By the year 1890 it was well-known that an isolated polished metal surface becomes positively charged when illuminated with ultraviolet light (UV). Presumably, the UV light somehow removed negative charges from the surface, leaving it with positive charge. This phenomena was named the *photoelectric effect*. **Lenard** (1900) showed that the ejected negative charge consists of *electrons* (discovered by J.J. Thompson in 1897).

1904 **Wilhelm Hallwachs** (Germany) discovered that a combination of copper and cuprous oxide was photosensitive.

1905 **Albert Einstein** explained the *photoelectric effect* in the framework of quantum physics.

1910 **Richard Willstätter** worked out the structure of *chlorophyll* ($C_{55}H_{72}MgN_4O_5$). It is the substance of plant leaves which, during *photosynthesis*, catalyzes the formation of carbohydrates from atmospheric CO_2 and H_2O through the action of *sunlight*, thus converting light into chemical energy — a living *solar cell*[727].

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[727] *Photosynthesis* is a process by which the energy of sunlight is used to form molecules such as glucose from CO_2 and H_2O:

$$\text{energy } + 6CO_2 + 6H_2O \rightarrow C_6H_{12}O_6 + 6O_2$$

Because this reaction produces O_2, a supply of oxygen built up in the atmosphere. This oxygen supply allowed primordial organisms to run the above reaction backwards (in which direction it proceeds *spontaneously*!) and oxidize the carbohydrates they devoured in order to obtain energy:

$$C_6H_{12}O_6 + 6O_2 \rightarrow 6CO_2 + 6H_2O + \text{ energy}$$

Those organisms who used the solar energy to change simple molecules into more complex ones are the ancestors of modern *plants*; those who ate carbohydrates and burned them to obtain energy are the ancestors of modern animals. As life evolved, it moved away from the above simple type of *organic chemistry* toward a more complex type of chemistry called *biochemistry*, in which highly specialized and complex molecules were "invented" by evolution to catalyze reactions, store and convert energy, build tissues, regulate structures and *heredity*; chlorophyll is an example of such an invented complex molecule.

The history of life also includes "inventions" in biological counterparts to fluid dynamics, material science, electrochemistry, mechanical engineering, "computer science", acoustics, quantum optics and nanotechnology.

1916 **Jan Czochralski** (1885–1953, Poland) developed a method for growing single crystal metal needles. In 1952, scientists at Bell Laboratories depended on Czochralski process to develop the first *crystalline silicon photovoltaic cell*.

1931 **Thomas J. Rhamstine** (1893–1975, USA) was first to use photovoltaic cell in *photography*, with the aim of measuring the luminosity of the object to be photographed and adapting the time of exposure and the aperture of the lens to the sensitivity of the film.

1933–1952 **Clarence M. Zenner** (1907–1993, USA) explained electrical breakdown in insulators in terms of the quantum-mechanical *tunneling effect*. Invented the *Zenner diode* voltage regulator to protect dc output from both rapid variations in supply input voltage and variation in load resistance. To avoid breakdown damage, a rectifying diode is connected with the load resistance. In the Zenner breakdown voltage regime, small changes in diode voltage are accompanied by large changes in diode current, producing voltages that compensate for changes in the voltage input and load resistance.

After the invention of the transistor, rectifying *p*-*n* junction diodes were used as Zenner diodes. Here a small increase in the reverse voltage in the breakdown range causes a substantial increase in the reverse current. Consequently, the voltage across the load resistance remains practically constant. Zenner's *avalanche breakdown mechanism* is the basis of *avalanche diodes* used for *light amplification* devices.

1938 **Chester Carlson** (USA) invented *electrophotography* or *xerography*, a method of photocopying utilizing the internal photoeffect in semiconductors.

A thin film of *high resistivity metal oxide* (usually ZnO) is deposited on a sheet of paper. Before the photographic process, the film is negatively charged by a gas discharge. When the image to be photographed is optically projected onto such paper, the surface charges from the *illuminated parts* leaks through the film much more readily than from the non-illuminated parts, and accordingly, an electric image of the object remains on paper after the exposition. To *develop* the electrical image, the paper is sprayed by a weak spray of special dry paint, or "toner". The particles of the toner are deposited on the negatively charged parts of the paper, thus *developing the image*. The image is *fixed* by heating the paper to the temperature at which the toner particles melt and adhere firmly to the paper.

The main advantage of electrophotography over normal photography is the exclusion of *chemical* development and fixation processes. This makes it possible to *increase the speed* of photographic processes drastically, reducing the necessary time to about a few seconds. However, electrophotography is as yet inferior to normal photography in accuracy and resolution.

1941 **Russell Ohl** (USA) invented a *silicon solar cell*. Its efficiency is about 1%.

1947–8 Invention of the *transistor* at the Bell Laboratories (USA) by **John Bardeen** (1908–1991), **Walter Houser Brattain** (1902–1987) and **William Bradford Shockley** (1910–1989).

1948–1956 The basic theory of the internal photoelectric effect, photoconductivity, semiconductor-metal junctions and photovoltaic cells, was developed in the framework of *semiconductor physics* by **Nevill Francis Mott** (1905–1996, England) and **Walter Hans Schottky** (1886–1976, Switzerland). In particular, Schottky discovered that aluminum in contact with an n-type material creates a *rectifying contact*, known as the *Schottky barrier diode*. Since in the Al-n diode there are only majority carriers (electrons in the n-region), *switching* is very fast because there is no wait for the recombination of injected minority carriers (holes in the n-region). Such diodes have switching times of the order of 10^{-11} sec. This makes them useful in radioelectronic pulsed circuits, and in computer and automation circuits where there is a need for high operational speeds.

The contact of aluminum with a p-type material creates an anti-barrier metal-semiconductor junction used to provide *ohmic contact* by means of which a semiconductor device is connected into an electric circuit (flow of hole current is easily accomplished by recombination with electrons supplied by the external circuit).

1952 **Gerald Pearson** (USA) invented the *alloy-junction diode*, thereby creating a *p*-*n* junction with controlled characteristics. First to discover the Zenner breakdown in semiconductors.

1952 Bell Lab engineers **G.K. Teal** and **J.B. Little** adopted the **Czochralski** process for producing *highly pure germanium single crystals*.

1954 **Paul Rappaport**, **J.L. Loferski** and **Dietrich A. Jenny** (RCA, USA) reported the PV effect in the element *Cadmium*.

Darryl Chapin, **Carl Fuller** and **Gerald Pearson** (Bell Labs, US) developed the first crystalline silicon photovoltaic cell. They refined the silicon solar cell, raising its efficiency to 6%, and then to 15%. This ushers an age of new solid state technology that pervades our lives today. The foundations have been laid for an industry of high-efficiency solar cells.

1958 The *space race* spurred improvement in solar cell design and efficiency. The U.S. Vanguard space satellite carried a small array of PV cells to power its radio. However, the drive to make space-grade qualified solar cells efficient and lightweight led to high costs, making them uneconomical for terrestrial applications where low price is the main concern.

With electricity, natural gas and oil being so cheap, U.S. government hesitated to promote the development of cheaper and more efficient cells; only few sensed the need for alternative energy sources.

1958–1974 The American space program created and sustained a solar cell industry; solar cells have powered every U.S. satellite from the first Vanguard to Skylab.

1959 *Explorer 6* was launched with a PV array of 9600 cells, each only of 1 cm $\times$ 2 cm.

1960 Hoffman Electronics achieved 14% efficiency in PV cells.

1962 Development of the *semiconductor laser*. It operates much the same way as the LED, except that the transitions are stimulated instead of spontaneous; *p*-*n* boundary layers between highly doped *p*- and *n*-regions (e.g. GaAs) are used. If a voltage of a few volts is applied for short periods in such a way that the *n*-type region's conduction electrons are lifted in energy and flow into the conduction band of the p-type region, its occupation by electrons can become higher than that of the p-type valence band (with a corresponding inversion occurring in the n-type region due to hole injection from the p-type side).

This process is called *electric pumping*[728]. Beyond a critical current density of this electron flow from the n-type region to the p-type region, induced transitions from the conduction band to the valence band of the p-type region are possible with emission of recombination radiation, since the electrons from the conduction band recombine with the positive holes of the valence band. Such an *injection laser* may consist of a tiny GaAs cube with a planar *pn* junction; Two parallel end faces of this cube carry electrodes for applying the pumping voltage, whereas two perpendicular end faces are polished so that they partly reflect, partly transmit the emitted radiation.

The photons of the recombination radiation are then partly reflected from these end faces back into the crystal where they induce further transitions until a narrow bandwidth, highly collinear intense light beam finally leaves the LASER. The efficiency of this transformation of electric energy into light is extremely high. Theoretically, it might approach 100% if incidental losses are neglected.

Semiconductor laser have evolved (1962–2004) into highly complex marvels of material science and opto-electronic engineering, involving engineering alloy crystal lattices, quantum effects, built-in optical cavities and optical feedback, thermo-electric cooling, advanced electrooptic modulation techniques, and molecular deposition manufacturing technologies.

Infra-red laser chips — with optical fibers running through them — are commonly used in the optical communication industry.

1963 Japan installed a 242 W PV array on a lighthouse.

1964 The *Nimbus* spacecraft was launched with a 470 W PV array.

[728] Other pumping mechanisms used in lasers are *chemical* and *optical* pumping.

1965 Invention of the *light-emitting diode* (LED), the inverse of the solar cell: electrical energy is fed into it to produce light energy. It is essentially a *pn*-junction semiconductor with a large *forward bias*, producing a large excess concentration of electrons that move to the p side and holes that moved to the n side of the junction. There they undergo radiative recombination and light is emitted. Since $E = h\nu$, we can obtain different frequencies of light by varying the energy difference through which the electron falls. We can vary the energy difference by using materials with different (even engineered) energy gaps, or cause the transition to be made to defect states within the forbidden energy gap.

Using different dopants and alloys, we can then obtain different light frequencies. LEDs are commonly used as displays for digital watches and calculators and in scanners; at higher luminosities they are used as everyday efficient light sources. The emitted radiation may be either invisible (infrared) or in the visible spectrum. Visible solid state lamps are used for long life indicator service. Infrared diodes have outputs carefully matched to silicon photoreceivers. They are used in conjunction with the receivers, for counting, sensing, and positioning applications. LEDs generally operate in the range of 1 to 3 volts at currents of 10 to 100 milliamperes. Their light output is not coherent as that of semiconductor lasers, yet it is nonetheless relatively monochromatic, collimated and bright.

1966 The Orbiting Astronomical Observatory was launched with a 1-kW PV array.

1968 The OVI-13 satellite was launched with two CdS panels.

1969–1990 *Carged-coupled device* (CCD) was first demonstrated at Bell Labs (1969), a *solid-state chip* which transforms light into electricity. Arrays of CCD's are used to capture images in video and digital cameras.

1975–1999 Rising energy costs, sparked by a worldwide oil crisis, renewed interest in making PV technology more affordable. Since 1975 the U.S. federal government, industry, and research organizations have invested billions of dollars in research, development, and production.

1979 NASA completed a 3.5 kW PV system in Arizona — the world's first village PV system.

1980 A 105.6 kW PV system established in Utah.

1982 A 1-MW plant was established in California.

1983 Worldwide PV production sales exceed 250 million dollars.

1995 Price of photovoltaics dropped from 200 dollars/Watt during the space program to 10 dollars/Watt or less.

1999 Commercial PV systems can convert from 7% to 17% of sunlight into electricity. They are highly reliable and last 20 years or more. The cost of PV-generated electricity has dropped 15–20 fold; PV modules cost about 6 dollars per Watt and produce electricity for as little as 25–50 cents per kWh.

The Plateau Problem — soap films and minimal surfaces

The theory of minimal surfaces *was initiated by* ***Lagrange*** *as an application of his studies in the calculus of variations (1760–1761).* ***Monge***, ***Meusnier***, ***Legendre***, ***Bonnet***, ***Riemann*** *and* ***Lie*** *contributed to the theory in the framework of differential geometry. It was Meusnier who discovered in* 1776 *the second* elementary *minimal surface, the right helicoid (the first one, the catenoid, which is the only curved minimal surface of revolution, was discovered by Euler in 1744).* ***Karl Weierstrass*** *(1866)* and ***H.A. Schwarz*** *developed the relationship between the theory of complex analytic functions and the real minimal surfaces.*

In the theory of capillarity the importance of minimal surfaces of least potential surface energy was demonstrated by the experiments of ***Plateau*** *(1873), who dipped a wire in the form of a closed space curve into a soap* *solution. Owing to the action of surface tension, a film of liquid is in stable equilibrium only if its area is a minimum. Plateau raised the mathematical question: can every closed curve in space be spanned by at least one minimal surface? This mathematical question became known as the Plateau problem. Many famous mathematicians of the* 19^{th} *and* 20^{th} *centuries*[729] *were intrigued by this problem, but its solution appeared to be difficult.*

Although the soap film shape is the solution to the corresponding physical problem, pure mathematicians do not admit empirical evidence in lieu of proof of mathematical existence. For a long time, all efforts to prove even the existence of such a minimal surface for every preassigned boundary curve, were unsuccessful. It was only in 1930–1931 *that the existence of the solution for the general case was proved.*

In recent years the problem of minimal surfaces has been studied when not only one but any number of contours is prescribed, and when, in addition, the topological structure of the surface is more complicated. For example, the surface might be one sided (non-oriented) or of genus different from zero. These more general problems produce an amazing variety of geometrical phenomena that can be exhibited by soap film experiments.

Consider for example soap bubbles; we know that out of all solids of a given volume, the sphere has the smallest surface area. Physically, the stable equilibrium of the bubble indicates a state of minimal potential energy (Bernoulli's law of virtual work). However, a sphere is a figure of constant mean curvature, i.e. not a minimal surface in the sense of the Plateau problem. (Inside the bubble the pressure is higher than outside it and the pressure difference is related to the surface tension T *and mean curvature* H *via the Laplace equation* $p = TH$*.) Another example is a cylindrical soap bubble between two coaxial rings. Beyond a certain critical separation of the rings, the cylindrical film becomes unstable and will decompose into two separate spherical bubbles of different sizes.*

This phenomenon was discovered by Plateau. He also found that there are exactly six different kinds of surfaces of constant mean curvature which are also surfaces of revolution: the plane and the catenoid (mean curvature zero), the sphere, the cylinder, the unduloid and the noboid (mean curvature

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[729] In the period 1975–1990, the number of published pages devoted to minimum surface type problems reached many thousands. One thus sees that the appeal of this problem to mathematicians has not diminished through the ages.

A relativistic Minkowski-space version of the problem turned out to be of some relevance to *String theories*; there, the surface represents the *world sheet* of an oscillating, fundamental string of typically Planck-scale sizes — the sheet being the 4-D locus of the *worldlines* of all points along the string.

nonzero). [Imagine the curve formed by one of the foci of the ellipse that rolls on a straight line. When this curve is rotated it generates the unduloid. The same setup with an hyperbola forms the noboid.]

The area of a surface $z = z(x, y)$ *over a region* D *of* $\mathbb{R}^2$ *spanned by a given contour, is determined by the double integral* $\iint_D \frac{dxdy}{\cos(\boldsymbol{n}, e_z)}$ *where* $\boldsymbol{n}$ *is the unit normal to the surface and* $\boldsymbol{e}_z$ *a unit vector in the direction of the* z *axis. Since* $\cos(\boldsymbol{n}, \boldsymbol{e}_z) = (1 + z_x^2 + z_y^2)^{-1/2}$, $z_x = \frac{\partial z}{\partial x}$, $z_y = \frac{\partial z}{\partial y}$, *the area functional is*

$$J[z] = \iint_D \sqrt{1 + z_x^2 + z_y^2}\, dx\, dy.$$

The Euler equation for the extremalization of a functional of a function of two independent variables, $J(z) = \iint_D F(z, z; z_x, z_y)\, dx\, dy$, *is*

$$\frac{\partial F}{\partial z} - \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial z_x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial z_y}\right) = 0.$$

For the case under discussion we want to find the surface of least area spanned by a given contour. Putting $F = (1 + z_x^2 + z_y^2)^{1/2}$ *in Euler's equation, we obtain the second-order* PDE,

$$Q = z_{xx}(1 + z_y^2) - 2z_{xy}z_xz_y + z_{yy}(1 + z_x^2) = 0.$$

This equation has a simple geometrical meaning since

$$H = \frac{1}{2}(\kappa_1 + \kappa_2) = \frac{Q}{F}$$

is the mean curvature *of the surface. This implies that the mean curvature of the required surface equals zero. Surfaces with zero mean curvature are called* minimal surfaces*. The* PDE $Q = 0$ *was discovered by* **Lagrange** *but* **Euler** *showed by a different method that every minimal surface, not part of a plane, must be saddle-shaped and that its mean curvature must be zero at every point.*

The mean curvature of a surface at a point is defined as follows: consider a normal vector to the surface at the point and a plane containing this normal. As this plane rotates *about the normal, the curvature,* κ*, of the curve defined by its intersection with the surface, varies. Half the sum of its principal values:* κ_1 *(minimum),* κ_2 *(maximum) is the mean curvature at the point. In general, the planes containing these extremal curvatures will be perpendicular to each other. The Gaussian curvature* $\kappa = \kappa_1\kappa_2$ *is obviously negative for a minimal surface.*

The nonlinear differential equation $Q = 0$, although it has been pursued with remarkable success, has proven to be both cumbersome and essentially inadequate. Surfaces are excluded if they cannot be globally represented by a function $z(x, y)$, while the geometrical minimum problem, formulated for arbitrary surfaces, in no way permits such a restriction. Thus it is advantageous to represent the minimal surface parametrically by a vector $\boldsymbol{r}(u, v)$ with components (x_1, x_2, x_3) which are themselves functions of the parameters u, v.

The area is expressed by the functional

$$A(\boldsymbol{r}) = \int\!\!\int_D w\, du\, dv$$

where

$$w = \sqrt{EG - F^2}, \quad E = \frac{\partial \boldsymbol{r}}{\partial u} \cdot \frac{\partial \boldsymbol{r}}{\partial u} = \sum_{k=1}^{3} \left(\frac{\partial x_k}{\partial u}\right)^2,$$

$$G = \frac{\partial \boldsymbol{r}}{\partial v} \cdot \frac{\partial \boldsymbol{r}}{\partial v} = \sum_{k=1}^{3} \left(\frac{\partial x_k}{\partial v}\right)^2, \quad F = \frac{\partial \boldsymbol{r}}{\partial u} \cdot \frac{\partial \boldsymbol{r}}{\partial v} = \sum_{k=1}^{3} \frac{\partial x_k}{\partial u} \cdot \frac{\partial x_k}{\partial v}$$

and D is the domain in the plane bounded by (u, v)-plane image of the closed Jordan curve γ that spans the minimal surface.

Euler's conditions for the functional w are a system *of differential equations*

$$\frac{\partial}{\partial u}\frac{\partial w}{\partial \alpha_k} + \frac{\partial}{\partial v}\frac{\partial w}{\partial \beta_k} = 0,$$

where $\alpha_k = \frac{\partial x_k}{\partial u}$, $\beta_k = \frac{\partial x_k}{\partial v}$, $k = 1, 2, 3$. These equations express again the fact that the mean curvature of the surface $\boldsymbol{r}(u, v)$ is zero.

Although the mathematical problem of proving the existence of a surface $\boldsymbol{r}(u, v)$ that solves the preceding differential equations and is bounded by a prescribed curve γ, has long defied mathematical analysis, the soap film experiments of Plateau have exhibited surfaces of stable equilibrium*, which corresponds to a* relative minimum area*.*

However, the general *Plateau problem can accommodate contours bounding* unstable *minimal surfaces, whose areas do not furnish relative minima.*

During the 19th century the Plateau problem was solved for many specific contours. Progress was made on the basis of one idea: taking advantage of the freedom of choice of the parameters u and v, one can simplify the nonlinear differential equations by introducing isometric parameters *u, v such that $F = 0$, $E = G = w$. The differential equation immediately becomes*

$\nabla^2 \boldsymbol{r} = 0$ *or* $\nabla^2 x_k = 0$ *and the corresponding surfaces are called Harmonic surfaces.*

Harmonic functions $x_k(u, v)$ *may be considered as real parts of analytic functions* $f(w) = x_k + i\widetilde{x}_k$ *of the complex variable* $w = u + iv$, *where* $\widetilde{x}_k$ *is the conjugate harmonic to* x_k. *It then follows from the Cauchy-Riemann relations that*

$$\phi(w) = \left(\frac{df_1}{dw}\right)^2 + \left(\frac{df_2}{dw}\right)^2 + \left(\frac{df_3}{dw}\right)^2$$
$$= \sum_{k=1}^{3} \left(\frac{\partial x_k}{\partial u} - i\frac{\partial x_k}{\partial v}\right)^2 = (E - G) - 2iF = 0.$$

This is known as the characteristic equation *of the minimal surface.*

As domain D for the variables u, v or for $w = u + iv$ *we may choose the disc* $u^2 + v^2 \leq 1$. *Plateau's problem is to solve the equation* $\nabla^2 \boldsymbol{r} = 0$ *for D under the additional conditions that* $\phi(w) = 0$ *and that* $\boldsymbol{r}$ *maps the boundary* $u^2 + v^2 = 1$ *onto the prescribed contour* γ. *After this simplification, the nonlinear character of the problem remains only in the boundary condition and in the condition* $\phi(w) = 0$.

A natural question that may be asked about a given contour is how many minimal surfaces can it bound? *It has long been known that planar curves bound only one minimal surface, a planar one. A second result was found (1932) by* ***Tibor Radó***[730] *(1895–1965, Hungary) who showed that if a contour has a simple projection onto a convex curve in a plane, then it can bound only one disc-type minimal surface. It was then shown that if a given contour has a* total curvature *of less than* 4π *(a circle of radius* 1 *has a total curvature of* 2π*), then there can be only one disc-type minimal surface bounded by that contour. If however, the total curvature of a curve is even slightly larger than* 4π, *then the curve can bound* infinitely many *disc-type minimal surfaces. An algebraic formula relating the minimal surfaces that span a fixed contour was derived by* ***Marston Morse*** *(1892–1977, USA) in 1936.*

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[730] The disc version of the Plateau problem was solved simultaneously by **Jesse Douglas** (1897–1965, USA) and **Tibor Radó** in 1930–1931.

1931 CE **Georges de Rham** (1903–1990, Switzerland). Mathematician. A leading topologist of the 20th century. Known especially for his "*de Rham Theorem*".

de Rham was born in Roche, Canton Vaud, Switzerland. He was educated at the Universities of Lausanne (1921–1925) and Paris (Ph.D. 1921) and held positions at the Universities of Göttingen (1930–1931), Lausanne (1932–1971) and Geneva (1936–1973).

1931 CE, May 01 *Empire State Building* officially opened at 350 5th Avenue in New York City. The building has 102 stories and is 381 meters high. It was the world's tallest building until 1972.

It cost about 41 million dollars and houses about 10,000 tenants. The building's steel structure creaks slightly when heavy winds cause it to sway and during winds of 160 km/hour, it yields a maximal sway of 3.7 cm from center.

1931 CE, May **Auguste Piccard** (1884–1962, France). Physicist. Ascended in his airtight *gondola* [which he attached to a huge hydrogen-filled balloon] to a height of 15,880 meters, the first venture into the *stratosphere*. In 1932 he ascended 16,800 meters and gathered information on cosmic rays and radioactivity.

1931 CE, June The *Yellow River* (Huang-ho) in Honan Province, China, overflowed, causing the worst flood in recorded history: over two million people perished out of a total of 180 million affected by the flooding.

1931 CE, June **Wiley Post** and **Harold Gatty**. Americans; *circled the globe* by airplane in 8 days, 15 hours elapsed time. In 1933 Wiley Post, alone, circled the globe in 7 days, 18 hours.

1931–1933 CE **Karl Guthe Jansky** (1905–1950, U.S.A.). Radio engineer. Discovered the cosmic radio waves — a radio emission from the Milky Way.

Jansky was the first person to detect radio waves from outside the solar system. His discovery led to the development of *radio astronomy*[731] (a branch

[731] The first radio telescope was built in 1937 by **Grote Reber** (1911–2002, U.S.A. and Tasmania). This lone amateur was the only radio astronomer until 1946, when wartime research provided a separate stimulus. A great opportunity was lost during these years, for since then radio communications have grown so rapidly that the longer wavelengths are all but closed to radio astronomy by overwhelming man-made radio interference.

of astronomy that studies radio waves from stars and other celestial objects). In 1931 he was carrying out for Bell Laboratories a study of the noise level to be expected when a sensitive short-wave radio is used with a directional aerial system in long-distance communication. He wanted to track down the source of inexplicable noises that were interfering with radio transmission to ships at sea and across the Atlantic. He was expecting crackling noises from thunderstorms but he heard only steady hissing sound, quite different in character from the crackles of thunderstorms.

He then found that the noise level never decreased below a certain level and that the greatest signal always occurred when the aerial pointed in a certain direction in space, fixed relative to the stars and not relative to the earth or even the sun. After extensive study, Jansky determined that the noise came from the neighborhood of constellation Sagittarius, near the center of the Milky Way galaxy. Thus, as often happens in science, Jansky discovered something completely unexpected.

Had 1932 been a year of high sunspot activity, Jansky would undoubtedly have found the radiation from the sun for which Edison had looked in vain. As it was, the sun was quiescent, and instead the radio waves coming from our galaxy were discovered.

Most technicians probably would not have spent much time fretting over a peculiar hissing. It was so faint it hardly seemed of any consequence. But Jansky would not let the puzzle rest. A frail, dedicated young man who suffered from a chronic kidney disease, he examined every conceivable cause for the 'flaws' in the antenna: disturbances by nearby power lines, electrical storms or stray signals from radio transmitters in the vicinity. Nothing seemed to explain the noise.

It was not until the 1950s' that the Russian physicist and Nobel Laureate (2003) **V.L. Ginzburg**, worked out the theory of *synchrotron radiation*, which explains the observed radio spectrum. Synchrotron radiation results from electrons moving at speeds close to the speed of light in magnetic fields.

Reber was born in Chicago. He received the B.Sc degree from the Illinois Institute of Technology, Chicago (1933). In 1962 he received an honorary D.Sc. degree from the Ohio State University, Columbus. In 1963 he received an Eliot Cresson gold medal from Franklin Institute of Philadelphia, Pennsylvania.

In the 1950s, Reber sought a field that seemed neglected by most other researchers and turned his attention to cosmic radio waves at very low frequencies (1–2 MHz, or wavelength 150–300 meters). Waves of these frequencies cannot penetrate the Earth's ionosphere except in certain parts of the Earth at times of low solar activity. One such place is Tasmania, where Reber lived for many years. He died in Tasmania on December 20, 2002.

Our galaxy is full of high speed charged particles, including electrons, known as "cosmic rays". We now believe that these particles were blasted into interstellar space as a result of supernova explosions. This is the origin of most of the radio radiation from the Milky Way that Jansky and Reber measured.

Jansky was born in Norman, Oklahoma and graduated from the University of Wisconsin.

Chemical Bonds[732] *— from Kekulé to Pauling (1858–1939)*

Comparatively few substances (among them the noble gases and mercury vapor) are composed of discrete atoms. More commonly, atoms combine into larger aggregates: molecules (Cl_2, H_2O, CO_2, O_2, N_2, H_2, C_8H_{18}); *ionic solids* [Na^+Cl^-, $Ca^{++}(F^-)_2$]; *metallic solids* (Na, Cu, Fe); *and covalent crystals* (*giant molecules such as diamond or quartz*).

The fundamental problem of chemistry is — which kind of forces cause the binding of atoms, either like or unlike, in a molecule. One has to explain, for example, why is there H_2 *and no* H_3, *CO and* CO_2 *but no* CO_3, H_2SO_4 *and no HSO, etc. Any attempt to interpret the course of a chemical reaction, such as the decomposition of nitrogen dioxide,* $2NO_2 \rightarrow 2NO+O_2$, *must start with the intimate details of molecular structure. These include not only (in this example) the strength of the* $N—O$ *and* $O—O$ *bonds, but also the sizes and shapes of the molecules.*

To date, molecular geometry (bond lengths, bond angles), has been determined for thousands of molecules via a variety of techniques, including microwave spectroscopy and electron and X-ray diffraction[733]. *In principle, if*

[732] To dig deeper, see:

- Pauling, L., *General Chemistry*, Dover, 1988, 959 pp.

[733] The gas-phase electron diffraction was developed in Germany in 1930 by **G. Mark** and **R. Wierl.** They built the first electron diffraction unit and studied the structure of several dozen simple molecules. Further work carried out in the 1930's and 1940's revealed important patterns of molecular geometry.

we know the structure of a molecule, all other properties, such as its melting point, its boiling point, its viscosity as a liquid and as a gas, its hardness and malleability as a solid, its solubility in other substances, and its color, should be predictable.

Consequently, the physical and chemical properties of matter are related to the molecular structure of its component parts and can best be understood in terms of one of the most powerful and pervasive theories of science — the theory of the chemical bond, which finalized the development of the electronic theory of valence.

The theory of valence originated in the middle of the 19^{th} *century. With the rapid development of organic chemistry, there arose the need for a unifying structural theory of organic compounds. It was provided by* ***A.S. Couper*** *and* ***F.A. Kekulé*** *(1858). Working independently, they both concluded that carbon has a common valence of four, and that carbon atoms could link to form chains. This theory culminated more than half a century of spectacular advances in analytical and synthetical chemistry and was inextricably linked to the simultaneous development of a rational scale of atomic weights. The theoretical explanation of the periodic table was the first major achievement of atomic physics for chemistry (1869). The valence theory was subsequently given a 3-dimensional form by* ***J.H. van't Hoff*** *and* ***J.A. Le Bell*** *who showed independently (1874), that the 4-valence bonds of the carbon atom are directed toward the corners of a regular tetrahedron; it deserves to rank as one of the outstanding intellectual achievements of the* 19^{th} *century.*

Soon after the discovery of the electron by ***J.J. Thomson*** *(1897), efforts were made to develop a more detailed structural theory of valence; the general ideas of electron transfer and electron sharing were developed at this time, but detailed electronic structures could not be assigned with confidence because of the lack of knowledge of the number of electrons in an atom and lack of information about atomic structures in general.*

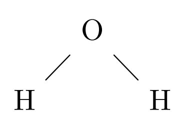
The determination of the atomic numbers of the element by ***H.G.J. Moseley*** *(1913) and the development of an early quantum theory of the atom by* ***N. Bohr*** *(1913), provided the basis for further progress. A most important contribution was made by* ***G.N. Lewis*** *(1916), who pointed out the significance of completed shells of 2 and 8 electrons. Lewis and* ***Irving Langmuir*** *(1919) identified the covalent bond with a pair of electrons shared by two atoms and jointly occupying an outer-shell orbital belonging to each atom.*

After the discovery of the theory of quantum mechanics (1925–1926), when ***E. Schrödinger*** *and* ***W. Heisenberg*** *were formulating the modern theory of the hydrogen atom, and* ***W. Pauli*** *was discovering his exclusion principle — a detailed quantitative theory of the covalent bond was developed. The* physical explanation of chemical bonds in homopolar molecules, achieved by the methods of quantum mechanics in 1927, completed the successful attempts at understanding the basic features of chemistry from the property of atoms and the matter-waves of electrons.

The chemical bond is the manifestation of the electrostatic interaction between charged units such as atoms and molecules; together with quantum effects. There is a binding force between atoms; consequently, there are polyatomic molecules in nature. Similarly, there is a binding force between molecules; consequently, there are inter-molecular forces and bonds, and aggregated states of matter. Were it not for chemical bonds, all matter would be gaseous and chemically inert, and compounds (let alone life) would not exist.

Clearly, the chemical bond must be something more specific than a simple attraction between atoms, or we would have to refer to all water in a glass as a single molecule, or to a table top as a single molecule. This specific property is really only one of degree, but generally if the attraction between two atoms is such that an energy of at least 10 kcal/mole at room temperature is required to break it, a chemical bond exists and that energy is called bond energy or dissociation energy.

The idea of a chemical bond is an important and logical hypothesis that is overwhelmingly supported by experimental evidence. The assumption that a molecule of many atoms is a structure connected by bonds, each joining two atoms, is an oversimplification, although it is usually justified. For example, consider the water molecule, H_2O. The chemist designates its structure by



where there is a single chemical bond between the oxygen atom and each of the hydrogen atoms. Yet there is no bond between the two hydrogen atoms.

In principle, the Schrödinger equation and the postulates of quantum mechanics are all that is required to calculate all properties of any molecule. In practice, however, the exact solution of the Schrödinger equation for complicated molecules has not been achieved; analytical and numerical approximations, coupled with experiments, are required to determine the structure and behavior of molecules. Although approximate methods of solution lead us to important conclusions, the theoretical predictions seldom have a precision as good as 10 percent. The difficulty is not in the theory; rather, it is a mathematical problem. Even for the hydrogen molecule, the difficulty of solving the fundamental equations of the theory is formidable, and tedious calculations *are required. For other molecules, the mathematical difficulties preclude an exact solution even with the aid of high-speed electronic computers.*

The major conclusions of quantum mechanical analysis are generally in agreement with the following experimental observations:

I. *Except in special cases, electrons constituting a bond are* localized *to the region of the bond and the two atoms it connects. Shifting of electrons between bonds can be important but is generally a second-order effect. The poor electrical conductivity of nonmetallic crystals such as diamond is an experimental demonstration of this fact. It is this* localization of electrons *that enables us to think of a chemical bond as something real.*

II. *Electrons in bonds are described by* molecular orbitals. *These orbitals are one-electron wave-functions that are approximated by* weighted sums of atomic orbitals *centered at the (two or more) atoms of interest. Such molecular orbitals are extremely useful in developing a descriptive picture of the nature of molecules. These sums, or* linear *combinations of atomic orbitals are merely approximations, not exact representations.*

Chemical bonds may be divided into two groups:

A. Intramolecular Bonds

From one point of view a molecule is a stable arrangement of a group of nuclei and electrons. The exact arrangement is determined by electromagnetic forces and the laws of quantum mechanics (Heisenberg's uncertainty principle, Pauli exclusion principle). This concept of a molecule is a natural extension of the concept of an atom.

Another view regards a molecule as a stable structure formed by the association of two or more atoms. In this view the atoms retain their identity. In general, the structure and properties of molecules are best described as a combination of both views.

When a molecule is formed from two atoms, the inner-shell electrons of each atom remain tightly bound to its nucleus and are barely disturbed at all. The outermost loosely bound electrons, however, are strongly disturbed *and are influenced by all the particles (ions + electrons) of the system. Their wave functions are significantly modified when the atoms are brought together. Indeed, it is this very interaction that leads to binding, i.e., to a* lower total *energy, when the nuclei or ions are close together. This bond, or interatomic force, is of electromagnetic (mainly electrostatic) origin.*

There are two principal types of intra-molecular binding, the ionic bond *and the* covalent bond*.*

I. IONIC BOND *is caused by the* Coulomb electrostatic attraction *between positively and negatively charged ions, and molecules thus bound can form ionic crystal lattice. Each ion in the crystalline compound has as its nearest neighbors, ions of the opposite sign; ions of the same sign are more distant from each other. The crystal is held together by the* relatively strong *electric attractions between the ions, the* electrovalent bond*. The cations and anions exist as discrete, not paired, units. The strength of the bond accounts for the high melting point, transparency, nonvolatility, hardness, brittleness and generally high stability of these substances.*

A typical example is common salt, $Na^{+}Cl^{-}$*. The sodium atom has lost its outer-shell electron (or valence electron) to the chlorine atom*[734]*. Since the sodium atom has now* 11 *protons and* 10 *electrons it became electropositive, while the chlorine atom now has* 17 *protons and* 18 *electrons, which makes it electronegative. Moreover, the electronic configuration of the* Na^{+} *ion is the same as that of an atom of neon, the noble gas preceding sodium; while the electronic configuration of the* Cl^{-} *ion is the same as that of an atom of argon, the noble gas following chlorine. In many such ionic compounds the atoms have the* stable electronic configuration *of noble gas atoms. Other examples are:* KBr, CaF_2, $AgCl$, $Ba(NO_3)_2$, $CaCO_3$*.*

Each ionic lattice *is characterized by two fundamental entities: plan of the elementary cell and the packing of the ions. Thus, in the NaCl crystal lattice, we have a* 3*-dimensional checker-board arrangement in which each sodium ion has* 6 *equivalent chlorine ions as nearest neighbors while each chlorine ion similarly has* 6 *equivalent sodium ions as nearest neighbors. The edge of the elementary cell, according to X-ray diffraction measurements, is* 5.64 Å*. It was also possible to determine that the chlorine ion is approximately* twice as large *as the sodium ion.*

Each ion in the lattice has ions of opposite charge as nearer neighbors and ions of like charge as more distant neighbors. The resultant effect is a net attractive electrostatic configuration*, with* inter-ionic forces *generally far stronger than the Van der Waals forces between neutral molecules containing comparable number of electrons. Consequently, ionic crystals, like covalent*

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734 More accurately, the electron is *shared* but spends more time in the chlorine component of the superposition of single-atom orbitals.

crystals, are far more stable physically than molecular crystals having molecular weights of similar magnitude. Computations show that about 90 *percent of the total dissociation energy of NaCl (183 kcal/mole) is attributed solely to electrostatic potential energy of Coulomb forces between ions; the balance is attributed to Van der Waals interaction.*

II. Covalent bond *exists between two atoms that* share *an electron-pair, jointly occupying an outer-shell orbital belonging to both of them. The atoms in these molecules are held tightly together in a bond that is nearly universally present in substances. It is conveniently represented by a connective line or* valency bond *drawn between the two atoms. This device was introduced as early as* 1864 *by the Scottish chemist* ***Alexander Crum Brown*** *(1838–1922) long before* ***G.N. Lewis*** *interpreted each line as representing a shared pair of electrons (1919).*

The simplest example of a covalent molecule is the hydrogen molecule, in which the two electrons are held jointly *by the two nuclei, and enable a firm attachment between them. This strong bond holds the nuclei at an average distance of* 0.74 *Å apart (they oscillate with an amplitude of a few hundredths of an Ångström at room temperature). A large energy of* 103.4 *kcal/mol is required to break the bond.*

In general, stable molecules (or complex ions) having covalent bonds have structures such that each atom achieves a noble-gas electronic configuration or some other stable configuration, the shared electrons being counted for each of the bonded atoms. The exact solution of the SE, even for a molecule as simple as H_2*, is not feasible and several approximate methods are used instead.* The valence bond method *regards the molecule as made up of atoms, slightly distorted to produce a bond. It is consistent with the traditional chemical concept of* localized bonds *between atoms, a point of view supported by the additivity of bond lengths and bond moments.*

Another approximation, the molecular orbital method*, considers the molecule as a collection of nuclei and electrons, the individuality of the atoms having largely disappeared. The bonding electrons belong to the molecule as a whole and need not be regarded as localized between pairs of nuclei. The interpretation of molecular spectra, for example, does not require any assumptions about localized bonds. This scheme is backed up mathematically by the* Born-Oppenheimer approximation *(1927).*

A covalent bond serves as a basis to a covalent lattice *in which a network of covalent bonds extends in fixed patterns throughout a crystal to form* a single giant molecule*, such as* diamond*: the unit cell shows a cubic symmetry, each atom having* 4 *equivalent nearest neighbors, located at positions outlining* *the corners of a regular tetrahedron about it. Since the carbon atom has just 4 electrons and 4 vacancies in its valence shell, it seems clear that in diamond the atoms are bonded to each other by covalent chemical bonds. Since the diamond crystal shows exceptional physical stability and is one of the hardest solids known, it is logical to regard the entire crystal as a single, gigantic molecule. Silicon and germanium crystallize in the same manner. Other examples are SiC (carborundum), SiO_2 (quartz), BN (borazon).*

As distinguished from the ionic bond, the covalent bond is often directional. *The directional property is not present in the H_2 molecule since the probability density of the valence electron in each separated H atom is spherically symmetrical, so that the only defined direction in the H_2 molecule is the one connecting the two nuclei, and the covalent bond acts along that direction, whatever it may be. In a more typical case the probability of a valence electron* has its own directional dependence *and certain preferred directions for forming covalent bonds. The directional properties of covalent bonds are manifested in the structural properties of covalently bonded molecules, and so form the basis of organic chemistry.*

The bond energies *for single covalent bonds range (at 25° C) from about 30 to 135 kcal per mole [e.g., C—H (99); N—H (93); H—H (104); H—F (135); C—O (84); C—C (83)]. Double and triple bonds (with each* bonding electron pair *represented by a line) may be higher [e.g., C=C (147); C=O (174); C ≡ C (194); N ≡ N (226)].*

Metallic solids exhibit a binding that can be thought of as a limiting case of covalent bonding in which electrons are shared by all *ions in the crystal.*

III. METALLIC BOND *exist between positive metal ions and mobile valence electrons.*

Although copper and argon atoms crystallize in precisely the same geometric pattern — a close-packed cubic structure — their physical properties are entirely different. A typical metal, copper is opaque with a highly reflecting surface. It is an excellent conductor of both heat and electricity. Its melting point is rather high at 1083° C and the liquid boils at 2582° C. Argon crystals are transparent, resembling Dry Ice in appearance. They are poor conductors of heat and electricity. The crystals melt far below room temperature at −189.3° C, and the liquid boils at −185.9° C. The nature of the forces in the two systems must therefore be totally different.

Consider the metals silver and copper: these have one *valence electron per atom which is responsible for the bonds to* all *neighbors in the lattice. Now, most metals crystallize with an atomic arrangement in which each atom has surrounded itself with the maximum number of atoms that is geometrically* *possible (cubic closest-packed structure, hexagonal closest-packed structure, body-centered cubic structure, etc.). Thus, for a silver atom which is symmetrically surrounded by 12 nearest neighbors, only $\frac{1}{6}$ of one electron is responsible for the bond between two neighboring atoms, in contrast to the 2 full electron charges between each two carbon atoms in diamond.*

Quantum-mechanically, the wave-function describing the behavior of the valence electrons is approximated by a linear combination of the wave functions describing the 12 possible bonds to the 12 partners. This is as if one *"real" electron-pair rotates and successively form the individual bonds. What counts in the final analysis is only the* average electron density *between the atoms to be bound, due to the overlap of atomic orbitals. Therefore, the valence electrons occupy a partially filled bond of delocalized molecular orbitals with closely spaced energies. The relatively high concentration of shared valence electrons in the regions between neighboring nuclei is associated with a lowered electrostatic potential energy through screening of nuclear charges.*

The overall effect is not unlike a covalent bond, except that there are vacant molecular orbitals *at energies slightly above the highest occupied ones. The bonding electrons are therefore* comparatively mobile: *little energy is required for them to transfer from one atom to another throughout the crystal.*

The physical picture of metallic binding is thus a negatively charged free-electron gas permeating a structure of positive ion cores. Their mobility accounts for the high electric and heat conductivities of the metals. The metallic bond per pair of nearest neighbor atoms is not as strong as ionic or covalent bonds. Nevertheless, ordinary metals are quite strong because of the large number of nearest neighbors.

B. Intermolecular Bonds

Molecular solids consist of molecules which are so stable that they retain much of their individuality when brought in close proximity. The electrons in the molecule are all paired so that atoms in different *molecules cannot form covalent bonds with one another.*

One kind of intermolecular binding force is the weak Van der Waals attraction *that is present between such molecules in the gaseous, liquid and solid phases. This attraction is the result of the mutual* interaction of the electrons and nuclei *of the molecules; it has its origin in the electrostatic attraction of the nuclei of one molecule for the electrons of the other.*

At small distances (about 4 Å *for argon, for example) this force of attraction is completely compensated (balanced) by a force of repulsion of electrons by electrons and nuclei by nuclei due to interpenetration of the outer electron shells of the molecules.*

It is these intermolecular Van der Waals forces that enable substances such as the noble gases, the halogens, etc., to condense to liquids and to freeze into solids at sufficiently low temperature. The physical mechanism involved in the Van der Waals attraction is an interaction between fluctuating electric dipoles. Because of the fluctuating quantum mechanical behavior of the electrons in a molecule[735]*, all molecules have a fluctuating electric dipole moment, even though for many of them symmetry considerations require that it fluctuates about an average value of zero. At a time when a molecule has a certain instantaneous electric dipole moment, the external electric field that it produces will induce in the charge distribution of a nearby molecule a dipole moment such that the resulting force between the inducing and the induced electric dipole is always attractive.*

The binding energies are of the order of 10^{-2} *eV and the force, known as London force (1930) generally varies approximately as the inverse seventh power of the intermolecular separation at large distances. In the solid, successive molecules have electric dipole moments which alternate in orientation so as to produce nearest-neighbor attractions between them. Many organic compounds, inert gases, and ordinary molecular gases such as oxygen, nitrogen, and hydrogen form molecular solids in the solid state.*

The Van der Waals bond is the weakest form of binding, but it occurs with all atoms. Only in the inert gas crystals (which solidify at temperatures of only a few degrees above absolute zero) is it the predominant form of binding. Inert gas atoms are closed shell, neutral atoms with strongly held valence electrons. With no valence electrons about, their closed shells rule out the other types of binding.

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[735] The *Heisenberg uncertainty principle* does not allow the centroid of the negative charge distribution (the center of the electron cloud) to be at the same place as the center of the positive charge distribution (the nucleus) all the time. Therefore, at some instant of time, the plus and minus charge centers of atom 1 will be separated by some distance. This separation gives atom 1 an electric dipole moment and sets up at atom 2 an electric dipole *field* centered at atom 1 which is proportional to r^{-3} (r is the separation of the centers of the atoms). This field *induces* an average dipole moment $\sim r^{-3}$ in atom 2. Therefore the electric binding energy due to this mechanism is proportional to r^{-6}, and the force to r^{-7}.

Because the binding is weak, solidification takes place only at very low temperatures, where the disturbing effects of thermal agitation are very small. The weak binding makes molecular solids easy to deform and compress, and the absence of free electrons makes them very poor conductors of heat and electricity. Intermolecular bonds of the Van der Waals type occur among:

I. NON-POLAR MOLECULES *with the appropriate molecular composition and symmetry such as Cl_2, have the center of the positive charge halfway between the nuclei, which is also the center of the negative charge. Yet, a second order Van der Waals attraction is formed between the non-polar covalent molecules. Other examples are CH_4, CF_4, CCl_4, CI_4, I_2, Br_2, H_2, CO_2, C_6H_6, where the vector sum of all bond moments in the molecule average to zero either due to linearity or spatial symmetry.*

II. POLAR MOLECULES *with non-zero vector sum of all dipole moments in the molecule. Such is the HCl molecule, in which the electron pairs are not shared equally in the covalent bond. Consequently, the centers of positive and negative charge do not coincide. Between such molecules we may have either a dipole-dipole interaction (which is either attractive or repulsive, depending on the orientation of the two dipoles) or dipole-induced dipole interaction, through which a molecule with a permanent dipole moment is able to induce a dipole moment in neighboring molecule, which always results in an attractive force between them. Other examples are: N_2O_5, P_4O_{10}, HNO_3.*

III. HYDROGEN BOND: *The covalently bonded hydrogen is unique among other atoms in chemical combination, because it has no inner shell of non-bonding electrons partially screening the nucleus. Particularly when it is bonded to one of the highly electronegative atoms (such as fluorine, oxygen or nitrogen), the pair of shared electrons is closer to the other nucleus, causing the whole molecule to become polarized.*

Consider, e.g., the water molecule[736] $H\text{—}\ddot{\underset{..}{O}}\text{—}H$; *due to the covalent bond between oxygen and hydrogen, an oxygen atom will form a 2-electron shared*

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[736] For many purposes it is convenient to represent the outer-shell electronic configuration succinctly by means of electron-dot formulas: The chemical symbol for the element is written with as many dots around it as corresponds to the *number of valence electrons present in the outer shell*, e.g.:

1. Neutral *hydrogen* has a positive proton in the nucleus and one valence electron in its outer shell; its symbol will be $H\cdot$

2. Neutral *oxygen* has 8 protons and 8 neutrons in its nucleus, two *paired* electrons (opposite spins ⇅) in its inner shell, and 6 electrons in its outer shell (2*s* and 2*p* subshells), of which 4 are paired (⇅, ⇅) and 2 are unpaired. Its symbol

pool with each hydrogen atom. But the oxygen atom, having the stronger hold on valence electrons, will hold them more firmly, so to speak, in its own outermost shell. These electrons will occupy the electron shells of the hydrogen atom to a correspondingly lesser degree.

Since the oxygen atom has more than an equal share of the pooled (bonding pair) electrons, it has more than an equal share of the negative charge of those electrons. There will be therefore a fractional negative charge on the oxygen atom. The hydrogen atom, deprived of its fair share, will have a small balancing positive charge.

Two such molecules can thus attract each other, mainly by an electrostatic interaction between a proton belonging to one and the unshared pair of electron belonging to the other; for example, two molecules $H\!-\!\ddot{\underset{\cdot\cdot}{F}}:$ *will join into a dimer* H_2F_2 *by the hydrogen bond* $H\!-\!\ddot{\underset{\cdot\cdot}{F}}: - - - H - \ddot{\underset{\cdot\cdot}{F}}:$ *, indicated by a dashed line. Likewise, the molecule of ammonia*

$$\begin{array}{c} H\!-\!\ddot{N}\!-\!H \\ | \\ H \end{array}$$

is capable of joining another ammonia molecule through a hydrogen bond.

Most common hydrogen bonds are of medium energies between 3 and 7 kcal/mole (a few tents of eV per bond), which is about one tenth as strong as a typical covalent bond. This fact is of crucial importance to the chemistry of

is $:\dot{\underset{\cdot\cdot}{O}}\cdot$

3. Neutral *fluorine* has 9 protons in its nucleus, two *paired* electrons in its inner shell, and 7 electrons in its outer shell, 6 of which are paired. Its symbol is $:\ddot{\underset{\cdot\cdot}{F}}\cdot$

4. Neutral *nitrogen* has 7 protons in its nucleus, two *paired* electrons in its inner shell, and 5 electrons in its outer shell, 2 of which are paired. It symbol is $:\dot{\underset{\cdot}{N}}\cdot$

5. Neutral *carbon* has 6 protons in its nucleus, two *paired* electrons in its inner shell, and 4 electrons in its outer shell, 2 of which are paired. Its symbol is $\ddot{\underset{\cdot}{C}}\cdot$

6. Neutral *neon* has 10 protons in its nucleus, two *paired* electrons in its inner shell, and 8 electrons in its outer shell, all paired. Its symbol is $:\ddot{\underset{\cdot\cdot}{Ne}}:$

life[737] *because it allows for weak, but highly specific, intermolecular interactions that can be altered without affecting the covalently bonded structures of the molecules in question.*

The fact that ammonia, water, and hydrogen fluoride are all capable of hydrogen bonding to themselves *has varied consequences and of great importance, especially for water. For example, the melting and boiling points of these compounds are abnormally high relative to other hydrogen compounds in the same groups. Without it water would be a gas at room temperature.*

Most substances diminish in volume and hence increase in density, with decrease in temperature. Water has the very unusual property of having a temperature at which its density is maximum. This temperature is $4°C$. *With further cooling below this temperature the volume of a sample of water* increases *somewhat. A related phenomenon is the increase in volume which water undergoes on freezing. Without this anomaly, life in lakes and rivers would not be possible during severe winter conditions, since they would freeze (throughout — not just a surface layer, as actually occurs). All these unusual properties of water are due to the* hydrogen bond *for the following reason: each molecule has two attached hydrogen atoms and two unshared electron pairs and hence can form* 4 *hydrogen bonds with its neighbors.*

Indeed, X-ray diffraction studies of ice *show that each oxygen atom is surrounded by* 4 *other oxygen atoms located about it at the corners of a* regular tetrahedron. *The hydrogen atoms presumably lie on the lines between the centers of the oxygen atoms. This structure, in which each molecule is surrounded by only* 4 *immediate neighbors, is a very open structure, and accordingly ice is a substance with abnormally low density.*

When ice melts, this tetrahedral structure is partially destroyed and the water molecules are packed more closely together, causing water to have greater density than ice. Many of the hydrogen bonds remain, however, and aggregates of molecules with the open tetrahedral structure persist in water above the freezing point. With increase in temperature, some of these aggregates break up, causing a further increase in density of the liquid; only at $4°C$ *does the normal thermal expansion due to* increase in molecular agitation *overcome this effect, and cause water to begin to show the usual decrease in density with increasing temperature.*

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[737] The hydrogen bonds in DNA can be "unzippered" open for replication of gene structures, then closed again until needed. The hydrogen bond is also important in certain ferroelectric and polymerization processes.

A great many salts are soluble in water, whereas they are insoluble in most other common solvents[738]*. The high solubility of salts in water is the result of two closely related properties of water:*

(1) *Water has an extremely high dielectric constant (80). This arises not only from the polarity of the individual molecules, but also from the correlated mutual orientations of the molecules. So the force of attraction between oppositely charged ions in water, as given by Coulomb's law is only* $\frac{1}{80}$ *the force that would exist between them in vacuum (or in air). Thus, a great many salts dissociate into ions upon contact with water.*

(2) *Water molecules have a large dipole moment (1.86 D; 1 D(Debye) = 20.85 e · picometer), which interacts strongly with the ions, especially the cations, to form hydrated ions. This process is accompanied by release of thermal energy. Typical hydrated ions are* $[Be \cdot (H_2O)_4]^{2+}$, $[Mg \cdot (H_2O)_6]^{2+}$, $[Al(H_2O)_6]^{3+}$ *in which the relatively negative end of the water molecule (the oxygen) interacts with the positive central ion.*

This striking power of water to dissolve ionic substances is partly due to its power to form hydrogen bonds[739]*.*

A few other liquids are ionizing solvents like water, resulting in salt solutions that conduct electricity because of the mobile ions in the solution. Some such liquids are hydrogen cyanide (HCN), hydrogen peroxide, liquid ammonia (NH_3) *and liquid hydrogen fluoride (HF). They all have high dielectric constants and large dipole moments, and are therefore called polar solvents. Hydrogen bonding is the factor primarily responsible for solubility of organic compounds in water.*

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738 If water were like most other liquids, the salt-like minerals would be practically insoluble in it and it would be far above its boiling point at the mean temperatures of the earth's surface. Therefore, life as we know it is completely dependent on the peculiar physical properties of water.

739 In some cases, however, some hydration actually has the opposite effect, increasing the stability of ionic solids or even allowing them to crystallize. Such hydrated crystals spontaneously pulverize into powder at low enough ambient humidity levels.

The Mystery of Aqua Regia

The ancient metallurgists — as well as brewers, dyers, and potters – accumulated much empirical knowledge that is now incorporated into the science of chemistry. Thus, the resistance of gold to corrosion by atmospheric oxygen, water, and acids has long been known. Indeed, Electrum, a natural mixture of gold and silver, was used for the early coins minted in Lydia in the eighth century BCE. Copper was added to coins by the Roman emperors to conserve the more precious metals.[740]

Aqueous acids "dissolve" most of the free metals by oxidation. Though none of the common laboratory acids are individually able to dissolve gold, a combination of 4 *parts of* HCl *to* 1 *part of* HNO_3 *does it. This fact was known already to medieval alchemists and the early chemists [e.g.* ***Lully*** *(1305),* ***Libau*** *(1611),* ***Van Helmont*** *(1648)]. The mixture is known as aqua regia, since gold was considered the "king of metals".* ***Gay-Lussac*** *(1844) tried to explain the phenomenon through the atomic theory known in his time. It was not until the theory of oxidation-reduction reactions through electron transfer became well-understood [****G.N. Lewis*** *(1916);* ***I. Langmuir*** *(1919);* ***G.A. Perkins*** *(1921);* ***W. Pauli*** *(1925);* ***L. Pauling*** *(1931)] that chemists finally resolved the aqua regia phenomenon.*

[740] Gold coins have long since been withdrawn from circulation in the United States, and the silver coins are being replaced by sandwich alloys of nickel and copper.

Table 5.7: PROPERTIES OF THE COINAGE METALS

	Cu	**Ag**	**Au**
Electron configuration	$[Ar]3d^{10}4s^1$	$[Kr]4d^{10}5s^1$	$[Xe]4f^{14}5d^{10}6s^1$
Atomic radius, 10^{-12} m	128	144	144
First ionization energy, kJ/mol	745	731	890
Electrode potential, V			
$M^+(aq) + e^- \to M(s)$	+0.520	+0.800	+1.83
$M^{2+}(aq) + 2e^- \to M(s)$	+0.340	+1.39	—
$M^{3+}(aq) + 3e^- \to M(s)$	—	—	+1.52
Oxidation states	+1, +2	+1, +2	+1, +3

The data in Table 5.7 helps us understand why this is so: The metal ions are easy to reduce to free metals, which means that the metals are difficult to oxidize.

In Mendeleev's periodic table, the alkali metals (group 1) and the coinage metals (group 11) appear together as group I. The only similarity between the two subgroups, however, is that both have a single s electron in the valence shells of their atoms. More significant are the differences between the group 1 and group 11 metals. For example, the first ionization energies for the group 11 metals are much larger than for the group 1 metals, and the standard electrode potentials are positive for the group 11 metals and negative for the group 1 metals.

Like the other transition elements that precede them in the periodic table, the group 11 metals are able to use d electrons in chemical bonding. Thus they can exist in different oxidation states, exhibit paramagnetism and color in some of their compounds, and form complex ions. They also possess to a high degree some of the distinctive physical properties of metals — malleability, ductility, and excellent electrical and thermal conductivities.

Copper, silver, and gold – the coinage metals – are used in jewelry making and the decorative arts. Gold, for instance, is extraordinarily malleable and can be pounded into thin translucent sheets known as gold leaf. The coinage metals are valued by the electronics industry for their ability to conduct electricity. Silver has the highest electrical conductivity of any pure element, but *both copper and gold are more often used as electrical conductors because copper is inexpensive and gold does not readily corrode. The most important use of gold is as the monetary reserve of nations throughout the world.*

The group 11 metals do not react with $HCl(aq)$, but both Cu and Ag react with concentrated $H_2SO_4(aq)$ or $HNO_3(aq)$. The metals are oxidized to Cu^{2+} and Ag^+, respectively, and the reduction products are $SO_2(g)$ in H_2SO_4 and either $NO(g)$ or $NO_2(g)$ in $HNO_3(aq)$.

Au does not react with either acid, but it will react with "royal water" – aqua regia. The $HNO_3(aq)$ oxidizes the metal and Cl^- from the $HCl(aq)$ promotes the formation of the stable complex ion $[AuCl_4]^-$

$$Au(s)+4H^+(aq)+NO_3^-(aq)+4Cl^-(aq) \rightarrow [AuCl_4]^-(aq)+2H_2O(l)+NO(g).$$

1931–1946 CE **Robert Jemison Van de Graaff** (1901–1967, U.S.A.). Physicist. Invented the electrostatic generator named after him, designed to accelerate particles in the realm of '*low energy*' nuclear physics (below about 10 MeV).

Van de Graaff produced the first generator of this kind at the Massachusetts Institute of Technology (MIT). In the generator, a continuous belt composed of an insulating material moves past a source of negative electricity. This source sprays electrons on the belt. The belt then goes into a hollow metal dome were a fine metallic brush moves the electrons onto the dome surface. When the charge at the top of the dome is high enough, electrically charged particles are hurled at targets at the bottom of the generator. The great merit of the Van de Graaff generator lies in its extremely steady potential when it is suitably stabilized.

Van de Graaff was born in Tuscaloosa, Alabama. He graduated from the University of Alabama in 1922 and did postgraduate work at the Sorbonne in Paris and Oxford University in England. He joined M.I.T. in 1931. In 1946 he helped found the High Voltage Engineering Corporation, where developments led to the production of the "*tandem*" machine — which is effectively two accelerators in series. Here, *negative ions* are first accelerated to the high-voltage terminal, then stripped of their electrons and accelerated a second time to the target.

Worldview XLIV: Paul Valéry[741]

* * *

"One has to be a Newton to note that the moon is falling when everyone sees that it does not fall."

* * *

"The science of mathematics is to a large extent, only a science of pure repetitive patterns. It grasps the mechanism of patterns and summarize it."

* * *

"All our inventions tend either to save our energies, or to save repetition"

* * *

"Everything that requires no effort is a waste of time"

* * *

"History will justify anything. It teaches precisely nothing, for it contains everything and furnishes examples of everything."

* * *

"History is the science of things that do not repeat themselves."

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741 **Paul Valéry** (1871–1945, France). Thinker, poet and essayist. Born in France of an Italian mother and a Corsican father. A rationalist who was deeply versed in 19^{th} century scientific ideas. He wrote on mathematics and physics from a viewpoint of a philosopher-poet.

* * *

"The past is an entirely mental thing. It is nothing but images and beliefs."

* * *

"In 1887, the air was strictly reserved for the birds. Electricity has not yet lost its wires. Solid bodies were still fairly solid. Opaque bodies were sill quiet opaque. Newton and Galileo reigned in peace. Physics was happy and its references absolute. Time flowed by in quiet days; all hours were equal in the sight of the universe. Space enjoyed being infinite, homogeneous, and perfectly indifferent to what went on in its august bosom. Matter felt that it had good and just laws... Could the greatest scholar, the profoundest philosopher of 1887 even have dreamed of what we know and see, after a mere fifty-five years?"

* * *

"Never has humanity combined so much power with so much disorder, so much anxiety with so much playthings, so much knowledge with so much uncertainty."

* * *

"Hitherto, all politics gambled on the isolation of events. *History was made up of events that could be localized. Any disturbance had at one point on the globe as it were, a boundless medium in which to reverberate; its effects were nil at a sufficient distance; everything went in Tokyo, as though Berlin were at infinity. It was therefore possible (it was even reasonable) to predict, to calculate, to act. There was room in the world for one or several great policies well planned and carried out. That time is coming to an end – the age of the finite world has began"*

(1931)

* * *

"One of man's most extraordinary inventions is the invention of the past *and the* future: *by expanding the moment, by using imagination to generalize the*

present, man creates time; *and in doing so he not only sets up perspectives before and after his intervals of reaction but, what is more, he lives but very little in the moment itself ... he continually feels the need of what does not exist."*

(1932)

* * *

"Nature is indifferent to individuals. If man prolongs or betters his life he is acting against nature"

(1932).

* * *

*"*Living *is an essentially monotonous practice, based on the regular recurrence of a few reflexes ...* Knowledge, *on the other hand, tends to absorb the particular and singular mundane into the general law".*

(1932)

* * *

"Modern life tends to spare us intellectual effort just as it does physical effort; for example, it replaces imagination by images, reasoning by symbols and writings, or by machines ... and often by nothing. It offers us every short cut for arriving at our goal without making the journey. It thus combines to produce a certain diminution of value and effort in the realm of the mind."

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5. *Demise of the Dogmatic Universe*

1895 CE–1950 CE

Maturation of Abstract Algebra and the Grand Fusion of Geometry, Algebra, Arithmetic and Topology

Logic, Set Theory, Foundation of Mathematics and the Genesis of Computer Science

Modern Analysis

Electrons, Atoms, Nuclei and Quanta

Einstein's Relativity and the Geometrization of Gravity; The Expanding Universe

Preliminary Attempts to Geometrize Non-Gravitational Interactions; Kaluza – Klein Models With Compactified Dimensions

Subatomic Physics: Quantum Mechanics and Electrodynamics; Nuclear and Particle Physics

Reduction of Chemistry to Physics; Condensed Matter Physics; The 4^{th} State of Matter

The Conquest of Distance by Automobile, Aircraft and Wireless Communication; Cinematography

The 'Flaming Sword': Antibiotics and Nuclear Weapons

Unfolding Basic Biostructures: Chromosomes, Genes, Hormones, Enzymes and Viruses; Proteins and Amino Acids

Electromagnetic Technology: Early Laser Theory; Holography; Magnetic Recording and Vacuum Tubes; Invention of the Transistor

'Big Science': Accelerators; The Manhattan Project

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Science 'Progress Reports'

Worldviews

Environmental Events that Impacted Civilization

1898–1923 *Bubonic plague* pandemic kills 20 million people in China, India, North Africa and South America

1900 The Galveston (TX, USA) *hurricane* kills 8000 persons

1902 The eruption of the Mt. Pelée *volcano* (Martinique) kills 30,000 people

1906 The San-Francisco *earthquake*

1908 The Messina *earthquake* kills 160,000 people

1908 The Tunguska *bolide explosion*

1912 The 'Titanic' disaster

1917–1920 Worldwide Influenza pandemic kills 80 million people, mostly in Europe and Asia

1921–1930 *Cholera*, *smallpox* and *typhus* pandemic in India kills ca 2 million people

1923 The Tokyo *earthquake*

1931–1950 *Floods* of the Yellow and Yangtze Rivers in China (1928, 1929, 1931, 1936, 1938, 1950) kill 22 million people

1942 *Hurricane* at the Bay of Bengal kills 40,000 people

1970 *Cyclone storm* and *tsunami* kill 500,000 people at the Bay of Bengal

1976 *Earthquake* in Tangshan (China) kills 650,000 people

1983–1985 Famine in Ethiopia kills ca 1 million people

Political and Religious Events that Impacted World Order

1904–1905	Russia and Japan at war
1917	The Bolshevik Revolution
1914–1918	World War I
1936–1939	The Spanish Civil War
1939–1945	World War II and the Holocaust
1947–1949	The rebirth of Israel
1949	Independence of India
1949	Foundation of the Republic of China
1949–1989	The 'Cold War'
1950–1953	The Korean War
1965–1975	The Vietnam War
1991	The Soviet Union officially ceased to exist
2001	The 'Nine-Eleven' event – Muslim terror hits the USA

1931–1949 CE **Lars Onsager**[742] (1903–1976, Norway and U.S.A.). Theoretical physicist. Discovered certain symmetry relations, known as *Onsager reciprocal relations*, which form the basis for the entire discipline of macroscopic irreversible thermodynamics. Onsager relations apply mostly to the *linear*, *near-equilibrium* regime, i.e., as long as the changes (temporal and spatial) of temperature, pressure, chemical potentials, etc. are small on the molecular scale.

The reciprocity relations are rigorous consequences of the *principle of microscopic reversibility*, or alternatively the *time-reversal invariance* property of the irreversible phenomena at the microscopic scale. Indeed, the principle of microscopic reversibility hinges on *time-symmetry* of the mechanical equations of motion (classical or quantum-mechanical) of the individual particles and fields of the system. It implies that fluctuations which take the system away from equilibrium occur at the same rate as those taking it toward equilibrium. However, *microscopic* reversibility in no way contradicts the fact that in a *macroscopic* description physical systems exhibit irreversible behavior (e.g., heat and material diffusion). Irreversible processes result from the statistics of the 2^{nd} law of thermodynamics, and (ultimately) from the nature of the initial state — and have nothing to do with violations of time-reversal invariance.

Consider the following 'thought experiment': a gas, originally occupying a certain volume V_2, is confined to a smaller box V_1 at time $t = 0$. At time $t = t_2$ it is allowed to diffuse to all the available volume V_2. It is clear that the time-asymmetry crept in through the fact that we *forced* the gas to occupy the smaller box at time $t = 0$, but *allowed* the gas to relax back to occupy the full volume at later times. Had we waited for the equilibrium gas configuration to *spontaneously* contract to the box and expand back again, there would have been complete temporal symmetry, but the expected wait time would exceed the age of the universe by many orders of magnitude.

In other words, the time asymmetry stems from the peculiar fact that we are able to impose special initial conditions on the gas, but do not know how

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[742] For further reading, see:

- Reif, F., *Fundamentals of Statistical and Thermal Physics*, McGraw-Hill, 1965, 651 pp.
- Rumer, Yu.B. and M.Sh. Ryvkin, *Thermodynamics, Statistical Physics, and Kinetics*, Mir Publisher: Moscow, 1980, 600 pp.
- Wannier, G.H., *Statistical Physics*, Dover, 1987, 532 pp.
- Rocard, Y., *Thermodynamics*, Pitman, 1961, 681 pp.

to arrange (in advance) that at some *future* time the gas will be confined to the box! Another way of saying it: all that asymmetry means is that the *conditional probability*, given a gas volume V_1 at $t = 0$, to achieve a volume V_2 at $t = t_2 > 0$, is *not* a symmetric function of V_1, V_2. This in itself *does not* contradict time-symmetry and the ensuing Onsager symmetry relation, *because* that symmetry only says that the *joint probability* of $V(0) = V_1$ and $V(t_2) = V_2$ *is indeed* symmetric in V_1 and V_2. Thus the real mystery is that we are able to impose macroscopic initial (but not final) conditions on a system.

The simplest example of Onsager's reciprocity is this: a system accommodates two simultaneous processes: a mass flow driven by a concentration gradient force and a heat flow driven by a temperature gradient. This mixed regime will then exhibit '*cross influences*' in the form of a mass flow induced by the temperature gradient and a heat flow induced by the concentration gradient, in such a way that the coupling coefficients of these cross influences are equal.

A molecule that moves by Brownian motion from position A to position B in the direction of a concentration gradient, has as much probability of moving from B to A against the concentration gradient, although the overall macroscopic state for all molecules of the species in question spontaneously evolves irreversibly in the direction of increasing global entropy (disorder). In this case, therefore, *diffusion* occurs (a tendency for concentration to become more uniform, and thus for their gradients to decrease).

Likewise, the random (thermal) component of molecular kinetic energy tends, macroscopically, to spread more equally over the available volume. The cross-influence coupling coefficients quantify how these two tendencies interact.

The importance of the Onsager relations lies in their generality. It is immaterial, for instance, whether the irreversible processes take place in a gaseous, liquid or solid medium. *The reciprocity expressions are valid independently of any model-dependent microscopic assumptions*. Their validity showed, for the first time, that nonequilibrium thermodynamics leads, as does equilibrium thermodynamics, to general results independent of any molecular model.

The discovery of the reciprocity relations can be considered to have been a turning point in the history of thermodynamics, as it marked a crucial point in the shift of interest away from equilibrium toward nonequilibrium. Classical thermodynamics alone does not make it possible to establish such relations, since it either ignores the irreversible changes that the system undergoes, or analyzes them indirectly (via changes of the state variables from an *initial* to a *final* equilibrium state). Whenever we have a precise molecular model which permits to set up a theory, then we have an exact expression for the coupling coefficients such that the *symmetry* implied by Onsager's relations holds.

Even before the development of the thermodynamics of irreversible processes, a few linear phenomenological laws had been known for a long time:

Fourier (1822) discovered the simple linear law relating the thermal flux to the temperature gradient: $\boldsymbol{J}_q = -\lambda \,\mathrm{grad}\, T$ where λ is the coefficient of *thermal conductivity* in solids.

Fick (1856) established that the diffusive flux of matter is proportional to the concentration gradient which generates the flux, $\boldsymbol{J}_d = -D \,\mathrm{grad}\, c$, where c is the solute concentration (number of moles) per unit volume and D represents the *diffusion coefficient* of the dilute mixture.

In a *thermodiffusion cell*, a nonreacting dye (solute) of (possibly inhomogeneous) concentration c is dissolved in a tube containing water (solvent). The tube is sealed and then subjected to a temperature gradients at its ends. The temperature gradient generates a concentration flow, and vice versa; thus, interference effects between the diffusion of matter and heat flow take place. The system, therefore, operates under the combined influence of Fourier and Fick laws and the cross-influences:

$$\boldsymbol{J}_d = -D \,\mathrm{grad}\, c - D^T \,\mathrm{grad}\, T, \qquad \boldsymbol{J}_q = -\lambda^D \,\mathrm{grad}\, c - \lambda \,\mathrm{grad}\, T.$$

The two new *phenomenological coefficients* are:

$$\lambda^D = \frac{L_{qd}}{T}\left(\frac{\partial \mu}{\partial c}\right)_T \quad (\textbf{Dufour} \text{ coefficient}),$$

$$D^T = \frac{L_{qd}}{T^2} + \frac{L_{dd}}{T}\left[\left(\frac{\partial \mu}{\partial T}\right)_c - \frac{\mu}{T}\right] \quad (\textbf{Soret} \text{ coefficient}).$$

where $\mu = \mu(c, T)$ is the *chemical* potential[743] of the solute. Setting also

$$D = \frac{L_{dd}}{T}\left(\frac{\partial \mu}{\partial c}\right)_T, \qquad \lambda = \frac{L_{qq}}{T^2} + \frac{L_{qd}}{T}\left[\left(\frac{\partial \mu}{\partial T}\right)_c - \frac{\mu}{T}\right],$$

$$\boldsymbol{X}_d = -\,\mathrm{grad}\left(\frac{\mu}{T}\right), \qquad \boldsymbol{X}_q = \mathrm{grad}\left(\frac{1}{T}\right),$$

[743] According to equilibrium thermodynamics, it is μ, rather than the concentration c, which tends to equalize between different samples in diffusional contact. If the solution may be approximated as an ideal gas, $\mu \approx k\,T \ln\left(\frac{c}{c_0}\right)$, with k Boltzmann's constant and c_0 an undetermined constant.

the flux equations assume their Onsager form $(d \equiv 1,\ q \equiv 2)$:

$$\boldsymbol{J}_1 = L_{11}\boldsymbol{X}_1 + L_{12}\boldsymbol{X}_2, \qquad \boldsymbol{J}_2 = L_{21}\boldsymbol{X}_1 + L_{22}\boldsymbol{X}_2.$$

The *Onsager reciprocity relation* $L_{12} = L_{21}$ relates the coefficient which gives the heat flux caused by a concentration gradient, to the coefficient yielding the mass flux caused by a temperature gradient. This, as well as the numerous other reciprocity relations when other gradients and flows interact, follows from the fact that the *microscopic* dynamics are time-reversal invariants discussed above.

In certain situations — such as external *magnetic fields*, *Coriolis forces* or certain types of *weak nuclear interactions* — microscopic reversibility *breaks down*, and then the Onsager reciprocity relations may be invalid or require modifications.

In general, if a system is subject to different flows and gradient "forces", every flow J_i is dependent to some degree on all other forces, X_j, and conversely, each force is dependent upon all flows appearing in the system:

$$J_i = J_i(X_1, \ldots, X_n) \quad \textit{and} \quad X_i = X_i(J_1, \ldots, J_n).$$

Here n is the total number of forces (or fluxes). In general, the explicit functional relationships between these quantities are not known. If the system is in equilibrium, all forces and consequently all flows vanish. Therefore, it is reasonable to assume that if we introduce weak forces (i.e. small gradients) into the system, the ensuing flows will be proportional to the forces. Also, the properties of the new nonequilibrium system are not drastically different from those of the equilibrium state. The new system remains in the neighborhood of the equilibrium state. As forces are weak in this regime, we may expand J_i in a Taylor series around the equilibrium state, and consider only first-order corrections:

$$J_i = (J_i)_{eq} + \sum_k \left(\frac{\partial J_i}{\partial X_k}\right)_{eq} X_k + \cdots.$$

By definition all fluxes $(J_i)_{eq}$ vanish at equilibrium, and therefore

$$J_i = \sum_k L_{ik} X_k.$$

Onsager, in a general proof, assuming microscopic reversibility, established that the coefficients L_{ik} of the phenomenological equations are symmetrical, provided $\boldsymbol{X}_i = \boldsymbol{\nabla}\left(\frac{\partial s}{\partial e_i}\right)$, with s the local entropy per unit volume, and e_i the local density of the extensive thermodynamic quantity, the flow (current density) of which is J_i.[744] In that case, the symmetry (reciprocity) relations read $L_{ik} = L_{ki}$, or alternatively:

$$\left(\frac{\partial J_i}{\partial X_k}\right) = \left(\frac{\partial J_k}{\partial X_i}\right).$$

It means that the increase in the flux J_i caused by a unit increase in the force X_k (while the remaining forces are held fixed) is equal to the increase of the flux J_k due to unit increase in the force X_i. For non-isotropic media, each index i or k includes both a flow/force type index and a spatial index.

Long before 1931, the symmetry relations were suggested by experiments on the conduction of heat in *anisotropic crystals*. **Charles Soret** (1854–1904, France) noticed in 1893 that the conductivity exhibits a much greater symmetry than exists in the crystal itself.

Indeed, the tensor relation (a generalization of Fourier's law)

$$J_i = \sum_{k=1}^{3} L_{ik} \frac{\partial T}{\partial x_k} \quad (i, k = 1, 2, 3)$$

with i, k *spatial* indices and $L_{ik} = L_{ki}$, expresses the fact that in a general anisotropic crystal, the *static equilibrium properties* (elastic constants, refractive index, etc.) do not have the symmetry of its dynamic irreversible diffusive properties. An entirely analogous situation is exhibited by the electric conductivity of single crystals (**H.B.G. Casimir**, 1945).

The Onsager relations can be applied also to *thermoelectric phenomena*, including the *Seeback effect* (1822), the *Peltier effect* (1834), and the *Thomson* (*Lord Kelvin*) *effect*.

Since their formulation, Onsager's reciprocal relations have been tested for a wide range of flows and forces, and their validity seems to be universal — to such an extent that they are sometimes labeled as the fourth law of thermodynamics.

They apply to a gamut of systems, involving chemical reactions, heat conduction, viscous flow, electrical conduction, polarized matter, and nonlinear dissipative systems which may be seen as prototypes for biological cells and membranes. Although the validity range of Onsager's relations is still a matter of debate, his theory is a good approximation in the linear response region.

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[744] For *chemical reactions* the situation is slightly different: e_i is the density of a particular reagent of the reaction, $\boldsymbol{X}_i$ the reaction-causing variation δS per $\delta e_i = 1$, and J_i the reaction-caused piece of $\dot{e}_i$.

Onsager's other major contribution to theoretical physics was his solution (1942–1949) of the two-dimensional *Ising problem*[745], which was the first exact

[745] In some metals, e.g., Fe and Ni, a finite fraction of the spins of the atoms becomes spontaneously polarized in the same direction, giving rise to a *macroscopic* magnetic field. This happens, however, only when the temperature is lower than a characteristic temperature known as the *Curie temperature*. The transition from the non-ferromagnetic state to the ferromagnetic state is a *phase transition*. The *Ising model* is a crude toy model in which each *domain* in a ferromagnetic substance is replaced by a single microscopic magnet, of fixed magnetic moment, which can point in either of two opposite directions. Its main virtue lies in the fact that a two dimensional Ising model yields to an exact treatment in *statistical mechanics*. The three-dimensional problem is already so difficult that it has so far defied an exact solution.
Consider a solid consisting of N identical atoms in a regular lattice. Each atom has a net electronic spin $\boldsymbol{S}$ and associated magnetic moment $\boldsymbol{m}$ related to its spin by $\boldsymbol{m} = g\mu_0\boldsymbol{S}$, where μ_0 is the *Bohr magneton* and the g (the *gyromagnetic ratio*) is of order unity. If no external magnetic field is present there are two possible interactions between atoms:

(1) a *magnetic* dipole-dipole interaction, which is far two small to account for ordinary ferromagnetism.

(2) an *electrostatic* interaction between the valence-electrons and cations of neighboring atoms, including *exchange interactions* between electrons of neighboring atoms which are a quantum-mechanical consequence of the Pauli exclusion principle. Since the exchange interaction between two atoms depends on the degree to which their electrons can overlap as to occupy approximately the same region in space, this interaction is negligible except when the atoms are sufficiently close to each other unlike the ordinary (non-exchange) electrostatic inter-atomic interactions, *the exchange forces mimic inter-atomic dipole-dipole magnetic forces*, except they are several orders of magnitude stronger. Thus each atom will interact "pseudo-magnetically" only with its nearest neighbor atoms. A two-atom exchange interaction Hamiltonian is written in the form $H_{jk} = -2J(\boldsymbol{S}_j \cdot \boldsymbol{S}_k)$, where J is a parameter (depending on the separation between the atoms) which measures the strength of the exchange interaction.

To simplify the problem, the *Ising model* leaves the essential physical situation intact by adopting the approximation $H_{jk} = -2JS_{jz}S_{kz}$, where the x and y terms of the scalar product have been neglected. The total Hamiltonian representing the interaction energy between the atoms can be written in form $H' = \frac{1}{2}\sum_{j=1}^{N}\sum_{k\neq j} H_{jk}$ where J is the exchange constant for neighboring atoms and the index k refers to atoms in the *nearest neighbor shell* surrounding the j-th atom.

solution of a nontrivial problem in the statistical mechanics of the equilibrium between different phases of matter.

Onsager was born in Oslo, Norway, and was trained as a chemical engineer. His work on electrolyte solutions (with **P. Debye** in Zürich) and his interest in chemical reactions in solutions led him to ponder general issues in irreversible thermodynamics. In 1928 he moved to the United States, where he spent the rest of his career, mostly at Yale (1934–1972, professor from 1945). He was awarded the Nobel Prize for chemistry in 1968.

1931–1951 **Linus Carl Pauling** (1901–1994, U.S.A.). Distinguished chemist. Applied quantum mechanics to the study of molecular structures and *chemical bonding*, and showed (1931) how quantum mechanics could yield results of broad chemical significance that went well beyond earlier theories of valency. His book *The Nature of the Chemical Bond* (1939) is one of the turning points in modern chemistry. He received the Nobel prize for chemistry in 1954.

Pauling utilized X-ray diffraction, electron diffraction, magnetic effects and the heat involved in forming chemical compounds for the calculation of interatomic distances and angles between chemical bonds, and related intermolecular geometry to molecular characteristics and to interactions between molecules.

He introduced the concepts of *hybrid electron orbitals*, *covalent bonds* (atoms sharing electrons) and *resonance hybrids* (1931–1934) vital for the understanding of the directional character of chemical bonding and the well-defined shapes of bonded atomic aggregates. He thus proposed (1931) that the phenomenon of *resonance* causes the stability of the *benzene ring*. In 1934 Pauling began to apply his knowledge of molecular structure to the complex molecules of living tissues, particularly in connection with proteins, and became interested in proteins involved in *immunological reactions*.

He recognized the importance of *hydrogen bonding* in protein structure and the interaction between macromolecules. He suggested (1946) that *enzymes* work by lowering the energy-barrier of a reaction through binding to a transitional state as the atoms in a compound move from position to position about the central core. This mechanism was later established for many enzymes.

From the observed shapes of amino acids and small peptide molecules, Pauling and **Robert Corey** (1951) formulated a set of structural conditions that any model of a *polypeptide chain* must satisfy. This led to the α-helix structure of proteins.

Pauling was born in Portland, Oregon. He took his Ph.D. at the California Institute of Technology (Caltech), Pasadena, in physical chemistry

(1925). He later worked under **Sommerfeld** in Münich, **Bohr** in Copenhagen, **Schrödinger** in Zürich and **Bragg** in London. He returned to Caltech in 1927 and became a full professor there in 1931. He left Caltech in 1963 to devote himself to the study of problems of war and peace.

1931–1953 CE The advent of *industrial fiberglass*: Fiberglass is glass[746] in the form of fine fibers (threads). The fibers may be many times finer than human hair, and may look and feel like silk. The flexible glass fibers are stronger than steel and will not burn, stretch, rot, or fade. The ancient Egyptians used *coarse* glass fibers for decorative purposes.

For a long time fiberglass was thought of as a curiosity without much of a future. However, in experiments conducted from 1931 to 1939 by the American firm Owens Illinois Glass Company, fiberglass material began to be used on a large scale for its heat-insulation properties. It was first used to make the entire bodywork of cars by the American firm Chevrolet (1953). The construction of fiberglass *streamlining* quickly won over the pleasure boats industry.

1931–1953 CE **Henry John Kaiser** (1882–1967, USA). Prominent industrialist, engineering administrator and innovator, master of creative entrepreneurial improvisations, builder and founder of giant businesses in cement, aluminum, magnesium, steel, tourism and health care. Known as the father of modern shipbuilding.

His many projects put thousands to work during the Depression in the 1930's and his massive shipbuilding during WWII (The *Liberty Ship Project*[747]) was a decisive factor in winning the war.

Kaiser was born in Sprout Brook, N.Y. He left school at 13 to go to work. Later, he went to the Pacific Coast, where he became a road builder.

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[746] *Glass* is a complex network of silicon and oxygen atoms. Because of its *irregular* arrangements of atoms, glass is not a bona fide crystalline solid. It is often regarded as a supercooled liquid and has no definite melting point as crystalline solids do but softens over a wide range of temperatures. Even at ordinary temperature it will *flow* appreciably over long periods of time.

[747] He ignored the usual methods of building from keel up, and used assembly-line methods. His ships were built in separate sections and welded together in a few days. During WWII, Kaiser yards constructed more than 1500 cargo ships: The first 'Liberty Ship' took 196 days to deliver. Kaiser cut the time to 27 days and by 1943 he was turning one out every 10.3 hours! The concept he developed for mass production of commercial and military ships are still used today. The USNS tanker *Henry J. Kaiser*, built in 1986 (205 m long, 30 m wide; deadweight tonnage 27,561 long tons), carried 180,000 barrels of JP-5 fuel.

As one of the executants of the early New Deal, especially the TVA, he had been outstanding not merely for thinking big but for producing an endless succession of ingenious small ideas too — putting wooden tires on wheelbarrows and having them drawn by tractors, replacing petrol engines in tractors and earth-shovels with diesels, and so on.

His major building and construction enterprises were:

- Organized the combine that built the Hoover Dam[748] (1931).
- Built the piers for the Oakland – San Francisco Bay Bridge.
- Erected the Permamente cement plant, then the world's biggest, in six months; produced the cement for the Shasta Dam (1935).
- Built the West's first steel plant, at Fontana, California, and when the government demanded 50,000 aircraft, he constructed *Kaiser Aluminum*, then *Kaiser Magnesium* in California. (It was a tragedy for the South that it had no capitalist leader comparable to Kaiser: that is why its own thrust into the modern world was delayed by nearly two decades.)
- Constructed one of the first commercially practicable *geodetic domes*.
- With Joseph W. Frazer founded the Kaiser-Frazer Corporation to build automobiles (1946).
- Advanced *medicine* with the development of hospitals, medical centers, clinics, and medical schools. Founded the largest American Health Maintenance Organization (now known as *Kaiser Permamente*). As founder of a medical care program, he worked with partnerships of physicians and established nursing schools and contributed to medical education.
- Built civic centers, roads, tunnels, housing industries.
- Spent much of his later years developing the urban landscape of Oáhu.

[748] It was the biggest dam in the world, followed by three other linked dams: Parker, Bonneville, and Grand Coulee. In building Grand Coulee, he had devised a special trestle, costing $1.4 million, to pour 36 million tons of concrete. Its cheap power made possible the vast manufacturing industries which flourished in CA during WWII, and transformed the entire West Coast, enriching it still further. Kaiser became chairman of the building consortium (1933). Federal money paid for most of it.

Since the 1930's Kaiser, more than any other man, was building the economic infrastructure of the modern Western USA, making the West the principal supplier of mass-produced weaponry and advanced technology. Some of his quotations reflect his personality, motives and achievements:

I always have to dream up there against the stars. If I don't dream I will make it, I won't even get close.

* * *

I make progress by having people around me who are smarter than I am and listening to them. And I assume that everyone is smarter about something than I am.

* * *

Live daringly, boldly, fearlessly. Taste the relish to be found in competition — in having put forth the best within you.

* * *

Problems are only opportunities in work clothes.

* * *

When your work speaks for itself, don't interrupt.

* * *

1931–1968 CE **Sewall Wright** (1889–1988, USA). Geneticist and statistician. One of the founders of the mathematical theory of *population genetics*. Helped modernize Darwin's theory of evolution, using statistics to model the behavior of population of genes; showed that within small isolated populations, certain genetic features may be lost randomly if the few individuals possessing the genes happen not to pass the genes on to the next generation, This is known as the *Sewall Wright genetic drift effect* (1945) that allows evolution to occur without the influence and involvement of natural selection (*Wright's formula*).

Wright was born in Melrose MA. He took his doctorate at Harvard, worked at the US Department of Agriculture (1915–1925), where he conducted experimental work in animal genetics, then became a professor at the universities of Chicago (1926–1954) and Wisconsin (1955–1960).

Wright introduced stochastic processes to models of population structure and evolution. He showed that when populations were small, *chance* could play an important role in changing the frequency of genes in populations, which is the essence of the evolutionary process. The phenomenon is known as *genetic drift*. The extent, however, to which drift contributes to long-term evolution depends upon whether *alternative genes* affect reproduction and survival and are thus subject to *natural selection*, or whether they are *neutral*. There is still substantial controversy on this point; while some genes are clearly adaptive, many others appear to behave as though they were neutral.

The population structure of *man*, throughout most of human history, has been ideally suited for drift effect with small population size and geographical isolation, but it still remains problematic how important drift has been in human evolution.

Wright's major book is *Evolution and the Genetics of Populations* (1968).

1931–1968 CE **Karol Borsuk** (1905–1982, Poland). Mathematician. Introduced fruitful new ideas in *metric differential geometry* (1931) and the notion of *cohomotopy groups* into topology (1936). He created the concept of a divisor of a map (1936) and initiated *shape theory* (1968). Shape theory grew at the same time as *infinite-dimensional topology* and the interaction between the two fields was of great mutual benefit.

Borsuk was born in Warsaw and was educated at the University of Warsaw (Ph.D. 1930). He held positions at the Universities of Warsaw (1931–1978), Princeton (1946–47), Berkeley (1959–60), Madison (1963–64).

Where did all the water come from? (1931–1951)

"And Steam would go up from the Earth and water the whole face of the ground".

Genesis **2**, 6

"All the rivers run into the sea; yet the sea is not full".

Ecclesiastes **1**, 7

One of the first methods used to determine the age of the ocean was to divide the total salt content of the world ocean by the annual increment of salt discharged into the sea by rivers. This procedure was suggested by **Edmund Halley** *in* 1715 *but not implemented until* 1899 *when* **John Joly** *(1857–1933, Ireland) made the first estimate using data on the abundance of sea salt obtained by the Challenger expedition.*

The result of several such computations of the salt age is somewhat less than 100 *million years. This period of existence is clearly too brief, because marine organisms, as well as present-day species, have been found in early Cambrian rocks of an age of the order of* 500 *million years. The source of the discrepancy stems from a recycling mechanism through which part of the salt leaves the ocean's surface, only to return via the rivers of the world. It is generally assumed that the oceans are at least as old as marine fossils.*

Closely associated with the age of the world ocean is the question of the origin of such a vast quantity of water (about 1 *billion* km^3*) and of the salt (about 3%) that it contains. Early speculations were concerned with a deluge. When* **James Hutton** *(1785) and* **John Playfair**[749] *(1802) proposed the uniformitarian doctrine, such ideas as the cataclysmic appearance of the land and the sea were in general currency, partly due to the lingering influence of religious dogma.*

There are only two possible sources for the water. Either it is the residue of a vast, dense primordial atmosphere and is almost as old as the earth, or

[749] Scottish mathematician and geologist (1748–1819).

else there was little or no ocean when the earth formed and it has since leaked out from the interior. But it can be shown that at present temperatures the atmosphere, fully saturated, can hold no more than some $13,000$ km^3 of water at any one time.

As water vapor over a molten earth (at perhaps $1200\,^\circ C$), only 16% of the present ocean volume would have remained in gaseous equilibrium. Moreover, if the earth had been much hotter during its early history than it is at present, the velocity of escape[750] of both molecular and dissociated water would have been exceeded and the earth would have lost any volatiles originally accumulated. The depletion of noble gases[751] suggests that the surface of the earth must have been very much hotter than it is today. Thus, the

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750 The earth's retention of volatile components such as air and water depends on its temperature. Because of their thermal motions, gases are continuously diffusing outward, but this tendency is counteracted by the gravitational attraction of the earth. In order to escape from the earth, a molecule, like a spaceship, must have a velocity that is greater than the escape velocity from earth, namely $11.2\ \frac{\text{km}}{\text{sec}}$. The average kinetic energy of a gas molecule is given by $\frac{1}{2}m\overline{v}^2 = \frac{3kT}{2}$ (T is the absolute temperature). Therefore $\overline{v} = \sqrt{\frac{3kT}{m}}$. Consequently, the lighter the molecule, or the higher the temperature, the more likely it is that gas can escape from the earth's surface. Under present conditions, hydrogen and helium are rapidly lost from the atmosphere while the heavier gases such as oxygen and nitrogen are retained.

751 A clue to the early thermal history of the earth is offered by the relative abundance of the noble gases on earth and in stars. These gases, unlike water, do not combine chemically, and so have always been in a gaseous state. We must compare the abundances of these permanent volatiles with those of elements that are chemically bound in the solid matter of the earth, such as silicon, the major metallic element in rocks.

Since the earth originally accumulated from stellar material, the original ratio of the noble gases to silicon was probably similar to the ratio that is currently observed in stars. If the material of the primitive earth was then heated, there would be a loss of the volatile components resulting in a decrease in the ratio of noble gases to silicon. The depletion of noble gases will be greatest for those elements of low atomic weight, helium and neon, and less for those of increasing atomic weight.

The data on the abundance of noble gases suggest that all the water in the ocean and the gases of the atmosphere must once have been held within the solid earth: The mantle of the earth has a volume of 10^{27}cm^3. Assuming a density of 4, this amounts to a mass of 4×10^{27} g, while the water of the ocean has a mass of 1.4×10^{24} g. Thus the mantle must have lost 0.035 percent of its weight in the form of water, on the average. We must compare this figure with *atmosphere and the ocean cannot be a remnant of the primordial earth; these gases must originally have been chemically combined within the solid earth; they could have accumulated on the surface only since the earth cooled to near its present temperature, perhaps by being released slowly within the earth and at temperatures lower than that of molten rock.*

In 1931, ***R.W. Goranson***[752] *found that water dissolve readily in molten silicate, basaltic and granitic rock. These are large constituents of volcanic lava. Molten rock can contain approximately* 5 *percent of its weight as dissolved water under temperatures and hydrostatic pressures approaching those supposed to exist in volcanic pipes and intrusive magmas within the crust of the earth.*

In 1951, ***W.W. Rubey***[753] *showed that the geological record was incompatible with a primordial origin for the water, and that volatile substances, including water, pour out of volcanoes in amounts much greater than are necessary to form the ocean. Although it was later found that most of the water emerging from volcanoes was recycled rain, Rubey's theory remained valid; geological time has been so long that only a miniscule amount of juvenile water from each volcanic eruption would be sufficient to have produced the ocean.*

It thus seems that the water on the surface of the earth flowed out of the interior along with volcanic rocks, but when and at what rate?

The simpler possibilities are that the flow was constant during geological time, or faster than average to begin with, or faster than average recently. All

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the average water content of the mantle. Although the mantle is not accessible for sampling, *meteorites* offer us samples of mantle-like material.

Meteorites are fragments of a planet-like object in the solar system that broke up. Pieces of this material are frequently captured by the earth's gravitational field. Some meteorites consists mainly of iron with a high nickel content and are believed to resemble the material of the core of the earth. Others, the stony meteorites, contain silicates and are believed to resemble the mantle. By examining the water content of the stony meteorites, we can therefore obtain an estimate of the water content of the mantle.

The average water content is about 0.5 percent or 10 times as much as the loss from the mantle that is required to account for the present ocean. Thus the mantle could be an adequate source for the water in the ocean.

752 Goranson, R.W., The solubility of water in granitic magmas, *Am. J. Sci.* **22**, 481–502, 1931.

753 Rubey, W.W., Geologic history of sea water, *Bull. Geol. Soc. Am.* **62**, 1111–1147, 1951.

possibilities have been proposed. Rubey assumed a constant flow, essentially on the philosophical grounds that it is the simplest possibility.

In conclusion, the Goranson-Rubey scenario is this: as the rock solidified, the water were expelled as steam. The quantity of steam from the volcanoes of a cooling earth can account for the volume of water in our oceans. Chemical similarities between volcanic steam and ocean water indicate that this steam, rising from a cooling earth, was very likely the source of our oceans and the atmosphere.

Once water was present at the surface, weathering *of crystalline rocks could commence. As a result, crystalline rocks were transformed to sediments and the salts of seawater. The dynamics of the lithosphere then led to the formation of continental crust so that, as the volume of water at the surface increased, the difference in elevation between the floor of the ocean and the surface of the continents also increased.*

Originally, the atmosphere was devoid of oxygen, and the ultraviolet irradiation of the sea surface led to the synthesis of complex organic molecules. Life evolved from, and was originally nourished by, the organic matter produced by solar radiation near the surface of the sea. The evolution of the first marine plants led to the biological emission of free oxygen and the gradual oxidation of the surface environment. Eventually this led to the present oxygen-containing atmosphere. Thus life processes have transformed the surface environment of our planet.

1932 CE *A Wonder Year* for elementary particles:

- **James Chadwick** discovered the *neutron*
- **Carl David Anderson** discovered the *positron*
- **Harold Clayton Urey** discovered the *deuterium*
- New particle accelerator technology
- New nuclear physics

1932 CE **James Chadwick** (1891–1974, England). Physicist. Discovered the neutron and determined its mass. In 1932 Chadwick observed that beryllium, when exposed to bombardment by alpha particles, released an unknown radiation that in turn ejected protons from the nuclei of various substances. Chadwick interpreted this radiation as being composed of particles of mass approximately equal to that of the proton, but without electrical charge — neutrons.

This discovery provided a new tool for inducing atomic disintegration, since neutrons, being electrically uncharged, could penetrate undeflected into the atomic nucleus.

Chadwick was born in Manchester and was educated at the Universities of Manchester, Cambridge and Berlin. From 1923 he worked with **Ernst Rutherford** in the Cavendish Laboratory, Cambridge, where they studied the transmutation of elements by bombarding them with alpha particles and investigated the nature of the atomic nucleus, identifying the proton, the nucleus of the hydrogen atom, as a constituent of the nuclei of other atoms. He won the Nobel prize for physics in 1935.

1932 CE **August Dvorak** (1894–1975, USA). Inventor and scholar. Designed a scientific, ergonomically designed typewriter keyboard that is twice as fast, makes half the errors and enables the typists to move their fingers 20 times less for as with the QWERTY keyboard. Yet *his revolutionary invention was never adopted*! The reason: the commitment to QWERTY of tens of millions of typists, teachers, sales people, office managers, and manufactures.

Dvorak (born in Glenco, MN) was a professor of education at the University of Washington in Seattle and a distant relative of the Czech composer Antonin Dvořak. Around 1914, Augustin's brother-in-law, William Dealy, attended some industrial efficiency seminars and watched slow-motion films of typists, and reported what he saw to Dvorak. The brothers in law then devoted *two decades* to enormously detailed studies of typing, typists errors, hand physiology and function, and the relative frequencies of letters, pairs of letters, and words in English. Finally they assembled all they had and in 1932 designed a new keyboard.

August died a bitter man: "I'm tired of trying to do something worthwhile for the human race", he complained. "They simply don't want to change."

The ‘QWERTY’ Syndrome — or: A Comedy of Errors

All human societies have many apparently arbitrary *practices that persist for centuries or even millennia — writing systems, counting systems, sets of number signs, calendars, to name just a few examples. At one time there existed alternatives to the system that was eventually adopted. Were some of these alternatives better than others? Did we in fact end up committed to the best ones? Are our alphabets, decimal counting, Arabic numerals, and Gregorian calendar really superior to Chinese logograms, Babylonian base-*60 *counting, Roman numerals, and the Mayan calendar?*

The origins of many other commitments are now lost in remote history. How did China become committed to its hard-to-memorize writing system? Chinese children can master pinyin *(a Roman alphabet adapted to Chinese) in one-tenth of the time required to learn the traditional writing system.*

Why do Americans cling to the awkward English measuring system of pounds, inches, and gallons? How did we become committed to decimal counting and a 24-hour clock? These questions are tantalizing but perhaps academic, because there is no prospect of our abolishing the 60 *minute hour or reverting to base-*60 *counting, even if such changes did prove advantageous.*

In modern times, commitments have shaped the history of technology and culture, often selecting which innovations become entrenched and which are rejected. In the 19th century United States, for example, those who profited from canals, barges, stagecoaches, and the pony express resisted the construction of railroads; In England, electric *street-lighting spread slowly, partly because of opposition from local governments with heavy investment in* gas *lighting. Even today, commitments influence railroad gauges and television technology, and whether we mark our rulers with centimeters or inches and drive on the right or on the left.*

The transistor was invented and patented in the United States in the 1940s, but Japan today dominate the world market for transistorized consumer electronic products. The reason: the company that became Sony bought transistor licensing rights from Western Electric at a time when the American consumer electronics industry was committed to churning out vacuum tube *models and reluctant to compete with its own products.*

There is, however, one example which serves to demonstrate this absurdity of conventions and commitments better than others because it is a comedy of errors that keeps going on to the very present day — the curse of QWERTY. It *started in 1874 when **Christopher Sholes** designed the typewriter keyboard to took like*

Q W E R T Y U I O P
A S D F G H J K L ← *'home row'*
Z X C V B N M

To overcome the problem of invisible jamming, Sholes applied antiengineering *principles with the goal of slowing down the typist and thus preventing the second bar from jamming the falling first bar. The idea of eight-finger touch-typing was still unknown. Typists rummaged around with one of two fingers while looking at the keyboard, and Sholes was ecstatic if the resulting typing rate reached* 20 *or* 30 *words per minute, the rate of writing by hand. To this end he scattered the most common letters or letter combinations in English texts as widely as possible over the keyboard.*

Why did QWERTY prevail even after improvements in typewriter technology (reducing the jamming problem) and the demand for fast typing had removed the original motivation for it? The reason was that its early head start and success created a commercial dominance and hence a commitment of its manufacturers (Remington) to the original layout.

When typing, one rests his finger on the 'home-row'. The more typing one can do without having to move the fingers from the home row, the faster one is able to type, the fewer errors one will make and the less one strains his fingers. Motion-picture studies prove that typing is fastest on the home row and slowest on the bottom row.

When Augustin Dvorak started his detailed studies of typing (1914) he immediately noticed the following shortcomings of QWERTY:

- *Not more than 100 English words can be typed without leaving the home row because QWERTY perversely puts the most common English letters on the other rows: The home row nine letters include two of the least used (*`J` *and* `K`*) but none of the three most frequently used (*`E`*,* `T` *and* `O`*) and only one of the five vowels (*`A`*), even though* 40 *percent of all letters in a typical English text are vowels. Thus, when typing the* *words pumpkin or minimum, ones fingers must not only reach from the home row to the top or bottom but must at times hurdle completely over the home row, moving directly from top to bottom and back again! These awkward hurdles and reaches slow one down and introduce typing errors and finger strain.*

- *In typing, whenever the left and right hands type alternative letters, one hand can be getting into position for the next letter while the other is typing the previous one. One can thereby fall into a steady rhythm and type quickly. A good positioning of letters in the keyboard should therefore strive to avoid typing strings of consecutive letters by the same hand. The longer the string, the slower the typing and the more frequent the errors. QWERTY typing tends to degenerate into long one-handed strings of letters, especially strings for the weaker left hand.*

  More than 3000 *English words utilize QWERTY's left hand alone (e.g. exaggerated, greatest; million, monopoly). The underlying reason for this shortcoming is that most English syllables contain both vowels and consonants, but QWERTY assigns some vowels (*`A` *and* `E`*) as well as some common consonants (*`R`, `S`, *and* `D`*) to the left hand, and others (*`I`, `O`, *and* `U` *plus* `H`, `L`, *and* `N`*) to the right hand. Hence for about half of all digraphs (two consecutive letters) in a typical English text, QWERTY allocates both letters to the same hand.*

- *Most people are right-handed with a weaker left hand. Yet QWERTY allocates to the weaker hand the most common English letter (*`E`*), the second common (*`T`*), and the* 4*th most common (*`A`*), thus making the left hand perform more than half of all typing strokes (*56 *percent).*

- *On each hand the* 5*th finger (pinkie) is the weakest, and finger-strength increases from the* 5*th to the* 2*nd (index). Yet QWERTY makes almost as much use of our weakest finger (left* 5*th) as of our* 2*nd stronger (right* 3*rd).*

- *When one must type two successive strokes with the same hand, it's fastest to do so with two remote fingers, next fastest with two adjacent fingers, slower with the same finger on the same row, and slowest of all with the same finger on different rows. Yet with the QWERTY keyboard,* 20 *percent of all English digraphs are typed by adjacent fingers, and more than* 4 *percent by the same finger.*

The results of these shortcomings is that typing on a QWERTY keyboard is unnecessarily tiring, slow, inaccurate, hard to learn and hard to remember. The infinitely superior Dvorak keyboard, the end result of twenty years of an arduous study, proposed the plan

P Y F G C R L
A O E U I D H T N S ← *'home row'*
Q J K X B M W V Z

with the following merits and advantages over QWERTY:

- *Devotes the home row 9 out of 12 most common English letters — including all 5 vowels and the 3 most common consonants (`T`, `H`, `N`), while the 6 rarest letters (`V`, `K`, `J`, `X`, `Q`, `Z`) are relegated to the bottom row. As a result, 70 percent of tying strokes remain on the home row, only 22 percent are on the upper row, and a mere 8 percent are on the hated bottom row; thousands of words can be typed with the home row alone; reaches are 5 times less frequent than in QWERTY typing, and hurdles hardly ever happen.*

- *The Dvorak keyboard forces one to alternate hands. It does so by placing all vowels plus `Y` in the left hand, but the 13 most common consonants in the right. As a result, not a single word or even a single syllable can be typed with the right hand alone, and only a few words can be typed with the left hand alone.*

- *56 percent of all strokes are given to the right hand. Only 2 percent of all English digraphs are typed by adjacent fingers and only 1 percent by the same finger.*

In a normal workday a good typist's fingers cover up to 30 km on a QWERTY keyboard, but only 1.5 km on a Dvorak keyboard! and all this with twice the speed and half the errors that QWERTY typists make.

Whatever the original reasons for adopting QWERTY, we now seem firmly committed to it. The typewriter, and its successor the computer are among the most widely used office machines in the world, and keyboard-related repetitive-strain injuries are among the most common industrial accidents. But if we were to overcome the fear of long-held commitments, millions of people would be able to type with increasing speed, greatly lowered finger fatigue, greater accuracy, and reduced sense of frustration — altogether ending a bad marriage that has long outlived its original justification.

1932–1935 CE **Gerhard Domagk** (1895–1964, Germany). Chemist, pathologist and physician. Discovered the first of the *sulfanilamide* drugs (*Prontosil*). In 1935 he used it successfully on his youngest daughter to prevent her death from a streptococcal infection, the first use on a human being.

Following **Ehrlich**'s spectacular success with Salvarsan (1910), researchers began to test virtually every substance that might be effective against infectious diseases. Although there were some successes — antimalarial drugs and those that fought protozoal infections such as amoebic dysentry — everything else proved either as destructive to the patient as to the bacteria, or unable to kill the germs once they had started to multiply. Scientists began to despair of finding any more 'magic bullets'.

However, in 1931, the American Bacteriologists **René Jules Dubos** (1901–1982) and **Oswald Theodore Avery** (1877–1955), of the Rockefeller institute, announced that they had discovered an *enzyme* derived from a soil bacteria that could break down the capsule that protected one particular type of pneumococcus. Although this proved too toxic for human use, the discovery gave new life to the search for anti-bacterial drugs.

In 1927, Domagk was appointed research director of the German chemical company, I.G. Farbenindustrie. Its main products were azo dyes used for color textiles, and Domagk decided to find out whether they had any adverse effect on streptococci. In 1932, he found that one azo compound — Prontosil red — cured mice injected with a lethal doze of hemolytic streptococci.

Domagk won the Nobel prize for physiology or medicine in 1939, but was prevented by the Nazis from accepting[754, 755].

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[754] **Carl von Ossietzky** (1889–1938, Germany). German journalist and pacifist, won the Nobel prize for peace (1935). He fought for Germany in WWI. In his capacity as a staff member of *Berliner Volks-Zeitung* and later (1928) as the editor of *Weltbühne*, he wrote vigorously in defense of pacifism, denouncing Nazi rearming. Consequently he was imprisoned in a concentration camp on charge of being an enemy of the state (1933–1936). He contracted tuberculosis in prison and was in a sanitarium when the Nobel prize was announced. The German government considered the award as a "challenge and an insult" and prohibited Germans thenceforth from accepting such awards.

[755] Domagk did not publish his findings until February 1935, and scientists were surprised that even then, his report failed to mention sulphanilamide, the component of Prontosil responsible for its bacteriostatic action. It is believed that Domagk and his associates knew that sulphanilamide was unpatentable because it had already been synthesized by a Viennese student, **Paul Gelmo**, who had published his findings in his doctoral dissertation (1908).

It is surmised that Domagk spent the years between 1932 and 1935 in a vain

Sulfanilamide possesses a rather simple chemical structure, and this, together with the later proof that it owed its action to a similarity to and competition with an essential bacterial metabolite, was an immense stimulus to pharmaceutical industrial research (sulphanilamide did not kill bacteria like an antibiotic, but prevented them from multiplying).

1932 CE **Harold Clayton Urey** (1893–1981, U.S.A.). Chemist. Discovered *deuterium* (heavy hydrogen). Awarded the Nobel Prize in chemistry (1934).

1932–1933 CE **Marian Rejewski**[756] (1906–1980, Poland). Cryptoanalyst and mathematician. A key figure in breaking the code of the German *Enigma* machine, by taking advantage of *repetition* in the Enigma encryption (a message key was enciphered *twice* at the beginning of every message). Applying certain theorems from the theory of permutations, he was able to separate the effect of *plugboard settings*[757] from those of the *scrambler settings*, thus reducing the total number of possible keys from 10^{16} to $105,456$. This eventually enabled the Polish team to find the day key in about two hours.

Rejewski studied mathematics at the University of Poznan. He was recruited into the *Biuro Szyfrów* (Polish Cipher Office) in 1929.

The Poles successfully used Rejewski's technique for several years. When Herman Göring visited Warsaw (1934), he was totally unaware of the fact that his communications were being intercepted and deciphered. As he and the other German dignitaries laid a wreath at the Tomb of the Unknown Soldier next to the offices of the Biuro Szyfrów, Rejewski could stare down at them from his window, content in the knowledge that he could read their most secret communications.

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attempt to find a drug similar to but better than sulphanilamide and one that could be patented. In the meantime, thousands of patients had suffered, and some had died — all for the sake of profits. The integrity of Domagk was therefore called into question. He was, however, awarded the Nobel prize (1939) despite the rumors about possible suppression of research results.

[756] For further reading, see:

- Singh, S., *The Code Book*, Anchor Books, 1999, 411 pp.

[757] *Plugboard settings*, i.e. number of ways of *swapping*, say, 6 pairs of 26 letters in $100,391,791,500$. But on its own, the plugboard would provide a trivial cipher, because it would do nothing more than act as a monoalphabetic substitution cipher, having no effect on the *frequency* of letters. The *scramblers* contribute a smaller number of keys, but their setup is continually changing, which means that the resulting ciphertext cannot be broken by *frequency analysis*.

Rejewski received little monetary compensation for his efforts, not much in the way of promotion, and only a minor Polish decoration. He merited, however, the highest accolade of all the Allied Nations. Perhaps his satisfaction came from a job well done.

1932–1936 CE **Carl David Anderson** (b. 1905, U.S.A.). Physicist. Discovered the *positron* (anti-electron), the first known particle of anti-matter (1932), as predicted by **P.A.M. Dirac** in 1928.

In 1936 he participated in the discovery of the *mu-meson* in cosmic rays at Pikes Peak, Colorado (an elementary fermionic particle about 207 times as massive as the electron; at first erroneously thought to be the carrier of the strong nuclear force) predicted by **H. Yukawa** (1935).

Carl Anderson was born in New York city and received his doctorate at the California Institute of Technology, Pasadena (1930), where he spent his entire career. In 1930 he began research on gamma rays and cosmic rays, utilizing the magnetic cloud chamber. In 1932 Anderson discovered the positron in the course of cosmic-ray studies, and one year later succeeded in producing positrons by gamma irradiation. He shared the Nobel prize for physics in 1936.

1932–1937 CE **Hans Adolf Krebs** (1900–1981, England). Distinguished biochemist. A pioneer of the field of *bioenergetics*: studied energy transformations of intermediate metabolic processes in living matter.

Discovered (1932) the cycle of reactions whereby urea is formed from ammonia and carbon dioxide in the livers of ureotelic organisms (*urea cycle*). This was the first biosynthetic pathway and metabolic cycle to be discovered[758]. Formulated (1936–1937) the citric acid cycle (known as the *Krebs cycle*), the final common pathway for the oxidation of all foodstuff.

Krebs' cycle is one of the most important metabolic pathways and energy producer in living organisms. It consists of a cyclic series of stepwise oxidation

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[758] The over-all reaction is: $2NH_3+CO_2 \rightarrow NH_2CONH_2+H_2O$; but ammonia is highly toxic and there is no ammonia to speak of anywhere in the body at anytime. It was not until the 1950's, however, that the *details* of the conversion were worked out; the nitrogen-containing groups are added in the form of *amines* obtained from amino acids. In going from *ornithine* to *citrulline*, *glutamic acid* (one of the commonest amino acids) donates both the amino group and the equivalent carbon dioxide. In doing so it must make use of a *high-energy phosphate bond* obtained from ATP. In going from citrulline to arginine, another amino acid, *aspartic acid*, donates an amine group, again at the expense of ATP. Thus, the production of urea is an energy-consuming reaction. Each turn of the urea cycle consumes two molecules of ATP.

reactions that occur in mitochondria and represents the final phase in the oxidation of nutrients to CO_2 and H_2O, with the release of large amounts of energy.

All the major nutrients of cells, notably carbohydrates, fats, and proteins ultimately pass through the Krebs' cycle.

Awarded (with **Fritz Lipmann**) the Nobel prize for physiology or medicine (1953).

Krebs was born in Hildsheim, Germany to Jewish parents. Following the Nazi rise to power (1933), he was dismissed from his post at Freiburg University and moved to England, where he became a naturalized citizen (1939). He later became professor of biochemistry at Sheffield (1945–1954) and Oxford (1954–1967).

1932–1939 CE **Eduard Cech** (1893–1960, Czechoslovakia). Mathematician. A foremost contributor to modern topology. Introduced (1932) the topic which today is called *Cech homology theory*[759].

Cech was born in Stracov, Bohemia (now Czech Republic). He studied at the Charles University of Prague (1912–1920) and Turin (1921–1922) and continued to work with **Fubini** in Turin (1922–1923). He then became a professor at the University of Brno (1928–1945) and Prague (1945–1959).

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759 *Homology theory*: If a closed curve C is the boundary of a region on a surface, C is homologous to zero, which is symbolized by $C \sim 0$. If motion on C are orientable (clockwise and anticlockwise), then $-C$ has a meaning. Since one can go round and round a closed curve, $\pm nC$ also have a meaning (cycles).

Cycles may be added and subtracted. Thus $C \pm C'$ have meaning too.

If one selects a convention concerning the bounded region (e.g. having the area to the *left* of the bounding curves), then $C - C'$ bounds an annulus such that $C - C' \sim 0$. In this case C is said to be homologous to C', i.e. from the point of view of homology, C and C' are equivalent. If all cycles homologous to a particular one are considered equivalent, then they can be classified as a single type.

Thus homology is an *equivalence relation* for the classification of cycles, just as *homeomorphism* is an equivalence relation for spaces. The number of types of cycles, that is, the number of *homology classes*, on a manifold, is an *invariant* of the manifold and the cycle types on a manifold form a *commutative* (abelian) group w.r.t. the operation of addition. This is known as the *Betti group*.

The concept of cycles and their homologies can be extended to n-dimensional manifolds. On higher dimensional manifolds one has not only closed curves but also closed surfaces, etc.

1932–1944 CE **Alexander Yakovlevich Khinchin**[760] (1894–1959, USSR). An outstanding mathematician. Established the general theory of stationary random processes, their spectral representations and correlation functions (*Wiener-Khinchin theorem*).

Studied the convergence of discrete Markov chains to continuous diffusion. With Kolmogorov he founded the Moscow School of Probability Theory.

Khinchin was born in Kondrovo, Russia, to Jewish parents. He graduated from Moscow University in 1916, and from 1927 onwards served as a professor of mathematics there. In 1944 he became an Academician of the Soviet Academy of Sciences.

1932–1947 CE **Edwin Herbert Land** (1909–1991, U.S.A.). Inventor and physicist. Invented the *polaroid* material (1932) and the polaroid Land camera that takes and prints a finished picture in seconds (1947). Both inventions resulted in numerous commercial, military and scientific applications.

The polaroid polarizes light through *selective absorption* by aligned submicroscopic crystals of iodoquinine-sulfate that are embedded in a sheet of plastic. The molecules *absorb* light whose electric-field-vector is parallel to their length, while they *transmit* light whose electric-field-vector is perpendicular to their length. In 1963 Land began to use polaroid material in sunglasses and other optical devices such as infrared filters, lightweight range finders, night adaptation goggles and many others. In 1947, he invented a one-step process for developing and printing photographs that produced a revolution in photography unparalleled since the advent of the roll-film.

Land was born to a Jewish family in Bridgeport, CT. He attended Harvard University, but never graduated. He nevertheless issued more than 500 patents for his innovations in light and plastics, and won honorary degrees and awards from numerous scientific institutions and organizations.

1932–1949 CE **Karl Theodor Jaspers** (1883–1969, Germany and Switzerland). Philosopher and psychiatrist. His central idea is that Pure Being (existence) inevitably escapes our efforts at apprehension; Existence eludes the conceptual intellect and all attempts at inclusive intellectual systematization must fail; man constantly tries to transcend his limitations through science, religion and philosophy, but he experiences failure or "shipwreck".

Jaspers believed that man learns most about himself in "limit situations" such as death, guilt, suffering, conflict and failure. He held that *philosophy* is

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[760] For further reading, see:

- Khinchin, A., *A course of Mathematical Analysis*, Hindustan Publishing Corp.: Delhi, 1960, 668 pp.

not a set of doctrines, but an *activity* trough which each individual can become aware of the nature of his own existence: He was not primarily interested in the philosophers' conclusions, because he held that in philosophy all context and *all conclusions are unimportant.*

Jaspers urged the study of other philosophers as a way to disturb and stimulate us so profoundly that we would be compelled to engage in the activity of philosophizing.

Jasper's major work, *Philosophy* (1932), gives his view of the history of philosophy and introduces his major themes. He identified philosophy with philosophical thinking itself, not with any particular set of conclusions. His philosophy is an effort to explore and *describe the margins and limits of experience.*

Jasper was born in Oldenburg, Lower Saxony. He began his intellectual career as a medical student and went on to carry out research in a psychiatric clinic (1916–1920). He then became a professor of philosophy at Heidelberg (1920–1937). Dismissed by the Nazis and barred from teaching for having a Jewish wife (1938–1945). In 1948 he accepted a professorship in philosophy in Basel, Switzerland.

1932–1956 CE **Lev Davidovich Landau** (1908–1968, Russia). Among the top-level physicists of the 20^{th} century. A master of the modern theoretical physics techniques of his time. A major contributor to theories of superfluidity[761], superconductivity and phase-transitions. Created the quantum Fermi-liquid theory (1956). Landau was awarded the 1962 Nobel prize for his work on the superfluid properties of Helium II. Landau left his mark

[761] Superfluidity of Helium 4 – the so-called Helium II phase was discovered by **P.L. Kapitsa** in 1938. It is the property of flow without viscosity in narrow capillaries or gaps. It was long thereafter believed that only one isotope of Helium, He^4, is a superfluid. But in 1972 it was discovered [**D.M. Lee** et all] that liquid He^3 also becomes a superfluid, at much lower temperatures (2 millikelvin, as opposed to 2.2 °K for He 4, at 1 atm pressure). Helium was the last of the elements to be liquefied, and is the most remarkable of all liquids. At temperatures other than absolute zero, Helium II behaves as if it were a mixture of two different liquids. One of these is a superfluid, and moves with zero viscosity along a solid surface. The other is a normal viscous fluid. No friction occurs between these two parts of the liquid in their relative motion.

He 4 atoms each contain 2 electrons and 4 nucleons and therefore an even number of fermions, and are thus *bosons*, and their Bose-Einstein condensation is manifested as superfluidity. He 3 atoms are *fermions* which can pair up, with each pair being a boson, at low enough temperatures; Superfluid He 3 is thus a condensate of He 3-atom pairs.

on a wide range of fields including low temperature, atomic, nuclear, plasma, high energy and cosmic-ray physics. His contributions are partly reflected in such terms as *Landau diamagnetism*[762], *Landau levels*, *Landau damping*, *Landau energy-spectrum*, *Landau cuts*, *Landau-Ginsburg potential, Landau-Pomeranchuk-Migdal effect*.

Landau suggested (1932), for the first time, the possibility of cold dense stars, composed principally of neutrons. He later suggested (1938) that every star has a neutron core. He gave a value for the limiting mass, above which an ordinary star becomes a neutron star.

However, Landau maintained that some stabilizing effect prevents the great mass of neutrons in the core to collapse indefinitely, and therefore he did *not* accept the concept of a 'black hole'. In his astrophysical work Landau used Newtonian gravitational theory.

Landau was born in Baku, Azerbaijan, to Jewish parents. His father was an engineer who worked in the Baku oil industry, and his mother a doctor who had at one time done physiological research. Landau was graduated at 13 from the Gymnasium and, because he was too young for the university, attended the Baku Economical Technical School. He matriculated in 1922 at Baku University, after studying physics and chemistry, and transferred in 1924 to the Leningrad State University, which at that time was the center of Soviet physics. Graduating in 1927, he continued research at the Leningrad Physico-Technical Institute.

At that time there were practically no outstanding senior theoretical physicists in the Soviet Union, and, since the younger men had to teach themselves and each other, it was important for them to go abroad and be in touch with the western theoretical physics schools that were flourishing in such centers as Copenhagen and Münich.

In 1929 Landau visited Göttingen and Leipzig and then stayed at Copenhagen's Institute for Theoretical Physics, where he came under the influence of **Niels Bohr**. After his stay in Copenhagen he visited Cambridge and Zürich, before returning to the Soviet Union. Apart from short visits to Copenhagen in 1933 and 1934, Landau spent the remainder of his life in his own country.

In 1932 Landau went to Kharkov to become the head of the Theoretical Division of the Ukrainian Physico-Technical Institute, a position he combined in 1935 with that of head of the Department of General Physics at the Kharkov

[762] In addition to the spin paramagnetism, there is an *orbital diamagnetism* of free electrons. This effect, unlike the paramagnetic spin susceptibility, is inversely proportional to the effective electron mass, in simple cases. [It is especially strong for Bismuth, where the electrons have abnormally low effective mass.]

A.M. Gorky State University. In Kharkov Landau began to build a Soviet school of theoretical physics, so that the city soon became the center of theoretical physics in the U.S.S.R. It was also in Kharkov that, with his friend and former student **E.M. Lifshitz**, he started to write the well-known *Course of Theoretical Physics*, a set of nine volumes that together span the whole of the subject. His great interest in the teaching of physics is also shown in his plans for a "*Course of General Physics*" and even a series entitled "*Physics for Everybody*".

Landau required that his students master all necessary mathematical techniques before coming to him. After that he expected them to master the so-called theoretical minimum, which included a basic knowledge of all the domains of theoretical physics. Only the ablest of the students were able to pass this minimum. In this way his students became proper physicists, rather than narrow specialists.

In 1937 **Pyotr Leonidovich Kapitsa** (1894–1984, England and U.S.S.R.), a low-temperature experimentalist, persuaded Landau to move to Moscow and to head the Theory Division of the S.I. Vavilov Institute of Physical Problems, which had been created by the U.S.S.R. Academy of Sciences.

Landau's attitude to physics and physicists was critical; he did not suffer fools gladly. While always willing to help anybody, he hated pomposity. People either adored him or were his bitter enemies. He was imprisoned during the Stalin era (1938) and only a personal intervention by Kapitsa freed him.

In 1937 Landau married K.T. Drobanzeva, and in 1946 they had a son, Igor, who became an experimental physicist.

On Jan. 7, 1962, Landau was involved in a car accident. He was unconscious for six weeks and was several times declared clinically dead, but he somehow revived. Distinguished specialists from several countries helped to save his life. After Landau had regained consciousness his faculties slowly returned to him, but he was no longer able to perform creative work. His physical condition never returned to normal, and he died six years later.

The Landau School (1932–1962)

Landau envisioned theoretical physics as one indivisible science with its own logic based on certain general principles. Later he converted this vision into a course on theoretical physics he developed with Evgeni M. Lifshitz at the Institute of Physical Problems at Kharkov University. The plan of the course became the "theoretical minimum" for the students; it also involved a number of mathematical problems which Landau regarded as indispensable knowledge for every theorist. By imparting this philosophy to his students, he set the tone of 20th century Soviet theoretical physics.

Virtually all his students and associates were tested on the theoretical minimum: The first exam Landau gave anyone eager to become his student was in mathematics: the exam required the applicant to be able to calculate any indefinite integral *that could be expressed in terms of elementary functions, to be able to solve any ordinary* differential equation *and to have knowledge of* vector analysis, tensor algebra *and the principles of functions of* complex variable.

Landau believed that tensor analysis and group theory should be studied together with the fields of theoretical physics in which they find application. Only after passing this exam could the applicant move successively on to study the seven sections of the "theoretical minimum". This study demanded basic knowledge of all fields of theoretical physics. Landau thought that all theorists should master this basic knowledge, regardless of their eventual specialty fields.

Of course, not everybody had the ability or the persistence to complete the study of the theoretical minimum. All in all, 43 physicists have passed the exam[763].

Attendance in the 'Landau seminar' was compulsory of all Landau's students. There, articles from authoritative scientific journals were surveyed.

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[763] In a recent article (Amer. Math. Soc. Transl. (2), **212**, 2004), **S.P. Novikov** (Field Medalist, 1970) reviewed the declining state of higher education in Russia and the West. He stated

"I can clearly see that contemporary education cannot produce a theoretical physicist capable of passing Landau's theoretical minimum.

...The purely democratic evolution of education, when people freely choose courses, works poorly in these sciences... physics and mathematics education is not a democratic structure in nature; it is not like a free economy... Thus we are entering the 21st century in a state of profound crisis."

Landau himself marked the articles he thought of particular interest, which ranged over all fields, from solid-state physics to GTR. *Presenting a report at the seminar was very time consuming and required an extensive background. One was to summarize the contents of the chosen paper based on a complete understanding of the subject. This is where the training ensured by the 'theoretical minimum' manifested itself. Landau was grounded in all fields of theoretical physics, and he required the same of his students and colleagues.*

After the presentation Landau would give an evaluation of the results obtained in the review article. If the results were outstanding, they were inscribed into the "Golden Book". If in the course of the discussion there arose problems requiring further investigation, these were introduced into the "Book of Problems".

Some articles Landau denounced as "pathological", which implied that principles of scientific analysis were violated, either in the solution of the problem or in its formulation. Landau himself did not read scientific journals, and thus the seminar was converted into a "creative laboratory", where Landau's students, while feeding him scientific data, were taught his deep critical analysis and understanding of physics.

Each physicist who passed the theoretical minimum acquired both rights and duties. He acquired the right to be backed by Landau, but at the same time, he made a commitment to give reports at the seminars. If a speaker failed to give intelligible answers to the questions pertaining to the reviewed material or could not clearly expound his thoughts, his situation was not enviable. Sometimes the unlucky fellow's name was excluded from the list of speakers, and he was deprived of the right to review articles from scientific journals (this happened rather seldom). In Landau's circle this measure was regarded as capital punishment: Landau despised such a theorist and immediately denied him backing.

Not all seminars were devoted to reviewing articles. Landau's students and physicists from other institutes and cities also made reports on original work. As a rule Landau would acquaint himself with original papers before the seminar; if he found a paper interesting, it would be presented. Landau personally spoke on all his own works at the seminar.

It was difficult, but a great honor, to deliver a talk at the seminar. The speaker was subjected to severe interrogation. The audience had the right to interrupt. The presentation was more a dialogue between the speaker and the audience (led by Landau) than a report. Often if the course of the dialogue, errors, gaps of logic, discrepancies and points of disagreement on basic assertions of the paper were brought to light. Landau was a man of great critical intellect: His criticism always helped find the truth.

If an author was a success at the seminar, he could be sure that his work was not logically inconsistent and that he had new results. This is why theorists were anxious to report their work at Landau's seminar: They knew they would always get an impartial, unprejudiced assessment of their work, and that from the highest possible authority.

Critical analysis of research is important in any field of science, and particularly so in theoretical physics. Investigation in theoretical physics is a chain of logical constructions that can sometimes be ruptured. In beginning his work an author may make assumptions whose validity is not always confirmed at the end; often these assumptions are not explicit. At Landau's seminar it would sometimes happen that after exhausting all his arguments, an author would unsuccessfully resort to his trump card: showing that his results coincided with the observed experimental data. This argument invariable provoked only laughter in the audience, since no coincidence of theory with experiment can justify logical gaps in the theorist's work.

Landau's working day always started with a visit to the experimental laboratories on the ground floor of the Institute for Physical Problems. There he rushed through the laboratories, found out the latest news and lingered on in case anyone wanted his immediate theoretical assistance. Landau believed the problems experimenters were currently solving had priority over the problems of theorists. He was always willing to cut short any activity whenever an experimenter asked him for a calculation however minor. His cooperation with experimenters gave rise to many of Landau's outstanding works. Indeed, his magnum opus — the formulation of the theory of superfluidity — was the fruit of his close cooperation with the experimenter Kapitsa.

For the sake of "economy of thought" Landau would often employ fundamental general principles, rejecting anything that could not be confined within these principles. But any new nontrivial result plunged him into deep thought. In such a case, Landau would apply his methods to the problem, either confirming or rejecting the result. It was in just this fashion that Landau became interested in the kinetic equation for elementary excitations in a quantum liquid; soon he found its exact solution.

Landau never did for his pupils what he believed they should do themselves. Sometimes, after many unsuccessful attempts to solve a problem, a student would ask Landau for help and hear the reply: "This is your problem. Why should I do it for you?" After Landau's flat refusal it become clear that no outside help would arrive; if the student was lucky, enlightenment would dawn and the problem would soon be solved. Neither did Landau formulate problems for his students, or supply thesis topics to his postgraduates: They were responsible for these tasks themselves. He thus trained them to be independent, educating them as future leaders of science.

Clear-cut logic and simplicity were characteristic of Landau's work. He thoroughly thought over his lectures and articles. He did not write his articles himself: His associates — most often Lifshitz — were entrusted with this respected task.

In mathematics, Landau always set greater store by methods that enable one to solve concrete physical problems than he did by existence theorems. He had however underestimated abstract mathematics which was not yet widely adopted in physics.

He used to say in jest: "We know that the mathematics of the 21^{th} *century is nothing but theoretical physics". Yet, by the 1970's, a growing list of modern mathematical disciplines such as topology, algebraic geometry, algebraic topology and differential geometry – have joined group theory and* 19^{th}*-century applied mathematics at the forefront of modern physics.*

Superconductivity[764]

Lev Landau *and* ***Vitaly L. Ginzburg***[765] *cast the phenomenology of superconductivity in the language of quantum mechanics (1950); they introduced a macroscopic wave-function* (ψ) *whose square modulus represents the Cooper-pair density* (n)*:* $|\psi|^2 = n$. *This wave-function satisfies a Schrödinger-like equation called the Ginzburg-Landau equation, which may be written in one dimension as* $\frac{d^2\psi}{dx^2} = \frac{2m^*a}{\hbar^2}\psi$, *where* m^* *is the effective mass of a Cooper-pair and* a *is a temperature-dependent real parameter. The solution is in the form* $\psi = Ce^{-x/\xi}$, *where* C *is a constant and* $\xi = \frac{\hbar}{\sqrt{2m^*a}}$ *is a coherence length.*

A microscopic theory of superconductivity was developed (1957) by ***John Bardeen***, ***Leon Cooper*** *and* ***Robert Schrieffer***, *known as the BCS theory. It provided a major conceptual breakthrough in the quantitative understanding of the mechanism of superconductivity. The central ingredient of the BCS theory is that electrons form Cooper pairs. By interaction with the ionic lattice, the conduction electrons develop a weak attraction for each other, mediated by quanta of lattice vibrations (phonons). This may be thought of in the following way: One electron passes through the lattice and the positive ions are attached to it, causing a distortion in their nominal positions. The second electron of the Cooper pair is then attracted by the clustered ions. The*

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[764] To dig deeper, see:

- Kittel, C., *Introduction to Solid State Physics*, Wiley, 1986, 646 pp.
- Epifanov, G.I, *Solid State Physics*, Mir Publishers: Moscow, 1979, 333 pp.
- Sychev, V.V., *Complex Thermodynamic Systems*, Mir Publishers: Moscow, 1981, 240 pp.
- Rumer, Yu.B. and M.Sh. Ryvkin, *Thermodynamics, Statistical Physics, and Kinetics*, Mir Publisher: Moscow, 1980, 600 pp.
- Feynman, R.P., *Statistical Mechanics*, Perseus Books, 1998, 354 pp.

[765] **Vitaly L. Ginzburg** (b. 1916) was awarded the Nobel Prize in physics (2003) with **A.A. Abrikosov** (b. 1928) and **A.J. Leggett** (b. 1930) for their pioneering contributions to the theory of superconductors and superfluids.

Cooper pairs (each of which consists of electrons of anti-aligned momenta vectors and spin states) behave like bosons and form a condensate *analogously to photons in a laser beam. The distance over which the electron pair is correlated is several thousand Å, much greater than the typical distance between neighboring conduction electrons.*

The attraction between the electrons is extremely weak. In spite of the weakness of the interaction, the superconductivity phase transition occurs because the electrons close to the Fermi energy *have a net attraction preventing them from being scattered by lattice defects or thermal phonons.*

At temperatures just above the critical temperature *(T_c) the material is in the normal (non-superconducting) phase. At temperatures just below T_c, however, a small energy gap develops where there are no electronic states. The energy gap is caused by the binding energy due to the formation of the Cooper pair.*

In this superconducting phase, *whenever an electron hits a defect or phonon, it is unlikely that its Cooper-pair partner – typically many lattice-spacings away – would simultaneously encounter an identical obstruction; so the attraction between these two electrons suffices to keep the pair from being slowed down.*

If external influences (e.g. electromagnetic fields) induce a current *within the superconductor, the momenta of a Cooper-pair's electrons no longer cancel, but the energy gap still protects pairs from dissociation; the currents then fail to dissipate and become* permanent *(provided a flow-circuit is available within the superconductor).*

The formation of the condensed Cooper pairs is hindered by the thermal excitation of the electrons above the critical temperature, or by too-high current densities and/or external magnetic fields even when $T < T_c$. As the temperature is lowered beneath T_c, the number of electrons that can cross the gap is significantly reduced, a greater number of Cooper pairs are formed, and the energy gap becomes larger. At a temperature of zero Kelvin, all the states below the gap are filled, and the total energy is lower than in the normal state.

1932–1951 CE **Allan Balcom Du Mont** (1901–1965, USA). Engineer, inventor and pioneer in the practical development of television. Invented the first commercial television oscilloscope by perfecting the cathode-ray tube, devised the first TV guidance system for missiles, and invented the "tuning eye".

Du Mont was born in Brooklyn, NY and obtained a B.Sc. in electrical engineering from Rensselaer Polytechnic Institute (1924). He founded the Du Mont Laboratories (1931) in his garage with $1000 — half of it borrowed. By 1939 he became the first television millionaire and by 1951 his company was doing a gross business of about $75 million a year. Regular television broadcasts were initiated on April 30, 1939.

1932–1959 CE **Francis Thomas Bacon** (1904–1992, England). Engineer and inventor. Direct descendant of **Francis Bacon** (1561–1626). Developed the first practical hydrogen-oxygen *fuel cell*, which convert air and fuel into electricity through electrochemical processes.

Although **William Grove** discovered the principle of fuel cells in 1842, they were considered a scientific curiosity until the early 1940's, when Bacon proposed their use in submarines.

Bacon was born in Billericay Essex, UK. He graduated from Eton College and from Trinity College, Cambridge.

Bacon began experimenting with alkali electrolytes in the late 1930s, settling on potassium hydroxide (KOH) instead of using the acid electrolytes known since Grove's early discoveries. KOH performed as well as acid electrolytes and was not as corrosive to the electrodes. Bacon's cell also used porous "gas-diffusion electrodes" rather than solid electrodes as Grove had done. Gas-diffusion electrodes increased the surface area in which the reaction between the electrode, the electrolyte and the fuel occurs.

Also, Bacon used pressurized gases to keep the electrolyte from "flooding" the tiny pores in the electrodes. Over the course of the following twenty years, Bacon made enough progress with the alkali cell to present large scale demonstrations.

He continued his research with the Anti-submarine Experimental Establishment, then returned to Cambridge (1946), where he demonstrated a successful six-kilowatt fuel cell (1959). The first practical application of this high-efficiency, pollution-free technology was in the *Apollo space vehicles* of the United States, which used the alkaline fuel cells to provide in-flight power, heat, and clean drinking water, the latter a by-product of the electrochemical reaction.

Bacon sought new applications for fuel cells as a principal consultant to National Research Development Corp. (1956–62), Energy Conservation Ltd. (1962–71), and the U.K. Atomic Energy Authority (1971–73).

By the end of the century, the technology was being developed internationally. He was made an Officer of the Order of the British Empire (1967), elected a fellow of the Royal Society (1973), and awarded the first Grove Medal (1991).

Electrochemical Technologies — Origin, Legacy and Perspectives

A. Introduction

Aristotle postulated that all matter is comprised of four basic elements: earth, water, air, and fire. The idea dominated science until the late 18th century, when revolutionaries from rival nations transformed chemistry from a jumble of medieval alchemy into a true science. The pace of discovery accelerated rapidly as chemists on the frontiers of knowledge established the theories and methodologies of modern science. Electrochemical systems have played a determinant role in the history of mankind. They are an intrinsic part of our evolution on this Planet.

The whole of electrical technologies is based on magnetic and electrical phenomena and no history of the subject can ignore the origins of these two groups, remote and sometimes uncertain as these origins may be. For many centuries man has observed magnetic effects in natural minerals found in the ground and electrical effects in lightning, the aurora borealis, St. Elmo's fire, the electric eel and the attraction of light objects by natural resins when rubbed.

Some of these observations have been put to practical use from the very earliest recorded times — the lodestone for navigation, the electric eel for medicinal purposes — so that, if electrical engineering is the practical application of electrical and magnetic science, there is a sense in which it has not only its roots in the remote past but actually existed as a human activity even in those far-off days. The two sides, magnetism and electricity, however, remained quite apart until the beginning of the nineteenth century, when the discovery of the close relationship between them brought the two streams of thought together and opened the way to the establishment of their *interrelationship. The great surge forward in understanding the foundations of electromagnetism made modern electrical engineering possible.*

Electrochemistry plays a dominant role in a vast number of research and applied areas. This is basically a consequence of a unique combination of different features of electrochemical reactions.

Today, electrochemical processes comprise a substantial part of chemical industry and consume about seven percent of the industrial electricity use. They are of increasing importance w.r.t. environmental protection, safety and energy technology.

Electrochemical reactions are known for a wide range of materials such as metals, semiconductors, polymers, and biological systems. Electrochemistry currently plays a large role in a number of rather diverse areas such as preparative chemistry, analytical chemistry, energy storage, energy conversion, biochemistry, solid state chemistry, materials science, and microelectronics.

At the beginning of the twentieth century, electrochemistry was mainly dominated by studies of the transport of charged species and thermodynamic considerations.

Kinetic aspects of electrochemistry have become more important in electrochemical research since the middle of the twentieth century with an increased understanding of the chemical and electronic structure of the solid/solution interface. These studies have been accelerated by the application of numerous in-situ and ex-situ spectroscopic techniques, which have been combined with electrochemical experiments over the last thirty years. Recently,the introduction of in-situ scanning probe techniques has allowed us to follow electrochemical reactions on an atomic or molecular scale.

Based on theoretical and experimental results and methods gathered by electrochemists for many decades, electrochemistry is now used in many fundamental fields, such as the study of new organic and inorganic compounds biological systems. In more applied arenas, it is used to shape materials from the macroscopic to the microscopic scale, to accurately analyze for chemical impurities, to understand and prevent the corrosion of materials at low and extremely high temperatures, to probe the functioning of living cells, and to directly convert chemical energy into electricity.

Electrical energy does not exist naturally in any convenient form and it must be converted from some other energy form when needed. Chemical energy is the most practical source and is generally used in one of two ways. Fuel can be burnt in a heat engine, such as a petrol or diesel engine, or a gas turbine, which then drives an electrical generator. This process is inherently inefficient. Alternatively, the fuel may be consumed in an electrolytic reaction in a battery or fuel cell.

For low power (less than 50 W) and short duration missions, batteries are the logical choice. Engine generators and fuel cells are the preferred choices for applications demanding greater than 500 W. The following list highlights the salient points of each technology:

Diesel (or gasoline) generators produce electricity in a multi-step process. The energy in the fuel is first converted into rotary mechanical energy in the engine. Then, the rotary mechanical energy is converted to DC or AC electrical energy by a generator.

As this is a multi-step energy conversion process, involving a heat engine with moving parts and, thus, frictional losses, it is an intrinsically inefficient method for producing electricity. Diesel generators typically have lower efficiencies, in the range of 20% to 30%. Gasoline generators usually exhibit lower efficiencies, in the 10% to 15% range.

Diesel and gasoline generators are also characterized by a high operating temperature, noise, air pollution, and their requirement for regular maintenance. On the other hand, diesel and gasoline generators are inexpensive and readily available. They are also easily, even continuously, refuelable. The energy available from the generator is limited only by its supply of fuel.

A battery is an electrochemical energy conversion device. It converts the energy in the fuel (active material of the electrodes) directly into DC electricity. There is no combustion, no multi-step energy conversion, and no frictional loss. The battery is intrinsically efficient, silent and non-polluting. In most embodiments, the battery is also a low temperature device that produces power immediately upon demand.

However, batteries suffer from the limitation that all the available fuel is contained within the battery case. When a non-rechargeable (primary) battery has consumed its fuel, it is discarded and, with it, the energy conversion device and the "fuel tank". In a rechargeable (secondary) battery, one can reuse the energy conversion device and the "fuel tank", but one must wait for several hours for the recharging process to be completed. One cannot operate a battery continuously as one can operate the diesel generator.

The fuel cell is a device that converts fuel and an oxidant directly into electricity by an electrochemical process. The fuel is not burned to produce heat and the efficiency of energy conversion is not limited by the Carnot Cycle limits placed upon heat engines. The fuel cell, like a battery, has no moving parts in the energy conversion device, and thus suffers no frictional losses.

Realistic energy efficiencies in excess of 50% can be achieved with fuel cell systems in generator set applications. Fuel cells combine the advantages of both diesel generators and batteries, while eliminating the major drawbacks of both. A fuel cell is essentially the electrochemical energy conversion device *for the battery, engineered in such away that it is continuously refuelable, like the diesel generator.*

A fuel cell is intrinsically energy efficient, non-polluting, silent, and reliable. In some embodiments, it is a low temperature device that provides power instantly upon demand, and exhibits a long operating life with minimal maintenance.

B. Timeline History of Electrochemistry

1766 **Henry Cavendish** (1731–1810, England) studied the properties and the processes of preparation of hydrogen (without knowing its elemental character) in great detail and gave it the name "inflammable air".

1781 **Joseph Priestley** (1733–1804, England) produced water by igniting hydrogen in oxygen.

1783 **Henry Cavendish** observed combustion of hydrogen to produce water. Lavoisier repeated the experiments conducted by Cavendish and realized that this was a new gas. He gave it the name 'hydrogen' from the Greek Words for 'water former'.

1791 **Luigi Galvani** (1737–1798, Italy). Discovered that an animal muscular tissue could be induced to twitch if two different metals (later called 'electrodes') were brought into contact with it. His experiments eventually laid the foundations for the principle of *storage batteries*.

1800 **Alessandro Volta** (1745–1827, Italy). Discovered the principle of *electrolysis*: the production of a chemical reaction by passing an electric circuit through an *electrolyte* (a liquid that conducts electricity as a result of the presence of positive or negative ions, such as solutions of ionic salts).

Invented the first battery (the *voltaic pile*). He described two arrangements that produced an electric current: one was a pile of silver and zinc discs separated by cardboard moistened with brine, and the other — a series of glasses of salty or alkaline water in which bimetallic curved electrodes were dipped; it enabled electric currents to be produced, and was the first method of artificially producing a reasonable and controllable electric current.

1800 **William Nicholson** (1753–1815, England). Broke water into its components hydrogen and oxygen through *electrolysis.*

1800–1803 **Johann Wilhelm Ritter** (1776–1810, Germany). Invented *electroplating*[766] when he passed current through a copper-sulfate solution (1800). Discovered that water consists of two parts of hydrogen and one part of oxygen. Invented the *accumulator* or *rechargeable* battery.

1802 **William Cruickshank** (England) designed the first electric battery capable of mass-production.

1807 **Humphry Davy** (1778–1829, England). Isolated potassium and sodium through electrolysis. He thus established the use of electrochemistry for isolating highly active elements which had proved unresponsive to traditional chemical techniques.

1815–1828 **William Hyde Wollaston** (1766–1828, England) made (1815) improvements to the voltaic pile. Further improvements were made by **Robert Hare** (1781–1858, USA) during 1820–1831, and in 1829 by **Antoine-César Becquerel** (1788–1878, France).

1832 **Michael Faraday** (1791–1867, England) developed the *quantitative theory* of electrochemistry. His basic laws are:

- The amount of a substance deposited on each electrode of an electrolytic cell is directly proportional to the quantity of electricity passed through the cell.
- The quantities of different elements deposited by a given amount of electricity are in the ratio of their chemical equivalent weights.

1836 **John Daniell** (1790–1845, England). Improved the *voltaic cell*, replacing it with the *Daniell cell* which more reliably produced a steady current.

[766] The origins of electroplating go back to the *alchemists* of the 16th century: an iron rod decomposed when soaked in blue-vitriol ($CuSO_4 \cdot 5H_2O$). The *Parthians* (250–224 BCE) may have used an electric battery to supply electricity for a silver-plating process. This artifact was discovered in 1957 in the Baghdad museum. Some Biblical scholars have speculated that the *Ark of the Covenant* (I Sam *4*, II Sam *6*), was indeed a powerful high-voltage battery or a charged capacitor.

1837 **Moritz Hermann Jacobi** (1801–1874, Germany and Russia). Improved the process of *electroplating*; the object, a conductor of electricity, was placed (as a cathode) in a bath in which the electrolyte was the salt of a metal to be deposited. His first undertaking was *silverplating*.

1839 **William Robert Grove** (1811–1856, England). Constructed the first *fuel-cell* through the reverse process of electrolysis; he placed test tubes of hydrogen and oxygen gases over two respective platinum stripes in a vessel containing a dilute sulfuric acid with a connecting wire.

In such a device, the reactions producing the electric current are obtained from substances contained *outside* and not inside the casing. The chemical energy of a fuel is converted directly into electric energy, i.e. the energy released in the *oxidation* of a conventional fuel is made *directly* available in the form of an electric current. It thus avoids the wasteful detour of conventional, thermal power stations.

1841 **Robert Wilhelm Bunsen** (1811–1899, Germany). Replaced the expensive platinum electrode used in Grove's battery with a *carbon* electrode.

1859 **Gaston Planté** (1834–1889, France). Invented the first practical *accumulator* or *rechargeable battery*.

1868 **Georges Leclanché** (1839–1882, France). Devised the *dry-cell* with zinc alloy and manganese dioxide electrodes in an ammonium chloride electrolyte. This is the type of battery used in flashlights, radiosets etc. This cell was later improved (1888) by **Carl Gassner**.

1870 **Jules Verne** predicted the use of hydrogen fuel in his science-fiction book *Mysterious Island*. Verne describes a process whereby "*...water will one day be employed as fuel, that hydrogen and oxygen which constitute it, used singly or together, will furnish an inexhaustible source of heat and light, of an intensity of which coal is not capable... Water will be the coal of our future.*"

1875–1879 **Friedrich Kohlrausch** (1840–1910, Germany) found that Ohm's Law also applies to dissolved electrolytes. Discovered the law of the independent migration of ions, i.e.: each type of migrating ion has a specific resistance no matter what its original molecular combination may have been, and therefore a solution's electrical resistance is due only to the migrating ions of a given substance.

1877–1896 **Svante August Arrhenius** (1859–1927, Sweden), **Friedrich Ostwald** (1853–1932, Germany) and **Jacobus Van't Hoff** (1852–1911, Holland) expounded the theory of electrolytic dissociation and conduction of dilute solutions of electrolytes in the framework of general chemical kinetics.

1888–1906 **Hermann Nernst** (1864–1941, Germany). Elucidated the theory of voltaic cells by assuming an *electrolytic pressure of dissociation* (1889) which forces ions from electrodes into solution and which is opposed to the *osmotic pressure* of the dissolved ions.

He applied the *principles of thermodynamics* to the chemical reactions proceeding in a battery and showed how the characteristics of the current produced, could be used to calculate the *free energy change* in a chemical reaction producing the current. The *Nernst Equation* relates the cell's voltage to its chemical properties (1906). Nernst also invented the Nickel-Cadmium battery (1899) and worked out the thermodynamics of the 'dry cell' (1891).

1906 **Joseph John Thomson** (1856–1940, England) discovered that a hydrogen atom has a single electron.

1923 **Johannes Nicolaus Brönsted** (1879–1947, Denmark) developed the donor-acceptor theory of acids and bases.

1923 **John B.S. Haldane** (1892–1964, England) presented a paper to Cambridge University in which he proposed to meet the increasing demand for energy by using wind energy to electrolyze water into hydrogen and oxygen. The gases, first liquefied, can be stored until needed. They can then be recombined in combustion motors or 'oxidation cells'.

1929 **Rudolf Erren** (Germany) advanced the concept of injecting hydrogen into the air-fuel mixture of combustion engines, serving to heighten the output of the combustion process. He converted buses, vans, rail cars, and even *submarines* to be powered by hydrogen or any combination of hydrogen-fuel mixtures.

1930 **James J. Drumm** (1897–1974, Ireland) invented the *zinc-nickel alkaline battery*. The cell is an alkaline cell and the only metals which enter into its construction are stainless steel and pure nickel. Its mechanical strength is therefore quite satisfactory. The positive-plate system consists of the hydroxides of nickel mixed with nickel flakes. This electrode was first developed by **Edison** (1905).

The negative plate is a grid of nickel gauze and the electrolyte is a solution of zinc oxide in potassium hydroxide (potassium zincate). During charge, zinc is plated on to the nickel grid, and during discharge this zinc dissolves readily in the potassium hydroxide.

1931 **Hermann Honnef** (Germany) designed huge wind-power generators which could theoretically produce up to 100 megawatts of power, stored as hydrogen.

1932–1959 **Francis Thomas Bacon** (1904–1992, England). Invented the first successful *fuel-cell*. He improved on the expensive platinum catalysts employed by **Ludwig Mond** (1839–1909) and **Charles Langer** (1889) with a hydrogen-oxygen cell using less corrosive alkaline electrolytes and inexpensive nickel electrodes (1932). In 1959, Bacon and his coworkers were able to demonstrate a practical 5-kilowatt system capable of powering a welding machine.

1932 **Harold Clayton Urey** (1891–1981) discovered deuterium (heavy hydrogen).

1934 **Marcus Laurence Elwin Oliphant** (1901–2000) discovered tritium.

1938 **Igor I. Sikorski** (1889–1972, USA) suggested the use of liquid hydrogen as an aircraft fuel.

1943 **Samuel Ruben** (1900–1988, USA) invented the *Mercury-oxide cell* [*Anode*: Zn; *Cathode*: HgO; *Electrolyte*: KOH or NaOH aqueous solution]. It revolutionized battery technology by packing more capacity in less space and by being durable enough for the harsh climates of wartime theaters like North Africa and the South Pacific — places where ordinary zinc-carbon batteries (used in flashlights, mine detectors, and walkie-talkies) could not hold up. Ruben joined efforts with the P.R. Mallory company to manufacture millions of mercury cells for the WWII war effort. Both Ruben and Mallory then created the *Duracell* company.

1948–1955 **R.O. King** (Canada) and his associates at the University of Toronto showed that combustion engines can be converted to run on hydrogen, simply and cheaply.

1950's The U.S. Air Force was using hydrogen fuel in experimental, high-altitude, long-range reconnaissance aircraft.

1956–1965 **Rudolph Arthur Marcus** (USA). Established the modern theory of chemical kinetics, including theories of *electron transfer reactions*, unimolecular reactions, *electrode reactions*, semiclassical theory of collisions and bound states, intermolecular dynamics, solvent dynamics, and chemical reaction coordinates.

Application of his theories include such phenomena as photosynthesis, electrically conducting polymers, *chemiluminescence*[767] and corrosion. Awarded the Nobel Prize for Chemistry (1992).

1957 **Carl Walton Lillehei** (USA), a cardiovascular surgeon, developed the long-life miniature mercury battery.

1959–1980 **Lew Urry** (USA) and **Karl Kordesch** (USA) independently developed the *alkaline primary battery* which replaced the zinc-carbon flashlight batteries. In 1966, Kordesch developed a 150 kW *Alkaline fuel cell.* In 1970 he built an alkaline fuel cell/battery hybrid electric car based on an A-40 Austin. The fuel cell was installed in the trunk of the car and hydrogen tanks on the roof, leaving room for 4 passengers in the 4-door car.

1960's

- Lockheed (USA) was developing a high altitude supersonic spy plane to run on liquid hydrogen fuel.
- NASA (USA) developed the use of the hydrogen fuel-cell for use in the Apollo missions to the moon. The fuel-cells, utilizing expensive platinum electrodes, provided on-board electrical power, as well as generating drinking water for the crew's consumption.

767 Photon emission of a substance resulting from a chemical reaction (such as slow oxidation of phosphorus). The light emitted by the fire-fly or glow-worm, and luminous combustion, are examples of this common phenomenon.

- **John O'M. Bockris** at General Motors (USA), began advancing the idea of '*hydrogen economy*'. In this ambitious energy project, the cities of the United States could be supplied with *energy derived from the sun*, and the energy could be stored using hydrogen.

1960–1980 **Allen J. Bard** (USA). Fostered the development of electroanalytical methods and instruments that deepened the fundamental understanding of *electron-transfer reactions* and electrogenerated *chemiluminescence*.

1966–72 **Roger Billing** (USA) converted many late automobile models to run on hydrogen, using their internal combustion engines.

1974 **Joseph Lindmayer** (USA) developed a silicon photovoltaic cell for harnessing solar power.

1975 **Alan J. Heeger** (USA), **Hideki Shirakawa** (Japan) and **Alan G. MacDiarmid** (USA) shared the 2000 Chemistry Nobel Prize for the discovery and development of organic conducting polymers ("synthetic metals").

C. Thermochemistry

When a chemical reaction occurs some chemical bonds are broken and others are formed. Thus, the energy associated with chemical bonds and intermolecular attraction (chemical energy) changes, and some of this energy change appears as the intake or release of heat, i.e.: microscopic kinetic energy associated with random molecular motion.

A heat of reaction *is the quantity of heat exchanged between a system and its surrounding when a chemical reaction occurs within the system at constant temperature (e.g.: heat combustion).*

If reaction occurs in an isolated system *(neither matter nor energy are exchanged with its surroundings), the reaction produces a change in the thermal energy of the system, accompanied by a change in temperature. An* *exothermic reaction* produces a temperature increase in an isolated system or, in a nonisolated system, gives off heat to the surrounding (*negative* heat of reaction). The reverse is true for an *endothermic reaction.*

Internal energy, U is the total energy of a system (both kinetic and potential) including:

- Translational, rotational and vibrational energy of molecules.
- Energy stored in chemical bonds and intermolecular interactions.
- Energy associated with the interaction of protons and neutrons in atomic nuclei (unchanged in a chemical reaction).

Heat transfer and *work* are means by which a system exchanges energy with its surrounding, and occur only during a *change* in the system.

The first law of thermodynamics dictates the relationship between heat supplied (q), external work done (W) and changes in internal energy:

$$\Delta U = q + W$$

The convention followed is that energy *entering* the system carries a *positive* sign. Thus, if heat is *absorbed* by the system, $q > 0$. If work is done *on* the system, $W > 0$. Any energy *leaving* the system carries a *negative* sign. Thus, if heat is *given off* by the system, $q < 0$. If work is done *by* the system, $W < 0$.

If, on balance, energy enters the system, $\Delta U > 0$. If more energy leaves the system, $\Delta U < 0$.

The *state* of a system is indicated by a complete set of *state variables*, such as its *temperature* (T), *pressure* (P) and the types and amounts of substances present.

One useful state variable is the *enthalpy* $H = U + PV$ $(V =$ volume). If a process is carried out at a constant temperature and pressure, $\Delta H = \Delta U + P\Delta V$. For example, the equation for the combustion of sucrose is

$$C_{12}H_{22}O_{11}\ (\textit{solid}) + 12O_2\ (\textit{gas}) \rightarrow 12CO_2\ (\textit{gas}) + 11H_2O\ (\textit{liquid})$$

for which

$$\Delta H = -5.65 \times 10^6\ \textit{Joule/mole} \qquad (\textit{heat is produced}).$$

This reaction is *exothermic*. On the other hand, for the reaction

$$N_2\ (\textit{gas}) + O_2\ (\textit{gas}) \rightarrow 2NO\ (\textit{gas})$$

$$\Delta H = +1.8 \times 10^5 \ Joule/mole$$

and heat is absorbed from the surroundings (endothermic).

The exothermic ionic reaction

$$H^+ \ (aq) + OH^- \ (aq) \rightarrow H_2O \ (liquid), \qquad \Delta H = -5.58 \times 10^4 \ J/mole$$

describes the neutralization of a strong acid by a strong base.

It is useful to define the Gibbs free energy $G = H - TS$, *where* S *is the entropy function. For a process occurring at constant* T, *the free energy change is* $\Delta G = \Delta H - T\,dS$.

We might think that the quantity of energy available to do work in the surroundings as a result of an exothermic chemical process as $-(\Delta H)$. *This would be the same as the quantity of heat that an exothermic reaction releases to the surroundings (including any volume-changing mechanical work, if* P *is held fixed). However, that quantity of heat must be adjusted for the heat requirement in reversibly producing the accompanying entropy change in the system* ($q_{\text{rev}} = T\Delta S$).

If an exothermic reaction is accompanied by an increase in entropy, the amount of energy available to do work in the surroundings is greater than $-\Delta H$. *If entropy decreases in the exothermic reaction, the amount of energy available to do work is less than* $-\Delta H$. *But in either case, this amount of energy is equal to* $-\Delta G$. *Thus, the amount of work that we are free to extract from a chemical process is* $-\Delta G$, *so the Gibbs function* G *is called the free energy function.*

Notice also that this interpretation of free energy allows for the possibility of work being done in an endothermic process if $T\Delta S$ *exceeds* ΔH[768]. *We will see next how the free energy change accompanying a reaction can be converted to electrical work. In any case we must not think of free energy as being "free" energy. Costs are always involved in tapping an energy source.*

Because we cannot establish absolute values for G *or* H *we must refer these state variables to a certain standard cell, with standard potential difference* ΔE° *(for an electrochemical process) and with corresponding* ΔH° *and* ΔG° *(for any chemical reaction or process).*

Now, we know that the entropy change for an isothermal expansion of one mole of an ideal gas is simply $\Delta S = R \ln \frac{V_f}{V_i}$ *(R the gas constant), where* V_f *is its final volume and* V_i *is its initial volume. Since for an ideal gas*

[768] A reaction for which $\Delta G < 0$ is called *exergonic*, whereas, if $\Delta G > 0$, it is referred to as *endergonic*.

$\Delta H = \Delta H^\circ$ and $\Delta S = \Delta S^\circ - R \ln Q$, where Q is the reaction quotient[769]*, we obtain $T\Delta S = T\Delta S^\circ - RT \ln Q$ and thus*

$$\Delta G = \Delta H^\circ - T\Delta S^\circ + RT \ln Q = \Delta G^\circ + RT \ln Q.$$

Hence the Nernst Equation (1889):

$$\Delta G = \Delta G^\circ + RT \ln Q.$$

Fossil Fuels

Energy-source materials, called fuels, liberate heat through the process of combustion. The bulk of Carnot cycle needs are met by petroleum, natural gas and coal, so-called fossil fuels. These fuels are derived from remains of plant and animal life from millions of years ago.

The original source of the energy locked into these fuels is solar energy; in the process of photosynthesis, CO_2 and H_2O, in the presence of enzymes, the pigment chlorophyll, and sunlight, are converted into carbohydrates which are compounds with formulas $C_m(H_2O)_n$, where m and n are integers [e.g. in sugar glucose $m = n = 6$, that is, $C_6(H_2O)_6 = C_6H_{12}O_6$. Its formation through photosynthesis is an endothermic process, represented as

$$6CO_2\ (gas) + 6H_2O\ (liquid) \xrightarrow[\text{sunlight}]{\text{chlorophyll}} C_6H_{12}O_6\ (solid) + 6O_2\ (gas),$$

$$\Delta H = +2.8 \times 10^6 \text{ Joules/mole of } C_6H_{12}O_6.$$

When this reaction is reversed (combustion of glucose), heat is produced in an exothermic process.

The complex carbohydrate cellulose, with molecular masses ranging up to 500,000 amu, is the principal structural material of plants. When plant life decomposes in the presence of bacteria and out of contact with air, O and H atoms are removed and the approximate carbon content of the residue increases in the progression (percentage by mass)

$$\text{peat} \to \text{lignite } (32\%\ C) \to \text{sub-bituminous coal } (40\%\ C) \to \text{bituminous coal } (60\%\ C) \to \text{anthracite coal } (80\%\ C)$$

[769] Q is defined as the product of $n_i^{a_i}$ for all *reagents* and *products* with n_i the molar concentrations, and a_i = number of molecules of species i in the balanced reaction ($a_i > 0$ for products, $a_i < 0$ for reagents).

For this process to proceed all the way to anthracite coal may take about 300 *million years. Coal, then, is a combustible organic rock consisting of carbon, hydrogen, and oxygen, together with small quantities of nitrogen, sulfur, and mineral matter (ash). (One proposed formula for a "molecule" of bituminous coal is* $C_{153}H_{115}N_3O_{13}S_2$*.)*

Petroleum and natural gas have formed in a somewhat different way. The remains of plants and animals living in ancient seas fell to the ocean floor, where they were decomposed by bacteria and covered with sand and mud. Over time, the sand and mud were stacked in layers and converted to sandstone by their own weight. The high pressures and temperatures resulting from this overlying sandstone rock formation transformed the original organic matter into petroleum and natural gas. The ages of these deposits range from about 250 *million to* 500 *million years.*

A typical natural gas consists of about 85% *methane* (CH_4), 10% *ethane* (C_2H_6), 3% *propane* (C_3H_8), *and small quantities of other combustible and noncombustible gases. A typical petroleum consists of several hundred different hydrocarbons that range in complexity from* C_1 *molecules* (CH_4) *to* C_{40} *or higher* (*such as* $C_{40}H_{82}$).

One way to compare different fuels is through their heats of combustion: In general, the higher the magnitude of the heat of combustion, the better the fuel. Table 5.8 lists approximate heats of combustion for the fossil fuels. These data show that biomass (living matter or materials derived from it — wood, alcohols, municipal waste) is a viable fuel, but that fossil fuels yield more energy per unit mass.

Table 5.8: APPROXIMATE HEAT OF COMBUSTION OF SOME FUELS

FUEL	HEAT OF COMBUSTION, KJ/GRAM
Municipal waste	−12.7
Cellulose	−17.5
Pinewood	−21.2
Methanol	−22.7
Peat	−20.8
Bituminous coal	−28.3
Iso-Octane	−47.8
Natural gas	−49.5

Methanol (methyl alcohol), CH_3OH, *can be obtained from coal by the reaction*

$$CO\ (gas) + 2H_2\ (gas) \rightarrow CH_3OH\ (liquid).$$

It can also be produced by thermal decomposition (pyrolysis) of wood, manure, sewage, or municipal waste. The heat of combustion of methanol is only about one-half that of a typical gasoline on a mass basis, but methanol has a high octane number — 106 — compared with 100 for the gasoline hydrocarbon iso-octane and about 92 for premium gasoline. Methanol has been tested and used as a fuel in internal combustion engines and is cleaner burning than gasoline. Methanol can also be used for space heating, electric power generation, fuel cells, and as a reactant to make a variety of other organic compounds.

Ethanol (ethyl alcohol), C_2H_5OH, *is produced mostly from ethylene,* C_2H_4, *which in turn is derived from petroleum.*

Ethanol production by fermentation is probably most advanced in Brazil, where sugarcane and cassava (manioc) are the plant matter (biomass) used. In the United States, ethanol fuel is used chiefly as a 90% gasoline–10% ethanol mixture called gasohol. Ethanol admixture is also used to raise the octane number of gasoline.

Another fuel with great potential is hydrogen. Its most attractive features are the following:

- *On a per gram basis, its heat of combustion is more than twice that of methane and about three times that of gasoline.*
- *The product of its combustion is* H_2O, *not* CO *and* CO_2 *as with gasoline.*

Currently, the bulk of hydrogen used commercially is made from petroleum and natural gas. (Alternative methods of producing hydrogen, and the prospects of developing an economy based on hydrogen are current subjects of research and development.)

Combustion reactions are only one means of extracting useful energy from materials. An alternative, for example, is to carry out reactions that yield the same products as combustion reactions in electrochemical cells called fuel cells. The energy is released as electricity rather than as heat. Solar energy can be used directly, without recourse to photosynthesis. Nuclear processes can be used in place of chemical reactions.

Alternative energy sources, including those intended for automobiles, are likely to become increasingly important in the twenty-first century.

D. Electrochemistry[770]

"A conventional gasoline-powered automobile is only about 25% efficient in converting chemical energy into kinetic energy (energy of motion). An electric-powered auto is about three times as efficient. Unfortunately, when automotive technology was first being developed, devices for converting chemical energy to electrical energy did not perform at their intrinsic efficiencies. This fact, together with the availability of high-quality gasoline at a low cost, resulted in the preeminence of the internal combustion automobile. Now, with concern about long-term energy supplies and environmental pollution, there is a renewed interest in electric-powered buses and automobiles."

"Chemical reactions can be used to produce electricity and electricity can conversely be used to drive chemical reactions. The practical applications of electrochemistry are countless, ranging from batteries and fuel cells as electric power sources, to the manufacture of key chemicals, the refining of metals, and the methods of controlling corrosion. Also important, however, are the theoretical implications. Because electricity involves flow of electric charge, a study of the relationship between chemistry and electricity gives us additional insight into reactions in which electrons are transferred in oxidation-reduction reactions.*"*

A basic concept of electrochemistry is that of oxidation number. *It is a number related to the number of electrons that an atom loses or gains, or otherwise appear to use in joining with other atoms in compounds. The following (somewhat arbitrary) conventions or rules are assigned:*

- *In its compounds,* hydrogen *has an oxidation state (O.S.) of* $\boxed{+1}$, *except in metal hydride compounds, where it has O.S. of* $\boxed{-1}$.
- *In its compounds,* oxygen *has an O.S. of* $\boxed{-2}$.
- *The O.S. of an individual atom in a* free element *(uncombined with other elements) is zero,* $\boxed{0}$.
- *The total oxidation states of all atoms in* neutral species *(isolated atoms, molecules) is zero; In* ions*, the O.S. number is equal to the charge of the ion.*

[770] Section D includes quotations and Figures (5.5, 5.6, 5.7, 5.8, 5.9) from ch.21 of: Petrucci, R.H., W.S. Hardwood and F.G. Herring, *General Chemistry*, Prentice-Hall, 2002, 1160 pp.

A reagent which gains oxygen (O) atoms is said to undergo oxidation. A reagent which loses oxygen (O) atoms is said to undergo reduction. An oxidation and a reduction must always occur together and such a reaction is called an oxidation-reduction or redox reaction.

In a broader sense: oxidation is a process in which the O.S. of some element increases and in which electrons appear on the r.h.s. of a half-equation. Accordingly, reduction is a process in which the oxidation number of some element decreases and in which electrons appear on the l.h.s. of a half-equation. For example,

$$\mathrm{Zn}\ (solid) + \mathrm{Cu}^{2+}\ (aq) \to \mathrm{Zn}^{2+}\ (aq) + \mathrm{Cu}\ (solid)$$

is rewritten as two half-equations that hold simultaneously:

$$\begin{array}{lrl} \textit{oxidation} & \mathrm{Zn}\ (s) & \to \mathrm{Zn}^{2+} + 2e^- \\ \textit{reduction} & \mathrm{Cu}^{2+}\ (aq) + 2e^- & \to \mathrm{Cu}\ (s) \end{array}$$

Explanation:

> *In the first half-reaction,* Zn *is oxidized — its oxidation state increases from* 0 *to* +2. *This change corresponds to a loss of two electrons by each zinc atom.*
>
> *In the second half-reaction,* Cu^{2+} *is reduced — its oxidation state decreases from* +2 *to* 0. *This change corresponds to the gain of two electrons by each* Cu^{2+} *ion.*

Consider a strip of metal (called an electrode) immersed in a solution containing ions of the same metal. This combination is known as a half-cell. Two kinds of interactions are possible between metal atoms in the electrode and metal ions in solution (assuming the metal does not interact with the water solvent):

1. *A metal cation* M^{n+} *from the solution may collide with the electrode, gain n electrons from it, and be converted to a metal atom* M. *The ion is reduced.*
2. *A metal atom* M *on the surface may lose n electrons to the electrode and enter the solution as the ion* M^{n+}. *The metal atom is oxidized.*

An equilibrium is quickly established between the metal and the solution, which we can represent as

$$M(s) \underset{reduction}{\overset{oxidation}{\rightleftarrows}} M^{n+}(aq) + ne^-$$

However, any changes produced at the electrode or in the solution as a consequence of this equilibrium are not easily measured. Instead, our measurements must be based on a combination of two different half-cells. *Specifically, we must measure the tendency for electrons to flow from the electrode of one half-cell to the electrode of the other. Electrodes are classified according to whether oxidation or reduction takes place there. If oxidation takes place, the electrode is called the anode. If reduction takes place, the electrode is called the cathode.*

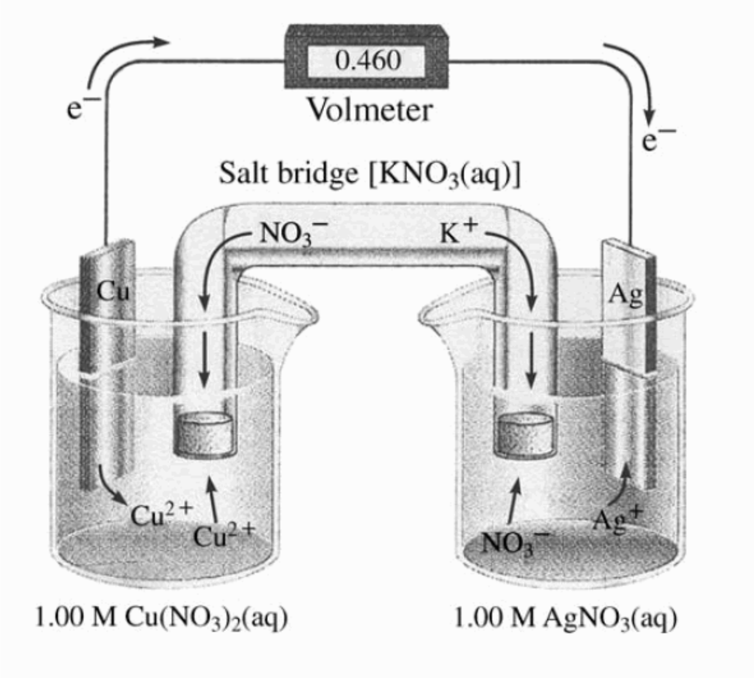

Fig. 5.7: Electrochemical cell

Example (Figure 5.7):

An electrochemical cell consists of two half-cells, one with Cu *electrode in contact with* Cu^{2+} *(aqueous), and the other with* Ag *and* Ag^{+} *(aqueous). The two electrodes are joined by wires and the solutions are joined by a third solution in a U-tube called a salt-bridge. The ends of the salt-bridge are plugged with a porous material that allows ions to migrate but prevents the bulk flow of liquid.*

Now, Cu *atoms on the anode release electrons and enter the* $Cu(NO_3)_2$ *aqueous solution as* Cu^{2+} *ions. These electrons pass through the wires to the* *cathode, where they are gained by the* Ag^+ *ions from the* $AgNO_3$ *solution, producing a deposit of metallic silver. Simultaneously, anions* (NO_3^-) *from the salt-bridge migrate into the copper half-cell and neutralize the positive charge of excess* Cu^{2+} *ions; cations* (K^+) *migrate into the silver half-cell and neutralize the negative charge of the excess* NO_3^- *ions. The overall reaction that occurs as the electrochemical cell* spontaneously *produces electric current is*

oxidation (at anode)	$\mathrm{Cu}\ (solid) \rightarrow \mathrm{Cu}^{2+}(aq) + 2e^-$
reduction (at cathode)	$2\left\{\mathrm{Ag}^+\ (aq) + e^- \rightarrow \mathrm{Ag}\ (solid)\right\}$
Overall	$\mathrm{Cu}\ (solid) + 2\mathrm{Ag}^+\ (aq) \rightarrow \mathrm{Cu}^{2+}\ (aq) + 2\mathrm{Ag}\ (s)$

The reading of the voltmeter $(0.460\ \mathrm{V})$ *is the cell voltage or the potential difference between the two half-cells. The unit of cell voltage Volt* (V) *is an energy per unit charge. Thus, a potential difference of one volt signifies a energy of one Joule for every coulomb of charge passing through an electric circuit. This voltage is the driving force for electrons through the circuit.*

The cell described above which produces electricity as a result of a spontaneous ($\Delta G < 0$, *exergonic) chemical reaction is called voltaic or galvanic cell. An electrochemical cell in which electricity is used to accomplish a non-spontaneous chemical change* ($\Delta G > 0$, *endergonic) is known as electrolytic cell.*

When a reaction occurs in a voltaic cell, the cell does electrical work. It is the work of moving electric charges. The total work (W_{elec}) *done is the product of three terms:* (*a*) E_{cell}*;* (*b*) n, *the number of moles of electrons transferred between the electrodes; and* (*c*) *the electric charge per mole of electrons, called the Faraday constant* (F). *The Faraday constant is equal to* $96,485$ *coulombs per mole of electrons* $(96,485\ \mathrm{C/mol}\ e^-)$. *Because the product* Volt $\times$ Coulomb $=$ Joule, *the unit of* W_{elec} *is joules* (J). *Thus*

$$W_{\mathrm{elec}} = nFE_{\mathrm{cell}}$$

This expression applies only if the cell operates reversibly. The maximal amount of available energy (work) that can be derived from a process is equal to $-\Delta G$, *namely*

$$\Delta G = -nFE_{\mathrm{cell}},$$

where G *is the Gibbs free energy function. It then follows from* $\Delta G^\circ = -nFE^0_{\mathrm{cell}}$*:*

$$E_{\mathrm{cell}} = E^\circ_{\mathrm{cell}} - \frac{RT}{nF}\ln Q,$$

which is the Nernst equation applied to electrochemistry.

Batteries

A battery is a device that stores chemical energy for later release as electricity. Some batteries consist of a single voltaic cell with two electrodes and the appropriate electrolyte(s); an example is a flashlight cell. Other batteries consist of two or more voltaic cells joined in series fashion — plus to minus — to increase the total voltage; an example is an automobile battery. We will consider three types of cells and batteries.

- *Primary batteries (or primary cells). The cell reaction is not reversible. When the reactants have been mostly converted to products, no more electricity is produced and the battery is dead.*
- *Secondary batteries (or secondary cells). The cell reaction can be reversed by passing electricity through the battery (charging). Such a battery can be used through several hundred or more cycles of discharging followed by charging.*
- *Flow batteries and fuel cells. Materials (reactants, products, and electrolytes) pass through the battery, which is simply a converter of chemical energy to electric energy.*

Batteries are vitally important to modern society: In developed nations, annual production has been estimated at over 10 *batteries per person per year.*

The most common form of voltaic cell is the Leclanché cell, invented by the French chemist ***Georges Leclanché*** *(1839–1882) in 1868. Popularly called a dry cell (because no free liquid is present) or flashlight battery, the Leclanché cell is diagrammed in Figure 5.8. In this cell, oxidation occurs at a zinc anode and reduction at an inert carbon (graphite) cathode. The electrolyte is a moist paste of* MnO_2, $ZnCl_2$, NH_4Cl, *and carbon black (soot). The maximum cell voltage is* 1.55 *V. The anode (oxidation) half-reaction is simple:*

$$\textit{Oxidation}: \qquad Zn\ (s) \rightarrow Zn^{2+}\ (aq) + 2e^-$$

The reduction is more complex. Essentially, it involves the reduction of MnO_2 *to compounds having* Mn *in a* $(+3)$ *oxidation state, for example,*

$$\textit{Reduction}: \qquad 2MnO_2\ (s) + H_2O\ (l) + 2e^- \rightarrow Mn_2O_3\ (s) + 2OH^-\ (aq)$$

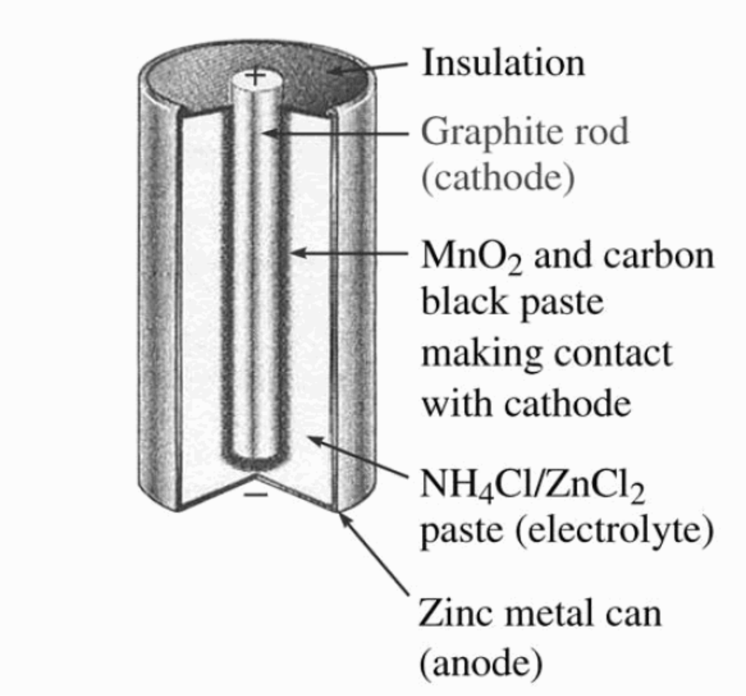

Fig. 5.8: Leclanché (dry) cell

An acid-base reaction occurs between NH_4^+ *(the cation from the salt* NH_4Cl*) and* OH^-.

$$NH_4^+\ (aq) + OH^-\ (aq) \rightarrow NH_3\ (g) + H_2O\ (l)$$

A buildup of NH_3 *(g) cannot be permitted to occur around the cathode because it would disrupt the current by adhering to the cathode. That buildup is prevented by a reaction between* Zn^{2+} *and* NH_3 *(g) to form the complex ion* $[Zn(NH_3)_2]^{2+}$, *which crystallizes as a chloride salt.*

$$Zn^{2+}\ (aq) + 2NH_3\ (g) + 2Cl^-\ (aq) \rightarrow [Zn(NH_3)_2]Cl_2\ (s)$$

The Leclanché cell is a primary cell; it cannot be recharged. This cell is cheap to make, but it has some drawbacks. When current is drawn rapidly from the cell, products such as NH_3 *build up on the electrodes, causing the voltage to drop. Also, because the electrolyte medium is acidic, the zinc metal slowly dissolves.*

A superior form of the Leclanché cell is the alkaline cell, which uses NaOH *or* KOH *in place of* NH_4Cl *as the electrolyte. The reduction half-reaction is the same as that shown above, but the oxidation half-reaction involves the*

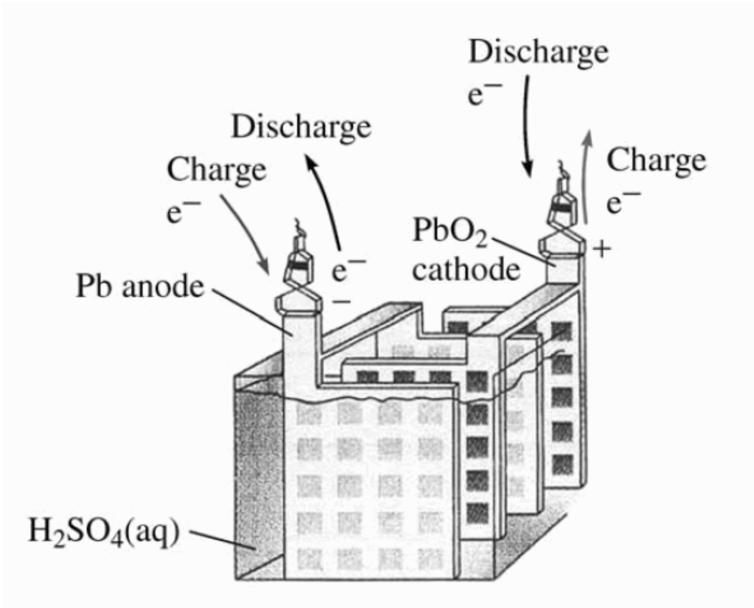

Fig. 5.9: Lead-acid cell

formation of $Zn(OH)_2$ *(s), which we can think of as occurring in two steps.*

$$\begin{array}{rrcl} & \text{Zn (s)} & \rightarrow & \text{Zn}^{2+}\ \text{(aq)} + 2e^- \\ & \text{Zn}^{2+}\ \text{(aq)} + 2\text{OH}^-\ \text{(aq)} & \rightarrow & \text{Zn(OH)}_2\ \text{(s)} \\ \hline \textit{Overall}: & \text{Zn (s)} + 2\text{OH}^-\ \text{(aq)} & \rightarrow & \text{Zn(OH)}_2\ \text{(s)} + 2e^- \end{array}$$

The advantages of the alkaline battery are that zinc does not dissolve as readily in a basic (alkaline) medium as in an acidic medium and the battery does a better job of maintaining its voltage as current is drawn from it.

The most common secondary *battery is the lead-acid battery or* storage battery, *used in automobiles since about 1915 (Figure 5.9). A storage battery is capable of repeated use because it uses chemical reactions that are reversible. That is, the discharged energy can be restored by supplying electric current to recharge the cell.*

The reactants in a lead-acid battery are spongy lead packed into a lead grid at the anode, red-brown lead (IV) oxide packed into a lead grid at the cathode, and dilute sulfuric acid with about 35% H_2SO_4, *by mass. In this strongly acidic medium the ionization of* H_2SO_4 *does not go to completion. Both* HSO_4^- *(aq) and* SO_4^{2-} *(aq) are present, but* HSO_4^- *predominates. The half-reactions and overall reaction are*

$$\begin{aligned} PbO_2\ (s) + 3H^+\ (aq) + HSO_4^-\ (aq) + 2e^- &\rightarrow PbSO_4\ (s) + 2H_2O\ (l) \\ Pb\ (s) + HSO_4^-\ (aq) &\rightarrow PbSO_4\ (s) + H^+\ (aq) + 2e^- \\ \hline PbO_2\ (s) + Pb\ (s) + 2H^+\ (aq) + 2HSO_4^-\ (aq) &\rightarrow 2PbSO_4\ (s) + 2H_2O\ (l) \end{aligned}$$

$$E_{\text{cell}} = E_{\text{PbO}_2/\text{PbSO}_4} - E_{\text{PbSO}_4/\text{Pb}} = 1.74V - (-0.28V) = 2.02V$$

When an automobile engine is started, the battery is at first discharging. Once the car is in motion, an alternator powered by the engine constantly recharges the battery. At times, the plates of the battery become coated with $PbSO_4$ *(s) and the electrolyte becomes sufficiently diluted with water that the battery must be recharged by connecting it to an external electric source. This forces the (non-spontaneous) reverse of the above reaction:*

$$2PbSO_4\ (s) + 2H_2O\ (l) \rightarrow Pb\ (s) + PbO_2\ (s) + 2H^+\ (aq) + 2HSO_4^-\ (aq) \qquad E_{\text{cell}} = -2.02\ V$$

To prevent the anode and cathode from coming into contact with each other and causing a short circuit, sheets of insulating material are used to separate alternating anode and cathode plates. A group of anodes is connected together electrically, as is a group of cathodes. This parallel connection increases the electrode area in contact with the electrolyte solution and thus increases the current-delivering capacity of the cell. Several such cells are then joined in a series fashion, positive to negative, to produce a battery. The typical $12\ V$ *battery consists of six cells, each cell potential of about* $2\ V$*.*

An important modern variant of the dry cell is the miniature button battery *— the* silver-zinc cell *used in watches, hearing aid, cameras, and other portable electronic equipment. In addition it fulfills the requirements of space craft, satellites, missiles, rockets, torpedoes, underwater vehicles, and life-support systems.*

The cell diagram of a silver-zinc cell *(Figure 5.10) is*

$$Zn\ (s),\ ZnO\ (s)\ |\ KOH\ (satd)\ |\ Ag_2O\ (s),\ Ag\ (s)$$

The half-reactions on discharging are

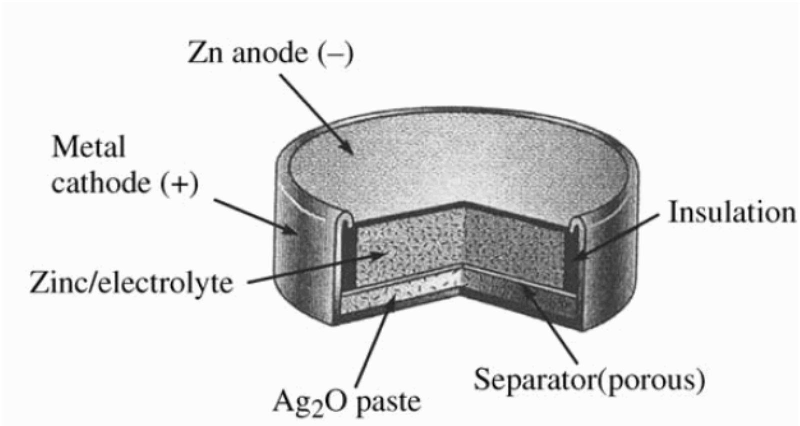

Fig. 5.10: Silver-Zinc cell

$$\begin{array}{ll} \textit{Reduction}: & Ag_2O\ (s) + H_2O\ (l) + 2e^- \rightarrow 2Ag\ (s) + 2OH^-\ (aq) \\ \textit{Oxidation}: & Zn\ (s) + 2OH^-\ (aq) \rightarrow ZnO\ (s) + H_2O\ (l) + 2e^- \\ \hline \textit{Overall}: & Zn\ (s) + Ag_2O\ (s) \rightarrow ZnO\ (s) + 2Ag\ (s) \end{array}$$

Because no solution species is involved in the overall cell reaction, the quantity of electrolyte is very small and the electrodes can be maintained very close together. The cell voltage is 1.8 *V, and its storage capacity is six times greater than that of a lead-acid battery of the same size.*

Rechargeable Batteries

In the field, a battery that suddenly goes dead is unacceptable. Secondary cells can be charged and discharged many times, making it economic to use, but requiring a more costly construction.

The nickel-cadmium cell is commonly used in cordless electric devices, such as electric shavers and handheld calculators. The anode in this battery is cadmium metal, and the cathode is the Ni *(III) compound* NiO(OH) *supported on nickel metal. The half-cell reactions for a nickel–cadmium battery*

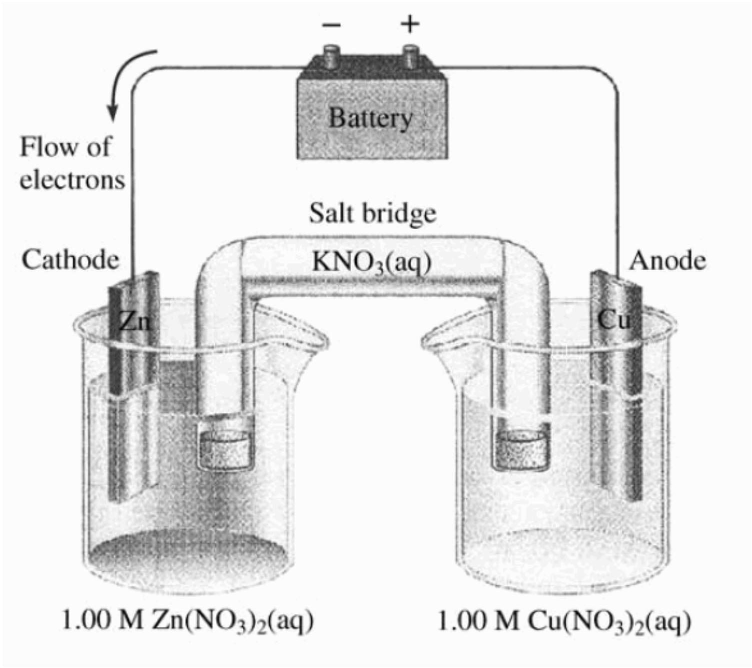

Fig. 5.11: An electrolytic cell

during discharge are

$$\textit{Reduction:}\quad \begin{matrix} 2NiO(OH)\ (s) \\ +\,2H_2O\ (l) + 2e^- \end{matrix} \rightarrow 2Ni(OH)_2\ (s) + 2OH^-\ (aq)$$

$$\textit{Oxidation:}\quad Cd\ (s) + 2OH^-\ (aq) \rightarrow Cd(OH)_2\ (s) + 2e^-$$

$$\textit{Overall:}\quad \begin{matrix} Cd\ (s) + 2NiO(OH)\ (s) \\ +\,2H_2O\ (l) \end{matrix} \rightarrow 2Ni(OH)_2\ (s) + Cd(OH)_2\ (s)$$

This battery gives a fairly constant voltage of 1.4 *V. When recharged by connecting the battery to an external voltage source, the reactions above are reversed. Nickel–cadmium batteries can be recharged many times because the solid products adhere to the surface of the electrodes.*

Note that assigning the terms anode *and* cathode *is not based on the electrode charges; it is based on the half-reactions at the electrode surfaces. Specifically,*

- Oxidation *always occurs at the* anode *of an electrochemical cell. Because of the buildup of electrons freed in the oxidation half-reaction, the anode* *of a voltaic cell is* $(-)$*. Because electrons are withdrawn from it, the anode in an electrolytic cell is* $(+)$*. For either type of cell, the anode is the electrode from which electrons exit the cell.*

- *Reduction always occurs at the cathode of an electrochemical cell. Because of the removal of electrons by the reduction half-reaction, the cathode of a voltaic cell is* $(+)$*. Because of the electrons forced onto it, the cathode of an electrolytic cell is* $(-)$*. For either type of cell, the cathode is the electrode at which electrons enter the cell.*

The following table summarizes the relationship between a voltaic cell and an electrolytic cell. Note that the sign of each electrode in an electrolytic cell is the same as the sign of the battery electrode to which it is attached. (However, in electronics a battery's anode is defined as "+".)

Electrolysis

Until now, we have emphasized voltaic (galvanic) cells, electrochemical cells in which chemical change is used to produce electricity. Another type of electrochemical cell — the electrolytic cell — uses electricity to produce a nonspontaneous reaction. The process in which a nonspontaneous reaction is driven by the application of electric energy is called electrolysis.

When the Zinc-Copper cell functions spontaneously, electrons flow from the zinc to the copper and the overall chemical change in the voltaic cell is (Fig. 5.11)

$$\mathrm{Zn}\ (s) + \mathrm{Cu}^{2+}\ (aq) \rightarrow \mathrm{Zn}^{2+}\ (aq) + \mathrm{Cu}\ (s) \qquad E^\circ_{\mathrm{cell}} = 1.103\ V$$

Now suppose we connect the same cell to an external electric source of voltage greater than $1.103\ V$*. That is, the connection is made so that electrons are forced into the zinc electrode (now the cathode) and removed from the copper electrode (now the anode). The overall reaction in this case is the inverse of the voltaic cell reaction, and* E°_{cell} *is negative.*

$$\begin{array}{ll} \textit{Reduction:} & \mathrm{Zn}^{2+}\ (aq) + 2e^- \rightarrow \mathrm{Zn}\ (s) \\ \textit{Oxidation:} & \mathrm{Cu}\ (s) \rightarrow \mathrm{Cu}^{2+}\ (aq) + 2e^- \\ \hline \textit{Overall:} & \mathrm{Cu}\ (s) + \mathrm{Zn}^{2+}\ (aq) \rightarrow \mathrm{Cu}^{2+}\ (aq) + \mathrm{Zn}\ (s) \end{array}$$

$$E^\circ_{\mathrm{cell}} = E^\circ_{\mathrm{Zn^{2+}/Zn}} - E^\circ_{\mathrm{Cu^{2+}/Cu}} = -0.763\ V - 0.340\ V = -1.103\ V$$

Table 5.9: RELATIONSHIP BETWEEN A VOLTAIC CELL AND AN ELECTROLYTIC CELL[(*)]

	Voltaic Cell			*Electrolytic Cell*	
Oxidation	$A \rightarrow A^+ + e^-$	Anode (negative)	Oxidation	$B \rightarrow B^+ + e^-$	Anode (positive)
Reduction	$B^+ + e^- \rightarrow B$	Cathode (positive)	Reduction	$A^+ + e^- \rightarrow A$	Cathode (negative)
Overall	$A + B^+ \rightarrow A^+ + B$ $\Delta G < O$ Spontaneous redox reaction releases energy		Overall	$A^+ + B \rightarrow A + B^+$ $\Delta G > O$ Nonspontaneous redox reaction absorbs energy to drive it	
	The system (the cell) does work on the surroundings			The surroundings (the source of energy) do work on the system	

(*) Table 5.9 and Figs. 5.5–5.9 were taken from Ch. 21 (pp. 823–856) of '*General Chemistry*' by Petrucci, Harwood and Herring, Prentice Hall, 1997.

Thus, by reversing the direction of the electron flow, we change the voltaic cell into an electrolytic cell.

E. Fuel-Cell Technology

In 1839, the Welsh-born jurist-physicist ***William Robert Grove*** *realized that if electrolysis, using electricity, could split water into hydrogen and oxygen, then the opposite could also be true, i.e.: combining hydrogen and oxygen in a suitable way, would produce electricity. To test his reasoning he indeed built a device that would do just that — the world's first fuel-cell. Grove's work thus advanced the idea of* energy's conservation and reversible conversion.

However, at that time, society was not intrigued with the fuel-cell and had little grasp of its technological potential. In fact, interest in Grove's invention diminished as the dawn of cheap fossil fuels *approached and the soon to be discovered* internal combustion engine *captivated the late 19th century populace; a larger segment of Western Society was enjoying a higher standard of living through the utilization of cheap energy and machines to do work. Against this background, Grove's invention was little more than a curiosity as the internal combustion engine and petroleum enthralled the age.*

Fossil fuels and the internal combustion engine reigned supreme from Grove's day to the present and they probably will into the near term future. However, looming on the horizon is the day when the world will run out of fossil fuels. This event will threaten energy prices and energy security. To avoid the turmoil a smooth transition to a hydrogen-based economy seems necessary.

Let us briefly survey the evolution of the fuel-cell concept since its inception by Grove. Since then, a variety of visionaries have worked to develop the technology to use hydrogen as an energy medium, and thus to create a non-polluting, cold-combustion, hydrogen economy.

First articulated by the French science fiction master Jules Verne, this vision has animated generations of engineers and scientists seeking a source of hard energy that is renewable and non-polluting. Hydrogen, when burned, yields water vapor energy; it is a viable replacement for the coal, petroleum and methane that are causing a dangerous rise in the CO_2 *content of the earth's atmosphere.*

Hydrogen is not a primary source, but rather an 'energy carrier' which must be 'broken' from water using electrolysis or chemical processes and recombined to generate electricity, heat or mechanical energy. Solar, wind or hot nuclear hydrogen fusion are candidates primary energy sources. If primary energy source is clean and cheap enough, a 'hydrogen economy' makes sense, and adapting cars, aircraft, electrical generation and heating becomes a manageable goal of commercial development engineering.

From 1889 until the early 20th century, many people tried to produce fuel cells (FC) that could convert coal or carbon directly to electricity. These attempts failed because not enough was known about metals or electricity. In 1932, ***Francis T. Bacon*** *developed the first successful FC. He used hydrogen, oxygen, an alkaline electrolyte, and nickel electrodes. In 1952, Bacon and a co-worker produced a* $5\ kW$ *fuel cell system.*

The large boost to FC technology came from NASA; In the late 1950's NASA needed a compact way to generate electricity for space missions. Nuclear energy was deemed too dangerous, batteries too heavy, and solar power too cumbersome. The answer was fuel cells.

There are presently five major fuel cell types:

- *alkaline fuel cell (AFC),*
- *molten carbonate fuel cell (MCFC),*
- *phosphoric acid fuel cell (PAFC),*
- *polymer electrolyte fuel cell (PEFC),*
- *solid fuel cell (SOFC).*

Both alkaline and polymer fuel cells have demonstrated their capabilities in the Apollo, Gemini and Space Shuttle manned space vehicle programs. The major efforts are presently focused on developing electric vehicles.

In order to provide an example of the electrochemical process that occurs in a fuel cell we consider the proton exchange membrane fuel cell (PEMFC) which uses one of the simplest reactions of any fuel cell:

The pressurized hydrogen gas molecules H_2 *(assumed to be available) are fed into the anode side of the cell where it encounters a catalyst. The catalyst is usually made of platinum powder very highly coated onto carbon paper or cloth. It is rough and porous so that the maximum surface area of the platinum can be exposed to the hydrogen. The platinum coated side of the catalyst faces the Proton Exchange Membrane (PEM) which plays here* *the role of electrolyte. It is a specially treated material, which looks like kitchen plastic wrap. The membrane blocks electrons and only conducts (lets through) positively charged ions.*

Now, when an H_2 *molecule comes in contact with the platinum of the catalyst, it splits into two* H^+ *ions and two electrons* (e^-)*:*

$$2H_2 \longrightarrow 4H^+ + 4e^-.$$

The electrons are conducted through the anode, where they make their way through an external circuit *doing useful work (such as: lighting a bulb, turning a motor, etc) and return to the* cathode *side of the fuel cell. The hydrogen ions* (H^+) *travel through the electrolyte contained in the fuel cell until they too reach the cathode. Meanwhile, on the cathode side of the fuel cell, oxygen gas* (O_2) *is being forced through the catalyst, where it forms two oxygen ions* (O^-)*. Altogether, on the cathode side*

$$O_2 + 4H^+ + 4e^- \longrightarrow 4H^+ + [2O^{2^-} - 4e^-] + 4e^- \longrightarrow 4H^+ + 2O^{2^-} \longrightarrow 2H_2O$$

The net reaction is therefore

$$2H_2 + O_2 \longrightarrow 2H_2O \qquad \text{(liquid)}$$

The by-products of the electrochemical reaction that occurs in the cell are: electricity, water vapor and heat. Theoretically, the water vapor can be recycled to produce additional hydrogen. The waste heat can be utilized for heating. Since this direct conversion of fuel (hydrogen) into electricity is not limited by the Carnot's law of thermodynamics, fuel cells can achieve substantially higher efficiencies then combustion. Fuel cells achieve efficiencies of 35 *percent to* 90 *percent depending on whether the waste heat is employed. These efficiencies are about* 2 *to* 3 *times higher than that of a combustion engine which converts fuel to heat, then into mechanical energy and finally into electricity.*

Another popular fuel-cell is the direct methanol fuel-cell *(DMFC), in which the working fuel is not* H_2 *but* CH_3OH *(methanol). Here again the* proton exchange membrane *is employed; a thin membrane covered on both sides with a sparse layer of platinum based* catalyst *and sandwiched between two electrodes. The methanol acts as an ideal hydrogen carrier because it readily frees its hydrogen to react in the fuel cell.*

How does the cell work? — A methanol/water solution is introduced to a negatively charged anode electrode that spontaneously reacts by breaking the methanol molecules apart. Once broken up, the carbon atoms combine with the oxygen atoms from methanol and water at the negative electrode to form *carbon-dioxide. At the same time the hydrogen atoms are further divided into protons and electrons.*

Altogether, the oxidation process at the anode yields

$$CH_3OH + H_2O \rightleftharpoons CO_2 + 6H^+ + 6e^- \qquad \textit{Anode}$$

Meanwhile, the hydrogen electrons are forced to flow to the positively charged electrode (cathode), forming an electrical current, while the protons pass through the membrane to the cathode.

The reduction process at the cathode is

$$\frac{3}{2}O_2 + 6H^+ + 6e^- \rightleftharpoons 3H_2O \qquad \textit{Cathode}$$

in which the two parts of the hydrogen atom are reunited and combine with oxygen to produce water. The overall reaction is thus

$$2CH_3OH + 3O_2 \rightarrow 2CO_2 + 4H_2O + \textit{waste heat.}$$

The reaction takes place at temperatures in the range 50°–85° C *with an efficiency of about* 50 *percent.*

It can be seen that the overall cell reaction consumes 4 *moles of methanol* (128 g) *with* 6 *moles of oxygen to produce* 4 *moles of carbon-dioxide gas and* 8 *moles of water* (144 g) *with a transfer of* 24 *moles of electrons* (24 *faradays*). *The potential of a single cell is* 0.6 *volts.*

The theoretical maximum electric energy available in any electrochemical cell is the free energy change for the cell reaction. The maximum thermal energy release when a fuel (at fixed ambient temperature and pressure) is burned is the enthalpy change, ΔH°. *One of the measures used to evaluate a fuel cell is the efficiency value* $\epsilon = \Delta G^\circ / \Delta H^\circ$. *For a hydrogen-oxygen cell* $\epsilon = -474.5$ kJ$/(-571.6$ kJ$) = 0.83$. *When methane (natural gas) is used,* $\Delta H^\circ = -890$ kJ, $\Delta G^\circ = -818$ kJ, $\epsilon = 0.92$.

Air Batteries

In a fuel cell, O_2 *(g) is the oxidizing agent that oxidizes a fuel such as* H_2 *(g) or* CH_4 *(g). Another kind of flow battery is known as an air battery, because it uses* O_2 *(g) from air. The substance that is oxidized in an air battery is typically a metal.*

One heavily studied battery system is the aluminum-air battery. In this battery oxidation occurs at an aluminum anode and reduction at a carbon-air cathode. The electrolyte circulated through the battery is NaOH *(aq). Because it is in the presence of a high concentration of* OH^-*, the* Al^{3+} *ions produced at the anode forms the complex ion* $[Al(OH)_4]^-$*. The half-reactions and the overall cell reaction are*

$$\textit{Reduction:}\quad 3\{O_2\ (g) + 2H_2O\ (l) + 4e^- \rightarrow 4OH^-\ (aq)\}$$

$$\textit{Oxidation:}\quad 4\{Al\ (s) + 4OH^-\ (aq) \rightarrow [Al(OH)_4]^-\ (aq) + 3e^-\}$$

$$\textit{Overall:}\quad \begin{array}{c} 4Al\ (s) + 3O_2\ (g) \\ + 6H_2O\ (l) + 4OH^-\ (\mathrm{aq}) \end{array} \rightarrow 4[Al(OH)_4]^-\ (aq)$$

The battery is kept charged by feeding chunks of Al *and water into it. A typical air battery can power an automobile several hundred miles before refueling is necessary. The electrolyte is circulated outside the battery, where* $Al(OH)_3$ *(s) is precipitated from the* $[Al(OH)_4]^-$ *(aq). This* $Al(OH)_3$ *(s) is collected and can then be converted back to aluminum metal at an aluminum manufacturing facility.*

The reaction in a single fuel cell produces only about 0.7 *Volts. To get the voltage up to a reasonable level, many separate fuel cells must be combined to form a fuel-cell stock.*

PEMFC's operate at a fairly low temperature (about $80°C$*), which means they warm up quickly and do not require expensive containment structures. Constants improvements in the engineering and materials used in these cells have increased the* power density *to a level where a device about the size of a small piece of luggage can power a car.*

Since most households appliances operate on high-voltage AC power, a final major compound of a fuel cell requires a converter from low-voltage DC power into high-voltage AC power.

Certain fuel cells are renewable*, that is, can accomplish the electrochemistry associated with both the production of electricity from fuel and oxidant* and *the production of fuel and oxidant from water when supplied with electricity, i.e. accomplishing* both *electrolysis and reverse electrolysis in the* same *cell. This allows one to consider the completely renewable production of electricity by using a renewable energy supply (e.g. solar, wind) to produce from water hydrogen and oxygen which can subsequently be used to produce electricity through the* same *fuel cell from the fuel and oxidant previously produced.*

A fuel-cell never 'runs down': it continues to produce electricity as long as fuel is present. When a battery 'runs down', it has to undergo a lengthy recharge time to replace the spent electricity. Depending on where the recharging current originates, pollution, costs and efficiency problems are then transferred from the batteries location to the central generating point.

When most people think of oil, they think of gasoline and other fuels. But in fact, oil and its hydrocarbon products are intertwined with many things we take for granted today. Plastics, chemicals, fertilizers and many other common products are based on oil and its by-products. When utilizing a finite, rapidly dwindling resource the question becomes: how can that resource best be utilized?

The answer that comes to mind is: use it for the things where there is no other substitute. In terms of chemicals, fertilizers and others there is no substitute for oil as a raw material. Regarding energy, however, there are alternatives. These alternatives include renewables, with hydrogen as the energy carrier and storing medium, and domestically produced hydrogen from natural gas or off-peak electricity. The largest hurdle for the hydrogen economy to overcome is a lack of infrastructure. This problem is by no means insurmountable with the proper investment by industry and government.

The United States is extremely dependent on other, politically unstable and unfriendly, countries for its supply of oil. According to the US Department of Energy (DOE) reports, the United States imports more than 50% *of its oil supply and this figure is expected to increase to* 65% *by 2020. The global demand for oil is increasing at* 2% *per year.*

The fuel flexibility inherent in fuel cells and the ability to produce hydrogen domestically would result in a decline in the dependence on foreign energy sources, greater national energy security, a reduction of the military forces now poised to defend energy interests at a moment's notice in the Persian Gulf and a decrease in foreign trade debt.

Internationally, the demand for energy is expected to increase by 50% *over the next ten years. Fuel cells and distributed generation will allow developing nations to undergo the "cellular phenomenon" when structuring their utility grids. In many developing countries, phone lines are almost non-existent. Instead, cell towers have been erected and people communicate via cell phone. This allows phone system operators to avoid the staggering cost of running a phone line to every residence and building.*

Fuel cells could have the same effect by allowing developing countries the opportunity to install smaller community and industrialized based energy generation sites. This avoids many of the costs associated with establishing a large utility distribution and grid system (distributing hydrogen via pipeline *has been estimated to be around $\frac{1}{4}$ as expensive as transmitting an equal amount of energy in the form of electricity over transmission lines) in addition to elimination of the pollution associated with fossil fuel energy plants.*

In conclusion, fuel cells and a potential hydrogen economy are riding some powerful historical trends. Throughout history, mankind's energy use has moved towards a higher hydrogen ratio in chemical composition of fuel and a reduction in the other components. Starting with wood, then to coal, oil and natural gas, society's shift in type of fuel is simply a movement along a hydrocarbon chain.

As the form of the fuel changed, more of the carbon, from which a significant percentage of the pollution associated with fossil fuels originates, was eliminated. Hydrogen fuel cells complete the process of eliminating the dirty carbon and finish the task of employing pure, clean hydrogen. Aside from historical forces, fuel cells and hydrogen are riding the momentum created by an increased environmental awareness, the inevitable eventual extinction of our fossil fuel reserves, and sound economic policy.

The forthcoming 'Hydrogen Economy' will eventually transform everyday life; the future will bring:

- *Reduced anthropogenic air pollution from fossil fuel combustion.*
- *Cell phones and laptops whose battery life is measured in days instead of hours.*
- *Vehicles operating silently and emitting harmless water vapor.*
- *Individual homes generating their own electricity and heat independent from the utility grid.*
- *Hydrogen power for sensitive electronic equipment, computer centers, cellular towers, mining equipment, banks, schools, hospitals, jails, sophisticated manufacturing, entertainment complexes, communication centers, navigation equipment, airports, road signs, defense installations, hotels, urban transit systems, heavy trucks, personal vehicles and remote sites requiring power.*

F. Hydrogen Power — From Science-Fiction to Science

Hydrogen is the lightest and most abundant element in the universe as well as the ultimate source of virtually all energy-release processes known to man, both in the solar system and beyond. Deep within the sun and stars, nuclear fusion converts hydrogen into helium[771]*. The energy that is released when four hydrogen nuclei become a helium atom is the energy which lights main-sequence stars and fuels all life. Evidence of the incredible amount of energy contained within a hydrogen atom is the thermonuclear or hydrogen bomb, which exploits nuclear fusion to release its destructive power.*

In our natural environment, hydrogen exists primarily in chemical combination with other elements. In order for hydrogen to be useful as a chemical fuel, it must exist as H_2 *or "free hydrogen".* H_2 *must therefore be produced, unlike fossil fuels such as natural gas, coal and oil which can be directly mined or extracted. In this sense, hydrogen is a secondary source of energy, analogous to electricity.*

The energy used to produce H_2 *is stored, after some losses, within the* H_2 *molecule. This energy can then be kept in storage, used on-site, or transported to a remote location for energy conversion. The fact that hydrogen must be produced is a major consideration when examining its effectiveness as an energy carrier, and is the biggest stumbling-block to widespread use in commercial applications.*

Free hydrogen exists at normal atmospheric conditions as an odorless, colorless gas. It is stable and will co-exist harmlessly with free oxygen (O_2) *until an input of energy drives the exothermic (heat-releasing) reaction which forms water. This reaction from a higher energy state to a lower one generates a positive output of energy.*

For over a century it has been predicted that a system will be developed in which hydrogen, extracted from pure water using energy derived from the

[771] Once the amount of hydrogen in a star's core become depleted enough, nuclear fusion of hydrogen into helium can no longer sustain pressure required to oppose gravitational collapse, and the latter forces other fusion processes; these produce nuclei of all elements and isotopes (beryllium, carbon, oxygen, nitrogen, etc) not initially produced in the Big Bang explosion. Latter-generations stars, such as our sun, are assembled via gravitational collapse of interstellar gas and dust enriched with elements produced in earlier stars. In some stars (but not our sun) the nuclear conversion of hydrogen to helium is mostly *indirect*, catalyzed by a series of intermediate processes involving carbon and other elements present within the stellar core.

sun, will be used as a fuel or as an "energy-carrier", and will serve to provide all of society's power requirements. The beauty of the system being that solar energy and water, the sources, are practically limitless and that the resulting energy conversion is relatively pollution-free with the only waste product being again pure water. A seemingly perfect cycle, beginning and ending with energy and water.

Since the early 19th century, scientists have recognized hydrogen as a potential fuel. Current uses of hydrogen are in industrial processes, rocket fuel, and spacecraft propulsion. With further research and development, this fuel could also serve as an alternative source of energy for heating and lighting homes, generating electricity, and fueling motor vehicles. When produced from renewable resources and technologies, such as hydro, solar, and wind energy, hydrogen becomes a renewable fuel.

Unlike most other fuels, hydrogen cannot be produced directly by digging a mine or drilling a well. It must be extracted chemically from hydrogen-rich materials such as natural gas, water, coal, or plant matter. Accounting for the energy required for the extraction process is critical in evaluating any hydrogen use option. Production techniques now used include steam reforming of natural gas, cleanup of industrial by-product gases, and electrolysis of water. A number of other technologies are being studied, including several that produce hydrogen from water or biomass using solar or other renewable energy.

Hydrogen is the most abundant of all the elements in the universe, and makes up more than three-quarters of the mass of the universe. Based on the "Big Bang" theory of cosmology, it is believed that most of the heavier elements were built up from hydrogen and helium inside stars, and that this process is still ongoing. Hydrogen is found in our sun and other stars and plays an important role in the reactions that account for their energy.

Hydrogen ranks ninth of all the elements in order of abundance on Earth, and makes up about 0.76% of the weight of the Earth's crust. The most important naturally occurring compound of hydrogen is water, which is the principal source of the element. In the quest for new and improved energy sources and uses, interest has been aroused in employing hydrogen as an energy currency.

It has been suggested that solar, wind, hydro, nuclear, or even coal conversion could be used to produce hydrogen. The hydrogen would be stored as a compressed gas or liquid and subsequently utilized in a fuel cell, or combusted to return the stored energy when needed. This is the basis for the hydrogen economy, or hydrogen energy research carried out throughout the *world. This research recognizes that fossil fuels will not last forever, and that the hydrogen cycle is simple and non-polluting.*

There are four processes which must be considered when developing a hydrogen-fuel system. These processes are:

- *Production*
- *Storage*
- *Transportation*
- *Energy conversion*

There are many alternatives from which to choose when developing a hydrogen system. The factors based on which each alternative is evaluated, involve efficiency, economic feasibility, and environmental impacts. How these factors are weighted against each other is open for debate. Currently the prevailing trend is to consider cost-effectiveness above all else. Recent trends in legislature and public concern are shifting emphasis towards renewability and pollution-free considerations as a priorities for development of hydrogen technology.

Hydrogen is a secondary source of energy, not a primary source like oil or natural gas. Therefore, in order to be utilized hydrogen must first be produced. There are many ways in which this can be done. Methods of production include chemical, electrochemical, photochemical, biological, and thermochemical processes.

The simplest method to produce hydrogen is to dissolve metals in acid. For example, when zinc (Zn) *is placed in a solution of hydrochloric acid, it reacts to produce zinc chloride and hydrogen:*

$$\mathrm{Zn} + 2\mathrm{HCl} \quad \rightarrow \quad \mathrm{ZnCl_2} + \mathrm{H_2}$$

This reaction can be reproduced simply in the laboratory, although the amount of hydrogen produced is minimal. Still, this method was used to a large extent during World War II when scrap aluminum was dissolved in sodium hydroxide in order to generate hydrogen. The hydrogen was then used to inflate unmanned balloons for weather observation and raising radio antennas.

This method is relatively expensive, and is not considered suitable for mass production (today, research is being done with scrap iron to produce hydrogen, for use in transportation as a method of producing hydrogen onboard vehicles). Small amounts of hydrogen can, however, be economically *produced via this method to provide the needs of a small hydrogen-fuel system.*

The cheapest, and by far the most widely used method for producing hydrogen is steam reformation. Steam, and a carbon-based feedstock (usually methane or natural gas), are combined under high temperature and pressure to produce carbon dioxide and hydrogen. It is estimated that 95% of the hydrogen produced in the US is created by the steam methane reformation method. Most of this hydrogen is used in industrial applications. Although hydrogen can be produced in this manner for about $0.65 per kilogram, the environmental consequences of the use of hydrocarbons are still a concern.

The production of carbon dioxide, a "greenhouse gas", as well as nitrogen oxides (NOx) contribute to the pollution of the earth's atmosphere. Also, the use of limited resources can only drive their costs up as the supplies of fossil fuel sources decrease. A newly developing renewable option is the use of biomass, or recycled carbonaceous material, as the feedstock in the steam reformation process. The air pollution problems still exist, but it will be an intelligent use of a waste product.

Another method for producing hydrogen is electrolysis. Electrolysis involves the application of a small voltage (approx. 2 V DC) to pure water. The electrical energy decomposes the water molecule into its constituent elements, hydrogen and oxygen. This technique has the advantage of producing hydrogen directly from water, with none of the environmental drawbacks which accompany processes using fossil-fuels. Still, the relatively low-efficiency (currently 60–65% with a theoretical maximum of 85%) of the process, and the high cost of electricity make this an expensive option. The cost of producing hydrogen via electrolysis is about $3.00 per kg.

The method of electrolysis is the most attractive for those interested in a completely clean, renewable process using solar energy to produce the electricity. Photovoltaic cells, hydropower, and wind turbines are currently being used to generate the electricity required to electrolyze water for hydrogen production.

Other renewable options include geothermal, tidal, wave action, and thermal gradients in the ocean. Although most of these processes do not produce sufficient amounts of energy to provide hydrogen on a large scale, on-site electricity production coupled with a small on-site electrolyzer can produce enough energy to provide for the energy needs of a household, along with fuel for the family automobiles. This allows hydrogen to be produced easily without having to wait for an infrastructure to develop.

Other attempts at water-splitting have involved super-heating water to temperatures high enough to liberate the hydrogen from the water molecule *(thermochemical). The temperatures required are in the range of* 5000°–6000° F. *Adding chemicals such as sulfuric acid can lower the required temperature but the bottom line is that the only feasible way of generating the heat required is by way of a nuclear reaction. Nuclear power generation, needless to say, has severe environmental and safety implications. There is still research being done on thermochemical production of hydrogen which doesn't require nuclear power plants.*

An example would be solar power plants in which the sun's infra-red radiation focused into a tiny point where the heat accumulates, much like a magnifying glass. Yet there are still environmental concerns due to the chemicals involved, and the nitrogen oxides which are formed from a heat reaction in air (which has a high concentration of nitrogen).

Photoprocesses involve the use of light energy for the production of hydrogen. These methods in one way or another, attempt to mimic the natural phenomena of photosynthesis. In plants, chlorophyll captures light energy and uses it to produce complex sugar-phosphate compounds. The most astonishing fact is that this chemical reaction, basically

$$CO_2 + H_2O + \textit{light energy} \quad \rightarrow \quad \textit{sugars} + O_2$$

occurs at room temperature! Much research has been done to reproduce this feat. Photobiological techniques which coax photosynthetic plants, algae, and bacteria into respiring hydrogen, photochemical techniques which synthetically duplicate the photosynthetic process, and photo electrochemical techniques which use layers of semiconductors separated by water are all being studied today. These are promising technologies, although still in the experimental stage. If efficiency improves, then photoprocesses may play a part in the future of hydrogen.

Storage and Transportation

Hydrogen is typically stored as a liquid, or gas. There are advantages and disadvantages to each of these storage options, the choice depending upon the ultimate use.

Hydrogen becomes a liquid at temperatures below −252.9°C. *Liquification of hydrogen is very energy-intensive, with one third of the energy content of the hydrogen used in the liquification process. This is offset by a reduction of volume requirements for hydrogen storage, with much less storage space required for a liquid than a gas. Less volume needed for storage, makes liquid* *hydrogen the preferred form of hydrogen used in the Aerospace industry with NASA being one of the largest consumers of liquid hydrogen in the world.*

Once in liquid form, hydrogen can be transported in pressurized tanks by truck, barge, or rail. Due to the very low boiling temperature of hydrogen, losses due to boil-off can be considerable. Insulation of the tanks is of utmost importance to reduce these losses. If insulated properly, hydrogen can be stored for as much as five years without significant losses.

Hydrogen can also be stored as a pressurized gas. It can then be transported via pipelines, using existing natural gas distribution lines. A concern would be possible embrittlement of the lines due to absorption by the metal fittings. Storage of hydrogen as a gas is the most economical method, but due to the necessity for larger tanks, weight and space requirements can be a problem. It is estimated that the mass of a pressure tank is 100 times the mass of the hydrogen stored within it. Higher pressure means less volume required, but the walls then need to be reinforced to withstand the greater pressure. Although hydrogen is extremely light, the containers necessary to store gaseous hydrogen can be heavy and bulky.

Another method of storing gaseous hydrogen involves metal hydrides. Certain metals such as magnesium, titanium, or iron, have an affinity for hydrogen. Under certain conditions, these metals will adsorb gaseous hydrogen, and store it within its molecular structure. When the hydride is heated, the hydrogen is released. Although energy is required to store and to release the hydrogen, this option has proved attractive for use as a storage medium onboard automobiles. The main reason is that it is much less energy-intensive than the liquification process, although heat energy is required to release the hydrogen. Also, safety and space concerns are reduced when metal hydride storage is used in automobiles.

There are a variety of other methods being developed for hydrogen storage. These include carbon adsorption, glass microspheres, onboard partial oxidation reactors, and recyclable liquid carriers. Some of these options appear promising, but they will still take some time to develop.

Power Conversion

There are two ways of using hydrogen to generate power. One is simple combustion. The use of hydrogen in internal combustion engines has been extensive. The other is the conversion of hydrogen into electricity in a fuel cell, which is essentially electrolysis in reverse. Both of these have their advantages and disadvantages.

Internal combustion engines can be easily converted to run on hydrogen, or a hydrogen-fuel mixture. The noxious emissions are greatly reduced, with water being the only by-product if pure hydrogen and oxygen are used. Nitrogen oxides are still formed from the high heat of combustion, and are still a source of air pollution.

Over the past two decades, most research has gone into the development of the fuel cell. The operation of a fuel cell involves the combination of hydrogen (anode) and oxygen (cathode) in the presence of an electrolyte. Output voltages range from 0.7 *to* 1.12V*. The type of fuel cell varies depending on the electrolyte used. Fuel cell types include the Phosphoric acid fuel cell, the alkaline fuel cell, and the solid oxide fuel cell.*

The most common type, the alkaline fuel cell, is still used by NASA on board spacecrafts. Another type of electrolyte being developed is the proton-exchange membrane which uses a solid polymer to facilitate the reverse electrolysis process. This solid polymer, which is much like plastic kitchen wrap, conducts protons, and is very conducive to the purpose of an electrolyte. Although membrane costs are high, this type of fuel cell appears very promising, and is currently being used in advanced research.

The use of hydrogen is at an all-time high. It is possible to convert any car sitting in the driveway to run on hydrogen. It is being proven every day that hydrogen can be used as a replacement not only for gasoline, but natural gas in heaters and stoves in the home. Hydrogen could some day replace high-voltage electrical power lines as the primary energy-carrier via high-voltage power lines, being transported in pipelines and converted to electricity on-site.

Production of hydrogen is also becoming easy to do for any one with access to about 2 V *of DC electricity. Many homesteads generate enough electricity using windmills and solar panels to supply the household's needs and even sell back some power to the regional utility company. A small electrolyzer added to this system could easily produce enough hydrogen to fuel a vehicle. It is clearly possible that anyone with a little ingenuity and skill can convert the household to use hydrogen, convert the car to run on hydrogen, and generate the electricity for hydrogen production using only solar energy, all for about the cost of a mid-sized American sedan.*

Any in-depth study of hydrogen reveals the vast array of possible system configurations for hydrogen power. The bottom line is that any system which utilizes hydrogen in any capacity is going to be better off for it. Harmful emissions are reduced, efficiency is increased and water(the original source), is reproduced. On a larger scale, it would seem possible that use of hydrogen alone or in conjunction with other fuels would be a major step in the right direction, and bring us a little closer to a more harmonious cycle of energy use.

G. Harnessing Solar Energy

The sun is an average main-sequence star with a mass equal to nearly one-third of a million earths. It is made up of almost 80% *hydrogen by mass and is entirely gaseous and plasma-phase, although the gas near its center is under such tremendous pressure that it behaves like a fluid. Because of this gaseous state, the sun rotates unevenly. Its equatorial section turns on its axis once every 25 days while the higher latitudes on the sun take over* 27 *days to rotate. The sun derives its energy from the fusion reaction of two hydrogen nuclei joining together to make one helium nucleus. This is basically the same nuclear reaction as that enabling the hydrogen bomb (although the latter makes use of several hydrogen isotopes).*

The fusion reaction began a few billion years ago as a result of the high temperatures in the sun's core, caused by the gravitational contraction of the huge mass of gas and dust (mostly hydrogen) from which the sun formed. The production of heat from the sun is quite small per unit of volume, because like all large bodies, its surface is relatively small compared to its volume. Also, the sun has had billions of years to heat up.

The sun's relatively small core is very hot (approximately $14,000,000°C$*) and it is in the core that the fusion reaction occurs. Even though the sun's energy output is small compared to its huge size, it turns out energy on an enormous scale by earth's standards, some* 5 *million megatons of energy per second. There is estimated to be enough hydrogen in the sun convertible to helium to allow the sun to shine at its present rate for 5 billion years to come.*

Man sees only the glowing surface (photosphere) of the sun, which emits an approximate blackbody radiation spectrum at $5,800\,°C$*. During solar eclipses when the main disc of the sun is blotted out, one can also see the solar atmosphere or corona. The corona is made up of ionized hydrogen, a hydrogen atom with its one electron knocked off. However, the corona also contains all of the elements common on earth. The corona is much more diffuse than the rest of the sun and much hotter that the surface —* $2,000,000°C$ *just above the sun's surface.*

This corona is thought to extend in diffuse form to the outer limits of the solar system. Even during quiet periods of the sun, there are coronal streamers of very hot gas out to ten solar diameters. The sun's diameter is $1,400,000$ *km. The surface of the sun is covered with bright granules, but the most noticeable features are the sun spots, darker and cooler regions* ($4,500\,°C$)*. Sun spots have tremendous local magnetic fields* ($3,000$ *to* $4,000$ *Gauss compared with about one Gauss for the rest of the sun's surface*).

Sun spots are believed to be places where doughnut-shaped magnetic field knots emerge from the sun's surface, and they usually occur in pairs of opposite magnetic polarity. At the maximum of solar activity in 1947, a sun spot of five billion square miles was seen. The sun alternates from very quiet to very turbulent and active, then back to quiet, over 11-year cycles. At the peak of a cycle, numerous solar flares occur which throw out huge masses of hot gas and energetic particles.

These masses of hot, ionized hydrogen hit the earth's magnetic field, bend it out of shape, disrupt radio communications, cause the aurora borealis, and feed the Van Allen radiation belts. Flares are $1,000$ *to* $100,000$ *times more dense than the surrounding material on the solar surface; the corona around the flares gets four times hotter than normal; and gas thrown out travels at around* $3,600,000$ *kmph.*

The cause of solar flares is not known, but they are generally believed to be related to sunspot magnetic fields. In addition to flares, the sun constantly throws out ionized hydrogen in the solar wind. This material moves at nearly 1.6 *million kmph and contains one to ten particles per cubic centimeter. This is still more diffuse than the "hardest" vacuum yet made on earth.*

The earth lives and feeds on energy from the sun, with little help from its own radioactivity. In addition there is a contribution from its own gravitation and the gravitation of the moon and the sun, (via tides) and a tiny component of cosmic radiation from the rest of the universe. Of these, the flow of radiation from the sun is paramount.

Over the lifetime of the earth this radiation has induced life in the earth's surface and that life has laid down a fossil record of the solar energy it once received hundreds of millions of years ago as chemical energy stored in coal, gas, oil and peat. At the present time, that fossilized solar radiation is our civilization's main source of exploitable energy. These fossil fuels are the life savings of the earth. As it is used, much of the fossil energy, inevitably, is lost as heat. But some is converted into metals, plastics, chemicals, electronic crystals and buildings.

In combination with gravity and the earth's rotation, solar energy drives the hydrological cycle which is responsible for floods, atmospheric circulation, hurricanes, thunderstorms, and other climatic and weather phenomena. Only 13 *percent of the total solar energy available to earth is responsible for evaporating water (mostly in the tropics) needed to drive the hydrological cycle.*

The rate of solar energy incident upon earth just outside the atmosphere is $1.35\frac{\mathrm{kW}}{\mathrm{m}^2}$. *This energy falls upon a circular disc of radius* 6380 *km which has the area of* $1.27 \times 10^{14}\ \mathrm{m}^2$. *Hence the total rate over the daylight hemisphere is* 1.72×10^{11} *megawatt. Of this* 7.5×10^{10} *megawatt is absorbed at the surface, and* 3.7×10^{10} *megawatt is absorbed in the atmosphere.*

Radiation reaching the surface is mostly converted into heat, but a fraction is used by vegetable life on land and sea to photosynthesize carbohydrates out of CO_2 *and water, and a fraction could be used by man to generate electricity in solar cells made out of semiconductors such as silicon or gallium arsenide.*

A realistic covering of 0.1 *percent of the earth with solar cells operating at* 10 *percent efficiency would produce a power of* 7.5×10^6 *megawatt.*

The power 7.5×10^{10} *MW is the average rate of surface ground-level absorption by the earth as a whole, and it corresponds to a continuous surface flux of* $590\ Wm^{-2}$. *A given location on the earth's surface is not exposed to this flux continually because of the earth's rotation and because the incident energy of solar radiation is spread over a hemisphere and not a disc. Rotation alone causes the average flux to be halved since there is exposure to sunlight during, on average, only* 12 *hours of the day.*

Another factor of two is introduced by the curvature of the earth since the area of a hemisphere is twice that of a disc of the same radius. Thus the global average insolation at a point on the earth's surface is $150\ Wm^{-2}$ *(roughly* $300\ Wm^{-2}$ *during the day and zero at night). In Europe the average is about* $120\ Wm^{-2}$, *in the U.S.A. it is* $200\ Wm^{-2}$, *and in the Sahara* $260\ Wm^{-2}$.

We summarize the global solar energy flow:

Solar radiation, incident	1.72×10^{11}	*MW*
absorbed in atmosphere	3.7×10^{10}	*MW*
absorbed at surface	7.5×10^{10}	*MW*
Atmospheric circulation	1×10^{10}	*MW*
Photosynthesis (Land + Ocean)	3×10^{8}	*MW*

The tapping of solar energy

Humans have used sunlight to perform a variety of tasks for centuries, but a serious scientific approach to the subject began in the wake of the industrial revolution in Europe. The Swiss scientist ***Horace de Saussure*** *invented (1767) the world's first solar collector. It was then used by* ***John Herschel*** *to cook food during his expedition to Southern Africa (1830).*

There are a variety of technologies that have been developed to take advantage of solar energy. These include:

- *Photovoltaic (solar cell) systems:*
  Producing electricity directly from sunlight.
- *Concentrating solar systems:*
  Using the sun's heat to produce electricity.
- *Passive solar heating and day lighting:*
  Using solar energy to heat and light buildings.
- *Solar hot water:*
  Heating water with solar energy.
- *Solar heating and cooling*
  Industrial and commercial uses of the sun's heat.

Solar thermal systems concentrate heat and transfer it to a fluid. The heat is then used to warm buildings, heat water, generate electricity, dry crops or destroy dangerous waste. Solar thermal collectors are divided into three categories:

Low-temperature collectors provide low grade heat, less than $43°C$, through either metallic or nonmetallic absorbers for applications such as swimming pool heating and low-grade water and space heating.

Medium-temperature collectors provide medium to high-grade heat (greater than $43°C$, usually $60°C$–$82°C$), either through glazed flat-plate collectors using air or liquid as the heat transfer medium or through concentrator collectors that concentrate the heat to levels greater than "one sun". These include evacuated tube collectors, and are most commonly used for residential hot water heating.

High-temperature collectors are parabolic dish or trough collectors primarily used by independent power producers to generate electricity for the electric grid.

Concentrating Solar Thermal Systems use three different types of concentrators:

Central receiver systems use heliostats (highly reflective mirrors) that track the sun and focus it on a central receiver.

Parabolic dish systems use dish-shaped reflectors to concentrate sunlight on a receiver mounted above the dish at its focal point.

Parabolic trough systems use parabolic reflectors in a trough configuration to focus sunlight on a tube running the length of the trough.

Technology Examples

Pool Heating — These systems can be as simple as water running through a black hose and specially manufactured systems are more efficient modifications on this concept.

Domestic Water Heaters — They come in a variety of styles but all of them collect heat in some liquid, usually water or water mixed with an anti-freeze, that runs through pipes in a box with glass on the front. The box helps keep temperatures inside around the pipes higher, so more heat transfers to the liquid. The hot liquid gives its heat to another loop of pipes through a heat exchanger and this new loop is used for home hot water use or heating the space with a radiator.

Commercial Scale Heaters — These can be designed to heat or cool a large commercial space or to make steam which can turn a turbine to produce electricity.

Photovoltaic systems

Solar electric or photovoltaic systems convert some of the energy in sunlight directly into electricity. Their history dates back to 1839. Photovoltaic (PV) cells are made primarily of silicon, the second most abundant element in the earth's crust, and the same semiconductor material used for computer chips.

When the silicon is combined with one or more other materials, it exhibits unique electrical properties in the presence of sunlight. Electrons are excited by the light and move through the silicon. This is known as the photovoltaic effect and results in direct current (DC) electricity. PV modules have no moving parts, are virtually maintenance-free, and have a working life of 20–30 *years.*

There are three basic categories of photovoltaic systems with several types in each category.

Crystalline Photovoltaic Materials: flat plate collectors are the most developed and prevalent type in use today. These include single crystal silicon and polycrystalline silicon which is either grown or cast from molten silicon and *later sliced into its cell size. They are then assembled onto a flat surface; no lenses are used.*

Thin Film systems are inherently cheaper to produce than crystalline silicon but are not as efficient. They are produced by depositing a thin layer of photovoltaic material to a substrate like glass or metal. This category includes amorphous silicon like the kind found in calculators and watches.*

Concentrators use much less of a specialized photovoltaic material and employ a lens or reflectors to concentrate sunlight on the photovoltaic cell and increase its output. They can be produced more cheaply than either of the other categories due to the reduced amount of expensive PV material required. But they can only use direct sun light, so they must track the sun precisely and do not work when it is cloudy.*

Solar cells (Figure 5.12)

To understand the operation of a PV cell, we need to consider both the nature of the material and the nature of sunlight. Solar cells consist of two types of material, often p-type silicon and n-type silicon. Light of certain wavelengths is able to ionize the atoms in the silicon and the internal field produced by the junction separates some of the positive charges ("holes") from the negative charges (electrons) within the photovoltaic device.

The holes are swept into the positive or p-layer and the electrons are swept into the negative or n-layer. Although these opposite charges are attracted to each other, most of them can only recombine by passing through an external circuit outside the material because of the internal potential energy barrier.

Therefore if a circuit is made, power can be produced from the cells under illumination, since the free electrons have to pass through the load to recombine with the positive holes. The amount of power available from a PV device is determined by:

- *The type and area of the material*
- *The intensity of the sunlight*
- *The wavelengths of the sunlight*

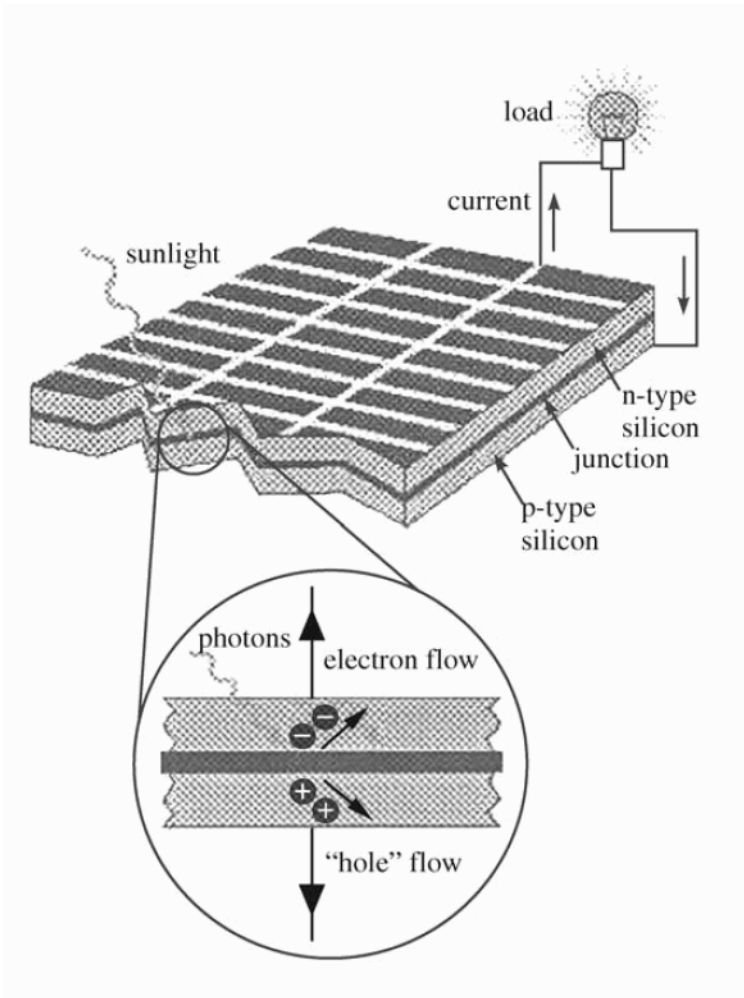

Fig. 5.12: Solar cell

Single crystal silicon solar cells, for example cannot currently convert more than 25% of the solar energy into electricity, because the radiation in the infrared region of the electromagnetic spectrum does not have enough energy to separate the positive and negative charges in the material. Polycrystalline silicon solar cells have an efficiency of less than 20% at this time and amorphous silicon cells, are presently about 10% efficient, due to higher internal energy losses than single crystal silicon.

A typical single crystal silicon PV cell of $100\ cm^2$ *will produce about* 1.5 *watts of power at* 0.5 *volts DC and* 3 *amps under full summer sunlight* ($1000\ Wm^{-2}$). *The power output of the cell is almost directly proportional to the intensity of the sunlight.*

An important feature of PV cells is that the voltage of the cell does not depend on its size, and remains fairly constant with changing light intensity. However, the current in a device is almost directly proportional to light intensity and size. When people want to compare different sized cells, they record the current density, or amps per square centimeter of cell area.

The power output of a solar cell can be increased quite effectively by using a tracking mechanism to keep the PV device directly facing the sun, or by concentrating the sunlight using lenses or mirrors. However, there are limits to this process, due to the complexity of the mechanisms, and the need to cool the cells. The current output is relatively stable at higher temperatures, but the voltage is reduced, leading to a drop in power as the cell temperature is increased.

1933 CE **Fritz Walter Meissner** (1882–1974, Germany). Physicist. Discovered that when a material is cooled into the superconducting phase in the presence of an external magnetic field, the magnetic flux is completely expelled from the interior of the superconductor, provided that the magnetic field is not too strong.

The phenomenon is called the *Meissner effect*. This is analogous to the exclusion of an *electric field* from the interior of an ordinary conductor, where charges in the conductor move and arrange themselves to create a component of electric field inside the conductor that exactly cancels the contribution of the external field. Similarly, an external *magnetic field* does not penetrate a superconductor because eddy currents flow on the surface in such a manner that the net magnetic field inside the superconductor is zero.

1933–1946 CE **Max Delbrück** (1906–1981, Germany and USA). Physicist and biologist. Founded molecular biology. Tested experimentally one of the QED nonlinear processes, known as *Delbrück scattering*[772] (1933). Joined Salvador Luria (1943) to demonstrate that bacteria adopt to new conditions (such as the presence of a virus) by Darwinian mechanisms, just as higher forms do. They concluded that virus resistant mutants preexist in a population, and are not induced by the selective agent (the virus) that is applied to isolate the mutants. Their demonstration of adaptation established bacteria as suitable objects for the study of genetic mechanisms, so that principles applicable to *all* life could be discovered.

Delbrück and Luria believed that they could better understand genetic mechanisms by studying one of nature's simplest creatures: the *bacteriophage* (or simply, *phage*). The phage is a virus that infects bacteria. Viruses reproduce in a living cell by using the cell's apparatus for reproduction of their DNA or RNA. By definition, viruses contain either DNA (*deoxyribonucleic* acid: a large, string-like molecule found in living cells that carries genetic information), which acts to redirect the bacterium's own biosynthetic systems to make more virus phage, or RNA (*ribonucleic* acid: single- or double-stranded molecules).

The final event in the infection process is usually a breakdown of the bacterial wall (called *lysis*), which frees the newly reproduced phage particles. Some phage are tadpole shaped, with a head containing DNA within a wall of protein. The phage's hollow tail, also made of protein, can attach to bacteria and facilitate the transfer of DNA into them. Because of their simplicity,

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[772] A *photon* is scattered by the electric Coulomb field of the nucleus [in contradistinction to the *Compton scattering* (1922) through which an electron (or a proton, or a whole nucleus) absorbs and reemits a photon]. The Delbrück scattering process is a succession of two *virtual* (where energy and/or momentum are not conserved over short distances or small time intervals) quantum processes:

(1) pair-creation in the nuclear Coulombic field $(\gamma \rightarrow e^+ + e^-)$;

(2) The inverse process $(e^+ + e^- \rightarrow \gamma)$ in the Coulombic field.

Note that the first process cannot occur as a *real* process (energy and momentum conserved), even in the presence of the nuclear Coulombic field, unless the photon energy is at least $2m_ec^2 \simeq 1.02$ MeV, but as a *virtual* process — there is no energy threshold and it can occur at any energy of the photon. [In general a stable particle cannot emit another particle, nor split into several particles – nor can the reverse process occur – without either external influences or one of the particles involved being virtual.]

phages seemed ideal for studying how genetic material reproduces, mutates, and expresses genetic information.

Delbrück was born in Berlin and did his Ph.D. in quantum mechanics under Max Born at Göttingen (1930). He then went to Copenhagen (1931) to work with Niels Bohr, who became his mentor. He returned to Berlin (1932) to work with **Lise Meitner**. He left for the US (1937) on a Rockefeller Fellowship to CalTech, and shortly after he left, Meitner discovered nuclear fission. In the US Delbrück's interests shifted to biology[773]. Following Bohr's line of thinking he thought perhaps new laws of physics may come out of study of biological systems.

At Pasadena he met Emery Ellis, who introduced him to bacteriophage. The phage appealed to Delbrück's physics-talented mind — he likened it to the hydrogen atom of biology, the simplest genetic system known.

Delbrück then took a faculty position at Vanderbilt University (1940–1947). In 1941 he met **Salvador Luria** and they began to collaborate on phage experiments. With **Alfred Hershey**, they discovered (1946) recombinations of viral DNA and received the Nobel Prize for medicine and physiology (1969). Delbrück moved back to CalTech and remained there (1947–1977).

1933–1956 CE **Alfred Tarski** (1902–1983, Poland and USA). Logician and mathematician. Made important contributions to mathematical logic, set theory, measure theory, model theory and general algebra.

Tarski was born in Warsaw and educated there. Excluded, as a Jew, from a faculty university post, Tarski taught concurrently in a high school and at Warsaw University until 1939. Fortunate to escape the holocaust, he came to the US (1939) and joined the faculty of the University of California at Berkeley (1942–1960).

A most notable achievement was his monograph (1933) "*The concept of truth in formalized languages*". This has been the starting point for all logically serious discussions of the subject, ever since. In it Tarski rehabilitates the classical correspondence theory of truth in contemporary logical guise. His later work has served to legitimize semantic discourse about the relations

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773 During 1932–1944 influential views of life have come from physicists rather than biologists: **Niels Bohr**, in his essay "*Light and Life*" (1932), urged applying the "complementarity" principle from quantum mechanics to biology. **Erwin Schrödinger** in "*What is Life*" (1944) identified the crucial question of how the cell is governed by a 'code-script' inscribed in the genes. The latter book inspired **Francis Crick** and **James Watson**'s investigation of the molecular structure of DNA.

between language and the world. Tarski had much influence on contemporary philosophers of science, especially Popper.

1933–1966 CE **Sergei Pavlovich Korolev** (1906–1966, The Soviet Union). Aerospace engineer. Founder of the Soviet space program. Responsible for the development of the world's first ICBM and artificial earth satellite (SPUTNIK 1). A brilliant engineer and superb organizer, who possessed the political cunning necessary to get his work done and protect his staff from a government so paranoid, he was forced to work in anonymity (known only as the 'Chief Designer') and kept under tight security almost until his death.

Korolev would go in the annals of aerospace technology to make these space records:

- First man-made orbiting satellite (Oct 04, 1957)[774].
- First man to orbit the earth.
- First craft to orbit the moon and photograph its back side.
- First craft to impact Venus.

Korolev was born in Zhitomir, Ukraine and became interested in aviation since early age. He was educated at the Kiev Polytechnic Institute (1924–1926) and the Moscow Bauman High Technical School (1926–1929), at that time the best engineering college in Russia. In 1982, Korolev was appointed chief of Jet Propulsion Research Group, one of the earliest state-sponsored centers for rocket development in the USSR. In 1933, Korolev led the development of cruise missiles and of a manned rocket-powered glider.

His work at that time culminated in designing Russia's first rocket propelled aircraft.

Thus, he was involved in pre-World War II studies of rocketry in the USSR.

On June 27, 1938, at the height of Stalin's purges, Korolev was arrested and sent to the GULAG camps in Siberia. In March 1940, Korolev was returned to Moscow and Imprisoned in the infamous Butyrskaya prison. On July 10 the same year, a special commission chaired by Lavrenti Beria, chief of Stalin's secret police, sentenced Korolev to eight years in labor camps on phony allegations of sabotage. "Fortunately" for Korolev, in September 1940,

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[774] During the Cold War, Americans were amazed that a culture, supposedly technologically inferior to the West, could excel where they lagged behind. Only after the fall of the Soviet Union could the identity and biography of Korolev be divulged.

he was transferred to "sharashka" – an aviation design bureau in prison. Officially called KB-29, Korolev's sharashka was led by **Andrei Tupolev**, also a GULAG prisoner.

On July 27, 1944, the authorities "paroled" Korolev and on Sept. 8, 1945, Korolev traveled to Germany for evaluation and restoration of V-2 ballistic missiles. In August 1946, while still in Germany, Korolev was appointed chief of a department in the newly created NII-88 in Podlipki, northeast of Moscow. This organization was made responsible for the development and industrial production of missile technology based on German hardware.

In the following years, Korolev led the development of several generations of ballistic missiles, launch vehicles, military and communications satellites, interplanetary probes and manned spacecraft. He died at the height of his career as a result of a botched surgical operation on January 14, 1966.

Due to secret nature of the Soviet space industry, Korolev's contribution to the space program was only recognized by the authorities after his death. For several more decades, Korolev's personality remained a subject of distortions by the official Soviet press. Only in 1994, Yaroslav Golovanov, a Russian journalist and historian, published the first uncensored biography of Sergei Korolev.

His incredible energy, intelligence, belief in the prospects of rocket technology, managerial abilities and almost mythical skills in decision-making made him the head of the first Soviet rocket development center, known today as RKK Energia. He deserves the most credits for turning rocket weapons into an instrument of space exploration and making Russia the world's first space-faring nation.

1933 CE, Jan. 30 Adolf Hitler was appointed chancellor by German President Paul von Hindenburg. Brought into office by a right-wing coalition, he rapidly disposed of his partners and liquidated all opposition to become (1934) dictator (*Führer*) of Germany. With enthusiastic support from a majority of the German people he used the power of the state to gradually strip Germany's Jewish citizens of their livelihood, freedoms and human rights; restored the country to a dominant position in Europe, repudiating the Versailles Treaty (1935), reoccupying the Rhineland (1936), forming the Rome-Berlin axis (1936), intervening in the Spanish civil war (1936–38), invading and annexing Austria[775] (1938), occupying Czechoslovakia (1938), making a pact with the USSR and invading Poland (1939).

[775] The *Anschluss* of March 11, 1938 — when the nation that produced Bach annexed the nation that produced Mozart.

1933, March 02 CE **O**ne of the most energetic earthquakes of the 20^{th} century hit Japan. Its epicenter was at 39.25 °N, 144.5 °E and its submarine fault unleashed a giant tsunami with a visual run-up of 23 m, killing some 3000 people in Sanriku, Honshu. The generating fault extended over an area of 370 km × 100 km accompanied by a displacement of 7.4 m. The energy release of this seismic event was equivalent to 150 Megatons of TNT having a seismic moment of 10^{29} dyn-sec.

1933–1985 CE **Hans Jonas** (1903–1993, Germany and USA). Philosopher. One of the most original and prominent thinkers of his generation. Contributed to the philosophy of biology, anthropology, theology and problems of ethics in a technological age.

Jonas' life spanned 90 years of the 20^{th} century, and like few other Western intellectuals he was able to see the deep and dramatic changes that have taken place in the entire so-called civilized world. His most significant works are: *The Gnostic Religion* (1934, 1958, 2001); *The Imperative of Responsibility* (1979); *The Phenomenon of Life* (1966); *Morality and Mortality* (1979); *On Technology, Medicine and Ethics* (1985).

Jonas was born to Jewish parents in Münchengladbach, Germany. He studied in Freiburg (1921), Berlin (1921–1923) and *Marburg* (1924–1928) under **Husserl**, **Heidegger** and **R. Bultmann** and received his Ph.D. in philosophy (1928). After Hitler had come to power, he first emigrated (1933) to England, then to the Hebrew University in Jerusalem (1934–1940) and finally to Canada (1949). There he taught for six years at McGill and Carleton Universities, before settling down permanently in New York, teaching at the philosophy department of the New School for Social Research.

The range of his topics was extremely wide — from early gnosticism to the philosophy of biology, from ethics to social philosophy, from cosmology to Jewish theology.

Shaped by his exile from Nazi Germany, the murder of his mother in the Auschwitz extermination camp, his participation as a soldier in WWII (the Jewish Brigade of the British Army) during 1940–1945 and the Israeli War of Independence (1948–1949) — he set himself the task of uncovering the intellectual origins of the crisis of Western civilization and proposing a new, positive orientation for humanity.

Jonas' work will become increasingly significant in the years ahead as we face the problems produced by current developments in technology such as biological engineering. Such issues were of particular interest to him, and he was unique among his philosophical contemporaries in devoting attention to them. His eloquent writings on these themes bring wisdom and common sense anchored in Jonas' own historical and biographical experience of the fragility of human life and the common good.

Worldview XLV: Hans Jonas

* * *

"Modern technology, informed by an ever deeper penetration of nature and propelled by the forces of market and politics, has enhanced human power beyond anything known or even dreamt of before. It is a power over matter, over life on earth, and over man himself; and it keeps growing at an accelerated pace. Modern technology is marked by a radical departure from everything previously known. It has disturbed the balance between humanity and nature in ways that are long-range, cumulative, irreversible, and planetary in scale. It has permanently altered the biosphere of the earth, it has challenged our definitions of "life" and "death". It has created a freedom without values.

Traditional ethics presumed that the effects of our actions are limited. All this has changed with modern technology. In its view, nature is a machine, not an end in itself. We may matter to ourselves, but there is no larger system of values to which we belong. In the end humans become the objects of their own fabrications, to be shaped according to the design of biotechnology.

As we are deprived of any consistent image of humanity, we are unable to answer the fundamental ethical question: Why should we care about the distant future of mankind on this planet?"

* * *

"Care of the future of mankind, is the overruling duty of collective human action in the age of technical civilization that has become 'almighty', if not in its productive then at least in its destructive potential. This care must obviously include care for the future of all nature on this planet as a necessary condition of man's own survival. We line in an apocalyptic situation, that is,

under the threat of a universal catastrophe if we let things take their present course. The danger derives from the excessive dimensions of the scientific-technological-industrial civilization.

The danger of disaster through scientific technology arises not so much from any shortcoming of its performance as from the magnitude of its success."

* * *

"The altered nature of human action calls for a change in ethics as well. All previous ethical concepts had as their content the relationship between human beings only. The good or bad of human actions was decided within the short-term context of the here and now or the immediate foreseeable future.

All this has decisively changed: modern technology has introduced actions of such novel scale, objects, and consequences that the framework of former ethics can no longer contain them.

A new dimension of responsibility, never dreamt before, is forced upon ethics. No previous ethics had to consider the global condition of human life and the far-off future, even existence, of the race. Those are now an issue and demand, in brief, a new conception of duties and rights, for which previous ethics and metaphysics provide not even the principles, let alone a ready doctrine.

In order to fulfill this new imperative of responsibility, a scientific futurology is required. An imaginative heuristic of fear must tell us what is possibly at stake and what we must beware of. The prophecy of doom must take priority over the prophecy of bliss. As mankind has no right to suicide, the existence of man must never be put at stake: mankind's existence becomes the First Commandment of a new ethical order."

* * *

The Imperative of Responsibility (1979)

1933–1996 CE **Paul Erdös**[776] (1913–1996; Hungary, USA and Israel). Legendary mathematician. Contributed to: *Analysis* (including Ergodic Theory), *Combinatorics* (including Graph Theory, Combinatorial Algebra, Combinatorial Geometry and Theoretical Computer Science), Number Theory, Probability Theory, and Set Theory.

He founded the field of *discrete mathematics*, which is the foundation of computer science. One of the most prolific mathematicians in history, publishing more than 1600 papers in which he collaborated with 507 people[777]

Erdös was regarded by fellow mathematicians as the most brilliant, if eccentric, mind in the field, being a *problem solver* rather than a builder of theories.

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[776] For further reading, see:

- Hoffmann, P., *The Man Who Loved Only Numbers*, Fourth Estate: London, 1999, 302 pp.

[777] Around 1965, Casper Goffman concocted the idea of an '*Erdös number*': If you had written a joint paper with Erdös, your Erdös number was 1. If you had written a joint paper with someone with Erdös number 1, your Erdös number was 2, and so on. There is now an *Erdös Number Project* home page on the web where one can see the results:

Erdös number 0	—	1 person
Erdös number 1	—	507 people
Erdös number 2	—	5713 people
Erdös number 3	—	26422 people
Erdös number 4	—	62136 people
Erdös number 5	—	66157 people
Erdös number 6	—	32280 people
Erdös number 7	—	10431 people
Erdös number 8	—	3214 people
Erdös number 9	—	953 people
Erdös number 10	—	262 people
Erdös number 11	—	94 people
Erdös number 12	—	23 people
Erdös number 13	—	4 people
Erdös number 14	—	7 people
Erdös number 15	—	1 person
Erdös number 16	—	0 people

Thus the median Erdös number is 5; the mean is 4.69, and the standard deviation is 1.27.

For more than 50 years, Erdös wandered the globe visiting mathematicians, attending meetings, teaching and lecturing. He had become the center of an enormous web of collaboration.

Erdös was the *supreme problem poser and problem solver of modern times.* His interests were mainly in number theory and combinatorics, though they ranged into topology and other areas of mathematics. He was fascinated by relationships among numbers, and numbers served as the raw material for many of his conjectures, questions, and proofs.

Paul Erdös was born in Budapest to Jewish parents, the original family name being Engländer. Despite the restrictions on Jews entering universities in Hungary, Erdös, as a winner of a national examination (1929), was allowed to enter in 1930.

In 1933 he gave a *simple* proof to *Bertrand's conjecture*[778] (1845) [proved by Tchebyshev, 1850]: for every positive integer n, there is a prime between n and $2n$.

He was awarded a doctorate in mathematics in 1934. He then held appointments at the universities of Manchester (1935–1938), Princeton (1939), Madison (1940–1943), Purdue (1943–1948), Notre Dame (1952–1954) and Jerusalem (1954–1964).

In 1949 he gave a proof (with Atle Selberg[779]) of the *Prime Number Theorem* (= the number of primes less than or equal to the real positive number x is asymptotically equal to $\frac{x}{\log x}$) that avoided using complex analysis.

Stooped and slight, often wearing socks and sandals, Erdös stripped himself of all the quotidian burdens of daily life: finding a place to live, driving a car, paying income taxes, buying groceries, writing checks.

Concentrating fully on mathematics, Erdös traveled from meeting to meeting, carrying a half-empty suitcase and staying with mathematicians wherever

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778 News about his success was passed around Hungarian mathematicians, accompanied by the rhyme:

> "*Chebyshev said it, and I say it again;*
> *There is always a prime between n and $2n$*"

779 Selberg and Erdös agreed to publish their work in back-to-back papers in the same journal, explaining the work each had done and sharing the credit. But at the last minute Selberg raced ahead with his proof and published first. The following year Selberg won the Fields Medal for his work. Erdös was philosophical about the episode.

he went. His colleagues took care of him, lending him money, feeding him, buying him clothes and even doing his taxes. In return, he showered them with ideas and challenges — with problems to be solved and brilliant ways of attacking them.

He wrote no best-selling books, and showed a stoic disregard for worldly success and personal comfort, living out of a suitcase for much of his adult life. The money he made from prizes he gave away to fellow mathematicians whom he considered to be needier than himself. "*Property is a nuisance*," was his succinct evaluation. The winners would often frame his checks without cashing them. (Solving a $1000 problem would make you internationally famous.)

During the Cold War, Erdös was persecuted by the US immigration and the FBI because of his travels to mathematical conferences behind the Iron Curtain and his correspondence with Chinese mathematicians.

Erdös won many prizes, including the $ 50,000 Wolf Prize of in 1983.

Paul Erdös – The man who loved only numbers

* * *

"My mother said; 'Even you, Paul, can be in only one place at one time.'
Maybe soon I will be relieved of this disadvantage.
Maybe, once I've left, I'll be able to be in many places at the same time.
Maybe then I'll be able to collaborate with Archimedes and Euclid."

* * *

"I am 2 billion years old because when I was in high-school I was taught that the earth was $2\frac{1}{2}$ *billion years old, but now we know that it is* $4\frac{1}{2}$ *billion years old."*

* * *

*On one occasion, **Erdös** met a mathematician and asked him where he was from. "Vancouver," the mathematician replied. "Oh, then you must know my good friend Elliot Mendelson," Erdös said.*

The reply was: "I AM your good friend Elliot Mendelson."

* * *

A salesman was knocking on Erdös' door. Paul, busy with the solution of a mathematical problem, called from his table: "Please come some other time, and at somebody else's door."

* * *

He observed one day that the audiences at his talks had been getting larger and larger, to the point where they filled halls so big that his old and feeble voice could not be heard. Erdös speculated as to the cause of this.

"I think," he said, "it must be that everyone wants to be able to say 'I remember Erdös; why, I even attended his last lecture!' "

* * *

This is my vision of the 'perfect death': It would occur after a lecture, when I have just finished presenting a proof and a cantankerous member of the audience would have raised a hand to ask: "What about the general case?" I would then reply: "I think I'll leave that to the next generation," and fall over dead.

* * *

"Mathematical truths are discovered, not invented."

Mathematicians on Paul Erdös

* * *

"A mathematical genius of the first order, Paul Erdös was totally obsessed with his subject — he thought and wrote mathematics for nineteen hours a day until the day he died. He traveled constantly, living out of a plastic bag, and had no interest in food, sex, companionship, art — all that is usually indispensable to a human life."

(Oliver Sacks)

* * *

"One never knew where Erdös was, not even the country. However one could be sure that during the year, Erdös was everywhere. He was the nearest thing to an ergodic particle that a human being could be."

(Richard Bellman)

* * *

"Because he seemed to be in a state of Brownian motion, it was hard to locate him at any given time.
With his death we have lost one of the great mathematicians and free spirits of this century, and it is hard to imagine that we will see anyone like him again. I feel fortunate to have the privilege of knowing him and working with him."

(Melvin Henriksen)

* * *

"In our century, in which mathematics is so strongly dominated by 'theory doctors', he had remained the prince of problem solvers and the absolute monarch of problem posers, the Euler of our time."

(Ernst Straus)

* * *

"Erdös' driving force was his desire to understand and know. You could think of it as his magnificent obsession. It determined everything in his life."

* * *

"He died with his boots on, in hand-to-hand combat with one more problem. It was the way he wanted to go."

(Ronald L. Graham)

* * *

"He was always searching for mathematical truths. He had an ability to inspire. He would take people who already had talent, that already had some success, and just take them to an entirely new level. His world of mathematics became the world we all entered."

(Mark Spencer)

1933–1941 CE **Fritz Zwicky** (1898–1974, U.S.A.). One of the first to suggest the existence of *dark matter* (1933) and *Neutron stars* (with **W. Baade**, in 1934). Made valuable contributions to the theory and understanding of *supernovae*.

Zwicky was born in Varna, Bulgaria. He received his doctorate in physics from the Swiss Federal Institute of Technology in Zürich (1922) and served on the faculty of the California Institute of Technology, Pasadena during 1925–1972. In 1933 he observed a group of galaxies orbiting one another and estimated the gravity needed to keep the cluster from flying apart. From the required gravitational pull and the size of the cluster, Zwicky could further calculate the mass contained within the cluster: about 20 times what could be generated by the visible stars and gas.

For more than forty years, most astronomers tried to ignore the possibility of dark matter. In 1934 he proposed that supernovae are a class of stellar explosions completely different from the ordinary novae, and occurring less often (ca once every 500 years in our galaxy). From 1937 to 1941 he discovered 18 of them (about 12 had been recorded previously in the history of astronomy).

The neutron was discovered via laboratory experiments in 1932. Within two years **F. Zwicky** and **W. Baade** developed a theory according to which stars too massive to become white dwarves will collapse into a highly compact ball of degenerate neutrons, *a neutron star*[780]. The degenerate neutron pressure could support a stellar 'corpse' against further gravitational collapse.

This prophetic proposal was politely ignored by most scientists for many years because a neutron star was considered a weird object — having, as theory showed, nuclear density of order 10^{15} g/cm^3, relatively small size (some 30 km in diameter) and an escape velocity equal to half the speed of light. All these properties seemed so outrageous in 1934 that few astronomers paid any serious attention to this theory, and the proposal was shelved until the discovery of the first pulsar in 1967.

[780] Neutrons, like electrons, obey the Pauli exclusion principle and can become degenerate if crowded into a sufficiently small volume. At white-dwarf densities, the electrons are degenerate, but not the nuclei. Thus, a star could collapse into degenerate neutrons if, once its nuclear fuel has been exhausted, it is too heavy to be stabilized by forming a white dwarf. In that case, the star's self gravity forces all electrons and protons (except in the star's crust) to combine, pairwise, into neutrons, which causes a massive neutrino burst (ca 10^{57} of them) and leaves behind only degenerate neutrons. The neutrons in such a condition cannot decay into protons and electrons for reasons of energetics — although a *free* neutron is unstable. In a sense, a neutron star is *one gigantic atomic nucleus*!

Extreme Earth – Chronology of Events[781]

We address the subject of singular terrestrial events in recorded history and their impact upon civilizations. Natural disasters have been the scourge of mankind since time immemorial. We survey their history throughout the past 6000 years, including such calamities as volcano eruptions, earthquakes, floods, tsunamis, major storms, hurricanes, droughts, pandemics, collision of earth with bolides, climatic changes and ecological collapses. Not only had many of theses singular events changed the course of civilization, but they have at the same time influenced both the advent and advance of science. It is estimated that ca 500 million people were killed since 4000 BCE by all catastrophes combined. This amounts to about 1.5 percent of all people who ever lived since 4000 BCE. The total fatal energy unleashed on the earth's surface against its inhabitants throughout the said time window is estimated at 2×10^7 *MT.*

The following consist of a chronological list of some 150 terrestrial events which had the most significant impact (and consequences) on human civilizations, since 5000 BCE.

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[781] For further reading, see:

- Ben-Menahem, A., *Extreme Earth – History, Chronology, and Energetics*, Keynote Address to the XXVI General Seismological Commission (ESC), Aug 23–28, 1998, Tel-Aviv, Israel.
- Robinson, A., *Earth Shock*, Thames and Hudson, 1993, 304 pp.
- Knauer, K. (ed.), *Nature's Extremes*, Time, 2005, 138 pp.
- Simkin, T. and L. Siebert, *Volcanoes of the World*, Geoscience Press, Tucson AZ, 1994, 349 pp.
- Watterau, B., *The New York Public Library Book of Chronologies*, Prentice-Hall: New-York, 1990, 634 pp.
- Brooks, C.E.P., *Climate Through the Ages*, Dover, 1970.
- Cartwright, F.F., *Disease and History*, Dorset Press, 1991, 248 pp.
- Gutenberg, B. and C.F. Richter, *Seismicity of the Earth.*, Princeton University Press, 1954.

Prehistory

4895 BCE *Crater Lake (Oregon, U.S.A.; VEI=7=Volcanic Explosion Index).*

4400–4000 BCE *Neoglaciation* and severe *climatic* depressions on a global scale: severe episodes of climatic deterioration manifest through glacial advance, increased rainfall, decline of average temperatures, rise of a sea-level and major flooding.

4350 BCE *Kakai* (Japan) volcano eruption (VEI=7).

3580 BCE *Vesuvius* (Italy) volcano eruption (VEI=6).

Ancient History

ca 3200 BCE

- Millennial-scale warming terminates with flooding in the lower latitudes followed by drought. Survivors organize into centrally directed and hierarchical culture.
- History begins at Sumer with the invention of the art of writing. Emergence of urban irrigation-based cultures centered on city-states.

  Sea-level changes.

  Index of tree-ring narrowness corresponds to temperature changes. Ice-core samples from Greenland (sulfate concentration, oxygen-isotope ratio indicative of ambient temperature).

  Flood-myths of most civilizations and many oceanic islands.

Ca 2920 BCE *Black Peak* (Alaska) volcano eruption (VEI=6).

Ca 2880 BCE *Taupo* (New Zealand) volcano eruption (VEI=6).

Ca 2297 BCE Massive rains burst the Hwang Ho (Yellow River), Wei and Yangtze rivers, flooding almost the entire North China Plain and turning it into a huge inland sea.

Ca 2180–2130 BCE *Sharp climatic changes* cause rise and fall of civilizations and mass-migration in Europe and Asia.
Failure of Nile annual floods; *drought* and *famine* in Egypt cause collapse of central government and an ensuing chaos.

Ca 2040 BCE *Long Island (New Guinea) volcano eruption (VEI=6).*

1855 BCE *St.Helens volcano eruption (VEI=6).*

ca 1800 BCE

- *Seismo-volcanic upheaval in the Dead-Sea region. Biblical allusions.*
- *Ecological collapse of city-state civilization in Southern Mesopotamia due to salination of cultivated soils. Mass-migration. Age of biblical patriarchs.*

1750 BCE *Veniaminof (Alaska) volcano eruption (VEI=6).*

May 1627 BCE *Thera volcanic paroxysm; ca 100,000 people perish. Ash identified as coming from the eruption has been found in coastal cities as far away as Israel and Sardi in Anatolia. Climatic and economic disruption of late Bronze-age Minoan civilization and Egyptian middle kingdom. Subsequent impact on civilizations in the Eastern Mediterranean. Event linked to myth of Atlantis and biblical account of Israel in Egypt.*

ca 1320 BCE *Bolide fireball explosion over Apasa, Asia Minor.*

ca 1250 BCE *Possible visitation of a comet as "pillar of fire"during the exodus of Israelites from Egypt.*

ca 1200–850 BCE *Sharp climatic changes – northward displacement of arid zone at late Bronze-age cause mass southward migration in Europe and the Aegean zone. Indo-European invasion into Greece and Asia Minor by sword-bearing people. Disruption of agriculture in Crete, Greece and Eastern Mediterranean. Homeric wars, Sea-People in Egypt, Cyprus, Israel and Italy. Decline of Mycenaen and Hittite civilizations. Severe droughts in China (ca 1122 BCE, 842–771 BCE).*

ca 950 BCE *Hekla (Iceland) volcano eruption (VEI=6). Radiocarbon evidence.*

767 BCE *First recorded pandemic in Europe and the Mediterranean world.*

600–500 BCE *Cooler weather, increase of rainfall and floodings in Europe and the Middle-East.*

430 BCE *Decline and fall of Athens by the plague.*

217 BCE *Vesuvius (Italy) volcano eruption.*

210 BCE *Raoul (Kermades Is.) volcano eruption (VEI=6).*

44 BCE *Etna volcano eruption.*

50 CE *Ambrym (New Hebrides) volcano eruption (VEI=6-7).*

65 CE *Bonna-Churchill (Alaska) volcano eruption (VEI=6).*

79 CE**, Aug. 24** *Vesuvius (Italy) volcano eruption (VEI=6). Ruin of Pompeii, Herculanum and Stabiae.*

ca 100 CE *Climatic change forces the abandon of the formerly flourishing (since 300 BCE) cities of Palmyra and Petra (now in the deserts of Syria and Jordan respectively). In these cities vine and olive were cultivated without much recourse to artificial irrigation. This implies higher water table than now and a climate that supplied more dependable rain.*

155 CE *Ksudach (Kamchatka) volcano eruption.*

250–594 CE *Decline and fall of Roman Empire aided by severe climatic changes causing droughts, plague and malaria.*
Plague of Justinian (542); millions perish in Africa and Mediterranean area.

260 CE *Ilopango (El Salvador) volcano eruption (VEI=6).*

536 CE *Rabaul (New Britain) volcano eruption (VEI=6). The greatest aerosol-producing eruption in recorded history.*

550–600 CE *Prolonged drought in the Peruvian Andes put an end to the 1000 year old Nazca Indian Culture, who in their great despair drew impressive geoglyphs to implore their gods for rain.*

Middle Ages

600–650 *Dry period in Arabia preceded the great wave of Arab outburst through the advent of Islam.*

626 *Volcano eruption in the Mediterranean region.*

700 *Bonna Churchill (Alaska) volcano eruption (VEI=6).*

934 *Eldgia (Iceland) volcano eruption.*

1000 *Collapse of the Andean Tiahuancco Empire due to a prolonged drought lasting some 80 years.*

1006 *Merapi (Java) volcano eruption destroyed Hindu-Javanese state of Mataram and its unique civilization.*

1022 *Drought in India; population decimated.*

1054 *Baitoushan (China) volcano eruption (VEI=7).*

1064–1072 *7-year failure of Nile flooding; widespread famine.*

1099 *Sea flood in Thanes Estuary and Holland; 100,000 estimated drowned.*

1104 *Hekla (Iceland) volcano eruption (VEI=5).*

1164 *Floods in northwestern Germany; 100,000 estimated perished.*

1200 *Floods in Friesland (Holland); 100,000 estimated drowned.*

1200–1250 *Drought and moist conditions drive Mongols into China, Europe and Middle-East. Decline of Islamic Empire.*

1212 *Floods in North Holland. Enormous loss of life: estimated 306,000 drowned.*

1218–1287 *Catastrophic floods in the Netherlands; 200,000 victims (coasts of Holland sank 2 m since Roman times). Strong tides and storms in the North Sea.*

1258 *A volcano eruption affects global weather.*

1332 *The Yellow River burst and drowned about 7 million people with possibly a further 20 million dying of famine. The Black Death plague appeared in China at the same time.*

1332–1351 *Black-Death Pandemic in India, China and Europe. History's greatest natural disaster; ca 50 million die in Europe and Asia (about 10 % of world's population at that time). Parching drought with consequential famine in Central Asia have caused rodent migration westward. End of feudal system and advent of the Renaissance in Europe.*

1400–1650 *'Little Ace Age' in Europe.*
Floods in Holland (1421); 100,000 perish. Again: 50,000 die in 1530 and 100,000 in 1646.

1452 *Kuwae (New Hebrides) volcano eruption (VEI=6).*

Modern Era

1507–1595 *Smallpox and typhus pandemics in the New World decimated the native population of Inca and Aztec empires; ca 50 million die.*

1556, Jan. 23 *Earthquake in Shensi (China); 830,000 perish.*

1563 *Bubonic Plague in London; 25% of the population perish.*

1570, Nov. 02 *Floods in the Netherlands; 400,000 perish.*

1577 *Apparition of the 'Great Comet' (0.63 au from earth). Turning point in astronomy.*

1580 *Billy Mitchell (Bougainville) volcano eruption (VEI=6).*

1600 *Plague in Russia; ca 250,000 victims.*
Huaynaputina (Peru) volcano eruption (VEI=6).

1630(1656) *Plague in Venice (Naples); ca 250,000 (100,000) die.*

1631 *Vesuvius (Italy) volcano eruption; more than 4000 perish.*

1641, Jan. 04 *Parker (Philippines) volcano eruption (VEI=6).*

1660 *Long Island (New Guinea) volcano eruption (VEI=6).*

1664 *Plague in London; 100,000 die.*

1669 *Etna (Sicily) volcano eruption; 100,000 perish. Destruction of Catania, Sicily.*

1672 *Plague in Naples; 400,000 perish.*

1693 *Earthquakes hit Naples and Catania; 150,000 victims.*

1703, Nov. 26 *Sea-tempest in the British Channel; 8000 perish. Greatest storm in 2000 years in the North Sea. Worst storm in British history.*

1711 *Plague in Germany and Austria; 500,000 die.*

1717 *Floods in Holland; 12,000 people drowned.*

1730, Dec. 30 *Earthquake hit Tokyo (Japan); 140,000 perish.*

1737, Oct. 11 *Cyclone at the mouth of the Ganges River at Calcutta (India); 300,000 die. Avachinsky (Kamchatka) volcano eruption. Great tsunami.*

1755, Nov. 01 *The Lisbon earthquake; 60,000 killed. Advent of modern seismology.*

1766 *Hekla (Iceland) volcano eruption (VEI=4).*

1769–1778 *Drought causes famine in India; ca 6 million perish.*

1780 *Hurricane hits Cuba and Central America; 25,000 perish by floods.*

1783, June 08 *Laki (Iceland) volcano eruption; 10,000 die; ash fall-out destroyed crops and livestocks causing famine and starvation.*

1792 *Unzen (Japan) volcano eruption; tsunami kills 14,300 persons. Plague in Egypt; 800,000 die; pandemic spread to North Africa (1799) and killed 300,000 more.*

1799 *Plague in North Africa; 300,000 die.*

1803, Apr. 26 *Meteor shower over the village of l'Aigle in France finally established extraterrestrial nature of meteors.*

1812–1813 *Dysentery decimates the Napolean army in Russia; ca 400,000 die.*

1815, Apr.10 *Tambora (Sunda Is.) volcano eruption; perhaps the most energetic single geophysical event in the past 5000 years (20,000 MT); 92,000 died from tephra, tsunami and starvation. Epicenter at Sumbawa 8°15'S 118°00'E. 150* km^3 *ash ejected into the atmosphere and affected world climate. Caused extreme cold winters in many parts of the world; 1816 – the year without summer; famine.*

1826–1837 *Cholera pandemic in Asia and Europe; 900,000 perish in 1831 alone.*

1829–1833 *Malaria epidemic kills 150,000 Indians in the Pacific Northwest.*

1835, Feb. 20 *Major earthquake in Chile; witnessed by Darwin.*

1837–1838 *Drought in India; some 800,000 die.*

1840–1894 *Worldwide Cholera pandemic; mainly Eastern Europe and India (due to crop failures); fatalities in the millions.*

1851–1855 *Tuberculosis ravaged England; 250,000 die.*

1857 *Earthquake in Tokyo (Mar.21; 110,000 die) and Napoli (Dec. 16). First attempts to describe source-mechanism; advent of instrumental seismology.*

1864 *Hurricane in the Calcutta harbor.*

1866 *Drought-related famine in Bengal; 1.5 million die.*

1868 *Earthquake shakes Ecuador, Colombia and Peru; 40,000 die.*

1876 *Tsunami at the Bay of Bengal; 215,000 perish in Calcutta; caused by an earthquake in the Andaman Islands.*

1876–1879 *Prolonged drought in India and Northern China; 18 million perish.*

1881, Oct. 08 *Typhoon hits Haiphong (North Vietnam); 300,000 die.*

1883, Aug. 27 *Krakatoa (Sunda Straits) volcanic eruption; E=200 MT; tidal wave kills 36,000 people. First evidence of jet-stream circulation. Explosion heard 5000 km away. Ejecta reached 80 km high, above the ozone layer.*

1887. Sept.–Oct. *Floods of the Yellow River (Hwang Ho, China); 6 million people reported lost; caused by rains. This river killed more people than any other river in the world. Worst flood in recorded history.*

1889–1890 *Influenza pandemic in the world; millions die.*

1891, Oct. 28 *Earthquake in Mino-Owari (Japan); first documentation of surface faulting; 7300 people die.*

1892–1900 *Drought, famine and plague in India and China; ca 8 million perish.*

1897, June 12 *Earthquake in Assam (India); first observations of P, S, R waves on seismograms.*

1900, Sept. 08 *Hurricane at Galveston (Texas, U.S.A.); 8000 killed. Wind speed of 170* $\frac{\text{km}}{\text{sec}}$ *sent 5 m tidal wave through town.*

1902, May 08 *Peleé (Martinique) volcano eruption; 29,000 perish at St.Pierre in 2 minutes by a fire-storm (nueé ardente).*

1902, Oct. 24 *Santa Maria (Guatemala) volcano eruption; ca 1500 perish.*

1906, Apr. 18 *Earthquake of San-Francisco; M=*$7\frac{3}{4}$*; 700 killed; elastic rebound theory.*

1908, June 30 *The 'Tunguska event'; asteroid explosion over Siberia; on 00:14:28 GMT,* $60°55$*'N,* $101°57$*'E a 100 m bolide mimicked a high-altitude nuclear explosion with yield of ca 12 MT. It arrived with a velocity of ca 40* $\frac{\text{km}}{\text{sec}}$*, having an estimated mass of 50,000 ton. The radius of total destruction was 2000* km^2*. The explosion was heard to a distance of 1270 km, seen at distance 500 km; heat was felt 70 km away and matter was burnt in a radius of 15 km. Had it arrived 4 hours* *and 27 minutes later it would hit Petrograd and may have changed the course of history of the* 20^{th} *century. Nuclear explosion may have been triggered by neutron production as the heated object sped through the atmosphere. Frequency of event – ca once every 300 years (over the entire earth's surface).*

1908, Dec. 28 *Earthquake of Messina (Italy); 160,000 killed in Calabria and Sicily.*

1911 *Floods of the Yangtze River (China); 200,000 die.*

1911, June 06 *Katmai (Alaska) volcano eruption (VEI=6).*

1912 *Novarupta (Alaska) volcano eruption (VEI=6).*

1916, Jan. 14 *Sea-floods in Holland; 10,000 die.*

1916–1919 *Influenza pandemic; 80 million die worldwide, dwarfing the toll of combat in WW1 (ca 10 million).*

1920, Dec. 16 *Earthquake in Kansu and Shensi Provinces (China); 100,000 perish.*

1921–1930 *Cholera and smallpox epidemic in India; 1,300,000 victims.*

1923, Sept. 01 *Earthquake in Yokohama and Tokyo; 100,000 killed by shock and fire. Advent of earthquake-engineering.*

1925, Mar. 18 *'Tri-State-Tornado' in U.S.A.; 43 billion $ damage; 700 dead.*

1927, May 22 *Earthquake in Nanshan (Kansu, China); 200,000 victims.*

1928 *Hurricane in Florida; 1800 die.*

1930, Aug. 13 *Bolide explosion over Curucá River (Brasilian Amazon;* $5°S$, $71.5°W$*; 12:04 MT). Yield =1 MT. A miniature 'Tunguska'. Exploded about 9 km above ground. Explosion heard 240 km away and seismic waves were recorded at La Paz, 1322 km away. Fine red ash fell on the forest but nothing else reached ground level.*

1931, June–Aug. *Floods of the Yangtze River due to rain; more than million people drowned, 180 million affected in Hoanan Province (China).*

1932 *Hurricane in Cuba; 2000 perish.*

1933, Mar. 02 *Seismic tsunami off coast Honshu (Japan); waves 23 m high; 3000 perish.*

1933, Mar. 10 *Long-Beach (U.S.A.) earthquake. Systematic study of the effect of earthquakes on buildings begins.*

1935, Sept. 04 *'Labor-day Hurricane' in the Florida keys; 400 die. Winds 320 km per hour. Barometer 669.3 mm.*

1935, Dec. 11, 10:30 am LT *Bolide explosion over British Guiana (now Guyana); another "'mini Tunguska".*

1937 *"Near miss" of asteroid Hermes (distance of 800,000 km from earth).*

1938 *Yellow River flooding; 1 million victims.*

1956, Mar. 30 *Bezymianny (Kamchatka) volcano eruption.*

1960, May 22 *Earthquake in Chile; experimental verification of propagating rupture of faults and the earth's free oscillations.*

1965, Mar. 31 *Bolide explosion over British Colombia. Explosion heard over 140 km. Estimated yield 4 KT.*

1970, May 31 *Earthquake in Peru; 66, 000 killed.*

1970, Nov. 12 *Tropical cyclone in Bay of Bengal (Bangladesh); 500,000 victims.*

1974 *Hurricane in Central America; 5000 perish by floods.*

1975, Mar. 04 *Bolide explosion over Western New Guinea (3°42'S, 133°17'E). Large area of jungle knocked down. Felled trees point away from center.*

1976, July 27 *Earthquake in Tangshan (China); 655,000 victims (M=7.8).*

1978 *Bolide explosion over South Pacific; yield ≃ 100 KT.*

1980 *St. Helens (Washington, U.S.A.) volcano eruption; yield = 5 MT.*
El Cichón (Mexico) volcano eruption; more than 2000 die.

1985 *Ruiz (Colombia) volcano eruption; 25,000 perish.*

1988, Sept. 13 *Hurricane 'Gilbert'; strongest Atlantic cyclone on record.*

1989, Mar. 23 *Asteroid 1989FC with energy of 1000 MT nearly missed the earth (2 moon-distances away).*

1992, Dec. 12 *Tsunami flooded the South Pacific volcanic islands of Flores and Babi; 2000 killed. Waves, 25 m high swept ashore.*

1995 *Bolide explosion over Northeastern Brazil.*

1998 *Great hurricane induced floods in Central America. 20,000 perish in Honduras and Nicaragua. Million homeless and a third of all homes in these countries destroyed.*
Devastating floods in China.

Hazardous events – background and data

The chronology presented in the last section includes a variety of natural disasters caused by the interaction of life-systems on earth with internal and external dynamic environmental systems such as the earth's crust, its oceans and atmosphere and outer space. We divided these events into a number of categories, each of which will be discussed in some detail. Table 5.10 lists the most destructive catastrophes on earth during the second millennium CE, with the corresponding estimates of human death-toll.

Table 5.10: MOST DESTRUCTIVE GEOPHYSICALCATASTROPHES ON EARTH DURING THE 2nd MILLENNIUM CE

Date	*Location*	*Disaster*	*Estimated death-toll*
1139, Oct. 12	*Syria, Aleppo*	*Earthquake*	*230,000*
1228	*Netherland coast*	*Flood*	*100,000*
1290, Sept. 27	*China, Chihili (Hope Province)*	*Earthquake*	*100,000*
1421	*Netherland coast*	*Flood*	*100,000*
1530	*Netherland coast*	*Flood*	*100,000*
1556, Jan. 26	*China, Shansi*	*Earthquake*	*830,000*
1642	*China, Yellow River*	*Flood*	*300,000*
1646	*Netherland coast*	*Flood*	*100,000*
1693, Jan. 11	*Italy, Naples and Catania*	*Earthquake*	*153,000*
1730, Dec. 30	*Japan, Hokkaido*	*Earthquake*	*137,000*
1737, Oct. 11	*India, Calcutta*	*Cyclonic storm*	*300,000*
1815, Apr. 05	*Indonesia, Tambora*	*Volcano eruption*	*92,000*
1857, May 08	*Japan, Tokyo*	*Earthquake*	*107,000*
1876	*Bay of Bengal (Bangladesh)*	*Seismic tsunami*	*215,000*
1882	*India, Bombay*	*Flood*	*100,000*
1887	*China, Yellow River*	*Flood*	*2,000,000*
1908, Dec. 28	*Italy, Messina*	*Earthquake*	*160,000*
1911	*China, Yangtze River*	*Flood*	*200,000*
1920, Dec. 16	*China, Kansu*	*Earthquake*	*200,000*
1923, Sept. 01	*Japan, Kwanto*	*Earthquake*	*143,000*
1927, May 22	*China, Nanshan*	*Earthquake*	*200,000*
1931	*China, Yellow and Yangtze Rivers*	*Flood*	*3,000,000*
1938	*China, Yellow River*	*Flood*	*1,000,000*
1970, Nov. 12	*Bay of Bengal (Bangladesh)*	*Flood*	*500,000*
1976, July 27	*China, Tangshan*	*Earthquake*	*655,000*

EARTHQUAKES

Plate tectonics refers to the movement and deformation of segments of the earth's lithosphere (plates). There are seven major plates: North American, South American, Nazca, Pacific, African, Indian-Australian and Eurasian.

There are three types of plate boundaries on the surface of the earth:

- *Divergent boundaries:* two plates which are moving away from each other, leaving room for material from the mantle to seep into the space and form new sea floor (e.g., *Mid-Atlantic Ridge*, at the rate of ca 1.5 $\frac{cm}{yr}$. Such zones are characterized by active volcanism, shallow-focus earthquakes, tensile (stretching) stresses and high rates of heat flow.
- *Convergent boundaries:* two plates move *toward* each other, causing one plate to submerge beneath the other (e.g., formation of the Himalayas by the underthrusting of the Indian plate relative to the Eurasian plate).
- *Transform boundaries:* two plates slide past each other (e.g., along the San-Andreas fault system in California at the borders of the North American and Pacific plates), with neither creation nor destruction of lithosphere. In general, each plate is bounded by some combination of these three kinds of zones. The global sum of plate creation and consumption is approximately zero: the plates form and disappear in size and shape as they evolve.

Much of the earth's landscape has been shaped by plate tectonics. As oceanic plates have been created and subducted, and the continents have collided and broken apart, mountains have been built, rift valleys formed, ocean ridges and trenches created, and volcanoes constructed.

Plate tectonics is driven by convective cooling of earth's interior: the continental lithosphere is made largely of rocks such as granite that are less dense than the mantle. The continents thus stay afloat and remain at the surface, although they have drifted together and broken apart many times. By 80 million years ago, most of the continents we know today were isolated and had began moving toward their current positions.

The continents move about 5 to 10 cm per year and it takes millions of years to build a mountain range. Around 250 million years ago, when North America collided with Africa, the ensuing large-scale crustal shortening generated the Appalacian Mountains. Erosion has worn them down, but they were once comparable to the present-day Himalayas. The Himalayas, however, were formed only about 35 million years ago, when India plowed into southern

Asia. The large-scale horizontal shortening that resulted, built up the highest mountains on earth.

Plates not only 'collide' with one another, they can also slip *past each other along a* fault *which is a thin boundary layer of crushed rock between the two moving blocks. Here, stresses build up, the rock breaks. This sudden slip shakes the earth, causing an* earthquake*. Thus, for example, the result of the slip of the Pacific plate and the North American plate created the San Andreas fault in California. It slices through California more than 1100 km and its activity accounts for more than 10,000 earthquakes (most of them minor) each year.*

Deep earthquakes *are associated with the subduction of oceanic plates. Some have been detected at depths exceeding 600 km, indicating how far the plates may plunge into the mantle.*

Earthquakes may release enormous amounts of energy that travel along the surface and through the interior to great distances. Much of what we know about the earth's interior, we have learned by studying how these waves travel through the earth.

During earthquake episodes, plate-motions release stored elastic energy in the earth's crust and upper mantle up to depths of 700 km. Most of this energy is expended, however, in the upper 70 km or so.

Table 5.11: 'Killer earthquakes' during the 2^{nd} millennium CE

(A) 1000–1800 CE *(21,000 or more deaths)*

Date	*Location*	M_s	*Comments*	*Approximate death-toll**
1038	*China: Shensi*			*33,000*
1042, Aug. 21	*Syria: Palmyra*	*7.2*	*City ruined; felt in Iran & Egypt*	*50,000*
1042, Nov. 04	*Iran: Tabriz*	*7.6*	*City ruined*	*40,000*
1057	*China: Chihli*			*25,000*
1139, Oct. 12	*Syria*	*7.4*	*Aleppo destroyed*	*230,000*
1157, Aug. 15	*Northern Dead-Sea Rift*	*7.3*	*Destruction in Syria & Lebanon*	*80,000*
1202, May 30	*Northern Israel*	*7.5*	*Macroseismic area engulfs the entire Middle-East; damage in Israel, Lebanon and Syria*	*30,000*
1268	*Turkey: Silicia*			*60,000*
1290, Sept. 27	*China: Hopeh*	*6.7*		*100,000*
1293, May 20	*Japan: Kamakura*			*22,000*
1444	*Turkey*			*30,000*
1455, Dec. 05	*Italy: Naples*	*7.5*		*40,000*
1458	*Turkey: Erzincan*	*7.6*		*32,000*
1481, Mar. 04	*Turkey: Erzurum*	*7.7*		*30,000*
1481	*Egypt*		*Felt in Israel, Syria & Arabia*	*30,000*
1498	*Japan: Totomi*	*8.0*		*41,000*
1522	*Iran: Tabriz*			*70,000*
1531, Jan. 26	*Portugal: Lisbon*			*30,000*

Table 5.11: (Cont.)

Date	*Location*	M_s	*Comments*	*Approximate death-toll*
1556, Jan. 23	*China: Shensi*		*Greatest recorded number of people ever killed by an earthquake*	*830,000*
1626, July 30	*Italy: Naples*			*70,000*
1641, Feb. 05	*Iran: Tabriz*			*30,000*
1662	*China*			*300,000*
1667, Nov.	*Caucasus: Azerbaijan*	*6.9*		*80,000*
1668, July 25	*China: Shandong*			*50,000*
1693, Jan. 11	*Italy: Naples & Catania*			*153,000*
1695, May 18	*China*			*30,000*
1703, Jan. 14	*Italy: Norcia, Aguila*			*40,000*
1703, Dec. 31	*Japan: Tokyo*			*200,000*
1707, Oct. 28	*Japan*			*30,000*
1718, June 19	*China*			*43,000*
1727, Nov. 18	*Iran: Tabriz*			*77,000*
1730, Dec. 30	*Japan: Hokkaido*			*137,000*
1731, Nov. 30	*China: Beijing*		*City destroyed*	*100,000*
1739, Jan. 03	*China*			*50,000*
1754, Sept.	*Egypt: Cairo*		*Half of city dwellings destroyed*	*40,000*
1755, June 07	*Iran: Tabriz*			*50,000*
1755, Nov. 01	*Portugal: Lisbon*	*8.0*	*Great tsunami (17 m); epicenter ca 100 km offshore Liston*	*70,000*
1759, Nov. 25	*Lebanon: Baalbec*	*7.4*		*30,000*
1773, June 07	*Guatemala*		*Santiago destroyed*	*58,000*

Table 5.11: (Cont.)

Date	*Location*	M_s	*Comments*	*Approximate death-toll*
1778–1780	*Iran: Tabriz*	*7.7*	*A series of earthquakes*	*100,000*
1783, Feb. 05	*Italy: Calabria*			*30,000*
1797, Feb. 04	*Ecuador: Quito*			*40,000*
1797, May 26	*Italy: Calabria*			*50,000*

* *Magnitudes and death-tolls in this table were evaluated by Prof.* ***Markus Båth*** *(1916–2000; Uppsala, Sweden).*

(B) 1800–2002 CE *(1500 or more deaths)*

1805, July 26	*Italy: Napoli*			*6500*
1810, Feb. 16	*Crete: Iraklion*		*Tsunami*	*2000*
1812, Mar. 26	*Venezuela Caracas*	*7.7*	*Heavy destruction in Caracas*	*26,000*
1815, Oct. 23	*China*			*13,000*
1815, Nov. 27	*Indonesia: Bali*			*10,250*
1819, June 16	*India: Kutch*	*8.05*	*Great damage; ca 10,000 houses destroyed*	*1540*
1822, Aug. 13	*Turkey: Antioch, Aleppo*	*7.2*	*Antioch destroyed*	*20,000*
1825, Mar. 02	*Algeria: Blida*			*7000*
1828, Dec. 28	*Japan: Honshu*	*8.0*		*30,000*
1829, Mar. 16	*Spain*	*7.0*		*6000*
1830 May 26	*China: Guangzho*			*6000*
1830, June 12	*China: Hebei*			*7000*
1831, Aug. 11	*Antilles: Barbados*			*3000*
1833, Sept. 06	*China*			*6700*
1835, Feb. 20	*Chile: Conception*	*8.0*	*Tsunami observed by Charles Darwin during cruise of the "Beagle"*	*thousands*

Table 5.11: (Cont.)

Date	*Location*	M_s	*Comments*	*Approximate death-toll*
1837, Jan. 01	*Israel: Safed*	*6.8*	*Destruction in Northern Israel*	*4100*
1840, July 02	*Armenia*	*6.8*		*2100*
1842, May 07	*Haiti: Dominican Republic,*		*Puerto Rico & Jamaica*	*5000*
1843, Feb. 08	*Antilles*			*5000*
1844, May 10	*Iran*			*1500*
1847, May 08	*Japan: Zenkoji*	*7.4*		*34,000*
1850, Sept. 12	*China*			*20,650*
1851, Aug. 14	*Italy: Melfi*			*14,000*
1851, Dec. 12	*Albania*			*2000*
1852, Feb. 22	*Iran*			*2000*
1853, May 04	*Iran: Shiraz*			*12,000*
1854, Dec. 23	*Japan: Honshu, Nankaido*	*8.0*		*31,000*
1855, Feb. 28	*Turkey: Tayabas*	*6.7*		*1900*
1855, Nov. 11	*Japan: Tokyo*	*6.9*		*7000*
1857, Dec. 16	*Italy: Napoli*	*6.5*		*12,000*
1859, Mar. 22	*Ecuador: Quito*			*5000*
1859, June 02	*Turkey: Erzurum*	*6.1*		*15,000*
1861, Mar. 21	*Argentina: Mendoza*			*18,000*
1868, Aug. 13	*Chile, Peru, Ecuador*	*8.3*	*Great damage at Arequipa, Quito, Ibarra, Esmeraldas*	*85,000*
1870, Apr. 11	*China*	*6.7*		*2300*
1872, Jan. 06	*Iran*	*6.3*		*4000*
1872, Apr. 03	*Turkey: Antioch*	*7.2*	*City destroyed*	*1800*
1875, May 18	*Colombia, Venezuela*	*7.5*		*16,000*
1879, Mar. 22	*Iran: Ardabil*			*3200*
1879, July 01	*China: Kansu*			*10,400*
1881, Apr. 03	*Greece: Aegean Sea*	*7.3*		*7880*

Table 5.11: (Cont.)

Date	*Location*	M_s	*Comments*	*Approximate death-toll*
1883, July 28	*Italy: Casamicciola*			*2300*
1885, May 30	*India: Srinagar*			*3000*
1887, Dec. 16	*China*	*6.8*		*2000*
1891, Oct. 27	*Japan: Mino-Owari*	*8.0*	*Great damage*	*7270*
1893, Nov. 17	*Iran: Khorasan*	*6.6*		*18,000*
1895, Jan. 17	*Iran: Quchan*			*11,000*
1896, June 15	*Japan: Riku-Ugo*	*7.6*	*Tsunami, Sanriku leveled*	*27,120*
1897, June 12	*India: Assam*	*8.2*	*Flooding, landslides; Shillong ruined*	*1540*
1898, Nov. 17	*Turkmenistan*			*18,000*
1899, Sept. 29	*Indonesia*	*7.8*		*3860*
1902, Apr. 19	*Guatemala*	*7.4*		*2000*
1902, Aug. 22	*China: Tien-Shan*	*7.6*		*2500*
1902, Dec. 16	*Kirgistan, Turkestan*	*6.4*		*4500*
1903, Apr. 28	*Turkey: Malazgirt*			*3560*
1905, Apr. 04	*India: Kangara*	*7.4*		*20,000*
1905, Sept. 08	*Italy: Calabria*			*2500*
1905, Nov. 08	*Greece: Chalkidiki*	*6.8*		*2000*
1906, Aug. 17	*Chile*	*8.1*		*1500*
1907, Oct. 21	*Tajikistan, Pamir*	*7.1*		*12,000*
1908, Dec. 28	*Italy: Messina, Reggio*	*7.0*	*Cities leveled; tsunami (11 m)*	*160,000*
1909, Jan. 23	*Iran: Silakor*			*5500*
1912, Aug. 09	*Marmara Sea, NAFS*	*7.5*		*1950*
1915, Jan. 13	*Italy: Arezzano*	*6.8*		*29,980*
1917, Jan. 21	*Indonesia: Bali*			*15,000*
1917, July 30	*China: Sechuan*	*6.4*		*1800*
1918, Feb. 13	*China: Guandong*	*7.2*		*10,000*

Table 5.11: (Cont.)

Date	*Location*	M_s	*Comments*	*Approximate death-toll*
1920, Jan. 04	*Mexico: Veracruz*			*4000*
1920, Dec. 16	*China: Kansu, Shensi*	*8.1*		*200,000*
1923, May 25	*Iran: Torbat-Haklari*	*5.8*		*5000*
1923, Sept. 01	*Japan: Kwanto, Tokyo, Yokohama*	*8.0*	*Destruction in Tokyo and Yokohama*	*143,000*
1925, Mar. 16	*China: Yunnan*	*6.9*	*Talifu destroyed*	*5000*
1927, Mar. 07	*Japan: Tanso*	*7.4*		*3020*
1927, May 22	*China: Nan-Shan*	*7.7*	*Extreme damage*	*320,900*
1929, May 01	*Iran: Shirwan*	*6.8*		*12,860*
1930, May 06	*Iran*			*2500*
1931, Mar. 31	*Nicaragua: Managua*	*6.0*		*2450*
1931, Apr. 27	*Iran: Zangezur*			*2890*
1932, Dec. 25	*China: Kansu*	*7.5*		*70,000*
1933, Mar. 02	*Japan: Sanriku*	*8.2*	*Tsunami*	*3000*
1933, Aug. 25	*China: Sechuan*	*7.3*		*10,000*
1934, Jan 15	*India: Bihar-Nepal*	*8.1*		*10,700*
1935, Apr. 20	*Taiwan*	*6.9*		*3270*
1935, May 30	*Pakistan: Quetta*		*City destroyed*	*30,000*
1935, July 16	*Taiwan*	*6.5*		*2740*
1939, Jan. 25	*Chile: Chillan*	*7.6*	*Damage in Conception and Chillan*	*28,000*
1939, Dec. 26	*Turkey: Erzincan*	*8.0*	*Macroseismic area: 800,000* km^2	*32,700*
1942, Nov. 26	*Turkey: Havza*			*4000*
1942, Dec. 20	*Turkey: Niksar*	*7.1*		*3000*
1943, Nov. 26	*Turkey: NAFS*	*7.4*		*4000*
1944, Jan. 15	*Argentina: San-Juan*	*7.0*		*8000*
1944, Feb. 01	*Turkey*	*7.2*		*5000*

Table 5.11: (Cont.)

Date	*Location*	M_s	*Comments*	*Approximate death-toll*
1945, Jan. 12	*Japan: Mikawa*			*1900*
1945, Nov. 27	*Pakistan: Makran*	*7.8*		*4000*
1946, Nov. 10	*Peru: Ancash*	*7.1*	*Great destruction; landslides;* $L = 100$ *km,* $U = 5$ *m*	*1500*
1948, June 28	*Japan: Fukui*	*7.1*		*5390*
1948, Oct. 05	*Turkmenia: Kopet Dag*	*7.1*		*19,800*
1949, Aug. 05	*Ecuador*			*6000*
1950, Aug. 15	*India: Assam*	*8.3*	*Landslides, floods; seiches in Norway*	*1530*
1957, Dec. 13	*Iran*			*2000*
1960, Feb. 29	*Morocco: Agadir*	*5.8*		*13,100*
1960, May 22	*Chile*	*8.5*	*Tsunamis, floods, volcanic activity. Visible fault rupture along 800 km; active area 256,000* km^2*; 199 killed in Hawaii by tsunami*	*5900*
1962, Sept. 01	*Iran: Qazvin*	*6.9*	*Landslides, rockfalls;* $L = 100$ *km, 21,309 houses in 324 villages destroyed*	*12,230*
1966, Aug. 19	*Turkey: Varto*	*6.9*		*2960*
1968, Aug. 31	*Iran: Dasht-E-Baiaz*	*7.1*	$L = 27$ *km,* $U = 4$ *m*	*20,000*
1969, July 25	*China: Hong-Kong*	*5.9*		*3000*

Table 5.11: (Cont.)

Date	*Location*	M_s	*Comments*	*Approximate death-toll*
1970, May 31	*Peru: Ancash*	*7.6*	*Landslides, floods, avalanches; 1.7 million homeless; great damage; 500 million dollars damages*	*66,790*
1972, Apr. 10	*Iran: Zagros Mts.*		*Landslides; major damage*	*5370*
1972, Dec. 23	*Nicaragua: Managua*	*6.2*	*80% of Managua ruined; fire; 800 million dollars damage*	*10,000*
1974, May 10	*China: Yunnan, Sechuan*	*6.8*		*20,000*
1974, Dec. 28	*Pakistan: Polas, Patan*	*6.4*		*5300*
1975, Feb. 04	*China: Yingtao*	*7.2*		*10,000*
1975, Sept. 06	*Turkey: Lice*	*6.8*	*Lice destroyed*	*2700*
1976, Feb. 04	*Guatemala: Montagua*	*7.5*	*Guatemala city damaged; landslides;* $L = 200$ km*; 1.3 billion dollar property damage*	*22,830*
1976, June 25	*Indonesia: Irian Jaya*		*Flooding, landslides*	*6000*
1976, July 27	*China: Tangshan*	*7.8*	*City leveled; great economic damage*	*655,000*
1976, Aug. 16	*Philippines: Mindanao*	*7.8*	*Tsunami*	*8000*
1976, Oct. 29	*Indonesia: Irian Jaya*			*6000*
1976, Nov. 24	*Turkey: Muradiye*	*7.1*		*3620*

Table 5.11: (Cont.)

Date	*Location*	M_s	*Comments*	*Approximate death-toll*
1978, Sept. 16	*Iran: Tabas*	*7.2*		*25,000*
1980, Oct. 10	*Algeria: El Asnam*	*7.1*		*11,000*
1980, Nov. 23	*Italy*	*6.8*		*4580*
1981, June 11	*Iran: Golbart*	*6.8*		*3000*
1981, July 28	*Iran: Kerman*	*7.1*		*8000*
1982, Dec. 13	*Yemen: Dhamar*	*6.0*	*300 villages destroyed*	*2800*
1985, Sept. 19	*Mexico: Michoaran, Mexico City*	*8.0*	*30,000 injured; tsunami; 3 billion dollars damages in Mexico City*	*9500*
1988, Dec. 07	*Colombia: Armenia*	*6.8*		*25,000*
1990, June 20	*Iran: Qazvin*			*50,000*
1990, July 16	*Philippines: Luzon, Baguio*			*1620*
1991, Oct. 19	*India*			*2000*
1992, Dec. 12	*Indonesia: Flores*	*7.5*	*Tsunami ran 300 m inland; 25 m waves*	*2500*
1993, Sept. 29	*India*	*6.3*		*9740*
1995, June 16	*Japan: Honshu*	*6.9*	*Landslides*	*5502*
1995, May 27	*Sakhalin Is.*	*7.5*		*1980*
1997, May 10	*Iran*	*7.5*	*60,000 homeless; 4460 injured*	*1560*
1999, Jan. 25	*Western Colombia*	*6.0*	*City of Armenia badly damaged*	*2000*
1999, Aug. 17	*North-Western Turkey: Izmit*	*7.4*		*17,110*
1999, Sept. 20	*Taiwan; Nantou*	*7.6*		*2100*
2001, Jan. 26	*India; Gujarat*	*7.7*	*339,000 buildings destroyed*	*20,085*

Volcanoes

Volcanism occurs when magma (molten rock) beneath the surface of the earth breaks through the surface. A volcano is thus a hole in the earth's crust which serves as a vent for magma and gases from below the earth's surface. The volcano forms from a buildup of ash and lava around the hole. Energy is released during a volcanic eruption as heat, partly as explosive energy, and partly in earthquakes. One example of volcanic land-form is a stratovolcano (a cone shaped mountain built by alternating layers of lava and volcanic ash.) Volcanic activity reflects ways in which heat is lost from the interior; ocean-ridge volcanism *creates new oceanic lithosphere.* Convergence-zone volcanism *produces stratovolcanoes in plate margins.* Interplate volcanism *produces shield volcanoes and flood basalt provinces in the interior of plates.*

In convergence-zone volcanism, very steep stratovolcanoes are formed when magma rises through the continental crust. The lava usually contains large amounts of gases that are often released explosively, forming ash. Alternating eruptions of lava and ash build a cone-shaped mountain with steep slopes. In interplate volcanism, magma comes from hot spots *in the mantle. The basalt lava has only small amounts of gas and produces little ash. The relatively free-flowing lava spreads widely, building a broad, domed mountain with gentle slopes. Hot spots are also responsible for the* flood basalt *areas on the continents. Magma from hot spots fractured the continental crust and rose through it.*

About 10,000 volcanoes have erupted in the past 10,000 years, and are thus considered potentially active. Current activity is limited to 500–600 volcanoes, distributed nonuniformly on the plates. Most are found on plate boundaries where lithosphere is being created or destroyed. Many are on the margins of the Pacific plate, forming the so-called Ring of Fire. *Interplate volcanoes account for only a small number of the active volcanoes.*

Volcanism has contributed enormous amounts of water, carbon dioxide, and other gases to the atmosphere. Sunlight broke water molecules into its components, hydrogen and oxygen. Photosynthesis by plants removed carbon dioxide and added oxygen to the primitive atmosphere. Volcanism was also an important factor in the growth of continents. The overall beneficial aspects of volcanism can be summarized as follows:

- *Outgassing the earth to produce the atmosphere and hydrosphere.*
- *Renewal of soil.*
- *Production of rich ore deposits by hydro thermal processes.*

- *Enabling the tapping of geothermal energy.*

The potency of volcanoes is graded on a scale known as the Volcanic Explosivity Index (VEI). It is based on a number of parameters that can be observed during an eruption.

There are six distinguished volcano eruptions that merit special mention, because of the role they played in human history: (E = energy in units of one megaton of TNT)

1. *Yellowstone (USA) VEI = 8,* $E = 2 \times 10^6 MT$.
   The eruption cycle is ca 600,000 years. Last eruption occurred 640,000 years ago.

2. *Toba (Sumatra) VEI = 8,* $E = 6 \times 10^6 MT$.
   Last eruption occurred 74,000 years ago. Lake Toba was discovered in 1949. It is surrounded by a vast layer of ignimbrite rocks. Rhyolite ash, similar to that found around Toba, was discovered 3000 km away in India. Oceanographers discovered a vast dusting of Toba ash on the floor of the Eastern Indian Ocean and the Bay of Bengal. It is estimated that the Toba eruption was perhaps the greatest since Yellowstone. The eruption caused a great collapse to occur, forming a caldera filled with water, creating a lake.

3. *Thera (Santorini) VEI = 6-7,* $E = 10^3 MT$.
   This titanic volcanic explosion (May 1627 BCE) contributed to the downfall of the Minoan in the Eastern Mediterranean. Its echoes resound in the literature of the ancient Hebrews and Greeks even 1200 years (50 generations!) after the event.

   This event changed the whole course of civilization in the Eastern Mediterranean. The Achaean civilization of the Greek mainland took over the Minoan culture and power of Crete. Casualties were estimated at 100,000, believed to be caused by a mega-tsunami. It was the greatest volcanic tsunami to have occurred within the last few thousand years. The ash fall-out due to this event may have been the cause of the great total Eastern Mediterranean famine in the days of Jacob and Joseph, described so vividly in the Old Testament Book of Genesis.

4. *Vesuvius VEI = 6, Aug 24, 79 CE.*
   Mount Vesuvius is but 17,000 years old. It has been intimately involved with mankind for at least 3000 years and is surrounded by the largest population – 2 million people – ever to dwell in the immediate vicinity

of an active crater. No other volcano played so definitive role in history, or has so dramatized the perils that are visited by those who choose to live among that flanks of an eruptive peak.

This mountain was so quiescent during ancient history that it was totally ignored by the gods of Greece and Rome and, again, by chroniclers of superstition during the Dark Ages. It did not achieve the full measure of world fame until archaeologists uncovered the lost cities of Pompeii, Herculaneum and Stabai during the 18th *and* 19th *centuries.*

5. *Tambora* $VEI = 7$, $E = 2 \times 10^4 MT$, *April 05, 1815*
   Perhaps the greatest and deadliest eruption in recent times was that of volcano Tambora on Lesser Sunda Is. In parts of Java, the blanket of ash produced a daytime darkness *more profound then the blanket of nights (Exodus* **10**, *23). A total volume of 150 km*^{3} *(*1.7×10^6 *tons) of ash were ejected into the atmosphere and the released energy reached a value of* 8.4×10^{26} *erg. It affected world climate, causing extreme cold winters in many parts of the world for 3 years (1816 — 'the year without a summer').*

6. *Krakatoa* $VEI = 6$, $E = 200MT$.
   In 1883 (Aug. 26 and 27) the volcano island of Krakatoa in the Sunda Straits, was blown to pieces. A volume of 20 km^3 *of material was emitted during a paroxysmal eruption, unleashing an energy of about* 10^{25} *erg. Material was shot 80 km high (above the ozone layer). Its global effects lasted for 4 years.*

   Certain aspects of the destruction mechanisms of volcano eruptions were clarified, on close scrutiny, only in the 20^{th} *century. One of these is the fire-storm known as* Nueé ardente *that caused the death of some 30,000 people on May 08, 1902 at St. Pierre, near Mt. Peleé, Martinique: A hot gas hurricane avalanche swept the city at a speed of 100 km/hr. The hot gas was found to be composed of* CO_2*, glass and dust at ambient temperature of about* $800^\circ C$*.*

Table 5.12: 'Killer volcanic eruptions' in the past two millennia

Date, CE	Volcano	Comments	Death-toll and cause
50	*Ambrym, New Hebrides*	*VEI=6.7*	
65	*Bona Churchill, Alaska*	*VEI=6*	
79, Aug. 24	*Vesuvius, Italy*	*Pompei rediscovered (1595)*	*ca 3360; Ash flow and falls*
155	*Ksudach, Kamchatka*	*VEI=6*	
186	*Tampo, New Zealand*	*Mountain exploded*	
472	*Vesuvius, Italy*		
536	*Rabaul, New Britain*	*VEI=6*	
626	*Mediterranean*		
1006	*Merapi, Indonesia*	*VEI=7*	
1054	*Baittoushan, China*		
1104	*Hekla, Iceland*	*VEI=5*	
1169	*Etna, Sicily*	*50 nearby cities destroyed*	*ca 15,000*
1452	*Kuwae, New Hebrides*	*VEI=6*	
1471	*Sakurajima, Japan*		
1477	*Bardarbunga, Iceland*		
1536	*Etna, Sicily*		*Thousands*
1540	*St. Helens, North America*		
1580	*Billy Mitchell, SW Pacific*		

Table 5.12: (Cont.)

Date, CE	**Volcano**	**Comments**	**Death-toll and cause**
1586	*Kelut, Java*		*ca 10,000; Lahar*
1591	*Taal, Philippines*		*Thousands*
1593	*Raung, Java*		
1600, Feb. 19–Mar. 05	*Huaynaputina, Southern Peru*	*One of the largest eruptions in historic times. Global weather changes*	
1616	*Mayon, Philippines*		*Thousands*
1631, Dec. 13	*Vesuvius, Italy*		*ca 18,000; mud and lava flows*
1638	*Raung, Java*		*ca 1500; Lahar*
1640	*Kamagatake, Japan*		*ca 700; tsunami*
1641, Jan. 04	*Parker, Philippines*		
1660	*Long Island, New Guinea*		
1669, Mar. 11–July 15	*Etna, Sicily*	*37.7°N, 15.0°E; city of Catania partially destroyed*	*ca 10,000; lava flows*
1672	*Merapi, Indonesia*	*7.54°N, 110.44°E*	*ca 300; pyroclastic flow*
1673	*Gamkonora, Indonesia*		
1711	*Awu, Indonesia*		*ca 3200; debris flow*

Table 5.12: (Cont.)

Date, CE	**Volcano**	**Comments**	**Death-toll and cause**
1739, Aug.	*Tarumai, Japan*		
1741	*Oshima, Japan*		*1480; tsunami*
1741	*Cotopaxi, Ecuador*	*World's* 2^{nd} *highest volcano (5897 m); villages below destroyed*	*ca 1000; avalanche of lava and ice*
1755	*Etna, Sicily*	*Conjunction with earthquake*	*ca 36,000*
1760	*Maklan, Indonesia*		*ca 1000; Lahar*
1766, Oct. 23–30	*Mayon, Philippines*		*ca 2000; floods*
1772	*Papandijan, Java*		*ca 3000; ash flows*
1783	*Asama, Japan*		*ca 1400; ash and mud flows*
1783, Dec.	*Laki, Iceland*	*Fifth of local population perish*	*9340; starvation*
1792, Apr. 01	*Unzen, Japan*		*15,110; volcano's collapse; tsunami*
1793	*Miyi-Yama, Japan*		*ca 50,000*
1794	*Tunquraohua, Ecuador*		*ca 40,000*
1814	*Mayon, Philippines*		*ca 1200; pyroclastic flow*

Table 5.12: (Cont.)

Date, CE	**Volcano**	**Comments**	**Death-toll and cause**
1815, Apr. 05	*Tambora, Indonesia*	*Greatest known eruption; world climate affected; temperature dropped 2°–4°C in Europe and North America for 3 years*	*ca 92,000; tsunami and starvation*
1822, Oct. 08	*Galunggung, Indonesia*	*Over 1000 villages destroyed*	*ca 4000; pyroclastic flow*
1835, June 20	*Cosiguina, Nicaragua*		
1845	*Nevado del Ruiz, Colombia*		*ca 1000; mud flows*
1854, Feb. 17	*Sheveluch, Russia*		
1856	*Awu, Indonesia*		*ca 3000; pyroclastic flow*
1877	*Cotopaxi, Ecuador*	*Mud flow traveled 240 km*	*ca 1000; mud flows*
1883, Aug. 27	*Krakatau, Indonesia*		*36,420; tsunami*
1888, July 15	*Bandaisan, Japan*		*ca 400*
1888	*Ritter, Papua, New Guinea*		*ca 3000; tsunami*
1892	*Awu, Indonesia*		*1530; pyroclastic flow*
1902, May 07	*Soufriere, St. Vincent Is.,*	*100 km^2 of the island devastated*	*1680; hot mud and ash flows*

Table 5.12: (Cont.)

Date, CE	**Volcano**	**Comments**	**Death-toll and cause**
1902, May 08	*Mt. Peleé, Martinique*	*City of St. Pierre demolished*	*ca 29,000; ash flows*
1902, Oct. 24	*Santa Maria, Guatemala*		*ca 6000; tephra and malaria*
1911, Jan. 30	*Taal, Philippines*	*13 villages destroyed*	*1330; ash flows; tsunamis*
1912, June 06	*Katmai, Alaska*	*More than 16 km^2 of ash and pumice were ejected; explosion heard 1600 km away; darkness at noon 160 km away; ash flow, gases and acid rain; damage to vegetation 600 km away*	
1914	*White Is., New Zealand*		*11; hot mud flow*
1919	*Kelut, Java*		*ca 5500; water and mud flows*
1929	*Vesuvius, Italy*	*Destroyed nearby villages*	*lava flow*
1930	*Merapi, Indonesia*	*15 km^2 of land and a number of villages covered with flowing avalanche*	*ca 1300; pyroclastic flow*

Table 5.12: (Cont.)

Date, CE	**Volcano**	**Comments**	**Death-toll and cause**
1937	*Rabaul, Papua, New Guinea*		*ca 3000; pyroclastic flow; tephra; tsunamis; starvation*
1943, June 10	*Paricutin, Mexico*		*ca 3500*
1951	*Merapi, Indonesia*		*ca 1300*
1951, Jan. 18–21	*Lamington, Papua, New Guinea*	*230 km^2 covered with*	*ca 3000; ash flow glowing avalanches*
1951, Dec. 04	*Hibok-Hibok, Philippines*	*Red-hot avalanche of gas and dust*	*ca 500; ash flows and gases*
1953	*Ruapehu, New Zealand*	*Mud flow derailed the Wellington-Auckland express*	*Several*
1956, Mar.30	*Bezymianny, Kamchatka*	*Returned to activity in 1955*	
1960, May 21–30	*Chilean volcanoes*	*In connection with the great earthquake of May 22, 1960*	*ca 5700; tsunamis*
1963, Mar. 17–21	*Agung, Indonesia*	*Leaving 200,000 homeless; dust and ash produced red sunsets in US*	*ca 1100; lava flows*
1968, July 29	*Arenal, Costa Rica*		*ca 80; Nuée ardente*
1973, Jan. 23	*Helgafell, Iceland*	*After 7000 years of inactivity*	

Table 5.12: (Cont.)

Date, CE	**Volcano**	**Comments**	**Death-toll and cause**
1979, Feb. 20	*Sinila, Java*		*175; poison gases*
1980, May 18	*St. Helens, Washington, US*		*60; asphyxiation and blast*
1982	*El Chicon, Mexico*		*ca 2000; ash flows*
1985, Nov. 13	*Nevado del Ruiz, Colombia*		*ca 25,000; mud flows*
1986, Aug. 21	*Lake Nios, Cameroon*		*ca 1750; toxic gas*
1991	*Pinatubo, Philippines*		*ca 800; Roofs collapse and disease spreads.*

Tsunamis

"This is what the Lord says: 'See how the waters are rising in the north; they will become an overflowing torrent. They will overflow the land and everything in it, the towns and those who live in them. The people will cry out; all who dwell in the land will wail at the sound of the hoofs of galloping steeds, at the noise of enemy chariots and the rumble of their wheels. Fathers will not turn to help their children; their hands will hang limp. For the day has come to destroy all the Philistines and to cut off all survivors who could help Tyre and Sidon. The Lord is about to destroy the Philistines, the remnant from the coasts of Caphtor."

Jeremiah **47**, *2–4 (ca 625 BCE)*

"... who calls for the waters of the sea and pours them out over the face of the earth."

Amos **9**, *5–7 (ca 780 BCE)*

It speeds across the sea as fast as a jet airplane. On reaching land, it can suck all the water out of a harbor. Then the creature may grow more than 30 meters tall and flatten whole villages. This sea monster is a tsunami. They are the most destructive waves in the ocean. Tsunamis, often wrongly called tidal waves, are not caused by tides or even by the wind, but by underwater earthquakes, landslides, volcanic eruptions, or worst of all — an asteroid impact in the ocean. These disturbances cause the sea bed to move very quickly, which shifts a large amount of water and disrupts the sea surface. A train of waves is set in motion traveling away from the source of disturbance. The resulting long waves can be devastating to low-lying coastal areas.

About four out five tsunamis happen within the "Ring of Fire", a zone of frequent earthquakes and volcanic eruptions roughly matching the borders of the Pacific Ocean. Along the ring's edges, giant slabs of the earth's crust, called tectonic plates, grind together. Sometimes the plates get stuck, and pressure builds, causing the plates to suddenly come apart and slam into a new position. This jolt causes an earthquake. If an earthquake lifts or drops part of the ocean floor, the water above it starts moving, too, triggering a tsunami.

A tsunami can race across the ocean at 800 km an hour. Oddly, in deep water its waves are only a few feet high, but when the waves approach shore, they increase in energy and height. Often before a tsunami hits, water is sucked from harbors and beaches. People see the bare sea bottom littered with flopping fish and stranded boats. That is because waves are made out of crests, or high points, and troughs, or dips between crests. When a trough hits land first, the water level drops drastically. Usually another wave blasts ashore about 15 minutes later, then another and another, for two hours or more.

The damage caused by a tsunami is due not just by a heavy wall of water hitting things, but much more due to the solid debris carried by up the powerful, churning deep water wave as it hits the continental shelf — the solid debris rams and batters anything in its way.

For example, the tsunami from the 1960 Chile earthquake created a deep water wave of only 20 cm above sea level, but when it hit the shore it had risen to an average height of ten times its ocean size — over 2 meters, and in some places much higher (10 m).

It is not easy to determine the frequency of tsunamis in the world historically. Unusual debris has been found in high places in many parts of the world which could be the result of a tsunami, though it is not easy to determine what happened for sure and when, by the ordinary nature of the material. There has been little effort to date to systematically assess the frequency and nature of tsunamis well before the 20^{th} *century. Recorded history by civilizations along the Atlantic Ocean has not noted major tsunamis. Yet, it has been even* speculated *that the old stories of Atlantis, and even Noah's Arc, may have origins in some prehistoric tsunami which wiped out coastal settlements.*

Searches for tsunamis in the geological record have mostly been started only in the 1990's. Of particular interest are tsunami along the Atlantic coast, where earthquake-induced tsunami are rare, so that any detected tsunami would probably be due to an asteroid. The results of these ongoing efforts will shed some light on the frequency of asteroid hits into the oceans.

A mainstream scientific analysis currently estimates that a tsunami exceeding 100 meters in height along the entire coast probably occurs once every few thousand years, which slightly exceeds written history in most of these ocean coastal regions. Such a 100 meter tsunami would cause unprecedented damage to low lying areas all along the U.S. east coast, and may totally submerge vast areas in Europe such as in Holland and Denmark. A 100 meter tsunami would travel inland about 22 km and a 200 meter tsunami would travel inland about 55 km.

Tsunamis have killed more than 275,000 people since 1870. To save lives, scientists established the Pacific Tsunami Warning System, based in Hawaii, in the U.S.A. Its network of earthquake detectors and tide gauges detects earthquakes that may cause a tsunami.

Table 5.13 lists some of the deadliest tsunamis that plagued the coasts of the world since 1628 BCE.

Table 5.13: Some notable seismic and volcanic tsunamis of the world 1628 BCE–1964 CE

Date	Source region	Visual run-up (m)	Location of hit area	Comments
1628 BCE	*Thera eruption*	*35*	*Crete*	*Devastation of Mediterranean coasts*
1273	*Japan*			*30,000 killed*
1640	*Japan*		*Komogataka*	*1700 killed*
1741	*Japan*		*Oshima*	*1500 killed*
1755, Nov. 01	*Eastern Atlantic*	*5–10*	*Lisbon, Portugal*	*30,000 killed*
1792	*Unzen eruption*		*Japan*	*14,300 killed*
1815, Apr. 10	*Tambora eruption*		*Indonesia*	
1837, Nov. 07	*Chile*	*5*	*Hilo, Hawaii*	
1841, May 17	*Kamchatka*	*4*	*Hilo, Hawaii*	
1854, July 09	*Japan*		*Hilo, Hawaii*	*2400 drowned*
1868, Apr. 02	*Hawaii Island*	*2*	*Hilo, Hawaii*	
1868, Aug. 13	*Peru, Chile, Ecuador*	*12*	*Arica, Peru*	*Observed in New Zealand; damage in Hawaii*
1876	*Andaman Ils.*		*Bay of Bengal*	*215,000 killed*

Table 5.13: (Cont.)

Date	**Source region**	**Visual run-up (m)**	**Location of hit area**	**Comments**
1877, May 10	*Peru, Chile*	*2–6*	*Japan*	*Destructive in Iquique, Peru*
1883, Aug. 27	*Krakatau eruption*		*Java*	*Over 30,000 drowned*
1896, June 15	*Honshu*	*24*	*Riku-Ugo, Japan*	*About 27,000 drowned*
1923, Feb. 03	*Kamchatka*	*5*	*Waiakea, Hawaii*	
1933, Mar. 02	*Honshu*	*23*	*Sanriku, Japan*	*3000 killed*
1946, Apr. 01	*Aleutians*	*10*	*Wainaka, Hawaii*	
1952, Nov. 04	*Kamchatka*	*4*	*Hilo, Hawaii*	
1957, Mar. 09	*Aleutians*	*3*	*Hilo, Hawaii*	
1960, May 23	*Chile*	*14*	*Waiakea, Hawaii*	
1964, Mar. 28	*Alaska*	*6*	*Crescent City, California*	*120 deaths*

Extreme Universe

There are about a hundred billion stars in the Milky Way galaxy. One of these stars is the sun, which gives our earth the light and warmth that it needs to sustain life. However, even though the time scales are long by human standards, the galaxy is a dynamic, changing place, with new stars being born and old stars evolving and eventually dying. New stars are born out of collapsing clouds of gas and dust that float between the stars. Initially, these clouds are cold, but as they contract, they heat up. The temperature at the core of such as "protostar" eventually becomes so high that nuclear fusion reactions set in. Hydrogen atoms are fused into helium atoms, releasing energy which makes the newborn star shine. Once born, the life stories of the stars are different depending on how much material they started out with at their births.

Light-weight stars like our sun live a long and sedentary life. After 5 billion years, the Sun still generates light and heat from fusing hydrogen into helium. Later in its life it will become a red giant star, fusing helium into carbon. The higher temperatures in the core will cause the sun to puff up to 100 or more times its current size, engulfing the earth; but that will not happen for another 5 billion years! Finally, the nuclear reactions in the sun will run down and it will become a white dwarf star. Without a power source at its center, the sun will slowly cool and grow dimmer and dimmer. There are many stars like our sun in the galaxy.

Massive stars, such as Wolf-Rayet stars, are rare and differ from our sun in several important ways. These heavy-weight stars live relatively short but very intense lives. They have very strong stellar winds, about 10 billion times stronger than the solar wind. The solar wind can sometimes be noticed on earth when it creates the Northern lights or disrupts radio communications, but it has very little direct effect on the sun's evolution. The winds from massive stars are so substantial that matter from the stars is carried away by the wind; they are evaporating as we watch.

Massive stars burn much hotter than the sun and are strong sources of UV radiation. They are capable of fusing progressively heavier elements, such as carbon, nitrogen, oxygen and so on up to iron after which nuclear fusion becomes endothermic.[782] *If you have ever wondered where all the chemical elements came from, the answer is that nearly all of them were made inside*

[782] The elements heavier than iron are thought to have been created in stellar explosions, via endothermic nuclear reactions.

of massive stars. The very material in our bodies was once inside of massive stars, with the exception of hydrogen. We are all made of stardust!

The strong stellar winds of massive stars carry away so much material that they peel away layer after layer from the star like the different layers of an onion. Different chemical abundances become exposed at the surface of the star, allowing us to study the material that was once inside it. At the ends of their lives, massive stars explode as supernovas.

Thus, when massive stars die, they scatter their ashes back out into the galaxy. This material mixes with gas and dust in interstellar space and provides the building materials out of which new and different stars, the next generation of stars, may form.

During the past 70 years (1933–2003) great advances were made in both physics and technology: the advent of relativistic cosmology, astrophysics and astrochemistry, plasma physics and magnetohydrodynamics, quantum physics, nuclear and elementary particle physics, solid state physics, rocket and satellite technology, communication and guidance technology, nuclear power technology and computer technology – all gave us new tools and capabilities to probe deep space and monitor a host of phenomena and events extending to the edge of the observable universe. Data associated with the birth and death of stars are unfolding daily before the 'eyes' of our telescopes over the entire range of the electromagnetic spectrum[783]*, enabling us to understand and reconstruct the complicated physical process that take place at the core of galaxies, in exploding stars, and in previous cosmological epochs.*

Among the bizarre astronomical objects that were thrusted upon astronomers, astrophysicists and cosmologists in the course of the 20th century are: neutron-stars, pulsars, black holes, white dwarfs, accretion discs, supernovae, hypernovae, quasars *and* gamma-ray bursters. *Let us briefly describe these in chronological order:*

Supernovae

Stars which are five times or more massive than our sun end their lives in a most spectacular way; they go supernova. A supernova explosion will occur *when there is no longer enough fuel for the* fusion *processes in the core of the*

[783] As well as though the new window of *neutrino astronomy* and the imminent one of *gravitational-wave astronomy.*

star to create an outward pressure which balances the inward gravitational *pull of the star's great mass.*

First, the star will swell into a red supergiant, at least on the outside. On the inside, the core yields to gravity and begins shrinking. As it shrinks, it grows hotter and denser. A new series of nuclear reactions begin to occur, temporarily halting the collapse of the core.

When the core contains essentially just iron, it has nothing left to fuse (because of iron's stable nuclear structure, it cannot fuse into heavier elements without a net intake of energy). Fusion in the core ceases. In less than a second*, the star begins the final phase of* gravitational collapse*. The core temperature rises to over 100 billion degrees as the iron atoms are crushed together. The repulsive force between the nuclei overcomes the force of gravity, and the core recoils out from the heart of the star in an explosive* shock wave*. As the shock encounters material in the star's outer layers, the latter is heated, fusing to form new elements and radioactive isotopes. The shock then propels the* matter *out into space. The material that is exploded away from the star is now known as a supernova remnant.*

The hot material, the radioactive isotopes, the free plasma *moving in the strong* magnetic field *of the* neutron star *— all of these produce X-rays and* gamma-rays.

All that remains of the original star is a small, super-dense core composed almost entirely of a degenerate "Fermi sea" of neutrons *–a neutron star; the electrons and protons of the core were squeezed together to form neutrons and* neutrinos*, the latter escaping out to interstellar space at essentially light-speed. Or, if the original star was very massive indeed (say 15 or more times the mass of our sun) even the neutrons cannot survive the core collapse, and a* black hole *forms.*

Another type of supernova involves the sudden explosion of a white dwarf star in a binary star system. A white dwarf is the endpoint for stars of up to about 5 times that of the sun. The remaining white dwarf has a mass of about 1.4 times the mass of the sun, and is about the size of the earth.

A white dwarf star in a binary star system will draw material off its companion star if they are close to each other. This is due to the strong gravitational pull of an object as dense as a white dwarf.

Should the infalling matter from the companion star cause the white dwarf to exceed a mass of 1.4 times that of the sun (a mass called the Chandrasekhar limit *after the scientist who discovered it) the white dwarf will have enough mass to collapse and restart the fusion process. The oxygen and carbon nuclei making up the star begin to fuse uncontrollably, resulting in a thermonuclear* *detonation of the entire star. Nothing is left behind, except a cloud of whatever elements were left over from the white dwarf or forged in the supernova blast. Among the new elements is nickel, which then undergoes fission, liberating huge amounts of energy, including visible light.*

When a star goes supernova, it can be seen across the entire electromagnetic spectrum — including visible light, radio waves, X-rays and gamma-rays.

Supernovae are extremely important for understanding our galaxy. They heat up the interstellar medium. distribute heavy elements throughout the galaxy, and accelerate cosmic rays.[784]

The Crab Nebula in Taurus is the remnant of a supernova whose light reached earth in 1054 CE. The last one observed in our galaxy was seen by Kepler in 1604. They probably occur in the Milky way at a rate of about 1 in 500 years.

In 1987, a supernova explosion just outside our galaxy — ca 10^5 *LY away, in the Large Magellanic Cloud — was observed; besides its electromagnetic radiation, a synchronous burst of neutrinos was observed — the first such neutron star formation signal ever intercepted by man.*

Supernova in other galaxies are routinely observed.

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[784] Recently, the discovery of a mass-extinction event in the earth's ocean(s) [ca 2 Myr ago] was correlated with unusual iron-isotope abundances. It is thought to have been possibly caused by a supernova in our region of the Galaxy.

One must, therefore, add encounters with nearby supernovae to the list of potential cosmic dangers and catastrophes. However, during the 1 to 2 million years (Myr) that passed since the extinction event in question, the sun has moved thousands of light years (LY) and acquired new stellar neighbors (at our distance from the galaxy center, it takes the sun and nearby star clusters about 200 Myr to circumnavigate the galaxy's axis).

Consequently, astrophysicists now believe that the solar system has been surrounded by a 'protective bubble' of relative vacuum that protects the biosphere (in addition to the protective ozone layer and the earth's magnetic field).

More and more it seems that it is not accurate to say (as people like Carl Sagan have been saying) that we sit in a very typical region of the galaxy. Like much else (the fine-tuned constants of nature, the existence of the moon and key events that led to the evolution of humans) it seems that our galactic environment and history have been unusually friendly and "presaged" our existence, although that probably will not last.

Neutron Star

The central star which is left behind after the supernova explosion is so collapsed that it may become even denser than a White Dwarf. In such a case it is called a neutron star, because the electrons and protons which would normally be present in atoms are squashed together to form neutrons under enormous pressure. The diameter of a neutron star is about 10 km, and its density is that of the nucleus of an atom, around 10^{15} *times higher than the density of ordinary matter. Because of its small size and high density, a neutron star possesses a surface gravitational field of order* 10^{12} *times that of earth.*

Neutron stars may appear in solitary supernova remnants or in x-ray binaries with a normal star. When a neutron star is in an x-ray binary, astronomers are able to measure its mass from the orbital dynamics. From a number of such x-ray binaries, neutron stars have been found to have masses of about 1.4 times the mass of the sun. Astronomers can often use this fact to determine whether an unknown object in an x-ray binary is a neutron star or a black hole, since black holes are more massive than neutron stars.

The energy released through a collapse forming a neutron star of mass M *and final radius* R *is (ignoring GTR corrections)*

$$E \approx \frac{3GM^2}{5R}.$$

With $G = 6.67 \times 10^{-8}$ *cgs,* $M = 1$ *solar mass* $= 2 \times 10^{33}$ *gram and* $R = 5$ *km, we obtain* $E \approx 3 \times 10^{53}$ *erg. About 99% of this energy is emitted as neutrinos in the first few seconds of the collapse, and the observation of the expected fraction of them from the 1987 supernova SN1987A helped confirm the theory of gravitational collapse and pulsar/neutron-star formation. The rest appears as visible and UV light, radio waves, X-rays and gamma rays from a supernova, or is expended in the collapsed star's vicinity (spin kinetic energy, magnetic fields, plasma processes, nebula formation and excitation, etc.).*

Black Holes

It is believed that the mass of a neutron star cannot exceed 2 solar masses.

Astrophysicists have calculated, using GTR, that a neutron star any heavier than this will be crushed into an ever-decreasing volume by its own gravity.

As the star shrinks, the surface gravitational field strength ($g = GM/r^2$) continues to increase and so the escape velocity at its surface ($v = \sqrt{2GM/r}$) gets larger (these formulae ignore GTR effects). Eventually the escape velocity equals the speed of light. This has the intriguing consequence that nothing whatever can escape from the collapsed star by any means. Such a star is therefore totally invisible and is known as a black hole. *It has been suggested that a few binary star systems contain black holes; these particular systems are very powerful X-ray sources.*

Even back in Newton's time, scientists speculated that such objects could exist, even though we now know they are more accurately described using Einstein's General Theory of Relativity. According to GTR, black holes are fascinating objects where space and time become so warped that time practically stops in the vicinity of the "event horizon" of a black hole (as viewed by external observers), and even stranger things happen inside *that horizon.*

There is a great deal of observational evidence for the existence of three types of black holes; those with masses of a typical star, *those* with masses of a typical galaxy, *and those at* centers *of galaxies (of order* 10^6–10^7 *solar masses).*

The first type have measured masses ranging from 4 to 15 suns, and are believed to be formed during supernova explosions. *The after-effects are observed in some* X-ray binaries *known as black hole candidates.*

On the other extreme, galaxy-mass black holes are found in Active Galactic Nuclei *(AGN). These are thought to have the mass of about 10 to 100 billion suns. The mass of one of these* supermassive black holes *has recently been measured using radio astronomy.* X-ray *observations of iron in* accretion discs *may actually be showing the effects of such a massive black hole as well.*

X-ray Binaries

Binary star *systems contain two stars that* orbit *around their common center of* mass. *Many of the stars in our* galaxy *are part of* binary systems.

A special class of binary stars *is that of* X-ray *binaries, so called because they emit X rays.* X-ray *binaries are made up of a normal star and a collapsed star (a* white dwarf, neutron star, or black hole). *These pairs of stars produce* X rays *if the stars are close enough together such that material is pulled off the normal star by the* gravity *of the dense, collapsed star. The X rays are* *emitted from the area around the collapsed star where the material that is falling toward it is heated to very high temperatures (over a million degrees!).*

Pulsars

Pulsars are rotating neutron stars. They were first discovered (1967) as radio sources that blink on and off at a certain constant frequency.

The brightest pulsars are observable at almost every wavelength of light.

Pulsars are spinning neutron stars that have jets of particles moving almost at the speed of light streaming out their two magnetic poles. These jets produce very powerful beacons of EM radiation.

For a similar reason that "true north" and "magnetic north" are different on earth, the magnetic and rotational axes of a pulsar are also misaligned. Therefore, the beam of light from the jet sweeps around as the pulsar rotates, just as the spotlight in a lighthouse does.

Wolf-Rayet stars (WR)

Observed Wolf-Rayet stars form a rare group of $\approx$ 200 *supergiants with spectacular emission line spectra. They have exceptional surface temperatures approaching, in some cases, 100,000K, and about 50 solar masses. Although very brilliant, they are all so remote that a telescope is needed to view them. They suffer from a high mass loss through a turbulent atmosphere and show very strong, broad emission lines of ionized* He *and* O *in addition to nitrogen [WN type] and carbon [WC type] respectively. The Doppler broadening of the lines is only evident at higher resolution.*

These lines rise as bright 'peaks' above the star's background spectral continuum. Some of these stars are binaries and the relative intensity of the emission lines is affected by the continuum contribution of the companion star. A concentration of bright WR stars in Cygnus makes it the best site for Northern Hemisphere observers

Wolf-Rayet stars are linked to hypernovae, which in turn are associated with gamma-ray bursters.

Quasars, Active galaxies and double radio sources

A quasar (or quasistellar object) is an object in the sky that looks like a star but has a huge redshift corresponding (by the Hubble law) to a very great distance from the earth.

To be seen from earth, a quasar must be very luminous – typically about 100 times brighter than an ordinary bright galaxy.

Relatively rapid fluctuations in the brightnesses of quasars indicate that they are unlikely to be much larger in size than the diameter of our solar system.[785]

An active galaxy is an extremely luminous galaxy that has one or more unusual features: an unusually bright, star-like nucleus; strong emission lines in its spectrum; extreme variations in luminosity; or jets or beams of radiation emanating from its core.

Most double radio sources seem to have an active galaxy located between the two radio lobes that distinguish this type of radio source.

The strong energy emission from quasars, active galaxies, and double radio sources may be produced as matter falls toward a supermassive black hole at the center of the object.

Gamma-Ray Burster stars (GRB)

Short-lived bursts of gamma-ray photons are associated with a special type of supernovae — the explosions marking the death of especially massive stars ("Gamma-Ray Bursters", or GRBs).

Lasting anywhere from a few milliseconds to several minutes, gamma-ray bursts shine hundreds of times brighter than a typical supernova making them briefly the brightest source of cosmic gamma-ray photons in the observable

785 This is so because the time patterns of brightness variations at two quasar points, a distance r apart, are unlikely to vary in phase unless they have a common cause. This in turn makes it likely that $r \lesssim ct$, with c the speed of light and t the typical fluctuation timescale.
This line of argument (which is admittedly model dependent) depends on the *local causality* principle of STR.

universe. GRBs are detected roughly once per day from wholly random directions of the sky.

It is highly likely that GRB are not located within our galaxy, but are at cosmological distances.

This makes them extremely interesting, because for them to be seen at such large distances they must correspond to events in which as much as 100 times the energy of a supernova is being liberated in a short period in the form of gamma rays. Furthermore, the mechanism producing the gamma rays must be such as to allow the gamma rays to escape without too much interaction with surrounding matter, because that interaction would convert the gamma rays to radiation of longer wavelength.

Although the exact picture has not been worked out, astronomers think the gamma-ray photons are probably produced inside the star. The explosion originates at the center of these massive stars. While a black hole forms from the collapsing core, this explosion sends a blast wave moving through the star close to the speed of light. The gamma rays are created when the blast wave collides with stellar material still inside the star. These gamma rays burst out from the star's surface just ahead of the blast wave. Behind the gamma rays, the blast wave pushes the stellar material outward.

Erupting through the surface, the blast wave of stellar material sweeps through space at nearly the speed of light, colliding with intervening gas and dust to produce additional emission of photons. These emissions are believed responsible for the "afterglow" of progressively less energetic photons, starting with X-rays and then visible light and radio waves. (Whether additional gamma rays are also produced in this "afterglow" phase is still not settled, although some evidence indicates they are.)

The afterglow phase can last for days or even weeks. Under this model, we detect both the GBR and the afterglow when the earth happens to lie along or very near the axis of the blast. In general, there are many more GRBs than are detected simply because we are not favorably aligned to see them.

Observations have allowed a distance to be estimated to the gamma ray bursters because spectral lines and their Doppler shift have been observed in the transients after the burst. Assuming these transients to be near the center of the gamma ray burst and the Doppler shifts to be Hubble redshifts, these observations have almost conclusively shown that gamma ray bursters are at cosmological distances, rather than in the halo of the Milky Way galaxy.

The enormous amounts of energy implied by such large distances suggest a gravitational source. Two popular candidate mechanisms are the merger of two black holes or the merger of two neutron stars. However, although such events might yield the required energy, it is not clear that they can be made *consistent with all the observations. Thus, the source of gamma ray bursts remains one of the most important mysteries in modern cosmology.*

GRB may be linked to the mass extinction that occurred 444 million years ago at the end of the Ordovician period. Such a cosmic explosion, a few thousand light years away, could have altered the environment and extinguished much of life on earth. In general, a supernova explosion could flood our planet with deadly radiation if it happens within about 100 light years of us. (In absolute power output, supernovae are mere firecrackers in comparison with GRBs).

Water would protect marine organisms from the heat of a GRB, but not from its other effects. Its gamma-rays would convert some nitrogen and oxygen in the atmosphere into nitrogen dioxide, the brownish gas present in urban smog.

Nitrogen dioxide would filter out sunlight, turning the skies dark. The cooling effect could trigger an ice age – there is evidence of widespread glaciation 440 million years ago. Nitrogen oxides also cause acid rain and destroy the ozone layer, exposing earth to more of the sun's harmful ultraviolet rays.

Ultraviolet radiation can penetrate tens of meters of water, so it could harm marine organisms at these depths. Indeed shallow water dwelling species, or those that spend their early lives in shallow water, seem to have suffered more than deep species in the Ordovician extinction.

In short, a nearby GRB might first have showered harmful radiation onto the exposed face of the planet, killing more or less indiscriminately, and may then have exposed the other hemisphere to increased ultraviolet radiation, damaging marine life decreasingly with increasing depth.

Table 5.14: Evolution of a massive star

Stage	Radius (approximation)	Balance against gravitational collapse
Young star	10^6 *km*	*Radiation pressure from proton fusion*
White dwarf	10^4 *km*	*Electron degeneracy pressure (Pauli Exclusion Principle)*
Neutron Star	10 *km*	*Neutron degeneracy pressure (Pauli Exclusion Principle)*
Black hole	1 *km*	*None*

Table 5.15: TIMETABLE OF OBSERVATIONS OF OUTSTANDING ASTROPHYSICAL AND COSMOLOGICAL PHENOMENA

130 BCE ***Hipparchos*** *observed a few WR stars, but parallax was insufficient for useful luminosity calibration. In 1989 CE, a satellite of the European Space Agency, bearing his name, was launched to survey the positions of more than 100,000 stars.*

1867 CE ***C.J.E Wolf*** and ***G.A.P Rayet*** *discovered the first three 'WR stars'.*

1871 CE The Italian astronomer ***Lorenzo Respighi*** *(1824–1889) was first to see the remarkable spectrum of the Southern WR star Gamma Velorum, an interacting binary of magnitude 1.74*[786].

1884 CE The Astronomer-Royal of Scotland, ***Ralph Copeland*** *(1837–1905) led an expedition to Lake Titicaca (Peru, altitude 4000 meters) to record the spectrum of Gama Velorum. In his own words:*

"Its intensely bright line in the blue, and the gorgeous group of three bright lines in the yellow and orange, render its

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786 Astrophysicists express the brightness of stars in visible light in two related forms.

Apparent visible magnitude, mv, measures the light that actually reaches us on earth. But that is not the true measure of a star or galaxy's brightness because distance makes objects appear dimmer. *So, absolute magnitude – Mv – is used to compare how bright stars would be if they all were 32.6 light years (10 parsecs) away.*

Bright stars are ranked first and are assigned the low numbers, followed by dim stars with higher numbers (6^{th} magnitude is the faintest that the naked eye can detect; reaching $mv = 25$ requires extremely sensitive telescopes and instruments). A few have negative magnitudes given when modern instruments showed them to be brighter than the initial magnitudes given by astronomers just using their unaided vision. A magnitude 1 object is about 2.5 times brighter than the next dimmer magnitude. A 100-fold difference in brightness makes a difference of 5 in magnitude.

At $mv = -1.45$, Sirius A has the greatest apparent magnitude of any star in our sky. At $Mv = +1.41$, it's still in the Top 20 for absolute magnitude. But that parade is led by *Deneb*, a blue supergiant in Cygnus (the Swan) with a whopping $Mv = -7.3$. The closest competitors are *Antares* and *Mimosa* (beta Cruces) at $Mv = -4.7$

spectrum incomparably the most brilliant and striking in the whole heavens. To a great extent it was the extraordinary beauty of this spectrum that led me to devote a considerable part of my time to more or less systematic sweeps of the neighborhood of the Milky Way."

1918 CE ***H.D. Curtis*** *discovered that the spiral nebula Andromeda was far beyond the reaches of our galaxy.*

1924–1928 CE *Observation of light from distant galaxies showed systematic red shifts, indicating that they are receding from us.*

1929 CE ***Edwin Hubble*** *discovered that the universe as a whole is expanding isotropically and homogeneously like a 3D version of a balloon surface.*

1932 CE ***Karl Jansky*** *(1905–1950) accidentally discovered radio waves from the Milky Way.*

1932 CE ***Bengt Edlen*** *correctly identified the observed spectral lines of Wolf and Rayet with laboratory spectra of highly ionized carbon, nitrogen and oxygen. He also explained that the Doppler broadening within strong stellar winds contributes to line width. It was later found that not all of the broadening could be attributed to this effect.*

1933 CE ***F. Zwicky*** *(1898–1974) and* ***W. Baade*** *(1893–1960) predicted the existence of neutron stars. This was verified in 1967 upon the discovery of pulsars.*

1936 CE ***Albert Einstein*** *and* ***Rudolf W. Mandl*** *proposed the theory of gravitational lensing as a testable prediction of GTR.*

F. Zwicky *stated (1937) that galaxies could act as gravitational lenses.*

1943 CE ***Carl Seyfert*** *(1911–1960, USA) identified a small number of galaxies whose nuclei show unusual spectra. Theses features will be later associated with Quasars.*

1960–1963 CE ***Alan Sandage*** *and* ***Maarten Schmidt*** *observed Quasar 3C 48, a qusistellar radio source with X-ray luminosity of* 10^{47} *erg/sec, estimated to be located* 5×10^9 *LY away. It was later suggested (1973) that a quasar is generated by a supermassive black hole at the center of a galaxy.*

1967 CE **Jocelyn Bell** and **Anthony Hewish** *discovered the first pulsar ($CP191$), a radio source in the middle of the Crab Nebula: now believed to be a rotating neutron star with period* 1.337 *sec emitting beams of synchrotron radiation. It is a remnant of the 1054 CE supernova explosion. Its total energy rate in synchrotron radiation is* 3×10^{38} *erg/sec [the sun's total EM emission is* 4×10^{33} *erg/sec].*

1967 CE *Gamma-ray bursts (GRB) discovered serendipitously by U.S. military satellites which were on the lookout for Soviet clandestine nuclear testing in the atmosphere.*

1971 CE *An X-Ray binary source known as Cygnus X-1 is identified as a Black Hole.*

1974–1978 CE **Russel Hulse** and **Joseph Taylor** *discovered the first binary pulsar* $PSR\ 1913 + 16$*: a pulsar orbiting around a companion star. Its behavior is in accordance of Einstein's GTR.*

1979–1983 CE **Dennis Walsh**, **Robert Carswel** *and* **Ray Weymann** *discover a gravitationally lensed Quasar.*

In 1980, an Einstein Ring, a gravitational lens distortion effect, was observed for the first time.

In 1987, **Roger Linds** and **Vahe Petrosian** *discovered an image of a distant unseen galaxy that has been formed by a gravitational lens. By 1993, the existence of compact dark objects in our galaxy, was confirmed via their gravitational lensing of more distant light sources.*

1987 CE *A supernova detected in the Large Magellanic Cloud, a satellite of the Milky Way galaxy. The supernova was the first in almost 400 years that could be seen without the aid of a telescope.*

1990 CE *Hubble Space Telescope launched. The HST has produced images of breathtaking clarity and has allowed astronomers to see light from more distant objects than ever before.*

1991 CE *The Compton Gamma Ray Observatory (GRO), a satellite, is launched by NASA to study the universe. It is more sensitive than earlier gamma ray telescopes. By 1995, the survey recorded about 3000 gamma-ray bursts isotropically distributed, suggesting a cosmological distribution.*

1992 CE ***Felix Miraber*** *and* ***Luis Rodriguez*** *discovered radio sources near our galactic center that resemble quasars. One of these sources, SS 433, shows signs of high-speed relativistic jets.*

1995–1996 CE *Hubble Space Telescope (HST) images of quasar PKS2349 support the hypothesis that quasars occur in the cores of galaxies. The images revealed that the environment surrounding a quasar is far more complex than first suspected. These new observations suggest galactic collisions and mergers between quasars and companion galaxies can reignite the supermassive black holes that drive quasars.*

In 1996 HST recorded a Quasar a billion LY from earth.

1997 CE *Japan's institute of Space and Astronautical Sciences linked a satellite in space with earth-based radio antennae to create an effective telescope larger than earth, called the Very Long Base Interferometry Space Observatory (VLBI). This telescope's resolving power is equivalent to reading a newspaper headline in Tokyo from Los Angeles.*

1997 CE, Feb 28 *Beppo SAX, equipped with both gamma-ray and X-ray detectors, spotted an X-ray afterglow signature associated with the gamma-ray burst of event GRB 970228.*[787]

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[787] Not until astronomers were able to make afterglow observations could they develop a working hypothesis on what caused gamma-ray bursts. And while the *Compton Gamma Ray Observatory's* Burst And Transient Source Experiment (BATSE) detector catalogued 2,704 GRBs during the observatory's nine year lifetime (1991 -2000), it was not equipped to make afterglow observations. Furthermore, it had not been possible to get either a ground or space-based telescope look up quickly enough to a spot where a GRB had been detected.

As a result, the first afterglow observation did not come until the Beppo SAX *satellite*. Beppo SAX, an Italian satellite, was equipped with both a *gamma ray* and an *X-ray* detector. It spotted the X-ray afterglow signature associated with the gamma-ray burst on February 28, 1997.

Today a worldwide network called the Gamma-ray Burst Coordinates Network (GCN) coordinates space-based observations and ground-based follow-through observations of GRB afterglow. NASA satellites include the High Energy Transient Explorer (HETE) operated by the Massachusetts Institute of Technology, and the Rossi X-ray Timing Explorer (RXTE). The European Space Agency operates *Integral*, a new gamma-ray mission launched in 2002. And there is the Interplanetary Gamma-Ray Burst Timing Network (IPN), which consists of a group of space probes with gamma-ray detectors at different locations in the

1999 CE, Jun 27 *'Blast from the past': one of the most powerful cosmic explosion ever recorded, bathed the earth in gamma rays. This gamma-rays burst called GRB 990123 – was so intense that its visible light could have been seen through common binoculars*[788]*. The duration of the event was about 110 seconds. The redshift measurement implied that it took the signal about 10 billion years to reach the earth.*

For the first time, scientists have witnessed a burster's visible light emitted at the same time as a gamma-ray burst.

2003 CE, Mar 29 *NASA's High energy Transient Explorer (HETE-II) detected a very bright gamma-ray burst (designated GRB 030329) is a sky region within the constellation Leo.*

Following identification of the "optical afterglow" by a 40-inch optical telescope of the Siding Spring Observatory (Australia), the redshift of the burst was determined as 0.1685 *by means of a high-dispersion spectrum analyzer of the UVES spectrograph of the 8.2 m telescope at Parnal Observatory (Chile). The corresponding distance is 2650 MLY.*

The optical spectrum was nearly identical to that of a supernova*, and* X-ray *observations also showed a signature associated with oxygen heated to high temperature. Such a pattern occurs when the supernova blast wave excites oxygen atoms in the vicinity of the star.*

*All this evidence pointed to a connection between the GRB and a "*hypernova*" explosion of a very massive, highly evolved star.*

This is caused by a very heavy star — presumably 25 times heavier than the sun.

These observations therefore indicate a common physical source of hypernova explosion *and the associated emission of strong gamma radiation.*

Solar System.

By timing the arrival of gamma-ray photons at each satellite, the location of the burst can be "triangulated". The *GCN sends out automatic notices* by email to astronomers worldwide; enabling both professional and amateur astronomers to make follow-up afterglow observations.

788 The peak gamma-ray *power* was estimated at that of 10^{16} suns, namely 10^{50} erg/sec. This is 100 times more energetic than a supernova explosion and is comparable to burning up the entire mass-energy of the sun in a few hours.

Glossary of Cataclysmic events

Huge explosions and other sorts of cataclysmic events are a natural part of the life-cycle of stars. Stars formed in swirling clouds of gas move along the H-R diagram, incorporating such occurrences into their evolution and forming in the process the elements needed to form new stars. The main 'cast of characters' and the fundamental concepts in this drama are the subjects of this small dictionary:

parsec — *The distance you would have to be from the solar system for the angular separation between the earth and the sun to be one arcsecond. That distance is*

$$206,265\ AU = 3.1 \times 10^{13}\ km = 3.26\ light\ years.$$

Magnitude — *A system for classification of stars according to apparent brightness. The human eye can detect stars with magnitudes up to 6 (the faintest). The Hubble space telescope is capable of imaging a magnitude 30 star, which has been compared to detecting a firefly at distance equal to the diameter of the earth (ca 12,800 km).*

Spectral classification — *A system for classifying stars according to their surface temperatures.*

Red dwarfs — *The most common stars, accounting for about 80 percent of the star population in the universe.*

Binaries — *Two-star systems, in which the stars orbit one another. The way the companion stars move can tell astronomers much about the individual stars, including their masses. Visual binaries can be resolved from earth. Spectroscopic binaries are too distant to be seen as distant points of light, but their relative motions can be studied with a spectroscope. In this case, a binary system is manifested via Doppler shifting of spectral lines as the stars receded and advanced along the line of sight. Eclipsing binaries: periodic, sharp changes in light intensity due to one star eclipsing the other. This yields information about orbital motion, masses and radii.*

Core hydrogen burning	*The principal fusion reaction process of a star. The hydrogen of the star's core is fused into helium (sometimes in nuclear reaction cycles catalyzed by carbon, nitrogen and oxygen nuclei), producing enormous amounts of energy in the process.*
Red giant	*The last stage in the evolution of stars about as massive as the sun. Its relatively low surface temperature produces its red color.*
White dwarf	*The remnant core of a red giant after it has lost its outer layers as a planetary nebula. Since fusion has now halted, the carbon-oxygen core is supported against further collapse only by the degeneracy pressure supplied by densely-packed electrons.* *Their small size makes them relatively faint objects despite their high surface temperatures.*
Core-collapse supernova	*The extraordinarily energetic explosion that results when the core of a high-mass star collapses under its own gravity.*
Neutron star	*Superdense compact remnant of a massive star, one possible survivor of a supernova explosion. It is supported by degenerate neutron pressure, not fusion. It is a star with the density of an atomic nucleus.*
Pulsar	*A rapidly rotating neutron star whose magnetic field is oriented such that its synchrotron-radiation "searchlight" sweeps across earth with a regular period. When pulsars were first detected (1967), their signals were so regular that some astronomers suspected they might be a sign of extraterrestrial intelligence. Further observations provided a more mundane explanation.*
Stellar mass black hole	*The end result of the core collapse of a high-mass star. It is an object from which no matter or even light can escape (apart from negligibly feeble Hawking radiation). Although space behaves strangely very close to a black hole, at astronomical distances the black hole's only signature is a normal Newtonian gravitational field, plus an unusually broad EM spectrum emitted by infalling matter in its accretion disc.*

	Only main-sequence stars of at least 20–30 solar masses will ever collapse into a black hole. *The Schwarzschild radius of any given mass is the radius it must be compacted to, to become a black hole (although the precise quantitative measure is tricky, due to the high GTR space curvature near that radius). The event horizon is a surface (spherical for a non-rotating black hole) whose radius is the Schwarzschild radius; the escape velocity equals light speed on that surface. No information of the events occurring within the event horizon can be communicated to the outside.* *A black hole will draw in matter that wanders too near to the event horizon. With such accretion, the star's mass will increase, even as its collapse continues, causing its Schwarzschild radius and event horizon to grow.*
SINGULARITY	*The infinitely dense remnant of a massive core collapse.*
EMISSION NEBULAE	*Glowing clouds of hot, ionized interstellar gas, located near young, massive stars.*
GIANT MOLECULAR CLOUDS	*Huge collections of cold (10 K to 100 K) gas that contains mostly molecular hydrogen. The cores of these clouds are often the sites of the most recent star formation.* *The expanding shock wave of a nearby supernova explosion might be sufficient to cause a cloud to collapse. A ripple in a galaxy (called a density wave) could also be a trigger. A fast-moving massive star, punching through a molecular cloud, could also cause part of it to collapse.*
BROWN DWARF	*A failed star, i.e. a star in which the hydro-thermodynamical forces reached equilibrium with its self-gravity before the core temperature rose sufficiently to trigger nuclear fusion.*
VARIABLE STAR	*A star that periodically changes its brightness.*
LOCAL GROUP	*A galaxy cluster; a gravitationally bound group of galaxies which includes the Milky Way, Andromeda, and other galaxies.*

SUPERCLUSTER — *A group of galaxy clusters. The Local Supercluster contains some* 10^{15} *solar masses.*

QUASAR — *Bright, distant, tiny objects, which produce luminosity of 100 to 1000 galaxies within the size of a solar system. The first quasars were detected at radio frequencies, though most quasars do not emit large amounts of radio energy. Quasars are among the most luminous objects in the universe, having luminosities in the range* $10^{45} - 10^{49}$ *erg/sec. These numbers average out to the equivalent of 1000 Milky Way Galaxies. (The sun has a luminosity of* 4×10^{33} *erg/sec). Quasar's brightness fluctuations are explained by fluctuations in the accretion-disc – the swirling disc of gas spiraling toward the black hole.*

ACTIVE GALAXIES — *Galaxies that have more luminous centers than normal galaxies.*

SEYFERT GALAXIES — *A subset of spiral galaxies characterized by a bright central region containing strong broad emission lines. Some show violent activities in their cores.*

RADIO GALAXIES — *A class of active elliptical galaxies characterized by strong radio emission, and in some cases, narrow jets and wispy lobes of emission located hundreds of thousands of light years from the nucleus. When radio-emitting blobs are moving at high-velocity toward us, there is an apparent superluminal motion (faster than the velocity of light due to a well-understood STR kinematical effect).*

SYNCHROTRON RADIATION — *Arises when charged particles are accelerated by strong magnetic fields. The emission from radio galaxies is mostly synchrotron.*

Science Progress Report No. 15
The Bookburners

"Where they burn books, they end up burning people".
Heinrich Heine[789], 1848 (1797–1856)

411 BCE Public burning of the books of Protagoras in Athens, Greece.

213 BCE Science books and savants were burned in China.

48 BCE The Romans burn part of the Alexandria library.

53 CE The apostle Paul burns books of pagan lore in Ephesus.

70 CE The Romans burn the library of the Temple in Jerusalem.

273 CE The Romans damage the Alexandria library.

295 The Roman emperor Diocletian ordered the burning of all books on the working of gold, silver and copper.

325 Constantine had the Arian writings burnt.

373 Roman emperor Valens ordered the burning of non-Christian books.

391 Emperor Theodosius I ordered the burning of the remnants of Alexandria library. Bishop Theophilus carries out the order.

431 Theodosius II burned the books of the Nestorians.

440 Valentinian III burned the books of the Manichaeans.

646 The Arabs (under caliph Omar) burn to ashes the remnant of the Alexandria library.

1109 Crusaders burn over 100,000 Muslim books in Tripoli.

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789 Indeed, the Nazi burned his books too. Since it was impossible to remove his *Lorelei* — so great was its popularity — Heine's name was obliterated and replaced by the words "*Author Unknown*".

1204 Mass-burning of classical books by Crusaders in Constantinople.

1242, Jun 17 Public burning of the Talmud and other Hebrew books in Paris under the order of pope Gregory IX.

1244, Mar 09 Pope Innocent IV orders that the Talmud be burned.

1415 The Inquisition burned **Jan Hus** in Constance.

1481, Feb 06 First Auto-da-fe of the Spanish Inquisition.

1484 Grand Inquisitor, Tomas de Torquemada, burned Marranos and their books in Toledo.

1507 Grand Inquisitor, Cardinal Ximenes, burned 24,000 Jewish books in Granada.

1509, Aug 19 Maximillian I ordered the burning of Jewish books.

1528 The first auto-da-fe in America, held in Mexico City when Jews and their books are burned on the stake. Additional Jews are burned in the New World on 28/29 Feb 1574.

1553 Burning of the Talmud in Italy.

1559 Pope Paul IV burns 12,000 Jewish books in Cremona, Italy.

1562 The Bishop of Yucatan burned to ashes almost the entire native literature of the Maya culture.

1600 The Inquisition burned **Giordano Bruno** in Rome.

1601, Jan 14 The Church burned Jewish books in Rome.

1635, Aug 11 Auto-da-fe in Lima, Peru for Marranos who escaped the Spanish Inquisition.

1649 One hundred and nine Jews and their books burned in Mexico City.

1682 Three Jews were publicly burned in Berlin as a result of a blood libel.

1731, May 28 Pope Clement XII orders the confiscation and burning of all Hebrew books in Papal States.

1757, Nov 13 The Bishop of Kamenets-Podolski burned all copies of the Talmud in his diocese.

1796, Oct 17 Tzar Paul I ordered censorship of Jewish books in Russia.

1836 Tzar Nikolai I decreed the burning of Jewish books. Most Jewish printing presses were closed.

1933, May 10 Nazis burn books of **Albert Einstein**, **Sigmund Freud** and other Jewish scientists in Germany. Later (1940–1945), the Germans perpetrated an industrial genocide through which they gassed[790] and burned 6,000,000 Jews in special ovens in Poland [German scientists developed an efficient *mass-murder technology*].

1966–7 Mao Tse-tung's 'Cultural Revolution': 'Red Guards' march across China burning books, libraries, museums, laboratories, art galleries and university campuses. A cultural heritage of 6000 years is destroyed. About 500,000 intellectuals are murdered in the biggest witch-hunt in history.

1975–6 Khmer-Rouge communists under Paul Pot burn books and exterminate 1,200,000 intellectuals in Cambodia.

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[790] *Hydrocyanic acid* (HCN) was discovered in 1782 by **Carl Wilhelm Scheele** (1742–1786, Sweden).

Salute to books — mankind's noblest creation

What is a book? Part matter and part spirit; part thing and part thought — however you look at it, it defies definition. Its outward form, essentially unchanged in nearly 2000 years, is a design as functional as, say, the pencil or the glove: you can't improve on it. Yet, by its nature the book is loftier than the common objects of this world. It is a vehicle of learning and enlightenment, an open sesame to countless joys and sorrows. At a touch, our book springs open, and we slip into a silent world — to visit foreign shores, to discover hidden treasure, to soar among the stars.

Sometime ago, by a unanimous decision of its 128 member nations, the United Nations Educational, Scientific and Cultural Organization (UNESCO) designated 1972 as the first International Book Year. The fact that the ensuing salutes were world-wide was only too appropriate, for the book is the end product of a unique conjunction of endeavors, made independently in far-flung corners of the globe. It is as if all mankind had conspired to create it.

The Chinese gave us paper. Phoenicia brought forth our alphabet. To Rome we owe the format of the book; to Germany, the art of printing from movable type. Britain and the United States perfected book production. Today, 15,000 finished books roll off high-speed presses in just one hour, and we find it hard to visualize the bookless world of our forebears, hard to imagine the enormous effort that lies behind the saga of the book.

In the beginning, there was only the spoken word. Then, to entrust his thoughts to a more lasting medium than mere memory, man took to drawing pictures representing things. Perhaps the oldest picture-script originated some 6000 years ago in Mesopotamia. Its images — bird, ox, ear of barley — were scratched into soft clay tablets, which were then baked hard for preservation.

But such writing was a cumbersome affair, mainly used for priestly documents and public records. What "literature" there was — such as heroic poems — depended almost totally on word-of-mouth transmission. The quick Mediterranean mind, awakening to a new culture, demanded a better way of harnessing the spoken language. Shortly before the 9th century BCE, the Phoenicians — swift seafarers, sharp traders and good record-keepers — began breaking spoken sounds into their basic elements, and shuffling the resulting "letters" to form words.

When wandering among the ruins of the Phoenician port of Byblos — one may still see the rudimentary inscription, hewn into the rock wall of a royal *grave shaft, which stands as the world's oldest known alphabetic writing. Soon the alphabet was seized upon by the Greeks, who gave letters more convenient shapes and added the still-missing vowels.*

No sooner had man taught himself to spell than a new problem raised its head. What to write on? Leather, tree bark, leaves and wax tablets had all proved unsatisfactory. In Egypt, for some 2500 years before Year One, texts had been inscribed on brittle sheets made from the pith of a Nile Delta water plant, papyrus.

The use of this material gradually spread through the Mediterranean world. Usually, several papyrus sheets were glued together to form a scroll that could accommodate a lengthy text. (One 40 m scroll containing the picture-script account of the deeds of Pharaoh Ramses III is still extant.) But what a clumsy thing to read! The scroll, wrapped around a wooden stick, had to be held in the right hand, while the left slowly unwound it to reveal the next column of writing. Nevertheless, the royal library at Alexandria — destroyed in the 4th century CE – is believed to have had no fewer than 700,000 scrolls.

Relatively fragile, papyrus invited rivalry. In wealthy Pergamum, on the coast of Asia Minor, scribes wrote on specially prepared sheep, goat or calf skins. This fine, pellucid stationery, tougher than papyrus and foldable, came to be known as parchment. Shortly after Year One, an unknown Roman scribe with a sense of compactness took a stack of thin parchment sheets, folded them and fastened them together at the spine. Thus, the book was born.

Likely as not, its earliest promoters were Rome's Christians. To them, it was essential to preserve the Scriptures in the most lasting medium — and parchment didn't wilt when handled. Moreover, when one wanted to hunt up a reference, chapter and verse, a book was a lot handlier than a scroll.

So it came about that, all through Europe's Dark Ages, an army of devoted monks, ensconced behind monastery walls, hand-copied the torn and shredded writings of the past on sturdy parchment sheets. Without their toil, the literary glories of ancient Greece and Rome, along with vital texts that shaped the Christian faith, might have been lost forever. It frequently took years to finish copying a thick tome, and many a sore-eyed monk, before putting away his goose quill, penned a sigh of relief on the final page: "Thank God I have finished!"

Meanwhile, in distant China, tradition has it that a gentleman named Ts'ai Lun, vexed at the wasteful use of costly silk as a writing material, reported to Emperor Ho-ti that a far cheaper substance could be made by pounding rags, tree bark and old fishing nets into a pulp, skimming thin layers off the top, and drying them.

Thus, in the year 105 CE, paper enters our story — to remain for six centuries, a closely guarded secret of the East. It wasn't until some Chinese papermakers were captured by marauding Arabs that the pliant blossom-white enduring marvel took the world by storm.

The Occident saw the next major breakthrough. In 1439, a stubbornly determined German craftsman, Johann Gutenberg, began experimenting with a substitute for hand-writing. If he could cast the letters of the alphabet in reusable metal type, then arrange them, in a mirror pattern, into words, lines and columns on an even-surfaced plate, an imprint taken from this plate would make one page. In place of one painstakingly handwritten book, he would be able to run off on his "press" as many imprinted books — exact copies of each other — as he wished.

Laboriously, Gutenberg put together his first page plates, each one composed of more than 3700 signs and letters. With the help of a hand-worked wooden press that he had adapted from the wine press of his native Rhineland (and which remained unchanged for the next 350 years), he started printing in a rented workshop in Mainz.

It took three years to turn out some 190 copies of the Gutenberg Bible of 1455. (Today 47 copies still survive, 14 of which are in the United States. One of the finest, worth an estimated $3 million to $10 million, is on display in the Library of Congress.)

Before the printed book, memory *ruled daily life; The memory of individuals and of communities carried knowledge through time. For millennia, personal memory reigned over entertainment, information, perpetuation and perfection of crafts, the practice of commerce and the conduct of professions. Memory was a faculty which everyone had to cultivate. Thus, the epics of the* Iliad *and the* Odyssey *were perpetuated by word of mouth. Laws were preserved by memory before they were preserved in documents. Rituals and liturgy, too, were preserved by memory, of which priests were the special custodians. By the time the printing press appeared, the arts of memory had been elaborate into countless systems.*

After Gutenberg, realms of everyday life, once valued and served by memory, would be governed by the printed page. Since then, the technology of memory retrieval, played a much smaller role in the higher realms of religion, thought, and knowledge. Spectacular feats of memory became mere stunts. The printed book, however, had its own drawbacks: it made it less necessary to shape ideas and things into vivid images and then store them in memory-places.

In the centuries after printing, interest has shifted from the technology of memory to its pathology. By the late 20th century, interest in memory was *being displaced by the interest in amnesia, hysteria, hypnosis and psychoanalysis. Pedagogic interest in the arts of memory came to be displaced by interest in the arts of learning, which were increasingly described as a social process.*

With Gutenberg's remarkable invention, book prices dropped 80 percent overnight, and learning to read became worthwhile. A mere half-century after Gutenberg's exploit, every major European country except Russia was printing its own books. It was as if floodgates had been opened. Some 520,000 titles were published in the 16th century, 1.25 million in the 17th, two million in the 18th and eight million in the 19th. Today, more than 500,000 titles come off the presses in a single year, adding up to an estimated seven billion *individual books.*

These eye-popping figures notwithstanding, there are those today who predict the disappearance of the reading habit. Canadian professor and commentator Marshall McLuhan, for one, has argued that mass media — films, radio, television — involve us more completely, and hence impart their message more directly, than the familiar lineup of black letters on the printed page[791]*.*

Be this as it may, the book has shown considerable fighting spirit in the face of the new threats. Book-club business has erupted into a stampede, and paperbacks are bought off store shelves as fast as they are put there. Indeed, exposure to the electronic media seems to have created a new desire to "curl up with a good book." And, as we turn its pages at our convenience, going back in leisurely fashion over a passage we've especially enjoyed, or skipping a little here, a little there, we are "involved" more intimately and completely than we could be with any other medium yet invented.

Man's thoughts and dreams, his knowledge and his aspirations, are stored in books — wealth to be tapped by all who so desire. From the first wobbly picture-script to quicker-than-the-eye offset presses, the book has come a long, arduous way, propelled by the genius and persistence of many individuals and nations, Indeed, all humanity has reason to be proud of the book, for it shows us at our very best. Long live the book!

[791] Indeed, the increasing role played by the *digital media* and the *internet* in the domains of contemporary communication, education, business and entertainment seem to fulfill the prophecies of McLuhan.

1934 CE First Hetch-Hetchy water delivered to the city of San-Francisco. A major engineering feat brought water (and electric power) along 240 km from the Tuolumne River in east-central California to the city of San-Francisco. The project called for a construction of a dam, built of concrete (277 m long and 131 m high) with a reservoir capacity of 444 million cubic meters of water. It is known as the *O'Shaughnessy Dam* after **Michael Maurice O'Shaughnessy** (1864–1934), the SF city engineer who directed the original construction. 444 miles of pipeline were laid to bring water from Sierra to SF reservoirs and 40 km of tunnel through Coast Range was built (longest in the world at that time). Cost (to 1934) was about $100 million and 89 men were killed in construction accidents.

San-Francisco (SF) water supplies became inadequate as the city grew. First water was shipped across the Bay or piped from local springs. By 1890 SF started to look for water sources in the Sierra. The Hetch-Hetchy Valley in Yosemite National Park was picked as the best site for the Dam, but early SF applications were turned down because site is in a national park. Civic organizations and the SF Examiner (W.R. Hearst) supported the dam while **John Muir** and the Sierra Club led the opposition. Gradually the dam supporters gained more influence in the US Congress and in the cabinet and in 1913 the project was passed by Congress. The O'Shaughnessy Dam was finished in 1923.

1934 CE **William Beebe** (1877–1962, USA) and **Otis Barton** (USA) set a depth record by diving to 1001 m below the ocean's surface in a tethered sphere called a *bathysphere*.

1934 CE **Wallace Hume Carothers** (1896–1937, U.S.A.). Chemist. Synthesized *nylon*, a synthetic fiber stronger and more durable than silk. Depressed by the untimely death of his twin sister, he committed suicide 3 years after his great invention.

1934–1937 CE **Pavel Alexeevich Cherenkov** (1904–1990, Russia). Physicist. Discovered (*experimentally*) the '*Cherenkov effect*'[792] at the Lebedev Institute, Moscow. It may have been the last important basic phenomenon

[792] The velocity of light in a medium c' is expressed by the formula $c' = \frac{c}{n}$, where n is the refraction index of the medium. Since $n > 1$, a high-energy particle can move in the medium with velocity v such that it exceeds the speed of light there, namely $c > v > c' = \frac{c}{n}$. This particle, if it carries a charge, will radiate light even if it travels with a fixed velocity.

The wave front of the Cherenkov radiation is the envelope of spherical waves emitted by the particle, and has the *shape of a cone* with its vertex at the in classical electrodynamics that remained to be discovered. When a charged particle travels close to the speed of light through a transparent liquid or solid, it emits light. This phenomenon occurs when the velocity of the particle is *greater* than that of light in the dense medium (but still less than c, light's speed in vacuum).

Cherenkov was born near Voronezh, Russia and graduated from the University there in 1928. In 1930, after teaching in a high school, Cherenkov moved to Leningrad and entered the Institute of Physics and Mathematics of the Academy of Sciences of the U.S.S.R. as a postgraduate student. In 1932 he started his research under Sergei I. Vavilov on luminescence activated by gamma rays in different liquids. When the Cherenkov effect was discovered, the majority of Cherenkov's colleagues did not show particular interest in his results.

Nobody recollected the calculation by **Arnold Sommerfeld** (1905) of the energy losses of an electron whose velocity exceeds the velocity of light, or the incredible intuition of **Oliver Heaviside**, who actually predicted the Cherenkov effect in 1888! Even as distinguished a physicist as **Leonid Isaakovich Mandelstam** (1879–1944) did not show much interest in Cherenkov's results, being quite sure that an electron moving with constant velocity could not emit radiation. Only Cherenkov's discovery of the asymmetry of the radiation pattern made (partly by chance) in 1936 after several

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particle. The situation is similar to that of water waves in the wake of a ship or the shock waves generated in air by missiles or aeroplanes which exceed the speed of sound.

The Cherenkov light is emitted forward such that the angle between the shock-wave and the particle's line of motion is given by $\cos\theta = \frac{c}{nv}$. [In lucite $n = 1.5$, the lowest electron energy for Cherenkov emission is 0.17 MeV and the lowest proton energy is 320 MeV.] This simple formula is used to derive the particle's velocity, which is equivalent to the measurement of its energy.

A Cherenkov counter is a particle detector that employs the Cherenkov radiation emitted in a thin conical layer at an angle θ to the direction of motion. The counter usually consists of a lucite block shaped in such a way that the generated light is sent forward and converges on to the cathode of a *photomultiplier tube*. Since no light at all is produced for $\frac{v}{c} < \frac{1}{n}$, these counters have a built-in threshold discrimination, i.e. they are completely insensitive to low velocity background radiation.

The Cherenkov technique is now an important tool for distinguishing particles of different masses in accelerator experiments; the momentum of the particle is measured by magnetic deflection and its velocity by using the angle or intensity of the Cherenkov light.

years of intense experimenting, assured him and his fellow researchers of the reality of the phenomenon and gave them the key to understanding it. But the acceptance of the effect did not come easy. In the middle of 1937, the editor of *Nature* declined to publish Cherenkov's paper. Later in 1937 *Physical Review* published the paper, and soon the phenomenon was confirmed and accepted.

He shared the Nobel prize for physics in 1958 with **Ilya M. Frank** (1908–1990) and **Igor Y. Tamm** (1895–1971) who worked out the theory of the effect in 1937.

The Russians used the Cherenkov effect in making a *cosmic-ray counter*, which their satellite *Sputnik 3* carried around the earth. Today, Cherenkov detectors are routinely used in the study of high energy particle collisions at accelerator laboratories.

1934–1953 CE **Igor Yevgenyevich Tamm** (1895–1971, Russia). Outstanding theoretical physicist. Made significant research in the fields of crystal optics, quantum theory of diffused light in solids, theory of cosmic rays and control of thermonuclear reactions. Developed (with **I.M. Frank**) the theoretical interpretation of the radiation of electrons moving through matter faster than the speed of light in that matter (the *Cherenkov effect*). Worked with his student **Andrei D. Sakharov** on the Soviet nuclear weapons program.

Tamm was born in Vladivostok to Jewish parents. He graduated from Moscow State University (1918), awarded the degree of Doctor of Physico-Mathematical Sciences (1925); Professor (1930); Academician (1933). Appointed (1934) head of the theoretical division of the Lebedev Institute of Physics of the USSR Academy of Sciences.

A decisive influence on his scientific activity was exercised by **L. Mandelstam** (1879–1944), under whose guidance he worked for a number of years, and with whom he was closely associated since 1920.

Tamm was also a member of the American Academy of Arts and Sciences.

1934–1974 CE **Ilya Mikhailovich Frank** (1908–1990, Russia). Distinguished physicist. Made significant research on photoluminescence, photochemistry, gamma rays and neutron physics. Winner of the Nobel Prize for physics (1958) jointly with **Igor Yevgenyevich Tamm** for the interpretation of the Cherenkov effect, leading to the development of the Cherenkov detector.

Frank was born in St. Petersburg (Leningrad) to Jewish parents. He graduated from the Moscow State University (1930) and joined (1934) the

Lebedev Institute of physics; D.Sc (1935); Professor (1944); Academician (1946). He married Ella Abramovna Beilikhis (1937).

1934–1944 CE **Geoffrey Ingram Taylor** (1886–1975, England). Physicist and hydrodynamicist. Introduced statistical mechanics into the analysis of *turbulence and diffusion* of vortex motion (1935–1938), thus initiating the *statistical study of turbulence*. His work was continued by A. Kolmogorov (1941) and W. Heisenberg (1947).

Taylor contributed to the *Manhattan Project* (1944) by solving a problem associated with the hydrodynamics of nuclear explosions. The problem was to calculate the interaction of several shock waves as they evolve through time: the so-called *Rayleigh-Taylor instability*, formed at the boundary between two materials, causes the two materials to mix in way that is extremely difficult to predict. This in turn made it difficult to predict the *yield* of the atomic bomb[793].

Taylor was born in London. Graduated from Trinity College, Cambridge (1908). Except during the world wars, when he provided assistance to the government, he was based at Cambridge throughout his career.

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793 G.I. Taylor was able to deduce the yield of the first nuclear explosion from a series of photographs of the expanding fireball in *Life* magazine. He realized that he was seeing a strong shock expanding into an undisturbed medium. The pictures gave him the *radius as a function of time*, $r(t)$. All that could be important in determining $r(t)$ was:

- E, the initial energy release.
- ρ, the density of the undisturbed medium (air).

The radius, with the dimension of length, depends on E, ρ and t, and he constructed the distance out of these quantities. Now, E and ρ had to come in the form E/ρ to cancel the mass. But E/ρ has the dimension $(\text{length})^5/(\text{time})^2$, so the only possible combination was $r(t) \propto \left(\frac{E}{\rho}t^2\right)^{1/5}$. A log–log plot of r versus t (measured from the pictures) gave a slope of $\frac{2}{5}$, which checked with the theory, and E/ρ could be obtained in any chosen unit system from extrapolation to the value of $\log r$ when $\log t = 0$. Since ρ was known, E was determined to within a factor of order one. For the practitioner of the art of *dimensional analysis*, the nation's deepest secret had been published in *Life* magazine. However, the energy release of the first nuclear device was not obvious to every reader of *Life* magazine, not even to those who were aware of the technique of dimensional analysis. The key success is to identify the essential features of the problem, which can be hard to do until you have been shown how.

Taylor proposed (1934) the idea of *dislocation* in crystals (a form of atomic misarrangement which enables the crystal to deform at a stress less than that of a perfect crystal). His many original investigations on the *mechanics of fluids and solids* were applied to meteorology, oceanography, aerodynamics and the study of Jupiter's Great Red Spot.

1934–1947 CE **Marcus Laurence Elwin Oliphant** (1901–2000, England). Nuclear physicist. Discovered *tritium*[794] (1934), the radioactive isotope of hydrogen, by bombarding heavy water with deuterons. Built a 60 inch cyclotron particle accelerator (1937) and proposed the idea of *proton synchrotron* already in 1943.

Oliphant was born near Adelaide, Australia. Received his Ph.D. from the University of Cambridge, England (1929). He then worked under Rutherford

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[794] The *deuteron* is a bound state of one neutron and one proton with total spin $s = 1$. The binding energy of the deuteron is 2.22 MeV. The atom made up of a bound state of a deuteron and an electron is called *deuterium* (^{2}H, or D). Deuterium exists on earth with natural abundance of about 1.5×10^{-4} times that of hydrogen. The deuterons that are found on earth (inside deuterium atoms) were mostly made in nuclear reactions in the early universe during the first 3 minutes following the Big Bang.

The rest-masses of the proton (m_p), the neutron (m_n) and deuteron (m_d) are related by $m_p + m_n - m_d = \frac{2.22}{c^2}$ MeV $= 3.95 \times 10^{-27}$ g $\cong 2 \times 10^{-3} m_p$. The *heavy water* molecule, D_2O, has deuterium instead of hydrogen.

Tritium (^{3}H) is an atom with a nucleus (*triton*) comprising a bound state of two neutrons and a proton with an orbiting electron.

If two light nuclei are combined into a heavier nucleus (*fusion*), the sum of the masses of the lighter nuclei is greater than the mass of the heavier nucleus. Therefore energy is released when the heavier nucleus is formed. In the fusion of deuterium and tritium

$$^2\mathrm{H} + {}^3\mathrm{H} \quad \rightarrow \quad {}^4\mathrm{He} + n + 17.6 \text{ MeV};$$

This is a very important *thermonuclear reaction*. The Oliphant reaction is:

$$^2\mathrm{H} + {}^2\mathrm{H} \quad \rightarrow \quad {}^3\mathrm{H} + {}^1\mathrm{H}$$

and was effected by bombarding deuterated compounds with low-energy deuteron beams. The half-life of the triton is 12.3 y; the first lower bound was measured by **E.M. McMillan** (1936), and the first reasonably accurate actual value was measured by **A. Novick** (1947).

in the Cavendish Laboratory and became a professor at Birmingham University (1937). He worked on the Manhattan Project at Los Alamos (1943–1945). After the war he was a professor at Canberra University (1950–1961) and served as Governor of South Australia (1971–1976).

1934–1941 CE **Rudolf Schoenheimer** (1898–1941, USA). Biochemist and physician. With his associates at Columbia University developed (from 1934) a technique of "tagging" molecules with radioactive isotopes to trace paths of organic substances through plants and animals, thus revolutionizing metabolic studies. Also studied relation of cholesterol to atherosclerosis.

Schoenheimer was born in Berlin to a Jewish family of physicians. He studied medicine at the University of Berlin, where he received his M.D. degree (1922). He then became interested in Biochemistry and taught at Freiburg (1926–1933). There he encountered the idea of using isotopes through Georg de Hevesy (1885–1966) who had applied radioactive isotopes as tracers in botanical studies (1923) to observe the distribution of lead in bean plants. But at that time few elements were available that could be used as isotopes in biological research.

In 1933 he was forced to leave Germany and went to the United States, joining the biochemistry department of Columbia University medical school. Harold Urey's discovery of deuterium (the isotope of hydrogen) at Columbia University (1932) provided Schoenheimer an ideal opportunity to label organic compounds without changing their chemical properties. Soon afterwards (1935), an isotope of nitrogen, ^{15}N, became available.

With David Rittenberg (1906–1970), a physical chemist, he developed the whole field in which stable isotopes were used as labels for the investigation of many important reactions in intermediary metabolism. Schoenheimer's pioneer work and ideas left a profound impact on biochemistry, biology and medicine.

In spite of his family assimilationist background Schoenheimer reacted vigorously to the growing antisemitism in Germany by a proud identification with his Jewish heritage. After WWI, in which he participated as a soldier he became firmly convinced of the need to establish a Jewish homeland in Israel and visited there in 1926. He intended to go there after WWII and help build up science at the Hebrew University in Jerusalem.

1934–1955 CE **Edmund Germer** (1901–1987, Germany and USA). Physicist, engineer and inventor. Invented and developed the *fluorescent lamp*

(1935) and the *high-pressure mercury-vapor lamp*[795] (1934).

Germer was born in Berlin and studied at the University of Berlin during the 1920's, earning a doctorate in light technology. His continued goal was to invent a better light source with higher lumen output and lower energy consumption compared to the incandescent lamp. After WWII he emigrated to the United States.

The *fluorescent lamp* is a low pressure gas discharge source. It consists of a tube that is coated on the inside with a fluorescent material (powder) and filled with *mercury vapor*. An electric discharge in the vapor produces *ultraviolet light*[796], which in turn causes the coating to fluoresce. By adjusting the coating, fluorescent lamps can be produced to give light with wavelengths that are closer to natural sunlight than can be achieved with the incandescent bulb.

The lamps were introduced commercially in 1938.

1934–1957 CE **Karl (Raimund) Popper** (1902–1994, England). Philosopher of natural and social science. In his key work '*The Logic of Discovery*'

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[795] The first practical mercury-vapor lamp was the Cooper-Hewitt lamp developed by Peter Cooper Hewitt in 1901. This was a tubular source about 4 feet long which produced light that was distinctly bluish green in color. The first high pressure mercury lamps similar to the ones used today, were introduced in 1934 in the 400 watt size. Today available, mercury lamps range in size from 40 watts to 1000 watts. Mercury lamps produce approximately 55–60 lumens per watt.

Operation: the arc tube of the mercury lamp has argon gas and a little pearl of mercury as filling ingredients. Its electrodes are made of tungsten and carry an emitter paste, e.g. a barium-yttrium compound, that reduces the ignition voltage required to start the lamp. Within three to five minutes after ignition, the mercury is completely vaporized and the characteristic blue-green spectrum of the mercury discharge is emitted. It contains strong ultraviolet radiation at wavelengths of 254 nm and 365 nm. Radiation in the red region of the spectrum is virtually negligible. A mercury lamp's color temperature ranges between 4000 K and 4500 K. Applying phosphor coatings to the outer bulb increases the light output by 10 to 15 percent.

[796] Thermal electrons, emitted by the filament of the negative electrode, are accelerated back and forth by an alternating voltage. Some of them collide with the mercury vapor, releasing energy in the form of ultraviolet light. The ultraviolet light causes the phosphorous coating to glow. Typically, a fluorescent lamp must efficiently generate 253.7 millimicron ultraviolet radiation to excite the phosphorous coating inside the tubular glass bulb. Modern fluorescent lamps have an efficiency of about 65–80 lumen per watt.

(1934) he criticized Francis Bacon's inductive method of scientific reasoning. He asserted that a statement is not necessarily scientific just because it can be confirmed by experience; it is essential for such a statement to be capable of being disproved by some possible event which, where it to occur, would exemplify a possibility that the statement itself excludes. This means that scientific theories can never be verified, only falsified, and that falsification is the true aim of scientific endeavor. In other words: falsification rather than verification is the true characteristic of science. It is this feature of *falsifiability*, Popper claimed, that separates science from metaphysics.

Popper asserted that science advances by way of deductive hypotheses created by man's imagination and the eventual test of its particular predictions through experiments and observations. According to this method, a scientist seeks to discover an observed exception to his postulated rule. The absence of contradictory evidence thereby allows tentative acceptance of his theory. According to Popper, such pseudo-sciences[797] as astrology, metaphysics, Marxist

[797] Theorists who insist that they *must* be right, that no experiment or data could disprove the logic and grandeur of their hypothesis are *pseudoscientists*. One of these was Immanuel Velikovsky, who developed a complex hypothesis that the earth had undergone near-collisions with the planets Mars and Venus, that these encounters could explain many of the narratives of the Bible and in other ancient documents, and that our view of the solar system must be largely revised to take account of his *worlds in collision*, planets wandering from what we take to be stable orbits.

Velikovsky was not a scientist, for the simple reason that he was "always right". Indeed, his world view did not allow for the possibility of error on his part: He could not, and did not, state the facts that would disprove his hypothesis if discovered. He stood *outside science*, never being able to present his theories in a way that allowed reasonable people to test them, and refusing to accept the critical ability of science to refute some theories at the expense of others.

By constructing an environment — the world of science — in which theories survive because they fit into the existing framework more successfully than competing theories, scientists have created the potential for anyone within their purview to make important advances in our collective knowledge.

Individuals make the theories; the social structure of science does the testing. He who does not accept this principle, does not belong to the scientific community. This does not mean that his ideas must be wrong, only that he and his ideas, will not be taken seriously. No one guarantees that anybody's ideas will always receive serious considerations in any case, but there certainly is no hope if one does not "think like a scientist".

history, Freudian psychoanalysis and creationism are not empirical sciences, because of their failure to adhere to the principle of falsifiability[798].

Velikovsky, besides being a pseudoscientist, was also dead wrong already in 1950, when he first published his theory; his scheme called for planets to have wandered tens of millions of kilometers from their present orbits within the past few thousand years, and for Mars and Venus to have been expelled from Jupiter shortly before that. But by 1950 we already knew that astronomical records from Mesopotamia showed that Venus must have had its present orbit, or one close to it, before its alleged collision with earth, and our knowledge of planets composition and motion dynamics, ruled out the careening motions that Velikovsky demanded.

798 The importance of falsification was already made clear by the French biologist **Claude Bernard** (1865). In real life, scientists often do *not* conform to this formula for doing science and have rather an unstated set of criteria for choosing one theory rather than another — and these encapsulate some of the main aims of science: accordingly, for a subject to qualify as science it needs at least to satisfy a number of criteria:

- deal satisfactorily with the phenomena it tries to explain;
- its ideas should be self-consistent;
- have as broad a scope as possible and so encompass a wide range of phenomena, i.e., capable of being linked with other branches of science;
- it should be quantitative and its ideas expressible by mathematics;
- predict new relationships and offer scope for further development;
- be as simple as possible, with a minimum number of hypotheses (Ockham Razor).

There are a number of excellent examples to show that neglect of falsifiability was beneficial to science:

(I) Copernicus' theory about the movement of planets had difficulties with the phases of Venus, and these difficulties were resolved only with Galileo's telescope, more than fifty years later. Galileo considered it praiseworthy in Copernicus that he had not permitted one unexplained puzzle to worry him.

(II) Around 1910 there arose a famous disagreement between Robert A. Millikan in Chicago, and F. Ehrenhaft in Vienna: the latter had reported finding charges of only a fraction of that expected to be carried by the electron. Millikan indeed rejected data that did not fit his basic assumption, but justified it on the basis of his experimental skills, and we know that he was right.

Thus falsification may fail as a criteria because experiments are sometimes wrong. In this connection, one must keep in mind the remark of Francis Crick, In his philosophical and autobiographical work, Popper credits himself for having invented the idea that we acquire knowledge by trial and error, yet the relation between philosophy and psychology in Popper's work is always fraught with tension. The earliest traces to Popper's '*searchlight theory*' of mind and knowledge are to be found in his unpublished dissertation *Zur Methodenfrage der Denkpsychologie* (1928).

According to Michel ter Hark[799] (2004), scrutiny of this manuscript reveals the formative influence of **Otto Selz**[800] (1913). Indeed he claims that Popper borrowed his crucially important 'Searchlight theory' from Selz, and demonstrates that Popper's philosophy of science, with its emphasis on the method of trial and error, is largely based on the psychology of Otto Selz, whose theory of problem solving and scientific discovery laid the foundation for much of contemporary cognitive psychology. By arguing that Popper's famous defense of the method of falsification as well as his elaboration of an evolutionary theory of knowledge are equally indebted to German psychology, Michel ter Hark challenges the received view of the development of Popper's philosophy.

Popper was born in Vienna to converted Jewish parents. After studying mathematics, physics and psychology at the University of Vienna, he taught philosophy at Canterbury University College, New Zealand (1937–1945). He was a professor of logic and scientific method during 1949–1969.

"*A theory that fits all the facts is bound to be wrong as some of the facts will be wrong*". In addition, falsification can itself be false; there is no guarantee that the experimental falsification will not itself turn out to be flawed; for example, the initial experiments carried to test the Weinberg-Salam theory (on the unification of electromagnetism and weak nuclear forces) showed that the theory was wrong. Only later experiments showed that the initial experiments were themselves wrong and the theory was confirmed.

Falsifiability is therefore a necessary but not a sufficient criterion. It is just one aspect of science.

799 **Michel ter Hark**:"Popper, Otto Selz and the Rise of Evolutionary Epistemology", Cambridge University Press 2004, 262 pp.

800 **Selz, Otto**: *Über die Gesetze des geordneten Denkverlaufs* Spemann, Stuttgart (1913); *Zur Psychologie des produktiven Denkens und des Irrtums*, Friedrich Cohen Bonn (1922).

Modern and Modernistic Philosophy (1750–1930)

In the medieval world, original meaning was processed by authority *(e.g. the Catholic Church) and the individual was dominated by* tradition*. The collapse of this worldview was caused by a variety of different factors:*

- *The perceived corruption of the* Catholic Church *hierarchy.*
- *The rise of market economy following the increase of wealth and power of a new '*merchant*' class.*
- *The new* astronomical *discoveries of Copernicus and Galileo.*
- *The new* geographical *discoveries of the Spanish, Portuguese, Dutch and British voyagers.*

All these played their part in undermining the religious and metaphysical foundations of Christian Europe. With the scientific revolution, Europeans lost faith in religion and found new hope in science *and* humanism*. Both of these seemed to offer more rational ways of achieving salvation. The new world was to be governed by the truths of science, based on careful examination, manipulation and observation of nature within a mathematical epistemological framework.*

There was to be no place for philosophy *in this new world of scientific modernity. From now on,* analysis*,* method *and* technique *would be the new gods, and the wonder and curiosity of the philosophers was to be replaced with a new kind of question. The question "*What is it?*" was now subordinated to the more technical question: "*How does it work?*". So philosophy became concerned with providing the metaphysical and epistemological supports for the new secular order of modern technocracy.*

The intellectual opposition to the medieval worldview began in the 15th century, during the Renaissance*, with the Humanist ideas of* ***Desiderius Erasmus*** *(1466–1536),* ***Niccolo Machiavelli*** *(1469–1527), and* ***Michel de Montaigne*** *(1533–1592). Each of these thinkers accepted the idea of* ***Protagoras*** *(c. 491–421 BCE) that our own human world is the only world in existence, and that we can make of this world what we will.*

At the start of the 16th century, the growing problems facing the Catholic Church were further compounded by ***Martin Luther*** *(1483–1546), who was about to begin a personal crusade to reform Christianity. Luther added to* this a new sense of individualism when he claimed that Christian belief was a matter of personal faith rather than objective truth.

The "modern" era was boosted by the European Enlightenment and the advent of the Industrial Revolution in England anchored in the development of the steam-driven motor at ca 1750. Although historians have traced elements of enlightenment back to the Renaissance, one can argue that Enlightenment thinking begins with the 18^{th} century.

The basic ideas of the Enlightenment are roughly the same as the basic ideas of humanism. Its main premises are:

- Man's self is knowable, rational, autonomous and universal. This self knows itself and the world through reason, or rationality, posited as the highest form of mental functioning, and the only objective form.
- The mode of knowledge produced by the objective rational self is science, which can provide universal truths about the world, regardless of the individual status of the knower. The knowledge provided by science is truth, and is eternal. The knowledge produced by science will always lead toward progress and perfection. All human institutions and practices can be analyzed by science and improved. Science thus stands as the paradigm for any and all socially useful forms of knowledge. Science is neutral and objective; scientists (those who produce scientific knowledge through their unbiased rational capacities) must be free to follow the laws of reason, and not be motivated by other concerns (such as money or power).
- Reason is the ultimate judge of what is true, and therefore of what is right and what is good (what is legal and what is ethical). Freedom consists of obedience to the laws that conform to the knowledge discovered by reason. In a world governed by reason, the true will always be the same as the good and the right (and the beautiful); there can be no conflict between what is true and what is right.
- Language, or the mode of expression used in producing and disseminating knowledge, must be rational also. To be rational, language must be transparent; it must function only to represent the real (perceivable) world which the rational mind observes. There must be a firm and objective connection between the objects of perception and the words used to name them.

These tenets served to justify and explain virtually all social structures and institutions, including democracy, law, science, ethics, and aesthetics. Moreover, the ensuing powerful and successful approach to nature and culture has *come to dominate the modern university and our social, economic, moral, and cognitive structures. Human reason, as exemplified in the deductive thought of mathematics and physics, would come to replace the superstitious world-views of religion and other forms of irrationality. Reason, science, technology, and bureaucratic management would improve our knowledge, wealth, and well-being through the rational control of nature and society.*

The most important philosopher to emerge from the 18th century philosophical movement known as the Enlightenment was ***Immanuel Kant*** *(Table 5.16). His philosophy tries to combine* scientific rationalism *(i.e. the prerequisite that science be built upon rational principles with the notion that the mind is* actively involved *in the objects it experiences. That is, it organizes experience into definite patterns, creating its own world through the powers of autonomous rational judgment.*

In the wake of Kant's philosophy came the romantics, who viewed both mind and nature as unified and saw art and aesthetic interpretation as the source of all true knowledge. For the romantics, such as ***Friedrich von Schelling*** *(Table 5.16) and the poet* ***Friedrich Schiller*** *(1759–1805) the genuine bringer of knowledge was the artistic genius rather the experimental scientist.*

*By the beginning of the 19th century, various philosophers made new attempts to combine Rationalism and Romanticism into a new '*higher order*' philosophy. The most famous attempt to achieve this new synthesis between seemingly competing philosophies can be seen in the works of the German philosopher* ***Georg Wilhelm Friedrich Hegel*** *(Table 5.16). Hegel accepted Kant's idealism and viewed reality as the product of the activity of a rational mind. however, for Hegel mind was a kind of* universal spirit *('geist') that moved through time and space. Reason was viewed as the underlying principle that governed the movement of this spirit through history. According to Hegel,* history *is a social process driven by contradictions between competing systems of ideas.* Historical changes *produce new forms of knowledge via a process of* thesis*,* antithesis*, and, finally, a new* synthesis*.*

Table 5.16: Leading Western Philosophers and Thinkers[801] 1637–1992

Philosopher	Nationality	Life-span	Publication of Major Work	Philosophy System
Thomas Hobbes	British	1588–1679	1651	Early Rationalism
Rene Descartes	French	1596–1650	1637	Rationalism
Blaise Pascal	French	1623–1662		
Baruch Spinoza	Jewish	1632–1677	1677	Rationalism
John Locke	British	1632–1704	1690	Empiricism
Isaac Newton	British	1642–1727		of physics
Gottfried Leibniz	German	1646–1716	1687	Rationalism
George Berkeley	British	1658–1753	1710	Empirical Idealism
Giambattista Vico	Italian	1668–1744		of historicism
Baron de Montesquieu	French	1689–1755		
Voltaire	French	1694–1778	1764	Empiricism
David Hume	British	1711–1776	1748	Empirical Skepticism
Jean J. Rouseau	French	1712–1778	1762	Naturalism
Adam Smith	American	1723–1790		of Political Economy

801 This Table includes not only *traditional philosophers* but also *thinkers* in the fields of sociology, political science, economics, history, mathematics, logic, physics, anthropology, linguistics and biology, as well as a few novelists and poets who impacted Western society via their deep insight into the human condition.

Table 5.16: (Cont.)

Philosopher	Nationality	Life-span	Publication of Major Work	Philosophy System
Immanuel Kant	German	1724–1804	1781	Idealism
Edmund Burke	British	1729–1804	1757	
Jeremy Bentham	British	1748–1832	1789	Utilitarianism
Johann Wolfgang von Goethe	German	1749–1832		poet-novelist
Friedrich Schiller	German	1759–1805		poet
Johann G. Fichte	German	1762–1814	1792	Idealism
Georg W.F. Hegel	German	1770–1831	1807	Idealism
Friedrich Hölderlin	German	1770–1843		poet
Friedrich W.J. von Schelling	German	1775–1854	1809	Idealism
Heinrich von Kleist	German	1777–1811		novelist
Bernard Bolzano		1781–1848		of mathematics
Arthur Schopenhauer	German	1788–1860	1836	Idealism
Thomas Carlyle	British	1795–1881		of history
Auguste Comte	French	1798–1857	1842	Classical Positivism

Table 5.16: (Cont.)

PHILOSOPHER	NATIONALITY	LIFE-SPAN	PUBLICATION OF MAJOR WORK	PHILOSOPHY SYSTEM
Ralph Waldo Emerson	American	1803–1882	1844	Transcendentalism
John S. Mill	British	1806–1873	1843	Utilitarianism
Charles Darwin	British	1809–1882	1859	Evolutionary Naturalism
Soren Kierkegaard	Danish	1813–1855	1844	Existentialism
Karl Marx	Jewish	1818–1883	1867	Dialectical Materialism
Herbert Spencer	British	1820–1903	1851	Evolutionary Naturalism
Oswald Spengler	German			
Leo Tolstoy	Russian	1828–1910		novelist
Ernst Mach	German			of physics
James Clerk Maxwell	British	1831–1879		of physics
Charles S. Peirce	American	1839–1914	1878	Pragmatism
William James	American	1842–1910	1907	Pragmatism
Friedrich Nietzshe	German	1844–1900	1886	Protestantism and Paganism
Ludwig Boltzmann	German	1844–1906		of physics

Table 5.16: (Cont.)

Philosopher	Nationality	Life-span	Publication of Major Work	Philosophy System
Georg Cantor	Jewish	1845–1918	1872	of Mathematics
Gotllob Frege	German	1848–1925	1879–1903	of Logic
Sigmund Freud	Jewish	1856–1939	1904, 1930	Science of the Mind
Georg Simmel	Jewish	1858–1918	1900–1918	of Sociology
Edmund Husserl	Jewish	1859–1938	1913	Phenomenologism
Henri Bergson	Jewish	1859–1941	1907	Evolutionary Naturalism
John Dewey	American	1859–1952	1910	Pragmatism
Benedetto Croce	Italian	1866–1952	1909	Neo-Idealism
Bertrand Russell	British	1872–1970	1903–1913	Critical Realism
Rainer Maria Rilke	German	1875–1926	1907–1923	poet
Albert Einstein	Jewish American	1879–1955	1905–1917	of physics
Oswald Spengler	German	1880–1936	1918–1922	of history
Ludwig von Mises	Austrian	1881–1973		of Economics and Libertarianism
Moritz Schlick	German	1882–1936	1936	Logical Positivism

Table 5.16: (Cont.)

Philosopher	Nationality	Life-span	Publication of Major Work	Philosophy System
Franz Kafka	Jewish	1883–1924	1915–1924	novelist
John Maynard Keynes	British	1883–1946	1936–1946	of Economics
Ortega y Gasset	Spanish	1883–1955	1929	of history
Karl Jaspers	German	1883–1969	1935	Existentialism
Erwin Schrödinger	Austrian	1887–1961		of physics and biology
Fernando Pessoa	Jewish-Portuguese	1888–1935	1914–1928	poet
Ludwig Wittgenstein	Jewish	1889–1951	1921	of Language
Martin Heidegger	German	1889–1976	1927	Existentialism
Friedrich A. von Hayek	Austrian	1889–1992		of Economics and Libertarianism
Rudolf Carnap	German	1891–1970	1928	Logical Positivism
Max Horkheimer	Jewish	1895–1973	1947	of Social Science
Herbert Marcuse	Jewish American	1898–1979	1954–1964	of Social Sciences
Leo Strauss	Jewish American	1899–1973	1959	of Social Science

Table 5.16: (Cont.)

Philosopher	Nationality	Life-span	Publication of Major Work	Philosophy System
Werner Heisenberg	German	1901–1976		of physics
Eric Hoffer	Jewish American	1902–1983	1951	of Social Science
Karl Popper	Jewish British	1902–1994	1937	of Science
John von Neumann	Jewish	1903–1957		of mathematics
Hans Jonas	Jewish American	1903–1993	1966–1979	of biology and technology
Jean–Paul Sartre	French	1905–1980	1946	Existentialism
Ayn Rand	Jewish American	1905–1982		of Objectivism
Kurt Gödel	German	1906–1978	1930	of Mathematics
Marshall McLuhan	Canadian	1911–1968	1964	of Communication Technology
Emil Cioran	Romanian	1911–1995	1956–1974	of history and life
Alan Turing	British	1912–1954		of mathematics
Barbara Tuchman	Jewish American	1912–1989	1938–1988	of history
Allan Bloom	Jewish American	1930–1992		of Political Science
Alfred J. Ayer	British	1910–1989		of Logical Positivism

Modernity is fundamentally about order, *creating order out of chaos, and about the pursuit of ever increasing levels of order. The more ordered a society is, the better it will function. This inherent quest for order led to search for* fundamental base-superstructures *in mathematical, physical, social and biological systems. Indeed,* **Charles Darwin** *theory of evolution (1859) presented a base-superstructure through which all 'higher' form of life are necessarily based, in causative formation, on 'lower' forms of life, which are all structured by hidden laws that are not self-evident.*

Karl Marx *(1867) introduces a base-superstructure in his discussion of the fundamental economic laws of a society and the consequent social institutions and philosophies built upon that materialistic foundation.*

Sigmund Freud *(1904) also used a base-superstructure metaphor in his explication of the foundational structures of the human psyche from which the limits and possibilities of human life ensue.*

Thus, just as science was able to prove much in nature that was counterintuitive, like the earth moving around the sun, the new social sciences of economics, psychology, anthropology, *and* sociology *would unveil the true nature of individual beliefs and social structures as causationally derived from some foundational base.*

The modernists were and are mostly hostile to religion, *because it represents a form of immovable unreason and dangerous irrationality. They envision a world freed of religious superstition. This vision profoundly influenced the culture of modern science and the secular university.*

What began with **Darwin**, ***Marx*** *and* **Freud**, *continued in the* 20^{th} *century intellectual history, as new disciplines and sub-disciplinary schools asserted their own foundational causative categories from which all else ensues.*

In conclusion, modernity *is equated with the enlightenment-humanist rejection of tradition and authority in favor of reason and natural science. This was founded upon the assumption of the autonomous individual as the sole source of meaning and truth – the Cartesian 'cogito'. Progress and novelty are valued in a 'real' world that evolves towards a state of increased objectivity. According to Sociologists, this new secular order began with the rise of a* market economy *and the growth of a* centralized bureaucratic state. *Both of these institutions had no need for philosophical wisdom as such. Instead, the burgeoning state and market required accurate information and faster means of communication and energy transport.*

Friedrich Nietzsche *was the first great philosopher after the revolution in Western thought brought about by the work of* **Charles Darwin** *(1859), who claimed that humans, rather than being created in God's image, were the evolutionary cousins of monkeys and apes. For Nietzsche this was devastating* *news and his whole philosophy can be seen as an attempt to answer this one question: How do we live in a world without something (a God) that guarantees that life has meaning?*

Nietzsche was one of the chief prophets of the modern age. He accurately predicted that life in the 20^{th} century would be a Perilous time; in a world without God, people would follow anyone or anything that offered them some sense of personal worth in a universe increasingly perceived to be devoid of significance. Nietzsche warned of the dangers that lay in the future, but he harbored little faith in the moral abilities of the majority of ordinary people. Unlike Marx he denied that any hope for a better future lay with the proletariat. For Nietzsche, the 20^{th} century would be the age of the false prophet who offers salvation but merely manipulates people to their doom. In the light of the atrocities committed by Hitler and Stalin, this is a truly remarkable prediction. Nietzsche's death in 1900 marked the end of philosophical romanticism.

***Sigmund Freud** (1856–1939) accepted the romantic critiques of modern society, but he believed that had found a scientific way of dealing with them. What we need, he thought, was a* new science of the mind *(psychoanalysis) that was more sensitive to hidden, irrational depths of the psyche. He believed that the rationalists and empiricist had only offered models of the* conscious mind *(ego) and had ignored the existence of the* unconscious mind *(id). Freud saw the rational part of the psyche as a thin veneer covering an older and much more unpredictable irrational part. For Freud, the unconscious represents the 'animal' within us: the part of us that is impulsive, instinctive, and demands immediate gratification. According to Freud, the mind is in conflict with itself and that all we can hope for is to accept the "demon within us" and bring it under rational forms of control.*

In his philosophical work: "Civilization and its Discontents" (1930) he argues that our most basic desires can never be satisfied and that the Marxist idea of an ideal communist society is nothing more than a silly wish; There is no easy way to live with our unconscious desires and that every man must find out for himself in what particular fashion he can be saved. Thus, in the end, psychoanalysis became an odd kind of moral philosophy.

Freud's ideas represent a technocratic answer to the problem raised by the 19^{th} century. During part of the 20^{th} century, this was the trend of much of intellectual thinking, and many philosophers believed that science could meet all the difficulties of modern life that the romantics had identified.

An important development in philosophy was stimulated by questions about the foundations of mathematics: *the system of logic laid down by Aristotle remained unaltered in its essentials until the 19^{th} century. By that time* *logic had come to be thought of as consisting of laws that govern thought. The works of* ***Boole*** *(1847),* ***A. de Morgan*** *(1847),* ***Cantor*** *(1872),* ***C.S. Peirce*** *(1878) and* ***Peano*** *(1886) heralded the merger of logic and mathematics at the fringes. Since the time of* ***Kant****, who had considered logic as complete, great changes had occurred in the study of logical theory. In particular, new forms of treating logical arguments by means of mathematical formulae had been developed. The first systematic account of this new way of dealing with logic in due to* ***Frege*** *(1892).*

Many of Frege's ideas were first transmitted by other people, including Peano. As a founder of symbolic logic*, Peano created his own logic notation (as did Frege) and established the basic elements of geometric calculus. Peano also invented his own international auxiliary language 'Iterlingua', which was a fusion of vocabulary from Latin, French, German and English.*

The axioms of Peano, for all their economy, were nevertheless unsatisfactory from a logical point of view, for it seemed somewhat arbitrary that it should be these rather than some other statements that were the basis of mathematical science. Frege then set to exhibit the axioms of Peano as a logical consequence of his symbolic system.

Frege is also one of the founders of linguistic philosophy *(*philosophy of language[802]*) which is the attempt to uncover the logical structure of* all *human languages. The fundamental recognition here is that logical relationships are independent of human thought, ergo: logical prepositions are* objective *truths, the existence of which has nothing to do with any feature of human thinking. Once all meaningful human languages could be reduced to logical formulas (i.e. abstract symbolic expression that look something like algebra), human language could be systematized, opening the way for the modern science of languages (linguistics).*

When this insight was applied to general philosophy it had momentous consequences. Since Descartes, Western philosophy grappled with the question: "What can I know?". Theory of knowledge (epistemology) had been at the center; and this was taken to mean that what went on in people's minds was the main subject of investigation. But Frege's insight had the consequence of de-psychologizing philosophy. If what is the case, and what followed from

[802] *Philosophy of language* is the study of philosophical questions about language, especially about *meaning and truth* (of words, phrases and sentences) in general. Grammar, books and dictionaries only *codify* how we use language, but meaning and truth of linguistic statements shape an entire view of the universe and our place in it. Thus, in arriving at our present philosophical outlook, questions about meaning play a centrally important role.

What, are both independent of the human mind, then our attempts to understand the world cannot legitimately center on epistemology.

The clear implication is that philosophy ought to be logic-based, not epistemology-based[803]*. Indeed, Frege's work precipitated changes in that direction which continued unabated in many of the main areas of philosophy throughout the 20th century.*

The work of Frege was completely ignored for twenty years and in his own country he long remained an obscure professor of mathematics. However, his new philosophy of language would influence ***Bertrand Russell*** *(1913) and* ***Ludwig Wittgenstein*** *(1921).*

Modernistic Philosophy

Modernism started as a movement in visual arts, music, architecture, literature and drama at the turn of the 20th century. It rejected the Victorian standards of how art should be made, consumed and what it should mean. Figures like ***Picasso****,* ***Proust****,* ***Kafka****,* ***Rilke*** *and* ***e.e. Cummings*** *are some of the founders of the 20th century modernism. From a literary perspective, the main characteristics of modernism include:*

- *An emphasis on impressionism, subjectivity and on* how *seeing and perception takes place rather than on* what *is perceived. A movement away from the apparent objectivity provided by omniscient third-person narrators, fixed narrative point of view, and clear-cut moral positions.*
- *A blurring of distinction between genres, so that poetry seems more documentary and prose seems more poetic.*
- *An emphasis on fragmented forms, discontinuous narrative, and random-seeming collages of different materials.*

[803] He helped to change the agenda of modern philosophy from the problem of *knowledge* to the even more fundamental one of *meaning*. Frege stressed that everyday grammatical language is not logical and that logic itself in independent of psychology. Language itself has *two* different functions: First, it consists of '*sense*' or meaning, that which we understand. Second, it '*refers*' to things and concepts. The *sense* of a language is a public phenomenon based on convention and can change, but *reference* is to truth or falsehood. Frege went on to found a complex system of logic based on this insight.

- *A tendency toward reflexivity, or self-consciousness, about the production of the work of art, so that each piece calls attention to its own status as a production.*

The modernistic movement fed on socio-economical changes induced by the major advancements made in physics and astronomy, WWI, monopoly capitalism (associated with electric and internal-combustion-motor industries) and the impact of Freud's new psychoanalysis. Its manifestations in philosophy came through the Analytic Philosophy *founded during the period between the two World Wars (1924–1937) in Western Europe.*

The 20th century was the first since the Middle Ages in which all the leading philosophers were academics. Partly as a result of this, there was a growth of concern with analysis. *In logical analysis and linguistic analysis important development occurred. Otherwise the biggest advances were on two fronts: one was a response to 20th century science, which compelled a reappraisal of the nature of human knowledge as such. The other was an attempt to understand the human condition in a universe no longer seen as created by God, or as having any meaning or purpose of its own.*

A revolutionary school of 20th century philosophy, rejecting traditional points of view is that of Analytic Philosophy *pioneered by* ***Russell*** *(1910–1913) and* ***Wittgenstein*** *(1921). Its adherers contend that the entire business of philosophy is that of analysis. As such, philosophy is devoid of content in the sense that* it does not add to the scope of scientific knowledge, *but instead consists of linguistic activity designed to eliminate problems and perplexities arising form intellectual confusion or misunderstanding and thus* to clarify knowledge which we already possess. *It is no longer to be the task of philosophy to search for ultimate or metaphysical truth; the metaphysical quest which originated with* ***Descartes*** *is to be replaced by the radically different task of philosophical analysis undertaken by Analytic Philosophy as a* nonmetaphysical *school of thought. Thus, Analytic Philosophy represents a reaction against the Idealists' synthesis and concepts of Absolute Reality originating with* ***Hegel***.

Analytic Philosophy came close to dominate philosophy in the English-speaking world for most of the 20th century. In the course of this time it took different forms, but common to them all was the close analysis of propositions, *or of the individual terms and concepts they employed, or of their logical implications both internal and external, with a view to bringing everything that was hidden in them to the surface. The overall question always was: "What are we really saying when we say so-and-so?".*

Among the groups that took up Russell's approach and developed it was one that came into existence in Vienna (1924–1936) and became known as *the Vienna Circle. It consisted more of scientists and mathematicians than philosophers, and its chief concern was to establish the philosophical foundations of a scientific worldview. Theirs was a philosophy that became known as Logical Positivism*[804]*. It considered that the true meaning of a statement was uncovered when we asked ourselves: "What would we have to do to establish the truth or falsehood of this statement?". In other words, what observable difference does its truth or falsehood make to the way things actually are? Only statements that are empirically verifiable are empirically meaningful.*

On the contrary – a statement that purports to be about reality but whose truth or falsehood makes no observable difference to anything, has no meaning[805]*. They thus concluded that all philosophy, especially Hegelian idealism, was metaphysical nonsense. They also thought that the "surface grammar" of language had led philosophers into endless, unsolvable pseudo-debates about imaginary entities like the "substances" of Spinoza and Leibniz.*

Logical Positivists thought that there was no such thing as "philosophical knowledge" – that road to real knowledge was only via science. Philosophy could only be an analytic activity which clarifies concepts and cleared up linguistic conclusions.

However, as a theory of meaning, the 'verification principle' of the Logical Positivists collapsed fairly quickly, partly because a lot of modern science is conceptual *and untestable in a simple "look and see" way. Meaning also has to be prior to testing, not a result of it. How can we test something if we don't understand it first?*

With the rise of the Nazis to power in Austria and Germany, the members of the Vienna Circle were scattered, mostly to the United States and Britain, where they exercised a major influence over a whole generation.

In his later works, **Wittgenstein** *set out to resolve the difficulty that the Logical Positivism had with their 'verification principle'. He showed (1951) that the great 20th century search for 'Meaning of Meaning' is futile because it is founded on the misconception that 'Meaning' is something separate from language. Instead, meaning is the result of socially agreed convention produced by 'forms of life' and cannot possibly be established outside a language.*

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[804] **Otto Selz** (1881–1943); **Moritz Schlick** (1882–1936); **Otto Neurath** (1882–1945); **Rudolf Carnap** (1891–1970); **Ludwig Wittgenstein** (1889–1951); **Alfred Tarski** (1902–1983); **Kurt Gödel** (1906–1978); **Karl Menger** (1902–1985); **Alfred Jules Ayer** (1910–1989); **Karl Popper** (1902–1994).

[805] So, "*God is absolute and eternal*" looks like sense, but is wholly untestable and therefore gibberish. Assertions about God, souls, immorality, moral values, aesthetic values, and universal substances (matter or spirit) cannot be accepted as a valid or invalid, true or false. Science in the only form of knowledge.

To him, philosophy is a critique of language — an activity which seeks the 'logical clarification of thoughts', the elucidation of propositions. Therefore, he argued that the way in which a word is used, *not its meaning as a name for some object, gives language and statements their validity: "Don't ask for the meaning, ask for the use".*

A different solution to the 'verification principle' was suggested by the philosopher of science ***Karl Popper*** *(1902–1994) in his work* Logic der Forschung *(The logic of Scientific Discovery, 1935); According to Popper, knowledge of the natural world never advances by direct confirmation of scientific theories, but only indirectly, through the systematic* falsification *of their alternatives by reference to our experience. He defended a realistic epistemology in his* Objective Knowledge *(1966).*

In the wake of the Einsteinian revolution (1905–1917) that toppled the Newtonian worldview, Popper realized that if the centuries of corroboration received by Newtonian science had not proved it to be true, nothing was ever going to prove the truth of a scientific theory. The so-called scientific laws were not incorrigible truths about the world after all; they were theories, and as such they were products of the human mind. If they worked well in their practical application then that meant they approximate the truth, yet it was always possible even after hundreds of years of pragmatic success, for someone to come along with a better theory that was closer to whatever the truth was. As ***Einstein*** *himself put it: "only daring speculation can lead us further, and not accumulation of facts".*

Popper developed this insight into a full-fledged theory of knowledge. *According to him, physical reality exists independently of the human mind, and is of a radically different order from human experience – and for that very reason can never be directly apprehended.*

It is impossible to prove, finally and forever, the truth of any scientific theory. But a theory can be disproved, *and this means that it can be tested. Thus, although no number of observations, however large, will ever prove the statement "All swans are white", a single observation of a black swan is enough to disprove it. So we can test general statements by searching for contrary instances. This being so, criticism becomes the chief means by which we do in fact make progress.*

A statement that no potential observation would falsify cannot be tested, and therefore cannot count as scientific, because if everything that could possible happen is compatible with its truth, then nothing can be regarded as *evidence for it*[806] *(e.g. the statement "God exists" is not a scientific statement).*

Postmodernism[807]

In the second half of the 20th *century (especially towards the mid 1980's) we witnessed the rise of an eclectic, nihilistic, irrational and* anti-science *movement known as* postmodernism. *It is a cultural formation which accompanies a particular stage of Capitalism, namely, consumer capitalism with its emphasis on marketing, selling and consuming commodities, not on producing them.*

While Modernism upheld the idea that works of science, art and literature can provide unity, coherence, and meaning (which had been lost in modern life), postmodernists claim that since the world is meaningless (to them), we should not pretend that art and science can endow it with meaning.

Until 1945, science could be been as the friend of mankind (with the exception of the 'gas chambers' annihilees, for whom chemistry was certainly not a best friend). Moderns medicine brought about a great increase not only in longevity but in the capacity to enjoy life physically. However, after the

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806 But as a scientific method, falsificationism has its own problems. If our observations of the world are themselves always "theory-laden", why should one observation immediately invalidate a complex scientific theory? How do we know which to trust? Scientific theories are complex and interdependent, so it is not always easy to falsify them with a single observation. History also reveals that scientists have often been very reluctant to jettison their pet theories because of one contradictory observation. Sometimes they have been quite right to be stubborn – but not always.

At any rate, as Popper himself argued, "Science is perhaps the only human activity in which errors are systematically criticized and in time corrected."

807 Few scholars have attempted to reduce the term 'Postmodernism' to an objective definition for fear that such definition becomes the immediate target for a Postmodern critique. The evasiveness of the leaders of this movement on this subject has been notorious: Pressed for an answer, one of their proponents, **Jacques Derrida**, a leftist avant-garde French theorist, said: "It is impossible to respond; I can only do something which will leave me unsatisfied". He was apparently following the advice of Wittgenstein: "*Of what one cannot speak, one must remain silent.*" .

explosion of the first atomic bomb its evident capacity to destroy humanity turned science into a potential enemy: for some, the prospect of a sudden cataclysmic end to all human life has destroyed the hope, slowly engendered through the 18th and 19th centuries, that science and research would bring a progressive increase in happiness.

Most generally, Postmodernism *is the abandonment of Enlightenment confidence in the achievement of human knowledge through reliance upon reason in pursuit of foundationalism, essentionalism, and realism. In philosophy, antimodernists typically express grave doubt about the possibility of universal objective truth, reject artificially sharp dichotomies, and delight in the inherent irony and particularity of language and life.*

Starting in the 1960's, computer technology emerged as a dominant force in many aspects of social life: The advent of electronic computer technologies began to revolutionize the modes of knowledge *production, distribution and consumption in our society. Social philosophers worried that anything which is not able to be translated into a form recognizable and storable by a computer (digitalizable) – will cease to be knowledge.*

In this paradigm, the opposite of 'knowledge' is not 'ignorance', but rather 'noise'. This attitude led eventually into a science phobia*, with the rise of religious fundamentalism in the Muslim world as one of its consequences.*

The horrors of WWII, coupled to fears of a nuclear annihilation of mankind, led certain philosophers (mostly European academicians) to the viewpoint that culture in modern times no longer provided meaning and purpose to people's lives, and that the most serious offender is empirical science. *This attitude which took shape in the 1970's might have just remained a European academic fad, were it not for other successive developments which gave it real substance:*

(1) The emerging of chaos *and* complexity *and their manifestation in nature.*

(2) The new cosmology, quantum field theories and their aim – the Theory Of Everything.

(3) The new progress in genetics, molecular biology, and neuroscience and their aims – the Human Genome project*, genetic therapy and engineering, etc.*

(4) The popularity of neo-conservatism *and rise of the Respectable Right in the U.S.A. in the 1970's.*

(5) The collapse of the Berlin Wall symbolizing the complete triumph of a free market economy *over a socialist command economy.*

Items (1), (2) and (3) have come under attack by critics from the disciplines of sociology, philosophy and history. In the academic world, most *professors of humanities are known to have little experience in mathematics, theoretical physics and theoretical biology, and hence have no credibility in passing judgment on highly specialized issues associated with physics, quantum theory, cosmology, nonlinear dynamics, complexity, computer science, molecular biology, etc.*[808]

Nevertheless such non-scientist as ***T. Adorno***[809]*;* ***G. Ryle*** *(1900–1976);* ***J. Derrida****;* ***R. Barthes*** *(1915–1980);* ***J. Lacan*** *(1901–1981);* ***F. Lyothard*** *(1924–1996);* ***M. Foucault*** *(1926–1984), and* ***J. Baudrillard****, pompously announced that the scientific method is little short of myth and that scientific knowledge is in fact manufactured. Furthermore, the self-styled Dadaist* ***Paul Feyerabend*** *claimed that science can be said to be in a condition of* anarchy *(sic!)*

No branch of human knowledge escapes from this radical, nihilistic and irrational corrosive outlook, according to which science and logic are accused of being "constructs" – merely interpretations of experience. There is no timeless and universal reality, and no certain knowledge of it either.

It is obvious that the postmodern anti-science movement stands on feet of clay and will eventually take its place in the dustbin of intellectual history.

Neo-Marxism

In the third quarter of the 20th century there arose a new trend in Western philosophy which integrated into a comprehensive framework of social theory and philosophy the following disciplines:

- *German philosophical thought of* ***Kant****,* ***Schelling****,* ***Hegel*** *and* ***Husserl****.*

808 Today, the sheer mass of scientific knowledge is beyond individual comprehension, despite the far higher level of general education. We have to some extent returned to the situations of primitive man who required myths and mysteries as protection against forces which he could not fully understand or control.

809 **Theodor Adorno** (1903–1969, Germany), born Theodor Wisengrund in Frankfurt, was a radical Marxist Jewish intellectual. He set up the *Frankfurt School for social research* (1923). This school tried to blend Marxist philosophy and Freudian philosophy into a critical theory of society. Adorno fully expected a more free and just society to emerge from the economic ruins of 1930's Germany; the rise of Nazism both shocked and horrified him. After the Holocaust his philosophy became very influenced by Judaism.

- *Marxist tradition as well as the critical neo-Marxist theory of the Frankfurt School as expounded by* ***Horkheimer***, ***Adorno*** and ***Marcuse***.
- *The sociological theories of* ***Weber*** and ***Durkheim***.
- *The linguistic philosophy of* ***Wittgenstein*** and ***Searle***.
- *The American pragmatist tradition of* ***Peirce*** and ***Dewey***.

The proponent of this scheme was ***Jürgen Habermas*** *(b. 1929), a social philosopher at the Frankfurt School, who developed the concept and theory of communicative rationality in his magnum opus, The Theory of Communicative Action (1981). It distinguishes itself from the rationalist tradition by locating rationality in structures of interpersonal linguistic communication rather than in the structure of either the cosmos or the knowing subject. He carries forward the tradition of Kant and the Enlightenment and of democratic socialism through his emphasis on the potential for transforming the world and arriving at a more humane, just, and egalitarian society through the realization of the human potential for reason.*

While postmodernists have "deconstructed" such long-treasured notions as "reason" and "justice" [claiming that "reason" is a name the powerful give to their rationales for holding power and "justice" is just an excuse for the majority to impose its morality on the minority], neo-marxists such as Habermas, disagree.

However, in the aftermath of 9/11, Derrida and Habermas *established a political solidarity by issuing together the book "Philosophy in a Time of Terror" — a plan for common European foreign policy.*

Modern Conservatism

Modern conservatism[810] *is a politico–economical philosophy rooted in the economic theories of the libertarians* ***Ludwig von Mises*** *(1881–1973, Austria),* ***Friedrich Hayek*** *(1889–1992, Austria) and* ***Milton Friedman*** *(1912–2006, USA).*

810 To dig deeper, see:

- Kurtz, P.W. (Ed), *American Philosophy in the Twentieth Century*, Macmillan, 1966.
- Brown, S. (Ed.), *Biographical Dictionary of Twentieth–Century philosophers*, Routledge, 878 pp.

It emerged from the rejection of social liberalism and the New Left counterculture of the 1960's, developing in the works and thoughts of the Americans intellectuals ***Leo Strauss*** *(1899–1973),* ***Ayn Rand***[811] *(1905–1982),* ***Barry Goldwater***[812] *(1909–1998) (who fused libertarianism with conservatism in*

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[811] **Ayn Rand** (1905–1982, USA; born Alisa Rosenbaum) was an American novelist and social philosopher; founder of *objectivism*. Immigrated from Russia (1926).

The social philosophy evolved by Rand, and explicated in her novels and essays, became known as *Objectivism*, which prescribed individualism with secular morality. It is a particularly pure form of small–government (*minarchic*) Libertarianism, which extends beyond economics and the proper functions of democratic governance to encompass all aspects of the Social Contract (as Rousseau called the contract among individuals that defines the State), and even aspects of morality, ethics, aesthetics, epistemology (the theory of knowledge) and metaphysics.

Rand emphasized the sanctity of the inviolate, rational mind of the *individual*; free individuals interact and form voluntary associations, in which they trade their best intellectual and economic achievements to mutual profit and satisfaction. There are also personal relationships of other kinds, of course – friendship, family, love – in which what is being traded is less concrete. But in all cases, *enlightened, rational self–interest* serves as a guide, and no person should be coerced by others (individually or collectively). Unlike most philosophers, Rand was entirely and implacably opposed to *religion* and *altruism*, seeing no redeeming value in them. Altruism is merely a scheme for making individuals into 'sacrificial animals', for various nebulous collective purposes (or to support other individuals who prefer to live off their more capable, motivated or talented fellows by inducing feelings of *guilt*).

[812] Fascist and communist tyranny has been supported all during the past 75 years by the modern left through its intellectuals and their newspapers, journals and periodicals; The *New York Times*, the *Guardian*, *The New Statesman* and the *Daily Mail* supported Stalin and are now calling for appeasement of Islamic fascism. *The Guardian* even supported the Khmer Rouge.

Among the ardent supporters of Lenin, Stalin and the Soviet mass–killings and purges: **Pablo Picasso**, **G.B. Shaw**, **Eric Hobsbaum**, **Pablo Neruda**, **Charles Chaplin**, **J-P Sartre**, **Graham Greene**, **Bertolt Brecht**, and many other leftist philosophers, intellectuals, journalists, writers and artists. **Clement Atlee** praised Stalin's Soviet Union in the 1930's and **Hobsbaum** supported Stalin's invasion of Finland and the soviet crushing of the Hungarian and Czech's Revolutions.

Among the supporters of Hitler were **Martin Heidegger**, **Knut Hamson**, **Salvador Dali**, **Ezra Pound** (actually worked for the fascists in WW2) and **Werner Heisenberg**. In France **J. Derrida** bemoaned the end of Soviet occu-

the 60's and 70's. His most famous quote: "Extremism in the defense of liberty is no vice, and moderation in the pursuit of justice is no virtue."), ***William F. Buckley*** *(1925–2008) and* ***Allan Bloom*** *(1930–1992). The ideas of these thinkers influenced the administrations of President* ***Ronald Reagan*** *and Premier* ***Margaret Thatcher****, representing a re-alignment in Western politico–economical philosophy, and the defection of many liberals to the right–hand side of the political spectrum.*

One accomplishment was to make criticism from the Right acceptable in the intellectual, artistic and journalistic circles, where conservatives had long been regarded with suspicion. In the U.S, neo-conservatism emphasizes foreign policy as paramount responsibility of government, seeing the need for the U.S. acting as the world sole superpower as indispensable to establishing and maintaining global order.

The Modern Left[813]

The historical Left pioneered on social issues of civil liberties such as rights for women, blacks, gays, atheists and other minorities. It opposed religious and sexual censorship, and enabled sexual freedom. All this however is uncontroversial now, since most of the modern right *agrees with this basic agenda.*

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pation of Eastern Europe, as did many other tenured academics living in comfort in the free West. **Harry Belafonte**, **Jesse Jackson**, **Norman Mailer** and H**arold Pinter** supported Cuban Communist dictatorship. **Nelson Mandela** supported Libya and Cuba. **Desmond Tuto** attacked the democracies of Israel and USA.

[813] To dig deeper, see:

- Lilla, Mark, *The Reckless Mind: Intellectuals and Politics*, Review Books: New York, 2001.
- Lilla, Mark, *The Stillborn God: Religion, Politics, and the Modern West*, Knopf, 2007.
- Bloom, Allan, *The Closing of the American Mind*, Simon and Schuster, 1987, 392 pp.
- Mirsky, Y., *From Fascism to Jihadism.*
- Johnson, Paul, *Modern Times.*

The main thing that defines the left in the modern world (new left) is its approach to foreign policy, with the economy *and* crime *being the lesser issues that distinguished them.*

*The left in France (**J.P. Sartre**; **M. Foucault**; **Barthes**; **A. Gide**; **J. Derrida**; **M. Merleau**; **G. Genet**; **G. Marcel**) was the cradle of late* 20^{th} *century tyranny:*

- *The Khmer Rouge trace their intellectual origins to France:* ***Khiew Samphan*** *and* ***Pol Pot*** *were both educated in France in the 1940s–1950s.*
- *The Baath Arab Socialist party of Saddam Hussein in Iraq traces it intellectual origins to France.*
- *The Iranian Islamist revolution traces its intellectual origin to human rights intellectuals in France: Ayatolla Khomeini was granted exile in France, where he openly denounced human rights and human freedom, plotted the Islamic takeover of Iran, and from where he returned triumphant in 1979.*
- *Ho Chi Minh learnt his Marxism in Paris, and was a member of the French Communist Party.*

Worldviews XLVI–XLVIII: Poet-Philosophers of the 20th Century: Rilke, Pessoa, Cummings

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“Who twines green leaves, worthless as common clods,
To wreaths of honor that stay always fresh?
Secure Olympus and unites the gods?
The strength of man, in poets become flesh.”
*(**Goethe**, Faust I, 154–157. Translated by Walter Kaufmann. Anchor Books, 1963)*

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***I: Rainer Maria Rilke** (1875–1926). German language’s greatest poet of the 20th century. His poetry is characterized by a dense, lyrical style and startling images that portray the complexities of modern life and their effect on human beings. It emphasizes human longing for completeness and for the absolute in face of deficiency, transience and emptiness due to changing flux of values and standards in an age of disbelief, solitude, and profound anxiety.*

Rilke was born in Prague. His father, Josef Rilke (1838–1906), became a railway official after an unsuccessful military career. His mother, Sophie (“Phia”) Entz (1851–1931) came from a well-to-do Prague manufacturing family (originally Jewish but later converted to Christianity to escape anti-semitism). His maternal grandmother Caroline née Kinzelberger (1828–1927) was also born in Prague[814].

[814] In his book: “*Rilke-Sein Leben, seine Welt, sein Werk*,” W. Leppmann states that Caroline’s grandmother, Theresia Mayerhof, was Jewish, and lived in Prague around 1775. We recall that all Jews were expelled from Prague in 1745 by Empress Maria Theresa, and Anti-Jewish riots broke again in 1848. Some Jews, at that time, converted to Christianity to escape persecution (see “*A History of Habsburg Jews*” by W.O. McCagg Jr.).
Rilke’s biographer, Ralph Freedman (“*Life of a Poet: Rainer Maria Rilke*,” Northwestern University Press, 1996) pointed out that Rilke’s contradictory relation to Jews was nurtured by a deeply prejudiced mother, with her pro-German dogmatism and cloudy religiosity.

Rilke attended a military academy (1886–1891) and studied literature and philosophy in Prague and Munich. He traveled to Italy (1898), Russia (1899, 1900) and Paris (1902–1910). On these trips he met ***Leo Tolstoy*** *and* ***Paul Cezanne****, and worked as a secretary for the sculpture* ***Auguste Rodin****. He married (1901) the sculptress Clara Westhoff (1878–1954) and they had a daughter Ruth (1901–1972). He spent the greater part of WWI in Munich, excluding six months of military service in Vienna.*

To escape the post-war chaos, he settled in Switzerland, where he died in 1926 of leukemia.

Rilke separated from his wife already in 1902, and during 1900–1926 he seeked intimate relations with many available free women. Some became his lovers and other remained just beloved; yet all of them served to stimulate and inspire his poetic creativity in one way or another.

[Lou Andreas-Salomé (1861–1937); Valeri von David-Rohnfeld (1871–1946); Paula Modershon-Becker (1876–1907); Magda von Hattinberg (1883–1959); Sidone Nadherny (1885–1950); Baladine Kossowska (1886–1969); Claire Studer-Goll (1891–1977); Mariana Tsvetaeva (1892–1941); Nimet Eloui Bey (1903–1943); Harriet Cohen (1895–1967); Loulou Albert-Lazard (1891–1969); Nany Wunderly-Volkart (1878–1962); Regina Ullmann (1884–1961); Ellen Delp (1890–1990); Marie Louise Dobrzensky (1889–1970); Helene von Hostitz (1876–1944); Katherina Kippenberg (1876–1947); Manon Solms-Laubach (1882–1975); Eva Solmitz-Cassirer (1884–1974); Yvone von Wattenwyl (1891–1976); Adelmina Romanelli; Mia (Maria) Mattauch; Elia Maria Nevar; Marthe Hennebert (b. 1887); Auguste (Gudi) Nölke; Jenny de Margerie; Jean de Sepibus; Julie von Nordeck; Lily Ziegler; Margot Sizzo-Noris; Key Ellen; Anna de Noailles; Clara Westhoff-Rilke; Maria von Thurn und Taxis.]

* * *

The only journy is the one within.

* * *

Many of the women Rilke loved were Jewish and he was attracted by a Jewish temperament and responded to it not only sexually but also intellectually. It can therefore be rightly assumed that this ambivalence is due to an inner atavistic conflict between two opposing elements of his personality — the Jewish and the German.

If your daily life seems poor, do not blame it; blame yourself that you are not poet enough to call forth its riches.

* * *

This is the miracle that happens everytime to those who really love: the more they give, the more they possess.

* * *

Live your questions now, and perhaps even without knowing it, you will live along some distant day into your answers.

* * *

There is more to marriage then four legs in a bed.

* * *

The future enters into us, in order to transform itself in us, long before it happens.

* * *

God, with me goes your meaning too.

* * *

In nature there are no loosers and winners — just survivors.

* * *

He who does not at some time, with definite determination, consent to the terribleness of life, or even exult in it, never takes possession of the inexpressible fullness of the power of our existence, but walks on the edge and will, sometime when the decision is made, have been neither alive nor dead.

* * *

God, are you the one who is living life?

* * *

The beast is free and has its death always behind it and God before it, and when it walks it goes toward eternity, as springs flow.

* * *

There are no classes in life for beginners; it is always the most difficult that is asked of one right away.

* * *

Works of art are indeed always products of having been in danger, of having gone to the very end in an experience, to where man can go no further.

* * *

It is our fate to be opposite and nothing else, and always opposite.

* * *

There is an ancient hatred between our normal life and the great work.

* * *

Who speaks of victory — enduring is everything.

* * *

This fleeting world keeps calling to us: once for each thing. Just once — no more, just once — and never again.

* * *

In one of my poems that is successful, there is much more reality than in any relationship or affection that I feel; where I create, I am true.... I may ask and seek for no other realizations than those in my work; there my house is, there are the women I need, and the children that will grow up and live a long time.

* * *

Only because we exclude death, when it breaks suddenly into our thoughts, has it become increasingly a stranger, and because we have kept it an alien, it has become an enemy.... It is conceivable that it is infinitely closer to us than life itself....

It has become more and more clear to me over the years that our effort can only be directed toward presuming the unity of life and death.... Death is a friend, our most intimate and only friend, who is never, never confused by our attitudes and vacillations, especially at a time when we most passionately and profoundly embrace earthly existence, activity, nature and love.

Life always says at the same time: Yes and No. He, death is the actual yes-sayer. He only says yes. In the face of eternity.

* * *

II: Fernando Antonio Nogueira Pessoa *(1888–1935, Portugal). The greatest Portuguese poet of the 20th century. Poet-philosopher Pessoa wrote his poetry under his own name and under that of three heteronymous selves whose biographies he invented. These heteronymous represent Pessoa's voyage of discovery of himself in the course of which he realized that extraordinary range of poetic values. Thus, his personality is diffused through the heteronyms. The strength of his overall poetry rests in his ability to convey a sense of loss, of sorrow for what can never be.*

The work which many consider his masterpiece, Livro do Desassossego *(The Book of Disquiet) was not published until 1982, almost 50 years after his death.*

Pessoa brings to mind the great solitaries: ***Kierkegaard***, ***Rousseau***, ***Schopenhauer*** *and* ***Montaigne***.

"Contradiction is the essence of the universe," Pessoa once wrote. He seems to have lived this apothegm. He was indeed a living contradiction. The dramatic coherences were for Pessoa ultimately the means by which he gave vent to the contradiction that was his life.

Pessoa was born in Lisbon, Portugal. He was descended from New Christians, i.e. Jews forcibly converted to Christianity in the 15^{th} century. His father died when he was five, a loss of lasting influence. In 1895 his mother remarried, and the family moved to Durban, South Africa, where his stepfather was Portuguese consul. He matriculated at the University of Cape Town, where he won the Queen Victoria Prize for English Essay. In 1905 he returned alone to Lisbon where he matriculated at the University. In Portugal he continued to read and write in English, which was the language of his youthful erotic verse. In 1918 he published in Lisbon his 35 Sonnets and in 1922 the three parts of his English Poems, all composed many years before. The Times Literary Supplement spoke of the 'ultra-Shakespearian Shakespearianisms' of this Portuguese writer.

The rest of Pessoa's life passed uneventfully in Lisbon. He earned a pittance from a number of commercial firms, composing free-verse or classical odes at the typewriter in the intervals of translating the firm's foreign correspondence. Many of the poems he wrote were published in literary reviews such as Orfeu *and* Portugal Futurista. *He also wrote much prose on questions of aesthetics, and sketches for detective novels. The only book published in his lifetime was* Mensagem, *a collection of poems on patriotic themes, which won only a consolation prize in a national competition.*

* * *

Contradiction is the essence of the universe.

* * *

Love is essential. Sex, mere accident.

* * *

To think about God is to disobey God, because he wanted us not to know.

* * *

I do not have a philosophy; If I speak of nature, it is not because I know what it is, but because I love it.

* * *

Wise is he who enjoys the show offered by the world.

* * *

Success consists in being successful, not in having a potential for success. Any wide piece of ground is the potential site of a palace, but there's no palace till it's built.

* * *

No intelligent idea can gain general acceptance unless some stupidity is mixed in with it.

* * *

Could it think, the heart would stop beating.

* * *

I have no ambitions or wants; to be a poet is not an ambition of mine. It is way of staying alone.

* * *

If, after I die, they should want to write my biography, there is nothing simpler. I have just two dates — of my birth, and of my death; In between the one thing and the other — all the days are mine.

* * *

After I was born, they locked me up inside me. But I left. My soul seeks me, through hills and valley; I hope my soul never finds me.

* * *

The light of the sun is worth more than the thoughts of all the philosophers and all the poets.

* * *

III: Edward Estlin Cummings *(1894–1962, USA). American poet, painter and essayist. His poetry often deals with themes of love and nature, as well as the relationship of the individual to the masses and the world. He wrote, however, in an unusual style, which includes unorthodox usage of both capitalization and punctuation, in which unexpected and seemingly misplaced punctuation sometimes interrupt sentences and even individual words.*

He was born in Cambridge, Massachusetts and raised in a liberal family. After graduating with a M.A. degree from Harvard (1916) he went to France (1917) as a volunteer for the Ambulance Corps in WWI. During 1921–1931 he traveled throughout Europe (including the Soviet Union) and spent several years in Paris.

After his father's tragic death (1926) he began to focus on more important aspects of life in his poetry.

Throughout 1919–1932 Cummings married and divorced three times. In 1952 Harvard University awarded him an honorary appointment as a guest professor.

* * *

It takes two to be serious.

* * *

Life is not a paragraph and death I think is no parenthesis.

* * *

Tomorrow is our permanent address.

* * *

I feel that (false and true are merely to know)
Love only has ever been, is, and will ever be, So.

* * *

Now you are and I am now and we are a mystery that will never happen again, a miracle which has never happened before.

* * *

Time's a strange fellow; more he gives than takes (and he takes all).

* * *

It's you are whatever a moon has always meant and whatever a sun will always sing is you.

* * *

A pretty girl who is naked, is worth a million statues.

* * *

Never mind a world with its villains or heroes (for god likes girls and tomorrow and the earth.

* * *

Life never grows old.

* * *

Life and day are only loaned: whereas night and death are given.

* * *

Greatness is alone.

* * *

I like the thrill of under me you so quite new.

* * *

Whenever men are right they are not young.

* * *

Spring — you and I may not hurry it with a thousand poems my darling, but nobody will stop it with all the policemen in the world.

Science Progress Report No. 16
Pseudoscience, Antiscience, Nonscience, Postmodernism, Scientific Illiteracy and Superstition, or — the absence of evidence is not evidence of absence

Science thrives on errors, cutting them away one by one. False conclusions are drawn all the time, but they are drawn tentatively. Hypotheses are framed so they are capable of being disproved. A succession of alternative hypotheses is confronted by experiment and observation. Science gropes and staggers toward improved understandings and disproof of scientific hypotheses is recognized as central to the scientific enterprise.

Pseudoscience is just the opposite. Hypotheses are often framed precisely so they are invulnerable to any experiment that offers a prospect of disproof, so even in principle they cannot be invalidated. Practitioners are defensive and wary. Skeptical scrutiny is opposed. When the pseudoscientific hypotheses fails to catch fire with scientists, conspiracies to suppress it are imagined and decried.

Perhaps the sharpest distinction between science and pseudoscience is that science has a far keener appreciation of human imperfections and fallibility than does pseudoscience. If we resolutely refuse to acknowledge where we are liable to fall into error, profound mistakes will forever be our companions.

In ancient China and Rome, astrology was the exclusive property of the emperor; any private use of this "potent" art was considered a capital offense.

In Russia, under the Tsars, religious superstition was encouraged, but scientific and skeptical thinking (except for a few tame scientists) was ruthlessly expunged.

Under Communism, both religion and pseudoscience were systematically suppressed, except for the superstition of the state ideological religion — which was advertised as scientific. Critical thinking — except by scientists in hermetically sealed compartments of knowledge — was recognized as dangerous, was not taught in the schools, and was punished where expressed.

When the lid was finally lifted, the subterranean bubblings of pseudoscience and superstition (as well as virulent ethnic hatreds) were exposed to view.

The region is now awash in UFOs, poltergeists, faith healers, quack medicines, magic waters, and old-time superstition. A stunning decline in life *expectancy, increasing infant mortality, rampant epidemic diseases, subminimal medical standards, and ignorance of preventive medicine all work to raise the threshold at which skepticism is triggered in an increasingly desperate population.*

In China, after the death of Mao tse-tung and the gradual emergence of market economy, UFO's, channeling and other examples of Western pseudoscience emerged, along with such ancient Chinese practices as ancestor worship, astrology and fortune telling — a revival of feudal ideology in the Chinese countryside. Individuals with "special powers" gained enormous followings[815]*.*

Asian Rhinos are being driven to extinction because their horns, when pulverized, are said to prevent impotence; the market encompasses all of East Asia.

Perhaps the most successful recent global pseudoscience (by many criteria, already a religion) is the Hindu doctrine of transcendental meditation (TM). This worldwide organization has an estimated valuation of three billion dollars. For fee they promise to be able to walk you through walls, to make you invisible, to enable you to fly. By thinking in unison they have, they say, diminished the crime rate in Washington D.C., and caused the collapse of the Soviet Union, among other secular miracles. No real evidence has been offered for any such claims. Yet, TM sells folk medicine, runs trading companies, medical clinics and "research universities", and has successfully entered politics.

Pseudoscience in America is part of the global trend. Here, psychics ply their wares on extended television commercials, personally endorsed by entertainers. They have their own channel, the "Psychic Friends Network"; a million people a year sign on and use such guidance in their everyday lives.

For CEOs of major corporations, financial analysts, lawyers and bankers — there is a species of astrologer / soothsayer / psychic ready to advise on any matter. Furthermore, TM seem to have attracted a large number of accomplished people, some with advanced degrees in physics or engineering.

The continuum stretching from ill-practiced science, pseudoscience, and superstition, all the way to respectable mystery religions, based on revelation, is indistinct.

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[815] Some have claimed they can project the "energy field of the universe" out of their bodies to change the molecular structure of a chemical 2000 kilometers away, to communicate with aliens, or cure diseases. An amateur chemist claimed to have synthesized a liquid, small amounts of which, when added to water, could convert it to gasoline. For a time he was funded by the army and the secret police, but when his invention was found to be a scam, he was imprisoned for his unwillingness to reveal his "secret formula" to the government!

Typical offerings of pseudoscience and superstition are:

- *Astrology.*
- *The Bermuda Triangle.*
- *"Big Foot" and the Loch Ness monster.*
- *Ghosts.*
- *"Evil Eye".*
- *Extrasensory perception (telepathy, precognition, telekinesis, "remote viewing" of distant places).*
- *Triskaidekaphobia: a fear of the number* 13 *or the belief that* 13 *is an "unlucky" number. This is the reason why so many no-nonsense office buildings and hotels in America pass directly from the* $12th$ *to the* $14th$ *floors (why take chances?). No contract or treaty signed on the* $13th$ *day which falls on a Friday* (14 *times in* 28 *years).*
- *Crop circles*[816].
- *The conviction that carrying the severed foot of a rabbit around with you brings good luck.*
- *The prophecies of Nostradamus.*
- *The notion that more crimes are committed when the moon is full.*
- *Palmistry.*
- *Numerology.*
- *Polygraphy.*
- *Comets, tea leaves (plus the ancient divinations accomplished by viewing entrails, smoke, shadows, shape of flames and excrement) — as harbingers of future events.*
- *"Photography" of past events (e.g. the crucifixion of Jesus).*
- *"Sensitives" who can blindfolded read books with their fingertips.*
- *Faith healing.*

[816] Scientists dismissed it as fraud on the basis that *real* extraterrestrials would choose to exhibit something much more exciting than mere dull unimaginative circles in wheat.

- *Water "remembering" which molecules used to be dissolved in it.*
- *Ouija boards.*
- *"Prophets", sleeping and awake.*
- *Out-of-body (e.g. near-death) experiences interpreted as real events in the external world.*
- *Telling character from facial features or bumps on the head (phrenology).*
- *3-cycle biorhythms.*
- *Perpetual motion machines.*
- *Professional "psychics" and their inept predictions.*
- *Jehovah's Witnesses' predictions.*
- *Dianetics and Scientology.*
- *Claims of finding the remaining of Noah's Ark*

Indeed, some claims are hard to test, e.g., if expedition fails to find the ghost of the brontosaurus, allegedly crashing through the rain forests of the Congo Republic, that does not mean it does not exist.

Astrology *has been with us for more than 4000 years. The fraction of U.S. school children believing in astrology rose from* 40 *percent to* 59 *between 1974 and 1984. Yet this takes place at an age when astrologers and scientists alike are aware that astrology*

- *accepts the precession of the equinoxes in announcing an "Age of Aquarius" while* rejecting *the precessing of the equinoxes in casting horoscopes;*
- neglects *atmospheric refraction;*
- *lists supposedly significant celestial objects that are mainly limited to naked eye objects known to Ptolemy in the second century, but* ignores *an enormous variety of new astronomical objects discovered since (e.g. near-earth asteroids);*
- Fails *to pass the identical-twin test;*
- inconsistently requires *detailed information on the time as compared to the latitude and longitude of birth;*

- *ignores major differences in horoscopes cast from the same birth information by different astrologers.*

Science carries us toward an understanding of how the world is, rather than how we wish it to be. When we shy away from it, we surrender the ability to take charge of our future. We are disenfranchised.

This plight is perhaps demonstrated most vividly in the case of scientific illiteracy. *It is hard to believe that more than 450 years after Copernicus:*

- *About* half *of American and Chinese adults do not know that the earth revolves around the sun and takes a year to do so.*
- Most *people on earth still think that our planet sits immobile at the center of the universe, and we are profoundly "special".*
- *Bright students at leading American universities did not know that stars rise and set at night, or even that the sun is a star.*
- *In the US more money is spent on quack medicine than on all of medical research.*
- *Only 9 percent of Americans accept the central findings of modern biology that human beings (and all other species) have slowly evolved by natural processes from a succession of simpler organisms with no divine intervention needed along the way.*
- *A* quarter *of Americans believe in reincarnation.*
- *Astrology seems more popular than ever. At least a quarter of all Americans "believe" in astrology. A third think sun-sign astrology*[817] *is "scientific". There are perhaps ten times more astrologers than astronomers in the United States. In France there are more astrologers than Roman Catholic clergy.*

1934–1971 CE **George Wald** (1906–1997, USA). Biochemist. Unraveled the nature of light-sensing molecules found in photoreceptor cells and

[817] The art of stereotyping people according to their time of birth and then using this bit of information to place them in a small number of previously constructed pigeonholes.

discovered that vitamin A is a vital ingredient of the pigments in the retina and, hence, important in maintaining vision.

With **Ragnar Granit** (1900–1991, Sweden) and **Haldan K. Hartline** (1903–1983, USA) awarded the Nobel Prize in Physiology or Medicine (1967) for his work on the *chemistry of vision*.

George Wald was born in New York, of immigrant parents. He received his B.Sc. from the New York University (1927) and his Ph.D. in Zoology at Columbia University (1932). He spent the next two years in Europe, doing postgraduate research under **Otto Warburg** (Berlin), **Paul Karrer** (Zürich) and **Otto Meyerhoff** (Heidelberg). It was there (1934) that Wald first identified vitamin A in the retina.

When Hitler came to power (Jan 1933), Germany was fast becoming a hostile country, especially for Jews, and both Meyerhoff and Wald were Jewish. He left Germany (Summer 1934) and came to Harvard University as a tutor in Biochemistry, remaining there to the end of his career. He became a Professor of Biology in 1948.

With his coworkers at Harvard he pioneered our understanding of the molecules responsible for the first steps in the vision process. Wald's group was the first to elucidate the molecular structure of the rod cell's functional protein *rhodopsin*. Prior to his work, rhodopsin was thought to be a chunk of molecular material. He determined that the protein consists of two molecular parts: a colorless amino acid sequence called *ospin* and a yellow organic chromophore called *retinal*.

When exposed to light, the rhodopsin releases retinal that is converted into vitamin A[818].

By the early 1950s Wald had succeeded in elucidating the chemical reactions involved in the vision process in the rods. In the late 1950s, with **Paul K. Brown**, he identified the pigments in the retina that are sensitive to yellow-green light and red light and in the early 1960s, the pigment sensitive to blue light. Wald and Brown also discovered the role of vitamin A in forming the three color pigments and showed that color blindness is caused simply by the absence of one of them. Wald became professor emeritus at Harvard in 1977.

[818] **S.T. Ball** et all in Liverpool have shown (1946) that retinal is vitamin A aldehyde.

Timeline: the Physiology of vision (up to 1938)

ca 380 BCE **Plato** taught that the eye is the *source* of illumination. i.e.: the viewer's eye sent out emissions to the object, and those emission enabled vision to occur.

ca340 BCE **Aristotle** put forward an alternative theory of human vision: the object being looked at, somehow altered the "medium" between the object itself and the viewer's eye. This alteration of the medium propagates to the eye, allowing the object to be seen.

ca 170 CE **Galen** expounded and developed the *visual ray theory*, using it for detailed description of the anatomy of the eye. This theory was given credibility with **Euclid**'s geometry (ca 280 BCE) and **Ptolemy**'s optics (ca 150 CE). The advantage of this theory was that its geometrical analysis of the visual fields (i.e. a perspective cone with its apex in the eye and its base on the object) provided a solution to the problem of the size and distance of objects in relation to the eye. The principal organ of vision was the crystalline lens.

ca 1000 CE **Alhazen**, through his *experimental* work, related geometrical optics to the anatomy of the eye, but still considered the lens as the sensitive part. Contested Plato's idealism which made the eye a source of illumination, and appeared to have recognized the eye as what we now call *camera obscura*. He seems to have been well acquainted with the projection of images of objects through small apertures, and to have been the first to show that the arrival of the image of an object at the retina, corresponds with the passage of light from an object through an aperture in a darkened box. He also investigated the problem of *image-inversion* and the uprightness of the perceived object, placing binocular vision in the common optic nerve.

ca 1250–1580 CE No major advance beyond **Alhazen** throughout the Middle Ages and the Renaissance: the problems identified by him remained central in the investigations of **Roger Bacon** (1214–1292), **Witelo** (1230–1275) and **Leonardo da Vinci** (1452–1519).

1583 CE The physician **Felix Platter** (1536–1614, Switzerland) first promoted the idea that it was the *retina* and not the lens which was sensitive to light. He argued that the optic nerve ought to be viewed as the primary organ of vision.

1604 CE **Johannes Kepler** offered the first theory of *retinal image* and firmly established the inverted and reversed point-by-point representation of the image on the retina analogous to the *camera obscura*. Kepler argued that the crystalline lens re-focused rays on the retina where vision is made possible.

1619–1625 CE The astronomer **Christopher Scheiner** (1575–1650, Germany) provided an experimental verification to Kepler's theory of retinal image.

1637 CE **René Descartes** (1596–1650) first suggested point-by-point projection of *retina* onto *brain*.

1684 CE First microscopic observation of the retina: **Leeuwenhock** noticed structures now known to be the *rods* and the *cones*.

1704 CE **Isaac Newton** (1642–1727), in his book '*Optiks*', developed a theory of color vision.

1801 CE Based on Newton's work, **Thomas Young** (1773–1829), physicist and physician, proposed the *trichromatic* theory of color vision: nerve fibers in the retina are capable of reacting to each of the three primary colors (red, green, violet). This helped explain color-blindness. Young carried out a number of studies on the eye that resulted in an understanding of how the lens focused images onto the retina. He also showed that *astigmatism* results from an improperly curved cornea.

1826–1856 CE **Johann H. Müller** (1801–1858, Germany) explained (1826) structure and functions of the compound eye of lower animals. He is regarded as the founder of modern physiology. He noted visual purple in *rods* (1851) and proved that *photoreception* occurs in *rods* and *cones*.

1856 CE **H. von Helmholtz** (1821–1894) conjoined the work of Young and **Maxwell** in a comprehensive work on color vision.

1866 CE Biologist **Max Schultze** (1825–1874, Germany) discovered that the *retinal cones* are the color receptors of the eye and the *retinal rods*, while not sensitive to color, are very sensitive to light at low levels (night-vision). He was also one of the first to establish that the cells of all organisms are composed of *protoplasm* and contain a nucleus. His theory of vision was later amplified by the physician **Henri Parinaud** (1844–1905, France) and by the physiologist **Johannes A. von Kries** (1853–1920, Germany), in 1905.

1876 CE The physiologist **Franz Boll** (1849–1879, Germany) discovered the rod visual pigment.

1878 CE The physiologist **Wilhelm Friedrich Kühne** (1837–1900, Germany) isolated the reddish-purple rod pigment, termed *visual purple*, later to be called *rhodopsin*. Kühne showed it to be a protein.

1938 CE The biophysicist **Selig Hecht** (1892–1947, USA) showed that rods respond to single quanta, i.e.: the absorption of a single photon was sufficient to excite a rod. This suggested that large amplification must occur when rhodopsin is excited.

The Photobiochemistry of Vision, or – How do we see?

From time immemorial humans have tried to understand the phenomenon of vision.

Vision is indeed a complicated process that requires numerous components of the human eye and brain to operate in unison, and the accompanying Timeline bears evidence to the lengthy, slow accumulation of scientific lore in this field.

Although the microscope was first used in scientific research in the late 16th and early 17th centuries, both the tools and the techniques of its use *reached a sufficient level of sophistication by the 19^th^ century to make it invaluable in examination of the structures of the eye.*

During 1826–1878, several German physiologists used the microscope to closely examine the retina. Through their observations, they discovered two different cells in the retina: the rod cells *and the* cone cells*, so named because of their shapes as viewed in the microscope.*

Additional research during 1866–1938 showed that the rod and cone cells were responsive to light[819]*.*

During the 1800's, the visual pigments were discovered in the retina. Scientists, working by candlelight, dissected the retinas from frog eyes. When the retinas were exposed to day light they changed color. These scientists had discovered that the retina is photosensitive. They realized that the color they were observing was due to presence of a visual pigment, which was given the name rhodopsin*. Later studies showed that rhodopsin is a protein that is found in the discs of the rod cell membrane.*

Pigments are also found in cone cells. There are three types cone cells, each of which contains a visual pigment. These pigments are called the red, blue or green visual pigment. The cone cells detect the primary colors, and the brain mixes these colors in seemingly infinitely variable proportions so that we can perceiver a wide range of colors.

The original theory of color vision was introduced by **Thomas Young** *around 1790, prior to the discovery of the cone cells in the retina. Young was the first to propose that the human eye sees only the three primary colors, red, blue and yellow and that all of the other visible colors are combinations of these. It is now known that color vision is more complicated than this, but Young's work formed the foundation of color vision theory for the scientists that followed. The photoreceptor proteins of the cone cells have not yet been isolated. This may possibly be due to the difficulty in obtaining them. There are many fewer cone cells than rod cells in the retina. Also many animals do not have cone cells and hence do not see in color.*

During the early part of the 20^th^ century, work continued on the frontier of research aimed at understanding vision. It was also around this time that

819 In the human eye there are many more rod cells in the retina than there are cone cells. The number of rod cells and cone cells in animals is often related to the animal's instincts and habits. For example, birds such as hawks have a significantly higher number of cones than do humans. This let them to see small animals from a long distance away, allowing them to hunt for food. Nocturnal animals, on the other hand, have relatively higher numbers of rod cells to allow them better night vision.

the relationship between vision and proper nutrition began being studied at universities and agricultural schools. It had been shown during World War I that a vitamin A deficiency caused night blindness. The link between vitamin A and night blindness, however, did not become clear until ***George Wald*** *and his coworkers isolated vitamin A from the retina in 1933. Prior to this finding the importance of vitamins was poorly understood. Additionally, the complete role of vitamins in physiological processes was unknown. It is now understood that the human body makes* retinal *from vitamin A through the following process:* Enzyme-catalized oxidation *converts vitamin A (retinol) into* trans-retinal.

Trans-retinal is present in the light-receptor cells of the human eye, but before it can fulfill its biological function, it has to be isomerized by an enzyme, retinal isomerase, *to give cis-retinal. This molecule fits well into the active site of a protein called opsin (approximate molecular weight 38,000). Cis-retinal reacts with one of the amine substituents of opsin to form the* imine rhodopsin, *the light-sensitive chemical unit in the eye.*

When a photon strikes rhodopsin, the cis-retinal part isomerizes extremely rapidly, in only picoseconds (10^{-12} *s), to the trans isomer. This isomerization induces a tremendous geometric change, which appears to severely disrupt the snug fit of the original molecule in the protein cavity. Within nanoseconds* (10^{-9} *s), a series of new intermediates form from this photoproduct, accompanied by conformational changes in the protein structure, followed by eventual hydrolysis of the ill-fitting retinal unit. This sequence initiates a nerve impulse perceived by us as light. The trans-retinal is then reisomerized to the cis form by retinal isomerase and re-forms rhodopsin, ready for another photon. A schematic flow-diagram is shown in Fig. 5.13.*

What is extraordinary about this mechanism is its sensitivity, which allows the eye to register as little as one photon impinging on the retina. Curiously, all known visual systems in nature, even though they might have a completely different evolutionary history, use the retinal system for visual excitation. Evidently, this molecule offers an optimal solution to the problem of enabling organisms to see.

Thus, the photoreceptor neurons in the retina collect the light and send signals to a network of neurons that then generate electrical *impulses that go to the brain. The brain processes those impulses and yields information about what we are seeing. i.e., it decodes the retina images into information that we known as* vision.

Scientists continue to study the role and mechanisms of photoreceptors in vision both to better understand the mechanism of human vision and to try to understand and remedy eye disease and blindness. Additionally, studies on

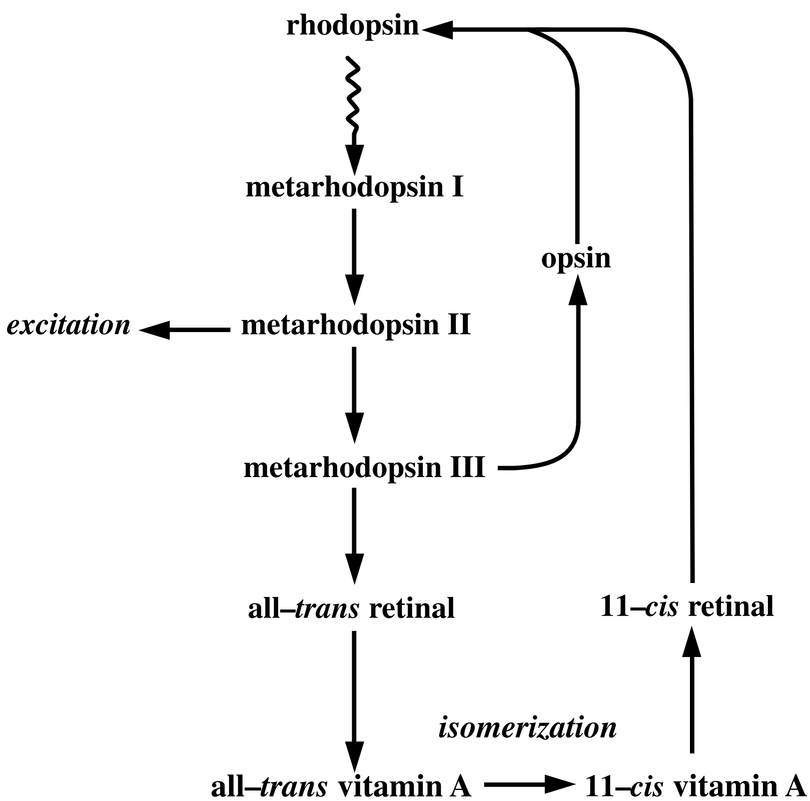

Fig. 5.13: When a photon strikes a rhodopsin

Scheme of the sequence of events that occurs following the absorption of a quantum of light by the rod visual pigment, rhodopsin. Light initiates the conversion of rhodopsin to retinal and opsin through a series of metarhodopsin intermediates. Metarhodopsin II is the active intermediate leading to excitation of the photoreceptor cell. Eventually, the chromophore of rhodopsin, retinal, separates from the protein opsin and is reduced to vitamin A (retinol). For the resynthesis of rhodopsin, the vitamin A must be isomerized from the all-trans to the 11-cis form, and this isomerization takes place in the pigment epithelium overlying the receptors. Vitamin A is replenished in the eye from the blood.

photoreceptors can lead to the development of better electronic and optical devices, as well as improvements in the field of robotics and artificial sensing.

1934–1987 CE **Emile Cioran** (1911–1995, Romania and France). Philosopher. Realized very early that the sense of existential futility can best be cured by the belief in a *cyclical concept of history*, which excludes any notion of the arrival of a new messiah or the continuation of techno-economic progress.

Historical pessimism and the sense of the tragic are recurrent motives in European literature. From Heraclitus to Heidegger, from Sophocles to Schopenhauer, the exponents of the tragic view of life point out that the shortness of human existence can only be overcome by the heroic intensity of living. The philosophy of the tragic is incompatible with the Christian dogma of salvation or the optimism of some modern ideologies. Many modern political theologies and ideologies set out from the assumption that "the radiant future" is always somewhere around the corner, and that existential fear can best be subdued by the acceptance of a linear and progressive concept of history. It is interesting to observe that individuals and masses in our post-modernity increasingly avoid allusions to death and dying. Processions and wakes, which not long ago honored the postmortem communion between the dead and the living, are rapidly falling into oblivion. In a cold and super-rational society of today, someone's death causes embarrassment, as if death should have never occurred, and as if death could be postponed by a deliberate "pursuit of happiness." The belief that death can be outwitted through the search for the elixir of eternal youth and the "ideology of good looks", is widespread in modern TV-oriented society. This belief has become a formula for social and political conduct.

Born in Romania in 1911, Cioran very early came to terms with the old European proverb that geography means destiny. From his native region which was once roamed by Scythian and Sarmatian hordes, and in which more recently, secular vampires and political Draculas are taking turns, he inherited a typically "balkanesque" talent for survival.

Cioran's political, esthetic and existential attitude towards being and time is an effort to restore the pre-Socratic thought, which Christianity, and then the heritage of rationalism and positivism, pushed into the periphery of philosophical speculation. In his essays and aphorisms, Cioran attempts to cast the foundation of a philosophy of life that, paradoxically, consists of total refutation of all living. In an age of accelerated history it appears to him senseless to speculate about human betterment or the "end of history."

"Future," writes Cioran, "go and see it for yourselves if you really wish to. I prefer to cling to the unbelievable present and the unbelievable past. I leave to you the opportunity to face the very Unbelievable."

Before man ventures into daydreams about his futuristic society, he should first immerse himself in the nothingness of his being, and finally restore life to what it is all about: a working hypothesis.

The feeling of sublime futility with regard to everything that life entails goes hand in hand with Cioran's pessimistic attitude towards the rise and fall of state and empires. His vision of the circulation of historical time recalls Vico's *corsi e ricorsi*, and his cynicism about human nature draws on Spengler's "biology" of history. Everything is a merry-go-round, and each system is doomed to perish the moments it makes its entrance onto the historical scene.

One can detect in Cioran's gloomy prophecies the forebodings of the Roman stoic and emperor Marcus Aurelius, who heard in the distance of the Noricum the gallop of the barbarian horses, and who discerned through the haze of Panonia the pending ruin of the Roman empire. Although today the actors are different, the setting remains similar; millions of new barbarians have begun to pound at the gates of Europe, and will soon take possession of what lies inside.

Cioran's philosophy bears a strong imprint of Friedrich Nietzsche and Hindu Upanishads. Although his inveterate pessimism often recalls Nietzsche's "Weltschmerz", his classical language and rigid syntax rarely tolerates romantic or lyrical narrative, nor the sentimental outbursts that one often finds in Nietzsche's prose. Instead of resorting to thundering gloom, Cioran's paradoxical humor expresses something which in the first place should have never been verbally construed.

When one reads Cioran's prose the reader is confronted by an author who imposes a climate of cold apocalypse that thoroughly contradicts the heritage of progress. Real joy lies in non-being, says Cioran, that is, in the conviction that each willful act of creation perpetuates cosmic chaos. There is no purpose in endless deliberations about higher meaning of life. The entire history, be it the recorded history or mythical history, is replete with the cacophony of theological and ideological tautologies. Everything is "éternel retour," a historical carousel, with those who are today on top, ending tomorrow at the bottom.

Cioran's most important books are:

- *On the Heights of Despair* 1934
- *Tears and Saints* 1937
- *A Short History of Decay* 1949
- *The Temptation to Exist* 1956
- *History and Utopia* 1960
- *The Trouble with Being Born* 1973
- *The Bad Demiurge* 1974
- *Anathemas and Admirations* 1987

In "*The Trouble with Being Born*" he explored how our troubles began with the act of being born and the anguish that pure consciousness and lucidity inflicts upon us.

In "*A Short History of Decay*," Cioran explores man's decay, the necessity and futility of rebellion against God and life itself. How we should seek our true hope in nothingness.

In "*Tear and Saints*" he equates religious fanaticism with 'delirium of self-aggrandizement hidden beneath meekness and the will to power masked by goodness.'

Cioran was born as Octavian Goga in the village of Rasinari near Sibiu (Austria-Hungary, present-day Romania), the son of an orthodox priest. He studied philosophy and letters at the Universities of Bucharest (1928–1932) and Berlin (1933–1935) and then worked as a high-school teacher at Brasov (1935–1936). In 1928 he also began an association with the *Iron Guard*, a nationalistic organization which he supported until the early years of *WWII*. This enabled him to obtain a scholarship from the French Institute in Bucharest, that brought him to Paris and he became a cultural councilor at the Romanian Embassy there (1940–1945). He later renounced the organization and frequently expressed regret and repentance for his participation in it. (Some critics have seen his remorse at his participation in the Iron Guard as the source of pessimism which characterized his later work, although others trace it back to events in his childhood[820].)

From 1944 on, Cioran wrote exclusively in French. Sometimes during the 1950's he met Simone Bouc, who became his lifelong companion.

He led a quite and solitary life of study and composition. Though he was highly regarded by Parisian literary circles, he was not well known to the rest of the world, and he sustained himself through his work as a translator or reader for various publishing houses.

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[820] In 1935 his mother is reputed to have told him that if she had known he was going to be so unhappy, she would have aborted him.

He lived in the same small apartment in the Latin Quarter of Paris from 1960 on. In his later life, Cioran withdrew from social life and gave up writing altogether in 1987. Alzheimer's disease began slowly deteriorating his mind in the 1990's, just as his native country began rediscovering him after the fall of Communism. He fell ill in 1994 and, after a yearlong battle, Emile Cioran finally found his long sought bliss in nothingness.

Worldview XLIX: Emile Cioran[821]

* * *

Life inspires more dread than death — it is life which is the great unknown.

* * *

If, at the limit, you can rule without crime, you cannot do so without injustices.

* * *

In a republic, that paradise of debility, the politician is a petty tyrant who obeys the laws.

* * *

Life is possible only by the deficiencies of our imagination and memory.

[821] The English translation in this section was done by **Tomislav Sunic**. His is also the source of appreciation of Cioran's philosophical life-work as quoted in our article.

* * *

Chaos is rejecting all you have learned, Chaos is being yourself.

* * *

Man starts over again everyday, in spite of all he knows, against all he knows.

* * *

We inhabit a language rather than a country.

* * *

I'm simply an accident. Why take it all so seriously?

* * *

I have no nationality — the best possible status for an intellectual.

* * *

I have always lived with the awareness of the impossibility of living. And what has made existence endurable to me is my curiosity as to how I would get from one minute, one day, to the next.

* * *

To act is to anchor in the imminent future.

* * *

Isn't history ultimately the result of our fear of boredom?

* * *

To want fame is to prefer dying scorned than forgotten.

* * *

Pursued by our origins... we all are.

* * *

For you who no longer possess it, freedom is everything, for us who do, it is merely and illusion.

* * *

To Live signifies to believe and hope — to lie and to lie to oneself.

* * *

Glory — once achieved, what is it worth?

* * *

What does the future, that half of time, matter to the man who is infatuated with eternity?

* * *

Knowledge subverts love: in proportion as we penetrate our secrets, we come to loathe our kind, precisely because they resemble us.

* * *

In most cases we attach ourselves to God in order to take revenge on life, to punish it, to signify we can do without it, that we have found something better, and we also attach ourselves to God in horror of men.

* * *

We understand God by everything in ourselves that is fragmentary, incomplete, and inopportune.

* * *

A people represents not so much an aggregate of ideas and theories as of obsessions.

* * *

We are born to Exist, not to know, to be, not to assert ourselves.

* * *

Knowledge, having irritated and stimulated our appetite for power, will lead us inexorably to our ruin.

* * *

Each of us must pay for the slightest damage he inflicts upon a universe created for indifference and stagnation; sooner or later, he will regret not having left it intact.

* * *

Whenever I happen to be in a city of any size, I marvel that riots do not break out everyday: Massacres, unspeakable carnage, a doomsday chaos. How can so many human beings coexist in a space so confined without hating each other to death?

* * *

Utopia is a mixture of childish rationalism and secularized angelism.

* * *

That history just unfolds, independently of a specified direction, of a goal, no one is willing to admit.

* * *

Society becomes consolidated in danger and it atrophies in peace.

* * *

Authority, not verity, makes the law.

* * *

"What to think of other people? I ask myself this question each time I make a new acquaintance. So strange does it seem to me that we exist, and that we consent to exist."

* * *

"Existing is plagiarism."

* * *

"A self-respecting man is a man without a country. A fatherland is birdlime..."

* * *

"Illusion begets and sustained the world; we do not destroy one without destroying the other. Which is what I do every day. An apparently ineffectual operation, since I must begin all over again the next day."

* * *

History teaches us that: violence and destruction are the main ingredients of history, because the world without violence is bound to collapse.

* * *

In those places where peace, hygiene and leisure ravages, psychoses also multiply. Thus, not "peace and love" will determine the course of tomorrow's history. Unable to put up resistance against tomorrow's conquerors, the fate of Western Europe is doomed.

* * *

"Suffering makes you live time in detail, moment after moment. Which is to say it exists for you. For the others, the ones who don't suffer, time flows, so that they don't live in time; in fact they never have."

* * *

I cannot excuse myself for being born. It is as if, when insinuating myself in this world, I profaned some mystery, betrayed some very important engagement, made a mistake of indescribable gravity.

* * *

1935 CE **Hideki Yukawa** (1907–1981, Japan). Physicist. Proposed that the exchange of virtual quanta (particles) of a hitherto unknown field (*mesons, meson fields*) is the cause of the complex interactions among nucleons (protons and neutrons) in the atomic nucleus. [The name *meson* was coined by

Homi Bhabha (India).] He predicted the rest-mass and spin of this class of particles[822], which were later discovered and named *pions* or π-mesons.

Upon graduating from Kyoto Imperial University in 1929, Yukawa became a lecturer there. In 1933 he moved to Osaka Imperial University, where in 1938 he was awarded the doctorate. He rejoined Kyoto University as professor of theoretical physics (1939–1950), held faculty appointments at the Institute for Advanced Study in Princeton, NJ and at Columbia University in New York city, and became director of the Research Institute for Fundamental Physics in Kyoto (1953–1970). Yukawa was awarded the Nobel prize for physics in 1949.

1935 CE **Arthur Jeffrey Dempster** (1896–1950, U.S.A.). Physicist. Built the first mass spectrometer (1918). Discovered uranium 235, an isotope of uranium that would be used (1942) in producing the first sustained nuclear chain-reaction. Born in Toronto, Canada; taught at the University of Chicago (1919–1950).

1935 CE **Wendell Meredith Stanley** (1904–1971, USA). Biochemist. Worked on purification and crystallization of viruses, thus demonstrating their molecular structure; crystallized tobacco mosaic virus (1935); also studied influenza viruses, for which he developed a preventive vaccine. Corecipient, with **J.H. Northrop** and **J.B. Sumner** of the 1946 Nobel Prize for Chemistry.

Stanley was born in Ridgeville, Indiana. Worked at the Rockefeller Institute for Medical Research (1932–1948) and became professor at the University of California, Berkeley (1948–1971).

Stanley chose to work with the *tobacco mosaic virus* because it was not dangerous to human beings and because characteristic infection could be demonstrated easily and rapidly. From the juice squeezed out of a ton of infected tobacco leaves, Stanley was able to isolate a few milligrams of a *crystalline* material that appeared to him, at first, to be a protein.

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[822] The successful description of the electromagnetic force in terms of virtual photons suggested to him that the strong nuclear force might be accounted for in a similar manner.

It was known that this force does not decrease gradually toward zero with increasing distance; rather, its range ends abruptly at distance of order 10^{-13} centimeter. From this fact and the uncertainty principle, Yukawa concluded that the particles whose virtual exchange was associated with the strong force field should be all of one, *finite* mass, $m \sim \frac{\hbar}{(10^{-13}\,cm)c} \sim \frac{100\,MeV}{c^2}$, of order two hundred electron masses. Since particles having a mass intermediate between the electron and proton were unheard of at the time this prediction was made, it was received with considerable skepticism.

The substance was a pure, homogeneous *non-cellular* material that could be stored, apparently indefinitely, in a dry state; however, if even one millionth of a milligram of the pure material was suspended in water and spread on a living tobacco leaf, the mosaic disease was produced. Careful chemical analysis of the pure virus showed that it was composed of 95 percent protein and 5 percent *ribonucleic acid*, and nothing else.

Stanley's discovery changed the course of biological thought and research. The virus was not a small bacterium; it was a large molecule. Thus, the problem of viruses fell within the domain of chemistry and physics as well as biology.

The line of demarcation separating the living from the non-living became cloudy, and a new group of scientists (including **Max Delbrück**, **Alfred Hershey** and **Salvador Luria**) entered the field of virology. This new school of virologists turned their attention to a group of viruses that attack bacteria, celled *bacteriophages*.

Microwave Technology

In 1935, British scientists led by **Robert Alexander Watson-Watt** *(1892–1973), developed the first pulse-radar* (Radio Detection and Ranging), *capable of detecting aircraft at ranges up to 27 km. Spurred by the growing threat of war, the British quickly recruited many of their best scientists and developed* PPI *radar (Plane Position Indicator) and efficient multicavity magnetrons to generate microwaves.*

By the time WWII *began in September 1939, the British had installed a chain of large radar warning stations on their shores. They also developed* GCI *(Ground Controlled Interception) radar and Airborne Interception radar for night fighter planes. During the Battle of Britain, these radars enabled a small force of British fighter planes and antiaircraft artillery to fight off the massive German air attacks.*

Magnetrons are vacuum tube devices which generate or amplify high-frequency electromagnetic waves. In ordinary vacuum tubes the period of the voltage or current cycle is long compared with the transit time required for an electron to travel from cathode to anode. However, at frequencies of *the order of 100 MHz the transit time becomes comparable with the period of the oscillations themselves. It then becomes dependent upon the space charge, and hence modulated by the oscillations.*

This effect, in conjunction with the effect of the interelectrode capacitance, will tend to nullify the control action of the grid. Thus, if the frequency of the signal is so high that the electric field experienced by an electron changes significantly during the time of transit, ordinary vacuum tubes cannot be used.

The magnetron *(like the* klystron *and the* traveling-wave tube*) utilizes the transit-time in their operation to sustain and amplify oscillations above 1000 MHz; in each of these devices, a stream of electrons is subjected to electric* and *magnetic fields precisely arranged so that an exchange of energy occurs between the signal and the electrons. The net result is that power supplied by a d–c source is converted to a–c power at the signal frequency and amplification is obtained.*

The Hull magnetron *(1921) underwent two basic modifications on its way from serving as a mere diode to becoming an efficient generator of microwave radiation. In its first use as an oscillator, the anode of the valve was split into two segments (1927) and a high-frequency resonant circuit was connected between the anode segments. This was known as the '*Split-Anode Magnetron*'. An electron emitted from the cathode at the center of the cylinder was then subjected to 3 different fields:*

(1) a d–c radial electric field E accelerating it toward the peripheral anode;

(2) a d–c axial magnetic field $B > B_c$, (B_c being a critical field value) affecting radial and azimuthal motion in a plane perpendicular to the axis;

(3) intense high-frequency field across the gaps of the anode segments.

Assuming first that both segments of the anode are at the same potential V_b and that the cathode is negligibly thin, each electron moves with an approximately constant speed $v = \sqrt{\frac{2e}{m}V_b}$ in a circular orbit of radius $r = \frac{mv}{eB}$ and period of revolution ('transit-time') $\tau = \frac{2\pi r}{v} = \frac{2\pi m}{eB}$. Under these conditions the electron neither gives nor takes energy from its surrounding.

Now suppose we activate the resonant tank circuit across the anode gaps; since $B > B_c$, an incoming electron in its circular orbit nearly grazes the anode. Depending on the polarity of the tank circuit, the incoming electron will be either accelerated or decelerated by the momentary field across the gap. If it is accelerated it will induce a current in the resonant circuit that will extract *energy from the circuit. If it is retarded at the gap, it will instead induce a current which delivers energy to the tank circuit.*

If, when the electron approaches the gap again, the tank circuit has oscillated through one cycle, then the electron is again retarded at the gap, and the whole process is repeated. In successive cycles the electron approaches the anode less closely, owing to its repeated losses of kinetic energy, and finally it comes to rest somewhere between the anode and the cathode and is removed by special arrangements.

Over time, the external circuit gains more AC energy from the electrons which are retarded at the gaps than it losses to electrons which are accelerated at the gap. The magnetron can therefore maintain oscillations in the tuned circuit connected across its anode segments. The AC oscillation energy comes from the high DC voltage supply, which sets up the radial accelerating field near the cathode. As the electrons pass through this field, the current which they induce flows between the cathode and the anode as a whole, resulting in a transfer of energy from the battery to the electrons, which in turn transfer part of this energy, at the frequency of the transit-time*, to the tank circuit.*

In the next stage in the development of the magnetron (1941), the tank circuit was replaced by a cavity, or several cavities. The anode now consisted of a solid block of copper, in which an even *number of identical cylindrical cavities were drilled, symmetrically arranged. In such a configuration, cavity communicates by means of a slot, with a central cylindrical hole housing an oxide-coated nickel cathode.*

These cavities are capable of entering into resonance at the desired oscillation frequency; a coupling loop, inserted in one of the cavities, suffices to energize a coaxial cable waveguide. Oscillations of the charge take place around the inner circumference of the cylinder, and set up an intense electric field over the narrow gap. The anode potential V_b is over 10,000 volts, and the axial magnetic flux density B is about 1000 Gauss.

Under static conditions no electrons would reach the anode, and there would be a cylindrical space-charge distribution of diameter less than the inner diameter of the anode.

This space-charge would, in effect, circulate round the cathode with an angular velocity depending on the anode potential and the magnetic flux density. But theory suggests that, when the cavity system produces a transient oscillation, the electric field of the resonators draw out the space-charge into a form rather like the spokes of a wheel. Then, the electrons swinging past the gaps in the cavities interact with the electric fields across those gaps.

Under certain conditions of flux density and anode potential the resonators gain energy from the electrons, and their oscillations grow to a steady amplitude. Neighboring segments of the anode must be at potentials of opposite sign at any given instant, each segment changing from maximum positive potential to maximum negative potential, and back again, once per cycle; in *other words — the configuration of the fields and currents rotates at* $\left(f/\frac{1}{2}N\right)$ *revolutions per second, where* f *is the frequency and* N *the number of cavities.*

The cavity magnetron is not suitable for producing an amplitude-modulated signal, because the high anode voltage is fixed by the conditions for oscillation. It was developed as a generator of pulses, for use in RADAR. Each pulse lasts for a few microseconds, with wavelengths between 10 cm and 3 cm (frequencies between 3000 and 10,000 MHz). Typical peak power of 150 kilowatts at a pulse rate of 1000 per sec were attained during WWII.

Today, high operating efficiencies are possible and microwave tubes find application in television transmission, satellite communication, industrial heating, home cooking (e.g. microwave oven), medical imaging, radiation therapy and high-energy physics.

1935 CE, Sept. 04 '*Labor Day*' Hurricane[823] struck the Florida Keys. One of the greatest storms to hit the United States in the 20^{th} century. The barometer fell to 669.3 mb, the lowest reading ever recorded in the Western Hemisphere. More than 400 persons were killed in the storm. Winds reached 320 km/hr in the Florida Keys.

Violent Storms

As with many other natural catastrophes, hurricanes and tornadoes attract a great deal of attention. Although these phenomena are relatively rare, they command a fascination that ordinary weather events cannot provide. Furthermore, because of the death and destruction that these storms leave

[823] The Caribbean word *huracan* was introduced by the Portuguese, Spanish and Dutch explorers of the 15^{th} and 16^{th} centuries into many European languages.

in their wake[824]*, they have been and continue to be an important focus of atmospheric research.*

Storms vary in areal size, wind velocity and total energy input:

- *In a tornado, wind may exceed* 650 *km/hr. It is usually less than* 2 *km wide and lasts for a few minutes. Its kinetic energy is typically* 4×10^{17} *erg, and its total energy is* 10 *to* 100 *times greater.*
- *In a hurricane (tropical cyclone), wind may be above* 300 *km/hr, its areal size may be* 800 *km in diameter and it may last for as much as a week or so. Its kinetic energy is about* 4×10^{23} *erg, and its total energy may reach* 4×10^{24} *erg.*
- *A cyclone (nearly circular area of low pressure which commonly form outside the tropics in the middle latitudes and slowly moves inland across the coast) usually has winds below* 80 *km/hr, an areal size of more than* 1500 *km in diameter and duration of perhaps a week. Its kinetic energy is on the average* 4×10^{24} *erg, while its total energy is about* 4×10^{25} *erg.*

The major portion of the energy in atmospheric systems is expended in overcoming the effects of friction and in heating the air inside and outside the systems. Thus, the input energy of an average hurricane may be equivalent to more than 10,000 *atomic bombs of the kind that destroyed Nagasaki (10 KT).*

There are various sources of energy for atmospheric vortices. Heat contained in the air and earth's surface, and the sinking of heavier air when it moves over lighter air are important factors. But the major contribution comes from heat released when water vapor condenses to form clouds[825]*. In hurricanes and cyclones there are indeed widespread areas of cloudiness and*

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[824] Enormous amounts of destruction and damage can be caused by one tropical cyclone, particularly in coastal areas. For example, on 12 November 1970 a tropical cyclone moving up the Bay of Bengal hit the coast of Bangladesh, sending a 6-meter surge ahead of it, which moved into a low-lying convergent coastline at the time of high-tide. The resultant hurricane storm tide destroyed the island of Bhola, over 500,000 people died and some 4.7 million people were affected by the disaster.

[825] In a single *thunderstorm* about 5 km in diameter, there may be 500,000 tons of condensed water in the form of water droplets and ice crystals. In the course of producing these particles, there would have been released about 3×10^{14} calories. This is equal to about 10^{22} erg.

rainfall. They indicate the release of enormous amounts of heat of condensation (latent heat).

Tornadoes appear as pendent funnels which dip downward from the base of existing clouds, and approach the ground in an irregular fashion. These are intense centers of low pressure having a whirlpool-like structure of winds rotating around a central cavity, where centrifugal forces produce a partial vacuum. Pressures within the center of some tornadoes have been estimated to be 100 *millibars less than immediately outside the storm. Because of such tremendous pressure gradients, it is estimated that wind speeds may reach* 650 *km/hr or more. Air sucked into the vortex of the storm is rapidly lifted and cooled adiabatically. The resulting condensation creates the pale and ominous-looking funnel cloud, which may darken as it moves across the ground, as it picks up dust and debris.*

The most striking feature of the tornado is the velocity of the wind, which in localized regions of the funnel may reach peak speeds close the speed of sound. The circulation of these winds is always counterclockwise (dust-devils usually spin clockwise!). The nearly total destruction wrought by tornadoes is linked to the combined effects of the exceedingly strong winds and the partial vacuum in the center of the storm. The winds may rip apart everything in the path of the storm, and the abrupt pressure drop may cause some building to literally explode[826].

Although meteorologists still do not know how and why tornado funnels form where they do, they can specify the conditions usually associated with their development throughout the spring: continental Canadian polar air from the arctic may still be very cold and dry, whereas maritime tropical air from the Gulf of Mexico is very warm and moisture laden. The greater the contrast, the more intense the storm. Since these two contrasting air masses are most likely to meet in the central United States, it is not surprising that this region generates more tornadoes than any other area in the country, and, in fact, the world.

Tornado warning is effected by visual sightings, conventional radar and recently by Doppler radar[827] *through which the frequency of the reflected*

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826 *Tornado stories*: in 1931 a tornado carried an 83-ton railroad coach and its 117 passengers 24 meters high through the air and dropped them in a ditch; The force of the wind during a tornado in Clarendon, Texas, in 1970, was enough to drive a wooden stick through a 4-centimeter metal pipe; Turkeys and chickens were stripped clean of their feathers, but remained alive! Thin pieces of straw were blown into three trunks or fence posts.

827 Nowadays, *Doppler radar* is used to detect the initial formation and subsequent development of a *mesocyclone*, an intense rotating wind system in the lower

signal is compared to that of the original pulse. These frequency changes are then interpreted in terms of speed forward or away from the radar unit.

Tropical cyclones develop in very humid air with temperature over $26\,^{\circ}C$*, so they happen mostly in the summer and early autumn, when the seas and the air above them are at their warmest. First, the warm sea heats the air above. A current of very warm, moist air rises quickly above the sea, creating a center of very low pressure on the surface below. Trade winds rush in towards this low-pressure center and whirl upwards. As they rise they cool, and the huge amounts of water vapor they contain condense and form towering cumulus and cumulonimbus clouds.*

Hurricanes are whirling tropical cyclones having wind speeds reaching 320 *km/hr — the greatest storms on earth. Out at sea they can generate* 15*-meter waves capable of inflicting destruction hundred of kilometers from their source. Should a hurricane smash into land, strong winds coupled with extensive flooding can cause great loss of life and catastrophic damage. [These awesome storms form in all tropical waters (except those of the south Atlantic) between the latitudes* $5\,^{\circ}$ *and* $20\,^{\circ}$*, and are known in each region by a unique name: in the western Pacific they are called typhoons, and in the Indian Ocean they are called cyclones.]*

Although hurricanes are most noted for their destruction, some parts of the world, especially eastern Asia, rely on them for much of their precipitation. (Consequently, while a resort owner in Florida dreads the coming of the hurricane season, a farmer in Japan welcomes its arrival.)

Hurricanes average 600 *kilometers in diameter and often extend* 12 *kilometers above the ocean surface. From the outer edge of the hurricane to the center, the barometric pressure on occasion drops* 60 *mb. This steep pressure gradient generates the rapid, inward spiraling winds. As the inward rush nears the core of the storm, it is deflected upward. Upon ascending the air condenses, generating the cumulonimbus clouds that constitute the doughnut-shaped inner structure of the hurricane called the eye-wall.*

Near the top of the hurricane the airflow is outward, carrying the rising air away from the storm center, thereby providing room for more inward flow at the surface. At the center of the storm is the spectacular eye. Averaging 20 *kilometers in diameter, this zone of calm and scattered cloud cover is*

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part of a thunderstorm that precedes tornado development. It can provide an average *warning* time of 21 minutes before tornado touchdown, as compared to 2 minutes at most by visual observations. Doppler radar also helps meteorologists gain new insights into thunderstorm development and air hazards that plague aircraft.

unique to the hurricane. The air within the eye slowly descends and heats by compression, making it the warmest part of the storm.

A hurricane can be described as a heat engine that is fueled by the energy liberated during the condensation of water vapor (latent heat). The enormous amounts of energy involved in a single storm is evident when we consider that it is equal to the total amount of electricity consumed in the United States over a 6-month period.

The release of latent heat warms the air and provides buoyancy for its upward flight. The result is to reduce the pressure near the surface — which encourages a more rapid inward flow of air. To get this engine started, a large quantity of warm, moisture-laden air is required and a continual supply is needed to keep it going.

Hurricanes develop most often in late summer, when the water has reached $27\,^{\circ}C$ *or more and is thus capable of providing the warm, moist air required. This fact is thought to account for the fact that the coolest tropical ocean, the South Atlantic, does not experience hurricanes.*

For the same reason, hurricane formation is confined to the warm sectors of the oceans, which are located not more than 20 *degrees on either side of the equator. On the other hand, hurricanes are not known to form within* 5 *degrees of the equator, because at low latitudes the Coriolis force is too weak to initiate the necessary rotary motion.*

Although the exact mechanism of formation is not completely understood, it is known that smaller tropical storms initiate the process. *These initial disturbances are regions of low-level convergence and lifting. Many tropical disturbances like these occur each year and move westward across the warm oceans, but only few develop into full-fledged hurricanes. It is believed that the upper-level airflow acts to further* intensify selected *storms by "pumping out" the rising air as it reaches the top of the storm, thus encouraging influx of warm moist air at the surface, which ascends and releases latent heat to fuel the storm.*

It seems that if the air is "pumped out" at the top faster than it is being replaced at the surface, the storm intensifies. However, if the rising air is not removed, the convergence at the surface will "fill" the storm center, equalizing the pressure differences, and the storm will die. Whenever a hurricane moves onto land, it loses its punch rapidly, for its source of warm water-laden air is cut off. Also, the added frictional effect of land causes the wind to move more directly into the center of the pressure low, helping to eliminate the large pressure differences.

North Atlantic hurricanes develop in the trade winds, which generally move these storms from east to west at about 25 *km/hr. Then, hurricanes* *curve poleward and are deflected into the westerlies, which increase their forward motion.*

Damage caused by hurricanes can be divided into three categories: (1) wind damage, (2) storm surge, and (3) inland fresh water flooding. Most of the devastating damage is caused by the storm surge, which is a dome of water 65 *to* 80 *kilometers long that sweeps across the coast near the point where the eye makes landfall. The torrential rains that accompany most hurricanes represent a third significant threat — flooding: as it moves inland, the storm can yield* 30 *centimeters of rain and drop* 100 *cubic kilometers of rain water over a radius of some* 200 *kilometers.*

One of the most awesome of natural atmospheric phenomena, similar in many respects to the tornado, is the firestorm. This often develops when a forest wild fire becomes organized by a cyclonic circulation. The highly unstable air (greatly exceeding the dry adiabatic rate[828]*) pulls in the surrounding air and develops a massive fire of terrifying proportions. An organized firestorm*

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828 Near the earth's surface most processes are non-adiabatic, namely: heat is readily exchanged between the ground and the air above. When the ground looses heat during the night, the air in contact with it is cooled. When there is enough moisture in the air, and there is little and no wind, the air temperature may be lowered to the point where *dew forms* (the dew-point temperature).

If there is just enough air motion to produce gentle stirring, so that additional air can be cooled by mixing or direct contact with the cold ground, the condensed moisture — *fog* — can attain a thickness of 100 m or more. However, because air is a poor conductor of heat, this exchange is virtually nonexistent above a few thousand meters. Thus, some other mechanism must operate during *cloud formation*.

As an airmass *rises*, pressure in it decreases and in response it expands. The expansion requires an expenditure of energy; since temperature is a measure of internal energy, this use of energy causes the airmass temperature to drop; this drop is 10 °C for each kilometer of ascent. A mass of rising air cools at this rate, known as the *dry adiabatic lapse rate* (DALR).

Conversely, sinking air is compressed and warms at the same rate. This activity determines the altitude at which a cloud will form or evaporate. When water molecules in the gaseous state condense, they lose some of their kinetic energies to the air; they release what is called *latent heat*. With water, this is a large amount of energy — nearly 600 kilocalories for each kilogram of condensed water, depending on the temperature and increasing as the temperature decreases.

Now, if air rises long enough, it will inevitably cool sufficiently to cause condensation. From this point along its ascent, latent heat stored in the water vapor *creates its own localized wind pattern which often becomes so violent as to fell trees, tear burning limbs from them, and scatter ambers from the upper levels of the convective column far and wide, thus starting new fires.*

When an intensive low-level jet-stream is over the region, an extremely dry subsiding air with a persistent wind velocity of 15–25 *m/sec can initiate a* blowup. *This may become a firestorm if a convective column develops and generates its own inflow winds. This occurred, for example, in* 1910 *in western Montana and eastern Idaho.*

Forecasting the evolution and movement of these systems remains one of the major unsolved problems of meteorology today.

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will be liberated. Although the air will continue to cool after condensation begins, the released latent heat works against the adiabatic process, thereby reducing the rate at which air cools from $10\,^\circ$C/km to the *saturated lapse rate* (SALR). It may be as low as $6\,^\circ$C/km, depending on the rate of release of latent heat. These lapse rates are referred to as being *adiabatic*, which means a temperature exchange process where there is no loss or addition of heat to or from an element of air molecules by its surroundings. No exchange of heat, momentum or water occurs with the environmental air molecules, and a parcel of air will heat or cool at a rate which is predictable and independent of environmental temperature.

For a vertical motion of air in the atmosphere, the temperature changes that take place are approximately adiabatic, as air is a poor conductor of heat, and diffusional mixing of a parcel with its surrounding is usually low. The air parcel will therefore tend to retain its own thermodynamic identity, which distinguishes it from the surrounding air.

The change of temperature of the environment with respect to the height is referred to as the *environmental lapse rate* (ELR, measured in $^\circ$C/km).

The atmosphere is said to be *absolutely stable* if $\text{ELR}<\max\{\text{DALR},\text{SALR}\}$, so that the rising parcel of air is always cooler and heavier than the surrounding air, and convection is suppressed.

On the other hand, the atmosphere is said to be *absolutely unstable* if $\text{ELR}>\max\{\text{DALR},\ \text{SALR}\}$, so that any air parcel cooling at the DALR will always be warmer and less dense than the surrounding air. Its buoyancy will give it an upward impulse, so that convection is encouraged. This motion will stop at a level of equilibrium where $\text{ELR}=\text{SALR}$.

1935–1937 CE **Nathan N. Rosen** (1909–1992, USA and Israel). Theoretical physicist. Worked with Albert Einstein on the foundations of quantum mechanics. Together with **Boris Podolsky**[829] (1896–1966) they concocted the ‘ *Einstein-Podolsky-Rosen Paradox*’ (1935) based on an idea of Rosen. It is a logically impeccable conclusion which casts a spotlight on some peculiarities of quantum mechanics.

Behind the title of the EPR paper, *Can the Quantum Mechanical Description of Physical Reality Be Regarded as Complete?*, is an attempt to demonstrate that quantum mechanics represents only an *incomplete* description of physical reality and therefore is unable to get beyond the formulation of statistical regularities.

Rosen was born in Brooklyn, New York, obtained his ScD at MIT (1932) and was a member of the Institute of Advanced Study (1934–1935). He collaborated with **Einstein** on the singularity-free solutions of the combined gravitational and electromagnetic field equations (1935), on the GTR two-body problem (1936), and on gravitational lenses (1937).

[829] Born in Taganrog, Russia. Emigrated to the United States (1913) and did his Ph.D. at CalTech (1928). He was a member of the Institute of Advanced Study (1934–1935) when the Einstein-Podolsky-Rosen collaboration took place. He later became research professor at the Xavier University in Cincinnati.

1935 CE '*The Einstein–Rosen Bridge*'[830]

Barely a few months after Einstein wrote down the field equations of the General Theory of Relativity, the first exact solution was found by Karl Schwarzschild (1916). He had shown that it is always possible to find a coordinate system in which the most general spherically symmetric solution of the vacuum field equation is obtained via the metric line element.

In relativistic units ($c = 1$),

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + sin^2\theta\, d\,\varphi^2) \qquad (1)$$

with

$$m = G\,M$$

Thus, if we interpret the Schwarzschild solution (SS) as due to a point particle situated at the origin, then the constant m is simply the mass of the particle in relativistic units. It is clear from (1) that m has the dimensions of length.

We notice that the SS becomes *singular* at $r = r_s = 2GM$, known as the *Schwarzschild radius* of the mass M.

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830 To dig deeper, see:

- Schwarzschild, K., *On the Gravitational Field of a Point Mass in Einstein's Theory*, Proc. Prussian Academy of Science, 1916, 424.
- Einstein, A. and N.J., Rosen, *The Particle Problem in the General Theory of Relativity*, Physical Review, 1935, **48**, 73.
- Kruskal, M.D., *Maximal Extension of Schwarzschild Metric*, Physical Review, 1960, **119**, 1743–1745.
- Fuller, R.W. and J.A., Wheeler, *Causality and Multiply–Connected Space–Time*, Physical Review, 1962, **128**, 919–929.
- Kerr, R.P., *Gravitational Field of a Spinning Mass as an Example of Algebraically Special Metrics*, Phys. Rev. Lett., 1963, **II**, 237–238.
- Hawkings, S. and R., Penrose, *The singularities of Gravitationa; Collapse and Cosmology*, Proc. Roy. Soc.: London, 1970, **A314**, 529–548.
- Morris, M.S., K.S., Thorne and U., Yurtserer, *Wormholes, time Machines, and the Weak Energy Condition*, Phys. Rev. Lett, 1988, **61**, 1446.
- Visser, M., *Lorentz Wormholes — From Einstein to Hawkings*, AIP Press: New York, 1995.

Imagine a body so small and massive that the radius r_s lies outside it, in empty space. The SS then holds down to this radius and actually displays a singularity. The question then arises as to whether this singularity is real or just an artifice of the coordinate system used. **Kruskal** (1960) found a coordinate system that allows us to avoid talking about a *Schwarzschild singularity*, if we are willing to allow the world an *unusual topology*. To exhibit this reinterpretation of the Schwarzschild singularity, he introduced a new set of coordinates r', θ, φ, t', defined by

$$\left.\begin{aligned} r'^2 - t'^2 &= T^2\left(\tfrac{r}{2GM} - 1\right)e^{\left(\frac{r}{2GM}\right)} \\ 2r'\,t'/(r'^2 + t'^2) &= tan\,h\left(\tfrac{t}{2MG}\right) \end{aligned}\right\} \qquad (2)$$

where T is a arbitrary constant. The SS (1) then becomes

$$ds^2 = F^2(dt'^2 - dr'^2) - r^2(d\theta^2 + sin^2\theta\, d\varphi^2); \qquad F^2 = \frac{\varphi r_s^3}{T^2\, r}\, e^{-\frac{r}{r_s}} \qquad (3)$$

where r is now to be understood as a function of $r'^2 - t'^2$ defined by (2). The metric is nonsingular as long as $r'^2 > t'^2 - T^2$.

Hence, during the time interval $0 < t' < T$, the metric is a perfectly smooth finite function of r' for all real r'. The space described by (3) is therefore *singularity–free*, but consists of two identical sheets $r' > 0$ and $r' < 0$, joined in a smooth way by a branch point at $r' = 0$. When t' reaches the time T, the two sheets detach from each other, and thereafter the metric has a real singularity at $r' = \pm\sqrt{t'^2 - T^2}$, that is, at $r' = 0$. This is unavoidable since the curvature becomes infinite at that point. However, even so, the metric has no singularity at the radius $r' = t'$ that corresponds to the Schwarzschild radius $r = 2\,G\,M$.

If we consider the submanifold $t' = 0$ in the Kruskal solution (3), then the line–element is given by $ds^2 = -F^2 dx'^2 - r^2(d\theta^2 + sin^2\theta\, d\varphi^2)$. As we move along the x'–axis from $+\infty$ to $-\infty$, the value of r decreases to a minimum $2m$ at $x' = 0$ and then increases again as $x' \longrightarrow -\infty$. We can draw a cross–section of this manifold corresponding to the equatorial plane $\theta > \frac{\pi}{2}$, in which case (3) reduces further to $ds^2 = -(f^2 dx'^2 + r^2\, d\varphi^2)$.

In their joint paper (1935), Einstein and Rosen showed that implicit in the formalism of GTR is a curved–space structure that can join two distant regions of space–time through a tunnel–like curved spatial shortcut. The purpose of their paper was not to promote faster–than–light travel, but to attempt to explain fundamental particles like electrons as space–tunnels threaded by electric lines of force.

Kerr (1968) found an exact solution of Einstein equation describing a realistic dying star – a spinning black hole. Because of the conservation of angular momentum, as a star collapses under gravity it spins faster. A spinning star could collapse like a particle into an infinitely thin disc which would remain stable because of the intense centrifugal force pushing outwards, canceling the inward force of gravity. The astonishing feature of such a black hole was that if an object fell directly into the thin disc perpendicular to its axis it would not be crushed but instead be sucked completely through the Einstein–Rosen bridge to a parallel universe. However, it is not clear how safe it would be to enter the bridge or how stable the doorway would be. **Steven Hawking** and **Roger Penrose** have studied (1970) the effects of these strange Kerr black holes. They have found, for example, that the neck of the Einstein–Rosan bridge may actually bend around and come out somewhere else in the universe. Thus, if rockets are sent directly through the black hole, at right angles to the disc, they would not emerge on the other side of the black hole, but on the other side of the universe! In this sense, the bridge could serve as a convenient passageway to the other side of space.

Special Relativity only applies locally. Wormholes allow superluminal (faster–than–light) travel by ensuring that the speed of light is not exceeded locally at any time. While traveling through a wormhole, subluminal (slower–than–light) speeds are used. If two points are connected by a wormhole, the time taken to traverse it would be less the time it would take a light beam to make the journey if it took a path through space outside the wormhole. However, a light beam traveling through the wormhole would always beat the traveler.

However, it has never been proven that wormtails exist and there is no experimental evidence for them. They are currently more *science–fiction* than they are science fact. Indeed, they play pivotal roles in science–fiction where faster–then–light travel is possible though limited, allowing connection between regions that would be otherwise unreachable within conventional timelines.

Yet, one should always keep in mind that the wormhole idea came from Einstein Theory of General Relativity using a spacetime coordinate system on a highly curved space in the vicinity of a black hole (Schwarzschild geometry).

‘WORMHOLE’ TIMELINE (1915–1988)

1915 **Albert Einstein** created his General Theory of Relativity.

1916 **Karl Schwarzschild** produced the first exact solution of Einstein field equation for a non–rotating spherically symmetric small mass (star) in empty space. His solution exhibits two singularities: one at the mass’ center (origin, $r = 0$) and the other at the critical radius $r_s = \frac{2GM}{c^2}$. For many years this solution was thought to apply only to ordinary stars and the critical radius remained a mere mathematical curiosity in the minds of most theorists. In retrospect, Schwarzschild’s solution is now called ‘the Schwarzschild *black hole*’.

1916 **Ludwig Flam** (1885–1964, Austria) realized that Einstein’s equations allowed (via the Schwarzschild solution) a second solution, now known as a *white hole*[831] (a black hole running backwards in time just as a black hole swallows thing irretrievably, so a while hole spits them out).

1930 **Subrahmanyan Chandrasekhar** (1910–1995, USA) discovered the *Chandrasekhar limit* — the minimum mass above which a star will ultimately collapse into a neutron star or a black hole. The ensuing prediction of the inescapable collapse of massive stars lead eventually to accept the concept of black holes and their associated structures.

1935 **Albert Einstein** and **Nathan Rosen** (1909–1995, USA and Israel) used Schwarzschild black hole solution as a model for elementary particles (e.g., the electron). In this way, they thought, GTR could be used to explain the mysteries of the quantum world in the framework of a unified field theory. They started with the standard

[831] GTR is time–symmetric: it does not know about the second law of thermodynamics, and it does not know about which way cause and effect go. The white hole arises from the negative square root solution inside the horizon. The negative square root solution outside the horizon represents *another universe*.

black holes solution (Schwarzschild), which resemble a large vase with a long throat. They then cut the throat, and merged it with a another black hole solution that was flipped over (a white hole). This smooth configuration is free of the singularity at the origin and might act like an electron. This concoction is known as the *Einstein–Rosen bridge*.

While the Einstein–Rosen idea of representing an electron as a virtual black hole was not accepted, cosmologists today speculate that in theory the 'Einstein–Rosen bridge' can act as a "tunnel" in space–time between *two different universes* (inter–universe) or between two remote points in the same universe (intra–universe). These connecting passage are now called 'wormholes'.[832]

[832] The Kruskal metric does not have a direct physical meaning. The name "wormhole" comes from an analogy used to explain the phenomenon. If a worm is traveling over the skin of an apple, then the worm could take a shortcut to the opposite side of the apple's skin by burrowing through its center, rather than traveling the entire distance around, just as a wormhole traveler could take a shortcut to the opposite side of the universe through a topologically nontrivial tunnel.

In physics, a wormhole is a hypothetical topological feature of spacetime that is basically a 'shortcut' through space and time. Spacetime can be viewed as a 2D surface, and when 'folded' over, a wormhole bridge can be formed. A wormhole has at least two mouths which are connected to a single throat or tube. If the wormhole is traversable, matter can 'travel' from one mouth to the other by passing through the throat. While there is no observational evidence for wormholes, spacetimes-containing wormholes are known to be valid solutions in General Relativity.

The term wormhole was coined by the **John Wheeler** in 1957. However, the idea of wormholes was invented already in 1921 by the **Hermann Weyl** in connection with his analysis of mass in terms of electromagnetic field energy.

Intra-universe wormholes connect one location of a universe to another location of the same universe (in the same present time or unpresent). A wormhole should be able to connect distant locations in the universe by creating a shortcut through spacetime, allowing travel between them that is faster than it would take light to make the journey through normal space. Inter-universe wormholes connect one universe with another. This gives rise to the speculation that such wormholes could be used to travel from one parallel universe to another. A wormhole which connects (usually closed) universes is often called a *Schwarzschild wormhole*. Another application of a wormhole might be time travel. In that case, it is a shortcut from one point in space and time

The 'wormhole' joining the two separate universes is the 'Einstein–Rosen bridge'.

The Einstein–Rosen work was disturbing to many physicists of the time because such a 'tunnel' through space–time, could in principle allow the transmission of information faster then the speed of light in violation of one of the key postulates of Special Relativity.

1960 **Martin David Kruskal** (1925–2006, USA) invented *Kruskal Coordinates*, used in GTR to explain black holes and lay out their complete space–time structure. Since then, progress in the theory of black holes has been very rapid and they are presently accepted as real phenomena almost universally.

1962 **R.W. Fuller** and **J.A. Wheeler** has given a topological interpretation of the Einstein–Rosen metric and shown that the Einstein–Rosen bridge space–time structure was dynamically unstable in field–free space. They proved that if such a wormhole somehow opened, it would close up again before even a single photon could be transmitted through it, thereby preserving Einsteinian causality.

1963 **R. Kerr** found an exact solution of Einstein's equations describing a *spinning black hole*.

1988 **K.S. Thorne** and his associates suggested that traversable wormholes could exist and that exotic[833] forms of energy threaded through a wormhole might keep it open.

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to another. In string theory, a wormhole has been envisioned to connect two D-branes, where the mouths are attached to the branes and are connected by a flux tube. Finally, wormholes are believed to be a part of spacetime foam. There are two main types of wormholes: *Lorentzian wormholes* and *Euclidean wormholes*. Lorentzian wormholes are mainly studied in General Relativity and semiclassical gravity, while Euclidean wormholes are studied in particle physics. Traversable wormholes are a special kind of Lorentzian wormholes which would allow a human to travel from one side of the wormhole to the other.

[833] Here "exotic" means that the stress energy must exceed the equivalent rest mass energy density: for a specific gravity of unity, the required stress is a little over 13×10^{15} psi!

They also concluded that what is needed is a field with *negative* equivalent mass density. This could be provided by employing the Casimir effect[834]. It remains, however, unclear whether such arrangements are physically feasible.

[834] Two identical, perfectly conducting spherical plates are placed one on each side of the throat. Each carries a homogeneous electric charge, so that they repel each other. According to the quantum–mechanical analysis of *Casimir*, the phenomenon of virtual particle pair–production causes the time–averaged energy density of the region between the plates to be *negative*.

Quantum Mechanics – Conceptual problems, Philosophical issues[835]

"I am convinced that quantum mechanics is not a final theory. I believe this because I have never encountered an interpretation of the present formulation of quantum mechanics that make sense to me. I have studied most of them in depth and thought hard about them, and in the end I still can't make real sense of quantum theory as it stands."

(Lee Smolin, 1997)

I. Introduction

Quantum Mechanics grew out of a series of anomalies in the picture of matter and light offered by Newtonian classical physics – in particular associated with blackbody radiation, the photo-electric effect, and the need to devise a model of the atom consistent with atomic spectra and the newly discovered subatomic particles.

Important aspects of Quantum Mechanics include its inherently statistical nature, at the heart of which lie the uncertainty and complementarity principles which sets limits on our knowledge of physical systems. Other key aspects are:

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[835] To dig deeper, see:

- Pagels, H.R., *The Cosmic Code*, Bantam Books, 1990, 333 pp.
- Heisenberg, W., *Physics and Philosophy*, Prometheus Books, 1999, 206 pp.
- Blinder, S.M., *Introduction to Quantum Mechanics*, Elsevier, 2004, 319 pp.
- Rae, A., *Quantum Physics: Illusion or Reality*, Cambridge University Press, 1986, 123 pp.
- Polkinghorne, J.C., *The Quantum World*, Princeton University Press, 1985, 100 pp.
- Treiman, S., *The Odd Quantum*, Princeton Press, 1999, 262 pp.

- *the ability of particles and systems to behave in two or more classically-contradictory ways (including e.g. "being in two places at the same time")*[836]*;*
- *a certain* irreducible disturbance *inflicted upon a system by observing it;*
- *very accurately reproducible values for certain observable dynamic variables (a curious counterpoint to the uncertainty principle); and*
- *high* correlations *between some observables that seem to (but do* not*) contradict their high* individual *randomness.*

The implications of the theory for the nature of reality have been and are much discussed. Most quantum theorists accept an intrinsic element of probability in the empirical predictions of fundamental physics; the need to see systems as wholes rather than merely dissecting them into their simplest components; and that not all of a quantum system's classical dynamical variables actually exist as numbers (even unknown *numbers!) except if, where and when they are actually measured.*

The empirical basis for the development and acceptance of quantum physics lies in such phenomena as blackbody radiation, the photoelectric effect, the specific heats of solids and the robustness and discreteness of the structure and the emission spectra of atoms and molecules, chemical reactions rates and energetics, condensed matter physics, and a host of other phenomena whose ranks swells yearly. All of these remain unexplainable in terms of classical physics.

In 1901, ***Max Planck*** *solved the blackbody problem by proposing that emitted or absorbed EM energy is quantized: it is released or taken up in discrete, not continuous, amounts. The quantization of light as 'photons' by* ***Albert Einstein*** *in 1905 explained the photoelectric effect, and the similar quantization of elastic waves as "phonons" allowed him to use a blackbody-like derivation to explain the specific heat of crystalline solids two years later. Furthermore, the particle/wave duality introduced into electromagnetic theory by this development — as well as the fact that photons move at the speed of light and are thus inherently relativistic — precipitated the discovery of both* non-relativistic quantum mechanics *and* relativistic quantum field theories.

[836] When an experiment is actually performed to *decide* the issue, though, it is resolved only at the price of destroying the original state and replacing it by a stochastic ensemble of new states.

In 1913 **Niels Bohr** *predicted the emission spectrum for hydrogen with a simple 'planetary' model of the atom in which the angular momentum of the orbiting electron, and thus the sizes and energies of its orbits, are quantized.*

In 1924, **Louis de Broglie** *attributed wave-like behavior to material particles and showed how this can lead to Bohr's quantization conditions. Based on this idea,* **Erwin Schrödinger** *developed the wave equation which has proved to be foundational for quantum mechanics;* **Werner Heisenberg** *formulated the uncertainty principle (and an alternative, but mathematically equivalent formulation to that of Schrödinger); and* **Wolfgang Pauli** *discovered the exclusion principle. The empirical discovery and theoretical understanding of the* spin *concept (***Stern***,* **Gerlach***,* **Goudsmit***,* **Uhlenbeck***,* **Dirac***,* **Landé** *and others) extended quantum mechanics to internal (non-spacetime) degrees of freedom.*

By the end of the 1920's (nonrelativistic) quantum mechanics was basically complete, and the 1930's and 1940's saw the development of Quantum Electrodynamics (QED) – the first, and most successful, relativistic QFT (Quantum Field Theory).

Still, almost a century later, major conceptual problems in interpreting quantum mechanics stubbornly persist:

- *The Schrödinger wave propagates continuously in time but 'collapses' discontinuously (in a process not described by the Schrödinger equation) when a few-particle quantum system interacts with a classical (i.e. mesoscopic or macroscopic) system (often called 'the* measurement problem*');*

- *The Schrödinger equation describes the propagation of the wave function, but this is a complex variable; itself not directly measurable, whose squared modulus represents statistical information about the quantum system potentially obtained via interactions of a whole ensemble of identical quantum systems with a measuring device;*

- *A composite quantum system displays a holistic character entirely unlike classical composite systems: once interacting, now vastly separated, particles continue to act in some ways as though they remained part of a single system — a feature underscored by the "EPR" paradox in the 1930s and* Bell's theorem *in the 1960s and now referred to as "quantum entanglement";*

- *'Chance' in quantum mechanics (i.e., quantum uncertainty and quantum statistics) works in strikingly different ways from classical chance. It actually gives rise, in a 'bottom-up' way, to the basic features of the* *classical world, including the robustness of atoms, the periodic table, and other properties of matter, chemistry, optics, and life.*

II. Historical Timeline of Quantum Mechanics 1925–1989

1926 *While Shrödinger's equation for the quantum wave function serves admirably as a vehicle by which to predict the outcome of laboratory experiments, the wave function itself has defied all attempts to give it an interpretation in terms of physically observable entities in a single (as opposed to an ensemble of copies of a) quantum system. It remains today as much of an ontological mystery as it was the day Schrödinger first wrote his equation.*

In fact, Schrödinger himself became so exasperated with Niels Bohr's persistent attempts to get him to admit that his wave function had no physical interpretation that he once blurted out, "I am sorry I ever started to work on atomic theory" — strong words from the man who in 1933 was awarded the Nobel Prize in Physics for "new insights into atomic theory."

1930 *Bohr's Copenhagen Interpretation (CHI).*

Probably the biggest mystery of the quantum world unveiled by ***Schrödinger***, ***Bohr***, ***Heisenberg***, ***Born***, ***Wigner*** *and others in the 1920s is what we now call the quantum measurement problem.*

The values of attributes (position, momentum and spin) of quantum objects such as photons, electrons, atoms, etc. are all arguments of the Schrödinger wave function. But until a measurement of one of these attributes is actually made, the wave function describes merely the likelihoods of the possible outcomes of such a measurement. Once a measurement is actually taken, of course, the range of potential outcomes of the corresponding attribute is replaced by a single outcome, which we term the result of the measurement.[837]

[837] *Other* attributes may remain "fuzzy", and even the just-measured attribute becomes progressively more fuzzy (uncertain) after the measurement.

The problem here is that prior to the measurement, the wave function exists as a kind of mathematical wave of probability. Yet as soon as we make an observation, this wave "collapses" to a single point with respect to the measured attribute – the outcome of the measurement. The essence of the measurement problem is to ask how this collapse comes about and what it actually means in physical terms. In short, what's so special about the act of a measurement?

The CHI makes a distinction between the observer and the observed; when no one is watching, a system evolves deterministically according to the Schrödinger wave-equation (though its classical attributes are fuzzy, correlated *via classically-impossible quantum statistics, and do not usually enjoy objective existence as numbers — even as* unknown *numbers). When, however, someone*[838] *is observing, the complex waveform (state) of the system "collapses" to an ensemble of other, observed states, which is why the act of observing changes the system. In this way the observer is accorded a* special status, *not given to any other object in quantum theory.*

Quantum mechanics can be interpreted philosophically in a variety of conflicting ways, and so far we know of no experimental basis for choosing definitively between them. These include ontological indeterminism *(**Heisenberg**),* ontological determinism *(**Einstein**, **David Bohm**),* many worlds *(**Everett**), or as involving* consciousness *(**von Neumann**, **Eugene Wigner**, **Roger Penrose**).*

All of these interpretations challenge classical-physics ontology, with its core concepts of waves, particles and locality, as well as a critical realist philosophy of nature.

The Copenhagen Interpretation cannot explain the observer itself, but it is now generally agreed that any large enough (mesoscopic or macroscopic) sample of matter, while obeying its own (vastly complicated) Schrödinger equation, interacts with a few-particle (nanoscopic or sub-atomic) quantum system as would the "ideal observer" of the Copenhagen Interpretation.

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[838] The "observer" need not be sentient, or even a recording device. Any classically-describable (i.e. many-mody) body which interacts with the "observed" quantum system and is irreversibly affected by this interaction, qualifies as an observer.

_1935_ *The EPR Paradox:* **Einstein** *and his colleagues claimed to have demonstrated the existence of* hidden variables *(fundamental elements of reality) which quantum theory fails to take into account, thus showing that the theory is* incomplete.

At the basis of EPR is the notion that if two systems are in isolation from each other for some time, then a measurement on the first can produce no real instantaneous change *of the second (no causal influence travels faster than light). This is Einstein's '*locality principle*' (separateness).*

Bohr*'s reply to this was that 'locality' was not allowed, i.e. quantum mechanics does not permit an ontological separation between the observer and the observed, even after they have separated. They are parts of a* single system. *In other words, Bohr's claim was that the EPR thought experiment does not demonstrate the incompleteness of quantum theory, but rather the naiveté of assuming* local *conditions in atomic systems. Once connected ("entangled"), atomic systems never separate.*

_1952–1966_ ***David Bohm*** *(1917–1992, USA) developed a* hidden variable[839] *interpretation of quantum mechanics which works just as well as the CHI, but gives a completely different view of quantum theory. According to his view, particles always have well-defined positions and velocities, but any attempt to measure these properties will destroy information about them by altering the* pilot wave *associated with the particles. Thus, measuring the position of an electron will immediately alter the shape of the pilot wave everywhere, affecting the future behavior of the electron.*

Bohm further developed the idea that everything is connected to everything else, and affected instantaneously by everything that happens to everything else, through the pilot-wave.

In later developments of his idea, Bohm proposed that the basic underlying order of the world consists of a field made up of an infinite number of overlapping waves, and that the overlapping of waves produces local effects which we perceive as particles. These ideas are strongly reminiscent of ***Feynman****'s*

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[839] *Hidden variables*: A hidden level of deterministic dynamics as opposed to genuinely intrinsic uncertainty. In recent years a number of key experiments have been performed to test this point. They have confirmed that uncertainty is indeed inherent in quantum systems.

sum-over-histories ("path integrals") approach to quantum mechanics[840].

1957 **Hugh Everett III** *(1930–1982, USA) developed his 'many-worlds' interpretation (MWI), which is a daring proposal to reconcile the continuity of the Schrödinger equation with the discontinuity of the quantum measurements process.*

According to this interpretation, whenever multiple viable possibilities exist for the results of the measurement of a particular attribute — whether the set of possibilities is finite, discrete-infinite or continuum-infinite in its size — the world – and in particular the measured and measuring systems – splits into a multiplicity of worlds, one world for each different possibility (in this context, the term "worlds" refers to what most people call "universes"). In each of these worlds, everything is identical, except they each have a distinct outcome of the measurement of the attribute in question. Note that the entangled quantum wavefunction of both observed and observing system split individually, although their combined ("holistic") wavefunction continues to evolve continuously, in accordance with its Schrödinger equations.

From the moment a given split occurs, the branch universes – at least for ideal measurements – develop independently, and no communication is possible between them, so the people

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[840] The formulation of quantum mechanics developed by **Feynman** in terms of path integrals builds on the familiar Lagrangian concept of the action of a trajectory in space and time and appears to be much closer to classical concepts than the Schrödinger or Heisenberg formulations. In Feynman's formulation, the *probability amplitude* of any quantum-mechanical process can be represented as a coherent superposition of contributions of all possible spatio-temporal paths that connect the initial and the final state of the system. The weight of each path is a complex number whose phase is equal to the classical action along the path, divided by Planck's constant. Even though this approach turned out to be very useful in quantum mechanics and almost indispensable in quantum field theory, it is nevertheless difficult to work with except in certain approximations — due to mathematical and computational difficulties with the quantum version of the *Wiener measure* (in path-space) which underlies path integrals.

Instead of saying that '*the photon*' travels by every possible route to a mirror and then up to the observer to make a reflected image, **Bohm** says that 'the pilot-wave' travels by every possible route, and then 'tells' the photon which path to actually follow.

living in those worlds (and splitting along with them) may have no idea that this is going on.

An advantage of the MWI is that it does away with the need for either an intelligent observer or a measuring device 'outside the system' to collapse wave functions and make reality real; indeed, the wave function of an observer (be it sentient or mechanical) is entangled (continuously and in accordance with the overall Schrödinger equation) with that of the observed system, and their mutual entanglement appears *to the observer as a collapse of the observed* system's *wavefunction — and with the same probability distribution of the measurement outcome as predicted by CHI (i.e., the squared modulus of the relevant Hilbert-space projection of the observed system's wavefunction).*

The MWI consists of two parts:

i. *A mathematical theory which yields evolution in time of the quantum state of the (single) universe.*
ii. *A prescription which sets up a correspondence between the quantum state of the universe and our (sense-perception and instrumental) experiences.*

Part (i) is essentially summarized by the Schrödinger equation. It is a rigorous mathematical theory and is not problematic philosophically although in practice it is always necessary to make various approximations — due to the immense number of quantum degrees of freedom of any realistic measuring device. Part (ii) involves, for any realistic case, a detailed quantum-thermodynamical treatment of the irreversible measurement process.

Everett demonstrated that observations in each world obey all the usual conventional statistical laws predicted by the probabilistic Born interpretation, by showing that the Hilbert space's inner product (and corresponding norm) has a special property which allows us to makes statements about the worlds where quantum statistics break down. The norm of the vector of the subset of entangled worlds where experiments contradict the Born interpretation ("non-random" or "maverick" worlds) provides a measure of these worlds which vanishes in the limit as the number of probabilistic trials goes to infinity, as is required by the definition of probability.

Thus we, as observers, are overwhelmingly likely to observe the familiar, probabilistic predictions of quantum theory to hold. Everett-worlds, where the Copenhagen Interpretation of probability rules breaks down, are never realized — or rather, *are as likely to be realized as a tepid glass of teas is likely to spontaneously re-heat at the expense of the ambient air's thermal fluctuations!*

From the MWI viewpoint, the universe is like a tree that branches and re-branches into myriads of new sub-branches with every passing zeptosecond (zepto $= 10^{-21}$*), and each of these new branch universes has a*

slightly different sub-atomic "history". Because an observer happens to have followed one particular path through the diverging branches of this universe-tree, he never perceives the splitting. Instead he interprets the resolution of the myriad of possibilities into one particular outcome as a Copenhagen-style collapse. But the observer plays no active role in the splitting. Events at the quantum level, of course, must lead to consequences in the every-day world, and one set of such consequences happens to be the irreducible quantum randomness of the sequence of empirical measurements recorded by our brains and instruments.

It seems that the majority of the opponents of the MWI reject it because, for them, introducing a very large number of worlds that we do not see is an extreme violation of Ockham's principle: "Entities are not to be multiplied beyond necessity".

However, in judging physical theories one could reasonably argue that one should not multiply physical laws beyond necessity either (such versions of Ockham's Razor has been applied in the past), and in this respect the MWI is the most economical theory (since it avoids wavefunction collapse as a separately-posited process). Indeed, it has all the laws of the standard quantum theory, but without the collapse postulate.

It is commonly thought that Many-Worlds is an un-falsifiable — and hence unprovable hypothesis, experimentally indistinguishable from the Copenhagen Interpretation.

1964–1966 **John S. Bell** *(1928–1990, Ireland). Developed an ingenious inequality principle* to test the questions raised by the EPR *paradox. The Bell inequality sets an upper limit upon the statistical correlations of certain observable pairs, each pair measured upon two quantum-entangled systems, instantaneously, and once they are too far to classically interact.*

To derive his inequality, Bell used certain facts and ideas on which everyone could agree, except for Einstein's condition of locality, which he assumed to be true.[841]

Now, if experiments showed the inequality was violated, this would mean that one of the premises in his derivation was false. Bell chose to interpret this to mean that nature is non-local if experiments would (as they did, starting in the 1980's) show the inequality to be violated; but his theorem also allows the alternate interpretation, according to which a set of attributes (dynamical variables) of a quantum system does not always even exist as numbers in case they are not measured. (This implies a violation of classical physical reality.)

The problem of the nature of locality, raised by EPR, clearly demanded some form of empirical investigation. To bring it into a form suitable for testing involves a modest degree of reformulation. The basic principles are threefold:

1. *Reality: Regularity of phenomena is due to an underlying physical reality. This requires that regularity should be the touchstone for telling reality from illusion.*
2. *Locality. This is what we are particularly keen to probe. In accord with STR it states that any influence of A upon B must not propagate between them faster than the velocity of light.*
3. *Induction: It is possible to reach conclusions valid for all systems of a given type from a consistent set of observations on a large sample of systems of that type.*

   Whatever may be the logical difficulties of a principle of induction, as a methodological strategy it is essential for science. Since we can never investigate all protons (say), any general statement about them whatsoever must depend upon a principle of this sort.

Note that the empirical violation of Bell's inequality imply that if one insists upon 1 and 3, quantum mechanical experiments produce results that are inconsistent with classical notions of causality.

[841] A position with which **Bohm** et al. would disagree.

1983 ***Alain Aspect*** *and his collaborators in Paris, obtained experimental verification of the* violation *of Bell's Inequality. This could be interpreted to mean that in spite of the local appearances of phenomena, the fabric of our world is actually supported by an invisible "infrastructure" of quantum reality which is unmediated and allows communication faster than light, even instantaneously. However, such communication is not necessarily capable of being used to* send information *or otherwise* effect causation *faster than light-speed, and communications of the latter two types can be proven to be* impossible *in mathematically consistent relativistic Quantum Field Theories*[842]*. No physical experiment to date – despite occasional claims to the contrary, which are based on misconceptions – has succeeded in configuring any causation at super-luminal (faster than light) speeds.*

1989 ***Steven Weinberg*** *suggested an experiment that would conclude whether or not there is a nonlinear term in the Schrödinger equation; the experiment was performed, and stringent upper bounds on the magnitude of such a term were thence deduced.*

1935–1939 CE **Gerti Theresa Cori, nee Radnitz** (1896–1957, U.S.A.) and **Carl Ferdinand Cori** (1896–1984, U.S.A.). Biochemists. A Prague-born American man-and-wife team. Studied *carbohydrate metabolism* and discovered (1936) how cells use and convert food into energy — a process now called the *Cori cycle*. Shared the Nobel prize in physiology or medicine (1947) with **B.A. Houssay**.

Gerti Radnitz was born in Prague to Jewish parents. She entered the Medical School of the German University of Prague and received her M.D. in 1920. Carl Cori was also born in Prague. His father — Dr. Carl Cori, was director of the Marine Biological Station in Trieste. His grandfather, Ferdinand Lippioh was a professor of Theoretical Physics at Prague. Carl

[842] Many physicists ask themselves: can we live with the preposterous concept of *action-at-a-distance* — even if only *correlation*, not information transfer or event causation, are involved?

met Gerti when they studied medicine together. He also got his M.D. in 1920 and married Gerti in the same year.

The couple emigrated to the U.S. (1922). They were on the staff of N.Y. State Institute for Study of Malignant Diseases (1922–1931), and then on the faculty of Washington University (1931–1957).

They discovered (1936) a *phosphated* form of glucose, known as the *Cori ester*; discovered (1938) the enzyme *phosphorylase*, and synthesized glycogen (1939). Their most important discovery was that glycogen in the body is not *hydrolyzed* (breaking the chemical bond by the addition of the elements of water), but is instead broken down by the use of phosphoric acid (*phosphorolysis*)

1935–1945 CE **Konrad Zuse** (1910–1995, Germany). Mechanical and civil engineer. Pioneer digital computer builder, who worked with electromechanical relay machines (having no *electronic* components). Because of his war-time isolation and his own reluctance to publish his work he was, for a long time, largely unknown.

Thus, his early machines had essentially no influence on the field although they included, in an elementary way, many of the features of modern computers. Throughout his life he received little understanding and support from the German government, industry, or academia. Not until 1960 (when it finally became clear to German leaders in the field that computing machinery was important) was he recognized as Germany's chief claim to fame in the modern history of computing.

Zuse was born in Berlin-Wilmersdorf to the family of a Prussian civil servant. He entered (1927) the Technical College in Berlin-Charlotterburg, majoring in both mechanical and civil engineering (1935).

During 1935–1938 he built two successive machines. The first, Z1, had the size of a large dining-room table, including metal sheets, glass plates, crank arms, gear-wheels and a program-cylinder — the first program-controlled computing machine. The second, Z2, was built with electromagnetic relays. In working with these machines, Zuse was able to develop and test in both theory and practice the basic laws of switching techniques, and his concepts of the design of a computer. During WWII he was developing remote-controlled flying bombs.

His first machine that really worked, the Z3, was completed in 1941. It was based on 2064 relays, used a 22-bit word and was controlled via an

8-track punched-celluloid film tape[843]. *Input* was through a keyboard and *output* was displayed on a lamp strip including a binary point. Its speed was approximately 3 sec for multiplication, division, or taking the square root of a number. The machine was demonstrated but never put into continuous operation. It was destroyed in a 1944 air raid. Zuse reconstructed it in 1960.

In his book *Calculating Space* (1969), Zuse proposed that the physical laws of the universe are discrete by nature, and that the entire universe is just an output of a giant deterministic *cellular automaton*.

1935–1948 CE **Frits** (**Frederik**) **Zernike** (1888–1966, Holland). Physicist. Invented the method of *phase contrast* and applied it in the optical *phase-contrast microscope*. The method is used to render visible a *transparent* object whose index of refraction differs slightly from that of a surrounding transparent medium. Phase contrast is particularly useful in microscopy for examination of living organisms; sections of biological materials examined under an optical microscope are often almost, or totally, transparent[844].

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843 The same ideas were occurring at the same time to other pioneers who were making similar plans and inventions, e.g. **H.H. Aiken** (1937–1944 USA), **Stibitz** (1940) and **Atanasoff** (1942, USA).

844 Because they do not absorb any of the indecent light. An object can be "seen" because it stands out from its surroundings — it has a color, tone, or lack of color which provides contrast with the background. This kind of structure is known as an *amplitude object* because it is observable by dint of variations which it causes in the amplitude of light waves. The wave which is either reflected or transmitted by such an object becomes *amplitude* modulated in the process. In contradistinction, it is often desirable to "see" *phase objects*, i.e., ones which are transparent, thereby providing practically no contrast with their environs and altering only the phase of the detected wave. The *optical thickness* of such objects generally varies from point to point (periodically or otherwise) as either the *refractive index* or the actual thickness, or both, vary.

As such materials have almost no effect on the amplitude on the light that passes through them, and since the eye (or any similar observing instrument) only distinguishes changes in intensity, such objects are invisible. (Strictly speaking, *some* details of the phase structure are always seen due to the finite size of the aperture. These can even be enhanced by a slight defocusing of the instrument.)

This is the problem which led biologists to develop techniques for *staining* transparent microscope specimens, thereby converting phase objects into amplitude objects. But this approach is unsatisfactory in many respects as, for example, when the stain kills the specimen whose life processes are under study, as is all too often the case.

A direct method for distinguishing between regions of different optical thickness (and therefore different biological compositions) is by *phase contrast microscopy*, for which Zernike was awarded the Nobel prize for physics in 1953:

The phase contrast microscope changes the phase between the light waves passing through the specimen and those not passing through it. This action turns phase objects into amplitude objects. Consequently some parts of the specimen appear brighter and the other parts darker than normal. Thus, the parts of a transparent object that vary in thickness or have different refractive index (or both), can be seen.

Zernike was born in Amsterdam. During 1915–1920 he was assistant astronomer at the University of Groningen, and afterwards a professor of physics there.

In 1938, Zernike gave a significant augmentation of the theory of *partial coherence* for quasi-monochromatic fields.

Theoretically, the phase-contrast method is a special case of *Abbe's theory* of image formation in paraxial optical systems. According to Abbe, the object acts as a diffraction grating, so that not only every element of the *aperture of the objective*, but also every element of the *object* must be taken into account in determining the complex disturbance at any particular point in the *image plane*.

Expressed mathematically, the transition from object to image involves two *Fraunhofer-type integrations*. The first over the area A of the object plane (x, y) covered by the object:

$$U(\xi, \eta) = C_1 \iint_A F(x, y) e^{-ik\left[\frac{\xi}{f}x + \frac{\eta}{f}y\right]}\, dx\, dy,$$

where F is the given transmission function of the object, U is the diffraction pattern over the back focal plane of the objective, f is the distance of this focal plane from the objective lens, and C_1 is a constant. Every point in the focal plane may be considered to be a center of a coherent secondary disturbance, whose strength is proportional to the amplitude at that point. The light waves that proceed from these secondary points will then interfere with each other and will give rise to the image $V(x', y')$ of the object in the image plane of the objective, whose typical coordinates are (x', y'). The image is given by the double Fourier integral

$$V(x', y') = C_2 \iint_B U(\xi, \eta) e^{-ik\left[\frac{x'}{D'}\xi + \frac{y'}{D'}\eta\right]}\, d\xi\, d\eta.$$

Here B is the object's aperture in the focal plane, while D' is the distance between the focal and image planes. The approximation

$$F(x, y) = e^{i\Phi(x,y)} = 1 + i\Phi(x, y) + O[\Phi^2]$$

leads to $U = U_0 + U_1$, where $U_0 = C_1 \iint_A e^{-\frac{ik}{f}(\xi x+\eta y)} dxdy$ represents the light distribution that would be obtained in the focal plane if no object were present, whilst $U_1 = C_1 \iint_A (F-1) e^{-\frac{ik}{f}(\xi x+\eta y)} dxdy$ represents the effect of *diffraction* (i.e. the interaction of the light with the object under study).

Now, the *direct light* U_0 corresponds to the central order of diffraction, and will be concentrated only in a small region B_0 in the focal plane around the axial point $\xi = \eta = 0$. On the other hand, a very small fraction of the diffracted light will, in general, reach this region, most of it being diffracted to other parts of the plane.

Suppose that the region B_0 (through which the *direct light passes*) is covered by a *phase plate* (a thin transparent material by means of which the *direct wave* is retarded or advanced by one quarter of a period relative to the diffracted spectra). The effect of this plate may be described by a transmission function $h = ae^{i\alpha}$, where $a = 1$ for a non-absorbing plate and $a < 1$ for an absorbing plate. Consequently, the new form of U will be $U'(\xi, \eta) = hU_0(\xi, \eta) + U_1(\xi, \eta)$ and $V(x', y') = V_0(x', y') + V_1(x', y')$ where

$$V_0 = hC_2 \iint_B U_0(\xi, \eta) e^{-\frac{ik}{D'}(x'\xi + y'\eta)} d\xi d\eta,$$

and

$$V_1 = C_2 \iint_B U_1(\xi, \eta) e^{-\frac{ik}{D'}(x'\xi + y'\eta)} d\xi d\eta.$$

Now, the aperture B greatly exceeds in size the region B_0, and U_0 is practically zero outside B_0. Thus both V_0 and V_1 may be given infinite integration limits, with $U_0 = 4\pi^2 C_1 \delta\left(\frac{k}{f}\xi\right)\delta\left(\frac{k}{f}\eta\right)$. Therefore $V_0 = Ch$; $V_1 = C\left[F\left(\frac{x'}{M}, \frac{y'}{M}\right) - 1\right]$ on the strength of the Fourier integral theorem, with $C = C_1 C_2 f^2 \frac{4\pi^2}{k^2}$. Finally, since $\frac{f}{D'} = -\frac{1}{M}$ (M = magnification between object plane and image plane) we find $V_1 = C\left[F(x, y) - 1\right]$. Inserting $F(x, y) - 1 \approx i\Phi(x, y)$, it follows that the intensity in the image plane is given by

$$I(x', y') = |V|^2 \approx |Ch + V_1|^2 \sim |C|^2 \left[a^2 + 2a\Phi(x, y)\sin\alpha\right].$$

For a phase plate $\alpha = \pm\frac{\pi}{2}$, and hence $I(x', y') = |C|^2\left[a^2 \pm 2a\Phi(x, y)\right]$. The intensity changes are directly proportional to the phase variations of the object. With a plate that absorbs a fraction a^2 of the direct light, the ratio of the second term to the first term has the value $\pm\frac{2\Phi}{a}$, so that the contrast of the image is enhanced.

In general, when the *phase of the central order is retarded* w.r.t. the diffraction wave, regions of the object which have greater optical thickness will appear brighter than the mean illumination. When the phase of the central order is advanced, regions of greater optical thickness will appear darker.

Optical Coherence[845] *(1865–1938)*

Light from a real physical source is never strictly monochromatic, since even the sharpest spectral line has a finite width. Moreover, a physical source is not a point source, but has a finite extension consisting of many elementary radiators (atoms); because of the quantized nature of the radiation process, light is emitted via electron transitions in the form of individual photons, which for our present purpose can be represented by finite wave trains.

Moreover, since the atoms are in random thermal motion, the frequency spectrum will be broadened by the Doppler effect. In addition, the atoms suffer collisions, which interrupt the wave trains and again tend to broaden the frequency distribution. The total effect of all of these mechanisms is that each spectral line has a bandwidth $\Delta\nu$ *rather than one single frequency. The temporal extent of the pulse is of the order* $\Delta t \sim \frac{1}{\Delta\nu}$ *and is referred to as the coherence time. The associated length* $\Delta x = c\Delta t$ *is the coherence length*[846].

In an ideal monochromatic wave field the amplitude of the vibration at any point is constant, while the phase varies linearly with time. A real source can

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[845] To dig deeper, see:

- O'Neill, E.L., *Introduction to Statistical Optics*, dover, 1992, 179 pp.
- Baldwin, G.C., *An Introduction to Nonlinear Optics*, Plenum Press, 1969, 155 pp.
- Beran, M.J. and G.B. Parrent, Jr., *Theory of Partial Coherence*, Prentice-Hall, 1964, 193 pp.

[846] *White light* has a frequency range from 0.4×10^{15} Hz to about 0.7×10^{15} Hz, that is, a bandwidth of about 0.3×10^{15} Hz. The coherence time is then roughly 3×10^{-15} sec, while the coherence length is 9×10^{-5} cm — a spatial extent only a few wavelengths long.

be visualized in the time domain as being composed of wave packets, bearing random phase relation to each other. In general, the Fourier spectrum of a single packet will differ from that of the light beam composed on N wave packets, because in the latter the amplitude and phase undergo irregular fluctuations. If, however, we restrict the time interval of the observed pulse to be small compared to the reciprocal of the effective width of the spectrum (i.e. the coherence time), then, within such a time interval the amplitudes of the spectral components will remain substantially constant; in this time interval the light behaves like a monochromatic wave with the mean frequency. This will happen whenever the source is quasi-monochromatic, i.e. if the bandwidth is small compared to the mean frequency. In effect, the coherence time is loosely the temporal interval over which we can reasonably predict the phase of the light wave at a given point in space.

The same characterization can be viewed somewhat differently. Imagine that we have two separate field points P_1 and P_2 to which light arrives from a quasi-monochromatic point source. If the coherence length $\Delta x = c\Delta t$ is much larger than the longitudinal separation between the points, the disturbances at P_1 and P_2 will be highly correlated. On the other hand, if this longitudinal separation were very much greater than the coherence length, many wave trains, each with an unrelated phase, would span the gap between P_1 and P_2. In that case, the disturbance at the two points in space would be independent at any given time. The degree to which a correlation exists is sometimes spoken of alternatively as the amount of longitudinal coherence. Whether we think in terms of coherence time (Δt) or coherence length $(c\Delta t)$, the effect still arises from the finite bandwidth of the source.

The idea of spatial coherence is most often used to describe effects arising from the finite extent of ordinary light sources.

The theory of optical interference is based on the principle of linear superposition of electromagnetic vector fields. According to this principle, the electric field $\boldsymbol{E}$ produced at a point in empty space jointly by several different sources is equal to the vector sum $\boldsymbol{E} = \boldsymbol{E}_{(1)} + \boldsymbol{E}_{(2)} + \boldsymbol{E}_{(3)} + \cdots$, where $\boldsymbol{E}_{(j)}$ are the fields produced at the point in question separately by the different sources. The same is true for magnetic fields. Consider the monochromatic, linearly polarized plane waves with corresponding fields $\boldsymbol{E}_{(1)} = \boldsymbol{E}_1 e^{i(\boldsymbol{k}_1\cdot\boldsymbol{r}-\omega t+\varphi_1)}$, $\boldsymbol{E}_{(2)} = \boldsymbol{E}_2 e^{i(\boldsymbol{k}_2\cdot\boldsymbol{r}-\omega t+\varphi_2)}$ etc. Here the quantities φ_1 and φ_2 have been introduced to allow for any phase differences between the sources of the two waves, so that $\boldsymbol{E}_j$ are real.

If the phase difference $\varphi_1 - \varphi_2$ is constant, the two sources are said to be mutually coherent. The intensity of radiation at a point between two sources is proportional to the square of the amplitude of the light field at the point *in question. Therefore, the intensity of the interference pattern between two sources is proportional to*

$$I = |\boldsymbol{E}|^2 = \boldsymbol{E}\cdot\boldsymbol{E}^* = (\boldsymbol{E}_{(1)} + \boldsymbol{E}_{(2)})\cdot(\boldsymbol{E}^*_{(1)} + \boldsymbol{E}^*_{(2)})$$

$$= |\boldsymbol{E}_1|^2 + |\boldsymbol{E}_2|^2 + 2\boldsymbol{E}_1\cdot\boldsymbol{E}_2\cos\theta = I_1 + I_2 + 2\boldsymbol{E}_1\cdot\boldsymbol{E}_2\cos\theta;$$

$$\theta = \boldsymbol{k}_1\cdot\boldsymbol{r} - \boldsymbol{k}_2\cdot\boldsymbol{r} + \varphi_1 - \varphi_2.$$

Since θ depends on $\boldsymbol{r}$, periodic spatial variations in intensity occur. These variations are the familiar interference fringes that can be seen when two mutually coherent beams of light are combined. If the sources of the two waves are mutually incoherent, then the quantity $\varphi_1 - \varphi_2$ varies with time in a random fashion. The result is that the mean value of $\cos\theta$ is zero, and there is no interference. (This will also happen if the polarizations are mutually orthogonal, i.e. $\boldsymbol{E}_1\cdot\boldsymbol{E}_2 = 0$.)

The classical experiment that demonstrates interference of light from a point source was first performed by **Thomas Young** *in 1802. In his 2-slit experiment $\theta \simeq -\frac{kyh}{x}$, where h = slit separation, x = slit-screen distance, and y = coordinate of a point on the far screen. If $|\boldsymbol{E}_1|^2 = |\boldsymbol{E}_2|^2 = I_0$, we shall have $I = 2I_0\left[1+\cos\left(\frac{kyh}{x}\right)\right]$ with bright fringes maxima at $y = 0, \lambda\frac{x}{h}, 2\lambda\frac{x}{h}, \cdots$ where $\lambda = \frac{2\pi}{k}$ = wavelength.*

In order to adequately describe a wave field produced by a finite polychromatic source, it is desirable to introduce some measure for the correlation that exists between the vibrations at different points in the field. We must expect such a measure to be closely related to the sharpness of the interference fringes which would result on combining the vibrations from the two points. We expect sharp fringes when the correlation is high (e.g. when light arrives at two separate field points from a very small source of narrow spectral range), and no fringes at all in the absence of correlation.

We describe these situations by the terms coherent and incoherent respectively. In general, neither of these situations is realized and we may speak of vibrations which are partially coherent. In this case, the amplitudes and phases usually vary with time such that the instantaneous light intensity at a given point fluctuates rapidly. It would then seem meaningful, to define the intensity as a time average.

In the case of two fields $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$, the intensity I can accordingly be expressed as

$$I = \langle\boldsymbol{E}\cdot\boldsymbol{E}^*\rangle = \langle(\boldsymbol{E}_1 + \boldsymbol{E}_2)\cdot(\boldsymbol{E}^*_1 + \boldsymbol{E}^*_2)\rangle = \langle|\boldsymbol{E}_1|^2 + |\boldsymbol{E}_2|^2 + 2Re(\boldsymbol{E}_1\cdot\boldsymbol{E}^*_2)\rangle.$$

The angular brackets denote the time average $\langle f \rangle = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} f(t)dt$*. It will be assumed that all quantities are* stationary *(time average is independent of the origin of time). Also, for convenience, the optical fields will be assumed to have the same polarization so that their vectorial nature can be ignored. Then* $I = I_1 + I_2 + 2\text{Re}\langle E_1 E_2^* \rangle$, *where* $I_1 = \langle |E_1|^2 \rangle$, $I_2 = \langle |E_2|^2 \rangle$.

Consider the general problem of coherence at some given field-point between waves from two source-points $\boldsymbol{x}_1$ *and* $\boldsymbol{x}_2$ *emitted at different times. Then, define the* mutual coherence function *(Zernike, 1938)*

$$\Gamma_{12}(\tau) = \Gamma_{12}(\boldsymbol{x}_1, \boldsymbol{x}_2, \tau) = \langle E_1(\boldsymbol{x}_1, t) \cdot E_2^*(\boldsymbol{x}_2, t+\tau) \rangle,$$

where E_1 *is the optical disturbance at the point* $\boldsymbol{x}_1$, E_2 *the optical disturbance at the point* $\boldsymbol{x}_2$, *and* $\tau = t_2 - t_1$ *is given by* $\Delta\ell/c$ *(where* $\Delta\ell$ *is the optical path difference between the two beams and* c *is the speed of light in vacuum).*

A special case of interest arises when Young's experiment is repeated with a partially coherent source (say, quasi-monochromatic with a spectral width $\Delta\nu$ *that is very small compared to the mean frequency* ν*, and also such that* $\Delta x \ll c/\Delta\nu$*). In this case one can consider two fields arriving from the same point* $\boldsymbol{x}_1 = \boldsymbol{x}_2$*) over different optical paths, which makes* E_1 *different from* E_2.

Defining the self-coherence *functions*

$$\Gamma_{11}(\tau) = \langle E_1(\boldsymbol{x}_1, t) \cdot E_1^*(\boldsymbol{x}_1, t+\tau) \rangle,$$

$$\Gamma_{22}(\tau) = \langle E_2(\boldsymbol{x}_1, t) \cdot E_2^*(\boldsymbol{x}_1, t+\tau) \rangle$$

as the complex autocorrelation of the fields due to the two beams, we find $\Gamma_{11}(0) = I_1$, $\Gamma_{22}(0) = I_2$. *Therefore,*

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\text{Re}\{\gamma_{12}(\tau)\}$$

where

$$\gamma_{12}(\tau) = \frac{\Gamma_{12}(\tau)}{\{\Gamma_{11}(0)\Gamma_{22}(0)\}^{1/2}}$$

is known as the complex degree of coherence *(in general, an approximately periodic function of* τ*). In terms of* $|\gamma_{12}(\tau)|$, *we have the following types of coherence:* $|\gamma_{12}| = 1$, *complete coherence;* $|\gamma_{12}| = 0$, *complete incoherence;* $0 < |\gamma_{12}| < 1$, *partial coherence (by the Cauchy–Schwarz inequality we always must have* $0 \le |\gamma_{12}| \le 1$*).*

Michelson *(1890) introduced the concept of* visibility *of the interference fringes — defined as the ratio*

$$\mathcal{V} = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}.$$

In terms of the degree of coherence it is equal to $\frac{2\sqrt{I_1 I_2}}{I_1+I_2}|\gamma_{12}|$. *In particular, if* $I_1 = I_2$, *then* $\mathcal{V} = |\gamma_{12}|$, *i.e., the fringe visibility is equal to the degree of coherence, and the latter is then simply measurable.*

For the Young experiment, one can specify the analytic form of the fields E_1 *and* E_2 *as:* $E_1 = K_1 V_1\left(t - \frac{r_1}{c}\right)$, $E_2 = K_2 V_2\left(t - \frac{r_2}{c}\right)$, *where* K_1 *and* K_2 *depend on the size of the slits and* r_1 *and* r_2 *are the respective distances from slits* S_1 *and* S_2 *to the field-point* P, *and* $V_1(t)$, $V_2(t)$ *are* stationary *random functions which describe the light oscillations at* S_1 *and* S_2 *respectively. Using the previous definitions, the intensity at* P *is given by*

$$I(P) = \left\langle\left|K_1 V_1\left(t - \frac{r_1}{c}\right) + K_2 V_2\left(t - \frac{r_2}{c}\right)\right|^2\right\rangle = I_1 + I_2 + 2\sqrt{I_1 I_2}|\gamma_{12}|\cos\Phi_{12},$$

where

$$\Phi_{12} = \arg\left[\Gamma_{12}(0)\right] + \frac{2\pi}{\lambda}(r_1 - r_2).$$

Because of the stationarity condition; γ_{12} *depends only on* $\tau = \frac{r_1}{c} - \frac{r_2}{c}$ *and not on* $\frac{r_1}{c}$ *and* $\frac{r_2}{c}$ *explicitly.*

Also, since only quasi-monochromatic light is being considered, the experiment is restricted to small path differences $r_1 - r_2 \ll \frac{c}{\Delta\nu}$.

When Young's experiment is performed with very narrow-bandwidth extended sources, spatial coherence *effects will predominate. The optical disturbances at the slits* S_1 *and* S_2 *will differ, and the fringe pattern will depend on* $\Gamma_{12}(0)$. *By examining the region about the central fringe where* $r_1 = r_2$, $\gamma_{12}(0)$ *can be determined. In this case, spatial coherence results primarily from* angular source size considerations. *Thus,* $\Gamma_{12}(0)$ *plays a central role in the measurement of stellar diameters. On the other hand,* $\Gamma_{11}(\tau)$ *is a measure of* temporal coherence.

The first investigations which had a close bearing on the subject of partial coherence appear to be due to ***Emile Verdet*** *(1865 to 1869), who demonstrated that the light from 2 pinholes in a screen illuminated by the sun will interfere in Young's experiment if the separation of the pinholes is less than about* $\frac{1}{20}$ *mm.* ***Laue*** *(1907) gave a quantitative measure for partial coherence. Theoretical interest in this subject lay dormant until it was revived in the* 1930*'s by* ***P.H. van Cittert*** *(1934, 1939) and* ***Zernike*** *(1938).*

These authors determined the theoretical complex degree of coherence $\gamma_{12}(P_1, P_2; \tau = 0)$ *for two points* P_1 *and* P_2 *on a planar screen illuminated by an* extended *quasi-monochromatic primary source, having the shape of a portion of a plane* σ *parallel to the screen. Assuming that the medium between the source and the screen is homogeneous, that the linear dimensions of* σ *are small compared to the source-screen distance, and that the angular* *dimensions of the source as seen from any point on the screen are small, it is found that*

$$\gamma_{12}(P_1, P_2; \tau = 0) = \frac{1}{\sqrt{I(P_1)I(P_2)}} \int_\sigma I(S) \frac{e^{i\overline{k}(r_1 - r_2)}}{r_1 r_2} dS,$$

where

$$I(P_1) = \int_\sigma \frac{I(S)}{r_1^2} dS, \quad I(P_2) = \int_\sigma \frac{I(S)}{r_2^2} dS.$$

Here $\overline{k}$ is the average wave-number in the medium, $I(S)$ is the intensity flux per unit area (flux) of the source, and r_1, r_2 are the respective distances between a typical source point S and the points P_1 and P_2 on the screen. The above result also incorporate the assumption that the coherence time from all point sources is larger than the time difference for all paths between the source and the points of observation.

It can be shown that γ_{12} is the spectral amplitude diffraction-pattern at P_1 (centered on P_2) when we replace the source by a diffraction aperture in an opaque screen of the same size and shape as the source (Huygens-Fresnel diffraction).

This result is known as the van Cittert-Zernike theorem[847]*. Establishing a coordinate system $O(\xi, \eta)$ at the source S, and a parallel one $O'(x, y)$ on the screen, the above integral can be reduced to the form*

$$\gamma_{12} \approx \frac{\iint_\sigma I(\xi, \eta) e^{-i\overline{k}(p\xi + q\eta)} d\xi d\eta}{\iint_\sigma I(\xi, \eta) d\xi d\eta} e^{i\Psi},$$

where

$$p = \frac{x_1 - x_2}{r}, \quad q = \frac{y_1 - y_2}{r}, \quad \Psi \simeq \overline{k}(OP_1 - OP_2);$$

$$r = OO'; \quad P_1 = (x_1, y_1); \quad P_2 = (x_2, y_2).$$

For a uniform circular source of diameter d, we find $\gamma_{12} \approx \frac{2J_1(v)}{v} e^{i\Psi}$, J_1 being the Bessel function of the first kind and first order, and $v = \frac{\pi}{\lambda} \frac{d}{r} \overline{P_1 P_2}$.

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[847] Apart from its theoretical significance, the theorem is particularly important from a computational point of view because the coherence calculation is normally more difficult than the calculation of the corresponding diffraction pattern. Note that the theorem ties up two Fourier transform pairs, namely, the *coherence-intensity pair* (visibility-brightness) to the *diffraction pattern-aperture pair*.

The function $\left|\frac{2J_1(v)}{v}\right|$ *decreases steadily from the value of unity when* $v = 0$ *to the value zero when* $v = 3.83$*; thus, as the points* P_1 *and* P_2 *are separated more and more, the degree of coherence steadily decreases and there is a complete incoherence when* P_1 *and* P_2 *are separated by the distance* $\overline{P_1P_2} = 1.21\ \frac{\lambda}{(d/r)}$. *For the sun* $\frac{d}{r} \sim 0\,^\circ 32' \sim 0.009$ *radian, and consequently* $\overline{P_1P_2} = 0.06$ *mm, as found by* ***Verdet*** *(1869) for* $\lambda = 5500$ *Å.*

1935–1952 CE **Alan Mathison Turing**[848] (1912–1954, England). Mathematician and logician. One of the founders of modern automata theory, computer logic and artificial intelligence.

During 1935–1937, Turing developed theories for an idealized computing device which can be considered as the theoretical prototype of present day digital computers (known today as a *Turing machine*). His specifications were:

(1) The machine has a *finite*, discrete set of different internal states.

(2) The input data is not restricted in size, i.e. no limit is placed on the amount of information that the machine can process.

(3) The calculational procedure (algorithm) is the same *finite* set of instructions no matter how big the data stream.

(4) The machine must be allowed to call upon an unlimited external storage space for its calculations[849]. [The question of whether the storage space be regarded as internal or external is just a technicality. The internal part could be the *hardware* and the external part the *software*.]

(5) The device 'reads' a *tape* with marks on it. These marks comprise a linear sequence of squares. Each square is either blank or a single mark (say, the symbols '1' and '0'). The machine reads one square at a time,

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848 For further reading, see:

- Hodges, A., *Alan Turing, The Enigma*, Simon and Schuster: New York, 1983, 587 pp.

849 The marvels of modern computer technology have provided us with electronic storage devices which can be treated as unlimited for most practical purposes.

and after each operation moves just one square to the right or left (as determined by the algorithm).

(6) The device could also place new marks on the tape where required and could obliterate old ones. The tape will keep running back and forth through the device so long as further calculations need to be performed. When the calculation is finally completed, the device comes to a halt and the answer to the calculation is displayed on that part of the tape which lies to one side of the device.

Problems that could be solved by the machine, were called by Turing '*computable*'.

In 1936 Turing solved Hilbert's 23^d problem by showing that there is no unique way to prove or disprove all logical statements.

In 1952 Turing originated the mathematical theory of *morphogenesis*, the development of pattern and form in living organisms. The main goal was to show how a uniform and symmetric structure could evolve, through a combination of diffusion and chemical reaction, into a strongly asymmetrical structure with a definite pattern. Though this work was left unfinished, it is a major contribution to mathematical biology. Through his equations[850], Turing introduced the concept of *diffusion-driven instability* (known today as *Turing instability*). He showed that the presence of diffusion induces small spatial perturbations into a uniform-mixture steady state. It is this instability that eventually determines how the pattern or mode are selected. His work, however, went unnoticed by the embryologists and chemists of his time.

In 1953, Turing suggested a new method for the determination of the number of zeros of the Riemann zeta function in a given range.

Alan Turing was born in London, the son of a British member of the Indian Civil Service who was away from his children during most of their childhood. Turing was educated at King's College, Cambridge. He completed his Ph.D. at Princeton University (1938). During WWII he played a significant role in breaking the German "Ultra" codes.

In 1945 he joined the staff of the National Physical Laboratory to lead the design, construction, and use of a large electronic digital computer that was named the Automatic Computing Engine (ACE). In 1948 he became deputy director of the Computing Laboratory at the University of Manchester, where the Manchester Automatic Digital Machine (MADAM, as referred to by the press), the computer with the largest memory capacity in the world at that

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[850] A.M. Turing, The chemical basis for morphogenesis, *Phil. Trans. Roy. Soc. Lond. B* **237**, 37–72, 1952.

time, was being built. *His efforts in the construction of early computers and the development of early programming techniques were of prime importance.*

He also championed the belief that computers could be constructed that would be capable of thought, and even proposed that machine thought could more closely resemble human thought if a random element could be introduced. Turing's papers on this subject are widely acknowledged as the foundation of research in artificial intelligence.

Turing was arrested for violation of British homosexuality statutes in 1952. He died of potassium cyanide poisoning while conducting electrolysis experiments. An inquest concluded that it was self-administered but it is now thought by some to have been an accident.

Chemical Patterns, Clocks and Waves[851]

"Has it a clock? Or is it a clock?"

Collin S. Pittendrigh (1957)

Nonlinear systems, that is, systems governed by a set of nonlinear equations (algebraic, functional, ordinary differential, partial differential, integral, stochastic or a combination of these) are used to describe a great variety of phenomena, in the social and life sciences, as well as the physical sciences and engineering. The theory of nonlinear systems has applications to problems of economics, population growth, ecosystems, the propagation of genes, the physiology of nerves, the regulation of heart-beats, chemical reactions, phase

[851] For further reading, see:

- Nicolis, G. and I. Prigogine, *Self–Organization in Nonequilibrium Systems*, Wiley, 1977, 491 pp.
- Prigogine, I., *From Being to Becoming*, W.H. Freeman, 1980, 272 pp.
- Prigogine, I. and I. Stengers, *Order Out of Chaos*, Bantam Books, 1984, 349 pp.

transitions, elastic buckling, the onset of turbulence, celestial mechanics, earth sciences, electronic circuits and many other phenomena.

Nonlinear system display certain characteristics that differ radically from those of linear systems. One finds, in particular, that as a parameter changes slowly a solution may change either slowly and continuously or abruptly and discontinuously. In many applications of the theory of nonlinear systems we are interested in enduring rather than transient phenomena, and so in steady states. Thus, steady solutions *of the governing equations are of special importance.*

Of these steady solutions, only the stable *ones, (i.e., those which, when slightly disturbed, are little changed afterwards), correspond to states which persist in practice, and so are usually the only ones observable. It follows that a state may change abruptly not only if it ceases to exist but also if it becomes unstable as parameters change.*

Another general phenomenon associated with nonlinearity is that as a result of an instability, a small cause may have a large effect *— in the sense that a small disturbance at a given instant may grow and become significant, to the extent that after a long time the behavior of the system depends substantially on the nature of the disturbance, however small it was.*

For example, a spherical pendulum with the bob finely balanced directly above its point of suspension, may be destabilized by a gentle breath on it; further, the direction and timing of the ensuing motion of the pendulum depend strongly on the very small disturbance of the unstable position of equilibrium[852]*.*

A bifurcation *occurs where the solutions of a nonlinear system change their qualitative character as a parameter changes. In particular, bifurcation theory is about how the number of steady solutions of a system depends on parameters. The theory of bifurcations, therefore, concerns all nonlinear systems and thus has a great variety of applications. Bifurcations of a nonlinear system and the onset of instability of a solution usually occur at the same critical value of a parameter governing the system.*

The word bifurcation *was coined by* ***J.H. Poincaré*** *(1885). In the study of self-gravitating spinning fluids (relevant to planetary formation) he found that a sequence of pear-shaped figures of equilibrium branches off the sequence of* Jacobi ellipsoids*, just as Jacobi ellipsoids branch off the* Maclaurin spheroids*. However, the significance of bifurcations first came to be recognized in the*

852 **E.N. Lorenz** described this in a metaphor in which an unstable prairie atmospheric condition might be triggered by the flutter of the wings of a butterfly in a distant jungle, and thereby a devastating tornado might arise.

18^{th} *century. The work of* **L. Euler** *(1744) on the equilibrium and buckling of an elastic column under load, and the work of* **J. le Rond d'Alembert** *(1747) on the figures of equilibrium of a rotating mass of self-gravitating fluid, are the foundations of bifurcation theory.*

The ideas of bifurcation theory arose slowly and imperceptibly at first, being almost as old as algebra itself. At the simplest, we may view the quadratic equation $x^2 - a = 0$ *as an example. If we examine the real solutions of this equation as a* function of the parameter a *(real), we find that there are two for* $a > 0$, *one for* $a = 0$ *and none for* $a < 0$. *This situation can be visualized by a drawing of a parabola in the* (x, a) *plane (*bifurcation diagram*). The point* $a = 0$ *is a* bifurcation point*, since there the qualitative character of the solutions change (in this case, the* number *of solutions).*

A less trivial example is the ODE $\frac{dx}{dt} = a - x^2$, *where the* number *and* character *of the solutions depend critically both on* a *and on the initial value* $x(0) = x_0$.

If we define an equilibrium point *(or* steady-state point*) as that point in the* (a, x_0) *plane for which* $x(t)$ *is time-independent, then these points lie on the parabola* $x^2 = a$. *The branch* $x = +\sqrt{a}$, $a > 0$ *corresponds to* stable *solutions, while the branch* $x = -\sqrt{a}$, $a > 0$ *consists of unstable solutions. The point* $(0, 0)$ *is* unstable*. It is obvious from the explicit expression*

$$x(t) = \begin{cases} \frac{x_0 + \sqrt{a}\tanh(t\sqrt{a})}{1 + x_0\tanh(t\sqrt{a})/\sqrt{a}} & , \quad a > 0 \\ \frac{x_0}{1 + x_0 t} & , \quad a = 0 \\ \frac{x_0 - \sqrt{-a}\tan(t\sqrt{-a})}{1 + x_0\tan(t\sqrt{-a})/\sqrt{-a}}, & \quad a < 0 \end{cases}$$

that the solution may become infinite after a finite time, and the value of t *at the singularity will depend on* x_0 *as well as on* a. *The point* $(0, 0)$ *is a* bifurcation*, since the number of steady solutions and their character changes as* a *increases through zero.*

The previous equation can be generalized into the form $\frac{dx}{dt} = F(x)$. *In the case where the variable* x *represents a position, the latter equation may be considered as an appropriate expression of the second law of dynamics in the presence of very high damping. Similar equations are often encountered in the thermodynamics of irreversible processes and in this case the term* $F(x)$ *is often called a 'generalized force'.*

If an explicit form of the potential function $V(x)$ *can be found such that* $F(x) = -\frac{dV}{dx}$, *then the stability can be determined from the sign of* $\frac{d^2V}{dx^2}$. *For example,* $F(x) = -Kx - K_1x^3$ $(K_1 > 0)$ *corresponds to the potential* $V(x) = \frac{K}{2}x^2 + \frac{K_1}{4}x^4$. *If* $K > 0$, *the origin* $x = 0$ *will be a point of stable* *equilibrium. If, however* $K < 0$, *the origin will be unstable, and two new stable points will be located at* $x_0^{(1,2)} = \pm\sqrt{\frac{|K|}{K_1}}$. *Clearly, there is a bifurcation at* $K = 0$ *of the single stable solution into two stable and one unstable solutions.*

*An important branching of a time-periodic solution from a steady state is known as a Hopf bifurcation (**E. Hopf**, 1942). For example, the system* $\dot{x} = -y + (a - x^2 - y^2)x$, $\dot{y} = x + (a - x^2 - y^2)y$, *is stable at its steady-state solution* $x = 0$, $y = 0$ *provided* $a < 0$. *The exact solution of this system is*

$$r^2(t) = \begin{cases} \frac{ar_0^2}{r_0^2 + (a - r_0^2)e^{-2at}}, & a \neq 0 \\ \frac{r_0^2}{1 + 2r_0^2 t} \quad , & a = 0 \end{cases}$$

where $x + iy = re^{i\theta}$, $\theta = \theta_0 + t$, *and* $r = r_0$, $\theta = \theta_0$ *at* $t = 0$. *The solution is written as* $x(t) = r(t)\cos(t + \theta_0)$, $y(t) = r(t)\sin(t + \theta_0)$. *For* $a \leq 0$, *all solutions vanish as* $t \to \infty$. *However, for* $a > 0$ *the origin becomes an unstable focus while a new stable periodic solution* $x = \sqrt{a}\cos(t + \theta_0)$, $y = \sqrt{a}\sin(t + \theta_0)$ *arises as a increases through zero. Such a solution is called a limit cycle, because it is a periodic solution approached by other solutions in the limit as* $t \to \infty$. *It is represented by a closed curve, in this case the circle* $x^2 + y^2 = a$, *in the phase plane of the orbits. Thus, the periodic solutions bifurcate from the null solution as a increases through zero.*

Suppose that $\ddot{x} = f(x, \lambda)$, *represents the equation of motion of a unit mass under the influence of a force* f, *where* x *is the displacement of the particle and* λ *is a parameter. Equilibrium points of the mass are given by* $f(x, \lambda) = 0$. *If there exists a function* $V(x, \lambda)$ *such that* $F(x, \lambda) = -\frac{\partial V}{\partial x}$ *for each value of* λ, *then* $V(x, \lambda)$ *is the potential energy of the system and equilibrium points correspond to stationary values of the potential energy.*

We expect a minimum of the potential to correspond to a stable equilibrium point, and other stationary values (the maximum and points of inflexion) to be unstable. In fact, V *is a minimum at* $x = x_1$ *if* $\frac{\partial V}{\partial x}$ *changes from negative to positive on increasing through* x_1; *this implies that* $f(x, \lambda)$ *changes sign from positive to negative as* x *increases through* $x = x_1$. *Hence the curve* $f(x, \lambda) = 0$ *in the* (λ, x) *plane is the locus of all equilibrium points.*

Suppose we shade the entire domain in which $f(x, \lambda) > 0$. *If a segment of the curve has shading below it, the corresponding equilibrium points are stable, since for fixed* λ, f *changes from positive to negative as* x *increases. Bifurcation points mark transition from stable to unstable points on* $f(x, \lambda) = 0$. *As* λ *varies through such points, the equilibrium point may split into two or more equilibrium points, or several equilibrium points may merge into a single one.*

Bifurcation points are therefore diagnostic of the dynamic behavior of a parameter-dependent system in which both the number and stability of equilibrium points may vary with the parameter(s).

Bifurcation is often associated with what is called symmetry breaking. A symmetry of a nonlinear system manifests itself as invariance of the set of all solutions under some group of transformations, but it does not necessarily follow that each solution is itself invariant. Symmetry is broken at a bifurcation if all solutions are symmetric where a parameter is greater (or less) than a critical value but some are asymmetric when the parameter is less (or greater) than that value.

Also, oscillations occur in many applications, so periodic solutions and their stability are important too. Further, unsteady solutions may occur which are seemingly random functions of time with stationary statistical properties; these are chaotic solutions or, more precisely, strange attractors. A chaotic solution also may be stable in the sense that it persists even (with changed detailed behavior but unchanged statistical properties) when the solution is perturbed slightly at some time.

The observed patterns and ordered structures of living organisms have long been a puzzle to biologists and were crying out for an explanation based on the physicochemical laws of nature.

A nonlinear partial differential equation of major importance in modern theoretical biology and nonequilibrium thermodynamics is the reaction-diffusion equation. The equation of conservation of matter for the flux of material $\boldsymbol{J}$*, concentration* c *and sources* f*, reads*

$$\frac{\partial c}{\partial t} + \operatorname{div} \boldsymbol{J} = f(c, \boldsymbol{r}, t).$$

Substituting $\boldsymbol{J} = -D \operatorname{grad} c$ *(Fick's law), we find an equation for* $c(\boldsymbol{r}, t)$*:*

$$\frac{\partial c}{\partial t} = f + D\nabla^2 c$$

for fixed diffusivity D*. This equation is referred to as the reaction-diffusion equation.*

Such a mechanism was proposed as a model for the chemical basis of morphogenesis by Turing (1952), and has been widely studied since 1970*. Examples for its application will be discussed next. They involve the complex behavior of those chemical systems exhibiting self-organization phenomena, such as the formation of stationary spatial structures or periodic oscillatory states.*

• *Spatial pattern formation*. *Morphological order is the most conspicuous attribute of living species. Leaves, branches, skin, hair, legs, all appear with a well-defined order and relationship with respect to each other. Moreover, this morphological order and its sequential unfolding in space and time is engraved in the tiny seeds and minute fertilized eggs.*

Whatever pattern one chooses to focus on in the animal or plant worlds, it is almost certain that the process that produced it is unknown. Although the mechanism must be genetically controlled, the genes themselves cannot create the pattern. They only provide a blueprint, or recipe, for the pattern generation. The problem is then to discover how genetic information is physically translated into the necessary pattern and form as manifested in the phenotype.

H.A.E. Driesch *(1867–1941, Germany), a biologist and philosopher, proposed (1895) that morphogenesis (the birth of forms) is the consequence of the onset of various gradients of unspecified nature in the developing embryo. In order to substantiate these ideas,* ***Turing*** *(1952) suggested that during stages in the development of an organism, chemical constituents generate a* prepattern *that is later interpreted as a signal for cellular differentiation. In other words, cells are pre-programmed to react to chemical concentration so that the cell can read out its position in the coordinates of chemical concentration and differentiate, undergo appropriate cell shape change, or migrate accordingly. Such chemical substances have been called* morphogens.

According to Turing, the prepattern *is generated through the processes of reaction and diffusion such as to produce a steady state heterogeneous spatial patterns.*

Consider two chemicals with the respective concentrations $C_1(\boldsymbol{r}, t)$ *and* $C_2(\boldsymbol{r}, t)$. *Let* $R_1(C_1, C_2)$ *be the generally nonlinear rate of production of* C_1, $R_2(C_1, C_2)$ *the rate of production of* C_2, *and* $\{D_1, D_2\}$ *the respective diffusion coefficients of the chemicals.*

The governing system of equations is then of the form

$$\frac{\partial C_1}{\partial t} = R_1(C_1, C_2) + D_1\nabla^2 C_1,$$

$$\frac{\partial C_2}{\partial t} = R_2(C_1, C_2) + D_2\nabla^2 C_2.$$

Turing explained why this system is expected to form spatial patterns: If in the absence of diffusion $(D_1 = D_2 = 0)$ C_1 *and* C_2 *tend to a stable uniform steady state* $\left(\frac{\partial C_1}{\partial t} = \frac{\partial C_2}{\partial t} = \nabla C_1 = \nabla C_2 = 0\right)$, *then this equilibrium may be disturbed by the addition of the diffusion terms, provided* $D_1 \neq D_2$ *and certain other conditions hold. This instability,* driven by diffusion, *is precisely the cause of the formation of the spatial patterns.*

To see intuitively how this pattern is formed due to diffusion, it is sufficient to consider the one-dimensional case and with the simplest form of R_1 *and* R_2*, namely* $R_1 = aC_1 + bC_2, \quad R_2 = cC_1 + dC_2;$ *i.e.,*

$$\frac{\partial C_1}{\partial t} = aC_1 + bC_2 + D_1\frac{\partial^2 C_1}{\partial x^2},$$

$$\frac{\partial C_2}{\partial t} = cC_1 + dC_2 + D_2\frac{\partial^2 C_2}{\partial x^2}.$$

Assume small deviations from equilibrium: $C_1 = C_{10} + U, \quad C_2 = C_{20} + V$*. Clearly, if at time* $t = 0$*,* $U = V = 0$ *for all values of* x*, then* U *and* V *will continue to be zero. If this equilibrium is disturbed, then for most values of the constants* $\{a, b, c, d, D_1, D_2\}$ *equilibrium will be restored and* $\{U, V\}$ *will tend to zero everywhere as* t *increases. Consequently, no spatial pattern will emerge.*

Surprisingly, however, there are values of the constants for which the homogeneous equilibrium is unstable: Let $a > 0$*,* $c > 0$*,* $b < 0$*, and* $D_2 > D_1$*. These inequalities will guarantee the following traits: if the concentration* U *rises above its equilibrium level, the rate of synthesis of both* U *and* V *will rise; if the concentration of* V *rises, it leads to destruction of* U*;* V *diffuses faster than* U*.*

Suppose now that the homogeneous equilibrium is disturbed by a small local rise *in* U*. This will lead to further rises in both* U *and* V*, but* V *has diffused out further. At those new regions where* V *has penetrated, but* U *hasn't yet,* C_1*, will be reduced and fall below its equilibrium value* ($U < 0$)*. This in turn will lead to destruction of morphogen* V*, so that a 'trough' will developed on either side of the initial peak.*

These troughs will cause the developments of further peaks, and so on until a standing wave *has developed, whose 'chemical wave-length' will depend on the values of the constants defining the rates of reaction and diffusion. The morphogen* C_1 *is known as* activator*, while* C_2 *is the* inhibitor*. A reaction-diffusion system thus exhibits diffusion-driven instability or* Turing instability *if the homogeneous steady state is stable to small perturbations in the absence of diffusion but unstable to small* spatially varying *perturbations when diffusion is present. A particular pattern will depend on the analytical form of* $R_1(C_1, C_2)$ *and* $R_2(C_1, C_2)$*.*

For the case of boundary conditions enforcing spatial periodicity with period $\Delta x = 1$*, Turing gave an explicit general solution to the one-dimensional* *linear PDE set* ((X_0, Y_0) *are concentrations at uniform equilibrium);*

$$\frac{\partial X}{\partial t} = a(X - X_0) + b(Y - Y_0) + M_x \frac{\partial^2 X}{\partial x^2},$$
$$\frac{\partial Y}{\partial t} = c(X - X_0) + d(Y - Y_0) + M_y \frac{\partial^2 Y}{\partial x^2},$$

in the form of an infinite Fourier-series

$$X(x,t) = X_0 + \sum_{n=-\infty}^{\infty} (A_n e^{p_n t} + B_n e^{p'_n t}) e^{inx},$$
$$Y(x,t) = Y_0 + \sum_{n=-\infty}^{\infty} (C_n e^{p_n t} + D_n e^{p'_n t}) e^{inx}$$

where $\{p_n, p'_n\}$ *are the roots of the quadratic*

$$(p - a + M_x n^2)(p - d + M_y n^2) = bc$$

and

$$A_n(p_n - a + M_x n^2) = bC_n,$$
$$B_n(p'_n - a + M_x n^2) = bD_n.$$

The Fourier coefficients can be considered as the spectral amplitudes of the standing wave pattern, and depend on the six parameters $\{a, b, c, d, M_x, M_y\}$.

• *Chemical clocks. In* 350 *BCE an officer in the army of Alexander the Great noted that the leaves of certain plants were open during daytime and closed at night. Until the* 1700*'s such rhythms were viewed as passive responses to a periodic environment, i.e., the succession of light and dark cycles due to the natural day length. In* 1729*, the astronomer* ***Jean Jacques d'Ortous de Mairan*** *(1678–1771, France) conducted experiments with a plant and reported that its periodic behavior persisted in a total absence of light cues. Although his results were disputed at first, further demonstrations and experiments by the botanist* ***Wilhelm Friedrich Philipp Pfeffer*** *(1845–1920, Germany), during* 1875–1915*, gave clear evidence in support of the observations that many physiological rhythms are endogenous (independent of any external environmental influences).*

We now know that most organisms have innate 'clocks' that govern peaks of activity: seasonal periodicities of plants, heart-beat of animals and circadian rhythms (periodic phenomena which appear with a period of approximately one day), and menstrual cycles are familiar manifestations of temporal *organization in nature. These large-scale rhythmic behaviors require the co-operation of a great many cells.*

In the last decades of the 19^{th} *century, a mathematical formalism for the description of* self-organized states *was developed for the study of planetary motion by* ***J.H. Poincaré*** *(1881 to 1889);* ***A.M. Lyapunov*** *(1892 to 1906), and* ***I. Bendixon***[853] *(1901). Concurrently, interest grew in the study of non-linear differential equations that originated in the fields of engineering and the applied sciences [****G. Duffing*** *(1918),* ***B. van der Pol*** *(1922),* ***V. Volterra*** *(1926)].*

Until the beginning of the 20^{th} *century, all laboratory chemical reactions were performed in closed systems: Various reactants were combined in a vessel, forming a closed thermodynamic system, in such proportions as to keep the reaction* initially *very far from equilibrium. In such conditions the quantity of the new-formed products increased gradually until the mass action law is satisfied at an* equilibrium state*. However, in* 1916 ***T.H. Morgan****, while experimenting with a reaction medium that contained hydrogen peroxide, formic acid, and sulfuric acid, observed a* periodic *release of carbon monoxide.*

Earlier (1910), ***A.J. Lotka*** *suggested a theoretical reaction which exhibits damped oscillations. Indeed,* ***W.C. Bray*** *(1921) discovered decaying oscillations in the concentrations of iodine in a hydrogen peroxide – iodate ion (*IO_3^-*) reaction. This interesting and important work was dismissed and widely disbelieved since, among other criticisms, it was mistakenly thought to violate the second law of thermodynamics.*

Thus, these few 'abnormal' oscillating chemical reactions went almost unnoticed by most chemists until the 1960*'s, when a few oscillatory biochemical processes were discovered that operate on the cellular level. For example, the synthesis of some proteins by cells follows an oscillatory pattern.*

The development by ***Prigogine*** *(1947) of the thermodynamics of far-from-equilibrium processes was a great leap forward in our understanding of biological systems. It showed that thermodynamic methods can predict the onset of spatio-temporal order in an open chemical system. Moreover, such systems must necessarily evolve according to nonlinear kinetics, and therefore must be described by nonlinear coupled partial differential equations of the type that produces* limit cycles*.*

Therefore, the fact that a great number of chemical processes in living organisms produce the observed spatio-temporal order is in complete accord with the macroscopic laws of nonequilibrium thermodynamics.

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[853] **Ivar Otto Bendixon** (1861–1935, Sweden). Mathematician. His important memoir (1901) supplemented some of Poincaré earlier work.

The fact that predator-prey type equations hold for certain autocatalytic chemical reactions, leads to the conclusion that chemical 'clocks' must exist. However, in the Lotke-Volterra model, the period of oscillation is a function of the initial prey and predator populations, and therefore essentially arbitrary (because it is amplitude dependent). The oscillation frequency of any clock worthy of the name, must be independent *of the externally determined initial conditions and is, rather, self-determined by the intrinsic characteristics of the system itself.*

The initial conditions may only influence the behavior of the system during the transient which precedes the establishment of the oscillating state. Periodic oscillations, when they correspond to a limit cycle *behavior, may be considered as* self-organization *phenomena.*

One mathematical model of reactions which exhibited an oscillatory behavior corresponding to a limit cycle is the so-called Brusselator *(Prigogine and Lefever, 1968). This trimolecular model is the most widely known theoretical model of chemical instability phenomena, although it does not represent a actual chemical reaction. It corresponds to the scheme of reaction in four steps:*

$$\overline{A}\xrightarrow{k_1}\overline{X},\qquad \overline{B}+\overline{X}\xrightarrow{k_2}\overline{Y}+\overline{D},\qquad 2\overline{X}+\overline{Y}\xrightarrow{k_3}3\overline{X},\qquad \overline{X}\xrightarrow{k_4}\overline{E},$$

where the k's are rate constants.

The concentrations of the initial and final products ($\overline{A}$, $\overline{B}$, $\overline{D}$ and $\overline{E}$) are kept constant, whereas the two components $\overline{X}$ and $\overline{Y}$ have concentrations that change with time, and also have different observable properties (e.g., colors). Since $\overline{A}+\overline{B}\to\overline{D}+\overline{E}$ in the complete process, $\overline{X}$ and $\overline{Y}$ have the role of intermediates. Their only function is that of mediating the conversion of the reagents $\overline{A}$ and $\overline{B}$ into the products $\overline{D}$ and $\overline{E}$ without undergoing any permanent transformation at the end of the reaction cycle.

Note that the third step is a trimolecular autocatalytic step which is necessary for oscillatory behavior. In the above scheme, the reverse reactions were ignored because the conditions of irreversibility are realized by holding the concentrations of $\overline{A}$, $\overline{B}$, $\overline{D}$, $\overline{E}$ far from their equilibrium values.

The reaction-diffusion equations corresponding to the above scheme are (t being ordinary time)

$$\frac{\partial\overline{X}}{\partial\bar{t}}=\overline{D}_1\nabla^2\overline{X}+\left\{k_1\overline{A}-(k_2\overline{B}+k_4)\overline{X}+k_3\overline{X}^2\overline{Y}\right\},$$

$$\frac{\partial\overline{Y}}{\partial\bar{t}}=\overline{D}_2\nabla^2\overline{Y}+\{k_2\overline{B}\,\overline{X}-k_3\overline{X}^2\overline{Y}\}.$$

With the scalings $t = k_4\bar{t}$, $X = \sqrt{\frac{k_3}{k_4}}\overline{X}$, $Y = \sqrt{\frac{k_3}{k_4}}\overline{Y}$, $B = \frac{k_2}{k_4}\overline{B}$, $A = \frac{k_1}{k_4}\sqrt{\frac{k_3}{k_4}}\overline{A}$ $D_1 = \frac{1}{k_4}\overline{D}_1$, $D_2 = \frac{1}{k_4}\overline{D}_2$, *the reaction-diffusion equations acquires the nondimensional form*

$$\frac{\partial X}{\partial t} = \left\{A - (B+1)X + X^2Y\right\} + D_1\nabla^2 X,$$
$$\frac{\partial Y}{\partial t} = \{BX - X^2Y\} + D_2\nabla^2 Y.$$

These equations are first investigated without the diffusion terms ('well-stirred' Brusselator). It is then found that the system has only one stationary state at $X_0 = A$, $Y_0 = \frac{B}{A}$. *This steady state becomes unstable (via a Hopf bifurcation) for* $B > 1 + A^2$, *provided that B remains fairly close to* $1 + A^2$. *Thus, for* $A = 1$, $B = 2$, *the equilibrium point* $\left(A, \frac{B}{A}\right)$ *is stable. However, as soon as B is increased to 2.2 it is seen that this point is unstable, a limit cycle appears, and a chemical oscillation develops.*

As B is further increased to the value 3, the limit cycle is still present although its form has changed. The period of the Brusselator in the vicinity of the bifurcation $(B \approx 1 + A^2)$ *is* $\frac{2\pi}{A}$.

In the presence of one-dimensional diffusion (unstirred Brusselator), in the x direction say, it can be shown that instability may arise in different ways: In addition to the above Hopf bifurcation there is the possibility of a Turing bifurcation in which a pattern of a standing wave arises. Finally, the limit cycle may also be space dependent and lead to a traveling concentration wave $\Big[B \geq 1 + A^2 + (D_1 + D_2)\pi^2\frac{m^2}{L^2}$ *where m is an integer and L is the length of the system*$\Big]$.

As a chemical clock (absent diffusion), the Brusselator mimics the experimentally observed chemical oscillations in a somewhat more complicated system, found by **B.P. Belousov** *and* **A.M. Zhabotinsky** *(1958, 1964). Their system shows a spectacular oscillatory change of colors from red to blue. Since chemical reactions are at the foundations of biological events, this model may offer also an understanding of the action of biological clocks (such as the control mechanism of the heart).*

Indeed, a biochemical clock which seems to behave like a limit cycle oscillator occurs in glycolysis and is readily studied in yeast cells. The question of whether these and other rhythms, such as opening and closing of plants, are limit cycles or harmonic oscillations has not been resolved yet.

• *Chemotaxis: The diffusion-reaction equation is also adequate to model natural phenomena of chemical signaling that occurs on all levels of the living world (insects, animals, leukocyte cells, single-cell amoebae, etc.). The chemically directed motion of cells up the gradient of concentration of a chemical is known as chemotaxis. For example, a large number of insects and animals rely on an acute sense of smell for conveying information between members of the species. When a bacterial infection invades the body it may be attacked by movement of cells toward the source as a result of chemotaxis.*

Let $n(\boldsymbol{r},t)$ *be the concentration of the moving cells, and* $a(\boldsymbol{r},t)$ *the concentration of an attractant chemical which gives rise to the cell motion up its gradient. We start from the conservation equation for* $n(\boldsymbol{r},t)$, *namely* $\frac{\partial n}{\partial t} + div\boldsymbol{J} = f(n)$, *where* $f(n)$ *represents the growth term for the cells, and* $\boldsymbol{J} = D\,\mathrm{grad}\,n + n\chi(a)\,\mathrm{grad}\,a$ *is the total flux of the cells: it has the ordinary diffusion contribution* $\{D\,\mathrm{grad}\,n\}$, *plus the extra chemotactic flux due to the presence of the attracting chemical that increases with the number of cells, with the gradient of the chemical attractant concentration and with a factor* $\chi(a)$ *that is a function of the attractant concentration.*

Since the attractant is a chemical, it also diffuses and is produced, partially perhaps, by the moving cell itself (amoebae, say). It obeys the equation $\frac{\partial a}{\partial t} = g(a,n) + \mathrm{div}(D_a\,\mathrm{grad}\,a)$, *where* D_a *is the diffusion coefficient of* a *and* $g(a,n)$ *is the source term. The combined set of equations for* n *and* a *define a diffusion-chemotaxis process. The balance between the stabilizing diffusion term and the destabilizing chemotaxis term could result in some steady-state spatial patterns in* n *and* a, *and in some unsteady wave-like spatially heterogeneous structures.*

Turing Machines, or — What is a Computation?

Although people have been computing for millennia, it has only been since 1936 CE that we have possessed a satisfactory answer to the above title question. Along with the development of modern computers, has emerged a new branch of applied mathematics — theory of computation: the application of mathematics to the theoretical understanding of computation. The foundations of this theory were laid by ***Emil Post*** *(1920),* ***Alan Turing*** *(1935) and* ***Alonzo Church*** *(1936).*

There are many different styles of mathematics. At one extremity is the pure existence proof, asserting that an object with certain properties must necessarily exist, *but giving no method to find it. This is part of* dialectic mathematics *— a rigorously logical science, where statements are either true or false and where entities either do or do not exist. It is an intellectual game played according to rules about which there is a high degree of consensus. Dialectic mathematics invites* contemplation *and generates* insight. *It originated with the Greeks. Throughout most of the 19th century, mathematics has been existence-oriented.*

At the other end is algorithmic mathematics[854] *— a tool for solving problems by means of a perfectly definitive procedure guaranteed to calculate*

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[854] *Algorithm*: a step-by-step recipe (instruction set) for performing some kind of calculation: e.g. the algorithmic solution to the equation $x^2 = 2$ is $x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right)$, $n = 1, 2, \ldots$. The corresponding dialectic solution uses the function $y = x^2 - 2$. Since $f(1) = -1$ and $f(2) = 2$, as x moves continuously from 1 to 2, y moves continuously from negative to positive values. Hence, *somewhere* between 1 and 2 there is an $x = x_0$ such that $y = 0$, i.e. $x_0^2 = 2$.

The details of the argument are supplied by the properties of the real number system and of continuous functions defined on that system. The first significant algorithm is to be found in **Euclid's** Book Seven where he taught us how to calculate the highest common factor of two numbers.

For some particularly simple Diophantine equations (**Diophantos of Alexandria**, ca 250 CE), there are known algorithms (e.g. for linear and quadratic equations in at most two unknowns). The name *algorithm* is associated with **Al-Khowarizmi**, who outlined (ca 825 CE) the rules for performing basic arithmetic operations, using numbers expressed in the Hindu decimal notation that we use today.

exactly what is required, if and only if one waits long enough. The rules of the game may vary according to the urgency of the problem at hand (we never could have put a man on the moon if we had insisted the trajectories should be computed with dialectic rigor). Algorithmic mathematics invites action *and generates* results. *The mathematics of Babylon and the ancient orient was all of the algorithmic type.*

Between the two extremes are constructive techniques *which provide a more explicit description of the desired results or objects then in pure existence proofs.*

The problem of transcendental numbers *illustrates all three styles well. Cantor's existence proof exhibits not a single transcendental. It merely observes that since there are more reals than algebraic numbers, transcendentals must exist. At the intermediate level are transcendence proofs for* specific *numbers, such as:* π, e, $2^{\sqrt{2}}$. *An algorithm for transcendence would be a general method of deciding, for any number whatsoever, whether it does or does not satisfy an algebraic equation. No such technique is known.*

There have been extensive philosophical arguments about the value and nature of these types of result. Does a pure existence proof really convey any useful information? One school of modern mathematics, the Constructionism, *take a very restrictive position and refuses to consider any non-constructive arguments at all. Others believe that in order to calculate something, it is* useful *to know in advance that it exists.*

Throughout most of the 19th century, mathematics has been existence-oriented. Then, at the turn of the 20th century (and especially since 1936) there occurred a partial shift back to constructive, or algorithmic, points of views.

Indeed, **David Hilbert** *(1928) proposed a problem (known as the* "Entscheidungs problem" *— the decision problem) as a fundamental problem of the newly developing field of mathematical logic. It can be stated as follows: a finite list of statements called* premises *is given together with an additional statement called the* conclusion. *The logical structure of the statements is to be explicitly exhibited in terms of "not", "and", "or", "implies", "for all" and "there exists".*

Hilbert wanted a computing procedure (algorithm) for testing whether or not the conclusion can be deduced (using the rules of logic) from the premises. Hilbert regarded this problem as especially important because he expected *that its solution would lead to a purely mechanical technique for settling the truth or falsity of the most diverse mathematical statements. (Such statements could be taken as the conclusion, and an appropriate list of axioms as the premises to which the supposed computing procedure could be applied). The end result of his programme would thus lead to a rigorous proof of the consistency of systems such as: logic; logic combined with set theory; or arithmetic.*

However, research during 1930–1936 showed that there are problems that had no algorithmic solution! First, ***Gödel*** *(1930) showed that there are true statements in arithmetic that can never be proved. In other words — one cannot prove that arithmetic is consistent. At about the same time,* ***Alan Turing*** *was working in mathematical logic, with a view to clarifying the notion of computability. He discovered that certain very natural questions have no answer whatsoever, i.e. — there is no solution using any method available to human beings.*

Turing based his precise definition of computation on an analysis of what a human being does when he computes. Such a person is following a strict set of rules which can be carried out in a completely mechanical manner.

In this process (be it long division, algebraic manipulations or steps in solving a calculus problem) there are irrelevant parts which are logically unnecessary and are included only to speed up specific operations. Turing described an idealized computer (Turing machine) with the most rudimentary structure possible.

Imagine an infinite tape (one dimensional paper) divided into square cells, passing under a head which can be in a finite number of initial states. The head can read what is on the tape, and optionally write symbols 0 *and* 1 *on it. Only* 0 *and* 1 *are required since any information can be encoded using just* 0*s and* 1*s (e.g. a Morse code with* 0 *representing a dot and* 1 *a dash).*

The machine behavior is controlled by a kind of computer program (Turing-Post program). This consists of a sequence of numbered steps, each step being one of seven instructions:

Instruction	*Programming Language Statement*
Write the symbol 1	`PRINT 1`
Write the symbol 0	`PRINT 0`
Move the tape forward one cell	`GO RIGHT`
Move the tape backward one cell	`GO LEFT`
{ Observe the symbol currently scanned and *choose* the next step accordingly	`GO TO instruction` i `IF 1 IS SCANNED` `GO TO instruction` j `IF 0 IS SCANNED`
Stop	`STOP`

In order that a particular Turing-Post program begin to calculate, it must have some "input" data. That is, the head begin scanning at a specific square of a tape already containing a sequence of zeros and ones. The machine processes the data according to the program, and then stops.

Turing showed that anything *a computer can calculate, can also be computed by a Turing machine. Moreover, he showed (1936) how to construct a* universal *machine capable of simulating the action of* any *program in* any *Turing machine. This is equivalent to saying that any computer, given enough time and memory, can be programmed to simulate any other computer. So, for theoretical purposes, we may think of all computers as capable of being emulated by Turing machines [Church-Turing thesis]. This also implies that there exists a single Turing-Post program which can compute everything* that *is computable.*

At this point we may define *an algorithm to be a Turing-Post program that eventually* halts *no matter what input is presented to the Turing machine who runs it. Clearly, an important property of an algorithm is that it should eventually* stop *with a definite answer. A calculation that may go on forever is not of much use. However, a Turing-machine may, in principle go on forever (e.g. 'move right looking for* 1*s' will not stop if there are no* 1*s' to the right of the head).*

In light of this possibility there arises the Halting Problem *for a particular Turing-Post program: to distinguish between initial inputs which lead to the program's eventually halting and initial inputs which lead the program to run forever. In other words, is there a method for determining* in advance *which input data leads the program to halt and which do not.*

The answer is no*. The halting problem is* unsolvable *(undecidable). Turing proved this in 1936 indirectly by* reductio ad absurdum*. Thus, a problem is* undecidable *if there exists no algorithm to solve it, and the undecidability of the Halting Problem places definite limits on the applicability of algorithms.*

Another problem which proved to be unsolvable was the 'word problem' of ***Axel Thue*** *(1908): Is there an algorithm for deciding, for any given alphabet and a set of production rules, whether or not any given two words are equivalent*[855]*? Emil Post proved (1947) that the unsolvability of the halting problem leads to the existence of an unsolvable word problem. Work on unsolvable word problems has turned out to be extremely important, leading to unsolvability results in different parts of mathematics such as group theory and topology.*

Yet another problem that eventually turned out to be unsolvable first appeared as the $10th$ *in a famous list of* 23 *problems posed by David Hilbert in 1900: he challenged to determine whether or not any given Diophantine equation has integer solutions*[856]*. Here the name refers to any algebraic equa-*

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855 Any string of letters is called a *word* of the alphabet. Given two arbitrary words on the alphabet, the problem is of determining whether one can be transformed into the other by a sequence of substitutions that are legitimate using the given rules. Unsolvability means here that no computational process exists for determining whether or not two words can be transformed into one another using the given rules.

Example: given an alphabet of three symbols a, b, c, and three rules encoded by the equations $ba = abc$; $bc = cba$; $ac = ca$, we can obtain other equations by substitution such as

$$\mathbf{ba}c = abcc$$
$$b\mathbf{ac} = \mathbf{bc}a = c\mathbf{ba}a = \mathbf{ca}bca = \mathbf{ac}bca = cab\mathbf{ca} = cab\mathbf{ac} = ca\mathbf{bc}a = cacbaa$$

(the letter strings in boldface type are the symbols about to be replaced). In this context questions can be raised such as: Can we deduce from the three equations listed above that

$$bacabca = acbca?$$

856 *Examples*: The equation $4x - 2y = 3$ has no solution in integers because the l.h.s. would have to be even while the r.h.s. is odd. On the other hand the equation $4x - y = 3$ has infinitely many solutions in integers (e.g. $x = 1$, $y = 1$; $x = 2$, $y = 5$). The Pythagorean equation $x^2 + y^2 = z^2$ has also infinitely many integer solutions, but $x^n + y^n = z^n$ for $n > 2$ has none. Likewise $x^2 + y^2 = 2$, considered as a Diophantine equation, has only 4 solutions $(1, 1)$; $(1, -1)$; $(-1, 1)$; $(-1, -1)$. If we change the equation just slightly, say to $x^2 + y^2 = 3$ there are no integer solutions at all. Thus, a Turing machine might work forever on this equation without producing any solution. From the point of view of a computer — the equation is unsolvable.

tion in one or more variables, with integer coefficients (the adjectival use of the word Diophantine refers not to the equation so much as to the kind of solution which is sought). The problem stayed open until 1970, when ***Yuri Matijasevich*** *used the Fibonacci numbers to prove that there cannot be an algorithm of the kind requested by Hilbert.*

There is perhaps no better example of the limited computability by computers than a mathematical problem [raised by ***Karl Menger***[857]*(1930)] which has resisted all attempts at a general solution via traditional methods. It is known as the Traveling Salesman Problem (TSP): given a network of cities and roads, find a tour that takes the salesman (tourist) to each city exactly once such that the total distance traveled is minimized. The order in which the cities are visited is of no importance (sometimes it is required that the tour begins and ends at the same location).*

The only known algorithm for solving the problem is the laborious uninsightful one of trying every possibility, and a computer can be easily programmed to do just that. To date, no faster algorithm has been found, not even a theory, except for special cases. Nor has anyone been able to prove the impossibility of a faster algorithm (this field of endeavor is known as complexity theory).

But the listing of all possible routes is practical only for a modest number of locations; If there are N *locations to visit, then there are* $N! \approx \left(\frac{N}{e}\right)^N \sqrt{2\pi N}$ *possible different itineraries, which will obviously lead to an exponential-time algorithm*[858]*. Already for* $N = 10$ *there are* $3,628,800$ *possible routes. This can be achieved by a modern computer, but when we go up to* $N = 25,$ *the*

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[857] (1902–1985, Austria and USA). Mathematician. Born in Vienna and completed his Ph.D. on *dimension theory* (1924). Forced by the Nazis (1938) to leave his chair of geometry at the University of Vienna and immigrated to US. Even after the war, the ex-Nazis at the University of Vienna told him that he was unwanted there... He worked at the University of Notre Dame (1938–1948) and then spent the rest of his career at the University of Illinois.

[858] An *algorithm* (a Turing-machine program) is said to run in *polynomial time* if there are fixed integers A and k such that for input data of length n, the computation is complete in at most An^k steps (for any value of n). For example: the *addition* of two numbers, each with $\frac{n}{2}$ digits (input data of length n), involves exactly n steps (allowing for carries). Thus the above definition is valid with $A = k = 1$. In the *multiplication* of two $\frac{n}{2}$-digit numbers there are $\frac{n^2}{4}$ basic digit multiplications, $\frac{n^2}{4}$ carry–and–add operations, and the finally $n-1$ 3 digits addition steps as encountered in an addition problem; this add up to $\frac{n^2}{2}+n-1$ steps in all. Since $\frac{n^2}{2}+n-1 < n^2$, the above definition holds with $A = 1$, $k = 2$.

number of routes to consider mounts to 16×10^{25} *while for* $N = 100$, *of order* 3×10^{161} *single–digit decimal arithmetic calculations must be performed for all possible tours, requiring of order* 10^{145} *years on today's fastest supercomputers. While a tour of* 25 *cities is quite realistic for a real-life salesmen, not many salesmen need to visit* 100 *cities, yet the TSP is important because it has far wider applications*[859] *than just in the travel industry.*

Algorithms which require n^n or $n!$ steps to handle data of length n (say, the number of cities in the TSP) are *exponential time* algorithms.

For an algorithm to be efficient (fast), it should run in polynomial time, provided of course that A and k are modest (values like $A = 10^{10}$ and $k = 100$ are hardly likely to be 'efficient' in any real sense). Assuming that a computer performs one basic operation in 10^{-6} sec, the dependence of the computation time on the data size for a number of exponential and polynomial time models is roughly as follows:

Size of data:	Run times for given computation complexity formula:				
n	n	n^2	n^3	2^n	3^n
10	10^{-8} s	10^{-7} s	10^{-6} s	10^{-6} s	$5.9 \cdot 10^{-5}$ s
20	$2 \cdot 10^{-8}$ s	$4 \cdot 10^{-7}$ s	$8 \cdot 10^{-6}$ s	10^{-3} s	3.5 s
30	$3 \cdot 10^{-8}$ s	$9 \cdot 10^{-7}$ s	$2.7 \cdot 10^{-5}$ s	1 s	57 hours
40	$4 \cdot 10^{-8}$ s	$1.6 \cdot 10^{-6}$ s	$6.4 \cdot 10^{-5}$ s	18.3 m	389 years
50	$5 \cdot 10^{-8}$ s	$2.5 \cdot 10^{-6}$ s	$1.25 \cdot 10^{-4}$ s	13 days	23 My
60	$6 \cdot 10^{-8}$ s	$3.6 \cdot 10^{-6}$ s	$2.16 \cdot 10^{-4}$ s	36.6 years	1.4 trillion years

Note that 1.4×10^{12} years exceeds by about three order of magnitude the current estimate of the age of the universe.

859 *For example*, electronic circuit board manufacturers have to drill as many as $65,000$ holes on their boards using laser drills. Finding the best way to drill the holes turns out to be a TSP, since it involves finding the shortest tour that visits each hole exactly once. And in fact, a team of academic and industrial researchers recently established a record for the TSP by finding the exact minimal distance path for visits of 3038 cities, where the "cities" were indeed holes drilled on a printed circuit board.

So, arriving at a solution by listing all the possibilities is obviously out of the question except when a small number of locations are involved. Modern computers, however, were able to solve specific *instances of the problem using special ad-hoc methods. Thus, a smart algorithm (1970) needed to examine only* 61 *relevant cases out of a vast number of* 33×10^{49} *possibilities. Another specific problem was solved (1979) for* 318 *locations.*

The Catastrophe Paradigm (1937–1972)[860]

Classical physics is essentially a theory of smooth behavior according to Newton's laws of motion. This is certainly manifested in the awe-inspiring stable motion of the planets in their courses around the sun. However, in our daily experience here on earth we meet other phenomena: water suddenly boils, ice melts, the earth quakes, bridges collapse, buildings fall, hearts fail, stock markets crash, wars erupt. The back of a camel is stable, we are told, under the load of n *straws, but breaks suddenly under a load of* $n + 1$.

All these are sudden *changes caused by* smooth *alterations in the situation. Such changes are far more difficult to predict and analyze than the stars in their courses and the sciences (from physics to economics and psychology) are still developing the analytical techniques to handle such abrupt behavior.*

There are numerous kinds of "jump" phenomena. There are forces that build up until friction can no longer hold them: when friction gives way an earthquake results. There is a critical population density below which certain creatures grow up as grasshoppers, above which as locusts: this is why locusts, when they do *occur, do so in a huge swarm.*

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[860] To dig deeper, see:

- Poston, T. and I. Stewart, *Catastrophe Theory and Its Applications*, Dover, 1996, 491 pp.
- Woodcock, A. and M. Davis, *Catastrophe Theory*, Avon, 1980, 151 pp.

A cell suddenly changes its reproductive rhythm and doubles and redoubles, cancerously. In engineering, for example, the gradual buildup strain on the structure of a bridge can eventually result in a sudden collapse. These phenomena occur also in economical and psychological events. They are named catastrophes[861]*, to convey the feeling of abrupt and dramatic changes, and the subject has (since 1973) become known as catastrophe theory.*

It is an attempt to describe those situations in which small gradually changing forces lead to large abrupt changes, or otherwise stated — the treatment of a continuous action producing a discontinuous result. It is capable of dealing with evolution of many natural phenomena that proceed by a series of gradual changes that are triggered by, and in turn trigger, large-scale changes.

Consider the potential function of a system having two variables $V(x, y; \alpha, \beta)$, *where* α, β *are some parameters which characterize the strengths of interaction between various parts of the system (properties of materials, amount of heat being supplied or extracted, etc.).*

For fixed values of α, β, *an equilibrium state of the system is determined by the equation* $\operatorname{grad} V = 0$ *(i.e.* $\frac{\partial V}{\partial x} = \frac{\partial V}{\partial y} = 0$*) and by the behavior of the scalar* $\Delta = ab - h^2$, *where* $a = \frac{\partial^2 V}{\partial x^2}$, $b = \frac{\partial^2 V}{\partial y^2}$, $h = \frac{\partial^2 V}{\partial x \partial y}$ *at the equilibrium point(s). In the regular case* $(\Delta \neq 0)$ *there are two possibilities: either the system stays near equilibrium and the potential* V *increase from its equilibrium state for every conceivable tiny change in* x, y ($\Delta > 0$, $\frac{\partial^2 V}{\partial x^2} > 0$; *stable) or there is no increase in* V *for some small alternation to* x, y *and the system tends to move away from equilibrium* ($\Delta \neq 0$, $\frac{\partial^2 V}{\partial x^2} < 0$ *or* $\Delta < 0$; *unstable).*

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[861] The word's overtones of disaster are, for most applications, misleading. 'Catastrophe theory' is in fact a model (paradigm), not yet an explanation. It is, in fact, an assemblage of mathematical and physical ideas.

'Catastrophe theory' is *not* the first mathematical method capable of treating divergent phenomena. The buckling of a beam one way or the other as stress increases is just such a phenomena, and was first analyzed by **Euler** (1744). Many similar problems of elasticity were well understood in their own terms by engineers long before 'catastrophe theory' was proposed, and for these we cannot expect the new mathematics to render new practical information — only a reformulation. 'Catastrophe theory' will, however, shed light on more complicated cases.

Choosing (without loss of generality) the origin at equilibrium,

$$V(0,0) = V_x(0,0) = V_y(0,0) = 0,$$

the Taylor-series expansion for V near the origin will take the form

$$V = \frac{1}{2}(ax^2 + 2hxy + by^2) + \textit{ higher order terms.}$$

Now, it is well known that the curve $ax^2 + 2hxy + by^2 = F$, where F is constant, is a conic section. If $\Delta = ab - h^2 > 0$ then it is either an ellipse (if $aF > 0$) or has no real points (if $aF < 0$). If $\Delta = ab - h^2 < 0$ then it is a hyperbola, with the sign of aF determining which of the two principal axes is the transverse axis. By considering the intersections of the surface $z = V(x, y)$, with planes $z = \pm\epsilon$, where $|\epsilon|$ is small, the above conditions follow.

In the neighborhood of a non-degenerate critical point $(\Delta \neq 0)$, the function of one variable *can be closely approximated by a parabola, opening upwards for a minimum or downwards for a maximum. The generalization for a function of two variables is that near a non-degenerate critical point it can be closely approximated either by an elliptic paraboloid $(\Delta > 0$, for a maximum or a minimum) or a hyperbolic paraboloid (for saddle point, $\Delta < 0)$.*

Consider, however, the physical situation where the parameters α, β are not fixed any longer: A slight adjustment in the environment will result in small deviations of α, β from their original values. Now, there are two possibilities; either the state of the system will be modified a little, or it will be transformed dramatically. In the latter case a catastrophe *occurs and the set of values for which this happens is said to be the* catastrophe set*. In the former case the state may be called* regular *because it is not affected much by small perturbations.*

Let this state of equilibrium be perturbed by a small variation of the parameters $\alpha \to \alpha + \delta\alpha$, $\beta \to \beta + \delta\beta$, and let the corresponding induced disturbance in the variables at the equilibrium state shift the system to a new equilibrium state *$x \to x + \delta x$, $y \to y + \delta y$. A Taylor expansion then yields:*

$$\begin{aligned}
&\frac{\partial}{\partial x}V(x+\delta x, y+\delta y; \alpha+\delta\alpha, \beta+\delta\beta) \\
&\quad = \frac{\partial}{\partial x}V(x,y;\alpha,\beta) + \delta x\frac{\partial^2}{\partial x^2}V(x,y;\alpha,\beta) + \delta y\frac{\partial^2}{\partial x\partial y}V(x,y;\alpha,\beta) \\
&\quad + \delta\alpha\frac{\partial^2}{\partial\alpha\partial x}V(x,y;\alpha,\beta) + \delta\beta\frac{\partial^2}{\partial\beta\partial x}V(x,y;\alpha,\beta) + \text{higher order terms;}
\end{aligned}$$

$$\frac{\partial}{\partial y}V(x+\delta x, y+\delta y; \alpha+\delta\alpha, \beta+\delta\beta)$$
$$= \frac{\partial}{\partial y}V(x,y;\alpha,\beta) + \delta x\frac{\partial^2}{\partial y\partial x}V(x,y;\alpha,\beta) + \delta y\frac{\partial^2}{\partial y^2}V(x,y;\alpha,\beta)$$
$$+ \delta\alpha\frac{\partial^2}{\partial\alpha\partial y}V(x,y;\alpha,\beta) + \delta\beta\frac{\partial^2}{\partial\beta\partial y}V(x,y;\alpha,\beta) + \text{higher order terms.}$$

The first term on the r.h.s. of each equation vanishes on account of the equilibrium condition. The new *equilibrium will be worthy of its name if in each equation, the sum of the remaining terms will vanish for the* given *changes $\delta\alpha$ and $\delta\beta$. This leaves us with a set of two equations in the two unknowns δx and δy in terms of $\delta\alpha$ and $\delta\beta$ (higher terms in the Taylor expansion being neglected),*

$$\delta x\frac{\partial^2 V}{\partial x^2} + \delta y\frac{\partial^2 V}{\partial x\partial y} = -\delta\alpha\frac{\partial^2 V}{\partial\alpha\partial x} - \delta\beta\frac{\partial^2 V}{\partial\beta\partial x}, \qquad \textit{and}$$

$$\delta x\frac{\partial^2 V}{\partial x\partial y} + \delta y\frac{\partial^2 V}{\partial y^2} = -\delta\alpha\frac{\partial^2 V}{\partial\alpha\partial y} - \delta\beta\frac{\partial^2 V}{\partial\beta\partial y}.$$

If the determinant of the coefficients matrix $\Delta = \begin{vmatrix} V_{xx} & V_{xy} \\ V_{xy} & V_{yy} \end{vmatrix}$ *on the l.h.s. (known as the Hessian) is non-zero, there is a unique solution of the above system:* $\delta x = a_{11}\delta\alpha + a_{12}\delta\beta$, $\delta y = a_{21}\delta\alpha + a_{22}\delta\beta$, *where a_{ij} are constants. Thus, the smaller the perturbations $\delta\alpha$ and $\delta\beta$, the smaller δx and δy, and the equilibrium state is regular. In contrast, if* $\Delta = 0$, *a general solution of the above system contains an arbitrary element which does not depend on $\delta\alpha$, $\delta\beta$, because it satisfies the homogeneous system of equations. Hence δx, δy cannot be forced to go to zero with $\delta\alpha$, $\delta\beta$ and there is a* catastrophe.

Thus, an equilibrium state is catastrophic or regular according as the Hessian of the potential function does or does not vanish. The points for which $\Delta = 0$ *therefore have an* inherent structural instability, *since there will be functions arbitrarily close to them with* $\Delta > 0$ *and with* $\Delta < 0$ *and these will be in general of different type.*

Now, Δ can vanish for two quite distinct reasons: first, all three *second order partial derivatives may be zero at the origin, and it is then clear that $V(x,y)$ is degenerate in both the x-direction* and *the y-direction. Second, we could have* $\Delta = 0$ *because* $V_{xx}V_{yy} = (V_{xy})^2$ *but not all the derivatives vanish separately. In that case* $|ax^2 + 2hxy + by^2|$ *is a* perfect square, *which allows us to write*

$$V(x,y) = \pm\frac{1}{2}(px+qy)^2 + \textit{ higher order terms,}$$

where $p = \sqrt{|a|}$, $q = \sqrt{|b|}$.

The form of the expansion suggests that we rotate the axes, transforming to new coordinates u, v *given by* $u = \frac{px+qy}{\sqrt{p^2+q^2}}$, $v = \frac{qx-py}{\sqrt{p^2+q^2}}$. *In the* u, v *system* $V(u,v) = \pm\frac{1}{2}(p^2+q^2)u^2$ *(parabolic cylinder). Consequently, the values of the first and second partial derivatives at the origin are*

$$\frac{\partial V}{\partial u} = \frac{\partial V}{\partial v} = \frac{\partial^2 V}{\partial u \partial v} = \frac{\partial^2 V}{\partial v^2} = 0,$$

$$\frac{\partial^2 V}{\partial u^2} = \pm(p^2+q^2) \neq 0.$$

Thus, V *has either a minimum or a maximum in the* u *direction. In any other direction* $w = u\sin\theta + v\cos\theta$, *we have at the origin* $\frac{dV}{dw} = \sin\theta\frac{\partial V}{\partial u} + \cos\theta\frac{\partial V}{\partial v} = 0$ *and*

$$\begin{aligned}\frac{d^2V}{dw^2} &= \sin^2\theta\frac{\partial^2 V}{\partial u^2} + 2\sin\theta\cos\theta\frac{\partial^2 V}{\partial u\partial v} + \cos^2\theta\frac{\partial^2 V}{\partial v^2} \\ &= \sin^2\theta\frac{\partial^2 V}{\partial u^2} = \pm\sin^2\theta(p^2+q^2).\end{aligned}$$

Hence V *has the same behavior in the* w*-direction as in the* u *direction, provided that* $\theta \neq 0$. *Thus, the problem has been reduced, by a single coordinate transformation, to the study of a function of one variable only.*

This result can be extended to a potential depending on n *variables and* k *parameters* $V(x_1,\dots,x_n;\alpha_1,\dots,\alpha_k)$ *with a critical point at the origin. We make an expansion in terms of the state variables near equilibrium*

$$V = V_0 + \sum_i a_{ij}x_ix_j + \sum_{i,j,\ell} b_{ij\ell}x_ix_jx_\ell + \sum_{i,j,\ell,m} c_{ij\ell m}x_ix_jx_\ell x_m$$

correct to the 4^{th} *order. The coefficients* a_{ij}, $b_{ij\ell}$, $c_{ij\ell m}$ *will depend on* $\alpha_1,\dots,\alpha_k$ *in general. It can be shown that the axes can be rotated until only the squares among the quadratic terms are left. (There are standard techniques for doing so based on matrices, the details of which are not of immediate concern.) After the rotation*

$$V = V_0 + \sum_{i=1}^{n}\lambda_iX^2 + \sum_{i,j,\ell} B_{ij\ell}X_iX_jX_\ell + \sum_{i,j,\ell,m} C_{ij\ell m}X_iX_jX_\ell X_m$$

where $\lambda_1,\lambda_2,\dots,\lambda_n$ *are known as eigenvalues. The number of non-zero eigenvalues depends on the rank of the Hessian matrix of* V, *namely* $\left[\frac{\partial^2 V}{\partial x_i\partial x_j}\right]$.

If the rank is n (i.e. its determinant does not vanish), then all $\lambda_i \neq 0$ and V is structurally stable.

If, on the other hand, the rank of the Hessian is $n-r$ for some $r>0$, then there exists a coordinate transformation which permits us to write V in the form $V=\lambda_{r+1}x_{r+1}^2+\lambda_{r+2}x_{r+2}^2+\cdots+\lambda_n x_n^2+$ higher order terms. The structural instability is confined to the variables $x_1, x_2, \ldots, x_r$ and can be analyzed in terms of these variables alone. The remaining variables can be ignored. The number r, known as the corank of the Hessian, is the number of directions in which the function is degenerate. It also denotes the number of kinds of catastrophes which can occur.

It can be shown that when only one of the eigenvalues vanishes, the potential near equilibrium is reducible in most cases to one of four canonical potentials:

$$V = c_1 x + x^3 \qquad \textit{(fold)}[862];$$

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[862] Example: *Equilibrium of a picture frame*. A rectangle of width $2a$ and depth $2h$ is suspended by an inextensible string of length 2ℓ (with $\ell > a$) attached to its upper corners and passing over a smooth pin c. Since the only force which does any work is gravity, the potential energy of the frame is determined by the depth of the frame's center of gravity below the suspension point.

Hence the potential energy

$$V(x,\theta;a,\ell,h) = -mg\left\{\left[\sqrt{\ell^2-x^2}\sqrt{1-\frac{a^2}{\ell^2}}+h\right]\cos\theta + x\sin\theta\right\},$$

where x is the distance of the center of side $2a$ (facing the suspension point c) from the perpendicular passing through the suspension point on this side, and θ is the angle at c between this perpendicular line and the vertical; m is the mass of the frame and g is the acceleration of gravity. There are two state variables x and θ because each can be varied independently of the other.

From the equilibrium equation $\frac{\partial V}{\partial x}=0,\ \frac{\partial V}{\partial \theta}=0$ we deduce that equilibrium is reached whenever $\sin\theta = 0$ or $\sqrt{1-\frac{x^2}{\ell^2}} = \frac{h}{a^2}\sqrt{\ell^2-a^2}$. The condition $\sin\theta=0$ implies that $x=0$, which is the expected symmetrical configuration where the frame is horizontal.

The second condition can be realized only if $h\sqrt{\ell^2-a^2}\le a^2$. Then there are two configurations, one on either side of the vertical. There are, therefore, 3 positions of equilibrium. It can be proved that the side positions, whenever they exist, are stable, but the central position is stable only when there are no side positions ($h\sqrt{\ell^2-a^2}>a^2$). When $h\sqrt{\ell^2-a^2}=a^2$, the Hessian determinant vanishes for the central position and a fold catastrophe ensues.

$$V = c_1x + c_2x^2 \pm x^4 \qquad (cusp);$$
$$V = c_1x + c_2x^2 + c_3x^3 + x^5 \qquad (swallow\text{-}tail);$$
$$V = c_1x + c_2x^2 + c_3x^3 + c_4x^4 + x^6 \qquad (butterfly).$$

If two eigenvalues can pass through zero, three additional catastrophes can occur:

$$V = x^3 + y^3 + axy + bx + cy \qquad (hyperbolic\ umbilic);$$
$$V = x^3 - 3xy^2 + a(x^2 + y^2) + bx + cy \qquad (elliptic\ umbilic);$$
$$V = x^2y + y^4 + ax^2 + by^2 + cx + dy \qquad (parabolic\ umbilic).$$

When three eigenvalues disappear, a further four catastrophes can be present — but when more than three can vanish, unlimited possibilities become available.

Thus, Catastrophes are bifurcations *between different equilibria, or fixed point attractors. Due to their restricted nature, catastrophes can be classified based on how many control parameters are being simultaneously varied. For example, if there are two controls, then one finds the most common type, called a "cusp" catastrophe. If, however, there are more than five controls, there is no classification.*

The foregoing ideas had their origin at the end of the 19th *century, when* ***Henri Poincaré*** *(1881 to 1895) linked calculus and topology (then called "*analysis situs*", analysis of location) to create* qualitative *dynamics and then applied it to unsolved problems of planetary motion. This may seem strange; after all, dynamics had been a firmly quantitative field since Newton. But Newton's methods yielded explicit solutions only for the interaction of two bodies — for example, the sun and the earth, or the earth and the moon. When three or more bodies are involved, the equations of motion cannot be solved directly, and even approximate solutions require tedious, complex procedures. Around* 1800 ***Pierre Simon de Laplace*** *had tried at length — but without success — to show that all the two-body attractions of the solar system added up to a stable dynamic system, a grand perpetual-motion machine that would run forever.*

Poincaré set out to show that even if quantitative solutions were impossible, it was still possible to make progress on important questions: does a complex, many-body system return periodically to the same arrangement? Does a slight perturbation simply "nudge" the whole system, or does it lead eventually to qualitatively different behavior, such as a planet spiraling into the sun or colliding with another planet? Though he did not fulfill Laplace's earlier hopes, Poincaré inaugurated a valuable new approach.

His fellow mathematicians saw its value, but thought it arbitrary because it was adapted to a particular physical problem rather than being part of a general method. Indeed, as late as 1937 *E.T. Bell, a historian of mathematics, summed up the state of the art in the words: "...few have mastered his weapons, and some, unable to bend his bow, insinuate that it is worthless in a practical attack".*

But in the very same year the mathematicians ***A.A. Andronov*** and ***L.S. Pontryagin*** *built on Poincaré's ideas in their general definition of structural stability. They made mathematical questions out of Poincaré's physical ones. Given the equations describing any dynamical system, they said, the crucial question was how the stable solutions for these equations were distributed topologically. Was a stable state of the system part of a continuous range, or an "island" surrounded by instability? Would a small quantitative change alter the solutions slightly, or produce very different new ones, or perhaps leave none at all?*

At about the same time, ***Marston Morse*** *was renewing the topological approach to the* calculus of variations*, making it possible to find the maxima and minima of whole family of curves.*

Interestingly enough, the ideas of Poincaré found fertile ground in biology. ***Conrad Hall Waddington*** *(1905–1975, England) was the first scientist of stature to acclaim catastrophe theory (1930). He came to these ideas through his exploration of the evolution of embryos before birth*[863]*. He studied processes of* morphogenesis *that transform an apparently uniform ball of cells into a layered structure of differentiated tissues. These investigations revealed much about the chemical signals of morphogenesis, including the surprising discovery that many substances — even some not normally found in organisms — can act as* triggers *for the same complex sequence of events. Waddington suggested (1940) the desirability of a theory of a generally topological kind, which would be appropriate to biological forms. Such a theory was eventually developed by* ***René Frédéric Thom***[864]*; he used it to illuminate singularities of differentiable mappings.*

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863 As early as 1917, **D'Arcy Thompson** had shown that the shape of a fish or of an animal's skull, drawn on a rectilinear grid, could be altered by a continuous, smooth transformation to that of a related fish or skull in the animal's evolutionary predecessor. Although Thompson did not develop quantitative mathematics for this visual relationship, he exerted pervasive influence on three generations of scientists.

864 French mathematician (b. 1923). Developed the theory (1966) and expounded it in his book "*Structural stability and Morphogenesis*" (1972). It has attracted publicity as well as some controversy.

Catastrophe theory is a special branch of dynamical systems theory. It has been applied to widely differing situations such as embryo development, stability of ships at sea and social interactions between human beings and animals. Although catastrophe theory was intended primarily as a mathematical language for biology, it turned out to be an efficient mathematical tool that also provides a common language for physical and psychological processes.

1935–1972 CE **Israel Moiseyevich Gelfand** (b. 1913, Russia). Mathematician. Generalized and extended classical mathematics by the use of infinite dimensional, yet geometric ideas.

Gelfand was born to Jewish parents in Krasnye Okny, near Odessa, in the Ukraine. He went to Moscow at the age of 16, before completing his secondary education. There he took a variety of different jobs (such as a door keeper at the University library). In 1932 he was admitted as a research student under Kolmogorov and presented his thesis (1935) on abstract functions and linear operators. He became a professor at Moscow State University (1943). In 1990 he emigrated to the United States, becoming Distinguished Visiting Professor at Rutgers University.

His achievements:

- Developed (1938) the theory of *commutative normed rings* which are of crucial importance in functional analysis and modern physics. Revealed close connections between Banach's general functional analysis and classical analysis.

- Developed the representation theory of *locally compact groups*[865], important in relativity theory and quantum mechanics (e.g. the Lorentz groups). His work in this area unifies the treatment of classical Lie groups (ubiquitous in physics) with their analogues in algebraic geometry. In particular, his work has led to essential mathematical methods in *the study of symmetries* of fundamental particles.

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[865] For further reading, see:
 – Gelfand, I.M., R.A. Minlos and Z.Y. Shapiro, *Representation of the Rotation and Lorentz Groups and their Applications*, Pergamon Press: New York, 1963.

- Contributed to the life sciences through his mathematical studies of *neuro-physiology* and *cell biology.*
- Developed *integral geometry*, which studies in geometric terms transformations by integrals of functions on a given space.
- Worked (with Naimark) on *non-commutative normed rings* and the theory of representations of *non-commutative groups.* He later advanced the theory of *group representations* and the *cohomology* of infinite dimensional Lie Algebras.
- Contributed to the theory of *generalized functions* (used in solving differential equations of mathematical physics).

1935–1973 CE **Leo Strauss** (1899–1973; Germany and USA). Political philosopher. Created the '*Straussian*' school of political science. No other conservative thinker has inspired a following remotely comparable, in size, continuity, and influence, to that of Leo Strauss. This school has its own interests, ideas, and purposes, which are clearly distinct from mainstream conservatism. His extraordinary influence as a leader has been demonstrated by his many students who have succeeded him in the field of political philosophy.

Strauss was born in Kirchheim, Hesse, Germany. He was brought up in an orthodox home, where the 'ceremonial' laws were rather strictly observed, but where there was little Judaic knowledge. Further identification as a Jew came early to Strauss, when refugees from the Russian Pogroms passed through his village.

In the Marburg Gymnasium (1905–1916) he was exposed to the message of *German Humanism* and read Schopenhauer and Nietzsche. He was exposed to political Zionism in 1917. During 1917–1918 he was conscripted into the German Army. He then studied philosophy, natural science and mathematics (1919–1921) at the Marburg University under the guidance of **Husserl**, **Heidegger** and **Hermann Cohen**, and received his Ph.D. there (1921). In 1932 he left Germany and eventually came to the United States. He taught at the new New-York School for Social Research during 1938–1949, then at the University of Chicago from 1949 to 1967, Claremont Men's College in 1968–69, and St. John's College until his death in 1973. Leo Strauss was the Robert Maynard Hutchins Distinguished Service Professor Emeritus in Political Science at the University of Chicago.

Strauss is the author of fifteen books, in which he set himself to investigating the fundamental problems of political philosophy. Among them: *Philosophy and Law* (1935); *Natural Right and History* (1953); *What is Political Philosophy* (1959); *Liberalism Ancient and Modern* (1968).

Strauss, an ethnic Jew and refugee from Nazi Germany, looked at the regnant liberalism of mid-century America, and saw the Weimar Republic: morally weak, incapable of self-preservation. His prophecy was fulfilled by the ignominious collapse of the liberal establishment, both political and academic, in the face of the New Left.

In the Straussian view, philosophy inadvertently exposed men to certain hard truths, truths too hard for them to bear: that there are no gods to reward good or punish evil; that no one's *patria* is really any better than anyone else's; that one's ancestral ways are merely conventional. This leads to nihilism, epitomized by the listless, meaningless life of bourgeois man, or to dangerous experiments with new gods – gods like the race and the *Fuehrer*.

Straussianism poses questions that need to be asked: what is the relation of nature to culture? Can society be founded on rational principles? Has Enlightenment brought about its own fall? How did this happen? What can be salvaged from the wreck?

Strauss' own answers are:

(1) A return to treating old books *seriously*, reading them carefully and with an effort to understand them as their *authors* did, rather than as History does.

(2) A recognition of the *political* nature of philosophy, that most philosophers who wrote did so with a political purpose.

(3) A recognition that the greatest thinkers often wrote both *exoteric* and *esoteric* teachings, either out of fear of persecution or a general desire to present their most important teachings to those most receptive to them. This leads to an attempt to discern the *esoteric* teachings of the great philosophers from the clues they left in their writings for careful readers to find.

(4) A recognition of the dangers that historicism, relativism, eclecticism, scientism, and nihilism pose to philosophy and to Western culture generally, and an effort to steer philosophy away from these devastating influences through a return to the seminal texts of Western thought.

(5) Careful attention paid to the dialogue throughout the development of Western culture between its two points of departure: Athens and Jerusalem. The recognition that Reason and Revelation, originating from these two points respectively, are the two distinct sources of knowledge in the Western tradition, and can be used neither to support nor refute the other, since neither claims to be based on the other's terms.

(6) A constant examination of the most drastic of philosophic distinctions: that between the Ancients and the Moderns. An attempt to better understand philosophers of every age in relation to this distinction, and to learn everything that we as moderns can learn about ourselves by studying both eras.

Worldview L: Leo Strauss

* * *

"The philosopher who, transcending the sphere of moral or political things, engages in the quest for the essence of all beings, has to give an account of his doings by answering the question 'why philosophy?' That question cannot be answered but with a view to the natural aim of man which is happiness, and in so far as man is by nature a political being, it cannot be answered but within a political framework."

* * *

"But let us hasten back from these awful depths to a superficiality which, while not exactly gay, promises at least a quiet sleep ..."

* * *

"Because mankind is intrinsically wicked, he must be governed. Such governance can only be established, however, when men are united — and they can only be united against each other."

* * *

Worldview LI: Allan Bloom

* * *

"As it now stands, students have powerful images of what perfect body is and pursue it incessantly. But deprived of literary guidance, they no longer have any image of a perfect soul, and hence do not long to have one. They do not even imagine that there is such a thing."

* * *

"The substance of my being has been informed by the books I learned to care for."

* * *

"Education is the movement from darkness to light."

* * *

"The most successful tyranny is not the one that uses force to assure uniformity but the one that removes the awareness of other possibilities, that makes it seem inconceivable that other ways are viable, that removes the sense that there is an outside."

* * *

"Music is the medium of the human soul in its most ecstatic condition of wonder and terror. Nietzsche, who in large measure agrees with Plato's analysis, says...that a mixture of cruelty and coarse sensuality characterized this state... Music is the soul's primitive and primary speech... without articulate speech or reason. It is not only not reasonable, it is hostile to reason."

* * *

"Rock music has one appeal only, a barbaric appeal, to sexual desire - not love, not eros, but sexual desire undeveloped and untutored."

* * *

Straussianism vs. Liberalism

The modern world is held to be the deliberate creation of the modern philosophers – namely, the Enlightenment*, which gave birth to both scientific-technological progress and the liberal ideology of social-political progress.*

The Enlightenment argued that instead of hiding philosophy, philosophers should reform society to make it more hospitable to philosophy: in particular, by undertaking the project of modern science*, by which reason masters nature and provides material gratifications (safety, health, wealth) to common men; Thus, physical science and technology would provide the know-how, while liberalism would provide the conditions of liberty and equality enabling men to pursue their self-interest.*

The great significance of Strauss for mainstream conservatives is that his is the deepest philosophical analysis of what is wrong with liberalism. Technocratic, legalistic, and empirical criticism of liberalism are all very well, but it is not enough. He believes that contemporary liberalism is the logical outcome of the philosophical principles of modernity, taken to their extremes. In some sense, modernity itself is the problem. Strauss believed that liberalism, as practiced in the advanced nations of the West in the 20th century, contains within it an intrinsic tendency towards cultural and moral relativism, which leads to nihilism. He first experienced this crisis in his native Germany's Weimar Republic of the 1920s, in which the liberal state was so ultra-tolerant that it tolerated the Communists and Nazis who eventually destroyed it and tolerated the moral disorder that turned ordinary Germans against it.

Strauss believed that America is founded on an uneasy mixture of classical (Greco-Roman), Biblical, and modern political philosophy. Conservatives have not failed to note that a significant part of the mischief of liberalism consists in abandoning the biblical element; this story has been told many times and is well represented in Washington. Where Strauss comes in is that he is the outstanding critic of the abandonment of the classical element. His key contribution to fighting the crisis of modernity was to restore the intellectual legitimacy of classical political philosophy, especially Plato and Aristotle.

Strauss' first move, which came as a stunning shock to a 1950s academic world sunk in scientism and desirous of making "political science" substitute for political philosophy, was to reactivate the legitimacy of ancient philosophy as real political critique. It is hard to overstate how unlikely this seemed at the time, it having been a casual article of faith since ancient philosophy had no more to say about modern political problems than ancient physics about modern engineering. Nevertheless he succeeded. When leftists today *feel obliged to denounce Great Books curricula, it is because they know, consciously or unconsciously, that classical thought is very much alive and is a real threat to them.*

The holy grail of Straussian scholarship has been to understand the ancient philosophers not from a modern point of view but from their own. The implication is that we then become free to adopt the ancient point of view towards modern political affairs, freeing us from the narrowness of the modern perspective and enabling us to step back from the distortions and corruptions of modernity. Strauss contends that the modern view of politics is artificial and that the ancient one is direct and honest about the experience of political things.

Strauss was not ignorant of the reasons modern political philosophy had come about. He saw it as a grand compromise made when the demands of virtue made by ancient political philosophy seemed too high to be attainable. Modern political philosophy provides no rational basis for higher human achievement, but it provides a very solid basis for the moderate human achievement of stability and prosperity.

Strauss not only believed that the great thinkers of the past wrote Straussian texts, he approved of this. It is a kind of class system of the intellect, which mirrors the class systems of rulers and ruled, owners and workers, creators and audiences, which exist in politics, economics, and culture. He views the original sin of modern political philosophy, which hundreds of years later bears poisonous fruit in the form of liberal nihilism, to be the attempt to abolish this distinction. It is a kind of Bolshevism of the mind.

The key hidden step in the Machiavellian view, a bold intellectual move that is made logically rigorous and then politically palatable by Thomas Hobbes and John Locke, is to define man as outside nature. Strauss sees this as the key to modernity. Man exists in opposition to nature, conquering it to serve his comfort. Nature does not define what is good for man; man does. This view is the basis for the modern penchant to make freedom and comfort the central concerns of political philosophy, whereas the ancients made virtue the center. Once man is outside nature, he has no natural teleology or purpose, and therefore no natural virtues. Since he has no natural purpose, anything that might give him one, like God, is suspect, and thus modernity tends towards atheism. Similarly, man's duties, as opposed to his rights, drop away, as does his natural sociability. The philosophical price of freedom is purposelessness, which ultimately gives rise to the alienation, anomie, and nihilism of modern life.

Strauss' elitist view of the 'good society' is strikingly similar to the view cultivated for centuries by the Catholic and Orthodox Churches and by Orthodox Judaism, not to mention other religions: there is a small number of *men who know the detailed truth; the masses are told what they need to know and no more.*

What is then the answer to nihilism? Does the restoration of classical political philosophy really re-establish convincing values? Are Aristotle's virtues really virtues? Is Plato's critique of democracy true?

Strauss believed that the great competitor of philosophy is revealed religion. He believed that reason and revelation cannot refute each other. He believed that religion was the great necessity for ordinary men. For him, religion is in essence revealed law, and he took his native Judaism to be its paradigm. Strauss had an ambivalent attitude towards Christianity. On the one hand, Christianity is the only practicable religion for America. On the other hand, Christianity has troubling strands within it, like St. Aquinas' claim that reason and revelation are compatible, for him the precise opposite of the most important truth. It is a commonplace that Christianity is a synthesis of Greek philosophy with biblical theism; Strauss rejects the idea that such a synthesis is possible. For him, religion is at bottom simply dogmatic and unapologetic about it.

Although Strauss was a critic of the natural-right teachings on which the US society is based, he only criticizes modern *natural-rights because he thinks it destroys itself and becomes untenable. As Strauss says, "just because we are friends of liberal democracy does not entitle us to be flatterers of liberal democracy." In his public utterances on contemporary politics he was a conventional conservative patriot who backed the United States against Nazi Germany in WWII and Soviet Russia in the Cold War. He was boldly anti-Communist at a time when most Western intellectuals were dangerously equivocal, if not outright sympathetic.*

What is undeniable is that he did see the United States as the most advanced case of liberalism and therefore the most susceptible to the nihilism he dedicated his life to fighting. But he also saw the United States as partly founded on the classical and Biblical political wisdom that offered an answer. There is no doubt that he saw the United States as the world's only hope. One of the lessons we can draw from him is that the essence of liberal modernity is so problematic that America cannot afford for its essence to adopt liberal modernity, whether that liberalism takes Lockean, classical (in the sense of the 19th century) or postmodern form.

*Strauss held that '*globalism*' (the liberal project of modernity of a universal society consisting of free and equal nations, each consisting of free and equal men and women, with all these nations fully developed as regard to their power of production, thanks to science) is* not *the inevitable culmination of modernity, as its proponents believe, but a* perversion *which would first make nations unfree and then abolish them outright.*

He believed that world citizenship is impossible, as citizenship, like friendship, implies a certain exclusivity, and universal love is a fraud. (If it exists, it is the province only of God.) Good men are patriots or lovers of their patria or fatherland, which must by definition be specific. The United Nations has failed in its fundamental mission: to prevent war.

Strauss' work was extended by his pupil ***Allan David Bloom*** *(1930–1992, USA), social philosopher, humanities scholar and academic educator. His seminal work,* The Closing of the American Mind[866] *(1987) is an open sharp criticism of the leftish analytic philosophy as a movement that originated in post WWII American Universities. To a great extent, Bloom's criticism revolves around the devaluation of the Great Books of Western Thought as a source of Wisdom. However, Bloom's critique extends beyond the university to speak to the general crisis in American Society.* Closing of the American Mind *draws analogies between the United States and the Weimar Republic.*

Bloom saw these social and cultural developments as stemming from a wholesale importation (by European intellectuals that arrived on American shores as emigres and refugees) of the latter day, nihilistic stages of German idealistic philosophy, ultimately traceable to ***Hegel****,* ***Nietzsche****,* ***Marx****,* ***Freud*** *and* ***Heidegger****. The anti-democratic pessimistic and nihilistic ideas of Nietzsche and Heidegger were transformed and 'laundered' by* ***J.P. Sartre*** *and the New Left. When combined with Freud's shallow pseudo–scientific theory of the mind (which forever seeks to explain higher, noble impulses and achievements as repressions and sublimations of lower urges), the result was a worldview and society that would have horrified* both *Nietzsche and the rationalist Enlightenment philosophers who created modernity to begin with. This hybrid post WWII worldview of the West was made possible by a paradoxical combination of:*

- *Advancing and indiscriminate democracy.*
- *Eroding stature of the ideals of the Enlightenment.*
- *An alliance of the revolutionary Left and the old, authoritarian Right – both of which despised that epitomy Democratic Man – the Bourgoise.*

[866] Bloom was born in Indianapolis to Jewish parents. He entered the University of Chicago (1946) and earned his Ph.D. there (1955) under **Leo Strauss**. He later taught at the Universities of Yale, Cornell (1963), Tel-Aviv and Toronto (1978) before returning to Chicago. After Blooms passing, Saul Bellow, his associate at the University of Chicago, implied that Bloom was gay and died of complications of AIDS.

Since the average Everyman is the ultimate ruler in a democracy, he needs to be flattered. And so the Nietzschean right to "kill God", create new gods, and dispense with ordinary right and wrong, morality and cultural standards – rights that Nietzsche meant to reserve for the "inner directed" superior man – were now, in an ironic twist, copiously bestowed upon every person who wanted to rebel against society's 'values' (formerly described with adjectives such as 'right', 'wrong', 'just', 'noble', 'learned' etc.) and 'do his own thing'. People were encouraged to 'find themselves', 'speak truth to power', etc. – anything but be boring Bourgoise, i.e. a 'cog in the machine'. When they follow their untutored whims they are said to be "creative"; every and any predilection that is not at present criminal or contrary to the secular fetish or moralism of the day, is merely "a lifestyle choice" – and the pressure is on to extend compassion, legality or even full societal sanction to "lifestyles" that were formerly merely tolerated – up to and including ideological terrorism (provided, of course, its perpetrators are some flavor of accepted 'Other'!).

This bizarre deification of the undeserving also explains, in Bloom's view, the phenomenon of Rock music — a way for savvy business moguls to tap the immense disposable income–resources adolescents are able to syphon from their parents, by offering a low art form whose drumbeats appeal directly to youngster's tribalism and untutored sexual awakening. Like drugs, much of this music offers a pale simulacrum of the 'highs' normally earned by great and noble achievements. Yet these satisfactions, delivered with the electronic and digital technology achieved by humanity's best thinkers and true rebels over a period of centuries, are readily available to anyone.

And the worst aspect of this European cultural brew, according to Bloom, is that the mental universe of most Americans has come to be delimited by the loan concepts enshrined in the above words. The people who coined these terms and imported them hither were familiar with their Greek, Judeo–Christian and Enlightenment origins, but today's Americans by and large are not. So their intellectual horizons have narrowed. In America's ultra–democracy, the utterances of Rock stars, basketball players, or purveyors of the latest environmental, atheistic or multicultural fads are treated on par with the greatest intellectual canons of Western tradition. Someone raised in this America and schooled in its universities, finds it exceedingly difficult to read the words of Plato, Aristotle, Plutarch, Shakespeare, Locke or Kant and absorb their meaning directly. Rather, they become filtered through the mental fog created by the neologisms of the later German nihilists and their disciples. The deification of any and every individual and his or her rights, development and pleasures in our society, at the expense of common projects (whether a cultural heritage or raising a family), led to inevitable problems in married life – and high divorce rates. That, in combination with the easy, early and consequence–free availability of sexual intimacy, which becomes separated *from* eros *and the cultural longings with which it was formerly associated has – in Blooms experience as a teacher of the young – completely changed the college. It made many students duller and more cynical human beings.*

Bloom hewed to the Straussian approach to social and political philosophy, which contrasted the views of the ancients with those of modernity – and traced the latter to the Enlightenment. In antiquity, the roots of the West's religions *(revealed truth, justice, piety and morality) sprouted in Judea (Moses, Samuel, Elijah, Amos, Hillel, Ben Zakai, Jesus); while* rational philosophy *(logico-mathematical, scientific, aesthetic, moral, economic and socio–political) came into being in Greece, Ionia and the Hellenistic world (Euclid, Pythagoras, Thales, Socrates, Aplaton, Protagoras, Aristotle, Archimedes, Aristarchos, Heron, Ptolemy and the rest).*

When the Church achieved supremacy in Europe, it borrowed logico-rhetorical tools from Aristotle and the Neo–Platonists to codify its theology, but forbade any further independent thinking. The Enlightenment natural and social philosophers adopted ancient Greek rationality and science as their own, further developing them. While their democratic inclinations and morality were Christian (and thus, ultimately, Judaic) in origin, they rebel against the Church's Aristotle-fortified and ossified authority – but not against Aristotle himself (though his theories of physics turned out to be wrong). These thinkers were avowed rationalists; *and while they argued and remonstrated with their ancient colleagues (Francis Bacon replacing the* new republic *for Plato's* republic*), they had no quarrel with the basic Greek approach. Thus, according to him, the Enlightenment was a gigantic, multi–century project to diminish clerical power and educate the European political elites in the ways of democracy.*

Over time, however, several things went awry in this program. The University, which the rationalists converted from theological strongholds to bastions of the liberal arts and sciences, did manage to stand up to the aristocracy and Church. Yet later, it became too eager to please its new master – the common man *– to the point where its properly elitist role gradually eroded. And along with it, so did the kind of liberal education that used to produce thoughtful, responsible national leaderships. Indeed, rationality itself came to be viewed as merely one point of view. And what is worse, many shallow sociological, economic, political and even pseudo-scientific 'theories' came to be spawned, adopted and championed by nominal rationalists. They defend these notions as fervently as their predecessors-in-spirit would champion a religious dogma*[867]*.*

867 As Bloom points out, a rationalized dogma is the only tool that can root out reason itself.

1936 CE *Gravitational lensing*: In the December 04, 1936 issue of *Science*, Albert Einstein published a note entitled "*Lens-like Action of a Star by the Deviation of Light in the Gravitational Field*". In it he predicted a General-Relativistic phenomenon, the observation of which in 1979 provided a new experimental test of GTR. Moreover, it helped astrophysicists and cosmologists to estimate Hubble's constant and determine the amount of invisible matter in the universe (1998). It thus took almost half a century before the cosmos known to us finally reached the dimensions (in the wake of the discovery of *quasars*), and astrophysics the technical sophistication, to render *gravitational lensing* an observable reality.

According to General Relativity, an object in space that is sufficiently massive acts as a *lens* for light coming from more distant objects in the same line of sight with respect to observers on earth. If the configuration of these background and foreground objects is right, the lens effect could produce two or more identical images of the more distant object or even distort its image into an *arc* or complete *ring* about the foreground object's image.

The first actual optical gravitational lens was discovered in 1979 by a British astronomer, Dennis Walsh. The lens in this case is a giant elliptical *galaxy*, and it produces a double image of a more distant *quasar*, since the light of the quasar is deflected to either side of the galaxy located between us and the quasar, thereby producing *two point images*. Both quasar images have the same spectral lines and redshifts, as predicted by GTR. The angular separation between the two quasar images is 6 seconds of arc.

A few other such phenomena have since been observed, including the 1998 discovery of an "Einstein ring" produced by a perfect alignment of nearer and farther objects. Astronomers used a network of radio telescopes in Britain and the *Hubble Space Telescope* (HST) to discover the Einstein ring. This *radio telescope array*, called MERLIN, found the ring, and the HST produced an image of it. The ring occurs because two distant galaxies happen to line up almost perfectly with Earth. The closer galaxy's gravitational lens spreads out the light of the more distant galaxy into a complete circle. Vast luminous arcs, or imperfect rings, have also been observed.

Astronomers can use gravitational lenses to determine the amount of matter in a galaxy by measuring how much the galaxy's gravitational field distorts the light of a more distant galaxy behind it. By comparing the gravitational effect of the nearer galaxy with the amount of visible matter in the same galaxy, astronomers can estimate the amount of invisible matter, or *dark matter*, that the nearer galaxy must hold.

The idea was not new: firstly, it is based on Einstein's own second test of GTR, namely the deflection of light by a massive star. Secondly, on his meeting with the astronomer **Freundlich** in Berlin (April 1912), Einstein discussed with him the possibility of a gravitational lensing effect (notes from that period, including his early conclusions on the subject, were indeed found).

Einstein, however, did not publish anything on the subject because he believed that there was no hope in observing this phenomenon. Indeed, the universe, as known to astronomers in the 1910's, essentially consisted of our own galaxy. Under these conditions, the observability of gravitational lensing was, per theory, almost impossible.

Nevertheless, there soon appeared precursor discussions of the idea in published works by others, among them **Oliver Lodge** (1919), **Arthur Eddington** (1920), and **O. Chwolson** (1924). Yet, Einstein himself was reluctant to publish his calculations, and did so only after much hesitation and under persistent badgering by the amateur scientist **Rudi W. Mandl** (1936).

1936 CE, summer First regular television broadcasts were made by NBC in the United States and the BBC in England. FM broadcasts from the Olympic games in Berlin. These constitute the first radio messages from planet earth that made their way past the ionosphere and into outer space (due to their high frequency). By the year 2000, they would reached receivers (if any) 64 light-years away (a distance of ca 6×10^{14} km).

Gravity's Lens – or, the triumph of an amateur's fantasy

In early April 1936, ***Rudolf W. Mandl****, an engineer from the Czech Technical University and amateur astronomer (and a Jewish immigrant refugee from Nazi Europe) walked into the offices of the Science Service in the building of the National Academy of Science in Washington DC. He came with a new idea — a proposed new test of GTR, based on observations during eclipses of stars.*

He was looking for someone to help him publish his ideas and persuade professional astronomers to take up the investigation of his proposal. He was *then invited to plead his case before professor Einstein himself: if the latter found his ideas worthwhile, he could return and seek further help from the NAS.*

Mandl indeed visited Einstein in Princeton on April 17, 1936, presenting him with a quaint combination of ideas from GTR, optics and astrophysics. He proposed a simple model according to which a sufficiently massive astrophysical object could act as a lens *for light coming from a more distant object along the same line of sight w.r.t observers on earth. He speculated that the effects of such a focusing might already have been observed, though not hitherto recognized as such.*

Among the possible effects that Mandl took into consideration were the recently discovered annular shaped nebulae — which he interpreted as gravitational images of distant stars.

The basic GTR consequence that Mandl pointed out to Einstein was that if an observer is perfectly aligned with a both 'near' and a 'far' star, then he will observe the image *of the far star as an* annular ring*, resulting from the bending of its light by the near star.*

Though Mandl's idea was daring, at its core was a valid insight that would eventually (several decades later) become not only an astrophysical confirmation of GTR but an "applied GTR" tool of astrophysics and cosmology.

To grasp the key idea, there is no need to use GTR proper, if one wishes to obtain a quantitative result up to a constant of order unity. In fact, one may employ the same arguments used by **Soldner** *(1804) to derive the deflection of light by a massive star, using only* classical Newtonian gravitational theory *and treating light as consisting of Newtonian non-relativistic corpuscles ("photons").*

Consider a distant point light–source, emitting a beam of earth–directed photons that travel along almost parallel trajectories (rays) with velocity c. Between this lensed object *and the* observer *on earth (eye or telescope) there is a mass M (lensing mass) that bends the rays and focuses them toward the observer. Assume the lensing mass to have the shape of a sphere, with a mass density distribution possessing axial symmetry along the observer's line of sight to the distant mass.*

Assume further that the angles of deflection are small enough such that their tangents can be replaced by the angles themselves (measured in radians). Let: (see Fig. 5.14)

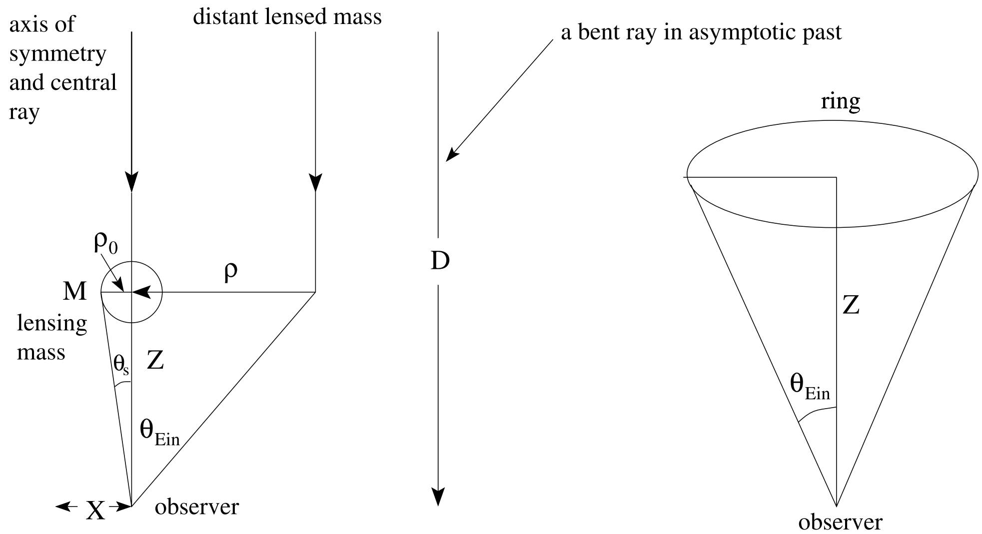

Fig. 5.14: Geometry of a 'gravitational lens'

$Z =$ *distance from center of the lensing mass M to observer on earth;*

$\rho =$ *asymptotic pre-bending distance from unperturbed central ray to a particular bent ray outside the lensing mass (called the "impact parameter" in scattering theory);*

$A_p = \dfrac{GM\rho}{(\rho^2+\xi^2)^{3/2}} =$ *instantaneous transverse gravitational acceleration component (toward the axis of symmetry) per unit photon mass;*

$\delta V_p =$ *photon's impulse (change of transverse momentum) per unit photon mass due to gravitational bending;*

$\xi =$ *variable spatial distance of central ray photon from center of lensing mass.*

In figure (5.14) light rays from a distant object in the background strike a lensing mass in the foreground. The bent rays refers to a cone extrapolated back into asymptotic past. The ring is the image seen by the observer.

In this 'Newtonian approximation' there is no dispersion *because according to the Galilean version of the Equivalence Principle, ray deflections are independent of the photon energy, assuming*[868] *(as we must) that all photons*

[868] A GTR version of this calculation, ignoring optical diffraction effects (permissible since $\lambda_{light} \ll \rho$), would show that the ray's deflection is independent of its frequency, which is to say, of its wavelength λ_{light}. In this calculation, each

move at speed c. It then follows that (in the impulse approximation, i.e. integrating along an undeflected ray):

$$\delta V_p = \int_{-\infty}^{\infty} A_p dt \approx \int_{-\infty}^{\infty} \frac{1}{c} A_p d\xi = \frac{2GM}{c} \int_{0}^{\infty} \frac{\rho d\xi}{(\rho^2 + \xi^2)^{3/2}} = \frac{2GM}{\rho c}.$$

Since the angular deflection is

$$\theta_{Ein} \approx \frac{\delta V_p}{c} \approx \frac{2GM}{\rho c^2} \approx \frac{\rho}{Z},$$

we have in this approximation:

$$\theta_{Ein} = \sqrt{\frac{2GM}{Zc^2}}, \qquad \rho = \sqrt{\frac{2GMZ}{c^2}}.$$

In his GTR, Einstein had already predicted in 1911 that

$$\theta_{Ein} \approx \frac{4GM}{\rho c^2}, \qquad \rho = \frac{2}{c}\sqrt{GMZ}$$

showing that the Newtonian value for θ is in error by a factor of $\sqrt{2} \approx 1.4$.

The above results show that there is just one ray, having one angle and one distance ρ, that bends into the observing instrument on earth for any given value of the azimuth angle. Hence the rays reaching the observer appear to be coming from a circle in the lensing–mass plane (known as the Einstein ring) with radius $Z\theta_{Ein}$. It can be shown that in the general case, when the distant light source is not a point but has some angular extent (and need not be axially symmetric) and rays penetrating through the lensing–mass periphery are considered, the lens creates either a finite-width ring image or two or more separate distorted images, on various sides of the source.

Now, since $\theta_{Ein} \propto \frac{1}{\rho}$, the angle of deflection θ_0 for light rays just grazing the edge of a deflecting object of radius ρ_0 (Fig. 5.14) is related to θ and ρ for a general *ray as follows:*

$$\theta_{Ein} \approx \theta_0 \frac{\rho_0}{\rho} \approx \frac{\rho}{Z},$$

ray's spacetime trajectory is a light-like geodesic in the curved metric induced by the foreground (lensing) mass.

and consequently

$$\theta_{Ein} \approx \sqrt{\frac{\theta_0 \rho_0}{Z}} = \sqrt{\theta_0 \theta_s}, \qquad (1)$$

where $\theta_s \approx \frac{\rho_0}{Z}$ *is the apparent angular radius of the lensing mass as it appears to an earthbound observer. Note that the fiducial grazing ray will not reach earth unless* $\theta_0 = \theta_s = \theta_{Ein}$, *which only holds when the lensing object is at distance* $Z = \rho_0/\theta_0$ *from earth.*

Mandl, who knew from Einstein's GTR result (1911) that $\theta_{Ein} \propto 1/\rho$, *now suggested to Einstein to use (1) as a new independent test of GRT, provided of course that* θ_{Ein}, θ_0, ρ_0 *and* Z *be determined from observations.*

Clearly, θ_{Ein} *could in principle be determined from the radius of the observed Einstein ring and* θ_0 *(the deviation of the grazing ray) could perhaps be determined during an* eclipse *of the lensing mass (assumed to be a star), as done by* **Eddington** *in 1919 for the sun*[869].

In his 1936 note, Einstein was rather skeptic about the experimental confirmation of 'Mandl's equation' on the ground that

"... θ_{Ein} *being of the order of magnitude of one second of arc, the angle* ρ_0/Z *is much smaller. Therefore, the light coming from the luminous circle can not be distinguished by an observer as geometrically different from that coming from the star, but simply will manifest itself as increased apparent brightness of the lensing star".*

Mandl, however, did not yield. His idea was that the lensing effect by the foreground star would result in a considerable brightness increase of the background star's light at the locus of the terrestrial observer, despite the fact that – unlike the case of a common optical lens – gravitational deflection by a massive star does not (even approximately) collect parallel light rays in one single focal point, but rather smears them out along a focal line.

Einstein obliged, and set forth to calculate this intensification. To this end he placed the observer not on the axis of symmetry, but a small distance x *away from it. It can be shown by simple geometrical arguments*

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869 But then the distant light source used to determine θ_0 cannot, in general, be along the same line–of–sight as are the lensing star and the distant star used to measure θ_{Ein}. If M and ρ_0 are known for the lensing star in question, one could also calculate θ_0 from GTR: $\theta_0 \approx \frac{4G\,M}{\rho_0\,c^2}$.
In modern astrophysics, ρ_0 and/or M are often *deduced* from measurements of magnification and luminosity changes as the lensing mass – sometimes not optically visible at all – passes through earth's line of sight to a distant object (e.g a quasar or galaxy).

that in this case the observer will not see a ring but, instead, two point-like images. The amplification factor will then be approximately (assuming $x/Z << 1, \quad \rho_0/Z << 1$*) proportional to* $1/x$*, and hence cause infinite intensity on the system's axis of symmetry itself (still assuming the lensed object to be infinitely far away, and ignoring wave optics). Furthermore, Einstein noted the curious effect that the amplification at a given point* x increased *with the* increasing *distance from the lensing star.*

Einstein's published note became the classical starting point for the officially recorded research of gravitational lensing. He opened it with the words:

"Some time ago, R.W. Mandl paid me a visit and asked me to publish the results of a little calculation, which I had made at his request..."

In the final section Einstein concluded:

"Therefore, there is no great chance of observing the phenomenon..."

This is the peculiar story of Einstein's double encounter with the idea of gravitational lensing (1911, 1936), and of Mandl's role in the second episode. Einstein's note triggered a number of other papers which further developed the idea, taking it much more seriously than Einstein himself had done.

However, the next major step came a year later, when ***Fritz Zwicky****, then at CalTech, published (1936) two notes in the* Physical Review.

*Referring specifically to Mandl's idea, he claimed that "*extragalactic nebulae*, as a consequence of their masses and apparent dimensions, were much more likely candidates for observation of gravitational lenses" and furthermore: "Present estimates of masses and dimensions of* cluster nebulae *are such that the observability of gravitational lenses effects among the nebulae would be seen"*[870].

In retrospect, Mandl's role in establishing this subject was crucial since he had helped turn gravitational lensing into a theoretical reality long before it became an observed one. What Mandl achieved, in the end, was to introduce a single idea into the canon of accepted scientific knowledge, an idea which

870 Zwicky's "extragalactic nebulae" means, in today's parlance, *galaxies* outside our sun's own (*milky way*) galaxy. By the end of the 20^{th} century, gravitational lensing became a routine tool of "applied GTR". It is used to better observe distant galaxies via the lensing of their light by foreground objects (including brightness and shape changes *and* magnification), as well as to survey dark matter in our galaxy (by the lensing of background stars which they cause), and other applications.

was rejected before only because the effect was deemed to be inaccessible to observations.

It thus happened that Mandl's initiative, persistence[871] *and vision, together with the fact that Einstein lent prominence to it with his 1936 publication, stimulated a broad discussion among astronomers and astrophysicists, and this discussion lasted until the effect was eventually confirmed by observations.*

Mandl's success was a victory of both theory and fantasy, since Einstein's publication stimulated his contemporaries to imagine the strange world of gravitational lensing, and thus to take the effect seriously and explore the conditions under which it might be observable, after all.

Furthermore, this episode shows that science develops not only by the exclusive combination of outstanding scientists working under the auspices of institutions of professional learning. In addition to – and often preceding – specialized theories, it may receive impetus from elementary ideas supplied by non-professionals outside this system.

Mandl's role in the history of gravitational lensing illustrates that innovative explorations in the natural sciences is not necessarily the privilege of a few distinguished scientists and other team members, and that even amateurs can sometimes contribute. After all, Mandl's expectations about the promise of gravitational lensing to provide new observational confirmations of GTR, have obviously been amply fulfilled.

[871] There is evidence that Mandl rather obsessively attempted to enlist professional scientists to his cause. Among them were **William Francis Gray Swann**, director of a center of cosmic ray studies; the Nobel laureate physicist **Arthur Holly Compton** and **Robert Andrews Millikan**; **V.K. Zworykin**, research scientist at the Radio Corporation of America (RCA) and one of the inventors of the first all-electronic television system. Some of them reacted with interest and gave Mandl's ideas some brief consideration, while others excused themselves pleading lack of time or understanding. None of them, in any case, pursued the matter seriously.

1936 CE

- 'Penguin Books' introduced the first *paperback*.
- Fluorescent lighting was introduced.

1936 CE The first *Fields Medal* [872] was awarded at a World Congress in Oslo. It is regarded as mathematics' closest analogue to the Nobel Prize[873] (which does not exist for this field), and awarded every four years by the international Mathematical Union to one or more outstanding researchers. Up to four medals could be awarded at each congress. A prize of 15,000 Canadian dollars goes with the Medal.

The Fields Medal is made of gold, and features the head of Archimedes (287–212 BCE) together with a quotation attributed to him (in Latin rather than his own Greek):

"TRANSIRE SUUM PECTUS MUNDO QUE POTIRI"

("Rise above oneself and grasp the world")

The following table summarizes Field Medals winners during 1936–2002. An asterisk indicates a mathematician of Jewish origin.

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[872] For further reading, see:

- Monastyrsky, M., *Modern Mathematics in the Light of the Fields Medal*, AK Peters LTD: Wellesley, Massachusetts, 1997.
- Devlin, K., *The Millennium Problems*, Basic Books, 2002, 237 pp.

[873] Alfred Nobel did not create a prize in mathematics because he believed only in inventions or discoveries of great *practical* benefit to mankind. It is commonly stated, however, that Nobel decided against a Nobel Prize in mathematics because of anger over the romantic attention of the Swedish mathematician **Gosta Mittag-Leffler** to a woman in his life (perhaps his Viennese mistress, Sophie Hess). There is no historical evidence to support this anecdote.

Table 5.17: FIELD MEDALISTS (1936–2002)

MATHEMATICIAN	YEAR	COUNTRY	LIFE-SPAN
Jesse Douglas*	1936	USA	1897–1965
Lars Ahlfors	1936	USA	1907–
Laurent Schwartz*	1950	France	1915–
Atle Selberg	1950	USA	1917–
Jean-Pierre Serre*	1954	France	1926–
Kunihiko Kodaira	1954	USA	1915–
Klauss Friedrich Roth*	1958	GB	1925–
René Thom	1958	France	1923–
Lars Hörmander	1962	Sweden	1931–
John Milnor	1962	USA	1931–
Stephen Smale	1966	USA	1930–
Paul Cohen*	1966	USA	1934–
Alexander Grothendieck*	1966	France	1928–
Michel F. Atiyah	1966	GB	1929–
Alan Baker	1970	GB	1939–
Sergei P. Novikov	1970	USSR	1938–
John Thompson	1970	GB	1932–
Heisuke Hironaka	1970	USA	1931–
David Mumford	1974	USA	1937–
Enrico Bombieri	1974	Italy	1940–
Pierre Deligne	1978	France	1944–
Danil Quiller	1978	USA	1940
Grigorii A. Margulis*	1978	USSR	1946–
Charles Fefferman*	1978	USA	1949
Alain Connes	1983	France	1947–
William Thurston	1983	USA	1946–
Shing Tung Yau	1983	USA	1949–

Table 5.17: (Cont.)

Mathematician	Year	Country	Life-span
Simon K. Donaldson	1986	GB	1957–
Gerd Faltings	1986	Germany	1954–
Michael Freedman*	1986	USA	1951–
Vladimir G. Drinfeld*	1990	USSR	1954–
Edward Witten*	1990	USA	1951–
Vaughan Jones	1990	USA	1952–
Shinfumi Mori	1990	Japan	1951–
Jean Bourgain	1994	France	1954–
Pierre-Louis Lions	1994	France	1956–
Jean C. Yoccoz	1994	France	1957–
Efim Zelmanov*	1994	Russia	1955–
Richard E. Borcherds	1998	USA	1959–
Timoty W. Gowers	1998	GB	1963–
Maxim Kontsevich*	1998	Russia	1964–
Curtis T. McMullen	1998	USA	1958–
Laurent Lafforgue	2002	France	1966–
Vladimir Voevodsky	2002	Russia	1966–

1936 CE **Heinrich Karl Johann Focke** (1890–1973, Germany). Aircraft designer. Developed the first practical airworthy *helicopter*, FW 61. Began building aircraft (1908); built monoplanes (1919–1920).

1936–1939 CE *The Spanish Civil War.* The prelude to WWII. Started as an internal conflict between a 'popular front' of Socialists, Syndicalists and Communists (known as *Republicans*) against the traditionally pro-monarchist forces — clergy, army and aristocracy (known as *Nationalists*). Had the Spaniards been left alone, the war would hardly have been the major tragedy it turned out to be. But intervention by other countries followed appeals for help by both sides and the war was not to remain a purely Spanish affair: Germany and Italy supported the Nationalists and seized the opportunity to test weapons and men in the field.

The Soviets, in turn, gave materiel and ideological support to the Republicans, but were incapable of matching the aid supplied by the fascists. To assure the survival of the Republicans, the wholehearted cooperation of the democracies was needed. Although public opinion in general supported the Republicans, the governments of France, England and the United States were not willing to risk a general war and remained neutral. (The U.S., for example, prohibited export of arms and munitions to either side in the conflict.) Volunteers from 50 countries did, however, join on the side of the Republicans, but supplies from Germany and Italy finally tipped the scales.

The war lasted almost three years: much of Spain lay in ruins and more than 700,000 died. The war did work for the Nazis in Germany as a diversion to cover their bloodless expansion in central Europe: WWII broke out five months after the Spanish Civil War ended. The big winner of the war was of course El Caudillo himself whose dictatorship lasted for nearly 40 years.

1936–1941 CE **Leopold L. Infeld** (1898–1968; USA, Canada and Poland). Theoretical physicist. Collaborated with **Albert Einstein** at Princeton (1936–8) on the N-body problem in GTR, and with **Max Born** on non-linear corrections to Maxwell's equations in the presence of strong fields.

Infeld was born in Cracow, Poland, and received his Ph.D. in 1921. He came to Princeton's Institute of Advanced Study in 1936, on a scholarship, after a few months in Edinburgh, Scotland, where he worked with Max Born. He later became a professor at the University of Toronto (1938–1950) and Warsaw (1950–1968).

In his work with Albert Einstein and **Banesh Hoffmann** (1906–1986) on the N-body problem, the gravitational field is no longer treated as an external field. Instead, it and the motion of its sources are treated *simultaneously*.

Newtonian celestial mechanics in its axiomatic structure consists of two clearly separate parts: the law of motion, and the law of gravity, which gives rise to the forces that keep the heavenly bodies in their orbits. The two parts stand alongside each other, unconnected. This separation into two strictly distinct sets of laws had not been overcome even by Einstein's initial treatment of motion in the general theory of relativity.

However, by the early 1920s investigations by Lorentz, Eddington, and Levi-Civita suggested that in the general theory of relativity these two sets are not really separate. After two years' work (1938) Einstein, Hoffmann, and Infeld were able to show, in a voluminous publication, that the field equations do in fact contain everything — not only the generation of the gravitational field as spacetime curvature caused by energy-momentum distribution, but also the motions and evolutions of this distributions in response to the gravitational fields.

This obviated the earlier separate "law of motion", according to which masses follow *spacetime geodesics* in the absence of non-gravitational forces; this former axiom of GTR henceforth became merely an *approximation.* Thus the general theory of relativity now described not only space, time, and gravitation, but also, for the first time, the dynamics of matter in response to gravitational fields.

The equations derived by EHI are widely used in analyses of planetary orbits in the solar system.

In the same year as this publication, there also appeared, in April 1938, a book by Einstein and Infeld, *The Evolution of Physics.* This work, reflecting the history of a discipline through the eyes of its greatest representative, owed its genesis not to Einstein's desire to communicate, but to economics at the Institute for Advanced Study.

"Infeld is a splendid fellow. We've done a very pretty thing together," Einstein reported after six months of joint research, but despite his fervent support the institute would not extend Infeld's modest scholarship. "The Institute has treated him badly. But I'll help him prevail here." Einstein, who felt he too had been badly treated by the refusal of a scholarship for his esteemed collaborator, wanted to defray the small sum from his own pocket.

But Infeld, who was embarrassed by that suggestion, had an original idea: How about writing a book together for a wide readership? With Einstein as one of the two authors, it could not fail, so that Infeld's share in the proceeds would secure his livelihood. Infeld found a publisher who paid him an advance, while Einstein planned the contents and the basic structure of the book, which eventually helped Infeld financially.

1936–1944 CE Advent of *information transmission by coaxial cables*. Bell Telephone engineers **Lloyd Espenshied** and **Herman A. Affel** invented the *coaxial cable*. The first line connected New York to Philadelphia (1936). Since 1944, the coaxial cable has been widely used for information transmissions such as long-distance telephone lines, coded, typed, or handwritten information, facsimile (maps, pictures, charts), television programs and computer data. Vast transcontinental and international networks were established[874].

A coaxial cable contains from 8 to 20 *coaxials*. A coaxial contains a *copper tube* with a *copper wire* held in the tube's center by plastic insulators. The *tube* (about 1 cm in diameter) shields the signal from outside electrical interference and prevents the signal from losing strength. The coaxial is wrapped with steel tape for strength, protection, and electrical shielding. The cable includes serial insulated wires, as well as the coaxials. The wires are used for control and maintenance. Amplifiers that strengthen the signals may be placed about 3 km apart. Coaxials may work in pairs: one carries signals in one direction, while the other handles signals in the other direction. In 1975, a fully equipped 20-tube cable could carry 32,400 two-way conversations simultaneously.

1936–1946 CE **John Maynard Keynes** (1883–1946, England). Economist. Most influential world figure in economics since Adam Smith, Ricardo and Marx. His *General Theory of Employment, Interest, and Money* (1936) ranks among the most important books on economics. It changed economic theory and policy, and is the basis of the economic policies of most nations today. His ideas helped to shift emphasis away from *laissez faire*, the classical capitalist economic theory that maintains that government should not interfere in economic affairs.

The basis of Keynesian economics is this: the level of economic activity depends on the total spending of consumers, business, and government. If business expectations are poor, investment spending will be cut, causing a series of reductions in total spending. Consequently, the economy will be led into a depression and stay there. To avoid depression, Keynes surged *increased government spending* and *easy money* (lower interest rates and making money more available for loans). These actions he argued, would encourage investment, increase employment, and enable consumers to spend more. The

[874] Early undersea cables could transmit telegraph signals, but they could not carry the wide frequency range that make up speech. Coaxial cables with built-in amplifiers made long-distance telephony by cable possible. In the US, coaxial cable's reach extended coast to coast in 1951. The first *transatlantic* coaxial telephone cable was laid in 1956. It stretched 3621 km from Clarenville, Newfoundland, to Oban, Scotland.

analysis showed that high levels of demand were essential for both full employment and economic growth.

Keynes was born in Cambridge, England and studied at Cambridge University. He served in the British Treasury (1915–1919) and became a member of the British Peace delegation after WWI (1919). During 1920–1922 he launched a polemical attack on the attitudes and approach of the Victorious Allies, especially in the matter of German reparations[875]. His books on the subject have encouraged appeasement policy pursued by the UK government towards Germany in the 1930s.

Between the world wars Keynes was a financial adviser in London and a professor at Cambridge. He played a large part in the formulation of UK economic strategy during WWII. After the war he helped create a new basis for international monetary and economic cooperation. The outcome was the establishment of the '*World Bank*'.

Today he is best known for the *Keynesian revolution*: the economic theory that recovery from recession is best achieved by a government-sponsored policy of full employment. The integrated economic scheme — based on aggregate demand, short-run relationships among the markets for labor, consumption goods, capital goods and money — is known as *macroeconomics*.

The flip side of Keynes' formula–tightening government spending (including payrolls and social welfare programs) to curb inflationary pressures and stabilize currencies, and *tax cuts* to stimulate the private sector and saving, which in turn can actually increase the tax base – have also been practiced quite effectively, both by international bodies such as the IMF (International Monetary Fund) in developing countries, and – most dramatically – in the UK and the USA starting in the 1980s.

Many Keynsians have blames the 1929 *Stock Market Crash* and subsequent *Great Depression* on too much *laisez faire*. However, the *initial* late–1920s market correction was healthy as long as it affected mainly speculative investment. That it spread to fundamentally solid stocks – and caused widespread hardship and unemployment – may be attributed to both the protectionist tariff polices then in vogue in the U.S, and the pre–Keynsian policies of the US Federal Reserve and the Bank of England. In a series of secret meetings in

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875 Germany's total obligations were fixed at 29 billion dollars. The total amount of reparation paid was about 6 billion dollars. In return Germany had received a far larger amount in foreign loans. The history of reparations has been compared to a merry-go-round: Germany borrowed American funds to pay reparations to the Allies, who used the money to repay their debts to the United States, who lent the money back to Germany.

Long Island, these institutions formulated a policy of *wage and price freeze*, which may have vastly exacerbated the market correction.

Furthermore, the New Deal policies of the Roosevelt administration were mainly *psychological* in their ameliorative impact in the US; it was only the outbreak of WWII – a decade after the crash and seven years after Roosevelt became president – that rallied Wall Street again and began to ramp up employment.

Mathematics and Economy[876]

Economy[877] *is a study of mankind in the ordinary business of life. Economy can also be defined as the study of human wants and their satisfaction or as the science of wealth and welfare.*

Economics describes the nature of behavior of an economic system. The economic system consists of the rules, principles[878] *and customs, which govern the operation of an economy (e.g. capitalistic system, socialistic system etc.). Economics investigates economic problems with the object of offering solutions. Such problems are the questions and situations arising from the operation of the system, e.g.: labor problems, the problem of taxation, etc.*

The social sciences have an uneasy relationship with mathematics: To some extent, they seek a Newtonian goal of quantification and prediction. Yet the human and environmental variables they must deal with are so many and varied, the possibility of meaningful experiment so limited, and the data so questionable, that the greatest achievements of economics so far are chiefly descriptive rather than analytic. In addition, any theory in the social sciences faces a special problem: the widespread fear that if knowledge is power, then

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876 For further reading, see:

- Wilmott, P. et al., *The Mathematics of Financial derivatives*, Cambridge University Press, 1999, 317 pp.

877 Derived from the Greek *oikos* (house) + *nemien* (to manage) and means literally: 'household management'.

878 Examples of *economic laws*:

- *Gresham's law* (1553): 'Bad coin drives out good coin'; if someone has two coins of equal face value and debt-paying power, but one is full-bodied and the other a debased coin, the owner's inclination will be to hoard the full-bodied coin and pass on the debased coin. This was how Chancellor Thomas Gresham explained to Queen Elizabeth I why only bad coins remain in circulation whereas newly issued good coins quickly disappeared.

- *Principle of diminishing returns*: If more of one factor (e.g. labor in this case) is increased, while other factors (e.g. capital) are held constant, eventually the marginal product of that factor (labor) in this case must fall.

knowledge in the social sciences could reinforce the power of those who may already have too much.

The most widely used mathematical tools in economy are statistical, *and the prevalence of statistical methods has given rise to abstract and hugely complicated theories. Statistical theories usually assume that the behavior of large number of people is a smooth, average "summing-up" of behavior over a long period of time.*

It is difficult for them to take into account the sudden, critical points *of important qualitative change. The statistical approach leads to models that emphasize the quantitative conditions needed for equilibrium — a* balance *of wages and prices, say, or of imports and exports. These models are ill suited to describe qualitative change and* discontinuity.

Thus, economists of the 19th century believed that in a system of goods and of demand for those goods, prices will always tend toward a level at which supply equals demand. In short, the negative feedback *from the supply/demand relationship to prices leads to a stable equilibrium. Little, however, is said about how equilibria are actually attained.*

Recently, however, economists have argued that is not at all the way the real economy works. Rather, they claim, what we see more often is positive feedback *in which the price equilibria are* unstable. *Economic examples of this situation*[879] *show how paradoxical, unpredictable and surprising behavior can*

[879] Example:

When video cassette recorders (VCRs) started becoming a household item, the market began with two competing formats — VHS and Beta — selling at about the same price. By increasing its market share, each of these formats could obtain increasing returns since, for example, large numbers of VHS recorders would encourage video stores to stock more prerecorded tapes in VHS format.

This in turn would enhance the value of owning a VHS machine, leading more people to buy machines of that format. So by this mechanism a small gain in market share could greatly amplify the competitive position of VHS recorders, thus helping that format to further increase its share of the market. This is the characterizing feature of positive feedback — small changes are amplified instead of dying out.

The feature of the VCR market that led to the situation described above is that it was initially unstable. Both VHS and Beta systems were introduced at about the same time and began with approximately equal market shares. The fluctuations of those shares in the early stages were due principally to things like "luck" and corporate maneuvering.

In a positive-feedback environment, these seemingly random factors eventually *emerge even in simple systems when the components of the system interact in ways that we don't fully understand. Sometimes the complex behavior is due to* nonlinearities *in which the outcome is disproportional to the input; sometimes the problem lies with inherent,* hidden instabilities *in the system. Unlike the behavior of physical and biological systems, the awareness of the participants in the system of the rules of the game, itself influences the system in a very fundamental way, introducing nonlinearities and instabilities.*

To cope with these difficulties, mathematicians during the second half of the 20th century began to view economic processes from new angles, applying methods of the newly developed game theory*,* catastrophe theory *and* deterministic chaos theory*.*

Consider a system such as the national economy. Suppose we're monitoring some measure of the performance of the economy, say the gross national product (GNP). This observed output of the economic system is determined by many factors — interest rates, employment levels, productive capacity and the like. We can think of the economy as a kind of machine; we feed in the value of each of these input quantities and the machine then produces a level of GNP as its output.

Since the economy is a dynamical process, it's reasonable to consider the level of GNP as being a fixed-point attractor of the economic process. Thus for every set of values of the inputs, the economy moves to a particular level of GNP, which can be envisioned as a point in the state-space of the economy. And since every setting of the inputs produces such a point, there is a whole surface of GNP points that the economy may produce — at least one for every level of interest rates, money supply, production facilities and all the rest. Catastrophe theory *is designed to study the geometrical structure of this surface in an overall effort to quantify the stability of changes in the systems.*

Generally speaking, if we change the inputs just a bit, the corresponding level of GNP will also shift by only slightly. But occasionally we will encounter a combination of input values such that if we change them by only a small amount, the corresponding output will shift discontinuously to an entirely new region of the GNP surface. Such a value of the inputs is called a catastrophe

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tilted the market toward the VHS format until it acquired enough of an advantage to take over essentially the entire market. But it would have been impossible to predict at the outset which of the two systems would ultimately win out. The two systems represented a pair of unstable equilibrium points in competition, so that unpredictable chance factors ended up shifting the balance in favor of VHS. In fact, if the common claim that the Beta format was technically superior holds any water, then the market's choice did not even reflect the best outcome from an economic point of view.

point. In colloquial terms, we might think of the catastrophe points as the straws that break the economy's back.

As it turns out, these catastrophe points arise at just those input levels where there is more than one possible fixed point to which the system can be attracted. And the jump discontinuity is a reflection of the system's "deciding" to move from the region (attraction basin) of one attractor to that of another. Catastrophe theory *shows that there are only a small number of inequivalent ways in which these jumps can take place, and it provides a standard picture for each of the different geometries that the surface of attractors can display.*

Another example is the problem of competition and prices: if production is cheaper in large quantities, the industry tends to be dominated by one company (monopoly) *or by a few* (oligopoly)*. Oligopoly can adopt one of the three strategies. Firstly, it can form a cartel (production and prices determined by negotiation) which as far as the consumers are concerned, acts like a monopoly.*

Second, one or more firms can initiate a round of predatory price-cutting to force weaker companies out of the market.

Third, in some circumstances, firms can merge, reducing the level of competition to the benefit of the remaining members. Even when these strategies are employed, manufacturers are generally unable to raise prices indefinitely since prices are affected by the elasticity of demand *too, and prices depend simultaneously on competition between producers* and *elasticity of demand. The dependence of the price on the elasticity of demand and number of producers is mathematically represented by an* equilibrium surface *displaying a* bifurcation set of the 'cusp' type.

The most interesting feature of this model[880] *is its prediction of two price ranges, one high and one low, in conditions of low to moderate elasticity of demand and moderate competition.* Transitions *between these ranges would be dramatic in cases of very low elasticity of demand (e.g. when alternative*

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[880] An example in which the formation of an oligopoly had a dramatic impact on price was the establishment of OPEC (association of oil producing and exporting countries). These nations had previously sold their oil competitively, but in 1973 they began to set prices in concert as an oligopoly. The price of crude oil went from $2.12 per barrel in January 1973 to $7.61 per barrel a year later, and to $10.50 per barrel by January 1975. Consumption fell by 14 percent in Belgium and the Netherlands, 10 percent in West Germany, and 3.5 percent in the United States. But the elasticity of demand for oil, in the short run at least, was low, and the merchants of OPEC had little difficulty in selling all they chose to produce.

source of energy becomes readily available). A more complex model, one based on the 'butterfly' catastrophe surface, would be needed in order to describe the interaction of monopolies and oligopolies.

A third economical example concerns inflation and unemployment*, which is inevitably bound up with politics: Empirical studies have suggested that there is a trade-off between these levels, and political parties accuse each other of sacrificing those out of work to the goal of lowered inflation, or conversely of sacrificing economic stability to the goal of full employment.*

More recently, both economists and politicians have recognized that the expectation *of future inflation is an important factor. If a high level of inflation is the norm, then workers begin to demand higher wages to offset the increased cost of living they believe will come during the period of contract. These higher wages themselves have an inflationary impact. This phenomenon suggests that a qualitative model for inflation should include the* expected inflation rate *as one control factor and* unemployment *as the other. We meet again the 'cusp' bifurcation set, which predicts a dramatic drop in inflation rate under conditions of a drastic increase of unemployment coupled with high inflation rate.*

Catastrophe theory is also useful in assessing the "fine tuning" of quantities such as money supply (which affects inflation via loan rates) and the government's expenditure for goods and services (which increases employment by raising total demand). The value of models based on this theory lies in its indication that the sequence *in which the control factors are altered can, at any given moment, be at least as important as their quantitative levels.*

The above examples serve to show that there is an enormous gulf between the highly simplified static *economists' models and the complex world to which they are applied.*

Another important problem concerns the interpretation of price – fluctuations: When analyzing prices of commodities, securities, or financial instruments in a variety of markets, a commonly used assumption is that many of the fluctuations observed in the market prices (known as "noise") are the result of purely stochastic (i.e. random) processes. One recognizes, of course, the effects on prices of external influences such as political developments, weather (especially important in commodities markets), and a variety of macroeconomic factors. In addition, there are other well-understood, time-dependent influences such as time to delivery of a future contract. But aside from these effects, the prevailing wisdom can be represented by a stochastic process; once the underlying trends are subtracted out, the remaining price fluctuations often appear to be random.

Furthermore, there was a conviction that small, transient changes had nothing to do with large, long-term changes: small-scale ups and downs during *a day's transactions are just noise, unpredictable and uninteresting; long-term changes are determined by deep macroeconomic forces, such as the trends of war and recession. Yet, that dichotomy was found to break down in many cases*[881]*. Evidence was found that tiny changes and grand ones were bound together across all scales and that within the most disorderly realms of data lurked an unexpected kind of order.*

A simple example serves to show how chaos may enter an economic system: suppose we have a simple market with just one commodity with time-dependent price $p(t)$ *such that*

$$p(t+1) = Ap(t) - Ap^2(t).$$

This is the well-known logistic map, which for $A > A_c \approx 3.57$ *renders* $p(t)$ *chaotic*[882] *except in isolated A-intervals. For* $A > 4$*, the iteration of the map diverges for most initial conditions. This behavior can easily be mistaken for randomness, for even if we apply some sophisticated statistical methods to the iterations of this map, we are not assured of finding any structure! Many chaotic systems pass as random under common statistical tests*[883]*.*

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881 **Bachelier** (1900) claimed that successive price changes are statistically independent and follow, in the first approximation, the one-dimensional Brownian motion.

Mandelbrot (1963) analyzed cotton price data at the New York City Cotton Exchange, 1880–1940, and interpreted this data as having a *fractal structure* with dimension $D = 1.7$, thus bearing the first evidence for *scaling* in economic, i.e. curves of daily changes and monthly price changes corresponded perfectly! Moreover, the degree of variation had remained constant over a tumultuous period of 60 year period that saw two world wars and a depression.

882 The discretization of the first-order *differential equation* $\dot{x} = F(x)$ leads to a first-order *difference equation* $x_{n+1} = F(x_n, c)$. In this form, the dynamics can be equally well viewed as a sequence of mapping $F\colon x_n \to x_{n+1}$. The dynamics of the difference equation is much richer than the first-order ODE because it is free from the continuity restrictions of differential equations; x_n can 'jump around' on the real axis, whereas a non-periodic $x(t)$ can only pass a point once if $\dot{x} = F(x)$. This freedom makes it possible for the difference equation to exhibit several interesting types of bifurcation sequences, leading to various forms of coherent or 'chaotic' behavior, depending on the magnitude of the control parameter c.

883 A simple test, though, will help to differentiate between randomness and deterministic chaos in the case of the logistic map; we construct a two-dimensional graph of $p(t+1)$ against $p(t)$ for a set of integer t-values in a given interval. If the values of $p(t)$ were really random, then our 2-dimensional plot would look like a scatter of points. On the other hand, a plot constructed from a lo-

Thus, depending on the value of A, *the nonlinear regularity mechanism (i.e., the term* $-Ap^2(t)$ *in the equation) could create all kinds of interesting price movements as a function of time, even random-looking ones.*

It may be that in a nonlinear market, the price movement may not be solely due to the new information affecting the market but may result partly from the nonlinear dynamics of the market itself, and these may indeed be vastly more complicated than that of the simple logistic map.

To complicate matters still more, the environment in which a market exists is not static. Changes take place in societies and economies on all time scales, from seconds to millennia. A financial market, coupled to other markets and to the society at large, will, in some enormously complex way, reflect in its prices all these changes over all time scales. These markets have exquisitely intricate self-regulatory mechanisms, reflecting the effects of human psychology, social behavior and, to some extent, rational thought.

A final example involves an economic system which defies any mathematical formulation:

Consider the curve $y = ax^{\alpha}(100 - x)^{\beta}$ *which purports to relate the percentage tax-rate* x *to the government's revenue* y *(known as the Laffer Curve). If there is too much taxation, people will not work as hard for a salary and look for other non-cash benefits, so the revenue will drop. However, data for the U.S. economy over the past* 50 *years reveals a 'neo-Laffer curve' which resembles the former only near the trivial points* $x = 0,\ 100$, *but looks totally chaotic in between these points.*

These results show that there is no Tax Rate – Government Revenue map, because there is no causal connection; human nature is presumably too complicated for such a connection to be deterministic. What is indeed fortunate, and often amazing, is that some complicated systems do appear to have dynamics which can be approximated by simple first order models, and we often do not have any profound physical understanding as to why this occurs.

1936–1949 CE **Eugène Louis Felix Néel** (1904–2000, France). Physicist. Predicted a fourth type of magnetism — *antiferromagnetism* (in addition

gistic map will render a *parabola*! A fair number of financial and economic data series have been analyzed using these methods, and there is significant evidence for the existence of underlying nonlinear processes in economics and finance.

to dia- , para- , and ferromagnetisms). He argued for a crystal model in which two lattices having their magnetic fields acting in opposite directions are interlaced. Their opposite magnetic fields would cancel, leaving the crystal with little observable magnetic field. His predictions were verified by experiment (1938), and further confirmed by neutron diffraction techniques (1949).

Later, Néel successfully predicted magnetization configurations in thin films and near surfaces. He also explained the strong magnetism found in ferrite materials such as magnetite (1948), demonstrating that if the magnetic field of one of the two lattices (mentioned above) were stronger than the other, there would be an observable magnetic field (*ferrimagnetism*). His work on ferromagnetic materials saw great application in the *coating of magnetic tape*, as well as in the processing of permanent magnets for motors and of *magnetic storage media* used by computers. He was awarded the Nobel Prize for Physics jointly with **Alfven**.

Néel was born in Lyons and graduated from the Ecole Normale Supérieure. He was later Professor of Physics at Strasbourg University (1937–1940) and Grenoble (1940).

1936–1949 CE **Matvei Petrovich Bronstein** (1906–1938, Soviet Union), **Jacques Solomon** (1908–1942, France) and **Carl Bryce Seligman DeWitt** (1923–2004, Belgium) — theoretical physicists and pioneers of *quantum gravity*, made the first approaches to quantize general relativity in an overall effort to unify gravity with quantum theory.

Bronstein entered Leningrad University (1926), graduated (1929) and joined the Leningrad Physico-Technical Institute (1930), where he worked with **Ioffe**, **Frenkel**, **Fock**, **Tamm** and **Landau**. In 1935 he presented his Doctoral Thesis on "Quantizing Gravitational Waves", to Vladimir Fock and Igor Tamm. In 1936, the work was published in Russian[884], as well as in a condensed German version[885].

On Aug 6, 1937, Bronstein was arrested in Kiev on the charge of "active involvement in a Leningrad counterrevolutionary organization" and was ex-

[884] Bronstein, M.P., "*Kvantovanie gravitatsionnykh voln [Quantization of gravitational waves]*", Zhurnal Eksperimentalnoy i Teoreticheskoy Fiziki G, pp. 195–236, 1936.

[885] Bronstein, M.P, "*Quantentheorie schwacher Gravitationsfelder*", Physikalishe Zeitschrift der Sowjetunion, 9, pp. 140–157, 1936.

ecuted by a Soviet NKVD firing squad on Feb 18, 1938[886,887]. His grave is unknown and references to his name disappeared from the annals of Soviet physics for several decades. Only 20 years later did his widow, the writer Lydia Chukovskaya, learned the exact date of his death.

Bronstein was first to claim that there is an essential difference between quantum electrodynamics and the quantum theory of the gravitational field. He indeed showed that general relativity and quantum theory are *fundamentally* difficult to unify. In that he disagreed with **Pauli** and **Heisenberg** (1929) who believed that the gravitational field could eventually be easily quantized along the lines of QED.

Indeed, Bronstein realized the intrinsic difference: in QED, an infinite charge density in the test body is, in principle, conceivable, whereas in the case of gravitation, the test body's gravitational radius should not exceed its real linear dimensions. Bronstein then derived [long before **Wheeler** (1955)] the intrinsic limitations of the quantization of the gravitational field, related to *Planck's scales*[888].

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[886] Gorelik, G.E. and V. Frenkel, "*M.P. Bronstein and Soviet Theoretical Physics in the Thirties*", Birkhauser Verlag Basel/Boston, 1994.

[887] Gorelik, G.E., *Matvei Bronstein and quantum gravity: 70th anniversary of the unsolved problem*, Physics – Uspekhi, 48(10), pp. 1039–1053, 2005.

[888] In his doctoral thesis (1935), Bronstein carried out the quantization of the linearized Einstein equations by the Fermi method, developing the quantum analogue of Einstein's quadrupole radiation formula, and deducing the Newtonian law of attraction from the interchange of longitudinal gravitational quanta. He then proceeded to some critical reflections on the physical significance of his results.

He carries out an analysis of the measurability of the (linearized) Christoffel symbols, which he takes to be the components of the gravitational field. By analogy with the then-recent Bohr-Rosenfeld analysis of the measurability of the electromagnetic field components, he shows that there are limitations on the measurability of the gravitational field components implied by the uncertainty relations between position and momentum of a test body, the acceleration of which is used to measure the gravitational field. *But he notes that there is an additional gravitational complication, which has no electromagnetic analogue:* To measure the components of the electromagnetic field, it is permissible to introduce electrically neutral test bodies, which have no effect on the field being measured. But in the gravitational case, due to the universality of gravitational interactions, the effect of the energy-momentum of the test bodies on the gravitational field cannot be neglected — *even in the linear approximation.* Bronstein derives an expression for the minimum uncertainty in a measurement of a component of the Christoffel symbols that depends inversely on the mass

At least one physicist outside the Soviet Union acknowledged, and indeed extended, Bronstein's views. In 1938 the French physicist **Jacques Solomon** after summarizing Bronstein's argument concluded:[889]

"In the case *when the gravitational field is not weak, the very method of quantization based on the superposition principle fails, so that it is no longer possible to apply a relations such as [the equation setting a lower limit on the measurability of the linearized field strength] in an unambiguous way... Such considerations are of a sort to put seriously in doubt the possibility of reconciling the present formalism of field quantization with the non-linear theory of gravitation.*"

In one of the many tragic ironies of history, both of these pre-war advocates of the need for a radically different approach to quantum gravity perished prematurely. Jacques Solomont was a Communist militant active in

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density of the test body, just as Bohr-Rosenfeld's corresponding result does on the charge density r of the test body. He then states what he sees as the crucial difference between the two cases:

"*Here we should take into account a circumstance that reveals the fundamental distinction between quantum electrodynamics and the quantum theory of the gravitational field. Formal quantum electrodynamics that ignores the structure of the elementary charge does not, in principle, limit the density of ρ. When it is large enough we can measure the electric field's components with arbitrary precision. In nature, there are probably limits to the density of electric charge... but formal quantum electrodynamics does not take these into account... The quantum theory of gravitation represents a quite different case: it has to take into account the fact that the gravitational radius of the test body ($k\rho V$) must be less than its linear dimensions $k\rho V < V^{1/3}$.*

...The elimination of the logical inconsistencies connected with this result requires a radical reconstruction of the theory, and in particular, the rejection of a Riemannian geometry dealing, as we have seen here, with values unobservable in principle, and perhaps also rejection of our ordinary concepts of space and time, modifying them by some much deeper and nonevident concepts."

In summary, he raised the problem of the measurability of the quantized gravitational field, arguing that, in addition to limits imposed by the uncertainty principle, in general relativity there is an 'absolute limit' to the accuracy with which the components of the linearized affine connection within a given volume can be measured. He suggested that the application of the formalism of quantum field theory might not yield the desired fusion of quantum theory and gravitation, calling for 'a radical reconstruction of the theory'.

889 Solomon, Jacques, "*Gravitation et Quanta*", Journal de Physique et de Radium, 9, pp. 479–485, 1938.

the underground resistance to the German occupation of France. He was arrested together with his wife, Helene Langevin, in March 1942 and was killed by the Germans on May 23, 1942; she was sent to Auschwitz, but survived the war.

Between them, Stalin and Hitler saw to it that the post-World-War-II discussion of quantum gravity took place without what could have been two significant voices.

Carl Bryce Seligman (1923–2004, USA) (changed his name to **DeWitt** in the 1950's) was a theoretical physicist best known for formulating canonical quantum gravity. It was one of the first approaches to quantizing general relativity[890]; for formulating the Wheeler-deWitt equation for the wavefunction of the universe. He is also known for advancing the formulation of the Hugh Everett's many-worlds interpretation of quantum mechanics.

Seligman was born to Jewish parents in Dinuba, CA. He received his bachelor's, master's and doctoral degrees from Harvard University. Afterwards he worked at the Institute for Advanced Study, the University of North Carolina at Chapel Hill and the University of Texas at Austin.

1936–1956 CE **Alonzo Church** (1903–1995, USA). Mathematical logician. Made important contributions in mathematical logic and theoretical computer science. Published the first precise definition of a calculable function, thus advancing the systematic development of the theory of algorithms. The mathematical analysis of computation is generally credited independently to Alan Turing and Alonzo Church. The *Church Theorem* (1936) extends the work of Gödel (1930) by showing that there is no decision procedure for arithmetic, i.e., there is *no algorithm* for a class of quite elementary arithmetical questions. In this sense, Church's theorem connects the *Turing halting problem* in computer science to Gödel's incompleteness theorem. The *Church-Turing Thesis* (not yet proved false or true) maintains that all computers are equally powerful problem solves (notwithstanding the fact that some computers will be able to solve problems *faster* than others). Thus, the Church-Turing thesis says that, given sufficient resources (time and memory), there is nothing that one computer can do, that any other can not (eventually).

Church was educated in Princeton and remained there for 40 years (Ph.D. 1927). He spent a year at Harvard (1925) and another year at Göttingen,

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[890] Seligman [DeWitt], C.B., "*The Theory of Gravitational Interactions and the Interaction of Gravity with Light*", Thesis, Harvard University, December 1949.

Germany (1926). He created the λ-*calculus*[891] (1930), which today is an invaluable tool for computer scientists, and then showed that any other computational scheme could be described in terms of it.

1936–1956 CE **Samuel Eilenberg** (1913–1998, Poland and USA). Mathematician. A leading topologist in the 20th century. Introduced the concepts of *functor* (1942), *category* (1945), *homological algebra* (1956) and contributed to the theories of *homotopy groups* and *fiber bundles* (1952).

Eilenberg was born in Warsaw to Jewish parents. He was educated at the University of Warsaw (Ph.D. 1936). He left Poland for the United Stated (1939) and held positions at the universities of Princeton, Michigan, Indiana and Columbia (1947–1998).

The New Geometry — Algebraic and Differential Topology[892] *(1900–1970)*

(A) Overview

Topology is an area of pure mathematics that deals with those properties of objects which are not affected by continuous deformation. This discipline is

[891] A logical language used to formalize the very general concept of a function. It is a formal mathematical system to investigate functions, function application and recursions. It has influenced many programming languages and has a good claim to be the prototype programming language ('denotational semantics').

[892] To dig deeper, see:

- Flegg, H.G., *From Geometry to Topology*, Dover, 2001, 186 pp.
- Stewart, I., *Concepts of Modern Mathematics*, Dover, 1995, 339 pp.
- Henle, M., *A Combinatorial Introduction to Topology*, Dover, 1994, 310 pp.
- Arnold, B.H., *Intuitive Concepts in Elementary Topology*, Prentice-Hall, 1963, 182 pp.
- Alexandroff, P., *Elementary Concepts of Topology*, Dover, 1961, 57 pp.

also called "rubber sheet geometry" since the properties that are of interest to a topologist are those that would be invariant on a stretchable rubber sheet.

Thus, geometric properties such as length *and* local curvature *are not of interest to a topologist, but the number of* holes *in an object* is *a topological property.*

In ordinary Euclidean geometry*, one may move objects around, rotate and flip them over, but one is not allowed to stretch or bend them. This leads to the concept of '*congruence*' in that geometry. Two objects in plane geometry are congruent if one can be laid on top of the other such that they match exactly.*

In projective geometry *(invented during the Renaissance to understand perspective drawing), two objects are considered the same if they are both views of the same object. For example, if one looks at a plate on a table from directly above the table, the plate looks round like a circle. But if one walks away a few feet and looks at it, it looks elongated, like an ellipse*[893]*. The ellipse and circle are projectively equivalent.*

In topology, any continuous change which can be continuously undone is allowed. So a circle is the same as a triangle or a square, because you just 'pull on' parts of the circle to make corners and then straighten the sides, to change a circle into a polygon. Then you just 'smooth it out' to turn it back into a circle. These two processes are continuous in the sense that during each of them, nearby points at the start are still nearby at the end.

The circle is not *topologically the same as a figure 8, however, because although one can squash the middle of a circle together to make it into a figure 8 continuously, when one tries to undo it, one has to* break *the connection in the middle and this is discontinuous: a set of points that are all infinitesimally near the center of the 8 end up split into two branches, on opposite sides of the circle, a finite distance apart.*

Another example: a plate and a bowl are the same topologically, because one may just flatten the bowl into a plate. At least this is true if one uses clay which is still soft and has not been fired yet. Once they are fired they become Euclidean rather than topological, because one cannot flatten the bowl any longer without breaking it.

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893 This is one reason it is hard to learn to draw. The eye and the brain work projectively. They look at the elliptical plate on the table, and think it's a circle because they know what happens when one looks at things at an angle. To learn to draw, one has to learn to draw an ellipse even though the brain is saying 'circle', so one can draw what one really sees, instead of 'what one knows it is'.

Thus, what distinguishes different kinds of geometry from each other are the types of transformations *that are allowed before one considers that something has changed.*

In two dimensions, topologists imagine that figures can be stretched and pulled as though they were drawn on an infinitely thin, infinitely stretchable material that can be deformed in any way (not including tearing, perforating, or gluing). Some properties *that are important in Euclidean geometry such as distance, measurement of angle, or straightness, have no topologically invariant meaning. Other properties, such as whether lines intersect or whether figures are* closed *(like the letter "O") or* open *(like the letter "U") remain important.*

In order to deal with these problems that don't rely on the exact shapes of objects, one must be clear about just which properties these problems do rely on. From this need arises the notion of topological equivalence. *In two dimensions, a triangle, a square, and a circle are all topologically equivalent.*

In three dimensions, the surfaces of a cube, a pyramid, and a sphere are topologically equivalent. A stretchable "skin" that covers any one of them can be readjusted to cover any of the others. The surfaces of a doughnut and a coffee mug are topologically equivalent — each is a three-dimensionally embedded, closed oriented (2-sided) surface with a single hole in it (the doughnut hole or mug handle).

Indeed, a topologist is sometimes said to be a person who does not know the difference between a coffee mug and a doughnut, since if these are made out of plastic clay, either of them can be continuously deformed into the other without tearing apart contiguous regions or pinching together disjoint regions: the hole in the handle of the mug corresponds to the hole in the doughnut, while the cupped part can be flattened and rounded.

Similarly, the set of all possible positions of the hour hand of a clock is topologically equivalent to a circle (i.e., a one-dimensional closed curve with no intersections – which can be embedded in two-dimensional space); the set of all possible positions of the hour and minute hands taken together is topologically equivalent to the surface of a torus *(i.e., a two-dimensional surface that can be embedded in three-dimensional space by sweeping a circle such that its center traces along a closed curve), and the set of all possible positions of the hour, minute, and second hands taken together is topologically equivalent to a closed three-dimensional manifold (the* three-torus*).*

Topologists do not limit themselves to the two- and three-dimensional worlds with which we are familiar via direct sense-perception. Many of the concepts and theorems in topology deal with multi-dimensional manifolds *which may exist only in the imagination, or as formal mathematical constructs.*

Topologists, then, are interested in a variety of surfaces and, more generally, "hyper-surfaces" (and their intrinsically-defined versions, manifolds). They try to understand exactly what it is that distinguishes one hypersurface from another, and to understand the relationships that a hypersurface can have with the higher-dimensional ("host-" or "target-") space that it is embedded in. The mathematical study of knots *is a branch of topology.*

Topology is relevant to the study of spatial objects such as curves, surfaces, maps, electrical or neural circuits, organizational or flow charts, configurations of electromagnetic fields, fluid flow patterns, the space we call our universe, the spacetime of general relativity, fractals, knots, manifolds (smooth intrinsic spaces that can be continuously patched from regions of a Euclidean space), configuration and phase spaces that are encountered in physics (such as the space of hand-positions of a clock or the space of possible positions and velocities of a system of stars and planets), symmetry groups such as the collection of ways of rotating a rigid body, *etc.*

Topology is divided into several (overlapping) parts, including graph theory *(or, more generally,* combinatorial topology*); general* point-set topology *(including* real analysis*);* measure theory*; and the* topology of manifolds*. The latter is further subdivided into* algebraic topology *and* differential topology*, and* low-dimensional topology*. In algebraic topology, tools from* abstract algebra *are used to study the global property of manifolds.*

One of the strengths of algebraic topology has always been its wide applicability to other fields. Nowadays that includes physics, differential geometry, algebraic geometry, and number theory.

As an example of this applicability, we present a simple topological proof that every non-constant polynomial $p(z)$ *with complex coefficients has at least one complex zero. Consider a circle of radius* R *and center at the origin of the complex plane. The polynomial function transforms this into a closed curve in the complex plane. If this image curve ever passes through the origin, we have our zero. Otherwise, suppose the radius* R *is very large. Then the highest power of* $p(z)$ *dominates and hence* $p(z)$ *transforms the circle into a curve (the* image curve*) which winds around the origin the same number of times as the degree of* $p(z)$.

This is called the winding number *of the image curve around the origin. It is always an integer and it is defined for every closed curve which does not pass through the origin. Winding numbers of closed curves form an* additive group*; if we concatenate two closed curves, their winding numbers add to yield that of the resultant curve. This is an example of how abstract algebra gave Algebraic Topology its name.*

If we deform the image curve subject to this last constraint, the winding number has to vary continuously but, since it is constrained to be an integer, it cannot change and must be a constant – unless the curve is deformed through the origin. The above argument thus shows that if $p(z)$ has no root (zero), the winding number of the image curve equals the degree of $p(z)$ for any *finite value of R. Now deform the image curve, by shrinking the radius R to zero and suppose again that the image never passes through the origin, that is to say we never hit a zero of the polynomial.*

The image curve becomes very small and if, as $R \to 0$, it tends to a (non-origin) point, it must have winding number 0 around the origin, in that limit. If the image curve does shrink to the origin, the origin is a zero of $p(z)$. If not, the winding number is 0 for sufficiently small R. Combining this fact with the large-R limit treated above, we deduce that the polynomial must have degree 0, in other words it is a constant. We have thus used algebraic topology *to prove, by contradiction, that a polynomial of nonzero degree must have at least one zero (root) in the complex plane.*

The winding number of a curve illustrates two important principles of algebraic topology. First, it assigns to a geometric object, the closed curve, a discrete-valued invariant, the winding number, which in this case is an integer. Second, when we deform the geometric object subject to certain constraints, the winding number does not change; hence it is called an invariant of deformation or, synonymously, an invariant of homotopy.

Modern algebraic topology *is the study of the global topological properties of spaces (especially manifolds) by means of abstract algebra. Poincaré was the first to link the study of spaces to the study of algebra by means of his fundamental group. This is a generalization of the concept of winding number which applies to any manifold.*

To get an idea of what algebraic topology is not *about, consider the fact that we live on the surface of a sphere (namely the earth), but locally this is difficult to distinguish from living on a flat plane. One way of telling that we live on a sphere is to measure the sum of the three angles of a triangle, the vertices of which are 3 points (loci) on the earth's surface, and the* sides *of which are segments of great circles (shortest-length curves) between the 3 possible pairs among these vertices. For a small triangle on earth's surface, this sum is slightly more than 180 degrees. For a large triangle, it is much more. This tells us that we live on a surface with what is called positive curvature. But, since we can use arbitrarily small triangles to measure curvature, it is a local property, not a global one. It properly belongs to the field known as* differential geometry.

Algebraic topology is concerned with the whole surface, and starts with the obvious fact that the surface of a sphere is a finite area with no boundary, *while the flat plane does not have this property. It expresses this fact by assigning various deformation-invariant groups to these and other spaces.*

One class of such groups consists of homotopy groups; another kind comprises homology groups[894]*. These groups are all discrete, and they are invariant in the sense that they do not change if the space is continuously deformed.*

One of the homotopy groups assigned to a sphere is an infinite Abelian group which corresponds to the topologically distinct ways in which a sphere can be wound about another sphere. The corresponding group for a plane, for example, is the trivial group (consisting of the identity element alone). Other homotopy groups have elements representing the various (deformation invariant) distinct ways in which an n-dimensional hypersphere (the n-sphere – denoted S^n) can wrap around the manifold in question.

The fact that these discrete groups may be different for different topological spaces, tells us that the spaces are globally different. Algebraic topology includes, but is not confined to, the study of spaces of dimensions two or three. It includes, for example, the contemplation of the shape of the three dimensional universe itself, or of the shape of the four dimensional spacetime of General Relativity, or whether two closed curves (or other embedded manifolds) can "link" each other in n dimensions; or the shape of the group manifold of all rotations in $\mathbb{R}^n$; the global study of phase-space trajectories of nonlinear dynamical systems; etc.

The concept of continuous deformation can be illustrated by the following examples. Consider again a coffee cup (with a handle) and a doughnut. If they are both made of some pliable substance like modeling clay, they can be deformed continuously (without ripping) one into the other. This is reflected in the fact that they have the same homotopy and homology groups, that is, their homotopy groups and homology groups are topological invariants. On

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[894] For a manifold M of dimension n, there are $n + 1$ additive *homology groups*, denoted $H_m(M, \mathbb{Z})(m = 0, ..., n)$, with $\mathbb{Z}$ the ring of integers (signed or zero); $\mathbb{Z}$ is sometimes replaced with a different ring or field (e.g. the rationals or reals). H_m consists of a set of m-dimensional *closed* and *oriented* submanifolds of M, for which group addition is defined in the sense of set union. The additive inverse of a submanifold is the same submanifold, but with its orientation reversed; the zero (unit) element of the additive group H_m is represented by any m-dimensional submanifold continuously deformable to infinitesimal neighborhoods of a point in M. Two-m-submanifolds are considered *equivalent* if their difference is the boundary of some $(m+1)$-dimensional submanifold; thus the elements of each homology group H_m are actually *equivalence classes* of m-submanifolds.

the other hand, a doughnut cannot be continuously deformed into a sphere. This means that their homotopy and/or homology groups are different.

(B) History[895]

Topology as a subject began to take shape between 1850 and 1900 in the works of these mathematicians: ***G.F.B. Riemann*** *(1826–1866),* ***J.B. Listing*** *(1808–1882),* ***A.F. Möbius*** *(1790–1868),* ***E. Betti*** *(1823–1892),* ***C. Jordan*** *(1838–1922),* ***Gustav Roch*** *(1839–1866),* ***F. Klein*** *(1849–1925) and* ***H. Poincare*** *(1854–1912).*

Although concepts that we now consider part of topology, were expressed and used by these mathematicians, algebraic topology as a part of rigorous mathematics (i.e., with precise definitions and correct proofs) only began in 1900 with the works of: ***M. Fréchet*** *(1878–1973),* ***M. Dehn*** *(1878–1952),* ***H. Lebesgue*** *(1875–1941),* ***E. Cartan*** *(1869–1951) and* ***F. Hausdorff*** *(1868–1942).*

At first, algebraic topology grew very slowly and did not attract many mathematicians; until 1920 its applications to other parts of mathematics were very scant and often shaky. This situation gradually changed with the introduction of more powerful algebraic tools, and the vision of Poincare (1895) of the fundamental role topology should play in all mathematical theories began to materialize. The main characters in this saga were: ***O. Veblen*** *(1880–1960),* ***L.E.J. Brouwer*** *(1881–1966),* ***S. Lefschetz*** *(1884–1972),* ***H. Weyl*** *(1885–1955),* ***J.W. Alexander*** *(1888–1971),* ***L. Vietoris*** *(1891–2002),* ***M. Morse*** *(1892–1977),* ***K. Reidemeister*** *(1893–1971),* ***E. Cech*** *(1893–1960),* ***H. Hopf*** *(1894–1971),* ***P.S. Alexandrov*** *(1896–1982),* ***K. Kuratowski*** *(1896–1980),* ***P. Uryson*** *(1898–1924),* ***J. Schauder*** *(1899–1943),* ***O. Zariski*** *(1899–1986),* ***K. Menger*** *(1902–1985),* ***W. Hodge*** *(1903–1975),*

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[895] Some enlightening statistics:

(i) A remarkable longevity among topologists: of 36 men born between 1878–1911, *nine* (25 percent) reached age above 92 and one reached the ages of 110!

(ii) Of the 56 'top topologists' listed here, at least 12 are European Jews. Hausdorff and Schauder were murdered by the Nazis (1942–3) and Hurewicz fell from atop a Mexican pyramid (1956). Thus, the advice "be a good topologist and live longer" must be heeded with caution.

***B.L. van der Waerden** (1903–1996), **G. de Rham** (1903–1990), **H. Cartan** (b. 1904), **J.W.C. Whitehead** (1904–1960), **W. Hurewicz** (1904–1956), **H. Freudenthal** (1905–1990), **C. Ehresmann** (1905–1979), **K. Borsuk** (1905–1982), **J. Leray** (1906–1998), **J.A.D.E. Dieudonné** (1906–1992), **A. Weil** (1906–1998), **K. Seifert** (1907–1996), **H. Whitney** (1907–1989), **L. Pontryagin** (1908–1988), **C. Chevalley** (1909–1984) and **N. Steenrod** (1904–1960).*

*Since 1945, the growth of algebraic and differential topology and its applications has been exponential and shows no sign of slackening. Some of the leading exponents were (are): **S. MacLane** (b. 1909), **S.S. Chern** (b. 1911), **S. Eilenberg** (1913–1998), **K. Kodaira** (1915–1997), **E.S. Spanier** (1921–1996), **A. Borel** (b. 1923), **R. Thom** (1923–2002), **J.P. Serre** (b. 1926), **A. Grothendiek** (b. 1928), **S. Smale** (b. 1930) and **J.W. Milnor** (b. 1931).*

The growth of general point-set topology, which we review next, has been largely driven by the ever-deepening studies of the real-number system starting in the 19th century. This field is an indispensable foundation of measure theory — which in turn is needed in defining and generalizing the concept of integration, as well as in the various branches of Functional Analysis. Measure theory also underlies the theory of probability and stochastic processes. Point-set topology underlies the topology of manifolds, as well as: the theory of function spaces, transforms, differential and integral equations and linear operators — as well as the rest of Functional Analysis.

(c) Basic Concepts

Topological spaces are structures which enable the formalization of intuitive concepts such as *convergence*, *connectedness* and *continuity*.

Formally, a topological space is a set X together with a set T of so-called *open subsets* of X satisfying these axioms:

1. The *union* of any collection of open sets an element of T is also an element of T.

2. The *intersection* of any finite set of elements (sets) in T is also in T.

3. X itself and the *empty set* are in T.

The set T is also called a topology on X. The sets in T are referred to as open sets, and their complements in X are called closed sets. Roughly speaking, open sets are thought of as neighborhoods of points; two points are considered "close" in a pre-metric sense if there are "many" open sets that contain both of them.

A function (also called a map) between topological spaces is said to be continuous if the inverse image of every open set is open. This definition is an attempt to capture & generalize the intuition that points which are "close together" get mapped to points which are likewise "close together".

Examples of topological spaces:

- *The set (also algebraic field) of real numbers $\mathbb{R}$: its open sets are unions of (possibly infinitely many) open intervals. This is in many ways the most basic topological space, and the one that guides most of our human intuition.*
- *More generally, every interval in $\mathbb{R}$ is a topological (sub) space, and so are the product Euclidean spaces $\mathbb{R}^n$.*
- *The set (field) of complex numbers $\mathbb{C}$: the open sets are (finite or infinite) unions of open discs.*
- *Any metric space can be turned into a topological space if we define a set to be open if and only if it is a (possibly infinite) union of open balls. This applies to $\mathbb{R}^n$, but also to such useful infinite dimensional spaces as Banach spaces and Hilbert spaces studied in functional analysis. Such infinite-dimensional spaces are very important in both classical and quantum physics.*
- *Manifolds (whether intrinsic or embedded); in particular, surfaces.*
- *A simplex – a type of convex object that is very useful in computational geometry. In 0, 1, 2 and 3 dimensional spaces the simplices are, respectfully: the point, line segment, triangle and tetrahedron.*
- *Simplicial complexes. A simplicial complex is made up of simplices. Many geometric objects can be modeled by simplicial complexes, and such discretizations are a cornerstone in the computerized numerical simulation of mechanical, quantum-mechanical, electromagnetic, thermal, and chemical behavior of physical systems on all scales of space and time, from sub-nuclear to cosmological.*

In topology, two geometrical objects (or "spaces") are called homeomorphic if, roughly speaking, each may be reversibly deformed into the other by a sequence of stretching and bending operations; cutting is also sometimes allowed, but only if the two parts are later glued back together along exactly the same cut. For example, a square and a circle are homeomorphic. A partially hollowed solid ball containing a smaller solid ball is homeomorphic to a hollowed cube with a solid cube (or ball) embedded inside of it.

If two objects are homeomorphic, there exists a continuous, one-to-one function (also called a map, mapping or transformation) which maps points from the first object to corresponding points of the second object, such that every point in the 2nd ("target") object is reached (an onto map) and the inverse map is continuous as well. Such a function is called a homeomorphism; intuitively, it maps points in the first object that are "close together" to points in the second object that are close together, and points in the first object that are not close together to points in the second object that are not close together.

Topology is the study of those properties of objects that do not change when homeomorphisms are applied.

For a formal definition, suppose X *and* Y *are topological spaces, and* f *is a function from* X *to* Y. *Then* f *is a homeomorphism iff all the following hold:*

1. f *is a bijection (i.e. one-to-one and onto),*
2. f *is continuous,*
3. *the inverse function* f^{-1} *also is continuous.*

If there exists a homeomorphism $f : X \to Y$, *then* Y *is said to be homeomorphic to* X *(or to be a homeomorph of* X*). In this case,* Y *is also homeomorphic to* X, *since* f^{-1} *is a homeomorphism as well — and we say that* X *and* Y *belong to the same homeomorphism class.*

Topological spaces can be broadly classified according to their degree of connectedness, their degree of compactness and the degree of separability of their points.

A space is metrizable if it is homeomorphic to a metric space. A space is locally metrizable if every point has a metrizable neighborhood.

Metric spaces were defined and investigated by **Fréchet** *in 1906, and Hausdorff spaces by* **Felix Hausdorff** *in 1914, and the current concept of topological space was described by* **Kuratowski** *in 1922.*

An n-dimensional manifold is a topological space that is locally homeomorphic to the "ordinary" space $\mathbb{R}^n$. An example is the surface of the ordinary two-dimensional sphere, which modern mathematicians often refer to as the 2-sphere (denoted S^2). It is topologically distinct from a plane, although simply–connected open patches of it are homeomorphic to $\mathbb{R}^2$. To make precise the notion of "locally homeomorphic" one uses overlapping local coordinate systems or charts, as will be described in detail below. Every manifold has a dimension — the number of coordinates needed in every local coordinate system.

The (partially) overlapping local charts on a manifold are assumed to be compatible in certain senses; in consequence one can talk about directions, tangent spaces, curves, submanifolds, differentiable functions and tensors on that manifold — as well as other optional attributes such as orientation, metric, metric signature, complex- and spin-structures, connections, etc. Manifolds on which directions and differentiable tensors exist are called differentiable. In order to measure lengths and angles, to parallel-transport a tensor or to define intrinsic curvature, a metric tensor is needed and one defines Riemannian manifolds.

Differentiable manifolds are used in mathematics to describe geometrical objects and geometrical (or other dynamical) degrees of freedom; they are also the most natural and general setting in which to study differentiability. In physics, examples of differentiable manifolds are the configuration and phase spaces in classical mechanics, four-dimensional pseudo-Riemannian manifolds used to describe spacetimes in the General Theory of Relativity (GTR), and fiber bundles with spacetime manifolds as their bases. Fiber bundles — of which more below — are differentiable manifolds, and they are also the natural topological and geometric frameworks for describing the space of possible field configurations in space and time — e.g. in GTR; QED (Quantum Electrodynamics); adiabatically evolving quantum-mechanical systems; and in quantum field dynamics of Non-Abelian Gauge Theories (important in sub-nuclear physics and Big-Bang cosmology).

Manifolds inherit many of the local properties of Euclidean space. In particular, they are locally path-connected, locally compact[896], and locally metrizable. The idea of the n-dimensional manifold was introduced by ***Riemann*** *in*

896 A manifold is locally compact *iff* every *open cover* of an open neighborhood homeomorphic to $\mathbb{R}^n$ can be replaced with a *finite* open cover. An "open cover" of a neighborhood N is a collection of open sets U_α, $\alpha \in I$, whose union covers N; i.e. N is a subset of this union. The *finite open cover* in question must be a finite subset of $\{U_\alpha\}$, i.e. a set $\{U_{\alpha_1}, \dots, U_{\alpha_k}\}$ with $k \geq 1$ a finite integer and $\alpha_j \in I$ for $j = 1, 2, \dots, k$.

1854. He also introduced the topological concept of connectivity *(1851, 1857) in the context of Riemann surfaces.*

In order to discuss differentiability of functions, one needs more structure than a topological manifold provides. We start with a topological manifold M *without boundary. An* open set *of* M *together with a homeomorphism between that open set and an open ball in* $\mathbb{R}^n$ *is called a coordinate* chart[897]*. A collection of charts which cover* M *is called an* atlas *of* M*. Suitably composing the defining homeomorphisms of two overlapping charts in the intersection of their open sets provides a* transition map *from a subset of* $\mathbb{R}^n$ *to some other subset of* $\mathbb{R}^n$*. If all these maps are* k *times continuously differentiable, then the atlas is termed a* C^k *atlas.*

Example: The unit two-dimensional sphere S^2 *(commonly embedded in* $\mathbb{R}^3$*) can be covered by two charts: the complements of the north and south poles*[898] *with their associated coordinate maps —* stereographic projections *centered at the south and north pole, respectively.*

Two C^k *atlases are called* equivalent *if their union is also a* C^k *atlas. This is an* equivalence relation*, and a* C^k manifold *is defined to be a manifold together with an equivalence class of* C^k *atlases. If all the transition maps are infinitely differentiable, then one speaks of a* smooth *or* C^∞ *manifold; if they are all* analytic*, then the manifold is an* analytic manifold*.*

A smooth atlas provides local coordinate systems such that the change-of-coordinate functions (within chart overlaps) are smooth. These coordinate systems allow one to define differentiability and integrability of functions on M *(e.g. maps from* M *into* $\mathbb{R}$*, or* $\mathbb{C}$*, or any* $\mathbb{R}^n$[899]*).*

On differentiable manifolds per se*, there are no notions of length, volume or angle. In order to introduce these, one needs a way to measure the lengths*

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[897] In most definitions of charts, the homeomorphism is between an open set of M and all of $\mathbb{R}^n$. This is equivalent: many open subsets of $\mathbb{R}^n$, including all open balls, are themselves homeomorphic to the full $\mathbb{R}^n$. Thus for instance, the open interval $(0,1)$ in $\mathbb{R}^1 = \mathbb{R}$ can be mapped onto all of $\mathbb{R}$ via

$$x \to f(x) = \ln x + \frac{1}{1-x} \quad ,$$

and f is one-to-one, onto, continuous, and its inverse is also continuous.

[898] By the complement of a pole we mean the (open) set of all points of S^2 except for that pole.

[899] Manifolds "locally looks like" *Euclidean space* R^n and are therefore inherently finite-dimensional objects. To allow for infinite dimensions, one may consider *Banach manifolds* which locally look like *Banach spaces*; or *Fréchet manifolds*, which locally look like *Fréchet spaces*.

and angles of and between tangent vectors of curves on M. A Riemannian manifold is a differentiable manifold (at least C^3) on which the tangent spaces are equipped with inner products in a differentiable fashion.

In differential geometry, one attaches to every point P of a differentiable manifold a tangent space: a real vector space (denoted T_p) which intuitively contains the possible "directions" in which one can pass through the given point. For example, if the given manifold is a 2-sphere embedded in $\mathbb{R}^3$, one can picture the tangent space at a point as the plane which touches the sphere at that point and is perpendicular to the sphere's radius through the point. In general, as in this example, all the tangent spaces have the same dimension, and it equals the manifold's dimension. However, the definition just provided for tangent spaces is inadequate since it is non-intrinsic — i.e. it relies on a particular embedding of the manifold. The following intrinsic definition is thus used instead: T_p is the n-dimensional vector space of linear differential operators (directional derivatives), acting on any differentiable function $f : M \to \mathbb{R}$ on a chart to which the point $p \in M$ belongs. It can be shown that the local tangent-spaces, thus defined, do not depend upon the particular chart or atlas used.

Once the local tangent spaces of the differentiable manifold have been introduced, one can define vector fields[900], which are abstractions of the velocity field of hypothetical particles moving on the manifold. A vector field attaches to every point of the manifold a vector from the tangent space at that point, in a smooth manner. Such a vector field serves to define (in any given chart) a system of first-order (and in general nonlinear) ordinary differential equations on the manifold: a particular solution to such a system is a differentiable curve on the manifold, $g : \mathbb{R} \to M$, such that for any differentiable function $f : M \to \mathbb{R}$, the ordinary derivative of the composite function $f \circ g : \mathbb{R} \to \mathbb{R}$ at a given argument $t \in \mathbb{R}$, $(f \circ g)'(t)$, is the above-mentioned element of T_p, where $p = g(t)$. While the system of differential equations defining the intersection of the curve with a given coordinate-chart is chart-dependent, the curve itself does not depend on the atlas or its charts.

[900] Vector fields, in turn, can be used to define a general *tensor field* on the differentiable manifold, with arbitrary numbers of *covariant* and *contravariant* indices. If the manifold is Riemannian, any covariant (subscript) index can be *raised* into a contravariant (superscript) index by using the metric tensor, and vice versa. A vector field is a particular case of a tensor field with a single, contravariant index. A skew-symmetric (i.e. completely antisymmetric) with m covariant indices ($0 \le m \le n = \dim M$) is called an *m-form*; Cartan's *exterior calculus* deals with these forms, as do the cohomology theory of de Rham and Hodge theory. A 0-form is simply a scalar field (a mapping $\varphi : M \to R$).

The ordered pairs of manifold points p and corresponding tangent spaces T_p for all points $p \in M$, can be "glued together" to form a new differentiable manifold of twice the dimension: the tangent bundle of the manifold. This is an example of a vector fiber bundle. The original manifold M is called its base while T_p are its fibers. A given vector field is a particular section of the tangent bundles. More generally, any type of tensor field is a section of the corresponding tensor bundle — a vector fiber bundle over the base manifold (M), whose fiber is again a vector space. The tangent bundle can be constructed for any differential manifold, whether or not it is Riemannian (i.e. endowed with a metric); if a metric exists, it can be viewed as an internal product in each tangent space. The tangent bundle — or indeed any other fiber bundle — can be endowed with a connection, by which is meant a rule specifying how to glue together (in a smooth manner) the fibers erected above neighboring base-manifold points. A connection provides atlas-independent definitions of how to parallel-transport and covariantly differentiate a vector (or any other tensor) field. For a Riemannian base-manifold M, one possible connection on a bundle is the Christoffel connection, constructed from the metric tensor.

Differential geometry is the study of Riemannian manifolds and manifolds with additional or alternative local geometrical structures (e.g. fiber bundles). Differential geometry deals with metrical notions on manifolds, while differential topology deals with those nonmetrical, global attributes of manifolds which are expressible in terms of local entities (including connections and metric–tensor related quantities, when those exist).

Differential topology and differential geometry are both intimately linked to the theory of differential equations.

*The fundamental group, also called the Poincaré group or the first homotopy group, (**Poincare**, 1895) is one of the basic concepts of algebraic topology.*

To grasp the general idea, take some manifold and some point in it, and consider all the loops at this point — directed paths which start at this point, wander continuously about the manifold, and eventually return to the starting point. Two loops can be combined together in an obvious way: travel along the first loop, then along the second. The set of all the loops with this method of combining them forms the fundamental group, provided we consider two loops to be the same if one can be deformed into the other without breaking.

Thus, each element in the fundamental group is an equivalence class of loops. It is an additive group, with the "sum" of two loops being their combination (composition) as described above. The unit (zero) group element is the class of loops that can be continuously shrunk to the starting point (base-point).

Although the fundamental group in general depends on the choice of base-point, it turns out that, up to an isomorphism, this choice makes no difference if the manifold X is path-connected.

In many spaces, such as R^n, there is only one homotopy class of loops, and the fundamental group is therefore trivial. A path-connected space with a trivial fundamental group is said to be simply connected.

A more interesting example is provided by the circle. It turns out that each loop homotopy class of the circle consists of all loops which wind around the circle a given number of times (which can be zero, positive or negative, depending on the direction of winding). The sum (composition) of a loop which winds around m times and another that winds around n times is a loop which winds around the circle $m + n$ times. So the fundamental group of the circle is isomorphic to $\mathbb{Z}$, the additive group of integers. Knowledge of the fundamental group of the circle can be used to provide a topological proof of the Fundamental Theorem of Algebra, as sketched above.

Unlike many of the other groups associated with a manifold or other topological space, the fundamental group need not be Abelian. An example of a space with a non-Abelian fundamental group is a figure 8 (two circles joined at one point). The fundamental group of a figure 8 is just the free group with two generators — roughly speaking, each loop of the 8 corresponds to one of the generators. The figure 8 is not a manifold (there is no chart containing the intersection point at which the two loops meet); but the closely related 2-torus (the direct product of two circles), which is a manifold, has the same non-Abelian fundamental group as does the figure-8 space.

It can be shown that the Abelianization of the fundamental group of a nonempty path-connected space is isomorphic to the first homology group of the space — which is another important concept in algebraic topology. Thus, the first homology group of the figure 8 space (or the 2-torus) is isomorphic to the direct product $\mathbb{Z} \times \mathbb{Z}$, i.e., the additive group of integer-coordinate square lattice points in the plane.

A topological group G is a group which is also a topological space such that the group multiplication

$$G \times G \quad \rightarrow \quad G$$

and the taking of inverses

$$G \quad \rightarrow \quad G^{-1}$$

are continuous maps. Here, $G \times G$ is viewed as a topological space by using the product topology – in which the direct product of two open sets of G is defined as an open set of $G \times G$.

Examples:

The real numbers $\mathbb{R}$, *together with addition as group operation and its ordinary topology, form a topological group. More generally, the Euclidean* n*-space* $\mathbb{R}^n$ *with addition and standard topology is a topological group. More generally still, all topological vector spaces, such as Banach spaces or Hilbert spaces, are topological groups.*

The above examples are all Abelian. Important examples of non-Abelian topological groups are given by most Lie groups (topological groups that are also manifolds), for instance the group $GL(n,\mathbb{R})$ *of all invertible* n*-by-*n *matrices with real entries. The topology on* $GL(n,\mathbb{R})$ *is defined by viewing* $GL(n,\mathbb{R})$ *as a subset of the Euclidean space* $\mathbb{R}^{n\times n}$.

All the examples above are Lie groups (if one views the infinite-dimensional vector spaces as infinite-dimensional "flat" Lie groups). An example of a topological group which is not a Lie group is the rational numbers $\mathbb{Q}$ *under addition. This is a countable space and it has the discrete topology. For a non-Abelian example, consider the subgroup of rotations of* $\mathbb{R}^3$ *generated by two rotations by rational multiples of* 2π *about two different axes, or the group of allowed operations on a Rubick's Cube (for which the generating rotations on any one of the cube's 6 faces are by angles that are integer multiples of* $\frac{\pi}{2}$*).*

In mathematics, an isomorphism is a type of bijective (onto and one-to-one) mapping between two abstract structures, in a manner such as to preserve the structure (whether group or algebra operations, internal products, etc).[901]

If there exists an isomorphism between two structures, we call the two structures isomorphic. Isomorphic structures are essentially the same; they are equivalent, in the abstract sense.

For example, if one object consists of a set X *with an ordering* $\leq$ *and the other object consists of a set* Y *with an ordering* $\sqsubseteq$, *then an isomorphism from* X *to* Y *is a bijective*[902] *function* $f\colon X \to Y$ *such that*

$$f(u) \sqsubseteq f(v) \qquad \text{iff} \qquad u \leq v.$$

Such an isomorphism is called an order isomorphism.

Or, if on these sets the binary operations $*$ *and* $\circ$ *are defined, respectively, then an isomorphism from* X *to* Y *is a bijective function* $f\colon X \to Y$ *such that*

$$f(u) \circ f(v) = f(u * v)$$

[901] If the one-to-one hypothesis is relaxed, one speaks of a *homomorphism*.

[902] i.e. one-to-one and *onto* — the latter means every element in Y is $f(x)$ for *some* x in X.

for all u, v in X. When the objects in questions are groups, such an isomorphism is called a group isomorphism.

In 1904, ***H. Poincare*** *put forward the Poincare Conjecture: Any compact orientable 3-dimensional manifold with trivial fundamental group must be homeomorphic to a sphere S^3.*

The conjecture admits a natural extension to n dimensions for any positive integer n. For $n \geq 5$, this conjecture was proved by ***Stephen Smale*** *(1959) and for $n = 4$ by* ***Michael Freedman*** *(1981).* ***Simon Donaldson*** *proved (1982) that $\mathbb{R}^4$ admits more than one differentiable structure (i.e. it admits non-equivalent C^k atlases for some $k \geq 1$). Donaldson's result is of interest to physicists, as it sheds light on the possible structures of 4-dimensional spacetime.*

(D) Global Analysis (1828–1968)

Differential geometry studies the local properties (e.g. curvature) of smooth manifolds. Differential topology studies the global properties of manifolds by reducing them to local properties. The study of differential operators on a smooth manifold reveals deep relationships between the geometry and the topology of the manifold on the one hand; and the (local and global) solutions of differential equations on this manifold, on the other.

Global analysis studies the global nature of differential equations on Riemannian manifolds (including infinite-dimensional manifolds and manifolds with singularities). In addition to local tools from ordinary and partial differential equation theory, global techniques include the use of fiber bundles and topological spaces and mappings, as well as optimization procedures. Analysis here means the study of ordinary and partial differential operators on vector bundles over differentiable manifolds.

The ideas of global analysis evolved over more than a century, from Green's theorem (1828), Gauss' divergence theorem (1839) and Stokes' theorem[903]

[903] Given an oriented surface S bounded by a directed curve C encircling S in a positive (counter-clockwise) sense, and a vector field $\boldsymbol{F}$,

$$\iint_S \boldsymbol{n} \cdot \operatorname{curl} \boldsymbol{F} dS = \int_C \boldsymbol{F} \cdot \frac{d\boldsymbol{r}}{ds} ds = \int_C \boldsymbol{F} \cdot d\boldsymbol{r},$$

where $\boldsymbol{n}$ is a unit vector normal to S and $\boldsymbol{r}(s)$ is a parametric representation of C.

*(**Kelvin**, 1850; **Stokes**, 1854) to the Atiyah-Singer index theorem (1963–1968 and later works), through the intermediary steps of:*

- *Cauchy's residue theorem (1831).*
- *Gauss-Bonnet theorem for the Euler characteristic of surfaces (Bonnet 1848; Chern's generalization, 1943–1945).*
- *Riemann-Roch theorem*[904] *for algebraic curves (Roch, 1864) and its generalization by **F. Hirzebruch** in his Signature theorem (1954) and by **A. Grothendieck** (1957) to arbitrary projective varieties in n dimensions.*
- *The Betti connectivity numbers (1870).*
- *The Poincare topological theory of nonlinear differential equations (1881).*
- *The Brouwer fixed-point theorem (1907).*
- *The Lefschetz fixed-point*[905] *theorem (1927).*
- *The Pontryagin*[906] *abstract harmonic analysis of functions defined on locally compact groups (1934).*
- *The Hodge index theorem and theory of harmonic integrals (W.V.D. Hodge 1903–1975; 1935).*

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904 *Riemann-Roch theorem* (1864): deals with functions on a Riemann surface of genus p. Essentially, the theorem determines the maximal number of linearly independent meromorphic functions on the surface that have at most a specified finite set of poles. It thus links complex analysis with algebraic geometry.

905 A *fixed point* of a mapping $F : X \to X$ from a set X to itself is a point $x \in X$ for which $F(x) = x$. Proofs of the existence of fixed points and methods for finding them are important mathematical problems having applications in physics and engineering. Depending on the structures defined on X, and the properties of F, there arise various fixed-point principles. Of greatest interest is the case when X is a topological space and F is continuous.

906 **Lev Semenovich Pontryagin** (1908–1988, Russia). Blinded by accident at the age of 14, his mother *Tatyana Andreevna Pontryagina* dedicated her life to help him became a mathematician. She worked as his secretary, reading scientific works to him. The contingent development of mathematics depends on a wide array of influences, other than the talents of the mathematicians themselves — such as economic and social factors and the acts of non-mathematicians such as Tatyana Andreevna.

Topological invariants in two dimensions

A 2D cell is a figure in two dimensions that is topologically equivalent to a discs. Figures that can be constructed from cells by gluing and pasting them together along their edges are called complexes. *In this way, complicated figures can be built from simple ones. Clearly, the number of cells that constitute a given figure is not uniquely determined: e.g., a spherical surface can be built from just two cells, for instance the northern and southern hemispheres of a globe or (alternatively) two hourglass-shaped cells such as those sewn together to make a baseball; or alternatively by eight cells, each having the shape of a quarter of a hemisphere.*

We call a cell a polygon *when a finite number of points on its boundary are chosen as* vertices. *The vertices on a polygon's boundary partition it into* edges. *A* polyhedron *is a 3D complex that is topologically equivalent to a ball (interior of a 2-sphere). Given a polyhedron, let F stand for the number its 2D cells (faces), E the number of distinct edges, and V the number of distinct vertices.* Euler's formula *for polyhedra then states that $V-E+F=2$ (**Descartes** 1639, **Euler** 1751). This formula states that no matter how a sphere may be divided into polygons, the sum $V-E+F$* always *equals 2. This number is a topological property of the sphere and is called its* Euler characteristic. *Clearly, this applies to any* connected map *on a sphere that has V vertices, E edges and F faces. If a map on a sphere with p handles has V vertices, E edges and F faces, and if each face is simply connected, then the above formula becomes: $V-E+F=2-2p$.*

If we remove one of the faces of a polyhedron, the remainder is topologically equivalent to a 2D cell (disc) and so may be flattened into a plane. By removing one face and leaving edges and vertices intact, the sum $(V-E+F)$ has been decreased by one; and it can be proved in general for a complex equivalent to a disc that $V-E+F=1$.

Both the sphere and the torus can be triangulated, *i.e., covered with a finite number of topological triangles (cells with 3 edges and 3 vertices each) which fit together along their edges. (It does not matter topologically that the triangles are not flat, or that the edges are not straight.) More generally, a* surface *(2D manifold) is a topological space which is triangulable, connected, compact and without any boundary, i.e. it has no edge belonging to only a single triangle. (Examples: sphere, torus, Klein bottle, projective plane.) The Möbius strip is not a surface in this sense, because it has an edge. The plane is not a surface, because it is not compact — i.e. it cannot be built up from a finite number of triangles. On any compact surface we can draw maps (partitions into polygonal cells) and we can count its faces, edges, and vertices. For a given surface S the number $V-E+F$ can be shown to*

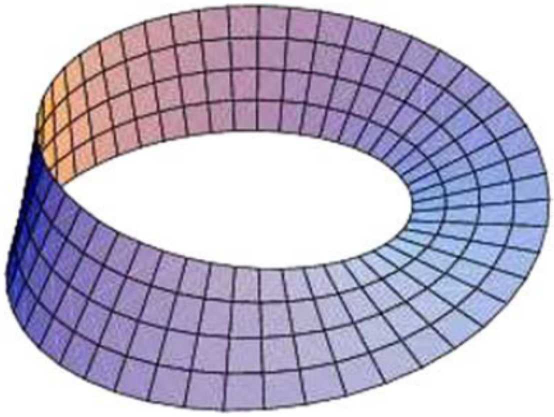

Fig. 5.15: The Möbius strip

be independent of the map we choose. It is known as the Euler (or Euler-Poincaré) characteristic of the surface, and is denoted by $\chi(S)$. *It is a topological invariant* because it is the same for all topologically equivalent spaces.

In particular, for any polyhedron homeomorphic to a sphere $\chi = 2$. *For closed orientable surfaces* $\chi = 2 - 2g$, *where* g *is the genus.*

For closed Riemannian orientable manifolds of even dimension, the Euler characteristic can also be found by integrating a scalar constructed from the curvature tensor, via the Gauss-Bonnet theorem for the 2-dimensional case and its generalization for higher dimensions.

Other topological invariants for closed (boundary-less) surfaces are:

- *First Betti number* (b_1): *The maximum number of simultaneous cuts that can be made without dividing a surface into disjoint pieces. This number — just one of the Betti numbers — can be defined for manifolds of any dimension, and is related to the homology and cohomology groups of algebraic topology & differential topology. The following surfaces have the corresponding first Betti numbers: plane lamina (0); sphere (0); cylinder (1); Möbius strip (Fig. 5.15) (1); projective plane (1); torus (2); Klein bottle (2).*

  Note that $b = h - 1$, *where* h *is the connectivity,*

- *genus* (g): *for a connected orientable surface it is equal to the number of handles (e.g., sphere has* $g = 0$ *and a torus has* $g = 1$*).*

- *orientability.*

Extensions to higher dimensions.

The building blocks of 3-dimensional complexes are points, edges, polygons and polyhedral solids. The latter is any subset of 3-dimensional space that is topologically equivalent to a spherical ball and whose boundary surface has been divided into faces, edges and vertices (i.e., the boundary surface of a polyhedron). A 3-dimensional complex is a topological space made up of polyhedral solids glued together along faces. The concept of complex can similarly extended iteratively to any higher dimension.

It can then be shown that Euler's formula $V - E + F = 1$, *valid for an open region of disc topology in two dimensions, can be generalized to an* n*- dimensional manifold having the ball topology; it then becomes the Euler-Poincaré* formula,

$$F_0 - F_1 + F_2 - \cdots \pm F_n = 1,$$

where F_m *is the number of* m*-dimensional 'faces' of the complex (map) into which the manifold has been partitioned. Thus, in 3 dimensions*

$$\chi = V - E + F - S = 1$$

where $S = F_3$ *is the number of polyhedral solids in the complex. For a tetrahedron we have:*
$V = F_0 = 4, \quad E = F_1 = 6, \quad F = F_2 = 4, \quad S = F_3 = 1.$

The Euler-Poincaré characteristic and Betti numbers

Another, related expression for the Euler characteristics of an n*-dimensional manifold — one that does not depend on the segmentation of the manifold into a complex — relates* χ *to the manifold's* $n+1$ *Betti numbers* $b_m (o \leq m \leq n)$*. These non-negative integers are, like the Euler–Poincaré index* χ*, topological invariants of the manifold.*

b_m *is the cardinality of two isomorphic groups — the m-th homology group and its m-th cohomology group. It can be shown that*

$$\chi = b_0 - b_1 + \cdots + (-1)^n b_n$$

This applies to any boundary-less and oriented manifold of dimension n*.*

Gauss-Bonnet Theorem and its generalizations (1848–1943)

This theorem in differential geometry and differential topology is an important statement about surfaces which connects their intrinsic geometry *(local curvature) to their* topology *(Euler characteristic).*

Let M *be a compact, orientable two-dimensional Riemannian manifold with boundary* ∂M*. Denote by* K *the* Gaussian curvature *at a general point on the surface, and by* K_g *the* geodesic (extrinsic) curvature *at a general point on* ∂M*. Then*

$$\int_M K dA + \int_{\partial M} K_g ds = 2\pi\chi(M) \tag{1}$$

where dA *is an area-element on* M, ds *is an* arc *element on* ∂M, *and* $\chi(M)$ *is the* Euler characteristic *of* M*. If the manifold does not have a boundary, the integral* $\int_{\partial M} K_g ds$ *can be omitted. Thus, the* integral curvature *of a closed orientable surface* M *of genus* g *does not depend on the shape of the surface and is equal to*

$$\int_M K dA = 4\pi(1-g). \tag{2}$$

This result makes it possible to express topological *properties of the surface — in this case the genus* g *(which remains invariant under arbitrary continuous deformations) — in terms of quantities of differential geometry (here, the integral curvature).*

If M *is open (i.e. has a boundary), the geodesic (extrinsic) curvature at a given point along the (closed) curve* ∂M *is the reciprocal of the radius of curvature of* ∂M *at that point, in a locally flat 2D coordinate system. Thus,* $K_g \equiv 0$ *if* ∂M *is a geodesic of the Riemann surface* M*. For example, if* M *is a portion of a unit-radius sphere defined by* $0 \leq \theta \leq \alpha < \frac{\pi}{2}$ *(in spherical polar coordinates),* ∂M *is the circle of latitude* $\theta = \alpha$ *(not a geodesic);* $K_g = \cot\alpha$ *uniformly on* ∂M, *and* $K = 1$ *uniformly on* M*. We thus have in this case* $\int_M dA = 2\pi(1-\cos\alpha)$, $\int_{\partial M} ds = 2\pi\sin\alpha$*. Thus, the Gauss-Bonnet theorem for this open surface yields:*

$$\int_M K dA + \int_{\partial M} K_g ds = 2\pi(1-\cos\alpha) + 2\pi\sin\alpha \cdot \cot\alpha = 2\pi = 2\pi\chi(M).$$

And indeed, since M *is topologically equivalent to a disc, its Euler characteristic is* $\chi(M) = 1$.

If one bends or deforms the manifold M, its Euler characteristic will not change, while its curvature at given points will. The theorem then requires, somewhat surprisingly, that the integral of all curvatures will remain the same.

In 1943, **Carl B. Allendoerfer** *and* **André Weil** *generalized the Gauss-Bonnet theorem to n-dimensional spaces. Also in that year, it was further generalized by* **S.S. Chern** *for a closed Riemannian manifold of dimension $2n$. He presented the Euler characteristic as an integral of a certain formal polynomial derived from its Riemann curvature tensor:*

$$\int_M Pf(\Omega) = 2^n \pi^n \chi(M), \tag{3}$$

where $Pf(\Omega)$ is the Pfaffian polynomial[907] *of Ω and Ω is the curvature 2-form matrix (in Cartan's exterior calculus). For $n = 1$ (2 dimensions)*

$$\Omega = \begin{bmatrix} 0 & K \\ -K & 0 \end{bmatrix} \omega, \quad Pf \begin{bmatrix} 0 & K\omega \\ -K\omega & 0 \end{bmatrix} = K\omega,$$

where $\omega = \sqrt{\det g}\ dx^1 \bigwedge dx^2$ is the volume 2-form, g the surface's metric tensor, (x^1, x^2) coordinates in the local chart, and "$\bigwedge$" the Cartan wedge

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[907] The *determinant* of a skew-symmetric square *matrix* X with elements x_{ij} can always be written as the square of a *polynomial* of degree n in the variables x_{ij}. This polynomial is called the *Pfaffian* of the matrix, and denoted $Pf(X)$. The Pfaffian is non vanishing only for $2n \times 2n$ skew-symmetric matrices. Thus

$$\det(X) = \begin{vmatrix} 0 & x_{12} & x_{13} & \cdots & x_{1n} \\ -x_{12} & 0 & x_{23} & \cdots & x_{2n} \\ -x_{13} & -x_{23} & 0 & \cdots & x_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ -x_{1n} & -x_{2n} & -x_{3n} & \cdots & 0 \end{vmatrix} = \begin{cases} [Pf(X)]^2 & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

For example:

$$Pf \begin{bmatrix} 0 & x_{12} \\ -x_{12} & 0 \end{bmatrix} = x_{12};$$

$$Pf \begin{bmatrix} 0 & x_{12} & x_{13} & x_{14} \\ -x_{12} & 0 & x_{23} & x_{24} \\ -x_{13} & -x_{23} & 0 & x_{34} \\ -x_{14} & -x_{24} & -x_{34} & 0 \end{bmatrix} = x_{12}x_{34} - x_{13}x_{24} + x_{23}x_{14}$$

product $(dx^2 \bigwedge dx^1 = -dx^1 \bigwedge dx^2)$. *Since* $\int_M K\omega = \int_M KdA$ *in standard (non-Cartan) notation, we recover equation (1).*

Atiyah-Singer Index Theorem (ASIT)

In the beginning, mathematics was used to count (arithmetic), e.g. for bookkeeping, planning and trade; or to describe shapes (geometry), e.g. for measuring a plot of land, for cutting out fabric for a dress, or for building a bridge. Modern applications of mathematics are often concerned with modeling — and thereby predicting — the evolution over time of complex, composite systems, such as oil and gas flow in porous rocks under the North Sea, how queues of text messages in a cellular network can best be resolved, or what the weather will be like this weekend.

Since the time of Newton and Leibniz, these mathematical models have often been described by systems of differential equations. *To use mathematics for the intended application, one seeks to find the* solutions *of such systems. The Atiyah-Singer Index Theorem (ASIT) is a fundamental insight that essentially says that we can find out* how many solutions *the system has by just knowing some simple pieces of information about the geometry of the space being modeled. Thus although the index theorem is a purely mathematical result, which links together analysis and topology, it can be used as a tool in many applications of mathematics.*

The subject of mathematics can coarsely be divided into four areas: algebra, analysis, topology/geometry, and logic. Mathematics is a diverse language that can describe, discuss and model many different objects, processes and problems, and the four areas tend to focus on different aspects of these objects. Nonetheless, there are no clear boundaries between them.

In analysis, *an entity is studied by first partitioning it into small pieces, and thereafter reassembling them (synthesis). Emphasis is put on the limiting case when the pieces become arbitrarily small, and simultaneously arbitrarily numerous. Keywords: differentiation, integration and calculus.*

In topology *and* geometry *one studies how an object can have a shape, or a spatial aspect. In particular, in topology one emphasizes properties of the whole global shape and properties unchanged under continuous, reversible deformations, rather than the local or deformation-sensible attributes of the object. If the shape is described by some notion of distance, angle, straightness etc., then we usually talk about* geometry.

The Index theorem (actually a class of related theorems) marked the birth of the mathematical field of global analysis. *One reason for seeking such a* *theorem was the attempt to unify all the aforementioned types of concepts into one formula. Such a formula had been conjectured earlier (1960) by* ***Israel Gelfand*** *in the context of the homology invariance of the index of a Fredholm operator. Given the examples from Hodge theory, Cauchy-Riemann operators in several variables, and the topologists' work on the Riemann-Roch Theorem at the time, the required concepts were perhaps 'in the air' by 1960.*

Thus for instance, from 1946, ***Salomon Bochner*** *(1899–1982) and his colleagues & students found novel connections between homology & cohomology theories on the one hand, and the local differential geometry of the corresponding manifold on the other.*

For example, Bochner proved that if a smooth Riemannian manifold M *has a positive-semidefinite Ricci curvature scalar which is positive at a point, then the manifold's first Betti number,* b_1*, vanishes. This has two implications. From the homology point of view,* $b_1 = 0$ *means there is no closed curve along which the manifold* M *can be cut while remaining connected. From the (de Rham) cohomology viewpoint,* b_1 *is the number of linearly independent 1-forms on* M *that are closed modulo exact forms. In other words, in Cartan's exterior calculus,* b_1 *is the dimension of the vector space of 1-forms* V *that are closed (i.e. satisfy* $d\mathrm{V} = 0$*), provided we regard two solutions* $\mathrm{V}_1, \mathrm{V}_2$ *of this differential equation as identical if their difference is exact - i.e.* $\mathrm{V}_1 - \mathrm{V}_2 = d\phi$ *for some scalar (0-form)* ϕ*.*

Before launching into technical examples of the use of ASIT in contemporary mathematical physics, we define a few necessary general concepts. We also provide the reader with a very simple and easily visualized example that will set the stage for what follows.

A brief statement of the simplest version of ASIT is:
Let $E(f) = 0$ *be an elliptic system of differential equations, defined over a closed, smooth, oriented* n*-dimensional manifold* M*. Then*

$$\text{analytical index } (E) = \text{topological index } (E)$$

where the analytical index is

$$\text{analytical index } (E) = \begin{array}{l} \text{dimension of the kernel of } E \\ - \text{dimension of co-kernel of } E \end{array}$$

and the topological index of E *is an explicit expression that characterizes the topology of the manifold.*

In formulating the theorem we start with two vector bundles over the base manifold M*, and an elliptic operator* E *mapping smooth sections of one vector bundle into smooth sections of the other vector bundle. If the sections are tensor fields then the fibers of the corresponding bundles are related to* *the local tangent-spaces of M, as discussed above. But, in general, a bundle's fiber need not have anything to do with its base manifold. Thus, for instance, in particle-physics applications, the electromagnetic or chromodynamic field tensors are curvature 2-forms in bundles whose fibers are* internal *(non-spacetime) spaces, acted upon by (Abelian or non-Abelian) Lie-group symmetries. The elliptic property of E is expressed in terms of its* symbol *— an algebraic entity derived from the coefficients of the highest-order derivatives in the operator E. Thus, if E is a generalized Laplacian, its symbol is a positive-definite quadratic form. The symbol is itself a section of a fiber bundle and is required to be non-singular.*

The differential (or pseudo-differential) operator E is a Fredholm operator. As such it has an index (called its analytical *index above). This is the difference between two integers. The first is the dimension of the* kernel *(or* null-space*) of E — i.e. the number of linearly independent solutions of the system of differential equations $Ef = 0$, where f is a section of the first bundle. This integer is denoted as $\dim\ker(E)$. (The solutions f can be viewed as generalized harmonic functions.) The second integer, denoted $\dim\operatorname{coker}(E)$, is the dimension of the vector space of linear constraints on sections $g = Ef$ of the* second *bundle, where f is now an arbitrary section of the first bundle.*

The index problem *is the following: compute the index of E using only the symbol s and* topological *data derived from the manifold and the vector bundles. The Index Theorem solves this problem. Its precise statement requires K-theory, as well as a background in* functional analysis *and pseudo-differential operators in the manifold setting (*global analysis*).*

A simple case is illustrated by a famous paradoxical etching of ***M.C. Escher****, "Ascending and Descending" (Fig. 5.16), where the people, always going uphill along the parapet, still manage to circle the castle courtyard. The index theorem would have told them this was impossible.*

To see this, we follow a hooded walker as he goes around, up or down the square staircase. Here the spatial (base) manifold M is a square, while the walker trajectory (section of the first vector-bundle) is a function $f(x)$ that equals the height above ground at each point x along M. The square M is parameterized by a coordinate x (at least two charts are needed, and we may choose $0 \leq x < 2\pi$ in one of them). The operator E is taken to be differentiation: $Ef = df/dx = f'(x)$. The differential equation $Ef = 0$ then has the 1-dimensional space of solutions $f(x) = C$, with C an arbitrary real constant; thus $\dim\ker(E) = 1$. On the other hand, $\dim\operatorname{coker}(E)$ also equals 1, since the differential equation $g(x) = f'(x)$ imposes the single constraint $\int_0^{2\pi} g(x)dx = 0$ (this followed from the fact that $f : M \to R$ is a single-valued function along the square). Thus, the analytical *index of E is $1 - 1 = 0$; and it can be shown that the corresponding* topological *index vanishes as well. In*

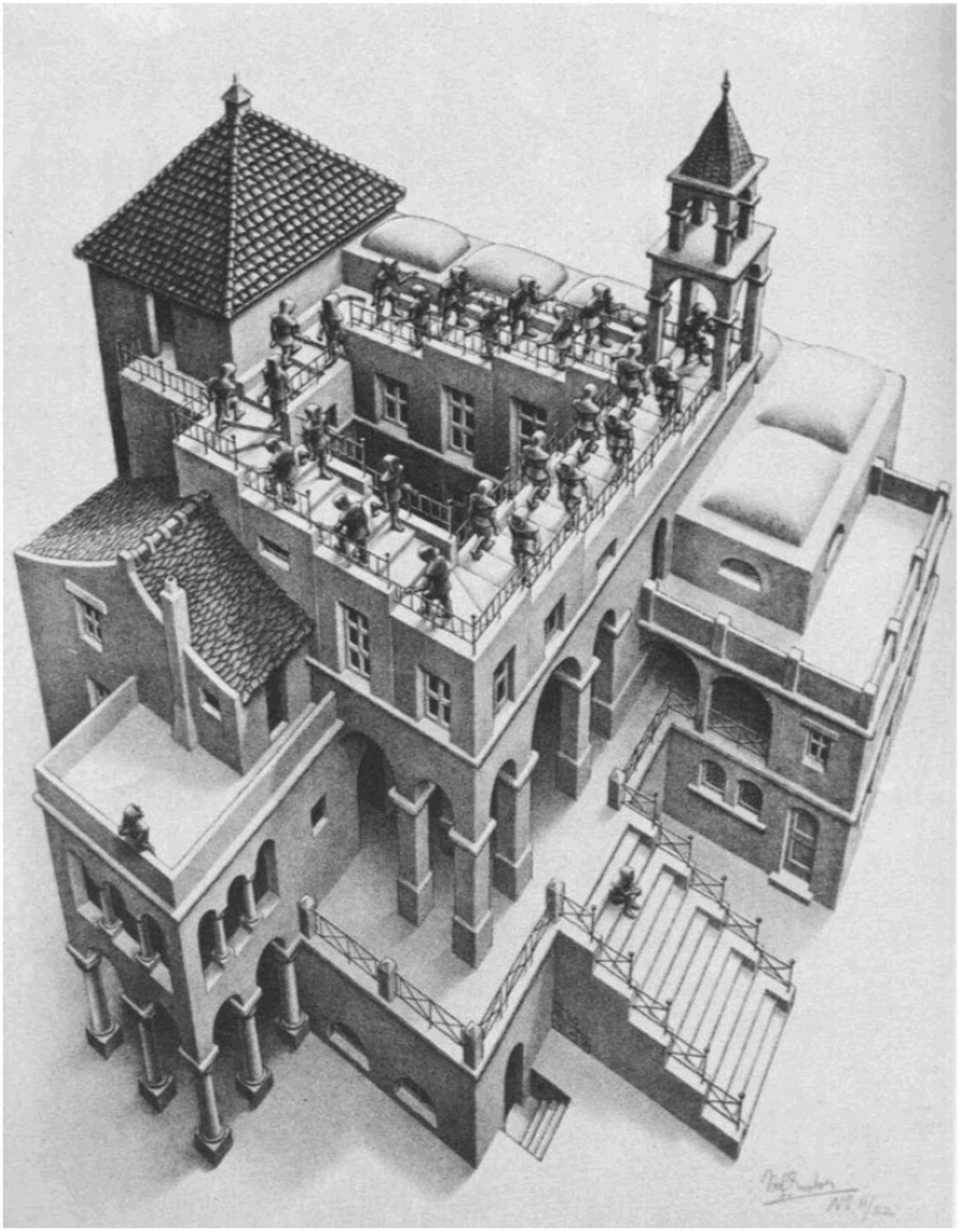

Fig. 5.16: M.C. Escher's "Ascending and Descending"

particular, the wanderer cannot ascend nor descent all the time, since that would imply $f'(x) > 0$*, contradicting the constraint* $\int_0^{2\pi} f'(x)dx = 0$.

Michael Francis Atiyah *(b. 1929) and* ***Isadore M. Singer*** *(b. 1924) shared the Field Medal (2004) for their discovery. They brought together topology, geometry and analysis and built new bridges between mathematics and theoretical physics.*

The ASIT is a topological formula for the Fredholm index of a linear, elliptical pseudo-differential operator (e.g. a "Dirac Operator") in terms of characteristic classes (differential-topological invariants) of the underlying bundle. The ASIT covers all the particular cases mentioned above, by means of specific choices of the linear operator and associated fiber bundle.

In the particular case of the Dolbeault Operator on complex manifolds this is the theorem of Riemann-Roch; while for the Chern-Gauss-Bonnet formula, the ASIT is applied to a vector bundle of smooth differential forms, with the (Dirac) operator being the square-root of the Hodge Laplacian.

We have seen that for an even-dimensional, closed (boundary-less), oriented, smooth and compact Riemannian manifold M*, the manifold's Euler characteristic,* $\chi(\mathrm{M})$*, can be expressed as an integral over* M *of an* n*-form constructed from the matrix curvature 2-form, where* $n = \dim \mathrm{M}$.[908] *On the other hand, we have seen that* $\chi(\mathrm{M})$ *can* also *be expressed as an alternating sum of the manifold's Betti numbers. The equality of these two expressions for* $\chi(\mathrm{M})$ *is one of the many useful consequences of the Atiyah-Singer Index Theorem — as we now proceed to discuss. The mathematical framework for this particular application of ASIT is the theory of exterior forms (skew-symmetric covariant tensors) and their cohomology classes on Riemannian manifolds, as developed by* **Cartan**, **de Rham**, *and* **Hodge**.

We will describe how ASIT works for this case, presenting more details for the 2-dimensional (Gauss-Bonnet) case. We will follow the more recent heat-kernel approach to proving ASIT, which utilizes ideas from physics. (The original proof of the Atiyah-Singer and related Index Theorems relied on K-theory and the theory of characteristic classes).

The Cartan exterior derivative operator, denoted "d", maps the vector space of m*-forms into that of* $(m+1)$*-forms; Thus for instance, if* $\phi : \mathrm{M} \to \mathrm{R}$ *is a scalar field (0-form), then* $d\phi$ *is a 1-form (in any particular chart* $d\phi = \frac{\partial \phi}{\partial x^k} dx^k$ *where the summation convention is understood). Or, if* ϕ_1, ϕ_2 *are two 0- forms, then* $\phi_1 d\phi_2$ *is a 1-form and we have* $d(\phi_1 d\phi_2) = d\phi_1 \bigwedge d\phi_2$*, a 2-form. Here we have used the fact that the operator* d^2 *vanishes identically (which,*

[908] The requirement that M be *compact* can be relaxed if the metric is chosen such that its deviation from the Euclidean case has *compact support* on M.

in ordinary vector calculus in flat 3-D space, yields the well-known results: $\operatorname{div}\operatorname{curl} = 0$ *and* $\operatorname{curl}\operatorname{grad} = 0$*).*

Another key mapping defines the Hodge dual. If V *is an* m*-form* $(0 \le m \le n)$*, then its Hodge dual* $^*V$ *(an* $(n-m)$*-form) is defined so that* $V \to {}^*V$ *is a linear map at any point* $p \in M$ *and* $^*(dx^1 \wedge dx^2 \wedge \ldots \wedge dx^m) = dx^{m+1} \wedge \ldots dx^n$ *in any positive-orientation chart for which the metric is Euclidean at* p*. (Such a chart can be added to any atlas for any point* p*, since* M *is* orientable *and* Riemannian*). This Hodge duality (or star)operation, like exterior differentiation, is chart-independent, and thus they are covariant (and truly geometric) operations. They can be combined to yield a third important operation (map), the* δ derivative*, defined as follows: if* W *is an* m*-form,* δW *is an* $(m-1)$*-form and*

$$\delta W = (-1)^{mn+n+1}\ {}^*d({}^*W).$$

It is easily shown that $^*({}^*\mathrm{W}) = (-1)^{m(n-m)}W$ *and therefore,* δ^2 *vanishes identically (just as* d^2 *does).*[909]

Hodge duality is also used to define an internal product *on exterior forms: if* V, W *are two forms,* $\langle V, W\rangle = 0$ *unless they have the same rank (i.e. they are both* m*-forms for some* m*); and if they do,*

$$\langle V, W\rangle \equiv \int_M V \wedge\ {}^*W,$$

where $\wedge$ *denotes the Cartan wedge-product. (Here the integrand* $V \wedge {}^*W$ *is an* n*-form; only* n*-forms can be integrated on an* n*-dimensional manifold). It is easy to prove that* δ *is the* adjoint *of the operator* d *relative to this internal product.*

For any m*-form* V*,* $\langle V, V\rangle \equiv \|V\|^2 \ge 0$*, and vanishes* iff $V = 0$*.*
The Hodge Laplacian *is defined as follows:*

$$\Delta = (d+\delta)^2 = d\delta + \delta d$$

Acting on any smooth m*-form, it yields another such form. For any* m*-form* V[910]*, we have (using that* d, δ *are each other's adjoint):*
$\langle V, \Delta V\rangle = \langle dV, dV\rangle + \langle \delta V, \delta V\rangle = \|dV\|^2 + \|\delta V\|^2$*. Therefore, due to the properties of the norm listed above,* Δ *is a* positive semi definite elliptic *operator, and any* harmonic m*-form* V *(i.e. satisfying* $\Delta V = 0$*) also satisfies*

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909 d acting on an n-form yields 0, as it must since no $(n+1)$-forms, exist – and for a similar reason, $\delta\phi = 0$ for any 0-form (scalar) ϕ.

910 From now on all forms and the metric will be assumed smooth.

$dV = \delta V = 0$[911]. *If the manifold M is compact, it can be shown that the spectrum of Δ (i.e. the set of its eigenvalues $\lambda_j; j = 0, 1, \ldots$) is discrete and infinite. If nonzero harmonic form(s) exist for any m then the lowest eigenvalue is $\lambda_0 = 0$. Since Δ is self-adjoint, any two of its m-forms eigenfunctions V, W are orthogonal ($\langle V, W\rangle = 0$) if they correspond to distinct eigenvalues. And the m-form eigenfunctions having the same eigenvalue (if more that one) can also be rendered mutually orthogonal, via the familiar Gramm-Schmidt procedure.*

The mathematical tools just described readily lead to some powerful results. The Hodge decomposition theorem guarantees that any m-form U can be written as $U = dV + \delta W + U_0$, where V, W are an $(m-1)$- and $(m+1)$-form respectively, and U_0 is a harmonic m-form.[912] *This, in turn, can be used to show that b_m, the m-th Betti number of M (an integer topological invariant which counts the generators of the m-th homology group), is also the number of linearly-independent harmonic m-forms.*

In order to apply the ASIT to the Betti numbers, we must first select the operator E and the (first) vector bundle it acts upon. We erect this bundle upon the base manifold M, with its fiber being the vector space of formal sums of all types of even rank forms (0-, 2-, … through n-forms, if $n = \dim M$ is assumed even). And we choose: $E = d + \delta$.

*E maps any section of the above-defined bundle into a formal sum of odd-rank forms; thus, if v is a 4-form, Ev is the formal sum of a 3-form and a 5-form. E is a type of generalized Dirac operator, and is (in a sense) the "square root" of the Hodge Laplacian operator Δ. E maps the above–defined bundle (whose sections are formal sums of even-rank forms) into a second bundle which the ASIT requires: namely, the vector bundle whose sections are formal sums of odd-rank forms (1-forms, 3-forms, … $(n-1)$-forms).
The adjoint of E, which we denote $E^\dagger$, is also $d + \delta$, but it acts on sums of odd-rank forms to yield even-rank forms.*

As noted above (from here on we assume $n = \dim M$ to be even),

$$\chi(M) = b_0 - b_1 + \ldots + b_n = \sum_{k=0}^{n/2} b_{2k} - \sum_{k=0}^{n/2-1} b_{2k+1} \equiv b_{even} - b_{odd}$$

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[911] Any form satisfying $dV = 0$ is called *closed*; any m-form V which can be written $V = dW$ for some $(m-1)$-form W is said to be *exact*. All exact forms are closed, since $d^2 = 0$ as noted above.

[912] In flat 3-D space this implies the *Helmholtz* theorem – familiar from ordinary vector calculus – according to which any vector field vanishing at infinity is the sum of a *curl* and a *grad*.

Here b_{even} is the number of linearly-independent, even-rank harmonic forms; while b_{odd} is the same for odd-rank harmonic forms. Since $\Delta = E^2$, it is easily shown that:

$$b_{even} = \dim\ker E, \qquad b_{odd} = \dim\operatorname{coker} E,$$

and we thus conclude that the Euler characteristic $\chi(M)$ is the analytical index of the generalized Dirac operator E:

$$\chi(M) = \text{analytical Index}(E)$$

It remains to express the RHS in terms of a topological index – in this case, the Euler-Poincaré characteristic of the tangent bundle $T(M)$ of the base manifold; this characteristic , in turn, is an integral over M of an n-form built from the Riemann-curvature matrix 2-form, as we saw earlier.

*We conclude with a demonstration of how the Heat Kernel method (**E.Getzler** 1983) can be used to express the analytical index of E as an integral over a curvature-dependent n-form. To that end, we restrict our attention to $n = 2$, i.e. M is a closed, compact, 2-D orientable surface; in this case the ASIT should reduce to the Gauss-Bonnet theorem for the boundary-less case.*[913]

A well-known theorem in 2-D differential geometry guarantees that an atlas exists on M such that, in each chart, the metric tensor is

$$g_{ij} = e^{\varphi(\xi,\eta)}\delta_{ij} \qquad 1 \le i \le 2, \quad 1 \le j \le 2$$

where δ_{ij} is the Kronecker delta and φ is a function (assumed smooth) of the chart's two coordinates:

$$x^1 \equiv \xi, \quad x^2 \equiv \eta.$$

We then have $\det g = \exp(2\varphi)$, and the local Ricci scalar is

$$R = e^{-\varphi}\left(\frac{\partial^2\varphi}{\partial\xi^2} + \frac{\partial^2\varphi}{\partial\eta^2}\right)$$

Now, it can be shown[914] *that for any number $t > 0$,*

$$\textit{Analytical Index}(E) = \operatorname{Tr}(e^{-tE^\dagger E}) - \operatorname{Tr}(e^{-tEE^\dagger}), \tag{4}$$

[913] Versions of the ASIT for manifolds *with* boundary have also been proven; for instance, the *Atiyah–Singer–Patodi* theorem.

[914] Eq. (4) follows from the fact that the nonzero spectra of the operator $E^\dagger E$ and $EE^\dagger$ are *positive* and *identical*, so all contributions to the two traces *cancel* except the zero-eigenvalue (i.e. *harmonic*) terms. These terms are t-independent, and thus the RHS of (4) is also t-independent.

Where the trace of any matrix-differential operator A acting on vector functions $v_j(\xi,\eta) \quad (i,j=1,\dots s$ for some integer $s\geq 1)$, is

$$\mathrm{Tr}(A)=\sum_{k=0}^{\infty}\Big\langle \Psi_k, \quad A\Psi_k\Big\rangle. \tag{5}$$

Here $\Psi_k(k=0,1,\dots)$ is an infinite, orthogonal basis of scalar function on the chart:

$$\sum_{k=0}^{\infty}\Psi_k(\xi,\eta)\Psi_k(\xi',\eta')=\frac{I}{\sqrt{\det g}}\delta(\xi-\xi')\delta(\eta-\eta'), \tag{6}$$

with $\delta()$ the Dirac delta-function.

Now, for any vector function $v_j(\xi,\eta)$ (a section of the first bundle over the given chart), the vector

$$w_j(\xi,\eta|t)\equiv\sum_{j'}(e^{-tE^{\dagger}E})_{jj'}v_{j'}(\xi,\eta)$$

solves the following parabolic PDE (generalized heat equation):

$$(\frac{\partial}{\partial t}+\Delta)w_j(\xi,\eta|t)=0 \tag{7}$$

with the initial condition: $w_j(\xi,\eta|0)=v_j(\xi,\eta)$. (Here $\Delta=d\delta+\delta d$ is the Hodge Laplacian. A similar PDE results for the second term on the RHS of (4).)

For a flat chart ($\varphi=0$) Eq. (7) is the standard heat equation (in two dimensional space). But a general surface need not be flat (in any chart); in that case, (7) reduces — for each m-form component of the vector bundle — to a heat equation with inhomogeneous heat conductivity. Such PDE's do not, in general, admit exact closed-form solutions. Fortunately, however, the RHS of (4) — and thus the sought-after index formula — does not depend on t; so the heat kernel method consists of finding small-t asymptotic expansions in t for the solutions of (7), and then taking the $t\to 0^+$ limit — in which the leading few terms in the asymptotic expansion become exact! Using these techniques, it can be proven without too much difficulty that the Gauss-Bonnet formula is indeed reproduced[915]*:*

[915] Eq. (7) is solved for initial conditions $v_j(\xi,\eta)$ that are Dirac delta-functions at a point P on M; and then the trace operation (5), together with (6), results in an integral over P — as seen in Eq. (8).

$$\chi(M) = b_0 - b_1 + b_2 = b_{\text{even}} - b_{odd} = \frac{1}{2\pi} \sum_{charts} \int_{chart} \left(\frac{\partial^2 \varphi}{\partial \xi^2} + \frac{\partial^2 \varphi}{\partial \eta^2} \right) d\xi d\eta$$
$$= \frac{1}{2\pi} \sum_{charts} \int_{chart} R\sqrt{\det g}\, d\xi \wedge d\eta = \frac{1}{2\pi} \int_M k dA, \tag{8}$$

where $k(P)$ is the Gaussian curvature at a point P of the surface M, and the sum over charts is done in such a manner as to compensate for multiple counting where charts overlap.

However, it is important to notice that the ASIT is much more general than all the more specialized theorems above. For example, a typical corollary which is not contained in the previous results is the index theorem for the spinor Dirac operators on the spin bundle of a curved spacetime manifold in the presence of electromagnetic (and other gauge) fields. This index theorem and the others mentioned above are of key importance in particle physics, Quantum Field Theory (QFT), quantum gravity and string theory.

Nowadays there are several extensions of the ASIT in many different directions. But index theorems are not the only object of research in global analysis. For example: scattering theory on noncompact manifolds, holomorphic torsion, trace formulas and analysis on locally symmetric spaces are all active fields of research. Applications of global analysis can be observed in almost all fields of pure mathematics, theoretical physics *and equilibrium theory in* microeconomics.

(e) Physics and Topology

In the 1970's and 1980's, theoretical physics underwent a significant transformation: the traditional tools of mathematical physics (real and complex analysis), which deal with space-time configurations and phase-space manifolds mainly locally, *were supplemented by topological approaches (more precisely, methods from* differential and algebraic topology*), that account for the* global *(holistic) structure of spacetimes, configuration and phase-spaces, function spaces, and other manifolds based upon spacetime (such as e.g. fiber bundles, whose* local *differential geometry also became important in physics, and whose fibers often involve non-spacetime ("internal") spaces). This trend was seen in:*

- *Geometrical phases in adiabatic Quantum Mechanics.*
- *The rise of Non Abelian Gauge theories.*

- *Solitons, instantons, path integrals, functional differential equations, symmetry breaking and gauge-fixing and anomalies in gauge theories and condensed-matter systems.*
- *Theory of defect-mediated phase-transition in condensed matter, QFT (Quantum Field Theory) and the particle-physics era in early-universe cosmology.*
- *Kaluza-Klein theories (microscopic extra dimensions and geometrization of non-gravitational forces), supergravity theories, and string and superstring theories.*
- *Attempts to develop Grand Unified QFT's and a theory of Quantum gravity.*
- *Nonlinear dynamics and chaos theory.*

As before, mathematicians forged the tools for the new physics. Thus, for example, De Rham's theorem *in cohomology (1931) and the work of* ***Chern*** *on* characteristic classes *and* fiber-bundles *(1943), became an important tools for understanding non-Abelian gauge theories (Yang-Mills theories) and quantized fields in curved spacetime. The work of* ***E. Calabi*** *(1954) on Kähler manifolds with Ricci flat metric and vanishing first Chern class became of importance in* superstring theory.

While most of the algebraic, differential-geometric and topological tools adapted for use in quantum physics are classical in nature, some — such as Quantum Groups *(Hopf algebras),* Virasoro *and* Kac-Moody *algebras, and the so-called* graded Lie algebras *of supersymmetry and supergravity — are inherently quantum-mechanical insofar as their relevance to physics goes.*

1936–1960 CE **Hans Adolph Rademacher** (1892–1969, Germany and USA). Mathematician. Made significant contributions to analysis and number theory. His most famous result (1936) is his derivation of an explicit asymptotic formula for the growth of the *partition function* (the number of representations of a number as a sum of natural numbers). This answered questions posed by **Leibniz** and **Euler** and followed results obtained by **Hardy** and **Ramanujan**. In addition he derived other important results in measure theory, complex analysis, geometry and numerical analysis.

Rademacher was born in Wandsbeck (near Hamburg) and was a student of Carathéodory and Landau at Göttingen (Ph.D. 1916). He then held academic appointments in Berlin, Hamburg and Breslau. Because of his pacifist views he was forced by the Nazis out of his professorship in Breslau and left Germany (1934). He spent the rest of his life in the United States, mostly at the University of Pennsylvania.

1936–1962 CE **Harold Marston Morse** (1892–1977, USA). Mathematician. Established connections between topology and equilibrium points in the calculus of variations, and applied such ideas to *minimal surfaces*. Contributed to what is now called *Global Analysis*, i.e. the study of differential equations, ordinary and partial, from a topological point of view. This development owes much to the calculus of variations since the problems of this field have an especially global character from the outset. *Morse function*, *Morse Lemma*, *Morse equality* and *Morse theory* are named after him.

Marston Morse was born in Waterville, Maine, and educated at Colby College (B.A., 1914) and Harvard (Ph.D., 1917). He taught at Harvard briefly before entering military service in World War I and again immediately thereafter. He then held positions at the universities of Cornell (1920–1925) and Brown (1925–1926). In 1926 he returned to Harvard, where he remained until his appointment to the Institute for Advanced Study in 1935. In 1962 he was appointed Emeritus Professor at the Institute and in 1965–1966 served as Visiting Professor at the Graduate Center of City University of New York.

Euler's relation for polyhedra inscribed in a sphere is $V - E + F = 2$ (V = number of vertices; E = number of edges; F = number of faces). For the hemisphere (topologically equivalent to a sphere with one hole) we have $V - E + F = 1$ (e.g. for a three-sided pyramid that is missing its base $V = 4$, $E = 6$, $F = 3$). On the other hand Morse discovered that for a hemispherical bowl with n equilibrium points $\{P_1, P_2, \dots, P_n\}$ with the corresponding *characteristic numbers*[916] $\{\lambda_1, \lambda_2, \dots, \lambda_n\}$ we have $\sum_{i=1}^{n}(-1)^{\lambda_i} = 1$ (Morse's Equation).

Morse connected this equation with the existence of n *minimal surfaces* (surfaces of locally-minimal area) that span a given contour. First, he assigned to each of the minimal surfaces a characteristic number λ_i analogous in meaning to the characteristic numbers in the bowl, in such a way that $\lambda_i = 0$ corresponds to a *stable* minimal surface with minimal area. A consequence

[916] *Characteristic number* of an equilibrium point is equal to the maximal number of mutually perpendicular directions in which the height from equilibrium point diminishes. Thus, for example, a stable point (minimum potential energy) has $\lambda = 0$, while a saddle point has $\lambda = 1$.

of Morse's equation is that the existence of n minimal surfaces of minimal area that span a given contour implies that there must be another $(n-1)$ *unstable* minimal surfaces bounded by the same contour.

The relation $\sum_i(-1)^{\lambda_i} = V - E + F$ reflects a profound statement that connects topology with the concept of equilibria in the calculus of variations. Morse's equation is forced upon us by some mathematical structure hidden within the topological nature of the bowl.

1936–1965 CE **Max Horkheimer** (1895–1973, Germany and USA). Social philosopher. Regarded philosophy as culture criticism and maintained that the enlightened spirit of modern man destroys itself by striving for a pleasant and fair existence. Accordingly, the ills of modern society are caused by the misuse and the misunderstanding of reason. If people use true reason to critique their societies, they will be able to identify and solve their problems.

Horkheimer was born in Stuttgart to an assimilated Jewish family. He left High School (1911) to work in his father's factory and then participated (1917–1918) in WWI. He enrolled in the University of München (1919) and studied philosophy and psychology, obtaining his Ph.D. degree in 1925. There he struck a lasting friendship with **Theodor Adorno** with whom he collaborated throughout his life. In 1930 he founded, with **Herbert Marcuse** the Frankfurt (Main) *Institute for Social Research*, and was appointed the full professor for social philosophy. The Nazis closed the institute (1933) and Horkheimer emigrated to the USA via Switzerland. In 1949 he returned to Frankfurt, where the Institute was reopened (1950). He returned to America to lecture at the University of Chicago (1954–1959). He died in Nürnberg (1973). His most important publications are

- *Authority and the Family* (1936).
- *Traditional and Critical Theory* (1937).
- *Eclipse of Reason* (1947).
- *The Dialectic of Enlightenment* (1947).
- *Critique of Instrumental Reason* (1967).

The last book deals with the concept of "reason" within the history of Western philosophy. Horkheimer defines true reason as rationality. He details the difference between objective and subjective reason and states that we have moved from the objective to the subjective. Objective reason deals with universal truths that dictate that an action is either right or wrong. Subjective reason takes into account the situation and social norms. Actions that produce the best situation for the individual are "reasonable" according to subjective reason. The movement from one type of reason to the other occurred when thought could no longer accommodate these objective truths or when it judged them to be delusions. Under subjective reason, concepts lose their meaning. All concepts must be strictly functional to be reasonable. Because subjective reason rules, the ideals of a society, for example democratic ideals, become dependent on the "interests" of the people instead of being dependent on objective truths.

Horkheimer's programmatic essay on 'Traditional and Critical Theory' (1937) enshrined the ambitions of the Institute. It described the necessity of integrating philosophy and social science, and of developing a relationship of integrity between critical theory and political practice. In later years, Horkheimer's vision became increasingly dark and gloomy. His later writings evince the difficulty — even the impossibility — of fulfilling the original ambition and programme of the Frankfurt School[917]. The result is an increasingly sharp critique of 'enlightened' reason and Western rationality. The *Dialectic of Enlightenment* (1947), written in collaboration with Adorno, was the first and most powerful statement of this theme.

1937 CE **Arne Wilhelm Kaurin Tiselius** (1902–1971, Sweden). Biochemist. Developed new methods of separation of colloids though *electrophoresis*[918] and used it for studying proteins.

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917 The "*Frankfurt School*" aimed to put philosophical ideas to the task of diagnosing social problems. It was comprised extraordinarily by a distinguished collection of leftists, mostly Jewish, philosophers and social thinkers that gathered in the *Institute of Social Research* at the University of Frankfurt. After the war, when most of its members returned from exile, the institute was led by **Theodor Adorno** and **Max Horkheimer**. Horkheimer's chair was given to **Jurgen Habermas** (1964).

918 *Electrophoresis*: A technique for the analysis and separation of colloids, based on the movement of charged colloidal particles in an electric field. There are various experimental methods. In one the sample is placed in a U-tube and a buffer solution added to each arm, so that there are sharp boundaries between buffer and sample. An electrode is placed in each arm, a voltage applied, and the motion of the boundaries under the influence of the field is observed. The rate of migration of the particles depends on the field, the charge on the particles, and on other factors, such as the size and shape of the particles. More simply, electrophoresis can be carried out using an adsorbent, such as a strip of filter paper, soaked in a buffer with two electrodes making contact. The sample is placed between the electrodes, and a voltage applied. Different components of

Tiselius was born in Stockholm. He studied at the University of Uppsala (1925–1930), specializing in chemistry under **Svedberg** (Ph.D. 1930). He was a professor at Uppsala, 1938–1968. Awarded the Nobel Prize for chemistry (1948) for his studies concerning the nature of *serum proteins.*

Electrophoresis became widely developed in the 1940s and 1950s when the technique was applied to molecules ranging from the largest proteins down to amino acids or even inorganic ions.

1937–1940 CE **George Robert Stibitz** (1904–1995, USA). Mathematician, computer scientist and inventor. Father of the modern digital computer. Designed and built the *Complex Number Calculator*, the world's first *electrical* digital computer. The design began in April 1939 and the end product first ran on January 08, 1940. Its "brain" consisted of 450 electromechanical telephone relays and 10 cross-bar switches, and it could find the quotient of two eight digit complex numbers in about 30 seconds. Three teletypewriters provided input to the machine.

Born in York, PA, Stibitz earned a Ph.D. in applied mathematics (1926) and a Ph.D. in physics from Cornell University (1930). He then joined Bell Telephone Laboratories (1930), serving as a mathematical consultant. Stibitz's interest in computers arose from an assignment to study the magneto-mechanics of telephone relays; he turned his attention to the binary circuits controlled by the relays, to the arithmetic operations expressible in binary form, and, in November 1937, to the construction of a two-digit binary adder based on relays, flashlight bulbs, and metal strips cut from tin-cans.[919]

In 1940, Stibitz performed a spectacular demonstration at the Dartmouth (NH) meeting of the American Mathematical association: Leaving his computer in New York City, he took a teleprinter to the meeting and proceeded to connect it to his computer via telephone — the world's first demonstration of remote computing. During WWII he designed program-controlled calculations for the military, but these were soon to be outmoded by the development of *electronic* digital computers.

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the mixture migrate at different rates, so the sample separates into zones. The components can be identified by the rate at which they move. This technique has also been known as *electrochromatography.*

Electrophoresis is used extensively in studying mixtures of proteins, nucleic acids, carbohydrates, enzymes, etc. In clinical medicine it is used for determining the protein content of body fluids.

919 The machine, called "Model K" (because most of it was constructed on his kitchen table), worked on the principle that if two relays were activated, they caused a third relay to become active, where this third relay represented the resultant of a binary operations.

Thereafter he developed a precursor of the electronic digital microcomputer (1954) and eventually applied computer systems development to a wide variety of topics in biomedicine. He became professor of physiology at the medical school of Dartmouth College (1966).

1937–1944 CE **Isidor Isaac Rabi** (1898–1988, U.S.A.). Experimental physicist. Perfected the technique of *molecular beams* and made it into a potent tool for measuring magnetic properties of molecules and atomic nuclei *with great accuracy*. For this he was awarded the Nobel prize for physics (1944).

Rabi was born in Rymanov, Poland, to Jewish parents and was brought at the age of one year to the United States. He studied at Cornell and then at Columbia University (Ph.D. 1927). Rabi spent the next two years in Europe, including some time in the laboratory of Otto Stern, where he acquainted himself with the technique of molecular beams. In 1929 he joined the faculty at Columbia. From 1940 to 1945 he was associate director of the *M.I.T. Radiation Laboratory*. He returned to Columbia after WWII.

Rabi and his collaborators introduced for the first time the induction of radio-frequency resonance in magnetic moment measurements. In their original experiments on nuclear moments, the beam was first deflected in an *inhomogeneous* field $\boldsymbol{B}_A$ and a particular component was selected by passing the beam through a slit.

This component then traversed a *homogeneous* field $\boldsymbol{B}_B$, which introduced no deflection, and was finally deflected back to the axis of the apparatus by a third field $\boldsymbol{B}_C$, which was inhomogeneous with the gradient opposed to that of $\boldsymbol{B}_A$. A beam detector was placed to record the arrival of atoms which traversed the three fields *with their spin and magnetic moment fixed throughout*.

A small magnetic field oscillating with a high frequency was then imposed on the beam in the homogeneous field region ($\boldsymbol{B}_B$) and the frequency was varied until the beam signal showed a sharp "resonance" dip. This dip was due to *defocusing* of the beam and indicated that the spin orientation was significantly affected in the $\boldsymbol{B}_B$ region.

1937–1949 CE **William Webster Hansen** (1909–1949, USA). Physicist. Co-inventor[920] of the *Klystron tube* and founder of microwave electronic technology. Contributed to developments on Doppler radar, aircraft blind-landing systems, electron acceleration (SLAC), and nuclear magnetic resonance.

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920 With the **Varian** brothers. **Edward Leonard Ginzton** (1915–1998, USA) applied the Klyston in satellite communications, airplane and missile guidance systems, telephone and television transmission, and other important applications (1942–1959). He succeeded Hansen in the Directorship of the Stanford

Hansen was born in Fresno, California. He received his doctorate in physics from Stanford University (1933) and joined the faculty there (1934). Hansen was a professor of physics and Director of the Microwave Laboratory at Stanford (1939–1949).

He invented the high-quality cavity resonator on which the linear electron accelerator depends. During WWII he worked in New York on defense applications of physics and electronics, including *radar*.

1937–1955 CE **Emilio Gino Segrè**[921] (1905–1989, Italy and USA). Experimental nuclear physicist. Discovered the first synthetic element *technetium* (1937), the new element *astatine* (1940) and the *antiproton* (1955).

Segrè was born in Tivoli, near Rome, and educated at Rome University, studying engineering and then physics (Ph.D. 1928). He remained at the university, working with Fermi until 1936, when he became a professor at Palermo. But in 1938 he was dismissed from this post and forced into exile by the Fascist government.

Apart from wartime research at Los Alamos on the Manhattan Project he worked from 1938 at the University of California at Berkeley, where he became professor (1947). In that year Segrè started work on proton-proton and proton-neutron interaction, using a cyclotron accelerator at Berkeley. This was how he created and detected the antiproton (antiparticle of the proton with identical mass but negative electric charge), which reconfirmed the relativistic quantum theory of Paul Dirac. He shared the 1959 Nobel Prize for Physics with his co-worker **Owen Chamberlain** (b. 1920).

1937–1964 CE **Arthur Erdélyi** (1908–1977, U.S.A. and England). Mathematician. A leading figure in American mathematics in the post-war development of the subject. Contributed to many fields in mathematical analysis. Among them: special functions[922], operational calculus, asymptotic expan-

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Microwave Laboratory (1949–1959). Under Ginzton's supervision, the preliminary design of the SLAC accelerator was completed in 1961.

921 **Segre** (also **Segri**) — an Italian Jewish family of rabbinical scholars since the 14th century. Members of the family in modern times include: **Corrado Segre** (1863–1924, Mathematician); **Arturo Segrè** (1874–1928, Historian); **Beniamino Segrè** (1903–1977, Mathematician).

922 He derived (1937) the integral representation of a multipole spherical eigenfunction: $h_\ell^{(2)}(k_c r)P_\ell^m(\cos\theta) = \frac{i^{m-\ell+1}}{k_c}\int_0^\infty J_m(k\Delta)e^{-\nu_c|z-z_0|}P_\ell^m\left(\frac{i\epsilon\nu_c}{k_c}\right)dk$, where $r^2 = \Delta^2 + (z - z_0)$, $\nu_c = \sqrt{k^2 - k_c^2}$, $\epsilon = \mathrm{sgn}(z_0 - z)$, J_m is the Bessel function of the m^{th} order, P_ℓ^m is the associated Legendre function, and $h_\ell^{(2)}$ is the spherical Hankel function of the second kind.

sions, integral equations and integral transform theory. Headed an international team of applied mathematicians that issued the encyclopedic source books on Higher Transcendental Functions and Integral Transforms (five volumes), which summed up all the accumulated lore in that field since Euler.

Erdélyi was born in Budapest, Hungary to Jewish parents. He then continued his studies in Brno, Czechoslovakia. The German occupation put him in mortal danger, but thanks to **Edmund Whittaker**, he was able to come to Edinburgh in 1939. In 1949 he moved to the California Institute of Technology, where he directed the famous Bateman project. He returned to the University of Edinburgh in 1964.

1937–1966 CE **Nahum Il'ich Akhiezer** (1901–1980, Russia). Mathematician. Made important contributions to function theory and approximation theory. Created the Kharkov school of mathematics.

Akhiezer was born in Cherikov, Belarus, to a Jewish family. He graduated (1924) from Kiev University an taught there until 1933. He then moved to the Kharkov Polytechnic Institute (1941–1956). From 1956 till the end of his life he worked at Kharkov State University.

1938 CE **Ladislau** and **Georg Biro** (Hungary) patented the ball-bearing point pen.

History of Writing Instruments — from the bamboo reed to the ball-point pen (3100 BCE–1938 CE)

ca 100,000 BP Early humans developed *symbolic thinking* manifested in linguistic speech capacity.

ca 3100 BCE First fully developed system of *word-writing*: Sumerians in Mesopotamia used *split bamboo reed* as stylus to write words and numbers on *wet clay tablets*. Only a few hundred words were used.

ca 3000 BCE Advent of *pictographic writing* on papyrus in Egypt; soot mixed with water served as ink.

ca 2700 BCE **Tien-Lcheu** (China) invented "Indian Ink" — a mixture of soot from pings smoke, lamp oil mixed with gelatin of donkey skin and musk.

ca 2200 BCE Oldest extant document written on *papyrus*.

ca 1500 BCE Origin of the oldest *phonetic alphabet* in the Sinai peninsula.

ca 1313 BCE The Phoenician **Cadmus** invented the *Phoenician alphabet* and the *written letter*.

ca 1300 BCE Chinese's pictographic language has a vocabulary of ca $50,000$ words.

ca 1200 BCE *Ink* becomes common in China.

ca 600 BCE Final stages of current *Hebrew alphabet* and writing.

ca 530 BCE A *library* in Greece.

ca 400 BCE Development of *Greek alphabet*. They employed a writing stylus made of either *metal*, *bone* or *ivory* to place marks upon *wax-coated tablets*. These tablets were made in hinged pairs that could be closed to protect the scribe notes.

Chinese write on silk.

ca 300 BCE *Greeks* created the *calamos* ($\kappa\alpha\lambda\alpha\mu o\varsigma$): a *reed pen* suitable for *parchment* and *ink*. They converted bamboo stems (of marsh grasses) into a form of fountain pen, cutting one end into a form of pen nib or point. A writing fluid (ink) was poured into the stem, then the reed was squeezed, forcing fluid to the nib.

ca 200 BCE *Parchment* developed in the city of Pergamum (now in Turkey); a superior writing material made of animal skin.

ca 50 BCE Men discovered that the sharpened goose *quills* (large feathers) made excellent writing instruments. [The word *pen* comes from the Latin *penna* which means *feather*.]

ca 79 AD A *bronze pen* in Pompei.

ca 100 AD *Wood-fiber paper* invented in China.

ca 400 AD A stable form of *ink* was developed — a composite of iron salts, nut-galls and gum; this basic formula was to last for centuries.

ca 600 AD *Books* printed in China.

ca 700–711 Chinese *wood-fiber paper* became known in Japan (700 CE); brought to Spain by the Arabs (711 CE).

ca 700 AD *Quill-pen* introduced in Europe; it was to remain predominant for over a thousand years.

The strongest quills were those extracted from living birds in the spring – from the five outer left-wing feathers (carved outwards and away when used by right-handed writer). Birds used were: goose, swan, eagle, owl, hawk, turkey. Quill-pens lasted for only a week before it was necessary to replace them.

The early European *parchment* was made from animal skin, which required much scraping and cleaning.

During the centuries that followed, metal points, often called *nibs*, were added to the quill. The nib-tips did not wear out as fast as quill-tips.

ca 765 AD *Picture books* printed in Japan.

ca 1000 AD Mayas in Yucatan, Mexico, make writing paper from tree bark.

1436 AD Invention of the *printing press* with replaceable wooden or metal letters by **Johannes Gutenberg** (Germany).

1565 AD Invention of the *pencil.*

1650–1685 Some pens were made entirely out of *metal*, sometimes with *precious stones* as the tip.

ca 1700 AD Advent of *fountain pens.*

1750–1850 The Industrial Revolution heralded the doom of the quill-pen. It began with *steel nibs* that could be inserted into a holder (1750). Then, the whole pens were made in the form of a *metal tube or barrel* (1809); *machine-made pens*, *gold-pens* (1810–1820); In 1822 horn and tortoise-shell were patented to the formation of pen-nibs, the points of which were rendered durable by small pieces of diamond or ruby, or by attaching small piece of thin gold sheet over the end of the tortoise-shell. By 1850, pen manufacturers were using alloys of *rhodium*, *osmium* and *iridium* to make very hard tips.

1884 AD **Lewis Edson Waterman** (US) patented one of the first *practical fountain pens*. The pen was filled with ink squeezed from an eyedropper.

1886 AD **John Loud** (US) invented a ball-point pen.

1913 AD **W.A. Sheaffer** (US) developed a lever-fill fountain pen.

1927 AD Disposable ink cartridges for fountain pens were developed.

1938 AD **Ladislau and Georg Biro** (Hungary) patented the *ball-bearing point pen*.

1945 AD First commercially successful ball-point pens.

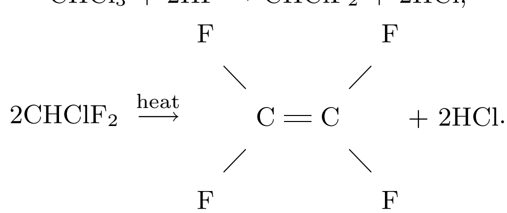
1951 AD Marker pens.

1938 CE **Roy J. Plunkett** (1910–1994, USA). Engineer at Du Pont, U.S.A., accidentally invented *Teflon*[923] (or PTFE), a polymer of tetra-fluoro-ethylene[924] (Poly-Tetra-Fluoro-Ethylene) that does not occur in nature. Plun-

[923] The name Teflon is derived by combining the chemists' name for tetrafluoroethylene — *tef* — with an arbitrary suffix — *lon* — that Du Pont chose for its products, as in nylon, orlon, etc.

[924] Tetrafluoroethylene is prepared in the following manner:

$$CHCl_3 + 2HF \rightarrow CHClF_2 + 2HCl;$$



$$\text{Polimerization yields} \quad n(CF_2 = CF_2) \xrightarrow[\text{pressure catalyst}]{\text{heat}} (CF_2CF_2)_n.$$

kett had been looking for a new kind of refrigerant, a gas to be used in air conditioners, to suck out the heat. He thought he was mixing together a batch of *tetrafluoroethylene* and *freon* (CCl_2F_2). But instead of finding the gas in his steel reactor bottle as expected, he found a white powder; inside the tank the gas molecules had formed into long chains.

Teflon became known to the public only in 1946 because it had been kept as a military secret during WWII; scientists working on the first nuclear bomb — the Manhattan Project — needed it for making gaskets that could resist the corrosive uranium hexafluoride (UF_6), used in the separation of uranium isotopes.

Teflon is an unusual plastic, which is extremely resistant to flames, oxidation and strong acids or bases. Its chemical and thermal stability ($-240\,°C$ to $260\,°C$) can be traced to two features. One is the considerable strength of the $C{=}C$ and $C{-}F$ bonds, which keeps the molecules from decomposing even when moderately heated. The second feature is the match between the sizes of the fluorine and carbon atoms, which result in the fluorine atoms forming an almost continuous sheath around the carbon atom chain, protecting it from chemical attack. In effect, the fluorine atoms act as chemical insulation around the carbon-atom "wire".

PTFE consist of very long chains, composed of about 50,000 $\{— CF_2 —\}$ groups each, with very little cross-linking between them. As a result, the molecules pack together to give a dense, compact solid with a high melting point. Grease and oil do not form bonds with PTFE, so surfaces coated in it are "nonstick" and PTFE feels slippery to the touch. Its molecules pack together so densely that the solid does not absorb water, and hence it is an excellent electrical insulator.

The chemical industry uses Teflon in corrosion-resistant gaskets, valve packing, cable insulation, bearings, fry pan coating, clothing and artificial body parts.

1938 CE, Oct 24 *Wages and Hours Law* became operative in the United States of America. It provided for humane wages, an 8 hours working day and a maximum of 40 weekly hours in industries affecting interstate commerce. It also prohibited child labor.

1938–1939 CE **Hans Albrecht Bethe** (1906–2005, U.S.A.). Physicist. Proposed a new theory for the energy production mechanism, important in some main-sequence stars: the nuclear fusion of hydrogen into helium, catalyzed by carbon nuclei[925]. This theory was confirmed by a combination of laboratory nuclear collision experiments and astrophysical observations. Moreover, it is believed today that nuclear reactions in the interior of very hot stars are the source of continual heavy-element build up; once the star's hydrogen is all fused into helium, ^4H nuclei proceed to fuse into Beryllium, Carbon and heavier elements.

Bethe also applied classical mathematical methods to the calculation of electron densities in crystals, the order-disorder states of alloys, the operational conditions of reactors and the ionization processes in shock waves.

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[925] Our sun is a nuclear furnace that turns mass into energy. Every second it converts over 657 million tons of hydrogen into 653 million tons of helium. The missing 4 million tons of mass are discharged into space as radiant energy. The earth receives only about one two-billionths of this. It has been estimated that in 15 minutes our sun radiates as much energy as mankind consumes in all forms, during an entire year. Solar fusion of ^1H into ^4He is mostly *direct*, with Bethe's CNO (Carbon-Nitrogen-Oxygen) cycle accounting for only a few percent of fusion reactions; but the CNO mechanism dominates in hotter main-sequence stars.

The sun is approximately 150×10^6 km from earth, 1.392×10^6 km in diameter and has a mass of approximately 2×10^{30} kg. It is an 'average' star in size, brilliance and age. There are more than 10^{11} stars in our sun's own galaxy, the Milky Way. Light energy, with a temperature of about 5800 degrees Kelvin is received on earth, from the sun. It takes light 8.3 min to travel from sun to earth. Supposing no major change in the sun's stability will occur until about 10^{-3} of its present total mass has melted away in nuclear reactions, it could keep going for 1.7×10^{10} years at the present rate of generating radiation. However, astrophysical theory predicts a gradual increase in solar power output, which will wreak havoc with earth's biosphere as early as 10^9 y from now. It also predicts that solar-core hydrogen will run out ca 5×10^9 y from now, leading to a collapse that will cause hydrogen in outer shell into fusion, followed by a dramatic *expansion* and *cooling* of the sun, and core burning of helium. In this *red giant phase* (starting ca 7×10^9 y from now) the expanding sun will engulf and incinerate the inner planets, possibly including earth. The present power output of the sun is 3.6×10^{23} kilowatts. The upper limit of the solar power available on the surface of the earth is about 1.6×10^{14} kilowatts. Solar power alone should thus be sufficient, in principle, to satisfy the energy requirements of man for the foreseeable future. In fact, most of our present energy consumption, whether in the form of hydroelectric power, food, lumber, or fossil fuels, can be traced back to solar energy.

Bethe was born in Strasbourg. He was educated at the Universities of Frankfurt and Münich, where he obtained his Ph.D. in 1928. He worked with **Fermi** in Rome (1931) and lectured on physics at the University of Tübingen (1933). Because his mother was Jewish, he was forced to leave Germany at the beginning of the Nazi regime. After a stay in Manchester, he emigrated to the United States in 1935 and settled at Cornell University, where he was a professor from 1937 to 1975, and a professor emeritus thereafter.

During 1943–1946 he headed the Division of Theoretical Physics of the Manhattan Project in Los Alamos, NM, and made important contributions to the development of the theory of the atomic nucleus and nuclear reactions. He received the Nobel prize for physics in 1967.

1938–1939 CE **Russell Harrison Varian**[926] (1898–1959, U.S.A.) and his brother **Sigurd Fergus Varian** (1901–1961, U.S.A.). Physicists. Invented the *klystron* for the generation and amplification of microwaves, during their research into RF electronics at Stanford University. Russell and Sigurd were born in Washington, D.C. and Syracuse, N.Y., respectively; Russell became president of *Varian Associates* (1948–1959).

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[926] For further reading, see:

- Varian, Dorothy, *The Inventor and the Pilot*, Pacific Books, 1983, 314 pp.

The Klystron[927]

Two separate principles are combined in the klystron oscillator: the velocity modulation of an electron beam, and the use of cavity resonators as tuned circuits. When the wavelength to which a circuit is tuned is not much greater than the physical dimensions of the circuit, a coil and a condenser arrangement becomes impracticable, and must be replaced by a length of transmission line in a coaxial-line triode. At still shorter wavelengths even quarter-wave lines become awkwardly small and, in their turn, are replaced by cavity resonators. In general the wavelength corresponding to the fundamental mode of a cavity is of the order of magnitude of its linear dimensions.

A resonant cavity is a high frequency descendant of the parallel resonant LC *circuit. Such an* LC *circuit will resonate at a very high frequency* ω_0 *if the inductance* L *and capacitance* C *are very small since* $\omega_0 = \frac{1}{\sqrt{LC}}$. *A relatively high frequency* LC *circuit might consist of two parallel plates joined by a single inductive turn of wire.*

In order to reduce the inductance, more turns may be added in parallel. The closed surface may be thought of as an infinite number of turns in parallel and represents a minimum value of inductance. The resulting cavity is a region of space surrounded by conducting walls.

The high-frequency currents in a cavity flow effectively only in a narrow layer of the inner surface, the width of which is known as the skin depth. (At 10,000 MHz the skin depth is of the order of 10^{-4} *cm for copper.) The energy is stored in the electric and magnetic fields within the cavity. Unlike an* LC *circuit, a cavity may have many resonant frequencies, each associated with a different mode of oscillation. These frequencies are functions of the cavity geometry, as are the fundamental frequency and harmonics of an acoustic organ pipe.*

The electromagnetic energy involved in the excitation of a mode of oscillation, oscillates between energy stored in the magnetic field and energy stored in the electric field. The oscillating fields induce currents in the cavity walls, which may dissipate energy in the form of heat since the walls are resistive. In a steady-state condition, the supply of exciting energy must be equal to the

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[927] To dig deeper, see:

- Hemenway, C.L. et. al., *Physical Electronics*, Wiley, 1962, 396 pp.
- Parker, P., *Electronics*, Edward Arnold: London, 1950, 1050 pp.

power lost in the cavity walls. The ratio of the stored energy to the energy dissipated in a cycle of oscillation is called the Q *of the circuit. The* Q*'s of cavities have typical values of* 10^4*. A high* Q *value also means a sharp tune or narrow bandwidth response.*

Coupling of electromagnetic power into and out of resonant cavities is sometimes achieved by means of a small loop of wire, which couples with the magnetic field within the cavity and joins to an external coaxial transmission line.

When an electron beam passes near an electrode connected to an impedance, a current may flow through the impedance only if the velocity of the electrons, as they pass near the electrode, is time dependent (modulated). This is so because an electron beam is equivalent to a current, and a d–c current delivers no a–c power by induction.

Velocity modulation is achieved by impressing a small a–c component of velocity on a d–c electron beam. This may be done by allowing the beam to pass through two grids across which a small a–c voltage is applied. As the electrons leave the modulating grids, the faster electrons move away from the slower electrons behind and overtake the slower electrons ahead. The numerical density of electrons further along the beam is no longer uniform, and the beam is said to be bunched*. Because of this bunching the beam current has acquired an a–c component. Thus, the velocity modulation imparted to the beam in passing through the grids gives rise to current modulation further along the beam.*

A–c power may be extracted *from the current-modulated electron beam by allowing it to pass through a second pair of grids connected to an external load impedance. The beam induces currents in the impedance and loses energy. In* klystrons*, the grids and impedances have the physical form of resonant cavities, in order to velocity – modulate an electron beam and extract useful a–c power at high frequencies.*

Details of the mechanism are as follows: An electron gun sends a beam of electrons into a tube connecting of two resonant cavities. The cavities are separated by a region called the drift space. The ends of the reentrant parts of the cavities may be wire grids. The cavity nearer to the gun is called the input or buncher cavity*. The signal couples to the RF magnetic field of the input cavity by means of a coaxial line, which has its center conductor joined to a small loop within the cavity. The input signal appears as an a–c voltage between the grids of the buncher cavity, and velocity-modulates the beam.*

Let us suppose that the electrons, upon entering the input cavity have been accelerated through a potential V_0 *and that a small a–c voltage* $V_1 \cos \omega t$ *is established between the grids and the cavity. The velocity* v *of the electrons as they emerge from the cavity is* *given by* $v = \sqrt{2\eta V} = \sqrt{2\eta}(V_0 + V_1 \cos\omega t)^{1/2} \approx v_0 \left(1 + \frac{V_1}{2V_0}\cos\omega t\right)$, *where* $v_0 = \sqrt{2\eta V_0}$, $\eta = e/m_e$ *and* $V_1 \ll V_0$ *is assumed, provided the time for the electron beam to pass between the grids is small compared with a period of an a–c cycle.*

In the drift space, bunching occurs: faster electrons start to pass slower ones, with the result that the beam current reaches a large value once each cycle*. But the current is* not *sinusoidal and contains many harmonics. For that reason, the position of the maximum a–c current of the fundamental mode is* not *located at the position of maximum bunching.*

The modulated beam passing through the output cavity induces a net a–c current on the inside surface of the cavity walls. This induced current initiates the oscillations of the cavity when the modulation is first turned on. The amplitude of the output cavity oscillations builds up at the expense of the electron's d–c kinetic energy. In a steady-state condition, the energy lost by the electrons equals the sum of the power delivered to the load and the energy dissipated in the cavity walls.

The relative phase is such that when the number of electrons crossing the gap is largest, the oscillating electric field of the cavity retards the electrons and extracts energy from the beam. On the average, power is delivered to the output cavity because more than half *the electrons pass through the cavity when the phase of the electric field is such as to* decelerate *the electrons and* less than half *the electrons pass through the cavity when the field is* accelerating*.*

To summarize, the klystron is an energy converter: The d–c kinetic energy of the electrons is converted to the a–c energy of the electromagnetic fields in the cavity. [In contradistinction, the magnetron *converts the d–c* potential energy *of electrons to a–c energy.] Power amplification in the klystron is achieved because only a small amount of a–c power is required to velocity modulate the electrons, and a large amount of output power is supplied by the kinetic energy of the electrons. The power amplification of a klystron can reach the order of 1000 or greater.*

The output cavity is connected to a load impedance by means of a coaxial line so that when the cavity is excited, a–c power is delivered to the load impedance. A collector electrode, maintained at output cavity potential, intercepts the beam as it emerges from the output cavity. Both cavities generally have the same resonant frequency.

1938–1943 CE Swiss chemists **Albert Hoffmann** and **Arthur Stoll** discover and name (1938) *lysergic acid diethylamide*, or LSD. Hoffman discovered (1943) that LSD is hallucinogenic. LSD became widely used as hallucinogen in the 1960's.

1938–1947 CE **Chester Carlson** (1906–1968, U.S.A.). Physicist and inventor. Invented *xerography* — an electrostatic process for copying printed material, using photo conductive materials (e.g. selenium compounds) to form an image. Carlson was born in Seattle, Washington.

By the age of 14, Carlson was supporting his invalid parents, yet he managed to earn a college degree from the California Institute of Technology, Pasadena, in 1930. After a short time spent with the Bell Telephone Company, he obtained a position with the patent department of P.R. Mallory Company, a New York electronics firm.

Plagued by the difficulty of getting copies of patent drawings and specifications, Carlson began in 1934 to look for a quick, convenient way to copy line drawings and text. Since numerous large corporations were already working on photographic or chemical copying processes, he turned to electrostatics for a solution to the problem. Four years later he succeeded in making the first xerographic copy.

Carlson obtained the first of many patents for the xerographic process in 1940, and over the next four years tried unsuccessfully to interest someone in developing and marketing his invention. More than 20 companies turned him down. Finally, in 1944, he persuaded Battelle Memorial Institute, Columbus, Ohio, a non-profit industrial research organization, to undertake developmental work. In 1947 a small firm in Rochester, NY, the Haloid Company (later the Xerox Corporation), obtained the commercial rights to xerography, and 11 years later Xerox introduced its first office copier. Carlson's royalty rights and stock in Xerox Corporation made him a multi-millionaire.

1938–1979 CE **Oscar (Ascher) Zariski**[928] (1899–1986, U.S.A.). Mathematician. Developed an abstract theory of *algebraic geometry*; abandoning topological and analytical methods he turned to modern algebra as a means of elucidating basic geometric ideas, introducing the notions of *valuation*, *integral closure* and *saturation*. He found strong links between the algebra of polynomials and the geometry of curves.

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[928] For further reading, see:

- Parikh, C., *The Unreal Life of Oscar Zariski*, Academic Press: New York, 1991, 264 pp.

Zariski was born in Kobrin, White Russia, son of a Jewish Talmudic scholar in what was then known as the Pale of Settlement. He spent his first eleven years in a traditional, almost exclusively Jewish society. During 1914–1918 he attended the gymnasium (high school) in Chernigov and from 1918 to 1920 he was a student at the University of Kiev.

In 1921 he moved to the University of Rome, Italy, which was at that time the most important center of algebraic geometry in the world. There he came under the influence of the algebraic geometers **Guido Castelnuovo** (1865–1952), and **Federigo Enriques** (1871–1946) [Enriques, brother-in-law of Castelnuovo, was a descendant of Spanish Jews, whose ancestors were expelled from Spain in 1492].

In 1926 Zariski received a Rockefeller foundation fellowship[929], but the rise of Fascism in Italy forced him to leave Rome. He applied to Zürich and Jerusalem, but both universities hired older men. On the recommendation of **S. Lefschetz** he joined in 1927 the faculty of mathematics of the John Hopkins University of Baltimore, U.S.A. In 1940 he moved to Harvard, where he remained for the rest of his life.

Zariski was a man caught up in many of the central conflicts of the 20^{th} century. He was torn between an allegiance to an intellectual world that ignored the politics of race and his emotional need to find safety for those members of his family who escaped the Nazi Holocaust. Intellectually, he was torn between a love of the free-spirited, creative Italian vision of geometry and his appreciation of the need for strict logical rigor which he found in the Bauhaus-like school of the abstract German algebraists. Zariski called geometry "the real life" and he lived intensely in the world of mathematics. This commitment led him safely through the turbulence of his 'unreal life'.

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[929] A member of the Harvard Mathematics Department, **G.D. Birkhoff**, had been asked by the Rockefeller Foundation to stop in Rome to discuss the work of the fellows. Zariski remembered, however, that their discussions were not confined to mathematics:

"*Is it difficult for a Jew to become a student at Harvard*?" Zariski asked one evening. "*No, not at all*", Birkhoff replied with no trace of embarrassment, "*although of course we naturally keep a certain proportion. The Jewish population is about* 3% *and we admit only* 3%". "*Then you must have very large classes*", said Zariski, but Birkhoff didn't smile.

Sources of Stellar Energy – the Advent of Nuclear Astrophysics[930] (1939–1957)

The main problem that baffled all astrophysicists concerned with stellar structure and evolution in the 1920*'s and the* 1930*'s, was the unknown source of stellar energy. It was generally agreed that energy is derived from transformation of mass into energy, according to Einstein's law* $E = mc^2$, *where* E *is the amount of energy released,* m *is the mass transformed, and* c *is the velocity of light in vacuum. However, the actual process involved was not known. In* 1919, ***Jean Perrin*** *put forward the notion that the synthesis of Helium from hydrogen is capable of supplying the quantity of energy that is required for the permanent radiation of the sun.*

In 1927, ***A. Eddington*** *showed that the temperatures near the center of the sun must be much greater than had previously been thought. Yet Eddington rejected the transmutation of elements as a possible source of stellar energy. As research into stellar structure and nuclear processes progressed, the alternative rejected by Eddington was shown to be feasible.*

When a gas is heated to temperatures of the order of 10^7 *to* 10^8 *degrees Kelvin, its particles gain (according to the Maxwellian energy distribution) so much energy that the collisions of nuclei can initiate thermonuclear reactions. Such temperatures occur in the interior of the sun and other stars, as astrophysicists have found. Indeed,* ***George Gamow*** *(1904–1968, U.S.A.),* ***Robert d'E Atkinson*** *(England) and* ***Fritz Houtermans*** *(1903–1966, Germany)*[931] *hypothesized already in* 1929 *that thermonuclear processes produce*

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930 To dig deeper, see:

- Shirokov, Yu.M. and N.P. Yudin, *Nuclear Physics*, Mir Publication: Moscow, 1982, Vols I-II (445 pp. + 303 pp.)
- Harwit, M., *Astrophysical Concepts*, Wiley, 1973, 561 pp.

931 All three met at Göttingen in 1928: Gamow arrived from Leningrad, Atkinson was a young British physicist and Houtermans had finished his Ph.D. thesis in 1927 under **James Franck**. In their joint 1929 paper, Atkinson and Houtermans put forward the idea that nuclear reactions are the source of stellar energy, namely, that near the sun's center, hydrogen nuclei might fuse together to produce helium nuclei in a reaction that would convert a tiny amount of mass into a very large amount of energy.

Houtermans' own account of the denouement: 'That evening, after we had finished our essay, I went out for a walk with a pretty girl. As soon as it grew *the heat and light from the sun. A more detailed account was given by* ***Carl von Weizsäcker*** *(b.* 1912*, Germany) in* 1936*.*

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dark the stars came out, one after another in all their splendor. "*Don't they shine beautifully*?" cried my companion. But I simply stuck out my chest and said proudly: "*I've known since yesterday why it is that they shine*"'.

Friedrich Georg Houtermans was born in Zoppot, near the then-German Baltic port of Danzig. He was reared in Vienna as an only child by his mother, who was half-Jewish. Fleeing Nazi Germany in 1933, he went to England. Soon, driven by idealism, he emigrated to the Soviet Union (1933), but fell victim to one of Stalin's purges (1934). He spent a couple of years in prison, where the NKVD had knocked out all his teeth and kept him in solitary confinement for months; his wife, with their two small children, managed to escape to the U.S.A.

When Germany made its temporary pact with the Soviet Union in 1939, it included an exchange of prisoners, and Houtermans was handed to the Gestapo. **Max von Laue**, one of the few German scientists with the prestige and courage to stand up to the Nazis, managed to free Houtermans and arrange for him to work with a wealthy German inventor, Baron **Manfred von Ardenne**, who had studied physics and who maintained a private laboratory in Lichterfelde, outside Berlin. Ardenne was pursuing uranium research independently of Heisenberg and the War Office. To raise funds for the work, he had approached the German Post Office, which commanded a large unused budget for research. The Minister of Posts, imagining himself handing Hitler the decisive secret weapon for the war, had funded the building of a million-volt van de Graaff and two cyclotrons, all under construction in 1941. Until they came on line, Houtermans turned his attention to the *theory of nuclear chain reactions*.

By August 1941, he had independently worked out all the basic ideas necessary to create a bomb. He discussed them in a 39-page report, "*On the question of unleashing chain reactions*", that considered fast-neutron chain reactions, critical mass, U^{235}, isotope separation, and element 94, which the Americans had already secretly named "*plutonium*". Houtermans discussed his ideas privately with **von Weizsäcker** and **Heisenberg**, but kept his report out of reach of the War Office, for whom Heisenberg was working on the very same problem!

However, Houtermans redeemed himself from Heisenberg's fate by making surreptitious contact with the Fermi group in Chicago. Long before completing the first nuclear reactor in December 1942, the Chicago group received a cable from Switzerland sent by someone at Houtermans' direction. "*Hurry up*", it said tersely. "*We are on the track*".

After the war, physics research in Germany was severely hindered by the Allied Control Commission. These restrictions went so far as to decree an upper limit of 10^9 ohm on resistors. In 1952 Houtermans became a professor at the University of Bern.

From the equation $\frac{1}{2}mv^2 = \frac{3}{2}kT$, *it is seen that the average kinetic energy of a nucleus at the sun's center* $(T \simeq 1.4 \times 10^7\ K)$ *is equivalent to only* 2000 *eV. In spite of the corresponding low value of the mean particle energy, as compared with the millions of eV of particles accelerated in laboratories, protons at that temperature can still initiate a sufficient number of nuclear reactions per second, for three reasons. Because of the Maxwellian distribution, there is always a small fraction of particles with kinetic energy far higher than the mean thermal energy. Furthermore, the very large volumes of stellar cores lead to such a high number of collisions that even reactions with a small probability occur with sufficient frequency. And – last but not least – when two hydrogen-isotope nuclei collide,* quantum tunneling *has a finite probability of occurring: this is a process via which, even if the nuclei's relative speed is insufficient for them to classically overcome their mutual Coulomb potential barrier, the* uncertainty *principle sometimes allows them to "borrow" the requisite kinetic energy for a brief time. Once safely past the Coulomb repulsion barrier, the nuclei's short-range nuclear attraction forces take over and allow the exothermic fusion reaction to proceed. Two protons fuse into a deuteron hydrogen isotope (with a positron and neutrino emitted), as well as other stellar nuclear fusion reactions.*

Two strongly exothermal nuclear reactions were definitely established. The balance of the first reaction cycle *results in the fusion of* 4 *protons to a* ${}^4_2He$ *nucleus with the simultaneous emission of* 2 *positrons and* 2 *neutrinos:*

$$4{}^1_1H \rightarrow {}^4_2He + 2e^+ + 2\nu_e.$$

Energy is released since the mass of four protons is $4 \times 1.00723 = 4.02892$ *atomic mass units, which is larger by* 0.02741 *than the mass of the product* ${}^4He$ *nucleus (4.00151). Therefore, in this reaction, an energy of about* 25 *MeV per helium nucleus is liberated, which corresponds to* 1.5×10^8 *kCal/gram or* 6×10^8 *kCal/mole. The neutrinos released during the reaction are responsible for about ten percent of the energy flux from the sun.*

The next step was the realization that only reactions with capture of protons by light nuclei play a notable part in the interior of stars. The final insight was supplied by ***H.A. Bethe*** *(1939). He showed that the most important source of energy in the heaviest and hottest main-sequence stars is the so-called "carbon-nitrogen-oxygen cycle", in which carbon, nitrogen and oxygen serve as catalysts for the conversion of four hydrogen atoms into one helium atom.*

The CNO cycle occurs in our sun, but is negligible there due to the sun's core temperature, which is low enough to favor direct fusion of hydrogen into helium. But the CNO-catalyzed fusion rate increases extremely rapidly with core temperature, and thus is dominant for main-sequence stars somewhat *hotter than the sun. There are several variants of the CNO cycle; they all occur in stars with relative rates determined by core temperatures. One common CNO cycle proceeds as follows:*

$$\begin{aligned} &{}^{12}_{6}C + p \rightarrow {}^{13}_{7}N, && {}^{13}_{7}N \rightarrow {}^{13}_{6}C + e^{+} + \nu_e, \\ &{}^{13}_{6}C + p \rightarrow {}^{14}_{7}N, && {}^{14}_{7}N + p \rightarrow {}^{15}_{8}O, \\ &{}^{15}_{8}O \rightarrow {}^{15}_{7}N + e^{+} + \nu_e, && {}^{15}_{7}N + p \rightarrow {}^{12}_{6}C + {}^{4}_{2}He, \end{aligned}$$

with the overall *reaction being again*

$$4p \rightarrow {}^{4}He + 2e^{+} + 2\nu_e \ .$$

In addition to dominating helium production in many stars, the CNO mechanism also plays a role in nova explosions, and in the production of heavier elements in post-main-sequence stars.

The presence of these elements in the sun and stars was proved spectroscopically.

1938–1939 CE **Robert Julius Oppenheimer** (1904–1967, U.S.A.). Theoretical physicist and scientific administrator. Laid the foundation for a 'general-relativistic' theory of stellar structure and gave first concrete description of a 'black hole'.

With his graduate student **Robert Serber** (1909–1997, U.S.A.) he studied the relative influence of nuclear and electromagnetic forces in neutron stars (1938).

With his graduate student **George M. Volkoff** (b. 1914, U.S.A.) he put forward (1938) a model of a static spherical star consisting of an ideal high-density 'Fermi fluid' of neutrons[932]. He found that the star is *stable* whenever its mass is not greater than one-third of the solar mass [present day value is 0.7 solar masses, known as the *Oppenheimer-Volkoff limit*]. The model incorporated an equation of state with the GTR equation of hydrostatic equilibrium.

With his graduate student **Hartland Snyder** (1913–1962, U.S.A.), he initiated the study of black-hole physics (1939). They formulated the process

[932] Neutrons, like electrons, when closely packed into a sufficiently small volume, resist further compression because of the *Pauli exclusion principle*.

of *gravitational collapse* of a neutron star and showed that when all thermonuclear sources of energy are exhausted, a sufficiently heavy neutron star will reach a state where its weight will overcome the sum of all opposing forces [centrifugal force due to fast spin, radiation and degeneracy pressures, thermal motion and blast-off of outer material].

On the other hand, the increasing density reduce the energy carried off by escaping radiation due to Doppler period-lengthening caused by the inward motion of collapse, gravitational red-shift due to increased gravity and bending of light rays seeking to escape[933].

An outside observer perceives the collapse as slowing down when the *Schwarzschild radius* $\left(R_S = \frac{2GM}{c^2}\right)$ is approached[934]. As perceived by such an observer, the collapse virtually comes to a standstill — a state of equilibrium. For an observer riding the collapsing surface, however the whole mass will shrink to a point at the center in finite time. No message or object sent by him, even via electromagnetic waves, is capable of ever reaching *outside* of the sphere $r = R_S$, if he is already *inside* that sphere when sending it [such a sphere in GTR is known as an *event horizon*].

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933 Two months before Oppenheimer and Snyder submitted their paper on stellar collapse (*Phys. Rev.* **56**, 455, 1939), Einstein submitted a paper (translated) "On a stationary system with spherical symmetry consisting of many gravitating masses" (*Ann. Math.* **40**, 922, 1939), in which he sought to prove, using his own GTR, that black holes were impossible.

His belief in the inadmissibility of singularities was so deeply rooted that it drove him to show that "*the Schwarzschild singularity at* $r = \frac{2GM}{c^2}$ *does not appear* [*in nature*] *for the reason that matter cannot be concentrated arbitrarily... because otherwise, the constituting particles would reach the velocity of light*". Ironically, the modern study of black holes, and more generally, that of collapsing stars, builds on a completely different aspect of Einstein's legacy-namely, his invention of quantum-statistical mechanics.

Without the effects predicted by Fermi–Dirac *quantum statistics*, every astronomical object would eventually collapse into a black hole, yielding a universe that would bear no resemblance to the one we actually live in.

After 1939, Oppenheimer never worked on the subject of black holes again. In 1947 Oppenheimer became the director of the Institute for Advanced Study in Princeton, NJ, where Einstein was still a professor. There is no record of their ever having discussed black holes. Further progress would have to await the 1960s, when discoveries of quasars, pulsars and compact X-ray sources reinvigorated thinking about the mysterious fate of stars.

934 As always in GTR, care must be taken to identify which coordinate system is meant, when coordinate values are specified.

As perceived by an external observer $(r > R_S)$, the comoving observer's fall is infinitely slowed down as he approaches the Schwarzschild radius. Any matter or radiation that becomes trapped inside $r = R_S$, simply increases the mass of the resulting *black hole*[935].

Oppenheimer was born in New York, the son of a Jewish emigrant from Germany, who had made his fortune by importing textiles. His mother, a painter and teacher, died when he was 9 years old. He was a child prodigy. After graduating from Harvard University (where he excelled in Latin, Greek, physics, chemistry and oriental philosophy) in 1925, he continued his studies at Cambridge and Göttingen.

In 1927 he received his doctorate at Göttingen, where he met **Niels Bohr** and **Paul Dirac**. He then accepted professorial positions at the University of California at Berkeley and the California Institute of Technology at Pasadena. Oppenheimer was a brilliant teacher, intense and dedicated — reading no newspapers, owning no radio, and learning Sanskrit as a diversion. In 1939 he began to seek a process for the separation of the isotope uranium-235 from natural uranium and the determination of the critical uranium mass required to make a nuclear bomb.

He became director at Los Alamos in 1943. There, a joint effort of outstanding scientists culminated in the first nuclear explosion on July 16, 1945. It was detonated at Alamogordo, NM, when Germany had already surrendered. In October 1945 he resigned as director of Los Alamos, and in 1947 became director of the Institute of Advanced Study at Princeton (1947–1966). He also served (1947–1952) as chairman of the General Advisory Committee of the Atomic Energy Commission (AEC), and in October 1949 *opposed* development of the hydrogen, thermonuclear, bomb.

After a heated debate with physicist **Edward Teller** and the AEC chairman Lewis Strauss, his security clearance was canceled in 1954 because of his early association with communists in the late 30's and delaying the naming of Soviet agents. A security hearing declared him not guilty of treason, but ruled that he should not have access to military secrets. He retired from Princeton in 1966 and died the following year of throat cancer.

Isadore Rabi said of him:

> "If he had studied the Talmud and Hebrew, rather than Sanskrit, he would have been a much greater physicist. I never ran into anyone who was brighter than he was. But to be more original and profound, I think, you have to be more focused."

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[935] The term 'black hole' was coined in 1967 by **John Archibald Wheeler** (b. 1911, U.S.A.).

1938–1939 CE **Otto Hahn** (1879–1968, Germany), **Lise Meitner** (1878–1968) and **Fritz Strassmann**[936] (1902–1980, Germany), using chemical techniques, collaborated in the discovery of '*nuclear fission*'.

The history of fission reads like a first-class adventure story:
Soon after the discovery of the neutron (1932), **Fermi** began systematic studies of reactions induced by the bombardments of heavy nuclei with neutrons (1934). However, these experiments, especially those where uranium was used, gave puzzling results. Thus, when bombarding uranium atoms with neutrons, isotopes of lighter elements, such as barium and krypton, were detected in the products of the reaction.

In the experiment of Hahn and Strassmann at the Kaiser Wilhelm Institute for Chemistry in Berlin, uranium was bombarded with neutrons, and it broke into two parts, or "fissioned". In the process it released extra neutrons, but the experimenters did not understand the mechanism of the process.

In 1939, **Lise Meitner**[937] and **Otto Frisch**, working in Sweden, explained the results of Hahn and Strassmann as being due a splitting of the heavy uranium atom into two roughly equal parts[938]. They predicted that the fission

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[936] **Fritz Strassmann**, physical chemist. His mastery of analytical chemistry contributed to the team's recognition of the lighter elements that resulted from uranium splitting. From 1945 to 1953 he was the director of the chemistry department at the Max Planck Institute for Chemistry.

[937] In November 1945, three months after the end of WWII, a narrow margin of the members of the Swedish Academy of Sciences decided to award the 1944 Nobel Prize in *Chemistry* to Otto Hahn for the discovery of nuclear fission. Hahn's Berlin colleagues, the chemist Fritz Strassmann and the physicist Lise Meitner, were excluded. Probably Strassman was ignored because he was not a senior scientist. Meitner's exclusion, however, points to other flaws in the decision process, and to four factors in particular:

- difficulty of evaluating an interdisciplinary discovery;
- a lack of expertise in theoretical physics;
- Sweden's scientific and political isolation during the war;
- a general failure of the evaluation committees to appreciate the extent to which German persecution of Jews skewed the published scientific record.

The subsequent effort by prominent physicists to reward Meitner and Frisch with the Nobel Prize in *Physics*, failed.

[938] One of the possible reactions is:

$$n + {}^{235}\mathrm{U} \rightarrow {}^{236}\mathrm{U} \rightarrow {}^{139}\mathrm{La} + {}^{95}\mathrm{Mo} + 2n$$

would release large amounts of *binding energy*. [A heavy nucleus is held together by nuclear forces and its stability against deformation is a result of its tendency to keep its surface area to a minimum. However, the electric charges carried by the protons repel one another, and thus tend to magnify any distortion of the nucleus that may occur. In a sufficiently heavy nucleus, the disruptive effects of the Coulomb repulsion may overcome the surface tension, and the nucleus then *spontaneously splits* into smaller fragments. In a nucleus that is somewhat less heavy than this critical value, splitting may be induced *artificially* by letting it absorb a slow neutron.]

Hypothetically, when free nucleons are brought from infinity (fiducial level of zero energy) to form a nucleus, the total energy of the system must *decrease* by a positive amount ΔE, which is radiated away as photons and/or nuclear fragments. This energy deficit, known as the binding energy of the nucleus, is equivalent, by STR, to a *mass deficit* $\Delta E/c^2$. Upon fission, a similar process takes place: the sum of the rest energies of the two fragments is less than that of the original nucleus. This energy deficiency ΔE, about 200 MeV, is of such size that 1 kg of ordinary uranium is equivalent in its heating effect to 2.5 *million* kg of coal, and the cost of fuel is about 400:1 in favor of uranium.

Another important property of fission is that the resulting fragments are relatively rich in free neutrons. On the average, 2 or 3 neutrons are actually emitted, which makes possible a *chain reaction* involving fissionable materials. The neutrons emitted in the fission of U^{235} have an average energy of 2 MeV, which corresponds to a velocity of $\sim 2 \times 10^9$ cm/s. Therefore, the time that elapses between the emission of a neutron and its capture by a new fissionable nucleus is very small, and the process of multiplication of neutrons in a fissionable substance is quite rapid.

The theoretical understanding of the fission process was advanced by **N. Bohr** and **J.A. Wheeler** in their '*liquid-drop model*' (1939). On his visit to the US (1939), Bohr bore the momentous news that fission of uranium had been demonstrated in Berlin and confirmed in Sweden. The news also reached **Leo Szilard**, who already in 1933 developed the idea of a nuclear '*chain reaction*' and the concept of a '*critical mass*' to create it. (He even

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The ^{235}U captures a neutron, and the compound nucleus ^{236}U is formed. *In the ground state* ^{236}U is essentially stable; it has a half-life of 2.4×10^7 yr. Such a nucleus can perform vibrations about its equilibrium state without fissioning. However, when ^{235}U captures a neutron, the compound nucleus ^{236}U is *excited*, and the amplitude of the vibration can became so large that the nucleus separates into two. The Coulomb force between the two fission products then drives them apart with considerable energy; part of it contributes to the fragment nuclei's own kinetic energy and part goes to evaporate some neutrons.

patented these concepts in 1934, without specifying the element capable of effecting such a reaction!) Szilard immediately realized the military potential of fission for the creation of a nuclear bomb, and from that moment on started feverish activities to convince both scientists and politicians to develop the bomb ahead of the Germans.

This eventually led to the Manhattan Project. In the summer of 1939 Szilard collaborated with Fermi to design the first nuclear reactor and in Dec 2, 1942 he put into operation, with Fermi, the world's first chain-reaction atomic "pile" (reactor) of their design.

Indeed, already in 1939, **Leo Szilard** (1898–1964, U.S.A.) and **Walter Zinn** (1907–2000, U.S.A.) had confirmed that fission reactions can be self-sustaining.

Table 5.18: Timeline of Nuclear Reactors and Weapons

1934	**Leo Szilard** patented in London the idea of an *atomic bomb*.
1934	**Ida Noddack** suggested the idea of *nuclear fission* (which attracted scant attention).
1934	**Enrico Fermi** discovered *induced radioactivity* by slow neutrons.
1936	**Niels Bohr** proposed the *liquid-drop model* of the atomic nucleus.
1936	**Aston** proposed energy production by conversion of hydrogen to *helium*.
1938–1939	*Uranium nuclear fission* after irradiation of uranium 235 by slow neutrons: **Otto Hahn**, **Lise Meitner**, **Fritz Strassmann** and **Otto Frisch**; Idea of *nuclear reactor*(Pile) by **Enrico Fermi** and **Leo Szilard** (1939).
1939	**Hans Bethe**'s theory of energy production in stars.
1942	First self-sustaining nuclear chain reaction in Chicago, headed by **E. Fermi**.
1945	• July 16, 05:29:45 GMT : First *atomic bomb explosion* test at Alamogordo New Mexico, USA ("TRINITY"). Yield = 18.6 kilotons of TNT. • Aug 06 08:16:45 : bombing of Hiroshima; Yield = 12.5 kilotons of TNT, from 60 kg of uranium 235. Bomb's gross weight = 4 tons; 140,000 civilian citizens were killed. • Aug 09 11:02:00 : bombing of Nagasaki; Yield = 22 kilotons of TNT, using 8 kg of plutonium; Bomb's gross weight = 4.5 tons; 70,000 civilian citizens were killed.
1949, Aug 29	The *Soviet Union* explodes an atomic bomb – (an implosion type plutonium bomb) at Semipalatinsk.

1950	USA explodes a *megaton atomic bomb* at Eniwetok atoll.
1952 Nov 01	USA explodes first H-bomb; Yield = 10.4 MT. Gross weight 65 tons. Elugelop island at Eniwetok disappeared.
1954, Jan 21	USA launched *Nautilus*, a nuclear-powered submarine.
1964, Oct	China exploded an atomic bomb at Lop Nor.
1973	USA developed miniature nuclear warhead; Yield 50 tons TNT.
1977	USA decided to develop *neutron bomb*. It differs from standard nuclear weapons insofar as its primary lethal effect come from the radiation damage caused by neutrons it emits.
1983	USA exploded to date 1051 nuclear bombs including 204 secret small scale (below 20 kT) underground tests.
1986, Mar 26	Chernobyl nuclear power-plant disaster.
1995	Nuclear warhead stockpile totaled 9000: 7000 in USA; 480 in Europe; 1500 with submarines.
1996	Nuclear explosion tests on earth have totaled 1452.

Nuclear Power (1932–1945)

Soon after the discovery of the neutron (1932), ***E. Fermi****,* ***E. Segré****, and others began systematic studies of reactions induced by the bombardment of heavy nuclei with neutrons (1934). Since experiments with uranium showed the presence of β particles, researchers thought that they have produced elements with Z (number of protons) > 92. However, In 1938,* ***O. Hahn*** *and*

***F. Strassmann** detected barium after the reaction, indicating that elements with far smaller atomic number than uranium are produced. **L. Meitner** and **O.R. Frisch** suggested (1939) that the uranium nucleus, after neutron capture, must divide itself into two nuclei of roughly equal size, and borrowed the name* fission *from biology. They also pointed out the analogy between the fission process and the division of a small liquid drop into droplets. Further theoretical understanding of the fission process was provided by **N. Bohr**[939] and **J.A. Wheeler** (1939).*

The discovery of fission was missed, for various reasons[940], by quite a few scientists. When it was finally confirmed, the news was carried to the United States by Niels Bohr, and it sparked feverish activity in many laboratories.

When the proton and neutron are well separated from each other, they are completely unbound. However, when they are together in a single nucleus, they are both parts of a bound system; *in the case of the* deuteron *(nucleus of the hydrogen isotope $^2H = D$), the nucleus comprises just one proton and one neutron. The sum of the potential and kinetic energies (w.r.t. infinite separation at rest) of any bound system is negative. Therefore the energy of the deuteron is* less *than the energy of the separated proton and neutron. Because of the relativistic mass-energy equivalence $E = mc^2$, less energy means less mass.*

The binding energy, *B is defined as the minimum energy* released *when a system becomes bound. It is also the minimum energy needed to break a bound system into its constituent parts.*

The parts of a nucleus are its Z protons and N neutrons. Therefore, conservation of mass-energy tells us that

$$M_N c^2 + B = Z m_p c^2 + N m_n c^2. \tag{1}$$

Here, M_N is the mass of the nucleus, m_p is the rest mass of the proton, and m_n is the mass of the neutron. For practical reasons it is customary to recast (1) in terms of atomic[941] *and not nuclear masses, where the atomic mass includes*

939 Previously, **C.F. von Weizsäcker** (1935) and **N. Bohr** (1936) patterned one of the early nuclear models after *liquid drops* which led to an understanding of the dependence of *binding energies* on atomic number, and consequently gave a physical picture of the fission process.

940 **L. Fermi** "Atoms in the Family", University of Chicago Press, Chicago, 1954.

941 The unit of atomic mass has been defined to be 1/12 of the mass of the atom ^{12}C; it is called atomic *mass unit* and abbreviated a.m.u.
In terms of MeV and gram (g), u is given by

$$1\ amu = 931.5\ MeV/c^2 = 1.66043 \times 10^{-24}\ g.$$

the mass of all electrons. So we can add the Z electron masses to Z proton masses to obtain $Z\,{}^1_1H$ atomic masses (ignoring the relatively small electron binding energy). Thus we arrive at the binding energy equation

$$B = [(ZM_H + Nm_n) - M_A(Z, N)]c^2 \tag{2}$$

where B is now the total binding energy of the entire nucleus; M_H is the mass of a neutral ${}^1_1H$ atom (1.007825 u); m_n is the mass of the neutron (1.008665 u); M_A is the mass of the neutral ${}^A_ZE$ atom[942]*, where $A = Z + N$ and $c^2 = 931.5$ Mev/u in the units used. Eq. (2) then states:*

"The binding energy equals the sum of the mass of the parts minus the mass of the whole, all times c^2".

The binding energy per nucleon, B/A, is an important quantity in the theory of fission.

To describe the fission process in a simplified model, we consider one possible fission reaction[943]

$${}^1_0n + {}^{235}_{92}U_{143} \to {}^{236}_{92}U_{144} \to {}^{144}_{56}Ba_{88} + {}^{89}_{36}Kr_{53} + 3\,{}^1_0n. \tag{3}$$

Here the ${}^{235}U$ nucleus captures a low-energy neutron, and the composite nucleus ${}^{236}U$ is formed. In the ground state, ${}^{236}U$ is essentially stable; it has a half-life of 2.4×10^7 y. Such a nucleus can perform vibrations about its equilibrium state without fissioning.

However, when ${}^{235}U$ captures a neutron, the compound nucleus ${}^{236}U$ is highly excited (not at ground state), and the amplitude of vibrations can

942 A semi-empirical formula for the *calculation* of M_A in terms of A, m_n, m_H, and Z was given by **von Weizsäcker** (1935) and **H. Bethe** (1936), and known as the *Bethe-Weizsäcker relation*.

943 A *nuclide* is a particular nuclear species with a given number of protons and neutrons; *Isotopes* are nuclides with the same number of protons (Z); *isotones* are nuclides with the same neutron number (N); and *isobars* are nuclides with the same total number of nucleons (nucleon = neutron or proton). Atoms built around *isotopes* behave the same way chemically, since chemical reactions are determined by the numbers of electrons filling the electron shells, which in turn are determined by the number of protons.
We use the notation ${}^A_ZE_N$ (E for 'element') where $A = Z + N$ (atomic mass number) is the *nucleon number*, Z is the *proton number* and N is the *neutron number*. Z, which is also the number of electrons in a neutral normal atom (atomic number), identifies the chemical element in the periodic table of elements.

become so large that the nucleus separates into two parts. The Coulomb force between the two fragments drives them apart with considerable energy. Not all the available energy goes into kinetic energy; some is stored as internal (excitation) energy of the two fragments. This energy is released primarily by evaporation of neutrons. The main products of the fission process are therefore two roughly equal nuclei and a few neutrons.

An estimate of the energy Q released in fission is made through eq. (2), with M_A for each nucleus calculated via the Bethe-Weizsäcker semi-empirical relation, yielding[944]

$$Q \ (in \ MeV) \approx -4.5A^{2/3} + 0.26Z^2A^{-1/3}. \tag{4}$$

For ^{235}U, $Q \approx 180$ *MeV.*

One can get the same result also by considering the binding energy per nucleon (B/A) which is about 7.8 MeV/nucleon for the heavy elements such as uranium. It increases to about 8.5 MeV/nucleon for the medium-mass fission fragments. Therefore, with 236 nucleons involved, the reaction (3) increases the total binding energy by about

$$(236 \ nucleons) \times (8.5 - 7.6)\frac{MeV}{nucleon} \approx 200 \ MeV \ .$$

But an increase in the binding energy means that the total potential energy of the system has decreased. Thus, the internal energy of the nuclides has decreased by about 200 MeV per fission, thereby releasing 200 MeV of kinetic energy for each fission. About 85 percent of this energy goes to the kinetic energy of the fission fragments. The remaining 15 percent is divided among the neutrons and the electrons, antineutrinos, and γ rays of the radioactive decays of the two heavy fission fragments.

A chemical reaction (e.g. a combustion: hydrocarbon $+O_2 \rightarrow CO_2 + H_2O$*) involves the valence electrons of the atoms, and cannot release more than the energy changes of the valence electrons, so a release of 20 eV per molecular*

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944 The *first* term in (4) is the *surface tension* term in the liquid-drop model, proportional to the nucleus' *area* ($r_{\text{nucleus}} \propto A^{1/3}$ is roughly true empirically, as determined by nuclear scattering cross-sections). This term is negative since surface tension helps to hold a nucleus together.
The *second* term in (4) is due to the Coulomb repulsion between protons, and is proportional to $(Ze)^2/r_{\text{nucleus}}$. This repulsion helps to *destabilize* a nucleus, which is why this term is *positive*. The liquid-drop model is a reasonable approximation because inter-nucleon forces are short-range, fairly non-directional and allow a nucleon freedom of movement — quite similar to inter-molecular forces in a liquid.

reaction would be considerable. However, about 200×10^6 eV per fission yields 10^7 times the energy output per reaction! This explains the immense energy output of fission processes from a relatively small amount of material.

On average, 2.5 neutrons are produced in every slow neutron-induced fission of ^{235}U. The neutrons in one fission process may go on to be absorbed, causing more fissions in what is called a chain reaction. *The average number of neutrons from one fission that actually cause another fission is called the* multiplication factor. *For example, if on average, less than 1 of the 2.5 neutrons produces another fission, the chain reaction will eventually die out. If so, we call the system* subcritical.

If the multiplication factor is exactly 1, the reaction will continue at a constant rate. We then call the system critical. *Finally, if the multiplication factor is greater than 1, the reaction rate increases and we call the system* supercritical. *A nuclear fission bomb (the "atom bomb") is a terrifying example of a deliberately caused supercritical reaction.*

A self-sustaining critical reaction requires that 1.5 of the 2.5 neutrons be removed from the reactions for ^{235}U fission. Some neutrons will simply leak out of the boundary surface of the bulk fissile material. Because of this loss, a certain minimum mass, called the critical mass, *must be present for a self-sustaining critical reaction. The critical mass depends on the fissible material mix and its geometry. Other neutrons will be absorbed within the system in nonfission reactions.*

A nuclear reactor *is a system in which chain reactions can be initiated and controlled. Most present-day reactors in the United States utilize fission of ^{235}U. The fission cross-section of ^{235}U increases as the speed and kinetic energy of the bombarding neutrons decrease. Slow-moving neutrons therefore have the best chance of causing fission before they escape or cause nonfission reactions.*

Let us consider a sphere composed of a pure fissile (fissionable) material ^{235}U, ^{233}U, or ^{239}Pu, unstable with respect to the capture of neutrons. If the diameter of this sphere is larger than the mean free path of the neutrons set free by nuclear fission, a single slow neutron or one spontaneous fission process will initiate an explosion. Namely, two or three neutrons are released on average by each fission process of a ^{235}U nucleus, and each of these neutrons may activate another nucleus to fission, which again releases two or more neutrons, and so on. Such a sequence of reactions is called a chain reaction. The product neutrons initiate further reactions so that the fission reaction spreads through the total fissionable mass by an avalanche-like multiplication. We emphasize that fission in the uranium bomb occurs by fast neutrons in contrast to reactors, where controlled nuclear fission in general is caused by thermal neutrons.

Now the fission of one ^{235}U *nucleus liberates*

$$200 \times 1.6 \times 10^{-6}\ erg = 3.2 \times 10^{-4}\ erg.$$

When this quantity is multiplied by the Avogadro number, the product gives the energy released in the fission of all nuclei in one gram-atom (235 grams *of* ^{235}U*), and is equal to* 1.93×10^{20} *ergs. The energy liberated by a complete fission of* 1 *kg of* ^{235}U *would therefore be* 8.21×10^{20} *ergs, which is roughly equivalent to the energy released in the explosion of 20,000 tons of TNT.*

The first task in developing the nuclear bomb was computation of the important nuclear data and the mean free path of the neutrons in the "bomb material" in order to obtain the correct dimensions for a bomb made from uranium 235 or plutonium. The bomb material, of course, is not actually pure at all, and the computations were based on experimental data which were sparse and obtained from very small quantities of material. ***Heisenberg*** *estimated the critical radius in* ^{235}U *to be 8.4 cm in his "Theorie des Atomkerns" (1949). Without the application of neutron reflectors, this value would require a minimal mass of about 50 kg* ^{235}U *for a nuclear bomb. The critical mass actually seems to be considerably smaller.*

The second condition for the production of an uranium bomb was the isolation of the fissionable ^{235}U*, previously achieved only in microscopic quantities. This* ^{235}U*, which represents only 0.72% of natural uranium, had to be separated from the dominant* ^{238}U *isotope. The expenditures in equipment, development work electric energy, and money necessary for this task were enormous. The official reports*[945] *give an idea of the problems and their actual solution in the USA, where the first bomb was exploded by* ***Robert Oppenheimer*** *and co-workers on July 16, 1945 near Alamogordo in the desert of New Mexico.*

A further problem was how to prevent with certainty the self-ignition of the bomb before the planned moment. For an explosion of the total fission mass must occur automatically as soon as the critical mass necessary for an explosion is united at any spot, since uranium nuclei can split spontaneously and a sufficient number of neutrons is always present, for instance from cosmic radiation.

Thus, self-ignition can only be prevented if the fissionable material in the bomb before its ignition is kept spatially separated in the form of several parts

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[945] Glasstone S., ed. *The effect of Nuclear Weapons*, Government Printing, Washington D.C., 1964.
Smyth, H.D. *Atomic Energy for Military Purposes*, Princeton University Press, 1946.

of subcritical size. The ignition is then initiated by the sudden mechanical union of the subcritical parts to one piece of supercritical size.

This mechanical union must be done so rapidly and completely that as many nuclear fissions occur as possible before the bomb explodes mechanically due to its large internal energy production – a process that would interrupt the chain reaction.

To this end bombs consist[946] *of a relatively large number of subcritical masses, which are brought together in the ignition process by the explosion (actually implosion) of appropriately shaped explosive "lenses" acting concentrically toward the center. Since the continuously multiplying number of fission processes in the bomb material is stopped by the mechanical explosion of the bomb, its effect is the greater the faster the fission processes follow one another.*

In order to avoid the capture of neutrons by non-fissionable nuclei, extremely pure material is used and the bomb proper is surrounded by a shield of a suitable material of high density. This reflector is made to scatter back at least part of the neutrons that would normally leave the fissionable material toward the outside.

According to official reports, during the explosion proper about 3% of the total energy released by a bomb is emitted as γ-radiation and another 3% as fast neutrons. This nuclear radiation emitted by the nominal bomb would kill the majority of people exposed to it at any distance less than 1 km. But its effect declines rapidly with increasing distance from the center of the explosion (assumed to occur in free atmosphere) so that the primary nuclear radiation would not be an essential danger at a distance of more than 2 km.

An additional 85% of the total energy of the bomb appear as kinetic energy of the fission products and thus serve to heat up the central vapor mass that originally comprised the bomb. The temperature obtained in this way is said to be of the order of $10^7\,{}^\circ K$. This means that nuclear physicists actually have manufactured a real although short-lived small "star" as far as characteristic core temperatures are concerned. After the end of the explosion proper, this fireball, very small in the beginning, expands very rapidly so that the radiating surface increases quickly and at the same time cools off. The maximum of the bomb's heat radiation will therefore be reached after a few tenths of a second, when the surface temperature of the fireball, now having a diameter of over 100 m, has decreased to $7000\,{}^\circ K$, comparable to the surface temperature of the sun.

[946] Rhodes, R. *The Making of the Atomic Bomb*, Simon and Schuster Inc., New York, 1986.

Depending on the transmittance of the atmosphere, this radiation may cause extremely dangerous burns at distances up to several kilometers, i.e., far beyond the range of the direct nuclear radiation. The absorption of γ-radiation and neutrons as well as that of the short-wavelength part of the heat radiation spectrum may initiate in the surrounding atmosphere a large number of photochemical effects such as dissociation of molecules and ionization of gases.

The remaining 9% of the energy released by the explosion of a typical fission bomb becomes liberated some time after the explosion proper in the form of β-radiation and γ-radiation of radioactive fission products. Together with the radioactive decay of the radionuclides produced by (n, γ)-processes in the immediate vicinity of the explosion center, this radiation causes the dangerous after-effects of a nuclear bomb explosion that are rightly dreaded by mankind.

1938–1949 CE **Claude Elwood Shannon** (1916–2001). Pioneer of the mathematical theory of communication and of modern digital technology. He was born in Gaylord, MI, educated at the University of Michigan and at the Massachusetts Institute of Technology, and then joined the Bell Telephone Laboratories (1941–1972).

His work, *A symbolic analysis of relay and switching circuits* (1938) is a founding document of the mathematical theory of *information*. In it he applied symbolic logic to relay circuits, helping transform circuit design from an art into science. In 1948 he published a classic paper: *The mathematical theory of communication*, seeking therein to render a coherent treatment of all forms of information transmission systems[947], whatever their physical nature.

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947 *Shannon's Sampling Theorem* (1949) is an important theorem on transmission of information. It states that if the Fourier transform of a function $f(t)$ is zero above a certain frequency ω_c, $F(\omega) = 0$ for $|\omega| > \omega_c$, then $f(t)$ can be uniquely determined from its sampled values $f_n = f\left(n\frac{\pi}{\omega_c}\right)$ at a sequence of equidistant time points. In fact, $f(t)$ is then given by $f(t) = \sum_{n=-\infty}^{\infty} f_n \frac{\sin(\omega_c t - n\pi)}{\omega_c t - n\pi}$. The corresponding *sampling theorem in the frequency domain* is as follows: if a function $f(t)$ is time-limited, $f(t) = 0$ for $|t| > T$, then its Fourier transform $F(\omega)$ can be uniquely determined from its values $F\left(n\frac{\pi}{T}\right)$ at a sequence of equidistant points, $F(\omega) = \sum_{n=-\infty}^{\infty} F\left(n\frac{\pi}{T}\right) \frac{\sin(\omega T - n\pi)}{\omega T - n\pi}$.

Developed further in a series of papers, Shannon's work is fundamental to all modern communication systems.

Shannon's theorem was first proved by the mathematician **J. Whittaker** (1935), before being applied to communication theory by Shannon in 1948. The theorem asserts that if the range of frequencies of a signal is zero through f, then the signal can be represented with *complete accuracy* by measuring its amplitudes $2f$ times a second. This is really a remarkable theorem considering that ordinarily a continuous curve can be only *approximately* characterized by stating any finite number of points through which it passes, and an infinite number would in general be required for complete information about the curve. But if the curve is *band-limited* (composed of a limited range of frequencies) — it can be reproduced *exactly* from a finite number of samples.

The result, which follows directly from Fourier analysis, has enormous consequences for the transmission and processing of information. It was no longer necessary to reproduce an entire analog signal — a limited number of samples suffices. E.g. the range of frequencies transmitted by a telephone is about zero to 4000 cycles per second. In a digital system, at least one voice sampling is thus performed 8000 times a second. Reproducing music with fidelity on a compact discs requires about $44,000$ samples a second. Measuring the signal more often,, or reproducing it continuously (as with old-fashioned records) does not gain anything.

Another important consequence is that high frequencies must be samples more often than low frequencies (audio frequency doubles every time one goes up an octave). The sampling theorem opened the door to *digital technology*: a sampled signal could be expressed as a series of digits.

Although communication theory grew out of the study of electrical communication, it attacks problem in a very abstract and general way. It provides, in the *bit*[948], a universal measure of *amount of information* in terms of choice or uncertainty. Specifying or leaving the choice between two equally probable alternatives, for a portion of a data stream to be transmitted, involves one *bit of information*. Communication theory tells us how many bits of information

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Shannon acknowledged that **Leo Szilard's** paper (1929) had proposed the basis for his new field of study. Indeed, the key elements of information theory, which Szilard first wrote about in 1922, were even transmitted to John von Neumann during his interaction with Szilard in Berlin in the late 1920s. Physicist Leon Brillouin, in turn, learned of Szilard's 1929 paper only in 1951, hearing about it from Warren Weaver at the Rockefeller Foundation.

948 A contraction of *binary digit*. One "bit" refers to a choice between two alternatives (for a computer, $0/1$, or a circuit that is off or on). The word was coined by **John Tukey**. Eight "bits" make a "*byte*".

can be sent per second over perfect and imperfect communication channels in terms of a rather abstract description of the properties of these channels.

In 1949, Shannon built the first chess-playing machine at the Massachusetts Institute of Technology.

Communication and Information[949] *(1894–1948)*

The range of problems involving the concepts of message and information is very broad. They have long been drawing close attention from physicists, engineers, mathematicians, linguists, and philosophers.

Communication, in the human sense, includes all the procedures by which one mind may affect another. This encompasses not only written and oral speech, but also music, the pictorial arts, the theater, the ballet, and in fact all human behavior. In the 20th century the concept has been broadened to include the procedures by means of which several mechanisms may exchange state-modifying instructions.

Shannon's work harks back to an observation of ***Ludwig Boltzmann*** *in some of his works on statistical physics (1894) that entropy is related to missing information, inasmuch as it is related to the number of alternatives which remain possible to a physical system after all the macroscopically observable information concerning it has been recorded. Shannon's work connects more directly with certain ideas developed by* ***H. Nyquist*** *and* ***R.V.L. Hartley*** *in the late* 1920*'s.*

Communication theory is also indebted to ***Norbert Wiener*** *for much of its basic philosophy, but while Shannon has been especially concerned with applications to engineering communication, Wiener has been more concerned with applications related to or inspired by biology (central nervous system phenomena, cybernetics, etc.).*

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[949] To dig deeper, see:

- Pierce, J.R., *An Introduction to Information Theory*, Dover, 1980, 305 pp.
- Woodward, P.M., *Probability and Information Theory*, McGraw-Hill, 1957, 128 pp.

Communication problems can be viewed on three levels. Thus it seems reasonable to ask:

- *How accurately*[950] *can the symbols of communication be transmitted? This is the* technical *problem, involving only the engineering details of good design.*
- *How well do the transmitted symbols convey the desired meaning? This is a* semantic *problem, concerned with the faithfulness of the interpretation of meaning by the receiver, as compared with the intended meaning of the sender.*
- *How* effectively *does the received meaning affect conduct in the desired way? This is an* effectiveness *problem, concerned with the success with which the meaning conveyed to the receiver leads to the desired conduct on its part. In the human sphere, this aspect involves aesthetic considerations in the case of the fine arts. In the case of speech, written or oral, it involves considerations which range all the way from the mere mechanics of style, through all the psychological and emotional aspects of propaganda theory, to those value judgments which are necessary to give useful meaning to the words* success *and* desired *mentioned earlier.*

There is overlap between all of the suggested categories of the problem. Shannon's mathematical theory of the engineering aspects of communication applies only to the first level. But any limitations discovered in the theory at the first level necessarily apply to the other two levels, and in this sense Shannon's theory affects the other levels as well.

The overall operation of a communication system on the first level is as follows: The information source *selects a desired* message *out of a set of possible messages. The selected message may consists of written or spoken words, or of pictures, music, etc.*

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[950] In the exact sciences, a measurement has a high *accuracy* if it has small *systematic* errors. If a measurement has small *random* errors we say that it has *high precision*. Systematic errors are errors associated with the particular instrument or technique of measurement, and are usually caused by biased or improperly calibrated instruments.

Random errors are produced by a large number of unpredictable and unknown variations in the experimental situation. They can result from small errors of judgment on part of the observer, unpredictable fluctuations in temperature, line voltage, etc.; or indeed from any kind of parametric fluctuation in the equipment. Since such random errors are frequently distributed according to a known *statistical* law, they can be dealt using statistical methods.

The transmitter changes this message into a signal *which is actually sent over the* communication channel *from* transmitter *to* receiver *[e.g.: in oral speech, the information source is the brain, while the transmitter is the voice mechanism producing the varying sound pressure (the signal) which is transmitted through the air (the channel). In radio, the channel is simply the air or vacuum, and the signal is the electromagnetic wave which is transmitted].*

The receiver is a sort of inverse transmitter, changing the transmitted signal back into a message, and handing this message on to its destination [e.g.: the ear and the auditory nerve in vocal communication]. During transmission, unwanted additional signals, known as noise*, distort the signal. The kinds of questions which one seeks to quantify and answer concerning such a communication system are:*

(1) How does one measure amounts of information*;*

(2) How does one measure the capacity *of a communication channel*[951]*;*

(3) What are the characteristics of an efficient coding process *(the action of the transmitter in changing the message into the signal according to a specific pattern known only to the transmitter and the receiver);*

(4) What are the general characteristics of noise*? How does noise affect the accuracy of the message finally received at the destination? How can one minimize the undesirable effects of noise?*

(5) If the signal being transmitted is continuous *(as in oral speech or music) rather than formed of* discrete *symbols (as in written speech, telegraphy, binary bits, etc.), how does this affect the problem?*

The word information *in this context must not be confused with meaning, and the semantic aspects of communication are irrelevant to the engineering*

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951 Let N_t denote the number of possible message sequences of duration t. Shannon defined the *capacity* C of the channel as

$$C = \lim_{t\to\infty}\left[\frac{\log_2 N_t}{t}\right],$$

where $\log_2$ denotes the logarithm to base 2. The rationale behind the definition is this: if a single selection is to be made from a number of equally probable alternatives, and if information is transmitted which reduces the number of alternatives by a factor of 2, then this amount of information is 1 *bit*, the unit of information. Thus, the logarithm to base 2 of the number of alternatives N_t varies in steps equal to the amount of information transmitted; division by time elapsed yields the *rate* of information transmittal (bits/second).

aspects (two messages, one of which is heavily loaded with meaning and the other of which is pure nonsense, can be exactly equivalent, from the present viewpoint, as regards information). But this does not mean that the engineering aspects are necessarily irrelevant to the semantic aspects.

The word information *in communication theory relates not so much to what you* do *say, as to what you* could *say, i.e., information is a measure of one's freedom of choice when one selects a message.*

In 1949, just one year after Shannon published his work establishing the field of information theory, linguist ***Georg Zipf*** *published the book* Human Behavior and the Principle of Least Effort. *In this volume, Zipf announced an empirical rule specifying how the frequency of occurrences of a word in a long stretch of text varies with the word's* ranking *(suitably defined) for given total vocabulary used. This relationship, now termed* Zipf law, *is very closely related to the principles laid down by Shannon.*

Mathematically

$$f(r) \approx \frac{1}{r \log_2[1.78R]}$$

where r *is the word rank and* $(1.78R)$ *is the vocabulary size*[952].

1938–1972 CE **Robert King Merton** (1910–2003, U.S.A.). Historian and sociologist of science. His work suggested important innovations, both theoretical and empirical, in the study of science as a *social process*. In his doctoral dissertation (1938): "*Science, Technology and Society in Seventeenth*

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952 *Example*: Conan Doyle's *The Hound of the Baskervilles* contains a total word vocabulary of $\sum_k t_k = 59,498$ words of which $R = 6307$ are different, yielding a vocabulary size of $1.78R \cong 10,000$ words. The breakdown according to rank is

$r =$ rank	word	t_k (times of occurrence)
1	the	3328
2	and	1628
5	to	1429
10	in	911
20	for	420
50	would	192
500	hours	13
5000	galleries	1

Century England" he has shown the need for a marriage between history of science and sociology, and has established the sociology of science as a discipline in its own right.

Merton's work took up the thesis of the sociologist Max Weber (1846–1920, Germany), on the relationship between Protestantism and Capitalism, by examining the explosion of scientific activity in 17^{th}-century England as a central part of social and cultural change. He showed that those sciences justifiable in terms of their utility in mining, navigation and warfare were more vigorously supported and pursued than others. For instance, technical problems relating to the drainage and ventilation of mines required for their solution extensive improvements in the knowledge of aerostatics and hydrostatics. The preoccupations and rhetoric of men of science reflected the high values placed upon useful application. Merton quantitatively analyzed papers offered to the Royal Society of London during the 17^{th} century, claiming to confirm the preponderance of scientific topics directly or indirectly related to capitalist and military technical requirements.

Secondly, Merton argued for significant positive links between Puritan forms of English Protestantism and the institutionalization of science. Rejecting the Victorian tradition of seeing science and religion in 'conflict', Merton showed that men of science justified their activities in terms of accepted Puritan values, such as their demonstration of God's existence and attributes from the study of nature, and their suggestion that natural knowledge was ultimately useful. The Puritanism-science link was supported by the correlation found between membership in the Society and Puritan sentiments. Merton concluded that Puritan values were congenial to scientific culture and that religion provided resources by which the pursuit of 17^{th}-century England science was 'positively sanctioned'.

In other national settings, the 'functional role' of providing social legitimacy for science may have been performed by different cultural constellations, or not performed at all.

Merton was born in Philadelphia, Pennsylvania to poor Jewish parents as Mayer Schkolnick. In 1924, he changed his name to Robert King Merton to enhance his chances to acquire higher education. After finishing high school he studied at the Temple University, founded for poor boys and girls of Philadelphia. Then he went to Harvard University in Cambridge, MA, obtaining his Ph.D. in 1936.

1938–1985 CE **Menahem Max Schiffer** (1911–1997; Israel and USA). Mathematician. His work opened up the possibility of applying variational methods in a systematic way to geometric problems in complex analysis. The 'Schiffer variation' is named after him. Made important contributions to the study of eigenvalue problems, to PDE, and to the variational theory of 'domain functionals' that arise in many classical boundary value problems.

Schiffer was born in Berlin to Jewish parents. He entered (1930) the Friedrich-Wilhelm University with the intention of becoming a physicist and studied physics under **Max von Laue**, **Walther Nernst** and **Erwin Schrödinger** and mathematics under **Issai Schur**. Schrödinger finally pushed him to become a mathematician. As a result of Nazi Persecutions, the Schiffer family emigrated to Israel and Menahem continued his studies at the Hebrew University in Jerusalem (MA, 1934; Ph.D. 1938). In 1952 he settled as a professor of mathematics at Stanford University and remained there thereafter. He was famous worldwide for his remarkable lectures in applied mathematics and mathematical physics: each lecture was a perfect set piece – no pauses, no slips, and no notes.

1938–1988 CE **Barbara Wertheim Tuchman** (1911–1989, The United States). Historian and author. Wrote eleven books about men of war and on brink of war. Of these, the most influential are:

- *Bible and Sword* (1956), about English involvement in the Middle East over the centuries.
- *The Guns of August* (1962), covering the outbreak of WWI and the events leading to that war.
- *A Distant Mirror* (1978), on the calamitous 14^{th} century: a time of ferocity, plague and spiritual agony when a world plunged into chaos.
- *The March of Folly* (1984), a meditation on un-wisdom as a force in history (wisdom = "the exercise of judgment acting on experience, common sense, and available information").

Her central themes are:

- Meaning in history emerges *not* from preconceived design, but from the aggregation of details and events that fall into a pattern; In this process, good is often crushed or subverted.
- The psychology of governing classes is often fatally flawed and the power to command frequently causes failure to think.
- Holders of high office act contrary to the way reason points and enlightened self-interest suggests. Consequently, intelligent mental processes seem often not to function.

Barbara Tuchman was born in New York to an illustrious Jewish family; her uncle, Henry Morgenthau Jr. was secretary of the Treasury under President Franklin D. Roosevelt.

She received a B.A degree from Radcliff College (1933) and in 1937 went to Madrid to report on the Spanish Civil War (which she saw as the end of the liberal world). In 1939 she married Dr. Lester R. Tuchman, a New York Internist, and they had three daughters. In her later years she was a lecturer at Harvard University and the U.S. Naval War College. In 1979 she was appointed the chairperson of the American Academy of Arts and Letters.

1939 CE *Elements de Mathematique*, by Nicholas Bourbaki, pseudonym for a group of young mathematicians at the Ecole Normale in Paris, is begun. This extended set of works aims to set down in writing, on a formal, abstract and rigorous footing, the established branches of modern mathematics.

1939 CE FM radios are sold commercially for the first time.

1939–1940 CE Bitterly cold weather killed 500,000 Russian soldiers invading Finland.

1939 CE **Lise Meitner**[953] (1878–1968, Austria and Sweden). Physicist. Born in Vienna as one of the eight children of a Jewish Viennese lawyer. She studied at Vienna University under **L. Boltzmann** and was one of the first women in Austria to earn a doctorate degree at that University (1906). In 1907 she moved to Berlin University and worked for some time with **Max Planck**. In 1913, she began research into radioactive substances with **Otto Hahn** (1879–1968) and in 1918 they discovered the element *protactinium*. Her major achievement at that time was the determination of the relationship between β and γ radiation of radioactive materials.

Lise Meitner was one of the first women to become a professor at the University of Berlin (1926). From 1917 on, she served for over 20 years as head of the physics department in the Kaiser Wilhelm Institute for chemistry in Berlin.

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[953] For further reading, see:

- Sime, R.Lewin, *Lisa Meitner: A Life in Physics*, University of California Press, Berkeley, 1996.
- Rife, P., *Lise Meitner and the Dawn of the Nuclear Age*, Birkhäuser, 1999.

After the *Anschluss* in 1938, she escaped[954] from Germany and settled in Stockholm, working on the staff of the Nobel Institute. There she received a letter from Hahn describing his discovery with Fritz Strassmann that, when an uranium atom absorbed a neutron, an atom of barium was sometimes produced thereby.

While vacationing near Gothenburg in December 1938, she discussed this with her nephew, **Otto Frisch** (b. 1904) who was working in Denmark with **Niels Bohr**. The two physicists immediately realized the significance of the discovery, which meant that the *uranium atom was split* into roughly equal parts, accompanied by a tremendous release of energy[955]. Frisch called this "fission", a term borrowed from biology. Lise Meitner visited the United States after 1945, but returned to Sweden and became a citizen there in 1949. Both before and after WWII she received many honors. She eventually retired to Cambridge, England, where she died.

1939 CE, March **Frederick Joliot Curie** (Paris, France) and **Enrico Fermi** (New York) discover the *nuclear chain-reaction*[956]: An uranium nucleus (U^{235} isotope), when split by a neutron, releases (as a rule) two or more neutrons.

Joliot-Curie's second paper in *Nature* (April 22, 1939) triggered two initiatives in Germany. A physicist at Göttingen alerted the Reich Ministry of Education. That led to a secret conference in Berlin on April 29, which led in turn to a research program, a ban on uranium exports and provisions

[954] On June 16, 1938 she was forbidden to leave. With the aid of Bohr, Dirk Coster, Adriaan Fokker, Hahn, Max von Laue and Paul Rosbaud (a scientific publisher) she escaped by train, on July 12, 1938 across the Dutch border to freedom.

[955] Hahn won the 1944 Nobel prize in chemistry for splitting the atom (nuclear fission). Meitner's share in this discovery was at least as great as his, but her contribution was discredited because she was a woman and a Jew. The two were working together on the fission experiment when Lise had to flee Nazi Germany. The fission occurred while she was away, but even so she gave the correct physical explanation of the result.

Hahn, however, dissociated himself from their partnership, and took all the credit for himself.

[956] The theory of *chemical* chain-reactions was established by the physical chemist **Max Bodenstein** (1871–1942, Germany) in 1913. Bodenstein was born in Magdeburg to a Jewish family and perished in the Nazi Holocaust.

for supplies of radium from the Czechoslovakian mines at Joachimsthal. The same week, a young physicist working at Hamburg, **Paul Harteck**, and his assistant **Wilhelm Groth** jointly wrote a letter to the German War Office:

> "We take the liberty of calling your attention to the newest development in nuclear physics, which, in our opinion, will probably make it possible to produce an explosive many orders of magnitude more powerful than the conventional ones... That country which makes use of it has an unsurpassable advantage over the others."

The letter was delivered to **Hans Geiger**. Geiger recommended pursuing the research and the War Office agreed. In June 1940, Paul Harteck in Hamburg tried to measure neutron multiplication in an arrangement of uranium oxide and dry ice (frozen CO_2), but was unable to convince **Heisenberg** to lend him enough uranium to guarantee unambiguous results.

1939 CE **John Vincent Atanasoff** (1903–1995, USA). Father of the electronic digital computer. Inventor, electrical engineer and mathematical physicist. Invented and built the world's first operational prototype electronic digital computer, known as the ABC (Atanasoff Berry Computer). It used vacuum tubes[957] to perform mathematical and logical operations and employed binary numbers that were stored in capacitors mounted on a rotating drum. Data were entered via punched cards. The cost of the entire project was about 1000 dollars. He was assisted by his graduate student Clifford Berry.

Atanasoff saw others take credit for his discovery: many of his ideas were used in the design of the ENIAC which is falsely considered by most people as the world's first electronic digital computer. A long trial ensued and it was not until 1973 that Dr. Atanasoff was given the recognition he deserved.

When Atanasoff invented the computer, he could not imagine the impact it would have on people's lives; the electronic age is a direct result of the invention of the computer. Never before has an invention mushroomed so quickly as the computer. Within the last 60 years, its speed and power has grown at an exponential rate.

John Vincent Atanasoff was born in Hamilton, New York to a family of Bulgarian immigrants. He graduated from the University of Florida (1925),

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[957] He recognized the usefulness of the fact that a vacuum tube could be turned on and off in about a millionth of a second — thousands of times faster than the sluggish relay.

and received his Ph.D. in theoretical physics at the University of Wisconsin (1930). He became an associate professor of mathematical physics at Iowa State College (1936–1942), and it is there that he built the ABC computer. During 1942–1952 Atanasoff worked in the U.S. Naval Ordnance Lab. He never made money off his invention.

1939 CE **Charles Stark Draper** (1901–1987, USA). Aeronautical engineer and inventor. Developed the first inertial guidance system for launching long-range missiles based on his earlier gyroscopic systems that stabilized and balanced gunsights and bombsights (1939). Draper subsequently developed the SPatial Inertial Reference Equipment (SPIRE) system for automatic aeronautical navigation — a system. He later refined and miniaturized it for use in the Polaris submarine missile system.

Draper was born in Windsor, Missouri. He obtained a D.Sc. from M.I.T. (1938), becoming a professor there. He continued to be a pace-setter in the space age as head of MIT's Department of Aeronautical and Astronautical engineering. His Instrumentation Lab was awarded the Apollo project contract for guiding spacecraft to the moon.

1939–1957 CE **Carl Gustav Rossby** (1898–1957, Sweden). Oceanographer and meteorologist. Provided the ideas and leadership necessary for the progress of synoptic and dynamic meteorology in the 40's and 50's.

After early work on the dynamics of ocean currents, he began to make fundamental contributions to meteorology, notably in studies of long circumpolar waves and the *jet stream*[958], on the application of the vorticity equation of cyclonic development, and on the barotropic model atmosphere for use in numerical forecasting. His name is perpetuated in meteorology by means of the *Rossby diagram* in thermodynamics, *Rossby waves*[959], and the *Rossby number*.

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958 The atmospheric layer between 8–13 km in altitude, with temperature between $-45\,^{\circ}$C and $-65\,^{\circ}$C and pressures of about 100 mb, accommodates the *jet stream*. These winds move with velocities above 100 km/hr, reaching in extreme cases even 500 km/hr. Commercial jet aircraft today fly in this layer, and even when they travel at an average speed of 800–900 km/hr, jet streams play a decisive role as far as navigation, flying time, and convenience are concerned.

959 Near-barotropic wavy nature of the upper troposphere flow: the upper *westerlies* propagate around both hemispheres in a series of long waves (4000–6000 km). These waves change only slowly in number and amplitude compared with surface systems, and they travel more slowly than the winds blowing through them. There are normally some 3 to 6 long waves around the northern hemisphere on any given day.

Rossby waves were determined by the trajectory of a balloon launched from

Rossby was born in Stockholm. He developed three outstanding university departments of meteorology — at M.I.T. in the 1930's, at Chicago (1941–1947), and at Stockholm from 1947.

New Zealand on 30 March 1966. It drifted at an altitude of 12 km, with mean speed of over 110 km/hr and for 49 days, in the upper westerlies. The movement of the balloon (tracked by satellite) traced out the shape of Rossby waves at a latitude range 30°–50°S during this 7-week period. On average, there were about 4 long waves per revolution, although the trough positions changed with time.

The existence of Rossby waves follows directly from the basic equations of inviscid atmospheric fluid dynamics in an earth–co–rotating frame:

$$\frac{Du}{Dt} - fv + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0;$$

$$\frac{Dv}{Dt} + fu + \frac{1}{\rho}\frac{\partial p}{\partial y} = 0,$$

which arise from approximation in which gravity, sound waves, and vertical motion are excluded. One then makes the additional assumptions:

- Regarding the atmosphere as an incompressible and horizontally homogeneous fluid $\left(\frac{\partial \rho}{\partial x} = \frac{\partial \rho}{\partial y} = 0\right)$.
- Waves travel in the x direction, v is y-independent and u is constant ($u = \overline{u}$; $\frac{\partial f}{\partial y} = \beta =$ constant = Rossby parameter).

Here $f = 2\Omega \sin\theta$ where $\Omega = 7.29 \times 10^{-5}$ rad/sec is the angular frequency of the earth's rotation, and $\theta =$ latitude. Under these conditions, the equations of motion reduce to

$$\frac{\partial^2 v}{\partial x \partial t} + \overline{u}\frac{\partial^2 v}{\partial x^2} + \beta v = 0,$$

with the harmonic wave solution

$$v = Ae^{ik(x-ct)}, \qquad c = \overline{u} - \frac{\beta}{k^2}, \qquad k = 2\pi/\text{wavelength} = \frac{\omega}{c}.$$

The waves are dispersed, since their phase velocity c depends on their wavelength. The transverse-horizontal motion of the Rossby waves is thus established. Such solutions were contained implicitly in the *theory of tides* of **Laplace** (1774), but Rossby was first to recognize their meteorological importance and to isolate them in pure form.

In contrast to sound and gravity waves, Rossby waves are always propagated westward *relative to the medium*, and travel at speeds that depend on their wavelength. They also travel relatively slowly: for wavelengths of 4500 km, their speed relative to the medium is of order 15 km/hr. In addition, their associated horizontal speeds are at least 100 times greater than their vertical speeds.

1939–1968 CE **Max Ferdinand Perutz** (1914–2002, Austria and England). Biochemist. Determined the atom-by-atom structure of *hemoglobin*. Introduced (1939) a technique of adding an atom of heavy element (such as gold) to an organic molecule to improve the X-ray diffraction pattern of the molecule and this method was used by him to obtain the hemoglobin structure (1960). Perutz predicted the presence of the alpha helix in hemoglobin (1951).

Perutz was born in Vienna to a Jewish family. After graduating at Vienna University, he emigrated to Cambridge, England. He became Director of the Medical Research Council (MRC) Unit for Molecular Biology (1947–1962) and after 1962 was Director of the MRC Laboratory for Molecular Biology.

Shared the 1962 Nobel Prize for Chemistry with **John Cowdery Kendrew** (1917–1997, England) who determined, by X-ray crystallography, the structure of the muscle protein *myoglobin*.

The shape of Hemoglobin (1960)

Hemoglobin is the protein in red blood cells that carries oxygen from the lungs to the tissues, and also helps transport carbon dioxide on its return trip. Obviously, it plays an extremely important role in the body. Abnormalities in its structure can lead to such life-threatening diseases as sickle cell anemia or thalassemia.

Besides being important, hemoglobin is also one of the most plentiful proteins in vertebrates. In the human bloodstream some 5 billion red blood cells, each containing 280 million molecules of hemoglobin, can be found in each milliliter of blood.

To decipher the structure of this enormous molecule, ***Max Perutz*** *had to determine the positions of 10,000 atoms, each one located in space by three coordinates. The 574 amino acids formed by these atoms were strung together in four separate, but connected, polypeptide chains. Perutz grew his own crystals of hemoglobin, took his own X-ray pictures, and did millions of computations without the help of a computer, looking for some regular underlying features that would simplify his problem. He thought the molecule* *might have four straight layers, each layer being a polypeptide chain. But he never found anything that clear-cut.*

*His first break came in 1953 when he and his colleagues invented a method that has been called the "*Rosetta Stone*" for interpreting X-ray diffraction patterns of large proteins: the use of certain "heavy" metal atoms, which are added to the hemoglobin crystals to serve as atomic markers. These metal atoms are called heavy because they have an extremely large number of proteins, neutrons, and electrons, and the advantage of using them is that they scatter X-ray beams very strongly.*

Perutz realized that if atoms of mercury or other heavy metals were added to the protein before it was crystallized, they would attach themselves to specific sites in the hemoglobin molecule and produce measurable differences in the intensities of the spots in the X-ray diffraction pattern. By comparing the X-ray patterns of hemoglobin with and without these heavy atoms, he could more precisely determine the intensity and angle of the X-ray reflections. This would allow him to analyze the molecule in three dimensions for the first time.

Six more years of effort were required to make the method work. Finally, all the pieces came together and Perutz stood before the completed structure of the hemoglobin molecule — a bizarre mass of twisted chains with four separate pockets, each one containing the dark red, iron-bearing component called heme *that is essential for the binding of oxygen and gives blood its color.*

Shocked by the result of his labors, Perutz puzzled about its meaning. "Could the search for ultimate truth really have revealed so hideous and visceral-looking an object?" he wondered.

John Kendrew*, who had joined Perutz in Cambridge in the 1940's, had a similar reaction after deciphering the structure of a smaller protein, myoglobin, 2 years earlier. Kendrew had chosen to study myoglobin in the hope that it would have a fairly simple structure, but to his surprise the molecule turned out to have a complex, asymmetrical shape.*

Myoglobin's job is to take oxygen from the red cells in the bloodstream and store it in muscle tissue until needed. This storage occurs by the binding of an oxygen molecule to the iron atom at the center of myoglobin's heme group. Since diving birds and mammals that spend much time underwater have abundant supplies of myoglobin in their muscles, Kendrew worked with myoglobin from sperm whales. After Perutz devised the heavy-metal method *of X-ray crystallography, Kendrew succeeded in solving myoglobin's structure. This was the first three-dimensional model that showed the full complexity of a protein molecule.*

At first there seemed to be neither rhyme nor reason to myoglobin's shape, but when Perutz determined the structure of hemoglobin the obvious similarities between the two models began to offer some clues to the significance of their design. Myoglobin has one pocket for a heme group; hemoglobin, four times its size, has four. Each of hemoglobin's four subunits looks like one molecule of myoglobin. And the interior of all these units consists almost entirely of hydrophobic (water-avoiding) amino acids. This allows the iron atoms in the buried heme group to bind with oxygen without becoming oxidized (rusted).

In 1962, Perutz and Kendrew won the Nobel Prize in chemistry for developing the heavy-metal method of X-ray diffraction and for solving the structures of hemoglobin and myoglobin.

Since this achievement, scientists have deciphered the three-dimensional atomic structure of well over 500 additional proteins. In fact, the structures of most of the proteins that scientists have succeeded in growing into crystals for study by X-ray diffraction are now known. But only a fraction of proteins, which normally exist in solution, crystallize like sugar or salt when the surrounding water is removed. The others either form an amorphous gel, with their molecules pointing in different directions, or turn into a powder. Little is known about the shapes of these tens of thousands of proteins that do not crystallize, although a new technique called 2-D NMR may soon come to the rescue.

Although the amount of data on protein sequences is increasing at an astounding rate because of advances in molecular biology, the folding problem remains a major challenge to scientists. Being able to predict the detailed three-dimensional structure of a protein from a given sequence of amino acids is "the most fundamental problem at the chemistry-biology interface," according to a report by the National Academy of Sciences, "and its solution has the highest long-range priority."

Science Progress Report No. 17

Martyr of Genetics

"We shall go to the pyre, we shall burn, but we shall not renounce our conviction."

(Nikolai Ivanovich Vavilov, 1940)

Joseph Stalin hated the idea that fate (even the fate of a fruit-fly eye) was determined by biology. It behooved his political system to claim that by changing the environment it is possible to do anything. He thus declared total war on "The capitalistic plot of bourgeois Mendelism–Morganism". Under his direct orders, his Director of Agriculture, T.D. Lysenko, started (1939) a hate campaign against genes and chromosomes.

It went a long way; the entire Soviet agriculture was planned on the false theory that exposing parents to a new environment (e.g. a cold spring in Siberia) meant that the offspring would inherit the ability to cope with icy water. This was an expensive disaster for both farming and genetics. Soviet biology thus fell into the hands of the fanatical eccentric Lysenko, who preached a theory of inherited acquired characteristics and what he called 'vernalization' (the transformation of wheat into rye, pines into firs, and so on) — essentially medieval stuff.

All in all, Soviet science went back to the Middle Ages; thousands of intellectuals lost their jobs. Thousands more went to the notorious Siberian slave camps. Their places were taken by creatures still more pliable, cranks and frauds[960]*. Stalin in person edited in advance Lysenko's presidential address of 31 July 1948 to the Academy of Agricultural Science, which launched the witch-hunt in biology. Scientific genetics was savaged as a 'bourgeois pseudo-science', 'anti-Marxist', leading to 'sabotage' of the Soviet economy: those who practiced it had their laboratories closed down. But all this was not enough; the Communist party needed a public show to terrorize the masses and close the lid on all potential dissidents. They needed a 'traitor'.*

He soon materialized in the form of ***Nikolai Ivanovich Vavilov*** *(1887–1943), a Soviet plant geneticist whose research into the origins of cultivated plants (1916–1933) won him a world acclaim.*

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[960] In medicine, a woman called O.B. Lepeshinskaya preached that old age could be postponed by bicarbonate of soda enemas — an idea that appealed to Stalin.

Vavilov was born in Moscow and graduated at the Agricultural Academy at Petrovsko-Razumovskoe, and later studied under W. Bateson at Cambridge University and at the John Innes Horticultural Institution in London (1913–1914). Returning to Russia, he became professor of botany at the University of Saratov (1917–21), and then head of what was finally called the Lenin All-Union Academy of Agricultural Sciences.

Vavilov made a comprehensive study of the origin of cultivated plants and proposed that there were several world centers of origin at which the greatest concentration of diversity in cultivated plant species occurred. He made expeditions to many parts of the world, including Iran, Afghanistan, Ethiopia, China, and Central and South America, amassing an immense collection of cultivated plants intended to be used for further study and the breeding of new varieties. He brought to the Soviet Union, for further study and breeding, samples of 50,000 varieties of wild plants and 31,000 wheat specimens[961].

Observations made during Vavilov's world-wide studies led him to postulate that a cultivated plant's center of origin would be found in the region in which wild relatives of the plant showed maximum adaptiveness. These conclusions were summarized in The Origin, Variation, Immunity and Breeding of Cultivated Plants *(Eng. trans. by K.S. Chester, 1951). In 1920 he expanded the theory, stating that the region of greatest diversity of a species of plant represents its center of origin. He eventually proposed 12 world centers of plant origin.*

From this prodigious labor emerged a new synthesis of the origin of cultivated plants, the first great advance since de Candolle[962].

Vavilov was arrested (1940) for opposing the views of Lysenko. After hours of interrogation Vavilov was found guilty in a five-minute trial of "belonging to a rightist conspiracy, spying for England, and sabotage of agriculture". He was sentenced to death and died in the Magadan labor-camp, Siberia, on Jan 26, 1943 of starvation and maltreatment by prison guards. The man who had done more than any other to feed Russia, died of starvation. He may have never known of his election as a foreign member of the Royal Society in 1942.

1939–1968 CE **Hannes (Olof Gösta) Alfvén** (1908–1995, Sweden). Astrophysicist. First to bring plasma physics into astronomy, and a pioneer

[961] Sadly, all those collections were *eaten* during the siege of Leningrad in WWII.

[962] **Augustin-Pyrame de Candolle** (1778–1841, Switzerland).

in the study of plasmas in magnetic fields, known as magnetohydrodynamics (MHD)[963]. Won the 1970 Nobel prize in physics for his work on the behavior of ionized gas in the solar magnetic field, through which he explained the puzzling phenomenon that the sun's photosphere is cooler than its chromosphere and corona. Explained the origin of the *cosmic rays* and introduced the new concepts of *Alfvén velocity* and *Alfvén waves*.

Alfvén suggested that *antimatter* may power the *quasars* and the violent explosions in the cores of galaxies [matter and antimatter collide and annihilate each other, thus releasing all of their rest-mass energy content].

Cosmic Rays (1899–1945)

Cosmic rays were first detected by **Elster** *and* **Geitel** *(1899) and* **C.T.R. Wilson** *(1900). In measuring the rate of discharge of a carefully shielded electroscope, they found that pure, dry air possesses a small conductivity, which presumably resulted from the presence of ionizing radiation in the laboratory or in the air (assumed to be caused by small amounts of radioactive substances).*

However, in 1911 **Hess** *showed that the ionizing radiation is entering the earth's atmosphere from above, and these rays must have great penetrating power, since their effects have been detected underground and in deep lakes. It is possible to arrange Geiger counters in coincidence arrays and to show that the detected rays travel predominantly in the vertical direction. Similar techniques reveal the occurrence of cosmic-ray showers, that is, groups of particle trajectories covering a large horizontal area within a short period of time.*

It is also found that the numbers of rays reaching ground level depends on the magnetic latitude; hence the earth's magnetic field must affect the trajectories of the original or "primary" particles entering the upper atmosphere. It is believed that most of the primary particles are protons, but there are also heavier particles in the primary rays, as shown by the heavy-ion tracks

[963] Energy is transferred via hydromagnetic waves from the magnetic field (lines of force) to electrons and ions that spiral along them.

produced in photographic emulsions flown to a great height. Measurements have proved that primary cosmic-ray particles with a kinetic energy up to 10^{20} *eV (corresponding to* 10^{11} *times the rest mass of the nucleon) occur, far greater than the energies achievable in any man-made accelerator.*

The cosmic rays observed at sea level are produced almost entirely by collisions and disintegrations occurring in the atmosphere following the entry of primary particles; most of these primary particles possess high energy, spread subsequently over a large number of particles. Most of the particles produced by the complex high-energy reactions are either intrinsically unstable, or are capable of producing new particles in "cascade" processes.

One important component of cosmic rays is called the "soft" component, because it is easily absorbed in lead, and this consists of electrons of both signs (electrons an positrons) as well as high-energy protons. A single, high-energy electron can generate photons in the presence of matter by the Bremsstrahlung process, and high-energy photons impinging upon nuclei may in turn produce electron-positron pairs, which may then cause further emission of photons.

Thus the soft component forms showers of the cascade type, and a single shower may spread over a wide area at sea level. The total energy found in some showers is of the order 10^{17} *eV, all of which must have come from a single primary particle entering the atmosphere.*

In addition to the soft component, cosmic rays contain a component which can penetrate several feet of lead and which consists of charged particles of both signs. These particles do not lose appreciable amounts of energy by the Bremsstrahlung process, but, at the same time, they are much more penetrating than massive particles like the proton.

Calculations of the energy losses by fast particles in matter suggest that moat of the penetrating particles must be intermediate in mass between the electron and the proton. In 1938 ***C.D. Anderson*** *and* ***S.H. Neddermeyer*** *published evidence strongly suggesting the existence of such* mesons*, with rest mass about* 200 *times that of the electron.*

During the years 1939–1945*, theoretical physicists tried to reconcile the cosmic-ray data with the meson theory of nuclear forces, despite the many difficulties which attend a rigorous theory of strong interactions. Slowly it became clear that the mesons observed in cosmic rays could* not *be responsible for nucleon-nucleon forces, since their interaction with nucleons is too weak. Today, the penetrating meson of Anderson and Neddermeyer are known to be the muon — a heavier analog of the electron, which has its own species of neutrino.*

The mesons thought to mediate the strong nuclear force, on the other hand, are known as pions *or pi-mesons. These latter were predicted by*

***H. Yukawa** (1935), and the confusion arose because pions and muons have similar masses.*

Later in the 20^{th} century, it became clear that pions, like nucleons, are composed of quarks, and that a more accurate description of strong nuclear forces – of both nucleon and pions, as well as their heavier "cousins" – is via exchange of "gluons" between quarks and anti-quarks.

1939–1987 CE **Walter Maurice Elsasser** (1904–1991), Germany and USA). Physicist and 'biological philosopher'. Produced the '*dynamo model*' to account for the earth's magnetic field (1939), and pioneered analysis of the earth's past magnetic fields patterns, now 'frozen' in rocks. He also foresaw the fact that the *genome* does not encode all the information that goes into the biology and functionality of an organism ('*Physical Foundation of Biology*', 1958).

Elsasser was born to a Jewish family in Manheim and educated at Göttingen. He left Nazi Germany (1933) for the Paris Sorbonne and then emigrated to the USA (1936). He worked at Caltech (1936–1941) and for the US Signal Corps in war research on radar (1941–1946). He subsequently held professorships at the Universities of Pennsylvania, Utah, Princeton and Maryland until 1974.

Considering the earth as a having a core of molten iron above the Curie temperature (and therefore no longer able to retain any permanent magnetism), Elsasser suggested that the earth's rotation sets up eddy currents in the liquid core, causing it to behave as an electromagnet. This theory provides an explanation of the terrestrial permanent magnetic field and the presence of secular variations.

In a series of books: *Atom and Organism* (1965), *The Chief Abstractions of Biology* (1975) and *Reflection on a theory of Organisms* (1987), Elsasser expounded his biological thoughts and general philosophy.

All of his biological writings refer to the immense complexity of the organism based on the number and types of atoms in a cell and the number of possible bonds connecting the atoms in organic molecules. *Complexity* is taken as an intrinsic aspect of the living state. Another important consideration is the near *reversibility* of most biochemical reactions with their *relatively small energy changes*.

Furthermore, the closeness of energy exchange in biochemical reactions to *thermal noise* is necessary for the *decision-making ability* of the organism, allowing it to choose between available states without need for more than a *minimal supply of energy.* This condition enables the system to respond with large-amplitude changes to small perturbations, and is also characteristic of the processes involved in the development and differentiation of the organism. These thoughts led Elsasser to formulate a set of principles to represent the living state:

- *The principle of ordered heterogeneity*: Combinatorial analysis shows that the number of structural arrangements of atoms in a cell is immense; that is, much greater than 10^{100}, a number that is itself much larger than the number of elementary particles in the observable universe (10^{80}). But biology also manifests regularity in the large, in addition to heterogeneity in the small.

- *The principle of creative selection*: Nature, through processes compatible with the laws of physics (but not uniquely determined by them!) makes a choice among the immense number of possible patterns inferred from the first principle. However, no mechanism can be specified by the operations of which the selected patterns differ from those not selected. The selection of a relatively small number of organisms from the immerse number[964] of possibilities allowed by quantum mechanics, physics and chemistry is a primary expression of *biological order.*

- *The principle of holistic memory*: It provides the *criterion of choice* not expressed in the second principle. The organism chooses patterns that resemble earlier patterns. This holistic "memory" secures the *stability of information* in time. It is dynamic memory without stable storage which secures transmission of morphological features through time without a dedicated material memory device.

- *The principle of operative symbolism*: a material carrier of information (namely DNA) acts as a releaser for the capacity of the whole organism to

[964] The number of different patterns is also immense in the physical science of *statistical mechanics*, but in that case the variation of structure from pattern to pattern averages out. The patterns of inorganic systems are either fully random or else repeat themselves over and over again ad infinitum, while those of each organism are unique and evolving.

reconstruct a complete message that characterizes the adult of the next generation.

These principles are not scientific laws in the usual sense since they are not derivable from the fundamental laws of physics, and therefore not determined directly by atomic and molecular physics. The basic assumption in this holistic interpretation is that an organism (or a cell) is a source of causal chains which cannot be traced beyond a terminal point because they are lost in the complexity of the organism.

1940 CE, Nov. 07, ca 10 am *The catastrophic collapse of the Tacoma Narrows suspension bridge* (near Seattle, State of Washington, U.S.A.) due to twisting motion induced by gusting winds. The bridge, with the third longest span in the world (853 m), collapsed only four months after its inauguration. One of the surprising facts of the accident was the relatively low speeds of the winds causing it, only $67\,\frac{\text{km}}{\text{hr}}$ $(19\,\frac{\text{m}}{\text{sec}})$ at the moment of collapse. Excluding tornadoes, the worst storms on land may reach $160\,\frac{\text{km}}{\text{hr}}$, but $120\,\frac{\text{km}}{\text{hr}}$ winds are observed frequently enough even in Europe).

Startling scenes of rippling pavements, featured in a classic film that captures this event, rank among the most dramatic and widely known images in science and engineering. This staple of most elementary physics courses, has left an indelible impression on countless students over the years.

The technique of suspension bridges started with the development of metallurgy at the turn of the 19^{th} century. Trial and error taught the method of calculation and indicated possible modes of construction, but several accidents restrained the builders. To begin with, there was the collapse of bridges induced simply by excessive amplitudes of vibration under an imposed alternating load in *resonance* with one of the natural frequencies. For instance, a small bridge erected in 1829 at Broughton near Manchester (England) collapsed due to troops marching in step (1831); a French bridge over the Loire also broke down under a battalion of infantry marching in step; several decades later (1886), an Austrian bridge over the river Ostrawitza collapsed under charging cavalry although the static load (26 soldiers, 16 horses, 2 carts) was very light.

Orders to break step prevented recurrence of these accidents and bridge engineers were led to think that only the properties of the steel used for the suspension cables would limit the possibilities of suspension bridges: Evidently, if the length of the span increases indefinitely, the weight of the cable grows more rapidly than the weight of the roadway it supports, and finally even without a roadway the cable becomes unable to support itself. Between 1900 and 1910 the mechanical properties of the available metals seemed to permit a maximum span of 3 km under these conditions; in 1953 a much greater length appeared admissible.

Nevertheless, experience has shown that the *wind* imposes other limitations: the 10 suspension bridges that have failed in gales[965] during the 19^{th} century illustrate the relentless regularity of these accidents and their apparent independence of technical progress.

Textbooks usually attribute the above events to the phenomenon of resonance. In the case of the Tacoma Narrows bridge, so the explanation goes, the wind blowing past the bridge generated a train of vortices that produced a fluctuating force in tune with the bridge's natural frequency, steadily increasing the amplitude of its oscillation until the bridge finally failed.

Recently[966], however, a new analysis challenged this common explanation. According to the new theory, large scale oscillations are caused by the inherent *nonlinearity* of suspension bridges, through which a bridge could go into large oscillations as a result of a single gust and at other times remain motionless even at high winds. The new theory also explains how large *vertical oscillations* could rapidly change into *twisting motion*.

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965 Dryburgh Abbey (Scotland, 1818); Union (England, 1821); Nassau (Germany, 1834); Brighton Chain Pier (England, 1836); Montrose (Scotland, 1838); Menai Straits (Wales, 1839); La Roche-Bernard (France, 1852); Wheeling (U.S.A., 1854), Niagara-Lewiston (U.S.A., 1869); Niagara-Clifton (U.S.A., 1869).

966 Linear theory predicts that if you stay away from resonance, then in order to create a large motion, you need a large exciting force. Nonlinear theory says that for a wide range of initial conditions, a given push can produce either small or large oscillations. Nonlinear theory predicts that a suspension bridge can respond to a whole range of forcing frequencies. Also, the nonlinear equations yield mathematical solutions of *waves traveling up and down a bridge's roadbed*. Such waves were indeed observed on windy days on roadbeds of a number of large suspension bridges.

According to the model of **Alan C. Lazer** and **P. Joseph McKenna** (*Science News* **137**, 344–346), gusts of winds initially act as a random buffeting force on a suspension bridge, causing the towers and main cable to go into a high-frequency periodic motion. That motion initiates low-frequency, vertical oscillations that ripple the roadbed. This may be followed by sudden transition to a twisting mode.

Thus an impact, due either to an unusually strong gust of wind or a minor structural failure, may provide sufficient energy to send the bridge from one-dimensional to torsional modes of oscillation. The resulting twisting destroys the bridge.

Suspension bridges built or remodeled after 1940, including the Golden Gate bridge, are unlikely to suffer the same fate as the Tacoma Narrows bridge. Civil engineers responded to the Tacoma disaster by *stiffening* existing bridges and building new bridges heavy and rigid enough to resist wind-induced motion. Because such bridges naturally flex very little, a linear analysis suffices. Only when flexibility becomes an issue and a bridge moves so much that it start loosening does the nonlinear theory come into play.

An *earthquake*, however, is precisely the sort of energy source that may push a suspension bridge into the nonlinear mode. (Indeed, during the 7.1 magnitude *Loma Prieta earthquake*, the Golden Gate bridge oscillated for about a minute. The stays connecting the roadbed to the main cables were alternatively slackening and tightening — a sign that the bridge was in a nonlinear stage. Fortunately, the bridge did not start twisting, because the earthquake wave hit it head-on rather than obliquely.)

1940 CE **Howard Walter Florey** (1898–1968, England), pathologist, and **Ernst Boris Chain**, biochemist (1906–1979, England) isolated and purified *penicillin*[967] as the first powerful antibiotic.

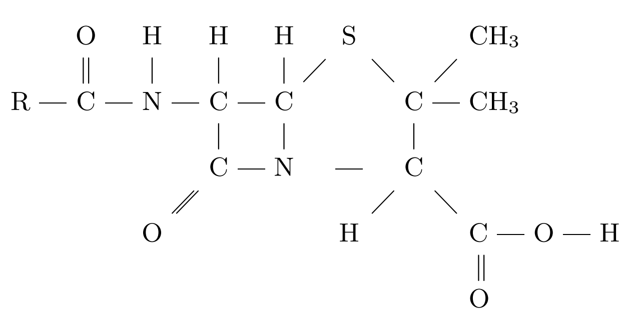
Florey was born in Australia. Professor at Sheffield University (1931–1935), Oxford (1935–1962).

Chain was born in Germany. Researcher at Charité Hospital, Berlin (1930–1933). Fled to England (1933) on account of Nazi persecution of the Jews; Cambridge University (1933–1935), Oxford University (1935–1948), Imperial college, London (1961–1973).

Both were awarded the Nobel prize for physiology or medicine (1945).

Louis Pasteur noted (1877) that some bacteria give off substances that kill other bacteria. It was not until 1939 that this observation was put to use

967 *Penicillin*:



R may be any of a number of hydrocarbon groups, as there are different types of penicillin.

when **René Jules Dubos** (1901–1982, U.S.A.) discovered two antibiotics in a substance produced by soil bacterium.

The details of antibiotic action are poorly understood; it is presumed that in general they block some vital metabolic process of the bacterial organism.

Certain phenomena make the use of antibiotics something less than a panacea for bacterial diseases:

- All antibiotic drugs are capable of producing a wide variety of allergic reactions.
- All antibiotics upset the ecological balance existing among microorganisms present in the human body, which can result in overproduction of particularly resistant bacteria and fungi. These secondary infections can on occasion be worse than the original one.

Bacteria may adjust to the presence of the antibiotic by devising an alternative metabolic pathway, where there is less interference from the antibiotic. In other cases a mutant strain of bacteria develops, which has greater resistance to the drug.

Advent of Modern Pharmacology[968] *(1800–1940)*

1805 ***Friedrich Sertürner*** *isolated the first alkaloid from opium.*

1821 ***Francois Magendie*** *(1783–1855) isolated alkaloids such as emetine and strychnine.*

[968] From the Greek pharmakos=medicine or drug. *Pharmacology* studies the effects of drugs and *how* they exert their effects. In actual use, however, its meaning is limited to the study of action of drugs (how they operate).

1847 *Pharmacology emerged as a separate science when the first university chair was established. This occurred when* ***Rudolf Buchheim*** *(1820–1879) was appointed professor of pharmacology at the University of Dorpat in Estonia (then a part of Russia). Buchheim is credited with turning the purely descriptive and empirical study of medicines into an experimental science.*

1869–1912 ***Oswald Schmiedenberg*** *(1838–1921,* Germany*), a student of Buchheim, is recognized as the founder of modern pharmacology. Introduced urethane as a hypnotic (1885). He was largely responsible for the preeminence of the German pharmaceutical industry up to WWII.*

1897–1926 ***John Jacob Abel*** *(USA). Trained under Schmiedenberg and went to John Hopkins University. Isolated epinephrine from adrenal gland extracts (1897) and histamine from pituitary extract (1919). Also prepared pure crystalline insulin (1926).*

1897–1912 ***Paul Eherlich****'s (1854–1915) chemotherapy, immunotherapy and 'side-chain theory' have influenced 20th century pharmacology.*

1899–1941 *The discoveries of* aspirin *(1899),* vitamins *(1912–1922),* hormones *(1902–1931) and* antibiotics *(1941) have revolutionized the pharmaceutical industry.*

1932 ***Gerhard Domagk*** *(1895–1964) discovered the first of the* sulfanilamide *drugs (1899).*

1940 First color television broadcast in the United States using a system developed by **Peter Goldmark** (1906–1977).

1940 **Igor I. Sikorsky** flew the first practical single-rotor *helicopter* at Stratford, CT.

1940–1947 **William (Wolfe) Frederick Friedman** (1891–1969; USA). Cryptography pioneer. U.S. Army Intelligence Colonel and Chief Cryptoanalyst of the War Department in Washington D.C. (1941–1947). Led the U.S. Army team (Special Intelligence Service) which broke the Japanese code in 1940 (*Purple* Code) and subsequently remained a key member of the Operation Magic teams which decoded Japanese ciphers and enabled U.S. military commanders to read Japanese intercepts on Japanese military movements.

Magic was the code name for the joint Army and Navy operation, first set up in 1939, to break Japanese diplomatic and military codes. Magic provided the U.S. military and political chiefs with much important intelligence throughout the war and its contribution to major Allied operational successes has until recently been largely underestimated. The Navy Special Intelligence Unit, Communications Security Unit, with a staff by 1942 of about 300, worked with Army Signals Intelligence Section (SIS), deciphering and relaying enormous amounts of traffic in coded messages sent by the Japanese government to their agencies worldwide and by Imperial Headquarters to their commanders at sea and in the field.

Probably the most important contribution made by Magic to the U.S. victory in the Pacific was the decoding of ciphers that revealed the Japanese attack plan for the Battle of Midway in mid-1942. Informed in advance of the Japanese objectives, Admiral Nimitz was able to preempt Japanese strategy and fight off a superior Japanese force, decisively halting the thrust of the Japanese offensive in the Pacific.

Friedman was born in Kishinev (now Chisinau, capital of Moldova). The following year, the family emigrated to the United States to escape increasing persecution of the Jews in Russia. The family settled in Pittsburgh (1893). After graduating from Cornell University, Friedman became interested in the study of codes and ciphers and during WWI became a cryptographic officer with the U.S. War Department in Washington.

Friedman introduced mathematical methods into cryptology and produced training material used by several generations of pupils. His work affected for the better both signals intelligence and information systems security, and much of what is done today at NSA may be traced to Friedman's pioneering efforts.

He was buried in Arlington National Cemetery, and his wife, Elisabeth Smith Friedman (1892–1980), a gifted codebreaker in her own right, is buried with him.

Friedman coined the term "cryptoanalysis".

1940–1948 **Martin David Kamen** (1913–2002, USA). Discovered carbon-14, a radioactive isotope of carbon with half-life of about 5700 years, and used it to study primary processes in *photosynthesis*.

Kamen was born in Toronto, Canada and studied in Chicago. In 1960 he was appointed Professor of Biochemistry at the University of California, San Diego. He pioneered the application of several radio-isotopes in a diversity of biochemical, particularly bacterial, systems. Confirmed the hypothesis that all the oxygen released in photosynthesis comes from water and not from CO_2.

He determined (1945) the initial state of 'fixed' CO_2 in photosynthesis; showed that bacteria, which carry out the photosynthetic conversion of CO_2 to carbohydrate (without oxygen release) require the presence of reducing substances (such as hydrogen sulfide); and that illumination increases phosphorus turnover in photosynthesis (1948).

1940–1942 **Hedy Lamarr** (1913–2000, Austria and USA). Movie star and inventor. Incepted the revolutionary idea of *frequency-hopping*, namely of sending radio signals without being detected, deciphered or jammed by simply modifying the carrier frequency according to random preassigned scheme (code) of frequencies, synchronized at both the sending and receiving ends. This she first suggested to apply to submarine torpedoes controlled by radio signals[969].

Hedy Lamarr's invention eventually influenced, impacted, and changed the field of communications. Originally intended to protect U.S. radio-guided torpedoes from interception, jamming, and miscalculated ocean drift, it ended up catalyzing a world-wide wireless revolution utilizing radio frequencies. Used in such devices as traffic signals, cellular phones, pagers, wireless Internet, and the Milstar Defense Satellite, it allows communications to be more secure,

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969 In order to reconstruct the original message, there is a need for a frequency-synthesizer, able to perform fast-hopping over the carrier frequencies. The faster the "hopping-rate", the higher the processing gain.

quicker, and cheaper than ever before. Radio frequencies were once thought to be a limited resource, but spread spectrum technology effectively allows simultaneous use by multiple users sharing several frequencies.

One of the most fascinating chapters in Lamarr's life and career had nothing to do with her film career and everything to do with her brain power. How many movie stars can you name, who hold the patent on a significant technological breakthrough? It's a story even Hollywood couldn't have invented.

At the age of 84, Hedy Lamarr was finally honored for her invention from the 1940's. She received an award for her and George Antheil's invention.

In 1997 Hedy received an award at the Computers, Freedom, and Privacy conference for "blazing new trails on the electronic frontier."

All that and brains too — the birth of spread-spectrum technology

Silver Screen actress **Hedy Lamarr** *(1914–2000) enjoyed one of the more memorable careers in Hollywood. Her name still ranks among the brightest lights in the history of the movies. But what many people do not know is that in 1941, as a 27 year-old this glamorous Hollywood star created an idea that revolutionized communication technology into the 21st century.*

She was born as Hedwig Eva Maria Kiesler in Vienna, Austria on November 9, 1914. The daughter of a bank director and a concert pianist, she had been well groomed in her native Vienna: Instructed by private tutors, who taught her three languages as well as ballet and piano. She then received her finishing touches at a private boarding school in Switzerland.

She made her first film in 1931 as an extra. In the 1932 Austrian production, Ecstasy, her teenage nudity made her world-famous and set standards for erotic pictures for decades to come. The film was extremely daring for its time and won her the Grand Prize in the 1934 Vienna Film Festival. The part also won her a millionaire husband, Fritz Mandl (1933), one of Europe's *largest armament manufacturers, who supplied Benito Mussolini with arms for his invasion of Abyssinia (1936), and later sold bombs, bullets and airplanes to Hitler. Mandl was also conducting research in weapons control systems, being especially interested in radio-controlled torpedoes. Radio signals, however, had a serious flaw: enemies could access the same radio-wave frequency and jam it. As Mandl's wife, Lamarr was exposed to military technology ideas, being at his side during business meetings. Although she had no formal technical education, she possessed a mind capable of understanding what she heard.*

In 1937, Lamarr fled her husband to London, where she came in contact with Louis B. Mayer of MGM, who arranged for her to come to the U.S. On the voyage across the Atlantic, Mayer gave her the name "Hedy Lamarr", in part inspired by the sea! In Hollywood, the beautiful actress found success, and had the world at her feet[970]*. But she anticipated the perils of Nazism, and as WWII was brewing in Europe she was determined to do something to help the war effort; recalling the 'Torpedo Problem', it occurred to her that jamming could be avoided if the frequency of the radio carrier wave could be quickly changed, taking on values in a prearranged discrete random sequence,* synchronized *at both the sending and receiving ends. Thus, the simple but revolutionary idea of* frequency-hopping *was born.*

To realize this idea technically, Lamarr had to solve the synchronization problem. To this end she engaged (1941) the ultramodern experimental Hollywood composer ***George Antheil*** *(1900–1959) who had a good deal of expertise with sound synchronization. Antheil proposed controlling the frequencies for the transmitter and receiver with paper rolls, perforated with a pseudo-random pattern to delineate the frequency path: two rolls with the same pattern would be installed in the transmitter and receiver; if the two rolls were started at the same time, and one stayed at the launch point while the other launched with the torpedo, and if there existed good rotary stability in the motors driving the paper rolls, one could maintain the synchronization right on down to where the torpedo hit the ship. The two inventors designed their system to use* 88 *frequencies — exactly the number of keys on a piano.*

[970] Hailed as the "Modern Eve", she looked stunning in all of her 26 films (1929–1958), most notably in *Algiers* (1938), in which she became the most alluring lady in American films; *Tortilla flat* (1942), as a Mexican beauty; and *Samson and Delilah* (1950), in which Cecil B. De Mille had the inspired idea of using her sex-appeal straight, and the public loved her as an elegant, cool and adventurous woman — the world's most renowned seductress. Unfortunately, she turned down many good film roles (Ingrid Bergman got most of them) and had little understanding of how best to advance her own career.

On June 10, 1941 Lamarr and Antheil received Patent No. $2,292,387$ *for their invention of a classified communication system that was especially useful for submarines. It was based on radio frequencies changed at irregular periods that were synchronized between transmitter and receiver: signals could be transmitted without being detected, deciphered or jammed — an unbreakable code!*

Rather than develop the patent commercially, they gave it away to the government for the war effort. Despite the fact that they stood to gain financially by holding onto the patent, they were both committed to helping defeat the Nazis.

The Navy, however, refused to take the Secret Communication System *seriously. Technologists questioned whether the paper rolls would hold without breaking, whether the rotary motor that synchronized the rolls would be accurate enough, and whether the paper rolls could be made small enough to fit inside a torpedo. Despite the inventors' active lobbying and insistence that the US Navy needed the invention to compete with Germany's sophisticated military technology, the Secret Communication System was never used during WWII*[971]*. Not only was the invention rebuffed, but so were Lamarr's efforts to contribute her considerable technical abilities to the task of defeating Hitler. When she offered to come to Washington, D.C. and work at the National Inventors Council, she was told she'd be of greater service to the war effort by remaining in Hollywood and using her star status to raise war bonds. She obeyed, selling $7 million worth of bonds in a single day by offering kisses, at $50,000 a kiss. After the patent expired (1958), the government began to use it*[972]*, but Hedy Lamarr received neither money nor recognition for her accomplishments. In the Cold War era, the military expanded its use of frequency hopping technology, relying on it to secure communications during the Cuban missile crisis (1962), but the idea was not yet widely known in the civilian world. In the mid 1980s, the US military declassified the use of frequency-hopping, also known as spread-spectrum technology, and the commercial sector began to develop it for consumer electronics.*

By the 1990s, as telecommunications became a bigger part of everyday life, interest in frequency-hopping grew as it became one way to enable multiple users to share a single radio frequency — an important spectral-efficiency measure as more and more pagers, cellular phones and other devices crowd into limited airwave spectra.

Hedy Lamarr's life is a remarkable story of survival, adventure[973] *and achievement. Her invention was 50 years ahead of its time.*

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[971] The Navy's reticence was due to an anti-cultural bias: In their patent Lamarr and Antheil attempted to better elucidate their mechanism by explaining that certain parts of it worked like the fundamental mechanism of a player piano. The top military brass in Washington who examined the invention read no further than the words 'player piano'. "My God," they said, "we shall have to put a piano in a torpedo..."

[972] E.g. the *Sonubuoy* was designed for the US Navy (1955); it is a device which had a two-way radio and antenna and was dropped from an airplane and floated on the surface of the ocean. It allowed airplanes to communicate with each other. With the use of Hedy Lamarr's invention of frequency hopping, the planes were able to communicate without interference and in complete security.

[973] She became famous for other things besides films, including a sizzling autobiography (*Ecstasy and Me: Life as a Woman*), over which she sued her collaborators for $21 million; six husbands, most of them millionaires; and a widely-publicized shoplifting incident. She had been short of money through-

The next time you pick up a cellular phone, give a brief thought to the improbable woman who first patented some of its underlying technology — "the most beautiful girl in the world", actress Hedy Lamarr.

1940–1951 CE **Edwin Mattison McMillan** (1907–1991, U.S.A.). With **Philip Hauge Abelson** (b. 1913) discovered *neptunium* (June 08, 1940), element 93, the first known transuranium element; exposed uranium to accelerated neutrons; some of the neutrons stuck to the uranium nucleus and then underwent beta-decay (changing from neutrons to protons by emitting an electron and an anti-neutrino).

Later that year they showed that a similar process also produced element 94, *plutonium*. Exhausting the supply of planets beyond Uranus as names, later elements would be named for places and persons [americium, curium, berkelium, californium, einsteinium and fermium — elements 95 to 100; all of those were created by adding protons to the nuclei of previously existing elements]. Off all these, only plutonium was found to be a fissible (fissionable) element and suitable for making nuclear weapons.

In 1945 McMillan [and independently, Soviet physicist **Vladimir I. Veksler** (1907–1966) invented the *synchrocyclotron*, an accelerator that produces particle energies in excess of 20 million electron volts. Overcoming the limitations of the *cyclotron* (1932), they worked out the theory of *phase stability* which guided the design of all future high-energy accelerators.

Awarded (1951) the Nobel prize for Chemistry, with **Glenn T. Seaborg** (1912–1999) for their discovery of plutonium and research on transuranium elements.

1940–1958 CE **Abraham Selman Waksman** (1888–1973, USA). Microbiologist. Awarded the Nobel Prize in physiology or medicine, 1952, for his discovery of *streptomycin*, the first antibiotic effective against tuberculosis.

Waksman was born in Priluka, Ukraine (near Kiev) to Jewish parents. He received his early education primarily from private tutors, obtained his high-school diploma as an extern in Odessa (1910) and left for the US immediately afterwards. There he studied at Rutgers (B.Sc. 1915; M.Sc. 1916) and at the University of California (Ph.D. in Biochemistry 1918). He returned to

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out the years: she told police she had jewels stolen but they were found; she filed false rape charges to gain money.

Rutgers University, where he became Professor of microbiology (1930), Head of the Department (1940) and Director of the Institute of Microbiology (1949).

He had isolated, together with his students and associates, a number of new antibiotics, including *actinomycin* (1940), *streptothricin* (1942), *streptomycin* (1943), *neomycin* (1948) and others. Of these, the last two have found extensive application in the treatment of numerous infectious diseases of men, animals and plants.

He has published more than 400 scientific papers and has written, alone or with others, 18 books.

1940–1962 CE **Pierre-Michel Duffieux** (1891–1976, France). Physicist. Originator of the field of *Fourier optics* (1940). Concepts such as "frequency response function", "transmission function" and "transfer function" of an optical system are associated with his name. Those concepts put on a broader foundation some ideas which **J.C. Maxwell** (1856), **Ernst Abbe** (1873) and **Lord Rayleigh** (1879) had conceived much earlier in connection with the resolving power of optical systems and the theory of optical imaging.

Duffieux showed, however, that the role of the Fourier transform is not confined to image formation, but that it is also an indispensable tool in image post-processing, whether the technique employed be analogue or digital.

With the aid of the Fourier transform one tries to simulate a kind of "reverse path", canceling some of the unwelcome features of the image forming process along the way, with a view to restoring the original image function. His theory was of major importance in the development of optical communication in the last third of the 20th century.

Duffieux was born in Saint-Macaire (Gironde), studied at Bordeaux and received a D.Sc. from the Ecole Normale Superieure (1915). He then held academic positions at Marseilles (1916–1927), Rennes (1927–1929), Grignon (1941) and became a professor of physics at the University of Besancon (1945–1962).

1940–1965 CE **Giulio Yoel Racah** (1909–1965; Italy and Israel). Physicist and mathematician. Made important contribution in the fields *atomic spectroscopy*. Named after him are: *Racah's symbol*, *Racah's V-coefficient*, *Racah's W-coefficient*, *Racah-Wigner calculus*. A crater on the moon was named after him. Wrote (with **U. Fano**)[974]: "*Irreducible Tensorial Sets*" (1959).

[974] **Ugo Fano** (1912–2001, Italy and USA). Atomic physicist. Since 1937 – at the University of Chicago with **E. Fermi**. Son of the mathematician **Gino Fano** (1871–1952).

Born in Florence, Italy, he took his Ph.D. from the University there (1930), and later studied under **E. Fermi** in Rome. Emigrated to Israel (1939) and appointed professor of Theoretical physics at the Hebrew University, Jerusalem. Published (1941–1949) three seminal papers on energy levels in many-electron atoms.

Died in Florence, on a way to the Zeeman centennial celebration in Amsterdam, as a result of a gas leak in an old heating installation.

1940–1969 CE **Alfred Day Hershey** (1908–1997, USA). Bacteriologist. Confirmed through experiments that DNA, not protein was the genetic material.

In one of the most famous experiments in 20^{th} century biology, Hershey and his assistant, **Martha Cowels Chase**[975] (1927–2003) marked bacteriophages with radioactive isotopes and then were able to trace protein and DNA to determine which is the molecule of heredity.

He was born in Owosso, Michigan and received his B.S. in chemistry at Michigan State University in 1930 and his Ph.D. in bacteriology in 1934, taking a position shortly thereafter at the Department of Bacteriology at Washington University in St. Louis.

He began performing experiments with *bacteriophages* with Italian-American **Salvador Luria** and German **Max Delbrück** in 1940, and observed that when two different strains of bacteriophage have infected the same bacteria, the two viruses may exchange genetic information.

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[975] Hershey and Chase announced their results in a paper: A.D. Hershey and M. Chase, 1952. *Independent functions of viral protein and nucleic acid in growth of bacteriophage*. Journal of General Physiology **36**: 39–56.

The experiment inspired American researcher **James D. Watson**, who along with England's **Francis Crick** figured out the structure of DNA at the Cavendish Laboratory of the University of Cambridge the following year.

Hershey shared the 1969 Nobel Prize in Physiology or Medicine with **Salvador Luria** and **Max Delbrück**. Chase, however, did not reap such rewards for her role. A graduate of The College of Wooster in Ohio (she had grown up in Shaker Heights, Ohio), she continued working as a laboratory assistant, first at the Oak Ridge National Laboratory in Tennessee and then at the University of Rochester before moving to Los Angeles in the late 1950s. There she married and earned her Ph.D. in 1964 from the University of Southern California. A series of personal setbacks through the 1960s ended her career in science. She spent decades suffering from a form of dementia that robbed her of short-term memory. She died in 2003.

He moved to Cold Spring Harbor, New York, in 1950 to join the Carnegie Institution of Washington's Department of Genetics, where he performed the famous Hershey-Chase blender experiment with **Martha Chase** in 1952. This experiment provided additional evidence that DNA, not protein, was the genetic material.

He became director of the Carnegie Institution in 1962 and was awarded the Nobel Prize in Physiology or Medicine in 1969, shared with Luria and Delbrück for their discovery on the replication of viruses and their genetic structure.

The Helicopter

In the helicopter [from the Greek: $\epsilon\lambda i\xi$ (helix) = screw, $\pi\tau\epsilon\rho o\nu$ (pteron) = wing], a large propeller (rotor), rotating horizontally, provides the lift that holds up the craft.

When the lift is exactly equal to its weight, the helicopter hovers motionless. If the pilot wants to go forward, he causes the blades to tilt forward at an angle. Then, the reaction of the air on the blades has a vertical component to counterbalance the weight, and a horizontal component to provide a forward motion.

The power-driven rotors serve the helicopter in lieu of fixed wings and a propeller. The helicopter is able to take off and land vertically, to move in any direction, or to remain stationary in the air. The lift is determined both by the rotary speed and angle of attack *(pitch) of the blades. For a certain speed, the generated lift will counterbalance the weight of the craft.*

The rotation of the rotors causes the fuselage of the aircraft to rotate in the opposite direction. To prevent this, a single-rotor helicopter is provided at its tail with a small vertical propeller that produces a counteracting sideways thrust. Alternatively, the helicopter may have two rotors which revolve in opposite directions and thus counterbalance each other[976].

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[976] The shape of the blade is such that the air streaming above it moves faster than the air below it. This difference creates a pressure gradient that lifts the wing. The helicopter has three controls:

Collective Pitch Stick: controls the main rotor blades to make the helicopter hover and fly straight up or down. To fly *upward*, the pilot pulls the stick up to increase the pitch of all the blades. To fly *downward*, he pushes the stick down to decrease the pitch of the blades. To *hover*, he sets the blades at a medium pitch. The collective pitch stick also increases the power sent to the rotor for upward flight, and decreases power for downward flight.

Cyclic Pitch Stick: controls the main rotor blades to make the helicopter fly *forward*, *backward*, or *sideways*. The pilot pushes the stick in the direction he wants to fly. The stick continuously and periodically changes the pitch of each blade to make it rise highest at a spot directly opposite the direction of flight. For example, to drive the helicopter forward, the blades rise highest at the rear. The blades get their greatest pitch one-fourth of a revolution ahead of the spot where they rise highest.

Tail Rotor Pedals: control the pitch of the tail rotor blades to turn the helicopter

The idea of the helicopter first appeared in the notebooks of ***Leonardo da Vinci*** *(ca 1500), who drew sketches of a flying machine of this type. At about the same time the Chinese made toys that flew like helicopters.*

The earliest attempts to build model helicopters that flew were made by two Frenchmen, ***M. Launoy*** *and* ***M. Bienvenu****, in* 1784. *The British inventor* ***George Cayley*** *designed a steam-powered model in* 1843. *Another British inventor,* ***W.H. Phillips****, built in* 1842 *a steam-powered model helicopter, the rotor of which was driven by jets of steam coming from the tips of the rotor blades.*

In 1878, ***Enrico Forlanini*** *(1848–1930,* Italy*) built a steam-driven helicopter model which hoisted to about* 12 *m above the ground, and stayed up for* 20 *sec.*

The first manned helicopter flight was achieved (1907) by ***Paul Cornu*** *(1881–1944,* France*) and* ***Louis Bréguet*** *(1880–1955,* France*). Both of these machines made short flights, but neither was practical. They were* unstable *(wobbly) and their flights could not be controlled.*

During and after WWI, helicopter designers gradually began to solve the problems of stability and control. In 1937, *a two-rotor helicopter built by* ***Heinrich Focke*** *(1890–1979,* Germany*) flew for more than an hour at an altitude of* 2400 *meters.* ***Sikorsky*** *promoted the development of a big helicopter industry in the United States, which proved its value in WWII and the Korean War. Helicopters rescued countless wounded and trapped troops and brought water, food, ammunition and medical equipment to the fighting men.*

right or *left*. For straight flight, the pitch of the tail rotor is set to prevent the helicopter's body from turning to the right as the main rotor turns to the left. The pilot pushes the left pedal to increase the pitch of the tail rotor and turn the body to the left. He pushes the right pedal to decrease the pitch of the tail rotor and turn the body to the right.

Science Progress Report No. 18
World-War II (Sep 01, 1939 – Sep 02, 1945)

"In the arts of life man invents nothing; but in the art of death he outdoes Nature itself"

George Bernard Shaw, the Devil's words to Don Juan, 1903 (1856–1960)

"There occur from time to time in the course of history, men who have a special genius for conquest and destruction. Think of Alexander the Great, Attila (properly named flagellum Die = "the scourge of God"), Ghingiz Khan, Hulagu (who sacked Baghdad in 1258). The methods of these men were very much the same "Blitzkrieg" accompanied by ruthlessness. Of course, these monsters did not think of themselves as 'destroyers' but of builders of a "new order". They did establish a kind of order, for when their work was done, there ruled in their dominion the order of exhaustion and death. We may apply to them the incisive words which the British chieftain Calagus applied to the Romans: 'To plunder, butcher, steal — these things they misname empire; they make a desolation, and they call it peace"'.

George Alfred Léon Sarton, 1948 (1884–1956)

"Science in the 20^{th} century was doubly warned by history: the First World War and the coup de grace of the second World War with its associates, genocide and vile totalitarianism".

Anon

A. Forerunner — the decline of science in Germany

*The main trend in German science at the turn of the century became one of ever-increasing nationalism. In 1914 a manifesto was issued, signed by 93 eminent German scientists and scholars, including such men as **Ehrlich**, **Haber**, **Ostwald** and **Planck**, which accused the contemporary British and French scientists of plagiarism, and claimed that the priority of German scientists* *to a number of discoveries had not been acknowledge outside Germany. A number of eminent German-speaking scientists did not associate themselves with the manifesto, notably* ***Einstein****, but they were in the minority.*

From 1930 *on, German nationalism began to weaken German science, and the decay in the quality of German science was an inevitable consequence of the degeneration of German culture under the Nazi regime. First, there was the "brain-drain" of Jewish and anti-Nazi scientists who left Germany for Britain, the United States and other countries. No less serious was the infusion of national-socialism into German science. Nevertheless, the theories of science kept contradicting the tenants of national socialism: In physics there was a postulate of the equivalence of all observers of nature (no privileged race!) whilst the findings of biologists and anthropologists did not agree with the racial pseudoscience.*

The spirit of national socialism gradually infected German scientists, and there appeared a tendency toward self-glorification both on the national and personal levels. With the rise of Germany's military power came the decline of fundamental research and its applications.

From 1933 *on, Germany and with it all of central Europe was gradually emptied of its best mathematicians, physicists, biologists and chemists (****Einstein****,* ***Bohr****,* ***Franck****,* ***Stern****,* ***Schrödinger****,* ***Born****,* ***Wigner****,* ***Teller****,* ***Noether****,* ***Weyl****,* ***Courant****,* ***Gödel****,* ***von Neumann****,* ***Bloch****,* ***Brauer****,* ***Artin****,* ***Ulam****,* ***Neugebauer****,* ***Wald****,* ***Neyman****,* ***Eilenberg****,* ***Chain****,* ***Krebs****,* ***Haber****,* ***Delbrück****,* ***Hertzberg****,* ***Lipmann****,* ***Meyerhof*** *and others).*

In toto, about 1150 Jewish scientists, many of them of great brilliance and in high positions, left Germany between 1933 and 1935 — a clear indication of the major role played by the Jews in German science before the rise of Hitler.

This exodus of intellectuals from Europe was mostly directed towards the U.S., which was very fortunate to be able to integrate the European scientific heritage into the ranks of its growing young sciences[977]*. Most of the refugees*

[977] The growth of the number of Jewish students in the American Ivy League universities during the 1920's was viewed with alarm by the WASP leadership of these institutions. At Harvard, for example, president A.L. Lowell struggled with his faculty to institute formal quotas for entering undergraduates. His policies resulted in a sizable reduction of the proportion of Jewish students, from 25–27% in the 1920's to 10–16% in the 1930's. Thus, for example, **Norbert Wiener** was black-listed by **George Birkhoff**. Harvard's concern over what Lowell termed "the Jewish problem" was not unique. When M.I.T. made trouble over the appointment of Norman Levinson to an assistant professorship, **G.H. Hardy**, who was visiting M.I.T. that year (1932), threatened Vannevar Bush, then M.I.T. president, to spread the news that M.I.T. stood for the

were Jews. They belonged to that part of the population that contributed about 30 *percent to overall German achievements in the sciences during* 1800–1933.

The physics brain-drain was accompanied by the rise to power of two strongly antisemitic physicists[978], *Johannes Stark (1874–1957) and Philipp Lenard (1862–1947). They considered the whole field of atomic physics as a Jewish fraud*[979]. *Quantum theory and relativity were for them "Jewish" physics, in contrast to "German" (or "Aryan") physics. German physicists who accepted and taught Jewish physics were called "white Jews". The detrimental effects on physics in Germany were obvious.*

In the biological sciences, those who had inspired the greatest developments in the field had left Germany and were to die in exile: ***Fritz Haber***, ***Richard Willstätter***, ***Otto Meyerhof***, ***Carl Neuberg***, ***Max Bergmann*** *and* ***Rudolf Schoenheimer***.

During the 1933 *business meeting of the Mathematische Reichsverband (MR) [Reich Mathematical Association], the 'leader' principle was accepted, and the former chair and now 'leader' of the group, Georg Hamel, made the*

Massachusetts Institute of Theology.

The influx of European scholars during the 1930's boosted the xenophobia in academic circles to unprecedented levels; in his address to the AMS in 1938, on the occasion of its 50^{th} anniversary celebration, G.D. Birkhoff warned his audience that eminent researchers from abroad were reducing the number of available positions for young Americans with "the attendant probability that some of them will be forced to become 'hewers of wood and drawers of water'."

Four years earlier, on 18 May 1934, the same Birkhoff had written a letter to R.G.D. Richardson to discourage the candidacy of Solomon Lefschetz for president of AMS: "*I have a feeling that Lefschetz will try to work strongly for his own race. They are exceedingly confident of their own power and influence in the good old U.S.A.... He will get very racial and use the Annals as a good deal of racial perquisite. The racial interests will get deeper as Einstein's and all of them do*". In spite of Birkhoff's antisemitic opposition, however, Lefschetz became the first Jewish president of the AMS in 1934.
Eventually, the great influx of European talent had transformed academic and *artistic* life in America.

978 On Lenard's office door in Heidelberg was a note: "Entrance to Jews not permitted".

979 In 1945, Stark and Lenard must have considered themselves very lucky that the *fraudulent Jewish device* was dropped on Hiroshima in lieu of Berlin.

following statement: "We want to cooperate sincerely and loyally in accordance with the total state. Like all Germans, we place ourselves unconditionally and happily in the service of the National Socialist movements, behind the Führer".

The expulsion of mathematicians from their positions began with the "Law for the Restoration of Civil Service" of 7 April 1933. Richard Courant, Edmund Landau, Felix Hausdorff, and Otto Toeplitz were among the victims[980]*. In 1937, the "New Official's Law" affected even those who were "related to Jews by marriage" like Erich Kamke or Emil Artin. By 1938 more than one-fourth of the 227 mathematics instructors in German universities were expelled*[981]*. These loses could hardly be replaced. Of the 7319 students of physics and mathematics in 1931, only 1270 remained by 1939.*

In 1934, the secretary of the Deutsche Mathematiker-Vereinigung (DMV) [German Mathematicians Union], Ludwig Bieberbach, who became the Nazi ideologue of mathematics, extended the Nazi racial theory into the realm of mathematics. Justifying the student boycott against Landau he divided mathematics into Jewish and Aryan mathematics. The former he characterized as "mental arrogance", devilish cleverness, "juggling with concepts" and the "cunning of Jewish mathematicians like Jacobi".

In 1934, ***David Hilbert*** *was sitting next to the Nazis' newly appointed minister of education, at a banquet. When asked: "And how is mathematics in Göttingen, now that it has been freed of the Jewish influence?" Hilbert replied: "Mathematics in Göttingen, there is really no such thing any more".*

Most German scientists considered public expression of any political allegiance to be inconsistent with the dignity of their profession. Einstein was exceptional in his open avowal of socialism and pacifism, as were Lenard and Stark in their vociferous devotion to the 'Führer'. The German universities, however, both staff and students, had been the strongest supporters of antisemitism, and the Weimar Republic had few friends among the professors.

980 After the purge of the Göttingen Mathematical Institute from all of its eminent Jewish mathematicians, the Nazis appointed in 1934 one of their devoted members, **Helmut Hasse** (1898–1979) to succeed Richard Courant as head of the institute. By irony of fate, the spirit of what the Nazis called *Jewish mathematics* continued to live in the works of Hasse, who learned all he knew from his Jewish teachers **Kurt Hensel**, **Edmund Landau** and **Emmy Noether** and propagated their ideas in his own papers during the pre-Nazi period 1923–1931. After the exodus, his contributions to mathematics became rather mediocre and some of his conjectures in number theory have now even proven wrong.

981 This program was overseen by the virulent Nazi mathematician **Oswald Teichmüller** (1913–1943) who perished on the eastern front.

As early as 1930, *the University of Jena appointed the virulently antisemitic Hans Gunther as Professor of Racial Science (Rassenkunde) and his inaugural address was wildly cheered by the students.*

Several efforts were made to persuade Max Planck to make a public protest against the Nazi dismissals of Jewish and socialist professors. Otto Hahn asked him to support a statement by a dozen eminent physicists, but Planck refused. Marie-Elisabeth Lüders, a former member of the Reichstag and old family friend, twice visited Planck in January, 1933, *and asked him to support a protest in which scientists would withdraw from all teaching and research. In later years (1963) she recalled their last meeting:*

"I hoped that this general strike of the intellect would tear from the eyes of millions half-blinded the veil behind which the Nazis concealed their dangerous abyss toward which they were leading Germany. I left Planck with the dreadful knowledge that there was no way to stop the coming downfall of German Science, and no remedy against the disgraceful readiness of so many to sell out teaching and research. Planck himself felt that what was to come, must come. After a long silence, he went to his grand piano and played a Bach chorale, then he offered me his hand, and I left".

In fact, Planck, in a way, cooperated with the Nazi regime: he uttered "Heil Hitler" and raised his arm in the Nazi salute whenever required of him. He even signed his letters to the minister of education with a "Heil Hitler". In his lectures and seminars he refrained from mentioning Einstein's name[982,983]

Planck's personal situation was so prestigious that he could have spoken against some of the excesses of the regime without fear of reprisals, but he chose to remain silent.

Years after the Second World War Einstein told a friend who was going to visit Germany: "Give my regards to Laue"; "And Planck?", the friend asked. Einstein shook his head sadly.

There is no known instance in which a professor of physics or chemistry without any Jewish family ties ever made an open protest against Nazi activities. During 1933 *and* 1934, *the scientific establishment, led by Max Planck and Walther Nernst, washed its hands of the growing terror and concentrated on defending its own special privileges.*

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[982] In contradistinction, **Laue** was teaching relativity theory in his seminars, always assuring his students that it had originally written in Hebrew!

[983] Yet, the Nazis reciprocated in their own satanic way: On July 23, 1944, three days after the failure of the plot to assassinate Hitler, Planck's son Erwin was arrested on charges of friendship with some of the conspirators. He was then executed on January 23, 1945.

On the 25^{th} *anniversary of Kaiser Wilhelm Gesellschaft, in* 1936*, Planck was able to send a telegram to Hitler thanking him for his "benevolent protection of German science".*

The only notable German scientist who was conspicuous in his disapproval of the Nazis was **Max von Laue***, and even his actions were taken within the physics establishment and not in open criticism of the regime. Laue tried without success to persuade Heisenberg to help resist the worst Nazi excesses.*

This unbroken record of collaboration had its hues and shades: Many were merely opportunists who welcomed the chance to advance their careers by taking the positions of dismissed Jews, while the more eminent were either mildly anti-Nazi like Planck and Hahn, or moderately pro-Nazi like Heisenberg.

During the war (1939–1945) the Nazis have systematically and scientifically exterminated in Poland 6×10^6 *Jews from all over Europe. Among them were many thousands of talented potential scientists, most of which vanished anonymously into oblivion. The ashes of the Auschwitz ovens must have been permeated with the remnants of the brains of future geniuses*[984].

Unpardonable were also the crimes of omission*: The allies and most of the democratic world did very little to offer refuge and save those who could be saved; thus, the allies refused to bomb the gas chambers at Auschwitz and the railroads leading to it during the war.*[985] *During the years between 1933 and*

984 *Example*: Ela Chaim Cunzer (pronounced *Tsunzer*) was born on June 6, 1914 in Lubcz, a small town near Wilno (present day Vilnius, capital of Lithuania). During 1932–1937 he studied for his master degree in mathematics under Antoni Zygmund (1900–1992) at the Stefan Batory University, Wilno, Poland. His master's thesis was *On convex and subharmonic functions* (1937).

After the Nazi occupation of Wilno (June 1941), many Jews were murdered in Ponar, peaceful woods outside Wilno. Cunzer, then working on his Ph.D. thesis, was herded, with others, into the Wilno ghetto. There, in the face of day-to-day treat of death by starvation, disease and violence, he continued in unimaginably crowded and squalid quarters to work on his doctoral thesis. When asked by his cousin whether studying mathematics under such horrifying conditions did not make him feel crazy, he replied: "I would go insane if I did not do it".

Cunzer was deported from the Wilno ghetto to a concentration camp where he perished during the winter of 1943–1944.

His thesis was found intact in 1993 in the archives of the Vilnius University.

985 The American President **Franklin Delano Roosevelt** (1882–1945) and the British Premier **Winston S. Churchill** (1874–1965) refused to do it, despite the fact that Allied bombers were regularly dropping bombs in military targets in the vicinity of the death camps. It is incomprehensible how such an enormous

1948, of all the nations in the Western world, Canada had the worst record in providing sanctuary to European Jewry. In fact, Canada was hermetically closed to the Jews of Europe. After 1945, however, Canada opened its gates to... Nazi war criminals (!) who settled there by the thousands.

It is a story summed up best in the words of a Canadian official who when asked how many Jews would be allowed into Canada after the war, replied: "None is too many"[986]*. Had Canada been admitting European Jews, her cultural record could have looked better. But it is a matter of record that no Canadian won the Nobel prize for Physics, Economics, Chemistry or Literature during the entire 20th century until 1971, and very few afterwards.*

The behavior of the 'neutral' countries in Western Europe did not fare better: they were willing to accept blood money for economic advantages. This was particularly true of Switzerland[987]*.*

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moral crime could have been perpetrated by Christian men deeply versed in the ancient Biblical tenet: "*Thou shalt not stand against the blood of thy neighbor: I am the Lord.*" (Leviticus **19**, 16).

Ironically, Roosevelt's mother, **Sara Delano** (1855–1941) was of Jewish origin (de-Leone) and his ancestors (who arrived from Holland in 1649) were also Jewish (Rosenfeld).

986 *"NONE IS TOO MANY": Canada and the Jews of Europe 1933–1948*, by Irving Abella and Harold Troper, Lester and Orpen Dennys, Publishers Toronto, 1983, 285 pp.

From 1933 to 1945, the United Kingdom opened its doors only to 70,000, and allowed another 125,000 into British-administered Palestine. Argentina took 50,000, Brazil 27,000 and Australia 15,000, The United States took only 200,000 Jews, including the select of European intellectual, cultural and scientific life.

987 The majority of Americans, when referring to Switzerland, think of exquisite chocolates, fine timepieces and visions of Heidi chasing goats across alpine meadows. Nonetheless, Switzerland was a much different nation during the 1940's despite that idyllic view. While the legend of the fierce Swiss neutrality lives on, it is more of myth considering Swiss policy during WWII was balanced heavily in favor of the Nazis. Much of Switzerland's complicity with the Nazis has only recently (1997) come to the forefront from the efforts of President's **Clinton** action regarding the return of the gold of the holocaust victims, allegedly held in the cellars of the Union Bank of Switzerland.

In Aug 1938, The Federal Council of Switzerland petitioned the Nazi government in Berlin to affix a "J" stamp on all passports for Jews in order to make it easier for their border guards to turn away Jews. Over 30,000 Jewish refugees were thus turned away. In August 1942, the Federal Council passed an additional law to seal the border to Jewish refugees. At the same time Switzerland made fortunes in dealing with the Nazis: Swiss manufacturers provided the

A fact that transcends all others is that the Jews of Europe were not so much trapped in a whirlwind of systematic mass murder as they were abandoned to it. The Nazis planned and executed the Holocaust, but it was made possible by an indifference to the suffering of the victims which sometimes bordered on contempt. The Nazis read rejection of the Jews by the democracies and the Catholic Church[988] *as tacit approval of their policies. If no one wanted them, then the Nazis felt free to offer their own solution. The Holocaust followed.*

B. The military conflict*: Sept. 1, 1939–Sept. 2, 1945*

More than 50 countries, representing the majority of world population took part in the war, which caused previously unheard of human misery. It killed more persons, cost more money, damaged more property, affected more people than any other war in history. It heralded the atomic age and brought about

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Nazis with ball-bearings, timers, locomotives, arms, ammunition, aluminum, electric power and other manufactured goods used in the production of war equipment. Credit Suisse and Union Bank supplied the Nazis with foreign currency. In return the Nazis deposited in Swiss Banks large amounts of looted gold, gems and art objects.

988 Neither the Vatican nor its German prelates even condemned the foul principles and practices of the Nazi Government. On the contrary, both Pius XI and Pius XII helped Hitler to attain power and made repeated and increasing efforts to contact an alliance with him. In fact, Pius XII was the ideal Pope for Hitler's unspeakable plan. In his book: '*Hitler's Pope*' (Viking, 1999), **John Cornwell** (a practicing Catholic) asserted that Eugenio Pacelli, alias Pope Pius XII (1876–1958), the Pope during World War II, brought lasting shame on the Catholic Church by failing to denounce the *Final Solution*. According to Cornwell, Pacelli was "a ruthless cynic and a secret anti-semite who was more interested in the Vatican's stockholdings, than the fate of the Jews." Cornwell charges that the pope was so intent on signing the Reich Concordant that he facilitated Hitler's rise to power, suppressed the Catholic Center Party in Germany, gulled Catholic resistance to Nazism, refused to support the Allies against Hitler, kept silent when by speaking out he could have saved European Jews, allowed Nazis to send Roman Jews to concentration camps and supported the Nazi puppet regime in Croatia.

Similar accusations were made by **Rolf Hochhuth**'s drama 'Der Stellvertreter' (*The Deputy*, 1963).

sweeping changes in warfare and geopolitics, as well as science, technology and their relation to society.

The cost of the war and its damages totaled 1600 *billion dollars. Ca.* 75 *million servicemen fought on either side, of which* 16 *million were killed*[989], 10 *million were wounded and* 3 *million were missing. In addition, about* 20 *million civilians were killed*[990].

Scientific inventions and discoveries helped shorten the war. Among them: radar[991]*, guided missiles, jet engines, early computers, and the atomic bomb.*

Germany surrendered on May 7, 1945. *It took two atom bombs (Hiroshima, Aug.* 6, 1945*; Nagasaki, Aug. 9, 1945) to bring about Japan's surrender on Sept. 2, 1945.*

C. German Secret Weapons

Between 1936 and 1944, Germans mere achieving a quite revolutionary level of design and development in aeronautical science. Although both Britain and the United States achieved great success during pre-war and war years in designing and developing fighters and bombers of an entirely conventional type (the Spitfire, Flying Fortress, Lancaster, Mosquito and P-51 Mustang), equal or superior of their German equivalents — Germans built and flew the first practical helicopter (the Focke-Achgelis FW-61), the first turbo-jet aircraft (the Heinkel He 179), the first cruise missile (the FZG-76 or V-1) and the first extra-atmospheric rocket (the A-4 or V-2).

It was an astonishing achievement, largely conducted in complete secrecy. Only the small size of Germany's industrial base, compared to that of the United States, prevented it from dominating the skies during WWII. Of all four achievements: helicopter, jet aircraft, cruise missile, rocket, the development of the V-2 was by far the most impressive.

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[989] Weather vicissitudes also played an important role in WWII: In 1941, a million German soldiers invading Russia succumbed to the winter, thus changing the course of the war. On Dec 13, 1944, the U.S. Pacific 3rd fleet under Admiral Halsey lost 3 destroyers with 7 more damaged, 100 aircraft and 800 men during a typhoon — one of the worst losses of the U.S. Pacific fleet during the war.

[990] Russia claimed to have lost 27 million of its citizens. Of these 300,000 soldiers were killed during the battle of Berlin.

[991] British historians maintain that "The atomic bomb ended the war but radar won it." This reflects the opinion that radar decided the 1940–1941 'Battle of Britain'.

Honors in the V-weapons campaign, if that word can be used about a method of making war on civilians, go to the Germans. Both the V-1, the first cruise missile, and the V-2, the direct technical ancestor of all extra-atmospheric missiles and of the space rockets, were far in advance of any aeronautical weapon produced by their enemies in 1939–45. ***Wernher von Braun****, who was to become an American citizen and to be celebrated as "the father of the space programme," was a scientific genius. The men who produced the V-1 were aeronautical technicians of the first class. Had Hitler had the vision to devote a proportion of Germany's scientific effort similar to that given to other weapon programmes to nuclear weapons, it is possible that, with the V-weapons, he could have won the war.*

Let us go in some more technical detail into the V-weapons: up to 1940, the Germans made good progress in the fields of aerial radio navigation, radar, tank technology and underwater warfare. These developments were manifested in the following weapon-systems, graded in ascending order of importance:

- *The anti-ship glider bomb (the HS 293)*
- *The acoustic torpedo ("Zaunkönig")*
- *The rocket-propelled shell (V-2, father of the ballistic missile; developed by the German Army)*
- *The Flying bomb (FZG-76 or V-1; father of the modern cruise missile, known as Flakzielgerät; developed by the German Airforce)*

The V-1 had the shape of cylinder, pointed at the nose, containing a ton of high explosives and was detonated by an impact fuse. Two short wings were attached at the point of launch. At the rear, mounted above the tail assembly, was the tube for a pulse-jet, fueled by low-grade petrol fed from an on-board tank. A shutter system caused the injected fuel to burn in regular bursts, giving the missile a speed over $640\ \frac{\text{km}}{\text{hr}}$, *its characteristic drone and a range of* $240-320$ km. *A single cut-out device shut off the fuel at a selected point, leaving it to dive vertically to earth.*

It was either launched from a mother aircraft or from a ramp. It was reliable and cheap, costing about 150 sterlings in 1944 values.

Had it been given priority, and been mass-produced in large numbers during 1943, it would have caused terrible damage to London and other southern British cities; it might have disrupted shipping in British southern ports as to have set back the June 1944 invasion.

But its production schedule (16,000 units during Jan-Sept. 1944) was not met because the V-2 programme diverted most of the secret weapons efforts.

The V-2 was expensive (12,000 sterling in 1944 values), complex (65,000 separate modifications had to be carried out before reliable performance was achieved), too difficult to mass-produce, and delivered too small a warhead to achieve decisive results. Its development was plagued by guidance system failures, disintegration of rocket body and explosion of the fuel.

However, it needed no elaborate launching system, it achieved stability without rotation, it had on board an autonomous guidance system, it was liquid-fueled and was single-stage — a truly revolutionary weapon.

D. The Breaking of the German and Japanese Codes

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"Amid the torrent of violent events one anxiety reigned supreme. Battles might be won or lost, enterprises might succeed or miscarry, territories might be gained or quited, but dominating all our power to carry on war, or even keep ourselves alive, lay our mastery of the ocean routes and the free approach and entry to our ports."

Winston Churchill

* * *

Two words describe the German WWII fighting machine successes: organization, and communication. Their lightning "blitzkrieg," which allowed them to roll over Europe almost unopposed, was a well-coordinated operation employing panzers (tanks) and Stukas (dive bombers). At sea, their efforts were aimed at cutting England's supply line from North America by well-directed submarine "wolf pack" attacks on convoys. For communications, the Germans relied almost entirely on messages sent by radio. These messages could be heard, of course, by anyone equipped with a receiver.

To ensure that the enemy would not intercept vital information, they used an electro-mechanical device called Enigma to encode the data. They believed that even if the enemy were to capture a machine, it would be useless unless both sender and receiver were also in possession of the same "key" which described how the message was encoded.

The Germans used different radio frequencies and keys for messages sent to their various units. This ensured that messages meant for the Luftwaffe (Air Force) were not readable by the Kriegsmarine (Navy). By assigning different keys to different units, communication could be directed to the appropriate unit. Not only would there be no point in a submarine decoding a message meant for a panzer unit, but some ultra-secret messages (for example to the SS) were confidential.

Scherbius' *Enigma, in a modified and improved form, was later used widely throughout the German armed forces as the standard method of encrypting messages prior to radio transmission.*

The British ULTRA and the American MAGIC are names applied to Allied Intelligence obtained from cryptoanalysis. It played a decisive role in many of the major battles in the European, Mediterranean, Atlantic and Pacific theaters. Most of ULTRA was derived from German traffic, after the Poles (1933) and the British (1940) have broken the Enigma Code. Likewise, most of MAGIC came after American cryptoanalysts breached the Japanese 'RED' and 'PURPLE' codes in 1940.

Had the Enigma Code not been broken, the Battle of Britain and the Battle of the Atlantic might have been lost, and England forced to capitulate[992].

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992 Prior to the outbreak of hostilities, England had established a balance of trade with the Western hemisphere. Manufactured goods were traded for raw materials such as oil, iron and grain. With the outbreak of war, this balance shifted. Imports increased while exports decreased as manufacture moved to the production of weapons. This excess of imports over exports was paid for by transferring England's gold reserves to the West. But those reserves, not unlimited, were rapidly becoming exhausted. German submarines were taking a fearful toll of merchant shipping. Prime Minister Winston Churchill realized that England's only salvation was to involve the United States in the conflict. Franklin D. Roosevelt, President of the United States, was sympathetic to the British cause, but his hands were tied by the complexities of American politics. It was a delicate situation — how could America supply vital war materiel to Great Britain and, at the same time, maintain its neutrality?

Roosevelt knew that America's involvement in the global conflict was inevitable, but the country was in no mood for another war. Furthermore, there was no guarantee that England would not be defeated. Roosevelt not only had to face an isolationist Congress, but worried that he would not be elected to a third term, realizing that switching national leaders on the eve of war would be most unwise.

The Irish Free State (Republic of Ireland, or Eire) had been established in 1921, leaving Northern Ireland still closely tied to Great Britain. As part of the settlement, some naval bases in the Irish Republic were occupied by the British. At

The Americans would have been denied a staging ground for the invasion of Europe. The War might have been dragged on for another two years, with many more millions of lives lost. Far worse, given this respite, Hitler might have developed the atomic bomb, and, most unthinkably, mated it to a 3-stage intercontinental ballistic missile capable of crossing the Atlantic.

So, already by 1940, the Allies were reading their enemy's mail on a regular basis and by the end of the War, some 10,000 people with sophisticated computers were decoding Axis messages, which they never could without the pioneering work of the master-codebreakers ***Marian Rejewski*** *(1906–1980, Poland),* ***Alan Turing*** *(1912–1954, UK) and* ***William F. Friedman*** *(1891–1969, USA).*

Cryptology, the science of ciphers, has applied since the very beginning some mathematical methods, mainly the elements of probability theory and statistics.

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the start of World War II, the Irish Republic flatly refused to become involved unless the whole island was united under their rule. The British declined, and relinquished their bases. However, they still maintained facilities in Northern Ireland, which guarded the vital Atlantic approaches. (The Republic of Ireland managed to stay neutral throughout World War II.)

In view of this, it was not surprising that Irish-Americans resisted the United States siding with the British. Irishman Joseph Kennedy, America Ambassador to Great Britain, was convinced that England could not hold out much longer. Some industrialists were looking forward to lucrative sales to a Nazi dominated Europe. Other Americans holding anti-British sentiments were German-Americans and American Communists (in view of Germany's non-aggression pact with Russia). Irishman John L. Lewis of the powerful United Mine Workers Union, controlled 5 million anti-Roosevelt votes. Besides, if England's fall was a foregone conclusion, why send military aid that would only fall into German hands?

Finally the U.S. arms embargo was modified to allow a "Cash and Carry" policy, declaring certain arms "surplus," which Britain could purchase for cash. There were other ways to get around the arms embargo; in September, 1940, fifty American four-stacker destroyers were traded for the U.S. use of British bases.

As British supplies of U.S. dollars and gold reserves were rapidly depleted, technology was traded for credit. The magnetron, an essential radar component, had been invented by the British, but was traded to the U.S. for arms.

Nov. 5, 1940, Roosevelt was elected to a third term, the first ever for a U.S. President, his hands now free to send vitally needed aid to Great Britain. On Jan. 10, 1941, the "Lend Lease" program was instituted. By War's end 30 billion dollars had been lent to Great Britain.

Mechanical and electromechanical ciphering devices, introduced to practice in the 1920s, broadened considerably the field of applications of mathematics in cryptology. This is particularly true for the theory of permutations (substitutions), developed in the second half of the 19^{th} *century. Its application by Polish cryptologists enabled, during 1932–1933, to break the German Enigma cipher, which subsequently exerted a considerable influence on the course of WWII (1939–1945).*

In the period between the World Wars, all the major powers and some of the minor ones were routinely decoding each other's messages. The Polish Cipher Bureau was among the best, and in 1932 embarked on a determined mission to break Enigma.

They were assisted by documents passed to them by the French stolen by Hans-Thilo Schmidt, an avaricious German cipher clerk with the chilling codename ASCHE (Ashes). Schmidt was arrested, interrogated and shot for his treachery in 1943.

At the Cipher Bureau, three university mathematics students, ***Marian Rejewski*** *(1906–1980),* ***Henryk Zygalski*** *(1906–1978)* and ***Jerzy W. Rozycki*** *(1909–1942), succeeded in breaking the Enigma in 1933. In one of the greatest-ever feats of cryptoanalysis, Rejewski deduced the internal wiring of the Enigma's rotors and Umkehrwalze. Zygalski invented a cryptanalytical method using perforated sheets which exploited the German procedural error of repeating the encipherment of the message-setting. Rózycki devised the clock method which was sometimes able to determine which of the Enigma's rotors was in the fast position. The Poles invented two rotary electro-mechanical machines, the cyclometer and the bomba, to assist in their work.*

Prior of the invasion of Poland by the Nazis, the Poles handed to the British cryptoanalysts the gear they developed for cracking the Enigma messages, and during the autumn of 1939, the scientists and mathematicians at Bletchley learned the intricacies of the Enigma cipher and rapidly mastered the Polish techniques. The Bletchley group was a bizarre combination of mathematicians, scientists, linguists, classicists, chess grandmasters and crossword addicts. A figure, who deserves to be singled out of this group was ***Alan Turing****, who identified Enigma's greatest weakness and ruthlessly exploited it*[993].

993 Turing focused on what would happen if the German military changed their system of exchanging message keys. Bletchley's early successes relied on Rejewski's work, which exploited the fact that Enigma operators encrypted each message key twice (for example, if the message key was YGB, the operator would encipher YGBYGB). This repetition was supposed to ensure that the receiver did

Yet, Bletchley still failed to crack the Naval Enigma and the Allies had no idea of the location of the German U-boats. Between June 1940 and June 1941 the Allies lost an average of 50 ships each month, and there was also a terrible human cost. (50,000 Allied seamen died during the War.) Britain was in danger of losing the Battle of the Atlantic, which would have meant losing the war.

Thus, an alternative strategy for cracking the Naval Enigma depended on stealing keys. Combined with the Rejewski-Turing techniques, the German Naval Code was finally breached.

The most valuable intelligence produced by the British intelligence services during WWII was derived from the interception and decryption of enciphered enemy signals.

By December 1940, with the help of recently acquired IBM card-sorting machines, Friedman's group at the SIS broke the Japanese diplomatic traffic enciphered on the "Purple machine." This machine was designed to achieve the same effect as Enigma. It was less mechanical, having no rotors, but instead a set of telephonic switches, connected to two typewriters. The first was used to input the text, the second to print out the encipherment for transmission. In between, the switches moved the incoming electrical current to achieve alphabetic substitutions.

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not make a mistake, but it created a chink in the security of Enigma. British cryptanalysts guessed it would not be long before the Germans noticed that the repeated key was compromising the Enigma cipher, at which point the Enigma operators would be told to abandon the repetition, thus confounding Bletchley's current codebreaking techniques. It was Turing's job to find an alternative way to attack Enigma, one that did not rely on a repeated message key.

As the weeks passed, Turing realized that Bletchley was accumulating a vast library of decrypted messages, and he noticed that many of them conformed to a rigid structure. By studying old decrypted messages, he believed he could sometimes predict part of the contents of an undeciphered message, based on when it was sent and its source. For example, experience showed that the Germans sent a regular enciphered weather report shortly after 6 a.m. each day. So, an encrypted message intercepted at 6:05 a.m. would be almost certain to contain **wetter**, the German word for "weather." The rigorous protocol used by any military organization meant that such messages were highly regimented in style, so Turing could even be confident about the location of **wetter** within the encrypted message. For example, experience might tell him that the first six letters of a particular ciphertext corresponded to the plaintext letters **wetter**. When a piece of plaintext can be associated with a piece of ciphertext, this combination is known as a *crib*.

E. The Manhattan project *(1942–1945)*

The magnitude of the potential technical revolution introduced by the discovery of nuclear fission, (especially its potentialities as a weapon in the face of the imminence of WWII), drove ***Eugene Paul Wigner*** *(1902–1995) and* ***Leo Szilard*** *(1898–1964, U.S.A.) to seek the help of* ***Albert Einstein****, then at the Princeton Institute for Advanced Study.*

The events that followed heralded the nuclear age: Einstein agreed to write a letter to U.S. President Franklin D. Roosevelt (1882–1945), in which he suggested that the fission of uranium could be used to produce an atomic bomb (Aug. 2, 1939).

The result of this letter was the establishment of the Manhattan project by the U.S. Government for the production of an atomic bomb. The work started in 1942*. It was the largest single enterprise in the history of science and technology, and also the costliest single weapons project.*

It comprised 37 *installations in* 19 *states and Canada, and employed* 43,000 *people with an overall budget of* 2.2 *billion dollars. A large number of physicists, among them famous scientists who had fled totalitarian Europe, such as* ***Niels Bohr****,* ***Enrico Fermi*** *and* ***Leo Szilard****, joined the project. New cities sprouted in the wake of the project: Oak Ridge, TN, for the gaseous diffusion plant for the separation of* U^{235} *from* U^{238} *(p. 79,000), and Hanford, WA, for the nuclear reactors that transformed* U^{238} *to* Pu^{239} *(p. 60,000). The fissionable material was shipped to Los Alamos, NM where* ***Robert J. Oppenheimer*** *directed the design and assembly of the bomb itself.*

At 3 : 20 *pm on Dec.* 2, 1942*,* ***Fermi*** *and his co-workers at the University of Chicago succeeded in producing the first man-made chain reaction, in an uranium and graphite pile set on the floor of a squash court. As soon as this happened,* ***Arthur Compton*** *sent the cryptic message, "The Italian navigator has landed in the new world", to other American scientists working on nuclear research.*

The bomb was finally tested on July 16, 1945*, at a secret site near Alamogordo, NM. A later War Department news release described the resulting phenomenon in those words: … "the whole country was lighted by a searing light with an intensity that of the midday sun. It was golden, purple, violet, grey, and blue. It lighted every peak, crevasse, and ridge of the nearby mountain range with a clarity and beauty that cannot be described … The explosion was followed by the strong, sustained, awesome roar which warned* *of doomsday, and made us feel that we, puny things, were blasphemous to dare tamper with the forces reserved for the Almighty"*[994].

The Germans underrated the value of science in general during World War II, and drafted many of their young scientists into the armed forces.

They did not appreciate the value of 'operational research', the use of scientific methods for ascertaining the most effective way of deploying limited military resources, a development which brought about a great saving of men and materials on the allied side.

No one in Germany (apart from a few individuals who were ignored) appears to have thought of an atomic bomb composed of an element heavier than uranium, until very near the end of the war. The possibility of developing a bomb composed of the light uranium isotope of mass 235 *was suggested, but the idea was dropped, as the German scientists thought that the separation of the uranium isotopes was impossible. However, the separation was carried out in America, and so too was the preparation of elements heavier than uranium, by means of the uranium pile. The German scientists conceived only of an uranium pile for use as a source of energy. By mid-*1945 *they had not yet constructed such a pile, that is, they had not reached the stage attained in America by the end of* 1942.

In addition, the Nazi nuclear research programme was dissipated between too many research organizations. There was no von Braun, no Peenemmünde and never enough money. The world, nevertheless, had a very narrow escape.

It is remarkable that the atomic bomb was mainly created (both theoretically and experimentally) by Jewish scientists from Europe and the USA, who were motivated by their desire to win the nuclear race with Nazi Germany. Many of them were deeply disappointed when the allies decided to drop it instead on Japan and later use it as a threat in the Cold War against the Soviets.

In the field of radar, the German did not go beyond the generation of radio waves of the order of a meter[995]*. In Britain, the development of the magnetron rendered waves of few centimeters or so in wavelength, affording*

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[994] Incidentally, **Robert Andrews Millikan** (1868–1953) along with the scientific advisers of Harry S Truman and Winston S. Churchill did not believe that the bomb would explode. Even **Albert Einstein** had his doubts; to a question of a reporter of the Pittsburgh Gazette (Dec 29, 1934) whether the huge amounts of energy corresponding to his formula $(E = mc^2)$ might be released by bombarding an atom, he replied: "It is as unpromising as firing at birds in the dark, in a neighborhood that has few birds." **Leo Szilard** knew better.

[995] The Germans had set up their own radar stations, but employed them in anti-shipping operations. German radar operated on a much shorter wavelength

much greater precession in the location of objects and less interference from extraneous sources.

F. Aftermath

WWII boosted the development of American applied science much more than WWI. Out of WWII came not only atomic energy, computers and radar, but many other advances:

WWII brought an end to the development of the airplane as had been conceived by Orville and Wilbur Wright. War-stimulated propulsion advances in the form of turbojet and rocket engines, brought manned flight *beyond the speed of sound within man's reach. Continued development of rocket-powered airplanes for aerodynamic research led the technology of manned flight to the threshold of space, at altitudes above atmospheric densities which allow sufficient lift to airfoils.*

Because of wartime shortages, American chemists worked to produce various substitute materials, such as plastics and synthetic rubber[996]*. Many of*

($1\frac{1}{2}$ meters to 50 cm) than the British (an unbelievable 10 meters!) and was transmitted from bowl-shaped parabolic antennas. To determine if the British had radar, the *Graf Zeppelin* (a large dirigible) made several flights up the coastline, listening for signals. The British were tracking the largest blip they had ever seen on their radar scopes. The Germans found nothing and concluded there was no British radar, which would cost them dearly once they started their attacks. They had been searching for the wrong wavelength! They had observed British coastal installations sporting some rather old-looking antennas, none of which were bowl-shaped, and decided they couldn't possibly be connected with radar. Besides, they were difficult to attack. This was to be their downfall in the Battle of Britain.

996 The importance of rubber in warfare had been demonstrated by the Germans in World War I. The Germans had been cut off from their foreign rubber supply by the British blockade. Without rubber their trucks ran out of tires while the troops did not have enough boots. In an effort to salvage the situation Germany began experimenting with synthetic rubber. However they could never find a formulation that worked well enough or could be produced in large enough quantities.

In World War II, Japan rapidly captured rubber producing lands in the far east, depriving America of 90% of its natural rubber sources. Suddenly America found itself in the same undesirable position that had confronted the Germans a generation before.

these proved superior to the material they replaced. In metallurgy, when imports of tungsten from China had been cut off during the war, molybdenum was successfully substituted for hardening steel. Because of its enormous contributions, American science seemed finally to have overcome the Government's established reluctance to support basic research. In 1950 *Congress established the National Science Foundation (NSF), the first U.S. Government-supported body created to sponsor basic research. It was small compared to such giants as the Atomic Energy Commission (AEC) and the Defense Development, which together accounted for* 3/4 *of the Research and Devel-*

However, due to their new educational emphasis, American chemical engineers were in a position to make great contributions to the synthetic rubber effort.

The unit operations concept, combined with mass and energy balances and thermodynamics (which had been stressed in the 30's), allowed the rapid design, construction, and operation of synthetic rubber plants. Chemical engineers now had the training to build industries from the ground up. With funds from the government, the chemical industry was able to increase synthetic rubber production over a hundred fold. This synthetic rubber found uses in tires, gaskets, hoses, and boots, all of which contributed to the war effort.

As German tanks and bombers swept across Europe using *Blitzkrieg* tactics, it became evident that World War II would be a highly mechanized conflict. The Allies needed tanks, fighters, and bombers, all supplied with *large quantities of high quality gasoline*. In supplying this fuel the American petroleum industry was stretched to its limit.

However, the development of Catalytic Reforming in 1940 by the Standard Oil Company had given the Allies an advantage. The reforming process produced high octane fuel from lower grades of petroleum (and it also made *Toluene* for TNT). Because of the performance edge given by better fuel, Allied planes could compete with better designed German fighters.

During World War II, American chemical engineers where called on to build and operate many new facilities, some never before conceived. After the war, Germany's massive chemical industry lay in ruins while the Americans were still operating at full production. Nevertheless, the United States Government still feared the German chemical complex. They therefore dismantled Hitler's enormous *I.G.Farben* and out of it, three new companies where created: BASF, Bayer, and Hoechst.

With America firmly leading the world in chemical technology, chemical engineering education began to change. Suddenly, the best way to discover the latest events in chemical technology was not to pick up a German technical journal, but instead to make those discoveries for yourself. Chemical engineering was becoming more focused on science than on engineering tradition.

opment (R&D) budget at that time. The R&D budget covered such elements as weapons developments by the AEC and the DOD.

WWII terminated the dominance of European science, and the center of science moved west to the United States.

In February 1950, Washington learned that for seven crucial years (1942–1949) ***Klaus Fuchs****, a Soviet nuclear spy, passed along secret information to the Soviet Union, on the basis of which they could explode their own fission bomb on Sept 22, 1949.*

Military research after 1930 led to inventions which were incorporated into decoding machines, analog computers and eventually stored-programme digital computers. British work took place at Manchester, Cambridge and the Natural Physical Laboratory. German work was done in Berlin, while in the USA work centered at Harvard and Princeton.

1941–1975 CE **Charles Ehresmann** (1905–1979, France). Mathematician. One of the creators of *differential topology*. Participated in the creation and development of current view of *fiber spaces*, *manifolds*, *foliations* and *jets*. After 1957 he became a leader in *category theory* and structures defined by *atlases*, and *germs of categories*.

Ehresmann was a native of Alsace[997]. He studied at the Ecole Normale Supérere (1924–1927), and the universities of Göttingen (1930–1931), Princeton (1932–1934) and Paris where he was awarded his Ph.D. (1934). From there on, his career was associated with the University of Strasbourg (1939–1955) and the University of Paris, where he held the chair of topology (1955–1975).

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997 *Alsace*, which was originally French, had come under German rule in 1871 but by 1902 had effective self-government. After 1911 it had its own constitution and progress was made toward Germanization in the region. After 1919 it was returned to France. The Germans invaded Alsace in 1940 and occupied it until 1945, whereupon it returned to the French again.

Science Progress Report No. 19

From the Cross to the Swastika

(1543–1943)

* * *

"First, their synagogues should be set on fire[998]*, and this ought to be done for the honor of God and of Christianity... Secondly, their homes should likewise be broken down and destroyed... we ought to drive the rascally bones out of our system: drive them out of the country for all time... so that you may be free of this insufferable devilish burden – the Jews."*

(Martin Luther, 1543)

* * *

"If the international Jewish financiers in and outside Europe should succeed in plunging the nations once more into a world war, then the result will be the annihilation of the Jewish race in Europe!"

Adolf Hitler, Reichstag speech (Jan. 30, 1939)

* * *

"How evil are these Jews? They are responsible not just for the blood of Jesus and the blood of all his messengers but also for the blood of all the righteous men who were ever murdered...

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[998] On Nov 10, 1938, on Luther's birthday, Jewish synagogues burned throughout Germany ("**Kristallnacht**"). On that very day, from his Church's pulpit, Bishop Martin Sasse of Thuringia, a leading Protestant Churchman, said:
"*The German people ought to heed these words of the greatest antisemite of his time, the warner of his people against the Jews.*"
In the light of the eliminationist antisemitism that pervaded the Protestant churches, it is no great surprise that even many prominent Church leaders threw their moral weight behind anti-Jewish measures that were still more radical than those of *Kristallnacht*.

These people produce idea after idea for the benefit of the World, but whatever it takes up becomes poisoned, and all that it ever reaps is contempt and hatred because ever and anon the world notices the deception and revenges itself in its own way."

(Martin Niemöller[999], "Here Stand I," 1937)

* * *

"The German State would be justified in taking steps to ward off the calamitous influence of one race on the national community. Wherever they were, Jews caused trouble for the host nation. The Jews had harmed Germany and a solution is necessary that would prevent future harm to Germans. . . . In the future perhaps it would be possible to allow Jews back into Germany because the number of Jews surviving and returning to Germany will not be so large that they could still be regarded as a danger to the German Nation."

(Dietrich Bonhoeffer[1000], "Proposals for a Solution to the Jewish Problem in Germany," 1943)

* * *

[999] **Martin Niemöller** (1892–1984) was a Protestant minister and an opponent of Nazism. As did many who were opponents of Nazism, despite his hatred of the regime, he concurred with the Nazi view of the world in one foundational respect: the Jews were eternally evil. However, by the time of his imprisonment in the Sachsenhausen concentration camp in 1939, he had overcome his antisemitism.

[1000] **Dietrich Bonhoeffer** (1906–1945) was a Protestant theologician who joined the unsuccessful wartime conspiracy to assassinate Hitler (hanged on 9 April 1945).

German 'Faustian' Ideology since Luther

There is a continuity in German ideology over a period of four centuries[1001]. ***Luther****, who had sown the seed of hatred that reached its horrible climax in the Third Reich, gave legitimation to the absolute evil. In 1517 he exempted the 'good Christian' from the need to make good deeds which, a-la-Luther, do not lead to redemption. Man's soul, he preached, is not affected by his deeds – it remains pure whatever he does. (No wonder that the Nazi leaders at the Nuremberg trials insisted that their conscious was clear!).*

Goethe *(1808) adopted a character, created in Luther's time, and rendered full legitimation to its crimes – Faust is never punished; on the contrary, it is received into Heaven!*

Faust, the archetype "everyman" figure, was accepted by all; we all identify with him and his unattainable goal. In that, Goethe gave legitimation to absolute evil; the Devil is the legitimate ruler of the real world. While all other European cultures set limitations to absolute freedom by means of laws, Germans since Luther were taught to create their own destiny, irrespective of an external binding law, bowing only to authority.

Kant *(1781) spoke of absolute freedom; man alone determines the laws and his own destiny. There is no external measure for morality. Consequently, the Germans held any external pragmatic morality in total contempt. Kant also introduced the concept of "People's Will", thus granting a philosophical foundation to the will of the people*[1002].

This tradition continued to flow through the teachings of ***Fichte*** *(1762–1814),* ***Hegel*** *(1770–1831),* **Bauer**[1003] and **Daumer**[1004], *in each of whom the anti-Jewish elements became more pronounced and more virulent, until reaching its climax in Nazi Germany.*

[1001] **Rivka Shechter**: "*Cosmic Enemy*", Jerusalem, 1979.
Hyam Maccoby: "*A Pariah People*", Constable: London, 1966, 236 pp.

[1002] In Nazi Germany, the will of Hitler and his cohorts was the will of the German people. The German War Criminals justify claimed that they carried out orders according to the general will.

[1003] **Bruno Bauer** (1809–1882). Leader of the *Hegelian Left*. In his '*Die Juden Frage*' (1843) radically criticized Judaism in the spirit of Voltaire.

[1004] **Georg Friedrich Daumer** (1800–1875). Vitriolic in his most-influential attacks on Judaism and Jews.

Gloom and Doom – Annihilation of the Jews in Western and Central Europe (1939–1945)

Toward the end of the 19^th century we come face to face for the first time with a unique phenomenon which, more than any other single factor, has influenced the course of European history since 1850. This is the phenomenon of anti-Semitism *which together with nationalism and racism created the barbarism of our age: some 9 million people died in Nazi-controlled concentration camps during 1934–1945; six million of them were Jews. To the earnest Nazi, the Jew was a non-man.*

Before it was institutionalized by the Nazis, antisemitism usually figured not as an active treat but as a dull, throbbing annoyance, casual and anonymous – an ugly possibility, rearing its head when least expected.

Already in 1922 Hitler declared that if he gained power "the annihilation of the Jews will be my first and foremost task". In Mein Kampf *(1925–1926) Hitler openly announced his political program: "might is right", racial supremacy of the* Aryans *and specifically the German people as a 'master race' (Herrenfolk) with a natural right to living space (Lebensraum) at the expense of inferior races as the Slavs, and the total elimination of the Jews.*

From the start the racial policies of the Third Reich were applied with equal ferocity to all Jews: orthodox and the assimilated, bankers and beggars, Nobel laureates, department-store clerks and school children; to the president of the Academy and to the German women's fencing champion (who won two Olympic medals for Germany); to the 100,000 Jewish veterans, many of them wounded or crippled, who had fought for Germany during WWI and had earned their 31,500 iron crosses as bravely as the next man.

At the same time, the Nazis proceeded with the willful destruction of everything that had to do with the Weimar Renaissance. During the early years of the regime, the world was treated to the unusual spectacle of a whole nation deliberately committing cultural suicide. *In the prevailing mindless frenzy to follow – the Führer, the mere possession of intellect became grounds for suspicion, and "Aryans" who persisted in trying to exercise it were denounced in the Nazi press as* weisse Juden *(white Jews).*

Nazism from the first had been essentially a revolt of the Know-Nothings: as a cultural revolution *it aimed at nothing less than the annihilation of the German intelligentsia. It was as if the Nazis could hardly wait to get their hands on the machinery of state so that they could begin smashing works of art and burning books.*

Indeed, within months after taking over the central government of the Reich, they had succeeded in paralyzing the literary and scientific life of the *nation. More than 1100 "non-Aryan" faculty members of universities and technical institutes lost their jobs in the initial purge.*

A traveling exhibition that made the rounds of German schools at the same time displayed Einstein's picture in the form of a "wanted" poster that identified him as an exiled subversive who remained at large and "still unhanged". The refugee intellectuals[1005] *constituted the greatest intellectual migration in history.*

During the World-War that followed (1939–1945), Hitler ruthlessly realized his campaign to 'eliminate' the Jews from German life, which led to the deliberate premeditated and methodical extermination of more than six million Jewish people. The pseudobiological ideology behind this holocaust was not new. Hitler's originality lay in the literal mindedness with which he was prepared to put these ideas into effect.

Part of the Jewish mathematicians in Europe were fortunate to reach safe havens in England, the United States, or South America. Many, however, perished in the Holocaust. Notably, ***Alfred Tauber*** *(1866–1942);* ***Felix Hausdorff*** *(1868–1942);* ***Alexander Rajchman*** *(1890–1940);* ***Juliusz P. Schauder*** *(1899–1943);* ***Stanislaw Saks*** *(1897–1942)* and ***Ela Chaim Cunzer*** *(1914–1942).*

[1005] Among them the non-Jews **Heinrich** and **Thomas Mann**, **Robert Musil**, **Erich Maria Remarque**, **Paul Hindmith**, **Walter Gropius**, **Erwin Schrödinger**, **Paul Klee**, **Bela Bartok**, **Max Beckmann**, **Fritz** and **Adolf Busch** and many others.

Contribution of the Jews to Modern Science and Western Culture

During the 165 years that elapsed from their Emancipation (1778) up to the German Holocaust (1943), Jews played a major role in the service of world culture; They helped extend the frontiers of mathematics, logic, physics, chemistry, biology, medicine, economy, psychology, to name just the leading sciences.

They became viceroys, prime ministers, statesmen, generals, explorers, historians, chess-champions, composers, soloists, conductors, painters, sculptures, educators – an avant-garde intellectuals who helped shape the map of Europe and chart the course of world history. All this in spite of the fact that the European non-Jews had a head-start of more than 300 years, in spite of the fact that the Jews in Central and Western Europe constituted about one percent of the total population, and in spite of adverse conditions of growing anti-semitism.

*Indeed, in spite of all that, 22.5 percent of all Nobel prize winners in the Science in the 20th century are of Jewish origin! Among the ten greatest physicists born during 1879–1918 [**Einstein**, **Born**, **Bohr**, **Schrödinger**, **de-Broglie**, **Pauli**, **Fermi**, **Heisenberg**, **Dirac**, **Feynman**], five* are Jews. *Among the ten greatest mathematicians born during 1777–1887 [**Gauss**, **Abel**, **Jacobi**, **Hamilton**, **Galois**, **Riemann**, **Cantor**, **Hilbert**, **Hardy**, **Ramanujan**], two are Jews.*

However, the contribution of the Jews to modern mathematics was much more decisive: During the activity period 1850–1950, 57 percent of the world's leading mathematicians were Jews, while their minuscule percentage in the combined populations of Europe and North America was less than one percent. All this was taking place at a time when Jewish Scholars were severely discriminated in Western Europe, Russia and the United States.

Not less impressive is their contribution to the nascent of the modern life-sciences: Botanist **Ferdinand Julius Cohn** *(1828–1898) became the founder of bacteriology;* **Paul Ehrlich** *(1854–1915) produced the first practical form of chemotherapy;* **Julius Sachs** *(1832–1897) was the creator of modern plant physiology and experimental botany;* **Leopold Auerbach** *(1828–1897) was a pioneer in the domain of cellular biology and histology.* **Eduard Adolf Strassburger** *(1844–1912) was a pioneer of the emerging science of cell biology;* **Robert Remak** *(1815–1865) pioneered neurology, embryology and histology.* **Julius Cohnheim** *(1839–1884) opened the field of modern pathology.*

Apart from the above-mentioned luminaries, the Jews produced during the post-emancipation era, many distinguished contributions in all active domains *of Western culture. We list below the most prominent among them born between 1778 and 1955:*

1. Mathematics and Logic

James Joseph Sylvester	*1804–1851*
Siegfried Aronhold	*1819–1884*
Leopold Kronecker	*1823–1891*
Gothold Max Eisenstein	*1823–1852*
Antonio Cremona	*1830–1903*
Julius Richard Dedekind	*1831–1916*
Rudolph Lipschitz	*1832–1903*
Lazarus Immanuel Fuchs	*1833–1902*
Julius Weingarten	*1836–1910*
Paul A. Gordan	*1837–1912*
Giulio Ascoli	*1843–1896*
Moritz Pasch	*1843–1930*
Amandus K.H. Schwartz	*1843–1921*
Georges-Henri Halphen	*1844–1889*
Georg Cantor	*1845–1918*
Cesare Arzela	*1847–1912*
Alfred Pringsheim	*1850–1941*
Salvatore Pincherle	*1853–1936*
Arthur M. Schönflies	*1853–1928*
Adolf Hurwitz	*1859–1919*
Vito Volterra	*1860–1940*
Kurt Hensel	*1861–1941*
Corrado Segré	*1863–1924*
Hermann Minkowski	*1864–1909*
Guido Castelnuovo	*1865–1952*
Jacque Solomon Hadamard	*1865–1963*
Alfred Tauber [(H)]	*1866–1942*
Felix Hausdorff [(H)]	*1869–1942*
Emanuel Lasker	*1868–1941*
Gino Fano	*1871–1952*
Boris Galerkin	*1871–1945*
Ernst Steinitz	*1871–1928*
Tullio Levi-Civita	*1873–1941*
Issai Schur	*1875–1941*
Edmund Landau	*1877–1938*
Max Dehn	*1878–1952*

[(H)] H = perished in the Holocaust.

Guido Fubini	*1879–1943*	
Sergei Bernstein	*1880–1968*	
Frigyes Riesz	*1880–1956*	
Lipot Fejer	*1880–1959*	
Otto Toeplitz	*1881–1940*	
Emmy Noether	*1882–1935*	
Harry Bateman	*1882–1946*	
Ernst Hellinger	*1883–1950*	
Richard Von Mises	*1883–1953*	
Eduard Helly	*1884–1943*	
Solomon Lefschetz	*1884–1972*	
Alfred Haar	*1885–1933*	
Marcel Riesz	*1886–1969*	
Hugo Steinhaus	*1887–1972*	
George Polya	*1887–1985*	
Harald Bohr	*1887–1951*	
Louis Joel Mordell	*1888–1972*	
Richard Courant	*1888–1972*	
Hermann Kober	*1888–1973*	
Alexander Rajchman $^{(H)}$	*1890–1940*	
Abram Besicovitch	*1891–1970*	
Hans A. Rademacher	*1892–1969*	
Avraham Halevi Fraenkel	*1891–1965*	
Alexander Ostrowski	*1893–1986*	
Jerzy Neyman	*1894–1981*	
Heinz Hopf	*1894–1971*	
Alexander Khinchin	*1894–1959*	
Nobert Wiener	*1894–1964*	
Tibor Rado	*1895–1965*	
Gabor Szegö	*1895–1985*	
Jesse Douglas	*1897–1965*	*FM*
Emil L. Post	*1897–1954*	
Pavel Samuilovich Uryson	*1898–1924*	
Oscar Zariski	*1899–1986*	
Otto Neugebauer	*1899–1990*	
John von Neumann	*1903–1957*	
Alfred Tarski	*1902–1983*	
Stanislaw Saks $^{(H)}$	*1897–1942*	
Juliusz P. Schauder $^{(H)}$	*1899–1943*	
Avraham Plessner	*1900–1961*	
Antoni Zygmund	*1900–1992*	

FM = Field Medal.

Richard Brauer	*1901–1977*	
Nahum I. Akhiezer	*1901–1980*	
Reinhold Baer	*1902–1979*	
Abraham Wald	*1902–1950*	
Kurt Mahler	*1903–1988*	
Benjamino Segré	*1903–1977*	
Withold Hurewicz	*1904–1954*	
Olga Taussky-Todd	*1906–1995*	
Andŕe Weil	*1906–2000*	
A.O. Gelfond	*1906–1968*	
Max Zorn	*1906–1993*	
Mark Krein	*1907–1989*	
Arthur Erdelyi	*1908–1977*	
M.A. Naimark	*1909–1978*	
Stanislaw Ulam	*1909–1984*	
Fritz John	*1910–1994*	
Joseph L. Doob	*1910–*	
Manahem Max Schiffer	*1911–1997*	
Norman Levinson	*1912–1975*	
L.V. Kantorovich	*1912–1986*	
Paul Erdös	*1913–1996*	
Samuel Eilenberg	*1913–1998*	
I.M. Gelfand	*1913–*	
George B. Danzig	*1914–2005*	
Ela Chaim Cunzer $^{(H)}$	*1914–1943*	
Marc Kac	*1914–1984*	
Laurent Schwartz	*1915–2002*	*FM*
Avraham Robinson	*1918–1974*	
Vladimir Rokhlin	*1919–1984*	
Benoit Mandelbrot	*1924–*	
Klaus F. Roth	*1925–*	*FM*
Martin David Kruskal	*1925–2006*	
Jean-Pierre Serre	*1926–*	*FM*
Alexander Grothendieck	*1928–*	*FM*
Kenneth Appel	*1932–*	
Paul Cohen	*1934–*	*FM*
Grigorii Margulis	*1946–*	*FM*
Charles Fefferman	*1949–*	*FM*
Michael Freedman	*1951–*	*FM*
Edward Witten	*1951–*	*FM*
Vladimir Drinfeld	*1954–*	*FM*
Efim Zelmanov	*1955–*	*FM*

2. Physical Sciences (Physics, Chemistry, Engineering)

Heinrich Gustav Magnus	*1802–1870*	
Siegfried Marcus	*1831–1898*	
Jacob Philipp Reis	*1834–1874*	
Adolph von Baeyer	*1835–1917*	*NP*
David Schwarz	*1845–1897*	
Gabriel Jonas Lippmann	*1845–1921*	*NP*
Otto Wallach	*1847–1931*	*NP*
Victor Meyer	*1848–1897*	
Eugen Goldstein	*1850–1931*	
Emile Berliner	*1851–1926*	
Henry Moissan	*1852–1907*	*NP*
Abraham Albert Michelson	*1852–1933*	*NP*
Heinrich Hertz	*1857–1894*	
Ernst Pringsheim	*1859–1917*	
Charles Proteus Steinmetz	*1865–1923*	
Fritz Haber	*1868–1931*	*NP*
Max Bodenstein $^{(H)}$	*1871–1942*	
Richard Willstätter	*1872–1942*	*NP*
Karl Schwarzshild	*1873–1916*	
Leonor Michaelis	*1875–1949*	
Lise Meitner	*1878–1968*	
Albert Einstein	*1879–1955*	*NP*
Paul Ehrenfest	*1880–1933*	
Theodore von Karman	*1881–1963*	
James Frank	*1882–1964*	*NP*
Max Born	*1882–1970*	*NP*
Niels Bohr	*1885–1962*	*NP*
George de Hevesy	*1885–1966*	*NP*
Gustav Hertz	*1887–1975*	*NP*
Victor Moritz Goldschmidt	*1888–1947*	
Alexander Friedmann	*1888–1925*	
Otto Stern	*1888–1969*	*NP*
Mikhail Gurevich	*1889–1973*	
Leopold Infeld	*1893–1968*	
Mariette Blau	*1894–1970*	
Peter Kapitsa	*1894–1984*	*NP*
Igor Tamm	*1895–1971*	*NP*
Leo Szilard	*1898–1964*	
Isaac Isidor Rabi	*1898–1988*	*NP*

NP = Nobel Prize Winner.

Wolfgang Pauli	*1900–1958*	*NP*
Dennis Gabor	*1900–1079*	*NP*
Fritz London	*1900–1954*	
Hyman Rickover	*1900–1986*	
Eugene P. Wigner	*1902–1995*	*NP*
Emilio Segré	*1905–1989*	*NP*
Felix Bloch	*1905–1983*	*NP*
Hans A. Bethe	*1906–2005*	*NP*
Rudolf Peierls	*1907–1995*	
Lev Landau	*1908–1968*	*NP*
Ilya M. Frank	*1908–1990*	*NP*
Giulio Racah	*1909–1965*	
Melvin Calvin	*1911–1997*	*NP*
William H. Stein	*1911–1980*	*NP*
Herbert C. Brown	*1912–2004*	*NP*
Max F. Perutz	*1914–2002*	*NP*
Robert Hofstadter	*1915–1990*	*NP*
Vitali Lazarevich Ginzburg	*1916–*	*NP*
Cristian B. Anfinsen	*1916–1995*	*NP*
Ilya Prigogine	*1917–2003*	*NP*
Herbert A. Hauptman	*1917–*	*NP*
Richard Feynman	*1918–1988*	*NP*
Julian Schwinger	*1918–1994*	*NP*
Jerome Karle	*1918–*	*NP*
Frederick Reines	*1918–1998*	*NP*
Morton Kaplon	*1921–2002*	
Arthur Schawlow	*1921–1999*	*NP*
Jack Steinberger	*1921–*	*NP*
Leon M. Lederman	*1921–*	*NP*
Walter Kohn	*1923–*	*NP*
Rudolph A. Marcus	*1923–*	*NP*
Georges Charpak	*1924–*	*NP*
Ben Roy Mottelson	*1926–*	*NP*
Donald A. Glaser	*1926–*	*NP*
Paul Berg	*1926–*	*NP*
Aaron Klug	*1926–*	*NP*
Martin L. Perl	*1927–*	*NP*
George A. Olah	*1927–*	*NP*
Alexi Abrikosov	*1928–*	*NP*
Murray Gell-Mann	*1929–*	*NP*
John Polanyi	*1929–*	*NP*
Leon Cooper	*1930–*	*NP*
Jerome Friedman	*1930–*	*NP*

Zhores Alferov	*1930–*	*NP*
David Lee	*1931–*	*NP*
Burton Richter	*1931–*	*NP*
Sheldon Glashow	*1932–*	*NP*
Melvin Schwartz	*1932–*	*NP*
Walter Gilbert	*1932–*	*NP*
Claude-Cohen Tannoudji	*1933–*	*NP*
Steven Weinberg	*1933–*	*NP*
Arno Penzias	*1933–*	*NP*
Alan Heeger	*1936–*	*NP*
Roald Hoffmann	*1937–*	*NP*
Sidney Altman	*1939–*	*NP*
Harold Kroto	*1939–*	*NP*
Jean-Marie Lehn	*1939–*	*NP*
Brian Josephson	*1940–*	*NP*
Douglas Osheroff	*1945–*	*NP*

3. Biochemical Sciences (Physiology, Medicine, Biochemistry, Bacteriology, virology, Immunology)

Jacob Henle	*1809–1885*	
David Gruby	*1811–1898*	
Gottlieb Gluge	*1812–1898*	
Robert Remak	*1815–1865*	
Moritz Schiff	*1823–1896*	
Natanael Pringsheim	*1823–1894*	
Leopold Auerbach	*1828–1897*	
Ferdinand Julius Cohn	*1828–1898*	
Samuel von Basch	*1837–1919*	
Julius Chonheim	*1839–1884*	
Eduard Strassburger	*1844–1912*	
Elie Metchnikoff	*1845–1916*	*NP*
Paul Ehrlich	*1854–1915*	*NP*
Sigmund Freud	*1856–1939*	
Waldemar Haffkine	*1857–1930*	
Rudolf Schoenheimer	*1858–1941*	
Oscar Minkowski	*1858–1931*	
Georges Fernand Isidore Widal	*1862–1929*	
Wilhelm Weinberg	*1862–1937*	
August von Wassermann	*1866–1925*	*NP*
Karl Landsteiner	*1868–1943*	*NP*
Aaron Phoebus Levene	*1869–1940*	

Alfred Adler	*1870–1956*	
Alex Bersredka	*1870–1940*	
Otto Loewi	*1873–1961*	*NP*
Gustav Emden	*1874–1933*	
Joseph Erlanger	*1874–1965*	*NP*
Robert Bárány	*1876–1936*	*NP*
Bela Schick	*1877–1967*	
Carl Neuberg	*1877–1956*	
Max Wertheimer	*1880–1943*	
Otto Warburg	*1883–1970*	*NP*
Casimir Funk	*1884–1967*	
Otto Fritz Meyerhoff	*1884–1951*	*NP*
Herbert Spencer Gasser	*1888–1963*	*NP*
Abraham Selman Waksman	*1888–1973*	*NP*
Herman Joseph Muller	*1890–1967*	*NP*
Gerty T. Cori	*1896–1957*	*NP*
Tadeus Reichstein	*1897–1996*	*NP*
Rudolf Schoenheimer	*1898–1941*	
Fritz Albert Lipmann	*1899–1986*	*NP*
Georg von Bekesy	*1899–1972*	*NP*
Charlotte Auerbach	*1899–1994*	
Hans Krebs	*1900–1981*	*NP*
Andre Lwoff	*1902–1994*	*NP*
Gregory Godwin Pincus	*1903–1967*	
Ernst B. Chain	*1906–1979*	*NP*
George Wald	*1906–1997*	*NP*
Rita Levi-Montalcini	*1909–*	*NP*
Bernard Katz	*1911–2003*	*NP*
Konrad Bloch	*1912–2000*	*NP*
Salvador E. Luria	*1912–1991*	*NP*
Julius Axelrod	*1912–2004*	*NP*
Robert F. Furchgott	*1916–*	*NP*
Arthur Kornberg	*1918–2007*	*NP*
Gertrude B. Elion	*1918–1999*	*NP*
Francois Jacob	*1920–*	*NP*
Baruch Benacerraf	*1920–*	*NP*
Edmond Fischer	*1920–*	*NP*
Rosalyn Yalow	*1921–*	*NP*
Stanley Cohen	*1922–*	*NP*
Joshua Lederberg	*1925–*	*NP*
Paul Greengard	*1925–*	*NP*
Baruch Blumberg	*1925–*	*NP*
Martin Rodbell	*1925–1998*	*NP*

Andrew V. Schally	*1926–*	*NP*
Marshall W. Nirenberg	*1927–*	*NP*
Cesar Milstein	*1927–2002*	*NP*
Sydney Brenner	*1927–*	*NP*
John Vane	*1927–*	*NP*
Daniel Nathans	*1928–1999*	*NP*
Gerald M. Edelman	*1929–*	*NP*
Eric R. Kandel	*1929–*	*NP*
Howard Temin	*1934–1994*	*NP*
David Baltimore	*1938–*	*NP*
Harold Varmus	*1939–*	*NP*
Joseph Goldstein	*1940–*	*NP*
Alfred Gilman	*1941–*	*NP*
Michael Brown	*1941–*	*NP*
Stanley Prusiner	*1942–*	*NP*
Richard Axel	*1946–*	*NP*
Herbert Horvitz	*1947–*	*NP*

4. Economics, Finance, Law, Journalism, Government, Peace

David Ricardo	*1778–1823*	
Adolphe Isaac Cremieux	*1796–1880*	
Lionel Nathan Rothschild	*1808–1879*	
Paul Julius von Reuter	*1816–1899*	
Karl Marx	*1818–1883*	
Ferdinand Lassalle	*1825–1864*	
Tobias Michael Asser	*1838–1913*	*NP*
Louis Brandeis	*1856–1941*	
Theodor Herzl	*1860–1904*	
Daniel Isaacs (1^{st} Marquis of Reading)	*1860–1935*	
Maximilian Harden	*1861–1927*	
Alfred Fried	*1864–1921*	*NP*
Paul Hymans	*1865–1941*	
Walter Rathenau	*1867–1922*	
Benjamin Cardozo	*1870–1938*	
Maxim Litvinov	*1876–1951*	
Leon Trotzky	*1879–1940*	
Felix Frankfurter	*1882–1965*	
Otto Bauer	*1882–1938*	
Emmanuel Shinwell	*1884–1986*	
Ivan Maisky	*1884–1975*	
Rene Samuel Cassin	*1887–1976*	*NP*

Lazar Moiseyevich Kaganovich	*1894–1991*	
Leslie Hore-Belisha	*1896–1957*	
Simon Kuznets	*1901–1985*	*NP*
Wassily Leontief	*1906–1999*	*NP*
Milton Friedman	*1912–2006*	*NP*
Leonid Vitaliyevich Kantorovich	*1912–1986*	*NP*
Paul A. Samuelson	*1915–*	*NP*
Herbert A. Simon	*1916–2001*	*NP*
Franco Modigliani	*1918–2003*	*NP*
John C. Harsanyi	*1920–2000*	*NP*
Lawrence R. Klein	*1920–*	*NP*
Kenneth J. Arrow	*1921–*	*NP*
Merton H. Miller	*1923–2000*	*NP*
Robert M. Solow	*1924–*	*NP*
Robert W. Fogel	*1926–*	*NP*
Harry M. Markowitz	*1927–*	*NP*
Gary Becker	*1930–*	*NP*
Reinhard Selten	*1930–*	*NP*
Daniel Kahneman	*1934–*	*NP*
George Akerlof	*1940–*	*NP*
Myron Scholes	*1941–*	*NP*
Joseph Stiglitz	*1943–*	*NP*
Robert C. Merton	*1944–*	*NP*

5. Literature (Poets, Writers, Philosophers, Critics, Philologists)

Heinrich Heine	*1799–1856*	
Berthold Auerbach	*1812–1882*	
Paul von Heyse	*1830–1914*	*NP*
Georg Brandes	*1842–1927*	
Ludwig Lejzer Zamenhof	*1859–1917*	
Peter Altenberg	*1859–1919*	
Edmund Husserl	*1859–1938*	
Henri Bergson $^{(H)}$	*1859–1941*	*NP*
Italo Svevo	*1861–1928*	
Arthur Schnitzler	*1862–1931*	
Richard Baer-Hoffmann	*1866–1945*	
Felix Salten	*1869–1945*	
Else Lasker-Schüler	*1869–1945*	
Marcel Proust	*1871–1922*	
Jacob Wassermann	*1873–1934*	

Hugo von Hofmansthal	*1874–1928*	
Robert Walser	*1878–1956*	
Otto Weininger	*1880–1903*	
Emil Ludwig	*1881–1948*	
Stefan Zweig	*1881–1942*	
Franz Kafka	*1883–1924*	
Georges Duhamel	*1884–1966*	
Lion Feuchtwanger	*1884–1958*	
André Mauroit	*1885–1967*	
Franz Rosenzweig	*1886–1929*	
Fernando Pessoa	*1888–1935*	
Shmuel Yosef Agnon	*1888–1970*	*NP*
Ludwig Wittgenstein	*1889–1951*	
Franz Werfel	*1890–1941*	
Boris Pasternak	*1890–1960*	*NP*
Nelly Sachs	*1891–1970*	*NP*
Herbert Marcuse	*1898–1979*	
Karl R. Popper	*1902–1994*	
Eric Hoffer	*1902–1983*	
Isaac Bashevis Singer	*1904–1991*	*NP*
Ayn Rand (Alice Rosenbaum)	*1905–1982*	
Simone Weil	*1905–1943*	
Elias Canetti	*1905–1994*	*NP*
Samuel Beckett	*1906–1989*	*NP*
Eugene Ionesco	*1909–1994*	
Alfred Ayer	*1910–1989*	
Saul Bellow	*1915–2006*	*NP*
Nadine Gordimer	*1923–*	*NP*
Imre Kertész	*1929–*	*NP*
Jacques Derrida	*1930–*	*NP*
Joseph Brodsky	*1940–1996*	*NP*

6. Psychology, Sociology, Anthropology

Joseph Popper-Lynkeus	*1838–1921*
Sigmund Freud	*1856–1939*
Emile Durkheim	*1857–1917*
Georg Simmel	*1858–1918*
Franz Boas	*1858–1942*
Alfred Adler	*1870–1937*
Max Wertheimer	*1880–1943*
Otto Selz	*1881–1943*

Edward Sapir	*188?–1939*
Erich Fromm	*1900–1980*
Bruno Bettelheim	*1903–1969*
Theodor Adorno	*1903–1969*
Viktor Frankl	*1905–1997*
Claude Levi-Strauss	*1908–*
Robert King Merton	*1910–2003*

7. World Chess Champions

Wilhelm Steinitz	*1836–1900*	*(1886–1894)*
Emanuel Lasker	*1868–1941*	*(1894–1921)*
Mikhail Botvinnik	*1911–1995*	*(1948–1957; 1958–1960; 1961–1963)*
Vasily Smyslov	*1921–*	*(1957–1958)*
Mikhail Tal	*1936–1992*	*(1960–1961)*
Boris Spassky	*1937–*	*(1969–1972)*
Robert Fisher	*1943–*	*(1972–1975)*
Garry Kasparov	*1963–*	*(1985–2000)*
Vladimir Kramnik	*1975–*	*(2000–)*

8. Prime Ministers (1868–1991)

Benjamin Disraeli	*(1868–1880)*	*1804–1881*	*England*
Julius Vogel	*(1873–1876)*	*1835–1899*	*New Zealand*
Luigi Luzzatti	*(1910–1911)*	*1841–1927*	*Italy*
Sydney Sonnino	*(1906–1910)*	*1847–1922*	*Italy*
Kurt Eisner	*(1918–1919)*	*1867–1919*	*Bavaria*
Leon Blum	*(1936–1938; 1946–1947)*	*1872–1950*	*France*
Bela Kun	*(1919)*	*1886–1937*	*Hungary*
Matyas Rakosi	*(1952–1953)*	*1892–1971*	*Hungary*
René Mayer	*(1953)*	*1895–1972*	*France*
Pierre Mendes-France	*(1954–1955)*	*1907–1982*	*France*
Roy Welensky	*(1958–1963)*	*1907–1991*	*Rhodesia*
Bruno Kreisky	*(1970–1988)*	*1911–1990*	*Austria*
Michel Debre	*(1959–1962)*	*1912–1996*	*France*
Joshua Hassan	*(1964–1987)*	*1915–1997*	*Gibraltar*
Petre Roman	*(1889–1991)*	*1946–*	*Romania*
Laurent Fabius	*(1984–1986)*	*1946–*	*France*

9. Explorers

Herman Vambery	*1832–1913*
Emin Pasha	*1840–1892*
Marc Aurel Stein	*1862–1943*
Sven Hedin	*1865–1952*

10. Painters

Camille Pissarro	*1830–1903*
Lionel Feininger	*1871–1956*
Franz Marc	*1880–1916*
Amadeo Modigliani	*1884–1920*
Jules Pascin	*1885–1930*
Marc Chagall	*1887–1985*
Man Ray (Emmanuel Radnitzky)	*1890–1976*
Moise Kisling	*1891–1953*
Chaim Soutine $^{(H)}$	*1894–1943*
Ben Shan	*1898–1969*

11. Composers

Giacomo Jacob Meyerbeer	*1791–1864*
Jacques Eli Halevy	*1799–1862*
Johann Strauss Sr.	*1804–1829*
Felix Mendelssohn	*1809–1847*
Jacques Offenbach	*1819–1880*
Carl Goldmark	*1830–1915*
Henryk Wieniawski	*1835–1880*
Gustav Mahler	*1860–1911*
Paul Dukas	*1865–1935*
Oscar Strauss	*1870–1954*
Arnold Schönberg	*1874–1951*
Fritz Kreisler	*1875–1962*
Reinhold Glière	*1875–1956*
Ernst Bloch	*1880–1959*
Emmerich Kalman	*1882–1953*

Jerome Kern	*1885–1945*
Irving Berlin	*1888–1989*
Arthur Honegger	*1892–1955*
Oscar Hammerstein	*1892–1955*
Darius Milhaud	*1892–1968*
Wolfgang Korngold	*1897–1957*
George Gershwin	*1898–1937*
Kurt Weill	*1900–1950*
Aaron Copland	*1900–1990*
Richard Rodgers	*1902–1979*
Morton Gould	*1913–1996*

The Social and Political Context of Science (1925–1950) – ***or, the road from exceptional prominence to prominent exception***

The 1944 Nobel Prize for chemistry went to the German Otto Hahn for "his discoveries in atomic fission."

The 1946 Nobel Prize for physiology medicine went to the American Hermann Joseph Muller for "his discovering that X-rays can produce mutations."

The 1950 Nobel Prize for physics went to the Briton Cecil Frank Powell for "his photographic method of studying atomic nuclei."

Yet, ***Lise Meitner***[1006] *(1878–1968), mother of nuclear shell physics, played a crucial role in the experiments that led to the fission discovery in December 1938. But she had to escape secretly to Holland on 13 July 1938. She was not included as a coauthor in Hahn and Strassman's publication (politically that would be impossible in Nazi Germany) — and, as a result, her part in the discovery was not recognized.*

Her exclusion from the fission discovery itself damaged her reputation, casting doubt on the work she had done before. Adding to the damage, Hahn was afraid to admit to his ongoing collaboration with a "non-Aryan" in exile and soon began to claim that Meitner and physics had contributed nothing to the discovery. Those who did not understand the science or the political situation concluded that the chemists had discovered fission while the physicists had merely explained it, and in 1945 the Nobel Prize in chemistry for 1944 was awarded to Hahn alone. With that, Meitner largely lost her place in the history of science.

As president of the newly formed Max-Planck-Gesellschaft, Hahn was the spokesman for the postwar rehabilitation of German science. Himself a "pure" scientist, a Nobel laureate, and a non-Nazi, Hahn projected an image of science as inherently excellent and untouched by the Nazi regime. Hahn never set the record straight with respect to Meitner, and for decades a chorus of his associates and other scientists, none of them close to the discovery, echoed his contention that Meitner had done nothing for the fission discovery except, perhaps, to impede it. Their stridency suggests a political motivation. A fair examination of the circumstances of the discovery would have called attention to the racial prosecution, political oppression, and moral compromises that permeated the scientific establishment, including Hahn's own institute, and that was just what Hahn and much of his generation were trying to suppress and forget.

Mariette Blau *(1894–1970) did pioneering work in the photographic method of studying particle tracks. She created emulsions with characteristic and development conditions that allowed for observations and measurement of proton tracks. She was the first physicist to show that proton tracks could*

[1006] For further reading, see:

- Sime, Ruth Lewin, *Lise Meitner: A Life in Physics*, University of California Press.
- Galison, Peter L., *Marietta Blau: Between Nazis and Nuclei*, Physics Today, 5, 42, 1997.
- *Biographical Memoirs of the Fellows of the Royal Society*, London, 1995 (*Charlotte Auerbach*).

be separated from α-particle tracks in emulsions. She was first to use nuclear emulsions to detect neutrons by observing recoil protons. All this she did during 1925–1942, years ahead of Powell. She must have been deeply frustrated that he was awarded the Nobel Prize for a discovery using her method.

She was nominated several times for the Nobel Prize by Erwin Schrödinger, Born and Einstein, but to no avail.

She was a prominent scientist who had the misfortune to live in a hostile environment, a victim of a sick society.

***Charlotte Auerbach** (1899–1994). Founded the science of mutagenesis — the study of gene mutation by chemicals — ahead of Muller. But, the Nobel committee, blinded by political considerations and social discrimination, did not consider this Jewish refugee woman worthy of the prize she deserved.*

1941–1979 CE **Friedrich August von Hayek** (1899–1992, Austria, England). Economist and political philosopher. *A philosopher of freedom.* Made major contributions to scientific methodology, psychology and the history of ideas. Largely concerned with the problem of individual values in a world of increasing economic controls. Argued the case for an economic system based on free markets and a political system granting individual freedom within the law. The importance of prices in controlling the functioning of the economy has been a constant theme in his works.

Hayek was born in Vienna, where he was a civil servant and a teacher (1921–1931); Professor of economics at London University (1931–1950). Naturalized British citizen (1938); Professor of social and moral sciences at the University of Chicago (1950–1962) and Professor at the Universities of Freiburg (1962–1968) and Salzburg (1968–1977). Nobel Prize for economics (1974). His major works: *The Pure theory of Capital* (1941); *The Road to Serfdom* (1944); *Law, Legislation and Liberty* (3 vols, London 1973–1979).

Hayek's name is virtually synonymous with the cause of *libertarianism*, the modern successor to the political *liberalism* of the 19th century. Through analysis of the relationships between economic and social factors he sought an answer to the question: "Is social justice feasible within the framework of the capitalistic economy?"

1942 CE, Nov 25 A small article appeared on *page 10* of the *New York Times* reporting the first official news that up to then, two million Jews had been killed in Europe.

1942 CE The beginning of the modern development of *radio astronomy*. During WWII, over the period February 26–28, 1942, British radars were being jammed and it was discovered that storms of radio-emission from the sun were responsible. At the time it was considered a top secret project, but in 1946 it was reported by J.S. Hey in *Nature* (*156*, 47–48). **Bracewell** (1956) succeeded in mapping the regions of emitted microwave radiation from the sun's disc by a tomographic method.

1942 CE **Bengt Edlen** (1906–1993, Sweden). Astrophysicist. Resolved the identification of certain lines in spectra of the solar corona[1007] that had misled scientists for the previous 70 years.

During the eclipse of 1869, astronomers recorded unexpected spectral lines in the sun's corona that they ascribed to the presence of a new element which they called 'coronium'. Similar lines were later discovered to originate nearer the earth; these were attributed to 'geocoronium'.

In the early 1940s, Edlen showed that, if iron atoms are deprived of many of their electrons, they can produce spectral lines like those of 'coronium'. Similarly ionized atoms of nickel, calcium, and argon produced even more lines. It was determined that such high stages of ionization would require temperatures of about 1,000,000 °C and when, in the 1950s, it was verified that such high temperatures did indeed exist in the solar corona, it became accepted that 'coronium' did not exist.

The lines thought to be caused by 'geocoronium' were found to be produced by atomic nitrogen emitting radiation in the earth's upper atmosphere.

Edlen was born in Gusum in Ostergotland, south-eastern Sweden. He was educated at Uppsala University. In 1944 he became Professor of Physics at Lund University, a post which he held until 1973.

[1007] During a total eclipse of the sun, when for a few minutes the moon completely covers the sun's face, a glow appears around the darkened sun – the solar corona, the sun's outermost atmosphere.

Structures visible in the corona at such times suggest that they are shaped by magnetic fields, and therefore that the corona consists of plasma. For instance, short "plumes" rising from the polar regions of the sun look very much like field lines coming out of the end of a bar magnet, and they therefore suggest that the sun, in addition to the intense fields of sunspots, also has a global magnetic field like the earth's.

Structures observed in the corona above sunspots often have horseshoe-shaped outlines, again suggesting that they follow magnetic field lines. From the tops of such "arches" long streamers may extend, to distances of the sun's diameter or even more, looking like pulled taffy, as if some process was pulling material away from the tops of the arches into space.

The most remarkable aspect of the corona is its high temperature. Much of that is sunlight scattered by coronal dust, but some light is also produced by the corona itself, in narrowly defined colors ("spectral lines") characteristic of its emitting atoms. In the 19^{th} century, some of the spectral lines of sunlight did not match the lines of any substance on Earth, and it was proposed that they came from a new unknown chemical element, named *helium* (from the Greek *helios* = sun). Later, in 1895, **Norman Ramsey** actually discovered helium on earth.

The source of the corona's heat remains a puzzle. It is almost certain that its energy comes from the sun's internal furnace, which also supplies the rest of the sun's heat. However, as a rule, temperatures are expected to drop the further one gets from the furnace, whereas the million-degree corona lies *outside* the surface layer where sunlight originates, whose temperature is less that 6000 C.

The space station *Sky lab* (1973–1974) observed soft X-rays emitted by the corona. The corona in such pictures appears quite uneven. It is brightest near sunspots, whose arched field lines apparently hamper the outflow of solar wind which carries away energy and helps cool the corona. It is darker in "coronal holes" in between, where field lines apparently extend out to distant space, making it easier for the solar wind to escape.

1942 CE **William Edward Hanford** (1908–1996, USA) and **Donald Fletcher Holmes** (1910–1980, USA). Chemists. Invented a process for making and modifying *polymeric products*. This method is today the basis for manufacture of all *polyurethanes*.

Flexible polyurethane foam is used as an upholstery material, and the rigid foam is commonly used as heat-insulating material in homes and refrigerators. Polyurethane is also used in life-saving *artificial hearts*, as safety padding in modern automobiles, and in carpeting.

Hanford was born in Bristol, Pennsylvania and received his B.S. from the Philadelphia College of Pharmacy (1930). Holmes received his B.S. in Organic Chemistry from Amherst College (1931). Both received their Ph.D. degrees from the University of Illinois and teamed up at the Dupont company.

1942–1952 CE **Norman Earl Steenrod** (1910–1971, USA). Mathematician. One of the leading topologists of the 20th century. Codified and solidified the theories of *homology* and *cohomology* in the framework of algebraic topology and played a crucial role in the development of the theory of *fiber bundles*. Named after him are: '*Steenrod algebra*', '*Steenrod volumes*'.

Steenrod was born in Dayton, Ohio and was educated at the Universities of Michigan (1932), Harvard (1934) and Princeton (Ph.D., 1936). He held positions at the University of Chicago (1939–1942), Michigan (1942–1947) and Princeton (1947–1971). His books: "*Topology of Fiber Bundles*" (1951), "*Foundations of Algebraic Topology*" (1952, together with Samuel Eilenberg).

1942–1969 CE **Salvador Edward Luria** (1912–1991, Italy and USA). Biologist. Pioneer in molecular biology, especially the genetic structure of viruses[1008]. Awarded the Nobel Prize for Physiology or Medicine (jointly with Delbrück and Hershey, 1969) for his discoveries related to the role of DNA in bacterial viruses.

Luria was born in Turin, Italy to a Jewish family. He graduated in medicine at Turin University. He left Fascist Italy (1938), and went to the Radium Institute in Paris to study medical physics, radiation and techniques of working with *bacteriophage*, the bacterial virus. When Italy entered WWII, Luria emigrated to the USA (1940), where he taught at Indiana University. In 1959 he became professor at MIT.

Luria obtained the first good electron photographs of a bacteriophage (1942). He then showed (1945) that the same spontaneous *mutations* occur in bacteriophages and in the bacteria on which the phages prey, suggesting that the genetic material of the phage gets mixed into the genetic material of the bacteria.

1942–1976 CE **Charlotte Auerbach** (1899–1994, Scotland). Geneticist. Founded the science of *mutagenesis* — study of gene mutation by chemicals.

[1008] Viruses that attack bacteria are called *bacteriophages*. This word means: *bacteria eater*. Bacteria, like plants, have tough cell walls. To penetrate these walls, most bacteriophages have a structure that resembles a hypodermic needle and works in a similar manner. This structure consists of a sphere-shaped head that contains *nucleic acid*, and a hollow, rod-shaped tail made of *protein*. When a bacteriophage enters a bacterium, the tail first penetrates the cell wall. Then the nucleic acid in the head moves through the tail and into the cell.

Viruses are such simple organisms that scientists can easily study them to gain more knowledge about life itself. Thus, research on bacteriophages has helped biologists understand *genes*, DNA, and other basic cell structures. In general, studies of the biological properties of bacteriophages contributed greatly to our understanding of the chemical and biological interactions of viruses and living cells.

She was born in Berlin into a learned scientific and artistic Jewish family[1009]. During 1919–1924 she studied biology, chemistry and physics at the Universities of Berlin, Wurtzburg and Freiburg. She fled Nazi Germany (1933) and joined the Institute of Animal Genetics at Edinburgh as a Ph.D. student. In 1942 she discovered (with A.J. Clark and J.M. Robson) that mustard gas (a highly toxic substance that had been used in trench warfare), caused genetic mutation in fruit flies (Drosophila). She was awarded Ph.D. (1935) and D.Sc. (1947) by the University of Edinburgh. Elected FRS (1957) and received the Darwin Medal (1977). Published: *Mutation* (1962), *Mutation Research* (1976).

Her approach was biological rather than chemical in that, while she acknowledged that mutation took place in the chemistry of the gene, she adhered to the idea that it was the biological interaction that gave the process its complexity. She used Drosophila, and later, microorganisms such as Neurospora and yeasts to test the mutagenic properties of other agents (mustard gas had proved too dangerous). From this she pursued several lines of inquiry: the patterns of combinations called mosaics (mutant and non-mutant cells), that increased mutation occurred in delayed or stored genes affected by a mutagen, 'replicating instabilities', i.e., that mosaics produced more mosaics in later generations, and that in 'specificity' — parts of the gene were affected differently by mutagenesis.

In 1947, she published a book of fairy stories titled *Adventures with Rosalind* under the pen-name of Charlotte Austen.

1943 CE **Oswald Theodore Avery** (1877–1955, U.S.A.). Biochemist. Announced the chemical nature of the gene, showing that hereditary characteristic could be induced by pure DNA, without a protein involved. Working at the Rockefeller institute with his colleagues **Colin M. MacLeod** and **Maclyn McCarty**, they were able to show that the gene was nucleic acid — and only nucleic acid. Moreover, they were able to transform one strain of bacteria into another by using a solution of the nucleic acid without any protein at all. They thus proved that the DNA, a hitherto unexplained substance, in the nucleus of living cells, was the very material of the gene, i.e. — that DNA is the hereditary material that carries the genetic information for almost all living organisms.

1943 CE **Willem J. Kolff** (b. 1911, Netherlands and USA). Physician and inventor. Invented the kidney dialysis machine (1943). Headed a team which invented and tested an artificial soft shell mushroom shaped heart.

[1009] She was the grandchild of **Leopold Auerbach** (1828–1897), the neuroanatomist and discoverer of *Auerbach's plexus*, and the daughter of the chemist, **Friedrich Auerbach** (1870–1925).

Kolff was born in the Netherlands and received his M.D. in Leiden (1938) and a Ph.D. degree from the University of Groningen (1946). Since 1967 he has been professor of surgery and head of the Division of Artificial Organs at the School of Medicine of the University of Utah.

1943 CE **Shin-ichiro Tomonaga** (1906–1979, Japan). Physicist. Developed (parallel to **R.P. Feynman** and **J.S. Schwinger**) a renormalizable, covariant quantum electrodynamics (QED), for which he shared with the aforementioned the Nobel prize for physics in 1965. In establishing this theory he had resolved the inconsistencies of the old QED theory of Heinsenberg, Pauli and others, without making any drastic changes and in a manner fully consistent with the special theory of relativity.

Tomonaga was born in Kyoto, Japan. He became professor of physics at Tokyo in 1941. His work, completed in 1943, came to the attention of the West only in 1946, since he was isolated from Western scientists during WWII.

1943–1950 CE **Abraham Wald** (1902–1950, USA). Mathematician. Originated the powerful optimization method of *dynamic programming*[1010], to address an array of questions arising in optimal control theory, game theory, production and scheduling processes.

Developed a new statistical method of *quality control*[1011] known as *sequential analysis* (1947), in response to the demand for more efficient methods of

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[1010] The extension of *linear programming* to *nonlinear* optimization problems. One class of such problems concerns the passage of people or information rather than commodities – for example, when people move from one part of a network to another at minimal cost — either in time, money, energy, or some other resource (*routing problems* that are reflected in the way airline ticket prices are set nowadays). Optimal control processes, like getting a satellite into orbit with minimal energy, problems with random influences such as investments with risky payoffs, "minimax" methods of game theory — are all within the purview of the theory.

[1011] *Quality control* methods permit us to regulate product quality by testing; A lot of items is sampled according to a scheme guaranteed to reject a good lot with probability α ("*supplier's risk*") and to accept a defective lot with probability β ("*consumer's risk*").

A lot is considered *good* if the parameter that negatively characterizes its quality does not exceed a certain limiting value and *defective* if this parameter has value not smaller than another limiting value. There are different methods of control: *single sampling*, *double sampling* and *sequential analysis*. The sequential Wald analysis for a variable sample size n and a random value of the controlled parameter in the sample, the likelihood coefficient γ, is computed and the control lasts until γ leaves the limits of the interval (B, A) where industrial quality control during WWII. In this connection he also developed the topic of *decision functions*.

Wald was born to a Jewish family in Kolozsvár, Hungary (now Cluj, Romania) and studied at Vienna. After the Nazis occupied Austria (1938) he fled to the USA, the only survivor of his family, who perished in the gas chambers of Auschwitz. Wald and his wife were later both killed in a plane crash in India.

1943–1960 CE **Shiing-Shen Chern** (1911–2004, USA). Mathematician. Made significant contribution to *global differential geometry*.

The now named '*Chern characteristic classes in fiber spaces*' are important not only in pure mathematics but also in mathematical physics.

Introduced the key concepts of *Secondary Invariants*, *Fiber Bundles*, *Sheaves* and *Foliated Leaves*. Gave a new proof to the Gauss-Bonnet formula[1012]. A large share of the credit for transforming differential geometry into a major subject in mathematics belongs to him.

Chern was born in Jiaxing, Zhejiang province, China. Received his D.Sc. from Hamburg University (1936) and studied under **Cartan** in Paris (1936–1937). Worked under **Weyl**, **Veblen** and **Lefschetz** at Princeton, USA (1943–1945). Held the chair of geometry at the University of Chicago (1949–1960) and then went to the University of California, Berkeley (1960–1980).

$B = \frac{\beta}{1-\alpha}$; $A = \frac{1-\beta}{\alpha}$; if $\gamma \leq B$, then the lot is accepted, if $\gamma \geq A$, the lot is rejected, and for $B < \gamma < A$ the test continues.

1012 For a closed orientable surface S of genus g,

$$\text{Integral curvature } = \iint_S K\,dA = 4\pi(1-g),$$

where K is the Gaussian curvature. [E.g. $g = 0$ (sphere), $g = 1$ (torus), $g = 2$ (pretzel).]

The Mathematics of Europe and USA (1850–1950): Fusion of Geometry, Algebra nd Analysis

Moritz Cantor's history of mathematics, which terminates with the end of the eighteenth century, consists of four large volumes averaging almost a thousand pages each.

It has been conservatively estimated that if the history of the mathematics of the nineteenth century should be written with the same detail, it would require at least fourteen more such volumes!

No one has yet hazarded an estimate of the number of such volumes needed for a similar treatment of the history of the mathematics of the twentieth century, which is by far the most active era of all. Little of this additional material could be properly appreciated by the ordinary undergraduate; indeed, an understanding of much of the material would require the deep background of a mathematical expert.

The almost explosive growth of mathematical research in modern times is further illustrated by the fact that prior to 1700 there were only 17 periodicals containing mathematical articles. In the eighteenth century there were 210 such periodicals, in the nineteen century 950 of them, and the number has increased enormously during the first half of the twentieth century.

Furthermore, it was not until the nineteenth century that there appeared journals devoted either primarily or exclusively to mathematics. Very few of the present-day articles can be read by anyone but the specialist.

Table 5.19 lists the leading mathematicians born between 1842 and 1919, 158 in number. The biographies of most them are included in chapters 4 and 5 of our Encyclopedia. The table includes the major contributions, life-span and national affiliation of the individuals.

The ethno-national affiliation of each biography are part and parcel of the person's origin, milieu, background and the grand scheme of the historical evolution of mankind.

In this day and age, where the history of diversity of the human species and its impact on genetics *are the subject of intense research*[1013]*, it is of great interest to weave human history, culture, and language in one grand sweep, and discover the hidden connectivity of scientific creativity and the ethno-cultural background of the individual scientists.*

[1013] e.g. **L.C. Cavalli-Sforza** and **F. Cavalli-Sforza**: "*The Great Human Diasporas*", Addison Wesley, Reading, Mass. USA, 1995; 300 pp.

Table 5.19: LEADING MATHEMATICIANS BORN BETWEEN 1842–1919

NAME	NAT.*	LIFE-SPAN	MAJOR CONTRIBUTION
Gaston J. Darboux	*F*	*1842–1917*	*Differential geometry of curves and surfaces; Algebraic manifolds; 'repermobile'; Darboux Theorem.*
Moritz Pasch	*J*	*1843–1930*	*Axiomatic projective geometry.*
Amandus K.H. Schwartz	*J*	*1843–1921*	*Theory of functions; minimal surfaces; conformal mapping calculus of variations.*
Erhard Schmidt	*G*	*1845–1921*	*Functional analysis.*
Georg Cantor	*J*	*1845–1918*	*Set theory; transfinite numbers.*
Alfred Pringsheim	*J*	*1850–1941*	*Analysis.*
William Burnside	*E*	*1852–1927*	*Finite order group theory, Burnside Lemma (1897).*
A.M. Schönflies	*J*	*1853–1928*	*Crystallographic point-groups.*
Salvatore Pincherle	*J*	*1853–1936*	*Functional analysis; Abstract linear spaces.*
R.H. Mellin	*S*	*1854–1933*	*Mellin transform.*
A.A. Markov	*R*	*1856–1922*	*Theory of linked probability; Markov process and chain.*
Walther F.A. von Dyck	*G*	*1856–1934*	*Group theory (group representation); topology; potential theory.*

Table 5.19: (Cont.)

NAME	NAT.	LIFE-SPAN	MAJOR CONTRIBUTION
Luigi Bianchi	*I*	*1856–1928*	*Differential geometry and tensor analysis ('Bianchi Identity').*
Karl Pearson	*E*	*1857–1936*	*Modern statistics: Standard Deviation; chi-square.*
Adolf Hurwitz	*J*	*1859–1919*	*Special functions; Modular functions; ODE; number theory.*
Vito Volterra	*J*	*1860–1940*	*Theory of functionals; Integro-differential equations.*
Ivar O. Bendixon	*S*	*1861–1935*	*ODE near singularities: 'Poincare-Bendixon Theorem'.*
Kurt Hensel	*J*	*1861–1941*	*p-adic arithmetic (non-Archimedean mathematics)*
David Hilbert	*G*	*1862–1943*	*Algebraic number theory; Foundation of geometry; Calculus of variations; Integral equations.*
Axel Thue	*S*	*1863–1922*	*Diophantine equations (Thue Theorem).*
Stanislaw Zaremba	*P*	*1863–1942*	*PDE, Potential Theory.*
Abram G. Miller	*J*	*1863–1951*	*Combinatorics.*
Paul Painleve	*F*	*1863–1933*	*Nonlinear ODE in the complex plain.*

Table 5.19: (Cont.)

Name	Nat.	Life-span	Major Contribution
Hermann Minkowski	*J*	*1864–1909*	*Geometrical theory of numbers; 4D relativistic space-time.*
Guido Castelnuovo	*J*	*1865–1952*	*Algebraic geometry.*
Jacques S. Hadamard	*J*	*1865–1963*	*Prime Number Theorem; Functions of complex variable; Theory of determinants; Functional analysis; Integral equations; Theory of variations; Theory of matrices.*
Ivar E. Fredholm	*S*	*1866–1927*	*Modern Integral equations theory.*
Alfred Tauber	*J*	*1866–1942*	*Tauberian Theorems for the operational calculus.*
Felix Hausdorff	*J*	*1868–1942*	*Topological and metrical spaces (Hausdorff's dimension; Forerunner of concept of 'fractal dimension').*
Emanuel Lasker	*J*	*1868–1941*	*Algebraic number fields and Ideals.*
Eli J. Cartan	*F*	*1869–1951*	*Calculus of differential forms; Spinors; GTR with torsion; finite continuous groups; theory of subalgebras.*
Helge N.F. von Koch	*S*	*1870–1924*	*Koch 'snowflake' curve (1906).*
Frederigo Enriques	*J*	*1871–1946*	*Algebraic geometry.*

Table 5.19: (Cont.)

Name	Nat.	Life-span	Major Contribution
Ernst Steinitz	*J*	*1871–1929*	*Algebraic geometry; Theory of polyhedra; Theory of fields.*
Emil E.J. Borel	*F*	*1871–1956*	*Measure of set points; Summability; Functions of real variable.*
Ernst F.F. Zermelo	*G*	*1871–1953*	*Axiomatic set theory; Axiom of choice; well-orderness.*
Tullio Levi-Civita	*J*	*1873–1941*	*Absolute differential calculus: parallel transport, intrinsic derivative; Differential geometry of generalized spaces; n-body problem.*
Rene Louis Baire	*F*	*1874–1932*	*Real functionals; semicontinuity; Baire functions.*
Gerhard Hassenberg	*G*	*1874–1925*	*Tensor analysis; Foundation of geometry.*
Thomas J.I. Bromwich	*E*	*1875–1929*	*Operational calculus; Infinite series.*
Henry L. Lebesgue	*F*	*1875–1941*	*Measure theory; Generalization of Riemann Integral.*
Issai Schur	*J*	*1875–1941*	*Number theory; Compact groups; Matrices; Schur Lemma.*
William S. Gosset	*E*	*1876–1937*	*Small-sample statistics; Student t-distribution; t-ratio*

Table 5.19: (Cont.)

Name	Nat.	Life-span	Major Contribution
Luther P. Eisenhart	*US*	*1876–1965*	*Non-Riemannian geometry.*
Edmund Landau	*J*	*1877–1938*	*Analytic number theory; Theory of functions; Distribution of primes; Prime Ideals.*
G.H. Hardy	*E*	*1877–1947*	*Analytic number theory; Diophantine analysis; Divergent series; Inequalities; Distribution of primes; Riemann Zeta function.*
Maurice R. Frechét	*F*	*1878–1973*	*Geometry of abstract metric spaces; Functional calculus. Functional derivative.*
Max W. Dehn	*J*	*1878–1952*	*Foundations of geometry; Theory of groups; Topology.*
Jan Lukasiewicz	*P*	*1878–1952*	*Mathematical logic; 3-value propositional calculus.*
J.L. Fatou	*F*	*1878–1929*	*Fractal geometry (1917).*
Hans Hahn	*J*	*1879–1934*	*Pioneer in set theory and functional analysis (Hahn-Banach theorem, 1922).*
Leopold Fejer	*J*	*1880–1954*	*Fourier series at a discontinuity; Fejer theorem.*
Oscar Perron	*G*	*1880–1975*	*Continued fractions; Differential equations.*
Sergi N. Bernstein	*J*	*1880–1968*	*Probability; approximation of functions (B. Polynomial).*
Frigyes Riesz	*J*	*1880–1956*	*Functional analysis.*

Table 5.19: (Cont.)

NAME	NAT.	LIFE-SPAN	MAJOR CONTRIBUTION
Oswald Veblen	*US*	*1880–1960*	*Non-Riemannian geometry; Differential and projective geometry; Topology.*
L.E.J. Brouwer	*D*	*1881–1967*	*Topology; 'Fixed-point theorem' (B. Theorem).*
Otto Toeplitz	*J*	*1881–1940*	*Functions of infinitely many variables; quadratic forms; Integral equations; Theory of matrices.*
Lewis Fry Richardson	*E*	*1881–1953*	*Forerunner of fractal geometry; Integration of the Navier-Stokes equations (weather prediction).*
Emmy Noether	*J*	*1882–1935*	*Founder of modern algebra: greatest woman mathematician ever: non-commutative algebra, hyper-complex numbers and systems; general theory of Ideals.*
J.H.M. Wedderburn	*US*	*1882–1948*	*Division rings; semi-simple algebras; finite projective geometries; Matrix theory.*
Harry Bateman	*J*	*1882–1946*	*Special functions; Integral and partial differential equations.*
Waclaw Sierpinski	*P*	*1882–1969*	*Set theory; Number theory.*
Richard M. von Mises	*J*	*1883–1953*	*Probability, statistical functions; elasticity; aerodynamics.*

Table 5.19: (Cont.)

Name	Nat.	Life-span	Major Contribution
Ernst David Hellinger	*J*	*1883–1950*	*Integral equations.*
Eduard Helly	*J*	*1884–1943*	*Functional analysis.*
George David Birkhoff	*US*	*1884–1944*	*Ergodic Theorem.*
Solomon Lefschetz	*J*	*1884–1972*	*Algebraic topology; stability of non-linear control systems.*
Leonida Tonelli	*I*	*1885–1946*	*Functional analysis.*
Alfred Haar	*J*	*1885–1933*	*Topological groups; H. measure; H. wavelet basis.*
John E. Littlewood	*E*	*1885–1977*	*Theory of numbers; Theory of functions; Fourier series; Inequalities; Summability of series.*
Hermann Weyl	*US*	*1885–1955*	*Non-Riemannian geometry; Continuous groups.*
Paul P. Levy	*J*	*1886–1971*	*Functional analysis.*
George Polya	*J*	*1887–1985*	*Random walks; Combinatorics; Number theory; Probability theory.*
Johann Radon	*G*	*1887–1956*	*Measure Theory; The Radon Transform.*
Srinivasa Ramanujan	*H*	*1887–1920*	*Most original mathematician since Jacobi, Lagrange and Euler; Theory of numbers; Infinite sums, products and Integrals; Continued fractions; modular equations and elliptic functions.*

Table 5.19: (Cont.)

Name	Nat.	Life-span	Major Contribution
Albert T. Skolem	*S*	*1887–1963*	*Mathematical logic; axiomatic set theory; lattice and modal theories; Diophantine equations.*
Harald A. Bohr	*J*	*1887–1951*	*Almost periodic functions.*
Stefan Mazurkiewicz	*P*	*1888–1945*	*Point-set topology; locally connected spaces.*
Paul Bernays	*J*	*1888–1977*	*Foundation of Mathematics.*
Richard Courant	*J*	*1888–1972*	*Analysis and numerical analysis (finite element method).*
Herman Kober	*J*	*1888–1973*	*Functional analysis (K. Theorem); Approximation theory.*
Louis Joel Mordell	*J*	*1888–1977*	*Number theory (M. Conjecture).*
James W. Alexander	*US*	*1888–1971*	*Topology.*
H. Nyquist	*US*	*1889–1976*	*Information theory (N. Criterion).*
R.V.L. Hartley	*US*	*1890–1970*	*Information theory (capacity of message); ‘Hartley Transform”.*
Ronald A. Fisher	*E*	*1890–1962*	*Statistical inference; test of significance.*

Table 5.19: (Cont.)

Name	Nat.	Life-span	Major Contribution
Walter A. Shewhart	*US*	*1891–1967*	*Statistical methods for Quality Control.*
Avraham Halevi Fraenkel	*J*	*1891–1965*	*Set theory.*
Abram S. Besicovitch	*J*	*1891–1970*	*Theory of fractals ('Hausdorff - Besikovitch dimension').*
John R. Kline	*US*	*1891–1955*	*Foundations of geometry.*
L. Vietoris	*G*	*1891–2002*	*Topology; Fourier series.*
Stefan Banach	*P*	*1892–1945*	*Functional analysis; Topological vector spaces; Theory of measure and integration.*
Hans A. Rademacher	*J*	*1892–1969*	*Number theory (partition functions); R. functions.*
Harold M. Morse	*US*	*1892–1977*	*Functional topology; variational theory: ('Morse theory').*
Gaston M. Julia	*F*	*1893–1979*	*Fractal geometry.*
Eduard Cech	*G*	*1893–1960*	*Topology; Homology theory.*
Alexander M. Ostrowski	*J*	*1893–1986*	*p-adic numbers; Algebraic geometry; Quasi-analytic functions. (O. Theorem).*

Table 5.19: (Cont.)

NAME	NAT.	LIFE-SPAN	MAJOR CONTRIBUTION
Norbert Wiener	*J*	*1894–1964*	*Cybernetics (Control mechanisms and transformation of information); Set theory, group theory, probability; Mathematical logic; The Fourier integral link of statistics and analysis; Stochastic processes; Prediction of stationary time-series; Brownian motion. W. Integral.*
Jerzy Neyman	*J*	*1894–1981*	*Statistics; quality control; confidence intervals.*
Paul Finsler	*G*	*1894–1970*	*Generalized Riemannian geometry. (F. metric).*
A.Y. Khinchin	*J*	*1894–1959*	*Stationary random processes.*
Heinz Hopf	*J*	*1894–1971*	*Algebraic topology; Cohomology.*
Tibor Rado	*J*	*1895–1965*	*Algebraic topology; Integration theory; Calculus of variations.*
Gabor Szegö	*J*	*1895–1985*	*Orthogonal polynomials; Extremal problems (limit theorems).*
Joseph Leonard Walsh	*US*	*1895–1973*	*Binary orthogonal systems: W. functions; W. series, Walsh-Hadamard transform.*
R.H. Nevanlinna	*S*	*1895–1980*	*Harmonic measure: Theory of value distribution.*

Table 5.19: (Cont.)

Name	Nat.	Life-span	Major Contribution
Pavel S. Alexandrov	*R*	*1896–1982*	*Algebraic topology; Homological theory of dimensions.*
Carl L. Siegel	*US*	*1896–1981*	*Number Theory; functions of complex variable; geometry of numbers.*
Emil L. Post	*J*	*1897–1954*	*Automata theory (P. machine); Mathematical logic; Modern proof theory; Recursive functions.*
Jesse Douglas	*J*	*1897–1965*	*Minimal surfaces ('Plateau Problem').*
Francesco C. Tricomi	*I*	*1897–1978*	*Differential and integral equations; functional transforms; probability theory.*
Stanislaw Saks	*J*	*1897–1942*	*Theory of real functions.*
Pavel S. Uryson	*J*	*1898–1924*	*Topology; Normal spaces (metrization theorems); U. Lemma.*
Emil Artin	*US*	*1898–1962*	*Class field theory; non-commutative rings; Algebra of associative rings; braids; algebraic number theory.*
Oscar Zariski	*J*	*1899–1986*	*Algebraic geometry.*
Juliusz P. Schauder	*J*	*1899–1943*	*Topology; fixed-point theorem; semilinear and quasilinear elliptic PDE.*

Table 5.19: (Cont.)

Name	Nat.	Life-span	Major Contribution
Otto Neugebauer	*J*	*1899–1990*	*History of ancient mathematics and astronomy.*
M.A. Lavrentev	*R*	*1900–1980*	*Quasi-conformal mapping; Non-linear waves.*
Antoni Zygmund	*J*	*1900–1999*	*Harmonic analysis; Trigonometric series.*
Nahum I. Akhiezer	*J*	*1901–1980*	*Function theory; Approximation theory.*
Abraham Wald	*J*	*1902–1950*	*Modern statistics: sequential analysis, quality control and non-linear optimization.*
Alfred Tarski	*J*	*1902–1983*	*Mathematical logic; set theory; measure theory.*
Karl Menger	*J*	*1902–1985*	*Dimension theory.*
W.V.D. Hodge	*E*	*1903–1975*	*Algebraic geometry.*
Georges de Rham	*SW*	*1903–1990*	*General theory of manifolds (de Rham theorem).*
Alonso Church	*US*	*1903–1995*	*Mathematical logic; Theoretical computer science.*
Andrei N. Kolmogorov	*R*	*1903–1987*	*KAM Theory; Topology; probability; Information theory; functional analysis; random stationary processes.*
John von Neumann	*J*	*1903–1957*	*Game theory; Computer science; topology groups; logic; set theory; ergodic theory; operator theory; C and C^* algebras.*

Table 5.19: (Cont.)

Name	Nat.	Life-span	Major Contribution
Charles Ehresmann	*F*	*1905–1979*	*Fiber bundles; differential geometry of groups; topology.*
André Weil	*J*	*1906–1998*	*Abstract algebraic geometry; Algebraic topology; Abelian varieties.*
Jean Leray	*F*	*1906–1998*	*Algebraic topology and PDE; turbulence; functional analysis.*
Jean A.E. Dieudonné	*F*	*1906–1992*	*Topological vector spaces; Algebraic geometry; Abstract analysis; Invariant theory; group theory.*
A.O. Gelfond	*J*	*1906–1968*	*Transcendental numbers; Interpolation and approximation of functions of complex variables.*
A.N. Tikhonov	*R*	*1906–1993*	*Topology and functional analysis (embedding theorem); Infinite-dimensional spaces; Fixed-point theorem for continuous maps.*
Max Zorn	*J*	*1906–1993*	*Infinite set theory (Zorn's Lemma).*
Olga Taussky-Todd	*J*	*1906–1995*	*Matrix theory; Number Theory.*
Raymond Paley	*US*	*1907–1933*	*Fourier-series and integrals; quasi-analytic functions.*

Table 5.19: (Cont.)

Name	Nat.	Life-span	Major Contribution
Arthur Erdélyi	*J*	*1908–1977*	*Special functions; operational calculus; asymptotic expansions; dual integral equations.*
Stanislaw M. Ulam	*US*	*1909–1984*	*Monte-Carlo method (statistical sampling method).*
M.A. Naimark	*J*	*1909–1978*	*Theory of group representations; functional analysis.*
Claude Chevalley	*F*	*1909–1984*	*Class field theory; local rings; semi-simple algebraic groups, algebraic geometry.*
Joseph Leo Doob	*J*	*1910–*	*Stochastic processes.*
Norman E. Steenrod	*US*	*1910–1971*	*Algebraic topology; 'Steenrod Algebra'.*
Nathan Jacobson	*J*	*1910–1999*	*Division rings; Lie Algebras ('Jacobson Radical').*
S.C. Chern	*US*	*1911–*	*Global differential geometry: fiber bundles; sheaves.*
Alan M. Turing	*E*	*1912–1954*	*Modern automata theory; Computer logic; Artificial Intelligence ('Turing Machine'). Non-linear PDE: diffusion driven instability.*
Norman Levinson	*J*	*1912–1975*	*Linear and non-local differential equations. Inverse scattering. Analytic number theory.*

Table 5.19: (Cont.)

Name	Nat.	Life-span	Major Contribution
L.V. Kantorovich	*J*	*1912–1986*	*Mathematical economy (Nobel Prize, 1975).*
Paul Erdös	*J*	*1913–1996*	*Number theory; Graph theory; Combinatorics.*
Samuel Eilenberg	*J*	*1913–1998*	*Topology: homology and cohomology theory.*
Jan G. Mikusinski	*P*	*1913–*	*Modern operational calculus (algebraic approach).*
I.M. Gelfand	*J*	*1913–*	*Commutative normal rings; locally compact groups; Integral geometry; generalized functions.*
Georg B. Danzig	*J*	*1914–*	*Linear programming; simplex method; operational research.*
Ela Chaim Cunzer	*J*	*1914–1943*	*Subharmonic functions.*
Kunihiko Kodaira	*N*	*1915–1997*	*Algebraic geometry; sheaves.*
Laurent Schwartz	*J*	*1915–2002*	*Theory of distributions; Stochastic differential calculus.*
Kiyosi Ito	*N*	*1915–*	*Stochastic calculus.*
Richard W. Hamming	*US*	*1915–1998*	*Error-detecting and error-correcting codes for computer systems; digital communication and data storing.*

Table 5.19: (Cont.)

Name	Nat.	Life-span	Major Contribution
John W. Tukey	*US*	*1915–2000*	*Fast Fourier Transform; Mathematical statistics.*
Claude E. Shannon	*US*	*1916–2001*	*Father of modern digital signal–processing technology; Mathematical theory of communication. Symbolic analysis of switching circuits.*
Avraham Robinson	*J*	*1918–1974*	*Non-standard (non-Archimedean) analysis; Mathematical logic; aerodynamic wing theory.*
Julia B. Robinson	*US*	*1919–1985*	*Number theory (Hilbert's 10^{th} problem); recursive functions.*

* *Nat. (Nationality)*

D=Dutch	*I=Italian*	*R=Russian*
E=English	*J=Jewish*	*S=Scandinavian*
F=French	*N=Japanese*	*SW=Switzerland*
G=German	*P=Polish*	*US=American*
H=Hindu		

1943–1962 CE **Benjamin Levich** (1917–1987, Russia and USA). Physicist. Opened the new field of *physicochemical hydrodynamics* which refers to phenomena governed by the interaction of fluid mechanics, heat and mass transfer, and chemical reactions. In particular, he studied *electrochemical kinetics*. An equation describing the current at a *rotating disc electrode*[1014] is named after him.

[1014] During 1945–1960 Levich collaborated with the electrochemist **Alexander Naumovich Frumkin** (1895–1976, Russia). The latter studied the funda-

Levich was born in Kharkov, Russia. He became a pupil of Lev Landau (1937) and worked with him on interfacial phenomena. He emigrated (1979) to the USA, where he established the Institute of Applied Chemical Physics at the New York City College.

1943–1968 CE **Jean-Paul Sartre** (1905–1980, France). Philosopher and political leader. Absorbed and amalgamated the existentialist ideas of the German philosophers **Hegel**, **Marx**, **Nietzsche**, **Husserl**, **Heidegger** and **Jaspers** into a form that fitted the political millieau of post WW2 world. Then, spent much of his literary life attempting to reconcile these existentialist views[1015] about free will with communist principles.

Existentialism tends to focus on the question of human existence – the feeling that there is no purpose at the core of existence. Finding a way to counter this nothingness, by embracing existence, is the fundamental theme of existentialism, and the root of the philosophical name.

Through the wide dissemination of the postwar literary and philosophical output of Sartre and his associates – existentialism became identified with a cultural movement that flourished in Europe during 1945–1970. Among the major philosophers identified as existentialists were Karl Jaspers, Martin Heidegger, **Ortega y Gasset** and **Miguel de Unamuno**. the 19^{th} century philosophers **Soren Kierkegaard** and **Friedrich Nietzsche**, came to be seen as precursers to the movement.

Existentialism was as much a literary phenomenon, and a very diverse coterie of writers and artists linked under the term: **Dostoevsky**[1016], **Ibsen**, **Kafka**[1017], **André Gide**, **Andre Malraux**, **Samuel**, **Albert Camus**,

mental theory of electrode reactions, considering the influence of the structure of the electrode/solution interface on the rate of electron transfer.

1015

- Satre J.-P., *Being and Nothingness (L'Etre at al n'eant)*, 1948
- Satre J.-P., *Existentialist is a Humanism (L'Existentialisme est un Humanisme)*, 1946
- Satre J.-P., *Critique of Dialectical Reason (Critique de la raison dialectique)*, 1960

1016 Kafka created often surreal and alienated characters who struggle with hopelessness and absurdity.

1017 Many of Dostoevsky's novels covered issues pertinent to existential philosophy.

Samuel Beckett, **Knut Hamson**, **Eugene Ionesco**, **Alberto Giacomneti** and **Emil Cioran**.

Sartre himself failed to write a great novel, or remarkable play, but used his novels and plays to transmit his political and social messages.

According to the existential view neither scientific nor moral inquiry can fully capture what it is that makes me *myself*, my ownmost self. Thus, a further set of categories is needed to grasp human existence. These categories are characterized by flight from the iron cage of reason. Thus, existential philosophy cannot be practiced in the disinterested manner of an objective science. Indeed, all themes popularly associated with existentialism – dread, boredom, alienation, the absurd, freedom[1018], commitment, nothingness, and so on – find their philosophical significance in the context of the search for a new categorical framework.

As a culture movement, existentialism belongs to the past. As a philosophical inquiry it has continued to play an important role in contemporary thought[1019], often bringing it into confrontation with more recent movements such as structurism, deconstruction, hermeneutics and feminism.

Sartre was born in Paris. His mother, of Alsatian origin, was the cousin of **Albert Schweitzer**. He graduated from the Ecole Normal Superieur (1929) with a doctorate in philosophy, drafted into the French army (1935) and held by the German as a war prisoner in Nancy and Trier (1940). He was released and given a civilian status (1941) and then settled in Paris and found a position at the Lucée Condorcet, replacing a Jewish teacher who was forbidden to teach by Vichy law.

Sartre's lack of political commitment during the German occupation and his further struggles for liberty was considered by his critiques as an attempt to redeem himself. According to **Camus** Sartre was "a writer who resisted, not a resistor who wrote".

1018 Existentialism generally postulates the absence of a transcendental force (such a God). It means that the individual is entirely free, and, therefore, ultimately responsible. It is up to humans to create an ethos of personal responsibility outside any belief system. Personal articulation of being is the only way to rise above humanity's absurd condition of much suffering and inevitable death. Existentialism is a reaction against traditional philosophies, such as rationalism and empiricism.

1019 To dig deeper, see

- Kaufmann, W., *Existentialism from Dostoevsky to Sartre*, Cleveland Meridian Books, 1968.
- Hayman, R., *Sartre: a life*, Simon and Schuster: New York, 1987.

Until 1944, when Sartre was almost forty, he had to earn his living as a schoolteacher. When the war ended he established a monthly literary and political review, and started writing full-time as well as continuing his political activism.

At that time France was a political quagmire; it was torn, radicalized by war and occupation; the movement of colonial liberation was spreading and the world was soon to split by the Cold War. The time was ripe for a socio–political upheaval in France and history put Sartre in the right place at the right time. He soon found himself famous and existentialism was *the* philosophy to study.

Curiously, as existentialism grew in popularity – to a point of becoming a pop–culture term – Sartre slowly left the philosophy that had brought him to fame and claimed a conversion to Marxism (1953).

As the Cold War developed, Sartre adhered to the Stalinist French Communist Party and became an orthodox Stalinist, ending up as a servant of one of the most oppressive regimes of all time. He accepted, at face value, the claims of the Soviet Union to be a peace–loving, democratic and socially just society. – a philosopher in service of totalitarism.

Jean-Paul Sartre also supported Mao, Castro and Che Guevara. He took prominent role in the struggle against French rule in Algeria and became an ardent supporter of the FLN in the Algerian War.[1020] By the 1970's, Sartre was reduced to being an apologist for tyranny and terror[1021].

The events in Cambodia in the 1970's, in which between $\frac{1}{5}$ and $\frac{1}{3}$ of the nation was starved to death or murdered, were entirely the work of a group of intellectuals who were for the most part pupils and admirers of J–P Sartre.

Paris university students rebelled in 1968, calling for various reforms. Sartre's support of the students caused him problems with both the left and the right in France. The Beat Generation owes a great deal to him.

Sartre political views and activities present, at best, the philosopher's lack of consistency: after making his philosophical debut as an impassioned advocate of individual freedom, denouncing Marxism as deterministic and Communism as undemocratic, he aligned himself with Marxism and relegated Existentialism as being a mere ideology. Marxism, he declared, was the only valid philosophy for our time. But in the seventies he announced that he was no longer a Marxist. Indeed, his Marxism had really shrunk back within the

[1020] He had an Algerian mistress (Arlette Elkayam) who became his adopted daughter (1965).

[1021] He justified the massacre of the Israeli Olympic team by the PLO in Munich (1972) as well as killing of European civilians by FLN in Algeria.

confines of the traditional hatred of the affluent capitalist West. The working–class Marxists still thought about surplus value but French intellectuals were obsessed by culture and found themselves without a proletariat.

1943–1976 CE **Mark Aronovich Naimark** (1909–1978, Russia). Mathematician. Contributed to functional analysis and the theory of group representations. Proved (1943) the Gelfand-Naimark theorem on self-adjoint algebras of operators in Hilbert space and generalized von Neumann's spectral theorem to locally compact Abelian groups. Made a detailed analysis of the infinite-dimensional representation of the *semi-simple Lie groups* and wrote with Gelfand a treatise on irreducible representations of the classical matrix groups (1950). This work formed the basis for later work on representations of semi-simple Lie Groups.

Naimark also contributed to the theory of Banach spaces. He wrote books entitled *Normed Rings* (1956) and *Theory of group representations* (1976).

Naimark was born in Odessa to Jewish parents. He was educated at the Odessa State University (1933) and received his doctorate from the Steklov Mathematical Institute of the USSR Academy of Sciences, Moscow (1941). Appointed professor at the Moscow Physical-Technical Institute (1954).

Life and the Laws of Physics[1022] ***— or, why are we here now? (The Anthropic Principle)***

"Life feeds on negative entropy"[1023].

Erwin Schrödinger, 1943 (1887–1961)

[1022] For further reading, see:

- Barrow, J.D. and F.J. Tipler, *The Anthropic Cosmological Principle*, Oxford University Press, 1990, 706 pp.
- Goldsmith, D. and T. Owen, *The Search for Life in the Universe*, Addison-Wesley, 1992, 530 pp.
- Bartusiak, M. (ed.), *Archives of the Universe*, Vintage Books, 2004, 695 pp.
- Resenberger, B., *Life Itself*, Oxford University Press, 1996, 290 pp.
- Barrow, J.D., *The Universe that Discovered Itself*, Oxford University Press, 2000, 448 pp.
- Hoyle, Fred and C. Wickramasinghe, *Evolution from Space*, A Touchstone Book, 1981, 176 pp.
- Davies, Paul, *The* 5^{th} *Miracle*, Simon and Schuster, 1999, 304 pp.
- Gribbin, J. and Martin Rees, *Cosmic Coincidences*, Bantam Books, 1989, 302 pp.
- Cohen-Tannoudji, G., *Universal Constants in Physics*, McGraw-Hill, 1993, 116 pp.
- Rees, M., *Just Six Numbers*, Basic Books, 2000, 195 pp.
- Davies, Paul, *The Accidental Universe*, Cambridge University Press, 1993, 139 pp.

[1023] This statement has a very profound meaning: contrary to popular belief, the essential purpose of eating, drinking and breathing is not merely to provide energy for vital body functions, but also to rid the system of the entropy it cannot avoid producing while being alive.

Since negative entropy may be considered as a measure of order, it is legitimate to say that an organism maintain a steady state by continually extracting order from its surroundings.

* * *

"We exist only in portions of the universe where the energy levels in carbon and oxygen nuclei happen to be correctly placed."

Fred Hoyle, 1965 (1915–2001)

Although the nature of living things has concerned natural philosophers since antiquity, 'life' as a general concept emerged in the early 1800's. Its emergence coincided with the introduction of biology as a new scientific field and with the growing conviction that the essential nature of animals and plants was the same, lying in their organization *rather than in their visible structures.*

18th and 19th century investigators attempted to identify common properties of living beings that distinguished them from inanimate objects.

Scientists divided into two groups: Vitalists *insisted that the phenomena of life cannot be explained adequately without ascribing to living organisms properties neither physical nor chemical. Among them were* ***Francois-Xaviar Bichat*** *(1771–1802),* ***Georges Cuvier*** *(1769–1832) and* ***Justus von Liebig*** *(1803–1873).*

In contrast, mechanists *and* reductionists *believed that the phenomena of life can be potentially explained by ordinary physical and chemical laws and that only physicochemical forces were at work in living organisms.*

Such were e.g. ***Helmholtz*** *(1813–1894) and* ***Karl Ludwig*** *(1816–1895). Others, such as* ***Claude Bernand*** *(1813–1878) and* ***Rudolf Virchow*** *(1821–1902) held intermediate views. Thus Bernard denied that living bodies were distinguished from non-living bodies by their physicochemical properties, however complex these might be. Rather, he held that the distinguishing feature of the living organism was the 'definite idea' directing its development.*

Virchow insisted that the cell was the ultimate locus of life and also disease. Others soon suggested that life might be a property of something less than

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In the case of human beings and other higher animals it is clear how this process is realized. Food stuffs consisting of highly organized (entropy-poor) organic molecules are taken in by the body, their energy partly utilized, and finally returned to the environment in a highly disorganized, or entropy rich form.

the cell, namely protoplasm [e.g. **T.H. Huxley** *(1868)]. The first physicists to expound influential views of life were* **Niels Bohr** *(1932),* **Max Delbrück** *(1943)* and **Erwin Schrödinger** *(1943).*

In February 1943, *at a bleak moment in the history of mankind, the physicist Schrödinger (then exiled from the Nazi Third Reich) gave a course of lectures at Trinity College, Dublin, Ireland. The lectures were published in a little book with the title What is Life? This book addressed the question: "How can the events in space and time which take place within the spatial boundary of a living organism be accounted for by physics and chemistry"? In short: is life based on the laws of physics? Schrödinger's own conclusion was that from all we know about the structure of living matter, it functions in a manner that cannot be reduced to the ordinary laws of physics. This meant that the laws already discovered in the analysis of matter were not enough in themselves and that new laws had to be found. Schrödinger's own conviction was that living organisms do involve other laws of physics, hitherto unknown. He described heredity in terms of molecular structure, inter-atomic bonds and thermodynamic stability.*

His message was clear — to continue investigation on the structure and function of living matter, biology must change its historical course and cooperate closely with physics and chemistry. Thus, his vision and prophecy opened the floodgates of molecular biology: the quest for these 'new laws' fired the enthusiasm of young physicists and later led (1962) to the breaking of the genetic code by **J.D. Watson**, **F.H. Crick** *and* **M.H.F. Wilkins**.

Consequently, it is now possible to understand something of the mechanism of evolutionary change at the molecular level.

In Darwin's theory, evolution is driven by random mutation and natural selection; mutations occur when genes, which are groups of molecules that can be studied directly, become randomly rearranged within an organism's DNA. Natural selection is the process whereby, in the continual struggle for resources, badly adapted individuals or groups (mutants or not) compete poorly and tend to die out. Thus organisms which are better suited to their environment are more likely to survive and reproduce than their less well-adapted competitors.

Although scientists do not doubt the fact of evolution — the adequacy of the Darwinian mechanism (i.e., random mutation and natural selection) has been questioned.

First, the principle of natural selection is essentially tautological ('Those organisms better suited to survive will survive better'). Also problematic is *the claim that evolutionary change is driven by* random fluctuations[1024]*. Some of the objections raised are:*

- *How can an incredibly complex organism, so harmoniously organized into an integrated functioning unit, perhaps endowed with exceedingly intricate and efficient organs such as eyes and ears, be the product of a series of pure accidents?*
- *How could random events have successfully maintained biological adaptation over millions of years in the face of changing conditions?*
- *How can chance alone be responsible for the emergence of completely new and successful structures, such as nervous system, brain, eye, etc. in response to environmental challenge?*
- *How could life have been started on earth by a series of random chemical reactions when simple minded probabilistic estimates show that* there was not enough time *for random reactions to get life going as fast as the fossil record shows that it did*[1025]*?*

[1024] In spite of the complexity of individual proteins, often containing specific combinations of as many as 300 individual amino acids, there are several proteins that appear in most forms of life in nearly-identical forms. This equivalence is strong evidence for a single source of all life forms.

It is implausible that this similarity arose by chance: There are 20 different types of amino acids used in forming proteins. The probability of *random* duplicating of two identical protein chains, each with 100 amino acids, is one in $20^{100} \sim 10^{130}$.

Since there are of order 10^{18} seconds in the (approximately) 14 billion years that elapsed since the Big Bang, we would need 10^{112} trials each second since the start of time just to reach a condition allowing a single protein to arise by chance with reasonable probability.

With these odds, it is impossible to explain the existence of similar proteins in bacteria and humans as due to mere chance.

[1025] *Metaphor*: Consider the likelihood that a troupe of monkeys, hammering away at typewriters, will eventually reproduce Shakespeare's 18^{th} sonnet, that ends with the lines:

> SO LONG AS MEN CAN BREATH OR EYES CAN SEE,
> SO LONG LIVES THIS, AND THIS GIVES LIFE TO THEE.

There are 488 letters in the sonnet. Neglecting the spacing between the words, the chance of randomly typing the 488 letters to reproduce this one sonnet is one in $26^{488} \sim 10^{690}$. Even with all the monkeys (plus every other animal) on earth typing away on typewriters over a period of time that exceeds that

And how about accidental copying errors during reproduction? The more intricately and delicately a complex system is arranged, the more vulnerable it is to degradation by random changes (even a tiny error in the blueprint of an aircraft or spacecraft might well lead to disaster!)

Thus, one would suppose that random mutations in biology would tend to degrade, rather than enhance, the complex and intricate adaptedness of organisms. Yet, it is still asserted that random 'gene shuffling' is responsible for the emergence of eyes, ears, brains, and all the other marvelous paraphernalia of living things. How can this be?

The above considerations suggest that biological evolution may require additional organizing principles *if the existence of the plethora of complex organisms on earth is to be satisfactorily explained.*

Schrödinger's ideas also impacted the ill-formulated Darwin's principle of evolution by natural selection. During the second half of the 20^{th} century many attempts have been made to prove *evolution by natural selection on the basis of mathematical models and physical laws. Although it has been accomplished in a few idealized cases, these share a common deficiency — scant physical input and lack of control over the range of values of many phenomenological parameters involved in their description.*

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elapsed since the Big Bang (10^{18} seconds) – still the probability of a sonnet appearing would be vanishingly small.

At one random try per second, with even a simple sentence having only 16 letters, it would take 2 million billion years to exhaust all possible combinations.

This calculation is based on the premise that the monkeys type independently of each other and that typewriters and monkey-groups do not evolve.
Hence the following caveats in the 'typing-monkeys' paradigm:

- Group of monkeys may evolve cooperative strategies.
- Typewriters may themselves evolve and become programmable (*Turing Machine*), thus enabling a monkey to reproduce a whole sensible word (e.g. 'Men') with a *single* key-stroke-like a key of the computer keyboard — having once programmed this key by accident.
- The environment could differentially reward literary outputs of varying quality ('selective pressure'), e.g. one could imagine an editor rewarding a monkey 'best-seller' by copious supplies of bananas or mates.
- Hierarchies could naturally evolve with some monkeys assuming managerial positions. In effect, such monkey-bosses are equivalent to higher order programmers.

Each of these points has its counterpart in molecular biology, where e.g. the sole task of some genes is to supervise complex patterns of activation and deactivation of lower-level genes (and similarly for neurons).

Another approach is based on the assumption that natural selection has led to the optimization of living beings according to various criteria of merit, which one tries to identify based on theoretical and experimental information. Several criteria of optimization have been proposed, such as maximal 'average fitness', maximal efficiency in resource utilization, minimum metabolized energy per unit biomass, etc. Of greater interest are the attempts to characterize fitness in thermodynamic terms. A fundamental tendency of non-equilibrium systems towards stationary states of maximal organization and minimal dissipation constitutes a potentially solid bridge between thermodynamics and Darwin's principle.

This has a direct bearing on the problem of the origin of life, namely: how does non-life beget life. The sort of conditions under which life is believed to have emerged were far from equilibrium and under these circumstances highly non-random behavior is expected. Quite generally, matter and energy in far-from-equilibrium open systems have a propensity to seek out higher and higher levels of organization and complexity. Thus, the primeval soup could have undergone successive leaps of non-random self-organization bifurcations along a very narrow pathway of chemical development. It could perhaps be that there are as yet unknown organizing principles operating in prebiotic chemistry that greatly enhance the formation of complex organic molecules relevant to life.

Is life a rare accident, an irrelevant fluke in a mindless and hostile cosmos?

Stellar lifetimes are a straightforward consequence of the physical laws and constants; biological evolution, on the other hand, is an immensely complex multistage process. There seems no conceivable reason why these times should be closely comparable. In typical cases, even if biological evolution got started on a planet, it might not have proceeded very far before that planet's star died.

Indeed, life as we know it is so special, so complexly organized and so fragile that it can flourish only within the narrowest finely-tuned environmental conditions; The continuation of life on earth and its successful development from the simple forms that we find in 3.3 *billion-year-old fossils to the complex organisms of today — reflect the earth's extraordinarily suitable conditions for life. It is as if the earth were especially made for life's eventual appearance and maintenance*[1026].

[1026] **James E. Lovelock** (1975) has introduced a concept known as '*Gaia* (the Greek earth goddess). Accordingly, planet earth is viewed as a holistic self-regulating system, in which the activities of the biosphere cannot be untangled from the chemical and physical processes that take place in the solid earth, its oceans and its atmosphere.

Carbon, oxygen and hydrogen are required in abundance to form the intricate and varied molecules of life. Yet neither carbon nor oxygen is abundant

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Moreover, the *Gaia* hypothesis suggests that life acted in such a way as to maintain the conditions needed for its own survival and progress. It provides an illustration of how a highly complex non-linear feedback system can display stable modes of activity in the presence of drastic external perturbation. It seems as if the earth "seeks" an optimal physical and chemical environment for life on this planet.

Example: Over the earth's history the internal structure of the sun changed due to its burn-up of hydrogen fuel. This, in turn, affected its luminosity, which increased by about 30 percent over the earth's history. In spite of this, the temperature of the earth's surface has remained remarkably constant over this time – since we know that the oceans have neither completely frozen, nor boiled [the very fact that life has survived over the greater part of the earth's history is itself testimony to the equability of conditions].

How has the earth's temperature been regulated? The primeval atmosphere contained large quantities of carbon dioxide, which acted as a blanket and kept the earth warm in the relatively weak sunlight of that era. With the appearance of life, however, the CO_2 in the atmosphere began to decline as the carbon was synthesized into living material.

In compensation, oxygen was released. As the sun grew hotter, so the CO_2 was gradually eaten away by life. On the other hand the oxygen produced an *ozone layer* in the upper atmosphere that blocked out the dangerous ultraviolet rays. With this ozone protection life was no longer restricted to the oceans, but could flourish in the exposed conditions on land.

The perturbations mentioned above do *not* necessarily include *man-made* alterations of some of the planet's major chemical cycles due to an ever increasing industrialization: we have increased the *carbon cycle* by 20 percent, the *nitrogen cycle* by 50 percent, and the *sulphur cycle* by 100 percent! We have increased the flow of toxins into air, water, and food chains. We have reduced the planet's green cover, while our factory outpourings reach the upper atmosphere and far into the oceans.

In the words of James Lovelock (1988): "*We shall have to tread warily to avoid the cybernetic disasters of a runaway positive feedback or of sustained oscillation between two or more undesirable states. We could wake one morning to find that we have landed ourselves with a lifelong task of planetary maintenance engineering. Then at last, we should be riding in that strange contraption,* Spaceship Earth.

People sometimes have the attitude that 'Gaia will look after us'. But that's wrong. If the concept means anything at all, Gaia will look after herself. *And the best way for her to do that might well be to get rid of us*".

in the universe, nor were they produced in the Big Bang. To nurture life, the Universe needed the nuclear alchemy that would change the primeval building blocks of hydrogen and helium into the heavier elements employed by life.

Following WWII, a group of physicists at the California Institute of Technology created a new field called Nuclear Astrophysics, which is the study of nuclear processes in stars. It seeks to learn how stars produce their energy, how the stars evolve, and how the stars produce the particular distribution of chemical elements throughout the universe.

*The specific nuclear reaction that is needed to make carbon is a rather improbable one: it requires 3 nuclei of helium to come together to fuse into a single nucleus of carbon. It was soon recognized (**Hoyle**, 1952) that making carbon in stars by this process was difficult: first, it were difficult to get three alpha-particles to meet, and then, even if this was accomplished, the fruits of their liaison might be short-lived, since all carbon could quickly get consumed by interacting with another alpha-particle to create oxygen.*

Hoyle realized (1954) that the only way to explain why there was a significant amount of carbon in the universe was to posit that the production of carbon went much faster and more efficiently than had been envisaged, so that the ensuing burning to oxygen did not have time to destroy it all.

There was only one way to achieve this carbon boost. A nuclear reaction may occasionally experience a special circumstances where its rate is dramatically increased. It is said to be 'resonant' if the sum of the energies of the incoming reacting nuclei is very close to a natural excited energy level of a new heavier nucleus. When this happens the nuclear reaction rate can be greatly enhanced, especially if the resonance is weakly damped (i.e. narrow[1027]).

Hoyle saw that the presence of a significant amount of carbon in the universe would be possible only if the carbon nucleus possessed a natural energy level at about 7.65 MeV above its ground state (i.e. 0.07% heavier). Only if that was the case could the cosmic carbon abundance be explained, Hoyle reasoned. Unfortunately no energy level was known in the carbon nucleus at the required place.

Hoyle decreed, in effect, "Since we exist, then carbon must have an energy level at 7.65 MeV!".

*In 1957, **William Fowler** led a team of nuclear physicists in search of the energy level that Hoyle was proposing, persuading himself that all the past*

[1027] Resonances in classical physics are tuned to particular *frequencies*. Due to Planck's relation $E = h\nu$, *quantum mechanical* resonances are tuned to *both frequency and energy*. And by dint of STR this implies a tuning of *mass* as well $(E = mc^2)$.

experiments could have missed the 7.65 MeV level. The result was dramatic: there was a new energy level in the carbon nucleus at 7.656 MeV, just where Hoyle had predicted it would be.

The whole sequence of events for the production of carbon by stars then looked delicately balanced *(fine tuned): Three helium nuclei (alpha-particles) have to interact at one place*

$$3\,{}^{4}_{2}He \rightarrow {}^{12}_{6}C + 7.656\ MeV. \tag{1}$$

This reaction proceeds in two steps: first, two α-particles fuse for a very short time ($\approx 10^{-16}$ sec) into an unstable beryllium ${}^{8}_{4}Be$ isotope nucleus

$$(99 \pm 6)keV + {}^{4}_{2}He + {}^{4}_{2}He \rightarrow {}^{8}_{4}Be. \tag{2}$$

Fortunately, beryllium has a peculiarly long lifetime, ten thousand times longer than the time required for two helium nuclei to interact; and so it stays around long enough to have a good chance of combining with another helium nucleus to produce a carbon nucleus and two gamma-ray photons:

$${}^{4}_{2}He + {}^{8}_{4}Be \rightarrow {}^{12}_{6}C + 2\gamma. \tag{3}$$

The 7.656 MeV energy level in the carbon nucleus lies just above the energies of the beryllium plus helium (which are equivalent to an excitation of 7.3667 MeV), so that when the thermal energy of the inside of the star is added, the nuclear reaction can become resonant and abundant carbon is produced (in some *stars and in* some *stages of their lives). But that is not the end of the story. The next reaction threatening to burn up all the carbon is*

$${}^{12}_{6}C + {}^{4}_{2}He \rightarrow {}^{16}_{8}O + \gamma. \tag{4}$$

Once reactions (1) and (4) have been completed the core of the star will be made up of carbon (${}^{12}_{6}C$ isotope) and oxygen (${}^{16}_{8}O$ isotope).

Clearly, if reaction (4) should turn out to be resonant as well, then all the rapidly produced carbon would disappear and the carbon resonance level would have been to no avail.

Remarkably, this last reaction just fails to be resonant. The oxygen nucleus has an excited energy level at 7.1187 MeV that lies just below *the total energy of carbon plus helium at 7.1616 MeV. So even when the extra thermal energy in the star is added, the oxygen-producing reaction can never be resonant and the carbon survives.*

Hoyle recognized that his finely balanced sequence of apparent coincidences was what made carbon-based life a possibility in the Universe.

The positioning of the nuclear energy levels in carbon and oxygen is the result of a very complicated interplay between nuclear and electromagnetic forces that could not be calculated easily when the discovery of the carbon resonance level was first made.

Today, it is possible to make very good estimates of the contributions of electromagnetic and nuclear forces to the levels concerned. One can see that their positions are a consequence of the fine structure constant *and strong* nuclear force (QCD) coupling constant *taking the values that they do, and that slight departure from these values would cause a "cosmic carbon bottleneck" and make carbon-based life impossible anywhere in the universe!*

If the fine structure constant, that governs the strength of electromagnetic forces, were changed by more than 4 per cent or the strong-interaction coupling constant by more than 0.4 of one per cent then the production of carbon or oxygen would be reduced by factors of between 30 and 1000.

Geoffrey *and* ***Margaret Burbidge***, ***Fred Hoyle*** *and* ***William Fowler*** *coauthored (1957) the paper: "*Synthesis of the Elements in the Stars*". In it they showed that* all *the elements from carbon to uranium could be produced by nuclear processes in stars, starting only with the hydrogen and helium produced by the Big Bang.*

Yet another fine tuning of the fine-structure and nuclear coupling constant involves the cosmological production of helium itself *during the first three minutes after the Big Bang. Were the strong nuclear force slightly weaker,* deuterium *nuclei would not exist — and the early-universe production of helium from hydrogen depends on deuterium as an intermediate state. And were the strong nuclear force strong enough for a* diproton *(a two-proton nucleus) to exist, again (almost) no deuterium would be produced, and very little helium — and thus no carbon.*

Water in its liquid state is needed as the medium within which the reactions of life are to occur (at least, this is so for terrestrial life). Yet liquid water exists only within a narrow range of temperatures and pressures. A long-term source of energy with constant output for billions of years is necessary to warm the water and to fuel the development of life from the simple to the complex. Such a long-term energy source can probably only originate with a star, but accompanying the warming stellar light is the devastating flux of ultraviolet and cosmic radiation. The potential abode of life required a window that allows light to enter but keeps out the ultraviolet radiation. This same home must have an umbrella that effectively deflects the continual shower of lethal cosmic radiation.

Earth's average distance from the sun is approximately 150 *million kilometers. Venus is only* 30 *percent closer than we are but the difference is crucial; typical surface temperatures on Venus are about* $500\,^{\circ}C$*. At this temperature, zinc and lead melt, wood burns spontaneously and glass is almost soft putty. There is not much chance for life under these conditions.*

The annual variation in distance from earth to the sun is only 4.5 *million kilometers, that is, only* 3 *percent of the total distance. (We are* 3 *percent closer in January.) This small annual variation means that the earth's orbit is almost circular. The orbit of Mars is quite elliptical, causing a variation in the distance to the sun of* 50 *million kilometers during the year. If the earth had such variance, our crust would deep-fry each January.*

In fact, if our distance from the sun were only 10 *million kilometers less (a change of less than* 7 *percent), the increased solar heat would prevent water vapor from condensing. There would be neither rain nor oceans. Ultraviolet radiation is able to penetrate only a few millimeters of water. This gave life a chance to develop within the oceans* prior *to the presence of oxygen (and thus UV-blocking* ozone*) in the atmosphere. Oxygen produced by aquatic algae formed the initial ozone screen. Life could then emerge from its protected berth in water and populate the land.*

The fine balance of the earth's composition is made manifest by another characteristic of the planet we inhabit: its radioactivity. The young earth contained enough radioactivity to have heated and melted it during its early development. Evidence of this internal heating is the increasing temperature experienced as we dig into the earth.

Because of the decrease in radioactivity over time, the earth now has a solid crust, but still a molten core. The motion of the molten iron mass within the earth's core produces the magnetic field with which we are familiar. The Lorentz forces due to this field divert much of the potentially lethal ionizing cosmic radiation that reaches the vicinity of the earth. We live under a literal magnetic umbrella. Were this cosmic radiation not deflected, it would bathe the surface of the earth with a continual shower of life-devastating ionization.

These constraints deal with the macroscopic characteristics of the universe required at life's abode. At the subatomic level, the demands of life are equally rigorous.

The forces that bind protons and neutrons into nuclei of atoms must be sufficiently strong to form the stable units we refer to as the elements, but weak enough to allow the spontaneous fission of some of the nuclei of the heavier among elements. The radioactivity caused by this fission supplied the heat that fueled the volcanoes that released trapped vapors and gases which formed the biosphere, the thin film of water and air in which all of life thrives.

Electromagnetism, which binds electrons to nuclei and repels them from each other and thus defines the properties of atoms — including all of chemistry, material science, optics and molecular biology — is also finely balanced. It must be weak enough to free electrons for occasional passage through and into neighboring atoms, yet strong enough to organize and join these adjacent atoms into ions and molecules, the basis for the solid and liquid structures of matter.

Gravity, the most enigmatic of the four fundamental forces of the universe, is the weakest of the four. Yet on large scales (planets, stars, galaxies and the universe as a whole) it is the most powerful force. As such it shapes the macrostructure of the universe and through gravitational instabilities and collapse, forms galaxies, stars and planets. If gravity were significantly more powerful, the life-times of the stars would be too short to allow life to flourish. Increasing or decreasing gravity, which binds planets, stars and galaxies in their flight through space, would result in unstable orbits, with planets spiraling toward, or away from their star; it also changes planets' sizes and chemical compositions.

To see this, we briefly summarize the, so-called coincidence of large numbers*, first exploited by* ***Arthur Eddington*** *(1921),* ***Paul Dirac*** *(1937) and* ***Robert Dicke*** *(1961).*

Eddington believed that he could create a theory that would weave together the macroscopic world of astronomy and cosmology with the subatomic world of protons and electrons. To this end he used the experimental values of his day to choose four dimensionless numbers:

- *The ratios of the masses of the proton and electron*

$$m_p/m_e \approx 1840. \tag{5}$$

- *The inverse of the* fine-structure constant

$$\frac{2\epsilon_0 hc}{e^2} \approx 137, \tag{6}$$

  (h = Planck's constant; c = velocity of light in vacuum, e = charge of the electron).

- *The ratio of the electromagnetic force to the gravitational force between an electron and a proton*

$$\frac{e^2}{4\pi\epsilon_0 G m_p m_e} \approx 2 \times 10^{39}. \tag{7}$$

- *The number of protons in the visible universe*

$$N_{Edd} \approx 10^{80}. \tag{8}$$

Note that since $e^2 \approx \frac{2\epsilon_0}{137}hc$, *we may use* (5) *to recast* (7) *in the form*

$$\frac{\hbar c}{Gm_p^2} \approx 1.5 \times 10^{38}; \qquad \hbar = \frac{h}{2\pi}. \tag{9}$$

Eddington then concocted a numerological derivation of the quadratic equation

$$10m^2 - 136m + 1 = 0, \tag{10}$$

which has two roots, the ratio of which is 1847.6. This equation thus ties up the first two of his constants (since m *is interpreted as* $\frac{m_p}{m_e}$, *and the coefficient* $136 = 137 - 1$ *is related to the fine-structure constant in his derivation). Then he claimed that* $\sqrt{N_{Edd}} \approx 10^{40} \approx \frac{e^2/4\pi\epsilon_0}{Gm_pm_e}$ *is justified on the premise that the ratio of the electric force to the gravitational force between two protons was a* statistical fluctuation *of a collection of* N *particles, given by the square root of* N.

Eddington's attempt to produce a unified explanation for the constants of nature attracted few adherents. The great physicists of his day, such as **Einstein**, **Dirac**, **Bohr** *and* **Born**, *found it useless and politely confessed that they could not understand it. Yet, his efforts drew the attention of physicists and created a new frontier to strive for.*

Unpersuaded by Eddington's numerological approach to dimensionless constants and the presence of large numbers amongst the constants of nature, **Dirac** *(1937) nonetheless argued that very large dimensionless numbers taking values like* 10^{40} *and* 10^{80} *are most unlikely to be* independent; *He then expounded the new conjecture: 'Any two very large dimensionless numbers occurring in nature are connected by a simple mathematical relation, in which the coefficients are of the order of unity'.*

The large numbers that Dirac marshalled to motivate this daring new hypothesis drew on Eddington's work and were three in number: (t = *present age of universe; modern, 1990's value used)*

$$\begin{aligned} N_1 &= \text{(size of the observable universe)/(classical electron radius)} \\ &= \frac{ct}{e^2/4\pi\epsilon_0 m_e c^2} \approx 5 \times 10^{40} \end{aligned} \tag{11}$$

$$\begin{aligned} N_2 &= \text{electromagnetic-to-gravitational force ratio between proton and electron} \\ &= \frac{e^2}{4\pi\epsilon_0 G m_e m_p} \approx 2 \times 10^{39} \end{aligned} \tag{12}$$

$$\begin{aligned} N &= \textit{number of protons in the observable universe}^{1028} \\ &= c^3 t/G m_p \approx 10^{80}. \end{aligned} \tag{13}$$

It then follows from (11)–(13) and Dirac's hypothesis that

$$N_1 \approx N_2 \approx \sqrt{N} \propto t \tag{14}$$

which implies

$$\frac{e^2}{4\pi\epsilon_0 G m_p^2} \propto t \tag{15}$$

at any *cosmological epoch. Dirac chose to accommodate (15) by abandoning the constancy of Newton's gravitation constant, G. He suggested that it was decreasing in direct proportion to the age of the Universe over cosmic time scales, as*

$$G \propto 1/t \tag{16}$$

Thus in the past G was bigger and in the future it will be smaller than it is measured to be today. One now sees that the huge magnitude of the three Large Numbers is a consequence of the great age of the universe: they all get larger as time wears on.

We define a nuclear time *t_N as the travel time of light across the* Compton wavelength *of the proton*

$$t_N = \frac{1}{c}\left(\frac{h}{m_p c}\right) = \frac{h}{m_p c^2}. \tag{17}$$

Combining (5), (11) and (17) we obtain for $t = t_{now}$

$$\frac{t_{now}}{t_N} \approx 10^{41}, \tag{18}$$

meaning that the age of the universe bears about the same numerical relation to t_N (up to a factor ≈ 50) as the electric force between a proton and electron in the hydrogen atom bears to their gravitational attraction.

One of the earliest specific demonstrations that biology can be used to explain the coincidence of large numbers *is due to* **Robert Henry Dicke** *(1916–1997, USA). In 1961 he declared that both Eddington and Dirac had been misguided in searching for new fundamental principles of physics to explain the apparent coincidences (14).*

Clearly, the present *age of the universe is defined by the recent nascence of the human technological society, occupying a minute fraction of its life-span. Dicke reasoned that t_{now} is not a randomly selected instant of time,*

[1028] Assuming a spatially closed and flat universe with no cosmological constant.

but intimately connected with the timescales of certain physical processes in the universe that are themselves prerequisites for the existence of intelligent life, and hence technology.

One could imagine a variety of such prerequisites, but the one chosen by Dicke concerns the existence of elements heavier than hydrogen. Life on earth is based on the element carbon, while nitrogen and oxygen are also vital. These elements did not exist in the primeval universe. Their presence in reasonable abundance is attributed to the nucleosynthesis which occurs inside stars.

Dicke presented these ideas (1961) in a more quantitative and cogent form specifically geared to explaining (in part) the large-number coincidences: The heavier elements are synthesized in the late stages of stellar evolution and are spread through the universe by supernovae explosions *which follow the* main-sequence *evolution of stars. Only universes of roughly the main-sequence stellar age could produce the heavy elements, like carbon, upon which life is based. Only those universes could evolve '*observers*'.*

Quantitatively, the argument shows that the main-sequence stellar lifetime *is roughly*

$$t_{ms} = \frac{\begin{pmatrix}\text{nuclear energy}\\ \text{available from}\\ \text{hydrogen fusion}\end{pmatrix}}{\begin{pmatrix}\text{nuclear-released energy}\\ \text{trapped within a star}\\ \text{at any given moment}\end{pmatrix}} \cdot \begin{pmatrix}\text{time for radiation}\\ \text{to diffuse out of}\\ \text{the star}\end{pmatrix}.$$

Now, the mean free path for photon diffusion is $\lambda \approx \frac{1}{\sigma n}$, *with* n *the hydrogen atom stellar density and* σ *the Thomson cross section* $\sim e^2/4\pi\epsilon_0 m_e c^2$; *the available fusion energy per hydrogen nucleus is of order of the nuclear self-Coulomb energy* $\sim e^2/4\pi\epsilon_0\lambda_c$, *with* $\lambda_c = h/m_p c$ *the proton Compton wavelength; while the thermal nuclear-derived energy inside the star at any moment is of order of its self-gravity potential energy. Thus*

$$t_{ms} \approx \left(\frac{hc}{Gm_e^2}\right)\left(\frac{h}{m_p c^2}\right)\alpha^3 \approx 4 \text{ billion years},$$

with $\alpha = e^2/4\pi\epsilon_0\hbar c$ *the fine structure constant.*

We could not expect to be observing the universe at times significantly in excess of t_{ms}, *since all stable stars would have expanded, cooled and died (i.e. all stars would be white dwarves, neutron stars and black holes). Nor would we be able to see the universe at times much less than* t_{ms} *because we could not exist! (i.e., no late-life stars, nor heavy elements like carbon).*

Living beings are therefore most likely to exist when the age of the universe is roughly equal to t_{ms}, *with* $t = 0$ *designating the Big Bang. Thus the value of Dirac's Large Number* $N(t)$ *is by no means random. It must have a value close to the value taken by* $N(t)$ *when* $t = t_{ms}$*. At that time we must inevitably observe the Dirac coincidence* $N_1 \approx N_2$ *to hold. It is a prerequisite for our existence, and no hypothesis of varying constants is necessary to explain it.*

Indeed, at time t_{ms} *after the beginning of the expansion of the universe it is inevitable that we observe* N_1 *to have the value*

$$N_1 \approx 4\pi\epsilon_0 \frac{m_e c^3}{e^2} t_{ms} \approx \left(\frac{hc}{Gm_e^2}\right)\left(\frac{m_e}{m_p}\right) 2\pi\alpha^2 \approx (2\pi)^2 \alpha N_2 \approx N_2. \qquad (20)$$

All that Dirac's coincidence is saying is that we live at a time in cosmic history after the stars have formed and before they die. This is not surprising. Dicke is telling us that we could not fail to observe Dirac's coincidence: it is a prerequisite for life of our sort to exist.

Note that since $N_1 N_2 = N$ *identically, Dicke's argument also explains the other Dirac coincidence,* $N_2 \approx \sqrt{N}$.

There is no need to give up Einstein's theory of gravitation by requiring G *to vary, as Dirac implicitly required, nor do we need to deduce some numerological connection between the strength of gravity and the number of particles in the universe as Eddington had thought. The Large Number coincidence is no more surprising than the existence of life itself. Nevertheless, Dicke's argument does* not *explain why gravity is so weak, nor why the universe is so old now that we exist — it only* reflects *the two facts. The separate explanation of either one may lie in grand-unified or string theories of particle physics, through the so-called "running coupling constant" phenomenon. If so, then physics will eventually explain why the timescale* t_{ms} *of stars is so huge in units of molecular interaction times — a fact that allowed life enough time to evolve.*

Dirac's response, his first written remarks about cosmology for more than twenty years, to this unusual perspective upon cosmological observations was rather bland:

'On Dicke's assumption habitable planets could exist only for a limited period of time. With my assumption they could exist indefinitely in the future and life need never end. There is no decisive argument for deciding between these assumptions. I prefer the one that allows the possibility of endless life.'

Although he was willing to admit that life would be unlikely to exist before the stars had formed he was unwilling to concede that it could not continue long after they had burnt out. With Dirac's idea of varying G the coincidences would continue to be seen at all times but on Dicke's hypothesis would only be seen near the present epoch.

Dirac didn't think there was any problem with having habitable planets in the far future on his theory. However, if gravity is getting weaker it is not clear that stars and planets would be able to exist in the far future. At the very least, other constants would need to vary to maintain the balance between gravity and the other forces of Nature that make their existence possible.

Dirac's hypothesis, however, did not survive for long. Other notable cosmologists did not believe that any 'Fundamental Theory' a-la Eddington could possible hope to explain coincidences between large numbers precisely because the large numbers involved the present *age of the universe.*

Since there was nothing special about the present time we were living at (apart from our existence), no theory of physics could predict it or pick it out so it could explain the coincidences. Yet, Dicke's arguments strikingly support the notion that the observable universe must be at least ten billion years old. Since it is expanding, it must be at least *of order ten billion light years in size. We could not exist in a universe that was significantly smaller.*

Thus, modern cosmology provide an illuminating response to the question why we are here at the time and place that we are.

Furthermore, the blend of the natural laws of the universe, along with the relative strength of the fundamental forces that operate among matter and energy, cannot vary by much if the universe is to develop such complicated things as multibillion-year-old stars and *living cells.*

The Anthropic Principle seeks to link aspects of local and global structure of the universe to those conditions necessary for the arousal and existence of *living* observers.

The expulsion of Man from his self-assumed position at the center of nature owes much to the Copernican principle that we do not occupy a privileged position in the universe. Although we do not regard our position in the universe to be central or special in every way, this does not mean that it cannot be special in any *way.*

This led **Brandon Carter** (1974) to limit the Copernican dogma by an *Anthropic Principle*, to the effect that our spacetime location in the universe and perhaps the very laws of nature, are necessarily privileged to the extent of being compatible with our existence as observers.

In other words: the measured values of many cosmological constants and physical quantities that define our universe (and our location and epoch therein) are circumscribed by the necessity that we observe from a site and time where conditions are appropriate for the occurrence of biological evolution and at a cosmic epoch exceeding the astrophysical and biological timescales required for the development of life-supporting environments and biochemistry[1029].

Put another way, the Anthropic Principle states that the structure of the universe is restricted by the fact that we are observing this structure; by the fact that, so to speak, the universe is observing itself. These ideas are phrased in the definition of the Weak Anthropic Principle (WAP):

> *The observed values of all physical and cosmological quantities are not equally probable but they take on values restricted by the requirement that there exist sites where carbon-based life can evolve and by the requirement that the universe be old enough for it to have already done so.*

The existence of a number of a priori unlikely coincidences and fine-tunings between dimensionless numbers (some of them of enormous magnitude) that are, superficially, either completely independent or determined from some life-unrelated mathematical principles, and the further fact that these unlikely relations appear essential to the existence of carbon-based observers in the universe – led some scientists to propose a stronger version of the WAP, namely the strong Anthropic Principle (SAP):

> *The universe must have those properties which allow life to develop within it at some stage in its history.*

This is clearly a more metaphysical and less defensible notion, it implies that the constants and laws of nature must be such that life can exist. This speculative teleological hypothesis admits a number of distinct interpretations of a radical nature:

- *Nature organizes itself in such a was as to make the universe self-aware.*
- *There exists a restricted set of possible universes 'designed' with the goal of generating and sustaining 'observers'.*

[1029] The observer's *scale* is privileged too: the mass of an adult *human* (ca 60 kg) is roughly the geometric mean of a *stellar mass* (ca 10^{30} kg) and an atomic mass (ca 10^{-27} kg). And the *size of a living cell* is about the geometric mean of the size of the observable universe (ca 10^{10} LY $\approx 10^{28}$ cm) and the Planck length ($\approx 10^{-33}$ cm)!

- *Observers are necessary to bring the universe into being.*
- *Intelligent information-processing must come into existence in the universe, and, once it comes into existence, it will never die out (known as the Final Anthropic Principle or FAP).*

While FAP and SAP are quite speculative, WAP is just an extension of the well-established principle of science that it is essential to take into account the limitations of one's measuring apparatus when interpreting one's observations.

Note that the SAP can be regarded as a meta-principle of organization, because it arranges the fundamental laws themselves so as to permit complex emergent organization to arise.

1944 CE **Howard Hathaway Aiken** (1900–1973, U.S.A.). Mathematician. Built one of the first automatic electro-mechanical digital calculating machine (1944) dreamed up by Babbage in 1833.

In 1937, Aiken had the idea of using the techniques and components developed for punched-card machines to produce a fully automatic calculating machine. At that time he was a graduate student at Harvard, and getting fed up with the tedious calculations required for his Ph.D. thesis. To implement his idea he approached the I.B.M. Corporation, one of the largest manufacturers of punched-card machinery. The result of their collaboration was the Automatic Sequence Controlled Calculator (A.S.C.C.), also known as Mark I, which was completed in 1944 and presented to Harvard University in August of that year.

In 1940, Aiken had his attention called to Babbage's pioneering efforts. When he read Babbage's charge to his successors he felt that Babbage was speaking directly to him from the past.

When the Harvard Mark I was in operation, it sounded "like a roomful of ladies knitting". The muted clicking noises were made by thousands of electromechanical relays opening and closing. Unlike most modern computers, the Mark I included electro-mechanical counters, descendants of those designed by Pascal.

1944 CE **Archer John Porter Martin** (1910–2002, England) and **Richard Lawrence Millington Synge** (1914–1994, England). Biochemists.

Developed *paper chromatography*, a new tool in identifying organic compounds using absorbent paper[1030]. They were awarded the Nobel prize for Chemistry (1952).

In this technique, a drop of mixture (amino acids, say) is placed at one end of a piece of very porous paper and is allowed to dry. The amino acid molecules remain firmly bound to a thin and invisible film of water on the paper. Now, an organic liquid such as butyl alcohol is allowed to creep up the paper, by capillary action. As the alcohol passes the dried mixture of amino acids, each amino acid moves at a different rate and is found at a different spot on the paper. In this way the individual components of even a very complex mixture can be separated and analyzed individually.

By this method, the amino acid analysis of a number of different proteins was carried through. For instance, the albumin of human blood plasma was found to be made of 510 proteins, each composed of 19 amino acids. This method is of course incapable of revealing the *order* in which the amino acids appear[1031] in the original protein molecules.

Their technique revolutionized analytical biochemistry and enabled rapid separation of small amounts of complex mixtures of *biochemicals* not possible by ordinary chemical methods.

1944 CE **Rudolf Karl Luneburg** (1902–1949, Germany and USA). Mathematician. Accomplished a systematic and fundamental development of ray and diffraction optics from Maxwell's equations.

Luneburg was born in Volkersheim, Germany. Received his doctorate from Göttingen University (1930) and was a Research Associate in Mathematics there (1930–1933). Emigrated to the United States (1935), holding a series of short term university appointments and worked for the American Optical Company during 1938–1945. His book: *Mathematical Theory of Optics*, pub-

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[1030] Thus extending the *chromatography* of **Willstäter**. Martin and Synge used *ninhydrin* to reveal the position of the *amino acids*. The developed strip is called a *chromatogram*.

[1031] Nevertheless, **Frederick Sanger** (b. 1918) was able (1953) to determine the order in which amino acids compose the molecule of *insulin*. The number of possible arrangements for these amino acids is greater than 10^{100}. For this he was awarded the Nobel Prize for Chemistry in 1958, and a second one in 1980 for work on the chemical structure of *genes*.

lished posthumously, was based on notes of lectures given in 1941 at Brown University.

He was killed in an automobile accident in Montana.

The chief contribution Luneburg made through his theory lies in having shown how the two main mathematical disciplines of instrumental optics, namely *geometrical optics* and *scalar-diffraction optics*, may be developed in a systematic manner from the basic equations of Maxwell's electromagnetic theory.

Prior to Luneburg's work these two disciplines were treated as self-contained fields, with little or no contact with each other and less with electromagnetic theory. The starting point of Luneburg's investigation was the observation of the formal equivalence of the basic equation of geometrical optics (the eikonal equation) and the equation that governs the propagation of discontinuous solutions of Maxwell's equations (the equation of characteristics).

By identifying the geometrical optics field with the electromagnetic field on a moving discontinuity surface, Luneburg was led to a complete formulation of geometrical optics as a particular class of *exact* solutions of Maxwell's equations. This formulation is by no means based on traditional ideas; for traditionally geometrical optics is regarded as the short wavelength limit of the monochromatic solution of the wave equation.

Luneburg was, of course, aware of this more traditional viewpoint, and he devoted considerable time to the interrelation between the two approaches. Some of his ideas became the nucleus from which a systematic theory of asymptotic series solutions of Maxwell's equations has gradually been developed.

1944–1951 CE **Hendrik Christoffel van de Hulst** (1918–2000, Holland). Astronomer. Suggested that interstellar hydrogen must emit radio waves at wavelength 21.2 cm (the key to mapping the spiral arms of our galaxy).

The ground state of the hydrogen atom is split by the hyperfine magnetic interaction of the electron and proton spins into two quantum states, with separation 0.047 cm^{-1} in inverse wavelength. In one of the states the spins of the electron and the proton are parallel, in the other antiparallel. When a spin reversal occurs in a ground-state atom due to an external disturbance, a photon of wavelength 21.2 cm is eventually emitted[1032], thus relaxing the atom back to its ground state spin configuration.

Even though in any individual excited interstellar atom, such emission (and accompanying spontaneous reversal) occurs, on the average, once in 11 million years, the cumulative effect of the large number of hydrogen atoms in interstellar space might be observable.

Confirmation of van de Hulst's prediction was provided on March 25, 1951, by the first observation of the interstellar hydrogen absorption line by **Harold Ewen** (U.S.A.) and **Edward Mills Purcell** (1912–1997, U.S.A.). An extremely important line of investigation has resulted from this discovery, enabling astronomers for the first time to map with some success the structure of the Milky Way.

1944–1952 CE **Stanislaw Marcin Ulam** (1909–1984, Poland and USA). Mathematician. Solved the problem of how to initiate fusion in the *hydrogen (thermonuclear) bomb*[1033]. Devised the '*Monte-Carlo method*' (1945) which

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[1032] This is in the radio region, and corresponds to a transition between two levels, 10^{-17} ergs apart in energy. Ewen and Purcell showed, in a laboratory experiment, that it is possible to stimulate the transition in a beam of hydrogen atoms, and thereby to measure the corresponding radio frequency 1420.403 Mc/sec, giving a wavelength of just over 21 cm.

[1033] An important thermonuclear reaction between the hydrogen-isotope nuclei is

$$\text{Deuterium} + \text{Tritium} \quad \rightarrow \quad (\alpha\text{-particle}) + \text{neutron} + 17.4 \text{ MeV}.$$

But tritium is radioactive with a half-life of 12 years and would be a troublesome permanent ingredient of a bomb, generating unwanted heat with the necessity of frequent replenishment.

The use of ^{6}LiD (Lithium deuteride with the isotope lithium-6) solved two crucial problems at once: as a chemical molecule it holds the deuterium in solid form without the need for refrigeration, and at the instant of detonation the ^{6}Li present provides the tritium needed for the $\mathrm{D}+\mathrm{T}$ (i.e., ${}^2_1H_1 + {}^3_1H_2$) reaction through the reaction $^6\mathrm{Li} + n \rightarrow \alpha + T + 5$ MeV.

It is because this reaction has a larger cross-section than the corresponding ^{7}Li reaction, that the separated isotope ^{6}Li is used in the lithium deuteride. So this design produces more tritium to increase the intensity of the thermonuclear reaction. Since lithium is cheap and tritium is expensive, this has economic as well as technical advantages.

We may think of an H-bomb as consisting mainly of an A-bomb (fission nuclear bomb) surrounded by, or close to, a mass of ^{6}LiD. When the A-bomb trigger explodes, and instantaneously before it has a chance to blow the H-bomb ingredients apart, it emits a very intense burst of neutrons that bombard the searches for solutions to mathematical problems using a statistical sampling method with pseudo–random numbers. Invented the concept of *cellular automata* at the Los Alamos laboratory (1948).

Ulam was born in Lemberg, Poland (now Lviv, Ukraine) to Jewish parents. He obtained his Ph.D. from his hometown Polytechnic Institute (1933). A common interest in set theory led to contact with von Neumann, who invited him to the USA (1936) and later involved him in the atomic bomb project at Los Alamos (1944). From 1946 he collaborated with Edward Teller in the design of the hydrogen bomb. His work required massive calculations; Ulam utilized existing calculation machines and applied probabilistic (so-called 'Monte Carlo') methods.

After WWII, Ulam continued to pursue his interest in using machines to solve mathematical and scientific problems, and held several professorships. At the time of his death he was a professor of biomathematics at the University of Colorado (from 1965).

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^{6}Li to produce the tritium needed, mixed with the deuterium already there, so that the heat produced by the A-bomb can detonate the $D + T$ reaction. This in turn gives off more heat to speed up this and other reactions and at the same time makes more neutrons to hit ^{6}Li nuclei and make more tritium, further speeding up the $D + T$ reaction.

All these interactions amongst the reactions conspire to make the explosion proceed very suddenly and to render a high energy yield, using up a considerable part of the materials present before the explosive expansion proceeds far enough to stop the reaction.

Cellular Automata[1034]

"God has put a secret art into the forces of Nature so as to enable it to fashion itself out of Chaos into a perfect world system."

(Immanuel Kant, 1781)

"If we wish to understand the nature of the universe, we have an inner hidden advantage: we are ourselves little portions of the universe and so carry the answer within us."

(Jacques Boivin, 1988)

The idea of living organisms as machines has proven irresistibly attractive to scientists and philosophers since the time of ***Aristotle***.

The evolution of science up to the middle of the 20th century has taught us that the human mind, when aided by numbers and symbols, is capable of expressing and understanding concepts of great complexity. Yet, expression *of complicated relations and equations is one thing; —* insight *gleaned from these relations is quite another.*

Today, powerful computers with advanced graphics can be used to produce representations of observed, processed and simulated multi-dimensional data from a number of perspectives, and to characterize and elucidate natural phenomena and mechanisms (man-made or naturally occurring) with increasing clarity and usefulness — thus vastly aiding the human mind in generating insights. Indeed, cellular automata *– classes of simple mathematical systems with exotic behavior – are not only at the core of the inner workings of computers, but are also beginning to show promise as models for a variety of physical processes. Though the rules governing the dynamics of these systems are simple, the patterns they produce are complicated and often pseudo-random, like a turbulent flow or the output of a cryptographic system.* Cellular automata

[1034] For further reading, see:

- Shatten, A., *Cellular Automata*, Vienna University of Technology, Vienna, 1997.
- Wolfram, S. (ed), *Theory and Applications of Cellular Automata*, World Scientific Press, Singapore, 1986.

are characterized by the fact that they operate in a discrete state space *(or* grid*) as opposed to a continuum.*

The history of cellular automata *(CA) began in 1947, when the mathematician* ***Stanislaw Ulam*** *was interested in the evolution of graphic constructions generated by simple rules. The basis for his construction was a 2-dimensional space divided into "cells", a sort of grid. Each of these cells has two possible states: ON* and *OFF.*

Starting from a given pattern, the pattern of the following "generation" was determined according to a neighborhood-based majority rule: e.g. if a cell was adjacent to two or more "ON" cells, it would switch to become ON too; otherwise it would switch OFF.

Note that in this scheme, the state transitions are local *in both space and time. This means that the next value of a given cell depends only upon the current value of that cell and the values of cells in an immediately adjacent neighborhood. So there are no fundamental time-lag effects, nor are there any direct nonlocal spatial interactions affecting the state transition.*

Another property is homogeneity*: each cell of the system is the same as any other cell, in the sense that they can each take on exactly the same set of possible values (on and off) at any moment, and each change their state in accordance with the same set of formal (deterministic and/or stochastic) rules.*

Ulam, who used one of the first computers, quickly noticed that this mechanism permitted the generation of complex and graceful figures and that these figures could, in some cases, self-reproduce: *copious and repetitive application of extremely simple rules resulted in the emergence of very complex patterns.*

Let us give here a few examples. Consider first the simplest CA – an infinite string of cells changing values according to a given rule. Let this system be specified by two numbers k and R at each cell, together with a rule determining the next value at each cell. The first number, k, specifies how many values are possible for each cell, while R refers to the range of the neighborhood used to compute the next value of a cell. So, for example if $k = 2$, $R = 1$, there are two possible states per cell (say 0 and 1), and a cell's neighbors are defined as the two cells on either side of it. A cell and its two neighbors form a neighborhood of 3 cells, so there are $2^3 = 8$ possible patterns for any given neighborhood at time t, namely

$$111, 110, 101, 011, 100, 010, 001, 000$$

A possible rule, known as the 'mod 2 rule', determines the central cell value of each neighborhood by adding the values of its two neighbors, dividing that number by 2, and keeping the remainder. The central cell of the above octet *will then have the values* $\{0, 1, 0, 1, 1, 0, 1, 0\}$. *Each time the rule is applied to the whole line, a new generation is produced.*

*The behavior of some one-dimensional CA can be quite complicated. It has been shown (***Wolfram***, 1980) that, depending on the rule chosen, the long-term behaviors of a CA are counterparts in discrete time and space of continuous-time patterns exhibited by dynamical systems exhibiting* fixed points, limit cycles, strange attractors *and patterns exhibited by quasi-periodic orbits.*

These kinds of simple CA have been used to model a bewildering variety of processes — ranging from the sequences of nucleotide bases on a strand of DNA to the dynamics of both human and computer languages.

In the late 1960s, Dutch biologist ***Aristid Lindenmayer*** *proposed a CA model for the* development of filamentous plants, *such as the blue-green algae* Anabaena. *His model contains the novel feature that the number of cells is allowed to increase with time according to a recipe laid down by the state-transition rule. In this way the model "grows" in a manner mimicking the growth of a filamentous plant.*

Another example is a 2-dimensional cellular grid, say, an infinite sheet of graph paper, where each square is a cell. Let each cell have two possible states (say, black and white), and the neighborhood of a cell consist of itself and the 8 squares touching it. Then there are $2^9 = 512$ *possible patterns for a cell and its neighbors. The transition rule for this CA could be given as a table. For each of the 512 possible patterns, the table would state whether the central square will be black or white in the next time step.*

A special case of this example is known as the Game of Life *(***J.H. Conway***, 1970). This automaton too uses two colors: black and white, with these rules:*

- *A cell that is* white *at one instant becomes* black *at the next if it has precisely 3 black neighbors.*
- *A cell that is* black *at one instant stays black at the next iff it has either 2 or 3.*

In all other cases, cells maintain their color.

Finally we present a simple CA known as the Langton ant, *after its inventor. An ant moves either north, south, east or west on a square grid of black and white cells, following three simple rules:*

- *If it is on a black cell it makes a* 90° *turn to the left.*
- *If it is on a white cell it makes a* 90° *turn to the right.*

- *As it moves to the next square, the one that it is on changes color from white to black, or the reverse.*

You may think that Langton's ant must be a remarkably simple animal, but such is not the case. In fact, it poses a problem that is currently baffling mathematicians.

Suppose you start the ant in an eastward direction on a completely white grid. Its first move takes it to a white square, and the square it started from turns black. Because it is on a white square, the ant's next move is a right turn, so that it is then facing south. That takes it to a new white square, and again the square it has just vacated turns black. After a few such moves it starts to revisit earlier squares that have previously turned black. If you try out the rules you'll find that the ant's motion gets quite complicated — and so does the ever-changing pattern of black and white squares that trails behind it. Every so often during the first few hundred moves, the ant produces a nice, symmetrical pattern. Then things get rather chaotic for about ten thousand moves. After that, the ant gets locked into a cycle that repeats the same sequence of 104 *moves, whose net result is to move it two squares diagonally. It continues like this indefinitely, systematically building a broad diagonal "highway."*

This behavior is curious enough, but computer experiments suggest something even more striking: If you scatter any number of black squares around the grid before the ant sets off, then it still ends up building a highway. For example, when the ant starts inside a particular solid rectangle, it builds a "castle" with straight walls and complicated crenelations at the corners. It keeps unbuilding and rebuilding these structures in a curiously "purposeful" way until it gets distracted and wanders off . . . building a highway. The problem that is baffling mathematicians is that nobody can prove *that the ant always ends up building a highway, for any initial configuration of (finitely many) black squares.*

Cellular automata are often simulated on a finite grid rather than an infinite (i.e. infinitely extensible) one. In two dimensions, the universe would then be a rectangle instead of an infinite plane. The edges are usually handled with a toroidal *arrangement: when you go off the top, you come back in at the corresponding position on the bottom, and when you go off the left edge you come back in on the right (This essentially simulates an infinite periodic tiling). This can be visualized as taping the left and right edges together to form a tube, then taping the top and bottom edges of the tube together to form a* torus *(doughnut shape). Universes of other dimensionalities are handled similarly. This is done in order to eliminate complications due to boundary conditions.*

Returning to the historical evolution of these ideas, it was ***John von Neumann*** *(1948) (relying on* ***A. Turing****'s work) who first asked the question: "What kind of logical organization and functional activities would an object have to posses, to be able to build a copy of itself?" Thus, von Neumann wanted to abstract from the processes of* self-replication *in nature the* logical *form of the reproduction process — independent of its realization in any particular material structure. This "recipe" could then be fed into an automaton which von Neumann named "kinematon". Such a machine was supposed to be able to reproduce (replicate) any machine described in its program, including a copy of itself.*

*Ulam then suggested to von Neumann to use what he named "*cellular space*" to build his self-replicating machine. They succeeded in proving that an abstract pattern* could *create a copy of itself by following a set of fixed rules.*

In fact, **von Neumann** *created a mathematical blueprint for a universal Turing machine consisting of a two-dimensional CA having 29 states per cell. His idea was to represent the initial machine as a particular pattern in this CA array. Self-replication would then be said to have occurred if a rule of state transition (using 5-cell von Neumann neighborhood) could be found that would cause the initial pattern to be duplicated* elsewhere *in the array.*

Von Neumann showed that his 29-state CA could be capable of universal construction*, from which self-replication follows as a special case when the machine described on the constructor's input is the constructor itself.*

There was yet another difficulty that von Neumann had to surmount: suppose we have succeeded in building a universal constructor*. We then feed the plans for the constructor back into it as input, and it will then replicate itself.* But it will not reproduce the instructions describing how to build itself. *Without these* the reproduction will not perpetuate *and will be a useless model for living cells.*

How, then, do we arrange it so that the blueprint, as well as the constructor, are faithfully reproduced?

*Von Neumann's way out of the "*blueprint dilemma*" was to build a supervisory unit into the constructor. This unit functioned in the following manner: Initially the blueprint is fed into the constructor as before, and the constructor reproduces itself. At this point the supervisory unit switches its state from construction-mode to copy-mode and proceeds to copy the blueprint as raw, uninterpreted data. The copy is then appended to the previously produced constructor (which includes a new supervisory unit), and the self-reproducing cycle is complete. The key element in this scheme is to prevent the description of the constructor from becoming a part of the constructor* *itself (i.e. the blueprint is located outside the machine and is then appended to the machine at the end of the construction phase by the copying operation of the supervisory unit).*

The crucial point to note about von Neumann's solution is the way information on the input blueprint is used in two fundamentally different ways. It's first treated as a set of instructions to be interpreted. *These instructions, when executed, cause the construction of a machine somewhere else on the CA grid. Thereafter, the blue print information is treated as* uninterpreted *data, which must be copied and attached to the new machine. These two different uses of information are also found in biological self-reproduction: the interpreted instructions correspond to the process of genetic* translation, *while the blind copying of the uninterpreted data corresponds to the process of genetic* replication. *The separation of these two types of processes prevents an infinite regress of self-referential instructions.*

These are exactly the processes involved in the operation of every living cell, and it's worth noting that von Neumann came to discover the need for these two different uses of information several years before their discovery by biologists working on the mysteries of DNA. The only difference between the way von Neumann set things up and the way nature does it is that he arbitrarily chose to have the copying process carried out after the construction phase, whereas nature copies the DNA early on in the cellular reproduction process.

In conclusion: a von Neumann machine is a cellular automaton capable of automatically replicate itself. When provided with a blueprint, it can build anything and replicate indefinitely, like a living cell, including a new copy of the blueprint itself as an "attachment" in each successive CA "generation".

It turns out that the automaton invented by **Conway** *(described earlier) has much simpler rules and is able of doing the same kind of thing. Suppose that in this* Game of Life *one starts with an object made up of black cells, and the rest of the board white. Then you follow the rules and watch how that object changes. For example, a 2×1 block dies out at the first move. A 2×2 block doesn't do anything, so it survives indefinitely. More interesting is a simple shape called a glider: It moves. It changes shape in a four-step cycle, after which it has moved one cell diagonally. More complicated shapes, termed "spaceships", move horizontally or vertically. A "glider gun," which changes through a fixed cycle of thirty shapes, fires an endless stream of gliders.*

Conway's Life *evolves in a reductionist and deterministic universe, which really does have a Theory of Everything, namely Conway's rules. Given a starting shape, its future is completely determined by those rules. But in practice it may be very hard to predict what will happen, even though it's all implicit in the rules. Big shapes can collapse, small ones can grow, and* *there are always surprises. Three of the features that emerge from* Life *are* programmability, undecidability, *and* replication.

First, Conway proved the existence of an initial configuration that acts like a programmable computer, using pulses of gliders instead of electrical impulses to carry and manipulate information.

From this he deduced that the outcome of the game is inherently unpredictable, in the following sense: There is no way to decide in advance whether a given object will survive indefinitely, or disappear entirely. He did this by appealing to a theorem in mathematical logic which states the following: There cannot exist a computer program that can decide in advance whether any given program, when run on a given machine, will go on forever, or will stop. The only way to find out is to run the program and watch. If it stops, you know; if it keeps going, you have no idea whether it will continue going, or whether it's just about to stop as soon as you give up and go away. The theorem is called the undecidability of the halting problem, *and the proof was discovered by* ***Alan Turing***.

Once you have programmable computers, it's not hard to pinch von Neumann's trick and design self-reproducing machines. So Life – a two-state cellular automaton with only three rules – has implicit within it, self-reproducing computers. Given that, you could set up self-reproducing "animals" with "genetic programs" that interact with each other (just program the Life computer to simulate such a system). You could "irradiate" the board with gliders to cause random mutations. Then you could sit back and watch evolution at work.

Cellular automata applications are diverse and numerous: Fundamentally, CA constitute completely known 'universes'. Our universe is subject to the laws of physics. These laws are partly known and appear to be highly complex. In a cellular automaton laws are simple and completely known. One can then apply the concepts of CA to gain a better understanding of the global behavior of a simplified universe, of elementary particles, of complex chemical molecules and perhaps the organization of living organisms.

Indeed, CA are being used for the following purposes, inter alia:

(1) *Simulation of* gas behavior: *A gas is composed of a set of molecules whose behavior depends on neighboring molecules ("lattice gas").*

(2) *Study of ferromagnetism according to the* Ising model: *this model (1925) represents the ferromagnetic – domain crystal lattice as a network in which each node is in a given magnetic state. This state is binary – representing the two possible quantum orientations of the spin of a radical or molecule – and the probabilities of any spin value being "up" or "down" depends self-consistently on the state of neighboring nodes.*

(3) *Simulation of percolation processes.*

(4) *Simulation of forest-fire propagation.*

(5) *Numerical solutions of partial differential equations.*

(6) *Configuration of massive parallel computers.*

(7) *Simulation and study of urban development.*

(8) *Simulation of crystallization processes: studies have shown that, starting from one occupied cell (which may be thought of as a single defect, or a nucleation center) in a lattice, the pattern will continue to "grow" in size as time progresses. In some experiments, two different background lattices with adjacent boundaries were used, and the defect propagated from its beginning point in a centered rectangular lattice through the interface of the second lattice.*

Adding a defect to these two-phase systems bears some similarity to seeding supersaturated solutions and watching the crystallization process grow and "hit" the boundary of a solution with a different composition.

(9) *Used as graphics generators.*

(10) *Investigations of ornaments and decorations of various cultures by consideration of their symmetry groups. Indeed, from a purely artistic standpoint, some of the figures produced by CA's are reminiscent of Persian carpet design, ceramic tile mosaics, Peruvian striped fabrics, brick patterns from certain Mosques, and the symmetry in Moorish ornamental patterns. This artistic resemblance is due to the complicated symmetries produced by algorithms.*

(11) *Study of complicated and ordered structures arising spontaneously from "disordered" states, such as snowflakes, patterns of flow in turbulent fluids, and biological systems (e.g. patterns of schools of fishes and flocks of birds).*

(12) *Game theory.*

(13) *Artificial Intelligence.*

(14) *Economics (automata may be used to simulate "agents" seeking to maximize individual advantage through mutual trade and commerce).*

(15) *Non-linear dynamics.*

(16) Emergence phenomena in studies of artificial life:

The notion of emergence first appeared with general system theory. It states that the global behavior is more than the sum of the behaviors of the individual parts. In other words — complex association of elements induces the appearance of new phenomena and mechanisms. It implies that novel, hard–to–predict and complex behavior results from simple interactions among a system's component parts. Thus, the property of emergence is linked to complexity.

At each level (of the prebiotic, biotic and social evolution) of life, new properties appear that cannot be explained by the properties of each part that constitutes the whole. The increase in the diversity of elements, and in the number of links between these elements, and the nonlinear interactions lead to unpredictable behaviors.

The so-called "global" emergence then characterizes the properties of a system that are new in the framework of the properties of its isolated components. Life is undoubtedly such an emergent phenomenon, as are intelligence and social organization.

In 1983 ***Stephen Wolfram*** *published the first of a series of papers systematically investigating cellular automata. The unexpected complexity of the global behavior stemming from simple rules — and the failure of mathematical methods to meaningfully describe them — led Wolfram to suspect that complexity in nature may be due to similar mechanisms, and that it, too, might not be amenable to traditional mathematical analysis.*

1944–1960 CE **Gregory Goodwin Pincus** (1903–1967, USA). Physiologist. Developed the oral contraceptive pill [with **John Rock** (1890–1984) and **Min Chueh Chang** (1908–1991)], using synthetic hormones to inhibit ovulation in mammals; the hormones mimic the condition of pregnancy in women, thus effectively preventing impregnation.

Pincus was born in Woodbine, NJ to Jewish parents and studied at Cornell and Harvard. In 1944 he co-founded the Worcester Foundation for Experimental Biology in Shrewsbury, MA. There he began his research on steroid hormones, which was encouraged by birth-control pioneer **Margaret Louise Sanger** (1883–1966, USA).

Synthetic hormones became available in the 1950s, and Pincus organized field trials of their anti-fertility effects in Haiti and Puerto Rico in 1954. The results were successful and oral contraceptives ('*the pill*') have been widely used since their first marketing in 1960, despite concern over some side effects.

Their success is a pharmaceutical rarity; synthetic chemical agents do not usually show nearly 100 per cent effectiveness in a specific physiological action, or have such remarkable social effects.

1944–1971 CE **Robert Burns Woodward** (1917–1979, U.S.A.). Organic chemist. Known especially for work in determining structures of complex organic compounds. Synthesized *Quinine* (1944), *Penicillin* (1945), *Strychnine* (1947), *Cortisone* and *Cholesterol* (1951), Lysergic acid (1954), *Reserpine* (1956), *Chlorophyll* and *Oleandomycin* (1960), *Tetracycline* (1962), *Vitamin* B_{12} (1971).

Woodward was born in Boston, MA. Entered M.I.T. at the age of 16 and received his Ph.D. there at the age of 20. He went to Harvard (1937) and remained there for the rest of his life. Received the Nobel prize in chemistry (1965) for his contribution to organic synthesis.

Woodward made fundamental contributions to organic chemistry covering structural elucidation, total synthesis, biosynthesis and *reaction mechanism*. His work was based on meticulous attention to detail, a logical and highly analytical approach, a profound understanding of the electronic and stereochemical behavior of molecules and a prodigious memory. During his work on vitamin B_{12} he recognized the role of *orbital symmetry* in the determination of the stereospecifity of concerted reactions[1035]. The generality of these ideas (1971) represents one of the fundamental advances in organic chemistry since WWII.

1945 CE First radar signals reflected from the *moon* by U.S. Army Signal Corps.

1945 CE, July 16 First successful test of the atomic bomb, near Alamogordo, NM.[1036]

1035 Together with **Ronald Hoffman** (Nobel prize for chemistry, 1981) he extended the range of applicability of simplified quantum-mechanical calculations to cover all organic molecules — the *Woodward-Hoffman rules* which explain why some reagents react easily while other pairs do not do so at all. The basis for the rules lies in the symmetry properties of the molecules concerned, and particularly in the disposition of their *electrons*. New chemical bonds are formed when the electrons involved form a complete circuit.

1036 For further reading, see:

- Rhodes, R., *The Making of the Atomic Bomb*, Simon and Schuster: New York, 1986, 886 pp.

Table 5.20: Destructive power — man vs. nature

$$(1 \text{ kT} = 4.2 \times 10^{19} \text{ erg}, \qquad 1 \text{ MT} = 4.2 \times 10^{22} \text{ erg})$$

Event	Yield
First atomic bomb, July 16, 1945	19 kT
Device exploded over Hiroshima	13 kT
All explosive of WWII, combined	2 MT
Hydrogen bomb	10 MT
Nuclear arsenal of the world, combined	10^5 MT
Thunderstorm	1 kT
Tunguska bolide explosion (1908)	10 MT
Hurricane (average kinetic energy)	10 MT
Magnitude 8 earthquake	100 MT
Cyclone (average kinetic energy)	100 MT
Tambora volcanic Explosion (1815)	2×10^4 MT
Tuba volcanic Eruption (73,500 BCE)	6×10^5 MT
Yucatan asteroid impact (65 million years ago)	2×10^6 MT

1945 CE First atomic bomb dropped on Hiroshima, Japan (Aug. 6, 8: 15 AM; nuclear fission bomb based on uranium-235). By sudden incineration and lingering death some 200,000 people died. A plutonium-based fission-bomb was exploded over Nagasaki (Aug. 9). This bomb killed only 70,000 because hills deflected the blast and radiation. Its yield was about 20 kT of TNT. This marked the end of the *military* engagement with Japan; the *economic* conflict would, however, enter a new phase.

Paradoxically, the adoption of **Shewhart's** philosophy and practice of quality control of industrial processes by the Japanese during the five decades after WWII, hampered American economic supremacy.

Before the war, Japan's vital export markets had accepted the shoddy products supplied. But only few Japanese understood the productive strength of the US. Thus, the Japanese High Command could not foresee that the US overwhelming productive capacity and its progressive management techniques would be a major factor in winning the war.

However, the Japanese surrender aboard the USS Missouri (2 September 1945) had only marked the end of military hostilities. Before this (in mid-August 1945), secret discussions were taking place at ministerial level in Tokyo about a recovery strategy that would, in the words of the Foreign Minister Shigeru Yoshida, ensure that:

> "*we could indeed rebuild Imperial Japan out of this way of defeat... science will be advanced, business will become strong with the introduction of American capital, and in the end our Imperial country will be able to fulfill its true potential. If that is so, it is not so bad to be defeated in this war.*"

With General MacArthur's acceptance of Japan's surrender, there arose the imminent need to enable the defeated people of Japan to support themselves at a time when the occupation powers had removed the 1,500 or so top managers of the powerful traditional zaibatsu (vertical groupings) of industry (by imprisonment or enforced retirement). It was then necessary for the middle managers to take over and rebuild their companies along new, more efficient, lines. But management training was non-existent in the years immediately following the end of the war.

An American electronics engineer, **Homer Sarasohn**, in his late twenties and fresh from wartime service with MIT Radiation Labs and Raytheon (where he had distinguished himself in rapidly converting experimental electronic equipment into production-line ready hardware) established (1946) in Tokyo a radio receiver industry so that the occupation powers could broadcast to the Japanese people.

By the end of the forties his efforts had so impressed MacArthur that he supported Mr Sarasohn's recommendation that, together with another engineer, **Charles Protzman**, they set up a university level management training programme so as to spread their experience further into Japan's slowly reviving economy. This course drew upon the work of Shewhart in ensuring that only reliable and successful products were produced.

Thus did two young but insightful engineers shape the early course of postwar economic history by passing on simply and clearly what they had been taught during the war by the Western Electric training programme. Little did they realize that their efforts would slowly undermine American economic supremacy.

Indeed, just 50 years later the chairman of the world's largest manufacturing company, GM, formally admitted defeat to the superior economic and manufacturing practices of the now globally dominant Japanese companies (such as Toyota, NT&T and Matsushita) and modern vertical groupings, or *keiretsu*, (such as Matsui, C Itoh and Mitsubishi).

Table 5.20 lists yields of natural vs. man-made events.

From Atomic theory to Nuclear Technology (460 BCE–1945 CE)

ca 450 BCE **Leucippos of Miletos** introduced the idea of the *atom* — an indivisible unit of matter.

ca 420 BCE **Democritos of Abdera** stated that all matter is made of indivisible particles called *atoms*.

The Greeks gave much to the development of physics by developing the basis of fundamental modern principles such as the conservation of matter, atomic theory, and the like. Very few new developments occurred for about 22 *centuries.*

1473 **Lucretius'** *De rerum natura* (*on the nature of things*) is translated back into Latin, making the *atomic theory* of Democritos and Leucippos known to scholars in the West.

1649 **Pierre Gassendi's** study of Epicuros *Syntagma philosophice Epicuri*, asserts that matter is made up of *atoms*.

1666 **Robert Boyle's** *The origine of formes and qualities* contains his view that everything is built up of *atoms* and reflects his mechanical view of nature.

1781 Planet *Uranus* discovered by **William Herschel**.

1789 Element *Uranium* discovered by **Martin Klaproth**.

1803 **John Dalton's** atomic theory of matter: since chemicals combine only in integral proportions, *atoms* must exist.

1812 **Jöns Jacob Berzelius** asserted that *atoms* have electrical charges. This is based on the assumption that electrical and chemical forces are identical.

1828 Element *Thorium* discovered by **Berzelius**.

1872 **James Clerk Maxwell** maintained that *atoms* remain in the precise condition in which they first began to exist.

1874 **George Stoney** developed a theory of the *electron* and estimated its mass: He proposed that electricity was made of negatively charged particles he called "*electrons*".

1881 **Hermann Ludwig von Helmholtz** showed that the electrical charges in *atoms* are divided into definite integral portions, suggesting the idea that there is a smallest unit of electricity.

1890 **Hendrik Antoon Lorentz** proposed that *atoms* may consist of charged particles that produce visible light by oscillating.

1896 **Henri Becquerel** discovered *radioactivity*.

1897 **Joseph John Thomson** discovered the *electron*, the first known particle smaller than an *atom*.

1898 **Marie and Pierre Curie** discovered and separated the radioactive elements Radium and Polonium.

1905 **Albert Einstein** proved the equivalence of mass and energy: $E = mc^2$, within the framework of the *special theory of relativity.*

1911–1914 **Ernst Rutherford** established experimentally that the atom was made of a very small dense *nucleus*, positively charged, surrounded by electrons (1911). Discovered the *proton* (1914).

1913 **Niels Bohr** constructed a planetary model theory of *atomic structure* based on *quantum* ideas.

1919 **Francis William Aston** discovered the existence of *isotopes.*

1925 **Patrick Blackett** made the first photograph of *nuclear reactions* mediated by neutrons.

1931 **James Chadwick** discovered the *neutron.*

1933 **Leo Szilard** concocted the first idea of *nuclear chain reaction* mediated by *neutrons.*

1935 **Hideki Yukawa** initiated the *meson* theory of nuclear forces.

1942–1945 The Manhattan Project.

July 16, 1945 First successful test of the atomic bomb, near Alamogordo NM.

1945 First atomic bomb dropped on *Hiroshima* (Aug 6). A second dropped on *Nagasaki* (Aug 9).

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The Gas Centrifuge Story, or — Stealing the Fire[1037] *(1945–2002)*

Fission (exothermic splitting of a heavy nucleus, such as uranium, into two fragments of comparable size) as a *natural* process is very rare. Thus, the uranium *isotopes* $^{235}_{92}U$ and $^{238}_{92}U$ that we find in uranium ores have been sitting for billion of years, their nuclei securely held by the surface tension and without enough energy to vibrate and become long enough to go over the fission barrier.

The usual method of producing fission *artificially* is to excite the nucleus. The threshold (minimum activation energy) required for fission of a heavy nucleus is from 4 to 6 MeV. One of the most effective means of inducing fission is by neutron capture. This is, for example, the case of the nucleus $^{235}_{92}U$, which undergoes fission after capturing a *slow* (or thermal) neutron.

For other cases, in order for fission to take place, the neutrons must have some kinetic energy – of the order of 1 MeV – in addition to the binding energy. This is what occurs with $^{238}_{92}U$, which fissions only after capturing a *fast* neutron. The reason for this different behavior lies in some details of the structure of the different nuclei. The nucleus $^{235}_{92}U$ is even-odd, with 143 neutrons, and when a neutron is captured, an even-even nucleus, $^{236}_{92}U$, is formed. The captured neutron is paired with the last odd neutron of $^{235}_{92}U$, releasing the additional pairing energy $\delta \sim 0.57$ MeV. On the other hand, $^{238}_{92}U$ is an even-even nucleus, with 146 neutrons, all paired, and when a neutron is captured, an even-odd nucleus, $^{239}_{92}U$, results, with no extra pairing energy available. For the same reason $^{239}_{94}Pu$, with 145 neutrons, undergoes fission by slow neutron capture.

Thus it is much easier to cause fission in $^{235}_{92}U$ than in $^{238}_{92}U$ and the former in the more valuable isotope for producing nuclear weapons[1038]. Unfortunately from the standpoint of producing power, but perhaps fortunately

[1037] **John S. Friedman** and **Eric Nadler**, film documentary, Oct 2002.

[1038] The fact that for each neutron absorbed in order to produce one fission, more than two new neutrons are emitted (on the average) suggests the possibility of a chain reaction. That is, if things are arranged in such a way that, after each fission, on average more than one of the new neutrons produces another fission, and of the neutrons released in this fission, again more than one produces a fission, and so on, then a self-sustaining exponential process, or chain reaction, results. (Chain reactions are very common in chemistry. Combustion *for the interim stability of world politics, the more useful isotope is much more rare: uranium in nature occurs as 0.7 percent* $^{235}_{92}U$ *and 99.3 percent* $^{238}_{92}U$. *There are also some exceptions*[1039].

We now come to the question of separating the two isotopes of uranium from one another or at least obtaining uranium that is richer in the rare isotope than the uranium occurring in nature. The main point here is to appreciate the enormous difference between the case of separating elements in chemical reactions and the difficulty of separating isotopes of the same element that all behave almost the same way chemically because they have the same number of electrons. Because of their different number of electrons, most atoms of different chemical elements are easily separated from one another, many of them in large-scale industrial processes in which are prepared many of the substances used in everyday life.

The separation of isotopes is much more costly and difficult. However, separation is less difficult for light elements than for heavy elements. The usual separation processes depend on the fact that a light molecule moves

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is a chain reaction. Burning requires that a molecule have a certain activation energy so that it can combine with an oxygen molecule. But once the first molecules are excited and combine with oxygen, the energy liberated is enough to excite more molecules of the fuel, and burning results.) If in each stage of the process more than one neutron per fission produces a new fission, the number of fissions increases exponentially and a divergent chain reaction results. This is what happens in a nuclear fission bomb. But if, on the average, only one neutron of each fission produces a new fission, a steady chain reaction is maintained under controlled conditions. This is what happens in a *nuclear reactor*.

In *fast* nuclear reactors the neutrons are used at the same energies (1 to 2 MeV) at which they are released in the fission process. But in a *thermal* nuclear reactor the neutrons are first slowed down by allowing them to collide with the nuclei of some other substance, called a *moderator*, until they come to thermal equilibrium with the substance. The neutrons are then called *thermal*. The moderator must be a substance which has small mass number and a small neutron capture cross section. Water, heavy water, and graphite are the substances most used as moderators.

The energy released in a nuclear reactor is extracted by means of a circulating fluid called a coolant. In power reactors this energy is used for heating or for the generation of electric power. In research reactors the neutrons are used for different kinds of experiments, or for isotope production.

[1039] Natural mineral veins with 70 percent of $^{235}_{92}U$ were found (1972) in the mines of Oklo in the Central African country of Gabon, as well as significant amounts of Plutonium-239 in natural state.

about more quickly and easily than a heavy molecule. When two kinds of molecules are mixed together in a gas at a given temperature, they have the same average kinetic energy, $\frac{1}{2}mv^2$ which means that the ones with the larger m have smaller v.

Originally, isotopes were studied for the sake of increasing our knowledge of the structure of matter. Today, isotopes are an indispensable aid to industry as well as to molecular physicists, chemists, and biologists, for isotopes are being used for identifying and tracing individual atoms among a large number of chemically similar ones. Therefore, science and technology are greatly interested in obtaining pure isotopes, i.e., in separating as completely as possible the various isotopes of an element. This interest was strongly stimulated by the modern utilization of nuclear energy.

The best method of separation, though suited only for small quantities, is that of mass spectroscopy. *For this purpose, mass spectrographs are used with large high-intensity ion sources and with collectors for the various isotopes. Some tenths of 1 gram of pure isotopes can be produced per hour in such "electromagnetic separators".*

For technical purposes however, relatively pure isotopes are needed in ton quantities. To satisfy this need, methods of enrichment by numerous repeated steps are applied. Each of these steps produces a relatively small change in the relative abundance of isotopes; but sufficiently numerous repetitions lead to high enrichment of the desired isotope. In this way it was possible to produce the technically very important heavy water, D_2O, in a concentration of better than 99.8%.

Essentially, 3 different methods of separation are used.

- Ultra-centrifuge: *in a high-rpm centrifuge, the heavy isotope is moving outwards, the lighter one toward the axis. This method is used, e.g. for separating gaseous uranium hexaflouride (UF_6)*

- Pore-diffusion: *through filters with very small pores, light isotopes diffuse better than heavy ones. In the* thermal-diffusion *method an isotope mixture streams along a temperature gradient.* ***Gustav Hertz*** *(Germany)conducted the first experiment (1932) on separation of neon isotopes by the pore-diffusion method.*

  With uranium the problem is much more difficult because the two isotopes differ in mass by only a little over 1 percent, about 238 nucleon masses for ^{238}U as compared with about 235 for ^{235}U. In this case the difficult separation is carried out in an enormous and very expensive gaseous diffusion plant that also uses a great deal of electric power. The first large-scale plant using pore-diffusion was built in Oak Ridge, *Tennessee*[1040]*, during World War II and others have been built since in the US (at Paducah, Kentucky, and Portsmouth, Ohio) as well as in France and China. The fact that these plants are extremely costly and sophisticated has been an important obstacle to the easy dissemination of nuclear bomb-making capability to various nations.*

- *The electromagnetic method* *consists of accelerating molecular ions and sending them between the poles of a magnet. This requires using gases at extremely low pressures which makes it difficult to handle large amounts of material. For this reason, this method is usually used only for further concentration or complete separation of the concentrated output from the gaseous diffusion plant.*

The Centrifuge Timeline

1940 *German physicist* **Fritz Lange** *(1899–1987) conducted the first experiment with centrifuges. His centrifuge was a mammoth structure, weighting about a ton. Because the future applications for Lange's centrifuge were unclear, all centrifuge research in Germany was suspended during the WWII years 1940–1945.*

1941 ***Jesse Wakefield Bears*** *(1898–1977; USA), pioneered centrifuge method for separation of uranium at the University of Virginia. There he conducted with his colleagues the first known separation of* $^{235}_{92}U$ *and* $^{238}_{92}U$ *using a gas centrifuge: the slightly heavier 238-isotope containing molecules in the* UF_6 *gas are forced closer to the inside wall of the rotor than the 235-isotope. The radial separation factor is proportional to the absolute mass difference between the two isotopes.*

[1040] As a commentary on the way government institutions can work, it is interesting that the diffusion plant at Oak Ridge was built in the period 1942–44 with almost half a billion dollars that was appropriated by Congress without Congress knowing anything about it or about the highly secret atomic bomb project of which it was part. The whole atomic project involved expenditure of two billion dollars during the war and was hidden in the even larger general military budget under the distracting code name "Manhattan District" of the United States Army Corps of Engineers.

1946–1956 *Austrian physicist* **Gernot Zippe** *(1917–) the German physicist* **Max Steenbeck** *(1904–1981) and the German physicist* **Manfred von Ardenne** *(1907–1997) built for the Soviets a light-weight inexpensive gas-centrifuge. They were captured by the Russians in Germany (1945) and brought to Sukhumi with a group of about 300 German scientists*[1041] *to help the Soviets built an atomic bomb. The Zippe–Steenbeck machine reached a 30 percent degree of enrichment*[1042].

1956–1958 *On Zippe's return form Russia, the CIA immediately snatched him to work on US centrifuge technology with Jesse Beams.*

1957 *Zippe returned to West Germany and signed up to do centrifuge work with Degussa*[1043] *(Deutsch Gold-Und Silver-Scheide Anstalt). He conceived a large array of high-speed*

1041 Among the leading German scientists who worked in Russia (1945–1955) on the first Russian bomb were: **Nikolaus Riehl**, **Gustav Hertz**, **Max Vollmer**, **Peter Thiessen**, **Wilhelm Menke**, **Reinhold Reichmann**, **Gerhard Krueger**, **Heinz Barwich**, **Werner Schuetze**, **Gunther Wirths** and **Robert Doepel**.

1042 One of Germany's most important contributions to the Soviet bomb program was the uranium confiscated from Germany. It greatly accelerated the pace of the Soviet nuclear project: Despite all its efforts, the Soviet Union was catastrophically short of uranium for its nuclear project (10 kg of metallic uranium and 300 kg of uranium oxide and nitrate). The Germans had large amounts of uranium, including some acquired from the Belgian Congo: 300 tons of uranium oxide and other uranium compounds were brought to the USSR from Germany.

In conclusion, while the Soviets did not need the Germans to built a nuclear weapon, their contributions certainly accelerated the Soviet's push to become a nuclear weapons state: The Soviets benefited considerably from German technology, expertise, and raw materials. The German contributions undoubtedly accelerated the program by *several years* and enhanced the Soviet's stature on the world's stage.

1043 *Degussa* was a large German firm engaged in metal refining and production of chemicals including Zyklon-B cyanide tablets used by the Nazis to liquidate millions of Jews in the gas chambers. *Degussa* was also the company that supplied the uranium for the Nazi atomic bomb project. Degussa held an exclusive contract with the Nazis for re-smelting items taken from the Jews in the concentration camps including *dental gold*. *Degussa* built a smelter at Auschwitz, where the *daily* yield of gold at the camp was 12 kg. While looting the Holocaust victims of their gold and silver, *Degussa* supplied the poison gas to the concentration camps used to annihilate the owners of these metals.

centrifuges, each feeding the next with gas that is successively richer in uranium-235. Since 1958 his centrifuges remain the primary means for obtaining uranium-235. As the technology improved both power production and bomb-making got easier.

1994 *Trial of* ***Karl-Heinz Schaab*** *in Munich for treason. Accused and convicted for illegal selling of stolen classified blueprints of the Zippe ultracentrifuge to Iraq. The first person in the world convicted of 'nuclear espionage' in an open trial in the past 50 years. Schaab was linked to Degussa and Leybold, a Degussa subsidiary. He received an extremely light sentence upon conviction of 100,000 German marks fine and 5 years' probation. In 1990, Degussa was fined $ 800,000 for illegally re-exporting nuclear weapons-related material to North Korea. If Iraq had not invaded Kuwait, it could have managed to build a gas centrifuge facilities by the mid-1990s. The Scud-B technology Iraq used in the Gulf-War was 90 percent German and its nuclear technology was 60 percent German.*

1945–1950 CE **Wilfred Thesiger** (b. 1910–2003, England). Explorer of the Arabian desert. Mapped the *Empty Quarter* and other areas (over a total distance of $16,000$ km) between Yemen and Oman, never before explored by a European. In his book *Arabian Sands* (1958), Thesiger recorded the many journeys he has made by camel through and around the parched sands of Arabia's *Empty Quarter*.

Following in the tradition of **Richard Francis Burton** (1821–1890), **Charles Montagu Doughty** (1843–1926), **Thomas Edward Lawrence** (1888–1935), **Harry St. John Bridger Philby** (1885–1960) and **Bertram Thomas** (1892–1950), Thesiger is perhaps the last of the Great British explorers of the terra incognita.

Thesiger was born in Addis Ababa and educated at Eton and Oxford, spending the WWII years in the Sudan. Since the war he traveled in Southern Arabia, Kurdistan, the Marshes of Iraq, the Hindu Kush, the Karakorams, Morocco, Abyssinia, Kenya and Tonganaika, always on foot or with animal transport.

1945–1952 CE **Alan Lloyd Hodgkin** (1914–1998, England) and **Andrew Fielding Huxley**[1044] (b. 1917). Biophysicists. Described the ionic mechanism by which neurons transmit electrical impulses[1045] (1952). Awarded the Nobel prize for medicine (1963). Their model relates the response of the action potential to the changes in membrane permeability that accompany a change in voltage. The model does not explain why the membrane permeability changes; it relates the shape and conduction velocity of the impulse to the observed changes in membrane permeability. Nonetheless, the work was a triumph.

Most of their experiments were carried out on the giant axon of the squid. This is *single cell*, several cm long and 0.5 mm in diameter. The removal of axoplasm from the preparation and its replacement by electrolytes has shown that the critical phenomena all take place in the membrane.

Hodgkin and Huxley were educated at Trinity College, Cambridge. They started their work in 1939, but it was interrupted by WWII. Hodgkin was first to implant electrodes into squids' giant nerve fibers. Their Nobel prize was shared with **John Carew Eccles** (1903–1997, Australia) who developed techniques for intracellular recording from fine neurons.

1945–1972 CE **Wernher von Braun** (1912–1977, Germany and USA). Rocket engineer. Directed teams that built the rockets that sent the first American into space (1961) and made possible man's first landing on the moon (1969).

Von Braun was born in Wirsitz, Germany (now Wyrzysk, Poland). He became advisor in the rocket program of the German Army (1932), and played a major role in developing the V-2 rocket, with which Nazi Germany bombed Allied cities during WWII. In particular, he was technical director of the German test facility at Peenemünde (1936–1945) that launched V-2 rockets into London[1046].

[1044] The Huxley "tribe": **Thomas Henry Huxley** (biologist; 1825–1895) was the father of **Leonard Huxley** (editor and author; 1860–1933). Leonard's first wife was Julia Arnold. Their sons were: **Julian Sorell Huxley** (neo-Darwinist zoologist; 1887–1975) and **Aldous Leonard Huxley** (novelist and critic; 1894–1963). Leonard's second wife, Rosalind Bruce, begot him **Andrew Fielding Huxley**. Someone said of them: "*Not a dynasty, nor a clan, but an élite.*"

[1045] Hodgkin, A.L. and A.F. Huxley, A quantitative description of membrane current and its applications to conduction and excitation in nerve, *J. Physiol.* **117**, 500–544, 1952.

[1046] He was quoted as saying: "I aim at the stars, but sometimes I hit London".

In 1945 von Braun surrendered (with 116 other scientists) to the American Forces and was sent to the US to work on guided missile systems. His team developed ballistic missiles for the army, including the four-stage Jupiter-C rocket that launched *Explorer I*, the first United States earth satellite.

Another of the group's rockets, the *Redstone*, launched the flight of America's first astronaut, Alan B. Shepard Jr. (1961).

Other von Braun projects included the *Saturn* rockets. In 1969, a Saturn V rocket launched the astronauts who made man's first landing on the moon. In 1970, NASA (National Aeronautics and Space Administration) appointed von Braun deputy associate administrator for planning.

1946 CE **John Prosper Eckert** (1919–1995) and **John William Mauchly** (1907–1980) at the University of Pennsylvania built the ENIAC (Electronic Numerical Integrator and Computer). It was based on ideas borrowed from the world's first digital electronic computer, the ABC, built in 1939 by **John Vincent Atanasoff**.

The ENIAC used 18,000 vacuum tubes, was programmable, but programs could not be stored in a memory. In 1952, its successor, the EDVAC (Electronic Discrete Variable Computer) incorporated ideas of **John von Neumann**, such as program storage memory. It processed binary numbers serially, and its functioning was based on Boolean logic.

Algorithmic Vs. Dialectic Mathematics

The mathematics of Egypt, of Babylon, and of the ancient Orient was all of the algorithmic *type. The Babylonian, for example, found (ca 1700 BCE) an excellent approximation for* $\sqrt{2}$ *in their* base-60 *notation, which is equivalent to* $\sqrt{2} = 1.414\ 212\ 963$ *in decimals.*

Dialectic *mathematics — strictly logical, deductive mathematics — originated with the Greeks. But it did not displace the algorithmic variety.* ***Pythagoras*** *(550 BCE) was perplexed by the existence of* $\sqrt{2}$ *as the diagonal of the unit square on one hand, and by its non-existence as a fraction on the other. In the work of* ***Euclid****, the role of dialectics is to justify a construction — i.e., an algorithm.*

It is only in modern times that we find branches of mathematics with little or no algorithmic content, which we could call purely dialectic.

One of the first investigations to exhibit a predominantly dialectic spirit was the search for the roots of a polynomial of degree n*. For 300 years, mathematicians searched in vain for algorithms that would render closed formulae for* $n > 4$*. The theorems proved by* ***Gauss*** *(1799) and* ***Galois*** *(1829) are dialectic, providing no algorithm for the actual location of the roots. Since then, until WWII, pure mathematics has been existence-oriented rather than algorithm-oriented.*

However, the advent of electronic computers has led since the 1970s, to renewed interest in and need for numerical algorithms and their mathematical analyses within the framework of the new computer science, as well as in new computer-driven branches of applied mathematics.

The intrinsic features of each approach could be summarized as follows:

dialectic mathematics	*algorithmic mathematics*
• Rigorously logical, statements are either true or false, objects with specified properties either do or do not exist.	A tool for solving problems.
• An intellectual game played according to rules about which there is a high degree of consensus.	The rules of the game vary according to the urgency of the problem at hand and the computing equipment available.
• Invites contemplation.	Invites action.
• Generates insight	Generates results.

Annals of Computers (1642–1950)

Man's first conscious mathematical operations probably involved only simple counting: the number of faces in a tribe, the number of cattle in the herd, etc. When the numbers involved exceeded the number of fingers (or perhaps toes), some new form of reckoning had to be invented. Piles of sticks and stones and marks in the sand or on cave walls would have been logical calculating aids for primitive man. Addition and subtraction could be carried out merely by adding and erasing symbols from the crude tally sheets. (Today, modern electronic digital computers count in this same simple way. The cave wall is replaced by arrays of iron rings strung on wires or magnetized domains on a rotating disc or electrical activity in circuits. Instead of drawing or crossing out a mark on the wall, we now magnetize metalic domains or change currents and voltages in semiconductor devices.

One method of proceeding beyond the anatomical limit of hands, said to be still in use in Africa, is to enlist the aid of a second man. The first counts the units up to ten on his fingers, while his partner counts the numbers of groups of ten so formed. The next major step, taken by the first civilizations of Egypt and the Asian river valleys, was to represent numbers by means of pebbles arranged in heaps of ten. This in turn led to the development of the abacus*, or counting frame.*

The abacus was in use in so many widely separated cultures that many authorities believe it was invented independently in several centers.

Man's mathematics has progressed far beyond the simple notion of counting to algebra and the calculus. The electronic digital computer, moreover, with counting as its only stock in trade, has followed closely behind mathematical advances. It seems that any calculational problem, whether it is figuring the best move in a chess game or checking airline seat reservations, can be reduced to counting alone.

Simple counting *is not the only way to compute. One can* measure *too. Surely early man must have noticed how the shadows of trees swung slowly around as the sun moved across the sky. The passage of time could be measured by the progress of shadows on such natural sundials. With no trains to catch, primitive people needed no refinements. Later, of course, early civilizations developed more accurate natural clocks. The arrangement of huge stones at Stonehenge, England, for example, told the ancient priests when Midsummer's day, eclipses, and other key events of religious significance were about to occur.*

The basic idea behind sundials, modern clocks, slide rules, thermometers, and automobile speedometers is the measurement of some secondary quantity whose variations mimics or simulates the thing we wish to know. Thus, distance around the rim of a sundial simulates time, and the height of mercury in a thermometer is the analog of temperature.

Two families of computers, the analog *and* digital *machines, have evolved from the simple concepts of measuring and counting, respectively. The word* computer *is commonly reserved for a* digital computer*.*

By the beginning of the 17^{th} *century the victory of the Arabic system of enumeration — for both calculation and recording — was complete in most of Europe. As a result the abacus went out of use in the countries west of Russia. It was a long time, however, before even the basic processes of calculation became either commonly understood or widely practiced. Even in the second half of the* 17^{th} *century, multiplication and division of large numbers required the skill of professional mathematicians!*

The blockage was cleared by two inventions — one quite minor and the other of the very first importance — which effectively reduced all arithmetical calculations to addition and subtraction. Both these inventions are due to ***John Napier*** *of Murchiston, near Edinburgh.*

His minor invention (1617) was a simple mechanical device known colloquially as 'Napier's Bones'. It was just an improvement of an old method that had been in use in the East for multiplication of large numbers. Napier's major achievement, which really took the sting out of multiplication and division, was the invention of logarithms (1614). The importance of Napier's invention was immediately recognized by the practicing human computers of his day and within a few years the first steps were being taken to mechanize the process. Logarithms were plotted along a straight line and multiplications and divisions were performed by adding or subtracting the corresponding lengths with the aid of a pair of dividers. Since numbers are represented on the slide rule by lengths on a certain scale (on a logarithmic scale), the device is an analog computer[1047]*.*

An analog machines is restricted in the kind of calculation it does. This is true even of the versatile slide rule — which cannot help much with addition or

[1047] Applying *Ohm's law* in electricity, a simple electrical circuit can be used as an analogue of multiplication. According to this law, the difference in electric potential (in volts) between two points on a wire is the product of the resistance (in ohms) between the points and the current (in amperes) flowing along the wire. Thus we could obtain the product of two numbers (x and y) by arranging for a current of x amperes to flow through a resistance of y ohms, and measuring the voltage difference.

subtraction. The more complicated analog machines are even more restricted; most of them are designed to deal with specialized calculations such as arise in science and engineering — for example, harmonic analysis or the solution of certain types of differential equations. Digital machines, on the other hand, since they operate directly on numbers in the same way as does a human being when he calculates with pencil and paper, can be used for any kind of computation which can be broken down into arithmetical or logical steps.

The first mechanical calculating machine was completed in 1642 *by the French philosopher and mathematician* ***Blaise Pascal*** *(1623–1662). Some of his models are preserved in Paris. He conceived the design at the age of seventeen in order, so the story has it, to assist his father, who was a tax collector. Pascal's machine is digital, decimal and operated with a stylus*[1048]*. Numbers are carried to adjacent wheels by gears inside the machine. It was essentially an adding (and subtracting) device; multiplication had to be treated as repeated addition, the number to be added at each step being set separately.*

The next major advance was due to ***G.W. Leibniz*** *(1646–1716), the famous philosopher and mathematician and co-inventor (with Newton) of the calculus. He took the next logical step and mechanized multiplication. His first machine was constructed about 1671. The crucial feature of the whole design was the stepped wheel*[1049]*, an elegant device which is still used in the*

1048 One of the fundamental problems of the machine designer is that of arranging for *carry* from one digital position to the next, more significant position. This can be done in several ways, one of the simplest being the method adopted for recording domestic gas consumption – i.e.through the direct gearing of successive shafts with *gear ratios* of ten to one. Another possible scheme is the *stripped gear* method for carrying: a gear with 20 teeth all round its edge is engaged with another wheel of equal size that is stripped of all but two of its teeth. Each time the stripped wheel makes a complete revolution, the 20-teeth wheel turns through 1/10 of a revolution.

1049 The Leibniz *stepped wheel* consists of a cylindrical drum containing nine teeth of graduated lengths. A smaller pinion wheel engages a varying number of teeth, depending on its position. The two wheels are mounted on parallel axles and the pinion wheel can be displaced along its axle by means of a pusher. We may assign a length of 9 units to the longest tooth on the cylinder, 8 units to the next longest, and so on, in decreasing sequence. Thus one revolution of the cylinder will cause the pinion wheel to engage 0, 1, 2, etc., up to 9 teeth, depending on its axial position as determined by the pusher. We have in effect, therefore, a gearwheel containing a variable number of teeth.

The 'wheels of multiplication' consist, then, of a set of Leibniz stepped wheels *mounted on a common axle*; the digits of the multiplicand are set by moving the pushers associated with the appropriate pinion wheels. Each stepped wheel *form Leibniz left it, in some contemporary calculating machines.*

The crucial steps were taken, then, by Pascal and Leibniz in the 17^{th} *century. Since then the story has been one of continuous improvement in detailed design to give greater convenience of use, increased speed and improved reliability. During the* 18^{th} *century, many attempts were made to design a machine that could be mass-produced, but the degree of mechanical precission needed was beyond the capabilities of the production engineering techniques of that time. It was not until* 1810 *that the first successful commercial machine was made by* ***Charles Thomas*** *of Colmar, Alsace.*

Some 1500 *machines which embodied the Leibniz stepped wheel mechanism, are believed to have been made over a period of about* 60 *years. A variant of the Leibniz wheel was patented by* ***F.J. Baldwin*** *in* 1875, *and a number of machines using Baldwin's device were made by* ***W.T. Odhner*** *a little later. A vast number of Odhner type machines have been made in many countries since then.*

The first man to put forward detailed proposals for an automatic all-purpose calculating machine was ***Charles Babbage*** *(1792–1871). The various calculating devices discussed hitherto are non-automatic in the sense that they require the frequent attention of a human operator. An automatic calculating machine is able to carry out extensive calculations without human intervention.*

Babbage was born into an England where mathematics (during the century since Newton's death), had all but stagnated. The few who were literate could not, for the most part, figure sums accurately. Financial accounts were snarled, logarithm tables were full of errors, and insurance data were

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is connected to the corresponding 'wheel of addition'. It is clear that multiplication by an arbitrary multiplier can be achieved with this device by rotating the stepped wheels, for each digit of the multiplier, a number of times corresponding to that digit, axially offsetting any two consecutive stepped wheels such as to effective add one more tooth at each digital stage. This procedure is, in fact, adopted in most of the simpler calculating machines today.

Leibniz went a step further, in the direction of *fully automatic operation*, by adding a third set of nine wheels, 'the wheels of the multiplier'. The mechanism is so contrived that the result of multiplying the complete multiplicand by any digit of the multiplier can be transferred to the result register (i.e., the set of 'wheels of addition') by means of a single turn of the appropriate multiplier wheel. To multiply by more than one digit requires only a single shift-and-turn operation for each digit of the multiplier. The final answer is obtained as the sum of various partial products, just as in a pencil-and-paper calculation.

grotesquely jumbled. Babbage was incensed at this state of affairs and resolved to correct it — using computing machines instead of people.

He surmised that machines can be built which can add and subtract large numbers at high speeds, and he set himself to the task of building them.

Babbage completed his 'Difference Engine' in 1822. *It was specially designed to compute polynomials for the preparation of mathematical tables. It was essentially a collection of gears and levers, similar to mechanical desk calculators of the mid* 20^{th} *century, accurate to* 6 *places. With this success, Babbage tried to construct a better Difference Engine, accurate to* 20 *places. He even talked the British government into contributing* 17,000 *pounds (an enormous sum in those days) to the project probably, because of the military value of a device to prepare good ballistic tables. This project, however, quickly became mired in manufacturing problems.*

The metal-working industry in the early 1800*'s could make smooth-bore cannons and good plowshares, but it wasn't competent for the precision gears and linkages described in Babbage's marvelously detailed drawings. So the project died, but not before Babbage had conceived of a new dream — and not before he had trained a few machinists to make metal parts with more precision and detail than the world had ever seen (some of those Babbage-trained men later founded machine-tool companies that made important contribution to England's industrial capability).*

Charles Babbage called his new dream the 'Analytical Engine' (1833). It was designed to do all kinds of computations with the flexibility of a modern electronic computer. A vast assemblage of cogs, levers, and gears would be run by steam power (electricity was still a laboratory curiosity). The Analytical Engine memory was to consist of banks of wheels engraved with the ten digits. One thousand 50*-digit numbers would be available upon demand by the 'mill', where the arithmetic was done.*

Answers were to be automatically printed out just as computers do today. And, even more prophetically the Analytical Engine was to control itself internally by punched cards. (The punched card idea came from the mechanized loom of the Frenchman **Joseph Marie Jacquard** *(1805), in which punched cards controlled the pattern-wearing apparatus.) It was all a beautiful idea, but it was born a hundred years too soon. Despite his ingenuity and the investment of his personal fortune, Babbage could not leapfrog the century of industrial development still ahead.*

A name associated with Babbage is that of Lady **Augusta Ada Lovelace**, *Lord Byron's daughter, who was the first computer programmer (1842–1843) and the chief chronicler of Babbage's exploits.*

Our story now moves to America, where the next major advance was made about twenty years after Babbage's death. In 1890, ***Herman Hollerith***, *a statistician on the staff of the U.S. Bureau of the Census, invented the electromechanical punched-card calculating machine*[1050]*. During the first half of the* 20^{th} *century, punched-card equipment has been extensively applied to the ever increasing mass of clerical work in commerce, industry, and administration — and to a lesser extent, to scientific and technical calculations.*

The next advance, this time in the field of analog computers, was made at M.I.T. by ***Vannevar Bush*** *and his associates. As in Babbage's time, much of the impetus for this project was military in origin. Bush had started work on analog computers in* 1925*. The idea was to simulate a physical quantity, like shell velocity, with an easily measurable analog, such as the angle through which a gear rotates, just as clock hands simulate time. By present standards, these early analog computers were slow, only about* 100 *times faster than a human operator using a desk calculator.*

By 1930*, Bush's computer, known as the differential analyzer*[1051] *was busy spinning out artillery firing tables. The analyzer was mainly used to obtain*

[1050] A typical punched-card installation consists of a number of self-contained machines, each of which can perform a single type of operation. Information to be processed is first converted into pattern of holes in standard cards. When a card is fed into any punched-card machine, the hole pattern is converted into a pattern of *timed electric currents*: The exact time at which the circuit corresponding to a particular column is completed determines the row, and hence the digital value, of a hole punched in that column. The machine then responds according to its design and to the way it has been set for the particular problem.

[1051] The principle of the *differential analyzer* was proposed by **William Thomson** (later **Lord Kelvin**) in a paper in the *Proceedings of the Royal Society* for 1876. It was stimulated by his brother's interest in the mechanical integrator. The proposal was unfortunately neglected.

The central portion of the differential analyzer is, in principle, merely an elaborate gearbox in which shafts and other items can be installed, to suit the problem to be solved. The longitudinal shaft can transmit rotations from one end of the machine to the other, or as far as necessary, and carry the gears and differential gears demanded by the problem. It may contain integrators, torque amplifiers, helical gears, motors, relays etc. The operation of differentiation is awkward to deal with in mechanical computers and is avoided whenever possible. Furthermore, the problems are set up on the machine by building up mathematical expressions term by term, which is hardly a process of analysis. On both counts, the name 'differential analyzer' is a misnomer.

solutions of ordinary differential equations. The need to develop a more versatile calculating machine, free from the limitations of the differential analyzer, drove **H.H. Aiken** *to realize Babbage's dream. Thus, in the late* 30*'s, while Bush worked on his analog computers at M.I.T., only a few miles away the young Harvard graduate student was planning his future Mark I automatic (non-electronic) digital computer. To speed his work up, he first invented (1937) a series of small, very specialized digital computers. He soon noticed that all his machines had common logical operations and many other similar features, such as memories and control units. In short, Aiken began plowing the same field Charles Babbage had tilled a century earlier. Aiken, however, was more fortunate in having electrical gadgets, like relays, to help him.*

Independently, **Konrad Zuse***, a German engineer working in Berlin, built an electromechanical digital computer, the Z3 (completed 1941). Unaware of the achievements of Aiken and Atanasoff in the USA, he created a fully automated, program-controlled, and freely programmable computer for binary floating-point calculations.*

The first electronic digital computer, the ABC, was invented and built by **John Vincent Atanasoff** *(1939) at Iowa State College. The term electronic computer implies that the storage and manipulation of numbers inside the machine, and also the control of the sequence of operations, were done by means of electronic circuits. Indeed, apart from the input and output mechanisms, the machine had no moving parts. The use of electronic techniques enabled the operating speed to be increased enormously.*

The second electronic digital computer, the ENIAC, was completed by 1946 *by* **J.P. Eckert** *and* **J.W. Mauchly** *at the Moore School of Electrical Engineering of the University of Pennsylvania. Although the ENIAC was a completely 'universal machine', it was primarily designed to meet a specialized military need; the calculation of trajectories of bombs and shells.*

With the success of the ENIAC the victory of the thermionic valve over the relay and the counter wheel became almost complete. Nearly all the automatic computers built since 1950 *have been electronic, although the valve has since been ousted in its turn by the more reliable and compact transistor.*

The year 1946 *may thus be taken as marking the end of the pioneer stage of automatic computer development.*

Computer theory was pioneered by **Emil L. Post** *(1920–1949).* **A.M. Turing** *(1935–1937),* **A. Church** *(1936–1956) and* **John von Neumann** *(1942) followed. Turing gave a theoretical description of a universal machine (Turing machine), and contributed to the construction of early computers and the development of early programming techniques. Von Neumann contributed to the design and construction of digital computers. The idea of instructions,*

Table 5.21: Timeline of the evolution of the computer 1943–2008

Year	Hardware	Software
1943	relay machines — 2000 relays	
1946	ENIAC operational — 18,000 electron tubes	stored program concept
1947	invention of the transistor	
1949	EDSAC, Cambridge UK — first stored-program computer	
1951	UNIVAC I, first commercial machine (sold to the US Bureau of Census)	the first assembler (UNIVAC I)
1953	invention of core memory	
1954		Fortran programming language (IBM)
1956	introduction of hard disc (IBM)	LISP (functional language)
1959	IBM 7090 — first commercial transistor machine (20,000 transistors)	COBOL
1960		Algol 60
1961	First integrated circuit — "the second industrial revolution"	
1964	CDC-66 — large-scale scientific computer	

Table 5.21: (Cont.)

YEAR	HARDWARE	SOFTWARE
1972		C: general-purpose programming
1976	Cray-1 supercomputer	
1976	Local Area Networks: Ethernet (Xerox)	
1977	first commercial microcomputer Apple II, using 8-bits microprocessors	
1980	Motorola 68000 processor: 68,000 transistors on a single chip	
1981	Introduction of the IBM PC; first commercial application of windows with mouse-based interaction	
1984	Apple Macintosh	
1990	Connection Machine Parallel computer with 64,000 basic processors	
1991	Cray Y-MP parallel supercomputer	
1992	DEC Alpha processor: 1.7 million transistors on a single chip	
1993	Alpha, Power PC, Pentium: 32-bits microprocessor	
2020	a chip will have 10^{11} components (comparable to number of neurons in the human brain)	

as distinct from data*, to be stored in the computer's memory was von Neumann's landmark idea. It became a reality when a computer using internally stored instructions was built at the Institute of Advanced Study at Princeton, N.J., in 1952.*

The evolution of computer technology since the ENIAC was dramatic: Before 1945, the word 'computer' had the meaning of 'a human performing computations', whether or not using mechanical equipment. The ENIAC was built to do the same kind of work, but faster and without human intervention. It was programmable, and it featured a very high logical complexity as compared to earlier information-processing machines, including those of Zuse and Aiken. Table 5.21 lists milestones in the history of the computer.

The computer revolutionized the modalities of scientific research: prior to 1945, physical scientists did not have computers at their disposal (even ***Jules Verne*** *did not contemplate the computer!). This compelled them to delve deeper into matters, and thus identify underlying operative* principles*. Whereas, had they lived after 1945, they might be tempted to* simulate *rather than* cogitate*.*

The 10-Billionth Hexadecimal Digit[1052] *of* π *is* 9

There has been a drive to compute π to more and more decimal places. This drive has been going on for thousands of years. The direct utility to problems of measurement *or* engineering *of ultra-accuracy computations of π*

[1052] For further information, see:

- Beckmann, P., *A History of π (PI)*, St. Martin's Press, 1971, 200 pp.
- Posamentier, A.S. and I. Lehmann, *pi*, Prometheus Books, 2004, 324 pp.

is nil[1053]*. What therefore, is behind the drive? Many things; not the least of which is the fact that there are many people in this world who like to break records, and many who love to compute just for the sake of computing.*

There is, however, a more profound reason. Symbolically, π*'s irrationality represents an irrationality to the universe that we do not like. Pi represents an omniscience which we can never possess, but that we can approach ever closer. Calculating* π *is a quest parallel to trying to fully understand our universe. It is for this reason that we wish to calculate* π *to millions of places and beyond.*

[1053] The ten digits $(3.141\ 592\ 654)$ that are built into most scientific calculators are sufficient for nearly any real-world calculations: one can calculate the circumference of the earth's orbit around the sun, and be off by less than 100 meters! Even extremely precise scientific work never requires more than 20 decimal places. Clearly, hundreds of thousands of digits of π have no practical value.

Some argue that by calculating many digits of π mathematicians can empirically verify theoretical ideas about π.

One theory that mathematicians were trying to prove empirically is that the statistical distribution of the digits of π is uniform. That is, the frequency of each digit $(0, 1, ..., 9)$ approaches $1/10$ as the number of sampled digits approaches infinity.

As early as 1960 this was shown to be the case for the first $16,000$ digits of π, up to expected statistical uncertainties. Since then the growing number of known digits of π have passed this test every time it has been run.

Five thousand digits of pi.

3.

1415926535	8979323846	2643383279	5028841971	6939937510	5820974944	5923078164	0628620899	8628034825	3421170679
8214808651	3282306647	0938446095	5058223172	5359408128	4811174502	8410270193	8521105559	6446229489	5493038196
4428810975	6659334461	2847564823	3786783165	2712019091	4564856692	3460348610	4543266482	1339360726	0249141273
7245870066	0631558817	4881520920	9628292540	9171536436	7892590360	0113305305	4882046652	1384146951	9415116094
3305727036	5759591953	0921861173	8193261179	3105118548	0744623799	6274956735	1885752724	8912279381	8301194912
9833673362	4406566430	8602139494	6395224737	1907021798	6094370277	0539217176	2931767523	8467481846	7669405132
0005681271	4526356082	7785771342	7577896091	7363717872	1468440901	2249534301	4654958537	1050792279	6892589235
4201995611	2129021960	8640344181	5981362977	4771309960	5187072113	4999999837	2978049951	0597317328	1609631859
5024459455	3469083026	4252230825	3344685035	2619311881	7101000313	7838752886	5875332083	8142061717	7669147303
5982534904	2875546873	1159562863	8823537875	9375195778	1857780532	1712268066	1300192787	6611195909	2164201989
3809525720	1065485863	2788659361	5338182796	8230301952	0353018529	6899577362	2599413891	2497217752	8347913151
5574857242	4541506959	5082953311	6861727855	8890750983	8175463746	4939319255	0604009277	0167113900	9848824012
8583616035	6370766010	4710181942	9555961989	4676783744	9448255379	7747268471	0404753464	6208046684	2590694912
9331367702	8989152104	7521620569	6602405803	8150193511	2533824300	3558764024	7496473263	9141992726	0426992279
6782354781	6360093417	2164121992	4586315030	2861829745	5570674983	8505494588	5869269956	9092721079	7509302955
3211653449	8720275596	0236480665	4991198818	3479775356	6369807426	5425278625	5181841757	4672890977	7727938000
8164706001	6145249192	1732172147	7235014144	1973568548	1613611573	5255213347	5741849468	4385233239	0739414333
4547762416	8625189835	6948556209	9219222184	2725502542	5688767179	0494601653	4668049886	2723279178	6085784383
8279679766	8145410095	3883786360	9506800642	2512520511	7392984896	0841284886	2694560424	1965285022	2106611863
0674427862	2039194945	0471237137	8696095636	4371917287	4677646575	7396241389	0865832645	9958133904	7802759009
9465764078	9512694683	9835259570	9825822620	5224894077	2671947826	8482601476	9909026401	3639443745	5305068203
4962524517	4939965143	1429809190	6592509372	2169646151	5709858387	4105978859	5977297549	8930161753	9284681382
6868386894	2774155991	8559252459	5395943104	9972524680	8459872736	4469584865	3836736222	6260991246	0805124388
4390451244	1365497627	8079771569	1435997700	1296160894	4169486855	5848406353	4220722258	2848864815	8456028506
0168427394	5226746767	8895252138	5225499546	6672782398	6456596116	3548862305	7745649803	5593634568	1743241125
1507606947	9451096596	0940252288	7971089314	5669136867	2287489405	6010150330	8617928680	9208747609	1782493858
9009714909	6759852613	6554978189	3129784821	6829989487	2265880485	7564014270	4775551323	7964145152	3746234364
5428584447	9526586782	1051141354	7357395231	1342716610	2135969536	2314429524	8493718711	0145765403	5902799344
0374200731	0578539062	1983874478	0847848968	3321445713	8687519435	0643021845	3191048481	0053706146	8067491927
8191197939	9520614196	6342875444	0643745123	7181921799	9839101591	9561814675	1426912397	4894090718	6494231961
5679452080	9514655022	5231603881	9301420937	6213785595	6638937787	0830390697	9207734672	2182562599	6615014215
0306803844	7734549202	6054146659	2520149744	2850732518	6660021324	3408819071	0486331734	6496514539	0579626856
1005508106	6587969981	6357473638	4052571459	1028970641	4011097120	6280439039	7595156771	5770042033	7869936007
2305587631	7635942187	3125147120	5329281918	2618612586	7321579198	4148488291	6447060957	5270695722	0917567116
7229109816	9091528017	3506712748	5832228718	3520935396	5725121083	5791513698	8209144421	0067510334	6711031412
6711136990	8658516398	3150197016	5151168517	1437657618	3515565088	4909989859	9823873455	2833163550	7647918535
8932261854	8963213293	3089857064	2046752590	7091548141	6549859461	6371802709	8199430992	4488957571	2828905923
2332609729	9712084433	5732654893	8239119325	9746366730	5836041428	1388303203	8249037589	8524374417	0291327656
1809377344	4030707469	2112019130	2033038019	7621101100	4492932151	6084244485	9637669838	9522868478	3123552658
2131449576	8572624334	4189303968	6426243410	7732269780	2807318915	4411010446	8232527162	0105265227	2111660396
6655730925	4711055785	3763466820	6531098965	2691862056	4769312570	5863566201	8558100729	3606598764	8611791045
3348850346	1136576867	5324944166	8039626579	7877185560	8455296541	2665408530	6143444318	5867697514	5661406800
7002378776	5913440171	2749470420	5622305389	9456131407	1127000407	8547332699	3908145466	4645880797	2708266830
6343285878	5698305235	8089330657	5740679545	7163775254	2021149557	6158140025	0126228594	1302164715	5097925923
0990796547	3761255176	5675135751	7829666454	7791745011	2996148903	0463994713	2962107340	4375189573	5961458901
9389713111	7904297828	5647503203	1986915140	2870808599	0480109412	1472213179	4764777262	2414254854	5403321571
8530614228	8137585043	0633217518	2979866223	7172159160	7716692547	4873898665	4949450114	6540628433	6639379003
9769265672	1463853067	3609657120	9180763832	7166416274	8888007869	2560290228	4721040317	2118608204	1900042296
6171196377	9213375751	1495950156	6049631862	9472654736	4252308177	0367515906	7350235072	8354056704	0386743513
6222247715	8915049530	9844489333	0963408780	7693259939	7805419341	4473774418	4263129860	8099888687	4132604721

Table 5.22: Milestones in the π-Race[1054]

Act I

ca 2000 BCE	Babylonian used	$\pi = 3\frac{1}{8} = 3.1250$
	Egyptians used	$\pi = \left(\frac{16}{9}\right)^2 = 3.1605$
ca 1150 BCE	Chinese used	$\pi = 3$
ca 950 BCE	Hebrews used	$\pi = 3(I\ Kings;\ \mathbf{1},\ 23)$
ca 250 BCE	Archimedes established	$3\frac{10}{71} < \pi < 3\frac{1}{7}$
	and used	$\pi = 3.14163$
ca 450 CE	Tsu Chung-Chi established	$3.141\,592\,6 < \pi < 3.141\,592\,7$
ca 150 *CE*	Ptolemy used	$\pi = \frac{377}{120} = 3.14166\ldots$
ca 1420	Al-Kashi of Samarkand calculated π to 14 decimals	

Act II

1593	Adriaen van Roomen	15	correct	decimals
1596	Ludolph van Ceulen	32	"	"
1621	Snell	35	"	"
1705	Sharp	72	"	"
1706	Machin	100	"	"
1719	de Lagny	127	"	"
1794	Vega	137	"	"
1844	Strassnitzky and Dase	201	"	"
1853	Rutherford	441	"	"
1855	Richter	500	"	"
1873	Shanks	527	"	"
1947	Ferguson (desk calculator)	808	"	"

Act III

1949	ENIAC computer	2036	"	"
1955	NORC computer	3089	"	"
1959	Genuys and Felton	10,000	"	"
1961	Shanks and Wrench	100,000	"	"
1976	Gilloud and Bouyer	1,000,000	"	"
1982	Tamura and Kanada (2^h, 53^m)	4,194,293	"	"
1991	David and Gregory Chudnovsky (U.S.A.)	1,000,002,260	"	"
1995	Bailey, the Borweins and Plouffe (Canada)	10,000,000,000	"	"

But there have been more theoretical reasons for computing high-accuracy approximations to π. Prior to 1766, when Lambert established that π is an irrational number, a reason for computing π to many figures was the hope that such a calculation might show periodic patterns of its decimal digits and hence its fractional form would stand revealed.

In recent years, many mathematicians have interested themselves in the distribution of the decimal digits of π. Statistical analysis of the many digits of π produced by computers tends to confirm the conjecture that the digits are random. Table 5.22 records some milestone in the π-race throughout the past 4000 years.

Traditionally, computations of π during 1706–1982 have employed trigonometric formulas such as:

$$\pi = 16\tan^{-1}\left(\tfrac{1}{5}\right) - 4\tan^{-1}\left(\tfrac{1}{70}\right) + 4\tan^{-1}\left(\tfrac{1}{99}\right),$$

$$\pi = 24\tan^{-1}\left(\tfrac{1}{8}\right) + 8\tan^{-1}\left(\tfrac{1}{57}\right) + 4\tan^{-1}\left(\tfrac{1}{239}\right),$$

$$\pi = 48\tan^{-1}\left(\tfrac{1}{18}\right) + 32\tan^{-1}\left(\tfrac{1}{57}\right) - 20\tan^{-1}\left(\tfrac{1}{239}\right),$$

$$\pi = 32\tan^{-1}\left(\tfrac{1}{10}\right) - 4\tan^{-1}\left(\tfrac{1}{239}\right) - 16\tan^{-1}\left(\tfrac{1}{515}\right),$$

$$\pi = 12\tan^{-1}\left(\tfrac{1}{4}\right) + 4\tan^{-1}\left(\tfrac{1}{20}\right) + 4\tan^{-1}\left(\tfrac{1}{1985}\right),$$

with

$$\tan^{-1} x = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \frac{1}{9}x^9 - \cdots$$

With such formulas, one million decimals were calculated by 1976 with the aid of the CDC–7600 electronic computer.

*The tradition of 'arctangent formulas' was broken with the discovery of a new method (1976) of computing π using an iterative algorithm derived from the works of **Legendre** and **Gauss** in the early 19^{th} century. This algorithm, based upon the arithmetic-geometric mean, and related to the theory of elliptic integrals, is extraordinarily rapidly convergent. Here, the n^{th} approximant to π is*

$$\pi_n = \frac{4a_{n+1}^2}{1 - \sum_{j=1}^{n} 2^{j+1}(a_j^2 - b_j^2)}$$

[1054] For further information, see:

- Blatner, D., The Joy of π, Walker Publishing Company, 1997.

with

$$a_n = \frac{1}{2}(a_{n-1} + b_{n-1}), \qquad b_n = \sqrt{a_{n-1}b_{n-1}}$$
$$a_0 = 1, \qquad b_0 = \frac{1}{\sqrt{2}}.$$

Then

$$\pi = \lim_{n\to\infty} \pi_n$$

and

$$|\pi - \pi_n| < \left[\frac{\pi^2 2^{n+4}}{(agm)^2}\right] e^{-(\pi 2^{n+1})}, \qquad agm = \lim_{n\to\infty} a_n.$$

The number of correct decimals is essentially doubled at each iteration. Here, agm = arithmetico-geometrico mean (Gauss).

With such iterative methods, coupled to the rapid improvement of the speed and memory capacity of electronic computers, the number of calculated digits of π *passed the billion mark in* 1991. *If the numbers were typed, they would stretch more than 5000 kilometers. Had we stored these digits in books, each containing* 400 *pages, with* 5000 *digits per page, this would have formed a library of* 500 *such books.*

Quite recently, a 4000-year quest changed direction: a totally new formula for π *was discovered*[1055]*, remarkable in its simplicity and deadly in its precision:*

$$\pi = \sum_{k=0}^{\infty}\left(\frac{1}{16}\right)^k \left(\frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6}\right).$$

One of the charms of mathematics is that it is possible to make elementary discoveries about objects that have been studied for millennia.

Up to now, it was generally believed that to compute the n^{th} *digit of a transcendental number like* π *was as difficult as calculating the first* n *digits. This is not true. The new algorithm can easily be implemented, does not need multiple precision arithmetic, requires virtually no memory, and features run-times that scale linearly with the order of the digit desired. This makes it feasible to compute, for example, the* 10*-billionth digit of* π *on a modest work station in a few days of run-time.*

To prove the above identity, one first shows that

$$\sum_{k=0}^{\infty}\left(\frac{1}{b^{nk}}\sum_{i=1}^{n}\frac{a_i}{nk+i}\right) = \sum_{i=1}^{n} a_i b^n \int_0^1 \frac{x^i}{b^n - x^n}dx$$

[1055] D. Bailey, P. Borwein and S. Plouffe, "*On the rapid computations of various polylogarithmic constants*", *Mathematics of Computation*, 1996.

by expanding the right-side using the geometric series formula. Then, one evaluates the integral on the right side using the complex roots of $b^n - x^n$, *which come in conjugate pairs.*

In 1976, ***Kenneth Appel*** *(U.S.A.) and* ***Wolfgang Haken***[1056] *(U.S.A.), after years of intense work and* 1200 *hours of computer time, were finally able to announce that they had proven the* 4*-color conjecture. Large and crucial parts of their argument were carried out by a computer, using ideas which had themselves been formulated as a result of computer-based evidence. So great was the amount of computing required that it was not feasible for a human mathematician to check every step. This means that the whole concept of a mathematical proof had suddenly changed.*

Something that had been threatening to occur ever since electronic computers were first developed in the early 1950*s had finally happened: the computer had taken over from the human mathematician part of the construction of a real mathematical proof. Until then, a* proof *had been a logically sound piece of reasoning by which one mathematician could convince another of the truth of some assertion. By reading a proof, a mathematician could become convinced of the truth of the statement in question, and also come to understand the reasons for its truth.*

The use of the computer in this case is in principle quite different from its uses in applied mathematics and in number theory.

In applied mathematics, the computer serves to calculate an approximate answer, when theory is unable to give us an exact answer. We may try to use our theory to prove that the computed answer is in some sense close to the exact answer. But in no way does the theory depend on the computer for its conclusions; rather, the two methods, theoretical and mechanical, are like two independent *views of the same object.*

In the study of distributions of primes or similar number-theoretical problems, the computer serves to generate data. *But by studying these data, the mathematician may be able to form a conjecture, such as the prime number theorem. The rigorous mathematics of the proof remains uncontaminated by the machine. The latter helps us to decide what to believe, and even how strongly to believe it, but it still does not affect what is proved. In the Haken-Appel* 4*-color theorem, the situation is totally different: the computer*

[1056] To dig deeper, see:

- Wilson, R., *Four Colors Suffice*, Princeton University Press, 2002, 262 pp.

became absolutely essential and in order to accept the proof one has to believe that the computer program used, does what the authors claim of it.

From the philosophical point of view, the use of a computer as an essential part of the proof involves weakening of the standards of mathematical proof. It introduces grounds for skepticism and involves a certain act of faith; since the reader's belief in the proof of the 4-color theorem depends, not only on his confidence in his own ability to understand and verify mathematical reasoning, but also his belief that computers work and do what they are supposed to do. This is a belief of a totally different order.

A mathematician may view the matter in a different light: to him, the fallibility of reason is such a familiar fact of life that he might welcome the computer as a more reliable calculator than he himself can hope to be.

Magnetic Resonance (1946)

Introduction

Certain dynamical magnetic effects are associated with the quantum mechanical spin angular momentum of nuclei and of electrons. By magnetic resonance one usually means resonant absorption and emission of electromagnetic radiation by electrons or atomic nuclei in the presence of a certain magnetic field configuration. The principles of magnetic resonance are applied in the laboratory to analyze the atomic and nuclear properties of matter.

The principal relevant phenomena are often identified by their acronyms, such as: NMR (nuclear magnetic resonance); NQR (nuclear quadrupole resonance); EPR or ESR (electron paramagnetic, or spin resonance); FMR (ferromagnetic resonance); SWR (spin wave resonance); AFMR (antiferromagnetic resonance); CESR (conduction electron spin resonance).

The information that can be obtained about solids by resonance studies may be categorized thus:

- *Electronic structure of single defects in crystals, as revealed by the fine structure of absorption spectra.*

- *Motions of a spin or of its surroundings, as revealed by changes in the spectral line width.*
- *Internal magnetic fields sampled by the spin, as revealed by the position of the resonance line.*
- *Collective spin excitations.*

Nuclear magnetic resonance is used to measure nuclear magnetic dipole moments, which characterize magnetic behavior of specific nuclei. Because these values are significantly modified by the immediate chemical environment, however, nuclear magnetic resonance measurements provide information about the molecular structure of various solids and liquids.

NMR techniques have had a great impact in organic chemistry and biochemistry, where they provide a powerful tool for the identification and structure determination of complex molecules. A very important medical application is NMR tomography, which utilizes spatially varying magnetic fields and RF electromagnetic pulses to enable the resolution in 3D of abnormal growths, configurations, and reactions in the whole body.

The fundamentals of physics required to understand the principles of NMR will next be presented. A rigorous derivation of the underlying theory requires the use of quantum mechanics, but acceptable models of the process can be built using classical electrodynamics.

NMR is based on the measurement of radio-frequency electromagnetic waves as a nucleus returns to its equilibrium spin-orientation state. Any nucleus with an odd number of particles (protons and neutrons) has a magnetic moment, and, when the atom is placed in a strong magnetic field, the magnetic moment of the nucleus tends to line up with the field.

If the atom is excited by an external magnetic field, it emits a radio-frequency signal as the nucleus returns to its equilibrium state. Since the frequency of the signal is dependent not only on the type of atom but also the magnetic field present, the position and type of each nucleus can be detected by combining a spatially inhomogeneous magnetic field with appropriate signal processing.

A more detailed elaboration of this basic idea runs as follows: In quantum mechanics the magnitude of the orbital angular momentum $\boldsymbol{L}$ *has the allowed values* $L = \sqrt{\ell(\ell+1)}\,\hbar$, *where the integer* ℓ *is the orbital angular momentum quantum number*[1057]*. Moreover, the* z *component of* $\boldsymbol{L}$ *along any fixed axis,*

[1057] The relation $|\boldsymbol{L}| = L = \sqrt{\ell(\ell+1)}\,\hbar$ agrees with experimental results, whereas the semiclassical Bohr assumption $L = n\hbar$ with integer n, does not.

namely L_z, is $L_z = m_\ell \hbar$, where m_ℓ (known as the orbital magnetic quantum number) can have values from $-\ell$ to $+\ell$ in steps of 1.

Consider a particle (say an electron) of charge $(-e)$ and mass m in a closed orbit due to a central force field. In classical physics, its angular momentum is given by $\boldsymbol{L} = 2m\frac{\boldsymbol{A}}{T}$, where $\boldsymbol{A}$ is the vectorial area swept during one orbital period of T sec. On the other hand, its average magnetic dipole moment is $\boldsymbol{\mu} = i\boldsymbol{A}$, and $i = -\frac{e}{T}$ is the equivalent orbital electric current. The elimination of $\boldsymbol{A}/T$ yields $\boldsymbol{\mu} = -\frac{e}{2m}\boldsymbol{L}$. The vectors $\boldsymbol{\mu}$ and $\boldsymbol{L}$ are in opposite directions because the electron has a negative charge.

A quantum-mechanics derivation gives the corresponding result

$$\mu_\ell = |\boldsymbol{\mu}_\ell| = \frac{e\hbar}{2m}\sqrt{\ell(\ell+1)},$$

showing that the orbital magnetic moment ($\boldsymbol{\mu}_\ell$) of the electron is also quantized. Taking the z-component of the corresponding vector equation, we find $L_z = m_\ell \hbar$, $\mu_{\ell z} = -\frac{e\hbar}{2m} m_\ell$. The constant $\frac{e\hbar}{2m_e}$ is called the Bohr magneton[1058]*, μ_B. Clearly, $\mu_\ell = \mu_B \sqrt{\ell(\ell+1)}$ and $\mu_{\ell z} = -\mu_B m_\ell$.*

From classical physics, we know that applying a magnetic field $\boldsymbol{B}$ to a magnetic dipole causes a torque on the dipole, $\boldsymbol{T} = \boldsymbol{\mu} \times \boldsymbol{B}$. Taking the z-axis to coincide with the direction of $\boldsymbol{B}$, and recalling that $\boldsymbol{T} = \frac{d\boldsymbol{L}}{dt}$ and that $\boldsymbol{\mu}$ is proportional to $\boldsymbol{L}$, the magnetic field is seen to cause the magnetic dipole to precess about the z-axis. For an electron, both $\boldsymbol{\mu}$ and $\boldsymbol{L}$ (anti–parallel vectors) precess about $\boldsymbol{B}$. In quantum physics, this precession results in our inability to determine exactly either $\mu_{\ell x}$ or $\mu_{\ell y}$, for a state of the electron for which we have measured $\mu_{\ell z}$. However, $(\mu_{\ell x}^2 + \mu_{\ell y}^2) = \mu_l^2 - \mu_{lz}^2$ can be determined.

The energy of interaction of the magnetic dipole and the magnetic field is

$$U = -\boldsymbol{\mu} \cdot \boldsymbol{B} = -\mu B \cos\theta = -\mu_z B = +\mu_B m_\ell B,$$

and is the change of the energy of the system caused by the interaction. Since m_ℓ can have $(2\ell+1)$ distinct values, the application of a magnetic field to an orbital state of a given ℓ splits the state into $(2\ell+1)$ different energy levels; the maximum value of m_ℓ (e.g., +2 for the $\ell = 2$, d state) occurs when $\boldsymbol{L}$ is parallel to $\boldsymbol{B}$ ($\boldsymbol{\mu}_\ell$ and $\boldsymbol{B}$ antiparallel), and this corresponds to the highest-energy split level. In the lowest energy state $(m_\ell = -2,\ \ell = 2)$ $\boldsymbol{\mu}_\ell$ and $\boldsymbol{B}$ are parallel.

[1058] Since $\hbar = \frac{h}{2\pi} = 1.0546 \times 10^{-34}$ J·sec $= 6.582 \times 10^{-16}$ eV·sec one finds that $\mu_B = 9.274 \times 10^{-24}$ J/T $= 9.274 \times 10^{-21}$ erg/gauss where J = Joule; 1T = 1 Tesla = $1Wb/m^2 = 10^4$ gauss; eV = electron Volt.

In the state $m_\ell = 0$, $\boldsymbol{\mu}_\ell$ *and* $\boldsymbol{B}$ *are perpendicular, corresponding to null interaction energy* $U = 0$. *Since any system tends, if possible, to inhabit its lowest energy state,* $\boldsymbol{\mu}_\ell$ *tends to line up with* $\boldsymbol{B}$ *much as a compass needle tends to line up with the earth's magnetic field.*

In quantum physics, we can calculate the probability that a transition from one state to another will occur. Quantum mechanics shows that some types of transitions are very improbable under normal conditions. These are called, with some exaggeration, forbidden transitions. *The more favorable transitions are called* allowed transitions, *and occur when certain quantum–number* selection rules *hold. These rules are obtained from probability calculations, which involve integrals that include both the original state and the final state wave functions.*

For example, these integrals will be small or zero under normal conditions unless $\Delta\ell = \pm 1$ *and* $\Delta m_\ell = 0$ *or* $\Delta m_\ell = \pm 1$. *Thus, when a magnetic field* B *is applied to a* d *state* $(\ell = 2)$, *three possible energies are allowed for the photon emitted during the transition:* $\Delta E = \Delta E_0$ $(\Delta m_\ell = 0)$, *the emitted photon energy in the absence of magnetic fields;* $\Delta E = \Delta E_0 - \mu_B B$ *(when* $\Delta m_\ell = +1$*); and* $\Delta E = \Delta E_0 + \mu_B B$ *(when* $\Delta m_\ell = -1$*). Even in strong magnetic fields one usually has* $\mu_B B \ll \Delta E_0$.

Nevertheless, the frequency $\nu_0 = \Delta E_0/h$ *splits into three lines. For* $\Delta m_\ell = 0, \pm 1$, *respectively, the frequencies of the three lines will be* ν_0, $\nu_0 \mp \frac{\mu_B}{h} B$ *(normal Zeeman effect). The corresponding change in wavelength is*

$$\Delta\lambda \approx \pm\lambda_0^2 \frac{eB}{4\pi mc}.$$

Every electron has, in addition to its orbital angular momentum, an intrinsic spin angular momentum, $\boldsymbol{S}$, *which has no classical analog. This was first suggested by* ***Goudsmit*** *and* ***Uhlenbeck*** *(1925) to explain experimental results. Spin involves two further quantum numbers,* s, *and* m_s, *of which* s *is fixed at 1/2 in the case of electrons, and* m_s *takes the values* $-\frac{1}{2}$ *or* $\frac{1}{2}$. *The number* m_s *is called the* spin angular momentum magnetic quantum number.

For electrons $s = \frac{1}{2}$, $m_s = \pm\frac{1}{2}$ *and therefore*

$$S = \sqrt{s(s+1)}\,\hbar = \frac{\sqrt{3}}{2}\,\hbar,$$

$$S_z = m_s \hbar = \pm\frac{1}{2}\hbar.$$

The electron state with spin parallel to the z-axis is often called spin up $\left(m_s = +\frac{1}{2}\right)$, while the state with spin antiparallel to the z-axis is called spin down $\left(m_s = -\frac{1}{2}\right)$[1059].

In analogy to the relation $\boldsymbol{\mu}_\ell = -\left(\frac{e}{2m}\right)\boldsymbol{L}$ for the orbital vectors, it was found experimentally and from relativistic quantum theory, that the spin magnetic dipole moment, $\boldsymbol{\mu}_s$ is related to its spin angular momentum by $\boldsymbol{\mu}_s = -g_e\frac{e}{2m}\boldsymbol{S}$, where, again, $\boldsymbol{\mu}_s$ and $\boldsymbol{S}$ are antiparallel because of the negative charge of the electron. The factor g_e is called the gyromagnetic ratio of the electron.

The experimental value for g_s is 2.0023193044, but for most practical purposes we can take $g_s = 2$. Using $\left(\frac{\sqrt{3}}{2}\right)\hbar$ as the magnitude of $\boldsymbol{S}$, we then have $\mu_s = \mu_B\sqrt{3}$. In a similar manner $\mu_{sz} = \mp(1.00116)\mu_B \approx \mp\mu_B$. Thus, the z component of the spin magnetic dipole moment is only about a tenth of a percent greater than the Bohr magneton (in absolute value).

The Schrödinger wave equation is consistent with, but does not predict the electron spin in its solutions, while Dirac's relativistic wave equation predicts for g_s exactly 2, not 2.00232… [The part of quantum physics that predicts the minute correction $(g_s - 2)$ is quantum electrodynamics (QED).]

When we apply an external magnetic field, there will be a torque on the spin magnetic dipole moment of the electron. This torque will cause precession of $\boldsymbol{S}$ and $\boldsymbol{\mu}_s$ about the z-axis (the direction of the magnetic field $\boldsymbol{B}$). The corresponding change of energy will be

$$\Delta U_s = -\mu_{sz}B \approx +2m_s\mu_B B.$$

The fundamental mechanical properties of a nucleus can be listed as: mass, size, binding energy, and spin. Its main electromagnetic properties are: charge, magnetic dipole moment, and electrical quadrupole moment.

***Pauli** (1924) suggested that the hyperfine structure in atomic spectra might be explained by a small magnetic moment of the nucleus. The interaction of this magnetic dipole with the motion of the electrons would produce a hyperfine multiplet of energy levels in a similar way as a (fine structure) multiplet is produced by interaction of the intrinsic (spin) magnetic moment of the electron with the magnetic field due to its orbital motion. The introduction of the electron spin concept (1925) made it possible to explain many*

[1059] One of the bizarre aspects of quantum mechanics is that S_z and L_z have *the same* discrete set of allowed values no matter *which* spatial direction is selected to be the z axis.

hitherto mysterious details of the spectra of hydrogen and other atoms, ions and molecules.

It thus appeared appropriate to connect the magnetic moment of the nuclei too, with rotating charges and to attribute to the nucleus a mechanical spin, known as nuclear spin*. It is designated by* $\boldsymbol{I}$*. Both* protons *and* neutrons*, like electrons, have intrinsic spin* $\frac{1}{2}$*. In addition, protons and neutrons possess orbital angular momentum associated with their motion in the nucleus. The resultant nuclear intrinsic angular momentum (spin) is obtained by combining, in a proper way, the orbital angular momenta* and *the spins of the nucleons (protons and neutrons) composing the nucleus.*

The nuclear spin is designated by the quantum number I *such that the magnitude of the nuclear spin is* $\sqrt{I(I+1)}\,\hbar$*. The component of the nuclear spin in a given direction is* $I_z = m_I\hbar$*, where* $m_I = \pm I,\ \pm(I-1), \ldots$*. There are thus* $2I+1$ *possible orientations of the nuclear spin*[1060]*. The values of* I *are integers (if the mass number* A *is even) or half-integers (if* A *is odd).*

It has been noted that practically all even-even *nuclei (i.e., nuclei that have an even number of neutrons* and *of protons) have* $I = 0$*, which indicates that identical nucleons tend to pair their angular momenta in pairs of opposite directions. This is called the* pairing effect*.* Even-odd *nuclei (i.e., nuclei that have an odd number of either protons or neutrons) all have half-integral angular momenta, and it is reasonable to assume that the nuclear spin coincides with the angular momentum of the last or unpaired nucleon, a result which seems to hold in many cases.*

Odd-odd nuclei have two unpaired nucleons (one neutron and one proton) and the experimental results are a little more difficult to predict, but their angular momenta are integers, since there is an even total number of fermions.

Consider next the internal orbital *magnetic dipole moment of the nucleus: Obviously neutrons have no charge and do not have an orbital magnetic dipole moments. For the case of protons* $\boldsymbol{\mu}_\ell = \left(\frac{e}{2m_p}\right)\boldsymbol{L}$*, and the component of the magnetic moment along the* z*-axis is* $\mu_{\ell z} = \left(\frac{e}{2m_p}\right)L_z = \left(\frac{e\hbar}{2m_p}\right)m_\ell = \mu_N m_\ell$*. The constant* $\mu_N = \frac{e\hbar}{2m_p} = 5.0504 \times 10^{-27}\ JT^{-1} = 5.0504 \times 10^{-24}$ *erg/gauss is called a* nuclear magneton*.*

If a particle of charge q *and mass* m *has spin* $\boldsymbol{S}$*, it also has a* spin *magnetic moment* $\frac{q}{2m}g_s\boldsymbol{S}$*, where* g_s *is a constant characteristic of the particle,*

[1060] I_x and I_y are quantum-uncertain for a state in which I and I_z are known with certainty; thus, each "orientation" of I is actually a *cone* of possible classical orientations.

called its spin gyromagnetic ratio. The value for its proton is $g_{s,p} = +5.5855$, *indicating that* $\boldsymbol{\mu}_s$ *is parallel to* $\boldsymbol{S}$.

The magnetic moment of the proton is quite different from that expected by substitution of the protons mass in Dirac's formula for the magnetic moment of the electron. The difference is thus known as the anomalous magnetic moment.

It has been observed (1939) that the neutron, although it has no net electric charge, has a spin magnetic moment corresponding to $g_{s,n} = -3.8263$. *The negative sign*[1061] *indicates that* $\boldsymbol{\mu}_s$ *is antiparallel to* $\boldsymbol{S}$.

The resultant magnetic dipole moment of a nucleus can be written as $\boldsymbol{\mu} = g_I \left(\frac{e}{2m_p}\right) \boldsymbol{I}$, *where* g_I *is the nuclear gyromagnetic ratio. The component of the resultant magnetic dipole moment of a nucleus along a given direction may be expressed by* $\mu_z = g_I \left(\frac{e}{2m_p}\right) I_z$. *Since* $I_z = m_I \hbar$, *it follows that* $\mu_z = \mu_N g_I m_I$.

Nuclear magnetic moments are listed as multiples of μ_N, *and for* $m_I = I$ *(the maximum value of* m_I*),* $\frac{\mu_z}{\mu_N} = g_I I$.

Taking into account the selection rule: $\Delta m_I = \pm 1$ *or 0, the change in energy due to a transition between two nuclear energy levels belonging to the*

[1061] The *quark model* tells us that the neutron's magnetic moment is not zero, and also that the proton's gyromagnetic ratio is not the Dirac value of 2. Our understanding of QCD (Quantum Chromodynamics: the current quantum-field theory for strong interactions) is not yet good enough to compute these numbers. However, the naive quark model gives a ratio of $-\frac{3}{2}$ for the quotient proton magnetic moment/neutron magnetic moment against the observed value of -1.46.

A neutron is made of one 'up' quark, of charge $+\frac{2}{3}$ (in electronic units), and two 'down' quarks with charge $-\frac{1}{3}$ each. Since two of the three quarks must have their spins aligned one way, and the third has its spin aligned the other way, the procedure to compute the neutron's overall moment is simply to add the quark spins' z components weighted by the corresponding quark charges, and then compute an expectation value in the simplest three-quark wave function. We must take into account that quarks come in three distinct "colors" inside each nucleon, that the nucleon is "color neutral" (a result of QCD) *and* that quarks obey the *Pauli exclusion principle*.

The result is nonzero, because the moments of the positively-charged quarks do not cancel those of the negatively charged quarks. (The quarks themselves are assumed to have the Dirac values for their gyromagnetic ratios). A similar calculation yields the non-Dirac moment of the proton.

Even before the advent of the quark model, it was realized that nucleons were not point-like.

same Zeeman multiplet, is given by $\Delta U = \pm g_I \mu_N B_0$, *where* B_0 *is the applied magnetic field. A quantum energy* $h\nu_0$ *can therefore resonantly excite transitions between energy levels if it has the same energy as the level spacing:* $h\nu_0 = \Delta U \approx g_I \mu_N B_0$, *where* ν_0 *is the frequency of the electromagnetic radiation supplying or absorbing the quantum of energy.*

For a proton, with $g_I = 5.58$ *and an applied field of* $B_0 = 5000$ *gauss, the resonance frequency* ν_0 *turns out to be 21.3 MHz, which is in the radio frequency band. The corresponding photon energy is of the order* 10^{-7} *eV. Although this energy is too small to induce electronic, rotational or vibrational transitions in atoms or molecules, it is sufficient to affect the magnetic moment and spin of the nuclei of atoms, so that resonant absorption of radio-frequency radiation by the nucleus occurs when atoms are placed in a magnetic field and irradiated with properly tuned radio waves. A spectrum is produced by observing resonances for a compound as the magnetic field or radio frequency is scanned.*

Now, in order to excite the said resonant transitions, it is necessary to supply radiation in such a way that its magnetic vector is polarized in a plane perpendicular to the steady magnetic field B_0. *To understand this, we show that this requirement of circular polarization is just what one would expect by classical argument in the case of conventional optical atomic spectroscopy (Zeeman effect).*

If a magnetic dipole $\boldsymbol{\mu}$ *is placed in a magnetic field* $\boldsymbol{B}_0$, *the dipole precesses about the direction of the applied field, where the rate of precession is given by the Larmor angular frequency*[1062] $\omega_0 = \gamma B_0$ (γ = *magnetogyric ratio of the dipole* $= \frac{\mu}{I} = g_I \mu_N$).

Suppose now that an additional small oscillating magnetic field $\boldsymbol{B}_1$ *is applied at right angles to* $\boldsymbol{B}_0$. *The dipole will experience a torque* $\boldsymbol{T} = (\boldsymbol{\mu} \times \boldsymbol{B}_1)$ *tending to change the angle* θ *between* $\boldsymbol{\mu}$ *and* $\boldsymbol{B}_0$. *If the small field* $\boldsymbol{B}_1$ *is made to rotate about* $\boldsymbol{B}_0$ *in synchronism with the precession of the dipole, this torque will cause the angle* θ *to resonantly increase, leading to a reorientation of the vector* $\boldsymbol{\mu}$.

In quantum-mechanical language the effect will be described as an EM transition between two energy levels. If, however, $\boldsymbol{B}_1$ *and the dipole rotate in different directions, or if they do not rotate with the same frequency, the*

[1062] Note that the frequency of precession depends on the magnitude of the external field B_0 *and, through the magnetogyric ratio* γ, on the chemical binding of the atom, since γ can change slightly due to the induces magnetic field contributed by electrons within the chemical environment of the given nucleus. These small changes in γ are known as *chemical shifts* and are used in NMR spectroscopy to identify the compounds in a sample.

torque will soon get out of phase and after a short time interval change its sign, so that the average effect over many Larmor periods will be small.

We thus see that as with quantum theory, so also classically, a condition for optimal observation of the resonance is that the electromagnetic radiation be circularly polarized with the magnetic vector rotating in a plane perpendicular to the steady (DC) magnetic field[1063].

Experimental basis

C.J. Gorter *(1936) was first to point out how the phenomenon, described above, could be used to detect nuclear magnetism. The first successful experiment, however, was performed by* ***I. Rabi*** *(1939) using a molecular beam technique*[1064]*; a magnetic dipole experiences a force when placed in an inhomogeneous magnetic field. Atoms or molecules which possess a magnetic moment are therefore deflected on passing through such a field. The beam method was powerfully improved upon by Rabi and his colleagues by the*

1063 It is usually much simpler to provide a linearly oscillating field; it may be regarded as the superposition of two rotating fields in opposite senses, each with half the amplitude of the linear oscillator. Resonance will be obtained with the component which has the correct sense, the other component having a negligible effect.

Conventionally, the z-axis is chosen along the axis of the static magnetic field $\boldsymbol{B}_0$ used to align the magnetic moments $(I_z = I$ before the RF pulse is applied). Let the radio-frequency magnetic field pulse be applied for t_p seconds in the x direction: $\boldsymbol{B}_1 = [2B_1 \cos\omega_0 t]\,\boldsymbol{e}_x$, where $\omega_0 = \gamma B_0$ is the resonance frequency.

The degree of tipping (nutation) that occurs is $\theta = \gamma B_1 t_p$. Generally, B_1 and t_p can be varied so that the moment can be tipped to any desired angle. By tipping the moment 90° the maximum signal is obtained as the system returns to equilibrium $(\theta = 0)$, while a 180° flip will change the *sign* of the moment.

1064 This technique was used with great success in the experiments of **Stern** and **Gerlach** (1921, 1924) to prove experimentally that the measurable values of the component of an atomic magnetic moment do not form a continuous range.

Instead they form a discrete set corresponding to the spin quantization of a valence electron of the atom in the magnetic field. From the magnitude of the deflection of the beam these workers were able to evaluate the atomic magnetic moment.

introduction of the resonance method: *Molecules evaporated from a furnace pass through some diaphragms to define a beam.*

The beam is split in the inhomogeneous field of a first magnet, passes through a homogeneous field B_0, and is refocused onto a detector by another inhomogeneous field (of opposite gradient), which deflects it in the opposite direction. The refocusing condition is fulfilled only if no reorientation of the nuclear spin occurs in the zone of the homogeneous field.

But if in addition to the homogeneous field B_0 a radio-frequency magnetic field is applied, perpendicular to B_0, and either the radio frequency or B_0 is slowly scanned, the current reaching the detector will pass through a minimum when the above resonance condition is fulfilled: at the resonance frequency the beam is subjected to electromagnetic radiation of just the right amount as to induce transitions between their quantized energy levels by a process of absorption or stimulated emission of quanta of energy.

If the nuclear spin number is I, each energy level is split by the steady *magnetic field into $(2I+1)$ approximately equally spaced sublevels. If the maximum measurable components of the nuclear magnetic moment is μ, the separation between the lowest and the highest sublevels is $2\mu B_0$; roughly speaking, these two levels correspond respectively to alignments of the nuclear moments with and against the magnetic field. The separation between two successive sub-levels is therefore $\frac{2\mu B_0}{2I}$, with the corresponding frequency $\frac{\mu B_0}{Ih}$.*

At this frequency there is a sharp reduction in the number of molecules reaching the detector, since molecules that suffer a change of energy (and thus of magnetic moment) do not have the correct deflection in the second inhomogeneous magnetic field. The resonances are usually sharp, and enable the magnetic moment of nuclei to be obtained with an accuracy of a few parts in 10^4. Gyromagnetic ratios of many nuclei have been measured in this way.

A special and independent application to neutrons *was made by* ***Bloch*** *and* ***Alvarez*** *(1940).*

The resonant exchange of energy between the $(2I+1)$ energy levels of a nuclear magnetic moment in a magnetic field is not restricted to matter in the form of molecular beams, but should also occur for matter in its ordinary *solid, liquid or gaseous states. The first successful nuclear magnetic resonance experiments with bulk matter were carried out independently at the end of* 1945 *by* ***Purcell***, ***Torrey*** *and* ***Pound*** *(1946) and* ***Bloch***, ***Hansen*** *and* ***Packard*** *(1946).*

While in the beam method each particle can be considered as free, in the case of solids, liquids and gases the interaction between the nuclei and their surroundings *cannot be neglected. They are, in fact, essential. To see this we* *consider matter in its normal physical and chemical states, where the nuclei are present in their usual role as central particles in atomic systems.*

The material in which the nuclear magnets are embedded is generally referred to as the 'lattice', whether it be solid, liquid, or gas. For simplicity we will assume that the nuclear spin number is $I = \frac{1}{2}$ *and ignore the interaction between the nuclei, and so take the energy levels discussed above for an isolated nucleus as those for each nucleus in the assembly. At the same time* some *coupling between the nuclei has to be assumed so that the assembly may be considered to be in* thermal equilibrium *at temperature* T_s.

Since $I = \frac{1}{2}$, *each nucleus has two possible energy levels separated by a gap* $2\mu B_0$. *If we now apply EM radiation at the resonant frequency, transitions between the two levels take place. From the simple theory of the Einstein coefficients (1917) we know that the probability per unit time of transitions upwards by absorption, per given RF radiation flux, is equal to the probability per unit time of transitions downwards by* stimulated emission. *In comparison with these probabilities, the probability of transitions downwards by* spontaneous emission *is quite negligible. If the number of nuclei in each energy level were equal, the average rate of transitions up and down would therefore be equal, and there would be no net effect on the system.*

Actually, however, since the nuclear spins are in equilibrium at some temperature T_s, *the population of the lower level* (n_1) *exceeds that of the upper level* (n_2) *by the Boltzmann factor* $e^{(2\mu B_0/kT_s)}$, *where* k *is the Boltzmann constant. At room temperature* ($\approx 300\,^\circ K$), *for protons in a field of* 3000 *gauss, this factor has the value*

$$e^{(2\mu B_0/kT_s)} \simeq 1 + \frac{2\mu B_0}{kT_s} \approx 2.0 \times 10^{-6},$$

or $\frac{n_1 - n_2}{n_1} = 2.0 \times 10^{-6}$. *On account of this typically small, but finite,* excess of population in the lower energy state, there is a net absorption of energy from the *radio frequency field.*

Moreover, a difference in population of 2 *parts in a million is detectable, a result which reveals the high sensitivity of the NMR technique. It is more sensitive than* chemical *techniques, for example, in identifying magnetic impurities in a crystal. Principally, however, it enables us to use the nucleus as a probe to get information about solids, much as radioactive tracers are used in biological systems.*

The absorption of energy corresponds to the transfer of some of the excess population in the lower level to the upper level. If there were no interaction between the system of nuclear spins and the lattice, the fractional excess of population, $2\mu B_0/kT_s$, *would steadily dwindle. Consequently, the temperature* T_s *of the spin system would steadily rise.*

This, however, is avoided due to the weak, but finite, thermal interaction between the lattice and the spin system. Such interaction tends to bring both into thermal equilibrium at the same temperature. This common temperature is almost identical with the lattice temperature on account of the latter's much greater heat capacity (except at extremely low temperatures). Thus, while the radio frequency radiation is reducing the excess population in the lower energy state, the interactions with the lattice tend to restore the excess to its original value.

Relaxation

Clearly, the power absorption by the sample would stop as soon as the two magnetic sublevels are equally populated (corresponding to an infinite *nuclear-spin temperature!). If the nuclei were isolated from their surroundings, they would remain in such a state for a long time. However, the temperature of the surrounding (the* lattice*) is finite, and the nuclei tend try to reestablish thermal equilibrium by interaction with the lattice. This interaction will lower the* infinite *spin temperature and slightly raise the temperature of the lattice.*

The time-characteristic of this spin-lattice relaxation *can be measured in various ways, for instance by determining the time it takes the protons to return to thermal equilibrium after the external rf power has been turned off. The spin-lattice relaxation time depends, for instance, on the chemical binding, and relaxation techniques have therefore become indispensable analytical tools of physical chemistry. The semi-classical theory of relaxation theory will next be described.*

Consider an assembly of identical, weakly interacting nuclei of spin number I, in thermal equilibrium at a spin temperature T_s in a steady magnetic field $\boldsymbol{B}_0$. Nuclei having magnetic quantum number $m = m_I$ are found in the energy level $-mg_I\,\mu_N\,B_0$.

The population of the level is therefore weighted by the Boltzmann factor $e^{\frac{-mg_I\,\mu_N\,B_0}{kT_s}} \approx 1 + \frac{-mg_I\,\mu_N\,B_0}{kT_s}$. Hence the population, $N(m)$, of each level (per cm^3, say) is

$$N(m) \approx \frac{N}{2I+1}\left(1 + \frac{mg_I\,\mu_N\,B_0}{kT_s}\right),$$

with the proviso that $\sum_{m=-I}^{I} N(m) = N =$ *total populations in all* $(2\ell + 1)$ *levels.*

The total magnetic moment per cm^3*, namely the magnetization* M_z*, is therefore approximated by*

$$M_z = \sum_{-I}^{I} m g_I \mu_N N(m) = \frac{N(g_I \mu_N)^2 B_0\, I(I+1)}{3kT_s}.$$

The static susceptibility is therefore given by

$$\chi_0(T_s) = \frac{M_z}{B_0} = \frac{N\, g_I^2 \mu_N^2 I(I+1)}{3kT_s}.$$

If we define $M_z = N\langle\mu_z\rangle$, *where* $\langle\mu_z\rangle$ *is the average nuclear magnetic moment, the above calculations yield*

$$\langle\mu_z\rangle = \frac{g_I^2 I(I+1)}{3kT_s} B_0 \mu_N^2 .$$

Bloch equations

A remarkable result of quantum mechanics is that the expectation value of the magnetic moment of an otherwise free spin precessing in an external magnetic field obeys the classical *vector equation* $\frac{d}{dt}\langle\boldsymbol{\mu}\rangle = \gamma\,[\langle\boldsymbol{\mu}\rangle \times \boldsymbol{B}]$*, with* γ *the spins' magnetogyric ratio. Multiplying by* N *we obtain* $\frac{d\boldsymbol{M}}{dt} = \gamma(\boldsymbol{M} \times \boldsymbol{B})$*, valid for both steady and time-dependent magnetic fields. But this equation needs modification to include the changes in* $\boldsymbol{M}$ *which occur because of effects other than the magnetic field. Suppose that in the absence of the rotating (RF) magnetic field and with the spin system and the lattice in thermal equilibrium,* $\boldsymbol{M}$ *is aligned with* $\boldsymbol{B} = \boldsymbol{B}_0$ *along the* z *direction such that* $M_z = M_0 = \chi_0 B_0$.

If the spin system and lattice are not *in thermal equilibrium, then in the absence of a radio-frequency field,* M_z *approaches* M_0 *exponentially with some characteristic time* τ_1*. In this case the* z *component of the equation of motion is* $\frac{dM_z}{dt} = \frac{M_0 - M_z}{\tau_1}$*, where* τ_1 *is termed the* longitudinal relaxation time.

The transverse components M_x *and* M_y *represent the rotating components of the precessing magnetization vector* $\boldsymbol{M}$*.* Local irregularities *of the magnetic field cause the individual precessing nuclei to get out of phase with each other in a relaxation time,* τ_2*, of the order of the* spin-spin interaction time *. In absence of a radio-frequency magnetic field, any phase coherence of the nuclear spins' precession would be destroyed in a time of the order* τ_2*, thus bringing* M_x *and* M_y *to zero.*

It is then reasonable to assume that the approach to zero is exponential, with characteristic time τ_2*. In the absence of either static magnetic field or the RF field (i.e., if* $B_x = B_y = B_z = 0$*), we then have*

$$\frac{dM_x}{dt} = -\frac{M_x}{\tau_2}, \qquad \frac{dM_y}{dt} = -\frac{M_y}{\tau_2}.$$

τ_2 *is known as the* transversal relaxation *time. Assuming an applied field of the form* $B_x = B_1 \cos\omega t$, $B_y = -B_1 \sin\omega t$, $B_z = B_0$, *the semiclassical equations obeyed by the magnetization vector components (often referred to as the Bloch equations) are:*

$$\frac{dM_x}{dt} = \gamma[M_y B_0 + M_z B_1 \sin\omega t] - \frac{1}{\tau_2} M_x,$$

$$\frac{dM_y}{dt} = \gamma[-M_x B_0 + M_z B_1 \cos\omega t] - \frac{1}{\tau_2} M_y,$$

$$\frac{dM_z}{dt} = \gamma[-M_x B_1 \sin\omega t - M_y B_1 \cos\omega t] + \frac{1}{\tau_1}(M_0 - M_z).$$

These equations may be solved to give the power absorption per sample unit volume from the rotating magnetic field, $P(\omega) = \frac{\omega\gamma M_0 \tau_2}{1+(\omega_0-\omega)^2\tau_2^2} B_1^2$, *where as before* $\omega_0 = \gamma B_0$ *and we assume* $\gamma^2 B^2 \tau_1 \tau_2 \ll 1$. *The half-width of the resonance at half-maximum power is* $(\Delta\omega)_{1/2} = \frac{1}{\tau_2}$. *The Bloch equations are plausible, but not exact. They do not describe all spin phenomena, yet this semi-macroscopic and semiclassical theory is broadly consistent with quantum mechanics.*

Note that the transformation

$$M_x = u\cos\omega t + v\sin\omega t,$$
$$M_y = -u\sin\omega t + v\cos\omega t$$

reduces the above Bloch equations into a system of three linear ODE's with constant coefficients in the variables (u, v, M_z):

$$\frac{du}{d\tau} + \beta u + \delta v = 0,$$

$$\frac{dv}{d\tau} + \beta v - \delta u - M_z = 0,$$

$$\frac{dM_z}{d\tau} + \alpha M_z + v = \alpha M_0,$$

with the abbreviations

$$b = \gamma B_1, \qquad \delta = \frac{\omega - \omega_0}{b}, \qquad \tau = bt, \qquad \alpha = (b\tau_1)^{-1}, \qquad \beta = (b\tau_2)^{-1}.$$

The precession of an individual nuclear spin is modified by its interaction with the fluctuating magnetic field due to neighboring nuclei and the electrons in paramagnetic atoms. From the point of view of quantum mechanics, changes in spin state can be either by absorption *of a photon at approximately the Larmor frequency* ω_0 *or* emission *of such a photon, depending on whether the interaction manifests itself in increase or decrease of energy, respectively.*

Emission can be either spontaneous *or* stimulated *by the presence of other photons at the Larmor frequency. The relative probabilities can be calculated using quantum mechanics. (Stimulated emission or absorption is much more probable than is spontaneous emission in the case of NMR.) If the random magnetic field at the nucleus changes rapidly enough due to molecular motion, it will have Fourier components at the Larmor frequency which can induce transitions that cause* M_z *to change.*

To get an idea of the strength of the spin-spin interaction, consider the field at one proton in a water molecule *due to the other proton. The field due* *to a magnetic dipole μ along the z axis is given by*

$$B_r = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{2\mu}{r^3}\right)\cos\theta, \qquad B_\theta = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{\mu}{r^3}\right)\sin\theta, \qquad B_\phi = 0,$$

where (r, θ, ϕ) are the spherical coordinates of one proton relative to the other. The order of magnitude of either B_r or B_θ is about 3×10^{-4} Tesla. When the water molecule tumbles, as in liquid or gas, the field changes with time through $\theta(t)$.

In this case, the fluctuating magnetic fields are best described by their autocorrelation functions. *The simplest assumption one can make is that the autocorrelation function of each magnetic field component is of the form $\phi_{11}(\tau) \propto e^{-|\tau|/\tau_c}$ and that each field component has the same* correlation time *τ_c. In this case one can show that*

$$\frac{1}{\tau_1} = \frac{C\tau_c}{1+\omega_0^2\tau_c^2}, \qquad C = 5.43 \times 10^{10} \text{ sec}^{-2}.$$

The general solution of the Bloch equations *for arbitrary external fields involves a rather complicated dynamics. Special cases of interest are:*

- $B_1 = 0$; *no relaxation* $(\tau_1 = \tau_2 = \infty)$, $\boldsymbol{B}_0 = B_0\boldsymbol{e}_z$, $M_z(0) = M_0$, $M_y(0) = 0$, $M_x(0) = M_\perp$, $\omega_0 = \gamma B_0$ *(Larmor frequency). The solution yields just the Larmor precession*

$$M_x = M_\perp\cos(\omega_0 t), \qquad M_y = -M_\perp\sin(\omega_0 t), \qquad M_z = M_0.$$

The vector $\boldsymbol{M}$ rotates around $\boldsymbol{B}_0$ (z-axis) such that its z-component is fixed, and its projection on the x–y plane rotates with frequency ω_0. If $M_\perp = 0$, the vector always remains parallel to $\boldsymbol{B}_0$.

- $B_1 = 0$, $\boldsymbol{B}_0 = B_0\boldsymbol{e}_z$, $M_z(0) = 0$, $M_y(0) = 0$, $M_x(0) = M_0$. *The solution describes a* relaxation motion *about a static field:*

$$M_x = M_0e^{-t/\tau_2}\cos(\omega_0 t), \quad M_y = M_0e^{-t/\tau_2}\sin(\omega_0 t), \quad M_z = M_0(1 - e^{-t/\tau_1}).$$

Thus, $\boldsymbol{M}$ starts at $t = 0$ on the x-axis and then its tip spirals about the z-axis such that it aligns itself with the direction of $\boldsymbol{B}_0$ at asymptotic times.

- $\boldsymbol{B}_1 = [B_1\cos(\omega_1 t)]\,\boldsymbol{e}_x$, $\boldsymbol{B}_0 = B_0\boldsymbol{e}_z$, *relaxation neglected. In this case, the precessional motion due to the static field $\boldsymbol{B}_0$ can be "transformed away" by describing the motion of $\boldsymbol{M}$ in a new coordinate system (x', y', z') co-rotating about the z-axis $\boldsymbol{e}_z$ with the Larmor frequency ω_0. In this rotating frame there is no static magnetic field.*

If $\omega_1 \neq \omega_0$, *the motion is complicated, but averaged over many Larmor periods, the r.h.s. of each equation vanishes, and consequently the net effect is* $\{\boldsymbol{M}_\perp\}_{\text{average}} = 0$.

If, however, $\omega_1 = \omega_0$, *and if in addition* $\boldsymbol{M}(0) = M_0\boldsymbol{e}_z$, *the solution will be:*

$$M_{x'} = 0, \qquad M_{y'} = M_0 \sin \Omega t, \qquad M_{z'} = M_0 \cos \Omega t,$$

where $\Omega = \gamma B_1$.

Choosing the duration Δt *of the applied oscillating pulse to be such that* $\Omega \Delta t = \frac{\pi}{2}$, *the end orientation of* $\boldsymbol{M}$ *in the rotating frame will be* $(0, M_0, 0)$. *If, on the other hand, we take* $\Omega t = \pi$, *the vector* $\boldsymbol{M}$ *will flip over into* $(0, 0, -M_0)$. *It may seem strange that an oscillating magnetic field, pointing along the* x*-axis, which is fixed in the laboratory frame, causes motion about the* x' *axis, which is fixed in the rotating frame. The reason is that* $\boldsymbol{B}_1$ *is also oscillating at the Larmor frequency, so that its amplitude changes in just such a way as to change* $\boldsymbol{M}$ *appropriately.*

Another interesting solution is one for which the initial value of $\boldsymbol{M}$ *is not along the* z*-axis but in the* x*–*y *plane:* $M_{z'} = 0$, $M_{x'}(0) = M_0 \cos \alpha$, $M_{y'}(0) = M_0 \sin \alpha$. *The solution for this case is:*

$$\begin{aligned} M_{x'}(t) &= M_0 \cos \alpha, \\ M_{y'}(t) &= M_0 \sin \alpha \cos \Omega t, \\ M_{z'}(t) &= -M_0 \sin \alpha \sin \Omega t. \end{aligned}$$

• *Suppose now that a small sample under examination, initially magnetized along the* z *direction, is placed at the origin and subjected for a time* Δt *to the combined effects of* $\boldsymbol{B}_0 = B_0 \boldsymbol{e}_z$, *and a coil in the* y*–*z *plane* $[\boldsymbol{B}_1 = B_1 \cos(\omega_0 t)\boldsymbol{e}_x]$ *which generates magnetization* $M_{y'} = M_0 \sin(\Omega \Delta t)$; *Let* $\Delta t = \frac{\pi}{2\gamma B_1}$. *This arrangement rotates the magnetization of the sample into the* x*–*y *plane. If the generator is then turned off, the same coil can be used to detect the relaxation motion about the static field, that is, the voltage induced by the ensuing change of flux through the coil.*

It then follows from the above examples that the resulting voltage signal V *is an exponentially damped sine wave, known as the free induction decay,* $V = v_0 e^{-t/\tau_2} \sin(\omega_0 t)$. *It remains to find* v_0. *To this end we first calculate the magnetic flux through the coil, which is the flux through a hemispherical cap bounding the coil (radius* a*):*

$$\Phi = a^2 \int_0^{2\pi} d\varphi \int_0^{\pi/2} \sin\theta d\theta B_r \qquad (r = a),$$

in spherical coordinates in which the z direction is replaced with the x direction. Substituting our previous result $B_r = \frac{\mu_0}{4\pi}\left(\frac{2\mu_x}{a^3}\right)\cos\theta$ *(since the y-component of $\boldsymbol{\mu}$ contributes no net flux through the coil), the flux for a magnetic moment* $\mu_x = M_x \Delta V$ *(where* $M_x = M_0 e^{-t/\tau_2}\sin\Omega t$*) is*

$$\Phi = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{2\pi M_0 \Delta V}{a}\right) e^{-t/\tau_2}\sin\omega_0 t.$$

The induced voltage, by Faraday's law, is $\left\{-\frac{\partial\Phi}{\partial t}\right\}$*, which under the condition* $\frac{1}{\tau_2} \ll \omega_0$ *simplifies to*

$$V \approx -\left(\frac{\mu_0}{4\pi}\right)\left(\frac{\omega_0}{a}\right) 2\pi M_0 \Delta V e^{-t/\tau_2}\cos\Omega t.$$

The value of M_0 *is that calculated previously as*

$$M_z = N\langle\mu_z\rangle = \frac{N\gamma^2\hbar^2 I(I+1)}{3kT_s} B_0.$$

Therefore, for $I = \frac{1}{2}$*, we have the approximate result*

$$V \approx -\left(\frac{\mu_0}{4\pi}\right)\left(\frac{\pi N\Delta V\gamma^3\hbar^3 B_0^2}{2kT_s a}\right) e^{-t/\tau_2}\cos\omega_0 t.$$

Here $(N\Delta V)$ *is the total number of nuclear spins in the sample,* B_0 *is the static field along the z axis, and a is the radius of the coil that detects the free induction decay (FID) signal. If the sample fills the coil, as in most laboratory spectrometers, then the sensitivity is proportional to a. Note that FID signals are proportional to the density of the magnetic moment M. If*[1065] $\tau_2 \gg \frac{2\pi}{\omega_0}$*, the FID signal can be written as* $V(t) \simeq AM(x, y, z)\cos\omega_0 t$*, where A is some constant.*

- *Bloch's equations can be extended to include the effect of diffusion of the molecules carrying the nuclear spin in an inhomogeneous external magnetic field. Let* $\boldsymbol{B}_0 = (B_0 + G_{zz}z)\boldsymbol{e}_z$*. If the processes are linear, this diffusion can be added to the other terms in the Bloch equations.*

In the rotating system $(\omega_0 = \gamma B_0)$ *there is no precession and the motion is affected only by relaxation, diffusion and the gradient of* $\boldsymbol{B}_0$*. Therefore,*

[1065] In tissues the typical times for τ_1 and τ_2 are 0.5 s and 50 ms, respectively, whereas $\frac{2\pi}{\omega_0} \approx 5 \times 10^{-8}$ sec.

the ordinary Bloch differential equations are replaced by a system of coupled partial diffusion-reaction type equations:

$$\frac{\partial M_{x'}}{\partial t} = \gamma G_{zz} z M_{y'} - \frac{M_{x'}}{\tau_2} + D\nabla^2 M_{x'},$$

$$\frac{\partial M_{y'}}{\partial t} = -\gamma G_{zz} z M_{x'} - \frac{M_{y'}}{\tau_2} + D\nabla^2 M_{y'}.$$

In the absence of diffusion, the system is solved by

$$M_{x'} = M_x(0)e^{-t/\tau_2}\cos(\gamma G_{zz} z t),$$

$$M_{y'} = -M_y(0)e^{-t/\tau_2}\sin(\gamma G_{zz} z t).$$

NMR Spectroscopy

The practice of NMR began in 1946; a spectrum is produced by observing resonances for a compound as the static magnetic field or the radio frequency is scanned. In analytical chemistry, NMR spectroscopy detects the shape and structure of molecules. Molecules containing hydrogen atoms have the strongest effect, and most instruments are designed to produce NMR spectra of hydrogen atoms. Both qualitative and quantitative information is provided, since each different type of bond of the hydrogen atom gives its own unique resonance.

NMR became of vital importance in scientific laboratories as the first commercial spectrometer, appearing in 1953, was followed by a number of other instruments. Some of these are quite complex and designed for research, while other, simpler systems are used for routine analytical work.

In conventional NMR spectroscopy, the specimen is homogeneous from a macroscopic point of view. It may be a pure crystal, powder, liquid, or gas. A small sample of the specimen is placed in a very uniform magnetic field (the uniformity may be one part in 10^9 *over the sample). The excitation is achieved by a short RF pulse, at the resonant frequency, which is applied to the coil surrounding the sample in such a way as to create a magnetic field perpendicular to the constant field* B_0*.*

When the deexcitation (decay or relaxation) occurs, the emitted radiation can be detected by the voltage induced in the receiver coil, which may be *the same coil used to cause the initial excitation. If the specimen is not uniform and has heterogeneous internal structure, the NMR spectrometer gives a superposition of the properties of the various materials.*

To see what can be learned from even a simple proton NMR spectrum, consider the case of ethyl alcohol (CH_3–CH_2–OH). The power spectrum of this molecule shows three distinct groups: a single line for OH, a split line with two small side lobes for CH_2 and a triple line for CH_3. Three features can be discerned:

(1) *The ratio of areas under the three groups is $1:2:3$, indicating the number of protons in each group.*

(2) *The protons in the three groups are not equivalent; their frequencies are slightly different. The reason is the NMR chemical shift: The external RF magnetic field induces electron–orbital currents in the molecule. These currents in turn produce magnetic fields at the proton sites. In general, the induced fields will be opposed to the external one and the resulting field at a proton will be smaller than the applied one. The size of this effect permits conclusions concerning the electronic surrounding.*

(3) *Where two or three protons are in the same group, the NMR line is split. The splitting is caused by the spin-spin interaction between the protons in the same group; the magnetic field of one proton at the site of another can either add to or subtract from the external field. In the above example the magnetic dipole interaction does not act directly between the protons but is instead mediated by the electrons of the carbon atom that lies between the hydrogen atoms.*

In many molecules, the dominant splitting is not caused by magnetic interaction (Zeeman-type) but is rather due to electric field gradients; p electrons can produce very large electric field gradients at a nuclear site. The proton resonance is not sensitive to such field gradients, because the proton has no electric quadrupole moment. Nuclides with quadrupole moment such as ^{35}Cl, ^{79}Br, and ^{127}I, experience a splitting of the nuclear ground state due to field gradients of the atomic and molecular electrons.

The quadrupole-split spectrum can be explored in essentially the same way as the magnetically split one. If the quadrupole moment is known, conclusions about the nature of the chemical bond and the symmetry class of the site can be obtained.

In the absence of atomic motion in rigid lattices (crystals), NMR makes it possible to determine molecular structures not observable by other means. In many solids, even at low temperatures, there occur atomic diffusion and *rotation of groups of atoms. These movements affect the shape of the NMR absorption peak. A study of these effects as a function of temperature can supplement other physical measurements.*

In metals, the nuclei are influenced by an interaction between the spins of the conduction electrons (electrons, not bound to atoms, which move freely through the metal) and the applied field. This condition results in a shift of the resonant frequency from the value observed for the same nucleus when it is present in an insulator.

These so-called metallic shifts *provide important information on the magnetic susceptibility, the quantum mechanical wave functions that describe energy states, and the density of states of conduction electrons in the metal. In superconductors, the shape of the NMR spectral peaks provide detailed information on the penetration and internal distribution of the magnetic field.*

In ferromagnets or antiferromagnets (crystals in which not all atomic electrons are paired), the NMR is influenced by the internal magnetic fields produced by the array of ordered electronic spins. In ferromagnets the shift is a measure of the lattice magnetization; in an antiferromagnet there are at least two shifts that give the magnetization of each antiferromagnetic sublattice separately, a result unattainable by conventional magnetic measurements.

High-resolution nuclear magnetic resonance has become one of the most prized tools in the fields of organic chemistry and biochemistry. On the experimental side, the requirements to be met by the equipment are severe. The applied magnetic fields must have a relative stability and homogeneity throughout the sample better than one part in 10^8.

Special magnets that give uniform, stabilized fields, devices that twirl samples in order to smooth out the magnetic inhomogeneity, and sophisticated radio-frequency detection equipment are commercially available. The trend toward higher fields (over 100 *kilogauss), resulting from superconducting solenoids, improves the resolution by increasing the chemical shift splittings and the signal-to-noise ratio.*

The measurement of the precession frequency of proton spins in a magnetic field can yield the value of the field with high accuracy and is widely used for that purpose. For low fields, such as the earth's magnetic field, the NMR signal is expected to be weak because the nuclear magnetization is small, but special devices can enhance the signal 100 *or* 1000 *fold. Incorporated in existing portable magnetometers, these devices make them capable of measuring fields to an accuracy of about one part in* $1,000,000$ *and detecting field variations of about* 10^{-8} *gauss.*

Apart from the direct measurement of the magnetic field on earth or in space, these magnetometers prove to be useful whenever a phenomenon is *linked with vibrations of magnetic field in space or in time, such as anomalies arising from submarines, skiers buried under snow, archaeological remains, or mineral deposits.*

ESR (Electron-Spin-Resonance)

In elements with unfilled inner electronic shells [e.g., free radicals (molecular fragments), metals, and various paramagnetic defects and impurity centers], the relevant total interaction energy includes:

(1) the energy of coupling between magnetic moments due to the electron spins and the external magnetic field, and

(2) the electrostatic energy between the electronic shells and the ligand field, which is independent of the applied magnetic field.

The energy levels give rise to a spectrum with many different resonance frequencies (fine structure).

Another important feature of electron-spin resonance results from the interaction of the electronic magnetization with the nuclear moment, causing each component of the fine-structure resonance spectrum to be further split into many so-called hyperfine components.

If the electronic magnetization is spread over more than one atom, it can interact with more than one nucleus; and, in the expression for hyperfine interaction energy (Hamiltonian), the hyperfine coupling of the electrons with a single nucleus must be replaced by the sum of the coupling with all the nuclei. Each hyperfine line is then further split by the additional couplings into what is known as superhyperfine *structure.*

ESR differs from NMR in one essential point: the resonance frequencies in NMR are in general shifted from those of bare nuclei by very small amounts because of the influence of conduction electrons, chemical shifts, spin-spin couplings, and so on.

However, the ESR frequencies in bulk matter may differ greatly from those of free spins or free atoms, because the unfilled subshells of the atom are easily distorted by the interactions occurring in bulk matter.

A model that has been highly successful for the description of magnetism in bulk matter is based on the effect of the crystal lattice on the magnetic *center under study. The effect of the crystal field, particularly if it has little symmetry, is to reduce the magnetism caused by orbital motion. To some extent the orbital magnetism is preserved against ligand fields of low symmetry by the coupling of the spin and orbital moments.*

The key theoretical challenge in electron-spin resonance is, on the one hand, to construct a mathematical description of the total energy of interaction in the ligand field plus the applied magnetic field and on the other hand, to deduce the parameters of the theoretical expression from an analysis of the observed spectra.

The comparison of the two sets of values permits a detailed quantitative test of the microscopic description of the structure of matter in the compounds studied by ESR.

1946 CE **Willard Frank Libby** (1908–1980, U.S.A.). Chemist. Discovered carbon-14 (radiocarbon)[1066] and found a way to use it for dating ancient objects such as prehistoric plant and animal remains. Awarded the Nobel prize in chemistry (1960).

1946 CE The first synchro-cyclotron was built at the University of California at Berkeley. It produced α-particles with kinetic energy of 380 MeV.

[1066] Carbon-14 (^{14}C) is a rare radioactive isotope that occurs naturally in the atmosphere and in living plants and animals. Its half-life of 5730 years is so low that ^{14}C has not been generally measurable in organic material older than about 40,000 years. No existing ^{14}C is primordial since its half-life is too short. Instead, it is continually being created in the upper atmosphere (at altitudes of about 15 km) as a by-product of cosmic-ray bombardment. In the relevant nuclear reaction, an atmospheric ^{14}N nucleus absorbs a neutron, emits a proton and changes to ^{14}C. The newly created carbon isotope is quickly incorporated into CO_2, and thus is assimilated into earth's carbon cycle.

The age of carbon-bearing material is determined from the ratio of ^{14}C to all other carbon in the sample. The method depends on the special assumptions (1) that the rate of ^{14}C production in the upper atmosphere is nearly constant and (2) that its rate of assimilation into living organism is *rapid* relative to its rate of decay. These assumptions appear to be valid.

1946–1950 CE **André Weil** (1906–1998, France). A leading mathematician and member of the *Bourbaki*[1067] *group*. Contributed mainly to the fields of *algebraic geometry* and *algebraic topology*. Developed (1946) a theory of polynomial equations in any number of indeterminates and with coefficients in an arbitrary field. In a paper bearing the title *The Future of Mathematics* (1950) he specified important unsolved problems and incompletely developed subjects of pure mathematical research. Therein he said: "*Great mathematicians of the future... will solve the great problems which we shall bequeath to them, through unexpected connections, which our imagination will not have succeeded in discovering, and by looking at them in new lights*[1068]".

Weil made a number of conjectures concerning algebraic topology that were eventually proven true. [The last of these conjectures was proven (1974) by the Belgian mathematician **Pierre Deligne**; it concerns a generalized version of the Riemann hypothesis (1857) which in itself remains unconfirmed.]

Weil was born in Paris to Jewish parents. He studied in Paris, Rome and Göttingen.

The war was a disaster for Weil who was a conscientious objector and so wished to avoid military service. He fled to Finland as soon as war was declared in an attempt to avoid becoming forced into the army, but it was not a simple matter to escape from the war in Europe at this time. He was sent from Finland back to France where he was put in prison.

Weil was certainly in great danger at this time, partly because he was Jewish, partly because he had a sister, **Simone Weil** (1909–1943), who was a mystic philosopher and a leading figure in the French Resistance. The dangers of his predicament made Weil decide that being in the army was a better bet and he was able to argue successfully for his release on the condition that indeed he did join the army.

[1067] A pseudonym for a group of 10–20 French mathematicians who started (1939) to publish a survey of mathematics called *Elements de mathématique* which emphasizes logical structure and an axiomatic approach. Throughout the 25 Bourbaki volumes that have appeared thus far, axiomatics and the Hilbertian spirit prevail. The facts presented are not new discoveries, but the method of presentation is highly original.

Charles Bourbaki (1816–1897) was the son of a Greek colonel and in 1862 refused the offer of the Greek throne. After graduation from St. Cyr he joined the French Foreign Legion and later commanded Algerian troops in the Crimean War, eventually participating in Franco-Prussian War (1871).

[1068] His other famous saying is: "God exists since mathematics is consistent, and the Devil exists since we cannot prove it" (1977).

Having used the army as a reason to get out of prison, Weil had no intention of serving any longer than he possibly could. As soon as the chance to escape to the United States came, he took it at once. In the United States he went to Pennsylvania where he taught from 1941 at Haverford College and at Swarthmore College. In 1945 he accepted a position in Sao Paulo University, Brazil, where he remained until 1947.

In 1947 Weil returned to the United States and was appointed to the faculty of the University of Chicago, a position he continued to hold until 1958. From 1958 he worked at the Institute for Advanced Study at Princeton University. He retired in 1976, becoming Professor Emeritus at that time.

Algebraic geometry developed from the theory of *algebraic curves and surfaces* and the n-dimensional geometry of the Italian school. The first contribution to the theory of plane algebraic curves were made by **Isaac Newton** (1643–1727), **Colin Maclaurin** (1698–1746), **Leonhard Euler** (1707–1783) and **Gabriel Cramer** (1704–1752). The founder of algebraic geometry in the strict sense was **Max Noether** (1844–1921). The Italian geometers, principally **Corrado Segre** (1863–1924), **Francesco Severi** (1879–1961) and **Federigo Enriques** (1871–1946) brought this discipline to complete development. In the 20^{th} century an investigation of the foundations of the subject from the algebraic point of view was undertaken by the German school, particularly by **Emmy Noether** (1882–1935).

1946–1979 CE **Claude Levi-Strauss** (1908– , France). Anthropologist. Known for his development of *structural anthropology*[1069] (1958), which gave the scholar the opportunity to come into contact with the lives of men of different cultures, rather than just Western cultures.

Levi-Strauss was born in Belgium into an intellectual French Jewish family. He studied law and philosophy at the Sorbonne in Paris. During 1935–1939 he visited the University of Sao Paulo, Brazil, conducting periodic research forays into the Amazon Rainforest. He returned to France in 1939, but after France capitulated to the Germans, Levi-Strauss fled to New York where he spent most of the war years. He returned again to France (1948), receiving his doctorate from the Sorbonne.

[1069] *Structuralism* – a method of understanding human society and culture. Levi-Strauss maintains that the structure of any cultural organization has the specifics of a *language*. He thus insists that *myth* is a language because it has to be told in order to exist. He asked: “Why do myths from different cultures from all over the world seem so similar?” In general, structuralism emphasizes the underlying structure of relationship between the elements of a story rather than focusing on the content of the story itself.

He assumed the chair of Social Anthropology at the Collège de France in 1959. His published books are (English translations):

- Structural Anthropology (1958–1978).
- Totemism (1962).
- The Savage Mind (1966).
- Mythology (1970–1979).

In *Structural Anthropology*, Levi-Strauss considers culture as system of symbolic communication. His war-time sojourn in New York introduced him not only to *structural linguistics* but also to the pioneering work in cybernetics and information theory of **Shannon**. Levi-Strauss then combined the new insight of the mathematical theory of communication with the principles of the linguistics of **Saussure** and applied the result to the comparative study of societies and cultures. This was a creative synthesis that made a generalized '*structuralism*' possible. However, for all the originality of his approach his work has always lain in the heartland of traditional social anthropology.

Worldview LII: Claude Levi-Strauss

* * *

“By underrating the achievements of the past, we devaluate all those which still remain to be accomplished.”

* * *

“Just as the individual is not alone in the group, nor any one in society among the others, so man is not alone in the universe.”

* * *

“The scientist is not a person who gives the right answers, he is one who asks the right questions.”

* * *

“The world began without man, and it will complete itself without him.”

* * *

“Nothing exists except through language.”

* * *

“History does not belong to us; we belong to it.”

1946–1968 CE **Eric Hoffer** (1902–1983, USA). Social philosopher. One of the most incisive thinkers of his time who wrote some of the most insightful commentary on our society and trends in the world. Self-taught and independent thinker. Best known for his critical analysis of fanaticism and mass-movement psychology.

Hoffer was born to Jewish parents in New York city and grew up in the Bronx. His childhood was a blighted one. By the age of 7 he lost his eyesight in an accident and remained blind for 8 years. His mother died in 1909. Having spent several years in blindness when most other children were in school, Hoffer could do only manual labor after he regained his eyesight, but was determined to educate himself through avid reading. After the death of his father (1920) he moved to the West Coast seeking a living as a migrant farmworker, dishwasher and lumberjack, eventually becoming a stevedore on the waterfront of San-Francisco (1941) and doing the most difficult types of manual labor during the next 25 years.

During that time he both worked and published a number of books, the most influential being '*The True believer*' (1951). In this work he portrayed political fanatics as people who embrace a cause to compensate for their feelings of guilt and inadequacy; a potent analysis of the temptation to submerge the disappointed self in a 'larger' somehow – ennobling cause or movement.

His work was not only original, it was completely out of step with dominant academic trends. In particular, it was *completely non-Freudian*, at a time when almost all American psychology was confined to the Freudian paradigm. In avoiding the academic mainstream, Hoffer managed to avoid the straitjacket of established thought.

Hoffer was among the first to recognize the central importance of *self-esteem* to psychological well-being. While most recent writers focus on the benefits of a positive self-esteem, Hoffer focused on the consequences of a lack of self-esteem. He *finds in self-hatred, self-doubt, and insecurity the roots of fanaticism and self-righteousness.* He finds that a passionate obsession with the outside world or with the private lives of other people is merely a craven attempt to compensate for a lack of meaning in one's own life.

Contrary to the prevailing assumptions of his time, Eric Hoffer did not believe that revolutionary movements were based on the sufferings of the downtrodden. "Where people toil from sunrise to sunset for a bare living, they nurse no grievances and dream no dreams," he said. He had spent years living among such people and being one of them.

Hoffer's insights may help explain something that many of us have found very puzzling – the scions of wealthy families spending their lives and their inherited money backing radical movements. He said: "Unlimited opportunities can be as potent a cause of frustration as a paucity or lack of opportunities."

Worldview LIII: Eric Hoffer

* * *

"Absolute faith corrupts as absolutely as absolute power."

* * *

"A preoccupation with the future not only prevent us from seeing the present as it is but often prompts us to rearrange the past."

* * *

"To be engaged in a desperate struggle for food and shelter is to be wholly free from a sense of futility."

* * *

"Some, when they are alone, cease to exit."

* * *

"The game of history is usually played by the best and the worst over the heads of the majority in the middle."

* * *

"It is easier to love humanity than to love your neighbor."

* * *

"A savior who wants to turn men into angels is as much a hater of humans as the totalitarian despot who wants to turn them into puppets."

* * *

"No one has a right *to happiness."*

* * *

"To most of us nothing is so invisible as an unpleasant truth. Though it is held before our eyes, pushed under our noses, rammed down our throats – we know it not."

* * *

"The beginning of thought is disagreement – not only with others but also with ourselves."

* * *

"Power corrupts the few, while weakness corrupts the many."

* * *

"The new barbarism of the 20^{th} century is the echo of words bandied about by brilliant speakers and writers in the second half of the 19^{th}."

* * *

"It is the fate of every great achievement to be pounced upon by pedants and imitators who drain it of life and turn it into an orthodoxy which stifles all stirrings of originality."

* * *

"Propaganda does not deceive people; it merely helps them to deceive themselves."

* * *

"There is sublime thieving in all giving. Someone gives us all he has and we are his."

* * *

"The monstrous evils of the 20th century have shown us that the greediest money grubbers are gentle doves compared with money-hating wolves like Lenin, Stalin and Hitler, who in less than three decades killed or maimed nearly a 100 million men, women, and children and brought untold suffering to a large portion of mankind."

* * *

"A mass-movement attracts and holds a following not because it can satisfy the desire of self-advancement, but because it can satisfy the passion for self-renunciation."

* * *

"Take man's most fantastic invention – God. Man invented God in the image of his longing, in the image of what he wants to be, then proceeds to imitate that image, vie with it, and strive to overcome it."

* * *

"Where freedom *is real,* equality *is the passion of the masses. Where equality is real, freedom is the passion of a small minority."*

* * *

"A man is likely to mind his own business when it is worth minding. When it is not, he takes his mind off his own meaningless affairs by minding other people's business."

1946–1968 CE **Herbert Marshall McLuhan**[1070] (1911–1980, Canada). Educator, philosopher and commentator on communication technology. One of the greatest intellectual pioneers since Freud. Probed into the moral and psychological impact of contemporary Western means of communication. He held that technology is an extension of the human nervous system and that technological changes, by imperceptibly altering patterns of perception, create new environments of sense and feeling.

According to McLuhan, electronic communications had created a world of instant awareness to which the categories of perspective space and sequential time were irrelevant and in which a sense of private identity was untenable ('*Global village*'). McLuhan virtually established a new academic field with his theories about the impact of the media on our perception.

McLuhan was born in Edmonton, Alberta, Canada and was educated at the Universities of Manitoba and Cambridge (UK). He was a member of the Department of English at the University of Toronto (1946–1977).

[1070] For further reading, see:
Marchand, P., *Marshall McLuhann, The Medium and the Messenger*, Ticknor and Fields: New York, 1989, 320 pp.

Worldview LIV: Marshall McLuhan

* * *

"The medium is the message."

* * *

"Facts and truth don't really have to do with each other."

* * *

"The only way you can reach people, whether you are a preacher or a professor, is to hurt them."

* * *

"The invention of the print effected a profound transformation in the psyche of Western man, leading to an emphasis on the visualization of knowledge and the subsequent development of rationalism, mechanistic science and industry, capitalism, nationalism and so on."

* * *

"Enemies were to be cherished because they functioned as superb PR agents, indirectly promoting one's work far more effectively than friends did."

* * *

"When this circuit learns your job, what are you going to do?"

(1967)

* * *

"Violence is the unfailing remedy for those deprived of their identities. It is one method, often futile but always available, of grasping for the meaningful."

* * *

"Human artifacts turn human beings into "servo-mechanisms" of those artifacts."

* * *

"Capitalist industrialism is distorting human life and sexuality."

* * *

"The content of any medium or technology is its user."

* * *

"The name of a man is a numbing blow from which he never recovers. A person's name was a medium in itself, and it carried its own message."

* * *

"The 16th century had been sparked by the interplay between the old manuscript culture and the emerging print culture. The electronic media created a total field of instant awareness."

* * *

"A single English word is more interesting than the entire NASA space program."

* * *

"The decay of philosophy and religion, the pervasive North American Darwinian approach to success, and commercial and materialistic values had reduced adults to a kind of emotional and mental delinquency. In this state

they were extremely vulnerable to the crude daydreams fostered by the mass media. The violent sensationalism, the sadism and masochism reflected by cartoon figures like Superman were an essential ingredient of these daydreams. The educational system was helpless against the powerful onslaught of such daydreams, that children received their real education from the media and not from their schoolteachers."

* * *

"If something occurred to me, sooner or later it would occur to others – and then would come to pass."

* * *

"Anyone who truly perceives the present, could also see the future, since all possible futures are contained in the present."

* * *

"Words are most potent and unfathomable of all human artifacts."

* * *

"If the world really did seem to be dismal, it was perhaps because all of us were very far from perceiving it as it really existed."

* * *

"Fulton's steamboat anticipated the mini-skirt; we don't have to wait for the wind anymore."

* * *

"Whereas earlier technologies had extended one sense or one part of the body – the wheel extending the foot, for example – the new electronic technologies

extended the entire human nervous system. The very movement of information in these new technologies corresponds to the movement of the human mind."

* * *

"I don't necessarily agree with everything I say."

The emergence of the 'Global Village' civilization

As late as the French Revolution, Europe drew most of its energy from an estimated 14 million horses and 24 million oxen, as well as the natural power sources of wind, sun, water flow and human muscles. These energy sources were renewable: Nature could eventually replenish the forest they cut and the wind that filled their sails and the rivers that turned their paddle wheels. Even animals and people were replaceable "energy slaves".

During the Industrial Revolution*, societies began to draw their energy from coal, gas and oil — from irreplaceable fossil fuels. This revolutionary shift, coming after* ***T. Newcomen*** *invented a workable steam engine (1712), meant that for the first time a civilization was eating into nature's capital rather than merely living off the interest it provided. And from that day to our own, nations built towering technologies and economic structures on the assumption that cheap fossil fuels would be endlessly available.*

On this technological base a host of industries sprang up: at first coal, textiles, and railroads; then came steel, auto manufactures, aluminum, chemicals, *and appliances. Huge factory cities sprang into existence and from these industrial centers poured endless millions of identical products — shirts, shoes, automobiles, aeroplanes, watches, toys, soap, shampoo, cameras, machine guns, and electric motors.*

The new technology powered by the new energy system opened the floodgate of mass production. Thus, custom distribution gave way to the mass distribution and mass merchandising that became a central component of all industrial societies.

All societies (primitive, agricultural, or industrial) use energy: they make things and distribute them. In all societies the energy system, the production system, and the distribution system are interrelated parts of the techno-sphere*, and it has a characteristic form at each stage of social development.*

Thus, in the industrial techno-sphere, non-renewable energy sources were directly plugged into a mass production system which, in turn, spewed goods into a highly developed mass distribution system. This techno-sphere needed a socio-sphere *to accommodate it — a radical new form of social organization.*

Before the industrial revolution, when agriculture held sway, people tended to live in large, multigenerational house, all working together as an economic production unit.

As economic production shifted from the field to the factory, key functions of the family were parceled out to new specialized institutions: Education of the child was turned out to schools, care of the aged turned over to old-aged homes and workers began to follow jobs from place to place.

Torn apart by the migration to the cities, battered by economic storms, families stripped themselves of unwanted relatives, grew smaller, more mobile, and more suited to the needs of the techno-sphere. The so-called nuclear family — father, mother, and a few children, became the standard, socially approved model in all industrial societies.

Another central structure of the individual society was the mass education *in factory-style institutions called* schools*. As work shifted out of the fields and the home, children had to be prepared for factory life through a covert curriculum teaching punctuality, obedience and repetitive work.*

Built on the factory model, mass education taught (from ca 1850 on) basic reading, writing, arithmetic, a bit of history etc. This machined young people into a pliable, regimented work force of the type required by electromechanical technology and the assembly line. Taken together, the nuclear family and the factory-style school formed part of a single integrated system for the preparation of young people for roles in industrial society.

A third institution arose, extending the social control of the first two — the invention known as the corporation*: the new technology required giant pools of* capital *— more than a single individual or even a small group could provide. To encourage investment, the concept of limited liability was invented.*

By 1901, the world's first billion-dollar corporation — United States Steel*, appeared on the scene. By 1919 there were half a dozen such behemoths. Indeed, large corporations became a built-in feature of economic life in all industrial nations. Together these three (the nuclear family, the factory-style school, and the giant corporation) became the defining institutions of the industrial age.*

Around these three core institutions, a host of other organizations sprang up: government, ministries, sport clubs, churches, chambers of commerce, trade unions, professional organizations, political parties, academic institutions and many others, creating a complicated organizational ecology. The common denominator of all these units, whether schools, hospitals, prisons or government bureaucracies — was its division of labor, its hierarchical structure and its metallic impersonality.

But a civilization is more than simply a techno-sphere and a matching socio-sphere. All civilizations also require an info-sphere *for producing and distributing* information*, which in turn depends on person-to-person* communication*.*

During the pre industrial era, channels of communication were reserved for the rich and powerful only (e.g. the 'pony express' service of the House of Taxis*). In the industrial era, technology and mass production required massive movements of information that the old channels could no longer handle.*

The first wide open channel for industrial-era communications was provided by the post office*. By 1837, the* British Post Office *was carrying some* 88 *million pieces of mail a year. By 1960 that number had already climbed to* 10 *billion.*

But the information needs of industrial societies could not be met by writing alone. Thus the telephone *and* telegraph *were invented in the 19th century to carry their share in the ever-swelling communication load. By 1960 Americans were placing some* 256 *million phone calls per day (over* 93 *billion a year) and even the most advanced telephone systems and networks in the world were often overloaded. Postal services could carry the same message to millions, but not quickly. Telephones could carry messages quickly, but not to millions of people simultaneously.*

This gap came to be filled by the mass media*:* newspapers*,* radio*,* movies *and* television *and finally* e-mail*,* cellular phones *and the* internet*. Here we find again an embodiment of the basic principle of the factory: stamping identical* *messages into millions of brains, standardized, mass-manufactured products flow from a few concentrated image-factories out to millions of consumers.*

Thus there sprang up an elaborate info-sphere — communication channels through which individual and mass messages could be distributed as efficiently as goods or raw materials. This info-sphere serviced the techno-sphere and the socio-sphere.

1946–1970 CE **Jean Alexandre Eugène Dieudonné** (1906–1992, France). Mathematician. Contributed to many areas of abstract analysis, Lie groups, algebraic geometry, general topology, topological vector spaces, invariant theory and classical groups.

As a founder of the **Bourbaki** group, his ideas on the representation of mathematics, laying great emphasis on precise abstract formulation and elegance, have marked out a distinctive French school of mathematics whose influence has lasted for some 50 years.

Dieudonné was born in Lille. He studied at the École Normale Superieure receiving his doctorate in 1931. He then held chairs in Rennes, Nancy, Sao Paulo (Brazil), Michigan and Northwestern Universities (USA, 1952–1959), Paris and finally Nice (1964–1970).

1946–1973 CE **Hyman George Rickover** (1900–1986, USA). Father of the Nuclear Navy. Planned, developed and supervised the construction of the first nuclear-powered submarine, the *Nautilus* (1954).

Rickover was born in Makow, Russia to Jewish parents and emigrated to the USA (1906). He studied at the U.S. Naval Academy (1918–1922) and Columbia University (1929–1933). In 1946 he was assigned to the Atomic Energy Commission laboratory at Oak Ridge, Tennessee. He later became chief of the National Reactors Branch of the U.S. Atomic Energy Commission and head of the Nuclear Power Division of the U.S. Navy.

Rickover was promoted to the rank of Admiral in 1973 and retired from the US Navy in 1981, after over 63 years of service under 13 presidents. His name is memorialized in the attack submarine USS Hyman G. Rickover (SSN 709) and he is buried in Arlington National Cemetery.

Evolution of Submarine Design

1578 The first submarine design was drafted by **William Borne** but never got past the drawing stage. Borne's submarine design was based on ballast tanks which could be filled to submerge and evacuated to surface — these same principles are in use by today's submarines.

1620 **Cornelis Drebbel**, a Dutchman, conceived and built an oared submersible. Drebbel's submarine design was the first to address the problem of air replenishment while submerged.

1776 **David Bushnell** builds the one-man human powered *Turtle* submarine. The Colonial Army attempted to sink the British warship HMS Eagle with the Turtle, albeit unsuccessfully. The Turtle was the first submarine to dive, surface and be used in Naval combat.

1798 **Robert Fulton** builds the submarine *Nautilus* which incorporates two forms of power for propulsion — a sail while on the surface and a hand-cranked screw while submerged.

1870 French novelist **Jules Verne** brought submarines to full public consciousness with "20,000 Leagues Under the Sea".

1895 **John P. Holland** introduces the *Holland VII* and later the *Holland VIII* (1900). The Holland VIII with its petroleum engine for surface propulsion and electric engine for submerged operations served as the blueprint adopted by all the world's navies for submarine design up to 1914.

1904 The French submarine *Aigette* is the first submarine built with a diesel engine for surface propulsion and electric engine for submerged operations. (Diesel fuel is less volatile than petroleum and is the preferred fuel for current conventionally powered submarine designs.)

1943 The German U-boat U-264 is equipped with a snorkel mast. This mast which provides air to the diesel engine, allows the submarine to operate the engine at a shallow depth and recharge the batteries.

1944 The German U-791 uses Hydrogen Peroxide as an alternative fuel source.

1954 The U.S. launches the *USS Nautilus* — the world's first nuclear powered submarine. Nuclear power enables submarines to become true "submersibles" — able to operate underwater for an indefinite period of time. The development of the Naval nuclear propulsion plant was the work of a team Navy – government and contractor engineers – led by **Hyman G. Rickover**.

1958 The U.S. introduces the USS *Albacore* with a "tear drop" hull design to reduce underwater resistance and allow greater submerged speed and maneuverability. The first submarine class to use this new hull design is the *USS Skipjack*.

1959 The *USS George Washington* is the world's first nuclear powered ballistic missile firing submarine.

1946–1979 CE **Jule Gregory Charney** (1917–1981, U.S.A.). Meteorologist. Made major advances in *numerical weather prediction*, bypassing problems which had previously proved intractable. Made notable contributions to the theory of the *Gulf Stream*, hurricane formation, and the large-scale vertical propagation of energy in the atmosphere. During the 1960's and 1970's Charney played a leading role in the formulation and experimental design of the Global Atmospheric Research Programme and the 1978–9 Global Weather experiment. Charney was born in San Francisco to Jewish parents.

Numerical Weather Prediction

The reawakening of interest in the problem of dynamical weather prediction began soon after WWII as a direct result of the wartime expansion of weather services, and the development of high-speed electronic computers; Meteorological data were collected regularly from a dense network of stations covering a very large geographical area, and were extended upward through the widespread use of radiosonde equipment. Thus, for the first time, adequate data were available for detailed studies of the atmosphere's behavior on a macroscopic scale.

These studies strongly indicated that many aspects of general behavior of all fluids are not *essential to the operation of the atmosphere's weather-production mechanism and this, in turn, has suggested how the general hydrodynamical equations might be specialized or simplified, without sacrificing their essential meteorological content.*

By 1946*, there were several electronic computing machines in various stages of design and construction, all of them capable of carrying out numerical computations at about* 10,000 *times the speed of a trained human computer operating a desk calculator. With the realization that computations on the scale of Richardson's now could be carried out in a matter of hours, rather than months, a number of theoretical meteorologists turned their attention and effort to formulating the problem of dynamical weather prediction for high-speed computation.*

Thus, an organized research movement started in 1946 *at The Princeton Institute for Advanced Study under the leadership of* **John von Neumann** *(1903–1957)* and **Jule Charney** *(1917–1981). Within a few years, this initial stimulus was followed by the establishment of similar research groups in Europe and Japan. Aside from approximative errors in the hydrodynamic equations, there appeared to be two serious defects in Richardson's method (1922):*

(1) The atmosphere is always very close to a state of mechanical equilibrium; i.e., the horizontal pressure-gradient force is almost exactly in balance with the Coriolis force, and the vertical force of buoyancy is almost exactly balanced by the virtual *gravitational force (the resultant of the earth's centrifugal force and pure gravitational forces). Thus, the large-scale accelerations of air are very much smaller than the individual forces per unit mass, generally at least* 10 *times less. From the standpoint of computing, this implies that the accelerations or* time derivatives of

velocity are small differences between large terms of the same sign. Accordingly, since the individual forces per unit mass must be computed independently, each must be computed to within 1 percent accuracy in order to compute the accelerations to within 10 percent accuracy. Needless to say, winds and pressure gradients are not measured or reported to 1 percent accuracy. This difficulty alone was enough to guarantee the failure of Richardson's method;

(2) *In equations of the hyperbolic type (such as the wave equation), the increment of time over which the data are extrapolated must be less than a time of order that required for a wave or impulse to traverse the distance between adjacent points in the finite difference grid. Otherwise, certain bands in the spectrum of random error are amplified and the computation "blows up", i.e., the solution of the finite-difference equation will not converge toward the solution of the corresponding differential equation.*

This phenomenon of computational instability *may arise, for example, in the integration of the equation for the propagation of pure* sound waves*, a* special *form of the hydrodynamic equation. A fortiori, therefore, it will also arise in the integration of the* general *hydrodynamic equations. Thus, if one chooses the mesh size of the grid to be* 100 *km, the finite difference integration would have to be carried out in time stages of* 10 *min or less, and a* 24*-hr prediction would involve making* 144 *successive* 10*-min forecasts. To meteorologists, this seems an unnecessary price to pay for computational stability. Thus, for practical reasons it may not be desirable to use the hydrodynamical equations in their exact form.*

Since the mere existence of sound waves and other fast-moving disturbances can have little effect on the course of meteorological events, it was necessary to reformulate the basic equations such that high-speed waves are excluded, leaving solutions corresponding to large-scale weather disturbances intact.

This task was accomplished by ***Charney*** *(1948) who discovered that discriminate introduction of the geostrophic and hydrostatic approximations had precisely the effect of* excluding *the solutions corresponding to sound and gravity waves. It turned out that, under somewhat idealized conditions, the modified hydrodynamical equations had the same form as the equations for a "model" atmosphere in which* the density is uniform, the motion is purely horizontal*, and whose initial state is identified with real atmospheric conditions at a height of about* 6100 *meters. Density changes and vertical motions being absent, such a model fluid is obviously incapable of supporting waves whose existence depends on compressibility or gravitational restoring forces.*

Solutions of the equations governing this model were first computed in 1950 *by Charney and von Neumann, using the ENIAC electronic computer. The results showed that the large-scale motions of the atmosphere could indeed be predicted from the field equations in their* inexact form. *They also showed that the theoretical approach to the prediction problem, coupled with high-speed computing technique, is a practical and a feasible one.*

However, since the above model atmosphere is governed by the principle of vorticity conservation, *it implied that the number and intensity of cyclonic and anticyclonic vortices in this model cannot change and, accordingly, that method based on the equations of this simple model cannot predict the formation and growth of* new disturbances. *Since* 1951, *improved models capable of accounting for the transformation of disturbances, have been developed.*

In 1961, **Edward N. Lorenz** *devised a computer model of* 2*-dimensional convection in the atmosphere. He discovered that small changes in the initial conditions may lead to instability of the corresponding solutions of the governing equations* (chaos), *a fact which makes weather prediction inherently difficult.*

From Balloons to Weather Satellites (1950–1990)

The development of modern climatology has been accelerated by the capability of 20^{th} *century technology to monitor the global atmosphere. In particular, data derived from the radiosonde, weather radar, aircraft and weather satellites have provided valuable new information on world weather on a variety of spatial scales. The analysis of this and more conventional data has been facilitated and considerably improved by the advent of high-speed computers.*

In the 1930s *the development of the* radiosonde balloons *allowed the* 3*-dimensional structure of the atmosphere to be monitored on a regular basis in terms of temperature, pressure, wind and humidity. Reconnaissance aircraft* *flying through and around weather systems (particularly frontal depression, tropical cyclones and thunderstorms) have provided new information on their 3-dimensional structure.*

In the mid-fifties, cloud-structure photographs were taken from jet aircraft flying in the lower stratosphere. Even before this use of high-altitude aircraft became common, flights of a number of V-2 rockets, fitted with cameras and launched in New Mexico in the early fifties, provided a preview for later technological advances.

With the development of modern rockets, satellites *and* space platforms *put into orbit, a new dimension has been added to the observation of the atmosphere, its clouds and the precipitation they produce.*

The field of meteorology entered the space age on April 01, 1960, *when the United States launched the first artificial satellite equipped to provide photographs of the earth's weather conditions. Its two television cameras transmitted both broad and detailed pictures of the earth's cloud cover. Since this satellite rotated for stability, for nearly* 15 *percent of the time its cameras were pointed away from the earth. Nevertheless, in its short life span of only* 79 *days, TIROS 1 radioed back thousands of pictures to the earth. A year before the* 9^{th} *and last TIROS was launched in* 1965, *the* 1^{st} *of the second-generation* Nimbus *satellites was orbiting the earth. The later* Nimbus *satellites were equipped with infrared cameras capable of detecting cloud coverage at night.*

In 1963, *the World Meteorological Organization approved a plan for mapping the weather around the globe. The plan, known as World Weather Watch, called for artificial satellites and thousands of land and sea stations to gather weather information.*

Two types of weather satellites were placed in orbit: The polar satellites *travel around the earth in a polar orbit in about* 110 *minutes. By properly orienting the orbits, these satellites drift about* 15 *degrees westward per orbit over the earth's surface. Thus they are able to obtain photo coverage of the entire earth twice a day and have constant surveillance over the daily patterns of planetary waves, cyclonic storms, hurricanes, and other large-scale weather patterns. This information is highly useful for short-range weather forecasting.*

By 1966, geostationary satellites *were positioned at an altitude of* 35,880 *km over the equator. These satellites 'hover' over a fixed point on the earth's surface, sending to earth images of large sectors of the planet every 30 minutes or so.*

Thus, climatologists have had, since 1966, *the facility of complete global cover of the world's weather by satellite imagery, offering new insights into* *circulations, particularly in areas where conventional meteorological data are either scarce or absent (such as the oceanic areas of the world — where many significant weather systems are spawned).*

During 1980–1990, *the observational capabilities of satellites (particularly in terms of direct and indirect measurements of radiation, vertical temperature profiles, wind and precipitation), image quality (in terms of resolution and sampling frequency), and the processing of satellite imagery, have all improved dramatically. Weather satellite data have provided climatologists with a more complete and detailed view of global weather systems and circulations over both land and sea than has ever before been possible.*

The weather reconnaissance satellites are equipped with highly sophisticated telemetry devices that relay signals to ground stations. These are then used to produce photographs. By obtaining simultaneous photographs of the earth at 5 *or* 6 *specific wavelengths, variations in signals are obtained that can be converted to differences in radiative temperature of the various surfaces.*

This provides a wealth of geophysical information that, when properly interpreted, helps to differentiate between cirrus clouds, lower and warmer clouds, snow, lakes, sea ice, and such special features as the boundaries of the warm waters of the Gulf Stream, which often produce clouds when the colder air from Polar regions flows over them. The geostat A.T.S. Satellites (Application Technology Satellite) also provide the time-lapse cloud photography routinely displayed on television weather programs.

Satellite photographs of clouds and weather patterns show that they are all parts of a continuous global system. They are not random occurrences but rather are part of an energy interplay between solar radiation, night-time cooling, seasonal changes and the pressure patterns that develop from these interactions (the cloud patterns develop from large- and small-scale rising air motions that bring about cooling, and cause tongues of moisture to undergo condensation).

Finally, the availability of modern, high-speed electronic computer technology permits large quantities of meteorological data to be analyzed and mapped both accurately and speedily: it can be used to test and develop new models and theories of atmospheric circulations, and to produce short- and long-range weather forecasts.

Knot theory — from Kelvin's atom (1867) to the DNA molecules (1982), statistical mechanics (1987) and Quantum Theory (1989).

A. Basic concepts — Knots, Links and Braids

The mathematical theory of knots originated in the 19th century, but knots have been of interest since ancient times. Knots appear in illuminated manuscripts, sculpture, painting and other art forms from all over the world. As early as human beings used any kind of rope, they probably began inventing knots, and sailors and scouts alike can attest to their variety and usefulness.

A 'mathematical' knot is just slightly different from the knots that we see and use every day. Mathematicians envision knots as closed (boundary-less) loops. It is as though the two free ends of tangled rope have been spliced together. When knots are drawn or projected on paper, the places where the rope crosses itself are shown as a broken line and a solid line. The intent is to show that the part of the rope represented by the broken line is passing under the part represented by the solid line.

So, by definition, a knot is simply a closed piecewise curve in 3-dimensional *Euclidean space. Its projection on a plane is known as the knot diagram. Thus, the same knot may have different projections. If we make a knot out of a wire, we obtain a* 3-dimensional *configuration that defines a certain boundary of a surface in space, and this surface can be used to study the knot. If one dips the wire in a bubble-solution, the form of the stretched and twisted soap-film is called a Seifert Surface.*

Knots have been catalogued in order of increasing complexity. One measure of complexity that is often used is the crossing number, or the number of self-intersection points in the simplest planar projection of the knot. There is only one knot with crossing number three (ignoring mirror reflections), the trefoil or cloverleaf knot. The figure-8 knot is the only knot with a crossing number of four. There are two knots with a crossing number of five, three with a crossing number of six, and seven knots with a crossing number of seven. From there on the numbers increase dramatically. There are 12,965 knots with 13 or fewer crossings in a minimal projection and 1,701,935 with 16 or fewer crossings.

Knot theory is a branch of algebraic topology where one studies the embedding of one topological space into another (the 'Placement problem'). Two knots are considered equivalent if one can be smoothly deformed into the other, or equivalently, if there exists a homeomorphism on $\mathbb{R}^3$ *which maps the image* *of the first knot onto the second. Cutting the knot or allowing it to pass through itself are not permitted.*

It is not too difficult to see (but slightly more difficult to prove) that the trefoil is not equivalent to the unknot (i.e. trivial loop). Also, the right and left handed versions of the trefoil are only equivalent if the homeomorphism mapping one into the other includes a reflection (other knots, such as the Figure-8 knot, which are equivalent to their mirror images, are known as achiral knots.

A knot is a mathematical object, just like number is, and mathematicians ask many of the same questions about knots as they ask about numbers. One of these questions is: "Are two given knots equal?"

Fig. 5.17 shows all knots with seven crossing or less. The notation N_k *means that there are* N *crossings and* k *different knots of the* N^{th} *class. Here* 0_1 *is known as the unknot, alias a trivial knot. It is the simplest of all knots. Next comes the trefoil knot* 3_1 *(tre = three; foil = leaf). Its mirror image (its reflection) is a different knot, and no matter how one twists or deforms one of them, it cannot be made to look like the other unless one cuts and reties it.*

Not all knots are different from their mirror image: 4_1, *known as the 8-figure knot is the mirror-image of itself. Note that the two versions of the trefoil knot differ only in the over/under placement of the strands.*

The mathematical proof of the unequivalence of the left and right trefoil knots is far more complicated than the pair appears to be. It was given by ***Reidemeister*** *in 1926.*

In Fig. 5.18 we see two 6-knots known as the 'reef' and the 'granny', which cannot be transformed into each other.

The difficulty of demonstrating equivalence is illustrated by the 4 pairs of knots in Fig. 5.19: with some effort it is possible to deform the r.h.s. knot in (1) to appear untangled. On the other hand, no amount of effort seems sufficient to unknot the two knots in (2). However, some clever manipulation of the l.h.s. knot could transform it to look like the r.h.s.

The two knots in (3) were assumed to be distinct. Yet, in 1974 ***K. Perko*** *discovered a deformation that turns one into the other.*

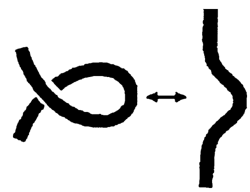
The central problem of knot theory is distinguishing between various knots and classifying them. A special case of this problem is one of the fundamental questions of Knot Theory: Given a knot, is it the unknot?

When we actually start trying to untangle and rearrange knots to look like one another we begin what can seem like a very complicated process. Mathematicians were perplexed at the seemingly unending number of ways a knot could be shaped and transformed. What was needed was a simple set of rules for working with knots. And indeed, in 1926 **Kurt Reidemeister** proved that if we have different presentations (or *projections*) of the same knot, we can get one to look like the other using just three simple types of moves.

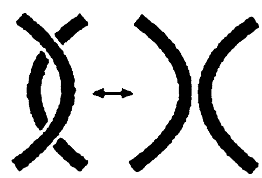
First, we must "simplify" the knot as much as possible. This means we use the Reidemeister moves to get as few crossings in the knot as possible. Once we simplify the knot so that we cannot remove any further crossings, the knot is classified by the number of crossings that remain. For example, the trefoil knot is classified by its fewest number of crossings — three.

The *Reidemeister moves* are the following:

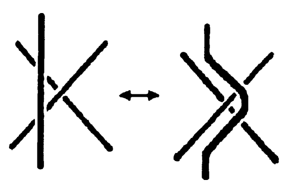
1. Take out (or put in) a simple twist in the knot:



2. Add or remove two crossings (lay one strand over another):



3. Slide a strand from one side of a crossing to the other:



We can change the way a knot looks so much that it can be hard to tell what we started with. *So, what stays the same about a knot in different projections?*

Knots have some properties that depend only on the knot itself and not on how it appears in any particular projection. These properties are called *invariants* of the knot.

One invariant is the *minimal crossing number*. The minimal crossing number of a knot is the least number of crossings that appear in any projection of the knot. For example, the unknot has a minimal crossing number of 0.

The trefoil knot has a minimal crossing number of 3. No matter how much we tangle a knot (without cutting), it can always be simplified to its minimal number of crossings using the Reidemeister moves.

Another invariant is the unknotting number. The unknotting number is the least number of crossing changes necessary to turn a knot into the unknot. By "crossing changes" we mean changing the orientation of two strings where they cross.

Similarly to how we think of counting numbers as being prime or composite, we use the same terms to refer to types of knots.

A composite knot is a knot which can be formed by the composition (joining) of two or more nontrivial knots. When we join knots to form a composite knot, the process can be referred to as a connected sum — we have combined two or more knots by connecting them. The knots that make up a composite knot are called factor knots.

If a knot is not composite, meaning it cannot be expressed as the connected sum of two other nontrivial knots, we call it a prime knot. The trefoil knot is a prime knot.

For example, the square knot (Fig. 5.20) is an example of a composite knot. It can be formed by cutting one side of two trefoil knots and joining the loose ends of each knot to the other. This forms what is known as a connected sum.

Note that the zero knot (the unknot whose crossing number is zero) is so named, not just because it looks like zero, but also because it behaves like the number zero: when one adds the zero knot to another knot, there is a little bit more rope, but the knot itself is unchanged.

What are the basic knot building blocks? Knot addition shows us how two knots can be added together to make a more complex knot. How does this work in reverse? Can you always break a complicated knot into two simpler ones that add together to form it? Of course the answer to that question implies that we know what complex and simple knots are!

This question is analogous to thinking about prime numbers. All numbers that are not prime can be produced by multiplying together a unique combination of prime numbers. Is the same thing true with knots? Indeed, numerous prime knots exist. Determining whether a given knot is equivalent to a connected sum of smaller building blocks is not always easy.

Links

A link is a collection of knots. Individual knots which make up a link are called components of the link. A specific link is known as the Borromean Rings (Fig. 5.21).

It can be proved that no deformation will separate the components. Note, however, that if one of the two components is removed, the remaining two can be split apart. Such a link is called Brunnian.

Just as mathematicians try to untangle knots to form the unknot, they try to separate links to form the "unlink". A link is referred to as splittable if the component loops can be separated without cutting.

In order to turn a link into two or more separate (un-linked) knots, you have to cut the rope. The number of times you would have to cut the rope to do this is called the link number. The link number can be thought of as the measure of how "linked up" the knots are.

Braids

A braid is a system of curves, which start from a straight line of points and points on end at a parallel line, but winding round each other on the way (Fig. 5.22). Depending on how the ends are spliced together, braids can be made into knots or links.

Braids are equivalent if you can deform one continuously into the other, just like knots and links; but now the curves have ends, and the ends have to stay fixed; moreover, one is not allowed to push curves over the ends and undo them. They have to stay between the two parallel lines. The new feature is that two braids can be combined, by joining the end of the first to the start of the second.

It turns out that braids form a group under this operation. The identity braid consists of parallel curves, not twisted in any way; and the inverse to a given braid is the same braid upside down. Notice that you only get a group if deformations of a given braid count as the same braid: the way to cancel out a braid is to combine it with its inverse and then straighten out the curves.

***Artin** found a complete symbolic description of the braid group. Suppose, for example, that the braids have four strands. Every braid can be built up from elementary braids* s_1, s_2 *and* s_3 *which just swap adjacent points* (s_j *swapping strands* j *and* $j+1$). *Their inverses are the same braids* *turned upside down: they look just the same except that overpasses become underpasses.*

We can symbolically express any braid as a sequence of powers of the s's, using negative powers for inverses: $s_1^3 s_2^4 s_1^7 s_3^5 s_2^{-8}$ *and so on.*

Braids, like knots, may be topologically the same even though they look different, and Artin captured this in the defining relations of his braid group, satisfied by 'adjacent' elementary braids:

$$s_1 s_2 s_1 = s_2 s_1 s_2$$

and so on.

He proved that these relations correspond precisely to topological equivalence of braids. That is, suppose two braids are represented as symbol sequences. Then they are topologically equivalent if and only if you can pass from one symbol sequence to the other by applying the defining relations over and over again.

Every knot is a closed circular braid (theorem). *This means that no matter how twisted, complex and entangled a knot might be, no matter how many crossings it has, the strands of rope can be rearranged into a single braided coil. When the knot is arranged this way and you follow the strand of rope all the way around, it will make a series of circles that cross over and under each other's strands. Every circle has the same center, there is no backtracking, and there are no extra loops.*

Because every knot is a closed circular braid, it is possible to describe any knot by listing the braid components, telling the order of their appearance, and telling how the ends are joined.

The mathematical theory of knots has made major advances in recent years. One of the most exciting developments has been the discovery of deep connections between knot theory and the branch of physics that studies the fundamental particles and forces that are the building blocks of the universe. It has also been found that DNA is sometimes knotted, and knots may play a role in molecular biology.

B. Historical survey

Kelvin *proposed (1867) that the atom of each chemical element should have unique signature based on how the element knotted up the ether surrounding it. He stated that the chemical properties of the elements were*

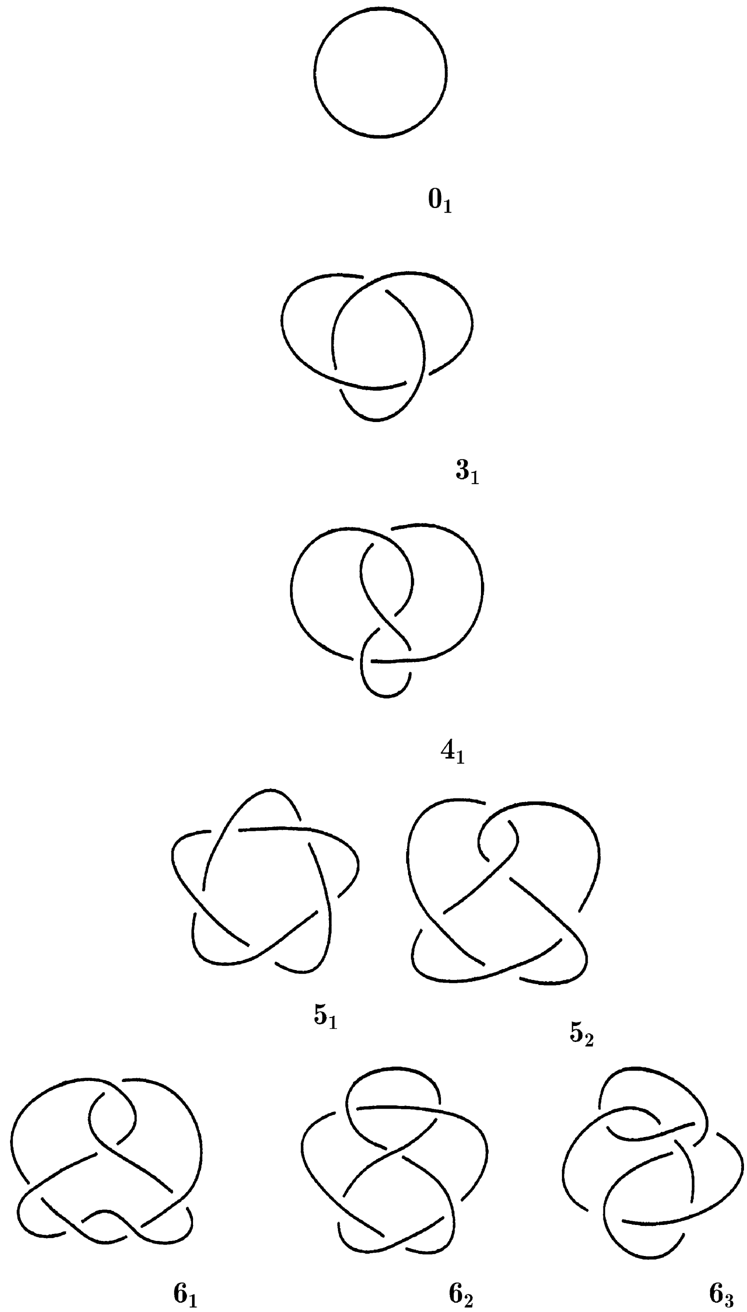


Fig. 5.17 Knots with seven crossings or less and their corresponding 'Alexander Polynomials'

$\mathbf{7_1}$ $\mathbf{7_2}$ $\mathbf{7_3}$ $\mathbf{7_4}$

$\mathbf{7_5}$ $\mathbf{7_6}$ $\mathbf{7_7}$

$\mathbf{3_1}$ $t^2 - t + 1$

$\mathbf{4_1}$ $t^2 - 3t + 1$

$\mathbf{5_1}$ $t^4 - t^3 + t^2 - t + 1$

$\mathbf{5_2}$ $2t^2 - 3t + 2$

$\mathbf{6_1}$ $2t^2 - 5t + 2$

$\mathbf{6_2}$ $t^4 - 3t^3 + 3t^2 - 3t + 1$

$\mathbf{6_3}$ $t^4 - 3t^3 + 5t^2 - 3t + 1$

$\mathbf{7_1}$ $t^6 - t^5 + t^4 - t^3 + t^2 - t + 1$

$\mathbf{7_2}$ $3t^2 - 5t + 3$

$\mathbf{7_3}$ $2t^4 - 3t^3 + 3t^2 - 3t + 2$

$\mathbf{7_4}$ $4t^2 - 7t + 4$

$\mathbf{7_5}$ $2t^4 - 4t^3 + 5t^2 - 4t + 2$

$\mathbf{7_6}$ $t^4 - 5t^3 + 7t^2 - 5t + 1$

$\mathbf{7_7}$ $t^4 - 5t^3 + 9t^2 - 5t + 1$

Fig. 5.17: (Cont.)

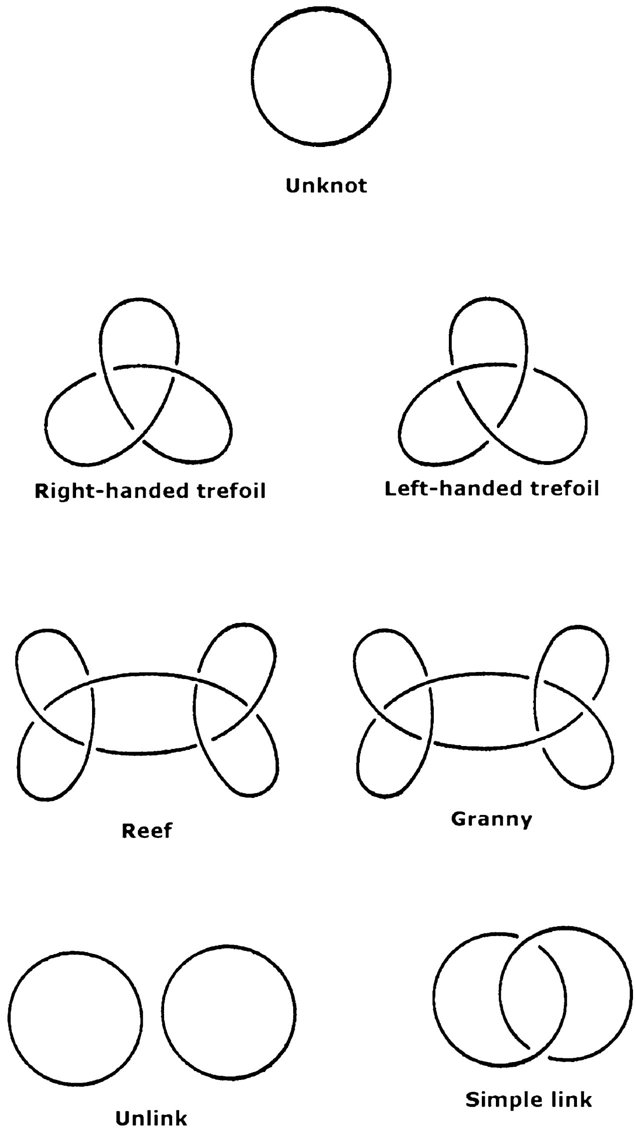
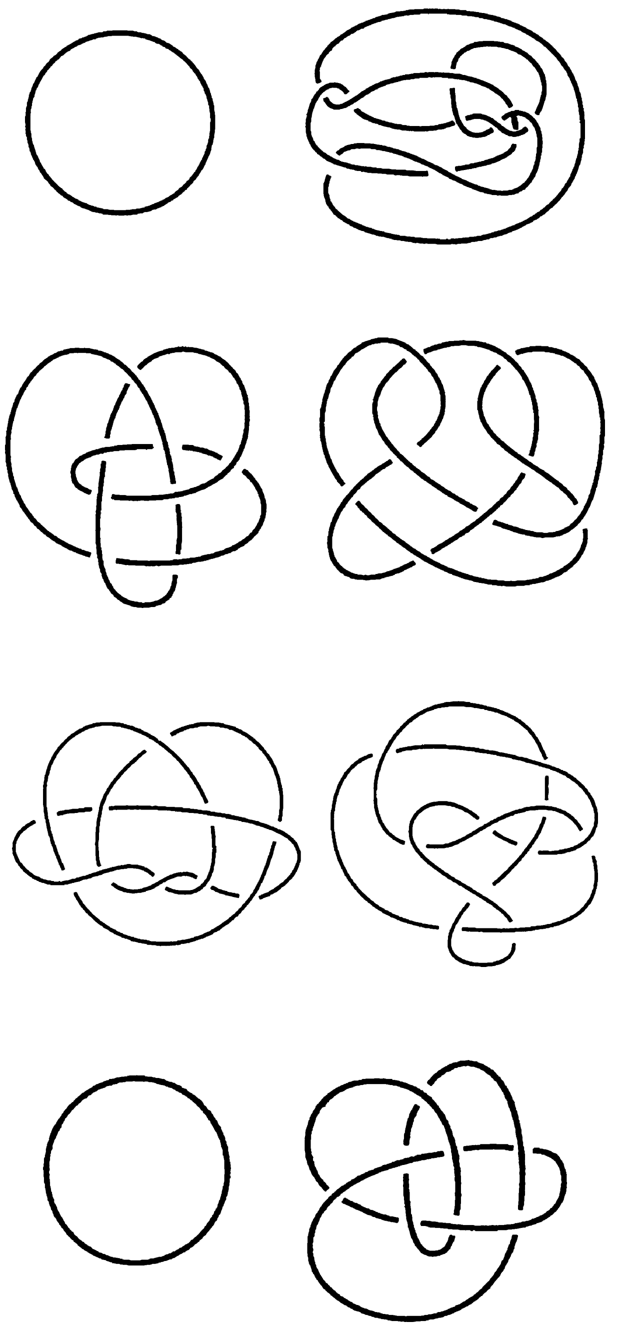


Fig. 5.18 Some basic knots and links



(1)

(2)

(3)

(4)

Fig. 5.19 4 pairs of equivalent knots

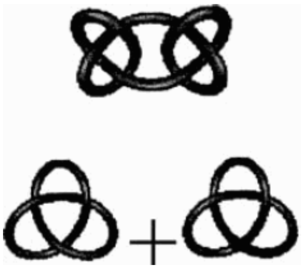


Fig. 5.20 The composite square knot

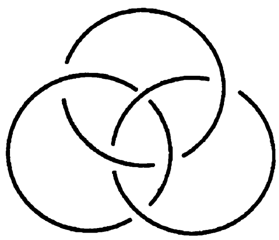
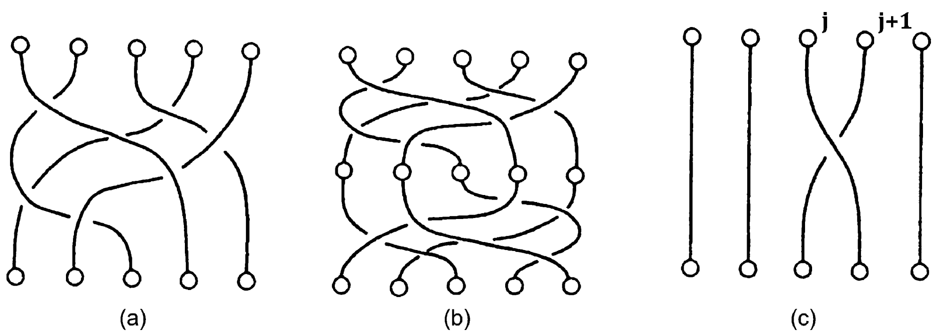
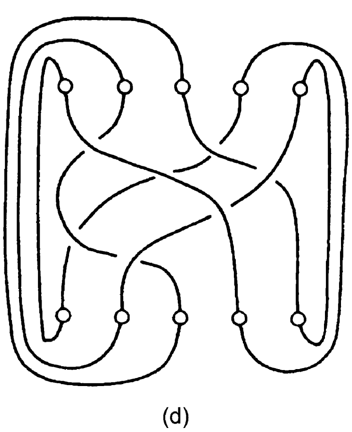


Fig. 5.21 The 'Borrumean ring'



(a) A braid. (b) Combining two braids to get a third.
(c) Elementary braid with a single crossing.



(d) Converting a braid into a link.

Fig. 5.22 Braids

related to knotting that occur between atoms, implying that insight into chemistry would be gained with an understanding of knots. This led many scientists to theorize that they could understand the elements by simply studying the knots, so mathematicians the world over began to construct tables of knots and their pictures.

However, soon enough the theory of the ether was dismissed, and mathematicians were left alone in pursuit of Knot theory for almost a century.

In the 1980's, biologists and chemists studying genetics found that deoxyribonucleic acid (DNA) can sometimes become tangled. Experiments suggested that how a DNA strand knots might have an impact on the properties of the resulting strand after replication.

Once geneticists became interested in knots, other scientists joined in again, as well. The fact that genetics is such a young and extremely interesting field of inquiry has led to a renewal of interest in studying Knot Theory from an applied mathematical perspective.

Motivated by Kelvin's idea, ***P.G. Tait*** *prepared (1877–1900) the enumeration of knots with* 10 *crossings or less. Tait viewed two knots as equivalent, or of the same type, if one could be deformed to appear as the other, and sought enumeration that included each knot type only once. When Tait began his work, the formal mathematics needed to address the study was unavailable, and the evidence that his listed knots are distinct was empirical.*

James Clerk Maxwell *wrote several papers (1868) on knots and links and set out the basic problem of the classification of knots and links. He, in fact, had defined the Reidemeister moves (!) which would be shown to be the fundamental moves in modifying and composing knots (1926).*

Work at the turn of the 20th century placed the subject of topology on firm mathematical ground, and it became possible to define the objects of knot theory precisely, and to prove theorems about them. In particular, algebraic methods were introduced into the subject, and these provided the means to rigorously established which pairs of knots were actually distinct.

Poincaré *(1895) introduced the algebraic entities known as fundamental groups and homology groups that can be associated with topological spaces in such a way that if two topological spaces differ w.r.t. any of these groups, then one can say for sure that these spaces are not equivalent (i.e., cannot be deformed into one another).*

Max Dehn *(1914) proved that the two simplest-looking knots, the right- and left-handed trefoils, represent distinct knot types; that is, there is no way to deform one to look like the other. He stated the Dehn Lemma:*

> *"If a knot is indistinguishable from the trivial knot using algebraic methods, then the knot is in fact trivial"*

J.W. Alexander *(1923) discovered a* polynomial invariant *of a knot. It is essentially a method of associating to each knot a polynomial (now called the* Alexander polynomial*), such that if one knot can be deformed into another, both will have the same associated polynomial.*

This invariant proved to be a powerful tool in the subject, but it has its limitations: it cannot distinguish handedness *[e.g., both left- and right-handed trefoils have the same polynomial* $(t^2 - t + 1)$*]. In addition it is not always* unique*:* 8 *out of* 87 *knots with* 9 *or fewer crossings share polynomials with others on the list.*

Alexander's initial definitions and arguments were combinatorial*, depending only on a study of the diagram of a knot, without reference to the algebra that had already proven successful.*

Kurt Reidemeister *(1893–1971) proved (1926) that if we have different plane projections of the same knot, we can get one to look like the other using just* 3 *simple types of* moves *(enumerated above). He showed, for example, that the* trefoil *is not equivalent to an unknotted loop (circle). He did this by breaking up any deformation of a knot into a series of standard* moves*, and finding a property of the trefoil that is preserved by each such more, but which fails to hold for any ordinary circle. Reidemeister wrote the first book on knot theory ('Knotentheorie', 1932).*

Emil Artin *invented (1925) the* braid theory*, entering* algebra *in a big way into topology of knots. To begin with, everything was* geometric*. This was followed by* modular arithmetic *and* combinatorics*.*

Finally knot theory became part of algebraic *topology. It turned out that braids form a* group *under the operation of joining the line of ends of one braid to the line of beginnings of another braid. The identity braid consists of parallel curves, not twisted in any way; and the inverse to a given braid is the same braid upside down. One only gets a group if deformations of a given braid count as the* same *braid. Artin found a complete symbolic description of the braid group.*

In 1934, ***Herbert Seifert*** *(1907–1996) demonstrated that if a knot is the boundary of a surface in 3-dimensional space, then that surface can be used to study the knot; he also presented an algorithm to construct a surface bounded by any given knot.*

This approach was certainly of practical importance, as it gave efficient means for computing many of the known invariants. Thus, using knots made out of wire*, one can see what happens when they are dipped into a* bubble *solution. The form of the stretched and twisted* soap film *is called a Seifert surface.*

In 1936, ***W. Burau*** *discovered how to find* matrices *that obey Artin's defining relations. It was found that the Alexander polynomial of a knot is related to the* Burau matrix *of the corresponding braid. Thus, the Alexander polynomial can be computed* algebraically *from the braid group.*

In 1917, ***H. Schubert*** *proved that any knot can be decomposed uniquely as the 'connected sum' of prime knots.*

Unlike the problem of distinguishing knots, the problem of developing general means for proving that one knot can *be deformed into another remained untouched. But in 1957,* ***C. Papakyriakopoulos*** *proved Dehn's Lemma, and it soon became the centerpiece of a series of major developments in the subject.*

In 1968, ***F. Waldhausen*** *proved that two knots are equivalent iff certain algebraic data associated to the knots are the same. The interplay between algebra and geometry was essential to this work, and the connection was provided by Dehn's Lemma.*

The late 1950's through the 1970's were also marked with by an extensive study of the classical knot invariants, and in particular, how properties of the knot were reflected in the invariants. For instance, ***K. Murasugi*** *(1958) proved that if a knot can be drawn so that the crossings alternate from over to under, then the coefficients of its Alexander polynomial alternate in sign. Marasugi's work (1971) also detailed relationships between knot invariants and* symmetries *of knots, another major topic in the subject.*

In a completely different direction, the investigation of higher dimensional *knots (such as knotted 2-spheres in 4-space), became a significant topic. By 1970 it had become a well-developed area of topology.*

Since 1970, knot theory has progressed at a tremendous rate: In 1970, ***John Conway*** *discovered a quick way to calculate Alexander polynomials, totally different from any classical method.*

In 1984, ***Vaughan Jones*** *discovered a new knot polynomial invariant that distinguishes* handedness. *In 1991,* ***Doll*** *and* ***Hoste*** *discovered the HOMFLY polynomial that generalizes both the Alexander and the Jones polynomials.*

During 1971–1989, knot theory had been applied to fields of theoretical physics and biology. It began in 1971, when ***H.N.V. Temperley*** *and* ***E.H. Lieb*** *linked knot diagrams to* statistical-mechanics *models via certain* von-Neumann matrix algebras. *In 1987* ***Louis Kauffman*** *interpreted the Jones' polynomials in statistical-mechanics terms.*

In 1982, geneticists discovered that DNA molecules can form knots and links. Thus, topology of knots became an important practical issue in biology.

When most of us think of deoxyribonucleic acid (DNA), we picture something like the neat, tidy double helix. In reality, this double helix consists of two very long curves intertwined millions of times. The DNA strand is only a few molecules wide, but several centimeters *long, tightly coiled inside the nucleus of every cell in our bodies.*

To give a more accurate picture of what real DNA looks like, imagine the nucleus of a cell scaled up to the size of a basketball. The DNA strand then scales to the width of thin fishing line about 200 km in length — packed inside our basketball nucleus. On this tremendous scale, we can easily imagine that the DNA strand could become tangled and knotted in such a cramped space.

In reality, that is exactly the case. The problem comes when it is time for the DNA to replicate to form another cell. Then the double helix of DNA has to split in two to complete the process of cell division. When the strand becomes knotted, the DNA cannot separate intact at crossings in the knot.

In order for the DNA to separate, replicate, and recombine, special enzymes in the nucleus actually "cut" the DNA strand so replication can occur and then reattach the loose ends once the crossing is resolved.

The particular fascination in this process for geneticists is the fact that chemical changes occur in the DNA strand as a result of this process. Changes in the DNA structure due to the actions of these enzymes have required geneticists to use knot theory in their study of molecular biology.

By understanding knot theory more completely, scientists are becoming more able to comprehend the massive complexity involved in the life and reproduction of the cell. More knowledge of knots and their properties may hold one of the keys unlocking the mystery of DNA in the new millennium.

In 1982, ***Simon Donaldson*** *proved that there is a topological space which is topologically equivalent to* $\mathbb{R}^4$ *and which is a differentiable manifold. It is particularly interesting because space-time is* 4 *dimensional.*

In 1989, ***E. Witten*** *applied topological ideas to Quantum Field Theory in such a way as to:*

- *give physical interpretation to* ***Donaldson's*** *work on* 4*-space*
- *find an intrinsically* 3*-D approach to Jones' polynomials*
- *generalize the Jones polynomials to knots that are tied in an arbitrary* 3*-D manifold.*

1947 CE, Oct. 14 **Charles Elwood Yeager** (b. 1923, U.S.A.). Pilot. Broke the "*sound-barrier*" in the Bell X-1 rocket-powered plane by flying faster than the speed of sound (Mach one) at Muroc Air Force Base in California soaring 96 km above the earth. He then set another record on Dec. 12, 1953 by flying $2\frac{1}{2}$ times the speed of sound in a Bell X-1A.

1947 CE **Willis Eugene Lamb** (1913–2008, U.S.A.). Physicist. Measured slight deviations from the predictions of Dirac's theory for the spectroscopy of the hydrogen atom. This *Lamb shift* boosted the development of *Quantum Electrodynamics* (QED). Lamb shared the Nobel prize in physics with **Polykarp Kusch** (1911–1993, U.S.A.) who arrived at the same discovery, independently.

In the Schrödinger equation for hydrogen, the only electromagnetic effect included is the Coulomb interaction between the proton and the electron. But the electron, when in an excited state, is a source of radiation. Furthermore, in any energy level (even the ground state), the electron may temporarily emit a *virtual* photon, and then re-absorb it. Furthermore, the virtual photon mediating the Coulomb interaction, may split into a virtual electron-positron pair, which quickly annihilate each other to become the virtual photon again. These effects produce a further splitting of energy levels, which are degenerate according to the *fine structure* (Dirac) formula.

Thus, this type of splitting (*Lamb shift*) is to be sought in the details of interactions between the real electron and the fluctuating electromagnetic and electronic quantum fields. Unfortunately, the latter interactions always leads mathematically to an infinite electron and photon self-energies (related to the infinite *radiation reaction* in pre-quantum electron theories), due to short-distance effects and the infinite number of high-frequency modes of the EM and electron fields.

In the scheme of QED, the infinite energy terms are systematically subtracted from the interaction and it is found that certain small finite terms remain. This small effect accounts for the Lamb-shift almost exactly, not only for hydrogen but also in *deuterium* and *ionized Helium* (He^+). One can think of these finite residual effects as due to perturbations to electron orbital states due to *vacuum polarization* (caused by virtual electron-positron pairs) and due to fluctuations in the real electron's motion due to its interactions with virtual fluctuations in the electromagnetic field.

Very similar effects are found for the motion of the electron in a *magnetic field*. Here again, the electron's virtual radiation can react back on the electron, and produce corrections to the electron's magnetic moment. Calculations within the framework of QED, to order α^6, (with $\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \simeq \frac{1}{137}$ the *fine structure constant*), predict a small change in the electron's *intrinsic* *magnetic moment* from the Dirac value of one Bohr magneton. These calculations are in complete agreement with experiment to within observational and theoretical uncertainties.

According to quantum mechanics as applied to Maxwell's theory of radiation, the interaction of electrons with the EM field occurs via the emission and absorption of photons. The probability for emission, which is a measure of the strength of the interaction, is proportional to the *fine structure constant* α — the dimensionless quantity which is constructed from the basic physical constants e, h and c. Maxwell's theory satisfies the postulates of Special Relativity, this quantum theory of radiation is a relativistic quantum theory. **Dirac** (1928) has modified the Schrödinger equation for the hydrogen atom, such as to make it too consistent with STR. He showed that the requirements of relativity as imposed on the quantum theory of the hydrogen atom, have the following consequences:

(1) The electron has *intrinsic spin* angular moment of $\frac{h}{4\pi}$.

(2) The interaction energy of the atom with a weak external magnetic field reveals an electron gyromagnetic ratio *twice* that predicted by classical magnetism (thus the **Dirac** intrinsic magnetic moment of the electron is one Bohr magneton rather then on *half* a magneton).

(3) There are '*fine structure*' corrections to the Bohr (E_n) formula for the hydrogen energy levels: each level previously specified by the quantum number n, splits into n different levels with the corresponding energies

$$E_{nj} = E_n \left[1 + \frac{\alpha^2}{n}\left(\frac{1}{j+1/2} - \frac{3}{4n}\right)\right],$$

where the *total* electron angular momentum j can take the values $j = \frac{1}{2}, \frac{3}{2}, \dots, n - \frac{1}{2}$ and the *orbital* angular momentum is $\ell = j \pm \frac{1}{2}$.

(4) There is a positively charged counterpart to the electron — the positron.

Results (1) and (2) were known previously, but had been semiempirically grafted onto the non-relativistic theory to obtain agreement with experiment. The Dirac equation shows that both results arise due to fundamental reasons. The fine structure formula had been derived already in 1916 by **A. Sommerfeld** on the basis of the Old Quantum Theory (but with incorrect interpretation of the quantum numbers). Result (4) was a novel prediction.

In spectroscopic notation, four quantum numbers are specified for each electron in an atom. In particular, a one-electron atom can have the following states: $n = 1,\ \ell = 0,\ j = \frac{1}{2}\ (1S_{1/2});\ n = 2,\ \ell = 0,\ j = \frac{1}{2}\ (2S_{1/2});\ n = 2,$ $\ell = 1,\ j = \frac{1}{2}\ (2P_{1/2});\ \ell = 1,\ j = \frac{3}{2}\ (2P_{3/2})$; etc.

The Dirac theory predicts a difference in energy between levels of different j for the same value of n (*fine structure*), due to the *spin-orbit coupling* and other relativistic effects. Thus, for the hydrogen atom $(Z = 1)$, the spin-orbit splitting between the $n = 2\ \ j = \frac{1}{2}$ and $j = \frac{3}{2}$ levels is

$$E(2P_{1/2}) - E(2P_{3/2}) \simeq \frac{m_e c^2 \alpha^4}{32} = 4.53 \times 10^{-5}\ \text{eV} = h \times 10960\ MHz.$$

However, Dirac's theory predicts that the states $2S_{1/2}$ and $2P_{1/2}$ are degenerate. But in 1947 Lamb and Rutherford measured the transition $2P_{3/2} - 2S_{1/2}$ as a function of an applied magnetic field. In the limit of zero field, the observed frequency value was approximately 1060 MHz *lower* than what could be expected from the above fine structure interval.

A more recent (1976) experimental value for the *Lamb shift*[1071] was

$$(\Delta E)_{\text{Lamb}} = E(2S_{1/2}) - E(2P_{1/2}) = 1057.862\ \text{MHz}.$$

A corresponding *theoretical* value (1975) [which takes into account vacuum polarization and the fluctuations in the electron's propagation in the Coulomb field due to emission and absorption of virtual photons, as well as nuclear recoil, effects of a finite nuclear radius and higher order so-called radiative corrections] yielded 1057.864 ± 0.024 MHz.

Since this figure is accurate to 10^{-1} MHz, theory and observations are in agreement to one part in 10^{-10} of the ground state binding energy.

1947 CE *Mount Palomar Astronomical Observatory* (California; altitude 1725 m) equipped with a 200 inch reflector telescope. It can collect 1 million times as much light as the human eye.

1947 CE *The Dead Sea Scrolls* were discovered in earthen jars in a cave near *Khirbet Qumran* (Israel). These scrolls contain religious texts offering insight into ancient Judaism and early Christianity.

[1071] For further reading, see:

- Park, D., *Introduction to Quantum Theory*, Dover, 2005, 601 pp.
- Itzykson, C. and Z-B. Zuber, *Quantum Field Theory*, McGraw-Hill, 1980, 705 pp.

1947–1949 CE *The rebirth*[1072] *of Israel*; The establishment of a Jewish home in the Land of Israel was the result of a British government decision made in 1917. Britain ruled the country under mandate from the League of Nations. The United Nations decided on 29 November 1947 to establish an independent Jewish state. On May 14, 1948 the British mandate came to an end, and the Jews, under the leadership of David Ben-Gurion, proclaimed the state of Israel. The same day Arab armies from Egypt, Lebanon, Syria, Jordan, Iraq, Saudi Arabia and Morocco invaded and attacked Israel who beat off the invaders. Armistice agreements were signed in 1949.

1947–1949 CE **Cecil Frank Powell** (1903–1969, England). Physicist. Discovered (with coworkers) a new subatomic particle: the charged *pi-meson* (or *pion*), denoted as $\pi^{\pm}$ (the positively charged π^{+} and the negative π^{-} are each other's antiparticle) the first true meson to be discovered. It was detected in cosmic rays in the Bolivian Andes. The particle was predicted by Hideki Yukawa (1935). In the interim period it was believed that the *muon* was the Yikawa meson, although theoreticians in 1942 and 1946 independently concluded that there must be two mesons.

Powell was a professor at the University of Bristol (1948–1963). Awarded the Nobel prize in physics (1950). Discovered the modes of decay of kaons (K-mesons).

1947–1960 CE **Robert Gaston André Maréchal** (b. 1916, France). Optical physicist. A pioneer in photographic image enhancement (optical processing for quality improvement) by the use of coherent spatial filtering techniques. His success with these techniques was to provide a strong motivation for future expansion of interest in the optical data information-processing field.

Maréchal regarded undesired defects in photographs as arising from corresponding defects in the optical system that produced them. He then combined *absorbing and phase-shifting filters* to reconstitute the detail in badly blurred photographs. These filters are transparent coating deposited on optical plates so as to retard the phase of various portions of the spectrum. His work led to the eventual replacement of the photographic stages, in increasingly many applications, by real-time electro-optical devices (e.g.: array of ultrasonic light modulators forming a multichannel input).

[1072] The first Commonwealth of Israel was established by King Saul in ca 1025 BCE and stabilized by King David in 990 BCE. It lasted until 586 BCE. The second Commonwealth was established by Sheshbatsar in 537 BCE in the wake of the Cyrus Declaration, and ended in 70 CE with the Roman destruction of Jerusalem.

Maréchal was born in La Garenne and educated at the University of Paris. He was a professor at the Paris Institute of Optics (1955–1985).

Nonequilibrium Thermodynamics

Thermodynamics can be divided into three distinct parts, the study of which corresponds to three successive stages in its development:

Equilibrium thermodynamics (ET)

Also known as thermostatics, zero-order thermodynamics or classical thermodynamics. It describes the end state *of thermodynamic evolution in an isolated system, when processes reach quasi-permanent states of equilibrium, and when the corresponding matter and energy fluxes vanish (e.g., chemical equilibrium of all reactions, thermal equilibrium, pressure equilibrium, etc.). Sequences of such states adequately describe both* reversible *and many types of irreversible phenomena*[1073]*. These sequences obey the first and second law of thermodynamics.* Reversible *sequences conserve both the entropy and internal energy of the isolated system, while* irreversible *sequences conserve energy but increase the entropy.*

The concept of equilibrium is central to all aspects of thermodynamic formalism: Even when a system undergoes a finite and continuous change, we may describe the phenomenon by a succession of a great number of quasi-equilibrium states, provided the rate of change is not too high.

Furthermore, for a reversible *sequence, the process does not furnish an "arrow of time"; that is to say, the time-reversal of a reversible process, is also an allowable reversible process (just as is the case in a classical mechanical or electromagnetic system with no dissipative effects, or in a quantum system between measurements and not involving the weak nuclear forces). ET generally yields an accurate description only for reversible processes or slow irreversible ones — an idealization which real processes can at best only approximate, in the limit in which they occur with infinite slowness.*

[1073] Only *sufficiently slow* irreversible phenomena are describable as sequences of equilibrium states.

Yet, ET provides a satisfactory explanation for a plethora of physicochemical and biochemical phenomena in a variety of systems: Thermodynamic methods are thus a daily tool for engineers, physicists, chemists, biologists and material scientists, as well as in other fields of the scientific endeavor.

Equilibrium thermodynamic was an achievement of the 19^{th} *century, and the first response of physics to the problem of natural complexity. During that century irreversible processes were viewed as nuisances — as being not worthy of study. They were obstacles to obtaining maximum yield in thermal engines. Therefore, the aim of engineers constructing thermal engines has been to minimize losses due to irreversible processes.*

The interest in irreversible processes started when it was recognized that most systems are not in thermal, mechanical or chemical equilibrium, and that the majority of events in biological systems operate under action of nonequilibrated forces which in turn produce fluxes (of concentrations, heat, volumes, charges, etc).

Nonequilibrium thermodynamics (NT) encompasses unidirectional time phenomena over a wide range — from simple irreversible processes like heat conduction, or the evaporation of an open perfume bottle, to complicated processes involving self-organization. A system in thermal equilibrium is dead (or immortal, depending on one's point of view); it is timeless and lacks history. Its fate, however, may be avoided with the help of external forces, that keep the system away from thermodynamic equilibrium.

Near-equilibrium thermodynamics (NET)

In the equilibrium state, entropy of an isolated system is at a maximum and is a function of any complete set of state variables, and is spatially uniformly distributed[1074] *within each of the media comprising the system. There is no need to know how this maximum value was reached, and the only changes in entropy result from an interaction with the surrounding during a reversible or irreversible (either slow or of finite duration) process.*

However, in systems which are not in thermodynamic equilibrium it is no longer justified to assume spatial or temporal homogeneity, and entropy, like any other thermodynamic variable, becomes a field function of space and time, obeying its own continuity equation.

In fact, the presence of forces, fluxes and chemical rate – imbalances in nonequilibrium systems imply that these systems are spatially non-uniform

[1074] As all other extensive and intensive state variables.

and/or undergo net chemical processes[1075]. *Therefore, for the most general cases the compositions and all other state variables of the system may be time- and space-dependent. We assume, however, that at each point of the nonequilibrium system, i.e., for each infinitesimal volume element, and during each infinitesimal time interval, thermodynamic state variables are the same functions of each other as in the equilibrium state of the medium in question (concept of local equilibrium).*

Ilya Prigogine *(1946) showed that within the range of validity of Onsager's relations, the entropy production of a nonequilibrium system not far from equilibrium takes its minimum value at steady states (Theorem of minimum entropy production). The system thus evolves toward a stationary state compatible with the constraints imposed on the system by the boundary conditions. (They may, for instance, correspond to two points in the system kept at different temperatures, or to a flux of matter that continuously supports a reaction and eliminates its products.)*

During its evolution the system transfers entropy to the outside world, and the particular stationary state toward which the system tends is the one in which the transfer of entropy to the environment is as small as is permitted by the boundary conditions. Thus, when the boundary conditions prevent the system from achieving equilibrium it evolves to a state as close to equilibrium as possible $\left(\frac{\partial S}{\partial t}\right.$ *=minimal;* S = *entropy). However, as in ET, the initial conditions are "forgotten" by the system. Whatever they were, the system will finally reach the state determined by the imposed boundary conditions.*

When fluctuations shift the system away from the minimum, the second law of thermodynamics imposes the return toward the attractor. Like in ET, the system follows an evolution that leads it to a stationary situation that is established once for all (up to small fluctuations).

For both ET and NET, some aspects of the dynamics and distribution of the fluctuations can be estimated from the equilibrium (or steady-state) theory itself.

Nonlinear thermodynamics — far from equilibrium

In NET there is a linear relation between forces and fluxes, the forces are weak, and the system reaches and remains in a state of least dissipation of

[1075] Here "forces" is meant in a generalized sense: mechanical forces, concentration gradients, heat sources and sinks, electromagnetic fields, etc.

free energy. Such restrictions exclude most chemical reactions and all biological systems. When the system is far from equilibrium, the dependence of fluxes upon forces is very complex and difficult to evaluate. In general, the magnitude of forces, and therefore the corresponding fluxes are large. Consequently, linear relationships are not available and the results derived for NET cannot be used. Yet, in spite of all these difficulties, evolutionary criteria can be derived: Prigogine[1076] *has developed methods for describing systems far from equilibrium which can evolve into* stable dissipative states*, that may show a variety of interesting behavior — including spatial and temporal oscillations. Prominent examples are:*

- *The transition from laminar flow to* turbulence*. Although turbulent motion appears irregular or chaotic on the macroscopic scale, it is highly organized on the mesoscopic scale. The multiple space and time scales involved in turbulence correspond to coherent behavior of numerous molecules. Part of the energy of the system, which in laminar motion resides in the thermal motion of molecules, is now transferred to macroscopic organized motion.*
- Bénard instability: *Bénard*[1077] *(1900, 1901) experimented with a thin layer of liquid with a free surface and heated from below, and observed a hexagonal convection pattern.*

  Consider a fluid layer maintained between two horizontal plates with separation h. The top plate is held at a constant and uniform temperature T_c, while the bottom plate is held at a constant and uniform temperature $T_H > T_c$. At steady state the constant force $X = \frac{1}{h}(T_H - T_c) = \mathrm{grad}\, T$ generates a constant heat flux, and there is also a linear relationship

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[1076] **Ilya Vicomte Prigogine** (1917–2003, Moscow) came to Brussels, Belgium in 1927; Ph.D., Free University of Brussels (1942) under **Theophile de Donder** (1893–1957, Belgium; a pioneer in the field of nonequilibrium thermodynamics). **Prigogine** has built up there, since WWII, one of the leading schools of statistical mechanics and thermodynamics. For more than 40 years his own contribution, for which he was awarded the Nobel prize for chemistry (1977), has been the extension of irreversible thermodynamics, and its application to physical and biological systems.

[1077] **Bénard, H.** and **D. Brunt**, The cellular vortices in a liquid sheet, *Ann. Chem. Phys.* (7) **23**, 62–144, 1901. This problem was also studied by **Lord Rayleigh** (1916), **H. Jeffreys** (1926) and **L. Prandtl** (1929). The unexpected result was obtained that with sufficiently large values of viscosity and thermal conductivity the cellular steady–state observed by **Bérnard** may be thoroughly stable. But at that time, no satisfactory mathematical treatment of the onset of the Bénard cells could be given. The phenomenon occurs in meteorology.

> *between the force and the flux via Fourier's law. One gradually and slowly increases $\Delta T = T_H - T_C$ such that the fluid may adjust at each moment to the given constraint.*
>
> *It is then observed that for a critical value $(\Delta T)_c$, the linear flux-force relation is no longer valid and the system enters a nonlinear regime. Below $(\Delta T)_c$, the fluid is at rest* and featureless, *but at $(\Delta T)_c$ the stationary state of heat conduction* becomes unstable and *convection* sets in; *a small increase in the gradient 'organizes' the fluid into regular convection cells of macroscopic* size *(the Bénard cells).*
>
> *This convection, corresponding to a coherent motion of ensembles of molecules in the cells, increases the rate of heat transfer from the bottom to the top of the fluid. At the molecular level, a complex spatial organization of immense numbers of molecules move coherently. Correlations between molecules extend over distances of the order of centimeter, whereas intermolecular attraction forces act only over distances of the order of 10^{-8} cm. Similarly, the time scales are different — they correspond not to molecular times (such as periods of vibration of individual molecules, of the order of 10^{-15} sec) but to macroscopic times: seconds, minutes, or hours.*

For a given fluid and at $(\Delta T)_c$, a wide variety of patterns (square, or hexagonal cells) may arise. As ΔT is increased beyond $(\Delta T)_c$, new self-organized phenomena appear.

The Bénard system is a dissipative structure in which viscous forces away from equilibrium play a basic role. For given values of the constraints (the gradient of temperature), the entropy production of the system is increased, in contrast with the theorem of minimum entropy production valid for NET. The phenomenon of Bénard instability is a striking example of how disorder may essentially contribute to the creation of organization and order[1078].

[1078] In a uniform fluid, all points of the system are equivalent. However, adjacent cells rotate in opposite directions, indicating that, locally, the symmetry of the fluid had been broken. This is therefore a *symmetry-breaking* instability which results from the energy dissipation in the system. Moreover, the motion of the fluid – cell clockwise or anticlockwise (w.r.t. a particular observer), is *unpredictable*, as the experimental apparatus is completely symmetric w.r.t. the two alternatives. The 'decision' depends upon *random fluctuations*, which first occur as the critical surface in parameter space [in this case, the point $(\Delta T)_c$] is reached.

In classical thermodynamics, as formulated by Gibbs (1876), the fundamental equation which relates the changes in extensive variables is (per unit volume)

$$dU = TdS - pdV + \sum_j \mu_j dN_j, \tag{1}$$

where S is the entropy, T the absolute temperature, U the internal energy, p the pressure, μ_k the chemical potential of the k^{th} chemical species, and N_k the corresponding number of moles. Under the assumption of the principle of local equilibrium, Eq. (1) is also valid for nonequilibrium states which are not too far from equilibrium.

This assumption is based on the idea that the macroscopic evolution of the system takes place over times which are long w.r.t. those necessary to establish an equilibrium in a very small portion of the system and that the spatial variation of the quantities considered is small over molecular mean free path.

This is equivalent to assuming that the functional dependences amongst p, T, S, U, V and N_j is the same as in thermostatics, even though we are now dealing with functions that vary with time and space.

For an isolated system which exchanges neither energy nor matter with the exterior, the second law of thermodynamics states that $dS/dt \geq 0$ where S is the total entropy of the system. In the case of non-isolated systems, it is assumed that the total change of entropy is the sum of two terms $dS = d_eS + d_iS$, where d_eS describes the change of entropy in the system caused by interaction with the environment and is of indefinite; sign, whereas d_iS describes internal dissipative processes $(d_iS \geq 0)$.

In a stationary state, $dS = 0$ this then implies that $dS_e = -d_iS < 0$, i.e., the system transfers entropy to the outside world. Therefore, at the stationary state the system's activity continuously increases the entropy of its environment.

Thus where there is an exchange of entropy between the system and the exterior, we can think of entropy as a fluid that can be destroyed or produced. In other words, we can talk about the entropy source density σ (entropy production per unit volume per unit time), the entropy flux density $\boldsymbol{J}_S$ (lies along the direction of entropy flow with a magnitude of entropy flow per unit area per unit time, i.e. $\left|\frac{S}{\text{area}\times\text{time}}\right|$), and the entropy density s (entropy per unit volume). All these entities are connected via a continuity equation

$$\frac{\partial s}{\partial t} + \operatorname{div} \boldsymbol{J}_S = \sigma, \tag{2}$$

where s, $\boldsymbol{J}_S$ *and* σ *are functions of position and time. Clearly, mass, molecular species populations and energy have similar conservation laws of their own. Eq. (2) can be considered as a differential formulation of the second law, adapted to open systems.*

We must next specify σ*. It can arise from three different sources: thermal, material and chemical. Upon using Eq. (1), and the vector identity* $-B \operatorname{div} \boldsymbol{b} \equiv -\operatorname{div}(B\boldsymbol{b}) + \boldsymbol{b} \cdot \nabla B$ *and summoning the laws of conservation of mass, energy and electric charge for a single type of reaction*[1079]*, we are able, via Eq. (2), to identity* $\boldsymbol{J}_S$ *and* σ *as follows*

$$\sigma = \boldsymbol{J}_q \cdot \operatorname{grad} \frac{1}{T} - \sum \boldsymbol{J}_j \cdot \operatorname{grad} \frac{\mu_j}{T} - \boldsymbol{J}_e \cdot \operatorname{grad} \frac{\phi}{T} + J_{\text{chem}} \frac{A_f}{T} \tag{3}$$

$$\boldsymbol{J}_S = \frac{1}{T}[\boldsymbol{J}_q - \sum \mu_j \boldsymbol{J}_j - \boldsymbol{J}_e \phi], \tag{4}$$

where $\boldsymbol{J}_q$ *is the density of heat flow,* $\boldsymbol{J}_j$ *the densities of material flow,* $\boldsymbol{J}_e$ *the density of electric current,* ϕ *the electrochemical potential,* J_{chem} *the chemical reaction rate per unit volume, and the affinity* A_f *is the 'driving force' of the reaction, which tends to zero when local chemical equilibrium is reached.*

Eq. (3) can be recast in the compact form $\sigma = \sum(\boldsymbol{J}_k \cdot \boldsymbol{X}_k)$ *where* X_k *are generalized forces. In general, every single flow* $\boldsymbol{J}_k$ *may depend upon all the generalized forces present:* $\boldsymbol{J}_k = \boldsymbol{J}_k(\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_N)$*. The simplest forms for the individual flows are governed by the empirical laws:* $\boldsymbol{J}_q = -\lambda \operatorname{grad} T$ *(Fourier),* $\boldsymbol{J}_j = -D \operatorname{grad} \mu_j$ *(Fick),* $\boldsymbol{J}_e = -\gamma \operatorname{grad} \phi$ *(Ohm).*

Consider, for instance, a system in which only heat flow takes place, i.e., $\sigma = \boldsymbol{J}_q \cdot \operatorname{grad}\left(\frac{1}{T}\right)$*. Substituting for* $\boldsymbol{J}_q$ *its value from Fourier's law, one obtains* $\sigma = \lambda(\operatorname{grad} T)^2/T^2 \geq 0$*. This illustrates the general fact that the entropy generated locally cannot be negative irrespective of whether the system*

1079 *Conservation of mass* for each component, per unit volume, is $\frac{\partial n_j}{\partial t} = -\operatorname{div} \boldsymbol{J}_j$, where $\boldsymbol{J}_j$ is the mass flux density for the j^{th} component and the concentration is $n_j = \frac{N_j}{V}$. *Conservation of energy* is given by $\frac{\partial u}{\partial t} = -\operatorname{div} \boldsymbol{J}_q$, where $u = \frac{U}{V}$ and $\boldsymbol{J}_q$ is the heat flux density. To take account of the flow of electricity and the occurrence of chemical reactions, we define the total specific charge per unit volume $e = \sum n_j e_j$, and add to μ_j the *electrochemical potential* times the charge e_j: ϕe_j. Gibbs' equation will then read $dU = TdS - pdV + \sum \mu_j deN_j + \phi de + A_f dN_r$, where deN_j stands for the part of dN_j that is due to ions or molecules entering or leaving the unit volume under consideration, while dN_r is the number of molecular chemical-reaction events (assumed to be of a single type) which occur. *Conservation of charge* reads $\frac{\partial e}{\partial t} = -\operatorname{div} \boldsymbol{J}_e$, $\boldsymbol{J}_e$ being the charge flux density. Here A_f is the *affinity* of the chemical reaction.

is isolated or not, and irrespective of whether the process under consideration is irreversible or not.

In the general case, we integrate Eq. (2) over the volume V of the system and substitute therein $\boldsymbol{J}_S$ from Eq. (4). The result is $dS = d_eS + d_iS$ where

$$d_iS = dt \int_V \sigma dV; \qquad d_eS = -dt \int_A \frac{1}{T} \boldsymbol{n} \cdot \{\boldsymbol{J}_q - \sum \mu_j \boldsymbol{J}_j - \boldsymbol{J}_e \phi\} dA, \quad (5)$$

and $\boldsymbol{n}$ is the unit normal to the surface A of the open system.

The interpretation of this result is as follows: the entropy increase of an open system is composed of a part d_eS due to the exchange of heat, matter, and charge between the system and its environment, and a part d_iS, which arises from processes occurring inside the system. (In the special case of a closed system $d_eS = \frac{dQ}{T}$). We now postulate *that $d_iS \geq 0$ is valid for an arbitrary volume, however small. It then follows from Eq. (5) that $\sigma \geq 0$. Thus, the assertion that the overall entropy produced in any process is zero or positive is expressed both in its global* and *local forms.*

This statement constitutes one of the basic postulates of irreversible thermodynamics. In a quasi-equilibrium process, $d_iS \approx \sigma = 0$ and $\frac{dS}{dt} \approx \frac{d_eS}{dt}$, confirming that in such processes, the only way that a system's entropy may change is for it to cross the boundaries of the system. If we isolate *the system from external entropy flux $(d_eS = 0)$, we are left with $\frac{dS}{dt} = \frac{d_iS}{dt} \geq 0$, which is compatible with our knowledge that the entropy of an isolated system may only increase.*

Next, consider the case of two fluxes $\boldsymbol{J}_q$ and $\boldsymbol{J}_e$. These vector functions may be expanded in a Taylor series about the state $(\boldsymbol{X}_q = 0, \ \boldsymbol{X}_e = 0)$. In this expansion, one can dispense with the zero-order terms $\boldsymbol{J}_q(\mathbf{0}, \mathbf{0})$ and $\boldsymbol{J}_e(\mathbf{0}, \mathbf{0})$, because the flows of heat and electricity vanish when the gradient vectors $\boldsymbol{X}_q$ and $\boldsymbol{X}_e$ become zero; One is thus left with

$$\boldsymbol{J}_q(\boldsymbol{X}_q, \boldsymbol{X}_e) \approx \boldsymbol{X}_q \cdot \left[\frac{\partial \boldsymbol{J}_q}{\partial \boldsymbol{X}_q}\right]_{\mathbf{0},\mathbf{0}} + \boldsymbol{X}_e \cdot \left[\frac{\partial \boldsymbol{J}_q}{\partial \boldsymbol{X}_e}\right]_{\mathbf{0},\mathbf{0}}$$

and a similar equations for $\boldsymbol{J}_e$. If $\boldsymbol{X}_q$ and $\boldsymbol{X}_e$ are small, products of their component may be neglected and the result of the approximation applies to near-equilibrium thermodynamics. In this linear region the components of $\boldsymbol{J}_q$, and $\boldsymbol{J}_e$ are linear functions of the components of $\boldsymbol{X}_q$ and $\boldsymbol{X}_e$. This can be written in the abbreviated notation, in the general case of n simultaneous fluxes:

$$J_i = \sum_{j=1}^{n} L_{ij} X_j, \quad L_{ij} = \left(\frac{\partial J_i}{\partial X_j}\right). \quad (6)$$

The corresponding entropy production function is

$$\sigma = \sum_i J_i X_i = \sum_i \sum_j L_{ij} X_i X_j \geq 0. \tag{7}$$

The above equations are readily amenable to analysis of stationary states of thermodynamic systems. A system is said to be in a stationary state if its macroscopic parameters — such as temperature, pressure, composition, and entropy — do not depend on the time (although these parameters may still vary from point to point in the system).

Thus, if heat is added at a constant rate to one end of a metal bar and withdrawn at an equal rate from its other end, the temperature at each point of the bar approaches a time-independent value. All the same, the temperature varies along the length of the bar, and entropy is produced continually as a result of heat conduction.

In an isolated system, or a system in contact with a uniform environment, the stationary state degenerates into a subclass of an equilibrium state where the macroscopic variables depend neither on time nor on position.

Nonequilibrium stationary states cannot endure unless the entropy-producing processes are sustained by a continual flux of energy, matter or both, between the system and its surroundings.

Consider a system characterized by n *independent forces* $X_1, X_2, \ldots, X_n$, *and let* k *of them, say* $X_1, X_2, \ldots, X_k$, *be kept at fixed values through the operation of external constraints. It is then found empirically that the remaining forces will also become constant with the passage of time.*

The stationary state reached by this method is known as a stationary state of order k. *It turns out that when the stationary state of order* k *is established, the fluxes* $J_{k+1}, J_{k+2}, \ldots, J_n$, *conjugate to the unconstrained forces, are individually zero. In the special case that no forces are held fixed* $(k = 0)$ *the system will continue to evolve until all fluxes and forces have vanished. Thus a stationary state of order zero is just a state of thermodynamic equilibrium.*

These observations are readily interpreted with the aid of Eq. (7). According to this formula, σ *is a positive definite quadratic form in the forces* X_i, *that is,* σ *is zero when all* X_i *are zero, and positive otherwise. Therefore, by keeping* k *forces fixed the function* σ *displays a minimum when the remaining* X_i *assume values satisfying the equations*

$$0 = \frac{\partial \sigma}{\partial X_i} = 2 \sum_j L_{ij} X_j = 2J_i, \qquad (i = k+1, k+2, \ldots, n;\ j = 1, \ldots, n),$$

where use has been made of the Onsager reciprocity relations $L_{ij} = L_{ji}$.

This result asserts the vanishing of the fluxes corresponding to the forces which are not held fixed. Therefore, in the domain of the validity of linear thermodynamics of irreversible processes, the steady states are characterized by a minimum entropy production. Consequently, the system reaches and remains in a state of least dissipation of free energy (Prigogine, 1947; de Groot, 1951).

Any stationary state represents a stable situation, that is to say, if a transient interference has caused perturbation of the stationary state, the system will return to its initial stationary condition. This can be shown with the aid of Eq. (7). Assume that the system reached a stationary state for which the forces $X_1, X_2, \ldots, X_k$ *were held fixed. Apply a virtual perturbation* δX_m *to one of the unconstrained forces* $(m > k)$. *The flux* $J_m = \sum L_{mj} X_j$, *which is zero in the unperturbed state, will assume the nonzero value* $\delta J_m = L_{mm}(\delta X_m)$ *in the perturbed state.*

Since $\sigma = \sum\sum L_{ij} X_i X_j \geq 0$, *we see that the coefficient* $L_{mm} > 0$, *so that* $\delta\sigma \approx (\delta J_m)(\delta X_m) = L_{mm}(\delta X_m)^2 > 0$. *Thus, the above principle of minimum entropy production will force* (δX_m) *to decrease. Then, the flux* (δJ_m) *will tend to nullify the perturbation* δX_m *which caused it, and eventually restore the unperturbed stationary state.*

This is exactly the principle of le Châtelier (1888) for thermostatic equilibrium $(k = 0)$; *It was extended to* $k = 1$ *by Prigogine (1947) and for arbitrary* k *by de Groot (1951).*

The application of some of the above ideas to biological phenomena started already in 1932, *when L. von Bertalanffy advanced the hypothesis that living organisms and cells should be treated as open thermodynamic systems. A similar notion was expounded by E. Schrödinger (1943), who drew attention to the fact that biological organisms only survive by continuously exchanging matter with their surroundings.*

Therefore, living structures need not necessarily follow the law of increasing disorder. But in order to perform vital tasks, cells must generate nonequilibrium conditions in their environment.

Prigogine's theory[1080] *(1946) accounted for several features of life which previously appeared to be inconsistent with the laws of physics, and for which*

[1080] For further reading, see:

- Babloyantz, A., *Molecules, Dynamics and Life*, Wiley, 1986, 345 pp.
- Nicolis, G. and I. Prigogine, *Self-Organization in Nonequilibrium Systems*, Wiley, 1977, 491 pp.

explanation was sought in terms of ideas foreign to physical science. Moreover, the theory of open systems provided quantitative laws regulating basic biological phenomena, such as metabolism and growth.

Like some inanimate matter (planets, for example), but much more so, the growth of living organisms and their cells is characterized by transitions leading to states of ever greater order and increasing differentiation; and once the adult stage is reached, the organism reverts to gradual decay to the state of equilibrium (a.k.a. death).

The apparent *conflict between the principles governing the behavior of animate and inanimate bodies may be easily resolved if one treats a living organism as an open system, which exchanges both energy and matter with its environment. For a system of this kind, an increase in the entropy s (per unit volume) may be avoided by an importation from outside of the* negative amount of entropy *(Schrödinger, 1943), with $\frac{d_e s}{dt}$ exceeding in absolute value the inescapable positive production of entropy $\sigma = \frac{d_i s}{dt}$ inside the living object.*

The main contribution to the local entropy production σ arises from metabolism, *that is, from the chemical and physical changes continuously occurring in living organisms and cells. Metabolism comprises processes by which assimilated food is built up into protoplasm and broken down into simpler substances or waste matter.*

During the period of growth $\left|\frac{d_e s}{dt}\right| > \left|\frac{d_i s}{dt}\right|$ and since $\frac{d_e s}{dt} < 0$ their sum $\frac{ds}{dt} < 0$. This decrease in the organism's entropy s manifests itself in improved organization and greater differentiation of the protoplasmic structure. The withdrawal of negative entropy *from the environment is a device whereby a living organism succeeds in keeping alive, or postponing the final state of equilibrium which is the fate of inanimate matter in isolation.*

Life is therefore an extreme case of a nonequilibrium process. Living species evolve toward states of minimum internal entropy production (Prigogine, 1946), or minimum metabolism *(thus in particular $\frac{\partial\sigma}{\partial t} \to 0$, not to be confused with the steady state $\frac{\partial S}{\partial t} = 0$). This hypothesis is supported by the following observations among animals resembling one another closely: (1) the intensity of metabolism per unit mass diminishes as the size of the animal increases; (2) migrant animals usually settle in environments allowing them to function with a minimum of metabolism.*

All these aspects of thermal physics were completely overlooked by classical thermodynamics.

1947–1948 CE **John Bardeen** (1908–1991, U.S.A.), **Walter Houser Brattain** (1902–1987, U.S.A.) and **William Bradford Shockley** (1910–1989, U.S.A.) invented the *point-contact transistor*. In 1948, Shockley developed the theory of the *junction transistor*. The three received the Nobel prize for physics in 1956[1081].

1947–1951 CE **Fritz Albert Lipmann** (1899–1986, U.S.A.). Biochemist. Opened the way to current understanding of *bioenergetics* and clarified the relationship between energy use and its storage in metabolism. Discovered the *coenzyme A*, a key substance in the human body metabolism.

Suggested that there were two types of *phosphate bonds*; the ordinary kind, such as existed in a sugar phosphate, was a "*low-energy phosphate bond*", while the pyrophosphate link was an example of a "*high-energy phosphate bond*". Proposed that the high-energy phosphate bond, and its transfer to and from ATP and other molecules, was a 'common currency' of energy transfer in biology.

Lipmann was born in Königsberg, Germany, to Jewish parents. Emigrated to the United States (1939). Professor at Harvard University (1941–1957) and at Rockefeller University (1957–1970). Awarded the Nobel prize for physiology or medicine (1953).

[1081] **John Bardeen** was *twice* a Nobel Prize winner (1965, 1972) as were **Marie Curie**, **F. Sanger** and **L.C. Pauling**.

The Chemistry of Life–
From Magendie to Lipmann (1816–1951)

Biochemistry is the study of the chemistry of living matter and the chemical changes that matter undergoes. Life in its many forms, whether plant or animal, is made up of (usually microscopic) units known as cells. *A living organism may be the single cell of bacteria, or it may be a complex organism such as man. Compounds present in every cell can be classified as* carbohydrates, *fats and* proteins. Vitamins, enzymes, hormones, *and* nucleic acids *are also present but in smaller quantities.*

Man gets his fuel supply primarily from sugars, cereal grains, vegetables, fruits, nuts, and to a lesser degree from animal sources.

1. History

Until the end of the 19^{th} *century, most advances in the chemistry of biological processes came from physiologists. Much of their biochemical discoveries were incidental to their major work. These physiologists were mainly concerned with the mechanics of bodily organs and to a lesser extent they investigated chemical processes, so an overall view of the biochemical functioning of the body was not obtained.*

The most important early result of the development of organic chemistry, from the viewpoint of biochemistry, was the demonstration that natural organic compounds are subject to the same laws as inorganic substances. The urea synthesis of **Friedrich Wöhler** *(1828) and the subsequent advances in organic syntheses by* **Marcellin Berthelot** *(1860) totally undermined the support for the* vitalistic hypothesis *that a special force controlled living matter.*

Many important discoveries were made in the 19^{th} *century, but they were like isolated pieces of a jigsaw puzzle. The science was called* physiological chemistry *at this period, since it was used to help understand specific physiological problems.*

It was only at the end of the 19^{th} *century that the pieces began to fit together so that a unified picture of the chemical changes in the cells and their significance for the body as a whole could be obtained. The* borderline *between physiology and chemistry then became a science in its own right, especially after the seminal discoveries of* ***Julius von Sachs*** *(1865) and* ***Richard Willstätter*** *(1910) in the field of photosynthesis.*

By about 1920, biochemistry possessed the basic principles upon which it is still developing. The chemical nature of the body constituents was fairly well understood, the nutritional requirements could be seen, and the enzymatic and hormonal mechanisms by which metabolic processes are enabled were at least known to exist.

2. The chemical elements of life

Of the 92 naturally occurring elements, 24 elements have been shown to be essential for the growth of young animals (Tables 5.23, 5.24). The background of the selection of these particular elements is as follows.

Three characteristics of the biosphere or of the elements themselves appear to have played a major part in establishing the chemistry of living forms and directly influenced the evolutionary selection of the elements essential for life:

- *The ubiquity of water (Table 5.25), the solvent base of all life on earth. Water is a unique compound; its stability and boiling point are both unusually high for a molecule of its simple composition. Many other compounds essential for life derive their usefulness from their response to water: whether they are soluble or insoluble, whether or not (if they are soluble) they carry an electric charge in solution and, not least, what effect they have on the viscosity of water.*

  Nearly all physical properties of water are either unique or are at the extreme end of the range of a property. Its extraordinary physical properties, in turn, endow it with a unique chemistry. The main physical characteristics of water from which follows its biological importance, are:

  I. *Water remains a liquid within the temperature range most suited to life processes. Water ice in the temperature range $0\,^\circ C$–$4\,^\circ C$ will float on top of liquid water. The fact that water freezes from the surface downward rather then from the bottom upward has important biological significance.*

Table 5.23: The chemical elements of life

Element		A	$(\%)_1$	$(\%)_2$	Comments
Hydrogen	H	1	(10)	(63)	Required for water and organic compounds. The most abundant *cosmic* element. Significant in plants.
Boron	B	5			
Carbon	C	6	(18)	(9.5)	Required for organic compounds. Due to its high valency and small radius is a constituent of many molecules.
Nitrogen	N	7	(3)	(1.4)	Required for many organic compounds.
Oxygen	O	8	(65)	(25.5)	Required for water and organic compounds.
Fluorine	F	9			Constituent of teeth and bones. Growth factor.
Sodium	Na	11	(.15)	(.03)	Principal extracellular cation.
Magnesium	Mg	12	(.05)	(.01)	Required for activity in many enzymes; in chlorophyll.
Aluminum	Al	13			
Silicon	Si	14			Structural unit of diatoms.
Phosphorus	P	15	(1.0)	(.22)	Essential for biochemical synthesis and energy transfer.
Sulfur	S	16	(.25)	(.05)	Required for proteins and other biological compounds.
Chlorine	Cl	17	(.15)	(.03)	Principal cellular and extracellular anion.
Potassium	K	19	(.35)	(.06)	Principal cellular cation.
Calcium	Ca	20	(1.5)	(.31)	Major component of bone. Required for some enzymes. (Lipid digestion.)
Vanadium	V	23			Essential in lower plants. Certain marine animals.
Chromium	Cr	24			Essential for higher animals. Action of insulin.
Manganese	Mn	25			Required for activity of several enzymes. (Pyruvate metabolism. Urea formation.)

Table 5.23: (Cont.)

Element		A	$(\%)_1$	$(\%)_2$	Comments
Iron	Fe	26	(.004)		Most important transition metal ion. Essential for hemoglobin and many enzymes.
Cobalt	Co	27			Required for activity of several enzymes; in vitamin B_{12}. (DNA biosynthesis. Amino acid metabolism.)
Nickel	Ni	28			
Copper	Cu	29			Essential in enzymes and hemocyanin. (Elasticity of aortic walls. Skin pigmentation. Photosynthesis.)
Zinc	Zn	30			Required for activity of many enzymes. (Alcohol metabolism. Protein digestion. CO_2 formation.)
Selenium	Se	34			Essential for liver function.
Bromine	Br	35			
Strontium	Sr	38			
Molybdenum	Mo	42			Required for activity of several enzymes. (Purine metabolism. Nitrate utilization.)
Rubidium	Ru	44			Bacteria and algae.
Cadmium	Cd	48			
Tin	Sn	50			Function unknown.
Iodine	I	53			Essential constituent of the thyroid hormones.
Barium	Ba	56			

A = Atomic number = number of protons in the nucleus of an atom or the number of electrons around the nucleus.
$(\%)_1$ = Percentage of weight of adult
$(\%)_2$ = Percentage of total number of atoms of human body
Percentage data is incomplete.

Table 5.24: Discoverers of the Elements (up to 1886)

(*arranged chronologically according to date of discovery*)

Element	*Symbol*	***Atomic*** *number*	*Discoverer*	*Country*	*Date*
Carbon	C	6	Known to the ancients	*Isaiah* **54**, 16	
Sulfur	S	16	Known to the ancients	*Genesis* **19**, 24	
Iron	Fe	26	Known to the ancients	ca 4000 BCE	
Copper	Cu	29	Known to the ancients	ca 4000 BCE	
Tin	Sn	50	Known to the ancients	ca 4000 BCE	
Phosphorus	P	15	Hennig Brand	Germany	1669
Cobalt	Co	27	Georg Brandt	Sweden	1737
Hydrogen	H	1	Henry Cavendish	England	1766
Nitrogen	N	7	Daniel Rutherford	Scotland	1772
Oxygen	O	8	Joseph Priestley	England	1774
			Carl Scheele	Sweden	1774
Chlorine	Cl	17	Carl Scheele	Sweden	1774
Manganese	Mn	25	Johann Gahn	Sweden	1774
Molybdenum	Mo	42	Carl Scheele	Sweden	1778
Strontium	Sr	38	A. Crawford	Scotland	1790
Chromium	Cr	24	Louis Vauquelin	France	1797
Sodium	Na	11	Humphry Davy	England	1807
Potassium	K	19	Humphry Davy	England	1807
Calcium	Ca	20	Humphry Davy	England	1808
Boron	B	5	Humphry Davy	England	1808
			J.L. Gay-Lussac	France	1808
Magnesium	Mg	12	Humphry Davy	England	1808
Barium	Ba	56	Humphry Davy	England	1808
Iodine	I	53	Bernard Courtois	France	1811
Cadmium	Cd	48	Friedrich Stromeyer	Germany	1817
Selenium	Se	34	Jöns Berzelius	Sweden	1817
Silicon	Si	14	Jöns Berzelius	Sweden	1823
Aluminum	Al	13	Hans C. Oersted	Denmark	1825
Bromine	Br	35	Antoine Balard	France	1826
Vanadium	V	23	Nils Sefström	Sweden	1830
Rubidium	Ru	44	Robert Bensen	Germany	1861
			Gustav Kirchhoff	Germany	1861
Flourine	F	9	Henri Moissan	France	1886

II. *Water has (almost) the highest* specific heat *among known liquids (the ability to store heat energy for a given increase in temperature). The same is true of water's* latent heat *of vaporization, which is a major energizer of the atmosphere.*

Its high specific heat means that, for a given rate of energy input, the temperature of a given mass of water will rise more slowly than the temperature of most other materials. Conversely, as energy is released, its temperature will drop more slowly. This slow warming and cooling, together with other important factors, affects yearly, daily and even hourly changes in the temperature of oceans and lakes, which are quite different from the corresponding changes in the temperature of land.

III. *Water has the greatest* thermal conductivity *of all liquids.*

IV–V. *The* dielectric *constant of water, with the exception of few other solvents, is greater than for any other substance. A related property is that (even distilled) liquid water is an ionic solution and one that always contains some hydrogen ions. Water's hydrogen bond*[1082] *structure is also responsible for the water having the* greatest surface-tension *of any liquid known (mercury excluded).*

1082 The forces between water molecules are very strongly *directional*: they arise from the interaction between electron-deficient hydrogen atoms in one molecule and electron-rich oxygen atoms in another. (The *hydrogen bond*, **Pauling**, 1931). In the solid state (ice) this hydrogen bonding leads to a very open structure in which every molecule of water is surrounded symmetrically (in the form of a regular *tetrahedron*) by 4 other water molecules. As the temperature is raised, the mean distance between molecules increases. There is thus a small decreasing in density with increasing temperature as with all 'normal' solids. At the melting point, however, the tetrahedral structure is partly *destroyed*, and the water molecules are packed more closely together, causing the water to have greater density than ice; with rising temperature this effect increases, but ceases at 3.98 °C when the tetrahedral lattice structure has been fully destroyed.

A further increase of temperature beyond 4 °C causes normal expansion closely related to an overall increase in mean distance between molecules by molecular agitation.

The abnormally large dielectric constant of water, which is responsible for the striking power of water to dissolve ionic substances, is also closely related to its power to form hydrogen bonds. But water is not just a collection of individual molecules; it is a cluster of mutually attracted molecules.

Water is involved in photosynthesis, the basis of all life on earth, in two ways: in transit (as part of the transpiration stream) and in residence (as its hydrogen is chemically bound into the plant structure).

- *The chemical properties of carbon, which evolution selected over silicon as the central building block for constructing giant molecules. Silicon is 146 times more plentiful than carbon in the earth's crust and exhibits many of the same properties. Silicon is in the same column and directly below carbon in the periodic table of the elements: like carbon it has the capacity to gain 4 electrons and form 4 covalent bonds.*

  Yet, the unusual stability of CO_2 (readily soluble in water and always remains a single molecule) and the unique ability of carbon to form long chains and stable rings with 5 or 6 members (this versatility of the carbon atom is responsible for the millions of organic compounds found on earth) — are the crucial differences that led to the preference by life for carbon compounds over silicon compounds. It may very well be that carbon chemistry is the only possible basis for life.

- *Atomic sizes and charge density: The four most abundant atoms in living organisms — hydrogen, carbon, oxygen and nitrogen — have atomic numbers of 1, 6, 8 and 7. This preponderance seems attributable to their being the smallest and lightest elements that can achieve stable electronic configurations by adding one to four electrons.*

  The ability to add electrons by sharing them with other atoms is the first step in forming chemical bonds leading to stable molecules. Many other elements are excluded on the basis of being too radioactive, too inert, unavailable or toxic.

Besides the three major classes of foodstuffs, the body needs the mineral elements listed in Table 5.23. While these elements do not provide energy, they are necessary for metabolism and other body processes. Over 96 percent of the human body is composed of O, C, H and N which make up the water and organic constituents of the body. The other elements are present in the form of inorganic salts. The general functions of inorganic salts are:

- *Maintain the rigid structure of the body.*
- *Build and repair tissues.*
- *Maintain the normal contractility of muscles and the irritability of nerves.*
- *Act as buffers and help maintain body neutrality.*
- *Maintain a constant osmotic pressure.*
- *Supply material for the production of digestive juices.*

Table 5.25: Physical properties of water

Property	*Discovered by*	*Compared to other substances*
Specific heat (ca 1 cal/g. °C at 1 atm and between 0 °C and 100 °C)	**Black** (1760)	Highest of all substances except liquid NH_3
Significance: Prevents extreme climatic temperature changes. Tends to maintain uniform body temperature.		
Latent heat of fusion (79.71 cal/g at 0 °C and 1 atm)	**Black** (1761)	Highest except NH_3
Significance: At freezing-point causes thermostatic effect due to release of absorption heat.		
Latent heat of vaporization (539 cal/g at 1 atm)	**Black** (1761)	One of the highest of all substances. The volume of water evaporated per unit of energy input is less than it would be for any other liquid
Significance: Aids in heat transfer between water and land. Aids in control of body temperature through evaporation of water as in perspiration.		

Table 5.25: (Cont.)

Property	*Discovered by*	*Compared to other substances*
Dielectric constant (dissolving tendency) ($\epsilon_0 \sim 80$ at 23 °C)	**Davy** (1812)	Surpassed only by liquid HCN, H_2O_2

Significance: All living cells depend on dissolved substances for nutrition.

Surface tension (water/air) $73(\times 10^{-3}$ N/m at 20 °C)	**Jurin** (1719) **Von Segner** (1751)	Greatest than any known liquid (except mercury)

Significance: Water uses its free energy to lift itself into the porous and cellular systems of soil and plants. In the soil, more liquid can be retained as water because of its high surface tension.

Hydrogen bond (3–6 Kcal/mol)	**Pauling** (1931)	Water forms an angular molecule because the 4 electron pairs (2 bonding and 2 nonbonding) repel each other. The protons serves as an *intermolecular* glue between the strongly electronegative oxygen atoms

Significance: Ubiquitous in all living cells. The relative weakness of the hydrogen-bond as compared with the *intra-molecular* covalent bond (20 times energetically weaker) makes it important in living tissues. Enzymes, genes and protoplasm are relatively *unstable* due to the weak hydrogen-bond.

- *Act as cofactor of certain enzymes.*

In plants*, the three elements C, H, O enter mainly as* CO_2 *and* H_2O*; all of the other required elements must be in a water-soluble salt or ionic form in the soil solution, being taken up with the water that all living cells require. These minerals originate in the soil as it formed from rock, and they are normally replaced in the soil as plants and animals decay. Small amounts arrive as rain washes down particulate matter suspended in the air.*

Thus there are mineral cycles *in which a molecule of, say, potassium phosphate moves from soil to plant to animal (in bacterium or fungus) and then back to soil. If the cycle is broken when a plant is harvested, a need to replenish the soil is established. Some minerals are present in such large concentrations that artificial renewal is rarely necessary; Ca, S, Na and Cl (as table salt) may sometimes present in excess.*

Of the required minerals, nitrogen is frequently limited in supply. Although 78% of air is nitrogen gas (N_2*), elemental nitrogen must be transformed into ionic form (ammonium ion* NH_4^+ *or nitrate ion* NO_3^-*) before plants can absorb and use it.*

Natural fixation of the element into ions can occur with lightning*, but the bulk of nitrogen is fixed by the biochemical activities of free-living* bacteria *and* blue-green algae*, or by bacteria in the root nodules of legumes and a few other plants. In all living cells, nitrogen atoms and ammonium ions are constituents of thousands of different compounds. In order to make a protein macromolecule, several thousand nitrogen-containing amino acids are linked together; 16% (by mass) of protein is nitrogen.*

Complex rings of oxygen and nitrogen atoms form the purine *and* pyrimidine *constituents of nucleic acids; the genetic code is a nitrogen-containing molecular arrangement; alkaloids, vitamins, and many other chemical constituents of life are either nitrogen-containing or have their synthesis and degradation controlled by proteinaceous enzymes. Cellular growth and development is thus directly limited by the supply of nitrogen.*

The phosphorus *atom has 5 electrons in its outer shell. It can share each of three of its electrons with other atoms, accepting a share in three of theirs, so that a stable configuration of 8 (a complete* $2s + 2p$ *shell) is obtained. The* unshared *pair can be* donated *to an atom that happens to be short of two electrons to make up the total of 8 that it requires, e.g., oxygen with only 6 electrons in its outer shell. This is known as a* coordinate bond*. The valence situation for phosphorus is thus 3 covalent plus one coordinate.*

The most important compound of phosphorus is the phosphoric acid in which all four bonds of the phosphorus atom are attached to oxygen atoms (arrow indicates a coordinate-bond),

$$\begin{array}{ccccc} & & O\text{—}H & & \\ & & | & & \\ H\text{—}O\text{—} & & P \rightarrow O & & \\ & & | & & \\ & & O\text{—}H & & \end{array}$$

This molecule is (H_3PO_4) denoted in shorthand as $H\text{—}O\text{—}\textcircled{P}$. *When combined with adenosine it will yield upon condensation (the reaction of two or more molecules to form larger molecules with or without the elimination of small molecules, such as water)*

$$R\text{—}O\text{—}H + H\text{—}O\text{—}\textcircled{P} \Rightarrow R\text{—}O\text{—}\textcircled{P} + H_2O.$$

Here $R\text{—}O\text{—}\textcircled{P}$ *is adenosine monophosphate. The addition of another phosphate yields adenosine diphosphate (ADP) which is a high-energy phosphate bond. This molecule is similar to a wound-up spring toy; as the spring is unwound, the energy is released. The addition of a third phosphate bond leads to adenomine triphosphate (ATP; $C_{10}H_{12}O_{13}N_5P_3$) which has two energy-rich bonds.*

Such compounds play an important role in energy transfer within the body and in the powering of biochemical reactions. They are also much used in the metabolism of foods and in the construction of proteins from the DNA template.

The structural formula of the ATP molecule (some carbons and hydrogens not labeled) is

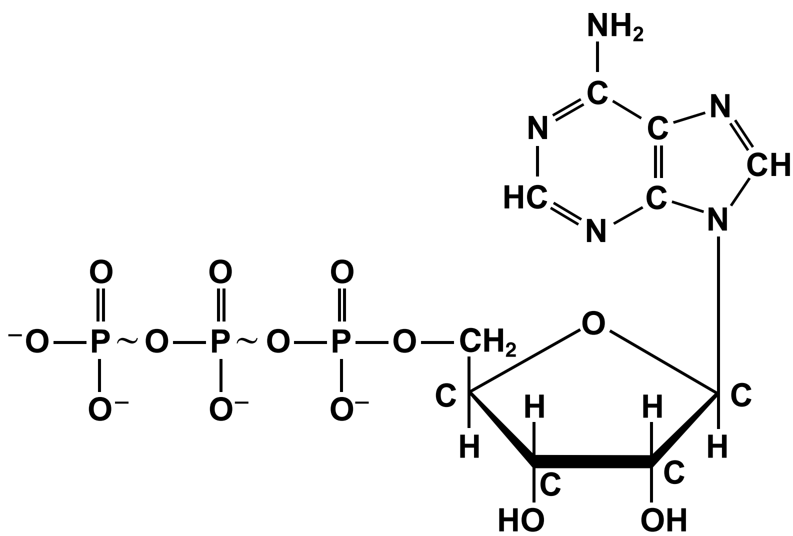

Fig. 5.23: Adenosine 5-triphosphate (ATP)

It is formed from three components. The central part is a sugar molecule (ribose) in the form of a five-member ring resembling one form of the fructose molecule. Attached to this are the two other components. One is the adenine base *consisting of linked five- and six-member rings of carbon and nitrogen atoms. The combination of the ribose and the base make up a unit called a* nucleoside. *Another atom of the ribose ring is attached to a string of three phosphate groups, and this string is where the "action" is.*

3. The strength of chemical bonds and chemical criteria for life

Heat makes a molecule vibrate, *rotate and move randomly; the probability that a molecule will come apart at temperature T, is proportional to $\{e^{-E/kT}\}$, where E is the binding energy and k is Boltzmann's constant; the*

Table 5.26: CHEMICAL BOND ENERGIES

Bond		*Bond strength at 15°C*	
		eV/molecule	*Kcal/mol*
O—O		1.44	33.12
N—N		1.66	38.18
C—N		3.02	69.46
C—C		3.60	82.80
C—O		3.64	83.72
C—H		4.28	98.44
H—H		4.51	103.73
N═N	Nitrogen	4.90	112.70
O═O	Oxygen	2.58	59.34
H—OH	Water	5.16	118.68
O═CO	Carbon dioxide	5.50	126.50
H—CH_3	Methane	4.51	103.73
H—NH_2	Ammonia	4.77	109.71
N—CHO	Formaldehyde	3.77	86.71

higher the temperature, the greater the amplitude of the random motions, and thus the more likely the components of the molecule are to dissociate. However, the rate of dissociation will also depend upon the frequency of the exciting vibration. A resonance occurs when the wavelength λ *of the driving oscillator is equal to the bond-length. If one takes* $\lambda = 10^{-8}$ *cm, c (velocity)* $= 10^5 \frac{\text{cm}}{\text{sec}}$ *(sound wave in solid), one finds that* $f = \frac{c}{\lambda} = 10^{13}$ *Hz, what is just the order of frequency of infrared radiation that is observed to excite vibrational modes in molecules.*

If the binding energy is $E = kT$, *the dissociation probability is* $\sim \frac{1}{3}$, *namely, a life time of about* 3 *oscillations – corresponding to a dissociation rate of just over* 10^{12} sec^{-1}, *or to a lifetime of just under one picosecond* (10^{-12} *sec). The lifetime reaches about a second when* $E = 30\ kT$, *and roughly one year when* $E = 47\ kT$. *A typical covalent bond binding energy is of order* $100kT$, *as can be deduced from Table 5.26.*

Another way of looking at the thermal stability of bonds is radiatively. At room temperature kT is about 0.025 *eV; the wavelength at the maximum of the blackbody emission at this temperature is in the infrared, with* $\lambda_m = 100,000$ *Å. This corresponds to a photon energy of* 0.12 *eV. It is thus clear that any atomic structures held together by binding energies less than a* *few tenths of an electron vole (about 7 Kcal/mol) are going to be unstable at ordinary temperature. Not surprisingly, the strength of the chemical bond in materials commonly found in animate and inanimate matter, is significantly greater than this (Table 5.26).*

The rate of chemical reactions between two molecules will depend upon all sorts of factors. Most important will be the frequency of collisions, for it is only when the molecules are close together that a chemical reaction can occur. The collision frequency will depend upon molecular concentrations and how fast molecules move around. Molecules in a solid may be present in extremely high concentrations but usually cannot move around, so chemical reactions within solids are virtually ruled out. In gases, molecules can move around very easily but are present in low concentration, whereas in liquids movement is impeded but the concentration is high. Chemical reactions in biology occur mostly in liquid states.

Not all collisions result in chemical reaction. There is sometimes a barrier to overcome before atoms can be exchanged between, or added to molecules. If this energy barrier is very large compared with the energy involved in the collision, which itself is determined by the temperature, reaction will be very rare.

If the environment is to be chemically active, the prevailing temperature must not be so high that few stable molecules can exist, nor so low that chemical reactions are inhibited. Outside of this temperature range chemically-based life cannot exist. This range is largely determined by the quantum mechanical forces of electromagnetism, which determine the strength of chemical bonds.

To creatures for whom a microsecond is an age or for whom a thousand years is a mere moment, the problem would look very different. But to us, and our fellow life-forms, with our circadian, monthly and annual rhythms, our common carbon chemistry and our dependence, directly or indirectly, on solar energy, it seems necessary that molecules remain stable for a matter of years, but are to indulge in reactions as rapidly as perhaps a 1000 times per second.

With these parochial criteria the dissociation rate must not greatly exceed 10^{-7} *sec*$^{-1}$*, and the reaction rate must not fall much below 1000 sec*$^{-1}$*. With a binding energy of, say,* $E = 3$ *eV, the dissociation rate constraint corresponds to an upper temperature of* $500^\circ C$*. If we relax our stability criterion by a factor of 100, so that a molecule with a binding energy of 3 eV is stable over a day rather than a year, the upper limit rises to about* $750^\circ C$*. But we cannot push the lower limit down much below, say* $-20^\circ C$*, on the grounds that very few bulk aggregates of polyatomic molecules remain liquid, and chemical reactions between solids occur only very slowly. A temperature of*

$-73\,^{\circ}C$, which is just above the points at which CO_2 and NH_3 become solids, is about the lower limit.

On the other hand, even if we relax the condition that molecules can last at least a day or so, chemically-based life cannot exist under conditions in which the dissociation rate equals the reaction rate; taking again $E = kT = 3$ eV, we obtain $T = 6000\,^{\circ}K$. Thus, if we predicate the possibility of life upon the possibility of chemical processes, we see that the environment must offer a temperature range between about $200\,^{\circ}K$ and $6000\,^{\circ}K$. Since the latter is of the order of the surface temperature of our sun and many other stars, it is probable that life (at least the chemistry-based category) can only exist on planets, and only on those warmed to at least 200 K by their local star and/or their geochemistry.

This theoretical temperature range seems large, but it is narrow compared with the range from near $3\,^{\circ}K$ in deepest space to over 10^9 K in the interior of massive stars. This cosmically narrow range, so important for life, is a remarkable consequence of electromagnetic interaction combined with quantum mechanics.

4. The Molecular Structure Of Life

A. CARBOHYDRATES

Sugars, starches, and cellulose are carbohydrates. *Sugars and starches serve as energy sources for cells; cellulose is the main structural component of the walls that surround plant cells. Carbohydrates contain carbon, hydrogen, and oxygen atoms in a ratio of approximately one carbon to two hydrogens to one oxygen $(CH_2O)_n$. The term* carbohydrate, *meaning "hydrate (water) of carbon," reflects the 2:1 ratio of hydrogen to oxygen the same ratio found in water (H_2O). Carbohydrates contain one sugar unit (monosaccharides), two sugar units (disaccharides), or many sugar units (polysaccharides).*

Monosaccharides *typically contain from three to seven carbon atoms. In a monosaccharide, a hydroxyl group is bonded to each carbon except one; that carbon is double-bonded to an oxygen atom, forming a carbonyl group. If the carbonyl group is at the end of the chain, the monosaccharide is an aldehyde; if the carbonyl group is at any other position, the monosaccharide is a ketone. (By convention, the numbering of the carbon skeleton of a sugar begins with the carbon at or nearest the carbonyl end of the open chain.)*

The large number of polar hydroxyl groups, plus the carbonyl group, gives a monosaccharide hydrophilic properties.

The simplest carbohydrates are the three carbon sugars (trioses): glyceraldehyde and dihydroxyacetone. Ribose and deoxyribose are common pentoses, sugars that contain five carbons; they are components of nucleic acids (DNA, RNA, and related compounds). Glucose, fructose, galactose, and other six-carbon sugars are called hexoses. (Note that the names of carbohydrates typically end in -ose.)

Glucose ($C_6H_{12}O_6$), *The most abundant monosaccharide, is used as an energy source in most organisms. During cellular respiration, cells oxidize glucose molecules, converting the stored energy to a form that can be readily used for cell work. Glucose is also used as a component in the synthesis of other types of compounds such as amino acids and fatty acids. Glucose is so important in metabolism that mechanisms have evolved to maintain its concentration at relatively constant levels in the blood of humans and other complex animals.*

Glucose and fructose are structural isomers: They have identical molecular formulas, but their atoms are arranged differently. In fructose (a ketone) the double-bonded oxygen is linked to a carbon within the chain, rather than to a terminal carbon as in glucose (an aldehyde). Because of these differences, the two sugars have different properties. For example, fructose, found in honey and some fruits, tastes sweeter than glucose.

Glucose and galactose are both hexoses and aldehydes. However, they are mirror images (enantiomers) because they differ in the arrangement of the atoms attached to asymmetrical carbon atom 4.

Molecules are not 2-D; in fact, the properties of each compound depend largely on its 3-D structure. Thus, 3-D formulas are helpful in understanding the relationship between molecular structure and biological function. Molecules of glucose and other pentoses and hexoses in solution are actually rings, rather that extended straight carbon chains.

Glucose in solution (as in the cell) typically exists as a ring of five carbons and one oxygen. It assumes this configuration when its atoms undergo a rearrangement, permitting a covalent bond to connect carbon 1 to the oxygen attached to carbon 5. When glucose forms a ring, two isomeric forms are possible, differing only in orientation of the hydroxyl (—OH) group attached to carbon 1. When this hydroxyl group is on the same side of the plane of the ring as the $—CH_2OH$ *side group, the glucose is designated beta glucose* (β*-glucose). When it is on the side (with respect to the plane of the ring) opposite the* $—CH_2OH$ *side group, the compound is designated alpha glucose* (α*-glucose). Although the differences between these isomers may seem small, they have important consequences when the rings join to form polymers.*

A disaccharide (two sugars) contains two monosaccharide rings joined by a glycosidic linkage, consisting of a central oxygen covalently bonded to two carbons, one in each ring. The glycosidic linkage of a disaccharide generally forms between carbon 1 of one molecule and carbon 4 of the other molecule. The disaccharide maltose (malt sugar) consists of two covalently linked α-glucose units. Sucrose, common table sugar, consists of a glucose unit combined with a fructose unit. Lactose (the sugar present in milk) consists of one molecule of glucose and one of galactose.

A polysaccharide is a macromolecule consisting of repeating units of simple sugars, usually glucose. The polysaccharides are the most abundant carbohydrates and include starches, glycogen, and cellulose. Although the precise number of sugar units varies, thousands of units are typically present in a single molecule. A polysaccharide may be a single long chain or a branched chain. Because they are composed of different isomers and because the units may be arranged differently, polysaccharides vary in their properties. Those that can be easily broken down to their subunits are well suited for energy storage, whereas the macromolecular 3-D architecture of others makes them particularly well suited to form stable structures.

Starch, the typical form of carbohydrate used for energy storage in plants, is a polymer consisting of α-glucose subunits. These monomers are joined by α 1—4 linkages, which means that carbon 1 of one glucose is linked to carbon 4 of the next glucose in the chain. Starch occurs in two forms: amylose and amylopectin. Amylose, the simpler form, is unbranched. Amylopectin, the more common form, usually consists of about 1000 glucose units in a branched chain.

Plant cells store starch mainly as granules within specialized organelles called amyloplasts; some cells, such as those of potatoes, are very rich in amyloplasts. Virtually all organisms have enzymes that can break α 1—4 linkages. When energy is needed for cell work, the plant hydrolyzes the starch, releasing the glucose subunits. Humans and other animals that eat plant foods also have enzymes to hydrolyze starch.

Glycogen (sometimes referred to as animal starch) is the form in which glucose subunits, joined by α 1—4 linkages, are stored as an energy source in animal tissues. Glycogen is similar in structure to plant starch but more extensively branched and more water soluble. Glycogen is stored mainly in liver and muscle cells.

Carbohydrates are the most abundant group of organic compounds on earth, and cellulose is the most abundant carbohydrate; it accounts for 50% or more of all the carbon in plants. Cellulose is a structural carbohydrate. Wood is about half cellulose, and cotton is at least 90% cellulose. Plant cells are surrounded by strong supporting cell walls consisting of mainly of cellulose.

Cellulose is an insoluble polysaccharide composed of many glucose molecules joined together. The bonds joining these sugar units are different from those in starch. Recall that starch is composed of α-glucose subunits, joined by α 1—4 glycosidic linkages. Cellulose contains β-glucose monomers joined by β 1—4 linkages. These bonds cannot be split by the enzymes that hydrolyze the α linkages in starch. Because humans, like other animals, lack enzymes that digest cellulose, we cannot use it as a nutrient. The cellulose found in whole grains and vegetables remains fibrous and provides bulk that helps keep our digestive tract functioning properly.

Some microorganisms digest cellulose to glucose. In fact, cellulose-digesting bacteria live in the digestive systems of cows and sheep, enabling these grass-eating animals to obtain nourishment from cellulose. Similarly, the digestive systems of termites contain microorganisms that digest cellulose.

Cellulose molecules are well suited for a structural role. The β-glucose subunits are joined in a way that allows extensive hydrogen bonding among different cellulose molecules, and they aggregate in long bundles of fibers.

Many derivatives of monosaccharides are important biological molecules. Some form important structural components. The amino sugars galactosamine and glucosamine are compounds in which a hydroxyl group (—OH) is replaced by amino group (—NH_2). Galactosamine is present in cartilage, a constituent of the skeletal system of vertebrates. N-acetyl glucosamine (NAG) subunits, joined by glycosidic bonds, compose chitin, *a main component of the cell walls of fungi and of the external skeletons of insects, crayfish, and other arthropods. Chitin forms very tough structures, such as the shell of a lobster, are further hardened by the addition of calcium carbonate ($CaCO_3$, an inorganic form of carbon).*

Carbohydrates may also combine with proteins to form glycoproteins, *compounds present on the outer surface of cells other than bacteria. Some of these carbohydrate chains allow cells to adhere to one another, whereas others provide protection. Most proteins secreted by cells are glycoproteins. These include the major components of mucus, a complex protective material secreted by the mucous membranes of the respiratory and digestive systems. Carbohydrates combine with lipids to form* glycolipids, *compounds on the surface of animal cells that allow cells to recognize and interact with one another.*

B. LIPIDS

Unlike carbohydrates, which are defined by their structure, lipids are a heterogeneous group of compounds that are categorized by the fact that they are soluble in nonpolar solvents (such as ether and chloroform) and are relatively insoluble in water. Lipid molecules have these properties because they consist mainly of carbon and hydrogen, with few oxygen-containing functional groups. Hydrophilic functional groups typically contain oxygen atoms; therefore lipids, with little oxygen, tend to be hydrophobic. Among the biologically important groups of lipids are fats, phospholipids, carotenoids (orange and yellow plant pigments), steroids, and waxes. Some lipids are used for energy storage, other serve as structural components of cellular membranes, and some are important hormones.

The most abundant lipids in living organisms are triacylglycerols. These compounds, commonly known as fats, are an economical form of reserve fuel storage because, when metabolized, they yield more than twice as much energy per gram as do carbohydrates. Carbohydrates and proteins can be transformed by enzymes into fats and in some seeds and fruits of plants.

A triacylglycerol molecule (also known as a triglyceride) *consists of glycerol joined to three fatty acids.* Glycerol *is a three-carbon alcohol that contains three hydroxyl (—OH) groups, and a* fatty acid *is long, unbranched hydrocarbon chain with a carboxyl group (—COOH) at one end. A triacylglycerol molecule is formed by a series of three condensation reactions. In each reaction, the equivalent of a water molecule is removed as one of the glycerol's hydroxyl groups reacts with the carboxyl group of a fatty acid, resulting in the formation of a covalent linkage known as an* ester linkage. *The first reaction yields a* monoacylglycerol(monoglyceride)*; the second, a diacylglycerol (diglyceride); and the third, a triacylglycerol. During digestion triacylglycerols are hydrolyzed to produce fatty acids and glycerol. Diacylglycerol is an important molecule for sending signals within the cell.*

About 30 different fatty acids are commonly found in lipids, and they typically have an even number of carbon atoms. For example, butyric acid, present in rancid butter, has four carbon atoms. Oleic acid, with 18 carbons, is the most widely distributed fatty acid in nature and is found in most animal and plant fats.

Saturated fatty acids *contain the maximum possible number of hydrogen atoms. Palmitic acid, a 16-carbon fatty acid, is a common saturated fatty acid. Fats high in saturated fatty acids, such as animal fat and solid vegetable shortening, tend to be solid at room temperature. This is because even electrically neutral, nonpolar molecules can develop transient regions of* weak positive charge and weak negative charge. This occurs as the constant motion of their electrons causes some regions to have a temporary excess of electrons, whereas others have a temporary electron deficit. These slight opposite charges result in attractions, known as *van der Waals interactions*, between adjacent molecules. Although van der Waals interactions are individually weak, they can be strong when many occur among long hydrocarbon chains.

Unsaturated fatty acids include one or more adjacent pairs of carbon atoms joined by a double bond. Therefore they are not fully saturated with hydrogen. Fatty acids with one double bond are *monounsaturated fatty acids*, whereas those with more than one double bond are *polyunsaturated fatty acids*. Oleic acid is a monounsaturated fatty acid, and linoleic acid is a common polyunsaturated fatty acid. Fats containing a high proportion of monounsaturated or polyunsaturated fatty acids tend to be liquid at room temperature, This is because each double bond produces a bend in the hydrocarbon chain that prevents it from aligning closely with an adjacent chain, thereby limiting van der Waals interactions.

Food manufacturers commonly hydrogenate or partially hydrogenate cooking oils to make margarine and other foodstuffs, converting unsaturated fatty acids to saturated fatty acids and making the fat more solid at room temperature. This process makes the fat less healthful because saturated fatty acids in the diet are known to increase the risk of cardiovascular disease. The hydrogenation process has yet another effect. Note that in the naturally occurring unsaturated fatty acids oleic acid and linoleic acid, the two hydrogens flanking each double bond are on the same side of the hydrocarbon chain (the *cis* configuration). When fatty acids are artificially hydrogenated, the double bonds can become rearranged, resulting in a *trans* configuration. *Trans* fatty acids are technically unsaturated, but they mimic many of the properties of saturated fatty acids. Because the *trans* configuration does not produce a bend at the site of the double bond, *trans* fatty acids are more solid at room temperature and, like saturated fatty acids, they increase the risk of cardiovascular disease.

At least two unsaturated fatty acids (linoleic acid and arachidonic acid) are essential nutrients that must be obtained from food because the human body cannot synthesize them. However, the amounts required are small, and deficiencies are rarely seen. There is no dietary requirement for saturated fatty acids.

Phospholipids belong to a group of lipids, called *amphipathic lipids*, in which one end of each molecule is hydrophilic and the other end is hydrophobic. The two ends of a phospholipid differ both physically and chemically. A *phospholipid* consists of a glycerol molecule attached at one end to two fatty *acids, and at the other end to a phosphate group linked to an organic compound such as choline. The organic compound usually contains nitrogen. The fatty acid portion of the molecule (containing the two hydrocarbon "tails") is hydrophobic and not soluble in water. However, the portion composed of glycerol, phosphate, and the organic base (the "head" of the molecule) is ionized and readily water soluble. The amphipathic properties of phospholipids cause them to form lipid bilayers in aqueous (watery) solution. Thus they are uniquely suited to function as the fundamental components of cell membranes.*

The orange and yellow pigments called carotenoids *are classified with the lipids because they are insoluble in water and have an oily consistency. These pigments, found in the cells of all plants, play a role in photosynthesis. Carotenoid molecules, such as β-carotene, and many other important pigments, consist of five-carbon hydrocarbon monomers known as* isoprene units.

Most animals convert carotenoids to vitamin A, which can then be converted to the visual pigment retinal. *Three different groups of animals — the mollusks, insects, and vertebrates — have eyes and use retinal in the process of light reception.*

Notice that carotenoids, vitamin A, and retinal all have a pattern of double bonds alternating with single bonds. The electrons that make up these bonds can move about relatively easily when light strikes the molecule. Such molecules are pigments; they tend to be highly colored because the mobile electron cause them to strongly absorb light of certain wavelengths and reflect light of other wavelengths.

A steroid *consists of carbon atoms arranged in four attached rings; three of the rings contain six carbon atoms, and the fourth contains five. The length and structure of the side chains that extend from these rings distinguish one steroid from another. Like carotenoids, steroids are synthesized from isoprene units.*

Among the steroids of biological importance are cholesterol, bile salts, reproductive hormones, and cortisol and other hormones secreted by the adrenal cortex. Cholesterol is an essential structural component of animal cell membranes, but when excess cholesterol in blood forms plaques on artery walls, it leads to an increased risk of cardiovascular disease. Plant cell membranes contain molecules similar to cholesterol. Interestingly, some of these plant steroids are able to block the intestine's absorption of cholesterol. Bile salts emulsify fats in the intestine so they can be enzymatically hydrolyzed. Steroid hormones regulate certain aspects of metabolism in a variety of animals, including vertebrates, insects, and crabs.

Animal cells secrete chemicals to communicate with each other or to regulate their own activities. Some chemical mediators are produced by the modification of fatty acids that have been removed from membrane phospholipids. These include prostaglandins, *which have varied roles, including promoting inflammation and smooth muscle contraction. Certain hormones, such as the juvenile hormone of insects, are also fatty acid derivatives.*

C. PROTEINS

Proteins, *macromolecules composed of amino acids, are the most versatile cell components. Scientists have succeeded in sequencing virtually all the genetic information in a human cell, and the genetic information of many other organisms is being studied. Some people might think that the sequencing of genes is the end of the story, but it is actually only the beginning. Most genetic information is used to specify the structure of proteins, and it has been predicted that most of the 21st century will be devoted to understanding this extraordinarily multifaceted group of macromolecules that are of central importance in the chemistry of life. In a real sense, proteins are involved in virtually all aspects of metabolism because most* enzymes *(molecules that accelerate the thousands of different chemical reactions that take place in an organism) are proteins. Proteins are assembled into a variety of shapes, allowing them to serve as major structural components of cells and tissues. For this reason, growth and repair, as well as maintenance of the organism, depend on proteins. As shown in Table 5.27, proteins perform many other specialized functions.*

The protein constituents of a cell are the clues to its lifestyle. Each cell type contains characteristic forms, distributions, and amounts of protein that largely determine what the cell looks like and how it functions. A muscle cell contains large amounts of the proteins myosin and actin, which are responsible for its appearance as well as its ability to contract. The protein hemoglobin, found in red blood cells, is responsible for the specialized function of oxygen transport.

Amino acids, *the constituents of proteins, have an amino group (*$—NH_2$*) and a carboxyl group (—COOH) bonded to the same asymmetrical carbon atom, known as the* alpha carbon. *Twenty amino acids are commonly found in proteins, each uniquely identified by the variable side chain (R group) bonded to the* α *carbon. Glycine, the simplest amino acid, has a hydrogen atom as its R group; alanine has a methyl (*$—CH_3$*) group.*

Amino acids in solution at neutral pH are mainly dipolar ions. This is generally how amino acids exist at cell pH. Each carboxyl group (—COOH)

Table 5.27: Major classes of proteins and their functions

Protein Class	**Functions and Examples**
Enzymes	Catalyze specific chemical reactions
Structural proteins	Strengthen and protect cells and tissues (e.g., collagen strengthens animal tissues)
Storage proteins	Store nutrients; particularly abundant in eggs (e.g., ovalbumin in egg white) and seeds (e.g., zein in corn kernels)
Transport proteins	Transport specific substances between cells (e.g., hemoglobin transports oxygen in red blood cells); move specific substances (e.g., ions, glucose, amino acids) across cell membranes
Regulatory proteins	Some are protein hormones (e.g., insulin); some control the expression of specific genes
Motile proteins	Participate in cellular movements (e.g., actin and myosin are essential for muscle contraction)
Protective proteins	Defend against foreign invaders (e.g., antibodies play a role in the immune system)

donates a proton and becomes ionized ($—COO^-$), whereas each amino group ($—NH_2$) accepts a proton and becomes $—NH_3{}^+$. Because of the ability of their amino and carboxyl groups to accept and release protons, amino acids in solution resist changes in acidity and alkalinity and therefore are important biological buffers.

Amino acids classified as having nonpolar *side chains tend to have hydrophobic properties, whereas those classified as* polar *are more hydrophilic. An acidic amino acid has a side chain that contains a carboxyl group. At cell pH the carboxyl group is dissociated, giving the R group a negative charge. A basic amino acid becomes positively charged when the aminogroup in its side chain accepts a hydrogen ion. Acidic and base chains are ionic at cell pH and therefore hydrophilic.*

In addition to the 20 common amino acids, some proteins have unusual ones. These rare amino acids are produced by the modification of common ones after they become part of a protein. For example, after they have been incorporated into collagen, lysine and proline may be converted to hydroxylysine and hydroxyproline. These amino acids can form cross links between the peptide chains that make up collagen. Such cross links produce the firmness and great strength of the collagen molecule, which is a major component of cartilage, bone, and other connective tissues.

With some exceptions, bacteria and plants synthesize all their needed amino acids from simpler substances. If the proper raw materials are avail-

able, the cells of animals can manufacture some, but not all, of the biologically significant amino acids. Essential amino acids are those an animal cannot synthesize in amounts sufficient to meet its needs and must obtain from the diet. Animals differ in their biosynthetic capacities; what is an essential amino acid for one species may not be for another. The essential amino acids for humans are isoleucine, leucine, lysine, methionine, phenylalanine, threonine, tryptophan, valine, and histidine. For children arginine is added to the list because they do not synthesize enough to support growth.

Amino acids combine chemically with one another by a condensation reaction that bonds the carboxyl carbon of one molecule to the amino nitrogen of another. The covalent carbon-to-nitrogen bond linking two amino acids together is a peptide bond. When two amino acids combine, a dipeptide is formed; a longer chain of amino acids is a polypeptide. A protein consists of one or more polypeptide chains. Each polypeptide has a free amino group at one end and a free carboxyl group (belonging to the last amino acid added to the chain) at the opposite end. The other amino and carboxyl groups of the amino acid monomers (except those in side chains) are part of the peptide bonds.

A polypeptide may contain hundreds of amino acids joined in a specific linear order. The backbone of the polypeptide chain includes the repeating sequence
N—C—C—N—C—C—N—C—C
plus all other atoms except those in the R groups. The R groups of the amino acids extend from this backbone.

An almost infinite variety of protein molecules is possible, differing from one another in the number, types, and sequences of amino acids they contain. The 20 types of amino acids found in proteins may be thought of as letters of a protein alphabet; each protein is a very long sentence made up of amino acid letters. The polypeptide chains making up a protein are twisted or folded to form a macromolecule with a specific conformation, or 3-D shape. Some polypeptide chains form long fibers. Globular proteins are tightly folded into compact, roughly spherical shapes. There is a close relationship between a protein's conformation and its function. For example, a typical enzyme is a globular protein with a unique shape that allows it to catalyze a specific chemical reaction. Similarly, the shape of a protein hormone enables it to combine with receptors on its target cell (the cell the hormone acts on). Scientists recognize four main levels of protein organization: primary, secondary, tertiary, and quaternary. The sequence of amino acids, joined by peptide bonds, is the primary structure of a polypeptide chain. This sequence is specified by the instructions in a gene. Using analytical methods investigators can determine the exact sequence of amino acids in a protein molecule. The primary structures of thousands of proteins are known. for example glucagon, *a hormone secreted by the pancreas, is a small polypeptide, consisting of only 29 amino acids.*

Primary structure is always represented in a simple, linear, "beads-on-a-string" form. However, the overall conformation of a protein is far more complex, involving interactions among the various amino acids that comprise the primary structure of the molecule. Therefore, the higher orders of structure — secondary, tertiary, and quaternary — ultimately derive from the specific amino acid sequence (the primary structure).

Some regions of a polypeptide exhibit secondary structure*, which is highly regular. The two most common types of secondary structure are the α-helix and the β-pleated sheet; the designations α and β refer simply to the order in which these two types of secondary structure were discovered. An* α-helix *is a region where a polypeptide chain forms a uniform helical coil. Each hydrogen bond forms between an oxygen with a partial negative charge and a hydrogen with a partial positive charge. The oxygen is part of the remnant of the carboxyl group of one amino acid; the hydrogen is part of the remnant of the amino group of the fourth amino acid down the chain. Thus 3.6 amino acids are included in each complete turn of the helix. Every amino acids in an α-helix is hydrogen bonded in this way.*

The α-helix is the basic structural unit of some fibrose proteins that make up wool, hair, skin, and nails. The elasticity of these fibers is due to a combination of physical factors (the helical shape) and chemical factors (hydrogen bonding). Although hydrogen bonds maintain the helical structure, these bonds can be broken, allowing the fibers to stretch under tension (like a telephone cord). When the tension is released, the fibers recoil and hydrogen bonds reform. This is why you can stretch the hairs on your head to some extent and they will snap back to their original length.

The hydrogen bonding in a β-pleated *which takes place between different polypeptide chains, or different regions of a polypeptide chain that has turned back on itself. Each chain is fully extended, but because each has a zigzag structure the resulting "sheet" has an overall pleated conformation (much like a sheet of paper that has been folded to make a fan). Although the pleated sheet is strong and flexible, it is not elastic. This is because the distance between the pleats is fixed, determined by the strong covalent bonds of the polypeptide backbones. Fibroin, the protein of silk, is characterized by a β-pleated sheet structure, as are the cores of many globular proteins.*

It is not uncommon for a single polypeptide chain to include both α-helical regions and regions with β-pleated sheet conformations. The properties of some complex biological materials result from such combinations. A spider's web is composed of a material that is extremely strong, flexible, and elastic.

Once again we see function and structure working together, as these properties derive from the fact that spider silk is a composite of proteins with α-helical conformation (providing elasticity) and others with β-pleated sheet conformations (providing strength).

The tertiary structure *of a protein molecule is the overall shape assumed by each individual polypeptide chain. This 3-D structure is determined by four main factors that involve interactions among R groups (side chains) belonging to the same polypeptide chain. These include both weak interactions (hydrogen bonds, ionic bonds, and hydrophobic interactions) and strong covalent bonds.*

(1) Hydrogen bonds form between R groups of certain amino acid subunits.

(2) An ionic bond can occur between an R group with a unit of positive charge and one with a unit of negative charge.

(3) Hydrophobic interactions result from the nonpolar tendency of R groups to be excluded by the surrounding molecules and therefore to associate in the interior of the globular structure.

(4) Covalent bonds known as disulfide bonds *or* disulfide bridges *(—S—S—) may link the sulfur atoms of two cysteines molecules; the two hydrogens are removed, and the two sulfur atoms that remain become covalently linked.*

Many functional proteins are composed of two or more peptide chains, interacting in specific ways to form the biologically active molecule. Quaternary structure *is the resulting architecture of these polypeptide chains, each with its own primary, secondary, and tertiary structure. The same types of interactions that produce secondary and tertiary structure also contribute to quaternary structure; these include hydrogen bonding, ionic bonding, hydrophobic interactions, and disulfide bridges.*

A functional antibody molecule, for example, consist of four polypeptide chains joined by disulfide bridges. Disulfide bridges are a common feature of proteins secreted from cells, such as antibodies. These strong bonds stabilize the molecules in the extracellular environment.

Hemoglobin, the protein in red blood cells responsible for oxygen transport, is an example of a globular protein with a quaternary structure. Hemoglobin consists of 574 amino acids arranged in four polypeptide chains: two identical chains called alpha chains *and two identical chains called* beta chains.

Collagen, mentioned previously, has a fibrous type of quaternary structure that allows it to function as the major strengthener of animal tissues. It consists of three polypeptide chains wound about each other and bound by cross links between their amino acids.

D. NUCLEIC ACIDS

Nucleic acids transmit hereditary information and determine what proteins a cell manufactures. Two classes of nucleic acids are found in cells: ribonucleic acid and deoxyribonucleic acid. *Deoxyribonucleic acid (DNA)* comprises the genes, the hereditary material of the cell, and contains instructions for making all the proteins, as well as all the RNA the organism needs. *Ribonucleic acid (RNA)* participates in the complex process in which amino acids are linked to form polypeptides. Some types of RNA, known as *ribozymes*, can even act as specific biological catalysts. Like proteins, nucleic acids are large, complex molecules. The name *nucleic acid* reflects the fact that they are acidic and were first identified, by **Friedrich Miescher** in 1870, in the nuclei of pus cells.

Nucleic acids are polymers of *nucleotides*, molecular units that consist of (1) a five-carbon sugar, either *deoxyribose* (in DNA) or *ribose* (in RNA); (2) one or more phosphate groups, which make the molecule acidic; and (3) a nitrogenous base, a ring compound that contains nitrogen. The nitrogenous base may be either a double-ring *purine* or a single-ring *pyrimidine.*

DNA commonly contains the purines adenine (A) and guanine (G), the pyrimidines cytosine (C) and thymine (T), the sugar deoxyribose, and phosphate. RNA contains the purines adenine and guanine, and the pyrimidines cytosine and uracil (U), together with the sugar ribose, and phosphate.

The molecules of nucleic acids are made of linear chains of nucleotides, which are joined by phosphodiester linkages, each consisting of a phosphate group and the covalent bonds that attach it to the sugars of adjacent nucleotides.

Note that each nucleotide is defined by its particular base and that nucleotides can be joined in any sequence. A nucleic acid molecule is uniquely defined by its specific sequence of nucleotides, which constitutes a kind of code. Whereas RNA is usually composed of one nucleotide chain, DNA consists of two nucleotide chains held together by hydrogen bonds and entwined around each other in a double helix.

In addition to their importance as subunits of DNA and RNA nucleotides perform other vital functions in living cells. *Adenosine triphosphate (ATP)*, composed of adenine, ribose, and three phosphates, is of major importance as the primary energy currency of all cells. The two terminal phosphate groups are joined to the nucleotide by covalent bonds. These are traditionally indicated by wavy lines, which indicate that ATP can transfer a phosphate to another molecule, making that molecule more reactive. In this way ATP is able to donate some of its chemical energy. Most of the readily available *chemical energy of the cell is associated with the phosphate groups of ATP. Like ATP,* guanosine triphosphate (GTP)*, a nucleotide that contains the base guanine, can transfer energy by transferring a phosphate group and also has a role in cell signaling.*

A nucleotide may be converted to an alternative form with specific cellular functions. ATP, for example, is converted to cyclic adenosine monophosphate (cyclic AMP) *by the enzyme adenylyl cyclase. Cyclic AMP regulates certain cell functions and is important in the mechanism by which some hormones act. A related molecule,* cyclic guanosine monophosphate (cGMP)*, also plays a role in certain cell signaling processes.*

Cells contain several dinucleotides, which are of great importance in metabolic processes. For example, nicotinamide adenine dinucleotide *has a primary role in biological oxidation and reduction reactions in the cells. It can exist in an oxidized form* (NAD^+) *that is converted to a reduced form (NADH) when it accepts electrons (in association with hydrogen). These electrons, along with their energy, are transferred to other molecules.*

5. The cycle of matter on earth; the Biosphere

For the last 4×10^9 *years, terrestrial matter has existed under rather soft and easy conditions, far from cosmic extremes:*

- *The terrestrial* gravitational field *is much weaker than that of any star, especially the very dense ones such as neutron stars or black holes. On the other hand it is not as weak as that prevailing in dust-gas clouds.*
- Temperatures *are not as high as in the stars and not as low as in the interstellar clouds.*
- Age *of the planet is not as great as the age of the galaxies, but is long in comparison with the half-life of many radionuclides.*

The atmosphere *shields the earth's biosphere from harmful solar radiation; it is the most important carrier of heat energy in two directions —* horizontally *across the continents and oceans and* vertically *from the surface to cosmic space; it contains the elements most important for life: hydrogen (in the form of water), oxygen (in water, free molecules of* O_2 *and* $C)_2$ *gas), carbon (in* CO_2*), and nitrogen (in free molecular form).*

The biosphere is defined as that part of the earth in which life exists. It is a region in which liquid water can exist in substantial quantities, it receives an ample supply of energy from an external source (sun), and within it there are interfaces between the liquid, solid and the gaseous states of matter.

All actively metabolizing organisms largely consist of elaborate systems of organic macromolecules dispersed in aqueous medium. The energy of solar radiation can enter the biological cycle only through the photosynthetic production of organic matter by chlorophyll-bearing organisms, namely green and purple bacteria, blue-green algae, phytoplankton and the vast and varied population of higher plants.

From the standpoint of the day-to-day running of the biosphere what is important is the continual oxidation of the reduced part, living or dead, by atmospheric oxygen to produce CO_2 *(which can be employed again in photosynthesis) and a certain amount of energy (which can be used for physical activity, growth and reproduction). The production of utilizable fossil fuels is essentially an accidental imperfection in this overall reversible cycle, one upon which we have come to depend all too confidently.*

In addition to H_2O *and* CO_2*, the movement of material through living organisms involves many more elements (Table 5.23); if the biosphere is to continue in running order, the biologically important materials must undergo cyclical changes so that after utilization they are put back, at the expense of some solar energy, into a form in which they can be reused.*

The rate at which this happens is quite variable:

- *The rate of circulation of organic matter of terrestrial organisms (derived from* CO_2 *of the atmosphere) is measured in decades. The rates of circulation of carbon and nitrogen are of this order.*
- CO_2 *itself, respired by animals and plant cells, enters the atmosphere and is fixed again by plant cells after an average atmospheric residence time of about 300 years.*
- *Oxygen, generated in the process of the biosphere exchange with the atmosphere and hydrosphere, is recycled in about 4000 years.*
- *All the earth's water is split by plant cells and reconstituted by animal and plant cells about every 2,000,000 years.*
- *Calcium is carried from continental rocks in rivers as calcium bicarbonate [Ca* $(HCO_3)_2$*] and precipitates as calcium carbonate [*$CaCO_3$*] in the open ocean largely in the form of tiny shells of foraminifera. Most of the replacement is due to the movement of the ocean floors toward coastal*

mountain building belts. This rate of replacement is measured in hundreds of millions of years. Phosphorus behaves in a similar way.

Without taking too seriously any of the estimates that have been made of the expectation of the life of the sun *and the solar system, it is evident that the biosphere could remain habitable for a very long time*[1083]*, many times the estimated length of the history of the genus* Homo *(which might be a few million years old).*

The water cycle

Water is the most abundant cosmic compound. It is the medium of life processes, and the source of their hydrogen*. It flows through living matter mainly in the stream of transpiration: from the roots of plant through its leaves. It is by far the most abundant single substance in the biosphere. The earth, oceans, ice caps, glaciers, lakes, rivers, soils and atmosphere contain* $1.5 \times 10^9\ km^3$ *of water in one form or another. Since each year some* $5.13 \times 10^5\ km^3$ *of water are vaporized, the* residence-time *of water in the ocean is* 2670 *years. Roughly calculated, the amount of heat required for the overall vaporization is* 1.26×10^{24} *Joule per year or* 4.08×10^{16} *watts. This is found to correspond to* $\frac{1}{3}$ *of the solar energy absorbed by the earth.*

In a nutshell, the water cycle is stated as follows: water in the atmosphere condenses in the air and falls to earth as rain or snow. Warmed by the sun, water evaporates back into the atmosphere[1084]*.*

In fact, the process is somewhat more complex: First, the cycle requires that worldwide evaporation and precipitation be equal; hydrogen losses to space are presumably replaced by juvenile water; ocean evaporation, however,

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[1083] At present, the *artificial* injection of some elements in a mobile form into the ocean and atmosphere is occurring much faster than it did in preindustrial days; new cycles have come into being that may distribute very widely (and in *toxic* quantities) compounds such as *lead* and *mercury*, as well as fairly *stable* elements such as *insecticides* and *defoliants*. Consequently, some environmental scientists are concluding that the expected future life of the biosphere as an inhabitable region for organisms is to be measured in decades!

[1084] This cycle was already recognized by the ancients as is evident from *Ecclesiastes* **1**, 7: "*All the rivers run into the sea; yet the sea is not full; unto the place from whence the rivers come, thither they return again*".

is greater than return precipitation; the reverse is true on land. Excess land precipitation may end up in ice caps and glaciers that contain 75 percent of all fresh water and may also replenish supplies taken from the water by transpiring plants, or may enter lakes and rivers, eventually returning to the sea as runoff. Once in the air, water vapor may circulate locally or become part of the general circulation of the atmosphere.

The general circulation is one of three important ways of moving water across the earth. Major ocean currents and the discharge of rivers comprise the remaining routes. Both have substantial effects on the biosphere: the ocean currents carry energy surpluses or deficits over great distances[1085]*; the rivers of the world are not only long-distance movers of water but also serve as conduits for dissolved and suspended material. Because of its chemical and physical properties, water is a very efficient erosive agent: erosion, transport and deposition have to be recognized as geological processes associated with water motion in the biosphere.*

The carbon cycle *(von Liebig, 1840)*

This process is a chain directly related to the flow of energy in the biosphere and technosphere.

The main cycle is from CO_2 *to living matter and back to* CO_2*. Some of the carbon, however, is removed by a slow epicycle that stores huge inventories in sedimentary rocks.*

In this cycle, CO_2 *is consumed through photosynthesis by plants and certain microorganisms. In this process,* CO_2 *and water react to form carbohydrates, with the simultaneous release of free oxygen, which enters the atmosphere. Some of the carbohydrates are directly consumed to supply plants with energy; the* CO_2 *so generated (by respiration in the absence of light) is released through the plant's leaves or through its roots. Part of the carbon fixed by the plants is consumed by animals (herbivorous), which also respire and release* CO_2*.*

[1085] If the cold Labrador Current had replaced the Gulf Stream, the history of civilization would have been very different!

Plants and animals die and are ultimately decomposed by microorganisms in the soil[1086]*. The carbon in their tissues is oxidized to* CO_2 *and return to the atmosphere. A similar carbon cycle takes place within the sea, where it takes at least* 1000 *years for the water in the deepest basins to completely replenish the* CO_2 *by deep ocean circulation.*

Only a few tenths of a percent of the immense amount of carbon at or near the surface of the earth (ca 2×10^{16} *tons) is in rapid circulation in the biosphere (atmosphere + hydrosphere + upper portions of earth's crust + biomass). The overwhelming bulk of near-surface carbon consists of inorganic deposits (chiefly carbonates) and organic* fossil *deposits (oil-shale, coal, petroleum) that required hundred of millions of years to reach their present levels*[1087]*.*

The living world has profoundly altered the primordial lifeless earth, gradually changing the composition of the atmosphere, sea and top layers of the

[1086] We can get an approximate idea of the rate at which organic matter in the soil is being transformed by measuring its content of the radioactive isotope ^{14}C: At the time carbon is fixed by photosynthesis, its ratio of ^{14}C to the non-radioactive isotope ^{12}C is the same as the ratio in the atmosphere, but after an organism's death ^{14}C *decays* and becomes less abundant w.r.t. ^{12}C. Measurements of this ratio yield rates for the oxidation of organic matter in the soil, ranging from decades in tropical soils to several hundred years in boreal forests.

[1087] Since around 1850 man has inadvertently been conducting a global geochemical experiment by burning large amount of fossil fuel and destroying forests, thereby returning carbon to the atmosphere that was fixed by photosynthesis hundreds of millions of years ago.

At present the total amount of carbon available in the atmosphere is 7×10^{14} kg. The total carbon release to the atmosphere is 7.5×10^{12} kg/year with a current (2004 CE) CO_2 content of 400 ppM (by volume). It is expected that during the next 100 years of fossil fuel burning the amount of carbon in the atmosphere could be doubled, increasing the concentration to 800 ppM.

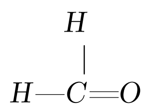
One of the most significant results of the increase in $\mathrm{CO_2}$ level is the reduction in the transparency of the atmosphere to infrared radiation, which is a critical feature of the heat balance of the globe; the mean global temperature could rise by two or three degrees, enough to cause polar-ice melting and rise of ocean levels. Much will depend on the capacity of the oceans to absorb $\mathrm{CO_2}$. But a new equilibrium in the biosphere will be reached that will gradually affect the deep oceans. These, with their turnover time of 1000 years, will become involved and their rate of exchange with bottom sediments will control the ultimate partitioning of carbon.

solid crust both on land and under the ocean. Thus, a study of the carbon cycle in the biosphere is fundamentally the study of the overall global interactions of living organisms and their physical and chemical environment.

The engine for the organic processes that reconstructed the primitive earth is photosynthesis. Regardless of whether it takes place on land or in the sea, it can be summarized by the single reaction:

$$CO_2 + 2H_2A + light \implies CH_2O + H_2O + 2A + energy.$$

The formaldehyde molecule



is one of the simplest organic compounds. The reaction stores energy in chemical form. H_2A is commonly water (H_2O), in which case $2A$ symbolizes the release of free oxygen (O_2). There are however bacteria that can use compounds in which A stands for sulfur (S), for some organic radical or for nothing at all.

There are organisms that are able to use CO_2 as their sole source of carbon (autotrophs); others use light energy for reducing CO_2 (phototrophic); still others use energy stored in inorganic chemical bonds, such as nitrates and sulfates (chemolithotrophics). Most organisms however, require preformed organic compounds for growth (heterotrophs). The non-sulfur bacteria are an unusual group that is both phototrophic and heterotrophic. Chemoheterotrophic organisms (e.g., animals) obtain their energy from organic compounds without need for light.

An organism may be either aerobic or anaerobic regardless of its source of carbon or energy. Thus some anaerobic chemoheterotrophes can survive in the deep ocean and deep lakes in the total absence of light or free oxygen.

The oxygen cycle

When free oxygen began to accumulate in the atmosphere 1.8 billion years ago, it was originally put there by plants. Hence the early plants made possible the evolution of higher plants and animals that require free oxygen for their metabolism. Yet, since the high rate of energy associated with oxygen *metabolism was potentially destructive to early forms of carbon-based life*[1088], *the origin of life and its subsequent evolution was contingent on the development of systems that shielded it from, or provided chemical defenses against ordinary molecular and atomic oxygen [oxygen-breathing life is more energy-efficient; e.g., fermentation of glucose yields only* 50 *Kcal per mole against* 686 *Kcal/mole obtained by direct oxidation].*

The oxygen cycle is complicated because oxygen appears in so many chemical forms and combinations, primarily as molecular oxygen (O_2)*, in water and in organic and inorganic compounds.*

Free oxygen is an extremely active element and reacts with almost all other elements. How can such an active element be so abundant in the atmosphere? Why is it not removed by chemical reactions? The total amount of free oxygen at present is 1.8×10^{18} *kg, which is equivalent to* 20.96 *mol percent of the total atmosphere.*

This oxygen content has remained stable in measurements made over the last 70 *years in spite of the increase in amounts of fuel, oil and coal burned in the technosphere. In addition to this technosphere activity, other chemical processes consume* O_2 *in the lithosphere and hydrosphere (e.g., oxidation of iron oxide in ores and elemental sulfur to the sulphate ion). To this we must add, of course, the biological oxidation of molecules such as carbohydrates [e.g.,* $CH_2O+O_2 \Rightarrow CO_2+H_2O+$*energy] and the oxidation of volcanic emissions*[1089] ($O_2 + 2CO \Rightarrow 2CO_2$)*.*

Against this oxygen-depletion processes there are oxygen-producing processes such as

[1088] Molecular oxygen reacts spontaneously with organic compounds and other reducing substances. This reactivity explains the toxic effects of oxygen above tolerable concentrations. **Louis Pasteur** discovered that very sensitive organisms such as *obligate anaerobes* cannot tolerate oxygen concentrations above about 2 percent of the present atmospheric level. Recently the cells of higher organisms have been found to contain organelles called *peroxisomes*, whose major function is thought to be protection of cells from oxygen; the peroxisomes contain enzymes that utilizes H_2O_2 (hydrogen peroxide), as a hydrogen acceptor in the oxidation of lactic acid. The rate of reduction of oxygen by peroxisomes increases proportionally with an increase of oxygen concentration, so that an excessive amount of oxygen in the cell increases the rate of its reduction by peroxisomes.

[1089] It could have led to massive oxygen decrease during great volcanic episodes in the remote past.

- *photolysis of water molecules by solar ultraviolet radiation in the upper atmosphere*

$$\text{water} + \text{solar ultraviolet} \implies \text{hydrogen} + \text{oxygen.}$$

  The hydrogen molecule is dissipated into space due to its low mass but oxygen remains;

- *photosynthesis*

$$\text{water} + \text{solar light} \implies \text{hydrogen} + \text{oxygen.}$$

  The hydrogen is bonded by CO_2 to form formaldehyde $[2H_2 + CO_2 \Rightarrow CH_2O + H_2O]$ and oxygen remains. The net amount of oxygen produced by photosynthesis is 2.07×10^{14} kg/year. This corresponds to an annual net production of 1.72×10^{14} kg of dry organic matter through a net absorbed solar energy of 2.9×10^{21} Joule/year. Since this annual production rate of oxygen is equivalent to 2.6×10^{-4} parts per years of the total atmospheric free oxygen, a full recycling time of oxygen is estimated at 3800 years[1090].

Compared to the total amount of oxygen in the atmosphere, the annual rate of consumption of oxygen in the technosphere (ca 2.3×10^{13} kg/year, needed to burn 7.5×10^{12} kg of oil and coal) is only 19 ppM. At the present rate, only after 50,000 years will all the atmospheric oxygen be burned by technological processes.

The nitrogen cycle *(von Liebig, 1840)*

Nitrogen exists in the atmosphere in molecular form (N_2) and is a rather inert gas, except to the few organisms that have the ability to convert the element to a combined form. Approximately 79 out of every 100 molecules in the atmosphere (that is 75.5% by weight) are nitrogen. It would appear to be merely a dilutant for the active agents of the atmosphere: oxygen, CO_2, water vapor etc., but this is not so.

Each living organism on this planet is made up of proteins, which play an essential role in the processes of life. The proteins are polymers, built from

[1090] This relatively fast recycling rate prompted some biogeochemists to say that "*The next breath you inhale will contain atoms exhaled by Jesus of Nazareth or by Adolf Hitler of Münich*".

20 different monomers, the amino acids; each amino acid consists of at least one amino group, NH_2*. There is no "life" without nitrogen.*

The transmutation of the inert molecular nitrogen to the life-carrying amino acid group is one of the greatest wonders of the biosphere. Each year about 70 *billion kg of nitrogen are involved in* biogenic *processes. A further* 8 *billion kg are fixed*[1091] *by* electrical discharges *in the atmosphere.*

Whereas in the case of the carbon cycle man's activity is influential in one direction only (combustion of fossil fuels which transfers carbon from the earth's crust to the atmosphere), in the case of the nitrogen cycle man's impact is in the other direction — the industrial fixation of nitrogen from the atmosphere by means of synthesis of ammonia *from atmospheric molecular nitrogen (Haber process, 1909;* $N_2 + 3H_2 \Rightarrow 2NH_3$*). Presently it runs at levels of some* 60 *billion kg of nitrogen per year, compared with approximately* 54 *billion kg of nitrogen fixed by biological activity and a further* 8 *billion kg by electrical discharge. This technological contribution is unsurpassed in the other material cycles; here man's activity*[1092] *is equal to that of the biosphere!*

The essential features of the nitrogen cycle are as follows: Nitric acid (HNO_3*) is formed by electrical discharges in the atmosphere, and is washed down by rain; only a small amount of this falls on fertile soil, and is utilized by plants. Besides the* HNO_3 *produced by electric discharges (which is absorbed by the soil in the form of* nitrates *by plants), leguminous plants can take up atmospheric nitrogen which is converted into organic nitrogen by the agency of micro-organisms which occur in nodules on the root. Algae, fungi, mosses and bacteria, present in the soil, are also capable of utilizing elementary nitrogen.*

The organic nitrogen compounds elaborated by plants serve as food for herbivorous animals, and the proteins of the latter are utilized in turn by carnivora.

When the bodies of animals and plants decay, decomposing bacteria produce ammonia*. In the soil this ammonia is oxidized by* nitrosifying bacteria *to* nitrites[1093]*, and these by the* nitrifying bacterium *to* nitrates*, the latter again serving for the nourishment of plants. A portion of the nitrogen, however, is*

[1091] By "fixed" is meant nitrogen incorporated in a chemical compound that can be utilized by plants and animals.

[1092] Clearly, it benefits mankind through enhanced food production arising from the use of *nitrogen fertilizers*, but it also damages the flora and fauna of lakes, rivers and estuaries caused by the influx of these same fertilizers because of *eutrophication* (deficiency in oxygen).

[1093] *Nitric* acid = HNO_3; its salts are *nitrates*, e.g., KNO_3, $NaNO_3$, etc. *Nitrous* acid = HNO_2; its salts are *nitrites*, e.g., KNO_2, $NaNO_2$, etc.

again returned to the atmosphere by the action of denitrifying bacteria *in the soil (these bacteria carry the* nitrogenase *enzyme complex).*

Nitrogen is able to play its complicated role in life processes because it has an unusual number of oxidation levels *(valences). An oxidation level indicates the number of electrons that an atom in a particular compound has "accepted" or "donated". In plants and animals most nitrogen exists either in the form of the ammonium ion or the amino* ($-NH_2$) *compounds. In either case it is highly reduced: it has* acquired *three electrons by its association with three other atoms and thus is said to have a valence of* $\textcircled{-3}$.

At the other extreme, when nitrogen is in the highly oxidized form of the nitrate ion (the principal form it takes in the soil), it shares 5 *of its electrons with oxygen atoms and has a valence of* $\textcircled{+5}$. *To convert nitrogen as it is found in the ammonium ion or* amino acids *to nitrogen as it exists in the* soil nitrates *involves a total valence change of* 8, *or* the removal of 8 electrons. *Conversely, to convert* nitrate nitrogen *into* amino nitrogen *requires the addition of* 8 *electrons.*

By and large the soil reactions that reduce nitrogen (or add electrons to it) release considerably more energy than the reactions that oxidize nitrogen (or remove electrons from it). Thus, for almost every reaction in nature where the conversion of one compound to another yields an energy of at least 15 *kcal/mole, some organism has arisen that can exploit this energy to survive.*

The fixation of nitrogen requires an investment of energy. Before nitrogen can be fixed it must be activated, *which means that molecular nitrogen must be split into two* atoms *of free nitrogen. This step requires at least* 160 *kcal per mole of nitrogen (28 grams). The actual fixation step, in which two atoms of nitrogen combine with three molecules of hydrogen to form two molecules of* NH_3, *releases* 13 *kilocalories.*

Whether nitrogen-fixing organisms actually invest this much energy, however, is not known: Reactions catalyzed by enzymes *involve the penetration of activation barriers and not a simple change in energy between a set of initial reactions and their product.*

Sulfur and phosphorus cycles

Although the biosphere is mainly composed of $\{O, C, H, N\}$ *other elements are essential constituents of living matter. Notable among them are* phosphorus *and* sulfur. *Together they comprise* the essential six.

Table 5.28: Chemical abundance of the elements
in the earth's crust (by weight in one ton crustal rocks)

Element	Z	Kg	Element	Z	Grams
Oxygen	8	466	Nickel	28	72
Silicon	16	277.3	Zinc	38	80
Aluminum	13	81.3	Copper	29	55
Iron	26	50	Cobalt	27	28
Calcium	20	36.3	*Nitrogen*	7	20
Sodium	11	28.3	Lead	82	10
Magnesium	12	27.7	Boron	5	10
Potassium	19	16.8	Tin	50	1.5
Titanium	22	8.6	Uranium	92	2.4 –4
Hydrogen	1	1.4			
Phosphorus	15	1.1			
Sulfur	16	0.3			
Carbon	6	0.2			

The biosphere is mainly wood, not protein but the carbohydrate cellulose. Nitrogen, a major constituent of protein, seems surprisingly scarce — about 5 parts per 1000 by weight. The rest, 12 parts per 1000 of the total, contain: {Ca, K, Si, Mg}, elements of important biochemical function: One atom of magnesium, for instance, lies at the center of every molecule of chlorophyll. The 9^{th} and 11^{th} place in the abundance list of the biomass (Table 5.28) is occupied by sulfur {S} and phosphorus {P}. Yet no protein can be made without sulfur. In fact, sulfur is the "stiffening" in protein: A protein cannot perform its function unless it is folded and shaped in a particular way. This 3-dimensional structure is maintained by bonds between sulfur atoms that link one segment of a protein molecule to another. Without these sulfur bonds a protein would coil randomly, like a carelessly dropped rope.

How is sulfur recycled? It has been known for many years that rocks containing the element in sulfate form deliver it to the oceans via the world rivers. The waste of industrial sources takes the same route. Sulfur is recycled back from the sea to the land via of the atmosphere. Furthermore, industrial sulfur-dioxide pollutes the atmosphere and is washed down in rain as sulfate. A certain bacterium metabolizes the sulfates in sea water and releases the sulfur as H_2S.

Our model of the biosphere has been constructed on two explicit assumptions:

- *the biosphere necessarily contains the five elements* $\{O, C, H, N, S\}$*;*
- *all five are both soluble and volatile.*

If we now add phosphorus as a sixth necessary element, we can safely assume its solubility in water. However, phosphorus is unknown in the atmosphere; none of its ordinary compounds has any appreciable vapor pressure. It therefore tracks the hydrologic cycle only partway, from the lithosphere to the hydrosphere.

6. The cell[1094]

Cells are dramatic examples of the underlying unity of all living things. This idea was first expressed by two German scientists, botanist ***Matthias Schleiden*** *in 1828 and zoologist* ***Theodor Schwann*** *in 1839. Using their own observations and those of other scientists, these early investigators used inductive reasoning to conclude that all plants and animals consist of cells. Later,* ***Rudolf Virchow****, another German scientist, observed cells dividing and giving rise to daughter cells. In 1855, Virchow proposed that new cells form only by the division of previously existing cells.*

The work of Schleiden, Schwann, and Virchow contributed greatly to the development of the cell theory, the unifying concept that (1) cells are the basic living units of organization and function in all organisms and (2) that all cells come from other cells. About 1880 another German biologist, ***August Weismann****, added an important corollary to Virchow's concept by pointing out that the ancestry of all the cells alive today can be traced back to ancient*

[1094] The relative size of chemical and organismic levels are as follows:
($1nm = 10^{-9}m = 10^{-7}cm = 10^{-3}\mu m$)
atom $\sim 0.1nm = 1\mathring{A}$ ($1\mathring{A} = 10^{-8}cm$)
aminoacid $\sim 1nm = 10\mathring{A}$
protein $\sim$ 2–10nm
virus $\sim$ 50–100nm
small bacteria $\sim 200nm$
typical bacterium $\sim 8\mu m$
red blood cell $\sim 10\mu m$
human egg cell $\sim 0.13mm$. (Approximately the size of the period at the end of this sentence.)

times. Evidence that all living cells have a common origin is provided by the basic similarities in their structures and in the molecules of which they are made. When we examine a variety of diverse organisms, ranging from simple bacteria to the most complex plants and animals, we find striking similarities at the cell level. Careful studies of shared cell characteristics help us trace the evolutionary history of various groups of organisms and furnish powerful evidence that all organisms alive today had a common origin.

Each cell is a microcosm of life. It is the smallest unit that can carry out all activities we associate with life. When provided with essential nutrients and an appropriate environment, some cells can be kept alive and growing in the laboratory for many years. By contrast, no isolated part of a cell is capable of sustained survival. Composed of a vast array of inorganic and organic ions and molecules, including water, salts, carbohydrates, lipids, proteins, and nucleic acids, most cells have all the physical and chemical components needed for their own maintenance, growth, and division. Genetic information is stored in DNA molecules and is faithfully replicated and passed to each new generation of cells during cell division. Information in DNA codes for specific proteins that in turn determine cell structure and function.

Cells exchange materials and energy with the environment. All living cells need one or more sources of energy, but a cell rarely obtains energy in a form that is immediately usable. Cells convert energy from one form to another, and that energy is used to carry out various activities, ranging from mechanical work to chemical synthesis. Cells convert energy to a convenient form, usually chemical energy stored in adenosine triphosphate, or ATP. Although the specifics vary, the basic strategies cells use for energy conversion are very similar. The chemical reactions that convert energy from one form to another are essentially the same in all cells, from bacteria to those of complex plants and animals.

Cells are the building blocks of complex multicellular organisms. Although they are basically similar, cells are also extraordinarily diverse and versatile. They can be modified in a variety of ways to carry out specialized functions.

Every complete cell has a small inner portion, the *nucleus*, marked off from the rest of the cell (the *cytoplasm*) by a thin membrane. The primary concern of the nucleus is in cell reproduction and in the accurate transfer of the *genes* controlling chemical characteristics from mother to daughter cells. The nucleus is an anaerobic system, and thus cannot be involved in energy production.

In 1898, a German cytologist, **C. Benda** discovered in the cytoplasm small granules which he called *mitochondria* (Greek for 'cartilage-threads'). Electron microscopy later revealed that mitochondria were bodies only one to three microns in diameter (**Albert Claude**, 1945).

Suspensions of mitochondria were found to catalyze all the reactions of the Krebs' cycle. It became clear that the mitochondria are the "powerhouses" of the cell; that their membranes are actually conglomerations of all enzymes and coenzymes needed for catabolizing foodstuffs and producing high-energy phosphate bonds. It has been estimated that an individual mitochondria may contain as many as $10,000$ *separate assemblies; each ripping off hydrogen atoms and producing higher energy phosphate-bonds.*

7. Carbohydrates and biochemical energetics

Living cells require chemical energy to synthesize the molecules necessary for their growth. Since cell division is the process necessary for the propagation of life, the synthesis of the molecules of life is a continuous process.

Life, when viewed as a vast and fabulously intricate system of ongoing chemical reactions, is a thermodynamically spontaneous process, so the net free energy change associated with cell-growth and division must be negative. Common sources of chemical energy for sustaining life are the carbohydrates. *Glucose* ($C_6H_{12}O_6$) *is a typical example. Cells ranging from simple bacteria to human cells utilize glucose as a source of both carbon and energy.*

Carbohydrates are classified as monosaccharides, disaccharides and polysaccharides. The most important monosaccharides *are glucose, galactose and fructose, having the same molecular formula* $C_6H_{12}O_6$, *but different structural formulas.* Glucose *(or dextrose) is found in fruits and honey. It is the sugar of the blood. A* disaccharide *is formed when two molecules of monosaccharide join together with the loss of a molecule of water. The most important member of this group* ($C_{12}H_{22}O_{11}$) *are sucrose, maltose, and lactose. Polysaccharides* $(C_6H_{10}O_5)_n$, *like starch, dextrin, glycogen and cellulose — are essentially polymers of glucose. In the formation of these polymers, a molecule of water is split off from* $C_6H_{12}O_6$ *as each unit adds to form the long polymer unit* $(C_6H_{10}O_5)_n$.

The energy source for most cells is the oxidation of glucose, fatty acids from fats, and amino acids from proteins. In simple combustion, rapid oxidation is simply a direct reaction with oxygen in which heat and light are released. In the cells, some of the energy that might be converted to heat must *be retained for other uses such as building chemical bonds and controlling muscle contraction; heat energy is produced as a by-product.*

Heat energy is not the only need of the body. Other mechanisms must be able to channel some of the heat to other forms of energy. For example, *some of the energy can be used to synthesize molecules of high energy such as adenosine triphosphate (ATP), which is found in all cells.*

The carbohydrates include the sugars, starches, cellular and other closely related substances. These compounds are named as they are because they were initially thought to be hydrates of carbon, since besides carbon, they contain hydrogen and oxygen in the ratio of two to one, as in water. (There are, however, some sugars that do not fit this general formula and a wider definition must be used.)

Carbohydrates are formed in cells of plants from CO_2 *in the air and water in the ground. In the presence of sunlight and chlorophyll (the magnesium-containing pigment of leaves), these two compounds react to form carbohydrate (represented by* $C_6H_{12}O_6$ *in the equation) and oxygen,*

$$6CO_2 + 6H_2O \xrightarrow[\text{sunlight}]{\text{chlorophyll}} C_6H_{12}O_6 + 6O_2.$$

This process by which plants transform radiant energy of the sun into chemical energy stored in food material is called photosynthesis. It is actually a series of complicated reactions. Photosynthesis is regarded as the most important chemical reaction on earth because it returns oxygen to the air as well as manufacturing and storing food material.

The most important chemical reactions of carbohydrates are: hydrolysis, dehydrogenation and fermentation.

Hydrolysis

As far as hydrocarbons are concerned, hydrolysis is a reaction through which complex organic compounds will react with water to form a simpler compound, e.g.

$$\underset{\text{sucrose}}{C_{12}H_{22}O_{11}} + H_2O \longrightarrow \underset{\text{glucose}}{C_6H_{12}O_6} + \underset{\text{fructose}}{C_6H_{12}O_6}$$

In hydrolysis (from the Greek: "to loosen with water"), water break the chemical bond of the carbohydrate while its elements H and OH, attach themselves to the split parts.

Dehydrogenation *(oxidation)*

Oxidation occurs if a substance either gains oxygen atoms, loses hydrogen atoms (dehydrogenation) or else loses electrons. The complete combustion of one mole (180 grams) of glucose liberates 686 *kcal of heat*

$$C_6H_{12}O_6 + 6O_2 \longrightarrow 6CO_2 + 6H_2O + 686 \ kcal/mole.$$

This is the well known respiration process through which stored energy is released and carbon dioxide and water are liberated (the reverse of photosynthesis).

Most biological oxidations are better described as dehydrogenation processes in which a compound is oxidized by the removal of two hydrogen atoms; oxygen's role is indirect. In other words, oxidation processes do not need to have oxygen as the direct oxidizing agent associated with it. Consider, for example, the oxidation of lactic acid during respiration: $C_3H_6O_3 + 3O_2 \Rightarrow 3CO_2 + 3H_2O$*+energy). We write this as the virtual sum of two half-reactions, each of which is balanced electrically,*

$$\begin{array}{ll} \textit{oxidation half-reaction} & C_3H_6O_3 + 3H_2O \ \Rightarrow \ 3CO_2 + 12H^+ + 12e^- \\ \textit{reduction half-reaction} & 3O_2 + 12H^+ + 12e^- \ \Rightarrow \ 6H_2O \\ \hline \textit{sum} & C_3H_6O_3 + 3O_2 \ \Rightarrow \ 3CO_2 + 3H_2O + energy. \end{array}$$

In the oxidation stage, lactic acid is dehydrogenized by removing from it all its hydrogen atoms[1095]*. The water molecules are also split to serve as sources*

[1095] Hydrogen atoms never actually exist free in the cell but are *transported* by specific *carriers*. Molecules which serve as hydrogen acceptors are proteins called *coenzymes*. One important example is *nicotinamide adenine dinucleotide* (NAD) made out of sugar, phosphoric acid and an organic base. Thus, the biochemical oxidation, with only the main characters present, is written as

$$\mathrm{CH_3} - \overset{\overset{\displaystyle \mathrm{H}}{|}}{\underset{\underset{\displaystyle \mathrm{OH}}{|}}{\mathrm{C}}} - \mathrm{COOH} + \mathrm{NAD} \Longrightarrow \mathrm{CH_3} - \underset{\underset{\displaystyle \mathrm{O}}{\|}}{\mathrm{C}} - \mathrm{COOH} + \mathrm{NADH_2},$$

where $\mathrm{NADH_2}$ is the hydrogen removing agent that stores (temporarily) the released free energy.

The free energy stored in the reduced components $\mathrm{NADH_2}$ and $\mathrm{FADH_2}$ (another *of oxygen and additional hydrogen. The second half-reaction emphasized the reduction of oxygen through its combination with the hydrogens of the first stage and the recombination of water. A total of 12 electrons are transferred between reactants in the oxidation of lactic acid. In the real process this oxidation occurs in six steps (two electrons at a time) in a stepwise oxidation process known as the Krebs' cycle.*

It is the dehydrogenation step that produces the energy released by the reaction (and utilized by the body in metabolism). The mere elimination of CO_2 from the compound does not produce the energy necessary for the formation of high-energy phosphate bonds. Consequently, it can be said that the body obtains its energy by burning hydrogen and that the burning of carbon is only incidental. This is not surprising since the burning of hydrogen liberates far more heat per unit weight than does the burning of carbon.

Fermentation

In fermentation (from the Latin: "to boil"), the degradation of the carbohydrate is done by means of anaerobic microorganisms (yeast) which secretes the enzyme zymase, e.g.

$$\underset{\text{glucose}}{C_6H_{12}O_6} \xrightarrow{\text{enzymes}} \underset{\text{alcohol}}{2C_2H_5OH} + 2CO_2$$

This schematic equation represent a much more complex process: The fermentation of glucose can be considered to occur in two stages. In the first stage, one molecule of sugar is broken down into two molecules of pyruvic acid

$$CH_3{-}\underset{\underset{O}{\|}}{C}{-}COOH$$

biochemically important oxidizing agent) is used by the cell in the formation of ATP from ERAP, a process called *oxidative phosphorylation.* ATP is a high-energy compound that plays a key role in many metabolic processes. Thus, in anaerobic glycolysis, a molecule of glucose is converted to two molecules of lactic acid with the net production of two higher energy phosphate bonds in the ultimate form of ATP. It is the lactic acid now that must be catabolized further with the formation of additional high-energy phosphate bonds.

and the equivalent of 4 hydrogen atoms (hydrogen atoms never actually exist free in the cell, but are transported by specific carrier molecules).The formation of pyruvic acid from glucose actually entails at least ten distinct and sequential reactions. Each of these reactions is catalyzed by a specific enzyme. The enzymes work in tandem to produce two pyruvic acid and two ATP molecules for each glucose molecules consumed. All this is written symbolically as

$$\text{glucose} + 2ADP \implies 2ATP + 2 \text{ pyruvic acid} + 4H. \qquad (A)$$

The second stage consists of the addition of hydrogen to pyruvic acid in one or more steps. The specific way in which these constituents are metabolized depends on the cells in which metabolism takes place:

- *Some anaerobic microorganisms add the hydrogen directly to pyruvic acid to produce lactic acid ($C_3H_6O_3$):*

$$2 \text{ pyruvic acid} + 4H \implies 2\ CH_3 - \underset{\displaystyle OH}{\overset{\displaystyle H}{\overset{|}{\underset{|}{C}}}} - COOH \qquad (B)$$

  (The same process takes place in muscle cells during anaerobic glycolysis).

- *Yeasts first split off CO_2 from the pyruvic acid and then add the hydrogen to form ethyl alcohol (the basis for the making of wine and beer)*[1096]

$$2 \text{ pyruvic acid} + 4H \overset{\text{yeast}}{\implies} 2 \text{ ethyl alcohol } (C_2H_5OH) + 2CO_2. \qquad (C)$$

- *Cells that are able to accommodate the process of respiration, burn pyruvic acid all the way to CO_2 and water.*
- *Other microbes convert the pyruvic acid to acetic acid, glycerol, butyl, alcohol, etc.*

When processes (A) and (B) are added together, the result is the net process of anaerobic glycolysis:

$$\text{glucose} \xrightarrow{\text{enzymes}} 2 \text{ lactic acid} + 36 \text{ kilocalories per mole}, \qquad (D)$$

which it is a spontaneous exergonic process ($\Delta G < 0$); i.e. it proceeds spontaneously. In anaerobic glycolysis lactic acid is a metabolic "dead

[1096] Note that the muscle "could" have converted lactic acid further to ethyl alcohol and CO_2 as in yeast fermentation, and thus liberate further energy for its use. But alcohol is more toxic to the body than lactic acid and its discharge into the bloodstream would be harmful to the body.

end". However, in the presence of oxygen, aerobic cells (those requiring oxygen for life) are able to utilize a much larger portion of the original free energy of the glucose by respiration

$$\text{lactic acid} + 3O_2 \implies 3CO_2 + H_2O \; + 320 \text{ kcal per mole.} \qquad (E)$$

If, however, no oxygen is available, the anaerobic glycolisis of (D) continues in two steps:

$$\begin{aligned} \text{lactic acid} &\implies \text{pyruvic acid} + 2H^+ + 2e^- && (D-1) \\ \text{pyruvic acid} + H_2O &\implies \text{acetic acid } (CH_3COOH) && \\ & \qquad + CO_2 + 2H^+ + 2e^- && (D-2) \end{aligned}$$

In the first step, lactic acid is virtually oxidized by a removal of two hydrogen atoms (dehydrogenation). In the second step known as oxidative decarboxylation, both CO_2 and two more atoms of hydrogen are removed from the pyruvic acid, producing acetic acid. It is the dehydrogenation step that produces the energy utilized by the body.

8. Metabolism; intermediate steps, sequences and cycles

Metabolism refers to the chemical changes that the absorbed products of digestion undergo in the tissues of the living body. The resulting products of digestion enter the various cells via the bloodstream.

Two main types of metabolism are:

- *Anabolism (from Greek: "to throw upward") comprises building-up processes whereby simple products of digestion are assembled into complex molecules, such as proteins and nucleic acids, to form new tissue, repair old tissue, store food supplies, and synthesize enzymes, pigments, hormones, etc. Anabolism requires energy.*
- *Catabolism (Greek: "to throw downward") is a breaking-down process in which absorbed products of digestion and worn-out tissues are reduced to simple waste products such as CO_2, water and urea, with the simultaneous release of energy. In some cells the metabolites may undergo a variety of transformations whereby some of available free energy may be channeled into muscular activity and other functions inside the body.*

Specific metabolic processes take place in or on specific intracellular structures. The main respiratory centers of the cells are the mitochondria, which contain many interacting oxidative enzymes. Only the mitochondria are capable of respiratory oxidation, which involves converting pyruvic acid to CO_2, H_2 and energy to form high energy phosphate bonds. For this reason, each mitochondrion is considered to be a functioning metabolic machine with a highly ordered pattern of enzyme molecules, and each of these minute mitochondria contain all of the enzymes required in the respiratory cycle.
Metabolic patterns have evolved to satisfy fundamental requirements:

(a) *Well-organized balance of anabolism and catabolism.*

(b) *Step-wise oxidation with the release of energy in small quantities.*

(c) *The ability to transfer released energy to high-energy storage compounds known as ATP. Every time a cell needs energy, a high-energy phosphate bond is broken in an ATP molecule, and about* 8000 *calories per mole are liberated.*

If glucose were oxidized to CO_2 in one step, little if any of the large amount of released free energy could be used efficiently, and most of the energy would be lost as heat. The oxidation of glucose in the body occurs in a number of steps. ADP molecules absorbs the energy produced by the oxidation of glucose to form ATP molecules. Later, at another location, at the demand of an enzyme, the ATP can be hydrolized back to ADP, and the energy made available for a variety of purposes such as to produce the desired muscular activity,

$$\text{relaxed muscle} + ATP \implies \text{contracted muscle} + ADP.$$

The muscle contraction is caused by the tendency of ATP to undergo a change to reduce the number of high-energy bonds by forming ADP and a phosphate ion. Thus muscle contraction consumes ATP, and the body then regenerates the ATP by oxidizing glucose units.

Digestion is enzyme-catalyzed hydrolysis. It converts the unabsorbable foodstuffs into absorbable structural units. (The foodstuffs in all their complexity are not absorbed.)

Each organism has its own variety of carbohydrates, lipids and proteins — differing in small details from other organisms. Thus, in the human body, foreign carbohydrates in food are broken into glucose to build up human carbohydrates. Foreign lipids are broken down to glycerol and fatty acids to build up human lipids. Then, glucose and fatty acids must undergo further catabolic changes to the still simpler CO_2 and water. The amino acids must *be catabolized to CO_2 and urea in the urine, while water is exerted via breath, urine and perspiration.*

When a meal high in carbohydrates is digested (catabolism), we obtain a supply of glucose that exceeds our immediate needs. It is carried by the blood into the liver, where it is stored as the polymer glycogen[1097] *(anabolism) to answer our future needs. The blood emerges from the liver with normal concentration levels of glucose, which is absorbed by body cells, where it is broken down to CO_2 and water (catabolism) for energy use in the muscles.*

This drain of blood glucose is signaled to the liver, which begins the reverse process — the breaking down, bit by bit, of its stored glycogen (catabolism). This is done in just sufficient quantities to replenish the blood with glucose[1098]*. The glucose balance is maintained by the hormones insulin and glucagon.*

Aerobic glucose metabolism *(respiration)*

If 180 *grams of glucose are burned in a calorimeter to CO_2 and water, a net heat energy in the amount of* 686 *kcal is released. The body, however, to*

[1097] The highly *branched* structure of glycogen make it possible for several glucose residues to be released at once to meet energy needs, rather than one at a time as would be the case with a *linear* polymer. This feature is useful to an organism in meeting short-term demands for energy by increasing the glucose supply as quickly as possible. It has been shown by mathematical modeling that the structure of glycogen is *optimized* for its ability to store and deliver energy quickly and for the longest amount of time possible. The key to this optimization is the average chain length of the branches. If the average chain-length were much greater or much shorter, glycogen would not be as efficient as a vehicle for energy storage and release on demand. Experimental results support the conclusions reached from the mathematical modeling.

[1098] The process is known as *the Cori cycle*: It involves the complementary processes of *glycolysis* in the muscle and *gluconeogenesis* in the liver. In the first, lactate (lactic acid) is formed from glucose in the muscle. It is transported by the blood to the liver. In the second stage of the cycle the lactate ions are converted back to glucose, which can be carried back to the muscles by the blood. Glucose can be converted to glycogen in the liver and the muscle with the aid of insulin by a process known as *glycogenesis*. The reverse process in the liver through which glycogen is degraded, with the aid of adrenalin, back to glucose is named *glycogenolysis*. All four processes are multistage reactions with many intermediate stages involving enzymes and the phosphate ion. In the Cori cycle there is a division of labor between liver and muscle, with different reactions and different enzymes in the different organs.

its own advantage, has developed an alternative metabolic route composed of three steps:

- *Aerobic metabolism of glucose to pyruvic acid and acetyl coenzyme A (a sequence of at least* 10 *enzyme-catalyzed steps).*
- *Aerobic metabolism of acetyl coenzyme A (the citric acid cycle or Krebs cycle*[1099]*).*
- *'Burning' hydrogen atoms from the first two stages and storing part of this energy in ATP molecules (oxidative phosphorylation).*

In the first step, which is common to both fermentation and respiration, sugar is converted to pyruvic acid. In respiration, however, pyruvic acid is further broken into a pair of hydrogen atoms and two-carbon molecules, the acetyl group

$$CH_3 - \overset{\displaystyle O \atop /\!\!/}{C} \diagdown \quad ,$$

which is the only one that can enter the citric acid cycle.

In the second stage this two-carbon molecule combines with another cellular product that contains 4 *carbon atoms, forming a* 6*-carbon product, citric acid. Citric acid is then broken down in a series of reactions with the step-wise*

The overall reaction in *anaerobic glycolysis* may be summarized by the following equation:

$$\underset{\text{glucose}}{C_6H_{12}O_6} + 2\text{ADP} + 2H_3PO_4 \Longrightarrow 2\ \underset{\text{lactic acid}}{CH_3 - \overset{\displaystyle OH \atop |}{\underset{\displaystyle | \atop H}{C}} - COOH} + 2\text{ATP} + 2H_2O$$

[1099] This cycle was formulated in the early 1940s from biochemical data obtained primarily by two refugees of the Hitler regime, **Albert Szent-Györgi** (1893–1986, Hungary, Holland, U.S.A.) and **Hans Krebs** (1900–1981; England). Szent-Györgi was awarded the Nobel prize for physiology or medicine (1937) for his discoveries in connection with *cellular energy metabolism* (oxidation of tissues), *vitamin C* and *fumaric acid*. Later he became interested in the *biochemistry of the muscle* and discovered the protein *actin*.

release of 4 pairs of hydrogen atoms and two CO_2 molecules, which can then again combine with the 2-carbon molecule and start the cycle all over.

The two principal benefits that the cell derives from the Krebs cycle are the creation of building blocks and the production of energy.

Some of the compounds formed during this cycle can be used to manufacture amino acids and other essential cellular components. For example, the amino acid aspartic acid ($C_4H_7O_4N$) is made by the enzyme-controlled addition of ammonia (NH_3) to $C_4H_4O_4$, an intermediate compound. In general, the Krebs cycle serves as the hub of the cell. Intermediate products can be siphoned off at different points in the cycle and utilized to form the variety of building blocks needed for biosynthesis; conversely, molecules that are not needed as building blocks can be fed into the cycle at different places, thus allowing the cell a greater diversity of usable foodstuffs.

With the Krebs cycle the aerobic conversion of glucose is not yet complete; a third stage of oxidative phosphorylation[1100] *is needed. Hence the coenzyme bound hydrogen atoms released in stages 1 and 2 go through a series of interlocking cyclic reactions in which they combine with oxygen to form water and liberate energy.*

It is now possible to compute the total number of ATP molecules formed for each molecule of glucose consumed. In the first step, the conversion of glucose to pyruvic acid, two ATP molecules and two pairs of hydrogen atoms are formed. When the two pyruvic acid molecules are burned into CO_2 via the Krebs cycle, ten pairs of hydrogen are produced, making a total of two ATP and twelve pairs of hydrogen atoms. Since each pair of hydrogens yields three ATP, a grand total of 38 ATP molecules are manufactured for each glucose that is oxidized into CO_2 and water.

A mole of ATP can be experimentally shown to liberate 8 kcal of energy as it forms ADP. Since 38 ATP are formed by the oxidation of a mole of glucose, 304 kcal are liberated. This represents 45 percent of the total of 686 Kcal available as a result of the complete combustion of glucose; the remainder is liberated as heat (Note that since a mole contains 6.023×10^{23} molecules, a molecule of ATP liberates the puny amount of 1.32×10^{-20} cal.)

[1100] The importance of the *oxidative phosphorylation* chain to living organisms is demonstrated by the lethal effect of two well-known poisons, CO (carbon monoxide) and HCN (cyanide). Both toxic materials exert their effects by combining with carrier molecules used in the oxidative phosphorylation chain, thus preventing transfer of hydrogens and interrupting the chain. This results in an immediate halt in ATP production, and unless the poison is removed, death ensues.

Anaerobic glycolysis

Under conditions in which oxygen is not available, glucose can still be used as an energy source. However, much less energy is released and far fewer ATP molecules are made, because oxidative phosphorylation is not occurring.

As in fermentation, pyruvic acid is formed to begin with, but instead of being converted to acetyl coenzyme A, it breaks down to either lactic acid or ethyl alcohol, depending on the organism.

All cells in animals receive sufficient oxygen for aerobic glycolysis while the animal is at rest. However, in muscles undergoing strenuous exertions, the muscle cells may not receive an adequate supply of oxygen and so must use anaerobic glycolysis as a source of energy; lactic acid (instead of acetylcoenzyme A) will then be formed from pyruvic acid, which loses two hydrogen atoms by reduction.

Clearly, muscular work (contraction and expansion) is done at the expense of the energy liberated by the conversion of glucose to lactic acid.

The rising level of lactic acid causes fatigue and muscular ache, stopping muscular motion altogether. Eventually, when further breakdown of glucose to lactic acid is no longer possible, oxygen is brought in by the bloodstream to produce the needed energy and make up for the "oxygen debt", which accumulated by using the less efficient process.

As the muscle rests, oxidative metabolism is resumed in all cells. In general, $\frac{4}{5}$ *of the lactic acid formed during anaerobic glycolysis in the contracting muscle is carried by the blood back to the liver where it is reconverted to glycogen (the Cori cycle). The remaining* $\frac{1}{5}$ *is reconverted in the muscle into pyruvic acid, which is again ready to enter the Krebs cycle.*

An aerobic conversion of glucose to CO_2 *and water produces much more free energy than the anaerobic use of glucose. When glycolysis is anaerobic and lactic acid is produced, only* 56 *kcal of energy are released. This represents* $\frac{56}{686} \times 100$ *percent of the energy released when glucose is completely oxidized. Anaerobic glycolysis produces* 2*ATP moles (per mole of glucose), which liberates* 16 *kcal of energy as they are converted to ADP. The efficiency of energy use is* $\frac{16}{56} \times 100 = 29$ *percent compared to* 45 *percent for the aerobic system. Anaerobic glycolysis therefore not only released far less energy than aerobic glycolysis but its efficiency is lower*[1101].

1101 The incomplete metabolism of glucose without oxygen has important evolutionary consequences. The early forms of life on earth developed an anaerobic glycolitic system because there was no oxygen in the atmosphere. Eventually oxygen accumulated as it was produced in photosynthesis from water by simple plant life.

Metabolism of fats and proteins

This process produces twice the amount of energy per gram than yielded by either carbohydrates or proteins. Fats consist of triester glycerol and three fatty acids. The glycerol and fatty acids are metabolized by different routes to acetyl coenzyme A, which then enters the Krebs cycle.

The mechanism of breakdown of the fatty acids make up a stepwise process that sequentially releases acetyl coenzyme A molecules until the fatty acid is completely broken down. Acetyl coenzyme A holds an important position in metabolism: In addition to its central role in intermediary metabolism of glucose, fatty acids and certain amino acids, it serves as the essential building units in many biosynthetic reactions.

Proteins are not normally metabolized to produce energy. However, there are metabolic pathways for the interconversion *of amino acids and carbohydrate and fat metabolism.*

9. Metabolic role of enzymes and hormones

Enzymes: A biochemical reaction takes place under two principal conditions:

- *The overall process is energetically favorable, i.e., the total free energy of the reaction is greater than the total free energy of the products, therefore leading to a more stable state.*
- *A significant percentage of the reacting molecules have an average energy greater than the rest; they have become* activated*, and can now cross some energy barrier [if there were not an energy barrier, every reaction that could happen, would happen!]*

Consequently, to secure control*,* selectivity *and* specificity*, there are energy barriers to be bridged and a measurable energy of activation is needed to bring reactants to the top of the barrier, after which they can then spontaneously slide downhill to yield products, without any further assistance.*

Consider a reaction in which substance AB reacts with substance C, producing new reaction products. One such example is the reoxidation of $NADH_2$

to pyruvate in muscle cell

$$NADH_2 + CH_3 - \underset{\underset{O}{\|}}{C} - COOH \Longrightarrow NAD + CH_3 - \overset{\overset{H}{|}}{\underset{\underset{OH}{|}}{C}} - COOH.$$

There is some transition state that describes the condition of the reacting molecules at the top of the energy barriers. The model for the reaction would involve a complex of AB and C molecules

$$AB + C \rightleftharpoons [AB \cdots C] \Longrightarrow A + BC.$$

The transition state complex, perched precariously at the summit of the energy curve, can fall apart in two ways; it can reverse itself, sliding back down the slope into a mixture of reactants, or it can role down the other side of the energy barrier, yielding products. The rates of these two alternative outcomes depends on the population of active molecules in the transition state. The rate of the forward reaction can be enhanced either by temperature increase or by catalysts who effectively lower the energy barrier, thereby making it possible for greater number of molecules to get to the top of the transition state and then over to the other side.

The burning of glucose with oxygen without the necessary enzymes, will literally burn it up, losing all specificity and control along the way. However, the presence of an enzyme catalyst lowers the activation energy barrier such that the same reaction proceeds at physiological temperatures. Not only are enzymes catalysts specific, but they are speedy and efficient. It is not uncommon for a typical enzyme molecule to catalyze a million reactions per minute. Some enzymes are single proteins; others are very complicated structures involving metal ions and other smaller proteins called coenzymes.

Glucose reacts with the enzyme-rich ATP molecule in the very first stage in glycolysis, resulting in phosphorylation. The reaction is catalyzed by the enzyme hexokinase and takes place on the surface of a specific protein catalyst. Further, there is a 3-dimensional fit of all the pieces of the puzzle. Molecular shapes and bonding sites allow only the right pieces to be correctly oriented. Schematically ('S' represents the substrate)

$$\underset{\text{enzyme}}{E} + \underset{\substack{\text{reactant}\\\text{molecules}}}{\substack{glucose\\ and\ ATP}} \rightleftharpoons \underset{\substack{\text{transition}\\\text{complex state}}}{[E \ldots S]} \rightarrow \underset{\text{products}}{E + \substack{glucose\text{-}6\text{-}phosphate\\ and\ ADP}}$$

where the enzyme catalyst is regenerated for continued use.

Hormones: The principal hormones that affect carbohydrates metabolism are:

- *Insulin is produced by the pancreas; it facilitates the transfer of glucose into the cell; it removes glucose from the bloodstream by hastening the conversion of glucose to glycogen in the liver and muscle (glycogenesis), by speeding up the oxidation of glucose in the tissues, by inhibiting the breakdown of liver glycogen, and by promoting the formation of fat from glucose [isolated* 1921 *by* **Frederick Grant Banting** *(1891–1941, England)].*
- *Adrenalin* (epinephrine) *is produced by the adrenal glands and discharged into the bloodstream when the individual is under stress. It accelerates the conversion of liver glycogen to glucose (glycogenolysis) thereby increasing the blood sugar level and providing quick energy to help the body meet an emergency. Its action is antagonistic to that of insulin.*
- *Glucagon is produced by the pancreas and, like adrenalin, causes a breakdown in liver glycogen and a rise in blood sugar levels.*

Machines and their Ghosts: Reductionism vs. holism, vitalism, dualism, emergence, teleology and all the rest

Science has shown that the behavior of a macroscopic body can be reduced to the states and motion of its constituent atoms, ions, molecules and subatomic particles and fields, all evolving in accordance with the quantum-mechanical and relativistic extensions of Newton's mechanistic laws. The procedure of breaking down physical systems into their elementary components and looking for an explanation of their behavior at the lowest level is called reductionism, and it has exercised a powerful influence over scientific thinking.

So deeply has reductionism penetrated physics that the ultimate goal of the subject remains the identification of the fundamental fields (and associated particles) and their dynamical behavior in interaction. Indeed, in the second half of the 20th century there has been spectacular progress toward this goal[1102].

Until the middle of the 20th century, there had been a strong belief among theologians, philosophers and many biologists that life was not reducible to the laws of physics and chemistry, that there was a "vital force" that made the difference between living things and inanimate matter. The world's religions were invoked: God breathed life, soul-stuff, into inanimate matter.

There are two main aspects of reductionism:

(1) *Ontological reduction* (ontology = *study of the nature of being) claims that every phenomenon is capable of being described (at least in principle) by physics, i.e.: the "stuff" comprising reality is, at its most basic level, nothing but forces and particles studied by physics. Such reductionism is not a mere human construct but actually a property of nature, independent of human culture. It is a claim about the way things actually* are.

(2) *Epistemological reduction* (epistemology = *a theory of the nature of knowledge) holds that theories and experimental laws in some fields of science can* always *be shown to be special cases of laws formulated in other areas of science. This is a claim not about nature itself but about the way human being see the world; e.g. the approximate reduction of classical thermodynamics to the level of statistical mechanics of atoms. We say that the Second Law is valid "at a higher level" of analysis; we use the Second Law because it generally gives correct answers without the vastly more complicated calculations which would be required if we used the lower-level theories (statistical mechanics). In the 20th century, chemistry has been epistemologically reduced to physics.*

1102 Technically speaking, the aim of the theorist is to provide a *Lagrangian* density for the given system under study; for the particle theorist or quantum cosmologist, this system is the universe at large or its local ground state, the *vacuum*, which contains virtual excitations of all possible particles and fields. Once this action functional is known, it accounts for all the observed fields and particles, as well as all composite structures, from hadrons and atoms to galaxy clusters. The implication is that given such a universal Lagrangian, theoretical physics would have reached its culmination, leaving only technical elaboration: The world will be explained.

Vitalism is an old idea which assigns to human life, particularly consciousness, a special quality which must forever remain outside conventional science. Vitalism is usually associated with an anti-reductionist stance, the view being that life cannot be reduced to mere physics and chemistry and that a more holistic approach is required.

While there is a genuine problem abut how to relate different levels of organization (such as atomic, chemical, cellular and organistic) to each other, and about which level is the most appropriate on which to tackle a particular set of problems, this was not what the anti-reductionists and vitalists had in mind. Any philosophy that is at its core holistic must tend to be anti-science, because it precludes studying parts of a system separately, or isolating some parts and examining their behavior without reference to everything else.

If every process were crucially dependent upon its embedding as part of a larger whole, then science could not have succeeded. We can study cells outside the body and particular biochemical reactions outside of cells. Indeed, the success of biochemistry is due to just such isolation of parts. That does not to deny the importance of also studying systems as a whole.

The unwillingness of holists to consider explaining life in terms of molecular biology and their desire to invoke some special life-force, effectively restores the concept of the soul and renders the concept of an afterlife conceivable.

Joseph Priestley *(c. 1776) tried to find the "vital force". He weighed a mouse before and just after it died; it weighed the same. All such attempts have failed. If there is soul-stuff, it is not made of matter.*

Helmholtz *(1847), together with* ***Karl Ludwig*** *(1816–1895),* ***Ernst von Brücke*** *(1819–1892) and* ***Emil du Bois-Reymond*** *(1818–1896) initiated a plan for a research programme to elevate physiology to equal rank with physics. Rejecting vitalism, this group proposed to analyze processes such as urine secretion or nerve conduction in physicochemical terms. Although the programme was naively optimistic (as the four subsequently recognized), modern biochemical science is still reductionist in intent.*

Claude Bernard *(1813–1878) rejected German reductionism and* ***Louis Pasteur*** *(1822–1895) always attributed unique functions to living cells.*

Vitalists [e.g. ***Henri Bergson*** *(1859–1941)] stated that life is an autonomous function controlled by its own laws of physics and chemistry. Vitalism maintains that the laws of physics and chemistry will never adequately explain life, for the reason that life is not material.*

Furthermore, reason itself is unable to explain life processes because its rational activity cannot go beyond the mechanistic explanations based on physicochemical laws, whereas life and consciousness, being independent of *physicochemical laws, cannot be completely understood by means of logical, scientific, or mathematical analyses.*

The chief constituent quality of living organisms is (á la Bergson) a vital impulse (élan vital) — a quality which can be understood only by means of man's intuition. However, the continuity between atomic physics, molecular biology and the nature of reproduction, heredity, metabolism, respiration, and other biological functions – even aspects of brain functionality – have by now been established.

And no new principle of science needed to be invoked anywhere in this vast, multi-scale programme!

It looks as if there are a small number of simple facts that can be used to understand the enormous variety and intricacy of living things. Thus, the discovery of the molecular structure of the gene achieved what Bergson and most geneticists only 30 years previously had thought impossible.

Reductionism is even better established in physics and chemistry. We have known for centuries that a handful of comparatively simple laws not only explain but quantitatively and accurately predict a variety of phenomena, not just on earth but throughout the entire observable universe.

We detect and identify light from distant quasars only because the laws of electromagnetism are the same ten billion light-years away as here. The spectra of those quasars are recognizable only because the same chemical elements are present here and there. The motion of galaxies around one another follows familiar Newtonian gravity. Gravitational lenses and binary pulsar spin-down reveal general relativity in the depths of interstellar and intergalactic space, just as Mercury's orbit and the behavior of light rays and radar beams skirting the sun reveal its working in our own solar system.

All in all, the greater part of Western science has been founded on the method of reductionism, whereby the properties of a complicated system are understood by studying the behavior of its component parts[1103].

That the universe is ordered, seems self-evident. Everywhere we look, from far-flung galaxies to the deepest recesses of the atom, we encounter regularity and intricate organization. We do not observe matter or energy to be distributed chaotically. They are arranged instead in a hierarchy of structure: atoms and molecules, condensed matter, crystals, living things,

[1103] *Example*: there is probably nobody who understands all the systems of a Boeing 747 airliner, but every part of it is understood by somebody. Thus, the airliner behavior as a whole is understood, because we believe that an airliner is just the sum of its parts.

planetary systems, star clusters, and so on. Moreover, the behavior of physical systems is not haphazard, but lawful and systematic.

We can distinguish between different sorts of order; first there is the order of simplicity, seen for example in the regularities of the solar system, or the periodic oscillations of a pendulum. Then there is the order of complexity, such as the arrangement of gases in the swirling atmosphere of Jupiter, or the complex organization of a living creature. This distinction emphasizes two different approaches: reductionism versus holism.

Reductionism seeks to uncover simple elements within complex structures, while holism directs attention to the complexity as a whole. The order of complexity suggests to many an element of purpose, in which all the component parts of a system fit together harmoniously in a cooperative way to achieve some particular end.

The world abounds with complex structures that amalgamate regularity and irregularity: coastlines, forests, mountain chains, ice sheets, star clusters. Matter is manifested in a seemingly limitless variety of forms. How does one go about studying them scientifically?

A fundamental difficulty is that, by their very nature, complex forms have a high degree of individuality. We recognize a snowflake as a snowflake, but no two of them are the same. Conventional science attempts to explain things exactly, in terms of general principles. Any sort of explanation for the contingent shape of a particular snowflake or a coastline could not be of this kind.

The Newtonian paradigm, which is rooted in that branch of mathematics — the differential calculus — that treats change as smooth and continuous, is not well adapted to deal with irregular things. The traditional approach to complicated, irregular systems is to model them by approximation to regular systems. The more irregular the real system is, the less satisfactory this modeling becomes. For example, galaxies are not distributed smoothly throughout space, but associate in clusters, strings, sheets and other forms that are often tangled and irregular in form. Attempts to model such features using Newtonian methods involve enormous computer simulations that take many hours even on modern machines.

When it comes to very highly organized systems, such as a living cell, the task of modeling by approximation to simple, continuous and smoothly varying quantities is hopeless. Belatedly attempts by sociologists and economists to imitate physicists and describe their subject matter by simple mathematical equations are rarely convincing.

Generally speaking, complex systems fail to meet the requirements of traditional modeling in four ways. The first concerns their formation. Complexity often appears abruptly rather than by slow and continuous evolution. There are many examples of this. Secondly, complex systems often (though not always) have a very large number of components (degrees of freedom). Thirdly, they are rarely closed systems; indeed, it is usually their very openness to a complex environment that drives them. Finally, such systems are predominantly nonlinear in their dynamics.

A wide range of physical systems can be satisfactorily approximated as regular, continuous or linear. *A* linear system *is one in which cause and effect are related in a proportionate fashion. An elastic string is a simple example. If the string stretches by a certain length for a certain pull, it stretches by twice that length for twice the pull. This is called a linear relationship because if a graph is plotted showing the length of the string against the pulling force it will be a straight line. The line can be described by the equation* $y = ax + b$, *where* y *is the length of the string,* x *is the force, and* a *and* b *are constants.*

If the string is stretched too much, its elasticity will start to fail (onset of plasticity) and the proportionality between force and stretch will also cease. The graph deviates from a straight line as the string stiffens; the system becomes nonlinear. *Eventually the string snaps, a highly non-linear response to the applied force.*

A great many physical systems are described by quantities that are approximately linearly related. An important example is wave motion. A particular shape of wave is described by the solution of some equation (mathematically this would be a partial differential equation, which is typical of nearly all dynamical systems). The equation will possess other solutions too; these will correspond to waves of different shapes. The property of linearity concerns what happens when we superimpose two or more waves. In a linear system one simply adds together the amplitudes of the individual waves.

Most waves and oscillations encountered in physics are linear to a good approximation, at least as long as their amplitudes remain small. In the case of sound waves, musical instruments depend for their harmonious quality on the linearity of vibrations in air, on strings, etc. Electromagnetic waves such as light and radio waves are also approximately linear, a fact of great importance in telecommunications. Oscillating currents in electric circuits are often linear too, and most electronic equipment is designed to operate linearly. Non-linearities that sometimes occur in equipment can cause distortions in the output, although carefully controlled non-linearities are sometimes crucial to circuit functionality.

A major discovery about linear systems was made by the French mathematician and physicist Jean Fourier. He proved that any periodic mathematical function can be represented by a (generally infinite) series of pure sine waves, whose frequencies are exact multiples of each other. This means that any periodic signal, however complicated, can be analyzed *into a sequence of simple sine waves. In essence, linearity then means that wave motion, or any periodic activity, can be decomposed into simple signals and put together again without distortion.*

Linearity is not a property of waves and oscillations alone; it is approximately possessed by electric and magnetic fields, currents, voltages, weak gravitational fields, stresses and strains in many materials, heat flow, diffusion of gases and liquids and much more. The greater part of natural phenomena and technology stems directly from the fortunate fact that so much of what is of interest and importance in natural phenomena involves linear systems. Roughly speaking, a linear system is one in which the whole is simply the algebraic sum of its parts.

Thus, however complex a linear system may be it can always be understood as merely the conjunction or superposition or coexisting simple elements that are present together but do not 'get in each other's way'.

Such systems can therefore be decomposed, analyzed or reduced to their independent component parts. It is not surprising that the major burden of scientific research so far has been towards the development of techniques for studying and controlling linear systems.

But by and large, non-linear systems have been largely neglected, although this has been gradually changing in recent decades. In a non-linear system the whole is much more than the sum of its parts, and it cannot be reduced or analyzed in terms of simple subunits acting independently. The resulting properties can often be unexpected, complicated and mathematically intractable.

There is a tendency to think of complexity in nature as a sort of annoying aberration which holds up the progress of science. Only very recently has an entirely new perspective emerged, according to which complexity and irregularity are seen as the norm, and smooth regularities the exception.

The new approach treats complex or irregular systems as primary in their own right. They simply cannot be 'chopped up' into lots of simple components yet still retain their distinctive qualities.

We might call this new approach synthetic or holistic, as opposed to analytic or reductionist, because it treats systems as wholes[1104]*. Just as there are idealized simple systems (e.g. the hydrogen atom, Keplerian orbits, etc.) to use as building blocks in the reductionist approach, so one must also search for idealized complex or irregular systems to use in the holistic approach. Real systems can then be regarded as approximations to these* idealized *complex or irregular systems.*

Reductionist biologists take the position that once the basic physical mechanisms operating in a biological organism have been identified, life has been explained as 'nothing but' the processes of ordinary physics. They argue that because each component of a living organism fails to reveal any sign of peculiar forces at work, life has already effectively been reduced to ordinary physics and chemistry (and chemistry itself reducible to physics).

Since animate and inanimate matter experience exactly the same sorts of microscopic forces and changes, and since many of life's processes can be conducted in a test tube (in vitro), any outstanding gaps in knowledge are attributed solely to technical limitations. As time goes on, it is claimed, more and more details of the workings of organisms will be understood within the basic mechanistic paradigm.

It is worth pointing out that the claim that animate and inanimate matter are both subject to the same physical forces is very far from being fully tested in practice. What the biologist means is that he sees no reason why the sort of molecular activity he studies should not be consistent with the operation of normal physical forces, and that should anyone decide to investigate more closely, the biologist would not expect any conflict with conventional physics and chemistry to emerge,

Let us nevertheless grant that the biologist may be right on this score. It is still far from being the case, however, that life has then been 'explained' by physics. It has, rather, simply been defined away. For if animate and inanimate matter are indistinguishable in their behavior under the laws of physics then wherein lies the crucial distinction between living and non-living systems?

The mystery of life, then, lies not so much in the nature of the dynamics that govern the individual molecules that make up an organism or their local, microscopic interactions but rather in how the whole assemblage operates collectively in a coherent and cooperative fashion. Biology will never be truly

[1104] However, in this "scientific holism" complex phenomena are actually still reduced to simpler sub-processes and are – in principle if not always in practice – further reducible to simple, microscopic degrees of freedom.

reconciled with physics until it is recognized that each new level in the hierarchical organization of matter may bring into existence new qualities that are simply irrelevant at the lower (smaller) scales of the hierarchy.

In recent years scientists have come to recognize more and more systems that must be understood holistically (in the above-defined, scientific sense); these systems are usually highly nonlinear and characterized by abrupt (random or pseudo-random) relationships among quantities. It may thus be an accident of history that the first scientists were preoccupied with linear physical systems, such as simple harmonic oscillations or low-amplitude waves, or else such simple, predictable nonlinear systems as two-body systems, atoms and small molecules, laminar fluids, equilibrium thermodynamics, etc., which are especially amenable to analytical techniques and a reductionist approach.

Reductionism has been under attack from yet another quarter: already the rise of quantum physics in the 1920s put paid to the idea of the universe as a deterministic machine. But the more recent work on chaos, self-organization, *and nonlinear system theory has been more influential. These topics have forced scientists to think more and more about* open systems, *which are not rigidly determined by their component parts because they can be influenced by their environment. This makes their behavior* unpredictable, *bestowing upon them a type of "freedom".*

What has come as a surprise is that open systems can also display ordered and law-like behavior in spite of being indeterministic and at the mercy of seemingly random outside perturbations.

There appear to exist general organizing principles that supervise the behavior of complex systems at many organizational levels, *principles that exist alongside the laws of physics (which operate at the bottom level of individual particles and/or coherent, predictable field patterns). These organizing principles are consistent with, but cannot be reduced to or derived from, the laws of physics; they point to a state of* contingent order.

Thus nature is attributed a sort of freedom (in the philosophical sense of free will) *which was absent in the clockwork universe of Newton and Laplace. This freedom may arise through partial sacrifice of reductionism; the world is – in effect if not in principle –* more *than the sum of its parts, and a physical system, while "merely" a collection of atoms, has emergent collective dynamical qualities that bear no simple mapping to its microscopic degrees of freedom. We must recognize the existence of many different levels of structure.*

A human being, for example, certainly is a collection of atoms, but there are many higher levels of organization that are not easy to deduce from this microscopic description yet which are essential for defining what we mean by the word "person".

By viewing complex systems as a hierarchy of organization levels, the simple "bottom-up" view of causality in terms of elementary particles and fields interacting with each other must be replaced by a more subtle formulation in which higher levels can act downward upon lower levels too.

This serves to introduce elements of teleology, or purposive behavior, into the affairs of nature.

The multi-level emergence of structure in biological organisms may be viewed either as a temporal process, the creation of novelty through growth or evolution, or non-temporally, as a thing possessing properties not possessed by any of its parts. [The smell of ammonia, for example, is neither present in hydrogen nor nitrogen, nor easily predictable from the laws of chemistry.]

While the "emergenist" approach acknowledge the genuine novelty in nature at successive level of organization and pure reductionism maintains that nothing really new can emerge at higher levels, there are those who take the middle road. To them, the higher form is new and different, not a mixture or a compounding of lower forms, and yet it is understandable in terms of them; one might say that the higher form actualizes the potentiality of lower forms.

For example, the form of sodium chloride can be understood by referring to the form of sodium and the form of chlorine. It does not have the same properties that they do, but its properties are based on theirs and are developments of them at a new level.

Thus, the emergence of table salt from the union of sodium and chlorine is neither hollow nor irrational. The salt is a genuinely novel substance and yet eminently intelligible. Likewise, the form of sodium is intelligible in terms of the potentialities of protons, neutrons, and electrons.

These principles are readily applied to living things. The organism is best understood if seen as the culmination of a long hierarchy of natural forms.

In the progression from subatomic particles to elements, to molecules, to compounds and minerals, to viruses, and to full-fledged organisms, we notice (as we proceed from small to large) that there arises more actuality, more stability, more perfect agency, and greater variety of kinds at each successive stage.

For example, in the standard model of particle physics, all ordinary matter and energy comprises of electrons, six "flavors" of quark (each in 3 colors), two heavy copies of the electron (muon and tau-lepton), three neutrinos, and antiparticles of all the above; as well as the graviton, photon, 8 gluons, three massive gauge bosons, and one or more scalar massive bosons. These do not grow or reproduce. They do not have an "inside," and like all nonliving things, they act only when they are acted upon from without. Their sphere of agency is severely limited though within it great power is available.

Stars exploit this power source and through thermonuclear combustion produce light and heat with heavy elements as by-products.

Astronomers speak of the "life cycle" of stars, but a star's "life" is strictly determined by the amount of matter it begins with.

A cloud of hydrogen with a mass of one-twentieth or less of the sun's mass coheres but its internal gravity is too weak to generate pressures and temperatures sufficient to trigger thermonuclear combustion.

This results in a failed star like the planet Jupiter which generates more energy than it receives from the sun but falls short of the conditions needed to set off nuclear reactions. A star cannot truly grow or reproduce itself. It is more an aggregate than a unity.

The interaction of subatomic particles produces the nearly hundred naturally occurring elements which exhibit far more variety and agency than the protons, neutrons and electrons of which they are composed. At a higher level of organization we find compounds, organic molecules, and minerals, each with its own special properties and powers. On the large scale, quantum uncertainty disappears, resulting in more stability.

By virtue of its structure, a complex organic molecule has an "inside" of sorts, as well as a wider range of activities; and sometimes (as in the case of enzymes) it can interact with other molecules without losing its identity.

Crystals increase in size by mere addition from the outside, involving no transformation of substance as in plant and animal growth. Also, a crystal requires the same spatial pattern to be repeated as the crystal grows. It is a regular arrangement of atoms from the bulk to the surface. Being inaccessible, the interior of the structure has no function. The crystal can develop only by the addition of components to its surface. It does not reproduce.

Crystals are made up of exceedingly small structural units, repeated, side by side, indefinitely in all directions. A perfect crystal is a homogeneous *body. Any small bit of it is just like any other small bit.*

Finally, the structures of crystals are limited in number, dictated by geometry. Mathematics states that there are only thirty-two possible classes of crystal symmetry. The 230 *types of crystal structures that occur in nature each fall into one of them.*

Viruses represent a much higher level of organization. About a thousand times larger than a protein molecule, the average virus is visible only through the techniques of electron microscopy. Viruses prefigure certain life functions and are considered by some to be rudimentary living things. Closer inspection, however, indicates otherwise.

Viruses carry out no true life activities. After the particles are formed they do not grow. They do not ingest food nor carry on any metabolic processes. So far as can be told by use of the electron microscope and by other methods of investigation, the individual particles of the virus are identical with one another, and show no change with time — there is no phenomenon of aging, of growing old.

The virus particles seem to have no means of locomotion, and seem not to respond to external stimuli in the way that larger living organisms do. Viruses have no cell membrane to receive materials selectively from without, no way to assimilate food, and no way to produce energy — all functions of even the simplest cell. Hence, the virus is closed in on itself.

It seems that reproduction is the only living activity viruses perform. But here also it is not genuine reproduction as found in animals and plants where the parent, without self-destruction, produces another being like itself, either by changing itself as when a paramecium divides into two, or by producing a seed or egg that can independently develop into an adult of the same species.

Viruses have no eggs or seeds, and they do not multiply by division. They are necessarily parasitic. Because they have no metabolism, viruses have no control over themselves and therefore cannot replicate themselves outside of a living cell.

The process of replication occurs not by the virus devouring the cell and changing its materials into more viruses. On the contrary, the virus, or at least its nucleic acid, is absorbed into the cell whose materials and energy sources the foreign nucleic acid commandeers. And, unlike reproduction in plants and animals, replication in viruses requires the disintegration of the "parent" virus.

If we require that living organisms have the property of carrying on some metabolic reactions, then the plant viruses would be described simply as molecules (with molecular mass of the order of magnitude of $10,000,000$*) that have such a molecular structure as to permit them to catalyze a chemical reaction, in a proper medium, leading to the synthesis of molecules identical with themselves.*

Thus, viruses are nonliving nanorobots. They have the machinelike capacity of being reassembled without loss. The form of the virus is determined by the requirements of physics and chemistry. In contradistinction, true growth in animals and plants produces forms not determined by known physical laws alone.

Viruses, like machines, are constructed from the outside. Living things grow from within as does, even the simplest bacterium.

It is easily demonstrated that only things that have grown from within can incorporate the kind of fivefold symmetry found in starfishes and sea urchins. Such symmetry is geometrically impossible for anything that increases from without.

Viruses take on mathematically predictable shapes. On geometrical and energetic grounds a viral coat of identical particles can be constructed in either of two arrangements: a cylinder having helical symmetry or a self-closing shell. Thus the adenovirus *that infects the human respiratory tract is an icosahedron, while the tobacco mosaic virus is a helix of RNA protected by about* $2,000$ *identical protein subunits.*

Viruses, then, fall just short of life. They are too small to incorporate life functions, and they do not have a sufficient diversity of parts. Thus in biology, as in physics, a quantum principle obtains: below a certain degree of organization, life cannot exist.

Life's unique kind of organization is widely recognized.

Activity is closer to the essence of life than structure, since structure exists for the sake of activity. The key to the living thing is the excellence of its agency. An organism can change itself; it can act or not act on its own initiative, not as determined by outside forces.

The animal or plant is not always growing or reproducing, even when food is abundant. Nonliving things do not have control over their activities; they are either always in action or are put into action from the outside. No machine turns itself on. It must be switched on, or plugged in, or at least put into contact with its energy source. Even mechanisms with built-in thermostats and timers must be set in advance, either by the manufacturer or by the user.

One of the striking things about living creatures is that they do no more (but also no less) than is required. Unlike most machines, they do not have to be switched on and off by an outside manipulator; something is built into them that does this at the proper time.

With the organism, acting or not acting, however conditioned by outside circumstances, comes from within. Living things move themselves, not merely with local motion of parts but by producing qualitative changes in those parts. Animals and even plants display a surprising degree of self-regulation regarding temperature, for example.

Inanimate objects simply take on the temperature of the environment. Living things, on the other hand, show their autonomy by balancing metabolic heat with evaporative cooling to suit their own requirements. A living thing can change itself *and thus exert control over its actions in ways never found in the inanimate world. Even the lowliest living being, is able to direct its own operations from within.*

In sum, the organism can reproduce itself without destruction. It can grow — that is, increase in quantity — while retaining its characteristic form. It can grow new diversity of parts. It can change other things into its own substance without losing its identity.

All these actions — reproduction, growth, self-regulation, nutrition — demonstrate the organism's agency. In a very real way, even the plant is a master of the material world, utilizing physical laws and inorganic powers to achieve its own goals. The animal is superior to the plant since the animal moves itself, not only through growth but through local motion directed by a sense of self awareness of the world around it.

For these reasons, among all natural objects, the organism is the highest.

What then is life? To the physicist, the two distinguishing features of living systems are complexity *and* organization. *Even a lowly bacterium reveals a complex network of function and form. It may interact with its environment in a variety of ways, propelling itself, attacking enemies, moving towards or away from external stimuli, exchanging material in a controlled fashion. Its highly organized internal working resemble a vast city in complexity of structure and form.*

Much of the control inheres within the cell nucleus, wherein is also contained the chemical blueprint that enables the bacterium to replicate. The chemical structures that control and direct this activity may involve molecules with as many as 10^6 *atoms strung together in a complicated yet highly specific way.*

Although a biological organism is made from perfectly ordinary atoms, a multi-component system may collectively possess qualities that are absent for individual components, without invoking concepts such as vitalism and holism. Atoms do not need to be 'animated' to yield life, they simply have to be arranged in appropriate complex ways.

Life itself may have arisen purely from the random self-organization of complex organic molecules. *It is not hard to envisage a prebiotic soup containing all the necessary ingredients of biology, driven by outside disturbance into interlocking self-organizing, self-reinforcing "feedback' loops, thereby fantastically increasing the odds in favor of crossing the life threshold.*

1947–1972 CE **Donald Olding Hebb** (1904–1985, Canada). Psychologist. Pioneer of physiological psychology and a founder of modern neuroscience. His theory prefigured modern mathematical models of *neural networks*, both in the actual brain and in artificial neural nets designed to solve engineering problems. Made an ambitious attempt to account in neurological terms for many of the phenomena of perception, learning and thinking. He taught that there is spontaneous brain activity which modifies and interacts with incoming stimuli.

On the neurological level, elemental neural patterns known as cell assemblies develop as a result of experience, and then combine to form a more complicated neural structure known as a 'phase sequence'.

Hebb's interest in mental processes led him to reject behaviorism. Behaviorists maintained that ideas, and thus mentalism, had no place in scientific psychology. Hebb showed that ideas could have just as firm a physical basis as muscle movements. They could consist of learned patterns of neuronal firing in the brain, initially driven by sensory input but eventually acquiring autonomous status.

He was first to claim that random neural nets could organize themselves to store and retrieve information. Hebb's studies culminated in 1949 with the publication of *The Organization of Behavior*. Although this work was not firmly grounded in physiology, it became possible later, as knowledge of the brain grew, to frame his ideas in more concrete neural terms.

Hebb was born in Chester, Nova Scotia. He graduated in English from Dalhousie University (1925). He wanted to become a novelist; to this end he set out to travel across Canada to see life, and later (1928) became a student of psychology at McGill University. Certain developments in his personal life made him leave Montreal for Chicago (1934) to continue his doctoral research under **Karl S. Lashley** (who, in 1930, had become convinced that memories could not be stored in a single region of the brain but must be spread throughout).

Hebb received his Ph.D. from Harvard (1936) and returned to McGill (1937) to work with **Wilder Penfield**, a surgeon who established the Montreal Neurological Institute. Hebb finally found a permanent position at Queen's University in Kingston, Ontario.

A turning point in his work came when he read about the work of **Rafael Lorente de Nó**, a neurophysiologist at the Rockefeller Institute for Medical Research, who had discovered *neural loops*, or feedback paths, in the brain. Hebb recognized that Lorente's looping paths were just what he needed to develop a more realistic theory of the mind.

1947–1963 CE **George Bernard Dantzig** (b. 1914, USA). Mathematician. Forged novel tools to solve linear optimization problems[1105] (*linear programming*) in the fields of economics, natural sciences and technology. Central to his work is an algorithm, known as the *simplex method*[1106]. His computational procedures laid the foundation for much of the field of system engineering and is widely used in the managerial sciences and in *operations research*.

Danzig was born in Portland, Oregon to Jewish parents of illustrious rabbinical ancestry from Eastern Europe. He was educated at the Universities of Maryland and Michigan. During WWII he was attached to the Statistical Control Headquarters of the US Air Force. He was later research mathematician with the Rand Corporation at Santa Monica, CA (1952–1966) and Stanford University (1967–1979).

[1105] Problems that involve an optimal allocation of some sort of resource (energy, money, materials) that is limited in supply; hence there are resource constraints that must be satisfied by any solution.

[1106] An n-dimensional *simplex* is the geometrical figure consisting of $(n + 1)$ points (or vertices) and all their interconnecting line segments, polygonal faces, etc. In 2 dimensions, a simplex is a *triangle*. In 3 dimensions it's a *tetrahedron* (not necessarily regular). The *simplex method* of linear programming also makes use of the geometrical concept of a simplex. Otherwise the latter is completely unrelated to the algorithm that is associated with it.

Mathematics for the Social Sciences: Linear Programming and the Simplex Method

Optimization problems were already formulated by **Euclid** *(ca 400 BCE), but only with the development of the differential calculus and the calculus of variations in the 17th and 18th centuries was a mathematical tool forged for the solution of such problems.*

By means of the classical calculus, the recipe for finding local extrema[1107] *is well-known: for functions of a single variable, one sets the first derivative to zero and then finds the root of this equation, yielding the position of the minima, maxima and other points with horizontal tangents. For functions of two variables or more, one finds candidate non-boundary local maxima and minima points by setting the gradient vector to zero and solving. Here, functions often display peculiar behavior [e.g.* $f(x, y) = y^2 - 3x^2y + 2x^4$*] and many apparently simple problems have no mathematically defined minimum values.*

Frequently, it is required to minimize a function of m *variables, where there are* k *constraints among the variables.*

In principle the k *constraint equations can be solved for* k *of the variables* x_i *and these can be used to eliminate those variables from the original function* $y = f(x_1, x_2, \ldots, x_m)$.

In practice this is apt to be impossible to carry out, and an ingenious trick known as the method of Lagrange multipliers[1108] *can be used instead.*

[1107] An *extremum* (maximum or minimum point) can be either *global* (truly the highest or lowest function value) or *local* (the highest or lowest in a finite neighborhood). Although some standard heuristics are used, virtually nothing is known about finding global extrema in general. The analytical algorithms for finding local extrema are subjected to two assumptions:

- the function and its derivatives are continuous (no cusps)
- the extreme values do not occur on the boundary

Both assumptions are significant in practice. Local extrema that *do* lie on the boundary can be found by restricting the function to that boundary and (unless the boundary is a union of discrete points) repeating the procedure for the restricted-domain function.

[1108] For example, the values of the points $(x_1, x_2, \ldots, x_m)$ may be required to lie on the surface of a sphere $\phi = x_1^2 + x_2^2 + \cdots + x_m^2 - a^2 = 0$. It is then shown that the problem reduces to optimizing the new function $L = f + \lambda\phi$ as a

There, is, however a wide class of optimization problems where this method will fail: namely, problems in which some of the constraints are inequalities. Thus, let it be required to minimize the linear function $f(x) = \sum_{i=1}^{N} c_i x_i$ *subject to the linear constraints* $\sum_{i=1}^{N} a_{ij}x_i - b_j \geq 0, \quad j = 1, \dots m.$ *These are called linear programming problems, and they are important because of the enormous variety of application which they have found in practical situations. We work out a specific example: determine*

$$x_1 \geq 0 \qquad x_2 \geq 0$$

so that

$$z = 10x_1 + 11x_2$$

is a maximum, and so that

$$3x_1 + 4x_2 \leq 9; \qquad 5x_1 + 2x_2 \leq 8; \qquad x_1 - 2x_2 \leq 1.$$

It is evident that the set of points satisfying all five constraints (inequalities) lie in the first quadrant of the x_1–x_2 *plane, and inside or on the boundary of a convex polygon of* 5 *sides bounded by the* 5 *lines:*

$$x_1 = 0; \qquad x_2 = 0; \qquad 3x_1 + 4x_2 = 9; \qquad 5x_1 + 2x_2 = 8; \qquad x_1 - 2x_2 = 1.$$

These lines intersect at the vertices of the polygon, namely at

$$O(0, 0); \qquad A(1, 9); \qquad B(1.5, 0.25); \qquad C(1, 1.5) \qquad D(0, 2.25).$$

The problem now is to select from the above 2-*dimensional continuum of points in and on the polygon the one point which renders* z *maximum. This*

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function of the x_i with λ as an additional variable. One then obtains $m + 1$ equations:

$$\frac{\partial f}{\partial x_i} + \lambda \frac{\partial \phi}{\partial x_i} = 0, \qquad i = 1, 2, \dots, m; \qquad \phi(x_1, x_2, \dots, x_m) = 0$$

in the $m + 1$ variables $(x_1, x_2, \dots, x_m; \lambda)$.
As an illustration consider the problem of finding the maximum rectangular block that will fit inside the ellipsoid $\phi = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0$. The volume to be maximized is $V = 8xyz$ subject to the constraint of the point (x, y, z) lying on the ellipsoid.
Here $L = V + \lambda\phi$ and straightforward analysis yields

$$x = \frac{a}{\sqrt{3}}, \qquad y = \frac{b}{\sqrt{3}}, \qquad z = \frac{c}{\sqrt{3}},$$

with a maximum volume of $V = \frac{8}{3\sqrt{3}}abc$.

task can be readily accomplished by plotting a family of parallel straight lines representing $10x_1 + 11x_2 = \text{constant}$ *and watching for that particular line which renders the highest value of the constant and still includes points of the polygon or its interior.*

Starting from a line which passes through $(0, 0)$, *the value of the constant steadily increases until its extreme value is obtained at the vertex* $C(x_1 = 1,\ x_2 = 1.5)$. *Hence* $z = 10 \times 1 + 11 \times 1.5 = 26.5$ *is the sought extreme value.*

The polygonal area is known as the feasible region; *a little thought will make it intuitively clear that the solution of a linear program lies on the boundary of the feasible region, usually on a vertex, but sometimes along one segment of a constraint line.*

The algebraic *counterpart of the above geometric scheme is as follows: one first converts the constant inequalities into corresponding equations:*

$$\begin{aligned} 3x_1 + 4x_2 + x_3 \qquad\qquad &= 9 \\ 5x_1 + 2x_2 \qquad + x_4 \qquad &= 8 \\ x_1 - 2x_2 \qquad\qquad + x_5 &= 1 \end{aligned}$$

where $\{x_3, x_4, x_5\}$ *are additional, so called* slack variables. *Clearly all five variables are non-negative:* $x_i \geq 0 \quad i = 1, 2, \ldots, 5$. *The above three equations involve five unknowns and, therefore, have an* infinite *number of solutions.*

However, if we arbitrarily assign the value zero to two of the five variables and solve for the other three, then there are 10 *possible choices, of which five yield negative values of some variables and are therefore not feasible.*

The remaining five are called basic feasible solutions, *since they satisfy all constraints. They also have a one-to-one correspondence to the five vertices of the polygonal domain. It remains to choose out of these five options, the optimal solution that maximizes* $z = 10x_1 + 11x_2$.

In general, when the number of variables and equations is large, the graphical-geometrical method presented earlier becomes useless and other methods must be used. Of the several computational schemes available, Dantzig's simplex method *is the most widely used. It is basically a numerical algorithm employing the* Gauss-Jordan elimination procedure *for preparing what is known as a* simplex tableau.

In a nutshell it amounts to starting at some vertex (basic feasible point) and, by a sequence of exchanges, proceed systematically to other such points *in a way which steadily reduces (or increases) the value of* $f(x)$ *until a solution point is found*[1109].

Linear programming applies to solutions of 2-person games. Let the payoff matrix, consisting of positive numbers a_{ij}, *be*

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

by which we mean that when player R *has chosen a row* i *of this matrix and player* C *has (independently) chosen column* j, *a payoff of amount* a_{ij} *is made from* R *to* C. *This constitutes one play of the game. The problem is to determine the best mixed strategy for each player in the selection of rows or columns. Let* C *choose one of three columns with probabilities* (p_1, p_2, p_3) *respectively. Then*

$$p_1, p_2, p_3 \geq 0 \qquad \text{and} \qquad p_1 + p_2 + p_3 = 1.$$

Depending on R's choice of row, C *now has one of the following three*

1109 Because of the enormous number of possible paths around the edges of a polytope, the *simplex algorithm* is known to be, in theory, an *exponential time algorithm*, but when used in practice (on problems involving hundreds or even thousands of variables) it works extremely well, homing in on the optimal vertex in a relatively small number of steps.

The indications are that it tends to run in *linear time*. [Indeed, Dantzig (1962) was able to solve a problem with $32,000$ constraints and 2 million variables in justifiable computer time.]

A group of Russian mathematicians (1976) used a modification of the simplex method, known as the *ellipsoidal method*, in which the direction of the path to be followed across the *interior of the polytope* is determined with the aid of a sequence of ellipsoids drawn to 'approximate' the polytope.

Yet, although the ellipsoidal method runs *theoretically* in polynomial time, the simplex method proved to be superior when applied to real-world problems.

A new polynomial-time linear programming algorithm was developed by **Narenda Karmarker** (1984, USA) which outperformed the simplex method on many occasions. In his method the sophisticated *geometrical ideas* of the simplex method are suppressed in favor of a series of *arithmetic* operations on *matrices*. In the case of *dynamic programming*, allocated computer resources grow *exponentially* as the problem size increases.

quantities for his expected winnings:

$$P_1 = a_{11}p_1 + a_{12}p_2 + a_{13}p_3;$$
$$P_2 = a_{21}p_1 + a_{22}p_2 + a_{23}p_3;$$
$$P_3 = a_{31}p_1 + a_{32}p_2 + a_{33}p_3.$$

Let P *be the least of these three numbers. Then, no matter how* R *plays,* C *will have expected winnings of at least* P *on each play and therefore asks himself how this amount* P *can be maximized. Since all numbers involved are positive, so is* P*; and we obtain an equivalent problem by letting* $x_1 = \frac{p_1}{P}$, $x_2 = \frac{p_2}{P}$, $x_3 = \frac{p_3}{P}$ *and minimizing* $F = x_1 + x_2 + x_3 = \frac{1}{P}$.

The various constraints may now be expressed as

$$x_1,\ x_2,\ x_3\ \geq\ 0$$
$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \geq 1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \geq 1$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \geq 1$$

Looking at things from R's point of view, we maximize

$$G = y_1 + y_2 + y_3 = \frac{1}{Q}$$

under the constraints,

$$y_1,\ y_2,\ y_3\ \geq\ 0$$
$$a_{11}y_1 + a_{21}y_2 + a_{31}y_3 \leq 1$$
$$a_{12}y_1 + a_{22}y_2 + a_{32}y_3 \leq 1$$
$$a_{13}y_1 + a_{23}y_2 + a_{33}y_3 \leq 1$$
$$y_1 = \frac{q_1}{Q}, \qquad y_2 = \frac{q_2}{Q}, \qquad y_3 = \frac{q_3}{Q}$$

where (q_1, q_2, q_3) = *probabilities with which* R *chooses the three rows.*

Thus for instance with $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}$, *the simplex method yields*

$$p_1 = \tfrac{1}{2}, \qquad p_2 = \tfrac{1}{6}, \qquad p_3 = \tfrac{1}{3}$$
$$q_1 = \tfrac{1}{6}, \qquad q_2 = \tfrac{1}{2}, \qquad q_3 = \tfrac{1}{3}$$

If either player uses the optimal strategy for mixing his choices, the average payoff will be $\frac{5}{6}$.

A second example illustrates the usefulness of the simplex method in solving real-life problems: A university department has a faculty of N *members and a yearly budget* B, *for salary increases. The chairman must decide how this money is to be distributed among the faculty. He sets up a committee through which all members are subjected to peer evaluation; each professor is ranked in three categories of activity:* research, service *and* teaching *and is labeled* outstanding *(O),* strong *(G) or* satisfactory *(S) in each of the activity categories, implying that each of the faculty are placed in one of* $3^3 = 27$ *overall categories.*

Let the serial number of a category be denoted by k *and let the number of faculty in the k-th category be denoted by* n_k*; k ranges over* $1, 2, ..., 27$ *and* $N = \sum n_k$.

Further, let the classification O, G, and S be labeled through the index i *as* $i = 1, 2, 3$ *respectively and let the classification research, service and teaching be labeled through the index* j *as* $j = 1, 2, 3$ *respectively.*

Construct a 3×3 *merit matrix,* M, *whose integer elements are* achievement factors:

$$\begin{array}{cccccc} & \text{Res.} & \text{Ser.} & \text{Teach.} & & \\ M = & \left[\begin{array}{c} m_{11} \\ m_{21} \\ m_{31} \end{array}\right. & \begin{array}{c} m_{12} \\ m_{22} \\ m_{32} \end{array} & \left.\begin{array}{c} m_{13} \\ m_{23} \\ m_{33} \end{array}\right] & \begin{array}{c} \text{O} \\ \text{G} \\ \text{S} \end{array} & \begin{array}{c} i=1 \\ i=2 \\ i=3 \end{array} \\ & j=1 & j=2 & j=3 & & \end{array}$$

The chairman has decided to solve the allocation problem by paying each faculty member an additional ν_j *dollars for every unit of achievement factor in the* j*-column. Since each value of* k *defines a mapping* $i = i(k, j)$ *where* $i, j = 1, 2, 3,$ *a faculty-member in category* k *will receive a total salary increase* $S_k = \sum_{j=1}^{3} m_{i(k,j)j}\nu_j$ *[e.g. if category* $k = 9$ *is {S, O, O} (*satisfactory *in research and outstanding in both service and teaching), then* $S_9 = m_{31}\nu_1 + m_{12}\nu_2 + m_{13}\nu_3$*]. Clearly,* $B = \sum_{k=1}^{27} n_k S_k = \beta_1\nu_1 + \beta_2\nu_2 + \beta_3\nu_3$ *is the* budget constraint, *where* $\{\beta_1, \beta_2, \beta_3\}$ *are constants calculable from* n_k *and* m_{ij}.

Defining $u_j = \frac{\nu_j}{B}$, *the normalized budget constraint assumes the form*

$$\beta_1 u_1 + \beta_2 u_2 + \beta_3 u_3 = 1,$$

which can be replaced with the two inequalities,

$$\sum_{j=1}^{3} \beta_j u_j \leq 1 \quad \text{and} \quad \sum_{j=1}^{3} \beta_j u_j \geq 1 \, .$$

The chairman next puts constraints on the relative magnitudes of the ν_j *according to what he thinks are the best interests of the department [e.g. if he is concerned about the departments research activity, he would like to provide an incentive to excel in it, etc.]. These considerations will be manifest in the constraints*

$$\frac{1}{\lambda}\nu_1 \leq \nu_2 \leq \lambda\nu_1, \qquad \frac{1}{\lambda}\nu_1 \leq \nu_3 \leq \lambda\nu_1, \qquad \frac{1}{\lambda}\nu_2 \leq \nu_3 \leq \lambda\nu_2$$

where $\lambda > 1$; λ *quantifies the maximal acceptable disparities between pay-raise incentives for the three activities.*

A further necessary constraint guarantees a minimum amount of increase that a professor can receive (for the sake of department morale!) i.e. $\nu_1 + \nu_2 + \nu_3 \geq \epsilon B$ *where* ϵ *is a small number. Combining all the relevant equations, the mathematical formulation takes the form of a linear program:*

find $\boldsymbol{u} = (u_1, u_2, u_3)$, *which maximizes* $J(\boldsymbol{u}) = \sum_{j=1}^{3} m_{1j} u_j$ *(the fraction of allocated salary-increase budget going to a professor ranked "outstanding" in all activity categories) subject to* $u_j \geq 0$, $\sum_{j=1}^{3} a_{ij} u_j \leq b_i$, *where* $\{a_{ij}, b_i\}$ *are given real numbers, and* $i = 1, 2, ..., 9$. *The maximized function is the salary increase to professors in category (O,O,O), as a fraction of B.*

The solution will depend on the values preassigned to the parameters λ *and* ϵ.

The previous examples shows that linear programming provide a mathematical model of a real-life problem in which something needs to be maximized (e.g. profit or security) or minimized (e.g. costs or risks).

The required optimization is achieved by a suitable choice of the values of a number of variables.

Both the factor to be optimized and some or all the variables (parameters) will be subject to one or more constraints[1110]. *For example, optimization problems in economics are extreme value problems with auxiliary conditions, which are often characterized by the fact that the number of variables is very large and that non-negative solutions are sought.*

The investigation of such problems began to enter the mathematical consciousness as a recognizable discipline during the latter part of the 1930s, gaining considerable visibility and attention during and immediately after WWII.

Many types of business and military questions involving things like the best way to schedule aircraft maintenance, allocate money to investments or process parts in an assembly-line operation, came to be part of a field now called operational research.

A fundamental problem in economics is the optimum allocation of scarce resources among competing activities — a problem that can be expressed in mathematical form. Linear optimization can be applied to that problem, as well as to many areas of natural sciences and technology.

1947–1967 CE **Richard Buckminster Fuller** (1895–1983, USA). Engineer, inventor, humanist and visionary. A creative maverick figure. Solved many design problems in such diverse fields as automobile designs, city planning, and architectural engineering. Best known for the design of huge 3-D structures which he developed to achieve maximum spans with minimum use of material.

Fuller sought to expand man's ability to control larger areas of his environment and still have close relationship with nature according to his principle: "Maximum gain of advantage from minimal energy input".

[1110] There are, however, *unconstrained* optimization problems such as the *Traveling Salesman Problem* and the nonlinear *Gropius Housing Problem*. Nonlinear optimization problems are solved within the framework of *dynamic programming* (**Abraham Wald**, 1950).

His ideas show the influence of such natural molecular structure as the *tetrahedron*[1111] and the *truncated icosahedron*. He became a guru to generations of architectural students because of his irrepressible optimism and his belief in technology as a tool for improving the quality of life.

Fuller was born in Milton, MA. After WWI he spent several years working for industry before producing his first important design, the *dymaxion house*[1112] (1927).

His dymaxion house was meant to be a high technology response to the chronic housing shortage of the depression era. This idea initiated the prefabricated housing market. *The dymaxion car* (1932) and the dymaxion *air-ocean city* (1943) followed.

He is best known, however, for his *geodetic dome*[1113] (1947), a huge structure approximating a spherical sector and composed of light, strong triangular parts. (There are now more than $200,000$ geodetic domes around the world, the most famous one at Epcot Center, Disney world, Florida.)

2500 years after Pythagoras and Plato, Fuller revived the connection between polyhedra and the natural world. Indeed, the C_{60} carbon molecule[1114] (1991, named after him) has the shape of a truncated icosahedron (Fig. 5.24).

[1111] For a given surface area S_0, the *sphere* encloses the greatest volume $V_{\max} = S_0^{3/2}/6\sqrt{\pi}$. For a given volume V_0, the sphere has the least surface area $S_{\min} = V_0^{2/3}\left\{3\left(\frac{4\pi}{3}\right)^{1/3}\right\}$. Of all polyhedra with a given surface area S_0, the *tetrahedron* has the least volume $V_{\min} = S_0^{3/2}/6\sqrt{6}\sqrt[4]{3}$, and of all polyhedra with a given volume V_0, the tetrahedron has the greatest surface area $S_{\max} = \left(6\cdot 3^{\frac{1}{6}}\right)V_0^{2/3}$.

[1112] Dymaxion = Dynamic plus maximum efficiency.

[1113] Since the sphere has the least surface area for a given volume, the geodetic dome both creates a great amount of internal space and minimizes heat loss because of its decreased outer skin surface. The geodetic dome has the property that its strength need only increase as the $\log$ of its size; its weight is only about 10 kg per m^2 of covered area. In this connection, Fuller coined the word *tensegrity* = tension + integrity.

[1114] Known as *buckminsterfullerene*, fullerene, or simply the *buckyball*: A unique molecular oddity, discovered (1991) by **Richard E. Smalley**, at Rice University, Houston, USA. The molecule is composed of 60 atoms of carbon, linked together to form a truncated icosahedron having 12 regular pentagons and 20 regular hexagons, looking like a soccer ball. It has the following properties:

- The largest possible symmetric ("rounded") molecule.
- Does not bond readily to other atoms or molecules, yet because it is hollow on the inside, all elements of the Periodic Table could fit inside. [However,

The icosahedron itself appears as one of the geodetic forms of *viruses* (e.g.: bacteriophage M52; Polio virus; Herpes virus; the AIDS virus HTLV-1; the K-virus, etc.].

Fuller was a research professor at Carbondale, Southern Illinois University (1959–1968). In 1968 he became a university professor and retired in 1975.

Twice expelled from Harvard University, business disasters and the death of his four year old daughter brought him close to suicide. However, he recovered and decided to devote himself to proving that technology could save the world from itself, provided it is properly used.

$C_{60}H_{60}$, the hydrogenated buckyball is generated from C_{60} by adding hydrogen atoms — it is known as the "*fuzzyball*".]

- *Spins* at a rate of 10^8 times per second; has 174 different modes of *oscillations*.
- When compressed to 70% of its original size becomes more than twice as hard as diamond. This compressibility points to its possible use as a shock absorber.
- Can withstand slamming into a stainless steel plate at a speed of $6\ \frac{\text{km}}{\text{sec}}$. This resilience could be an asset in creating rocket fuels, as they must undergo extreme pressure; another possible use of the buckyball, involving its pressure resistance, is in the armor industry.
- Can serve as a conductor, insulator, semiconductor or superconductor.
- Exhibits ferromagnetic properties.
- Can be made into a battery by stripping away electrons from the buckyball. Other fullerene molecules have been synthesized by capping a *carbon nanotube* (rolled graphite sheet, a few nanometers across and 100's of nm long) with a half-buckeyball at either end.

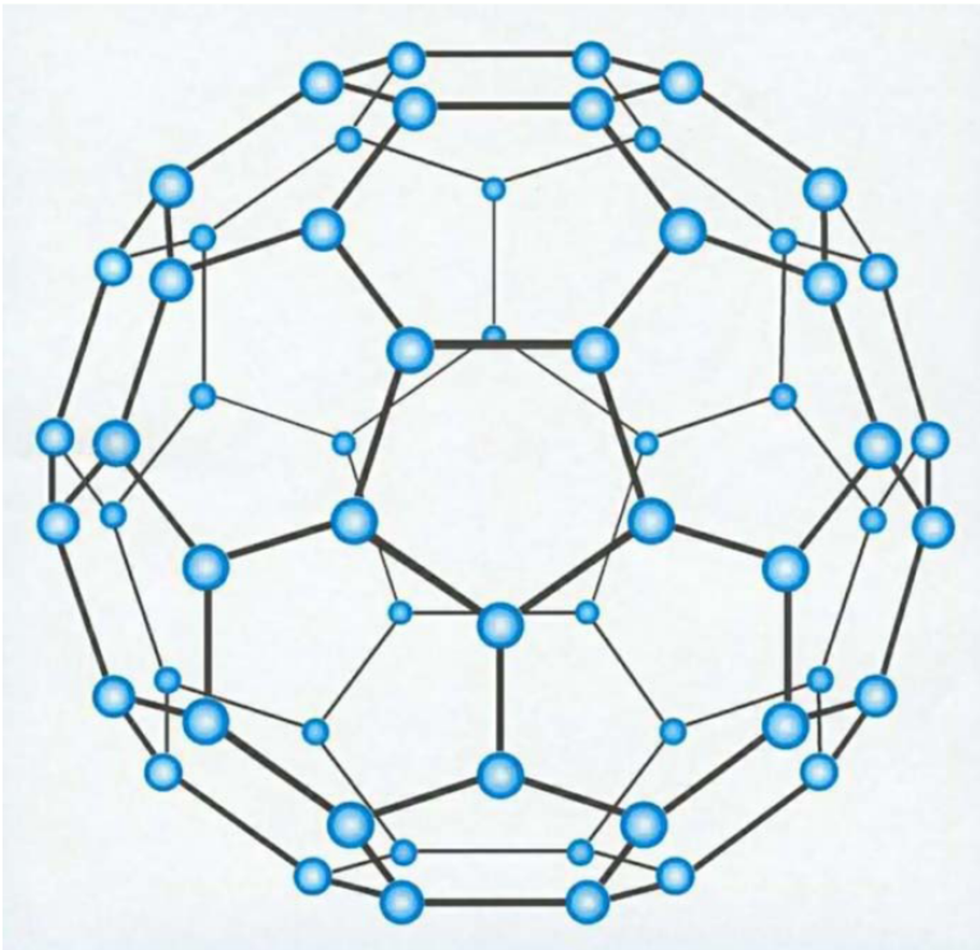


Fig. 5.24: Geometry of the molecule C_{60}

Mathematics and Architecture — from the Pyramids to the Geodetic Dome

Historically, architecture was part of mathematics, and in many periods of the past, the two disciplines were indistinguishable. In the ancient world; mathematicians were architects, whose constructions — the pyramids, ziggurats, temples, stadia, and irrigation projects — we marvel at today.

In Classical Greece and ancient Rome, architects were required to also be mathematicians. When the Byzantine emperor ***Justinian*** *wanted an architect to build the Hagia Sophia as a building that surpassed anything ever built before, he turned to the two geometers* **Isidoros** *and* ***Anthemios****, to do the job. This tradition continued into the Islamic civilization. Islamic architects created a wealth of two-dimensional tiling patterns centuries before western mathematicians rendered a complete classification of such patterns.*

Some historians of science believe that the concept of the Golden ratio $\left[\frac{1}{2}(1+\sqrt{5}) = 1.618\ 033\ldots\right]$ *had been used in the construction of the Giza pyramid of King Khufu (2575 BCE). There is, however, no proof that sophisticated geometry lies behind the construction of the Pyramids.*

The first definite mathematical influence on architecture is that of ***Pythagoras*** *and his Pythagoreans. The discovery that beautiful harmonious sounds depend on ratios of small integers led to architects designing buildings using ratios of small integers. This, in turn, led to the use of a* module*, a basic unit of length for the building, where the dimensions were now small integer multiples of the basic length.*

To Pythagoras, numbers also had geometrical properties. The Pythagoreans spoke of square numbers, oblong numbers, triangular numbers etc. Geometry was the study of shapes and shapes were determined by numbers.

But more than this, the Pythagoreans developed a notion of aesthetics based on proportion. In addition geometrical regularity expressed beauty and harmony and this was applied to architecture with the use of symmetry.

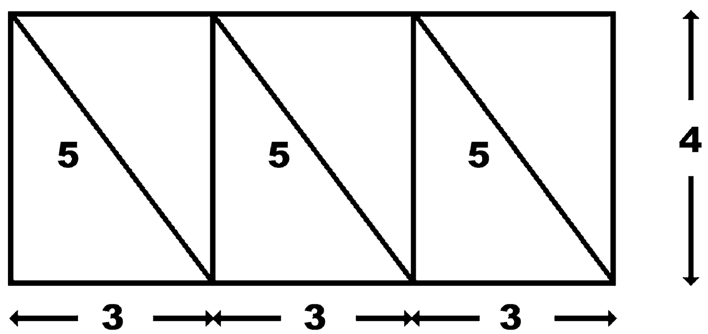
To a mathematician today, symmetry suggests an underlying action of a group on a basic configuration, but it is important to realize that the word comes from the ancient Greek architectural term "symmetria" which indicated the repetition of shapes and ratios from the smallest parts of a building to the whole structure. It should now be clear what the belief that "all things are numbers" meant to the Pythagoreans and how this influenced ancient Greek architecture.

Let us look briefly at the dimensions of the Parthenon to see how its lengths conform to the mathematical proportion principles of the Pythagoreans. In 480 BC the Acropolis in Athens was totally destroyed by the Persians in the Second Persian War. This was about the time of the death of Pythagoras.

After the Greek victory over the Persian at Salamis and Plataea, the Greeks did not begin the reconstruction of the city of Athens for several years. Only after the Greek states ended their fighting in the Five Years' Truce of 451 BC did the conditions exits to encourage reconstruction.

*Pericles, the Head of State in Athens, set about rebuilding the temples of the Parthenon in 447 BC. The architects **Ictinos** and **Callicrates** were employed, as was the sculptor **Phidias**.*

In the construction of the Athena Parthenos, ratios of small numbers were used in the following way:



The ratio $2 : 3$ *and its square* $4 : 9$ *were fundamental to the construction. A basic rectangle of sides* $4 : 9$ *was constructed from three rectangles of sides* 3 *and* 4 *with diagonal* 5*. This form of construction also meant that the* $3 : 4 : 5$ *Pythagorean triangle could be used to good effect to ensure that right angles in the building were accurately determined.*

The length of the temple is $69.5\ m$*, its width is* $30.88\ m$ *and the height at the cornice is* $13.72\ m$*. To a fairly high degree of accuracy this means that the ratio width: length* $= 4 : 9$ *while also the ratio height: width* $= 4 : 9$*. The greatest common divisor of these measurements leads to the ratios*

$$\text{height} : \text{width} : \text{length} = 16 : 36 : 81$$

which gives a basic module of length $0.858\ m$*.*

The length of the temple is then 9^2 *modules, its width* 6^2 *modules and its height* 4^2 *modules. The module length is used throughout; for example the overall height of the temple is* 21 *modules, and the columns are* 12 *modules high.*

The naos, which in Greek temples is the inner area containing the statue of the god, is 21.44 *m wide and* 48.3 *m long which again is in the ratio* 4 : 9*. One may note that the columns are* 1.905 *m in diameter and the distance between their axes is* 4.293 *m; again the ratio* 4 : 9 *is being used.*

***Plato** was much influenced by the ideas of Pythagoreans. He saw mathematics as providing the most fundamental of all ideas and therefore maintained that buildings, which last longer than life-forms, should be designed on mathematical principles.*

We may learn about the mathematical methods of ancient architects through the work De architectura *by the Roman engineer and architect **Vitruvius**, written shortly before 27 BCE. He was in charge of building projects in Rome during the reign of Octavianus. The ten books of his treatise are*

1. *Principles of architecture.*
2. *History of architecture, and architectural materials.*
3. *Ionic temples.*
4. *Doric and Corinthian temples.*
5. *Public buildings, theaters, music, baths, and harbors.*
6. *Town and country houses.*
7. *Interior decoration.*
8. *Water supply.*
9. *Dials and clocks.*
10. *Mechanical engineering with military applications.*

It is interesting (particularly given the details above on how the Temple of Athena on the Parthenon was constructed) to look at what Vitruvius says in Book 3 on designing temples.

The book begins with an essay on symmetry and then describes the use of symmetry and proportion in the design of temples. For Vitruvius the proportions of the human body were fundamental in achieving beauty and he *says that the proportions of the temple should follow these human proportions. He suggests that the circle and the square are perfect figures for generating architectural designs because they approximate the geometry of the spread-eagled human body.*

There is a religious significance here, since Vitruvius believed that the human body was made in God's image and was therefore perfect. Of course many have argued that the golden number can be found in the proportions of the human body, so it may be that the evidence found today for the golden number in ancient Greek temples is explained by its relation to human proportions.

In Book 5 Vitruvius explains sound as a displacement of air in waves which he compares with the waves that can be observed on the water's surface when a stone is thrown into a pond. What is more remarkable was Vitruvius' application of the wave theory to architectural acoustics. The wave theory of sound was Greek, while its application to the acoustics of a hall was typically Roman.

Vitruvius analyzes the acoustics of a theater and the phenomena that may spoil it, which today we call interference, reverberations and echoes.

In Book 10 Vitruvius describes hoisting machines, engines for raising water, water wheels and water mills, water screws, Ctesibios' pump, water organs and odometers, and passes from civil engineering to engines of war, catapults and scorpions, ballistae, stringing and tuning of catapults, siege engines and tortoises for filling ditches.

In Europe there was little further progress in mathematics and architecture until the 14th and 15th Centuries. Architecture was modeled on the teachings of Vitruvius and on the classical architecture which was still plentiful, particularly in Greece and Italy.

The next person we want to mention is ***Brunelleschi*** *(1377–1446) who was trained as a goldsmith. There were really no professional architects at this time and Brunelleschi learnt his skills in architecture by visiting Rome.*

He made drawings of a great many ancient buildings, including baths, basilicas, amphitheaters, and temples, particularly studying the construction of architectural elements, such as vaults and cupolas. The object of his architectural researches, however, was not to learn to reproduce Roman architecture, but to enrich the architecture of his own time and to perfect his engineering skills.

Brunelleschi made one of the most important advances with his discovery of the principles of linear perspective. Classical scholars had understood some of the principles of perspective but no text seems to have been written on the topic.

We think of an understanding of perspective as being essential for a realistic two dimensional representation of a three dimensional scene when painting on a canvass. However Brunelleschi's understanding of perspective was used in his design of buildings as he created his designs to ensure that the visual effect he wanted was visible from all possible positions of the observer.

Following the rules of proportion and symmetry of the ancients was important to Brunelleschi but he also wanted these mathematical principles of beauty to be those seen by all observers. In some sense he was trying to achieve a certain invariance of proportion, independent of the angle of view, and to ensure that it was the apparent proportion which was right rather than the actual proportion.

Many of the famous mathematicians from the time of Brunelleschi on, made contributions to architecture. ***Alberti*** *(1404–1472) wrote a text on the topic, as well authoring an important text on perspective in which he wrote down Brunelleschi's discoveries for the first time. He was one of a number of mathematicians to develop a general theory of proportion which was motivated by his architectural studies.*

The great ***Leonardo da Vinci*** *(1452–1519) was fascinated by mathematics. Architecture was another of his specialities and he learnt about it, in particular the mathematical principles behind it, from studying* ***Alberti's*** *texts.*

Leonardo was a man of wide ranging abilities and interests and, at one stage in his career, earned his living advising the Duke of Milan on architecture, fortifications and military matters. He was also well regarded as a hydraulic and mechanical engineer and worked for Cesare Borgia as a military architect and general engineer. Later the French King Francis I appointed him first painter, architect, and mechanic to the King.

Another mathematician from Renaissance times was ***Rafael Bombelli*** *(1526–1572) who was taught by* ***Pier Francesco Clementi****, himself an engineer and architect. With this training* ***Bombelli*** *was soon working on his own as both engineer and architect, employing his mathematical skills both in his work and in his investigation of complex numbers.*

Another to combine his skills in both mathematics and architecture was the painter ***Leonaert Bramer*** *(1596–1674, Delft) who was employed directing constructions of fortifications and castles. He published a work on the calculation of sines, prompted by the practical work in which he was involved.*

He followed Alberti (1435), ***Dürer*** *(1525) and* ***Bürgi*** *(1604) when in 1630 he constructed a mechanical device that enabled one to draw accurate geometric perspective.*

La Faille *(1597–1652) was a contemporary of* ***Bramer*** *who taught mathematics and military engineering. He worked as an architect advising on fortifications, and wrote an architectural treatise as well as important works on mechanics*[1115].

Later in the 17th century lived the English architect ***C. Wren****, in many ways the best known architect in English history. A well-rounded scientist, he solved a number of important mathematical problems before taking up architecture as a profession. Although he is better known as an architect than as a mathematician he was considered one of the leading mathematicians of his day by* ***Newton****.*

It was clear that ***Wren*** *saw mathematics as being a subject which had applications to a wide variety of scientific disciplines and his mathematical skills played an important role in his architectural achievements.*

One of the architects with whom he worked, ***Robert Hooke****, is better known as a mathematician and scientist than as an architect. Again, that mathematics and architecture were closely related disciplines was considered natural at this time.*

Another 17th century mathematician was ***Philippe de La Hire*** *(1640–1718) whose interest in geometry arose from his study of architecture. In 1687 he was appointed to the chair of architecture at the Academie Royale. His interest in geometry arose from his study of perspective and he went on to make important contributions to the theory of conic sections.*

In the 18th century ***Giovanni Poleni*** *(1683–1761) made contributions to hydraulics, physics, astronomy and archaeology. He held university chairs in astronomy, physics and mathematics as well as working as an architect.*

The nineteenth century saw a change of attitude which led to a separation in people's minds of the scientific and the artistic. From this period on, the roles of mathematician and architect were seen as distinct in a way they were not in previous centuries. This is not to say that the connections between mathematics and architecture vanished, just that the scientific and artistic aspects were seen as complementary skills not to be found in the same person.

Of course there were still those who did excel in both mathematics and architecture; it was only perceptions which had changed.

An example of a person who excelled in architecture and mathematics was ***Siegfried Heinrich Aronhold*** *(1819–1884,* Germany*) who taught at the Royal Academy of Architecture at Berlin from 1851. Aronhold was appointed professor there in 1863. He made outstanding contributions to geometry.*

[1115] He was first (1632) to determine the center of gravity of the sector of a circle.

Others from this period who combined the two skills include ***Francesco Brioschi*** *(1824–1897, Italy) and* ***Ludwig Christian Wiener*** *(1826–1896, Germany).*

From 1852 to 1861 ***Brioschi*** *was professor of applied mathematics at the University of Pavia. There he taught mechanics, architecture and astronomy.*

Wiener *studied engineering and architecture at the University of Giessen from 1843 to 1847. With this training he went on to become a teacher of physics, mechanics, hydraulics and descriptive geometry at the Technische Hochschule in Darmstadt.*

There were a number of late 19th century and 20th century mathematician who began their careers as architects before turning to mathematics, for example the Frenchman ***Jules Joseph Drach*** *(1871–1941) and the American* ***Samuel Stanley Wilks*** *(1906–1964).*

Drach *worked as an architect before becoming a mathematician.*

Wilks *studied architecture at North Texas State Teachers College, receiving a B.A. in architecture in 1926. However his eyesight was poor, and he feared that this would be a handicap if he pursued architecture as a profession; he thus decided upon a career in mathematics.*

Two unique talents from the 20th century were ***Escher*** *and* ***Buckminster Fuller****.*

Escher *was never a mathematician, despite his fascination with the subject and the deep mathematical ideas which underlay his art. He trained at the School of Architecture and Decorative Arts in Haarlem, Holland, and only at age 21 did he give up architecture in favor of art.*

Buckminster Fuller was an engineer, mathematician and architect who applied geometric principles to introduce a totally new concept in building design during the second half of the 20th century. He made an art out of structural purity, using simple geometric forms for aesthetic as well as functional purposes.

1947–1977 CE **Richard Wesely Hamming** (1915–1998, USA). Mathematician. Invented and applied *error-detecting and error-correcting codes*

(ECC)[1116] for computer systems, digital communication and data storage (1947–1950) thereby launching a new subject within information theory.

[1116] In the transmission of digital information errors are likely to occur due to *noise* in the information channel. This noise might be inherent in the physical nature of the channel, or due to technical failures or negligence on the part of the sender or receiver. In coding theory it is assumed that errors occur randomly and independently; it is equally likely for a bit (= binary digit) of value 0 to be incorrectly received as bit 1 and vice versa. In this basic setup there is no malicious adversary acting on purpose. The overall purpose of coding theory is to introduce *redundance* in such a way that even if errors occur in the transmission, the received message can still be correctly interpreted with very high probability. Of course, some assumption has to be made concerning the expected rate of errors; no amount of redundancy is sufficient to always correct any number of errors.

Coding theory and *cryptography* have opposite aims. In coding theory one tries to write the message in such a form that reasonably many errors can be tolerated in the transmission. In this sense the clarity of the message is increased.

In cryptography, on the other hand, one tries to decrease the clarity in order to make the message incomprehensible to an eavesdropper, but in such a manner as can be deduced by a legitimate recipient with privileged knowledge of the coding/decoding procedures.

Because of these opposite aims it is difficult to combine the two approaches, although it would be very important to translate messages into a form protected both against eavesdroppers *and* random noise.

When we wish to store, search for, or send information electronically in the presence of noise, *efficiently* and with least error — sophisticated coding operations are required in order to achieve efficiencies as close as possible to the theoretical bounds. One must first distinguish between *error-detection* and *error-correction*:

Consider a *binary code* which consists of a set of *codewords*, each being a string of n bits.

Suppose we wish to send the messages *north*, *south*, *east*, *west* which are coded as follows:

	north	south	east	west
code C_1	00	01	10	11
code C_2	000	101	011	110

The code C_1 requires transmission of only two bits, but this code cannot detect any errors, since if errors occur in either or both bits, then an incorrect message is received. For example, if 01 (*south*) is sent and there is a transmission error in the first bit so that 11 is received, then this is interpreted as *west*.

He also worked on numerical analysis and automatic coding system. Named after him are: *Hamming distance*; *Hamming bound*; *Hamming code*; *Hamming spectral window* (used in computation for smoothing data before Fourier-analyzing it).

Hamming was born in Chicago and received his Ph.D. in mathematics (1942) from the University of Illinois. He joined the Manhattan project at Los Alamos (1945–1946) and worked at the Bell Telephone Laboratories throughout the major part of his academic career (1946–1976). He then accepted a chair of computing science at the Naval Postgraduate School at Monterrey, California.

In 1947, Hamming was one of the earliest users of primitive computers at Bell Laboratories. Frustrated by their lack of fundamental reliability, he therefore puzzled over the problem of how a computer might check and correct its own results. Within several months Hamming discovered that extra bits could be added to the internal binary numbers of the computer to redundantly encode numerical quantities. This redundancy enabled relatively simple circuitry to identify and correct any single bit that was bad within the encoded block of bits (typically one word of data).

The second code C_2 can detect any single error. For example if 101 is sent and an error occurs in a single bit, then either 001, 111 or 100 is received. None of these is a codeword, so the receiver is aware of a transmission error. Although this code cannot correct errors, it reveals how redundancy can be added to the original message, in the form of extra bits, in such a way that transmission errors can be detected. Clearly, a code which detects, but does not correct errors is only useful if the receiver can obtain a *repetition* of the message in which an error has been detected.

Assume, for example, that a message consists of a single bit, 0 or 1 ('Yes' or 'No'). The *repetition code* of length 3 simply consists of transmitting the message three times

Message	0	1
Codeword	000	111

If, for example, 100 is received (assuming that at most one error has occurred in transmission), we deduce that 000 was sent. By considering all the other 7 cases of possible received messages, it can be shown that this code corrects all single errors.

The field of error-correcting codes was stimulated by **Shannon**, when he showed that error-free transmission is possible *in principle*. People then began to ask: "How can we achieve it?"

This encoding scheme, now known as Hamming Code, also detects the condition of any two bits in the encoded block that fail simultaneously.

Hamming's achievement enormously improved the practical application of early computers by substantially increasing their reliability. But it is even more remarkable that many modern computers still use Hamming's techniques to correct errors in main memory.

Although modern computers have very reliable fundamental components, the huge number of such components, e.g. the bits in a computer's main memory, means that the probability of an erroneous result would be significant without Hamming Codes and similar codes that he inspired.

It is not an exaggeration to say that modern graphical computing, which requires large main memories, would be impractical without his invention. Furthermore, computers in critical control applications cannot be allowed to have any significant probability of an erroneous result.

1948–1956 CE **Jan G. Mikusinski**[1117] (1913–1987, Poland). Mathematician. Developed an operational calculus that casts a new light on *Laplace transform* methods, in effect freeing it from considerations of convergence introduced by the improper integral $\int_0^\infty e^{-pt} f(t)dt$, and bringing the essential theory into the realm of algebra[1118, 1119].

1117 To dig deeper, see:

- Mikusinski, J.G., *Operational Calculus*, Pergamon Press, 1959.
- Erdelyi, Arthur, *Operational Calculus and Generalized Functions*, Holt, Rinehart and Winston, 1962, 103 pp.
- Hoskins, R.F., *Generalized Functions*, Ellis Horwood: Chichester, England, 1979, 192 pp.
- Jones, D.S., *Generalized Functions*, McGraw-Hill, 1966, 482 pp.
- Lighthill, M.J., *Introduction to Fourier Analysis and Generalized Functions*, Cambridge University Press, 1962, 79 pp.

1118 The first papers (1949) of Mikusinski were: Sur le calcul opératoire, *Časopis Pest. Mat. Fys.* **74**, 89–94; Sur le fondaments du calcul opératoire, *Studia Math.* **11**, 41–70.

1119 One of the drawbacks of the Laplace transform method is that the rules for transforming derivatives presuppose properties of a function which in actual practice are not known in advance; for the function to be transformed is often the unknown solution of a differential or an integral equation.

Mikusinski accomplished this change of approach by setting up a commutative

1948 CE Introduction of the *atomic clock* (see Table 5.27).

From Cathode Rays to Transistors (1869–1947)

The history of electronics starts with the discovery of cathode rays. While ***Hittorf*** *and* ***Crookes*** *were studying cathode rays (1869),* ***Maxwell*** *was developing his mathematical theory of electromagnetic radiation. Soon after* ***Edison*** *observed electronic conduction in vacuum (1886),* ***Hertz*** *demonstrated (1888) the existence of radio waves predicted by Maxwell. At the time of* ***J.J. Thomson****'s discovery of the electron and measurement of its charge–to–mass ratio* $\frac{e}{m}$ *(1897),* ***Marconi*** *was becoming interested in wireless communication and succeeded in spanning the Atlantic (1901).*

While ***Einstein*** *was explaining the photoelectric effect,* ***J.A. Fleming*** *was busy inventing the first electron tube (1904), a sensitive diode detector utilizing the Edison effect.* ***De Forest****'s invention of the triode (1906) made*

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ring in which the elements are the class $\mathcal{C}$ of continuous real- or complex-valued functions over the interval $0 \leq t < \infty$, in which the operations are the addition and convolution of functions.

Thus from the functions a and b we obtain by addition and convolution the functions $a+b$ and ab: $(a+b)(t) = a(t) + b(t)$, $ab(t) = \int_0^t a(x)b(t-x)dx$. Under these operations the set of functions form a *commutative ring* (an algebraic system closed under two operations).

The absence of divisors of zero in this convolution ring in conjunction with the existence of a convolution *inverse* to any nonzero ring element, enables one to extend the ring to a field Q, whose elements are *convolution quotients* which are sometimes functions and sometimes *operators*.

Thus functions form a special class of operators.

The field Q of operators a/b contains numerical operators, continuous and discontinuous functions, the integral operator $hf(t) = \int_0^t f(x)dx$, and the differential operator $p = \frac{1}{h}$, $pf(t) = f'(t)$. That these diverse entities may be handled from the same point of view and with common rules of operation is an important virtue of Mikusinski's operational calculus.

it possible to amplify signals electronically and led to ***Armstrong****'s sensitive regenerative detector (1912) and the related oscillator.*

The invention by ***Zworykin*** *of the picture tube (1928) and the idea of* ***Watson-Watt*** *for radio detection and ranging (radar), were developed rapidly under the pressure of WWII. Postwar demands and increased knowledge of semiconductor physics, led to the invention of the transistor by* ***Shockley****,* ***Bardeen****, and* ***Brattain*** *(1947) and of the silicon solar cell by* ***G. Pearson*** *(1954). The invention of the* integrated circuit *(1958) by* ***J.S. Kilby*** *(b. 1923) permitted placing a complete network containing many semiconductor devices on a single monolithic chip.*

Conduction occurs in a vacuum if free electrons are available to carry charge under the action of an applied field. In an ionized gas positively charged ions, as well as electrons, contribute to the conduction process.

The multielectrode vacuum tube, as exemplified by the pentode and the cathode-ray tube, is a versatile means for precisely controlling electron flow. Such tubes have been widely used since 1904 *in a host of applications. However, the high power necessary for* thermionic emission *and the large surfaces required for practical operation are important disadvantages. In contrast, semiconductor electronic devices require no cathode power and their dimensions are very small. As a result, the vacuum tube has been displaced from many applications in which it was formerly preeminent by semiconductor devices based on* electron conduction in solids*, where electron motion is influenced by the fixed ions of a crystal to which doping atoms were added.*

The use of impurity semiconductors in electronics is based upon four basic devices: the rectifier *(1936), the* transistor *(1947), the* solar cell *(1954), and the* tunnel diode *(****Leo Esaki****, 1958). These devices, coming under the common generic name of transistors, are all about us: The* 10^9 *or so transistors in a large electronic computer enable it to carry out operations that would have been impractical with vacuum tubes. Transistors have increased the performance of long-distance telephone transmission and are an indispensable part of communication satellites. They are used in radios, television sets, cell phones, calculators, PDA's, laptop, cars, and other electronic devices. A transistor can be so small that manufacturers are able to put millions of them on a flat chips no larger than a postage stamp.*

Electronic equipment has been revolutionized by transistors, and almost all such equipment made today uses them instead of vacuum tubes. Without transistors, manufacturers could not make pocket calculators, or high-speed computers. Battery-operated radios and TV sets would be much larger and cost more to operate.

Both ideas and technology were necessary for the invention of the transistor. Theoretical understanding of the behavior of semiconductors was needed, *and only quantum mechanics could provide this. Indeed,* ***N.F. Mott*** *and* ***H. Jones*** *(1936) published a book "Properties of Metals and Alloys" which contained most of the quantum ideas needed to understand the transistor.*

A second essential was extremely pure semiconducting materials, in which free negative electrons and positive charges (holes) could coexist together for appreciable lengths of time. The radar receivers of WWII used silicon or germanium "crystal detectors", and during and after the war methods were worked out for producing extremely pure silicon and germanium. (In impure semiconductors opposite charges quickly recombine.)

The third requirement for the invention of the transistor was the application of able and inquiring minds to grapple with the dual problem of making a new type of amplifier and understanding the puzzling and peculiar phenomena encountered.

At the beginning of the search, the investigators were armed with the new understanding provided by quantum mechanics, with materials of unprecedented purity and with one idea of how an amplifier might be made[1120]*. During the search, the investigators encountered and then understood the transistor effect, and so invented an amplifier quite different in principle from that which they were seeking.*

To grasp the modus operandi of the transistor, the modern atomic view of electric conduction must be first surveyed:

The classical Drude model supposes that the thermal and electric properties of a metal can be calculated by considering the free electrons to constitute a "gas" that obeys the ***Maxwell–Boltzmann*** *(M–B) distribution law. The results obtained with this model are typically in error by a factor of* 100 *or so, although in some cases (e.g., thermal conductivity) acceptable predictions result from fortuitous cancellations.*

During the early decades of the 20th century, the achievements of the Drude model were accompanied by considerable confusion.

The great success of the kinetic theory in explaining the properties of gases led to the attempt to apply the similar free electron gas model to the case of electron conductivity. Kinetic theory is successful because the atoms and molecules in a gas (at ordinary temperatures) behave very much like Newtonian particles. Electrons, however, are definitely not Newtonian particles in solids (although they do behave like ones in cathod-ray tubes and other vacuum tubes). During the 1920*'s it was finally realized that electrons are quantum entities exhibiting particle-wave duality, and obey quantum mechanical rules.*

[1120] This sort of amplifier was finally realized as the *field effect amplifier*.

As a result, the gas of free electrons does not *obey the Maxwell-Boltzmann distribution law; instead these electrons are described by the drastically different* ***Fermi–Dirac*** *(F–D) distribution law. In* 1928, ***Arnold Sommerfeld*** *modified the Drude theory of metals accordingly*[1121].

The electrons in an isolated, electrically neutral atom are bound to that atom by the electric attraction of the positive nuclear charge. Each electron in the atom occupies a discrete, well-defined energy state (provided the inter-electron coulombic and Pauli exclusion repulsions are treated as a perturbation). Bound *electrons in an atom all have* negative *total energies with respect to a zero of energy defined for an infinite separation of an electron from the atom.*

In the case of the copper atom, the minimum energy that must be supplied to the atom to remove the least bound electron (first ionization energy) is $U_a = 7.72$ eV; *this removal creates a* Cu^+ *ion.*

The copper atoms in bulk matter have a separation between their nuclear centers of approximately 2.5 Å. When two neighboring atoms have this separation, the sum *of the two individual atomic potential energy functions is the effective potential that acts on their valence electrons. We then say that this electron is* shared *by the two atoms. However, the potential just outside the* surface *of the material remains at the normal (zero) level.*

Thus, a potential barrier *is formed which acts to confine the electron gas within the conductor. The energy* E_B *measures the surface barrier height relative to the minimum energy in the conduction band. The* least *energy required to remove a conduction electron from the conduction band is* E_ϕ, *called the* work function *of the material. Evidently* $E_\phi = E_B - \epsilon_F$.

[1121] According to the F–D distribution law, the number of electrons per unit volume that have kinetic energies between E and $E + dE$ is $dN(E) = C_e E^{1/2} dE$, $E < \epsilon_F$, where $C_e = \frac{8\pi}{h^3}\sqrt{2m_e^3}$ and the energy ϵ_F is the electronic chemical potential or *Fermi energy* $\epsilon_F = 3.65 \times 10^{-19} N^{2/3} eV$ (N is the overall conduction electron density, measured in m^{-3}). The expression for $dN(E)$ is strictly true at $0\,^\circ$K, but it also closely describes the distribution for room temperatures. The Fermi energy ϵ_F is the maximum electron energy at a temperature of $0\,^\circ$K, measured relative to the bottom of the *conduction band*. At the Fermi energy, one-half of the states with an energy ϵ_F will be occupied (on average). According to the M–B distribution law, the *average energy* of the electrons is $\langle E\rangle_M = \frac{3}{2}kT$, whereas according to the F–D distribution law, this energy is $\langle E\rangle_F = \frac{3}{5}\epsilon_F = \frac{3}{5}kT_F$ where T_F (the *Fermi temperature*) is approximately $81600\,^\circ$K for copper. Thus, at room temperature ($293\,^\circ$K), the ratio of the root-mean-squared speeds for the two distributions in the case of copper is $\frac{U_{\mathrm{rms}}(D)}{U_{\mathrm{rms}}(B)} = \sqrt{\frac{2T_F}{5T}} = 10.6$.

When two dissimilar metals are placed in contact, there develops between them a contact potential difference*, typically of the order of a Volt; it arises as a direct result of the difference in the work functions of the two metals*[1122].

At very high temperatures, the high energy tail of the F–D energy distribution may contain a significant number of electrons with energies greater than the barrier energy $E_B = \epsilon_F + E_\phi$*. These* thermionic electrons *are able to escape from the conductor surface. A nearby electrode that is maintained at a positive potential w.r.t. the conductor will collect the liberated electrons. The current so produced is called the* thermionic current*.*

This thermionic effect is used in electron guns, where the electrons are emitted from an electrically heated filament. (To enhance the thermionic emission, these filaments are often coated with a metal that has a particularly low work function, such as cesium.)

The Sommerfeld model was an important step in the development of the theory of metals, but it left a number of problems only partially resolved. The model was quite good for the alkali metals *(Li, Na, K, Rb, Cs), but had low accuracy for the* noble metals *(Cu, Ag, Au) and for the alkali earths, Be, Mg, Ca, Sr and Ba. The most glaring deficiency of the model was that it failed to explain why some elements are good electric conductors while others are not.*

Why, for example, is aluminum a good conductor, whereas boron, which is one row higher in the same column of the periodic table, is an electric insulator?

The next important improvement in understanding the physical properties of solids was the development of band theory*: When NV (N per unit volume, V = volume) atoms of a metal are collected together to form a bulk sample, the discrete energy states of an isolated atom are broadened into bands. The*

[1122] Let the work function $E_{\phi 1}$ of metal 1 be less than that of metal 2; that is, the electrons at the top of the conduction band in metal 1 have higher energies than the corresponding electrons in metal 2. Consequently, when the metals are brought into contact, electrons in metal 1 seek lower energy states by transferring to metal 2. This transfer lowers the Fermi level in metal 1 while raising it in metal 2.

The number of electrons actually transferred in this process is exceedingly small compared with the total number of electrons present; consequently, the values of E_ϕ and ϵ_F for each metal remain approximately unchanged. When equilibrium is reached, with as many electrons per unit time crossing the junction in one direction as in the other, the *Fermi levels coincide*. Clearly, the contact potential difference is given by $\delta\phi = \frac{1}{e}(E_{\phi 2} - E_{\phi 1})$.

degeneracy[1123] *of each band is just NV times the degeneracy of the corresponding atomic energy state. That is, each band actually consists of NV states crowded into a narrow range of energies.*

Because NV is very large for a sample of bulk matter (10^{23} or so), the spacing between the states is so small that the band represents an essentially continuous distribution of energies. In a bulk sample the shared electrons can possess only energies that fall within the allowed energy bands; all other energies are forbidden. The highest fully occupied band is called the valence band.

[1123] In an isolated lithium atom, for example, 2 electrons completely fill the lowest energy level (the $1s$ orbital) so that the third valence electron must occupy a state in the second lowest energy level (the $2s$ orbital). The maximum number of electrons that can occupy a given energy level is the *degeneracy* of that level. The level $1s$ is full, the level $2s$ is half-filled and the third level $2p$ is empty. The respective degeneracies are 2, 2, and 6.

The 1s and 2s bands form because the atomic orbitals of neighboring atoms overlap, allowing the electrons in these levels to hop between atoms throughout the lattice. Thus hopping causes the degenerate atomic energy levels (1s or 2s) to broaden into bands, for the same reason that coupling a row of identical tuning forks creates a dense band of eigenfrequencies (becoming a continuum in the limit of infinitely-many forks). Different energy levels within a given band (1s or 2s) can be thought of as electrons moving at different speeds in their hopping motions. The filled 1s band is called a *valence band*, while the partially-full 2s band is the *conduction band*.

This energy is negligible in comparison with the energy separation between the $1s$ and $2s$ bands (ca 48 eV). Because the $1s$ band is completely filled, no energy state within the band is available for the promotion (that is, movement to a higher state) of a lower energy electron. The *minimum* energy that a $1s$ electron can absorb is an energy that would lift it to the unfilled portion of the $2s$ (conduction) band; this energy is unlikely to be acquired in a collision with a conduction electron, and even if it happens, the 1s and 2s electrons would merely trade places. Thus, the $1s$ electrons remain attached to their parent atoms and do not contribute to the current flow in the sample.

The $2s$ band, on the other hand, is only half filled. At low temperatures ($\lesssim 2000\,^\circ$K) the $2s$ electrons occupy the lowest available energy states in the band. The spacing of the unoccupied energy states just above the Fermi energy ϵ_F is $\Delta E = \frac{2}{3NV}\epsilon_F$ which is $\Delta E = 6.8 \times 10^{-23}$ eV for a 1–cm^3 sample of lithium. Therefore, $2s$ electrons can easily be promoted by collisions into the numerous vacant energy states above the Fermi energy. These electrons are the conduction electrons that participate in the current flow.

All good electric conductors have the feature that the highest occupied energy band is only partially filled (it is then called the conduction band, *the band just beneath it being the* valence band*). However, in a poor conductor (or* insulator*) the highest occupied energy band is completely filled (valence band) and there is a substantial energy gap between this filled band and the nearest unfilled band. Thus, in* diamond *(a crystalline form of carbon), the lowest three bands (1s and the band-broadened* bonding *molecular orbitals formed from each carbon atom's sp^3-hybridized 2s and 2p orbitals) are completely filled.*

*The minimum energy required to promote an electron from this filled band into an unoccupied state in the lowest empty (*antibonding *sp^3-hybridized 2s and 2p) band is equal to the* gap energy *E_g, which is 5.5 eV for diamond. Any realistic applied electric field is totally inadequate to impart this amount of energy to an electron. Thus, electric conduction of the type discussed so far does not occur, and diamond is not a conductor.*

However, in graphite *(another crystalline form of carbon), in which the atoms have geometric arrangement and spacing different from that in diamond, the gap energy is reduced almost to zero. Electrons can be promoted across the gap into the empty conduction band by* thermal *effects[1124].*

When an electron is promoted from the valence band into the conduction band, it leaves behind an unoccupied hole *in the valence band. When another valence electron moves in to occupy this hole, it creates a hole at its previous site. Thus, a hole can "move" and act as a carrier of* positive charge*. Both holes and electrons contribute to the current flow when an external electric field is applied.*

The conductivity of an intrinsic semiconductor depends on the density of thermally excited negative and positive charge carriers[1125] (electrons and holes).

1124 The F–D distribution contains a high-energy tail of the order kT that extends above the Fermi energy ϵ_F. At room temperature $kT = 0.0252$ eV. Thus, if the energy gap E_g is small enough, ($\lesssim kT$) some of the electrons in the valence band will be thermally excited into the empty conduction band. Those elements that have small energy gaps ($E_g \leq 2eV$) are called *intrinsic semiconductors.*

1125 One of the results of the theory is that at an absolute temperature T the number density (per cm^3) of these carriers is given approximately by $N_+ = N_- \simeq (4.83 \times 10^{15}\ cm^{-3})T^{3/2} exp\,(-\epsilon_g/2kT)$. The gap energy for the intrinsic semiconductor germanium is 0.67 eV. Thus, at room temperature, we find for germanium $N_+ = N_- \simeq 4.1 \times 10^{13}\ cm^{-3}$. For copper, on the other hand, the number density of conduction electrons is $N = 8.5 \times 10^{22}\ cm^{-3}$. When the different electron mean free paths (and thus mobilities) for two cases are taken into consideration, the ratio of the resistivity values of germanium

However, thermal excitation is not the only way to enhance the conductivity of a semiconductor material:

Real semiconductors can be very pure, but none are perfect crystals. Defects include missing atoms (vacancies), atoms in noncrystalline sites (interstitials), and impurity atoms anywhere in the crystal. These defects may result in extra electrons, missing electrons, or no change at all in the number of electrons.

An important type of defect is the substitutional impurity, *an impurity atom that replaces a regular atom in the solid. The tremendous advances in solid state electronics are largely the result of our ability to selectively add various amounts of impurity atoms to create different* extrinsic semiconductor *regions. The process of adding the impurities is called* doping.

For example, silicon has 4 *valence electrons (as do carbon and germanium) and forms a diamond-like crystalline lattice in which each atom is connected to* 4 *other atoms by covalent bonds. Now, suppose that an arsenic (*As*) atom, which has* 5 *valence electrons, is substituted for one of the silicon atoms in the lattice. Four of the arsenic electrons are used to duplicate the silicon bonds, and there is one surplus electron left over.*

Thus, the crystal contains an As^+ *ion, and an extra electron is so loosely bound (by about 0.05 eV) to the arsenic ion that it can be readily released by thermal excitation. In effect, the arsenic impurity atom has donated a negative charge carrier (an electron) to the material. When introduced into silicon, arsenic is a* donor *atom, and the result is referred to as an* n-type (negatively doped) *material.*

*On the other hand, if an element with 3 valence electrons, such as gallium (*Ga*), is introduced into a silicon crystal, each dopant atom lacks one electron to complete the bonds to the four neighboring silicon atoms.*

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to copper at room temperature is found to be approximately 3×10^7.

The resistivity of a semiconductor depends strongly upon the temperature. The usual increase in resistivity with temperature due to *lattice vibrations* is completely obscured by the increase in the number of charge carriers due to the rapidly changing exponential factor in the above expression for $N_\pm$.

The number density of charge carriers at room temperature in diamond (no impurities) is (with $E_g = 5.5$ eV) $N_+ = N_- \simeq 9.8 \times 10^{-29}$ cm^{-3}. Thus, an increase in the gap energy from 0.67 eV (germanium) to 5.5 eV (diamond) results in a decrease in the density of charge carriers by a factor of about 10^{42}. Diamond is a very good insulator indeed!
(In substances like diamond, conduction actually takes place by the *diffusion of ions* through the solid.)

To compensate for this deficiency, a gallium atom "steals" an electron from a silicon atom, thereby becoming a Ga^- ion. This leaves an Si^+ ion or hole, which then acts as a positive charge carrier. The gallium impurity is called an acceptor *atom, and the material is a p-type (positively doped)* semiconductor.

When p-type and n-type crystals are brought into contact (p–n junction), some interesting physical phenomena take place. Some of the 'hole gas' from the p side diffuses[1126] *into the n side, while at the same time, some of the electron gas from the n side diffuses into the p side; thus the p side of the junction acquires a negative charge.*

*An electron diffusing into the p side finds many holes to fall into (*recombine *with), and a hole diffusing into the n side soon* recombines *with an electron.*

At equilibrium, some of the results of these diffusions processes are:

- *A built-in electric field is set up in the region of the junction, directed from the n side (+) to the p side (−). This field creates a built-in voltage. The force on an electron acts opposite to the direction of the electric field. Therefore, to move an electron from the n side to the p side now requires external work. An electron's potential energy is raised as it travels from the n side to the p side – an "energy-hill" has been set up.*
- *The p side energy is raised and the n side energy is lowered until the* Fermi energy *is constant across the junction.*
- *The* recombination *process leave the junction region depleted of charge carriers compared to the remainder of the material. The diffusion of electrons and holes across the junction because of the initial concentration gradient followed by recombination result in a* recombination current.
- *The electron-hole pairs constantly being generated in the junction region by* thermal processes, *are separated and swept out of this region by the built-in field. The result is a* generation current.
- *At equilibrium, the recombination and generation currents are equal and opposite.*

Now, suppose that a battery is connected across a p–n junction such that its (+) terminal is connected to the p side and its (−) terminal to n side. This arrangement creates a forward bias *voltage V across the junction. The*

[1126] Not unlike the diffusion of two gases into each other when the barrier that separates them (in a container) is removed. In both cases, the diffusion results from an initial concentration gradient of either gas between the two parts of the container.

doped semiconductor material is a reasonably good conductor, except in the depleted region of the p–n junction. There the battery sets up a strong electric field which opposes *the built-in electric field. The vector sum of the two fields is* smaller *than before the battery was connected, and so is the new 'energy hill'.*

The smaller electric field and the smaller energy hill do not greatly affect the rate at which electrons and holes are thermally generated in the junction region (assuming constant temperature).

Therefore the generation current with a potential difference V is approximately the same as with no potential difference. But now, the electrons and hole gases do not have to climb as high an energy hill to diffuse *(electrons into the p side, holes into the n side).*

Thus, the two (still equal) diffusion rates increased by the lowering $\Delta E = -eV$ of the energy hill, will change by the Boltzmann factor $e^{-\Delta E/kT} = e^{eV/kT}$. It can be shown that the total current through the p–n junction is $i = i_s(e^{eV/kT} - 1)$, with i_s V-independent.

Had we instead connected the (+) battery terminal to the n side of the junction and its (−) terminal to the p side, a reverse bias would be created. The energy hill is then increased *by $\Delta E = eV$, and the recombination current is* decreased *by the corresponding Boltzmann factor – whereas the generation current will hardly change.*

The total current through the junction in this case is again

$$i = i_s(e^{eV/kT} - 1),$$

except that V and i are now negative (reverse bias). Since the resistance R equals V/i, the forward resistance becomes small for large forward currents, while the reverse resistance becomes very large for large reverse voltages[1127]. A p–n junction with two terminals that is used to provide a small forward resistance and a large reverse resistance is called a junction diode.

The property of allowing current to flow easily in the forward direction while almost stopping the current in the reverse direction gives the junction

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1127 The physical explanation is as follows: Forward bias drives holes in the p side and electrons in the n side toward the p–n junction. There the electrons can easily fall into the holes (recombine), and the forward resistance is low.

However, reverse bias pulls electrons and holes in the opposite direction, or *away from the junction*. For the current to continue, electrons must be continually pulled out of holes at the p–n junction — a hard thing to do, requiring an expenditure of energy. Therefore a large reverse resistance is created.

diode an application as a rectifier[1128]*. A semiconductor rectifier has many advantages over a diode vacuum-tube rectifier, including longer life and much smaller size.*

Like the vacuum-tube diode, the p–n junction is a non-ohmic element, the current-voltage relation being nonlinear*. Unlike a vacuum tube, there is no need for a power-consuming filament in the semiconductor device, so that its efficiency is greater.*

pnp junction transistor *is formed with a thin (< 0.1 mm) and lightly doped n-type region, known as* base*, placed between two more heavily doped p-type regions, known as* emitter *and* collector*.*

The base is so lightly doped that most (typically, > 98 percent) of the holes from the emitter diffuse right across the base to the collector.

Although typically fewer than 2 percent of the holes from the emitter recombine with electrons in the base, the quantity is enough to quickly drive the base positive, thereby setting up a strong electric field to repel any further hole current from the emitter.

Thus, the hole current will be cut off quickly unless the positive charge in the base portion of the pnp *transistor is somehow neutralized. This can be achieved by adding electrons from an external source to the base. Consequently,* a small current from the base will allow a large emitter-collector current to flow*. Hence, a transistor can be used to* switch *electric currents.*

Suppose that one p-type region (emitter) was connected to a battery terminal positive relative to the base, while the other p-type region of the pnp *transistor (the collector) is connected to the negative terminal. The emitter-base p–n junction is thus forward biased and therefore has a small resistance. The base-collector p–n junction, however, is reverse biased and therefore has a large resistance.*

Now, connect an a–c source in series with the battery in the emitter circuit. When the source then produces a small change in voltage ΔV_{in}, the emitter current will change by Δi. As this current change will also pass through the base-collector junction, the large R of this junction will yield a large ΔV_{out};

[1128] Suppose that the saturation current of a junction diode is $i_s = 10\mu A$. At room temperature $kT = \frac{1}{40}$ eV. Applying forward and reverse biases of 0.20 V, we find:

$$R(\text{forward}) = \frac{|V|}{i_s(e^{e|V|/kT} - 1)} = 6.7 \text{ ohm},$$

$$R(\text{reverse}) = \frac{|V|}{i_s(1 - e^{-e|V|/kT})} = 20,000 \text{ ohm}$$

consequently a small input voltage change gives a large output voltage change; a transistor can thus be used to amplify a–c signals.

Toward the Absolute Zero[1129] *(1898–1995)*

The absolute zero represents that state of matter at which all random microscopic motion has ceased. By 'motion' is meant all mechanical and molecular motion, as well as electronic motion. Not included, however are large-scale organized motion (e.g. fluid flows, rigid motions or electric currents) or the quantum mechanical "zero-point motions"; the latter cannot be stopped without destroying the assembly of molecules and their constituent atoms. The absolute zero can never be reached in any actual experiment, but it has been approached to within a few billionths of a degree.

All the random microscopic motions taking place within matter (over and above the core zero-point motions) are called thermal motions. They are not visible as such to a macroscopic observer (except via Brownian motion), but the extent of these motions determines the numerous temperature-dependent properties of matter.

The science of cryogenics[1130] *is the study, attainment and use of very low temperatures. It may be concerned with practical engineering problems, such as producing lots of liquid oxygen for manufacturing high-quality steel or burning rocket fuel.*

It may be used to quick-freeze a surgical tissue specimen for a medical researcher or to freeze-dry a lightweight dinner for a mountaineer. Or it may help physicists study some of the most basic properties of matter.

Cryogenic researchers work with temperatures down to within several billionth of a degree of the absolute zero (273.16 degrees below zero on the Centigrade scale or -459.72 *degrees on the Fahrenheit scale).*

[1129] For further reading, see:

- Zemansky, M.W., *Temperatures Very Low and Very High*, Van Nostrand, 1964, 127 pp.

[1130] From the Greek *kruos* = frost.

Modern cryogenics originated in 1898 *with the first liquefying of hydrogen by* ***J. Dewar****, using the Joule-Thomson effect and a counterflow heat exchanger. The next breakthrough occurred in* 1926 *with the invention of magnetic cooling via adiabatic demagnetization, by* ***Giauque*** and ***Debye****, which allowed the lowering of the temperature threshold to within a fraction of a degree away from the absolute zero.*

In 1956, ***Franz Eugen Francis Simon***[1131] *(1893–1956,* Germany and *England) and* ***N. Kurti*** *conducted the first* nuclear cooling experiment*, using adiabatic demagnetization at the* nuclear *level in a* paramagnetic salt*, to reach a temperature of* $10^{-5}\,^{\circ}K$*. In 1960,* ***N. Kurti*** *applied nuclear cooling methods to reach a record low*[1132] *of* $3 \times 10^{-6}\,^{\circ}K$.

We now know that at ordinary temperatures the atoms, molecules and electrons in all matter are in a constant state of random motion or agitation. This motion often masks the fundamental interactions between atoms, nuclei, and electrons. Lowering the temperature usually reduces the interference caused by this motion.

Thus, low-temperature studies have contributed greatly to our understanding of the forces between atoms and molecules, of the mechanisms by which electric currents are carried in metals and in semiconductors, and of the nature of that well-known, yet still mysterious force – magnetism.

Cryogenics has also been essential in the discovery of two completely unexpected phenomena, superfluidity *and* superconductivity*. Since* 1925*, almost*

[1131] A pupil of **Walther Nernst**. Fled Nazi Germany in 1933 and established a flourishing laboratory of low temperature research in Oxford. He used to say, rather wryly, that he was probably the only man who had both an Iron Cross of Imperial Germany and a Knighthood of the British Empire.

[1132] A temperature of $3 \times 10^{-6}\,^{\circ}\mathrm{K}$ represents a fraction of room temperature ($300\,^{\circ}\mathrm{K}$) equal to 10^{-8}. Cryogenics has therefore enabled us to get to one-hundred-millionth of room temperature. The surface temperature of the sun, $6000\,^{\circ}\mathrm{K}$, is only 20 times room temperature, and the temperature in the interior of the hottest star, about $3 \times 10^{9}\,^{\circ}\mathrm{K}$, is ten million times room temperature. 1960 cryogenics is still ahead by a factor of ten! and by the 1990's, laser-cooling of monoatomic gases was combined with magnetic and RF techniques to produce *MOT*'s (magneto-optical traps) capable of cooling tiny samples of some gases to tens of nK (nano-kelvins), at which temperatures they form a novel state of matter – the Bose-Einstein Condensate (BEC), probably found nowhere else in nature. By 2004 the lowest BEC temperature achieved crossed below the 500 pK (pico-Kelvins)!

every theoretical physicist of note has struggled to explain these 'super' properties, yet it was only during 1955–1965 *that consistent answers have been formulated to the questions they pose.*

The explanation for superfluidity and superconductivity can be given only in the abstract language of quantum mechanics, although both scientists and engineers make daily practical use of these amazing properties: At ordinary temperatures it would be unbelievable if a liquid ran uphill or flowed freely through virtually airtight barriers, yet this is just what superfluid helium *does, at the extremely low temperatures achievable via cryogenics.*

It would be equally amazing if an electric current kept flowing through an electric circuit after all contacts with the power source had been broken, yet this is what happens in a superconductor. These unusual motions are nonetheless very natural and comprehensible, albeit surprising, phenomena associated with low temperatures.

The large-scale industrial technology of cryogenics resulted mainly from the exigencies of WWII and the space-exploration program. Just as the study of nuclear physics had led to the development of nuclear medicine, nuclear electric power plants, and the study of solid state physics to transistor television sets and radios, personal computers, digital telecommunications and the rest of the microelectronics revolution, so today's cryogenic research may lead to tomorrow's engineering marvels and even to new kind of consumer goods.

Cryogenic techniques are now employed to produce ultra-high vacuums. Cryogenic computers *and cryogenic high-field magnets for high-energy particle physics and controlled thermonuclear fusion research are also in use. One application of cryogenic research in space technology is the possible future use of superconducting magnets as shields for the protection of spacecraft from space radiation.*

Still another important future use of supermagnets may be in magnetohydrodynamic (MHD) *electric generators, which operate by passing a hot ionized gas, or plasma, rapidly through a magnetic field, thus converting the heat energy to electricity without the necessity of a boiler or turbogenerator.*

1948 CE **Hendrick Brugt Gerhard Casimir** (1909–2000, Holland). Physicist. Predicted the *Casimir effect* — a small attractive force which acts between two close parallel *uncharged* conducting plates. It is due to *quantum virtual vacuum fluctuations* of the electromagnetic field.

In the 1930s, **Paul Dirac** proposed that the vacuum actually teems with virtual electromagnetic waves – a quantized-field version of the *zero point energy already familiar then from non-relativistic quantum mechanics*[1133]. This energy would be contained in *virtual photons*, or light particles that are constantly winking in an out of existence, making the electromagnetic energy fluctuate in time and space. When external energy is supplied, the virtual light quanta can materialize as real (actual) photons – as happens when an excited bound (or virtual free) electron emits a real photon. Dirac's electron theory also led to the realization that the vacuum seethes with virtual electron-positron pairs.

Casimir proposed an ingenious way to *observe* the virtual-photon energy directly: Two perfectly reflective metal plates are placed a micrometer apart forming, in effect, a narrow channel in the electromagnetic 'ocean' that allows only certain wavelengths of light, and their respective virtual photons to exist there. But the 'ocean' outside the channel would have virtual photons of all wavelengths. This would create an ever-so-slight discrepancy between the energy density inside and that outside the channel, causing a tiny force pushing the plates together.

In fact, only those virtual photons[1134] (between the plates) whose half-wavelengths fit a whole number of times into the gap should be counted when calculating the vacuum energy.

[1133] Both types of zero-point fluctuations represent *nonthermal* energy, most evident as the ambient temperature approaches absolute zero.

[1134] Virtual particles other than the photon also contribute a small effect but only the photon force is measurable. All Bosons such as photons produce an attractive Casimir force while Fermions make a repulsive contribution. If electromagnetism was supersymmetric there would be fermionic *photinos* whose contribution would exactly cancel that of the photons and there would be no Casimir effect. The fact that the Casimir effect exists shows that if supersymmetry exists in nature it must be a broken symmetry.

According to the theory of QED the total zero point energy density in the vacuum is infinite when summed over all possible photon modes. The Casimir effect comes from a *difference of energies in which the infinities cancel*. The energy of the vacuum is a puzzle in theories of quantum gravity since it should act gravitationally and produce a large cosmological constant (much larger than actually observed) which would cause space-time to curl up to tiny dimensions. The solution to the inconsistency is expected to be found in an eventual theory of quantum gravity (perhaps some version of 'string theory'). The Casimir effect is primarily a low frequency (long wavelength), nonrelativistic effect, and it shows that the *changes in the zero point energy* of the electromagnetic vacuum (which is *infinite* in extent) can be finite and *observable*.

The energy density decreases as the plates are moved closer, which implies there is a small force drawing them together.

This force per unit area of a plate can be calculated to be $F = -\frac{\pi^2 \hbar c}{240 d^4}$, where d is the separation between the plates[1135]. It amounts to about one billionth of a newton for two plates, each a square mm in area separated by $d = 1$ micron[1136]. The tiny force was measured in 1996 by **Steven Lamoreaux**, and his results were in agreement with the theory to within the experimental uncertainty of 5 per cent.

Other contributions of Casimir are:

- Introduced the phenomenological theory of *superconductivity* (1934).
- Contributed to the theories of *paramagnetic relaxation*, *irreversible thermodynamics* and quantum mechanics (*Casimir operators*, 1931).

Casimir was born in the Hague. He studied physics at Leiden University beginning in 1928 and received his Ph.D. there in 1931. During that period he also spent some time in Copenhagen with **Niels Bohr**.

After receiving his Ph.D., Casimir worked as an assistant to **Wolfgang Pauli** at Zürich, but returned to Leiden until 1942 when he joined the *Research Laboratories of the Phillips Company.* He became a co-director of these laboratories in 1946 and a member of the board of directors of the company in 1956. He retired from Phillips in 1972.

[1135] This formula ceases to be valid when d is of the order of intermolecular distances (ca $\frac{1}{3}$ nm)

[1136] *Example*: the force per unit area between two large square parallel perfectly conducting plates of size L at distance $d \ll L$ apart is

$$F = -\frac{\pi^2}{240}\frac{\hbar c}{d^4} = -\frac{0.013}{[d(\text{micron})]^4}\frac{\text{dyn}}{\text{cm}^2}$$

and its signs corresponds to attraction. The existence of this force has been demonstrated experimentally (1958) by **M.J. Sparnay**.

Another example of a Casimir-like effect is furnished by Van der Waals forces among tiny, yet macroscopic aggregates of neutral atoms or molecules. It is believed that Casimir type *vacuum-fluctuation* forces are responsible for the existence of the *cosmological constant* in GTR cosmology, and that the cosmological constant today (a.k.a. *dark energy*) comprises most of the energy density of the observable universe.

1948 CE **Denis Gabor** (1900–1979, Hungary and England). Electrical engineer, physicist and inventor. Won the Nobel prize for physics in 1971 for his invention of *holography*. This is a technique which he developed while seeking to improve the electron microscope during his work at the Research Laboratory of the British Thomson-Houston Company. It made possible the recording of 3-dimensional photographic images without a lens.

The image itself, a *hologram*, appears as an unrecognizable pattern of stripes and holes until illuminated by *coherent light*. Gabor's proposal was of limited practical interest until the development of *lasers* in the early 1960s made possible the widespread application of holography in medicine, print-making, communications and computer technology.

Gabor was born to Jewish parents in Budapest. His father inspired him with stories about Thomas A. Edison and other inventors. Gabor graduated from the Technical College in Berlin in 1924. He fled to England as a refugee from Nazi Germany in 1934 and worked in industry, becoming a British subject in 1946. His invention of holography was followed by a move to Imperial College, London (1949–1967), where he became professor of applied electron physics in 1958. He later moved to the CBS Research Laboratories in Stamford, CT, U.S.A.

1948 CE **Peter Carl Goldmark** (1906–1977, Hungary and U.S.A.). Physicist and inventor. Developed at the Columbia Broadcasting Systems, U.S.A. the first 30-cm wide long-playing ($33\frac{1}{3}$ rpm) record. The playing time of each side had changed from about 5 minutes for the large 78 rpm records to 25 minutes and the sound quality was considerably improved. The 78 rpm records vanished quickly from the market.

The arrival of the long-playing record in 1948 followed much technical and theoretical experimentation. In 1931 the Americans **H.C. Harrison** and **H.A. Frederick** proved that the sound quality could be improved by using a softer recording material and a lighter cartridge.

This discovery, however, entailed an unwelcome commercial upheaval, for all the equipment for recording and reproducing sound, including the discs, then had to be replaced; due to the crisis resulting from the Wall Street Crash (1929), people were unwilling to spend money on this type of commodity, and the tens of millions of records already on the market were liable to be made obsolete. In addition, the radio was competing ever more strongly with the gramophone.

In 1933, the Americans **F.V. Hunt**, **J.A. Pierce** and **W.D. Lewis** made progress toward the long-playing record, demonstrating the advantage of *very fine grooves* (ca 100 per radial centimeter), as well as of a light cartridge and a much more delicate stylus which rested on the edges rather than the bottom of the groove.

Goldmark, who came to the U.S. in 1933, became president of CBS (1972). He demonstrated the first color television system in 1940.

Holography[1137]

An ordinary photograph records the amount of light reflected or scattered from each point of an object to the camera lens and thence to the photographic film (or, in the case of a digital camera, the recording takes place opto-electronically on a CCD chip).

During exposure, only the distribution of the mean square amplitude (intensity) of the electromagnetic field of a light wave is recorded, and that in a 2-dimensional projection of the object onto the plane of the photograph[1138].

[1137] For further reading, see:

- DeVelis, J.B. and G.O. Reynolds, *Theory and Applications of Holography*, Addison-Wesley Publishing Company, Reading, MA, 1967, 196 pp.
- Ostrovsky, Yu.I., *Holography and its Applications*, Mir Publishers: Moscow, 1977, 267 pp.
- Lizuka, K., *Engineering Optics*, Springer-Verlag: New York, 1987, 489 pp.
- Fowles, G.R., *Introduction to Modern Optics*, Dover: New York, 1975, 328 pp.
- Goodman, J.W., *Introduction to Fourier Optics*, McGraw-Hill: New York, 1968, 287 pp.
- Steward, E.G., *Fourier Optics*, Ellis Horwood, Wiley, 1987, 269 pp.

[1138] In analog (non-digital) photography, the recording of this projection is effected by a photographic emulsion consisting of silver halide particles suspended in a gelatin base. This emulsion is applied to a glass substrate or an acetate film. When light falls on a silver halide particle, it creates centers of reduced silver. In the *developing process*, particles containing this reduced silver are converted to metallic silver. In regions devoid of reduced-silver centers, the particles remain in halide form.

After developing, the recorded image provides an approximately linear mapping of this intensity into amplitude transmittance. The information concerning the phase *of the wave is lost in this process. Thus, a photograph conveys only partial information about the object. In particular, there is no information about the distances of various parts of the object from the photographic plate. For this reason, when examining the photograph from various directions, we do not obtain new angles of approach, and we cannot see, for example, what is happening behind objects in the foreground.*

By contrast, if both amplitude and *phase of the original wavefronts emanating from the object could somehow be reconstructed, the resulting light field (assuming the frequencies are the same) would be indistinguishable from the original. This means that we would then see (and could photograph from various angles) the re-formed image in perfect* 3*-dimensionality, exactly as if the object were there before us, actually generating the waves.*

Such a process, known as holography[1139]*, has in fact been invented. It records the* interference *between coherent light wave-trains striking a film directly, and the corresponding secondary wave-trains which are (almost) simultaneously scattered from the object being photographed (or rather "holographed"). No lens is needed in front of the film.* Every section of the film records information from the entire non-occluded portion of the object. *The coherent light is usually provided by a laser; it must be intense enough to expose the film in a time short enough so that vibrations do not disturb the relative positions of light source, object, and film.*

The resulting photograph (hologram) displays a picture that appears unintelligible when viewed with ordinary light. What has been recorded is an interference pattern containing information about both amplitude and phase differences between light scattered from the object and light that reached the same mesoscopic regions of the film directly.

During the process of *fixation* that follows development, silver halide particles are removed and the plate contains metallic silver only, in small particles which form dark spots on the plate. These particles comprise the *negative* image.

On a logarithmic scale, the density of plate blackening is approximately $Q = \Gamma \log_{10}\left(\frac{E}{E_0}\right)$, where Γ and E_0 are characteristic of the photographic material, and E is the product of the exposure time t and the light intensity I. After development, the *amplitude transmittance* of the plate has the form $\tau = \tau_0 \left(\frac{E}{E_0}\right)^{-\Gamma}$.

1139 From the Greek word *holos* meaning *the whole* — to indicate that it contains the whole of the information about the object, through amplitude *and* phase of the light reflected or scattered from it.

Both types of information are "scrambled" together and converted to intensity variations for recording purposes, since all recording media available respond only to light intensity. This photographic record (hologram) bears little resemblance to the object, and upon visual observation contains a seemingly meaningless jumble of diffraction patterns.

If, however, the hologram is viewed with transmitted coherent light from a laser (or even with nearly coherent light from a strong point source), the original scene is approximately reconstructed *and can be seen as a* real image *on the near side of the film, or as a* virtual image *on the far side*[1140]. *The*

1140 The basic mathematical theory of the *wave-front reconstruction process* is as follows: Set up a Cartesian coordinate system (x, y) in the plane of the photographic plate. A reference plane wave (laser beam) of complex amplitude $U_0(x, y) = a_0 e^{i(\mu x+\nu y)}$ is scattered from an object onto the plate, where a_0 is a real constant, and (μ, ν) are the spatial frequency components of the reference beam in the xy plane. They are given by $\mu = k \sin\alpha,\ \ \nu = k\sin\beta,$ in which k is the wavenumber $\left(= \frac{\omega}{c}\right)$ of the laser light, and the angles (α, β) specify the direction of the beam. Let $U(x, y) = a(x, y)e^{i\phi(x,y)}$ denote the complex amplitude of the scattered wavefront in the xy plane, where $a(x, y)$ is real.

The intensity $I(x, y)$ that is recorded by the photographic plate is thus given by the expression ($* =$ complex conjugation)

$$I(x, y) = (U + U_0)(U + U_0)^* = a^2 + a_0^2 + 2aa_0 \cos[\phi(x, y) - \mu x - \nu y].$$

This is an *interference pattern*. It contains information in the form of (relatively slowly-varying) amplitude and phase modulations of the *spatially periodic* reference beam — analogous to the impression of *temporal* information on the higher-frequency radio carrier wave by means of amplitude *and* phase modulation.

In the above expression for $I(x, y)$ it is assumed that the *scattering of the secondary wave* U is neglected (Born's first approximation), and that U is of the same frequency as the incident wave — i.e. one ignores Raman scattering, Doppler effect, and other frequency (color) conversion mechanisms.

In the *reconstruction stage*, the developed hologram (called the *positive* hologram) is illuminated by the *coherent background* U_0 alone. The resulting *transmitted wave* $U_T(x, y)$, which goes through the hologram and reaches the eye, will be approximately proportional to U_0 times the transmittance of the hologram at the point (x, y). The transmittance, in turn, will be approximately proportional to $I(x, y)$. Hence, except for a constant proportionality factor that we ignore, $U_T(x, y) = U_0 I(x, y) = (a^2 + a_0^2)U_0 + a_0^2 U + U^{-1}U_0^2 a^2,$ where the negative exponents signify phase reversal (since $U^* = a^2U^{-1}$).

The hologram acts somewhat like a *diffraction grating*. It produces a *zeroth-order* direct beam $(a^2 + a_0^2)U_0 \approx a_0^2 U_0$, and two *first-order* diffracted beams *light reaching the eye has been influenced by the interference pattern, with more light passing through where there was* destructive *interference and less where there was* constructive *interference. The resulting light forms an interference pattern between the hologram — which is itself a frozen interference pattern*[1141] *— and the illuminating light. This new pattern is almost exactly proportional to the one that would have formed had one been looking directly at the object in the first place (at least through the solid angle allowed by the geometry of the holography process).*

Furthermore, this pattern is different in the left eye from what it is in the right eye just as it would have been if one had viewed the 3-dimensional object directly. Hence one sees the object in 3 dimensions. By moving one's head with respect to the illuminated hologram within some solid angle determined by the geometry of the recording procedure, one can look around the object to the same extent that one could if one had been in that position looking at the object directly.

One way to understand why such a two-stage interference procedure allows us to see the original object is to analyze the interference pattern produced by a very simple geometry. Suppose that the object being photographed is a long rod. The coherent light from an incident coherent plane wave scatters from the rod, interferes with the plane wave striking the film directly. The resulting interference pattern is a series of variably-spaced opaque and transparent bands covering the entire film.

If a positive transparency is made of the film and is then illuminated with the original plane wave of coherent light, the cylindrical wave fronts originating from the transparent bands on the film will produce virtual reinforcement maxima at a distribution of points approximating the original rod's light-scattering centers. The waves diverging from these points will look as if they

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on either side of the direct beam: The term a_0^2U reproduces the *scattered* light from the object and forms the *virtual image* (reconstructed wave). On the other hand, the term $U^{-1}U_0^2a^2$ has the same amplitude as the reconstructed wave but with reversed phase; it may be regarded as being due to a fictitious object of similar nature as the true object, but situated in a different plane (*real image*).

1141 If plane-wave light is used and the interference recorded at a far screen (Fraunhofer diffraction), a diffraction pattern of a diffraction pattern is the original source pattern! Thus, a diffraction pattern of a pinhole is a series of alternating dark-light concentric circles. The diffraction pattern of this configuration is a point image. The diffraction pattern of a single slit furnishes another example. Mathematically, this is simply because the 2D Fourier transform is its own inverse.

had come from the original rod. Another set of wave crests converge from the film to form a real *image of the original illuminated rod on the same side of the film as the viewer.*

Note that the information about the rod is not localized on the film but is distributed over its entirety; thus, the position of the rod is determined by the spacing of the light and dark fringes. If the rod is close to the film, the spacing is large at first and then decrease rapidly. If the rod is far away from the film, the spacing is almost uniform.

Thus, holography has succeeded in removing a drawback of conventional photography through recording on a photographic plate (or any other medium) an image of the whole field *(amplitude and phase).*

Already at the beginning of the 19^{th} *century,* ***Young***, ***Fresnel*** *and* ***Fraunhofer*** *had sufficient knowledge to formulate the fundamental principles of holography. Moreover, many scientists in the second half of the* 19^{th} *century and the beginning of the* 20^{th} *century —* ***Kirchhoff***, ***Rayleigh***, ***Abbe***, ***Lipmann***, ***W.L. Bragg***, ***M. Wolfke*** *(1883–1947,* Poland, *1920), and* ***Hans Boersch*** *(1909–1986,* Germany, *1938) were very close to discovering the principles of holography.*

The invention of holography in 1948 *by* ***D. Gabor*** *stemmed from his work on improving the quality of images obtained in electron microscopy. In the* 1940s *the results obtained with electron microscopes (invented in 1929) were disappointing because although a* 100*-fold improvement on the resolving power of the best light-microscope had been obtained, the resolution fell far short of the theoretical limit. The fast electrons used in electron microscopy have a de Broglie wavelength of about* $\frac{1}{20}$ $\mathring{A}$*, so that atoms should in principle have been resolved; but the practical limit at that time was approximately* $12\mathring{A}$.

A major reason for the shortfall was the presence of aberrations associated with the electromagnetic electron lenses used[1142]*. It was in thinking about how to solve that problem that Gabor devised the technique he called* wavefront reconstruction*. His inspiration came partly from the principles involved in W.L. Bragg's* microscopy*. He reasoned that if he could record the phases as well as the intensities in an electron microscope image, then perhaps he could complete the image formation in an optical system which at the same time could be designed to correct the aberrations in the electron optics.*

1142 Mainly *spherical aberrations* that result from the fact that the focal points of rays far from the optic axis of a spherical lens (or mirror) are different from the focal points of rays of the same wavelength passing near the axis.

He was also influenced by Zernike's successful use of a coherent background wave in phase-contrast microscopy. However, Gabor's light-optics results were bedeviled by the inadequate coherence length (only about 0.1 mm) of the light from the high pressure mercury vapor lamp that was used, and by the low level of illumination available after the introduction of a small pinhole (3 microns in diameter) to secure adequate spatial coherence.

For these and a number of other reasons, the envisaged application to electron microscopy was unsuccessful, and so holography went into hibernation. It was not until 1962 that the modern revolution in holography began. ***Emmet N. Leith*** and ***Uris Upatnieks*** *recognized the similarity of Gabor's wavefront reconstruction process to their theoretical results on 'side-looking' radar.*

Lasers (1917–1969)

A laser (acronym: Light Amplification by Stimulated Emission of Radiation) is a device that emits highly monochromatic, well-collimated beams of coherent light. In other words, the light waves from a laser have only a small variation about a single frequency or wavelength, the waves do not spread out in space much more than the minimum dictated by the diffraction limit, and the wave trains retain their phase coherence with one another.

Photons can be absorbed by electrons in atomic, molecular or band orbitals, exciting them to higher-energy state. Once an electron is excited, it can naturally return to a lower state in a random exponentially-distributed process with an average characteristic lifetime. This natural decay is called spontaneous emission; light from common sources such as electric bulbs, fluorescent lamps, and the sun belong to this category. The resulting radiation is highly irregular, namely: polychromatic, incoherent and multidirectional. The excited electron can also be induced to return to a lower state in a shorter time e.g via collisions with other atoms or molecules.

In stimulated emission, photons of energy $h\nu = \Delta E$ *(where* ν *is close to the frequency of some spontaneous emission of the excited state) pass through the excited atom and, because of a resonance effect (also interpretable as due to the Einstein-Bose statistics of photon gases), stimulate transition to a state with lower energy. This transition yields the emission of a further photon of almost the same energy and frequency as the incident photons. In short,* N *photons go in but* $N+1$ *photons come out. Therefore the light's energy will be progressively amplified, which is the basis for operation of the laser.*

The theoretical mechanism of stimulated emission was given by ***Einstein*** *already in 1917, based on the old quantum theory and before quantum mechanics had been discovered. By considering a gas of photons in dynamic equilibrium with a gas of atoms, Einstein was able to show that the probability of a stimulated transitions is directly proportional to the average population of electron per state in the initial excited level, as well as to the intensity (or photon density) of the pre-existing radiation field.*

Consider two levels of energies $E_1 < E_2$, *occupied by* N_1 *and* N_2 *atoms respectively. Let* A_{21} *represent the spontaneous emission transition probability per unit time from level 2 to level 1. If radiation of a frequency range including overlapping the emission/absorption band surrounding the resonant* $\nu_0 = (E_2 - E_1)/h$ *is present and its intensity spectral density is* $I(\nu)d\nu$, *absorption transitions from* E_1 *into* E_2 *are produced.*

Let B_{12} *be the absorption transition probability per unit time and unit intensity of radiation, and* B_{21} *the stimulated emission transition probability per unit time and unit intensity of radiation.*

When radiation and matter are in thermal equilibrium, no net absorption or emission occurs (the total number of absorption and emission transitions per unit time is the same).

Einstein used this fact to derive the blackbody-radiation law, but his argumentation also contributed to the discovery of Bose-Einstein statistics (which applies in particular to a gas of photons).

Although the induced (stimulated) absorption probability rate may be smaller than the spontaneous transition probability, absorption can match emission because of the larger population of the lower level at thermal equilibrium. In the general case, in which radiation interacts with matter without necessarily being in equilibrium, we have

$$\frac{\text{Emission rate}}{\text{Absorption rate}} = \frac{[A_{21} + B_{21}\ I(\nu_0)]N_2}{B_{12}\ I(\nu_0)N_1} = \left(1 + \frac{A_{21}}{B_{21}\ I(\nu_0)}\right)\frac{N_2}{N_1},$$

where use has been made of time-reversal symmetry: $B_{12} = B_{21}$.

If the energy difference $E_2 - E_1$ *is sufficiently small, so that the ratio* $h\nu/kT$ *is very small (as occurs, for example, in the microwave region at room temperature),* $A_{21}/B_{21}\mathrm{I}(\nu)$ *is shown to be negligible compared with unity for ambient radiation at or above the classical Rayleigh-Jeans (thermal equilibrium) spectral distribution. In this case, we may write* $\frac{\text{Emission rate}}{\text{Absorption rate}} \approx \frac{N_2}{N_1}$.

If the substance is in thermal equilibrium, N_2 *is smaller than* N_1 *and the emission rate is smaller than the absorption rate. But if, by some means, the relative population of the excited and ground levels is inverted, so that* N_2 *is larger than* N_1*, making the ratio* N_2/N_1 *larger than* 1*, then the emission rate is larger than the absorption rate.*

In other words, if electromagnetic radiation of energy density $\mathrm{E}(\nu)$ *passes through this system, the radiation that comes out has more photons of frequency* ν_0 *than the incident radiation — resulting in an "amplification" of the radiation at that frequency. This is only true, of course, if*[1143] $E_2 - E_1 = h\nu_0 \approx h\nu$*. Since more atoms are de-excited than excited, the upper level begins to be depleted, so that the amplification is decreased until thermal equilibrium is re-established.*

Thus, to sustain a steady–state amplification, it is necessary to continuously replenish the atoms in the upper level, or to remove atoms from the lower level by some other means.

Several means have been devised to overpopulate the upper level in a steady fashion. All these methods require some expenditure of energy, and the efficiency of a maser (microwave laser) or (optical) laser is the ratio between the energy output and the energy input. One typical method is optical pumping*, in which light energy is supplied, either continuously or in bursts, to excite the atoms to higher energies. Electrical or chemical pumping schemes are also in use*

In masers and lasers the stimulated, coherent, monochromatic radiation is very intense, in comparison with the spontaneous incoherent radiation, which is treated as noise *in these devices.*

Due to the strong predominance of induced transitions, the noise is relatively smaller in masers and lasers than in conventional amplifiers and oscillators. Maser amplifiers are used whenever very low noise is of prime importance, such as in radioastronomy work, satellite communication, and microwave spectrometry.

[1143] More precisely, the spectral band of ambient radiation must overlap the spectral band of the two-level transition under consideration. The bandwidth of the latter is the *natural* (isolated atom) linewidth, plus environmental broadening terms due to pressure, collisions and thermal Doppler shifts.

Basic parts of the laser include a power source and a light-amplifying (lasing) substance. Stimulated emission results when energy from the power source pumps a majority of atoms in the substance into excited states. In the case of the ruby crystal laser a powerful flash tube sends intense light through the ruby.

The atoms excited thereby radiate light as their electrons drop back to low-energy orbitals. Part of this light travels along the axis of the ruby as laser light. This light is reflected back and forth by mirrors and stimulates other excited atoms into releasing their energy, which amplifies the laser light manifold.

Alfred Kastler (1902–1984, France; Nobel prize for physics, 1966), developed (1950) optical pumping, a system using light or radio waves to excite atoms, which then emit coherent electromagnetic waves (a precursor to the laser).

Charles Hard Townes (b. 1915, U.S.A.; Nobel prize for physics, 1964) first proposed the idea of the maser (Microwave Amplification by Stimulated Emission of Radiation) in 1953. The idea of the laser first occurred to **Gordon Gould** in 1957, then a graduate student at Columbia University.

It was independently conceived by **Arthur L. Schawlow** (b. 1921, U.S.A.; Nobel prize for physics, 1981) and C.H. Townes in 1958. In 1960, **Theodore Harold Maiman** (b. 1927, U.S.A.) developed and constructed the first laser, in which synthetic ruby was the light-amplifying substance. Semiconductor lasers were first operated in 1962. The first liquid laser was operated in 1966.

In 1969, astronauts on the Apollo 11 lunar mission placed a cubic-mirror laser reflector on the moon. Scientists used this device to measure precisely the distance between the earth and the moon, by measuring the time required for the laser beam to travel to the reflector and back.

History of Gyroscopic Phenomena and Technologies, II
(1913–2001)

Gyroscopes have traditionally relied on the spinning of a mass and the fact that a fast spinning mass points toward a fixed direction in space unless disturbed by a force. This makes the gyroscope a very important navigation instrument.

It can also be used to determine the angular rate of change in the direction of the vehicle on which it is mounted. Thus, gyroscopes (or gyros*) are used for* guidance*,* navigation*, and stabilization of the carrying vehicle. They are used for guidance and orientation of aircraft and missiles, tracking the deviation of flights from set patterns, to determine the bearing of automobiles as they turn on streets, etc.*

Optical gyros

Optical gyros depend upon the Sagnac effect *(1913) to detect any rotation. The effect manifests itself in an experimental setup called* ring interferometry*. It hinges upon Einstein's STR principle that the speed of light in vacuum is independent of the motion of the source and is equal to c in all frames of reference. Sagnac discovered that if two identical beams of wavelength λ travel in opposing directions along a closed path undergoing rotation at an angular speed Ω rad/sec, then the light beam traveling in the same direction as the rotation takes a longer time to travel around the path than the other beam. This results in a change in the interference pattern between the counter–rotating beams. The phase shift $\Delta\phi$ produced is given by*[1144]

[1144] The effect is best explained in the framework of *Special Relativity*: Let the beams traverse a circular path of radius R. In a local instantaneous rest frame at any point the light-conducting fiber (if any), moves with speed $\frac{c}{n}$, with n the refractive index ($n = 1$ if only mirrors are used to bend beams along their paths).
Therefore, if one could define a *global* co-rotating Lorentz frame, there will be no phase-shift.
However, unlike the Michelson-Morely effect, one *cannot* have a globally rotating frame for the following reason: the rotating-frame angle-time coordinate (θ', t') would be related to the laboratory-frame's (θ, t) via the angular Lorentz

$$\Delta\phi \approx \left(\frac{4\pi AN}{\lambda c}\right)\Omega(\text{radians});\quad \Delta\phi = \frac{2\pi c}{\lambda}\Delta t'$$

where A is the area enclosed by the beams path and N is the number of times the beam has gone around the path. The term in parenthesis is called the scale factor of the gyro. Clearly, the larger the area enclosed by the beams, the better the performance.

Although Sagnac and other scientists demonstrated the concept in the laboratory, it was not until 1963, with the advent of the laser beam with its unique properties, that the principle could be used in a practical gyroscope.

The key properties of the laser that make the gyroscope possible are the laser's coherent light beam, its sharply–tuned frequency, its small amount of spreading, and its ability to be easily focused, split, and deflected. With advances in solid-state technology (detectors, modulators, etc.), optical gyros have now become highly reliable and compact.

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transformation (θ, θ' in radians):

$$\theta' = \frac{\theta - \Omega t}{\sqrt{1 - \frac{\Omega^2 R^2}{c^2}}};\quad t' = \frac{t - \frac{\Omega R^2}{c^2}\theta}{\sqrt{1 - \frac{\Omega^2 R^2}{c^2}}},$$

which are transcribed form the one-dimensional Lorentz transformation equations with $x \to R\theta$, $v = \Omega R$.
Such a globally rotating frame (θ', t'), however, cannot be globally defined because the laboratory coordinate θ undergoes a discontinuity in some arbitrary direction $\theta = 0$, which therefore causes a discontinuity in the time assignment t'. In other words, no matter where the angular discontinuity is chosen, there will be some space-time events for which the t' coordinate will be ambiguous – like the international date line, but *not* avoidable by using the Greenwich Mean–Time!
The jump in t' upon completing the circular circuit around the fiber is (with $\Delta t = 0$, $\Delta\theta = 4\pi$ – not 2π since one is comparing two counter-rotating beams):

$$\Delta t' = \frac{4\Omega\pi R^2}{c^2\sqrt{1 - \frac{\Omega^2 R^2}{c^2}}} \approx \frac{4\Omega\pi R^2}{c^2}. \tag{1}$$

Note that this derivation implies that the Sagnac effect is purely due to the *topology* of the Minkowski space-time. Note also that $\Delta t'$ is independent of the refractive index n, which makes (1) applicable for *fiber optic gyros*.

Optical gyros include the ring laser gyros (RLG) and the fiber optic gyros (FOG). Both are inertial rotation sensors using the Sagnac effect.

Ring gyro configuration

The device consists of a three- or four-sided block which defines a closed optical cavity. The light path is defined by mirrors mounted on corners. Light travels through holes in the block. The cavity is filled with a gas, usually a helium-neon mixture which lases when excited. Thus there is no need for an external laser (as in the fiber gyros).

The laser light propagates clockwise and counterclockwise in the cavity. We now have two beams in the cavity with an optical path of $\sim 8cm$ *to* $40cm$. *The counter–rotating beams interfere and a detector quantifies any changes in the interference pattern.*

In the above discussion, the platform rotation mentioned is rotation with respect to an inertial reference frame. Since this experiment does not involve a relativistic velocity the results are valid both in the context of classical electrodynamics and special relativity.

The Sagnac effect is the electromagnetic counterpart of the dynamics of rotation. A spinning gyroscope that is mounted on appropriate gimbals can be used to measure the rotation of the mounting, and likewise, a Sagnac interferometer measures its angular velocity with respect to the local inertial frame.

Note that the phase-difference of the Sagnac effect given above, can be interpreted as a frequency-difference that arises between counter-rotating modes when the whole system is rotating: the co-rotating mode shifts its frequency to the red, while the counter-rotating node shifts its frequency to the blue (a Doppler shift). Measuring the frequency difference renders the rotation rate. Thus,

$$\Delta\phi = \frac{2\pi P\Delta f}{c} = \frac{4A\Omega}{\lambda c} \qquad \therefore \qquad \Delta f = \frac{4A\Omega}{2\pi\lambda P}, \tag{2}$$

where P *is each beam's path-length and* λ *is the vacuum wavelength.*[1145]

[1145] Δf is known as the *beat frequency*. One may interpret the Sagnac effect as due to an interference of two wave systems: one is a standing wave, stationary w.r.t. the fixed stars, and another rotating with the earth. They create beats that are $\frac{\lambda}{2}$ apart. Thus, for a rotation angle of $\frac{\lambda}{2R}$ radians, one beat is recorded.

The idea of a ring laser was put to test by ***W. Macek*** *of Sperry-Rand in 1963 using a square ring. Further theoretical investigations (1986–1993) have shown that* large *rings have basically the potential to detect* earth rotation variations. *With $A = 0.75m^2$, a beat frequency of $\Delta f = 17Hz$ results from this rotation.*

Fiber optic gyros (FOG)

In fiber optic gyros, the optical wave-propagation takes place within an optical fiber coil, which could be as long as 2km. Two beams of laser light are sent in opposite directions around the coil. Because the speed of the laser light is constant, the motion of the optic ring itself, the laser, and the detector, have no effect on the individual light beams. An interference effect is created when the two counter-rotating laser beams are recombined at the detector. Imagine a FOG sensor that is rotating clockwise as seen from the top. A solid-state laser creates a single laser beam. The laser light is split into two beams, one going clockwise, and one counter-clockwise. After traveling through the fiber optic loops, the laser beams are recombined at the detector. The beam going clockwise will have to travel a little farther in going from the laser to the detector, because the detector has rotated away from it some; the beam going counterclockwise travels a little less from the laser to the detector because the detector has rotated into it. The difference in distance traveled creates a phase shift between the two beams.

FOG rate sensor have extremely low levels of bias drift with time or temperature. Solid-state laser diodes provide a very stable source of laser light at a constant frequency. This translates into very stable operation.

FOG sensors have all of the advantages of reliable solid-state technology over mechanical gyroscopes:

- *no moving parts;*
- *high stability over time;*
- *high stability over a temperature range;*
- *reliability;*
- *low sensitivity to environmental factors (vibration, shock, acceleration).*

For a fixed angular velocity $\Omega\frac{\text{radians}}{\text{sec}}$ one sees $\frac{\Omega}{\lambda/2R}$ beats/second, or Hertz. This beat frequency Δf can then be written as $\Delta f = \frac{2R\Omega}{\lambda} = \frac{4A\Omega}{\lambda P}$, $P = 2\pi R$, $A = \pi R^2$.

These advantages endear FOG to military users, to whom accuracy, long-term stability, low cost, high reliability, low maintenance, high tolerance to accelerations and vibrations, small size, light weight, and low power requirements are important.

One of the significant attributes of the laser gyro is its use of very few moving parts. Indeed, it is theoretically possible to build laser gyros without any moving components. Unlike the conventional spinning gyroscope with its gimbals, bearings, and torque motors, the laser gyroscope uses a ring of laser light, together with rigid mirrors and electronic devices. Thus the laser gyroscope is more rugged than conventional gyros, offering the obvious advantages of much greater reliability and lower maintenance requirements. Typically, laser gyros have a mean-time between failures about twice that found in conventional gyros. Not only does the greater reliability of the laser gyro mean lower life-cycle costs, but such gyros potentially could be less costly to produce in the first place. Current technological efforts are under way to get production costs down. Indeed, some of the advanced work on very small solid-state devices portends substantial reduction in cost and increases in reliability. The miniature laser gyros that may result could be used in such applications as low-cost tactical missiles and even a "guidance" system issued to the individual foot soldier to replace his compass.

Because the laser gyro uses solid-state components and "massless" light, it is insensitive to variations in the earth's magnetic and gravity fields. Likewise, shock and vibration have little impact. The laser gyros are especially attractive for high-performance aircraft, remotely piloted vehicles, and missiles. High-speed turns, dives, and jinking maneuvers do not represent a real problem to a laser gyro. Unlike a conventional gyro that requires a finite time for wheels to spin up and bearings to come up to operating temperatures, the laser gyro is essentially ready instantaneously when turned on. Again, because of the absence of moving parts and its solid-state components, a typical laser gyro has much lower power requirements than a conventional gyro and requires half as much cooling.

In regard to the important matter of accuracy, the laser gyro has the potential to provide accuracy equivalent to that offered by mechanical gyroscopes, even up to the accuracy levels required for ballistic missile guidance.

In an actual application, such as an aircraft autopilot, three laser gyroscopes would be used to sense changes in pitch, roll, and yaw. In addition, three accelerometers are used to measure longitudinal, lateral, and vertical motion.

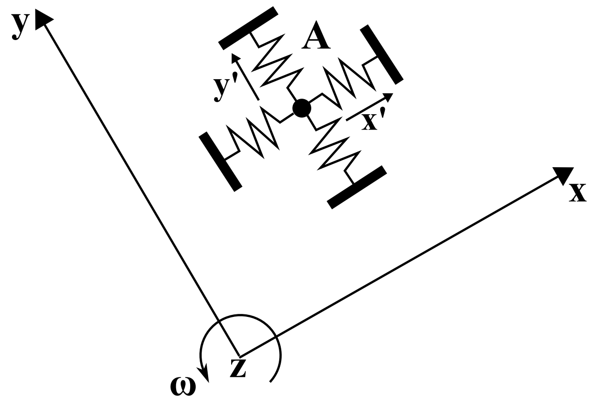

Fig. 5.25: Mass-spring system – vibrating MEMS gyroscope operating principle

Micromachined-Electro-Mechanical-System gyros (MEMS)

MEMS gyroscopes make use of vibrating mechanical elements to sense rotation. They are fabricated by using micromachining techniques in silicon or piezoelectric materials. In these gyroscopes a resonant primary excited mode contributes, together with Coriolis accelerations produced by the platform rotation, to a secondary resonant mode which gives the measure of the rotation.

The operating principle of vibrating gyros is the same for the different kinds of devices. In particular, those gyros can be modeled by a mass-spring system as shown in Fig. 5.25.

The elementary sensing unit is represented by a particle A and the spring represents the elasticity of the particle-supporting structure. The particle has two degrees of freedom; at any time instant its movement is defined by the displacements x' along the x-axis and y' along the y-axis. A rotation of the plane xy, the reference frame, around an orthogonal z-axis is characterized by an angular rate. To measure it, a vibration of the particle along the x-axis must be first induced; the vibration amplitude has to be constant. This oscillation is indicated as the primary motion *of the gyroscope, or* drive mode*. The vibration is produced by a feedback control system, which excites the particle at its resonant frequency while maintaining the vibration amplitude at a set value.*

When the gyroscope rotates, the particle experiences a Coriolis force, $\boldsymbol{F_c}$, which has an amplitude proportional to the applied rotation rate ω, and *its direction is, in the rotation frame, perpendicular to the primary motion direction:*

$$\boldsymbol{F_c} = 2m\boldsymbol{v} \times \boldsymbol{\omega},$$

where m is the vibrating mass and $\boldsymbol{v}$ is the velocity in the direction of the primary motion.

The Coriolis force will induce a particle vibration along the y-axis, indicated as the secondary motion of the gyroscope or sense mode. A measurement of its amplitude allows estimation of the angular velocity of the reference frame. The Coriolis acceleration is proportional to the primary velocity, so the amplitude and frequency of the drive oscillation have to be as large as possible. At the same time, it has to be ensured that the frequency and the amplitude remain constant; the amplitude control is accomplished by an automatic gain control loop while frequency stability is obtained by a phase locked loop.

The angular rate of the frame can be measured by means of a closed-loop control over the secondary motion. The measurement is used to generate a control force able to annul the motion along y-axis; the control force magnitude represents a measure of the rotation rate.

A large number of vibrating gyroscopes has been proposed whose configurations are rather complicated. They can be broadly classified with reference to their structure as follows: (a) *vibrating beams* (prismatic, triangular); (b) *tuning forks* (single, dual, multi-tone); (c) *vibrating shells* (hemispherical, ring, cylinder); (d) *vibrating plates* (linear disc, angular disc, linear plate).

Vibrating shells with hemispherical and cylinder configurations are macro-sized devices while vibrating beam, tuning fork, ring and plate gyroscopes are micro-sized devices manufactured from silicon or quartz.

Neutron Interferometry and the Sagnac Effect *(1988–1994)*

A neutron interferometer is an interferometer capable of diffracting neutrons, allowing the wave-like nature of neutrons to be explored. Like X-ray interferometers, neutron interferometers are typically carved from a single large crystal of silicon, often 10 to 30 or more centimeters in diameter and 20 to 60 or more in length. Modern semiconductor technology allows large single-crystal silicon boules to be easily grown. Since the boule is a single crystal, the atoms in it are precisely aligned, to within small fractions of a nanometer or an angstrom, over the entire boule. The interferometer is created by carving away all but three slices of silicon, held in perfect alignment by a base. Neutrons impinge on the first slice, where, by diffraction from the crystalline lattice, they separate into two beams. At the second slice, they are diffracted *again, with two beams continuing on to the third slice. At the third slice, the beams recombine, interfering constructively or destructively, completing the interferometer.*

Around 1988, physicists began to investigate the interference phase-shift induced by the rotation of a neutron interferometer. The result consisted of a Sagnac term, *which is due to the coupling of the orbital angular momentum of the neutron with the rotation of the frame, and a* new term *which arises from a similar coupling because of the* neutron spin. *The latter effect is generally smaller than the Sagnac phase-shift by the ratio of de Broglie wavelength of the neutron to the dimension of the interferometer.*

Experiments (1994) involving the interference of neutron de Broglie waves ($\lambda \sim 2\AA$)*, extending over distances of order* $10cm$*, were conducted in a rotating frame, establishing the existence of the Sagnac effect.*

Quantum Field Theory[1146]

Non-relativistic quantum mechanics elucidates the relation between observable particles (e.g., electrons) and their corresponding probability wave-field; it has been so successful that the results of a vast range of possible experiments could be predicted and explained in an unambiguous way. Thus,

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[1146] For further reading, see:

- Itzykson, C. and J-B. Zuber, *Quantum Field Theory*, McGraw-Hill, 1980, 705 pp.
- Chang, S.J., *Introduction to Quantum Field Theory*, World Scientific, 1990, 382 pp.
- Penrose, Roger, *The Road to Reality*, Alfred A. Knopf: New York, 2005, 1099 pp.

the wave-particle duality (part of the complementarity principle) was clarified and confirmed.

However, quantum mechanics in its initial form proved unsuitable for clarifying a similar duality relation between an electromagnetic field and the photons which correspond to it in the particle concept.

Let us compare two closed stationary systems: the hydrogen atom and a rectangular 3D cavity with ideally reflecting walls, which may be filled with radiation. For the first system, Schrödinger and Heisenberg showed that only certain discrete modes of the electron's probability-amplitude complex wave function (the eigenfunctions or eigenstates of the electron in the atom) with corresponding discrete eigenvalues (energy levels) are compatible with the boundary conditions (which are the requirement of a localized normalized bound state).

For the second example, the radiation-filled cavity, the boundary condition requires that only such waves occur for which each cavity dimension is an integral multiple of one-half the wavelength (spatial period) along the corresponding direction.

With the help of the relation $E = h\nu$, *quantum mechanics then yields the energy of the associated photons from the frequencies of the stationary waves. But this analogy does not tell us anything about the number of photons which populate a given cavity eigenmode having a given frequency. Furthermore, the modal wave functions are of the EM fields, which are real, observable quantities – not complex probability amplitudes like the Schrödinger wave functions. A comparison with the first example shows that something is still missing. This 'missing link' turns out to be the quantization of the electromagnetic field*[1147]*. Since we have associated the wavelength (or frequency) of the quantized electromagnetic cavity modes with the energy of the corresponding photons, we expect the second property of the cavity waves, i.e., their amplitude, to correspond to the number of photons.*

We thus come to the conclusion that the amplitudes of the individual discrete modal waves of the cavity must also be quantized (so-called second quantization), so that the electric and magnetic field vectors at each spatial cavity

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[1147] Also known as '2$^{\text{nd}}$ *quantization*': '*first quantization*' applies to nonrelativistic particles. The second quantization is associated with quantum mechanics of particles *and fields*; It is consistent with STR (once the electron's *Dirac field* is also quantized) and the number of particles (quanta) is variable (dynamical).

The so-called '3$^{\text{rd}}$ *quantization*' applies to a putative quantum field theory which would *quantize spacetime* (GTR in quantum gravity) and in which the number of *universes* is variable.

point become hermitian Hilbert-space Heisenberg operators, just like a particle trajectory in ordinary quantum mechanics. To describe this situation in a "Schrödinger picture", we need a Schrödinger equation whose solutions are complex wave functionals, i.e. complex numbers depending upon the cavity field:

$$\Psi\left[\boldsymbol{E}_0(\boldsymbol{r})|t\right],$$

where $|\Psi|^2$ *is a probability density (in some suitable functional measure) that measuring* $\boldsymbol{E}(\boldsymbol{r},t)$ *will yield a field pattern within an infinitesimal neighborhood of* $\boldsymbol{E}_0(\boldsymbol{r})$*. In this language, there is an uncertainty relation precluding exact knowledge of both* $\boldsymbol{E}(\boldsymbol{r},t)$ *and* $\boldsymbol{B}(\boldsymbol{r},t)$*; the wave functional* $\widetilde{\Psi}\left\{\boldsymbol{B}_0(\boldsymbol{r})|t\right\}$ *is thus an infinite-dimensional Fourier-transform of* $\Psi\left\{\boldsymbol{E}_0(\boldsymbol{r})|t\right\}$ *just as the momentum wavefunction in the* 1^{st}*-quantized case,* $\widetilde{\Psi}(\boldsymbol{p}|t)$*, is a 3D Fourier transform of* $\Psi(\boldsymbol{r}|t)$*, the spatial Scrödinger wave function.*

Indeed, field strength and number of photons prove to also be complementary to one another in the sense of the uncertainty and complementarity principles, since $\boldsymbol{E}$ *and* $\boldsymbol{B}$ *belong to the wave concept, whereas the number* n *of the photons belongs to the particle picture. This is in addition to the just-mentioned complementarity between,* $\boldsymbol{E}$ *and* $\boldsymbol{B}$*, and yet another complementarity between the* phase *of a light wave and the* number of photons *it comprises. It is most interesting that the theory leads to the same result that had been* postulated *by* ***Planck*** *in* 1900 *when he tried to describe theoretically the black-body radiation and thus initiated the entire development of quantum theory*[1148].

The stationary electromagnetic modes of the cavity (which may or may not be a black body) behave exactly like linear oscillators. Consequently, their energy (apart from the zero-point energy $\frac{1}{2}h\nu_0$*)*[1149] *can only be a discrete integral multiple of* $h\nu_0$ *if* ν_0 *is the eigenfrequency of the corresponding stationary wave. In the particle picture, a cavity eigenvibration of excitation energy* $nh\nu_0$ *thus corresponds to* n *photons of energy* $h\nu_0$ *in each.*

The differences between quantum-theoretical computation and classical computation diminish with increasing quantum number n*. Thus, if the mean*

[1148] It is also interesting that **Einstein** was able to give a *theoretical derivation* of Planck's heuristic result (1917) before even *first-quantized* Quantum Mechanics was developed. These adumbrations can be traced to the fact that the blackbody law follows from just the wave/particle duality of light, the existence of atomic energy levels and general thermodynamical principles. It does not require the full machinery of quantum field theory.

[1149] It is precisely the sum of zero-point energies of all the infinity of cavity modes – the vacuum (ground state) virtual-fluctuations EM energy – that is responsible for the *Casimir effect*.

squared amplitude of a certain cavity vibration is associated to a large *quantum number n via field quantization or in the particle language, if there are* many *photons with the corresponding energy $h\nu$ in the cavity, then no substantial deviations from the classical theory are expected, and none are found. This holds true, for instance, for the long EM waves (down to the visible-light waves), where the energy $h\nu$ of the individual photons is small compared to the typical radiation energies recorded in radiation measurements, and where most phenomena can be described satisfactorily without making use of quantum theory. In the ultraviolet and yet shorter wavelengths, however, the energy of the individual photon is often large enough that considerable deviations from the classical theory are to be expected since individual photons may be measured here*[1150]*. Indeed, it is known that the earlier formula for the spectral energy distribution of the black-body radiation (**Rayleigh–Jeans**) agrees satisfactorily with measurements in the regime of waves long relative to the (temperature-dependent) Wien's Law* value, *but results in discrepancies which become larger and more fundamental the shorter the wavelength of the radiation is. Of course these are generalities: X-ray beams could exist, for instance, and conversely, in some detectors, radio waves are absorbed one photon at a time (and thus non-classically).*

A tiny grain of sand, perhaps 10^{-3} cm across, behaves in almost every way like an object of the large scale world. But in the realm of atoms (10^{-8} cm) particles arrange themselves in smeared-out, yet discretely denumerable configurations; observable changes often occur in abrupt quantum jumps and even the modified laws of motion determine only the probabilities of events, not the individual events themselves. These profound changes in behavior are due primarily to differences in the relative size of objects and their de Broglie waves; large and multi-atom objects are enormous compared to their associated matter waves; atoms and their waves are similar in size at low enough temperatures and/or for light enough atoms; and electron waves are vastly bigger than the electron particle itself.

As we descend to the scale of subatomic particles[1151] *(10^{-13} cm and smaller), the de Broglie relation between wavelength and momentum, to-*

1150 Individual photons are sometimes measured for longer wavelengths. Rhodopsin molecules in the human eye detect single visible-band photons.

1151 Subatomic particles include the *photons*, *leptons*, *hadrons* , non-Abelian gauge bosons and other, more exotic species. The photon mediates the electromagnetic interaction, despite the fact that it has no electric charge. Thus, for instance, photons are emitted by accelerated charges.

Neutrinos, *electron*, *muon* and tau-leptons and their anti-particles, are grouped together under the name 'leptons'. All leptons have weak interactions. The charged leptons, in addition, are also subject to the electromagnetic force. Strongly interacting particles, including all nuclei, are hadrons, and their be-

gether with the short range of strong and weak nuclear forces, implies that large values of energy, brief interaction times and relativistic velocities become the norm. Therefore, particle phenomena are necessarily rapid and violent — so violent that matter and energy interconvert freely, and matter loses the stability it displays under less drastic conditions.

Molecular, atomic and subatomic phenomena (as well as many bulk-matter phenomena, especially in solids) are successfully dealt with by the methods of quantum mechanics. *The theoretical methods dealing with* particles *that may move with velocities close to the velocity of light require, in addition, relativistic considerations*[1152]. *One of the great syntheses of* 20^{th} *century physics was the incorporation into quantum mechanics (QM) of special relativity (STR); and some of the surprising conclusions of this synthesis were that:*

- *Every particle species either has a distinct antiparticle or is its own antiparticle.*
- *Interactions can create and destroy particles and/or antiparticles.*
- *Even the vacuum is thus a many-body system, seething with zero-point* virtual *particles and particle-antiparticle pairs.*

The powerful combination of STR and QM gave rise to Quantum Field Theory (QFT) *which are concerned with the relations between* quanta and fields *and between* matter and fields. *The 4 known interaction classes in nature have been described (albeit incompletely thus far) via QFT: the strong and weak nuclear forces, the electromagnetic interactions (EM) and gravitation. Of these, the quantum theory of gravity is the least well understood at present. The current, makeshift QFT incorporating the other three classes of interactions, as well as the leptons, quarks, gauge particles and some other auxiliary particles and fields, is called the* Standard Model of particle physics *and consists of QED; QCD (the Yang-Mills gauge theory of quarks, gluons*

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havior is governed by the strong, the electromagnetic, and the weak interactions. Hadrons are subdivided into *mesons* and *baryons*, and are composed of *quarks*. The lightest of the hadrons is the *pion*. All particles are affected by the gravitational force.

1152 Since light particles (photons) move at the speed of light, it is now obvious why the quantum field theory (QFT) of the EM field is necessary to understand atomic transitions. QED (*Quantum Electro Dynamics*) is an extension of this QFT, in which the *Dirac field* is also quantized, allowing description of relativistic electrons, electron-positron pair production/annihilation, vacuum polarization (resulting e.g. in the Lamb shift) and other effects.

and the strong nuclear forces); and the unified electroweak QFT gauge theory, of which QED actually forms a subset.

The sub-theory describing the EM interactions of photons, electrons and positrons, is known as quantum electrodynamics (QED). This discipline explains properties of these particles and their interactions in terms of fields (the 'fermionic' electron field and the 'bosonic' EM field, both quantized) and results from the union of classical electrodynamics and quantum mechanics, modified to be compatible with the principles of relativity. The three particles with which it deals, are well suited to theoretical treatment because they are point-like, stable, their properties are well understood, and they interact mainly through the familiar electromagnetic force and are, to the limits of present-day empirical precision, found to have no internal structure[1153].

To physicists of the mid 19^{th} *century, fields meant a condition of strain in the ether, a tenuous elastic "jelly" filling all space. These strains were thought to produce the forces acting upon electric charges. There was also the luminiferous ether, possibly different from the electric and magnetic ethers, which transmitted oscillatory strains as light waves.*

In his synthesis of electromagnetism and optics, ***Maxwell*** *(1864) erected the electromagnetic theory of light (in which light appears as oscillating electric and magnetic fields propagating together through space) with no mention of the ether model. The ether was finally banished from physics by* ***Einstein*** *(1905).He showed that the idea of an entity filling all space and acting as a stationary reference frame, relative to which all motions could be described in an absolute manner, is untenable, and that only the relative motions of objects have meaning (absent accelerations).*

Yet fields, in particular the traveling electromagnetic fields of light and radio waves, still retained a measure of reality. These carried energy and momentum and could cause electric charges to oscillate. ("A tension in the membrane, but without the membrane" as Steven Weinberg put it.) Again, it was Einstein (1905) who robbed them of these trapping of reality by postulating the photon.

It was, however, quantum field theory that has wrought a revival in the status of fields. Although they are still largely mathematical conceptions, they

[1153] QED has been applied heavily (and with striking success) to atomic physics, with nuclei treated as point-particles. Nuclei are hadronic, have a messy internal structure and are not ideally suited for tests of QED.

Fortunately, however, nuclei are much smaller than atoms, and they can mostly be replaced, for atomic (and molecular) physics purposes, by classical, non-relativistic point particles possessing only mass, charge and a few EM multipoles.

have acquired strong overtones of reality. In fact, this theory (or rather, class of theories) asserts that fields alone are real, and that particles are merely the momentary manifestations of interacting fields. Thus, the solutions of the quantum field equations lead to quantized energy levels which manifest all the properties of particles. The dynamics of the fields can seem particle-like because quantized fields may be localized and interact very abruptly and in very minute regions of space and time.

The interactions of the electromagnetic fields, whose energy is carried by photons, and the electron fields, which manifest themselves as electrons (and, at high energies or very short distances and durations, also as positrons) is already familiar in the production of photons by the quantum transition of atomic electrons. It is, however, not apparent how photons, which travel through space with the highest possible velocity, might be involved in static electric fields such as those which bind electrons to the atomic nucleus.

Here a new concept is needed, that of virtual photons. Their existence is due (in a remarkable, yet logical manner) to the Heisenberg uncertainty principle. One form of this principle asserts that the uncertainty ΔE in the energy possessed by a system and the uncertainty Δt in the time during which it has this energy are related by the formula: $\Delta E \times \Delta t \geq \hbar$.

Because of the relativistic correspondence between energy and mass, this relation applies as well to the uncertainty Δm in mass, which is $\Delta E/c^2$. Applied to an electron, this means that its mass, in effect, does not maintain one precise value; rather, it fluctuates, the magnitude of the fluctuations being in inverse proportion to the time interval during which they persist.

In general, the uncertainty principle allows, for short durations, processes which violate classical (but not quantum) energy-momentum conservation. Free electrons may thus emit photons, but these exist only on the sufferance of the uncertainty principle.

When their time Δt is about up, they must be re-absorbed, e.g. by an electron (the emitter or another) or by a nucleus[1154]. They cannot leave the electron permanently, carrying off energy-momentum, nor can they deliver energy to any detection device, including the human eye. It is impossible for them to be seen or detected; therefore they are called virtual, not real. Yet theories in which they are postulated yield results in agreement with experimental observation. In the language of quantum field theory the interaction of the electron and photon fields brings about a condition in which, by permission of the uncertainty principle virtual photons are continually created and destroyed.

[1154] It is an elementary result in classical, special-relativistic kinematics that a free electron cannot emit a true (propagating) photon and itself remain free.

Virtual photons of greater energy and momentum exist for shorter times and travel shorter distances before they are annihilated; those of lesser energy reach out farther. In fact, they travel a distance of order the length of their associated waves (radio waves, light waves and others), which may vary over the whole range of values from zero to infinity. This swarm of short-lived borrowed-energy virtual photons darting outward from the central electron in all directions constitutes the quantum EM fields surrounding the electron. They can also be thought of as the local modification made by the electron (or positron) to the zero-point quantum fluctuation of the $\boldsymbol{E}$ *and* $\boldsymbol{B}$ *fields discussed above – a single-electron version of the Casimir effect.*

Calculations based on this concept show that the field is strongest close to the electron and drops off in inverse proportion to the square of the distance, in agreement with Coulomb's law of electric force. Virtual photons are the quanta of all classical electrostatic fields. For large charged objects, the virtual photons are so numerous that they produce a sensibly smooth and continuous effect, identical with the classical field.

Two electrically charged objects exchange virtual photons. Furthermore, the virtual photon mediating the Coulomb interaction, may split into a virtual electron-positron pair, which quickly annihilate each other to become the virtual photon again. This makes the vacuum behave like a polarizable (dielectric) medium near an electron (or nucleus), giving rise e.g. to the Lamb shift in hydrogen-atom spectroscopy. These virtual e^+e^- *pairs can also produce an exchange force between two photons (virtual or real), a result which follows directly from the principles of quantum electrodynamics — but which unfortunately has no analog in classical physics and cannot be visualized in terms of familiar experience. But these photon-photon interactions have been observed (e.g. Delbrück-scattering of gamma rays by atomic nuclei).*

There are, however, further complications. The virtual photons, produced by the electron, interact with the electron field in the nearby vacuum to produce virtual electrons and positron, which in turn yield virtual photons, and so on. Thus the theory, starting with one electron, ends up with an infinite number of virtual electrons, positrons and photons – each of which can be made real (propagating) if external energy and momentum are suitable supplied. Fortunately, the magnitudes of the successive steps in this sequence drop off rapidly, so that the results of all this complex virtual activity can be calculated very precisely[1155] *via asymptotic (albeit diverging) power expansions in the fine structure constant* $\alpha = e^2/4\pi\epsilon_0\hbar c$.

[1155] This fortuitous circumstance arises since the fine-structure constant is small: $\alpha = \frac{1}{137}$. Things are much more complicated for some other Quantum Field Theories, notably QCD (= Quantum Chromodynamics), which is believed to describe the strong nuclear forces, and for which the expansion parameter is of order 1 for some processes.

For situations in which sufficient energy or momentum is made available, some of the virtual photons, electrons or positrons surrounding an electron may be "promoted" to real ones. This explains real photon emission when atoms release energy by making transitions to lower energy states or, when an electron passes near a heavy nucleus ('bremsstrahlung'). It also explains e^+e^- pair production by a gamma ray photon passing by a heavy atomic nucleus, the production of two or three gamma rays when an e^+e^- "atom" (positronium) annihilates, and other effects.

According to the principles of quantum field theory, particles are associated fields extending throughout space, which means that fields exist even where there are no permanent particles, that is, where there is a vacuum. From these principles it follows that a vacuum is not an empty space. Rather, it is a seat of continuous activity, with virtual particles (or alternatively described, virtual field fluctuations) of many kinds winking in and out of existence.

Physicists thus speak of a physical vacuum as distinct from the bare vacuum of classical physics. Although these vacuum phenomena briefly violate classical mass-energy-momentum conservation, they are in accord with the many other conservation laws of charge, spin, baryon number and all the rest, and also obey the quantum version of energy and momentum conservations (no 'perpetuum mobile' here!)

QED is an extremely successful theory: It resolves many of the problems that led to the downfall of classical physics at the turn of the century, explains atoms (except nuclear structure) and their spectra, and in principle accounts for all of chemistry[1156]*, condensed matter and even life and brain function.*

QED also gave rise to the unforeseen concepts of antimatter and vacuum structure. It is the best tested and most accurate theory devised by man, and its validity is still being verified to ever increasing precision and over a span of distance scales ranging from astronomical to subnuclear. Thus, it has been applied with fantastic success over a photon-wavelength range of 24 orders of magnitude, from 10^{-15} cm out to an outer limit of about 80 earth radii ($\sim 5 \times 10^{10}$ cm).

It gives precise answers to questions involving the interactions between leptons and photons; and in the framework of the partially unified standard model, has been verified down to even smaller distances [although not to the spectacular levels of accuracy as for pure or almost-pure QED effects, such as the Lamb shift, the electron gyromagnetic ratio, and the spectrum and

[1156] Through quantum mechanics, physics has finally established a primacy over chemistry.

lifetime of positronia (metastable bound states of an electron-positron pair)]. Problems remain, but they are much deeper and lie outside the framework of QED or even the Standard Model of particle physics:

- *Why is charge quantized?*
- *What determines the charge and mass of the electron?*
- *Why are there three charged leptons? Why only three?*

Questions of this type will probably have to await deeper insight into the nature of all interactions, the presumed GUT (Grand Unified Theory) or even Quantum Gravity.

1948–1964 CE **Imre Lakatos** (1922–1974, Hungary and England). Philosopher of science and mathematics, and avid campaigner for academic values. Argued that a formalist presentation of mathematics obscures the real nature of living mathematical discovery and invention, which is a quasi-empirical process involving conjectures, the discovery of counter-examples, concept-stretching, and the search for more discriminating proofs.

In connection with the empirical sciences he developed his methodology of scientific research programmes (MSRP): the basic units of appraisal are competing programmes, rather than theories, a programme being characterized by a *hard core* of fundamental assumptions and an associated *heuristic* which indicates how the protective belt of subsidiary assumptions should be progressively modified in a content-increasing way that generates novel predictions.

MSRP called for a more penetrating kind of historiography for science, involving case-studies to identify the competing research programmes at work and to assess their relative progress.

Lakatos was born as Imre Lipschitz into a Jewish family in Debrecen, Hungary. His life would be dominated by the chaos that resulted from Nazi rise to power and WWII[1157]. He fled to England after the Hungarian uprising

[1157] More than 550,000 of Hungary's 750,000 Jews were murdered by Nazis during the war, including Imre's mother and grandmother who both died at Auschwitz. To avoid the gas chambers, Imre changed his name to Imré Molnár. After the war, being an active communist, he changed his name to Imre Lakatos. In 1950 he was arrested and served three years in a Stalinist prison.

(1956) and taught at the London School of Economics (1960), becoming a professor of logic (1969). Lakatos published *Proofs and Refutations* (1963–4), a work based on his doctoral thesis at Cambridge University (1961).

1948–1968 CE **Alexander Il'ich Akhiezer** (1911–2000, Russia). Theoretical physicist. One of the pioneers of many-body quantum theory. Contributed to the theory of resonance nuclear reactions, beam instability in plasma physics, stability criteria for MHD waves, interacting magnetrons in solid state physics, microscopic theory of magnetic relaxation, magneto acoustical resonance, and absorption of ultrasound in metals. Initiated the field of electron acoustics.

Akhiezer was born in Cherikov, Belarus to Jewish parents (younger brother of the mathematician **Nahum Il'ich Akhiezer**). After graduating from Kiev Polytechnic Institute (1934), he began graduate research with **Lev D. Landau** in Kharkov. With **Alexander S. Kompaneets**, **Evgeny M. Lifshitz** (1915–1985), **Isaak Ya. Pomeranchuk** (1913–1966) and **Laszlo Tisza** (b. 1907), Akhiezer formed the first generation of Landau's Jewish students who passed the demanding *Theorminimum* exam and established the core of the famous Landau School of theoretical physics. Distinguished by its strong esprit de corps, style and universality of approaches to problems arising in diverse areas in physics, the Landau School exerted great influence on the discipline of 20-th century theoretical physics.

Akhiezer received his Ph.D. (1936) and D.Sc. (1940) at Kharkov. In 1937, when Landau fled the city to escape Stalinist purges, a bulk of Landau's students also relocated to Moscow, except Akhiezer who remained in Kharkov. He stayed there for the rest of his life, producing groundbreaking studies in QED, as well as solid-state, plasma, and nuclear theories.

1948–1972 CE **Yakov Borisovich Zeldovich**[1158] (1914–1987, Russia). Astrophysicist. A pioneer of *Quantum Cosmology*.

During the 1940s he investigated the problems of flame propagation and gas dynamics. In the 1950s he turned to cosmology and studied the production of primordial hydrogen-to-helium ratio and the degree of isotropy in the early universe.

[1158] For further reading, see:

- Zel'dovich, Ya.B., *Stars and Relativity*, Dover, 1996, 522 pp.
- Zel'dovich, Ya.B., *Higher Mathematics for Beginners*, Mir Publishers: Moscow, 1973, 494 pp.
- Zel'dovich, Ya.B. et al., *The Almighty Chance*, World Scientific, 1990, 316 pp.

In 1959 he postulated the existence of *parity-nonconserving weak interactions* involving neutral currents and parity violation in atomic transitions (which were experimentally demonstrated two decades later).

He predicted that it would be possible to find *black holes* associated with X-ray emitting binary stars.

In 1972 he discovered how the total energy carried by the Cosmic Microwave Background Radiation (CMBR) could be increased as it passes through the medium between the galactic clusters. This effect has important cosmological applications and can be used to independently estimate the *Hubble constant* (which measures the manner in which the expansion rate of the universe varies with distance scale).

Zeldovich was born to Jewish parents in Minsk. He graduated from the University of Leningrad (1931) and moved to the Soviet Academy of Sciences, becoming a full Academician in 1958.

1948–1988 CE **Julian Seymour Schwinger** (1918–1994, USA). Distinguished theoretical physicist. One of the formulators of modern Quantum Electrodynamics.

Calculated the theoretical Lamb shift and the anomalous magnetic moment of the electron (1948). named after him: *Schwinger action principle*; *Schwinger model* for QED in one space dimension and one time dimension; *Schwinger terms* in *current algebra*.

Schwinger was born in New York City to Jewish parents. He earned his B.Sc. (1936) and Ph.D. (1939) from Columbia University; worked at Berkeley (1939–1941) under J.R. Oppenheimer, and later joined the faculty of Harvard University (1945–1972) and UCLA (1972–1994).

Working independently of R.P. Feynman and S. Tomonaga, he developed modern Quantum Electrodynamics — the relativistic, quantum mechanical theory of electrons, positrons and EM fields. He was a joint winner of the Nobel Prize for Physics (1965) for his fundamental work in QED.

1948–1988 CE **Richard Phillips Feynman**[1159] (1918–1988, U.S.A.). Distinguished theoretical physicist.

[1159] For further reading, see:

- Feynman, R.P., *Lectures on Computation*, Perseus Publishing: Cambridge MA, 1999, 303 pp.
- Feynman, R.P., *QED: The Strange Theory of Light and Matter*, Princeton University Press: Princeton, NJ, 1985, 158 pp.
- Feynman, R.P., *Quantum Electrodynamics*, Addison-Wesley Publishing Company: Reading, MA, 1961, 198 pp.

A key figure in the development of modern Quantum Electrodynamics (QED) and the taming of its divergences through *renormalization*[1160]. Introduced a universal diagrammatic description of fundamental quantum processes[1161] (*Feynman diagrams*, 1949) applicable in both particle physics and condensed matter physics.

Developed (1942) the *path-integral* approach[1162] to quantum mechanics (a useful alternative to the canonical operator approaches).

- Feynman, R.P., *Statistical Mechanics*, Perseus Books, 1998, 354 pp.
- Feynman, R.P., R.B. Leighton and M. Sands, *The Feynman Lectures on Physics*, 3 Volumes, Addison-Wesley Publishing Company: Reading, MA, 1963–1965.
- Feynman, R.P., *The Pleasure of Finding Things Out*, Penguin Books, 1999, 270 pp.
- Feynman, R.P., *The Character of the Physical Law*, BBC Corporation: London, 1965, 173 pp.
- Feynman, R.P., *Surely You're Joking, Mr. Feynman!*, W.W. Norton, 1997, 350 pp.
- Feynman, R.P., *What Do You Care What Other People Think?*, W.W. Norton, 1988, 255 pp.
- Mehra, J., *The Beat of a Different Drum*, Oxford University Press, 1996, 630 pp.
- Gleick, J., *Genius — The Life and Science of Richard Feynman*, Pantheon Books: New York, 1992, 531 pp.

1160 *Renormalization* = redefinition of the original parameters of the theory, in order to absorb infinities, so that physical [observable] quantities be finite; e.g. the 'bare' charge e_0 of the electron is taken negatively infinite, so that the effective charge e one sees at large distances ($|e| < |e_0|$ due to screening by vacuum polarization) is finite.

1161 Early attempts in this direction were made by the Swiss physicist **Ernst Stückelberg** (1941).

1162 In contrast to the *Schrödinger equation*, which is a differential equation determining the properties of a quantum state at a given time from its known properties at an infinitesimally earlier time, *path integrals* yield the quantum mechanical amplitudes in a global approach involving the superposed pseudo-classical histories of a system over a *finite interval of time*.

Thus, in contradistinction to the operator formalism of quantum mechanics and quantum statistics, which may not always lead to the most transparent

Provided a quantum mechanical underpinning for Landau's theory of *Helium superfluidity* (1953–1957). Participated in establishing the universal V–A (parity-violating superposition of *Vector* and *Axial* terms) structure of the four-Fermi weak interaction (1958).

Originated the *parton* picture of *hadron* structure (1969) to account for the scaling observed in deep inelastic scattering experiments at the Stanford Linear Accelerator Center (SLAC).

Feynman's name is also associated with the advent of *nanotechnology*, *reversible computing* and *quantum computing*.Feynman shared the 1965 Nobel prize in physics for his work on QED and Feynman diagrams.

Richard Feynman was born in Far Rockaway, a town on the outskirts of New York City, to Jewish parents: Melville and Lucille Feynman. His father, a sales manager for a uniform manufacturer, was interested in the natural sciences, and encouraged Richard's inquisitiveness. The young Feynman set up a makeshift laboratory at home, earned pocket money by repairing radios, and utilized simple chemical principles in neighborhood magic shows.

Feynman attended Far Rockaway high school, where he displayed a fondness for solving puzzles and an ability to solve mathematical problems in unconventional ways. After graduating from high school in 1935 he enrolled at M.I.T., where he earned a B.Sc. in physics in 1939. While at M.I.T., he became aware of the most pressing theoretical challenge of that period — the conceptual and computational problems besetting the nascent theory of quantum electrodynamics, or QED.

In 1939 Feynman began graduate studies at Princeton University, under J.A. Wheeler[1163]. He published his dissertation, "*The Principle of Least Action in Quantum Mechanics*" and received his Ph.D. in 1942. Thereupon,

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understanding of quantum phenomena, the path-integral formalism offers an equivalent method in which operators are avoided by the use of *infinite dimensional (functional) integrals*.

1163 During Feynman's schooldays at Princeton, nuclear physicists, quantum theorists, and even pure mathematicians were consumed by the *lawn sprinkler mystery*: What would happen if this familiar device were placed under water and made to *suck* water instead of spewing it out? Would it spin in the *reverse direction* (because the direction of the flow was now reversed, pulling rather than pushing), or would it spin in the *same direction* (because the same twisting force was exerted by the water, on the curved sprinkler pipes whichever way it flowed, as it was bent around the pipes' *S*-shaped curve)?

When Wheeler was asked for his own verdict he said that Feynman had absolutely convinced him the day before that it went around backward; that Feynman had absolutely convinced him today that it went around forward;

he joined a select group of Princeton scientists in isotope-separation work, as part of the *Manhattan Project*. During 1942–1945 he was a group-leader under Hans A. Bethe at the project's Los Alamos installation, while continuing to work on QED in his spare time[1164]. His years at Los Alamos brought him to contact with such physics luminaries as Niels Bohr, Enrico Fermi and Robert J. Oppenheimer.

After the end of WWII, Feynman accepted an offer by Bethe and came to Cornell University as an associate professor of theoretical physics (1945–1949). Following a sabbatical in Brazil (1949–1950), he moved to the California Institute of Technology in Pasadena, where he spent the rest of his career. After an unsuccessful marriage to Mary Louise Bell (1952–1954) he married (1960) Gweneth Howarth (1934–1989). Of this marriage he had a son (Carl, b. 1962), and an adopted daughter (Michelle, b. 1968).

Feynman was a unique figure in his generation: a master calculator, an acclaimed lecturer and an unconventional mind obsessed with originality[1165]; he had to always create from first principles. He read almost nothing, resented art and melody, rejected tradition, religion, history, and literature.

Unlike his great faith – sake Albert Einstein, he completely turned his back on his own ethnic and cultural heritage: Judaism, Zionism, the Holocaust and the revival of the Jewish homeland in Israel meant nothing to him. Even the gentle Reform Judaism of his parents left him cold. Feynmann left his own indelible personal mark on 20^{th} century physics.

He lived his life in the search for truth about nature through physics, and he was a physicist's physicist.

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and that he did not yet know which way Feynman would convince him the next day. [Years later, a friend of Feynman said to him "*It was clear to me at first sight*", to which Feynman shot back: "*It was clear to everybody at first sight. The trouble was, some guy would think it was perfectly clear one way, and another guy would think it was perfectly clear to him the other way*".] When no consensus could be reached, Feynman resorted to experiment. The experiment revealed that *the sprinkler does not turn at all*, which is the correct answer.

1164 Feynman was married to his high-school sweetheart Arlene H. Greenbaum in 1941. She contracted T.B., and during much of his stay at Los Alamos he would visit her, when possible, at the nearby sanatorium. She died in 1945.

1165 This is best demonstrated through Feynman's work on the Space Shuttle Challenger investigation. There, his masterful detective work zeroed in on the brittleness of frozen O-rings that led to the disaster. Then already dying of cancer, he single-handedly and doggedly dug at the truth until it became evident to all.

Worldview LV: Feynman

* * *

"If you believe that atoms are like little solar systems, then you are back in 1910."

* * *

"I think it is safe to say that no one understands quantum mechanics. Do not keep saying to yourself: 'But how can it be like that?' because you will go into a blind alley from which nobody has yet escaped. Nobody knows how it can be like that."[1166]

* * *

"In mathematics, everything can be defined, and then we do not know *what we are talking about. In fact, the glory of mathematics is that we do not have to know what we are talking about."*

* * *

"Any simple idea is approximate; as an illustration, consider an object... what is an object? Philosophers are always saying, 'well, just take a chair for example'. The moment they say that, you know that they do not know what they are talking about any more... To define a chair precisely, to say exactly which atoms are chair, and which atoms are air, or which atoms are dirt is impossible.

[1166] Some physicists believe that quantum mechanics should be viewed principally as a computational artifice, the justification of which rests mainly on its success, rather than on underlying ontological content. There is a general agreement that quantum mechanics is correct as a mathematical system, but its physical basis is obscure. This opinion was shared by Albert Einstein and Erwin Schrödinger.

It says in some books that any science is an exact subject, in which everything *is defined. If you insist upon a precise definition of force, you will never get it! First, because Newton's Second Law is not exact, and second, because in order to understand physical laws, you must understand that they are all some kind of approximation."*

* * *

"Newton's ideas about space and time agree with experiment very well, but in order to get the correct motion of the orbit of Mercury, which was a tiny, tiny difference, the difference in the character of the theory needed was enormous."

* * *

"The physicist is always interested in the special case; he is never interested in the general case. He is talking about something*. He is not talking abstractly about anything."*

* * *

"I always find it mysterious, and I do not understand the reason why it is that the correct laws of physics seem to be expressible in such a tremendous variety of ways."

* * *

"Mathematics is a language plus reasoning; it is like a language plus logic. Mathematics is a tool for reasoning."

* * *

"When you speak about only the most overall general qualities of nature, the topic has a tendency to become too philosophical. A person talks in generalities such that everybody could understand him. It is then considered to be some deep philosophy."

* * *

"Physics is the most fundamental and all-inclusive of the sciences, and has had a profound effect on all scientific development. It is the present day equivalent of what used to be called natural philosophy, *from which most of our modern science arose.*

Mathematics is not a science from our point of view, in the sense that it is not a natural *science. The test of its validity is not experiment.*

If a thing is not a science, it is not necessarily bad. For example, love is not a science."

* * *

"It always bothers me that, according to the laws as we understand them today, it takes a computing machine an infinite number of logical operations to figure out what goes on in no matter how tiny a region of space, and no matter how tiny a region of time. How can all that be going on in that tiny space? Why should it take an infinite amount of logic to figure out what one tiny place of space/time is going to do?"

* * *

"It's impossible to learn very much by simply sitting in a lecture, or even by simply doing problems that are assigned. The best teaching can be done only when there is a direct individual relationship between the student and the good teacher — a situation in which the student discusses the ideas, thinks about the things and talks about the things."

* * *

"Scientific knowledge is an enabling power to do either good or bad — but it does not carry instructions on how to use it."

* * *

"The imagination of nature is far, far greater than the imagination of man."

* * *

"Scientific knowledge is a body of statements of varying degrees of certainty — some most unsure, some nearly sure, but none absolutely certain."

* * *

"Our freedom to doubt was born out of a struggle against authority in the early days of science."

* * *

"Throughout all ages of our past, people have tried to fathom the meaning of life. They have realized that if some direction or meaning could be given to our actions, great human forces would be unleashed. So, very many answers have been given to the question of the meaning of it all... If we take everything into account — not only what the ancients knew, but all of what we know today that they did not know, then I think we must frankly admit that we do not know *the meaning of the mystery of our existence."*

* * *

"The electron does anything it likes. It just goes in any direction at any speed, forward or backward in time, however it likes, and then you add up the amplitudes and it gives you the wave function."

* * *

"Falling in love with a theory, like falling in love with a woman, is only possible if you do not know much about her, so you cannot see her faults...

So, what happened to the old theory that I fell in love with as a youth? Well, I would say it's become an old lady, that has very little attractive left in her and the young today will not have their hearts pound when they look at her anymore. But, we can say the best we can for any old woman, that she has been a good mother and she has given birth to some very good children."

* * *

"It's enough of a miracle that there are laws at all, but what's really a miracle is to be able to find them."

* * *

"Those are atoms! This is religion. You shouldn't be asking questions, you should look at the pictures. You don't have to say anything. Just look at it. That's God you know! Atoms right there."

(On looking at the images of atoms in the *scanning tunneling microscope*)

* * *

"In fundamental physics, the thing that doesn't fit is the thing that's the most interesting – the part that doesn't go according to what you expected."

* * *

"There are two ways of doing physics: the Babylonian way and the Greek way. The Greeks were very logical and worked on things from first principles, form axioms, where one thing depended on the other. The Babylonians just related one thing to the other. I am a Babylonian."

* * *

"The real work in a field is always done by a limited number of people."

* * *

"Artists are lost: they don't have any subject! They used to have religious subjects, but they lost their religion and now they haven't got anything. They don't know anything about the beauty of the real *world – so they don't have anything in their heart to paint."*

* * *

"The problem is not to find the best *or most efficient method to proceed to discovery, but to find any method at all. Theories of the known which are described by different physical ideas may be equivalent in all their predictions*

and hence scientifically indistinguishable. However, they are not psychologically identical when trying to move from that base into the unknown.”

* * *

“You can recognize truth *by its beauty and simplicity. Nature has simplicity and therefore a great* beauty.*”*

* * *

“I can live with doubt and uncertainty. I think it’s much more interesting to live not knowing than to have answers which might be wrong.”

* * *

“What I am going to tell you about is what we teach our physics students in the third or fourth year of graduate school... It is my task to convince you not to turn away because you don’t understand it. You see my physics students don’t understand it... That is because I don’t understand it. Nobody does.”

* * *

“I am interested not so much in the human mind as in the marvel of a nature which can obey such an elegant and simple law as this law of gravitation. Therefore our main concentration will not be on how clever we are to have found it all out, but on how clever nature is to pay attention to it.”

* * *

“Nature uses only the largest threads to weave her patterns, so each small piece of her fabric reveals the organization of the entire tapestry.”

* * *

“There is plenty of room at the bottom.”

* * *

"We have a habit in writing articles published in scientific journals to make the work as finished as possible, to cover up all the tracks, to not worry about the blind alleys or describe how you had the wrong idea first, and so on. So there isn't any place to publish, in a dignified manner, what you actually did in order to get to do the work.

* * *

"Equilibrium is when all fast things have happened and the slow ones have not."

* * *

"It is scientific only to say what is more likely and what is less likely. Science proceeds by informed guesses whose implications are compared with experiment."

* * *

"Fundamental theoretical chemistry is really physics."

* * *

"Principles of physics, as far as I can see, do not speak against the possibility of maneuvering things atom by atom. It is not an attempt to violate any laws; it is something, in principle, that can be done; but in practice, it has not been done because we are too big."

* * *

"There is enough room on the head of a pin to put all of the Encyclopedia Britannica."

* * *

"There is nothing that I can see in the physical laws that says the computer elements cannot be made enormously smaller than they are now."

* * *

"How many times when you are working on something frustratingly tiny like your wife's wrist watch, have you said to yourself: "If I could only train an ant to do this!"

* * *

"Small automobile would only be useful for the mites to drive around in, and I suppose our Christian interests don't go that far."

* * *

"The problem of chemistry and biology can be greatly helped if our ability to see what we are doing, and to do things on the atomic level, is ultimately developed – a development which I think cannot be avoided."

* * *

"I offer a prize of $ 1,000 to the first guy who can take the information on the page of a book and put it on an area 1/25,000 smaller in linear scale in such manner that it can be read by an electron microscope."

* * *

"I offer another $ 1,000 to the first guy who makes an operating electric motor – a rotating electric motor which can be controlled from the outside and, not counting on lead-in wires, is only 1/64 inch cube."

* * *

"Philosophers say a great deal about what is absolutely necessary for science, and it is always, so far as one can see, rather naive, and probably wrong."

* * *

"For a successful technology, reality must take precedence over public relations, for Nature cannot be fooled."

* * *

"I believe that a scientist looking at nonscientific problems is just as dumb as the next guy."

* * *

"I was born not knowing and have had only a little time to change that here and there."

* * *

"There are 10^{11} stars in the galaxy. That used to be a huge number. But it's only a hundred billion. It's less than the national deficit! We used to call them astronomical numbers. Now we should call them economical numbers."

* * *

"We are at the very beginning of time for the human race. It is not unreasonable that we grapple with problems. But there are tens of thousands of years in the future. Our responsibility is to do what we can, learn what we can, improve the solutions, and pass them on."

* * *

"You can know the name of a bird in all the languages of the world, but when you're finished, you'll know absolutely nothing whatever about the bird... So let's look at the bird and see what it's doing – that's what counts. I learned very early the difference between knowing the name of something and knowing something."

Quantum Electrodynamics (QED)[1167]

QED was initially developed around 1930*, largely through the work of* ***Paul Dirac****. It describes the relativistic behavior of electrons, positrons and photons – virtual and real – and of the vacuum – under the stimulus of a given externally applied electromagnetic field. The theory yielded two important results: it showed that the electron has an* alter ego*, the positron, and it gave the electron its spin in a manner arising naturally from the union of quantum mechanics and STR. (Previously it had to be arbitrarily grafted into the theory.) Also, as the first Quantum Field Theory (QFT) to be developed, QED cast the vacuum as the ground state of all that can be, and pointed the way towards developing a host of other QFT's, including the empirically successful QCD and electroweak sectors of the* standard model*.*

When QED was applied to the old problem of the fine structure of the hydrogen spectrum (the small differences between the observed wavelengths and those given by the Bohr theory), it produced improved values in good agreement with existing measurements.

Nevertheless, the theory in its original form suffered from lack of manifest Lorentz (relativistic) covariance, and was plagued by mathematical divergences, appearing in such physical quantities as the effective charge and mass of the electron, and by inelegant and often ambiguous mathematical procedures.

At Princeton (1939–1942), while experimenting with various mathematical and physical approaches to both classical and quantum electrodynamics, ***Feynman*** *strove to eliminate the bothersome short-distance infinities of the theory.*

One particularly imaginative approach sought to eliminate the infinite self-action of the electron by replacing the standard delayed *electrodynamic field*

[1167] For further reading, see:

- Power, E.A., *Introductory Quantum Electrodynamics*, Longmans, 1964, 147 pp.
- Feynman, R.P., *QED*, Princeton University Press, 1988, 158 pp.
- Sokolov, A.A. et al., *Quantum Electrodynamics*, Mir Publishers: Moscow, 1988, 335 pp.

emanating from it, by a suitable linear combination of delayed and advanced fields. To preserve causality and the arrow of time, Feynman posited a cosmological absorbing shell at a large distance. The effect is to restore the purely delayed action of one point charge on another, while retaining both advanced and retarded actions of an electron on itself, needed to eliminate the divergence.

While spurious, this work illuminates the mode of thought that later led Feynman to his propagators, which can project backwards as well as forward in time.

Meanwhile QED was facing empirical challenges, thanks to advances in atomic spectroscopy and the new microwave technology spawned by the war. Thus, in 1947, ***Willis E. Lamb*** *and* ***Robert Retherford*** *made highly precise measurements of the small differences in two hydrogen energy-levels, predicted by the Dirac theory to be degenerate (i.e., to have the same energy). They used the quanta of radio waves, which are needed to induce the requisite low-energy transitions, and discovered the Lamb shift. Similar techniques were used by* ***P. Kusch*** *in a precision measurement of the intrinsic magnetic moment of the electron.*

In both cases, small deviations from the Dirac values were found[1168]*. These results stimulated renewed theoretical efforts; the QED of that period did contain the physical mechanisms that account for these deviations from Dirac's theory, but due to the theoretical problems mentioned above, this was unclear at the time. These mechanisms are basically two:*

(I) An electron (or positron) occasionally emits a virtual photon, thus entering a short-lived virtual state as an electron-photon composite, until the photon is reabsorbed (by the same, or different, charged particle). Since the electron has a finite probability at any time to be such a composite system, its EM properties – including, inter alia, its magnetic moment and its hydrogen-atom orbitals – are not those of a point particle; and

[1168] The *anomalous magnetic moment* of the electron – deviation of its gyromagnetic ratio (magnetic moment divided by intrinsic spin) from the Dirac value of 2 – was measured to be on the order of a tenth of a percent. Ever more refined measurements of this entity continue to furnish high-precision tests of QED which it has, to date, passed with flying colors. Some of these tests involve other (non-EM) subnuclear interactions, since virtual quantum processes result in the creation, for brief periods of time, of quarks, pions, muons, nucleons, etc., along with their respective anti-particles, nearby the single electron under study.

this accounts in part for the anomalous magnetic moment and the Lamb shift.

(II) The virtual photons, whose exchange between electron and proton hold them together to form the hydrogen atom, occasionally create a short-lived virtual electron-positron pair out of the vacuum[1169].

Thus, in QED, the vacuum is not structureless, but is rather a medium! Since this medium consists of positive and negative charges (virtual positrons and electrons), it is polarizable ("vacuum polarization"), and partially screens the classical electromagnetic forces between electron and proton in hydrogen[1170]*. Effects (I) and (II) (in the context of Dirac's relativistic electron quantum mechanics) suffice, in principle, to account for the Lamb shift.*

[1169] These creation-annihilation processes, like the virtual emission-absorption of a photon by the electron, cannot by themselves be real processes, since this is forbidden by the STR conservation of energy and momentum: *at least* one electron, positron or photon in a fundamental QED vertex event must be virtual. But if one of the participating particles is "anchored" to another charged system, or if more than one real photon is involved, such processes *can* be real. Even if not 'real', however, virtual processes indirectly give rise to observable results, as is also true in ordinary quantum mechanics. Thus, virtual excitations of an electron in an atom or molecule contribute to bulk refractive indices of material, even when no *actual* absorption of light occurs. Several QED examples of real multi-stage processes mediated by virtual events: annihilation $e^+ + e^- \to 2\gamma$ (predicted by Dirac, 1930); the inverse process: $\gamma + \gamma \to e^+ + e^-$ (Dirac, 1931); pair creation by a gamma ray falling on a nucleus (Oppenheimer, 1933); $e^+ + e^- \to \gamma$, where the electron is bound to a nucleus (Fermi and Uhlenbeck, 1933). The scattering of a real photon by an atomic nucleus' electrostatic field (*Delbrück scattering*) is, like the Lamb shift, an example of an observable effect of virtual e^+e^- pairs and virtual photons.

[1170] Unlike the *Ether*, this medium is *Lorentz invariant* and does not furnish a preferred inertial frame. In the vicinity of matter and energy, however, the QED vacuum becomes 'anchored' to these particles and fields, and gives rise to covariant modifications of Maxwell's equations in vacuo – including a field-dependent refractive index and dispersion. Like a normal dielectric medium, the QED vacuum can also undergo *electric breakdown* – albeit at a much higher critical field (the *Schwinger field*), of order

$$E_{sch} = \frac{m_e^2 c^3}{e\hbar} \approx 1.3 \times 10^{18}\ Volt/meter.$$

Although understood in principle, these theoretical mechanisms were difficult to treat in the pre-Feynman "messy QED". This was due to two main reasons:

(A) *The field quantization procedure was not manifestly Lorentz covariant: even though Maxwell's and Dirac's equations* are *covariant, the Schrödinger equation (for the wave-functional of the quantized EM and Dirac fields) is* not *– for,* time *plays a special role there. One symptom of the lack of manifest covariance is that transitions to virtual intermediate states were still described as conserving momentum but not energy ("old-fashioned perturbation theory") — just as in non-relativistic quantum mechanics.*

(B) *The summation over all virtual states, mandated by the rules of quantum mechanics, includes an integral over the total energy of these states, which typically diverges at high energy (ultraviolet divergences). (A) itself is merely an inconvenience, as covariance should still be retained, albeit hidden by the formalism. But point (B) renders the mathematics meaningless. And to add insult to injury, the all-important* quantum gauge invariance *– tied to relativistic causality, local charge conservation, Lorentz covariance* and unitarity *(essentially the principle that probabilities of quantum events add up to unity) – is also vulnerable to the divergences*[1171].

Such was the state of QED *in* 1947, *when Feynman strode upon its stage. Aided by Bethe's pioneering work at Cornell, he continued to tackle these fundamental problems, but entered a period of stagnation. Soon enough,*

[1171] Gauge invariance – an elegant mathematical curiosity related to local charge conservation and of limited significance in classical electrodynamics – is crucial and nontrivial in QED, since it is the unobservable, gauge-*variant* vector potential that enters the interaction Hamiltonian in a natural way, rather than the observable, gauge *invariant* $\boldsymbol{E}$ and $\boldsymbol{B}$ fields. This is dramatically illustrated in the empirically-verified *Aharonov-Bohm* effect, in which an electron 2-slit diffraction pattern is *shifted by the vector potential* of a current solenoid located just behind and between the slits, even though no electron ever enters the magnetic field region. The extension of the gauge-invariance principle to local conservation of a *vector* of charges ("isospin") in the 1950's led theorists to a class of QFT – termed *Yang-Mills* or Non-Abelian Gauge Theories – that turned out (in the 1970's) to explain *all* thus–far known non-gravitational interactions (Standard Model of particle physics). The *gauge principle*, like the *general covariance* of GTR, is also related to deep principles of differential geometry and topology.

however, his mental vigor was restored and in 1948–49 *his reformulation of QED*[1172] *came to fruition.*

It cast any process as a sum of progressively smaller complex contributions from an infinite number of (progressively more complicated) discrete networks of space-time points (interaction vertices), connected by lines (propagators). A propagator represents the quantum amplitude for a free particle's worldline connecting two given spacetime interaction points. Positive-energy modes of the electron and photon quantum fields, which are physical particles, travel forward in time, whereas negative-energy modes travel to the past *— which does not violate causality, since negative-energy modes are interpreted as positive-energy* anti-particles *moving forward in time.*

Thus, the photon is its own anti-particle, but an electron propagator may represent either an electron or a positron, depending on whether the propagator points forward or backwards in time. In the latter case, the positron travels forward in time, as a physical particle should.

At each vertex, two electron lines and one photon line meet. Depending on their time directions, the vertex may represent pair creation, pair annihilation or the emission (or absorption) of a photon by an electron (or by a positron).

Since these Feynman Diagrams treat time and space on equal footing, by describing a quantum amplitude through its space-time history, they are easier to represent and compute in a manifestly covariant way. Thus, in this approach, both energy and momentum are conserved during virtual processes.

[1172] Feynman also, at this time, developed an alternative formulation of quantum mechanics to add to the pair of formulations produced by Schrödinger and Heisenberg. He defined the probability amplitude as a functional integral over possible classical histories (paths).

Probability amplitudes were normally associated with the likelihood of a particle or a system arriving at a certain place, or a certain state (momentum, or orbital, etc.) at a certain time. Feynman associated the probability amplitude with the entire motion of a particle (or, more generally, a system of quantum particles and/or quantum fields) along a path. He stated the central principle of his formulation of quantum mechanics: *The probability of an event which can happen in several different ways is the absolute square of the sum of complex contributions, one from each alternative history path.*

These complex amplitudes were written in terms of the classical action. He showed how to calculate the action for each path as a certain integral, and how to integrate over all paths via a functional (**Wiener**-type) *integral*. He established that this approach was mathematically equivalent to the standard Schrödinger wave functions. Feynman's path-integral view of nature, his vision of a "sum of histories" was, in essence, the principle of least action reborn.

But since we know that these processes are kinematically impossible, something must give; and indeed, it develops that in the Feynman theory, Einstein's relation between mass, energy and momentum, namely $p_\mu p^\mu = m^2c^4$, *is violated for virtual states ("off-shell particles").*

The new QED was developed, almost simultaneously, by ***Feynman***, ***Tomonaga*** and ***Schwinger***, *who jointly received the* 1965 *Nobel prize for this feat. Using the revamped theory, Feynman could account for all the order-*α *corrections*[1173] *to QED known then — including the Lamb shift* and *anomalous magnetic moment.*

In accord with second (field) quantization, the electron field (now not a wavefunction but an operator in Hilbert space) obeys the covariant Dirac equation, while the photon quantum field (i.e. the EM 4-vector potential, or alternatively, the EM fields themselves), also an operator, obeys Maxwell's equations – with source charge-density and current density operators that are bilinear in the electron's Dirac field. However, the grand "wave-functional" of the entire system – which depends on a (mutually commuting) subset of these fields as well as upon time – still obeys a Schrödinger equation (SE).

We now review the history of attempts to render the SE Lorentz invariant, since it is these attempts that led to Dirac's relativistic electron equation, field quantization, QED, and indeed the entire framework of QFT's and the modern theory of particles and fields. At the time when Schrödinger developed his nonrelativistic wave equation, he also proposed a charge-flow form of it. He defined a probability currents density: corresponding to the time-dependent SE

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V(\boldsymbol{r})\psi,$$

there is a complex conjugate SE equation

$$-i\hbar\frac{\partial\psi^*}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi^* + V(\boldsymbol{r})\psi^*,$$

assuming the potential to be real. Multiplying the first equation by ψ^*, *the second by* ψ, *subtracting one from the other and integrating over some arbitrary* volume, *yields:*

$$\begin{aligned} i\hbar\frac{\partial}{\partial t}\int \psi^*\psi\, d\boldsymbol{r} &= -\frac{\hbar^2}{2m}\int (\psi^*\nabla^2\psi - \psi\nabla^2\psi^*)\, d\boldsymbol{r} \\ &= -\frac{\hbar^2}{2m}\int \ \mathrm{div}\ [\psi^*\nabla\psi - \psi\nabla\psi^*]\, d\boldsymbol{r}. \end{aligned}$$

[1173] α is the dimensionless QED coupling constant, $\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137}$.

We now define the probability current density $\boldsymbol{j} = \frac{\hbar}{2mi}(\psi^* \nabla\psi - \psi\nabla\psi^*)$ *and note that*

$$\psi^*\psi = |\psi|^2 = \rho = \text{ probability density.}$$

With these definitions, we obtain $\int \left(\frac{\partial\rho}{\partial t} + \text{ div } \boldsymbol{j}\right) d\boldsymbol{r} = 0$, *which leads to the Lorentz-invariant law of local conservation of probability density:*

$$\frac{\partial\rho}{\partial t} + \text{ div } \boldsymbol{j} = 0$$

If this equation were to be multiplied by the charge e *of a particle, it would be describing a relationship between an electric charge density* $(e\rho)$ *and an electric current density,* $e\boldsymbol{j}$. *The probability interpretation tells us only that a quantum-mechanical charged particle can be thought of as a smeared-out charge distribution.*

Such a picture is consistent with the uncertainty principle. For a plane wave $\psi = Ae^{i\boldsymbol{k}\cdot\boldsymbol{r}}$, *we can calculate* $\boldsymbol{j}$ *using the above definition:*

$$\boldsymbol{j} = \frac{\hbar}{2mi} \times 2i\boldsymbol{k}|A|^2 = \frac{\hbar\boldsymbol{k}}{m}|A|^2 = \boldsymbol{V}_g|A|^2$$

where $\boldsymbol{V}_g$ *is the group velocity of a corresponding wave-packet. On the other hand,* $\rho = \psi\psi^* = |A|^2$, *and so* $\boldsymbol{j} = \boldsymbol{V}_g\rho$ *for any plane-wave solution – as expected in a classical flowing, charged medium. Note, however, that the SE is not Lorentz covariant, nor do* $(\rho, \boldsymbol{j})$ *transform as a four-vector.*

The Dirac theory, unlike the SE, is relativistically covariant. It describes the behavior of electrons and also their positron antiparticles, either free or under the stimulus of a given externally applied electromagnetic field. The wave-function ψ *had to be replaced by an operator* ψ *in Hilbert space (the quantum Dirac field), because otherwise paradoxes plagued the theory — e.g., an extended EM field could cause probabilities to flow back in time, or to not add up to unity; electrons can have arbitrarily-large negative energies and thus be unstable, etc. These problems were solved by quantizing the Dirac field, which now described both electrons and positrons. But the EM fields in the Dirac theory were still classical, external fields. However, it is well known that electrons are themselves the principal contributors to such fields.*

In particular, an electron which undergoes acceleration in an electromagnetic field radiates real photons by virtue of this acceleration, and the EM radiation of excited atomic systems is brought about by the dynamics of the electronic charge cloud.

Thus if we could find, as a complement to the Dirac quantized-field equation, an equation which describes the behavior of the quantized (operator) *electromagnetic fields under the stimulus of given quantum electronic motions, these two sets of equations should provide a quite broad description of the behavior of electrons and radiation*[1174].

The natural way is to begin with Maxwell's equations in terms of the 4-vector potential A_μ *in a Lorentz gauge,*

$$\nabla^2 A_\mu - \frac{1}{c^2}\frac{\partial^2 A_\mu}{\partial t^2} = -J_\mu. \tag{1}$$

In this expression the 4-current J_μ *— which is the source of the field — must be evaluated in terms of the Dirac field which encodes the quantum motions of relativistic electrons (and positrons). Classically, the current density* $\boldsymbol{j}$ *due to a charge distribution* ρ *moving at velocity* v *is*

$$\boldsymbol{j} = \rho \boldsymbol{v}. \tag{2}$$

However, in a quantum field theory, the charge of an electron is "smeared out" into a operator–valued density $e\psi^*\psi$, *while its current density is similarly smeared into the operator* $J_i = \psi^* v_i \psi$.

Here ψ *is the Dirac field operator and* v_i, $i = 1, 2, 3$ *are constant matrices acting on the Dirac index* $a = 1, 2, 3, 4$.

Thus the proper expressions for the charge and current density operators are

$$\begin{aligned} &(a) \qquad \rho = e\psi^*\psi \\ &(b) \qquad J_i = e\psi^* v_i \psi. \end{aligned} \tag{3}$$

In terms of the four-component electron field of the second-quantized Dirac theory, $\psi_a^*(x)$ *is no longer the complex conjugate of a number, but rather the hermitian adjoint of an operator field; and (3) becomes*

$$\frac{1}{c} J_0 = \rho = e\psi^*\psi = e(\psi_1^*\psi_1 + \psi_2^*\psi_2 + \psi_3^*\psi_3 + \psi_4^*\psi_4) \tag{4}$$

and

$$J^i = ec\psi^*\alpha_i\psi, \qquad i = 1, 2, 3 \tag{5}$$

where we have used the relation $v_i = c\alpha_i$ *from Dirac theory [note that* (J_0, J^i) *are a contravariant four-current in the Dirac theory]. Inserting these into Eq.*

[1174] A simple thought-experiment, devised by Heisenberg in the 1930's, demonstrates that the *electric* and *magnetic* fields must obey an uncertainty (and hence complementarity) principle analogous to that obeyed by *position* and *momentum*. Thus, the EM fields *must* be quantized!

(1) and appending the Dirac field equations themselves, we obtain the operator field equations of quantum electrodynamics (Heisenberg representation):

$$\begin{aligned}
&(a) \qquad \left[c\boldsymbol{\alpha}\cdot\left(\frac{\hbar}{i}\boldsymbol{\nabla}-e\boldsymbol{A}\right)+\beta m_0c^2+e\phi\right]\psi=-\frac{\hbar}{i}\frac{\partial\psi}{\partial t}\\
&(b) \qquad \nabla^2\boldsymbol{A}-\frac{1}{c^2}\frac{\partial^2\boldsymbol{A}}{\partial t^2}=-ce\psi^*\boldsymbol{\alpha}\psi \qquad (6)\\
&(c) \qquad \nabla^2\phi-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}=-e\psi^*\psi.
\end{aligned}$$

In the above equations $m_0 = m_e$ is the electron rest–mass; $\{\beta;\alpha_i\}$ (also denoted $\{\gamma^0;\gamma^0\gamma^i\}$, respectively) are the Dirac (4×4) matrices, and ψ is the 4-component Dirac-spinor electron field operator. Equations (6) must be augmented by appropriate canonical commutation relations[1175] *(CCR) between ψ and ψ^* and between $\boldsymbol{A}$ and $\dot{\boldsymbol{A}}$. These CCR establish the relevant* particle statistics *(Fermi-Dirac for electrons/positrons; Bose-Einstein for photons). They also embody the* uncertainty principle *obeyed by the quantized Dirac and Maxwell fields.*

Unlike in 1^{st}-quantized quantum mechanics – in which time is a numerical label but spatial positions are time-dependent Heisenberg operators – the spacetime coordinate $(\boldsymbol{x},\boldsymbol{t})$ upon which all the quantum fields in (6) depend, are all *numerical, classical labels – no longer operators. Because of the nonlinearity and mathematical complexity of Eqs. (6), coupled with the operator nature of the fields and the commutation and anti-commutation relations obeyed by them, it is not possible to write down exact solutions even for simple cases. And in any event,* particular *solutions are irrelevant in solving quantum-mechanical operatorial (as opposed to Schrödinger) equations of*

[1175] EM-field CCR in Coulomb gauge:

$$A_j(x)\dot{A}_k(y)-\dot{A}_k(y)A_j(x)=\hbar\, i\,\delta^{\perp}_{jk}(\boldsymbol{x}-\boldsymbol{y})\ ,$$

$$\Psi_a(x)\Psi^*_b(y)-\Psi^*_b(y)\Psi_a(x)=\delta_{ab}\,\delta(\boldsymbol{x}-\boldsymbol{y})\ ,$$

when $x^0=y^0$. Here j,k are spatial indices; a,b Dirac indices; δ_{ab} the Kronecker delta; $\delta(\boldsymbol{x})$ the 3D Dirac delta function; and

$$\delta^{\perp}_{jk}(\boldsymbol{x})=\left(\delta_{jk}-\frac{\nabla_j\,\nabla_k}{\boldsymbol{\nabla}^2}\,\delta(\boldsymbol{x})\right)$$

$$\frac{1}{\boldsymbol{\nabla}^2}\delta(\boldsymbol{x})=\frac{1}{4\pi|\boldsymbol{x}|}$$

motion such as (6). Instead, it has so far proved necessary to solve these equations by techniques involving successive approximations *(usually some variant of perturbation theory). And even this involves quite formidable mathematics.*

An electromagnetic (EM) field (classical or quantum) may produce virtual electron-positron pairs as a consequence of quantum effects. This means that the dynamics of an effective classical EM field contain quantum corrections *to the Maxwell equations, even in empty space.*

One may define an effective Lagrangian *density* $\mathcal{L}_{\text{eff}} = \mathcal{L}_0 + \delta\mathcal{L}$, *where* $\mathcal{L}_0$ *includes the classical field-quadratic terms while* $\delta\mathcal{L}$ *includes the quantum corrections; the latter can be expanded perturbatively as a joint Taylor expansion in* $\alpha = \frac{1}{4\pi\epsilon_0}\frac{e^2}{\hbar c}$ *(fine-structure constant),* eE *and* eB.

Heisenberg *and* ***H. Euler*** *have derived (1936) an analytical approximation to the nonlinear correction term – valid to all orders in* eE, eB *but only to zeroth order in* α *(at fixed* eE, eB*). To lowest (4th order) in the fields,* $\delta\mathcal{L}$ *has the form (in "absolute" units in which* $\hbar = c = 1$*):*

$$\delta\mathcal{L} = \frac{2\alpha^2}{45m_e^4}\left[(E^2 - B^2)^2 + 7(E\cdot B)^2\right]$$

Their approximation applies only for EM field wavelengths which are much longer than the electron's Compton wavelength[1176]*. The correction becomes non-negligible only at field strengths of the order of the* Schwinger critical *field,* $\frac{m_e^2c^3}{e\hbar} \sim 10^{16}\,\frac{\text{Volt}}{\text{cm}}$.

The latter quantity is named after ***Julian Schwinger****, since he reanalyzed (1951) the problem using his elegant functional method. The nonquadratic correction* $\delta\mathcal{L}$ *to the classical Lagrangian results, to lowest order in* α *and to all orders in* eE *and* eB, *from the virtual production and annihilation of a* single virtual electron-positron pair *in the external EM field.*

Three examples of observable physical effects which result from the Euler-Heisenberg correction are: 'light by light' scattering (the collision of two γ

[1176] The electron's (reduced) Compton wavelength

$$\frac{\lambda_{\text{compton}}}{2\pi} = \frac{\hbar}{m_e c},$$

has the interesting property that if an electron or positron is confined to a box having a size of this order or less, its zero-point motional energy suffices to create new e^+e^- pairs. Thus, measuring an electron's position to that accuracy renders not only the electron's *momentum* uncertain, but even the *number* of electrons!

*rays); The Delbrück effect (1933 – scattering of a photon by the Coulomb field of the nucleus); and photon splitting (into several longer-wavelength photons) in a strong magnetic field (**S. Adler**, 1971 – not yet confirmed empirically).*

The fundamental reason that QED engenders nonlinear corrections to the Maxwell empty-space equations is this: the coupled Maxwell-Dirac operatorial field equations (6) are themselves nonlinear. Even in the absence of physical ('on shell') electrons or positrons, the vacuum itself has a finite quantum amplitude to occasionally produce virtual $\{e^+, e^-\}$ *pairs for short periods of time (of order* $\frac{\hbar}{m_e c^2} \sim 10^{-21}$ *sec).*

Upon integrating out and averaging over these electronic vacuum fluctuations, the nonlinearity of the original Maxwell-Dirac equations is manifested as effective nonlinear corrections to the purely EM sector of the theory.

At the classical level, Maxwell's equations in vacuo thus receive nonlinear corrections. At the level of quantized EM fields, the nonlinearities result in effects such as the three mentioned above.

We note in passing that the nonlinear modifications to classical EM theory are not all quantum in origin. The minimal framework needed to encompass both Maxwell's theory and Einstein's GTR is that of the Maxwell-Einstein coupled field equations, which are nonlinear and yet completely classical in origin.

Feynman's Formulation of Wave Mechanics: Path Integrals[1177]

Some 20 years after it was first discovered, Feynman proposed (1948) an alternative formulation of wave (quantum) mechanics. Although his formulation can be shown to be identical to Schrödinger's (1925) and Heisenberg's, it gave the theory a new, attractive physical picture. Furthermore, the QFT– sums over histories of field configurations – has proven very useful in finding non-perturbative (large field) effects, and in verifying the quantum versions of gauge symmetries in the standard model, of particle physics, and in Candidate Grand Unified Theories. The QFT version of Feynman's approach, also called path integrals*, has proven useful in taking the tentative first steps toward a theory of quantum gravity.*

If we are given the values of a wave-function at any possible spatial point x_a *at the time* t_a, *the SE in one dimension enables us to find the value of the wave function at a point* x_b *at another time* t_b *since by means of a differential equation we may proceed with infinitesimal increments to neighboring points in space and time.*

If we were interested in the corresponding classical *problem [that is — given the initial state of a system at* (x_a, t_a) *to find the path by which it reaches the point* (x_b, t_b)*], we would say that the system will proceed along a path for which the integral action* $\int_{t_a}^{t_b} L\,dt$ *will be a extremal. This is the classical principle of* least action*, and we note that the path of the particle is unequivocally determined.*

Feynman's formulation of quantum mechanics says that there is a contribution to the probability amplitude $\psi(x_b, t_b)$ *from all possible paths that can be drawn between* (x_a, t_a) *and* (x_b, t_b). *The contribution from each path is weighted by a factor proportional to* $\exp\left[\frac{i}{\hbar}\int_{t_a}^{t_b} L\,dt\right]$ *taken along the path*

1177 To dig deeper, see:

- Feynman, R.P. and A.R. Hibbs, *Quantum Mechanics and Path Integrals*, McGraw-Hill, 1965, 365 pp.
- Schulman, L.S., *Techniques and Applications of Path Integrals*, Wiley, 1981, 359 pp.

in question[1178]*. We can write the contribution* $\Delta\psi(x_b, t_b)$ *to* $\psi(x_b, t_b)$ *due to a particular path integral as*

$$\Delta\psi(x_b, t_b) = A\psi(x_a, t_a)e^{\frac{i}{\hbar}\int_{t_a}^{t_b} L\,dt},$$

where A *is some constant. The value of* $\psi(x_b, t_b)$ *is given then by*

$$\psi(x_b, t_b) = A\int_{\substack{\text{all}\\ \text{paths}}} \psi(x_a, t_a)\exp\left[\frac{i}{\hbar}\int_{t_a}^{t_b} L\,dt\right] D[x(t)].$$

This integral is symbolic: it is a formal expression to be taken over all possible paths, including different initial positions x_a *at the fixed time* t_a*. It cannot be carried out in the conventional manner until we find a method for characterizing the paths. This is usually done by discretizing time* t*, then carefully taking the limit* $\Delta t \to 0$*.*

For any given x_a *value in the above path integral, the particle moves along path which – when the semiclassical (WKBJ) approximation holds – do not much differ from the classical path given by the principle of least action between* $x(t_a) = x_a$ *and* $x(t_b) = x_b$*. The weighting function, an imaginary exponential function, is generally an oscillating functional of its argument, the trajectory history* $\{x(t)\}$*.*

If the action integral is large compared to $\hbar$*, the net effect of most of the paths is to cancel each other out. The configurations (histories) that will contribute most are the ones in whose vicinity all exponentials will contribute in approximately the same phase and will therefore add. But this is the case*

[1178] The appearance of the real action in the standard phase factor $e^{iS/\hbar}$ of the path integral (which stems from the unitarity of the SE evolution operator) ensures that the stationary – phase (i.e. extremal action) path, or paths – subject to the boundary conditions $x(t_a) = x_a$, $x(t_b) = x_b$ – is the strongest contributor to the probability of transition between initial and final states, if such a path exists.

Non-stationary – phase paths will also contribute to the probability of a quantum process, but with decreasing magnitude the more they deviate from the stationary path – due to the increasingly rapid fluctuations in $\exp\left[\frac{i}{\hbar}(S - S_{\text{stationary}})\right]$. This leads to *destructive interference*, unless $|S - S_{\text{stationary}}|$ is of order $\hbar$. In some cases, *complex* solutions of the action's Euler-Lagrange equations dominate the path integral (an infinite-dimensional version of saddle-point integration!). Such complex "classical solutions" are called *instantons*, and are important in path-integral evaluation of *quantum tunneling* problems – in both the 1^{st}–quantized and QFT versions of Feynman's formulation.

for classical motion — the path that contributes most is the path of extremal action.

Classical motion, therefore, is along that path for which the variations of the action integral are zero when the path is varied to neighboring paths. Extreme wave-mechanical properties are found from contributions along paths where the calculated deviations of the action integral from its classical – path value is $\gg \hbar$ in magnitude, or when there are several (or none) of those paths.

Feynman invented a new method for the quantization of classical systems: given a classical system described by a Lagrangian, which is a function of velocities and coordinates only, a description of a quantum mechanical version of the system may be written down directly, without working out a Hamiltonian. The Lagrangian method can easily be expressed relativistically, on account of the action function being a relativistic invariant.

Feynman then proceeded to derive the Schrödinger wave-equation using his action–based path integral. His argument is as follows:

The trajectory of a particle, moving in one dimension, can be specified by mean of a function $x(t)$. If a particle at an initial time t_a starts from the point x_a and arrives at a final point x_b at a later time t_b, we shall say that the particle goes from a to b.

In quantum mechanics we shall associate to such a transition an amplitude $G(a, b)$ to get from the spacetime point a to the spacetime point b. It will be the integral (over all the trajectories that go between the end points a and b), properly weighted, of the contributions from each.

This is to be contrasted with the situation in classical mechanics in which there is only one specific and particular trajectory which goes from a to b, the so-called classical trajectory. Thus in quantum mechanics we have to specify how each trajectory contributes to the total amplitude to go from a to b.

The phase of the contribution from a given path is the action S for that path in units of the quantum of action $\hbar$. That is, the probability $P(b, a)$ to go from point $x_a = x(t_a)$, at time t_a to the point $x_b = x(t_b)$, at time t_b is the squared magnitude $P(b, a) = |G(b, a)|^2$ of an amplitude $G(b, a)$ to go from a to b. This amplitude is the sum of the contributions $\psi[x(t)]$ from each path

$$G(b, a) = \sum_{\substack{\text{over all paths from} \\ a \text{ to } b}} \psi[x(t)],$$

where the contribution of each path has a phase proportional to the action S:

$$\psi[x(t)] = \text{const} \cdot \exp\left\{\frac{i}{\hbar} S[x(t)]\right\}.$$

The action is that for the corresponding classical system. The constant is chosen to properly normalize the probabilities.

Thus, we shall write the sum over all paths [which go through the specified points (x_i, t_i)*] as*

$$G(b,a) = \int_a^b e^{\frac{i}{\hbar}S(b,a)}\, Dx(t),$$

which we shall call a path integral.

In three dimensions the differential is written as

$$D[\boldsymbol{r}(t)] = Dx(t)\, Dy(t)\, Dz(t)$$

and the path integral for a particle in a field with potential $V(\boldsymbol{r}, t)$ *will have the form*

$$\int D[\boldsymbol{r}(t)] \exp\left\{\frac{i}{\hbar}\int_{t_a}^{t_b} dt \left[\frac{1}{2}m\dot{\boldsymbol{r}}^2 - V(\boldsymbol{r}, t)\right]\right\}.$$

A special case of interest arises when a particle goes between two points separated by an infinitesimal time interval $t_2 - t_1 = \epsilon = \frac{(t_b - t_a)}{N}$. *It then follows from the definition of the action integral that* $S = \int_t^{t+\epsilon} L[\dot{x}(t), x(t), t]\, dt \approx \epsilon L$, *correct to first order in* ϵ. *Consequently*

$$G(x_2, t_2; x_1, t_1) \approx \frac{1}{A}\exp\left[\frac{i\epsilon}{\hbar}L\left(\frac{x_2 - x_1}{\epsilon}, \frac{x_2 + x_1}{2}, \frac{t_2 + t_1}{2}\right)\right],$$

where A *is a (possibly complex) normalization factor.*

We know that if the above formal path integral is defined as a standard Riemann multiple integral

$$\int_{j=1}^{N-1} \pi d(x_j)\ ,$$

where $x_0 = x_a, \quad x_N = x_b$ *and* $x_j = x(t_j) \qquad t_0 = t_a + j\,\epsilon,$ *then recursion on N yields the equation*

$$\psi(x_2, t_2) = \int_{-\infty}^{\infty} G(x_2, t_2; x_1, t_1)\psi(x_1, t_1)\, dx_1$$

expressing the wave function at a time t_2 *in terms of the wave function at a time* t_1. *Using the above approximation for* G, *we have*

$$\psi(x, t+\epsilon) = \int_{-\infty}^{\infty} \frac{1}{A}\exp\left[\epsilon\frac{i}{\hbar}L\left(\frac{x-y}{2}, \frac{x+y}{2}, t\right)\right]\psi(y,t)\, dy$$

We shall now apply this to the special case of a particle moving in a potential $V(x,t)$ *in one dimension, for which* $L = \frac{1}{2}m\dot{x}^2 - V(x,t)$.

In this case the quantity $\frac{(x-y)^2}{\epsilon}$ *appears in the exponent due to the first (kinetic energy) term of L.*

Here ϵ *is assumed very small, so unless* y *is near* x *this factor oscillates rapidly and the integral over* y *will give a very small value, because of the smooth behavior of the other factors.*

For this reason we make the substitution $y = x + \eta$, *with the expectation that appreciable contributions to the integral will occur only for small* η *(of order* $\sqrt{\epsilon}$*). We obtain*

$$\psi(x,t+\epsilon) \approx \int_{-\infty}^{\infty} \frac{1}{A} \exp[\frac{im\eta^2}{2\hbar\epsilon}] \cdot \exp[-\frac{i\epsilon}{\hbar}V(x+\frac{\eta}{2},t)]\psi(x+\eta,t)\, d\eta$$

The phase of the first exponential changes by the order 1 *radian when* η *is of the order* $\sqrt{\frac{\epsilon\hbar}{m}}$, *so that most of the integral is contributed by values of* η *of this order. We may expand the LHS in a power series, keeping only terms of order* ϵ. *Expanding the l.h.s. to first order in* ϵ *and the r.h.s. to first order in* ϵ *and second order in* η, *we obtain*

$$\psi(x,t) + \epsilon\frac{\partial\psi}{\partial t} \approx \int_{-\infty}^{\infty} \frac{1}{A} e^{\frac{im\eta^2}{2\hbar\epsilon}} \left[1 - \frac{i\epsilon}{\hbar}V(x,t)\right] \left[\psi(x,t) + \eta\frac{\partial\psi}{\partial x} + \frac{1}{2}\eta^2\frac{\partial^2\psi}{\partial x^2}\right] d\eta$$

where the error in this equation tends to zero faster then order ϵ *as* $\epsilon \to 0$. *Comparing terms of the same order of* ϵ *on both sides, and using two known (Fresnel) integrals, we obtain* $A = \sqrt{\frac{2\pi i\hbar\epsilon}{m}}$ *and a differential equation for* ψ:

$$-\frac{\hbar}{i}\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x,t)\psi,$$

which is recognized as the ID Schrödinger equation *for a particle moving in a general potential.*

It is likewise possible to start *with this time–dependent Schrödinger equation (SE) and* derive *the path integral representation of the quantum amplitude, by evolving the solution to the SE over a succession of short time–steps, which one then allows to become infinitesimal.*

When using a Feynman path integral, one can never be sure that it converges – or even well-defined until it is worked out either exactly using Fresnel–type integrals, or (as happens far more often) as an asymptotic expansion[1179].

In contradistinction, the Wiener path integrals are free from this difficulty, since their convergence is secured[1180]*. Nevertheless, physicists have used Feynman's integrals widely and successfully.*

1179 A few examples of exact evaluation of some simple path integrals are instructive at this point. The simplest path integrals are those in which all the variables appear up to the second degree in an exponent. We shall call them *Gaussian integrals*. In quantum mechanics this corresponds to the case in which the action functional S involves the path $x(t)$ up to and including the second power.

Consider the one-dimensional Lagrangian of the form:

$$L(\dot{x}, x, t) = a(t)\dot{x}^2 + b(t)x\dot{x} + c(t)x^2.$$

We wish to determine

$$G(b, a) = \int_a^b Dx(t) \exp\left[\frac{i}{\hbar}\int_{t_1}^{t_2} L(\dot{x}, x, t)\, dt\right]$$

For a free particle $V = 0$, $L = \frac{1}{2}m\dot{x}^2$ and the path integral reduces exactly to

$$\left[\frac{m}{2\pi i\hbar(t_b - t_a)}\right]^{1/2} \exp\left\{\frac{im(x_b - x_a)^2}{2\hbar(t_b - t_a)}\right\}$$

which is identical with the Green's function of the Schrödinger equation for a free particle

$$\frac{\partial\psi_0}{\partial t} = \frac{i\hbar}{2m}\frac{\partial^2\psi_0}{\partial x^2}.$$

For a *harmonic oscillator* the Lagrangian is $L = \frac{1}{2}m\dot{x}^2 - \frac{m\omega^2}{2}x^2$, and the result is

$$G(b, a) = \left[\frac{m\omega}{2\pi i\hbar\sin\omega T}\right]^{1/2} \exp\left\{\frac{im\omega}{2\hbar\sin\omega T}\left[(x_a + x_b)^2\cos\omega T - 2x_a x_b\right]\right\},$$

where $T = t_b - t_a$. It coincide, of course, with the result obtained through more traditional means.

1180 Wiener path integrals are encountered in *classical* stochastic processes and are equivalent to the *Ito calculus*. Their convergence in secured by rigorous theorems; they are integral over function–spaces of possible trajectories, with the integrand being a real functional (rather then a complex phase as in the quantum case).

The pioneering work of Feynman and his successors brought to light the close mathematical and physical analogies between quantum mechanics (QM) on the one hand, and (classical or quantum) equilibrium statistical mechanics on the other. Specifically, the sum over histories (or 'Path Integral') of a quantum system is directly analogous to the partition sum in statistical mechanics.

A key difference is that in QM, one sums a complex phase, $e^{iS/\hbar}$, *where* S *is the classical action for a given history of the system and* $\hbar$ *is Planck's (reduced) constant; whereas in statistical mechanics, the partition sum is* $Z = \sum_{\text{microscopic states}} e^{-E/kT}$, *a sum over real numbers, with* E *the microscopic state energy.*

Mathematically, however, it is rather straightforward to relate the two via a suitable analytical continuation. (This procedure is technically known as a Wick rotation; *it involves an analytical continuation of real physical time into imaginary time).*

This transformation is similar to that used in STR to recast real, pseudo–euclidean Minkowski space as a four dimensional euclidean space with imaginary time; in fact, the two are the same in the case of the path integral of relativistic QFT's. In consequence, versions of QM (with or without quantized fields) utilizing statistical-mechanics like partition sums are often referred to as euclidean time *versions.*

In contemporary physics, when a quantum system (whether in a known state or a statistical-mechanical ensemble), or a classical statistical-mechanics system, has an infinity of degrees of freedom, its mathematical description usually belongs to the class of Quantum Field Theories (QFTs, for short). (The most celebrated, and empirically successful, example of a QFT is Quantum Electrodynamics, the theory of electrons, positrons and quantized electromagnetic fields.)

By employing the above-mentioned mathematical mapping – as well as various versions of Feynman's mapping between a Schrödinger equation and a path integral or *partition sum – great advances were made in the understanding of physical phenomena in condensed matter physics (especially critical phenomena) using the machinery of QFT and vice versa.*

In a zero-temperature, zero chemical potential QFT, the basic unperturbed state is the vacuum — which, far from being nothing at all, is the seat of all possible quantum processes.

These processes are virtual, *however — i.e. do not result in observable changes — except in two cases: when a suitable external disturbance ('source') is applied, or when the initial vacuum state is metastable (as is thought to* *have occurred in the evolution of the Universe, a fraction of a second after the Big Bang).*

The (complex) quantity corresponding to the partition sum in this case is the so-called vacuum persistence amplitude, expressed as the following path integral:

$$W\{J\} = \int [d\phi] e^{i(S\{\phi\} + \int d^4x J(x)\phi(x))/\hbar}$$

This definition is a functional version of the Fourier transform. In it, $J(x)$ *represents an external 'source' distribution (which can be viewed as either a current or field) at the spacetime point* x*; the classical path is* $\{\phi(x)\}$*, with the spatial distribution* $\phi(x) = \phi(\boldsymbol{x}, t)$ *for all* $\boldsymbol{x}$ *at given* t*, being one point of this path;* $S\{\phi\}$ *is the classical action functional of the vacuum for a particular field configuration; and* $\int d^4x J(x)\phi(x)$ *is the additional action term reflecting the interaction of the vacuum with the given external source.* $W\{J\}$ *and* $S\{\phi\}$ *are functionals, i.e. 'functions of functions', while the notation* $\int [d\phi]$ *implies functional integration over all possible spacetime configurations of the field* ϕ*, while the symbol* $\int d^4x$ *represents ordinary Riemann integration over four dimensional spacetime.*

QFTs are also routinely studied in other dimensions; in lower dimensional space times because the theory is easier to solve there (or because certain condensed matter systems, and a string "world-sheet" in string theory, do "live" in lower dimensional spaces); and in higher dimensions because some theories of particle physics envision our familiar four dimensional spacetime as having arisen from 'compactification' of more fundamental, higher-dimensional spaces.

Indeed, there are condensed matter mechanisms that effectively take place in two spatial dimensions — such as degenerate electron systems in two-dimensional layers; solid-on-solid interfaces; quantum wells; et cetera – or even one dimensional (quantum wires).

The current $J(x)$ *need not be an actual external source; it is often used as an auxiliary field.* $W\{J\}$ *is then merely a generating function(al); that is, it encodes all possible physical processes in the given vacuum, through its all-order functional (Frechet) derivatives at* $J(x) = 0$*.*

In addition, the persistence amplitude $W\{J\}$ *is mapped into* $Z(H)$*, the partition sum of a statistical mechanics system in the presence of an external field.*

Mathematically, there are only two differences between the two cases:

(A) H *is a uniform field, while* $J(x)$ *has an arbitrary spacetime dependence; (although variable external fields are sometimes treated in classical statistical mechanics, too).*

(B) W and Z are related by analytical continuation.

Just as W encodes all possible physical processes in a given vacuum (and even decays from metastable to more stable vacua!), so Z (as a functional of suitable external sources) contains information on all possible responses of a condensed matter system to external macroscopic stimuli – even the onset of phase transitions and the nucleation of more stable matter phases.

Feynman and Molecular Nanotechnology (1959–2008)

Nanotechnology[1181] *is a term used to describe a wide array of approaches to engineering tiny*[1182] *machines. Everything from devising microscopes to resolve atomic-scale distances or displacements, to envisioning molecular robots that could swim through our bloodstream and fight disease, falls within its purview. Physicists, chemists, material scientists, molecular biologists, mathematicians, engineers and programmers around the world are working in the fields collectively called nanotechnology.*

People working in this field today are also be divided into:

- *Those working from the "bottom up", mostly chemists attempting to create structures by connecting molecules.*
- *Those working from the "top down" — taking existing devices, such as transistors, and making them smaller. Top-down or mechanical nanotechnology — which itself involves many disciplines – chemistry, optics, material science, charged–particle beams electrical engineering, Computer Assisted Design (CAD) etc. — will have the greatest impact on our life in the near future.*

Biology already involves both modalities of nanotechnology, but works already! Photosynthesis, after all, involves a molecular scale solar energy collection device, while enzymes are essentially nanosize factories; microbial flagella and red–blood–cell hemoglobin molecules are intricate new–machines, actuated and powered by difference of solution pH (acidity) values; the various electrochemical signals and clocks regulating life's processes involve molecular kinetics of picogram samples. The challenge for nanotechnologists is to

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1181 *Nano* from the Greek word for *dwarf*;

$$\begin{aligned}1 \text{ nanometer } &= 10^{-9}m = 10^{-6}mm = 10^{-3} \text{ micron}\\ &= 1 \text{ millimicron } = 10\mathring{A} \simeq 3-5 \text{ atoms}\end{aligned}$$

$$1 \text{ virus size} \simeq 100 \text{ nanometers } \sim 0.1 \text{ micron}$$

1182 The characteristic dimensions in nanotechnology are less than about

$$1000 \text{ nanometers } = 1 \text{ micron } = 10^{-3}mm.$$

The *human eye* can resolve about $0.2mm$ and an *electron microscope* can resolve about $10\mathring{A} = 10^{-6}mm$.

learn to design and control such processes. Once they do, huge advances in everything from microelectronics *to chemical engineering will be possible.*

Manufactured products are made from atoms. The properties of those products depend on how those atoms are arranged. If we rearrange the atoms in coal we can make diamond; if we rearrange the atoms in sand (and add a few other trace elements) we can make computer chips. If we rearrange the atoms in dirt, water and air we can make potatoes. The atoms and molecules making up plants and minerals, could be reshuffled to render optical fibers, plastics and audio tapes; and so on.

Today's manufacturing methods are very crude at the molecular level. Casting, grinding, milling electroplating and chemical reactions, and even plasma, Uv, X-ray or acid etching and lithography, all move atoms in great thundering statistical herds.

In the future we will be able to snap together the fundamental building blocks of nature easily, inexpensively and in almost any arrangement that we desire. This will be essential if we are to continue the revolution in computer hardware beyond the first few decades of the 21^{st} *century, and will also let us fabricate an entire new generation of products that are cleaner, stronger, lighter, and more precise.*

For example, continued improvements in lithography have resulted in integrated–circuit strip widths that are several tents of nanometers. Submicron lithography is clearly very valuable, but it is equally clear that lithography will not let us build semiconductor devices in which individual dopant atoms are located at pre-specified lattice sites. Many of the exponentially improving trends in computer hardware capability have remained steady for the last 50 years. There is fairly widespread confidence that these trends (in memory, speed, power–consumption reduction, logic–gate density) are likely to continue for at least another ten years, but then lithography starts to reach its fundamental limits.

If we are to continue these trends we will have to develop a new "post-lithographic" manufacturing technology, which will let us inexpensively build computer systems with mole quantities of logic elements that are molecular in both size and precision — and are interconnected in complex and highly idiosyncratic patterns. Nanotechnology will let us do this, it seems.

It should allow engineers to:

- *Get essentially every atom in the right place.*
- *Produce almost any structure which is consistent with the laws of physics and chemistry and that can be specified in atomic detail.*

- *Incur manufacturing costs not greatly exceeding the cost of the required raw materials and energy.*

There are two main automation concepts commonly associated with nanotechnology:

- *Positional control.*
- *Self replication.*

Indeed, attainment of the above three desiderata seems difficult without using some form of atomic–scale positional control (to get the right molecular parts in the right places) and some form of self replication (to keep the costs down).

The need for positional control implies an interest in molecular robotics, *e.g., robotic devices that involve both sensor and actuator components and are molecular both in size and in terms of spatiotemporal and spectral precision. These molecular–scale positional devices are likely to resemble very small versions of their everyday macroscopic counterparts. Positional control is frequently used in normal macroscopic manufacturing today, and provides tremendous advantages. The idea of sensing* manipulating individual atoms and molecules *is still new; yet a rudimentary version of it already exists in STM's (scanning tunneling microscopes) and AFM's (atomic force microscopes). And devices have been built which can control even* electrons *and* photons *one particle at a time (e.g. in carbon nanotubes, quantum dots, few-qbits quantum computers, or RF–cavity QED experiments).*

We need to apply at the molecular scale the concept that has demonstrated its effectiveness at the macroscopic scale: making parts go where we want by putting *them where we want!*

The requirement for low cost creates an interest in self–replicating manufacturing systems, *studied by* **von Neumann** *in the 1940's. Such systems are able both to make copies of themselves and to manufacture useful products. If we can design and build one such system, the manufacturing costs for more such systems and the products they make (assuming they can make copies of themselves in some reasonably inexpensive environment) will be very low.*

Timeline

1959 The science of building small was first introduced by **Richard P. Feynman**. At that time, most scientists were thinking big — about interplanetary spacecraft and ever larger telescopes and particle accelerators – to probe both the cosmos and the subnuclear. Feynman awakened them to the possibilities of controlling single molecules, or even atoms and electrons, and creating nanoscopic machines with them.

1980 **Heinrich Rohrer** and **Gerd Binnig** (Switzerland and Germany) invented the *scanning tunneling microscope* (STM) which can produce images of individual atoms on the surface of a conducting solid material.

With a magnification factor of 10^8 it can resolve a distance of $10^{-2}\mathring{A} = 10^{-3}$ nanometer. This device works by holding a fine conducting probe to the surface of a sample. The probe's tip tapers down to a single atom. As electrons tunnel between the metallic sample and the probe, the probe's raster–scan movement, actuated electronically via piezoelectric motors, yields a contour map of the surface[1183].

1990 **Don Eigler** used an STM and AFM (Atomic Force Microscope) at IBM's Zurich Research Laboratory to reposition 35 individual Xenon atoms on a nickel surface at temperature $4K^\circ$ thus producing the world's smallest graffiti — the initials `IBM` spelled out in atoms. The STM was then used to image the result.

1995 • **Nadrian Seeman** (USA) built cubes (7 nanometers across) and more complex structures, out of synthetic DNA,

[1183] The tunneling current is an extremely short-range and sensitive function of the tip's distance to the surface, so it is used as the position–sensing element in a feedback loop to control the tip's variable vertical displacement as it scans. The electronic control circuit, maintaining constant current, causes the probe's tip to faithfully follow the sample "atomic terrain" — producing a digital false-color contour map, representing the electron density at the metallic surface; this density is determined by the interaction of conduction orbitals and electronic wave-functions pinned to surface impurity atoms.

attempting to create building blocks of molecular mechanical devices and super-resistant 'smart' molecules.

• Engineers at Cornell University (USA) built a *nanoguitar*: a guitar 10 microns long (0.01 mm), about as big as a human white blood cell — the perfect size for a bacterial rock star. Each of its 6 silicon strings is 100 *atoms wide*. It cannot be seen without an electron microscope — let alone strummed.

1996 • An IBM team, led by **James K. Gimzewski**, built the world smallest *abacus*, each bead having a diameter of less than 1 nanometer. The finger used to move each bead is the ultrafine tip of an AFM.

• **George Whitesides**, a chemist at Harvard (USA), patented computer-chip circuits just 30 nanometer wide. His circuits could give a single chip the ability to perform at speeds *exceeding* 1 *teraflop* (10^{12} floating point operations per second).

• Chemist **James Tour** at the University of Southern California (USA) and his team created the first *quantum wire* — a single molecular chain that completed a circuit between a gold leaf surface and the tip of an STM. They were testing a *molecular transistor*.

• **Richard E. Smalley** discovered "Buckminister Fullerenes" (named after the architect who invented the geodesic dome). These soccer – ball shaped pure carbon molecules, dubbed "*buckyballs*", and their cylindrical quantum–wire counterparts named "*nanotubes*" (essentially tightly–rolled graphite sheets with different discrete–valued helical pitches) are likely to be the strongest (highest tensile strengths) substances in existence. Nanotubes are created by vaporizing carbon with a laser and then letting it reassemble in an inert gas such as Helium. Aside from creating super-strong polymers that could replace the graphite used in everything from tennis racquets to airplanes, nanotubes could be used as circuits elements in the nanoelectronic devices of the future. It is foreseen that aggregates of *nanometer sized solar cells* could be built to provide the world's energy needs in the year 2050.

1997 The NASA – Ames nanotechnology group modeled *molecular gears* that could be powered by a laser. The gears, which exist only in computer designs (but are physically possible) would rotate at 100 *billion turns per second*. Because the devices lofted into space must be light, consume very little power and be immune to cosmic radiation, nanoelectronics and nanomachinery may be vital to future NASA programs.

1999 Intel's Pentium processor already has parts measuring just

$$350 \text{ nanometers } \cong \frac{1}{2850} \text{ mm}$$

Images of Time[1184]

I. Measures of physical time[1185] *(see Table 5.28)*

Time is one of the deepest mysteries known to man. No one can say exactly what it is. Yet the ability to measure time makes man's way of life possible.

[1184] For further reading, see:

- Eddington, A., *The Nature of the Physical World*, 1928.
- Jeans, J., *The Universe Around Us*, 1929.
- Weinberg, S., *Dreams of a Final Theory*, Vintage, 1993, 260 pp.
- Whitrow, G.J., *The Natural Philosophy of Time*, Oxford University Press, 1990, 399 pp.

[1185] This English word comes from the root *ti*, to stretch. The early word for everyday *time* was tide. As tide took on its more limited application to the ebb and flow of the oceans, back came *time* into the more general sense. *Time* is not directly related to the Latin *tempus*. The Greek word for time was *chronos*. The Latin *Aeon* evolved from the Greek *aion* = age.

Many of his activities involve groups of people acting together in the same place and at the same time. People could not do this if they did not all measure time in the same way. Thus, time is not an article of faith but a datum of observation and experience. Indeed, this utilitarian approach is found already in the opening chapter of the Hebrew Bible:

> *"And God said, let there be lights in the firmament of the heaven to divide the day from the night; and let them be for signs, and for seasons, and for days, and for years" (Genesis* **1**, *14).*

Indeed, the principle of measuring physical, external time by means of successive phenomena[1186]*, recurring at regular intervals, has not changed through the entire history of technology, up to the present day.*

The most striking astronomical phenomenon which rigorously fulfills this condition — the apparent daily revolution of the celestial sphere caused by the rotation of the earth — has from remotest antiquity been employed as a measure of time.

The problem of determining the exact time at any moment is practically identical with that of determining the apparent position of any known point on the celestial sphere with regard to one of the fixed (imaginary) great circles appertaining to the observer's station: the meridian or the horizon. The point selected is either the sun or one of the (so–called "fixed") stars.

A sequence of times thus determined serves to calibrate the rate of the clock, chronometer, or watch employed and also to estimate its error.

In 1884, World-time was standardized in terms of the mean solar day*. Thus the* mean solar second*, representing the basic unit of time, was defined as* $\left(\frac{1}{60}\right)\left(\frac{1}{60}\right)\left(\frac{1}{24}\right)$ *of a mean solar day. Time that is referenced to the rotation of the earth about its axis is called* universal time *(UT). It is defined as the mean solar time of the Greenwich meridian and is reckoned on a twenty-four hour basis starting at midnight. [In practice, a telescope is pointed at the local zenith. The time intervals between successive passages of a given distant galaxy through the center of the telescope field of view, are averaged over one year to yield one unit (1 day) of UT.]*

Another measure of time is the time interval between two successive north to south earth crossings of the plane of motion of Jupiter *about the sun. [The*

1186 The activities of many plants and animals are timed to the cycle of day and night. These natural rhythms are called *circadian rhythms*. The most obvious example is the sleep cycle.

Many plants and animals are sensitive to other natural time cycles: e.g., some sea animals time their activities to the changing tides.

earth crosses that plane twice per orbit, but only once from north to south.] This is known as ephemeris time (ET) and the unit is one year.

The atomic clock was developed at the National Bureau of Standards, Washington, D.C. in 1948 *under the direction of* **Willard Frank Libby** *(1908–1980, U.S.A.). The clock consists of a precise radio-frequency source which serves both to excite a population of atomic or molecular resonators in a cavity, and measure their frequency very accurately, by giving rise to slow beats that can easily be determined. Time measured in this way is denoted as atomic time (AT).*

With the aid of atomic clocks it has been possible to determine that the rotation of the earth about its axis is slowing down – its period lengthening by the amount of 1.8–3.2 *milliseconds/day. Thus, the availability of atomic clocks of unprecedented accuracy led scientists to abandon the rotation of the earth as the fundamental measure of time.*

Accordingly, in 1967*, the second was redefined as the time required by a cesium atom to emit* 9,192,631,770 *cycles of microwave electromagnetic radiation*[1187]*. It was later found that the earth's rotation slowed down since* 1967*, such that it was occasionally necessary to add a 'leap second' to the atomic-clock year, to let the earth catch up.*

The new standard has the distinct advantage of being "indestructible" and widely reproducible. In addition, being based upon a radio frequency, it has the advantage of being transmittable by radio to any place where there is a suitable receiver. Its accuracy is ± 1 *sec per* $100,000$ *years. Atomic clocks have both short–term–noise errors – which can be reduced (up to a point) by averaging over many cycles – and longer – term drifts.*

Modern UT – called UTC (Universal Time Coordinated) – is obtained by averaging AT's of a worldwide ensemble of atomic clocks. Apart from UT,

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[1187] One second = time required for a cesium-133 atom to undergo $9,192,631,770$ vibrations (wavelength ≈ 3 cm). The physics behind this device is as follows: A cesium atom has a heavy nucleus surrounded by a number of full electronic shells. But the outermost shell has only a single electron with its quantum spin (but no orbital) angular momentum. The nucleus, having an odd number of nucleons (protons and neutrons), also has spin and there are two possibilities: the electron may spin in the same sense as the nucleus, or in the opposite sense. By supplying RF electromagnetic waves at the above–quoted resonant frequency, the electron may absorb a photon and flip its spin relative to that of the cesium nucleus (a *hyperfine transition*). When the valence electron's spin eventually flips back again, the energy difference [corresponding to a transition between two hyperfine levels of the ground state] is reemitted as an RF photon, again at the above frequency (ca 9.2 GHz).

ET and AT there exists another measure of time: measuring the amount of tritium (^3H) that beta-decays into the helium isotope ^3He. If the tritium is kept at temperature about $10\,^\circ K$, the helium will diffuse out as it is formed. The tritium is weighed. When the mass has dropped to half its initial value, we say that the time is one unit of nuclear time *(NT), equal to about 12 mean solar years. A clock can be set that strikes each time the remaining mass is reduced by a factor of two.*

The basic difference between the various methods are: NT uses the weak nuclear interaction as its basic mechanism. AT uses an electromagnetic process, UT uses the earth's rotation and ET makes use of the law of universal gravitation.

II. Absolute time

Time has the peculiar quality which makes us feel intuitively that we understand it perfectly so long as we are not asked to explain what we mean by it[1188].

The first question to consider is the origin of the idea that time is a kind of linear progression *measured by clock and calendar. In modern civilization this conception of time so dominates our lives that is seems to be an inescapable necessity of thought.*

The first serious attempt to define *time is due to* **Aristotle** *(ca 340 BCE). This he achieved through the association of time with the* motion *of bodies. The time of which Aristotle speaks is* physical time *(or,* external *time), susceptible to quantification via* periodic *kinematic phenomena (such as the apparent daily revolution of the celestial sphere).*

Aristotle's notion, however, was devoid of mathematical *formulation, and totally innocent of the concept of acceleration. His concept of physical time remained in limbo until the* 17^{th} *century, when motion could be properly formulated with the aid of the differential calculus and Newtonian dynamics*[1189].

The invention of the first successful pendulum clock by **Huygens** *(1656), and the progressive increase in the precision, stability and portability of timekeeping that followed, fostered the image of a mechanical and predictable side of nature. The technological development of clocks disentangled time from*

[1188] The story is told of the Russian poet Samuel Marshak (1887–1964), that when he first visited London in 1912 and did not know English, he went up to a man in the street and asked: "*Please, what is time*?" The man looked up very surprised and replied: "*But that's a philosophical question. Why ask me*?"

We are all familiar with the feeling that time has a qualitative character about it. While the Newtonian view of time as a regular oscillation of some giant pendulum in the sky seems to work well in classical physics, everyday life is filled with far more subjective aspects to our perception of time than Newton ever conceived of. Quantum Mechanics does not really change this Newtonian concept; even the theories of Relativity (STR and GTR) retain it in their concept of (albeit locality – and observer dependent) *inertial frames*.

[1189] The Greeks sought *forms of nature* derived solely from *uniform motion* and simple spatial geometries, whereas classical Newtonian physics looked for *forces* and accelerated motion. The failure of Greek mathematics to recognize or measure accelerations is matched by a practical handicap: sundials, water-clocks and even sand-clocks are inadequate for precise measurement of short time intervals.

human events and helped to create belief in an independent world of science (The Clockwork Universe).

We can trace the birth of a truly scientific concept of time back to ***Newton****. The time incorporated in his equations was* absolute time*, measured by the apparent motion of the celestial bodies, as well as by terrestrial clocks. His absolute time was an ideal scale of time that made the laws of mechanics simplest, and its discrepancy with* apparent time *was attributed to such things as irregularities in the motions of the earth.*

Insofar as these motions were explained by Newton's mechanics, the entire procedure was vindicated. All of the physical universe was imbued with the same temporal element, as Newton said in his Principia*: "Absolute, true, and mathematical time of itself and from its own nature... flows equably without relation to anything external".*

For any given initial conditions, Newton's equations can in principle be integrated backwards and forwards in time to any desired point in the past or the future. Moreover, since his equations are (absent dissipative processes) invariant w.r.t. the transformation $t \to -t$*, they fail to decide which* direction of time *constitutes the actual past and future of any system governed by these equations, including the universe at large; i.e.* time is stripped of its sense of direction[1190]*. In this deterministic world past and future are preordained (or postordained!).*

This naive deterministic dream was sharpened by Laplace's conjecture*, according to which everything would be predictable if only we would know the positions and velocities of all particles in the universe at any single instant of time.*

Since Newton's mechanical equations have no intrinsic arrow of time, there is no reason to choose one direction in time in preference to the other. But things are worse still; there is a theorem due to ***Poincaré*** *which shows that, given a long enough interval of time, any (classical)* isolated*, non–dissipative system of masses (e.g., the universe itself) will return to its initial state to*

[1190] This *symmetrical time* could, for example, be highlighted with a hypothetical film of planetary motion taken by, say, the *Voyager 2* space probe, which was launched to explore the outer solar system in 1977; the film would be approximately consistent with Newton's laws of celestial mechanics whether we run the film forwards or backwards. The main "breaking" (violation) of this past–future symmetry would stem from tidal friction within the planets and their moons – which cause slow secular variations in their orbits and spin rates (such as the gradual lengthening of the solar day noted above).

any given level of accuracy (again, assuming no dissipative forces[1191]*). In fact, given an unlimited duration of time, it will do so an unlimited number of times ("Poincaré recurrences").*

Clearly, for many systems of interest to us there is such a large number of particles (e.g., atoms, molecules) present, that this recurrence time, is inconceivably larger than the present age of the universe ($\sim 10^{10}$ *years).*

*Nonetheless, these endless almost cyclic recurrences undermine the essential notion of time's arrow, and negates the concept of evolution. The concept of 'Poincaré return'*or recurrence, *in spite of its limitations, has proven to be one of the most potent paradigms in the mind of theoretical physicists.*

The notion of space and time as absolute metaphysical entities (among his "categories") formed an important part of the philosophy of ***Immanuel Kant****, who came to the conclusion that time is one of the forms of our 'intuition'. By this he meant that time does not characterize external objects but has do to with subjective phenomena or processes in the brain.*

Consequently, Kant believed that the idea of scientific, linear time is an automatic consequence of the fact that we are rational creatures. (This line of thought later led some scientists to believe that the arrow of time, as manifested in the Second Law of Thermodynamics, is an illusion.)

Newtonian mechanics, as studied in the 18^{th} *century, was largely concerned with reversible systems – often periodic; even when irreversible, they could be wound up and prepared anew, and their motions repeated, at any time. Particularly notable was the proof of the stability of the solar system that was formulated by* ***Laplace****.*

The motion of heavenly bodies was now predictable. God, most learned people believed, had set the cosmic "clockwork" in motion at the beginning of time, and no further divine intervention was required. Indeed, it was by no means clear that there had been any beginning of time.

The 19^{th} *century introduced conflicting views on the nature of time: The advent of electromagnetism which culminated with the discovery of* ***Maxwell****'s field equations (1873), led to technological applications which heralded the end of local time, which depended on the accuracy of local solar and mechanical timekeeping, and the beginning of national and international time, a worldwide sense of 'now' and of the uniform flow from 'earlier' to 'later'.*

1191 Poincare's theorem applies even to dissipative effects, such as friction, provided they can be described as reversible *classical* processes at the microscopic level as indeed they are, except effects such as weak nuclear radioactive decays.

Radio waves could be used to synchronize the time given by many clocks spread across the globe. Furthermore, electromagnetic devices contributed to the ever-increasing precision of clocks. But, as with Newton's equations of motion, Maxwell's field equations are time-symmetric and make no distinction between the past and the future that precludes the interchanging of their roles.

Yet it is clear that many electromagnetic phenomena, have *a temporal direction: (as do everyday dissipation – riddled phenomena and, most notably, life itself). One never sees light waves converging from a brightened room onto the filament of a lamp where they are all absorbed, nor is light emitted from our eyes and absorbed by the sun or other conventional source – although such so–called "advanced wave" pattern are just as much solutions of the electromagnetic equations as are the standard (and observed) "retarded–wave" patterns.*

This strange duality (time–symmetric equations, time–asymmetric solutions of only one type observed) was reinforced by yet another idea that took hold among growing intellectual circles: biological evolution. *As a result of the theories of* **Lyell** *(1830) and* **Darwin** *(1859), interest grew in systems that* evolve *through time. So, in contrast with the symmetric time of Newtonian physics, there was now also the unidirectional time of geological*[1192] *and biological systems.*

III. Time's arrow

While classical mechanics portrayed the universe as a perfect machine, thermodynamics *appears to imply that the machine is running down toward complete disorganization. [It seems to contradict Darwin's theory of evolution*

[1192] The question of just how many years are represented by rock layers in the stratigraphic time scale has been around for at least 2500 years: **Xenophanes of Colophon** (ca 530 BCE) was the first of the early philosophers to recognize the significance of fossils as remnants of former life on the sea bottom. **Herodotos** (ca 450 BCE) estimated the Nile delta to be many thousands of years old.

When modern geology started gathering momentum, through the works of **Hutton** and others, it was recognized that rocks are very old, and the earth much older. During the 19^{th} century the estimated age of the earth rose steadily from 75,000 (**Buffon**) to 75 million years (**Helmholtz**, **Kelvin**). Finally the discovery of *radioactivity* (1895) enabled geologists to make more accurate determinations and put the age of the earth at about 4.6 billion years.

which tends to show that life[1193] *has become more — not less — organized through time, as simple creatures evolved into more complex ones – though the work of Prigogine and others in the 20th century established that there is no contradiction.*

These gross violations of temporal symmetry ('Time's Arrow') that are apparent in the observable world were first analyzed by ***Boltzmann*** *(1878) who attempted to explain the asymmetry in terms of atomic and molecular behavior and give the Second Law of Thermodynamics a statistical–mechanics interpretation.*

One obvious asymmetry is that there are traces of the past (footprints, fossils, tape recordings, memories) and not of the future. There are spontaneous mixing processes in molar physics but no comparable spontaneous unmixing processes: milk and tea easily combine to give a whitish brown liquid, but it requires ingenuity, energy and complicated apparata to separate the two liquids again.

A cold saucepan of water on a hot brick will soon become a tepid saucepan on a lukewarm brick; but the heat energy of a tepid saucepan never spontaneously flows into the warm brick to produce a cold saucepan and hot brick.

Even though the relevant laws of nature are assumed to be time symmetrical at the atomic level it is possible to explain these asymmetries by means of suitable boundary conditions — which, however, themselves want explaining.

Another striking temporal asymmetry on the macroscopic level, mentioned earlier, is the absence of time-reversed (advanced–wave) electromagnetic radiation[1194]*. Electromagnetic phenomena are covered by the statistical Boltzmann*

[1193] Conventional, quasi–equilibrium thermal physics views any macroscopic change as a necessary regression, as devolution toward equilibrium or temporary *steady-state*. Here on earth, our planet derives nearly all its supply of *free energy* (a thermodynamically defined entity) from the sun, in the form of electromagnetic radiation. After 5 billion years, it should be quite close to steady state, its temperature constant, all chemical reaction halted — like the moon – or at least uniformized. However, under the influence of *life*, earth has moved steadily away from steady state; energy and entropy flow patterns on earth have become *more* complex, organized and out–of–equilibrium over time.

[1194] Waves moving outward from a source but backward in time would be precisely equivalent to waves that moved *inward* (advanced potential field) to meet their source in the future. The two descriptions are nothing more than two different ways of looking at the same thing. It is no more possible to distinguish between them than it is to distinguish (in Feynman's Quantum Electrodynamics) between a positron that moves forward in time and a negative-energy electron *principle of the increasing entropy of photons and the atoms and molecules that emit and absorb them; and so this arrow of time is not really different from the previously discussed asymmetry*[1195].

These considerations also provide some justification for the common-sense idea that the cause-effect *relation is a temporally unidirectional one, even though the laws of nature themselves allow for retrodiction no less than for prediction.*

A third striking asymmetry on the macroscopic level is that of the cosmological mutual recession of galaxy clusters, which can be deduced from the red shifts observed in their spectra. It is still not clear whether, or to what extent, this asymmetry can be reduced to the two asymmetries already discussed, though some suggestions have been made. This cosmological expansion (along with other observed data) is well explained by the Big Bang theory, in which the universe – or part of it – began as a super–hot and super–dense soup of elementary particles and their fields; from there on, the thermodynamic arrow suffices. But the ultimate cause *for these cosmic initial conditions is still at large (it could have its root in Quantum Gravity).*

A fourth time arrow is furnished by certain types of weak nuclear interactions, as manifested e.g. in the rare two-particle decay of the neutral K_L *meson, which exhibits a minute violation of time-reversal symmetry even at the microscopic level*[1196].

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that moves backward. We cannot observe motion into the past.

From our point of view, the past is already gone. All that we can see is behavior that looks *as though* it were taking place in a film or a videotape that was being run in the wrong direction. If we do see something like this, it is always possible to interpret it in two different ways, depending upon whether we want to view matters in our habitual forward-in-time way or adopt the opposite viewpoint. Another way of stating this is to note that any such observed anomaly – e.g. observing a lightbulb suck in light from a room, or pieces of a broken vase bounce up from the floor and reassemble on a shelf – would be evidence for the coexistence of opposite arrows of time. No such phenomena were ever seen.

1195 There exists another possible explanation of the origin of the electromagnetic time arrow, which has nothing to do with thermodynamics or with probabilities: **Feynman** and **Wheeler** (1945) put forward a theory according to which we do not see advanced-potential radiation because emission and absorption processes by a future absorber cause it to be canceled out by *destructive interference*. Their theory is based on the assumption that electromagnetic radiation in the present is always emitted symmetrically in *both* time directions.

1196 As originally discovered by **A. Sakharov**, this weak–force violation of time

A fifth arrow of time is furnished by the measurement process in quantum mechanics – which seems (at least in the Copenhagen interpretation) to irreversibly collapse the entangled wave-functions of "observe" and "observing" systems. This time arrow is probably closely connected with the thermodynamic and electromagnetic arrows, although some researchers have suggested it is independent and stems from quantum gravity effects.

It is also possible that the thermodynamic, electromagnetic, quantum-measurement and cosmological time-arrows will all be eventually understood in terms of quantum gravity.

A sixth arrow of time, the psychological one, is associated with man's cognitive reaction to temporal order in his life; the distinction between past, present, and future is basic to our experience of consciousness — we are conscious in the now, we remember the past, but we cannot know the future. It also is central to our idea of free will, for it implies that our actions in the present affect the future, that the past is immutable but the future can be changed.

The nature of the relationships among the five arrows of time (thermodynamic, cosmological, weak interactions, and electromagnetic) defined by physics is a topic that has been the object of considerable controversy. However, the relationship between these five "objective" arrows of physics and the psychological arrow of time, with its subjective durations and 'moving now', is even more mysterious. It is so mysterious that some philosophers have been led to conclude that time does not really exist.

How is it then, that all the microscopic equations of physics relevant to macroscopic observations (apart from some tiny weak interaction effects) are symmetrical in time, and can be used equally well in one direction of time as in the other — future and past seemingly on completely equal footing? Newton's laws, Hamilton's and Lagrange's equations, Maxwell's equations,

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symmetry, in conjunction with the cosmological and thermodynamic time arrows, could explain the mysterious surplus of matter over antimatter in our universe – to which we owe the existence of galaxies, stars, planets and people. This mechanism also explains minute microscopic violations of other symmetries, such as the distinction and mirror symmetry between particles and antiparticles. Modern theories of particle physics – and observations of *neutrino oscillations* and rare meson decays at laboratories such as Kamiokonde (Japan) and SLAC (USA) – indicate that Sakharov–type mechanisms may well succeed in explaining the matter surplus. Thus, it now appears that the cosmological, thermal and weak–interaction time–arrows are at least partially tied to each other, as well as to other baffling and subtle violations of symmetries in physics.

Einstein's general relativity, Dirac's equation, the Schrödinger equation — are all covariant under reversal of the direction of time.

Time-symmetry extends into modern physical theories. The laws of quantum mechanics (both 1^{st}-quantization and quantum field theories) are time-reversible for the electromagnetic and strong–nuclear interactions as well as for the dominant weak–nuclear effects; they define no unique direction of time. In relativity theory, for example, time is simply the 4^{th} dimension — there is not much more difference between past and future than between left and right; and all the equations would look the same if time were reversed. Yet the following subtle points must be considered:

- *STR does not contradict ordinary ideas of* causality*: the temporal order of events along a time-like or light-like worldline is Lorentz-invariant, and therefore no observer, in any state of motion, will ever describe a nail as being driven into a piece of wood before it was struck by a hammer.*
- *In STR, the "time" at which a distant event takes place is dependent upon the state of motion of the observer. Thus "time" cannot be defined in an unambiguous way throughout all of space. Since different times will be computed by different observers, the concept of absolute time is really inapplicable. If we look at time in subjective terms, we can say that "now" does not extend beyond "here".*
- *Rapidly moving objects exhibit a* time dilation effect*. This effect is real, not illusory, as the "twin paradox" (confirmed in laboratory experiments) demonstrates.*
- *If two events, A and B, are so close in time or so widely separated in space that no signal traveling even at the speed of light can possibly get from one to the other before the latter event takes place (i.e. if A and B are* space-like *in their separation), then their time ordering is ambiguous — some observers will conclude that even A happens first, while other will conclude that event B took place earlier in time.*
- *Whereas Einstein had banished the notion of* absolute time*, independent of the observer, he has not imagined a physical system's causal history reversing its course, nor does STR allow this any more than Newtonian physics. GTR does allow some solutions with "closed time-like curves", but in the absence of a theory of quantum gravity, it is impossible at present to ascertain whether such solutions are spurious or not.*
- *It is widely believed that black holes emit a thermal-like* Hawking radiation*, and some have argued that this is a new, quantum-gravity arrow of time, closely related to the cosmological arrow.*

- *The above-mentioned "weak–nuclear arrow", in conjunction with the cosmological and thermal arrows, are believed to be necessary enabling conditions that allowed* matter *to avoid total annihilation into* radiation *in the first instants after the Big Bang (the "matter surplus" discussed above).*

In all, nothing in the fundamental *laws of physics (except certain subnuclear processes of doubtful influence upon to the arrow of time, though they* interact *with it) seems to mandate a distinction between past and future.*

And what of the present*? We are all aware of the subjective "flow" of time. We are conscious of a moment we call "now" that seems to move inexorably toward the future. But physics has no need of the concept of "now". Its laws deal only with the* continuum of time *and event–defined instanced along it, and say nothing about the present moment. In other words, there is no "flow" of time in physics; all that physics really tells us about it is that some videotape recordings of physical reality represent impossible (or extremely unlikely) chain of events when they are played backwards.*

If we were to introduce the idea of a "flow" of time into physics, we would immediately encounter problems. Physics can answer questions about how an object moves in time, but to the question of how fast does a "now" move, physics has no answer. In physics, time is a dimension with only minor privileges over the other dimensions, and there is no objective description of time as a moving "now".

Indeed, according to relativity, there is not really such a thing as a 'now' that extends beyond 'here': as we saw before, the 'now' according to one observer would not agree with that for another. STR emphatically states that whatever time is, it does not flow at an even rate throughout the universe.

Faced with the apparent conflict between the time-symmetry of the relevant basic physical laws and the six (partially interconnected) time-arrows specified above, we must look elsewhere to find where the distinction between past and future must lie.

A clue is afforded when we consider the evolutionary view[1197] *of the universe; all complicated forms of matter and energy, without exception, evolved from a simple state in the early universe.*

[1197] The evolutionary view is commonly held by various civilizations: all the innumerable 'things' in the present Universe are held to have evolved from a primordial "*one*". The evolutionary view resolves the old "paradox" of which comes first, the chicken or the egg; according to the evolutionary view this question is pointless, for neither the chicken nor the egg was there in the beginning; they both evolved gradually from simpler things.

Thermodynamics without gravity leads to a thermal death (maximal entropy) doomsday scenario. Thus, only gravitation and thermodynamics together can accommodate a process of evolution that avoids thermal death. Indeed, without gravity, the sun and other stars would not even exist; there could be no shining stars at all without the gravitation, that is needed in order to hold its material together and to provide the temperature and pressure that are needed for nuclear ignition in stellar cores. There would be a cold, uniform and diffuse gas in place of the sun and its retinue of planets, and of course — no life!

The deep significance of time must therefore be somehow linked to the thermodynamics of the expanding universe. Let us consider the following simplified model of an adiabatically expanding, radiation-dominated universe[1198]*; there is no heat exchange with the 'exterior' because no other system exists outside the universe, and there is no difference between any typical region and its exterior – on account of the overall uniformity.*

Under these conditions $dE = -PdV$ *where* E, P, V *are the energy, pressure and volume of a radius –* R *region, respectively. In the above equation we set* $V = \frac{4\pi}{3}R^3$, $P = \frac{1}{3}\epsilon_r$, *(thermodynamic equation of state of pure radiation),* $E_r = V\epsilon_r$ *where* $\epsilon_r = c^2\rho_r$ *is the radiation energy density and* E_r *is the total radiation energy in a sphere of radius* R*;* ρ_r *is the equivalent mass density. A simple differential equation is thus obtained, the solution of which is* $\epsilon_r \propto \frac{1}{R^4}$*. But since* $\epsilon_r \propto T_r^4$ *by Stefan–Boltzmann law, we find* $T_r \propto \frac{1}{R}$ *where* T_r*, the temperature of the radiation, falls in inverse proportion to the scale factor* R *as the universe expands. We now re-introduce particulate matter, assume a matter-dominated universe (as exists today) and use a similar method to discuss the thermal behavior of the matter (electrons, protons, neutrons, etc.) constituents in any finite–volume subsystem.*

For simplicity, we assume only a single species of matter particles. Let P_m *be the pressure of the (nonrelativistic) particulate matter; it obeys the ideal gas law,* $P_m = nkT_m$*, where* n *is the number density of the particles,* T_m *the temperature and* k *the Boltzmann constant. Let the particle energy density be* $\epsilon_m = c^2\rho_m = nmc^2 + \frac{3}{2}nkT_m$ *(as for a monotonic gas), with* m *the particle rest–mass.*

1198 The Big Bang theory tells us that the universe was radiation–dominated until about 300,000 years after the initial singularity and approximately matter–dominated thereafter. This time is reckoned in local inertial frames in which the microwave background radiation – originating in that same "recombination" transition – appears approximately isotropic.

We then obtain from the above adiabatic energy equation

$$d(R^3\epsilon_m) = -P_m d(R^3),$$

which together with the above equations of state and the law of conservation of total particle number $d\left(\frac{4\pi}{3}R^3 n\right) = 0$ *[from which* $n \propto R^{-3}$*], yields the following result:* $\frac{3}{2}\frac{dT_m}{T_m} = \frac{-d(R^3)}{(R^3)}$. *This leads us to the solution* $T_m \propto \frac{1}{R^2}$. *Consequently, as the universe expands the particle temperature* T_m *decreases, but the manner of decrease is inversely proportional to the square of the scale factor* R *(as opposed to* $T \sim \frac{1}{R}$ *for the radiation–dominated era).*

Thus, even if at the beginning we have $T_r = T_m$, *after a period of expansion we must have* $T_r > T_m$ *(assuming the co-extensive matter and radiation have decoupled after matter became gravitationally dominant).*

Now, if the time required to achieve uniform temperature after the decoupling is longer than the time scale of the cosmic expansion, then there will never again be thermal equilibrium between radiation and particles, and both components are separately *in thermal quasi-equilibrium*[1199].

Thus, cosmic expansion saves the universe from thermal death. Since the expansion of the universe is linked in an essential way to the gravitational interaction, we may say that the combination of gravitation and *thermodynamics can produce a possible mechanism to avoid thermal death and explain the evolution from the simple to the complex.*

Let us next explore this hypothesis in the context of a smaller-scale system, namely — the solar system: A planet of mass m *moves in a circular orbit of radius* r *around the sun of mass* $M \gg m$. *From* $\frac{GMm}{r^2} = \frac{mv^2}{r}$ *we find* $v = \sqrt{\frac{GM}{r}}$ *for the planet's orbital velocity. Since its potential energy is* $U = -\frac{GMm}{r}$ *and its kinetic energy* $T = \frac{1}{2}mv^2 = \frac{1}{2}\frac{GMm}{r}$, *the total* New*tonian energy of the planet is* $T + U = -\frac{1}{2}\frac{GMm}{r}$. *This energy is* negative

[1199] After the Recombination Era, the left–over radiation bath–initially near the visible but now in the millimeter–wave RF spectrum–fell out of equilibrium even with itself, and only retains a spatially–uniform temperature ($\sim 2.7^\circ K$ today) thanks to free–streaming adiabatic expansion (cosmological red shifts). Meanwhile, T_m is so low now that matter thermodynamics is completely dominated by local astrophysics. The last (especially stars) produced photons at all wavelengths, but these – though crucial for astronomy – are negligible in numbers compared with the CMBR photons. There are, on average, $\sim 10^{10}$ CMBR photons per proton in the observable universe in our present era.

because work must be invested to remove the planet away to infinity, freeing it from the influence of the sun.

Suppose that we had a mechanism through which we could inject orbital kinetic energy into the planet (e.g. giant nuclear–powered rockets attached to its soil at various carefully chosen positions). This will manifest itself kinematically in an immediate increase *of its orbital radius, followed by an eventual* decrease *in its orbital velocity, on account of the relation* $v = \sqrt{\frac{GM}{r}}$.

Now, from a thermodynamic point of view, planetary energy is synonymous with heat (provided one views planets and stars as microscopical particles). By the same token (velocity)^2 *is proportional to the temperature. The "translation" then yields: Add heat* $(\Delta Q > 0)$ *to the solar system and the temperature of the system will be lowered* $(\Delta T < 0)$. *Extract heat from the solar system — and the temperature of the system is raised.*

In brief, the heat capacity $\frac{\Delta Q}{\Delta T}$ *of the solar system is negative.*

This startling conclusion applies not only to the solar system but to all systems maintained by gravitation: the thermal capacity of all self-gravitating systems is negative.

It can easily be shown that, as long as self-gravitating systems are present, a stable thermal equilibrium cannot exist because the existence of systems with negative thermal capacity is thermodynamically destabilizing[1200].

To see this we consider a system comprised of a body A *with a positive heat capacity and a body* B *with a negative heat capacity. In the beginning, the system is in thermal equilibrium and the temperatures of* A *and* B *are equal; the equilibrium is a dynamic one, that is, energy emitted by* A *is absorbed by* B*, and vice versa. The two flows cancel out and equilibrium is maintained.*

There always are, however, small fluctuations about an equilibrium. For example, the radiation that flows from A *to* B *may be temporarily slightly larger than that from* B *to* A*, and so* B *absorbs a small net energy.*

[1200] Indeed, according to Hawking's theory, a GTR black hole emits quantum radiation at a effective blackbody temperature that varies inversely with its mass. As it radiates and loses mass–energy, then, the black hole *heats up!* This is a runaway thermodynamic process which, theoretically, should lead to an explosion after a finite time. Thus a hypothetical *primordial black hole* of mountain–size ($\sim 10^{15}$ gram) mass and packed within a nuclear–sized *event horizon* ($\sim 10^{-15}$ meter), is predicted to explode after several billions of years.

If B had a positive heat capacity, then B's temperature would rise, its radiation output increasing thereby, soon canceling out the excess absorption of energy and returning the system to equilibrium.

However, since B was actually assumed to have a negative heat capacity, then an excess in the energy it absorbs will lower its temperature, and its radiation output will become weaker — making its net energy absorption even higher. In the resulting runaway process, B's temperature will keep getting lower while that for A will also be lowered. If body A is large enough, it is easy to see that $T_A - T_B$ *will keep increasing in a runaway reaction; the original equilibrium is destroyed – it was* unstable.

When, to the contrary, the fluctuation is such that B absorbs a little less energy, then the outcome is an ever-increasing temperature of B, again destroying the original equilibrium.

Thus, systems in which gravitation plays a decisive role cannot be in a state of stable equilibrium and thus tend to leave thermal equilibrium; they spontaneously become far–from–equilibrium thermodynamical systems.

One of the immediate important consequences of this state of affairs is that it provides for a mechanism for the formation of spatio–temporal structures in the universe: We already know that in an isolated non-gravitating system, the evolution of the distribution of matter and energy is from non-uniform to uniform (or structured to structureless) on account of the second Law of Thermodynamics.

In a system with gravitational interaction, however, we have just the opposite scenario; as soon as some local region acquires some slightly higher *energy (matter densities) through fluctuations, its orbital speeds tend to be reduced, while its gravitational attraction increases. The system thus attracts* more *exterior mass–energy, and the process reinforces itself.*

Likewise, if the density in some region is slightly lowered *by fluctuation, its gravitation is weakened, its internal orbital speeds increased, and still more mass–energy will escape, forming a still lower density. In short, a small fluctuation will completely destroy the homogeneous state and its direction of evolution will be from structureless (uniform) toward a structured (non-uniform) state.*

Throughout the universe, gravitation[1201] *is the dominant large-scale force. Therefore, even if the initial universe is uniform and structureless, it will spon-*

[1201] In this context, gravitation is governed by the Einstein equations of the *general theory of relativity*; When we reach the truly vast distances which arise at the scale of the whole universe, the Newtonian picture of gravity breaks down.

Although the average density of matter of the universe is extremely low (and thus, so is the average curvature of spacetime), the distances involved are *taneously generate a non-uniform and structured state. Clusters of galaxies of various scales owe their formation to this process of inhomogenization, as does the geological stratification of the earth.*

Gravitation is therefore responsible for the universe being so structurally and thermally complex, ordered and out of equilibrium.

In the eventual cosmology (the one which will include a correct description of quantum gravity – including the so-called 'Planck Era', that is, the first 10^{-43} *seconds or so after the big Bang, during which one needs a Schrödinger wave function of the entire universe to understand its evolution), the* thermal *state of the hot big bang will somehow have to be explained in terms of a pure (non-ergodic)* initial *wave-function.*

In such a theory of quantum gravity the thermodynamic time arrow *— which may be thought of as a peculiar initial condition having very weak correlations between microscopic degrees of freedom — will be understood in terms of the* cosmological arrow of time*.*

But even while we are still ignorant of what the link between these two arrows is, the following is *clear: at an epoch much later than the Planck Era, namely the epoch in which stars and planets formed, gravity continued to play an important role vis a vis the arrow of time because:*

- *Instabilities in a homogeneous Dust Universe naturally lead to accretion of galaxies (the Jeans instability) and stars; i.e., to the formation of 'order out of chaos'. Even on a planetary scale, gravity was the agent that heated the early earth and provided for differentiation of its layers.*

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so enormous that curvature effects become of overriding importance. GTR enabled physicists, for the first time, to consistently probe the behavior of our world on the grandest of all scales and to think in a scientific manner about the origin of the Universe.

However, close to the moment of the big bang singularity we are dealing with a scenario in which Einstein's theory must fail; for there are certain built-in suppositions in GTR which are known to be incorrect at very short distances. According to Einstein's equations, the present age t_0 of a flat (zero spatial curvature) universe model (that is — time since the Big Bang) is related to the present *mass density* ρ_0 via the relation $t_0 = \frac{1}{\sqrt{6\pi G\rho_0}}$, where $G = 6.67 \times 10^{-8}$ $\mathrm{cm^3 s^{-2} g^{-1}}$ is Newton's gravitational constant.Thus, a present mass density of $\rho_0 \approx 3 \times 10^{-30}$ g cm^{-3} (about two hydrogen-atom masses per cubic meter) is compatible with an age of order 15 billion years in this simple model – close to the observed value.

- *The nuclear ignition of main–sequence stars (and, to some small extent, slow radioactive decay, within planets) provides suitable planets with a constant flow of energy, over billions of years; this creates Prigogine's 'open thermodynamic system far from equilibrium'. In this regime there occur bifurcations, limit cycles, spontaneously emerging spatial structure, and, perhaps, life itself.*

To summarize: gravitation influences arrows of time at several different levels, and some of these influences are more speculative than others; but even if it turns out that gravity is not the ultimate 'culprit' in creating the thermodynamic arrow, it is almost certainly the agent that caused, indirectly, through stars (including supernovae) and planet accretion, the establishment of the biological arrow of time.

But the biological arrow is dependent for its continuation on the maintenance of thermodynamic non-equilibrium; i.e., if one removes the sun (hauling it away to alpha centauri, say), life and evolution on earth will cease. And since astrophysics teaches us that all stars eventually expand, explode or otherwise become unsuitable for sustaining life – and new materials for new stars eventually run out – earth's biosphere, in its present form, cannot continue for more than ca another 5×10^9 *years. However,* human volition and intelligence *should also entered into the equation. We know, for example, that man's intervention has already modified the thermodynamics of our planet to some small, and perhaps significant, extent. It is not unreasonable to expect that our descendants of the remote future will be able to protect the biosphere against ice ages, the aging of our star, an even – perhaps by altering earth's orbit – against the sun's death.*

If intelligent life is capable of spreading or arising spontaneously throughout the observable universe, it could conceivably alter the fate of the entire universe by some suitable (planned or unplanned) action! Thus, intelligent life can represent a new arrow, apart from mere life, that will modify the cosmology we live in.

IV. Chronons

Quantum field theorists routinely investigate models where space and/or time are rendered discrete*, for computational ease. Many of the difficulties in achieving a consistent theory of* quantum gravity *are due to the continuity of spacetime. It is expected that at the* Planck scale *(*$\sim 10^{-43}$ *sec, or* 10^{-33} *cm), wild fluctuations in spacetime topology render geometry (whether 3D* *or 4D) meaningless. In view of all that, it is tempting to conjecture that the spacetime continuum may be a mere coarse–grained approximation.*

In antiquity, the idea of indivisible atoms of time may have been advocated by ***Zenocrates****, a pupil of Plato, as well as by Indian philosophers in the* 2^{nd} *century BCE.*

In the Middle Ages the atomicity of time was maintained by various thinkers, notably by ***Maimonides*** *(1190 CE) who postulated an indivisible unit of* 5×10^{-15} *sec.* ***Descartes*** *(1641) adopted this view and postulated that temporal existence was like a line composed of separate dots, a repeated alternation of the state of being and the state of non-being. Descartes' contemporary* ***Torricelli*** *also regarded time as 'granular', a succession of discrete segments which he calls 'instants'.*

The idea of temporal atomicity does not necessarily imply that there must be gaps between successive instants. The essential criterion for atomicity is that there is a limit to the division of any duration into constituent parts: time would be like a line which can be divided into a denumerable sequence of adjacent segments with no intervals between them. It would mean that, from the temporal aspect, there are minimal processes in nature, no process occurring in less than some shortest unit of time, or chronon.

Most speculations concerning the chronon have often been related to the idea of a smallest natural length. This may e.g. be given by the effective diameter of the proton, that is to say, at most of order 10^{-13} *cm. If this were a minimal natural length scale and we divided it by the fastest possible speed, that of light in vacuo* (3×10^{10} *cm/sec), the resulting interval of time would be about* 10^{-23} *sec. However, empirical data on dynamical short–range processes (collisions, decays etc.) at such scales is explained quite well by the so-called Stanford Model, a quantum field theory predicted upon a continuous spacetime. Current empirical knowledge thus implies that the chronon, if it exists, must correspond to a duration of less then* 10^{-25} *seconds. A time of this order characterizes the normal weak-interaction virtual quantum fluctuations responsible for radioactive beta-decay (although the half-lives of these decays are usually many orders of magnitude larger); strong nuclear interactions half-lives are of order* 10^{-24} *sec or so.*

A purely theoretical unit of time much shorter than 10^{-25} *sec can, however, be constructed from the three fundamental constants G, h and c. The Planck Length is* $\sqrt{\frac{Gh}{c^3}}$*, and is of the order* 10^{-33} *cm. If this is divided by the velocity of light, it gives a time of the order of the Planck time, i.e.,* 10^{-43} *sec, which might be the a candidate for the chronon scale.*

Feynman *noted that a discrete structure of spacetime would imply an anisotropic speed of light, which can be accepted in principle, provided it is* *not too large (modern precision experiments utilizing atomic clocks have put stringent bounds upon any possible anisotropy).*

Until there is general agreement concerning the chronon (or any other non-continuous theory of time), the concept of mathematical time underlying physical science, including microphysics, will continue to be based on the hypothesis of continuity (or infinite divisibility) of both the spatial dimensions and of time.

In our quest for understanding as to why time seems to flow in just one direction and not the other, we have had to travel to the very beginnings of time, and look at both the rarest of quantum processes and those that probe the spacetime manifold at the highest resolutions. At such epochs and scales, the very notions of space and time might well dissolve away. We have learned that our theories are not yet adequate to provide answers to the question, 'what is time?'[1202]

[1202] Apart from our scientific ideas about time (*physical time*, *biological time*, *geological time* and *cosmological time*), this concept has occupied the minds of philosophers, poets, psychologists and thinkers from antiquity to the present, throughout the entire history of human culture. The ultimate efforts of the human spirit to comprehend the meaning of time is best reflected in the writings of the Hebrew Bible, the Greek Tragedies, the plays and sonnets of Shakespeare and, during the 20^{th} century in the novel of **Marcel Proust** (1871–1922, France) *Remembrance of Things Past* (1908–1918). To Proust, reality remains elusive. It is constantly changing, because the passing of time alters not only his only perspective, but also the nature of what is perceived. He finally recognizes that reality is not external but something stored in the depths of man's unconscious memory. There it is preserved from the changes of time, but is accessible only in rare and happy moments. On this view, artists can reveal reality to mankind because their sensitivity enables them to dig deeply into their own unconscious memory. But Plato, in his cave metaphor, has warned us against the distortions of the human mind – a theme that **Francis Bacon** built upon in his Novum Organum. And, as the poet **William Blake** wrote: "The mind, altering, alters all".

Table 5.29: Horology — from Sundials to Atomic Clocks[1203]

c. 3500 BCE	*Gnomon*[1204] — the first system for telling the time; *Shadow clock*. It consisted of a horizontal surface with a vertical pillar stuck into it, the shadow of which indicated the movement of the sun. In the morning the clock was pointed east into the sun, and in the afternoon it was turned around to point west; known to exist in Egypt 2000 BCE.
c. 2000 BCE	Early *sand-timers*, the earliest timekeeper independent of the celestial bodies. Inefficient for measuring more than a limited duration.
1500–1451 BCE	Thutmosis III erected the '*Needle of Cleopatra*'; its shadow was used to calculate the time, seasons and solstices.
c. 1400 BCE	Egyptians construct crude *water clocks* (*clepsydras* = water stealer; water trickled trough a hole in the bottom of a stone bucket; the time was indicated by the level of the water against a scale marked on the inside. It had the advantage of continuing to indicate time after sunset. The length of the hours changed according to the length of the day (longer summer hours) and the temperature.
c. 750 BCE	Gnomons in Egypt were improved to become the proper *sundial*: on its base it had 6 divisions, each corresponding to an *hour*.
c. 520 BCE	*Sundial* was introduced into Greece by **Anaximander**.
c. 300 BCE	Further improvements of the *sundial* by the Babylonian **Berosos**, the Greek **Apollonios of Perga** and **Ptolemy of Alexandria** made this clock more accurate during daytime. These measures, however, did not enable the difference between average midday and true midday to be established. Rome's first sundial was established in 190 BCE, when a Samnite clock was captured in war.

1203 Horology: From the Greek *horolegein* (= that which tells the time). Hence the word *hour*.

1204 Latin; from the Greek *gignoskein* = to know; i.e. one who knows = indicator.

c. 270–50 BCE — Greeks and Romans improve the *water clock*. Romans improved the flow of water by installing larger tanks. Then they invented a mechanism for reading the hour, consisting of a float equipped with a toothed stem connected to a pawl wheel: the lowering of the float engaged the wheel, which itself had a needle to designate the corresponding hour on a graduated dial.

This mechanism described by **Vitruvius** was already in use by the 1st century BCE and so the principle of the modern analog clock display, the *reading of a dial*, can be said to date back to that time. Earlier (c. 270 BCE) the Greek **Ctesibius of Alexandria** developed improved water clocks using a siphon system to replenish the reservoir automatically.

Clearly, water clocks were incapacitated in freezing conditions.

c. 100 CE — Romans used *sandglass clocks*: when the sand in the top bulb had emptied into the bottom one, it meant that a fixed time had elapsed.

725–1092 CE — Earliest known *mechanical clock*, with escapement mechanism, was built in China by **I-Hsing** (**Yi Xing** ca 725 CE). It was derived from the water-clock (i.e. its driving power was hydraulic) but it was more precise than the holed vessel. It consisted of a wheel with strictly identical paddles into which the water flowed; each time a paddle filled up, it rotated the wheel a 36th of an arc.

A huge gearing system (the clock was about 10 m high) caused the rotation of a celestial sphere around which the sun and moon were represented, so that there was one complete rotation of the sun every 365 days and one complete rotation of the moon after slightly more than 29 days.

The level of a part representing the horizon also enabled the exact hours of sunrise and sunset to be determined, as well as the dates of the new and full moons and the hours and quarter-hours, which were read and announced by bell and drumbeats.

A later version (976 CE) by **Chang Su-Hsun** (**Zhang Xu Xun**), using a *chain-drive*, showed the movements of the five then-known planets, the Pole star and the Great Bear.

A third version, designed by **Su Sung** and built in 1092, showed the movement of the stars and certain special days and hours.

c. 850 CE *Candle clock* was reputedly invented in England by King Alfred the Great: A candle marked with hours. As it burned the time could be read off the scale.

1310–1319 CE *The first weight-driven mechanical clock* appeared in Europe; it had no hands and was used only to mark proper time for ringing church bells. Its invention is attributed to **Gerbert** (c. 1000 CE). The principle of this clock probably arrived in the West from China in the form of descriptions or drawings. It constitutes the first model of the *escapement* system in Europe, which was later improved in the form of *cylinder escapement* or *verge escapement*. (In China, however, water was still used as a power source.)

c. 1335 CE First public clock that *struck hours* was put up in Milan, Italy.

c. 1354 CE Mechanical clock at the Strasbourg Cathedral.

c. 1370 CE Clock faces with a *single hand*, to show hours, appeared in Western Europe.

c. 1400 CE First small, weight-driven clock for use in household appeared in Europe. Continent is soon taken with novelty of *timekeeping*.

c. 1470 CE The *mainspring* (a spiral whose gradual unwinding powers a clock) was invented in Germany. Adoption of this principle makes it possible to construct more compact mechanisms, and indeed it paved the way for the *portable timepiece*, but demanded a device to compensate for the diminishing force of the spring as it uncoiled; this problem did not arise with weight-driven mechanisms, as the force exerted by the weight at the end of the descent is the same as that at the beginning.

The necessary compensation was effected by the *fusee*, essentially a conical drum with helical groove so cut in it that as the spring uncoiled the connecting cord exerted the same moment on the shaft carrying the fusee (Leonardo da Vinci left us a sketch of such a mechanism in his 1490 notebook).

1494 CE **Leonardo da Vinci** made a drawing of a clock with a *pendulum.*

1504 CE **Peter Henlein** (Germany) made the first *domestic* spring-driven clocks, with a horizontal dial on top and a single hand, the one which indicates the hours. He used a coiled steel mainspring to drive them. It was the first *watch*. Being too large for the pocket, it were frequently hung from the girdle.

1533 CE **Gemma Frisius** was first to point out that by knowing the correct time according to a mechanical clock and comparing it with the sun — time can be used to find the local *longitude*.

1615 CE **Marine Mersenne** pointed attention to the geometrical properties of the *cycloid curve*; it is later used by **C. Huygens** in his *pendulum clock*.

1656 CE **Christian Huygens** (Holland) build the first *pendulum clock* by applying the pendulum theory of Galileo (1583) to the existing cylindrical balance escapement, thus ushering in a new era of precision timekeeping. He also developed a *cylindrical pendulum clock*, the period of which is independent of the amplitude.

1658 CE **Robert Hooke** (England) invented the *anchor or recoil escapement* which had the particular advantage of being able to transfer some of its energy to the pendulum. Also, it restricted the swing of the pendulum to a small arc, improving accuracy.

1670 CE **William Clements** (England) added the *minute hand* to the pendulum clock.

1704 CE **Nicolas Fatio de Duiller** used *gems* for bearings in clocks.

1735–1759 AD **John Harrison** (England) constructed the first practical *marine chronometer* which enabled sailors to calculate longitude accurately. With the work of Harrison, the problem of finding the longitude at sea was finally solved. It was accurate to within 0.1 second a day.

1754 CE **Thomas Mudge** (England) introduced the lever escapement, used by most mechanical clocks and watches today.

1790 CE Earliest known *wristwatch* made in Switzerland.

1799 CE Post-revolution French astronomers define the *solar second* as $\frac{1}{86400}$ of a mean solar day.

1841 CE **Alexander Bain** (Scotland) made the first *electric watch*, where the electric energy served only to move the pendulum.

1875 CE About $2\frac{1}{2}$ million people in the world own pocket watches.

1884 CE International conference in Washington D.C. divided the world into 24 *Time Zones*, changing then-current practice of each locality keeping its own time. This act was motivated by the rapid growth of railroads. The mean solar time at *Greenwich* Observatory (England) became *Greenwich Mean Time* (GMT).

1904 CE Radio stations began broadcasting time signals to ships at sea to aid navigation.

1929 CE **W. Marrison** (Canada) produced the first *Quartz crystal clock*. The mineral quartz (silicon dioxide; in hydrate form, $SiO_2 \cdot nH_2O$) exhibits a piezoelectric effect. A quartz slab acts as a high-Q stable tuned oscillator, when excited by an AC electromagnetic field. These resonant vibrations are then used to electromagnetically control the speed of an electric motor which drives the clock hands. A quartz clock can keep time with accuracy of about 0.1 sec/day.

1948 CE The *atomic clock* was introduced; the steady coherent oscillations of a superposition of two quantum states of a single–species population of atoms (e.g.: caesium) or molecules (e.g.: ammonia, NH_3) in a tuned resonance cavity, are harnessed to regulate clocks with accuracy much greater than that of quartz clock. The principle was worked out by **W. Libby** (USA) in 1946. In 1969, the US Naval Research Laboratory built the first *ammonia atomic clock*, accurate to within one second per $1,700,000$ years.

1956 CE The fundamental time standard — the *second* – is redefined to represent $(31,556,925.974\,7)^{-1}$ of the time it takes the earth to orbit the sun (solar second). Before 1956, the second was defined as $\left(\frac{1}{60}\right)\left(\frac{1}{60}\right)\left(\frac{1}{24}\right)$ of a *mean solar day*, where a solar day is the year–averaged interval between successive meridional crossings.

1962 CE *Telstar* became the first satellite used to *synchronize time* internationally (to an accuracy of 1 microsecond), between the USA and Britain.

1967 CE *The atomic second* replaces solar second as fundamental time standard: The International Bureau of Weights and Measures redefined the second as the time that microwaves (emitted by RF–excited caesium-133 atoms) execute $9,192,631,770$ oscillations.

The Seiko Company (Japan) produces the first *electronic quartz wristwatch*.

1972 CE Electronic wristwatches were equipped with *liquid-crystal digital display*.

1948–1967 CE **Andrei Dmitrievich Sakharov** (1921–1989; Soviet Union). Distinguished nuclear physicist and most ardent and unrelenting champion of human rights and freedoms. Regarded as "father of the Soviet hydrogen bomb". Suggested (1948) a new principle for a thermonuclear device, and proposed the idea of the *Tokamak thermonuclear reactor* (1950). In a pioneering paper he suggested (1967) an explanation for the mysterious asymmetry of matter in the universe (surplus of matter over antimatter) in terms of a combination of three effects in the early universe: baryon–number nonconservation, time–reversal asymmetry in the weak nuclear force, and out–of–equilibrium thermodynamic processes.

Sakharov was born and educated in Moscow. In 1938 he enrolled in the physics department of Moscow University where he was quickly recognized to be an outstanding student. In June 1948 he was recruited to work on the Soviet nuclear weapons program by his professor **Igor Tamm** (1895–1971). During the same year they outlined a principle for the magnetic isolation of high-temperature plasma, and their subsequent work led directly to the explosion of the first Soviet hydrogen bomb (1953).

By 1950 they also formulated the theoretical basis for controlled thermonuclear fusion – which could also be used for the generation of electricity and other peaceful ends.

In the early 1960's, Sakharov was instrumental in breaking biologist **Trofim Lysenko**'s hold over Soviet science.

While contributing more than anyone else to the military might of the USSR, he gradually became one of the regimes most courageous critics, a defender of human rights and democracy. He could not be silenced, and helped bring down one of history's most powerful dictatorships.

Unlike his American colleague **Robert Oppenheimer**, Sakharov did not feel physicists had "learned sin" by working on nuclear weapons. Nor was he like Edward Teller, proud to have persuaded political leaders of the necessity of building the hydrogen bomb (Soviet leaders did not need any persuasion).

Sakharov was awarded the Nobel Peace Prize in 1975.

1949 CE **Erwin Chargaff** (1905–2002, Austria and USA). Biochemist. First to discover an important clue to DNA structure[1205].

Revealed the very striking diversity in chemical composition of nucleic acids from different sources and suggested that nucleic acid could function as genetic material. Four years later his findings were shown to follow from the double-helix structure of DNA.

He was born in Czernowitz (now in the Ukraine) to a Jewish family and studied in Vienna and at Yale (1928–1930). He went to Berlin (1930–1933) and came to Columbia University (1935), where he was appointed professor (1952).

1949–1961 CE **Melvin Calvin** (1911–1997, USA). Chemist. Major contributor to elucidation of the chemistry of plant life. Using carbon-14 as a tracer, he determined the biochemical processes of *photosynthesis*, in which green plants use chlorophyll to convert CO_2 and H_2O into sugar and O_2. Awarded the Nobel Prize in Chemistry (1961).

Calvin was born in St Paul, Minnesota to Russian immigrant Jewish parents and studied at the University of Minnesota. He joined UCLA (1937) where he became professor of chemistry and head of the Lawrence Radiation Laboratory (1963–1980).

[1205] He showed that the number of *adenine* molecules in DNA equals the number of *thymidine* molecules, and that the number of *guanine* equals the number of *cystosine*. This is now understood as a simple mathematical relationship between the proportions of molecules that connect the two strands of the DNA molecule.

When **J. Watson** and **F. Crick** published their seminal work on the double helix (1953), they claimed to have been unaware of Chargaff's work. This illustrated once more that "what counts in science is to be not so much the first as the last". (Chargaff, *Science*, 1971).

Calvin started to investigate the process of photosynthesis in the single-celled alga *chlorella* and showed that there is a cycle of reactions (now called the *Calvin cycle*[1206]) involving an enzyme as catalyst.

Science Progress Report No. 21
The Great Soviet Encyclopedia (GSE)

It was published in 1949 in the Soviet Union. Through it, the ex-seminarist Joseph Vissarionovich Dzhugashvili (Stalin) stage-managed his own apotheosis as the embodiment of human wisdom. It was full of gems e.g.:

> *"In 1751–2, Leonty Shamshugenkov, a peasant in the Nizhny-Novgorod province, constructed a self-propelled vehicle operated by two men."*

Stalinist "historians", who contributed to the GSE claimed that many discoveries made by Russians were plagiarized by foreign capitalist scientists. For example:

- *The steam engine was* not *invented by* **Watt***, but by a Siberian laborer named Polzonov.*
- *The electric bulb was* not *invented by* Edison *but by the Russian Yablochkov.*
- *The first successful flight in a power-driven, heavier-than-air machine was* not *made by the Wright brothers in 1903 but by the Russian engineer Mozaisky.*

1206 When *Chlorella* was exposed to radioactive CO_2 in the dark, radioactivity was transferred to *succinate*, *fumerate*, *malate* and other compounds before being found in *glucose*.

A brief spell of illumination caused the radioactivity to appear in the triose phosphates and sugar phosphates that are now associated with the pentose phosphate pathway. This led to the elucidation of the *Calvin cycle*, whereby CO_2 interacts with ribulose diphosphate giving (via several reactions)various sugar phosphates and *regenerating* ribulose diphosphate, ready to repeat the cycle. The pathway occurs in all photosynthesizing organisms.

In fact, according to GSE, whatever was not invented by Russians in the 20th century, had already been invented by Mikhail Lomonosov in the 18th century.

In this perverse pseudo-science manifesto of Soviet Cultural Revolution, theoretical physics, cosmology, chemistry, genetics, medicine, psychology and cybernetics were all systematically presented as a capitalistic plot against Communism.

Relativity theory was condemned not (as in Nazi Germany) because Einstein was a Jew but for equally irrelevant reasons: Marx had said the universe was infinite, and Einstein had got some ideas from Mach, who had been proscribed by Lenin.

Behind this lay Stalin's suspicion of any ideas remotely associated with Western or bourgeois values. It was an attempt to change fundamental human attitude over the whole gamut of knowledge by the use of naked police power.

1949 CE **Derek Harold Richard Barton** (1918–1988, England). Organic chemist. First to demonstrate that chemical properties of complex organic molecules depend strongly on their 3-D shape; showed that the biological activity of natural compounds often depends on positions and orientation of key functional groups.

Barton studied (Harvard, 1949) the different rates of reaction of certain steroids and their triterpenoid isomers (substances with the same composition but differing in the way their atoms are joined and arranged in space).

He deduced that the difference is spatial orientation of their functional groups accounts for their behavior, and so developed a new field in organic chemistry which became known as *conformational analysis*. He then went on to examine many natural products, concluding that the structures of many phenols and alkaloids could be explained and predicted.

Barton was born in Gravesend Kent, and studied at Imperial College, London. Professor at the University of London from 1978. He shared the Nobel Prize for Chemistry (1969).

Biotechnology Chronicles II

III From genes and proteins to DNA 1900–1953

The end of the nineteenth century was a milestone of biology. Microorganisms were discovered, Mendel's work on genetics was accomplished, and institutes for investigating fermentation and other microbial processes were established by Koch, Pasteur, and Lister.

Biotechnology at the beginning of the twentieth century began to bring industry and agriculture together — a result that, in many areas of Europe and North America, irreversibly changed the face of the land. Even before WWI, cancer-causing viruses were discovered, and bacteria were used for the first time to treat sewage[1207].

During WWI, fermentation processes were developed that produced acetone from starch and paint solvents for the rapidly growing automobile industry.

Vitamins were identified as key growth factors and ***Sutton*** *coined the term "gene". The first chemotherapeutic agents were employed medically and* ***Thomas Hunt Morgan*** *commenced work on the fruit fly — an animal which has been indispensable in unraveling the fundamental mechanisms of heredity.*

In 1912, Bragg reported the use of X-rays as a method for studying molecular structures of simple crystalline substances. During the war years (1914–18), industrial processes were used for the mass production of chemical weapons, and the existence of bacterial phages proposed.

[1207] In 1918, this process was used to clean the River Seine in Paris. This river was an open-air sewer, the stench from which suffocated the city dwellers: it was caused by the disposal of the excrement and waste water directly into the river, a situation made worse by the increasing population. This increase was a result of an increase in human life expectancy, which was partly due to medical advances. The Seine did not begin to be odor-free again until around 1920. From that time onwards it was possible to have a picnic on the banks of the Seine without fainting. A *sewage treatment* processes toxic domestic and industrial waste into less harmful materias, i.e. H_2O and sludge: Organic wastes are *degraded* by the action of a complex community of microbes. Sewage treatment facilities provide an optimized environment for the organisms to process the waste, while at the same time allowing containment and monitoring of the process.

In 1920 **H.M. Evans** *(1882–1971)* and **J.A. Long** *(1879–1953) discovered human growth hormone* and *throughout that decade, plant hybridization was practiced with significant gains being made in the agricultural sector. In 1928* **Flemming** *discovered the first antibiotic — penicillin, still in use to this day.*

The first controlled reproduction of a cultured animal — a teleost — was reported in the same year and **Linus Pauling** *finally elucidated the physical laws governing the arrangement of atoms in 1935.*

Between 1940–1945 the large-scale production of penicillin became reality, cortisone was *manufactured in large quantities and* **Sanger** *described a method for examining the amino acid sequence of bovine insulin — chromatography.*

In 1941, a Danish researcher coined the phrase "genetic engineering".

Throughout the period 1940–1950, agricultural practice went through the transition from animal to machine power, jumping genes were recorded and artificial insemination of livestock was accomplished.

Between 1951–1960, the electron microscope came *into its own, and* **Watson** and **Crick** *elucidated the double-stranded helical nature of DNA.*

The "cold war" years were dominated by work with microorganisms in preparation for biological warfare, as well as work on antibiotics and fermentation processes.

The following timetable summarizes the milestones of progress in biotechnology during the first half of the 20th century.

Table 5.30: Milestones in the progress of biotechnology
1910–1953

1910:	Thomas H. Morgan	Proved that genes are carried on chromosomes. "*Biotechnology*" term coined
1918:		Germans use acetone produced by plants to make bombs
		Yeast grown in large quantities for animal and glycerol
		Using activated sludge for sewage treatment process
1920:		Boom or rayon industry
1927:	Herman Mueller	Increased mutation rate in fruit flies by exposing them to X-rays
1928:	Alexander Fleming	Discovered antibiotic properties of certain molds
1920–1930:		Plant hybridization
1938:		Proteins and DNA studied by X-ray crystallography
		Term "*molecular biology*" coined
1941:	George Beadle	Proposed "one gene, one enzyme" hypothesis
	Edward Tatum	
1943–1953:	Linus Pauling	Described sickle cell anemia calling it a molecular disease. *Cortisone* made in large amounts
		DNA is identified as the genetic material
1944:	Oswald Avery	Performed transformation experiment with Griffith's bacterium
1945:	Max Delbrück	Organized course to study a type of bacterial virus that consists of a protein coat containing DNA
Mid-1940's:		*Penicillin* produced
		Transition from animal power to mechanical power of farms
1950:	Erwin Chargaff	Determined that there is always a ratio 1 : 1 adenine to thymine in DNA of many different organisms
		Artificial insemination of livestock
1952:	Alfred Hershey Margaret Chase	Used radioactive labeling to determine that it is the DNA, not protein, which carries the instructions for assembling new phages
1953:	James Watson	Determined the double helix structure of DNA
	Francis Crick	

1949–1958 CE **Robert Hanbury Brown** (1916–2002, England and Australia). Radio Astronomer. Invented a new type of stellar interferometer (known as *intensity* or *correlation interferometer*) capable of measuring the angular diameter of stars with a resolution of 0.000 1 seconds of arc.

The principle of the intensity interferometer was then applied successfully by Hanbury Brown and his collaborators to *optical astronomy*.

In this field, measurements of correlation were not significantly affected by *atmospheric turbulence* which was a serious handicap in Michelson's method for measuring stellar diameters (1890).

The theory of the optical intensity interferometer (Hanbury Brown and Twiss, 1958) is complicated by the quantum nature of the photoelectric effect.

Brown was born in England and educated at the University of London. He worked at the Air Ministry (1936–1945), University of Michigan (1949–1964) and then became professor of astronomy at the University of Sydney, Australia (1964–1981).

Due to their enormous distance, the angular diameter of stars are extremely small, of the order of hundredths of a second of arc, even for nearby stars.

Michelson was the first to determine stellar diameters by interferometry. He employed mirrors to increase the optical path between the slits. One of the largest stars, Betelguese, was found to have an angular diameter of 0.047 seconds (the disc of this red giant was optically resolved by the *Hubble space telescope* in the 1990's). From the known distance, this corresponds to a linear diameter of about 280 times that of the sun.

Hanbury Brown's method makes it possible to determine much smaller stellar angular diameters than those measurable by Michelson's method.

The essential features of his intensity interferometer are as follows: Parallel light rays are collected by two curved searchlight mirrors of diameter 1.56 m and a variable baseline up to 14 m.

The light of each mirror is focused onto photocells the outputs of which are proportional to the instantaneous intensities $|E_1|^2$ and $|E_2|^2$ at the two mirrors. The signal from one mirror is fed into a variable delay line to meet the other undelayed signal from the second mirror in an electronic multiplier and integrator, the output of which is proportional to the time-average of the product: $|E_1|^2|E_2|^2$.

This quantity is known as the *second-order coherence function* of the two fields. It can be shown that for a distant extended source, a measurement of the second-order coherence between two receiving points yields the *lateral coherence width* and hence, the *angular diameter of the source*[1208].

The main advantage of the method of intensity interferometry is that high quality optical components and rigid mountings are not necessary.

1949–1966 CE **Avraham Robinson** (1918–1974, Israel and USA). Logician and mathematician. Invented *nonstandard analysis* (1966). Also did pioneering work in model theory and the metamathematics of algebraic systems.

Nonstandard analysis, a new branch of mathematics, marks a new stage of development in several famous ancient paradoxes. At its kernel is a revived notion of the "*infinitesimal*" which has roots stretching back to antiquity.

An infinitesimal is defined as a number that has zero as a limit i.e. infinitely small in absolute value yet greater than zero.

Leibniz had thought of them as being infinitely small positive or negative numbers that still had "*the same properties*" as ordinary numbers of mathematics.

On its face the idea seems self-contradictory. If infinitesimals have the same "properties" as ordinary numbers, how can they have the "property" of being positive yet smaller than any ordinary positive number?

It was by using a formal language that Robinson was able to resolve the paradox.

He showed how to construct a system containing infinitesimals that was identical with the system of "real" numbers w.r.t. all those properties *expressible in a certain formal language.*

Naturally the "property" of being positive yet smaller than any ordinary positive number will turn out *not* to be expressible in the language although the formal language can be "enlarged" to accommodate this novel new property – thereby escaping the paradox[1209].

1208 Let λ be the wave-length, L the distance earth-star, D the star's diameter and a the base-line separation. Then, the method of intensity interferometry is valid if $\lambda L \sim Da$. If $\lambda = 5000\mathring{A}$, $D = 10^6\ km$, $L = 50\ LY$, we must have $a \simeq 200$ m [1 LY=9.46×10^{12} km].

1209 The situation is familiar to users of computer machines; A computer accepts as input only symbols from a certain list that is given in advance to the user, and the symbols must be used in accordance with certain given rules. Computers are "stupid" because unlike humans they work in a formal language with a given vocabulary and a given set of rules. In contradistinction, humans work in a natural language, with rules that have never been made fully explicit.

Thus Robinson elevated the method of infinitesimals from the heuristic to the rigorous level.

The approach of formal logic succeeds by totally evading the question that excited **Berkeley** and all the other controversialists of former times, that is, whether or not infinitesimal quantities really exist in some objective sense.

From the viewpoint of the working mathematician, the important thing is that he regains certain methods of proof and certain lines of reasoning that had been fruitful since before Archimedes.

The notion of an infinitesimal neighborhood is no longer a self contradictory figure of speech but a precisely defined concept, as legitimate as any other in analysis, and the cumbersome *limit* procedure can be eliminated from many formal proofs.

Nonstandard approaches spread to almost all mainstream disciplines of mathematics:

- Non-Euclidean *geometry* and topology
- Non-Archimedean *arithmetic* and *analysis* (p-adics)
- Version of *set theory* where the axiom of choice or the continuum hypothesis, or both, are suppressed
- Nonstandard *analysis* where infinitesimal are actual numbers rather than limits
- Fuzzy (or quantum) *logic*

Robinson was born in Waldenburg, Germany to Jewish parents and emigrated to Israel (1933) on account of Nazi persecution. He studied mathematics in Jerusalem under **A.A. Fraenkel** and **Jacob Levitzki** (1904–1956). Went to the Sorbonne (1939) but was forced to flee when Germany invaded.

He reached England on one of the last small boats to evacuate refugees, and worked on aerodynamics during WWII.

After the war he attended London University and received there his Ph.D. (1949). He then held professorial appointments in Toronto (1951–1956), Jerusalem (1957–1965), Princeton (1966) and Yale (1967–1974), where he died of cancer at the age of 55.

1949–1973 CE **John Wilder Tukey** (1915–2000, USA). Mathematician. One of the most influential statisticians of the second half of the 20th century. Contributed to mathematical statistics, with an emphasis on its computational aspects.

Introduced modern technique for the estimation of spectra of time series and is particularly known (with **J.W. Cooley**) for the important *Fast Fourier Transform algorithm* (1965), known as FFT. Coined the words '*bit*' and '*software*'.

Tukey was born in New Bedford, MA. Received his doctorate from Princeton University (1939) and joined the AT&T Bell Laboratories (1945). He then spent decades as both a professor at Princeton and a researcher at the Bell Labs. Tukey was awarded the US National Medal of Science (1973).

FFT is one of the most frequently used mathematical tools for digital signal processing.

One can immediately see from a table of Fourier transforms that only a limited number of functions can be transformed into closed analytical forms. When a transform is not available in analytic form, it must be estimated by numerical computation.

The numerical approximation to the *Fourier Transform integral*, is time consuming even when a high speed computer is used. The algorithm of Tukey and Cooley reduces the computation time by decreasing the number of computations necessary to compute a one-dimensional Fourier Transform of a signal with n values from n^2 to order $n \log n$.

For instance an 8192 point discrete Fourier transform, which takes about 30 minutes of computer time when conventional integration programming is used, can be computed in less than 5 seconds with the algorithm.

As often happens, it appears to have been discovered a number of times before that, with the idea going back to **Gauss**! (1805), predating Fourier[1210] Analysis itself (**Fourier**, 1807).

[1210] For further reading, see:

- Titchmarsh, E.C., *Introduction to the Theory of the Fourier Integrals*, Oxford University Press, 1948, 394 pp.
- Sneddon, I.N., *Fourier Transforms*, McGraw-Hill, 1951, 542 pp.
- Booth, A.D., *Fourier Technique in X-Ray Organic Structure Analysis*, Cambridge University Press, 1948, 106 pp.
- Carslaw, H.S., *An Introduction to the Theory of Fourier's Series and Integrals*, Dover, 1950, 368 pp.
- Papoulis, A., *The Fourier Integral and its Applications*, McGraw-Hill, 1962, 318 pp.
- Lighthill, M.J., *Fourier Analysis and Generalized Functions*, Cambridge University Press, 1962, 79 pp.

The FFT transformed entire industries, as well as areas of research (e.g. *crystallography*) that rely heavily on the Fourier Transform.

The Fast Fourier Transform

Even with the sampling theorem, the number of samples in realistic physical signals is such that the calculation of the Fourier coefficients by direct methods is time-consuming even when electronic computers are used. To address this problem, **J. Cooley** and **J. Tukey** *introduced the Fast-Fourier-Transform (FFT) (1965). With it, calculations could be done in seconds that previously were too costly to do at all.*

The key to FFT is an algorithm that reduces the number of computations necessary to compute a Fourier transform of a signal with n *values from* n^2 *to* $n \log n$*. The idea goes back to* **Gauss** *(1805), predating Fourier analysis itself (1807).*

The Fourier transform of a time signal $g(t)$ *is* $G(\omega) = \int_{-\infty}^{\infty} g(t) e^{-i\omega t}\, dt$. *With* $\omega = 2\pi f$, *we have the pair*

$$G(f) = \int_{-\infty}^{\infty} g(t) e^{-2\pi i f t}\, dt, \qquad g(t) = \int_{-\infty}^{\infty} G(f) e^{2\pi i f t}\, df.$$

Because a digital computer works only with discrete data, the numerical computation of the Fourier transform requires discrete sampled values of $g(t)$, *which we call* g_k. *In addition, a computer can compute the transform* $G(f)$ *only at discrete value of* f, *that is, it can provide discrete samples of the transform,* G_l.

- Wiener, N., *The Fourier Integral and Certain of its Applications*, Dover, 1958, 201 pp.
- Byerly, W.E., *Fourier Series*, Dover, 1959, 287 pp.

If $g(kT)$ *and* $G(lf_0)$ *are the* k*th and* l*th samples of* $g(t)$ *and* $G(f)$*, respectively, and* M *is the number of samples in* a *signal of length* L*, then one defines*

$$g_k = T_0 g(t_k) = \frac{L}{M} g(kT_0), \qquad G_l = G(lf_0), \qquad f_0 = \frac{1}{L}.$$

The Discrete Fourier Transform (DFT) is then defined as

$$G_l = \sum_{k=0}^{M-1} g_k w^{kl}, \qquad w = e^{-\frac{2\pi i}{M}}, \qquad l = 0,1,2,3,\ldots,M-1 \tag{1}$$

$$g_k = \frac{1}{M} \sum_{l=0}^{M-1} G_l w^{-kl}, \qquad k = 0,1,2,3,\ldots,M-1 \tag{2}$$

Since the sum has to be performed for every $0 \leq l < M-1$ *and each sum has length* M*, the direct summation requires at least* M^2 *multiplications. But this is not actually necessary, due to the redundancy in the values of the rotating unit vector (in the complex plane)* $w^{kl} = \exp(-2\pi i \frac{kl}{M})$*.*

We can see this in either of the two following ways. First, algebraically (in case M *is even)*

$$\begin{aligned} G_l &= \sum_{k=0}^{M-1} e^{-\frac{2\pi i kl}{M}} g_k \\ &= \sum_{k=0}^{\frac{M}{2}-1} e^{-2\pi i (2k)\frac{l}{M}} g_{2k} + \sum_{k=0}^{\frac{M}{2}-1} e^{-2\pi i (2k+1)\frac{l}{M}} g_{2k+1} \\ &= \sum_{k=0}^{\frac{M}{2}-1} e^{-2\pi i (2k)\frac{l}{M}} g_{2k} + w^l \sum_{k=0}^{\frac{M}{2}-1} e^{-2\pi i (2k)\frac{l}{M}} g_{2k+1} \\ &= G_l^{\text{even}} + w^l G_l^{\text{odd}} \end{aligned} \tag{3}$$

Thus, in order to perform a Fourier transform of length M*, one needs to do two Fourier transforms:* G^{even} *and* G^{odd} *of lengths* $\frac{M}{2}$*, on the even and odd elements, respectively.*

These two, so called subtransforms can then be combined with the appropriate factors w^l *to give the desired Fourier transform* G*.*

Note that in (3), the index l *may be restricted to the interval* $0 \leq l \leq \frac{M}{2} - 1$*, because* G_l^{even} *and* G_l^{odd} *are periodic in* l *with length* $\frac{M}{2}$*. Now, by a reapplication of this principle, the two transforms are themselves a sum of two transforms of length* $\frac{M}{4}$*.*

Finally, a recursive scheme reduces the problem down to successive even and odd subdivisions of the data, until reaching the one-point transform.

It can be shown that in one dimension the FFT requires $M\log_2 M$ *operations (multiplications) as compared to* M^2 *by the direct FT.*

In two dimensions an array of M_1M_2 *points requires* $(M_1M_2)\log_2(M_1M_2)$ *for the FFT as compared to* $(M_1M_2)(M_1+M_2)$ *for the DFT. For an array of* 1024×1024 *this represents a speed gain of* $50,000$*!*

An alternative way to show the same thing goes back to Gauss (1805) [in the matrix notation of Cayley (1858)]. It hinges on the simple observation that (1) can be recast as the linear transformation

$$\begin{bmatrix} G_0 \\ G_1 \\ G_2 \\ \vdots \\ G_{M-1} \end{bmatrix} = \begin{bmatrix} w^0 & w^0 & w^0 & \dots & w^0 \\ w^0 & w^1 & w^2 & \dots & w^{M-1} \\ w^0 & w^2 & w^4 & \dots & w^{2(M-1)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ w^0 & w^{M-1} & w^{2(M-1)} & \dots & w^{(M-1)(M-1)} \end{bmatrix} \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ \vdots \\ g_{M-1} \end{bmatrix} \tag{4}$$

or

$$[G_M] = [F_M][g_M] \tag{5}$$

If there are only M *sampled points, then there are only* M *spectrum points. It thus seems, at first sight that the generation of all* M *coefficients* G_{M-1} *requires at least* M^2 *multiplication. But this is not the case; all elements of the matrix* F_M *in (4) have a peculiar character which can be used for simplification of the matrix.*

Indeed, considering $w^{kl} = \exp(-2\pi i \frac{kl}{M})$ *as a rotating unit vector in the complex plane, and choosing* $M = 2^N$, *the redundancy of* w^{kl} *for all permissible values of* k *and* l *leads to the factorization*

$$F_{2^N} = \begin{bmatrix} I_{2^{N-1}} & D_{2^{N-1}} \\ I_{2^{N-1}} & -D_{2^{N-1}} \end{bmatrix} \begin{bmatrix} F_{2^{N-1}} & 0 \\ 0 & F_{2^{N-1}} \end{bmatrix} \begin{bmatrix} \textit{shuffle} \end{bmatrix} \tag{6}$$

where $F_{2^{N-1}}$ *is the matrix in (4),* $I_{2^{N-1}}$ *is the unit matrix with* 2^{N-1} *elements in its diagonal,* [*shuffle*] *is a column vector with shuffled components of* $g_{2^{N-1}}$, *and* $D_{2^{N-1}}$ *is a diagonal matrix of order* 2^{N-1} *with diagonal values of* $(w^0, w^1, w^2, \dots, w^{2^{N-1}})$, $w = e^{-2\pi i \frac{s}{2^N}}$, $s = 0, 1, 2, \dots, 2^{N-1}$.

This factorization cuts the work of computing a Fourier transform almost in half. But $F_{2^{N-1}}$ *can again be factorized in the same way, giving new submatrices with* 2^{N-2} *elements in each, which themselves can be factorized. At the end, this systematic reduction of the number of multiplication leads to* $M\log_2 M$ *basic multiplications.*

1949–1978 CE **Michael James Lighthill** (1924–1998, England). Mathematician. A prominent applied mathematician of the 20th century. A pioneer in supersonic aeronautics, in oceanographic studies and astrophysics.

He created the field of bio-fluid-dynamics (the study of how animals move through air or water), as well as the study of the fluid mechanics of the cardiovascular system.

Lighthill won a scholarship to Trinity College, Cambridge when he was just 15.

After WWII he went to teach at Manchester University, where he became a professor. He soon became known for his theoretical work on jet engines, discovering the *Lighthill law* which states that the acoustic power radiated by a jet is proportional to the 8th power of the jet speed.

In 1953 he became a fellow of the Royal Society and in 1959 moved to be Director of the Royal Aircraft Establishment at Farnborough for five years. There, his work in wind-tunnels was to prove critical to the development of the Concorde.

During 1969–1979 he served as Lucasian Professor of Mathematics at Trinity College, Cambridge, continuing to publish on fluid dynamics (particularly the theory of waves in ocean and atmosphere) and on chaos theory and the unpredictability of large systems. In 1979, Lighthill became a provost of University College, London.

He died on July 17, 1998 while swimming around the Channel Island of Stark.

1950–1980 CE **Heinz Kohut** (1913–1981, USA). Psychoanalyst and psychiatrist. Developed *Self–Psychology*[1211], a school of thought within psychoanalytic theory that transformed the modern practice treatment approach. Veering away from the psychoanalysis dogma, he submitted a new idea that went beyond Freud's conceptualization.

In the aftermath of WWII and the Holocaust, Freudian analysis was too focused on individual guilt and failed to reflect the emotional interests and

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- Kohut, H., *The Analysis of Self*, 1971
- Kohut, H. and A., Goldberg, *How Does Analysis Care?*, 1984

needs of people struggling with issues of identity, meaning, ideas, and self–expression. Though he initially tried to remain true to the traditional analytic viewpoint with which he had become associated and viewed the self as a separate but coexistent to the ego, Kohut later rejected Freud's structural theory of the id, ego, and superego. He then developed his ideas around what he called the *tripartite self*.

According to Kohut, this three-part self can only develop when the needs of one's *self states*, including one's sense of worth and well–being, are met in relationships with others. In contrast to traditional psychoanalysis that focused on *drives* (instinctual motivations of sex and aggression), internal conflicts and fantasies, *self psychology* thus placed a great deal of emphasis on the vicissitudes of relationships.

Kohut expanded on his theory during the 1970's, a time in which aggressive individuality, overindulgence, greed, and restlessness left many people feeling empty, fragile and fragmented.

A key concept in Kohut's psychology is *abstract empathy*. By that he meant the role of empathy in defining the science of psychoanalysis. According to Kohut, any science is defined by an object of study and a method by which the data of that science is collected.

For example, the physical sciences have as their object of study the discernible world that can be observed via the senses and those instruments that enhance the senses.

On the other hand, psychoanalysis has as its object of study the inner life of man (the data of human experience) while the method by which the analyst makes his observations is introspection into oneself and vicarious introspection or empathy into another.

In other words, empathy is nothing more than the "tool" or "instrument" that permits psychoanalysts to collect their data, which over time can be translated into explanations in the clinical setting and abstract constructs in the theoretical realm. It was this methodology that made it possible for Freud to discover transference, countertransference, defenses, and resistance.

As Freud moved away from the empathic mode of data collecting, he introduced constructs and assumptions that belong to other sciences.

One example is that of the "drive", which was assumed to be on the borderland between the psyche and the soma. Thus "drive theory" psychoanalysis could no longer be viewed as a pure psychology but rather as an amalgam of psychology and biology, that is, a psycho-biology or bio-psychology.

By the operational definition of empathy Kohut is referring to the clinically relevant definition of empathy as "the capacity to think and feel oneself into the inner life of another person".

Derived from the German term *Einfühlung*, empathy evolved in its meaning to connote “feeling into” or “searching one’s way” into the experience of another.

For Kohut, empathy is simply what allows an individual to know another’s experience without losing one’s objectivity.

In other words, empathy is experience-near observation and nothing more.

By *selfobject* Kohut (1971, 1984) means the experience of another – more precisely, the experience of impersonal functions provided by another – as part of the self.

Selfobject transference, therefore, is the patient’s experience of the analyst as an extension or continuation of the self, that is as fulfilling certain vital functions that had been insufficiently available in childhood to be adequately transformed into reliable self structure.

Because of its positive, open, and emphatic stance on human nature as a whole as well the individual, self psychology is considered one of the “four psychologies” (the others being Drive Theory, ego psychology, and object relations); that is, one of the primary theories on which modern dynamic therapists and theorists rely.

Though dynamic theory tends to place emphasis on childhood development, Kohut believed that the need for such self–object relationships does nor end at childhood but continues throughout all stages of a person’s life.

Kohut was born to Jewish parents and received his M.D in neurology at the University of Vienna. He fled the Nazi occupation of Austria and settled in Chicago as a member of Chicago Institute of Psychoanalysis.

1951–1973 CE **Leonard Bernstein** (1918–1990, USA). Distinguished musician and scientific musicologist. An influential figure in the history of classical and 20-th century music. In “The Unanswered Question-six lectures at Harvard” (1973), he used contemporary linguistics to analyze and compare musical construction to language. His other important publications are: “The Joy of Music” (1959), “The Infinite Variety of Music” (1966) and “Young People’s Concerts” (1969).

Bernstein was born in Lawrence MA to a Russian Jewish family. He studied music at Harvard University and the Curtis Institute of Music in Philadelphia. At the age of 25 he was already an assistant conductor of the New York Philharmonic Orchestra. He later conducted the Vienna Philharmonic (1970), the Israel Philharmonic (1978) and the Berlin Philharmonic (1979) and was appointed professor at Harvard University (1973).

1951–2008 CE **Martin Deutsch** (1917–2002, USA). Physicist. Discoverer of *Positronium*. Measured and confirmed the existence of a substance composed of a pair of electron and positron (positronium) whirling about each other.[1212]

Deutsch was born to Jewish parents in Vienna. He moved to Zurich (1934) and then to Cambridge MA (1935) where he enrolled at MIT. He received his Ph.D. there (1941) and joined the Manhattan Project in Los Alamos, N.M (1943). In 1946 he joined the physics faculty at MIT, retiring in 1987.

Positronium is a bound state of an electron and a positron which is formed when these two particles are brought together at low enough relative speed. They attract and orbit each other in a "dance of death".

This metastable quasi–atom exists for a fraction of a second before the e^+e^-–pair annihilate into γ rays; its two possible decay modes are 2γ and 3γ, depending on the orbital and spin of the initial positronium state. Mathematically, the positronium is exactly equivalent to an ideal hydrogen atom (no nuclear structure) with two differences:

- The electron mass in all formula (Rydberg, Dirac etc.) is replaced by the reduced mass of the e^+e^- pair; since the electron and positron have equal masses, their reduced mass is exactly $\frac{1}{2}m_e$
- The hyperfine splitting is much stronger than in hydrogen, because the gyromagnetic ratio of *both* particles involves the Bohr magneton (no nuclear magneton).

Since the dynamics of Positronium involve only QED phenomena (no nucleus!), Positronium physics is an ideal test of QED. Its discovery and study thus played a key role in the verification of modern QED theory of Feynman et al.

Positronium has two important analogues: (1) the *charmonium* mesons (composed of a mutually orbiting pair of charmed and anticharmed quark), which decays into 2 or 3 gluons; (2) an *exciton* – a metastable state of an electron and hole in a semiconductor; The two annihilate to produce one or more lattice phonons.

[1212] Positronium's properties were predicted by **Carl D. Anderson** of Caltech in 1932.

History of Biology and Medicine, V – The 20th century

In the 20th century, the rediscovery of ***Mendel****'s work led to the rapid development of genetics by* ***T.H. Morgan*** *and his students. By the 1930's, the combination of popular genetics and the natural selection hypothesis let to the 'neo-Darwinian synthesis' and the rise of the discipline of evolutionary biology. New biological disciplines developed rapidly, especially after the discovery of the DNA structure.*

Following the cracking of the genetic code, biology has split between organizational biology (consisting of ecology, ethology, systematics, paleontology, evolutional biology, developmental biology and other disciplines that deal with whole organisms or groups of organisms) — and the constellation of disciplines related to molecular biology (including: cell biology, biophysics, biochemistry, neuroscience, immunology, and many other overlapping subjects).

In about 1902, the chromosome was identified as being the site of the genes, and its central position in heredity and development was finally realized. By the end of the 19th century all of the major pathways of drug metabolism had been discovered. In the early decades of the twentieth century, minor components of foods in human nutrition, the vitamins, began to be isolated and synthesized. Then in the 1920's and 1930's the metabolic pathways of life, such as the citric acid cycle and glycolysis, finally began to be worked out by biochemists. This work continued to be very actively pursued for the rest of the century and into the next. During 1939–1941 ***Fritz Lipmann*** *showed that ATP is the universal carrier of energy in the cell, and in the mid-1950's the power generators of the cell, the mitochondria, also began to be understood.*

Oswald Avery *conclusively showed in 1943 that DNA was the genetic material of the chromosome, not its protein. By 1953* ***James D. Watson*** *and* ***Francis Crick*** *showed that the structure of DNA was a double helix and showed its probable connection to replication. The nature of the genetic code was unraveled experimentally starting with the work of* ***Nirenberg****,* ***Khorana*** *and others in the late 1950's.*

The history of molecular biology begins in the 1930's with the convergence of various, previously distinct and unrelated branches of biology: biochemistry, genetics, microbiology, and virology. Numerous physicists and chemists also took an interest in this new material.

As its name indicates, this new branch of biology attempts to explain the phenomena of life starting from the macromolecular properties that generate them. Two categories of macromolecules in particular are the focus of the *molecular biologist: 1) nucleic acids, among which the most famous is deoxyribonucleic acid (or DNA), the constituent of genes, and 2) proteins, which are the active agents of living organisms. The scope of molecular biology therefore is to characterize the structure, function and relationships between these two types of macromolecules. This relatively limited definition will suffice to allow us to establish a date for the so-called "molecular revolution", or at least to establish a chronology of its most fundamental developments.*

In 1940, ***George Beadle*** *and* ***Edward Tatum*** *demonstrated the existence of a precise relationship between genes and proteins. In 1944,* ***Oswald Avery****, working at the Rockefeller Institute of New York, demonstrated that genes are made up of DNA. In 1952,* ***Alfred Hershey*** *and* ***Martha Chase*** *confirmed that the genetic material of the bacteriophage, the virus which infects bacteria, is made up of DNA. In 1953,* ***James Watson*** *and* ***Francis Crick*** *discovered the double helical structure of the DNA molecule. In 1961,* ***Francois Jacob*** *and* ***Jacques Monod*** *hypothesized the existence of an intermediary between DNA and its protein products, which they called messenger RNA. Between 1961 and 1965, the relationship between the information contained in DNA and the structure of proteins was determined: there is a code, the genetic code, which creates a correspondence between the succession of nucleotides in the DNA sequence and a series of amino acids in proteins. At the beginning of the 1960's, Monod and Jacob also demonstrated how certain specific proteins, called regulative proteins, latch onto DNA at the edges of the genes and control the transcription of these genes into messenger RNA; they direct the "expression" of the genes.*

The chief discoveries of molecular biology took place in a period of only about twenty-five years. Another fifteen years were required before new and more sophisticated technologies, united today under the name of genetic engineering, would permit the isolation and characterization of genes, in particular those of highly complex organisms.

If we evaluate the molecular revolution within the context of biological history, it is easy to note that it is the culmination of a long process which began with the first observations through a microscope in the 18^{th} *century. The aim of these early researchers was to understand the functioning of living organisms by describing their organization at the microscopic level. From the end of the* 18^{th} *century, the characterization of the chemical molecules which make up living beings gained increasingly greater attention, along with the birth of physiological chemistry in the* 19^{th} *century, developed by the German chemist* ***Justus von Liebig*** *and following the birth of biochemistry at the beginning of the* 20^{th}*, thanks to another German chemist* ***Eduard Buchner****. Between the molecules studied by chemists and the tiny structures visible under the optical microscope, such as the cellular nucleus or the chromosomes, there was an obscure zone, "the world of the ignored dimensions," as it was* *called by the chemical-physicist Wolfgang Ostwald. This world is populated by colloids, chemical compounds whose structure and properties were not well defined.*

The development of molecular biology is also the encounter of two disciplines which made considerable progress in the course of the first thirty years of the twentieth century: biochemistry and genetics. The first studies the structure and function of the molecules which make up living things. Between 1900 and 1940, the central processes of metabolism were described: the process of digestion and the absorption of the nutritive elements derived from alimentation, such as the sugars. Every one of these processes is catalyzed by a particular enzyme. Enzymes are proteins, like the antibodies present in blood or the proteins responsible for muscular contraction. As a consequence, the study of proteins, of their structure and synthesis, became one of the principal objectives of biochemists.

The second discipline of biology which developed at the beginning of the 20th century is genetics. After the rediscovery of the laws of Mendel through the studies of ***Hugo de Vries****,* ***Carl Correns*** *and* ***Erich von Tschermack*** *in 1900, this science began to take shape thanks to the adoption by* ***Thomas Hunt Morgan****, in 1910, of a model organism for genetic studies, the famous fruit fly (Drosophila melanogaster). Shortly after, Morgan showed that the genes are localized on chromosomes. Following this discovery, he continued working with Drosophila and, along with numerous other research groups, confirmed the importance of the gene in the life and development of organisms. Nevertheless, the chemical nature of genes and their mechanisms of action remained a mystery. Molecular biologists committed themselves to the determination of the structure, and the description of the complex relations between, genes and proteins.*

The development of molecular biology was not just the fruit of some sort of intrinsic "necessity" in the history of ideas, but was a characteristically historical phenomenon, with all of its unknowns, imponderables and contingencies: the remarkable developments in physics at the beginning of the 20th century highlighted the relative lateness in development in biology, which became the "new frontier" in the search for knowledge about the empirical world. Moreover, the developments of the theory of information and cybernetics in the 1940's, in response to military exigencies, brought to the new biology a significant number of fertile ideas and, especially, metaphors.

The choice of bacteria and of its virus, the bacteriophage, as models for the study of the fundamental mechanisms of life was almost natural — they are the smallest living organisms known to exist — and at the same time the fruit of individual choices. This model owes its success, above all, to the fame and the sense of organization of ***Max Delbrück****, a German physicist,* *who was able to create a dynamic research group, based in the United States, whose exclusive scope was the study of the bacteriophage: the* School of the Phage.

The geographic panorama of the developments of the new biology was conditioned above all by preceding work. The US, where genetics had developed the most rapidly, and the UK, where there was a coexistence of both genetics and biochemical research of highly advanced levels, were in the avant-garde. Germany, the cradle of the revolutions in physics, with the best minds and the most advanced laboratories of genetics in the world, should have had a primary role in the development of molecular biology. But history decided differently: the arrival of the Nazis in 1933 — and, to a less extreme degree, the rigidification of totalitarian measures in fascist Italy — caused the emigration of a large number of Jewish and non-Jewish scientists. The majority of them fled to the US or the UK, providing an extra impulse to the scientific dynamism of those nations. These movements ultimately made molecular biology a truly international science from the very beginnings.

Table 5.31: Notable Biologists, Biochemists and Men of Medicine (1895–1950)

Key:

BI = Biochemistry
BP = Biophysics
CR = Crystallography
BO = Botany
B = Biology
S = Surgery
CY = Cytology
A = Anatomy
NU = Nutrition
P = Physiology
IM = Immunology
M = Medicine
BA = Bacteriology
H = Heredity
N = Neuroscience
V = Virology
PA = Paleontology
C = Chemistry
PR = Pharmacology
PG = Population Genetics
EM = Embryology
PA = Pathology
PC = Physical Chemistry
MB = Mathematical Biology
BT = Biotechnology
MB = Molecular Biology
EN = Endocrinology
EP = Epidemics

Name	*fl.*	*specification*		*NP*
Edward Buchner	*1892–1901*	*Fermentation*	*C*	*1907*
W.C. Röntgen	*1895*	*X-rays*	*M, BP*	*1901*
C.S. Sherington	*1895–1930*	*Neuron, synapse*	*N*	*1932*
M. von Gruber	*1896*		*M, BA*	
Walter Reed	*1896–1902*	*Yellow Fever*	*S, M*	
J.B.V. Bordet	*1898–1919*		*BA, IM*	*1919*
H. Dresser	*1899*	*Aspirin*		
Jacque Loeb	*1899–1913*		*B*	
Carl Correns	*1900*	*Heredity*	*BO, G*	
Eric von Tschermak	*1900*	*Heredity*	*BO, G*	
Hugo de Vries	*1900–1907*		*BO, G*	
Karl Landsteiner	*1901–1940*	*Blood types*	*IM, PA*	
W.S. Sutton	*1902–1903*		*M, G, H*	
A.E. Garrod	*1902*		*M, BI, G*	
W.M. Bayliss	*1902–1908*	*Hormones*	*P*	
E.N. Starling	*1902–1908*	*Hormones*	*P*	
W. Einthoven	*1903*	*Electro-cardiograph*	*M, P*	*1924*
Robert Barany	*1903–1910*	*Inner ear*	*P, M*	*1914*
Carl Neuberg	*1903–1911*		*M, B, BI*	
Arthur Harden	*1905*		*BI*	

Table 5.31: (Cont.)

Name	*fl.*	*specification*		*NP*
R.M. Willstätter	*1905–1925*	*Photosynthesis*	*BI*	*1915*
Alexis Carrel	*1905*	*Heat Surgery*	*B, S*	
Herman Nernst	*1906*	*'Nernst Equation'*	*PC, N*	
Emil Hermann Fischer	*1907*	*Peptide bond*	*C, BI*	
Axel Holst	*1907*	*Scurvey*	*NU, BI*	
Theodore Frölich	*1907*	*Scurvey*	*NU, BI*	
G.H. Hardy + W. Weinberg	*1908*		*M, PG*	
W.L. Johannsen	*1908–1909*	*Gene*	*P, H*	
Aaron Levene	*1909–1929*	*RNA, DNA*	*BI*	
Hans Fischer	*1910*	*Chlorophyll; hemin*	*P*	*1930*
T.H. Morgan	*1910–1917*		*H, G*	*1933*
Casimir Funk	*1912*	*Vitamin*	*BI*	
F.G. Hopkins	*1912*	*Vitamin*	*BI*	
O.H. Warburg	*1912*		*CY, BI*	
A.V. Hill	*1913*		*P*	*1922*
L. Michaelis	*1913*		*P*	
E.V. McCollum	*1913–1922*		*BI, M*	
George de Hevesy	*1913–1935*	*Radioactive tracing*	*M, BP*	*1943*
Bela Schick	*1913–1942*		*M, IM*	
Clement von Pirquet	*1913*	*Allergy*	*M, IM*	
E.C. Kendall	*1914*		*BI*	*1950*
Adolf O.R. Windaus	*1915–1938*	*Cholesterol*	*BI*	*1928*
F.W. Twort	*1915–1917*	*Bacteriophage*	*BA*	
Felix d'Herelle	*1915–1917*	*Bactoriophage*	*BA*	
D. da Roche Lima	*1916*		*BA*	
H.T. Ricketts	*1916*		*BA*	
O.F. Meyerhof	*1918*		*M, B, P, BI*	*1922*
B.A. Houssay	*1919–1943*	*Pituitary gland*	*P, EN*	*1947*
Karl Ereky	*1919*	*Biotechnology*	*BT*	
E.D. Adrian	*1919*		*M, P*	*1932*

Table 5.31: (Cont.)

Name	fl.	specification		NP
A.J. Lotka	1920		MB	
Herbert M. Evans	1920	Human growth hormone	BI, P	
Otto Loewi	1920–1929	Neurobiology	N	1936
Henry H. Dale	1920–1929	Neurobiology	N	1936
Joseph Erlanger	1921–1935	Neurobiology	N	1944
H.S. Gasser	1921–1935	Neurobiology	N	1944
F.G. Banting + J.J.R. Macleod	1921	Insulin	M, P	1923
R.A. Fisher	1921–1942		G	
E.B. Harvey	1924	Induction	B, EM	
Hans Spemann	1924	Induction	B, EM	1935
J.B.S. Heldane	1924	Induction	B, EM	1935
J.D. Bernal	1924–1968	X-ray crystallography	CR, BP, MD	
Hans Berger	1924–1929	EEG	P, N	
A.I. Oparin	1924–1957	Origin of Life	BI	
Reymond Dart	1925	Origin of Man	A, PA	
Vito Volterra	1926–1931		MB, PG	
Vladimir Varadansky	1926	Biogeochemistry	BI	
Hermann J. Muller	1927	X-ray, gene-mutation	G	1946
Bernhard Zondek	1927	Sex hormones	P, M, G	
W.O. Kermack	1927		EP	
A.G. McKendrick	1927		EP	
Barbara McClintock	1927–1950	‘Jumping Genes’	G	1983
Alexander Fleming	1928–1940	Penicilin	M, BA	1945
Ernst Boris Chain	1928–1940	Penicilin	M, BA	1945
Howard W. Florey	1928–1940	Penicilin	M, BA	1945
Georg von Bekesy	1928–1960	Physiology of hearing	P, BP	1961

Table 5.31: (Cont.)

Name	*fl.*	*specification*		*NP*
W.C. Rose	*1930*	*Essential amino acids*	*BI, P*	
Adolf Butenandt	*1931–1934*	*Sex hormones*		*1939*
Edward A. Doisy	*1931–1934*	*Sex hormones*		
L. Ruzicka	*1931–1934*	*Sex hormones*		*1939*
Linus Pauling	*1931–1946*	*Hydrogen bond*	*C*	*1954*
Sewell Wright	*1931–1968*		*PG*	
Rene J. Dubos	*1931–1939*	*Enzymes, antibiotics*	*BI, BA*	
Oswald T. Avery	*1931–1943*	*Enzymes, gene, DNA*	*BI, BA*	
Gerhard Domagk	*1932*	*Prontosil*	*M, PA*	*1939*
Walter B. Cannon	*1932*	*Homeostasis*	*P*	
H.A. Krebs	*1932–1937*	*Krebs' cycle*	*BI*	*1953*
Fritz Lipmann	*1932–1937*	*Coenzyme A*	*M, P*	*1953*
Ragnar Granit	*1932–1956*	*Vision*	*P*	*1967*
Haldan K. Hartline	*1932–1956*	*Vision*	*P*	*1967*
N. Reshevsky	*1933–1946*		*MB*	
Max Delbruck	*1933–1946*	*Molecular Biology*	*MB, B, P*	*1969*
Rudolf Schoenheimer	*1934–1941*		*M, BI*	
George Wald	*1934–1971*	*Vision*	*BI*	*1967*
Albert Györgi	*1935*	*Cellular energy metabolism*	*P, BI*	
W.M. Stanley	*1935*	*Enzymes*	*BI*	*1946*
J.H. Northrop	*1935*	*Enzymes*	*BI*	*1946*
J.B. Summer	*1935*	*Enzymes*	*BI*	*1946*
Gerti and Carl Cori	*1935*	*Cori cycle*	*BI*	*1947*
A.N. Belozersky	*1935–1939*	*DNA*	*BI*	
Arthur Tansley	*1935*	*Ecosystems*	*B*	
F.C. Bawden	*1937*	*RNA*	*BI*	
A.W.K. Tiselius	*1937*	*Serum proteins*	*BI*	*1948*

Table 5.31: (Cont.)

Name	fl.	specification		NP
Selig Hecht	1938	Vision	BP	
Max F. Perutz	1939–1968	Hemoglobin	BI	1962
M. David Kamen	1940	Carbon 14	BI	
Abraham S. Waksmann	1940	Streptomycin	MI, M	1952
George Beadle	1940	Genes and proteins	BI, G	1958
Edward Tatum	1940	Genes and proteins	BI, G	1958
Dorothy M. Hodgkin	1942–1945	X-ray crystallography	BI, CR	1964
S.E. Luria	1942		BI, M	1969
W.J. Kolff	1943	Kidney machine	M	
A.J.P. Martin	1944	Paper chromatography	BI	1952
R.L.M. Synge	1944	Paper chromatography	BI	
G. Pincus	1944	Contraceptives	M	
A.L. Hodgkin	1945–1952	Neurobiology	N	1963
Andrew F. Huxley	1945–1952	Neurobiology	N	1963
Julius Axelrod	1945–1965	Neurobiology	PR	1970
Bernard Katz	1945–1965	Neurobiology	BP	1970
F.A. Lipmann	1947	Bioenergetics	BI	1953
A.R. Todd	1947	ADP, ATP	BI	
Arthur Kornberg	1947–1953	DNA	BI	1959
Severo Ochoa	1947–1953	RNA	BI	1959
Tadeus Reichstein	1948	Hormones	BI	1950
P.S. Hence	1948	Hormones	BI	1950
E. Chargaff	1949	DNA	BI	
Melvin Calvin	1949	Calvin's cycle	BI	1961
Peter Medawar	1949–1957	Imm. tolerance	Z, IM	1960
Alick Isaacs	1951–1957	Interferon	V	
Jean Lindemann	1951–1957	Interferon	V	

Table 5.31: (Cont.)

Name	*fl.*	*specification*		*NP*
Robert B. Woodward	*1951*	*Cortisone*	*C*	*1965*
Charles B. Huggins	*1951*	*Hormones*	*S, P*	*1966*
Alfred Hershey	*1952*	*DNA*	*BA*	*1969*
Martha C. Chase	*1952*	*DNA*	*B*	
Ulf von Euler	*1952*	*Neurobiology*	*PR*	*1970*
F. Sanger	*1953*	*Insulin*	*BI*	*1958*
Frederick Hopkins		*Vitamins*		*1929*
Carl von Voit		*Caloric energy*	*P*	
Max Rubner		*Caloric energy*	*P*	
Christiaan Eijkman		*Vitamins*	*M, P*	*1929*

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6. *Deep Principles – Complex Structures*

1950–2008

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Overview

The roots of the evolutionary progress of mathematics, physics and technologies based upon applied science during the second half of the 20th century are anchored in the early days of the Industrial Revolution.

*Science interacted strongly with political, economic and other social changes in Western society since the Industrial Revolution in the middle of the 18th century. Major events such as the advent of modern industrialization in England (1749–1760) and the consequent rise of the first British Empire, the nascence and rise of the United States from British North America (*18th *century on), the French Revolution (1793), the Unification of Germany (1871), WWI (1914–1918) and WWII (1939–1945), impacted science and were impacted by it in return.*

In 1700, England was still chiefly a rural land – there were no big towns except London, and the economy was primarily based on agriculture. Previously, the revocation (1685) of the edict of Nantes (1598) caused extensive emigration of French Huguenots and their dispersal in various countries, enriching these with their talents. The ensuing industrialization was partly a result of the oppression of the Puritans. Until 1800, science in England was the business of gentlemen and noblemen, mostly occupied with Evolution and Geology. Science and philosophy were empirical. *At the turn of the 19th century British science as a whole became more theoretical again:* ***Thomas Young*** *(1773–1829) revived the wave theory of light (1801) and* ***John Dalton*** *(1803) introduced atomic theory into chemistry.*

In France, during 1740–1819, philosophers sowed the seeds of a rationalist *revolt which,* inter alia *began the French Revolution. In a complementary way, the French then became more empirical, spurred on to experimental and applied science by the needs of the Napoleonic Wars.*

German science, with its metaphysical roots, influenced by the philosophy of ***Kant***[1] *(1781) and* ***G.H.F. Hegel*** *(1817), began to blossom in the first half of the 19th century and reached its peak during 1870–1930.*

The industrial revolution had changed the status of science and scientists: in the first place it caused scientists to be considered as representatives of radicalism (i.e. progressive). Then it shifted the emphasis towards the applied side of science: The rapid growth of applied mathematics *in the wake*

[1] **Berkeley** and **Kant** in turn were influenced by **D. Hume** while Berkeley influenced **Mach** and **Einstein**.

of the industrial revolution called for the establishment of a discipline of approximations through which algorithms could be systematized and developed methodically to answer the growing needs of the exact sciences.

Above all, since ***J.C. Maxwell*** *and* ***Hertz****, science began to become a dominant force in Western society, in times of peace as well as war.*

The 19th century was rushing toward its close, propelled by steam and electricity. The railroad, the steamship, the telegraph, the telephone, the phonograph, photography, radio and the internal combustion engine took their places in human life in bewildering succession.

It had been a magnificent century for mathematics; most of the mathematical tools later needed for GTR and quantum mechanics were forged by 19th century mathematicians. In fact, 20th century physics is essentially based on 19th century mathematics! Thus GTR is based on Riemannian geometry and tensor analysis; Quantum mechanics and quantum field theory are based upon abstract algebra and functional analysis – and specifically the theories of matrices, Lie algebras and Lie groups, differential geometry, differential and algebraic topology, as well as Hilbert spaces and c^*–algebras. Many of these mathematical fields also supported Nonlinear Dynamics.[2]

The 20th century was an epoch of Revolutions in Physics and Biology. Physics was the first of the natural sciences to become fully modern and highly mathematical. Chemistry followed in the wake of physics, but biology, the retarded child, lagged far behind. Even in the time of ***Newton*** *and* ***Galileo****, men knew more about the moon and other heavenly bodies than they did about their own.*

It was not until the late 1940's that this situation changed. The postwar period ushered in a new era of biological research.

World-War I made chemistry respectable (even ***Ernest Rutherford*** *often presented himself as a chemist). World-War II made physics respectable (Atomic bomb, Radar, Rockets, Jet planes, Computers).*

During 1916–1928 there emerged the two most revolutionary physical theories ever – the General Theory of Relativity and Quantum Mechanics, neither

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[2] *Nonlinear Dynamics* – a field of applied mathematics that started in the 19th century by the advent of computer simulations in the second half of the 20th century. And all of these fields, of course, benefited from 19th century work on differential and integral equations, integral transforms and complex analysis. The needs of 20th century developments in technology and in the natural and social sciences also spurred many extensions and ramifications of pre-existing mathematical fields – e.g. the theory of stochastic processes (from probability theory), algorithms and computational complexity, *index theorems*, etc.

of which had been adumbrated by any scientist, philosopher or even science-fiction writer. In terms of breadth of applicability and accuracy of predictions, Quantum Mechanics (augmented by STR in the 1930's and 1940's to become QFT) proved itself the most fruitful theoretical framework ever developed by man.

After WWII the USA emerged as a political, military, economic, technological and scientific superpower. It supported the largest scientific establishments in the history of mankind (accelerators, Manhattan Project, NASA). New discoveries are constantly being made, and many of these discoveries have important political, economical and social overtones.

But the 20th century left in its wake an overcrowded, polluted and dangerous world – disillusioned by global wars and conflicts, and with the very concepts of progress and individual responsibility that gave rise to it; a morally rudderless world stricken by major environmental problems, social disorders and the rise of a new barbarism.

Perhaps, no words are more becoming then those of ***Leonard Bernstein*** *in his succint 'Requiem-like' summation of the 20th century (1973):*

"Ours is the century of death: the end of faith. Why is our century so uniquely death-ridden? Couldn't we say this of other centuries as well? Yes, true: all human histories have been a long record of the struggle to survive, to deal with the problem of mortality. Yes; but never before has mankind been confronted by the problem of surviving global death, total death, the extinction of the whole human race. In fact, all the truly great works of art, music and literature have been born of despair and protest or refuge from death: think of Tolstoy, Rilke, Kafka, Sibelius, Mahler, Picasso, Sartre and Camus.

The 20th century has been a badly written drama from the beginning: Greed and hypocracy leading to genocidal world – wars, totaliterianism, post-war hysteria, existentialism, galloping technology, the flight into outer space, the doubting of reality, sub cultures and counter – cultures, new religions movements – all under the aegis of planetary death."

Front-lines of Science 1950–2001

Mathematics

- Continuation of Abstraction and Unification of Pure Mathematics:
  - Combinatorial Geometry;
  - Pure topology and graph theory (e.g. 4-color Theorem);
  - Abstract Algebra and Group Theory; Affine Lie Algebras; classification of finite groups;
  - Functional Analysis; Categories, Functors;
  - Fusion of Differential Geometry, Algebraic Topology and Algebraic Geometry; K-theory; Homotopies, Homologies and Cohomologies; Knot Theory, Topology of Smooth 4-manifolds; Fiber Bundles and Non-abelian Gauge Theories; Complex and almost-complex structures on manifolds; Spinors on manifolds and spin structures; sheaves; Cobordism; characteristic classes and index theorems;
  - Modular Functions and analytic Number Theory; Elliptic Curves;
  - Logic, Set Theory, Foundations of Mathematics and the genesis of Computer Science (Halting problem, foundations of mathematics, meta-mathematics proofs, Axiom of Choice, Continuum Hypothesis);
  - p-adic Analysis;
  - Non-standard Analysis: 'emancipation' from infinitesimals.
- Discrete Mathematics and the Ubiquitous Algorithm.
  - Digital Signal Processing.
- Computation Beyond Algorithms:
  - Soft Decisions and Fuzzy Logic;
  - Bayesian Inference;
  - Artificial Neural Networks;
  - Genetic Algorithms;
  - Simulated Annealing.

- Nonlinearity, Stochasticity, Optimization, Multiple Scales and the Science of Complexity:
  - Nonlinear Dynamics, Chaos and Fractals, Self-Organizing Criticality and Cellular Automata;
  - Algorithmic Complexity;
  - Optimization and Control Theories;
  - Integrable Systems, Invariant Tori and the KAM Theorem;
  - Nonlinear Diffusion and Wave Propagation; Turbulence; Reaction-Diffusion-Advection equations;
  - Queuing Theory;
  - Homogenization, Multiscale Analysis and Singular Perturbation Theory;
  - Theory of Stochastic Processes and Measure Theory;
  - Ito Calculus and Stochastic Differential Equations.
- Mathematization of the Biological, Environmental and Social Sciences:
  - Game Theory and Microeconomics;
  - Macroeconomics and Stochastic Processes;
  - Population Dynamics and Population Genetics;
  - Morphogenesis; Epidemiology; Neuronal and Brain Modeling;
  - Modeling of Protein Folding and Molecular Biology.
- Abstract Algebra and group theory in modern Theoretical Physics:
  - Groups (Discrete, Continuous, finite, infinite) in Quantum Mechanics, Chemistry, Crystal lattices, Quantum Field Theories, Particle Physics; Classification of all Finite Groups;
  - Global vs. Local (gauge) symmetry groups;
  - Spontaneous symmetry breaking;
  - C^*-Algebras applied to QFT, condensed matter and string theory;
  - Affine Lie Algebras;
  - 2-D Conformal field theory;
  - Current Algebras;
  - Quantum Groups and Braid Groups;
  - Operator-product expansions.

Physics

- PARTICLE PHYSICS AND THE COMING OF GAUGE:
  - Weak and Strong Nuclear Forces;
  - Quantum Field Theory beyond QED;
  - The 8-Fold way, the Quark model and Current Algebra;
  - Non-Abelian (Yang-Mills) Gauge Theories and Differential Geometry of Fiber Bundles;
  - Path Integrals, Fiber Bundles and the Quantization of Gauge Theories;
  - Partons, Quarks, Gluons, Quantum Chromodynamics, Scaling, Confinement and Asymptotic Freedom;
  - Heavy-Ion Collisions and the Quark-Gluon Plasma;
  - Phase Transitions and Spontaneous Symmetry Breaking, Nambu-Goldstone Modes and the Higgs Mechanism;
  - The Standard Model of Particle Physics;
  - Solitons and Instantons in Quantum Field Theories;
  - Supersymmetry, Supergravity and Grand Unified Theories;
  - Quantum Effects in Background Gravitational Fields;
  - Extra Dimensions, Kaluza–Klein models and String Theories;
  - Attempts at a Quantum Theory of Gravity;
  - Non-Accelerator Experiments and Indirect Evidence.
- THE NEW COSMOLOGY: WHENCE, WHITHER AND WHY?
  - The Big Bang Theory and Hubble's Constant;
  - Cosmic Microwave Background Radiation, Large Scale structure, and "Precision Cosmology";
  - Dark Matter, Dark Energy and the Cosmological Constant;
  - The Early Universe as a Hot Quanta Soup;
  - CP violation and Baryogenesis in the Early Universe;
  - The "First Three Minutes" and Nucleosynthesis;
  - The Recombination Era;
  - The "Dark Ages", re-ionization and first Stars and Galaxies;
  - The Planck Era and the Question of Initial Conditions;

  - Stellar and Galactic Models, Pulsars, Neutron Stars, Black Holes, Quasars, Active Galactic Nuclei and Gamma Ray Bursters, Brown Dwarves, Dark Matter Surveys and Candidates, the Intergalactic Medium;
  - New Windows to the Heavens: Infrared, Ultraviolet, X-ray, Gamma Ray, Neutrino and Gravitational Wave, Gravitational Lensing of Light Astronomies.

- General Relativity Matures and Meets Particle Physics, Astrophysics and Quantum Mechanics:
  - The Unmanageable Infinities of Quantum Gravity;
  - Singularity Theorems and self-limitations of GTR;
  - Quantum Field Theories in Curved Background Metrics;
  - Radiation from Black Holes;
  - Supergravity and Models of Hidden Dimensions;
  - Strings, Branes and "M-Theory";
  - Gravitational Lensing and Advent of "Applied GTR";
  - The evolution of theories of fundamental strings from Dual Resonance Models to Theories Of Everything;
  - Effective (non fundamental) strings in QFT, Astrophysics and Cosmology;
  - New Tests of GTR: Pound-Rebka experiment, Shapiro's radar-ranging, atomic clocks, the binary pulsar, orbiting gyroscope, LISA, and LIGO;
  - Gravitational Waves from Astrophysical Cataclysms and from Early Universe Phase Transitions.

Biochemistry

- Biochemistry of Nature and Man:
  - Cell Structure and Signaling;
  - Chemical and Morphological Logic of Life;
  - Molecular Biology;
  - Non-Equilibrium Thermodynamics in Physical Chemistry and the Biosphere;

- Neuroscience and functioning of the Mind;
- The Human Information and Storage systems: Genome, Immune System and Brain.

Technology and Engineering

- Quantum Technology:
  - Semiconductors; superconductors, superfluids, lasers and holography; advances in Magnetics, Ferroelectricity, Quantum Optics and Electrooptics;
  - Quantum Electronics, Optronics and Photonics;
  - Quantum Metrology (precision physical measurements using quantum effects);
  - Quantum Computing, cryptography and Teleportation;
  - Reversible Computing;
  - Ultra-low Temperatures; Magneto-Optical Traps and the Bose-Einstein Condensate (BEC) technology, and the slowing and freezing of light waves;
  - Electromagnetics of a single atom or particle in an RF cavity, cavity QED, observation of single-atom decays and the Quantum Zeno effect;
  - Microdevices and Nanotechnology;
  - Designer's Atoms and Materials: Quantum Dots, Wires and Wells, and Quantum Hall Effect, Superlattices and heterostructures;
  - Advances in Atomic Clocks;
  - Few-Quanta experiments, Quantum Entanglement and empirical tests of the foundations of Quantum Mechanics;
  - Electromagnetically Induced Transparency, "Dark States" and Lasing without population inversion;
  - Carbon nanotubes and Buckeyballs;
  - Optical ponderomotive forces: optical tweezers and ratchets; Light Lattices and Laser Molasses;
  - Role Reversal: Control of Material Object with Light Structures.

- RF Engineering: Electron guns, Klystrons, Waveguides and Junctions and Particle-accelerator Technology; Charged Particle Traps; Magnetic focusing and guidance of Particles beams.
- Worldwide Sensing, Communication and Information Technology:
  - Recording and Reproduction of Light and Sound; Television and Video Technology;
  - Sonar, Radar and Laser Ranging, Satellite Telemetry and Reconnaissance, Remote Sensing;
  - Optronics: Diode Lasers and detectors, Electro-optical chips, Optical Fibers and Optical Amplifiers, Optical Isolators and Polarization Dependence, Interferometers, AO (Acousto-Optic, devices) and SAW filters, Digital Optical Fiber and HFC (Hybrid Fiber Coax) Networks;
  - Client-Servers, Routers; Cable, optical, twisted – wire and wireless Networks; GPS;
  - Computer Based Communication Systems and the Rise of the Internet;
  - RFID (Radio Frequency ID) tags for tracking goods and national security;
  - Geosatellites in service of Telecommunication.
- Modern Microscopy and Telescopy:
  - ***Microscopes***: Field Emission; Scanning Electron; Transmission Electron; Scanning Tunneling; Atomic Force; confocal; FRET (Fluorescent Resonant Energy Transfer); Near-field Optical (to resolve distances much smaller than a wavelength); TPM (Two-Photon Microscopy);
  - ***Telescopes***: Spaceborne; computerized; Terrestrial and Spaceborne Interferometers; Detection of planets of other stars; with adaptive optics; with charge coupled devices; Hubble; Observing the Universe via Infrared, ultraviolet, X-ray, Gamma-ray, radio and Neutrino radiations; Gravitational-wave telescopes (LIGO and others).
- The Klystron and Particle Accelerator Technology:
  - Application to particle-physics experiments, X-ray sources, material science, medical imaging, time-resolved spectroscopy in physical chemistry.

- Material Science:
  - Metals, ceramics, plastics and composite materials;
  - Research on Smart Materials;
  - Doping in Electronics and photonics;
  - Rare-Earth Magnets;
  - Designer's Atoms and Superlattices.
- Computer-Related and Computer-Enabled Technology:
  - The Microchip;
  - Electro-Optical and Microwave chips; MEMS (Micro Electro Mechanical Systems);
  - Robotics and Control, Machine Vision, Machine Learning and AI;
  - Applications of Abstract Algebra, Number Theory, Topology and other branches of mathematics: Error Correction Codes and Cryptography; Wavelet Transform and Multiresolution Compression, Signal Processing and Pattern Recognition; Dynamic Topology of Computer Networks; Computer design.
- Computer Simulation – the Third Mode of Science:
  - Monte Carlo Simulations;
  - Finite Element, Finite Difference and Particle-In-Cell Simulations;
  - Computerized Symbol Manipulation;
  - Special-Purpose Computer architectures;
  - Computers as an integrative tool of Science Research.
- 'Big Science':
  - The Age of Accelerators and the Particle Physics Frontier;
  - Observatories and Detectors for Exotic Particles from Space, Exotic Particle Decay Modes, and Gravitational Waves;
  - NASA and the Manned and Unmanned exploration and Observation of the Solar System and Beyond;
  - Satellites, Rockets and Missiles;
  - Mapping the Human Genome;
- Biomedical Informatics, Radiology, Imaging, Molecular Medicine and Biotechnology:
  - MRI (Magnetic Resonance Imaging) and fMRI (*functional* MRI);

- PET (Positron Emission Tomography) scans;
- CAT (Computer Assisted Tomography) scans;
- Nuclear medicine;
- Particle-beam radiology and therapy;
- Ultrasound Imaging and Therapy;
- Stem-cell research, gene therapy, telomerase research, cloning, and genetically modified crops;
- DNA Markers and Retinal Scans for identification; individual genome sequencing for health care;
- Prions and abnormal protein folding;
- Bioinformatics, genomics and *proteomics* (the study and quantification of gene expression);
- Remote and sensor-assisted surgery;
- Analysis of microscopic tissue samples using optics, microfluidics, gene chips, tagging, and digital processing.

The Trees of Knowledge and Life – New Trends in Contemporary Science

Certain trends in mathematics, the natural sciences and technology are apparent:

1. Inward-bound progress

Great progress in the "inward" direction. In Physics *this is manifested as ever deeper, more unified and more aesthetic mathematical principles underlying physical reality – matter, energy, space and time. This "inward" direction points both to the very* small *(in spatial and temporal extents) and the* large *(large energy concentrations at the high temperatures achieved in particle accelerators and inferred for the early epochs after the Big Bang, and large scales of time and space in Cosmology).*

In material science *and the* biological sciences *this "inward" trend is always towards smaller scales of space and mass (and sometimes of time and/or of temperature as well), and is manifested by dramatic improvements in small-scale imaging (of cells and their constituents, microscopic lifeforms, molecules, and individual atoms). It is also manifested by* indirect *elucidation of microscopic structures, as in the sequencing of genomes, identification of proteins, and molecular-dynamics computer simulations of physical, chemical and biological processes at the smallest scales.*

In material science *and the* biological sciences*, the "inward" journey involves no new fundamental principles of nature, but merely newly-discovered interplays of known physical principles.*

All these feats of analysis, imaging, simulation and deduction usually involve computerization*, in addition to analog scientific hardware and mathematical theories and models. Here 'computerization' refers to various interfaces with digital sensing, actuating, memory and computation circuits and devices – whether embedded within the hardware, controlling it, interfacing externally with it, or used for offline analysis of data.*

2. Outward-bound progress – the Aegis of 'Complexity'

Impressive progress also in the "outward" direction – the study of complex phenomena and structures in all the natural sciences (including population dynamics, evolution, ecology, the new fields of mathematical biology and bioinformatics, neuroscience and neuronal modeling, epidemiology, continuum mechanics, material science, meteorology and geophysics) as well as in traditionally humanistic fields (economics, sociology, etc.); and the study of complexity *itself as a new branch of applied mathematics.*

This "outward" progress is largely driven by the computing *and* information *revolution, but also involves experimentation, observations and mathematical modeling, as do all other scientific endeavors. The aegis of "complexity" covers such phenomena as* chaos *and* fractals *that were found to be ubiquitous in complex systems.*

New mathematical tools, as well as older ones, were pressed into service to help ferret out, model and understand complex structures; examples of such tools are nonlinear dynamics, multiscale analysis, wavelets, artificial neural networks, reaction-diffusion partial differential equations *and* stochastic differential equations.

3. Bridges and Unifications

The forging of a robust web of bridges between different structural and functional levels *of description* in different sciences, *resulted in a powerful trend towards convergence of our disparate "islands of knowledge" of nature.*

Thus, for instance, modern research in biology tends to gradually reduce biotic structures and functions to physical *and* chemical *ones, organized at all scales (including the so-called* nanoscale*); all this new knowledge is placed in computerized, online* databases *which are then* mined via sophisticated algorithms *(the new field of* bioinformatics*) for the purpose of developing new drugs and treatments, identifying new genes and their locations, sequences and functions, identifying new proteins, elaborating the "tree of life" (i.e., maps of* where *each organism that ever lived is placed in the tree of evolving DNA), et cetera.*

We see that science at the start of the 21^{st} century is on an inexorable *trend toward* unification of the growing "tree of knowledge" *with the "tree of* *life" – a trend with incalculable potential effects on human lifespan, health, cognitive powers, and material and mental well being.*

4. New Mathematical Structures

The demarcations between our chapters 4, 5 and 6 were largely driven by developments in the natural sciences and technology. Pure mathematics has its own intrinsic dynamics; although it undergoes dramatic new developments in response to impetus from the sciences, it then typically continues to hone and develop the resulting structures for very long periods of time – often generating, in the process, new tools that are then adopted for modeling physical reality.

As an example, the revolutionary theories of 20^{th} century physics – quantum mechanics and the theories of relativity – used pre-existing mathematical structures that are properly classified as belonging to the "Abstraction and Unification" phase of mathematics, even if the discovery of these structures is listed by us in both chapters 5 and 6. And in turn, work by physicists on relativity and quantum theory led to new types of mathematical structures.

There are many other instances that illustrate this type of two-way influence between mathematicians and scientists working on the two sides of the "Math-Science divide".

In view of the above, it should now be clear that the mathematics of the second half of the 20^{th} century can be roughly divided into two endeavors (with, however, interactions and hybrids abounding):

(A) The continuation *of the "Abstraction and Unification" themes of chapters 4 and 5 in such fields as analytic number theory, differential geometry, abstract algebra, topology, algebraic geometry, algebraic topology, functional analysis and the foundations of mathematics.*

(B) The development of new, computer-driven branches of mathematics, including studies of complexity (as outlined above) and of computer science itself (algorithms for data processing within and by computers). These computer-driven branches are just as rigorous as the older ones, but they tend to deal with discrete objects and processes (a representative paradigm being that of "cellular automata").

5. Man outdoing Nature

In addition to the ongoing and novel trends listed above, there appears a new feature of modern science: the growing capability of human technology to outdo nature itself in certain respects.

We do not, as yet, have the technology to shift stars from their courses, create new planets and stars, or create galaxies and quasars; nor to create life from non-life, or sentience from non-sentience. We simply cannot compete with inanimate nature at extremes of high energy, large size or long duration, at least not in the foreseeable future. Nor are we anywhere near to computing, as engineers, with Life's devices in terms of density, adaptivity, plasticity, miniaturization, 3-D packing and multiscale structure and function. However, we are already able*, on some specific fronts, to outdo both life and inanimate nature.*

The philosophical conclusion drawn from this observation can be stated as follows: In the sense of the Strong Anthropic Principle, sentient beings (which themselves presumably arose spontaneously from inanimate matter via molecular and biological evolution) are, in a sense, nature's vehicle for developing in certain directions that are extremely unlikely to occur without the agency of such beings. Nature's way of enabling this option was achieved by endowing man with intelligence and consciousness *which he acquired through the process of evolution.*

Let us now specify those directions along which human technology outdid nature:

(A) Low Temperatures: Using Magneto-Optical Traps (MOT) and Gravito-Magnetic Traps – a combination of magnetic fields, gravitation, laser beams and radio waves – physicists (since the 1990's) routinely cool small samples (several million atoms each) of various species of atoms down to temperatures as low as several nanodegrees kelvin or even below a nano-kelvin. At such low temperatures, the de Broglie wavelengths at the atoms' quantum wave-functions become larger than inter-atomic separation, in effect turning the entire mesoscopic sample into a single, coherent quantum probability wave.

Quite apart from the rich scientific and technological implications of the attainment of these so-called 'Bose-Einstein Condensates' (BEC), we note that as far as is known, such low temperatures and state of matter have never before existed in nature *– neither in the depths of intergalactic space, nor on any planet, nor in the remote reaches of our universe or in its early history – save, perhaps, in the laboratories of some other, non-human civilization of sentient beings.*

(B) Symbolic representation, *recording, transmission and processing: since the dawn of human culture, people have been using physical manifestations (cuneiform etchings, ink marks, et cetera) to represent ideas and convey information in a symbolic manner. As far as is known to science, inanimate nature never does that.*

Now, this particular direction in physical parameter space does not require hi-tech or even intelligence – bees, ants and many other low orders of animals and plants (and even individual cells) convey symbolic information, by means chemical, electric, optical and acoustical; and indeed, any form of life whatever on earth does so by means of the genetic code. *Nonetheless, we note that the* information technology revolution *of the late 20*th *century has introduced on our planet such complex mazes of symbolic information storage, retrieval, transmission and processing (mainly electromagnetic) as to rival – and, by many measures, to surpass – the main three information processing/storage facilities in the human body (genome, immune system, brain – in order of increasing storage capacity).*

The latter three systems, while 'natural' (i.e. not devised by humans), are once again hallmarks of animate *matter. Inanimate matter does not seem to have any uses for storing and manipulating information in a symbolic manner.*

Granted, the physical laws *which govern all matter and energy are, arguably, a sort of highly condensed logico-mathematical* symbolic code; *but this code is not physically stored anywhere (except again, in the writing and minds of human beings – and presumably in those of other advanced sentient beings in the cosmos, if they exist).*

(C) Efficient information processing: *granted that symbolic information processing/storage/transmission does not seem to exist outside the realm of life and its products and artifacts (such as genomes, brains, cuneiform tablets, abaci and computer hard-discs), the trends in modern computer technology also point to ever increasing* efficiency *of such symbolic manipulations.*

This enhanced efficiency either already exceeds, or will before too long, what animate, yet pre-technological, nature has wrought – in several directions in physical parameter space. These directions are high density *(bits per volume),* high speed *(access and processing speeds in bits per second),* low energy cost *(Joules per bit processed), and* efficient particle usage *(low number of elementary particles whose quantum state is affected by each bit change). We note, though, that the entire human genome (several Gigabytes of digital data, not counting partially-analog epigenetic information) is packed within almost each cell of the human* *body – indeed, most of it within the cell's nucleus; human technology cannot yet achieve such information storage densities.*

(D) *Design optimization: At its most fundamental, we know that nature's laws may invariably be cast as* optimum principles *(principles of least time, least action, etc). At the biological level, nature optimizes at a different level – efficient use of wax in honeycombs; optimizing paths of ants and birds; survival strategies of species at both the genomic and population levels; and so forth.*

This latter type of optimization is opportunistic and a rigorous 'global optimum' is almost certainly never achieved. Humans, however, are capable of modeling problems mathematically, and the resulting optimizations – especially when computers are utilized – may well be far more comprehensive than pre-technological nature is able to achieve. And even without advanced mathematical models, various engineering solutions to practical problems (such as streamlined design of vehicles and furniture) can easily exceed the efficiency found in animate, pre-technological nature. (However, nature seems at present to be better at optimizing at the 'systems level' – witness the self-inflicted ecological problems humanity is grappling with!)

(E) *Composite and smart materials:* *Advances in material and computational sciences – and even more importantly, the convergence of the two (the emerging branches of engineering sometimes referred to as nanotechnology) – are enabling another form of optimization: at the level of the organization of atoms into macroscopic matter.*

Thus, "designers' materials" can be planned and fabricated, taking the process of technological optimization [discussed in section (D)] to new heights. While human technology has already outstripped inanimate nature *along these directions in physical parameter space, it is still lagging behind what* animate *nature has accomplished (even a single living cell represents a far more sophisticated feat of 'natural nanotechnology' than any fashioned in a human lab).*

However, there is every reason to believe we will surpass even biology in these directions. This is even more apparent when one considers that molecular genetics (and the burgeoning biotechnology based upon it) is enabling humanity to 'co-opt' various naturally occurring biochemical and biophysical mechanisms, rearranging them almost at will – either with one another, or in conjunction with inanimate human technology (such as in the so-called "gene chips").

(F) Quantum Computing: Almost all of today's computers are based on simple Turing Theory and employ Boolean logic based on binary mathematics. Even "parallel" computers are really complex Turing engines employing multiple computing modules which deal with pieces of incoming and internally-generated data (digitized acquired data, instructions, etc). There has been some research into biological computing using enzymes or large-molecule systems as memory, shift registers, etc, but this has not yet proven to be very practical.

Quantum Computing is based on a different physics than ordinary digital computing. Instead of having two (or three) states per element: off, on, or in between ('hung'), quantum computers consist of elements each of which may at any given moment be in a superposition of "on" and "off" states. An eight bit digital computer can exist in only one of 256 states at a time while an eight bit quantum computer can exist in all 256 states at a time and theoretically, work on 256 calculations at once (quantum parallelism). Each of the 256 numbers in this 8-bit example has an equal probability of being measured, so that a quantum processor functions, in effect, as a random number generator. The actual register represents all these combinations of bit-values at once, but a single 8-bit value output only occurs at measurement.

While a classical digital computer would have to operate on each number from 0 to 255, a quantum computer requires only one pass through the "processor" — radically reducing calculation time. Of course, the larger the register size, the larger the reduction factor – even a simple 10-bit quantum computer could make a supercomputer pale in comparison.

Where the digital computer uses binary digits (bits), the quantum computer uses qubits ("quantum bits"), but qubits are extremely difficult to generate. A quantum switch may not be disturbed by anything – a single photon of light, a single impinging molecule, or ambient fields – for the proper operation of a quantum computer depends on the interaction of the various qubits without any outside influence (measurements included). When disturbed, the qubit temporarily "collapses" into a conventional 1-bit register.

A quantum computer can perform an arbitrary reversible classical computation on all its qubit data registers simultaneously, and also has some ability to produce interference, constructive or destructive, between various different computational paths. By doing a computation on many different input data-sets at once, then interfering the results to get a single answer, a quantum computer has the potential to be much more powerful than a classical computer of the same size.

> *The most famous example of the extra power of a quantum computer is an algorithm for factoring large numbers. Factoring is an important problem in cryptography; for instance, the security of public key cryptography depends on factoring being a hard problem. Despite much research, no efficient classical factoring algorithm is known.*
>
> *There are many proposals for how to build a quantum computer, with more being made all the time. The 0 and 1 of a qubit might be represented by the ground and excited states of an atom in a linear ion trap*[3]*; they might be represented by polarizations of photons that interact in an optical cavity; they might even be represented by the excess of one nuclear spin state over another in a liquid sample in an NMR machine. As long as it admits of a way to put the system in a quantum superposition and there is a way to store, couple and occasionally measure multiple qubits, a physical system can potentially be used as a quantum computer.*
>
> *In order for a system to be a good choice, it is also important that many operations may be performed before losing quantum coherence. It may not ultimately be possible to make a quantum computer that can do a useful calculation before decohering, but if we can get the error rate low enough, we can use a* quantum error-correcting code *to protect the data even when a certain fraction of the individual qubits in the computer decohere.*

In conclusion: As of the 1990's, quantum and computer technologies reached the ability to produce states of matter and energy that (barring other

[3] Here, the charges, voltages and magnetized domains which represent data bits in conventional digital computers, are replaced with photons and ions trapped by electromagnetic fields. Because they are encased in these fields, the ions are in a coherent state for fractions of a second. The obstacle in using ions is that they may lose their quantum coherence too quickly to be useful as a computing resource. Los Alamos researchers have been able to apply a single laser pulse to a single ion in an electromagnetic trap.
From their demonstration, the researchers said that as many as 100,000 logic operations could be applied to registers that consist of up to 50 trapped ions. It would take but a few microseconds for a register to complete a single operation. Although it is smaller and faster than any silicon-based device, quantum computers are not expected to replace desktop PCs or supercomputers, at least not at first. Instead, these machines would be dedicated to specialized tasks such as generating keys for strong cryptography, an operation that requires a computer to factor very large numbers.

intelligent species) could almost certainly never be produced in nature outside of human laboratories. These frontier states can be grouped into three classes:

- *Low-temperature frontier (BEC)*
- *Mesoscopic quantum coherence frontier (e.g. Quantum Computing)*

Nature finds it hard, if not impossible, to reach these niches in physical parameter-space (as well as the niches described in (B), (C), (D) and (E) above) except through the agency of sentience. By the anthropic principle, we exist, as part of nature, to enable nature to accomplish such tasks.

6. "Lest he take also of the tree of life" (*Gen*: **3**, 22)

The current state of development of the theory of stochastic processes, as well as our ability to realistically simulate the dynamics of complex biotic macromolecules, is too primitive to compute with any reliability the probability of the spontaneous arousal of living from non-living molecules.

But we have HINTS such as:

- ***I. Prigogine**'s theory (1947) of far-from-equilibrium chemical processes,*
- *The Jacob-Monod theory (1961) of the operon mechanism (genes controlling other genes),*
- *Knowledge about various chemical pathways and point mutations in biology,*

which indicate that naive probability models may be completely useless in estimating the probabilities and likely timescales involved in a spontaneous genesis of life.

Thus, Prigogine's models show that complex spatiotemporal structures can arise from homogeneous mixtures in open thermodynamic systems far from equilibrium.

The evidence from molecular biology (such as the operon mechanism) illustrates that, since the proverbial typewriting monkeys can sometimes accidentally learn to program a sequence of keystrokes into a single keypunch, *and since this process is hierarchical, the expected time for a homogeneous mixture of pre-biotic molecules (under primordial-earth conditions) to produce living organisms – might be* drastically shorter *than what the simplest (**Hoyle**-like) combinatorial arguments would suggest.*

*No single example better illustrates the primitive state of our bio-stochastic modeling and simulation tools than the humbling fact that present-day (2008) optimization algorithms (whether stochastic or deterministic) and current computing power, are unable even to predict the correct shapes of folded proteins of known amino-acid sequences in aqueous solutions – a problem that nature herself "integrates empirically" (as **Albert Einstein** would say) in a matter of minutes!*

Science vs. Technology

Science may be loosely defined as the well-ordered, systematic and programmatic gathering of positive knowledge concerning the universe and ourselves. The history of science, therefore, is concerned with the story of gradual unveiling of objective truths and the conquest of matter, energy, space and time by mind; it also describes an age-long and endless struggle for freedom of thought – freedom from violence, intolerance, error and superstition. This process of gathering and explanation of systematized positive knowledge is the only human activity which is truly cumulative and progressive.

Science is also the systematic attempt to understand and comprehend the deep principles, hidden order, complex structures and beauty of operation that lies behind all natural phenomena. It aims to discover the true facts about, and the rational relationships between, observable phenomena in nature, and to establish theories that serve to organize these facts and relationships in the language of *mathematical symbolism*.

The scope of science is not just the external world, but ourselves as well – our bodies, perceptions, thoughts, imaginings, emotions and actions. Science deals with ideas and is a curiosity-driven, abstract, cultural activity, but unlike philosophy – it always concerns itself concrete phenomena, within which the general, abstract principles and patterns are reified. It is motivated primarily by intrinsic interest, not by utility.

Science flourishes best under special conditions of society, under which there is freedom to exercise to the full the two aspects of the *scientific method:* on the one hand, creative imagination aided by rigorous logic must seek to build and examine *hypotheses* extending beyond existing knowledge; on the other hand, *experimental* investigation must subject these hypotheses to the most rigorous empirical testing, employing the most elaborate scientific instruments available to the scientist. Yet, in the end, no matter how complex the apparatus is, the information that it delivers has to be observed by a scientist, who must examine it critically in relation to his hypotheses, and then, perhaps, reformulate his hypotheses.

Technology (or *Applied Science*, as it is sometimes called) is an effort to apply empirical scientific knowledge to some useful purpose. It is science applied to the business of life. It deals with tools, techniques and procedures that people use for utilizing the findings of science.

There are technologists in many countries where there are no scientists. In fact, all countries have technologists; even counties in the Stone Age, for example, had experts in the manufacture of stone axes.

Technologists (who, however, are sometimes also scientists) are responsible for all the marvelous inventions *that have transformed the conditions of our life. Think of the spectrum of revolutions in the means of communication, in the materials for all purposes, and in electronics; think of all the new inventions relating to medical practice, to agriculture and food, to the chemical industry, and to computers. This human activity is quite different from that of science, although it utilizes and exploits discoveries of science and, in turns, aids scientific research. Of course, a technologist has to have wide-ranging knowledge, imagination, and high intelligence, just as does a scientist.*

Faraday's laws of electricity and magnetism are science. Marconi invented wireless technology. Clausius contributed to the science of thermodynamics. Watt invented the modern steam engine. It was science that clarified the nature of nuclear binding energy; but it was technology that, in an astonishingly short time, converted it to a weapon of unimaginable power.

Whereas the first clear records of scientific concerns date back to about 600 BCE, the history of technology is much older. There is evidence that toolmaking goes back as far as one million years. Invention that produced technology did not require scientific reasoning until relatively modern times.

Progress in technology was an important component of natural selection as our human ancestors learned to cope with recurrent ice ages and other conditions hardly conducive to creative contemplation. By the time of the apparently abrupt appearance of scientific thought, a considerably sophisticated technology was at hand: fire; wheels; metalworking; agriculture; weights and measures; elements of arithmetic, algebra and geometry; an astronomical data base; navigation; land surveying; medicine and surgery; calendars.

The history of science is constantly interwoven with the history of technology and it is impossible to fully separate one from the other, especially in recent centuries: industrial requirements are always putting new questions to science while the progress of science continually gives birth to new industries or brings new life to old ones. Let us review some examples:

Science has been progressing at breakneck speed during the last century, allowing us to regard the universe from a much loftier vantage point than had ever been possible before. Thus viewed, it is infinitely greater, infinitely more complex and yet amenable to a more unified description; infinitely more harmonious, more beautiful, and yet much more mysterious. Although not all veils have been lifted, many have been during the past decades.

On the other hand, the very progress of science created new riddles, stirred up new contradictions, which seem more difficult than ever to explain. One revolutionary discovery after another seemed to put everything into question.

The natural sciences have made enormous strides in discerning and intuiting the fundamental laws of nature, as well as in explaining a myriad of complex structural and operational mechanisms (both artificial and naturally-occurring) predicated upon these laws.

The riddles cracked, or in the process of being cracked, by science – a relatively new human activity – are extremely diverse, ranging over some many tens of orders of magnitude in terms of space, time, mass and energy. Thus, physics – the most fundamental of sciences – directly explain atoms, stars, and galaxies, and provide us – through a sub-discipline dubbed cosmology – with an embryonic theory of the universe on the largest possible scale.

Furthermore, physics also explains (either on its own or through successive layers of emergent *disciplines, such as chemistry and the life sciences) a good deal of what we observe on scales intermediate between those of elementary particles and the universe as a whole.*

Microbes and DNA, salt crystals, quasicrystals and critical opalescence, flowers and hurricanes, brains and ecosystems have all become subjects of meticulous examination under the mental microscope of the scientific enterprise. Perhaps surprisingly, some of the most complex (and thus most difficult of disentangling) systems known to us lie at comfortably human scales of distance and time – well away from the extremes of the observable universe on the one hand (about 10^{28} *cm), or the Planck length (*10^{-33} *cm) on the other.*

Indeed, the human brain – a marvel of complexity and function – occupies rather 'mundane' dimensions (circa 10 cm). The consideration of complex systems could thus be a compensation of sorts for a humanity that has been rudely evicted from any semblance of centrality in the cosmic scheme of things, by successive scientific discoveries. But beyond such comforts, it points to an essential duality in the nature of science: it seeks to analyze *all objects and systems down to their basic components and underlying principles. And having analyzed them, it then strives to assemble the pieces back again, and explain how composite systems may arise (or be engineered) from its building blocks.*

The scope, activity and growth of a scientific field can be evaluated from the size of the population of scientists who work in it or from the number of scientific publications that describe their research results.

In 1910 all the German and British physicists and chemists put together amounted to perhaps 8000 people. By 1990 the number of scientists and engineers actually engaged in research and development in the world was estimated at 10 million, of whom almost 2 million were in the USA, a slightly larger number in the states of Europe, and about 3 million in Russia. Statistics shows that, by the above measures, science as a whole now doubles every 20 years.

It follows that at the end of a professional career of about 40 years, an aging scientist finds that there are 4 times as many scientist and scientific books and journals as when he was a student. In subjects such as molecular biology, particle physics, oceanography, and few other fields, there is a doubling every 4 years! During a professional career there are thus 10 doublings, which is to say, a 1000-fold expansion.

The scientist nowadays is always surfing up the swelling wavefront, just trying to keep his head above a flood of scientific literature. This enormous expansion cannot continue forever, but it does not yet appear to be slackening. This is because it is constantly refueled by revolutionary scientific discoveries and by the need for additional information as an exploding human population exploits nature.

Discovery vs. Invention

An invention *is the design and creation of a* teleonomic *(i.e., purpose-oriented) device or procedure:*

The purpose of an invented device or procedure can be anything desired by the inventor, including e.g. labor-saving, entertainment, edification, or assistance in achieving some other human goal. The terms "design" and "creation" refer to a key characteristics of an invention *that distinguished it from a* discovery *– namely, that an invention is a pattern of energy, matter, information and/or organization that cannot be naturally thought of as having existed "out there" before it was brought into being by its inventors.*

Thus, a statue hewn of rock cannot be said to have existed within the original slab of rock; nor is it reasonable to regard a newly-fabricated Lexus automobile as having been somehow "discovered" in the patterns of the plastics, metals, chemicals and composite materials of which it consists. (In fact, many of those components are themselves *quite elaborate inventions – plastic polymers, new materials, electronics, hydrocarbon fuels, etc.) Even something as abstract as a computer algorithm qualifies as an invention, provided it is complex enough, arbitrary (contingent) enough, and sufficiently oriented towards and informed by a specific purpose.*

If it doesn't fulfill these conditions, the algorithm might be regarded as a 'discovered fact' of mathematics (an example is the Euclidean algorithm ubiquitous in number theory and abstract algebra). And this brings us to the concept of discovery – often confused with invention.

A discovery *is the elucidation of a physical pattern, structure or effect in nature, or of a mathematical pattern, structure or fact (theorem) within some set of axioms. Unlike an* invented *device or procedure, a discovered fact or effect is neutral vis-a-vis any possible application – i.e., it is not inherently teleonomical.*

Furthermore, a "discovery" has the essential hallmark suggested by the word itself: to wit, it emerges from the basic fabric of reality (or from a given set of mathematical axioms) in a manner sufficiently natural and non-arbitrary that it can be regarded as being in some sense "out there" (covered, as it were) in the universe before its discovery. (Thus, the Pythagorean theorem, the irrationality of the number $\sqrt{2}$*, or any of the laws of mechanics, electromagnetism, or the rest of physics and chemistry, cannot be reasonably viewed as having been* invented *by the persons who happened to first realize or demonstrate them.)*

With regard to strictly mathematical *discoveries, one must add the caveat that the theory (a set of hopefully-consistent axioms) within which the discovery is made, is* itself *the arbitrary creation of the human mind – and this circumstance seems prima facia to preclude the assignment of any a priori existence to a discovered mathematical theorem.*

But anyone who has had sufficient experience with pure and applied mathematics would agree that "interesting" mathematical theories are those that keep coming up and interconnecting in novel and unexpected ways; these interconnections are "inside" and "amongst" the mathematical theories themselves, and also between these theories and physical reality. Branches of "invented" mathematics that lack this kind of connectivity do not seem to last very long or draw the interest of many practitioners.

Likewise, discovered laws of nature – if they withstand the test of time – are invariably seen to be gradually refined and forged into general principles of great aesthetic appeal, which are also deeply interrelated with those mathematical theories having the above-mentioned enhanced-connectivity property.

Finally, we note that inventions are often thought of as not having been realized in the natural world before humans conceived of and reified them; discoveries, on the other hand, are commonly thought of as being realized in nature quite apart from humanity's (or other sentient civilizations) endeavors.

But this is not always the case! For instance, life – with its intricate, teleonomic mechanisms, patterns and structures – clearly qualifies, by our definition above, as a vast trove of "inventions" – although modern science realizes that there were likely no inventors *of these inventions; rather, they arose through random interactions of molecular assemblies, incessantly pruned by the mathematical patterns of nonlinear reaction-diffusion-advection partial differential equations.*

Thus, life is a striking example of natural laws and patterns of physics, chemistry and mathematics – "discovered" facts – giving rise to spontaneous, "inventorless inventions", including human beings and their brains.

And just as blind natural forces can "invent" (thus forms of heavier-than-air flight and symbolic processing of information were instantiated in biology long before humans and their technology) so is the converse true: many physical effects that qualify as having been "discovered" (in the sense defined above) are thought never to have actually been instantiated *during the universe's lifetime, outside of human-devised laboratories (or those of other sentient beings elsewhere).*

Examples of discoveries in this category are legion: the Mössbauer effect and the Bose-Einstein condensation of weakly-interaction clouds of Alkaline atoms are two likely examples. Such effects or laws of nature were, of course, *always in existence – but only as unrealized potential phenomena; it took sentient, biotic beings ("invented" by inanimate nature!) to reify these effects and principles into actual, material reality.*

Invention characterizes the living being; it attests to an effort of adaptation to the environment. Therefore it is found in the animal kingdom as well as in humankind. In Africa monkeys have been seen to use a stick to get at food which is out of reach: they have invented a tool. This ability is not restricted to mammals: birds can be watched dropping thick shells on rocks in order to break them and then eating the mollusks inside. Perhaps the Californian mosquito shows proof of invention too, as it increases by 200 times its production of the gene which enables it to synthesize the enzyme antagonistic to an insecticide.

Animal inventions are limited, however, and traditionally the history of inventions starts with the emergence of Homo sapiens sapiens. *Hardly had he appeared but* Homo sapiens sapiens *not only adapted to the environment but also adapted the environment to himself.*

Early humans were at the mercy of hostile predators and the climate. Without their ability to invent, it is doubtful whether our relatively weak and slow ancestors would have survived for long. Spears with fire-hardened points served first of all as hunting weapons and then for building fences.

Some eight thousand years BCE human populations were no longer content with hunting, fishing and gathering. They captured wild animals which they domesticated: horses for draught and for riding, dogs for protection, sheep for meat and poultry for eggs and meat; they also selected crops to cultivate, and therefore founded settled communities. So agriculture and farming began.

The countryside changed drastically, through the burning of the woodlands and then by repeated cultivation. Man invented pottery, affixed shaped flints onto shafts of wood, and made different tools according to his needs, such as the scraper, axe, pruning knife and adze.

At approximately the same time the settlers in Anatolia and on the banks of the Danube invented a rational organization of their shelters which were to become permanent. The food stores were wisely situated in the middle of the communities. Such was the origin of town planning.

During the Copper, Bronze and finally Iron ages, in the 5^{th}, 3^{rd} *and* 1^{st} *millennia BCE respectively, the techniques and consequently the inventions began to diversify.*

One or more unknown persons invented successively the technique for working iron, then iron with carbon to make steel, then how to make needles, then metal wheels, armour, and cooking utensils which could withstand *high temperatures. The historical age, very roughly taken as starting in the 5^{th} century BCE, took over from the prehistoric ages.*

Then great waves of invasion and commercial trading played a role in the spreading of inventions, comparable to that of swarms of insects in the pollination of plants. The conqueror and the tradesman placed instruments and products from one end to the other of the ancient continents, Africa and Eurasia.

Tables 1–3 summarize man's major inventions up to the 9^{th} century CE. It was around this time that northwestern Europe began to climb to an ascendancy in technology that it has held to for centuries since. The poorer climate of this region, combined with the need to develop a new form of agriculture, was responsible for the emergence, around the 8^{th} century, of the crop rotation methods still used today.

The technologies which have had the most profound effects on human life are usually simple. A good example of a simple technology with profound historical consequences is hay. Nobody knows who invented hay, the idea of cutting grass in the autumn and storing it in large enough quantities to keep horses and cows alive through the winter. This technology was unknown to the Romans but was known to every village of medieval Europe. It was a decisive event which moved the center of gravity of urban civilization from the Mediterranean basin to Northern and Western Europe. The Roman Empire did not need hay because in a Mediterranean climate the grass grows well enough in winter for animals to graze. North of the Alps, great cities dependent on horses and oxen for motive power could not exist without hay. So it was hay that allowed populations to grow and civilizations to flourish among the forests of Northern Europe.

In Northwestern Europe, the wind was put to use in both sea-going vessels and land-based mills. The region grew more populous and, by the 11^{th} century, Northern Europeans were extending their influence into the Mediterranean and Middle East.

With few exceptions, most inventions made until the 16^{th} century are anonymous. But even later, despite the work of historians and documenters, it is impossible to compile a complete *history of inventions dating from ancient times to the end of the 19^{th} century, for the following three reasons:*

- *No large-scale control of production existed until well into the 19^{th} century; the work of each inventor was carried out to fill his own needs and wishes and so did not resemble that of anyone else. (Thus, hundreds of types of lifting instruments have existed throughout the centuries, and one just gets lost in speculation as to the one which was used in the* *building of the fabulous Colossus of Rhodes in the 3^{rd} century BCE, which was 35 m high.*

- *Many inventions which were designed on paper, were never realized, so the principle of selection cannot be applied to unrealized inventions.*
- *Many inventions are undoubtedly lost forever and other have been lost and found again, or reinvented. For example, not all the works of the inventors of the famous Alexandrian School are known. Before printing, the only documents describing inventions were manuscripts, of which often only a very few copies existed.*

The Ultimate Machine

At first all machines were mechanical, whatever their motive power; mechanical design requires strict specifications of parts' dimensions, material properties, relative positions, and mechanical contact, rolling, sliding etc. between parts. This can be thought of as the initial stage in the evolution of engineering – a stage in which machine functionalities and structure are very tightly correlated with the physical properties and configuration of the machine's parts.

Later, when electromagnetic theory was put on a sure footing, electrical circuitry and their standardized electronic components were developed (capacitors, resistors, inductances, batteries, vacuum tubes, transistors, and a variety of thermal, mechanical and optical transduction devices). This allowed the modular design and hookup of devices in which functionality and structure was partially liberated from the tyranny of geometry and mechanics; instead, function and structure tended to inhere to a large extent in circuit connectivity.

In the middle of the 20th century, analog electronic circuits were for the first time used to perform digital *(i.e. discrete logical) functions; thus was born the field of* digital design. *The ubiquitous* digital computer *is an extreme case of a machine whose entire function (except actuators and transducers interfacing with peripheral equipment) is to manipulate digital information; but digital components have since been interwoven into many other machines types as well (automobiles, wristwatches, camcorders, stereo systems, televisions, telescopes, microscopes...).*

Digital design is even more flexible than analog-electric one, because in addition to being liberated from geometric and physical constraints, it is also tolerant (up to a point) of major distortions of the electrical wave-forms and signals themselves (a signal representing the bit "1" can be recognized as such even after undergoing significant distortion from its original shape – much as human handwriting can be recognized across a fairly wide spectrum of individual penmanship, orthographic styles, font types, script size, orientations, etc).

Digital design need not be implemented electronically, of course – or even electrically; thus, an old fashioned, Pascal-type calculating machine is a digital computer implemented mechanically or electro-mechanically, while a pre-digital telephone exchange performs rudimentary digital computations via analog electric circuitry and electromechanical switches.

In the second half of the 20^{th} century, electronic circuits became increasingly miniaturized – especially digital computation circuits, but also to a certain extent analog circuits, and even some electromechanical devices). In such miniaturized devices, individual components are crammed together in dense configurations – which demands close attention to issues of heat dissipation, distributed analog electromagnetic effects and, of course, mechanical design and material science. This trend culminated (thus far!) in the VLSI silicon chip, and extended to electro-optical and optronic devices ("photonics"), Field Programmable Gate Arrays (FPGAs), Acousto-Optical (AO) devices, Micro Electro-Mechanical devices (MEMs), a variety of high-density digital memory devices, miniaturized RF devices, etc.

Despite the fact that shrinking chip sizes re-introduce physical and geometric constraints, once a batch of chips is successfully manufactured – and assuming chip operation is reasonably resistant to thermal, chemical, mechanical and external electromagnetic disturbances and stresses – the chips can be treated as if they were simple electronic components with known logic functionality and assembled onto motherboards to render machines whose overall (digital and analog) functionality is, once again, flexible and modular.

Looking ahead into the coming decades, advances in photonics, electronics, material science and nanotechnology promise to both continue the miniaturization trend, and allow machine functionalities to be controlled by light alone, or by a combination of photons and electrons – in some cases, only one photon or electron at a time – as opposed to the millions of electrons whose motion states must change to flip a single bit from '0' to '1' in present-day computer chips.

This raises the further issue of how robust can a machine be when controlled by a single quantum particle! Even contemporary chips are susceptible to random bit-flips due to stray cosmic rays, natural radioactivity or external electromagnetic interference; this problem is customarily resolved via error correction codes *(the modern mathematical theory of which involves abstract algebra and Galois fields!).*

There are further challenges ahead to digital robustness, stemming from quantum uncertainties and superpositions of states*; researchers have already begun to tackle those, extending standard computer science into the new uncharted waters of* quantum computing.

Nanoscopic, and even quantum-coherent, machines now being contemplated – in which the "gears" are single quanta, atoms or molecules – mark man's final efforts (so far) in his long way of emancipating structure and functionality from physical and geometrical constraints of matter and energy.

The Iterative Nature of Scientific Knowledge

The manner in which ***Kepler*** *used the empirical astronomical data available to him (consisting of antiquity's accumulated lore plus* ***Tycho Brahe****'s observations) is an instructive case study in how science's knowledge of laws of nature is actually abstracted from experience – through sequences of* interactive, iterative *and* convergent *interplays between empirical investigations on the one hand, and theoretical speculations and modeling on the other.*

Various (sometimes conflicting) misapprehensions about how this process operates are rampant among the general lay public and certain types of historians and sociologists of science: on the one hand, one often hears that laws of nature are discovered by painstaking accumulation of empirical observations, followed by suitable generalizations; then further experiments are carried out to verify new predictions of the generalized laws; and so on. (This is the view of the "Scientific Method" whose earliest formal champion was ***Francis Bacon****.)*

But on the other hand, some modern philosophers and academics in the social sciences are of the opinion that general laws of nature are "underdetermined" by empirical data, and are thus (in part or even wholly) cultural constructs – which these scholars then, of course, gleefully rush to deconstruct *(a term enjoying unfortunate ubiquity in today's best universities).*

But both these schools of thought are simply wrong; the actual progress of our understanding of how the universe works is a far more interesting – and more robust – process than "dreamed of in their philosophies". And the resulting laws of nature – though always subject to eventual re-interpretation and modification from the vantage point of the next, deeper-level round of understanding – are real, not culturally-relative, and are never "repealed" or "go out of style" as, say, do legislations or dress fashions or styles of musical composition and architecture.

Thus, ***Newton****'s theory of gravitation was not repealed by the newer, deeper understanding furnished by* ***Einstein****'s theory of General Relativity – only its range of validity was circumscribed; after all, the Newtonian theory is still the coin of the realm in NASA's computations of satellite orbits, deep-space probe trajectories and the like.*

Nor do we cease to apply Newtonian mechanics to the modeling of aircraft flight, mechanical gears, engines, water spouts, golf-ball physics and all other manner of macroscopic machinery and motion, just because this theory breaks down for very small or very fast (or large or massive) objects – in which regimes, we now know, it must be replaced by quantum mechanics and (special and general) relativistic physics, respectively.

There is an iterative, back-and-forth interplay between theory and experiment in science which always seems to converge sooner or later! This is an amazing fact, given the ever-increasing length of the logical chains used by science in deciphering Nature.

These long chains of reasoning, deductions and modeling used to "interpolate" between experimental data points are necessitated by the limitations imposed upon humans by many constraints: their size, duration, particular location in the universe; by the limited spatial and temporal resolution of their sense-perception apparata; and by their limited ability to process and interpret concepts and data.

These limitation are, of course, vastly compensated for by human-invented technology, *which itself benefits greatly from scientific discovery – thus introducing another element into the dynamics of theory/experiment iterative interplay.*

Not least among humanity's technological aids to scientific research are those enabling it to summarize, record and propagate to subsequent generations its ever-increasing storehouse of data, theories, models and conclusions. Among such technologies are writing, printing, and (of late) electronic data storage; even the university *may be viewed as such an invention – for it perpetually trains new cadres of scientists who can process, add to, and (where necessary) modify the accumulated knowledge and understanding of the ages).*

Chronological Lists of Inventions

Table INV-1: Major inventions up to 3000 BCE

Invention	*Inventor(s)*	*Date*	*Use*
Tools made from sharp-edged stones; creating fire	Homo Habilis Homo Erectus	ca 2 million years ago ca 400,000 years ago	Fight off predators; cook and hunt for food. Surviving the second ice-age
• Collecting seeds and plants for 'next-year' planting; cereal crops	Agrarian valley-civilizations and societies	8000–3000 BCE	
• Distillation			
• Systematic crop-rotation (Europe)			Preserving soil fertility
• Producing wine, beer, mead, and bread, using fermentation			
• Producing yogurt, cheese and vinegar			
• Domestication of millet and rice			
• Domestication of animals			
• The wheel (ca 3400 BCE)			Transportation, milling devices Irrigation systems
• Smelting metals; glass; firing of bricks and pottery (potters wheel); alloyed bronze			
• Ploughshares, sickles; spinning wheel, saw, mirrors, balance; *lock*; *ball-bearings*			Organized city-life and agriculture
• River boats; *irrigation systems*			
• Calendar; sundial; hieroglyphics			
• Bridge (China); corbeled arches and domes (Mesopotamia) • Loom (Egypt, 4400 BCE) • Cosmetics (Egypt, 4000 BCE) • Sailing boats (Sumer, 4800 BCE)			

Table INV-2: Major inventions 3000 BCE–100 CE

Invention	*Inventor(s)*	*Date*	*Use or comment*
• Silk	China	ca 2700 BCE	
• Cartography	Mesopotamia	ca 2300 BCE	Earliest surviving map (city map of Lagash, Mesopotamia created in stone)
• Bitumen	Mesopotamia	ca 2400 BCE	
• Wheeled vehicle on tracks	Babylon	ca 2250 BCE	Parallel lines of grooved stone blocks
• Horses tamed for transport	Near East	2000 BCE	
• Plant medicines	Egypt	2000 BCE	
• Toilet flush	Crete (Minoan)	2000 BCE	
• Pitch	Mesopotamia		Watertight boats
• Rubber (Latex)	Maya	ca 1600 BCE	Rubber human figurines;
• Alphabet (23 letters)	Phoenicians	ca 1700 BCE	
• Waterclock	Egypt	ca 1450 BCE	
• Wrought iron	Mesopotamia	ca 1400 BCE	Warfare and agriculture
• Natural gas	China	ca 900 BCE	Cooking and light
• Arches	Etruscans	ca 900 BCE	Used by Romans for bridges and aqueducts
• Iron ploughshares		ca 900 BCE	
• Stone canal and aqueduct		ca 700 BCE	
• Bridge (Euphrates)		ca 600 BCE	
• Coins	Asia Minor	ca 650 BCE	
• Papyrus	Egypt	ca 650 BCE	Paperlike writing material
• Seasonal harvesting	Mesopotamia	ca 600 BCE	
• Windmills	Persia (Neh)	ca 644 BCE	
• Mining of silver	Greece	ca 580 BCE	At Laureion
• Rails	Greece	ca 470 BCE	Grooved stone wagonway 8.8 km Athens-seaport of Piraeus

Table INV-2: (Cont. [a])

Invention	*Inventor(s)*	*Date*	*Use or comment*
• World map	Herodotos	ca 450 BCE	
• Automaton	Archytas of Tarentum	ca 350 BCE	Consists of hydraulic circuits and counterweight activating levers
• Connecting-rod crank system; propeller; hydraulic pump; levers; screws; pulleys	Archimedes	ca 250 BCE	Warfare
• Water clock	Ctesibios	ca 270 BCE	Measurements of time-intervals
• Chain drive	Philo of Byzantium	ca 250 BCE	Catapults (warfare)
• Astrolabe	Hipparchos	ca 150 BCE	Astronomy
• Concrete		1^{st} century BCE	Road construction network
• Aqueducts	Vitruvius and	1^{st} century CE	Water supply systems
• Urban water supplies	S.J. Frontinus		
• Water mill			Grinding corn
• Coal			Fuel in Northern Europe
• Commercial book-publication			
• Water wheel			Grounding grain and olives
• Arch		19 BCE	Seen in the aqueduct of Pont du Gard (Niem)
• Differential gear	Geminos of Rhodes	80 BCE	
• Production of cogwheel system			Gear box
• Steam engine	Hero of Alexandria	ca 60–80 CE	
• Turbine			Powered by steam or water devices
• Float regulator			Automatic regulation of thermomechanical devices

Table INV-3: Major inventions in the Far East 400 BCE–1400 CE

Invention	*Inventor(s)*	*Date*	*Use or comment*
• Use of coal for iron making; cast iron; fuel oil; fire wells (CH_4)		ca 350 BCE	Cooking, lighting and heating metallurgy
• Animal harness		300 BCE	Animal power agriculture and transportation
• The crossbow		300 BCE	
• Explosives		150 BCE	
• Universal (Cardan) joint		140 BCE	
• Roller bearings (wooden)		100 BCE	
• Suspension bridge		25 CE	Span = 15 m
• Porcelain		50 BCE	
• Paper from pulp		100 CE	
• Abacus	China	300 CE	First mechanical calculator
• Hydraulic piston mill	400 BCE–1400 CE	550 CE	For steam piston engine
• Block book printing		700 CE	
• Gun powder		700 CE	
• Printed newspaper		750 CE	
• Escapement in a mechanical clock		725 CE	Clock driven by hydraulic power; warfare
• Chain driven mechanical clock		970 CE	
• Magnetic compass		1086 CE	Navigation
• Mechanical water-clock		1092 CE	
• Multicolor printing		1107 CE	
• Rockets		1150 CE	Warfare
• Bombs		1221 CE	Warfare
• Small cannon		1288 CE	Warfare
• Chess	Indo-China	ca 450 CE	Modern version of the game "Goose-egg"
• The zero	India	ca 660 CE	Sign for a *number* (not just a placeholder)

Table INV-4: MAJOR INVENTIONS IN EUROPE CA 800 CE–1800 CE

Invention	*Inventor(s)*	*Date*	*Use or comment*
• Crop-rotation methods			Increase of food supply
• Hay	Northwestern Europe	Middle Ages	Motive power for horses and oxen
• Blast furnace			Cast-iron production
• Treatment of ores			Metals for housing and coinage
• Spinning wheel			and weapons
• Crossbow		ca 1050 CE	
• Spectacles	Roger Bacon S. Armati A. della Spina	ca 1250–1285 CE	Lenses for improved vision and reading
• Water-powered saw-mill	Villard de Honnecourt	1235 CE	
• Movable-type printing press	Gutenberg	1450 CE	Speeding up transfer of technology and scientific knowledge between people
• Concave lenses	Nicolas of Cusa	1451 CE	Treat nearsightedness
• Parachute			
• Flying machines	Leonardo da Vinci	1480–1497 CE	
• Military engineering			Warfare
• Centrifugal pump			
• Gas turbine			
• Hydraulic cooling system			
• Domestic spring-driven clock	Peter Henlein	1504 CE	
• Compound microscope	Zacharias Janssen	1590 CE	
• Thermometer	Galileo Galilei	1593 CE	
• Regular newspaper	Abraham Verhoeven	1605 CE	
• Use of coke	Hugh Plat	1603 CE	
• Telescope	Hans Lippershey	1608 CE	First optical refracting telescope
• Slide rule	William Oughtred	1622 CE	
• Micrometer	W. Gascoigne	1636 CE	
• Mechanical calculator	Blaise Pascal	1642 CE	

Table INV-4: (Cont. [a])

Invention	*Inventor(s)*	*Date*	*Use or comment*
• Barometer	E. Torricelli	1643 CE	
• Pendulum clock	Christiaan Huygens	1656 CE	
• Improved microscope	Robert Hooke	1662	
• Free escarpment		1666	Pendulum clock
• Matches	Robert Boyle	1680	
• Inoculation	Giacomo Pylarini	1701	Advent of immunology
• Dead-beat escapement	George Graham	1715	Mechanical clock
• Hammer klavier	Bartolomeo Christofori	1709	Father of the modern piano
• Achromatic lens	Chester Moor Hall	1733	Refracting telescope
• Marine chronometer	John Harrison	1737	
• Free escapement	Pierre Le Roy	1748	
• Lever escapement	Thomas Mudge	1765	First modern clock
• Balloon flight	Montgolfier brothers	1783	
• Gold-leaf electroscope	Abraham Bennet	1787	
• Vacuum pump	Otto von Guericke	1645	
• Optical refracting telescope	Isaac Newton	1668	
• Clinical thermometer	Santorio Santorio	1615	
• Mercury thermometer	G.D. Fahrenheit	1714	
• Centigrade thermometer	A. Celsius	1742	
• Sextant	John Hadley	1731	
• Machine tools	John Wilkinson Henry Maudsly Marc I. Brunel	1775–1801	
• Appendectomy	Claudius Aymand	1763	First successful
• Pianoforte with pedals	John Broadwood	1783	
• Railway		1681	4 km track in Stourbridge, England
• Railway	Charles Brandlin	1758	First railway for a colliery wagonway
• Solar heating	Antoine Lavoisier	1780	

Table INV-5: Major inventions in Europe and North America throughout the 18^{th} and 19^{th} centuries

Field	*Inventor(s)*	*Date*	*Invention*
	Abraham Darby	1704–1709	Pig-iron production using coke in blast furnace
	Benjamin Huntsman	1740	Cast-steel production (crucible steel process)
	Henry Cort	1783–1784	Purifying iron by puddling
Iron Metallurgy	Henry Bessemer	1856	
	The brothers Siemens	1858–1864	Open-hearth steel process; cheap mass-
	P. and E. Martin	1864	production for civil and military engineering projects
	F.W. Geissenhainer (US)	1833	Hard coal for metal production
Civil Engineering	Thomas Telford	1786–1826	
Projects:	James Finley (US)	1800	Rapid transit systems for faster
	Pierre Tresaguet	1764–1784	travel on land and water to answer
Fast roads;	Marc Seguin	1764–1784	the needs of an advanced technological society
Suspension iron bridges;	John McAdam	1806–1827	
Canals; Aqueducts;	Ithiel Town (US)	1820	
Harbors and Tunnels	Marc Brunel	1825–1843	Sub-Thames tunnel
Architecture	William Le Baron Jenney	1885	First skyscraper built in Chicago (with iron frame)
	Hans C. Oersted	1825	First extraction of *Aluminum*
	J.L. Lambot, F.J. Monier	1845	Reinforced concrete
	W.H. Perkins	1856	Synthetic dye from coal tar
	Alexander Parks	1862	Parkesine
	Daniel Spill	1867	Xylonite
Material Science	John W. Hyatt	1869	Celluloid
	Hilaire Chardonnet	1884	Rayon; first synthetic fiber
	Charles Goodyear (US)	1839	Vulcanization
	Justus von Liebig	1840	Artificial fertilizer
	Robert W. Thomson	1845	Pneumatic rubber tire
	J.B. Dunlop	1887	Air-inflated rubber tire
	Joseph Aspdin	1824	Portland cement

Table INV-5: (Cont. [a])

Field	*Inventor(s)*	*Date*	*Invention*
MATERIAL SCIENCE	John Walicer	1832	Corrugated iron
STEAM-ENGINE TECHNOLOGY	Denis Papin	1687	Draining of mines (for increase of metal and coal production) and land areas to create more agricultural land (to supply the needs of cities); tractor; steamboat
	Thomas Savery	1698	
	Thomas Newcomen	1705	
	James Watt	1769	
	Nicholas Cugnot	1770	
	John Fitch (US)	1787	
	William Siemens	1847	Regenerative steam engine
	John Kay	1733	Flying shuttle
	Richard Arkwright	1769	Water-powered spinning machine
	James Hargreaves	1770	'Spinning Jenny'
	Jacques de Vaucanson	1745	
	Edmund Cartwright	1785	Steam-powered loom with chain-transmission – first automatic weaving loom. Advent of mass production of wool and cotton factories – the Industrial Revolution begins
	Joseph-Marie Jacquard	1801	
	Johann A. von Segner	1750	Reaction turbine
	C. de Jouffroy	1783	Early steamships
	J. Rumsey (US)	1787	
	O. Evans (US)	1804	
	Richard Trevithick	1800	High-pressure non-condensing steam engine; valve steam engine; Motive power for locomotive on steal-rails, boats, and road vehicles
	G.H. Coreiss	1800	
	John Stevens (US)	1802	
	Robert Fulton	1807	
	George Stephenson	1825	
	Kingdom Brunel	1837–1858	Iron ships with screw propeller cross the Atlantic
	James Nasmyth	1839	Steam hammer for iron working
	Charles Pearson	1843	Underground railway

Table INV-5: (Cont. [b])

Field	*Inventor(s)*	*Date*	*Invention*
Steam-Engine Technology	Robert Stirling	1816	Single-cycle heat engine
	Jean Lenoir	1860	Fuel-powered engine
	Sigfried Marcus	1875	Internal combustion engine
Internal Combustion Engine	Nikolaus Otto	1877	Gas-engine
	Gottlieb Daimler	1888	
	Karl Benz	1885	Gasoline-powered automobile
	Rudolph Diesel	1892–1896	Diesel-engine
	Coulomb	1777	Torsion balance
	L.S. Lenormand	1783	Parachute
	Joseph Bramah	1795	Hydraulic press
	Alois Senefelder	1798	Lithography
	N.F. Appert	1809	Preservation of food by canning
		1814	Large scale gas lighting (London)
	Karl D. Sauerbronn	1816	Bicycle
	Augustus Siebe	1819	Diving suit which receives air pumped down from the surface
Non-Electric Machines and Devices	B. Fourneyron	1827	Water turbine
	John Ericsson	1833–1837	Screw propeller; wind powered generator
	Gaston Plante	1834	Rechargable battery
	Linus Yale	1851	Lock
	J.B.L. Foucault	1852	Gyroscope
	J.E. Lundstrom	1855	Safety matches
	Elisha G. Ottis	1857	Steam-driven elevator
	Christopher Shoals	1867	Typewriter
	Pierre Michaux	1862	Steel ball-bearings
	I.W. McGaffey	1869	Vacuum cleaner
	A. Mouchet	1878	Solar generator to power a printing press
	John Milne	1880	Seismograph

Table INV-5: (Cont. [c])

Field	*Inventor(s)*	*Date*	*Invention*
Non-Electric Machines and Devices	Ottmar Mergenthaler	1886	Linotype machine
	Tolbert Langston	1887	Monotype printing
	W.L. Judson	1891	Zipper
	Benjamin Holt	1900	Tractor
	King C. Gillette	1901	Shaving with double-edged safety razor
	Jacob Perkins	1834	Mechanical compression system for air-conditioning
	James Harrison; A. Twining	1850	Refrigerator
	Ferdinand Caree	1854	Ammonia absorption system for air-conditioning
	Thomas S. Mort	1861	Artificial refrigeration of food
	Carl von Linde	1874	Ammonia compression system
Calculating Machines	Charles Babbage	1822–1834	Principle of programmable computer
	Ada A. Lovelace (Byron)	1842	Programming
	William S. Burroughs	1885	Mechanical calculating machine
	Hermann Hollerith	1890	Punched cards
Chemical Industry	August W. von Hofmann	1845	Aniline dye from benzene and *nitric acid* (HNO_3)
	C.F. Schönbein	1846	Nitrocellulox from cellulose and *citric acid*
	W.H. Perkins	1856	First synthetic dyestuff; quinine

Remarks:
HNO_3 used for fertilizers, explosives, dyestuffs.
H_2SO_4 used for fertilizers, explosives, dyestuffs, petrochemicals, dehydration, iron and steel pickling.
HCl used for food preservation, metallurgy, oil industry.
NH_3 used for nylon, fertilizers, explosives, plastics, films.
Na_2SO_4 used for wood pulp, glass, detergents.
Na_2CO_3 used for glass, soap, detergents, textile treatment.

Table INV-5: (Cont. [d])

Field	*Inventor(s)*	*Date*	*Invention*
Chemical Industry	K. Graebe; K. Lieberman	1869	Synthesizing alizarin dye with the aid of *sulphuric acid* (H_2SO_4)
	Nicolas Leblanc	1789	
	Augustine Fresnel	1811	Production of sodium bicarbonate
	Heinrich A. von Vogel	1822	
	Ernst Solvay	1861	
	Ludwig Mond	1872	
	Joshua Ward	1749	Production of H_2SO_4
	C.L. Berthollet	1785	Bleaching ($CaOCl_2$)
	Charles Tennant	1799	Bleaching powder
	John B. Lawes	1834	Large-scale fertilizers industry
	Peter Spence	1845	Manufacturing *Alum*
	Walter Weldon	1866	Cheep production of chlorine to make bleaching powder
	Fritz Haber	1909	Synthesis of ammonia
Gas and Oil	P. Lebon	1791	Production of gas from charcoal
	M. Murdock	1792	Coal gas for domestic lighting
	J. Young	1850	Production of parafin from crude oil
	E. Drake (US)	1859	First oil well
	Benjamin Silliman	1855	Petroleum products: tar, naphthalene, gasoline
	G.L. Benton; C.M. Pielsticker	1885–1890	Petroleum cracking process
Optics and Photography	John Dollond	1757	Lens combination for chromatic aberration correction
	William Wollaston	1812	Camera lens system
	Charles Chevalier	1821	The crown-glass lens
	Joseph Petzval	1841	Achromatic lens with low spherical aberration
	Joseph Niepce	1822	Fixed positive image, using silver chloride
	Louis Daguerre	1839	Silver image on a copper plate

Table INV-5: (Cont. [e])

Field	*Inventor(s)*	*Date*	*Invention*
Optics and Photography	William Talbot	1839	Photographic paper for negatives
	James C. Maxwell	1861	
	Ducos de Hauron	1869	
	Charles Cros	1869	Color Photography
	Hermann Vogel	1873	
	George Eastman (US)	1888	Photographic film
	John Tyndall	1890	Fiber optics
	Gabriel J. Lippmann	1891	
	J.W. Draper	1840	Astronomical photography: first moon photographs
	Henry Draper	1872	Astronomical spectral photography: first spectrum photograph of Vega
Warfare	David Bushnell	1775–1776	Submarine
	Henry Shrapnel	1784	Gun Shell
	Robert Fulton	1798	First steam powered submarine ('Nautilus')
	William Congreve	1804	Rockets
	Samuel Colt	1835	Revolver
	Christian F. Schönbein	1845	Nitrocellulose (guncotton)
	Asanio Sobrero	1846	Nitroglycerin
	Claude Minié	1849	Conical bullet
	R.J. Gatling	1862	Machine-gun
	J. Wilbrand	1863	TNT
	Alfred Nobel	1867	Dynamite
	Robert Whitehead	1868	Self-propelled torpedo (produced by compressed air)
	H.S. Maxim	1884	Machine-gun
	F. Abel; J. Dewar	1889	Cordite
	P. von Mauser	1889	Bolt-action rifle
Health and Medicine	R.T.H. Laennac	1781	Stethoscope
	Edward Jenner	1796	Inoculation

Table INV-5: (Cont. [f])

Field		*Inventor(s)*	*Date*	*Invention*
HEALTH AND MEDICINE		H. Davy	1800	Pioneer of anaesthetics
		F. Serturner	1805	Morphine
		Crawford Long	1842	First surgical operation using anaesthetic
		Charles Pravaz	1853	Hypodermic syringe
		Alexander Wood	1853	
		Joseph Lister	1867	Pioneer of antisepsis
		Louis Pasteur	1868–1870	Pasteurization
			1882	Rabies vaccine
		Augustus Waller	1887	Electrocardiogram
		Paul R. Ehrlich	1891	Pioneer of chemoteraphy
		Charles Gerhardt	1893	Aspirin
		Wilhelm Conrad Röntgen	1895	X-ray photography
ELECTRIC AND ELECTROMAGNETIC DEVICES	POWER	Ewald G. von Kleist	1745	Leyden jar (condenser)
		C. van Mussahenbroek	1746	
		P. van Mussahenbroek		
		Alessandro Volta	1800	Electric battery
		Luigi Brugnatelli	1805	
		Moritz H. von Jacobi	1837	Electroplating
		G.R. Elkington	1896	
		H. Elkington	1896	
		Michael Faraday	1821–1831	Principles of the motor, generator (dynamo) and the transformer (induction coil)
			1834	Electrolysis
		Joseph Henry	1829–1832	Principle of autotransformer
		William Sturgeon	1823	Electromagnet
		James B. Francis	1849	Hydroelectric turbine
		Georges Leclanche	1868	Dry cell
		William Stonely	1885	AC Transformer

Table INV-5: (Cont. [g])

Field		*Inventor(s)*	*Date*	*Invention*
Electric and Electromagnetic Devices	Power	Z. Theophile Gramme	1859–1869	DC and AC electric generator
		Werner von Siemens	1881	Electric railway (Germany)
		Charles R. Parsons	1884	Reaction steam turbine
		Carl G. de Laval	1883	Impulse steam turbine
		George Westinghouse	1885	AC power transmission (long distance)
		Frank J. Sprague	1884	Electric locomotive (US)
		P. Heroult; C. Hall	1886	Extraction of aluminum by electrolysis
		Lester Allen Pelton	1884–1889	The Pelton wheel (water turbine with *curved buckets* fixed to its periphery)
		N. Tesla	1899	AC induction motor
		Thomas A. Edison	1882	First steam electric power plant
		Sebastian de Ferranti	1899	High voltage AC generation and transmission
		Charles G. Curtis (US)	1900	High-power turbine plant (5000 kw)
	Light	J. Starr	1845	Long-lasting light source powered by electricity; carbon filament in vacuum
		Joseph Swan	1878	
		Thomas A. Edison	1879	
		Charles F. Brush	1879	Carbon arc street lamp
	Visual Communication and media	Alexander Bain	1842	Facsimile telegraphy
		F.C. Bakewell	1847	Facsimile with rotating scanning drums
		Leon S. de Martinville	1855	Phonoautograph
		J. Plücker	1859	First gas-discharge tube (CRT)

Table INV-5: (Cont. [h])

Field			*Inventor(s)*	*Date*	*Invention*
Electric and Electromagnetic Devices	Visual Communication and media		Louis May; Willoughby Smith	1873	Photoconductivity of selenium
			Eadweard Muybridge	1872	Cinematography
	Communication	Recording, Reproduction and Transmission of Signals	Thomas A. Edison	1877	Phonograph
			Charles Cros	1877	Gramophone record of photo-galvanoplastic process
			Constantine Selencq	1878	Facsimile via selenium photo-conductivity
			Oberlin Smith	1878	Magnetic recording and reproduction of sound
			William Crooks	1879	Cathode-ray tube (CRT)
			Julius Elster; Hans Geitel	1880	First practical photocell
			Shelford Bidwelll	1881	Picture transmission via photocell and scanning
			Charles Fritts	1883	Solar cells
			Paul Nipkow	1884	Mechanical TV scanning system photocell
			Louis de Prince	1886	Camera and projector system for cinematography
			Emile Berliner	1887	Grooved audio disc in a gramophone system
			Heinrich Hertz	1888	Wireless communication over long distance
			The Lumiéré brothers	1895	The cinematograph

Table INV-5: (Cont. [i])

Field			*Inventor(s)*	*Date*	*Invention*
Electric and Electromagnetic Devices	Communication	Recording, Reproduction and Transmission of Signals	Guglielmo Marconi	1895	Radio waves transmission and reception
			Karl F. Braun	1897	Cathode-ray tube oscillograph
			Valdemar Poulsen	1898	Magnetic wire recording and reproduction of sound
		Telegraphy	C.F. Gauss; W.E. Weber	1833	Short-distance communication
			Charles Wheatstone	1837–1858	Long-distance communication
			Samuel Morse	1838	Signal alphabet code
			Lord Kelvin Cyrus W. Field (US)	1866	Transatlantic telegraph cable-line
		Telephony	Charles Wheatstone	1827	Microphone
			Philip Reiss	1855–1861	Microphone
			Alexander G. Bell	1876–1877	Microphone
			David Hughes	1877–1878	Carbon microphone
			Almon B. Stowager	1891–1906	Automatic exchange with dial switches
			Michael Pupin	1894	Long-distance telephony (New York–Chicago)

Table INV-6: MAJOR INVENTIONS IN THE 20^{th} CENTURY

Field		*Inventor(s)*	*Date*	*Invention*
TRANSPORTATION TECHNOLOGY	AUTOMOBILE	Henry Ford	1908	Assembly line method of automobile manufacturing
		Charles F. Kettering	1911	Self-starter; Engine ignition system
		Jean A. Gregoire Pierre Feneille	1926	Front wheel drive
		The Sturtevant brothers	1904	Automatic transmission
		Franz Wankel	1951	Rotary engine
	AVIATION	Samuel P. Langly Clement Ader	1896 1890	Airplane pioneers
		F.W. Lanchester	1897–1908	Wing theory and design
		Ferdinand von Zeppelin	1900	Built first successful dirigible
		The Wright brothers	1903	Self-powered controlled flight of heavier than air vehicles
		Rene Lorin A.A. Griffith Frank Wittle	1909 1926 1930	Turbojet engine
		Max von Opel	1929	Flight powered by rocket engine
			1947	First supersonic jet flight
			1970	Concord supersonic jet reaches Mach 2
		H. Anschutz-Kempfe Elmer Sperry (USA)	1908 1911	The gyrocompass
		Enrico Forlanini	1900	Hidrofoil boat
		Christopher Cockerell	1956	Hovercraft
		Paul Cornu	1907	Built first helicopter
		Juan de la Cierra	1923	Autogyro
		Heinrich Focke Rene Leduc Hans von Ohain Igor I. Sikorsky	1936 1938 1939 1939	Helicopter

Table INV-6: (Cont. [a])

Field		*Inventor(s)*	*Date*	*Invention*
Transportation Technology	Trains		1955	SNCF electric locomotive (France) on the Bordeaux line travels at speed 331 km/hr
			1990	TGV (France) on the Courtalain-Tour line travels at speed 515 km/hr
			1997	*Magnetic levitation* vehicle moves at 550 km/hr
Material Technology		Leo H. Baekland	1907	Bakelite
		Julius A. Nieuwland	1904–1924	Synthetic rubber
		J.E. Brandenburger	1912	Cellophane
		H. Brearly	1913	Stainless steel
			1917	X-ray crystallography for analysis of crystal structure and polymers
		Hermann Staudinger	1920	Birth of polymer science
			1927	PVC for pipes and bottles
		Eugene Freyssinet	1927	Prestressed concrete
		Richard Drew	1929	'Scotch' tape
		William Chalmers	1930	Plexiglass (perspex)
			1930	Polystyrene (styrofoam) for cups, packaging and thermal insulation
		Thomas Midgley	1930	Freon-refrigeration coolant
		Wallace Carothers	1931	Nylon: neoprene
		Roy J. Plunkett	1938	Teflon
		Thomas; W. Sparks	1937	Butyl rubber
			1941	Styrene butadiene rubber to make *tires*
		J.R. Whinfield; J. Dickson	1941	Terylene
			1941	Polyethylene for packing, piping and toys
		W.E. Hanford; D.F. Holmes	1942	Synthetic polyurethanes
			1952	Mylar polyester film
		Karl Biegler; G. Natta	1953	Polyethylene; polypropylene rubber

Table INV-6: (Cont. [b])

Field	*Inventor(s)*	*Date*	*Invention*
Material Technology	Michael Schwartz	1954	Polyisoprene rubber (imitation natural rubber)
	Edward Schmitt; R. Polistine	1963	Synthetic sutures (polyglyolic acid): tissue engineering
	Bruce Merrifield	1964	Synthetic proteins and peptides
	W.J. Buehler	1967	Shape-memory alloys(*)
	Ronald Rosenzweig	1968	Ferrofluids (magnetic liquid)
	James Fergason Hoffmann La Roche	1969–1970	Liquid crystal display (LCD)
	James Economy	1970	Ekonol (moldable high temperature polymer for electronic devices and aircraft engines)
	Stephanie Louise Kwolek	1971	Kevlar (polymer fiber stronger than steel) for bullet-proof vests and fire-proof garments (up to 300°C)
	A.G. McDiarmid; H.J. Heager	1977	Conductive plastics
	L.E. Lyons; Neville Mott; Hugh McDiarmid	1978	Molecular transistors (carbon based materials)
	Dan Shechtman et al.	1984	Quasicrystals(**)
	R. Smalley; H. Kroto	1985	C_{60} ('Bucky balls')
	G.J. Bendorz; Alex K. Müller	1987	High-critical temperature ceramic superconductors
	H. Naarmann; N. Theopilou	1987	Polyacetylene-Iodine compound: efficient conductor of electricity

(*) The study of shape-memory alloys takes off when **William J. Buehler** of the Naval Ordnance Laboratory, discovered the *shape-memory effect* in equiatomic Nickel-Titanium alloy (NITINOL: NIckel-TItanium Naval Ordnance Lab). The effect is governed by the thermoelastic behavior of the martensite phase.
Shape memory alloy have found applications in medicine (Vascular Stents), orthodontistry (braces), coffeepots, thermostats and other *smart materials*.
The Shape Memory Effect (SMF) was first discovered by the Swedish physicist **Arne Olander**, for a gold-cadmium alloy in 1932.
(**) *Discovered*, but may not exist in nature outside laboratories. Produced via invented annealing technique. May lead to many inventions and products.

Table INV-6: (Cont. [c])

Field		*Inventor(s)*	*Date*	*Invention*
World-Wide Communication and Information Technology	Radio	J.A. Fleming	1904	Diode tube
		Greenleaf W. Pickard	1906	Crystal radio detector
		R.A. Fessenden	1906	Amplitude-modulation; alternator
		Lee de Forest	1907	Triode tube
		Edwin H. Armstrong	1912	Frequency-modulation
		Irving Langmuir	1913	Gas-filled electron tube
		Edwin H. Armstrong	1920	Superheterodyne receiver
		Frank Conrad	1920	First radio broadcast station (Pittsburgh)
		Alan D. Blumlein	1932	Stereo sound system
		Seymour Benz	1942	Use of germanium crystal as radio detector: precursor to the transistor radio
	Television		1900	The word 'television' coined
		Manfred von Ardenne, Boris Rosing, A. Campbell-Swinton	1901–1923	Early experiments with television: CTR with fluorescent screen and electron scanning to replace Nipkow's mechanical system
		Charles F. Jenkins, Philo T. Farnsworth	1923–1930	Many major TV components: power, focusing systems synchronization, contrast and scanning
		Denes von Mihaly	1928	
		Vladimir Zworykin	1928–1935	All-electronic system: iconoscope and kinescope
		E.F.W. Alexanderson	1928	First home TV receiver
		John Logie Baird	1939–1945	Electromechanical TV transmitter
		Peter C. Goldmark	1940	Color TV transmission
		Henri de France	1961	SECAM = Sequenced Color And Memory
		Walter Bruch	1963	PAL = Phase Alternating by Line

Table INV-6: (Cont. [d])

Field		*Inventor(s)*	*Date*	*Invention*
World-Wide Communication and Information Technology	Recording and Reproduction of Sound and Light	Eugene A. Lauste	1906	Early sound motion pictures
		Samuel Waters	1920	Early stereophonic reproduction of sound
		T.W. Case	1922–1926	Sound motion pictures
		Paul M. Ramey	1924	Pulse code modulation (to be used subsequently in computers and audio CD)
		Fritz Pfleumer	1927	Sound recording on plastic magnetic tape
		J.A. O'Neill	1927	Sound recording on coated paper ribbon
		H.C. Harrison; F.V. Hunt A.H. Frederick; J.A. Pierce	1931–1933	Early attempts to produce long-playing records
		W.D. Lewis		
		H.J. von Braumühl Walter Weber	1940	Modern tape recorders: high-frequency para-magnetization and AC bias
		Hedy Lamarre (cinema star actress)	1940–1942	Frequency hopping (spread-spectrum); subsequently used for torpedoes and cellular phones
		Peter C. Goldmark	1947	Long-period high-fidelity records
		Emery Cook	1951	Stereophonic LP recording
		Charles Ginsburg; Ray Dolby	1956	First commercial VCR recorder (videotape)
		Klass Compaan	1969	The idea of digitally encoded disc
		Klass Compaan; P. Kramer	1970	Glass-disc prototype
		Joop Sinjou; Toshitada Doi	1979–1983	Audio compact disc (500 MB) (CD)
			1996	DVD technology (5 GB)
	Photography	Edwin H. Land	1932	Polaroid instant camera
		Chester Carlson	1938	Xerography
		Dennis Gabor	1947	Holography
			1969–1990	Video digital camera
			1969	Charged Coupled Device (CCD)

Table INV-6: (Cont. [e])

Field		*Inventor(s)*	*Date*	*Invention*
World-Wide Communication and Information Technology	Telecommunication	W. Chen	1977	Videophone
			1983	Advent of cellular phone
			1984	Cable TV; coaxial development
			1984	Single-mode optical fiber communication system deployed. Transmitters use multi-longitudinal-mode Fabry-Perot lasers in the 1.3 μm infrared wavelength band.
			1987	Light amplified by light: First practical wideband, high-gain EDFA (Erbium Fiber Amplifiers) demonstrated: 1.5 μm wavelength-band signals amplified by nonlinear quantum photonics, with the energy provided by a shorter-wavelength pump laser.
			1987	Advent of WDM (Wavelength Division Multiplexing) in optical fiber communications.
			1987	HFC (Hybrid Fiber Coax) technology introduced (AM based)
			1988	Cellular phone network developed
			1989	DSL (Digital Subscriber Line) data-communication technology over twisted copper wire is introduced. Originally planned for video transmission, in the late 1990's it became a leading high-speed domestic internet technology.
			1990	Domination of world-wide news coverage (CNN)
			1993	Internet merges with new WWW (World Wide Web)
			1993	US launched first Direct Broadcast Satellite (DBS)
			1994	Advent of Coaxial Cable Modem (for internet)
			1995	World-wide web explodes
			1995	VoIP (Voice over Internet Protocol): internet telephony
			1996	Digital cellular phone system takes off

Table INV-6: (Cont. [f])

Field		*Inventor(s)*	*Date*	*Invention*
World-Wide Communication and Information Technology	Telecommunication		1998	Telecommunications go wireless
				Iridium global satellite phone
				High definition TV
			2000	Military-grade GPS (Global Positioning System) position tracking and time signals become available to civilians, but can still be jammed by U.S. military in war zones or during global alerts.
			2001	Cable internet connections take off
Warfare and Weapons Technology			1901	Britain's first submarine launched
		Isaac Lewis	1911	Lewis gun
		John Browning	1918	Automatic rifle
			1918	Gyroscope-guided missile (US Navy)
		Wernher von Braun	1942	V-2 rockets, powered by liquid oxygen and alcohol, travel 200 km from Peenemünde to London
			1944	Electromagnetic method of "enrichment", separation of uranium isotopes U-235 and U-238, is achieved in Oak Ridge Tennessee. Necessary to produce fissionable material
		Le Prieur	1915	Air-to-air unguided rockets
		Herbert Wagner	1943	Air-to-air guided missile
		Leo Szilard; Enrico Fermi;	1945	US exploded two atomic bombs over Japan
		Hans Bethe, et al.	1952	US hydrogen *fusion* nuclear bomb
		E. Teller; Stanislaw Ulam	1956	US 'suit-case' hydrogen bomb
		Wernher von Braun	1958	US launched *Atlas*, ICBM (range: 15,000 km) powered by liquid oxygen and kerosene
			1959	US launched *Titan 1*, ICBM
			1960	Underwater firing of solid-fuel *Polaris*
			1970	Anti-tank missile (US 'Tow')

Table INV-6: (Cont. [g])

Field	*Inventor(s)*	*Date*	*Invention*
Warfare and Weapons Technology		1972	Laser-guided missile (‘smart bomb’)
		1977	US Neutron bomb
		1983	US *Tomahawk* cruise missile
		1985	US assault-rifle M16A2
			‘Laser-gun’, ‘Laser-pistol’, Laser-targeting
		1990	Stealth fighter aircraft F-17A
		1991	Advanced medium range air-to-air missile
		1993	“Star-wars” initiative (SDI) announced
		1999	SDI missile system
Medical and Bio-Technology	Willem Einthoven	1903	Electro-cardiograph
	Mikhail Tsvett	1906	Chromatography (for separation of organic compounds)
	Paul Gelmo	1908	Sulfa drugs
	Paul R. Ehrlich	1909	Chemoteraphy: salvarsan (‘magic bullet’)
	Chaim Weizmann	1915	Discovered the microbe C. Acetobutylicum which converts starch into acetone and butanol, and uses the process to produce *cordite* explosive for the British WWI war effort (artillery shells)
		1917–1920	Sewage treatment using bacteria. Germans use acetone produced from plants to make explosives
	Alexander Fleming	1928	Penicilin
	A.G. Weyman	1932	Cardiac pacemaker
	W.B. Kouwenhoven	1932	Cardiac defibrillator
	Arnold O. Beckman	1934	pH-meter
	Arne W.K. Tiselius	1937	Electrophoresis (biochemistry)
	W.H. Florey; E.B. Chain	1940	Antibiotics
	A. Martin; R. Synge	1944	Chromatography (biochemistry)
	Erwin Chargaff	1978	
	R. Burns; William Doering	1944	Synthetic quinine
	Robert Woodward	1951–1972	Synthetic cholesterol, cortisone, chlorophyll, vitamin B12

Table INV-6: (Cont. [h])

Field	*Inventor(s)*	*Date*	*Invention*
Medical and Bio-Technology	Jonas Salk; Albert Sabin	1954	Polio vaccine (AS = safe oral live virus)
	George Pincus; Carl Djerassi	1955	Oral contraceptive
	Wilson Greatbach	1956	Silicon pacemaker
	Clarence W. Lillehei	1957	Pacemaker (internal)
	Willem Kolff	1957	Artificial heart
	Ian McDonald	1958	Ultrasound as diagnostic and therapeutic aid
		1959	Laser assisted surgery
	René Faraloro	1967	First coronary artery bypass operation
	Christiaan Barnard	1967	First human heart transplant
	Arieh Aviram	1974	Human genome project proposed using carbon-based
	Mark Ratner	1974	molecules as computer switches – the *molecular switch*
	Paul Berg; H.W. Bayer; S. Cohen	1977	Recombinant DNA technology (gene transfer)
		1977	First human protein manufactured by a bacteria
		1978	Laparascopy
		1978	First ‘test-tube’ baby born
		1980	Genetic engineering: production of human insulin
	Kary B. Mullis	1983	Polymerase Chain Reaction (PCR)
	Alec Jeffreys	1984	DNA ‘fingerprinting’
	Robert Jarvik	1982	Artificial heart implant
		1986	Genetically engineered human vaccine
	Peter Schultz	1986	Combination of antibodies with enzymes (‘abzymes’)
		1988	Advent of the ‘Human Genome Project’
	Richard Friend	1988	Diode model of electrochemical circuits of living cells
	Steven Rosenberg	1989	Human gene transfer
	Michael Blease	1989	Human gene transfer
	French Anderson	1989	Human gene transfer
		1990	First ‘gene teraphy’
		1995	Immune system modulation

Table INV-6: (Cont. [i])

Field	*Inventor(s)*	*Date*	*Invention*
Medical and Bio-Technology		1997	Artificial human chromosoms
		1999	First cloning of a human embryo
Artificial Intelligence	Karel Capek	1917	Coined the word '*Robot*'
	John von Neumann	1928	*Game Theory*
	Alan A. Turing	1937	'*Turing Machine*'
	John P. Eckert John W. Mauchly	1946	*ENIAC*, the world's first fully electronic programmable electronic computer
	Norbert Wiener	1947	'*Cybernetics*': information and control theory
		1947	The discipline '*Artificial Intelligence*' is born
	Maurice Wilkins	1949	Built 'EDSAC' – the world's first *stored-program* computer
	Alan Turing	1950	'*Turing Test*' for machine intelligence
	Claude Shannon	1950	Proposed a computer *chess program*
	D. Devol; J.F. Engelberger	1956	First industrial robot '*Unimate*'
	John von Neumann	1966	Developed theory of self-replicating automata. Such systems can find applications in *nanotechnology* and planetary explorations and terraforming.
		1974	First *computer-controlled* industrial robot
	Gordon Matthews	1979	Patent filed for *voice-mail* invention
		1984	Wabot-2, a 100 kg robot (in Japan) reads sheet music through its camera eyes and plays an organ with its 10 fingers and two feet
		1986	A robotic ping-pong player wins against humans
		1992	*Smart Weapons* incorporate electronic copilots, pattern-recognition techniques and other advanced technologies for tracking, identification and destruction
		1996	Speech recognition systems enter corporate voice menus, facilitating the handling of customer calls
		1997	First continuous-speech dictation software released

Table INV-6: (Cont. [j])

Field	*Inventor(s)*	*Date*	*Invention*
Artificial Intelligence		1997	*DEEP BLUE* IBM computer wins a chess match against the world's champion
Computer Technology	J. Abraham; E. Bloch	1918	Binary electric calculating machine
	W.H. Eccles; F.W. Jordan	1919	First "flip-flop" circuit design
	Vannevar Bush	1928	Partly electronic analog computer
	Konrad Zuse	1935	Programmable electromechanical computer
	Alan A. Turing	1941	The '*BOMBE*' computer (*Enigma* project)
	George R. Stibitz	1940	Electronical digital computer
	Russell Ohl	1940	Silicon pn junction demonstrated at Bell Labs.
	John von Neumann	1942–1952	Stored programs – advent of computer science
	John Atanasoff; C. Berry	1942	Electronic digital computers
	T.H. Flowers; A.W.B. Coombes	1943	The '*COLOSSUS*' computer (*Enigma* project)
	H.A. Aiken; Grace Hopper	1944	Harvard Mark I – automatic digital computer
	John W. Mauchly; J.P. Eckert	1946	ENIAC I (20,000 vacuum tubes)
	John Bardeen William B. Shockley Walter H. Brattain	1947–1948	Doped semiconductor transistor
	Frederick Williams; T. Kilburn	1948	First stored-program computer, SSEM, built as a test for newly invented tube memory device, Williams storage CRT.
	Yoshiro Nakamata	1950	Floppy disc
	Jay Forester	1951	Development of RAM (Random Access Memory); magnetic core memory at M.I.T.
	G.W.A. Drummer	1952	The idea of integrated circuits
	John Backus	1954	High level programming language
	J.S. Kilby; Robert N. Noyce	1959	Integrated circuit – the "chip"
	Douglas Engelbart	1964–1965	The 'mouse'

Table INV-6: (Cont. [k])

Field		*Inventor(s)*	*Date*	*Invention*
Computer Technology		James T. Russell	1965	Created the digital Compact Disc (CD)
		Lofti Zadeh	1965	Fuzzy logic
		Marcian Hoff; F. Faggin; S. Mazo	1970	The microprocessor
		Alan Shugart	1971	Reintroduced the floppy disc (1.44 MB)
			1972	Magnetic disc; microcomputers based on microchips
		J.S. Kilby; J.D. Merryman	1972	Electronic pocket calculator
		Robert Metcalf	1973	Networking (Ethernet)
		David Ahl	1974	First PC computer
		Edward Roberts	1975	PC, the 'Altair'
		Stephen Wozniak	1976	Apple I, PC
		Steve Jobs	1977	Apple II, PC (first commercial PC)
		Seymor Rubenstein; R. Barnaby	1979	Word processor
		Kenneth H. Olsen	1986	Magnetic core memory
		Tim Berners-Lee	1989	World-Wide Web (WWW)
		Scott Fisher	1989	Virtual Reality (VR)
		Thomas Bein; C.G. Wu	1994	'Molecular wire' nano-electronic device
			1997	PC in 35% of US homes; 27 million US adults regularly use the internet
Imaging Technology	Sonar, Radar and Lidar	Paul Langevin	1917	Sonar echolocation system (tracing U-boats)
		Christian Hulsmeyer	1904	One-mile range electromagnetic echo system
		Albert W. Hull	1917–1921	Magnetron diode; LIDAR-Light Detection and Ranging
		G. Breit and M. Tuve	1925	First practical RADAR
		Robert Watson-Watt	1935	Microwave RADAR used in WWII
		The Varian brothers	1938	*Klystron* microwave generator
		J.T. Randall; Henry Boot	1940	Multicavity *magnetron* generator
			1945	Reflection of RADAR signals from the moon

Table INV-6: (Cont. [l])

Field		*Inventor(s)*	*Date*	*Invention*
Imaging Technology	Microscopy	Max Knoll; E.A.F. Ruska	1929	Electron microscope
		Frits Zernike	1935	Phase-contrast microscope
		James Hillier	1937	Improved electron microscope
		Erwin W. Mueller	1955	Field-ion microscope ($\times 10^6$)
		Albert Victor Crewe	1970	Scanning electron microscope
		Gerd Binnig; Heinrich Rohrer	1980	Scanning Tunneling Microscope (STM)
				Atomic-force microscope
				Field-emission microscope
				Near-field optical microscope
				Confocal microscope
			1988	Positron transmission microscope
	Tomography	Johann Radon	1917	The Radon transform
		Pierre-Michel Duffieux	1940–1966	Optical imaging
		André Maréchal	1946–1960	Photographic image enhancement
		Felix Bloch; E.M. Purcell	1946	Nuclear Magnetic Resonance (NMR)
		Norbert Wiener	1948	Mathematical basis for magnetic resonance imaging (Cybernetics = computer controlled systems)
		Godfrey Hounsfield; Alan Cormack	1967–1968	Computerized tomography imaging
		Godfrey Hounsfield	1972	Computed Tomography Scanner (CT)
		Raymond Damadian	1971	NMR imaging
		Paul Lauterbur	1973	Magnetic Resonance Imaging (MRI)
		J.E. Greenleaf; S.A. Johnson	1974–1980	Ultrasound computed tomography
		Louis Sokoloff	1978	Emission Computed Tomography (ECT)
		Douglas Boyd	1979	Computed Tomography Scanner

Table INV-6: (Cont. [m])

Field		*Inventor(s)*	*Date*	*Invention*
IMAGING TECHNOLOGY	TOMOGRAPHY			Electron Paramagnetic Resonance (EPR)*
				Ferromagnetic Resonance (FMR)*
				Anti-Ferromagnetic Resonance (AFMR)*
			1979	Single-Photon Emission Computed Tomography (SPECT)
			1985	Positron Emission Tomography (PET)
	EARTH-BASED TELESCOPY		1917	*Mount Wilson* 254 cm Optical Reflecting Telescope began operation (ORT)
		Karl Jansky	1930	Built a 30 m long rotating aerial radio telescope
		Bernhard V. Schmidt	1934	First 35.5 cm Schmidt Optical Reflecting Telescope (SORT)
			1936	*Palomar* 45.7 cm SORT began operation
		Grote Reber	1937	Built a 945 cm Radio Telescope (RT)
		Bernard Lovell	1947	*Jodrell-Bank* 66.5 m Non-Steerable Radio Telescope (NSRT)
			1949	*Palomar* 122 cm SORT began operation
			1949	*Palomar* 508 cm ORT began regular operation
		Robert Hanbury Brown	1949	Correlation interferometer for measuring angular diameters of stars
		Bernard Lovelll	1957	*Jodrell-Bank* 76 m Steerable Radio Telescope (SRT)
			1960	*Owen-Valley* 27 m RT
			1963	*Arecibo* (Puerto Rico) 300 m RT
		Martin Ryle	1964	Cambridge (England) 1.6 km radio interferometer
			1965	*Owen-Valley* 40 m RT
			1967	First VLBI images – 182 km baseline
			1969	*Big-Bear* solar observatory

* These were *discoveries*, but applied to imaging *inventions*.

Table INV-6: (Cont. [n])

Field		*Inventor(s)*	*Date*	*Invention*
IMAGING TECHNOLOGY	EARTH-BASED TELESCOPY		1970	*Cerro-Tololo* (Chile) 4 m ORT
			1970	*Kitt-Peak* (Tucson, AZ) 4 m ORT
			1974	Siding-Springs (Australia) 3.88 m ORT
		G. Smith; F. Landauer J. Janesick	1975	Used CCD to observe Uranus
			1979	*Amado* (Arizona) 4.47 meter Infrared Reflecting Telescope (IRT)
			1979	*Mauna Kea* (Hawaii) 3.81 m IRT
			1880	*Socorro* (NM) VLA
			1992	*W.M. Keck* (Mauna Kea Volcano, Hawaii) 10 m optical and infrared telescope: the position of its 36 hexagonal glass segments are continually aligned by computer controlled actuators (Honeycomb 'fly's-eye' pattern); equipped with *wide-field CCD imagers* and *adaptive optics* techniques to compensate for atmospheric distortion
	SPACE-BASED TELESCOPY		1962	First orbital telescope on GB's *Ariel1*: energy spectrum of cosmic rays
			1970–1973	First X-ray satellite
			1972	UV telescope on *Copernicus* satellite
			1979–1981	Gamma-ray telescope on the *Einstein* satellite
			1992	COBE (Cosmic Background Explorer) satellite discerns anisotropy in Cosmological Microwave Background Radiation; advent of *precision cosmology*
			1993	*Hubble space telescope*: 2.4 m aperture of primary mirror: images of celestial bodies
			1999	Chandra X-ray space observatory: 4.2 m diameter telescope: imaging of cosmic X-ray sources

Table INV-6: (Cont. [o])

Field		*Inventor(s)*	*Date*	*Invention*
NUCLEAR[4] TECHNOLOGY	MEDICINE	George Parthes; Walter B. Cannon Henri A. Danlos; E. Bloch	1798–1905	Early uses of X-rays and radium for diagnosis and treatment
		Frederick Proescher	1913	Intravenous radium injection
		Georg von Hevesy	1923	Radioactive tracing (e.g. Lead radioisotope)
		Herman Blumgart (US)	1927	Radioactive traces for diagnosis of heart diseases
		The Joliot-Curies	1934	Artificial radioactivity
		John H. Lawrence	1936–1939	Phosphoros-32 used for leukemia treatment
		John Livingood; S.M. Seidlin	1936–1946	Iodine-131 and Cobalt-60 used for cancer treatment
		Emilio Segré; Glenn Seaborg	1939	Technetium-99 (half-life = 6h)*
		Glenn Seaborg	1950	Californium (transuranium elements)*
		Benedict Cassen	1951	Radioisotope photo-scanner
		Gordon Bronwell; H.H. Sweet	1954	First positron-emission detector
		Hal Anger	1958	Scintillation camera (viewing an organ as a whole)
			1958	Bone scanning with technetium-99
		William Oldendorf	1960	Brain studies with radioisotopes
		Rosalyn Yalow; Solomon Berson	1961	Radio-Immuno-Assay (RIA)
		David Kuhl	1962	Emission reconstruction tomography (the father of SPECT, PET and CT)
		Henry Wagner	1963	Radiolabeled Albumin for imaging lung
			1973	SSRL (Stanford Synchrotron Radiation Laboratory) produced high-intensity X-rays used for medical and material research
			1975	Cardio-vascular imaging with Thalium-201
		Michel Ter-Pogossian	1975	PET scanner (Positron Emission Tomography)
		John Keyes	1976	Single Photon Emission Computed Tomography (SPECT)

* *discoveries* of elements not present in nature

Table INV-6: (Cont. [p])

Field		*Inventor(s)*	*Date*	*Invention*
Nuclear Technology	Medicine	R. Damadian; L. Minkoff M. Goldsmith	1977	Magnetic Resonance Imaging (MRI): first examination of a human body
		David Goldenberg; J.P. Mach	1978–1981	Radiolabeled antibodies for tumor imaging
		Steve Larson; J. Carrasquillo	1982	Iodine-131 labeled monoclonal antibodies used to treat cancer
	Particle Detectors and Accelerators	Boltwood	1907	Radioactive dating
		W.F. Libby	1955	Radiocarbon dating
		Charles Wilson	1894–1911	Cloud-chamber for tracking charged particle in supersaturated gases
		William Crookes	1903	Scintillation counter
		Johannes Geiger	1910	Geiger counter
		A.J. Dempster; F.W. Aston	1918–1919	Mass spectrometer
		Ralph Wideröe	1928	RF linear accelerator
		R.J. van de Graff	1931	Electrostatic accelerator
		J.D. Cockroft; E.T.S. Walton	1932	Linear proton accelerator
		Ernest O. Lawrence	1934	Cyclotron
		D.W. Kerst	1940	Betatron
		Marcus L.E. Oliphant	1943–1947	Proton-synchroton
		E.M. McMillan; V. Vexler	1945	Synchro-cyclotron
			1944	Scintillation counter (with photomultiplying tube)
		Donald A. Glaser	1952	Bubble chamber
			1952	Cosmotron (at BNL)
			1953	First “breader reactor” (US): U-235 is split to provide energy, while U-238 is changed into plutonium, to be used to fuel further nuclear fission reaction
		S. Fukui; S. Miyamoto	1959	Spark chamber
			1966	Stanford electron LINAC (LINear ACcelerator)

Table INV-6: (Cont. [q])

Field		*Inventor(s)*	*Date*	*Invention*
Nuclear Technology	Particle Detectors and Accelerators	G. Charpak; R. Bouclier	1968	Multiwire proportional particle detection chamber
			1965	Alternating Gradient Synchrotron (at BNL)
		Simon van der Meer	1982	Proton-cooling (at CERN)
			1984	Tevatron (Fermilab)
			1987	Storage ring collider (Fermilab)
			1989	Large Electron-Positron Collider (LEP, at CERN)
			1989	Stanford Linear Collider (SLC): single pass collider
			2000	Relativistic Heavy Ion Collider (RHIC) at BNL
Nuclear and Particle-acceleration Technology	Energy		1951	Nuclear fission produce useful electricity in Idaho
			1954	First nuclear power reactor (US)
			1955	First town (Arco, Idaho) to be powered by nuclear energy
			1967	Project 'Gasbuggy': release of natural gas into a nuclear explosion underground chamber
			1975	Laser separation of uranium isotopes
			1976	Full-Electron Laser (FEL; John Madey)
	Applications		1985	Lead-iron phosphate glass for durable containment of nuclear waste
			1991	Cesium atomic clock (1 second drift in 1.6×10^6 years)
				Free-electron laser
	Weapons	Lise Meitner; Otto Hahn	1938	Uranium fission achieved (Germany)
		Enrico Fermi	1942	First self-sustaining nuclear chain reactor (Univ. of Chicago)

Table INV-6: (Cont. [r])

Field		*Inventor(s)*	*Date*	*Invention*
NUCLEAR AND PARTICLE-ACCELERATION TECHNOLOGY	WEAPONS	Seth Neddermeyer	1945	"Fat Man" implosion atomic bomb (Nagasaki)
				"Little Boy" atomic bomb (Hiroshima)
		Edward Teller, Stanislaw Ulam et al.	1952	Thermonuclear hydrogen bomb
			1961	Largest man-made explosion: USSR H bomb, yield = 57 MT (Novaya Zemlya)
			1973	Miniature nuclear warhead (50 ton TNT); USA
			1977	Neutron bomb (radiation damage); USA
		Hyman G. Rickover	1958	Launching of the first nuclear submarine (USS 'Nautilus')
		Hyman G. Rickover	1961	Launching of the first nuclear aircraft carrier (USN)
GEOSATELLITES AND SPACE PROBES	ROCKETS	Konstantin Tsiolkovsky	1903	Rocket propulsion
		Robert Goddard	1926	Launch of first liquid-fuel rocket
		Herman Oberth	1929	Rocketry and space travel technology
		Charles S. Draper	1939	Inertial guidance system
		Theodore von Karman	1941–1963	Long-distance rockets and space travel
		W. von Braun; W. Dornberger	1942–1969	V-2 rockets; Atlas, Titan and Apollo rockets
	SPACE EXPLORATION	Sergei Korolev (project Head)	1957	Sputnik I – first orbiting satellite(USSR)
			1958	Explorer I – first US satellite
		NASA	1962	Mariner 2 – first mission to *Venus*
		Freeman Dyson	1963	Gravity-assist fly by maneuvers
		NASA	1973–2001	

Table INV-6: (Cont. [s])

Field		*Inventor(s)*	*Date*	*Invention*
GEOSATELLITES AND SPACE PROBES	SPACE EXPLORATION	NASA, USA	1965	Mariner 4 – first clear pictures of *Mars*
		NASA	1966	Luna 10 – first spaceship to orbit the *moon*
		NASA	1969	First manned mission to the moon; Apollo 11 launched by Saturn V rocket
		NASA	1971	Lunar rover
		NASA	1974	Mariner 10 photographs of *Mercury*
		NASA	1975	Venera 9 sent pictures of the surface of *Venus*
		NASA	1976	Viking I, II landed on *Mars*
		NASA	1979	Voyager I, II sent images of *Jupiter* and its system
		NASA	1980–1981	Voyager I, II sent images of *Saturn* and its system
		NASA	1981	First flight of space shuttle
		NASA	1986	Voyager II sent images of *Uranus* and its system
		NASA	1989	Voyager II sent images of *Neptune* and its system
		NASA	1989	COBE satellite launched
		NASA	1990	Hubble space telescope
		NASA	1998	'Deep space' ion-rocket prototype
		NASA	2000	US 'Image' space weather satellite
	COMMUNICATION	Arthur C. Clarke	1945	The idea of communication satellite in geosynchronous orbit
		NASA	1960	Weather satellite *Tirus 1*
		NASA	1962	*Telstar 1* re-transmits TV program watched by 200 million people in US and Europe
		NASA	1964	*Syncom 3* launched into a geostationary orbit and used to broadcast Olympic games from Japan
				INTELSAT also relayed Olympic games to North America
			1965	*EARLY BIRD* – first geostationary commercial satellite over the Atlantic Ocean

Table INV-6: (Cont. [t])

Field		*Inventor(s)*	*Date*	*Invention*
GEOSATELLITES AND SPACE PROBES	COMMUNICATION		1965–1992	*INTELSAT 2, 3, 4, 5, 6*. The last one weighing 2500 kg and relaying 24,000 telephone lines plus 3 television channels
		NASA	1969	First live television broadcast from the *moon*, by US *Apollo 11* moon-landing mission, is transmitted to some 600 million viewers worldwide
			1972–1992	*LandSat 1–6* launched to observe the earth, providing warning of crop disease, flooding, icebergs and monitoring pollution
			1977	*Eutelsat*, linked telecommunication networks of the European countries
			1987	Soviet radar satellite (20 tons) launched for map-making, oceanography, crop predictions, ice monitoring and prospecting for minerals
		NASA	1993	*ACTS* was launched from shuttle *Discovery*. It is a testbed for future communication satellite technology such as multi-beam antennae and advanced signal processing
			1996	*INTELSAT* has 18 satellites in orbit working for the world wide web
			2000	Civilian access to GPS satellite data.
QUANTUM TECHNOLOGY	LASERS	Albert Einstein	1916	Proposed the process of stimulated emission (in addition to absorption and spontaneous emission)
		Alfred Kastler	1950	Optical pumping
		Charles Townes; J.P. Gordon H.J. Ziegler	1953	Built the *Maser* which used ammonia to produce coherent microwave radiation
		A. Schawlow; C. Townes	1954	Proposed the *Laser*
		Gordon Gould	1957	

Table INV-6: (Cont. [u])

Field		*Inventor(s)*	*Date*	*Invention*
QUANTUM[5] TECHNOLOGY	LASERS	Theodore H. Maiman	1960	Built the first laser from synthetic ruby
		P.A. Franken; A.E. Hill C.W. Peters; G. Weinreich	1961	Harmonic generation of light by passing the pulse from a ruby laser through a quartz crystal
		A. Javan; W.R. Bennet D.R. Harriott	1961	First *gas laser* (Helium + Neon near infrared)
		M.I. Nathan	1962	Gallium arsenide (semiconductor) laser
		Kumar Patel	1963	CO_2 laser
		W.B. Bridges	1964	Noble-gas Ion lasers
		J.V. Kaspar; G.C. Pimental	1964	Iodine laser (used in holography)
		E. Snitzer	1964	First *optical fiber amplifier* using Neodynium doped fiber, near wavelength 1.06 micron
		P.P. Sorokin; J.R. Lankard	1966	Organic-dye laser (*molecular* energy levels)
		John M.J. Madey	1971–1976	Free Electron Laser (FEL)
		R.H. Stolen; E.P. Ippen	1973	*Raman amplification* of optical signals via stimulated Raman scattering
			1975	Laser separation of Uranium isotopes
		D.L. Mathews	1985	X-ray laser
		M. Nakazawa	1989	Erbium-Doped Fiber Amplifier (EPFA) pumped by high power laser diode of wavelength 1.48 micron
		R.H. Stolen; H.A. Haus	1989	Raman amplifier in silica-core optical fibers. Beginnings of all-photonic computation and communication
		John Bardeen; William B. Shockley Walter H. Brattain	1947	Semiconductor-based diodes and transistors
		Harold Lyons	1949	Atomic clock (based on the quantum-mechanical vibrations of the Ammonia molecule

Table INV-6: (Cont. [v])

Field	*Inventor(s)*	*Date*	*Invention*
QUANTUM TECHNOLOGY		1933–1952	*Zenner-diode* voltage regulator. Used subsequently for light-amplification devices
	Russell Ohl	1941	Silicon junction solar cell
	W.H. Schottky	1948–1956	Schottky barrier diode
	Gerald L. Pearson	1952	Alloy-junction diode
	Narinder S. Kapany	1955	*Optical fiber* communication
	Leo Esaki	1958	*Tunnel diode*
	George Heilmeier	1964	*Light-Crystal-Display* (LCD)
	Nick Holonyak	1965	*Light-Emitting-Diode* (LED)
	Charles Kao; G. Hockham	1966	Proposed using glass fiber for an optical transmission system
		1975–1977	Transmission of video signals by optical fibers
	C. Goy and D. Kleppner	1983–1987	Single-atom cavity[a] QED
		1988	First trans-Atlantic optical-fiber system
	George J. Bendorz Alex K. Müller	1986	High-temperature superconductor alloys
	Bernard Raveau		*Superconductor chip*
	Bennet; Brassand	1989	The idea of quantum computer
	Gerald Pearman	1991	Low-power silicon solar battery
	Sumio Iijima	1991	Carbon nanotubes and their uses as quantum-wire switches
	Peter Shor; N. Gershenfeld I.L. Chung; M.G. Kubinec	1994–1977	First attempts to build a two-bit quantum-mechanical computer using NMR
	Eric Cornell; Carl Wieman	1995	Bose-Einstein Condensate[b]
	L.K. Grover		

[a] Modification of spontaneous emission properties of single atoms in electromagnetic cavities due to coupling with macroscopic cavity modes (RF or optical).

[b] Using a combination of *laser-cooling* and *magnetic evaporative cooling*, they cooled a gas of about 2000 rubidium atoms to a record-breaking temperature of 20 nanoKelvin, enough for the atoms' de-Broglic wave to overlap – thus producing the first ever weakly interacting BEC. Further refinements of these cooling technique led to even lower temperatures (the record as of 2004 is about 500 picoKelvin).

Table INV-6: (Cont. [w])

Field	*Inventor(s)*	*Date*	*Invention*
	Anton Zeilinger	1997	Quantum teleportation
	W.H. Carrier	1902	Air conditioner
	P.C. Hewitt	1903	Mercury vapor lamp
	Georges Claude	1910	Neon light
	G. Claude	1911	Neon lamp
			Halogen light
	Irving Langmuir	1915	Tungsten filament
	Charles Strite	1919	Automatic electric toaster
	B. von Platen; C. Munters	1923	Electric refrigerator
	C.W. Rice; E.W. Kellog	1924	Loudspeaker
	Clarence Birdseye	1924	Frozen food
General Technology	Theodor Svedberg	1925	Ultra-centrifuge (for determination of molecular weights)
	Hans Wilsdorf	1927	First waterproof wristwatch (Rolex)
	Joseph Horton; Warren Morrison	1928	Quartz crystal clock
	Charles Beebe	1934	Bathysphere (depth = 924 m)
	Edward Germer	1934	Fluorescent light
	Jacques-Yves Cousteau	1943	Dived with compressed-air aqualung
	Percy L. Spencer	1947	Microwave oven
	Maria Telkes	1948	Solar heated home
	Douglas Ross	1952	Computer control of machine tools
	Felix Trombe	1954	Solar power plant with parabolic mirror
	D.M. Chaplin; C.S. Fuller	1954	Solar cell
	Gerald L. Pearson	1954	Solar cell
		1960	Large-scale desalination of sea water (US)

Table INV-7: Notable cultural structures on earth since 5000 BCE

Builder	*Location*	*Project*	*Date*	*Height*	*Details*
	Egypt	Sphinx (Giza)	~5000 BCE	20 m	Length = 73 m
	Mesopotamia	Irrigation canal system (Sumer)	~3000 BCE		
Imhotep	Egypt	Pyramid of Khafre (Giza)	~2700 BCE	60 m	Base = 121×109 m^2
Hemon	Egypt	Pyramid of Khufu (Giza)	~2600 BCE	146 m	Base = 230×230 m^2
	Egypt	Sad el Kafara Dam	~2500 BCE	11 m	
	England	Stonehenge (Wiltshire)	~1900 BCE		
	Egypt	Abu Simbel	~1250 BCE	20 m	
	Cambodia	Temple of Angkor Wat	~1150 BCE		
	Jerusalem	Temple of King Solomon	960 BCE	40 m	Base = 30×30 m^2
	Babylon	Marduk Ziggurate (Etemenanki)	~600 BCE	100 m	
	Babylon	Hanging Gardens	~600 BCE	23 m	Area= 120×120 m^2
Cherisphron; Metegenes	Asia Minor	Temple of Artemis (Ephesos)	~550 BCE	12 m	Base = 115×55 m^2
	Iran	Takht-e-Yamshid (Persepolis)	~550 BCE		Length=1100 m
	Jordan	Petra (Nabataean city)	~550 BCE		
	Peru	Geoglyphs and lines of Nasca	~550–500 BCE		Area = 450 km^2
Eupalinos	Greece	Rock-cut water supply tunnel (Samos)	~530 BCE		
	Jerusalem	The Second Temple	516 BCE	51 m	Base = 51×51 m^2
	Burma	Shwe-dagon Pagoda (Rangoon)	~500 BCE	112 m	
Pheidias	Greece	Statue of Zeus (Olymphia)	457 BCE	12 m	
Ictinus; Callicratos	Greece	Parthenon (Akropolis, Athens)	438 BCE	18 m	Area = 37×34 m^2
Satyrnos; Pythios	Greece	Mausoleum (Halicarnassos)	353 BCE	41 m	
Appius Claudius Caecus	Italy	Appian Way (and aqueduct)	312 BCE		Length = 589 km
Chares of Lindos	Greece	The Colossos (Rhodes)	283 BCE	37 m	
Sostratos	Egypt	Pharos Lighthouse (Alexandria)	270 BCE	134 m	
Romans	Spain	Baebolo Silver mining tunnel	250 BCE		Length = 2200 m
	China	Gukow River Dam	250 BCE		Length = 30 m
	China	Great Wall	214 BCE	8 m	Length = 2400 km
	Asia Minor	Closed-pipe water system (Pergamon)	170 BCE		Length = 378 m

Table INV-7: (Cont. [a])

Builder	*Location*	*Project*	*Date*	*Height*	*Details*
	Philippines	Banaue Rice Terraces (Ifuago)	~1 CE		Area = 100 × 100 km^2
	Italy	Narni River Bridge	14 CE		Length = 43 m
	Italy	Lake Fucinus drainage tunnel (Rome)	~45 CE		Length = 5600 m
	Italy	Subiaco River Dam	50 CE		Length = 39 m
	Italy	Colosseum (Rome)	80 CE	49 m	Area = 180 × 150 m^2
	China	Lan Chin Bridge (Yunnan)	65 CE		Length = 76 m
	Syria	Palmyra	~100 CE		
	England	Hadrian Wall	125 CE	6 m	Length = 117 km
	Italy	Pantheon (Rome)	128 CE		
	Mexico	Pyramid of the Sun (Teotihuacan)	300 CE		
	Easter Island	Moai Statues (Rapa Nui)	400 CE		
	Turkey	Hagia Sophia (Istanbul)	537 CE	56 m	
	Mexico	Pyramid and Temple of Inscriptions (Palenque)	~650 CE		
	China	Grand Canal	~650 CE		Length = 1735 km
	Japan	Horyu-Ji Temple (near Kyoto)	~690 CE		
	Spain	Cathedral of Cordoba	786 CE	91.5 m	
	Java	Borobudur Temple	~790 CE	42 m	Area = 123 × 123 m^2
	Japan	Toji Pagoda (near Kyoto)	~796 CE	57 m	
	Guatemala	Maya Temple (Tikal)	~870 CE	145 m	
	France	Mont Saint-Michel (Normandy)	~1050 CE		
	England	Westminster Abbey	1065 CE		
	Italy	St. Mark Cathedral (Venice)	~1071 CE		
	England	Tower of London	1086 CE	30 m	
	France	Reims Cathedral	1211 CE		
	France	Chartres Cathedral	1219 CE	114 m	
	France	Rouen Cathedral	~1220–1876 CE	151 m	
	Germany	Cologne Cathedral	1248–1880 CE	157 m	Length = 142 m (completed 1880)

Table INV-7: (Cont. [b])

Builder	*Location*	*Project*	*Date*	*Height*	*Details*
	France	Amien Cathedral	1270 CE	112 m	
	France	Notre Dame Cathedral (Paris)	1330 CE	69 m	Area = 130×55 m^2
	Spain	The Alhambra (Granada)	1354 CE		
	Italy	Leaning Tower of Pisa	1372	55 m	
	Tibet	Chak-Sem Bridge (Brahmaputra)	1420		Length = 137 m
Filippo Brunelleschi	Italy	Santa Maria Del Fiore Cathedral and Duomo (Florence)	1471	113 m	
	Mexico City	Great Aztec Temple	1487		
	Spain	Cathedral of Seville	1519	56 m	
Inca (Manco II)	Peru	Machu Picchu (High Andes: 2430 m)	1537		
	England	Lincoln Cathedral	1548	160 m	
	Mexico	Nochistongo drainage tunnel	1609		Length = 6400 m
	Italy	St. Peter Basilica (Vatican City)	1626	123 m	Size = 210×137 m^2 (cross)
Shah Jahan	India	Taj-Mahal (Agra)	1654	41 m	
Christopher Wren	England	St. Paul Cathedral (London)	1661	149 m	
	France	Palace of Versailles	1678		Length = 800 m
	England	Telford Bridge: first cast-iron major structure	1775	30 m	
	Italy	Cathedral of Milan	1813	108 m	Area = 149×55 m^2
	USA	Erie Canal	1825		Length = 600 km
	England	Menai Straight Bridge	1826		Length = 177 m
	Sweden	Göta Ship Canal	1832		Length = 382 km
	Italy	Mole Antonelliana (Turin)	1863	167 m	
Ferdinand de Lesseps	Egypt	Suez Canal	1869		Total length = 191 km
	Switzerland	St. Goothard Tunnel	1880		Length = 15 km
	India	Darjeeling Himalayan Railway	1881		
	USA	Brooklyn Bridge (NY)	1883		

Table INV-7: (Cont. [c])

Builder	*Location*	*Project*	*Date*	*Height*	*Details*
	USA	Washington Monument (Washington, DC)	1884	169 m	
	USA	Statue of Liberty (NY Harbor)	1886	46 m	
Gustave Eiffel	France	Eiffel Tower (Paris)	1889	300 m	Base = 101×101 m^2
	USA	Cathedral of St. John the Divine (NY)	1892–2001		Area = 183×98 m^2
Antoni Gaudi	Spain	Casa Mila in Barcelona	1905		
Ferdinand de Lesseps	Italy–Switzerland	Simplon Tunnel	1906		Length = 20 km (completed 1922)
G.W. Goethals	Panama	Panama Canal	1914		Length = 81.6 km
	USA	George Washington Bridge (NY)	1931		Length = 1067 m
R. Shreve, T.A. Lamb, F. Harmon	USA	Empire State Building (NY)	1931	381 m	Area = 86×86 m^2
Paul Landowsky	Brazil	Statue of Cristo Redentor (Rio)	1931	30 m	
	Russia	White Sea–Baltic Canal	1933		Length = 227 km
	Italy	Apennine Tunnel	1934		Length = 18.5 km
	USA	Hoover Dam (Arizona–Nevada)	1936	223 m	Capacity = $35,154 \times 10^6$ m^3
	USA	Golden Gate Bridge (San-Francisco)	1937		Length = 1280 m
	USA	Overseas Highway (Florida–Keywest)	1938		Length = 11 km
	Belgium	Albert Ship Canal	1939		Length = 130 km
Gutzon Borglum	USA	Mount Rushmore Memorial	1942	150 m	(above valley)
	USA	Grand Coulee Hydroelectric Plant (Washington)	1942		Capacity = 6494 MW
	USA	The Pentagon Building (Washington, DC)	1943		Area = 344,243 m^2 $\simeq$ (587 m)^2
	USA	Alaskan Highway	1946		Length = 2288 km
	USA	Delaware Aqueduct Tunnel	1944		Length = 169 km

Table INV-7: (Cont. [d])

Builder	*Location*	*Project*	*Date*	*Height*	*Details*
E. Saarinen	USA	Gateway Arch (St. Louis)	1948	192 m	
	Holland	Amsterdam–Rhine Canal	1952		Length = 72 km
	Russia	Volga–Don Canal	1952		Length = 101 km
	USA–Canada	St. Lawrence Seaway (Montreal–Duluth)	1959		Total length = 3840 km
Frank Lloyd Wright	USA	The Guggenheim Museum (NY)	1959		
	Canada	Daniel Johnson Dam	1960	214 m	Capacity = $141,852 \times 10^6$ m^3
Jacque Piccard Donald Walsh	USA	US Navy Bathyscaphe '*Trieste*'	1960		Deepest ocean descent 10,916 m in the Mariana Trench
	Russia	Bratsk Hydroelectric Plant	1961		4500 MW
	USA	Seattle Space Needle	1962	185 m	
	Switzerland	Grand Dixence High Dam (Valais)	1962	284 m	
	USA–Chile	Pan-American Highway	1962		Total length = 47,516 km
	USA	KTHI TV mast (steel) (Fargo, ND)	1963	629 m	
	Russia	Volga–Baltic Canal	1964		Total length = 1100 km
	USA	Verrazano–Narrows Bridge	1964		Length = 1298 m
	Israel	Shalom Tower (Tel-Aviv)	1966	140 m	
	England	The "Queen Elizabeth 2" Passenger Ship	1967	294 m	Speed = 53 km/hr; Passengers and crew = 2600
	Russia	Krasnoyarsk Hydroelectric Plant (Yenisei)	1968		6000 MW
	Egypt	Aswan High Dam	1970		
	Canada	Mica Dam	1970	243 m	Capacity = $24,670 \times 10^6$ m^3
	USA	World Trade Center Twin Towers (NY)	1972	417 m	(destroyed by terrorists Sept. 11, 2001)
	USA	New Cornelia Tailings Dam	1973		Capacity = $209,500 \times 10^3$ m^3
Joern Utzon	Australia	Sydney Opera House	1973	67 m	Area = 185×120 m^2
	USA	Sears Tower (Chicago)	1974	442 m	
	Canada	CN Tower (Toronto)	1975	533 m	

Table INV-7: (Cont. [e])

Builder	*Location*	*Project*	*Date*	*Height*	*Details*
	Pakistan	Tarbela Dam	1976		Capacity = $121,720 \times 10^3$ m^3
	Tajikistan	Nurek High Dam	1980	300 m	Capacity = $10,500 \times 10^6$ m^3
	England	Humber Bridge	1981		Length = 1410 m
	Brazil–Paraguay	Itaipu Hydroelectric Plant (Parana River)	1983		14,000 MW
	Tajikistan	Rogun High Dam	1985	335 m	Capacity = $11,600 \times 10^6$ m^3
	Japan	Seikan Tunnel	1988		Length = 54 km (23 km underwater)
	Russia	Sayano–Shushensk Hydroelectric Plant and Dam (Yenisei)	1989		6400 MW; Capacity = $31,300 \times 10^6$ m^3
	England–France	Channel Tunnel	1994		Length = 50 km (38 km underwater)
	Hong Kong	Tsing Me Bridge	1997		Length = 1377 m
	Sweden	Höga Kusten Bridge	1997		Length = 1210 m
	China	Civic Plaza Building (Guangzhou)	1996	391 m	
	Japan	Akashi Kaikyo Bridge (Hyogo)	1998	283 m	Length = 1990 m
Cessar Pelli	Kuala-Lumpur	Petronas Twin Towers	1998	452 m	
	Denmark	Storebaelt Bridge	1998		Length = 1624 m
	China	Jiangyin Bridge (Yangtze River)	1999		Length = 1385 m
	Shanghai	Jin Mao Building	1999	421 m	
	Norway	Laerdal Tunnel	2000		Length = 24.5 km
	Turkey	Izmit Bay Bridge	2001		Length = 1668 m
	Denmark–Sweden	Oresband Bridge	2001		
	Canada	Syncrude Tailing Dam			Capacity = $540,000 \times 10$ m^3
	Argentina	Chapeton Dam			Capacity = $296,000 \times 10^3$ m^3
	Argentina	Pati Dam			Capacity = $238,000 \times 10^3$ m^3
	Argentina–Chile	Tunnel through the Andes	2001		Length = 25 km

Footnotes in tables

4 Hans Dehmelt developed the *Penning trap*, with which electrons and ions – and (later) positrons, baryons and antibaryons, and even *anti-hydrogen* atoms – are confined by DC and AC electromagnetic fields to a small apace for long periods of time, during which the properties of the trapped particles can be studied. In 1973 Dehmelt succeeded in isolating a *single* electron in such a device. This feat enabled high-precision measurements of electron attributes such as the mass magnetic moment and the fine structure constant. The electron (and positron) gyromagnetic ratio was measured to an accuracy of a few parts in a trillion. Its comparison with QED theoretical predictions, within experimental and theoretical uncertainties, represents the *the best quantitative success yet of any scientific theory*. The Penning trap was also used to measure mass ratios and EM multipole moments of the positron, muons, protons and antiprotons and neutrons, and the neutron lifetime. It was used to measure atomic spectral frequencies and observe individual quantum jumps. A similar electromagnetic trap, the *Pauli trap*, was developed by Wolfgang Pauli in the 1950s.

5 In 1995 J. Prestage et al., at CalTech, by comparing precise Hg^+ (positive ion of mercury) ion-trap atomic clock with hydrogen-maser clock, derived an empirical upper bound of 0.07 parts per trillion annual variation of the dimensionless fine structure constant. Less stringent bounds were obtained from studying Samarium isotope ratios in a natural 2×10^9 (2 billion) year old fission reactor in Oklo mine, Gabon, and from spectroscopy of high-redshift cosmological objects.
In 2004, scientists at NIST (National Institute of Standards and Technology, formerly NBS) have unveiled a miniature cesium atomic clock. The heart of the clock is about the size of a grain of rice, capable of being mass-produced on semiconductor wafers using existing technology. The entire package, including external electronics, can be as small as a cubic centimeter. The device consumes less than 75 mW (and thus can be battery-powered). In this chip-scale clock, the difference between two infrared-laser frequencies is resonantly tuned to a cesium RF transition frequency, producing a "dark state" in which the cesium atoms cease absorbing and emitting light. This defines the atomic RF frequency standard, relative to which an external oscillator is then stabilized. This miniature atomic clock is not yet as accurate as standard atomic clocks, which are much larger (of order 1 meter) as well as expensive and power-guzzling. Chip-scale atomic clocks potentially offer a 1000 fold improvement in long-term timing precision as compared with quartz crystal oscillators.

Timelines of Contemporary Science and Modern Technologies

The Emergence of Worldwide Communications and Associated Technologies

Introduction

In 1844, four years after Samuel F.B. Morse proved that he could send coded messages along electrical wires, he managed to persuade the U.S. congress to appropriate $30,000 to build a telegraph line between Washington and Baltimore. It was on the opening of that earliest line that Morse sent his historic telegram[6] *– "What hath God wrought" (Numbers* ***23****, 23). With that Morse opened the age of telecommunications and triggered one of the most dramatic commercial developments of the 19th century. He started a powerful process that is still unfolding in our time.*

Because so much business now depends on getting and sending information, companies around the world have been rushing to link their employees through electronic networks. These networks – which today are digital and often have computers as nodes – form a key infrastructure of the 21th century, as critical to business and national economic development as the railroads were in Morses' era.

Some of these networks are "local area networks" (LAN), which merely hook up computers in a single building or complex. Other are globe-girdling nets that connect people the world over. For example, IBM alone connects 400,000 terminals around the world through a system called VNET, which in 1997 handled an estimated 20 trillion characters of data. By itself, a single part of that system, called PROFS, saved IBM the purchase of 10 million envelopes, and IBM estimated that without PROFS it would need nearly 50,000 employees to perform the same work. Networking has spread to the smallest businesses. With some 150 million PCs in use in the United States, many companies now advertise over the Internet.

Companies grow more dependent by the day on their electronic nets – for billing, ordering, tracking, and trading; for the exchange of design specification, engineering drawings, and schedules.

Regarded as purely administrative tools, networked information systems are increasingly seen as strategic assets, helping companies protect established markets and attack new ones.

The race to build these networks has taken on some of the urgency that accompanied the age of railroad construction in the 19th century, when nations

[6] The first telegraph message, sent on May 24, 1844 from the Supreme Court Room of the U.S. Capitol to the Mount Claire Station of the Baltimore and Ohio Railroad, Baltimore, Maryland.

became aware that their fates might be tied to the extensiveness of their rail systems.

The networks of Morse (telegraph), Bell (telephone) and others were unintelligent. Common sense taught that a straight line is the shortest distance between two points. So engineers sought this straight line, and messages sent from one city to another were always sent over this pathway. As this first-stage network *expanded, it was discovered that in the world of networks this is not necessarily the best way to get a message from one place to the other.*

Thus, when a network began to monitor its own performance *it could inject "intelligence" into the system and become, as it were,* "self-aware"*. Crisscrossing the entire planet with wires running into hundreds of millions of homes, and incorporating complex switching systems and transmission, these* second-stage networks*, constantly improved, and endowed with ever more intelligence, were among the true marvels of the industrial age.*

Nowadays, as millions of computers, from giant Crays to tiny laptops, are linked to form a dense interconnected mesh, a still higher level of intelligence or "self-awareness" is needed to process the incredibly vast volumes of information pulsing through them. As a result, researchers are racing to make network even more intelligent. Their goal is the third-stage neural networks*. These will not only route and reroute massages, but actually* learn from their own past experience*, forecast where and when heavy loads will be, and then automatically expand or contract sections of the networks to match the requirements.*

Yet even before this major effort comes to fruition, another, even more gigantic leap is being taken. We are moving not into a fourth–stage system but to another kind of intelligence altogether.

*Until now, even the smartest networks, including the hoped-for neural networks, had only what might be called "*intra-intelligence*" – with all its smartness* aimed inward*. It is akin to the intelligence embedded in our own autonomic nervous system, which regulates the involuntary operations of the body, such as heartbeat and hormonal secretion.*

One could, however, reach beyond intra-intelligence toward networks that do not just transfer data, but analyze, combine, repackage, or otherwise alter messages, sometimes creating new information along the way.

*Thus enhanced, what comes out the other end in such a network would be different from what is fed in – changed by software embedded in the network. These are termed "*Value Added Networks*" (VAN); they are "*extra-intelligent*".*

Combined with the third-stage neural-networks capacity, the advent of VANs would give communication networks not only self-awareness and the *ability to change themselves, but also the ability to intervene directly in our lives, beginning first with our businesses. If this comes to pass, networks will take on revolutionary new roles in business and society.*

I. The electromagnetic telegraph (1800–1902)

During the 16th, 17th and 18th centuries, steady but slow progress in the study of magnetic and electrical phenomena was recorded. Already ***William Gilbert*** *(1600) suggested a link between electricity and magnetism. However what progress occurred between 1600 and 1800 is nevertheless negligible when compared with the strides made between 1800 and 1840.*

The rapid advance which then occurred was not precipitated by any manufacturing process in which electricity or magnetism played a direct role. It was chiefly fostered by a rapidly growing interest in theoretical chemistry *during the rise of* chemical manufacture *and thus benefited form the outlook of new industrial leaders eager to exploit new discoveries.*

For example, the chemist ***Stephan Gray****, while experimenting with insulators and conductors (1729), transmitted static electrical charges (generated by an electrostatic friction-type generator) along a brass wire 100 meter long.*

On the other hand, experiments with frogs (1780) led the biologist-physician ***Luigi Galvani*** *to observations which quickly culminated with the discovery of the cell or battery, which later became the electrical energy source powering the telegraph.*

1800 ***Alessandro Volta*** *(Italy) invented the* voltaic pile *(primitive battery).*

1816 ***Joseph Henry*** *(USA) proposed a single-wire telegraph.* ***Francis Ronalds*** *(1788–1873, England) demonstrated such a device but the verdict of the British Admiralty was (1832): "Telegraphs of any kind are wholly unnecessary".*

1820–1821 *Hans C. Oersted (Denmark),* ***Ampère*** *(France) and* ***Michael Faraday*** *(England) created the science of electromagnetism.*

[Oersted discovered that an electric current creates a magnetic field; Faraday discovered that a changing magnetic field creates an electric current; and Ampère formulated some of the basic laws of electrodynamics.]

1825 ***William Sturgeon*** *(England) built the first electromagnet for practical technological use.*

1830–1833 ***Gauss*** *and* ***Weber*** *(Germany) developed a small scale working telegraph system in Göttingen (3-km line). In the U.S.,* ***Joseph Henry*** *experimented with a closed circuit in which a source of current activates an electromagnet which in turn activates a bell at a distance.*

1836–1837 ***John Daniell*** *and* ***Charles Wheatstone*** *(England) improved the voltaic cell, creating a stable current source.*

1837–1844 ***Samuel Morse*** *(USA) invented the practical telegraph. He set up a 60 km telegraph line between Washington DC, and Baltimore (1844).*

1838 ***Carl August von Steinheil*** *(1801–1870; Germany) discovered the possibility of using the* earth *for a return conductor in telegraphy (*grounding*).*

1851 *First telegraph cable laid across the English Channel.*

1855 ***David Edward Hughes*** *(England and USA) invented a keyboard telegraph with rotating type-wheel printer that grew into the modern* telex *industry.*

1865 *A telegraph system established between India and England. It took on average 6 days to telegraph a message overland between the two countries.*

1866 *First successful transatlantic telegraph cable laid. Promoted by the financier* ***Cyrus West Field*** *(1819–1892, USA) and* ***Lord Kelvin*** *(1824–1907, England).*

1884 ***Emile Baudot*** *(1845–1903, France),* ***J.B. Stearns*** *(USA) and* ***Michael Pupin*** *(USA), independently invented a* multiplex system *– a time sharing device to increase transmission speeds along the telegraph cable.*

1902 *First pacific telegraph cable between Canada and New-Zealand.*

With the advent of railroads, the telegraph became one of the causes and one of the consequences of the industrial revolution. It changed the face of international commerce, opening the door to worldwide communications and accelerating the flow of information.

II. The Telephone – adventure of mass communication (1854–1991)

Mark Twain had a weakness for new inventions; he was fascinated by them, and over the years he lost more than half a million dollars investing in various contraptions.
Once, after a series of bad investments had temporarily tempered his enthusiasm for technology, he was approached by a tall young man with a mysterious device under his arm.
Mark Twain listened politely to what the young man had to say, but explained that he had been burnt once too often and was not interested.

"But I'm not asking you to invest a fortune", said the young man.
"You can have as large a share as you want to for $500."

The author shook his head and the tall, stooped figure started away. Mark Twain, saddened by the sight of his pathetic young man, called after him. "What did you say your name was again?" "Bell", was the reply. "Alexander Graham Bell."

Telephone changed almost everything about business. It permitted operations over a greater geographical area. Top executives could now speak directly with branch managers or salesmen at distant regional offices to find out, in details, what was going on. Voice communication conveyed far more information, through intonations, inflection, an accent, than the emotionless dash-dots of Morse Code ever could.

The telephone made big companies bigger. It made centralized bureaucracies more efficient. Phones helped integrate the industrialized economy.

Capital markets became more fluid; commerce easier. Deals could be struck swiftly, with a confirming letter as a follow-up. Phones accelerated the pace of business activity, which, in turns, stepped up the rate of economic development in the more technically advances nations.

People did not lie awake through the centuries dreaming of making a call. Telephone history did not proceed in a straight line; it was a series of events – mostly technological, some accidental – that made the telephone possible.

As the electrical telegraph was making its way around the world, especially with the laying of the first transatlantic cable in 1866, a new technology was being developed. It was first called the "talking telegraph". Many scientists have conceived the telephone:

1854 ***Charles Bourseul*** *(France), a Belgian telegraph agent, discovered that the vibration of the human voice could be transmitted: a flexible plate would vibrate in response to a varying pressure of the air and used to open or close an electric circuit; a similar plate at the receiving station would be acted on electromagnetically, and thus produce as many pulsation as there are breaks in the current. His idea met with skepticism, and he never built a telephone.*

1855–1861 ***Philip Reiss*** *(Germany) Constructed a talking machine based on the ideas of Bourseul and called it das Telephon. He could not, however, reproduce the human voice with sufficient clarity. It was the first non-talking telephone.*

1876–1877 ***Alexander Graham Bell*** *(USA) and* ***Elisha Gray*** *(USA) independently invented the telephone. The system was based on the principle of the electromagnetic induction [Faraday 1831]: human speech caused a membrane to vibrate, which modulated the magnetic flux threading through an electric circuit and generated by a magnet situated in front of the membrane; the modulated magnetic flux in turn, generated alternating electric currents. At the receiving end, the sound was reproduced by a reverse process. Bell accidentally discovered the principle of the telephone when trying to improve telegraphy.*

Bell's telephone represented a powerful threat to the vested interest of the telegraph companies. This led Western Union to call in Edison to develop an alternative instrument. Since Edison's microphone turned out to be a more efficient transmitter of sound while Bell had the more efficient receiver, the situation was resolved when Western

Union assigned their rights on Edison's device to the Bell Company.

1877–1878 **David Edward Hughes** *(England and USA) invented the carbon microphone [the word* microphone *was coined by Charles Wheatstone in 1827].* **Edison** *(USA) and* **Emile Berliner** *(Germany and USA) hit upon the same idea at about the same time. All three arrived at the invention by way of their efforts to improve the telephone transmitter. Indeed, the device played a critical role in the development of the telephone by increasing its transmission capacity.*

1878 *The first commercial telephone exchange opened in New Haven, CT.*

1880 *The first UK national phone directory.*

1891–1892 **Almon B. Stowager** *(USA) patented the* automatic exchange dial system, *the first electromechanical switching system, which made possible the extension of the telephone network. By 1892, the Bell Telephone Company had 240,000 subscribers in the US.*

1893 **Oliver Heaviside** *(England) solved the problem of the electrical transmission line, enabling engineers to construct transoceanic cables and realize long-distance telephony.*

1894 **Michael Pupin** *(USA) invented the 'Pupin coil': he devised a means of greatly extending the range of distant telephone communication by placing loading coils at predetermined intervals along the transmission wire. It made long-distance telephony practical by amplifying the signal at intervals along the line without distortion.*

1903 *3,278,000 telephones in the United Sates. By the 20th century, the telephone had become the symbol of modern society. It was ubiquitous in most businesses and had impacted both urban and rural life.*

1904–1912 *The advent of* vacuum tubes: *rectifiers (***Fleming***; 1904), amplifiers (***Lee de Forest***, 1906) and feedback oscillators (***Edwin H. Armstrong***, 1912).*

Tubes improved telephone and radio communications and led to US national phone services.

1914 *9.7 telephone per 100 people in US.*

1915 *First transcontinental telephone line opened between New York and San Francisco.*

1916–1931 *The first teleprinter (which made it possible to send messages over the telephone line) was invented in the US. A teleprinter system becomes operational in 1928 and was extended nationally by Bell Labs (1931) under the name telex (= teleprinter exchange).*

1924–1941 *Pulse code modulation (PCM), invented and completely worked out by* ***Paul M. Ramey****. It is the basis of digital audio and used in voice transmission and reproduction. It incorporates the three stages of* Sampling*,* quantization *and* coding*. But this groundbreaking work was then apparently forgotten. The idea was reinvented in 1939 by* ***A.H. Reeves****, forgotten again and finally resurrected during WWII by Bell Labs during research into methods of encoding phone conversation.*

1929 ***H.A. Affel*** and ***L. Espenschied*** *of the AT&T/Bell Laboratories (US) patented a* coaxial telephony system*: the wide bandwidth enabled large number of telephone channels to be assembled in* frequency division multiplexing *– important for meeting the ever increasing telephone traffic. As an added bonus, the outer conductor of the cable screened the signals from interference of radio stations and powerlines.*

1947 *First* microwave relay station *for long-distance telephone communication was adopted between Boston and New York. It eliminated the need for expensive trunk lines.*

1948 ***Claude Shannon*** *created* Information Theory*, containing the basis for* data compression *(source encoding),* error detection and correction *(channel encoding), and estimation of* data channel capacity*.*

1950 ***Richard Hamming****'s work on* error detection *and* correction codes*.*

1950 *First terrestrial* microwave *telecommunication system, installed to support 2400 telephone circuits.*

1956 *First transatlantic telephone cable laid. It consisted of 7242 km of coaxial cable, laid in waters up to 4 km deep. The complete system provided 35 high-quality telephone circuits from London to New York (29) and Montreal (6).*

1963 *First push-button telephones were introduced.*

1965 *First commercial electronic telephone exchange brought into service by AT&T. It embodied for the first time the principle of stored program control, which gave greatly increased flexibility in the services provided by the exchange and better maintenance.*

1966 *First successful transatlantic direct-dial phone calls are made.*

1971 *Regular direct-dial phone call began from New York to Paris and London.*

1976 *High capacity transatlantic cable went into service; it carried 4000 conversations simultaneously.*

1975–1985 *First cellular telephone system launched in Sweden (1975) by the Ericsson Company. Within five years, some 200,000 Scandinavians were using these mobile phones. A cellular (cell) phone is essentially a portable analog or digital radio transmitter and receiver; cell phones are linked via microwave radio to base transmitter and receiver stations that connect the user to conventional telephone networks. They operate in the 829–949 MHz frequency band (wavelength at 900 MHz is 33 cm) or the 1850–1990 MHz band. The geographic region served by a cellular system is subdivided into areas called cells, each cell using frequencies different from those used by its surrounding cells. When the phone carrier moves from one cell to another, the telephone call is transferred from one base station (and its frequency) to the next using a computerized switching system. Cellular phone networks in Japan (1980) and the US (1983). Cellular phone went into cars (1985).*

1983 *565 million telephones in the world.*

1985 *A single optical fiber carries the equivalent of 300,000 simultaneous phone calls in a Bell Labs test.*

1987 *AT&T completed* digitization *of all its long-distance facilities. Voice transmission signals are converted into* compressed *digital computer codes for more efficient transmission trough lines, then 'decompressed' at the receiving end.*

1991 *The total annual volume of international telephone traffic is estimated at 35 billion telephone minutes, globally averaging about 6 telephone-minutes per person per year.*

III. Wireless communication and the birth of Radio (1864–1961)

Maxwell's research into electrodynamics began a few weeks after his graduation from Cambridge (1854), and ended just before his death (1879). By 1864 he had established both a dynamical theory of the electromagnetic field and the electromagnetic theory of light, accounting for the phenomenon of electromagnetic waves, *propagating in vacuo or in a medium at the velocity of light. With this discovery he opened the way for a future technological civilization in which electromagnetic effects play cardinal roles – including the propagation of electromagnetic waves which enables radio, cell phones, television, radar, and optical fiber communications. The first experimental confirmation of Maxwell's theory was rendered by* **Heinrich Hertz** *(1887) who produced and detected the first man-made radio wave signals*[7].

[7] Certain amateurs allegedly transmitted wireless signal before Hertz or even transmitted voice signals before Marconi.

Their results, however, are not true Hertzian radiation effects but rather *induction–field* phenomena. One such person was **Malcolm Loomis** (1826–1886, USA), a dentist by trade; At the close of the Civil War in 1865, he flew two kites, carrying wires, from mountain tops 23 km apart. The wire from one kite was attached to ground through a telegraph key; the other kite wire was grounded trough a galvanometer that could measure very small currents. When he operated

1874–1906 *Evolution of wave detectors:* ***Karl F. Braun*** *(1874, Germany) discovered 'one way conduction' properties in metal sulfide crystals. [In 1901 he introduced the use of a crystal detector as part a wireless receiver.]*

History of the Crystal Radio

The crystal radio receiver is a very simple kind of radio receiver. It needs no battery or power source except the power received from radio waves by a long outdoor wire antenna.

Simple crystal radios are often made with a few hand made parts, like an antenna wire, tuning coil of copper wire, crystal detector and earphones. Because crystal radios are passive radio receivers, they are technically distinct in many respects from ordinary radios containing active powered amplifiers. This is because they must receive and preserve as much electrical power as possible from the antenna and convert it to sound power whereas ordinary radios amplify the weak electrical energy "signal" from the radio wave.

A crystal radio receives programs broadcast from radio stations. Radio stations convert sound into radio waves and send out the waves everywhere. Radio waves travel across the crystal radio antenna all the time. Radio waves make radio wave electricity flow between the antenna wire and the ground wire. This electricity is connected to the crystal radio by the antenna and ground wire. The crystal radio uses a tuner to tune the electricity to receive just one station. Then it uses a crystal detector to convert this radio

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the key, detectable changes of current occurred in the other kite wire. He was granted a patent on his system in 1872, but no known attempt was made to make commercial use of this phenomenon.

It is believed that Loomis merely interrupted current in the antenna resulting from flying an antenna into a cloud, transmitting information between two points by conduction or induction but not via far-field electromagnetic waves.

Another, equally obscure experimenter lurking in the shadows of the early history of radio was **Nathan Stubblefield** (1858–1928, USA) a mendicant Kentucky melon farmer and telephone repairman who could allegedly send messages on a wireless telephone over a distance 800 m (1892–1902). It is assumed that he, too, was relying on the electromagnetic induction field *wave electricity back to sound electricity. It uses earphones to convert the sound electricity to sound you can hear.*

Crystal radio was invented by a long, partly obscure chain of discoveries in the late 1800s that gradually evolved into more and more practical radio receivers in the early 1900s; and constitutes the origin of the field of electronics. The earliest practical use of crystal radio was to receive dot and dash coded radio signals transmitted by early amateur radio experimenters using very powerful spark-gap transmitters. As electronics evolved, the ability to send voice signals by radio caused a technological explosion in the years around 1920 that evolved into today's radio broadcasting industry.

Early radio telegraphy used spark gap and arc transmitters as well as high-frequency alternators running at radio frequencies. At first a primitive detector called a Branley Coherer was used to indicate the presence (or absence) of a radio signal. However, these lacked the sensitivity to convert weak signals.

Around 1906, researchers discovered that certain metallic minerals, such as galena, could be used to detect signals. These devices were called "crystal detectors". ***Greenleaf Whittier Pickard*** *on August 30, 1906 filed a patent for a silicon crystal detector, which was granted on November 20, 1906. Pickard's detector was revolutionary in that he found that a fine pointed wire known as a "cat's whisker", in delicate contact with a mineral produced the best semiconductor effect. A crystal detector includes a crystal, a special thin wire that contacts the crystal and the stand that holds the components in place. The most common crystal used is a small piece of galena. Several other minerals also performed well as detectors. Another benefit of crystals was that they could demodulate amplitude modulated signals. This mode was used in radiotelephones and to broadcast voice and music for a public audience. Crystal sets represented an inexpensive and technologically simple method of receiving these signals at a time when the embryonic radio broadcasting industry was beginning to grow.*

In 1922 the (then named) U.S. Bureau of Standards released a publication entitled, Construction and Operation of a Simple Homemade Radio Receiving Outfit. This article showed how almost any family having a member handy with simple tools could make a radio and tune in to weather, *crop prices, time, news and the opera. More than any other system, the design contain therein, was responsible for bringing radio to the general public.*

While there were a number of earlier experiments with radio broadcasts to the general public, some historians consider the Autumn of 1920 to be the beginning of radio broadcasting for entertainment purposes. Pittsburgh, PA, station KDKA, owned by Westinghouse, received its license from the United States Department of Commerce just in time to broadcast the Harding-Cox presidential election returns. In addition to reporting on special events, broadcasts to farmers of crop price reports were an important public service, in the early days of radio.

In 1921, factory-made radios were very expensive. Many of them cost more than $2,000 USD (in year 2005 equivalent dollars), and less affluent families could not afford to have one. Newspapers and magazines in many countries urged readers interested in radio to acquire one of the inexpensive crystal sets or build their own. To minimize the cost, many of the plans suggested winding the tuning coil on an empty cylindrical oatmeal box. For years afterwards, home experiments used oatmeal boxes as coil forms for homemade radios. Even the crystal itself could be made by mixing powdered sulfur into molten lead to form the lead sulfide "crystal". The crystal radio did not require batteries, but it did require the user to purchase a commercially made set of headphones (or telephone receivers as they were called in those days), since that accessory was not suitable for home construction.

"Carbon amplifier" consisting of a carbon microphone and an electromagnetic earpiece sharing a common membrane and case. This was used in the telephone industry and in hearing aids nearly since the invention of both components and long before vacuum tubes. This could be readily bought or handcrafted from surplus telephone parts for use with a crystal radio. Unlike vacuum tubes, it could run with only a flashlight or car battery and had an almost infinite lifetime.

In the early 1920s Russia, devastated by civil war, young scientist ***Oleg Losev*** *was experimenting with applying voltage biases to various kinds of crystals, with purpose to refine the reception. The result was astonishing – with* *a zincyte (zinc oxide) crystal he gained amplification. This was negative resistance phenomenon, decades before the tunnel diode. After the first experiments, he built regenerative and superheterodyne receivers, and even transmitters. However, this discovery was not supported by authorities and soon forgotten and no device was produced in mass quantity beyond a few examples for research. This was partly due to the low education and overall ignorance of leadership, and partly due to the totalitarian nature of the USSR regime.*

The USSR opposed freedom of information, and registered all radio receivers until 1962, typewriters and copy machines until its demise. Crystadine was produced in primitive conditions; it can be made in a rural forge – unlike vacuum tubes and modern semiconductor devices. It was an unwanted discovery to the authorities, and was consigned to obscurity. Oleg Losev died 1943 in besieged Leningrad, abandoned and nearly forgotten.

When Allied troops were halted near Anzio, Italy, during the spring of 1944, personal portable radios were strictly prohibited, as the Germans had radio detecting equipment that could detect the local oscillator signal of superheterodyne receivers. Some resourceful GIs found that a crude crystal set could be made from a coil made of salvaged wire, a rusty razor blade and a pencil lead for a diode. By lightly touching the pencil lead to spots of blue on the blade, or to spots of rust, they formed what is called a point contact diode and the rectified signal could be heard on headphones or crystal ear pieces. The idea spread across the beachhead, to other parts of the war, and to popular civilian culture. The sets were dubbed "foxhole receivers" by the popular press, and they became part of the folklore of World War II.

In some Nazi occupied countries there were widespread confiscations of radio sets from the civilian population. This led to particularly determined listeners building their own "clandestine receivers" which frequently amounted to little more than a basic crystal set. However anyone doing so risked imprisonment or even death if caught and in most parts of Europe the signals from the BBC (or other allied stations) were not strong enough to be received on such a *set. However there were places such as the Channel Islands where it was possible.*

While it never regained the popularity and general use that it enjoyed at its beginnings, the circuit is still used. The Boy Scouts (who emerged as the unofficial custodians of crystal radio lore) kept construction of a set in their program since the 1920s. A large number of prefabricated novelty items and simple kits could be found through the '50s and '60s, and many children with an interest in electronics built one.

Building crystal radios was a craze in the 1920s, and again in the 1950s. Recently, hobbyists have started designing and building sophisticated examples of the instruments. As much effort goes into the visual appearance of these sets as well as their performance, and some outstanding examples can be found. Annual crystal radio DX contests and building contests allow these sets to compete with each other and help form a community of interest in the subject.

The long wire type antennas often used with crystal radios are monopoles. To receive signals from this type of antenna, a ground reference is needed to provide a place for the antenna signal electricity to flow into and out of. Because crystal radios have no other source of power than the electrical power they receive from the antenna, the grounds for crystal radios must be much better than those used by amplified radios. Amplified radios use energy detectors and as such do not need to take much raw power from the antenna and need little or no physical ground. Crystal radios rely on power detection and need to encourage as much antenna current as possible to flow. This requires effective grounding.

A crystal set *is the simplest radio receiver. There are a variety of circuit designs available. A common design consists of a long-wire antenna, a variable inductor and a variable capacitor forming a tuner or tank circuit to select the desired radio signal frequency, and a detector consisting of a diode demodulator usually consisting of a sharp wire called a cat's whisker pressing against a sensitive point on a mineral crystal in a holder.*

A semiconducting mineral crystal, typically lead sulfide (galena) is fixed inside a brass cup and the radio operator finds the loudest signal by touching the cat's whisker to *various points on the surface of the crystal. Alternately, a discrete semiconductor diode can replace a makeshift cat's whisker diode. The most expensive part can be the length of antenna wire.*

The detector extracts the amplitude modulation from the radio signal by rectifying it, and provides an audio output in proportion to the strength of the signal coming from the antenna. The entire set is passive, requiring no external power. Because no electrical amplification is used, sensitive earphones are required. These sets have no way to control the audio volume.

***Temistocle Calzechi Onesti** (1853–1922, Italy) observed that the electric resistance of a container packed with metal granules is decreased upon the passage of electromagnetic waves. This so called 'coherer' acts like a kind of a macroscopic semiconductor; the device was later developed into a detector of radio waves by **Edouard Eugene Desire Branly** (1844–1940, France) in 1890 and by **Oliver Joseph Lodge** (1851–1940, England) in 1894. [Lodge was also the first to suggest in 1894 that the sun might be a source o radio waves; this was confirmed in 1942.]*

***Greenleaf Whittier Pickard** (1877–1956, USA) discovered (1899) that a contact between a fine metallic wire ("cat whisker") and the surface of a certain crystalline material (notably silicon) rectifies and demodulates high-frequency radio waves. He patented such a device in 1906 and it became an essential component of the crystal radio set. The point-contact rectifier was the forerunner of the transistor (1948). **E.G Acheson** first produced carborundum (silicon carbide) in 1891. In 1906 **N.C. Dunwoody** observed that carborundum could be used as a radio detector.*

1897** Improvement of the Hertz oscillator by **Augusto Righi** (1850–1920, Italy) and **Adolf Slaby** (1849–1913, Germany). Righi designed the ball discharger (spark gap) which could generate stable electromagnetic waves with wavelengths as short as 2.5 cm (12,000 MHz). In Germany Slaby transmitted radio signals to a distance of 21 km using as a transmitter a spark coil connected to an antenna wire. At the receiving end the signal was picked by another antenna, passing through a coherer which in turn activated a bell. Thus, Morse-key pressed at the transmitting end was* *'heard' at the receiver. Slaby was assisted by* ***George Wilhelm von Arco *(1869–1940). The first antenna was used by* ***A.S. Popov*** *who devised it but never used it himself for radio communications.*

1901 *Marconi developed the first practical wireless telegraph system. He was the last in the long line of contributors during 1884–1901. He combined the* ***Ruhmkorff*** *induction-coil, the 3-spark* ***Augusto Righi*** *oscillator, the* ***Onseti-Branly-Lodge*** *coherer and the* ***Popov*** *antenna into a workable system that could transmit coded Morse signals over great distances. Finally in Dec 1901 he succeeded in broadcasting a signal over the Atlantic from Poldhu (England) to Glace-Bay Newfoundland, over a distance of some 3400 km.*

Human speech was first transmitted via radio waves by ***R.A. Fessenden*** *(US).*

1902 ***Wilhelm Schlömilch*** *(1870–1969, Germany) invented an electrolytic detector.*

Discovery of a radio-wave reflection layer in the upper atmosphere.

First radio chess-match: passengers on the American liner Philadelphia *and the Cunard liner* Campania*, 70 miles away in the Atlantic, played the first match by radio, transmitting their moves via wireless operators aboard the ships.*

Valdemar Poulsen *(Denmark) transmitted human voice via radio waves over a distance of 200 m.*

1904–1911 *Advent of the '*crystal set*' receiver based on the crystal detector of* ***Karl F. Braun*** *(1901) and* ***Pickard*** *(1906). Amateurs could from now on build their own wireless receivers and hear early radio broadcasts. It was popular until the crystal detector was superseded by the vacuum-tube radio.*

1904–1914 *High frequency alternators, vacuum tubes, amplitude modulation, feedback and heterodyne systems ushered in modern radio and improved reception:* ***J.A. Fleming*** *(England) and* ***Arthur Wehnelt***[8] *(1871–1944, Germany) introduced the thermionic rectifier valve (1904).*

[8] **Wehnelt** invented the oxide cathode; it consists of a metal wire or sheet which is coated with a mixture of metal oxides and is heated to incandescence directly or indirectly. Its thermal electron emission is by many orders of magnitude larger

R.A. Fessenden *(Canada and US) made the first AM radio broadcast (Dec 24, 1906): sound waves of speech modulate the amplitude of a transmitted radio-frequency carrier wave, creating a* band *of transmitted radio frequencies. The modulated wave is then demodulated by the receiver to recover the original sound wave*[9]*.*

Lee de Forest *(1906, USA) and* ***Robert von Leiben*** *(1878–1913, Germany) invented the triode amplifier tube. De Forest (1907) began regular radio music broadcasts.*

Edwin H. Armstrong *designed (1912, USA) the feedback oscillator vacuum-tube. It was used (1913) by* ***Alexander Meissner*** *(1883–1958, Germany) to generate a radio-frequency signal carrying a spoken conversation between Berlin and Nauen.*

R.A. Fessenden *had suggested (1900) that an* alternator *[a device that converts direct current into alternating current capable of producing continuous radio-frequency waves] could generate electromagnetic waves capable of carrying sound and music. He uses a spark generator to send a human voice to a distance of about 1600 m. In his 1906 broadcast he used the alternator invented by* ***E.F.W. Alexanderson*** *(1906)*

1910 *First successful radio communication from airplane to ground station.*

1911 *Method for locating a radio source by direction finding was developed.*

1912 *The 63 kW spark-transmitter distress call of the* Titanic *is received by the liner* Carpathia *93km away.*

1913 *The first trans-Atlantic two-way radio-telegraph service is lunched by Marconi between Nova Scotia and Ireland.*

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than that of pure metals of the same temperature. We know today that this empirical fact agrees well with *semiconductor physics*, But this was unknown to Wehnelt at the time.

[9] Already in 1899, **G.W. Pickard** had transmitted spoken message from the Blue Hills observatory (Milton, MA) over a distance of 15 km using this “cat whisker” receiver to receive audible signals that had been impressed upon (modulated) a radio-frequency wave.

***William David Coolidge** (US) invented a hot-cathode X-ray tube and introduced the tungsten filament into the incandescent lamp.*

***Irving Langmuir** (US) made the first gas-filled lamp at atmospheric pressure.*

1915 *Physicist **Manson Benedicks** (USA) discovered that a germanium crystal can rectify ac current.*

First major demonstration of long-distance voice communication (radio telephony) from Arlington, VA to Paris.

1918 *Development of the* superheterodyne *radio receiver.*

1919 *Development of the short-wave radio.*

1920 *First commercial radio station (KDKA) in Pittsburgh, PA began broadcasting.*

1921–1925 *Experiment revealing advantage of short-wave radio transmission for long–distance voice communication.*

1922–1926 ***Oleg Vladimirovich Losev** (1903–1942, Russia) developed novel kinds of crystal radio sets (with new crystals he fabricated himself) and was the first to study the effects of bias voltage upon the functioning of crystal diodes in circuits, essentially discovering "*negative resistance*" before the tunneling diode, as well as a pre-ATT version of the transistor and associated amplifiers. He then constructed completely solid-state radios that function up to 5 MHz, a quarter of a century before the transistor.*

1922 ***Edwin H. Armstrong** built the first* portable radio, *known as '*Operadio*'.*

U.S. President Harding had a radio installed in the White House. The Ford T-Model car was equipped with a radio.

1922–1924 ***E.A. Appleton** (1892–1965, England) confirmed the existence of the '*ionosphere*', a region of partially ionized air surrounding the earth at a height of 130–320 km. This acts like a great mirror, reflecting radio waves back to the earth, which also acts as a reflector. In consequence, medium and short-wave radio signals bounce back and forth between sky and earth in a two dimensional "channel" thousands of kilometers in lateral extent, enabling long-distance radio telecommunication.*

AM band was assigned; it spans 550–1550 kHz. More than 1000 radio stations operating in the US

1925 *Advent of short–wave radio broadcasts in the US.*

1927 *The Pentode, a vacuum tube with 5 electrodes, was introduced by* ***H.S. Black*** *of Bell Laboratories (US) who conceived the idea of 'negative feedback'. It is later found to be one of the most significant inventions in electronics and communications. It took 10 years for the patent to be approved.*

First commercial radio-telephone service operated between the US and Britain.

1928 *Crystal radio sets were being gradually replaced by home radio sets with vacuum tubes, loudspeakers and connection to main. A combined radio-gramophone appeared on the market.*

1929 ***Edwin H. Armstrong****, in collaboration with* ***Michael Pupin****, invented the Frequency Modulation (FM) method of radio broadcasting. According to this method, the transmitted signal is made to modulate the frequency of the carrier wave. This means that FM is static-free and capable of high-frequently sound reproduction. Despite its advantages, FM did not get off the ground until after WWII.*

1932 *Automatic Volume Control (AVC) was introduced.*

Abraham Esau *(1844–1955) increased the radio band to include ultra short waves.*

Karl Jansky *invented the radio telescope.*

1937 ***Grote Reber*** *invented the parabolic disc antenna*

1944 *57 million radio sets in the US.*

1945 *Arthur C. Clarke suggested using satellites to relay radio broadcasts.*

1952 *U.S. President Harry S Truman created National Security Agency (NSA), a part of the Department of Defense. It is an organization that monitors the telephone, radio and other communications of both friends and adversaries of the United States. Surreptitiously, it reads the world's mail.*

_1952_ *The first appearance of the Sony pocket sized radio transistor.*

_1958_ *First monolithic integrated circuit ('microchip') was demonstrated by* **Jack St.Clair Kilby** *and co-inventor* **Robert N. Noyce** *(1927–1990, USA). It revolutionized the design and manufacture of electronic components in radio and television.*

The first integrated circuit consisted of a slice of Germanium on which were formed a transistor, a capacitor and 3 resistors, constituting a simple phase-shift oscillator. The components where linked by fine gold wires and the connections embedded as a part of a manufacturing process. The integrated circuit put entire systems of tiny transistor switches, capacities, resistors, diodes and other electronic devices on one tiny microchip. Made chiefly of silicon, aluminum and oxygen – the three most common substances in the earth's crust – the microchip eventually reduced the price of electronic circuits by a factor of order one million.

_1961_ *FM stereo broadcasting was authorized in the US. Stereo radio systems became available.*

_1962_ *The first commercial satellite goes into orbit.*

_1963_ **James B. Gunn** *discovered the Gunn effect: a nonlinear deviation from Ohm's law in gallium arsenide. The presence of a negative differential conductivity region on the current-voltage characteristics of the GaAs crystal makes it possible to devise ultra–high frequency oscillator, knows as Gunn diodes. In 1966 a first commercial UHF generator, working at a frequency of 2–3 GHz with power output of approximately 100 W in pulsed operation, was produced. It was used (1968) in radar technology to measure the speeds of moving objects. These radars were small enough to be carried by hand.*

_1985_ *Sony built a radio the size of a credit card.*

_1987_ **Leo Esaki** *invented the tunneling diode, based on a quantum mechanical effect whereby electrons can travel through a region of electrostatic potential that they would be unable to penetrate classically. Consequently electrons are able to 'tunnel' from one region of a semiconductor to another (by their Schrödinger waves passing through a classical barrier),* *causing resistance to* decrease *rather then* increase *with increasing current. This results in a negative differential resistance which permits oscillations. The tunnel diode is thus used as a very high frequency oscillator at low voltage and power (ca. 100 MHz).*

IV Facsimile (1842–1980)

Smoke and drum signals are believed to have been the earliest form of rapid, long-range communications by humans.

We owe the development of the fax to a Scottish inventor, ***Alexander Bain*** *(1842). Even now, after the advent of electronics and computerized digital communications, Bain's original concept is still the basis of modern facsimile machines.*

Facsimile (fax) is a method of encoding data, transmitting it over telephone lines or radio broadcast channels, and receiving hardcopy text, line drawings, or even photographs.

A modern, digital fax machine scans an image, whether it be text, pictures or mixed, by reading a very small area of the image at a time. The fax machine decides whether the area it is reading is light or dark and assigns the area a number such as "0" for white and "1" for dark. Then the fax transmits the number to a remote facsimile receiver (usually via telephone lines). The receiver makes a mark on paper corresponding to the area on the original image.

This process continues as the transmitting machine scans a series of small areas horizontally across the image, and transmits that information to the remote receiver. The transmitting fax then scans the next lower line and so on until the entire image has been scanned, digitized, and transmitted. (In today's computerized fax machines, the scanned texts are often stored and queued for later transmission.) Facsimile telegraph is one of the oldest telegraph techniques.

Bain's invention was improved by six generations of scientists and engineers, the most important of which are:

1847 **Frederick Collier Bakewell** *(England). Physicist. The (analog) images were transmitted and received on cylinders that rotated at a uniform rate by means of a clock mechanism. A demonstration took place in 1851 at the World's Fair in London.*

1862 ***Giovanni Caselli*** *(1815–1891, Italy) Physicist. Improved on Bakewell's version. His pantelegraph used two extremely accurate clocks and made the synchronization timers independent of the current relayed by the telegraphic line itself. He could send handwritten messages as well as photographs. The first commercial facsimile service run between Paris and Lyon. He transmitted nearly 5,000 faxes in 1865.*

Giovanni Caselli was born in Siena in 1815; he studied literature and science. From 1841 to 1849 he lived in Modena as tutor of the sons of Marquis of San Vitale, but as he took part in the riots for the annexation of the Duchy of Modena to Piedmont, he was expelled from the Duchy. He spent all the money he had saved during his Modenese period in experiments which eventually led to his Pantelegraph.

1877–1880 ***Constantin Senlecq de Ardres*** *(1842–1934, France). Physicist. Made use of the photoconductive properties of selenium. The transmitted image was first focused onto a glass plate of a Camera Obscura. Then, the image was traced line by line with a selenium stylus which converted the amount of light at each spot on the image to an electric current. The electrical signals were sent by wires to the receiver where they magnetically controlled a pencil that re-recorded the document. In 1880 de Ardres published a book entitled "The Telectroscope", and in 1881 he outlined photo-telegraphy.*

1881 ***Shelford Bidwell*** *(England). Demonstrated a device that transmitted silhouettes using both selenium and a scanning system. It was called phototelegraph.*

1895 ***Ernest A. Hummel*** *(USA). Watchmaker in St. Paul, Minnesota.*

His system used synchronized rotating 8-inch drums, with a platinum stylus used as an electrode in the transmitter. The original image was drawn on tin foil using a non-conducting ink made from shellac mixed with alcohol. The image was received on carbon paper wrapped between two sheets of blank paper. When the electrode touched the tin foil in the transmitter the circuit was closed; when it touched the shellac the circuit was opened.

The system was known as the Telediagraph, one of several fax-line devices sending pictures via telegraph lines.

The first machines were installed in the office of the New York Herald in 1898. By 1899, Hummel had improved the machines, and they were in use in the offices of the Chicago Times Herald, the St. Louis Republic, the Boston Herald, and the Philadelphia Inquirer.

1902–1907 ***Arthur Korn*** *(1870–1945, Germany). Physicist. Father of telephotography (1902): sent photos on telephone lines over a distance of 1000 km from Munich to Nuremberg. In 1907 he sent the first wire photos from the continent to England. In 1922, May 06, he wired a picture from near Rome to Berlin, whence it was radioed across the Atlantic to a Navy radio station in Maine in about forty minutes.*

Korn put a sheet of photographic film on a revolving glass drum. Light scattered off the picture, traversed both film and glass, went through a prism and was projected on a selenium cell connected with a battery. The ensuing current were then transmitted by wire to the distant receiver, where it was decoded.

Korn was born in Germany and was appointed professor of Physics at the university of Munich (1903–1908). From 1914–1936 he was professor of Electro-Physics at the Berlin Institute of Technology. Emigrated to the United States in 1939. Korn's idea of scanning the object and transforming light signals into electronic signals, preceded the first meaningful research into television by just two years.

1921 ***Eduard Belin*** *(1876–1963, France). Engineer. First transatlantic transmission of photos by radio. His system, known as belinograph, is based on Korn's idea, but is automatic while the previous transmissions were manual.*

1922 *RCA provided the first transatlantic facsimile service.*

1927 *First commercially available equipment for phototelegraphy goes into operation. Radio transmission of pictures becomes an important tool for press news reporting and weather services around the world.*

Photo 1: Picture transmitted in the 1920s by electric telegraph

1939–1945 **John Logie Baird** *used television for facsimile transmission of maps and written material in WWII.*

1958 *Slow-Scan Television (SSTV).*

1971 *First prototype of laser fax.*

1980 *Public international fax services.*

1984 *Japanese introduce high-quality facsimile.*

V. Recording and Reproducing of Sound and Light (1796–1990)

(Photography, Audio and Video recording and playback, analog and digital methods, cinematography)

1796 *The Music Box was invented in Geneva by the watchmaker **A. Favre**.*

1839 ***Louis Jacques Daguerre** (France) announced his process of making photographs (a silver image on a copper plate), known as the daguerreotype. **William Talbot** (England) invented photographic paper for making negatives*

***Edmund Becquerel** discovered the electrochemical effects of light: the photovoltaic effect; he observed that shining light on an electrode in an electrolytic cell increased the generation of the electric current between the electrodes. His discovery, however, remained a curiosity of pure science for the next 65 years.*

1840–1856 ***Joseph Max Petzval** (Hungary, 1840) and **Phillipp Ludwig von Seidel** (Germany, 1856) lay the foundation for the design and construction of aberration-free objective lenses with large aperture and wide fields. It had great impact on the design of modern cameras and telescopes.*

1855–1857 ***Leon Scott de Martinville** (France) developed a device that produces a graphical image of sound - the Phonautograph. It enables people to "see" sound. The system used a mouthpiece horn and a membrane fixed to a stylus that recorded sound waves on a rotating cylinder with smoked blackened paper. It could not play the 'record' back.*

1856 *The first aerial photos of Paris taken by the French photographer **Cespard Felix Touranchon** (1820–1910), from a balloon.*

1873 ***Louis Joseph May** (England) and **Willoughby Smith** (1828–1891, England) discovered electrical photoconductivity of selenium, thus enabling the transformation of visual images into electrical signals. They found that the electrical conductively of a bar of the element selenium changes* *when it is exposed to light such that the ensuing current is proportional to the amount of light hitting the bar. May then used selenium to send a signal through the Atlantic telegraph cable.*

1875 ***John Kerr*** *(Scotland) discovered the electro-optical Kerr effect (1875) through which intensive electric fields cause certain isotropic amorphous substances to become doubly reflecting (anisotropic). In 1876 he discovered the magneto-optic Kerr effect.*

1877–1878 ***Werner Siemens*** *(1877) and* ***Oliver Joseph Lodge*** *(1878, England) independently patented the loudspeaker; music had yet to be converted into electrical signals that would enable a speaker to work.*

1877 ***Thomas A. Edison*** *and* ***John Kruesi*** *(USA) made the first recording of a human voice – the "talking machine", alias the 'tinfoil cylinder phonograph'. Edison then (1878) recorded sound onto discs and cylinders.*

In the same year, ***Charles Cros*** *(1842–1888, France) developed a gramophone record from which, through a photogalvanoplastic process, the sound could be reproduced.*

W.G. Adams *and* ***R.E. Day*** *observed the photovoltaic effect in solid selenium. Made the first selenium cell.*

1880 ***Constantin Selencq de Ardres*** *(France) announced his method of telegraphic transmission of images via his Telectroscope.*

1880–1904 ***Charles Summer Tainter*** *(England) constructed the first system that utilized a selenium cell to convert sound into light signals (1880).*

The first practical photocell was devised by ***Julius Elster*** *and* ***Hans Geitel*** *in Germany (1900–1904). They patented a system for "recording and reproduction of sound", using a magnetic induction sensing device (1886). This was a precursor of* ***Berliner****'s Gramophone.*

1881 ***Clement Ader*** *(France) built an ultra-sensitive microphone and with it discovered the stereo effect. He used 12 of these microphones to transmit sounds of the Paris Opera, via lines laid through the Paris sewers, to the Exhibition Hall at the "Palais de l'Industrie". Up to 48 listeners* *could hear the opera using two receivers each, one for each ear.*

1883 ***Charles Fritts*** *(US) described the first solar cell made from selenium wafers.*

1886 ***Louise Aimé Augustine Le Prince*** *(1842–1890) developed in Leeds, England, the first camera and projector system suitable for* cinematography. *Mysteriously disappeared in Sep. 1890 on the train from Dijon to Paris, and was never found.*

1887–1901 ***Emile Berliner*** (Germany and USA) *introduced the modern* gramophone *with the groovy audio discs, the needle, and the mechanical loudspeaker horn (1887). He later invented (1897) the shellac disc that could be mass-produced.*

1887–1905 ***Heinrich Hertz*** *discovered the* photoelectric effect. *Hertz found that electric sparks pass more readily between electrodes illuminated by ultraviolet light given off by another spark (i.e. light altered the lowest voltage capable of causing a spark to jump between two metal electrodes).*

Wilhelm Hallwachs *(1859–1922, Germany) discovered that a combination of copper and cuprous oxide is* photosensitive.

Augusto Righi *(1850–1920,* Italy) *related (1897) the proportionality of the current of liberated particles to the intensity of the incident light.*

Phillipp Lenard *(1902) found experimentally that the maximum velocity of the released electrons is independent of the intensity of the incident light but gave a wrong interpretation within the framework of classical physics.*

Albert Einstein *(1905) opened the floodgates of modern* quantum physics, *arguing from Planck's blackbody radiation law that the maximum energy of the released electrons is proportional to the* frequency *of the incident light.*

1887 ***Charles Cros*** *(France) innovated the essential ideas behind a disc-based audio recording and playback system. He also developed the idea behind the duplication system in the manufacture of records today. He was, however unable to construct his own machines because of the cost.*

1888 **Oberlin Smith** *(1840–1926, USA) was first to suggest the use of permanent magnetic impressions* for sound recording. *According to his plan, cotton or silk in which steel dust (or short clippings of fine wire) were suspended, will serve in the role of particle to be magnetized in accordance with the undulatory current delivered from a microphone. Thus, he reckoned, a magnetic pattern will be established which is a replica of the microphone current. Smith never built such an instrument.*

1895 **Auguste** and **Louis Lumiéré** (France) *invented a camera and projector system later patented and known as the Cinematographe. The first public showing to an audience of invited specialists was on March 22, at 44 Rue des Rennes, Paris.*

1898–1901 **Valdemar Poulsen** (Denmark) *developed the first practical magnetic sound recorder and reproducer, the telegraphon. It recorded (on a magnetized steel piano-wire), the varying magnetic fields induced by the sound, and played it back in a reverse process.*

William Bu Bois Duddell *(England) and* **E. Ruhmer** *(Germany) independently suggested the photographic recording of sound. Their progress, though incomplete, inspirited further development by* **Lee de Forest** *(1920), whose work, in turn, served as a basis for the 'talkies' of the cinematography industry.*

1902 **Enrico Caruso** *recorded his voice on a* **Berliner** *gramophone disc.*

1907 **Henry Joseph Round** *(1881–1966, England) discovered the phenomenon of electroluminescence (emission of light from a semiconductor diode: after the application of a potential of ten volts between two points on a crystal of Carborundum, the crystal gave out a yellowish bright light).*

1913 *Beethoven's Fifth symphony was recorded in its entirety on a gramophone disc; The performance was by the Berlin Philharmonic Orchestra under the baton of* **Arthur Nickish**. *It is the first full recording of a symphonic piece ever.*

1920 ***Lee de Forest*** *(USA) pioneered a process of* optical sound recording: *First, sound waves were recorded on magnetic tape which in turn produced an electrical waveform with corresponding amplitude variation. The electrical impulses were then transformed into mechanical vibration of a mirror. A source of light shone a beam of light on the vibrating mirror which then fell through a slit on an exposed moving sound track film. The pattern of light falling on the film was a transcript of the original sound. To play back, light from another lamp was controlled into a beam that shone through the sound track and then struck a photoelectric cell, converting the modulated beam into electric impulses. These reconstructed impulses were then amplified and fed into a loudspeaker behind the screen. The sound track could be synchronized with an accompanying motion picture.*

In a few years this method served as a basis for the 'talkies' of the cinematography industry. For the next 59 years, optical sound recording was used only in cinema. Then numerous inventors endeavored to apply it to music records.

1924 ***C.W. Rice*** and ***E.W. Kellog*** *(US) introduced the modern* loudspeaker.

1927 ***Oleg Vladimirovich Losev*** *(1903–1942, Russia). Discovered what we now know as LED (Light Emitting Diode) and foresaw its use in telecommunications. His discovery languished for half a century before being recognized in the late* 20^{th} *century and early* 21^{th} *century. Losev published his results in Phil. Mag,* ***6****, 1024–1044 (1928) and issued a Soviet Patent #12191 in 1929.*

In 1962, four research groups in the US simultaneously reported a functioning LED semiconductor laser based on gallium arsenide crystals, thus opening the field of solid-state optoelectronics.

1927 ***J.A. ONeill*** *(USA) replaced Poulsen's wire (1898) with a magnetically coated ribbon.* ***John Logie Baird*** *(England) first recorded TV pictures on shellac discs.*

Fritz Pfleumer *(1897–1945, Germany) built the Magnetophone, the first magnetic tape–recorder, using paper tape coated with steel-powder. The all-electric record-player* *with electronic amplifier and built-in loudspeaker went on sale.*

W.L. Carlson and ***G.W. Carpenter*** *(USA) invented the "a-c biasing" method, the next great milestone in magnetic recording.*

End of the silent film; The enormous success of the movie The Jazz Singer (starring Al Jolson), introduced the era of talking motion pictures with the projection of synchronized sound.

1928 ***Warner Brothers*** *in Hollywood adopt the Movietone: the direct photographic recording and reproduction of sound on film:*

Sound is picked up by a microphone, amplified, and the current from the power amplifier is passed through a gas discharge tube, which emits an amount of light proportional to the current. The light is focused into a narrow beam perpendicular to a rolling film, marking upon it a track of constant width but variable density. In the reproducer, a tungsten filament emits a narrow beam of light at right angles to the sound track, and after passing through the track the light is focused upon the cathode of a photocell. The current through the cell is proportional to the pressure of the original sound wave. The photo-current is then passed through a receiver and sets up across it a potential difference, whose fluctuations have the waveform of the original sound. The potential difference is amplified and used to drive a loudspeaker.

1929 *Ten million gramophone records are sold in Germany alone.*

1930–1935 *With the advent of the electric amplifier and a-c biasing it became possible to make quite satisfactory magnetic records and play them back at the desired volume. At that time however, the motion-picture industry was the main client, using magnetic records as a sound-recording medium for talking pictures. Magnetic sound recording was also used in dictating machines and recorders of phone conversations. Thus the magnetic recorder went under the guise names Dailygraph, Telegraphone and Textophone.*

1931 ***Alan D. Blumlein*** *(1903–1942, England) Produced the first stereophonic record and patented the stereo sound system. Bell Telephone Laboratories (USA) also produced stereo recordings for experimental use.*

RCA Victor USA introduced coarse groove discs that ran at $33\frac{1}{3}$ *rpm, but these failed to replace popular 78 rpm discs. The first such record: Beethoven's Fifth was conduced by* ***Leopold Stokowski****. For the first time a complete orchestral piece was issued on a standard long-playing record.*

1935 *Development of the Tweeter and Woofer in loudspeaker technology to reduce distortion.*

1936 ***Eduard Schüller*** *(1904–1976, Germany) produced a commercial version of the first true magnetic tape recorder.*

1937 *German Telefunken Company issued the first electromagnetic light-weight tone-arm with a sapphire stylus.*

Alec Reeves *(England) reinvented the principle of pulse code modulation (PCM) of digitally encoded signals. It later revolutionize the transmission, recording and processing of voice, fax, data and video signals. PCM protects signals against noise and interference. By using a defined digital format it enables differing types of signals to be assembled into a common multiplex, i.e. an integrated services digital network (ISDN).*

C.N. Nickman *(Bell Telephone, USA) demonstrated a magnetic tape recorder of excellent quality, with a tape speed of 40 cm/sec and a recording medium called Vicalloy. It marks the true beginning of the modern period in magnetic recording.*

1938 **Chester Carlson** *(1906–1968, USA) invented Xerography, the first method of photocopying.*

George Harold Brown *developed the vestigial sideband filter for use in television transmission, doubling the horizontal resolution of television picture at any given bandwidth.*

1940 ***Hans-Joachim von Braümuhl*** *(1900–1980, Germany) and* **Walter Weber** *(1907–1944, Germany) introduced the method of high-frequency premagnetization and ac bias of tapes for better sounding tape recording.*

1941 ***Russell Ohl*** *(USA) invented the silicon solar cell.*

1947 ***Peter C. Goldmark*** *(1906–1977, Hungary and USA) developed the first long-playing high-frequency record ($33\frac{1}{3}$ rpm) with 23 min per side capacity [compared to 5^m of the 78 rpm records]. In the new LP vinylite record sound quality was considerably improved. The old records soon disappeared from the market.*

Dennis Gabor *(1900–1979, Hungary and England) pioneered holography.*

Edwin H. Land *(USA) launched the first Polaroid Camera on the US market. It weighted 2.7 kg and took 8 prints per film pack, developing each picture in 60 seconds.*

Invention of the transistor at Bell Laboratories, USA by ***William B. Shockley***, ***John Bardeen*** *and* ***W. Brattain***.

1948 ***Claude Shannon*** *(USA) founded information theory.*

1958 *LP stereo records and record players*[10] *reached the market.*

1958–1964 *First video recorders developed by Ampex (1958) for use by television stations. It is a device to read pictures and sound signals from a television camera onto a magnetic tape. First domestic video recorder was produced by Sony (1964).*

1963 *First compact audio cassette (Phillips).*

Ray Dolby *(US) developed his noise-suppressing technique.*

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[10] LP *stereo sound reproduction*: The *stylus*, mounted in a *cartridge*, follows the molded contours of the record. These contours are analog representations of the sound waveform. The *pickup* converts the movement of the stylus (imparted to it by the groove) into corresponding electrical signals in the form of alternating voltage.

In a *magnetic pick up* (most commonly used) the electrical output is induced by the relative motion of a magnetic field and a coil (or 2 coils in the case of a stereophonic pick up) located in the field. The movable system in the pickup must be mounted so that the stylus can follow every rapid change of direction of the groove, virtually without resistance. For the same reason the mass of the moving parts must be as small as possible to reduce groove wear. The stylus must have a rounded tip suited to the cross-sectional dimensions of the groove, so as to assure that the tip is maintained in contact with the sloping sidewalls of the groove and clear of the bottom. The wavy pattern of the groove determines both *the frequency and the amplitude* of the sound vibration.

1965 **James T. Russell** *invented the digital compact disc. It has since then become an essential component of audio, video and computer systems.*

1969 **Klass Compaan** *(Holland) conceived the idea of a digitally encoded disc.*

James Fergason *(USA) invented liquid crystal display (LCD). It completely redefined many industries, such as computer displays, medical and industrial devices, and a vast array of consumer electronics.*

LCDs were originally based on a concept which used a large amount of power, provided a limited lifetime, and provided a poor visual contrast. Fergason overcame these obstacles in 1969 with his discovery of the twisted nematic field effect, which forms the basis of modern LCDs.

Digital watches were among the first consumer items to use LCDs. Other items included calculators and computer displays. LCDs are used in over five billion items a year.

1969–1975 *VCR (video cassette recorder) was produced by Phillips. The type of domestic video recorder most used now is the VHS (video home system) format, launched by JVC (Japan) in 1975.*

1969–1990 *Charged-coupled device (CCD) was first demonstrated at Bell Labs (1969). It is a solid-state chip which transforms light into electricity and decomposes an optical image into pixels, whose gray-scale values are then digitized. The CCD contains a bank of light detectors, each registering variation of intensity as small changes in voltage.*

It has the form of a small capacitor, composed of metal oxide and semiconductor layers, capable of both photodetection and memory storage. When the subtle changes in voltage (created by the photoconductor electrons) are applied to the metal layer (called the 'gate'), electron-hole pairs created in the semiconductor (by absorption of photons) are separated by an electric field, and the electrons become trapped in the region under the gate. This trapped charge represents a small piece of the image known as a pixel (picture element).

The complete image can be recreated by reading out a sequence of pixels from an array of CCD's. These arrays *are used to capture images in video and digital cameras included in telescopy and microscopy.*

1972 *First domestic* videodisc *demonstrated by Phillips.*

1975 *First laser printer introduced by IBM.*

1979 *Videodisc read by laser in Holland.*

1981 *Hologram technology improved; now in* video games

1982–1984 *The Camera-Recorder combination (camcorder): first coupling of the camera and tape (Japan).*

1982 *Kodak camera used film on a disc cassette.*

1983 *First* audio compact discs *(CD's) marketed in the UK and the US. It is a plastic disc, 12 cm in diameter, that can hold on a single side over an hour's worth of* digitally *encoded sound recording, stored as a succession of pits and plateaux in tracks. The disc is coated with a reflective material (usually aluminum) which either scatters or reflects back into the photoelectronic detector a* laser *beam used to 'read' the encoded sound when the disc is rotated at a high constant linear speed. The advantage of the compact discs over the LP records is its freedom from surface blemishes, so that it can approach perfection in sound reproduction.*

The CD player became available from Sony and Phillips, who collaborated in both research and production. The optical sound reading in compact discs differed from the de Forest process (1920) in that sound was no longer in analog form, *but instead it was coded in a* digital (binary) form; *the micro-hollows on the disc were processed at a rate of 4 million per second. Consequently it produces much better clarity of sound than the microgroove.*

The invention of the compact disc, launched in 1982, was the direct result of research on the videodisc, *also invented by Phillips and marketed in 1980.*

1990 *28% of U.S. homes own CD players; sales total 9.2 million players and 288 million compact discs in the US alone.*

1990–91 *The lossy compression standard JPEG (Joint Photographic Experts Group) is set, and commercial applications based upon it begin to appear.*

1990's *Digital cameras, digital scanners, storage of photos and movies on PC's, and email transmission of imagery become widespread among the general public in developed countries.*

VI. Sonar, Radar and Satellite Radionavigation Systems (1887–1971)

Communication using electromagnetic radiation (except for light) began early in the 20^{th} century, and most early practical systems used very long wavelengths which traveled great distances. Eventually, electronics were developed, including the vacuum tube (valve), which allowed controlled frequencies and modulation schemes. This led to the use of higher carrier *frequencies, many channels, and commercial and industrial radio. During the 1930's and 1940's, various experimenters discovered that higher frequencies could bring other advantages to communications. Some of these experimenters worked for government agencies and the military, some were at universities and some were private individuals. Quantitatively, the high frequency bands divide as follows: (λ = wavelength):*

Television and FM radio	*50–600 MHz*	λ = *6 m–50 cm*
Cellular Phones	*800–900 MHz*	λ = *37.5 cm–33.3 cm*
Microwaves (Radar)	*1000–300,000 MHz*	λ = *30 cm–1 mm*
Cordless Phones	*multi-GHz: 2.4 GHz, etc.*	

Thus every communication service uses a part of the spectrum that is suitable for its needs. Microwaves, for example are easier to control than *longer wavelengths because small antennas could direct the waves very well (i.e. the diffraction limit is less constraining). One advantage of such control is that the energy could be easily confined to a tight beam (expressed as narrow beam width). This beam could be focused on another antenna dozens of miles away, making it very difficult for someone to "eavesdrop" on the conversation. Another characteristic is that because of their high frequency, greater amounts of information could be put on them (expressed as increased modulation bandwidth). Both of these advantages (narrow beamwidth and modulation bandwidth) make microwaves very useful for radar.*

All of these qualities led to the use of microwaves by the telephone companies. They placed towers every 30 to 60 miles, each with antennas, receivers and transmitters. These would relay voice conversations across the country. The ability to modulate with a wide bandwidth permitted a large number of conversations on just one signal, and the reduction in beamwidth reduced interference and hampered casual eavesdropping.

Amateur radio interests in microwaves have mostly been for the challenge of working with such esoteric frequencies that require specialized techniques in design, fabrication and testing. Furthermore, in order to reach beyond LOS (line-of-sight) amateurs have spent countless hours carefully measuring propagation phenomena. Amateurs have carried on conversations using 10 GHz well over 1,000 miles, and have bounced signals at that frequency off the moon.

Radar (Radio Detection and Ranging) *is an instrument for object detection based on the principle of electromagnetic wave reflection: locating an object at a distance and studying its echo.*

It helps airplanes, ships and ground stations detect other objects before they can be visually identified. Radar can be used for military and civilian purposes.

It works by using a radio transmitter and a receiver. The transmitter sends out a high frequency radio wave. If the radio wave hits a metallic or other conducting object (such as a ship), some of it will reflect back to the receiver. By measuring the time it takes for the wave to return and the power of the scattered radio waves which are picked up by the receiver, the position and size of the plane or ship can be estimated and projected onto a radar screen. If a lot of the signal is bounced back then the object is close; if only a little bit of the signal is bounced back then the radar is far from the object, and/or the object is small. The distance is simply derivable from the round trip signal time, since the radio waves in air travel essentially at the speed of light in vacuum. This process is similar to a bat's use of echo-location to locate insects in the dark. The only difference is that echo-location uses sound waves as opposed to the radio waves sent out by the radar.

Radar can "see" through great distances despite fog, rain, snow, clouds or darkness: It can find and accurately locate missiles, aircrafts, ships, submarines, cities, rainstorm and mountains.

Radar also prevents planes from crushing into runways, control towers, and other planes, and is an essential tool for analyzing and predicting the weather.

Microwave radar, as well as LIDAR (Light Detection And Ranging) are routinely used for communications and to measure atmospheric pollution.

Radar was developed for military use in the 1930's by several countries. The invention of radar just prior to WWII changed forever the nature of warfare. Radar eliminated many of the limitations that were imposed on the human ability to see. It virtually eliminated the possibility of sneak attack by enemy aircraft. For those and many other reasons, radar has grown in prominence, so that today it is an absolute necessity.

During WWII radar was used widely on planes and ships. Battles were won by the side that was first to spot enemy airplanes, ships, and submarines. To give the Allies an edge, British and American scientists developed radar technology to "see" for hundred of kilometers, even at night. The research that went into improving radar helped set the stage for post-war invention of the transistor. During the war, radar was tremendously useful for:

- *Precision bombing of targets under conditions of poor visibility.*
- *Enhancing defense of convoys against German submarines in the Atlantic battles of 1943. Subs were detected by airborne radar and subsequently destroyed from the air.*
- *Communication between aerial defense headquarters and planes equipped with airborne radar receivers.*

Had it not been for radar, England might have been invaded by the Germans and Allied bombers would not have been able to attack Germany successfully. Indeed, it was radar that turned the tide of the war. Winston Churchill was quoted as saying: "The atomic bomb ended the war, but radar won it".

Sonar is an acronym for SOund Navigation And Ranging. It is a detection and ranging system based on the reflection of underwater sound waves.

A typical sonar system emits ultrasonic pulses by using a submerged radiating device. It listens with a sensitive microphone for reflected pulses from potential obstacles or submarines.

Modern submarines rely on sonar for detecting the presence of enemy vessels. The most advanced system, called a towed array, uses long cable to which microphones are attached, and the submarine drags this trailing cable far behind as it travels through the sea. Airplanes are used to deploy a different sonar. This system uses a device called a sonobuoy, consisting of a microphone mounted in a floating buoy. It is designed so that when a sound such as that of a submarine engine is picked up, the detector can transmit signals to patrolling antisubmarine planes.

Some animals use sonar to locate their prey and also for navigation. Thus, *bats* are equipped with a biotransmitter in the frequency range 15–40 kHz [wavelength 20 mm–3 mm], emitting ultrasonic squeaks that bounce back from obstacles. *Dolphins* also use sonar. Human sonars generally use sound frequency in the range 5–25 kHz.

In peacetime sonar can be used to determine water depths, or to locate fish schools. The signals are generated by a *transmitter* within the ship. A *transducer* changes the electric signals to sound waves and sends them through the water. These waves strike a target and are reflected back (or scattered) to the ship, where they are transformed again into electric signals which activate the *indicator* that calculates the target's range, direction and size. *Scanning sonar* employs a rotating beam for rapid search.

The Global Positioning System (GPS) is a space-based radio-navigation system, consisting of 24 satellites and ground support. GPS provides users with accurate information about their position and velocity, as well as the time, anywhere in the world and in all weather conditions.

GPS Satellites fly in circular orbits at an altitude of 10,900 miles (17,500 km) and with a period of 12 hours. The orbits are tilted to the earth's equator by 55 degrees to ensure coverage of polar regions. Powered by solar cells, the satellites continuously orient themselves to *point their solar panels toward the sun and their antennae toward the earth.* Each satellite contains four atomic clocks.

There are often more than 24 operational satellites, as new ones are launched to replace older satellites. The satellite orbits repeat almost the *same ground track once each day (as the earth turn beneath them). The orbit altitude is such that the satellites repeat the same track configuration over any point approximately each 24 hours (4 minutes earlier each day). There are six orbital planes [with nominally four space vehicles (SV) in each, equally spaced (60 degrees apart) and inclined at about 55° w.r.t. the equatorial plane. This constellation provides the user with between 5–8 SVs visible from any point on earth.]*

Precise positioning is possible using GPS receivers at reference locations, providing corrections and relative positioning data for remote receivers. Surveying, geodetic control, and plate tectonic studies are examples of applications of GPS.

Time and frequency dissemination, based on the precise clocks on board the SVs and controlled by the monitor station, is another use for GPS. Astronomical observatories, telecommunications facilities and laboratory standards can be set to precise time signals or controlled to accurate frequencies by special purpose GPS receivers.

GPS determines location by computing the difference between the time that a signal is sent and the time it is received. GPS satellites carry atomic clocks that provide extremely accurate time. The time information is placed in the coded broadcast by the satellite so that a receiver can continuously determine the time the signal was broadcast. The signal contains data that a receiver uses to compute the locations of the satellites and to make other adjustments needed for accurate positioning. The receiver uses the time difference between the time of signal reception and the broadcast time to compute the distance, or range, from the receiver to the satellite. The receiver must account for propagation delays, or decreases in the signal's speed caused by the ionosphere and the troposphere. With information about the ranges to three satellites and the location of the satellites when the respective signals were sent, the receiver can compute its own three-dimensional position.

An atomic clock synchronized with GPS is required in order to compute ranges from these three signals. However, by taking a measurement from a fourth satellite, the receiver avoids the need for an atomic clock. Thus, GPS is available in two basic forms: the standard positioning service (SPS) and the precise positioning service (PPS). The atomic-clock time measured on board the GPS satellites is so precise, that the software must correct for STR and GTR effects (!), which amount to about 38 μs per day.

Timeline

1886 ***Heinrich Hertz*** (Germany) *conducted experiments on reflection of electromagnetic waves in the microwave region (around 500 MHz with a wavelength of about 60 cm, and around 2 GHz, corresponding to a wavelength of 15 cm). Yet he did not see its potential application.*

1895 ***Guglielmo Marconi*** *(Italy) experimented with transmission of microwave radio signals.*

1900 ***N. Tesla*** *(USA) was first to describe the possibility of locating a moving object using continuous radio-wave echoes. However, technology at that time was insufficient to implement this detection system.*

1904 ***Christian Huelsmeyer*** *(Germany) patented a radio detector based on Tesla's principle. This engineer proposed the use of radio echoes in a detecting device to avoid collisions in marine navigation.*

1915 *First* sonar *invented during WWI by British, American and French scientists. It was used to locate submarines and icebergs via passive listening devices. The first submarine to be sunk after being detected by hydrophone was the German U-boat UC-3 in the Atlantic (1916). Active sonar, using a pinging device, was developed by* ***Paul Langevin*** *(France; 1918). It was installed in submarines and ships to detect underwater objects by echo sent and received by a transducer. The beam spread out until it detected an object and then reflected (or scattered) back an image echo to the ship. The reflected waves were then converted into electric pulses that formed an image of the object on a screen.*

1921 **Albert Wallace Hill** *(US) The* Magnetron diode *– an electron tube that produced* microwaves. *It underwent two basic modification (1927, 1941) to become an efficient generator of microwave radiation power.*

1922 **G. Marconi** *(Italy),* **A.H. Taylor** *(US), and* **L.C. Young** *(US) suggested detection of a moving object using* pulsed waves *to facilitate the determination of the times of emission and reception of echoes.*

1924 **Edward Victor Appleton** *(England) used radio echoes to determine the height of the ionosphere (an ionized layer of the upper atmosphere that reflects longer radio waves). It was the first successful radio range-finding experiment.*

1925–1935 *Great efforts by scientists around the world to find a practical method of detecting and locating objects by radio echoes. Most of this research was motivated and sponsored by the military.*

1927 *Split-anode magnetron was introduced in Japan.*

1931 **W. Butement** *and* **P. Pollard** *(England) built the first experimental radar system to measure the range of ships from shore.*

1934–1937 *British scientists, led by* **Robert Alexander Watson-Watt** *developed a working radar system and succeeded in detecting radio waves reflected from a flying aircraft (1935). At the time, Watson-watt suggested linking together radio-detection stations to a fighter-control network. His work was vital in developing radar for use in the Second World War in Britain. German scientists, working independently, had already tested their radar system successfully in 1934.*

The equipment used by **Watson-Watt** *in 1935 consisted of a high-voltage radio pulse generator (thermionic valves) and a cathode-ray tube display. However, since the initial beam could not be strongly focused*[11]*, the echo thrown back from the surface of the water or the earth sometimes overshadowed the much weaker echo from the target. Satisfactory focalization was achieved only in 1936 when Watson-Watt employed a* magnetron *with an* electron gun *installed in the cathode emission tube and an electronic amplifier for the echoes. In 1937 he was able to erect a chain of 20 radar station along the British east coast.*

1938–1940 *Telefunken in Germany built a 560 MHz radar system for the Luftwaffe. Over 8 military research centers and 200 German institutes were working on improving the radar, under the somewhat vague direction of Göring. In 1940,*

[11] The size and shape of the antenna depends on its function: e.g. a vertical antenna is used to find the height of aircraft, etc.

however, Hitler banned all electronic research, however fundamental it was, under the pretext that electronics was a 'Jewish science'. His total lack of scientific education and antisemitic zeal cost him the war.

1938–1939 **Russel Harrison Varian** *and* **Sigurd Fergus Varian** *(US) invented the klystron tube for the generation and amplification of microwaves. It basically converts the dc kinetic energy of cathode-emitted electrons into UHF electromagnetic energy. The klystron played a crucial role as part of radar systems during WWII. The invention spawned a whole new microwave industry, and came to play a key role in particle accelerators, used for high-energy fundamental physics research and to collimate x-ray synchrotron radiation for applied research.*

1940 **John T. Randall** *and* **Howard A.H. Boot** *(England) invented the multi-cavity magnetron. This tube is capable of generating high-frequency (microwave) radio pulses at high power levels in the wavelength band around 1 cm. In fact, the magnetron allowed a 20-fold increase in radiated power over the existing electron-valve system. Ground experiments in 1940 demonstrated that detection ranges of order 10 km were possible.*

The first radar stations used aerials over 100 m in height to produce a directional beam of radio waves. But if aerial were much smaller and could be steered, they would be much more useful. However, to make smaller aerials meant using radio waves of shorter wavelengths. The cavity magnetron was created to generate such waves. It converted the dc kinetic energy of electrons, accelerated by a dc potential difference, into UHF electromagnetic energy.

In the magnetron, the tank circuit of the valve oscillator was replaced by a cavity or several cavities. It generated pulses for the radar, each pulse lasting for a few microseconds with wavelengths of 1–10 cm and corresponding frequencies of 3,000–30,000 MHz, and typical peak power of 150 kW at a pulse rate of 1000 sec^{-1} *were attained during WWII. [The magnetron has found applications in television transmission, satellite communication, industrial heating, home cooking (microwave oven), medical imaging, radiation therapy and asteroid tracking.]*

The British had an excellent ground-based-radar system using the magnetron transmitters, but these were too heavy and big to be fitted into an aircraft. A compromise was sought in mounting only the radar receiver *in an aircraft, tuned to the same frequency as a ground radar to hopefully pick up reflected radar echoes. By 1940, when the Germans switched to night bombing, the British equipped their night fighters with* klystron radar *as part of their* airborne radar*, which helped win the Battle of Britain.*

The Germans had developed a ground-based radar which was rather bulky but essentially worked well for their defense *purposes. However, they payed little attention to the klystron know-how (found in their military archives after the war), depending instead on their V-2 rockets to defeat England.*

1940–1945 *Establishment (1940) of the MIT Radiation Laboratory (US) which became the center for most radar developments during WWII. In this Lab, United States and British radar scientists cooperated closely. American scientists gave the British the* klystron *generator and the* duplex switch*, and the British gave the United States the multicavity magnetron. The Allied scientists scored a big achievement by developing receivers, high-power transmitters, and microwave antennae. This made it possible to develop narrow-band, highly accurate radars with small antennae for aircraft, ships and mobile ground stations. The new equipment enabled detection of submarines in the Atlantic (1943), development of long-range navigation systems, early warning radar, and radar for night bombing of remote targets.*

Radar became so effective in aiming anti-aircraft guns that each side tried to jam the other's radar. Allied bombers carried radios to send signals that confused or blanked out enemy radar. They also dropped tons of aluminum foil strips that reflected false echoes to enemy radar screens. In the Pacific Ocean, radar gave the US Navy superiority over Japanese forces in night naval battles. In the Atlantic, airborne radar enabled the Allies to inflict crippling losses on the German submarine fleet.

The research that went into improving radar helped set the stage for the post-war invention of the transistor: radar *systems used a semiconductor crystal rectifier diode. These crystals often could not adjust to the quickness and intensity of a rapidly fluctuating radar signal, and they would frequently burn. A number of institutions including Purdue University, Bell Labs, MIT and the University of Chicago joined forces to build better crystals.*

1942 **Seymour Benzer** *at Purdue University (US) discovered that Germanium* crystals made the best detectors. *Germanium was used to make the first working transistor 5 years later. Scientists also learned new technique on how best to* grow and dope *the crystals. Within the decade, this superb understanding of crystal growth would pay dividends in unexpected areas, not the least of which were the insights necessary to allow the solid state researchers at Bell Labs to grow the germanium semiconductors that were at the heart of the first transistors.*

1943–1945 *Radar operators continually saw on their radar screens images of precipitation (like rain and snow). Scientists then realized, for the first time, that radar would be sensitive enough to detect precipitation. After the war radar become an essential tool for analyzing and predicting the weather.*

1945–1989 *Advances in sonar technology (used for tracking and identification of nuclear submarines) during the Cold War.*

1953 *Invention of the* ammonia maser *(Microwave Amplification by Stimulated Emission of Radiation). It emits ultra-short wave beams at the frequency 23,900 MHz and wavelength 1.3 cm.*

1960 *The US Navy, using the moon as a reflector, sent a radio messages from Pearl Harbor to Washington DC in a pioneering communication experiment.*

1968 *Asteroid* 1566 Icarus *tracked by radar.*

1970 *Pulsed* ultrasonic Doppler *used for blood-flow sensing.*

1971 *Doppler radar is used by meteorologists to study storm systems.*

VII. Television technology – or "A picture is better than ten thousand words" (1817–1990)

The possibility of vision at a distance had occupied scholars' minds long before the idea of sound broadcasting. Webster's dictionary defines television as "the process of transmitting images by converting light to electrical signals and then back again". Television is constructed of two elements – video and audio. Video comes from the Latin word "I see" and audio is derived from the Latin word "I hear". Historically, no single person or invention is credited with the development of television.

The modern day television set can be traced back to the discovery of selenium in 1817 *by* **Jons Berzelius** *(Sweden). Television is based on photoelectric technology. Television's initial developments were linked to pioneering attempts to improve the transmission of still images down a telegraph wire. In the mid 1800's, sending still images by telegraph wire was an electrochemical process. The concepts of synchronized scanning and the use of photoelectric technology evolved over a fifty year period.*

The idea of sending still images via the telegraph traces its roots to 1839. At that time **Edmond Becquerel**, *a French physicist interested in the study of light, found that when two pieces of metal were immersed in an electrolyte, an electrical charge developed when one of the pieces was illuminated. Although Becquerel had discovered the electrochemical effects of light he did not offer any practical suggestion for its use.*

In 1842, **Alexander Bain** *proposed a facsimile telegraph transmission system based on Becquerel's discovery. Bain proposed that metallic letters of the alphabet could be illuminated in solution, and the resulting modulated voltages scanned and transmitted electrically. The electrified metal letters could be scanned by a pendulum device and reproduced at the other end of the telegraph wire by a synchronized pendulum contacting a piece of chemical* *paper, which is stained at appropriate positions by the transmitted electric signal; the letter shapes are thereby reproduced.*

Historians normally associate Bain's idea's with the modern day facsimile (fax) machine. Bain is also credited with the idea of scanning an image, so it can be broken up into small parts for transmission. His invention drew attention to the need for synchronization between the transmitter and the receiver in order for the transmission system to work.

In 1847, **F. Bakewell** *of Great Britain patented a chemical telegraph. Bakewell improved Bain's proposal by replacing the pendula with synchronized rotating cylinders. Later, in 1861, Bakewell's system was improved by an Italian priest,* ***Giovanni Caselli****. Caselli wrapped tin foil around the rotating cylinders and was able to use it to send handwritten messages and photographs.*

In 1873, **Louis May**, *a British telegrapher, discovered what we consider today to be the basics of photoconductivity. He found that a selenium bar, when exposed to light, was a strong conductor of electricity. He also noted that the conduction of electrical current would vary depending on the amount of light hitting the selenium bars.*

The final links between telegraphs and television fell into place with **M. Senlacq de Ardres** *(France) in 1878. He proposed that selenium could trace documents. He proposed that the changes in electrical voltage produced by selenium scanning a document, could magnetically control a pencil at the receiving end of the transmission. By 1881, British pioneer* **Shelford Bidwell** *successfully transmitted silhouettes using both selenium and a scanning system. He called the device the* scanning phototelegraph.

1873 *Photoconductivity of* selenium *discovered.*

1878–1880 *Early notions of television* Scanning *– the breaking down of an image into picture elements which are then reassembled on the screen of a receiver:* **Carlo Peresino** *(1879, Italy);* **W.E. Sawyer** *(1880, USA);* **Maurice Leblanc** *(1880, France). Leblanc first proposed 'photoelectric scanning' to transmit moving pictures at a distance.*

1878 **William Crookes** *(England) developed the cathode-ray tube.*

1884 **Paul Nipkow** *(Germany) proposed the sequential* scanning disc: *a rapidly rotating spirally perforated disc is placed between the object and a light-sensitive selenium element, thus progressively revealing the image to the sensor. It is believed that a working model was never built.*

1885–1900 *Improvements of Nipkow's disc:* **Henry Sutton** *(1856–1914, Australia) devised the Telephane system (1885) – the first real television proposal involving scanning, synchronizing, a light sensitive cell and a control valve but no vision signal amplifiers.*

Lazare Weiller *(France) replaced Nipkow's disc by a mirrored drum (1889); As the drum spun, a light beam aimed at the drum's mirrored surface (each mirror being tilted slightly more than its predecessor) was reflected to progressively trace a path across the object to be 'televised'. In receiver node, the light beam was reflected onto the drum, which in turn reflected the beam in a raster-scan fashion onto a selenium screen. The resolution of the scanned image was directly related to the number of angled mirrors, just 30 lines per scan.*

1887 **Heinrich Hertz** *(Germany) discovered electromagnetic waves.*

1891 **Paul Eduard Liesegang** *(1838–1896, Germany) introduced the concept of 'television'.*

1895 **Julius Elster** *(1854–1920) and* **Hans Geitel** *(1855–1923, Germany) produced the first practical photoelectric cell.*

1897 **J.J. Thomson** *(England) discovered the electron.*

Karl F. Braun *(Germany) invented the Cathode-ray tube (CRT), the ancestor of the television picture tube.*

1900 **Constantin Persky** *(Russia) coined the word 'Television' when presenting a paper at the Paris International Electricity Congress (Aug 25).*

1902 **Otto von Bronk** *(1872–1951, Germany) suggested a scanning method for transmitting colored pictures.*

1904 **Arthur Rudolf Wehnelt** *(1871–1944) introduced a cylindrically shaped glow-cathode with oxide layering to improve the electron stream bundling and hence the overall performance of the Braun CRT.*

1906 **Lee de Forest** *(US) developed the 'Audion', a three-element vacuum tube (triode). This made possible amplification of video signals created by photoconductivity and photoemission.*

1907–1911 *Early attempts to combine mechanical scanning with cathode-ray tube by* **Boris Lwowitsch Rosing** *(1869–1933, Russia) and* **Alan Archibald Campbell-Swinton** *(1863–1930, England). Rosing (1907) used a rotating mirror drum for* scanning *and Braun's tube to reproduce television images. Swinton (1908–1911) utilized cathode ray tube as image converter for transmission, designing in effect, the first electronic camera tube.*

1922–1927 **Philo Taylor Farnsworth** *(1906–1971, USA) first invented the all-electronic television (1922). Produced (1927) the first successful television transmission by wholly electronic means.*

1923–1935 **John Logie Baird** *(1888–1946, England) developed a working electromechanical television system based on Nipkow disc. First to harness television for military uses.*

1925–1929 **Herbert Eugene Ives** *(1882–1952, US),* **Charles Francis Jenkins** *(1867–1934, US) and* **Dénes von Mihàly** *(1894–1953, Hungary) improve on the transmission of television images.*

1926–1928 **Kálmán Tihanyi** *(1897–1947, Hungary and USA). Patented a fully electronic television on the basis of continuous electron emission with accumulation and* storage *of released secondary electrons during the entire scanning cycle.*

1926 **Kenjito Takayanagi** *(Japan) operated a working electronic television system, using a CRT to transmit an image of Japanese writing.*

1927 *Early attempts by AT&T (US) to combine the television and the telephone into a system of* videophone *with no specific additional equipment.*

1928–1935 **Vladimir Kosma Zworykin** *(1889–1982, Russia and USA). Constructed a complete all-electronic television system composed of the* iconoscope *(an electronic tube that converts light rays into electric signals and acts as a television camera suitable for broadcasting, 1923) and the* kinescope *(the picture tube used in television receivers).*

By 1933 the system reached a resolution of 240 lines per inch. His television provided the final impetus for the development of modern television as an entertainment and *educational medium. He also developed a color television system (1928).*

1929–1931 **René Bartholemy** *(1889–1954, France) developed a mechanical television scanner (using 30-line scanning) with a disc receiver. He gave two demonstrations (1931).*

1934–1936 **Isaac Schoenberg** *(England) developed a camera tube similar to the iconoscope (1934).*

Advent of public television in Germany (1935) and England. The BBC employed a regular 405-line monochrome TV service using Schoenberg's Emitron camera tube, which converted light from a scene in a studio into a television signal (1936). The Nazis in Germany built a TV station in order to broadcast the Berlin Olympic games. But in both countries, mass-market manufacture of television receivers was halted when WWII broke out.

A videophone service is set up in Germany with overhead cables between Berlin-Witzleben, Leipzig, Nuremberg and Hamburg. Transmission took place using the Nipkow mechanical scanning system; the image to be transmitted was imaged by 90 objective lenses in 1/25-th of a second and consisted of 180 lines at reception.

1938 *Sideband filter used in television broadcasting is developed for doubling resolution of pictures.*

1940 **Peter Carl Goldmark** *(Hungary and USA) demonstrated a color television system of his invention. CBS broadcasted the world's first color TV signals.*

One million television sets in the United States alone.

1956 *The Ampex Company (US) demonstrated the first viable video recording*[12] *of television.*

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[12] *Video recording*: In the early days, film was the only medium available for recording television programmes. Owing to the specific needs of American television networks, however (broadcasting at different times on the Atlantic and Pacific coasts) researchers were led to investigating more flexible systems. Thoughts turned to magnetic tape,which was already being used for sound, but the greater quantity of information carried by the television signal demanded new studies. During the 1950s, a number of American companies began investigating the problem. In April 1956, the Ampex company demonstrated the first viable product.

1956 **Robert Adler** *(USA) invented the television remote control.*

1958 *Silicon controlled rectifier (SCR) introduced by General Electric (US). This semiconductor device was about to revolutionize dimming applications for theater and television lighting around the world. Previous to this time, dimming systems were large, generally inefficient and mechanically very complex.*

The SCR allowed the design of compact remote controlled dimming systems – with no moving parts in the dimmer. The typical modem SCR dimmer employs two PNPN semiconductor devices commonly known as silicon control rectifiers, or thyristors, connected in inverse parallel and in series with the lamp load. A signal applied to the control gates of these devices is utilized to control their conduction period. The dimmer thereby controls the effective power dissipated in the lamp load, and thus the intensity of the lamps. The dimmer is completely inert and requires no maintenance.

1961–1967 *Development of color television in Europe.*

In 1961, **Henri de France** *put forward the SECAM system (Sequentiel Couleur à Memoire) in which the two chrominance components are transmitted in sequence, line after*

It recorded in black and white. Its rival RCA followed suit in 1957, with equipment designed for color. The mechanical principle adopted was the same and was to remain in use for a long time. The system had four heads on a disc rotating obliquely across the width of the tape, thus tracing an oblique track pattern. The tape was 50.8 mm (2 inches) wide. With the development of editing equipment, the initial "delayed broadcasting" function gradually gave way to "production" functions. The first all-electronic editing equipment avoiding the need for splicing tape was introduced in the late 1960s. Slow-motion and variable-speed playback techniques were impossible with the "four head" system. The situation changed with the advent of helical-scan video recorders (Toshiba, 1959) which at last provided editing facilities analogous to those of film. Helical scanning is now used in all video recorders: each track contains one entire field (or a major part of it), and the tape can be "read" at different speeds, even when it is stationary. The magnetic tape is 25.4 mm (1 inch) wide. In 1986 the first digital video cassette recorder meeting the international digital television standards was presented by the Sony Corporation.

line, using frequency modulation. In the receiver, the information carried in each line is memorized until the next line has arrived, and then the two are processed together to give the complete color information for each line.

In 1963, **Walter Bruch** *(Germany) proposed a variant of the American NTSC system, known as PAL (Phase Alternation by Line). It differs from the NTSC system by the transmission of one of the chrominance components in opposite phase on successive lines, thus compensating for phase errors automatically. Both solutions found application in the color television services launched in 1967 in England, Germany and France, successively.*

1964–1971 *Picture-phone* system, *combining television and telephone, was exhibited by AT&T but failed commercially.*

1964 **Marshall McLuhan** *(Canada) foresaw the impact of television on man's social behavior; Television aims at our most immediate perception. Pictures are to see, almost to feel; they present the whole world before us. TV offers us entertainment games, sports and news. It offers something of everything for everybody.*

Our senses are assailed every day by the attraction of the visual message – "The medium is the message" as McLuhan succinctly put it. Its all-pervasiveness and instantaneity are tuned to our way of thinking, and vice versa. We expect from it effortless pleasure and hot news. But the stupefaction takes its toll and we thirst for more. Images pour over us in a never-ending torrent.

Television has already modified our social behavior. It fosters, for example, our taste for things visual through the impact of the picture and its colors. It encourages in us a yearning for the big spectacle and the forthright declaration. The effect can be seen in the way we react to one another and in the world of advertising.

But television cannot yet be said to have enriched our civilization, mainly because it has not yet become interactive.

Television cannot, on its own, serve as an instrument of culture. It must be appreciated that it is not well suited for detailed analysis or in-depth investigation. The way it operates and its hi-tech infrastructure are such that it cannot do justice to the words of the poet.

In the flood of images from the silver screen the less good accompanies the best, just as in the cinema or literature. The factor which distinguishes television from the cinema and books, however, is that the full quality range, down to the very worst, is offered to us round the clock, in our own homes. Unless we take particular care to preserve our sense of values, we let it all soak in. We have not yet become "diet conscious", as regards our intake of television fare. Without this self-control our perception becomes blurred and the lasting impression we have ceases to be governed by a strict process of deliberate reflection.

However, by the end of the 20^{th} century, TV seems destined to merge with the internet (if not be replaced by it altogether). The return path *in cable television networks, and VOD (Video On Demand) services – as well as commercial movie-download online databases and the World Wide Web itself – suggest that TV, or its replacement, will be as interactive as the internet.*

1964 *Color television sets become popular in the United States.*

1967 *There are in the US 3,895 AM stations, 1,336 FM stations and 770 television stations.*

BBC and ITV began regular color TV transmissions.

1970–1980 *High-definition television (HDTV) developed by Sony in Japan.*

1972 *Digital[13] TV operating in US laboratories.*

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[13] *Digital television*: The values of the brightness or color signals (*luminance* and *chrominance* respectively) of picture elements along a television line can be represented by a series of numbers. If these are expressed in base 2, each value can be transformed into a sequence of digital (binary) electrical pulses.

The conversion from the "analog" world to the "digital" world comprises two stages:

- *sampling*: in which the pixel values are measured at regular intervals
- *quantization*: in which each measurement is converted into a binary number.

The last operation is carried out by an Analogue to Digital Converter (ADC).

The series of "1" and "0"s obtained after quantization can be modified (i.e. coded) to counteract more effectively the disturbances the signal will undergo during transmission.

Digital television technology is an extension of computer and image processing technology. Advantages are high fidelity, easy storage and great scope for image

1977 *Renewed efforts to establish a low-cost videophony by digital transmission of images via telephone lines using optical fibers and image compression methods: the image is broken into pixels (=* **Pic***ture* **El***ements), as for television, each pixel being defined by one numerical value of luminance and two for chrominance (color). In order not to exceed the transmission capacity of optical fibers at that time, the image resolution was limited to 34 megabits per second.*

1978 *Rapid expansion of Cable Television systems.*

1979 *300 million television sets in the US.*

1982 *Japanese introduced wristwatch-size television with 3 cm screen.*

1983 *600 million television sets in the world.*

1985 *Sony produces a television set with a screen size of 24 m × 48 m.*

1991 *BBC World System Television is launched via satellite.*

1993–1995 *America's first high-power Direct Broadcast Satellite (DBS) is launched. It is followed by another in the same year and a third in 1995.*

1976–1990 *Advent of CNN television worldwide news network.*
1976 – Ted Turner delivered nationwide TV in the U.S. via satellite.
1980 – 24-hour news channel.
1990 – Domination of worldwide news coverage.

1990

- *1446 Television stations broadcasting in the United States.*
- *There are more television sets in the world than there are telephones.*
- *In Belgium, the average time spent watching television by children from 10 to 13 years was 210 minutes per day.*

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processing.
Each picture element is isolated and can be called up independently.

VIII. Fiber Optics (1870–1988)

1870 ***John Tyndall*** *(UK) demonstrates that light can travel along a curved transparent waveguide by total internal reflection.*

1876 ***Alexander Graham Bell*** *(USA) invented the* photophone, *an instrument for transmitting sound by its modulation of a beam of light.*

1880 ***William Wheeler*** *(USA). Engineer in Concord, MA. Patents a system of internally reflective pipes to guide light through a building.*

1887 ***Charles Vernon Boys*** *(1855–1944,* England) *describes the concept of guiding light through* glass fibers.

1893 ***Edward Drummond Libbey*** *(USA) introduced* fiberglass, *a material made of very thin strands of glass. The fibers may be many times finer than human hair, and may look and feel like silk. The flexible glass fibers are stronger than steel, and will not burn, stretch, rot or fade.*

1917 ***Albert Einstein*** *developed the theory of stimulated emission of radiation, a process by which an incoming photon beam can be* amplified *by stimulated emission of photons of the same frequency.*

1931–1939 *Experiments conducted by the Owens Illinois Glass Company and the Corning Glass Works, led to development of practical methods of making fiberglass commercially.*

1927 ***John Logie Baird*** *(UK) experimented with light propagation along strands of flexible glass. A pioneer in fiber optics.*

1955 ***Narinder S. Kapany*** *(UK). The father of optical fiber communication. First to develop the theory of light propagation by total internal reflection in transparent glass fibers which contain a core encased in a cladding of lower refractive index. Tests begin to demonstrate communication via fiber optics.*

1957 ***Basil Isaac Hirschowitz*** *(South Africa) first used a new kind of an endoscope – the fiberscope: a flexible tube, made of an optical fiber, to explore the inside of a human body.*

1957–1960 *Creation of the LASER (Light Amplification by Stimulated Emission of Radiation) by* ***Gordon Gould****,* ***Charles Townes****,* ***Arthur Schawlow*** *and* ***Theodore H. Maiman*** *(USA).*

The characteristic peculiar to the laser effect is that the photons are emitted exactly in phase with the stimulating beam. This coherence means that the laser beam is as directed (non divergent) as allowed by the theoretical diffraction limit, and can deliver considerable power to a point of microscopic dimensions. (The name laser was coined by **Gould***.)*

1966 ***Charles Kao*** *and* ***G. Hockham*** *of Standard Telecom Labs (UK) presented design principles of fiber-optic cables instead of copper wires to carry telephone conversations, using laser light and lossless hair-thin glass fibers. This concept was still a 'leap of faith' since at that time losses in the glass exceeded 10 db/km. At transmission a modulator transforms the electric signals into light signals, and at reception these are transformed by photo-diodes back into electrical signals.*

Optical fibers carry signals with much less energy loss than twisted copper-wire pairs or coaxial copper cables, and with much higher bandwidth. This means that fibers can carry more channels of information over longer distances and with fewer repeaters required. Optical fibers are much lighter and thinner than copper wires with the same bandwidth. This means that much less space is required, per unit bandwidth, in underground cable ducts. They are immune to electromagnetic interference from radio signals, car ignition system, lightning etc. They can be routed safely through an explosive or flammable atmosphere.

1970 *Corning Glass researchers* ***Robert Maurer****,* ***Donald Keck*** *and* ***Peter Schultz*** *designed and produced the first optical fiber with optical losses low enough for wide use in telecommunications (1 db/km). Their fiber-optic wire was capable of carrying 65,000 times more information than* *conventional copper wire. This technology would revolutionize communications in coming decades, replacing copper wires with cheaper and less bulky glass fibers. Normal electrical wiring limits the amount of telephone, television and computer traffic that can be carried by wire. But, in theory, a single laser beam should be able to carry all radio, television and telephone conversation of the entire world.*

1975–1977 *Transmission of* video signals *by optical fibers in UK and USA. Bell Telephone System installed the first working laser cable system beneath the streets of downtown Chicago (1977). It was the first to carry phone calls, computer data and video signals on pulses of light.*

1985–1986 *AT&T's Labs achieve the equivalent of sending 300,000 simultaneous telephone conversations (or 200 high-resolution TV channels) at once over a single optical fiber; optical fiber attenuation is reduced to a loss of 0.154* db/km*, close to the 0.13* db/km *silica theoretical limit.*

1988, Dec 14 *World's first trans-Atlantic optical fiber system. It extends 5800 km and has a capacity to carry 40,000 telephone conversations simultaneously, doubling trans-Atlantic cable capacity in a stroke.*

Because the information-carrying capacity of a signal increases with frequency, the use of laser light offers many advantages, and therefore fiber-optic laser systems are currently being used in military communications networks, cable television, and other applications.

One advantage of optical fiber systems is the long distances that can be maintained before signal repeaters are needed to regenerate signals. These are currently separated by about 100 km, compared to about 1.5 km for electrical systems. Newly developed optical fiber amplifiers can extend this distance even farther.

Optical fiber networks also offer a more private transfer of data, as it is quite difficult to 'tap' into a fiber, even compared to a 'private' phone line.

IX. Computer-based Communication Systems (1922–1998)

A. First Generation *(1822–1959):* Principle of programmable computer; mechanical, electromechanical and vacuum-tube based calculating machines; punched card and paper input/output, stored program, machine and assembler languages, high-level language.

1943 *The earliest programmable digital Electronic Computer runs for the first time in Britain. It contained 2400 vacuum-tubes ("electron valves") for logic, and was called the Colossus. It was built by* **Thomas Flowers** *to crack the German coding '*Enigma*' machines and used at Bletchly Park during WWII. It translated 5000 characters a second and used punched tape for input.*

1944 ***H.A. Aiken*** *and* **Grace Hopper** *created the Harvard MARK 1 computer.*

1946 *ENIAC (Electronic Numerical Integrator and Computer): the first totally electronic, electron-valve driven, digital computers. Development started in 1943 and finished in 1946 at the Ballistic Research Laboratory by* ***John W. Mauchly*** *and* ***John P. Eckert****. ENIAC weighed 30 tons and contained 18,000 electronic valves, consuming around 25 kW of electric power. It could do around 100,000 calculations a second and was used for calculating ballistic trajectories and testing theories behind the hydrogen bomb.*

1947 ***Norbert Wiener*** *(US) published Cybernetics.*

William B. Shockley*,* ***John Bardeen*** *and* ***Walter H. Brattain*** *(US) invent the transistor at Bell Laboratories. This invention greatly impacted the history of computers.*

1949–1952 *EDVAC (Electronic Discrete Variable Computer), the first computer to use magnetic tape. Proposed by* ***John von Neumann*** *and completed (1952) at the Institute of Advanced Study, Princeton, USA.*

1950 ***Yoshiro Nakamata*** *invented the Floppy Disc at the Imperial University in Tokyo. The sales license for the disc was granted to IBM, who introduced it only in 1971.*

1951 *UNIVAC 1, the first commercially available electronic computer designed to handle both numeric and textual information. Designed by* ***John P. Eckert*** *and* ***John W. Mauchly****.*

1953 *There are about 100 computers in the world.*

1957 *The development of FORTRAN (***For***mula* ***Tan***slation) completed by* ***John Backus*** *and his team at IBM – the first scientific computer programming language.*

1958 ***Jack St.Clair Kilby*** *(at Texas Instruments) and* ***Robert N. Noyce*** *invented the integrated circuit ('microchip').*

B. Second Generation *(1959–1965):* Advent of transistors, magnetic tape, printed circuit, high-level languages, resulting in much smaller, more powerful and 'user friendly' computers.

1960 *About 6000 computers are in operation in the US.*

1963 *The first PDP-8 minicomputer built.*

1965 *Fuzzy Logic designed by* ***Lofti Zadeh*** *at the University of Berkeley. BASIC (***B***eginners* ***A***ll-Purpose* ***S***ymbolic* ***I***nstruction* ***C***ode) developed at Dartmouth college, USA by* ***T.E. Kurtz*** *and* ***J. Kemeny****. Not implemented on minicomputers until 1975.*

James T. Russell *(USA) created the compact disc (CD) – the optical digital recording and playback process which has become an essential component of computer systems.*

C. Third Generation *(1965–1971):* Integrated circuits, monitors and keyboards, mouse and windows, hypertext, operating systems, families of computers (IBM 360 series etc).

1965–1968 ***Douglas Engelbart*** *of SRI (US), invented the computer* mouse *and* Windows*; pioneered work in the creation and design of modern interactive computer environments. His NLS (o***N***-***L****ine-****S****ystem) introduced two-dimensional computerized text editing using the* mouse *(to position a pointer into the text on the screen), a keyboard, a word processing program and a hypertext system. The mouse was conceived in 1965 but did not become popular until 1983 with the Apple computers, and was adopted by IBM only by 1987.*

The first supercomputer*, the Control Data CD6660, was developed (1965).*

ARPA Sponsored a study on 'cooperative networks of time-sharing computers' (1965). It then asked for bids to build a prototype link of 4 host computers in California and Utah (1968–1971).

Theodor Holm Nelson *coined the term 'hypertext' to describe nonsequential writing-text that branches and allows choices to the reader; best used at an interactive screen.*

The first supercomputers were first used for cryptography and nuclear physics, but have come into use for highly complex physical and mathematical computations in fields including oil and mineral prospecting, analysis of subatomic an subnuclear physics, studying the earth's ozone layer and simulating processes including weather systems and nuclear reactions.

1967 *First construction of a computer with parallel processor is proposed.*

1969

- *Bubble-memory* *system for computers was invented; it retains information even after computer is turned off.*
- *Work began on the* ARPAnet*, grandfather to the Internet. Computers began to talk to each other.*

1969–1971 *Advent the* microprocessor *– a computer on a chip:* ***Marcian Hoff*** *(US) proposed (at Intel) that a single chip general-purpose computer known as Central Processor Unit (CPU), would be programmed to perform most of the calculator function.*

Further refinement in architecture and logic design were made by ***Stanley Mazor*** *and* ***Frederico Faggin****. The first microprocessor, the 4004 was released in 1970. It contained the equivalent of about 2300 transistors and was capable of about 60,000 operations per second, running at a clock rate of 108 kHz. This single chip had as much computing power as the first computer, ENIAC (1946) which filled a room! The microprocessor is one of the most important developments of the second half of the* 20^{th} *century. It is now found virtually in every automobile, household appliance, and computer.*

1970–1971 *Development of the UNIX operating system started by* ***Ken Thomson*** *and* ***Dennis Ritchie****. It was created at Bell Laboratories (US) for multi-user computers. It can run on a variety of different computers and thus rapidly became a worldwide standard, especially in universities and research institutions. It also speeded the development of Unix to Unix computer communication via phone and high-speed data lines.*

Intel 1103 – the first available dynamic RAM chip. (RAM = Random Access Memory.)

1970–1999 *From ARPANET to INTERNET – story of the 'Information superhighway'.*

*A military network called ARPAnet (***A***dvanced* **R***esearch* **P***roject* **A***gency) was formed. It became open to non-military users in 1979–1980 when Academia was allowed to connect. Its name was then changed to INTERNET (1979) and many universities and large businesses went "on-line". The number of linked* host computers *increased exponentially during the years: 1969 (4); 1971 (23); 1981 (213); 1984 (1000), 1987 (10,000); 1988 (100,000); 1990 (300,000); 1992 (1,000,000); 1996 (10,000,000). Significant milestones in the development of the INTERNET are:*

- *In the first International Conference on Computer Communication at Washington DC (1972) the ARPANET was in the public eye for the first time.*
- *The engineer* **Ray Tomlinson** *(US) sent the first electronic mail between two computers at Cambridge, MA (1972). His software was rapidly incorporated into ARPANET's file-transfer protocol ('ftp') to facilitate communication via e-mail. He also created the @ symbol in the e-mail address to separate the user's name from that of his or her machine or domain.*
- *ARPANET became international with the first international link to University College, London (1973).*
- *BITNET (= '***B***ecause* **I***t's* **T***ime'* **Net***work) launched (1981) as a cooperative network of the City University of New York (CUNY). It used e-mail. One of the first wide-area networks.*
- *The term INTERNET used for the first time in 1982. It has grown to be an international network of networks.*
- **W***orld* **W***ide* **W***eb (WWW) was launched in 1992.*
- *INTERNET encompassed 13,000 regions and national networks; there are some 10 terabytes of publicly available data (1993).*
- *NETSCAPE browser 1.0 (1994).*
- *NETSCAPE 'NAVIGATOR' 2.0 (1996).*
- *150 countries are connected to the INTERNET (1996).*

1971 *The Floppy disc is reintroduced by* **Alan Shugart** *at IBM for storing data used by computers. It is a 20 cm disc coated with iron oxide.*

D. Fourth Generation *(1972–):* MAGNETIC DISC, MICROCOMPUTERS, LSI (= **L**ARGE **S**CALE **I**NTEGRATION) BASED ON MICROPROCESSORS, TYPICALLY 500 OR MORE COMPONENTS ON A CHIP; VLSI (**V**ERY **L**ARGE **S**CALE **I**NTEGRATION), TYPICALLY 10,000 COMPONENTS ON A CHIP.

Modern chips may now contain millions of components. This has led to very small, yet incredibly powerful computers. The fourth generation is generally viewed as running right up to the present time, since although computing power has increased, the basic technology has remained virtually the same. By the late 1990's people began to suspect that this technology has built-in limitations, and that further miniaturization could only proceed so much. Thus, 64 megabit RAM chips have circuit so small that it can be measured in atoms. Circuits this small pose many technical problems – notably the heat created, but they are also very susceptible to influence by temperature and radiation.

1972 *C programming language developed.*

First electronic pocket calculator (Texas Instruments).

Alan Kay *conceived the laptop computer (introduced by 1981).*

1973 *the microcomputer was born in France.*

1974 *The first computer with parallel architecture.*

1975–1981 *First personal (consumer) computer (PC), the Altair 8800 is introduced in kit form in the US (1975). It has 256 bytes of memory. About 100 computers connected to the ARPANET. IBM launches the 5100 portable computers.*

Apple computer, Inc. founded (1976). Designed by ***Stephen Wozniak*** and ***Steve Jobs****.*

Apple II, the first personal computer available in assembled form, was introduced (1976). In 1978, Apple brought out the first disc hard drive used in personal computers. In 1981, IBM announced their 5150 PC.

Its operating system soon became an industry-standard **D***isc* **O***perating* **S***ystem (DOS) with 64 kB of RAM and 40 kB of ROM.*

1973 **Robert Metcalfe** *at Xerox (US) created the Ethernet – a local area network for connecting computers within a building, using hardware running from machine to machine. The patent described it as "multinet data communication system with collision detection". It differs from the Internet* *which connect remotely located computers by telephone line, software protocol and some hardware.*

1976–1992 *CRAY 1, the first commercially developed* supercomputer*, built by* **Seymour Cray** *(US). It contained 200,000 integrated circuits and was freon-cooled. It could perform 150 million floating-point operations per second. The circuits of CRAY 2 (1992) featured* gallium arsenide *instead of silicon chips, and were submerged in a cooling bath of liquid fluorocarbon to prevent heat from the gallium arsenide from melting the machine. It could do about 250 million floating-point operations per second.*

1979 *Advent of* word processors*.*

1980 *More than 1 million computers are in use in the US.*

1981 *Microsoft's MS-DOS* operating system*.*

1983–1985 *The CD-ROM (Compact Disc Read Only Memory) is developed by Phillips (1983) and marketed by Sony (1985). It is an extension of audio CD technology for use in computers.*

Apple's Lisa brings the old mouse *(1965) and pull-down menu to the PC (a mouse is a devise that moves the cursor on the screen as a result of moving the mouse on a hard surface; pressing the button (or one of the buttons) on the mouse sends a command to the computer, depending on where the cursor is located).*

IBM's PC-XT is the first PC with a hard-disc drive built into it – a magnetic memory device then capable of storing 10 MB of information. By 2001 PC disc drives would hold many tens of gigabytes*.*

1984 *Apple Macintosh* released *(16 MB (megabytes) of RAM).*

1984 *Optical discs* for storage of computer data were introduced*.*

1985–1998 *Microsoft Windows* launched *(1985); it was followed by Windows 3.0 (1990), Windows 95 (1995), Windows 98 (1998), and later versions.*

1987 *A bandgap-engineered Heterojunction Bipolar Transistor made of epitaxial SiGe (Silicon-Germanium) alloys was demonstrated for the first time. Because the band structure and transport properties of electrons and holes can be* *engineered in these alloys, cutoff frequencies can be made larger than in devices based on doped Silicon alone; by 2004, unity-gain cutoff frequencies well above 200 GHz have been achieved for SiGe devices. SiGe HBT's can be used in electronic circuits for RF applications, as well as in high-speed optical networks.*

1987–1988 *Massive Connection Machine: a supercomputer which instead of integration of circuits operates up to 64,000 fairly ordinary microprocessors (using* parallel *architecture) at the same time. It can perform about 2 billion operations per second.*

1988–1990 *First optical chip, the* optical computer *developed by AT&T; it uses light (photons) instead of electricity (electrons) to carry data and can potentially reach calculation speed orders of magnitude faster than existing* electronic computers*.*

1989 *World Wide Web*, invented by **Tim Berners-Lee** *(at CERN, the European particle-physics laboratory). He saw the need for a global information exchange that would allow physicists to collaborate on research and exchange text, visual and other files interactively. The Web was a result of the integration of* hypertext *(coded in 'HTML': Hypertext Mark-up Language) and the internet.*

1992 *First U.S.* website *launched at SLAC (Stanford Linear Accelerator Center).*

1993–1998 *Pentium* chip *released (1993). The 166 MHz version contains the equivalent of 3.3 million transistors. The 1995 version of Pentium Pro achieved a clock speed of 200 MHz and contained 5.5 million transistors. In 1998, Intel produced the 333 MHz Pentium II processor.*

1996 *Texas Instruments announced they can now pack 125 million transistors into a single silicon chip the size of a thumbnail.*

E. Fifth Generation*:* CD-ROM, OPTICAL DISCS, PHOTONIC COMPUTERS.

1985 *CD-ROM can put 275,000 pages of text on a single CD record.*

1997 *Seiko Epson Japan invented a hand-cranked PC. When the battery starts to fade, a red-alert message alerts the user to crank the main spring.*

1998 *Sales of Personal Computers (percentage of all US homes): 1995 (27%), 1997 (43%), 1998 (50%); Driven by strong sales of lower-priced PC (ca 2100 dollar per unit), half of all US households have a PC. The PC is rapidly becoming a standard household appliance.*

1998 *Internet details:*

- *The total world wired population is 150 million people.*
- *The number of computer* hosts *in the global network is 36 million.*
- *Internet provides electronic mail, file transfer, a dynamic document distribution and presentation system (WWW). It is the greatest information and intellectual resource in the world, has a growing presence in entertainment, commerce, chats and other social interactions, and is the most visible manifestation of the information society.*

2004 *The Intel corporation released the LGA775 platform for the Pentium-4 chip. Its clock speed is 3.6 GHz, it packs 1 MB (MegaByte) of L2 cache, and the width of its circuit strips is 90* μ*m (900 Angstrom – one-sixth the wavelength of yellow light!). Chips based on bandgap-engineered devices – e.g. utilizing SiGe (Silicon-Germanium alloys) – could enter later generation CPU chips, because they will enable clock speeds of* hundreds *of GHz.*

X. Artificial Intelligence (AI) (1917–1997)

(*inter* = between; *legere* = to choose; Intelligence = to choose between)

1917–1921 **Karel Capek** *(1890–1938, Czechoslovakia) coined the term 'Robot' (robota = forced labor in Czech) to describe the mechanical people in his science fiction story, R.U.R. His intelligent machines, intended as servants for their human creators, end up taking over the world and destroying all mankind. "Robot" is defined as an automatic, autonomous device that performs functions normally ascribed to humans, or a machine in the form of humans.*

1928 **John von Neumann** *(USA) discovered the minimax theorem central to Game Theory.*

1937 **Alan Turing** *(England) introduced the abstract Turing Machine.*

1941 **Konrad Zuse** *(Germany) completed the world's first fully programmable* digital computer.

1942 **Isaac Asimov** *(1920–1992, US), Science Fiction author, introduced his fictional 'three laws of robotics'.*

1943 **Warren McCulloch** *and* **Walter Pits** *discuss neural-network* architectures for intelligence.

1946 ***John P. Eckert*** *(US)* and ***John W. Mauchly*** *(US) developed ENIAC, the world's first fully electronic, programmable digital computer.*

1947 **Norbert Wiener** *(US) published Cybernetics, a seminal book on information and control theory.*

1947 *The field of Artificial Intelligence (AI) began as the first computers were developed. It is often defined as a multidisciplinary field encompassing computer science, neuroscience, philosophy, psychology, robotics, and linguistics. It attempts to reproduce with machines the methods and results of human reasoning, speech and other brain activities.*

1949 ***Maurice Wilkins*** *(US) built EDSAC, the world's first stored-program computer.*

1950 **Alan Turing** *described a criterion for determining whether a machine is intelligent (the Turing Test).*

Claude E. Shannon *proposed a computer chess program.*

1956 **Stanislaw Ulam** *(US) wrote MANIAC 1, the first computer program to beat a human being in a chess game.*

The first use of the term Artificial Intelligence was given by **John McCarthy** *and* **Claude Shannon**, *at a computer conference at Dartmouth College, US.*

1956 **D. Devol** *and* **J.F. Engelberger** *(US): the first industrial robot 'Unimate'. It was later installed at a General Motors plant to work with heated die-casting machines.*

1957 **Allen Newell**, **Herbert Simon** *and* **J.C. Shaw** *(US) developed their Logic Theorist program, one of the first AI programs that allow a computer to reason abstractly (it could, for example, prove theorems in the 'Principia Mathematica' of Russell and Whitehead.*

1959 **Arthur Samuel** *of IBM wrote a checkers-playing program that performs as well as some of the best players at that time.*

1962 **Engelberger***'s 'Unimation' company began marketing industrial robots.*

1963 **Marvin Minsky** *(US) published "Steps Toward Artificial Intelligence".*

1964 **Marshall McLuhan** *(Canada) in his book 'Understanding Media', foresaw electronic media as creating a 'global village' in which "the medium is the message".*

1966 *The SRI (**S**tandard **R**esearch **I**nstitute) robot represents an attempt to combine learning programs, pattern recognition programs, problem-solving programs and programs that represent information about the outside world.*

1969 *First man landed on the moon.*

1971 *The first microprocessor was introduced in the US.*

1974 *The first computer-controlled industrial robot was developed.*

1977 *Voyagers 1 and 2 launched.*

1980 *Philosopher* **John R. Searle** *(USA) criticized computationalism and the pretensions of the strong Artificial Intelligence program. He maintains that computers cannot think because a system's behaving as if it had mental states is insufficient to establish that it does in fact have these states. Moreover, he claims that there is no fact intrinsic to the physics of computers that makes their operations syntactic or symbolic: rather, the ascription of syntax or symbolic operations to a computer program is a matter of human interpretation.*

1982 *Second-generation robots were built, with the ability to precisely effect movements with 5 or 6 degrees of freedom. They are used for industrial welding and spray painting.*

Defense robots used by Israel in Lebanon.

1984 *Waseda University in Tokyo completed Wabot-2, a 100 kg robot that reads sheet music through its camera eyes and plays organ with its ten fingers and two feet.*

Marvin Minsky *published The Society of Mind, in which he presents a theory of the mind in which intelligence is seen to be the result of proper organization of a very large number of simple mechanisms, each of which is by itself unintelligent.*

1986 *Third-generation robots with limited intelligence and some vision and tactile sensing.*

Dallas police used a robot to break into an apartment. The fugitive ran out in fright and surrendered.

1986 *'Optical character recognition' (OCR) technology is growing.*

New medical imaging systems are creating a revolution in medicine.

The university of Pennsylvania developed a robotic ping-pong player that wins against human beings.

1988 *Population of industrial robots has increased from a few hundred (1970) to several thousand, most of them in Japan.*

***1992**–* *Military strategies of the leading industrial nations increasingly rely on flying smart weapons which incorporate electronic copilots, pattern-recognition techniques, and advanced technologies for tracking, identification, homing and destruction.*

A multi-hundred-billion-dollar computer and information-processing industry is emerging, together with a generation of ubiquitous machine intelligence that works intimately with its human creators.

Reliable person identification, using pattern-recognition techniques applied to visual and speech patterns, replace locks and keys in many instances.

AI technology is of greater strategic importance than man-power, geography, and natural resources.

1997, May 11 *The IBM computer DEEP BLUE wins* $3\frac{1}{2} : 2\frac{1}{2}$ *against the world chess champion Garry Kasparov; first machine of human-competitive intelligence created by man. A computer is the world chess champion. The machine will be able to handle (ca 2000 CE) 1 billion nodes per second and will have a chess master rating of 3,400, about 500 points higher than the world's best human player.*

XI. Geo Satellite in the service of telecommunication (1945–1996)

1945 **Arthur C. Clarke** *proposed the idea of communication satellite that would appear to hover motionless over a single point on earth; called* synchronous satellite *(or satellite in a* geosynchronous orbit*). They became the principal means of intercontinental communication starting in 1965. Newtonian physics predicts a 24 hour period for a mass 35,880 km above the equator. Clarke also recognized that a net of three such satellites, suitably placed, could provide radio relays between any points on earth (except regions very close to the poles).*

1957, Oct 4 *Soviet satellite* Sputnik 1 *("fellow traveler") was launched: a little steel ball weighing 83.6 kg, containing a radio transmitter and batteries, transmitting a steady series of beeps, but doing nothing more.*

1958, Jan 31 *First US satellite* Explorer 1 *was launched: a two meter long body, 15 cm in diameter, weighing 14 kg. It carried a Geiger-Mueller counter to detect cosmic rays. It detected the Van Allen radiation belt girdling the earth.*

1960, Apr *The weather satellite* Tiros 1 *(military, in civilian guise) discovers the beginnings of a tropical storm in the South Pacific. It sent back nearly 23,000 photographs of earth's cloud cover.*

On Aug 12, NASA launched ECHO 1*, a spherical balloon with a metalized skin. Once in orbit the balloon was inflated until it reached its intended diameter of 30 m, and it was then used as a reflector to bounce radio signals across the oceans – simply a radio mirror in the sky.*

1962, July 10 *US "*Telstar 1*" was launched, the first commercially developed true communications satellite. It was a multifaceted sphere just under 1 m in diameter and weighing about 77 kg. It was able to receive messages from the ground, amplify them and then retransmit them immediately. This made it possible to send high-quality data, for instance television pictures, from place to place. On July 23 Telstar 1* *carried a TV program between USA and Europe that was watched by 200 million viewers. It ceased to function in March 1963.*

1964, Aug 19 *US launched "Syncom 3" into a geostationary orbit; the first successful demonstration of the principle of a geostationary communication satellite; It was used with great effect to broadcast the coverage of the 1964 Olympic games from Japan. It was a cylindric satellite with a built-in rocket motor. Upon reaching the altitude of 35880 km it fired its motor to circularize the orbit at that height and become geostationary.*

1964, Aug 20 *The formation, by 11 countries, of INTELSAT (International Telecommunications Satellite Organization) with headquarters in Washington DC (119 members in 1990). Tokyo Olympic Games are replayed to North America.*

1965, April 6 *INTELSAT 1 (or Early Bird) launched and placed in geostationary orbit over the Atlantic Ocean. It went into service on June 28 – the world's first commercial communication satellite.* Early Bird *was a cylinder, 72 cm wide and 52 cm high, with solar cells around its circumference. It was spun around the axis to provide stability in space. It could relay 24 telephone lines or one television channel and remained in service for* $3\frac{1}{2}$ *years.*

1967–1991 *INTELSAT developed bigger and bigger satellites in response to continuing increase in demand for services: INTELSAT 2 (1967) weighed 86 kg. INTELSAT 3 (1968) weighed 150 kg, could carry 1200 telephone circuits or four Television channels (or a combination of both) and its antenna could point continually toward the earth.*

INTELSAT 4 (1971) weighed 720 kg with an overall height (including extended antennas) reaching 5 m. It was able to carry 6000 telephone circuits and color television channels.

INTELSAT 5 (1980) weighed 1000 kg and was able to transmit 12,000 telephone channels plus color television. It was kept stable about all 3 axes by gas jets, and had a pair of large solar wings to generate electricity. This system allowed all the solar panels to be exposed to sunlight all the time and produced more power, which in turn increased the capabilities of the satellite.

INTELSAT 6 (1989–1992), weighed 2500 kg (11.7 m high), and could relay 24,000 telephone lines plus 3 television channels. It was launched into orbit by a commercial Titan rocket.

1969 *First live television broadcast from the moon, by the US Apollo 11 moon-landing mission, is transmitted to some 600 million viewers worldwide by satellite feed. The worldwide linkup cost $ 55 billion and involved some 40,000 TV technicians and personnel in 49 nations. Sophisticated transmitting and receiving equipment was part of the compact VHF communication system on board the Apollo command and lunar modules.*

1972–1984 *Landsat 1 (formerly named ERTS 1), dubbed 'eye-in-the-sky', was launched on 23 July, 1972, by a Delta rocket as part of a project to observe the earth, monitoring such problems as crop disease, flooding, icebergs and pollution. It was followed by Landsat 2 (1975), Landsat 3 (1978), Landsat 4 (1982), Landsat 5 (1984) and Landsat 6 (1992).*

Landsat 1 weighed 891 kg. It was placed at an altitude of 900 km in a polar orbit, from which it could observe the same area every 18 days. Landsat 4 and 5 weighed 2000 kg each. Since 1985, Landsat data have been marketed, supplying users with photographs or electronic images suitable for display and image processing on desktop computers.

1977 *Eutelsat links the telecommunication networks of the European countries via communication satellites.*

1987 *Soviet radar satellite (weighing 20 tons) was launched. It has applications in map-making, oceanography, crop predictions, ice monitoring and prospecting for minerals.*

1993 *NASA's* **A***dvanced* **C***ommunications* **T***echnology* **S***atellite (ACTS) is launched from the Shuttle Discovery during Mission STS-51 on Sept 12. ACTS is a testbed for future communications satellite technology, such as multi-beam antennae and advanced signal handling.*

1995 *World Wide Web has become big business as NETSCAPE Communications goes public. About 6.6 million computers are connected to the INTERNET.*

1996 *13 million host computers on the INTERNET.*
The INTERNET includes by now 18 satellites in orbit.

XII. Is Anyone out There? – Spacebound Odyssey in Search of Extraterrestrial Intelligence (1959–1989)

(a) Introduction

Until very recently, man's ancient dream to free himself of the chains of gravity and fly away from earth in search of other worlds, materialized only in myth, literature, and science fiction.

However, the rapid development of aeronautical science, nuclear physics, solid state physics, computer technology and fuel chemistry in the wake of WWII, to which the cold war added a strong element of competition – pushed the science of rocketry to the verge of realization.

In 1944, within two years of the historic first self-sustaining nuclear chain reaction (Dec. 2, 1942), **Stanislaw Ulam** *and* **Frederick de Hoffman** *mused at Los Alamos that the power of the atomic explosion might somehow be controlled to launch space vehicles. Ulam and de Hoffman were following the thoughts of the great master of science fiction,* **Jules Verne***, who, in his 1865 novel, De la Terre la Lune, wrote that the Baltimore Gun Club fired a manned projectile to the moon from a huge cannon emplaced near Cape Canaveral, now Cape Kennedy, Florida.*

But ideas rarely come to fruition unless a practical need beckons; near the close of WWII, German V-2 rockets had proved they could carry some 750 kg of amatol explosives 360 km from Nazi-held territory in Europe to London. What if the V-2's had carried atomic bombs? The thought was unsettling. Soon designs for Intercontinental Ballistic Missiles (ICBMs) began to take shape on drawing boards around the United States. Some of these big rocket *design were nuclear at both ends – they had nuclear warheads and nuclear engines. Secret reports issued in July 1946 by North American Aviation, Inc., and Douglas Aircraft Company are landmarks in the history of nuclear rockets. The reports underlined the great promise of the "heat transfer" nuclear rocket, noting its high exhaust velocity*[14]*, its attainment of very high temperatures, and its high rate of heat transfer.*

The military rocket work was naturally classified as secret by the U.S.

[14] Chemical rocket engines, jet engines, automobile engines – in fact, most of mankind's engines – extract heat from a fuel and turn it into macroscopic motion through the expansion of hot gases. The nuclear rocket also creates hot high-pressure gas and turns it into reaction thrust. The higher the propellant *velocity*, the more thrust we get from each kilogram of gas that roars out the nozzle each second. We want to have a high exhaust velocity for good rocket performance, because we can thereby accomplish a space mission with less propellant.

The nuclear rocket produces about *twice* the exhaust velocity of the best chemical rocket, for the following reason: the exhaust velocity V of any rocket is proportional to $\sqrt{\frac{T}{M}}$, where T is the temperature of the hot gases just before they enter the nozzle throat, and M is the average molecular weight of the exhaust gases. Now, chemical rockets already operate at temperatures close to $3000\,^{\circ}$K; nuclear rocket reactors operate at the same general temperature level. However, chemical rocket exhaust velocities are limited by the high molecular weight of the combustion products [water ($M = 18$), methane ($M = 16$), ammonia ($M = 17$)] whereas a nuclear rocket, where combustion is not required, can make use of a propellant with low molecular weight, such as molecular hydrogen ($M = 2$). [Fissile-material nucleus release kinetic energy without chemical stimulation, and the propellant is not an engine fuel but a separate substance that is heated by the fissioning nuclei in a nuclear reactor.] With $M = 2$ (instead of 18 as in hydrogen-oxygen chemical engines), the nuclear rocket exhaust velocity will be more than double that of the best chemical rocket for the same temperature. Using the equations: thrust $= \frac{dm}{dt}V$, power $= \frac{1}{2}\frac{dm}{dt}V^2$, we see that if the thrust is held fixed and exhaust velocity V is doubled, propellant flow $\frac{dm}{dt}$ will be halved, but the power required will double. The price of increasing the exhaust velocity is the need for increased power production by the engine. From this relationship arises another important advantage of the nuclear rocket: The great reservoir of energy contained in its nuclear fuel can be turned into high exhaust velocity.

The three basic facts about nuclear rockets are:

- They convert fission-generated heat into the kinetic energy of rocket propellant.
- Chemical combustion is not needed, and so they can use lower molecular weight propellant to attain high exhaust velocities.
- Their reactor fuel has a more potential energy packed into it.

Air Force. However, at the Applied Physics Laboratory of the John Hopkins University in Maryland a group of engineers, who were unaware of the ICBM work and did not have access to the secret reports, innocently proceeded to duplicate all the important findings of North American and Douglas. Their unclassified report was published in January 1947. It was obvious that any competent with a slide rule and a few scraps of paper could discover the essentials of the nuclear rocket without much help.

In 1948 and 1949, two British space enthusiasts, **A.V. Cleaver** *and* **L.R. Shepherd**, *again duplicated most of the secret nuclear rocket fundamentals in a classic series of papers published in the Journal of the British Interplanetary Society. Not long before the English report appeared, the American-educated Chinese scientist* **H.S. Tsien** *had reported his studies on the application of nuclear energy to rockets and other "thermal jets" at a M.I.T. seminar. The basic principles of the nuclear rocket could no longer be concealed. [It is interesting that Shepherd went on to become a key man in Britain's atomic energy program, and Tsien later returned to China where he was a principal figure in the development of the Beijing government's atomic bomb.]*

A nuclear rocket engine is considerably more than a heater of hydrogen. It is true that the engine is built around the reactor core, the wellspring of energy, but in addition hydrogen propellant must be transported from tank to reactor; heat must be partially converted to thrust. The engine, in fact, has five major segments:

- *The nuclear reactor heat source.*
- *The pump that pulls liquid hydrogen from its tanks and forces it through the reactor.*
- *The nozzle, which transforms heat to thrust.*
- *The structure that physically holds the pieces together.*
- *The controls that force all engine components to march in step at the command of the spacecraft pilot.*

In a nuclear rocket engine intended for space travel inside the solar system, the reactor must generate at least 5000 megawatts of thermal power (more than the output of 50,000 V.W. cars or 160,000 home-heating furnaces), and still not encumber the rocket with too much inertia.

Four requirements control reactor design:

- *The need to attain a critical mass (i.e., the smallest mass of fissionable material that will sustain a chain reaction).*

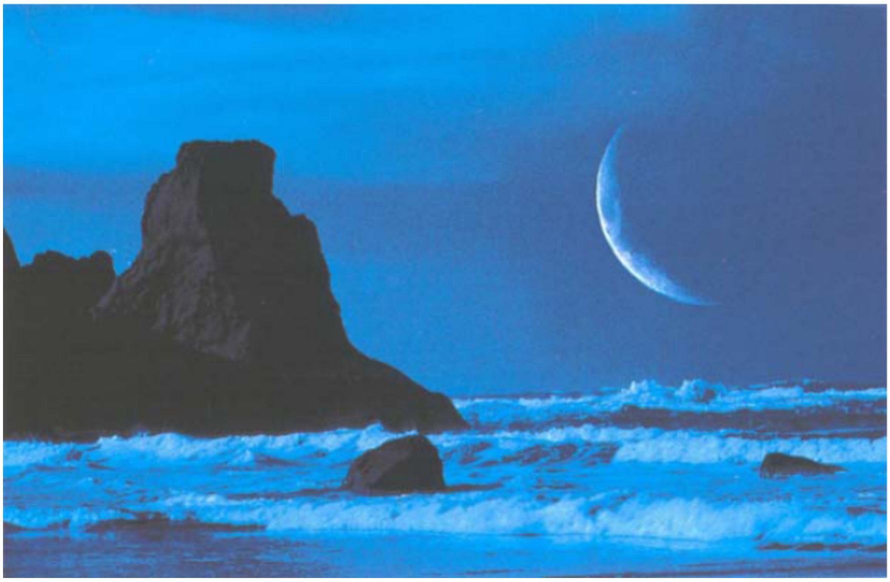

Photo 2: Elsewhere in the Universe (courtesy of Steve Satushek)

- *The need to remove all generated heat.*
- *The need to raise the power level at will, or control the reactor.*
- *The need to maintain structural integrity at high temperatures and under the forces exerted by the high thrust.*

Attainment of a critical mass is a matter of "neutron economics", because it is the cloud of invisible neutrons coursing through the reactor that stimulates nuclear fission and thus power production. Each fissioned atom of uranium-235 produces $2\frac{1}{2}$ *new neutrons, on the average. If the rate at which fission occur (and consequently the power level) is to remain constant, exactly one of these* $2\frac{1}{2}$ *neutrons (on average) has to go on and cause another fission. Reactor "criticality" occurs at just this point. This balance sheet leaves* $1\frac{1}{2}$ *neutron per fission, each of which may either escape the reactor altogether or be absorbed in nonfission nuclear reactions. To prevent too* many *neutrons from escaping, a material is placed around the reactor to reflect some errant neutrons back into the core.*

Excessive neutron absorption can be avoided by using core and reflector materials that have little appetite for neutrons; fortunately, graphite *has just such a low neutron absorption cross section. Naturally, enough uranium-235 atoms must be dispersed throughout the graphite core so that the neutrons can find and fission them. Core design requires a balancing of all these considerations, plus one more.*

Almost all the energy released by uranium is first incorporated in the kinetic energy of two large fission fragments that fly off in opposite directions as the nucleus splits. In a few millionths of a second, the kinetic energy of the heavy fragments is transferred to the nearby atoms and molecules, setting them in motion. A heat pulse *flows outwards to the boundary of the* nuclear fuel *(uranium carbide dispersed throughout the graphite), where it heats the hydrogen propellant. If this heat is not removed, core temperatures will quickly rise beyond the sublimation point of the fuel. The reactor core, therefore, has to be designed in such a way that all this fission-generated heat is transferred to the hydrogen gas that is to be driven through the reactor by the pump. This means that the reactor must be perforated with holes that carry the hydrogen (serving here as a reactor coolant) through and past the hot fuel to the nozzle. Coolant holes are coated with niobium carbide to prevent chemical corrosion of graphite by the hot hydrogen.*

To raise or lower the reactor power, the neutron economy must be upset, or altered. Control drums *help perform this task. These are cylinders covered on one side with neutron "poison", i.e. materials whose nucleus readily absorb neutrons at the typical speeds they acquire in the reactor.* Boron *is one such* *element. When all the drums' absorbing faces are turned inward, neutron that would otherwise be reflected back into the core to cause new fissions are absorbed by the poison instead. To start the reactor, motors slowly rotate the control drums, moving the poison away from the core regions, thus giving the neutron economy a boost. Unless the drums are returned to the exact point where criticality occurs, reactor power will rise exponentially. Because neutron generations are only milliseconds apart, neutron "population explosion" (and thus reactor power changes) can come about very quickly.*

A small portion of the hot hydrogen stream is "bled" off, diluted with a little cold hydrogen and directed through the turbine that powers the pump. This pump must raise the pressure of liquid hydrogen by about 94 atmospheres while delivering about 8 tons of it per minute to the reactor.

The hydrogen leaving the hot end of the nuclear rocket reactor is laden with the thermal energy that first must be converted into gas kinetic energy *and then into* rocket kinetic energy*. This is the job of the nozzle. A constriction called the nozzle "throat" starts the process. First the throat speeds up the hydrogen velocity until it is traveling at the speed of sound (Mach 1). Beyond the throat, the nozzle opens up into a carefully contoured divergent section. Here, the hydrogen expands and cools rapidly as heat energy is converted to gas velocity (kinetic energy). The hydrogen, now traveling at supersonic velocities, pushes against the divergent sides of the nozzle, thrusting the rocket in the opposite direction through airless space. The expansion of hot gas in the nozzle is analogous to gas expansion against a piston or a turbine blade. To achieve a high exhaust velocity, the exit area of the divergent section must be as large as possible in comparison to the throat area. This ratio (100: 1) is limited only by the length and weight of the nozzle. Powerful forces act on the nozzle because it has to carry the entire thrust load up to the rocket body proper. What material can withstand such stresses in the presence of super-hot hydrogen rushing past it at supersonic speeds?*

The solution is a high temperature alloy such as stainless steel, covered by a solid phalanx of cooling tubes that keep the nozzle temperatures well below the melting point of the alloy. The coolant that is pumped through these tubes is the supercooled liquid hydrogen which is returned to reactor's core. Without this cooling the nozzle would not survive more than a few seconds.

Besides the system unity imposed by the structure and controls, the engine parts have to fit together thermodynamically. To breath "life" into a nuclear rocket there must be a starter*, like that in an automobile, intrinsic in the system. The engine must "catch", become self-sustaining, and generate useful power. For this to happen, two energy sources must be found: One to start the engine and another to power the pump that keeps propellant flowing through the engine.*

In a nuclear rocket, the heavy reactor replaces the empty combustion chamber of the chemical rocket. Heavy though the nuclear engine may be, it is still but a small appendage on a much larger structure consisting mainly of the huge propellant tank.

The doubled exhaust velocity of the nuclear rocket in comparison with the chemical rocket, means that the nuclear rocket uses only half as much propellant each second of operation. Thus, the nuclear rocket's great economy in propellant consumption make it superior for missions in which most of the spacecraft weight is allotted to propellant. This is subject to the condition that the payload *(anything carried that is not necessarily for the flight of the vehicle) is not a great deal smaller than the weight of the nuclear engine itself; manned expeditions to Mars, or the ferrying of supplies to the moon, are right down the nuclear rocket's alley. Generally speaking, the more ambitious the mission, the better the nuclear rocket looks.*

On Oct. 04, 1957, the Soviet Union launched the first artificial satellite, SPUTNIK (Russian: fellow traveler). It was a metal sphere with diameter of ca 58 cm, circling the earth with a velocity ca 29,000 km/h, once every 95 minutes[15]*. Its remains fell to earth on Jan. 04, 1958. In 1959, a Russian*

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[15] Let a mass m be fired off a planet of mass M and radius R. To achieve orbit, the initial speed should exceed the *first cosmic velocity*, V_1:

$$\frac{mV_1^2}{R} = mg = G\frac{mM}{R^2}, \therefore V_1 = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

For earth $V_1 = 7.9$ km/sec.
Second cosmic velocity (=*escape velocity*):

$$\frac{1}{2}mv_2^2 = \frac{GmM}{R} \therefore V_2 = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

For earth $V_2 = 11.3$ km/sec. Above, G is the universal gravitational constant and g is the surface gravity.
For a mass m orbiting the said planet at radial distance $r > R$, Newton's 2^{nd} law, $\frac{GMm}{r^2} = \frac{mV^2}{r}$, yields $V = \sqrt{\frac{GM}{r}}$. The period of revolution is $T = \frac{2\pi r}{V} = 2\pi\sqrt{\frac{r^3}{GM}}$, namely $T^2 \propto r^3$ (Kepler's third law). Substituting $GM = gR^2$, we find $\left(\frac{r}{R}\right)^3 = \frac{gT^2}{4\pi^2 R}$. If T is chosen to be the planet's own period of rotation about its axis, we obtain for the earth $r = 6.618\ R$, or altitude $= r - R = 5.618\ R = 35,890\ km$. This is the suitable altitude for a *synchronous communication satellite*.

space probe televised the first pictures of the dark side of the moon hidden from earth. On April 12, 1961, the Russians launched Yuri Gagarin, the first man ever in space, on the spacecraft Vostok I. He circled the earth in 1^h48^m.

The United States responded to the Soviet challenge in space by establishing the National Aeronautics and Space Administration (NASA) in 1958 to conduct and coordinate the U.S. nonmilitary research into problems of flight within and beyond the earth's atmosphere. It has more than 10,000 scientists, engineers and technicians. Its installations include the John F. Kennedy Space Center (Cape Canaveral, FA), the Lyndon B. Johnson Space Center (near Houston, TX), the Goddard Space Flight Center (Greenbelt, MD), the George C. Marshall Space Flight Center (Huntsville, AL), Flight Research Center (Edwards AFB, CA), Jet Propulsion Laboratory (Pasadena, CA) and other centers.

The results of this gigantic effort culminated in the first lunar landing of Apollo 11 on July 20, 1969. On that day (10:56 UT pm), after 4 days of travel at an average speed of 4000 km/h, **Neil A. Armstrong** *set foot on the rocky plain of the moon's Sea of Tranquility.*

The commitment to land the first man on the moon, successfully fulfilled in 1969, again boosted scientists onto the crest of a wave of popularity. Caught up in the excitement of the space race, no fewer than 20 U.S. Federal agencies were supporting research and development in the 1960's.

This multiplicity of funding sources produced spectacular results. It yielded exciting new knowledge about the nature of the planets and of our place in the universe. Fundamental physics, astronomy, electronics and computer technology have benefited, as well as chemistry, material science and the life sciences. Not only did the U.S. decisively win the race to the moon, but it has acquired new and better electronic and medical services that owe their existence, at least in part, to the needs of the space exploration.

On March 03, 1972, Pioneer 10 was launched from Cape Canaveral, FL. The 260 kg craft made its way safely through the asteroid belt, a region between the orbits of Mars and Jupiter littered with rocky debris. It flew within 130,000 km of Jupiter's cloudtops on Dec. 02, 1973, returning the first close-up images of the sun's largest planet. It proceeded to traverse the orbits of Saturn, Uranus, Neptune and Pluto. On June 13, 1983, Pioneer became the first spacecraft to depart the realm of the known planets.

Note that the net energy of the orbiting mass (kinetic plus potential) is

$$\frac{1}{2}mV^2 - \frac{GmM}{r} = \frac{1}{2}m\frac{GM}{r} - \frac{GmM}{r} = -\frac{1}{2}\frac{GmM}{r} < 0.$$

It becomes *more* negative, the closer the mass is to the planet.

Radio messages from Pioneer tell us that it moves in cold, dark and empty space. 'Solar wind' particles, moving with speeds of 1.5 million km/h, blow at its tail while cosmic rays race inward past it. It may be able to detect gravitational waves or locate the putative Planet X perturbing Uranus and Neptune. As the years go by, its radio will go dead and its guidance sensors will lose sight of the sun. It will then cruise on, mankind's first emissary to the universe. About 10,506 years from now it will pass within 3.8 LY of Barnard's star. In 862,063 years Pioneer will approach the vicinity of Altair, a star nine times brighter than the sun. In case Pioneer is intercepted by intelligent beings, it carries a plaque with images of a man and a woman, a diagram of the solar system and other symbols that might enable 'others' to locate the origin of the little craft. Five billion years from now, Pioneer should be wandering about the outer rim of the Milky Way galaxy. The sun is expected to burn out and die in 5 billion years, and with it the earth.

At the turn of the 21^{th} *century, the engineering problems encountered in the design of interstellar space vehicles seem to present a number of apparently insurmountable obstacles. It is nevertheless quite interesting to examine the feasibility of interstellar travel from the point of view of the theory of relativity.*

The interstellar distances involved are of the order of a few light years ($\approx 10^{16}$ *m) to the nearest star, of order* 10^4 *light-years* ($\simeq 10^{20}$ *m) to the center of our own galaxy, and of order* 10^6 *light-years* ($\approx 10^{22}$ *m) to the nearest neighboring galaxies. In all cases the limited lifetime of the crew requires either space vehicles capable of attaining speeds close to the speed of light, or a multi-generational ('space ark') ship.*

Supposing one can solve the engineering problem of constructing a vehicle that can accelerate (as measured by the crew) at the rate $a = g \approx 10$ *m/sec*^{2} *for the entire duration of the trip, then the velocity reached by the vehicle after a time* t *has elapsed in an inertial frame stationary w.r.t. the earth is, according to a standard relativistic calculation,*

$$v = \frac{gt}{\sqrt{1 + (gt/c)^2}}$$

and as seen from earth it will have traveled the distance

$$d = \frac{c^2}{g}\left(\sqrt{1 + \left(\frac{gt}{c}\right)^2} - 1\right).$$

The crew benefits, however, from the time dilation effect. One finds for the time t' elapsed, as measured by the crew during earth time t, the expression

$$t' = \frac{c}{g} ln \left[\frac{gt}{c} + \sqrt{1 + \left(\frac{gt}{c} \right)^2} \right].$$

Assuming the crew wants to land at a distance D from earth, the ideal way of traveling would consist of accelerating at rate g up to $d = \frac{1}{2}D$ and then decelerating at that rate over the other half of the total distance, so that the total traveling time will be $T = 2t$ in earth time, and $T' = 2t'$ in crew time. By applying the formulae given above to this case one finds:

Distance traveled $D = 2d$	*Traveling time as measured on earth,* $T = 2t$	*Traveling time as elapsed for crew,* $T' = 2t'$
2×10^{16} *m*	*3.6 yr*	*2.7 yr*
2×10^{20} *m*	2.2×10^4 *yr*	*20 yr*
2×10^{22} *m*	2.2×10^6 *yr*	*29 yr*

Clearly, because of the relativistic time dilation effect, one cannot rule out absolutely the feasibility of interstellar or even intergalactic travel within a single current human lifetime, even without cryogenics-assisted suspended animation *for the crew.*

Besides nuclear-powered impulse rockets of the kind described here, other, even more revolutionary designs have been proposed and studied for interplanetary and interstellar space missions. Among these:

- ion rocket *(tested in a 1998 NASA experiment)*
- antimatter engines *(power and/or thrust via annihilation of stored antimatter with matter and subsequent particle production)*
- photon rocket *(shining backward – propagating light beams for thrust)*
- ramjet engine *based on nuclear fusion fueled by scooped interstellar hydrogen*

(b) Timeline

Scientists have been sending regular radio waves out into space since 1920. All of our radio, TV, satellite, and radar signals are currently spreading out, slowly sweeping through the ca. 10^{11} *(100 billion) stars of the Milky Way Galaxy.*

1959 *The birth of SETI (acronym for the* **S***earch for* **E***xtra***T***erres-trial* **I***ntelligence) – the science of searching the skies for signals from alien civilizations using radio and optical telescopes. Incepted with a paper in Nature by* **Giuseppe Cocconi** *and* **Philip Morrison** *in which detection by radio waves was suggested as the earliest method of communication.*

1960–1990 **Frank Drake** *(radio astronomer, US) embarks on Project Ozma: a search for extraterrestrial intelligence in the form of radio signals from other civilizations. With an antenna diameter of 25 m he* listened*, at the edge of the "spectral water hole", at frequency 1420 MHz (the natural frequency of the 21 cm – wavelength hyperfine – splitting line of atomic hydrogen). In 1975, Drake and* **Carl Sagan** *(1934–1996, US astronomer) listened by means of an antenna with diameter 300 m in the frequency range 1420–2380 MHz. An ongoing SETI program has been privately funded since the 1980s. So far, no alien signals have been detected. Considering that there are millions of frequency bands to sift through and the relatively short period of listening, it may be that we just have not yet looked at the right place at the right time.*

1972–1973 *The first space probes Pioneer 10 (launched March 02, 1972) and Pioneer 11 (launched 05 April 1973) to leave the solar system. Each carries a plaque that will allow whoever finds them to trace them back to earth, using pulsars as astronomical signposts in time and space. Each plaque, 155 mm × 229 mm in size, is of gold-anodized aluminum plate, into which is etched a diagram of the solar system, a pulsar map and a picture of man and a woman. The plaques are mounted in a position on the spacecraft's antenna mount which is expected to protect them from erosion by interstellar dust for at least 100 million years. By 1974 the Pioneers coasted Jupiter and by 1994 they were 9 billion km from the sun – far beyond the Solar system.*

1977–1989 *Voyager 1 (Sept. 05) and Voyager 2 (Aug 20) sent from USA to explore the edge of the solar system.*

Voyager 1 arrived at Jupiter (spring 1979), Saturn (Nov 1980) and its largest satellite Titan.

Voyager 2 flew by Jupiter (July 1979), Saturn (Aug 1981), past Uranus (1986), Neptune (Aug 1989) and Neptune's largest moon, Triton. On Nov 05 2003, Voyager 1 reached 90 AU and is the first human-built craft to explore and report back on the interstellar medium, with Voyager 2 close on its heel.

1990's–early 2000 Over 120 *exoplanets* (planet around other nearby stars) discovered indirectly. Indirect methods include:

- *Radial Doppler shifts* (periodical shifts in a star's spectrum caused by the wobble of the star about the COM of its planetary system).
- *Precision astrometry* (minute movements of a star perpendicular to its line-of-sight from earth, again due to planet-caused wobble).
- *Photometric detection* of the effects of exoplanet *transits* upon the parent star's spectrum.
- *Single-star gravitational lensing.*
- *Space-based interferometry.*

Numerous exoplanet-hunting space missions are in the works, scheduled to be launched during the first two decades of the 21^{th} century. Some of them will try to look *directly* at exoplanets, by either dimming their star's disc or looking in the infrared.

Most of the exoplanets thus far found are gas giants, and all are either too cold or too hot to be likely harbors of life. It is hoped that some of the planned missions will be able to identify *earth-like* exoplanets, and even analyze their chemistry (and possible biochemistry!) via their atmospheric spectroscopy.

A Timeline of Discoveries in Particle Physics

Bubble tracks are left in a bubble chamber by tiny electrically charged subatomic particles as they travel through the chamber's depressurized cryogenic fluid. The ions left along the track nucleate bubbles because the chamber's rapid de-pressurization – synchronized with the arrival of the primary (projectile) particle beam – renders the fluid superheated.

The bubble chamber functions as both a target *for the projectile particles (furnished by the liquid's nuclei-protons in the case of liquid hydrogen) and as a* detector *for all charged particle tracks, whether projectiles or reaction products.*

Bubble chamber images have the topology of branched trees: tracks (prongs) successively branch off at interaction vertices.

An electrically neutral particle leaves no bubble tracks, but its path can be reconstructed between the vertex which created it (if any) and the vertex at which it ceased to exist. An ambient magnetic field *bends the charged tracks; the curvature is used to calculate the particle's momentum and charge. The charged particle's* energy *affects its rate of energy and momentum loss along the track, and can thus be deduced from the bubble track's appearance. From a track's momentum $\boldsymbol{p}$ and energy E, the corresponding particle's* rest mass *can be deduced via the special-relativistic formula:*

$$m_0^2c^4 = E^2 - \boldsymbol{p}^2c^2.$$

Particle physicists thus have to decode cascades (vertex-track-vertex trees) – captured in bubble chamber images and other types of detectors arrayed around mammoth particle accelerators – in order to deduce basic information about the observed particles, such as electric charge, particle spin, mass, lepton number, baryon number, parity and other quantum numbers *that turn out to be useful in describing the elementary particle side of nature.*

Before the invention of the bubble chamber, particle tracks were traced and interpreted via other means, such as photographic emulsions and cloud chambers. *The latter is based on an inverse principle to that of the bubble chamber: the charged particle is allowed to enter a super-saturated vapor chamber, and its ionized track furnishes nucleation centers for liquid drops (instead of bubbles), which can then be photographed and the resulting visible tracks interpreted.*

Here is a short history of elementary particle observations, which began with the discovery of the electron in 1897:

1896 *X rays and other forms of radioactivity were observed.*

1897 *The* electron *was discovered. Electrons (e^-) were first called* cathode rays *by their discoverer.*

1899 *Alpha particles* *were discovered, and later shown to be helium nuclei consisting of two neutrons and two protons.*

1911 *Nuclear model of the* atom*, with heavy nucleus in the center and light electrons orbiting around it, was proposed, and became accepted. C.T.R. Wilson (Cambridge, England) observes for the first time the tracks of ionizing particles (alpha and beta particles) in a* cloud chamber*, a device he had invented already in 1896. The cloud chamber played an important role in early nuclear and particle physics.*

1911 *Electron charge* *measured in an oil drop experiment indicates that all electrons carry the same electric charge.*

1932 *The* neutron *directly observed for the first time.*

1932 *The* positron *(e^+), predicted by* ***Paul A.M. Dirac*** *in 1928, was discovered by* ***Carl D. Anderson*** *in a mountain-top* cloud chamber *tracks.*

1934 *Radioactive* nuclei *were produced in the laboratory.*

1937 *The* muon *(μ^+), a charged lepton[16] like the electron (only about 200 times heavier and unstable), was observed.*

[16] *Lepton*, *meson* and *baryon* are Greek-derived neologisms meaning, respectively: light (small), intermediate, and heavy (large). The muon was originally classified as a meson, as were the "pions" (charged and neutral π mesons), since they are all intermediate in mass between the electron and the *nucleons* (proton and neutron). Nowadays, the classification is based on quantum numbers and interaction types, not mass. The electro; muon; tau lepton; plus their neutrinos, and the antiparticles of all these, comprise the 12 *leptons*; *mesons* are particles made of a valence quark and antiquark; and a baryon has 3 valence quarks. An anti-meson is still a meson, while the anti-baryon is a distinct category. A *hadron* is a meson, baryon or anti-baryon.

1947 *Two charged π mesons, with positive and negative charge, were discovered.*

1950 *The neutral π meson was discovered.*

1952 *Invention of the bubble chamber charged-particle track detector by* ***D. Glaser****.*

1953 *The lambda baryon and K meson were discovered.*

1956 *The existence of the electron neutrino (ν_e) and its antiparticle ($\bar{\nu}_e$), predicted by weak-interaction theory in 1930, was confirmed by experiments in which the missing neutral track form the vertex of a weak nuclear decay (such as beta decay,*

$$n \text{ (neutron)} \rightarrow p \text{ (proton)} + e^- + \bar{\nu}_e$$

creates a vertex at which an inverse process occurs (e.g. $\bar{\nu}_e + p \rightarrow n + e^+$).

1950s–60s *Many baryons and mesons were discovered, and their properties recur in regular patterns that look as if baryons and mesons were made of smaller building blocks.*

1961 *The muon neutrino was discovered and shown to be a different particle from the electron neutrino..*

By the mid-1960's, physicists realized that their previous understanding, whereby all matter is composed of the fundamental protons, neutrons, and electron, was insufficient to explain the myriad new particles being discovered. Quark theory solved these problems. Over the last forty years, the theory that is now called the Standard Model of particle physics has gradually emerged and gained increasing acceptance with new evidence from new particle accelerators.

1964 **Murray Gell-Mann** *and* **George Zweig** *tentatively put forth the idea of quarks. They suggested that baryons and antibarions are respectively composed of 3 quarks and 3 antiquarks of types called up, down, or strange (u, d, s) with spin 1/2 and electric charges (in units of proton's charge) 2/3, -1/3, -1/3, respectively; mesons consist of quark-antiquark pairs. Since fractional charges had never been observed, the introduction of quarks was treated more as a mathematical explanation of patterns of particle masses,* *decays and reactions than as a postulate of actual physical objects. Later theoretical and experimental developments allow us to now regard quarks as real physical objects, even though they cannot be isolated.*

1968 *Particle physicist* ***G. Veneziano*** *(Italy, Israel) used the Euler beta function to create a class of models of the strong nuclear interactions called Dual Resonance Models (DRM).*

These models are at first heuristic, but exhibit a duality of high-energy scattering amplitudes which is in accord with experiments. The duality property is between unstable excited hadrons resonantly produced during collisions on the one hand, and virtual particles exchanged during scattering, on the other. Work by L. Susskind, Y. Nambu and others in the late 1960's showed that DRM's can be explained if hadrons are viewed as relativistic strings. But the high-energy interactions of hadrons with leptons, studied experimentally at accelerators at SLAC, Brookhaven, CERN and other labs, failed to agree with string theories, and they were abandoned in the 1970's in favor of local non-abelian (Yang-Mills type) gauge quantum field theories of the strong, weak and electromagnetic interactions.

Meanwhile, however, ***Michael Green***, ***John Schwartz*** *and other researchers noted that some string theories predict a spin-2 massless particle which couples to the energy-momentum 4-tensor – the graviton! These workers thus suggested that string theories might be related to grand unification and quantum gravity. These ideas were not then embraced by the particle physics community. But starting in 1984, another reversal occurred. Many classes of string theories with several curled-up (Kaluza-Klein) spatial dimensions were found to be mathematically consistent, and even free of the short-distance divergences that plague all quantum field theories.*

From that time on, various versions of String Theory have been viewed as leading contenders for an ultimate "Theory of Everything", although mathematical difficulties and lack of data at high enough energies has so far (2008) prevented this very active field of theoretical research from making any direct contact with experimental physics.

1968–1970s *Deep inelastic electron-proton scattering experiments revealed more of the quark structure inside protons and other hadrons, and that quarks' strong nuclear interactions decrease with decreasing distance (*asymptotic freedom*).*

1974 *A fourth* flavor *of quark (beyond u, d and s) – named* charm *– was detected in a newly discovered meson, confirming a theoretical prediction. The Glashow-Salam-Weinberg theory, unifying Quantum Electrodynamics (QED) with the weak nuclear forces in the framework of a* spontaneously broken, Non-Abelian Gauge Theory, *was shown to be mathematically consistent ('renormalizable').*

Another, unbroken, non-abelian gauge theory – called Quantum Chromodynamics (QCD) – was shown to account for accelerator experiment result such as 'asymptotic freedom', leading to increasing acceptance of QCD as the theory of the strong nuclear force. QCD predicts that each quark flavor *comes in three 'colors', but that only colorless combinations can exist as separate, free particles.*

The Glashow-Salam-Weinberg theory and QCD theories together compose the Standard Model.

1975 *The* tau lepton *was discovered, marking a third* generation *of leptons after the* electron *generation (e^-, ν_e and their antiparticles) and the* muon *generation (μ_-, ν_μ and antiparticles). The* tau neutrino *would only be observed in 2000.*

1979 *A fifth flavor of quark, named* bottom, *was found in the newly discovered Upsilon meson. This pattern leads particle physicists to believe they will find a sixth and final flavor of quark. This predicted last flavor of quark is called* top *and would only be detected experimentally in 1995.*

1982 *The massive gauge bosons that carry the weak nuclear force, called the W^+, W^- and Z^0, were discovered and their masses measured, confirming key predictions of the Glashow-Salam-Weinberg model.*

1989 *The lifetime of the Z^0 weak nuclear gauge boson was measured, and agrees precisely with the existence of only three kinds of neutrinos.*

1995 *The* top quark *was finally directly observed and measured, confirming the predictions of theorists that there are six flavors of quarks, as described in the current version of the Standard Model.*

Future *The search goes on for the* Higgs boson *(the only particle predicted by the Standard Model that hasn't been seen yet), for* supersymmetric particles *predicted by some extensions of the Standard Model, for proton decay and for* magnetic monopoles *predicted by Grand Unified Theories, and for* new kinds of exotic particles.

Timeline History of Quantum Theory

At the start of the twentieth century, scientists believed that they understood the most fundamental principles of nature. Atoms were solid building blocks of matter; people trusted Newtonian laws of motion; most of the problems of physics seemed to be solved.

However, starting with Einstein's theory of relativity which replaced Newtonian mechanics, scientists gradually realized that their knowledge was far from complete. Of particular interest was the growing field of quantum mechanics, which completely altered the fundamental precepts of physics.

1900 ***Max Planck*** *suggested that radiation as emitted and absorbed by atoms is quantized (it comes in discrete amounts of energy proportional to the frequency of the radiation).*

1905 ***Albert Einstein****, one of the few scientists to take Planck's ideas seriously, proposed a quantum of light (later named the photon) which behaves like a particle carrying energy, momentum and angular momentum. Einstein's other theories explained the equivalence of mass and energy, the particle-wave duality of photons, the equivalence principle, and special relativity.*

1909–1911 ***Johannes Geiger*** *and* ***Ernest Marsden****, under the supervision of* ***Ernest Rutherford****, scattered alpha particles off a gold foil and observe large angles of scattering, suggesting that atoms have a small, dense, positively charged nucleus.*

1912 ***Albert Einstein*** *explained gravitation as the curvature of space-time.*

1913 ***Niels Bohr*** *succeeded in constructing a provisional theory of atomic structure based on quantum ideas.*

1919 ***Ernest Rutherford*** *found the first evidence for a proton.*

1921 ***James Chadwick*** *and* ***E.S. Bieler*** *concluded that some strong forces holds the nucleus together.*

1923 ***Arthur Compton*** *discovered the quantum (particle) nature of X rays, thus confirming photons as particles.*

1924 ***Louis de Broglie*** *proposed that matter has wave properties.*

1925(Jan) ***Wolfgang Pauli*** *formulated the exclusion principle for electrons.*

1925(April) ***Walther Bothe*** and ***Johannes Geiger*** *demonstrated that energy and mass are conserved in atomic processes.*

1926 ***Erwin Schrödinger*** *developed wave mechanics, which describes the behavior of quantum systems in terms of a matter-wave equation which reduces to Newtonian dynamics in the short-wavelength limit.*

Max Born *gave a probability interpretation of quantum mechanics.*

G.N. Lewis *proposed the name "photon" for a light quantum.*

1927 *Certain materials had been observed to emit electrons (nuclear beta decay). Since both the atom and the nucleus have discrete energy levels, the observed continuous spectrum of emitted electrons implies that another, invisible particle is emitted as well.*

1927 ***Werner Heisenberg*** *formulated the uncertainty principle, which may be roughly formulated as follows: the more you know about a particle's location, the less you know about its momentum (and vice versa).*

1928 ***Paul A.M. Dirac*** *combined quantum mechanics and special relativity to describe the electron, and his new theory predicts the positively-charged positron,* e^+ *(the anti-electron).*

1930 *Quantum mechanics and special relativity are well established. There are thought to be just three fundamental particles: protons, electrons, and photons.*

Max Born*, after learning of the* ***Dirac*** *equation, said, "Physics as we know it will be over in six months."*

1930 **Wolfgang Pauli** *suggested the neutrino to explain the continuous electron spectrum for beta decay.*

Carl D. Anderson *discovers the positron (e^+) in cosmic-ray tracks left in a magnetic cloud chamber atop a mountain (Pike's Peak, Colorado).*

1931 **James Chadwick** *discovered the neutron. The mechanisms of nuclear binding and decay become primary problems of modern physics.*

1933–1934 **Enrico Fermi** *put forth a theory of beta decay that introduces the weak (nuclear) interaction. This is the first theory to explicitly incorporate neutrinos and what would later be called flavor-changing charged currents, which play the role that electric currents do in classical and quantum electrodynamics.*

1933–1934 **Hideki Yukawa** *combined relativity and quantum theory to describe nuclear interactions by an exchange of new particles (mesons called "pions") between protons and neutrons. From the size of the nucleus and the uncertainty principle, Yukawa concluded that the mass of the conjectured particles (mesons) is about 200 electron masses. This was the beginning of the meson theory of nuclear forces.*

1937 *A particle with mass about 200 electron masses was duly discovered in cosmic rays. While at first physicists thought it was Yukawa's pion, it was later discovered to be a muon, the second-generation charged lepton.*

1938 **E.C.G. Stuckelberg** *observed that protons and neutrons do not decay into any combination of electrons, neutrinos, muons, or their antiparticles. The stability of the proton cannot be explained in terms of energy or charge conservation; he proposes that heavy particles (what are now called baryons) are endowed with an independently conserved quantum number.*

1941 **C. Moller** and **Abraham Pais** *introduced the term "nucleon" as a generic term for protons and neutrons.*

1946–1947 *Physicists realize that the cosmic ray particle thought to be Yukawa's meson is instead a "muon," the first particle of the second generation of matter particles to be found. This discovery was completely unexpected –* **I.I. Rabi** *comments* *"who ordered that?" The term "lepton" was introduced to describe objects that do not interact via the strong nuclear force but do interact via the weak interactions (electrons and muons are both leptons, as well as their then-hypothetical associated neutrinos and the antiparticles of all these. Later (1975) the tau particles ($\tau^{\pm}$) and the corresponding neutrino ν_τ and antineutrino, $\bar{\nu}_\tau$, would complete the list of known leptons).*

1947 *A meson that does interact strongly was found in cosmic rays, and is determined to be the pion.*

1948 ***Willis Lamb*** *used molecular beam and RF technology to measure the small split (Lamb Shift) between the $2S_{1/2}$ and $2P_{1/2}$ energy hydrogen levels (transition frequency: 1058 MHz). This splitting results from QED "radiative correction" effects involving virtual electrons, positrons and photons in the atomic vacuum. Lamb's measurement spurred physicists to reformulate QED (Quantum Electrodynamics) in order to circumvent its inherent short-distance divergencies and extract unambiguous predictions for such radiative corrections. Their final version of QED not only agreed with Lamb's measured shift, but also led to other, extremely high precision predictions (such as for the electron-positron gyromagnetic ratio) that were found to be in agreement with increasingly accurate experiments ever since.*

1948 *Physicists* ***R. Feynman***, ***Julian Schwinger*** *and* ***S. Tomonaga*** *developed procedures to calculate electromagnetic properties of electrons, positrons, and photons. Introduction of* ***Feynman*** *diagrams.*

1948 *The Berkeley synchro-cyclotron produced the first artificial pions.*

1949 ***Enrico Fermi*** *and* ***C.N. Yang*** *suggested that a pion is a composite structure of a nucleon and an anti-nucleon. This idea of composite "elementary" particles was quite radical.*

1949 *Discovery of K^+ (charged strange meson) via its weak nuclear decay.*

1950 *The neutral pion was discovered.*

_1951_ *Two new types of particles are discovered in cosmic rays. They were discovered by looking at V-like tracks and reconstructing the electrically-neutral object that must have decayed to produce the two charged objects that left the tracks. The particles were named the* Λ *(lambda) and the* K^0 *(neutral K meson).*

_1951_ ***Erwin Wilhelm Mueller*** (Germany) *developed the field ion microscope.*

_1952_ *Discovery of particle called* Δ*: there were four similar particles with different electric charges (*Δ^{++}, Δ^{+}, Δ^{0}, *and* Δ^{-}*.)*

_1952_ ***Donald Glaser*** *invented the bubble chamber.*

The Brookhaven Cosmotron, a 1.3 GeV accelerator, started operation.

_1953_ *The beginning of a "particle explosion" – a proliferation of "elementary" particles.*

_1953–1957_ *Scattering of electrons off nuclei revealed a charge density distribution inside protons, and even neutrons. Description of this electromagnetic structure of protons and neutrons suggested some kind of internal structure to these objects, though they were still regarded as fundamental particles.*

_1954_ ***C.N. Yang*** *and* ***Robert Mills*** *developed a new class of nonlinear field theories called non-abelian gauge theories by combining ideas from differential geometry, electrodynamics and general relativity. Although not realized at the time, this type of theory now forms the basis of the Standard Model, underlying both the weak and strong nuclear forces.*

_1956_ ***C.N. Yang*** *(USA) and* ***T-D Lee*** *(USA) discovered that parity is not conserved for week interactions.*

_1957_ ***C.G. Wu*** *(China and U.S.A.) verified the* **Yang-Lee** *theory of parity violation in weak nuclear decay.*

She did this by detecting a spatial asymmetry in beta particle (electron) emission during the radioactive decay of a Cobalt-60 nucleus, and establishing that this asymmetry depends upon the nuclear spin in the precise manner predicted by the theory.

1957 ***Julian Schwinger*** *proposed the unification of weak and electromagnetic interactions.*

1957 ***John Bardeen*** *(USA),* ***L.N. Cooper*** *(USA)* and ***J.R. Schrieffer*** *(USA) explained the phenomenon of superconductivity, first observed by* ***Kamerling Onnes*** *(1911), using a quantum mechanical theory. Accordingly, the superconducting current is carried by electron pairs ("Cooper pairs") weakly bound together through lattice vibration quanta* (phonons). *At low enough temperatures, these pairs' kinetic energy cannot be dissipated through scattering (the usual mechanism for electrical resistance in conductors).*

1957–1959 ***Julian Schwinger***, ***Sidney Bludman***, and ***Sheldon Glashow***, *in separate papers, suggested that all weak interactions are mediated by charged heavy bosons (later called* W^+ and W^-*) and augmented by the neutral* Z*. Actually it was* ***Hideki Yukawa*** *who first discussed bosons exchange twenty years earlier, but he proposed the pion as the mediator of the strong nuclear force.*

1961 *As the number of known particles keep increasing, a mathematical classification scheme to organize the hadrons helps physicists recognize patterns of particle types and properties.*

1961 ***Robert Hofstadter*** *(USA) discovered the inner structure of protons and neutrons.*

1967 ***Steven Weinberg*** *(USA),* ***Abdus Salam*** (England) and ***Sheldon Glashow*** *(USA) developed a theory of unification of the weak force and the electromagnetic force.*

1975 *Gravitational physicist* ***Stephen W. Hawking*** *(Cambridge, England) investigates a quantized field in the background of a Schwartzschield spacetime metric. His theoretical calculations predict that, due to the uncertainty principle and quantum causality violations near the event horizon of a black hole, black holes are in fact not black; they should be sources of thermal radiation, with effective temperature proportional to the inverse of the black hole mass. Since the mass gradually decreases due to this Hawking radiation, the temperature increases as the black hole shrinks and evaporates (negative specific heat!), finally resulting in* *an explosion when the black hole mass is reduced to order 10 micrograms (the Planck mass). Actual black hole candidates thus far observed by astronomers (at galactic centers and remnants of heavy-star supernova explosions) glow for other reasons – infalling matter from companion stars and ambient dust and gas – and their predicted Hawking radiation is many orders of magnitude weaker than the EM radiation due to this infall; thus Hawking radiation has not yet been observed, and is not expected to until we either find a much smaller* primordial *black hole, or develop technology to manufacture one ourselves.*

Hawking's result does not depend upon the details of the (as yet unknown) theory of Quantum Gravity, and thus is a very robust prediction which should constrain any such theory, including string theories.

Hawking's calculation confirmed the earlier (1973–74) ideas of ***J. Bekenstein*** *(Israel), who used a purely classical argument – the increase of total black hole horizon area when two black holes merge – to suggest an analogy with the* second law of thermodynamics*, with the event-horizon area playing the role of entropy. Hawking's field-theoretical model indeed revealed that the entropy of the thermal radiation is proportional to the event horizon area – suggesting that* bits of information are stored on the horizon *at a density of order one bit per square Planck length (one bit per about* 10^{-66} *cm*^{2}*!). Black hole entropy remains an active field of (thus far only theoretical) research in string theories.*

1980 ***Klaus Von Klitzing*** *(Germany) discovered the quantum Hall effect: A plate kept in a transverse magnetic field near absolute zero shows changes in transverse (Hall) impedance in discrete steps instead of continuously. It is one of the few examples of quantum behavior that is directly observed.*

1984 String theory *was accepted by the mainstream of the theoretical particle physics community as a candidate for a theory unifying quantum mechanics the Standard Model and gravity.*

1986 *A team of physicists from the US National Bureau of Standards observed individual quantum jumps in individual atoms.*

1986–1987 ***Alex K. Müller*** and ***George J. Bendorz*** *developed materials that become superconductive at substantially higher temperature then liquid Helium. The new materials are ceramics that display superconductivity between 90° and 120° K, above the boiling point of liquid nitrogen.*

1995 *At the Stanford Linear Accelerator Center (SLAC), a high power compressed-pulse laser beam (multi-TeraWatt peak power) is made to collide almost head-on with the 50 GeV (Giga-electron-Volts) electron beam. The peak transverse electric fields experienced by electrons in their rest frame are of order the Schwinger critical field (*1.3×10^{18} *Volt/meter). Such high fields result in coherent production of multiple back-scattered gamma-ray photons and electron-positron pairs, and probe non-perturbative nonlinear quantum electrodynamics.*

Photo 3

Astronomer by Candlelight (1658 CE)

by

Geritt Dou (1613–1675, Holland)

Working by candlelight an astronomer measures the distance between two points on a celestial globe.

The preeminent candle and the hourglass are traditional symbols of the brevity of life, suggesting that the astronomer symbolizes the vanity of human ambition seeking to comprehend the infinite.

Dou was Rembrandt's first student. Always admired for his nightscenes and his depicting of figures in niches, the artist combined the two genres in this meticulously executed painting.

Relativistic Astrophysics and Cosmology

History and Structure of the Universe

As far as we can tell, the expansion of the universe started ca 14 billions of years ago from a very hot and dense state of uniform elementary-particle plasma, too hot for even nuclei to exist. From that initial state, it mushroomed and evolved into the universe we know today. Cosmologists call that process of expansion the Big Bang *because in some phases, especially in the beginning, the process was rather like an explosion.*

Much of understanding of the Big Bang is based on extrapolating between knowledge of particle physics today, and projections from the mathematical model of an expanding universe in general relativity. The Einstein field equations give us a mathematical model for calculating how fast the universal expansion would be accelerating or decelerating at a given age (epoch), given the energy density and equation of state of matter and radiation at that time. We base our estimates about the matter and radiation density of the early universe on ancient light and radio waves reaching us from the past and collected in telescopes, and what we have learned about elementary particle physics, through theory and experiment.

The history of the universe divides roughly into three regimes which reflect the status of our current understanding:

- *Standard cosmology*
- *Particle cosmology*
- *Quantum cosmology*

The standard cosmology *is the most reliably elucidated epoch, spanning the epochs from about $\frac{1}{100}$ of a second after the Big Bang through to the present day. The composition of the universe during this stage evolved from a soup of neutrons, protons, electrons, positrons, photons and neutrinos, through* nucleosynthesis *(formation of light nuclei) and positron annihilation, through the* recombination era *(ca 300,000 yr after the Big Bang) when the universe became* transparent *and the formation of stars and galaxies. The standard model for the evolution of the universe during this epoch has successfully faced many stringent observational tests, including the $2.7^{\circ}K$ blackbody* cosmic *microwave background radiation* and the abundances of hydrogen, helium, deuterium and lithium.

Particle cosmology builds a picture of the universe prior to this, but at temperature regimes which still lie within known physics. For example, high energy particle accelerators at CERN and Fermilab allow us to test physical models for processes which would occur only 10^{-11} *seconds after the Big* Bang. This area of cosmology is more speculative, as it involves at least some extrapolation from the Standard Model of particle physics, and often faces intractable calculational difficulties. Many cosmologists argue that reasonable extrapolations can be made to times as early as a grand unification phase transition (temperatures of order 10^{27} or 10^{25} °K, a time ca 10^{-23} sec after the Big Bang). This stage in the evolution of the universe includes the *baryogenesis* epoch, during which a symmetry developed between the abundances of *matter* and *antimatter*.

Quantum cosmology considers questions about the origin of the universe itself, including the spacetime manifold in which it is embedded. It endeavors to describe quantum processes at the earliest times that we can conceive of in a classical space-time, that is, the Planck epoch at ca 10^{-43} sec after the Big Bang. Given that we do not as yet have a theory of quantum gravity, this area of cosmology is extremely speculative.

The four key observational successes of the standard Hot Big Bang model are the following:

- *Expansion of the universe*
- *Origin of the cosmic background radiation*
- *Nucleosynthesis of the light elements*
- *Formation of galaxies and large-scale structure*

The Big Bang model makes accurate and scientifically testable predictions in each of these areas, and the remarkable agreement with the observational data gives us considerable confidence in the model.

Expansion of the Universe

The universe began about 14 billion years ago in a violent explosion, at temperatures in excess of 10^{30} °K and correspondingly high mass-energy densities. The fact that galaxies are receding from us in all directions is a consequence of this initial explosion, and was first discovered observationally by

Hubble. *There is now excellent evidence for Hubble's law, which states that the recessional velocity v of a galaxy is proportional to its distance d from us, that is, $v = Hd$ where H is Hubble's constant*[17]. *Projecting galaxy trajectories backwards in time means that they converge to a high density state – the initial fireball.*

The cosmological principle *states that the* universe appears the same in every direction from every point in space. *It amounts to asserting that our position in the universe – with respect to the largest scales – is in no sense preferred. There is considerable observational evidence for this assertion, including the measured distributions of galaxies and faint radio sources. The best evidence comes from the near-perfect isotropy of the relic* cosmic microwave background radiation *(microwave photons detected by us* now, *were redshifted from near-visible photon emitted by hydrogen atoms during the* recombination era, 3×10^5 *yr after the Big Bang). This means that any observer anywhere in the universe will enjoy much the same view as we do, including the observation that galaxies are moving away from them, in accordance with Hubble's law.*

The fact that the universe is expanding – about every point in space – can be a difficult concept to grasp. The analogy of an expanding balloon may be helpful: Imagine residing in a curved flatland on the surface of a balloon. As the balloon was inflated, the geodesic distance between any two points grew; the two-dimensional universe grew but there was no preferred center. For many decades it was unclear whether the universe is topologically closed *(a 3D version of the flatland balloon surface), or* open *(albeit curved). Data collected by observatories in the 1990's, including the* Hubble Space Telescope, *indicates that the universe we inhabit is* open, *i.e. infinite in spatial extent and in (future) temporal extent, too.*

Origin of the cosmic background radiation

About 300,000 years after the Big Bang, *the temperature of the Universe had dropped sufficiently for electrons and protons to combine into hydrogen atoms ("recombination era"). From this time onwards, the* matter *in the universe came to gravitationally dominate over* radiation *and the hydrogen-filled*

[17] This simple law requires nonlinear corrections due to gravitational, relativistic and time-delay effects; these corrections are computable, within a given cosmological model, from GTR.

universe became largely transparent *to electromagnetic radiation (decoupling of matter and radiation). The last photon emitted in that era have propagated freely ever since, while constantly losing energy and increasing in wavelength because its wavetrains are stretched by the expansion of the universe. At the recombination decoupling, the radiation temperature was about 3000 degrees Kelvin, whereas today it has fallen to only 2.7 °K.*

Observers detecting this radiation today are able to see the universe at a very early stage on what is known as the 'surface of last scattering'. Photons in the cosmic microwave background radiation (CMBR) have been traveling towards us for over 13 billion years.

Timeline – History

1576 ***Thomas Digges*** *modified the Copernican system by removing its outer edge and replacing the edge with a star-filled unbounded space.*

1610 ***Johannes Kepler*** *uses the darkness of the night sky to argue for a finite universe.*

1720 ***Edmund Halley*** *also formulated an early form of* **Olbers***' paradox.*

1744 ***Jean Philippe de Cheseaux*** *also formulated an early form of* **Olber***'s paradox.*

1862 ***Heinrich Olbers*** *enunciated* Olbers' paradox*: If stars are distributed throughout the universe they must have been shining for a finite time interval, or else our night sky on earth, here and now, would be uniformly ablaze with the brightness of the sun's disc.*

By the beginning of the 20th century, it was generally accepted that our galaxy was disc-shaped and isolated. But what about spiral nebulae like M31 (Andromeda) – were they inside or outside the Milky Way? **Immanuel Kant** *had speculated that they were 'island' universes.*

1912 ***Vesto Slipher*** *measured spectra from spiral nebulae, showing that many were Doppler-shifted. That is to say: the wavelengths of the observed line spectrum from many of them stretched (scaled up) relative to laboratory-observed spectra of the corresponding atoms, or relative to the sun's spectrum. And these wavelength stretchings were consistent with Doppler* red shifts *due to motions of the nebulae* away *from the solar system, at various speeds (different nebulae receding at different speeds). Such electromagnetic Doppler shifts are well known from Maxwell's theory, and are routinely encountered in the laboratory, satellite telemetry, traffic enforcement, meteorology, etc. The optical Doppler effect is analogous to the way the pitch of a train's whistle or a car's sound is modulated by its velocity relative to the listener.*

1915–1917 ***Albert Einstein*** *created the General Theory of Relativity. It became the cornerstone of massive stars' astrophysics and of all future studies of the large scale structure of the universe.*

1917 ***Willem de Sitter*** *derived from Einstein's GTR equations an isotropic static cosmology with a cosmological constant as well as an empty expanding cosmology with a cosmological constant.*

1918 *A key advance in cosmology came with the development of means to measure the distance to these nebulae.* ***Harlow Shapley*** *used Cepheid variables, bright stars which pulsate at regular intervals ranging from a few days to about a month. The period of their variation is correlated with their absolute luminosity, which he calibrated in the nearby Large Magellanic Cloud. Comparison of nebulae's apparent and absolute luminosities then gave him their distances from earth.*

1922 ***Vesto Slipher*** *summarized his findings on the spiral nebulae's systematic redshifts.*

1922 ***Alexander Friedmann*** *found a solution to the Einstein field equations which suggests the general expansion of space.*

1923–1929 ***Edwin Hubble*** *was able to resolve Cepheid variable stars in M31 (The Andromeda nebula or galaxy) with the 100″ telescope at Mt. Wilson. He developed a new distance calibration method using the brightest stars in more distant galaxies (as the nebulae now recognized as external to the Milky Way galaxy were now increasingly referred to). He correlated these measurements with Slipher's nebulae to discover a proportionality between velocity* v *and distance* d*, that is, Hubbles law* $v = Hd$*. The constant of proportionality* H *is called Hubble's constant (it was significantly overestimated by Hubble himself). He than concluded (1929) that the universe is expanding.*

1927 ***Georges-Henri Lemaître*** *discussed the creation event of an expanding universe governed by the Einstein field equations.*

1928 ***Harold Robertson*** *briefly mentioned that Vesto Slipher's redshift measurements combined with brightness measurements of the same galaxies indicate a redshift-distance relation.*

1933 ***Edward Milne*** *named and formalized the cosmological principle.*

1934 ***Georges-Henri Lemaître*** *interpreted the cosmological constant as due to a "vacuum" energy with an unusual, perfect fluid equation of state.*

1938 ***Paul A.M. Dirac*** *presented a cosmological theory where the gravitational constant slowly decreases so that the age of the universe divided by the time light takes to traverse the atomic nucleus is of order the ratio of the electric and gravitational forces between a proton and electron.*

1948 ***Ralph Alpher***, ***Hans Bethe***, *and* ***George Gamow*** *examined* element synthesis *(nucleosynthesis) in a rapidly expanding and cooling universe and suggested that the elements were produced by rapid neutron capture.*

1948 ***Herman Bondi***, ***Thomas Gold***, *and* ***Fred Hoyle*** *proposed steady state cosmologies based on the* perfect *cosmological principle, i.e. one in which the average* isotropy *and* uniformity *of the universe to observers in any galaxy is augmented by a* time invariance *(*static *universe).*

1960 ***Robert V. Pound*** *(USA) and* ***Glen A. Rebka*** *(USA) made a laboratory measurement of the change in frequency of gamma-ray photons as they fall in a gravitation field [gravitational 'Red Shift' of light], reconfirming the first of Einstein GTR predictions. Thus began the renaissance of interest in GTR.*

1960 ***Rudolph L.B. Minkowski*** *(USA). Using the 200 inch Palomar telescope, obtained spectra for a cluster of galaxies, receding at nearly half the speed of light. This would put their distance at some 7 billion light years. At this point the range to which galaxies could be observed is about, or somewhat less than, half the estimated "radius" (reckoned via light time-of-flight distance coordinate) of the observable universe.*

1960 **Thomas Mathews** *(USA) and* **Allan Sandage** *(USA), at Palomar, examined radio source 3C48 and discovered a 'quasar'. This discovery thrusts GTR to the forefront of astronomy. This quasar recedes with* $\frac{1}{3}$ *the speed of light and its brightness is 100 times that of our galaxy.*

The source of this power was suggested as gravity, which is the strongest force on a cosmic scale. The source has to be very compact; Since the source was varying coherently over a period of one hour, it could not be much longer then the distance traveled by light in one hour, in order for one side of the source to 'know' what the other side is doing, and thus behave in unison.

A strong gravitational field requires a very dense mass confined to a space with a diameter of the orbit of Jupiter.

1961 *Reflection (echo) of radar pulses from Venus (at time of 'inferior conjunction' (closest to earth)), enabled a precise determination of the average distance of the sun from earth: 149,500,000 km. This ended a 2250 year pursuit after the scale of the solar system from* **Aristarchos** *and* **Hipparchos** *through the parallax-seekers* **Richer-Cassini** *(1671),* **J.F. Encke** *(1835) and the international team of 1931.*

1963–1965 **R. Kerr** *discovered a family of exact solutions to Einstein's vacuum field equations; these solutions describe uncharged, rotating black holes, and reduce to the Schwartzschield solution as a special case.*

The charged generalization was found as a solution of the Einstein-Maxwell field equations by **E.T. Newman** *et al. (1965). Only later was the connection of theses mathematical solutions to black holes recognized.*

The Kerr-Newman geometry described by these solutions provides a unique and complete description of the external gravitational and EM fields of a stationary, rotating black hole.

1963 **Maarten Schmidt** *(USA) discovered that absorption lines in the spectra of object 3C273 were 'red shifted' by an extraordinary amount. This 'red shift' corresponds to a recession velocity of 47400 km/sec and constitutes the first recognition of quasars. It opened a new field of relativistic astrophysics.*

1965 ***Arno Penzias*** *(USA) and* ***Robert Wilson*** *discovered the cosmic microwave background radiation (CMBR), a radio-wave remnant of the 'Big-Bang' previously suggested by* ***George Gamow****.*

1965 ***Martin Rees*** *and* ***Dennis Sciama*** *analyzed quasar source count data and discover that the quasar density increases with redshift.*

1965 ***Edward Harrison*** *resolved Olber's paradox by noting the finite lifetime of stars.*

1966 ***Stephen W. Hawking*** *and* ***George Ellis*** *showed that any plausible general relativistic cosmology is singular.*

1966 ***Jim Peebles*** *showed that the Hot Big Bang predicts the correct helium abundance in the universe.*

1967 ***Andrei Sakharov*** *derived, from first principles, the requirements for cosmological generation of a baryon-antibarion asymmetry in the universe. Such an symmetry, is necessary to explain the universe in the present epoch, in which antimatter is very rare.*

1967 ***John Bahcall****,* ***Wal Sargent****, and* ***Maarten Schmidt*** *measured the fine-structure splitting of spectral lines in object 3C191 and thereby showed that the fine-structure constant does not vary significantly with time.*

1967 ***Jocelyn Bell*** *(England) and* ***Anthony Hewish*** *(England) discovered the first pulsar (CP1919) in the middle of the Crab Nebula. It is recognized as a neutron star rotating with period* $t = 1.3373011$ *sec and emitting beams of synchrotron radiation (coherent regular pulses, in this case in the radio wavelength region of the EM spectrum). Coherence and pulse-rate indicate a source small compared to normal stars. The theorized emission mechanism involves a scenario of synchrotron radiation emitted by charged particles near the star's magnetic poles: neutrons near the star's surface decay into protons and electrons and these are driven by powerful radial electric fields on the star's surface, along the curved intense magnetic field lines, with relativistic velocities.*

Thus strong directional beams in the direction tangential to the charged particles' motion are radiated. As the star *rotates, the narrow beams sweeps around the galaxy like a searchlight. If the earth happens to lie in the path of the beams, the pulsar can be detected every time it passes the earth. The star loses angular momentum during radiation. Hence its rotation and pulsation rates slow down [CP1919 increases its period by 42×10^{-9} sec/year]. Thus, pulsars are cosmic lighthouses with rotating beacons of radio waves (and in some cases of visible light, X-rays and gamma rays).*

The Crab Nebula pulsar has a period of 0.033 sec. It was formed in the 1054 CE supernova explosion.

1968 ***Brandon Carter*** *speculated that perhaps the fundamental constants of nature must lie within a restricted range to allow the emergence of life – first use of the weak anthropic principle.*

1968 ***Joseph Weber*** *(USA) made pioneering efforts to detect gravitational waves in the laboratory by means of a bar antenna. These experiments stimulated a worldwide search for this weak and elusive radiation. Improvement in sensitivity of detectors by several orders of magnitude will likely be needed before this goal is achieved.*

1970 ***Stephen W. Hawking*** *(England) and* ***Roger Penrose*** *(England) showed that the equations of GTR in their classical form (without allowing for quantum effects) absolutely require that there was a singularity at the birth of the universe. Hence, there is no way around the singularity problems within the framework of GTR. If singularities are to be avoid in the real universe, the only hope is to improve General Relativity theory by including the effects of quantum theory and developing a quantum theory of gravity.*

The singularity theorem assumes, in addition to the validity of GTR, that the following 4 conditions are met:

(1) The mass density and pressure of early cosmological matter never became negative.

(2) There are no closed time-like or light-like curves in the universe (i.e. it is impossible to visit one's own past, such as in Gödel's universe solution of GTR).

(3) The universe is either closed or there is enough matter in the universe to refocus light via 'light bending'.

(4) A reasonable mathematical condition is satisfied.

1972–1979 *Radio-wave deflection experiments (1972), lunar laser-ranging results (1976), radio wave time-delay results that came in through the missions of* Mariner *6, 7, 9 (1975–1978) and finally the* Viking *mission (1975) – all sided decisively with Einstein's GTR and against the rival Brans-Dicke theory.*

1973–1974 *USA astronauts made observations with a large telescope mounted on the Skylab space station.*

1973 ***Edward P. Tyron*** *(USA) suggested that the entire universe may have been created from absolutely NOTHING as a result of the probabilistic laws of quantum mechanics: an allowable fluctuation in a quantum vacuum could result in the creation of energy.*

The great medieval Hebrew philosopher-poet ***Shlomo Ibn-Gabirol (Avicebron)*** *(1021–1058, Spain) expounded the vision of the creation of the universe* ex-nihilo *in his metaphysics poem* The Royal Crown, *in the following words (1050 CE):*

> *"Calling unto the void and it was cleft,*
>
> *And unto existence and it was urged,*
>
> *And to the universe and it was spread out."*

1974–1978 ***Russell A. Hulse*** *(USA) and* ***Joseph A. Taylor*** *discovered the first* binary pulsar *PSR 1913+16 at the Arecibo Radio Telescope, Puerto Rico. The system consists of a neutron star pulsar, in orbit about a companion dark star, of nearly equal mass and probably also a neutron star. Its discovery opened up a new area for experimental relativity.*

After four years of observation it was concluded that the source consisted of two neutron stars, each of mass near the Chandrasekhar limit *(1.42 solar masses). They move in elliptical orbits about their common center of mass. Only one of them beams its pulses toward earth, while the other's radiation is beamed elsewhere, if at all. The intrinsic pulse period is 0.05902 99952 71 sec, increasing at the rate of* 0.273×10^{-9} *sec/year. The orbital period was 27906.98163 sec (sept 01, 1974) and the estimated distance of the system from earth is 16,000 LY. The radio frequency of the observations was* 430 megahertz.

Since the individual masses of the pulsars can not be measured independently , observations could not be used as a direct test of GTR. However, assuming the validity of GTR, all observed quantities could be explained and reconciled with known results.

On account of the precision of the pulsar's period, relativistic effects can be measured with great accuracy. Thus, the periastron advance of the orbit, 4.2263°/year, is some 36,000 times larger than the perihelion advance of Mercury! Other effects are:

(1) Ordinary Doppler-shift of pulsar's period (affects 5^{th} decimal place).

(2) Special-relativistic time dilation: pulsar's clock run slow (as seen by us) on account of its velocity. Because the orbital speed varies during the motion (being maximal at periastron and minimal at apoastron), the amount of slowing down will be variable but repeat itself each new orbital passage. (affects 8^{th} decimal).

(3) Gravitational red shift (equivalence principle). The pulsar moves in the gravitational field of its companion, while we observe it at great distance. Consequently the pulsars period is red-shifted (lengthened). However, this lengthening varies with the distance between the pulsar and its companion as the system changes from periastron to apoastron, and also repeats itself each orbital passage. The combined effect of (2) and (3) results in a periodic up and down variation of the pulsars period, by at most 58×10^{-9} sec.

(4) GTR predicts a decrease of the orbital period at a rate of 75×10^{-6} sec/year. Using data through Aug 1983, Taylor and Colleagues reported in 1984 an observed value of $(76 \pm 2) \times 10^{-6}$ sec /year.

The predicted decrease could be a result of continual energy loss due to gravitational wave emission, which in turn results from the orbital acceleration. This loss would manifest itself in the speed-up of the two bodies and decrease in their orbital separation. The combined effect of these two changes will cause the time required for a complete orbital period to decrease.

In 1978, Taylor announced what he claims to be the discovery of gravitational radiation, on the strength of the fit of the observed orbital decay compared with the theoretical *prediction of GTR. Although the agreement seemed impressive, alternative mechanisms have not been ruled out, candidates including tidal friction, presence of other stars, or the possible invalidity of the linear approximation in the process of* emission *of gravitational waves. Thus, one could envisage the possibility that the agreement is purely coincidental, resulting from a conjunction of nonlinear gravitational wave 'back reaction' combined with tidal effects involving the observed pulsar and its companion.*

1974 ***Robert Wagoner***, ***William Fowler***, *and* **Fred Hoyle** *showed that the Hot Bing Bang predicts the correct deuterium and lithium abundances.*

1975–1977 ***Vera Rubin*** *(USA) and* ***Kent Ford*** *(USA) established the anisotropy of the universe over a scale of 400 million LY. It is detected by measurements of different recession velocities for distant galaxies in different directions. The amplitude of this deviation from large scale uniformity amount to at most 10 percent of the* ***Hubble*** *recession velocity. Over larger scales, the astronomical evidence suggests isotropy. Rubin and Ford discovered that the net velocity of the Milky Way galaxy relative to the cosmological reference frame (as determined by the cosmic microwave background radiation) is about 600 km/sec [the velocity of the earth relative to the background radiation is 39 km/sec; the earth's motion around the sun is at 30 km/sec; the solar motion around the galactic center is 250 km/sec, and the motion of the Milky was galaxy toward the Andromeda Galaxy is 100 km/sec. A vector addition of these velocities will yield the above-quoted velocity of the Milky Way relative to the local CMBR frame].*

1976 ***A.I. Shlyakhter*** *used* samarium ratios *from the primordial natural fission reactor in Gabon to show that some laws of physics have remained unchanged for over two million years.*

1977 ***Gary Steigman***, ***David Shramm***, *and* ***James B. Gunn*** *examined the relation between the primordial helium abundance and number of neutrino types, and deduced that at most five lepton families can exist.*

1980 ***Alan H. Guth*** *(USA) proposed a new cosmological model of the birth of the universe, called inflationary universe. Accordingly, the universe expanded exponentially rapidly for a very short epoch during the early part of the "particle cosmology" stage of expansion following the Bing Bang.*[18]

1985 ***Mark Morris*** *(USA) discovered at the center the Milky-Way galaxy a number of strong string-shaped radio sources, candidates for low-energy 'cosmic strings' . These are long thin remnants of the original energy of the Bing-Bang. These 'cosmic strings' could supply part of the 'missing mass' needed to form the observed nearly-flat universe.*

1986 *A team of Astronomers discovered that our galaxy cluster (the local group), and other components of the local super-cluster of galaxies, move toward the 'Great Attractor' - a point in the direction of the Southern Cross.*

1986 *Deep redshift galaxy surveys demonstrated the existence of huge bodies, filaments and sheets on scales from 25 Mpc to over 100 Mpc. Subsequent galaxy surveys are providing detailed information about the distribution of large-scale structures. Radio galaxy and quasar surveys indicate that homogeneity (or uniformity) is approached only on scales of several hundred Mpc (that is, nearly a billion light years).*

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[18] Inflationary cosmology was motivated by several previously unexplained observations:

- The high degree of isotropy of the CMBR, in apparent violation of causality ("horizon problem").
- Why has the universe's spatial curvature, ever since the late "particle cosmology" era, been so small? ("flatness problem").
- How can the observed upper bound on the abundance of *magnetic monopoles*, predicted by many Grand Unification extensions of the Standard Model of Particle Physics, be so low? ("monopole problem").

The exponentially rapid expansion predicted by inflationary models – perhaps by a factor of ca. 10^{50} – explains all three puzzles. GTR predicts that inflation would occur if the expanding vacuum is initially trapped in a metastable "false vacuum" quantum state; the subsequent release of vacuum energy when the vacuum "rolls" to its lowest-energy state created a temporarily large cosmological constant, which caused the inflationary expansion. The "standard cosmology" stages of Big Bang cosmology are largely unaffected by inflation.

1987 **Francisco Paresce** *and* **Christopher Burrows** *discovered a disc of protoplanets (gas, dust and debris around the star Beta Pictoris, 53 light years away) that will eventually crash into each other and coalesce to form planets.*

1987 **Roger C. Lynds** *and* **Vahe Petrosian** *discovered in Abell 370 an image of a far-distant unseen galaxy; the image is believed to have been formed by gravitational lensing, an effect predicted by GTR.*

1987 *Neutrinos from a supernova explosion 1987A in the Large Magelanic Cloud reached earth (150,000 LY away).*

1992 *In April 1992, the COBE satellite team announced the discovery of anisotropies in the cosmic microwave background radiation at the level of one part in 100,000. These are thought to be a snapshot, at $t = 300,000$ years after the Big Bang, of the primordial fluctuations that led to galaxy formation. This map of the sky is also the best evidence for the high degree of isotropy (or spherical symmetry) of the universe.*

1992, Feb *The Hubble Space Telescope (HST) revealed a black hole in Galaxy M-87, in Virgo, at distance $R = 52$ million light years away.*

1995–1996 *The Hubble Space Telescope (HST) was able to resolve Cepheid variable stars in the Virgo cluster, ensuring a much better calibration of cosmological distance measures. This has allowed more accurate estimates to be made of Hubble's constant H, and thus, of the age of the universe. Early galaxies and quasars have also been observed by the HST, raising serious doubts about current structure formation models.*

Modern Microscopy

A microscope (Greek: micron = small, scopos = aim) is an instrument for viewing and magnifying very small, close objects, too small to be seen by the unaided eye. The first to be invented was the optical microscope*, containing one or more lenses that produce an enlarged image of an object placed in the focal plane of the lens(es).*

The principle of the simple microscope *(uses only one lens for magnification) was known already to the Romans in the form of water–filled glass bowls (1^{st} century CE).*

Microscopy is the technical field of using microscopes to view samples or objects. There are three well-known branches of microscopy: optical, electron and scanning probe microscopy.

Optical and electron microscopy involve the diffraction, reflection, or refraction of electromagnetic radiation incident upon the subject of study, and the subsequent collection of this scattered radiation in order to build up an image. This process may be carried out by wide field irradiation of the sample (for example standard light microscopy and transmission electron microscopy) or by scanning of a fine beam over the sample (for example confocal microscopy and scanning electron microscopy). Scanning probe microscopy involves the interaction of a scanning probe with the surface or object of interest. The development of microscopy revolutionized biology and remains an essential tool in that science, along with many others.

Optical microscopy techniques include:

- *Bright field optical microscopy*
- *Dark field optical microscopy*
- *Phase–contrast optical microscopy*
- *Differential interference contrast microscopy*
- *Fluorescence microscopy*

- *Confocal laser scanning microscopy*
- *Bright field optical microscopy*
- *Deconvolution microscopy*
- *X–ray microscopy*

For light microscopy, the wavelength of light limits the resolution to around 0.2 micrometers (2000 Å). In order to gain higher resolution, the use of an electron beam width for smaller wavelength is used in electron microscopes.

- *Transmission electron microscopy (TEM) is principally quite similar to the compound light microscope, by sending an electron beam through a very thin slice of the specimen. The resolution (2005) is around 0.05 nanometer (0.5 Å).*
- *Scanning electron microscopy (SEM) visualizes details on surfaces of cells and particles and gives a 3D view.*
- *The atomic de Broglie microscope* uses neutral Helium atoms as probe particles, could provide a resolution at nanometer scale and be absolutely non-destructive.

Scanning probe microscopy is a sub-diffraction technique. It includes:

- *The atomic force microscope (AFM)*
- *The scanning tunneling microscope (STM)*
- *The photonic force microscope (PFM)*

All such methods imply a solid probe tip in the vicinity (near field) of an object, which is supposed to be almost flat.

Modern Telescopy

By 2004, old mothballed Mount Wilson Observatory – which was used to revolutionize our understanding of the cosmos in the first half of the 20th century – was refurbished with some 21st century technology. On that year, the Georgia State University (USA) began to operate CHARA (Center for High Angular Resolution Astronomy) array. It uses optical interferometry to combine signals from six one-meter telescopes for a combined angular resolution (in IR) of 5×10^{-4} *arc-sec. The largest single terrestrial optical telescopes are presently the twin Keck telescopes (Mauna Kea, HI; ten meters diameter each; 1993, 1996). Each of their mirrors comprises 36 hexagonal segments, constantly realigned – via computerized control – to an accuracy of four nanometers. Both Keck telescopes are equipped with adaptive optics to cancel atmospheric turbulence-caused blurring. When operating together as the Keck-Interferometer, the twin telescopes can achieve an angular resolution of* 5×10^{-3} *arc-sec at a wavelength of 2.2 micron.*

Timeline – History

1267 ***Roger Bacon*** *described experiments with hand-held magnifying glasses.*

1608–1609 ***Hans Janssen*** *and his son* ***Zacharias Janssen*** *(1588–1630), Dutch lens grinders and spectacle makers, invented the first compound microscope. In its simplest form (as used by* ***Robert Hooke****) it would have a single glass lens of short focal length for the objective, and another single glass lens for the eyepiece or ocular.* ***Galileo Galilei*** *developed (1609) a compound microscope with a convex and a concave lens.* ***Christiaan Huygens*** *developed a simple 2-lens ocular system (late 1600's) that was achromatically corrected.*

1610 ***Johannes Kepler*** *invented the modern compound microscope.*

1674 ***Anton von Leeuwenhoek*** *was first to bring the simple microscope to the attention of biologists. His microscopes consisted of a single, small, convex lens mounted on a plate with a mechanism to hold the biological specimen. With a magnification of about 270, he was able to see highly detailed images. Thus, he was first to describe cells and bacteria.*

18*th* century *– Several technical innovations make microscopes better and easier to handle, which leads to microscopy becoming more and more popular among scientists. An important discovery is that lenses combining two types of glass could reduce the chromatic effect, with its disturbing halos resulting from differences in refraction of light.*

1826 ***Dames Smith*** *(ca 1800–1870, England) constructed a microscope with much reduced chromatic and spherical aberrations.*

1830 ***Joseph Jackson Lister*** *reduced the problem with spherical aberration by showing that several weak lenses used together at certain distances gave good magnification without blurring the image.*

1878 ***Ernst Abbe*** *formulates a mathematical theory correlating resolution to the wavelength of light. Abbes formula (sine condition) make calculations of maximum resolution in microscopes possible. The company of* ***Carl Zeiss*** *exploited this discovery and became the dominant compound microscope manufacturer of its era.*

Modern microscopes of this kind are usually more complex, with multiple lens components in both objective and eyepiece assemblies. These multi-component lenses are designed to reduce aberrations, particularly chromatic aberration and spherical aberration. In modern microscopes the mirror is replaced by a lamp unit providing stable, controllable illumination.

1903 ***Richard Adolf Zsigmondi*** *(1865–1929, Austria) invented the ultramicroscope, for seeing small particles in a colloidal solution. Improved by* ***Joseph Barnard*** *(1870–1949, England) in 1912.*

1932 ***Frits Zernike*** *(1888–1966, Holland) invented the phase–contrast microscope that allows the study of colorless and transparent biological materials.*

1933 ***Ernst A.F. Ruska*** *(1906–1988, Germany) built the first electron–microscope, using* ***de Broglie*** *(1924) electron waves: electrons are emitted from heated metal and accelerated through a vacuum. They are then focused by powerful magnets onto the specimen. The magnified image appears on a screen. Ruska obtained a magnification of 12,500. Modern electron microscopes can reach a magnification of 1 million.*

1936 ***Erwin Wilhelm Mueller*** *(1911–1977, Germany and USA). Invented the field–emission*[19] *microscope (FEM). It is a type of electron microscope in which a high negative voltage is applied to a metal tip emitter, placed in an evacuated vessel some distance from a detector: a glass screen*

[19] The emission of electrons from cold metals by electric fields. In order to build up sufficiently large electric fields, the metal is usually shaped to a sharp needle point. Field emission is an example of the “tunnel effect” in quantum mechanics, with an electron in the metal being in a potential barrier. In field emission the probability of tunneling, which can be calculated using the “semiclassical approximation” is related to the “work function” of the metal.

with a fluorescent coating. The tip produces electrons by "field emission". The emitted electrons form an enlarged pattern on the fluorescent screen, related to the individual exposed planes of atoms. Since the resolution of the instrument is limited by the vibrations of the metal atoms, it is helpful to cool the tip in liquid helium.

Although the individual atoms forming the point are not displayed, individual absorbed atoms of other substances can be, and their activity is observable.

1948 **Paul Kirkpatrick** *(1894–1992, USA) and* **Albert Baez** *developed X-ray reflection microscope.*

1955 **Erwin Wilhelm Mueller** *developed the field–ion microscope (FIM). The first instrument that can picture individual atoms. It is, again, a type of electron microscope that is similar in principle to the "field–emission microscope", except that a high positive voltage is applied to the metal tip, which is surrounded by a low pressure gas (usually helium) rather than a vacuum.*

The image is formed in this case by field ionization: ionization at the surface of an unheated solid as a result of a strong electric field creating positive ions by electron transfer from surrounding atoms or molecules. The image is formed by ions striking the fluorescent screen. Individual atoms on the surface of the tip can be resolved and, in certain cases, absorbed atoms may be detected.

1955 **George Nomarski** *published the theoretical basis of Differential interference contrast microscopy.*

1969 **Manfred von Ardenne** *(1907–1997, Germany) built the first Scanning electron microscope (SEM).*

In contradistinction to the transmission electron microscope (Ruska) the beam of primary electrons scans the specimen, and those electrons that are reflected, together with any secondary electrons emitted, are collected. This current is used to modulate a separate electron beam in a TV monitor, which scans the screen at the same frequency, consequently building up a picture of the specimen.

1981 **Gerd Binnig** and **Heinrich Rohrer** *invented the scanning tunneling microscope that gives 3-dimensional images of objects down to the atomic level. The surface of the specimen is scanned by measuring a current between a very small tip and the specimen. Individual atoms can thus be detected.*

1986 **Gerd Binnig** *and colleagues invented the atomic force microscope: a small probe, consisting of a tiny chip of diamond, is held on a spring–cantilever in contact with the surface of the sample. The probe is moved slowly across the surface and the tracking force between the tip and the surface is monitored. The probe is raised and lowered so as to keep this force constant, and a profile of the surface is produced. Scanning the probe over the sample gives a computer generated contour map of the surface. The instrument is similar to the "scanning tunneling microscope", but uses mechanical forces rather than electrical signals. It can resolve individual molecules and, unlike the scanning tunneling microscope can be used with nonconducting samples, such as biological specimens.*

1987 **Arthur Rich** and **James van Hoch** *develop the positron microscope, using positrons emitted from a radioactive source.*

From Cell to Biosphere —
Hierarchy of the Living World

* *
*

"Many worlds might have been botched and bungled, throughout an eternity, ere this system was struck out; much labor lost: Many fruitless trials made: And a slow, but continued improvement carried on during infinite ages in the art of world-making."

David Hume, 1779

* *
*

"If it could be demonstrated that any complex organ existed which could not possibly have been formed by numerous, successive, slight modifications – my theory would absolutely break down."

Charles Robert Darwin

* *
*

"No theory of evolution can be formed to account for the similarity of molecules, for evolution necessarily implies continuous change, and the molecule is incapable of growth or decay, of generation or destruction.
None of processes of Nature, since the time when Nature began, have produced the slightest difference in the properties of any molecule.
. . . They continue this day as they were created – perfect in number and measure and weight; and from the ineffaceable character impressed on them we may learn that those aspirations after accuracy in measurement, and justice in action, which we reckon among our noblest attributes as men, are ours because they are essential constituents of the image of Him who in the beginning created, not only the heaven and the earth, but the materials of which heaven and earth consist."

James Clerk Maxwell, 1873

* *
*

"The eternal mystery of the world is its comprehensibility."

Albert Einstein, 1915

* *
*

"As a result of a thousand million years of evolution, the Universe is becoming conscious *of itself, able to understand something of its past history and its possible future. This cosmic self-awareness is being realized in one tiny fragment of the universe – in a few of us human beings... "*

Julian Huxley

* *
*

"The probability of life originating at random is so utterly miniscule as to make it absurd."

Francis Crick

* *
*

The match between our intelligence and the intelligibility of the world is no accident. Nor can it properly be attributed to natural selection, which places a premium on survival and reproduction and has no stake in truth or conscious thought. Indeed, meat-puppet robots are just fine as the output of a Darwinian evolutionary process.

William A. Dembski

Contents

(*) *These two rubrics are relevant at both cellular and organism levels*

Preface

The following monograph is a bold attempt to present a compact and lucid outline of the vast field of biology and biological history in just 160 pages.

This is, no doubt, a pretentious and somewhat perilous undertaking through which important details are sometimes sacrificed for the sake of oversimplified generalizations.

Yet, this unavoidable pitfall, notwithstanding such an effort carries its own hidden benefits:

First, the attentive reader never looses temporal and spatial perspective of his field of view, and is seldom consumed with overwhelming details. He may always complete the missing information regarding a specific mechanism or particular substance (molecule, protein etc.), using a proper textbook, article or a computer database.

Second, I explicitly narrate the accomplishments of some 300 scientists, who during the past 2300 years since Aristo, have created and developed the medical and biological sciences to its present state.

It is hoped that with a constructive feedback flowing from our readers and reviewers, we would be able to improve this outline in the future.

Introduction

Biology is the branch of science dealing with the study of life. It encompasses a broad spectrum of fields that together address phenomena related to living organisms over wide range of scales. It is thus concerned with the characteristics, classification, and function of organisms, how species come into existence, and the interactions they have with each other and with the environment.

At the molecular scale, life is studied in the disciplines of molecular biology, biochemistry *and* molecular genetics.

At the next level, that of the cell, it is studied in cell biology. *At the multicellular scale, it is examined in* physiology, anatomy, *and* histology.

Developmental biology *studies life at the level of individual organism's development (ontogeny).*

Moving up the scale towards more then one organism, ethology *considers the behavior of groups of organisms.* Population genetics *operates at the level of an entire population, and* systematics *considers multi-species scale of lineages. Independent populations and their habitats are examined in* ecology *and* evolutionary biology. *This last discipline is concerned with the origin and descent of species, as well as their change over time.*

Evolutionary biology *is mainly based on paleontology, which searches for evidence in the fossil record.*

The two major traditional taxonomically-oriented disciplines are botany *and* zoology. *Botany is the scientific study of* plants. *Botany encompasses a wide range of scientific disciplines that study the* growth, reproduction, metabolism, development, diseases, *and* evolution *of plant life. Zoology involves the study of animals, including the study of their* physiology *within the fields of* anatomy *and embryology.*

Biology is subjected to the same physical laws operating in branches of science (such as the laws of chemical thermodynamics and conservation of mass). Yet, because of the inherent complexity of most biological systems a straightforward application of the physics and mathematical physics is not always feasible. Nevertheless, the biological sciences are characterized and unified by several major underlying principles and concepts: universality, evolution, diversity, continuity, genetics, homeostasis, *and* interactions.

Some striking examples of biological universality include life's carbon-based biochemistry and its ability to pass on characteristics via genetic material, using a DNA and RNA based genetic code with only minor variations across the range of living things.

Another universal principle is that all organisms (that is, all forms of life on Earth except for viruses) are made of cells. Similarly, all organisms share common developmental processes. For example, in most animals, the basic stages of early embryonic development share similar morphological characteristics and include similar genes.

The central organizing concept in biology is that all life has a common origin ancestor and has changed and developed through the process of evolution. This is thought to have led to the observed similarity of processes.

***Charles Darwin** established evolution as a viable scenario by articulating its driving force, natural selection. Genetic drift was embraced as an additional mechanism of evolutionary development in the modern synthesis of the theory.*

The evolutionary history of a species — which describes the characteristics of the various species from which it descended — together with its genealogical relationship to every other species is called its phylogeny. Widely varied approaches to biology generate information about phylogeny. These include the comparisons of DNA sequences conducted within molecular biology or genomics, and comparisons of fossils or other records of ancient organisms in paleontology.

Despite its underlying unity, life exhibits an astonishingly wide diversity in morphology, behavior, and life histories. In order to grapple with this diversity, biologists attempt to classify all living things. Scientific classification seeks to reflect the evolutionary trees (phylogenetic trees) of the organism being classified. Classification is the province of the disciplines of systematics and taxonomy. Taxonomy places organisms in groups called taxa, while systematics seeks to define their relationships with each other. This classification technique has evolved to reflect advances in cladistics and genetics, shifting the focus from physical similarities and shared characteristics to phylogenetics.

Classification systems generally begin with the three-domain system: Archaea, Bacteria, Eukaryote. These domains reflect whether the cells have nuclei or not, as well as differences in the cell exteriors. Further, each kingdom is broken down continuously until each species is separately classified.

The hierarchy is: 1) Kingdom, *2)* Phylum, *3)* Class, *4)* Order, *5)* Family, *6)* Genus, *7)* Species. *The scientific name of an organism is obtained from its Genus and Species. For example, humans would be listed as* Homo *sapiens. Homo would be the Genus and Sapiens is the species. Whenever writing the scientific name of an organism it is proper to capitalize the first letter in the genus and put all of the species in lowercase; in addition the entire term would be put in italics. The term used for classification is called Taxonomy.*

There is also a series of intracellular parasites that are progressively "less alive" in terms of metabolic activity: Virus, Viroid, Prions.

Up into the 19th century, it was commonly believed that life forms could appear spontaneously under certain conditions. This misconception was challenged by ***William Harvey's*** *dictum that "all life [is] from [an] egg" (from the Latin "Omne vivum ex ovo"), a foundational concept of modern biology. It simply means that there is an unbroken continuity of life from its initial origin to the present time.*

A group of organisms is said to share a common descent if they share a common ancestor. All organisms on earth are thought to have descended from a common ancestor or an ancestral gene pool. This last universal common ancestor of all of today's organisms is believed to have appeared about 3.5 billion years ago. Biologists generally regard the universality of the genetic code as definitive evidence in favor of the theory of universal common descent (UCD) for all BACTERIA, ARCHAEA, *and* EUKARYOTES.

Homeostasis is the ability of an open thermodynamical system to regulate its internal environment to maintain a stable condition by means of multiple dynamic equilibrium adjustments controlled by interrelated regulation mechanisms. All living organisms, whether unicellular or multicellular, exhibit homeostasis. Homeostasis manifests itself at the cellular level through the maintenance of a stable internal acidity (pH); at the organismic level, warm-blooded animals maintain a constant internal body temperature; and at the level of the ecosystem, as when atmospheric carbon dioxide levels rise and plants are theoretically able to compensate by removing more of the gas from the atmosphere. Tissues and organs can also maintain homeostasis.

Every living thing interacts with other organisms and its environment. One reason that biological systems can be difficult to study is that so many different interactions with other organisms and the environment are possible, even on the smallest of scales. A microscopic bacterium responding to a local sugar gradient is responding to its environment as much as a lion is responding to its environment when it searches for food in the African savanna. For any given species, behaviors can be co-operative, aggressive, parasitic or symbiotic. Matters become more complex when two or more different species interact in an ECOSYSTEM. *Studies of this type are the province of* ECOLOGY.

Whether we study a single one-celled organism or the world of life as a whole, we can identify a hierarchy of biological organization:

- *Atoms join to form molecules of varying size, including very large macromolecules such as proteins and DNA.*
- *Atoms and molecules form organelles, such as cell nuclei and mitochondria (the site of energy transformations).*
- *Many organelles work together to perform the various functions of the cell.*
- *Cells associate to form tissues, such as bone marrow, skin, etc.*
- *Tissues form organs, such as bones, heart, liver, etc, that in turn comprise organ systems.*
- *The skeletal system and other structural systems work together to make up the functioning organism.*
- *A population of different species that inhabit a particular area make up a community, which together with the nonliving environment form an ecosystem.*
- *Those parts of earth's atmosphere, bodies of water and crust that support life, together with all its ecosystems, constitutes the biosphere.*

That aspects of the natural sciences whose methodologies aim at structures by studying their parts is known as reductionism. However, the whole is more then the sum of its parts. Each level of structural scale & description reveals emergent properties — characteristics not found at lower levels.

Table 6.8 correlates the characteristic spatial size of each organizational level with the corresponding subfield of Biology which aims to study phenomena at this level. It spans 15 orders of magnitude ($10^{-9}m$ – $10^{6}m$).

Table 6.8: LIFE'S HIERARCHY

	LEVEL OF ORGANIZATION *(entities or system)*	TYPICAL LINEAR SIZE	SUBFIELD
Molecular Biology	*Molecule (protein, enzyme, sugar, fatty acid, nucleic acid, solutes, etc.)*	2–10nm	*Biochemistry, Biophysics, Molecular genetics, Molecular evolution*
	Virus	20–400nm	*Virology*
	Chromosome	300–700nm *(unfolded)*	
	Bacterium	200–8000nm	*Bacteriology, Microbiology*
	Living cell	1–50μm	*Origins of Life, cell-Biology (cytology), Mycology, Immunology*
Organizational Biology	*Multicellular systems*		*Histology, Physiology, Anatomy, Nuclear Medicine*
	Individual organism or organ	1mm–30m	*Botany, Zoology, Paleontology, Anatomy, Medicine, Systematics*
	Ecosystems and Populations	1m–1000kn	*Ecology and animal behavior (Ethology), Biogeography, Population genetics, Evolutionary Biology, Developmental Biology*

Life at the Molecular Level (Molecular Biology[20])

Molecular biology *is the study of biology at the molecular level. The field overlaps with other areas of biology and chemistry, particularly genetics and biochemistry. Molecular biology chiefly concerns itself with understanding the interactions between the various systems of a cell, including the interrelationship of DNA, RNA and protein synthesis and, learning how these interactions are regulated.*

Biochemistry *is the study of the chemical substances and thermodynamic processes occurring in living organisms.*

Genetics *is the study of the physico-chemical properties, variations, mutations, combinations, heredity and expression of genetic materials & the information & instructions they encode. Gene expression involves the extent and manner in which individual genetic differences affect individual organism (via protein synthesis). Often this can be inferred by the absence of a normal component (e.g. one gene). Genetics includes the study of "mutants" – organisms which lack (or posses an extra) one or more functional or structural components with respect to the so-called "wild type" or normal phenotype. Genetic interactions such as epistasis can often confound simple interpretations of such "knock-out" studies.*

Molecular biology is the study of molecular underpinnings of the process of replication, transcription and translation of the genetic material. The central dogma of molecular biology holds that genetic material is transcribed into RNA and then translated into protein, with each gene coding for a single & unique protein. This "dogma", despite being an oversimplified picture of molecular biology, still provides a good starting point for understanding the field. This picture, however, is undergoing revision in light of emerging novel roles for RNA.

Much of the work in molecular biology is quantitative, and recently much work has been done at the interface of molecular biology and computer science

[20] The term was first coined by **Warren Weaver** (1938).

in bioinformatics and computational biology. As of the early 2000s, the study of gene structure and function, molecular genetics, has been amongst the most prominent subfields of molecular biology.

Increasingly many other fields of biology focus on molecules, either directly studying their interactions in their own right such as in cell biology and developmental biology, or indirectly — as when the techniques of molecular biology are used to infer historical attributes of populations or species, as in evolutionary biology, population genetics and phylogenetics. There is also a long tradition of studying biomolecules "from the ground up" in biophysics.

Since the late 1950s and early 1960s, molecular biologists have learned to characterize, isolate, and manipulate the molecular components of cells and organisms. These components include DNA, the repository of stored genetic information; RNA, a close relative of DNA whose functions range from serving as a temporary working copy of DNA to actual structural and enzymatic functions as well as a functional and structural part of the translational apparatus; and proteins, the major structural and enzymatic type of molecule in cells.

The successes of MOLECULAR BIOLOGY *derive from the exploration of that unknown world via the new technologies & methodologies developed by chemists, physicists & mathematicians during the* 20^{th} *century; these studies revealed the structure and function of biotic macromolecules (as well as higher-level structures).*

The salient technologies and methodologies are:

Chromatography	*1901*	(***M.S. Tsvet***)
X-ray crystallography	*1912*	(***L. Bragg***)
Ultracentrifugation	*1925*	(***T. Svedberg***)
Electron microscopy	*1932*	(***Knoll and Ruska***)
Electrophoresis	*1933*	(***Arne Tiselius***)
Radionuclide imaging	*1938*	(***Glenn Seaborg***)
Ultrafast Laser spectroscopy	*1960*	(***T. Maiman***); *1987* (***Steven Chu***)
Single-photon emission tomography	*1964*	
CT Scanning	*1972*	(***Godfrey Hounsfield***)
NMR Spectroscopy	*1973*	(***Paul Lauterbur***)
Atomic force microscopy	*1986*	(***Gerd Binnig*** *et al.*)
Optical trapping nanometry		
Scanning tunneling electron microscopy	*1989*	
Optical trapping interferometry	*1993*	
Cryo-electron microscopy	*2000*	
Two-photon microscopy		
FRET (Fluorescent Resonance Energy Transfer) microscopy		
Fluorescence microscopy using chromophores		
Microarray chips & microfluidics		
Statistical methods (to reconstruct genome sequences from oligonucleotide fragments)		

I. Biophysics and Biochemistry

Biophysics

Biophysics (also biological physics) is an interdisciplinary science that applies the theories and methods of physical sciences, especially those of physics, to questions of biology.

Biophysics research today comprises a number of specific lines of biological studies, which don't share a unique identifying factor, or admit of clear-cut and concise definitions. This is the result of biophysics' relatively recent appearance as a scientific discipline. The studies included under the umbrella of biophysics range from sequence analysis through fluid mechanics (e.g. to study blood flow) to neural networks.

In the recent past, biophysics included creating mechanical limbs and nanomachines to regulate biological functions. Nowadays, these are more commonly referred to as belonging to the fields of bioengineering and nanotechnology respectively. We may expect these definitions to further refine themselves. Traditional studies in biology are conducted using statistical ensemble experiments, typically using femto- to micro-molar concentrations of macromolecules.

Because the molecules that comprise living cells are so small, techniques such as PCR amplification, gel blotting, fluorescence labeling and in vivo staining are used so that experimental results are observable with an unaided eye or, at most, optical magnification. Using these techniques, biologists attempt to elucidate the complex systems of interactions that give rise to the processes that make life possible.

By drawing knowledge and experimental techniques from a wide variety of disciplines, biophysicists are able to indirectly observe or model the structures and interactions of individual molecules or complexes of molecules.

In addition to things like solving a protein structure or measuring the kinetics of single molecule interactions, biophysics is also understood to encompass research areas that apply models and experimental techniques derived from physics (e.g. electromagnetism and fluid dynamics) to larger systems such as tissues or organs (hence the inclusion of basic neuroscience as well as more applied techniques such as fMRI).

The interdisciplinary nature of biophysics is manifested in the wide range of biological fields in which these researchers are active:

- Biology and Molecular biology *(gene regulation, single protein dynamics, bioenergetics, biomechanics).*

The effectiveness of these techniques has perhaps been best demonstrated in the field of molecular motors, particularly with the use of optical trapping techniques in conjunction with nanometer-precision position detection schemes (optical trapping nanometry). An optical trap is produced by highly focused laser light, and can be used to grab, move, and exert measurable forces (typically of order pico-Newtons) on micron-sized (and smaller) objects, such as dieletric microspheres. A microsphere, chemically coupled to a molecule of interest, provides a means of measuring the molecule's position and the force that it exerts. Previously, when single kinesin molecules were observed to have a step-size of 8 nm, it became clear that optical trapping nanometry had great potential to probe the molecular mechanisms of motor proteins.

Further demonstration came from the subsequent observation of forces and displacements produced by single myosin molecules using feedback-enhanced optical traps. Since then, optical trapping nanometry has revolutionized the field of molecular motors, and has become the technique of choice for many researchers in this field.

A number of single-molecule manipulation techniques exist, including optical trapping nanometry, magnetic bead, microneedle, micropipette, and some scanning probe microscopies. These techniques differ in their precision of position detection (∼ 1 Å to tens of nanometers) and force regimes (∼ 0.1–10000 pN).

- Structural biology - *angstrom-resolution structures of proteins, nucleic acids, lipids, carbohydrates, and complexes thereof.*
- Biochemistry and chemistry - *biomolecular free energy, reaction kinetics and structure, siRNA, nucleic acid structure, structure-activity relationships.*
- Computer science - *molecular simulations, sequence alignment, neural networks, databases.*
- Mathematics - *graph/network theory, population modeling, dynamical systems, phylogenetical analysis.*
- Medicine and neuroscience - *tackling neural networks experimentally (brain slicing, EMG, EKG, EEG & motor-control studies) as well as theoretically (computer models), membrane permittivity, gene therapy, understanding tumors.*

- Pharmacology and physiology - *channel biology, biomolecular interactions, cellular membranes, polyketides.*
- Physics - *biomolecular structures and dynamics, protein folding, stochastic processes, surface dynamics.*

Related fields are:

- *Animal locomotion*
- *Cellular biophysics*
- *Molecular biophysics*
- *Channels, receptors and transporters*
- *Electrophysiology*
- *Cell membranes*
- *Bioenergetics*
- *Molecular motors*
- *Muscle and contractility*
- *Nucleic acids*
- *Photobiophysics and biophotonics*
- *Proteins*
- *Signaling*
- *Supramolecular assemblies*
- *Spectroscopy, imaging, etc.*
- *Systems neuroscience*
- *Neural encoding*
- *Bionics*
- *Polysulphur membranes*
- *Biosensors and Bioelectronics*

Of these, the fields of cellular biophysics, molecular motors, protein unfolding, *biopolymer mechanics and receptor-ligand interactions have been very active in the past. Indeed, the advent of biophysical techniques for the manipulation of single biological molecules has made possible a large number of significant breakthroughs in biology.*

Biochemistry

During the 21^{st} *Century "big 4" of X-ray, radionuclide imaging, ultrasound and MRI continue to dominate, in their many variants, but many other interesting developments in other techniques are occurring, especially when we consider "imaging" to include microscopic as well as macroscopic biological structures (thermal imaging, electrical impedance tomography, scanned probe techniques, etc.) In addition, the emphasis in the future will increasingly be on obtaining functional and metabolic information simultaneously with structural (image) information. This can already be done to some extent with radioactive tracers (e.g. PET) and magnetic resonance spectroscopy.*

Biochemistry then, is the science dealing with the chemical constitutions of living systems and the dynamics of living chemical processes (metabolism). Although the word "biochemistry" was coined by ***F. Hoppe-Seyler*** *(1877), the full-fledged and institutionalized discipline emerged only in 1903. It was formed from:*

- *chemist's animal and vegetable chemistry*
- *biologists, cytologists and physicians physiological, zoological or biological chemistry.*

It rapidly also formed a base for immunologists, nutritionists and chemists working on fermentation.

Biochemistry interfaces with biology and chemistry and is concerned with the chemical processes that take place within living cells. Modern biochemistry developed out of and largely came to replace what in the nineteenth and early twentieth centuries was called physiological chemistry, which dealt more with extracellular chemistry, such as the chemistry of digestion and of body fluids. Biochemistry as such is largely, though not exclusively, a twentieth-century discipline.

Molecular biology, on the other hand, has come to mean the study of the function and the three-dimensional structure of such biologically important macromolecules as proteins and nucleic acids. Molecular biology is as much an interface of *biology with physics* as of *biology with chemistry*. In many respects biochemistry and molecular biology represent the realization of the dream of early twentieth-century mechanistic biologists, who were convinced that the most fundamental biological processes could ultimately be understood in terms of the laws of physics and chemistry.

Researchers in biochemistry use specific techniques native to biochemistry, but increasingly combine these with techniques and ideas from *genetics, molecular biology* and *biophysics*. There has never been a hard-line between these disciplines in terms of content and technique, but members of each discipline have in the past been very territorial; today the terms molecular biology and biochemistry are nearly interchangeable.

Biochemical research is involved in three main fronts:

- Components of cells: structure and function
- Energetics and metabolism
- Working of the genetic code

Living organisms, and even the individual cells of which they are composed, are enormously complex and diverse. Nevertheless, certain unifying features are common to all things that live. All make use of the same types of biomolecules, and all use energy. As a result, organisms can be studied via the methods of chemistry and physics. The belief in "vital forces" (forces thought to exist only in living organisms) held by 19th-century biologists has long since given way to awareness of an underlying unity throughout the natural world.

Disciplines that appear to be unrelated to biochemistry can provide answers to important biochemical questions. An example is the discovery, made by physicist in the early 20th century, that x-rays can be diffracted by crystals. The resultant experimental method of x-ray crystallography led to the elucidation of the three-dimensional structures of molecules as complex as proteins and nucleic acids. Biochemistry is a field that draws on many disciplines, and its multidisciplinary nature allows it to use results from many sciences to answer questions about the *molecular nature of life processes*. Enormously important applications of this kind of knowledge are made in medically related fields; an understanding of health and disease at the molecular level leads to more effective treatment of illnesses of all sorts.

The activities within a cell are analogous to the transportation system of a city. The cars, buses, and taxis correspond to the molecules involved in *reactions (or series of reactions) within a cell. The routes traveled by these vehicles are likewise comparable to the reaction & transport pathways that occur in the life of the cell. Note particularly that many vehicles travel more that one route — for instance, cars and taxis can go anywhere — whereas other, more specialized modes of transportation such as subways and streetcars are confined to single paths.*

Similarly, some molecules play multiple roles, whereas others take part only in specific series of reactions. And in terms of spatial transport, each cell's cytoplasm is criss-crossed by a complex 3-D network of myriad microtubules — nearly invisible "subway tunnels" along which molecules are pushed and pulled (between the membrane & interior of the cell) by molecular nano-motors. Also, all the routs (both spatial transport paths and chemical pathways) operate simultaneously.

The fundamental similarity of cells of all types makes it interesting and illuminating to speculate on the origins of life. Even the structures of comparatively small biomolecules consist of several parts. Large biomolecules such as proteins and nucleic acids have complex structures, and living cells are enormously more complex.

Even so, both molecules and cells must have arisen ultimately from very simple molecules such as water, methane, carbon dioxide, ammonia, nitrogen, and hydrogen[21]. *In turn, these simple molecules must have arisen from atoms. The way in which the universe itself, and the atoms of which it is composed, came to be, belongs to the discipline of* cosmology*.*

Since, life-forms alive today are believed to have descended from the same common ancestor, they certainly have similar biochemistries, even in matters which would appear to be essentially arbitrary, such as the genetic code or handedness of various biomolecules. It is unknown whether alternate biochemistries are possible or practical.

Biochemistry is the study of the structure and function of cellular components, such as proteins, carbohydrates, lipids, nucleic acids, and other biomolecules. Chemical biology aims to answer many questions arising from biochemistry by using tools developed within synthetic chemistry.

Although there are a vast number of different biomolecules, they tend to be composed of the same repeating subunits (called monomers), in different orders. Each class of biomolecules has a different set of subunits. Recently, biochemistry has focused more specifically on the chemistry of enzyme-catalyzed reactions, and on the properties of proteins.

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[21] The biochemistry of all life-forms on earth is carbon and water based.

The biochemistry of cell metabolism and the endocrine system has been extensively described. Other areas of biochemistry include the genetic code (DNA, RNA), protein synthesis, cell membrane transport, and signal transduction.

Originally, it was generally believed that life was not subject to the laws of science the way non-life was. It was thought that only living beings could produce the molecules of life (from other, previously existing biomolecules). Then, in 1828, ***Friedrich Wöhler*** *published a paper about the synthesis of urea, proving that organic compounds can be created artificially. The dawn of biochemistry may have been the discovery of the first enzyme, diastase (today called amylase), in 1833 by* ***Anselme Payen****.*

Eduard Buchner *contributed the first demonstration of a complex biochemical process outside of a cell in 1896: alcoholic fermentation in cell extracts of yeast. Although the term biochemistry seems to have been first used in 1882, it is generally accepted that the formal coinage of biochemistry occurred in 1903 by* ***Carl Neuberg****.*

Since then, biochemistry has advanced, especially since the mid-20th century, with the development of new techniques such as chromatography, X-ray diffraction, NMR spectroscopy, radioisotopic labeling, electron microscopy and molecular dynamics simulations. These techniques allowed the discovery and detailed analysis of many molecules and metabolic pathways of the cell, such as glycolysis and the Krebs cycle (citric acid cycle).

Today, the findings of biochemistry are used in many areas, from genetics to molecular biology and from agriculture to medicine.

In the 1940s, following up on ***Griffith****'s experiment,* ***Avery, MacLeod*** *and* ***McCarty*** *definitively identified deoxyribonucleic acid (DNA) as the "transforming principle" responsible for transmitting genetic information. In 1953,* ***Francis Crick*** *and* ***James D. Watson*** *published their famous paper on the structure of DNA, based on the research of* ***Rosalind Franklin*** *and* ***Maurice Wilkins****. These developments ignited the era of molecular biology and transformed the understanding of evolution by enabling a description of it as a molecular process: the mutation of segments of DNA.*

During this era of molecular biology, it also became clear that a major mechanism for variation within a population, once again, is mutations of DNA. In the mid-1970s, ***Motoo Kimura*** *formulated the neutral theory of molecular evolution, firmly establishing the importance of genetic drift as a major mechanism of evolution. The theory sparked the "neutralist-selectionist" debate, partially solved by the development of Tomoko Ohta's nearly neutral theory of evolution.*

II. Molecular Genetics — from Gene to Genome (1859–2008)

Genetics is the study of genes on all levels: from the level of molecules up to the level of populations.

It is the science of genes, heredity, and the variation of organisms. In modern research, genetics provides important tools in the investigation of the function of a particular gene, or the analysis of genetic interactions. Within organisms, genetic information generally is carried in chromosomes, where it is represented in the chemical structure (such as base sequences) of particular stretches of the DNA molecule.

Genes encode the information necessary for synthesizing proteins, which in turn play a large role in influencing (though, in many instances, not completely determining) the final phenotype of the organism.

Heredity and Genetics

In about 1902, the chromosome was identified as being the site of the genes, and its central position in heredity and development were finally realized. Linkage of genes and the crossing over of chromosomes during cell division were explored, particularly in ***Thomas Hunt Morgan****'s fly lab in Columbia University. Early in the twentieth century, a unification of the idea of evolution by natural selection with Mendelian genetics to produce the modern synthesis occurred. These ideas continued to be developed in the discipline of population genetics and in the second half of the century began to be applied in the new discipline of the genetics of behavior, sociobiology, and, especially in human's, evolutionary psychology.*

By the end of the 19th century all of the major pathways of drug metabolism had been discovered. In the early decades of the twentieth century, the role of minor components of foods in human nutrition, the vitamins, began to be isolated and synthesized. Then in the 1920s and 1930s, the metabolic pathways of life, such as the citric acid cycle, glycogenesis and glycolysis finally began to be worked out by biochemists.This work continued to be very actively pursued for the rest of the century and into the next. During 1939–1941 ***Fritz Lipmann*** *showed that ATP is the universal carrier of energy in the cell, and then in the mid-1950's the power generators of the cell, the mitochondria, also began to be understood.*

Oswald Avery *conclusively showed in 1943 that DNA was the genetic material of the chromosome, not its protein. By 1953* ***James D. Watson*** *and* ***Francis Crick*** *had shown that the structure of DNA was a double helix, and its probable connection to replication. The nature of the genetic code was* *unraveled experimentally starting with the work of* **Nirenberg Khorana** *and others in the late 1950's. This discovery and others — gave rise to the vigorous science that we know today as molecular biology.*

The largest, most costly single biological study ever undertaken, the Human genome project, began in 1988 under the leadership of James D. Watson, and a first draft of the human DNA sequence announced in 2000. By 2003, 99% of the genome had been sequenced to an accuracy of one part in ten thousand. The HapMap project to determine patterns of differences in the human genome began in 2002 and by 2005 completed its first phase work.

The advent of whole-genome sequencing and surveys of their variation in different populations (races), together with new statistical methods, permitted researches by 2006 to systematically identify candidate loci for recent natural selection during evolution in humans. Some of these genes were also shown to be ancestry-informative markers which came to be used in genealogical studies and to understand ancient human migrations.

The study of organisms, their reproduction, and the functions of their organs had increasingly become the study of molecules. Reductionism was triumphant. Even the methods of scientific classification of organisms, especially cladistics, began in the last quarter of the century to use RNA and DNA sequences as characters. By the mid 1980's even the overall division of the tree of life into three domains (as opposed to the classical two), the Archaea, the Bacteria, and the Eukaria, became generally accepted in the scientific community.

While cloning in plants was known for millennia it was only in 1951 that the first animal, the tadpole, was cloned by nuclear transfer. Within a few years, several other animals, including dogs, cats, horses and cattle were cloned by similar methods.

In 1965 it was shown that normal cells in culture divide only a fixed number of times. Then aged and died. About the same time, stem cells were shown to be exceptions to this rule and began to be studied in earnest. Toward the end of the century, stem cells came to be recognized as crucial for the understanding of developmental biology and raised hopes for new medical applications.

In 1983 the unity of much of the morphogenesis of organisms from fertilized egg to adult began to be unraveled, first in fruit fly, then in other insects and animals, including man.

Genetic Engineering (GE)

Genetic engineering, genetic modification (GM) and gene splicing are terms the process of manipulating genes, usually outside the organism's normal reproductive process.

It involves the isolation and reintroduction of DNA into cells or model organisms, usually to express a protein. The aim is to introduce new characteristics or attributes physiologically or physically, such as making a crop resistant to herbicide, introducing a novel trait or producing a new protein or enzyme.

Since a protein is specified by a segment of DNA called a gene, future versions of that protein can be modified by changing the gene's underlying DNA. One way to do this is to isolate the piece of DNA containing the gene, precisely cut the gene out, and then reintroduce (splice) the gene into a different DNA segment. ***Daniel Nathans*** *and* ***Hamilton Smith*** *received the 1978 Nobel Prize in physiology or medicine for their isolation of restriction endonucleases, which are able to cut DNA at specific sites. Together with ligase, which can join fragments of DNA together, restriction enzymes formed the initial basis of recombinant DNA technology.*

The first Genetically Engineered drug was human insulin, approved by the USA's FDA in 1982. Another early application of GE was to create human growth hormone as replacement for a drug that was previously extracted from human cadavers. In 1986 the FDA approved the first genetically engineered vaccine for humans, for hepatitis B. Since these early uses of the technology in medicine, the use of GE has expanded to supply many drugs and vaccines.

One of the best known applications of genetic engineering is the creation of genetically modified organisms (GMOs).

Although there has been a tremendous revolution in the biological sciences in the past twenty years, there is still a great deal that remains to be discovered. The completion of the sequencing of the human genome, as well as the genomes of most agriculturally and scientifically important plants and animals, has increased the possibilities of genetic research immeasurably. Expedient and inexpensive access to comprehensive genetic data has become a reality with billions of sequenced nucleotides already online and annotated.

Human Genome Project (HGP)

In biology the genome[22] *of an organism is its whole hereditary information and is encoded in the DNA (or, for some viruses RNA). This includes both the genes and non-coding sequences of the DNA.*

More precisely, the genome of an organism is a complete DNA sequence of one set of chromosomes; for example, one of the two sets that a diploid individual carries in every somatic cell. The term genome can be applied specifically to mean the complete set of nuclear DNA (i.e., the "nuclear genome") but can also be applied to organelles that contain their own DNA, as with the mitochondrial genome or the chloroplast genome.

When people say that the genome of a sexually reproducing species has been "sequenced," typically they are referring to a determination of the sequences of one set of autosomes and one of each type of sex chromosome, which together represent both of the possible sexes.

Even in species that exist in only one sex, what is described as "a genome sequence" may be a composite from the chromosomes of various individuals. In general use, the phrase "genetic makeup" is sometimes used conversationally to mean the genome of a particular individual or organism. The study of the global properties of genomes of related organisms is usually referred to as genomics, which distinguishes it from genetics which generally studies the properties of single genes or groups of genes.

Most biological entities more complex than a virus sometimes or always carry additional genetic material besides that which resides in their chromosomes. In some contexts, such as sequencing the genome of a pathogenic microbe, "genome" is meant to include this auxiliary material, which is carried in plasmids. In such circumstances then, "genome" describes all of the genes and non-coding DNA that have the potential to be present.

In vertebrates such as sheep and other various animals however, "genome" carries the typical connotation of only chromosomal DNA. So although human mitochondria contain genes, these genes are not considered part of the genome. In fact, mitochondria are sometimes said to have their own genome, often referred to as the "mitochondrial genome".

Comparison of different genome sizes is shown in Table 6.9

[22] The term coined in 1920 by **Hans Winkler** (Germany) as a portmanteau of the words *GEN*e and chromos*OME*.

Table 6.9: Comparison of different genome sizes

Organism	***Genome size (base pairs)***	***Note***
Virus, Phage -X174	*5386*	*First sequenced DNA-genome*
Virus, Phage λ	5×10^4	
Archaeum, Nanoarchaeum equitans	5×10^5	*Smallest non-viral genome Dec, 2005*
Bacterium, Buchnera aphidicola	6×10^5	
Bacterium, Wigglesworthia glossinidia	7×10^5	
Bacterium, Escherichia coli	4×10^6	
Amoeba, Amoeba dubia	6.7×10^{11}	*Largest known genome, Dec 2005*
Plant, Arabidopsis thaliana	1.2×10^8	*First plant genome sequenced, Dec 2000*
Plant, Fritillaria assyrica	1.3×10^{11}	
Plant, Populus trichocarpa	4.8×10^8	*First tree genome, Sept 2006*
Yeast, Saccharomyces cerevisiae	2×10^7	
Nematode, Caenorhabditis elegans	9.8×10^7	*First multicellular animal genome, December 1998*
Insect, Drosophila melanogaster aka *Fruit Fly*	1.3×10^8	
Mammal, Homo sapiens	3×10^9	

Note: The DNA from a single human cell has a length of 1.8 m (but at a width of 2.4 nanometers).

Genomes are more than the sum of an organism's genes and have traits that may be measured and studied without reference to the details of any particular genes and their products. Researchers compare traits such as chromosome number (karyotype), genome size, gene order, codon usage bias, and GC-content to determine what mechanisms could have produced the great variety of genomes that exist today.

Duplications play a major role in shaping the genome. Duplications may range from extension of short tandem repeats, to duplication of a cluster of genes, and all the way to duplications of entire chromosomes or even entire genomes. Such duplications are probably fundamental to the creation of genetic novelty.

The information generated by the human genome project is expected to be the source book for biomedical science in the 21st century and will be of immense benefit to the field of medicine. It will help us to understand and eventually treat many of the more than 4000 genetic diseases that afflict mankind, as well as the many multifactorial diseases in which genetic predisposition plays an important role.

III. Structural Biology — the Molecular Architecture of Life (1853–2008)

Structural biology is a branch of molecular biology concerned with the study of the architecture and shape of biological macromolecules – proteins and nucleic acids in particular – and what causes them to have the structures they have. This subject is of great interest to biologists, because macromolecules carry out most of the functions of a cell, and because typically it only is by coiling into a specific three-dimensional shape that they are able to perform their functions. This shape, which is called the "tertiary structure" of a molecule, depends in a complicated way on the molecule's basic composition, or "primary structure."

Biomolecules are too small to see in detail even with the most advanced light microscopes. The methods that structural biologists use to determine their structures generally involve measurements on vast numbers of identical molecules at the same time. These methods include crystallography, NMR, ultra fast laser spectroscopy, electron microscopy, electron cryomicroscopy (cryo-EM), and circular dichroism. Most often researchers use them to study the static "native states" of macromolecules. But variations on these methods are also used to watch nascent or denatured molecules assume or re-assume their native states.

Proteins are amino acid chains, made up from 20 different amino acids that fold into unique 3-dimensional protein structures. The shape into which it folds is determined by its sequence of amino acids (aa). Below about 40 aa the term peptide is frequently used. A certain amount of aa is necessary to perform a particular biochemical function, and about 40–50 aa appear to be the lower limit for a functional domain size. Protein sizes range from this lower limit to several thousand aa in multi-functional or structural proteins. However, the current estimated for the average protein length is about 300 aa.

A third approach that structural biologists take to understanding structure is bioinformatics to look for patterns among the diverse sequences that give rise to particular shapes. Researchers often can deduce aspects of the structure of membrane proteins based on the membrane topology predicted by hydrophobicity analysis.

In the past few years it has become possible for highly accurate physical molecular models to complement the study of biological structures. Rapid prototyping technologies such as those used by 3D Molecular Design, or the creation of molecular models in glass, are examples of recent advances in this field.

Some of the 20 standard proteinogenic amino acids are called essential amino acids because the human body cannot synthesize them from other compounds through chemical reactions, but therefore must be obtained from food. Histidine and arginine are generally considered essential only in children, because the metabolic pathways that synthesize these amino acids are not fully developed in children.

Structural biology aims to explain the activity of biological important molecules in terms of their atomic structure – and to use this knowledge to design new therapies or vaccines.

When scientists understand the precise structure of some components of the surface of cancer cells, for instance, they may be able to tailor new drugs to fit these components as accurately as a key in a lock. With a really tight fit, very low doses of drugs could kill the cancer cells effectively without harming the normal cells nearby.

Similarly, researchers are trying to understand the three-dimensional structure of some natural chemicals that attach themselves to the genetic material DNA. These chemicals selectively control the activity of specific genes at specific times and in specific places. Deciphering their structure could lead to laboratory-made chemicals that do exactly the same thing.

Structural biologists use information gathered from chemistry, physics, genetics, cell biology, and mathematics, plus supercomputers and other highly specialized equipment. In recent years these scientists have relied increasingly on the use of computer graphics to visualize how various subunits of molecules fit together or move.

They have made rapid strides in the analysis of proteins (the extraordinarily varied molecules that do most of the work of the body); nucleic acids (DNA and a related molecule, RNA); carbohydrates (sugars and starches); lipids (fats); and the complex combinations of these substances.

Structural biology burst upon public attention for the first time in 1953, when ***Francis H.C. Crick*** *and* ***James D. Watson*** *announced that they had deciphered the structure of DNA – work for which they later won the Nobel Prize. Their model, the famous double helix, paved the way for many advances in genetics, including the development of recombinant DNA technology, which allows scientists to cut and splice together pieces of DNA from different sources.*

According to this model, DNA is a twisting ladder made up of different sequences of four components called nucleotides. Each rung of the ladder consists of a pair of nucleotides (which can pair only in certain ways). The specific sequence of these pairs contains all the information necessary for the development and survival of an organism such as a human being. The model *explained for the first time how genetic information is transferred from the parent archive of DNA to daughter strands of nucleic acid: the nucleotide pairs separate in the middle of the rung, as if a zipper had opened, making two complementary strands, and each of these strands participates in forming a new, complete double helix.*

The structures of many other biologically important molecules have been revealed since then, particularly the structures of those marvelously versatile substances called proteins.

Nonscientists tend to think of proteins as just something we eat. But in fact we depend on tens of thousands of different proteins in our bodies to keep us alive. Each protein is a complex biochemical machine with its own specialty. The reason we eat proteins from plants or animals in order to make proteins of our own.

Some of the proteins we depend on are enzymes that increase the speed of chemical reactions up to a million times, without themselves undergoing any change. In this way, enzymes control the pathways and the timing of billions of chemical operations. They regulate our growth from a single cell to a mature organism. They make our cells differentiate into eyes, blood, or brain cells. They deftly break down or build up other proteins.

There are other proteins that sit on the surface membranes of our cells, where they receive messages from distant cells, or control the flow of molecules into or out of the cell. The surfaces of cancer cells are dotted with proteins that play a large role in cells' runaway growth. The coats of viruses are made up largely of proteins.

Some of the neurotransmitters that carry urgent massages from one nerve cell to another are also proteins. So are many of the hormones that regulate our growth, sex drive, and reactions to stress.

The thousands of antibodies that recognize and fight foreign substances such as viruses and bacteria are proteins. The chemicals that interact with DNA that turn specific genes on or off are proteins. One protein, hemoglobin has the key job of transporting oxygen throughout the body via the bloodstream.

Different kinds of proteins act as structural materials, making up our skin, hair, nails, muscles, tendons, and bones. They also maintain the inner structure of the millions of cells in our bodies.

Some proteins are relatively small and compact; others are bulky conglomerates. Some are globular; other are long and narrow. But all of these different types of proteins are polymers, substances composed of many smaller units linked in chains. The subunits of proteins are 20 different amino acids, which can be linked together in any order. Thus, for any protein made up *of 100 amino acids there can be a huge number of possible structures. But each protein has a specific order, and one protein can be distinguished from another by the order of its amino acids.*

Difficult as it was to "solve" the structure of DNA, protein structures are far more difficult to decipher – not only because proteins have more subunits, but because they have irregular shapes. DNA structures tend to be more regular, although variations can still occur. By contrast, each type of protein molecule has a different shape, which determines its function. This shape defines what other molecules will bind to the protein and what chemical activity will take place.

All proteins are constructed in accordance with instructions coded by the DNA inside our cells.

Nevertheless, we have only begun to understand how proteins function. The toughest problem has been to decipher their 3-D structure, which holds the key to their normal or abnormal activity.

Analyzing the structures of proteins and nucleic acids down to the precise arrangement of their atoms, and trying to understand the basis of each protein's activity, are major goals of structural biology. For a while these goals seemed very distant, but now the field is in a period of great excitement because of the new possibilities opened up by several recent developments:

- *Recombinant DNA technology is allowing scientists to obtain large quantities of specific proteins or nucleic acids for study. In the past, research was severely limited by the shortage of experimental material.*

- *Researchers can now determine the sequence of subunits of these substances – for example, the sequence of amino acids in specific proteins, or the sequence of nucleotides on specific stretches of DNA – without an extraordinary expenditure of time and effort.*

- *Major improvements in X-ray crystallography, nuclear magnetic resonance (NMR) spectroscopy, and other techniques are enabling researchers to decipher the 3-D structures of large molecules more precisely and rapidly.*

- *Faster and less expensive computers are giving scientists new powers to handle enormous quantities of data and – through computer graphics – to visualize the structures and movements of proteins.*

The coming together of these separate developments has made structural biology ready for a major leap forward. So promising is this work that many research laboratories in universities and drug companies are rushing into the field.

DNA and protein molecules are called "macro", or "large", in comparison with molecules of water, fats, nucleotides,or amino acids, which consist of *fewer that 50 atoms. A smaller protein molecule has about 1,000 atoms, while a more complex protein such as hemoglobin may have roughly 10,000 atoms. Some very large proteins contain as many as 100,000 atoms. Towering above them, an average molecule of human DNA has as many as 4.4 billion atoms.*

But in fact, even these molecules are still exceedingly small – so small that their images appear weak and fuzzy under the most powerful electron microscope. How can one determine the structure of anything so tiny?

The main sources of information have been the laws of chemistry that bear on the strength and geometry of the chemical bonds between atoms, and data from instruments developed by physicists. Some of the most useful data have come from X-ray crystallography, a technique based on the fact that the atoms in a crystal are arranged in a definite pattern that is repeated regularly in three dimensions.

As early as 1912, ***Max von Laue****, realized that the wavelengths of X-rays – 3,000 to 4,000 times shorter than those of visible light – were just about the same size as the spaces between atoms in a crystal. Therefore, he reasoned, X-rays that were passed through a crystal should be affected by the regularly spaced layers of atoms in just the same way as light waves are affected by the slats of a metal grid. Von Laue's idea proved correct: crystals scattered X-rays in specific patterns. But he mistakenly thought that some peculiarities in these patterns were the result of peculiarities in the X-rays.*

At about the same time, ***William Lawrence Bragg****, looked at the same patterns and concluded instead that their peculiarities were vital clues to the arrangement of atoms in the crystal. He realized that the orderly array of atoms scattered the X-rays in a orderly way, and that this caused a repeating series of overlapping circles of waves. When intercepted by a photographic plate, the peaks and troughs of these waves reinforced each other at some points and canceled each other out at other points, producing a pattern of spots of varying intensity – an interference pattern that could be translated into information about the structure that produced it.*

Bragg rapidly worked out equations with which scientists could get an image of the arrangement of atoms in a crystal from the X-rays diffracted by a particular axis of the crystal. If one keeps realigning the crystal so that a different axis of the crystal is parallel to the X-ray beam each time, one can make a set of photographs which, with the aid of Bragg's equations, allow one to deduce the 3-D arrangements of the molecules in the crystal.

Bragg soon tried out his technique on crystals of sodium chloride (table salt) and potassium chloride and succeeded in working out their atomic structures.

He thus invented X-ray crystallography and "solved" the first structures of crystals.

In order to do X-ray diffraction studies of a biological substance, one must get the molecules neatly aligned to form straight planes that can reflect the X-ray beams. This generally means that the substance must take the form of a crystal. But in some cases the fibers of substance will naturally be so neatly aligned that they behave as though they were well-ordered crystals. Fortunately, this is what happened in some of the first experiments with X-ray diffraction of DNA.

DNA is an extremely long, thin, and fragile thread that scientists have had great difficulty in extracting from cell nuclei in pieces that are large enough to study. While working with a highly viscous solution of DNA extracted from calf thymus cells in 1950, ***Maurice Wilkins*** *noticed that every time he touched this solution with the tip of a glass rod and then drew the rod away, he "had spun a very thin fibre of DNA, almost invisible, like a filament of spider web." The fibers seemed highly uniform, so Wilkins took them to a graduate student for X-ray crystallography. These first X-ray diffraction pictures of DNA offered many clues to its structure.*

Very little was then known about DNA's vital statistics – its diameter, length, density, chemical bonds, or the angle at which it twisted. About the only information one could rely on came from X-ray diffraction patterns. These showed the distance between the nucleotides in DNA and pointed to a large repeated structure of some kind. But the X-ray patterns did not reveal how many strands of nucleotides the DNA molecule contained, nor whether the backbone of the structure was on the inside or outside of the strands.

On the basis of these X-ray diffraction pictures, ***James Watson*** *and* ***Francis Crick*** *built models of what the DNA molecule might look like and then testing these models. Did every precisely scaled piece of the model fit what was known from the X-ray data? Did it obey standard rules about chemical bonds?*

Their first models failed the tests. But eventually – after learning more about DNA from some new X-ray pictures taken by ***Rosalind Franklin*** *at King's College – they devised a double helix whose basic structure repeated itself every 34 hundred-millionths of a centimeter (34 angstroms), exactly 10 times the distance between one nucleotide and the next. They put two backbones on the outside and had the nucleotides meet in the center.*

While shifting around cardboard cutouts of nucleotide bases (key component of nucleotides), Watson realized that one of these bases, adenine (A), could form two hydrogen bonds with another one, thymine (T). Moreover, the bond lengths were correct for the model. Another base, guanine (G), *could make similar hydrogen bonds with the fourth base, cytosine (C). When Watson compared the two cardboard pairs (A with T and G with C), they turned out to form nearly identical shapes that could fit snugly inside the backbones. This conformed with – and explained – a finding made 3 years earlier by* ***Erwin Chargaff*** *of Columbia University that number of A bases in DNA was equal to that of T bases, while the number of G bases was equal to that of C bases.*

From the way the bases attached to the backbones, which consisted of chains of sugars and phosphates, Crick then saw that the two chains must run in opposite directions. The bases could appear in any order on one strand of DNA, but this order determined the sequence of the complementary bases on the other strand. He also saw a built-in means of replication. Prior to the cell's duplication, the hydrogen bonds (connecting the base pairs) are broken, and the two chains unwind and separate. Each chain then acts as a template for the formation on to itself of a new companion chain, so that eventually we shall have two pairs of chains, where we only had one before. Moreover, the sequence of the pairs of bases will have been duplicated exactly.

The precise sequence of the bases is the code which carries the genetical information. This meant that the four bases were a kind of alphabet – a small set of "letters" with which an infinite number of instructions could be written.

The instructions contained in DNA sequences tell the cell how to manufacture the thousands of enzymes and other proteins on which life depends. But these instructions are not transmitted directly. The DNA remains safely in the nucleus, somewhat like the printing block in a printing press. Meanwhile a copy of one strand of DNA is made in the nucleus, leaves it, and directs the production of proteins in other parts of the cell. This "working copy" of the DNA is a nucleic acid called RNA.

The language in which the DNA's and RNA's instructions are transmitted – the genetic code – was deciphered by ***Marshall W. Nirenberg*** *and* ***H.G. Khorana****. The code is based on triplets of nucleotides, or "codons", which are read in sequence. Each codon specifies either one of 20 amino acids or a signal to start or stop constructing an amino acid chain. Each gene consists of a series of codons that contain the instructions for building a specific protein, which influences a specific trait, or that make the RNA used to carry out the DNA's instructions.*

For years scientists had thought that nucleotides occurred in regular, repeating sets. Watson and Crick's model made it clear that nucleotides occur in infinitely varied sequences. There are 3 billion pairs of nucleotides in the DNA of a typical human cell, and each person's genetic material has a unique nucleotide sequence, duplicated only in identical twins.

The sequence of nucleotides in a piece of DNA can now be determined through increasingly efficient techniques, some of which are being automated. Scientists know that these sequences contain the instructions for making proteins. They can "read" these instructions. They can also know how to determine the sequence of amino acids in a given protein. But to date nobody understands precisely how the sequence of amino acids leads to the remarkably complex and irregular three-dimensional structures of proteins.

IV. Biotechnology and Bioinformatics (1950–2008)

Biotechnology[23] *can be defined as the manipulation of organisms to do practical things and to provide useful products.*

Early cultures also understood the importance of using natural processes to breakdown waste products into inert forms. From very early nomadic tribes to pre-urban civilizations it was common knowledge that given enough time organic waste products would be absorbed and eventually integrated into the soil. It was not until the advent of modern microbiology and chemistry that this process was fully understood and attributed to bacteria.

The most practical use of biotechnology, which is still present today, is the cultivations of plants to produce food suitable to humans. Agriculture has been theorized to have become the dominant way of producing food since the Neolithic Revolution. The processes and methods of agriculture have been refined by other mechanical and biological sciences since its inception. Through early biotechnology farmers were able to select the best suited and high-yield crops to produce enough food to support a growing population.

Other uses of biotechnology were required as crops and fields became increasingly large and difficult to maintain. Specific organisms and organism byproducts were used to fertilize, restore nitrogen, and control pests. Throughout the use of agriculture farmers have inadvertently altered the genetics of their crops through introducing them to new environments, breeding them with other plants, and by using artificial selection. In modern times some plants are genetically modified to produce specific nutritional values or to be economical.

[23] The word "biotechnology" was first used by the agricultural engineer **Karl Ereky** (1919).

The process of Ethanol fermentation lead to one of the first forms of biotechnology. Cultures such as those in Mesopotamia, Egypt, and Iran developed the process of brewing which consisted of combining malted grains with specifics yeasts to produce alcoholic beverages. In this process the carbohydrates in the grains were broken down into alcohols such as Ethanol. Later other cultures produced the process of Lactic acid fermentation which allowed the fermentation and preservation of other forms of food. Fermentation was also used in this time period to produce leavened bread. Although the process of fermentation was not fully understood until ***Louis Pasteur's*** *work in 1857, it is still the first use of biotechnology to convert a food source into another form.*

Combinations of plants and other organisms were used as medications in many early civilizations. Since as early as 200 BC people began to use disabled or minute amounts of infectious agents to immunize themselves against infections. These and similar processes have been refined in modern medicine and have lead to many developments such as antibiotics, vaccines, and other methods of fighting sickness.

One can distinguish between:

- *Red biotechnology is applied to medical processes. Some examples are the designing of organisms to produce antibiotics, and the engineering of genetic cures to cure diseases through genomic manipulation.*

- *White biotechnology, also known as grey biotechnology, is biotechnology applied to industrial processes. An example is the designing of an organism to produce a useful chemical. White biotechnology tends to consume less in resources than traditional processes when used to produce industrial goods.*

- *Green biotechnology is biotechnology applied to agricultural processes. An example is the designing of transgenic plants to grow under specific environmental conditions or in the presence (or absence) of certain agricultural chemicals. One hope is that green biotechnology might produce more environmentally friendly solutions than traditional industrial agriculture. An example of this is the engineering of a plant to express a pesticide, thereby eliminating the need for external application of pesticides. Whether or not green biotechnology products such as this are ultimately more environmentally friendly is a topic of considerable debate.*

- *Blue biotechnology is used to describe the marine and aquatic applications of biotechnology.*

Traditional pharmaceutical drugs are small molecules that treat the symptoms of a disease or illness - one molecule directed at a single target. Biopharmaceuticals are large biological molecules known as proteins and these target the underlying mechanisms and pathways of a malady; it is a relatively young *industry. They can deal with targets in humans that are not accessible with traditional medicines. A patient typically is dosed with a small molecule* via *a tablet while a large molecule is typically injected.*

Small molecules are manufactured by chemistry but large molecules are created by living cells: for example, - bacteria cells, yeast cell, animal cells.

Modern biotechnology is often associated with the use of genetically altered microorganisms such as E. coli *or yeast for the production of substances like insulin or antibiotics. Genetically altered mammalian cells, such as Chinese Hamster Ovary (CHO) cells, are also widely used to manufacture pharmaceuticals. Another promising new biotechnology application is the development of plant-made pharmaceuticals.*

Biotechnology is also commonly associated with landmark breakthroughs in new medical therapies to treat diabetes, hepatitis B, hepatitis C, cancers, arthritis, haemophilia, bone fractures, multiple sclerosis. Cardiovascular as well as molecular diagnostic devices than can be used to define the patient population. Herceptin, is the first drug approved for use with a matching diagnostic test and is used to treat breast cancer in women.

A more recent field in biotechnology is that of genetic engineering. Genetic modification has opened up many new fields of biotechnology and allowed the modification of plants, animals, and even humans on a molecular level.

Informatics[24] *is defined as the study of the structure, behavior, and interactions of natural and artificial computational systems. It encompasses the study of systems that* represent, process *and* communicate *information, including all computational, cognitive and social aspects. The central notion is the transformation of information – whether by computation or communication, whether by organisms or artifacts. In this sense, informatics can be considered as encompassing computer science, cognitive science, artificial intelligence, information science and related fields, and as extending the scope of computer science to encompass computation in natural, as well as engineered, computational systems.*

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[24] In 1957 the **Karl Steinbuch** (1917–2008) published a paper called "Informatik: Automatisch Informationsverarbeitung" (i.e. "Informatics: automatic information processing").

The term was coined as a combination of "information" and "automation", to describe the science of automatic information processing. The morphology — *informat*-ion + -*ics* — uses "the accepted form for names of sciences, as conics, linguistics, optics, or matters of practice, as economics, politics, tactics", and so, linguistically, the meaning extends easily to encompass both the science of information and the practice of information processing.

Informatics includes the science of information the practice of information processing.

Informatics studies the structure, behavior, and interactions of natural and artificial systems that store, process and communicate information. It also develops its own conceptual and theoretical foundations. Since computers, individuals and organizations all process information, informatics has computational, cognitive and social aspects.

Used as a compound, in conjunction with the name of a discipline, as in medical informatics, bioinformatics, *etc., it denotes the specialization of informatics to the management and processing of data, information and knowledge in the named discipline.*

Informatics is broader in scope than: information theory *— the study of a particular mathematical concept of information;* information science *— a field primarily concerned with the collection, classification, manipulation, storage, retrieval and dissemination of information in human society; artificial intelligence — the study and engineering of intelligent behavior, learning, and adaptation, in machines; or* computer science *— the study of the storage, processing, and communication of information using engineered computing devices.*

Bioinformatics and computational biology involve the use of techniques including applied mathematics, informatics, statistics, computer science, artificial intelligence, chemistry, and biochemistry to solve biological problems usually on the molecular level. Research in computational biology often overlaps with systems biology. Major research efforts in the field include sequence alignment, gene finding, genome assembly, protein structure alignment, protein structure prediction, prediction of gene expression and protein-protein interactions, and the modeling of evolution.

The terms bioinformatics and computational biology are often used interchangeably. However bioinformatics more properly refers to the creation and advancement of algorithms, computational and statistical techniques, and theory to solve formal and practical problems inspired from the management and analysis of biological data.

Computational biology, on the other hand, refers to hypothesis-driven investigation of a specific biological problem using computers, carried out with experimental or simulated data, with the primary goal of discovery and the advancement of biological knowledge. Computational biology also includes lesser known but equally important subdisciplines such as computational biochemistry and computational biophysics.

A common thread in projects in bioinformatics and computational biology is the use of mathematical tools to extract useful information from data produced by high-throughput biological techniques such as genome sequencing.

Life at the Cellular Level (Cell Biology)

Cell biology *(also called cellular biology or cytology, from Greek kytos, "container") is an academic discipline that studies cells. This includes their physiological properties, their structure, the organelles they contain, interactions with their environment, their life cycle, division and death. This is done both on a microscopic and molecular level. Cell biology research extends to both the great diversity of single-celled organisms like bacteria and the many specialized cells in multicellular organisms like humans.*

Knowing the composition of cells and how cells work is fundamental to all of the biological sciences. Appreciating the similarities and differences between cell types is particularly important to the fields of cell and molecular biology. These fundamental similarities and differences provide a unifying theme, allowing the principles learned from studying one cell type to be extrapolated and generalized to other cell types. Research in cell biology is closely related to genetics, biochemistry, molecular biology and developmental biology.

Every cell typically contains hundreds of different kinds of macromolecules that function together to generate the behavior of the cell. Each type of protein is usually sent to a particular part of the cell. An important part of cell biology is the investigation of molecular mechanisms by which proteins are moved to different places inside cells or secreted from cells.

I. Circulatory fluid systems — Body's Internal Transport System

There are two main fluid systems in the body: blood and lymph. The blood and lymph systems are intertwined throughout the body and they are responsible for transporting the agents of the immune system.

The Blood System

The 5 liters of blood of a 70 kg (54 lb) person constitute about 7% of the body's total weight. The blood flows from the heart into arteries, then to capillaries, and returns to the heart through veins.

Blood is composed of 52–62% liquid plasma and 38–48% cells. The plasma is mostly water (91.5%) and acts as a solvent for transporting other materials (7% protein [consisting of albumins (54%), globulins (38%), fibrinogen (7%), and assorted other proteins (1%)] and 1.5% other stuff). Blood is slightly alkaline (pH $= 7.40 \pm .05$*) and somewhat heavier than water (density* $= 1.057 \pm .009$*).*

All blood cells are manufactured by stem cells, which live mainly in the bone marrow, via a process called hematopoiesis. The stem cells produce hemocytoblasts that mature into three types of blood cells: erythrocytes *(red blood cells or RBCs),* leukocytes *(white blood cells or WBCs), and* thrombocytes *(platelets).*

The leukocytes are further subdivided into granulocytes (containing large granules in the cytoplasm) and agranulocytes (without granules). The granulocytes consist of neutrophils (55–70%), eosinophils (1–3%), and basophils (0.5–1.0%). The agranulocytes are lymphocytes (consisting of B cells and T cells) and monocytes. Lymphocytes circulate in the blood and lymph systems, and make their home in the lymphoid organs.

There are 5000–10,000 WBCs per mm^3 *and they live 5–9 days. About 2,400,000 RBCs are produced each second and each lives for about 120 days (they are trapped by the spleen. Once there, that organ scavenges usable proteins from their carcasses). A healthy male has about 5 million RBCs per* mm^3*, whereas females have a bit fewer than 5 million.*

Normal Adult Blood Cell Counts are:

Red Blood Cells	$5.0 * 10^6/mm^3$	
Platelets	$2.5 * 10^5/mm^3$	
Leukocytes	$7.3 * 10^3/mm^3$	
Neutrophil		*50–70%*
Lymphocyte		*20–40%*
Monocyte		*1–6%*
Eosinophil		*1–3%*
Basophil		$< 1\%$

The proteins on RBCs are responsible for the usual ABO blood grouping, among other things. The grouping is characterized by the presence or absence of A and/or B antigens on the surface of the RBCs. Blood type AB means both antigens are present and type O means both antigens are absent. Type A blood has A antigens and type B blood has B antigens.

Some of the blood, but not red blood cells (RBCs), is pushed through the capillaries into the interstitial fluid.

HISTORY

As the most extensive visible fluid in the body, blood assumes major medical and symbolic significance. In many societies its shedding, both natural (as in menstruation) and deliberate, is unclean, impious and unlucky except in particular solemn and ritual circumstances. Blood is attributed responsibility not only for thought and sensation but also for life itself.

Greek medical writers confusingly considered it both one of the four humors and the fluid in which the humor blood predominated. It was the source of nutrient for many parts of the body, but any excess, plethora, either of it as humor or of one of its constituent humors was highly dangerous and might require phlebotomy (bloodletting).

The discovery of the circulation of the blood by **Harvey** *(1578–1657) established its primacy and many 18th-century authors attributed to it alone all the properties formerly associated with the other humors. From the end of the 18th century, physiological investigations concentrated more upon its constituent parts and assigned properties to them, and, although modern reliance on blood tests and transfusions has emphasized its role in diagnosis and therapy, it is now viewed primarily as a carrier and transmitter of other, more important, chemical substances around the body, e.g. hormones.*

It was traditionally observed that blood (one of the humors) was not a homogeneous substance, settling into a red clot and a colorless fluid ('plasma') separated by a thin white region non-clotting inside the living body often attributed to vitalism (e.g. by **John Hunter** *(1728–1793)).*

Microscopists like ***Jan Swammerdam*** *(1637–1680) and* ***M. Malpighi*** *(1628–1694) described red blood 'particles', the larger 'corpuscles' (leukocytes) being studied by Hunter's pupil* ***William Hewson*** *(1739–1774). The much-smaller platelets were described (1842) by* ***Alfred Donné*** *(1801–1878), their role in clot formation elucidated (1882) by* ***Giulio Bizzozero*** *(1846–1901).*

From the 1830s, the cell theory provided a sharper framework, ***J.H. Bennett*** *(1812–1875) and* ***Rudolf Virchow*** *(1821–1902) describing a pathological increase in leukocytes (leukemia), and* ***Thomas Addison*** *(1793–1860) observed pathological decrease in red blood cells (anemia).* ***Paul Ehrlich's*** *(1854–1915) staining techniques showed several different kinds of leukocytes, important in inflammation and defense mechanisms. The erythrocytes' function in respiration was uncovered from the 1850s, and the complicated cellular and chemical events in blood coagulation was studied by many, including* ***G. Hayem*** *(1841–1933) and* ***W.H. Howell*** *(1860–1945).*

The Lymph System

Lymph is an alkaline ($pH > 7.0$) fluid that is usually clear, transparent, and colorless. It flows in the lymphatic vessels and bathes tissues and organs in its protective covering. There are no RBCs in lymph and it has a lower protein content than blood. Like blood, it is slightly heavier than water (density $= 1.019 \pm .003$).

The lymph flows from the interstitial fluid through lymphatic vessels up to either the thoracic duct or right lymph duct, which terminate in the subclavian veins, where lymph is mixed into the blood. (The right lymph duct drains the right sides of the thorax, neck, and head, whereas the thoracic duct drains the rest of the body.)

Lymph carries lipids and lipid-soluble vitamins absorbed from the gastrointestinal (GI) tract. Since there is no active pump in the lymph system, there is no back-pressure produced. The lymphatic vessels, like veins, have one-way valves that prevent back-flow. Additionally, along these vessels there are small bean-shaped lymph nodes that serve as filters of the lymphatic fluid. It is in the lymph nodes where antigen is usually presented to the immune system.

The human lymphoid system has the following:

- *primary organs: bone marrow (in the hollow center of bones) and the thymus gland (located behind the breastbone above the heart), and*

- *secondary organs at or near possible portals of entry for pathogens: adenoids, tonsils, spleen (located at the upper left of the abdomen), lymph nodes* *(along the lymphatic vessels with concentrations in the neck, armpits, abdomen, and groin), Peyer's patches (within the intestines), and the appendix.*

History

Ancient writers (e.g. ***Herophilos****, fl 290 BCE) probably noted the lacteals (abdominal lymphatics) and* ***Gaspare Aselli*** *(1581–1625) definitely described them as ending in the liver.* ***Jean Pecquet*** *(1622–1674) noted the thoracic duct in 1647, its connection to the lacteals established (1652) by* ***Olaf Rudbeck*** *(1630–1702).* ***Francis Glisson*** *(c1597–1677) suggested they carried fluid lubricating the body cavities back to the blood vessels.*

Lymph glands' (nodes) were described by ***M.A. Severino*** *(1580–1656) and* ***Johann Peyer*** *(1653–1712), and associated with the lymphatic system by* ***Marcello Malpighi*** *(1628–1694). Claims for priority in establishing the absorbent functions of smaller lymph vessels (through injection experiments) were contested by* ***William Hunter*** *(1718–1783) and* ***Alexander Monro secundus*** *(1733–1817). The role of the lymphatic system in the body's defense mechanisms has been elucidated by modern immunology.*

II. The Immune System — Body's Internal Defense

Introduction

The immune system, our internal defense system, protects the body against disease-causing organisms and certain toxins. Disease-causing organisms, or pathogens, include certain viruses, bacteria, fungi, and protozoa. Pathogens enter the body with air, food, and water; during copulation; and through wounds in the skin. The immune system recognizes pathogens and toxins and responds to eliminate them. Derived from the Latin for "safe", the word immune *refers to the early observation that when a person recovered from smallpox and other serious infections, they were safe from contracting the same illnesses again. Immunology, the study of internal defense systems of humans and other animals, is one of the most rapidly changing, challenging, and exciting fields of biomedical research today.*

For more than 50 years, immunologists based their work on the hypothesis that internal defense depends on the animal's ability to distinguish between self *and* nonself*. Such recognition is possible because each individual is biochemically unique. Cells have surface proteins different from those on the cells* *of other species or even other members of the same species. An animal's immune system recognizes its own cells and can identify those of other organisms as foreign. Thus when a pathogen invades an animal, its distinctive macromolecules stimulate the animal's defensive responses. A single bacterium may have from 10 to more than 1000 distinct macromolecules on its surface.*

Immunologists are aware of several limitations of the self-nonself hypothesis. For example, the immune system does not typically respond to foreign molecules that are harmless. In 1994, the U.S. National Institutes of Health, proposed the danger model, which hypothesizes that the immune system does more than distinguish between self and nonself. It responds to danger signals from injured tissues, such as proteins released when cell membranes are damaged.

Most immunologists now agree that internal defense relies on a combination of factors, including the ability to identify foreign molecules and to respond to chemical clues from injured tissues. The immune system is a collection of many types of cells and of tissues scattered throughout the body. Immune responses require communication among cells, or cell signaling. Cells of the immune system communicate directly by means of their surface molecules and indirectly by releasing messenger molecules. Understanding the complex signaling systems of the immune system is a major focus of research.

Sometimes pathogens overcome the body's internal defenses, resulting in disease. Some diseases, as well as certain genetic mutations, prevent or compromise immune function. HIV, the retrovirus that causes AIDS, infects T cells, an important component of the immune system. The immune system may overfunction, as in allergic reactions, or it may respond in ways that are clinically important, such as in Rh incompatibility or the destruction of the cells of organ transplants. In about 5% of adults in highly developed countries, certain immune responses are directed against self tissues, resulting in autoimmune disease.

Among the greatest accomplishments of immunologists are the development of vaccines that prevent disease, and techniques for successful tissue and organ transplantations. Sophisticated research tools, such as gene transfer, have enabled immunologists to expand their knowledge of the cells and molecules that interact to generate immune responses, and to develop new approaches to the prevention and treatment of disease. Much has been learned, and many challenges lie ahead.

An immune response *is the process of recognizing foreign or dangerous macromolecules and responding to eliminate them. Two main types of immune responses protect the body: nonspecific and specific.* Nonspecific immune responses*, or* innate immunity*, provide general protection against pathogens, parasites, some toxins and drugs, and cancer cells.*

Nonspecific immune responses prevent most pathogens from entering the body and rapidly destroy those that do penetrate the outer defenses. For example, the cuticle or skin provides a physical barrier to pathogens that come in contact with an animal's body. Phagocytosis, another nonspecific defense, destroys bacteria that invade the body.

Some of the molecules important in nonspecific immune responses recognize and attack certain pathogen- associated molecular patterns, which are shared by whole groups of viruses, bacteria, or fungi.

Specific immune responses, also referred to as adaptive or acquired immunity, are highly specific. Any molecule that cells of the immune system specifically recognize as foreign is called an antigen. Proteins are the most powerful antigens, but some polysaccharides and lipids can be antigenic.

Antibodies are highly specific proteins that recognize and bind to specific antigens. Specific immune responses are directed toward particular antigens and typically include the production of antibodies. In complex animals, an important characteristic of specific immune responses is immunological memory, the capacity to respond more effectively the second time foreign molecules invade the body.

All invertebrates species that researchers have studied demonstrate the ability to distinguish between their own cells and those of other species. For example, sponge cells have specific glycoproteins on their surfaces that enable them to recognize their own species. When cells of two different species are mixed together, they reassort according to species. When two different species of sponges are forced to grow in contact with each other, tissue is destroyed along the region of contact. Cnidarians (such as corals), annelids (such as earthworms), arthropods (such as insects), and echinoderms (such as sea stars) reject tissue grafted from other animal, even from the same species.

Invertebrates have very efficient nonspecific immune mechanisms. For example, many invertebrates (cnidarians, annelids, and mollusks) are covered by mucus that traps and kills pathogens. Tough external skeletons, such as shells or cuticles, shield the body of many invertebrates. Most invertebrate coelomates (animals with a coelom) have amoeba-like phagocytes that engulf and destroy bacteria and other foreign matter. In mollusks, substances in the hemolymph (blood) enhance phagocytosis.

Researches have identified antimicrobial peptides in all eukaryotes (including plants) that have been studied, suggesting an early common origin of these molecules. More than 800 of these peptides that inactivate or kill pathogens have been described! When researches inject an antigen into an insect, as many as 15 antimicrobial peptides are produced within a few hours. Antimicrobial peptides are very effective because of their small size (a dozen or fewer amino acids), which facilitates their rapid production and diffusion.

What stimulates an animal to produce antimicrobial peptides and other immune defenses? Animal cells have receptors that recognize certain types of pathogen molecules, and then signal the cell to produce antimicrobial peptides.

One important family of these signaling receptors, the Toll group*, is a focus of current research. Immunologists first identified the Toll group in the fruit fly* Drosophila*. Toll receptors recognize some common molecular features of classes of pathogens called* pathogen-associated molecular patterns*, or PAMPs. Examples of PAMPs include the double-stranded RNA of certain viruses and peptidoglycan in Gram-positive bacteria.*

Certain cnidarians, arthropods, some echinoderms and simple chordates (such as tunicates) appear to remember antigens for a short period. As mentioned earlier, immunological memory enables the body to respond more effectively when it encounters the same pathogens again. Although certain invertebrates demonstrate some specificity and memory, their immune responses are primarily nonspecific.

A specialized lymphatic system, including lymphocytes*, (white blood cells specialized to carry out immune responses), evolved in the vertebrates. The lymphatic system performs the sophisticated specific immune responses of vertebrates.*

The Immune System: Cellular Aspects

A major component of the immune system is the class of white blood cells called lymphocytes*. Like all blood cells, they arise from common precursor cells (stem cells) in the bone marrow. Unlike other blood cells, however, they can leave the blood vessels and circulate in the lymphatic system. Lymphoid tissues such as lymph nodes, (as spleen, and, above all, the thymus gland) play important roles in the working of the immune system.*

Two kinds of lymphocytes can be distinguished: T cells and B cells. T cells *develop primarily in the thymus gland and* B cells *primarily in the bone marrow, accounting for their names. Much of the cellular aspect of immunity is the province of the T cells, whereas much of the molecular aspect depends on the activities of the B cells.*

T cells can have a number of functions. As T cells differentiate, each becomes specialized for one of the possible functions. The first of these possibilities, that of killer T cells*, involves surface receptors that recognize and bind to* antigens*, the foreign substances that trigger the immune response. The antigens are present to the T cell by other white blood cells called macrophages. The macrophages ingest and process antigens, and then present them to T cells.*

T cells that bind to a given antigen, and only to that antigen*, grow when these conditions are fulfilled. Note the specificity of which the immune system is capable. Many substances, including ones that do not exist in nature, can be antigens. The remarkable adaptability of the immune system in dealing with so many possible challenges is another of its main features. The process by which only those cells that respond to a given antigen grow in preference to other T cells is called clonal selection. The immune system can thus be versatile in its responses to the challenges it meets.*

As their name implies, killer T cells destroy antigen-infected cells. They do so by binding to them and by releasing a protein that perforates the plasma membrane of the infected cell. This aspect of the immune system is particularly effective in preventing the spread of viral infection by killing virus-infected host cells. In a situation such as this, the antigen can be considered to be all or part of the coat protein of the virus. When the infection subsides, some memory cells remain, conferring immunity against later attacks from the same source.

The Immune System: Molecular Aspects

Antibodies are Y-shaped molecules, consisting of two identical heavy chains and two identical light chains, held together by disulfide bonds. They are glycoproteins, with oligosaccharides linked to their heavy chains. Each light chain and each heavy chain has a constant region and a variable region. The variable region (also called the V domain) is found at the prongs of the Y and is the part of the antibody that binds to the antigen. The binding sites for the antibody on the antigen are called epitopes.

Most antigens have several such binding sites, so that the immune system will have several possible avenues of attack of naturally occurring antigens. Each antibody can bind to two antigens, and each antigen usually has several binding sites for antibody, giving rise to a precipitate that is the basis of experimental methods for immunological research. The constant region (the C domain) is located at the hinge and the stem of the Y; it is this part of the antibody that is recognized by phagocytes and by the complement system (the portion of the immune system that destroys antibody-bound antigen).

How does the body produce so many highly diverse antibodies to respond to essentially any possible antigen? The number of possible antibodies is virtually unlimited, as is the number of words in the English language. In a language, the letters of the alphabet can be arranged in countless ways to give a variety of words, and the same possibility for enormous numbers of rearrangements exists with the gene segments that code for portions of antibody chains.

Each B cell (and each progeny plasma cell) produces only one kind of antibody. In principle each such cell should be a source of a supply of homogeneous antibody by cloning. This is not possible in practice because lymphocytes do not grow continuously in culture. In the late 1970s ***Georges Köhler*** *and* ***César Milstein*** *developed a method to circumvent this problem, a feat for which they received the Nobel Prize in physiology or medicine in 1984.*

The technique requires fusing lymphocytes that make the desired antibody with mouse myeloma cells. The resulting hybridoma *(hybrid myeloma), like all cancer cells, can be cloned in culture and produces the desired antibody. Since the clones are the progeny of a single cell, they produce homogeneous* monoclonal antibodies. *In this way it is possible to produce antibodies to almost any antigen in quantity. Monoclonal antibodies can be used to assay for biological substances that can act as antigens. A striking example of their usefulness is in testing blood for the presence of HIV; this procedure has become routine to protect the public blood supply.*

We have been considering active immunity, *immunity that develops following exposure to antigens. Active immunity can be naturally or artificially induced. If someone with chickenpox sneezes near you and you contract the disease, you develop active immunity naturally.*

Active immunity can also be artificially induced by immunization, that is, by exposure to a vaccine. *When an effective vaccine is introduced into the body, the immune system actively develops clones of cells, produces antibodies, and develops memory cells.*

The first vaccine was prepared in 1796 by British physician ***Edward Jenner*** *against vaccinia, the cowpox virus. The term* vaccination *was thus derived. Jenner's vaccine provided humans with immunity against the deadly disease smallpox. Jenner had no knowledge of microorganisms or of immunology, and it remained for French chemist* ***Louis Pasteur*** *to begin to develop scientific methods for preparing vaccines 100 years later. Pasteur showed that inoculations with preparations of attenuated (weakened) pathogens could be used to develop immunity against the virulent (infectious) form of the pathogen.*

However, not until 20th century advances in immunology – for example, Burnet's clonal selection theory in 1957 and the discovery of T and B cells in 1965 – did scientists gain a modern understanding of vaccines. Effective vaccination stimulates the body to launch an immune response against the antigens contained in the vaccine. Memory cells develop, and future encounters with the same pathogen are dealt with rapidly.

Microbiologists prepare effective vaccines in a number of ways. They can attenuate a pathogen so it loses its ability to cause disease. When *pathogens are cultured for long periods in non-human cells, mutations adapt the pathogen to the nonhuman host so that they no longer cause disease in humans. This is how the* ***Sabin*** *polio vaccine and the measles vaccine are produced.*

Whooping cough and typhoid fever vaccines are made from killed pathogens that still have the necessary antigens to stimulate an immune response. Tetanus and botulism vaccines are made from toxins secreted by the respective pathogens. The toxin is altered so that it can no longer destroy tissues, but its antigenic determinants are still intact.

Most vaccines consist of the entire pathogen, attenuated or killed, or of a protein from the pathogen. Researches are investigating several approaches that would reduce potential side effects. For example, they are developing DNA vaccines (or RNA vaccines), made from a part of the pathogen's genetic material. The DNA of the pathogen is altered so that it transfers genes that specify antigens. When injected into a patient, the altered DNA is taken up by cells and makes its way to the nucleus. The encoded antigens are manufactured and stimulate both cell-mediated and antibody-mediated immunity. Several DNA vaccines, including vaccines to prevent and treat HIV infections, are in clinical trials.

Specific Immune Responses

An antibody molecule, also called immunoglobulin (Ig), has two main functions: It combines with antigen, and it activates processes that destroy the antigen that binds to it. For example, an antibody may stimulate phagocytosis. Note that an antibody does not destroy an antigen directly; rather, it labels the antigen for destruction.

The basic structure of the immunoglobulin molecule was clarified by ***Rodney Porter****, of the University of Oxford in England, and* ***Gerald Edelman****, of Rockefeller University in New York, during the 1960s. Porter used the plant enzyme papain, a protease, to split Ig into fragments. Based on his findings, Porter developed a working model of the structure of the Ig molecule and was the first to suggest that it is Y-shaped. These researchers then constructed an accurate model of the antibody molecule. Porter and Edelman won the 1972 Nobel Prize in Medicine for their contributions.*

Two fragments of the antibody molecule bind antigen and are referred to as Fab fragments (Fab stands for antigen-binding fragment). The fragment that interacts with cells of the immune system is the Fc fragment (c indicates that this fragment crystallizes during cold storage). Many cells of the immune system have Fc receptors.

A typical antibody is a Y-shaped molecule in which the two arms of the Y (the Fab portions) bind with antigen. This shape enables the antibody to combine with two antigen molecules and allows the formation of antigen-antibody complexes. While the arms of the Y bind to antigen, the tail of the Y, the Fc portion, interacts with cells of the immune system, such as phagocytes, or binds with molecules of the complement system.

The antibody molecule consists of four polypeptide chains: two identical long chains called heavy chains, and two identical short chains called light chains. Each chain has a constant region and a variable region. In the constant (C) region, of the heavy chains, the amino acid sequence is constant within a particular immunoglobulin class. One can think of the C region as the handle portion of a door key. Like the pattern of bumps and notches at the part of a key that slides into a lock, the variable (V) region has a unique amino acid sequence. The variable region of the immunoglobulin extends outward from the B cell, whereas the constant region anchors the molecule to the B cell.

At its variable regions, the antibody folds three-dimensionally, assuming a shape that enables it to combine with a specific antigen. When they meet, antigen and antibody fit together somewhat like a lock and key. They must fit in just the right way for the antibody to be effective. A given antibody can bind with different strengths, or affinities, to different antigens. In the course of an immune response, higher-affinity antibodies are generated.

In an antigen, specific sequences of amino acids make up an antigenic determinants, or epitope. These sequences give part of the antigen molecule a specific shape that is recognized by an antibody or T cell receptor. Usually, an antigen has many different antigenic determinants on its surface; some have hundreds.

How can the immune system recognize every possible antigen? In 1956, the clonal selection hypothesis was developed which proposes that before a lymphocyte ever encounters an antigen, the lymphocyte has specific receptors for that antigen on its surface. When an antigen bins to a matching receptor in a lymphocyte, it activates the lymphocyte, which then gives rise to a clone of cells with identical receptors. A major problem with this hypothesis was its suggestion that our cells must contain millions of separate antibody genes. Since each human cell has a large amount of DNA, it is not enough to provide a different gene to code for each of the millions of possible specific antibody molecules.

In 1976, immunologists demonstrated that three separate families of genes code for immunoglobulins and that each gene family contains a large number of DNA segments that code for V regions. Recombination of these DNA segments during the differentiation of B cells is responsible for antibody diversity.

Note that rearrangement of these DNA segment produces an enormous number of potential combinations! Millions of different type of B (and T) cells are produced. By chance, one of those cells may produce just the right antibody to destroy the pathogen that invades the body.

Before 1975, the only method for obtaining antibodies for medicine and research was immunizing animals and collecting their blood. Then, immunologists developed monoclonal antibodies – identical antibodies produced by cells cloned from a single cell.

There are three memory systems in the human body:

- *Central nervous system memory, located in the brain.*
- *Immunological memory, located in nonlymphatic tissues including the lung, liver, kidney, and intestine.*
- *Genetic memory.*

Immunological memory manifest itself in "memory B cells" that continue to live and produce small amounts of antibody long after the body has overcome an infection. If the same pathogen enters the body again, the antibody immediately targets it for destruction. At the same time, specific memory cells are stimulated to divide, producing new clones of plasma cells that produce the same antibody.

III. The Nervous System — Body's Communication Network

I. Introduction

Neurobiology is the study of cells of the nervous system and the organization of these cells into functional circuits that process information and mediate behavior. It is a subdiscipline of both biology and neuroscience. Neurobiology differs from neuroscience, a much broader field that is concerned with any scientific study of the nervous system. Neurobiology should also not be confused with other subdisciplines of neuroscience such as computational neuroscience, cognitive neuroscience, behavioral neuroscience, biological psychiatry, neurology, and neuropsychology despite the overlap with these subdisciplines.

Neuroscience is an interdisciplinary program of study that provides an interconnectedness of the sciences and requires the synthesis and integration of different focal areas of investigation such as, neuroanatomy, neuroimaging, neuropsychopharmacology, neurophysiology, molecular neurobiology, neuroendocrinology, and how brain structures function to regulate behavior.

The integrative nature of neuroscience requires the tools provided by experience and training in general biology, genetics, physiology, molecular biology, chemistry (general, organic and biochemistry), physics, psychology (behavior, memory, cognition, sensation & perception) and research design and analysis.

More information has been discovered about the brain in the last ten years of investigative study than has ever been known before, in part, due to the fact that the 1990s were proclaimed as the "Decade of the Brain." Neuroscience is one of the most exciting and rapidly expanding fields of study that is only limited by the boundaries of the advancements in technology.

Below, are listed some of the most notable discoveries and achievements in Neuroscience in the 20^{th} *century.*

Neuroscience subfields: Neurobiology, Cognitive Neuroscience, Computational Neuroscience, Neural Engineering, Neuroanatomy, Neurochemistry, Neuroimaging, Neurolinguistics, Neurology, Neuropharmacology, Neurophysiology, Neuropsychology, Psychopharmacology, Systems Neuroscience.

II. Neural signaling

An organism's ability to survive and to maintain homeostasis[25] *depends largely on how effectively it detects and responds to stimuli — changes in the environment. Stimuli within the body include internal signals such as hunger or lowered blood pressure. Stimuli from the outside world include changes in temperature, or light, an odor, or movement that may indicate the presence of a predator or of prey.*

In all animals except the sponges, responses to stimuli depend on cell signaling by networks of nerve cells, or neurons. These cells are specialized for transmitting propagating and receiving electrochemical impulses, electrical signals and chemical messages. In the human brain alone, there are over 10^{11} *(a hundred billion) neurons.*

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[25] The capacity of an organism to maintain internal stability of equilibrium. Coined (1926) by **W.B. Cannon** (1871–1945). Much modern physiological work on homeostasis concerns itself with *bio-feedback* mechanisms which are controlled by the autonomic nervous system.

In most animals, neurons and supporting cells are organized as a nervous system *that, like a computer takes in information, integrates it, and responds. Just how animals respond to stimuli depends on how their neurons are organized and connected to one another. A single neuron in the vertebrate brain may be functionally connected to thousands of other neurons. The endocrine system works with the nervous system to regulate many behaviors and physiological processes. The endocrine system generally provides relatively slow long-lasting regulation, whereas the nervous system typically permits more rapid, but brief, responses.*

Neurobiology is one of the most exciting areas of biological research. Many investigations are studying neurotransmitters*, the chemical messengers used by neurons to signal other neurons, and the* receptor *that bind with the neurotransmitters.*

Another active area of research is the role of glial cells *in the nervous system. These cells support and protect the neurons and have many regulatory functions. The glial cells provide glucose for neurons and also help regulate the composition of the extracellular fluid in the brain and spinal cord. Neurobiologists recently demonstrated that astrocytes induce and stabilize synapses (connections between neurons) in the brain. Although astrocytes can generate weak electrical signals, they communicate with one another and with neurons mainly with chemical signals.*

A nerve consist of many parallel, independent signal paths, each of which is a nerve cell or fiber. Each cell is capable of transmitting signals in only one direction; separate cells carry signals to or from the brain. Each cell has an input end, a long conducting portion or axon, and an output end. It is these ends that give the cell its unidirectional character. The input end may be a transducer (stretch receptor, temperature receptor, etc.) or a junction (synapse) with another cell. A threshold mechanism is built into the input end; when an input signal exceeding a certain level is received, the nerve fires and an impulse of fixed size and duration travels down the axon. There may be several inputs at the synapse which may either aid or inhibit each other, depending on the nature of the synapse.

The axon *is a long tail on the nerve cell that transmits the impulse without change of shape. It may be more than a meter in length, extending in the human, for example, from the brain to low in spinal cord or from the spinal cord to a finger or toe. Bundles of axons constitute what we usually think of as a nerve. The output end of the axon branches out in fine nerve endings, which appear to be separated by a gap from the next nerve or muscle cell which they drive.*

The long, cylindrical axon has properties similar to those of an electric cable. Its diameter may range from less than a micron to 500 μm for the giant *axon of a squid; in humans, the upper limit is about 20 μm. Pulses travel along it with speeds ranging from 0.6 ms^{-1} to 100 ms^{-1}, depending, among other things, on the diameter of the axon. The axon core may be surrounded by either a membrane (for an* unmyelinated *fiber) or a much thicker sheath of fatty material (*myelin*), wound on like electrical tape. A myelinated fiber has its sheath interrupted at intervals and replaced by a short segment of membrane similar to that on an unmyelinated fiber. These interruptions are called the* nodes of Ranvier.

The axon may be removed from the rest of its cells and still will conduct nerve impulses. Its conduction properties depend on the membrane; the interior fluid (axoplasm) has been squeezed out of squid giant axons and replaced by an electrolyte solution without altering appreciably the propagation of the impulses. The axoplasm does contain chemicals essential to the long-term metabolic requirements of the cell.

Most animal cells have a difference in electrical charge across the plasma membrane — a more negative electrical charge inside the cell compared with the electrical charge of the extracellular fluid outside. The plasma membrane is said to be electrically polarized*, meaning that one side, or pole, has a different charge from the other side. When electrical charges are separated in this way, a potential energy difference exists across the membrane. This difference in electrical charge across the plasma membrane gives rise to an electrical voltage gradient.*

The voltage measured across the plasma membrane is called the membrane potential. *If the charges are permitted to come together, they have the ability to do work. Thus the cell can be thought of as a biological battery. In excitable cells, such as neurons and muscle cells, the membrane can transmit signals to other cells.*

The membrane potential in a resting (not excited) neuron or muscle cell is its resting potential. *The resting potential is generally expressed in units called* millivolts *(mV). (A millivolt equals one thousandth of a volt.) Voltage is the force that causes charged particles to flow between two points. Like other cells that can produce electrical signals, the neuron has a resting potential of about 70 mV. By convention this is expressed as -70 mV because the cytosol close to the plasma membrane is negatively charged relative to the extracellular fluid.*

Biologists cam measure the potential across the membrane by placing one electrode inside the cell and a second electrode outside the cell, and connecting through a very sensitive voltmeter or oscilloscope. If one places both electrodes on the outside surface of the neuron, no potential difference between them is registered. (All points on the same side of the membrane have *the same charge.) However, once one of the electrodes penetrates the cell, the voltage changes from zero to approximately* $-70mV$.

Two main factors determine the magnitude of the membrane potential: (1) differences in the concentrations of specific ions inside the cell compared with the extracellular fluid, and (2) selective permeability of the plasma membrane to these ions. The distribution of ions inside neurons and in the extracellular fluid surrounding them is like that of most other cells in the body. The potassium ion (K^+) *concentration is about 10 times greater inside than outside the cell. In contrast, the sodium ion* (Na^+) *concentration is about 10 times greater outside than inside. This asymmetric distribution of ions across the plasma membrane at rest is brought about by the action of selective ion channels and ion pumps. In vertebrate neurons (and skeletal muscle fibers), the resting membrane potential depends mainly on the diffusion of ions down their concentration gradients.*

Ions cross the plasma membrane by diffusion through ion channels that are formed by membrane proteins. Net movement of ions occurs from an area of higher concentration of that ion to one of lower concentration. Typically, these channels allow only specific types of ions to pass.

Neurons have three types of ion channels: passive ion channels, voltage-activated channels, *and* chemically activated ion channels. *Passive ion channels permit the passage of specific ions such as* Na^+, K^+, Cl^-, *and* Ca^{2+}. *Unlike voltage-activated and chemically activated ion channels, passive ion channels are not controlled by gates.*

Potassium channels are the most common type of passive ion channel in the plasma membrane, and cells are more permeable to potassium than to other ions. In fact, in the resting neuron, the plasma membrane is up to 100 times more permeable to K^+ *than to* Na^+. *Sodium ion pumped out of the neuron cannot easily pass back into the cell, but* K^+ *pumped into the neuron easily diffuse out.*

Potassium ions leak out through passive ion channels following their concentration gradient. As these positively charged ions diffuse out of the neuron, they increase the positive charge in the extracellular fluid outside the cell relative to the charge inside the cell. The resulting change in the electrical gradient across the membrane influences the flow of ions. This electrical gradient forces some of the positively charged potassium ion back into the cell.

At the input end of a nerve cell, the response to chemicals from a synapse is often an increase in membrane permeability to sodium ions, which causes an increase in the interior potential. In other cases the interior potential becomes more negative and firing is inhibited. If the potential becomes high *enough (that is, more positive or less negative), the regenerative action of the membrane takes over and the cell initiates an impulse.*

If the input end of the cell acts as a transducer, the interior potential rises as the cell is stimulated. If the input is from another nerve, the signal may cause the nerve to fire, or it may cause the potential to increase by a subthreshold amount so that two or more stimuli must be received simultaneously to cause firing, or it may decrease the potential and inhibit stimulation by another nerve at the synapse. Comparison of the axoplasm with the interstitial fluid surrounding each axon shows an excess of potassium and a deficit of sodium and chloride ions within the axon.

A typical axon might have a radius of 5 μm = 5000 nm If the axon is not myelinated, the thickness of the cell membrane might be 5–10 nm; a myelin layer might be 2000 nm thick, with nodes of Ranvier spaced every 1–2 mm.

The inside of the cell has an electrical potential about 70 mV less than outside the cell.

As the pulse passes by, the potential at a fixed point on the axon rises in a millisecond or less to about +40 mV. Then it falls back to about –90 mV and finally recovers slowly to the resting value of –70 mV. The membrane is said to depolarize and then repolarize. The regenerative action that produces these sudden changes of membrane potential is caused by changing permeability of the membrane to sodium and potassium ions.

At the end of a nerve cell the signal passes to another nerve cell or to a muscle cell across a synapse or junction. There are gaps of 10–20 nm between presynaptic and postsynaptic nerve cells and gaps of 50–100 nm of the neuromuscular junction. In some cases such as the heart, the transmission may be electrical; yet in many cases the signal is carried by chemicals.

At most vertebrate neuromuscular junctions, the nerve impulse is followed by an electrical impulse which propagates throughout the muscle fiber and initiates contraction. There is good experimental evidence that acetylcholine is released by the nerve endings when the nerve fires. It increases the permeability of the nearby muscle membrane to sodium, which then leaks in and depolarizes the membrane.

III. Mathematical Models

Neuron physiology describes the electrical properties of nerve cell membrane. Models of nerves are based on the Nernst equation *(**W.H. Nernst**, 1864–1941) that determines a cell membrane's potential from the ion concentrations near it. A neuron consists of dendrites that receive signals, a cell body that synthesized incoming signals and generates new ones, an axon that* *transmits new signals away from the cell body, and a synapse that transmits the signals to other cells. The structure and function of synapses play important roles in this study. Neurotransmitters (chemical molecules) released at synapses in response to changes in membrane voltage communicate these changes to the neuron's environment.*

Attempts to describe a nerve cell's electrical behavior have been based on electrical circuit analogies and their mathematical models.

The Hodgkin-Huxley model of nerve membrane (1952), *was based on observations of neural signals propagating in the nerve axon of a giant squid. While it does not accurately describe all aspects of membrane behavior, it has been instrumental in suggesting and promoting understanding of a variety of important experiments.*

An electrical potential is established across a cell's membrane by having different concentrations of electrically charged chemical species inside and outside of a semipermeable membrane separates two regions of space that have concentrations of ions, *e.g.* C_1 *and* C_2, *inside and outside, respectively, then Nernst equation states that the resulting potential* E *is given by* $E = (\frac{RT}{q})\log(\frac{C_1}{C_2})$, *when* R *is a gas constant,* T *is the absolute temperature. This shows how to compute the membrane potential once the ion concentrations inside and outside are known.*

The ions most important to the cell membrane potential are sodium (Na^+) *and potassium* (K^+). *Each ionic species has associated with it a membrane potential that is maintained by a pump in the equilibrium, the Nernst equation shows that*

$$\begin{aligned} E_{Na} &= (\frac{kT}{q})\log(\frac{C_0^{Na}}{C_i^{Na}}) = 55mV, \\ E_K &= (\frac{kT}{q})\log(\frac{C_0^{K}}{C_i^{K}}) = -75mV. \end{aligned}$$

These are referred to as the sodium and potassium resting potentials.

The main parts of interest in the neuron are the dendrites, *which receive signals from impinging axons, the* cell body *that can generate electrical activity, usually in the from of a voltage pulse called an* action potential, *the* axon, *which carries an action potential from the cell body to the synapse, and the* synapse, *which causes chemical signals to be released outside the cell in response to the arrival of an action potential.*

Chemical signals are molecules called neurotransmitters. These diffuse across the synaptic gap to a dendrite and cause an electrical potential to be created across the dendrite's membrane. Eventually, this either causes the cell body to fire (i.e., creates an action potential) or inhibits it from *firing. Neuron membranes at rest are impermeable to* (Na^+)*. Therefore, the observed potential is near* E_K*. When excited,* (Na^+) *channels open rapidly and the membrane potential approaches* E_{Na}*. The* (K^+) *channels opens more slowly, but eventually returns the membrane potentials to near* E_K*.*

The mechanisms that control these channels remain unknown, although many models of membrane potentials and currents have been derived and used effectively to suggest experiments and to interpret data.

Action potentials arriving at a synapse, called presynaptic potentials, cause vesicles containing chemical neurotransmitters to migrate to the synapse membrane and release their contents into the synaptic gap.

The neurotransmitters diffuse across the synaptic gap, though some are lost from the gap. The molecules that arrive at the postsynaptic membrane interact with it to modify its membrane potential. If $c(t)$ *denotes the concentration of neurotransmitter in the gap at time* t*, this increases in response to the arrival of action potential and decreases because of chemical binding with the postsynaptic membrane and diffusion out of the gap. These chemical kinetics are modeled by the equation*

$$\mathrm{d}c/\mathrm{d}t = -k_{dif}\, c - k_{post}\, c + S$$

Basically the axon is a long cylindrical tube which extends from each neuron and electrical signals propagate along its membrane, about 50–70 Ångströms thick. The electrical pulses arise because the membrane is preferentially permeable to various chemical ions with the permeabilities affected by the currents and potentials present. The key elements in the system are potassium (K^+) *ions and sodium* (Na^+) *ions. In the rest state there is a transmembrane potential difference of about* -70 *millivolts (mV) due to the higher concentration of* (K^+) *ions within the axon as compared with the surrounding medium.*

The deviation in the potential across the membrane, measured from the rest state, is a primary observable in experiments. The membrane permeability properties change when subjected to a stimulating electrical current I *: they also depend on the potential. Such a current can be generated, for example, by a local depolarization relative to the rest state.*

Let us take the positive direction for the membrane current, denoted by I*, to be outwards from the axon. The current* $I(t)$ *is made up of the current due to the individual ions which pass through the membrane and the contribution from the time variation in the transmembrane potential, that is the membrane capacitance contribution. Thus we have*

$$I(t) = C\frac{dV}{dt} + I_i, \tag{1}$$

where C is the capacitance and I_i is the current contribution from the ion movement across the membrane. Based on experimental observation ***Hodgkin*** *and* ***Huxley*** *(1952) took*

$$\begin{aligned} I_i &= I_{Na} + I_K + I_L, \\ &= g_{Na} m^3 h (V - V_{Na}) + g_K n^4 (V - V_K) + g_L (V - V_L), \end{aligned} \tag{2}$$

where V is the potential and I_{Na}, I_K and I_L are respectively the sodium, potassium and "leakage" currents: I_L is the contribution from all the other ions which contribute to the current. The g's are constant conductances with, for example, $g_{Na} m^3 h$ the sodium conductance, and V_{Na}, V_K and V_K are constant equilibrium potentials. The m, n and h are variables, which are determined by the differential equations

$$\begin{aligned} \frac{dm}{dt} &= \alpha_m(V)(1-m) - \beta_m(V)m, \\ \frac{dn}{dt} &= \alpha_n(V)(1-n) - \beta_n(V)n, \\ \frac{dh}{dt} &= \alpha_h(V)(1-h) - \beta_h(V)h, \end{aligned} \tag{3}$$

where the α and β are given functions of V (again empirically determined by fitting the results to the data). α_n and α_m are qualitatively like $(1+\tanh V)/2$ while $\alpha_h(V)$ is qualitatively like $(1-\tanh V)/2$, which is a "turn-off" switch if V is moderately large.

If an applied current $I_a(T)$ is imposed the governing equation, using (1), becomes

$$C\frac{dV}{dt} = -g_{Na} m^3 h (V - V_{Na}) - g_K n^4 (V - V_K) - g_L (V - V_L) + I_a \tag{4}$$

The system (4) with (3) constitute the 4-variable model which was solved numerically by ***Hodgkin*** *and* ***Huxley*** *(1952).*

If $I_a = 0$, the rest state of the model (3) and (4) is linearly stable but is excitable. That is, if the perturbation from the steady state is sufficiently large there is a large excursion of the variables in their phase space before returning to the steady state. If $I_a \neq 0$ there is a range of values where regular repetitive firing occurs; that is – the mechanism displays limit cycle characteristics. Both types of phenomena have been observed experimentally.

*Because of the complexity of the equation system, various simpler mathematical models, which capture the key features of the full system, have been proposed, the best known and particularly useful one is the FitzHugh-Nagumo model (**FitzHugh** 1961, **Nagumo** et al. 1962).*

The time scales for m, n and h in (3) are not all of the same order. The time scale for m is much faster than the others, so it is reasonable to assume it is sufficiently fast such that it relaxes immediately to its value determined by setting $dm/dt = 0$ in (3). If we also set $h = h_0$, a constant, the system still retains many of the features experimentally observed. The resulting 2-variable model in V and n can then be qualitatively approximated by the dimensionless system

$$\frac{du}{dt} = f(u) - \omega + I_a, \qquad \frac{dv}{dt} = bu - \gamma v, \tag{5}$$

$$f(u) = v(a-u)(u-1), \tag{6}$$

where $0 < a < 1$ and b and γ are positive constants. Here u is like the membrane potential V, and v plays the role of all three variables m, n and h in (3).

It can be shown that this system of equations governs the existence of traveling pulses which only propagate if a certain threshold perturbation is exceeded. By a pulse here, we mean a wave which represent an excursion from a steady state and back to it – like a solitary wave on water.

If in (5) we assume $I_a = 0$ (no applied current) and allow spatial diffusion *in the transmembrane potential*

$$\frac{du}{dt} = f(u) - v + D\frac{\partial^2 u}{\partial x^2}, \qquad \frac{dv}{dt} = bu - \gamma v, \tag{7}$$

$$f(u) = u(a-u)(u-1), \tag{8}$$

Here, u is directly related to the membrane potential V and v plays the role of several variables associated with terms in the contribution to the membrane current from sodium, potassium and other ions. The "diffusion" coefficient D is associated with the axial current in the axon. The parameters $0 < a < 1$, b and γ are all positive.

IV. The Endocrine System — Body's Messengers and Regulators.

Introduction

A caterpillar becomes a butterfly. A crustacean changes color to blend with its background. A young girl develops into a woman. An adult copes with chronic stress. These physiological processes and many other adjustments of metabolism, fluid balance, growth and development, and reproduction are regulated by the endocrine system. *This system works closely with nervous system to maintain homeostasis, the steady state of the body.*

The endocrine system is a diverse collection of cells, tissues, and organs, including specialized endocrine glands *that produce and secrete* hormones, *chemical messengers responsible for the regulation of many body processes. Hormones excite, or stimulate, changes in specific tissues.*

Endocrine glands have no ducts; they secrete hormones directly into the interstitial fluid or blood. Hormones are typically transported by the blood and produce a characteristic response only after they reach target cells and bind with specific receptors. Target cells, the cells influenced by a particular hormone, may be another endocrine gland or in an entirely different type of organ, such as a bone or the kidney. Target cells may be located far from the endocrine gland. For example, the vertebrate thyroid gland secretes hormones that stimulate metabolism in tissues throughout the body. Several types of hormones may be involved in regulating the metabolic activities of a particular type of cell. In fact, many hormones produce a synergistic effect in which the presence of one hormone enhances the effects of another.

Endocrinologists extract the suspected compound from the endocrine tissue of one animal and inject it into an experimental animal from which the tissue producing the compound has been removed. Deficiency symptoms should be relieved by replacing the suspected hormone. Researchers then isolate the active compound and determine its chemical structure. Finally, the compound is synthesized in the laboratory and injected into experimental animals. If its effects are those predicted, the researchers have data to support their hypothesis.

Using such procedures, endocrinologists have identified about 10 discrete endocrine glands. More recently investigators have identified specialized cells in the digestive tract, heart. kidneys, and many other parts of the body that also release hormones. In addition, some neurons release hormones. As a result of these discoveries, the scope of endocrinology has been broadened to include the production and actions of chemical messengers produced by a wide variety of organs, tissues, and cells.

One active focus of research is the study of the mechanisms of hormone action, which includes characterizing receptors and identifying the molecules involved in signal transduction. *Some endocrinologists now use a reverse strategy for discovering new hormones and signaling pathways within the cell. They focus on "orphan" nuclear receptors, those for which ligands (the molecules that bind with them) are not yet known. Some of these "orphan" receptors are receptors for hormones that have not yet been identified. Using this strategy, researchers have identified intracellular signaling pathways for steroids, fatty acids, and several other compounds.*

Cell signal one another with neurotransmitters, hormones, and local regulators. Some chemical compounds function as all three of these types of signals. Thus a neuron, endocrine gland, or some other cell type all may secrete the same chemical message. However, the same message can have different meanings for various target cells.

Pheromones *are chemicals messengers that animals produce for communication with other animals of the same species. Because pheromones are generally produced by exocrine glands and do not regulate metabolic activities within the animal that produces them, most biologists do not classify them as hormones.*

Endocrine glands differ from exocrine glands *(such as sweat glands and gastric glands) that release their secretions into ducts. Endocrine glands have no ducts, and secrete their hormones into the surrounding interstitial fluid or blood. Typically, hormones diffuse into capillaries and are transported by the blood to target cells. biologists have discovered that in addition to the discrete classical endocrine glands, specialized cells in many tissues and organs (such as kidneys and heart) also release hormones or hormone-like substances.*

The complexity of animal physiology challenges simplistic definitions. As new chemical signals and their modes of action have been discovered, the traditional definition of a hormone as a substance secreted by an endocrine gland and transported by the blood has become inadequate.

Certain neurons, known as neuroendocrine cells, *are an important link between the nervous and endocrine systems. Neuroendocrine cells produce* neurohormones *that are transported down axons and released into the interstitial fluid. they typically diffuse into capillaries and are transported by the blood. invertebrate endocrine systems are largely neuroendocrine. In vertebrates, the hypothalamus produces several neurohormones that link the nervous system with the pituitary gland, an endocrine gland that secretes several hormones.*

A local regulator *is a signaling molecule that diffuses through the interstitial fluid and acts on nearby cells. Certain chemical compounds that are* *indisputably hormones because they are typically transported by the blood, under some conditions acts as local regulators. In* autocrine regulation*, a hormone, or other regulator, acts on the very cells that produce it. For example, the female hormone estrogen, which functions as a classical hormone, may also exert an autocrine effect that stimulates additional estrogen secretion. Estrogen can also act on nearby cells, a type of local regulation known as* paracrine regulation.

Local regulators include local chemical mediators such as histamine, growth factors, and prostaglandins.

Histamine *is stored in mast cells and is released in response to allergic reactions, injury, or infection. Histamine causes blood vessels to dilate and capillaries to become more permeable. More than 50* growth factors*, typically peptides, stimulate division and normal development in specific types of cells. Growth factors have autocrine or paracrine effects.* Nitric oxide *(NO), another local regulator, is produced by many types of cells, including those lining blood vessels. It relaxes nearby smooth muscle fibers, dilating the blood vessel.*

Prostaglandins *are modified fatty acids released continuously by the cells of most tissues. Biologists have grouped them into nine different classes. Although present in very small quantities, these local regulators affect a wide range of body processes. Prostaglandins are paracrine regulators that act on cells in their immediate vicinity. They modify cyclic adenosine monophosphate (cAMP) levels and interact with other hormones to regulate various metabolic activities.*

The major prostaglandin target is smooth muscle. Some prostaglandins stimulate smooth muscle to contract, whereas others cause relaxation. Thus some reduce blood pressure, whereas others raise it. Prostaglandins synthesized in the temperature-regulating center of the hypothalamus cause fever. In fact, nonsteroidal anti-inflammatory drugs (NSAIDs) such as aspirin and ibuprofen reduce fever and decrease inflammation by inhibiting prostaglandin synthesis.

Because prostaglandins are involved in the regulation of so many metabolic processes, they have great potential for a variety of clinical uses. Physicians use them to induce labor in pregnant women, to induce abortion, and to promote the healing of ulcers in the stomach and duodenum. Prostaglandins may someday be used to treat a wide variety of illnesses, including asthma, arthritis, kidney disease, certain cardiovascular disorders, and some forms of cancer.

Although hormones are chemically diverse, they generally belong to one of four different chemical groups: (1) fatty acid derivatives, (2) steroids, (3) amino acid derivatives, or (4) peptides or proteins.

The endocrine system provides an electrochemical connection from the hypothalamus *of the brain to all the organs that control the body* metabolism, growth *and* development, *and* reproduction.

There are two types of hormones secreted in the endocrine system: steroidal and non-steroidal, or protein based hormones. Signal transduction of some hormones with steroid structure involves nuclear hormone receptor proteins that are a class of ligand activated proteins that, when bound to specific sequences of DNA serve as on-off switches for transcription within the cell nucleus.

These switches control the development and differentiation of skin, bone and behavioral centers in the brain, as well as the continual regulation of reproductive tissues. They also bind to receptor sites, and activate second messenger systems for more rapid responses. Nonsteroidal hormones bind to receptor sites on the external surface of the cell membrane and use a second messenger method of altering internal cell functions, by altering the pathways already existing in the cells, by activating or deactivating enzymes which modify existing proteins.

The endocrine system regulates its hormones through negative feedback. *Increases in hormone activity decrease the production of that hormone. The* immune system *and other factors contribute as control factors also, altogether maintaining constant levels of hormones.*

There are three different kinds of hormones based on their chemical composition:

Amines

Amines, such as norepinephrine, epinephrine, and dopamine, are derived from single amino acids, in this case tyrosine. Thyroid hormones such as 3,5,3'-triiodothyronine (T3) and 3,5,3',5'-tetraiodothyronine (thyroxine, T4) make up a subset of this class because they derive from the combination of two iodinated tyrosine amino acid residues.

Peptide and Protein

Peptide hormones and protein hormones consist of three (in the case of thyrotropin-releasing hormone) to more than 200 (in the case of follicle-stimulating hormone) amino acid residues and can have molecular weights as large as 30,000. All hormones secreted by the pituitary gland are peptide hormones, as are leptin from adipocytes, ghrelin from the stomach, and insulin from the pancreas.

Steroid

Steroid hormones are derived from cholesterol and are subdivided into those with an intact steroid nucleus (gonadal and adrenal steroids) and those *with a broken steroid nucleus (vitamin D). Steroid hormones include estrogen and progesterone from the ovary, testosterone from the testes, and cortisol and aldosteron from the adrenal gland.*

Table 6.10 lists the human endocrine glands and their hormones.

Table 6.10: ENDOCRINE GLANDS AND THEIR HORMONES

A = Amino acid derivatives
S = Steroids
P = Polypeptides

GLAND	HORMONE	TARGET TISSUE	PRINCIPAL ACTIONS
Hypothalamus	*Releasing and inhibiting hormones*	*Anterior lobe of pituitary*	*Regulate secretion of hormones by the anterior pituitary*
Posterior pituitary	*Oxytocin (P)*	*Uterus*	*Stimulates contraction*
		Mammary glands	*Stimulates ejection of milk into ducts*
	Antidiuretic hormone (ADH)	*Kidneys (collecting ducts)*	*Stimulates reabsorption of water*
	Growth hormone (GH)(P)	*General*	*Stimulates growth of skeleton and muscle*
	Prolactin	*Mammary glands*	*Stimulates milk production*
	Thyroid-stimulating hormone (TSH)(P)	*Thyroid gland*	*Stimulates secretion of thyroid hormones*
	Adrenocorticotropic hormone (ACTH)(P)	*Adrenal cortex*	*Stimulates secretion of adrenal cortical hormones*

Table 6.10: (Cont.)

GLAND	HORMONE	TARGET TISSUE	PRINCIPAL ACTIONS
	Gonadotropic hormones (follicle-stimulating hormone[FSH](P); Luteinizing hormone[LH](P))	*Gonads*	*Stimulates gonad function and growth*
Thyroid gland	*Thyroxine* (T_4) *and triiodothyronine* (T_3)	*General*	*Stimulates metabolic rate; regulate energy metabolism*
	Calcitonin	*Bone*	*Lowers blood-calcium level*
Parathyroid glands	*Parathyroid hormone*	*Bone, kidneys, digestive tract*	*Regulates blood-calcium level*
Pancreas	*Insulin(P)*	*General*	*Lowers blood glucose concentration*
	Glucagon(P)	*Liver, adipose tissue*	*Raises blood glucose concentration*
Adrenal Medulla	*Epinephrine and Norepinephrine(A)*	*Muscle; blood vessels; liver; adipose tissue*	*Help body cope with stress; increase metabolic rate; raise blood glucose level;increases heart rate and blood pressure*
Adrenal cortex	*Mineralocorticoids (S)*	*Kidney tubules*	*Maintain sodium and potassium balance*

Table 6.10: (Cont.)

GLAND	HORMONE	TARGET TISSUE	PRINCIPAL ACTIONS
	Glucocorticoids (S)	*General*	*Help body cope with long-term stress; raise blood-glucose level*
Pineal gland	*Melatonin*	*Hypothalamus*	*Important in biological rhythms*
Ovary	*Estrogens(S)*	*General; uterus*	*Develop and maintain sex characteristics in female; stimulates growth of uterine lining*
	Progesterone	*Uterus; breast*	*Stimulates development of uterine lining*
Testis	*Testosterone*	*General; reproductive structures*	*Develops and maintains sex characteristics in males; promotes spermatogenesis*

Life at the Organizational Level (Developmental and Evolutional Biology)

The complicated reconciliation of Darwinian natural selection with Mendelian genetics involved contributions from naturalists, experimentalists and population geneticists. Naturalists, intimately acquainted with adaptation, geographic variation and speciation, helped promote the idea of species as populations rather than ideal types. Experimentalists provided the important distinction between genotype and phenotype, showed that mutations could be small and inherited in a Mendelian fashion, and demonstrated, that selection could work on continuous variations to modify the characters of a population. Population genetics constructed mathematical models demonstrating how mutations, selection, migration and other factors could affect the frequencies of genes in populations.

The genetic analysis of natural populations, pioneered in Russia by ***Sergei Chetverikov*** *(1880–1959), was developed in the West by his student,* ***Theodosius Dobzhansky*** *(1900–1975). With Dobzhansky,* ***Julian Huxley*** *(1887–1975),* ***Ernst Mayr*** *(1904–2005), and* ***George Gaylord Simpson*** *(1902–1984), the modern, "synthetic" theory of evolution took shape. This synthetic or "neo-Darwinian" theory of evolution sees natural selection, working on small, Mendelian variations produced by mutations and recombination, as the major agent in evolutionary change.*

Thus, in the early 20th century, the rediscovery of Mendel's work led to the rapid development of genetics by ***Thomas Hunt Morgan*** *and his students, and by the 1930s the combination of population genetics and natural selection led to the "neo-Darwinian synthesis" and the rise of the discipline of evolutionary biology. New biological disciplines developed rapidly, especially after* ***Watson*** *and* ***Crick*** *discovered the structure of DNA in 1953.*

Following the cracking of the genetic code and the establishment of the Central Dogma, biology was largely split between organismal biology – consisting of ecology, ethology, systematics, paleontology, evolutionary biology, developmental biology, and other disciplines that deal with whole organisms and groups of organisms – and the constellation of disciplines related to molecular biology – including cell biology, biophysics, biochemistry, neuroscience, immunology, and many other overlapping subjects.

Developmental biology studies the process by which organisms grow and develop. Originating in embryology, today developmental biology studies the genetic control of cell growth, differentiation and "morphogenesis," which is the process that gives rise to tissues, organs and anatomy. Model organisms *for developmental biology include the round worm Caenorhabditis elegans, the fruit fly Drosophila melanogaster, the zebrafish Brachydanio rerio, the mouse Mus musculus, and the weed Arabidopsis thaliana.*

Molecular data regarding the mechanisms underlying development started to accrue quickly during the 1980's and '90's. As scientists began to compare the developmental mechanisms in different organisms, they realized that these mechanisms are conserved through deep evolutionary time. By combining the disciplines of phylogenetics, paleontology and comparative developmental biology, scientists try to infer the way in which early organisms developed, thus spawning the new discipline of "evo-devo."

I. Botany and Zoology

Botany is the scientific study of plantlife. As a branch of biology, it is also sometimes referred to as plant science(s) or plant biology. Botany covers a wide range of scientific disciplines that study the structure, growth, reproduction, metabolism, development, diseases, ecology and evolution of plants.

As with other life forms in biology, plant life can be studied from different perspectives, from the molecular, genetic and biochemical level through organelles, cells, tissues, organs, individuals, plant populations, and communities of plants. At each of these levels a botanist might be concerned with the classification (taxonomy), structure (anatomy and morphology), or function (physiology) of plant life.

Historically, botany covers all organisms that were not considered to be animals. Some of these "plant-like" organisms include fungi (studied in mycology), bacteria and viruses (studied in microbiology), and algae. Most algae, fungi, and microbes are no longer considered to be in the plant kingdom. However, attention is still given to them by botanists, and bacteria, fungi, and algae are usually covered in introductory botany courses.

The study of plants has importance for a number of reasons. Plants are a fundamental part of life on Earth. They generate the oxygen, food, fibres, fuel and medicine that allow higher life forms to exist. Plants also absorb carbon dioxide, a significant greenhouse gas, through photosynthesis. A good understanding of plants is crucial to the future of human societies as it allows us to:

- *Feed the world*
- *Understand fundamental life processes*
- *Utilize medicine and materials*
- *Understand environmental changes*

Virtually all of the food we eat comes from plants, either directly from staple foods and other fruit and vegetables, or indirectly through livestock, which rely on plants for their nutrition. In other words, plants are at the base of nearly all food chains, or what ecologists call the first trophic level.

Understanding how plants produce the food we eat is therefore important to be able to feed the world and provide food security for future generations, for example through plant breeding. Not all plants are beneficial to humans, some weeds are a considerable problem in agriculture and botany provides some of the basic science in order to understand how to minimize their impact.

However, other weeds are pioneer plants which start an abused environment back on the road to rehabilitation, underlining that the term 'weed' is a very relative concept and that, broadly defined, a weed is simply a plant which is too successful. Ethnobotany is the study of this and/or other relationships between plants and people.

Plants are convenient organisms in which fundamental life processes (like cell division and protein synthesis for example) can be studied, without the ethical dilemmas of studying animals or humans. The genetic laws of inheritance were discovered in this way by ***Gregor Mendel****, who was studying the way pea shape is inherited. What Mendel learnt from studying plants has had far reaching benefits outside of botany.*

Additionally, ***Barbara McClintock*** *discovered 'jumping genes' by studying maize. These are a few examples that demonstrate how botanical research has an ongoing relevance to the understanding of fundamental biological processes.*

Many of our medicinal and recreational drugs, like cannabis, caffeine, and nicotine come directly from the plant kingdom. Aspirin, which originally came from the bark of willow trees, is just one example. There may be many novel cures for diseases provided by plants, waiting to be discovered. Popular stimulants like coffee, chocolate, tobacco, and tea also come from plants. Most alcoholic beverages come from fermenting plants such as barley malt and grapes.

Plants also provide us with many natural materials, such as cotton, wood, paper, linen, vegetable oils, some types of rope, and rubber. The production of silk would not be possible without the cultivation of the mulberry plant. Sugarcane and other plants have recently been put to use as sources of biofuels, which are important alternatives to fossil fuels.

Plants can also help us understand changes in on our environment in many ways:

- *Understanding habitat destruction and species extinction is dependent on an accurate and complete catalog of plant systematics and taxonomy.*

- *Plant responses to ultraviolet radiation can help us monitor problems like the ozone depletion.*

- *Analyzing pollen deposited by plants thousands or millions of years ago can help scientists to reconstruct past climates and predict future ones, an essential part of climate change research.*

- *Recording and analyzing the timing of plant life cycles are important parts of phenology used in climate-change research.*

- *Lichens, which are sensitive to atmospheric conditions, have been extensively used as pollution indicators.*

In many different ways, plants can act a bit like the 'miners canary', an early warning system *alerting us to important changes in our environment. In addition to these practical and scientific reasons, plants are extremely valuable as recreation for millions of people who enjoy gardening, horticultural and culinary uses of plants every day.*

A considerable amount of new knowledge today is being generated from studying model plants like Arabidopsis thaliana. *This mustard weed was one of the first plants to have its genome sequenced. The sequencing of the rice genome have made rice the de facto cereal/grass/monocot model. Another grass species,* Brachypodium distachyon *is also emerging as an experimental model for understanding the genetic, cellular and molecular biology of temperate grasses. Other commercially important staple foods like wheat, maize, barley, rye, millet and soybean are also having their genomes sequenced. Some of these are challenging to sequence because they have more than two haploid (n) sets of chromosomes, a condition known as polyploidy, common in the plant kingdom.*

The "Green Yeast" Chlamydomonas reinhardtii *(a single-celled, green alga) is another plant model organism that has been extensively studied and provided important insights into cell biology.*

History of Zoology — from Aristotle to Darwin

Zoology is the scientific study of animals. The word is derived from $\zeta\omega o\gamma$ *= a living thing.*

Throughout history, man has always lived with animals and sought to understand them. Early cave paintings from before the Ice Age depict antelopes, bison, giraffes, and other animals, some of which are now extinct. Ancient Egyptians were fascinated by animals and treated them with religious reverence. It was not until ***Aristotle*** *(382–322 BC) created his History of Animals that zoology became a science. In his work, he collected all the known facts about approximately 500 animals, and devised the first known classification system. Aristotle's system divided the animals kingdom between animals with blood (4-footed animals that bear their young; 4-footed animals that lay eggs; Birds; Fish) and animals without blood (Mollusks; Crabs; Insects).*

Other Greek writers such as ***Ctesias of Cnidos*** *(fl. ca. 480 BCE) and* ***Herodotos*** *(c.485–425 BCE) also contributed to the knowledge of animals in their writings. In Roman times, the main writer about natural history was* ***Pliny the Elder*** *(23–79 CE), the author of Historia Naturalis.*

After the fall of the Roman Empire, Christianity dominated the culture of western civilization. There was a focus on the Bible and the afterlife, rather than on science and the secular world. Nevertheless, a book called the Physiologus *became extremely widespread. It was written in Greek by an unknown source probably around 200–300 CE in Egypt, This book along with the Bible and the works of Aristotle and Pliny, was the source of many medieval bestiaries, in which stories about fictional creatures were widely spread. The Physiologus was widely used for more than a thousand years, with the hand written copies being made up to 1724.*

Stories of fictional beasts were added to as Europeans began to explore the world. For example, the travels stories of ***Marco Polo*** *(1254–1324) and* ***John Mandeville*** *(fl.1320–1370).*[26]

In the 1400's, universities began to form, and there was a renewed interest in science and knowledge as the Renaissance began. ***Leonardo da Vinci*** *(1452–1519) contributed by conducting autopsies on humans.*

Scientific zoology really started in the 16th century with the awakening of the new spirit of observation and exploration, but for a long time ran a separate course uninfluenced by the progress of the medical studies of anatomy and physiology.

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[26] The medieval attitude towards both plants and animals hold no relation to real knowledge, but was part of a peculiar and in itself highly interesting mysticism. A fantastic and elaborate doctrine of symbolism existed which comprised all nature; witchcraft, alchemy and medicine were its practical expressions. Animals as well as plants were regarded as "simples" and used in medicine, and a knowledge of them was valued from this point of view.

The active search for knowledge by means of observation and experiment found its natural home in the universities. Owing to the connection of medicine with these seats of learning, it was natural that the study of the structure and functions of the human body and of the animals nearest to man should take root there; the spirit of inquiry which now for the first time became general, showed itself in the anatomical schools of the Italian universities of the 16th century, and spread fifty years later to Oxford.

The discovery of the microscope in the 1500's contributed a lot to the study of biology. In 1555, ***Conrad Gesner*** *wrote the first of a series of several books called* Historia Animalum, *which became the new standard for the next two hundred years. During this time there were also number of books about specialized topics, such as birds, fish, and others. Gesner, and other writers like* ***Thomas Browne*** *(1605–1682) and* ***Ulisse Aldrovandus*** *(1522–1602) began to subject biology and zoology to scientific scrutiny.*

In the 17th century the investigators of nature by means of observation and experiment, banded themselves into academies or societies for mutual support and intercourse. The first founded of surviving European academies, the Academia Naturae Curiosorum (1651)[27] *especially confined itself to the description and illustration of the structure of plants and animals; eleven years later (1662) the Royal Society of London was incorporated by royal charter, having existed without a name or fixed organization for seventeen years previously (from 1645).*

A little later the Academy of Sciences of Paris was established by Louis XIV. The influence of these great academies of the 17th century on the progress of zoology had the effect of bringing together of the museum-men and the physicians or anatomists, which was just what needed for further development.

As the amount of knowledge grew quickly, it was necessary to develop a classification system. ***Carl Linnaeus*** *(1707–1778) created a classification system that used two Latin names – the species, and the genus. This is the system that is still used today. But it still took many years for scientists to understand how systems of biology and zoology worked.*

Whilst the race of collectors and systematizers culminated in the latter part of the 18th century in Linnaeus, *a new type of student made its appearance in such men as* ***John Hunter*** *and other anatomists, who, not satisfied with the superficial observations of the popular "zoologists", set themselves to work to examine anatomically the whole animal kingdom, and to classify its members by aid of the results of such profound study.*

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[27] The Academia Secretorum Naturae was founded at Naples in 1560, but was suppressed by the ecclesiastical authorities.

Under the influence of the touchstone of strict inquiry set by the Royal Society, the marvels of witchcraft, sympathetic powders, and other relics of medieval superstition disappeared like a mist before the sun, whilst accurate observations and demonstrations of a host of new wonders accumulated. Among these which were numerous contributions to the anatomy of animals, and none perhaps more noteworthy than the microscope observations, of ***Leeuwenhoek****, (1683).*

It was not until the 19th century that the microscope, thus early applied by ***Leeuwenhoek****,* ***Malpighi****,* ***Hooke****, and* ***Swammerdam*** *to the study of animal structure, was perfected as an instrument, and accomplished for zoology its final and most important service.*

The perfecting of the microscope led to a full comprehension of the great doctrine of cell structure and the establishment of the facts —
(1) that all organisms are either single cells of living material (microscopic animalcules, etc.) or are built up of an immense number of such units;
(2) that all organisms begin their individual existence as a single unit or corpuscle of living substance, which multiplies by binary fission, the products growing in size and multiplying similarly by binary fission; and
(3) that the life of a multicellular organism is the sum of the activities of the corpuscular units of which it consists, and that the processes of life must be studied in and their explanation obtained from an understanding of the chemical and physical changes which go on in each individual corpuscle or unit of living material or protoplasm.

Paleontology is a newer science than zoology. Some ancient Greek writers believed that fossils were from prehistoric creatures, but ***Aristotle*** *thought they were merely formed by a mud slide. For hundreds of years, nobody understood fossils, and thought they were freaks of nature. Paleontology did not become a science until the early 1800's when* ***George Cuvier*** *(1769–1832) founded comparative anatomy and brought about a synthesis of anatomy and physiology. This science studied the development, functions, and structure of internal organs, and allowed scientists to identify and understand fossilized remains.*

Meanwhile the astronomical theories of development of the solar system from a gaseous condition to its present form, put forward by ***Kant*** *and by* ***Laplace****, had impressed men's minds with the conception of a general movement of spontaneous progress or development in all nature.*

The science of geology came into existence, and the whole panorama of successive stages of the earths history, each with its distinct population of strange animals and plants (unlike those of the present day and simpler in proportion as they recede into the past) was revealed by ***Cuvier****,* ***Agassiz****, and others.*

*The history of the crust of the earth was explained by **Lyell** as due to a process of slow development, with no cataclysmic agencies and no mysterious forces differing from those operating at the present day.*

Thus he carried on the narrative of orderly development from the point at which it was left by Kant and Laplace – explaining by reference to the ascertained laws of physics and chemistry the configuration of the earth, its mountains and seas, its igneous and its stratified rocks, just as the astronomers had explained by those same laws the evolution of the Sun and planets from diffused gaseous matter of high temperature. The suggestion that living things must also be included in this great development was obvious.

The delay in the establishment of the doctrine of organic evolution was due, not to the ignorant and unobservant, but to the leaders of zoological and botanical science.

Knowing the almost endless complexity of organic structures, realizing that man himself with all the mystery of his life and consciousness must be included in any explanation of the origin of living things, they preferred to regard living things as something apart from the rest of nature, specially cared for, specially created by a Divine Being.

Thus it was that the so-called "Natur-philosophen" of the last decade of the 18th century, and their successors in the first quarter of the 19th, found few adherents among the working zoologists and botanists.

***Lamarck**, **Treviranus**, **Erasmus Darwin**, **Goethe**, and **Saint-Hilaire** preached to deaf ears, for they advanced the theory that living beings had developed by a slow process of transmutation in successive generations from simpler ancestors, and in the beginning from simplest formless matter, without being able to demonstrate any existing mechanical causes by which such development must necessarily be brought about.*

They were met by the criticism that possibly such a development had taken place; but, as no one could show as a simple fact of observation that it had *taken place, nor as a result of legitimate inference that it* must *have taken place, it was quite as likely that the past and present species of animals and plants had been separately created or individually brought into existence by unknown and inscrutable causes. It was held that scientific man would refuse to occupy himself with such fancies facts.*

*In 1859, **Charles Darwin** (1809–1882) placed the whole theory of organic evolution on a new footing. Indeed, he gave a new direction to morphology and physiology, by uniting them in a common biological theory: the theory of organic evolution. The result was a reconstruction of the classification of animals upon a genealogical basis, fresh investigation of the development of animals, and early attempts to determine their genetic relationships.*

Moreover, he discovered a mechanical cause actually existing and demonstrable by which organic evolution must be brought about.

After publication of the Origin of the Species, Darwin became interested in the animal and plant mechanism that confer advantages to individual members of a species.

Darwin's evolutionary theory resolved many zoological problems, not least because he established definite connections between extinct organisms (fossils) and those of today.

Darwin upset the theological vision of the "economy of nature"[28], *stimulating the development of disciplines like* biogeography, ecology *and* ethology.

Nowadays, zoology is particularly important for genetic investigations (genetic code, mutation, recombination) and for population studies.

[28] **Linnaeus'** (1707–1778), *Oeconomia Naturae* (1749) suggested that each creature has its allotted place in nature, having been assigned its peculiar food and geographic range. Competition with other creatures was thereby avoided, ensuring harmony and plenty. Different creatures were linked together in elaborate food chains: the excess of one species sustaining another. Thus, predation and high reproductive rates amongst prey species were functional to maintain a just proportion between all species (no competitive struggle for existence).

II. Dietetics, Hygiene, Metabolism and Nutrition

Dietetics[29] *is the systematic control of food and drink in order to conserve health and combat disease.*

Metabolism is the chemical process within an organism, whereby new substances are synthesized (anabolism) or broken down (catabolism) for purposes such as regulational (homeostasis), growth, tissue repair and energy supply.

Nutrition is the body of science that seeks to explain metabolic and physiologic responses to diet. To this end it studies the relationship between diet, health and disease. It is thus concerned with the process of eating, digesting and using food and the determination of an optimal diet for purposes of health, body building and other purposes.

Hygiene encompasses theories and activities for preserving individual, communal and public health discussed under prevention of disease.

With advances in molecular biology, biochemistry, and genetics, nutrition is additionally developing into the study of integrative metabolism, which seeks to connect diet and health through the lens of biochemical processes.

The human body comprises chemical compounds such as water, amino acids (proteins), fatty acids (lipids), nucleic acids (DNA/RNA), and carbohydrates (e.g. sugars). These compounds in turn consist of elements such as carbon, hydrogen, oxygen, nitrogen, and phosphorus, and may or may not contain minerals such as calcium, iron, and zinc. Minerals ubiquitously occur in the form of salts and electrolytes. All of these chemical compounds and elements occur in various forms and combinations (e.g. hormones/vitamins, phospholipids, hydroxyapatite), both in the human body and in organisms (e.g. plants, animals) that humans eat.

The human body necessarily comprises the elements that it eats and absorbs into the bloodstream. The digestive system, except in the unborn fetus, participates in the first step which makes the different chemical compounds and elements in food available for the trillions of cells of the body. In the digestive process of an average adult, about seven liters of liquid, known as

[29] Already **Hippocrates** (450–370 BCE), **Asclepiades** (13–140 BCE) and their followers elaborated a philosophy of living based on moderation, instructing on diet, exercise, massage, sex, dress, bathing etc. Ancient philosophers (e.g. **Plato** (427–347, BCE), the Pythagoreans, Stoics) were also concerned with rules of conduct and living. Early Christians eventually integrated Greek Hygienic precepts with Old and New Testament Instructions.

digestive juices, exit the internal body and enter the lumen of the digestive tract. The digestive juices help break chemical bonds between ingested compounds as well as modulate the conformation and/or energetic state of the compounds/elements. However, many compounds/elements are absorbed into the bloodstream unchanged, though the digestive process helps to release them from the matrix of the foods where they occur. Any unabsorbed matter is excreted in the feces. But only a minimal amount of digestive juice is eliminated by this process; the intestines reabsorb most of it; otherwise the body would rapidly dehydrate.

The body requires amino acids to produce new body protein (protein retention) and to replace damaged proteins (maintenance) that are lost in the urine. In animals, amino acid requirements are classified in terms of essential (an animal cannot produce them) and non-essential (the animal can produce them from other nitrogen containing compounds) amino acids. Consuming a diet that contains adequate amounts of essential (but also non-essential) amino acids is particularly important for growing animals, which have a particularly high requirement.

Mineral and/or vitamin deficiency or excess may yield symptoms of diminishing health such as goiter, scurvy, osteoporosis, weak immune system, disorders of cell metabolism, certain forms of cancer, symptoms of premature aging, and poor psychological health (including eating disorders), among many others.

As of 2005, twelve vitamins and about the same number of minerals are recognized as "essential nutrients", meaning that they must be consumed and absorbed - or, in the case of vitamin D, alternatively - to prevent deficiency symptoms and death. Certain vitamin-like substances found in foods, such as carnitine, have also been found essential to survival and health, but these are not strictly "essential" to eat because the body can produce them from other compounds. Moreover, thousands of different phytochemicals have recently been discovered in food (particularly in fresh vegetables), which have many known and yet to be explored properties including antioxidant activity (see below). Other essential nutrients include essential amino acid, choline and the essential fatty acids.

In addition to sufficient intake, an appropriate balance of essential fatty acids – omega-3 and omega-6 fatty acids – has been discovered to be crucial for maintaining health. Both of these unique "omega" long-chain polyunsaturated fatty acids are substrates for a class of eicosanoids known as prostaglandins which function as hormones. The omega-3 eicosapentaenoic acid (EPA) (which can be made in the body from the omega-3 essential fatty acid alpha-linolenic acid (LNA), or taken in through marine food *sources), serves as building block for series 3 prostaglandins. The omega-6 dihomo-gamma-linolenic acid (DGLA) serves as building block for series 1 prostaglandins, whereas arachidonic acid (AA) serves as building block for series 2 prostaglandins.*

Both DGLA and AA are made from the omega-6 linoleic acid (LA) in the body, or can be taken in directly through food. An appropriately balanced intake of omega-3 and omega-6 partly determines the relative production of different prostaglandins, which partly explains the importance of omega-3/omega-6 balance for cardiovascular health. In industrialized societies, people generally consume large amounts of processed vegetable oils that have reduced amounts of essential fatty acids along with an excessive amount of omega-6 relative to omega-3.

Because different types and amounts of food eaten/absorbed affect insulin, glucagon and other hormones to varying degrees, not only the amount of omega-3 versus omega-6 eaten but also the general composition of the diet therefore determine health implications in relation to essential fatty acids, inflammation and mitosis.

Obesity can unfavorably alter hormonal and metabolic status via resistance to the hormone leptin, and a vicious cycle may occur in which insulin/leptin resistance and obesity aggravate one another. The vicious cycle is putatively fueled by continuously high insulin/leptin stimulation and fat storage, as a result of high intake of strongly insulin/leptin stimulating foods and energy. Both insulin and leptin normally function as satiety signals to the hypothalamus in the brain; however, insulin/leptin resistance may reduce this signal and therefore allow continued overfeeding despite large body fat stores. In addition, reduced leptin signaling to the brain may reduce leptin's normal effect to maintain an appropriately high metabolic rate.

Antioxidants are another recent discovery. As cellular metabolism /energy production requires oxygen, potentially damaging (e.g. mutation causing) compounds known as radical oxygen species or free radicals may form. For normal cellular maintenance, growth, and division, these free radicals must be sufficiently neutralized by antioxidant compounds, some produced by the body with adequate precursors (glutathione, Vitamin C in most animals) and those that the body cannot produce may only be obtained through the diet through direct sources (Vitamin C in humans, Vitamin A, Vitamin K) or produced by the body from other compounds (Beta-carotene converted to Vitamin A by the body, Vitamin D synthesized from cholesterol by sunlight).

Different antioxidants are now known to function in a cooperative network, e.g. vitamin C can reactivate free radical-containing glutathione or Vitamin

E by accepting the free radical itself, and so on. Some antioxidants are more effective than others at neutralizing the free radicals. Some cannot neutralize certain free radicals. Some cannot be present in certain areas of free radical development (Vitamin A is fat-soluble and protects fat areas, Vitamin C is water soluble and protects those areas). When interacting with a free radical, some antioxidants produce a different free radical compound that is less dangerous or more dangerous than the previous compound.

Since the Industrial Revolution some two hundred years ago, the food processing industry has invented many technologies that both help keep foods fresh longer and alter the fresh state of food as they appear in nature. Cooling is the primary technology that can help maintain freshness, whereas many more technologies have been invented to allow foods to last longer without becoming spoiled. These latter technologies include pasteurization, autoclavation, drying, salting, and separation of various components, and all appear to alter the original nutritional contents of food.

Pasteurization and autoclavation (heating techniques) have no doubt improved the safety of many common foods, preventing epidemics of bacterial infection. But some of the (new) food processing technologies undoubtedly have downfalls as well.

Modern separation techniques such as milling, centrifugation, and pressing have enabled up-concentration of particular components of food, yielding flour, oils, juices and so on, and even separate fatty acids, amino acids, vitamins, and minerals. Inevitably, such large scale up-concentration changes the nutritional content of food, saving certain nutrients while removing others. Heating techniques may also reduce food's content of many heat-labile nutrients such as certain vitamins and phytochemicals, and possibly other yet to be discovered substances.

Because of reduced nutritional value, processed foods are often "enriched" or "fortified" with some of the most critical nutrients (usually certain vitamins) that were lost during processing. Nonetheless, processed foods tend to have an inferior nutritional profile than do whole, fresh foods, regarding content of both sugar and high GI starches, potassium/sodium, vitamins, fibre, and of intact, unoxidized (essential) fatty acids. In addition, processed foods often contain potentially harmful substances such as oxidized fats and trans fatty acids.

A dramatic example of the effect of food processing on a population's health is the history of epidemics of beri-beri in people subsisting on polished rice. Removing the outer layer of rice by polishing it removes with it the essential vitamin thiamine, causing beri-beri. Another example is the development of scurvy among infants in the late 1800s in the United States. It turned out *that the vast majority of sufferers were being fed milk that had been heat-treated (as suggested by Pasteur) to control bacterial disease. Pasteurization was effective against bacteria, but it destroyed the vitamin C.*

As mentioned, lifestyle- and obesity-related diseases are becoming increasingly prevalent all around the world. There is little doubt that the increasingly widespread application of some modern food processing technologies has contributed to this development. The food processing industry is a major part of modern economy, and as such it is influential in political decisions (e.g. nutritional recommendations, agricultural subsidizing). In any known profit-driven economy, health considerations are hardly a priority; effective production of cheap foods with a long shelf-life is more the trend. In general, whole, fresh foods have a relatively short shelf-life and are less profitable to produce and sell than are more processed foods.

Thus the consumer is left with the choice between more expensive but nutritionally superior whole, fresh foods, and cheap, usually nutritionally inferior processed foods. Because processed foods are often cheaper, more convenient (in both purchasing, storage, and preparation), and more available, the consumption of nutritionally inferior foods has been increasing throughout the world along with many nutrition-related health complications.

History of Dietetics, Hygiene, Metabolism and Nutrition

Humans are believed to have evolved as omnivorous hunter-gatherers over the past 250,000 years. Early diets were primarily vegetarians with infrequent game meats and fish where available.

Agriculture developed about 10,000 years ago in multiple locations throughout the world, providing grains such as wheat, rice, and maize, with staples such as bread and pasta. Farming also provided milk and dairy products, and sharply increased the availability of meats and the diversity of vegetables. The importance of food purity was recognized when bulk storage led to infestation and contamination risks. Cooking developed as an often ritualistic activity to strict recipes and procedures, and also contributed to demands for food purity and consistency (e.g. the laws of ceremonial purity and dietary restrictions given in the Old Testament book of ***Leviticus*** *chapters 11–16, which were established ca 1300 BCE and written down in the days of King Hezekiah, ca 710 BCE).*

The first recorded nutritional experiment is found in the Bible's Book of Daniel. Daniel and his friends were captured by the king of Babylon during an invasion of Israel. Selected as court servants, they were to share in the king's fine foods and wine. But they objected, preferring vegetables and water in *accordance with their Jewish dietary restrictions. The king's chief steward reluctantly agreed to a trial. Daniel and his friends received their diet for 10 days and were then compared to the king's men. Appearing healthier, they were allowed to continue with their diet.*

The idea that changes that we now call chemical, occur in the human body is very old. According to ***Aristotle*** *(384–322 BCE) and* ***Galen*** *(129–200), food entering the alimentary tract undergoes processes resembling fermentation and through it turn into blood. This theory was generally accepted for more than 20 centuries. On its basis, numerous diets designed to make easier food transformations into blood and provide it with higher nutritive properties were developed for healthy and sick subjects. According to this classical tradition alimentation was one of the exterior forces which acted upon the human body.*

Considered from the therapeutic point of view the proper regulation of these forces was subsumed under the general Greek term diaita *meaning "way of life" or regimen. Foodstuffs were classified in terms of their qualities of hotness, coldness, moistness and dryness in accordance with the dominant humoral theory of physiological action. As a consequence of their particular qualities foods might be purgative, constipating, strengthening or weakening. Their role in the production of the humors was also a major consideration.*

With their increasing search for chemical mechanisms, the iatrochemists of the 16^{th} *and* 17^{th} *centuries tried to explain body functions by purely* chemical reactions *such as the neutralization of acids and bases. Nevertheless, ideas of chemicophysiological mechanisms remain vague and disorganized even while* ***Vesalius*** *(1514–1564) was describing in a scientific manner the details of anatomy, and the physicomechanical details of physiology were developing toward the discovery of the circulation of the blood by* ***William Harvey*** *(1578–1657).*

In 1747, Dr. ***James Lind*** *(1716–1794), a physician in the British navy, discovered that lime juice saved sailors (who had been at sea for years) from* scurvy[30]*, a deadly and painful bleeding disorder. The discovery was ignored*

[30] Ascorbic acid, or Vitamin C, was discovered after scientists had searched centuries for a cure for the disease known as scurvy. The name Ascorbic acid comes from word "anti-scurvy" acid, because it was known to dramatically cure this disease. This disease was caused by a serious deficiency of Vitamin C, and it caused its victim's small blood vessels to rupture, bones to weaken, and joints to swell, among other symptoms. These symptoms were due to the fact that without a source of Vitamin C one developed severe problems concerning the body's connective tissues, which is found in bones, skin, muscles, teeth, blood vessels, and cartilage. This disease would eventually lead to death if it went untreated,

for 40 years. The essential Vitamin C within the lime juice would not be recognized by scientists until the 1930's.

During the 18^{th} *century, classical dietary theories began to be superseded by ideas derived from the new chemistry. Qualitative grades tended to be replaced by such considerations as "acidity or alkalinity". In 1770,* ***Antoine Lavoisier*** *(1743–1794), the "Father of Chemistry" discovered the details of metabolism, demonstrated that* oxidation *of food is the source of body heat. In 1790,* ***George Fordyce*** *(1736–1802) recognized, that* calcium *was necessary for fowl survival.*

Early 19^{th}*-century scientists distinguished "animals" and plants as complementary, the former only breaking down material synthesized by plants. From the middle of the* 19^{th} *century, the chemistry of food and biochemistry started to develop successfully.*

In the early 1800s, the elements carbon, nitrogen, hydrogen *and* oxygen *were recognized as the primary components of food, and methods to measure their proportions were developed.*

In 1816, ***Francois Magendie*** *(1783–1855) discovers that dogs fed only carbohydrates and fat lost their body protein and died in a few weeks, but dogs fed also protein survived, identifying protein as an essential dietary component.*

In 1840, the German chemist ***Justus Liebig*** *(1803–1873) discovered the chemical makeup of* carbohydrates *(sugars),* fats *(fatty acids) and* proteins

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and was not uncommon, especially during the winter months of the year. The disease often plagues armies, explorers, and crusaders, since these men's diets normally consisted of biscuits and salted meat that could easily be stored and kept unspoiled on a ship.

Lind published his findings as *Treatise on the Scurvy* in 1753, and as a result, in 1795 daily doses of lime juice were prescribed to all the sailors in the British navy and Scurvy quickly vanished. However, the British were the only people who accepted the idea that Scurvy was the result of a dietary deficiency, and Great Britain was the only place where there was a decline in the cases of Scurvy. In America, during the Civil war, many men on both sides of the war died from this disease due to the lack of a source of Vitamin C in their diet.

In 1907 **Axel Holst** and **Theodore Frolich**, proved that Scurvy's symptoms could be produced in a guinea pig when denied certain foods, and that these symptoms would vanish when the food was restored. However, even on into the early 20^{th} century, explorers were still dying of Scurvy. One example was Robert Scott's expedition to the South Pole, when he and his crew were affected by the lack of Vitamin C containing foods in their diets, not the harsh conditions and temperatures.

(amino acids). In 1842 he applied a "black-box" model to animal metabolism, analyzing intake of food (e.g. protein, sugar) and excretion (urea, carbon dioxide) and assuming a simple one-way path from one to the other.

In the 1860's, ***Claude Bernard*** *(1813–1878) discovered that body fat can be synthesized from carbohydrate and protein, showing that energy in blood glucose can be stored as* glycogen.

His discovery of glycogen synthesis and breakdown by the liver showed animal metabolism to be rather complicated. This induced further research of metabolism parameters by ***Carl von Voit*** *(1831–1908),* ***Max von Pettenkofer*** *(1818–1901),* ***T.L.W. von Bischoff*** *(1807–1882),* ***Max Rubner*** *(1854–1932) and others.* ***Edward Frankland*** *(1825–1899),* ***F.K.A. Stohmann*** *(1832–1887) and others established accurate calorific values for many foods and the discovery of "cell-free" fermentation by* ***Eduard Buchner*** *(1860–1917) implicated intracellular metabolic processes, regulated by enzymes.*

Controlled dietary studies by ***T.B. Osborne*** *(1859–1929),* ***L.B. Mendel*** *(1872–1935) and* ***E.V. McCollum*** *(1879–1967) showed that some amino acids (the building blocks of proteins) could not be synthesized by animals and were essential components of the diet, as were the vitamins.*

Modern biochemistry and molecular biology have elucidated many molecular mechanisms, including anabolic and catabolic pathways and the genetic control of enzyme synthesis ('one gene – one enzyme hypothesis'). The study of inherited metabolic disorders was pioneered by ***A.E. Garrod****'s (1857–1936) in 1909.*

Thus, from the second half of the 19^{th} *century dietetic theory took new forms based on an understanding of the role of food substances in body metabolism.*

The following timeline summarizes the progress made during 1800–1950:

- *1816* **Francois Magendie** *(1783–1855, France). Physiologist. Showed for the first time that nitrogenous foods were needed for life.*
- *1819–1820* **Henri Braconnot** *(1781–1854, France). Naturalist. Obtained* glucose *(1819) from sawdust, linen and bark of trees. Studied and isolated the first amino acid (1820). [Earlier (1812),* **Gottlieb Sigismund Constantin Kirchhoff** *(1764–1833, Russia), chemist, produced glucose by heating starch with a small amount of sulfuric acid as catalyst.]*

- *1824* ***Joseph Louis Gay-Lussac*** *(1778–1850, France). Wrote the chemical equation for glucose fermentation.*
- *1827* ***William Prout*** *(1785–1850, England). Chemist. Classified food components into* fats, carbohydrates *and* proteins.
- *1835* ***Anselm Payen*** *(1795–1871, France). Chemist. Isolated and studied the first plant enzyme* diastase *from grain. It catalyzes the breakdown of starch.*
- *1836* ***Theodor Schwann*** *(1810–1882, Germany). Discovered and isolated* pepsin, *a digestive enzyme in the human stomach.*
- *1843* ***Carl Schmidt*** *(1822–1894, Germany). Physiological chemist. Coined the name* carbohydrates. *Found that blood contains small quantities of glucose.*
- *1840* ***Justus von Liebig*** *(1803–1873, Germany). Published "Die Organische Chemie und ihre Anwendung auf Agrikultur und Physiologie", explaining his theory of the exchange of carbon and nitrogen in plants and animals — the first crude model of the carbon and nitrogen cycles in the biosphere.*
- *1843–1856* ***Claude Bernard*** *(1813–1878, France). Physiologist. Discovered (1843) that the liver serves as a source of blood sugar by converting glycogen to glucose. Coined the word* glycogen. *Isolated glycogen from the liver (1856). [This was done independently in the same year by* ***Viktor Hensen*** *(1835–1924, Germany).]*
- *1856* ***Louis Pasteur*** *(1822–1895, France). Discovered that* fermentation *is caused by microorganisms (yeast).*
- *1868–1902* ***Carl von Voit*** *(1831–1908, Germany). Physiologist. Showed (1868) that energy conversion in the body takes place through* intermediary substances *which are formed from the original food before final union with oxygen occurred. Discovered (1891) that various ingested sugars are converted to glycogen in the body for storage until needed.*
- *1870* ***Friedrich Miescher*** *(1844–1895, Switzerland) first identified* nucleic acid *in the nuclei of puss cells.*
- *1871* ***Ernst Hoppe-Seyler*** *(1825–1895, Germany). Physiologist and chemist. Discovered* invertase, *an enzyme that speeds up conversion of sucrose into glucose and fructose.*
- *1884–1894* ***Max Rubner*** *(1854–1932, Germany). Physiologist. Discovered that the body gets energy from carbohydrates, fats and proteins* after *stripping away* nitrogen *for other uses.* ***Voit*** *and* ***Rubner***, *independently measured caloric energy expenditure in different species of animals, applying principles of physics in nutrition.*

- *1889* ***Oskar Minkowski*** *(1858–1931, Germany), physiologist, and* ***Joseph von Mering*** *(1849–1908, Germany), physician, discovered that the pancreas supplies a hormone (insulin) essential to glucose metabolism.*

- *1890–1907* ***Emil Hermann Fischer*** *(1852–1919, Germany). Chemist. First to synthesize a simple protein molecule (1907) and demonstrate the peptide-bond. Pioneered in understanding nitrogen metabolism. Proposed the theory of lock and key to explain stereospecific interaction of enzyme with substrate.*

- *1896* ***Eugen Baumann*** *(1846–1896, Germany) discovered that the thyroid gland is rich in iodine.*

- *1897* ***Eduard Buchner*** *(1860–1917, Germany). Chemist. Demonstrated that alcoholic fermentation of sugars is due to action of enzymes contained in yeast. That meant that enzymes extracted from yeast are effective in converting sugar into alcohol. Awarded the 1907 Nobel Prize for chemistry.*

- *1897* ***Christiaan Eijkman*** *(1858–1931), a Dutch physician in Java cured natives of beriberi by feeding them brown rice. Over two decades later, nutritionists learned that the outer rice bran contains vitamin B1, also known as thiamine. Together with biochemist* ***Frederick G. Hopkins*** *(1861–1947) he was awarded the 1929 NP for Physiology or Medicine.*

- *1897–1901* ***Jokichi Takamine*** *(1854–1922, Japan), physiologist and* ***John Jacob Abel*** *(1857–1938, U.S.A.), physiological chemist, independently isolated adrenaline.*

- *1905* ***Arthur Harden*** *(1865–1940, England). Chemist. First to detect and identify inorganic phosphate in metabolic intermediates.*

- *1906* ***F.G. Hopkins*** *(1861–1947, England) recognized "accessory food factors" other than calories, protein and minerals, as organic materials essential to health but which the body cannot synthesize.*

- *1912* ***Casimin Funk*** *(1884–1967, Poland) coined the term vitamin, a vital factor in the diet, from the word "vital" and "amine", because the unknown substances preventing scurvy, beriberi, and pellagra, were thought then to be derived from ammonia.*

- *1913* ***Elmer V. McCollum*** *(1879–1967) discovered the first vitamins, fat soluble vitamin A, and water soluble vitamin C (as the then-unknown substance preventing scurvy). In 1922 he discovered vitamin D in cod liver oil, which prevents rickets.*

- *1913* **Archibald Vivian Hill** *(1886–1977, England). Physiologist. Discovered that muscle cells use oxygen after contraction is finished in anaerobic glycolysis.*
- *1918* **Otto Fritz Meyerhof** *(1884–1951, Germany and U.S.A.). Biochemist. Showed that muscular activity involves anaerobic conversion of glucose glycogen (glycolysis). During muscle rest, lactic acid combines with oxygen to restore glycogen level. First to note that metabolic pathways of all organisms are essentially similar.*
- *1921* **Frederick Grant Banting** *(1891–1941, Canada). Physician. Isolated the hormone insulin.*
- *1922* **H.M. Evans** *(1882–1971, USA) and* **K.S. Bishop** *discovered vitamin E in green leafy vegetables.*
- *1926* **James Batcheller Sumner** *(1887–1955, U.S.A.). Biochemist. Proved that the enzyme urease was indeed a protein. It quickly became apparent that all enzymes are proteins.*
- *1927* **Adolf O.R. Windaus** *(1876–1959, Germany) synthesized vitamin D (NP 1928).*
- *1928–1935* **Albert Szent-Györgi** *(1893–1986, USA). Physiologist and biochemist. Isolated ascorbic acid (1928) and proved that it is vitamin C (1932). Discovered fundamental processes in cellular energy metabolism. Concurrently elucidated much of the citric acid cycle. (NP 1937).*
- *1929* **Albert Lipmann** *(1899–1986, Germany and USA), biochemist, and* **K. Lohmann** *isolated adenosine triphosphate (ATP) from muscle tissue.*
- *1931–1939* **William C. Rose** *(1887–1985, USA) identified essential amino acids, necessary proteins which the body cannot synthesize.*
- *1931* **Linus Pauling** *(1901–1994, U.S.A.). Discovered the hydrogen bond.*
- *1932–1937* **Hans Adolf Krebs** *(1900–1981, England). Biochemist. Discovered the most important metabolic pathway and energy producer in living organism (Krebs' cycle), the second stage in the aerobic glucose metabolism.*
- *1933* **Walter N. Haworth** *(1883–1950, England) and* **Tadeus Reichstein** *(1897–1996, Poland and Switzerland) synthesized vitamin C, becoming the first vitamin to be artificially made. Haworth won the NP in 1937 jointly with* **Paul Karrer** *(1889–1971, Switzerland). Reichsten won his NP in 1950.*

- *1935* **E.J. Underwood** *(1905–1980, England) and* **H.R. Marston** *(1900–1965, England) independently discover the necessity of cobalt.*

- *1936* **Gerty** and **Carl Cori** *(1896–1957, 1896–1984, U.S.A.). Biochemists. Discovered the Cori cycle of carbohydrate metabolism: the complementary process of glycolysis in the muscle and gluconeogenesis in the liver and the process of phosphorolysis; showed that glycogen in the body is broken down by the use of phosphoric acid.*

- *1938* **Erhard Fernholz** *discovered the chemical structure of vitamin E.*

- *1938* **Paul Karrer** *(1889–1971) synthesized vitamin E.*

- *1946* **Linus Pauling** *suggested that enzymes work by lowering the energy-barrier of a reaction.*

- *1947* **Alexander Robertus Todd** *(1907–1997, England). Biochemist. Synthesized ADP and ATP.*

- *1947–1951* **Albert Lipmann** *discovered the coenzyme A, a key substance in human body metabolism which is involved in the control of energy in cells. Proposed that the high-energy phosphate-bond and its transformation to and from ATP was a 'common currency' of energy transfer in biology (1947). Discovered acetylcoenzyme A, an essential part of body chemistry that is especially important in breaking down carbohydrates, fats and proteins to obtain energy for cells (1951).*

III. Evolutionary Biology

(a) Introduction

Evolutionary biology is a sub-field of *biology* concerned with the origin and descent of species, as well as their change, multiplication, and diversity over time. One who studies evolutionary biology is known as an *evolutionary biologist*, or less formally, an *evolutionist*.

Evolutionary biology is an *interdisciplinary* field because it includes scientists from a wide range of both field and lab oriented disciplines. For example, it generally includes scientists who may have a specialist training in particular organisms such as *mammalogy*, *ornithology*, or *herpetology*, but use those organisms as *case studies* to answer general questions in evolution.

It also generally includes *paleontologists* and *geologists* who use *fossils* to answer questions about the tempo and mode of evolution, as well as theoreticians in areas such as *population genetics*.In the 1990s *developmental biology* made a re-entry into evolutionary biology from its initial exclusion from the modern synthesis through the study of *evolutionary developmental biology*.

Its findings feed strongly into new disciplines that study mankind's *sociocultural evolution* and *evolutionary behavior*. Evolutionary biology's frameworks of ideas and conceptual tools are now finding application in the study of a range of subjects from *computing* to *nanotechnology*.

Artificial life is a sub-field of *Bioinformatics* that attempts to model, or even recreate, the evolution of organisms as described by evolutionary biology. Usually this is done through mathematics and computer models.

Historically, when Mendel's work was "rediscovered" in 1900, it led to a conflict between Mendelians and biometricians, who insisted that the great majority of traits important to evolution must show continuous variation that was not explainable by Mendelian analysis.

Eventually, the two models were reconciled and merged, primarily through the work of the biologist and statistician **R.A. Fisher**. This combined approach, applying a rigorous statistical model to Mendel's theories of inheritance via genes, became known in the 1930s and 1940s as the modern synthesis of Darwin's theory.

Evolutionary biology as an *academic discipline* in its own right emerged as a result of the *modern evolutionary synthesis* in the 1930s and 1940s. It was not until the 1970s and 1980s, however, that a significant number of universities had departments that specifically included the term evolutionary biology in their titles. In the United States, as a result of the rapid growth of *molecular and cell biology*, many universities have split (or aggregated) *their biology departments into molecular and cell biology-style departments and ecology and evolutionary biology-style departments (which often have subsumed older departments in* paleontology, zoology *and the like).*

Microbiology *has recently developed into an evolutionary discipline. It was originally ignored due to the paucity of morphological traits and the lack of a species concept in microbiology. Now, evolutionary researchers are taking advantage of our extensive understanding of microbial physiology, the ease of microbial* genomics, *and the quick generation time of some microbes to answer evolutionary questions. Similar features have led to progress in* viral *evolution, particularly for* bacteriophage.

Notable contributors to evolutionary biology include:

Pierre Louis Maupertuis	*1698–1759*
Jean-Baptiste Lamarck	*1744–1829*
Charles Darwin	*1809–1882*
Alfred Russel Wallace	*1823–1913*
August Weismann	*1834–1914*
Ernst Haeckel	*1834–1919*
Sewall Wright	*1889–1988*
R.A. Fisher	*1890–1962*
J.B.S. Haldane	*1892–1964*
Theodosius Dobzhansky	*1900–1975*
Ernst Mayr	*1904–2005*
Gustave Malécot	*1911–1998*
James F. Crow	*1916–*
John Maynard Smith	*1920–2004*
Motoo Kimura	*1924–1994*
George C. Williams	*1926–*
Carl Woese	*1928–*
Edward Osborn Wilson	*1929–*
Richard Lewontin	*1929–*
Allan Wilson	*1934–1991*
W.D. Bill Hamilton	*1936–2000*
Danil Janzen	*1939–*
Stephen Jay Gould	*1941–2002*
Robert Trivers	*1943–*
Niles Eldredge	*1943–*
Richard D. Alexander	*1948–*

(b) Molecular Evolution and Population Genetics

Mendel (1866) cross-pollinated purebred pea plants RR with purebred wrinkled pea plants (rr). The first filial generation peas were all round (Rr), but those in the next generation included RR (round)*, Rr* (round)*, Rr* (round) *and rr* (wrinkled)*. Thus, the* genotype *Rr* produced the phenotype round peas.

This work of Mendel and later geneticists has shown that the development of the individual organism is controlled by hereditary regulation known as genes*. Genes are constructed of the nucleic acid called DNA (deoxyribonucleic acid) and are normally located in the cell nucleus, where they are organized into larger, paired, thread-like units called* chromosomes*, each of which may contain thousands of genes.*

The number of chromosomes is usually constant for each species, but varies between species, ranging from as few as one pair to as many as several hundred pairs. The usual number is between 5 and 30 pairs. Man, for example, has 23 pairs. When a cell divides during normal growth, the chromosomes reproduce themselves exactly to give, in the two new cells, the same number and kinds of chromosomes as in original parent cell. This process of exact chromosome reduplication is called mitosis*. In organisms with sexual reproduction, a more specialized kind of cell division called* meiosis *takes place in the organs where* gametes *(specialized reproductive cells, such as eggs and sperm) are produced.*

In this second process there are two cell divisions. In the first the chromosome pairs are divided in a random fashion so that each of the two new cells receives one number of each original pair, or exactly half the number of chromosomes found in the original cell. Then these two cells reproduce themselves exactly, each of which carries half the number of chromosomes necessary for the final organism.

When two gametes meet in the process of fertilization*, a new organism is produced, one that receives half of its chromosomes from each of the two parents. Thus, meiosis and subsequent fertilization provide a means of interchanging genetic material between organisms, whereas mitosis provides a means of exactly duplicating cells within an individual organism. In man, all cell divisions are achieved by mitosis except those in the testes of the male and ovaries of the female where sperm and eggs are produced by meiosis.*

The chromosomes are the larger units of heredity, but it is the smaller genes that ultimately determine the nature of the individual organism. Genes may exist in different expressions, called alleles*, each of which leads to a different hereditary result in the adult organism. In man, for example, blue eyes are the result of one eye-color allele, whereas brown eyes result from another. Recall that chromosomes normally occur in pairs. It is a fundamental fact of heredity that paired chromosomes have analogous sets of genes that control the development of the same structures in the adult organism.*

Cells produced by mitosis therefore have two complete sets of genes, one set contained in each member of the paired chromosomes. Such cells are called diploid *cells. In meiosis, on the other hand, only one member of each chromosome pairs is transmitted to the daughter cell, which thus has only half as many chromosomes as the diploid parent cell. Such cells are called* haploid *cells. Haploid cells contain* one complete set *of genetic instructions, whereas diploid cells contain* two complete sets*. Diploid cells, with their double set of chromosomes, may have the same or different alleles for a particular gene in each set. In man, for example, an individual may have the blue-eye allele in both chromosomes of the pair which contains the eye-color gene, or he may have the brown-eye allele in both chromosomes, or, finally, he might have the brown-eye allele in one chromosome and the blue-eye in the other.*

If both chromosomes contain the same allele, then all gametes produced by dividing the pairs during meiosis will also have the same eye-color allele. Individuals with the same allele on both chromosomes thus produce only one kind of gamete and are said to be homozygous *for that particular gene. When the two chromosomes of a pair contain different alleles, the gametes produced by dividing the pairs will be mixed; half will contain the blue-eye allele and half the brown-eye allele. In this case the individual is said to be* heterozygous *for that particular gene.*

Note that each character of the adult organism is not always controlled by a single gene. Some few characters, such as human eye color, are inherited in this simple way, but most characters are determined by the combined effects of many genes. As the eye-color example shows, predictions about the effects of differing combinations of alleles on the next generation are easily made when only a single gene is responsible for a character, but predictions become increasingly difficult when more genes, each with differing alleles, are involved.

We now return to our evolutionary theme and consider the actual causes if the variations found among individuals of the same species. Such variations are of two kinds: those due to heredity, and those due to environmental influences operating during the lifetime of the individual. In man, differences in eye color are hereditary variations, whereas the differences in muscle size between an athlete and an office worker of comparable physical build would be an example of variation caused by environmental influences.

Only those variations caused by hereditary differences are important in evolution, for they alone can be passed on to the next generation. The children of the athlete will inherit their eye color from him, but they will not have his physique if they choose to become office workers.

Inherited variations, in turn, result from two interrelated processes: mutation and genetic recombination. Mutation *is the sudden, spontaneous appearance of a new allele for a particular gene or group of genes. Apparently,* *mutations are continuously taking place in all organisms, but normally they occur at a very low rate. In the fruit-fly genus* Drosophila, *which has long been a favorite animal group for genetic study, there is about one gene mutation for every 20 gametes produced. Because each gamete includes about 20,000 genes, the rate of mutation is only about one per 400,000 genes. Mutations may have little or no effect on the adult organism; some, however, are lethal, while others lead to small but advantageous changes. The causes for mutations are obscure, but in most organisms the rate of production can be changed (usually accelerated) by artificial exposure to certain kinds of radiation (gamma, ultraviolet, cosmic), to various chemicals, or to changes in temperature. Apparently, these agents alter the chemical structure of the DNA which makes up the genes, thus producing new alleles.*

Mutations are the only source for new alleles but are too rare to be directly responsible for most of the constantly appearing variation found in individuals of the same species. These variations result from simple recombination, *during meiosis and fertilization, of the alleles already present in the parent organisms. Because the chromosomes of most kinds of organisms contain tens of thousands of genes, each of which may have several alleles, there are almost limitless possibilities for recombination of alleles to produce individuals with differing genetic patterns. It is these differing patterns that lead to the variations seen in adult organisms of the same species.*

The genotype *of any organism is total makeup of its genetic material, i.e the particular set of* genes *it possesses. Two organisms whose genes differ at even one position of their genome are said to have different genotypes. The term "genotype" refers, then, to the full hereditary information of an organism.*

The phenotype *of an organism, represent its actual physical properties, such as height, weight, hair color, and so on.*

Mendelian genetics has shown that because of the dominance of some characters, organism with the same phenotype may have different genotypes. Only by studying their offsprings can we distinguish between them. Likewise, organisms with identical genotype differ in their phenotypes (e.g identical twins).

Biological evolution seems to violate our common sense awareness that in general, disorder tends to increase as time passes. *All around us, we see configurations of matter move from order to disorder, from improbable states to more probable ones. Paint weathers; rocks crumble; iron rusts; wood decays; stars radiate away their energy. But here on Earth, life continues to combine elements into specific molecules and monomers into lengthy polymers, making ever-greater complexity and order from simplicity and disorder.*

A single DNA molecule, which may be a million times longer than it is wide, represents an exceedingly nonrandom bit of matter. Such molecules store the tremendous amounts of information needed to carry out life's activities, information that can be preserved undamaged through thousands of replications. Hence, a great degree of order is required to keep matter alive. This order not only persists but has actually increased *as life has evolved to ever more complex forms on earth.*

How can we explain this apparent contradiction – the maintenance, and even the increase, in the order and complexity of life in a universe that is inexorably evolving toward increasing disorder? The resolution lies in a consideration of the total system, of which life is just one part.

Life on earth does not form a closed system. Instead, life can maintain its highly improbable configuration only at the expense of its environment; that is, life can become highly organized only by increasing the disorganization of its surroundings. The disorder of the total system increases, *while the disorder of living creatures within it* decreases. *Here "disorder" refers not to pollution but to the way that life acquires the energy it needs.*

Molecular evolution *is the process of evolution at the scale of DNA, RNA, and* proteins. *Molecular evolution emerged as a scientific field in the 1960's as researchers from* molecular biology, evolutionary biology *and* population genetics *sought to understand recent discoveries on the structure and function of nucleic acids and protein. Some of the key topics that spurred development of the field have been the evolution of enzyme function, the use of nucleic acid divergence as a "*molecular clock*" to study species divergence, and the origin of non-functional or* junk DNA.

Recent advances in genomics, including whole-genome sequencing, high-throughput protein characterization, and bioinformatics have led to a dramatic increase in studies on the topic. In the 2000s, some of the active topics have been the role of gene duplication *in the emergence of novel gene function, the extent of adaptive molecular evolution versus neutral drift, and the identification of molecular changes responsible for various human characteristics especially those pertaining to* infection, disease, *and* cognition.

Mutations are permanent, transmissible *changes to the* genetic material *(usually DNA or RNA) of a cell. Mutations can be caused by copying errors in the genetic material during* cell division *and by exposure to* radiation, *chemicals, or* viruses, *or can occur deliberately under cellular control during the processes such as* meiosis *or* hypermutation. *Mutations are considered the driving force of* evolution, *where less favorable (or* deleterious*) mutations are removed from the gene pool by* natural selection, *while more favorable (or* beneficial*) ones tend to accumulate.* Neutral mutations *do not affect the organism's chances of survival in its natural environment and can accumulate* *over time, which might result in what is known as punctuated equilibrium; the modern interpretation of classic evolutionary theory.*

There are four known processes that affect the survival of a characteristic; or, more specifically, the frequency of an allele;

- *Mutation*
- *Genetic drift describes changes in gene frequency that cannot be ascribed to selective pressures, but are due instead to events that are unrelated to inherited traits. This is especially important in small mating populations, which simply cannot have enough offspring to maintain the same gene distribution as the parental generation.*
- *Gene flow: or gene admixture is the only one of the agents that makes populations closer genetically while building larger gene pools.*
- *Selection, in particular natural selection produced by differential mortality and fertility. Differential mortality is the survival rate of individuals before their reproductive age. If they survive, they are then selected further by differential fertility – that is, their total genetic contribution to the next generation. In this way, the alleles that these surviving individuals contribute to the gene pool will increase the frequency of those alleles. Sexual selection, the attraction between mates that results from two genes, one for a feature and the other determining a preference for that feature, is also very important.*

The production and redistribution of variation is produced mostly by three of the four agents of evolution: mutation, genetic drift, and gene flow. Natural selection, in turn, acts on the variation produced by these agents. One important goal is to understand (using both data from molecular biology and theory from population genetics) the main force driving molecular evolution.

*Currently, three main positions are defended. (1) Neutralism and near-neutralism (**Kimura**, 1983), where neutral or nearly-neutral mutations, along with random genetic drift and purifying selection, explain most of evolution. (2) Selectionism, where balancing selection is considered the main force, and finally, (3) mutationism, where mutational input and random genetic drift are thought to be more important.*

Population genetics is the study of changes in gene allele frequency distribution within interbreeding populations under the influence of the four evolutionary forces: natural selection, genetic drift, mutation, and migration. It also takes account of population subdivision and population structure in *space. As such, it attempts to explain such phenomena as* adaptation *and* speciation. *Population genetics was a vital ingredient in the* modern evolutionary synthesis, *its primary founders were* ***Sewall Wright***, ***J.B.S. Haldane*** *and* ***R.A. Fisher***, *who also laid the foundations for the related discipline of* quantitative genetics.

What was perhaps the most significant discovery concerning such changes was made independently in 1908 by ***G. Weinberg***, *a German geneticist, and* ***G.H. Hardy***, *a British mathematician, and has come to be known as the* Hardy-Weinberg law. *Hardy and Weinberg demonstrated by simple algebra that the* relative proportion of alleles within a randomly interbreeding population will remain constant unless outside forces work to change it. *Intuitively, one would assume that rare alleles would gradually be lost from the population, and that common would tend to become more common. Instead, Hardy and Weinberg showed that there is a natural* genetic equilibrium *which can preserve even the least common alleles in a randomly interbreeding population. Only through nonrandom processes, such as nutation or selective reproduction, can the proportion of rare alleles be increased or the proportion of common alleles decreased.*

This discovery was particularly significant in re-establishing natural selection as an evolutionary mechanism, because, as you will recall, the basis of natural selection is *nonrandom reproduction. Not every individual, but, on the average, more of the better fitted, will survive to produce the next generation. Natural selection is therefore an ideal mechanism for explaining changes in allele frequency and shift away from genetic equilibrium.*

Building on the Hardy-Weinberg law, *modern population geneticists have developed a body of refined mathematical models, often devised with help of high-speed computers, to stimulate changes in gene frequencies by natural selection in populations that differ in such features as selection pressures, size, original allele ratios, reproductive habits, mutation rates, migration rates, and recombination patterns. In addition to these mathematical formulations, there is now a large body of observational and experimental evidence that confirms the importance of natural selection as a means of changing gene frequencies.*

(c) Non–Darwinian Evolution; Genetic Drift (1968–1983)

In order to follow the changes in life through billions of years, one must look more closely at the property of life that are most distinctive: the capacity to reproduce and to evolve. At the molecular level, life's ability to reproduce begins with the replication of DNA, during which two new spirals are created that are exact replicas of the origin molecule.

Sometimes a change in the sequence of nucleotide bases, called a mutation, *occurs in the DNA polymer. Such changes arise basically at random, sometimes from the impact on the DNA molecules of high-energy gamma rays or of cosmic-ray particles, or from exposure to various chemical agents called* mutagens, *or even from rare* errors *made by cell's own DNA-coping machinery. We do not know which of these processes predominates in causing mutations throughout the history of life on earth. When a mutation arises in a part of the DNA where information for a protein is encoded, it can cause a different, "incorrect" amino acid to be inserted into the protein under construction.*

Many mutations are neither helpful nor hurtful. They are simply called "neutral" *mutations. If the new protein does its cellular job poorly, the organism may be less fit for survival and reproduction, or it might not survive at all. On other occasions, mutations may actually change a protein in such a way that it does its job* better *than the original protein did. The lucky organism with this mutation would have some advantage over its fellows – perhaps it can replicate its DNA more quickly, swim more rapidly, sense food more efficiently, have more brightly colored feathers, or smell predators at a greater distance.*

Such advantages would give the organism greater reproductive success than its fellows: Because they are healthier, more resistant to cold, prettier, better at escaping from predators, or superior for some other reasons, the more fit organisms will (by definition) produce more surviving offspring than their less fit relatives. As a result, organisms carrying favorable mutations will, over time, come to predominant in a population. "Differential reproduction" *– the greater or lesser success that organisms achieve in producing offspring – lies at the heart of the process called* natural selection.

Differential reproduction determines whether a given mutation becomes established, or "fixed" in the general population. Thus natural selection, operating through differential reproductive success, causes the characteristics of a species gradually to change when advantageous, or "adaptive", mutations sweep through the population. In this way, differential reproduction allows one species to evolve into a new species.

Sometimes groups within a species become isolated from each other for many generations. When this happens, different mutations, appearing at random, become fixed in the separated populations. Gradually, the populations differ more and more from each other. When populations differ significantly (usually, when they can no longer interbreed to produce fertile offspring) we call them two separate species.

In 1839, ***Charles Darwin*** *noticed such changes among bird species in the Galápagos Islands. Darwin studied populations of birds resembling, but not identical to, those that he knew well from his native England. He found* *different species and subspecies of finches on the different islands that he visited, and his speculations about how the differences arose led to his theory of evolution, first published in 1859. In this epochal work, Darwin identified what we have called natural selection (differential reproductive success) as the driving force that makes new species on earth.*

During the past 40 years much progress in understanding the process of evolution has resulted from combining Darwin's mechanism of natural selection with the discoveries of geneticists concerning the inheritance of individual variations. Because of the renewed emphasis on natural selection, this modern synthesis is often referred to as Neo-Darwinism. A fundamental theme of Neo-Darwinism has been the study of inheritance not merely in individual organisms, but in populations*, which are interbreeding groups of individuals of the same species. It is now recognized that mutation, recombination, and natural selection can lead to evolutionary change only as they act on such groups of individuals; this emphasis has given rise to the science of* population genetics*, a subject that has been responsible for many of the advances of Neo-Darwinism.*

The size of a population has important effects on allele frequencies because random events, or chance, tend to cause changes of relatively greater magnitude in a small populations. If a population consist of only a few individuals, an allele present at a low frequency in the population could be completely lost by chance. Such an event would be unlikely in a large population. For example, consider two populations, one with 10,000 individuals and one with 10 individuals. If an uncommon allele occurs at a frequency of 10%, or 0.1, in both populations, then 1900 individuals in the large population have the allele.[31] *That same frequency, 0.1, in the smaller population means that only about two individuals have the allele.*[32] *From this exercise, it is easy to see that there is a greater likelihood of losing the rare allele from the smaller population than from the larger one. Predators, for example, might happen to kill one or more individuals possessing the uncommon allele in the smaller population purely by chance so these individuals would leave no offspring.*

The production of random evolutionary changes in small breeding populations is known as genetic drift*. Genetic drift result in changes in allele frequencies in a population from one generation to another. One allele may be eliminated from the population purely by chance, regardless of whether that allele is beneficial, harmful, or of no particular advantage or disadvantage. Thus, genetic drift decreases genetic variation* within *a population, although it tends to increase genetic differences* among *different populations. Because*

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[31] $2pq + q^2 = 2(0.9)(0.1) + (0.1)^2 = 0.18 + 0.01 = 0.19; \quad 0.19 \times 10,000 = 1900$

[32] $0.19 \times 10 = 1.9$

of fluctuations in the environment, such as depletion in food supply or an outbreak of disease, a population may rapidly and markedly decrease from time to time. The population is said to go through a bottleneck *during which genetic drift can occur in the small population of survivors. As the population again increases in size, many allele frequencies may be quite different from those in the population preceding the decline.*

Scientists hypothesize that genetic variation in the cheetah was considerably reduced by a bottleneck at the end of the last Ice Age, some 10,000 years ago, Cheetahs nearly became extinct, perhaps from overhunting by humans. The few surviving cheetahs had greatly reduced genetic variability, and as a result, the cheetah population today is so genetically uniform that unrelated cheetahs can accept skin grafts from one another. (Normally, only identical twins accept skin grafts so readily.)

Whereas natural selection *describes the tendency of beneficial alleles to become more common over time (and detrimental ones less common), genetic drift refers to the fundamental tendency of any allele to vary randomly in frequency over time due to statistical variation alone, so long as it does not comprise all or none of the distribution.*

Genetic drift may be modeled as a stochastic process *that arises from the role of random sampling in the production of offspring. The genes of each new generation are not a simple copy of the genes of the successful members of the previous one, but rather a sampling, which includes some* statistical error*. Drift is the cumulative effect over time of this sampling error on the* allele frequencies *in the population.*

By definition, genetic drift has no preferred direction. A neutral allele may be expected to increase or decrease in any given generation with equal probability. Given sufficiently long time, however, the mathematics of genetic drift predict the allele will either die out or be present in 100% of the population, after which time there is no random variation in the associated gene*. In this regard, genetic drift tends to sweep gene variants out of a population over time, such that all members of a species would eventually be* homozygous *for this gene. Genetic drift is opposed in this regard by* genetic mutation *which introduces novel variants into the population according to its own random processes.*

Like selection, genetic drift acts on populations, altering the frequency of alleles (gene variations) and the predominance of traits*. Drift is observed most strongly in* small populations *and results in changes that need not be* adaptive*.*

Similarly, in a breeding population, if an allele has a frequency of p, probability theory dictates that (if natural selection is not acting) in the following *generation, a fraction p of the population will inherit that particular allele. However, allele frequencies in real populations are not probability distributions; rather, they are a random sample, and are thus subject to the same statistical fluctuations.*

When the alleles of a gene do not differ with regard to fitness, on average the number of carriers in one generation is proportional to the number of carriers in the previous generation. But the average is never tallied, because each generation parents the next one only once. Therefore the frequency of an allele among the offspring often differs from its frequency in the parent generation. In the offspring generation, the allele might therefore have a frequency p', slightly different from p. In this situation, the allele frequencies are said to have drifted. Note that the frequency of the allele in subsequent generations will now be determined by the new frequency p'.

The size of the breeding population (the effective population size*) governs the strength of the drift effect. When the effective population size is small, genetic drift will be stronger.*

Drifting alleles usually have a finite lifetime. As the frequency of an allele drifts up and down over successive generations, eventually it drifts until fixation – that is, it either reaches a frequency of zero, and disappears from the population, or it reaches a frequency of 100% and becomes the only allele in the population. Subsequent to the latter event, the allele frequency can only change by the introduction of a new allele by a new mutation*.*

The lifetime of an allele is governed by the effective population size. In a very small population, only a few generations might be required for genetic drift to result in fixation. In a large population, it would take many more generations. On average, an allele will be fixed in $4N_e$ *generations, where* N_e *is the effective population size.*

According to the Hardy-Weinberg Principle*, which holds that allele frequencies in a gene pool will not change over time, a population must be sufficiently large to prevent genetic drift from changing allele frequencies over time. This is why the law is unstable in a small population.*

Genetic drift and natural selection *rarely occur in isolation from each other; both forces are always at play in a population. However, the degree to which alleles are affected by drift and selection varies according to circumstance.*

In a large population, where genetic drift occurs very slowly, even weak selection on an allele will push its frequency upwards or downwards (depending on whether the allele is beneficial or harmful). However, if the population is very small, drift will predominate. In this case, weak selective effects may not be seen at all as the small changes in frequency they would produce are overshadowed by drift.

Drift can have profound and often bizarre effects on the evolutionary history of a population. These effects may be at odds with the survival of the population.

In a population bottleneck, *where the population suddenly contracts to a small size (believed to have occurred in the history of human evolution), genetic drift can result in sudden and dramatic changes in allele frequency that occur independently of selection. In such instances, many beneficial adaptations may be eliminated even if population later grows large again.*

Similarly, migrating populations may see founder's effect, *where a few individuals with a rare allele in the originating generation can produce a population that has allele frequencies that seem at odds with natural selection. Founder's effects are sometimes held to be responsible for high frequencies of some genetic diseases. The mathematical foundation of the* neutral theory of evolution *was first promulgated in 1968 by* ***Motoo Kimura*** *(1924–1994, Japan). It challenged the notion that natural selection was the sole directive force in evolution. Arguing that mutations and random drift account for variations at the level of DNA and amino acids, Kimura advanced a theory of evolutional change. The crux of his theory is in the motion that most genetic diversity is there because* it makes no difference, *not because it has been picked by natural selection for a purpose. Mutation pumps a continual stream of genetic changes that do not affect anything into the gene pool, and that they are gradually purged again by genetic drift – a random change. So there is constant turnover* without adaptive significance.

According to Kimura, when one compares the genomes *of existing species, the vast majority of molecular differences are selectively "neutral." That is, these differences do not influence the* fitness *of either the species or the individuals who make up the species. As a result, the theory regards these genome features as neither subject to, nor explicable by, natural selection. This view is based in part on the* degenerate genetic code, *in which sequences of three nucleotides (*codons*) may differ and yet encode the same* amino acid *(GCC and GCA both encode* alanine, *for example). Consequently, many potential single-nucleotide changes are in effect "silent" or "unexpressed"*

Such changes are presumed to have little or no biological effect. However, it should be noted that the original theory was based on the consistency in rates of amino acid changes, and hypothesized that the majority of those changes too were neutral.

A second assertion or hypothesis of the neutral theory is that most evolutionary change is the result of genetic drift *acting on neutral* alleles. *A new allele arises typically through the spontaneous* mutation *of a single nucleotide* *within the sequence of a gene. In single-celled organisms, such an event immediately contributes a new allele to the population, and this allele is subject to drift.*

In sexually reproducing multicellular organisms, the nucleotide substitution must arise within one of the many sex cells that an individual carries. Then only if that sex cell participates in the genesis of an embryo and offspring, does the mutation contribute a new allele to the population. Neutral substitutions create new neutral alleles.

Through drift, these new alleles may become more common within the population. They may subsequently decline and disappear, or in rare cases they may become "fixed" – meaning that their substitution becomes a universal feature of the population or species. When an allele carrying one of these new substitutions becomes fixed, the effect is to add a substitution to the sequence of the previously fixed allele. In this way, neutral substitutions tend to accumulate, and genomes tend to evolve.[33]

The process of change of the frequency of a gene over time in a large random-mating population can be treated as a stochastic process and approximated by a diffusion process. Kimura modeled the dynamic process of gene frequency change over time under different models for mutation and selection by making use of diffusion theory.

Consider a large random-mating population; two alleles exist at a locus with a selectively advantageous allele with frequency p_0 at generation 0. The probability density that the frequency of the favored allele is x at generation t, denoted by $\phi(x, t|p_0)$, satisfies the Kolmogorov forward equation.

The average change of allele frequency per generation under selection (with selection coefficient $s > 0$) is approximately $sx(1-x)$ according to a diffusion approximation if there is no dominance. The variance of the change of gene frequency due to random drift is $\frac{1}{2N}x(1-x)$ and $\phi(x, t|p_0)$ can be obtained by solving the following PDE,

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[33] As of the early *2000s*, the neutral theory is widely used as a "null model" for so-called *null hypothesis* testing. Researchers typically apply such a test when they already have an estimate of the amount of time that has passed since two species or lineages diverged – for example, from *radiocarbon dating* at *fossil* excavation sites, or from historical records in the case of human families. The test compares the actual number of differences between two sequences and the number that the neutral theory predicts given the independently estimated divergence time. If the actual number of differences is much less than the prediction, the null hypothesis has failed, and researchers may reasonably assume that *selection* has acted on the sequences in question. Thus such tests contribute to the ongoing investigation into the extent to which molecular evolution is neutral.

$$\frac{\partial\phi(x,t|p_0)}{\partial t}=\frac{1}{4N}\frac{\partial^2[x(1-x)\phi(x,t|p_o)]}{\partial x^2}-\frac{\partial[sx(1-x)\phi(x,t|p_o)]}{\partial x}$$

with boundaries $x=0$, and $x=1$ where N is the population size. Kimura obtained the explicit separation of variables solution,

$$\phi(x,t|p_0)=\sum_{k=1}^{\infty}C_k e^{-\frac{\lambda_k^2+c^2}{4N}t+2cx}\sum_{k=0,1}^{1}f_n^k\,T_n^1(1-2x), \qquad (1)$$

where
$r=1-2p_0$, $c=Ns$, $T_n^1=$ *the Gegenbauer polynomial,* λ_k *is the eigenvalue of the oblate spheroidal angular function,* f_n^k *is the intermediate coefficient of the spheroidal harmonics, and*

$$C_k=\frac{(1-r^2)e^{-c(1-r)}\sum'_{n=0,1}f_n^k\,T_n^1(r)}{\sum'_{n=0,1}((n+1)(n+2))/(2n+3)(f_n^k)^2},$$

The explicit solution of probability of fixation, or loss, of an allele by generation t, given the initial frequency p_0, can be derived by means of (1) and shown to be

$$f_0(t|p_0)=1-\frac{1-e^{-4cp_0}}{1-e^{-4c}}-\sum_{k=1}^{\infty}\frac{C_k}{\lambda_k+c^2}e^{-((\lambda_k+c^2)/4N)t}\sum_{n=0,1}{}'\frac{(n+1)(n+2)}{2}f_n^k,$$

and

$$f_1(t|p_0)=\frac{1-e^{-4cp_0}}{1-e^{-4c}}-\sum_{k=1}^{\infty}(-1)\frac{C_k}{\lambda_k+c^2}e^{-((\lambda_k+c^2)/4N)t+2c}\sum_{n=0,1}{}'\frac{(n+1)(n+2)}{2}f_n^k,$$

respectively. This is the solution to a genetic situation of selection without dominance complicated by random sampling. Kimura gave also an explicit solution to the problem of random drift only ($s=0$).[34] *To sum up, the neutral theory says that, through the history of life from beginning to end, random*

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[34] To dig deeper, see:

- Bharucha–Reid, A.T., *Elements of the Theory of Markov Processes and Their Applications*, McGraw-Hill Book Company: New York, 1960 *statistical fluctuations have been more important than Darwinian selection in causing species to evolve. Evolution by random statistical fluctuation is called genetic drift. Kimura maintains that genetic drift drives evolution more powerfully than natural selection.*

According to ***F. Dyson*** *(1985) Genetic drift and natural selection are both important, and there are times and places where one or the other may be dominant. In particular it is reasonable to suppose that genetic drift was dominant in the very earliest phase of biological evolution, before the mechanism of heredity had become exact.*

We know almost nothing about the origin of life. We do not even know whether the origin was gradual or sudden. It might have been a process of slow growth stretched out over millions of years or it might have been a single molecular event that happened in a fraction of a second. As a rule, natural selection is more important over long periods of time and genetic drift is more important over short periods. If one thinks of the origin of life as being slow, one must think of it as a Darwinian process driven by natural selection. If one thinks of it as being quick, then the Kimura picture of evolution by statistical fluctuation without selection is appropriate. In reality the origin of life must have been a complicated process, with incidents of rapid change separated by long periods of slow adaptation. A complete description needs to take into account both drift and selection.

If one wishes to examine seriously the double-origin hypothesis, the hypothesis that life began and flourished without the benefit of exact replication, then it is natural to imagine that genetic drift remained strong and natural selection remained relatively weak during the early exploratory phases of evolution. But this is not to say that Darwinian selection had to wait until life learned to replicate exactly. Darwinian selection is not logically dependent on exact replication. Indeed, Darwin himself knew nothing of exact replication when he invoked the idea of natural selection. Darwinian selection would have operated to guide the evolution of living creatures even at a time when those creatures may have lacked anything resembling a modern genetic apparatus. All that is necessary for natural selection to operate is that there be some inheritance of chemical constituents from an organism to its progeny. The inheritance need not be exact. It is sufficient if a cell splitting into two daughter cells transmits to each of its daughters with a high probability a population of molecules capable of continuing its own pattern of metabolism. Statistical inheritance, as Darwin well knew, can be good enough. Darwinian selection is unavoidable as soon as inheritance begins, no matter how sloppy the mechanism of inheritance may be.

Lewontin *(1974) outlined the theoretical task for population genetics. He imagined two spaces: a "genotypic space" and a "phenotypic space". The* *challenge of a complete theory of population genetics is to provide a set of laws that predictably map a population of genotypes (G1) to a phenotype space (P1), where selection takes place, and another set of laws that map the resulting population (P2) back to genotype space (G2) where Mendelian genetics can predict the next generation of genotypes, thus completing the cycle. Even leaving aside for the moment the non-Mendelian aspects revealed by molecular genetics, this is clearly a gargantuan task. Visualizing this transformation:*

$$G_1 \rightarrow^{T_1} P_1 \rightarrow^{T_2} P_2 \rightarrow^{T_3} G_2 \rightarrow^{T_4} G_1^{'} \rightarrow \ldots$$

T_1 represents the genetic and epigenetic laws, the aspects of functional biology, or development, that transform a genotype into phenotype. We will refer to this as the "genotype-phenotype map". T_2 is the transformation due to natural selection, T_3 are epigenetic relations that predict genotypes based on the selected phenotypes and finally T_4 the rules of Mendelian genetics.

In practice, there are two bodies of evolutionary theory that exist in parallel, traditional population genetics operating in the genotype space and the biometric theory used in plant and animal breeding, operating in phenotype space. The missing part is the mapping between the genotype and phenotype space. This leads to a "sleight of hand" (as Lewontin terms it) whereby variables in the equations of one domain, are considered parameters or constants, where, in a full-treatment they would be transformed themselves by the evolutionary process and are in reality functions of the state variables in the other domain. The "sleight of hand" is assuming that we know this mapping. Proceeding as if we do understand it is enough to analyze many cases of interest. For example, if the phenotype is almost one-to-one with genotype (sickle-cell disease) or the time-scale is sufficiently short, the "constants" can be treated as such; however, there are many situations where it is inaccurate.

(d) Panspermia and the Anthropic Principle

*Panspermia is the theory that life on earth was seeded by microbial life from space.There are several variations on this theme held by many historical advocates, including the Greek philosopher, **Anaxagoras** (500–428 BCE), **Hermann von Helmholtz** (1821–1894), and **William Thomson** (1824–1897).*

*More recently, **Svante Arrhenius** promulgated the theory of radio-panspermia, wherein microbes from space are transported by light pressure. **Fred Hoyle** and **Chandra Wickramasinghe** have advocated that DNA arrived on earth via meteorites (ballistic panspermia) or by comets (modern* *panspermia).* ***Francis Crick*** *has advocated the theory of directed panspermia, wherein RNA was transported by unmanned spaceships, or the space probes of intelligent extraterrestrial civilizations. He was led to argue for Panspermia by his belief that the chances of life accidentally originating on Earth were very low.*

Crick argued that the universality of the genetic code can only be explained by an "infective" theory of the origin of life. In this theory, life on earth would be a "clone" derived from a single set of organisms. Crick's radical theory of directed panspermia *suggests that RNA was the first replicator molecule on earth in an early biological era.*

The anthropic principle *is based on a biological argument: the minimum time required for the evolution of "intelligent observers." In this scheme, a billion years is required for the evolution of intelligence; therefore, a star must have been stable for at least that long. The anthropic timescale argument allows that the types of processes allowed in the Universe must be of such an age that "slow evolutionary processes will have had time to produce intelligent beings from non-living matter."*

Interestingly, the contemporary advocates for the existence of extraterrestrial intelligent life are primarily astronomers and physicists, while most leading experts in evolutionary biology contend that the earth is probably unique in harboring intelligence, and that human beings are alone in the universe because Darwinian evolution has told us so: intelligence was never there beforehand and can only be gotten through an incremental succession of steps involving pure random luck – an earth-based anomaly.

(e) Cosmology and Darwinian Evolution

The origin of life is one of the few scientific problems which is broad enough to make use of ideas from almost all scientific disciplines: ***E. Schrödinger*** *(1887–1961) brought his ideas from* physics; ***J. von Neumann*** *(1903–1957) from* mathematical logic; ***M. Eigen*** *(b. 1927) and* ***L. Orgel*** *(b. 1927) from* chemistry; ***L. Margulis*** *(b. 1938) from* ecology, *and* ***M. Kimura*** *(1924–1994) from* population biology.

Recently[35]*, Darwinian evolution was also linked to cosmology.*

[35] **Lee Smolin** (b. 1955) "The Life of the Cosmos", Oxford University Press, New York, 358 pp. 1997. A succinct popular account was given by John Gribbin in *Prospect* magazine, issue 19, May 1997.

If the universe began in the hot fireball of a big bang some 15 billion years ago, how did it evolve to produce galaxies and stars, planets and people?

The Big Bang theory does not provide a satisfying account for why the flying debris of the primordial blast congealed into stars and galaxies, and galaxies of galaxies – a hierarchy of structure. It is more likely that the universe would have turned out to be utterly random, a featureless fog.

More miraculous till is that the Big Bang seems to have produced a universe perfectly designed to support life. It is often argued that if gravity or electromagnetism were a little stronger or a little weaker, or if the nuclear forces were not precisely as they are, there would be no stars. And without stars, which cook hydrogen and helium into carbon and other complex atoms, there would be no chemistry, no biology, no complexity, no life. The question is "Why is the universe so interesting?", and the answer, given by Lee Smolin (1997) is:
"Since evolution so successfully explains why the biosphere is the way it is, why not apply the theory of Darwinian evolution by natural selection to the entire creation? – The universe is perfectly tuned to life because it evolved that way."

Smolin's thesis is then that the way the universe works can best be understood not simply by applying the rules of physics worked out by Newton and Einstein, but by taking account as well of the rules of evolution worked out by Darwin – the theory of natural selection.

The universe itself, and its main components (notably galaxies such as our own Milky Way) may have evolved through natural selection from a simpler state to produce the complexity we see around us. To take literally the equations of GTR, the big bang itself emerged from a point of infinite density, known as a singularity. There is, however, another place where singularities are known to occur – at the heart of black hole.

Indeed, as ***Roger Penrose*** *and* ***Stephen Hawking*** *proved in the 1960's, the expanding universe is described by exactly the same equations as a collapsing black hole, but with opposite direction of time.*

If all the complexity of galaxies, stars, planets and organic life has emerged from the singularity in which our universe was born within a black hole, could not something similar be happening to the singularities at the heart of other black holes?

The most basic view of what might happen to a collapsing singularity to turn it into the kind of expanding singularity that we see in our universe is that there is simply a "bounce," turning collapse into expansion. Unfortunately, that will not do as an explanation. A singularity forming from a collapse within our three dimensions of space and one of time cannot turn itself around *and explode back outwards in the same three dimensions of space and one of time.*

But in the 1980s relativists realized that there is nothing to stop the material that falls into a singularity from being shunted through a kind of spacetime warp and emerging as an expanding singularity in another set of dimensions — another spacetime.

Mathematically, this "new" spacetime is represented by a set of four dimensions (three of space and one of time), just like our own but with all of the new dimensions at right angles to all of the familiar dimensions of our own spacetime. Every singularity, on this picture, has its own set of spacetime dimensions, forming a bubble universe within the framework of some "super" spacetime, which we can refer to simply as "superspace."

One way to picture what this involves is to use the old analogy between the three dimensions of expanding space around us and the two-dimensional expanding surface of a balloon that is being steadily filled with air. The analogy is not with the volume of air inside the balloon, but with the expanding skin of the balloon, stretching uniformly in two dimensions, but curved around upon itself in a closed surface.

Imagine a black hole as forming from a tiny pimple on the surface of the balloon, a small piece of the stretching rubber that gets pinched off, and starts to expand in its own right. There is a new bubble, attached to the original balloon by a tiny, narrow throat – the black hole. And this new bubble can expand away happily in its own right, to become as big as the original balloon, or even bigger, without the skin of the original balloon (the original universe) being affected at all. There can be many bubbles growing out of the skin (the spacetime) of the original universe in this way at the same time. And, of course, new bubbles can grow out of the skin of each new universe, ad infinitum.

The dramatic implication is that many – perhaps all – of the black holes that form in our universe may be the seeds of new universes. And, of course, our own universe may have been born in this way out of a black hole in another universe. This means that the universe may not be unique. Instead, it may be one of a population of universes, interconnected by what physicists call wormholes. *The key element that Smolin has introduced into the argument is the idea that every time a black hole collapses into a singularity and* a new baby universe is formed, *the basic laws of physics are altered slightly as spacetime it-self is crushed out of existence and reshaped. The process is analogous (perhaps more than analogous) to the way mutations provide the variability among organic life forms on which natural selection can operate. Each baby universe is, says Smolin, not a replica of its parent, but a slightly mutated form.*

The original, natural state of such baby universes is to expand out to only about the Planck length, before collapsing once again. But if the random changes in the workings of the laws of physics-the mutations-happen to allow a little bit more expansion, a baby universe will grow a little larger. If it becomes big enough, it may separate into two, or several, different regions, that each collapse to make a new singularity, and thereby trigger the birth of a new universe. Those new universes will also be slightly different from their parents. Some may lose the ability to grow much larger than the Planck length and will fade back into the quantum foam. But some may have a little more inflation still than their parents, growing even larger, producing more black holes and giving birth to more baby universes in their turn. The number of new universes that are produced in each generation will be roughly proportional to the volume of the parent universe. There is even an element of competition involved, as the many baby universes are in some sense vying with one another, jostling for spacetime elbow room within superspace. Heredity is an essential feature of life, and this description of the evolution of universes works in a similar manner to living systems.

On this picture, universes pass on their characteristics to their offspring with only minor changes, just as people pass on their characteristics to their children with only minor changes. Universes that are "successful" are the ones that leave most offspring. Provided that the random mutations are indeed small, there will be a genuinely evolutionary process favoring larger and larger universes.

Once universes start to be big enough to allow stars to form, in succeeding generations of universes there will be a natural evolution, a drift in the laws of physics, to favor the production of the kinds of stars that will eventually form black holes.

The end product of this process should be not one but many universes which are all about as big as it is possible to get while still being inside a black hole, and in which the parameters of physics are such that the formation of stars and black holes is favored. Our universe exactly matches that description.

This explains the otherwise baffling mystery of why the universe we live in should be "set up" in what seems, at first sight, such an unusual way. Just as you would not expect a random collection of chemicals to suddenly organize themselves into a human being, so you would not expect a random collection of physical laws emerging from a singularity to give rise to a universe such as the one we live in.

To sum up, universes evolve in favor of the production of black holes. It is a theory of the origin of universes by means of natural selection. Smolin's answer to the question as why our universe is the way it is (i.e, the value of *its parameters) is: the parameters have the values we observe, because these make the formation of black holes much more likely than most other values. Hence, our universe is a product of mutation and selection analogous to the evolution of species described by Charles Darwin (1859).*

Cosmologists are now having to learn to think like biologists and ecologists, and to develop their ideas not within the context of a single, unique universe, but in the context of an evolving population of universes. Each universe starts from its own big bang, but all the universes are interconnected in complex ways by black hole "umbilical cords," and closely related universes share the "genetic" influence of a similar set of physical laws.

But the realization that our universe is just one among many, that it is alive and that no supernatural influences need be invoked to explain its existence, is still not the most dramatic conclusion we can draw from the new cosmology. Although it is now clear that the universe has not been set up for our benefit, and that the existence of organic life forms on Earth is a minor side effect of an evolutionary process involving universes, galaxies and stars, nevertheless it is clear that the existence of life forms such as ourselves is an inevitable side effect of those greater evolutionary processes.

The same laws of physics apply throughout our universe and throughout many other universes besides. Organic (carbon based) material occurs in profusion between the stars of a spiral galaxy such as our Milky Way. This carbon-rich material seems to be crucially involved in the processes which allow gas clouds to cool and new stars to form, so a universe that is good at making black holes will also be good at making carbon based compounds. Those compounds will undoubtedly seed any earth-like planet that forms with each new generation of stars.

Astronomers calculate that there may be as many as 10^{20} *planets suitable for life forms such as ourselves in our universe. We see the components of organic life everywhere in the universe, and the chances are that most of those* 10^{20} *planets actually are carriers of our kind of life, in the same way that earth is a carrier of life. The birth of the living universe inevitably gave rise to the birth of living planets. Which still leaves physicists the task of explaining just how complexity arose in a hot universe expanding out of a big bang.*

Note that in principle, life and Cosmological Natural Selection could be independent of each other. There are two reasons for this:

- *On the one hand there may be universes full of black holes were life as we know it couldn't evolve. For example it might be possible that there are only short-lived giant stars which collapse quickly into black holes,* *or that there are universes dominated either by helium or by neutrons (corresponding to the neutron/proton mass difference being either zero or negative), or that there are universes with many (more) primordial black holes and maybe without stars at all. Such universes might be very reproductive because of their giant stars or primordial black holes but are not able to produce earth-like life.*

- *On the other hand we can conceive a universe without black holes at all (if supernova lead to neutron stars only, or if there are not stars above a critical mass limit) but which could be rich in earth-like life nevertheless.*

Thus, there is not a (logically) necessary connection between black holes and life, unless, perhaps, black holes could be advantageous to life, or vice versa.

If Smolin is right, the implications for our view of nature would be enormous. Our universe would be only a grain of sand on the incredible large beach of the Multiverse. And it would be in no way special.

According to the Cosmological Principle, our universe is homogeneous and isotropic on the large scale, that is, it appears the same at all places and, from any one place, looks the same in all directions. If Smolin is right, we could accept a truly Perfect Cosmological Principle, that is, the Multiverse is literally "full" of universes like our own universe.

It has often been said that the Copernican revolution has catapulted the earth and hence, mankind, out of the center of the universe. In the twentieth century it became clear that neither the earth, nor the sun, nor the milky way, nor the local supercluster of galaxies are at the center of the universe, because there is no center at all. Nor does the baryonic matter of which we consist dominate the universe, i.e. in comparison with the assumed particles of dark matter even the material we are made of is quantitatively negligible.

This dramatic widening of our horizon and diminution of our role in the universe led to a radical cosmic expulsion. Our position in space-time is completely irrelevant, providing no evidence for a universal meaning or a cosmic value of mankind, nor a protection against contingency and absurdity.

In recent decades, some interpretations of the Anthropic Principle and the alleged fine-tuning of the physical parameters have been examples for a tendency to reverse this development. But the idea of the Multiverse made out of many different universes, which implies an Observational Selection effect, is once again sufficient to wipe out any romantic dreams of anthropocentrism.

But in this picture at least our life-bearing universe might still be special. In Smolin's account however, our universe has most properties in common with all other universes. It would be a very ordinary world indeed. Thus, if

Smolin is right, there is no reason to believe that (human) life is unique or that the constants of nature have a singular status.[36]

(F) CRITIQUE OF EVOLUTIONARY BIOLOGY (1978–2008)

Darwin's theory of evolution basically states that life on earth began with single-celled life-forms, which evolved into multicellular life-forms, which over countless aeons evolved into higher life-forms, including man – all as a result of the chance process of random mutation of desirable attributes followed by natural selection, without guidance or assistance from any intelligent entity. Thus, for example, evolution holds that the human eye came into existence purely by accident.

However, Darwin knew nothing of DNA and the vastly complex systems studied by molecular biologists, such as the information processing, storage, and retrieval in DNA.

Moreover, until recently, scientists did not know what the inside of a cell looked like: the cell was a mysterious "black box".

Michael Behe[37] *(1996) used discoveries in microbiology to cast serious doubts upon Darwinism: for example, a bacterial motor, that propels bacteria*

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[36] To dig deeper, see:

- Schrödinger, E., *What is Life*, Cambridge University Press, 1944
- Jacob, Francois, *The Possible and the Actual*, Pantheon Books: New York, 1982
- Hoyle, Fred, *The Intelligent Universe*, 1983
- Dyson, F., *Origins of life*, Cambridge University Press, 1985
- Barrow, J.D. and F.J. Tipler, *The Anthropic Cosmological Principle*, Oxford University Press: New York, 1986
- Kimura, M., *The Neutral Theory of Molecular Evolution*, 1986
- Smith, J. Maynard, *The Theory of Evolution*, 1997
- Rees, Martin, *Just Six Numbers*, Basic Book: New York, 2000

[37] **Michael J. Behe** (b. 1952, USA), biochemist. Advocates the idea that some structures are too complex at the biochemical level to be adequately explained as a result of evolutionary mechanisms and that these systems could not, even in principle, have evolved by natural selection. This is because the calculated

and sperm, called a flagellum, depends on the coordinated interaction of 30–40 complex protein parts. The absence of almost any one of the parts would render the flagellum useless. Likewise, an animal cell's whiplike oar, called a cilium, is composed of about 200 protein parts.

Now, it does not matter if 200 mutations happened a once or over a billion years. All 200 mutations would have to (1) occur, (2) be the "most fit", (3) survive long enough to exist at the same time and place, in order to (4) assemble themselves into a working cilium. The cell is as complicated as the entire city of New York. Natural selection has never been demonstrated to change anything fancier than the shape of a bird's beak.

Behe then argued that it is extremely unlikely for all 30 parts of the flagellum (or 200 parts of the cilium) to have been brought together by numerous, successive, slight modifications primed by natural selection. He then concluded that life at the molecular level is a "loud, clear, piercing cry of design".

Hence the contention that natural processes of mutation and natural selection cannot explain the complexity of living things.

In fact, the more we know about molecules, cells and DNA, the less plausible Darwin's theory of natural selection becomes. Indeed, Darwin himself

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probabilities of mutations required for evolution to succeed are too small. He termed this concept "*irreducible complexity*" [*Darwin's Black Box*, Free Press, 1996]. He argues that the eye, or the bacterial flagellum are nanotechnological machines and cannot have evolved by any number, however large, of small mutations that each confer reproductive advantage.

Behe's claims about the irreducible complexity of key cellular structures are strongly contested by the community of Darwin followers, and his claims about "intelligent design" have been characterized as pseudoscience. His adversaries have also pointed out that he offered no design theory, or attempted to model the design process, and in general failed to offer an alternative to evolution.

However, Behe's critics themselves tend to evade his (quite reasonable) statistical arguments; Furthermore, adducing an alternative theory is not a precondition for pointing out potentially fatal flaws in a widely accepted theory or dogma. The truth is that both sides in this debate have not the slightest idea how to compute the relevant probabilities; and that the neo-Darwinian synthesis is not really a quantitative scientific theory-since it consists entirely of non-refutable, non-quantitative *scenarios* (what the Harvard scientist R. Lewontin has duffed "just-so stories").

noted the difficulty of explaining the eye in The Origin of the Species, admitting he could not do it.[38]

In his book "The Design Inference" (Cambridge University Press, 1998) ***William A. Dembski*** *(b.1960) added his doctrine of Specified Complexity to Behe's assertion that irreducibly complex systems cannot evolve gradually. Again, Dembski's work was strongly criticized within the scientific community, who argued that there were a number of major logical inconsistencies and evidential gaps in Dembski's hypothesis. His writings were labeled as pseudoscience.*[39]

Evolutionary biologists argue that material mechanisms suffice to account for biological complexity, while intelligent design advocates reject this claim. Both sides are trying to determine the truth of some definite matter of fact – whether life is the result of mindless material mechanisms or whether, to the contrary, it demonstrably points to a designing intelligence.

A prominent critic of Evolutionary Biology from inside the scientific establishment is ***Richard Charles Lewontin*** *(b. 1929, USA), Alexander Agassiz Professor at Harvard. In "Biology and Ideology" (1991), Lewontin argued that while traditional Darwinism has portrayed the organism as passive receiver of environment influences, a correct understanding should emphasize*

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[38] Darwin hypothesized that the eye might have begun as a patch of light- sensitive cells upon which natural selection could then work its magic, making gradual improvements - creating an eye socket and slowly increasing focus and perspective and so on – until these special cells became a full – fledged eye. But this "explanation" explains nothing – it is just a story about how something might have happened: for light sensitive cells to work the cells would have to have the capacity to initiate an electric signal, a nerve capable of carrying the electric signal to a brain, and a brain capable of processing the signal and using it to emit other electric signals. No one disputes that organisms can develop small improvements on something that already exist. The interesting question is: How did the "light sensitive cells" come to exist in the first place?

[39] The mark of a *pseudoscience* is not that it is false but, in the words of physicist **Wolfgang Pauli**, that it is "*not even false*". In other words, with a pseudoscience there is no way to decide whether it is true or false.
Recently, many physicists and mathematicians have accused *string theory* of having become a pseudoscience. Psychoanalysis is doubtless in this category; and so is the so-called neo-Darwinian synthesis.
At present, the latter tells us various often-shifting, verbal, qualitative stories, albeit buttressed by results from exact sciences such as biochemistry & molecular genetics (as well as much less precise findings from fossil records). It is conceivable that one day evolution will truly merit the term "theory".

the organism as an active *constructor of its environment. Niches are not preformed, empty receptacles into which organisms are inserted, but are defined and created by organisms.*

Lewontin has also been a critic of traditional neo-Darwinian approaches to adaptation: *he emphasized the need to give an engineering characterization of adaptation separate from measurement of number of offspring. This grew out of his recognition that the fallacies of* sociobiology *reflect fundamentally flawed assumptions of adaptiveness of all traits in much of the modern evolutionary synthesis.*

Along with others, Lewontin has been a persistent critic of some themes in neo-Darwinism such as sociobiology *and* evolutionary psychology, *which attempt to explain animal behavior and social structures in terms of evolutionary advantage or strategy.*

Darwin *thought that "the mind of man developed from a mind as low as that possessed by the lowest animal". But if our abilities "evolved to let us get along in the cave, how can it be that they permit us to obtain deep insight into cosmology, elementary particles, molecular genetics, number theory?" asks molecular biologist and physicist* **Max Delbrück**.

None of these abstract enterprises has any direct relation to survival. Indeed, mans possession of such traits as morality, consciousness of morality, religion, or the ability to create & appreciate art – let alone his capacity to develop such intellectual constructs as relativity and quantum theory, does not seem likely to be the result of gradualistic evolution.

Darwinists and neo-Darwinists believe that humans evolved from bacteria. But for these who believe in intelligent design, this challenges our ideas of individuality, independence and the alleged uniqueness of human intelligent consciousness.

When Darwin first published The Origin of the Species *(1859), his most virulent opponents were paleontologists, for there was absolutely nothing in the fossil record to support his claims. Far from showing gradual change with one species slowly giving way to another, as Darwin hypothesized, the fossil record showed vast numbers of new species suddenly appearing out of nowhere, remaining largely unchanged for millions of years, and then disappearing.*

Darwin blamed the absence of fossil support for his theory on the extreme imperfection of the geological record, but was sure that paleontologists will soon produce the necessary evidence.

However, after 150 years of intense looking, the geologic record still does not yield a finely graduated chain of slow and progressive evolution!

If mutations are utterly random, as Darwinism claims, there ought to be an infinite variety of transitional animals *with small mutations that eventually led to a new attribute (like a wing or a lung). But we do* not *have fossils connecting the extinct to the extant along fine graduated steps. What the fossil record shows is sudden bursts of all manner of animals, modest change, and then sudden and total extinction. Dinosaurs appeared, lived for 150 million years, and then disappeared, only to be quickly replaced with mammals.*

We don't have fossils for the vast quantity of hapless creatures that ought to have died out in the survival-of-the fittest regime! If each one of the incremental mutations is more "fit" than what preceded it (which it had to be in order to survive), those transitional mutations should have stayed around long enough to appear in the fossil record, before mutating their way to something even better. But in the course of millions and millions of years, all we see are slight variations on the final product.

Indeed, Darwinian evolution is supposedly the completely accidental process *that created butterfly wings, bat radar, the human brain, and the millions of species alive today. The theory of evolution requires millions of mutations just to create an eye. A process that is supposed to have transformed an amoeba into Wolfgang Amadeus Mozart or Albert Einstein by "random mutations" must have produced some spectacular failures. Why can't we find any of them?*

For over a hundred years, evolutionists proudly pointed to the same sad birdlike animal, archaeopteryx, as their lone transitional fossil linking dinosaurs and birds. Discovered a few years after Darwin published The Origin of Species*, Archaeopteryx was instantly hailed as the transitional species that proved Darwin's theory. This unfortunate creature had wings, feathers, teeth, claws, and a long, bony tail. If it flew at all, it didn't fly very well. Alas, it is now agreed that poor Archaeopteryx is no relation of modern birds. It's just a dead end. It transitioned to nothing.*

But could Archaeopteryx be our one example of bad mutations eliminated by natural selection? Archaeopteryx can't fill that role either, because it seems to have no predecessors. The fossils that look like Archaeopteryx lived millions of years after Archaeopteryx, and the fossils that preceded Archaeopteryx look nothing at all like it. The bizarre bird is just an odd creation that came out of nowhere and went nowhere.

The more advances paleontologists make in uncovering the fossil record, the more absurd the evolution fable becomes.Most nettlesome for evolutionists is the Cambrian period, showing a vast quantity of plants and animals appearing on the scene in the blink of an evolutionary eye more than 500 million years ago. In a period of less than 10 million years, there is a sudden explosion of nearly all the animal phyla we have today. It is as though they *were just planted there, without evolutionary history. Darwin himself referred to the great difficulty of explaining the absence of "vast piles of strata rich in fossils" before the Cambrian explosion.*

In 1984, Chinese paleontologists discovered fossils just preceding the Cambrian era. The discovery showed that the dramatic transformation of life from primeval single-cell organisms to the complex multicellular precursors of modern fauna was sudden, swift and widespread within a mere 5 to 10 million years. Even the famously difficult-to-evolve eye appeared at the beginning of the Cambrian period. And there were no light-sensitive pits.

It seems now that traditional Darwinian evolution is a conjecture about how species might have arisen that is contradicted by the fossils record and by nearly everything we have learned about molecular biology since Darwin's day.

To 'save the phenomenon', modern revisionists of the Darwinian "theory" have concocted a sophisticated scheme called "punctuated equilibrium" *(**Stephen Jay Gould** and **Niles Eldredge**, 1993): Instead of gradual change occurring by random mutation and natural selection choosing the most "fit" to survive and reproduce, evolution could also happen really fast and then stop happening at all for 150 million years – all this occurring completely by chance!*

Darwinian evolution theory also run into other difficulties. To begin with, there is not a single observable example *of one species evolving into another by the Darwinian mechanism of variation and selection.*

Then, there is the problem of establishing progress *via mutation and natural selection: the successive appearance of more complex species does seem to show something that looks like progress. But that has nothing to do with the Darwinian mechanism of natural selection. The appearance of progress hardly establishes mutation and natural selection as the engine of change. To the contrary, the similarities, look more like the progress of a* designed object *than the result of a series of lucky accidents. Far from the competition of dog-eat-dog struggle to survive, we see a fossil record that reveals a rather clean, well-organized sequence.*

Fossils do not reveal a parent/descendal relationship. We certainly do not know whether any particular mammal descended from any particular reptile. But more important, the apparent progress from simple animals to more sophisticated higher animals – with no transitional species – looks more like planned, deliberate progress *than a series of random mutations.*

In this connection, another nasty question can be asked: if all species evolved from the same single-celled organism beginning in the same little mud puddle, why hasn't the earthworm made a little more progress? Was *it never, ever desirable in any of the worm's many dirt holes to mutate eyes or legs or wings or a brain? How could one clump of cells starting in the same little puddle become a human being while others never make it past the amoeba stage?*

In 1835, Darwin counted 13 species of finches on the Galapagos Islands. He then hypothesized that species evolved from one species.

Today, after more than 170 years of wild variation in the environment, mutation, and "natural selection", there are still 13 species – not one more. The finches beaks have moved back and forth in shape and nothing more.

In her book "Godless" (2006), Ann Coulter succinctly summarized the many setbacks, hoaxes, pranks, fakeries and frauds that beset the theory of evolution: "Finches on Galapagos Islands with deeper beaks begin to outnumber finches with shallower beaks during a drought – and then the population of shallow beaks finches immediately rebounds after a rainy season. Bacteria develop a resistance to antibiotics and viruses develop resistance to antiviral medication – but nothing new is ever created. A bacterium remains a bacterium, a virus remains a virus, a finch remains a finch"...

"Human breeders have not been able to produce one biologically novel structure in the laboratory – much less a new animal species – even under artificial conditions. No such demonstrations exists; none has ever been provided. The fruit fly has been abused, mutilated, and stressed over the course of thousands and thousands of generations. The poor dumb creature remains what it has always been, a fruit fly in the first instance, dumb in the second. This negative result is perfectly consistent with the long history of breeding experiments, which demonstrate beyond question that species may be changed only within very narrow margins of variability. No practical breeder imagines, for example, that he will ever succeed in creating a chicken with antennae or pig with a dorsal fin.

Amid this dismal record, there have been a few exciting developments for the Darwinians. There was the discovery of a manlike ape that looked like a transitional fossil between ape and man – the long-sought after "missing link". There were drawings of embryos demonstrating that vertebrates all looked alike in the earliest stages of development. There was the peppered moth that became darker – allegedly to better camouflage itself from predatory birds – when industrial air pollution blackened the trees in England. It wasn't terribly impressive in terms of "evidence," but it filled out a few pages in biology textbooks claiming evolution was a FACT.

And then, one by one, each of these pillars of evidence for evolution was exposed as s fraud. (Ironically, each appeared to have been an intelligently designed prank.) It's difficult to imagine that any other "scientific" theory *has been beset with as many hoaxes as the theory of evolution – always a good sign of a serious scientific endeavor."...*

"The only time "radiocarbon dating" was used in connection with the theory of evolution was the time it was used to expose the Piltdown Man as a hoax being pawned off as proof of evolution. It was one of the greatest scientific frauds of all time, right up there with the Pepsi challenge and that commercial where ordinary laundry detergent gets red wine out of a white blouse.

For half a century, Piltdown Man constituted a major piece of evidence for Darwin's theory. After decades of being embarrassed by the fossil record's stubborn refusal to come to Darwin's aid, in 1912 the Piltdown Man miraculously appeared in a gravel pit in Sussex, England. Amateur paleontologist Charles Dawson claimed to have discovered a skull with a human-like cranium and an apelike jaw in Piltdown Quarry. It was a creature that was not quite ape, not quite man, but a transitional species between the two, rather like the actor Pauly Shore. This Pauli Shore-like fossil wouldn't have proved evolution, but it would have given evolutionists a possible link between apes and man on their imaginary "tree of life."

It was almost uncanny how precisely Piltdown Man matched what prevailing scientific theory predicted the "missing link" would look like. The New York Times *headline for the article on the Piltdown Man proclaimed, "Darwin Theory Is Proved True."*

The Piltdown fossil was "peer-reviewed" – so we know it would pass muster with the editors of Scientific American*, still flush with success after triumphantly exposing the "Ohio flight hoax." Experts confirmed the age and origin of the bones. Indeed, the Piltdown Man received the approval of Arthur smith Woodward, the leading geologist at the British Museum (Natural History).* Eoanthropus dawsoni *was born.*

Dawson was showered with praise, fame, and awards. If only Vanity Fair *had been around, Dawson could have been photographed in his Jaguar and hailed for "speaking truth to power." He was made a fellow of the Geological Society and a fellow of the Society of Antiquaries. (He was even offered a position writing editorials for* Scientific American.*)*

For more than forty years, the Piltdown Man was taught as scientific fact. Then, in 1953, it was exposed as a complete and utter fraud – in part through the process of radiocarbon dating."

In conclusion, anti-Darwinists claim that Darwinism is a nondisprovable conjecture, and certainly not a science. In that sense it belongs to the same category as "String Theory" and psychoanalysis.

IV. Ecology — Living Organisms and their Environment

The science of ecology – the study of the interrelationships among the biological and physical components in the natural world – has emerged as a distinct discipline only in the twentieth century. While naturalists from ancient times to the late nineteenth century noted the interdependence among organisms and the adaptations of organisms to their environment, the attempt to study those interactions as part of a larger natural system has occurred – only recently.

Ecology is a multi-disciplinary science. Because of its focus on the higher levels of the organization of life on earth and on the interrelations between organisms and their environment, ecology draws heavily on many other branches of science, especially geology and geography, meteorology, pedology, chemistry, and physics.

Thus, ecology studies the distribution and abundance of living organisms, and interactions among organisms and between organisms and their environment. The environment of an organism includes both its habitat, which can be described as the sum of local abiotic factors such as climate, and geology, as well as the other organisms that share its habitat. Ecological systems are studied at several different levels, from individuals and populations to ecosystems and the biosphere.

*Ethology studies animal behavior (particularly of social animals such as primates), and is sometimes considered a branch of zoology. Ethologists have been particularly concerned with the evolution of behavior and the understanding of behavior in terms of the theory of natural selection. In one sense the first modern ethologist was Charles Darwin, whose book '*The Expression of the Emotions in Animals and Men*' influenced many ethologists.*

Biogeography studies the spatial distribution of organisms on the earth, focusing on topics like plate tectonics, climate change, dispersal and migration, and cladistics.

Agriculture, fisheries, forestry, medicine and urban development are among human activities that would fall within the definition of ecology.

Ecology is a broad discipline comprised of many sub-disciplines. A common, broad classification, moving from lowest to highest complexity, where complexity is defined as the number of entities and processes in the system under study, is:

- *Physiological Ecology* (or ecophysiology) and *Behavioral ecology examine adaptations of the individual to its environment.*

- *Population ecology (or autecology) studies the dynamics of populations of a single species.*
- *Community ecology (or synecology) focuses on the interactions between species within an ecological community.*
- *Ecosystem ecology studies the flows of energy and matter through the biotic and abiotic components of ecosystems.*
- *Landscape ecology examines processes and relationship across multiple ecosystems or very large geographic areas.*

Ecology can also be sub-divided according to the species of interest into fields such as animal ecology, plant ecology, insect ecology, and so on. Another frequent method of subdivision is by biome studies, e.g., Arctic ecology (or polar ecology), tropical ecology, desert ecology, etc. The primary technique used for investigation is often used to subdivide the discipline into groups such as chemical ecology, genetic ecology, field ecology, statistical ecology, theoretical ecology, and so forth. Note that these different systems are unrelated and often applied at the same time; one could be a theoretical plant community ecologist, or a polar ecologist interested in animal genetics.

Biosphere

For modern ecologists, ecology can be studied at several levels: population level (individuals of the same species in the same or similar environment), biocoenosis level (or community of species), ecosystem level, and biosphere level.

The outer layer of planet earth can be divided into several compartments: the hydrosphere (or sphere of water), the lithosphere (or sphere of soils and rocks), and the atmosphere (or sphere of the air). The biosphere (or sphere of life), sometimes described as "the fourth envelope", is all living matter on the planet or that portion of the planet occupied by life. It reaches well into the other three spheres, although there are no permanent inhabitants of the atmosphere. Relative to the volume of earth, the biosphere is only the very thin surface layer which extends from 11,000 meters below sea level to 15,000 meters above.

It is thought that life first developed in the hydrosphere, at shallow depths, in the photic zone. (Recently, though, a competing theory has emerged, that life originated around hydrothermal vents in the deeper ocean.) Multicellular organisms then appeared and colonized benthic zones. Photosynthetic organisms gradually produced the chemically unstable oxygen-rich atmosphere that characterizes our planet. Terrestrial life developed later, after the ozone layer protecting living beings from UV rays formed.

Diversification of terrestrial species is thought to be increased by the continents drifting apart, or alternately, colliding. Biodiversity is expressed at the ecological level (ecosystem), population level (intraspecific diversity), species level (specific diversity), and genetic level. Recently, technology has allowed the discovery of the deep ocean vent communities. This remarkable ecological system is not dependent on sunlight but bacteria, utilizing the chemistry of the hot volcanic vents, are at the base of its food chain.

The biosphere contains great quantities of elements such as carbon, nitrogen, hydrogen and oxygen. Other elements, such as phosphorus, calcium, and potassium, are also essential to life, yet are present in smaller amounts. At the ecosystem and biosphere levels, there is a continual recycling of all these elements, which alternate between the mineral and organic states.

While there is a slight input of geothermal energy, the bulk of the functioning of the ecosystem is based on the input of solar energy. Plants and photosynthetic microorganisms convert light into chemical energy by the process of photosynthesis, which creates glucose (a simple sugar) and releases free oxygen. Glucose thus becomes the secondary energy source which drives the ecosystem. Some of this glucose is used directly by other organisms for energy. Other sugar molecules can be converted to other molecules such as amino acids. Plants use some of this sugar, concentrated in nectar to entice pollinators to aid them in reproduction.

Cellular respiration is the process by which organisms (like mammals) break the glucose back down into its constituents, water and carbon dioxide, thus regaining the stored energy the sun originally gave to the plants. The proportion of photosynthetic activity of plants and other photosynthesizers to the respiration of other organisms determines the specific composition of the earth's atmosphere, particularly its oxygen level. Global air currents mix the atmosphere and maintain nearly the same balance of elements in areas of intense biological activity and areas of slight biological activity.

Water is also exchanged between the hydrosphere, lithosphere, atmosphere and biosphere in regular cycles. The oceans are large tanks, which store water, ensure thermal and climatic stability, as well as the transport of chemical elements thanks to large oceanic currents.

Ecosystems

Each living organism has an ongoing and continual relationship with every other element that makes up its environment. An ecosystem can be defined as any situation where there is interaction between organisms and their environment.

The ecosystem is composed of two entities, the entirety of life, the biocoenosis and the medium that life exists in, the biotope. Within the ecosystem, species are connected by food chains or food webs. Energy from the sun, captured by primary producers via photosynthesis, flows upward through the chain to primary consumers (herbivores), and then to secondary and tertiary consumers (carnivores), before ultimately being lost to the system as waste heat. In the process, matter is incorporated into living organisms, which return their nutrients to the system via decomposition, forming biogeochemical cycles such as the carbon and nitrogen cycles.

The concept of an ecosystem can apply to units of variable size, such as a pond, a field, or a piece of deadwood. A unit of smaller size is called a microecosystem*. For example, an ecosystem can be a stone and all the life under it. A mesoecosystem could be a forest, and a* macroecosystem *a whole ecoregion, with its drainage basin.*

The main questions when studying an ecosystem are:

- *Whether the colonization of a barren area could be carried out,*
- *Investigation the ecosystem's dynamics and changes,*
- *The methods of which an ecosystem interacts at local, regional and global scale,*
- *Whether the current state is stable,*
- *Investigating the value of an ecosystem and the ways and means that interaction of ecological systems provide benefit to humans, especially in the provision of healthy water.*

Ecosystems are often classified by reference to the biotopes concerned. The following ecosystems may be defined:

- *As continental ecosystems, such as forest ecosystems, meadow ecosystems such as steppes or savannas), or agro-ecosystems*
- *As ecosystems of inland waters, such as lentic ecosystems such as lakes or ponds; or lotic ecosystems such as rivers*
- *As oceanic ecosystems*

Another classification can be done by reference to its communities, such as in the case of an human ecosystem.

Ecosystems are not isolated from each other, but are interrelated. For example, water may circulate between ecosystems by the means of a river or ocean current. Water itself, as a liquid medium, even defines ecosystems. Some species, such as salmon or freshwater eels move between marine systems and fresh-water systems. These relationships between the ecosystems lead to the concept of a biome.

A biome is a homogeneous ecological formation that exists over a large region as tundra or steppes. The biosphere comprises all of the earth's biomes – the entirety of places where life is possible – from the highest mountains to the depths of the oceans.

Biomes correspond rather well to subdivisions distributed along the latitudes, from the equator towards the poles, with differences based on to the physical environment (for example, oceans or mountain ranges) and to the climate. Their variation is generally related to the distribution of species according to their ability to tolerate temperature and/or dryness. For example, one may find photosynthetic algae only in the photic part of the ocean (where light penetrates), while conifers are mostly found in mountains.

Though this is a simplification of more complicated scheme, latitude and altitude approximate a good representation of the distribution of biodiversity within the biosphere. Very generally, the richness of biodiversity (as well for animal than plant species) is decreasing most rapidly near the equator and less rapidly as one approaches the poles.

The biosphere may also be divided into ecozones, which are very well defined today and primarily follow the continental borders. The ecozones are themselves divided into ecoregions, though there is no agreement on their limits.

In an ecosystem, the connections between species are generally related to food and their role in the food chain. There are three categories of organisms:

- Producers *– plants which are capable of photosynthesis.*
- Consumers *– animals, which can be primary consumers (herbivorous), or secondary or tertiary consumers (carnivorous).*
- Decomposers *– bacteria, mushrooms which degrade organic matter of all categories, and restore minerals to the environment.*

These relations form sequences, in which each individual consumes the preceding one and is consumed by the one following, in what are called food chains or food network. In a food network, there will be fewer organisms at each level as one follows the links of the network up the chain.

These concepts lead to the idea of biomass (the total living matter in a given place), of primary productivity (the increase in the mass of plants during a given time) and of secondary productivity (the living matter produced by consumers and the decomposers in a given time).

These two last ideas are key, since they make it possible to evaluate the load capacity – the number of organisms which can be supported by a given ecosystem. In any food network, the energy contained in the level of the producers is not completely transferred to the consumers. Thus, from an energy – and environmental – point of view, it is more efficient for humans to be primary consumers (to subsist from vegetables, grains, legumes, fruit, cotton, etc.) than as secondary consumers (from eating herbivores, omnivores, or their products, such as milk, chickens, cattle, sheep, etc.) and still more so than as a tertiary consumer (from consuming carnivores, omnivores, or their products, such as fur, pigs, snakes, alligators, etc.). An ecosystem(s) is unstable when the load capacity is overrun and is especially unstable when a population doesn't have an ecological niche and overconsumers.

The productivity of ecosystems is sometimes estimated by comparing three types of land-based ecosystems and the total of aquatic ecosystems:

- *The forests (1/3 of the earth's land area) contain dense biomasses and are very productive. The total production of the world's forests corresponds to half of the primary production.*
- *Savannas, meadows, and marshes (1/3 of the earth's land area) contain less dense biomasses, but are productive. These ecosystems represent the major part of what humans depend on for food.*
- *Extreme ecosystems in the areas with more extreme climates – deserts and semi-deserts, tundra, alpine meadows, and steppes – (1/3 of the earth's surface) have very sparse biomasses and low productivity.*
- *Finally, the marine and fresh water ecosystems (3/4 of Earth's surface) contain very sparse biomasses (apart from the coastal zones).*

Humanity's actions over the last few centuries have seriously reduced the amount of the earth covered by forests (deforestation), and have increased agro-ecosystems (agriculture). In recent decades, an increase in the areas occupied by extreme ecosystems has occurred (desertification).

Dynamics and Stability

Ecological factors which can affect dynamic change in a population or species in a given ecology or environment are usually divided into two groups: abiotic and biotic.

Abiotic factors are geological, geographical, hydrological and climatological parameters. A biotope is an environmentally uniform region characterized by a particular set of abiotic ecological factors. Specific abiotic factors include:

- *Water, which is at the same time an essential element to life and a milieu*
- *Air, which provides oxygen, nitrogen, and carbon dioxide to living species and allows the dissemination of pollen and spores*
- *Soil, at the same time source of nutriment and physical support*
- *Soil pH, salinity, nitrogen and phosphorus content, ability to retain water, and density are all influential*
- *Temperature, which should not exceed certain extremes, even if tolerance to heat is significant for some species*
- *Light, which provides energy to the ecosystem through photosynthesis*
- *Natural disasters can also be considered abiotic*

Biocenose, or community, is a group of populations of plants, animals, micro-organisms. Each population is the result of procreations between individuals of same species and cohabitation in a given place and for a given time. When a population consists of an insufficient number of individuals, that population is threatened with extinction; the extinction of a species can approach when all biocenoses composed of individuals of the species are in decline. In small populations, consanguinity (inbreeding) can result in reduced genetic diversity that can further weaken the biocenose.

Biotic ecological factors also influence biocenose viability; these factors are considered as either intraspecific and interspecific relations.

Intraspecific relations are those which are established between individuals of the same species, forming a population. They are relations of co-operation or competition, with division of the territory, and sometimes organization in hierarchical societies.

Interspecific relations – interactions between different species – are numerous, and usually described according to their beneficial, detrimental or neutral effect (for example, mutualism (relation ++) or competition (relation –). The most significant relation is the relation of predation (to eat or to be eaten), *which leads to the essential concepts in ecology of food chains (for example, the grass is consumed by the herbivore, itself consumed by a carnivore, itself consumed by a carnivore of larger size).*

A high predator to prey ratio can have a negative influence on both the predator and prey biocenoses in that low availability of food and high death rate prior to sexual maturity can decrease (or prevent the increase of) populations of each, respectively.

Selective hunting of species by humans which leads to population decline is one example of a high predator to prey ratio in action. Other interspecific relations include parasitism, infectious disease and competition for limiting resources, which can occur when two species share the same ecological niche.

The existing interactions between the various living beings go along with a permanent mixing of mineral and organic substances, absorbed by organisms for their growth, their maintenance and their reproduction, to be finally rejected as waste. These permanent recyclings of the elements (in particular carbon, oxygen and nitrogen) as well as the water are called biogeochemical cycles. They guarantee a durable stability of the biosphere (at least when unchecked human influence and extreme weather or geological phenomena are left aside).

This self-regulation, supported by negative feedback controls, ensures the perenniality of the ecosystems. It is shown by the very stable concentrations of most elements of each compartment. This is referred to as homeostasis. The ecosystem also tends to evolve to a state of ideal balance, reached after a succession of events (for example a pond can become a peat bog).

Ecological crisis

Generally, an ecological crisis occurs when the environment of a species or a population evolves in a way unfavorable to that species survival.

It may be that the environment quality degrades compared to the species needs, after a change in an abiotic ecological factor (for example, an increase of temperature, less significant rainfalls). It may be that the environment becomes unfavorable for the survival of a species (or a population) due to an increased pressure of predation (for example overfishing). Lastly, it may be that the situation becomes unfavorable to the quality of life of the species (or the population) due to a rise in the number of individuals (overpopulation).

Ecological crises may be more or less brutal (occurring between a few months to a few million years). They can also be of natural or anthropic *origin. They may relate to one unique species or on the contrary, to a high number of species.*

An ecological crisis may be local (as an oil spill) or global (a rise in the sea level related to global warming).

According to its degree of endemism, a local crisis will have more or less significant consequences, from the death of many individuals to the total extinction of a species. Whatever its origin, disappearance of one or several species often will involve a rupture in the food chain, further impacting the survival of other species.

In the case of a global crisis, the consequences can be much more significant; some extinction events showed the disappearance of more than 90% of existing species at that time. However, it should be noted that the disappearance of certain species, such as the dinosaurs, by freeing an ecological niche, allowed the development and the diversification of the mammals. An ecological crisis thus paradoxically favored biodiversity.

Sometimes, an ecological crisis can be a specific and reversible phenomenon at the ecosystem scale. But more generally, the crises impact will last. Indeed, it rather is a connected series of events, that occur till a final point. From this stage, no return to the previous stable state is possible, and a new stable state will be set up gradually.

Lastly, if an ecological crisis can cause extinction, it can also more simply reduce the quality of life of the remaining individuals. Thus, even if the diversity of the human population is sometimes considered threatened, few people envision human disappearance at short span. However, epidemic diseases, famines, impact on health of reduction of air quality, food crises, reduction of living space, accumulation of toxic or non degradable wastes, threats on keystone species (great apes, panda, whales) are also factors influencing the well-being of people.

During the past decades, this increasing responsibility of humanity in some ecological crises has been clearly observed. Due to the increases in technology and a rapidly increasing population, humans have more influence on their own environment than any other ecosystem engineer.

Some usually quoted examples as ecological crises are:

- *Permian-Triassic extinction event 250 million of years ago*
- *Cretaceous-Tertiary extinction event 65 million years ago*

- *Global warming related to the greenhouse effect. Warming could involve flooding of the Asian deltas, multiplication of extreme weather phenomena and changes in the nature and quantity of the food resources.*
- *Ozone layer hole issue.*
- *Deforestation and desertification, with disappearance of many species.*
- *The nuclear meltdown at Chernobyl in 1986 caused the death of many people and animals from cancer, and caused mutations in a large number of animals and people. The area around the plant is now abandoned because of the large amount of radiation generated by the meltdown. Twenty years after the accident, the animals have returned.*

Human Ecology

Human ecology began in the 1920s, through the study of changes in vegetation succession in the city of Chicago. It became a distinct field of study in the 1970s. This marked the first recognition that humans, who had colonized all of the earth's continents, were a major ecological factor. Humans greatly modify the environment through the development of the habitat (in particular urban planning), by intensive exploitation activities such as logging and fishing, and as side effects of agriculture, mining, and industry.

Besides ecology and biology, this discipline involved many other natural and social sciences, such as anthropology and ethnology, economics, demography, architecture and urban planning, medicine and psychology, and many more. The development of human ecology led to the increasing role of ecological science in the design and management of cities.

Timelines

Timeline of Biophysics

1780–1794 ***Luigi Galvani*** *(1737–1798, Italy) discovered bioelectricity (causing muscular contraction in a frog's leg by application of static electricity). Pioneer of electrophysiology.*

1847–1894 ***Herman von Helmholtz*** *(1821–1894) First to measure the speed of nerve impulses. Studied human hearing and vision.*

1856 ***Adolf Eugen Fick*** *(1829–1901, Germany) Developed fundamental laws of diffusion in living organisms; discovered Fick's law of diffusion.*

Efforts to capture visions beyond the range of the human eye have long engaged scientists and engineers. By the mid-1880s ***George Eastman*** *had improved upon celluloid and at the turn of the 20th century used it with his new camera, the Brownie. That boxy little contraption is still remembered by many adults today, even as digital cameras record the world around us by harnessing electrons. The discovery of X-rays was only the first of many achievements leading to the development of imaging devices that today support all manner of endeavors in the medical sciences.*

1900 ***George Eastman*** *introduces the Kodak Brownie Camera.*

1895 ***Wilhelm Conrad Röntgen*** *discovered X-rays.*

1903 ***Willem Einthoven*** *founded electrocardiography (EKG)*

1913 ***William David Coolidge*** *invented the hot cathode x-ray tube, using a thermionic tube with a heated cathode electron emitter to replace the cold, or gas, tube. All modern x-ray tubes are of the thermionic type.*

1913 ***Albert Solomon****, a pathologist in Berlin, uses a conventional x-ray machine to produce images of 3,000 gross anatomic mastectomy specimens, observing black spots at the centers of breast carcinomas. Mammography, the resulting imaging, has been used since 1927 as a diagnostic tool in the early detection of breast cancer.*

1926 ***Hermann J. Miller*** *discovered that X-rays cause mutation in living cells.*

1928 ***Georg von Békésy*** *(1899–1972) expounded the physical mechanism within the Cochlea of the inner ear.*

1932 ***Max Knoll*** *and* ***Ernst A.F. Ruska*** *developed the transmission electron microscope. It can magnify objects one million times.*

1945 ***Bernard Katz*** *discovered how synapses work.*

1945 ***Alan Lloyed Hodgkin*** *(1914–1998) and* ***Andrew Fielding Huxley*** *(1917-) described the ionic mechanism by which neurons transmit electron pulses.*

1952 ***Rosalind Franklin*** *and* ***Maurice Wilkins*** *pioneered DNA crystallography.*

1953 *First application of* positron emission tomography *for medical diagnosis of brain tumor by* ***Gordon Brownell*** *and* ***William Sweat****.*

1953 ***Max Perutz*** *(1914–2002) and* ***John Kendrew*** *(1917–1997) pioneered* protein crystallography *(Hemoglobin).*

1954 ***David Kuhl*** *introduced* radionuclide emission tomography*.*

1950s ***Russel Morgan****,* ***Edward Chemberlain*** *and* ***John W. Coltman****, perfect a method of screen intensification that reduces radiation exposure and improves* fluoroscopic vision*. Their* image intensifier *is used in* medical fluoroscopy*.*

1959 ***Ian Donald*** *developed practical technology and applications of ultrasound as a diagnostic tool in obstetrics and gynecology. Ultrasound displays images on a screen of tissues or organs formed by echoes of inaudible sound waves at high frequencies (20 KHz or more) beamed into the body.*

1960 ***Powell Richards*** *and* ***Walter Tucker*** *invented a short halflife radionuclide generator that produces technetium-99m for use in diagnostic imaging procedures in* nuclear medicine *– a branch of medicine that uses radioisotopes for research, diagnosis, and treatment of disease. [Technetium-99m was discovered in 1939 by* ***Emilio Segré*** *and* ***Glenn Seaborg****]*

1964 *Single-photon emission computerized tomography (SPECT) methods become capable of yielding accurate information similar to PET.*

1972 **Godfrey Hounsfield** and **Alan Cormack** *invented Computer Assisted Tomography (CT scanners).*

1973 **Paul Lauterbur** adapted *Magnetic Resonance Imaging (MIR) for medical purposes, using high speed computers.*

1974 **J.E. Greenleaf** and **S.A. Johnson** *developed ultrasound computer tomography.*

1974 **Mikhail Volkenshtein** *(1912–1992) developed Quantum Biophysics:* a *quantum-mechanical model of enzyme catalysis, supporting a theory that enzyme catalysis depends on quantum-mechanical effects such as tunneling.*

1977 **Peter Mansfield** *developed the Echo-planar imaging (EPI) technique to produce movie of a single cardiac cycle.*

1978 *Emission Computer tomography (ECT) by means of radionuclide imaging.*

1981 **Gerd Binnig** and **Heinrich Rohrer** *designed and built the first scanning tunneling microscope (STM) with a small tungsten probe tip, about one or two atoms wide.*

1986 **Gerd Binnig**, **Cal Quate** and **Cristoph Gerber** *introduced the atomic force microscope (AFM) which is used in microbiology and cellular biology.*

1993 *EPI is used (with functional MRI) in mapping regions of the brain responsible for thought and motor control.*

1986 **Benoit Roux** *investigated dynamics and function of biological macromolecular systems (receptors, protein kinases); functioning of biological systems at the molecular level.*

Carlos Bustamante *(1951–) used novel methods of single-molecule visualization, such as scanning force microscopy to study structure & function of nucleoprotein assemblies. Used methods of single-molecule manipulations (such as optical tweezers) to characterize the elasticity of DNA, to induce mechanical unfolding of individual protein* *molecules and to investigate the machine-like behavior of molecular motors.*

Steven Chu *(1948–) helped develop techniques for cooling and trapping atoms using laser light.*

Steven Block *(1952–) and* ***Arthur Ashkin*** *pioneered the use of optical tweezers to study the motion of enzymes (kinesin and RNA polymerase) at a single-molecule level.*

Howard Berg *characterized properties of bacterial chemotaxis.*

Timeline of Genetics

Ca 1300 BCE *The Bible (**Leviticus** 18, 6–17 forbids one to mate with one's relatives.)*

470–322 BCE *The Greek philosophers **Hippocratos**, **Aristotle** and **Plato** wrote about the inheritance of human traits. They observed that certain traits are passed from parent to child. Although they did not understand the exact contribution of the male and female parent to the offspring, they believed that semen is in some way responsible for passing on traits.*

1814 CE ***Joseph Adams** published "A Treatise on the Supposed Hereditary Properties of Diseases", stating therein that there is some intrafamilial correlation of diseases.*

1839 ***Matthias Schleiden** and **Theodore Schwann** suggested that cells with nuclei are the fundamental units of life.*

1855 ***Rudolph Virchow** hypothesized that new cells can only be formed by the division of existing cells.*

1859 ***Charles Darwin** published "On the Origin of Species", proposing evolution by natural selection. His key premise was that evolution occurs through the selection of inherent and transmissible (rather than acquired) characteristics between individual members of a species. Darwin did not specify the means by which characteristics are inherited and the mechanism of heredity had not been determined at that time.*

1865 ***Gregor Mendel** discovered the fundamental* laws of inheritance*. He deduced that genes come in pairs and are inherited as distinct units, one from each parent. Mendel tracked the segregation of parent genes and their appearance in the offspring as dominant or recessive traits. He recognized the mathematical patterns of inheritance from one generation to the next.*
Mendel's results were not appreciated until 1900, when they were rediscovered. Today he is widely considered founding father of modern genetics. Moreover, his work eventually helped to partly explain Darwin's concept of evolution.

1869 ***Johann F. Miescher*** *isolated DNA (first called "nuclein") as an acidic substance found in cell nuclei of white blood cells (puss). We now know that he had discovered the material basis of heredity, but it took another 80 years before nuclein was shown to be DNA (it become known as* nucleic acid *after 1874, when Miescher separated it into a protein and an acid molecule. It was suspected of exerting some function in the heredity process.)*

1819–1882 ***Walther Flemming*** *used new staining techniques to see tiny threads within the nucleus of cells in salamander larvae that appear to be dividing. In so doing he discovered* chromosomes.

1889 ***August Weissman*** *theorized that the material basis of heredity is located on the* chromosomes.

1900 • *The science of* genetics *was finally born when* ***G. Mendel****'s work was* rediscovered *by* ***Hugo de Vries***, ***Erich von Tshchermak*** *and* ***Carl Correns***.

• *Major outbreaks of disease in overloaded industrial cities led to the introduction of large-scale sewage purification system based on* microbial activity *(first time in Manchester, England, 1911).*

• *It was first shown that key industrial chemicals (glycerol, and butanol) could be synthesized using* bacteria.

1901–1912 *Scientists suggested that food contains ingredients essential to life that are not proteins or carbohydrates:* ***E. Wildiers*** *(1901) discovered a new substance – a growth-factor indispensable for the development of yeast;* ***Frederick Hopkins*** *and* ***C.A. Pekelharing*** *(1906) discovered such substances in rice and citrus. In succeeding years they came to be called* vitamins *(****Casimir Funk****, 1912).*

1902 *The term '*immunology*' first appeared.*

1902–1912 *Scientists identified* chromosomes *as carriers of heredity.* ***Walter Sutton*** *(1902) stated that chromosomes are paired and that* genes *are carried by chromosomes. He argued that each egg or sperm contains only one of each chromosomes which accounts for the random factor in heredity and accords with* Mendel's *theory. He thus pointed out the interrelationships between cytology and Mendelism, closing the gap between cell morphology end heredity.*

__Theodor Boveri__ reached the same result independently (1903). __Edmund Wilson__ and __Nettie Stevens__ independently described the behavior of sex chromosomes: XX determines female; XY determines male.

1903–1909 *First experiments on quantitative traits in broad beans by __Wilhelm Johanssen__ and in wheat by __Herman Nilsson-Ehle__.*

1905 *The word "genetic" as coined by __William Bateson.__ proposed the idea that separate X and Y chromosome determine sex.*

1905–1908 *__William Bateson__ and __Reginald Punnett__ demonstrated that action of some genes modify action of other genes: The first time gene regulation was demonstrated.*

1908 *__Hardy__ – __Weinberg__ law was derived.*

1908–1909 *__Archibald Garrod__ proposed (1908) that some human diseases are due to inborn errors of metabolism that result from lack of specific enzyme[40]. He is now considered the founder of biochemical genetics. In 1909 he proposed that genes dictate phenotypes through enzymes that catalyze specific processes in the cell.*

1909 *__Wilhelm L. Johannsen__ coined the words gene, genotype, phenotype.*

1909–1929 *__A. Levene__ discovered that the sugar ribose is found in some nucleic acids, those we now call RNA (1909). He discovered (1929) deoxyribose in nucleic acids that do not contain ribose, those known today as DNA.*

1910 *__Thomas H. Morgan__ (1907–1911) proved that chromosomes have a definite function in heredity, established mutation theory, and led to a fundamental understanding of the mechanisms of heredity. He explained the separation of certain inherited characteristics that are usually linked*

[40] In 1929, **Richard Schönheimer** studied a patient with hepatomegaly due to massive glycogen storage and suggested that this disorder may by due to an enzyme deficiency. It was not until 1952 that **Cori and Cori** found glucose-6-phosphatase to be deficient in "von Gierke disease"; (glycogen storage disease type I). This observation marks the first time that an inborn error of metabolism was attributed to a specific enzyme deficiency.

as caused by breaking of chromosomes during the process of cell division, and began to map the positions of genes on chromosomes of the fruit fly. Morgan proved that the genes responsible for the appearance of a specific phenotype were located on chromosomes and that some genetically–determined traits are sex-linked. He also found that genes on the same chromosome do not always assort independently.

Morgan suggested that the strength of linkage between genes depended on the distance between them on the chromosome. The nearer two genes lie on a chromosome, the greater the chance of being inherited together. Likewise the farther away they are from each other, the more is the chance of being separated by the process of crossing-over. The genes are separated when a crossover takes place in the distance between the two genes during cell division.

*One of his students, **Calvin Bridges**, in 1913, established that genes are located on chromosomes. In the same year, another student of Morgan's, **Alfred Sturtevant**, determined that genes are arranged on the chromosomes in a linear fashion, much like beads on a necklace. Moreover, Sturtevant demonstrated that the gene for any specific trait is in a fixed location or locus. Yet another Morgan student, **Herman J. Muller**, in 1926 discovered methods for artificially producing mutants in fruit flies by ionizing radiation and other mutagens. In so doing, he discovered the origin of new genes by mutations, a theory first proposed by **Hugo de Vries** in the early 1900s.*

1912 ***Lawrence Bragg** discovered that X-rays can be used to study the molecular structure of simple crystalline substances. This discovery led to the development of X-ray crystallography, which made it possible to further explore the 3-dimensional structure of acids and proteins.*

While two World Wars killed millions and pushed medicine to new limits, various fields of science began to converge to explore the mechanism of reproduction – the nature and structure of the heredity-carrying materials (eventually found to be DNA molecules).

1915 ***Frederick W. Twort** discovered phages, viruses that prey on bacteria.*

1918 **R.A. Fisher** *publishes "The Correlation Between Relatives on the Supposition of Mendelian Inheritance" the modern synthesis of genetics and evolutionary biology starts. (See population genetics.)*

1920's *Nucleic acid found to be a major component of the chromosomes, but it was not considered good candidate for a carrier of genetic information.*

1927 *Physical changes in genes are called mutations.*

1928 **Frederick Griffith** *discovers that hereditary material from dead bacteria can be incorporated into live bacteria. (See Griffiths experiment.)*

1930 **W.C. Rose** discovered *essential amino acids.*

1930's *Chemical nature of nucleic acids was investigated. The ubiquitous presence of nucleic acid in the chromosome was generally explained in purely physiological or structural terms.*

1931 *Crossing over is identified as the cause of recombination.*

1933 **Arne Tiselius** *introduced electrophoresis, a new technique for separating proteins in solution.*

1935–1939 **Andrei Nicolaevitch Belozersky** *isolated DNA in its pure state for the first time (1936). Showed (1939) that both DNA and RNA are always present in bacteria.*

1937 **Frederick C. Bawden** *discovered that tobacco mosaic virus contains RNA.*

1938
- *The term 'molecular biology' was coined.*
- *Proteins and DNA were studied in various labs with the aid of X-ray crystallography.*

early 1940s *Nucleic acid still viewed as a uniform polymer unaffected by its biological source. Hereditary information commonly thought to reside in the chromosomal proteins.*

1941 **George Beadle** and **Edward Tatum** *show that genes code for proteins: they performed experiment that suggest that one gene codes for one enzyme.*

1941 *The term 'genetic engineering' was first used.*

1942 *The* electron microscope *was used to identify a* bacteriophage

Barbara McClintock *discovered* transposable *genetic elements: c.e. genes can move on chromosome and jump from one chromosome to another. Her findings were greeted with initial skepticism until the 1970's when molecular biologists confirmed the existence of an enzyme that enables jumping genes to hop around on the DNA. At age 81 (1983) she was awarded the NP. Scientists now think such transposons may be linked to some genetic disorders such a hemophilia, leukemia and breast cancer. They also think that transposons have played a crucial role in evolution.*

1944–1959 ***Oswald T. Avery***, ***Colin MacLeod*** *and* ***Maclyn McCarty*** *continued the work of* **Frederick Griffith** *and demonstrated that DNA is the material of genes, i.e. the molecule of genetic information. Most people were skeptical of these findings until 1952.*

1946 ***Joshua Lederberg*** *and* ***Edward Tatum*** *showed that material can be transferred laterally between bacterial cells, c.e: bacteria can exchange genetic material directly through* conjugation.

1950 ***Erwin Chargaff*** *discovered a one-to-one ratio of adenine to thymine and guanine to cytosine in DNA samples from a variety of organisms. DNA rather then protein carry genetic information.*

1951 *The first animal, the* tadpole *was cloned by nuclear transfer (cloning in plants was known for millennia).*

1952 ***Alfred Hershey*** *and* ***Martha Chase*** *show in bacteriophage labeling experiments that DNA is the molecule of heredity.*

1952 ***Rosalind Franklin*** *and* ***Maurice Wilkins*** *perform X-ray crystallography studies of DNA, providing sharp diffraction photographs that led to the elucidation of the structure of DNA. These diffraction patterns of the DNA molecule revealed the helical structure and the location of the phosphate sugar on the DNA molecule.*

• ***Jean Brachet*** *suggested that RNA, a nucleic acid, plays a part in the synthesis of proteins.*

1953 **Francis Crick** and **James D. Watson** *proposed, on the basis of* **Franklin***'s data, the double-stranded, helical, complementary, anti-parallel model for DNA.*

They determined that deoxyribonucleic acid (DNA) is a double strand helix of nucleotides. Each nucleotide consists of deoxyribose sugar molecule to which is attached a phosphate group and one of four nitrogenous bases: two purines (adenine and guanine) and two pyrimidines (cytosine and thymine). The nucleotides are joined together by covalent bonds between the phosphate of one nucleotide and the sugar of the next, forming a phosphate-sugar backbone from which the nitrogenous bases protrude. The two strands are linked by selective hydrogen bonds: the purin adenine bonds only with the pyrimidine thymine, and the purine cytosine only with the pyrimidine guanine.

DNA replication is possible through the complementary nature of the two strands. The chemical complexity of the molecule is thought to be sufficient to store the requisite information.

The precise manner in which the information in the DNA is activated to build an organism is still very poorly understood; what is firmly demonstrated is that so-called structural genes manufacture the proteins for living tissues.

1955–1959 *Biologists work out the mechanism by which DNA functions to make protein. They hypothesized that the DNA sequence specifies the amino acid sequence in a protein and that genetic information flows only in one direction, from DNA to messenger RNA to protein. The replication mechanism of DNA is demonstrated [***Francis Crick** and **George Gamow***, 1957;* **Arthur Kornberg***, 1958].*

1956 **Jo Hin Tjoi** and **Albert Levan** *established the correct chromosome number in humans to be 46.*

1957 **Crick** and **Gamow** *worked out the "Central Dogma" to explain protein synthesis from DNA; the DNA sequence codes for amino acid sequences and genetic information flows in one direction — from DNA to mRNA to protein.*

1958 *The* **Meselson** – **Stahl** *experiment demonstrates that DNA is semiconservatively replicated.*

1961–1967 ***Marshall Nirenberg***, ***Har Gobind Khorana***, ***Heinrich Mathaei*** *and* ***Severo Ochoa*** *cracked the Genetic Code. They demonstrated that a sequence of 3 nucleotide bases, a codon, determines each of the 20 amino acids. This means that there are 64 combinations possible for 20 amino acids. Nirenberg and* ***Philip Leder*** *found that there are extra redundant codons that serve as stop signs for RNA synthesizing protein.*

1965 ***Leonard Hayflick*** *(1928–) observed that cells dividing in cell culture divided about 50 times before dying. The human limit is around 52.*[41]

1961 ***Francois Jacob*** *(1920– , France) and* ***Jacques Monod*** *(1910–1976, France) advanced the OPERON MECHANISM theory for controlling enzyme activities in the cell.*

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[41] This is known as the *Hayflick limit*. It has been linked to the shortening of *telomeres*, a region of DNA at the end of chromosomes.

The only known way of circumventing the Hayflick limit is with the enzyme telomerase, which regenerates telomeres during DNA replication.

Stem cells, by definition, have not yet been fully differentiated, and therefore many of these cells may continue to regenerate new cells for the entire lifespan of the organism, without limit, thus constituting a notable exception to the Hayflick limit in humans and other organisms. While the manifestations of the constant regenerative effects of stem cells is most easily seen in tissues which must constantly produce replacements for existing cells, such as skin and blood cells, stem cells of one form or another are found in every tissue of the human body, even if only as dormant stem cells known as "spore-like cells".

Cancer cells constitute the other main exception to the limits on cell division. It is believed that the Hayflick limit exists principally to help prevent cancer. If a cell becomes cancerous and the Hayflick limit is approaching, the cell will only be able to divide a certain number of times. Once it reaches this limiting number of divisions, the formed tumor will no longer be able to reproduce and the cells will die off. Cancers become problems after having reactivated telomerase-encoding genes. Cells that have found a way around the limit are referred to as "immortal". Such immortal cells may still die, but the group of immortalized cells produced from cell division of an immortal cell has no limit as to how many times cell division might take place among the cells that constitute such a group of immortalized cells.

It is believed by some that some or all cancers start off as stem cells that become genetically damaged over their long lives. This would mean they already aren't limited by the Hayflick limit and can easily metastasize into the pool of cells in their final cell type destination.

*The experimental system used by them was the common bacterium **E. Coli**. They discovered in these cells a class of genes that control the activity of other genes. If for some reason, the controlling genes function improperly, the other genes may get out of control and damage the cell. This basic regulatory concept is fundamental to* cellular *regulation for* all *organisms.*

Lac operon *is a DNA sequence that governs the production of* enzymes *for metabolizing* lactose *(milk sugar) in bacteria such as E. Coli. The key idea is that E. coli does not bother to waste energy making such enzymes if there is no need to metabolize lactose, such as when other sugars like* glucose *are available.*

1967 • ***Mary Weiss*** and ***Howard Green*** *found a technique for combining human cells with mouse cells in one culture (somatic cell hybridization).*

• ***W.M. Fitch*** and ***E. Maroliash*** *set up the first evolutionary trees from protein sequences.*

1968 ***Gerold Edelman*** *(1929–)* and ***Rudney Porter*** *isolated the first DNA ligase.*

1970's *Important discoveries were made that led to modern techniques for studying genetics:*

1970 ***Howard Temin*** and ***David Baltimore*** *discovered how viruses affect the genes of cancer cells: Their work described how viral RNA that infects a host bacterium uses an enzyme to integrate its message to the host's DNA. This discovery allow scientists to create* clones

1970 ***Hamilton Smith***, ***Daniel Nathans*** and ***Kent Wilcox*** *isolated first* restriction enzyme *that could cut DNA molecules within specific recognition sites. The restriction enzymes were discovered in studies of a bacterium and were used to cut-up foreign DNA from invading organisms such as viruses (NP 1978).*

1972 ***Paul Berg*** *isolated and employed a* restriction enzyme *to cut DNA. He also used ligase to paste two DNA strands* *together to form a hybrid circular molecule. This was the first recombinant DNA molecule*[42]*. (NP 1981)*

1973 *Scientists for the first time successfully transferred DNA molecules from one life form into another:* ***Stanley H. Cohen****,* ***Annie Chang*** *and* ***Herbert Boyer*** *used a restriction enzyme to cut sections of viral DNA and bacterial DNA, "spliced" them together and inserted this recombinant molecule into the DNA of the bacterium Eschericia Coli, thereby introducing the first recombinant-DNA organism; this is the beginning of genetic engineering.*

1973–1977 ***Frederick Sanger*** *invented a DNA sequencing technique.*

1975 ***Cesar Milstein*** *discovered how to fuse cells together to produce monoclonal antibodies.*

1976 • ***Michael J. Bishop*** and ***Harold Varmus*** *showed that oncogenes* appear on animal chromosomes, and alterations *in their structure can result in cancerous growth.*

• ***Martin F. Gellert*** *discovered the enzyme gyraze that caused DNA to form supercoils (larger helix).*

1977 • *Advent of the Age of Biotechnology:* The production of the first human protein manufactured by a bacteria (human growth hormone – releasing inhibitory factor). *For the first time, a synthetic recombinant gene was used to clone a protein.*

• *The first genetic engineering company (Genentech) is founded, using recombinant DNA methods to make medically important drugs.*

• ***Walter Gilbert*** *and* ***Allan Maxam*** *devised a procedure for rapidly sequencing long sequences of DNA.*

• *Bacteriophage FX-174 was the first complete genome (DNA) to be sequenced.*

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[42] Beng realized the risks of his experiment and temporarily terminated it before the recombinant DNA molecule was added to *E.coli*, where it would have been quickly reproduced. He proposed a one-year moratorium on recombinant DNA studies while safety issues were addressed.

1978 • *Genetic engineering techniques used to produce rat insulin.*

• *Scientists show it is possible to introduce specific mutations at specific sites in a DNA molecule.*

• *Scientists successfully transplanted mammalian gene.*

•***Yuel Wai Kan***, ***Andree-Marie Dozy*** *and* ***David Botstein*** *discovered restriction-fragment-length polymorphisms.*

Individual humans differ one basepair in every 500 nucleotides or so. The most interesting variations for geneticists are those that are recognized by certain enzymes, called restriction enzymes. These enzymes, each of which cut DNA only when they see a specific sequence, for instance GAATTC in case of the restriction enzyme EcoR1. This sequence is called a restriction site. The enzyme will bypass the region if it has mutated to GACTTC. Thus, when a specific restriction enzyme cuts the DNA of different people, it may produce fragments of different lengths.

These DNA fragments can be separated according to size by making them move through a porous gel in an electric field. Since the smaller fragments move more rapidly than the larger ones, their sizes can be determined by examining their positions in the gel. Variations in their lengths are called restriction-fragment-length polymorphisms, or RFLPs.

1980 *Swiss researchers introduced a gene for human interferon into bacteria and then cloned millions of cells to produce an inexpensive and abundant supply of this previously rare protein. This was the first big success story in the commercial production of drugs by genetic engineering.*

1980–1986 • ***Kary B. Mullis*** *invented the polymerase chain reaction (PCR), a method for rapidly and easily cloning of DNA fragments: this method for multiplying DNA sequences in vitro uses heat and enzymes to make unlimited copies of genes and gene fragments. It later becomes a major tool in biotech research and product development worldwide.*

The purpose of PCR is to make a high number of copies of a specific DNA fragment, a gene for instance. This method amplifies fragments of DNA million of times to make sufficient quantities available for DNA sequence analysis.

1980 • *Researchers introduced a human gene (one that codes for the protein* interferon*) into a bacterium.*

1981 • *Three independent research teams announced the discovery of human oncogenes (cancer genes).*

• *Human mitochondrial DNA sequenced.*

• *Scientists produced the first transgenic animals by transferring genes from other mammals to mice.*

1982 • *Human* insulin *drug produced by genetically-engineered bacteria (using recombinant DNA methods) for the treatment of diabetes.*

• *A human cancer gene (isolated from bladder cancer cells) is cloned in* Eschericia coli*. The base sequence of the cancer gene is found to differ from the same locus in a normal cell by a single base pair, which causes a substitution of an amino acid in the resulting protein.*

1983 • *The first artificial chromosome was created.*

• *Discovery of the* homeobox *genes in the fruit fly.*

• *First genetic modified plant is created; a tobacco plant resistant to an antibiotic.*

1980–1993 *The work of* **Stuart Orkin** *in 1986,* **Lou Kunkel** *in 1987,* **Jim Gusella** *and* **Nancy Wexler** *in 1982 and* **Mary-Claire King** *in 1991, led to the birth of modern* clinical genetics*.*

1984 ***Alec Jeffreys*** *introduced a technique for* DNA fingerprinting *to identify individuals: it is based on identification of certain core sequences of DNA unique to each person. It is to be used for establishing family relationships.*

• *Scientists cloned and sequenced the entire genome of the HIV virus.*

• *The first genetically engineered vaccine was developed.*

• *successful cloning of sheep producing genetically identical animals. It is done by separating an embryo into separate cells and introducing a cell's nucleus into sheep ova that have had their nuclei removed: the altered eggs (ova) are then implanted in female sheep for development into fetuses and consequent birth.*

• British scientists mixed goat and sheep embryo cells and implanted them into a surrogate animal. This led to the birth of the first chimera, a cross between a goat and a sheep.

*• **McGinnis** discovered homeotic (Hox) regulatory genes, responsible for the basic body plan of most animals. In subsequent work, his team demonstrates that a single mutation in a Hox gene suffices to suppress all limb development in the thoracic region of fruit flies.*

1985 *• Genetically engineered plants resistant to insects, viruses, and bacteria were field-tested for the first time.*

*• **Lap-Chee Tsui** working in Massachusetts mapped the gene for cystic fibrosis to the long arm of chromosome 7.*

1986 *• Applied Biosystems introduced the first automated DNA fluorescence sequencer.*

• The Environmental Protection Agency (USA) approves the release of the first genetically engineered crop: a herbicide resistant tobacco plants.

1986 *• The first genetically engineered human vaccine – recombinant HB – was approved as a treatment for prevention of hepatatis B.*

*• **Peter Schultz** described how to combine antibodies and enzymes (creating "abzymes") to create pharmaceuticals.*

• Scientists and technicians at CalTech invented an automated DNA fluorescence sequencer *.*

1988 *• A US National Center for Biotechnology Information (NCBI) founded at NIH/NLM.*

U.S. Congress funded the Human Genome Project*, a massive effort to map and sequence the human genetic code as well as the genomes of other species. All human DNA was to be mapped and sequenced by 2005 CE. The program was launched in 1990 with an estimated cost of $13 billion.*

1989 *Scanning tunneling electron microscope is used to obtain, for the first time, a direct images of* pure DNA*. Eventually, this method may be used to observe active viruses or the action of molecules on the cell surface.*

1989 ***Francis Colins*** and ***Lap-Chee Tsui*** *identified the gene coding for the cystic fibrosis transmembrane conductance regulator protein (CFTR) on chromosome 7 that, when mutant, causes cystic fibrosis.*

1990 *The U.S. Human Genome Project launched at estimated cost of 13 billion dollars. The project started as a 15-year effort co-ordinated by the U.S. Department of Energy and the National Institutes of Health. The project goals were to:*

1) identify all the genes in human DNA,
2) determine the sequences of the 3 billion chemical base pairs that make up human DNA, store this information in databases,
3) improve tools for data analysis,
4) transfer related technologies to the private sector, and
5) address the ethical, legal, and social issues (ELSI) that may arise from the project.

To help achieve these goals, researchers also are studying the genetic makeup of several nonhuman organisms. These include the common human gut bacterium Escherichia coli, the fruit fly,the nematode Caenerhabditis elegans, the rat and the mouse.

1990's *DNA fingerprinting, gene therapy and genetically modified foods come onto the scene.*

1990 *The first transgenic dairy cow was created. It was used to produce human milk proteins for infant formula.*

1992 *The Institute for Genome Research (TIGR), associated with plans to exploit sequencing commercially through gene identification and drug discovery, was formed.* ***Mel Simon*** *introduced the use of BACs for cloning.*

1993 *The Sanger Centre, a genome research institute with the purpose to further the knowledge of genomes, was crated in Hinxton, UK*
The EMBL European Bioinformatics Institute, the center for research and service for bioinformatics was established in Hinxton, UK

1993 *FlavrSavr tomatoes, genetically engineered for longer shelf life, were marketed.*

1993 *The first gene therapy took place on a 4-year-old girl with an immune-system disorder called ADA deficiency. The therapy appeared to work.*

1993 *Scientists in Paris produced a rough map of all 23 pairs of human chromosomes.*

1993 **Dean Hamer** *and colleagues reported at least one gene related to homosexual orientation that appears to reside on the X chromosome and is inherited from the mother.*

1994 *The FDA approved the commercial use of bovine somatropin, also known as bovine growth hormone. This hormone increases the production of milk in cows, and became one of the first genetically engineered products available to farmers.*

1994 • *A multitude of genes, human and otherswise, were identified and their function determined [e.g a gene predisposing to obesity, a breast cancer susceptibility gene, a gene associated with* apoptosis (programmed cell death)*].*

• *Linkage studies identify genes for a variety of ailments including: melanoma, hearing loss, dyslexia, thyroid cancer, sudden infant death syndrome.*

1995 • *Further evidence is found to support the idea that RNA was the central molecule in the origin of life.*

• *Gene therapy, immune system modulation and genetically engineered antibodies enter the clinic in the war against cancer.*

1995 *Duke University researchers announced the transplant of hearts from genetically altered pigs into baboons.*

1995–2000 *First completed sequences of the following genomes:*

- *Bacteria Haemophilus influenza (1995)*
- *Yeast and E.Coli (1996)*
- *Soil Nematode Caenerhabditis (1998)*
- *Fruit fly Drosophila melanogaster (1999)*
- *Arabidopsis (2000)*

1996 • *Sottish scientists clone identical lambs from early embryonic sheep.*

• *Scientists sequenced the complete genome of baker's yeast – more than 12 million base pairs of DNA.*

• *The discovery of a gene associated with Parkinson's disease.*

• *Sequencing the genome of ancient microorganisms,* archaea, *found in inhospitable climates deep in thermal vents under the sea – a step in understanding of the evolution of life on earth.*

1996 *Genzyme Transgenics announced the birth of a goat carrying BR-96 monoclonal antibodies to be used experimentally to deliver conjugated anticancer drugs to humans.*

1997 • *Artificial human* chromosomes *created for the first time.*

• Clock, *the first gene providing the* circadian rhythm *of mammalian life, identified.*

• *Using a bit of DNA and some commonplace laboratory techniques, researchers engineered the first DNA* computer *"hardware" ever:* logic *made of DNA*[43].

• *A new DNA technique combined PCR,* DNA chips, *and computer programming, thus providing a new tool for the search for disease - causing genes.*

1997 ***Hunt Willard*** *and others working at Case Western Reserve created the first artificial human chromosome, opening the door to designer babies.*

1997 ***Stanley Prusiner*** *earned the Nobel Prize for his pioneering work on prion diseases such as Bovine Spongiform Encephalopathy, which is thought to have caused the outbreak of mad cow disease in Britain and Creutzfeldt-Jakob disease.*

1997 *A sheep named Dolly was cloned.*

1998 *Dolly gave birth to Bonnie, a lamb conceived by conventional means. The birth offered reassurance that cloned animals like Dolly can develop into healthy animals capable of reproducing.*

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[43] It is not a *general purpose* computer, thought: it is tailored to solve just a single mathematical problem (the *traveling salesman* problem).

1998 *A rough draft of the human genome map was produced, showing the locations of more than 30,00 genes.*

2000 *The Human Genome Project presents its preliminary results: each of the body's 100 trillion cells contains some 3.1 billion nucleotide units. Only 1% of these are thought to be transcriptional, clustered in possibly as few as 30,000 genes. An accurate chemical map of the genome tells us surprisingly little about how it functions. Targeted experimentation is now possible.*

2002 *Presentation of human genome by Celera Genomics and the collaborating group of laboratories supported by public foundation.*

2003 *(14 April) Successful completion of Human Genome Project with 99% of the genome sequenced to a 99.99% accuracy.*

Timeline History of Structural Biology

_1910_ **_Albrecht Kossel_** *(1853–1927) discovered the amino acids Histidine (1896), thymic acid* and *agmatine (1910) in the cell.*

_1925_ **_Theodor Svedberg_** *(1884–1971) developed the ultracentrifuge that could spin its samples with a force of over 100,000 g.*

_1932_ *Invention of the electron microscope. All electron microscopes suffer from a serious drawback: since no specimen can survive under their high vacuum, they cannot show the ever-changing movements that characterize a living cell.*

_1949–1956_ **_Christian de Duve_** *(1917–),* **_Albert Claude_** *(1899–1983)* and **_George E. Palode_** *(1912–) discovered and elucidated the subcellular biochemical structure and function of organelles in biological cells.*

_1953_ **_James D. Watson_** *(1928–),* and **_Francis Crick_** *(1916–2004) discovered the structure of the DNA molecule.*

_1955_ *Microscopists* and *biochemists began to communicate: the microscopists discovered that the biochemists' particles matched what they had seen under their microscopes, and the biochemists learned that the microscopists could actually see what they had been analyzing. The result was an avalanche of discoveries about the world within the cells.*

_1959_ **_Max Perutz_** *(1914–2002)* and **_John Kendrew_** *(1917–1997) determined the molecular structure of the protein hemoglobin.*

_1961_ **_Christian Anfinsen_** *(1916–1995) first discovered the folding of protein molecules. (These molecules begin their biological activity (life) only after having folded into intricate, convoluted shapes.) Once the protein has folded, its shape determines its physical and chemical properties and particularly its active sites.*

1964 **Cyrus Levinthal** *(1922–1990) and* **Robert Langridge** *(1933–) developed the first system that showed biological molecules on a computer screen.*

1965 **Alex Novikoff** *(1913–1987), first to show that lysosomes (their enzymes can digest substances) exist in eukaryotic cells.*

1972 **Michael S. Brown** *(1941–) and* **Joseph Goldstein** *(1940–) elucidated the genetic defects and role of the LDL (Low Density Lipoprotein) receptor in Familial Hypercholesterolemia. These studied led to development of a drug to treat FH.*

1982 **Thomas Coch** *(1947–) and* **Sydney Altman** *(1939–) discovered that the nucleic acid RNA can act as enzyme because it can cut and splice itself.*

1985 **Michael Rossmann** *(1930–) mapped the structure of human common-cold virus – the first animal virus to be seen at atomic resolution.*

1989 *Researches reported using scanning electron microscopy to obtain, for the first time, direct images of pure DNA. This method is used to observe living viruses, or the action of molecules on the cell's surface, and enables scientists to see the tiniest details of cell structure and activity in ways undreamed of a few years ago.*

1995 **J.M. Hogle**, **David Filman** *and* **Marie Chow** *reported the structure of the polio virus at atomic resolution.*

Timeline History of Immunology

_1796_ *Smallpox vaccination;* **Edward Jenner** *(1749–1823) discovered that cowpox induces protection against* smallpox. *Jenner had no knowledge of microorganisms or of immunology. This had to wait for Pasteur some 100 years later.*

_1857_ ***Louis Pasteur*** *(1822–1895) noted the existence of substances capable of exerting antimicrobial effects.*

Phase I: 1879–1910

Most of the major components of the immune response were described.

_1879_ ***Louis Pasteur*** *developed attenuated vaccines of* chicken cholera, anthrax *and* rabies. *He showed that* inoculation *with preparations of weakened pathogens could be used to develop immunity against infectious forms of the pathogen. (However, not until some 80 years later, did scientists gain a modern understanding of vaccines!)*

_1883_ ***Eli Metchnikov*** *(1845–1916), pioneer of immunology, established a cellular theory of vaccination, (NP 1908).*

_1888_ ***Emil Roux*** *(1853–1933)* and **Alexandre Yersin** *(1863–1943) proved that diphtheria bacilli produced a toxin which can be separated from the bacterial cells (*antitoxin *is the specific* antibody *capable of neutralizing the pathogenic toxin).*

_1890_ ***Emil von Behring*** *(1854–1917), pioneer of immunology, discovered that serum could be used to treat diphtheria (NP 1901); explained that diphtheria immunity depended on the capacity of the cell-free blood serum to neutralize the toxic substance produced by the diphtheria bacilli.*

1897 ***Paul R. Ehrlich*** *(1854–1915) laid the* chemical *theory accounting for the molecular basis of antibody–antigen reactions. Father of modern immunology, (NP 1908). Conceptualized the interactions between cells, antibodies and antigens as essentially* chemical *responses. Shared the 1908 NP with Metchnikov for their contributions to immunity and serum therapy.*

1901 ***Karl Landsteiner*** *(1868–1943) used the specific antibody–antigen reaction to identify major human blood groups A, B, AB, and O.*

Phase II: 1910–1938

Knowledge was increasingly applied in preparation of immune sera and diagnostic reagents for infection diseases: First vaccine for Diphtheria (1923), Pertussis (1926), Tetanus (1927), Yellow Fever (1935), Polio (1652, 1962); Measles (1964); Mumps (1967); Rubella (1970); Hepatitis B (1981).

1928 ***Alexander Fleming*** *(1881–1955) discovered Penicillin.*

1932 ***Gerhard Domagk*** *(1895–1964) found the* sulfonamide prontosil *to be effective against streptococcus. This was the first drug effective against bacterial infections. Sulfonamides became a revolutionary weapon at the time, but were later replaced by penicillin, which showed both better effects, and fewer side effects. (Kidney stones and changes in bone marrow). Domagk's work on sulfonamides eventually led to the development of the* antituberculosis *drugs.*

Phase III: 1938–1988*:*

Cellular and molecular aspects of the immune-system.

Immunology has benefited from the techniques and theories of molecular biology. *The* lymphocytes *of the blood have become an important focus of study and* clonal selection *theory had dominant research.*

_1938_ ***John Marrack*** *expounded the antigen-antibody binding hypothesis.*[44]

_1949–1957_ ***Peter Medawar*** *(1915–1987) and* ***Frank M. Burnet*** *(1899–1985) discovered how the immune system rejects or accept organ transplantation, thus creating a platform for developing methods of transplanting solid organs (Immunological Tolerance* hypothesis*). (NP 1960).*

_1953–1978_ ***Michael Heidelberger*** *(1888–1991) The father of modern immunology. Together with* ***Oswald Avery*** *showed that polysaccharides of pneumococcus are antigens, enabling him to show that antibodies are proteins.*

_1956_ ***Niels K. Jerne***, ***David Talmage*** *and* ***Frank M. Burnet*** *developed clonal selection hypothesis; it proposes that before a lymphocyte ever encounters an antigen, the lymphocyte has specific receptors for that antigen on its surface.*

_1956–1961_ ***Baruj Benacerraf*** *(1920–),* ***Jean Dausset*** *(1916–), and* ***George D. Snell*** *(1903–1996) discovered genetically–determined structures on the cell surface that regulate immunological reactions. (NP 1980).*

_1957_ ***Alick Isaacs*** *(1921–1967) and* ***Jean Lindenmann*** *(1924–) discovered interferon.*[45]

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[44] Antibodies are Y-shaped molecules, two arms of which are identical and have the ability to bind to certain proteins called antigens. The binding takes place by van der Waals forces. Because these forces are very short range, there must be a close fit between the atoms in the arm of the antibody and those in the antigen, and the bonding is very specific. An inactive part of the antibody can be tagged with fluorescein isothiocyanate. Antigens to which these tagged antibodies have become attached can then be identified by their green fluorescence when exposed to ultraviolet light. A high-energy photon is absorbed by the atom, which then loses energy in two or more successive steps. If the high-energy photon is in the ultraviolet, one or both of the lower-energy photons my be in the visible range. Typical lifetimes for the decay of the excited atoms are 10^{-9} to 10^{-7} s; if the decay is much longer than this, the phenomenon is called phosphorescence. Lifetimes can range up to hours.

[45] Its effects were noticed earlier (1954) by the Japanese virologists **Yasuichi Nagano** (1906–1998) and **Yasuhiko Kojima**. Interferons are a class of natural proteins produced by cells of the immune system of most animals in response to challenges of foreign agents such as *viruses, bacteria, parasites* and *tumor* cells. Interferons belong to the large class of glycoproteins known as *cytokines*.

1958–1962 **Gerald M. Edelman** *(1929–) and* **Rodney R. Porter** *(1917–1985) discovered human leukocyte antigens and antibody structure, thymus involvement in cellular immunity and T and B cell cooperation in immune response. (NP 1972).*

1965–1972 **Elvin A. Kabat** *(1914–2000) elucidated structure and genetic basis for specificity of antibodies. First to demonstrate that antibodies are γ-globulins.*[46]

1966 **Kimishige Ishizaka** *(1925–) discovered a new type of immunoglobulin IgE that develops allergy and elucidated the mechanism of allergy at molecular and cellular levels. Following this discovery,* **S.G.O. Johansson** *found an IgE myeloma, confirming Ishizaka's discovery. This achievement contributes to the clinical diagnosis and treatment of allergic diseases.*

1974 **Rolf M. Zinkernagel** *(1944–) and* **Peter C. Doherty** *(1940–) discovered how the immune system recognizes virus-infected cells. (NP 1996).*

1975 **Cesar Milstein** *(1927–2002),* **Georges J.F. Köhler** *(1946–1995) and* **Niels K. Jerne** *(1911–1994) developed theories concerning the specificity in development and control of the immune system and the discovery of the principle for production of monoclonal antibodies. (NP 1984). This discovery led to an enormous expansion in the exploitation of antibodies in science and medicine.*

1976 **Susumu Tonegawa** *(1939–) discovered a genetic principle for generation of antibody diversity.*[47] *(NP 1987).*

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[46] Memory B cells are a *B cell* sub-type that are formed following primary infection. When a B cell is activated, by recognizing a specific *antigen*, it proliferates to form *antibody* producing *plasma cells* and long-lived memory B cells. The memory B cells are specific for the antigen that first stimulated their production. If this antigen is encountered again, memory B cells can recognize it and quickly proliferate. This forms a new generation of antibody-producing plasma cells. This means that the antibody response is much more rapid in subsequent infections, than primary infection, reducing the chance of symptom development. This is the principle behind *vaccination*.

[47] To achieve the diversity of antibodies needed to protect against any type of antigen, the immune system would require millions of *genes* coding for different

1983 *Discovery of HIV.*

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antibodies, if each antibody was encoded by one gene. Instead, as **Tonegawa** showed in a series of experiments beginning in 1976, genetic material can rearrange itself to form the vast array of available antibodies. Comparing the *DNA* of *B* cells (a type of *white blood cells*) in embryonic and adult mice, he observed that genes in the mature B cells of the adult *mice* are moved around, recombined, and deleted to form the diversity of the variable region of antibodies.

Timeline History of Neurobiology

1873–1880 ***Camilo Golgi*** *(1843–1926), pioneer of modern neurophysiology. Discovered dendritic nerve cells called "Golgi cells" and "Golgi tendon spindle" (NP 1906)*

1889–1928 ***Santiago Ramón y Cajal*** *(1852–1934), pioneer of modern neurophysiology. First to formulate the* neuronal theory *(based on the individuality of the nerve cell. NP 1906)*

1895–1932 ***Charles Scott Sherrington*** *(1857–1952) formed the scientific basis of modern neurology; coined the terms* neuron *and* synapse*. Outlined the nature of communication between nerves and between nerves and muscles. (NP 1932)*

1903–1912 ***Robert Barany*** *(1876–1936) pioneered in the study of the human inner ear's balancing organ. (NP 1914)*

1913–1928 ***Edgar Douglas Adrian*** *(1889–1977). Pioneering studies of the electrophysiology of the brain and the nervous system. (NP 1932)*

1920–1929 ***Otto Loewi*** *(1873–1961) and* ***Henry Hallett Dale*** *(1875–1968) independently isolated* acetylcholine*, the substance released by the vagus nerve, thus providing evidence for chemical transmission of nerve impulses across the synapse. (NP 1936)*

1921–1935 ***Joseph Erlanger*** *(1874–1965) and* ***Herbert Spencer Gasser*** *(1888–1963) analyzed fundamental properties of neural conduction impulses and discovered that the velocity of the impulse was proportional to the diameter of the nerve fiber. (NP 1944)*

1928–1960 ***Georg von Békésy*** *(1899–1972). Discovered physical mechanisms of stimulation within the cochlea of the inner ear. (NP 1961)*

1932 ***Walter B. Cannon*** *(1871–1945, USA) coined the term* homeostasis*.*

1932–1956 ***Ragnar Granit*** *(1900–1991),* ***Haldan K. Hartline*** *(1903–1983) and* ***George Wald*** *(1906–1997), share the 1967 Nobel Prize for their discoveries in the neurophysiology of vision.*

1945–1952 ***Alan Lloyed Hodgkin*** *(1914–1998, England),* ***John C. Eccles*** *(1903–1997) and* ***Andrew Fielding Huxley*** *(1917– , England) described the ionic mechanism by which neurons transmit electrical impulses. (NP 1952)*

1945–1965 ***Julius Axelrod*** *(1912–2004),* ***Bernard Katz*** *(1911–2003) and* ***Ulf Svante von Euler*** *(1905–1983) share the 1970 Nobel Prize for their work on electrical simulation of nerves and the processes of neuromuscular transmission.*

1965–1977 ***David Hunter Hubel*** *(1926–) and* ***Torsten Nils Wiesel*** *(1924–), followed on from the work of Granit and Hartline to study the way in which the brain processes visual information. They demonstrated that there is a hierarchical processing pathway, of increasingly sophisticated analysis of visual information by nerve cells from the retinae to the cerebral cortex. Shared the NP in 1981.*

1968–1981 ***Stanley H. Cohen*** *(1917–) and* ***Rita Levi-Montalcini*** *(1909–) awarded the Nobel Prize (1986) for their work on the control of nerve cell growth.*

1976–1986 ***Erwin Neher*** *(1944–) and* ***Bert Sakmann*** *(1942–). Revolutionized cell physiology with the invention of the "patch–clamp" recording technique which made it possible to record the electrical activity of very small areas of membrane, by eliminating the membrane's electrical noise. This improved the sensitivity of previously available methods by a factor of a million. This was done by touching a cell membrane with the tip of a glass pipette filled with saline solution, and by applying suction through the pipette, creating a seal which isolated a small section of the membrane. This method was then applied to the study of nerve impulse propagation along axons.*

1969–1985 ***Alfred G. Gilman*** *(1941–) and* ***Martin Rodbell*** *(1925–1998), discovered G-proteins and the role of these proteins in signal transduction in cells. (G-proteins are a vital intermediary between the activation of receptors on the cell membrane and actions within the cell). NP 1994.*

Arvid Carlsson *(1923–),* ***Paul Greengard*** *(1925–) and* ***Eric Kandel*** *(1929–) share the 2000 NP for their discoveries concerning signal transduction in the nervous system.*

1973 ***Solomon H. Snyder*** *(1938–). Discovered the presence of opiate receptors in nervous tissues.*

Timeline History of Endocrinology

1902 ***William Bayliss*** *(1860–1924) and* ***Ernest H. Starling*** *(1866–1927) discovered secretin, the hormone secreted by the duodenum that stimulates pancreatic secretions. Previously, the process had been considered (e.g., by* ***Ivan Pavlov****) to be regulated by the nervous system. Starling and Bayliss demonstrated that injected duodenal extract into dogs rapidly increased pancreatic secretions, raising the possibility of a chemical messenger.*

1905 ***William B. Hardy*** *(1864–1934), a Cambridge physiologist coined the word "hormone" and suggested its use to Starling.*

1911 ***Bernardo Houssay*** *(1887–1971) discovered the part played by the hormone of the anterior pituitary lobe in the metabolism of sugar (NP 1947)*

1921 ***Frederick Grant Banting*** *(1891–1941) and* ***John J.R. Macleod*** *discovered the hormone Insulin. (NP 1923)*

1927 ***Bernard Zondek*** *(1891–1966) discovered the sex hormone gonadotrophine and developed the first reliable hormone pregnancy test.*

1929 ***Edward Doisy*** *(1893–1986) discovered the sex hormones oestrone, oestriol and oestradiol. (NP 1943 for discovering the chemical nature of vitamin K)*

1930 ***Adolf F.J. Butenandt*** *(1903–1995). Discovered the sex hormones androsterone and oestriol (NP 1939;* ***Zondek*** *should have shared it, but at that time he was a refugee in Palestine, driven out of Nazi Germany in 1933)*

1935 ***Karoly G. David*** *isolated pure crystalline testosterone from testicles.*

1948 ***E.C. Kendall*** *(1886–1972),* ***Tadues Reichstein*** *(1897–1996) and* ***P.S. Hench*** *(1896–1965), discovered the structure and biological effects of the hormones of the adrenal cortex. (NP 1950)*

1951 **Robert Woodward** *(1917–1979) synthesized cortisone. (NP 1965)*

1951 **Charles Brenton Huggins** *(1901–1997) won the Nobel Prize (1966) for his hormonal treatment of prostate cancer.*

1951 **Frederick Sanger** *(1918–) determined the exact sequence of amino acids comprising the insulin molecule. (NP 1958)*

1960 **Rosalyn Sussman Yalow** *(1921–) received the Nobel Prize for the development of radioimmunoassays for insulin (1977).*

1960–1976 **Sune K. Bergström** *(1916–2004)*, **Bengt I. Samuelsson** *(1934–)* and **John R. Vane** *(1927–2004) won the NP for 1982 for discoveries concerning prostaglandins and related biologically active substances.*

1942–1969 **Dorothy Crowfoot Hodgkin** *(1910–1994) determined the spatial conformation of the insulin molecule by means of X-ray diffraction studies.[She has been awarded earlier in 1964 the NP in chemistry for the development of crystallography in 1954].*

1970 **Roger Guillemin** *(1924–)* and **Andrew V. Schally** *(1926–) discovered the peptide hormone production of the brain. (NP 1977)*

Timeline History of Ecology

Ecology is generally spoken of as a new science, having only become prominent in the second half of the 20th Century. Nonetheless, ecological thinking at some level has been around for a long time, and the principles of ecology have developed gradually, closely intertwined with the development of other biological disciplines.

354–322 BCE *One of the first ecologists may have been* ***Aristotle*** *or perhaps his student,* ***Theophrastos****, both of whom had interest in many species of animals. Theophrastos described interrelationships between animals and between animals and their environment as early as the 4th century BCE.*

75–79 CE ***Pliny the Elder*** *(23–79 CE) writes: "Natural History"*

1555 CE ***Georg Bauer*** *(Agricola, 1490–1555, Germany) writes: "De re Metallica"*

Throughout the 18th and the beginning of the 19th century, the great maritime powers such as Britain, Spain, and Portugal launched many world exploratory expeditions to develop maritime commerce with other countries, and to discover new natural resources, as well as to catalog them. At the beginning of the 18th century, about twenty thousand plant species were known, versus forty thousand at the beginning of the 19th century, and almost 400,000 today.

1789–1804 *Ecology blossomed due to the new discoveries of Antoine Lavoisier (1743–1794, France) and* ***de Saussure*** *(1767–1845, France), notably – the nitrogen cycle.*

1789 ***James Hutton*** *(1726–1797) considered the earth to be a* super-organism *and that its proper study should be by physiology. He was thus the first to coin the term 'geophysiology'.*

1798 ***Thomas Malthus*** *(1766–1834, England) writes: "Essay of the Principle of Population Theory".*

The above-mentioned expeditions were joined by many scientists, including botanists, such as the explorer:

1802–1807 ***Alexander von Humboldt*** *(1769–1859, Germany). He is often considered a father of ecology.*

He was the first to take on the study of the relationship between organisms and their environment. He exposed the existing relationships between observed plant species and climate, and described vegetation zones using latitude and altitude, a discipline now known as geobotany.

In 1804, for example, he reported an impressive number of species, particularly plants, for which he sought to explain their geographic distribution with respect to geological data. One of Humboldt's famous works was "Idea for a Plant Geography" (1805).

1852 ***David Thoreau*** *(1817–1862, USA), first to coin the term ecology.*

1858 ***Charles Darwin*** *(1809–1882, England) published "The Origin of species". Ecology passed from a repetitive, mechanical model to a biological, organic and hence evolutionary model.*
Alfred Russel Wallace *(1823–1913, England), contemporary and competitor to Darwin, was first to propose a "geography" of animal species. Several authors recognized at the time that species were not independent of each other, and grouped them into plant species, animal species, and later into communities of living beings.*

1866 ***Ernst Haeckel*** *(1834–1919, Germany) recoined the term oekologie. The word derives from the Greek oikos="household" and logos="study". It meant to the Greek – the study of nature.*

The word "ecology" is often used in common parlance as a synonym for the natural environment or environmentalism. Likewise "ecologic" or "ecological" is often taken in the sense of environmentally friendly.

1875 ***Eduard Suess*** *(1831–1914, Austria) proposed the name biosphere for the conditions promoting life, such as those on earth, which induces flora, fauna, minerals, matter cycles, etc. The need for this term followed his observation that life developed only within strict limits of each compartment that makes up the atmosphere, hydrosphere and lithosphere.*

1879–1914 **John Muir** *(1838–1914, USA), explorer, naturalist and ecologist. Campaigned for the forest conservation in the United States. Founded the Sierra Club which became a leading conservation organization.*

1899 **Henry Chandler Cowles** *(1869–1939, USA). Ecological Pioneer. Invented the concept* Ecological succession *as a process by which a natural community moved from a simpler level of organization to a more complex community.*

1926 **Vladimir Vernadsky** *(1863–1945, Russia). Founder of modern* biogeochemistry. *Detailed the idea of the biosphere in his work "The biosphere" (1926) and described the fundamental principles of the biogeochemical cycles. He thus redefined the biosphere as the sum of all* ecosystems.

First ecological damages were reported in the 18th century, as the multiplication of colonies caused deforestation. Since the 19th century, with the industrial revolution, more and more pressing concerns have grown about the impact of human activity on the environment. The term ecologist has been in use since the end of the 19th century.

Over the 19th century, botanical geography and zoogeography combined to form the basis of biogeography. This science, which deals with habitats of species, seeks to explain the reasons for the presence of certain species in a given location.

1935 **Arthur Tansley** *(1871–1955, England) coined the term ecosystem, the interactive system established between the biocoenosis (the group of living creatures), and their biotope, the environment in which they live. Ecology thus became the science of ecosystems.*

1951–1962 **Rachel Carson** *(1907–1964, USA). Zoologist and marine biologist. Her book "*Silent Spring*" launched the global environmental movement. In the United States it spurred a reversal in national pesticide policy.*

1953 **Eugene P. Odum** *(1913–2002, USA). Known for his pioneering work on* ecosystem ecology. *Explored how one natural system can interact with another. Believed that* homeostasis *and stability in ecosystems was a result of evolutionary processes.*

1971 *Ecology became a central part of the World's politics as UNESCO launched a research program called Man and Biosphere, with the objective of increasing knowledge about the mutual relationship between humans and nature. A few years later it defined the concept of Biosphere Reserve.*

1972 *The United Nations held the first international conference on the human environment in Stockholm. This conference was the origin of the phrase "Think Globally, Act Locally".*

1979 ***James Lovelock*** *(1919– , England). Researcher and environmentalist. Proposed the* '*Gaia Hypothesis*'*, in which he postulated that the earth functions as a kind of superorganism. He popularized the term* "geophysiology" *(foreshadowed by* ***James Hutton*** *(1789) and* ***Huxley*** *(1825–1895)). While the* Gaia Hypothesis *was readily accepted by many in the* environmental community*, it has not been fully accepted within the* scientific community. *Critics point out that since natural selection operates on individuals, it is not obvious how planetary-scale homeostasis can evolve.*

1992 *Earth Summit in Rio de Janeiro; The concept of the* biosphere *was recognized by the major international organizations, and risks associated with reduction in biodiversity were publicly acknowledged.*

1997 *The dangers the biosphere was facing were recognized from an international point of view at the conference leading to the Kyoto Protocol. In particular, this conference highlighted the increasing dangers of the greenhouse effect – related to the increasing concentration of greenhouse gases in the atmosphere, leading to global changes in climate. In Kyoto, most of the world's nations recognized the importance of looking at ecology from a global point of view, on a worldwide scale, and taking into account the impact of humans on the earth's environment.*

Life Prospects on a Hazardous Planet

Cycles of matter and energy on earth, caused both by internal and external agents, have always been accompanied by catastrophes hazardous to life. The energy sources and energy content of the various natural processes are well known: In general, the combination of the internal heat flow and gravity results in plate tectonic movements and the associated earthquakes, volcanism, mountain uplift as well as forming a few types of tsunami. The combination of the sun's energy and gravity results in all weathering, erosion and mass wasting phenomena including rain-floods, blizzards, hurricanes, tornadoes, lightning, hail, avalanches, mudflows, waves, landslides and associated tsunami.

During the early history of the earth, interior energy *sources were contributed by:*

- *Bolide impacts*[48]*: these helped raise the temperature of the early earth and some of that heat was left over as the internal heat of the earth.*
- *Gravity: gravitational collapse of the earth raised the interior temperatures.*

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[48] Up to 1970's, there was little interest in asteroids, including near earth asteroids. They were considered low class astronomical objects. Indeed, the small asteroid which destroyed hundreds of square kilometers in remote Siberia in 1908 was an event little known to the general public. A small asteroid which skimmed the upper atmosphere in the 1970's, as detected by a U.S. military satellite, received little publicity.

In the 1970's, things started to change. A small but increasing number of astronomers interested in asteroids began to realize the abundance of asteroids which passed close to earth, by instituting processes to catalog asteroids accidentally seen on telescopic plates and previously not recorded.

Theoretical computer models revealed that the gravity of the planets caused a sizable number of asteroids from the Main Belt between Mars and Jupiter to cascade down into lower orbits approaching or crossing earth's. Further, a significant fraction of comets passing through the inner solar system would be diverted into orbits near earth due to gravitational encounters with the inner planets. As a result of these discoveries, the estimated number of near-earth objects dramatically expanded by about 1000 times. Finally, new telescope technology, emerging around 1900, increased the discovery rate of all asteroids. It is now estimated that there are about 300,000 near-earth asteroids which are over 100 m in diameter, and about 2000 over 1 kilometer in diameter.

- *Radioactivity: it continues to create tremendous heat in the earth, although more heat was generated earlier. Radioactivity allows for precise age dating of rock strata.*

Exterior energy sources[49] *are due to solar energy which surpasses heat flow from the interior by a factor of 5300; 30 percent of the sun's energy is reflected as short wavelength radiation, 47 percent is absorbed and re-radiated as long wavelength radiation (infrared) and 23 percent evaporates water (mostly in the tropics) to begin the hydrological cycle. The hydrological cycle in combination with gravity results in waterfalls, rivers, floods, atmospheric circulation, hurricanes, thunderstorms, lightning, tornadoes, mass movement, waves and landslide induced tsunami.*

Another exterior energy source is impact *energy of asteroids and comets with solar system velocities ranging from 11 to 70 $\frac{km}{sec}$. Temperatures of at least 10,000 degrees are produced during impact. These events have also been proposed to have produced global devastation and mass extinctions.*

Terrestrial catastrophism via bombardment episodes by 'cosmic bullets' impacted the mind of man since times immemorial: the same basic story repeated itself in many ancient cultures. The Bible, for example, tells us of darkness, pillar of cloud by day and fire by night, hail and fire dropped on earth, winged serpents in the sky, huge burning stars falling from heaven, dust from impact, earthquakes, large ecological upsets, tsunamis – all recognizable symptoms of asteroid collision with earth.

These phenomena, although faithfully observed by prehistoric people, could not have been documented prior to the invention of writing in ca 3200 BCE and certainly could not be understood before the advent of modern science in the early 16th century.

Natural disasters may be classified into the following categories:

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49 Earth receives from the sun the total of 1.75×10^{17} watt ($\frac{Joule}{sec}$) which amounts to an average flux of 1367 $\frac{watt}{m^2}$. Of this only 1.25×10^{17} watt is absorbed by the atmosphere, hydrosphere and surface (69%). The average *total solar* flux on the earth's surface amounts to 168 $\frac{watt}{m^2}$. The *non-solar* geothermal energy release (radioactive decay + primordial heat) reaches a level of 6.6×10^{13} watt $= 6.6 \times 10^{20}\ \frac{erg}{sec} = 2.07 \times 10^{28}\ \frac{erg}{year} = 5 \times 10^5\ \frac{MT}{year}$. This energy flow feeds continental drift, earthquakes, volcanoes and hot springs. Direct solar energy feeds atmospheric circulation and all accompanying meteorological processes. Appreciable global changes occur at the level of one percent of the solar input.

- *Climatic changes and ecological collapses; return of a severe glacial epoch*[50].
- *Collision of earth with asteroids, mini-planets*[51] *and comets.*
- *Volcanic eruptions and earthquakes.*
- *Floods, tsunamis, mega-tsunamis and major storms.*
- *Droughts, famines.*
- *Pandemics.*
- *Supernova explosion*[52].

Collision with Earth of Asteroids and Comets

How do asteroids arrive at earth? Detailed scenarios for the formation and transport of meteorites and other near-earth asteroids were clarified only

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[50] The frightening aspect of this 'option' is that the sun's smallest tantrum can influence our climate significantly in spite of the vast distance of 150 million km separating it from earth. *Sunspots* correspond to changes in solar activity: times of greatest activity correspond to warm periods on earth. In the middle of the 20^{th} century observations showed a great density of sunspots while recorded air temperatures were noticeably higher. In contrast, between 1430 to 1850, the sun seems to have gone through a calm period and sunspots were practically absent. This period coincided with a decrease in temperatures.

[51] With diameter$>$ 1 km.

[52] A *supernova explosion* could completely extinguish life on earth by high-energy gamma-ray bursts and particles from the explosion, provided it explodes within 200 light-years from the solar system. Among the 100,000 closest stars we expect one supernova to explode in a time-scale of roughly a billion years – of the order of the time interval during which life has thus far existed on earth!
The last one seen in our galaxy was Kepler's, in 1604. In general, supernovas derive their energy from the *gravitational collapse* of the central core of a heavy enough star: the core, having a mass of order that of the sun, collapses into a neutron star and thereby liberates enormous amounts of energy (ca 10^{53} erg).
Other known supernova explosions are the Crab Nebula explosion (1054 CE) and the 1987A explosion in the large Magellanic Cloud.

recently. The basic idea goes back to ***Kirkwood*** *(1866), who identified gaps in the distribution of asteroids at distances from the sun at which their orbital periods would be commensurable with Jupiter's period (such as 5.93 years), forming ratios of small whole numbers. Kirkwood proposed that the gaps arose as a consequence of perturbations caused by Jupiter: the effect of Jupiter's mass would be to force any asteroid that appeared in one of the asteroid-free zones into another orbit, with the result that it would immediately leave the zone (according to Kepler's law, a period of 5.93 years corresponds to a semimajor axis of 3.28 AU).*

It has been shown that such 'resonances' (as they are called) could raise asteroids' orbital eccentricities to a point that bodies swept away from the gaps could reach Mars. Scientists demonstrated that material could reach earth through chaotic dynamics at the 3:1 resonance. It was reported that numerous weak resonances and interactions among the resonances cause chaotic behavior capable of raising main-belt eccentricities to allow for a Mars crossing. Objects then break away from the resonances when they have close encounters with Mars and subsequently evolve as Mars crossers, until they become near-earth asteroids.

This scenario is sped up by a series of minor resonances that yield fairly chaotic evolution, lasting a few times 10^7 *years, during which a reasonable fraction is likely to hit earth. In short — several resonances provide the escape route from the main belt and allow bodies to reach earth in a few tens of million of years.*

*The most energetic – and best-documented – encounter with the earth of an earth-crossing body produced a great fireball over the Podkamennaya–*Tunguska River *region of Siberia on the morning of June 30, 1908. Traveling from southeast to northwest, the bolide nearly passed over the town of Kirensk; the endpoint of the trajectory was about 60 km northwest of the remote trading post of Vanovara, over a very sparsely inhabited part of the Siberian taiga.*

The bolide was observed from distances as great as 600–1000 km from the endpoint; the atmospheric shock was audible at a distance of 1270 km and heat was felt at a distance of 70 km. Trees were knocked down at distances up to 40 km from the endpoint, and circumstantial evidence suggests that dry timber was ignited by thermal radiation from the fireball at distances up to 15 km from the endpoint.

Intensive investigation by expeditions carried out over many decades has shown that the Tunguska bolide disintegrated in the atmosphere; it deposited most of its kinetic energy at an estimated altitude of $\sim$*8.5 km (****A. Ben-Menahem****, 1975). Only microscopic spheres of glass and magnetite, formed by ablation, reached the ground.*

Long-period atmospheric gravity and acoustic waves excited by the atmospheric shock were well recorded on weather-station barographs in Siberia and southern England; the passage of these waves in both the direct and reverse paths was recorded on a barograph at Potsdam, Germany. Coupling of the air wave to the ground near the endpoint produced seismic waves detected at Irkutsk, Tashkent, Tiflis, and Jena; local coupling of the air wave to the ground as it passed over some seismic stations was also recorded.

On the basis of a thorough study of the seismic and acoustic records, Ben-Menahem (1975) estimated the released energy at 12.5 ± 2.5 *megatons. The distance to which trees were felled near the endpoint is consistent with a* $\sim$*13-megaton explosion at an altitude of* $\sim$*8 km and scaling from the effects of nuclear weapons; the ignition of wood by the thermal radiation also suggests an energy of the order of the same yield.*

E. Shoemaker (1983) has shown that the 'best estimate' of the frequency of a 12-megaton encounter with earth is about once every 150–600 years. The explosion of the world's entire nuclear arsenal (ca 10^5 *MT ; see Table 6.12) could not match the energy released when a kilometer-size asteroid hits the earth.*

Stimulated by the renewed interest of scientists in the Tunguska explosion, Alvarez et al. (1980) hypothesized that the major dinosaur (and other fauna) extinction event at the Cretaceous-Tertiary boundary (65 My) was the result of an enormous impact of an asteroid or a comet. The heat from the impact caused forest fires, and lofted huge amounts of dust from the impact crater into the atmosphere. The smoke and ash from the fires and the dust particles together blocked the sun for at least several months. The temperature decreased since the sunlight was partially blocked from reaching the earth's surface.

Without sunlight and in the cold, plants began to die since they could not obtain nutrition. This in turn affected the plant-eating dinosaurs; lacking their source of food they too began to die. Since the plant-eating dinosaurs were dying, there was a lack of food for the meat-eating dinosaurs, and they perished as well. In other words, the effects of the impact caused the ecosystem to collapse. It is speculated that this impact led to the extinction of 75 percent of the species alive on earth during that time.

Such a 'relatively common' event would not show up much in geological records on a global scale. There have been many local tsunamis and brief climate changes in recorded history without any plausible explanation (e.g., Amos **9**, *5–7).*

If an asteroid of size 1 kilometer hit earth, its impact would wipe out life within proximity of the impact site. However, more serious is how it would *affect the whole world in indirect ways. The dust and/or vapor cloud created by an impact to either the land or the ocean could be big enough to create a "nuclear winter" like mini-ice age, and disrupt climatological wind patterns, adversely affecting major food-growing regions of the world, thus straining world food supplies, governments and civilization. This is comparable to what caused the dinosaur extinction, as well as other major extinctions of smaller creatures over geologic time scales.*

Earth's atmosphere gives protection against the vast majority of small asteroids which hit. Asteroids hit the atmosphere at typical speeds in excess of 10 km/sec — an average of about 20 km/sec for asteroids whose entire orbits reside within the inner solar system (with exact relative speed depending upon their angle of approach) and with speeds over 50 km/sec common for small cometary objects making a pass from the outer solar system. At such speeds, they usually break up due to severe shock pressures, and burn up due to friction with the atmosphere.

For asteroids coming in at 20 km/sec, it is generally thought that to penetrate the atmosphere and cause major damage by tsunami, an iron asteroid must be around 40 to 60 meters in diameter, and a stony asteroid 200 meters in diameter. However, a stony asteroid 60 meters in diameter can cause significant damage by airbursts. The exact damage caused by an asteroid or comet depends upon a number of factors: size, speed, composition, and whether it hits land or ocean.

For a land impact, it can be said in general that an object of roughly 75 meters in diameter can destroy a city, a 160 meter object can destroy a large urban area, a 350 meter object can destroy a small district or state, and a 700 meter object can destroy a small country.

For an ocean impact, the destruction is much greater. Smaller objects can cause far more widespread damage. The effects of an ocean impact are felt much further away than the effects of an airburst due to the more effective propagation of water waves, and the fact that human populations and assets are largely concentrated in coastal cities which historically became established due to water transport (i.e., shipping and trade) and the ports that serve it.

For example, the earthquake-induced tsunami in Chile in 1960 produced waves in Hawaii, 10,600 km away, of height up to over 10 meters, and up to 5 meters in Japan 17,000 km away with an average height of 2 meters, causing heavy damages and loss of lives.

The effects of an airburst are far more localized because the intensity of the phenomenon decreases with the inverse square of the distance, whereas the height of a water wave decreases only with the inverse of the distance, i.e., as the first inverse power, due to its circular, two-dimensional nature.

On March 23, 1989, *an asteroid with a kinetic energy of over 1000 one-megaton hydrogen bombs (i.e., about 5000 times more powerful than the bomb dropped on Hiroshima) was recorded to have passed very close to earth, using new equipment recently deployed. Named* 1989FC, *this asteroid was detected well after its point of closest approach, and astronomers found out it had passed so close only after calculating backwards its orbital path having once realized its nearness. This was a key event that brought near earth asteroids into the political arena.*

Later, the new Spacewatch Camera in Arizona, using the latest technology in electronic sensors and computerized automated detection, discovered four asteroids that came closer to the earth than the moon (actually within half the distance of the moon) in 1991–94.

It is thought that the very massive asteroid Geographos, a cigar-shaped body of 5.1 kilometers by 1.8 kilometers which passed near earth in 1969 and again in 1994, could collide with earth in the not too distant future. It is probably an iron or stony iron asteroid. It would make the Tunguska meteorite look like a trivial impact. Geographos would cause a global ice age for several years from the dust it would kick up. But it will not impact in the next few hundred years, at least. We can not project Geographos' orbit in the far future with enough precision to determine if it will impact earth or the moon, or whether it will have a close encounter which will fling it elsewhere.

Table 6.11 lists forthcoming close approaches to earth of certain minor planets and comets.

The regional and global effects from an impact of an asteroid of diameter $d > 1$km *are as follows:*

- *Massive* earthquake *with magnitude* $M \geq 13$ *(Richter scale).*
- Global winter *syndrome: dust pumped into the atmosphere blocks incoming solar radiation, producing* darkness *and* temperature depression *for months. Disruption of ecosystems by diminished photosynthesis.*
- Greenhouse gases H_2O *and* CO_2 *are formed if impact occurs in the oceans.* Warming effects *are created for many years after cooling.*
- Giant tsunamis, *1–3 km high, flood the interior of continents.*
- Shock waves *in the atmosphere produce* nitrogen oxides *(Nitrogen + Oxygen). Combined with water, nitric acid is formed, and precipitates as* Acid Rain.

Figure 6.1 exhibits the evidence concerning the dependence of the cumulative frequency distribution of earth-bolide encounter on the energy released during impact. It tells us that:

1 – year event has the energy equivalence of	*10 KT*
10 –	*100 KT*
100 –	*3 MT*
1,000 –	*50 MT*
10,000 –	*1,000 MT*
100,000 –	*20,000 MT*
1,000,000 –	6×10^5 *MT*
10,000,000 –	10^7 *MT*
100,000,000 –	10^8 *MT*

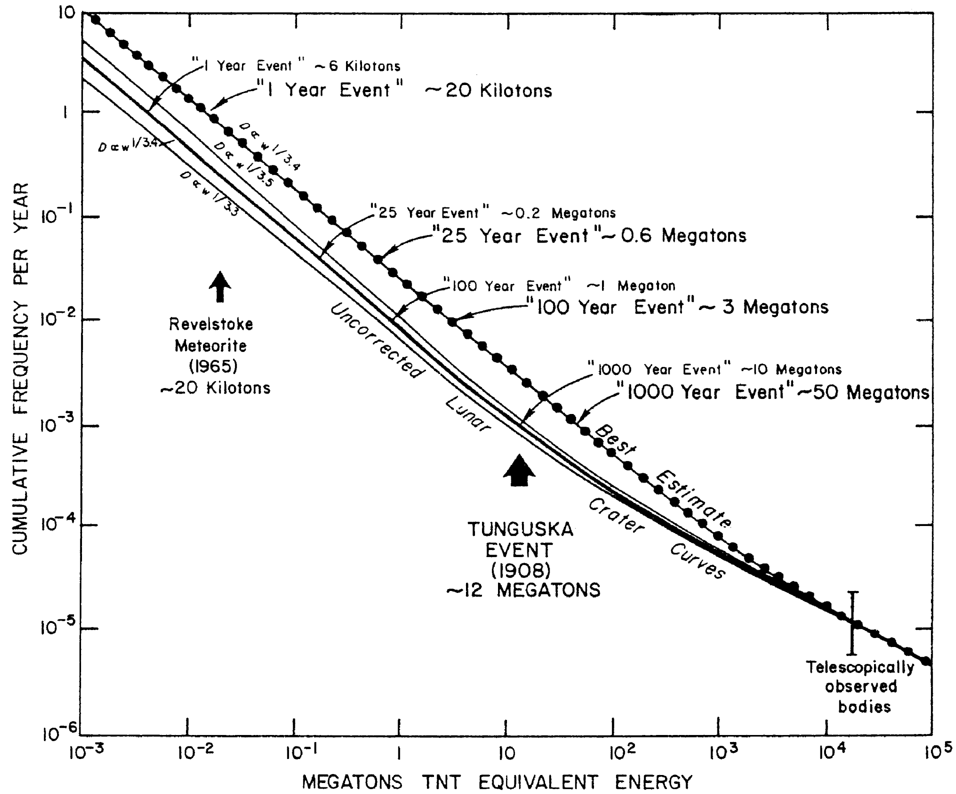

Fig. 6.1: Estimated cumulative frequency distribution of kinetic energy of bodies colliding with the earth

Table 6.11: Predicted minor-planet and comet encounters with earth to within 0.05 AU (Astronomical Unit = ca 150 million km) during 2005–2031. At 40 km/sec, an object covers a distance of about 3.5×10^6 km per day

Object	Date of encounter	Distance (AU)
1992 UK4	*Aug. 08, 2005*	*0.0404*
(4450) Pan	*Feb. 19, 2008*	*0.0408*
(1620) Geographos	*Mar. 17, 2008*	*0.1251*
	Aug. 12, 2026	*0.1704*
1991 VH	*Aug. 15, 2008*	*0.0458*
1998 CS1	*Jan. 17, 2009*	*0.0286*
1994 CC	*June 10, 2009*	*0.0169*
1998 HE3	*May 10, 2012*	*0.0340*
1998 KJ17	*May 27, 2012*	*0.0355*
1988 TA	*May 09, 2013*	*0.0411*
1998 FW4	*Sept. 27, 2013*	*0.0075*
(2340) Hathor	*Oct. 21, 2014*	*0.0482*
(5604) 1992 FE	*Feb. 24, 2017*	*0.0336*
(3122) Florence	*Sept. 01, 2017*	*0.0472*
1991 VG	*Feb. 11, 2018*	*0.0471*
1998 HL1	*Oct. 25, 2019*	*0.0416*
1997 BQ	*May 22,2020*	*0.0416*
1993 BX3	*Jan. 17, 2021*	*0.0473*
(3361) Orpheus	*Nov. 21, 2021*	*0.0386*
	Nov. 19, 2025	*0.0379*
(7335) 1989 JA	*May 27, 2022*	*0.0269*
(6037) 1988 EG	*Aug. 23, 2023*	*0.0407*
1998 KY26	*June 01, 2024*	*0.0309*
1997 QK1	*Aug. 03, 2025*	*0.0304*
1997 NC1	*June 27, 2026*	*0.0171*
1990 MU	*June 06, 2027*	*0.0309*
1998 ML14	*Aug. 03, 2028*	*0.0307*
1997 XF11	*Oct. 26, 2028*	*0.0064*
1994 WR12	*Nov. 26, 2030*	*0.0180*

Energetics

The energy sources and energy content of the various natural disasters are well known. Prominent features of energy conversion processes are:

- Internal *heat flow and gravity are responsible for the generation of earthquakes, volcanism and certain types of tsunamis.*

  Internal heat flow due to radioactivity, gravitational collapse *(when objects move closer to earth's center by falling, slipping, sinking – kinetic energy is released) and* bolide impact *(especially during early history) helped raise* earth's temperature.

- Exterior energy sources *are mainly due to solar energy. In conjunction with gravity and* earth's rotation *this drives the* hydrological cycle *which is responsible for floods, atmospheric circulation, hurricanes, thunderstorms, lightning, tornadoes, waves, and certain tsunami.*

- *Only 23% of total solar energy is available for evaporating water (mostly in the tropics) to drive the hydrological cycle.*

Table 6.12: Energy content of singular natural phenomena
(1 KT = 4.2×10^{19} erg; 1 MT = 4.2×10^{22} erg)

(a) Single Events *(BP = Before Present, KE = Kinetic Energy)*

Thunderstorm	*1 KT*
Atomic bomb (Hiroshima)	*10 KT*
St. Helens (1980; VEI=5)	*5 MT*
Hydrogen bomb	*10 MT*
Tunguska (1908)	*10 MT*
Hurricane (average KE)	*10 MT*
Barringer asteroid (diameter=50 m)	*20 MT*
Earthquake (total seismic wave energy; $M_s = 8.1$*)*	*100 MT*
Cyclone (total KE)	*100 MT*
Krakatoa explosion (1883; VEI=6)	*200 MT*
Thera (1500/1600 BCE)	*ca 1000 MT*
Asteroid 1989 FC (close encounter in 1989), KE	1.0×10^3 *MT*
Asteroid impact $d = 570$ *m; crater diameter=10 km*	1.0×10^4 *MT*
Tambora (1815; VEI=7)	2.0×10^4 *MT*
Crater Lake eruption (4895 BCE; VEI=7)	5.0×10^4 *MT*
World nuclear arsenal (10,000 warheads; 10 MT each)	1.0×10^5 *MT*
Tuba volcano, Indonesia (73,500 BP; VEI=8)	6.0×10^5 *MT*
Total seismic energy release over the past 6000 years	7.0×10^5 *MT*
Yellowstone, Wyoming (2My BP; VEI=8)	2.0×10^6 *MT*
Yucatan (Chicxulub) asteroid ($\sim 65 \times 10^6$ *years BP)*	10^8 *MT*
Hephaistos asteroid (d$\simeq$*10 km), KE*	10^8 *MT*

Table 6.12: (Cont.)

(b) Average global energy flow

	Erg/sec	*MT/year*
Volcanic eruptions	3.3×10^{16}	*25*
Earthquakes	1.6×10^{17}	*120*
Thunderstorms	3.0×10^{18}	*2400*
Total tidal dissipation per year	3.3×10^{19}	*26,000*
Atmospheric circulation	1.0×10^{23}	*80,000,000*

(c) Available terrestrial energy (MT/year)

Geothermal	5.0×10^5
Solar	8.4×10^8

Estimated Human Death-toll

Table 6.13 renders estimated cumulative death tolls since 4000 BCE due to all natural disasters combined, on the background of the evolution of world population Table 6.13.

In this connection it is of interest to estimate the total number of people who have ever lived: One begins by determining the mean population size for a birth-death stochastic process *(i.e., the average behavior of a population whose size varies stochastically, growing over time due to random occurrence of births and deaths). One then assumes a starting population of two persons 1.5 million years ago and divides the total time span into a number of smaller subintervals by using times for which estimates of world population have been made (e.g., N (8000 BCE)* $= 5 \times 10^6$*; N (0 BCE)* $= 250 \times 10^6$*; N (1750 CE)* $= 800 \times 10^6$*; N (1825 CE)* $= 10^9$*; N (1930)* $= 2 \times 10^9$*; N (1960)* $= 3 \times 10^9$*; N (1980)* $= 4.4 \times 10^9$*). The total number of people who ever lived since 1.5* *million years before present is then found to range from* 50 to 100 billion (10^{11}).

In the second category of Table 6.13, flood *is by far the worst of natural disasters, claiming some 40% of all deaths caused world-wide by acts of nature (epidemics excluded).*

Table 6.13: Estimated human death toll since 4000 BCE

Earthquakes and volcanoes and their immediate aftereffects	30×10^6
Floods, storms, tsunamis and droughts	100×10^6
Epidemics	370×10^6
Total:	ca 500×10^6

Table 6.14: World population growth (in units of 10^6 person)

2000 CE	6000
1991	5400
1960	3000
1930	2000
1800	1000
1750	800
1200	400
1000	250
500	210
0	250
-500	140
-1000	120
-1500	100
-2000	50
-8000	5
-12,000	4

Conclusion

Natural disasters have been the scourge of mankind since times immemorial.

It is estimated that ca 500 million people were killed since 4000 BCE by all catastrophes combined. This amounts to about one percent of all people who have ever lived since 4000 BCE. The total fatal energy unleashed on the earth's surface against its inhabitants throughout the said time window is estimated at 2×10^7 *MT.*

The direct effect of this onslaught on man's survival on earth was manifested through

- *Decimation and mass migration of populations.*
- *Destruction of habitat, agriculture and water systems on at least a regional scale.*

However, the interaction between nature and man is certainly ambivalent and not always unidirectional.

We know that life changed a primordial reducing *atmosphere into one rich in oxygen. On the other hand, comets could have brought an abundant amount of water to the early earth, and volcanic outgassing resulted in the formation of the primitive atmosphere and oceans. Mass extinctions due to asteroid encounters changed the course of evolution, promoting the eventual emergence of Homo Sapiens.*

Even today, nature's agents of destruction are at the same time a blessing in disguise. Take hurricanes, for example – one of the most powerful engines of death and destruction on earth. These storms are products of the tropical ocean and atmosphere: powered by heat from the sea, steered by the Easterly trades and temperate Westerlies, and their own fierce energy. But they are major source of rain and may have other hidden benefits as well for the planet.

In fact, Humans are now changing the planet in such a way that is already causing a wave of extinction on a scale unparalleled since the demise of the dinosaurs.

When we use the words 'Natural disaster' we are taking the narrow anthropocentric point of view. But in the grand scheme of things what is bad for us is not necessarily bad for the earth and its environment. In this wider context we should remember that although mankind had survived so far, there is no such guarantee for the remote future.

Cosmology and Biological Evolution: a Speculative Connection[53]

At the current stage in the development of mathematics, biochemistry and computer technology, Science is still far from being able to answer even the simplest questions regarding the feasibility of Darwinian evolution. Our level of understanding of stochastic processes, and our knowledge of the chemistry of macro-molecules, are insufficient to derive realistic estimates of the probabilities involved in a population of self-replicating biotic molecules arising in any given inanimate thermodynamic environment. We posses only the vaguest theoretical insights (and almost no empirical evidence) as to how Nature "invented" the genetic code; nor do we even know whether replication and metabolism preceded the advent of this code. And if our understanding of molecular evolution is so rudimentary, it goes without saying that we have almost no clue as to how (or indeed whether) random mutations pruned by natural selection can transform one species into another, or allow organisms of a given species to change their body plan or grow new organs.

However, as pointed out by the late astronomer Fred Hoyle, such naive probability estimates as we are able to derive, indicate that the timeline of terrestrial biological evolution, as outlined by evolutionary biologists, is almost miraculously compressed, especially in the period immediately following the onset of the so-called "Cambrian Explosion" (the emergence of virtually all presently-existing organism body plans, about 0.5 Gy ago, within the space of a few million years).

Indeed, Hoyle has pointed out that not only life and its molecules, but even the existence of carbon atoms (surely a prerequisite for life as we know it on Earth to emerge) seems a priori to have an extremely low probability of occurring within the framework of the currently accepted theories of cosmic origins and stellar evolution. Indeed, Hoyle adduced the cosmic abundance of carbon as a posteriori evidence that the constants of Nature – governing

[53] This essay explains how particle physics and cosmology might play a role in enabling biological evolution, partly aided (perhaps) by intelligent design (but not necessarily). These ideas are highly speculative, of course – no one has actually SEEN the constants of particle physics mutating, or one universe spawn baby-universes through narrow spacetime "necks", etc. But then again, it is important to keep in mind that no one has actually seen evolution happen, either (in either real time, the fossil record or even mathematical simulations).

elementary particle physics, their four types of fundamental interactions and the very structure of spacetime – have somehow been "rigged" (fine-tuned) to allow life and sentient beings to emerge. Other physicists have pointed out other apparent coincidences pointing to possible fine-tuning of the constants of nature. As an example, the following three large dimensionless numbers are all roughly ten to the power 40:

(A) The ratio of the electrostatic force between a proton and electron to the corresponding gravitational force;

(B) The square-root of the number of hydrogen atoms in the observable universe; and

(C) The ratio of the age of the universe to the time it takes light to traverse a proton[54]*.*

Another example involves the cosmological constant ("dark energy"), which governs the innate tendency of empty space to expand (a form of "antigravity"). Observations performed in the 1990s with the aid of the Hubble Space Telescope revealed that this constant, while very small (of order 10 to the negative 120th power!) on the natural scale provided by particle physics, is nonetheless not zero, and is causing a slight acceleration of the (Big-Bang caused) expansion of our universe.

It has been calculated that if this tiny constant were only ten times larger, the local "antigravity" effect it would cause would prevent galaxies from accreting, and thus possibly prevent life from emerging – although we cannot be sure about that last inference, since our knowledge about how, why and with what likelihood life emerges is so scant at present.

Two other unexplained coincidences which seem crucial to the emergence of life are: the longevity of the universe (sixty orders of magnitude longer than the Planck timescale!), and the longevity of current-generation main-sequence stars. The latter longevity is due to a fine balance (involving all four fundamental forces of Nature!) between the nuclear forces, tending to explode stars, and their self-gravitation, which tends to collapse them. A long-lived universe containing stable, long-lived stars is probably an important enabling

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[54] One of the two mathematical relations expressing these coincidences is equivalent to the statement that the total (gravitational plus kinetic) energy of the observable universe is approximately zero; or, in other words, that the Schwarzschild radius of the observable cosmos is of the order of its actual radius. Since this is also a property of black holes, this coincidence bears an intriguing similarity to the conjecture, discussed below, that our observed universe might have been spawned by the gravitational collapse of a relatively small and light lump of mass in a "parent" universe.

condition in the long (and still poorly understood) chain that gave rise to life (at least on our Earth).

Al these coincidences need not have been "engineered" by some intelligent super-being, although we lack sufficient knowledge to rule that possibility out. There are other logical possibilities, one of which (involving a Darwinian-like evolution of multiple universes and their natural constants) we now describe. Theorists attempting to develop a theory of quantum gravity have discovered that Einstein's General Theory of Relativity, in conjunction with Quantum Mechanics, may allow one universe to spawn others via gravitational collapse – the same phenomenon that forms black holes.

Such a "daughter universe" would appear, to beings in the parent universe, as a relatively small and light black hole; yet any observer that manages to safely "glide" into the daughter universe, would find himself in a vast, new universe which expands in accordance with its own "Hubble Law", and may eventually produce its own stars, galaxies and even life and intelligence.

The spacetime manifolds of the two universes are connected through a slender "neck", and the geometry of this neck explains the curious fact that in such solutions of Einstein's field equations, the daughter universe appears to have a more or less fixed event horizon from the parent-universe side, while undergoing cosmological expansion as viewed by observers on the other side of the neck (i.e., within the daughter universe).

Theorists have argued that such "reproduction" of universes may occur naturally, but also that daughter universes could be designed by intelligent beings in the parent universe. A universe may differ from those of its parent – depending, perhaps, on physical conditions (electromagnetic fields, temperature, spatial curvature, etc.) in the region of the parent universe in which the daughter was created.

Theorist Lee Smolin as argued that this combination of ensemble of reproducing universes and (possibly random) mutations in their natural constants might amount to a form of Darwinian evolution – in which universes play the roles of living organisms, while their natural constants are analogous to genomes!

However, in order for this meta-cosmology of mutating and spawning universes to undergo evolution in the Darwinian sense, a further ingredient is of course needed – namely, natural selection.

Now, what does it mean that a given universe in this ensemble is "fit"? Smolin's answer is simple: the more fecund a given universe is at producing daughter universes, the universes will be added to the ensemble which have constants of nature close to those of the given universe (assuming that a daughter universe's constants are not very different from those of its parent).

And since universe-spawning is mediated via gravitational collapse, it follows that in Smolin's scenario, a universe in which the probability of black hole formation is higher, will on average produce more universes similar to it.

Smolin further argues that a universe's rate of black-hole formation is positively correlated with its probabilities of developing life (and possibly intelligence), although of course (as with all such arguments) the last inference is the least sound. This is simply because – as emphasized above – our existing scientific knowledge does not allow us to derive any estimates, whatever, of the local constants of nature and other environmental factors.

Peering ahead into the far future of human science, we may speculate that some day, our computing technology, in addition to advances in pure and applied mathematics and in physics and chemistry, will allow us to reliably estimate such probabilities. When that stage is reached, we may finally be able to determine whether our existence is inevitable, or whether (to the contrary) many universe had to live and die in order that one allowing the likes of us to emerge, happened to be spawned. Or, in the prescient words of the British empiricist philosopher David Hume:

"But were this world ever so perfect a production, it must still remain uncertain, whether all the excellences of the work can justly be ascribed to the workman. If we survey a ship, what an exalted idea must we form of the ingenuity of the carpenter who framed so complicated, useful, and beautiful a machine? And what surprise must we feel, when we find him a stupid mechanic, who imitated other, and copied an art, which, through a long succession of ages, after multiplied trials, mistakes, corrections, deliberations, and controversies, had been gradually improving?

Many worlds might have been botched and bungled, throughout an eternity, ere this system was struck out; much labor lost, many fruitless trials made; and a slow, but continued improvement carried on during infinite ages in the art of world-making. In such subjects, who can determine, where the truth; nay, who can conjecture where the probability lies, amidst a greater which may be imagined?"

Epilogue

In the 20th century, a revolution occurred, one beyond the boldest dream of earlier scientists and engineers. In that century, science revolutionized technology to a degree that completely changed the daily life of man, society as a whole and in fact, the fate of mankind.

For three hundred years Western science pictured the world as a giant clock or machine, in which knowable causes produced predictable effects. It was a deterministic and totally ordered universe which, once set in motion, rendered all subsequent events meritable.

If this were an accurate description of the real world, the initial conditions of any process would determine its outcome — a machine-like universe set in motion by a Prime Mover, divine or otherwise.

If, on the other extreme, events were entirely random in an entirely accidental universe of completely random processes, there would be no law, regularity and predictability, and nature would have been nothing more than an endless series of random events, each with random consequences. It is unlikely that life, with its intricately regular processes, could ever arise – or indeed survive – in such a universe.

We now know that ours is a universe that combines both chance and necessity, chaos and order. Indeed, in accordance with the ideas of the 18th-century British empiricist philosopher David Hume, our *free will* and ability to detect causal relationships in the world around us, are both made possible by this dualistic nature of the universe we inhabit.

Appendices

- Philosophers of Nature
- Mathematicians
- Number Theorists
- Physicists
- Chemists and Biologists
- Mathematical Economists and Statisticians
- Historians of Science and Mathematics
- Earliest known use of Mathematical Symbols
- Earliest known use of Mathematical Terminology
- Photographs of celebrated scientists that flourished during 1800–1950
- List of Abbreviations and Acronyms
- The Greco – Latin Origins of Scientific Terminology
- Quotations

Table 6.15: Philosophers of Nature

fl.	Name	Life-span
585 BCE	Thales of Miletos	(624–548 BCE)
560 BCE	Anaximander of Miletos	(611–547 BCE)
500 BCE	Heraclitos of Ephesos	(540–475 BCE)
470 BCE	Parmenides of Elea	(504–456 BCE)
450 BCE	Philolaos of Tarentum	(480–420 BCE)
420 BCE	Democritos of Abdera	(460–370 BCE)
375 BCE	Plato	(427–347 BCE)
350 BCE	Aristotle	(384–322 BCE)
50 BCE	Lucretius	(94–55 BCE)
50 CE	L.A. Seneca	(4 BCE–65 CE)
1260	Roger Bacon	(1214–1292 CE)
1610	Francis Bacon	(1561–1626 CE)
1640	René Descartes	(1596–1650 CE)
1660	Baruch Spinoza	(1632–1677 CE)
1685	Isaac Newton	(1642–1727 CE)
1770	Immanuel Kant	(1724–1804 CE)
1800	Johann Wolfgang von Goethe	(1749–1832 CE)
1810	Jean de Lamarck	(1744–1829 CE)
1860	Charles Darwin	(1809–1882 CE)
1890	Henri J. Poincare	(1854–1912 CE)
1920	Albert Einstein	(1879–1955 CE)

Table 6.16: MATHEMATICIANS

NAME	LIFE-SPAN
Pythagoras	(580–500 BCE)
Eudoxos	(408–355 BCE)
Euclid	(330–260 BCE)
Archimedes	(287–212 BCE)
Bhaskara	(1114–1185 CE)
P. Fermat	(1601–1665 CE)
I. Newton	(1642–1727 CE)
G.W. von Leibniz	(1646–1716 CE)
L. Euler	(1707–1783 CE)
J. Lagrange	(1736–1813 CE)
Laplace	(1749–1827 CE)
C. Gauss	(1777–1855 CE)
F. Abel	(1802–1829 CE)
C.G.J. Jacobi	(1804–1851 CE)
Hamilton	(1805–1865 CE)
Galois	(1811–1832 CE)
B. Riemann	(1826–1866 CE)
G. Cantor	(1845–1918 CE)
D. Hilbert	(1862–1943 CE)
Hardy	(1877–1947 CE)
Ramanujan	(1887–1920 CE)

Table 6.17: Number-Theorists

fl.	Name	Life-span
535 BCE	Pythagoras	(580–500 BCE)
300 BCE	Euclid	(330–260 BCE)
250 CE	Diophantos	(206–290 CE)
1640	P. Fermat	(1601–1665 CE)
1750	L. Euler	(1707–1783 CE)
1780	J. Lagrange	(1736–1813 CE)
1800	A. Legendre	(1752–1833 CE)
1810	C. Gauss	(1777–1855 CE)
1830	C.G.J. Jacobi	(1804–1851 CE)
1860	B. Riemann	(1826–1866 CE)
1910	Hardy	(1877–1947 CE)
1917	Ramanujan	(1887–1920 CE)

Table 6.18: PHYSICISTS[55]

NAME	LIFE-SPAN	
Democritos	(460–370 BCE)	
Aristarchos	(310–230 BCE)	
Archimedes	(287–212 BCE)	
Eratosthenes	(276–194 BCE)	
Hipparchos	(180–110 BCE)	
Ptolemy	(85–165 CE)	
N. Copernicus	(1473–1543 CE)	
Galileo	(1564–1642 CE)	
J. Kepler	(1571–1630 CE)	
C. Huygens	(1629–1695 CE)	
I. Newton	(1642–1727 CE)	
C. Coulomb	(1736–1806 CE)	
J. Lagrange	(1736–1813 CE)	
W. Herschel	(1738–1922 CE)	
Laplace	(1749–1827 CE)	
Ampère	(1775–1836 CE)	
M. Faraday	(1791–1867 CE)	
Carnot	(1796–1832 CE)	
Hamilton	(1805–1896 CE)	
J. Maxwell	(1831–1879 CE)	
J.W. Gibbs	(1839–1903 CE)	
Boltzmann	(1844–1906 CE)	
M. Planck	(1858–1947 CE)	NL 1918

[55] Including greatest astronomers of antiquity and the Middle Ages.
NL = Nobel Laureate

Table 6.18: (Cont.)

Name	Life-span
M. Curie	(1867–1934 CE)
A. Einstein	(1879–1955 CE)
M. Born	(1882–1970 CE)
N. Bohr	(1885–1962 CE)
E. Schrödinger	(1887–1961 CE)
E. Hubble	(1889–1953 CE)
L. de-Broglie	(1892–1987 CE)
W. Pauli	(1900–1958 CE)
E. Fermi	(1901–1954 CE)
W. Heisenberg	(1901–1976 CE)
P. Dirac	(1902–1984 CE)
R. Feynman	(1918–1988 CE)

Table 6.19: Chemists[56] (1665–1965)

Name	Life-span	
Robert Boyle	(1637–1691 CE)	
Joseph Priestly	(1733–1804 CE)	
C.W. Scheele	(1742–1786 CE)	
Nicolas Le Blanc	(1742–1806 CE)	
A.L. Lavoisier	(1743–1794 CE)	
A. Volta	(1745–1827 CE)	
C.L. Berthollet	(1748–1822 CE)	
John Dalton	(1766–1844 CE)	
A. Avogadro	(1776–1856 CE)	
Humphry Davy	(1778–1829 CE)	
J.L. Gay–Lussac	(1778–1850 CE)	
J.J. Berzelius	(1779–1850 CE)	
F. Wöhler	(1800–1882 CE)	
C. Goodyear	(1800–1860 CE)	
J. von Liebig	(1803–1873 CE)	
Thomas Graham	(1822–1869 CE)	
Louis Pasteur	(1822–1895 CE)	
Joseph Lister	(1827–1912 CE)	
A. Kekulé	(1829–1896 CE)	
D.I. Mendeleev	(1834–1907 CE)	
J.W. Hyatt	(1837–1920 CE)	
W.H. Perkin	(1838–1907 CE)	
H. Chardonnet	(1839–1924 CE)	
W.K. Röntgen	(1845–1923 CE)	NL 1901

[56] NL = Nobel Laureate

Table 6.19: (Cont.)

Name	Life-span	
H.L. Le Chatelier	(1850–1936 CE)	
H. Becquerel	(1851–1908 CE)	NL 1903
Emil Fischer	(1852–1919 CE)	NL 1902
J.J. Thomson	(1856–1940 CE)	NL 1906
Svante Arrhenius	(1859–1927 CE)	NL 1903
L.H. Baekeland	(1863–1944 CE)	
W.H. Nernst	(1864–1941 CE)	NL 1920
Marie Curie	(1867–1934 CE)	NL 1911
F. Haber	(1868–1924 CE)	NL 1918
E. Rutherford	(1871–1937 CE)	NL 1908
Gilbert N. Lewis	(1875–1946 CE)	
F.W. Aston	(1877–1945 CE)	NL 1922
Hans Fischer	(1881–1945 CE)	NL 1930
Irving Langmuir	(1881–1957 CE)	NL 1932
Hermann Staudinger	(1881–1965 CE)	NL 1953
Alexander Fleming	(1881–1955 CE)	NL 1945
Henry Moseley	(1887–1915 CE)	
Frederick Banting	(1891–1941 CE)	NL 1923
James Chadwick	(1891–1974 CE)	NL 1935
H.C. Urey	(1893–1981 CE)	NL 1934
W. Carothers	(1896–1937 CE)	
E.O. Lawrence	(1901–1958 CE)	NL 1939
W.F. Libby	(1908–1980 CE)	NL 1960
F.H.C. Crick	(1916–2004 CE)	NL 1962
R.W. Woodward	(1917–1979 CE)	NL 1965

Table 6.20: Biologists and Men of Medicine (400 BCE–1950 CE)

Name	Life-span	
Hippocrates of Cos	(460–377 BCE)	
Aristotle	(384–322 BCE)	
Pliny the Elder	(23–79 CE)	
Galen	(129–200 CE)	
Alhazen	(965–1039 CE)	
Paracelsus	(1493–1541 CE)	
Vesalius	(1514–1564 CE)	
William Harvey	(1578–1657 CE)	
Anton van Leeuwenhoek	(1632–1723 CE)	
Carolus Linnaeus	(1707–1778 CE)	
Edward Jenner	(1749–1823 CE)	
Matthias J. Schleiden	(1804–1881 CE)	
Charles Darwin	(1809–1882 CE)	
Rudolf Virchow	(1821–1902 CE)	
Louis Pasteur	(1822–1895 CE)	
Gregor Johann Mendel	(1822–1884 CE)	
Jean–Henri Fabre	(1823–1915 CE)	
Joseph Lister	(1827–1912 CE)	
Robert Koch	(1843–1910 CE)	NL 1905
Walther Flemming	(1843–1905 CE)	
Camillo Golgi	(1843–1926 CE)	NL 1906
Ilya Mechnikov	(1845–1916 CE)	NL 1908
Charles L.A. Laveran	(1845–1922 CE)	NL 1907
Luther Burbank	(1849–1926 CE)	
Martinus Beijernick	(1851–1931 CE)	

Table 6.20: (Cont.)

Name	Life-span	
Santiago Ramon y Cajal	(1852–1934 CE)	
Emil von Behring	(1854–1917 CE)	NL 1901
Paul Ehrlich	(1854–1915 CE)	NL 1908
Sigmund Freud	(1856–1939 CE)	
Charles S. Sherrington	(1857–1922 CE)	NL 1932
Thomas Hunt Morgan	(1868–1945 CE)	NL 1933
Karl Landsteiner	(1868–1943 CE)	NL 1930
Jules Bordet	(1870–1961 CE)	NL 1919
Leonor Michaelis	(1875–1949 CE)	
Karl von Frisch	(1886–1982 CE)	NL 1973
Selman Waksman	(1888–1973 CE)	NL 1952
Sewall Wright	(1889–1988 CE)	
Ronald Fischer	(1890–1962 CE)	
Alexander Oparin	(1894–1980 CE)	
Gerhard Domagk	(1895–1964 CE)	NL 1939
Gerta F. Cori	(1896–1957 CE)	NL 1947
Carl F. Cori	(1896–1984 CE)	NL 1947
Howard W. Florey	(1898–1968 CE)	NL 1945
Hans Adolf Krebs	(1900–1981 CE)	NL 1953
Andre Lwoff	(1902–1994 CE)	NL 1965
Gregory G. Pincus	(1903–1967 CE)	
Max Delbrück	(1906–1981 CE)	NL 1969
George Wald	(1906–1997 CE)	
Jacques Monod	(1910–1976 CE)	NL 1965

Table 6.20: (Cont.)

Name	Life-span	
Salvador Luria	(1912–1991 CE)	NL 1969
Jonas Salk	(1914–1995 CE)	
Max F. Perutz	(1914–2002 CE)	NL 1962
John C. Kendrew	(1917–1997 CE)	NL 1962
Rosalind Franklin	(1920–1958 CE)	
François Jacob	(1920– CE)	NL 1965
Christiaan Barnard	(1922–2001 CE)	
Motoo Kimura	(1924–1994 CE)	

Table 6.21: Mathematical Economists and Statisticians (S)

fl.	Name	Life-span
1817	David Ricardo	(1772–1823)
1838	Antoine Cournot	(1801–1877)
1840	Adolph Quetelet (S)	(1796–1874)
1870	William Jevons	(1835–1882)
1880	Francis Edgeworth	(1845–1926)
1880	Wilhelm Lexis (S)	(1837–1914)
1880	Leon Walras	(1824–1910)
1890	Vilfredo Pareto	(1848–1923)
1890	Alfred Marshal	(1842–1924)
1900	Carl Pearson (S)	(1857–1936)
1900	William Gosset (S)	(1876–1937)
1920	Ludwig von Mises	(1881–1973)
1920	Irving Fisher	(1867–1947)
1920	George U. Yule (S)	(1871–1951)
1925	Ronald Fisher (S)	(1890–1962)
1935	John M. Keynes	(1883–1946)
1950	Ragnar Frisch	(1895–1973)

Table 6.22: HISTORIANS OF SCIENCE AND MATHEMATICS (M)

FL.	NAME	LIFE-SPAN
325 BCE	Eudemos of Rhodes (M)	(360–300 BCE)
70 CE	Pliny the Elder	(23–79 CE)
340 CE	Pappos of Alexandria (M)	(290–350 CE)
435 CE	Maritanus Capella	(390–455 CE)
520 CE	Severinus Boethius	(480–524 CE)
1835 CE	William Whewell	(1794–1866 CE)
1835 CE	Auguste Comte	(1798–1857 CE)
1890 CE	Moritz Cantor (M)	(1829–1920 CE)
1890 CE	Agnes Mary Clarke	(1842–1907 CE)
1900 CE	Jules Tannery (M)	(1848–1910 CE)
1935 CE	George Sarton	(1884–1956 CE)
1950 CE	Otto Neugebauer (M)	(1899–1980 CE)

Table 6.23: Earliest Known Use of Mathematical Symbols

Symbol	Meaning, Name, Use	Date Introduced	Inventor
	Zero as a placeholder	ca 1000 BCE	Meso Americans
Greek Letters	Lettering of points, lines, planes	ca 440 BCE	Hippocrates of Chios
	Zero as blank space on counting board	ca 350 BCE	Chinese
$\parallel$	Parallelism	ca 150 CE	Heron, Pappos
o (omicron)	Zero as a placeholder	ca 150	Ptolemy
Greek Letters	As variables	ca 250	Diophantos
◯ (circle)	Zero, both as a *number* and a placeholder	ca 505	Hindus
e.g. $\frac{2}{3}$	Horizontal fraction bar	ca 1200	Al-Hassar
$+, -$	Plus and Minus	1486	J. Wideman
$\sqrt{\ }$	Square root	1525	C. Rudolff
	Multiplication by juxtaposition	1544	M. Stifel
[ ]	Brackets	1550	R. Bombelli
( )	Parentheses	1556	N. Fontana
$=$	Equals	1557	R. Recorde
tangent, sine, cosine	Trigonometric functions	1583	Thomas Finck(e)
$\cdot$	Decimal point	1592	G.A. Magini

Table 6.23: (Cont.)

Symbol	Meaning, Name, Use	Date Introduced	Inventor
{ }	Braces	1593	F. Viéta
$>$	Greter than	1631	Thomas Harriot
$<$	Less than	1631	Thomas Harriot
$\cdot$	Multiplication (dot) [p=posthumously]	1631 (p)	Thomas Harriot
$\times$	Multiplication	1631	W. Oughtred
$\angle$	Angle	1634	Pierre Herigone
$\perp$	Perpendicularity	1634	Pierre Herigone
AB	Multiplication by juxtaposition	1637	R. Descartes
$a^1,\ a^2,\ a^3, \ldots$	Exponents (positive integers only)	1637	R. Descartes
$x,\ y,\ z$	Letters for unknown quantities	1637	R. Descartes
$a,\ b,\ c$	Letters for known quantities	1637	R. Descartes
$\overline{AB}$	Line segment	1647	B. Cavalieri
log	Logarithm	1647	W. Oughtred
∞	Infinity	1655	J. Wallis
$a^{-1},\ a^{1/2}$,...	Negative and fractional exponents	1656	J. Wallis
$\div$	The Obelus sign for division	1659	John Rahn

Table 6.23: (Cont.)

SYMBOL	MEANING, NAME, USE	DATE INTRODUCED	INVENTOR
$\therefore$	Therefore	1659	John Rahn
cot	Trigonometric cotangent	1674	J. Moore
$\int$	Integral sign	1675	G.W. von Leibniz
$dx,\ dy,\ \dfrac{dy}{dx}$	Infinitesimals and derivative	1675	G.W. von Leibniz
a^n	for any real number, n	1676	I. Newton
$\sim$	Geometrical similarity	1679	G.W. von Leibniz
$\simeq$	Congruence	1679	G.W. von Leibniz
:	Double-dot for both ratio and division	1698	G.W. von Leibniz
π	Ratio of circumference to diameter of circle	1706	W. Jones
/	Diagonal fraction bar	1718	Thomas Twining
e	2.718 281 828 149 045...	1728	L. Euler
A.S	Arcsine	1729	D. Bernoulli
$f(x)$	The function of x	1734	L. Euler
$\leq$	Less than or equal to	1734	P. Bouguer
$\geq$	Greater than or equal to	1734	P. Bouguer
$\neq$	Not equal	1740	L. Euler

Table 6.23: (Cont.)

Symbol	Meaning, Name, Use	Date Introduced	Inventor
sh, ch	Hyperbolic functions	1750	Vincento Riccati
$\sum$	Summation	1755	L. Euler
Ψ'	$\frac{d\Psi}{dx}$	1770	J.L. Lagrange
∂	Partial derivation	1770	A.N. Caritat
sinh, cosh	Hyperbolic function	1771	J. Lambert
i	$\sqrt{-1}$	1777	L. Euler
$\cong$	Approximately	1777	J.F. Häseler
$\left(\begin{array}{c} n \\ r \end{array}\right)$	Combination	1778	L. Euler
$\frac{\partial u}{\partial x}$	Partial derivative	1786	A.M. Legendre
lim	Limit	1786	S.A.J. L'Huuilier
$f'(x),\ f''(x)$	First and second derivatives of $f(x)$	1797	J.L. Lagrange
$D_x y$	Derivative of $y(x)$ w.r.t. x	1800	L.F.A. Arbogast
$\phi(m)$	Euler ϕ function	1801	C.F. Gauss
$\equiv$	Congruence of numbers	1801	C.F. Gauss
$n!$	Factorial	1804	C. Kramp
$\Gamma(n)$	Gamma function	1811	A.M. Legendre
sin^{-1}	inverse sine function	1813	J.F. Herschel

Table 6.23: (Cont.)

SYMBOL	MEANING, NAME, USE	DATE INTRODUCED	INVENTOR
$\int_a^b$	Limit of integration	1822	J.B. Fourier
$\delta - \epsilon$	In proofs	1823	A.L. Cauchy
∇	Nabla	1837	W.R. Hamilton
grad	Gradient operator	1860	J.C. Maxwell
B	Beta function	1839	P.M. Binet
$\mid\;\mid$	Determinant	1841	A. Cayley
$\|x\|$	Absolute value function of x	1841	K. Weierstrass
$a^2 + b^2$	Norm of $a + ib$	1842	L. Dirichlet
J	Bessel function	1843	P.H. Hansen
$y = mx + b$, $\frac{x}{a} + \frac{y}{b} = 1$	Equations of a line in analytic geometry	1848	G. Salmon
ζ	Zeta function	1857	B. Riemann
R	Rationals	1872	R. Dedekind
div	Divergence	1875	W.K. Clifford
$[x]$	Signum function (sign of x)	1878	L. Kronecker
Rho	Radian	1881	G.B. Halstead
∇^2	Laplacian operator	1883	R. Murphy
$\cap$, $\cup$	Intersection, union	1888	G. Peano
ln	Natural logarithm	1893	I. Stringham

Table 6.23: (Cont.)

Symbol	Meaning, Name, Use	Date Introduced	Inventor
$\aleph_0$	Aleph null	1893	G. Cantor
σ	Standard deviation	1894	K. Pearson
O	Big O	1894	P. Bachmann
ϵ	Membership	1894	G. Peano
N, n	Positive integer, integer	1895	G. Peano
$\exists$	Existence	1897	G. Peano
${}_nC_r$	Combination	1899	G. Chrystal
$\cdot$	Dot product	1902	J.W. Gibbs
$\times$	Vector product	1902	J.W. Gibbs
o	Little o	1909	E. Landau
$\pi(x)$	Number of primes less than x	1909	E. Landau
ϕ	Golden ratio	1914	Theodore Cook
$\oint$	Integral around closed path	1917	A. Sommerfeld
Z	The set of integers	1930	E. Landau
μ	Mean of normal distribution	1936	A. Fisher
C	Complex number	1939	N. Jacobson

Table 6.24: Earliest Known Mathematical Terminology

Year	Word	Mathematician
ca 1550 BCE	Pyramid	Ahmes Papyrus
ca 585 BCE	Geometry	Thales
ca 533 BCE	Odd, even and Prime numbers	Pythagoras
ca 380 BCE	Harmonic Mean	Archytas of Tarentum
ca 350 BCE	Axiom	Aristotle
ca 340 BCE	Logic, Number Theory	Xenocrates of Chalcedon
ca 300 BCE	Diameter, polygon, polyhedron, cube, trapezium	Euclid
ca 250 BCE	Helix	Archimedes
ca 230 BCE	Cylinder, asymptote, ellipse, parabola, hyperbole	Apollonios of Perga
ca 100 CE	Diagonal, Torus	Hero of Alexandria
ca 370	Analysis	Theon of Alexandria
ca 510	Radius, progression	Severinus Boethius
ca 550	Rational, irrational (numbers)	Cassiodorus
ca 630	Negative number	Brahmagupta
ca 825	Algebra	Al-Khowarizmi
ca 1145	Sinus (sine)	Robert of Chester
1202	Factor (noun), nominator, denominator, plus, minus	Fibonacci

Table 6.24: (Cont.)

Year, CE	Word	Mathematician
ca 1235	Product	Albertus Magnus
ca 1350	Zero	—
1386	Degree (angle)	G. Chaucer
1484	Natural number	N. Choquet
1543	Arithmetic and geometric progressions	M. Stifel
1557	Subtract, digit (number under 10)	Robert Recorde
1570	Vertex, discrete	John Dee
1571	Axis, orthogonal, convex	Thomas Digges
1580	Polynomial, coefficient	F. Viéta
1583	Secant, tangent (trigonometry)	Thomas Finck(e)
1594	Great circle	John Davis
1595	Trigonometry	B. Pitiscus
1596	Cosecant	Rheticus
1599	Cycloid	
1604	Focus (of an ellipse)	Johannes Kepler
1605	Pure Mathematics	Francis Bacon
1608	Operation (algebraic)	C. Clavius
1614–1617	Logarithm, decimal point	John Napier
1619	Isocahedron, dodecahedron (truncated)	Johannes Kepler

Table 6.24: (Cont.)

Year, CE	Word	Mathematician
1620	Cosine, cotangent	Edmund Gunter
1637	Real and imaginary numbers	René Descartes
1651	Spherical trigonometry	N. Mercator
1655	Continued fraction	J. Wallis
1657	Series	John Collins
1667	Convergent and divergent series	J. Gregory
1671–1672	Polar coordinates, ellipsoid	Isaac Newton
1673	Evolute	C. Huygens
1673–1694	Function (1673); Differential equation (1676); Calculus (1680); Differential calculus (1684); Algorithms (a systematic technique for solving a problem; 1684); Transcendental (1684); Differential (noun; 1690); Abscissa (1692); Coordinate (1692); Combinatorial (1694); Ordinate (1694); Variable (1694); Constant (1694)	G.W. von Leibniz
1685	Harmonic (conic)	Philippe de la Hire
1690	Integral calculus, Exponential function	Jakob Bernoulli
1694	Combination	Blaise Pascal

Table 6.24: (Cont.)

Year, CE	Word	Mathematician
1696	Normal	E. Scarburgh
1704	Vector	—
1706	Order (degree)	H. Ditton
1730	Degree	—
1738	Potential function	Daniel Bernoulli
1742	Binomial theorem	Colin Maclaurin
1753	Radius-vector (astronomy)	—
1756	Calculus of variations	L. Euler
1770	Statistics	W. Hooper
1779	Analytic geometry	S. Horley
1789	Conformal mapping	F.T. Schubert
1796	Formula	Richard Kirwan
1797	Analytic function, singular integral	J.L. Lagrange
1800	Definite integral	S-F. Lacroix
1801	Congruence (modular arithmetic)	C.F. Gauss
1811	Radix (base of a number system)	Peter Barlow
1812	Determinant	A.L. Cauchy
1814	Implicit function	—
1814	Commutativity, distributivity	F.J. Servois

Table 6.24: (Cont.)

YEAR, CE	WORD	MATHEMATICIAN
1820	Coordinate geometry	Matthew O'Brien
1821	Conjugate $(a \pm ib)$	A.L. Cauchy
1824	Algebraic number	N.H. Abel
1825	Torsion	L.T. Vallee
1825	Elliptic function	A.M. Legendre
1831	Separable variables	J.R. Young
1831–1840	Complex number, norm, potential function (1840)	C.F. Gauss
1839	Radius vector	J.R. Young
1840	Characteristic equation, absolute convergence	A.L. Cauchy
1843	Associative, Cartesian coordinate, vector, scalar, versor	W.R. Hamilton
1844	Inner product, outer product, vector space	H.G. Grassmann
1845	Linear transformation	A. Cayley
1846	X-axis	B. Peirce
1847	Topology	J.B. Listing
1848	Factor (verb)	J. Ray
1850	Geodesic	Joseph Liouville

Table 6.24: (Cont.)

Year, CE	Word	Mathematician
1851–1853	Invariant (1851); Canonical form (1851); Discriminant (1852); Jacobian (1852); Covariant (1853)	Joseph Sylvester
1854	Orthogonal matrix	Charles Hermite
1854	Bessel function	O.X. Schlömilch
1857	Order of Magnitude	I.T. Danson
1858	Field	R. Dedekind
1861–1863	Group theory, X-component etc (1863)	Joseph Sylvester
1870	Curl	J.C. Maxwell
1871	Radian	James Thomson
1871	Ideal (number theory)	R. Dedekind
1872	Limit point (accumulation point)	G. Cantor
1877	Covariant derivative	G. Ricci, T. Levi-Civita
1878	Vector product, scalar product, cross-ratio (of 4 point), divergence (of vector field)	W.K. Clifford
1878	Base of a vector space, rank of matrix	F.G. Frobenius
1878	Extremum, integral equation	P. Du Bois-Reymond
1878–1883	Graph (older sense, noun), latent root (1883)	Joseph Sylvester

Table 6.24: (Cont.)

Year, CE	Word	Mathematician
1879	Numerical analysis	S. Realis
1882	Abelian group (commutative)	H. Weber
1882	Isomorphism	W. Dyck
1883	Set (menge), closed set	G. Cantor
1884	Dyad	J.W. Gibbs
1885	Cofactor	W. Johnson
1888	Correlation, correlation coefficient	F. Galton
1890	Automorphic function	Felix Klein
1893	Circle of convergence, Integrand	John Harkness, F. Morley
1893	Standard deviation	Karl Pearson
1896	Group character	F.G. Frobenius
1896	Domain	—
1897	Gradient	Horace Lamb
1898	Tensor	W. Voigt
1900	Chi-square	Karl Pearson
1900	Class field	Teji Takagi
1901	Dot product	J.W. Gibbs
1901	Improper definite integral	E.H. Moore
1902	Adjoint of a matrix	L.E. Dickson

Table 6.24: (Cont.)

Year, CE	Word	Mathematician
1904	Kernel (as integral), Eigenvalue	David Hilbert
1904	Axiom of choice	E. Zermelo
1905	Positive definite	J. Pierpont
1906	Compactness	M.R. Fréchet
1907	Convergence in the mean	Ernst Fischer
1909	Cross product (vectors)	J.W. Gibbs
1911	Binomial distribution	G.U. Yule
1913	Hilbert space	F. Riesz
1916	Tensor Analysis	A. Einstein
1917	Stochastic	L.J. Bortkiewicz
1918	Gauge	H. Weyl
1919	Time Series	W.M. Person
1919	Central limit theorem	R. von Mises
1922	Saddle point	G.N. Watson
1922	Functional analysis (analytic properties of functionals)	Paul Levy
1923	Fourier transform	—
1927	Random number	—
1933	Convolution	N. Wiener
1934	Random variable	A. Winter
1942	Algebraic topology	Solomon Lefschetz

Table 6.24: (Cont.)

Year, CE	Word	Mathematician
1946	Bit	J.W. Tukey
1949	Programming	G.B. Dantzig
1956	Byte	W. Buchholtz
1975	Chaos	J.A. Yorke
1975	Fractal	B. Mandelbrot

Photo 4: Laplace (1749–1827)

Photo 5: Gauss (1777–1855)

Photo 6: Hamilton (1805–1865)

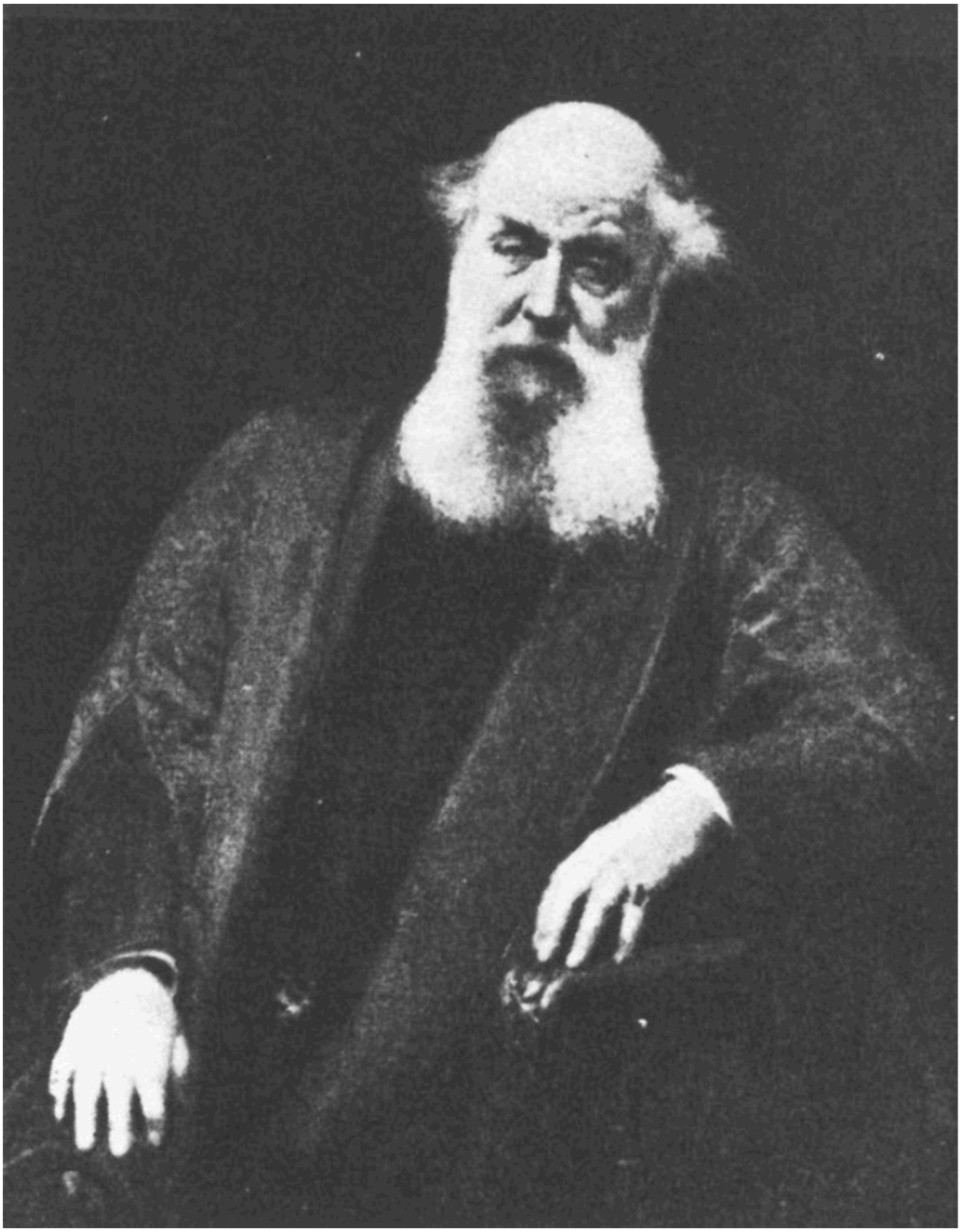


Photo 7: Sylvester (1814–1897)

Photo 8: G. Stokes (1819–1903)

Photo 9: A. Cayley (1821–1895)

Photo 10: B. Riemann (1826–1866)

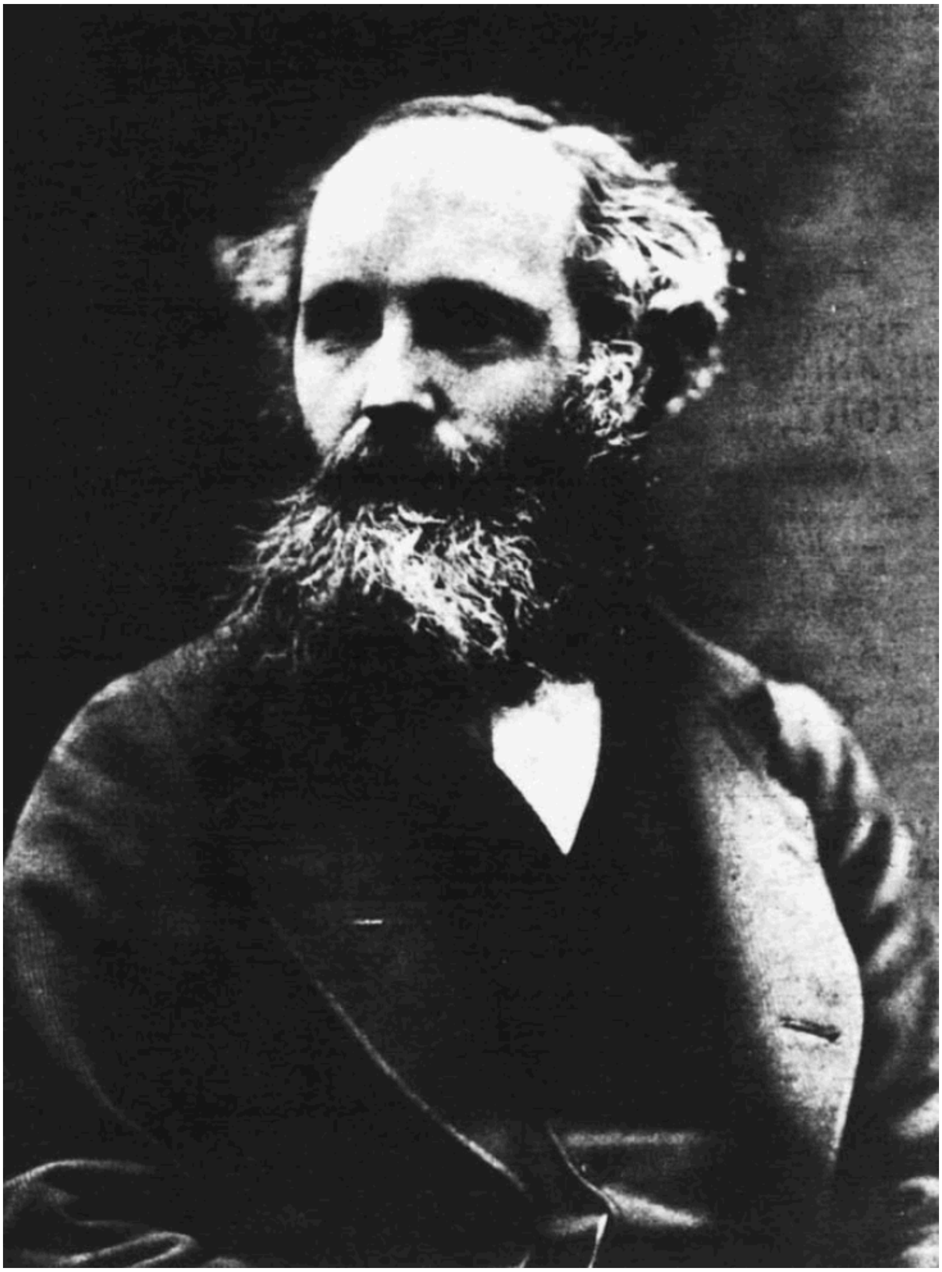


Photo 11: Maxwell (1831–1879)

Photo 12: Lord Rayleigh (1842–1919)

Photo 13: Hertz (1857–1894)

Photo 14: Minkowski (1864–1909)

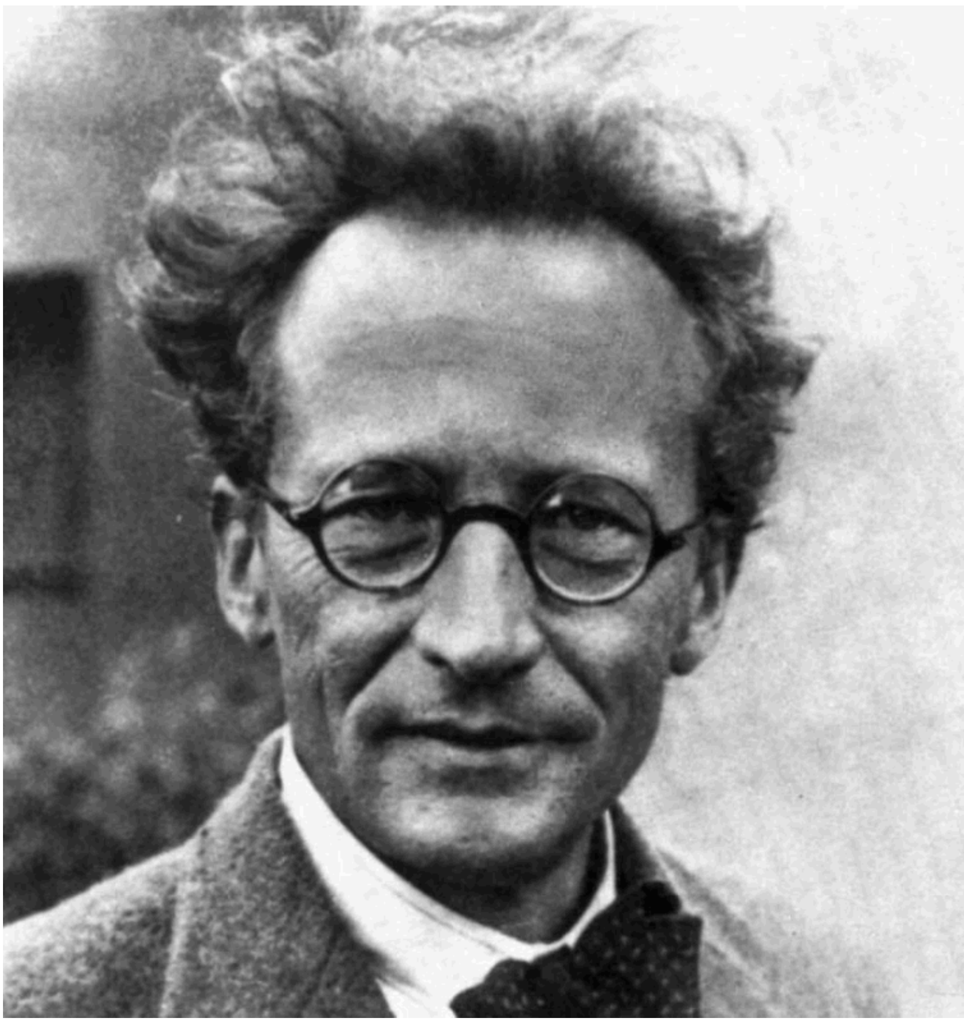


Photo 15: Schrödinger (1887–1961)

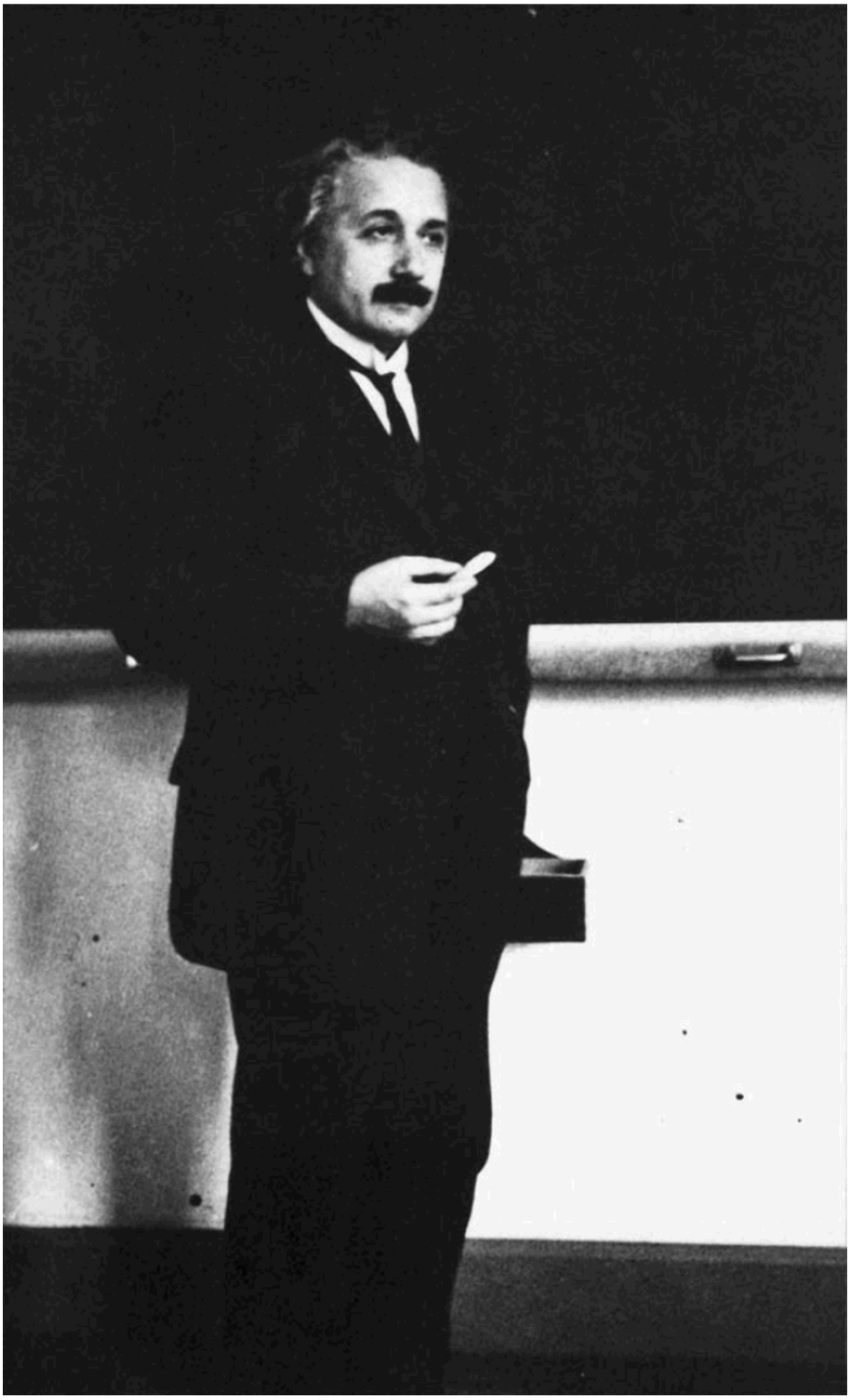
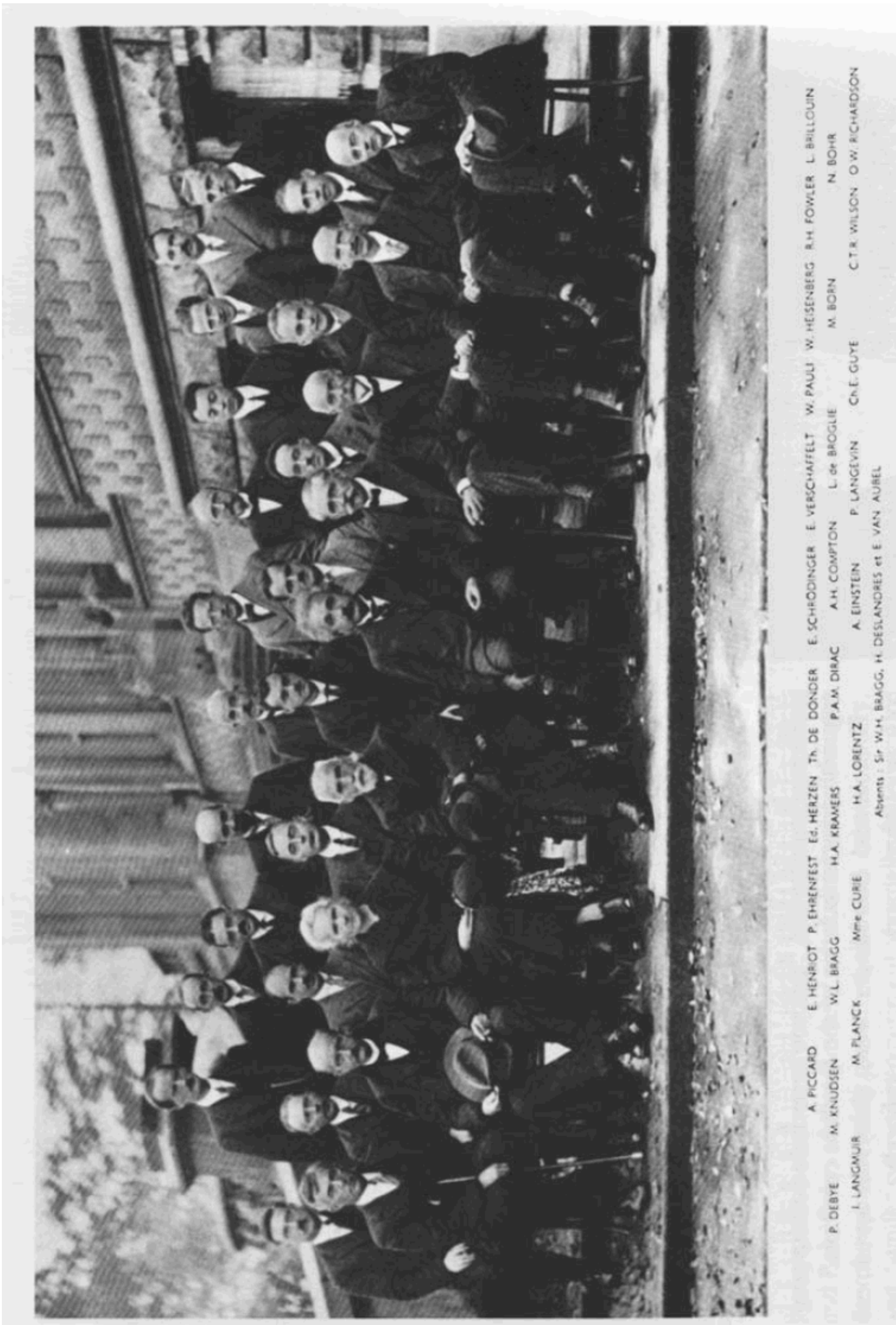


Photo 16: Einstein (1879–1955)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P. DEBYE M. KNUDSEN W.L. BRAGG H.A. KRAMERS P.A.M. DIRAC A.H. COMPTON L. de BROGLIE M. BORN N. BOHR
I. LANGMUIR M. PLANCK Mme CURIE H.A. LORENTZ A. EINSTEIN P. LANGEVIN Ch.E. GUYE C.T.R. WILSON O.W. RICHARDSON
Absents : Sir W.H. BRAGG, H. DESLANDRES et E. VAN AUBEL

Photo 17

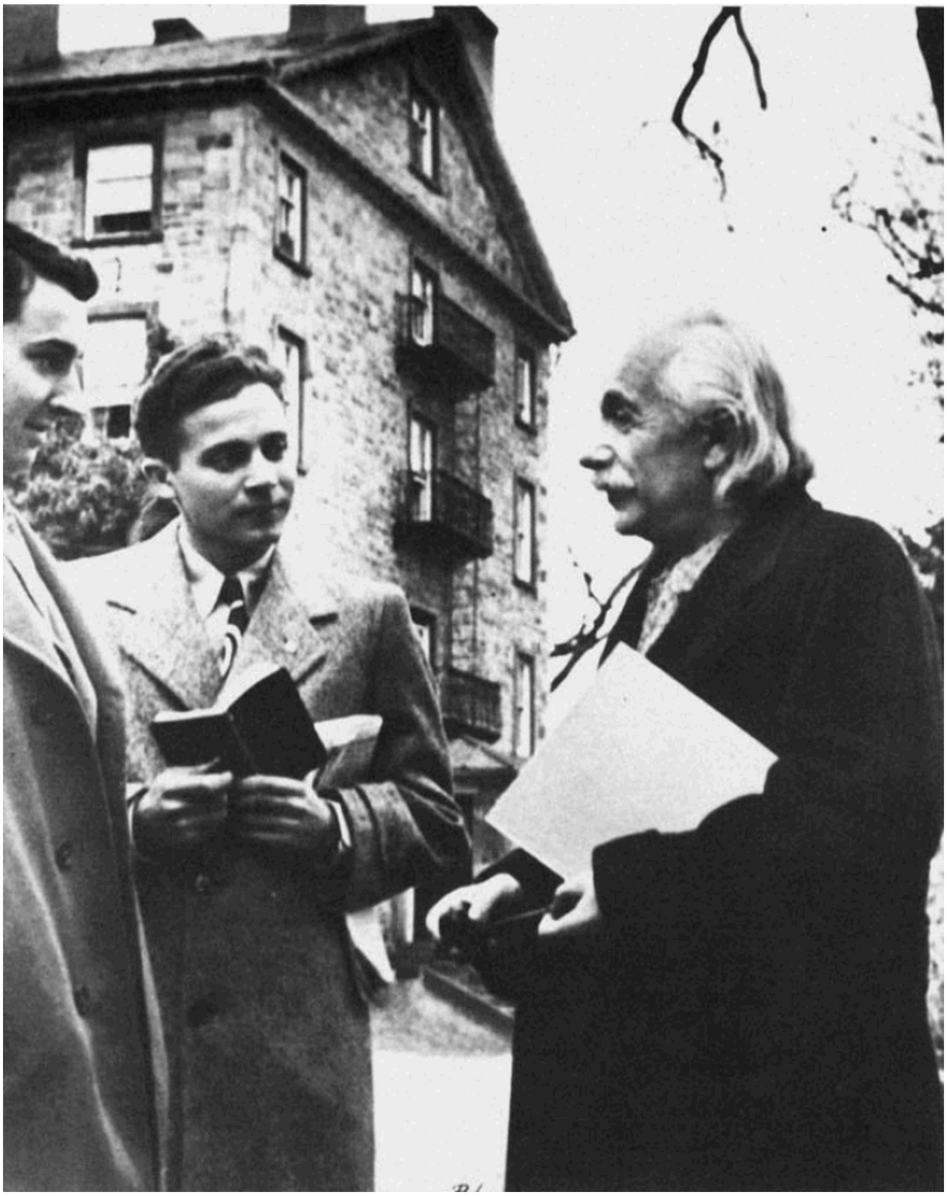

Photo 18

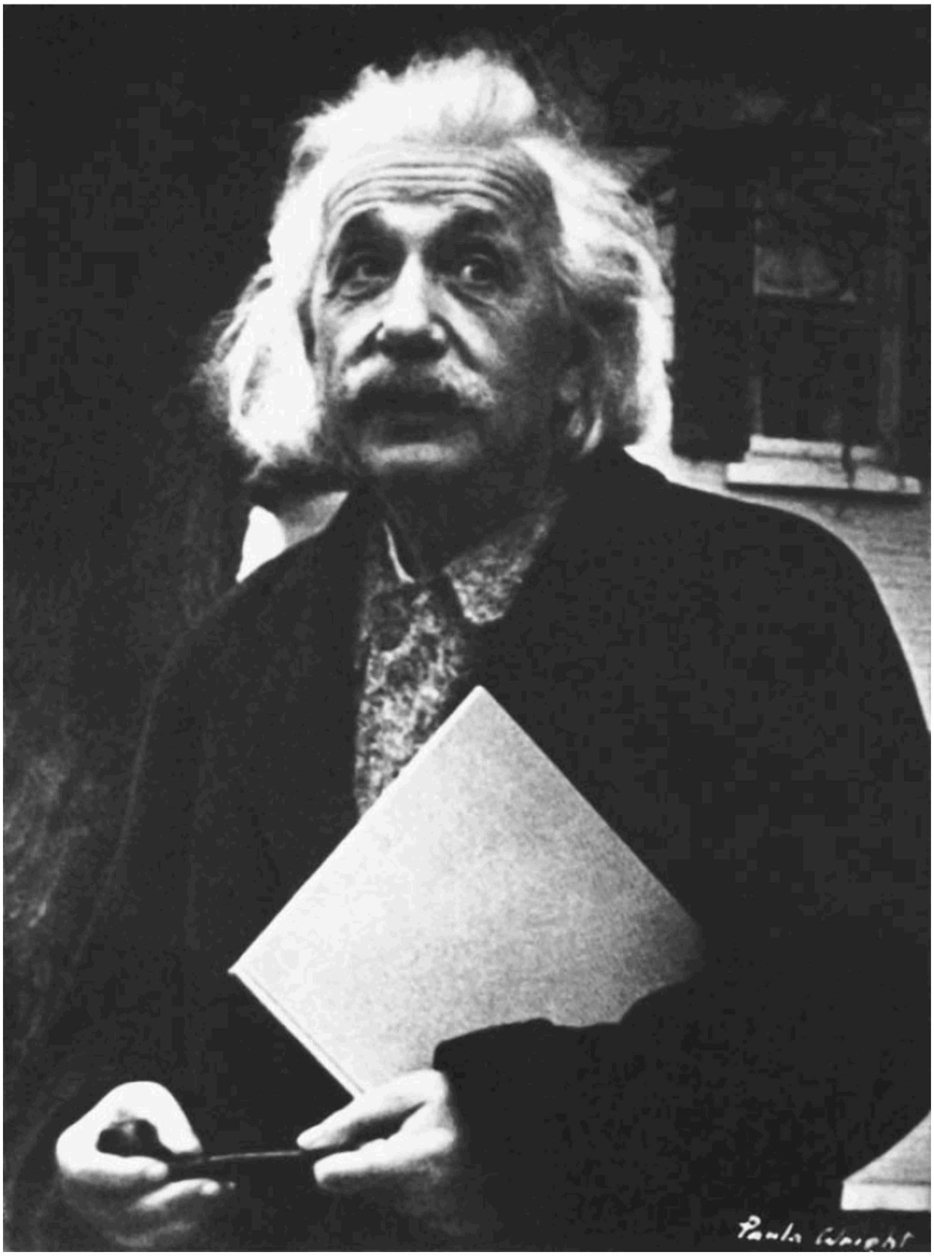

Photo 19

List of Abbreviations and Acronyms

Å	*Ångström unit (=* 10^{-8} *cm)*
ABC	*American Broadcasting Corporation*
AC, ac	*Alternating current*
AFM	*Atomic Force Microscopy*
AFMR	*Anti-Ferromagnetic Resonance*
AI	*Artificial Intelligence*
am (AM)	*ante meridian (= before noon)*
APFIM	*Atomic Probe Field Ion Microscopy*
ATP	*Adenosine 5-TriPhosphate*
au	*astronomical unit (ca 150 million km)*
b.	*born*
BA	*Bachelor of Arts*
BBC	*British Broadcasting Cooperation*
BCE	*Before the Christian Era*
BCS	*Bardeen-Cooper-Schrieffer*
BEC	*Bose-Einstein Condensates*
BEEM	*Ballistic Electron Emission Microscopy*
BeV	*Billion electron-volt*
BMNH	*British Museum of Natural History*
BS, BSc	*Bachelor of Science*
c	*velocity of light (ca 300,000 km/sec)*
$^\circ C$	*degrees Celsius*
CA	*California*
CALTECH	*California Institute of Technology*
CBI	*Cosmic Background Imager (telescope in Chile)*
CBS	*Columbia Broadcasting Service*
CBS	*Columbia Broadcasting System*
CCD	*Charged-Coupled Device*
CD	*Compact Disc*
CE	*Christian Era*
CERN	*Conseil Europeen pour Recherches Nucleaires*
c.g.s.	*centimeter, gram, second (system of units)*
cm	*centimeter*
CMB	*Cosmic Microwave Background*

CNN	*Cable News Network (Cellular Neural Network)*
COBE	*Cosmic Background Explorer*
COM	*Center of Mass*
CPU	*Central Processing Unit*
CRT	*Cathode-Ray Tube*
CT	*Computed Tomography*

d.	*died*
2D	*two-dimensional*
3D	*three-dimensional*
DALR	*Dry Adiabatic Lapse Rate*
DASI	*Degree Angular Scale Interferometer (at Antarctica)*
db	*decibel*
DBS	*Direct Broadcast Satellite*
DC, dc	*Direct Current*
deg.	*degree*
DNA	*Deoxyribo Nucleic Acid*
DVD	*Digital Video Disc*

e	*charge of the electron*
E	*East*
ECG	*Electrocardiogram*
ECT	*Emission Computed Tomography*
ed.	*editor*
EDSAC	*Electronic Delay Storage Automatic Computer*
EM	*ElectroMagnetic*
EMS	*Electron Momentum Spectroscopy*
ENIAC	*Electronic Numerical Integration and Calculation*
EPFA	*Erbium-doped Fiber Amplifier*
EPR	*Electron Parametric Resonance*
EPR	*Einstein-Podolsky-Rosen*
eq.	*equation*
eV	*electron-volt (atomic energy unit)*

°F	*degrees Farenheit*
FC	*Fermat's Conjecture (last theorem)*
FEL	*Free Electron Laser*
FEM	*Field Emission Microscopy*
FFT	*Fast Fourier Transform*
fig	*figure*

FIM	*Field Ion Microscopy*
fl.	*flourished*
FLT	*Fermat Little Theorem*
FMR	*Ferromagnetic Resonance*
FRS	*Fellow of the Royal Society*
g	*gram (unit of mass or weight)*
G	*Newton's universal gravitational constant*
GB	*Giga Byte*
GB	*Great Britain*
gcd	*= greatest common divisor*
GMT	*Greenwich Mean Time*
Gr.	*Greek*
GTR	*General Theory of Relativity*
h	*hour*
$\hbar$	$= \frac{h}{2\pi}$ *(h = Planck's constant)*
H_0	*Present-day value of the Hubble Constant*
hcf	*= highest common factor (= gcd)*
HTML	*Hypertext Markup Language*
IBM	*International Business Machines*
ICBM	*Intercontinental Ballistic Missile*
IRT	*Infrared Reflecting Telescope*
ISS	*Ion Scattering Spectroscopy*
IT	*Information Technology*
Jr	*Junior*
$^\circ K$	*degrees Kelvin*
kg	*kilogram* $(= 10^3$ *gram)*
km	*kilometer* $(= 10^3\ m = 10^5\ cm)$
L	*Length*
laser	*light amplification by stimulated emission of radiation*
LCD	*Liquid Crystal Display*
lcm	*= lowest common multiplier*

LED	*Light-Emitting Diode*
LEP	*Large Electron Positron Collider*
LHC	*Large Hadron Collider (at CERN)*
l.h.s.	*left hand side (of an equation)*
LIDAR	*Light Detection and Ranging*
LIGO	*Laser Interferometer Gravitational-wave Observatory*
LINEAC	*Large Linear Electron Accelerator*
LISA	*Laser Interferometer Space Antenna*
log_{10}	*logarithm to base 10*
log_e, ln	*logarithm to base e (natural logarithm)*
LSP	*Lightest Supersymmetric Partner*
LY	*= light year* $\approx 10^{13}$ *km*

m	*meter, minute*
MA	*Master of Arts*
MACHO	*Massive Compact Halo Object*
MAP	*Microwave Anisotropy Probe (satellite, 2001)*
maser	*microwave amplification by stimulated emission of radiation*
MB	*Mega Byte*
MD	*Doctor of Medicine*
MeV	*million electron-volt*
MFM	*Magnetic Force Microscopy*
mg	*milligram* $(= 10^{-3}$ *g)*
MHD	*MagnetoHydroDynamics*
MIR	*Multiple Internal Reflection*
MIT	*Massachusetts Institute of Technology*
ml	*milliliter*
mm	*millimeter* $(= 10^{-3}\ m = 10^{-1}\ cm)$
$m\mu m$	*millimicron* $(= 10^{-9}\ m = 10^{-7}\ cm = 10\mathring{A})$
mod	*modulus*
MOKE	*Magneto-Optic Kerr Effect*
MOND	*Modified Newtonian Dynamics*
MOSFET	*Metal-Oxide-Semiconductor-Field-Effect-Transistor*
MOT	*Magneto-Optical Traps*
MRI	*Magnetic Resonance Imagining*
M_s	*Surface-wave magnitude of earthquakes*
MT	*Megaton of TNT (Energy)*
M-theory	*An eleven-dimensional unification of Superstring Theory*
My	*Megayear* $(10^6$ *years)*
MYA, Mya	*millions years ago*

N	*North*
NAFS	*North Anatolian Fault System*
NASA	*National Aeronautics and Space Administration*
NBC	*National Broadcasting Corporation*
NBS	*National Bureau of Standards (renamed NIST)*
nm	*nanometer (=mμm)*
NMR	*Nuclear Magnetic Resonance*
NSOM	*Near-Field Scanning Optical Microscopy*
NY	*New York*
ODE	*Ordinary Differential Equation*
ORT	*Optical Reflection Telescope*
P	*Seismic Compressional Waves*
PAL	*Phase Alternating by Line*
PC	*Personal Computer*
PCR	*Polymerase Chain Reaction*
PDE	*Partial Differential Equation*
PEEM	*Photo Emission Electron Microscopy*
PEP	*Positron-Electron Project*
PET	*Positron Emission Tomography*
pH	*Potential of Hydrogen (level of acidity)*
Ph.D.	*Doctor of Philosophy*
pixel	*Picture Element*
pp.	*pages*
PST	*Pacific Standard Time*
PV	*PhotoVoltaic*
QCD	*Quantum ChromoDynamics*
QED	*Quantum ElectroDynamics*
QFT	*Quantum Field Theory*
QM	*Quantum Mechanics*
R	*Seismic Rayleigh Waves*
RADAR	*Radio Detection and Ranging*
RAM	*Random Access Memory*
RCA	*Radio Corporation of America*
ref.	*reference*
RF, rf	*radio frequency*

RHIC	*Relativistic Heavy Ion Collider (at Brookhaven N.Y.)*
r.h.s.	*right hand side (of an equation)*
RIA	*Radio Immuno Assay*
RNA	*RiboNucleic Acid*
ROM	*Read-Only Memory*
rpm	*revolutions per minute*
RSA	*Rivest-Shamir-Adelman (Public Key Cryptography)*
RT	*Radio Telescope*
R.V.	*Revised Version*

S	*South*
S	*Seismic Shear Waves*
SDI	*Space Defense Initiative*
SE	*Schrödinger's Equation*
sec.	*second (unit of time)*
SECAM	*Sequenced Color and Memory*
SEM	*Scanning Electron Microscopy*
SF	*Science Fiction*
SLAC	*Stanford Linear Accelerator Center*
SLC	*Stanford Linear Collider*
SLT	*The Second Law of Thermodynamics*
SNCF	*Societé Nationale de Chemins de Fer (France)*
SONAR	*Sonic Navigation and Ranging*
SORT	*Schmidt Optical Reflecting Telescope*
SPECT	*Single Photon Emission Tomography*
Sr	*Senior*
SRT	*Steerable Radio Telescope*
SSRL	*Stanford Synchrotron Radiation Laboratory*
STM	*Scanning Tunneling Microscopy*
S.T.P.	*Standard Temperature and Pressure*
STR	*Special Theory of Relativity*

tanh	*hyperbolic tangent*
TGV	*le Train à Grand Vitesse (France)*
TNT	*Tri-Nitro-Toluene (explosive)*
TPM	*Two-Photon Microscopy*
TV	*Television*

u (or amu)	*Atomic mass-unit (1/12 of the mass of an atom of Carbon 12)*

U	*Displacement on fault*
UFD	*Unique Factorization Domain*
UHV	*Ultra High Vacuum*
UK	*United Kingdom*
USA (US)	*United States of America*
USSR	*Union of the Soviet Socialist Republics*
UT	*Universal Time*
VCR	*Video Cassette Recorder*
VEI	*Volcanic Explosivity Index*
VLBI	*Very Long Baseline Interferometry*
VLSI	*Very Large Scale Integration*
VR	*Virtual Reality*
W	*west*
WIMP	*Weakly Interacting Massive Particle (a prime candidate for the Exotic Dark Matter in the universe)*
WKBJ	*Wentzel-Kramers-Brillouin-Jeffreys*
w.r.t.	*with respect to*
WWI	*World War I*
WWII	*World War II*
ya	*years ago*
μm	*micron* $(= 10^{-6}\ m = 10^{-4}\ cm = 10^{-3}\ mm)$
$[x]$	= *greatest integer not exceeding* x *(e.g. [7.61] = 7)*

$$
\begin{array}{lcllcl}
micro & = & 10^{-6} = \mu & mega & = & 10^{6} = M \\
nano & = & 10^{-9} = n & giga & = & 10^{9} = G \\
pico & = & 10^{-12} = p & tera & = & 10^{12} = T \\
femto & = & 10^{-15} = f & peta & = & 10^{15} = P \\
atto & = & 10^{-18} = a & exa & = & 10^{18} = E
\end{array}
$$

Algebra

$\mathbb{Z}$	*ring of all integers* $\{...-2, -1, 0, 1, 2, 3,...\}$
$\mathbb{Z}^+$	*set of all positive integers* $\{1, 2, 3,...\}$ *(natural numbers)*
$\mathbb{N}$	*set of all non-negative integers* $\{0, 1, 2, 3,...\}$
$\mathbb{Q}$	*field of all rational numbers*
$\mathbb{Q}[\sqrt{d}]$	*quadratic field over rational numbers* ($d \in Z$, *not a perfect square)*
$\mathbb{R}$	*field of all real numbers*
$\mathbb{L}^n(R)$	*vector field of real functions of a real variable over the real field, with the norm* $\int_{-\infty}^{\infty} dx\|f(x)\|^n$
$\mathbb{R}^n$	*vector field of real n-tuples*
$\mathbb{Z}[\sqrt{-1}]$	*a complex number of the form* $a + b\sqrt{-1}$, *where* $a,\ b\ \in\ \mathbb{Z}$ *(Gaussian integer)*
$\mathbb{Z}[\sqrt{d}]$	*quadratic ring of all numbers of the form* $a + b\sqrt{d}$, *where* $a,\ b\ \in\ \mathbb{Z}$ *and* d *is any integer other than a perfect square*
$\mathbb{C}$	*field of all complex numbers* $a + b\sqrt{-1}$, *where* $a,\ b\ \epsilon\ \mathbb{R}$
$\mathbb{S}^n$	*n-dimensional Riemannian manifold embedded in* $\mathbb{R}^{n+1}$ *by means of the surface equation* $\sum_{i=1}^{n+1} x_i^2 = 1$
$\mathbb{E}^n$	$= \mathbb{R}^n$ *with the Euclidean norm*
$\mathbb{C}^n(R)$	*vector-space of n times differentiable real-valued functions of real variable over the real field*
$\mathbb{GL}(n, R)$	*group of* $n \times n$ *real non-singular matrices*
$\mathbb{O}(n, R)$	*group of* $n \times n$ orthogonal *real matrices (subgroup of* $\mathbb{GL}$*)*

$\mathbb{SO}(n, R)$	*subgroup of* $\mathbb{O}(n, R)$ *consisting of those elements that have a unit determinant*
$\mathbb{SU}(n)$	*group of* unitary $n \times n$ *complex matrices of unit determinant*
$\mathbb{U}(n)$	*group of all* unitary $n \times n$ *complex matrices*
$\mathbb{SL}(n, C)$	*group of all* $n \times n$ *complex matrices of unit determinant*

All six groups are infinite, continuous, and are simultaneously groups and Riemannian manifolds and therefore called Lie groups. *To every Lie group there correspond a* Lie algebra, *denoted by lower case letters outside the parenthesis, e.g., the Lie algebra corresponding to the Lie group* $\mathbb{GL}(n, R)$ *is* $gl(n, R)$. *Any Lie algebra is isomorphic to the* tangent vector space *of the corresponding Lie group at any point (= element of the Lie group). Any element of a Lie algebra can be represented as a* differential operator *on the corresponding Lie group manifold, and this differential operator is a* Lie derivative.

The Greco–Latin Origins of Scientific Terminology

Introduction

Latin and Greek were the two common languages of Western scholars well into the 1800's. Much of this was influenced by the Catholic church which kept Latin alive in its ceremonies and in its illumination of the biblical scriptures. Ancient texts like the gospels and Greek myths were written in Greek and later translated into the Latin Vulgate by Church scholars and scribes. It was a natural place for those early scientists to go for a rich source of new descriptive words. Along with most any scholar, poets and other icons of literature up to the 20th century, were commonly schooled in Latin and Greek. They read in the great works of the ancients: Aristotle, Plato, Homer, Pythagoras, Lucretius, Marcus Aurelius in their original languages. Even well into the 20th century, a learned person was to some extent marked by his command of Latin and Greek.

Figure 6.2 shows the family tree of the Indo–European languages that includes Greek, Latin, French, English and German.

Table 6.25 lists the major historical events during 1750 BCE–1755 CE that triggered the gradual symbiosis of Greek and Latin with the English language.

Table 6.26 lists the leading progenitors of the Greco–Roman literary culture.

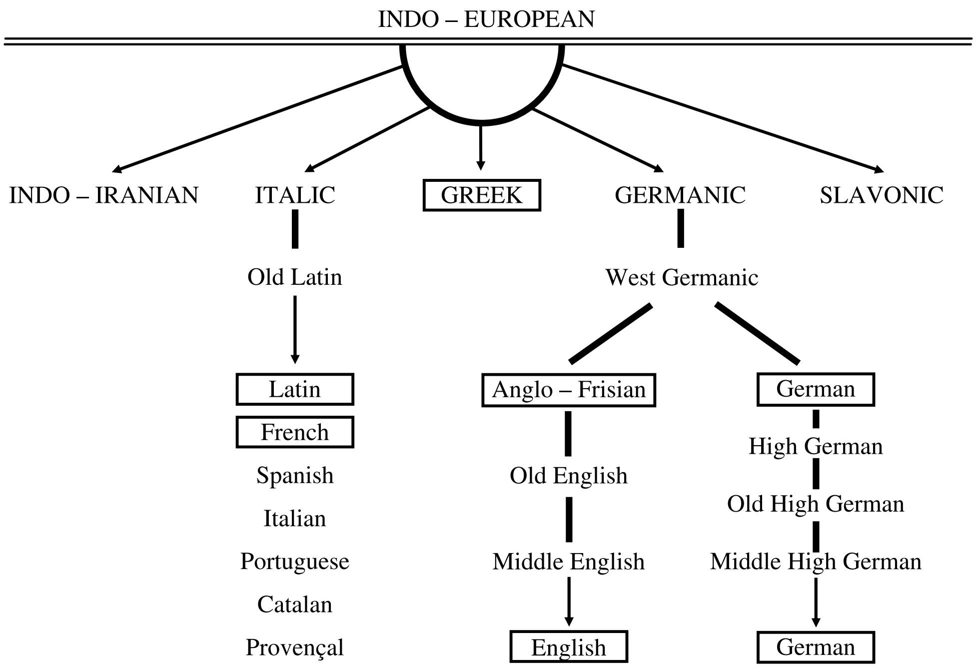

Fig. 6.2: The family of the Indo–European Languages relevant to Greek, Latin, English, German and French

Table 6.25: Man, Language and History

Date	Event
c.1750 BCE	*Appearance of the* Semitic alphabet
c.1450	*Earliest writings in the* Hebrew Bible
953	*Consecration of* **Solomon**'s Temple *in Jerusalem*
776	*First* Olympic games *in Greece*
c.770	*Latin first brought to Italy by migrants from the north*
753	*Alleged founding of* Rome *in Latium by* ***Romulus***
750–585	*Age of* hebrew prophets: ***Amos***, ***Isaiah*** *and* ***Jeremiah***
c.750	***Homer's*** Iliad and Odyssey *in Greek writing arrived in Greece*
500–400	*The Golden Age of* Athenian *democracy*
440–322	*Age of the* Greek philosophers: ***Socrates***, ***Aplaton*** and ***Aristoteles***
323	*Death of* ***Alexander the Great***
285–250	*Jewish scholars first translated the Torah into Greek (Septuagint)*
200–100	Rome *extends its power in the Mediterranean*
146	Greece *becomes a* Roman *colony*

Table 6.25: (Cont.)

DATE	EVENT
100 BC–200 CE	*The age of* "CLASSICAL LATIN" *of the* ROMAN LITERATI: *(**Cicero, Lucretius, Cato, Vergil, Horace, Ovid, Seneca, Quintilian, Lucan, Martial, Tacitus, Marcus Aurelius, Juvenal**)*
55–54	***Julius Caesar** raids Britain*
c.7	***Jesus of Nazareth** born in the* LAND *of* ISRAEL
43 CE	ROMANS *conquer Britain*
122	*Hadrian's Wall built*
300–900	*The Age of* "LATE LATIN"
328	*Constantinople becomes the capital of the Eastern part of the* ROMAN EMPIRE
390–405	***Jerome** (*HIERONYMUS*) translated the Hebrew Bible into Latin. The translation is known as the Vulgate*
410	ROMANS *leave Britain*
476	*"Official" collapse of the Western* ROMAN EMPIRE
400–600	*The* SAXONS, JUTES *and* ANGLES *invade Britain, bringing the anglo–saxon language with them*
597	***Augustine** brings a new wave of* CHRISTIANITY *to Britain*
711	*The* MOORS *invade* SPAIN, *bringing their* ARAB *translations of* GREEK *books*

Table 6.25: (Cont.)

Date	Event
900–1300	*Age of* "Medieval Latin"
1066	*The* Normans *invade Britain, bringing Norman French with them*
1265–1374	end of "Medieval Latin:" *the poets **Dante** and **Petrarch***
1266	***Roger Bacon** publishes:* "opus majus" *in Latin*
1300–1600	*Age of* "Renaissance Latin"
1453	*The Turks sack Constantinople*
1492	***Columbus** crossed the Atlantic and heralds the colonization of the* Americas
1500–1650	*Belated* "renaissance" *in Britain: rediscovery of the* Roman *civilization*
1543	***Andreas Vesalius** published:* "de humani corporis fabrica"
1543	***Copernicus** published* "de revolutionibus orbium coelestium" *in Latin*
1600–1900	*Age of* "New Latin"
1609–1621	***Kepler**'s publications:* "astronomia nova" *(1609),* "harmonice mundi" *(1619) and* "mysterium cosmographicum" *(1621) in Latin*
1610	***Galileo**'s publication:* "sidereus nunicus" *in Latin*

Table 6.25: (Cont.)

DATE	EVENT
1611	*Publication of the King James version of the Hebrew Bible*
1616	*Death of* ***William Shakespeare***
1620	***Francis Bacon*** *published:* "NOVUM ORGANUM" *in Latin*
1644	***René Descartes*** *published:* "PRINCIPIA PHILOSOPHIAE" *in Latin*
1675	***Baruch Spinoza*** *completed his* "ETHICA ORDINE GEOMETRICO DEMONSTRATA" *in Latin*
1687	***Isaac Newton*** *published:* "PRINCIPIA MATHEMATICA" *in Latin – the last great scientific work to be written in this language; Advent of the Scientific Revolution*
1755	***Samuel Johnson*** *published his dictionary of the English language*
1776	*The motto "E Pluribus Unum" ("Out of Many — One") was proposed for the first Great seal of the United States by* **John Adams**, **Benjamin Franklin** and **Thomas Jefferson**. *This phrase offered a strong statement of the American determination to form a single nation from a collection of States and from people of many different backgrounds and beliefs*

Table 6.26: Timeline of the leading progenitors and torchbearers of the Greco–Roman literary culture during 750 BCE–550 CE

I. Poets

1.	*Homer*	*c.* 8^{th} *century BCE*	*Greek*
2.	*Hesiodos*	*fl. c. 700 BCE*	*Greek*
3.	*Archilochos*	*mid* 7^{th} *century BCE*	*Greek*
4.	*Pindaros*	*c. 518–438 BCE*	*Greek*
5.	*Accius*	*c. 170–86 BCE*	*Latin*
6.	*Lucretius*	*c. 99–55 BCE*	*Latin*
7.	*Catullus*	*c. 84–54 BCE*	*Latin*
8.	*Vergil*	*c. 70–19 BCE*	*Latin*
9.	*Horace*	*c. 65–8 BCE*	*Latin*
10.	*Ovid*	*c. 43 BCE–17 CE*	*Latin*
11.	*Lucan*	*c. 39–65 CE*	*Latin*
12.	*Martial*	*c. 40–103 CE*	*Latin*
13.	*Statius*	*c. 45–96 CE*	*Latin*
14.	*Juvenal*	*c. fl. 127 CE*	*Latin*
15.	*Claudian*	*c.350–404 CE*	*Latin*

II. Thinkers (Philosophers[57]*, Natural philosophers, mathematicians)*

1.	*Thales*	*c. 624–546 BCE*	*Greek phil.*
2.	*Anaximander*	*c. 610–546 BCE*	*Greek phil.*
3.	*Anaximenes*	*c. 585–525 BCE*	*Greek phil.*
4.	*Pythagoras of Samos*	*c. 582–507 BCE*	*Greek phil. and mathematician*
5.	*Xenophanes of Colophon*	*c. 570–480 BCE*	*Greek phil. and mathematician*
6.	*Heraclitos*	*c. 535–475 BCE*	*Greek phil. and mathematician*
7.	*Parmenides of Elea*	*c. 515–450 BCE*	*Greek phil. and mathematician*
8.	*Anaxagoras of Clazomenae*	*c. 500–428 BCE*	*Greek phil. and mathematician*
9.	*Empedocles of Acragas*	*c. 490–430 BCE*	*Greek phil. and mathematician*
10.	*Zeno of Elea*	*c. 490–430 BCE*	*Greek phil. and mathematician*
11.	*Leucippos*	*c. 480–420 BCE*	*Greek phil. and mathematician*
12.	*Antiphon*	*c. 480–411 BCE*	*Greek phil. and mathematician*
13.	*Protagoras*	*c. 480–420 BCE*	*Greek phil. and mathematician*

[57] The Hellenistic schools of thought included: *Cynicism*, *Epicureanism*, *Hedonism*, *Eclecticism*, *Neo-Platonism*, *Skepticism*, *Stoicism*, *Sophism*.
In Europe, the spread of Christianity through the Roman world marked the end of the Hellenistic philosophy and ushered in the beginning of Medieval philosophy.

II. Thinkers (cont.)

14.	*Hippocrates of Chios*	*c. 470–410 BCE*	*Greek phil. and geometer*
15.	*Philolaos*	*c. 480–405 BCE*	*Greek phil.*
16.	*Socrates*	*c. 469–399 BCE*	*Greek phil.*
17.	*Democritos*	*c. 460–370 BCE*	*Greek phil.*
18.	*Plato (Aplaton)*	*c. 428–347 BCE*	*Greek phil.*
19.	*Diogenes of Sinope*	*c. 400–325 BCE*	*Greek phil.*
20.	*Stilpo*	*c. 380–330 BCE*	*Greek phil.*
21.	*Aristotle*	*384–322 BCE*	*Greek phil.*
22.	*Epicuros*	*341–270 BCE*	*Greek phil.*
23.	*Pyrrho*	*c. 365–275 BCE*	*Greek phil.*
24.	*Zeno of Citium*	*c. 333–263 BCE*	*Greek phil.*
25.	*Euclid*	*c. 325–265 BCE*	*Greek phil.*
26.	*Chrysippos*	*c. 287–212 BCE*	*Greek phil. and mathematician*
27.	*Eratosthenes*	*c. 276–194 BCE*	*Hellenistic astronomer*
28.	*Panaetios*	*c. 185–110 BCE*	*Greek stoic and Neo-Platonic phil.*
29.	*Posidonios*	*c. 153–50 BCE*	*Stoic phil. and Historian*
30.	*Cicero*	*106–43 BCE*	*Roman statesman and phil.*

II. Thinkers (cont.)

31.	*Philo Alexandrius*	*30 BCE–45 CE*	*Jewish philosopher*
32.	*Seneca*	*4 BCE–65 CE*	*Roman stoic phil.*
33.	*Epictetus*	*c. 55–135 CE*	*Roman phil.*
34.	*Claudius Ptolemy*	*c. 85–165 CE*	*Greek astronomer*
35.	*Marcus Aurelius*	*121–180 CE*	*Roman emperor and phil.*
36.	*Plotinus*	*c. 205–270 CE*	*Greco-Roman phil.*
37.	*Augustine of Hippo*	*354–430 CE*	*Church theologian*
38.	*Hypatia of Alexandria*	*c. 370–415 CE*	*Alexandrian mathematician*
39.	*Proclus Diadochus*	*411–485 CE*	*Greco-Roman phil.*
40.	*Boethius*	*472–524 CE*	*Christian phil.*

III. Prose authors (Historians, biographers, writers, playwrights)

a. Historians and Biographers

1.	*Herodotos*	*480–428 BCE*	*Greek hist.*
2.	*Thucydides*	*c. 460–399 BCE*	*Greek hist.*
3.	*Xenophon*	*c. 430–354 BCE*	*Greek hist.*

4.	*Cato the Elder*	*c. 234–149 BCE*	*Roman hist.*
5.	*Polybius*	*c. 230–118 BCE*	*Greek hist.*
6.	*Julius Caesar*	*c. 100–44 BCE*	*Roman Emperor and writer*
7.	*Cornelius Nepos*	*c. 100–25 BCE*	*Roman biographer*
8.	*Sallust*	*86–35 BCE*	*Roman hist.*
9.	*Strabo*	*64 BCE–c.30 CE*	*Greco-Roman geographer and hist.*
10.	*Diodoros Siculus*	*fl. 60–30 BCE*	*Sicilian Greek hist.*
11.	*Titus Livius (Livy)*	*59 BCE–17 CE*	*Roman hist.*
12.	*Josephus*	*c. 37–100 CE*	*Jewish hist.*
13.	*Plutarch*	*c. 45–128 CE*	*Greco-Roman hist.*
14.	*Tacitus*	*c. 56–117 CE*	*Roman hist.*
15.	*Pliny the Younger*	*c. 63–113 CE*	*Roman historian*
16.	*Suetonius*	*c. 71–135 CE*	*Roman biographer*
17.	*Dio Cassius*	*c. 150–235 CE*	*Roman hist.*
18.	*Ammianus Marcellinus*	4^{th} *century CE*	*Roman hist. and biographer*
19.	*Eusebius*	*c. 260–340 CE*	*Greek-Christian hist.*

b. Oratos and Rhetoricians

1.	*Demosthenes*	*384–322 BCE*	*Greek*
2.	*Cicero*	*106–43 BCE*	*Roman*
3.	*Cato the Younger*	*95–46 BCE*	*Roman*
4.	*Quintilian*	*35–100 CE*	*Roman*
5.	*Aelius Aristeides*	*117–189 CE*	*Greek*

c. Writers and playwrights

1.	*Aesop*	*c. 620–560 BCE*	*Greek writer*
2.	*Aeschylus*	*c. 525–456 BCE*	*Greek playwright*
3.	*Sophocles*	*c. 496–406 BCE*	*Greek playwright*
4.	*Euripides*	*c. 480–405 BCE*	*Greek playwright*
5.	*Aristophanes*	*c. 450–385 BCE*	*Greek playwright*
6.	*Terence*	*c. 190–158 BCE*	*Roman playwright*
7.	*Varro*	*c. 116–27 BCE*	*Roman writer*
8.	*Publilius Syrus*	*first century BCE*	*Roman writer*
9.	*Tertullian*	*c. 160–240 CE*	*Roman writer*
10.	*Vegetius*	*fl. 380–400 CE*	*Roman writer*

The Greek Heritage

- Numerals
- Prefixes
- General Vocabulary
- Phrases and Epigrams

The Greek language has contributed to the English vocabulary directly as an immediate donor and, indirectly, through other intermediate languages, mainly Latin and French. In a typical English dictionary of 80,000 words, which corresponds very roughly to the vocabulary of an educated English speaker, about 5 percent of the words are borrowed from Greek directly, and about 25 percent indirectly.

Since the living Greek and English languages were not in direct contact until modern times, borrowings were necessarily indirect, coming either via Latin (through texts or various vernaculars), or from ancient Greek texts, not the living Language.[58]

More recently, a huge number of scientific, medical and technical neologisms have been coined from Greek roots.

Until the 16^{th} *century, the few Greek words that were absorbed into English came through their Latin derivatives. Most of the early borrowings are for expressions in theology for which there were no English equivalents. In the late* 16^{th} *century an influx of Greek words were derived directly in the new science.*

In the 19^{th} *and* 20^{th} *centuries a few learned words and phrases were introduced using more or less direct transliteration of Ancient Greek rather than the traditional Latin-based orthography (e.g. nous, hoi polloi).*

Many English words and word elements (roots, prefixes, suffixes) can be traced back to the ancient Greek language. For example: Demon, Lexicon, Colon, Stigma, scheme, bishop, priest, metaphor, mathematica, encyclopedia, hemoglobin, edema, dynamo, kinematica, physics, mechanics, electron, hadron, galaxy, optics, acoustics, thermodynamics – are of Greek origin.

Tables 6.27–6.29 present a glossary of Greek numerals, prefixes and general vocabulary used in the vernacular of scientists and other scholars. Table 6.30 shows some common Greek phrases and epigrams used today in the scientific and the general literature.

[58] Moreover, Greek culture and language had a direct major influence on Roman art, literature and science; Indeed, the seven Greek Muses [**Clio** (history), **Urania** (astronomy), **Calliope** (epic poetry), **Melpomene** (tragedies), **Euterpe** (harmony), **Erato** (lyric and love poetry), **Terpsichore** (dancing), **Thalia** (comedy), and **Polyhymnia** (music)] were so absorbed by the Romans that Quintilian was prompted to say: "Satura quidem tota nostra est" (At least satire is completely ours).

Table 6.27: Greek Numerals

Ordinals	Cardinals
protos – first	*Hen – one*
deuteros – second	*Dyo – two*
tritos – third	*Treis, tria – three*
tetratos – fourth	*Tettara (tetra) – four*
	Pente – five
	Hex – six
	Hepta – seven
	Octo – eight
	Emea – nine
	Deka (deci) – ten
	Hendeka – eleven
	Dodeka – twelve
	Hekaton (hecto) – one hundred
	Chilioi (Kilo-) – one thousand
	Myrioi – ten thousand, innumerable (myriad)

Table 6.28: Greek Prefixes

Prefix	Meaning	Examples
A-, an-	*not, without, lack of*	*atom, anemia, achromatic, atypical, amoral, anesthesia, analgesic*
Amphi-	*around, about, both, in two ways*	*amphibious, amphitheater*
Ana-, ano-	*up, back, anew, again, throughout, against*	*analysis, anion, anamnesis, anabolism*
Anti	*against, opposed to, resisting*	*antibalistic,antitoxin, antiseptic, antiacid, antifreeze*
Ante	*before, in front of*	*antecedent*
Apo-	*away from, separation, lack*	*aphelion, apostasis, apogee, apologize*
Arche-, archi-	*first, chief, primitive (ancient)*	*archetype, architecture, archeology*
Cata-, Kata-, Cath-	*down, lower, under, complete, across*	*catastrophe, cathode, catarsis, catabolism, catatonia, catoptrics, catapult*
Di-	*twice, twofold, double*	*diatomic, dilemma*
Dia-	*through, across, apart, thoroughly*	*diameter, diagnosis, diabetes, diarrhea, diuretic*
Dicha-	*in two, double, asunder*	*dichotomy*
Dys-	*bad, difficult, hard, disorder, painful*	*dysentery, dystrophy, dyslexia*
Ecto-	*outer, outside*	*ectoplasm, ectozoa*
Ek-, ex-	*out from, outside, out of*	*eccentric, exit*

Table 6.28: (Cont.)

PREFIX	MEANING	EXAMPLES
En-, em-	*in, within, among*	*endemic (demos = people), embolism, enthusiasm*
Endo-, Ento-	*within*	*endocrine, entoplasm*
Epi-, eph-	*upon, on*	*ephemeral, eponym, epidemic, epidermis*
Eso-	*inward, within*	*esoteric*
Eu-	*well, good, normal, easy*	*eugenics, eulogy*
misos	*bad, hate*	*miscarriage, misanthropic, misogyny*
Exo-	*outside, outward, outer*	*exothermal, exogamy, exodus (gamos = marriage), exonerate*
Hemi-	*half, partly*	*hemisphere, hemin*
Hyper-	*above, over, excessive*	*hyperactive, hypersensitive*
Hypo-	*under, below, deficient, in bottom*	*Hippopotamus, hypothesis, hypoglycemia, hypodermic*
Meta-, meth-	*after, among, beyond, behind, change, transformation*	*metabolism, metaphysics, metamorphosis*
Pali(n)-	*back, again, once more, backwards*	*palindrome*
Para-	*by the side of, near, accessory, abnormal, prevented, beside*	*paranoia, paraxial, paragraph, paradigm*
Peri-	*all around, near*	*perimeter, periphery, pericardium, periscope*

Table 6.28: (Cont.)

Prefix	Meaning	Examples
Pre-,pro-	*before, in front of, forward*	*prognosis, prophylactic, precede, progress*
Syn-, Sym-, Sy-	*with, together*	*sympathy, symbiosis, symphony, syndrome, synagogue, synapsis*

Table 6.29: Greek General Vocabulary

Greek Word	English Meaning	Examples
Agora	*bringing together; assembly*	*category*
Aither	*the upper air*	*ether*
Akouein	*to hear*	*acoustic*
Algos, algesis	*pain, sense of pain*	*analgen, neuralgia*
Alos, allelon, allotrios	*other, different, external, foreign, of each other, another's*	*allergy, allotropic, parallelism*
Aner, andros	*a man, male*	*android, androgen*
Anthropos	*man, a human being*	*philanthropy, anthropoid, misanthrope*
Arthron	*joint, juncture of bones*	*arthritis*
Aster, astra	*star*	*astronomy*
Atmos	*air, breath, vapor, steam*	*atmosphere*

Table 6.29: (Cont.)

Greek Word	English Meaning	Examples
Autos	*self, by itself*	*automatic, autopsy, autism, autobiography*
Bakterion	*a little staff, rod, bacteria*	*bacteria*
Baros, barys	*weight, pressure, heavy*	*barometer, isobaric, baryon*
Bathys, bathos	*deep, inner, depth*	*bathysphere*
Bios, biosis	*a living, way of living, life*	*biology, microbe, symbiosis*
Bromos	*stench*	*bromid*
Brachys	*short*	*brachistochrone*
Breyin	*to be full, swell*	*embriology*
Chamai	*on the ground*	*chameleon*
Charakter	*to sharpen, engrave*	*character*
Chloros	*green, yellowish*	*chlorine, chlorophile*
Chole	*bile, gall, bitter, anger*	*cholic, cholera*
Choreia	*dancing*	*choreography*
Chromatos, chroma	*color of the skin, color*	*chromium, monochrome, chromosome*
Cyclo, gyro	*round*	*gyroscope, cyclone, cycloid*
Chronos	*time*	*chronic, synchronism, chronicle, chronology*
Crypto, kryptos	*hidden*	*cryptanalysis*

Table 6.29: (Cont.)

Greek Word	English Meaning	Examples
Daimon	*a divine power, demon*	*demoniac*
Demos	*country, land, people of a country*	*democracy, endemic, epidemic, demagogue*
Deuteros	*second, next*	*deuterium*
Dexios	*right (side)*	*dexterity*
Diaita	*life, way of living*	*diet*
Dosis	*a giving, dose*	*dose, antidote*
Dromos	*a running, course*	*syndrome, hypodrome*
Dynamis	*power, active force*	*hydrodynamics, dynamometer*
Eidsos, idea (id = to see)	*appearance, thought, mental impression*	*idea*
Eikon, eikonos	*image, likeness*	*icon*
Elektron	*amber, electricity*	*electron*
Enteron	*that within, the intestine*	*dysentery*
Eos	*morning red, dawn, an early age*	*eocene, eon*
Ergon	*work, functioning*	*erg, energy, allergy, synergic*
Ethnos	*race, nation*	*ethnic*
Gala, galactos	*milk*	*galaxy, galactose*
Ge	*earth*	*geode, geography, geology*

Table 6.29: (Cont.)

Greek Word	English Meaning	Examples
Gen, genesis, genos	*to become, be born, coming into, being, origination, birth*	*gene, genesis, glycogen, halogen, genetics, genealogy*
Gigas, gigantos	*giant*	*giant*
Glykis, glykeros	*sweet, sugar*	*glucose, glycogen*
Gnonai, gnosis	*to know, judge, knowledge*	*gnostic, diagnosis, prognosis*
Graphein, gramma	*to draw, write, inscribe, record, something written down*	*monograph, graphic, telegram, cardiogram*
Gymnos	*naked*	*gymnasium, gymnasitcs*
Gyros	*circle, ring, turn*	*gyrate, gyroscope*
Haima, haimatos, hemo	*blood*	*anemia, leukemia, ischemia, hemoglobin*
Helios	*sun*	*helium, heliocentric*
Hemera	*day*	*ephemeral*
Heteros	*other, different from*	*heterogenous*
Hodos	*road, way, path*	*hodograph*
Holos	*entirely, all, whole*	*hologram, holistic*
Homos, homoios	*like, the same as, equal*	*homosexuality, homeopathy*
Hormon	*setting in motion, arousing, exciting*	*hormone*
Hydor, hydatos	*water, fluid*	*hydrogen, hydrostatic, hydrate*

Table 6.29: (Cont.)

Greek Word	English Meaning	Examples
Hygros	*wet, moist, fluid*	*hygrometer, hygroscopic*
Hypnos	*sleep*	*hypnosis*
Hystera	*womb, uterus*	*hysteria*
Idios	*one's own, peculiar*	*idiosyncrasy*
Isos	*equal*	*isomorphism, isomer*
Kakos	*bad, distorted, abnormal*	*cacophonia*
Kalos	*beautiful*	*calligraphy, calisthenics*
Kausos, kaustos	*burning, heat, burnt*	*caustic*
Kephale, enkephalon	*head, the brain*	*encephalic*
Kineein	*to move*	*Kinematics, cinema*
Klimax	*ladder*	*clima, climate*
Klinein, klisis, klinikos	*to bent, turn, slope, make recline, lean, lie down, pertaining to bed*	*clinic, inclination*
Kolla	*glue*	*collagen, colloid*
Koma, komatos	*deep sleep, coma*	*comatos*
Kranion	*cranium, skull*	*migrain*
Krinein, krisis	*to separate, distinguish, decide, emit, secrete, point of decision*	*crisis, critical*

Table 6.29: (Cont.)

Greek Word	English Meaning	Examples
Kryos, kristallos	*ice-cold, frost, clear ice*	*crystal, cryogen*
Kyanos	*dark blue*	*cyanamide*
Kyklos	*circle, ring, wheel*	*cyclic, cyclotron, epicycle*
Leon, leontos	*lion*	*leopard*
Leptynein	*to make thin, thin, delicate*	*lepton, lepidoma*
Lethe, lethargos	*oblivion, forgetfulness, drowsiness*	*lethe, letharge*
Leukos	*white*	*leukemia, leucite*
Lexis, legein	*to speak, speech, word, phrase, diction, reading*	*lexicon, dyslectic*
Lipos	*fat, oily*	*lipids, lipoma*
Lithos	*stone*	*Lithosphere, neolithic*
Logos, -logy	*word, speech, thought, reason, treatise, body of knowledge*	*biology, hematology, monologue, travelogue*
Makros	*long, large*	*macroscopic*
Mania	*madness, frenzy, enthusiasm*	*maniac, egomania, kleptomania*
Megas, megalou	*large*	*megalopolis, megalomaniac*
Meion	*less, smaller*	*miosis, miocardia*
Melas, melanos	*black, dark*	*melancholia, melanin*

Table 6.29: (Cont.)

Greek Word	English Meaning	Examples
Men, menos, meniskos	*month, semi-lunar*	*menopause, menstruation, meniscus*
Meros	*part, segment, partition*	*isomer,polymer*
Mesos	*middle, intermediate*	*meson, mesopotamia*
Metron	*measure, measuring instrument*	*geometry, kilometer, perimeter, parameter*
Mikros	*small, little*	*microbe, micron, microscope, microcosmos*
Mnaesthai, mneme	*to remember, memory*	*amnesia, mnenothecnic*
Monos	*alone, single, one*	*monotonic, monolith, monotheism, monograph, monosyllable, monocrome, monopoly, monoplane*
Moros	*dull, sluggish, stupid*	*moron*
Morph	*form, shape, figure*	*amorphous, morphology*
Myein	*to close, shut*	*myopia, myosis*
Nanos	*dwarf, billionth*	*nanosecond*
Narke	*numbness, stupor*	*narcosis, narcomania*
Naus, nautes	*ship, sailor*	*nausea, aeronautic*
Nekros	*dead body, dead*	*necrophil*
Neos	*new, young, recent*	*neodymium, neophyte, neonatal*
Neuron	*nerve, tendon, sinew*	*neuralgia, neurosis*

Table 6.29: (Cont.)

Greek Word	English Meaning	Examples
Nomos	*law, ordering*	*autonomic, economy, taxonomy*
Noos (nous), noema	*mind, thought*	*paranoia*
Nosos	*disease, sickness*	*nosogenic, neuronosis*
Oikos	*house, dwelling*	*economy, ecology*
Oligos	*few, small, scanty*	*oligarchy*
Onoma, onomatos	*name*	*eponymic, onomatopoeia, anonymous*
Optikos, optos (op = to see)	*pertaining to vision or the eye*	*optical, optics, cataoptrics*
Organ, ergon	*something that does work, instrument, tool, organ of the body*	*organism, organic*
Orthos	*straight, correct, normal*	*orthopedics, orthonormal, orthogonal*
Osmos	*thrusting, pushing*	*osmosis*
Oxys	*sharp, swift, quick, sour, acid*	*oxygen, oxymoron, dioxide, paroxysm, anoxia*
Palaios	*old, ancient*	*paleontology*
Pas, pantos, pan	*all, entire*	*panacea, pancreatic, pandemic, panorama, pantheon*
Pathein, pathos	*to be affected, experience, suffer feeling, disease*	*sympathy, telepathy, empathy, pathology*
Peiraein	*to attempt, try, test*	*empiric*

Table 6.29: (Cont.)

Greek Word	English Meaning	Examples
Phagein	*to eat, devour*	*bacteriophage, phagocyte*
Phain, phasma	*to bring to light, show, appear, apparition*	*fantasy, phantom*
Phanai, phon	*to speak, speech, voice, sound*	*phone, telephone, phonetics, cacophony*
Pherein, phoros	*to bear, carry, bring*	*euphoria, periphery*
Philein, philia	*to love, affinity for*	*philosophy, philanderer, philanthropy, philharmonic*
Phlegein, phlegma	*to burn, became hot, heat, flame*	*phlogiston, flame*
Phlogos	*inflammation*	*phlogosis*
Phobos	*fear, flight*	*hydrophobia, xenophobia*
Phos, photos	*light*	*phosphorous, photograph*
Phyein, physis	*to be by nature, arise, grow*	*physics, physical*
Planos, planktos	*wandering*	*plankton*
Plasscin, plastos	*to form, mold*	*plastic, plasma*
Polys	*much, many, more that usual*	*polygonal, multiply, polymer, polystyrene*
Pragma, praxis, praktikos	*a thing done, deed, fact, practical, fit for doing*	*pragmatism, practice*
Protos, proteins	*first, primitive, simple, primary*	*protocol, protein, proton, protozoan, prototype*

Table 6.29: (Cont.)

Greek Word	English Meaning	Examples
Pseudeo	*false, imaginary*	*pseudoscalar, pseudoscience, pseudonym*
Psyche	*spirit, soul, mind*	*psychic, psychotic, psychiatry*
Pyros	*fire, heat, fever*	*pyrotechnics, pyromania*
Rheein, rhema	*flow, stream, current*	*rheology, diarrhea, gonorrhea*
Rhythm	*rhythm, measure*	*arrhythmia*
Sakcharon	*sugar*	*sacchrine*
Sarx, sarkos	*flesh*	*sarcoma*
Seirein, seismos	*to shake, earthquake*	*seismology*
Sepsis, sapros	*rotting, decayed, putrid*	*asepsis, antiseptic, saprophyte*
Skopeein	*to look at, view*	*stethoscope, microscope*
Skotos	*darkness*	*scotoma*
Soma, somatos	*body*	*psychosomatic*
Sphaira	*sphere, ball, globe*	*spheroid, spherula*
Sta, stasis	*to cause to stand, set, fix, arresting*	*ecstasy, apostasis*
Steno	*narrow*	*stenographer*
Stereo	*solid*	*stereogeometry, cholesterol*
Sthenos	*strength*	*calisthenics*
Stizein, stigma	*to prick, puncture, brand, dot, mark*	*stigma*

Table 6.29: (Cont.)

Greek Word	English Meaning	Examples
Tachys	*swift, quick, rapid*	*tachion, tachycardia*
Tauto	*the same*	*tautology*
Taxis	*arrangement, order*	*syntactic, taxonomy*
Techne	*art, skill, craft*	*technic, technique*
Tele	*far off, at a distance*	*telepathy, television, telephone, telemetry*
Thenai, thesis	*to put, place, set down, a proposition*	*thesis, synthesis*
Theos	*God*	*theist, theology*
Therapeuein	*to take care of*	*therapy*
Therapeia	*to heal, treat medically*	*physiotherapy*
Thermos, therme	*hot, warm, heat*	*thermodynamics, diathermy*
Tomos, tome	*cutting, a segment*	*anatomy, atom*
Topos	*place, region, spot*	*topology, isotopes*
Trephein, trophe	*nourish, nourishment*	*atrophy*
Tropein	*to turn, bend*	*allotropic*
Xanthos	*yellow*	*xanthophyll*
Xenos, xenia	*foreign, stranger, host, guest, hospitality*	*xenophobia*
Zoon	*something living, an animal*	*zoology, protozoon*
Zone	*girdle, belt, zone*	
Zyme	*a ferment, fermentation*	*enzyme, zymase*

Table 6.30: List of Greek Phrases and Epigrams

(1) ***Ageōmetrētos mēdeis eisitō***
(Let no-one without knowledge of geometry enter)
Motto over the entrance to Plato's Academy

(2) ***Aei ho theos geōmetrei*** *(Plato)*
(God always geometrizes)

(3) ***Anthrōpos metron*** *(Protagoras)*
(Man is the measure of all things)

(4) ***Andrōn epiphanōn pasa gē taphos*** *(Thucydides: Periclés Funeral Oration)*
(For illustrious men have the whole earth for their tomb)

(5) ***Ariston men hudōr*** *(Pindar)*
(Greatest however is water)

(6) ***Gnōthi seauton [Noce te ipsum]***
(Know Thyself)
Inscribed in the forecourt of the Temple of Apollo at Delphi. Attributed to at least six Greek sages: Heraclitos, Chilon of Sparta, Thales of Miletos, Socrates, Phytagoras, Solon of Athens.
The saying "Know thyself" may refer by extension to the ideal of understanding human behavior, morals, and thought, because ultimately to understand oneself is to understand other humans as well. However, the ancient Greek philosophers thought that no man can ever comprehend the human spirit and thought thoroughly, so it would have been almost inconceivable to know oneself fully. Therefore, the saying may refer to a less ambitious ideal, such as knowing one's own habits, morals, temperament, ability to control anger, and other aspects of human behavior that we struggle with on a daily basis.

(7) ***Diairei kai basileue***
(Divide and rule)

(8) ***Dōs moi pā stō, kai tan gān kināsō*** *(Archimedes)*
(Give me a place to stand and I will move the earth)

(9) ***Hen oida hoti ouden oida*** *(Socrates)*
(I know one thing, that I know nothing)

(10) ***Heurēka!*** *(Archimedes)*
(I found it)

(11) ***Ē tan ē epi tas***
(Either with your shield, or upon it)
Spartan mothers to their sons before they went to battle

(12) ***Lathe biōsas***
(Live in obscurity) *(Epicurean phrase)*

(13) ***Métron áriston***
(moderation is the best thing)

(14) ***Mē mou tous kyklous taratte*** *(Archimedes)*
(Do not disturb my circles)

(15) ***Mē cheiron vēltiston***
(The least bad (choice) is the best)
When there is no good option one should pick the one that does the least harm.

(16) ***Mēden agan*** *("Ne quid nimis" – st. Jerome)*
(Nothing in excess)
A carving from the temple of Apollo at Delphi

(17) ***Molōn labe****!*
(Come take them!)
King Leonides of Sparta, in response to King Xerxes of Persia's demand that the Greek army lay down their arms before the battle of Thermopylae.

(18) ***Hoper edei deixai***
(Quod Erat Demonstrandum – Q.E.D.)
Used by early mathematicians including Euclid and Archimedes to signify the proof as complete.

(19) ***Outis emoi ǵ onoma***
(My name is Nobody)
Odysseus to Polyphemos when asked what his name was.
(Homer, Odyssey)

(20) ***Pistis, elpis, agapē*** *(1 Corinthians* ***13****,13)*
(Faith, hope, (and) love).

(21) ***Rhododaktulos Ēōs***
(Rosy-fingered dawn)
Occurs frequently in the Homeric Poems.

(22) ***Speude bradeōs***
(Latin: festina lente = less haste, more speed)

(23) ***Ta Panta rhei kai ouden menei*** *(Heraclitos)*
(Everything flows, nothing stands still)

(24) • *What is hard? — to know yourself.*
• *What is easy? — to advise others.*
• *What is quite common? — Hope.*
(When all is gone, there is still hope)
• *What is the fastest? — mind; it travels through all media.*

Thales

(25) ***Phobou tous Danaous kai dōra pherontas***
(Quidquid id est) timeo Danaos et dona ferentes
(Beware of the Danaans (Greeks), even bearing gifts)

Virgil, "Aneid"

(26) ***Khalepa ta kala*** *(Plato)*
(The good things are hard to attain)

(27) *Oh stranger, tell the Spartans that here we lie, obedient to their laws.*
(Epigram by Simonides at Thermopylae)

(28) ***Hoī polloī*** *("Plebs urbanus" in Latin)*
(The many)

(29) ***Hoi āristoi***
(The aristocracy)

(30) ***Kalōs kīndyons***
(A beautiful risk)

(31) ***Melēt e tō pān***
(Practice is everything; practice makes perfect)

(32) ***Nike Somen***
(We shall overcome)
Said by the Greeks before battle.

(33) ***Ou pollā, allā polȳ*** *(Multum, non multa)*
(Not quantity, but quality)

The Latin Heritage

- History
- Prepositions and Conjunctions
- Numerals and Lengths
- Phrases and Epigrams
- Prefixes and Suffixes
- Phrases and Abbreviations
- Epigrams
- Root Groups
- Dictionary of Greco–Latin Word Origins

History

When we delve into the etymology of the English language we soon discover its affinity to the classical vernaculars of Latin. Just open Webster's Encyclopedic Dictionary under the word VOCAL *to see how from the single Latin root Vocare (to call) there evolved twenty English words:*

advocate
avocation
convocation
convoke
equivocate
evocation
evocative
evoke
invocation
invoke
irrevocable
provocateur
provocation
provocative
provoke
revocation
revoke
unequivocal
vocable
vocabulary
vocal
vocation
vocalist
vocalize
vociferous

Latin (lingua Latina) is the language of ancient Rome and the ancestor of the modern Romance languages: Italian, French, Spanish, Portuguese, Romanian, Catalan and so on.

Half of the English vocabulary comes from ancient Rome, and everyday communications are peppered with Latin phrases like et cetera, per capita *and* *cui bono? (Who benefits?). Scientific literature abounds with Latin words, roots, prefixes, suffixes, prepositions, conjunctions and the like.*

Indeed, signs of the Zodiac and names of the planets are of Latin origin. Mathematical terms like: Plus, minus, Q.E.D., matrix, invariant, divergence, calculus, derivative, maximum, minimum, residuum, covariant, curvature, intrinsic, genus are of Latin origin. So are the physical and chemical terms: Valence, oxygen, solid, liquid, momentum, longitude, latitude, equilibrium, flux, quanta, altitude, peninsula, and many others.

English used many Latin words without any change in spelling or any significant change in meaning. Many other Latin words involve the change of only a few letters. Here are some examples:

Latin	English
innera	*inner*
hydan	*hide*
donwel	*do well*
succedere	*succeed*
concelare	*conceal*
interior	*interior*
defendo	*defend*
signum	*sign*
copiosus	*copious*
memento	*remember*
gladiator	*gladiator*
senator	*senator*
consul	*consul*
schola	*school*
modor	*mother*

Latin	English
saed	*sad*
hete	*hate*
miser	*miserable*
imbibere	*imbibe*
maternus	*maternal*
radix	*root*
heah	*high*
waeccan	*wake (watch)*
drincan	*drink*
mercator	*merchant*
cantus	*chant*
taberna	*tavern*
elevare	*elevated*
observare	*observe*
accelerare	*accelerate*
descendre	*descend*
spowan	*speed*

As the Romans conquered most of Europe, the Latin language spread throughout the region.

In 1066 England was conquered by William, duke of Normandy, which is northern France. For several hundred years after the Norman invasion, *French was the language of court and polite society in England, It was during this period that many French words were borrowed into English. Linguists estimate that some 60% of our common everyday vocabulary today comes from French. Thus many Latin words came into English indirectly through French.*

Many Latin words came into English directly, though, too. Monks from Rome brought religious vocabulary as well as Christianity to England beginning in the 6^{th} century. From the Middle Ages onward many scientific, scholarly, and legal terms were borrowed form Latin.

Moreover, many of the early scientists spoke Latin or had learned it as part of their education. During the Dark Ages, the sciences and culture were segregated inside the monasteries where the spoken language was Latin. Thus, science started out using Latin as its universal language. Since the only real connections between countries through the Dark Ages were through the Roman Catholic Church, Latin became the standard lingual link between countries using different languages.

During the 17^{th} and 18^{th} centuries, dictionary writers and grammarians generally felt that English was an imperfect language whereas Latin was perfect. In order to improve the language, they deliberately made up a lot of English words from Latin words. For example, fraternity, from Latin fraternitas, was thought to be better than the native English word brotherhood.

Latin is a member of the family of Italic languages, and its alphabet, the Latin alphabet, emerged from the Old Italic alphabets, which in turn were derived from the Greek and Phoenician scripts (Table 6.31).

The Italic subfamily is a member of the Centum branch of the Indo-European language family. It includes the Romance languages (among others: French, Italian, Spanish, Portuguese, Romanian), and a number of extinct languages.

Latin was first brought to the Italian peninsula in the 9^{th} or 8^{th} century BCE by migrants from the north, who settled in the Latium region, specifically around the River Tiber, where the Roman civilization first developed. Latin was influenced by the Celtic dialects in northern Italy and the non-Indo-European Etruscan language in Central Italy, and by Greek in southern Italy.

Although surviving Latin literature consists almost entirely of Classical Latin, an artificial, highly stylized and polished literary language from the 1^{st} century BCE, the actual spoken language of the Roman Empire was Vulgar Latin, which significantly differed from Classical Latin in grammar, vocabulary, and eventually pronunciation. Also, although Latin remained the main written language of the Roman Empire, Greek came to be the language spoken by the well-educated elite, as most of the literature studied by Romans *was written in Greek. In the eastern half of the Roman Empire, which became the Byzantine Empire, the Greek Koine of Hellenism remained current and was never replaced by Latin.*

Fig. 6.3: Approximate distribution of languages in Iron Age Italy during the sixth century BCE

We may divide the age of Latin into seven epochs:

–75 BCE	*Old Latin*
75 BCE–200 CE	*Classical Latin*
300–900	*Late Latin*
900–1300	*Medieval Latin*
1300–1600	*Renaissance Latin*
1600–1900	*New Latin*
1900–present	*Recent Latin*

Old Latin (also called Early Latin or Archaic Latin) refers to the period of Latin texts before the age of Classical Latin.

Classical Latin is the form of the Latin language used by the ancient Romans in what is usually regarded as "classical" Latin literature. Its use spanned the Golden Age of Latin literature – broadly the 1^{st} century BCE and the early 1^{st} century – possibly extending to the Silver Age – broadly the 1^{st} and 2^{nd} centuries.

What is now called "Classical Latin" was, in fact, a highly stylized and polished written literary language selectively constructed from Old Latin, of which far fewer works remain. Classical Latin is the product of the reconstruction of early Latin in the prototype of Attic Greek. Classical Latin differs from the earliest Latin literature, such as that of Cato the Elder, Plautus, and to some extent Lucretius.

The spoken Latin of the common people of the Roman Empire, especially from the 2^{nd} century onward, is generally called Vulgar Latin. Vulgar Latin differed from Classical Latin in its vocabulary and grammar, and as time passed, it came to differ in pronunciation as well.

The golden age of Latin literature, in Latin Latinitas aurea, is a period consisting roughly of the time from 75 BCE to 14 CE, covering the end of the Roman Republic and the reign of Augustus Caesar. Many Classicists believe that this period represents the peak of Latin literature, and that its usage of the artificial and heavily stylized literary language known as Classical

Latin represents the ideal norm which other writers should follow. Classical Latin continued to be used into the Silver Age of Latin literature, 1st and 2nd centuries.

In reference to Roman literature, the Silver age covers the first two centuries CE directly after the Golden age (which was the first century BCE, and the start of the first century CE). Literature from the Silver age has traditionally, perhaps unfairly, been considered inferior to that of the Golden age. Silver Latin itself may be subdivided further into two periods: a period of radical experimentation in the latter half of the first century CE, and a renewed Neoclassicism in the second century CE.

Under the reigns of Nero and Domitian, writers like ***Seneca*** *the Younger,* ***Lucan*** *and* ***Statius*** *pioneered a unique style that has alternately delighted, disgusted and puzzled later critics. Stylistically, Neronian and Flavian literature shows the ascendance of rhetorical training in late Roman education. The style of these authors is unfailingly declamatory – at times eloquent, at times bombastic. Exotic vocabulary and sharply-polished aphorisms glimmer everywhere, though at times to the detriment of thematic coherence.*

Thematically, late 1st century literature is marked by an interest in terrible violence, witchcraft, and extreme passions. Under the influence of Stoicism, the gods recede in importance, while the physiology of emotions looms large. Passions like anger, pride and envy are painted in almost anatomical terms of inflammation, swelling, upsurges of blood or bile.

By the end of the 1st century, a reaction against this form of poetry had set in, and ***Tacitus****,* ***Quintilian*** *and* ***Juvenal*** *all testify to the resurgence of a more restrained, classicizing style under Trajan and the Antonine emperors.*

Vulgar Latin (in Latin, sermo vulgaris) is a blanket term covering the vernacular dialects of the Latin language spoken mostly in the western provinces of the Roman Empire until those dialects, diverging still further, evolved into the early Romance languages – a distinction usually assigned to about the ninth century.

This spoken Latin differed from the literary language of classical Latin in its pronunciation, vocabulary, and grammar. Some features of Vulgar Latin did not appear until the late Empire. Other features are likely to have been in place in spoken Latin, in at least its basilectal forms, much earlier. Most definitions of "vulgar Latin" mean that it is a spoken language, rather than a written language, because the evidence suggests that spoken Latin broke up into divergent dialects during this period. Because no one transcribed phonetically the daily speech of any Latin speakers during the period in question, students of vulgar Latin must study it through indirect methods.

Romance languages, a major branch of the Indo-European language family, comprise all languages that descended from Latin, the language of the Roman Empire. The Romance languages have more than 600 million native speakers worldwide, mainly in the Americas, Europe, and Africa, as well as in many smaller regions scattered through the world.

All Romance languages descend from Vulgar Latin, the language of soldiers, settlers, and slaves of the Roman Empire, which was different from the Latin of the Roman literati. Between 200 BCE and 100 CE, the expansion of the Empire, coupled with administrative and educational policies of Rome, made Vulgar Latin the dominant native language over a wide area spanning from the Iberian Peninsula to the Western coast of the Black Sea. During the Empire's decadence and after its collapse and fragmentation in 5th century, Vulgar Latin began to evolve independently within each local area, and eventually diverged into dozens of distinct languages. The oversea empires established by Spain, Portugal and France after the 15^{th} *century then spread Romance to the other continents – to such an extent that about 2/3 of all Romance speakers are now outside Europe.*

Medieval Latin refers to the Latin used in the Middle Ages, primarily as a medium of scholarly exchange and as the liturgical language of the medieval Roman Catholic Church. It is therefore largely synonymous with the term Ecclesiastical Latin (sometimes called Church Latin), which refers to the Latin language as used in documents of the Roman Catholic Church and in its Latin liturgies.

Renaissance Latin is a name given to the Latin written during the European Renaissance in the 14^{th}*–*16^{th} *centuries, particularly distinguished by the distinctive Latin style developed by the humanist movement.*

Ad fontes was the general cry of the humanists, and as such their Latin style sought to purge Latin of the medieval Latin vocabulary and stylistic accretions that it had acquired in the centuries after the fall of the Roman Empire. They looked to golden age Latin literature, and especially to Cicero in prose and Virgil in poetry, as the arbiters of Latin style. They abandoned the use of the sequence and other accentual forms of metre, and sought instead to revive the Greek formats that were used in Latin poetry during the Roman period. The humanists condemned the large body of medieval Latin literature as "gothic" – for them, a term of abuse – and believed instead that only ancient Latin from the Roman period was "real Latin".

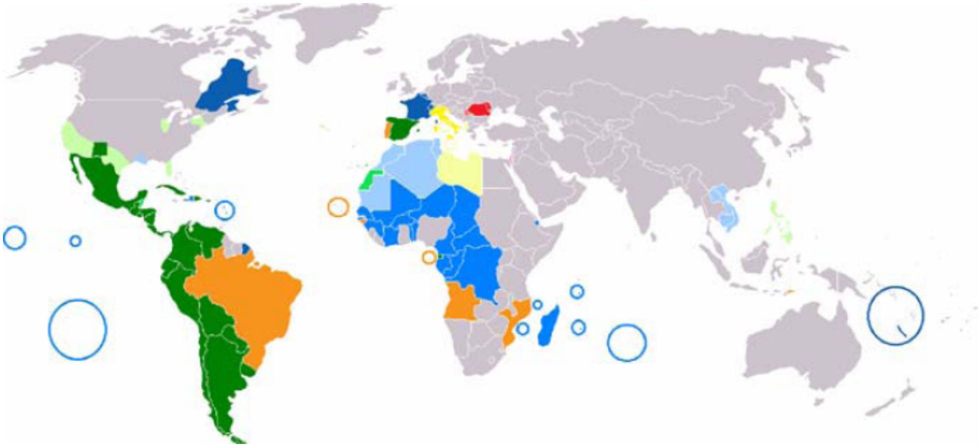

Fig. 6.4: Romance languages in the world: Blue – French; Green – Spanish; Orange – Portuguese; Yellow – Italian; Red – Romanian

The humanists also sought to purge written Latin of medieval developments in its orthography. They insisted, for example, that ae *be written out in full wherever it occurred in classical Latin; medieval scribes often wrote* e *instead of* ae*. They were much more zealous than medieval Latin writers that* t *and* c *be distinguished; because the effects of palatalization made them homophones, medieval scribes often wrote, for example,* eciam *for* etiam*. Their reforms even affected handwriting; Humanists usually wrote Latin in a script derived from Carolingian minuscule, the ultimate ancestor of most contemporary lower-case typefaces, avoiding the black-letter scripts used in the Middle Ages. Erasmus even proposed that the then-traditional pronunciations of Latin be abolished in favor of his reconstructed version of classical Latin pronunciation.*

The humanist plan to remake Latin was largely successful, at least in education. Schools now taught the humanistic spellings, and encouraged the study of the texts selected by the humanists, to the large exclusion of later Latin literature. On the other hand, while humanist Latin was an elegant literary language, it became much harder to write books about law, medicine, science or contemporary politics in Latin while observing all of the Humanists' norms about vocabulary purging and classical usage. Because humanist Latin lacked precise vocabulary to deal with modern issues, their reforms accelerated the process of turning Latin from a workday language to an object of antiquarian study. Their attempts at literary work, especially poetry, often *have a strong element of pastiche. Their efforts turned Latin from a classical, but still useful language, into a truly extinct language. Latin vocabulary continued to be used by the creators of New Latin, but extensive discourses on contemporary subjects in Latin gradually ceased to be written during this period.*

New Latin (or Neo-Latin) is a post-medieval version of Latin, now used primarily in International Scientific Vocabulary cladistics and systematics. The term came into widespread use towards the end of the 1890s among linguists and scientists.

Table 6.31: Latin Prepositions and Conjunctions

Alieni	*another's*
Aut	*either, or*
Cam	*with*
Cum	*since*
Circa	*about, approximately*
Donec	*until*
Dum	*while*
Enim, nam	*for*
Ergo, igitur	*therefore*
Et	*and*
Etiam, quoque	*also*
Ibi	*there*
Iuxta	*next to*
Modo, tantum	*only*
Neque	*nor*
Ob, propter	*because, because of*
Prope	*near*
Postquam	*after that*
Que	*and*
Quod	*because*

Si	*if*
Sed	*but*
Sine	*without*
Tanem, autem	*however*
Tandem	*at last*
Ubi	*where*
Ut	*so that*
Utinam	*may it happen*
Sui	*one's own*

Table 6.32: LATIN NUMERALS

ORDINALS	CARDINALS
Primus – first (primary)	*Unus – one (union, uniform, unit, univalent)*
Secundus – second	*Duo – two (duplex, duplicate)*
Tertius – third (tertiary, trident)	*Tres, tri – three*
Quartus – fourth	*Quattuor – four*
Quadrus – fourfold	
Quintus – fifth	*Quinque – five*
Sextus – sixth	*Sex – six*

Ordinals	Cardinals
Septimus – seventh (September)	*Septen – seven*
Octavius – eight (octavo)	*Octo – eight (octave)*
Nonus – ninth (Novembers)	*Novem – nine*
Decimus – tenth (December)	*Decem – ten*
Undecimus – eleventh	*Undecima – eleven*
Duodecimus – twelfth	*Duodecim – twelve (duodocimal)*
Vicesimus – twentieth	*Viginti – twenty*
Centesimus – hundredth	*Centum – one hundred*
Millesimus – thousandth	*Mille – one thousand*
Semi, demi – 1/2 (semicircle, semifinal)	*plus – more*
Sesqui – 3/2	*minus – less*
Bi, bis, bin – twice, twofold, double (bicycle, biped, binaural)	*equ – equal (equidistant, equilibrium)*

Table 6.33: Roman Lengths

cubit	:	*from the Latin* cubitum *(elbow). The unit represents the length of a man's forearm from his elbow to the tip of his outstretched middle finger (c. 52.35 cm)*
foot	:	*from the Latin* pes naturalis*. Based directly on the length of the human feet (31.3 cm). The modern foot (c. 30.5 cm) may have been the invention of Henry I (1100–1135)*

inch	:	*from the Latin uncia (twelfth part). Defined as the width of man's thumb at the base of the nail.*
Pace	:	*from the Latin passus (step). The distance between two successive falls of the same foot (c. 1.5 m).*
mile	:	*from the Latin milia or mille (a thousand paces): the distance a Roman legion could march in thousand paces. It's the measured distance between surviving milestones of Roman roads (close to 500 feet). Roman had set their mile equal to 8 stadia (c. 202.3 yards).*
league	:	*from the Latin leuga or leuca; it was intended to represent, roughly, the distance a person could walk in an hour (c. $1\frac{1}{2}$ Roman miles)*

Table 6.34: Latin Negation Prefixes

Prefix	English Meaning	Examples
In-, il-, ir-, im-	*not, beyond belief*	*infallible, impossible, immiscible, invalid*
Un-	*not*	*undo*
Non-	*not*	*non-portable, nonsense, nonstop*
Contra-	*against*	*contradict, contravene, contrarevolution*
Anti-	*against, opposed to, resisting*	*antiwar, antithesis, antisymmetrical, antibacterial, antitoxin, antiseptic*
Mis-	*badly, ill*	*miscast, misanthrope*
Mal-	*bad*	*malcontent, malpractice*
De-	*draw or remove from, down*	*devalue*

Table 6.34: (Cont.)

Prefix	English Meaning	Examples
Dis-	*apart, away from, not any*	*disport, dissect, dissonant, disbelief, discharge, disable*
Ab-	*from, away*	*abduct, abjured, aberrant*
A-, an-	*not, without, lack of*	*atom, anomaly, anoxia, amnesia, anemia, analgesic, anesthesia*
Ex-	*out of*	*export, exorbitant, ex officio, expatria*
Ob-, o-, op-	*against, in the way*	*obovate*

Table 6.35: Latin Spatial and Positional Prefixes

Prefix	English Meaning	Examples
Super-, sur-	*above, over, excessively*	*supervise, supernatural, superrealistic*
Extra-	*beyond, outside, outer, is addition to*	*extracurricular*
Dia-	*through, between*	*diameter, diaphanous*
Sub-, sup-	*under, below, near somewhat*	*submarine, suffix, subway*
Hypo-	*under*	*hypodermic, hypothesis*
De-	*down, off, from, undoing*	*descend, dementia, defrost, depart*
Epi-	*upon, beside, on*	*epidermis, epitome*

Table 6.35: (Cont.)

Prefix	English Meaning	Examples
Trans-	*across, beyond, through*	*transport, transformation*
Ambi-	*about, on all sides*	*ambition, ambidextrous, ambivalent*
Circum-	*around*	*circumnavigate, circumvent, circumpacific*
Peri-	*around*	*perimeter, perigee, pericardium*
Inter-	*between*	*intercom, internecine, interstate, intercourse, interatrial*
Ante-	*in front of, forward*	*anteroom*
Ultra-	*beyond*	*ultrasound*
Endo-	*within*	*endoscopy*
Supra-	*above, upon, upper*	*suprapelvic*
Se-	*apart, free from*	*secretion, segregation*
Retro-	*backward, behind, back*	*retrogress*
Re-	*again, back, backward*	*refraction, return, reappear, refrigerate, regurgitate*
Co-, com-, con-	*together with*	*cohesion, concert, coauthor, congress, coequal, conjugate, conspiracy, continue, cognate, colloquy, convene, concise, conduit, contravene*
A-, ad-	*at, to, toward, near*	*adhesion, admix, allegation, addict, ablate, abstract, annotate*

Table 6.35: (Cont.)

PREFIX	ENGLISH MEANING	EXAMPLES
In-, im-, ir-, il-	*in, into, on, against, onto*	*immerse, illicit, intangible, imbibe, irruption*
Infra-	*below, lower*	*infrared, infrastructure*
Intro-	*within, inward*	*introspect*
Intra-	*within, inside*	*intravenous*
Juxta-	*beside, near to*	*juxtaposition*
Pro-, pur-	*in front of, front part of, in favor of*	*prodigal, purport, protract, prospect*
Per-, pel-	*throughout, thoroughly, excessively, complete*	*pervade*
Sym-, syn-	*with, together*	*syntax, synthesis, syndicate*

Table 6.36: LATIN TEMPORAL PREFIXES

PREFIX	ENGLISH MEANING	EXAMPLES
Post-	*after, behind*	*postmortem, postpone, postnatal*
Pre-	*before*	*prepared, presume, prehistory, premeditated*

Table 6.37: Latin Suffixes

Suffix	Connotation	Examples
-or	*the agent which does*	*vector (vehera, vectum = carry)*
-io, -ura	*condition resulting from action*	*ligatio (ligare = tie), tensio (tendere = stretch)*
-men, -mentum	*action resulting from action*	*sedimentum (sedera = sit), tegmen (tegere = stand)*
-culum, -ulum	*instrument, means of action*	*curriculum, mandibula (mandare = chew)*
-ra, -itas, -or	*quality, condition, state*	*dementia (demens = out of one's senses), magnitudo (magnus = large)*
-arium, -orium, -ium	*place, apparatus, area for work*	*aquarium, sanatorium, auditorium, planetarium*
-ueas, -culus, -ellus, -illus	*small (diminutive)*	*calculus, globulus, capsula, ventriculus, flagellum, lamella*
-ilib, -bilis	*ability, capability, capacity*	*facilis, fissilis, fragilis*
-idus	*in a state of*	*fluidus, rigidus*
-eus, -ius	*made of, like, having the nature of*	*virilis, senilis*
-anus, -enus	*place, origin, belonging to*	*americanus, africanus*
-osus, -lentus	*full of*	*fibrosus, corpulentous*
-ate	*put into action, perform*	*elaborate, radiate*

Table 6.37: (Cont.)

SUFFIX	CONNOTATION	EXAMPLES
-sc	*to begin an action*	*calescence, deliquesce*
-ad	*in the direction of, toward*	*ectad, retrad*
-ase	*enzyme*	*oxidase*
-ose	*carbohydrate*	*dextrose*
-ate	*salt*	*sulphate*
-ite	*salt*	*sulphite*
-id (ide)	*a compound of two elements*	*ferric oxide*
-ol	*alcohol or phenol*	*glycerol*
-duc	*to lead, bring*	*deduce, produce, reduce*
-dict	*to say*	*contradict, edict, predict, dictate*
-gress	*to walk*	*progress, digress, transgress*
-ject	*to throw*	*eject, inject, project, reject, subject*
-pel	*to drive*	*compel, impel, repel*
-pend	*to hang*	*append, depend*
-port	*to carry*	*deport, export, import, report, support*
-tract	*to pull, drag, draw*	*attract, contract, extract, retract*
-vert	*to turn*	*convert, divert, invert, revert*

Table 6.38: LATIN TERMS, PHRASES AND ABBREVIATIONS

Mathematics is an ancient discipline, and consequently it has picked up a good deal of terminology over the years that is not commonly used in ordinary discourse. Phrases and terms from Latin make up a large part of this terminology, and reading mathematical texts - especially more advanced ones – is made easier if one is equipped with knowledge of these terms in advance.

We review below the Latin terms most commonly used in mathematics, and follow with a more extensive list of such terms and phrases as one may run into more rarely or in other contexts.

Note that when Latin or other non-English words are used in writing, they should be italicized except where they are abbreviated as single letters.

ab initio	*From the beginning.*
accessit	*Honorable mention.*
a.d.	*See anno domini.*
ad cautelem	*for safety sake: to be on the safe side.*
ad hoc	*For the immediate purpose. An ad hoc committee is appointed for some specific purpose.*
ad hominem	*"To the man." An argument is ad hominem when it attacks the opponent personally rather than addressing his arguments.*
ad infinitum	*Literally, "to infinity," indicates that a process or operation is to be carried out endlessly.*
ad nauseam	*Something continues ad nauseam when it goes on so long you become sick of it.*
ad rem	*to the point.*
a fortiori	*"With stronger reason." If every multiple of two is even, then a fortiori every multiple of four is even.*
alias	*also known as (at another time).*

alibi	*elsewhere.*
alma mater	*Your alma mater is the university or college which granted your degree.*
alumnus/alumna	*An alum, as it is sometimes shortly said, is a former member/student of a university or college. (The "us" ending is masculine, the "a" ending feminine. The plurals are alumni and alumnae, respectively.)*
agenda	*things that have to be done.*
a.m ante meridiem	*(before noon)*
anno domini	*"In the year of Our Lord." Indicates that a date is given in the Western or Gregorian calendar, in which years are counted roughly from the birth of Christ.*
annus mirabilis	*a wonderful year.*
a posteriori	*"From effect to cause." A thing is known a posteriori if it is known from evidence or empirical reasoning.*
a priori	*A thing is known a priori if it is evident by logic alone from what is already known.*
arguendo	*for the sake of argument; hypothetically.*
ars perdita	*a lost art.*
B.A.	*Baccalaureus Artium*
bona fide	*"In good faith." One's bona fides are documents or testimonials establishing one's credentials or honesty.*
B.Sc.	*Baccalaureus Scientiae*
carpe diem	*"Seize the day." A motto which says to live in the now, and/or to not waste time or opportunity.*
ceteris paribus	*other things being equal.*

cf. *See confer.*

circa *Approximately. Used with dates, abbreviated as c or ca.*

confer *"Compare." Usually abbreviated cf. and often used in footnotes, this indicates that one should compare the present passage or statement with the one referred to.*

cum laude *"With praise." Used on degree certificates to indicate exceptional academic standing.*

de facto *"In reality." Used to indicate that, whatever may be believed or legislated, the reality is as indicated here.*

de jure *"In law." Contrast to de facto.*

dixi *That settles it. Literally, "I have spoken."*

e.g. *See exempli gratia.*

emeritus *(feminine: emerita) Indicates someone who has served out his or her time and retired honorably.*

ergo *Therefore.*

erratum/errata *Literally, "error/errors," this term in fact refers to the corrections included in a paper or book after it is published to correct minor errors in the text.*

et al. *Abbreviation of et alia, meaning "and others." Used to indicate an unstated list of contributing authors following the main one, for instance.*

et cetera *And so forth.*

exempli gratia *"For example." Usually abbreviated to "e.g." and often confused with "i.e."*

ex paritate *by analogy.*

ex post facto *"From what is done afterward."*

fac simile *make a copy (fax).*

grand nimis *too great.*

hic of nunc *here and now.*

ibid. *See ibidem.*

ibidem *"In the same place." Used in footnotes to indicate that the reference is the same as the preceding one(s).*

id est *Literally, "that is." Usually abbreviated "i.e." and often confused with "e.g." The decision whether to use "i.e.," or "e.g." should be based on whether "that is" or "for example" is what is wanted in the sentence.*

i.e. *See id est.*

in extenso *in full.*

in globo *globally.*

in re *"In regards to." Often used to head formal correspondence. When only re is written, it should be translated as "regarding", or "concerning".*

inter alia *Among other things.*

in toto *Entirely.*

in vacuo *Literally, "in a vacuum." Should be taken to mean "in the absence of other conditions or influences."*

in vitro *taking place in the laboratory test tube, artificial.*

in vivo *taking place in a living organism.*

ipso facto *Literally, "by that very fact."*

lapsus calami *a slip of the pen.*

LL.D. *Legum Doctor*

loc. cit loco citato	*in the place quoted*
M.A.	*Magister Artium*
magna cum laude	*With great praise. See cum laude.*
magnum opus	*a masterpiece*
M.D.	*Medicinae Doctor*
modus operandi	*Manner or method of work characterizing a particular person's professional habits.*
M.Sc.	*Magister Scientiae*
mutatis mutandis	*With necessary changes. "Mutatis mutandis, this proof applies in more general cases."*
n.b.	*See nota bene*
non sequitur	*"Not following." Used to indicate a statement or conclusion that does not follow from what has gone before.*
nota bene	*Literally, "note well." Usually abbreviated "n.b.", this is a way of saying, "take note of this."*
para avis	*a rare bird.*
passim	*everywhere.*
per impossibile	*"As is impossible." Qualifies a proposition that cannot be true.*
per se	*"In and of itself." Example: "This argument does not force the conclusion per se, but with this added premise the result would follow."*
Ph.D.	*Philosophia Doctor*
plusve minusve	*more or less.*

p.m. post meridiem	*after noon*
post hoc, ergo propter hoc	*"After, therefore because of." A common fallacy in reasoning, in which causality is ascribed to preceding conditions which were in fact irrelevant to the supposed effect.*
post scriptum	*"Written after." Indicates an afterword or footnote to a main text, and is often used in written correspondence (where it is abbreviated p.s.).*
prima facie	*"On its face." Indicates that a conclusion is indicated (but not necessarily proved) from the appearance of things.*
pro forma	*"For form's sake." E.g., "It was a pro forma interview – the decision to hire her had already been made."*
Q.E.D	*See quod erat demonstrandum.*
Q.E.F	*See quod erat faciendum.*
qua	*"In the capacity of."*
quod erat demonstrandum	*"That which was to have been proved." Traditionally placed at the end of proofs, the QED is now usually indicated by a small square.*
quod erat faciendum	*"That which was to have been shown." Abbreviated QEF, it was traditionally used to mark the end of a solution or calculation. It is rarely used now.*
quod vide	*Usually abbreviated q.v., this is a scholarly way of directing the reader to a reference.*
q.v.	*See quod vide.*
R.I.P. requiescat in pace	*Rest In Peace*

semper idem *always the same.*

sic! *wrong, but that was how the original speaker said or wrote it.*

sine qua non *"That without which nothing." Indicates an essential element or condition.*

stet (editorial) *let it stand.*

summa cum laude *With greatest praise. See cum laude.*

summum opus *his greatest work.*

s.v. sub voce *under that word*

tabula rasa *"Blank Slate." Often refers to a person who has not yet formed prejudices or preconceptions on a given matter.*

temp tempore *in the period of*

terra incognita *unknown land.*

vade mecum *"go with me" – a favorite book, guidebook.*

verbatim *Word-for-word. Indicates a precise transmission of a phrase, discussion, or text.*

via *by way of.*

videlicet *Usually abbreviated viz., this is translated as "namely" or "in other words", or "that", or "to say".*

viz. *See videlicet.*

vs. versus *against*

Table 6.39: SELECTED LATIN EPIGRAMS, MAXIMS AND PROVERBS

While the majority of the entries in this Table date back to the classical times, some came into use in the Middle Ages.

One principle of selection was the inherent wisdom reflected in the thought. Another was the insight into a civilization implicit in a thought; Indeed, one is struck by the universality of people's problems throughout the ages and the satisfying solutions afforded, despite the often contradictory nature of these solutions. Finally, some of the entries have an inherent poetic beauty (and sadness!) of their own, reminiscent of the greatness of Biblical passages and lines from Shakespeare. To this category belong most of these epigrams.

1. ***In necessariis unitas, in dubiis libertas, in omnibus caritas.***

(In necessary things – unity;
in dubious things – liberty;
in all things – charity)

*[**st. Augustine**]*

2. ***Sic transit gloria mundi***

(at times of deep human shortcomings, we realize the transitory nature of grand projects)

*[**Thomas à Kempis**]*

3. ***Ave Imperator: morituri te salutamus!***

(Hail emperor: we who are about to die salute you!)

*[quoted by **Suetonius**]*

4. ***Fame in magnis, dignitas autem in homilitate habitat.***

(Fame lives in great things, but dignity lives in humility)

*[**Cicero**]*

5. ***Ingenita levitas et erudita vanitas.***

(Frivolity is inborn, conceit acquired by education)

*[**Cicero**]*

6. ***Longumeque illud tempus cum non ero magis me movet quam hoc exiguum, quod mihi tamen longum videtur.***

(That long time to come when I shall not exist has more effect on me than this short present time, which nevertheless seems endless)

*[**Cicero**]*

7. ***Utinam tam facile vera invenire possem quam falsa convincere.***

(I only wish I could discover the truth as easily as I can expose falsehood)

*[**Cicero**]*

8. ***Universus hic mundus sit una civitas communis deorum atque hominum existimanda.***

(We must conceive of this whole universe as one commonwealth of which both gods and men are members)

*[**Cicero**]*

9. ***Philosophic est ars vitae***

(Philosophy is the art of life)

*[**Cicero**]*

10. ***Prospeae res et in plebem ac vilia ingenia deveniunt; at calamitates terroresque mortalium sub iugum mittere proprium magni vivi est.***

(Success comes to the common man, and even to commonplace ability; but to triumph over the calamities and terrors of mortal life is the part of a great man only)

*[**Seneca**]*

11. ***Magna servitus est magna fortuna.***

(A great fortune is a great slavery)

*[**Seneca**]*

12. ***Labor optimos citat.***

(Toil summons the best men)

*[**Seneca**]*

13. ***Quicquid bene dictum est ab ullo, meum est.***

(Whatever is well said by anyone is mine)

*[**Seneca**]*

14. ***Non potest constare liberates. Hanc si magno aestimas, omnia parvo aestimanda sunt.***

(Liberty cannot be gained for nothing. If you set a high value on liberty, you must set a low value on everything else)

*[**Seneca**]*

15. ***Dum spiro spero.***

(While I breath I hope)

*[**Cicero**]*

16. ***Ipsa scienta potestas est.***

(Knowledge is power)

*[**Francis Bacon**]*

17. ***Rex regnat sed non gubernat.***

(The king rules but does not govern)

18. ***Aut viam inveniam aut faciam.***

(I'll either find a way or make one)

19. ***Dum vivimus, vivamus.***

(While we live, let us really live)

[motto of the Epicureans]

20. ***Poeta nascitur, orator fit***.

(A poet is born but an orator is manufactured)

*[**Publius Annius Florus**]*

21. ***Primum viveri diende philosophari***

(Live before you philosophize)

22. ***Forsan et hae olim meminisse iuvabit.***

(Some day perhaps it will be pleasing to remember these things too)

*[**Vergil**]*

23. ***Vivitur ingenio, caetera mortis erunt.***

(Intelligence lives on, the rest eventually dies)

*[**Vesalius**]*

24. ***Omnia mea mecum porto.***

(all that is mine, I carry with me)

25. ***Homo vitae comodatus non donatus.***

(Man is loaned to life not given to it)

*[**Publilius Syrus**]*

26. ***Vivere disce, cogita mori***

(Learn to live; Remember death)

27. ***Vox audita perit, littera scripta manet***

(The spoken word vanishes but the written letter remains)

28. ***Verba docent, exempla trahunt****.*

(Words instruct, examples lead)

29. ***Sutor, ne ultra crepidam.***

(Shoemaker, do not go further than the shoes)

[quoted by ***Pliny****]*

30. ***Jacio en aleam, librumque scribo, seu praesentibus, seu posteris legendum, nihil interest; expectet ille suum lectoremper annos centum; si Deus ipse perannorum sena millia contemplatorem praestolatus est.***

(The die is cast, and I am writing this book – whether to be read by my contemporaries or by posterity matters not. Let it wait its reader for a hundred years, if God Himself has been ready for his contemplator for six thousand years)

*[****Johannes Kepler****, 1619]*

31. ***Ubi materia, ibi geometria.***
(Where there is matter, there is geometry)

*[**Johannes Kepler**]*

32. ***Non nobis solum nati sumus.***
(We are not born just for our own sake)

[Cicero]

33. ***Qui non profit – deficit.***
(He who does not gain – loses)

34. ***Non multa sed multum***
(Not many but much)

35. ***Vulturum non capit muscam.***
(The eagle does not catch flies)

36. ***Qui desiderat pacem, praeparet bellum**.*
(Whoever desires peace, should prepare for war)

*[**Varro**]*

37. ***Amare et sapere vix deo conceditur.***

(Even a god can scarcely love and be wise at the same time)

*[**Publilius Syrus**]*

38. ***Qui usque tandem abutere, Catalina, patienta nostra?***

(How long, then, Cataline, will you abuse our patience?)

*[**Cicero**]*

39. ***Vita sine libris mors est.***

(Life without books is death)

40. ***Nullus est instar domus.***

(There is no place like home)

41. ***Esse quam videri***

(To be rather than to seem)

42. ***Docent omnia.***

(Everything teaches)

*[**Seneca**]*

43. ***In silvan ne ligna feras***

(Don't carry logs into the forest)

44. ***In medio tutissimus ibis***
(You will go safest in the middle)

*[**Ovid**]*

45. ***Homo sum, humani nil a me alienum puto.***
(I am a man; nothing human is alien to me)

*[**Terrence**]*

46. ***Per angusta ad augusta.***
(Through difficulties to great things)

47. ***Quieta non movere***
(Don't move quiet things; "Let sleeping dogs lie")

48. ***Ubi bene, ibi Patria***
(Where you feel good, there is your home)

*[**Pacuvius**]*

49. ***Non semper ea sunt qua videntur.***
(Things are not always what they seem to be)

50. ***Nulla dies sine linea.***
(Not a day without something done)

*[**Pliny the Elder**]*

51. ***Facilis descensus averni***
(Easy is the road to evil)

*[**Vergil**]*

52. ***Non est vivere sed valere vita est.***
(Life is not just to live but to be of value)

53. ***Per ardua ad astra***
(Through perils to the stars)

54. ***Docendo discimus***
(By teaching we learn)

*[**Seneca**]*

55. ***Oderint dum metuant***
(Let them hate, as long as they fear)

*[**Caligula**]*

56. ***Non omnis moriar***
(I shall not completely die)

*[**Horace**]*

Table 6.40: Latin Root Groups

Indication	Root	English meaning	Examples
Physical action			
	Vocare	*call*	*vocation, revoke, vociferous*
	Tractere	*draw, pull*	*tract, tractor, attractive, protracted*
	Spectere	*see, look*	*spectator, spectacles, aspect, spectrum*
	Cedere	*go, move, yield*	*cede, precede, acede, precedent*
	Loqui	*talk, speak*	*loquacious, oblique, soliloquy*
	Vertere	*turn*	*divert, revert, aversion*
	Capere	*take, seize*	*capture, captivate, captious*
	Facere	*do, make*	*factory, fraction, factitious*
	Tenere	*hold*	*tenant, tenacious, tentative*
	Sentire	*feel*	*sensation, sensual*
	Currere	*run, happen*	*current, cursory, precursor*
	Ponere	*put, place*	*component, posit*
	Cadore	*fall*	*cadence, decadence, cascade*
	Ferre	*carry, bear*	*ferry, inference, defer, tolerate*
	Jecere	*throw*	*reject, eject, projectile, deject*

Table 6.40: (Cont.)

Indication	Root	English meaning	Examples
	Pellere	*push, drive*	*dispel, repel, expel, propeller, impel*
	Caedere	*cut*	*excise, cadaver, incision, incidence, decadence*
	Saltare	*leap, jump*	*insult, salient, result, resilience*
	Mittere	*send*	*emit, commit*
	Ducere	*lead*	*duct, conductor, educe, induce*
	Venere	*come*	*convened, advent*
	Tangere	*touch*	*tangent, tangible, contiguous*
Activities of the intellect			
	Anima	*mind, breath, soul, equanimity*	*animation, magnanimous*
	Cognoscere	*to know*	*cognizant, recognize*
	Quaerere	*ask, seek*	*inquest, query, acquisitive, exquisite*
	Judicare	*judge*	*judicial, judicious, dictum, adjudge*
	Credere	*believe*	*credo, credit, credulity, credentials*

Table 6.40: (Cont.)

INDICATION	ROOT	ENGLISH MEANING	EXAMPLES
	Scribere	*write*	*prescribe, scribe, ascribe, circumscribe*
	Nomen	*name*	*nomenclature, nominal, nominative*
	Dicere	*talk*	*diction, dictum*
	Monere	*warn*	*monitor, admonish*
	Videre	*see*	*video, envision*
	Philos	*love*	*philosophy, bibliophile*
	Cantere	*sing*	*cantor*
	Psych	*mind*	*psychology*
	Gnos	*know*	*agnostic*
	Graph	*write*	*graphology, autograph, graphite*
	Lagos	*the study of*	*logic*
Growth, Change and movement			
	Genus	*birth, kind*	*genesis, eugenics, engender*
	Natura	*nature*	*supernatural, cognate, innate*
	Novus	*new*	*novel, novice, innovation*
	Crescere	*grow, increase*	*crescendo*

Table 6.40: (Cont.)

Indication	Root	English meaning	Examples
	Stare	*stand*	*distance, static*
	Gradus	*step*	*graduate, gradation, gradient*
	Movere	*move*	*promote, mobile, motile*
	Jungere	*join*	*junction, junta, enjoined, conjugal*
	Frangere	*break*	*fracture, fragile*
	Torquere	*twist*	*torsion, torque*
	Plicere	*fold*	*complicated, complicity, imply*
	Fluere	*flow*	*effluent, affluent, confluence, fluent*
	Mortis	*dead*	*mortal, morbid*

Table 6.41: Dictionary of Greco-Latin Origins

[(*) used in modern Hebrew]

Root	Core	Meaning	English Words
acer (acris)	***acri***	*sharp, bitter*	*acrimony, acrid*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
acere	***eas, jac, ject, jet***	*to lie, to throw*	*jet, trajectory, jettisoned, adjacent, object, conjecture, reject, objective, injection, ejecta, subject, adjective, abject, deject, eject, project, projectile, projection, projector*
aedes	***edif***	*a building, temple*	*edifice*
aequus		*level, equal*	*equality, equal, equilibrium,equivalence*
aevum	***ev***	*age*	*primeval, medieval, longevity, eternity*
ager	***gri***	*field*	*peregrine, pilgrim, agriculture*
agere	***act(actum)***	*do, act, drive, set in motion, function*	*action, active, actual, actuate, enact, redact, reactor, transact, actuary, exact, agent, reaction*
agere	***gate***		*castigate, divagate, expurgate, fumigate, fustigate, navigate, objurgate, variegate, levigate, litigate, mitigate*
agere	***gen***		*cogent, agendum, agency, exigent, intransigent*
agere	***gi, gu***		*ambiguity, agility, agitate, cogitate, assay, putge*
agora *(*)*	***gor***	*marketplace*	*category, agora, agoraphobia*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
akademia	***academ***	*park near Athens*	*academy, academic*
akros	***acro***	*a point, topmost*	*acrobat, acronym, acrostic, acme*
albus	***alb***	*white*	*albatross, albumen, album, auburn, albino, albedo, albion*
Alere	***al***	*to feed, nourish, grow*	*alimentary, coalition, alimony, coalesces, albuminus*
altus	***alt***	*high*	*altitude, altimeter, alt (voice)*
allos	***alle***	*other*	*allegory, parallel, allergy*
amare	***ama, amo***	*to love*	*amateur, amour, enamored*
amicus	***ami***	*friend*	*amity, amigo, amiable, (enemy)*
amplus	***ampl***	*large, wide*	*amplifier, amplitude, ample*
angelos	***angel***	*messenger*	*angel, evangelist, archangel*
angulus	***ang***	*angle*	*triangle*
animus	***anim***	*mind, soul*	*animal, unanimous, equanimity, anima, magnanimous, animated, animosity, animalcule*
annus	***anni, annu, enni***	*year*	*annual, perennial, anniversary, centennial, millenium, annuity, annals*
anthos	***ant***	*a flower*	*anthology*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
antiquus	***antiqu***	*ancient*	*antique, antiquity, antiquarian*
aristos		*best*	*aristocrat*
arkhein	***arch***	*rule, begin, ancient, first*	*monarch, oligarchy, anarchy, archetype, archaic, patriarch, matriarch, hierarchy, archbishop, architect, archipelago, archives, archaeology, archduke*
arctos		*a bear*	*arctic ocean*
astron	***astro, aster***	*star*	*astronomy, astrology, disaster, asterix, asteroid, astronaut*
atavus	***atav***	*ancestor*	*atavism*
athlon	***athl***	*a prize*	*athlete, decathlon, pentathlon, triathlon, decathlon*
audire	***audi***	*to hear*	*audible, auditory*
aura		*air, breath*	*aura*
ballein	***blem, bol***	*to throw*	*emblem, symbol, hyperbole, parabola, parabolic, hyperbolic, matebolism, anabolism, problem, diabolic*
bassus *(*)*	***base, bass***	*low, short*	*bass, bassoon, base, abase, debase*
battuere	***bat***	*to beat*	*battery, battalion, battle, battlement, combat, batten, abate, debate*
bellum	***bell***	*war*	*rebel (revel), bellicose, belligerent, duell, casus belli*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
bene, bonus	***bene, beni***	*good, well*	*benediction, benevolent, benign, beneficent, benefactor, benefit*
bibere	***bib***	*to drink*	*imbibe, bibulous, beverage, beer*
biblion	***bibli, biblio***	*book, papyrus, scroll*	*bible (from Byblos, a phoenician city from which papyrus was exported), bibliophile, bibliography, bibliotheca*
bombos	***bomb***	*a booming, humming, sound*	*bomb, bombard, bombastic*
brev	***brev***	*short*	*abbreviation*
calx (calcis)	***cal***	*limestone, pebbles*	*calculus*
capere	***cip, cup***		*anticipate, emancipate, incipient, municipal, participate, percipient, participle, principal, recipient, recipes, principle, occupy, occupation, preoccupy, prince, cable*
carus	***cheri, chari***	*dear*	*cherish, charity, caress*
cavus	***cave***	*hollow*	*care, concave*
censere	***cen***	*asses, judge*	*censuse, recension, censor*
chronos	***chron***	*time*	*chronism, chronicle, chronology, chronological, chronometer*
civis	***cit, civ***	*citizen*	*civilization, civil, city, citizen, civic*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
citare	***cit***	*arouse, motivate*	*excite, excitation*
clamere	***claim, clam***	*to call out, shout*	*claim, exclaim, declaim, reclaim, proclaim, acclaim, disclaim, clamant, clamor, acclamation*
claudere	***clos, clude, clus***	*to shut, close*	*conclude, disclose, exclude, enclose, foreclose, include, occlude, preclude, recluse, secluded, closet, cloister, claustrophobia*
colare	***col***	*strain, filter*	*percolate, coulee*
coquere	***coc, cot***	*cook*	*precocious, apricot, concoct, ricotta*
corpus		*body*	*corpse, corporal, corpulent, corpuscle*
caballus	***caval***	*horse*	*cavalry, cavalier, cavalcade*
cadere	***cad, cas, cid***	*to fall*	*accident, cadaver, causality, case, cascade, cadenza, cadence, caducity, escheat, chute, parachute, coincide, decadence, incidence, occasion, occident, recidivation*
calere	***cal, cha***	*to be warm, hot*	*caldron, chafing, nonchalant, chafe, chaff, calory*
calvi	***cal, chal***	*to deceive*	*calumny, challenge*
canere	***cant, chant***	*to sing*	*chant, cantata, cantor, canto, recant, accent, enchanting, incentive*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
capere	***cap, capt***	*to seize, contain, lay hold of*	*capable, capacity, capsule, capacitate, capstan, captive, caption, capture*
capere	***cas, chas***		*case, cassette, purchase, encase, cash, cashier, caskets, cask, chase*
captere	***ceit, ceiv***		*conceive, receive, deceive, perceive, conceit*
captere	***cept***		*accept, concept, deception, exceptional, inception, intercept, percept, precept, susceptible, receptionist*
cosmos		*order, arrangement*	*cosmetics, cosmonaut, cosmopolitan*
credere	***cred***	*to believe*	*credit, credible, credulous, credulity, accredit, discredit, creed*
dare	***dat, dit***	*give*	*date, dative, datum, data, addendum, add, edit, edition, tradition, perdition, traitor, render, rent, rendezvous*
decem, deka	***dec***	*ten*	*December, decade, decimate, decathlon*
derma	***derm***	*skin*	*epiderm, dermatology*
dexter	***dext***	*on the right side*	*dexterity, ambidextrous, dexter, dextrose*
dicare	***dic, dict***	*to tell, to say*	*abdicate, benedict, addict*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
dicere			*dedication, dictate, dictator, contradict, dictum, edict, dictionary, indicator, indices, indicative, indict, interdict, judicially, judiciary, predicament, predicate, prejudice, vindicate, valediction, digit, predict*
didonai	***dos, dot***	*to give*	*antidote, dose, overdose*
digitus	***digit***	*finger, toe*	*digit, digital*
discere	***disci***	*to learn*	*discipline, disciple*
docere	***doc***	*to teach*	*doctor, doctorate, document, documentary, indoctrinate, docile, docent*
dokein	***dox***	*to seem. think*	*dogma, dogmatism, orthodox, heterodox, paradox*
dolere	***dol***	*to feel pain, grieve, suffer*	*condole, condolence, doleful, dolorous, indolent, dolor*
dominus	***domin***	*master*	*dominate, dominant, dominion, domineering, dame, madam, madonna, don (*), domino, donna, anno domini, dominique, domingo, condominium*
donare	***don***	*to give*	*pardon, condone, donation, donor*
dromein	***drome***	*to run*	*aerodrome, hippodrome, syndrome, palindrome*
dunamis	***dyna***	*force, power, strength*	*dynamite, dynamo, dynamic, dynasty*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
duo	***dou, dub***	*two*	*doubt, dubious, doublet*
	duo, dup		*duplicate, duplicity, duplex, doubloon, dual, duel, duet*
durare	***dur***	*hard, continue to exist*	*endure, durable, duration, duress, indurate, obdurate, predurate*
edere	***edi***	*to est*	*edible*
ego	***ego***	*I*	*egoist, egotist, egocentric, egomania*
eidos	***ido***	*form*	*idol, idolatrous, kaleidoscope*
eikon *(*)*	***icon***	*image, likeness*	*icon, iconoclast*
ergon	***erg, urg***	*work*	*erg, ergomania, allergy, energy, liturgy, surgeon, metallurgy*
errare	***err***	*to wander*	*err, error, erratum, aberrant, erroneous, aberration*
esse	***sen, sent***	*to be, exist*	*absence, absentia, essence, entity, essential, quintessential, present, presentation, represent*
eus	***eu***	*good, well*	*euphoria, euphony, eulogies, eugenics, euthenics, euthanasia, Eunice, Eugene*
facree	***fac, face***	*to make, do*	*fact, factory, faction, factor, manufacture, benefactor, factotum*
facere	***feat, feas***		*feature, defeat, defeasance, feasible*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
facere	***fect***		*affect, effect, affectionate, confection, disaffect, defect, defective, infect, infection, perfect, prefect*
facere	***feit, fit***		*comfit, profit, benefit, counterfeit, forfeit, surfeit*
facere	***fic, fice***		*aficionado, artifice, beneficiary, deficit, deficient, difficult, efficient, edifice, magnificent, munificent, office, officer*
figere	***fix***	*to fasten, pierce, fix*	*affix, fix, fixture, fixative, fixer, prefix, suffix, infix, transfix, fixation, idee fix, crucifix*
fluere	***flu***	*to flow*	*affluent, confluent, effluent, effluvia, fluorescent, fluoride, flush, influence, influenza, superflous, superfluid, fluid, flux, fluctuate, fluent*
forma	***form***	*shape*	*conform, reform, conformist, formalist, formation, deform, deformity, form, formal, format, informal, formality, formula, formation, informer, information, disinform, transform, transformer, uniform, proforma*
fortis	***fort***	*strong*	*comfort, comfortable, comforter, effort, force, enforce, reinforce, fort, fortress, fortifications, fortitude, pianoforte, fortissimo*
fortuna	***fort***	*luck*	*fortune, fortuitous, misfortune, unfortunate*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
fallere	***fal, fail, faul***	*to deceive*	*false, falsification, falsetto, fallacy, fallible, fail, failure, fault, default, faulty, faucet*
felix	***feli***	*happy, lucky*	*Felix, felicity, felicitation, felicitous*
femina	***femin***	*woman*	*feminine, femininity, femme fatale*
fendere	***fen***	*to ward off*	*offence, offensive, offend, defence, fencing, fender*
ferre, ferere	***fer***	*to bear, bring, carry, produce*	*aquifer, confer, conference, defer, different, difference, differ, indifference, differential, differentiate, fertile, infer, offer, prefer, proliferate, refer, referendum, suffer, referee, insufferable, sufferance, transfer, vociferate, vociferous, fertile, ferry*
fervere	***ferv***	*to boil, be hot*	*fervent, fervid, fervor, fervency, ferment, effervescence*
fidere	***fid***	*to trust*	*confide, fiance, affiance, confidence, defiance, diffident, fidelity, fiduciary, affidavit, infidel, perfidy, perfidious*
gamos	***gam, gamy***	*marriage*	*bigamy, monogamy, polygamy, allogamy, autogamy, cryptogram*
gemin		*twin, paired, born together*	*gemini*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
genus, (generis)	***gen***	*race, kind, class, origin*	*general, generalize, generation, engender, generate, generator, degenerate, regenerate, generous, generic, genre, genus, gender, misce generation, gene*
gignoskein	***gnos***	*to know*	*gnosis, gnosticism, gnostics, agnosticism, agnostic, prognosis, diagnosis*
gladius		*sword*	*gladiator*
globus	***glob***	*ball, sphere*	*globule, hemoglobin, globus*
glossa	***gloss***	*tongue*	*glossary, gloss*
gnoscere	***cogn, gnor, noti***	*to get to know*	*cognitive, cognizance, recognition, precognitive, cognoscenti, ignorance, ignoramus, ignore, incognito, notice, notion, noticeable, reconnoiter, notify, reconnaissance*
gradi *(*)*	***grad, gress***	*to go, step, walk*	*egress, congress, digress, ingress, progress, regress, retrogress, transgress, aggression, grade, graduate, gradual, gradation, gradient, degrade, retrograde*
grandum	***gran***	*grain, seed*	*pomegranate, granary, granules, granite, granulate*
gratus	***grac, grat, gree***	*beloved, dear, pleasing*	*grace, disgrace, gracious, grateful, gratitude, gratify, gratuitous, gratuity, ingratiate, ingrate, agree, agreement, disagree, disagreeable, gratis*

Table 6.41: (Cont.)

ROOT	CORE	MEANING	ENGLISH WORDS
gravis	***grav, griev***	*heavy, serious, weighty*	*gravity, gravitation, grave, aggravate, grievance, grieve, aggrieve*
gregare, grex (gregis)	***greg***	*to herd, flock*	*gregarious, congregation, segregate, aggregate, egregious, gregarious, segregation*
gustus	***gust***	*a tasting*	*disgust, gusto, gustation*
haerere (haesum)	***her, hes***	*to stick, to cling*	*adhesion, coherent*
helios	***sol, helio***	*sun*	*Helliocentric, solar, parasol, insolation, solstice, helium*
hepta	***hepta***	*seven*	*heptagon, heptathlon, heptahedron*
heres	***heir, heri***	*on heir*	*heir, inherit, heritage, inheritance, disinherit*
hodos	***od***	*way, journey*	*period, periodic, episode, method, synod, exodus*
horrere	***horr***	*to bristle, dread, shudder*	*abhor, horrendous, horrific, horrible, horror, horrid*
hostis	***hostil***	*enemy, stranger*	*hostility, hostile*
humor (humidis)	***hum***	*liquid, moisture, damp, moist*	*humidity, humoral*
identi	***identi***	*same*	*identical, identity*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
insula	***sola, sula***	*island*	*peninsula, insulation, isolate, insular, isolation*
intimus		*to be close, private, within*	*intimate*
ire (itum)	***it***	*to go*	*ambition, circuit, initiate, circuitous, initial, initiative, concomitant, exit, coition, itinerary, itinerate, obit, obituary, transitory, seditious, transit, transitive*
jugum	***jug***	*yoke*	*jugular, conjugate, subjugate*
jungere	***join, junct***	*to join*	*adjoin, join, adjunct, conjoin, conjunction, enjoin, injunction, disjoin, joint, junction, juncture, rejoin, subjoin, subjunctive, conjunctive*
jurare	***jur, jus, juris***	*to swear, law*	*adjure, conjure, injure, objurgate, perjury, juridical, jurisdiction, jurist, jury, just, justify, justice*
juvenis	***juv***	*young*	*juvenile, rejuvenation*
kaiein	***cau***	*to burn*	*caustic, holocaust*
kamara	***cam, cham***	*vault*	*chamber, camarade, camera, chambermaid, chamberlain*
kosmos	***cosm***	*order, the world, the universe*	*cosmopolitan, cosmic, cosmology. cosmonaut, cosmetic, microcosm*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
krinein	***crit***	*distinguish, separate*	*criterion, critic, criticism, critical, hypocrite, criticize*
kyklos	***cycl***	*cycle, wheel*	*cycle, recycle, cyclotron, encyclopedia, cyclone, cycloid, cyclop, cyclic, recycle, motorcycle, tricycle, unicycle*
labor	***labor***	*labor, exertion, toil*	*labor, laborious, laboratory, collaboration, elaborate, laborious*
lacuna		*small pit, gap*	*lacuna*
latus (dilatatus, lateris)	***lat***	*carried, borne, wide, broad, expanded, side*	*ablate, collate, collateral, correlate, dilate, elate, oblate, relate, relation, relative, superlative, translate, lateral, latitude, legislator, dilatation, bilateral*
laudare	***laud***	*to praise*	*laud, laudatory, cum laude*
legare	***lega, lege, legi***	*to bind, choose, send*	*alleged, allegation, allegedly, college, collegial, legate, delegation, legation, legacy, relegate*
legein	***lexi, log, logue***	*to gather, speak*	*lexicon, lexicographer, dyslexia, catalog, dialog, monolog, decalogue, analog, prolog, epilog, travelog*
levare	***lev***	*to lighten, lift, raise*	*alleviate, elevate, elevator, elevation, lever, leverage, levant, levity, levitation, levy, levigate, relevant irrelevant*
lex	***leg***	*law*	*legal, legitimate, illegal, illegitimate, legislative, legislature, privilege*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
liber	***lib***	*free*	*liberty, liberate, liberation, liberal,*
limen (liminis)		*treshold*	*elimination, subliminal*
lingua	***lingu***	*tongue, language*	*linguist, bilingual, linguistics, lingua franc.*
liquere	***liqu***	*flow, be liquid*	*liquid, liquidity, liquidate, liquify, liqueur, liquor*
lithos	***lith***	*stone*	*monolith, megalith, paleolithic, neolithic, mesolithic, lithography, lithology, lithium*
littera	***liter***	*letter of the alphabet*	*illiteracy, literally, literary, literature, alliteration, obliterate, literati*
locare (locus)	***loca***	*to place, place*	*local, locate, allocate, collocate, dislocate, location, loci, localize*
logos	***log, logy***	*word*	*logic, logical, logistics, sylogism, logo, analogous, apologetic, apologia, eulogy, biology, tautology, anthology, etymology, seismology*
longus	***long***	*long*	*longevity, longitude, prolong, elongation, oblong, longueur, longanimity*
loqui	***locu, loqu***	*to speak*	*colloquium, grandiloquence, loquacious, ventriloquist, soliloquy, somniloquy, eloquence, interlocutor, locution, obloquy*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
lucere	***lumen, luc, lum***	*to shine, light*	*luminous, lucid, elucidate, illumination, illuminati, lucubration, luminosity, lucent, luminiferous*
luna, selena	***luna***	*the moon*	*lunatic, lunacy, lunar, lunation*
magister	***mas, mis***	*a master*	*master, mastermind, masterpiece, mister, mistress, magistrate, masterwork*
magnus	***magn, maj***	*great, large, big*	*majority, majesty, major, mayor, magnate, Magna Carta, magnifico, magniloquent, magnitude, magnificent, majestic, magnanimous*
malleus	***mall***	*hammer*	*mall, mallet, pall mall*
malus	***mal, mali, male***	*bad, evil*	*malignant, malevolent, malediction*
mandare	***mand, mend***	*to order*	*commandment, commander, commandant, commando, commandeer, commend, recommend, commendation, demand, demanding, mandate, mandatory, remand*
manere	***man***	*to remain*	*mansion, manor, immanent, permanent*
manus	***man***	*hand*	*manual, manage, manager, management, manure, maneuver, manner, mannerism, mannered, emancipate, manufacture, manifest, manifesto, manipulate, manicure*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
mappa		*napkin, cloth*	*map, napkin, apron*
margo (marginis)	***marg***	*edge, border*	*margin*
mater	***mat, metro***	*mother*	*maternal, matrimony, matron, maternity*
medius	***medi***	*middle*	*mediate, immediate*
meter			*matriarch, matrix, matriculate, alma mater, metropolis, metropolitan, metrodome, metro*
merx	***merc***	*goods, trade, traffic*	*gramercy, mercy, merci, mercenary, mercantile, merchant, commerce, mercury, mercurial, mercer*
mikros	***micro***	*small*	*microwave, microphone, microscope, micrometer, microbe, microcosmos, microorganism, microfilm*
mole	***mole***	*mass*	*molecule*
movere (motum)	***motio***	*to move*	*emotion, motion, locomotion, motile*
mutare (mutatum)	***mut***	*to change*	*mutation, commute, permute*
miscere	***misc, mix***	*to mix*	*miscellany, mix, mixture, intermix, admix, promiscuous, promiscuity, miscegenation, miscible*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
mittere	***mise, miss, mit, mitt***	*send*	*admit, admissible, admission, admittance, admittedly, commit, commitment, commissar, commissary, commission, compromise, demit, demise, dismiss, demit, emit, emission, emissary, intermittent, intermission, mission, missioner, missive, missile, omit, omission, permit, permissible, permission, permissive, premise, promise, remit, remiss, remittal, remission, unremitting, submit, submissive, submission, surmise, transmit*
misein	***misa, miso***	*to hate*	*misanthrope, misandry, misogynism, misogamist, misoneism*
nasci	***nat***	*to be born*	*cognate, nature, native, naturalization, naturally, nation, nationalist, natal, international*
naus *(*)*	***nau***	*ship*	*nausea, nautical, astronaut*
negativus	***nega, negl***	*no, negative*	*negate, neglect, negative, negligible*
nekros	***necro***	*corpse*	*necrology, necrophilia, necromancy*
nepos	***neptis***	*grandson, nephew, niece*	*nepotism, nephew*
nocere	***noka, noc, nox***	*to harm*	*innocent, innoxious, innocuous, noxious, obnoxious*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
nomen, nominare	***nom***	*name, to name*	*misnomer, nomenclature, nominate, nominator, denominator, ignominy, nominal, nominee, nom de plunc*
norm	***norm***	*measure, standard, pattern*	*normal, abnormal, normalization*
novus	***nov, novi***	*new*	*novel, novella, novelist, innovator, novelty, renovate, novice, novitiate, nova, nouveau*
nox	***noct***	*night*	*nocturnal, nocturne, noctilucent, equinoctial*
numerus	***numer***	*number*	*numerate, enumerate, numeral, numerical, numeration, innumerable, numerology, innumerate*
oculus	***oc***	*eye*	*pinochle, inoculate, monocle, oculist, ocular, binoculars*
odium	***no***	*hatred*	*annoy, noisome, annoying*
offici(um)	***fic***		*official, orifice, proficient, pontificate, sacrifice, significant, specific, suffice, sufficient, superficial, beatific, honorific, pacific, soporific, prolific*
oikos	***eco***	*house*	*ecology, economy, parochial, ecumenical, economize*
omnis	***omni***	*all, every*	*omnibus, omnipotent, omnivorous*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
onoma	***onum***	*a name*	*anonym, pseudonym, eponym, antonym, toponym, anonymous, heteronym, homonym, metonymy, onomatopoeia, patronym, synonym*
orbis	***orb***	*circle, wheel*	*orbit, orbital*
ordo	***ord***	*order*	*order, disorder, ordinary, ordained, ordination, ordinarily, ordinance, ordinate, coordinate, inordinate, subordinate, insubordination*
pais	***ped***	*child*	*pediatrics, pediatrician, pedodontics, encyclopedia, pedagogue, pedant, pedantry, pederasty, pedophile*
par	***par***	*equal, peer*	*compare, par, parity, disparity, disparage, comparative, au pair, parlay, peer, at par, comparable*
parere	***par***	*give birth, come in sight*	*parents, parental, parenthood, transparent, apparent, apparition, oviparous, viviparous*
paschein	***path***	*to suffer*	*pathetic, pathos, pathology, pathogen, apathy, antipathy, sociopath, psychopath, empathy, sympathy, telepathy, homeopathy, osteopathy, allopathy*
pater	***patri, patro***	*father*	*patronize, patronage, patriot, compatriot, expatriate, pater, patricians, patrimony, patriarch, patronymic, paternity, perpetrate*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
pax	***pac, peace***	*peace*	*peace, pacification, pacifist, pacific, pacifier*
peccare	***pecca***	*stable, sin*	*peccable, peccant, impeccable*
pecus	***pecu***	*cattle, properly*	*pecuniary, impecunious, peculate, peculiar, peculation, peculiarity*
pellere	***pel, pul***	*to drive*	*appeal, appellation, appellant, compel, compulsion, compulsive, compulsory, expel, expulsion, impel, impulse, impulsive, propel, propeller, propulsion, repel, repulse, repellant, repulsion, pelt, pulsate, pulse*
pendere	***pen, pend, pens, pond***	*weigh, hang, pay*	*ponder, perpend, preponderate, ponderous, append, appendage, appendix, appendant, penthouse, appendectomy, compensate, depend, dependable, dependence, independence, dispense, dispensable, dispensary, dispensation, expend, expense, expenditure, expensive, impend, impending, pendant, pendulant, pending, penchant, propensity, pendular, pendulum, perpendicular, pension, equiponderance, spend, suspend, suspense*
penis	***pen***	*tail*	*penicilin*
penna	***pen***	*feather*	*pen, pennant*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
pes	***ped***	*foot*	*impend, expedite, impediment, expedition, expedient, pedal, pedometer, pedicure, pedestal, centipede, millipede, pedigree, bipeds, quadruped, pedestrian*
pius	***pi***	*devoted*	*pieta, pious, piety, pity, pittance*
plebes, demos	***pleb, dem, demo***	*the people*	*demagogue, plebiscite, plebeian, endemic, epidemic, pandemic, democracy, democrats, demotic*
pleetere (plexum)	***plex***	*to plait, interweave*	*complex, plexus*
plenus		*full*	*plenty, plenary, replenish*
planetes		*a wanderer*	*planet*
polis	***poli***	*city*	*politics, political, polity, politburo, politic, apolitical, acropolis, metropolis, police, megalopolis, cosmopolitan*
porcus	***por***	*hog, pig*	*porcupine, porpoise, porcelain, pork*
portare	***port***	*to carry*	*sport, sportsmanship, porter, portmanteau, portfolio, portable, comportment, deportment, comport, import, export, importance, importunate, importune, port, portly, purport, report, rapport, support, transport*
potentis, potensi	***potent***	*having power*	*potent, potential*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
pungere	***pun, punc***	*to point, stab*	*punctuate, punctilious, punctuation, punctual, pungent, compunctions, expunge, pun*
quaerere	***quest, quir, quis***	*to seek*	*quest, inquire, questionable, question, inquest, inquiry, conquest, request, acquire, acquisition, acquirement, acquisitive, disquisition, exquisite, inquisitive, inquisition, requisite, prerequisite*
qualis	***qual***	*of what kind*	*quality, qualify, qualitative*
quantus	***qualt***	*how many, how much*	*quantify, quantity, quantum, quantitative*
qui, quam, quom		*who, how, when*	*quibble, quorum, quasi, quasar, quondam*
quid, quies	***qui***	*what, something, quiet, rest*	*quid, pro quo (something for something), acquiesce, acquiescence, acquit, quiet, quiescence, quit, quite, requiem, requite*
quot		*how many*	*quota, quotient, quotidian, quotation*
radere	***ras***	*to scrape*	*rascal, rash, abrasion, abrasive, eraser, razor*
radius	***radi***	*rod, spoke, ray, beam*	*irradiation, radial, radiology, radium*
radix	***radi***	*a root*	*radix, radish, eradicate, radical*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
regere, rectum	***rect***	*keep straight*	*erectile, rectum*
rete (retis)	***ret***	*net, network*	*retina*
rex	***reg, roy***	*king*	*royal, regal, royalist, royalties, Regina, regalia, regale, viceroy, regnant, regulus, regicide, regnum*
ridere	***rid, ris***	*to laugh*	*derision, ridicule, deride, derisive, ridiculous, risible*
rigor	***rig***	*stiffness*	*rigidity, rigorous*
rodere, rosum	***ro, ros***	*to gnaw*	*erosion, corrosion, erode, corrode, rodents*
rogare	***gat, gate***	*to ask*	*Roguery, abrogate, arrogance, derogate, interrogate, perogative, supererogation, surrogate, subrogate*
rota	***rot***	*wheel*	*rotator, rotunda*
ruber (rubris)	***rub***	*red*	*bilirubin, rubella*
rumpere	***rupt***	*to break*	*abrupt, bankrupt, corrupt, disrupt, interrupt, erupt, irrupt, rupture*
salire	***sal, sult, xult***	*to leap, spring forward*	*insult, desultory, exult, result, resilience, somersault, salient, salacious, salmon, saltant, saltation*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
salus	***salu***	*health*	*salute, salud, salutation, salutary, salubrious*
salvus	***sal***	*safety, health*	*salvage, salve, salvation, savior, save, salutary, salubrious*
sanguis	***sang***	*blood*	*sanguine, sangria*
sacer	***sac***	*sacred, holy*	*sacerdotal, sacred*
sapere	***sav***	*to taste, to be wise*	*savant,unsavy, savory, savor*
scala	***scal***	*ladder*	*escalade, escalator, upscale, escalate*
scandere	***scen, scend***	*climb, leap*	*ascend, descend, ascent, ascendance, descendant, condescend, transcend*
scire (scitum)	***sci***	*to know*	*science, conscience, unconscious, self-conscious, subconscious, plebiscite, omniscient, prescient, nescient, scientific, sciolism*
scribere	***scri***	*to write*	*scribe, manuscript, scripture, script, scribbler, postscript, inscription, inscribe, transcribe, descriptive, prescript, transcript, proscribe, circumscribe, ascribe, conscript, conscription, subscription, prescribe, describe, prescription, description, nondescript, indescribable*
secare (segmentum)	***seg, sect, sex***	*to cut, part, segment*	*segment, secant, bisect, dissect, sex, section*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
sedere	***sed, sess, sid***	*to sit, settle*	*preside, president, reside, resident, residence, residue, residuum, residual, assess, assiduous, sessions, assize, obsess, possess, dispossess, repossess, sedate, sedative, sediment, sedentary, subside, subsidiary, subsidy, dissident, superseded, insidious, sedulous*
semen	***semin***	*seed*	*seminary, seminal, insemination, semination, dissemination*
senex	***sen***	*old, old man*	*senator, senior, senile, senescent*
sequor	***secu, xeco, sequ, suit***	*to follow*	*consequence, consequential, consequently, inconsequent, obsequencious, sequel, subsequently, sequence, sequential, sequitur, nonsequitur, sequested, consecutive, execute, executive, persecute, prosecute, lawsuit, suitable, unsuited, pursuit, suite*
signum	***sign***	*a mark, seal, sign, indication*	*sign, signature, signal, signify, significance, signet, insignia, assign, designate, assignment, consign, design, designate, designer, ensign, resign, resignation*
similis	***simul, sembl, simil***	*like, at the same time, alike*	*assemble, assembly, assimilate, assimilation, similar, ensemble, resemble, semblance, simulacrum, verisimilar, similitude, simile, facsimile, simulation, simulcasts, simultaneous, dissimilar*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
sinister	***sin***	*on the left*	*sinistral*
skhizein	***schiz***	*to split*	*schizophrenic*
skopein	***scope***	*see, look at*	*scope, microscope, telescope, horoscope, periscope, kaleidoscope, spectroscope*
solus	***sol***	*alone*	*solo, solitude, solist, desolate, solifidian, sole, solipsism, soliloquy, solitaire, solitary*
solvere	***solu, solv***	*to loosen*	*absolute, resolute, solution, resolution, irresolute, unresolved, resolve, dissolve, soluble, insolvent, insoluble, dissolute, absolve, absolution*
sonare (sonatum)	***son, sound***	*to make a sound*	*sonnet, sonata, sound, sonorous, resound, resonant, resonance, assonance, consonant, consonant, dissonant, sonic, supersonic, transonic*
spargo	***sper***	*to scatter besprinkle*	*aspersions, asperse, disperse, sparse, intersperse, dispersion*
spondere	***spond, spons***	*to pledge*	*respond, respondent, correspond, correspondent, irresponsible, responsibility, responsive, sponsor, despondent*
stare (statum)	***stat, stet***	*to stand*	*status, substitution, obstetrics*
stinguere	***stinct, stingu***	*pierce, quench*	*instinct, distinct, distinction, distinctive, distinguish, indistinct, indistinguishable*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
struere (structum)	***stru, struct***	*construct, build, arrange*	*construct, construction, construe, misconstrue, destruction, indestructible, instruct, instrument, obstruct, structure, infrastructure, struma*
tacere	***tacit***	*to be silent*	*taciturn, tacit*
tangere (tactum)	***tact, tang, tag***	*to touch*	*contact, tangent, contagious*
tardus	***tard***	*slow*	*retard, retarded, tardy*
tekhne	***techn***	*art, craft, skill*	*technique, technician, technical, polytechnic, technicality, technology, technocracy*
tellus (telluris)	***tel***	*the earth*	*tellurian*
temnein	***tom, tomy***	*to cut*	*atom, epitome, dichotomy, anatomy, tome, vasectomy, mastectomy,*
temperare	***temper***	*to regulate, mix, moderate*	*temper, temperament, temperate, temperance, temperature tempera*
tempus (temporis)	***temp***	*time*	*temporary, temporal, tempo, tempestuous, extemporaneous, extempore, contemporary, contemporaneous*
terere	***tri, trit***	*rub, wear away*	*attrition, contrition, contrite, detriment, detrimental, tribulation, trite, triturate*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
terra	***terr, terra***	*earth, land*	*territory, terrain, terrace, territorial, mediterranean, subterranean, terrestrial, extraterrestrial, parterre, terriers*
theos	***the***	*god*	*theology, theocracy, atheist, pantheism, pantheon, apotheosis, Theo, Theodosius, Theodoric, Theodora, Theodore*
terrere	***terr***	*frighten*	*terror, terrible*
tithenai	***thes***	*put, place*	*thesaurus, thesis, antithesis, synthesis, hypothesis, parenthesis, prosthesis*
topos	***top***	*a place*	*utopia, dystopia, topography, toponymy, topic, topical*
torquere	***tor, tort***	*to twist, turn*	*torch, contort, torture, distort, extort, extortion, torment, tortious, torsion, torque, tortoise, tortes, retort, distortion*
torrere	***toast, torr***	*to parch*	*toast, torrential, torrid, torrent*
totus	***tot***	*all, entire*	*total, totalitarian*
trahere (tractum)	***tract***	*to draw*	*extract, tract, traction, contract*
tremere	***trem***	*to tremble, shake, quiver*	*tremor*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
tropos	***trop***	*a turn*	*trophies, tropics, tropical, entropy, trope*
tupos	***type***	*a blow, impression*	*type, archetype, phototype, typeset, stereotype, typecast, typewriter, stenotype, logotype*
turbane (turbatum)	***turb***	*to stirr, disturb*	*turbid, turbine*
ultra	***ult***	*beyond*	*ultimate, penultimate, ultra, ulterior, ultimatum*
umbra	***umbra***	*shadow*	*umbrage, umbra, penumbra, adumbra*
unda	***und***	*wave, billow*	*abound, abundance, inundation, redundant, surround, undulation*
uti	***uti***	*to use*	*utilize, use, useful, utile, utility, abuse, misuse, peruse, abusive*
vacare (vacuus)	***vac, voi****d*	*to be empty*	*vacant, avoid, vacous, vacuity, unavoidable, void, vacuum, vacate, devoid evacuate*
valere	***val***	*to be strong, be well*	*equivalent, valence, invalid, bivalent*
varius	***vari, vario***	*bent, changeable, crooked, diverse, manifold speckled, different*	*various, variety, vary, variance, variation, variant, variegated*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
vehere	***vec***	*to carry*	*vector, convect, vehicle, convex, convey, advect*
verbum	***verb***	*word*	*verbal, verbiage, verbose, verbatim, verb, adverb, proverb, proverbial*
venire (ventum)	***vent***	*to come*	*circumvent, intervention*
verei	***revere***	*to fear, feel awe*	*revere, reverence, reverent, irreverence*
vertere (versum)	***vert***	*to turn*	*version, vertebra, pervert, invert*
vertex (verticis)	***vert***	*summit, top*	*vertex, vertical, vertigo*
verus	***ver***	*true*	*veracity, veritable, veracious, verification, very, verify*
vestis	***vest***	*garment*	*vest, vesture, invest, investment, divest, transvestite, travesty, revest*
vestus	***vet***	*old, long-standing*	*veterinary, vet, veteran, veterate*
via	***via, vio, vium***	*way, road*	*trivia (trivium = three ways, crossroad), trivial, obviously, pervious, impervious, deviate, deviant, devious, previous, obviate, obvious, viaduct*
vicis	***vic***	*change, instead of*	*vicar, vicissitudes, vicarious, vice versa*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
videre (visum)		*to see*	*vision, television*
vigilare	***vig, veil***	*to watch*	*vigil, vigilant, vigilantes, surveillance, reveille*
villa	***villa***	*farmhouse*	*villa, village, villain*
vir	***vir***	*man, manliness, virtue, strength*	*virtue, virility, virtual, triumvirate, virtuoso, virtual*
virus	***virus***	*potent juice, poison*	*virulent, virus*
vita, vivere	***vit, via, viv***	*life, to live*	*vital, vitamin, viable, convivial, vivacious, revive, revitalize, revivify, revival, survive, victual, viviparous, vita, vitals, vitality, viva, vivid, vivacity, vivify, vivisection*
vitrum	***vit***	*glass*	*vitrina, vitriol, vitreous*
vocare	***vox, voc, vok***	*to call, voice*	*advocation, vocation, provoke, evoke, revoke, vociferous, provocative, evocative, convoke, invoke, invocation, revocation, irrevocable, advocate, equivocate, unequivocal*
volvere	***vol***	*to roll*	*volume, voluble, revolve, revolution*
vovare	***vor***	*to devour, oat*	*voracious, carnivorous, herbivorous, omnivorous, insectivorous*

Table 6.41: (Cont.)

Root	Core	Meaning	English Words
vovere	***vot, vow***	*to vow, wish, pledge*	*vote, votary, devotee, vow, votive, devote, devotion*
vulgare	***vulg***	*publish, make common*	*vulgate, vulgar, divulge*
xenos	***xeno***	*foreign, strange*	*xenophobe, xenon*
zelos	***zeal, jeal***	*ardor*	*jealous, jealousy, zeal, zealot, zealous*
zoion	***zoo***	*animal*	*zoology, zoo, zoolatry*

Quotations

Preface

The following collection of over 1000 aphorisms, maxims, sayings,truisms, dictums, adages, mottoes and anecdotes, was authored by some 430 persons during the past 2600 years or so, among them philosophers, scientists, scholars, sages, savants, statesmen artists, musicians, poets and lay people with a keen open mind, eye and heart.

My overall purpose in presenting this treasury of wit and wisdom has been to set down within the final volume of my Encyclopedia, an assembly of well–phrased thoughts and flashes of insight on scientific topics that I came across during a long course of reading and study.

It seems appropriate that this historical display of the development of science will conclude with man's critical self examination of his relation to nature and mankind.

Content

1. On Nature, Universe, and Man

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"What is man, that thou art mindful of him? And the son of man, that thou visitest him? For thou hast made him a little lower than God, and hast crowned him with glory and honor. Thou madest him to have dominion over the works of thy hands..."

Psalms **8**, 5–7 (ca 1000 BCE)

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"And now, rejoicing in the prosperous gales,
With beating heart Ulysses spread his sails:
Placed at the helm he state, and mark'd the skies,
Nor closed in sleep his ever-watchful eyes.
There view'd the Pleiads, and the Northern Team,
And great Orion more refulgent beam,
To which, around the axle of the sky,
The Bear, revolving, points his golden eye:
Who shines exalted on the etheral plain,
Nor baths his blazing forehead in the main.
Far on the left those radiant fires to keep
The nymph directed, as he sail'd the deep"

Homer, Odyssey **V**, 270–276 (ca 800 BCE)

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Nature does not hurry, yet everything is accomplished

Lao Tzu, 4th century BCE

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And the Lord answered Job out of the Whirlwind, and said,
"...
Where wast thou when I layed the foundation of the earth?... Who hath laid the measured thereof... Whereupon are the foundations thereof fastened? Or who laid the corner stone thereof?... Or who shut up the sea with doors, when it brake forth...? Hast thou commanded the morning...? Where is the way where light dwelleth?... Canst thou bind the sweet influences of Pleiades, or loose the bands of Orion? Canst thou bring forth Venus in his season? Or canst thou guide Arcturus with his sons?..."

(Job **38**), ca 600 BCE

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"What profit is there in my blood,
When I go down the pit?
Shall the dust praise thee?
Shall it declare thy truth?"

Psalms **30**, 10

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"It is the glory of God to conceal a thing; but the honor of king is to search out a matter."

Proverbs **25**, 2

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"He hate made everything beautiful in his time: also he hate set the world in their heart, so that no man can find out the work that God maketh from the beginning to the end."

Ecclesiastes, **3**:11

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"Opinions say hot and cold, but the reality is atoms and empty space."

"Everything existing in the Universe is the fruit of chance and necessity."

Democritos (ca 460–370 BCE)

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"Even God cannot change the past."

Agathon (448–400 BCE)

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"Do you think it a matter worthy of lamentation that when there is such a vast multitude of them (universes), we have not yet conquered one?"

Alexander the Great (356–323 BCE)
Wept when told by Anaxarchos of Abdera
that there exist an infinity of worlds.

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"Nature does nothing uselessly."

Aristotle (384–322 BCE)

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"The laws of nature are but the mathematical thoughts of God."

Euclid (ca 300 BCE)

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"The universe is not bounded in any direction. If it were, it would have a limit somewhere. But clearly a thing cannot have a limit unless there is something outside to limit it... It makes no odds in which part of it you make take your stand: whatever spot anyone may occupy, the universe stretches away from him just the same in all directions without limit."

Lucretius (ca 99–55 BCE)

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"Nothing which we can imagine about nature is incredible."

Pliny, the Elder (23–79 AD)

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"Nothing is difficult for nature, especially when she rushes to destroy herself. At the beginning of things she uses her strength sparingly and apportions herself out in imperceptible increases. For destruction she comes suddenly with all her violence. A long time is needed so that a child, once conceived, may come to be born. The tender infant is reared only with great toil. The frail body finally develops only with diligent nurture. But how with no effort it is all undone!
It takes an age to establish cities, an hour to destroy them. A forest grows for a long time, becomes ashes in a moment. Great safeguards may exist and all things may be flourishing, but quickly and suddenly they all fall apart."

"The day will come when diligent research over long periods will bring to light the mysteries of nature which now lie hidden. A single lifetime, even though totally devoted to the sky, would not be enough for the investigation of so vast a subject... And so this knowledge will be unfolded only through long successive ages. The day will yet come when our descendants will be amused that we did not know things that are so plain to them... Many discoveries are reserved for ages still to come, when memory of us will have been perished.
Our universe is a sorry little affair unless it has in it something for every age to investigate...
Nature does not reveal her mysteries once and for all."

"Any deviation by nature from the existing state of the Universe is enough for the destruction of mankind."

"What else is nature but God?"

Lucius Annaeus Seneca (4 BCE–65)

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"The rational soul goes about the whole universe and the void surrounding it and traces its plan and stretches forth into the infinitude of time."

Marcus Aurelius, "Meditations", **11**,1,2

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"Opened Rabbi Tanhuma and said: "He hate made everything beautiful in his time" – the world was created at the right time, for it was unworthy to be created earlier. Followed Rabbi Abbahu and said: Hence that the Lord was creating universes and destroying them, recreating and destroying them again, until he made the present universe."

(Genesis Rabba, **9** ca 450 CE)

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"Thou art wise, and from Thy wisdom
Thou hast set apart Thy appointed purpose,
Like a craftsman and an artist
To draw up the films of Being from Nothingness
As light is drawn that darteth from the eye:
Without bucket from the fountain of light
Thy workman drawn it up,
And without tool hath he wrought,
Hewing, graving, cleansing, refining.
Calling into the void – and it was cleft,
And unto existence – and its was urged,
And to the universe – and it was spread out."

Shlomo Ibn-Gabirol (1021–1070)

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Do there exist many worlds, or is there but a single world? This is one of the most noble and exalted questions in the study of nature.

Albertus Magnus (ca 1200–1280)

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If a man in the heavens ... could see the earth distinctly ... it would appear to him that the earth was moving in daily motion, just as to us on earth it seems as though the heavens are moving.
...One could then believe that the earth moves and not the heavens.

Nicole Oresme (ca 1325–1382)

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"Nature never breaks her own laws."

"The genius of man will never discover a more beautiful, a more economical, or a more direct one that nature's, since in her inventions nothing is wanting and nothing is superfluous."

"Oh God, you sell all things to men at the cost of their effort, but life is short for this kind of commerce!"

"Natura semper agit per vias brevissimas."

Leonardo da Vinci (1452–1519)

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"A dark flame issued forth from the innermost hiddenness, from the mystery of the infinite..."

The *Zohar* (13th century)

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“Pluritas non est pondera sine necessitate.” (‘It is in vain to do with more than can be done with less.’)

William of Ockham (1285–1349)

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“Let us permit nature to have her way. She understands her business better than we do.”

Michael de Montaigne (1533–1592)

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“The laws of nature ultimately are more important then nature.”

René Descartes (1596–1650)

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“Time and space are of the same essence.”

Yehudah Liwa of Prague (1512–1609)

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“Man can only conquer nature by obeying her.”

“Nature is often hidden, sometimes overcome, seldom extinguished.”

“God never wrought miracle to convince atheism, because his ordinary works convince it.”

Francis Bacon (1561–1626)

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“Ubi materia – ibi geometria.”

Johannes Kepler (1571–1630)

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“One touch of nature makes the whole world kin.”

*“Thou, Nature, art my goddess;
To thy law my services are bound.”*

William Shakespeare, *‘King Lear’, Act I, Scene II* (1564–1616)

“I could be bounded in a nut-shell, and count myself a king of infinite space.”

William Shakespeare, ‘*Hamlet*’ (1564–1616)

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“Nature hath no goal though she hath law.”

“A fountain breaks out in the wilderness, but that fountain cares not, whether any man may come to fetch water, or no; a fresh and fit gale blows upon the sea, but it cares not whether the mariners hoist sail or no; a rose blows in your garden, but it calls you not to smell it.”

John Donne (1572–1631), ca 1620

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“Beautiful are the things we see; More beautiful those we understand; But the most beautiful are those we do not understand.”

Niels Steno (1638–1685)

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“Nature has set no end before herself, and all final causes are nothing but human fictions.”

“Space and matter are not really different.”

“I believe that a triangle, if it could speak, would say that is eminently triangular, and a circle, that the divine nature is eminently circular; and thus would every one ascribe his attributes to God.”

Baruch Spinoza (1632–1677)

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"The ethernal silence of these infinite spaces frighten me."

"Man is equally incapable of seeing the nothingness from which he emerges and the infinity in which he is engulfed."

"The fabric of Nature has its center everywhere and its circumference nowhere."

Blaise Pascal (1623–1662)

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"Nature does nothing in vain, and more is in vain when less will serve; for Nature is pleased with simplicity, and affects not the pomp of superfluous causes."

Isaac Newton (1642–1727)

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"... I should think that anyone who considered it more reasonable for the whole universe to move in order to let the Earth remain fixed would be more irrational than one who should climb to the top of your cupola just to get a view of the city and its environs, and then demand that the whole countryside should revolve around him so that he would not have to take the trouble to turn his head."

"I did not feel obliged to believe that the same God who has endowed us with senses, reason and intellect has intended us to forego their use."

Galileo Galilei (1654–1642)

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"Accuse not Nature, she hath done her part; Do thou but thine."

John Milton, 'Paradise Lost' (1608–1674), 1667

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"Prior to the creation of the universe time did not exist. Time started at the moment of creation."

Jonathan Eibshitz (1690–1764)

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It seems not very likely, that a most Wise Agent should have made such vast bodies, as the sun and the fixed stars ... only or chiefly to illuminate a little globe.

Robert Boyle (1627–1691), 1688

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"Men argue, Nature acts."

Voltaire (1694–1778)

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"All nature is but art unknown to thee;
All chance, direction which thou canst not see;
All discord, harmony not understood;
All partial evil, universal good;
And, spite of pride, in erring reason's spite,
One truth is clear, Whatever IS, is RIGHT."

Alexander Pope (1688–1744)

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"The enormous and the minute are interchangeable manifestations of the eternal."

"To see a World in Grain of Sand
And a Heaven in a Wild Flower,
Hold Infinity in the palm of your hand,
And Eternity in an hour."

William Blake (1757–1827)

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"Give me matter, and I will construct a world out of it."

Immanuel Kant (1724–1804)

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“Nature has neither kernel nor shell; she is everything at once.”

“Nature goes her own way, and all that to us seems an exception is really according to order.”

“Nature! We are surrounded by her and locked in her clasp: powerless to leave her, and powerless to come closer to her. Without asking us or warning us she takes us up into the whirl of her dance, and hurries on with us until we are weary and fall from her arms.”

“Nature alone knows what she wants.”

“Every moment Nature starts on the longest journey, and every moment she reaches her goal.”

“The difficulty in nature is to see the law where it is concealed from us, and not to be misled by phenomena that contradict our senses. For in nature there is much that contradict our senses and nevertheless true. That the sun stands still, that he does not rise and set, but that the earth performs a diurnal revolution with incredible swiftness, contradicts to senses as much as anything; but yet no well-informed person doubts that this is the case.”

“I love him who yearns for the impossible.”

Johann Wolfgang von Goethe, ‘*Faust II*’ (1749–1832)

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“Space is the stature of God.”

Joseph Joubert (1754–1824)

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"After your death you will be what you were before your birth."

Arthur Schopenhauer (1788–1860)

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"Deep into the darkness peering, long I stood
there, wondering, fearing,
Doubting, dreaming dreams no mortal ever
dared to dream before."

Edgar Allan Poe (1809–1849)

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"When Man thinks, it is Nature which is thinking itself."

Arthur Schopenhauer (1788–1860)

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"Nature is visible thought."

"So we keep asking, over and over,
Until a handful of earth
Stops our mouth –
But is that an answer?"

"Like a great poet, Nature knows how to produce the greatest effects with the most limited means."

Heinrich Heine (1797–1856)

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"Nature is not embarrassed by difficulties of analysis."

Augustin Fresnel (1788–1827)

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"We are a sign that is not read."

Friedrich Hölderlin (1770–1843)

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"God hides in the minute details of Nature but is manifested in its grand design."

Alphonse de Lamartine (1790–1869)

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"I believe in the incomprehensibility of God."

Honore de Balzac (1799–1850)

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"I would rather believe that God did not exist than believe that He was indifferent."

George Sand (1804–1876)

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"The decisive events of the world take place in the intellect."

Henri Frédéric Amiel (1821–1881)

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"The chess-board is the world; the pieces are the phenomena of the universe; the rules of the game are what we call the laws of Nature. The player on the other side is hidden from us. We know that his play is always fair, just, and patient. But also we know, to our cost, that he never overlooks a mistake, or makes the smallest allowance for ignorance."

T.H. Huxley (1825–1895)

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"Nature teaches more than she preaches. There are no sermons in stones. It is easier to get a spark out of a stone that a moral."

"Nature is not benevolent; Nature is just, gives pound for pound, measure for measure, makes no exceptions, never tempers her decrees with mercy, or winks at any infringement of her laws."

John Burroughs (1837–1921)

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"When we try to pick out anything by itself, we find it is tied to everything else in the universe."

John Muir (1838–1914)

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"The God whom science recognizes must be a God of universal laws exclusively, a God who does a wholesale, not a retail business. He cannot accommodate his processes to the convenience of individuals."

William James (1842–1910)

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"Nature, in her indifference, makes no distinction between good and evil."

"Chance is the pseudonym of God when he did not want to sign."

Anatole France (1844–1924)

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"The true mystery of the world is the visible, not the invisible."

Oscar Wilde (1854–1900)

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"Repetition is the only form of permanence that nature can achieve."

George Santayana (1863–1952)

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"To believe *in* God *is to* desire *His existence, and what is more, to* act *as though He existed."*

Unamuno, y Jugo de Miguel (1864–1936)

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"The bee intentionally seeks for a method of economizing wax."

D'Arcy Wentworth Thompson (1860–1948)

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A man said to the universe:
"Sir, I exist".
"However", replied the universe,
"The fact has not created in me
A sense of obligations".

Stephen Crane (1871–1900)

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"The history of thought may be summed up in these words: it is absurd by what it seeks, great by what it finds."

"God made everything out of nothing. But the nothingness shows through."

Paul Valéry (1871–1945)

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"The Universe is full of magical things patiently waiting for our wits to get sharper."

"God, why did you make the evidence for your existence so insufficient."

Bertrand Arthur William Russell (1872–1970)

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"The true return to nature is the definitive return to the elements – death."

André Gide (1869–1951)

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"Seek simplicity, and distrust it."

"The aim of science is to seek the simplest explanation of complex facts."

Alfred North Whitehead (1861–1947)

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" Say something to us we can learn
By heart and when alone repeat.

Say something! And it says "I burn".
But say with great degree of heat.

Talk Fahrenheit, talk centigrade.
Use language we can comprehend.
Tell us what elements you blend.
It gives us strongly little aid,
But does tell something in the end."

Robert Lee Frost (1874–1963)

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“Rerum natura nullam nobis dedit cognitionem finitum.” (‘Nature has given to us no knowledge of the end of things.’)

Winston Churchill (1874–1965)

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“In nature there are neither rewards nor punishments – there are consequences.”

Robert G. Ingersoll (1839–1899)

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“Who speaks of victory, survival is all.”

“What will you do, When I die?
When I, your pitcher, broken lie?
When I, your drink, go stale or dry?
I am your garb, the trade you ply,
You lose your meaning, losing me.”

Rainer Maria Rilke (1875–1925)

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“The universe is not hostile, nor yet is it friendly. It is simply indifferent.”

John Haynes Holmes (1879–1964)

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"Everything should be made as simple as possible, but not simpler."

"The eternal mystery of the world is its comprehensibility. The fact that it is comprehensible is a miracle."

"The most incomprehensible thing about the universe is that it is comprehensible."

"Nature hides her secrets because of her essential loftiness, but not by means of ruse." ("Die Natur verbirgt ihr Geheimnis durch die Erhabenheit ihres wesens, aber nicht durch List.")

"Die Natur hat es nicht angelegen sein lassen, uns die Auffindung ihrer Gesetze bequem zu machen."

"Nature did not deem it her business to make the identification of her laws comfortable for us."

"Out yonder there is a huge world, which exists independent of us human being and which stands before us like a great, eternal riddle, at least partially accessible to our inspection and thinking. The contemplation of this world beckons like a liberation."

"God is subtle but he is not malicious."

"What I'm really interested in is whether God could have made the world in a different way; that is, whether the necessity of logical simplicity leaves any freedom at all."

"For the rest of my life I will reflect on what light is!"

Albert Einstein (1879–1955)

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"The God within creates the God in nature."

Arthur Stanley Eddington (1882–1944)

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"Today there is a wide measure of agreement . . . that the stream of knowledge is heading towards a non-mechanical reality; the universe begins to look more like a great thought than a great machine."

"The history of physical science in the twentieth century is one of progressive emancipation from the purely human angle of vision."

"The Universe can be pictured, although still very imperfectly and inadequately, as consisting of pure thought, the thought of what for want of a wider word, we must describe as a mathematical thinker."

James Jeans (1877–1946)

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— " 'Never will you draw the water out of the depths of this well'.
What water? What well?
— 'Who is asking?'
Silence.
— 'What silence?' "

"Man's fundamental weakness lies by no means in the fact that he cannot achieve victory, but in the fact that he cannot exploit his victory."

Franz Kafka (1883–1924)

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"I respect the idea of God too much to hold it responsible for a world as absurd as this one is."

George Duhamel (1884–1966)

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"Many people find that modern science is far removed from God. I find, on the contrary, that it is much more difficult today for the knowing person to approach God from history, from the spiritual side of the world, and from morals; for there we encounter the sufferings and evil in the world which it is difficult to bring into harmony with an all-merciful and all-mighty God. In this domain we have evidently not yet succeeded in raising the veil with which our human nature covers the essence of things. But in our knowledge of physical nature we have penetrated so far that we can obtain a vision of the flawless harmony which is in conformity with sublime reason."

Hermann Weyl (1885–1955)

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"To believe in God means to see that life has a meaning."

Ludwig Witgenstein (1889–1951)

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"The universe is not only stranger than we imagine; it is stranger than we can imagine."

John Burdon Sanderson Haldane (1892–1964)

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"The universe was dictated but not signed."

Christopher Morley (1890–1957)

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"Everything you've learned in school as "obvious" becomes less and less obvious as you begin to study the universe. For example, there are no solids in the universe. There's not even a suggestion of a solid. There are no absolute continuums. There are no surfaces. There are no straight lines."

"God is a verb."

"Humanity is acquiring all the right technology for all the wrong reasons."

"Nature is trying very hard to make us succeed, but nature does not depend on us. We are not the only experiment."

Richard Buckminster Fuller (1895–1983)

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"Nature always has the last word."

John Stewart Collis (1900–1984)

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"Atoms are not things.... When we get down to the atomic level, the objective world in space in time no longer exists, and the mathematical symbols of theoretical physics refer merely to possibilities, not to facts."

Werner Heisenberg (1901–1976)

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"The notion of structure is comprised of three key ideas: the idea of wholeness, *the idea of* transformation, *and the idea of self-regulation."*

Jean Piaget (1896–1980)

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"The Lord is a weak left-hander, but he still appears to be left-right symmetric when he expresses himself strongly."

Wolfgang Pauli (1900–1976)

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"By getting to smaller and smaller units, we do not come to fundamental units, or indivisible units, but we do *come to a point where division has no meaning."*

Werner Heisenberg (1901–1976)

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"The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth, space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality."

Herman Minkowski (1864–1909), 1908

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"Human destiny is bound to remain a gamble, because at some unpredictable time and in some unforeseeable manner nature will strike back."

Rene Dubos (1901–1982)

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"I am not interested in proofs, only what nature does."

Paul Dirac (1902–1984)

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"Laws of nature could not exist without principles of invariance."

Eugene Wigner (1902–1995)

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"We must expect the ice that retreated some 10,000 years ago to come back again."

George Gamow (1904–1968)

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"The universe is like a safe to which there is a combination. But the combination is locked up in the safe."

Peter de Vries (1910–1993)

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"Matter has reached the point of beginning to know itself . . . [man is] a star's way of knowing about stars."

"It would be a poor thing to be an atom in a universe without physicists, and physicists are made of atoms. A physicist is an atom's way of knowing about atoms."

George Wald (1906–1997)

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"In man's brain the impressions from outside are not merely registered; they produce concepts and ideas. They are the imprint of the external world upon the human brain. Therefore, it is not surprising that, after a long period of searching and erring, some of the concepts and ideas in human thinking should have come gradually closer to the fundamental laws of the world, that some of our thinking should reveal the true structure of atoms and the true movements of the stars. Nature, in the form of man, begins to recognize itself."

Victor Frederick Weisskopf (1908–2002)

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"Biology occupies a position among the sciences at once marginal and central. Marginal because — the living world constituting but a tiny and very "special" part of the universe — it does not seem likely that the study of living beings will ever uncover general laws applicable outside the biosphere. But if the ultimate aim of the whole science is indeed, as I believe, to clarify man's relationship to the universe, then biology must be accorded a central position..."

"Man at last knows that he is alone in the unfeeling immensity of the universe ... Neither his destiny nor his duty have been written down."

Jacques Monod (1910–1976)

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"Cosmologists are often in error, but never in doubt."

Yakov Borisovich Zeldovich (1914–1987)

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"We could have lived in a Universe with different laws in every province, but we do not. We might have lived in a Universe in which nothing could be understood by a few simple laws, in which Nature was complex beyond our abilities to understand, in which the laws that apply on earth are invalid on a distant quasar. But the evidence proves otherwise; Luckily for us we live in a Universe in which much can be "reduced" to a small number of comparatively simple laws of Nature. Otherwise we might have lacked the intellectual capacity to comprehend the world."

Carl Sagan (1934–1996)

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"Basic laws are simple only in their first approximation."

Martin Gardner (1914–)

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"My suspicion is that the universe is not only queerer than we suppose, but queerer than we can suppose."

J.B.S. Haldane (1892–1964), 1927

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"Nature is not economical of structures — only of principles."

Abdus Salam (1926–1996)

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"If you ask what is needed to work out the full consequence of the laws of physics, the answer is:

— Nothing less than the whole universe. —

It is not too much of a guess to say that is just what the universe is.

This explains the problem that has puzzled theologians, philosophers and scientists alike: "Why is there a universe at all?" The theologian, with his belief in an all powerful God, wanders why God did not simply perceive the universe. Why bother actually to have it?

The answer is that the universe is the simplest way of perceiving it."

Fred Hoyle (1915–2001), 1994

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"Thirty seconds after the explosion came, first the air blast pressing hard against people and things, to be followed almost immediately by the strong, sustained awesome roar which warned of doomsday and made us feel that we puny things were blasphemous to dare tamper with the forces heretofore reserved to the Almighty."

Thomas Farrell (1944–), official report
on the first atom bomb test,
Alamogordo, New Mexico, July 16, 1945

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"The is a rhythm and a pattern between the phenomena of nature which is not apparent to the eye, but only to the eye of analysis; and it is these rhythms and patterns which we call Physical Laws."

"The burden of (this) lecture is just to emphasize the fact that it is impossible to explain honestly the beauties of the laws of nature in a way that people can feel, without their having some deep understanding of mathematics."

"I think that it is much more likely that the reports of flying saucers are the results of the known irrational characteristics of terrestrial intelligence than of the unknown rational efforts of extra-terrestrial intelligence."

"Physicists like to think that all you have to do is say, these are the conditions, now what happens next?"

"One does not, by knowing all the physical laws as we know them today, immediately obtain an understanding of anything much."

Richard Feynman (1918–1988)

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'The very scale of the universe – more than a hundred billion galaxies, each containing more than a hundred billion stars – speaks to us of the inconsequentiality of human events in the cosmic context. We see a universe simultaneously very beautiful and very violent. We see a universe that does not exclude a traditional Western or Eastern god, but that does not require one either."

"The universe forces those who lie in it to understand it."

Carl Sagan (1934–1996), 1979

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"Man is a complex thing: he makes desert bloom and lakes die."

Gil Stern

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"The best (scientific) data we have are exactly what I would have predicted had I nothing to go on but the five books of Moses, the Psalms, the Bible as a whole. ... What we have ... is an amazing amount of order; and when we see order, in our experience it normally reflects purpose."

Arno Penzias (1933–)

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"The ability to reduce everything to simple fundamental laws does not imply the ability to start from those laws and reconstruct the universe."

Philip W. Anderson (1923–)

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"There is no reason that the universe should be designed for our convenience."

John D. Barrow (1952–)

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"Every snowflake is an avalanche pleads not guilty."

Stanislaw Lec (1909–1966)

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"Atoms of our body, were once part of stars."

"As astronomers you can't say anything except that here is a miracle. ... Can you go the other way, back outside the barrier and finally find the answer to the question why is there something rather than nothing? No, you cannot, not within science."

Allan Sandage (1926–)

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"Why is it that the world is structured in such a way that we can know something without knowing everything? If we could not understand limited parts of the universe without understanding the whole, science would be a hopeless enterprise."

"If science demonstrated an infinitely old *universe, science could disprove a Creator, because an infinitely old universe would never have been created! On the other hand, in a universe of* infinite extent*, anything that is possible must happen somewhere by pure chance. Obviously,* we *will find ourselves just where that fantastic happening has occurred."*

"As the universe is slowly choked to death by its own entropy, will God die too?
The alternative — gravitational collapse to a singularity, resulting in the total obliteration of the physical universe — seems even less promising.
Only a universe of the cyclic or steady-state varieties, would appear to offer scope for a natural God to be both infinite and eternal."

"A universe that came from nothing in the big bang will disappear into nothing in the big crunch, its glorious few zillion years of existence not even a memory."

Paul Davies (1946–)

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“If you see a formula in the Physical Review that extends over a quarter of a page, forget it. It’s wrong. Nature isn’t that complicated.”

Matthias Berndt (1918–1980)

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“No purported inconsistency with the General Theory’s predictions has ever stood the test of time. No logical inconsistency in its foundations has ever been detected. No acceptable alternative has ever been put forward of comparative simplicity and scope.”

John Archibald Wheeler (1911–2008)

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“Gods are born and die, but the atom endures.”

“Walking in space, man has never looked more puny or more significant.”

Alexander Chase

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“As we look out into the Universe and identify the many accidents of physics and astronomy that have worked together to our benefit, it almost seems as if the Universe must in some sense have known that we were coming.”

“If it should turn out that the whole of physical reality can be described by a finite set of equations, I would be disappointed. I would feel that the Creator had been uncharacteristically lacking in imagination.”

Freeman Dyson (1923–)

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"I do not pretend to have all the answers. But the questions themselves are worth talking about."

"There is no reason to assume that the universe has the slightest interest in intelligence – or even in life. Both may be random accidental by-products of its operations like the beautiful patterns on a butterfly's wings. The insect would fly just as well without them."

"Sometimes I think we're alone in the universe, and sometimes I think we're not. In either case, the idea is quite staggering."

Arthur C. Clarke (1917–2008)

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"The world began without man, and it will end without him."

Claude Lévi-Strauss (1908–)

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"Einstein's postulate that the laws of nature should appear the same to all freely moving observers was the foundation of the theory of relativity, so called because it implies that only relative motion is important."

"Why does the universe go to all the bother of existing?"

"We may now be near the end of the search for the ultimate laws of nature."

Stephen Hawking (1942–)

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"Astronomers discover God!"

Headlines in a well-known periodical (1982)

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"The Strong Anthropic Principle
Because there appear to exist such a large number of remarkable and apparently disconnected 'coincidences' which conspire to allow life to be possible in the universe, the universe must *give rise to observers at some stage in its history (i.e. the universe* must *be such as to admit the creation of observers within it at some stage)."*

"The Weak Anthropic Principle
The observed values of all physical and cosmological quantities take on values restricted *by the requirement that there exist sites where carbon-based life can evolve and by the requirement that the universe be old enough for it to have already done so (i.e. what we* can *expect to observe must be* restricted *by the conditions necessary for our presence as observers)."*

"The universe must be such as to admit the creation of observers within it."

Brandon Carter (1942–)

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"Has it a clock? Or is it a clock?"

Colin S. Pittendrigh (1918–1996), 1957

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"The improbable is bound to happen one day."

(Anon)

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"The familiar idea of a god who is omniscient (someone who knows everything) does not immediately ring alarm bells in our brains; it is plausible that such a being could exist. Yet, when it is probed more closely one can show that omniscience of this sort creates a logical paradox and must, by the standards of human reason, therefore be judged impossible or be qualified in some way. To see this consider this test statement:

THIS STATEMENT IS NOT KNOWN TO BE TRUE BY ANYONE.

Now consider the plight of our hypothetical Omniscient Being ('Big O'). Suppose first that this statement is true and Big O does not know it. Then Big O would not be omniscient. So, instead, suppose our statement is false. This means that someone must know the statement to be true; hence it must be true. So regardless of whether we assume at the outset that this statement is true or false, we are forced to conclude that it must be true! And therefore, since the statement is true, nobody (including Big O) can know that it is true. This shows that there must always be true statements that no being can know to be true. Hence there cannot be an Omniscient Being who knows all truths. Nor, by the same argument, could we or our future successors, ever attain such a state of omniscience. All that can be known is all that can be known, not all that is true."

John D. Barrow (1952–)

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"Why is there something rather than nothing at all?"

(Anon)

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"It is not so much that you are in the universe, as that universe is in you."

Meher Baba (1894–1969)

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"We can never tell whether the hand of God was at work in the moment of creation. ... In the searing heat of the first moment, all the evidence needed for a scientific study of the cause of the great explosion was melted down and destroyed."

Robert Jastrow (1925–2008)

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"It is the most persistent and greatest adventure in human history, this search to understand the universe*, how it works and where it comes from. It is difficult to imagine that a handful of residents of a small planet circling an insignificant star in a small galaxy have as their aim a complete understanding of the entire* universe*, a small speck of creation truly believing it is capable of* comprehending *the whole."*

Murray Gell-Mann (1929–)

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"The more the universe seems comprehensible, the more it seems pointless."

"Maybe nature is fundamentally ugly, chaotic and complicated. But if it's like that, then I want out."

Steven Weinberg (1933–)

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"Laws of science (physical laws), are at best approximation of the truth; they are inaccurate. Laws of Nature are some other laws (statements, principles), doubtless more complex (which are literally true), which govern the natural phenomena of the world. These are factual truths, not logical ones. They are true for every time and every place in the universe, are universal or statistical claims."

(Anon)

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"The whole of science is nothing more than a refinement of everyday thinking."

(Anon)

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"Only daring speculation can lead us further, and not accumulation of facts."

(Anon)

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"Scientific knowledge is the most reliable and useful knowledge that human being possess."

(Anon)

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"Nature does not care for the survival of individuals, only survival of species."

(Anon)

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"The bluebird carries the sky on his back."

Henry David Thoreau (1817–1862)

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"In order to see birds it is necessary to become a part of the silence."

Robert Lynd (1879–1949)

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"A bird thinks nothing of its flying or it would fall."

Leslie Sahler (1952–)

2. The Quantum World

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"Without mysticism man can achieve nothing great."

André Gide (1869–1951)

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"We have seen the truth, and the truth makes no sense."

Gilbert Keith Chesterton (1874–1936)

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"Anyone who is not shocked by Quantum Theory has not understood it."

"There is no quantum world. There is only an abstract quantum physical description. It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature."

Niels Bohr (1885–1962)

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"The electron is not as simple as it looks."

Lawrence Bragg (1890–1971)

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“Undoubtedly a piece of definitive truth – but not the whole truth, let alone the definitive truth.”

“Gott würfelt nicht.” (God casts the die, not the dice)

“The more success the quantum theory has, the sillier it looks.”

“Quantum physics formulates laws governing crowds and not individuals.”

“The statistical interpretation of quantum theory has led to important successes, but I still believe in the possibility of producing a model of reality which shall represent events themselves and not merely the probability of, their occurrence.”

“There is no doubt that quantum mechanics has seized hold of a beautiful element of truth and that it will be a touchstone for a future theoretical basis in that it must be deducible as a limiting case from that basis, just as electrostatics is deducible from Maxwell equations of the electromagnetic field or as thermodynamics is deducible from statistical mechanics. I do not believe that quantum mechanics will be the starting point in the search for this basis, just as one cannot arrive at the foundations of mechanics from thermodynamics or statistical mechanics.”

“All these fifty years of conscious brooding have brought me no nearer to the answer to the question ‘what are light quanta?’ Nowadays every Tom, Dick and Harry thinks he knows it, but he is mistaken.”

Albert Einstein (1879–1955)

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“Complementarity is a thoughtless slogan. If I were thoroughly convinced that Bohr is honest and really believes in the relevance of his – I do not say theory, but – sounding word, I shall call it intellectually wicked.”

Erwin Schrödinger (1887–1961)

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"Only metaphysics can inspire the hard work of theoretical physics."

"I don't like it, and I'm sorry I ever had anything to do with it."

Erwin Schrödinger (1887–1961)

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"By getting to smaller and smaller units, we do not come to fundamental units, or indivisible units, but we do come to a point where division has no meaning."

Werner Karl Heisenberg (1901–1976)

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"The theory of quantum electrodynamics describes nature as absurd from the point of view of common sense. And it agrees fully with experiment. So I hope you can accept nature as she is – absurd!"

"If people say they understand quantum mechanics they're lying."

"If you believe that atoms are like little solar systems, then you are back in 1910."

"I think it is safe to say that no one understands quantum mechanics. Do not keep saying to yourself: 'But how can it be like that?' because you will go into a blind alley from which nobody has yet escaped. Nobody knows how it can be like that."

Richard Phillips Feynman (1918–1988)

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“All of modern physics is governed by that magnificent and thoroughly confusing discipline called quantum mechanics invented more than fifty years ago. It has survived all tests. We suppose that it is exactly correct. Nobody understands it but we all know to use it and how to apply it to all problems: and so we have learned to live with the fact that nobody can understand it.”

“Niels Bohr brainwashed a whole generation of physicists into believing that the problem of the interpretation of quantum mechanics had been solved fifty years ago.”

Murray Gell-Mann (1929–)

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“When we are dealing with things as small as atoms and electrons, the observer or experimenter cannot be excluded from the description of nature. The laws of subatomic physics cannot even be formulated without some reference to the observer.”

Freeman Dyson (1923–)

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“I dreamt I died and went to heaven, and Saint Peter led me into the presence of God. And God said ‘You won’t remember me, but I took your Quantum Mechanics Course in Berkeley in 1947’.”

Robert Serber (1909–1997)

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"God not only plays dice. He also sometime throws the dice where they cannot be seen."

Stephen William Hawking (1942–), 1975

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"The basic concept and methods of quantum field theory are becoming more and more mathematical."

N.N. Bogoliubov (1909–1992)

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"Scientists have come to accept some pretty bizarre notions. The most precise theory ever created, quantum electrodynamics, starts with one of its fundamental postulates being a statement of uncertainty. We deal with concepts of curved space, of a vacuum that is rich in physical properties, and of pointlike particles that have a radius equal to zero but which, without embarrassment, carry spin, electric charge, mass and a plethora of other endowments."

Leon M. Lederman (1922–)

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"Most of the mathematics needed for quantum mechanics was forged in the 19^{th} century, but quantum mechanics itself was never predicted."

Shahar Ben-Menahem

3. Life, Evolution and Reductionism

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"The tree of life in the garden and the tree of knowledge of good and evil."

(Genesis **2**, 9)

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"It is remarkable that the stupidest ape differs so little from the wisest man, that the surveyor of nature has yet to be found who can draw the line between them."

Carl Linnaeus (1707–1778)

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"As many more individuals of each species are born than can possibly survive... it follows that any being, if it vary ever so slightly in a manner profitable to itself... will have a better chance of survival, and thus be naturally selected."

Thomas Malthus (1766–1834)

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"Life, as manifested to us, is a function of the asymmetry of the universe and of the consequences of this fact. The universe is asymmetrical; for, if the whole of the bodies which compose the solar system moving with their individual movements were placed before a glass, the image in the glass would not be superposed upon the reality. Even the movement of solar light is asymmetrical... Terrestrial magnetism, the opposition which exists between the north and the south poles in a magnet and between positive and negative electricity, are but resultants of asymmetrical actions and movements... Life is dominated by asymmetrical actions. I can even imagine that all living species are primordially, in their structure, in their external functions, functions of cosmic asymmetry."

Louis Pasteur (1822–1895), 1874

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"To suppose that the eye with all its inimitable contrivances for adjusting the focus to different distances, for admitting different amounts of light, and for the correction of spherical and chromatic aberration, could have been formed by natural selection, seems, I confess, absurd in the highest degree."

Charles Darwin (1809–1882)

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"The difference between a piece of stone and an atom is that an atom is highly organized, whereas the stone is not. The atom is a pattern, and the molecule is a pattern, and the crystal is a pattern; but the stone, although it is made up of these patterns, is just a mere confusion. It's only when life *appears that you begin to get organization on a larger scale. Life takes the atoms and molecules and crystals; but instead of making a mess of them like the stone, it combines them into new and more elaborate patterns of its own."*

"The great end of life *is not knowledge but* action.*"*

Thomas Henry Huxley (1825–1895)

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"Nothing in life is to be feared; It is to be understood."

Marie Curie (1867–1934)

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"Analogies from chemical experience will not, of course, any more than the ancient comparison of life with fire, give a better explanation of living organisms than will the resemblance, often mentioned, between living organisms and such purely mechanical contrivances as clockworks. An understanding of the essential characteristics of living beings must be sought, no doubt, in their peculiar organization, in which features that may be analyzed by the usual mechanics are interwoven with typically atomistic traits in a manner having no counterpart in inorganic matter."

Niels Bohr (1885–1962)

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"An organism has an astonishing gift of concentrating a 'stream of order' on itself and thus escaping decay into atomic chaos – of 'drinking orderliness' from a suitable environment."

Erwin Schrödinger (1887–1961)

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"The amoeba and the paramecium are potentially immortal. From time to time each divides itself into two, but... no new individual is ever produced-only fragments of the original individuals, whose life has thus been continuous back to the time when life itself was first created."

Joseph Wood Krutch (1893–1970)

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"It is selection, not mutation, that determines the direction of evolution... No mutant gene has the slightest chance of maintaining itself against even the faintest degree of adverse selection."

Gavin de Beer (1899–1972)

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"Nothing in Biology makes sense except in light of evolution."

T.G. Dobzhansky (1900–1975), 1973

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"To us as mammals, the insects seem to belong to some topsy-turvy world almost outside the reach of our understanding. They have the skeleton on the outside of the body, the main nervous system below the digestive tract... and use blood (body fluid) only for the transport of food materials."

Marston Bates (1906–1974)

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"Life is a cosmic event – so far as we know, the most complex state of organization that matter has achieved in our cosmos."

George Wald (1906–1997)

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"The apparent cause of illnesses is often an infection, an intoxication, nervous exhaustion, or merely old age. But actually a break-down of the hormonal mechanism *seems to be the most common ultimate cause of death in man."*

Hans Selye (1907–1982)

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"Strange though it may sound, it was a combination of Judeo-Greek ideas, amalgamated within the medieval church itself, which were to form part of the foundation out of which finally arose, in the eighteenth and nineteenth centuries, one of the greatest scientific achievements of all time: the recovery of the lost history of life."

Loren Eiseley (1907–1977)

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"Elements and stars, Planets and time, air and water – what would these things be without intelligent life to illuminate them with perception and insight?"

Preston Cloud (1912–1991)

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"The programming of a plant's form is based on the speed and direction of growth. To take a very simple example, the apple is round because growth continues equally in all directions as it matures; but a pear grows faster along its long axis than its radial one."

Anthony Huxley, 1974

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"Scrutiny of the organization of shells of many viruses with the electron microscope proves that their protein molecules are assembled according to well-known principles of solid geometry, the same ones employed by roof builders to construct quasi-spherical shells of maximum strength using uniform building elements. The shells of viruses bear close resemblance to Buckminster Fuller's domes."

"The perfect geometric shape of virus shells is in its way as remarkable as the symmetrical shape of a starfish or a sea urchin. But the shape of these animals and of all complex organisms is achieved through an elaborate process of development, involving cellular interactions whose complex mechanism is not yet understood. The shape of a virus is simply the outcome of the assembly of protein molecules tending, like all molecular structures, to reach a state of minimal energy."

Salvador Luria (1912–1991)

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"As far as the meaning of life in general, or in the abstract, as far as I can see, there is none. If all of life were suddenly to disappear from earth and anywhere else it may exist, or if none had ever formed in the first place, I think the Universe would continue to exist without perceptible change. However, it is always possible for an individual to invest his own life with meaning that he can find significant. He can so order his life that he may find as much beauty and wisdom in it as he can."

Isaac Asimov (1920–1992)

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"I feel that it is the very process of creating so many species which leads to evolutionary progress... Without speciation, there would be no diversification of the organic world, no adaptive radiation, and very little evolutionary progress."

"Attempts at a 'reduction' of purely biological phenomena or concepts to laws of the physical sciences has rarely, if ever, led to any advance in our understanding. Reduction is at best a vacuous, but more often a thoroughly misleading and futile, approach... System always have the peculiarity that the characteristics of the whole cannot (not even in theory) be deduced from the most complete knowledge of the components, taken separately or in other partial combination. The appearance of new characteristics in wholes has been designated emergence*: Species, competition, territory, migration, and hibernation are examples of organismic phenomena for which a purely physical description is at best incomplete and usually biologically irrelevant."*

Ernst Mayer (1901–1952)

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"Biology can neither reduced to physics, nor do without it."

"It is not inconceivable that in the future the thousands of chemical species contained in the bacterial cell may be synthesized one by one. But there is no chance of seeing all these compounds being assembled correctly and a bacterium emerging fully armed from a test-tube."

"It is natural selection that gives direction to changes, orients chance, and slowly, progressively produces more complex structures, new organs, and new species. Novelties come from previously unseen association of old material. To create is to recombine."

Francois Jacob (1920–), 1977

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"The greatest mystery is why there is something instead of nothing, and the greatest something is this thing we call life."

Allan R. Sandage (1926–)

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"Large, widespread, and successful species tend to be especially stable. Humans fall into this category, and the historical record supports such a prediction. Human body form has not altered appreciably in 100,000 years."

Stephen Jay Gould (1941–2002)

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"Consider the difference in size between some of the very tiniest and the very largest creatures on Earth. A small bacterium weighs as little as 0.00000000001 gram. A blue whale weighs about 100,000,000 grams. Yet a bacterium can kill a whale... Microbes, not macrobes, rule the world."

Bernard Dixon

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"Darwinism is not a theory of random chance. It is a theory of random mutation plus non-random *cumulative natural selection."*

"There is enough information capacity in a single human cell to store the Encyclopedia Britannica*, all 30 volumes of it, three or four times over."*

Richard Dawkins (1941–)

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"Chaos brings new challenge to the reductionist view that a system can be understood by breaking it down and studying each piece. This view has been prevalent in science in part because there are so many systems for which the behavior of the whole is indeed the sum of its parts. Chaos demonstrates, however, that a system can have complicated behavior that emerges as a consequence of simple, nonlinear interaction of only a few components. The problem is becoming acute in a wide range of scientific disciplines, from describing microscopic physics to modeling macroscopic behavior of biological organisms. For example, even with a complete map of the nervous system of a simple organism, the organism's behavior cannot be deduced.
Similarly, the hope that physics could be complete with an increasingly detailed understanding of fundamental physical forces and constituents is unfounded. The interaction of components on one scale can lead to complex global behavior on a larger scale that in general cannot be deduced from knowledge of the individual components."

James P. Crutchfield

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"One of the striking things about living creatures is that they do no more than is required. Unlike most machines, they do not have to be switched on and off by an outside manipulator; Something is built into them that does this at the proper time."

Niko Tinbergen (1907–1988)

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"Continuously dancing bees can even reproduce from memory the distance and the angle of these sites to the sun... I think that in this capacity for making delicate responses to the environment and keeping them in mind, lies a clue to the origins of intelligence and creative awareness."

Lyall Watson (1939–), 1976

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"The twenty-four-hour cyclical process is so basic from an evolutionary point of view that all plant and animal cells possess a basic metabolic circadian rhythm... The whole organism, in a sense, is the clock."

John E. Orme

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"The capacity to blunder slightly is the real marvel of DNA. Without this special attribute, we would still be anaerobic bacteria and there would be no music."

Lewis Thomas (1913–1993)

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"Left for themselves, things lead to disintegrate and must reach a state of chaos."

(The Second Law of Thermodynamics)

4. Time, History and Man

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"Very few things happen at the right time, and the rest do not happen at all; the conscientious historian will correct these defects."

Herodotos (ca 484–425 BCE)

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"The sun also ariseth, and the sun goeth down, and hasteth to his place where he arose. The wind goeth toward the south, and turneth about unto the north; it whirleth about continually, and the wind returneth again according to his circuits... The thing that hath been, it is that which shall be; and that which is done is that which shall be done; and there is no new thing under the sun... There is no remembrance of former things; neither shall there be any remembrance of things that are to come..."

Ecclesiastes **1**, 5–11

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"And I heard, but I understood not:
then said I, 'O my Lord, what shall be
the end of these things?'
And he said, 'Go thy way, Daniel:
for the words are closed up and sealed
till the time of the end'".

Daniel **12**, 8–9, (ca 165 BCE)

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"To be ignorant of what occurred before you were born is to remain always a child. For what is the worth of human life, unless it is woven into the life of our ancestors by the records of history?"

Cicero (46 BCE)

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"Nothing is ours except time."

Seneca (4 BCE–65)

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"What, then, is time? If no one asks me, I know; if I want to explain it to someone who does ask me, I do not know."

"The world was made, not in time, but simultaneously with time."

Aurelius Augustinus (354–430)

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"The moving Finger writes; and, having writ,
Moves on: nor all thy Piety nor Wit
Shall lure it back to cancel half a Line,
Nor all thy Tears wash out a Word of it."

Omar Khayyam (ca 1100)

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"The most precious thing in life is its uncertainty."

Yoshida Kenko (ca 1340)

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"What's past is prologue."

William Shakespeare (1564–1616), '*The Tempest*'

"When I do count the clock that tells the time
And see the brave day sunk in hideous night,
When I behold the violet past prime
And sable curls all silvered o'er with white,
When lofty trees I see barren of leaves,
Which erst from heat did canopy the herd,
And summer's green, all girded up in sheaves,
Borne on the bier with white and bristly beard:
Then on thy beauty do I question make
That thou among the wastes of Time must go,
Since sweets and beauties do themselves forsake
And die as fast as they see others grow;

And nothing 'gainst Time's scythe can make defense
Save breed, to brave him when he takes thee hence."

William Shakespeare (1564–1616), Sonnet **12**

"O time! thou must untangle this, not I;
It is too hard a knot for me to untie!"

William Shakespeare (1564–1616), 'Twelfth Night' **3**, (II)

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"Coming events cast their shadow before."

Thomas Campbell (1777–1844)

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"Prior to the creation of the universe, time did not exist. Time started at the moment of creation."

Jonathan Eibshitz (1690–1764)

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"To prevent an effect from occurring at all requires a force equal to the cause of that effect, but to give it a new direction often requires only something very trivial."

George Christoph Lichtenberg (1742–1799)

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"Greek civilization depended essentially on slave-labor but could not progress without the harnessing of natural forces to labor-saving machines. Only the free man, not a slave, has a disposition and interest to improve implements or to invent them. Accordingly, in the devising of a complicated machine, the workmen employed upon it are generally co-inventors. The eccentric *and the* governor*, most important part of the steam-engine, were devised by laborers. The improvement of established industrial methods by slaves, themselves industrial machines, is out of question."*

Justus von Liebig (1803–1873)

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"In analyzing history, do not be too profound, for often the causes are quite superficial."

Ralph Waldo Emerson (1803–1882)

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"Ideas have consequences."

F. Dostoyevski (1821–1881)

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"There is one thing stronger than all the armies in the world: and that is an idea whose time has come."

Victor Hugo (1802–1885)

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"Biography is the only history: The history of the world is but the biography of great men."

"Speech is of time, silence is of eternity."

Thomas Carlyle (1795–1881)

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"Anybody can make history. Only a great man can write it."

Oscar Wilde (1854–1900)

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"An idea isn't responsible for the people who believe in it."

Don Marquis (1878–1937)

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"If you want work well done, select a busy man – the other kind has no time."

Elbert Hubbard (1856–1915)

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"Time is neutral; but it can be made the ally of those who will seize it and use it to the full."

Winston Churchill (1874–1965)

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"Time is the stuff life is made of."

Benjamin Franklin (1706–1790)

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"With me goes your meaning too."

"Das ist der Sinn von allem, was einst war,
dass es nicht bleibt mit seiner ganzen Schwere,
dass es zu unserm Wesen wiederkehre,
in uns verwoben, tief und wunderbar."

"How can the least thing happen,
unless all future fullness,
time's completed sum,
move to meet us half-way?"

"The free animal
has its decease perpetually behind it
and God in front, and when it moves, it moves
into eternity, like running springs."

"The future enters into us, in order to transform itself in us, long before it happens."

Rainer Maria Rilke (1875–1925)

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"Nothing has really happened until it has been recorded."

Virginia Woolf (1882–1941)

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"The oldest of the old follows behind us in our thinking, and yet it comes to meet us."

Martin Heidegger (1889–1976)

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"We physicists work with time every day, but don't ask me to tell you what time is; it is too complicated to be thinking about it."

Richard Phillips Feynman (1918–1988)

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"Everything is a matter of chronology."

"In theory one is aware that the earth revolves, but in practice one does not perceive it. So it is with time in one's life."

Marcel Proust (1871–1922)

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"For the tribal man space was the uncontrollable mystery. For technological man it is time that occupies the same role."

Herbert Marshall McLuhan (1911–1980), (1951)

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"Time is in fact the hero of the plot. What we regard as impossible on the basis of human experience is meaningless here. Given so much time, the 'impossible' becomes the possible, the possible probable, and the probable virtually certain. One has only to wait: time itself performs the miracles."

George Wald (1906–1997), (1954)

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"Money-making, social achievements, family and posterity are nothing but plain nature, not culture. Culture lies outside the purpose of nature."

"The essence of culture is continuity and conservation of the past."

C.G. Jung (1875–1961)

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"Space and time are modes of our thinking and not conditions of our life."

"I never think of the future. It comes soon enough."

"Michael left this strange world just before me. This is of no importance. For us, devout physicists, the distinction between past, present and future signifies only an obstinate illusion."

Albert Einstein (1879–1955), (1955)

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"Time is a fluid condition which has no existence except in the momentary avatars of individual people."

William Faulkner (1897–1962), 1958

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"The invention of the mechanical clock was one of a number of major advances that turned Europe from a weak, peripheral, highly vulnerable outpost of Mediterranean civilization into a hegemonic aggressor."

David Landes (1924–), 1980

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"Our knowledge of time and space owes more to the labors of mathematicians and physicists than to those of professed philosophers."

Charles Broad

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"History is the study of the bones of civilizations that failed, as the pterodactyl and the dinosaur failed."

Colin Wilson (1931–)

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"Civilizations die of suicide not murder."

Toynbee (1889–1975)

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"Ideas shape the course of history."

John Maynard Keynes (1883–1946)

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"In the last 3421 years of recorded history only 268 have seen no war."

Will Durant (1885–1981)

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"A study of history shows that civilizations that abandon the quest for knowledge are doomed to disintegration."

Bernard Lovell (1913–)

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"Great events do not necessarily have great causes."

"All changes in history, all advance, comes from the non-conformists. If there had been no troublemakers, no dissenters, we should still be living in caves."

A.J.P. Taylor (1906–1990)

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"There is no such thing as history, there are only historians."

Walter Benjamin (1892–1940)

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"Civilizations rise and fall on ideas.*Thus, the "Sturm und Drang" philosophy of Goethe, Kant, Hegel, Rousseau ad Blake, which was essentially a reaction to rationalism, later ruined Europe and is now undermining the United States. The Liberal-Left understands this, but the Right has nothing to offer except religion."*

Alan Bloom (1930–1992)

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"When the anonymous masses enter history, it is chiefly to be slaughtered in battle, to die of famine or privation – to illustrate the failures of their betters. . . . We have the mighty pyramids, but no firsthand account of the feelings of the wretches who built them."

Herbert J. Muller (1905–1980)

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"Man is the only animal to be troubled by time, and from that concern comes much of his finest art, a great deal of his religion, and almost all his science. First was the temporal regularity of nature — the rising of sun and stars, the slower rhythm of seasons — which led to the concept of law and order and in turn to astronomy, the first of all sciences. Changeless environments like in the deep ocean or the cloud-wrapped surface of Venus provide no stimulus to intelligence and in such places it may never be able to arise."

Arthur C. Clarke (1917–2008)

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"The genius of Einstein leads to Hiroshima."

Pablo Picasso (1881–1973)

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"There is never time to do it right, but there is always time to do it over."

(Anon)

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"You are never given enough time or money."

(Anon)

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"What happens first is not necessarily the beginning."

(Anon)

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"Throughout all great adventures of men, the cowards stayed behind, the weak perished on the way, only the strong survived and reached their destination."

(Anon)

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"The Protestant families which were exiled from the Catholic countries of Europe during the sixteenth and seventeenth centuries and even during the eighteenth, have given birth to an extraordinarily high number of distinguished scientists. This is not to be wondered at. These people who preferred the misery of exile to moral servitude were certainly above the average in their conscientiousness and earnestness."

(Anon)

5. *On Mathematics, Zero and Infinity*

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"All things are numbers."

Pythagoras (580–506 BCE)

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"When the Greek philosophers found that the square root of 2 is not a rational number, they celebrated the discovery by sacrificing 100 oxen."

Herodotos (ca 484–424 BCE)

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"In fact, everything that can be known has number. For it is not possible to conceive of, or to know, anything that has not."

Philolaos of Croton (ca 480 BCE)

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"The mathematical sciences particularly exhibit order, symmetry and limitation; and these are the greatest forms of the beautiful."

Aristotle (384–322 BCE)

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"God ever geometrizes."

"He is unworthy of the name of man who is ignorant of the fact that the diagonal of a square is incommensurable with its side."

"How did Thales measure the height of the Pyramide of Cheops? — he measures the length of its shadow when a man's shadow was equal to his height."

" Let no one ignorant of geometry enter here."(Inscribed over the door of the Academy, ca 387 BCE)

Plato (427–347 BCE)

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"If we can approach the Divine only through symbols, then it is most suitable that we use mathematical symbols, for these have an indestructible certainty."

Nicolas of Cusa (1401–1464)

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"Nothing can be created from nothing."

Lucretius (ca 99–55 BCE)

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"Beauty — the adjustment of all parts proportionately so that one cannot add or subtract or change without impairing the harmony of the whole."

Leon Battista Alberti (1404–1472)

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"There is no royal road to Geometry."

"The laws of nature are but the mathematical thoughts of God."

Euclid (323–283 BCE)

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"But let us remember that we are dealing with infinities and indivisibles, both of which transcend our finite understanding: the former on account of their magnitude, the latter because of their smallness. In spite of this, men cannot refrain from discussing them, even if it must be done in a roundabout way."

"The universe stands continually open to our gaze but it cannot be understood unless one first learns to comprehend the language and interpret the characters in which it is written. It is written in the language of mathematics."

"The book of Nature is written in mathematical symbols."

Galileo Galilei (1564–1642)

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"No human investigation can be called real science if it cannot be demonstrated mathematically."

Leonardo da Vinci (1452–1519)

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"Ubi materia — ibi geometria."

Johannes Kepler (1571–1630), 1609

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"The mathematicians who are merely mathematicians reason correctly, but only when everything has been explained to them in terms of definitions and principles. Otherwise they are limited and insufferable, for they only reason correctly when they are dealing with very clear principles."

"All that transcends geometry transcends our comprehension."

Blaise Pascal (1623–1662)

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"The imaginary number is a fine and wonderful recourse of the divine spirit, almost an amphibian between being and not being."

"Music the pleasure the human mind experiences from counting without being aware that it is counting."

"Datis ordinatis etiam sunt ordinata."

Gottfried Wilhelm von Leibniz (1646–1716)

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"Eadem mutata resurgo."

Engraved on the tombstone of Jakob Bernoulli (1654–1705)

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"I could be bounded in a nutshell and count myself a king of infinite space."

William Shakespeare (1564–1616), "*Hamlet*"

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"What is man in nature? Nothing in relation to the infinite, everything in relation to nothing, a mean between nothing and everything."

"When I consider the small span of my life absorbed in the eternity of time, or the small part of space which I can touch or see engulfed by the infinite immensity of spaces that I know and that know me not, I am frightened and astonished to see myself here instead of there... now instead of then."

Blaise Pascal (1623–1662)

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"An infinitesimal is the spirit of a departed quantity."

George Berkeley (1685–1753)

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"Mathematics was born and nurtured in a cultural environment. Without the perspective which the cultural background affords, a proper appreciation of the content and state of present-day mathematics is hardly possible."

"Mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country."

"Mathematicians have tried in vein to this day to discover some order in the sequence of prime numbers, and we have reason to believe that it is a mystery into which the human mind will never penetrate."

Leonhard Euler (1707–1783)

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"Algebra is generous: she often gives more than is asked for."

"A quantity is something or nothing; if it is something, it has not yet vanished; if it is nothing, it has literally vanished. The supposition that there is an intermediate state between these two is chimera. "

Jean le Rond d'Alembert (1717–1783)

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"As long as algebra and geometry proceeded along separate paths, their advance was slow and their applications limited. But when these sciences joined company, they drew from each other fresh vitality and thenceforth marched on at a rapid pace towards perfection."

Joseph Louis Lagrange (1736–1813)

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"Nature laughs at the difficulties of integration."

"At bottom, the theory of probability is only common sense expressed in mathematical language."

"All the effects of nature are only mathematical consequences of a small number of immutable laws."

"...a profound and important idea which appears so simple to us now that we ignore its true merit. But its very simplicity and the great ease which it lent to all computations put our arithmetic in the first rank of useful inventions."

"It is India that gave us the ingenious method of expressing all numbers by means of ten symbols, each symbol receiving a value of position as well as an absolute value; a profound and important idea which appears so simple to us now that we ignore its true merit, but its very simplicity, the great ease which it has lent to all computations, puts our arithmetic in the front rank of useful inventions; and we shall appreciate the grandeur of this achievement when we remember that it escaped the genius of Archimedes and Apollonios, two of the greatest men produced by antiquity."

Pierre Simon de Laplace (1749–1827)

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"There is no branch of mathematics, however abstract, which may not some day be applied to phenomena of the real world."

Nikolai Lobachevsky (1792–1856)

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"The mathematician is only complete insofar as he feels within himself the beauty of the true."

Johann Wolfgang von Goethe (1749–1832)

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"The enormous and the minute are interchangeable manifestations of the eternal."

William Blake (1757–1827)

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"Nature is not embarrassed by difficulties of analysis."

Augustin Fresnel (1788–1827)

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"The profound study of nature is the most fertile source of mathematical discoveries."

Jean Baptiste Joseph Fourier (1768–1830)

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"It is difficult to say which of Fourier results is most to be praised: their uniquely original quality, their transcendently intense mathematical interest, or their perennially important instructiveness for physical science."

"Mathematics is the only good metaphysics."

Lord Kelvin (1824–1907)

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"It is not knowledge but the act of learning, not possession but the act of getting there, which grants the greatest enjoyment. When I have clarified and exhausted a subject, then I turn away from it in order to go into darkness again."

"Important propositions, with the impress of simplicity on them, are often easily discovered by induction, and yet are of so profound a character that we cannot find the demonstrations till after many vain attempts; and even then, when we do succeed, it is often by some tedious and artificial process, while the simple methods may long remain concealed."

"Pauca Sed Matura."

"All the measurements in the world are not the equivalent of a single theorem that produces a significant advance in our greatest of sciences."

Carl Friedrich Gauss (1777–1855)

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"Fourier's book is a great mathematical poem."

J.C. Maxwell (1831–1879)

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The success of Fourier "It was, no doubt, partially because of his very disregard for rigor that he was able to take conceptual steps which were inherently impossible to men of more critical genius."

Rudolph Langer

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"Looking back, we can see Fourier's memoir as heralding the surge of new mathematical methods and results which were to mark the new century. His ideas are built into the commonsense of our society."

T.W. Körner, 1988

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"Fourier's book was of paramount importance in the history of mathematics and pure analysis perhaps owed it even more than applied mathematics."

Poincare (1854–1912), 1895

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"I have had my results for a long time: but I do not yet know how I am to arrive at them."

"I protest against the use of an infinite quantity as an actual entity; this is never allowed in mathematics. The infinite is a manner of speaking, in which one properly speaks of limits to which certain ratios can come as near as desired, while others are permitted to increase without bound."

"Mathematics is concerned only with the enumeration and comparison of relations."

"The higher arithmetic offers an inexhaustible wealth of interesting truths, which are not isolated but stand in intimate relationship with each other, and which reveal, as the science develops, ever new, even unexpected, connexions."

Carl Friedrich Gauss (1777–1855)

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"Between Analysis and Number Theory, which have long been held to be quite separate disciplines, one has discovered, in recent times, with increasing frequency, interconnections which are often unexpected. A rich source of such connections, and one which will long remain unexhausted, is the analysis of elliptic functions."

"The theory of elliptic functions is a vast subject of research, which in the course of its development embraces almost all algebra, the theory of definite integrals, and the science of numbers."

"The deduction of these arithmetical theorems from developments in Analysis not only increases the supply of methods of proof in arithmetic, but the theorems themselves are found to acquire a new and striking form."

"God ever arithmetizes."

"Man muss immer generalisieren."

"For Gaussian rigor, we have no time."

"Problems of number theory are just as important as problems from the real world. The honor of the human spirit is at stake."

C.G.J. Jacobi (1804–1851)

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"There is no study in the world which brings into more harmonious action all the faculties of the mind then mathematics. It seems to raise them, by successive steps of initiation, to higher and higher states of conscious intellectual being."

"The object of pure physics is the unfolding of the laws of the intelligible world; the object of pure mathematics is that of unfolding of the laws of human intelligence."

James Joseph Sylvester (1814–1897)

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“The moving power of mathematical invention is not reasoning but imagination.”

Augustus de Morgan (1806–1871)

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“Time was when all the parts of the subject were dissevered, when algebra, geometry, and arithmetic either lived apart or kept up cold relations of acquaintance confined to occasional calls upon one another; but that is now at an end; they are drawn together and are constantly becoming more and more intimately related and connected by a thousand fresh ties, and we may confidently look forward to a time when they shall form but one body with one soul.
The world of ideas which it [mathematics] discloses or illuminates, the contemplation of divine beauty and order which it induces, the harmonious connection of its parts, the infinite hierarchy and absolute evidence of the truths with which it is concerned, these, and such like, are the surest grounds of the title of mathematics to human regard, and would remain unimpeached and unimpaired were the plan of the universe unrolled like a map at our feet, and the mind of man qualified to take in the whole scheme of creation at a glance.”

J.J. Sylvester (1814–1897), 1869

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“I see it but I do not believe it.”

“In re mathematica ars proponendi questionem pluris facienda est quam solvendi.”

Georg Cantor (1845–1918)

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"The teacher should let science develop before the eyes of his pupil. As it develops and takes form in the mind of the mature thinker, out of his fundamental ideas, so shall he present it, merely adjusting it to the youthful power of understanding."

"It is true that a mathematician who is not also something of a poet, will never be a perfect mathematician."

Karl Theodor Wilhelm Weierstrass (1815–1857), 1883

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"The integer numbers have been made by God, everything else is the work of man."

Leopold Kronecker (1823–1891)

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"In eternity, I plan to spend eight million years on mathematics."

Mark Twain (1835–1910)

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"In most sciences one generation tears down what another has built, and what one has established another undoes. In mathematics alone, each generation builds a new story to the old structures."

Hermann Hankel (1839–1873)

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"A mathematician may say anything he pleases, but a physicist must be at least partially sane."

"Mathematics is a language."

Josiah Willard Gibbs (1839–1903)

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"Among the minor, yet striking characteristics of mathematics, may be mentioned the fleshless and skeletal build of its propositions; the peculiar difficulty, complication, and stress of its reasonings; the perfect exactitude of its results; their broad universality; their practical infallibility."

"The one [the logician] studies the science of drawing conclusions, the other [the mathematician] the science which draws necessary conclusions."

"...mathematics is distinguished from all other sciences except only ethics, in standing in no need of ethics. Every other science, even logic, especially in its early stages in danger of evaporating into airy nothingness, degenerating, as the Germans say, into an arachnoid film, spun from the stuff that dreams are made of. There is no such danger for pure mathematics; for that is precisely what mathematics ought to be."

Charles Sanders Peirce (1839–1914)

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"One cannot escape the feeling that these mathematical formulae have an independent existence and intelligence of their own, that they are wiser than we are, wiser even than their discoverers, that we get more out of them than was originally put into them."

Heinrich Rudolf Hertz (1857–1894)

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"Geometrical properties are characterized by their invariance under a group of transformations."

"When I was a student, Abelian functions were, as an effect of the Jacobian tradition, considered the uncontested summit of mathematics and each of us was ambitious to make progress in this field. And now? The younger generation hardly knows Abelian functions. How did this happen? In mathematics, as in other sciences, the same process can be observed again and again. First new questions arise, for internal or external reasons, and draw researchers away from the old questions. And the old questions, just because they have been worked on so much, need ever more comprehensive study for their mastery. This is unpleasant, and so one is glad to turn to problems that have been less developed and therefore require less foreknowledge — even if it only a matter of axiomatics, or set theory, or some such thing."

Felix Christian Klein (1849–1925)

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"I know mathematical processes that I have used with success for a very long time, of which neither I nor anyone else understands the scholastic logic. I have grown into them, and so understand them that way."

"Shall I refuse my dinner because I do not fully understand the process of digestion? No, not if I am satisfied with the result."

"Euclid is the worst. It is shocking that young people should be addling their brains over mere logical subtleties, trying to understand the proof of one obvious fact in terms of something equally obvious."

Oliver Heaviside (1850–1925)

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"With the introduction of the infinitely small and infinitely large, mathematics, usually so strictly ethical, fell from grace. The virgin state of absolute validity and irrefutable proof of everything mathematical was gone forever; the realm of controversy was inaugurated, and we have reached the point where most people differentiate and integrate not because they understand what they are doing, but from pure faith, because up to now it has always come out right."

Friedrich Engels (1820–1895)

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"...without mathematical infinity*, there would be no science at all, because there would be nothing general."*

"It has been said that geometry is the art of applying good reasoning to bad diagrams. This is not a joke but a truth worthy of serious thought. What do we mean by a poorly drawn figure? It is one where proportions are changed slightly or even markedly, where straight lines become zigzag, circle acquire incredible humps. But none of this matters.
An inept artist, however, must not *represent a closed curve as if it were open, three concurrent lines as if they intersected in pairs, nor must he draw an unbroken surface when the original contains holes."*

"One geometry cannot be more true than another; it can only be more convenient. Geometry is not true, it is advantageous."

"Mathematics is the art of giving the same name to different things."

"If we wish to foresee the future of mathematics, our proper course is to study the history and present conditions of the science."

Henri Jules Poincaré (1854–1912)

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"The mathematician, carried along on his flood of symbols, dealing apparently with purely formal truths, may still reach results of endless importance for our description of the physical universe."

Karl Pearson (1857–1936)

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"The mathematician has reached the highest rung of the ladder of human thought."

Havelock Ellis (1859–1939)

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"The perfection of mathematical beauty is such...that whatsoever is most beautiful and regular is also found to be most useful and excellent."

D'Arcy Thompson (1860–1948)

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"The first comment I would make after waking at the end of a thousand year sleep would be: 'Is the Riemann hypothesis established yet'?"

"As long as a branch of science offers an abundance of problems, so long is it alive; a lack of problems foreshadows extinction or the cessation of independent development."

"From time immemorial the infinite *has stirred men's emotions more than any other question. Hardly any other idea stimulated the mind so fruitfully. Yet no other concept needs clarification more than it does."*

David Hilbert (1862–1943)

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"Do not work within two hours of a substantial meal; blood cannot be in two places at once."

"Any identity, once verified, is trivial."

John Edensor Littlewood (1885–1977)

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"Oriental mathematics may be an interesting curiosity, but Greek mathematics is the real thing. The Greeks first spoke a language which modern mathematicians can understand; as Littlewood said to me once, they are not clever schoolboys or 'scholarship candidates', but 'Fellows of another college'. So Greek mathematics is 'permanent', more permanent even than Greek literature. Archimedes will be remembered when Aeschylus is forgotten, because languages die and mathematical ideas do not. 'Immortality' may be a silly word, but probably a mathematician has the best chance of whatever it may mean."

"Here is to pure mathematics, may it never find an application."

"The 'seriousness' of a mathematical theorem lies, not in its practical consequences, which are usually negligible, but in the significance *of the mathematical ideas which it connects. We may say, roughly, that a mathematical idea is 'significant' if it can be connected, in a natural and illuminating way, with a large complex of other mathematical ideas. Thus a serious mathematical theorem, a theorem which connects significant ideas, is likely to lead to important advances in mathematics itself and even in other sciences."*

"Sometimes one has to say difficult things, but one ought to say them as simply as one knows how."

Godfrey Harold Hardy (1877–1947)

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‘If insight is the essential element in intelligent problem-solving, fixation is its archenemy. Fixation is overcome and insight attained by a sudden shift in the way the problem or the objects involved in it are viewed. The work described in this article has pointed to some of the factors that necessitate this sudden shift, but precisely what brings it about is still unknown. It remains the central problem of problem-solving.”

George Polya (1887–1985)

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“Mathematics is the most powerful technique for the understanding of patterns and for the analysis of the relations of patterns.”

“In the year 1500 *Europe knew less than Archimedes who died in the year* 212 *BCE.”*

“ I will not go so far as to say that to construct a history of thought without profound study of the mathematical ideas of successive epochs is like omitting Hamlet from the play which is named after him... But it is certainly analogous to cutting out the part of Ophelia. This simile is singularly exact. For Ophelia is quite essential to the play, she is very charming – and a little mad.”

“The science of pure mathematics may claim to be the most original creation of the human spirit.”

Alfred North Whitehead (1861–1947)

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“Deepest interrelationship in analysis are of an arithmetical nature.”

Hermann Minkowski (1864–1909)

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"Rigor merely sanctions the conquests of the intuition."

Jacques Solomon Hadamard (1865–1963)

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"A barber is presumed to shave each man in his town who does not shave himself, but not to shave anyone else. Does the barber shave himself?"

"Physics is mathematical not because we know so much about the physical world, but because we know so little: it is only its mathematical properties that we can discover."

"It must have required many ages to discover that a brace of pheasants and a couple of days were both instances of the number two."

"Mathematics, rightly viewed, possesses not only truth, but supreme beauty — a beauty cold and austere, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show."

"A good notation has a subtlety and suggestiveness."

"Mathematics is the science in which we do not know what we are talking about, and do not care whether what we say about it is true."

"Mathematics is, I believe, the chief source of the belief in eternal and exact truth, as well as in a super-sensible intelligible world."

Bertrand Russell (1872–1970)

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"Mathematics may explore the forth dimension but the Czar can be overthrown only in the third dimension."

Vladimir Ilyich Lenin (1870–1924)

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"I don't believe in mathematics."

"Mathematics offers the exact natural sciences certain measure of security which, without mathematics, they could not attain."

"Pure mathematics is, in its way, the poetry of logical ideas."

"Mathematics are well and good but nature keeps dragging us around by the nose."

"Since the basic equations of physics are nonlinear, all the mathematical physics will have to be done over again."

"How can it be that mathematics, being after all a product of human thought, independent of experience, is so admirably adapted to the objects of reality?"

"As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality."

"God does not care about our mathematical difficulties. He integrates empirically."

Albert Einstein (1879–1955)

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"Egyptian pyramids, Doric temples, and Gothic cathedrals are mathematics in stone."

Oswald Spengler (1880–1936)

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"Proof, as pure mathematicians understand it, is really quite uninteresting and unimportant. No one who is really certain that he found something good should waste his time looking for a proof."

"I know passages written in mathematical symbols which in their sublimity might vie with a sonnet of Shakespeare."

Arthur Stanley Eddington (1882–1944)

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"There is a certain loose continuity in all mathematics, clear back to Babylon and Egypt, but the interesting and fruitful points on the curve of progress are the discontinuities that appear when the curve is closely analyzed."

Eric Temple Bell (1883–1960)

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"God is a mathematician."

James Hopwood Jeans (1887–1946)

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'Zero' is the absolute negations of all attributes.
'Infinity' is the totality of all possibilities, manifested in a reality that is inexhaustible.
The product of infinity and zero supplies the while set of finite numbers.
Each act of creation could be symbolized as a particular product of infinity and zero. From each such product could emerge a particular individual of which the appropriate symbol was a particular finite number.

"An equation for me has no meaning unless it expresses a thought of God."

"As soon as I heard the problem it was clear that the solution should obviously be a continued fraction; I then thought, which continued fraction? And the answer came to my mind."

S. Ramanujan (1887–1920)

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"Mathematics is the art of problem solving."

"When you have satisfied yourself that the theorem is true you start proving it."

"If you cannot solve a problem, then there is an easier problem you can't solve: find it."

George Polya (1887–1985)

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"There are easier ways to make money than by proving Fermat's Last Theorem."

Louis Joel Mordell (1888–1972)

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"Empirical evidence can never establish mathematical existence — nor can the mathematician's demand for existence be dismissed by the physicist as useless rigor. Only a mathematical existence proof can ensure that the mathematical description of a physical phenomenon is meaningful."

Richard Courant (1888–1972)

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"When feeling burdened or downcast,... the human mind will gladly turn to the realms of Mathematics, where a lucid and precise grasp of objectivities is obtained and insight is gained so pleasantly through appropriate concept formation. Here the human spirit feels at home."

Paul Bernays (1888–1977)

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"Mathematics itself is only a particular formulation of the mathematical."

Martin Heidegger (1889–1976)

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"To be a scholar of mathematics you must be born with talent, insight, concentration, taste, luck, drive and the ability to visualize and guess."

Paul Halmos (1916–2006), 1985

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"When I am working on a problem, I never think about beauty. I think only how to solve the problem. But when I have finished, if the solution is not beautiful, I know it is wrong."

Richard Buckminster Fuller (1895–1983)

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"Greek pure mathematics covers a very short time indeed, beginning in the time of Plato (Theaetetos and Eudoxos, ca 400 BCE), condensed in the Elements *and appearing for the last time in the works of Archimedes and Apollonios (200 BCE). The main reason for this early interruption of pure mathematics can be found in the purely geometric type of expression which was adopted in order to gain the higher degree of generality which the geometrical magnitudes represent, in contrast to the field of rational numbers, which was the exclusive concern of oriental mathematics and astronomy. This geometrical language, however, very soon reached such a degree of complication that development beyond the theory of conic sections was practically impossible. As a result, the development of theoretical mathematics ended two centuries after its beginning.*
I think that the influence of this Greek pure mathematics on the general standard of mathematics in antiquity has been very much overestimated. Even Euclid's own works, other than the Elements*, are on a very different level; this can be simply explained by the remark that the* Elements *are concerned with very special group of problems, mainly concentrated on the theory of irrational numbers where the exactitude of definitions and conclusions is the essential point of the discussion.*
The main part of mathematical literature, however, was less rigorous and represented the direct continuation of Babylonian and even Egyptian method. The Babylonian influence is, for instance, mainly responsible for the general character of other groups of Greek mathematical literature, as, e.g., the work of Diophantos (ca 300 CE). This situation in the field of mathematics corresponds very much to the general character of the Hellenistic culture, with its mixture of very contradictory elements from all parts of the ancient world."

Otto Neugebauer (1899–1990)

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"The mathematician plays a game in which he himself invents the rules while the physicist plays a game in which the rules are provided by Nature, but as time goes on it becomes increasingly evident that the rules which the mathematician finds interesting are the same as those which Nature has chosen."

"In science one tries to tell people, in such a way as to be understood by everyone, something that no one ever knew before. But in poetry, it's the exact opposite."

"If there is a God, he's a great mathematician."

"Sensible mathematics involves neglecting a quantity when it is small – not neglecting it because it is infinitely great and you do not want it!"

"If there is a God, he's a great mathematician."

"It seems to be one of the fundamental features of nature that fundamental physical laws are described in terms of mathematical theory of great beauty and power, needing quite a high standard of mathematics for one to understand it. You may wonder: Why is nature constructed along these lines? One can only answer that our present knowledge seems to show that it is so constructed. We simply have to accept it. One could perhaps describe the situation by saying that God is a mathematician of a very high order, and He used very advanced mathematics in constructing the universe. Our feeble attempts at mathematics enable us to understand a bit of the universe, and as we proceed to develop higher and higher mathematics we can hope to understand the universe better."

"I am not interested in proofs, only what nature does."

"Mathematics is the tool specially suited for dealing with abstract concepts of any kind and there is no limit to its power in this field."

Paul Adrien Maurice Dirac (1902–1984)

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"If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is."

"In mathematics you don't understand things. You just get used to them."

"It is only proper to realize that language is largely a historical accident. The basic human languages are traditionally transmitted to us in various forms, but their very multiplicity proves that there is nothing absolute and necessary about them. Just as languages like Greek or Sanskrit are historical facts and not absolute logical necessities, it is only reasonable to assume that logics and mathematics *are similarly historical, accidental forms of expression. They may have essential variants, i.e. they may exist in other forms than the ones to which we are accustomed. Indeed, the nature of the central nervous system and of the message systems that it transmits indicate positively that this is so. We have now accumulated sufficient evidence to see that whatever language the central nervous system is using, it is characterized by less logical and arithmetical depth than what we are normally used to. The following is an obvious example of this: the retina of the human eye performs a considerable reorganization of the visual image as perceived by the eye. Now, this reorganization is effected on the retina, or to be more precise, at the point of entry of the optic nerve by means of three successive synapses only, i.e. in terms of three consecutive logical steps. The statistical behavior of the message system used in the arithmetics of the central nervous system and its low precision also indicate that the degeneration of precision cannot proceed very far in the message system involved. Consequently, there exist here different logical structures from the ones we are ordinarily used to in logic and mathematics. They are characterized by less logical and arithmetical depth than we are used to under otherwise similar circumstances. Thus logic and mathematics in the central nervous system, when viewed as languages, must structurally be essentially different from those languages to which our common experience refers."*

John von Neumann (1903–1957)

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"The zero is the most important digit. It is a stroke of genius, to make something out of nothing by giving it a name and inventing a symbol for it."

Bartel Leendert van der Waerden (1903–1996)

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"The problem is, when we try to calculate all the way down to zero distance, the equation blows up in our face and gives us meaningless answers – things like infinity. This caused a lot of trouble when the theory of quantum electrodynamics first came out. People were getting infinity for every problem they tried to calculate."

"Mathematics is a tool for reasoning. It is not a science from our point of view, in the sense that it is not a natural science. The test of its validity is not experiment."

"I love only nature, and I hate mathematicians."

Richard Feynman (1918–1988)

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"God exists since mathematics is consistent, and the Devil exists since we cannot prove it."

Andre Weil (1906–1998)

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"The danger of the mathematician making mistakes is an unavoidable corollary of his power to hit sometimes upon an entirely new method. This seems to be confirmed by the well-known fact that the most reliable people will not usually hit upon new methods."

Alan Mathison Turing (1912–1954)

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"When the answer to your question is 'yes' then you have asked the wrong question."

Fritz John (1910–1994)

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"The genesis of the majority of mathematical theories is obscure and difficult. Often, today's presentation of a classical topic, will be much more accessible and concise than it could even have been when it was developed. Any scientist's work, can only be understood within its contemporary scientific framework."

Walter K. Bühler (1944–)

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"Probability is a simple subject; all the answers are between zero and one."

Byron Goldstein

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"From long experience, all working mathematicians know that there is a preliminary period of rapid advancement in ideas without worrying about exact definitions and proofs, after which there is very hard work to go from that level of accuracy to finished mathematics, where the bugs in definitions and proofs are gone, and concepts are quite clear. A lot of things change in the process. This is the essence of finishing mathematical work."

Anil Nerode

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"Mathematics is the science of order. Its object is to find, describe, and understand the order that underlies apparently complex situations. The principal tools of mathematics are concepts which enable us to describe order. Precisely because mathematicians have been searching for centuries for the most efficient concepts for describing obscure instances of order, their tools are applicable to the outside world: for the real world is the very epitome of a complex situation in which there is a great deal of order."

Andrew Gleason (1921–)

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"It has been said that World War I was the chemist's war, World War II was the physicist's war, World War III (may it never come) will be the mathematicians' war."

(Anon)

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"The infinite is the mathematician's paradise."

(Anon)

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Q: What's the difference between a mathematician and a physicist?
A: A mathematician thinks that two points are enough to define a strait line while a physicist wants more data.

(Anon)

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"The equation $e^{\pi i} = -1$ *has been called the eutectic point of mathematics, for no matter how you boil down and explain this equation, which relates four of the most remarkable numbers of mathematics, it still has a certain mystery about it that cannot be explained away."*

(Anon)

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"An applied mathematician loves the theorem. A pure mathematician loves the proof."

(Anon)

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"The trouble with integers is that we have examined only the very small ones. Maybe all the exciting stuff happens at really big numbers, ones we can't even begin to think about in any very definite way. Our brains have evolved to get us out of the rain, find where the berries are, and keep us from getting killed. Our brains did not evolve to help us grasp really large numbers or to look at things in a hundred thousand dimensions."

Ronald L. Graham (1935–)

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"Mathematics has beauties of its own – a symmetry and proportion in its results, a lack of superfluity, an exact adaptation of means to ends, which is exceedingly remarkable and to be found only in the works of the greatest beauty. When this subject is properly ... presented, the mental emotion should be that of enjoyment of beauty..."

J.W.A. Young

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"Hiding between all the ordinary numbers was an infinity of transcendental numbers whose presence you would never have guessed unless you looked deeply into mathematics."

Carl Sagan (1934–1996)

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"Nature does not count nor do integers occur in nature. Man made them all, integers and all the rest, Kronecker to the contrary notwithstanding."

Percy William Bridgman (1882–1961)

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"The problem with linear theory is that it is not nonlinear."

John A. Adam (1735–1826)

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"Zero is powerful because it is infinity's twin. They are equal and opposite, yin and yang. They are equally paradoxical and troubling. The biggest questions in science and religion are about nothingness and eternity, the void and the infinite, zero and infinity. The clashes over zero were the battles that shook the foundations of philosophy, of science, of mathematics, and of religion. Underneath every revolution lay a zero – and an infinity.
Zero was at the heart of the battle between East and West. Zero was the center of the struggle between religion and science. Zero became the language of nature and the most important tool in mathematics. And the most profound problems in physics – the dark core of a black hole and the brilliant flash of the big bang, are struggles to defeat zero.
Yet, through all its history, despite rejection and exile, zero has always defeated those who opposed it. Humanity could never force zero to fit its philosophies. Instead, zero shaped humanity's view of the universe – and of God."

Charles Seife

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"The mathematical description of the world depends on a delicate interplay between continuity and discontinuous, discrete phenomena. The latter are perceived first. 'Functions, just like living beings, are characterized by their singularities'. Singularities, bifurcations and catastrophes are different terms for describing the emergence of discrete structures from smooth, continuous ones."

Vladimir Igorevich Arnol'd (1937–)

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"Mathematics is the grammar of science and order."

Lancelot T. Hogben (1895–1975)

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"We should now place the operational calculus with Poincaré discovery of automorphic functions and Ricci's discovery of the tensor calculus as the three most important mathematical advances of the last quarter of the 19^{th} century. Applications, extension, and justifications of it constitute a considerable part of the mathematical activity of today."

Edmund Taylor Whittaker (1873–1956), 1928

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"The whole form of modern mathematical thinking was created by Euler. It is only with the greatest difficulty that one is able to follow the writings of any author preceding Euler, because it was not yet known how to let the formulae speak for themselves. This art Euler was the first to teach.."

F. Rudio (1857–1929)

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"Mathematics is an exciting, dynamic field that continues to generate provocative ideas and novel concepts. Some notions quickly find their way into applications; others rest on their intrinsic beauty; still others occupy a modest place in the growing structure of mathematics itself."

Ivar Peterson

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"The mathematician is simultaneously a revolutionary and a conservative; a deep–rooted skeptic and an avowed optimist."

Max Dehn (1878–1952)

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"When, after a thousand-year stupor, European thought shook off the effect of the sleeping powders so skillfully administered by the Christian Fathers, the problem of infinity was one of the first to be revived."

"In the history of culture, the discovery of zero will always stand out as one of the greatest single achievements of the human race."

Tobias Dantzig (1884–1956)

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"Perhaps the most surprising thing about mathematics is that it is so surprising. The rules which we make up at the beginning seem ordinary and inevitable, but it is impossible to foresee their consequences. These have only been found out by long study, extending over many centuries. Much of our knowledge is due to a comparatively few great mathematicians such as Newton, Euler, Gauss, Cauchy or Riemann; few careers can have been more satisfying than theirs. They have contributed something to human thought even more lasting than great literature, since it is independent of language."

E.C. Titchmarsh (1899–1963)

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"I like mathematics because it is not human and has nothing particular to do with this planet or with the whole accidental universe - because like Spinoza's God, it won't love us in return."

Bertrand Russell (1872–1970), 1912

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“Geometry, however, supplies sustenance and meaning to bare formulas. Geometry remains the major source of rich and fruitful intuitions, which in turn lend creative power to mathematics. Most mathematicians think in terms of geometric schemes, even though they leave no trace of that scaffolding when they present the complicated analytical structures. One can still believe Plato’s statement that geometry draws the soul toward truth.’
Mathematics is a marvelous invention, but the marvel lies in the human mind’s capacity to construct understandable models of complex and seemingly inscrutable natural phenomena and thereby give man some enlightenment and power. Mathematics may be the queen of the science and therefore entitled to royal prerogatives, but the queen who loses touch with her subjects may lose support and even be deprived of her realm. Mathematicians may like to rise into the clouds of abstract thought, but they should, and indeed they must, return to earth for nourishing food or else die of mental starvation. They are on safer and saner ground when they stay close to nature. As Wordsworth put it “Wisdom oft is nearer when we stoop than when we soar.”

“The tantalizing and compelling pursuit of mathematical problems offers mental absorption, peace of mind amid endless challenges, repose in activity, battle without conflict, ‘refuge from the goading urgency of contingent happenings,’ and the sort of beauty changeless mountains present to sense tried by the present-day kaleidoscope of events.”

Morris Kline (1908–1992)

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“Mathematics was born and nurtured in a cultural environment. Without the perspective which the cultural background affords, a proper appreciation of the content and state of present-day mathematics is hardly possible.”

R.L. Wilder (1896–1982)

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"After having spent years trying to be accurate, we must spend as many more in discovering when and how to be inaccurate."

Ambrose Gwinett Bierce (1842–1914)

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"Occupation with the study of mathematics is the best remedy against the lusts of the flesh."

Thomas Mann (1875–1955)

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"I had a feeling once about mathematics — that I saw it all. Depth beyond Depth was revealed to me — the Byss and the Abyss. I saw a quantity passing through infinity and changing its sign from plus to minus. I saw exactly how it happened and why the tergiversation was inevitable — but it was after dinner and I let it go."

Winston Churchill (1874–1965)

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"The solution of problems is one of the lowest forms of mathematical research, ...yet its educational value cannot be over estimated. It is the ladder by which the mind ascends into higher fields of original research and investigation. Many dormant minds have been aroused into activity through the mastery of a single problem."

Benjamin Finkel (1865–1947), 1894

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"Mathematics is the language of the possible."

(Anon)

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"Topologist – a man who cannot tell the difference between a cup and a doughnut."

(Anon)

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"Mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country."

(Anon)

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"Part of the charm in solving a differential equation is in the feeling that we are getting something for nothing. So little information appears to go into the solution that there is a sense of surprise over the extensive results that are derived."

(Anon)

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"Black holes result from God dividing the universe by zero."

(Anon)

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"The infinite! No other question has ever moved so profoundly the spirit of man."

(Anon)

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"The true spirit of delight, the exaltation, the sense of being more than Man, which is the touchstone on the highest excellence is to be found in mathematics as surely as in poetry."

(Anon)

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"The Theory of Groups is a branch of mathematics in which one does something to something and then compares the result with the result obtained from doing the same thing to something else, or something else to the same thing."

(Anon)

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"I'm sorry to say that the subject I most disliked was mathematics. I have thought about it. I think the reason was that mathematics leaves no room for argument. If you made a mistake, that was all there was to it."

(Anon)

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"What is it that constitutes value or importance in mathematical research? To put it another way: Wherein lies the difference between valuable research and laborious trifling? Or, again, it may be asked: How is valuable research to be distinguished from the construction of examination questions or from mathematical recreations? The distinction is recognized to exist. It is recognized that some kinds of work are better worth doing than others... It is easy to note certain qualities by which valuable research work is characterized.

One of these qualities is novelty. In any valuable research this element of creation or novelty cannot be absent. The new thing, or the created thing, may be a new idea, or a new method, or a new result, or a new proof of a known result... It seems that the progress of mathematics needs many kinds of work. A worker who introduces a new idea may be compared with the exploring prospector who discovers that there is gold in a country; one who invents a new method, with the mechanician who devises the processes and perfects the tools by which the gold can be extracted; one who obtains new results, with the miner who extracts the gold; one who obtains new proofs of known results, with the metallurgist who refines the gold and uses it for making beautiful objects. Few of us can hope to play the part of the prospector or the mechanician, but their efforts would be fruitless without the work of the miner and the metallurgist.

After this all-important character of novelty, or artistic creation, we may note other qualities which valuable research must possess. One of these, which is not very easy to define, I propose to call "relevancy". A piece of work, to be valuable, must be a branch of the tree of knowledge; it must stand in a proper relation to the state of mathematical knowledge existing at the time when it is produced. If it is isolated or has no such relation, it is irrelevant. A proposition may be new and true and difficult to prove and yet it may be irrelevant. Let me give an example. Prof. Hobson, in his little book on the Squaring of the Circle, *has given us a series of most interesting surveys of the state of knowledge in regard to this problem existing at various periods. Anyone who should now spend time on developing new series for calculating approximate values for π, after the fashion of Gregory's series for the inverse tangent, or Newton's series for the inverse sine, would be doing work that might have been valuable in the seventeenth century but would be irrelevant now. This example is rather extreme, but the quality of relevancy or irrelevancy attaches in greater or less degree to all original work in mathematics.*

Another quality which characterizes valuable original research may be named "definiteness". A piece of research work should aim at giving a definite answer to a definite question. For example, the most famous work of Galois aimed at answering the question: What algebraic equations can be solved by means of radicals? We observe in regard to this question that when asked it was supremely relevant. . .

We have noted three qualities as characteristic of valuable research in mathematics: novelty, relevancy, definiteness, I would add to this catalogue a fourth: generality. The best work is never parochial, it is never restricted to a narrow outlook. The quality of generality may seem to be opposed to the quality of definiteness, but generality must not be confused with vagueness. As an example of a piece of work which shows conspicuously the mark of generality, I would cite Gauss' famous memoir on the hypergeometric series. At the time when this was published it could be said of it that it included the theory of almost all the functions which up to that time had been investigated by analysts. But there is nothing indefinite or vague about Gauss' work. . . Is there then such a thing as excessive generality? A story is told concerning a certain variety of roses which were in great demand among the makers of bouquets, not only on account of their beauty, but especially because the stalks were very long and stiff. The growers took steps to increase the length of the stalk, and were very successful, producing blooms with stalks as much as seven feet long. But unfortunately as the stalk lengthened the bloom dwindled, indeed most of the very long stalks bore no flowers at all. It may be treading on dangerous ground to suggest that there is such a thing as excessive generality, though even so convinced an analyst as Picard is not without misgivings on the subject. Yet it can sometimes be wished that writers who develop general theories at great length would pause to inquire how far they are available for the solution of special problems. . .

Finally, to revert to the aspect of mathematics as a creative art, I would urge that an essential element in the equipment of an investigator is a literary education, or, if you prefer it, a training in the means of expression. It is necessary to be articulate, but more than this is desirable. It is desirable to be mathematically articulate, to be able to express mathematical ideas in such a way that they can be comprehended easily by those who have the requisite training. Some great work is marred by obscurity. This charge has been brought against even so great an originator as ***Abel****. Others, such as Laplace, are models of lucidity. There is such a thing as style in mathematics, and it is worth cultivating. The mathematician is an artist; and every artist, we have been told by Mr. Bernard Shaw, must grow his own style out of himself. But there are points of style to which it is desirable to attend, such as clearness, arrangement, rigor, avoidance of haste, conciseness, notation. It* *is desirable to say exactly what one means, neither less nor more. It is desirable to introduce new ideas, or new relations, one at a time, so that each one seems to arise naturally just at the place where it makes its appearance in a piece of written work. No trouble is too great to secure rigor, if it can be secured. We have all heard how Newton kept back the publication of the work, which was ultimately embodied in the* Principia, *until he had obtained a conclusive proof that spheres attract as if their masses were condensed at their centers. If absolute rigor has occasionally to be sacrificed, it should be made perfectly clear at what points it is absent. A memoir should not bear marks of hurry; the argument should be developed in a straightforward fashion from the premises to the conclusion. On the other hand, it should not waste time, as, for instance, by undue restriction of conditions in the main argument, with the object of excluding exceptional cases, or by overloading the main argument with details; it should always be possible to distinguish the wood from the trees. The choice of notation is not to be despised; it may make all the difference to the ease with which a piece of work can be assimilated, or a new idea applied to new questions. It may be necessary to rewrite a memoir more than once or twice if these advantages are to be secured. It is worth while.”*

Augustus Edward Hough Love (1863–1940), 1914

6. Truth, Error, Probability and Statistics

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"It is never possible to step twice into the same river."

Heraclitos

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"Everything existing in the Universe is the fruit of chance and necessity."

Democritos of Abdera (ca 460–370 BCE)

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"The probable is what usually happens."

Aristotle (384–322 BCE)

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"Probability is the very guide of life."

"Probabilities direct the conduct of the wise man."

Marcus Tullius Cicero (ca 50 BCE)

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"The only certainty is that there is nothing certain."

Pliny the elder (23–79 CE)

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"Time and chance happeneth to them all."

Ecclesiastes **9**, 11

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"Truth emerges more readily from error than from confusion."

"If a man will begin with certainties he shall end in doubts; but if he will be content to begin with doubts he shall end in certainties."

Francis Bacon (1561–1626)

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"When it is not in our power to determine what is true, we ought to follow what is probable."

René Descartes (1596–1650)

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"One can have three principle objects in the study of truth: to discover it when one searches for it, to prove it when one possesses it and to distinguish it from falsity when one examines it".

"Contradiction is not a sign of falsity, nor the lack of contradiction a sign of truth."

Blase Pascal (1623–1662)

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"Almost all human life depends on probabilities."

Voltaire (1694–1778)

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"The finite mind cannot attain to the full truth about things through similarity. For the truth is neither more or less, but rather indivisible. What is itself not true can no more measure the truth than what is not a circle can measure a circle. Hence reason, which is not the truth, can never grasp the truth so exactly that it could not be grasped infinitely more exactly. Reason stands in the same relation to the truth as the polygon to the circle; yet even when the number of vertices grows infinite, the polygon never becomes equal to a circle, unless it becomes a circle in its true nature. The real nature of what exists, which constitutes its truth, is therefore never entirely attainable. It has been sought by all the philosophers but never really found."

Nicolas of Cusa (1401–1464), 1440

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"I cannot get over my amazement as to the mental inertia of our astronomers in general who, like credulous women, believe what they read in the books, tablets, and commentaries as if it were the divine and unalterable truth; they believe the authors and neglect the truth. It is necessary to keep the stars doggedly before one eyes, and to rid posterity from ancient tradition."

Regiomontanus (1436–1476)

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"There is no isolated truth."

Jean-Francois Millet (1814–1875)

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"It is easier to perceive error than to find truth, for the former lies on the surface and is easily seen, while the latter lies in the depth, where few are willing to search for it."

Johann Wolfgang von Goethe (1749–1832)

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"The most important questions of life are, for most part, really only problems of probability."

"But since he has unnecessarily complicated the method by the considerations of diagrams, I shall present it here in its simplest analytical form."

"It is remarkable that a science which began with the consideration of games of chance should have become important object of human knowledge."

Pierre Simon de Laplace (1749–1827)

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"Everyone wishes to have truth on his side, but not everyone wishes to be on the side of truth."

Richard Whately (1787–1863)

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"All the compliments that I have received from Arago, Laplace and Biot never gave me so much pleasure as the discovery of a theoretical truth, or the confirmation of a calculation by experiment."

Augustin Jean Fresnel (1788–1827)

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"In the field of observation, chance favors the prepared mind."

Louis Pasteur (1822–1895)

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"When you can measure what you are speaking about, and express it in numbers, you know something about it."

Lord Kelvin (1824–1907)

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"It is the customary fate of new truths to begin as heresies and to end as superstitions."

Thomas Henry Huxley (1825–1895)

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"Truth is stranger than fiction, but it is because fiction is obliged to stick to probabilities; Truth isn't."

Mark Twain (1835–1910)

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"Don't be consistent, but be simply true."

Oliver Wendell Holmes (1841–1935)

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"Great men's errors are to be venerated as more fruitful than little men's truths."

Nietzsche (1844–1900)

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"If a million people believe a foolish thing, it is still a foolish thing."

Anatole France (1844–1924)

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"He uses statistics as a drunken man uses lamp posts — for support rather than illumination."

Andrew Lang (1844–1912)

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"The Gaussian distribution — physicists think that it is a mathematical theorem while mathematicians believe that physicists have verified it experimentally."

Gabriel Lippman (1845–1921)

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"Man can believe the impossible, but can never believe the improbable."

"The pure and simple truth is rarely pure and never simple."

Oscar Wilde (1854–1900)

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"A new scientific truth does not triumph by convincing opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it."

Max Planck (1858–1947)

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"If your experiment needs statistics, you ought to have done a better experiment."

Ernest Rutherford (1871–1937)

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"You can only find truth with logic if you have already found truth without it."

G.K. Chesterton (1874–1936)

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"Man occasionally stumble over the truth, but most of them pick themselves up and hurry off as if nothing happened."

Winston Spencer Churchill (1874–1965)

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"Faith — an illogical belief in the occurrence of the impossible."

Henry Louis Mencken (1880–1956)

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"The conception of chance enters into the very first steps of scientific activity in virtue of the fact that no observation is absolutely correct. I think chance is more fundamental concept than causality; for whether in a concrete case, a cause-effect relation holds or not can only be judged by applying the laws of chance to the observation."

Max Born (1882–1970)

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"Truth exists, only falsehood has to be invented."

Georges Braque (1882–1963)

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"To call the statistician after the experiment is done may be no more than asking him to perform a postmortem examination: he may be able to say what the experiment died of."

R.A. Fisher (1890–1962)

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"If an effect is really there, it shouldn't take a statistician to bring it out."

Harold Jeffreys (1891–1989)

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"Facts do not cease to exist because they are ignored."

Aldous Huxley (1894–1963)

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"The progress of science implies not only the accumulation of knowledge, but its organization, its unification, and this involves the periodical invention of new syntheses, coordinating existing knowledge, and of new hypotheses, which give us methods of approaching the unknown. Science is essentially a system, but instead of being, as it was for the schoolmen, a closed system, it is never closed, but always subject to revision or even to complete discard. The true scientist considers his theories not as perfect and permanent, but as essentially incomplete and precarious; he is ever ready to abandon any part or the whole of them, should new experimental facts make it necessary. There are scientific methods; there are no scientific dogmas; there is no scientific orthodoxy. Of course this does not mean that there are no scientific doctrines; there are at any time many of them which are binding as long as they have not been shown to be erroneous, but not a moment longer; an orthodoxy as mobile as that is not a real orthodoxy as theologians understand it. Nor does it mean that men of science are never dogmatic, for being men, they are necessarily frail. In this sense, we might say that men of science are essentially heterodox. Their heterodoxy is not restricted to this or that doctrine; it extends potentially to every doctrine. We would never think of saying that a genuine man of science is loyal to this or that theory as he would be loyal to his church or country; he knows no such scientific loyalty; his only loyalty is to truth,a loyalty which causes him to abandon his most cherished opinions as soon as they are invalidated. This has been proved repeatedly during the last thirty years, for a number of revolutionary discoveries have put his scientific conscience into the crucible. For example, the discovery of radioactivity, the introduction of quanta, and the theories of relativity have obliged every physicist and chemist to change radically his ways of thinking about essential things; they have done it as soon as their conviction was completed, if not without reluctance (it is not easy, especially for older men, to change the intellectual habits of a lifetime), at least without struggle and without rancor. As for the younger men, the more revolutionary the theories, the more exhilarating; it gave them the impression of being the witnesses of a new revelation, of a new beginning. Science is not a being, but a becoming. Thus the love of science is not the love of this or that system, which is bound sooner or later to be superseded by a better one, but simply the love of truth."

George Sarton (1884–1956)

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"It must always be kept in mind that "true" values are unknown *and must remain unknown; so that the errors being deviations from an unknown value, are likewise unknown. True values must be* estimated *by appropriate substitutes, namely, "best" or optimal values, and errors by the deviations of the observed from the optimal values."*

Alexander Craig Aitken (1895–1967)

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"It is not so much important to be rigorous as to be right."

A.N. Kolmogorov (1903–1987)

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"Error can often be fertile but perfection is always sterile."

A.J.P. Taylor (1906–1990)

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"There is no part of mathematics *that is intimately connected with everyday experiences than the theory of* probability, *and recent developments in mathematical physics have emphasized the importance of this theory in every branch of science. Knowledge of* probability *is required in such diverse fields as quantum mechanics, kinetic theory, the design of experiments, and the interpretation of data. Operations analysis applies* probability *methods to questions of traffic control, allocation of equipment, and the theory of strategy. Cybernetics, another field of recent origin, uses the theory to analyze problems in communication and control."*

I.S. Sokolnikoff

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"Statisticians have an understandable penchant for viewing the whole of the history of science as revolving around measurement and statistical reasoning. This view, which stops very short of insisting that science is only *measurement, is not entirely parochial."*

Stephen M. Stigler

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"Statistics are like a bikini. What they reveal is suggestive, but what they conceal is vital."

A. Levenstein

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"An error doesn't become a mistake until you refuse to correct it."

Orlando A. Battista

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"If you don't make mistakes, you are not working on hard enough problems, and that is a big mistake."

Frank Wilczek (1951–)

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"The human being is an incurable optimist: he believes he has a pretty good chance to win a lottery prize, but that there is scarcely the slightest chance of his getting killed in a traffic accident."

(Anon)

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"Everyone believes that the Gauss distribution describes the distribution of random errors: Mathematicians — because they think physicists have verified it experimentally, Physicists — because they think mathematicians have proved it theoretically."

(Anon)

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"Every scientific truth is an approximation."

(Anon)

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"Truth is not determined by majority vote."

(Anon)

Some vital statistics:

- Only one out of 10,000 women have the classical measures of 35–25–35 or 36–25–36.
- 200,000 babies are born daily in the world (1993).
- A man of 60 has spent 20 years in bed and over 3 years in eating.
- Mathematicians publish more than 200,000 theorems every year.
- 40,000 persons die yearly of snake bites (mostly in India, from Cobra bites); only 12 them in the USA (1993).
- There are 37 million dogs in American homes. 5 million of them are turned each year to be annihilated (1993).
- 2 million people in the USA are in prison.
- More then a million American teen-agers get pregnant each year.
- About 100,000 people in the USA die of physician's errors each year.
- 1.2 billion people in the world are overweight and 1.2 billion are underweight.
- Close to 17,00 stores are selling and renting video cassettes in the USA. The x-rated biz has become a home and hearth industry. It finally took sex out of the theaters and put it back in the bedroom, where it belongs.
- There are 30 million singles over 40 in the US (2006).
- There are 100,000 people older then 100 years in the world (2000).
- An average US family earns $ 35,000 per year, to be spent as follows: Household (9,000), Insurance (4,800), Taxes (6,500), Payments (13,500), Savings (1,200).
- There are 27 million functionally illiterate Americans.
- An estimated 10–30 percent of American and British babies are illegitimate. The seducer usually proves to be the male next door.
- There are about 500,000 physicians in the US.
- In the US, a woman is raped every 6 minutes.
- There are 8,000 different words in King's John version of Bible, but 34,000 words in Shakespeare.

- There are 60,000 homeless people in New York city. They sleep in the streets. Many die in winter.
- There are one million millionaires in the US; every 39 minutes another millionaire is made there.
- In American homes, television sets are on an average of 42.7 hours per week.
- 5,000 young people commit suicide every year in the US.

7. On Philosophy, Philosophers and Man

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"Composite things decay. Drive diligently"

Buddha (ca 563–483 BCE)

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"All is foreseen and free will is given."

Rabbi Akiva Ben Yosef (ca 50–ca 135 CE)

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In 330 BCE ***Aristotle*** *taught as that a human being is a creature whose distinguishing feature and chief survival mechanism is its ability to consider the world* rationality*. In the end* reason *will sway emotion*

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Immanuel Kant *declared science incapable of solving the three fundamental problems of metaphysics: God, freedom of will, and immortality. He thus contended that physics can never determine if God exists, if we have free will, or if God will grant us immortal life.*

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David Hume *maintained that since divinity and metaphysics do not contain any abstract reasoning concerning quantity or number it does not contain any experimental reasoning concerning matter of fact.*

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"Man is measure of all things; of those which are – that they are, of those which are not – that they are not."

Protagoras of Abdera (ca. 490–420 BC)

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"The unexamined life is not worth living".

Socrates (470–399 BCE)

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"The ultimate purpose of philosophy is to lead man to the good life of true happiness."

Epicurus (341–270 BCE)

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"Philosophy is the science which considers truth".

Aristotle (384–322 BCE)

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"Of all existing things – some are in our power, and other are not. In our power are: thought, desire, will to chose and will to avoid, and, in a word, everything which is our own doing. Things not in our power include the body, property, reputation, office, and, in a word, everything which is not our own doing. Things in our power are by nature free, unhindered, untrammeled; things not in our power are weak, servile, subject to hindrance, dependent on others."

"All philosophy lies in two words, *sustain* and *abstain."*

Epictetos (ca. 55–ca. 135)

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"Astra inclinat, non trahunt"
(Stars impel, but they do not compel)

Latin maxim

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"Mankind must pray to God for fortune but obtain wisdom for themselves."

Marcus Tullius Cicero (106–43 BCE)

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"Philosophy – molds and constructed the soul, guides our conduct, shows us what we should do and what we should leave undone; it sits at the helm and directs our course as we waver amid uncertainties. Without it, no one can live fearlessly or in peace of mind".

Lucius Annaeus Seneca (4 BCE–65 CE)

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"But there was never yet philosopher that could endure the toothache patiently."

"Much Ado About Nothing"

"Hang up philosophy!
Unless philosophy can make a Juliet,
Displant a town, reverse a prince's doom,
It helps not, it prevail not, talk no more."

"Romeo and Juliet" **3**, 3

"There are more things in heaven and earth, Horatio, than are dreamt of in your philosophy."

William Shakespeare (1564–1616), "Hamlet" **1**-5, 191-2

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"Things do not pass for what they are but for what they seem."

Baltasar Gracián

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"Esse is percipi"
(nothing exists unless it is perceived by some mind)

George Berkeley

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"Is God willing to prevent evil, but not able? – Then he is impotent.
Is he able but not willing? – Then he is malevolent.
Is he both able and willing? – Whence then is evil."

David Hume (1711–1776)

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"Know thyself."? If I knew myself, I'd run away."

J.W. von Goethe (1749–1832)

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"There are two kinds of truths: those of reasoning and those of facts. The truths of reasoning are necessary and their opposite is impossible.
The truths of fact are contingent and their opposites are possible."

Leibniz (1646–1716)

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"Life can only be understood backwards, but it must be lived forwards."

Søren Kierkegaard (1813–1855)

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"The philosopher *is not interested in truth, but only in "my truth"."*

"There is more wisdom in your body than in your deepest philosophy.*"*

"All ethics begins when the individual is taken to be of infinite importance *– in contrast to nature, which behaves cruelly and playfully toward the individual."*

F.W. Nietzsche (1844–1900)

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*"*Physics *is what you know,* philosophy *is what you don't know."*

Bertrand Russel (1872–1970)

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"Life is at the same time freedom and fatality: we accept the fatality and within it we decide on a destiny."

Jose Ortega y Gasset (1883–1955)

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"Philosophy is not a theory but an activity."

Ludwig Wittgenstein (1889–1951)

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"If Napoleon had been as intelligent as Spinoza, he would have lived in a garret and written four books."

Anatole France (1844–1924)

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"Chemistry emerged from alchemy as astronomy from astrology and physics from philosophy."

"Every man of science somewhat of a cynic*, because he does not accept words and conventions at their face value, and of a* skeptic*, because he refuses to believe anything without adequate proof."*

George Sarton (1884–1956)

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"Where there is the necessary technical skill to move mountains, there is no need for the faith that move mountains."

Eric Hoffer (1902–1983)

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"Philosophy: unintelligible answers to insoluble problems."

Henry B. Adams (1803–1873)

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"It is easy to build a philosophy. It doesn't have to run."

C.F. Kettering (1876–1958)

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"Die Logic ist zwar unerschüterlich, aber einem menschen, der leben will, widersteht sie nicht."

Franz Kafka (1883–1924)

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"Faith – an illogical belief in the occurrence of the impossible."

"There is no record in human history of a happy philosopher."

H.L. Mencken (1880–1956)

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"To believe in God is to desire his existence, and what is more, to act as though he existed."

Miguel de Unamuno (1864–1936)

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"Logic does not apply to the real world."

M.L. Minsky

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"That which enters the mind through reason *can be corrected.*
That which is admitted through faith – hardly ever."

Santiago Ramón y Cajal (1852–1934)

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"'Thinkers' are people who re-think; who think that what was thought before was never thought enough*."*

Paul Valéry (1871–1945)

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"Truth is the object of philosophy, but not always of philosophers."

John Churton Collins (1848–1908)

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"What pride to discover that nothing belongs to you – what a revelation."

Emile M. Cioran (1911–1995)

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"Nothing matters very much, and very few things matter at all."

A.J. Balfour (1848–1930)

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"Idealism is what precedes experience; cynicism is what follows."

David T. Wolf (1943–)

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"Once there were philosophers – today there are only professors of philosophy."

(Anon)

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"Without discipline there is no knowledge and without philosophy there is no purpose."

(Anon)

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"No real problem has a solution."

(Anon)

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"If God is dead, everything is permitted."

Jean-Paul Sartre (1905–1980)

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Napoleon: "Why, in your entire treatise Celestial Mechanics*, you had not once mentioned God?"*
Laplace: "Sir, I had no need for that hypothesis."

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"The philosopher-mathematician Bernard Russell was once asked why he did not believe in God.
He replied: "Not enough evidence".

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"In his congressional testimony (1987), Steven Weinberg asked for money to build the SSC, a $10 billion device to be constructed in Texas (funding has since been cut off). A congressman asked Weinberg if the SSC would enable us to find God, and Weinberg (unlike Laplace) declined to answer."

8. On Science, Engineering and Technology[59]

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"It is easy to distinguish those who argue from fact and those who argue from notions... The principles of every science are derived from experience: thus it is from astronomical observations that we derive the principles of astronomical science."

Aristotle (384–322 BCE)

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"Science liberates man from the terror of the gods."

Lucretius (ca 99–55 BCE)

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"Entia non sunt multiplicanda praeter necessitatem." (Entities should not be multiplied unnecessarily.)

"Pluritas non est ponendra sine necessitate."

William of Ockham (1285–1349)

[59] *Technology*: from the Greek *techne* – practical skill. The Greeks relegated skills to a lower sphere; the ideal of a free man was leisure (*schole*), and the pursuit of wisdom which it permitted. Only in the modern world was *techne* made into a prodigious instrument for scientific investigation and material progress.

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"In dealing with a scientific problem, I first arrange several experiments, since my purpose is to determine the problem in accordance with experience, and then to show why the bodies are compelled so to act. That is the method which must be followed in all researches upon the phenomena of Nature... We must consult experience in the variety of cases and circumstances until we can draw from them a general rule that is contained in them. And for what purpose are these rules good? They lead us to further investigations of Nature and to creation of art. They prevent us from deceiving ourselves, or others, by promising results to ourselves which are not to be obtained."

"There is no certainty in science where one of the mathematical sciences cannot be applied."

"Science is the observation of things possible."

Leonardo da Vinci (1452–1519)

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"Science is not a belief to be held but a work to be done."

Francis Bacon (1561–1626)

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"In matters of science, the authority of a thousand does not stand against the humble opinion of one."

Galileo Galilei (1564–1642)

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"It (my book) may wait a century for a reader, as God has waited six thousand years for an observer."

Johannes Kepler (1571–1630)

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"The world is my country, science is my religion."

Christiaan Huygens (1629–1695)

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"I wish to direct all sciences to one end and aim, so that we may attain to have supreme human perfection which we have named; and, therefore, whatsoever in the sciences does not serve to promote our object will have to be rejected as useless."

Baruch Spinoza (1632–1677)

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"The first man of science was he who looked into a thing, not to learn whether it furnished him with food, or shelter, or weapons, or tools, or armaments, or playwiths but who sought to know it for the gratification of knowing."

Samuel Taylor Coleridge (1772–1934)

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"I have had my results for a long time: but I do not yet know how I am to arrive at them."

Carl Friedrich Gauss (1777–1855)

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"Science says the first word on everything, and the last word on nothing."

Victor Hugo (1802–1885)

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"In science one must search for ideas. If there are no ideas, there is no science. A knowledge of facts is only valuable in so far as facts conceal ideas: facts without ideas are just the sweepings of the brain and the memory."

Vissarion Belinskii (1811–1848)

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"Only a misguided mind tries to introduce religion into science. More misguided still, is he who attempts to introduce science into religion, because he entertains greater respect for the scientific method."

Louis Pasteur (1822–1895)

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"Accurate and minute measurements seem to the non-scientific imagination a less lofty and dignified work than looking for something new. But nearly all the grandest discoveries of science have been but the rewards of accurate measurements and patient long-continued labors in the minute sifting of numerical results.
I often say that when you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind: it may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the stage of science, whatever the matter may be."

"In science there are no paradoxes."

Lord Kelvin (1824–1907)

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"Science is meaningless because it gives no answer to our question, the only question important for us: 'What shall we do and how shall we be?'"

Lev Nikolayevich Tolstoy (1828–1910)

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"Science is for those who learn; poetry for those who know."

Joseph Roux (1834–1886)

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"There is something fascinating about science. One gets such wholesale returns of conjecture out of such a trifling investment of fact."

Mark Twain (1835–1910)

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"Do you believe that the sciences would ever have arisen and become great if there had not beforehand been magicians, alchemists, astrologers and wizards, who thirsted and hungered after abscondite and forbidden powers?
Indeed, infinitely more had to be promised *than could ever be fulfilled in order that anything at all might be fulfilled in the realms of knowledge."*

Friedrich Nietzsche (1844–1900), 1886

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"In science the credit goes to the man who convinces the world, not to the man to whom the idea first occurs."

Francis Darwin (1848–1925), 1914

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"Science is always simple and always profound. It is only the half-truths that are dangerous."

George Bernard Shaw (1856–1950)

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"Thus I saw that most men only care for science so far as they get a living by it, and that they worship even error when it affords them a subsistence."

Johann Wolfgang von Goethe (1749–1832)

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"Science is the knowledge of many orderly and methodically digested and arranged so as to become attainable by one."

A.F.W. Herschel

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"Culture is one thing and varnish is another."

Ralph Waldo Emerson (1803–1882)

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"Culture is to know the best that has been said and thought in the world."

Mattew Arnold (1822–1888)

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"There are no such things as applied science, only applications of science."

"In the field of observation, chance favors the prepared mind."

Louis Pasteur (1822–1895), 1872

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"Science has founded the only true religion. Science is the only redemption of this world."

Robert G. Ingersoll (1833–1899), 1906

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"An experiment is a question which science poses to Nature, and a measurement is the recording of Nature's answer."

"A new scientific truth does not triumph by convincing opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it."

"Scientists don't change their minds, they just die."

"Science cannot solve the ultimate mystery of nature. And that is because, in the last analysis, we ourselves are part of nature and therefore part of the mystery we are trying to solve."

"Science is a mountain from which we can see far and wide into the surrounding terrain, but the mountain itself, is not visible."

Max Planck (1858–1947)

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"Every great advance in science has issued from a new audacity of imagination."

John Dewey (1859–1952)

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"Scientific principles and laws do not lie on the surface of nature. They are hidden, and must be wrested from nature by an active and elaborate technique of inquiry."

John Dewey (1859–1952), 1920

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"One machine can do the work of fifty ordinary men. No machine can do the work of one extraordinary man."

Elbert Hubbard (1856–1915), 1923

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"Science is nothing but trained and organized common sense."

T.H. Huxley (1825–1895)

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"It is of the highest importance in the art of detection to be able to recognize out of a number of facts which are incidental and which are vital. . . .I would call your attention to the curious incident of the dog in the night-time." "The dog did nothing in the night-time." "That was the curious incident."

"It is a capital mistake to theorize before one has data. Insensibly one begins to twist facts to suit theories instead of theories to suit facts."

Arthur Conan Doyle (1859–1930)

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"The aims of scientific thought are to see the general in the particular and the eternal in the transitory."

Alfred North Whitehead (1861–1947)

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"Science is a cemetery of dead ideas."

Miguel de Unamuno (1864–1936)

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"In science we must be interested in things, not in people."

Marie Curie (1867–1934)

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"...the problem of all natural philosophy is to drive out qualitative conceptions and to replace them by quantitative relations."

Robert Millikan (1868–1953)

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"Nothing is more vulnerable and ephemeral than scientific theories, which are mere tools and not everlasting truths."

"Science is not the summa *of life. It is only one of the forms of human thought."*

Carl Gustav Jung (1875–1961)

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"Although this may seem a paradox, all exact science is dominated by the idea of approximation."

"In art nothing worth doing can be done without genius; in science, even a very moderate capacity can contribute to a supreme achievement."

"Science is what you know, philosophy is what you don't know."

"Man has existed for about a million years. He has possessed writing for about 6000 years, agriculture somewhat longer, but perhaps not much longer. Science, as a dominant factor in determining beliefs of educated men, has existed for about 300 years. In this brief period it has proven itself as an incredibly powerful revolutionary force. When we consider how recently it has risen to power, we find ourselves forced to believe that we are at the very beginning of its work in transforming human life."

"The science have developed in an order the reverse of what might have been expected. What was most remote from ourselves was first brought under the domain of law, and then, gradually, what was nearer: first the heavens, nest the earth, then animal and vegetable life, then the human body, and last of all (as yet very imperfectly) the human mind."

Bertrand Russell (1872–1970)

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"Scientists should be on top, but not at the top."

"Science, which now offers us a golden age with one hand, offers at the same time with the other the doom of all that we have built up inch by inch since the Stone Age and the dawn of any human annals. My faith is in the high progressive destiny of man. I do not believe we are to be flung back into abysmal darkness by those fiercesome discoveries which human genius has made. Let us make sure that they are servants, but not our masters."

"The Dark Ages may return on the gleaming wings of Science."

Winston Spencer Churchill (1874–1965)

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"The most beautiful thing we can experience is the mysterious. It is the source of all true art and science."

"There are no eternal theories in science, nearly every great advance in science arises from a crisis in the old theory, through an endeavor to find a way out of the difficulties created."

"No amount of experimentation can ever prove me right; a single experiment can prove me wrong."

"Science is the attempt to make the chaotic diversity of our sense-experience correspond to a logically uniform system of thought."

" Once the Teutonic barbarians had destroyed Europe's ancient culture, a new and finer cultural life slowly began to flow from two sources that had somehow escaped being altogether buried in the general havoc — the Jewish Bible and Greek philosophy and art. The union of these two sources, so different one from other, marks the beginning of our present cultural epoch, and from that union, directly or indirectly, has sprung all that makes the true values of our present-day life."

"Science without religion is lame, religion without science is blind."

"A theory is more impressive the greater the simplicity of its premises is, the more different kinds of things it relates, and the more extended its area of applicability."

"The whole of science is nothing more than a refinement of everyday thinking."

"No great discovery was ever made in science except by one who lifted his nose above the grindstone of details and ventured on a more comprehensive vision."

"The grand aim of all science is to cover the greatest number of empirical facts by logical deduction from the smallest number of hypotheses or axioms."

Albert Einstein (1879–1955)

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"The scientist describes what is: the engineer creates what never was."

Theodor von Kármán (1881–1963)

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"The moment man cast off his age-long belief in magic, Science bestowed upon him the blessings of the Electric Current."

Jean Giraudoux (1882–1944)

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"Culture is the system of vital ideas by which the age lives. It borrows from science *what is vitally necessary for the interpretation of our existence. There are entire portions of science which are not culture, but pure scientific technique. And vice versa, culture requires that we posses a complete concept of the world and of man: it is not for culture to stop, with science, at the point where the methods of absolute rigor happen to end. Life cannot wait until science may have explained the universe scientifically. We cannot put off living until we are ready. The most salient characteristic of life is its coerciveness: it is always urgent, 'here and now' without any possible postponement. Life is fired at us point-blank. And culture, which is but its interpretation, cannot wait any more than can life itself. Science is not something by which we live. The internal conduct of science is not a vital concern; that of culture is. Science is indifferent to the exigencies of our life, and follows its own necessities. Accordingly, science grows constantly more diversified and specialized without limit, and is never completed. But culture is subservient to our life here and now, and it is required to be, at every instant, a complete, unified, coherent system — the plan of life."*

José Ortega y Gasset (1883–1955)

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“Science is ‘the only human activity which is truly cumulative and progressive’.”

“Science is more than an accumulation of facts; it is not simply positive knowledge, but systematized positive knowledge; it is not simply unguided analysis and haphazard empiricism, but synthesis; it is not simply a passive recording, but constructive activity.”

George Alfred Léon Sarton (1884–1956)

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“Modern science is not relevant to the search for the underlying metaphysical and moral truths by which one lives. They must be intuitively, almost mystically arrived at.”

“Every second of our lives is saturated with the physical consequences of science or, as we could say, with excrements from the progress of research.”

“Modern science may be as far from revealing the underlying laws of the natural universe as was the science of ancient Greece.”

Erwin Schrödinger (1887–1961)

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“Physical models are as different from the world as a geographical map is from the surface of the earth.”

Leon Brillouin (1889–1969)

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"Don't confuse hypothesis *and* theory*. The former is a possible explanation; the latter, the correct one. The establishment of theory is the very purpose of science."*

"Facts are not science — as the dictionary is not literature."

Martin H. Fischer (1879–1962)

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"The fantastic advance in the field of electronic communication constitute a greater danger to the privacy of the individual."

Earl Warren (1891–1974)

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"Science is not technology, it is not gadgetry, it is not some mysterious cult, it is not a great mechanical monster! Science is an adventure of the human spirit. It is essentially an artistic enterprise, stimulated largely by the universe, served largely by disciplined imagination, and based largely on faith in the reasonableness, order, and beauty of the universe of which man is part."

Warren Weaver (1894–1978)

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"It is my belief that if a scientist cannot talk simply about his subject, he has not got to the bottom of it himself."

Edward Victor Appleton (1892–1965)

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"Technology: the invention, manufacture, and use of tools."

Arnold J. Toynbee (1889–1975), 1961

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"In science the credit goes to the man who convinces the world, not to the man to whom the idea first occurs."

William Osler (1849–1919)

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"Science is out of the reach of morals, for her eyes are fixed upon eternal truthes. Art is out of the reach of morals, for her eyes are fixed upon things beautiful and immortal and ever-changing. To morals *belong the lower and less intellectual spheres."*

Oscar Wilde (1854–1900), 1891

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"The simplest schoolboy is now familiar with truths for which Archimedes would have sacrificed his life."

Ernest Renan (1823–1892)

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"Science is built of facts the way a house is built of bricks; but an accumulation of facts is no more science than a pile of bricks is a house."

Henry Poincaré (1854–1912), 1905

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"In chess, if you make one wrong *move you are finished whereas in science if you can do one thing* right*, you are famous."*

Norbert Wiener (1894–1964)

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"Watch out for the engineers — they begin with sewing machines and end up with the atomic bomb."

Marcel Pagnol (1895–1974)

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"Whether we choose to call it pure or applied, the story of science is not something apart from the common life of mankind. What we call pure science only thrives when the contemporary social structure is capable of making full use of its teaching, furnishing it with new problems for solution and equipping it with new instruments for solving them. Without printing there would have been little demand for spectacles, without spectacles neither the telescope nor microscope, without these the finite velocity of light, the annual parallax of the stars, and the micro-organisms of fermentation processes and disease would never have been known to science. Without the pendulum clock and the projectile there would have been no dynamics nor theory of sound. Without the dynamics of the pendulum and projectile, no Principia*. Without deep-shaft mining in the sixteenth century, when abundant slave labor was no longer to hand, there would have been no social urge to study pressure, ventilation and explosion. Balloons would not have been invented, chemistry would have*

barely surpassed the level reached in the third millennium and the conditions for discovering the electric current would have been lacking."

Lancelot Hogben (1895–1975)

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"I do not hesitate to assert that I consider astronomy as the most important force in the development of science since its origin sometime around 500 BCE."

Otto Neugebauer (1899–1990)

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"Atomic energy bears that same duality that has faced man from time immemorial, a duality expressed in the Book of Books thousands of years ago: "See, I have set before thee this day life and good and death and evil ... therefore choose life."

David E. Lilienthal (1899–1981)

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"The danger of the past was that men became slaves. The danger of the future is that men may become robots."

Eric Fromm (1900–1980)

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"A theory is not an absolute truth but a self-consistent analytical formulation of the relations governing a group of natural phenomena."

Julius Adams Stratton (1901–1994)

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"In science one tries to tell people, in such a way as to be understood by everyone, something that no one ever knew before. But in poetry, it's the exact opposite."

Paul Dirac (1902–1984)

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"It is his intuition, his mystical insight into the nature of things, rather than his reasoning which makes a great scientist."

"Every truly scientific theory necessarily has to be couched in a form capable of being disproved."

Karl Raimund Popper (1902–1994)

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"The spiritual and intellectual decline which has overtaken us in the last thirty years ... [may be due] to the diversion of all the best brains to technology."

Kenneth Clarke (1903–1983)

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"To ask in advance for a complete recipe would be unreasonable. We can specify only the human qualities required: patience, flexibility, intelligence."

John von Neumann (1903–1957)

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"In the first-instance the work of science is co-operative; a scientist takes his colleagues as judges, competitors and collaborators."

Robert Oppenheimer (1904–1967)

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"Many civilizations invented different technologies, but science was invented only once."

Loren Eiseley (1907–1977)

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"Modesty befits the scientist, but not the ideas that inhabit him and which he is under the obligation of upholding."

"In science, self-satisfaction is death. Personal self-satisfaction is the death of the scientist. Collective self-satisfaction is the death of the research. It is restlessness, anxiety, dissatisfaction, agony of mind that nourish science."

Jacques Monod (1910–1976)

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"There is not a 'pure' science. By this I mean that physics impinges on astronomy, on the one hand, and chemistry and biology on the other. And not only does each support neighbors, but derives sustenance from them. The same can be said of chemistry. Biology is, perhaps, the example par excellence today of an 'impure' Science."

Melvin Calvin (1911–1997)

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"Technology — the knack of so arranging the world that we don't have to experience it."

Max Frisch (1911–1991)

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"No scientist is admired for failing in the attempt to solve problems that lie beyond his competence. The most he can hope for is the kindly contempt earned by the Utopian politician. If politics is the art of the possible, research is surely the art of the soluble. Good scientists study the most important problems they think they can solve. It is, after all, their professional business to solve problems, not merely to grapple with them."

Peter Medawar (1915–1987)

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"I am sorry to say that there is too much point to the wisecrack that life is extinct on other planets because their scientists were more advanced than ours."

John Fitzgerald Kennedy (1917–1963)

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"This is one of the obvious things that take a very long time to notice . . . "

"In scientific practice, the onus of the proof is always on the advocate of the more complicated hypothesis."

H. Jeffreys (1891–1989)

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"If it keeps up, man will atrophy all his limbs but the push-button finger."

Frank Lloyd Wright (1867–1958)

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"Space isn't remote at all. It's only an hour's drive away if your car could go straight upwards."

Fred Hoyle (1915–2001), 1979

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"Science may be described as the art of systematic over-simplification."

Karl R. Popper (1902–1994), 1982

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"The engineer is the key figure in the material progress of the world. It is his engineering that makes a reality of the potential value of science by translating scientific knowledge into tools, resources, energy and labor to bring them into the service of man ... To make contributions of this kind the engineer requires the imagination to visualize the needs of society and to appreciate what is possible as well as the technological and broad social age understanding to bring his vision to reality."

Sir Eric Ashby (1904–1992), 1958

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"The scientist is not a person who gives the right answers, he's one who asks the right questions."

Claude Levi-Strauss (1908–), 1964

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"It is a medium of entertainment which permits millions of people to listen to the same joke at the same time, and yet remain lonesome."

T.S. Eliot (1888–1965)

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"Science has promised us truth – an understanding of such relationships as our minds cam grasp; it has never promised us either peace or happiness."

Gustav Le Bon (1841–1931)

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"Technology means the systematic application of scientific or other organized knowledge to practical tasks."

"We are becoming the servants in thought, as in action, of the machine we have created to serve us."

John Kenneth Galbraith (1908–2006)

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“ Both science and art have to do with ordered complexity.”

Lancelot Law White (1896–1972)

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“The planet and mankind are in grave danger of irreversible catastrophe... wars of mass destruction, overpopulation, pollution, and the depletion of resources.”

Richard A. Falk

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“The true men of action in our time, those who transform the world, are not the politicians and statesmen, but the scientists. Unfortunately, poetry cannot celebrate them, because their deeds are concerned with things, not persons and are, therefore, speechless.”

Wystan Hugh Auden (1907–1973)

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“The medium is the message. This is merely to say that the personal and social consequences of any medium ... result from the new scale that is introduced into our affairs by each extension of ourselves, or by any new technology.”

Marshall McLuhan (1911–1981)

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"The danger always exists that our technology will serve as a buffer between us and nature, a block between us and the deeper dimensions of our own experience."

Rollo May (1909–1994)

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"The ideal engineer is a composite ... He is not scientist, he is not a mathematician, he is not a sociologist or a writer; but he may use the knowledge and techniques of any or all of these disciplines in solving engineering problems."

N.W. Dougherty, 1955

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"A first-rate theory predicts; a second-rate theory forbids; and a third-rate theory explains after the event."

A.I. Kitaigorodskii (1914–1985)

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"Modern technology
Owes ecology
An apology."

Alan M. Eddison

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"Almost all aspects of life are engineered at the molecular level, and without understanding molecules we can only have a very sketchy understanding of life itself."

Fracis Crick (1916–2004)

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"What was once thought can never be unthought."

Friedrich Dürrenmatt (1921–1990)

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"Any sufficiently advanced technology is indistinguishable from magic."

Arthur C. Clarke (1917–2008)

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"Science is wonderfully equipped to answer the question "how?" but it gets terribly confused when you ask the question "why?"."

Erwin Chargaff (1905–2002)

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"It is scientific only to say what is more likely and what is less likely: the existence of flying saucers was not impossible, just very unlikely."

Richard Phillips Feynman (1918–1988)

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“That which today calls itself science gives us more and more information, an indigestible glut of information, and less and less understanding.”

Edward Abbey (1927–1989)

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“Science is far more exciting than science fiction, far more intricate, far more subtle, and science has the additional virtue of being true.”

Carl Sagan (1934–1996)

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“The probability of success is difficult to estimate, but if we never search, the chance of success is zero.”

Giuseppe Cocconi (1914–) and Philip Morrison (1915–2005)

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“Electronic machines can solve problems which the man who made them cannot solve; but no government subsidized commission of engineers and scientists could create a worm.”

Joseph Krutch (1893–1970)

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“I’ve given up trying to be rigorous. All I’m concerned about is being right.”

Stephen J. Hawking (1942–)

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"Science is really the search for simplicity. William of Occam, a fourteenth-century philosopher made the dictum... "Entities should not be multiplied beyond necessity." This principle of parsimony ... means that no more forces or causes should be postulated than are necessary to account for the phenomenon observed."

Claude A. Villee (1917–2003)

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"The development of science is unlike the smooth and monotonically progressive path which one would hope for. Gains are so easily followed by loss; and what one generation builds, another tears down. And yet all is not lost. Man still by nature desires to know; and if he will but appreciate that the preservation *of knowledge is just as important as its* acquisition*, the spiral course of science may yet be directed always upward."*

Carl B. Boyer (1906–1976)

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"If nature attempts to conceal her tiny secrets, science bares them publicly with magnifications of five million fold. Neither the porcupine nor the mosquito can keep its love life to itself any longer. Scientists peep into their private familiarities and delight in detailed descriptions in lectures, papers and books. If God's molecular gifts are too bulky for human utility, science chops them into little pieces of useful chemicals. If the natural bits are too small, science joins them together into larger units. If the Thanksgiving turkey is too large, a small one is bred. If seeds are not wanted in fruits, seedless varieties are developed. Not satisfied with man's mundane three-dimensional world, science conjures up four- and six-dimensional phantasms. The curiosity of science and her bent on innovation seem uncontrollable. She pries into every heavenly nook and earth cranny. She respects neither the ancient sanctity of tombs nor the caressing intimacies of boudoirs. Science speeds on unabashed!"

R.G.H. Siu

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"Innovation occurs for many reasons, including greed, ambition, conviction, happenstance, acts of nature, mistakes, and desperation. But one force above all seems to facilitate the process. The easier it is to communicate, the faster changes happens. Every time there is an improvement in the technology with ideas and people come together, major changes ensues:

- *The Greek alphabet have birth to philosophy, logic and the democratic process.*
- *The printing press generated the entire Scientific Revolution.*
- *The telegraph brought modern business methods into existence and held empires together.*

Today, supercomputers and fiber-optic networks, with their ability to make unimaginable amounts of data instantly accessible to millions of people, are accelerating the process of change by many orders of magnitude. The tidal wave of the Information Age will soon break upon us."

James Burke (1936–)

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"No good *model ever accounted for* all *the facts since some data was bound to b misleading if not plain wrong."*

James Watson (1928–)

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"It would be a poor thing to be an atom in a universe without physicists, and physicists are made of atoms. A physicist is an atom's way of knowing about atoms."

George Wald (1906–1997)

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"Western society has accepted as unquestionable a technological imperative that is quite as arbitrary as the most primitive taboo: not merely the duty to foster invention and constantly to create technological novelties, but equally the duty to surrender to these novelties unconditionally, just because they are offered, without respect to their human consequences."

Lewis Mumford (1895–1990)

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"Science is not one success after another; it is one success in a desert of failure."

"You must swim ahead; science goes where you imagine it."

Yehuda Folkman (1933–2008)

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"Despite the dazzling success of modern technology and the unprecedented power of modern military systems, they suffer from a common and catastrophic fault. While providing us with a bountiful supply of food, with great industrial plants, with high-speed transportation, and with military weapons of unprecedented power, they threaten our very survival."

Barry Commoner (1917–)

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"Contrary to popular belief, what is important in science is as much its spirit as its product: it is as much the open-mindedness, the primacy of criticism, the submission to the unforeseen, however upsetting, as the result, however new that may be. Ages ago, scientists gave up the idea of an ultimate and intangible truth, the exact image of a "reality" waiting around the corner to be unveiled. They now know they must be satisfied with the incomplete and the temporary. This approach goes against the natural inclination of the human mind, which calls for unity and coherence in its representation of the world under the most varied aspects. In fact, this conflict between the universal and the local, the eternal and the temporary, reappears at regular intervals in certain controversies — for example, in the debate between the advocates of creation and those of evolution, where arguments already used more than a hundred years ago between Huxley and Wilberforce, Agassiz and Gray, are being used again.

Scientists have come under increased attack in recent years. They are accused of being heartless and conscienceless, of not caring about their fellow humans, even of being dangerous people who do not hesitate to discover new means of destruction and coercion and to use them. That is giving them too much credit. In any population sample there is a constant proportion of stupid people and of crooks, be it among scientists or insurance agents, writers or peasants, priests or politicians. And in spite of Dr. Frankenstein and Dr. Strangelove, catastrophes in history have been caused more often by priests and politicians than by scientists.

For people do not kill each other only for material benefit but also for reasons of dogma. Nothing is more dangerous than the certainty that one is right. Nothing is potentially so destructive as the obsession with a truth one considers absolute. All crimes in history have been the result of fanaticism of one type or another. All massacres have been carried out in the name of virtue, of true religion, of legitimate nationalism, of proper policy, of right ideology: in short, in the name of the fight against somebody else's truth, of the fight against Satan. The coldness and objectivity so often held against scientists are perhaps more suitable than fervor and subjectivity when it comes to dealing with some human matters. For scientific ideas do not generate passion. It is rather passion that exploits science to support its *cause. Science does not lead to racism and hatred. It is rather hatred that calls upon science to justify its racism. One can hold against scientists the ardor with which they sometimes champion their ideas. But no genocide has yet been committed for the triumph of a scientific theory. At the end of the twentieth century, it should be clear to each of us that no single system will ever explain the world in all its aspects and detail. The scientific approach has helped to destroy the idea of an intangible and eternal truth. This is not the least of its titles to fame."*

Francois Jacob (1920–)

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"Science deals with ideas and is a curiosity-driven, abstract, cultural activity. Technology deals with tools and other things that people use. Faraday's laws of electricity and magnetism are science. Marconi invented wireless technology. Clausius contributed to the science of thermodynamics. Watt invented the steam engine. It was science that clarified the nature of nuclear binding energy; but it was technology that, in an astonishingly short time, converted it to a weapon of unimaginable power. Whereas the first clear records of scientific concerns date back to about 600 BCE, the history of technology is much older. There is evidence that toolmaking goes back as far as one million years. Invention that produced technology did not require scientific reasoning until relatively modern times. The progress in technology was an important component of natural selection as our human ancestors learned to cope with recurrent ice ages and other conditions hardly conductive to creative contemplation. By the time of the apparently abrupt appearance of scientific thought, a considerably sophisticated technology was at hand: fire; metal working; agriculture; weights and measures; elements of arithmetic, algebra and geometry; an astronomical data base; navigation; land surveying; medicine and surgery; calendars. It is a matter for detailed historical scholarship to determine how much of the technology that was available in the 7^{th} *century BCE came about by curiosity and those drives that we now associate with pure research. If modern experience is any guide, a substantial component did come from the dreamers; it is hard to believe that stellar navigation, for example, was discovered by lost seamen desperately trying to find their way home on a dark night in the Aegean.*

Twentieth-century technology is essentially all derived from the results of science. There is an intimate interweaving and mutual enhancement of *these two disciplines that, in the past century, accounts for the ever-escalating pace of both: science begets technology, science uses technology to create more science. More science begets more technology.*

This litany deserves illustration:

1. *Science begets technology; for example, the quantum theory of solids leads to the transistor.*
2. *Science uses technology to create more science. The transistor provides fast, low-cost digital computers and electronic circuitry for data acquisition, controls and analysis. Electronic controls and computer guidance vastly improve the performance of electron accelerators and not only do research in particle physics but also produce synchrotron radiation.*
3. *More science begets more technology. The synchrotron X rays are used for lithographic etching of integrated circuits for more powerful application of transistors to the making of super-computers."*

Leon M. Lederman (1922–)

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"One has to look out for engineers – they begin with sewing machines and end up with the atomic bomb."

Marcel Pagnol (1895–1974)

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"What is the origin of the urge, the fascination that drives physicists, mathematicians, and presumably other scientists as well? Psychoanalysis suggests that it is sexual curiosity... This explanation is somewhat irritating and therefore probably basically correct."

David Ruelle (1935–)

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"Science cannot solve the ultimate mystery of nature. And that is because, in the last analysis, we ourselves are part of nature and therefore part of the mystery that we are trying to solve."

(Anon)

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"Impossible only means that you haven't found the solution yet."

(Anon)

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"Scientists do the work of God, engineers do the work of man."

(Anon)

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"The interactions between science and religion have often had an aggressive character. There has been, most of the time, a real warfare. But, as a matter of fact, it is not a warfare between science and religion *— there can be no warfare between them — but between* science and theology*. It is true that the man in the street does not easily differentiate between religious feelings and faith, on one side, and dogmas, rites and religious formalism, on the other. It is true also that the theologians, while affecting that religion itself was aimed at when they alone were criticized, have not ceased from aggravating these misunderstandings."*

(Anon)

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"In the 18^{th} century, science expounded the dynamics of ordered simplicity.
In the 19^{th} century, statistical mechanics expounded the science of disordered complexity.
In the 20^{th} century, foundations were laid for the science of ordered complexity."

(Anon)

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"Until the Scientific Revolution of the seventeenth century, meaning flowed from ourselves into the world; afterward, meaning flowed from the world to us."

(Anon)

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"Technology is the science of arranging life so that one need not experience it."

(Anon)

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"Through the telephone you loose privacy *and the charm of distance."*

(Anon)

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"Here men from the planet Earth first set foot on the moon, July 1969 A.D. We came in peace for all mankind."

Anonymous (American). Plaque marking the spot on the moon where the historic event took place

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"In Scientific practice, the onus of the proof is always on the advocate of the more complicated hypothesis."

(Anon)

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"Experimentalists observe things that cannot be explained and theoreticians explain things that cannot be observed."

(Anon)

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"Television brought the brutality of war into the comfort of the living room."

(Anon)

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“A theory can be proved by experiment; but no path leads from experiment to the birth of a theory.”

(Anon)

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“What is now proved was once only imagined.”

(Anon)

Ideas, Discoveries and Inventions

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"I haven't failed; I've found 10,000 ways that don't work."

Benjamin Franklin (1706–1790)

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"There is one thing stronger than all the armies in the world; and that is an idea whose time has come."

Victor Hugo (1802–1885)

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"The origin of an original work is always the pursuit of a fact which does not fit into accepted ideas."

Claude Bernard (1813–1878)

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"After previous investigation of the problem in all directions, happy ideas come unexpectedly without effort, like an inspiration."

Hermann von Helmholtz (1821–1894)

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"Ideas have consequences."

Dostoevsky (1821–1881)

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"In the field of observation, chance favors the prepare mind."

Louis Pasteur (1822–1895)

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"Choose one definite objective and drive ahead toward it. You may never reach your goal, but you will find something on the way."

Felix Klein's advice to a perplexed student (1849–1925)

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"Say what you know, do what you must, come what may."

Sonja Kovalevsky (1850–1891)

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"An idea that is not dangerous is unworthy of being called an idea at all."

Oscar Wilde (1854–1900)

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"It is through science that we prove, but through intuition that we discover."

Henri Poincare (1854–1912)

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"In the realm of thought, momentous discoveries and solution of problems are possible only to an individual, working in solitude."

Sigmund Freud (1856–1939)

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"The earth is the cradle of the mind, but one cannot live forever in a cradle.
To set foot on the soil of the asteroids, to lift by hand a rock from the moon, to observe Mars from a distance of several tents of kilometers, to land on its satellite or even on its surface, what can be more fantastic?
From the moment of using rocket devices, a new great era will begin in astronomy: the epoch of the more intensive study of the firmament."

Konstantin Eduardovitch Ziolkowski (1857–1935)

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"It requires a very unusual mind to undertake the analysis of the obvious."

Alfred North Whitehead (1861–1947)

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"Young man, you can be grateful that my invention is not for sale, for it would undoubtedly ruin you. It can be exploited for a certain time as a scientific curiosity, but apart from that it has no commercial value whatsoever."

Auguste Lumière (1862–1954). French chemist. On the motion-picture camera he invented in 1895

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"It is obvious that invention or discovery, be it in mathematics or anywhere else, takes place by combining ideas."

Jacques Hadamard (1865–1963)

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"Learning the secret of flight from a bird was a good deal like learning the secret of magic from a magician. Once you know the trick and know what to look for, you see things that you did not notice."

Orville Wright (1871–1948)

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"It isn't that they can't see the solution. It is that they can't see the problem."

G.K. Chesterton (1874–1936)

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"The scientist takes off from the manifold observations of predecessors, and shows his intelligence, if any, by his ability to discriminate between the important and the negligible, by selecting here and there the significant stepping stones that will lead across the difficulties to new understanding. The one who places the last stone and steps across to the terra firma of accomplished discovery gets all the credit."

Hans Zinsser (1878–1940)

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"It is the lone worker who makes the first advance in a subject: the details may be worked out by a team, but the prime idea is due to the enterprise, thought and perception of an individual."

Alexander Fleming (1881–1955)

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"Masterpieces are not single and solitary births; they are the outcome of many years of thinking in common, of thinking by the body of the people, so that the experience of the mass is behind the single voice."

Virginia Woolf (1882–1941)

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"All excellent things are difficult as they are rare."

"Ideas in science come into being as a response to new and better-phrased questions."

John R. Pierce (1910–2002)

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“We forget it but too often, and our histories are full of injustice, because we are almost always too generous toward those who made the last steps and reaped the result of all antecedent efforts, and too little generous to those who made the first and least profitable steps.”

“A deeper study of almost any discovery reveals that what we call the discovery is only the final clinching of an argument developed by many men throughout a long period of time.”

George Sarton (1884–1956)

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“The task is not so much to see what no one has yet seen; but to think what nobody has yet thought, about that which everybody sees.”

Erwin Schrödinger (1887–1961)

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“Discovery consists in seeing what everyone else has seen and thinking what no one else has thought.”

“A discovery is said to be an accident meeting a prepared mind.”

Albert Szent-Gyorgi (1893–1986)

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“Discoveries are not made by the rules of logic, but by guesswork and creative intuition.”

Vladimir A. Fock (1898–1974)

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"Reason is man's faculty for grasping the world by thought. Intelligence is man's ability to manipulate the world with the help of thought."

Erich Fromm (1900–1980)

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"Don't worry about people stealing your ideas. If your ideas are any good, you'll have to ram them down people's throats."

Howard Aiken (1900–1973)

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"The machine, the genie that man has thoughtlessly let out of its bottle and cannot put back again."

George Orwell (1903–1950)

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"What counts ... in science is not so much the first as the last."

Erwin Chargaff (1905–2002)

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"Scientific discoveries and *ideas* are produced by the intuition, creativeness and genius of a man. *Dollars of themselves don't produce this any more than they could be expected to produce another Mona Lisa."*

Harry Hess (1906–1969)

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"The technological breakthroughs leading to great inventions usually come from totally unrelated areas. For instance, if a queen of ancient Crete had launched a Minoan Manhattan Project to achieve mass literacy through improved printing, she would never have thought to emphasize research into cheese, wine and olive presses — but those presses furnished prototypes for Gutenberg's most original contribution to printing technology. Similarly, American military planners trying to build powerful bombs in the 1930's would have laughed at suggestions that they finance research into anything so arcane as transuranic elements.

We picture inventors as heroes with the genius to recognize and solve a society's problems. In reality, the greatest inventors have been tinkerers who loved tinkering for its own sake and who then had to figure out what, if anything, their devices might be good for. The prime example is Thomas Edison, whose phonograph is widely considered to be his most brilliant invention. When he built his first one, in 1877, it was not in response to a national clamor for hearing Beethoven at home. Having built it, he wasn't sure what to do with it, so he drew up a list of 10 uses, like recording the last words of dying people, announcing the time and teaching spelling. When entrepreneurs used his invention to play music, Edison thought it was a debasement of his idea.

Our widespread misunderstanding of inventors as setting out to solve society's problems causes us to say that necessity is the mother of invention. Actually, invention is the mother of necessity, by creating needs that we never felt before. (Be honest: did you really feel a need for your Walkman CD player long before it existed?) Far from welcoming solutions to our supposed needs, society's entrenched interests commonly resist inventions. In Gutenberg's time, no one was pleading for a new way to churn out book copies: there were hordes of copyists whose desire not to be put out of business led to local bans on printing.

The first internal-combustion engine was built in 1867, but no motor vehicles came along for decades, because the public was content with horses and railroads. Transistors were invented in the United States, but the country's electronics industry ignored them to protect its investment in vacuum-tube *products; it was left to Sony in bombed-out postwar Japan to adapt transistors to consumer-electronics products. Manufacturers of typing keyboards continue to prefer our inefficient* qwerty *layout to a rationally designed one.*

All these misunderstandings about invention pervade our science and technology policies. Every year, officials decry some areas of basic research as a waste of tax dollars and urge that we instead concentrate on "solving problems": that is, applied research. Of course, much applied research is necessary to translate basic discoveries into workable products — a prime example being the Manhattan Project, which spent three years and $2 billion to turn Otto Hahn and Fritz Strassman's discovery of nuclear fission into an atomic bomb. All too often, however, the world fails to realize that neither the solutions to most difficult problems of technology nor the potential uses of most basic research discoveries have been predictable in advance. Instead, penicillin, X-rays and many other modern wonders were discovered accidentally — by tinkerers.

So forget those stories about genius inventors who perceived a need of society, solved it single-handedly and transformed the world. There has never been such a genius; there have only been processions of replaceable creative minds who made serendipitous or incremental contributions. If Gutenberg himself hadn't devised the better alloys and inks used in early printing, some other tinkerer with metals and oils would have done so. For the best invention of the millennium, do give Gutenberg some of the credit — but not too much."

Jared Diamond (1937–)

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"A theory has only the alternative of being right or wrong. A model has a third possibility: it may be right, but irrelevant."

Manfred Eigen (1927–)

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"The greatest discovery of the 19th century was that the equations of nature were linear, and the great discovery of the 20th century is that they are not."

T.W. Körner

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"After developing retractable landing gear, flaps, navigation devices, autopilots, and active controls to provide stability for unstable vehicles, people realized that birds had been doing these things for 100 million years."

Paul MacCready (1925–2007)

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"To invent the automobile is easy; a really good science fiction would predict the traffic jam."

Frederick Pohl (1889–1991)

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"The four 'laws' of discovery:

- *Discoveries are rarely attributed to the correct person.*
- *Nothing is ever discovered for the first time.*
- *Everything of importance has been said before by someone who did not discover it.*
- *All great discoveries are made by mistake."*

Arthur Bloch (1948–)

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"Nothing was ever achieved by a reasonable man."

Anon

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"If we can really understand the problem, the answer will come out of it, because the answer is not separate from the problem."

Anon

9. Mind, Brain and the Computer

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"Generally, and device that can perform numerical calculations – even an adding machine, an abacus, or a slide rule – may be called a computer. Currently, however, the term usually refers to an electronic device that can use a list of instructions, called a program, to perform calculations or to store, manipulate, and retrieve information."

L.R. Shannon

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"One ought to know that on the one hand pleasure, joy, laughter, and games, and on the other, grief, sorrow, discontent, and dissatisfaction arise only from the brain. It is especially by it that we think, comprehend, see, and hear, that we distinguish the ugly from the beautiful, the bad from the good, the agreeable from the disagreeable..."

Hippocrates of Cos (460–377 BCE)

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"The Analytical Engine has no pretensions whatsoever to originate anything. It can do whatever we know how to order it to perform."

Augusta Ada Byron (1788–1824)

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"As machine get to be more and more like men, men will come to be more and more like machines."

Joseph Wood Krutch (1893–1970)

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"The complexity of the nerve-structures for vision is even in the insect something incredibly stupendous... The intricacy of the connections defies description. Before it the mind halts, abased."

"As long as our brain is a mystery, the universe – the reflection of the structure of the brain – will also be a mystery."

Santiago Ramón y Cajal (1852–1934)

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"The brain is wider than the sky
For, put them side by side,
The one the other will contain
With ease, and you beside.

The brain is deeper than the sea,
For, hold them, blue to blue,
The one the other will absorb,
As sponges, buckets do.

The brain is just the weight of God,
For, heft them pound for pound,
And they will differ, if they do,
As syllable from sound."

Emily Dickinson (1830–1886)

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"Computers are useless. They can only give you answers."

Pablo Picasso (1881–1973)

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"How could a mechanism composed of some 10^{10} *unreliable components function reliably while computers with* 10^4 *components regularly fail."*

John von Neumann (1903–1957)

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"Just as the electron does not have a precise position and motion, I believe that consciousness has no location."

George Wald (1906–1997)

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"Computers are circumscribed by their binary, yes or no, mode of operation, by their 'two-bit wit'. Subtleties such as yes and no, are beyond them."

Marshall McLuhan (1911–1980)

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"The Christian notion of the possibility of redemption is incomprehensible to the computer."

Vance Packard (1914–1996)

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“The real danger is not that computers will begin to think like men, but that men will begin to think like computers.”

Sydney J. Harris (1917–1986)

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“Computers make it easier to do a lot of things, but most of the things they make easier to do don’t need to be done.”

Andy Rooney (1919–)

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“Part of the inhumanity of the computer is that, once it is competently programmed and working smoothly, it is completely honest.”

Isaac Asimov (1920–1992)

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“No, I’m not interested in developing a powerful brain. All I’m after is just a mediocre brain, something like the president of the American Telephone and Telegraph Company.”

“I propose to consider the question, ‘Can Machine Think?’ ”

Alan Mathison Turing, 1936

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"You, your joys and sorrows, your memories and your ambitions, your sense of personal identity and free will are in fact no more than the behavior of a vast assembly of nerve cells and their associated molecules."

Francis Crick (1916–2004)

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"There is nothing that I can see in the physical laws that says the computer elements cannot be made enormously smaller than they are now."

Richard Feynman (1918–1988)

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"My fundamental premise about the brain is that its workings – what we sometimes call 'mind' – are a consequence of its anatomy and physiology and nothing more."

Carl Sagan (1934–1996)

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"Can mental states and processes be reduced *to brain (neurobiological) states and processes? Can one be a reductionist?"*

Patricia Churchland (1943–), 1986

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"I regard a human as nothing but a particular type of a machine, the human brain as nothing but an information processing device, the human soul as nothing but a program being run on a computer called the brain. Further, all possible types of living things, intelligent or not, are of the same nature, and subject to the same laws of physics as constrain all information processing devices. A human being is *a quantum mechanical object which can be exactly described by a computer program coding* 10^{45} *bits of information."*

Frank J. Tipler (1947–), 1994

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"The brain is a three-pound mass you can hold in your hand that can conceive of a universe a hundred-billion light years across."

Marian Diamond

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"One can search the brain with a microscope and not find the mind and can search the stars with a telescope and not find God."

Gustav J. White

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"I'm struck by the insidious, computer-driven tendency to take things out of the domain of muscular activity and put them into the domain of mental activity. The transfer is not paying off. Sure, muscles are unreliable, but they represent several million years of accumulated finesse."

Brian Eno (1948–), 1999

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"The question of whether computers can think is just like the question of whether submarines can swim."

Edsger W. Dijkstra (1930–2002)

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"There is no way that an unassisted human brain, which is nothing more than a dog's breakfast, three and a half pounds of blood-soaked sponge, could have written Beethoven's Ninth Symphony."

Kurt Vonnegut (1922–2007)

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"The great thing about a computer notebook is that no matter how much you stuff into it, it doesn't get bigger or heavier."

"E-mail is a unique communication vehicle for a lot of reasons. However e-mail is not a substitute for direct interaction."

Bill Gates (1955–)

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" 'Out of sight, out of mind', when translated into Russian (by computer), then back again into English, became 'invisible maniac' ".

Arthur Calder–Marshall (1908–1992)

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"There is no reason for any individual to have a computer in their home."

Ken Olsen (1926–), 1977

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"If you can make a machine that contains the contents of your mind, then that machine is you... Even if it doesn't last forever, you can always dump it onto tape and make backups... Everyone would like to be immortal... I'm afraid, unfortunately, that I am the last generation to die."

Gerald Jay Sussman

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"Mankind is a catalyzing enzyme for the transition from a carbon-based to a silicon-based intelligence."

Gérard Bricogne

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"The brain and the satellite glands have now been probed to the point where no possible site remains that can reasonably be supposed to harbor any physical mind."

Edmund O. Wilson (1929–)

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"If the brain were so simple we could understand it, we would be so simple we couldn't."

Lyall Watson (1939–)

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"Video won't be able to hold onto market it captures after the first six months. People will get tired of staring at a plywood box every night."

Darryl F. Zanuck (1902–1979), 1940s

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"The computer is no better than its program."

Elting E. Morison (1910–1995), 1966

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"Nobody predicted the computer – not even Joule Verne."

"The fact that prior to 1945 physical scientists did not have computers at their disposal compelled them to delve deeper into matters, and then come up with the 'pearl in the midst of the oyster', i.e. to identify underlying operative principles; whereas, had they lived after 1945, they might be tempted to simulate rather than cogitate."

Shahar Ben-Menahem

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"I think there is a world market for maybe five computers."

Thomas J. Watson (1874–1956)

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"The intelligence of the computer is syntactic *whereas human intelligence is* semantic *(i.e. assigns meaning to concepts)."*

(Anon)

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"Where a calculator on the ENIAC is equipped with 18,000 vacuum tubes and weighs 30 tons, computers in the future may have only 1,000 vacuum tubes and perhaps weigh 1.5 tons."

(Anon)

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"Someday, computers may not only be able to beat human beings at chess, but also at tennis and ice hockey and volleyball. Someday, computers may be able to marry Brooke Shields."

(Anon)

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"Computers have lots of memory but no imagination."

(Anon)

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"• *Brain cause minds.*

• *Syntax is not sufficient for semantics.*

• *Computer programs are entirely defined by their formal, or syntactical, structure.*

• *Minds have mental contents; specifically, they have semantic contents.*

Conclusion: No computer program by itself is sufficient to give a system a mind. Programs are not minds, and they are not by themselves sufficient for having minds."

"The computer, in contradistinction to the brain, has no self-consciousness, self-awareness – it does not know that it is a computer, it cannot think."

"For a long time many biologists and philosophers thought it was impossible, in principle, to account for the existence of life *on purely biological grounds. They thought that in addition to the biological processes some other element must be necessary, some* élan vital *must be postulated in order to lend life to what was otherwise dead and inert matter. It is hard today to realize how intense the dispute was between vitalism and mechanism even a generation ago, but today these issues are no longer taken seriously. Why not? I think it is not so much because mechanism won and vitalism lost, but because we have come to understand better the biological character of the processes that are characteristic of living organisms. Once we understand how the features that are characteristic of living beings have a biological explanation, it no longer seems mysterious to us that matter should be alive. I think that exactly similar considerations should apply to our discussions of consciousness. It should seem no more mysterious, in principle, that this hunk of matter, this grey and white oatmeal-textured substance of the brain, should be conscious than it seems mysterious that this other hunk of matter, this collection of nucleo-protein molecules stuck onto a calcium frame, should be alive. The way, in short, to dispel the mystery is to understand the processes. We do not yet fully understand the processes, but we understand their general* character,

we understand that there are certain specific electro-chemical activities going on among neurons or neuron-modules and perhaps other features of the brain and these processes cause consciousness."

"By 'mind' I just mean the sequence of thoughts, feelings and experiences, whether conscious or unconscious, that go to make up our mental life.But the use of the noun 'mind' is dangerously inhabited by the ghosts of old philosophical theories."

"On the traditional account of the brain, the account that takes the neuron as the fundamental unit of brain functioning, the most remarkable thing about brain functioning is simply this. All of the enormous variety of inputs that the brain receives – the photons that strike the retina, the sound waves that stimulate the ear drum, the pressure on the skin that activates nerve endings for pressure, heat, cold, and pain, etc. – all of these inputs are converted into one common medium: variable rates of neuron firing. Furthermore, and equally remarkably, these variable rates of neuron firing in different neuronal circuits and different local conditions in the brain produce all of the variety of our mental life. The smell of a rose, the experience of the blue of the sky, the taste of onions, the thought of a mathematical formula: all of these are produced by variable rates of neuron-firing, in different circuits, relative to different local conditions in the brain."

John Searle, 1984

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"One machine can do the work of fifty ordinary men. No machine can do the work of one extraordinary man."

Albert Hubbard, 1948

10. Science and Scientists — the Lighter Side

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Justus von Liebig *(1803–1873) was approached one day by his assistant who all excited informed him that he had just discovered a universal solvent. Liebig asked: "And what is a universal solvent?" Assistant: "One that dissolves all substances." Liebig: "Where are you going to keep that solvent, then?"*

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At a session of the Academy of Sciences of the (former) USSR, the notorious agronomist Lysenko gave a talk on the inheritance of acquired traits. When his report was over, the famous physicist ***Landau*** *asked: – So, you argue that if we will cut off the ear of a cow, and the ear of its offspring, and so on, sooner or later the earless cows will start to be born? – Yes, that's right. – Then, how can you explain that virgins are still being born?*

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When asked for his latest results, ***Edison*** *said: "Results! Why, man, I have gotten a lot of results. I know several thousand things that won't work."*

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On a paper submitted by a physicist colleague, ***Pauli*** *said: "This isn't right. This isn't even wrong."*

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The 67th Mersenne number $2^{67} - 1$*, claimed by Mersenne to be prime, was proven to be non-prime in 1903 by* ***F.N. Cole*** *(1861–1927). In the October meeting of the AMS, Cole announced a talk "On the Factorization of Large Numbers".*
He walked up to the blackboard without saying a word, calculated by hand the value of 2^{67}*, carefully subtracted* 1*. Then he multiplied two numbers (which were* 193707721 *and* 761838257287*). Both results written on the blackboard were equal. Cole silently walked back to his seat, and this is said to be the first and only talk held during an AMS meeting where the audience applauded. There were no questions.*
It took Cole about 3 years, each Sunday, to find this factorization.

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Theorem:
All positive integers are interesting.

Proof:
Assume the contrary. Then there is a lowest non-interesting positive integer. But, hey, that's pretty interesting! A contradiction.
QED

(Anon)

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Dirac *was working on an equation on the board. Turning around to a silent audience he asked for any questions. A person in audience raised a hand and said "I do not understand such-and-such equation". To which Dirac replies, "That's not a question, it's a statement."*

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Ernst Eduard Kummer *(1810–1893) was rather poor at arithmetic. Whenever he had occasion to do simple arithmetic in class, he would get his students to help him. Once he had to find* 7×9. *"Seven times nine," he began, "Seven times nine is er – ah – ah – seven times nine is..." "Sixty-one," a student suggested. Kummer wrote 61 on the board. "Sir," said another student, "it should be sixty-nine." "Come, come, gentlemen, it can't be both," Kummer exclaimed. "It must be one or the other."*

Another version:
Kummer said to himself: "Hmmm, the product cannot be 61, because 61 is prime, it cannot be 65, because 65 is a multiple of 5, 67 is a prime, 69 is too big – Only 63 is left."

In one of his lectures, ***Richard Feynman*** *said:*
"This is the third of four lectures on a rather difficult subject – the theory of quantum electrodynamics – and since there are obviously more people here tonight than there were before, some of you haven't heard the other two lectures and will find this lecture incomprehensible. Those of you who have heard the other two lectures will also find this lecture incomprehensible, but you know that that's all right: as I explained in the first lecture, the way we have to describe Nature is generally incomprehensible to us."

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What is set theory?

When there are five people in the room and seven are leaving, two have to enter the room, so it is empty.

Paul Erdös

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"Fast cars, fast women, fast algorithms... what more could a man want?"

Joe Mattis

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An MIT student cornered the famous **John von Neumann** *in the hallway:*

Student: "Er, excuse me, Professor von Neumann, could you please help me with a calculus problem?"

John: "Okay, sonny, if it's real quick – I'm a busy man."

Student: "I'm having trouble with this integral."

John: "Let's have a look." (insert brief pause here) "Alright, sonny, the answer's two-pi over 5."

Student: "I know that, sir, the answer's in the back – I'm having trouble deriving it, though."

John: "Okay, let me see it again." (another pause) "The answer's two-pi over 5."

Student (frustrated): "Uh, sir, I know the answer, I just don't see how to derive it."

John: "Whaddya want, sonny, I worked the problem in two different ways!"

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"Two trains 200 miles apart are moving toward each other; each one is going at a speed of 50 miles per hour. A fly starting on the front of one of them flies back and forth between them at a rate of 75 miles per hour. It does this until the trains collide and crush the fly to death. What is the total distance the fly has flown?

The fly actually hits each train an infinite number of times before it gets crushed, and one could solve the problem the hard way with pencil and paper by summing an infinite series of distances. The easy way is as follows: Since the trains are 200 miles apart and each train is going 50 miles an hour, it takes 2 hours for the trains to collide. Therefore the fly was flying for two hours. Since the fly was flying at a rate of 75 miles per hour, the fly must have flown 150 miles. That's all there is to it.

When this problem was posed to ***John von Neumann****, he immediately replied, "150 miles."*

"It is very strange," said the poser, "but nearly everyone tries to sum the infinite series."

"What do you mean, strange?" asked Von Neumann. "That's how I did it!""

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A student in ***Rutherford****'s lab was very hard-working. Rutherford had noticed it and asked one evening:*
– Do you work in the mornings too?
– Yes, – proudly answered the student sure he would be commended.
– But when do you think? – amazed Rutherford.

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"The traditional mathematics professor of the popular legend is absent-minded. He usually appears in public with a lost umbrella in each hand. He prefers to face the blackboard and to turn his back to the class. He writes a, he says b, he means c; but it should be d. Some of his sayings are handed down from generation to generation.
"In order to solve this differential equation you look at it till a solution occurs to you."
"This principle is so perfectly general that no particular application of it is possible."
"Geometry is the science of correct reasoning on incorrect figures."
"My method to overcome a difficulty is to go round it."
"What is the difference between method and device? A method is a device which you used twice."

George Polya (1887–1985)

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Niels Bohr *once commented on someone's lecture:*
"His theory is crazy... but it's not crazy enough to be true."

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"I aim at the stars, but sometimes I hit London."

Werner von Braun (1912–1977)

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David Hilbert *was invited to give a talk on any subject he liked during the early days of air travel. His subject:*

The Proof of Fermat's Last Theorem

Needless to say, his talk was eagerly anticipated. The day arrived, the talk was given, and it was brilliant – but it had nothing at all to do with Fermat's Last Theorem.
After the talk, someone asked Hilbert why he had picked a title that had nothing to do with the talk. His answer: "Oh, that title was just in case the plane crashed."

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Hilbert *has accepted an invitation to deliver a keynote address to a large engineering convention. The organizers subsequently learned that Hilbert was known for rather acerbic attitude towards engineering. Greatly concerned they decided to go back and talk to him.*

After beating around the bush for a while they managed to convey to him that they are worried that he may offend some people, and if he could sort of hold back during his speech.

When Hilbert realized what they were asking he grinned broadly and said, "You don't have to worry about that at all. How could I possibly offend anyone, for mathematics and engineering have absolutely nothing in common".

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J.E. Littlewood *(1885–1977) read in the proof sheets of Hardy on Ramanujan: "As someone said, each of the positive integers was one of his personal friends." His reaction was, "I wonder who said that; I wish I had." In the next proof-sheets he read (what now stands), "It was Littlewood who said..."*

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Hungarian mathematician ***Frigyes Riesz*** *needed two assistants for his lectures: one was reading aloud his (Riesz's) book, the second one was writing everything on the board, while Riesz was standing next to the board nodding.*

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When ***Edmund Landau*** *was asked for a testimony to the effect that Emmy Noether was a great woman mathematician, he said:*
"I can testify that she is a great mathematician, but that she is a woman, I cannot swear."

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In the period that ***Einstein*** *was active as a professor, one of his students came to him and said: "The questions of this year's exam are the same as last years!" "True," Einstein said, "but this year all answers are different."*

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"Do you realize the tremendous strategic importance of machines of this sort? They could be an effective straightjacket for that noisy shopkeeper Harry Truman. We must go ahead with it, comrades. The problem of the creation of transatlantic rockets is of extreme importance to us."

Premier I.V. Stalin, at a meeting of the Politburo (1947)

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"Give me a one-handed economist! All my economists say, on the one hand... on the other."

Harry S. Truman (1884–1972)

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"During the Paris peace negotiations between the Allies and some Eastern European countries, a Hungarian woman journalist came to me and wanted an interview. She asked me what I thought was the greatest progress in aviation in the last decade. I told her: 'Propulsion by reaction'. She said to me: 'Professor, could you express this in some other way? I cannot write in a progressive paper that progress is accomplished by reaction!'"

"Young students with athletic ability may have brilliant minds and still believe that to jump two inches farther than anybody else is an important contribution to human progress."

Theodore von Kármán (1881–1963)

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Theodore von Kármán *was a showman in the higher sense of the natural raconteur and purveyor of honest entertainment. his story-telling skills were enhanced by a rich and mysterious Hungarian accent that was so thick it provoked the late Dr.Hugh Dryden, Deputy Administrator of NASA, to comment jokingly that it must have been fabricated for "commercial reasons." Von Kármán's charming accent, coupled with hand gestures that carried extraordinary editorial comment, were enough to captivate any audience. He knew how to make a lasting effect. Like Bernard Baruch, von Kármán also carried an ornate hearing aid which he surreptitiously turned off whenever the conversation grew boring. He once confided to a friend that he thought his deafness since youth had been one of his most important assets because it enabled him to concentrate.*

He was also a master of the "squelch", which he used at times with withering effect. On one early occasion at the University of Aachen, the owner of a small tool factory came to him expressing concern about a problem of vibration. It seemed that one of the machines was in danger of shaking itself to destruction. Nobody could find the cause. Would the eminent Herr Professor take a look?

Von Kármán agreed and found the trouble in a few minutes. It was a small dislocation and he suggested turning a gear ninety degrees. When this was done, the vibration miraculously disappeared. The factory owner was overjoyed.

However, a few days later, he again sought out the professor. This time it was von Kármán's bill. "How can I pay so much money," the owner cried, "for making just a ninety-degree turn of the gear?"

"Very well," said von Kármán, "turn the gear ninety degrees back and I will tear up the bill."

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There is a story about the student who wanted to know, "Can one prove the ***Mandelstam*** *representation from field theory?" He went to* ***Weisskopf*** *who responded, "Field theory, what is field theory?" Then he sought out Wigner who said, "Mandelstam, who is Mandelstam?" Finally, our persistent student found his way to Chew, repeated the question, and heard, "Proof, what is proof?"*

Scientists Reflect on the World and Themselves

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"If I have been able to see further, it was because I stood on the shoulders of giants."

"I do not know what I may appear to the world; but to myself I seem to have been only like a boy playing on the seashore, and diverting myself in now and then finding a smoother pebble, or a prettier shell than ordinary; whilst the great ocean of truth lay all undiscovered before me.."

Isaac Newton (1643–1727)

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"When the storm rages and the state is threatened by shipwreck, we can do nothing more noble then to lower the anchor of our peaceful studied into the ground of eternity."

Johannes Kepler (1571–1630)

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"I do not feel obliged to believe that the same God who has endowed us with sense, reason, and intellect has intended us to forego their use."

Galileo Galilei (1564–1642)

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"I am truly a lone traveler and have never belonged to my country, my home, my friends, or even my immediate family, with my whole heart; in the face of all these ties, I have never lost a sense of distance and a need for solitude."

"Great spirits have always encountered violent opposition from mediocre minds."

"The trite objects of human efforts – possessions, outward success, luxury — have always seemed to me contemptible."

A. Einstein (1879–1955)

"The works of Archimedes are without exception, monuments of mathematical exposition; the gradual revelation of the plan of attack, the masterly ordering of the propositions, the stern elimination of everything not immediately relevant to the purpose, the finish of the whole, are so impressive in their perfection as to create a feeling akin to awe in the mind of the reader."

Thomas Heath (1861–1940), 1921

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"One cannot read Archimedes' complicated accounts of his quadratures and cubatures without saying to oneself, "How on earth did he imagine those expedients and reach those conclusions?"

George Sarton (1884–1956), 1952

"The greatest change in the axiomatic basis of physics — in other words, of our conception of the structure of reality — since Newton laid the foundation of theoretical physics, was brought about by Faraday's and Maxwell's work on electromagnetic phenomena.

Before Maxwell people conceived of physical reality as material points, whose changes consist exclusively of motions, which are subject to total differential equations. After Maxwell they conceived physical reality as represented by continuous fields, not mechanically explicable, which are subject to partial differential equations. This change in the conception of reality is the most profound and fruitful one that has come to physics since Newton."

Albert Einstein[60] (1879–1955), 1931

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"From a long view of the history of mankind there can be little doubt that the most significant event of the 19th century will be judged as Maxwell's discovery of the laws of electrodynamics. Even the American Civil War, will pale into provincial insignificance before this more powerful event of the 1860's."

Richard Feynman (1918–1998), 1964

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In a letter written near the end of his life (dated February 24, 1918) he recalled how he first learned of Maxwell's work one day while in a library:
"I remember my first look at the great treatise of Maxwell's when I was a young man. Up to that time there was not a single comprehensive theory, just a few scraps; I was struggling to understand electricity in the midst of a great obscurity. When I saw on the table in the library the work that had just been published (1873), I browsed through it and I was astonished! I read the preface chapter, and several bits here and there; I saw that it was great, greater and greatest, with prodigious possibilities in its power. I was determined to master the book and set to work. I was very ignorant. I had no knowledge of mathematical analysis (having learned only school algebra and trigonometry which I had largely forgotten) and thus my work was laid out for me. It took me several years before I could understand as much as I possibly could. Then I set Maxwell aside and followed my own course. And I progressed much more quickly."

Oliver Heaviside (1850–1925)

[60] Maxwell died in 1879, the year that Einstein was born.

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Von Kármán's *fabulously rich and varied life would be hard to duplicate in today's age of superspecialization. He was not modest or humble about his accomplishments. He followed Goethe's advice: "Nur die Lumpen sind Bescheiden" ("Only the loafers are modest").*

In science he saw himself among the immortals. Once he was asked to rate himself among the great scientists of the century. "If you define a great scientist as a man with great ideas," he replied, "then you will have to rate Einstein first. He had four great ideas. In the history of science perhaps only Sir Isaac Newton is ahead of Einstein, because he had five or six ideas. All the other major scientists of our age associated with just one, or at the most two, great ideas. In my case I have had three great ideas. Maybe more. Yes, perhaps three and a half great ideas."

Historians of science may assess von Kármán on a different scale. Obviously even "great ideas" are not "great" on the same level. Some scientists of our century, such as Bohr, Planck, Fermi, Dirac, and Schrödinger, have opened up vistas in physics whose impact on scientific thought may be far longer lasting, far more responsible for changing the future of civilization, than von Kármán's contribution to the conquest of air and space. But this impact is in part related to the social use being made of scientific discovery, such as the control of nuclear energy. In any evaluation of scientific genius as it deals in an inspired way with the apparently insoluble problems of the universe, there is no doubt that von Kármán rates a place among the first ten scientific minds of this century.

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"One never noticed what has been done; one can only see what remains to be done..."

Marie Curie (1867–1934), 1894

They Died Unconvinced

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"Inventions have long since reached their limit, and I see no hope for further development."

Julius Sextus Frontinus (40–103 CE)

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"The new astrologer wants to prove that the earth moves and revolves, instead of the sky, the sun and the moon. Just as if somebody moving in a carriage might hold that he was sitting still, at rest, while the earth and trees walked and moved. Ridiculous!"

Martin Luther (on Copernicus) (1483–1546)

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"To tell if a given number of 15 to 20 digits is prime or not, all time would not suffice for the test."

Marin Mersenne (1588–1648)

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"Alchemy is the only Art which might be able to complete and bring to light not only medicine but also a universal Philosophy."

Isaac Barrow (1630–1677)

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"There is no likelihood man can ever tap the power of the atom."

Robert Millikan (1868–1953), 1923

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"That one body may act on another through a vacuum, without the mediation of anything else, by and through – which their action and force may be conveyed from one to another, is to be so great an absurdity, that I believe no man who has in philosophical matters a competent faculty of thinking can ever fall into it."

Isaac Newton (1643–1727)

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"Atoms are the figment of a weak imagination."

Leibniz (1646–1716)

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"What, sir, you would make a ship sail against the wind and currents by lighting a bonfire under her decks. I pray you excuse me. I have no time to listen to such nonsense."

Napoleon, to Robert Fulton, who unsuccessfully tried
to interest the emperor in his idea to build a steamship, 1800

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“Paltry and unsubstantial papers... destitute of every species of merit... His theory have no other effect than to check the progress of science and renew all those wild phantoms of the imagination which Newton put to flight from her temple.”

The London Royal Society, regarding Thomas Young’s
epoch-making papers describing the wave nature of light; 1802

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“I could more easily believe that two Yankee professors would lie than that stones would fall from heaven.”

Thomas Jefferson, on a report on a meteorite shower
which fell in Weston, Connecticut in 1807

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“The laws of mechanics are identical with those of Nature.”

René Descartes (1596–1650)

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“It is quite impossible that the noble organs of human speech could be replaced by ignoble, senseless metal.”

Jean Bouillaud, member of the French Academy, before
viewing a demonstration of Edison’s phonograph; 1877.
After the demonstration, Bouillaud called Edison’s
invention a fake and attributed the demonstration he had
seen to “ventriloquism”

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"Such startling announcements as these should be deprecated as being unworthy of science and mischievous to its true progress."

William Siemens, England's most distinguished electrical engineer on Edison's electric light bulb; 1879. Siemens had been unsuccessfully working on electric light bulbs for a decade.

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"His claims are so manifestly absurd as to indicate a positive want of knowledge of the electric circuit and the principles governing the construction and operation of electrical machines."

Edwin Weston, a respected specialist in arc lighting on Edison's electrical light bulb; 1879.

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"Edison's ideas are good enough for our transatlantic friends... but unworthy of the attention of practical or scientific men."

Committee set up by the British Parliament to look into Edison's work on the incandescent lamp, c. 1878.

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"If the whole of the English language could be condensed into one word, it would not suffice to express the utter contempt those invite who are so deluded as to be disciples of such an imposture as Darwinism."

Francis Orpen Morris, British ornithologist (1810–1893)

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"Heavier-than-air flying machines are impossible."

1895

"X-rays are an elaborate hoax."

1896

"We find something at every turn to show the utter futility of Darwin's philosophy."

William Thomson (Kelvin) (1824–1907)

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"I am tired of all this thing called science... We have spent millions on that sort of thing for the last few years, and it is time it should be stopped."

Senator Simon Cameron, demanding that the funding of the Smithsonian Institution be cut off, 1861

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"...in a few years, all great physical constants will have been approximately estimated, and the only occupation which will be left to men of science will be to carry these measurements to another place of decimals."

James Clerk Maxwell (1831–1879)

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"How can he [Thomas Edison] call it a wonderful success when everyone acquainted with the subject will recognize it as a conspicuous failure, trumpeted as a wonderful success. A fraud upon the public."

Henry Morton, Professor of Physics and President of the Stevens Institute of Technology, on Edison's Incandescent light bulb, in *The New York Herald*, December 18, 1879

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"The abdomen, the chest, and the brain will be forever shut from the intrusions of the wise and humane surgeon."

John Erichsen (1818–1896)

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"All true scientific progress ceased around 1900."

Joseph Larmor (1857–1942)
(steadfastly opposed the new relativity and quantum theories)

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"Attempting to fly a heavier-than-air aircraft is simply absurd."

Rear-Admiral George Melville, chief engineer, US Navy, 1902

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"Heavier-than-air powered human flight is utterly impossible. Any form of powered flight require the discovery of an entirely new force."

Simon Newcomb, Professor of mathematics and astronomy at John Hopkins University; 1903, just a few weeks before the airplane flew.

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"Atoms cannot be perceived by the sense; like all substances, they are things of thought. Furthermore, the atoms are invested with properties that absolutely contradict the attributes hitherto observed in bodies. However well fitted atomic theories may be to reproduce certain groups of facts, the physical inquirer who has laid to heart Newton's rules will only admit those theories as provisional *helps, and will strive to attain, in some more natural way, a satisfactory substitute.*
The atomic theory plays a part in physics similar to that of certain auxiliary concepts in mathematics; it is a mathematical model *for facilitating the mental reproduction of facts. But these mental expedients have nothing whatsoever to do with the phenomenon* itself*."*

"This conclusion, that heat consists in mechanical processes, in motion, has spread over the whole cultivated world like wildfire. There is a huge mass of literature on this subject, and now people are everywhere eagerly bent on explaining heat by means of motions. They determine the velocities, the average distances, and the paths of the molecules, and there is hardly a single problem which could not, people say, be completely solved in this way by means of sufficiently long calculations. If, then, we are astonished at the discovery that heat is motion, we are astonished at something which has never been discovered. It is quite irrelevant for scientific purposes whether we think of heat as a substance or not."

"I can accept the theory of relativity as little as I can accept the existence of atoms and other such dogmas."

Ernst Mach (1838–1916)

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"Atoms are only hypothetical things."

Friedrich Wilhelm Ostwald (1853–1932)

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"...merely statistical validity of the Second Law is not good enough; irreversibility is a fundamental property of natural processes, and any molecular hypothesis – or perhaps all conceivable molecular hypotheses based on Newtonian mechanics – that permits any exception, must be wrong."

Ernst Zermelo (1871–1956), 1906

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"The popular mind often pictures gigantic flying machines speeding across the Atlantic carrying innumerable passengers. It seems safe to say that such ideas must be wholly visionary. Even if a machine could get across with one or two passengers, it would be prohibitive to any but the capitalist who could own his own yacht."

William Pickering (1910–2004), 1913

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"Everything that can be invented has been invented."

Charles H. Duell, commission of the US Patent Office, in a letter to President William McKinley, urging him to close the office, 1899.

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“By 1940 the relativity theory will be considered a joke.”

George Francis Gillette, 1929

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“The energy produced by the atom is a very poor kind of thing. Anyone who expects a source of power from the transformation of these atoms is talking moonshine.”

Ernest Rutherford, *The New York Herald Tribune*,
September 12, 1933

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“Professor Goddard does not know the relation between action and reaction and the need to have something better than a vacuum against which to react. He seems to lack the knowledge ladled out daily in the high schools.”

editorial in The New York Times, 1921, dismissing Robert Goddard,
who proposed that someday rockets could reach the moon.

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“There is no likelihood man can ever tap the power of the atom... Nature has introduced a few foolproof devices into the great majority of elements that constitute the bulk of the world, and they have no energy to give up in the process of disintegration.”

Robert Andrews Millikan (1868–1953), 1923

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"We can surely never hope to see the craft of surgery made much more perfect than it is today. We are at the end of a chapter."

Berkeley George Moynihan (1865–1936), 1930

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"There is not the slightest indication that [nuclear] energy will ever be obtainable. It would mean that the atom would have to be shattered at will."

Albert Einstein (1879–1955), 1932

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"Fortunately; uranium bombs cannot at once be adapted for war, as the apparatus needed is very heavy and also very delicate, so it cannot at present be dropped from an airplane"

J.B.S. Haldane (1892–1964), 1940

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"Real mathematics has no effect on war: No one has yet discovered any warlike purpose to be served by the theory of numbers or relativity, and it seems very unlikely that anyone will do so for many years."

G.H. Hardy (1877–1947), 1940

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"That is the biggest fool thing we have ever done... The (atomic) bomb will never go off, and I will speak as an expert on explosives."

Admiral William Leahy to President Harry Truman, 1945

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"Man will never reach the moon regardless of all future scientific advances."

Lee De Forest (1873–1961), 1957

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"Further investigation and experimentation have confirmed the findings of Isaac Newton in the 17th century, and it is now definitely established that a rocket can function in a vacuum as well as in an atmosphere. The Times *regrets the error."*

editorial in *The New York Times*, July 17, 1969

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"There is no reason for any individual to have a computer in their home."

Ken Olsen, President of Digital Equipment Corporation, USA, 1980

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"If one believes in science, one must accept the possibility – even the probability – that the great era of scientific discovery is over... Further research may yield no more great revelations or revolutions, but only incremental, diminishing returns."

John Morgan (1806–1871), 1996

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"The use of small quantities [of uranium], sufficient, say, to operate a car or an airplane, so far is impossible, and one cannot predict when it will be achieved. No doubt, it will be achieved, but nobody can say when."

Albert Einstein (1879–1955), 1945

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"I think we are not quite yet fit for Flying Machines and therefore there will be none."

Ralph Waldo Emerson (1803–1882), 1843

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"I do not hesitate to forecast that atomic batteries will be commonplace long before 1980."

David Sarnoff (1891–1971), 1955

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"The best way to teach that one should be suspicious of everything one holds dear is through the study of brilliant people of the past, who by modern standards are so wrong, and where it is easy to see that their errors were the result of cultural biases of their day."

Stephen Jay Gould (1941–2002), 1993

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"The ideas of Freud were popularized by people who only imperfectly understood them, who were incapable of the great effort required to grasp them in their relationship to larger truths, and who therefore assigned to them a prominence out of all proportion to their true importance."

Alfred North Whitehead (1861–1947)

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"It may be that the stars of heaven appear fair and pure simply because they are so far away from us, and we know nothing of their private life."

Heinrich Heine (1797–1856), 1833

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"The certain proof that intelligent life exists elsewhere in the Universe is that no one has bothered yet to make contact with us."

(Anon)

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"You know why there are so many whitefish in the Yellowstone River? Because the Fish and Game people have never done anything to help them."

Russell Chatham (1939–), 1978

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"With classical thermodynamics, one can calculate almost everything crudely; with kinetic theory, one can calculate fewer things, but more accurately; with statistical mechanics, one can calculate nothing."

Eugene Wigner (1902–1995)

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"First law of thermodynamics: You cannot win.
Second law of thermodynamics: You cannot break even.
Third law of thermodynamics: You cannot get out of the game."

(Anon)

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"Astronomers discover God!"

Headlines in a well-known periodical (1982)

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"The question is not so much whether there is life elsewhere in the universe as whether it will continue to be possible to live on earth."

(Anon)

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"If you want to see comets that are comets, you've got to get outside our solar system – where there's room for them."

"There is something fascinating about science. One gets such wholesale returns of conjecture out of such a trifling investment of fact."

Mark Twain (1835–1910)

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"Comets are the nearest thing to nothing that anything can be and still be something."

(Anon)

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"We owe a lot to Thomas Edison – if it wasn't for him, we'd be watching television by candlelight."

Milton Berle (1908–2002)

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"No one has ever complained of a parachute not opening."

(Anon)

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"Statistics prove that 50 per cent of the married people in the United States are women."

(Anon)

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"My great-grandfather looked at him severely. 'My man,' he said, 'don't you know that very few people ever die after the age of 99? Statistics prove it!'"

(Anon)

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"Never try to walk across a river just because it has an average depth of four feet."

Martin Friedman (1962–)

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"Amoebas at the start
Were not complex;
They tore themselves apart
And started Sex."

Arthur Guiterman (1871–1943)

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"When you have excluded the impossible, whatever remains, however improbable, must be the truth."

"You know my method. It is founded upon the observance of trifles."

"It has long been an axiom of mine that the little things are infinitely the most important."

"You see, but you don't observe."

'Is there any point to which you would wish to draw my attention?'
'To the curious incident of the dog in the night-time.'
'The dog did nothing in the night-time.'
'That was the curious incident,' remarked Sherlock Holmes.

Arthur Conan Doyle (1856–1930)

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"That theory is worthless. It isn't even wrong!"

Wolfgang Pauli (1900–1958)

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"The guy who invented the first wheel was an idiot. The guy who invented the other three, he *was a genius."*

Sid Caesar (1922–)

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"If it weren't for Philo T. Farnsworth, inventor of television, we'd still be eating frozen radio dinners."

Johnny Carson (1925–2005)

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Louise: "how did you get there?"
Johnny: "Well, basically, there was this little dot, right? And the dot went bang and the bang expanded. Energy formed into matter, matter cooled, matter lived, the amoeba to fish, to fish to fowl, to fowl to frog, to frog to mammal, the mammal to monkey, to monkey to man, amo amas amat, quid pro quo, memento mori, ad infinitum, sprinkle on a little bit of grated cheese and leave under the grill till Doomsday."

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"No committee could ever come up with anything as revolutionary as a camel — anything as practical and as perfectly designed to perform effectively under such difficult conditions."

Laurence J. Peter (1919–1990)

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"Physics is becoming so unbelievably complex that it is taking longer and longer to train a physicist. It is taking so long, in fact, to train a physicist to the place where he understands the nature of physical problems that he is already too old to solve them."

Eugene Wigner (1902–1995)

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"An expert is someone who knows some of the worst mistakes that can be made in his subject and how to avoid them."

Werner Heisenberg (1901–1976)

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"An expert is a man has made all the mistakes, which can be made, in a very narrow field."

Niels Bohr (1885–1962)

Name Index (quotations)

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In creating this treatise I have borrowed information and data from various source-media such as books, articles and Internet data bases. References to these sources have been placed in four distinct locations:

- *In the text proper (name of books, articles, historical events, quotations, etc.)*
- *Footnotes at the bottom of the pages.*
- *At the end of each chapter.*
- *A comprehensive bibliography (some 900 entries) at the end of chapter 6.*

The following bibliography is divided here into ten categories:

1. Source–media for Biographies, History and Chronologies (Encyclopedias, Lexicons, Dictionaries, Timetables, Archives, Atlases and Internet Data Bases)

2. History of Science and Technology (and Biographies of Scientists, Engineers, Inventors and Explorers)

3. Mathematics, Logic and Philosophy

4. Physics and Cosmology

5. Chemistry

6. Biology and Medicine

7. Earth–Sciences

8. Space–Science, Astronomy and Astrophysics

9. Social Sciences

10. Quotations (of Thinkers, Sages, Savants, and Scholars throughout the ages)

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